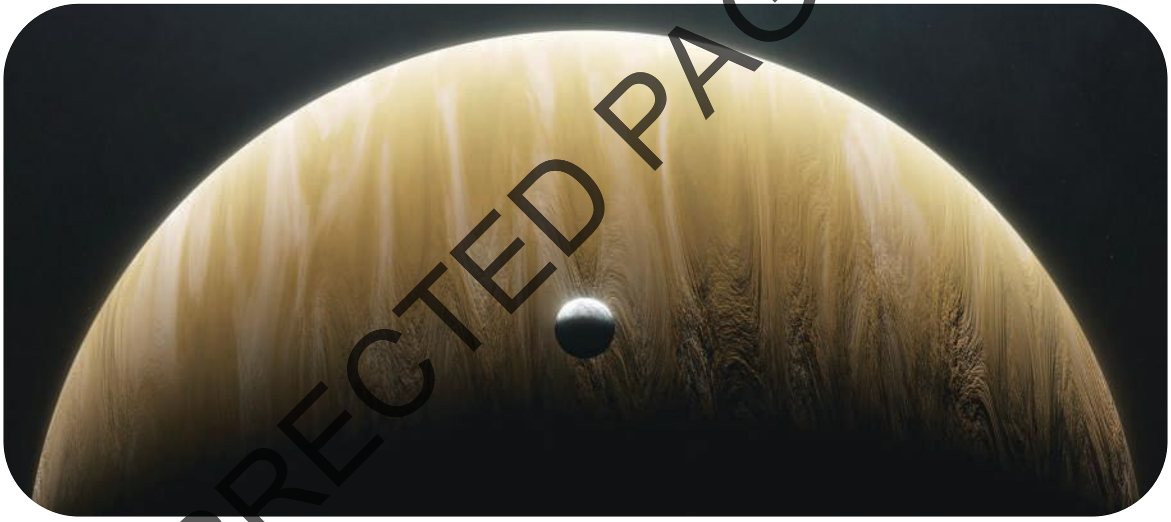
NELSON

QCE Physics

UNITS
3
4

LEARNING DISCOVERY

GRAVITATIONAL FORCE & ORBITAL MOTION



Insert text here to relate gravital force and orbital motion to book content. Insert text here insert text here Insert text here insert text here insert text here Insert text here insert text.

Scott Adamson
Shanee Conran
Matt Lourigan

Copyright Page

CONTENTS

 9780170483704

AUTHOR AND REVIEWER TEAM

Nelson QCE Physics is adapted from *Nelson QScience Physics Units 1 & 2* and *Nelson QScience Physics Units 3 & 4*, by Oliver Alini, Tara Kuhn, Neil Champion and Scott Adamson.

SCOTT ADAMSON

Scott Adamson has an academic background in education, including a Master of Education (ICT). An award-winning Physics teacher, recipient of the Queensland Teacher of the Year in 2011 with more than 25 years of experience in Queensland high schools, Scott has occupied various positions in secondary education, including school senior leadership team member and head of department science. An educational leader skilled in educational change management, e-Learning and Physics, Scott is a strong education professional who has led significant change in Physics in Queensland and is experienced in Queensland Curriculum and Assessment Authority assessor roles as Chief Marker, Lead Confirmer and Endorser in Physics, and previously as State Panellist and District Panel Chair for Physics in the Brisbane Central region. Lead author of the *Nelson Cengage QScience Physics* series of student texts and workbooks based on the new Australian Curriculum and Queensland Physics syllabus, Scott has authored Physics, Science and Mathematics text series over the past two decades.

SHANEE CONRAN

Shanee Conran is a Chemistry and Physics teacher with experience in the Queensland Curriculum and Assessment Authority, International Baccalaureate, Board of Senior Secondary Studies and Western Australian Certificate of Education systems. She has brought her knowledge of question writing and external exams to the team.

MATTHEW LOURIGAN

Matthew Lourigan has an academic background in biochemistry and engineering and is an award-winning Physics teacher with more than 20 years of experience in Queensland high schools. After working in industry, he has occupied various positions in secondary education, including head of department and deputy principal as well as being a course author and lecturer in science pedagogy in the tertiary sector. Matthew has held various roles at the Queensland Curriculum and Assessment Authority over many years and is currently a lead endorser for Physics.

REVIEWERS

<text to come>

ACKNOWLEDGEMENTS

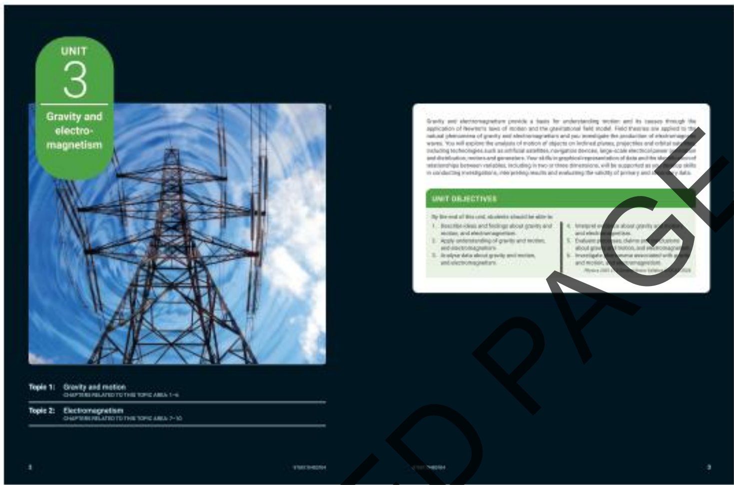
The publisher would like to acknowledge the authors for their expertise and devotion to supporting teachers and students engaging in Physics around Queensland.

ABOUT THIS BOOK

Nelson QCE Physics Units 3 & 4 is a comprehensive textbook specifically tailored to align with the 2025 QCAA Senior Secondary Science Syllabus – Physics v1.2. It has been thoughtfully developed to empower students by providing a strong foundation in essential concepts and equipping them with the necessary skills to excel in their studies. Emphasising the importance of making connections between topics and practising exam techniques, this edition is designed to support students in unlocking their full potential and achieving success in their journey.

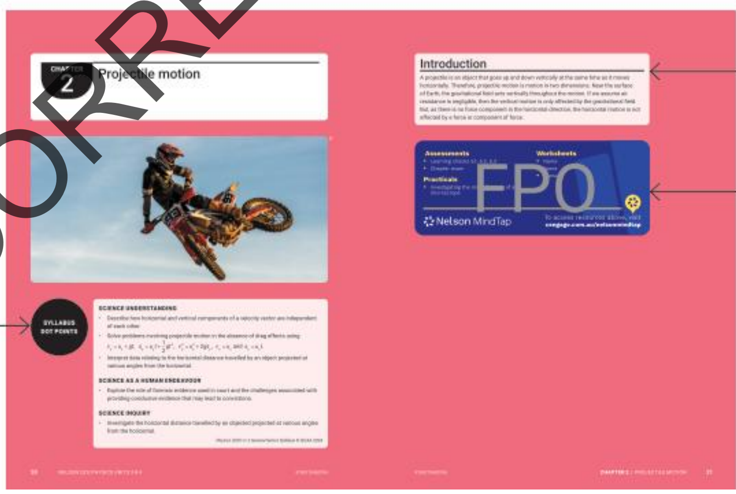
AT THE BEGINNING OF UNIT AND TOPIC

- Unit introductions are an overview of the key content in the unit.



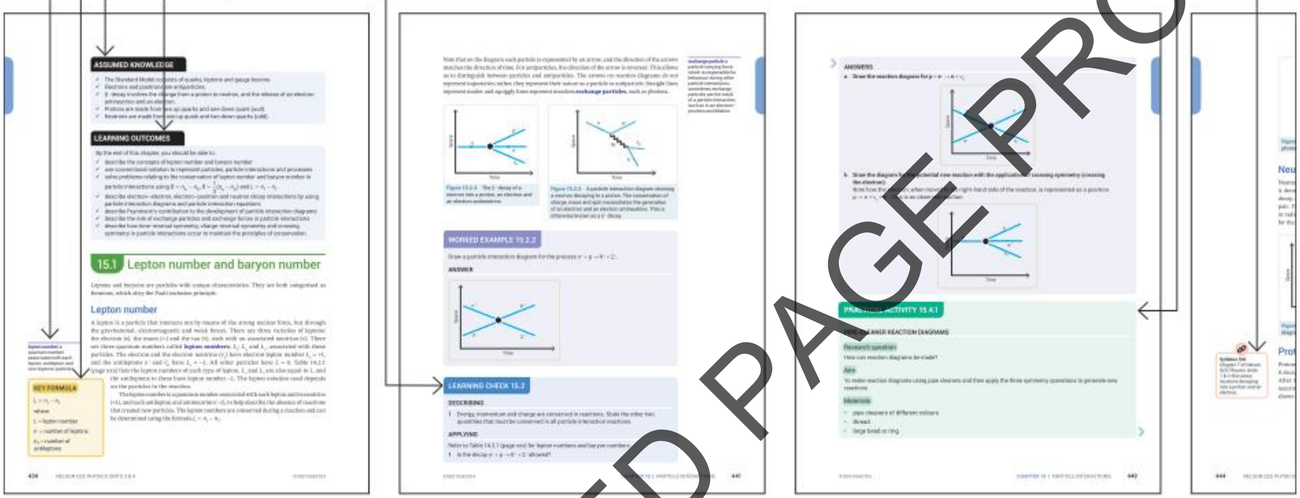
AT THE BEGINNING OF EACH CHAPTER

- Chapter introduction to set the context of the upcoming key content
- List of syllabus dot points being covered in the chapter
- List of resources available on Nelson MindTap



IN EACH CHAPTER

- **Assumed knowledge** – knowledge and skills students are expected to know coming into the chapter that relate to the chapter content
- **Learning outcomes** – highlights the key outcomes from chapter
- **Key terms** – defined in situ to help students deconstruct scientific language
- **Learning check** – written to the developmental levels highlighted in the syllabus objectives
- **Syllabus links** – highlighting links to other areas in the syllabus to help students make connections
- **Key formulas** – important formulas to remember
- **Practicals** – syllabus-aligned practicals with guided instructions on the materials, procedure, collection and analysis of results, and discussion



AT THE END OF EACH CHAPTER

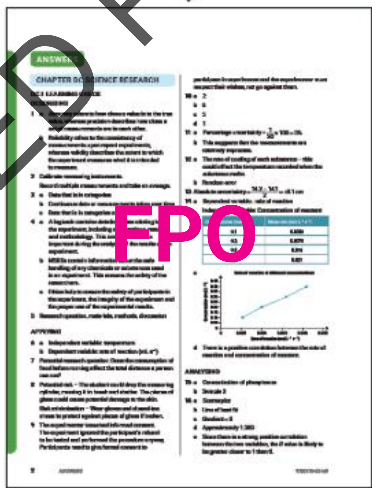
- **Chapter summary** – visual summaries to help summarise key concepts
- **Chapter exam** – exam-style questions to help students develop exam skills, including deliberate practice in data analysis and making connections across content

AT THE END OF EACH TOPIC

- **Science as a Human Endeavour** – a double-page deep dive on the evolution of science and how it has contributed to and influenced society

AT THE END OF THE BOOK

- **Glossary** provides explanations of all terms introduced in the text.
- **Answers** provide complete answers for student reference.



Nelson MindTap

An online learning space that provides students with tailored learning experiences.

- Access tools and content that make learning simpler yet smarter.
- **Flexible formats:** choose how you navigate using either the online eText or offline PDFs.
- Margin links in the student book signpost multimedia student resources found on Nelson MindTap.

Weblink
Error bars

For students:

- **Engage** with the **online eText** by adding notes, highlights, bookmarks and using the **Search** and **Read Aloud** (in Australian voice) functions.
- **Check** your understanding with assumed knowledge reviews.**
- **Explore** key concepts with worksheets and additional online-only practicals.
- Access **supporting materials** such as the chapter summary pack and weblinks.
- **Revise** with chapter tests to practice your skills and build confidence.**
- **Navigate** your own learning path, accessing the content and support resources as you need it.

** Available if assigned by your teacher in Cognero Assess.

For teachers*:

- **Tailor** content to different learning needs – assign directly to the student, or the whole class.
- Use **teaching plans** and curriculum guides to support easy program and assessment planning.
- **Prepare** students for assessment with practice versions of data tests, unit-level exams and the external assessment.
- **Answers** to student book questions help you grade efficiently.
- **Risk assessments** and technical notes are provided for each practical activity to ensure the safety of students.
- **Chapter summary** presentations provided in PowerPoint format let you edit and build upon them.
- Help build your students' exam readiness with **Cognero Assess** – a test bank containing hundreds of questions and answers to create, assign or export formative and summative tests.

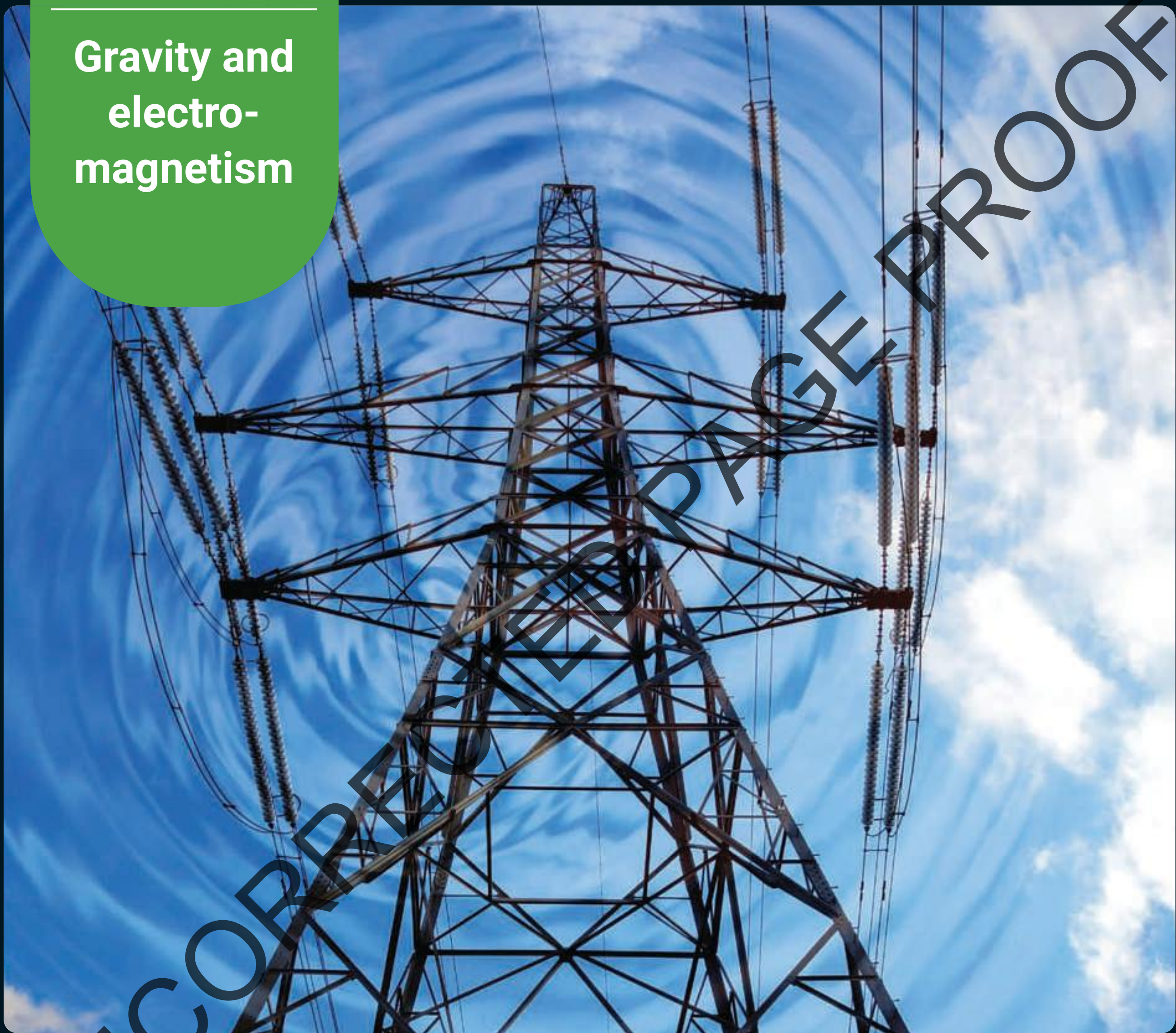
* Complimentary access to these resources is only available to teachers who use this book as part of a class set, book hire or booklist. Not available for single purchases. Alternatively, teacher access can be purchased as a separate subscription for schools. Contact your Cengage learning consultant for information about access and conditions.

Security & privacy:

Nelson MindTap joined the Safer Technologies 4 Schools (ST4S) Product Badge Program in 2024. The annual ST4S assessment is part of our commitment to supporting the online security and safety of students and schools. Learn more at st4s.edu.au

UNIT 3

Gravity and electro-magnetism



Topic 1: Gravity and motion
CHAPTERS RELATED TO THIS TOPIC AREA: 1–6

Topic 2: Electromagnetism
CHAPTERS RELATED TO THIS TOPIC AREA: 7–10

9780170483704

Gravity and electromagnetism provide a basis for understanding motion and its causes through the application of Newton's laws of motion and the gravitational field model. Field theories are applied to the natural phenomena of gravity and electromagnetism and you investigate the production of electromagnetic waves. You will explore the analysis of motion of objects on inclined planes, projectiles and orbital satellites, including technologies such as artificial satellites, navigation devices, large-scale electrical power generation and distribution, motors and generators. Your skills in graphical representation of data and the identification of relationships between variables, including in two or three dimensions, will be supported as you develop skills in conducting investigations, interpreting results and evaluating the validity of primary and secondary data.

UNIT OBJECTIVES

By the end of this unit, students should be able to:

1. Describe ideas and findings about gravity and motion, and electromagnetism.
2. Apply understanding of gravity and motion, and electromagnetism.
3. Analyse data about gravity and motion, and electromagnetism.
4. Interpret evidence about gravity and motion, and electromagnetism.
5. Evaluate processes, claims and conclusions about gravity and motion, and electromagnetism.
6. Investigate phenomena associated with gravity and motion, and electromagnetism.

CHAPTER 1

Gravity and motion

TK

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Apply vector analysis to resolve a vector into two perpendicular components.
- Solve vector problems by resolving vectors into components, adding or subtracting the components and recombining them to determine the resultant vector.

Physics 2025 v1.2 General Senior Syllabus © QCAA 2024

Introduction

A quantity can be specified completely by a single scale (scalar) or by two or more scales (vector). In symbol form, vectors are distinguished from scalars by placing an arrow on top of the letter symbol: $\vec{A}$, $\vec{B}$, $\vec{C}$ and so on.

Assessments

- Learning checks
- Chapter exam

Practicals

- Investigating the agnification o microscrope

Worksheets

- Name
- Nam
- Na e

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ Scalars are quantities that only have magnitude, and vectors are quantities with both magnitude and direction.
- ✓ Common examples of vector quantities in physics are displacement, velocity, acceleration and force.
- ✓ Average velocity can be calculated by $v_{av} = \frac{s}{t}$.
- ✓ Acceleration due to gravity on Earth is $g = 9.8$ m s^{-2}.
- ✓ Vectors can be symbolised graphically and algebraically; for example, F, $\tilde{F}$ and $\vec{F}$.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ apply vector analysis to resolve a vector into two perpendicular or resolute components
- ✓ solve vector problems by resolving vectors into rectangular components, adding or subtracting the components either mathematically or graphically and recombining them to determine the resultant vector
- ✓ describe the effect of multiplying a vector quantity by a scalar quantity
- ✓ describe compass bearings as true or quadrant.

1.1 Vector and directional analysis

When describing motion (kinematics), displacement, velocity and acceleration are all vectors comprising scalar magnitude and direction. Similarly, when explaining motion (dynamics), force and momentum are vectors. Magnitude is given in standard SI units, such as metres, kilograms, seconds or newtons. Direction is given with respect to an agreed axis system: compass points (quadrant or true/azimuth bearings), Cartesian axes (positive *x*-axis), one of the vectors involved, or the plane of a sloping surface.

Syllabus link
Chapter 12 of *Nelson QCE Physics Units 1 & 2*, introduces vectors in relation to one- and two-dimensional displacement, velocity and acceleration. Chapter 12 also discusses quadrant and true (azimuth) bearings.

Vector geometric addition in two dimensions: scale drawing

All vectors can be added and subtracted geometrically. Solutions can be found by careful scale drawing using Cartesian graph paper, a protractor, a ruler and a fine-point pencil.

Tip-to-tail method

Using the tip-to-tail method, the two force vectors form two sides of a triangle **(Figures 1.1.1** and **Figure 1.1.2)**.

Parallelogram method

parallelogram method
vector addition in which the tail of each vector is connected at the same position; the vectors are used as adjacent sides of a parallelogram and the resultant is the diagonal that starts at the tails of the vectors being added

Vector arrows are just representations of reality and can be moved to positions appropriate for the solution. In the **parallelogram method**, both vectors are aligned so that their tails are at the same position. A parallelogram is constructed using the vectors as adjacent sides. The resultant is the diagonal that starts at the tails of the vectors being added **(Figure 1.1.3)**.

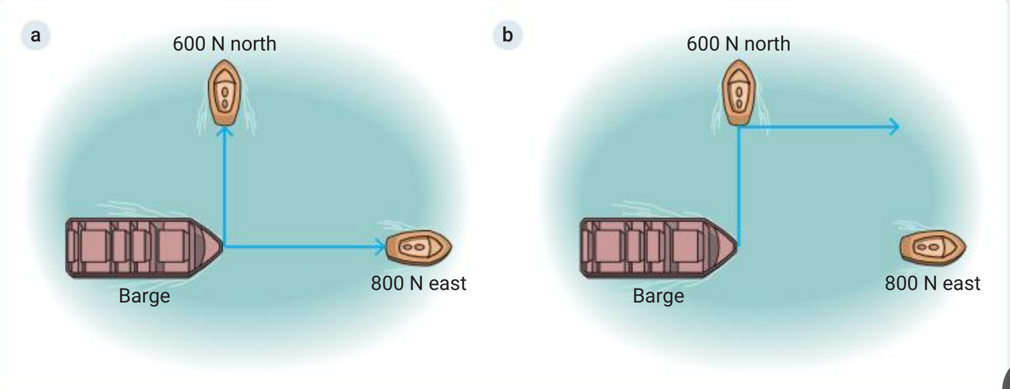


FIGURE 1.1.1 (**a**) An example of the tip-to-tail method. The vector sum of two vectors, $\vec{C} = \vec{A} + \vec{B}$. (**b**) In the tip-to-tail method, vectors are added with the tip of vector 1 being the tail of vector 2.

Weblinks
Vector addition
Adding and subtracting vectors

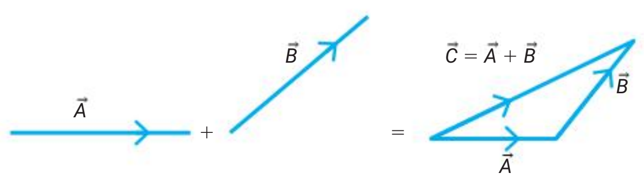


FIGURE 1.1.2 The vector sum of two vectors, $\vec{C} = \vec{A} + \vec{B}$. In the tip-to-tail method, vectors add from tip to tail.

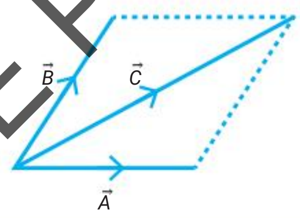


FIGURE 1.1.3 Adding vectors by the parallelogram method. The resultant starts on the tails of the vectors being added.

Vector subtraction

As in ordinary subtraction, vector subtraction is the addition of the negative vector. Thus:

$$\vec{C} = \vec{A} - \vec{B}$$
$$\Rightarrow \vec{C} = \vec{A} + (^{-}\vec{B})$$

KEY FORMULA

Subtraction of vectors: addition of the negative

$$\vec{C} = \vec{A} - \vec{B}$$
$$\Rightarrow \vec{C} = \vec{A} + (^{-}\vec{B})$$

Components of vectors

Any vector can be constructed as the sum of any two other vectors. Each of these two vectors is a **component** of the original vector. When a vector is resolved into two components, each component is called a **resolute**.

It is very useful to resolve vectors into **rectangular components** that are perpendicular to each other; for example, horizontal and vertical components or east–west and north–south components. This enables the use of the geometry and trigonometry of right-angle triangles. **Figure 1.1.4** shows the rectangular resolutes of the displacement vector 55 m, N37°E. The resolutes are taken in the north and east directions respectively.

On a Cartesian grid, resolutes are taken with respect to the x- and y-axes. The angle is taken with respect to the positive direction of the x-axis.

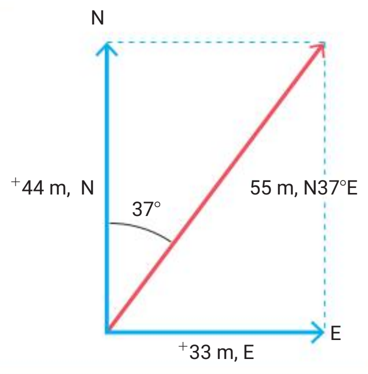


FIGURE 1.1.4 Resolutes of the vector 55 m, N37°E (or 55 m, E53°N): north component = 55 cos 37° m or 44 m; east component = 55 sin 37° m or 33 m.

component one of two or more vectors into which a vector can be resolved

resolute a component of a vector

rectangular components components of a vector that are at right angles to each other; perpendicular components

Not all vectors are defined by compass points or graphs. For example, resolutes for projectile motion are usually defined relative to horizontal and vertical directions. For motion on an inclined plane, the resolutes are usually taken parallel and perpendicular to the plane. For a variety of forces being added, it is often easier to measure all vectors from a reference direction, such as down an incline.

Adding vectors by components

Resolutes can be added algebraically in the *x*- and *y*-directions. This enables the magnitude of the resultant to be calculated by applying Pythagoras' theorem, and the angle by the tangent ratio from trigonometry.

Consider vector $\vec{A}$, which is oriented at an angle θ to the positive *x*-axis. The rectangular components are A_x and A_y **(Figure 1.1.5)**.

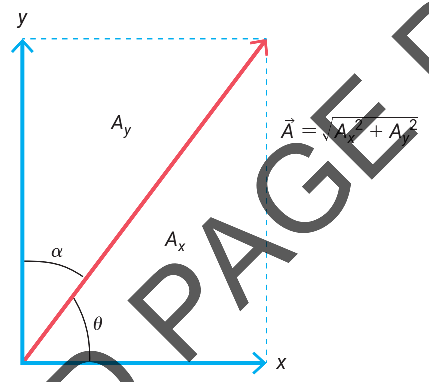


FIGURE 1.1.5 Vector $\vec{A}$ has rectangular components, A_x and A_y. The angle is taken relative to the positive direction of the *x*-axis.

KEY FORMULA

Rectangular components of vector $\vec{A}$:

$$x\text{-component: } A_x = A\cos\theta$$

$$y\text{-component: } A_y = A\sin\theta$$

The magnitude is found by Pythagoras' theorem:

$$A = \sqrt{A_x^{\,2} + A_y^{\,2}}$$

where:

A = magnitude of vector, $\vec{A}$

A_x = magnitude of the *x*-component

A_y = magnitude of the *y*-component

θ = angle relative to the positive direction of the *x*-axis

Angle:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{y\text{-component}}{x\text{-component}}$$

$$\theta = \tan^{-1}\left(\frac{y\text{-component}}{x\text{-component}}\right)$$

9780170483704

The addition of two vectors, $\vec{A}$ and $\vec{B}$, to form a resultant $\vec{R} = \vec{A} + \vec{B}$, uses the convention described above. The components of the resultant $\vec{R}$ are:

- *x*-component: $R_x = A_x + B_x$
- *y*-component: $R_y = A_y + B_y$

The magnitude of length, R, of the resultant vector, $\vec{R}$, is found by Pythagoras' theorem **(Figure 1.1.6)**:

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

Angle:

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$= \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$

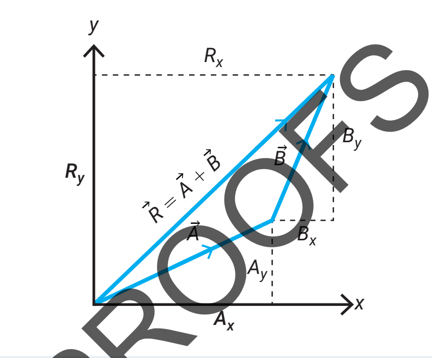


FIGURE 1.1.6 $\vec{R} = \vec{A} + \vec{B}$. Components in the *x*- and *y*-directions add algebraically to calculate the resultant vector.

Multiplication of a vector by a scalar

The magnitude of a vector is a scalar, which can be multiplied by a scalar factor. This **scalar multiplier** makes the vector longer or shorter by the scale factor without changing its direction. However, if a scalar multiplier is negative, the magnitude is changed and the direction is reversed **(Figure 1.1.7)**.

scalar multiplier a positive or negative number that can change the magnitude and/or the direction of a vector

Weblink
Multiplication of vectors with scalar

KEY FORMULA

A vector may be multiplied by a scalar quantity.

- Multiplication by a positive scalar changes the magnitude but not the direction of the vector.
- Multiplication by a negative scalar changes the magnitude *and* reverses the direction of the vector.

Division of a vector by a scalar is the same as multiplication of the vector by the inverse of the divisor.

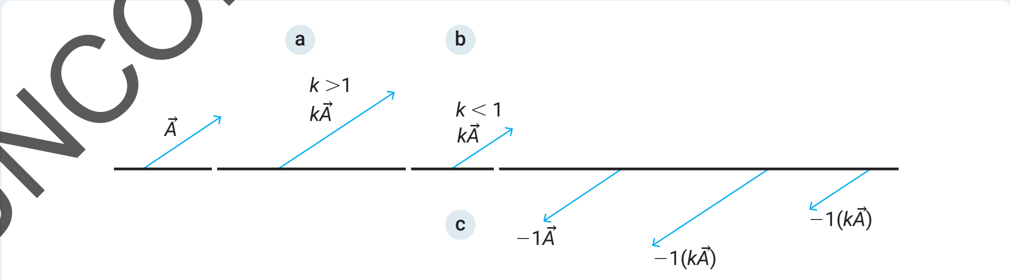


FIGURE 1.1.7 Multiplication of a vector by the scalar k. (**a**) If $0 < k < 1$, magnitude is reduced and the direction is *not* altered. (**b**) If $k > 1$, magnitude is increased and the direction is *not* altered. (**c**) If $k < 0$, magnitude is affected *and* the direction is reversed.

LEARNING CHECK 1.1

DESCRIBING

1 **Identify** the two methods of geometric vector addition.

2 **Explain** what must be done in order to add vectors geometrically.

3 **Describe** what happens to a vector when it is multiplied by a scalar k, when k:

a $k > 1$ b $0 < k < 1$ c $k < 0$.

4 a Write the components of vector $\vec{A}$, in terms of the angle relative to the x-axis.
b **Recall** how to find the angle in terms of components.

UNDERSTANDING

5 A vector represents a force of 150 N east 40° north. **Calculate** the east (horizontal) resolute and the north (vertical) resolute of the vector.

6 **Explain** how vector subtraction can be treated as vector addition.

7 **Explain** why a consistent scale is needed when constructing answers to vector equations, using a scale diagram method.

8 **Describe** the conditions that must be met before Pythagoras' theorem and trigonometric ratios can be used to solve vector additions and subtractions.

APPLYING

9 For vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, write vector *addition* equations to show:
a $\vec{C}$ as the resultant when $\vec{A}$ and $\vec{B}$ are added
b $\vec{C}$ as the resultant when $\vec{A}$ is subtracted from $\vec{B}$
c $\vec{A}$ as the difference between $\vec{B}$ and $\vec{C}$
d $\vec{C}$ as the resultant when $2\vec{A}$ and $3\vec{B}$ are added.

ANALYSING

10 $\vec{P}, \vec{Q}$ and $\vec{R}$ are all vectors on a Cartesian plane. Write an equation to show the components in x- and y-directions when $\vec{R} = \vec{P} + \vec{Q}$.

1.2 Solving problems: vectors

Vectors involving magnitude and direction can be added and subtracted geometrically. In order to do this accurately, use Cartesian graph paper. Use a ruler, protractor and a fine-point pencil to draw the vectors.

Tip-to-tail method

Follow these steps for geometrically adding vectors by the tip-to-tail method.

1. Decide on an appropriate scale for the drawing – state the scale in the form, p units represents q units or (p units: q units).
2. Draw one of the vectors to scale, showing tail and head clearly. Label it $\vec{A}$.
3. Draw the second vector to scale, so that its tail is on the head of the first vector, and its head is pointing in the correct direction. Label it $\vec{B}$.

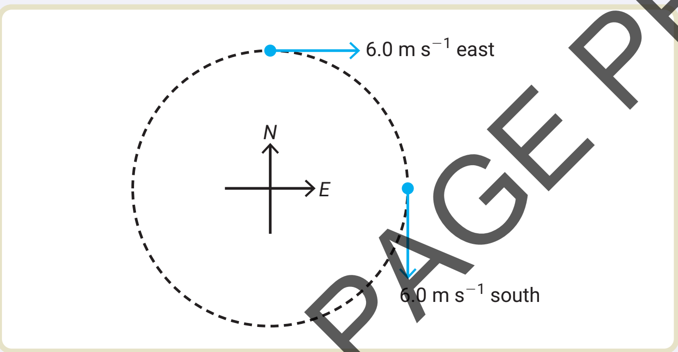
4. The arrow drawn from the tail of the first vector to the head of the second vector is the resultant (sum) of the two vectors: $\vec{R} = \vec{A} + \vec{B}$.
5. Measure the length of the resultant and use the scale to convert it to the correct value.
6. Measure the angle carefully with the protractor – angles may be related to the compass points (quadrant or azimuth bearings) or the x-axis, or relative to one of the vectors in the problem.

WORKED EXAMPLE 1.2.1

A ball-bearing slides around a horizontal circular track at a constant speed of 6.0 m s^{-1}. At one point, it is travelling east. One-quarter of a turn later, it is travelling south.

Use the tip-to-tail method to find the change of velocity for the ball-bearing.

Recall that $\Delta v = v - u$ or $u_{final} - u_{initial}$.



ANSWER

1. Determine the formula formula.

Find the change of velocity by subtracting 6.0 m s^{-1} east from 6.0 m s^{-1} south. This is done by adding the negative of the initial velocity.

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$
$$= \vec{v}_f + (-\vec{v}_i)$$

2. Substitute known values.

$$\Delta v = 6.0 \text{ m s}^{-1} \text{ south} - 6.0 \text{ m s}^{-1} \text{ east}$$
$$= 6.0 \text{ m s}^{-1} \text{ south} + 6.0 \text{ m s}^{-1} \text{ west}$$
$$\Delta\vec{v} = \sqrt{(6_2 + 6_2)}$$

3. Calculate the answer.

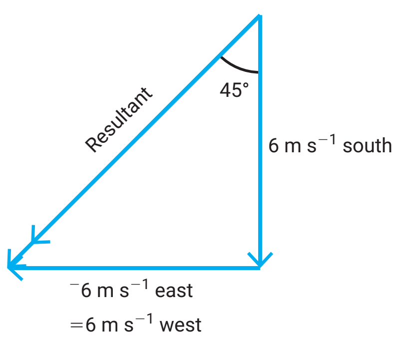
$= 8.40$ m s^{-1}

Scale: 1.0 cm = 1.0 m s^{-1}

The resultant, $\Delta\vec{v}$, is measured to be 8.49 cm. This converts to 8.49 m s^{-1}.

The angle is measured as W45°S.

Thus, $\Delta\vec{v} = 8.49$ m s^{-1}, W45°S.



The magnitude of the resultant may be found algebraically using Pythagoras' theorem.

Parallelogram method

Weblink
Parallelogram rule for vector addition

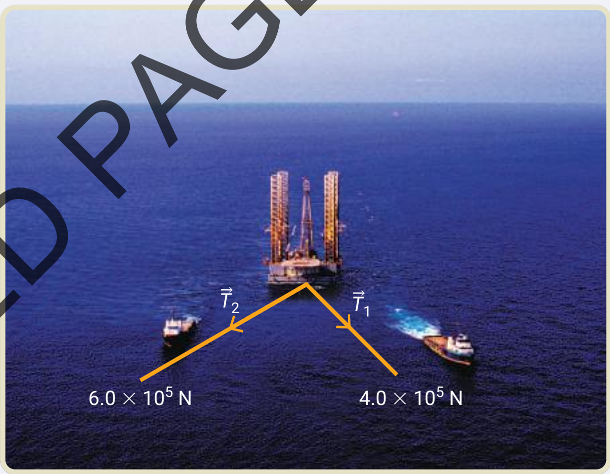
Follow theses steps for adding vectors geometrically by the parallelogram method.

1. Decide on an appropriate scale for the drawing – state the scale in the form, *p* units represents *q* units or (*p* units: *q* units).
2. Draw one of the vectors to scale, showing tail and head clearly. Label it $\vec{A}$.
3. Draw the second vector to scale, so that its tail is on the tail of the first vector (the origin), and its head is pointing in the correct direction. Label it $\vec{B}$.
4. Construct a parallelogram, using the two vectors as adjacent sides.
5. The arrow drawn along the diagonal that starts from the tails of the two vectors is the resultant (sum) of the two vectors: $\vec{R} = \vec{A} + \vec{B}$.
6. Measure the length of the resultant and use the scale to convert it to the correct value.
7. Measure the angle carefully with the protractor – angles may be related to the compass points (quadrant or azimuth bearings) or the *x*-axis, or relative to one of the vectors in the problem.

WORKED EXAMPLE 1.2.2

An oil rig is being towed into place by two tug boats, T_1 and T_2. T_1 pulls on the rig with a force of 4.0×10^5 N, 135° true. The true bearing of T_2 is 240° and it pulls with a force of 6.0×10^5 N.

Determine the resultant of these two forces.

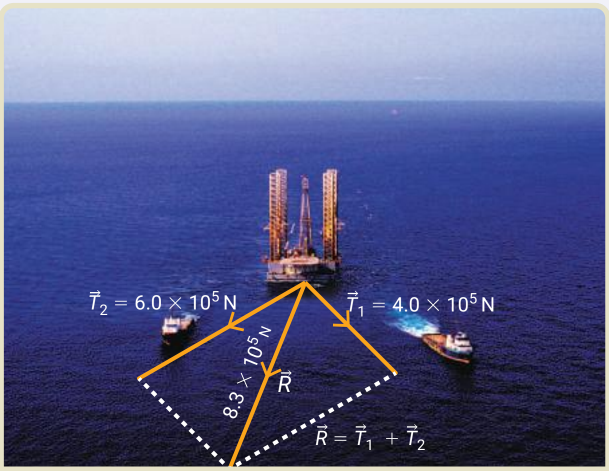


Peter Bowater/Science Source

ANSWER

Scale: 1.0 cm represents 1.0×10^5 N

1. **Determine the resultant.**
   The resultant, $\vec{R}$, is measured to be 12.6 cm.
   This converts to 8.3×10^5 N **(Figure 1.2.1)**.
2. **Determine the angle.**
   The angle is measured as 207° true.
3. **Calculate the resultant force.**
   Thus, $\vec{R} = 8.3 \times 10^5$ N, 207° true or 8.3×10^5 N, west 63° south.



Peter Bowater/Science Source

FIGURE 1.2.1 Addition of forces by parallelogram method

9780170483704

Rectangular components

Follow this general procedure for adding (or subtracting) two or more vectors using components.

1. Sketch a diagram that clearly shows the vector addition.
2. Choose rectangular *x*- and *y*-axes. Sometimes this is obvious, as in the case of using the compass points. Sometimes it is better to select the *x*-axis to be along one of the vectors so that one of the vectors has a single component.
3. Resolve each vector into rectangular resolutes (*x*- and *y*-components, compass points, horizontal and vertical etc.). Be sure to identify positive and negative values for the resolutes.
4. Calculate the magnitude of each component by trigonometry.

KEY FORMULA

For vector $\vec{A}$, which is oriented at an angle θ to the defined positive direction (*x*-axis), the magnitudes of the components are:

- along the *x*-axis: $A_x = A \cos \theta$
- along the *y*-axis: $A_y = A \sin \theta$

5. Find the magnitude of the components of the resultant vector.

KEY FORMULA

The *x*-component, R_x, of the resultant, $\vec{R}$, is the sum of all the *x*-components of the individual vectors being added. Similarly, the *y*-component, R_y, is the sum of all the *y*-components of the individual vectors being added.

6. Determine the magnitude of the resultant using Pythagoras' theorem.
7. Determine the angle using the tangent ratio.

KEY FORMULA

The magnitude of the resultant is determined using Pythagoras' theorem:

$$R = \sqrt{R_x^2 + R_y^2}$$

KEY FORMULA

The angle with respect to the axis is found by the trigonometric ratio for tan:

$$\tan\theta = \frac{R_y}{R_x}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

8. Check that the angle is given in the terms required by the question.

WORKED EXAMPLE 1.2.3

Solve the tug boat–oil rig problem from Worked example 1.2.2 by using the sum of component vectors.

ANSWER

1 Determine the formula to calculate *x*.
On initial inspection, the resultant is likely to be more towards the west.
Take west as positive *x*-direction:

$$R_x = T_{1x} + T_{2x}$$

2 Subsitute the known values.

$$R_x = {}^{-}4.0\times10^5\ \text{N}\times\cos 45° + 6.0\times10^5\ \text{N}\times\cos 30°$$

3 Calculate the *x*-components.

$$R_x = 2.368\times10^5\ \text{N}$$

4 Determine the formula to calculate *y*.
On initial inspection, the resultant is likely to be more towards the south.
Take south as positive *y*-direction:

$$R_y = T_{1y} + T_{2y}$$

5 Substitute the known values.

$$R_y = 4.0\times10^5\ \text{N}\times\sin 45° + 6.0\times10^5\ \text{N}\times\sin 30°$$

6 Calculate the *y* components.

$$R_y = 5.828\times10^5\ \text{N}$$

7 Determine the formula for the magnitude of resultant.

$$R = \sqrt{R_x^2 + R_y^2}$$

8 Substitute known values.

$$R = \sqrt{(2.368\times10^5)^2 + (5.828\times10^5)^2}$$

9 Calculate the magnitude of resultant.

$$R = 8.3\times10^5\ \text{N}$$

10 Determine formula to calculate angle.
Let θ be the angle opposite the *y*-component:

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

11 Substitute known values.

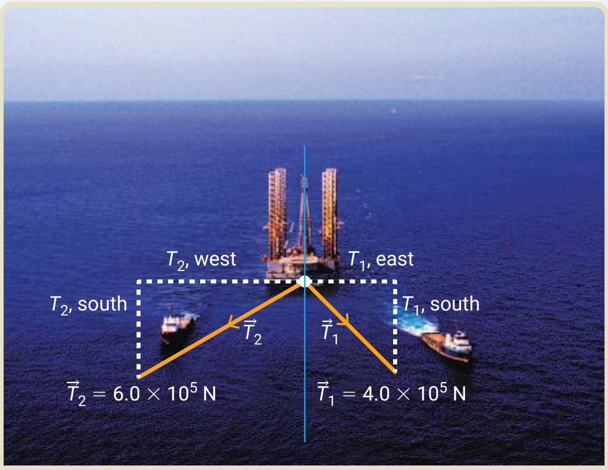
$$\theta = \tan^{-1}\left(\frac{5.828\times10^5\ \text{N}}{2.368\times10^5\ \text{N}}\right)$$

12 Calculate the angle.

$$\theta = 68°$$

13 Determine the quadrant or azimuth bearing.
Let α be the angle with respect to west (270°).
$\alpha = 68°$
$\alpha =$ west 68° south

14 Determine the true bearing.
True bearing = 270° − 68°
True bearing = 202°



Peter Bowater/Science Source

FIGURE 1.2.2 A free-body diagram showing forces and their components for the oil rig being towed by two tug boats (see Figure 1.2.1).

WORKED EXAMPLE 1.2.4

A mass slides without friction down a slope that is inclined at 30° to the horizontal. Find the component of the gravitational acceleration, $g = 9.8\ \text{m s}^{-2}$, parallel to the surface.

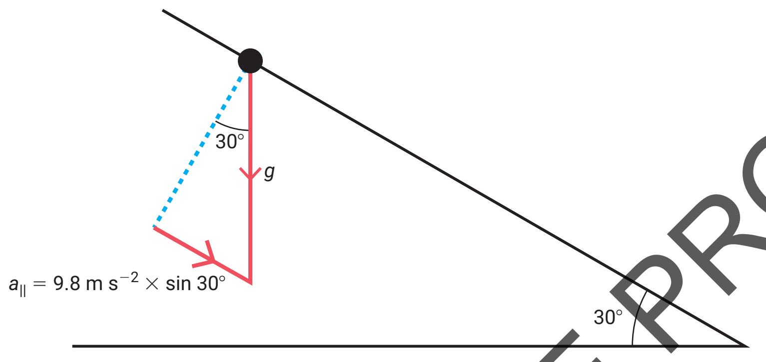


FIGURE 1.2.3 The component of the acceleration due to gravity down the inclined plane

ANSWER

The acceleration due to gravity is $g = 9.8\ \text{m s}^{-2}$ vertically down.

1 **Determine the formula for the gravitational acceleration.**
Parallel to the slope, the component of the gravitational acceleration, $a_{||}$, is:
$a_{||} = g \sin \theta$

2 **Substitute the known values.**
$a_{||} = 9.8\ \text{m s}^{-2} \times \sin 30°$

3 **Calculate the answer.**
$a_{||} = 4.9\ \text{m s}^{-2}$

LEARNING CHECK 1.2

DESCRIBING

1 **Contrast** the steps of the tip-to-tail method and the parallelogram method for solving vector problems.

2 State an advantage of adding vector components to determine a resultant compared to the scale diagram method.

3 State the equations for resolving a vector into the *x*- and *y*-components.

4 Using a diagram, **describe** the process of finding the net force acting on an object when two forces act at different angles to one another.

APPLYING

5 Use the tip-to-tail method to add the following vectors on a Cartesian plane.

a $\vec{A} = 50$ m, $\theta = 30°$; $\vec{B} = 80$ m, $\theta = 75°$

b $\vec{A} = 10$ m, $\theta = 30°$; $\vec{B} = 30$ m, $\theta = 45°$

6 Use the parallelogram method to add the following vectors on a Cartesian plane.

a $\vec{A} = 3 \times 10^3$ N, $\theta = 30°$; $\vec{B} = 4 \times 10$ N, $\theta = 60°$

b $\vec{A} = 320$ N, $\theta = 40°$; $\vec{B} = 250$ N, $\theta = 80°$

ANALYSING

7 Use the components method to subtract $\vec{B}$ from $2\vec{A}$ on a Cartesian plane, given the following values.

a $\vec{A} = 15$ m s^{-1}, $\theta = 30°$; $\vec{B} = 25$ m s^{-1}, $\theta = 60°$

b $\vec{A} = 35$ N, $\theta = 120°$; $\vec{B} = 25$ N, $\theta = 45°$

8 **Sketch** the vector sums, $\vec{P} + \vec{Q}$, then find the resultant, $\vec{R}$, using the component method.

a $\vec{P} = 20$ N, 120° true; $\vec{Q} = 50$ N, 300° true

b $\vec{P} = 300$ N, N30°W; $\vec{Q} = 450$ N, S60°W

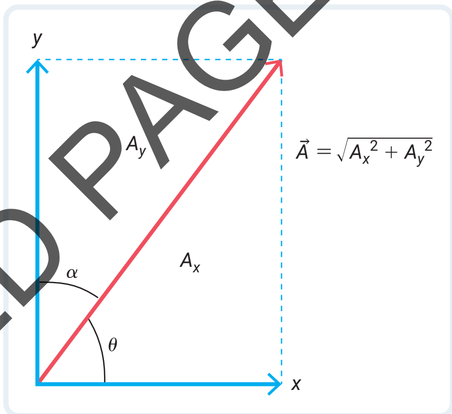
CHAPTER SUMMARY

Geometric vector addition

- All vectors can be added and subtracted geometrically.
- In the tip-to-tail method, the tail of a vector is placed at the head of another vector and the line connecting the tail of the first vector to the head of the second vector is the resultant vector.
- In the parallelogram method, the vectors are placed so their tails coincide to form a parallelogram with the vectors as adjacent sides. The diagonal of this parallelogram, from the common tail, represents the resultant vector.
- The tip-to-tail and parallelogram methods can be used for addition and subtraction of vectors by reversing the direction of the vectors.

Components of vectors

- All vectors can be constructed as the sum of any two other, component vectors.
- Rectangular components are components at right angles to each other.
- Vector $\vec{A}$ can be resolved into rectangular components using the general formula:
  x-component: $A_x = A\cos\theta$
  y-component: $A_y = A\sin\theta$
  The magnitude is found by Pythagoras' theorem:



$$A = \sqrt{A_x^2 + A_y^2}$$

where: A = magnitude of vector, $\vec{A}$
A_x = magnitude of the x-component
A_y = magnitude of the y-component
θ = angle relative to the positive direction of the x-axis

Angle:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{y\text{-component}}{x\text{-component}}$$
$$\theta = \tan^{-1}\left(\frac{y\text{-component}}{x\text{-component}}\right)$$

- Multiplication of a vector by a scalar:
  - Scalar multipliers are positive or negative numbers that can change the magnitude and/or direction of a vector.
  - Multiplication of a vector by a positive scalar changes the magnitude but not the direction of the vector.
  - Multiplication of a vector by a negative scalar changes the magnitude and reverses the direction of the vector.

CHAPTER EXAM

MULTIPLE CHOICE

1. Vectors and scalars differ because a:

A vector must have two scales; a scalar must have only one scale.
B vector must have at least two scales; a scalar must have only one scale.
C scalar must have only two scales; a vector must have only one scale.
D scalar must have at least two scales; a vector must have only one scale.

2. Which of the following statements is true?

A True bearings are taken with respect to the positive x-axis.
B Quadrant bearings are taken with respect to the positive y-axis.
C True bearings can take any value between 0° and 360°.
D Quadrant bearings can take any value between 0° and 360°.

3. A vector has a magnitude M and true bearing of θ, where $0° < \theta < 90°$. What is its component in the direction of east?

A $M\sin\theta$ **B** $M\cos\theta$
C $M\tan\theta$ **D** M, E$(90° - \theta)$N

4. A vector has a magnitude M and true bearing of θ, where $180° < \theta < 270°$. What is its component in the direction of east?

A $-M\sin(270° - \theta)$ **B** $M\cos(\theta)$
C $-M\cos(270° - \theta)$ **D** M, S$(270° - \theta)$W

5. On a Cartesian plane, a vector, $\vec{R}$, has an x-component of p and a y-component of q. What are the magnitude, R, and direction, θ, of $\vec{R}$ respectively?

A $\sqrt{p^2 + q^2}$; $\tan\left(\frac{p}{q}\right)$ **B** $\sqrt{p^2 + q^2}$; $\tan\left(\frac{q}{p}\right)$
C $\sqrt{p^2 + q^2}$; $\tan^{-1}\left(\frac{p}{q}\right)$ **D** $\sqrt{p^2 + q^2}$; $\tan^{-1}\left(\frac{q}{p}\right)$

6. If a force of 10 N is applied at an angle of 30° to the horizontal, what are the horizontal and vertical components of the force?

A 8.7 N, 5.0 N **B** 5.0 N, 5.0 N
C 5.0 N, 7.1 N **D** 10 N, 0.0 N

7. A boat moves with a velocity of 12 m s^{-1} east, while the current flows 5 m s^{-1} south. What is the resultant velocity of the boat?

A 13 m s^{-1}, 22.6° south of east **B** 10 m s^{-1}, 45° south of east
C 17 m s^{-1}, 45° south of east **D** 7 m s^{-1}, 22.6° south of east

8. Three displacement vectors are 5 m north, 7 m east and 2 m south. What is the resultant displacement?

A 7.8 m, 45° north-east **B** 5.0 m, 53.1° east of north
C 6.4 m, 46.4° east of north **D** 7.6 m north, 66.8° east

9. A vector has components 9 m at N70°W and 12 m at N10°E. What is the magnitude and direction of the resultant vector?

A 15.0 m, N20°W **B** 14.7 m, N25°E
C 16.2 m, N23°W **D** 13.4 m, N10°E

10. Three forces of 8 N act on an object at N60°E, S30°E and S45°W. What is the magnitude of the resultant force?

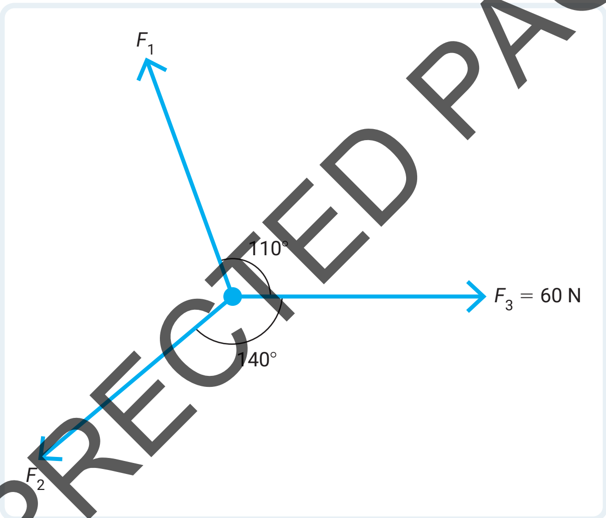
A	0 N	**B**	8 N
C	16 N	**D**	24 N

SHORT RESPONSE

11. A yacht travels at a velocity of 120 m s^{-1}, N50°E before changing speed and direction to 108 m s^{-1} in the direction 140° true. Find the average acceleration of the yacht if the change takes a total of 2 min 25 s.

12. A drone enthusiast practises on a circular course of radius 10.0 m. In one practice flight, the drone travels at a constant speed of 4.2 m s^{-1}.
    - **a** **Calculate** how long it takes for the drone to complete one circuit of the course.
    - **b** **Construct** the change of velocity vector for a one-quarter segment of the circular circuit. Use an initial velocity of u heading west.

13. Three friends are pulling on a round table from different directions, trying to move it. The forces are shown in the diagram below. If the third friend is pulling with a force of 60 N, **determine** the forces exerted by the other two friends, given that the table remains stationary.



14. A plane flies through the air and its jets exert a thrust force of 13 kN east. A significant wind is applying another force to the plane of 950 N from S70°E. **Determine** the acceleration of the plane.

15. Three forces act on a 2.0 kg object.
F_1 = 10 N at N30°E
F_2 = 8 N at S20°E
F_3 = 6 N at S45°W
The object starts from rest. **Calculate** the final displacement of the object if the forces act on it for 4 seconds.

CHAPTER 2

Projectile motion

TK

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Describe how horizontal and vertical components of a velocity vector are independent of each other.
- Solve problems involving projectile motion in the absence of drag effects using $v_y = u_y + gt$, $s_y = u_y t + \frac{1}{2}gt^2$, $v_y^2 = u_y^2 + 2gs_y$, $v_x = u_x$ and $s_x = u_x t$.
- Interpret data relating to the horizontal distance travelled by an object projected at various angles from the horizontal.

SCIENCE AS A HUMAN ENDEAVOUR

- Explore the role of forensic evidence used in court and the challenges associated with providing conclusive evidence that may lead to convictions.

SCIENCE INQUIRY

- Investigate the horizontal distance travelled by an object projected at various angles from the horizontal.

Physics 2025 v1.2 General Senior Syllabus © QCAA 2024

Introduction

A projectile is an object that goes up and down vertically at the same time as it moves horizontally. Therefore, projectile motion is motion in two dimensions. Near the surface of Earth, the gravitational field acts vertically throughout the motion. If we assume air resistance is negligible, then the vertical motion is only affected by the gravitational field. But, as there is no force component in the horizontal direction, the horizontal motion is not affected by a force or component of force.

Assessments
- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practicals
- Investigating the ma[illegible] of a microscrope

Worksheets
- Name
- Name
- [illegible]me

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ The kinematic variables displacement, initial and final velocity, acceleration and time can be represented by the variables *s, u, v, a* and *t*.
- ✓ Acceleration of near-Earth objects due to gravity is always vertically downwards and its value is independent of the objects' mass.
- ✓ The gradient of a linear graph can be calculated by $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe how horizontal and vertical components of a velocity vector are independent of each other
- ✓ solve problems involving projectile motion in the absence of drag effects using $v_y = u_y + gt,\ s_y = u_y t + \frac{1}{2}gt^2,\ v_y^2 = u_y^2 + 2gs_y,\ v_x = u_x$ and $s_x = u_x t$
- ✓ explain how $R = \frac{u^2 \sin 2\theta}{g}$, 0° < θ < 90°, is derived from the above equations
- ✓ interpret data relating to the horizontal distance travelled by an object projected at various angles from the horizontal
- ✓ explain the relationships between launch angles, maximum heights and ranges.

2.1 Horizontal and vertical vector components

A projectile is launched at speed, *u*, and angle, θ, relative to the horizontal.

Horizontal component of the launch velocity, u_x

The horizontal component of the launch velocity is:

$$u_x = u\cos\theta$$

The relationship can be seen in **Figure 2.1.1**.

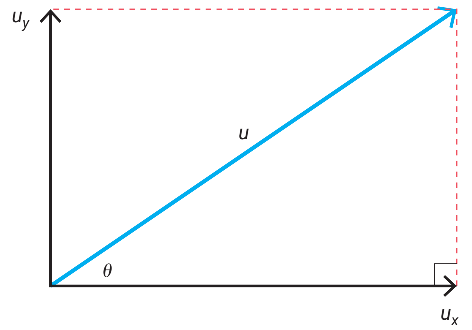


FIGURE 2.1.1 A projectile is launched at speed u and angle θ to the horizontal. The horizontal and vertical components of the launch velocity, u_x and u_y respectively are shown.

Vertical component of the launch velocity, u_y

Typically the direction upwards from the launch position is defined as positive. Then, the vertical component of the launch velocity is:

$$u_y = u\sin\theta$$

Weblink
Describing projectiles with numbers: (horizontal and vertical velocity)

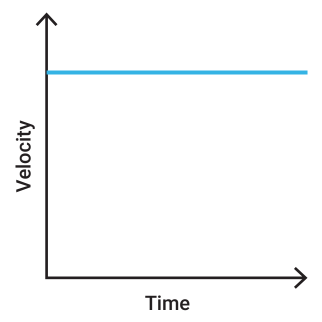


FIGURE 2.1.2 The horizontal component of motion of a projectile remains the same, neglecting air resistance; $u_x = v_x$.

KEY FORMULA

Components of the launch velocity

Horizontal component: $u_x = u\cos\theta$
Vertical component: $u_y = u\sin\theta$

where:
u = launch speed (typically m s^{-1})
u_x = horizontal component of the launch velocity (m s^{-1})
u_y = vertical component of the launch velocity (m s^{-2})
θ = angle of the launch velocity relative to the horizontal (typically degrees °)

WORKED EXAMPLE 2.1.1

A projectile is fired at an angle of 30° to the horizontal with a speed of 120 m s^{-1}. For the initial velocity, find the:

a horizontal component
b vertical component.

ANSWERS

a **1** **Determine the formula.**
$u_x = u\cos\theta$

2 **Substitute the known values.**
$u_x = 120\text{ m s}^{-1} \times \cos 30°$

3 **Calculate the answer.**
$u_x = 104\text{ m s}^{-1}$

b **1** **Determine the formula.**
$u_y = u\sin\theta$

2 **Substitute the known values.**
$u_y = 120\text{ m s}^{-1} \times \sin 30°$

3 **Calculate the answer.**
$u_y = 60\text{ m s}^{-1}$

The vertical motion is affected by Earth's gravitational field, which applies a force downwards on the object. As the vertical (upwards) direction is defined as positive, the constant acceleration due to Earth's gravitational field near Earth is negative:

$$g = -9.8\text{ m s}^{-2}$$

The speed–time graph of the vertical component of motion of a projectile is shown in **Figure 2.1.3**.

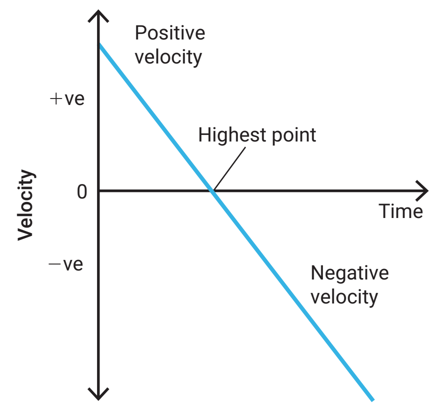


FIGURE 2.1.3 When drag is considered negligible, the vertical component of motion of a projectile proceeds at constant negative acceleration: $g = -9.8$ m s^{-2}.

Combining horizontal and vertical components of projectile motion

For a projectile, the complete set of motion variables can be specified by treating the horizontal component of the motion separately from the vertical component of the motion. Algebraically, the *suvat* kinematics equations can be rewritten using the symbols defined.

KEY FORMULAS

$v_y = u_y + gt$

$s_y = u_y t + \frac{1}{2}gt^2$

$v_y^2 = u_y^2 + 2gs_y$

$v_x = u_x$

$s_x = u_x t$

Horizontal component of motion

There is no horizontal component of force to affect the horizontal motion. The horizontal component of acceleration $a_x = 0$. Consequently, the horizontal component of the motion remains constant and $u_x = u_y$. Therefore, the horizontal distance, s_x, travelled in a time interval t is:

$$s_x = u_x t = v_x t$$

The velocity–time graph of the horizontal component of motion of a projectile is shown in Figure 2.1.2.

KEY FORMULA

Horizontal component of motion at zero acceleration

$$s_x = u_x t = v_x t$$

where:

s_x = horizontal distance travelled (m)
u_x = horizontal component of the launch velocity (m s^{-1})
v_x = horizontal velocity (m s^{-1})
t = time (s)

Vertical component of motion at constant acceleration, *g*

The vertical component of motion is affected by the constant gravitational field, *g*, which produces a constant acceleration of $g = -9.8\ \text{m s}^{-2}$ on every mass near the surface of Earth. Thus, the *suvat* equations can be used to analyse the vertical component of the motion. The initial and final vertical components of speed are, respectively, u_y and v_y. In the vertical direction, the object travels a distance interval of $s_y = y$ over a time interval *t*. The *suvat* equations can be written for projectile motion as follows:

$$v_y = u_y + gt$$

$$s_y = u_y t + \frac{1}{2}gt^2$$

$$v_y^2 = u_y^2 + 2gs_y$$

KEY FORMULAS

$$v_y = u_y + gt$$

$$s_y = u_y t + \frac{1}{2}gt^2$$

$$v_y^2 = u_y^2 + 2gs_y$$

where:
u_y = vertical component of the launch velocity (m s^{-1})
v_y = vertical component of the velocity some time after launch (m s^{-1})
$s_y = y$ = vertical height interval relative to the launch position (m)
g = acceleration due to gravity (m s^{-2})
t = time interval (s)

LEARNING CHECK 2.1

DESCRIBING

1 Consider a projectile launched at speed *u* and angle relative to the horizontal θ.
  a **Draw** a vector diagram to show the launch velocity and its horizontal and vertical components.
  b Write the equation for each of the components u_h and u_v.
2 **Define** all the variables in the kinematics equation $v_y^2 = u_y^2 + 2gs_y$.
3 Near the surface of Earth, the gravitational field is taken to be constant. **Explain** this approximation after calculating '*g*' using $g = \frac{Gm}{r^2}$, where $r = 6.371 \times 10^6$ m (at sea level) and where $r = 6.408 \times 10^6$ m (at the peak of Mt Everest).
4 **Explain** why the horizontal and vertical components of motion of projectiles are independent of each other.

APPLYING

5 Copy and complete the following table.

Launch speed (m s^{-1})	Angle to the horizontal (°)	Horizontal component (m s^{-1})	Vertical component (m s^{-1})
20	30		
15.6	45		
2.41	60		

6 A ball is thrown upwards with a speed of 12 m s^{-1} at an angle of 70° to the horizontal.

a **Calculate** the velocity of the ball when it reaches its highest point.
b **Determine** the acceleration of the ball at the top of its flight.
c Find the time when the ball is at a height of 4.0 m above its launch position.

7 A rocket leaves the launch pad with a speed of 300 m s^{-1} at an angle of elevation of 35°. **Calculate** the horizontal distance travelled, in kilometres, when it returns to the same height as the launch site.

ANALYSING

8 **Compare** the velocity–time graphs for vertical and horizontal motion of a projectile (Figures 2.1.2 and 2.1.3). **Determine** what the slopes represent and how they differ.

9 A projectile is launched at an angle of 60° above the horizontal. It rises to a maximum height of 25 m. Find the launch speed.

REFLECTING

10 Qualitatively **compare** projectile motion with and without the effect of air resistance.

2.2 The trajectory of projectiles 'near Earth'

Projectiles may be launched with initial velocities that are vertical (up or down), horizontal or at some angle to the horizontal. 'Near Earth' the magnitude of the acceleration due to gravity is 9.8 m s^{-2} directed vertically downwards.

Falling and horizontally projected objects

Two balls are released simultaneously from the same position above the ground. Ball A, initially at rest, falls vertically down. Ball B is projected horizontally at initial speed u_x (Figure 2.2.1).

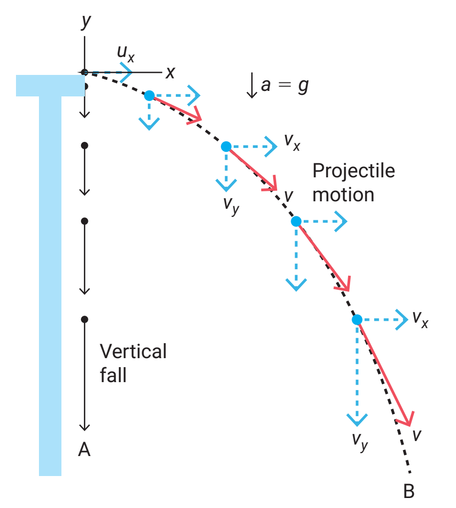


FIGURE 2.2.1 Two balls A and B are released simultaneously from the same height, one of which also has a horizontal velocity.

Both objects fall at the same rate, but the path of ball B is 'stretched' horizontally by the horizontal component of the velocity. After a time interval t, the velocity of each ball can be found by analysing the horizontal and vertical motions independently of each other, then combining the results.

Horizontal motion of ball B

The horizontal component of the velocity after a time interval t is the same as the original horizontal component of the launch velocity:

$$v_x = u_x \text{ for all time intervals}$$

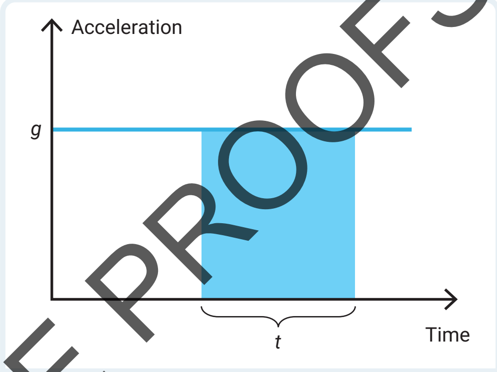


FIGURE 2.2.2 The area under an acceleration–time graph, such as this g–t graph, is the change in velocity.

Vertical motion of ball A

The vertical component of the launch velocity is changed by an amount equal to the area under the acceleration–time or g–t graph (Figure 2.2.2).

$$v_y - u_y = gt$$

$$v_y = u_y + gt$$

where $u_y = 0\,\text{m s}^{-1}$ if dropped

$$v_y = gt\ (u_y = 0)$$

This is the same vertical component of velocity as ball B.

Combining vertical and horizontal components for ball A

The velocity, $\vec{v}$, can now be specified by using Pythagoras' theorem (magnitude) and trigonometry (direction):

$$|\vec{v}| = \sqrt{u_x^2 + v_y^2}$$

$$= \sqrt{u_x^2 + (gt)^2} \quad (u_y = 0)$$

$$\tan\theta = \frac{v_y}{v_x}$$

Weblinks
Characteristics of a projectile's trajectory
Projectile motion

Projectile launched at an angle to the horizontal

For the motion of a projectile launched at speed u and angle θ, a general analysis, similar to that for horizontal projection ($u_y = 0$), can be undertaken. The velocity is tangential to the path of the projectile (Figure 2.2.3).

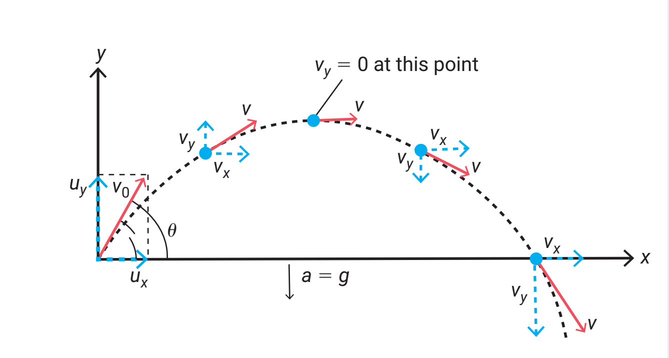


FIGURE 2.2.3 The path of a projectile fired with initial speed u and angle θ. At each point, the horizontal component of the velocity is the same. The vertical component changes according to the time for which the gravitational field acts.

Horizontal motion of projectile

The horizontal component of the velocity after a time interval t is the same as the original horizontal component of the launch velocity:

$$v_x = u_x \text{ for all time intervals}$$

Vertical motion of projectile

The vertical component of the launch velocity is changed by an amount equal to the area under the acceleration–time or g–t graph (Figure 2.2.2).

$$v_y - u_y = gt$$
$$v_y = gt + u_y \; (u_y > 0)$$

Combining vertical and horizontal components for a projectile

The velocity, $\vec{v}$, at any time can now be specified by using Pythagoras' theorem (magnitude) and some trigonometry (direction).

KEY FORMULAS

$$\vec{v} = \sqrt{u_x^2 + v_y^2}$$

$$\text{where } v_y = u_y + gt$$

$$\tan\theta = \frac{v_y}{u_x}$$

where:
$\vec{v}$ = magnitude of the final velocity (m s^{-1})
u_x = horizontal component of launch velocity, $u \cos \theta$ (m s^{-1})
u_y = vertical component of launch velocity, $u \sin \theta$ (m s^{-1})
v_y = vertical component of final velocity (m s^{-1})
θ = angle of the launch velocity relative to the horizontal (°)
g = the acceleration due to gravity (m s^{-2})
t = time interval (s)

WORKED EXAMPLE 2.2.1

A projectile is launched with an initial velocity of 25 m s^{-1} at an angle of 20° above the horizontal. The projectile lands on level ground. Assume air resistance is negligible.

a Determine the total time of flight.
b Calculate the horizontal range of the projectile.

ANSWERS

a 1 Consider the information presented.
Consider the variables provided in the question and required for the answer.
The vertical displacement is $s_y = 0$ s (since the projectile lands on level ground).

2 Determine the formula.
Use the vertical motion equation:

$$s_y = u_y t + \frac{1}{2}gt^2$$

where:
$s_y = 0$ m (vertical displacement)
$u_y = 25 \sin 20$ m s^{-1} (initial vertical velocity)
$g = -9.8$ m s^{-2} (acceleration due to gravity)
t is the total time of flight (s).

$$0 = 25 \sin 20 \times t - \frac{1}{2} 9.80\, t^2$$

3 **Subsitute the known values.**

4 **Calculate the answer.**

$t = 1.75$ s

a 1 **Determine the formula to calculate horizontal motion.**

This gives two solutions: $t = 0$ s (initial time) and $t = 1.75$ s (flight time).

The horizontal range is calculated using the horizontal motion equation:

$s_x = u_x t$

2 **Substitute the known values.**

The horizontal velocity is: $u_x = u\cos\theta$

$s_x = 25\cos 20 \times 1.75$

3 **Calculate the answer.**

$s_x = 41.1$ m

Range of a projectile over level ground

A projectile that is launched over level ground at 60° stays above ground for longer than one launched at 30°. However, they both travel the same horizontal distance. In general, for a particular launch velocity, there are two solutions to the horizontal range of the projectile. **(Figure 2.2.4)** At a projection angle of 45°, the two solutions coincide and the range is a maximum. This can be deduced from the equations for projectile motion.

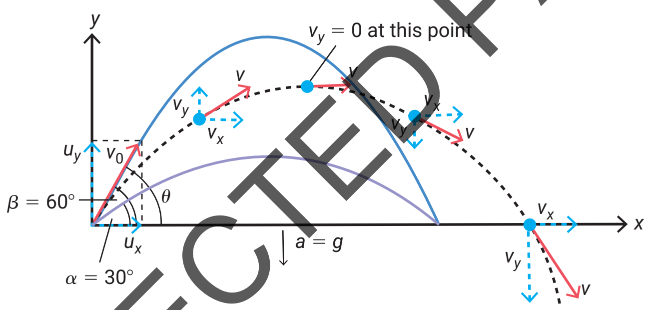


FIGURE 2.2.4 The range of a projectile over level ground depends on the angle of projection. There are two solutions for the same range, except where they coincide at 45°.

Vertical component of motion

The time of flight can be calculated from the vertical component of the motion. The position, y, above the ground is zero when the projectile is launched and when it lands.

$$y = 0$$

$$\frac{1}{2}gt^2 + u_y t = 0$$

$$t\left(\frac{1}{2}gt + u_y\right) = 0$$

Let $t = 0$ (initial condition); or

$$\frac{1}{2}gt + u_y = 0$$

$$t = \frac{2u_y}{g} \text{ (positive as } g < 0)$$

$$\frac{1}{2}gt^2 + u_y t = 0$$

$$t\left(\frac{1}{2}gt + u_y\right) = 0$$

$$\frac{1}{2}gt + u_y = 0$$

$$t = \frac{2u_y}{g} \text{ (positive because } g < 0)$$

Horizontal component of motion

The range, R, can be calculated by finding values for s_x from the constant horizontal component of the launch velocity and using the time of flight:

$$s_x = u_x t = R \text{ (range)}$$

$$R = u_x\left(\frac{2u_y}{g}\right) \text{ (from vertical analysis)}$$

$$= \frac{2u_x u_y}{g}$$

But $u_x = u\cos\theta$ and $u_y = u\sin\theta$.

$$R = \frac{2u^2 \sin\theta\cos\theta}{g}$$

$$= \frac{u^2 \times 2\sin\theta\cos\theta}{g}$$

Recall the trigonometric identity that $\sin 2\theta = 2\sin\theta\cos\theta$.
Hence

$$R = \frac{u^2 \sin 2\theta}{g}, 0° < \theta < 90°$$

KEY FORMULA

$$R = \frac{u^2 \sin 2\theta}{g}, 0° < \theta < 90°$$

where:
R = range over level ground from launch site to landing position (m)
u = initial launch speed (m s^{-1})
θ = launch angle (°)
g = acceleration due to gravity (m s^{-2})

The range is a maximum when:

$$\sin 2\theta = 1$$
$$2\theta = 90°$$
$$\theta = 45°$$

Note that the sine function is positive in the first and second quadrants. This means that, if $2\theta = \alpha$ is a solution, then $2\theta = 180° - \alpha$ is a solution. Both solutions for θ lie in the range $0° < \theta < 90°$.

A word of caution

This analysis only works for projectiles that travel over level ground or between two positions at the same vertical distance above the horizontal. If the projectile lands above or below the launch position, the analysis will not provide correct answers as the path is not symmetrical.

WORKED EXAMPLE 2.2.2

A projectile is launched with an initial velocity of 30 m s^{-1} at an angle of 35° above the horizontal. The projectile lands on level ground. Assume air resistance is negligible. Determine the horizontal range of the projectile.

ANSWER

1 Determine the formula.
The horizontal range may be calculated using the range equation.

$$R = \frac{u^2 \sin 2\theta}{g} \quad 0° < \theta < 90°$$

2 Substitute the known values.

$$R = \frac{30^2 \sin 2(35)}{9.80}$$
$$= \frac{900 \times 0.939690}{9.80}$$

3 Calculate the answer.

$$R = \frac{845.7}{9.80}$$

and $R = 86.3$ m
The horizontal range of the projectile is 86.3 m.

Range of a projectile over varied surface levels

A projectile that is launched to heights above or below their launch points still trace out a parabolic path, though the path is not symmetrical. Therefore, these cases require more specific application of the various kinematic formulas to account for their varied vertical displacements.

WORKED EXAMPLE 2.2.3

A projectile is launched with an initial velocity of 20 m s^{-1} at an angle of 40° to the horizontal. The projectile lands on a platform that is 3 m above the launch height. Assume air resistance is negligible.

a Determine the total time of flight.
b Calculate the horizontal range of the projectile.

ANSWERS

a 1 Determine the formula.

The time of flight is determined by analysing the vertical motion using kinematics equations.
Consider the variables provided within the question and required of the answer.

$$s_y = u_y t + \frac{1}{2} g t^2$$

where:

s_y = vertical displacement
$u_y = u \sin\theta$ (initial vertical velocity)
$g = -9.8 \text{ m s}^{-2}$ (acceleration due to gravity)
t = the total time of flight (s)

2 Substitute the known values.

$$3 = 20 \sin 40° \times t - \frac{1}{2} \times 9.8 \times t^2$$

$$3 = 14.9t - 4.9 \times t^2$$

$$0 = -3 + 14.9\,t - 4.9 \times t^2$$

Or in standard form

3 Solve using the quadratic formula or using the equation solver in a graphing calculator.

This provides two solutions where $s_y = 3$ m above the launch height, $t = 0.22$ s and $t = 2.83$ s.
The total time of flight is therefore the second one, where the projectile has come down from its maximum height to land on the platform, hence $t = 2.83$ s.
The horizontal range may now be determined using the time of flight and the constant horizontal velocity, $u \cos\theta$.

b 1 Determine the formula.

$$\text{Range} = s_x = u_x t$$

$$s_x = u \cos\theta \times t$$

2 Substitute the known values.

$$s_x = 20 \cos 40° \times 2.83\ s$$

3 Calculate the answer.

$$s_x = 43.36 \text{ m}$$

PRACTICAL ACTIVITY 2.2.1

PROJECTILE MOTION

Introduction

Projectiles with the same launch speed over level ground travel different horizontal distances depending on the angle of launch. When air resistance is negligible and the acceleration due to gravity near the surface of Earth is constant, the range, R, is related to the magnitude of the launch velocity, u, and the angle of launch, θ, by the equation:

$$R = \frac{u^2 \sin 2\theta}{g}$$

Research questions

How can the relationship between launch angle and distance travelled be shown?
How does the theoretical value for range and launch speed compare with experimental values?

Aim

For a projectile that travels over level ground:

1 to demonstrate the relationship between launch angle and distance travelled
2 to compare theoretical with measured values for:
   a range
   b launch speed

Materials

- flat, horizontal surface
- curved track, such as a toy car track, mounted on a solid rigid base so that the end of the track is parallel to the base
- small ball, such as a ball bearing or glass marble
- wedges of different angles: 15°, 30°, 45°, 60° and 75°
- ruler
- carbon paper and A3 plain paper or sand tray for recording the landing position

Risk assessment

What are the risks in doing this experiment?	How can you manage these risks to remain safe?
Small balls may roll across the floor and create a tripping hazard.	Keep balls in a secure container when not in use. Assign one group member to collect the projected balls and return them to the container immediately.

Procedure

1 Arrange the curved track on a table so that the ball can be projected at an angle to the horizontal.
2 Use the wedges to raise and lower the base of the track to different angles of elevation.
3 Arrange the recording system so that the landing position is horizontally opposite the launch position.
4 Release the ball from a position on the track that is the same height above the launch position for each launch angle.
5 Record the horizontal distance, R, travelled by the ball.
6 For each angle, θ, record the horizontal distance, R, at least three times.

Results

1 Ensure each data point is recorded in appropriately constructed data tables as it is produced.
2 Estimate the uncertainty in each of the variables.
3 Produce a summary data table of values measured for R and θ, including the uncertainty in each value.

Analysis of results

1 Plot the data from the summary table on a correctly constructed graph of R against θ, including uncertainty bars.
2 Draw the line of best fit.
3 Use the line of best fit to plot R versus $\sin 2\theta$.
4 Use the graph to find the experimentally determined launch speed.

Interpretation

5 Explain why the ball must always be released from the same vertical height above the launch position.

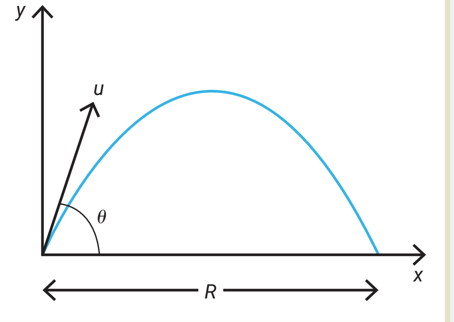


FIGURE 2.2.5 A ball is launched at an angle θ from a constant height, h, above the launch position. The landing position is at the same height above the ground as the launch position.

6 Explain why the landing position must always be the same height above the table as the launch position.
7 Explain the reason for taking three range measures for each angle.
8 Use conservation of energy to deduce the launch speed; hence, deduce the theoretical range for each of the launch angles.
9 Compare the theoretical values of R to the measured values of R, taking into account measurement uncertainties.

Evaluation

10 Explain how the experiment could be improved to collect more precise data.
11 Describe the experiment in two or three short sentences.
12 State any relationship that can be justified between measured variables.
13 Compare the measured values of R and u with the results predicted from theory.
14 Identify limitations in the experiment in two or three short sentences.

LEARNING CHECK 2.2

DESCRIBING

1 **Define** all the variables in the equation $|\vec{v}| = \sqrt{u_x{}^2 + v_y{}^2}$.
2 A projectile launched from ground level travels over a horizontal plane before landing. Write an equation for the range.
3 **Explain** how the range of a projectile is affected by changes in the initial launch angle while keeping the initial speed constant.
4 **Explain** why a projectile launched horizontally lands with the same vertical component of velocity as a projectile dropped from the same height.
5 **Describe** the conditions under which the range equation can be applied. **Justify** your answer.

APPLYING

6 **Calculate** the velocity of a projectile 5.0 s after it is launched at a speed of 50 m s^{-1} and an angle of 50° to the horizontal.
7 For a projectile that leaves and lands at the same horizontal height above ground, copy and complete the following table.

Launch speed (m s^{-1})	Smallest launch angle to the horizontal (°)	Largest launch angle to the horizontal (°)	Range (m)
20	30		
30			91
	39		48

ANALYSING

8 A paintball is fired horizontally at a hanging bag 12.0 m away. The paintball and bag are both initially 10.0 m above the ground. The bag is let drop at the same time as the shot is fired.
a **Explain** why the bag is still splattered with paint.
b Find the minimum launch speed of the paintball shot for it to connect with the bag before landing.
9 A projectile is launched at an angle of 30° above the horizontal from a given height. It lands on a platform that is 6 m below the launch height and a horizontal distance of 30 m away horizontally. Use this information to:
a **determine** the launch velocity (u)
b **calculate** the total time of flight (t).
Assume acceleration due to gravity is 9.8 m s^{-2} and neglect air resistance.

10 A golf ball is struck from a tee, making an initial angle of 20° with the horizontal. It travels a horizontal distance of 120 m and lands on the green, which is 4 m above the height of the tee. Neglecting air resistance and using $g = 9.8\ \text{m s}^{-2}$, use this information to:

a **determine** the launch velocity (u)

b **calculate** the total time of flight (t).

2.3 Solving problems in projectile motion

When solving problems involving projectile motion, follow the steps below.

1. Read the question carefully.
2. Sketch the real situation described.
3. On the sketch:
   - **a** show the path from launch to landing
   - **b** draw the initial velocity vector
   - **c** show the horizontal and vertical components of the initial velocity vector
   - **d** add any data provided in the question, including quantitative values for the vertical and horizontal components of the initial velocity
   - **e** show the direction and magnitude of the gravitational acceleration: g.
4. Separate the analysis into horizontal and vertical components and select appropriate formulas:
   - **a** horizontal motion

     $u_x = u\cos\theta$

     $s_x = u_x t = v_x t$
   - **b** vertical motion.

     $u_y = u\sin\theta$

     $v_y = u_y + gt$

     $s_y = u_y t + \frac{1}{2}gt^2$

     $v_y^{\,2} = u_y^{\,2} + 2gs_y$

     $|\vec{v}| = \sqrt{v_x^{\,2} + v_y^{\,2}}$

     $\tan\theta = \frac{v_y}{v_x}$
5. For the special case of projectiles that are launched from and return to the same horizontal height:

$$R = \frac{u^2 \sin 2\theta}{g},\ 0° < \theta < 90°.$$

   Pay attention to the two possible solutions.
6. Transpose formulas for the required unknown variable or substitute values directly into the equation.
7. Solve the equations.
8. Check to ensure the answers are those required.

WORKED EXAMPLE 2.3.1

1 Consider a projectile fired at an angle of 45° above the horizontal with a speed of 86 m s^{-1} (Figure 2.3.1). Find the:

a vertical component of the initial velocity
b maximum height reached by the projectile
c time taken to reach the maximum height
d acceleration at maximum height
e velocity at maximum height.

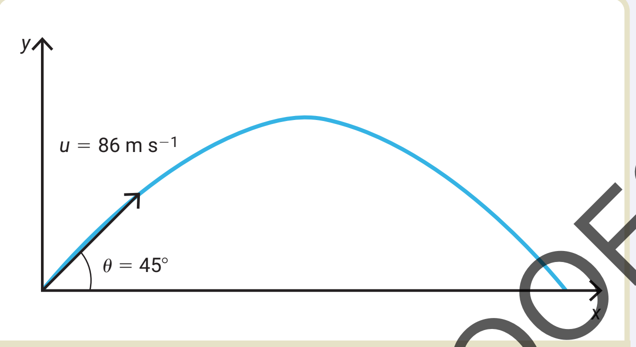


FIGURE 2.3.1 A sketch of projectile motion. Note the symmetrical path and the tangential initial velocity.

ANSWERS

a 1 **Determine the formula.**
$u_y = u \sin\theta$

2 **Substitute the known values.**
$u_y = 86\ \text{m s}^{-1} \times \sin 45°$

3 **Calculate the answer.**
$u_y = 61\ \text{m s}^{-1}$

b 1 **Determine the formula.**
$v_y^2 = u_y^2 + 2gs_y$

2 **Rearrange the formula to find the unknown.**
$$s_y = \frac{v_y^2 - u_y^2}{2g}$$

3 **Substitute the known values.**
$$s_y = \frac{0^2 - (61\ \text{m s}^{-1})^2}{2 \times (-9.8\ \text{m s}^{-2})}$$

4 **Calculate the answer.**
$s_y = 190\ \text{m}$

c 1 **Determine the formula.**
$v_y = u_y + gt$

2 **Rearrange the formula to find the unknown.**
$$t = \frac{v_y - u_y}{g}$$

3 **Substitute the known values.**
$$t = \frac{0 - 61\ \text{m s}^{-1}}{-9.8\ \text{m s}^{-2}}$$

4 **Calculate the answer.**
$t = 6.2\ \text{s}$

d $a = g = -9.8\ \text{m s}^{-2}$

e 1 **Determine the formula.**
$u_x = u \cos\theta$

2 **Rearrange the formula to find the unknown.**
$u_x = 86\ \text{m s}^{-1} \times \cos 45°$

3 **Substitute the known values.**
$u_x = 61\ \text{m s}^{-1}$

WORKED EXAMPLE 2.3.2

Find the distance covered over level ground by a projectile that is launched from ground level with a speed of 35 m s^{-1} and an angle 55° above the horizontal (Figure 2.3.2).

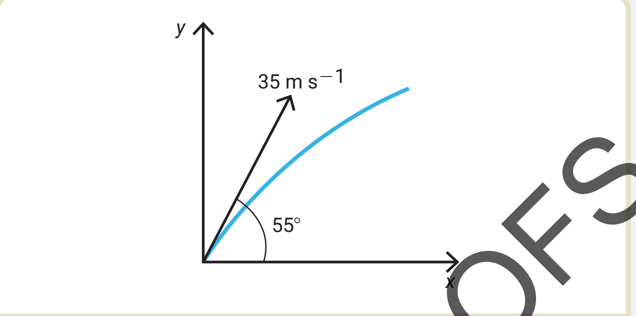


FIGURE 2.3.2 A sketch of projectile motion for a launch angle of 55° and initial velocity of 35 m s^{-1}

ANSWER

1 Determine the formula.

$$R = \frac{u^2 \sin 2\theta}{g}$$

2 Substitute the known values.

$$R = \frac{(35\text{ m s}^{-1})^2 \times \sin(2 \times 55°)}{9.8\text{ m s}^{-2}}$$

$$= \frac{(35\text{ m s}^{-1})^2 \times \sin(180° - 110°)}{9.8\text{ m s}^{-2}}$$

3 Calculate the answer.

$$R = \frac{(35\text{ m s}^{-1})^2 \times \sin 70°}{9.8\text{ m s}^{-2}}$$

$$= 117\text{ m}$$

WORKED EXAMPLE 2.3.3

Consider a projectile is launched from a 96 m high cliff at a speed of 32 m s^{-1} (Figure 2.3.3). The angle of launch is upwards at 10° to the horizontal. Find the:

- **a** maximum height attained above the cliff
- **b** time taken to reach the maximum height
- **c** the time taken to land
- **d** the distance from the base of the cliff to the landing position.

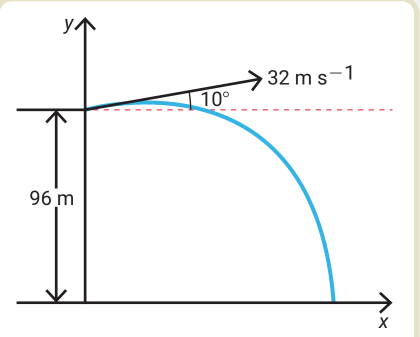


FIGURE 2.3.3 A sketch of projectile motion for a launch angle of 10° and initial velocity of 35 m s^{-1}

ANSWERS

a 1 Determine the formula.

$$v_y{}^2 = u_y{}^2 + 2gs_y$$

2 Rearrange the formula to find the unknown.

$$s_y = \frac{v_y{}^2 - u_y{}^2}{2g}$$

3 **Substitute the known values.**

$$s_y = \frac{(0\,\text{m s}^{-1})^2 - (5.557\,\text{m s}^{-1})^2}{2 \times (-9.8\,\text{m s}^{-2})}$$

4 **Calculate the answer.**

$s_y = 1.6\,\text{m}$

b 1 **Determine the formula.**

$v_y = u_y + gt$

2 **Rearrange the formula to find the unknown.**

$$t = \frac{v_y - u_y}{g}$$

3 **Substitute the known values.**

$$t = \frac{0\,\text{m s}^{-1} - 5.557\,\text{m s}^{-1}}{-9.8\,\text{m s}^{-2}}$$

4 **Calculate the answer.**

$t = 0.57\,\text{s}$

c 1 **Determine the formula.**

Vertically from top of flight:

$y = (96 + 1.6)\,\text{m} = 97.6\,\text{m}$, $u_y = 0\,\text{m s}^{-1}$, $v_y = ?$, $g = +9.8\,\text{m s}^{-2}$, $t = ?$

$$y = gt^2 + \frac{1}{2}u_y t$$

2 **Rearrange the formula to find the unknown.**

$$t = \sqrt{\frac{2y}{g}}, u_y = 0$$

3 **Substitute the known values.**

$$t = \sqrt{\frac{2 \times 97.6\,\text{m}}{+9.8\,\text{m s}^{-2}}}$$

4 **Calculate the time taken.**

$t = 4.5\,\text{s}$

5 **Calculate the total time taken.**

Time of flight, t:

$t = t(\text{to top}) + t(\text{from top to ground})$

$= 0.56\,\text{s} + 4.46\,\text{s}$

$= 5.0\,\text{s}$

d 1 **Determine the formula horizontally.**

To calculate the horizontal:

$u_y = u \cos\theta = 32\,\text{m s}^{-1} \times \cos 10° = 31.5\,\text{m s}^{-1}$

2 **Determine the formula to calculate the distance.**

$s_x = u_x t$

3 **Substitute the known values.**

$s_x = 31.5\,\text{m s}^{-1} \times 5.0\,\text{s}$

4 **Calculate the answer.**

$s_x = 158\,\text{m}$

LEARNING CHECK 2.3

DESCRIBING

1 State which aspect of a projectile's launch velocity controls the time of flight: the vertical component or the horizontal component.

2 A projectile is launched at an angle of 45°. **Identify** the point in the projectile's flight path where there is the greatest difference between the magnitude of its vertical velocity component and the magnitude of its horizontal velocity component.

3 **State** the magnitude of the vertical speed and acceleration at the top of a projectile's flight, for a projectile that was launched:

a vertically **b** at an angle to the horizontal.

4 **Compare** a projectile's vertical velocity to its vertical acceleration in terms of direction as the projectile is going:

a up **b** down.

5 **Identify** the conditions under which the following formulas can be applied.

a $s_x = u_x t = v_x t$ **b** $s_x = u_x t$ **c** $v_y^2 = u_y^2 + 2gs_y$

APPLYING

6 For a projectile launched with an initial velocity of 40 m s^{-1} at an angle of 30° above the horizontal, use the kinematics formula to **determine** the:

a maximum height above its launch height
b time taken to reach maximum height
c acceleration at the top of its flight
d velocity of the projectile after 1.0 s.

7 A river flows between two cliffs of equal height. A projectile is fired across the river from the edge of one cliff and lands on the edge of the other. If the rocket is launched with a speed of 50 m s^{-1} at an angle 75° above the horizontal, find the distance across the river.

ANALYSING

8 Near Earth, the speed of a projectile at any position after launch can be given in terms of three variables: the initial speed, *u*, launch angle, *θ*, and time of flight, *t*. **Demonstrate** this proposition, starting from the equation $|\vec{v}| = \sqrt{u_x^2 + v_y^2}$.

9 Find the range of a projectile that is launched with a speed of 70 m s^{-1} at an angle of 30° above the horizontal and which lands 30 m below its launch position. **Determine** if the range is the same as for a launch angle of 60°?

CHAPTER SUMMARY

Projectile motion

- Projectile motion is motion in two dimensions as a consequence of an object that goes up and down vertically at the same time as it moves horizontally.
- The projectile is launched at a speed, u, and angle, θ, relative to the horizontal.
- This launch velocity vector can be resolved into the horizontal component, u_x, and the vertical component, u_y.
  $u_x = u\cos\theta$
  $u_y = u\sin\theta$
- The vertical motion is affected by Earth's gravitational field, g, and therefore the vertical component of velocity varies while the projectile is in motion.
  $g = -9.8\ \text{m s}^{-2}$
  $v_y = u_y + gt$
  $s_y = u_y t + \frac{1}{2}gt^2$
  $v_y = u_y^2 + 2gs_y$
  where: u_y = vertical component of the launch velocity (m s^{-1})
  v_y = vertical component of the velocity some time after launch (m s^{-1})
  s_y = vertical height interval relative to the launch position (m)
  g = acceleration due to gravity (m s^{-2})
  t = time interval (s)
- However, there is no horizontal component of force to affect horizontal motion, so the horizontal component of velocity is constant while the projectile is in motion.
  $v_x = u_x$
  $s_x = u_x t$

Types of projectile motion

- Types of projectile motion include:
  - vertical motion only, where the projectile is released with no horizontal velocity component, only vertical component
  - falling horizontally, where the projectile is thrown sideways with no initial vertical velocity component
  - projectile launched at an angle to horizontal with symmetrical path
  - projectile launched at an angle to horizontal with an unsymmetrical path (starts at a higher point than it lands or vice versa).

Range of projectile

- At angles of equal distances from 45°, the angles are complementary to each other (sum to 90°) and yield the same horizontal range for a given initial velocity.
  where $\theta_1 = 45 + x$ and $\theta_2 = 45 - x$, $s_{x1} = s_{x2}$
  where the projectile lands at the same height it was launched, the range, R, or the horizontal distance travelled, can be derived using the formula:
  $$R = \frac{u^2 \sin 2\theta}{g}$$
  where: R = range
  u = initial velocity
  θ = angle of projection
  g = acceleration due to gravity

CHAPTER EXAM

MULTIPLE CHOICE

1. The vertical and horizontal components of a launch velocity u and angle of launch above the horizontal α, are, respectively:
   - **A** $u \sin \alpha$ and $u \tan \alpha$.
   - **B** $u \tan \alpha$ and $u \cos \alpha$.
   - **C** $u \sin \alpha$ and $u \cos \alpha$.
   - **D** $u \cos \alpha$ and $u \sin \alpha$.

2. What is the maximum height reached by a rocket, that is launched at 30° to the horizontal at 320 m s^{-1} closest to?
   - **A** 1300 m
   - **B** 1.3×10^4 m
   - **C** 0.13 km
   - **D** 1.3 km

3. A ball takes 1.5s to travel from the thrower to the catcher, who is 45 m away. The ball is caught at its maximum height, which is 5.0m above the launch height. What was the approximate speed at which the ball was thrown?
   - **A** 30 m s^{-1}
   - **B** 31 m s^{-1}
   - **C** 33 m s^{-1}
   - **D** 39 m s^{-1}

4. Find the launch speed of a projectile that travels a horizontal distance of 200 m in 5.0 s after being launched at an angle below the horizontal of 45°.
   - **A** 56 m s^{-1}
   - **B** 40 m s^{-1}
   - **C** 28 m s^{-1}
   - **D** 20 m s^{-1}

5. For an object that lands below its launch point, the acceleration due to gravity is taken to be negative when the positive launch velocity is:
   - **A** above the horizontal and the vertical displacement on landing is positive.
   - **B** above the horizontal and the vertical displacement on landing is negative.
   - **C** below the horizontal and the vertical displacement on landing is negative.
   - **D** below the horizontal and the vertical displacement on landing is positive.

6. A ball is thrown 40° above the horizontal at 4.0 m s^{-1}. What will the horizontal component of the ball's velocity be after 0.50 s?
   - **A** 2.6 m s^{-1}
   - **B** 3.1 m s^{-1}
   - **C** 3.4 m s^{-1}
   - **D** 5.5 m s^{-1}

7. A ball is thrown 40° below the horizontal at 4.0 m s^{-1}. What will the horizontal component of the ball's velocity be after 0.50 s?
   - **A** 2.6 m s^{-1}
   - **B** 3.1 m s^{-1}
   - **C** 3.4 m s^{-1}
   - **D** 5.5 m s^{-1}

8. What will the vertical component of the velocity of the ball in Question 7 be after 0.50 s?
   - **A** 2.6 m s^{-1}
   - **B** 3.1 m s^{-1}
   - **C** 4.9 m s^{-1}
   - **D** 7.5 m s^{-1}

9. A ball is thrown horizontally from a roof at 12.0 m s^{-1}. What will the ball's velocity be 2 s later?

   A 12.0 m s^{-1} **B** 19.6 m s^{-1}
   C 23.0 m s^{-1} **D** 31.6 m s^{-1}

10. A ball is launched in the air at an angle of 42° on flat ground. Ignoring the effects of air resistance, which statement is incorrect?

   A The ball will have a vertical velocity component of 0 m s^{-1} for an instant when it reaches its maximum height.
   B The time between launch and reaching maximum height is the same as the time between maximum height and striking the ground.
   C The ball's acceleration will change during its flight.
   D The ball's horizontal velocity will not change during its flight.

SHORT RESPONSE

11. A soccer ball is kicked off the ground at speed 30.0 m s^{-1} and angle 42.0°. It lands on the ground some distance away. **Calculate** the:

   a time of flight of the ball until it lands
   b maximum height above the ground attained by the ball
   c velocity of the ball at the top of its flight
   d acceleration of the ball at the top of its flight.

12. A shooter aimed her rifle horizontally at a large target. The bullet travelled at 452 m s^{-1} and struck the target 23.4 cm below the point at which the rifle was aimed. **Determine** the angle of elevation necessary if the shooter is to hit the target.

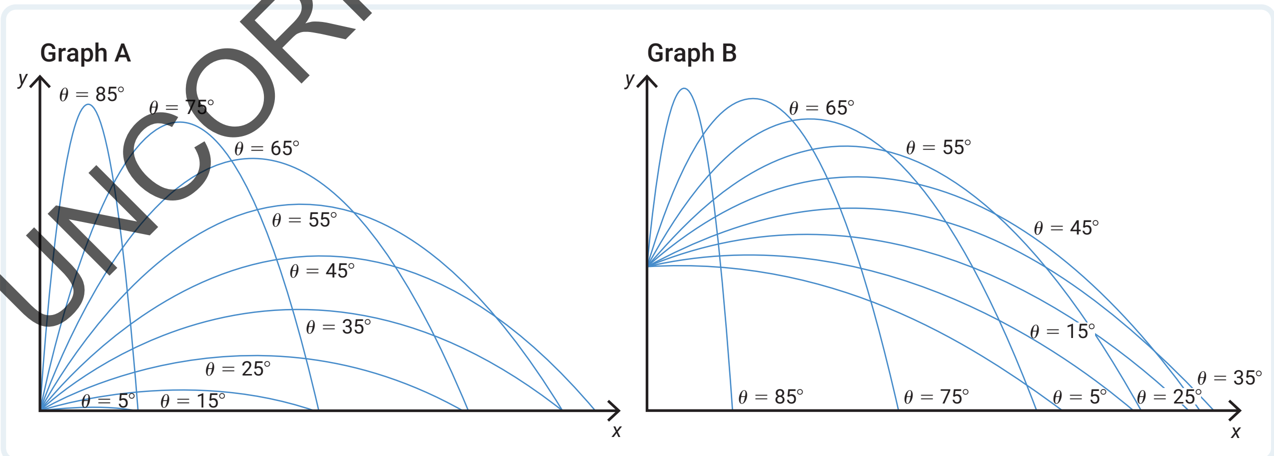
CROSS-CHAPTER QUESTION

13. A 100 g ball is launched at an angle of 35° and gains 12 J of gravitational potential energy to reach its maximum height. **Determine** the launch velocity of the ball.

DATA ANALYSIS

14. **Analyse data**

   Graphs A and B depict the paths of a projectile with a set launch velocity.



a **Identify** the relationship between launch angle and range evident in Graph A, using quantitative evidence to justify your answer

b **Identify** the relationship between launch angle and maximum height in Graph A, using quantitative evidence to justify your answer

c **Identify** the relationship between launch angles that attain the same range, using quantitative evidence to justify your answer

d Given there is no change in launch velocity, **infer** what condition has been changed to produce Graph B?

e Are the same patterns that you identified in Graph A evident in Graph B?

ANALYSE DATA

15. The data for a falling object over the first second of its movement is presented in the following table.

Time t (s)	Time squared t^2 (s^2)	Vertical distance fallen s_y (m)
0.10	0.01	0.049
0.20	0.04	0.196
0.30	0.09	0.441
0.40	0.16	0.784
0.50	0.25	1.225
0.60	0.36	1.764
0.70		
0.80	0.64	3.136
0.90	0.81	3.969
1.00	1.00	4.900

By analysing and interpreting the data in the table:

a **describe** the mathematical relationship between time squared t^2 and distance fallen s_y

b **determine** s_y at $t = 0.70$ s

c **infer** whether the object was dropped from rest or not. **Explain** your answer

d draw a conclusion about whether air resistance had a significant impact on the falling object. Use quantitative information to **justify** your answer

e **predict** if or how the data above would change if the object was given an additional horizontal component to its velocity as it was released. **Explain** your answer.

CHAPTER 3

Inclined planes

TK

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Solve problems involving force due to gravity (weight) and mass using $F_g = mg$.
- Describe the concept of the normal force.
- Describe the forces acting on an object on an inclined plane (e.g. force due to gravity, normal force, tension, frictional force and applied force) through the use of free-body diagrams.
- Determine the net force acting on an object on an inclined plane using vector analysis.

SCIENCE INQUIRY

- Investigate the parallel component of the weight of an object down an inclined plane at various angles.

Physics 2025 v1.2 General Senior Syllabus © QCAA 2024

Introduction

An inclined plane is one of the simplest machines and consists of a sloped surface. Inclined planes are used as wedges and levers and can be seen in winding roads and screw threads. Movement up or down a slope involves the gravitational force of weight and the forces applied by the surface – friction (parallel to the slope and acting against motion) and the normal force (perpendicular to the slope).

Assessments

- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practicals

- Investigating the magnification of a microscrope

Worksheets

- Name
- Name
- Name

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ The net force acting on an object can be calculated, using $F_{net} = ma$.
- ✓ The SI unit for mass is the kilogram (kg).
- ✓ Weight is the resultant force when gravity acts on mass and can be calculated, using $F_w = mg$.
- ✓ The normal force is the force applied by a surface at right angles to the surface.
- ✓ Friction force is a force that opposes motion and is parallel to the contact surface.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ solve problems involving force due to gravity (weight) and mass, using $F_g = mg$
- ✓ quantify normal force
- ✓ describe the forces acting on an object on an inclined plane (e.g. force due to gravity, normal force, tension, frictional force and applied force) through the use of free-body diagrams
- ✓ determine the net force or acceleration acting on an object on an inclined plane by using vector analysis
- ✓ describe and explain static friction and kinetic friction
- ✓ quantify static friction and kinetic friction
- ✓ analyse or interpret graphical representations of the forces or accelerations associated with inclined planes.

3.1 Inclined planes

A box will stay where it is on a horizontal floor unless it is forced to move. That is the consequence of Newton's first law of inertia. If the floor is tilted at an angle, the box will eventually start to slide down the slope, overcoming friction. It accelerates down the slope when the net force is greater than zero. That is a consequence of Newton's second law, $a = \frac{F}{m}$. The box accelerates when the effect of the gravitational force on the box along the surface overcomes the frictional force applied by the surface.

The inclined plane

angle of inclination the angle, θ, relative to the horizontal; $0° < \theta < 90°$

An inclined plane is a surface that is tilted at an angle to the horizontal. This angle is called the **angle of inclination** (Figure 3.1.1). The angle of inclination is always between zero and a right angle:

$$0° < \theta < 90°$$

Galileo Galilei famously used inclined planes to reduce the impact of acceleration due to gravity on balls rolling down inclines, slowing their motion and allowing a more accurate determination of the acceleration due to gravity at Earth's surface.

KEY FORMULA

Angle of inclination, θ

$0° < \theta < 90°$

Forces on an inclined plane

A mass on an inclined plane is subject to forces parallel to the plane and perpendicular to the plane. Some forces are gravitational in origin; others are electrostatic in origin. The combined effect of forces can be considered in terms of vector sums. These can be analysed as forces applied parallel to the surface and forces applied perpendicular to the surface.

Forces on a horizontal surface

An object that is not accelerating on a horizontal surface is subject to two forces:

- the normal force applied by the surface on the object perpendicular to the surface

$$\vec{F}_{\perp} \text{ (by surface on object) or } \vec{N}$$

- the friction force applied by the surface on the object parallel to the surface.

$$\vec{F}_{\parallel} \text{ (by surface on object) or } \vec{f}$$

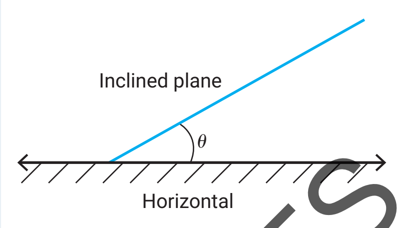


FIGURE 3.1.1 The angle of inclination, θ, is an angle relative to the horizontal.

Weblinks
Inclined planes
Inclines

KEY FORMULA

Normal force

$\vec{N} = \vec{F}_{\perp}$ (by surface on object)

where:

$\vec{N}$ = normal force applied by a surface on an object, perpendicular to the surface (N)
$\vec{F}_{\perp}$ (by surface on object) = force applied by a surface on an object, perpendicular to the surface (N)

KEY FORMULA

Friction force

$\vec{f} = F_{fr}$ (by surface on object) (N)

where:

$\vec{f}$ = friction force applied by a surface on an object, parallel to the surface (N)
$\vec{F}_{\parallel}$ (by surface on object) = force applied by a surface on an object, parallel to the surface (N)

When there is no acceleration, the net force on an object is zero **(Figure 3.1.2)**. In the direction perpendicular to the surface, the net force on the object is zero. It comprises the normal force, $\vec{N}$ up and away from the surface and the perpendicular component of the downwards weight force, $\vec{w}$. Applying Newton's second law:

$$\sum F = 0$$
$$N - w = 0$$
$$N = w$$
$$N = mg$$

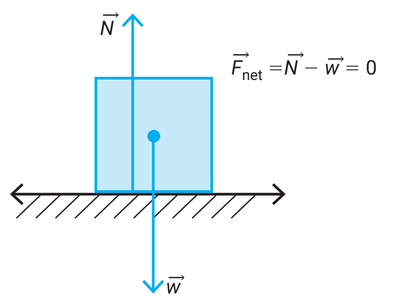


FIGURE 3.1.2 A horizontal plane: the net force acting on a stationary object is zero. In the vertical direction, $\sum\vec{F} = \vec{N} - \vec{w} = 0$.

Forces on a vertical surface

When the surface is tilted so that it is perfectly vertical, the object is no longer supported by the surface and hence the normal force is zero. Despite it looking as though there is contact between the object and the surface in this situation, there is no direct interaction between the particles that make up the object and the particles that make up the surface because of the absence of a normal force. This also means there is no friction. The object falls due to the gravitational force (weight). The net force is the weight **(Figure 3.1.3)**.

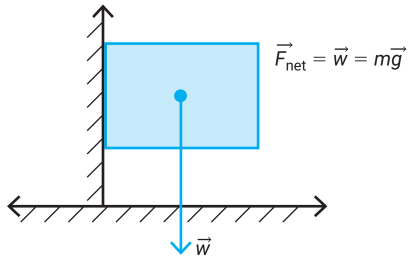


FIGURE 3.1.3 A vertical plane: in the vertical direction, the net force on the object is the weight force: $\Sigma\vec{F} = \vec{w} = m\vec{g}$.

Sliding on a frictionless inclined plane

If the surface is tilted at an angle somewhere between horizontal and vertical, and in the absence of friction, the normal force and the weight force combine to form a net force down the slope **(Figure 3.1.4)**. The object slides down the slope with increasing speed (acceleration).

FIGURE 3.1.4 An inclined plane: for a frictionless surface, the net force is the vector sum of the normal force and the weight force.

Applying trigonometry to the vector sum gives values for $\sum\vec{F}$ and $\vec{N}$ in terms of the weight. The net force is found as follows:

$$\frac{\sum\vec{F}}{\vec{w}} = \sin\theta$$

$$\sum F = mg\sin\theta$$

Thus, the net force down the slope is the rectangular component of the weight force acting down the slope. It is this component that causes the object to accelerate.

Similarly, the normal force is found as follows:

$$\frac{\vec{N}}{\vec{w}} = \cos\theta$$

$$N = mg\cos\theta$$

The normal force is the rectangular component of the weight force perpendicular to the slope. This component pulls the object into the surface at right angles to the surface, enabling the surfaces more or less to stick together. Gravitational force is responsible for the object being pulled into the surface. Electrostatic force is responsible for the surfaces sticking together. The net force perpendicular to the surface is zero, since the object does not leave or fall through the surface. Applying Newton's second law:

$$\sum F_{\perp} = 0$$
$$N - mg\cos\theta = 0$$
$$N = mg\cos\theta$$

KEY FORMULA

Parallel to surface

$\sum F_{\parallel} = mg \sin\theta = ma$

$\Rightarrow a = g\sin\theta$

where:

$\sum F_{\parallel}$ = net force parallel to the surface (N)
m = mass (kg)
g = gravitational force = 9.8 m s^{-2}
θ = angle of inclination (°)
a = acceleration along the slope (m s^{-2})

Sliding on an inclined plane with friction

Friction is a force that occurs when one surface affects the movement of another surface. At the microscopic level, the base of a box and an inclined plane both have bumps and hollows (Figure 3.1.5). When the two surfaces are pushed together, strong, attractive electrostatic forces stick the molecules of both surfaces to each other.

This **static friction** must be overcome before the box can move. Since static friction depends on the force applied by the box on the surface, static friction depends on the perpendicular component of the weight force. Static friction rises to a maximum value, at which point the box begins to accelerate.

static friction the force that impedes motion up to the point where motion begins

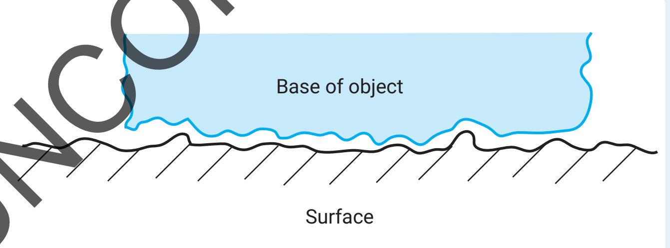


FIGURE 3.1.5 Microscopic and submicroscopic surface irregularities contribute to surfaces sticking together, producing friction.

KEY FORMULA

Perpendicular to surface

$\sum F_{\perp} = N - mg\cos\theta = 0$

where:

$\sum F_{\perp}$ = net force perpendicular to the surface (N)
N = normal force (N)
m = mass (kg)
g = gravitational force = 9.8 m s^{-2}
θ = angle of inclination (°)

kinetic or sliding friction the force that impedes motion once motion has begun; kinetic friction < static friction in magnitude.

Once the box begins to move, the friction force between the surfaces reduces. This **kinetic or sliding friction** can be considered to be a constant force that opposes the motion of the box. Kinetic friction is also dependent on the gravitational force component perpendicular to the surface because this component of the weight force pulls the surfaces together so that they more readily stick.

Figure 3.1.6 shows that static friction rises to a maximum as force is applied to the box. At this point, the kinetic friction becomes a constant, but lesser value, than the maximum static friction.

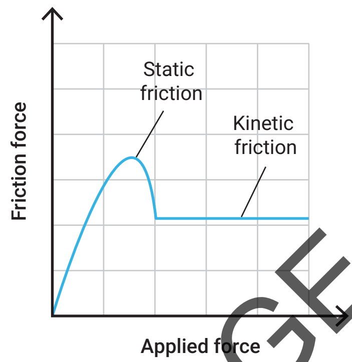


FIGURE 3.1.6 As force is applied to a box, the static friction rises linearly to a maximum. Once moving, the kinetic friction applied to the box is constant.

On surfaces involving friction, the kinetic friction opposes the motion caused by the component of the weight parallel to the slope. Newton's second law is used to find the net force:

$$\sum F_{\parallel} = ma$$

$$mg \sin \theta - f = ma$$

KEY FORMULA

When friction is involved

Parallel to the slope:

$\sum F_{\parallel} = ma$
$mg \sin \theta - F_{fr} = ma$

where:

$\sum F_{\parallel}$ = net force object parallel to the slope (N)
m = mass (kg)
g = gravitational force = 9.8 m s^{-2}
θ = angle of inclination (°)
$F_{fr} = f$ = kinetic friction (N)
a = acceleration of the mass (m s^{-2})

KEY FORMULA

When friction is involved

Perpendicular to the slope:

$\sum F_{\perp} = 0$
$N - mg \cos \theta = 0$

where:

$\sum F_{\perp}$ = net force object perpendicular to the slope (N)
N = normal force (N)
m = mass (kg)
g = gravitational force = 9.8 m s^{-2}
θ = angle of slope, $0° < \theta < 90°$ (°)

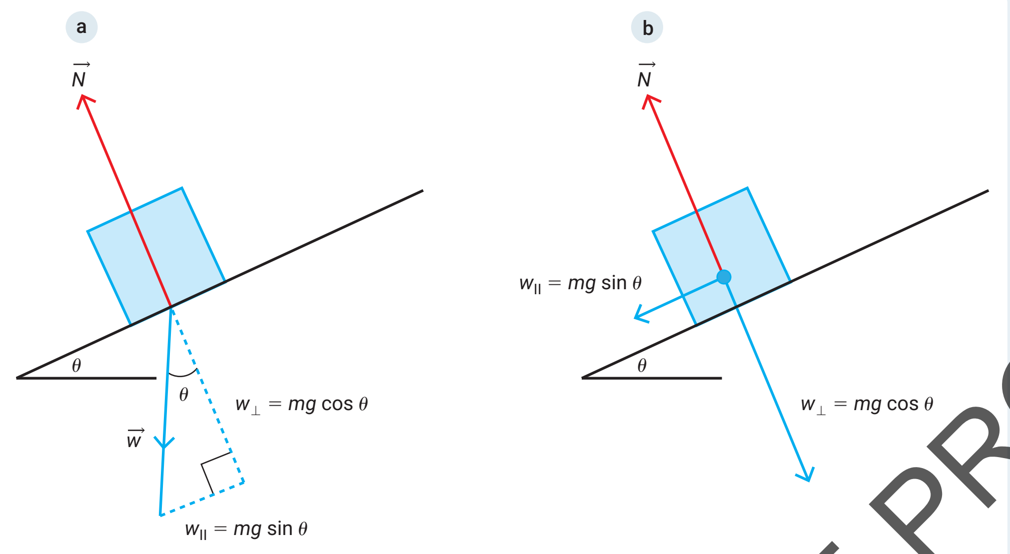


FIGURE 3.1.7 Perpendicular and parallel components of forces acting on a box on an incline. (**a**) The normal force and the weight force weight are drawn. (**b**) The normal force and the parallel and perpendicular components of the weight force are drawn. Both free-body force diagrams are valid and correct.

Sliding and rolling friction

Kinetic friction acts against the motion of an object sliding down a slope. The friction involved in rolling an object down a slope is typically less than sliding friction. This **rolling friction** causes an object to roll, even as the object moves down a slope under the effect of the weight component. Rolling friction contributes to the linear acceleration of a rolling object down the slope being less than that of a sliding object. The distribution of the mass throughout the object also contributes to the acceleration along the slope. These two reasons are why the linear acceleration of a rolling object down a slope is not equal to the linear acceleration due to the component of the weight acting parallel to the surface – rolling objects also have angular velocity.

rolling friction the force that acts to oppose motion and that rotates an object as it rolls along a slope

PRACTICAL ACTIVITY 3.3.1

MEASURING FRICTION

Introduction

An object on a plane will start to slide when the angle of inclination of the plane reaches a particular value. The angle can be used to find the static friction between the object and the plane. When sliding, the object will experience a force of sliding or kinetic friction that is less than the static friction.

Reseach question

How can the static and sliding friction of an object be measured?

Aim

To measure static and sliding friction between an object (e.g. a smartphone) and an inclined plane

Materials

- flat surface made from a solid material such as wood, melamine or plastic
- a range of rectangular blocks ranging in height from 1 cm to 10 cm
- smartphone with protective cover and accelerometer app
- ruler
- electronic balance

Procedure

1 Weigh the smartphone and record its mass.
2 Place the smartphone, cover down, horizontally on the plane and activate the accelerometer app.
3 Record the distance from the end of the plane to the front end of the smartphone.
4 Use the blocks to lift the end of the plane until the smartphone just starts to move.
5 Record the height of the front end of the smartphone above the horizontal at the angle for which the smartphone just starts to move.
6 Increase the angle by approximately 10° (do this five times).
7 Let the smartphone slide down the slope.
8 For each angle tested, collect data for the acceleration at a point halfway down the slope.
9 Repeat the steps 2–8 three times.

Results

1 Calculate the following quantities and their uncertainties:
   - a angle of inclination
   - b component of weight:
     - i parallel to the surface
     - ii perpendicular to the surface
   - c normal force
   - d static friction
   - e sliding friction.

2 Draw the graphs for:
   - a friction versus angle of inclination
   - b friction versus applied force parallel to the slope
   - c friction versus applied force perpendicular to the slope.

Analysis of results

1 Summarise the purpose of the experiment.
2 On the graphs, identify the sections for which:
   - a no motion occurred
   - b motion occurred
   - c there was the maximum static friction
   - d there was sliding friction.

3 Explain why the procedure was repeated three times.
4 Use a diagram to describe the method.
5 State precise values of:
   - a static friction
   - b sliding friction.

6 Specify any experimentally justifiable relationships between friction and other variables.
7 Show one representative set of calculations for all derived quantities.

Evaluation

8 Identify limitations in the experimental design and procedures.
9 Discuss how the experiment could be improved to produce more precise results.

LEARNING CHECK 3.1

DESCRIBING

1 **Describe** the following forces applied by a surface to an object it is supporting.
   a Friction force
   b Normal force

2 Write vector equations for the net force on a mass that is sliding on a frictionless surface when the surface is:
   a horizontal
   b inclined at an angle to the horizontal
   c vertical.

3 For an inclined plane where friction is involved, write equations for net force:
   a parallel to the surface
   b perpendicular to the surface.

APPLYING

4 **Calculate** $F_{w\parallel}$ acting on an 18 kg mass down a slope of incline 25°.

5 **Calculate** the force normal acting on a 15 kg object in equilibrium on an inclined plane at 30° with the horizontal.

6 **Compare** static friction and kinetic friction.

7 **Sketch** a vector diagram to show the forces applied to an object that is accelerating down a:
   a frictionless surface
   b surface where friction is involved.

8 A maximum static friction force of 15 N is applied to a 3.0 kg mass on an inclined plane.
   a **Calculate** the net force on the mass perpendicular to the surface.
   b **Calculate** the net force on the mass parallel to the surface.
   c **Determine** the angle of inclination of the slope.

9 A 120 kg toboggan and rider slide from rest for 120 m down an icy, frictionless surface at an angle of 25° to the horizontal.
   a **Calculate** the normal force applied by the surface on the toboggan and rider.
   b **Calculate** the acceleration of the toboggan.
   c **Determine** the final velocity of the rider at the end of the slope.

ANALYSING

10 A mass of 20 kg is placed on a rough horizontal surface. The surface is then rotated to an angle of inclination of 30°. At this angle, the object is just about to move.
   a **Explain** why this angle can be used to measure static friction, not kinetic friction.
   b **Calculate** the static friction.

3.2 Solving problems on inclined planes

When solving problems involving motion on an inclined plane, follow these steps.

1. Read the question carefully.
2. Visualise or sketch the real situation described.
3. Draw a free-body diagram.
   a Identify each force acting on the object in question.
   b Write each force in symbol form: normal, weight and, if appropriate, friction. It may be appropriate to show components of weight, $F_{\parallel}$ and $F_{\perp}$.

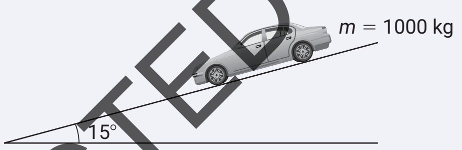
Weblink
Static and Kinetic Friction on an inclined plane simulation

c Add any data provided in the question.
d Define the direction of the net force (or acceleration).

4. Consider whether to draw a vector diagram.
   a For frictionless motion: $\vec{N} + \vec{w} = m\vec{a}$, noting that $\vec{a}$ must be parallel to the surface (the vector signs reduce to a directed number line).
   b For motion involving friction: $\vec{N} + \vec{w} - \vec{f} = m\vec{a}$, noting that $\vec{f}$ and $\vec{a}$ must be parallel to the surface (the vector signs reduce to a directed number line).
5. Separate the analysis into motion that is:
   a parallel to the surface. Use Newton's second law to set up equations along the surface:
      i without friction: $\sum F_{\parallel} = mg\sin\theta = ma$; $a = g\sin\theta$
      ii with friction: $\sum F_{\parallel} = mg\sin\theta - f = ma$
   b perpendicular to the surface. Use Newton's second law to set up equations at right angles to the surface: $\sum F_{\perp} = N - mg\cos\theta = 0$.
6. Transpose formulas for the required unknown variable or substitute values directly into the equation.
7. Consider whether any of the *suvat* kinematic formulas should be used to complete your answer to the question.
8. Solve the equations.
9. Check to ensure the answers are those required.

WORKED EXAMPLE 3.2.1

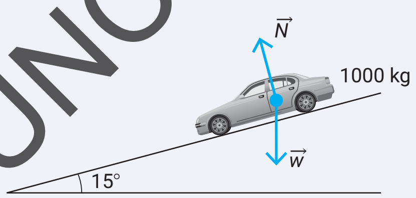
A 1000 kg car is sliding down a frictionless plane that is inclined at an angle of 15° to the horizontal.



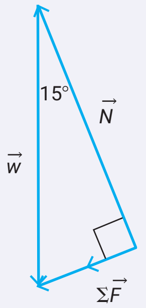
a On the diagram, use vectors to show the forces acting on the car.
b Sketch a vector sum diagram.
c Calculate values for the:
   i weight force acting parallel to the surface
   ii force that stops the car being pushed off the surface
   iii acceleration of the car.

ANSWERS

a **Draw free-body vectors on the diagram.**



b Sketch the vector sum diagram.



c i 1 **Determine the formula.**

Parallel to the surface:

$\Sigma F_{\parallel} = mg \sin \theta$

2 **Substitute the known values.**

$F_{\parallel} = 1000 \text{ kg} \times 9.8 \text{ m s}^{-2} \times \sin 15°$

3 **Calculate the answer.**

$F_{\parallel} = 2.5 \times 10^3 \text{ N}$

ii 1 **Determine the formula**.

Perpendicular to the surface:

$F_{\perp} = mg \cos \theta$

2 **Substitute the known values**.

$F_{\perp} = 1000 \text{ kg} \times 9.8 \text{ m s}^{-2} \times \cos 15°$

3 **Calculate the answer.**

$F_{\perp} = 9.5 \times 10^3 \text{ N}$

iii 1 **Determine the formula.**

$a = g \sin \theta$

2 **Substitute the known values.**

$a = 9.8 \text{ m s}^{-2} \times \sin 15°$

3 **Calculate the answer.**

$a = 2.5 \text{ m s}^{-2}$

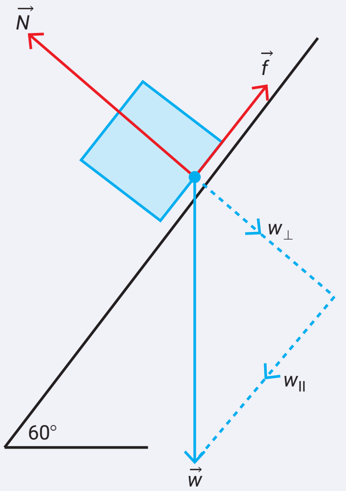
WORKED EXAMPLE 3.2.2

A 50 kg box slides down a ramp that has an angle of inclination of 60°. There is a 15 N frictional force between the ramp and the box.

a Find the net force acting on the box.

b Calculate the acceleration of the box.

c Determine the normal force acting on the box.



ANSWERS

a **1 Determine the relevant formulas.**

Parallel to the slope:

$\sum F_{\parallel} = ma$

$mg \sin\theta - f = ma$

Perpendicular to the slope:

$\sum F_{\perp} = ma$

$N - mg\cos\theta = 0$

2 Combine the formulas to determine the net force.

Therefore, the net force on the box is the net force parallel to the slope:

$\sum F_{\parallel} = mg\sin\theta - f$

3 Substitute the known values.

$\sum F_{\parallel} = 50\text{ kg} \times 9.8\text{ m s}^{-2} \times \sin 60° - 15\text{ N}$

$\sum F_{\parallel} = 424.3\text{ N} - 15\text{ N}$

4 Calculate the answer.

$\sum F_{\parallel} = 409.3\text{ N}$

b $a = \frac{\sum F}{m}$

$= \frac{409.34\text{ N}}{50\text{ kg}}$

$= 8.19\text{ m s}^{-2}$

c **1 Determine the formulas.**

Perpendicular to the slope

$N - mg\cos\theta = 0$

$N = mg\cos\theta$

2 Substitute the known values.

$N = 50\text{ kg} \times 9.8\text{ m s}^{-2} \times \cos 60°$

3 Calculate the answer.

$N = 245\text{ N}$

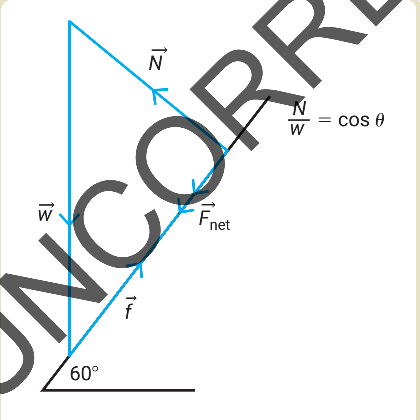


FIGURE 3.2.1 The net force, F_{net}, is the sum of the forces *w*, *N* and *f*.

LEARNING CHECK 3.2

REMEMBERING

1 **Explain**, using a drawing and the rules of geometry, why the angle of inclination, θ, must be the same as the angle between the *w* and *N* force vectors for an object on an inclined plane.

2 **Sketch** and label with force arrows a mass sliding down a frictionless inclined plane.

3 Write net equations in terms of forces acting on an object on an inclined plane:

- **a** perpendicular to the surface
- **b** parallel to a frictionless surface
- **c** parallel to a surface involving friction.

UNDERSTANDING

4 Use Newton's laws to **explain** how a mass can slide on an inclined plane at constant speed.

APPLYING

5 A frictionless plane is inclined at an angle of 20° to the horizontal. A 100 kg box slides down the plane. **Calculate** values for the:

- **a** force acting parallel to the surface
- **b** normal force
- **c** acceleration of the box.

6 A 400 kg motorcycle slides down a 6° incline at a constant 3.0 m s^{-1}. **Identify** the frictional force applied to the motorcycle.

ANALYSING

7 A 2.0 kg box on a plane inclined at 25° is at its limit of static friction. **Sketch** and label with force arrows a free-body force diagram of the situation.

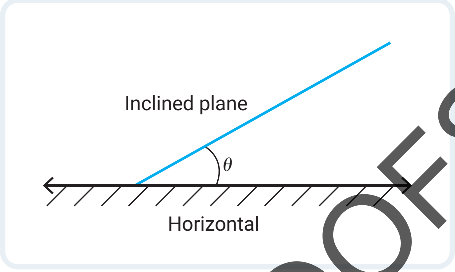
8 A 60 kg box slides on a frictionless plane inclined at 15°. **Identify** the magnitude and direction of the external force other than the weight that must be applied to the box to achieve an acceleration of 3.2 m s^{-2}:

- **a** down the slope
- **b** up the slope.

CHAPTER SUMMARY

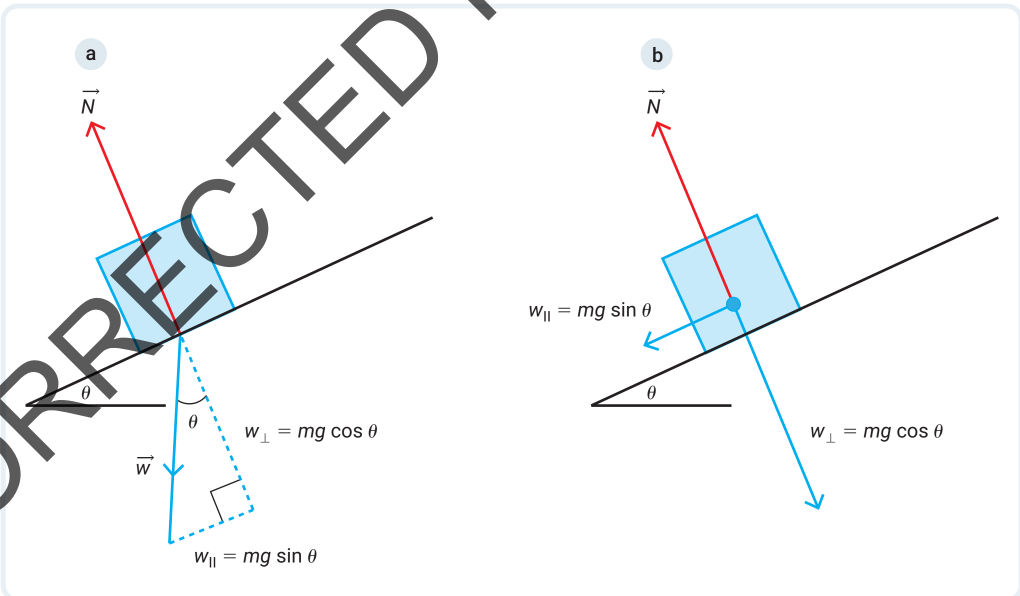
Features of inclined planes

- Inclined planes are surfaces that are tilted at an angle to the horizontal.
- The angle of inclination is the angle between the inclined plane and the horizontal surface and affects how much net force is acting on the object. The angle of inclination must be between 0° and 90°: $0° < \theta < 90°$



Gravitational force

- The gravitational force acts straight down due to gravity and is calculated as $F_w = mg$.
- Due to the inclined plane, the gravitational force is converted to its components, $F_{||}$ and $F_{\perp}$, so that forces only act in two perpendicular directions.
- The parallel component of the gravitational force, $F_{||}$, acts parallel to the inclined plane, causing the object to slide down. It is calculated as $F_{||} = mg \sin \theta$, where θ is the angle of inclination.
- The perpendicular component of the gravitational force, $F_{\perp}$, acts perpendicular to the inclined plane and is balanced by the normal force. It is calculated as $F_{\perp} = mg \cos \theta$, where θ is the angle of inclination.



Other forces

- The normal force (N) is the force exerted by the inclined plane that is perpendicular to its surface. This force counteracts the component of the gravitational force perpendicular to the plane.
- Frictional force (f or F_{fr}) opposes the motion of the object sliding down the plane which acts parallel to the surface of the inclined plane but in the opposite direction to the motion.

MULTIPLE CHOICE

CHAPTER EXAM

1. The angle of inclination of a plane is the angle:

A to the horizontal.
B to the vertical.
C perpendicular to the plane.
D parallel to the plane.

2. On an inclined plane, the normal force is the:

A reaction to the component of the weight parallel to the plane.
B reaction to the weight.
C force applied on a mass by the surface perpendicular to the plane.
D force applied on a mass by the surface parallel to the plane.

3. What is the acceleration of a 10 kg mass sliding down a frictionless plane inclined at 30°?

A 9.8 m s^{-2}
B 8.5 m s^{-2}
C 4.9 m s^{-2}
D 2.8 m s^{-2}

4. A 5.0 kg mass is situated 1.8 m from the lower end of an inclined plane. It is about to slide when it is 45 cm above the horizontal. The magnitude and type of force keeping the mass in place is:

A 13 N; static friction.
B 13 N; kinetic friction.
C 49 N; static friction.
D 49 N; kinetic friction.

5. A rope is tied to a 65 kg box on a 30° slope. The friction between the box and the slope is 65 N. What is the tension in the rope?

A 384 N
B 319 N
C 254 N
D 65 N

6. An object of mass m rests on an inclined plane angled at θ. Which equation represents the magnitude of the normal force?

A $F_N = mg \sin \theta$
B $F_N = mg \cos \theta$
C $F_N = mg \tan \theta$
D $F_N = mg$

7. A block slides down a frictionless incline at 30°. What is the net force acting along the plane if the block has a mass of 5 kg?

A 19.6 N
B 25 N
C 42.4 N
D 49 N

8. A box is being pushed up a 25° incline with an applied force F_a. The box experiences friction. Which of the following correctly describes the net force on the box?
   - **A** $F_{net} = F_a - (mg \sin \theta + f)$
   - **B** $F_{net} = F_a + mg \cos \theta + f$
   - **C** $F_{net} = F_a - mg \cos \theta$
   - **D** $F_{net} = F_a - f$

9. A 10 kg box rests on a 20° incline. What is the gravitational force component acting perpendicular to the incline?
   - **A** 33.5 N
   - **B** 47 N
   - **C** 92.0 N
   - **D** 98 N

10. If an object remains stationary on an incline, what must be true about the forces acting parallel to the incline?
   - **A** The parallel component of gravity is greater than the frictional force.
   - **B** The parallel component of gravity equals the frictional force.
   - **C** The parallel component of gravity equals the normal force.
   - **D** The normal force equals the frictional force.

SHORT RESPONSE

11. A 90 kg snowboarder accelerates uniformly from rest at 2.70 m s^{-2} down a 60° slope.
   - **A** **Calculate** the total resistance force on the snowboarder.
   - **B** **Calculate** the speed of the snowboarder after 100 m of travel down the slope.

12. A 31 kg child on a 4.0 kg luge accelerates uniformly from rest to a maximum speed of 20 m s^{-1} in 5.0 s down a 30° slope. The child maintains this speed for a further 5.0 s.
   - **A** **Calculate** the normal force on the child and luge system.
   - **B** Find the net force on the child 3.0 s after the start.
   - **C** **Determine** the maximum kinetic frictional force on the child and luge system.

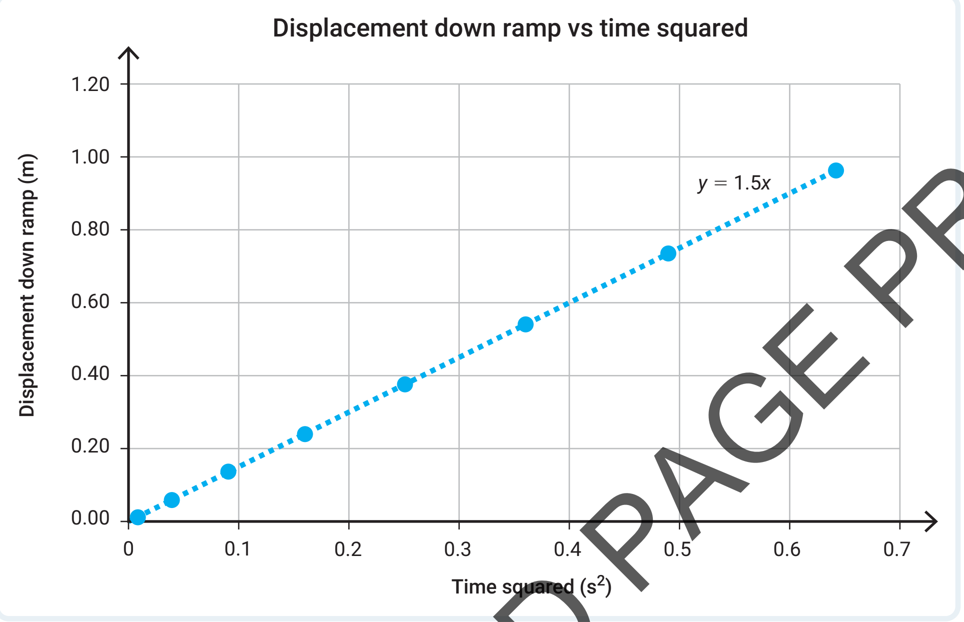
CROSS-CHAPTER QUESTION

13. A 100 g cart is initially at the bottom of a frictionless ramp. It is then accelerated up the length of ramp with a continuously applied force of 1.00 N. The angle of inclination of the ramp is 42° and its length is 1.0 m.
   - **A** At what velocity will the cart leave the top of the ramp?
   - **B** How far away from the ramp will the cart land if the 1.00 N force is no longer applied once it leaves the ramp?

DATA ANALYSIS

14. Analyse data

A digital motion sensor was used to progressively measure the displacement of a dynamics cart after it was released from rest and allowed to roll down a frictionless ramp. A plot of the data is presented below.



Determine the angle of inclination of the ramp by using this data.

15. Interpret evidence

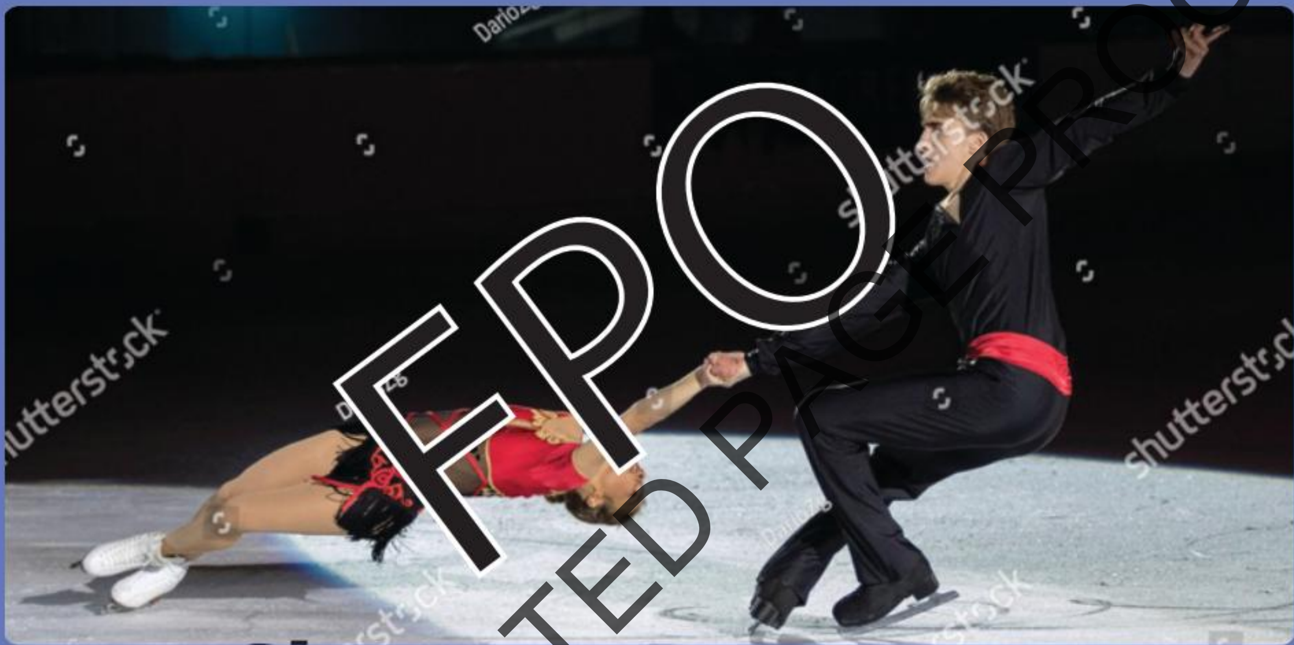
A 20.0 kg trolley that was initially at rest on a ramp was released. Its velocity was monitored as it rolled down the ramp, yielding the following data set.

Time (s)	Velocity (m s^{-1})
0.10	0.30
0.20	0.60
0.30	0.90
0.40	1.2
0.50	1.5
0.60	1.8
0.70	2.1
0.80	2.4
0.90	2.7
1.00	3.0

Draw a conclusion about the magnitude of the frictional force acting on the trolley if the ramp has an angle of inclination of 20°.

CHAPTER 4

Circular motion



TK

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Describe the concept of uniform circular motion.
- Describe the concepts of average speed and period.
- Solve problems involving objects undergoing uniform circular motion at a constant speed using $v = \frac{2\pi r}{T}$ and $a_c = \frac{v^2}{r}$.
- Describe the concepts of centripetal acceleration and centripetal force.
- Solve problems involving forces acting on objects in uniform circular motion using $F_c = F_{net} = \frac{mv^2}{r}$.

SCIENCE AS A HUMAN ENDEAVOUR

- Consider the international collaboration required to monitor the orbits of satellites, and the management of space debris.
- Consider the factors that contribute to positioning of satellites used for observation of weather, natural phenomena, traffic and military movements.

SCIENCE INQUIRY

- Investigate the net forces acting on an object undergoing horizontal circular motion on a string.

Introduction

An ice skater glides effortlessly in a straight line at constant speed. That is a consequence of Newton's first law of inertia: a moving object travels at constant speed in a straight line unless forced to change speed, direction, or speed and direction simultaneously. An ice skater travelling at constant speed can change direction without a change of speed. Note the difference between speed (a scalar) and velocity (a vector). In order to change direction, a force must be applied to the ice skater. If the force is applied continuously, the skater can change direction continuously; that is, follow a circular path.

Assessments

- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practicals

- Investigating the magnification of a microscrope

Worksheets

- Name
- Name
- Name

FPO

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ The circumference of a circle can be calculated from $C = 2\pi r$.
- ✓ The radius of a circle is half the diameter $D = 2r$.
- ✓ The frequency (f) of a cyclic or oscillating phenomena is the number of cycles or oscillations per second; units are hertz (Hz) or s^{-1}.
- ✓ The period (T) of a cyclic or oscillating phenomena is the time it takes for a single cycle or oscillation; units are seconds (s).
- ✓ Frequency and period of a wave can be calculated using the formula $f = \frac{1}{T}$.
- ✓ Average speed can be calculated from $v_{av} = \frac{\text{distance}}{\text{time}}$.
- ✓ Tension is a force that can be transmitted by a flexible medium such as a string or rope.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe the concept of uniform circular motion
- ✓ describe the concepts of average speed and period
- ✓ solve problems involving objects undergoing uniform circular motion at a constant speed using $v = \frac{2\pi r}{T}$ and $a_c = \frac{v^2}{r}$
- ✓ describe the concepts of centripetal acceleration and centripetal force
- ✓ solve problems involving forces acting on objects in uniform circular motion using $F_c = F_{net} = \frac{mv^2}{r}$
- ✓ explain how the centripetal equations are derived using Newton's second law and the vector analysis of circular motion
- ✓ use trigonometric analysis and centripetal theory to explain the advantage of having a curved section of road banked
- ✓ discriminate between the forces in action at the top and bottom points in vertical uniform circular motion
- ✓ explain the centripetal acceleration or force involved in contexts such as orbits, theme park rides, how electrons move around the nuclei of atoms or the curved paths of charged particles in magnetic fields.

4.1 Uniform circular motion

Weblink
Uniform circular motion: interactive

Ice skaters, cars, cyclists and runners as well as planets and satellites can all move at constant speed while changing direction. Each motion can be modelled in terms of a point particle for which the mass is concentrated at the centre of mass.

Period and frequency

period the time taken for an object undergoing circular motion to complete one revolution

frequency the number of times a circular motion is completed in a time period

Uniform circular motion of a point particle is described in terms of the radius of the circle, r, and the time taken for the particle to complete one revolution, T, called the **period**.

For repetitive circular motion, such as a ball being swung around on the end of a string, the time taken for one revolution, T, is related to the number of times per second that the object describes a circle in a unit of time. The number of times per second is called the **frequency**, f. It is measured in units of hertz (Hz) or s^{-1}. For example, if a ball takes 0.1 s to go once around a circle ($T = 0.1$ s), the number of times per second it goes around is 10 ($f = 10$ Hz). Thus:

$$f = \frac{1}{T}$$

Average speed and period

The speed of a particle, v, is related to the two fundamental variables of distance and time. For uniform circular motion, the instantaneous speed at any point, v_{inst}, is the same as the average speed, v_{av}, between any two points or measured over one complete revolution **(Figure 4.1.1)**. The distance covered in one revolution of a circle is the circumference. Thus, from the definition of average speed:

$$v_{av} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{\text{circumference}}{\text{period}}$$

$$= \frac{2\pi r}{T}$$

$$v = \frac{2\pi r}{T}$$

$$\text{But } f = \frac{1}{T}$$

$$v = 2\pi r f$$

Since v_{inst} and v_{av} are the same for uniform circular motion, the subscripts will be dropped and the symbol, v, will be used.

KEY CONCEPT

Circular functions and radians

Circular functions are projections of circular motion onto x- and y-axes. The role of the factors $\frac{2\pi}{T}$ or $2\pi f$ in circular functions relate directly to the way in which angles change. Angular measurement can be done in radians as well as degrees.

KEY FORMULA

Period and frequency

$$f = \frac{1}{T}$$

where:
f = frequency (Hz or s^{-1})
T = period (s)

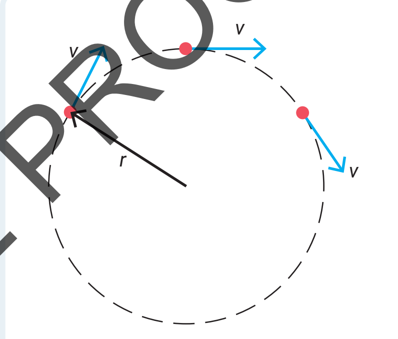


FIGURE 4.1.1 In circular motion, the velocity vector changes direction continuously. It travels the circumference of the circle in one time period.

KEY FORMULA

Average speed and period

$$v = \frac{2\pi r}{T} = 2\pi r f$$

where:
r = radius (m)
T = period (s)
f = frequency (Hz)
v = average and instantaneous speed (m s^{-1})

Solving problems in circular motion

Period and frequency

For questions requiring the conversion of period to frequency or frequency to period follow the steps below.

1. Read the question carefully.
2. Identify the variables and quantities provided.
3. Identify the appropriate equation:

$$f = \frac{1}{T} \text{ or } T = \frac{1}{f}$$

4. Substitute values, including units.
5. Solve the equation.
6. Check to ensure the question has been answered in the correct units.

Average speed, radius, period and frequency

For questions requiring the calculation of speed, radius, period or frequency follow the steps below.

1. Read the question carefully.
2. Identify the variables and quantities provided.
3. Write the appropriate equation:

$$v = \frac{2\pi r}{T} \text{ or } v = 2\pi rf$$

4. If necessary, transpose the equation to make the required variable the subject.
5. Substitute values, including units.
6. Solve the equation.
7. Check to ensure the question has been answered in the correct units.

WORKED EXAMPLE 4.1.1

A pendulum takes $T = 2$ s to complete one full oscillation. Calculate the frequency, f, of the pendulum's motion.

ANSWER

1 **Determine the formula.**
The relationship is given by $T = \frac{1}{f}$
where T = time period (time taken for one complete oscillation) (s)
f = frequency (number of oscillations per second) (Hz).

2 **Rearrange to find the unknown.**
$f = \frac{1}{T}$

3 **Substitute the known values.**
$f = \frac{1}{2}$

4 **Calculate the answer.**
$f = 0.5$ Hz

WORKED EXAMPLE 4.1.2

A satellite orbits Earth in a circular path with a radius of $r = 7.0 \times 10^6$ m and a time period of $T = 6000$ s. Calculate the velocity, v, of the satellite in its orbit.

ANSWER

1 **Determine the formula.**
The relationship is given by the formula:

$$v = \frac{2\pi r}{t}$$

where r = radius of the circular path (m)
T = time period for one complete revolution (s)
v = velocity (m s^{-1}).

9780170483704

3 Substitute the known values.

$$v = \frac{2\pi \times 7.0 \times 10^6}{6000}$$

4 Calculate and state the answer.

$v = 7330\ \text{m s}^{-1}$

WORKED EXAMPLE 4.1.3

A car moves along a circular track with a radius of $r = 50$ m. It completes 5 revolutions every minute. Calculate the velocity, v, of the car.

ANSWER

1 Determine the formula.

The relationship is given by the formula:

$v = 2\pi rf$

where r = radius of the circular path (m)

f = frequency of oscillation or revolution (Hz or s^{-1})

v = velocity (m s^{-1}).

2 Substitute the known values.

$$v = 2\pi \times 50 \times \frac{5}{60}$$

3 Calculate and state the answer.

$v = 26.2\ \text{m s}^{-1}$

LEARNING CHECK 4.1

DESCRIBING

1 **Define**:
 a frequency
 b period.

2 Write down the formula that links:
 a frequency to period
 b speed to radius and period
 c speed to radius and frequency.

3 **Explain** why speeds rather than velocities are used for uniform motion.

4 Transpose $V = \frac{2\pi r}{T}$ and $v = 2\pi rf$ to make equations with the following subjects.
 a r
 b T
 c f

APPLYING

5 Find the frequency for an object that has a period of:

- **a** 50 s
- **b** 0.34 s
- **c** 2.8×10^{-3} s.

6 Find the speed of an object moving at uniform circular speed for:

- **a** radius = 1.2 m; period = 0.3 s
- **b** radius = 0.37 m; period = 2.9×10^{-2} s
- **c** radius = 1.5×10^{8} km (Earth's orbital radius around the Sun), period = 365.25 days
- **d** radius = 0.2 m; frequency = 15 Hz.

7 **Determine** the period of an object with frequency:

- **a** 12 Hz
- **b** 4.9×10^{3} Hz
- **c** 2.5×10^{14} Hz.

8 **Determine** the period of an object moving at uniform circular speed with radius 35 m and speed 4.2×10^{7} m s^{-1}.

9 **Determine** the velocity, in m s^{-1}, at the equator of Earth given the radius is 6.4×10^{6} m and the period of one rotation of Earth is one day.

10 Find the radius of orbit for an object moving at uniform circular speed for:

- **a** speed = 3.0×10^{5} m s^{-1}; frequency = 3.0×10^{5} Hz
- **b** speed = 4.5 m s^{-1}; period = 28 days.

ANALYSING

11 From the point of view of an observer on Earth, the planet Jupiter appears to be moving away in a straight line along its orbit around the Sun. Ganymede, one of Jupiter's moons, is rotating towards the observer as it circles the planet. The observer on Earth wants to find the speed of Ganymede relative to them.

- **a** **Describe** the calculations the observer must undertake for both Jupiter and Ganymede.
- **b** **Identify** the data the observer needs to collect.

12 The factor $\frac{2\pi}{T}$ is the angular speed for an object moving with uniform circular motion. **Explain**.

13 A twelve-inch vinyl record rotates at 33.3 revolutions per minute (rpm). Point A is halfway from the centre to point B, which is on the edge of the record. **Calculate** the ratio of the speed of point A to the speed of point B.

4.2 Centripetal acceleration and force

Weblink
Visual understanding of centripetal acceleration formula

Uniform circular motion means that the speed is constant. But the speed is constantly changing direction. Consequently, the velocity changes. This velocity change occurs over a time interval. Thus, circular motion is accelerated motion. According to Newton's second law, this acceleration is caused by a net force and affected by the mass that is being forced to change its state of motion.

Centripetal acceleration

An object circling around a point must be forced (pushed or pulled) continuously inwards. Therefore, the acceleration is directed inwards towards the centre of the circle. In the case of uniform circular motion, the acceleration is always directed towards the exact centre of the motion. **Figure 4.2.1** justifies this assertion.

KEY FORMULA

Centripetal acceleration

Centripetal acceleration is the change in velocity over a period.

$$a_c = \frac{\Delta v}{t}$$

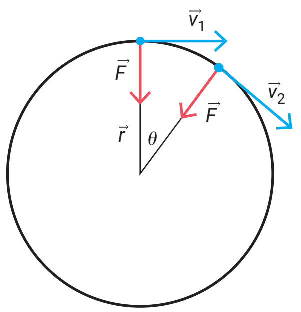


FIGURE 4.2.1 Derivation of centripetal acceleration: two velocity vectors of equal magnitude are separated by a time interval, which is represented by the angle between the equal magnitude radius vectors.

The change in velocity is:

$$\begin{aligned}\Delta v &= v_2 - v_1 \\ &= v_2 + (-v_1)\end{aligned}$$

This is shown geometrically as a vector subtraction in Figure 4.2.3.

The average acceleration, $\vec{a}_{av}$ is the change in velocity over the time interval:

$$\begin{aligned}\vec{a}_{av} &= \frac{\Delta \vec{v}}{\Delta t} \\ &= \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}\end{aligned}$$

The direction of the average acceleration vector is the same as the direction of the change in velocity vector. They both act towards the centre of the circle.

KEY CONCEPT

Centripetal force and acceleration

Centripetal forces and accelerations are not forces in themselves. A centripetal force may be caused by a tension force (as in a ball swung on a string), a gravitational force (as in a satellite circling Earth), an electrostatic force (as in an electron circling the positive nucleus of an atom) or any other force.

For uniform circular motion, the change in velocity is smooth. The instantaneous change of velocity vector for a time interval occurs approximately halfway through the time interval. The smaller the time interval, the closer this approximation comes to the actual value. In the limit, when the time interval approaches zero, the change in the velocity vector points directly towards the centre of the circle. The average and instantaneous acceleration vectors become the same. Acceleration always points to the centre of the circle. This pointing to the centre is called **centripetal**; the acceleration is a centre-seeking, centripetal acceleration.

centripetal centre-seeking; directed towards the centre

The relationship between the speed and the radius can now be deduced. In **Figure 4.2.2**, the length of the circumference interval, $v\Delta t$, is approximated by a straight line because the time interval is very small. The vector subtraction in **Figure 4.2.3** is similar to this triangle because both triangles are isosceles triangles with the same angle between the equal sides.

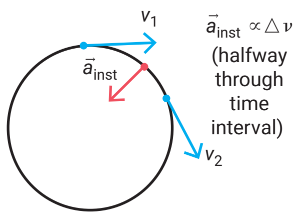


FIGURE 4.2.2 For smooth changes, the instantaneous acceleration occurs approximately halfway through the time interval. For very small time intervals, the approximation becomes $\vec{a}_{av} = \vec{a}_{inst}$. The acceleration is centripetal (centre-seeking).

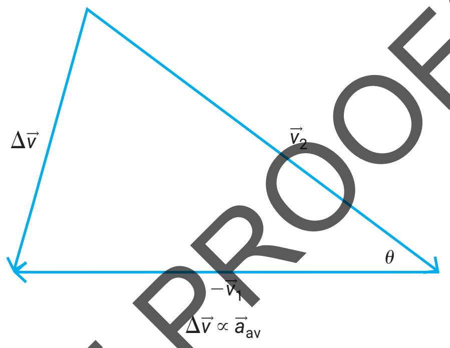


FIGURE 4.2.3 The change of velocity is a geometrical, vector difference.

For small Δt, $\frac{\Delta v}{v} = \frac{\Delta s}{r}$, where Δs = arc length

Therefore $\frac{\Delta v}{v} = \frac{v \times \Delta t}{r}$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r} \text{ and } a = \frac{v^2}{r}$$

KEY FORMULA

$$a = \frac{v^2}{r}$$

where:
a = acceleration (m s^{-2})
v = speed (m s^{-1})
r = radius (m)

KEY FORMULA

$$a = \frac{4\pi^2 r}{T^2}$$

where:
a = acceleration (m s^{-2})
r = radius (m)
T = period (s)

This equation links a, v and r. By substituting $v = \frac{2\pi r}{T}$ into this equation, a related equation can be deduced to connect a, r and T:

$$a = \frac{v^2}{r}$$

$$= \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{\left(\frac{2\pi}{T}\right)^2 \times r^2}{r}$$

$$= \left(\frac{2\pi}{T}\right)^2 \times r$$

$$= \frac{4\pi^2 r}{T^2}$$

Net force causes circular motion

An object moving in uniform circular motion undergoes centripetal acceleration. This means that the net force applied to the object is also directed towards the centre. Another name for this net force is **centripetal force**. Centripetal force is not a kind of force like the normal force, friction, tension, gravitational force, electrostatic force or magnetic force. These forces are real forces in their own right. Centripetal force is the name given to the sum of the real forces that act on a particle to cause it to undergo circular motion.

Newton's second law applies to circular motion as follows. The centre-seeking (centripetal) acceleration, *a*, on a mass, *m*, is caused by a net force, $\Sigma\vec{F}$:

$$\vec{a} = \frac{\Sigma\vec{F}}{m}$$

$$\Sigma\vec{F} = m\vec{a}$$

By substituting the expressions for acceleration derived in section 4.2, equations can be derived for the net force:

When *m*, *v* and *r* are provided:

$$\Sigma\vec{F} = m\frac{v^2}{r}$$

When *m*, *r* and *T* are provided:

$$\Sigma\vec{F} = m\frac{4\pi^2 r}{T^2}$$

Weblink
What is centripetal force?

centripetal force in uniform circular motion, the sum of real forces that point towards the centre of the circle

KEY FORMULA

Net force

$$\Sigma\vec{F} = m\frac{v^2}{r} \text{ and } \Sigma\vec{F} = m\frac{4\pi^2 r}{T^2}$$

where:
$\Sigma\vec{F}$ = net force directed towards the centre of the circle (N)
m = mass (kg)
v = speed (m s^{-1})
r = radius (m)
T = period (s)

Types of force that contribute to net force

Real forces are responsible for the net force that causes circular motion. Real forces include electrostatic forces (normal, friction, electric, tension), gravitational force (weight) and magnetic forces.

The normal force is always at right angles to a surface. For example, it is the normal force that causes a person to go around inside a fun-park rotor (Figure 4.2.4).

Friction by the ground on an object, such as an athlete's foot or the tyres of bicycles or the wheels of cars, causes the object to turn corners. In terms of Newton's third law, the action force applied by the object on the surface, which is directed away from the centre, is equal to the opposite reaction force by the surface on the object. It is this reaction force by the surface on the object that causes the object to change direction.

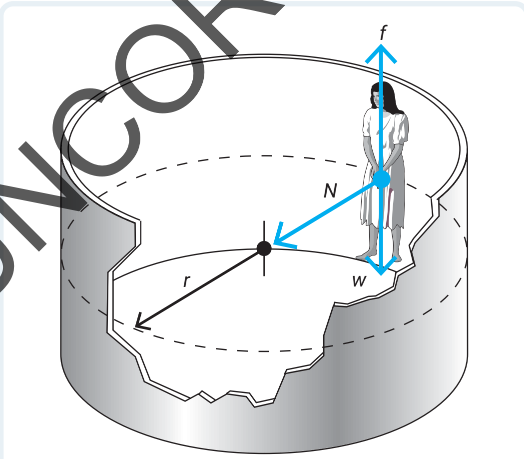


FIGURE 4.2.4 The normal force, *N*, applied by the rotor wall causes a person to move in a circular path.

Shutterstock.com/supergenijalac

FIGURE 4.2.5 The outwards directed action force by the car on the surface affects the surface. The inwards reaction force applied by the surface friction on the wheels of the car causes the car to move in a circular path.

Tension force by a string makes it possible to whirl an object around in a circle **(Figure 4.2.6)**.

Gravitational force is responsible for the motion of the Moon and other satellites around Earth, and the planets around the Sun **(Figure 4.2.7)**.

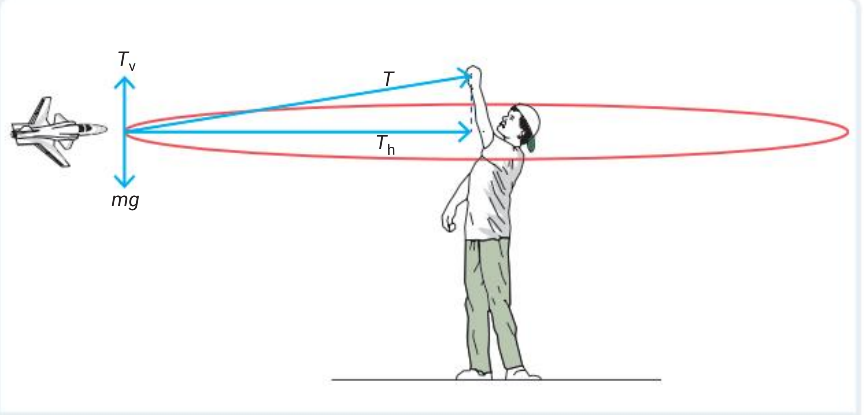


FIGURE 4.2.6 The horizontal component, T_h, of the force applied by the string in tension, T, on the toy plane causes it to move in a circular path.

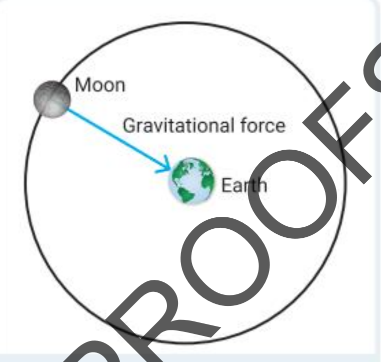


FIGURE 4.2.7 The force applied by the mass of Earth on the mass of the Moon causes the Moon to move in a circular orbit.

Electrostatic force causes moving charged particles to circulate around a central charge, such as the electron in a hydrogen atom, which circles around the proton **(Figure 4.2.8)**.

Magnetic forces act at right angles to the motion of charged particle streams to cause them to travel in circles. This occurs in a mass spectrometer where ions are given kinetic energy and then injected into a magnetic field **(Figure 4.2.9)**. The different circular paths depend on the charge and mass of the ions.

One or more of these forces may be applied to a particle in order to move it in a circle. For example, the normal force and the weight force may act on a cyclist to enable uniform circular motion around the banked track of a velodrome. These two forces combine to produce the net force on the cyclist. This sum of forces is centre-directed, hence centripetal. This centripetal force is the sum of different forces but is not itself a new kind of force.

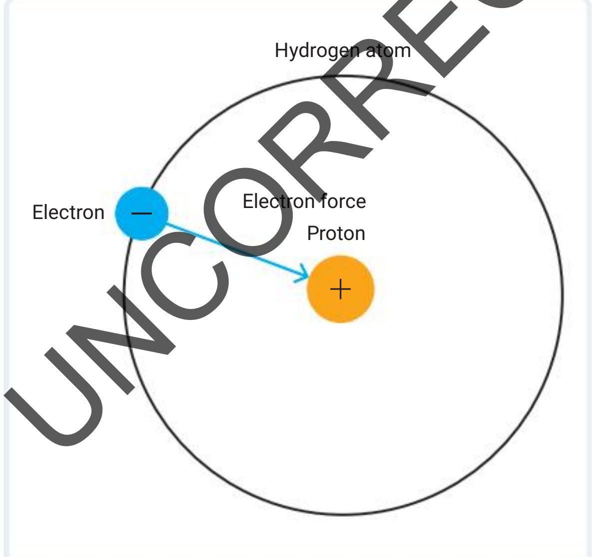


FIGURE 4.2.8 The electrostatic force applied by the central charge (proton) on the moving electron causes the electron to move in a circular orbit.

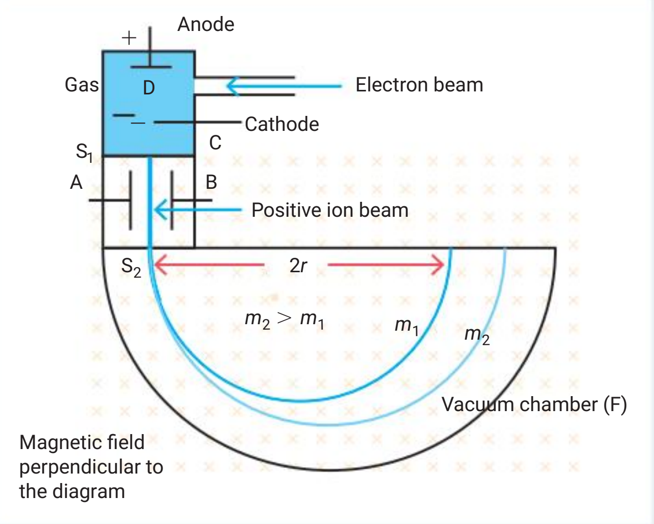


FIGURE 4.2.9 In a mass spectrometer, the force applied by the magnetic field on the flow of charged ions causes the ions to move in circular paths whose radius depends on their mass. The smaller the mass, the smaller the radius.

PRACTICAL ACTIVITY 4.2.1

NET FORCE AND CIRCULAR MOTION

Introduction

A mass can be whirled around in a horizontal circle on the end of a string. For a particular radius, the force applied to the mass is related to the frequency:

$$F(\text{by string on mass}) = \frac{2\pi r}{T} = 2\pi r f$$

$$\Rightarrow f = \frac{F(\text{by string on mass})}{2\pi r}$$

In this practical activity, the radius is kept at a constant length, the force is changed and the frequency is measured.

Research question

How does the force acting on an object travelling in uniform circular motion affect the frequency of rotation?

Aim

To determine the relationship between net force acting on an object travelling in uniform circular motion and its frequency of rotation

Materials

- two-hole rubber stopper
- 2 m of strong inextensible string (or fishing line)
- 15–20 cm long glass or plastic tube with polished ends
- 30 metal washers of equal mass
- crocodile clip
- metre ruler
- stopwatch

Risk assessment

RISK ASSESSMENT

What are the risks in doing this experiment?	How can you manage these risks to stay safe?

Copy and complete the risk assessment table in your write-up. Add any more risks you can think of, and ways to manage them. Ask your teacher to check your table before you proceed.

Procedure

1. Tie the rubber stopper securely to one end of the string.
2. Pass the string through the tube.
3. Measure 40 cm of string from the top of the tube to the stopper.
4. Attach the crocodile clip to the string just below the bottom of the tube.
5. Attach and secure 30 washers or a brass mass to the end of the string.
6. Hold the tube vertically and whirl the rubber stopper around in a horizontal circle (Figure 4.2.10).
7. Measure and record the time taken for 20 revolutions of the stopper for three trials.
8. Repeat the procedure with different numbers of washers or masses; for example, 25, 20, 18, 15, 10.

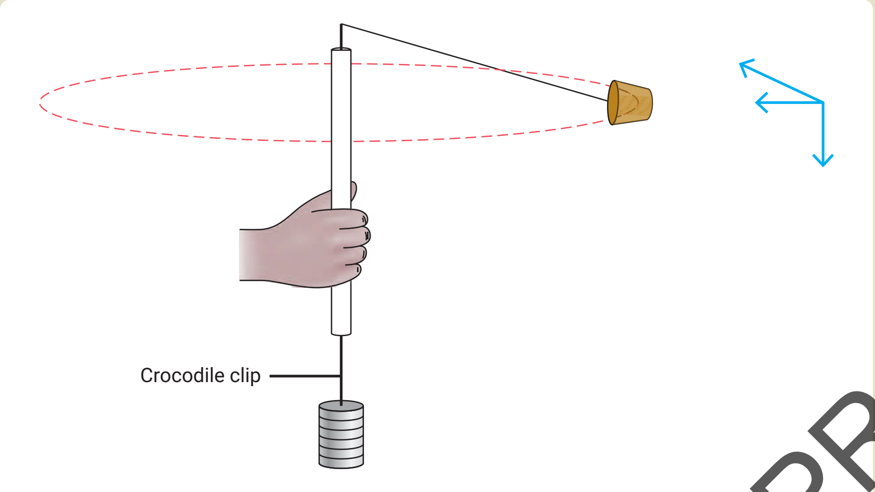


FIGURE 4.2.10 Whirl the rubber stopper in a horizontal circle. Note that the crocodile clip must always remain a fixed distance below the bottom end of the tube.

Data analysis

1 Plot a graph of frequency against mass.
2 Describe mathematically any justifiable relationship between frequency and force applied.

Analysis of results

1 Explain why it is important not to allow the crocodile clip to touch the tube.
2 Draw a free-body diagram to show the:
   a forces applied to the rubber stopper
   b net force on the rubber stopper.

Evaluation

3 Summarise the relationship identified.

KEY FORMULA

For forces when cornering on a banked road

$F_{net} = mg \tan\theta$

$mg \tan\theta = \frac{mv^2}{r}$

where:

F_{net} = vector sum of the normal force and the weight force (N)
m = mass (kg)
v = speed (m s^{-1})
r = radius (m)
θ = angle of banking (°)
g = acceleration due to gravity, 9.80 m s^{-2}

Uniform circular motion on a banked track

A car that travels horizontally around a banked roadway at constant speed is acted upon by two forces: the normal force and the weight force (Figure 4.2.11). Friction is considered to be negligible in this analysis.

The normal force and the weight force are vectors. The vector sum is equal to the net force, which is horizontally directed towards the centre of the curve of the road. The net force can be found in terms of the weight force and the angle of banking of the road:

$$\frac{F_{net}}{w} = \tan\theta$$

$$F_{net} = w\tan\theta$$
$$= mg\tan\theta$$

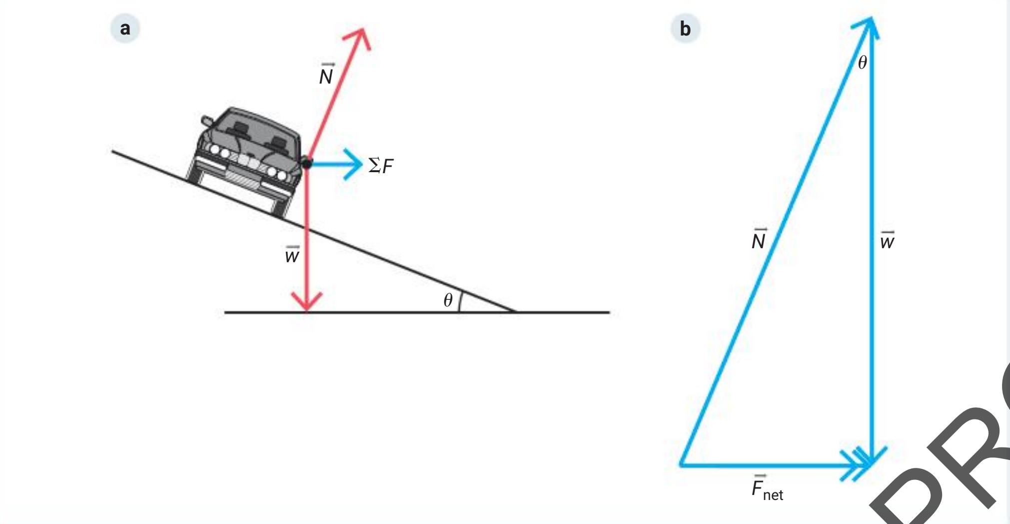


FIGURE 4.2.11 (**a**) Forces on a car cornering on a frictionless banked road. The net force is the horizontal component of the normal force. (**b**) The vector sum of the forces. Note the double-headed arrow indicating resultant or net force.

Weblink
Vertical circle simulation

$$mg \tan\theta = \frac{mv^2}{r}$$

$$g \tan\theta = \frac{v^2}{r}$$

Friction by the road is never really negligible. The friction force acts parallel to the slope and against the object's motion

Vertical circular motion

Motion in a vertical circle can be quite complicated to analyse; however, forces at the upper and lower positions are relatively straightforward. An object on the end of a string, such as a toy aeroplane (**Figure 4.2.12**), is subject to two forces: tension, T, and weight, w. These combine to form the net (centripetal) force on the object at the:

- top: $T - w = \frac{mv^2}{r}$
- bottom: $T + w = \frac{mv^2}{r}$.

If the object is travelling at constant speed, the tension at the bottom must be greater than at the top.

A car travelling at speed v is subject to the normal force, N, by the road and the weight force, w, on the car. These combine to form the net (centripetal) force on the car when travelling over a hill (**Figure 4.2.13**) or through a dip in the road.

Travelling over a rise:

$$w - N = \frac{mv^2}{r}$$

$$N = \frac{mv^2}{r} - mg$$

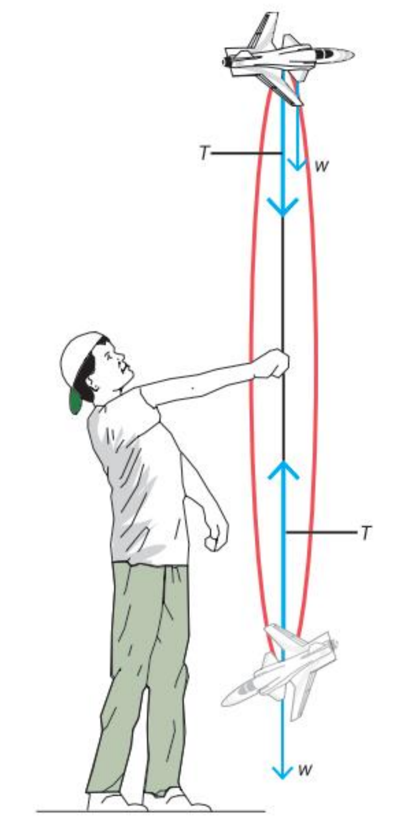


FIGURE 4.2.12 An object being whirled in a vertical circle. Note that the F_c is centre-seeking; F_w acts downwards and F_t acts towards the centre of the circle. F_w and F_t do not often act in the same direction in non-uniform circular motion.

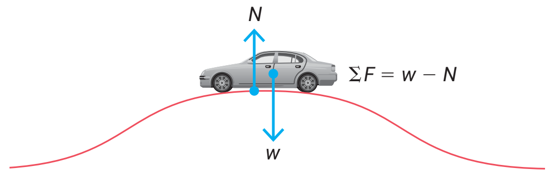


FIGURE 4.2.13 The net force for a car travelling over a rise is downwards.

This means that it is possible for the normal force applied by the road on the car to reduce to zero:

$$a = \frac{v^2}{r} - g = 0$$

$$0 = mg - \frac{mv^2}{r}$$

$$v = \sqrt{gr}$$

The faster the car travels over a rise, the less control the driver can exert on the car, because it is the normal force that governs the ability of the driver to exert a force on the road in order to change direction as the force of friction is the product of the coefficient of friction and the force normal. When the normal force is reduced to zero, the speed is $v = \sqrt{gr}$ and the car becomes airborne, therefore uncontrollable.

Travelling through a dip:

$$N - w = \frac{mv^2}{r}$$

$$N = \frac{mv^2}{r} + mg$$

This means that the faster the car travels through a dip, the greater the normal force applied on the car by the surface (Figure 4.2.14). Steering may become more difficult because the larger normal force means that more effort is needed from the driver to change direction.

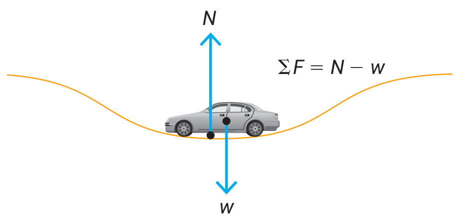


FIGURE 4.2.14 The net force for a car travelling through a dip is upwards.

LEARNING CHECK 4.2

REMEMBERING

1 **Define**:

  a centripetal force

  b uniform circular motion.

2 Use a diagram to **identify** the directions of velocity and centripetal acceleration acting on an object at both the top and bottom of a loop when it is being spun vertically in a circle.

3 Name three different forces that enable objects to travel in circular paths.

4 Draw a free-body diagram to show the forces applied on a car that is travelling on a banked track in a horizontal circle. From this diagram, sketch a vector diagram to show the net force on the car.

5 **Explain** why an object in vertical circular motion experiences varying tension forces throughout the motion.

6 **Explain** why there is a maximum speed at which it is possible to steer a car while travelling over a rise in the road.

APPLYING

7 Copy and complete the following table for centripetal acceleration and force.

m (kg)	v (m s^{-1})	r (m)	a (m s^{-2})	ΣF (N)
1.0	2.0	0.55		
400	20	150		
1.5×10^3	28	50		

ANALYSING

8 A 400 kg motorcycle travels around a corner of radius 80 m at 25 m s^{-1}. **Calculate** the net force applied by the motorcycle on the road.

9 A car of mass 1500 kg travels over a hump in the road that forms the arc of a circle with radius 25 m.

a **Calculate** the maximum speed the car can travel while just staying in contact with the road at the top of the hump.

b **Determine** what would happen if the car exceeded that speed.

10 Figure 4.2.12 illustrates a situation in which tension through a string is providing the centripetal force required to keep an object in a vertical circular path. Consider a rollercoaster travelling through a section of track in a vertical loop. **Deduce** what is providing the required centripetal force in this case.

4.3 Solving problems with centripetal force and acceleration

When solving problems involving acceleration, speed, radius and period for circular motion, follow the steps below.

1. Read the question carefully.
2. Visualise or sketch the real situation described.
3. Identify all variables provided.

   a Connect v, r and T with the equation:

   $$v = \frac{2\pi r}{T}$$

   b Connect a, v and r with the equation:

   $$a = \frac{v^2}{r}$$

   c Connect a, r and T with the equation:

   $$a = \frac{4\pi^2 r}{T^2}$$

4. Transpose the equation to make the required variable the subject.
5. Solve the equations.
6. Check to ensure the answers are those required.

WORKED EXAMPLE 4.3.1

An object accelerates at 25 m s^{-2} when swung in a circle of radius 80 cm. Calculate the:

a speed of the object

b period of rotation.

ANSWERS

a **1 Determine the formula.**

$$a = \frac{v^2}{r}$$

2 Rearrange the formula to find the unknown.

$$v = \sqrt{ar}$$

3 Substitute the known values.

$$v = \sqrt{25\ \text{m s}^{-2} \times 0.80\ \text{m}}$$

4 Calculate the answer.

$$v = 4.5\ \text{m s}^{-1}\ (4.47\ \text{m s}^{-1})$$

b **1 Determine the formula.**

$$v = \frac{2\pi r}{T}$$

2 Rearrange the formula to find the unknown.

$$T = \frac{2 \times \pi r}{v}$$

3 Substitute the known values.

$$T = \frac{2 \times \pi \times 0.80\ \text{m}}{4.47\ \text{m s}^{-1}}$$

4 Calculate the answer.

$T = 1.1$ s

When solving problems involving forces and circular motion, follow the steps below.

1. Read the question carefully.
2. Visualise or sketch the real situation described.
3. Draw a free-body diagram.
   - **a** Identify each real force acting on the object in question. (Do not make the mistake of adding centripetal force as a separate force – it is the vector sum of all the real forces acting on a single object.)
   - **b** Write each force in the form: F(by A on B) or use the symbols provided in the question.
   - **c** Identify the direction of the centripetal acceleration or net force.
   - **d** Add any data provided in the question.
4. Consider the application of Newton's laws to the question asked.
   - **a** Newton's second law: the equations relate to $\Sigma F = ma$.
   - **b** Newton's third law: the inwards force can be the reaction to an outward push.
5. Set up any equations, using the symbols from the free-body diagram.
6. Transpose the equation to make the desired variable the subject.
7. Solve the equations.
8. Check to ensure the answers are those required.

WORKED EXAMPLE 4.3.2

A 250 g aeroglider on the end of a string is swung in a horizontal circle with a radius of 1.2 m. The aeroglider makes a revolution every 2.0 s.

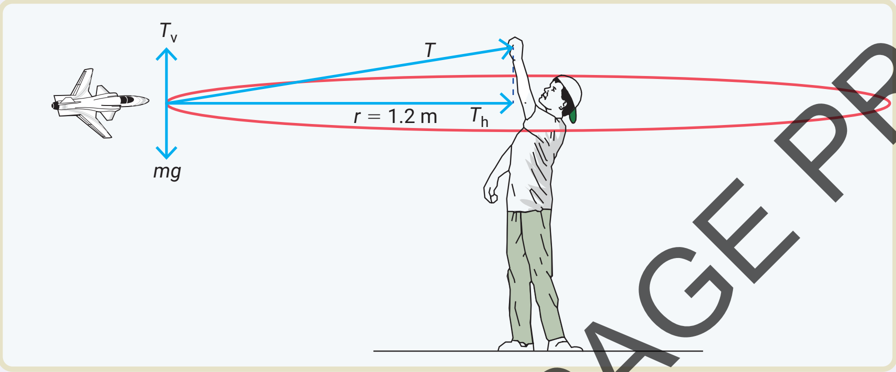
a Calculate the horizontal component of the tension force applied by the string to the aeroglider.

b Calculate the acceleration of the aeroglider.

c Find the force applied by the aeroglider to the string.

ANSWERS

a 1 Sketch the situation.



2 Determine the formula.

The horizontal component of the tension keeps the aeroglider in the circle (the vertical component of the tension balances the weight, but neither affects motion at right angles to their direction of action).

Set up the appropriate equation and solve it.

$\Sigma F = T_h$

$$T_h = m\frac{4\pi^2 r}{T^2}$$

3 Substitute the known values.

$$T_h = 0.250\ \text{kg} \times \frac{4 \times \pi^2 \times 1.2\ \text{m}}{(2.0\ \text{s})^2}$$

4 Calculate the answer.

$T_h = 3.0\ \text{N}$

b 1 Determine the formula.

Consider the application of Newton's second law.

$$a = \frac{\Sigma F}{m}$$

2 Substitute the known values.

$$a = \frac{3.0\ \text{N}}{0.250\ \text{kg}}$$

3 Calculate the answer.

$a = 1.20\ \text{m s}^{-2}$

c 1 Determine the formula.

Consider the application of Newton's third law.

The tension applied by the string in the aeroglider is equal and opposite to the force applied by the aeroglider on the string:

$$|F(\text{by aeroglider on string})| = |T|$$

$F(\text{by aeroglider on string}) = \sqrt{T_h^{\,2} + T_v^{\,2}}$

$T_h = 3.0\,\text{N}$ and $T_v - mg = 0$

2 **Substitute the known values.**

$T_v = 0.250\,\text{kg} \times 9.8\,\text{ms}^{-2}$

3 **Calculate the vertical tension.**

$T_v = 2.45\,\text{N}$

4 **Calculate the force.**

$F(\text{by aeroglider on string}) = \sqrt{(3.0\,\text{N})^2 + (2.45\,\text{N})^2}$
$= 3.9\,\text{N}$

Recall that the centripetal force is the resultant of actual forces acting. F_c may be represented by gravitational force, tension force, electromagnetic force, electrostatic force or any other force. Hence, each formula below may be used to determine F_c.

$$F_c = \frac{GMm}{r^2}$$

$$F_c = F_T$$

$$F_c = Bqv \sin\theta$$

$$F_c = \frac{kQq}{r^2}$$

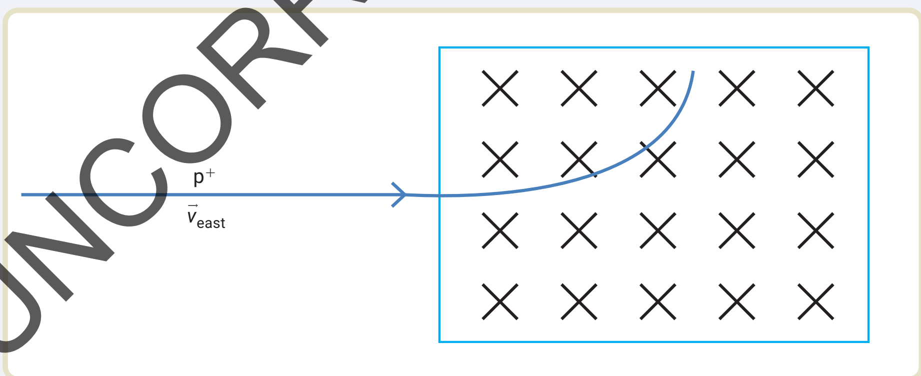
WORKED EXAMPLE 4.3.3

A proton of mass 1.67×10^{-27} kg travels east at 10 m s^{-1} into a magnetic field acting into the page and with a magnetic field strength of 50 μT.

a Draw a diagram of the situation.
b Calculate the net centripetal force acting on the proton, including its direction.
c Determine the radius of curvature of the proton.

ANSWERS

a



b 1 **Determine the formula.**

$F_c = Bqv\sin\theta$

2 **Substitute the known values.**

$F_c = 50 \times 10^{-6} \times 1.6 \times 10^{-19} \times 10 \times \sin 90°$

3 **Calculate the answer.**

$F_c = 8.00 \times 10^{-23}$ N

The direction is at right angles to the velocity and the magnetic field; that is, initially it is forced up the page. (This is Fleming's right hand rule applied.)

c 1 **Determine the formula.**

Let $F_c = \frac{mv^2}{r}$.

2 **Rearrange to find the unknown.**

$r = \frac{mv^2}{F_c}$

3 **Substitute the known values.**

$r = \frac{1.67 \times 10^{-27} \times (10)^2}{8.00 \times 10^{-23}\,\text{N}}$

4 **Calculate the answer.**

$r = 2.08 \times 10^{-3}$ m

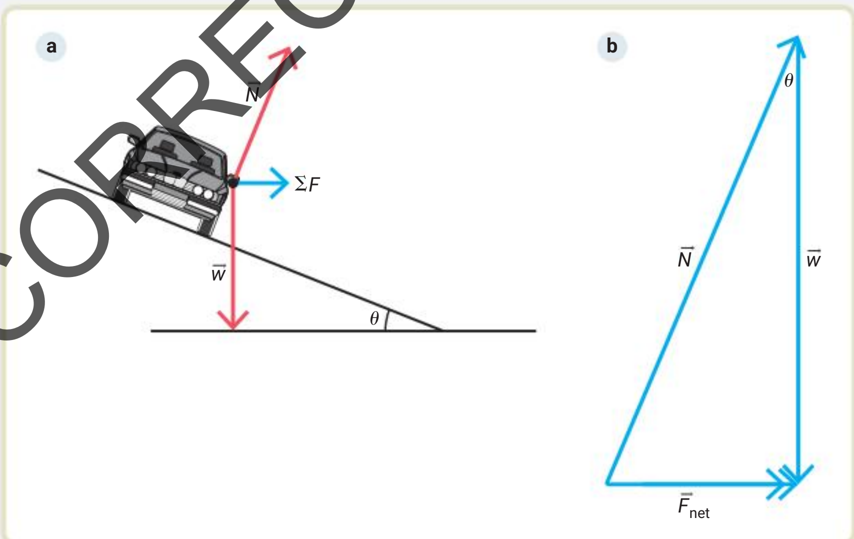
WORKED EXAMPLE 4.3.4

A car of mass 1500 kg travels horizontally at 20 m s^{-1} around a bend that is banked at 10° to the horizontal. Friction along the slope is negligible.

a Calculate the normal force acting on the car.
b Calculate the net force acting on the car.
c Determine the radius of curvature of the road.

ANSWERS

a 1 **Sketch a diagram to help visualise the scenario.**



2 **Determine the formula.**
From the vector diagram:
$\frac{w}{N} = \cos\theta$

3 Rearrange to find the unknown.

$$N = \frac{w}{\cos\theta}$$

4 Substitute the known values.

$$N = \frac{1500\ \text{kg} \times 9.8\ \text{m s}^{-2}}{\cos 10°}$$

5 Calculate the answer.

$$N = 1.49 \times 10^3\ \text{N}$$

b 1 Determine the formula.

$$\frac{\Sigma F}{w} = \tan\theta$$

2 Rearrange to find the unknown.

$$\Sigma F = w \tan\theta$$

3 Substitute the known values.

$$\Sigma F = 1500\ \text{kg} \times 9.8\ \text{m s}^{-2} \times \tan 10°$$

4 Calculate the answer.

$$\Sigma F = 2.59 \times 10^3\ \text{N, centre-directed}$$

c 1 Determine the formula.

$$a = \frac{v^2}{r}$$

2 Rearrange to find the unknown.

$$r = \frac{v^2}{a}$$

3 Substitute the known values.

$$r = \frac{(20\ \text{m s}^{-1})^2}{9.8\ \text{m s}^{-2} \times \tan 10°}$$

4 Calculate the answer.

$$r = 5.23\ \text{m}$$

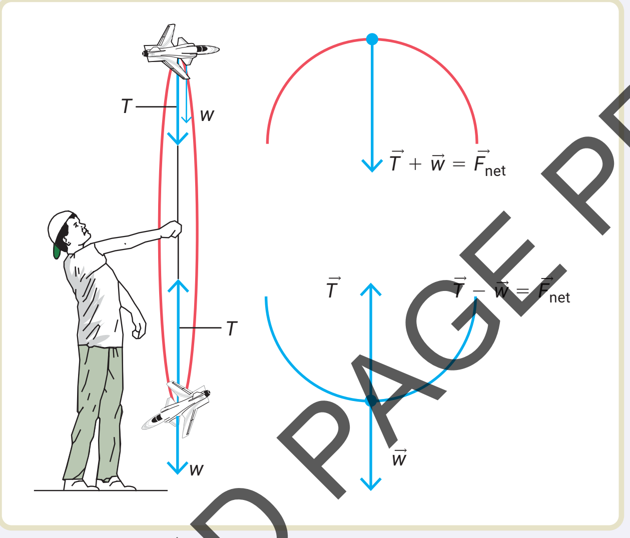
WORKED EXAMPLE 4.3.5

A 0.20 kg aeroglider is whirled in a vertical circle on the end of a string of length 0.60 m at a constant speed of 3.0 m s^{-1}. Calculate the tension in the string at the:

a top of the circle
b bottom of the circle.

ANSWERS

a 1 Sketch a diagram to help visualise the scenario.



2 Determine the formula.

$$\Sigma F = T + w = m\frac{v^2}{r}$$

$$T + w = m\frac{v^2}{r}$$

3 Rearrange to find the unknown.

$$T = m\frac{v^2}{r} - mg$$

4 Substitute the known values.

$$T = 0.20\,\text{kg} \times \frac{(3.0\,\text{m s}^{-1})^2}{0.60\,\text{m}} - 0.20\,\text{kg} \times 9.8\,\text{m s}^{-2}$$

5 Calculate the answer.

$T = 1.0\,\text{N}$

b 1 Determine the formula.

$$\Sigma F = T - w = m\frac{v^2}{r}$$

$$T - w = m\frac{v^2}{r}$$

2 Rearrange to find the unknown.

$$T = m\frac{v^2}{r} + mg$$

3 Substitute the known values.

$$T = 0.20\,\text{kg} \times \frac{(3.0\,\text{m s}^{-1})^2}{0.60\,\text{m}} + 0.20\,\text{kg} \times 9.8\,\text{m s}^{-2}$$

4 Calculate the answer.

$T = 5.0\,\text{N}$

LEARNING CHECK 4.3

DESCRIBING

1. **Recall** how to determine the radius of the path a charged particle of mass *m* will travel when entering into a magnetic field at velocity v.
2. Transpose the following equations to make *v* and *T* the subject respectively.

   a $a = \frac{v^2}{r}$

   b $\Sigma F = m\frac{4\pi^2 r}{T^2}$

APPLYING

3. A 1.2 t car travels around a horizontal curve of radius 120 m at 100 km h^{-1}.

   a **Calculate** the acceleration of the car.

   b **Determine** the force applied by the car on the road.
4. A velodrome is banked at 42°. A 75 kg cyclist travels horizontally at 18 m s^{-1} around the curved part of the track. Friction is negligible.

   a **Calculate** the net force acting on the cyclist.

   b **Calculate** the radius of curvature of the cyclist’s path.
5. The bottom of a rollercoaster ride has a track with a radius of curvature of 28 m. The ride passes this point with a speed of 12 m s^{-1}. Find the normal force on a 50 kg person at the lowest point.
6. A car goes over a crest in the road that has a radius of curvature of 34 m. **Determine the speed** at which the car will lose contact with the road.
7. A car travels through a dip in a road that has a radius of curvature of 45 m at 35 m s^{-1}. **Calculate** the force applied by the seat of the car on the 82 kg driver.

ANALYSING

8. The acceleration of an object travelling with uniform circular motion in a horizontal circle of radius *r* and period *T* is $25r^2$.

   Determine the period of this motion.
9. An alpha particle (a helium nucleus) with the mass of two protons and two neutrons travels to the right at 15 m s^{-1} into a magnetic field acting into the page and with a magnetic field strength of 35 μT. **Determine** the magnitude and direction of the force acting on the alpha particle as it enters the field.

CHAPTER SUMMARY

Variables in circular motion calculations

- Uniform circular motion of a point particle is described in terms of the radius of the circle, r, and the time taken for the particle to complete one revolution, T, called the period.
- The number of revolutions per second is called the frequency (Hz).

$$f = \frac{1}{T}$$

Speed versus velocity

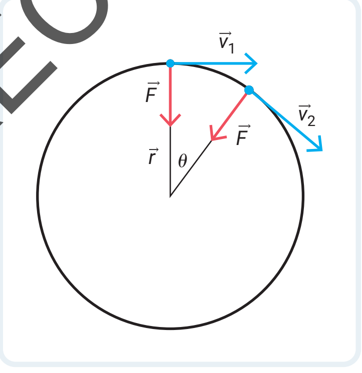
- Speed is a scalar quantity and therefore only measures the magnitude of how fast an object is moving along a circular path. However, velocity is a vector quantity that measures both the magnitude and direction of the object's motion.
- In uniform circular motion, the magnitude of how fast the object is moving remains constant as the object covers equal distances in equal time intervals. This means the speed is constant.
- However, the direction of the velocity is always tangential to the path of the motion, meaning the velocity vector continuously changes direction (to remain tangential to the circle), meaning the velocity varies:

$$v = \frac{2\pi r}{T} = 2\pi r f$$

Centripetal acceleration and force

- Centripetal acceleration is the acceleration of the object, always directed towards the centre of the circular path, resulting from the continuous change in the direction of the object's velocity in circular motion.

$$a_c = \frac{v^2}{r}$$



- Centripetal force is the force keeping the object in motion along the curved path, always directed towards the centre of the curved path. The force is the sum of the real forces that act on the object to cause it to undergo circular motion. This force can be provided through the sum of any real forces like weight, tension, electrostatic or electromagnetic forces.

$$F_c = \frac{mv^2}{r} = qvB\sin\theta = \frac{GMm}{R^2} = mg = \frac{kqQ}{d^2}$$

CHAPTER EXAM

MULTIPLE CHOICE

1. Which of the following types of force could be responsible for an object moving on a circular path?
   - **A** Tension force, magnetic force, centripetal force and gravitational force
   - **B** Centripetal force, electrostatic force, gravitational force and net force
   - **C** Net force, magnetic force, gravitational force and electrostatic force
   - **D** Tension, gravitational force, electrostatic force and magnetic force

2. A 25 g object takes 13 s to revolve 15 times around a fixed point. Calculate the speed of the object when it is 45 cm from the fixed point.
   - **A** $3.3\,\text{m s}^{-1}$
   - **B** $33\,\text{cm s}^{-1}$
   - **C** $0.82\,\text{m s}^{-1}$
   - **D** $8.2 \times 10^{-2}\,\text{cm s}^{-1}$

3. An object is rotating at 1200 Hz at a speed of $2.5 \times 10^3\,\text{m s}^{-1}$. What is its acceleration?
   - **A** $1.88 \times 10^7\,\text{m s}^{-2}$
   - **B** $1.88 \times 10^6\,\text{cm s}^{-2}$
   - **C** $1.88 \times 10^5\,\text{m s}^{-2}$
   - **D** $1.88 \times 10^3\,\text{m s}^{-2}$

4. A rollercoaster car turns right on a horizontal track at $10\,\text{m s}^{-1}$. At this point, the track has a radius of curvature of 25 m. What is the magnitude and direction of the force applied by the seat on a 50 kg person sitting in the car?
   - **A** 290 N, right
   - **B** 290 N, vertically up
   - **C** 690 N, vertically up
   - **D** 690 N, vertically down

5. A car travelling at $28\,\text{km h}^{-1}$ encounters a speed hump. At this speed, the car just leaves the road. What was the radius of curvature of the speed hump?
   - **A** 80 m
   - **B** 16.6 m
   - **C** 6.2 m
   - **D** 2.9 m

Questions 6–8 relate to the following information.
A 1200 kg car travelling at $6\,\text{m s}^{-1}$ rounds a turn of 30 m radius.

6. What is the centripetal force on the car?
   - **A** 48 N
   - **B** 147 N
   - **C** 240 N
   - **D** 1440 N

7. If the car rounds the same turn at $12\,\text{m s}^{-1}$, the required centripetal force is:
   - **A** halved.
   - **B** doubled.
   - **C** the same.
   - **D** quadrupled.

8. The maximum centripetal force that friction can provide the car on a rainy day is 8000 N. What is the highest velocity at which the car can round the turn?

A 14 m s^{-1}
B 77 m s^{-1}
C 200 m s^{-1}
D 40 km s^{-1}

9. Which of the following best defines the period of an object in circular motion?

A The time taken for the object to complete one full revolution
B The speed of the object at any point in the circular path
C The distance covered in one second
D The radius of the circular path

10. A 2 kg object is moving in a circle of radius 3 m at a speed of 6 m s^{-1}. What is its centripetal acceleration?

A 9 m s^{-2}
B 12 m s^{-2}
C 18 m s^{-2}
D 36 m s^{-2}

SHORT RESPONSE

11. A 65 kg motorcyclist rides a 350 kg motorcycle at a constant speed of 100 km h^{-1} around a horizontal bend of radius 85 m.

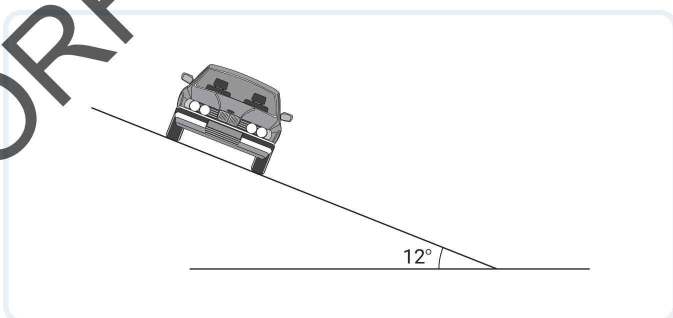
a **Calculate** the force applied on the motorcyclist.
b **Determine** the force applied by the motorcycle and rider on the road.

12. A woman swings a bucket of water at a constant velocity in a vertical circle 100 cm in radius

a If the water is not to spill, what is the minimum velocity the bucket can have?
b **Determine** the period of revolution.

CROSS-CHAPTER QUESTION

13. The diagram shows a 1.3 t car travelling in a horizontal circle of radius 265 m on a road that is banked at 12°.



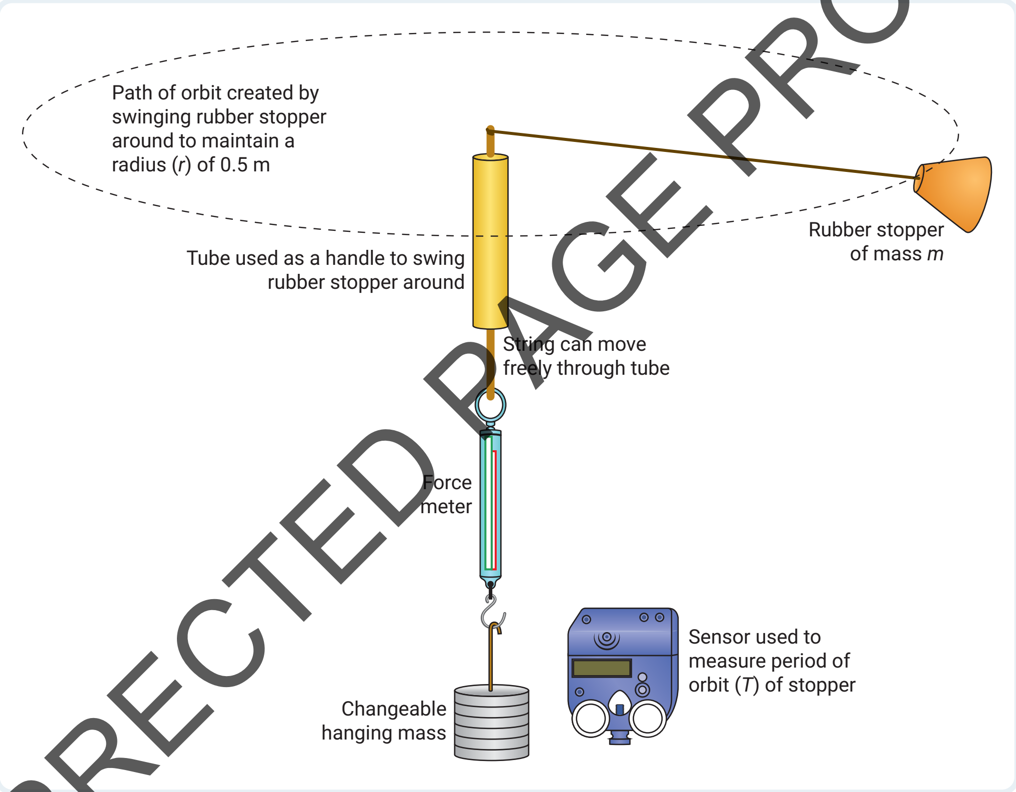
a Copy the diagram and draw force arrows to show all the forces acting on the car.
b **Calculate** the:
i acceleration of the car
ii speed of the car.

14. A hammer thrower can project the hammer at an angle of 42° and the hammer lands 86.5 m away. If the effective radius of the circular path of the hammer before being released was 2.17 m and the hammer has a mass of 1.65 kg, **determine** the force exerted by the hammer thrower in swinging the hammer.

DATA ANALYSIS

15. **Interpret evidence**

An experiment was set up as shown. In this set-up, the weight of the hanging mass being transmitted through the string provides the centripetal force required to keep the rubber stopper in a circular path.



The following data was collected with the objective of indirectly determining the mass of the rubber stopper using the laws of centripetal motion.

Trial number	Hanging mass (kg)	F_c reading on force meter (N)	Period of orbit T (s)	Velocity of rubber stopper v(m s^{-1}) $v = \frac{2\pi r}{T}$	Experimentally determined mass m of the stopper $F_c = F_{net} = \frac{mv^2}{r}$
1	0.100	0.981	1.41		
2	0.200	1.962	1.01		
3	0.300	2.943	0.81		
4	0.400	3.924	0.71		
5	0.500	4.905	0.66		

a **Draw** a conclusion about the accuracy and precision of the force meter. **Justify** your conclusion using quantitative aspects of the data.

b **Calculate** the values for the two last columns on the data table

c **Determine** the average experimentally determined mass m (kg) of the rubber stopper.

d **Determine** the percentage error of the experimental value obtained in part c if the mass of the stopper was accurately found to be 49.0 g using an electronic balance.

CHAPTER 5

Gravitational force and field strength

TK

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Describe the Law of Universal Gravitation.
- Solve problems using the magnitude of the gravitational force between two masses using $F = \frac{GMm}{r^2}$.
- Describe the concept of gravitational fields.
- Solve problems using the gravitational field strength at a distance from an object using $g = \frac{F}{m} = \frac{GM}{r^2}$.

SCIENCE AS A HUMAN ENDEAVOUR

- Explore the international collaboration required in the discovery of gravity waves and associated technologies, e.g. Laser Interferometer Gravitational Wave Observatory (LIGO).

Introduction

Newton's universal law of gravitation was developed from the prior observations and work of Brahe, Kepler, Copernicus and others. In this chapter, the nature of gravitational fields is explored, including the historical models of gravity and the development of Newton's universal law of gravitation. The inverse-squared nature of the gravitational force is demonstrated and the magnitude of the force is calculated for a range of scenarios.

Assessments

- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practicals

- Acceleration due to gravity using an inclined plane (online only resource)
- The inverse-square law (online only resource)
- The variation in gravitational force between objects due to mass and distance (online only resource)

Worksheets

- Name
- Name
- Nam

FPO

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ Force and acceleration are vector quantities with SI units of newtons (N) and $m\ s^{-2}$ respectively.
- ✓ The SI unit for mass is the kilogram (kg).
- ✓ The SI unit for length, distance or displacement is the metre (m).
- ✓ The average value of gravitational acceleration on the surface of Earth is $9.8\ m\ s^{-2}$.
- ✓ Gravitational force can be calculated using $F_g = mg$.
- ✓ Gravitational potential energy can be calculated using $E_p = mgh$.
- ✓ Kinetic energy can be calculated using $E_k = \frac{1}{2}mv^2$.
- ✓ Work done is a function of the force applied as well as the movement in the direction of the force: $W = Fs$.
- ✓ An inverse-square relationship between variables x and y means $y \propto \frac{1}{x^2}$.
- ✓ A non-contact force is a force that can act on an object without direct physical contact.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe the law of universal gravitation
- ✓ compare the scientific models of gravity postulated by Aristotle, Galileo, Brahe, Kepler, Copernicus and Newton
- ✓ solve problems involving the magnitude of the gravitational force between two masses using $F = \frac{GMm}{r^2}$
- ✓ describe the concept of gravitational fields
- ✓ solve problems involving the gravitational field strength at a distance from an object using $g = \frac{F}{m} = \frac{GM}{r^2}$
- ✓ describe the energy transformation occurring as an object moves in a gravitational field and perform the relevant calculations to quantify this
- ✓ apply the inverse-square law to interpret gravitational data.

Weblinks
The history of gravity
How to think about gravity
Misconceptions about gravity

5.1 The history of gravity

Why do things fall to the ground? To us, this seems obvious: gravity pulls objects down towards Earth. For Aristotle (384–322 BCE) and his contemporaries, the answer was also obvious: things fell to the ground because they were made of earth, and earth naturally moved towards Earth. They had a kind of heaviness or '**gravitas**' that enabled them to fall straight down.

gravitas the Aristotelian idea about the 'heaviness' of objects made of earth that allowed them to fall in straight lines towards Earth

empirical methods the central tenet of the scientific method whereby hypotheses are tested by observation and experimentation

The emergence of **empirical methods** to test Aristotle's ideas led to the development of kinematics (the relationship between measurements of distance and time) as a way of understanding motion. It was not until the 15th and 16th centuries that actual experiments on projectiles and the motion of falling objects were carried out. The most significant of these experiments were undertaken by Galileo Galilei (1564–1642), who showed that falling objects accelerated uniformly towards Earth. This was famously conducted using an inclined plane that decreased the acceleration due to gravity to just a fraction of the vertical acceleration (Figure 5.1.1).

From the mid-17th century, Isaac Newton (1643–1727) united the work in gravitational acceleration of those scientists who preceded him, including Nicolaus Copernicus (1473–1543) and Johannes Kepler (1571–1630). By 1687, Newton had developed a description of gravitational force to involve a relationship between force, mass and distance that obeyed an **inverse-square law**.

FIGURE 5.1.1 Galileo Galilei used the inclined plane to experimentally determine a value for acceleration due to gravity, '*g*'.

inverse-square law a law that describes a relationship in which the dependent variable is proportional to the square of the inverse of the independent variable, $y \propto \frac{1}{x^2}$ (e.g. $F = \frac{GMm}{r^2}$)

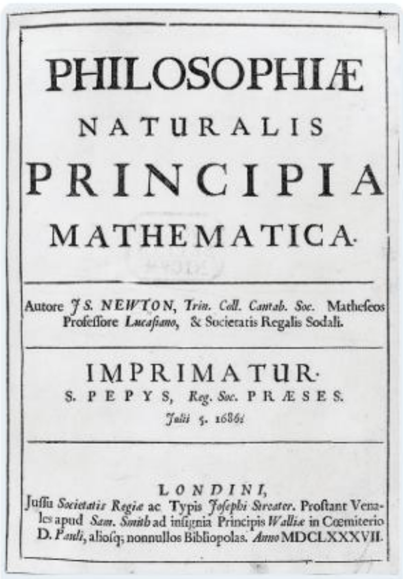
This law was 'universal' because it incorporated all motion and could be applied to Earth as well as across the universe. Newton's understanding of gravity was built on the impressive measurements of Tycho Brahe and the mathematical interpretation of these data by Johannes Kepler. Earlier work by Nicolaus Copernicus, itself indebted to the accuracy of Muslim astronomers such as Muhammad al-Battani (850–929) and Galileo on the motion of the planets around the Sun, as well as data from the Royal Observatory at Greenwich contributed to Newton's confidence in the universality of his gravitational theory.

Weblink
Newton's law of universal gGravitation

Newton's law of universal gravitation remained undisputed until Albert Einstein (1879–1955) made significant modifications in his 1915 article on the general theory of relativity. These changes, and the expansion in high-quality Earth- and space-based astronomical observations during the past 100 years have enhanced our knowledge of the universe immensely. Current theories of dark matter and dark energy are significantly based on our developing understanding of gravity.

Edmund Halley (1656–1742), Robert Hooke (1635–1703) and Newton all recognised that the elliptical orbits of planets, first described by Kepler, could be explained by a force that depended on the object's distance from the Sun. Newton wrote to Halley: 'It is now established that this force is *gravitas*, and therefore we shall call it gravitas from now on.' Newton used the Aristotelian idea of 'heaviness' to describe what we now refer to in English as gravity.

In 1687, Newton published *Philosophiae Naturalis Principia Mathematica* in which he described the law of universal gravitation **(Figure 5.1.2)**.



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostant Venales apud Sam. Smith ad insignia Principis Walliæ in Cœmiterio D. Pauli, aliosq; nonnullos Bibliopolas. Anno MDCLXXXVII.

FIGURE 5.1.2 *Philosophiae Naturalis Principia Mathematica* is a book by Isaac Newton in which he discusses the laws of motion and universal gravitation.

KEY FORMULA

Law of universal gravitation

$$F = \frac{GMm}{r^2}$$

where:
F = gravitational force (N)
G = Newtonian constant of gravitation (6.67×10^{-11} N m^2 kg^{-2})
M = mass of object 1 (kg)
m = mass of object 2 (kg)
r = radius or distance between the objects (m)

LEARNING CHECK 5.1

DESCRIBING

FORMULA AND DATA BOOK

1 State a contribution to the understanding of gravitational acceleration by each of these scientists.
   a Aristotle
   b Einstein
   c Galileo
   d Newton
2 **Identify** the term that Aristotle used to describe heavy objects falling towards Earth.
3 State two examples of inverse-squared relationships in physics. You may refer to the *Formula and Data Book*.
4 **Describe** the key difference between the scientific methods applied by Aristotle and Galileo.
5 **Determine** the force of gravitational attraction between a mass of 75 kg and Earth 5.97×10^{24} kg at the surface of Earth ($r = 6.38 \times 10^6$ m).

APPLYING

6 **Explain** why the term 'universal' is applicable to Newton's law of gravitation.

ANALYSING

7 **Predict** the net difference (increases, decreases, remains the same) for the value of the gravitational force, F, when the:
   a mass, M, increases
   b radius, r, increases.

REFLECTING

8 **Construct** a historical timeline for the development of our understanding of gravity, including the work of Aristotle, Copernicus, Galileo, Brahe, Kepler and Newton.
9 **Calculate** the gravitational force between two masses of 50 kg at distances of 10 m and 20 m to verify the inverse-squared relationship between force and distance.

5.2 Gravitational potential energy

potential energy energy stored in a system due to the interaction of components in the system via forces; energy stored in a field; gives a system the ability to do work (measured in joules (J))

work energy transferred due to the action of a force: $W = Fs$ (measured in joules (J))

Weblink
Gravitational potential energy

KEY FORMULA

Work

$W = Fs$

where:
W = work (J)
F = force (N)
s = displacement (m)

Energy may be classified in two distinct types: kinetic energy and **potential energy**. Energy is transferred when a force acts over a distance. When a force, F, acts on an object and moves it through some displacement, s, in the direction of the force, **work** is done.

Both force and displacement are vectors; however, work is done only when the force and the component of the displacement are parallel to each other.

When work is done on an object, its kinetic energy changes. When the component of the displacement is in the same direction as the force, the work done causes the kinetic energy of the object to increase. At the same time, the potential energy of the system must decrease so that energy is conserved. Conversely, when the component of the displacement is in the

WORKED EXAMPLE 5.2.1

Determine the work done when a force of 100 N is applied against gravity to raise an object 1.50 m.

ANSWER

1 **Determine the formula.**
$W = Fs$

2 **Substitute the known values.**
$W = 100\text{ N} \times 1.50\text{ m}$

3 **Calculate the answer.**
$W = 150\text{ J}$

opposite direction to the force, kinetic energy decreases and the potential energy increases. Thus, the change in kinetic energy is the opposite of the change in potential energy:

$$\Delta E_k = -\Delta E_p$$

KEY FORMULA

$$\Delta E_k = -\Delta E_p$$

When doing work within a field, the change in kinetic energy is the opposite of the change in potential energy.

Remember that potential energy belongs to a system of interacting objects. It is not meaningful to refer to the potential energy of a single object.

Gravitational potential energy is due to the interaction of objects via their **gravitational fields**.

KEY FORMULA

$$g = \frac{GM}{r^2}$$

where:
g = gravitational field strength (m s^{-2} or N kg^{-1})
G = Newtonian constant of gravitation (6.67×10^{-11} N m^2 kg^{-2})
M = mass of object (kg)
r = radius or distance between the objects (m)

It is the gravitational field that mediates or exerts the force on one object due to the mass of another. Hence, we say that the gravitational field does work when an object falls in Earth's gravitational field. In this case, the work is done by the **field** on the object because the object moves in the direction of the field. The kinetic energy increases and the potential energy decreases.

Consider a pencil allowed to fall in Earth's gravitational field, as in **Figure 5.2.1a**. If we define our system as the pencil and Earth, the gravitational field of Earth does work on the pencil, increasing its kinetic energy. The potential energy of the Earth–pencil system has decreased. If we then pick up the pencil and lift it through some height, we must apply a

gravitational potential energy the potential energy associated with the interaction of objects via the gravitational force; the potential energy is stored in the gravitational field

gravitational field the field that mediates the gravitational force between all objects with mass; the field surrounding all objects with mass: $g = \frac{GM}{r^2}$

field the means by which action-at-a-distance forces are exerted

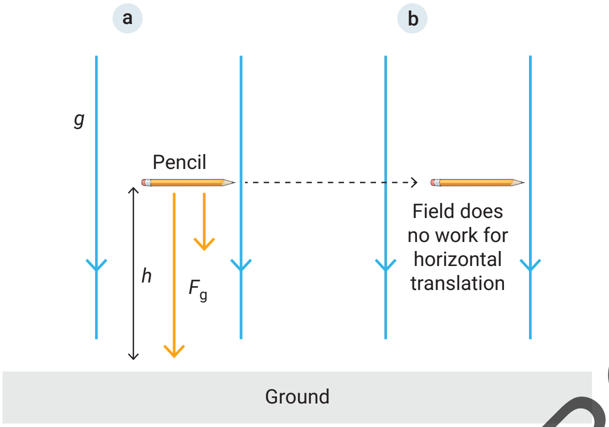


FIGURE 5.2.1 (a) When we lift a pencil at constant speed, we do work against the gravitational field: $W = Fs = F_{applied}h$. Work is done on the field and the kinetic energy is unchanged; the potential energy of the pencil–Earth system is increased. When the pencil falls through a distance h, the field does work $= F_g h$ on the pencil, increasing its kinetic energy. (b) When we move the pencil horizontally, no work is done on or by the field and there is no change in potential energy.

force in the direction opposite to the gravitational field (Figure 5.2.1a). We do work equal to Fs on the pencil, where F is the force we apply and s is the distance through which the pencil is moved.

If the pencil begins and ends at rest, there is no change in the pencil's kinetic energy, yet we have done work, so energy must have been transferred. The field was also doing work at the same time. In this case, the work done by the field was negative. The total work done *on the pencil* is zero. The total work done *on the Earth–pencil system* is the amount of work that we have done, and is equal to the change in potential energy of the Earth–pencil system. We say that this work was done *on* the field. So the energy transferred by the application of the force appears as an increase in the potential energy of the system.

When you hold the pencil stationary, both you and the gravitational field are exerting a force on it. However, the displacement is zero, so no work is done by either force. This is also the case when the displacement is perpendicular to the field; that is, it is moved in the horizontal direction. In this case, there is no component of displacement in the direction of the field, so the work done by or on the field is zero (Figure 5.2.1b).

The gravitational potential energy belongs to the *system*, which is both the object creating the field *and* the object experiencing a force due to the field. But, you might wonder, where is the energy stored? A pencil does not contain gravitational potential energy. Gravitational potential energy is *not* stored in the object; rather, it is stored in the field.

The field is able to do work because it exerts a force. The energy is distributed throughout all space where the field exists. The energy in a given volume (the energy density) depends on the field strength. In field theory, we model the action-at-a-distance forces, including gravity, as being mediated by a field. The field applies a force to objects in the field and because the field is able to do work, we say that the potential energy of the system is stored in the field.

The zero of gravitational potential energy

To be able to say how much potential energy a system has, we need to be able to define a zero energy position or configuration for the system. Note that we are again talking about *systems*, not isolated objects.

Both kinetic and potential energy cannot be defined for an isolated object. An object only has potential energy because a force is exerted on it, and the force must have some agent. Kinetic energy must be measured against some reference frame.

The potential energy of an object is always dependent on other objects, which generate the field. Even kinetic energy is not truly the property of a single object because it is due to motion, which is always relative to other objects in a frame of reference.

Choosing a zero for the potential energy of the Earth–pencil system may seem obvious. If we take the zero as being when the pencil is on the ground, then the potential energy of the system when the pencil is at any height, h, above the ground is simply $mg\Delta h = mg(h - 0) = mgh$.

The work done must be equal and opposite to the work done by the gravitational field if the pencil is to begin and end at rest.

KEY FORMULA

$$W = \Delta E_p = F_g s$$

$$\Delta E_p = mg\Delta h$$

where:
W = work (J)
ΔE_p = change in potential energy (J)
F_g = force due to gravity (N)
s = displacement (m)
g = acceleration due to gravity (9.80 m s^{-2})
Δh = change in height (m)

WORKED EXAMPLE 5.2.2

Determine the work done in increasing the potential energy of a 50.0 kg object by lifting it 1.20 m in Earth's gravitational field. Let $g = 9.80$ m s^{-2}.

ANSWER

1 **Determine the formula.**
$W = Fs$
$W = mgh$

2 **Substitute the known values.**
$W = 50.0 \text{ kg} \times 9.80 \text{ m s}^{-2} \times 1.20 \text{ m}$

3 **Calculate the answer.**
$W = 588$ J

This is a useful working definition when considering forces and motion close to Earth's surface. However, be careful to define exactly what you mean by the surface or ground level, as this may vary according to the situation. It also means that the potential energy of any Earth–object system becomes negative whenever the object falls below the defined zero level. There is nothing wrong with a negative potential energy: the negative sign simply means that the potential energy at the end is less than the potential energy at the beginning. As we only ever measure changes in potential energy, these changes can be positive or negative.

This 'ground level' definition for zero potential energy is not applicable to other planets or to the behaviour of objects that move a long way above the surface of Earth. The simplest way to define a meaningful zero value that is not based on any single particular object or position is to take the zero as being when all objects in a system are infinitely separated. Consider a system of massive objects very far apart from each other. When all the objects in the system are infinitely separated, the forces acting on them are zero and the potential energy of this configuration is defined as zero. If the objects are not moving, there is no kinetic energy, so the total energy of the system is zero.

KEY FORMULA

Law of conservation of energy

The law of conservation of energy means that no change occurs to the total energy in a system:

$$\Delta E_{total} = 0$$

$$\Delta E_k + \Delta E_p = 0$$

$$\Delta E_k = -\Delta E_p$$

where:
ΔE = change in energy (J)

KEY FORMULA

Kinetic energy

$$\Delta E_k = \frac{1}{2}mv^2$$

where:
m = mass (kg)
v = velocity (m s^{-1})

The gravitational force is always attractive. Any change from this zero configuration lowers the potential energy of the system to a negative value. The gravitational field due to each object does work on the other objects, bringing them closer together. The work done by the field decreases the potential energy of the system, as it attracts the objects closer together. This means that the kinetic energy must increase in accordance with the conservation of energy. As the objects accelerate closer together, their kinetic energy increases.

When an object moves in the direction of the gravitational field, the gravitational field does positive work – the kinetic energy of the object increases while the potential energy of the system decreases.

When an object moves against the gravitational field in an isolated system (where no other force is exerted on it), the gravitational field does negative work and the kinetic energy of the object decreases, while the potential energy of the system increases.

In an open system, work can be done on the objects in the system by an external agent; for example, lifting a pencil in the Earth–pencil system. In this case, if an object is moved against the field, the potential energy of the system again increases. These energy changes in a gravitational field are summarised in Table 5.2.1.

TABLE 5.2.1 A summary of energy changes in a gravitational field

System	Object moves	Work is done	Potential energy	Kinetic energy
Closed	With the field	By the field	Decreases	Increases
Closed	Against the field	On the field	Increases	Decreases
Open	With the field	By external agent	Decreases	Increases
Open	Against the field	By external agent	Increases	Either

Note that when work is done by an external agent to move an object in a field, the field may still do work. The work done by the field is positive if the object moves with the field, and negative if it moves against the field.

WORKED EXAMPLE 5.2.3

A 7.0 kg school bag is lifted onto a shelf 1.25 m above the ground.

a How much work was done on the bag?
b By how much did the gravitational potential energy of the bag change?
c The bag fell off the shelf to the floor. With what kinetic energy would the bag land on the ground?

ANSWERS

a **1 Determine the formula.**

$W = Fs$

$W = mg \times s$

2 Substitute the known values.

$W = 7.0 \text{ kg} \times 9.80 \text{ m s}^{-2} \times 1.25 \text{ m}$

3 Calculate the answer.

$W = 85.75 \text{ J} = 86 \text{ J}$

b Determine the potential energy.

$E_p = 86$ J

c Determine the kinetic energy.

$E_k = 86$ J

LEARNING CHECK 5.2

DESCRIBING

1 Write the law for the conservation of energy in terms of energy changes.

2 As an object falls in a gravitational field, potential energy is reduced. **Explain** where the energy goes.

3 **Define:**

a gravitational potential energy

b kinetic energy

c potential energy.

4 The gravitational potential energy in the field does not change as an object near Earth moves horizontally. **Explain**.

5 **Explain** why a falling object is having work done on it by the gravitational field.

APPLYING

6 How much work is done in raising a 200 kg mass through a vertical height of 30 m?

7 The gravitational potential energy associated with a stationary object is 2.352×10^3 J.

a What is its kinetic energy?

b **Determine** the height of the object above the ground (zero) if it has a mass of 60 kg.

8 a How much work is done by the gravitational field when a 60 kg diver falls through a vertical height of 3.0 m?

b Using the law of the conservation of energy law, **determine** the velocity of the diver as they enter the water.

9 A 400 kg rocket is launched from ground level. When it is at an altitude of 100 m its vertical velocity is 50 m s^{-1}.

a What is the kinetic energy of the rocket when it is at 100 m altitude?

b How much work was done on the rocket to change its gravitational potential energy?

c What is the minimum work done on the rocket?

10 What is the minimum work that must be done to move a 1500 kg car up a 300 m high hill?

ANALYSING

11 **Explain** why the gravitational potential energy of an asteroid is small compared to Earth's gravitational potential energy.

12 400 J of work is done on a stationary 5.0 kg mass to raise it from a position 100 m above the ground.

a What is its new height above the ground?

b The mass is dropped from its new height. What is its velocity as it passes its original position?

c What is its velocity when it strikes the ground, assuming all other forces are negligible?

5.3 Gravitational fields

Weblink
Gravitational fields

Each fundamental force (gravitational, electromagnetic, strong nuclear and weak nuclear) can be described as acting via a field. These fundamental forces are all action-at-a-distance forces. They allow us to explain how one object can exert a force on a second object without being in contact with it.

The gravitational field model allows us to explain how objects can exert forces without being in contact. It also allows us to:

- predict the acceleration of an object in any gravitational field
- calculate the mass of an object from the observed force it exerts on another object
- calculate the mass of distant objects, such as planets, by observing their orbits about the Sun.

Measuring the acceleration of objects dropped on the surface of the Moon tells us about the mass of the Moon. The radius of the Moon can be measured from astronomical observations, and then combining the size and mass information tells us that the density of the Moon is very similar to that of Earth's crust. This information was important in the development of modern theories of the formation of the Moon. These theories state that the Moon was actually formed when a massive object collided with Earth, breaking off some material that re-formed in orbit, becoming our natural satellite, the Moon.

Gravity 'near Earth'

At Earth's surface, objects are subject to the force of attraction applied by the combined mass of Earth. Unless otherwise constrained, all objects fall from a height to the surface with an acceleration of $9.80\,m\,s^{-2}$. This is the effect of Earth's gravitational field on the masses. 'Near Earth' is an approximation that can be applied because the field lines in a local area are very nearly parallel to each other, striking the surface at right angles (**Figure 5.3.1**). Up to several kilometres – well above the tallest buildings – the field strength varies by very little. We say that there is **negligible** variation in field strength over any local region.

negligible any value or variation in a value that is too small to be taken into account

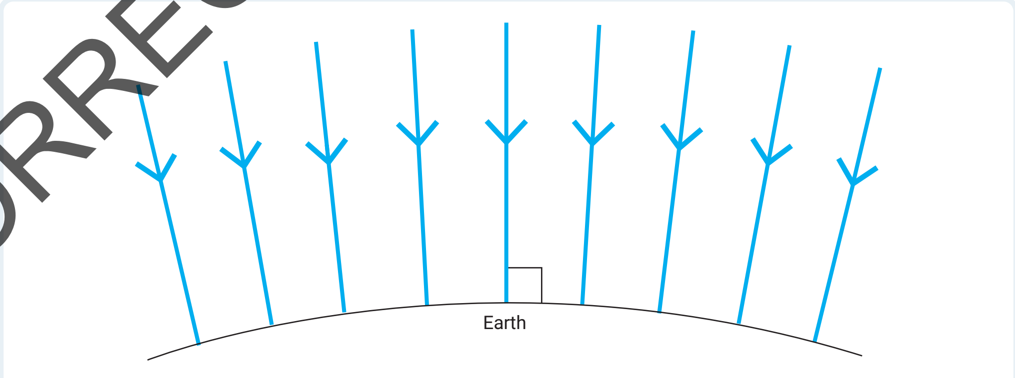


FIGURE 5.3.1 The gravitational field near Earth's surface is perpendicular to the surface and directed towards the centre of Earth.

Force of weight on a planet

All matter has mass; however, the weight associated with each mass varies. In fact, the force of weight on an object differs according to the gravitational field in which it lies. On the surface of Earth, an object's force weight is the product of its mass and the acceleration due to gravity at this radius ($g = 9.80\,\text{m s}^{-2}$): force weight, $F_w = mg$. On another astronomical body, such as the Moon ($g = 1.63\,\text{m s}^{-2}$) or Jupiter ($g = 23.1\,\text{m s}^{-2}$) the weights would be considerably different. **Table 5.3.1** lists the acceleration due to gravity on the Moon and various planets of the solar system.

The gravitational force of attraction has an infinite range – every object in the universe is attracted to every other object due to their masses. However, the gravitational force reduces quickly with distance because of its inverse-square Gravitational field strength relationship. We personally experience a noticeable gravitational force due to the significant combined mass of Earth and our mass. However, we do not experience a similar force of attraction from smaller objects, such as the person sitting next to us. This is simply because the force is negligible, due to our smaller masses.

KEY FORMULA

Force weight

Force weight is found using Newton's second law, $F = ma$.

$$F_w = mg$$

where:
F = force weight (N)
m = mass (kg)
g = acceleration due to gravity for the astronomical body, typically Earth, where $g = 9.80\,\text{m s}^{-2}$

TABLE 5.3.1 Acceleration due to gravity on the Moon and various planets of the solar system

Object	Mass (kg)	Radius (m)	Acceleration due to gravity (m s^{-2})
Sun	1.98×10^{30}	6.95×10^{8}	–
Moon	7.34×10^{22}	1.74×10^{6}	1.63
Mercury	3.28×10^{23}	2.57×10^{6}	3.70
Venus	4.83×10^{24}	6.31×10^{6}	8.89
Earth	5.97×10^{24}	6.37×10^{6}	9.80
Mars	6.37×10^{23}	3.43×10^{6}	3.69
Jupiter	1.90×10^{27}	7.18×10^{7}	23.10
Saturn	5.67×10^{26}	6.03×10^{7}	8.98
Uranus	8.80×10^{25}	2.67×10^{7}	8.71
Neptune	1.03×10^{26}	2.48×10^{7}	11.00

WORKED EXAMPLE 5.3.1

Calculate the force weight of a 60 kg mass on the surface of Mercury. Refer to Table 5.3.1 for the acceleration due to gravity on the surface of the planet.

ANSWER

1 **Determine the formula.**
According to Newton's second law, $F_w = mg$, where $g = 3.70$ m s^{-2}:
$F_w = mg$

2 **Substitute the known values.**
$F_w = 60 \text{ kg} \times 3.70 \text{ m s}^{-2}$

3 **Calculate the answer.**
$F_w = 222$ N

WORKED EXAMPLE 5.3.2

Determine the force weight of a 75 kg astronaut on the surface of the Moon. Refer to Table 5.3.1 for the acceleration due to gravity on the surface of the Moon.

ANSWER

1 **Determine the formula.**
According to Newton's second law, $F_w = mg$, where $g = 1.63$ m s^{-2}:
$F_w = mg$

2 **Substitute the known values.**
$F_w = 75 \text{ kg} \times 1.63 \text{ m s}^{-2}$

3 **Calculate the answer.**
$F_w = 122.25$ N

LEARNING CHECK 5.3

DESCRIBING

1 **Describe** the difference between force weight and mass.

2 List the planets of our solar system in order of their gravitational pull on their surface, from least to greatest (refer to Table 5.3.1).

3 **Explain** why the weight of an object may vary but its mass remains constant.

APPLYING

4 **Calculate** the force weight of a 1200 kg lunar lander on the surface of Earth and on the surface of the Moon (refer to Table 5.3.1).

5 **Calculate** the difference in the force weight of an 80 kg astronaut on the surfaces of Mars and Venus (refer to Table 5.3.1).

6 The Mars rover has a mass of 533 kg. **Determine** the force weight of the Mars rover on the surface of Mars (refer to Table 5.3.1).

7 The Cassini spacecraft had a mass of 2523 kg. **Determine** its force weight on the surface of Earth and on the surface of Neptune (refer to Table 5.3.1).

8 Titan (one of the moons of Saturn) has a mass of 1.35×10^{23} kg, a radius of 2.58×10^6 m and an acceleration due to gravity of 1.35 m s^{-2}. **Determine** the weight force of an 860 kg probe on the surface of Titan.

> **ANALYSING**
>
> **9** The gravitational field strength of a planet varies with distance above the surface. **Explain** why $g = 9.80$ m s^{-2} can be used as a value for 'near Earth's surface' for nearly all calculations.
>
> **10** Why is the gravitational field of a planet considered as 'acting at a distance' even though it may be applying forces to objects that are in contact?

5.4 Gravitational field strength

Every mass can be imagined as having a gravitational field, g, surrounding it. This field reaches to infinity; however, as distance increases, its strength decreases non-uniformly, in an inverse-square relationship. The gravitational field of a mass, M, exerts a force on another mass, m, as shown in **Figure 5.4.1**. Although the masses are not in contact, the force acts at a distance; we say that the force is mediated by the field.

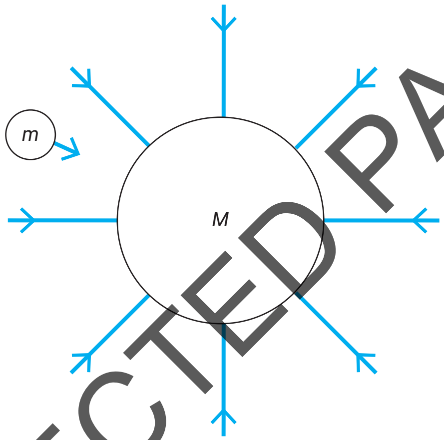


FIGURE 5.4.1 The gravitational field surrounding a mass mediates the gravitational force, accelerating the objects towards each other.

Although the gravitational field due to Earth's mass is non-uniform and radial in its nature, in everyday life on the surface of Earth the field experienced is very nearly uniformly vertical.

> **KEY FORMULA**
>
> **Kinetic energy**
>
> $$g = \frac{GM}{r^2}$$
>
> where:
> g = gravitational field (N kg^{-1} or m s^{-2})
> G = universal gravitational constant (6.67×10^{-11} N m^2 kg^{-2})
> M = mass of the planet (kg)
> r = radius of the planetary body (m)

WORKED EXAMPLE 5.4.1

Calculate the magnitude of the gravitational field at the surface of Earth.
Mass of Earth, $M = 5.97 \times 10^{24}$ kg
Radius of Earth, $r = 6.37 \times 10^{6}$ m

ANSWER

1 **Determine the formula.**

$$g = \frac{GM}{r^2}$$

2 **Substitute the known values.**

$$g = \frac{6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2} \times 5.97 \times 10^{24}\,\text{kg}}{(6.37 \times 10^{6}\,\text{m})^2}$$

3 **Calculate the answer.**

$g = 9.81$ m s^{-2}

WORKED EXAMPLE 5.4.2

Calculate the gravitational field strength on the surface of Mars.
Mass of Mars, $M = 6.37 \times 10^{23}$ kg
Radius of Mars, $r = 3.43 \times 10^{6}$ m

ANSWER

1 **Determine the formula.**

$$g = \frac{GM}{r^2}$$

2 **Substitute the known values.**

$$g = \frac{6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2} \times 6.37 \times 10^{23}\,\text{kg}}{(3.43 \times 10^{6}\,\text{m})^2}$$

3 **Calculate the answer.**

$g = 3.61$ m s^{-2}

WORKED EXAMPLE 5.4.3

Determine the gravitational field strength on the surface of the Moon.
Mass of Moon, $M = 7.34 \times 10^{22}$ kg
Radius of Moon, $r = 1.74 \times 10^{6}$ m

ANSWER

1 **Determine the formula.**

$$g = \frac{GM}{r^2}$$

2 **Substitute the known values.**

$$g = \frac{6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2} \times 7.34 \times 10^{22}\,\text{kg}}{(1.74 \times 10^{6}\,\text{m})^2}$$

3 **Calculate the answer.**

$g = 1.62$ m s^{-2}

KEY FORMULA

It follows from the definition of gravitational field, $g = \frac{F}{m}$, and Newton's universal law of gravitation, $F = \frac{GMm}{r^2}$, that the field of M at distance r is given by:

$$g = \frac{GM}{r^2}$$

Note that the field associated with M is independent of the mass m in the field. The gravitational field due to M exists whether or not another mass is put near it.

KEY CONCEPT

- Gravitational forces act at a distance.
- Gravitational force is modelled as acting at the centre of an object's mass (its centre of gravity).
- Gravitational fields are calculated as the force per unit mass, N kg^{-1}, directed towards the centre of mass of an object.
- Gravitational forces and gravitational fields are vector quantities (having both magnitude and direction) acting in the same direction.

When measuring the local value of the gravitational field, g, due to the mass, M, we use a small test mass, m. The force on the small test mass, m, due to the gravitational field of M causes m to accelerate towards it in accordance with Newton's second law of motion, $F = ma$. This acceleration is the gravitational field strength, g.

KEY FORMULA

$$g = \frac{F(\text{by } M \text{ acting on } m)}{m}$$

The gravitational field has the units of N kg^{-1}, which is equivalent to the units for acceleration, m s^{-2}.

$$1.0 \text{ N kg}^{-1} = 1.0 \text{ m s}^{-2}$$

Thus, if we can measure the acceleration of a mass, m, placed near the larger mass, M, we can find the value of the gravitational field. This means that all objects, independent of their mass, fall at the same rate of acceleration. At the surface of Earth this acceleration is approximately equal to 9.80 m s^{-2}, although this value does vary, depending on the height above sea level, among other factors.

Gravitational field near Earth's surface

At any point near Earth's surface, an object experiences the effect of its mass as an acceleration due to gravity and ultimately as a force weight. Newton was the first to realise that this same effect was the force that held the Moon in orbit around Earth and the planets in their orbits about the Sun. Newton's consideration of how Earth's gravitational field emanates through space led him to invent integration in the field of mathematics.

Because of Earth's nearly spherical shape, an object anywhere near Earth's surface is about the same distance from its centre of mass. In theory, this field applies a force to every object in the universe. However, the great distances within our own solar system and neighbouring galaxies means that, in reality, Earth's gravitational field only has an influence on objects within a few hundred million kilometres in any real sense.

The approximation that the gravitational field is constant is reasonable when close to the surface of Earth (or any other massive body or planet). However, the field decreases in accordance with the radius, following an inverse-square relationship, $F \propto \frac{1}{r^2}$. For an object at the height of the International Space Station (about 400 km above Earth), the gravitational field is approximately 90 per cent of that at Earth's surface. For satellites that may be several thousand kilometres above Earth's surface, the near-Earth approximation cannot be used because g is substantially less than 9.80 m s^{-2}.

The gravitational field strength around any mass is determined by the distance from the centre of mass and the mass itself. The force applied to any mass within the gravitational field is, in turn, determined by the strength of the gravitational field. Near Earth's surface, a 1.0 kg mass has a force of 9.80 N applied to it by Earth's gravitational field.

According to Newton's third law of equal and opposite reactions, Earth's mass and the mass of another object on its surface exert a gravitational force of equal magnitude on each other. These forces act in opposing directions, along a line joining their centres of mass.

KEY FORMULA

Gravitational field strength

$$g = \frac{F(\text{by mass of Earth on mass } m)}{m} \text{ N kg}^{-1}$$

KEY FORMULA

For any mass, *m*, the magnitude of the force applied to it by Earth's gravitational field (i.e. its weight) is given by:

$$F_w = mg$$

where:

F_w = force of weight (N)

m = mass (kg)

g = gravitational field strength (N kg^{-1} or m s^{-2}) The value of g near Earth's surface is 9.80 N kg^{-1}.

Newton's second law, $a = \frac{F_{net}}{m}$, can be applied to the gravitational force acting on any mass, m, near Earth's surface. Using $F_{net} = mg$, we get:

$$a = \frac{F_{net}}{m} = \frac{mg}{m} = g$$

Therefore, the acceleration of a mass free to move near Earth's surface is 9.80 m s^{-2}. As g is the acceleration due to gravity, the direction of the acceleration must be towards the centre of mass of Earth.

This gives a very simple way of measuring the gravitational field at any point in space. We simply need to ensure that no forces other than gravity are acting on a test mass, and then observe its acceleration.

KEY FORMULA

The gravitational field at any point is equal to the acceleration of a mass due to the gravitational force at that point:

$$g = \frac{F_{gravitational}}{m}$$

Close to Earth's surface, $g = 9.80\ \text{N kg}^{-1} = 9.80\ \text{m s}^{-2}$.

PRACTICAL ACTIVITY 5.4.1

EARTH'S GRAVITATIONAL FIELD STRENGTH

The period, T, of a pendulum is dependent on two variables: length of pendulum, ℓ, and the gravitational field strength, g, in which it swings. The relationship between the variables T, ℓ and g is given by $T = 2\pi\sqrt{\frac{\ell}{g}}$.

Research question

How can the strength of Earth's gravitational field by measured?

Aim

To measure the strength of Earth's gravitational field, g, near the surface by using a pendulum

Materials

- retort stand
- boss head and clamp
- length of string (approximately 1.0 m long)
- mass bob
- stopwatch or timing device
- ruler
- datalogging apparatus (optional)

Procedure

1 Set up the apparatus as shown in Figure 5.4.2.
2 Measure the effective length of the pendulum from the top of the string to the centre of the bob. Begin with a length of approximately 1.00 m.
3 Pull the bob back until it makes an angle of approximately 5° to the vertical.
4 Record the time taken for 10 complete oscillations of the pendulum, using the stopwatch (or datalogging device).

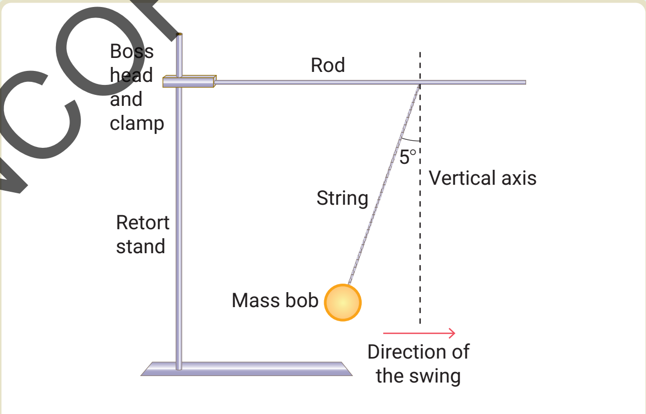


FIGURE 5.4.2 The pendulum apparatus to determine the acceleration due to gravity

KEY FORMULA

Period of a pendulum

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where:
T = period (s)
ℓ = length (m)
g = acceleration due to gravity (m s^{-2})

5 Change the length of the string to approximately 0.8 m and repeat steps 3 and 4.
6 Repeat again with string lengths of approximately 0.6, 0.4 and 0.2 m.
7 Calculate the period of the pendulum for each length.

Results

Record the data in a table similar to Table 5.4.1.

TABLE 5.4.1 Experimental pendulum data

Length of pendulum (ℓ) (m)	Time for 10 oscillations (s)	Period of pendulum (T) (s)

Two alternative methods may be used to determine the value of g.

Method 1: Algebraic – Using equations to model the pendulum

1 Each period and length measured may be substituted into the equation for period, to determine values for g:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T^2 = (2\pi)^2 \frac{\ell}{g}$$

$$g = \frac{4\pi^2 \ell}{T^2}$$

2 The multiple calculated values for g may then be used to determine an average.

Method 2: Graphical – using graphical representations to model the pendulum

1 Plot the relationship between ℓ and T. Note that it is non-linear.
2 Considering the equation, it is seen that $\ell \propto T^2$; hence, a graph of ℓ versus T^2 will exhibit a linear relationship. Be sure to plot ℓ (m) on the x axis, as it is the independent variable, and T^2(s^2), the dependent variable, on the y axis. Draw a line of best fit through the data points.

Using the equation for T:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T^2 = (2\pi)^2 \frac{\ell}{g}$$

$$= \frac{4\pi^2}{g}\ell$$

it can be seen that the value of the gradient of the graph of T^2 vs ℓ is $\frac{4\pi^2}{g}$.

3 Using any two points on the line of best fit, determine the gradient of the graph and hence calculate the value of g. This value of g may also be determined by finding the gradient of the relationship on a graphing calculator.

Analysis of results

1 Explain why the time for 10 oscillations was measured, and then divided by 10, to determine the period, T.
2 If the length of the pendulum was consistently overestimated, how might this affect the value of g obtained in each method of analysis?
3 Give your best estimate of g to the correct number of significant figures.

Evaluation

4 What are the sources of uncertainty in this experiment?

5 Suggest ways in which these uncertainties could be minimised.

Vertical motion under gravity

An object that is free to fall near Earth's surface experiences a gravitational force vertically, regardless of its direction of motion. This gravitational force results in the object accelerating vertically downwards with a value equivalent to g.

As the value of g at Earth's surface is approximately 9.80 m s^{-2} downwards, this gives:

$$a = \frac{v_y - u_y}{t} \text{ (the definition of acceleration)}$$

$$\Rightarrow v_y = u_y + at$$

so:

$$v_y = u_y + gt$$

Notice that the direction is indicated by the sign for each unit; that is, if upwards is taken as being the positive direction, then the value of g, as it is acting downwards, must be written as −9.80 m s^{-2}. Worked examples 5.4.4 and 5.4.5 demonstrate how the downwards direction may be treated as positive or negative. Whichever way this is defined at the outset of solving a problem, it needs to be consistently applied. Quantities with a downwards direction, such as acceleration and displacement, are typically assigned negative values. It is possible to assign the opposite – negative values to quantities with an upwards direction. If this is done, the answer will have the same physical meaning and value.

WORKED EXAMPLE 5.4.4

A parcel is dropped from a hot air balloon that is sitting momentarily at a height of 200 m above the ground.

a With what speed does the parcel hit the ground?

b How long does the parcel take to fall?

ANSWERS

a **1** **Identify known and required variables.**

$u = 0$ m s^{-1}, $a = 9.80$ m s^{-2}, $s = 200$ m, $v = ?$

Let the downwards direction be positive.

2 **Determine the formula.**

$v_y^2 = u_y^2 + 2as$

3 **Substitute the known values.**

$v_y^2 = 0^2 + 2 \times 9.80 \text{ m s}^{-2} \times 200 \text{ m}$

4 **Calculate the answer.**

$v_y^2 = 3920$

$v_y = 62.6$ m s^{-1} vertically down

b **1** **Identify known and required variables.**

$u = 0$ m s^{-1}, $a = 9.80$ m s^{-2}, $s = 200$ m, $t = ?$

Let the downwards direction be taken as positive.

2 **Determine the formula.**

$s = ut + \frac{1}{2}at^2$

3 **Substitute the known values.**

$200 = 0 \times t + \frac{1}{2} \times 9.80 \times t^2$

$\frac{200}{4.90} = t^2$

4 **Calculate the answer.**

$t = \sqrt{40.82}$

$= 6.39 \text{ s}$

It takes 6.39 s for the parcel to fall.

WORKED EXAMPLE 5.4.5

A ball is thrown vertically upwards at 20.0 m s^{-1}. Determine the maximum height it reaches.

ANSWER

1 **Identify known and required variables.**

$u = 20.0 \text{ m s}^{-1}$, $a = -9.80 \text{ m s}^{-2}$, $v = 0 \text{ m s}^{-1}$, $s = ?$

Let the downwards direction be taken as negative.

At the maximum height, the vertical velocity is 0 m s^{-1}.

2 **Determine the formula.**

$v_y{}^2 = u_y{}^2 + 2as$

3 **Substitute the known values.**

$0^2 = 20.0^2 + 2 \times -9.80 \text{ m s}^{-2} \times s$

4 **Calculate the answer.**

$0 = 400 + -19.6 \times s$

$s = \frac{400}{19.6}$

$= 20.4 \text{ m}$

The maximum height reached is 20.4 m.

WORKED EXAMPLE 5.4.6

A ball is thrown vertically upwards at 20.0 m s^{-1}. Determine total time of flight.

ANSWER

1 **Identify known and required variables.**

$u = 20.0 \text{ m s}^{-1}$, $a = -9.80 \text{ m s}^{-2}$, $v = 0 \text{ m s}^{-1}$, $t = ?$

Let the downwards direction be taken as negative.

At the maximum height, the vertical velocity is 0 m s^{-1}.

2 **Determine the formula.**

$v_y = u_y + at$

3 **Substitute the known values.**

$0 = 20.0 + -9.80 \times t$

4 Calculate the answer.

$$t = \frac{-20.0}{-9.80}$$

$$= 2.04\ \text{s}$$

The total time of flight is $2 \times 2.04\ \text{s} = 4.08\ \text{s}$. This is because the path is symmetrical and it takes the same time to travel up to the maximum height as it does to fall back down (ignoring air resistance).

LEARNING CHECK 5.4

DESCRIBING

1 What are the units of g?

2 **Recall** the kinematics formulas that relate the variables s, u, v, a and t.

3 **Identify** the purpose of using the notation v_y rather than simply v for vertical motion.

4 **Explain** why $a_y = g$ for free-falling objects near Earth's surface.

5 When finding the maximum height reached by a tennis ball hit vertically upwards, the value of v may be assigned as zero. Why is this possible?

APPLYING

6 A ball is dropped from a very tall building. How long does it take the ball to reach a velocity of $60.0\ \text{m s}^{-1}$?

7 A ball is thrown from a window with an initial downwards velocity of $2.40\ \text{m s}^{-1}$. It hits the ground after 1.40 s. **Determine** the height of the window above the ground.

8 With what minimum velocity must a student throw an object vertically upwards so that it reaches a point 5.5 m above its starting height?

9 Titan (one of the moons of Saturn) has a mass of 1.35×10^{23} kg and a radius of 2.58×10^{6} m. **Determine** the acceleration due to gravity on its surface.

ANALYSING

10 An object is thrown vertically upwards with an initial velocity of $30.0\ \text{m s}^{-1}$.

a What is the maximum height reached by the object?

b How long will it take for the object to fall back to its original position?

11 **Calculate** the gravitational acceleration, g, at a range of altitudes above Earth's surface and construct a table of values to demonstrate the relationship. Use 6.37×10^{6} m for the radius of Earth, 5.97×10^{24} kg for the mass of Earth and altitudes of 0.0 m (surface), 100 km, 1000 km and 10 000 km.

REFLECTING

12 Neptune's acceleration due to gravity ($11.0\ \text{m s}^{-2}$) is greater than that of Saturn ($8.98\ \text{m s}^{-2}$) even though Saturn is nearly five times as massive. **Explain** how this may be the case.

13 Given that the acceleration of objects near Earth's surface is directed vertically downwards, **explain** why a bullet shot horizontally from a height, h, hits the ground at the same time as one dropped from the same height.

5.5 Newton's law of universal gravitation and gravitational force

Weblink
Newton's Law of Universal Gravitation

centre of mass the average position of the mass in an object or group of objects; the point at which the gravitational force can be modelled as acting when the object is in a gravitational field

Newton determined that the gravitational force that keeps us on the ground and the planets in orbit about the Sun is a product of the masses of the objects and varies with an inverse-square relationship with the distance between the objects. That is, the force, F, is dependent on both masses M and m, as well as the inverse of the square of the distance, r, between them. The distance, r, is measured between the **centre of mass** of each object (Figure 5.5.1). This relationship is termed Newton's law of universal gravitation.

KEY FORMULA

$$F = \frac{GMm}{r^2}$$

where:
F = gravitational force (N)
G = Newtonian constant of gravitation (6.67×10^{-11} N m^2 kg^{-2})
M = mass of object 1 (kg)
m = mass of object 2 (kg)
r = radius or distance between the objects (m)

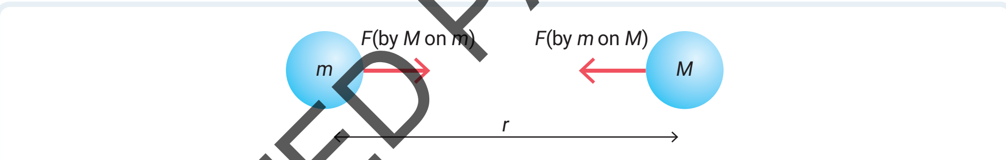


FIGURE 5.5.1 Gravitational force acts between bodies with mass with an equal and opposite force, in accordance with Newton's law of universal gravitation. Note that the forces may be drawn from the centre of mass also.

Although the force of gravity is an everyday phenomenon, it is actually relatively weak by comparison to the other fundamental forces (Table 5.5.1).

TABLE 5.5.1 A comparison of the four fundamental forces, in order of their relative magnitude

Type of fundamental force	Relative magnitude
Strong nuclear	$\times 10^{38}$
Electromagnetic	$\times 10^{36}$
Weak nuclear	$\times 10^{29}$
Gravitational	$\times 10^{0}$

Newton's third law and universal gravitation

Recall that the gravitational force acting on a mass, m, at a distance, r, from the centre of another mass, M, is given by Newton's law of universal gravitation.

$$F = \frac{GMm}{r^2}$$

The Cavendish experiment was the first experiment to measure the strength of the gravitational constant G.

We have seen that the field due to mass M acts on a mass m with a force F:

$$F(\text{on } m \text{ from } M) = \frac{GMm}{r^2}$$

But, what about the force applied by m on M? This force has the same magnitude:

$$F(\text{on } m \text{ from } M) = \frac{GMm}{r^2}$$

This is in accordance with Newton's third law of motion: for every action force there is an equal and opposite reaction force. That is, Earth is attracted towards you with the same magnitude of force, but opposite direction, as you are attracted towards Earth.

Weblink
The Cavendish experiment

KEY FORMULA

Gravitational acceleration

The gravitational acceleration of m is the field strength of M at distance r:

$$g_M = \frac{GM}{r^2}$$

The gravitational acceleration of M is the field strength of m at distance r:

$$g_m = \frac{Gm}{r^2}$$

Thus, m and M accelerate at different rates, even though the magnitude of the force applied to each is the same. For example, the gravitational field of Earth acts on a 0.1 kg (100 g) apple with a force of 0.98 N. The apple consequently accelerates at 9.80 m s^{-2}. The apple acts on Earth with the same force, 0.98 N, but due to Earth's mass of approximately 5.97×10^{24} kg, Earth accelerates at approximately 1.63×10^{-25} m s^{-2}. (Earth will not accelerate appreciably!)

The universal gravitational constant, *G*

In 1798, 71 years after Newton's death, Henry Cavendish (1731–1810) measured the value of the constant of proportionality, G, in Newton's law of universal gravitation. He placed massive lead balls near two much smaller balls at the end of a long rod, as in **Figure 5.5.2**. The forces applied to each of the smaller balls caused a rotation of the rod. This rotation was opposed by the torsion (twisting) in the metal suspension line. Cavendish used the amount by which the suspension line rotated in calculations to determine G. Cavendish's experimental method was extraordinarily accurate and this value of $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} remains applicable for all calculations of gravitational force.

from Cavendish,H. (1798), 'Experiments to determine the Density of the Earth' in McKenzie, A.S. ed. Scientific Memoirs Vol.9: The Laws of Gravitation, American Book Co. 1900, p.62

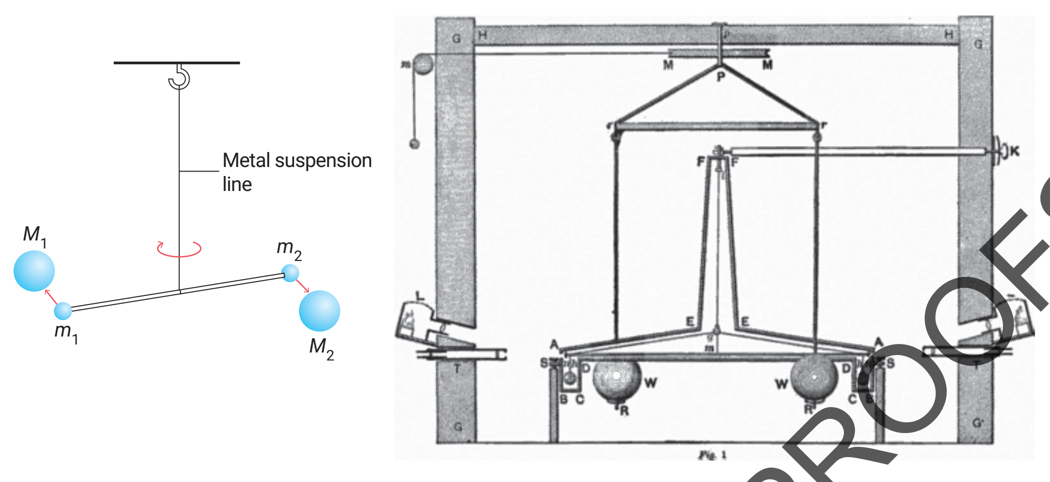


FIGURE 5.5.2 Cavendish's experimental apparatus. He measured the value of G to within about 1% of the currently accepted value of $G = 6.673\ 84 \times 10^{-11}$ N m^2 kg^{-2}.

WORKED EXAMPLE 5.5.1

Determine the gravitational force of attraction between Earth and the Sun.

Mass of Earth = 5.97×10^{24} kg

Mass of the Sun = 1.99×10^{30} kg

Radius of Earth's orbit around the Sun = 1.49×10^{11} m

ANSWER

1 Determine the formula.

$$F = \frac{GMm}{r^2}$$

2 Substitute the known values.

$$F = \frac{6.67 \times 10^{-11}\,\text{N}\,\text{kg}^{-2}\,\text{m}^2 \times 5.97 \times 10^{24}\,\text{kg} \times 1.99 \times 10^{30}\,\text{kg}}{(1.49 \times 10^{11})^2\,\text{m}^2}$$

3 Calculate the answer.

$F = 3.57 \times 10^{22}$ N

The gravitational force between various masses can be determined in a similar way. Table 5.5.3 contains a range of planetary data, including acceleration due to gravity values, for performing further calculations.

TABLE 5.5.3 Planetary data for the solar system

Object	Mass (kg)	Radius (m)	Mean orbital radius (m)	Mean orbital radius (AU)	Period of revolution (s)	Acceleration due to gravity (m s^{-2})
Sun	1.99×10^{30}	6.95×10^{8}	–	–	–	–
Moon	7.35×10^{22}	1.74×10^{6}	3.84×10^{8}	–	2.36×10^{6}	1.63
Mercury	3.28×10^{23}	2.57×10^{6}	5.79×10^{10}	0.387	7.60×10^{6}	3.70
Venus	4.83×10^{24}	6.31×10^{6}	1.08×10^{11}	0.723	1.94×10^{7}	8.89

Object	Mass (kg)	Radius (m)	Mean orbital radius (m)	Mean orbital radius (AU)	Period of revolution (s)	Acceleration due to gravity ($m\ s^{-2}$)
Earth	5.97×10^{24}	6.37×10^{6}	1.49×10^{11}	1.000	3.16×10^{7}	9.80
Mars	6.37×10^{23}	3.43×10^{6}	2.28×10^{11}	1.520	5.94×10^{7}	3.69
Jupiter	1.90×10^{27}	7.18×10^{7}	7.78×10^{11}	5.200	3.74×10^{8}	23.10
Saturn	5.67×10^{26}	6.03×10^{7}	1.43×10^{12}	9.540	9.30×10^{8}	8.98
Uranus	8.80×10^{25}	2.67×10^{7}	2.87×10^{12}	19.19	2.66×10^{9}	8.71
Neptune	1.03×10^{26}	2.48×10^{7}	4.50×10^{12}	30.07	5.20×10^{9}	11.00

KEY CONCEPT

Astronomical unit

The astronomical unit, AU, is a unit of length used to measure distances in the solar system. It is the average Earth–Sun distance, approximately 1.49×10^{11} m.

WORKED EXAMPLE 5.5.2

Earth and the Moon form a binary system – they are bound by their equal and opposite forces of gravitational attraction, orbiting around their common centre of mass. Determine the gravitational force of attraction between Earth and the Moon.

$\text{Mass}_E = 5.97 \times 10^{24}$ kg
$\text{Mass}_M = 7.35 \times 10^{22}$ kg
Radius of the Moon's orbit around Earth = 3.84×10^{8} m.

ANSWER

1 **Determine the formula.**

$$F = \frac{GMm}{r^2}$$

2 **Substitute the known values.**

$$F = \frac{6.67 \times 10^{-11}\,\text{N kg}^{-2}\,\text{m}^2 \times 5.97 \times 10^{24}\,\text{kg} \times 7.35 \times 10^{22}\,\text{kg}}{(3.84 \times 10^{8})^2\,\text{m}^2}$$

3 **Calculate the answer.**

$F = 1.98 \times 10^{20}$ N

The vector nature of gravitational force

The force exerted on an object within a gravitational field is applied in a direction towards the centre of mass of the object. Gravitational fields exist even for the smallest of masses. Therefore, we can say that a gravitational force is acting between you and the person nearest you. However, because this force is so small, it goes unnoticed. The very large mass of Earth results in a gravitational force on any object near it that cannot be ignored. The position of the centre of mass of Earth means that the gravitational force is exerted vertically downwards. The variations in the direction of this force caused by local influences such as mountains or dense bodies of rock beneath the surface are very small.

Weblink
Gravity force lab

Vector addition of forces

Newton's second law, $a = \frac{F_{net}}{m}$, allows for the fact that any number of forces may be acting on a mass, m, at any time. The symbol F_{net} signifies the resultant force (or net force) – the sum of all the forces acting.

To find the sum of the forces acting on an object, the magnitudes, or sizes, of the forces cannot simply be added. The vector nature of force means that the directions of the individual forces must be taken into account. When adding two force vectors acting on an object at the same time, the resultant force may be found either geometrically or by drawing a scale diagram.

WORKED EXAMPLE 5.5.3

An 8.0×10^3 kg spacecraft positioned 2.0×10^8 m from Earth and travelling directly to the Moon experiences a gravitational force from both Earth and the Moon, but in different directions. The net force acting on the spacecraft is the vector sum of the two gravitational forces.
Determine the net force acting on the spacecraft.

Earth–Moon distance = 3.84×10^8 m
$m_E = 5.97 \times 10^{24}$ kg
$m_M = 7.35 \times 10^{22}$ kg

a Determine the gravitational force and direction between Earth and the spacecraft.
b Determine the gravitational force and direction between the Moon and the spacecraft.
c Determine the net force acting on the spacecraft.

ANSWERS

a 1 Determine the formula.

$$F_E = \frac{GMm}{r^2}$$

2 Substitute known values.

$$F_E = \frac{6.67\times10^{-11}\times5.97\times10^{24}\times8.0\times10^3}{(2.0\times10^8)^2}$$

3 Calculate the answer.

= 79.64 N acting towards Earth

b 1 Determine the formula.

$$F_M = \frac{GMm}{r^2}$$

2 Substitute the known values.

$$F_M = \frac{6.67\times10^{-11}\times7.35\times10^{22}\times8.0\times10^3}{(1.84\times10^8)^2}$$

3 Calculate the answer.

F_M = 1.16 N acting towards the Moon

c 1 Determine the relationship between Earth and the Moon.

The net force acting on the spacecraft is the vector sum of the F_E and F_M.

2 Substitute the known values.

F_E = 79.64 N towards Earth, and F_M = 1.16 N towards the Moon
F_E = +79.64 N + (–1.16 N) towards Earth

3 Calculate the answer.

F_{net} = +78.48 N towards Earth

Gravitational equilibrium

Further to our calculation of net force, we can explore points of gravitational equilibrium between massive objects such as Earth and our natural satellite, the Moon (**Figure 5.5.4**).

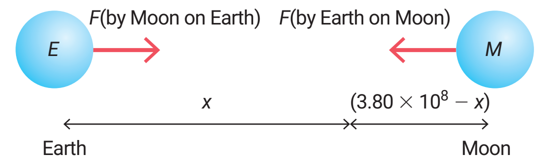


FIGURE 5.5.4 The Earth–Moon system. There is a point of gravitational equilibrium between Earth and the Moon where the forces are equal in magnitude and opposite in their direction.

WORKED EXAMPLE 5.5.4

Use Newton's law of universal gravitation to determine the point of gravitational equilibrium between Earth and the Moon.

Earth–Moon distance = 3.80×10^8 m
$m_E = 5.97 \times 10^{24}$ kg
$m_M = 7.35 \times 10^{22}$ kg

ANSWER

1 **Determine the formula.**
Let F_g of Earth acting on the Moon = F_g of the Moon acting on Earth

$$F_{g\,\text{Earth}} = F_{g\,\text{Moon}}$$

Distance from Earth to the point of gravitational equilibrium = x metres; therefore, the distance from the Moon to the point of gravitational equilibrium = $(3.80 \times 10^8 - x)$ metres:

$$\frac{Gm_E m_{\text{spacecraft}}}{x^2} = \frac{Gm_M m_{\text{spacecraft}}}{(3.80\times10^8 - x)^2}$$

2 **Cancel the mass of the craft and the gravitational constant (they appear on both sides of the equation)**

$$\frac{m_E}{x^2} = \frac{m_M}{(3.80\times10^8 - x)^2}$$

3 **Substitute the known values.**

$$\frac{5.97\times10^{24}}{x^2} = \frac{7.35\times10^{22}}{(3.80\times10^8 - x)^2}$$

4 **Cross-multiply the denominators.**

$$5.97\times10^{24}(3.80\times10^8 - x)^2 = 7.35\times10^{22}x^2$$

5 **Simplify the equation into the form of $ax^2 + bx + c = 0$.**

$$5.97\times10^{24}(1.444\times10^{17} - 7.60\times10^8 x + x^2) = 7.35\times10^{22}x^2$$

$$8.62068\times10^{41} - 4.5372\times10^{33}x + 5.97\times10^{24}x^2 = 7.35\times10^{22}x^2$$

$$8.62068\times10^{41} - 4.5372\times10^{33}x + 5.8965\times10^{24}x^2 = 0$$

6 **Solve the equation for both solutions (use a graphics calculator equation function; degree 2 polynomial).**
$x = 3.42 \times 10^8$ or $x = 4.27 \times 10^8$ m

7 **State the valid solution, including the direction from the source**
Reject $x = 4.27 \times 10^8$ m because this point of gravitational equilibrium is on the far side of the Moon (not between the two bodies). Therefore, the gravitational point of equilibrium is 3.42×10^8 m from Earth.

WORKED EXAMPLE 5.5.5

Using Newton's law of universal gravitation, determine the point of equilibrium between Jupiter and its moon Io. The distance between Jupiter and Io, the mass of Io and other values for Jupiter's Galilean moons are listed in Table 5.5.5.

TABLE 5.5.5 Table of data for Jupiter and its four Galilean moons

Object	Mean distance from Jupiter (m)	Mass (kg)	Orbital period around Jupiter (Earth days)	Orbital period around Jupiter (Earth seconds)	Mean diameter (km)
Jupiter	–	1.90×10^{27}	–	–	69 911
Io	4.22×10^{8}	8.93×10^{22}	1.769	5.58×10^{7}	1822
Europa	6.71×10^{8}	4.80×10^{22}	3.551	1.12×10^{8}	1561
Ganymede	1.070×10^{9}	1.48×10^{23}	7.155	2.26×10^{8}	2634
Callisto	1.883×10^{9}	1.08×10^{23}	16.689	5.27×10^{8}	2410

ANSWER

1 **Let F_g of Jupiter acting on Io = F_g of Io acting on Jupiter.**

$F_{g\,J} = F_{g\,Io}$

Distance from Jupiter to the point of gravitational equilibrium = x metres; therefore, the distance from the moon Io to the point of gravitational equilibrium = $(4.22 \times 10^8 - x)$ metres:

$$\frac{Gm_J m_{spacecraft}}{x^2} = \frac{Gm_{Io} m_{spacecraft}}{(4.22 \times 10^8 - x)^2}$$

2 **Cancel the mass of the craft and the gravitational constant (they appear on both sides of the equation).**

$$\frac{m_J}{x^2} = \frac{m_{Io}}{(4.22 \times 10^8 - x)^2}$$

3 **Substitute the known values.**

$m_J = 1.90 \times 10^{27}$ kg

$m_{Io} = 8.93 \times 10^{22}$ kg

$$\frac{1.90 \times 10^{27}}{x^2} = \frac{8.93 \times 10^{22}}{(4.22 \times 10^8 - x)^2}$$

4 **Cross-multiply the denominators.**

$$1.90 \times 10^{27}(4.22 \times 10^8 - x)^2 = 8.93 \times 10^{22} x^2$$

5 **Simplify the equation into the form of $ax^2 + bx + c = 0$.**

$$1.90 \times 10^{27}(1.781 \times 10^{17} - 8.44 \times 10^8 x + x^2) = 8.93 \times 10^{22} x^2$$

$$3.384 \times 10^{44} - 1.604 \times 10^{36} x + 1.90 \times 10^{27} x^2 = 8.93 \times 10^{22} x^2$$

$$3.384 \times 10^{44} - 1.604 \times 10^{36} x + 1.899 \times 10^{27} x^2 = 0$$

6 **Solve the equation for both solutions (use a graphics calculator equation function; degree 2 polynomial).**

$x = 4.35 \times 10^8$ or $x = 4.09 \times 10^8$

7 **State the valid solution, including the direction from the source.**

Reject $x = 4.35 \times 10^8$ m because this point of gravitational equilibrium is on the far side of Io (not between the two bodies). Therefore, the gravitational point of equilibrium is 4.09×10^8 m from Jupiter.

Other forces at a distance

Gravitational force is not the only force that acts at a distance. Other such forces are the electrostatic forces between charged particles, the magnetic force that is easily observed using magnets and compasses, and the strong and weak nuclear forces that act over very small distances within the nuclei of atoms. Without these two forces, atoms would not be stable, but they are not detectable over distances usually encountered in everyday life.

When forces act at a distance, the force is a result of the objects interacting with the surrounding field. It takes a finite time for the interaction to be transmitted from one object to the other; the effect is not instantaneous, as is sometimes thought. Field theory does not explain why interactions are not instantaneous. This limitation of field theory was one factor that led to the development of a different model of forces – the exchange-particle model.

LEARNING CHECK 5.5

DESCRIBING

1 **Contrast** gravitational field strength with gravitational force.

2 **Contrast** the formulas used for gravitational field and gravitational force.

3 Name the two variables that determine the acceleration due to gravity at the surface of a planet.

4 'Gravitational field does not depend on the mass, m, in the field.' **Explain** this statement.

5 **Explain** how Newton's third law applies to the force of gravitational attraction between any two objects.

6 The force of gravitational attraction and light intensity both exhibit the inverse-square law. **Explain** what this means.

APPLYING

7 Two masses, m and M, have a gravitational force of attraction, F, when they are a distance, r, apart. What is the relative magnitude of the force as a factor of F when this distance is increased to $4r$?

8 Using Newton's law of universal gravitation, **determine** the point of equilibrium between Earth and Venus. The distance between Earth and Venus is 4.10×10^{10} m. Refer to Table 5.5.3, for other planetary data values.

9 An astronaut on the Moon drops a ball from a height of 1.50 m. It takes 1.36 s for the ball to fall to the surface of the Moon. **Determine** the acceleration due to gravity on the surface of the Moon.

ANALYSING

10 Use the acceleration due to gravity on the surface of the Moon, 1.63 m s^{-2}, to determine the time taken for the hammer and feather from the famous Apollo 15 experiment to fall 1.2 m to the surface of the Moon when dropped.

11 At what distance from Earth's centre is the net gravitational field of Earth and the Moon zero? Take the distance between Earth and the Moon to be 3.84×10^5 km. ($m_E = 5.97 \times 10^{24}$ kg, $m_M = 7.35 \times 10^{22}$ kg)

REFLECTING

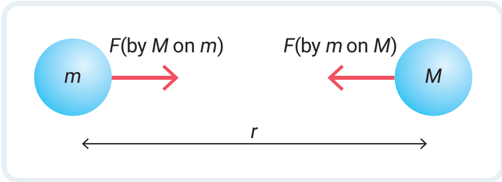
12 **Reflect** on the effect that a changing mass or distance has on gravitational force. Which variable has the greater impact?

13 **Define** 'centre of mass' and explain why it is necessary to consider it when calculating gravitational attraction between masses.

CHAPTER SUMMARY

Gravitational fields

- The region of space around a mass where another mass experiences a force of gravitational attraction is a gravitational field, g.
- Every point mass in the universe attracts every other point mass.



- The strength of the gravitational field is given by the formula:

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

- where G is the universal gravitational constant (6.67×10^{-11} N m^2 kg^{-2}), M is the mass of the planet (kg) and r is the radius (m) of the planetary body.
- Field lines are used to visually represent the direction and strength of a gravitational field.
- Field lines point towards the mass creating the field, indicating the direction of gravitational force a test mass would experience.
- The density of field lines indicates gravitational field strength.
- Gravitational fields store potential energy, which can be converted into kinetic energy as an object moves within the field, and represents the work the gravitational force can do on an object:

$$W = Fs$$

- where W is the work (J), F is the force (N) and s is the displacement (m)

Law of universal gravitation

- Newton's law of universal gravitation states that every point mass in the universe attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres:

$$F = \frac{GMm}{r^2}$$

CHAPTER EXAM

MULTIPLE CHOICE

1. Which of the following is correct for the gravitational potential energy of a 100 kg mass held 1.2 m above Earth's surface?
   A 1100 kg
   B 11 kg
   C 120 J
   D 1176 J

Questions 2 and 3 relate to the following information.
The gravitational force of attraction between two spheres (masses *M* and *m*) that are a distance *D* apart is found to be *F*.

2. If the distance between the spheres is increased to 2*D*, what is the new force?
   A F
   B $\frac{1}{2}F$
   C $\frac{1}{4}F$
   D $\frac{3}{4}F$

3. The distance between the spheres is decreased to $\frac{1}{2}D$ and the masses are doubled. What is the new force between the spheres?
   A $\frac{1}{4}F$
   B $2F$
   C $8F$
   D $16F$

4. An astronaut weighs 3200 N on a planet whose mass is the same as that of Earth but whose radius is half that of Earth. What is the astronaut's weight on Earth?
   A 300 N
   B 800 N
   C 1600 N
   D 3200 N

Questions 5 and 6 relate to the following information.
Two balls (A and B) of equal masses are placed a distance *D* apart and the gravitational force between them is found to be *F*. Another ball (C) of the same mass is placed midway between them.

5. What is the magnitude of the net gravitational force on C due to A and B?
   A $2F$
   B $4F$
   C F
   D 0

6. What is the magnitude of the net force on ball B from A and C?
   A $2F$
   B $3F$
   C $4F$
   D $5F$

7. The mass of Saturn is 5.11×10^{27} kg and its radius is 1.2×10^6 m. If Francesca's weight on Earth is 650 N, what would her weight on Saturn be?
   A $400 \leq W < 650$
   B $650 \leq W < 1000$
   C $1000 \leq W < 2000$
   D $2000 \leq W$

8. In what ways do g (gravitational field strength) and G (universal gravitational constant) change with increasing height above Earth's surface?
   A Both g and G decrease.
   B g decreases but G remains constant.
   C g remains constant but G decreases.
   D Both g and G remain constant.

9. A satellite orbits Earth at a height where the gravitational field strength is 6.0 N kg^{-1}. If Earth's mass is 5.97×10^{24} kg, what is the distance of the satellite from the centre of Earth?
   A 6.4×10^6 m
   B 7.8×10^6 m
   C 8.2×10^6 m
   D 1.0×10^7 m

10. Which of the following best describes Newton's law of universal gravitation?
   A All objects with mass attract each other with a force proportional to their masses and inversely proportional to the distance between their centres.
   B All objects with mass repel each other with a force proportional to their volumes and directly proportional to the square of the distance between them.
   C All objects attract each other with a force inversely proportional to their densities and the distance between their centres
   D All objects repel each other with a force proportional to their masses and inversely proportional to their separation distance.

SHORT RESPONSE

11. **Calculate** the gravitational force acting between two planets of mass 3.60×10^{25} kg and 6.29×10^{24} kg. The planets are at an average mean distance of 1.1 million kilometres.

12. Find the velocity (in m s^{-1}) that an artificial Earth satellite must have to pursue a circular orbit at an altitude of half an Earth radius.

13. Two planets, A and B, are separated by a distance of 1.0×10^7 m (centre to centre). Planet A has a mass of 6.0×10^{24} kg and Planet B has a mass of 6.0×10^{24} kg.

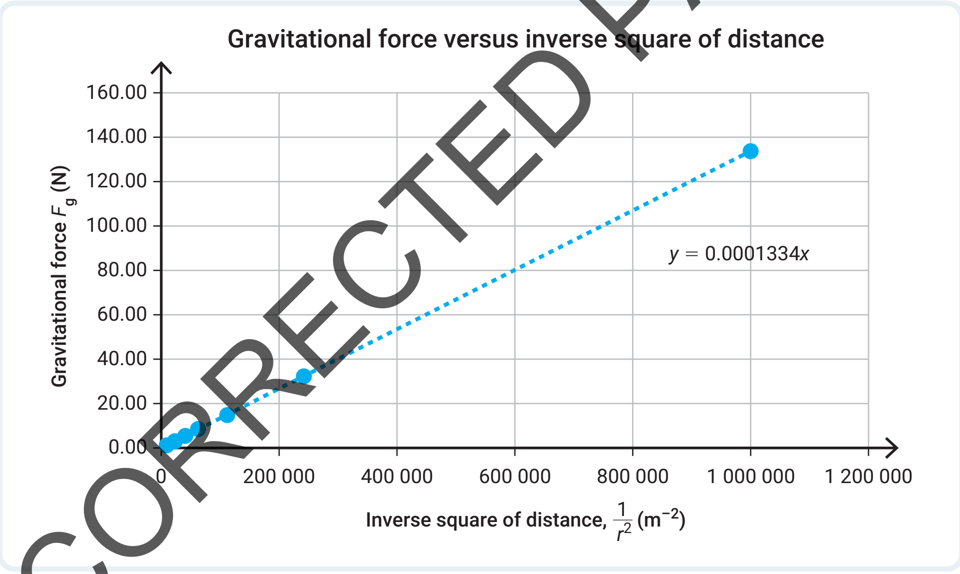
    Determine the position between the two planets where the gravitational forces exerted by the two planets on an object are equal. Express your answer as the distance from the centre of Planet A.

DATA ANALYSIS

14. Analyse data

The gravitational force between two objects was measured at various distances of separation. The raw data was processed and then presented in the following table and graph.

$\frac{1}{r^2}$ (m^{-2})	F_g (N)
1 000 000.00	133.40
250 000.00	33.35
111 111.11	14.82
62 500.00	8.34
40 000.00	5.34
27 777.78	3.71
20 408.16	2.72
15 625.00	2.08
12 345.68	1.65
10 000.00	1.33

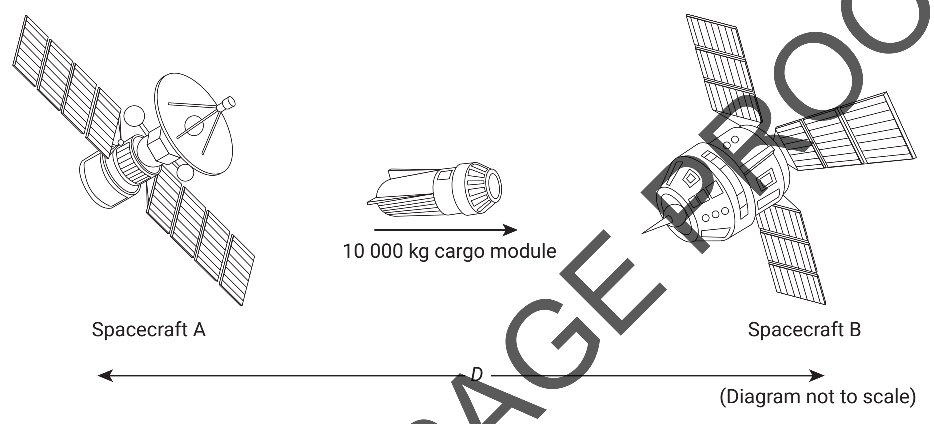


a If one of the objects under study had a mass of 1000 kg, use all of the data available to determine the mass of the other object.

b Considering the data set above and the relationship evident between F and r, sketch what the general shape of a graph of F versus r would look like.

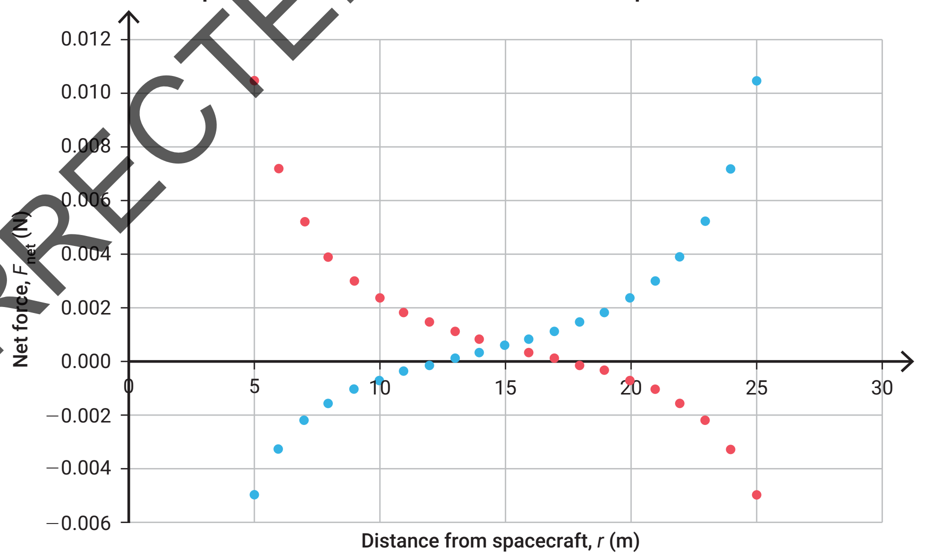
15. Analyse data

Two spacecraft, A and B, are separated by a distance, D, as shown below. A 10 000 kg cargo module is being transferred from Spacecraft A to Spacecraft B to replenish its supplies. So that its computer-controlled thrusters can react accordingly, the cargo module has a sensitive force meter on board that can measure the net gravitational force it experiences due to Spacecraft A and Spacecraft B. It is calibrated so that the positive direction for net force is towards Spacecraft A.



A graph of what the force meter measured is shown in Graph 1.

Graph 1: Net force versus distance from spacecraft



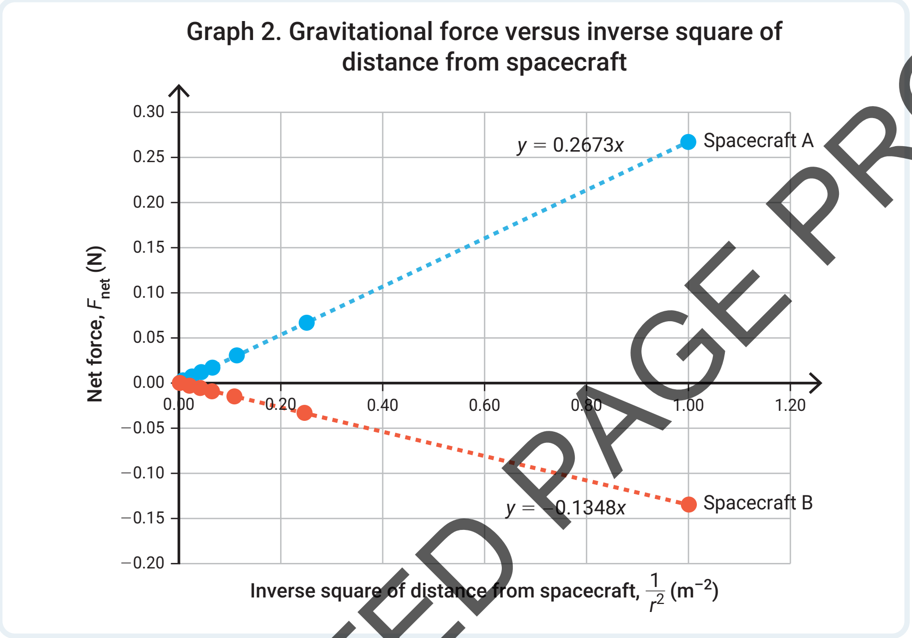
a **Deduce** which data set (red or blue) represents net force versus distance from Spacecraft A. **Justify** your response by referring to the data and using your knowledge of the law of gravitational attraction.

b **Deduce** which data set (black or green) represents the net force versus distance from Spacecraft B. **Justify** your response by referring to the data and using your knowledge of the law of gravitational attraction.

c **Deduce** the distance, D, between Spacecraft A and Spacecraft B.

d **Deduce** which spacecraft (A or B) has the greatest mass.

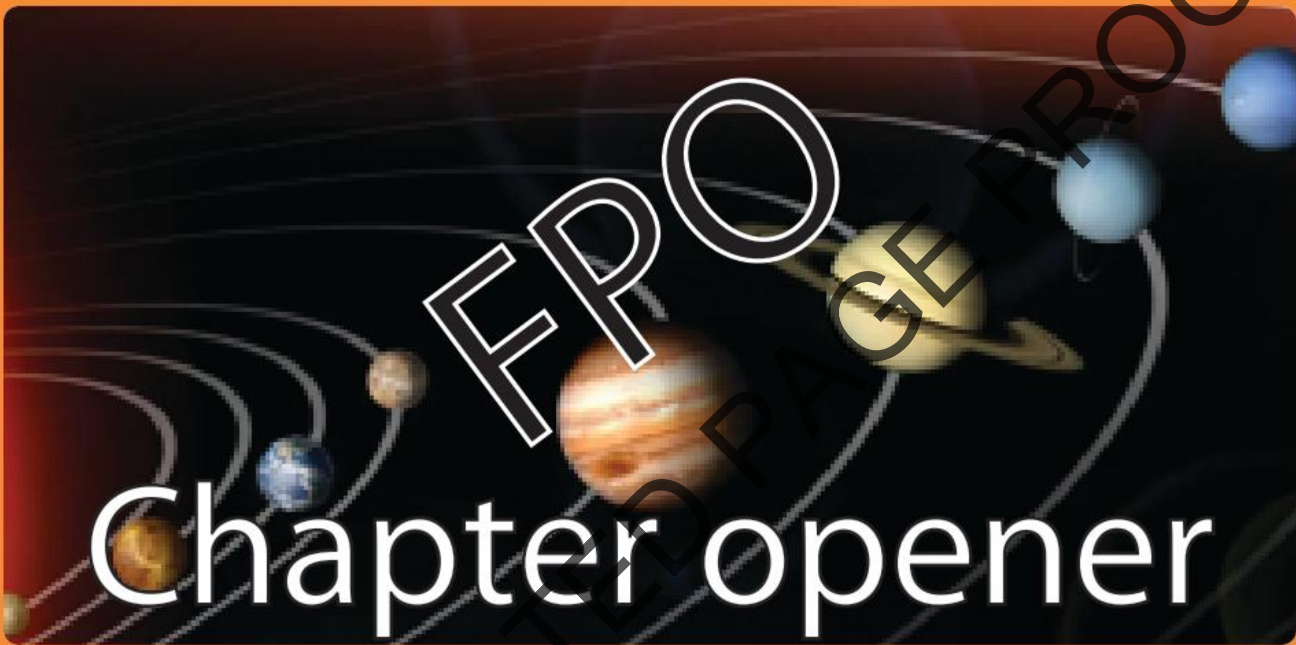
The data gathered from the force sensor was further processed to produce the following graph.



e **Determine** the masses of Spacecraft A and Spacecraft B to one significant figure, using the line equations in Graph 2 and $F = \frac{GMm}{r^2}$.

CHAPTER 6

Orbital motion



SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- State the three laws of planetary motion.
- Describe the relationship between the Law of Universal Gravitation and uniform circular motion and recognise this as the third law of planetary motion.
- Solve problems using the third law of planetary motion using $\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3} = \frac{4\pi^2}{GM}$.

SCIENCE AS A HUMAN ENDEAVOUR

- Appreciate how the accepted model of the solar system slowly shifted under the influence of carefully collected and analysed data.
- Explore the difficulties experienced by scientists who supported a heliocentric model of the solar system and the hindrances to the acceptance of their discoveries by society.

SCIENCE INQUIRY

- Investigate the relationship between orbital radius and mass for orbiting objects using a simulation.

Introduction

Orbital motion has dictated the revolutions of planets, comets and natural satellites for aeons. Humans have recently taken advantage of the relationships that guide this motion to place artificial satellites into orbit. In this chapter, the nature of orbits is explored, including Kepler's three laws of planetary motion as well as the relationship between centripetal force and gravitational force that allows satellites to orbit Earth.

Assessments

- Learning checks 5.1, 5.2, 5.
- Chapter exam

Practicals

- Investigating the magnification of a microscrope

FPO

Worksheets

- Name
- Name
- Name

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ Common astronomical bodies such as moons (satellites), planets, stars and galaxies have general features and arrangements.
- ✓ 1 year on Earth = approximately 365 days
- ✓ 1 day on Earth = approximately 24 hours
- ✓ The centripetal force acting on an object can be calculated from $F_c = \frac{mv^2}{r}$.
- ✓ The gravitational force between two objects with mass can be calculated from $F_g = \frac{GMm}{r^2}$
- ✓ The period (T) of a cyclic or oscillating phenomenon is the time it takes for a single cycle or oscillation.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ state the three laws of planetary motion
- ✓ describe the relationship between the law of universal gravitation and uniform circular motion and recognise this as the third law of planetary motion
- ✓ solve problems involving the third law of planetary motion using $\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3} = \frac{4\pi^2}{GM}$
- ✓ compare the historical models of planetary motion postulated by Ptolemy, Plato, Aristotle, Copernicus, Kepler and Newton
- ✓ contrast the geocentric model with the heliocentric model of the solar system
- ✓ perform calculations pertaining to the units used to quantify astronomical distances such as km, AU, Mpc and ly
- ✓ solve problems pertaining to satellite motion including geostationary and geosynchronous orbits using $F_c = \frac{mv^2}{r}$, $F_g = \frac{GMm}{r^2}$, $v_{orbit} = \sqrt{\frac{GM}{r}}$ and $v_{escape} = \sqrt{\frac{2GM}{r}}$.

6.1 Early models of planetary motion

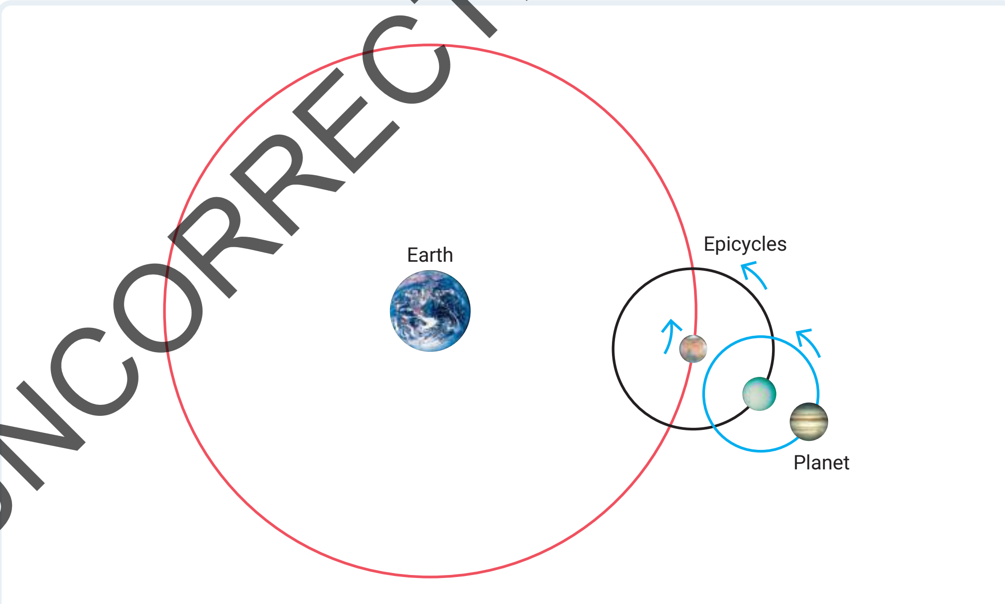
Models of planetary motion have been proposed, and revised, for centuries. The ancient Greek philosopher Plato had the heavenly bodies fixed in **concentric spheres** (Figure 6.1.1). The stars were fixed in one sphere while the Moon and Sun had different spheres to account for their different movements among the stars. The planets were observed to wander across the sky (the word 'planet' is Greek for 'wanderer'). This example of a model designed to explain natural observations is one of many developed over the centuries. As the precision of measurements, the number of points of data or the thinking changed, the models were improved. Claudius Ptolemy's (100–168 CE) later models retained the circles but added **epicycles** (circles on circles) (Figure 6.1.2). He added these to describe the retrograde motion observed in the motion of the planets in the sky. Better observations required better models. However, the models quickly became complex, with no explanation of how the epicycles were maintained.

concentric spheres spheres that share a common centre

epicycle a small circle whose centre is on the radius of larger circles; used by Ptolemy to describe the motion of planets in an early geocentric model

Royal Astronomical Society/Science Photo Library

FIGURE 6.1.1 The ancient Greek philosopher Plato thought the heavenly bodies were fixed in concentric spheres circling Earth.



TK

FIGURE 6.1.2 Epicycles, or circles on circles, were added to Plato's model of the geocentric universe to better describe the motion of planets at the time.

Weblink
Ptolemy's geocentric model of thes Solar system

geocentric model a superseded model of the solar system with the Sun, Moon and planets revolving about Earth at its centre

heliocentric model a current model of the solar system with the Sun (*Helios*) at its centre and all planets revolving about it; closely associated with the work of Copernicus and Galileo

TK

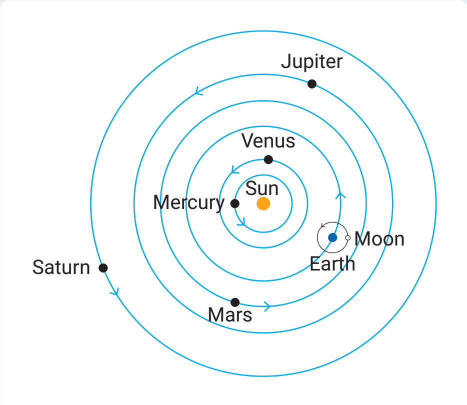


FIGURE 6.1.3 The Sun-centred heliocentric model of the universe, as determined by Nicolaus Copernicus

The scientific revolution transformed the key scientific ideas of the Aristotelian tradition. Aristotle's cosmological understandings positioned Earth in the centre of the known universe – the **geocentric model**. Ptolemy's subsequent model of planetary motion was also geocentric, predicting the positions of the Sun, Moon, planets and stars, but not always accurately.

Nicolaus Copernicus, Galileo Galilei, Johannes Kepler and Isaac Newton each determined an alternative explanation of the motions of heavenly bodies, placing the Sun at the centre of the cosmos in what was termed a **heliocentric model** (Figure 6.1.3).

KEY CONCEPT

Refinement of models and theories

Models and theories are contested and refined or replaced when new evidence challenges them, or when a new model or theory has greater explanatory power. This has certainly been the case for our model of the solar system and the universe, highlighted by the difficulties experienced by scientists, such as Galileo, who supported a heliocentric model of the solar system.

LEARNING CHECK 6.1

DESCRIBING

1 **Describe** one contribution to the understanding of our universe by:
   a Copernicus
   b Galileo
   c Kepler
   d Newton.
2 **Define** 'concentric'.
3 **Explain** the term 'epicycle'. Use a diagram to assist your explanation.

APPLYING

4 **Contrast** the geocentric model with the heliocentric model of the solar system.

ANALYSING

5 **Deduce** how the scientific method has enabled our understanding of the universe to change.

REFLECTING

6 **Explain** the process of how scientific knowledge develops and changes. Refer to the various models of the solar system to assist in your response.

6.2 Kepler's laws of planetary motion

Johannes Kepler inherited the volumes of Tycho Brahe's (1546–1601) meticulous observations of the motions of the planets, the Moon and the stars. Built up over many years, Brahe's measurements and recordings, all made before the invention of the telescope, enabled the mathematically minded Kepler to propose a new model for the motion of the planets.

Weblink
Orbits and Kepler's laws

orbit a regularly repeated elliptical path of one object about another massive object, such as a planet about a sun

ellipse a regular, curved shape that is a conic section (formed by cutting a cone obliquely); the path of satellites in orbit around larger bodies

Kepler's first law: the law of ellipses

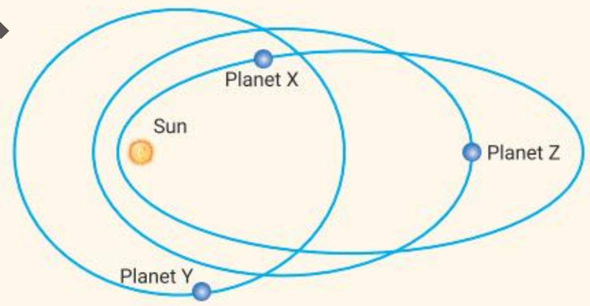
It had always been assumed that the planets orbited Earth, and in later models the Sun, in perfectly circular **orbits**. This was, in part, due to the belief that the heavens were perfect and that circles were considered to be a perfect shape. Moving in anything but a circle had not been proposed previously. Kepler found that, if the planets were considered as moving in elliptical orbits, then their observed positions in the sky could be predicted almost perfectly. Natural satellites, such as moons, and planets typically revolve about their planets or stars in elliptical, though at times, near circular orbits.

An **ellipse** is a curved shape with two focal points. Its major axis is the longest line between two points on the edge drawn through the geometric centre. The minor axis of an ellipse is the shortest line joining two points on the edge drawn through the geometric centre (Figure 6.2.1). An ellipse has two foci: one primary and one secondary. A circle is a special case of an ellipse in which the two axes are equal in length and the two foci are located at the same position. The orbits of many planets, moons and satellites are very close to circular, whereas the orbits of comets are highly elliptical. How pronounced an ellipse is can be indicated mathematically by its eccentricity, a value between 0 and 1. The elliptical eccentricity of Earth's path around the Sun is 0.167 while that of Halley's comet is 0.967.

KEY LAW

Kepler's first law: the law of ellipses

All planets move in elliptical orbits with the Sun at one focus.



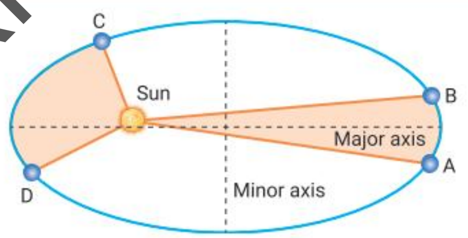


FIGURE 6.2.1 Kepler's first law, the law of ellipses, illustrates that the path of the planets about the Sun is elliptical in shape, the Sun being at one focus. (Kepler's first law). Segments AB and CD sweep out equal areas in equal time intervals (Kepler's second law).

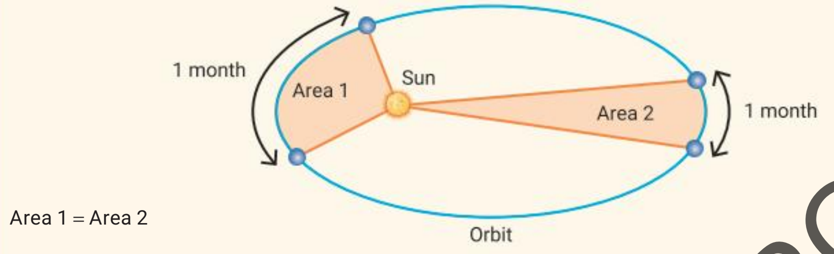
Kepler's second law: the law of equal areas

Kepler noticed that the speeds of the planets changed during their orbits. Nearer to the Sun, their speeds increased; further away, their speeds decreased. He was able to conclude, through application of the conservation of angular momentum, that the areas covered in equal time intervals were the same.

KEY LAW

Kepler's second law: the law of equal areas

A line that connects a planet to the Sun sweeps out equal areas in equal time periods.



Kepler's third law: the law of periods

mean orbital radius the average radius of orbit of one massive object about another, e.g. Earth revolving about the Sun

By doing further work on Tycho Brahe's data and using his own observations, Kepler showed that there is a relationship between the average radius of orbit of a planet (r) and its period of revolution around the Sun (T), so that the period of revolution squared is proportional to the **mean orbital radius** of the orbit cubed; that is, $T^2 \propto r^3$.

$$\frac{T^2}{r^3} = k \text{ (}k\text{ is a constant)}$$

This constant (k) for a given orbital system can be quantified by using $\frac{4\pi^2}{GM}$, where G is the gravitational constant, 6.67×10^{-11} N m^2 kg^{-2}, and M is the mass in kg of the central body the planets are orbiting around. In our solar system, this is the Sun with a mass of 1.99 × 1030 kg. The derivation of this expression will be outlined in section 6.3.

Thus, for a given system, the ratio of period of revolution squared to mean radius of orbit cubed is a constant for all planets in the same orbital system. For any two planets, a and b, in the same orbital system:

$$\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3} = \frac{4\pi^2}{GM}$$

It is important to note that *mean* radius must be applied to orbits because of their elliptical nature. An ellipse does not have a singular constant radius; thus a mean is used.

KEY LAW

Kepler's third law: the law of periods

The square of the period of a planet's orbit is proportional to the cube of its mean orbital radius:

$$T^2 \propto r^3$$

For any two planets, a and b, in the same orbital system:

$$\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3} = \frac{4\pi^2}{GM}$$

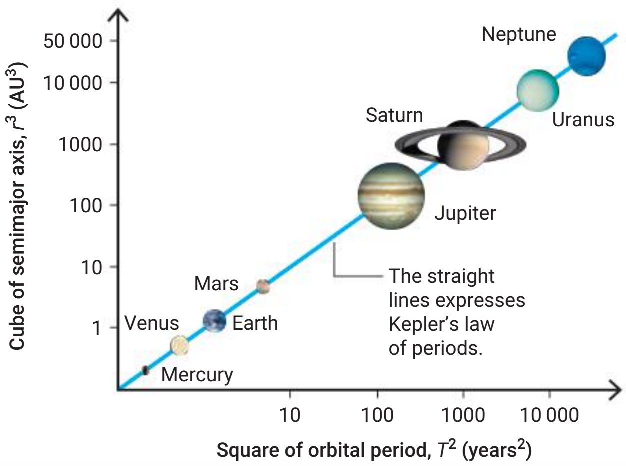


FIGURE 6.2.2 Kepler's third law, the law of periods, is constant for all orbiting bodies within a given system. This is illustrated for the planets of our solar system.

WORKED EXAMPLE 6.2.1

Kepler's third law, the law of periods, $\frac{T^2}{r^3}$ = constant, may be used to graphically illustrate the relationship that applies for the planets of our solar system. This may also be used to determine whether other bodies belong to this system, such as Halley's comet.

Manipulate and graph the selection of data in Table 6.2.1 to confirm that these planets may be classified as belonging within our solar system.

TABLE 6.2.1 Solar system planetary data tabulated for analysis

Planetary body	Mean orbital radius, r ($\times 10^9$ m)	Mean orbital radius^3 (m^3)	Orbital period, T (s)	Orbital period^2 (s^2)	$\frac{r^3}{T^2}$
Mercury	57.9		1.00×10^7		
Earth	149.6		3.16×10^7		
Mars	227.9		6.74×10^7		

ANSWER

The $\frac{T^2}{r^3}$ constant for the planetary system is consistent for the planets Mercury, Earth and Mars (see Table 6.2.2). The $\frac{T^2}{r^3}$ value is also the inverse of the gradient of the graph of r^3 versus T^2, confirming that the planets all belong to the same system.

Note that the units do not necessarily need to be SI units; that is, the mean orbital radius may be given in metres, kilometres or astronomical units and the orbital period may be given in seconds, days or years. However, it is important to ensure that the same units are used consistently for all bodies.

TABLE 6.2.2 Solar system planetary data tabulated analysis

Planetary body	Mean orbital radius ($\times 10^9$ m)	Mean orbital radius^3 (m^3)	Orbital period (s)	Orbital period^2 (s^2)	$\frac{T^2}{r^3}$
Mercury	57.9	1.94×10^{32}	1.00×10^7	1.00×10^{14}	5.15×10^{-19}
Earth	149.6	3.35×10^{33}	3.16×10^7	9.99×10^{14}	2.99×10^{-19}
Mars	227.9	1.18×10^{34}	6.74×10^7	4.54×10^{15}	3.83×10^{-19}

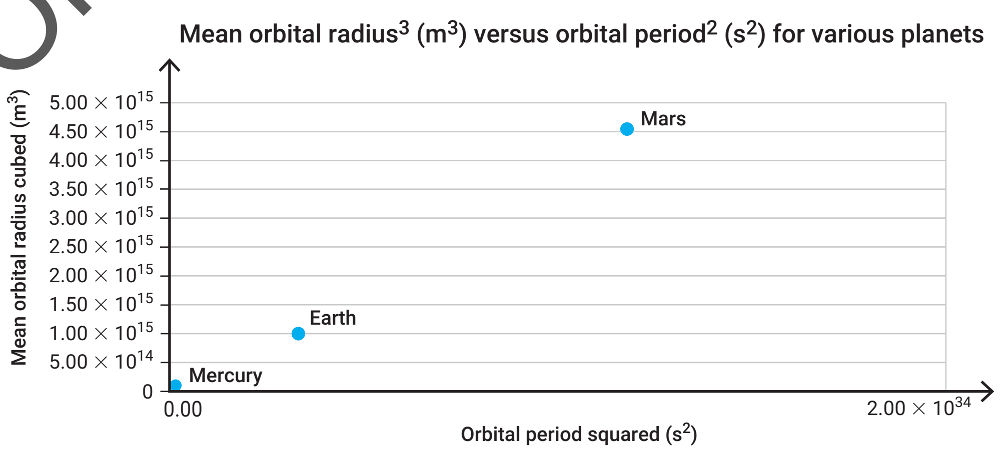


FIGURE 6.2.3 A graph of mean orbital radius cubed (m^3) versus orbital period squared (s^2) for various planets in a system

WORKED EXAMPLE 6.2.2

Kepler's third law applies for all bodies within a given planetary system. Use Kepler's law of periods, $\frac{T^2}{r^3}$ = constant, to determine whether Planet X is part of the same planetary system as planets A, B and C orbiting a central star.

TABLE 6.2.3 Planetary system data for analysis

Measurement	Planet A	Planet B	Planet C	Planet X
Mean orbital radius (m)	2.30×10^9	7.50×10^9	3.75×10^9	1.04×10^9
Mass (kg)	1.49×10^{24}	1.43×10^{24}	7.10×10^{23}	8.82×10^5
Orbital period (s)	2.09×10^5	1.25×10^6	4.35×10^5	8.82×10^5

ANSWER

1 **Substitute the known values for Planet A.**

The $\frac{T^2}{r^3}$ value for Planet A $= \frac{(2.09 \times 10^5)^2}{(2.30 \times 10^9)^3}$

2 **Calculate the answer.**

$\frac{(2.09 \times 10^5)^2}{(2.30 \times 10^9)^3} = 3.60 \times 10^{-18}$

3 **Substitute the known values for Planet B.**

The $\frac{T^2}{r^3}$ value for Planet B $= \frac{(1.25 \times 10^6)^2}{(7.50 \times 10^9)^3}$

4 **Calculate the answer.**

$\frac{(1.25 \times 10^6)^2}{(7.50 \times 10^9)^3} = 3.70 \times 10^{-18}$

5 **Substitute the known values for Planet C.**

The $\frac{T^2}{r^3}$ value for Planet C $= \frac{(4.35 \times 10^5)^2}{(3.75 \times 10^9)^3}$

6 **Calculate the answer.**

$\frac{(4.35 \times 10^5)^2}{(3.75 \times 10^9)^3} = 3.60 \times 10^{-18}$

7 **Substitute the known values for Planet X.**

The $\frac{T^2}{r^3}$ value for Planet X $= \frac{(8.82 \times 10^5)^2}{(1.04 \times 10^9)^3}$

8 **Calculate the answer.**

$\frac{(8.82 \times 10^5)^2}{(1.04 \times 10^9)^3} = 6.90 \times 10^{-16}$

9 **Compare the values for all planets.**

The $\frac{T^2}{r^3}$ constant for the planetary system is consistent for planets A, B and C (at about 3.63×10^{-18}); however, the value for Planet X is considerably different (at 6.90×10^{-16}); therefore, it does not form part of this planetary system.

Kepler arrived at his three laws empirically, basing them on an analysis of the data that Tycho Brahe had provided as well as his own observations. Kepler's laws had excellent predictive power, although they were not based on any underlying models or theoretical basis; they did not give any explanation of the observed behaviour of the planets. Newton's model for gravity and the principle of conservation of momentum provided the theoretical framework needed to explain *why* planets and other orbiting bodies moved as described by Kepler's laws.

LEARNING CHECK 6.2

DESCRIBING

1 State Kepler's first, second and third laws of planetary motion.

2 **Sketch** a diagram to illustrate Kepler's first law.

3 **Describe** Kepler's third law.

4 State the physical phenomena used to explain the motion of planetary bodies in Kepler's second law.

APPLYING

5 Being a large planet, Jupiter is at the centre of its own orbital system with many moons orbiting it. Just like the planets orbiting the Sun, the moons orbiting Jupiter also obey Kepler's laws, but on a smaller scale. Data values for several natural satellites (moons) of Jupiter are in Table 6.2.4. Use the data for the moon Io to determine the ratio of $\frac{T^2}{r^3}$ and hence **determine** the mean orbital radius of Jupiter's moons Europa, Ganymede and Callisto.

TABLE 6.2.4 Jupiter system data for analysis

Moon	Orbital period, T (days)	Mean orbital radius, r (m)
Io	1.78	4.22×10^8
Europa	3.56	
Ganymede	7.16	
Callisto	16.70	

6 Astronomers can detect distant planets in other solar systems. They are called 'exoplanets'. A recently discovered exoplanet, Planet X, is found to travel within our galaxy near a known star, Star Y. It has been suggested that Planet X orbits Star Y; however, this is yet to be confirmed. Use Kepler's law and the data provided for planets A and B in Table 6.2.5, which are known to be part of this system, to confirm whether Planet X should be classified as part of this system.

TABLE 6.2.5 Exoplanet system data

Measurement	Planet A	Planet B	Planet X (not yet confirmed)
Radius ($\times 10^3$ km)	21.1	6.50	42.2
Mass (kg)	1.96×10^{24}	1.84×10^{24}	2.65×10^{24}
Orbital period (Earth seconds)	4.15×10^6	2.45×10^7	4.42×10^8
Mean orbital radius (m)	4.52×10^{10}	1.44×10^{11}	10.2×10^{11}
Rotational period (days)	8.33	1.92	0.83

REFLECTING

7 **Define** 'mean orbital radius' and **explain** why this is required when applying Kepler's laws.

8 **Describe** the difference in motion of a comet compared to a planet, with reference to the eccentricity of its elliptical path.

6.3 Universal gravitation and Kepler's third law

In the case of the periodic circular motion of a planet around the Sun or a moon about a planet, the centripetal force that accelerates the orbiting body about the primary focal point is due to the gravitational force. Kepler's third law can be deduced from Newton's law of universal gravitation and the equation for uniform circular motion. For simplicity, we will assume that the planet follows a circular orbit, although a more complex geometrical analysis of elliptical orbits provides the same result.

KEY FORMULA

Centripetal force

$$F_c = \frac{mv^2}{r}$$

where:
F_c = centripetal force (N)
m = mass of planetary body (kg)
v = velocity (m s^{-1})
r = radius (m)

KEY FORMULA

Gravitational force

$$F_g = \frac{GMm}{r^2}$$

where:
F_g = gravitational force (N)
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
M = mass of body 1 (that being orbited) (kg)
m = mass of body 2 (orbiting body) (kg)
r = radius (m)

WORKED EXAMPLE 6.3.1

Determine the force required to keep a 1200 kg satellite in orbit around Earth at an altitude of 300 km.
$M_E = 5.97 \times 10^{24}$ kg
$r_E = 6.37 \times 10^6$ m

ANSWER

1 **Determine the formula.**
The centripetal force is equal to the gravitational force.

$$F_g = \frac{GMm}{r^2}$$

2 **Substitute the known values.**

$$F_g = \frac{6.67\times10^{-11}\times5.97\times10^{24}\times1.2\times10^3}{(6.37\times10^6+300\times10^2)^2}$$

$$= \frac{4.78\times10^{17}}{(6.40\times10^6)^2}$$

$$= \frac{4.78\times10^{17}}{4.096\times10^{13}}$$

3 **Calculate the answer.**
$F_g = 1.167 \times 10^4$ N

The equivalence between the centripetal force accelerating the orbiting body and the gravitational force can be further developed to determine Kepler's third law, the law of periods. Consider a planet (mass, m) orbiting the Sun (mass, M) with a mean orbital radius of r. The only force applied to the planet is the force mediated by the gravitational field of the Sun, providing the centripetal acceleration and hence uniform circular motion.

Figure 6.2.2 shows this constant relationship for the planets in our solar system. Note that, in this example, the radius is given in astronomical units (AU), which is Earth's mean orbital radius (approximately 1.50×10^{11} m). The orbital period is given in years. Also note that the equation remains valid regardless of the units used, as long as they are consistent. The equation is also valid if inverted.

KEY FORMULAS

F (due to Sun's gravitational field) $= G\dfrac{Mm}{r^2}$ (N)

F (due to circular motion) $= \dfrac{mv^2}{r}$ (N)

F (due to circular motion) $= m\dfrac{(2\pi r)^2}{T^2 r}$ (N)

F (due to circular motion) $= m\dfrac{4\pi^2 r}{T^2}$ (N)

Let the gravitational force = centripetal force

$$G\frac{Mm}{r^2} = m\frac{4\pi^2 r}{T^2}$$

Then: $\dfrac{T^2}{r^3} = \dfrac{4\pi^2}{GM} = k$ (constant)

Astronomical distances

Although the international standard (SI) unit of length is the metre (m), the typical distances between planets, stars and galaxies are enormous; hence, astronomers often find it more convenient to use a range of longer distance units to perform calculations. Typically, such units include the astronomical unit, the megaparsec and the light-year.

- The **astronomical unit (AU)** is a unit of measure equivalent to Earth's mean orbital radius about the Sun.
- The **megaparsec (Mpc)** is the distance subtended by an angle of 1 arcsecond $\times 1 \times 10^6$.
- The **light-year (ly)** (despite its name) is the distance that light would travel in 1 year.

KEY FORMULAS

Astronomical unit: 1.0 AU = 1.50×10^8 km = 1.50×10^{11} m
Megaparsec: 1.0 Mpc = 3.09×10^{19} km = 3.09×10^{22} m
Light-year: 1.0 ly = 9.47×10^{12} km = 9.47×10^{15} m
Speed of light in a vacuum: $c = 3.00 \times 10^8$ m s^{-1}

astronomical unit (AU) a unit of measure equivalent to Earth's mean orbital radius about the Sun (1.50×10^{11} m)

megaparsec (Mpc) the distance subtended by an angle of 1 arcsecond $\times 1 \times 10^6$ (3.09×10^{22} m)

light-year (ly) a measure of the distance that light would travel in 1 year (9.47×10^{15} m)

The analysis does not apply just to the planetary motion of our solar system, but to all systems where satellites (moons, planets) orbit larger masses; for example, the 95 known moons orbiting

WORKED EXAMPLE 6.3.2

The Andromeda galaxy, otherwise known as Messier 31, is the nearest large galaxy to the Milky Way. It is a spiral galaxy approximately 780 000 parsecs, or 0.78 megaparsecs, from Earth.

State the distance to the Andromeda galaxy in:

a kilometres
b light-years.

ANSWERS

a **1** **Substitute values into the formula to convert to kilometres.**

0.78 Mpc $\times 3.09 \times 10^{19}$ km per Mpc

2 **Calculate the answer.**

$= 2.41 \times 10^{19}$ km

b **1** **Substitute values into the formula to convert to light-years.**

$$\frac{2.41 \times 10^{19}\ \text{km}}{9.47 \times 10^{12}\ \text{km ly}^{-1}}$$

2 **Calculate the answer.**

$= 2.54 \times 10^6$ ly

Jupiter (at the time of writing!). The value of the constant $\frac{T^2}{r^3}$ is unique to each system, varying with each different central mass as the constant $\frac{4\pi^2}{GM}$ depends on the mass, M, around which the bodies are orbiting. Hence, the constant is not universal, but is unique to a given system.

WORKED EXAMPLE 6.3.3

Calculate the period T for a satellite of Earth with a mean orbital radius of 42 000 km.
Mass of Earth = 5.97×10^{24} kg
Gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

ANSWER

1 **Determine the formula.**

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

2 **Rearrange the formula to find T.**

$$T^2 = \frac{4\pi^2}{GM} \times r^3$$

$$= \sqrt{\frac{4\pi^2}{GM} \times r^3}$$

3 **Substitute the known values.**

$$T = \sqrt{\frac{4\pi^2}{6.67 \times 10^{-11}\text{ N kg}^{-2}\text{ m}^2 \times 5.97 \times 10^{24}\text{ kg}} \times (42\,000 \times 10^3)^3\text{ m}^3}$$

4 **Calculate the answer.**

$$T = \sqrt{7\,345\,264\,562}$$

Therefore, $T = 8.6 \times 10^4$ s (or 23.8 hours)

WORKED EXAMPLE 6.3.4

A small planet, Planet A, is observed to orbit a star every 30 days. A second planet, Planet B, orbits the same star at a distance that is nine times the orbital radius of Planet A. What is the period of Planet B?

ANSWER

1 **Determine the formula.**
Here, the mass M of the star is unknown. The term $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$ is the same for both planets, as they are in the same system and orbit the same star, so we can use:

$$\frac{T^2}{r^3} = \text{constant and } \frac{T_A^{\ 2}}{r_A^{\ 3}} = \frac{T_B^{\ 2}}{r_B^{\ 3}} = \frac{4\pi^2}{GM}$$

$$\frac{T_A^{\ 2}}{r_A^{\ 3}} = \frac{T_B^{\ 2}}{r_B^{\ 3}}$$

2 **Substitute the known values.**

$$\frac{T_A^{\ 2} \times r_B^{\ 3}}{r_A^{\ 3}} = T_B^{\ 2} \text{ where } r_B = 9 \times r_A \text{ and } T_A = 30 \text{ days}$$

$$T_B = \sqrt{656\,100}$$

3 **Calculate the answer.**
$T_B = 810$ days

LEARNING CHECK 6.3

DESCRIBING

1 State the formula used to derive:

a centripetal force

b gravitational force.

2 **Define** 'astronomical unit'.

3 State the speed of light in a vacuum.

4 **Contrast** the units of length measurements megaparsec and light-year.

5 Convert 12.8 Mpc into light-years.

6 **Explain** how Kepler's third law can be used to determine whether an unknown comet or planet should be classified as belonging to a given solar system.

APPLYING

7 **Calculate** the orbital period, T, for an artificial satellite orbiting Earth at an altitude of 300 km. Use Earth's mass as 5.97×10^{24} kg and Earth's radius as 6.37×10^{6} m.

8 The mean orbital radius of Saturn is 1.43×10^{12} m. **Determine** the orbital period of Saturn, given that Kepler's third law, the law of periods, for our solar system has an average value of $\frac{T^2}{r^3}$ of approximately 3.41×10^{18} s^2 m^{-3}.

9 The Whirlpool galaxy, otherwise known as Messier 51a, is approximately 23 million light-years from the Milky Way. **State** the distance to the Whirlpool galaxy in:

a kilometres

b megaparsecs

c parsecs.

ANALYSING

10 A newly identified exoplanet, Planet P, has been observed to orbit its nearby star with a period of 18 days. A second exoplanet, Planet Q, orbits the same star at double the orbital radius of Planet P. **Determine** the orbital period of Planet Q.

11 A natural satellite (moon) orbiting a planet of mass 7.5×10^{25} kg has a mean orbital radius of 29 000 km. Using $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, **determine** the period of revolution for this satellite.

12 **Calculate** the mean orbital radius of an Earth-orbiting satellite that has a period of 19 h.

REFLECTING

13 **Explain** why the astronomical unit, AU, is used for measuring astronomical distances rather than the SI unit of the metre.

14 State what occurs to the period of a satellite if its radius of orbit is decreased.

15 Could the $\frac{T^2}{r^3}$ value for Earth orbiting the Sun be applied for satellites orbiting another planet? **Justify** your response.

6.4 Satellite motion

The motion of a **satellite** can be modelled as uniform circular motion. Most satellites have circular or very nearly circular orbits around Earth. They are in a constant state of **free-fall**; the only force acting on them is gravity, or their **weight**. The gravitational force by Earth's mass on a satellite is directed towards Earth's centre, which is also the centre of the satellite's circular orbit. Therefore, the net force acting on the satellite is perpendicular to the velocity of the satellite.

satellite a natural (e.g. moon) or artificial (e.g. GPS or communications satellite) body that orbits a significantly larger mass

free-fall falling with the acceleration g, the local gravitational field strength weight the gravitational force that acts on an object, $F_w = mg = \frac{GMm}{r^2}$

orbit velocity the precise velocity required for an object to continue to orbit a mass at a given altitude

Orbital velocity

A satellite in orbit is still very much within Earth's gravitational field. It is falling to Earth with an acceleration equal to the gravitational acceleration, g, at that distance from Earth. An orbiting spacecraft or satellite is given a horizontal velocity, its **orbit velocity**, such that, as it falls, it is also moving horizontally with such a speed that its path is circular. If no force other than that due to the gravitational field acts on the satellite, the satellite will continue in its orbit forever.

The gravitational force acting on satellites provides the centripetal force to keep them travelling in their circular orbit (Figure 6.4.1). The gravitational force can be found from the formula $F_g = \frac{GMm}{r^2}$.

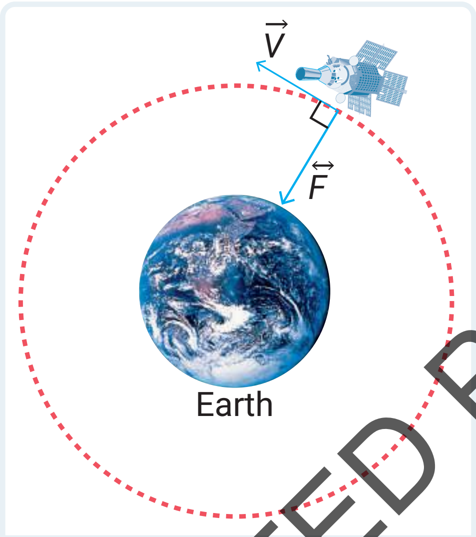


FIGURE 6.4.1 The net force on a satellite is perpendicular to its velocity and directed towards the centre of its circular orbit.

KEY FORMULAS

$$F_c = F_g$$
$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$
$$v^2 = \frac{GM}{r}$$
$$v = \sqrt{\frac{GM}{r}}$$

where:
F_c = centripetal force (N)
F_g = gravitational force (N)
$G = 6.67 \times 10^{-11}$ N m² kg⁻²
M = mass of body being orbited (kg)
m = mass of orbiting body (kg)
r = radius (m)
v = orbital velocity (m s⁻¹)

Weblinks
Catalogue of satellite orbits
Space flight: the application of orbital mechanics
Weightlessness

apparent weightlessness the experience of having no normal force exerted on you; this occurs during free-fall

This provides the centripetal force, $F_c = \frac{mv^2}{r}$ to determine the orbital velocity, $v = \sqrt{\frac{GM}{r}}$.

Travelling at the precise orbital velocity enables a satellite to fall towards Earth at the same rate as Earth curves away from it; hence, the satellite is able to continually accelerate towards Earth, yet remain at the same orbit altitude. It is in this situation that passengers and their craft undergo free-fall and experience **apparent weightlessness**.

WORKED EXAMPLE 6.4.1

Determine the velocity of a satellite orbiting Earth at an altitude of 630 km.
Mass of Earth = 5.97×10^{24} kg
Radius of Earth = 6.37×10^{6} m

ANSWER

1 **Determine the formula.**
Note that the radius of orbit is the sum of Earth's radius and the altitude above the surface.

$$v = \sqrt{\frac{GM}{r}}$$

2 Substitute the known values.

$$v = \sqrt{\frac{6.67 \times 10^{-11}\,\text{N kg}^{-2}\,\text{m}^2 \times 5.97 \times 10^{24}\,\text{kg}}{6.37 \times 10^6 + 630 \times 10^3\,\text{m}}}$$

$$= \sqrt{56\,885\,571}$$

3 Calculate the answer.

$v = 7542\text{ m s}^{-1}$ or 7.54 km s^{-1}

WORKED EXAMPLE 6.4.2

Determine the orbital period of a satellite orbiting Earth at an altitude of 23 000 km.
Mass of Earth = 5.97×10^{24} kg
Radius of Earth = 6.37×10^6 m

ANSWER

1 Determine the orbital velocity.

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{6.67 \times 10^{-11}\,\text{N kg}^{-2}\,\text{m}^2 \times 5.97 \times 10^{24}\,\text{kg}}{6.37 \times 10^6 + 23\,000 \times 10^3\,\text{m}}}$$

$$= \sqrt{13\,558\,018}$$

$v = 3682\text{ m s}^{-1}$ or 3.68 km s^{-1}

2 Determine the period using the formulas for velocity and the circumference of a circle.

$$v = \frac{s}{t}$$

and $s = 2\pi r$

$$v = \frac{2\pi r}{t}$$

where $v = 3682\text{ m s}^{-1}$

$$3682\text{ m s}^{-1} = \frac{2 \times \pi \times (6.37 \times 10^6 + 23\,000 \times 10^3)}{t}$$

$t = 50\,118$ s or 13.92 h

Satellite orbits

A **geostationary satellite** remains above one place on Earth. It must travel directly above a point on the equator. **Geosynchronous satellites** travel above any great circle. A great circle is any circle on Earth whose radius extends from Earth's centre. Both geostationary and geosynchronous orbits have approximately 24-hour periods (23 h 56 min 4 s or 86 164 s).

Satellites in **low Earth orbit (LEO)** are high enough to be moderately affected by atmospheric friction but low enough to be relatively easy to service from Earth. This region extends about 250–1000 km above Earth's surface. The International Space Station has been in LEO since November 1998. Its altitude averages 370 km. Even at this altitude, the friction of the very low density atmosphere there is sufficient to slow the International Space Station down; hence, its rocket motors must be fired every few weeks to boost it back into a higher orbit.

As a satellite in LEO slows down, its orbital radius decreases. At lower altitude, there is greater friction and the process, without booster rocket intervention, would result in the spacecraft eventually crashing back to Earth or burning up in the atmosphere during a fiery re-entry.

geostationary satellite a satellite positioned directly above a point on the equator; has periods of approximately 24 hours

geosynchronous satellite a satellite that completes one orbit of Earth in the same time as Earth completes one revolution; has a period of approximately 24 hours

low Earth orbit (LEO) a satellite orbit within the range of approximately 250–1000 km above Earth's surface

NASA/sts-114 Crew

FIGURE 6.4.3 The International Space Station has been in low Earth orbit since 1998.

PRACTICAL ACTIVITY 6.4.1

THE RELATIONSHIP BETWEEN RADIUS AND MASS FOR AN OBJECT IN ORBIT

Research question

How does the radius of orbit affect the velocity of an object orbiting another object of constant mass?

Aim

To simulate and examine the relationship between the radius of orbit and the mass of an orbiting object on its orbit velocity

Materials

- Simulation involving the use of the formula for orbit velocity:

$$v = \sqrt{\frac{GM}{r}}$$

Procedure 1

1 Consider the role of radius in the orbit velocity of a satellite of mass 2000 kg orbiting at varying altitudes above Earth (Table 6.4.1).

TABLE 6.4.1 Satellite orbit altitudes

Orbit altitude (km)	Orbit velocity (km s^{-1})
1000 (low Earth orbit)	
20 000 (medium Earth orbit)	
42 164 (geosynchronous Earth orbit)	

Be sure to add the radius of Earth, $r_E = 6.37 \times 10^6$ m, to the altitude. Use $M_E = 5.97 \times 10^{24}$ kg.

2 Using the orbital velocity formula, determine the velocity for this satellite at each altitude.
3 Compare the velocities for each altitude and draw a conclusion.

Procedure 2

1 Consider the role of mass in the orbit velocity of satellites of varying masses orbiting at the altitude of 3000 km (Table 6.4.2).

TABLE 6.4.2 Satellite orbit masses

Satellite mass (kg)	Orbit velocity ($km\ s^{-1}$)
7 (pico-satellite)	
3500 (communication satellite)	
419 455 (International Space Station)	

Be sure to add the radius of Earth, $r_E = 6.37 \times 10^6$ m, to the altitude. Use $M_E = 5.97 \times 10^{24}$ kg.
2 Using the orbital velocity formula, determine the velocity for each satellite of different mass at this altitude.
3 Compare the velocities for each mass. Consider the role of mass in the formula and draw a conclusion.

Analysis of results

1 Explain the effect of altitude on orbital velocity.
2 Explain the effect of mass on orbital velocity.

Escape velocity

In order to escape the effect of Earth's gravitational field, extra energy must be expended. A rocket fired into the atmosphere may reach a specific height and then fall back to Earth. Given enough energy, it may reach a critical velocity, allowing it to orbit Earth. Given further energy still, the rocket may exceed the gravitational attraction of Earth and escape it entirely. **Escape velocity** is the minimum velocity needed for an object to escape the gravitational field of a planet or other large mass so that it no longer experiences a gravitational force due to that large mass. For Earth, the escape velocity has been determined to be 11.2 km s^{-1}. The escape velocity may be derived by considering the initial and final kinetic energies of the object, where this change in kinetic energy is equal to the change in the potential energy of the system.

escape velocity the minimum velocity required for an object to escape the gravitational field of a planet or other large mass

KEY FORMULA

Escape velocity

$$v_{escape} = \sqrt{\frac{2GM}{r}}$$

where:
$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}
M = mass of the planetary body (kg)
r = radius of planet (m)

WORKED EXAMPLE 6.4.3

Determine the escape velocity required for a rocket to escape Earth's gravitational attraction.

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

Earth's mass = 5.97×10^{24} kg

Earth's radius = 6.37×10^6 m

ANSWER

1 **Determine the formula.**

Use the formula for escape velocity:

$$v_{escape} = \sqrt{\frac{2GM}{r}}$$

2 **Substitute the known values.**

$$v = \sqrt{\frac{2 \times 6.67 \times 10^{-11}\text{ N m}^2\text{ kg}^{-2} \times 5.97 \times 10^{24}\text{ kg}}{6.37 \times 10^6\text{ m}}}$$

3 **Calculate the answer.**

$$= \sqrt{125\,023\,234}$$

$v = 11\,181$ m s^{-1} or 11.2 km s^{-1}

LEARNING CHECK 6.4

DESCRIBING

1 State the formula for determining orbit velocity.

2 State the formula for determining escape velocity.

3 **Describe** the change in orbit velocity as the radius of orbit increases. Use two example calculations to determine the trend.

4 **Explain** why astronauts orbiting Earth experience weightlessness.

APPLYING

For Questions 5–7, use $M_E = 5.97 \times 10^{24}$ kg and $r_E = 6.37 \times 10^6$ m.

5 **Determine** the velocity of a satellite orbiting Earth at an altitude of 2300 km.

6 **Calculate** the gravitational force acting on a satellite of orbital radius 650 km and mass 4000 kg.

7 **Determine** the orbital period, in hours, of a satellite of mass 950 kg and altitude of 12 000 km.

ANALYSING

8 **Determine** the escape velocity required for a rocket to escape Mars' gravitational attraction. Use $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, $M_M = 6.37 \times 10^{23}$ kg and $r_M = 3.43 \times 10^6$ m.

9 **Determine** the gravitational field strength for a satellite orbiting Earth at an altitude of 36 000 km. Use $M_E = 5.97 \times 10^{24}$ kg and $r_E = 6.37 \times 10^6$ m.

10 Earth's gravitational field strength is found to be 4.90 m s^{-2} at the height of a satellite in orbit. **Determine** how far from Earth's centre the satellite must be.

REFLECTING

11 State one advantage and one disadvantage of placing satellites in a low Earth orbit.

12 **Explain** why the escape velocity from a planet is always greater than the velocity required to orbit the planet.

CHAPTER SUMMARY

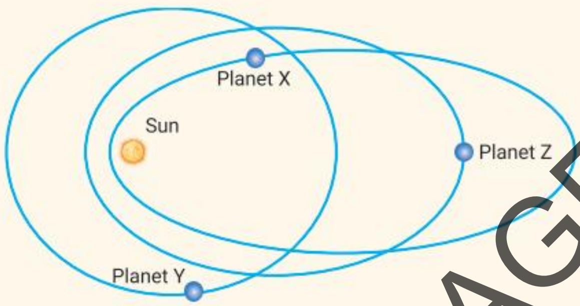
Kepler's laws of planetary motion

- First law: the law of ellipses which states planets orbit the Sun in elliptical paths, with the sun at one focus. This orbital path shape means that the distance between a planet and the sun varies throughout its orbit.

KEY LAW

Kepler's first law: the law of ellipses

All planets move in elliptical orbits with the Sun at one focus.

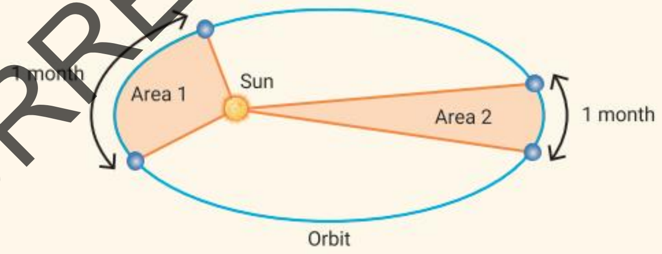


- Second law: a line segment joining a planet and the sun sweeps out equal areas during equal intervals of time. This means planets move faster in their orbital path when closer to the Sun and slower when farther from the Sun.

KEY LAW

Kepler's second law: the law of equal areas

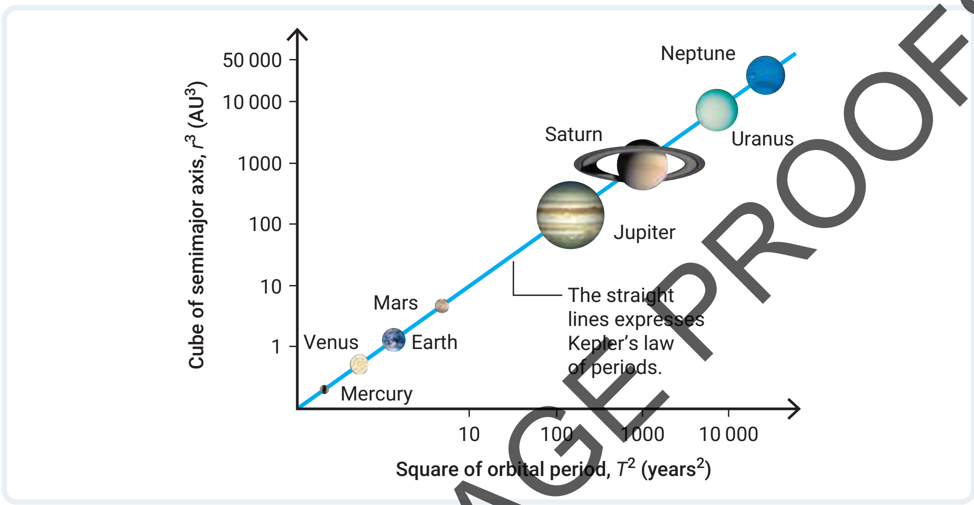
A line that connects a planet to the Sun sweeps out equal areas in equal time periods.



UNCORRECTED PAGE PROOFS

- Third law: the square of the period of a planet's orbit is proportional to the cube of its mean orbital distance. The formula below can also be derived from Newton's law of universal gravitation and the equation for uniform circular motion.

$\frac{T^2}{r^3} = \frac{4\pi^2}{GM} = k$, where k is a constant.

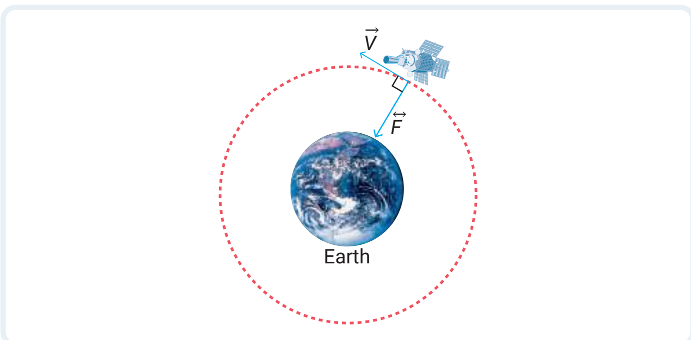


Astronomical distances

- Astronomical unit, 1.0 AU = 1.50×10^8 km = 1.50×10^{11} m
- Megaparsec, 1.0 Mpc = 3.09×10^{19} km = 3.09×10^{22} m
- Light-year, 1.0 ly = 9.47×10^{12} km = 9.47×10^{15} m
- The speed of light in a vacuum, $c = 3.00 \times 10^8$ m s^{-1}

Satellite motion

- Motion of satellites can be modelled as uniform circular motion.
- Gravitational force acting on satellites helps to keep them travelling in their circular orbit.



CHAPTER EXAM

MULTIPLE CHOICE

1. What would happen to the orbit velocity of a satellite if it were moved to an orbit of higher altitude?

- **A** It would decrease.
- **B** It would increase.
- **C** It would remain constant.
- **D** It would need to exceed the escape velocity of the planet.

2. Who proposed the heliocentric model of the solar system?

- **A** Nicolaus Copernicus
- **B** Galileo Galilei
- **C** Johannes Kepler
- **D** Tycho Brahe

3. The mechanism that provides the centripetal acceleration that satellites in orbit experience towards Earth is:

- **A** force tension.
- **B** mass.
- **C** force friction.
- **D** force gravity.

4. A satellite orbits Earth in a circular path with a speed of $7.5\,\text{km}\,\text{s}^{-1}$ at a radius of $6.7 \times 10^{6}\,\text{m}$. What is the orbital period of the satellite?

- **A** 560 s
- **B** 4500 s
- **C** 5610 s
- **D** 6700 s

5. What does Kepler's first law state about planetary motion?

- **A** Planets move in circular orbits around the Sun, with the Sun at the centre of the circle.
- **B** Planets move in elliptical orbits around the Sun, with the Sun at one focus.
- **C** The square of a planet's orbital period is proportional to the cube of its mean orbital radius.
- **D** The speed of a planet is constant throughout its orbit.

6. What does Kepler's second law (law of equal areas) state?

- **A** The distance a planet travels during its orbit is the same every year.
- **B** A line segment joining a planet and the Sun sweeps out equal areas in equal times.
- **C** The gravitational force between a planet and the Sun is constant throughout the orbit.
- **D** A planet's orbital period depends only on its mass.

7. Which force provides the centripetal force necessary for uniform circular motion in planetary orbits?

- **A** Electromagnetic force
- **B** Gravitational force
- **C** Frictional force
- **D** Tension force

8. The average orbital radius of Earth is 1.5×10^{11} m, and its orbital period is 1 year. If a planet has a mean orbital radius of 5.0×10^{11} m, what is its orbital period?
   A 2.0 years
   B 4.0 years
   C 8.0 years
   D 16.0 years

9. How does Newton's law of universal gravitation relate to Kepler's third law?
   A It describes how the speed of a planet depends on the distance to the Sun.
   B It provides the mathematical explanation for the relationship between the square of the orbital period and the cube of the orbital radius.
   C It explains why planets move in elliptical orbits around the Sun.
   D It describes how the Sun's gravity varies with time.

10. What is the orbital period T of a satellite orbiting a planet of mass 6.0×10^{24} kg at a distance of 4.0×10^{7} m from the centre of the planet?
   A 2.0 h
   B 5.5 h
   C 12.4 h
   D 24.0 h

SHORT RESPONSE

11. Find the altitude of a satellite in orbit around Earth when its orbital speed is 5.0 km s^{-1}. Use $r_E = 6.37 \times 10^{6}$ m and $M_E = 5.97 \times 10^{24}$ kg.

12. **Determine** the radius of a satellite orbit whose period is 12 h.

13. Titan, a moon of Saturn, has a mean orbital radius of 1.22×10^{6} km. It takes Titan 15 days 22 hours to revolve around Saturn. Use this data to **determine** the mass of Saturn.

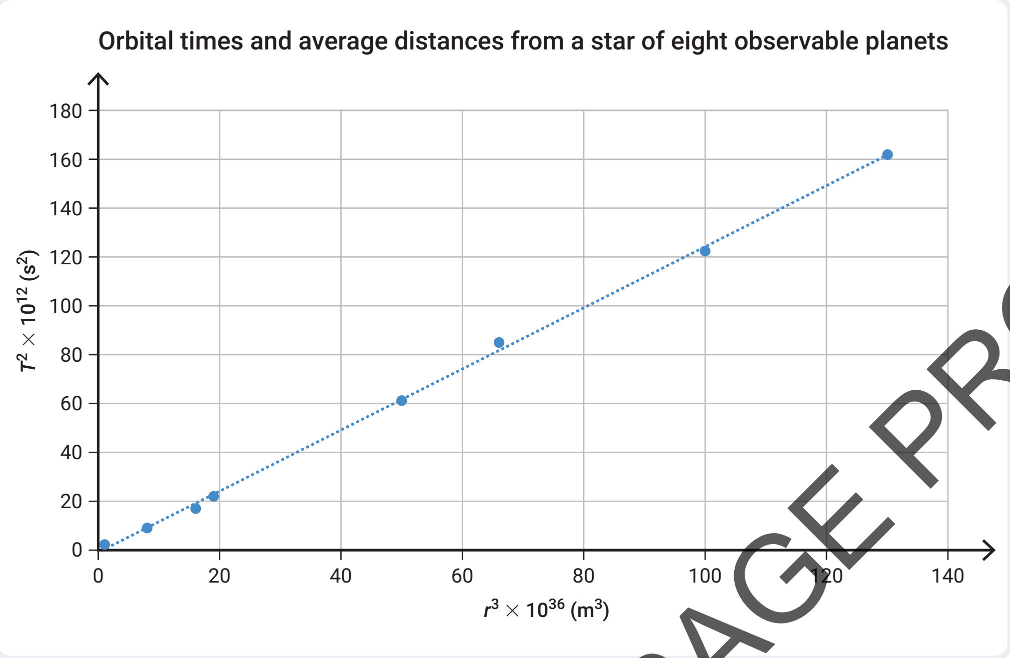
CROSS-CHAPTER QUESTION

14. Earth orbits the Sun with a slightly elliptical orbit, leading to variations in its orbital speed throughout the year. In January, Earth's orbital speed is approximately 30.3 km s^{-1}, and its distance from the Sun is 1.47×10^{11} m. In July, its orbital speed is 29.3 km s^{-1}, and its distance from the Sun is 1.52×10^{11} m. The mass of the Sun is 1.99×10^{30} kg.
   a **Calculate** the centripetal acceleration of Earth at the two given points in its orbit.
   b Using the law of universal gravitation, **calculate** the gravitational force between Earth and the Sun and the two given points.
   c **Explain** how these results align with Kepler's second law and the elliptical nature of Earth's orbit.

DATA ANALYSIS

15. **Analyse data**
   A distant star system was observed over a period of time and the orbital times (T) and average distances from the star (r) of the eight observable planets was recorded. The orbital times were measured in seconds, and orbital distances in metres.

The data was linearised according to Kepler's' third law and is shown below.



Calculate the unknown mass of the star.

SCIENCE AS A HUMAN ENDEAVOUR

Syllabus dot point

- **Consider the international collaboration required to monitor the orbits of satellites, and the management of space debris.**
- **Consider the factors that contribute to positioning of satellites used for observation of weather, natural phenomena, traffic and military movements.**

The science behind satellite positioning and its impact on society

Satellites have become indispensable tools for observing Earth and providing vital data across various domains, such as weather forecasting, monitoring natural phenomena, managing traffic, and even tracking military movements. Understanding the science behind satellite positioning and how it influences their tasks is crucial for appreciating the human endeavour in practising science. Let's dive deeper into the physics and principles that guide satellite placement and how they affect society.

The role of satellite orbits

The orbit of a satellite is one of the most important factors that determines its ability to carry out its mission. The selection of a satellite's orbit is influenced by its purpose, the physics of orbital motion, and practical constraints such as fuel efficiency and the need to avoid space debris. Satellite orbits are generally one of several major categories, each with their own set of uses.

Geostationary orbits

Satellites in geostationary orbit revolve around Earth at the same speed as Earth's rotation, so they remain fixed above a single point on Earth's surface. This makes them ideal for continuous monitoring and communication tasks because they provide real-time data over large areas. Geostationary satellite orbits are approximately 42 164 km in radius (or 35 786 km in altitude). Applications include:

- Weather monitoring: Satellites such as GOES (Geostationary Operational Environmental Satellite) monitor weather patterns and track large-scale phenomena such as hurricanes.
- Communication networks: These satellites support global communication systems by maintaining consistent contact with ground stations.

Low Earth orbits

Satellites in low Earth orbit (LEO) are much closer to Earth and complete orbits more quickly, meaning they pass over different regions throughout the day. LEO satellites are typically up to 2000 km in altitude. Applications include:

- High-resolution imaging: LEO satellites provide detailed images of specific regions, making them invaluable for monitoring natural disasters, traffic or even military activity.
- Constellations: To achieve continuous coverage, LEO satellites often operate in constellations, where multiple satellites work together to provide consistent data as they orbit.

Medium Earth orbits

Satellites in medium Earth orbit (MEO) are positioned between LEO satellites and geostationary orbits, typically at altitudes of 2000–35 786 km. These orbits strike a balance between coverage area and orbital period, making them ideal for specialised applications that require moderate coverage and latency. Applications include:

- Navigation systems: Satellites like those in the global positioning system operate in MEO, enabling precise positioning, navigation and timing for users worldwide.
- Data relay: MEO satellites can serve as intermediaries between LEO satellites and ground stations, supporting efficient data transmission for diverse uses.

Weather satellites

Weather satellites are critical for monitoring Earth's atmospheric conditions, collecting data on cloud cover, temperature, precipitation and other meteorological variables. They operate in various orbits, each suited to specific tasks. Applications include:

- Geostationary weather satellites: Positioned to continuously monitor large areas, these satellites, such as GOES, track large-scale weather patterns, enabling early warning for storms and hurricanes.
- Polar-orbiting weather satellites: Typically, in LEO, these satellites, such as the US Polar-orbiting Operational Environmental Satellite, provide detailed global coverage by orbiting from pole to pole, contributing to daily weather forecasts and climate studies.

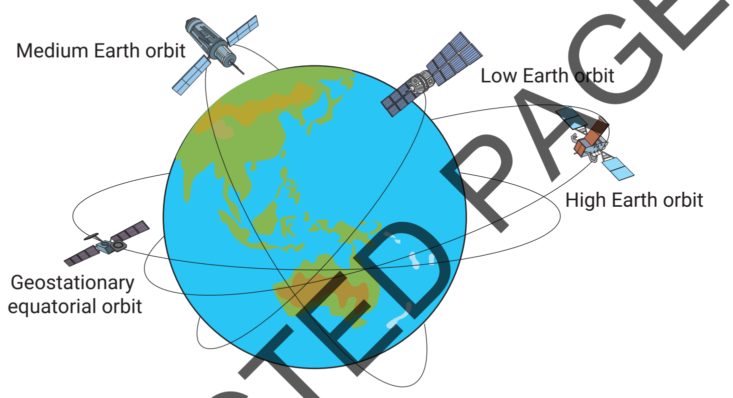


FIGURE 1 There are several types of satellite orbits, each with their own set of applications.

The physics of satellite motion

The principles that govern satellite orbits are based on the same physics concepts that describe planetary motion. These principles include:

- Gravitational force: A satellite stays in orbit because the gravitational pull of Earth provides the necessary centripetal force to keep it moving in a curved path.
- Orbital velocity: The velocity required to maintain orbit depends on the satellite's distance from Earth. A satellite in a lower orbit (such as LEO) needs a higher velocity to counteract the stronger gravitational pull at that altitude, whereas geostationary satellites, much farther away, move at lower speeds to stay in sync with Earth's rotation.
- Kepler's laws: These laws of planetary motion also apply to satellites, explaining how the satellite's orbital period depends on its distance from Earth, and why certain orbits are more stable than others for specific tasks.

Technological innovations in satellite systems

The advancement of satellite technology and positioning systems represents the intersection of physics, engineering and human endeavour. The ability to maintain a satellite in a precise orbit while ensuring it can communicate with ground stations and other satellites requires the application of several physics principles and engineering solutions:

- Tracking and telemetry: Ground stations use radar and other tracking technologies to monitor satellite orbits and ensure that they are functioning correctly. These technologies allow adjustments to be made to satellite orbits, compensating for factors such as atmospheric drag in LEO or minor gravitational perturbations from the Moon and Sun.
- Fuel efficiency and station-keeping: Satellites must carry fuel to adjust their orbits, especially in LEO, where atmospheric drag can gradually pull them out of orbit. Engineers use physics-based models to calculate the most efficient ways to maintain orbit, reducing the need for frequent adjustments and extending the satellite's operational life.

Impact on society

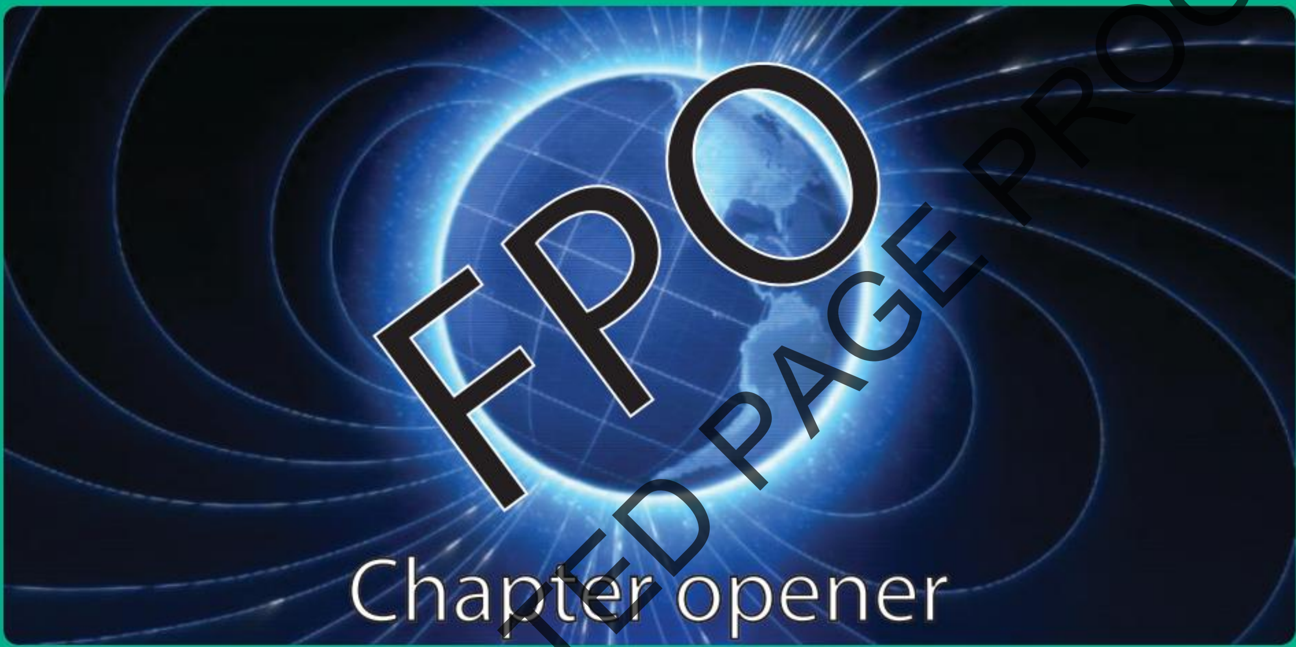
The careful positioning of satellites and the continuous innovation in satellite technology have profound implications for society. By observing weather patterns, tracking natural disasters, and supporting communications, satellite systems help improve public safety, environmental monitoring and global connectivity. Examples include:

- Weather forecasting: The real-time data provided by weather satellites helps meteorologists predict storms, monitor climate change and provide early warnings for severe weather events, potentially saving lives.
- Disaster response: Satellites play a critical role in monitoring natural disasters such as earthquakes, tsunamis and wildfires, providing real-time imagery that helps first responders assess damage and plan rescue operations.
- National security: Satellites used for military purposes help nations monitor borders, track potential threats and ensure global security.
- Global communication: The infrastructure for modern communication relies heavily on satellite networks, enabling internet access, television broadcasts and telephone services across the world, including in remote and rural areas.

The science behind satellite positioning is a perfect example of how physics and technological innovation can converge to serve humanity. By understanding the physical laws that govern orbital motion and applying advanced technologies for monitoring and control, satellites provide a wealth of information that benefits society on a daily basis. From weather forecasting to global communication and disaster management, satellite networks illustrate how human endeavour in science continues to shape the world around us.

CHAPTER 7

Electrostatics



TK

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Describe Coulomb's Law.
- Solve problems using $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{r^2}$.
- Describe the concepts of electric fields, electric field strength and electrical potential energy.
- Solve problems involving electric field strength using $E = \frac{F}{Q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$.
- Solve problems involving the work done when an electric charge is moved in an electric field using $V = \frac{\Delta U}{q}$.

SCIENCE INQUIRY

- Investigate the effects of electrostatic charge on various materials, e.g. on trickling water.

9780170483704

Introduction

There are four fundamental forces: the strong nuclear force, the weak nuclear force, the gravitational force and the electromagnetic force. The electromagnetic force is a combination of the electrostatic and magnetic forces. All objects with charge emanate an electric field. As charged particles come close to each other, their behaviour can be described by determining the type of charge on the particle, its electric field, and how far it is from another particle of charge.

Assessments
- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practicals
- Investigating the magnification of a microscrope

Worksheets
- Name
- Name
- Name

FPO

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ A non-contact force is a force that can act on an object without direct physical contact.
- ✓ Electric or electrostatic charge can be positive or negative.
- ✓ An electron is a negatively charged subatomic particle.
- ✓ A proton is a positively charged subatomic particle.
- ✓ An inverse-square relationship between variables x and y means $y \propto \frac{1}{x^2}$.
- ✓ The gradient of a linear graph can be calculated using $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$.
- ✓ SI units may be used with prefixes such as kilo (k), micro (μ) and milli (m).
- ✓ The law of conservation of energy states that energy in a system must be conserved.
- ✓ The ampere (A) is the unit for the current in a circuit and the volt (V) is the unit for the voltage in a circuit.
- ✓ $W = Fs$.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe Coulomb's law
- ✓ solve problems using $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{r^2}$
- ✓ describe and analyse the proportionality that exists between F and r in electrostatic interactions
- ✓ interpret experimental data to confirm the existence of an inverse-square relationship
- ✓ interpret or construct representations of electric fields by using field lines
- ✓ solve problems involving electric field strength, using $E = \frac{F}{Q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$
- ✓ solve problems involving the work done and electric potential when an electric charge is moved in an electric field, using $V = \frac{\Delta U}{q}$
- ✓ describe potential difference.

7.1 Coulomb's law

If a particle has a surplus or deficit of negatively charged electrons, the particle is considered to be negatively or positively charged respectively. If two charged particles are separated by a distance r, they exert an equal and opposite force on each other. This force can be classified as attractive or repulsive, depending on the nature of the charge on each particle.

Force exerted on charged particles

If two positively charged particles are brought close to each other, they experience a **repulsive force**; they will repel and move away from each other. If two negatively charged particles are brought close to each other, they will also repel. Conversely, if a positively charged particle is brought close enough to a negatively charged particle, they will experience an **attractive force** and move towards each other. This can be summarised as the **first law of electrostatics**.

The force that any two particles exert on each other is equal (Figure 7.1.1), and the magnitude of the force can be determined with **Coulomb's law**. Coulomb's law is also known as the **second law of electrostatics**

repulsive force when two particles of like charge are forced away from each other

attractive force when two particles of unlike charge are forced towards each other

first law of electrostatics like charges repel and unlike charges attract

Coulomb's law (the second law of electrostatics) the force exerted between two point charges is directly proportional to the product of their electric charges, inversely proportional to the square of the distance between them and inversely proportional to the permittivity of the surrounding medium; $F = \frac{kQq}{r^2}$

The movement of charges due to a repulsive force is described simply by Newton's third law. The forces F(by q on Q) and F(by Q on q):

- are equal in magnitude
- are opposite in direction
- have the same fundamental nature
- each act on a different object.

KEY FORMULA

Coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{d^2}$$

where:

F = electrostatic force q exerts on Q (N)
q = charge (C) of one point charge
Q = charge (C) of the other point charge
r = separation distance of q and Q (m)
ε_0 = permittivity of free space
$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$; also known as Coulomb's constant, k

Coulomb's law (simplified)

$$F = \frac{kQq}{r^2}$$

Weblinks
Coulomb's law
Coulomb's law video
Coulomb's law: calculating the electrostatic force.

Coulomb's law states that the force that one charged particle exerts on a second charged particle can be found if the magnitudes of both charges are known, and the distance between them can be measured. Negatively charged particles are substituted into Coulomb's law with a negative sign. If a force is calculated to be negative, it means the force between the two particles is attractive. If the force is calculated to be positive, it means the force between the two particles is repulsive.

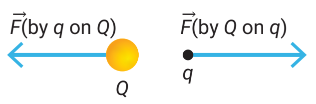


FIGURE 7.1.1 Two like-charged particles exert the exact same repulsive force on each other. This causes the particle Q to move to the left (from the force exerted by the interaction with charge q) and the particle q to move to the right (from the force exerted by the interaction with particle Q).

WORKED EXAMPLE 7.1.1

Calculate the force exerted by a proton on an electron at a distance of 8×10^{-11} m. This is the orbital radius of an electron in hydrogen.

ANSWER

1 Determine the formula.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

$$= \frac{kQq}{r^2}$$

2 Substitute the known values.

Recall that the charge on an electron is -1.6×10^{-19} C and the charge on a proton is 1.6×10^{-19} C.

$$F = 9.0 \times 10^{9} \times \frac{-1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(8 \times 10^{-11})^{2}}$$

3 Calculate the answer.

$F = -3.6 \times 10^{-8}$ N

The negative sign indicates a force of attraction between the proton and the electron.

PRACTICAL ACTIVITY 7.1.1

ELECTROSTATIC FORCE

Research question

How do charged objects move charged stationary objects by electrostatic force?

Aim

To observe how charged objects can move electrically charged stationary objects by electrostatic force

RISK ASSESSMENT

What are the risks in doing this activity?	How can you manage these risks to stay safe?
Depending on the size of the electroscope, it could cause injury if dropped.	Be sure to place the electroscope properly on the bench top.

Materials

- electroscope
- fur
- Perspex rod
- balloon

Procedure

Part A

1 Set up the electroscope. Wrap the fur around the Perspex rod and quickly move the fur up and down the rod. This will cause an electron transfer from the fur to the rod, making the rod negatively charged.
2 Remove the fur and bring the charged end of the Perspex rod close to the conduction plate of the electroscope.
3 Observe what happens to the leaves of the electroscope.

Part B

1 Blow up the balloon and rub it rapidly on your head until you notice that hairs are sticking up to the balloon.
2 Turn on a tap so there is a light stream of water coming from it.
3 Bring the charged part of the balloon you were just rubbing on your head close to the water stream without actually touching it to the water. Observe what happens.

Analysis of results

1 From your knowledge of electrons, explain why the electroscope leaves and the water move when a charged object comes close.
2 In Part A, do you think you would have noticed anything different if the rod was positively charged instead of negatively charged? Explain your answer.

LEARNING CHECK 7.1

DESCRIBING

1 **Contrast** an attractive force and a repulsive force.

2 State the values and units for charge on an electron and proton.

3 State the formula used to determine the force exerted on a charge from another charge over a distance.

4 Show that the permittivity of free space has a magnitude of 8.84×10^{-12}, using $k = 9 \times 10^9$ and the formula for Coulomb's law $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ or $F = \frac{kQq}{r^2}$.

APPLYING

5 A charged particle of 8 μC is separated from a charged particle of –7 μC by a distance of 20 mm. **Calculate** the force that these two particles exert on each other and state whether the force is attractive or repulsive.

6 Two charges of 9 μC are separated by 30 cm. **Calculate** the force of repulsion that these charges exert on each other.

7 **Calculate** the force on an electron due to a helium nucleus containing two protons and two neutrons at a separation distance of 8×10^{-11} m.

8 **Calculate** the force on the helium nucleus due to an electron at a distance of 31 pm. State the direction of the force.

7.2 Solving problems using Coulomb's law

Solving problems with Coulomb's law is more complicated than simply putting numbers into a formula. When experimental data is obtained for a known relationship, the data can be manipulated to find an unknown constant in the experiment. Additionally, formulas can be algebraically manipulated to acquire a deeper understanding of the relationships between variables.

Algebra: relationships between variables

It is possible to determine how much the force on a charged particle changes, even if we do not know specific values. For example, we can determine how the force on a charged particle changes if the distance between it and another charged particle increases or decreases by a given factor. Additionally, we can determine how the force of a charged particle changes if the magnitude of the charge on the other charged particle increases or decreases by a given factor.

Consider the following example. Two point charges q and Q exert a force F on each other when separated by a distance r. This can be represented as:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

$$F = \frac{kQq}{r^2}$$

If the distance separating these two charges doubles, this can be represented as:

$$F_{2r} = \frac{kqQ}{(2r)^2}$$

The denominator can be simplified as:

$$F_{2r} = \frac{kqQ}{4r^2}$$

This can be further separated as:

$$F_{2r} = \frac{1}{4}\frac{kqQ}{r^2}$$

By using our initial information that $F = \frac{kqQ}{r^2}$, we see that:

$$F_{2r} = \frac{1}{4}F$$

This shows that as we double the distance r between q and Q (i.e. increase the distance by a factor of 2), we quarter the force the charged particles exert on each other.

Graphing: relationships between variables

Relationships between variables are not always simple. In your study of physics, you have been exposed to a range of proportionalities. These include direct proportionalities, squared proportionalities and inverse proportionalities.

WORKED EXAMPLE 7.2.1

A charge q is separated from a charge Q by a distance r.

a How does the force on q change if the magnitude of the charge on Q is doubled?

b How does the force on q change if q and Q remain the same, but the distance between them decreases by a factor of 3?

ANSWERS

a

1 Determine the formula.

$$F = \frac{kqQ}{r^2}$$

2 Determine the impact of doubling Q.

$$F_{2Q} = \frac{kq2Q}{r^2}$$

Separating the 2 from the numerator gives:

$$F_{2Q} = 2 \times \frac{kqQ}{r^2}$$

and substituting in $F = \frac{kqQ}{r^2}$, we can conclude that:

$$F_{2Q} = 2 \times F$$

When the charge Q doubles in magnitude, the force exerted on q doubles as well.

b

1 Determine the formula.

$$F = \frac{kqQ}{r^2}$$

2 Determine the impact of decreasing the distance by a factor of 3.

$$F_{\frac{r}{3}} = \frac{kqQ}{\left(\frac{r}{3}\right)^2}$$

The denominator can be simplified as:

$$F_{\frac{r}{3}} = \frac{kqQ}{\frac{r^2}{9}}$$

which can be further simplified as follows:

$$F_{\frac{r}{3}} = 9\left(\frac{kqQ}{r^2}\right)$$

Substituting $F = \frac{kqQ}{r^2}$ into the equation, we can conclude that:

$$F_{\frac{r}{3}} = 9F$$

When the separation distance between q and Q is decreased by a factor of 3, the force is 9 times stronger.

This quantitatively verifies what we stated as the second law of electrostatics: that the force exerted between two point charges is directly proportional to the product of their strengths and inversely proportional to the square of the distance between them.

$F \propto Qq$ and $F \propto \frac{1}{r^2}$

An excellent way to determine the relationship between variables is to graph them. If a linear function is obtained when the two variables are plotted against each other, the relationship is direct. If a parabolic function is obtained, the relationship is squared. Sometimes it is hard to determine the type of function from a set of data; however, using a spreadsheet program to draw a regression line and find an appropriate function describing that line can be very helpful.

Experimentally, if q is separated by a fixed distance from Q and the charge on Q continues to increase in magnitude, we will obtain a graph similar to that in **Figure 7.2.1a**. If q and Q are fixed in charge and magnitude, and we increase the distance between them, we obtain a graph similar to that in **Figure 7.2.1b**.

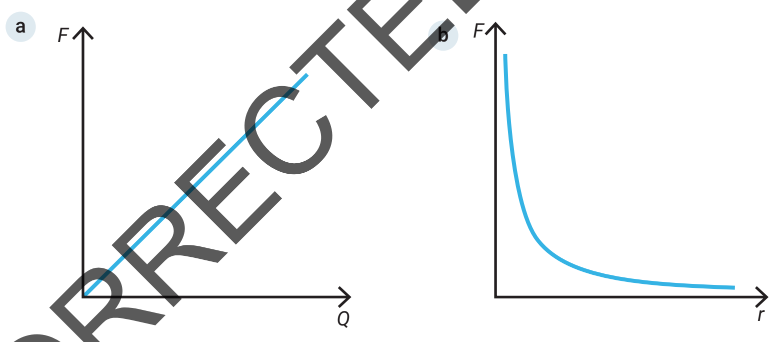


FIGURE 7.2.1 (**a**) A linear function. This function has a constant gradient, showing that the factor by which Q changes will directly affect the force F in the same fashion. (**b**) An inverse-squared function. This function shows that as the separation distance is increased, the force F between the two charged particles decreases dramatically and in an inverse-squared relationship.

In Coulomb's law, we can deduce the following relationships:

$$F \propto q$$

$$F \propto Q$$

$$F \propto \frac{1}{r^2}$$

These are found from the theoretical relationship of Coulomb's law. It is important to note that relationships are stated between variables, not constants. It does not make sense to say that $F \propto k$ because constants are used to help equate variables.

Manipulating data with Coulomb's law

When collecting data, it is very important to manipulate the data in order to find constants that equate the variables tested. At any given time, only one variable can be changed (the independent variable, usually plotted on the *x*-axis) to test its effect on another variable (the dependent variable, plotted on the *y*-axis); all other variables in the experiment remain constant.

The effect on *F* of changing *q*

As one point charge changes in magnitude, the other point charge and the distance between the point charges remains constant. It is possible to plot *F* against *q* in order to determine the type of relationship between *F* and *q*.

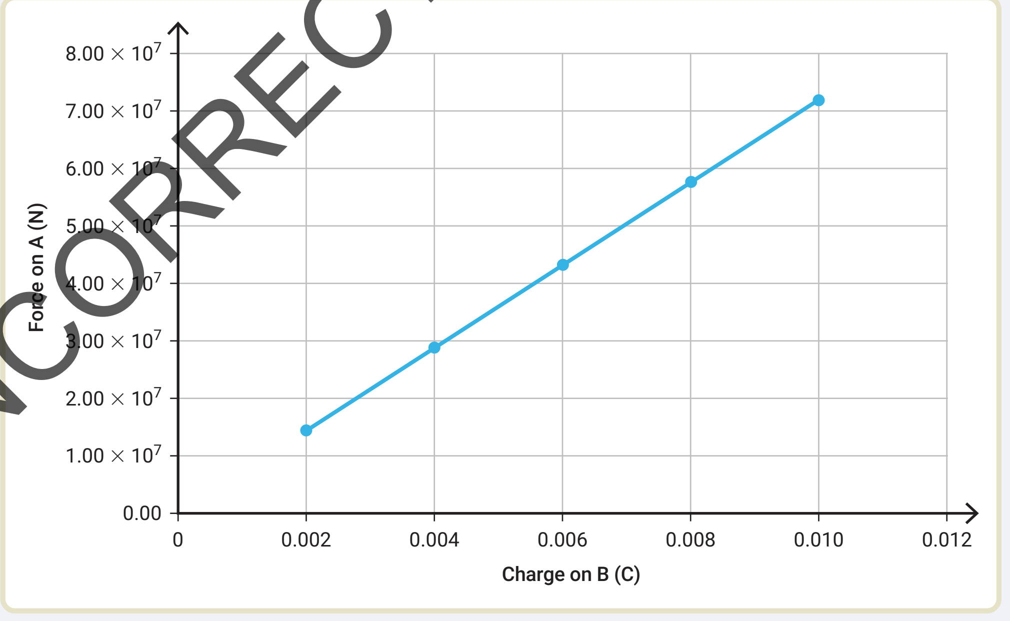
WORKED EXAMPLE 7.2.2

Two point charges A and B are separated by a distance of 50 cm. Point charge A has a fixed charge of 2 mC, and point charge B can have its charge varied. The force on point charge A was measured as the charge on point charge B increased and the data recorded in the table.

Charge on B (C)	Force on A (N)
0.002	1.44×10^7
0.004	2.88×10^7
0.006	4.32×10^7
0.008	5.76×10^7
0.010	7.20×10^7

Plot the force on A against the charge on B to determine the type of relationship between *F* and *q*.

ANSWER



Note that the relationship between *F* and *q* is directly proportional. As *q* increases, *F* increases proportionally. This is indicated by the linear trend line.

The effect on *F* of changing *r*

As both point charges remain constant but the distance between them changes, it is possible to plot the force F that the point charges exert on each other against the separation distance r, to determine the type of relationship between F and r. This relationship is reinforced when the data is organised linearly.

The inverse-squared law

When graphing data, there are many types of curves that look quite similar, and it is very hard to distinguish between them if a relationship is being looked for with no prior knowledge (**Figure 7.2.2**). This is where the process of manipulating data to demonstrate a linear relationship becomes helpful.

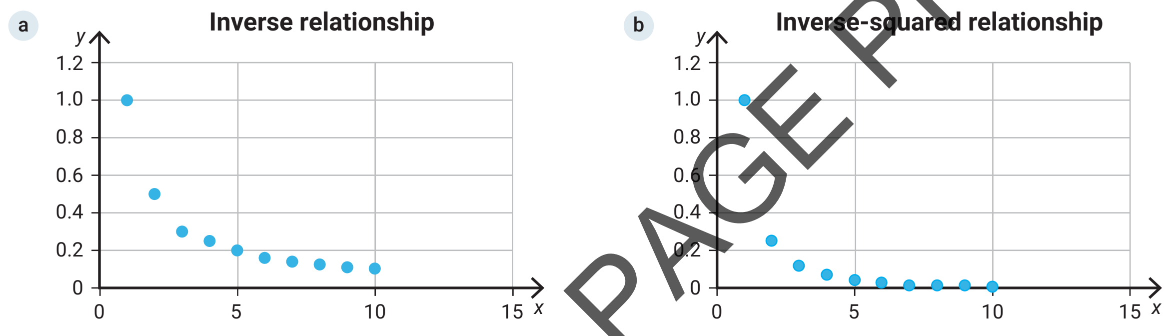


FIGURE 7.2.2 (**a**) An inverse relationship. (**b**) An inverse-squared relationship. Note how similar these curves look and how it may be difficult to distinguish between these relationships when no prior knowledge of the relationship or equation is known. A graph of $y \propto \frac{1}{x}$ and $y \propto \frac{1}{x^2}$ would assist in determining the nature of the relationship.

WORKED EXAMPLE 7.2.3

The following data was collected when like charges of 5 μC were separated by an increasing distance *r*. By manipulating data to show a linear relationship, find an experimental value for Coulomb's constant, *k*.

F (N)	*r* (m)
2250	0.01
563	0.02
250	0.03
141	0.04
90	0.05
63	0.06
46	0.07
35	0.08
28	0.09
23	0.10

ANSWER

From theory, we know that the relationship between *F* and *r* is an inverse-squared relationship. Making a new column for $\frac{1}{r^2}$ we obtain the following:

F (N)	*r* (m)	$\frac{1}{r^2}$ (m^{-2})
2250	0.01	10 000
563	0.02	2500
250	0.03	1111
141	0.04	625
90	0.05	400
63	0.06	278
46	0.07	204
35	0.08	156
28	0.09	123
23	0.10	100

1 Determine the formula.

Separating Coulomb's law, we can see that:

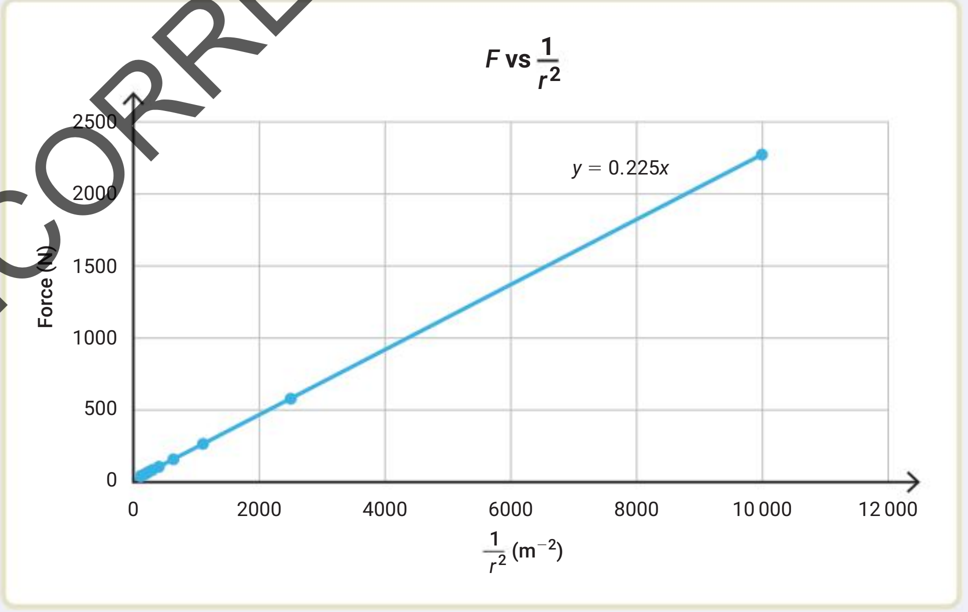
$$F = \frac{kqQ}{r^2}$$

$$F = kQ\frac{1}{r^2}$$

$$y = mx$$

2 Draw a graph to represent the relationship between *F* and $\frac{1}{r^2}$.

By plotting *F* against $\frac{1}{r^2}$, the gradient will be equal to *kqQ*.



3 Determine the formula to calculate *k*.

Gradient = m = 0.225 N m^2

$0.225 = kqQ$

$$k = \frac{0.225}{qQ}$$

4 Calculate the answer.

$$k = \frac{0.225}{5 \times 10^{-6} \times 5 \times 10^{-6}}$$

$\therefore\ k = 9 \times 10^9$ N m^2 C^2

As expected.

LEARNING CHECK 7.2

DESCRIBING

1. State the relationship between force and distance for two charged particles *Q* and *q*.

APPLYING

2. A charge of 4 mC is separated from a charge of −2 mC by 2.5 m. **Calculate** the force these particles exert on each other. Remember to also state the nature or direction of the force.
3. **Calculate** the distance between charges of 7 µC and 12 µC when the force they exert on each other is 1.89 N of repulsion.
4. Two charges of the same charge and magnitude are separated by a distance *r*. How does the force of repulsion change when the distance between these charges is tripled?
5. Two unlike charges, one double the charge of the other, are separated by a distance *r*. By what factor does the force of attraction between them change when the distance between them is halved?
6. For two point charges *Q* and *q*, how does the force on *Q* change when *q* is halved and the separation distance is doubled?
7. For two point charges *Q* and *q*, how does the force on *Q* change when the distance between *Q* and *q* is increased by a factor of 5?

ANALYSING

8. Two point charges of −50 µC each were separated by a distance *r*. The force exerted by each charge was measured with a Coulomb meter at increasing separation distances, as shown in the table.

r (m)	*F* (N)
0.05	9001
0.10	2250
0.15	992
0.20	566
0.25	365
0.30	251
0.35	180
0.40	147
0.45	120
0.50	92

Plot *F* on the *y*-axis against $\frac{1}{r^2}$ on the *x*-axis and use the gradient to determine Coulomb's constant, *k*.

SYNTHESISING

9 An experiment in which two point charges of 3.5 μC and −5.5 μC were separated by a distance *r*. The force exerted by each charge on the other was measured with a Coulomb meter at increasing separation distances, as shown in the table.

r (mm)	*F* (N)
5	7000
10	1742
15	203
20	431
25	281
30	192
35	137
40	106
45	89
50	70

Graphically **analyse** the data to determine an experimental value for k and ε_0. **Identify** any anomalies.

7.3 Electric fields

electric field the field due to an electric charge, which applies a force on other electric charges

All objects with electric charge emanate an **electric field** around themselves. This field depends on the size of the charge, and the distance from the charged object. In section 7.2, you used Coulomb's law to calculate the magnitude of the force that charged particles exerted on each other. The reason these particles exert either an attractive or repulsive force on each other is due to their electric field.

Weblinks
Electric charge and electric fields
NASA discovers a long-sought global electric field on Earth

Electric field strength

Electric fields emanate outwards from charged objects or particles. The electric field at a point is defined as the force per unit charge that acts on a small positive test charge at that point (Figure 7.3.1).

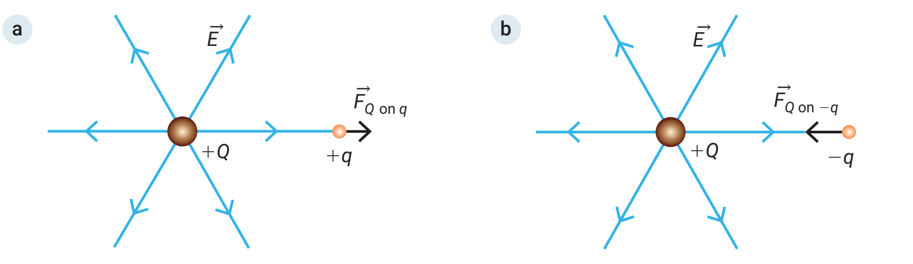


FIGURE 7.3.1 (**a**) A large positive charge repels a small positive test charge. (**b**) A large positive charge attracts a small negative test charge. Note how the electric field radiates out from the large positive test charge.

 9780170483704

The electric field strength at any point can be determined from:

$$E = \frac{F}{q}$$

Force is a vector, which means that the electric field is also a vector quantity and has both magnitude and direction. An electric field exerts a force in the direction of the field on a positive charge, and in the opposite direction on a negative charge. Table 7.3.1 shows the approximate electric field strength and direction of different appliances and electric sources.

KEY FORMULA

Electric field strength

$$E = \frac{F}{q}$$

where:
E = electric field strength ($N\ C^{-1}$)
F = force acting on test charge q (N)
q = charge of the test object in the field (C)

TABLE 7.3.1 Some typical electric field strengths at everyday distances

Electric field due to ...	Approximate field strength ($N\ C^{-1}$)
a hairdryer, 20 cm away	4
a thunderstorm	50, upwards
Earth's fair-weather field	100, downwards
high-voltage overhead power lines, 30 m away	10–1000
an electric blanket, 10 cm away	2000

WORKED EXAMPLE 7.3.1

A battery is connected across a piece of copper wire resulting in an electric field of $3.0 NC^{-1}$ in the wire. What is the force on an electron in this wire?

ANSWER

1 **Determine the formula.**
$E = \frac{F}{q}$

2 **Rearrange the formula to find *F*.**
$F = Eq$

3 **Substitute the known values.**
We know that $E = 3.0\ N\ C^{-1}$ and that $q = -1.6 \times 10^{-19}$ C.
$F = 3.0 \times -1.6 \times 10^{-19}$

4 **Calculate the answer.**
$F = -4.8 \times 10^{-19}$ N
The force is in the opposite direction to that of the field.

KEY FORMULA

Electric field strength when *q* is unknown

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

where:

Q = large charge (C)

r = distance (m) between q and Q

$\frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9\ \mathrm{N\ m^2\ C^{-2}}$ also known as Coulomb's constant, k

Electric field strength simplified:

$$E = \frac{kQ}{r^2}$$

The electric field strength can also be found if the force is unknown, but the charge emanating from the electric field is known, as well as the distance the test charge is from the source. The greater the distance between the test charge and the source of the electric field, the smaller the force the test charge will experience, and hence the smaller in magnitude the electric field is at that point.

As

$$E = \frac{F}{q} \qquad (1)$$

and

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \qquad (2)$$

We can substitute (2) into (1) to obtain the following:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\cancel{q}Q}{r^2} \times \frac{1}{\cancel{q}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

and

$$E = \frac{kQ}{r^2}$$

The electric field from Q at an infinite distance away from a test charge q is $0\ \mathrm{N\ C^{-1}}$. From this logic, if r is large enough, the electric field from Q is approximated to be zero.

WORKED EXAMPLE 7.3.2

What is the electric field strength on a test charge q if it is placed at a distance of 5.5 m from a source charge of 4.6 μC?

ANSWER

1 **Determine the formula.**

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

2 **Substitute the known values.**

$$E = 9 \times 10^9 \times \frac{4.6 \times 10^{-6}}{5.5^2}$$

3 **Calculate the answer.**

$E = 1369\ \mathrm{N\ C^{-1}}$

It is important to note that in all calculations between a large charge Q and test charge q, the field of Q is so much larger than that of q that the field of q is not represented in diagrams.

Electric field lines

electrostatic field model the model that assigns an electric field to stationary charges; it is this field that exerts forces on other charges

Fields can be represented by field diagrams. An electric field diagram has lines with arrows to show the direction of the force on a small positively charged test particle. This model is called the **electrostatic field model**.

To draw a field diagram, start by considering the force on a test charge at various points. Start with a single positive charge and think about what happens when you put a small positive test charge close to it. Like charges repel, so the test charge will be accelerated away from the positive charge. By looking at these force vectors in Figure 7.3.2, we are able to extrapolate these force vectors into **electric field lines** that demonstrate the direction a positive test charge will move when placed close to the source. Conversely, if we apply the same logic with the same force vectors with a positive test charge being acted on by a large negative charge, we obtain the field lines pointing in the opposite direction (Figure 7.3.3).

electric field lines net lines of force pointing in the direction a positive test charge will move when placed in the electric field due to a charge *Q*

Weblink
Electric field lines

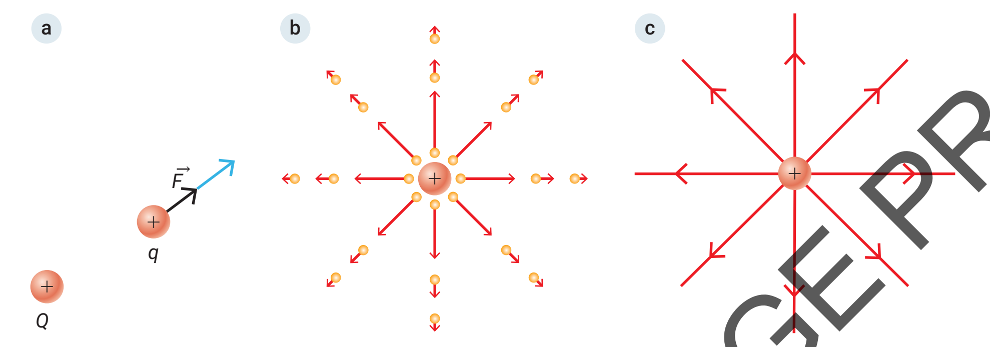


FIGURE 7.3.2 (a) A test charge is accelerated away from the positive source charge. (b) The size of the vector decreases as the distance between test charge and source charge increases. (c) Join up vectors to obtain electric field lines. Note that field lines always point *away* from a positive charge.

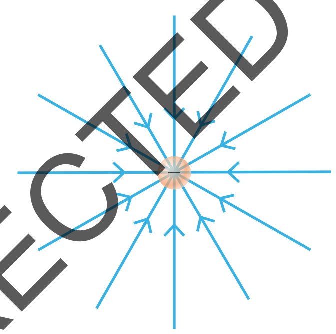


FIGURE 7.3.3 A field line diagram for a negative point charge. Note that the field lines always point *towards* a negative charge.

Field lines of two close charges

A positive and negative charge are attracted to each other. Coulomb's law allows us to calculate the magnitude of this attraction, and field lines allow us to visually represent how this attraction occurs.

Conventions for drawing electric field lines:

- Field lines point in the direction of the force acting on a positively charged particle due to the field.
- Field lines never cross, because they represent *net* lines of force.
- Field lines begin on positive point charges and end on negative point charges.
- Field strength is proportional to the density of the field lines.

Uniform electric fields

uniform electric field an electric field that has the same magnitude and direction at all points; characterised by parallel field lines

A **uniform electric field** is one that has the same magnitude and direction at all points. A uniform electric field exists close to any large, flat, uniform distribution of charge. On a small scale, a uniform electric field can be created by two charged parallel plates. This arrangement of charged plates is called a capacitor (Figure 7.3.4).

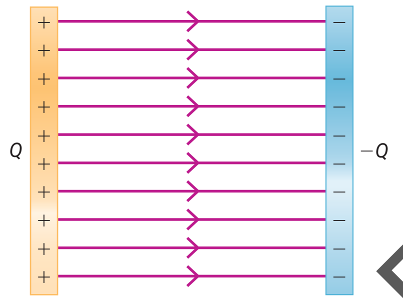


FIGURE 7.3.4 A uniform electric field formed between the plates of a parallel plate capacitor.

On a larger scale, we can look at the field very close to the surface of a large charged sphere, such as the dome of a van de Graaff generator. In this case, the field is approximately uniform. You have seen and used this useful approximation many times before for the gravitational field of Earth. Every time you write $F = mg$, you are making the approximation that Earth is flat. Most of the time this is perfectly reasonable. Close to the surface of Earth, the gravitational field is effectively constant.

This applies to Earth's electric field as well. You will be familiar with the effects of Earth's electric field in stormy weather (Figure 7.3.5). In sunny weather, Earth has an electric field of approximately 100 N C^{-1}, pointing downwards. Although the field varies over the surface of Earth, it can be treated as uniform for distances of a few hundred metres or less. The force due to Earth's electric field on a small charged particle (e.g. an electron) is many orders of magnitude greater than the force on it due to the gravitational field. For neutral objects such as humans, the gravitational force is much greater.

Weblinks
Earth's vertical electric field
Electrical potential energy

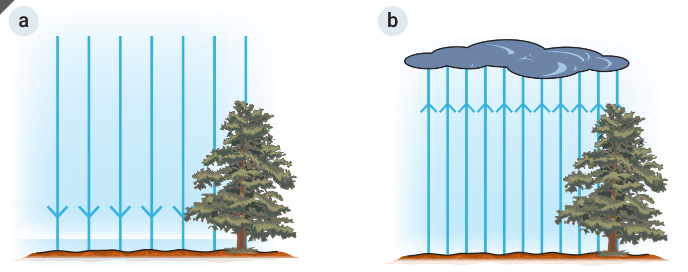


FIGURE 7.3.5 Earth's electric field in (a) sunny weather and (b) stormy weather. Note that in stormy weather, the field changes direction and is stronger, as is indicated by more densely drawn field lines.

Electrical potential energy

Just as the gravitational field does work on an object falling or being lifted in a gravitational field, the electric field does work on a charged object in an electric field. For example, when a pencil falls from a height h, the gravitational field does work on the pencil, and the pencil accelerates towards the ground. The same happens to a charged object when placed in an electric field. The charge will accelerate in the direction of the field. In both of these cases, the work done on the pencil or charge is equal to the change in potential energy of the system.

Although we refer to these objects as having potential energy, this is not strictly true. Rather, the objects within the field have potential energy. We could say Earth and the pencil combined have the potential energy, or even the gravitational field and the pencil. Potential energy exists whenever a force acts between objects, and hence a small isolated charge cannot be said to have potential energy on its own. It only has potential energy because of its interaction with an electric field.

Hence, fields are not only a way of exerting a force at a distance, *fields also store energy*. The gravitational field stores gravitational potential energy and the electric field stores **electrical potential energy**.

When considering a charge in an electric field, it is important to consider the **electrical potential** (or just 'potential' of that charge). Electrical potential has the same relationship to potential energy that the field has to force. The field is defined as force per unit charge $E = \frac{F}{q}$, and the potential is defined as the potential energy per unit charge $V = \frac{U}{q}$. The electrical potential, V, is measured in JC^{-1}, also known as volts.

The difference in potential for a charge at different points in the field is called the **potential difference** or, more commonly, voltage. The change in potential energy when a charged particle is displaced in a field is equal to the work done on the object. More specifically, the potential difference for a displaced charge in an electric field is equal to the work done per unit charge during the displacement.

electrical potential energy potential energy stored in an electric field; the change in potential energy of an object is also the work done on that object by the electric field

electrical potential potential energy per unit charge in an electric field, measured in volts (or $J\,C^{-1}$)

potential difference the difference in potential between two points in an electric field; work done per charge

KEY FORMULAS

Electrical potential

$$V = \frac{\Delta U}{q}$$

where:
V = electrical potential of a charge (V)
ΔU = change in potential energy (J)
q = magnitude of the charge in the field (C)

Potential difference

$$\Delta V = \frac{\Delta U}{q}$$

where:
ΔV = potential difference between two points (V)
ΔU = change in potential energy (J), also known as the work done on the charge
q = magnitude of the charge moving in the field (C)

WORKED EXAMPLE 7.3.3

An alpha particle of 3.2×10^{-19} C moves at a distance of 1.0 m along Earth's fair weather field lines. The field is 100 V m^{-1}, so the particle passes through a potential difference of 100 V.
What is the work done on the alpha particle?

ANSWER

1 Determine the formula.

$$\Delta V = \frac{\Delta U}{q} = \frac{W}{q}$$

$$W = q\Delta V$$

2 **Substitute the known values.**
$W = 3.2 \times 10^{-19}\,\text{C} \times 100\,\text{V}$

3 **Calculate the answer.**
$W = 3.2 \times 10^{-17}\,\text{J}$

The magnitude of the work done is 3.2×10^{-17} J as the alpha particle is moving with the field.

Positive and negative potential difference

A positively charged object released from rest in an electric field will be accelerated in the direction of the field. The force exerted on the object by the field acts to increase its kinetic energy. From conservation of energy, we know this kinetic energy must come from somewhere. It comes from a decrease in potential energy. Hence, the positive charge has moved from a point of higher electrical potential to one of lower electrical potential, so ΔV must be negative.

If a negatively charged object moves from higher to lower potential so that ΔV is negative, then the change in potential energy, ΔU, is positive. This can only happen if the charge has some initial kinetic energy, or if some external force is doing work on the system. This is shown in **Figure 7.3.6**. This is the case for an electron in a circuit passing through a battery. **Table 7.3.2** summarises the changes in potential and energy when a charge moves in a field. The field will do work on the charge when a positive charge moves with the field or a negative charge moves against the field. Work must be done on the system to make a positive charge move against the field or a negative charge move with the field.

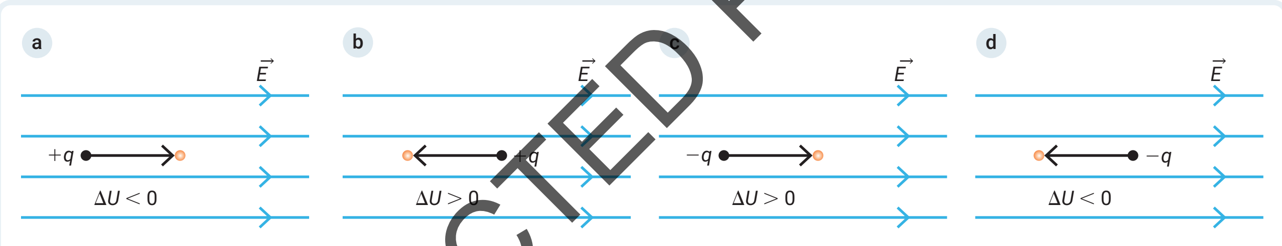


FIGURE 7.3.6 Kinetic energy and electric potential energy changes when a charge moves in an electric field. (**a**) A positive charge moving in the direction of the field experiences an increase in kinetic energy (E_k), and a decrease in potential energy, U, hence the potential difference is negative. (**b**) A positive charge moving against the direction of the field experiences a decrease in E_k, and an increase in U, and hence the potential difference is positive. (**c**) A negative charge moving in the direction of the field experiences a decrease in E_k, and an increase in U, and hence the potential difference is positive. (**d**) A negative charge moving against the direction of the field experiences an increase in E_k, a decrease in U, and hence the potential difference is negative. Note that if the change in potential is positive, the potential difference is also positive and vice versa.

TABLE 7.3.2 Changes in potential energy for a charge moving in an electric field

Charge	Movement with or against the field lines	Change in potential energy	Work done
+	With	Negative (decrease)	By the field
+	Against	Positive (increase)	On the field
–	With	Positive (increase)	On the field
–	Against	Negative (decrease)	By the field

WORKED EXAMPLE 7.3.4

An electron in an X-ray machine is accelerated through a potential difference of 100 kV before colliding with a target and emitting X-rays. What is the energy of the electron just before it hits the target?

ANSWER

1 **Determine the formula.**

If the electron accelerates from rest, before it hits the target all its potential energy will have converted into kinetic energy.

So:

$\Delta U = q\Delta V$

2 **Substitute the known values.**

$\Delta U = 1.6 \times 10^{-19}\,\text{C} \times 100\,\text{kV}$

3 **Calculate the answer.**

$\therefore\ \Delta U = 1.6 \times 10^{-14}\,\text{J}$

This is the energy the electron has just before it hits the target.

The zero of potential energy

The changes in the potential and potential energy of a system can be positive or negative, depending on how we define the **zero of potential energy**. Electrostatics uses the same convention as gravity in this circumstance, so that the zero of potential energy is when the components of the system are infinitely separated, and all the charges that make up the system are very far apart.

Consider the case of two positive charges very far apart. This arrangement has zero potential energy. If we want to bring them closer together into a final arrangement, we have to do work on them because they will repel each other. Work is applied to the system, so the final potential energy is positive. Now consider a positive and negative charge very far apart so that their arrangement has zero potential energy. These charges attract each other, and if we release them from very far apart, they will move towards each other. This causes an increase in kinetic energy and hence a decrease in potential energy. A system of a positive and a negative charge at any separation less than infinity has negative potential energy.

zero of potential energy when all charges in the system are infinitely far apart; any other arrangement has positive or negative potential energy

LEARNING CHECK 7.3

DESCRIBING

1 **Sketch** an electric field diagram round a point positive charge and around a point negative charge of twice the magnitude.

2 State the difference between electrical potential and electrical potential energy.

3 **Explain** how a change in potential energy for a charge moving in an electric field can be concluded to be positive or negative.

4 **Compare** a uniform electric field and a non-uniform electric field (such as a field produced from a point charge).

APPLYING

5 A current of 4.0 A flows through a 12 V battery for 2.0 s. What is the change in potential energy of the battery?

6 What is the electric field strength of a proton at a distance of 2 cm?

7.4 Solving problems in electric field strength

Solving problems involving electric field strength requires an understanding of the scenario. The electric field due to a point charge varies with the size of the charge, and decreases with the distance squared from the charge. Additionally, electric field strength can be determined by considering the force that one point charge Q exerts on a second point charge q.

WORKED EXAMPLE 7.4.1

Two conducting spheres each of charge 6.0 μC are hanging by insulating thread from a beam.

Find the electric field strength experienced by each sphere if they are 10 cm apart. Draw a diagram of the scenario showing the forces acting (F_e, F_t and F_w).

ANSWERS

1 **Determine the formula.**

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$$

2 **Substitute the known values.**

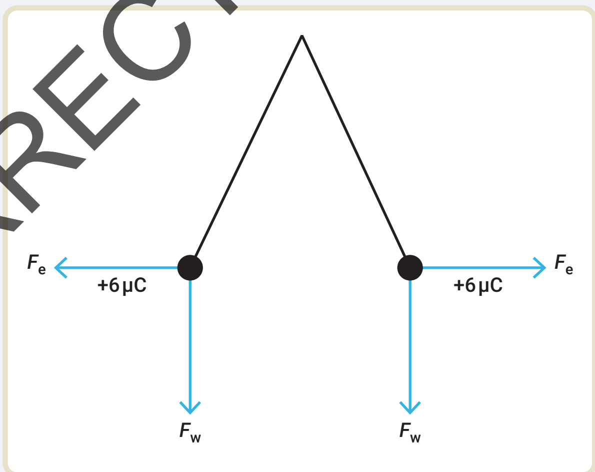
$$E = 9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-1} \times \frac{6.0 \times 10^{-6} \text{ C}}{0.1^2}$$

3 **Calculate the answer.**

$E = 5.4 \times 10^6 \text{ N C}^{-1}$

4 **Draw a diagram to represent the scenario.**

Electrostatic repulsion and force weight act and place the two charges into equilibrium.



WORKED EXAMPLE 7.4.2

If a charge of 4.6 μC is placed 11 m from a test charge q, what is the electric field strength the test charge experiences?

ANSWER

1 Determine the formula.

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$$

2 Substitute the known values.

$$E = 9 \times 10^9 \text{ N m}^2\text{C}^{-2} \times \frac{4.6 \times 10^{-6}\text{ C}}{\left(11\text{ m}^2\right)^2}$$

3 Calculate the answer.

$$E = 342 \text{ N C}^{-1}$$

Electric charges create electric fields, and electric fields store electrical potential energy. If a test charge is placed in an electric field, work will be done on or by the test charge. This is governed by the potential difference between two points in the electric field. Charges placed in electric fields can be initially stationary and move according to the field, or they can have some initial velocity.

Even though charges can be considered stationary when placed in electric fields, the fact that they have electrical potential energy means that they will end up moving. Charges have mass and in an electric field they experience a force, so they will behave according to Newton's laws of motion.

Newton's first law tells us that an object experiencing a net force will accelerate. Newton's second law quantifies this acceleration as $a = \frac{F}{m}$. From the definition of an electric field $E = \frac{F}{q}$, we can see that $F = Eq$; therefore, $a = \frac{Eq}{m}$. This is the acceleration a charge q will experience when in an electric field, due to the electrical potential.

KEY FORMULA

$$F = qE$$

$$a = \frac{qE}{m}$$

where:
F = force applied on a test charge (N)
a = acceleration of the test charge q (m s^{-2}, a vector quantity)
E = electric field strength (N C^{-1}, a vector quantity)
q = charge of the test charge q (C)
m = mass of the test charge (kg)

WORKED EXAMPLE 7.4.3

A positive calcium ion (Ca^{2+}) with mass of 6.7×10^{-26} kg experiences an acceleration of 4.8×10^{13} m s^{-2} as it moves through a channel in a cell membrane. What is the electric field in the membrane?

ANSWER

1 Determine the formula.

$$a = \frac{qE}{m}$$

2 Rearrange to find the unknown.

$$E = \frac{ma}{q}$$

3 **Substitute the known values.**

$$E = \frac{6.7 \times 10^{-26} \times 4.8 \times 10^{13}}{3.2 \times 10^{-19}}$$

4 **Calculate the answer.**

$E = 1.0 \times 10^{7}$ N C^{-1}

The electric field is in the direction of the acceleration of the positive charge.

WORKED EXAMPLE 7.4.4

Consider a sodium ion (Na^+) of mass 3.82×10^{-26} kg travelling through a channel in a cell membrane. If the electric field strength in the membrane is $1.0 \times 10^{7} \times$ N C^{-1}, what acceleration does the sodium ion experience?

ANSWER

1 **Determine the formula.**

$$a = \frac{qE}{m}$$

2 **Substitute the known values.**

$$a = \frac{1.6 \times 10^{-19} \times 1.0 \times 10^{7}}{3.82 \times 10^{-26}}$$

3 **Calculate the answer.**

$a = 4.19 \times 10^{13}$ m s^{-2}

LEARNING CHECK 7.4

APPLYING

1 **Calculate** the electric field strength 20 cm from a charge of 25 μC.

2 The electric field strength 12 cm from a charge is 1.0×10^{-3} N C^{-1}. **Determine** the size of the charge.

3 The force experienced by an electron in an electric field is 100 N. What is the magnitude of the electric field?

4 At what distance from an object carrying a charge of 4.0×10^{-8} C would the electric field strength be 2.3×10^{6} N C^{-1}?

5 It takes 155 J of work to bring a charge of +11 C from infinity to a positively charged conductive sphere. What is the electrical potential of this sphere?

6 What is the electrical potential at a distance of 2 cm from a charge of 5.0×10^{-7} C?

7 An electron moves in an electric field from a point at which the potential is 1.5 V to a point at which the potential is 3.5 V.

a **Determine** the change in the potential energy of the electron.

b Is this change positive or negative?

8 What is the acceleration of a sodium ion (Na^+) of mass 3.82×10^{-26} kg in an electric field of 5.0×10^{6} N C^{-1}.

ANALYSING

9 Two positive charges of 10 μC and 20 μC are situated 50 cm apart in air. **Determine** the electric field strength midway between these charges.

10 **Determine** the speed of particles accelerated from rest through a potential difference of 1000 V when the particle is:

a an electron?

b a proton?

9780170483704

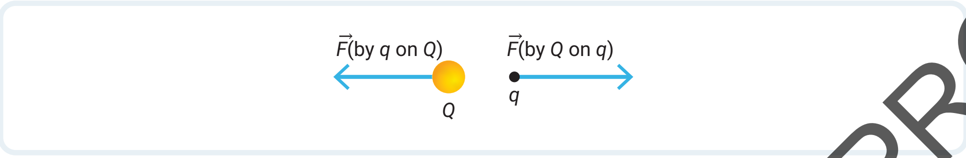
CHAPTER SUMMARY

Coulomb's law

- The electrostatic force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{kQq}{r^2}$$

- The force is attractive if the charges are opposite in sign and repulsive if the charges are of the same sign.

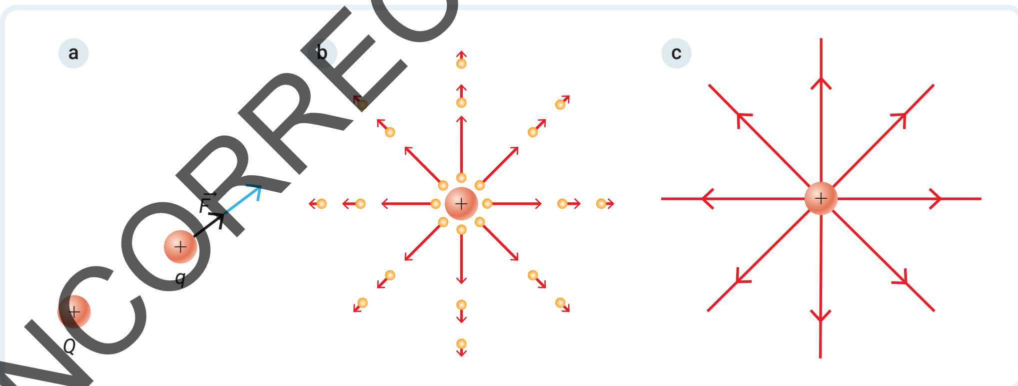


Electric fields

- An electric field, E, is a region around a charged particle where other charges experience a force.
- Electric field strength is the force per unit of charge experienced by a positive test charge placed in the field.

$$E = \frac{F}{q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$$

- Electric fields can be visually communicated by electric field lines, which point from positive charges to negative charges. The density of electric field lines indicates the strength of the electric field. Field lines never cross.
- Uniform electric fields have the same magnitude and direction at every point, often represented by equally spaced, parallel lines.



Electrical potential energy

- Electric fields do work on charged objects within the field.
- When a charged object is placed in an electric field, the charge accelerates in the direction of the field.

$$V = \frac{\Delta U}{q}$$

- The difference in electrical potential energy for a charge at different points in the field is called the potential difference, or voltage, V.
- When a positive charge moves against the direction of the electric field, or when a negative charge moves with the direction of the electric field, the electrical potential energy will increase as work is done on the field.
- When a negative charge moves against the direction of the electric field, or when a positive charge moves with the direction of the electric field, the electrical potential energy will decrease, as work is done by the field.

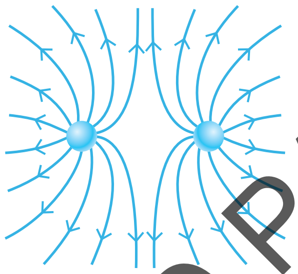
MULTIPLE CHOICE

CHAPTER EXAM

1. An attractive force would be observed in which of the following scenarios?
   A An electron placed near an electron
   B A proton placed near an electron
   C A proton placed near a proton
   D An alpha particle placed near a proton

2. A charge q is separated from a charge Q by a distance r. The force experienced between them is 1500 N. Which of the following must be true?
   A q and Q are attracted to each other.
   B q and Q have the same magnitude of charge.
   C q and Q have like charges.
   D q and Q have a magnitude of charge smaller than 1 C.

3. The field diagram below shows the interaction of two fields.



   These fields are due to:
   A two positive charges.
   B two negative charges.
   C two neutrally charged spheres.
   D a positive and negative charge.

4. Kinetic energy changes when a charge moves in an electric field. If the kinetic energy of the charge decreases, then the:
   A charge must be positive.
   B charge must be negative.
   C change in potential difference must be positive.
   D change in potential difference must be negative.

5. Which of the following are the units for an electric field?
   A $N\,C^{-1}$
   B $V\,m^{-1}$
   C E
   D kg mC

6. What work needs to be done to move a charge of 2.0×10^{-14} C across a potential difference of 6.0×10^{2} V?
   A 1.2×10^{-16} J
   B 1.2×10^{-11} J
   C 3.0×10^{-13} J
   D 3.6×10^{-15} J

7. Two charges, $q = 2.0\,\mu C$ and $q = -3.0\,\mu C$ are separated by 0.50 m. What is the magnitude of the electrostatic force between them?

A 0.22 N
B 1.08 N
C 2.16 N
D 3.60 N

8. If the distance between two charges is doubled, how does the electrostatic force between them change?

A It doubles.
B It is halved.
C It is quartered.
D It remains the same.

9. What is the electric field strength E at a point 0.25 m away from a point charge $q = 3.0\,\mu C$?

A 4.31×10^4 N C^{-1}
B 2.88×10^5 N C^{-1}
C 5.40×10^5 N C^{-1}
D 4.31×10^4 N C^{-1}

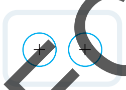
10. A charge $q = 5.0 \times 10^{-9}$ C moves through a uniform electric field of strength 500 N C^{-1} over a distance of 0.020 m. How much work is done on the charge?

A 5.0×10^{-9} J
B 5.0×10^{-8} J
C 1.0×10^{-7} J
D 2.0×10^{-7} J

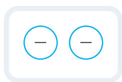
SHORT RESPONSE

11. **Sketch** the shape of the electric field due to the charges shown in the following regions.

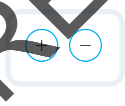
a



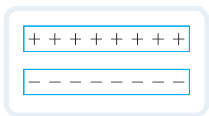
b



c



d



12. Within a helium nucleus, two protons are separated by a distance of 1×10^{-14} m. **Calculate** the size of the Coulomb force between them. Ensure you state whether it is attractive or repulsive.

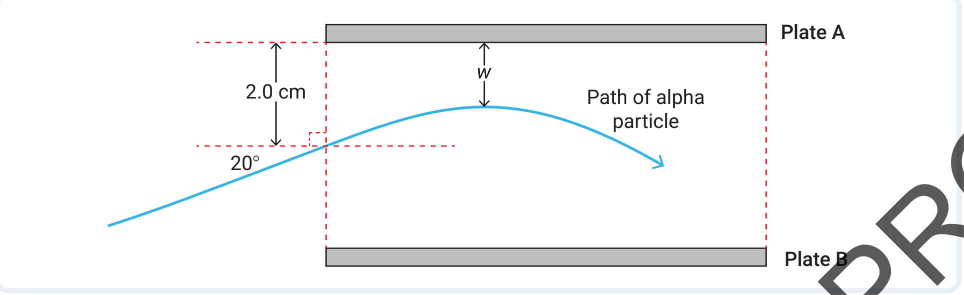
Note: The charge of a proton is of the same magnitude as a charge of an electron.

13. Four point charges, A, B, C and D, are arranged on corners of a square of sides 25 cm. If A and B each have a charge of $+1\,\mu C$ while C and D each have a charge of $+2\,\mu C$, what is the resultant force in a charge of $+1\,\mu C$ placed at the centre of the square?

9780170483704

CROSS-CHAPTER QUESTION

14. In a vacuum chamber, an alpha particle with a velocity of 1.200 m s^{-1} entered a region between two electrostatically charged plates (A and B) with a uniform electric field between them of 2.080×10^{-7} N C^{-1}. A top view (looking down on the apparatus) showing the path of the alpha particle is illustrated below.



a **Deduce** the charge (negative or positive) of plate A and plate B.

b **Describe** what the electric field lines would look like between the two plates.

c **Determine** distance *w* at the point in the alpha particle's path where it is closest to plate A.

DATA ANALYSIS

15. Analyse data

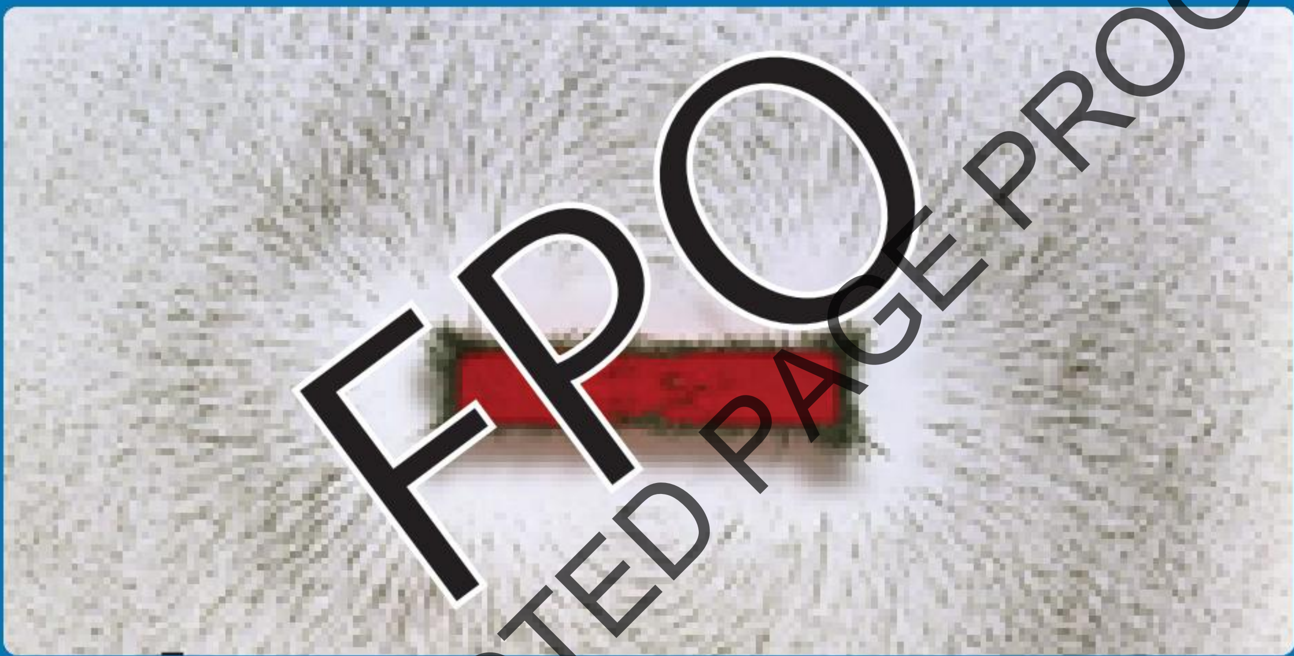
The following table shows how the electrostatic force changes between two 10 µC spheres for increasing separation distances *r*.

r (cm)	*F* (N)
5	3.60
10	0.95
15	0.40
20	0.22
25	0.15
30	0.10
35	0.05

Manipulate the data to obtain a linear relationship and use that to **determine** an experimental value for the constant *k* from the gradient.

CHAPTER 8

Magnetic fields



TK

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Describe the concept of a magnetic field.
- Sketch magnetic field lines due to a moving electric charge, electric currents and magnets.
- Describe the generation of a magnetic field from a moving electric charge.
- Solve problems involving the magnitude and direction of magnetic fields around a straight electric current-carrying wire and inside a solenoid using $B = \frac{\mu_0 I}{2\pi r}$ and $B = \mu_0 nI$.
- Describe the force experienced by electric current-carrying conductors and moving electric charges when placed in a magnetic field.
- Solve problems involving the magnetic force on an electric current-carrying wire and moving charges in a magnetic field using $F = BIL\sin\theta$ and $F = qvB\sin\theta$.
- Interpret data relating to the force acting on a conductor in a magnetic field.
- Interpret data relating to the strength of a magnet at various distances.

9780170483704

SCIENCE INQUIRY

- Investigate the force acting on a conductor in a magnetic field.
- Investigate the strength of a magnet at various distances.

Introduction

Magnetic fields are important to us in many ways. Organisms such as pigeons, sharks, bees and bacteria use Earth's magnetic field to navigate by, and humans have used it to navigate by for thousands of years. The properties of magnetic materials such as magnetite and iron in Earth's crust have only been explained in the last couple of hundred years. It is the properties and behaviour of fundamental particles such as electrons that give rise to the magnetic behaviour of metals such as iron. In order to understand magnetism, scientists need to extend the previous model of the electromagnetic field.

Assessments

FPO

- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practicals

- VISUALISING MAGNETIC FIELD LINES (online-only resource)
- Make your own electromagnetic (online-only resource)

Worksheets

- Name
- Name
- Name

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ Materials can be magnetic or behave like magnets.
- ✓ Magnets can exert forces on each other and other objects.
- ✓ A non-contact force is a force that can act on an object without direct physical contact.
- ✓ A field in physics is a region in which each point has a physical quantity associated with it.
- ✓ In physics, field lines are used to represent fields.
- ✓ A traditional directional compass is a device in which a movable needle orientates itself according to Earth's magnetic field.
- ✓ Electric current is the flow of electric charges and is measured in amperes (A).
- ✓ An alpha particle is a helium nucleus of known charge and mass.
- ✓ An electron is a subatomic particle of known negative charge and mass.
- ✓ A proton is a subatomic particle of known positive charge and mass.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe the concept of a magnetic field
- ✓ describe magnetic phenomena such as poles, domains, the laws of attraction and repulsion, the types of magnetic materials, Earth's magnetic field and aurora
- ✓ sketch magnetic field lines due to a moving electric charge, electric currents and magnets
- ✓ describe the generation of a magnetic field from a moving electric charge
- ✓ solve problems involving the magnitude and direction of magnetic fields around a straight electric current-carrying wire and inside a solenoid, using $B = \frac{\mu_0 I}{2\pi r}$ and $B = \mu_0 nI$
- ✓ describe the force experienced by electric current-carrying conductors and moving electric charges when placed in a magnetic field
- ✓ solve problems involving the magnetic force on an electric current-carrying wire and moving charge in a magnetic field, using $F = BIL\sin\theta$ and $F = qvB\sin\theta$
- ✓ interpret data relating to the force acting on a conductor in a magnetic field
- ✓ interpret data relating to the inverse-square relationship between the strength of a magnet and the distance from it
- ✓ use Maxwell's screw rule (the right-hand rule, or Ampere's right-hand rule) to predict the direction of concentric magnetic field lines around a current-carrying conductor or the direction of magnetic flux within a current-carrying solenoid
- ✓ use the right-hand rule to predict the relative orientation of the resultant force when a magnetic field acts on current-carrying conductor
- ✓ use the right-hand rule to predict the relative orientation of the resultant force when a magnetic field acts on a moving charge
- ✓ use the right-hand rule to predict the path of charged subatomic particles in a magnetic field.

8.1 Magnetic fields

magnetic field the field created by moving charges, including charges in magnetic materials and currents through conductors

A **magnetic field** is the field created by moving charges, including charges in magnetic materials (such as iron). The strength of a magnetic field depends on the source. If the magnetic field is produced from charges within a material (such as iron), the field depends on how much *alignment* there is within the atom. If the magnetic field is produced from moving charges, or a current, the strength is proportional to the current.

Electron alignment

Metals (e.g. iron, cobalt, nickel and ores such as magnetite) have natural magnetic properties. An object that is magnetised has its internal **magnetic domains** aligned. Each metal substance is abundant in electrons, and each electron can be thought of as having a north and south **magnetic pole** (Figure 8.1.1). This means that, for a magnet to be formed, all the small magnets within each material must point in the same direction. If the metal maintains these magnetic properties, it is referred to as a permanent magnet.

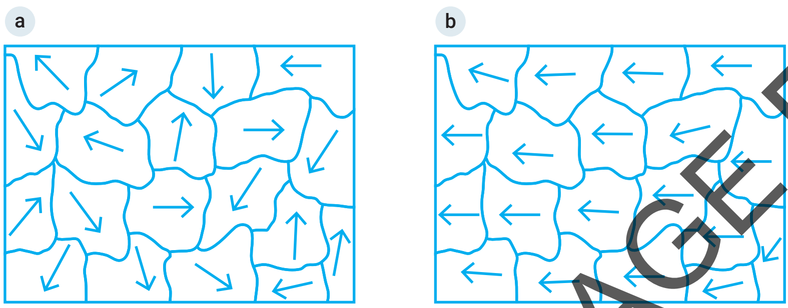


FIGURE 8.1.1 (**a**) Randomly aligned magnetic domains. The material is not magnetised as a result. (**b**) Magnetic domains are aligned. This material is magnetised and is therefore a permanent magnet.

Magnets are magnetic substances that have a large majority of their domains aligned. They exhibit magnetic fields and can, in turn, affect other magnetic materials nearby. Materials can be classified by the effect that nearby magnets have on them. It is important to note that if a magnetised material is cut in half, there would still be a north and south pole because each individual domain can be thought of as a miniature magnet. The strength of the magnetic field of a magnet depends on how many of the internal domains are aligned.

Types of magnetic materials

Michael Faraday (1791–1867) classified materials according to how they were influenced by nearby magnets. Materials can be **diamagnetic**, **paramagnetic** or **ferromagnetic**. Diamagnetic materials are weakly repelled by nearby magnets, paramagnetic materials are weakly attracted by nearby magnets, and ferromagnetic materials are strongly attracted to nearby magnets.

LEARNING CHECK 8.1

DESCRIBING

1 **Define**:
   a magnet
   b magnetic field.
2 **Explain** how a magnetic field is formed.
3 **Compare** diamagnetic, paramagnetic and ferromagnetic materials.
4 **Explain** how you might make a magnetic field with a length of conducting wire and a battery.

Weblink
Intro to magnetic fields

magnetic domain a region within a magnetic material where the magnetic properties point in the same direction

magnetic pole a point where magnetic field lines emerge from or return into

magnet a substance that has most of its magnetic domains aligned; produces a magnetic field

diamagnetic a material that is weakly repelled by nearby magnets (e.g. bismuth and copper)

paramagnetic a material that is weakly attracted to nearby magnets (e.g. aluminium and rare earth ions); does not retain permanent magnetism

ferromagnetic a material that is strongly attracted to nearby magnets (e.g. iron, nickel and cobalt); can retain permanent magnetism and this can be induced by other very strong magnets

Weblink
Magnetic properties

8.2 Representing magnetic fields

north pole the pole of a magnet where the field lines start; they are drawn coming out of the north pole

south pole the pole of a magnet where the field lines end; they are drawn entering the south pole

A magnetised material has a specific **north pole** and **south pole**. Magnetic field lines are defined from this property. Similar but not the same as electric field lines, magnetic field lines are drawn coming *out* from the north end of a magnet, and *in* to the south end **(Figure 8.2.1)**.

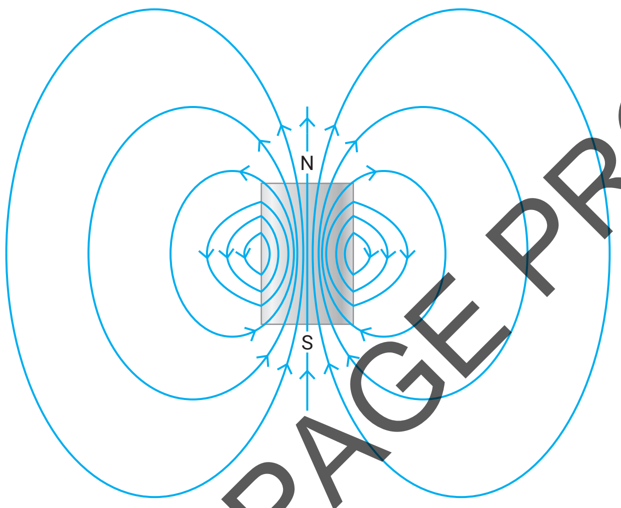


FIGURE 8.2.1 Magnetic field lines coming out of a bar magnetic. A bar magnet is a bar of metal in which all domains are aligned. Note that the arrows point out from the north pole, and in to the south pole.

KEY CONCEPT

Conventions for drawing magnetic field lines

1 Magnetic field lines do not cross.
2 Magnetic field lines are drawn in continuous loops.
3 Magnetic field lines are drawn from north to south outside the magnet (and from south to north inside the magnet).
5 The density of a magnetic field is shown by how close the field lines are drawn.

When a north pole of one magnet and a south pole of another magnet are brought in close proximity, they will *attract* and their field lines will align **(Figure 8.2.2a)**. When two like poles are brought into close proximity, they *repel* and their field lines will push away from each other **(Figure 8.2.2b)**.

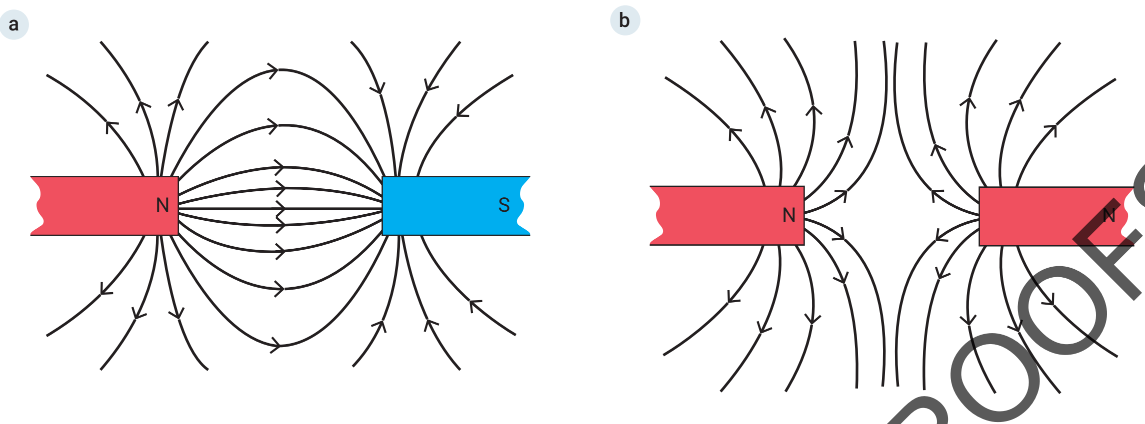


FIGURE 8.2.2 (a) Two unlike poles (north and south) brought together attract. (b) Two like poles brought together repel. Note the similarity between these field lines and the electric field lines in the previous chapter when like charges and unlike charges are brought near each other.

Earth's magnetic field

Earth is filled with many magnetic substances in its crust, mantle and molten core. Combined, these materials give Earth a magnetic field that many animals can sense and use to migrate. The magnetic poles of Earth are not in the exact same location as the geographical poles.

The magnetic pole closest to the north geographic pole is near Hudson Bay, Canada. The north compass needle points to the pole on Earth's surface, so it is actually the *south magnetic pole*. The magnetic pole closest to the south geographic pole is in Antarctica and about 2000 km from the south geographic pole. The south compass needle points to this pole on Earth's surface, so it is actually the *north magnetic pole* (Figure 8.2.3). The historical naming of the magnetic pole that the compass needle points to as 'north' was done in approximately 1600, well before the nature of Earth's magnetic field was fully understood.

Weblink

Earth's magnetic field is overdue a flip. Should we be worried?

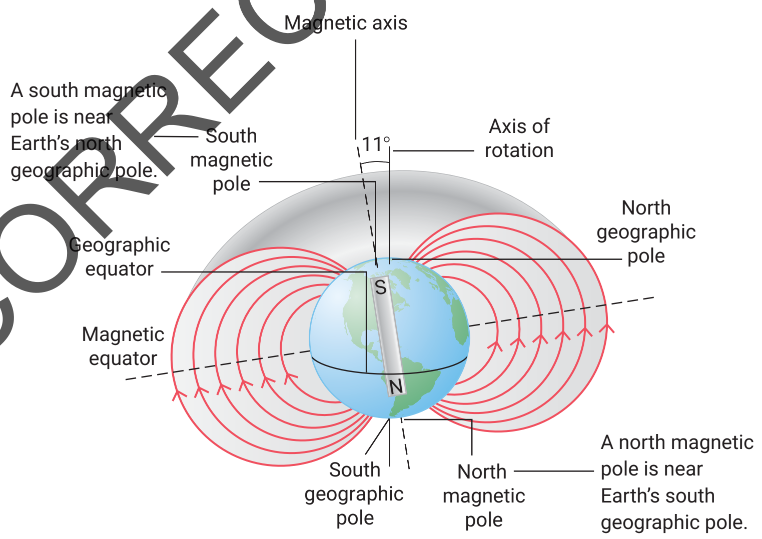


FIGURE 8.2.3 A two-dimensional representation of Earth's magnetic field. Notice that the geographic north pole is closer to the magnetic south pole, and conversely the geographic south pole is closer to the magnetic north pole. The field lines are all over Earth's surface as well as through its centre.

Wandering poles

Earth's magnetic poles 'wander' on a daily and annual basis. Explore this phenomenon, and suggest whether the geographic south pole and magnetic south pole will ever be at the same location.

Aurora Borealis and Aurora Australis

Weblinks
Auroras: The Northern and Southern Lights
Awesome aurora

Earth's magnetic field forms a shield that keeps out high-energy charged particles from space (mainly from the Sun). These particles would otherwise be extremely damaging to organisms on Earth. The few particles that manage to leak in through Earth's magnetic field follow helical paths towards the magnetic poles. When these high-energy charged particles reach the atmosphere, they collide with air molecules, which are then ionised. The energy released when the atoms and molecules re-form are the auroras – the Aurora Borealis or Northern Lights, and the Aurora Australis or Southern Lights **(Figure 8.2.4)**.

Getty Images/Stefan Christmann

FIGURE 8.2.4 The Aurora Australis (Southern Lights), like the Aurora Borealis, is generated by charged particles interacting with Earth's magnetic field. The field lines are more concentrated at the magnetic poles.

LEARNING CHECK 8.2

DESCRIBING

1 What pole do magnetic field lines come out of for diagrammatic purposes?
2 Consider three magnets at the corners of an equilateral triangle with their north poles facing inwards. Draw the magnetic field lines in this situation.
3 **Compare** the magnetic fields of bar magnets and horseshoe magnets.

APPLYING

4 If two magnets with different magnetic field strengths come near each other, which magnet will move more towards the other? **Explain** your answer.

8.3 Moving charge and magnetic fields

Magnetic fields emanate from magnets where the field lines point from north to south outside of the magnet. Magnetic fields are also formed from moving charges. If a current is flowing through a wire, a magnetic field is formed around the wire.

A magnetic field, also known as a ***B* field**, has a corresponding strength. In a natural magnet, this strength depends on how many of the domains align. In the case of current moving through a wire, the *B* field strength depends on the magnitude of the current through the wire, and how far away from the wire the *B* field is being measured. These observations were made by Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841), who performed many experiments with magnets and current-carrying wires. They found that for a point P some distance from a long, straight current-carrying wire, the:

- magnetic field is perpendicular to both the direction of the current and to a line between the wire and P
- magnitude of the field is inversely proportional to the distance from the wire to P (Figure 8.3.1)
- magnitude of the field is proportional to the current. Mathematically, this is modelled as follows: $B = \frac{\mu_0 I}{2\pi r}$

***B* field** a magnetic field; a region of space where a magnetic force is exerted on moving charged particles or magnetic materials, represented by magnetic field lines, which indicate the direction and strength of the field (measured in teslas (T))

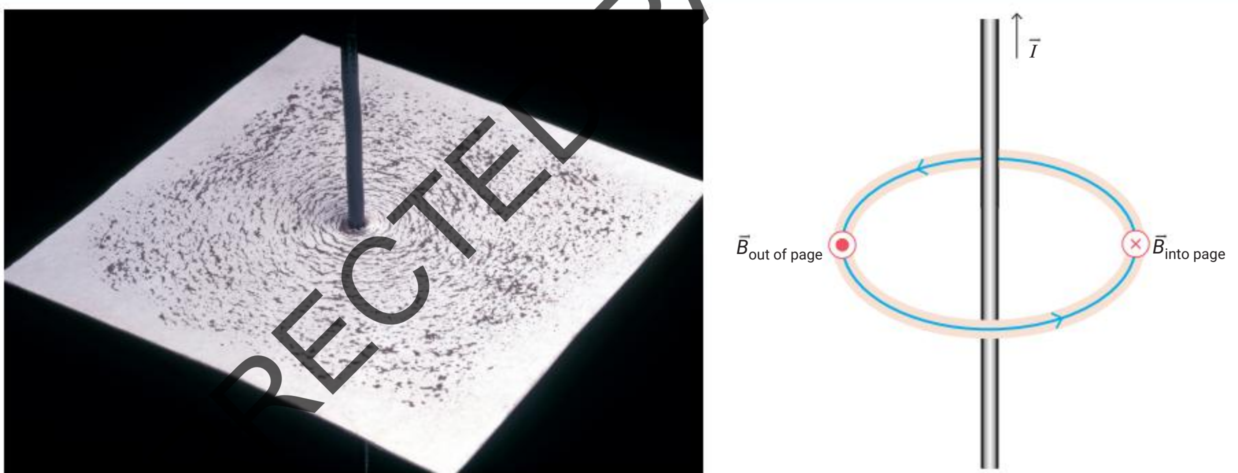


FIGURE 8.3.1 A current-carrying wire will cause a magnetic field to be induced around the wire in concentric circles about the wire. Note that the field is less defined further away from the wire as it is weaker.

KEY FORMULA

Magnetic field strength from current-carrying conductor

$$B = \frac{\mu_0 I}{2\pi r}$$

where:

B = magnetic field strength distance r from the current-carrying conductor (T)

μ_0 = permeability of free space ($4\pi \times 10^{-7}$ T m A^{-1})

I = current travelling through the conductor (A)

r = perpendicular distance from the current-carrying conductor (m)

conventional current the flow of positive charge in an electric circuit; moving from the positive terminal of a power source to the negative terminal, regardless of the actual direction of electron flow

Table 8.3.1 summarises the strengths of some magnetic fields. Most magnetic fields are measured in micro- or milli-tesla, as the tesla (T) is a very large unit. Only in the last few decades have scientists been able to create magnetic fields larger than a few tesla.

In each case of current flowing in a current-carrying conductor, it is customary to consider the direction of current flow as the direction that a positive charge would flow. This is known as **conventional current**.

Syllabus link
Chapter 9 of *Nelson QCE Physics Units 1 & 2* discusses that charge is conserved at all points in an electrical circuit.

TABLE 8.3.1 Magnetic field strengths of some typical magnets or current-carrying devices

Source of field	Approximate magnitude (T)
Earth (surface)	3×10^{-5} to 6×10^{-5}
Typical fridge magnet (surface)	5×10^{-3}
Modern rare earth magnet (surface)	1
Medical MRI system (inside)	3
Strong research facility field	100
Neutron star	$>10^{8}$

WORKED EXAMPLE 8.3.1

Calculate the magnetic field strength at a perpendicular distance of 10 cm from a long wire carrying a current of 10 A.

ANSWER

1 Determine the formula.

$$B = \frac{\mu_0 I}{2\pi r}$$

2 Substitute the known values.

$$B = \frac{4\pi \times 10^{-7} \times 10\ \text{A}}{2\pi \times 0.1}$$

3 Calculate the answer.

$B = 2 \times 10^{-5}$ T

The strength of the magnetic field is 2×10^{-5} T. It is typical to expect a magnetic field to be in this order of magnitude.

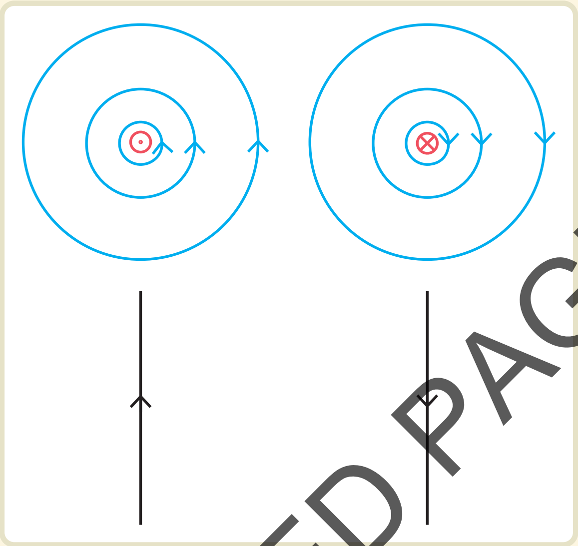
Magnetic fields around current-carrying wires may be described or drawn in various ways: from above (top view), in front (front view),or alongside (side view). It is important to become adept at reading each of the scenarios drawn in each way to analyse and solve problems.

KEY CONCEPT

Drawing current directions in wires

In a top view, conventional current is shown coming out of the page as a circle with a dot in the middle and conventional current is shown going into the page with a circle with a cross in the middle.

In a side view, conventional current is shown going up the page as a line with an arrow up the page and conventional current is shown going down the page with a line with an arrow down the page.



Maxwell's right-hand screw rule

Determining which way the *B* field is moving around a current-carrying conductor depends on the direction of the conventional current. An easy way to determine this is with the right-hand rule for current-carrying conductors. This is also known as Maxwell's screw rule. In this case, your right thumb points in the direction of conventional current flow (opposite to electron flow), and your fingers wrap around the wire in the direction of the *B* field (Figure 8.3.2).

In order to show this in a scientific diagram, we need to introduce some new symbols. As the *B* field acts in three dimensions, we need to identify symbols for into the page and out of the page. For Figure 8.3.2, we can represent the magnetic *B* field around the wire as shown in Figure 8.3.3a.

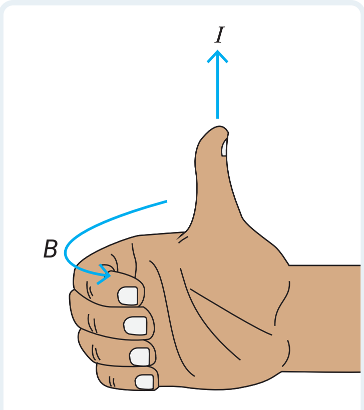


FIGURE 8.3.2 Maxwell's right-hand screw rule shows the direction of the magnetic field (*B* field) around a current-carrying conductor. Point your right thumb in the direction of the conventional current, and your fingers will wrap around in the direction of the external *B* field.

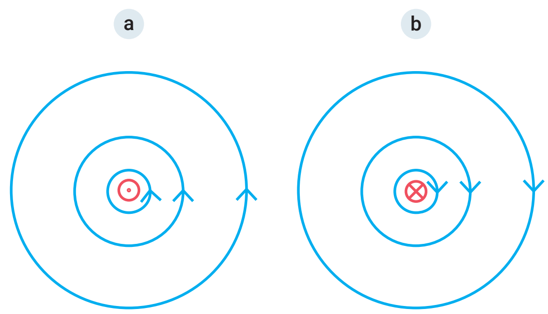


FIGURE 8.3.3 A top view of magnetic fields about two wires. (**a**) Conventional current coming out of the page, denoted by a circle with a dot in the middle. This means the right thumb points up, and the field is formed anticlockwise according to the direction of the fingers. (**b**) Conventional current going into the page, denoted by a circle with a cross in the middle. This means the thumb points down, and the field is formed clockwise according to the direction of the fingers. Think of it like an arrow. When it is coming towards you, you just see its point or dot. When it is going way from you, you see its cross feathers on the shaft.

WORKED EXAMPLE 8.3.2

A long wire is carrying a current directly upwards, out of the page.

Draw the magnetic field lines due to the current as seen from above. Draw arrows on your diagram showing the direction of the magnetic field lines.

ANSWER

1 **Determine the requirements when drawing magnetic fields.**
When drawing magnetic field lines, remember that:
- the density of the field lines indicates the field strength
- currents produce field lines that form concentric circles about the current
- the direction of the field is given by Maxwell's right-hand screw rule.

2 **Draw the magnetic field.**

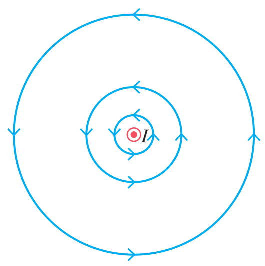


FIGURE 8.3.4 Note that the conventional current runs out of the page towards the viewer in this top view. The concentric circles indicate the magnetic field direction and that it is weaker further away from the current-carrying wire.

PRACTICAL ACTIVITY 8.3.1

MAGNETIC FIELD ABOUT A CURRENT-CARRYING WIRE

Research question

How can the *B* field around a current-carrying wire be observed?

Aim

To observe the *B* field around a current-carrying wire

Risk assessment

RISK ASSESSMENT

What are the risks in doing this activity?	How can you manage these risks to stay safe?
A high current can cause sparks if a short circuit occurs.	Ensure the voltage on the DC power supply is controlled.

Materials

- 12 V DC power pack
- very long wire (looped several times to mimic a large current)
- insulated platform
- set of small compasses or iron filings
- resistor

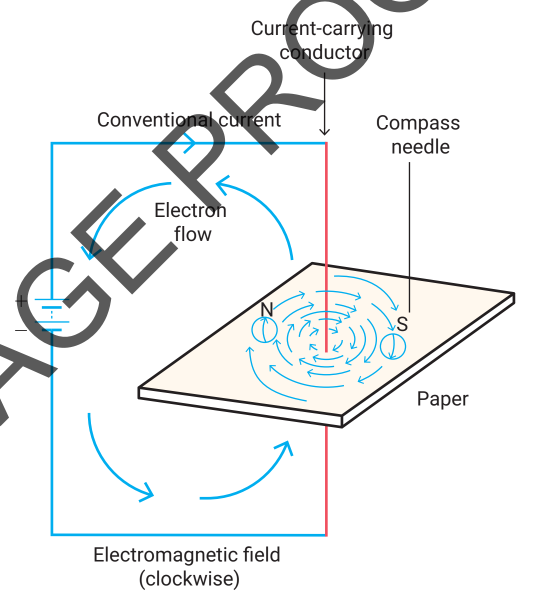


FIGURE 8.3.5 The experimental set-up to observe magnetic field about a current-carrying conductor

Method

1. Set up the circuit as shown in **Figure 8.3.5**.
2. Place the small compasses or iron filings on the platform and notice the field lines close to the wire and further away from the wire.
3. Change the direction of the current. Observe the compass needles change direction.

LEARNING CHECK 8.3

DESCRIBING

1. **Define** 'magnetic field strength'.
2. How does magnetic field strength vary with distance from a long, straight current-carrying wire?
3. **Describe** two sources of magnetic fields.
4. **Explain** how Maxwell's right-hand screw rule can be applied to find the direction of the *B* field around a current-carrying wire.
5. Draw the magnetic field lines due to a wire carrying current directly downwards, as seen from above.

APPLYING

6. **Calculate** the magnetic field strength 1.0 cm from a wire carrying a current of 1.0 A.
7. At what distance from a wire carrying a current of 12 A is the field strength 1.0 mT?

8 What current is necessary in a long, straight current-carrying wire to produce a magnetic field strength of 50 mT at a distance of 1 cm?

9 Find the B field strength at distances 20 cm, 40 cm, 60 cm, 80 cm and 1.0 m from a wire carrying 10 A of current.

10 Plot a graph of field as a function of distance from the wire for the data set 20 cm, 40 cm, 60 cm, 80 cm and 100 cm for a current of 5.0 A.

8.4 Solenoids and electromagnets

solenoid a coil of current-carrying wire that creates a large uniform field within the coil when a current is passed through it

electromagnet a magnet with a north and a south pole formed by a current in a solenoid

Weblink
Solenoid

A current-carrying wire creates a magnetic field. If we want to create a large field or control the magnitude of the field, we use many turns of wire or a single wire in a coil called a **solenoid**.

Each loop of the wire creates a magnetic field (**Figure 8.4.1b**). Inside the coils these fields add up to give a large and approximately uniform magnetic field. The more loops or turns of a wire, the greater the field strength. In a tightly wound solenoid, the internal field lines are straight and parallel. Outside the coil, the field lines are more spread out. The result is an extremely useful device called an **electromagnet**, as the field lines are now synonymous to a bar magnet with a north and a south pole. Solenoids are used in transformers, magnetic switches and many other applications where large magnets are required. Electromagnetics have a huge advantage over permanent magnets in that their magnetic fields can be switched on and off with the current.

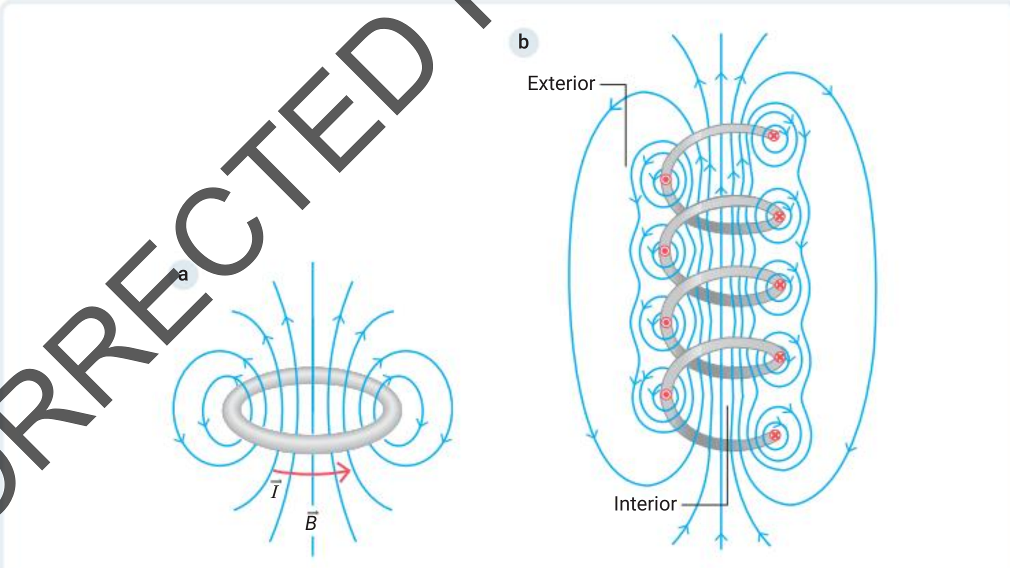


FIGURE 8.4.1 (a) Magnetic field due to a single loop of wire. (b) Magnetic field due to a solenoid. Note that the field lines resemble a bar magnet. Field lines run from north to south outside of the solenoid (and south to north inside the solenoid).

Magnetic field in a solenoid

The B field that is produced from a solenoid depends on a few variables:

- the magnitude of the current flowing through the wire
- the number of turns (coils) in the solenoid
- the magnetic permeability of free space, μ_0.

This is modelled mathematically as:

$$B = \mu_0 nI$$

KEY FORMULA

Magnetic field produced by a solenoid

$B = \mu_0 nI$
where:
B = strength of the field inside the solenoid (T)
μ_0 = permeability of free space ($4\pi \times 10^{-7}$ T m A^{-1})
n = number of turns in the solenoid per metre
I = current travelling through the solenoid (A)

KEY CONCEPT

Direction of field lines

Field lines run from north to south outside the solenoid (and south to north inside the solenoid).

Additionally, if the direction of the current through the coils is known, the magnetic field direction can be found using an adaptation of Maxwell's screw rule. If the fingers curl in the direction the conventional current is flowing in the coil, the thumb points in the direction of the north pole of the electromagnet **(Figure 8.4.2)**.

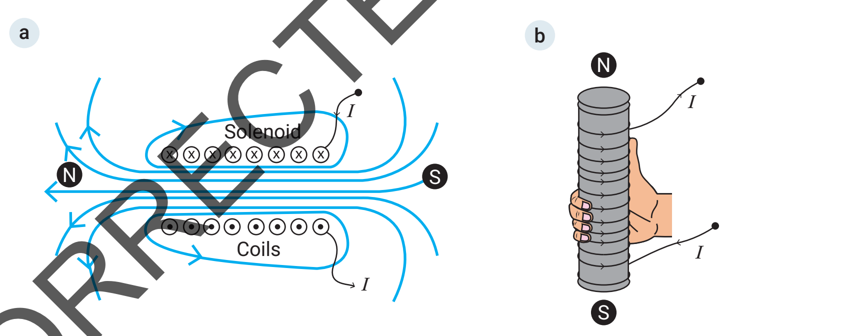


FIGURE 8.4.2 (**a**) A cross-section of the current in the coils of a solenoid, and the magnetic field lines. (**b**) The right-hand grip rule being used to determine the magnetic field direction in a solenoid.

WORKED EXAMPLE 8.4.1

A solenoid with a current of 3 A going through it produces a B field of 0.2 mT inside the coil. Determine how many turns are in the solenoid if it has a length of 75 cm.

ANSWER

1 Determine the formula.

$B = \mu_0 nI$

2 **Rearrange the formula to find the unknown.**

$$n = \frac{B}{\mu_0 I}$$

3 **Substitute the known values.**

$$n = \frac{2 \times 10^{-4}\ \text{T}}{4\pi \times 10^{-7}\ \text{T m A}^{-1} \times 3\ \text{A}}$$

4 **Calculate the answer.**

$n \approx 53$ turns per metre

The number of turns, N, in a 75 cm length is given by $n = \frac{N}{l}$.

$$53 = \frac{N}{0.75\ \text{m}}$$

$N = 39.75$ or approximately 40 turns in the solenoid

PRACTICAL ACTIVITY 8.4.1

INVESTIGATING PROPERTIES OF AN ELECTROMAGNET

Research question

What factors affect the strength and poles of an electromagnet?

Aim

To determine factors that change the strength and poles of an electromagnet

Risk assessment

RISK ASSESSMENT

What are the risks in doing this activity?	How can you manage these risks to stay safe?
Strong magnetic fields may damage nearby electronics or affect nearby magnetic materials by making them move.	Ensure all magnetic materials and electronic devices are not on the lab bench with the solenoid.

Materials

- DC power supply and connecting cables
- solenoid
- rheostat
- bar magnet

Method

1 Connect the solenoid and rheostat in series and attach to a DC power supply.

2 Turn on the power supply to the lowest voltage and bring the bar magnet close to the solenoid. Observe what happens.

3 Steadily increase the voltage, while keeping the resistance constant. Observe the interaction between the solenoid and the bar magnet.

4 Change the direction of the current by switching the terminals of the DC power supply. Observe how the interaction between the solenoid and the bar magnet changes.

Analysis of results

1 What would happen if the bar magnet were to be pulled inside the solenoid?

2 Explain why an electromagnet is considered more useful than a bar magnet.

PRACTICAL ACTIVITY 8.4.2

FORCE ON A CURRENT-CARRYING WIRE IN AN EXTERNAL MAGNETIC FIELD

Aim

To observe the force on a current-carrying wire in an external magnetic field

RISK ASSESSMENT

What are the risks in doing this activity?	How can you manage these risks to stay safe?
Strong permanent magnets can be damaged if not set at a fixed distance. They can also cause pinching of skin.	Ensure magnets are kept with keepers between them and are far enough apart that they will not fly together.
Alligator clips can be sharp.	Handle sharp objects with care.

Materials

- DC power supply
- long, light conductive material (such as a long piece of aluminium foil)
- 2 alligator clips
- large horseshoe magnet (large *B* field) or two neodymium magnets
- retort stands, boss heads and clamps
- connecting wires

Method

1 Set up the circuit as shown in Figure 8.4.3.

2 Turn on the power supply and observe what happens to the aluminium foil.

3 Reverse the terminals of the DC power supply and notice what happens to the aluminium foil when the current is reversed.

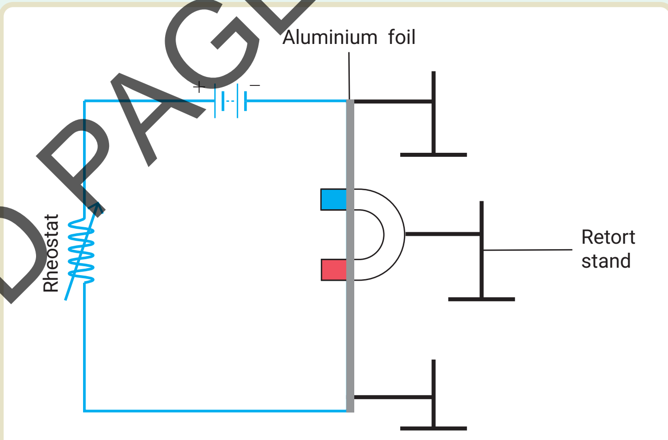


FIGURE 8.4.3 The experimental set-up of a current-carrying conductor in a magnetic field

Discussion

1 What variables determine how much the aluminium foil moves?

2 Describe how you could determine the north and south pole of the magnet if it was not indicated on the magnet.

PRACTICAL ACTIVITY 8.4.3

FORCE ON PARALLEL CURRENT-CARRYING WIRES

Research question

How do wires that are parallel to current-carrying wires behave when in close proximity?

Aim

To observe how two parallel current-carrying wires behave when in close proximity

RISK ASSESSMENT

What are the risks in doing this activity?	How can you manage these risks to stay safe?
Crocodile clips, pins and scissors can pierce skin.	Handle sharp objects with care.

Materials

- 2 long, thin wires
- 2 retort stands, boss heads and clamps
- 2 alligator clips
- DC power supply
- rheostat
- ammeter
- connecting wires

Method

1 Set up the circuit with the rheostat in series with the long, thin connecting wires hanging close together between the retort stands. Ensure the long wires are connected in parallel so the current flows through them in the same direction (Figure 8.4.4a).

2 Turn on the DC power supply and observe how the wires move.

3 Turn off the power supply and reconnect the wires in series but keep them hanging parallel. The current should now be flowing in opposite directions (Figure 8.4.4b).

4 Turn on the DC power supply and observe how the wires move.

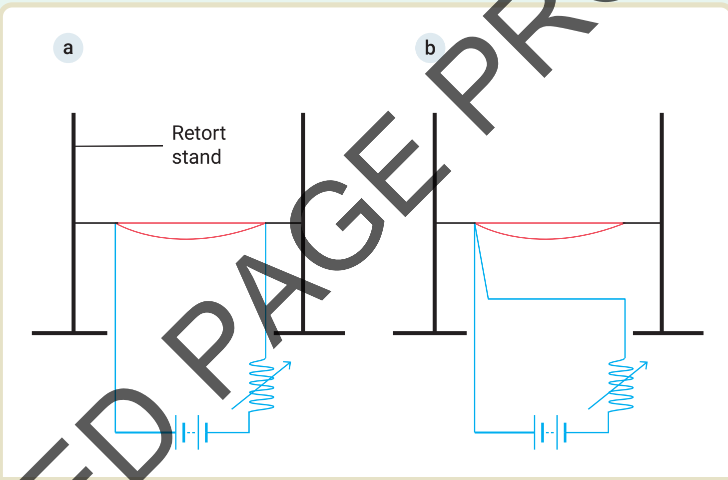


FIGURE 8.4.4 Long, thin wires connected (a) in parallel and (b) in series

Analysis of results

1 Why does changing the current change the direction of the forces on the wires?

2 Draw the magnetic field lines about each of the wires in both of these scenarios.

LEARNING CHECK 8.4

DESCRIBING

1 **Define** ‘electromagnet’.

2 **Explain** how you may determine the north pole and south pole of a solenoid (electromagnet) by using drawn magnetic field lines.

APPLYING

3 Find the magnetic field strength in a solenoid of 50 turns per metre, carrying a current of 80 mA.

4 The *B* field in a solenoid of 100 turns is found to 60 μT. **Determine** the magnitude of the current passing through the coil.

ANALYSING

5 State why electromagnets have much wider applications than permanent bar magnets do.

8.5 Solving problems in magnetic fields

The B field around bar magnets and electromagnets depends on several variables. Each of these variables needs to be carefully considered when solving problems about magnetic field strength. The variables that affect magnetic fields from magnets and electromagnets are:

- distance from magnet or electromagnet
- magnitude of current in wire or solenoid
- number of turns in a coil (electromagnet only).

If these variables are known, it is possible to determine the magnetic field at a given location. Additionally, if the B field and two of the above three variables are known, it is possible to calculate the unknown third variable.

WORKED EXAMPLE 8.5.1

A wire is carrying a current of 15 A.

a At what distance from the is the field 1.0 µT?

b If this same wire is then coiled into a solenoid with 25 turns over 50 cm, what is the B field strength inside the coil?

ANSWERS

a **1 Determine the formula.**

$$B = \frac{\mu_0 I}{2\pi r}$$

2 Rearrange the formula to find the unknown.

$$r = \frac{\mu_0 I}{2\pi B}$$

3 Substitute the known values.

$$r = \frac{4\pi \times 10^{-7} \times 15\text{ A}}{2\pi \times 1.0 \times 10^{-6}\text{ T}}$$

4 Calculate the answer.

$$B = \frac{\mu_0 I}{2\pi r}$$

$$r = \frac{\mu_0 I}{2\pi B}$$

$$= \frac{4\pi \times 10^{-7} \times 15\text{ A}}{2\pi \times 1.0 \times 10^{-6}\text{ T}}$$

$r = 3$ m

3 m is the distance from the wire where the field is 1.0 µT.

b **1 Determine the formula.**

If this wire was then coiled:

$$B = \mu_0 n I$$

2 **Substitute the known values.**

$$B = 4\pi \times 10^{-7} \times \frac{25}{0.5} \times 15$$

3 **Calculate the answer.**

$$B = \mu_0 nI$$
$$= 4\pi \times 10^{-7} \times \frac{25}{0.5} \times 15$$
$$= 9.42 \times 10^{-4}\ \text{T or } B = 94.2\ \text{mT}$$

WORKED EXAMPLE 8.5.2

If a coil with 0.5 A of current and 100 turns has B field of 7.6×10^{-5} T inside it, what must be the length of the coil?

ANSWER

1 **Determine the formula.**
$B = \mu_0 nI$ where n is number of turns per metre (N/L)

$$B = \frac{\mu_0 nI}{L}$$

2 **Rearrange the formula to find the unknown.**

$$L = \frac{\mu_0 nI}{B}$$

3 **Substitute the known values.**

$$L = \frac{4\pi \times 10^{-7} \times 100 \times 0.5\ \text{A}}{7.6 \times 10^{-5}\ \text{T}}$$

$$B = \frac{\mu_0 nI}{L}$$

$$L = \frac{\mu_0 nI}{B}$$
$$= \frac{4\pi \times 10^{-7} \times 100 \times 0.5\text{A}}{7.6 \times 10^{-5}\ \text{T}}$$

4 **Calculate the answer.**
$L = 0.826$ m

LEARNING CHECK 8.5

DESCRIBING

1 What is the effect on the magnetic field within a solenoid if the:

a current is increased?
b current direction is reversed?
c number of turns per metre is decreased?

APPLYING

2 What is the magnetic field a distance of 10 mm from a wire carrying a current of 1.6 A?

3 How large a current is necessary to produce a field of 0.15 a distance of 2.0 cm from a long, straight current-carrying wire?

4 A solenoid of 125 turns per metre has a B field of 0.5 T. **Determine** the current through the coil.

5 A coil with 0.34 A of current and 50 turns has a B field of 1.5×10^{-5} T. **Determine** the length of the coil.

8.6 Force on particles in a magnetic field

Scientists can measure a magnetic field by its effect on moving charges, such as a current in a wire. Experiments using lengths of current-carrying wire in magnetic fields show that the **magnetic force** on the wire depends on the:

1. magnitude of the current carried, I (A)
2. strength of the magnetic field, B (T)
3. angle between the direction of the current and the field, θ (°)
4. length of the wire in the magnetic field, L (m).

These observations can be summarised mathematically as $F = BIL \sin\theta$. This equation uses the magnitude of vector quantities, and so the resulting force calculated is the magnitude of the force acting on the wire. This equation tells us that the force is zero if L and B are in the same direction **(Figure 8.6.1a)** and that the force on a current-carrying wire is a maximum when it is perpendicular to the external field B **(Figure 8.6.1b, c)**. Otherwise, the force exerted on the wire is between 0 N and the maximum force experienced, governed by the angle θ the wire makes with the field **(Figure 8.6.1d)**. The closer θ is to 0 (i.e. the wire is lying in the same direction as the magnetic field), the smaller the magnetic force the wire experiences due to the external field.

magnetic force the force that a magnetic field exerts on a moving charge or current

Weblink
Magnetic force on a moving electric charge

KEY FORMULA

Force on a current-carrying conductor

$F = BIL\sin\theta$

where:

F = magnetic force on the wire (N)

I = current flowing through the wire (A)

B = magnitude of the external magnetic field (T)

L = length of conductor (m)

θ = angle the wire makes with the external magnetic field (°)

This can also be rearranged to express magnetic field strength:

$$B = \frac{F}{IL\sin\theta}$$

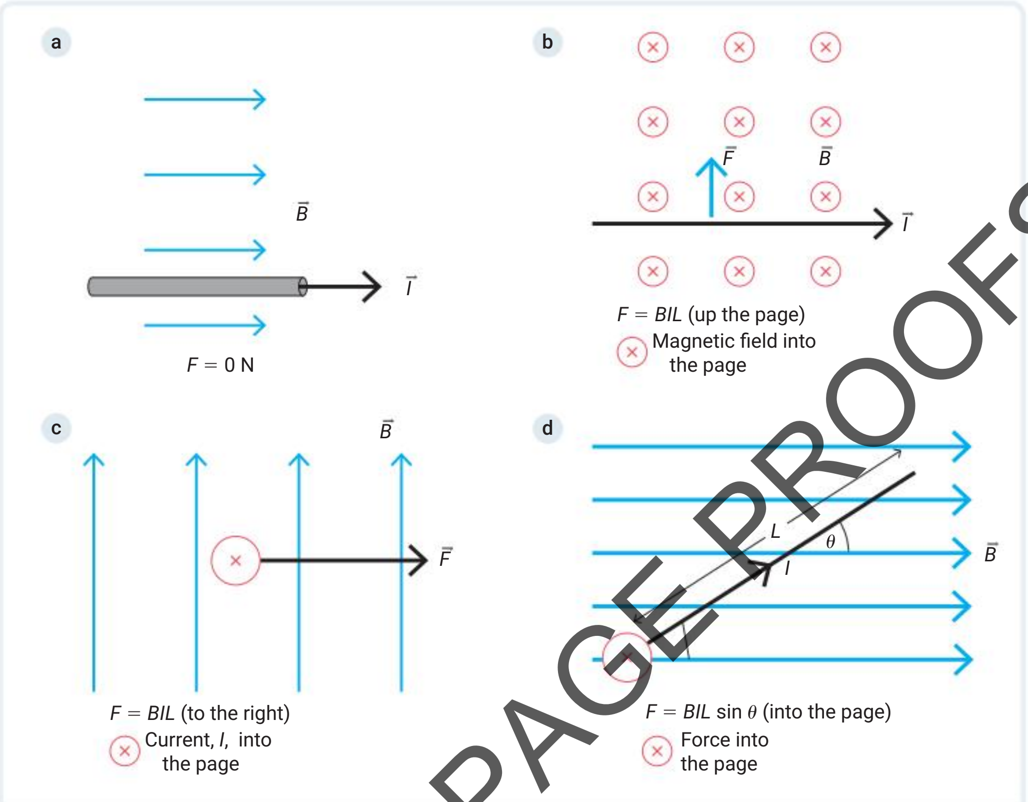


FIGURE 8.6.1 The force a wire of length L experiences in an external B field when the wire is (**a**) parallel to the field, (**b**) perpendicular to the field, (**c**) perpendicular to the field with a different orientation, (**d**) at an angle θ to the field where $\theta \neq 0$ or 90.

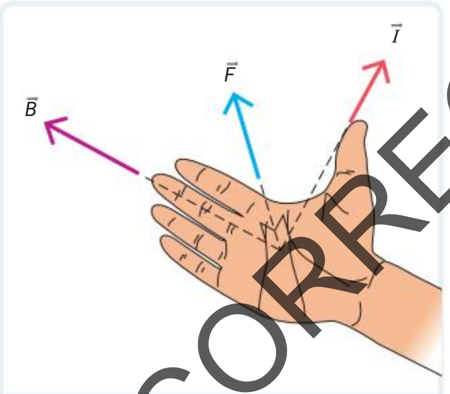


FIGURE 8.6.2 The right-hand palm rule for determining the direction of a magnetic force acting on a current-carrying wire in an external magnetic field

Right-hand palm rule for currents and charges in external magnetic fields

To find the direction of a force on a current-carrying wire (or charges) in external magnetic fields, we apply the right-hand palm rule. This rule is slightly different from Maxwell's screw rule. There are several ways to use your right hand for this rule.

Hold out your right hand flat and move your hand so that your thumb points in the direction of conventional current and your fingers point in the direction of the external magnetic (B). Your palm then points in the direction in which the magnetic force will push the wire (**Figure 8.6.2**). Try this for Figure 8.6.1b and c and note that your palm will point in the direction in which the force is indicated on each diagram. As the right-hand palm rule is the result of a vector cross product, there are different ways in which it can be applied, but the results will all be the same.

WORKED EXAMPLE 8.6.1

An overhead power line carrying a current of 1000 A is in a magnetic field of 30 μT. The field is perpendicular to the wire. What force per unit length does the wire experience?

ANSWER

1 Determine the formula.

$F = BIL \sin\theta$

2 Substitute the known values.

$$\frac{F}{L} = 30 \times 10^{-6}\ \text{T} \times 1000\ \text{A} \times \sin 90^\circ$$

3 Calculate the answer.

$F = 0.030\ \text{N m}^{-1}$

$F = BIL \sin\theta$

$$\frac{F}{L} = 30 \times 10^{-6}\ \text{T} \times 1000\ \text{A} \times \sin 90^\circ$$

$F = 0.030\ \text{N m}^{-1}$

This is quite a large force considering how long power lines are. However, power line current is alternating at 50 Hz, so the force also alternates direction, rather than pulling on the power line in a single direction.

LEARNING CHECK 8.6

DESCRIBING

1 State the equation to find the force on a current-carrying wire in an external magnetic field.

2 State the orientation of a current-carrying wire to the external magnetic field if no force is exerted on it from the field.

3 State the direction in which the magnetic force acts on a wire if the current is carried in the wire from left to right, and the magnetic field is coming out of the page. Draw a diagram to represent the scenario.

APPLYING

4 A 5 m long current-carrying wire is at an angle of 30° to a magnetic field. It carries a current of 30 A and experiences a force of 0.02 N. What is the strength of the magnetic field?

5 A straight wire 2.1 m long and carrying a current of 0.85 A has a force of 5.0×10^{-2} N exerted on it by a uniform magnetic field at right angles to the wire. What is the magnitude of the magnetic field?

ANALYSING

6 A wire is carrying a large current directly upwards, out of the page. Draw a diagram to represent the scenario.

7 What total force would be exerted on a span of wire 200 m long if the wire carries a current of 1000 A and is in a magnetic field of 20 μT? **Compare** this force with the gravitational force acting on the wire if the wire has a linear density of 750 kg km^{-1}.

8.7 Force on moving particles in a magnetic field

A current is a collection of charges moving in the same direction. A current experiences a force in a magnetic field, so we can expect that a single moving charge will also experience a force. The magnitude of the magnetic force that a charged particle experiences is written mathematically as $F = qvB\sin\theta$.

KEY FORMULA

Force on a moving charged particle

$F = qvB\sin\theta$

where:

F = magnitude of the force on the moving charge (N)

q = magnitude of the charge moving in the magnetic field (C)

v = magnitude of the velocity of the moving charge ($m\ s^{-1}$)

B = strength of the external magnetic field (T)

θ = angle between the vectors v and B (°)

The force is again perpendicular to both the magnetic field and the velocity of the charged particle. Using the right-hand palm rule allows us to find the direction of the magnetic force on a charged particle when it enters a magnetic field. In Figure 8.6.2, the thumb pointed in the direction of conventional current. This time, the thumb is pointed in the direction a positive charge will move in the field, your fingers point in the direction of the external magnetic (B) field, and your palm will again point in the direction of the magnetic force. If you are considering the force on a negatively charged particle, remember to point your thumb initially in the opposite direction as negative charges will travel in the opposite direction to positive charges in a magnetic field (Figure 8.7.1).

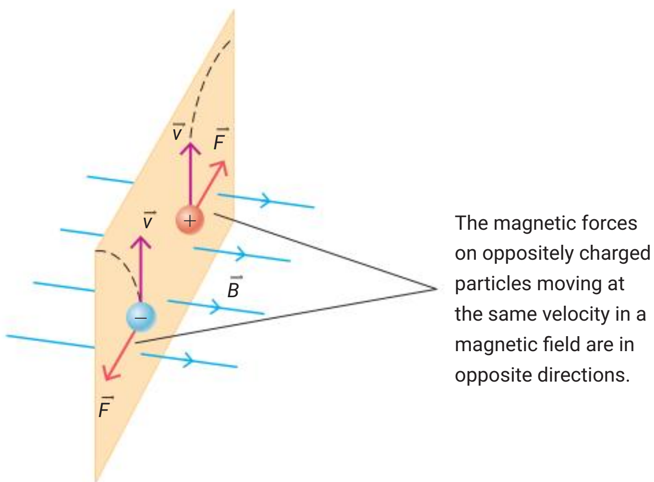


FIGURE 8.7.1 The forces on positive and negatively charged particles in a magnetic field. Note that they are in opposite directions. The dashed lines show the path the particles will take in this field.

WORKED EXAMPLE 8.7.1

An alpha particle enters Earth's magnetic field at a velocity of 55 000 m s^{-1}. The local field strength is 40 μT. What is the range of possible accelerations of the alpha particle as the angle, θ, varies?

ANSWER

1 Determine the formula.

First, consider that $F = ma$ and that $F = qvB \sin \theta$, where $-90° \leq \theta \leq 90°$

Now, $F = ma$

$F = qvB \sin\theta$

$\therefore ma = qvB \sin\theta$

2 Rearrange the formula to find the unknown.

$$a = \frac{qvB\sin\theta}{m}$$

where a will have a range of values depending on the value of sin θ.

The maximum magnitude a can have is when $\theta = 90$, i.e. when $\sin \theta = 1$. So:

$$a = \frac{qvB}{m}$$

3 Substitute the known values.

$$a = \frac{2 \times 1.6 \times 10^{-19}\,\text{C} \times 55000\,\text{m s}^{-1} \times 40 \times 10^{-6}\,\text{T}}{6.6 \times 10^{-27}\,\text{m s}^{-2}}$$

4 Calculate the answer.

$$a = \frac{qvB}{m}$$

$$= \frac{2 \times 1.6 \times 10^{-19}\,\text{C} \times 55000\,\text{m s}^{-1} \times 40 \times 10^{-6}\,\text{T}}{6.6 \times 10^{-27}\,\text{kg}}$$

$a = 1.1 \times 10^{8}$ m s^{-2}

The acceleration of the alpha particle can have any value between -1.1×10^{8} m s^{-2} and 1.1×10^{8} m s^{-2} depending on the angle between the direction of the velocity and the magnetic field.

Paths of particles in magnetic fields

Fast-moving charged particles experience large forces, even in small magnetic fields. These forces result in large accelerations, meaning that the direction of the velocity of the charged particle changes. The path of a charged particle in a uniform magnetic field depends on the angle between the initial velocity and the field. The path may be a straight line, a circle or a helix.

If the particle enters the field with velocity perpendicular to the external magnetic (B) field, it will experience the maximum magnetic force. This will cause the charged particle to follow a circular path, as per the right-hand rule **(Figure 8.7.2a)**. If instead the particle has initial velocity with a component in the direction of the field, this component of velocity is not altered by the field. However, the perpendicular component is altered by the acceleration due to the field (from the magnetic force acting on the particle). In this case, the particle follows a helical path, with the axis of the helix in the direction of the field **(Figure 8.7.2b)**. If the particle enters the field with velocity parallel to the field, no force will act on the charged particle and it will travel in a straight line **(Figure 8.7.2c)**.

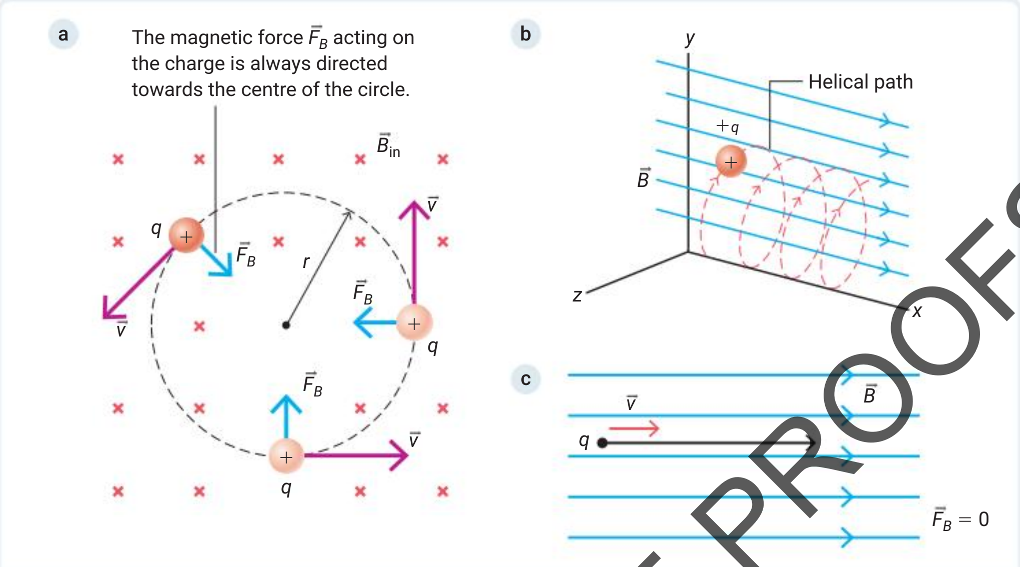


FIGURE 8.7.2 In a uniform B field, the path of a charged particle is (**a**) circular when it enters in a direction perpendicular to the field, (**b**) helical when it enters at an angle to the field strictly between 0° and 90°, and (**c**) not deflected when travelling parallel to the field.

WORKED EXAMPLE 8.7.2

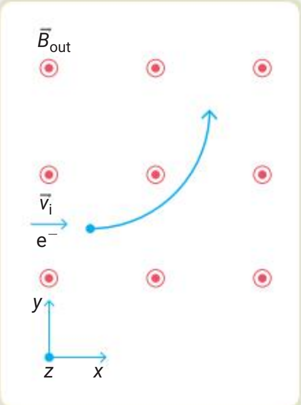
A magnetic field points in the positive *z* direction. Draw the path of an electron in the field with an initial velocity that is in the:

a positive *x* direction **b** negative *x* direction **c** positive *z* direction.

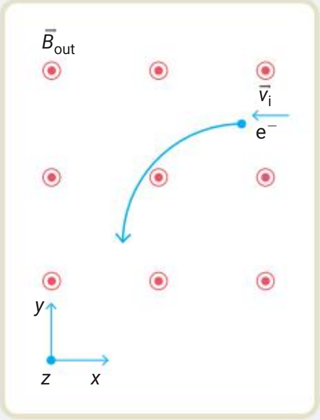
ANSWERS

Use the right-hand rule to solve these problems. In all cases, the *B* field is coming out of the page, and your thumb should point in the opposite direction to that in which the electron is travelling as it is negatively charged. Your palm then points in the direction the electron will move.

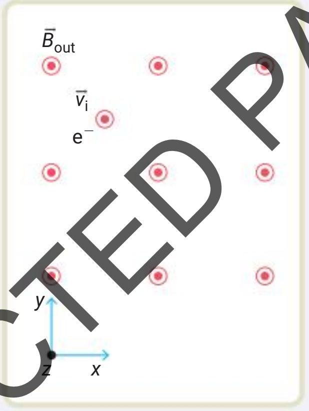
a **Draw an electron entering the *B* field in the positive *x* direction.**
The electron is coming in perpendicular to the field, so it will follow a circular path.



b Draw an electron entering the *B* field in the negative *x* direction.
The electron is coming in perpendicular to the field, so it will follow a circular path.



c Draw an electron entering the *B* field in the positive *x* direction.
The electron's path will not be altered because it is travelling parallel to the magnetic field.



LEARNING CHECK 8.7

DESCRIBING

1 State how the forces on moving negatively and positively charged particles differ in a magnetic field.
2 Which of the possible paths in **Figure 8.7.3** is not possible for a charged particle entering a region of uniform magnetic field?
3 **Determine** the initial direction of the deflection of the charged particles in **Figure 8.7.4** as they enter the magnetic fields shown.

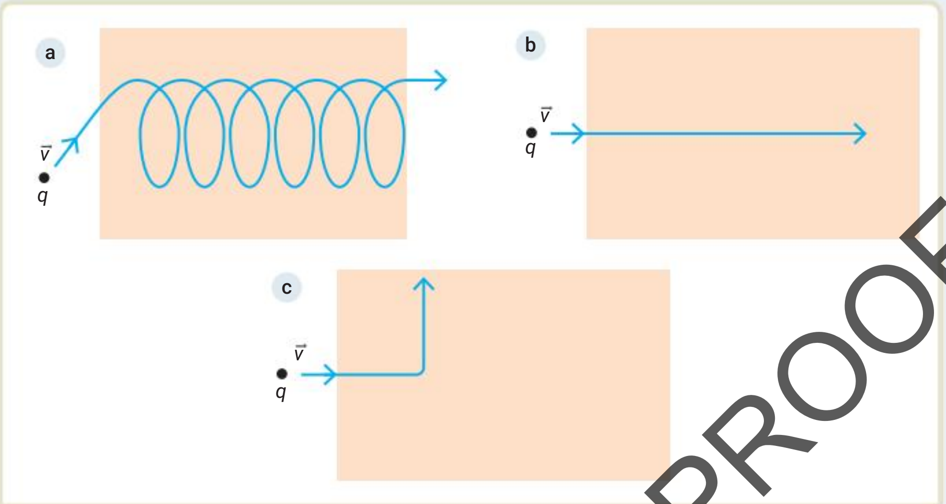


FIGURE 8.7.3 (a) A helical path; (b) a path with no deflection; (c) a path with a right angle

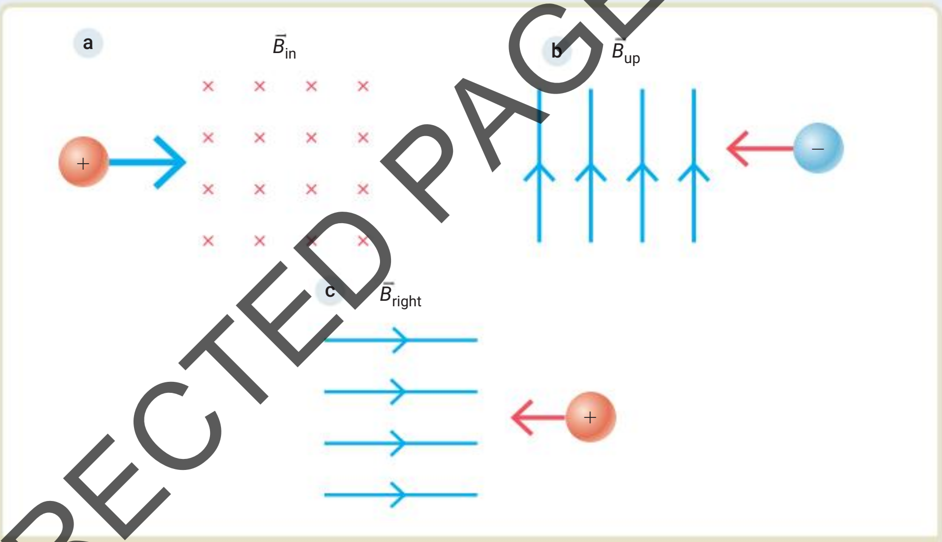


FIGURE 8.7.4 Charged particles entering external magnetic fields

PRACTICAL ACTIVITY 8.7.1

STRENGTH OF A MAGNET

Research question

How does the strength of a magnetic field change based on the distance from a bar magnet?

Aim

To determine how the strength of a magnetic field changes with distance from a bar magnet

Materials

- 2 bar magnets
- electronic scale
- retort stand, boss head and clamp
- ruler

Method

1 Set up the materials as shown in Figure 8.7.5. Ensure that the like ends of the magnets are pointed towards each other.
2 Beginning with a separation distance of 30 cm, record the mass of the bar magnet on the scale.
3 Decrease the separation distance by 5 cm and record the mass of the bar magnet in a suitable table. Repeat the procedure to obtain at least four more readings.
4 Convert all masses to weight in newtons and plot a graph of force against distance.

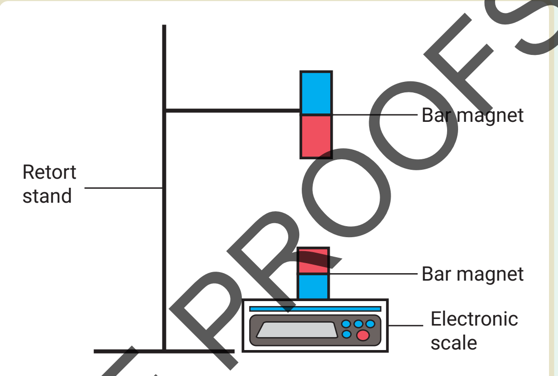


FIGURE 8.7.5 The experimental set-up to determine how magnetic fields exert different forces at different distances

Analysis of results

1 What happened to the mass of the bar magnet on the scale as the bar magnet in the retort stand was brought closer?
2 Why do you think this mass changed in this way?
3 Suggest what would happen if the magnetic field between the magnets was attractive instead of repulsive.
4 What kind of relationship exists between magnetic force and separation distance? Compare this to other forces you have studied so far.

Interpretation

5 Manipulate the data to draw a linear graph. Determine the relationship between magnetic force and separation distance.

PRACTICAL ACTIVITY 8.7.2

A CURRENT BALANCE

Research question

How can the magnetic force on a current balance be measured?
How can the strength of the magnetic field strength on a solenoid be measured?

Aim

To measure the magnetic force on a current balance and find the magnetic field strength in a solenoid

RISK ASSESSMENT

What are the risks in doing this activity?	How can you manage these risks to stay safe?
Strong magnetic fields may damage nearby electronics or cause nearby magnetic materials to move.	Ensure all magnetic materials and electronic devices are not on the lab bench with the solenoid.
Crocodile clips, pins and scissors can pierce skin.	Handle sharp objects with care.

Materials

- air core solenoid
- parts to make current balance (thin, stiff plastic or cardboard; stiff conducting wire; copper or zinc sheet; pin; fine sandpaper, scissors, sticky tape)
- 2 short pieces of wire (mass known)
- 2 DC power supplies
- 2 ammeters
- 2 rheostats
- 2 switches
- crocodile clips and leads

Method

Making the current balance apparatus

1 Cut a rectangle from the cardboard or plastic so that half will fit into the solenoid and half is outside.

2 Attach a small pin to the middle of one of the short sides of the rectangle, overhanging the outside end (Figure 8.7.6).

3 Make a rectangular half loop of conducting wire, to sit near the edges of the rectangle that goes into the solenoid. Make sure it is attached to the rectangle.

4 Cut two supports out of the metal sheet, bend them and attach them to the end of the solenoid as shown in Figure 8.7.6. Use the crocodile clips to connect them to the current balance circuit. Note that they should not make any electrical contact with the solenoid.

5 Bend the ends of the rectangular half loop so that they sit on the metal supports.

6 Use sandpaper to clean the metal and ensure a good electrical connection.

7 Measure the length of the current element that is perpendicular to the magnetic field of the solenoid (the short end of the rectangle).

8 Balance the current balance by hanging pieces of wire or small weights over the pin.

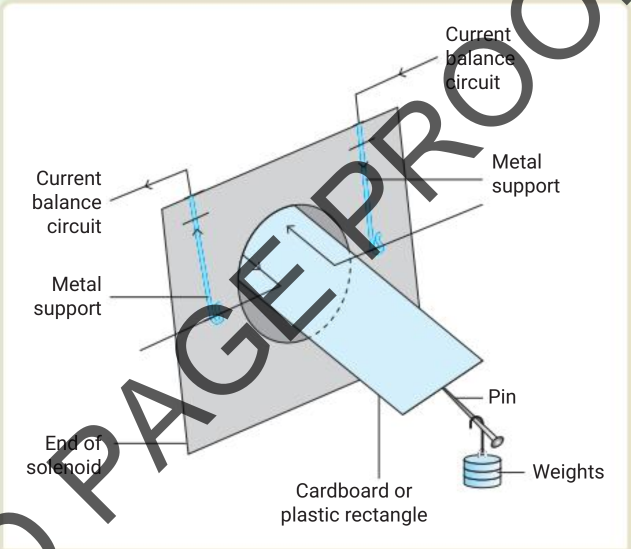


FIGURE 8.7.6 How to set up the current balance inside the solenoid

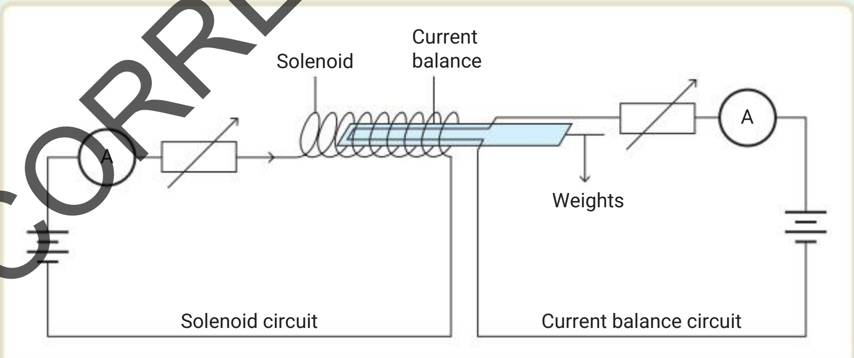


FIGURE 8.7.7 The circuit connections to connect the solenoid and current balance respectively

Force on wires due to an external field

1 Connect the balance and the solenoid circuits as shown in Figure 8.7.7.

2 Turn on both circuits and observe what happens to the current balance – it needs to act as a seesaw with the inner end being pushed down by the magnetic force. Alter the apparatus accordingly.

3 Adjust the number of weights and their positions on the current balance until the current balance is parallel with the laboratory bench.

4 Record the current in the solenoid, the current in the current balance, the distance from the pivot to the current element, and the distance from the pivot to the balancing weights. Weigh the masses that were added to make the current balance parallel to the bench and record this value.

Analysis of results

1 Calculate:

   a gravitational force on balancing masses

   b torque by weight force on current balance (product of gravitational force and distance from masses to pivot point)

   c magnetic force on current balance in the solenoid

   d magnetic field in the solenoid.

Evaluation

2 Comment on the quality of your data and how this affects your results.

3 How could you improve the quality of your data?

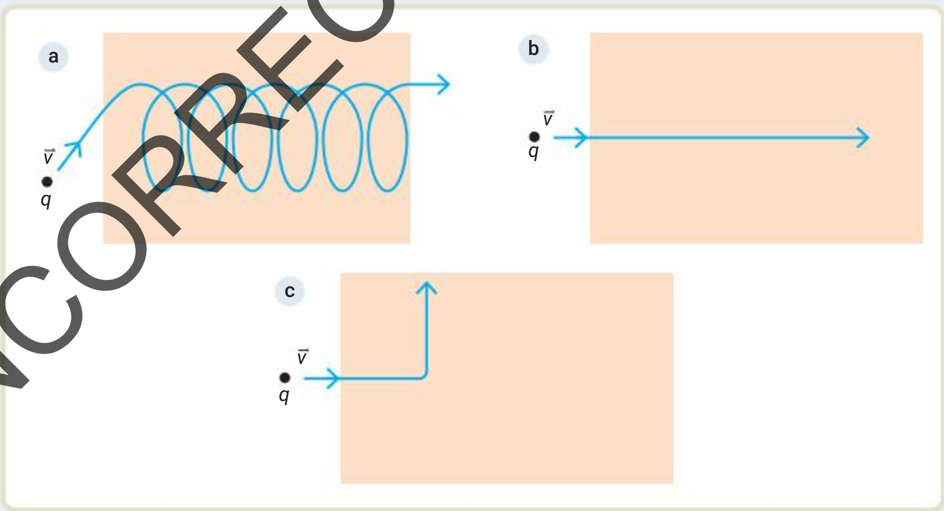
4 Redesign the experiment so that you can experimentally determine the relationship between the current in the solenoid and the strength of the magnetic field in the solenoid. Hint: You will need to use the magnetic force the field exerts on the current balance!

LEARNING CHECK 8.7

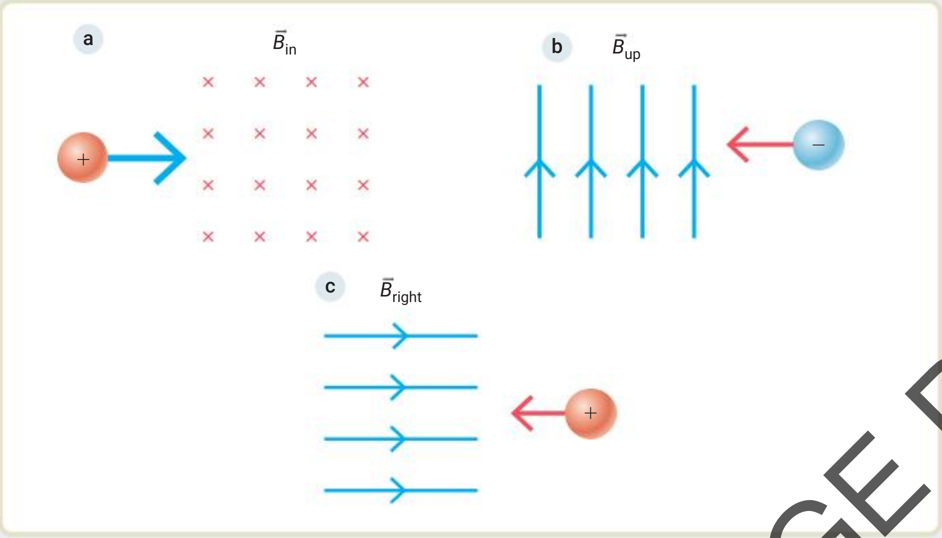
DESCRIBING

1 State how the forces on moving negatively and positively charged particles differ in a magnetic field.

2 Which of the following possible paths is not possible for a charged particle entering a region of uniform magnetic field?



3 Determine the initial direction of the deflection of the charged particles in the following diagrams as they enter the magnetic fields shown.



APPLYING

4 What is the minimum magnitude of a magnetic field necessary to apply a force of 1×10^{-12} N to an electron moving at a speed of 500 km s^{-1}?

5 An external magnetic field is uniform in the negative *z* direction. A proton enters this field from the positive *x* direction with velocity 55 m s^{-1}. What is the magnitude and direction of the force this proton experiences?

6 What acceleration would an electron entering a field of 20 µT experience at a speed of 55 000 m s^{-1} if it is travelling:

a perpendicular to the field?
b parallel to the field?
c at an angle of 45° to the field?

ANALYSING

7 A proton and an electron enter a uniform magnetic field. The particles are travelling at the same speed perpendicular to the field. Draw a diagram showing their paths and **explain** the differences of the paths each charged particle takes. Consider both $F = qvB\sin\theta$ and $F = \frac{mv^2}{r}$.

8 **Sketch** the path of a proton as it enters a *B* field from the positive *x* direction, if the *B* field is going into the page.

CHAPTER SUMMARY

Magnetic fields

- Magnet domains are small regions within a material where the magnetic moments (spins) of atoms are aligned in the same direction. In unmagnetised materials, these domains are randomly orientated, so their magnetic fields cancel each other out. In permanent magnets, the domains are aligned.
- A magnet is a substance in which most of its domains are aligned and produce magnetic fields. All magnets have a north pole and a south pole.
- A magnetic field is created by moving charges and can be described as a region around a magnet where magnetic forces can be detected.

Magnetic field strength

- The strength of a magnetic field from a point source decreases with the square of the distance.
- Magnetic field lines represent the direction and strength of the field. Field lines flow from the north pole to the south pole of a magnet. The density of the field lines indicates the strength of the field. Field lines never intersect.

Magnetic field in a solenoid

- A magnetic field (*B* field) that is produced from a solenoid depends on the:
  - magnitude of the current, I, flowing through the wire
  - number of turns (coils), n, in the solenoid
  - magnetic permeability of free space, μ_0.

  This is modelled mathematically as:

  $B = \mu_0 nI$

Magnetic field strength around a current-carrying conductor

- A magnetic field can also be formed from moving charges, so when a current flows through a wire, a *B* field is formed around the wire.

  $B = \frac{\mu_0 I}{2\pi r}$

Solenoids

- A solenoid is a coil of wire that produces a near-uniform magnetic field when an electric current flows through it.
- When current flows through a solenoid, it forms an electromagnet, a magnet with a north and south pole.

  $B = \mu_0 nI$

  where:

  B = strength of the field inside the solenoid (T)

  μ_0 = permeability of free space ($4\pi \times 10^{-7}$ T m A^{-1})

  n = number of turns in the solenoid per metre

  I = current travelling through the solenoid (A)

- If the direction of the current through the coils is known, the magnetic field direction can be found by using an adaptation of Maxwell's screw rule. If the fingers curl in the direction of the conventional current flow, the thumb points in the direction of the north pole.
- Outside the solenoid, the magnetic field lines flow from the north pole to the south pole. However, on the inside, the field lines are nearly parallel and run from the south to the north pole inside the solenoid.

Forces experienced by particles in magnetic fields

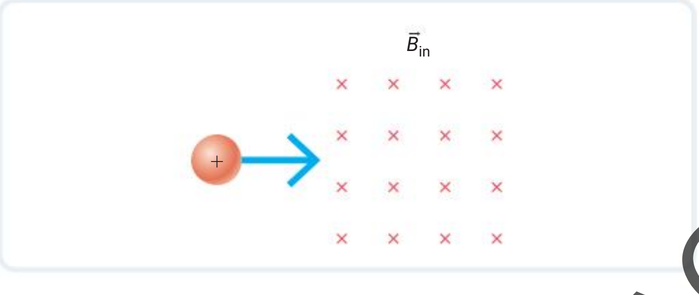
- A current-carrying conductor placed in a magnetic field experiences a force known as the Lorentz force.
  $F = BIL\sin\theta$
  where:
  F = magnetic force on the wire (N)
  B = magnitude of the external magnetic field (T)
  I = current flowing through the wire (A)
  L = length of wire within the magnetic field (m)
  θ = angle the wire makes with the external magnetic field (°)
- To find the direction of the force, use the right-hand rule. Hold out your right hand flat and move your hand so your thumb points in the direction of conventional current and fingers point in the direction of the *B* field. Your palm then points in the direction in which the magnetic force will push the wire.
- A single moving charge also experiences a force in a magnetic field. Its direction of motion is its direction of (single charge) conventional current.
- Charges moving into external magnetic fields also experience magnetic forces acting perpendicular to the field and to their motion. The magnitude of these forces is determined by:
  $F = qvB\sin\theta$
  where:
  F = magnetic force on the wire (N)
  q = magnitude of the charge moving in the magnetic field (C)
  v = magnitude of the velocity of the moving charge (m s^{-1})
  B = strength of the external magnetic field (T)
  θ = angle between the vectors of conventional current direction and external magnetic field (°)
- The direction of the force can be found similarly by using right-hand rule. The thumb points in the direction a positive charge will move in the field, fingers point in the direction of the *B* field and the palm points in the direction of the magnetic force.

MULTIPLE CHOICE

CHAPTER EXAM

1. A magnetic field exists around a current-carrying wire. At 2 cm from the wire, the field strength is 6.0×10^{-5} T. The field is 3.0×10^{-5} T at:
   A 4 cm from the wire.
   B 3 cm from the wire.
   C 1 cm from the wire.
   D 0.5 cm from the wire.

2. What is the direction of the magnetic force acting on the following charged particle as it enters the *B* field below?



   A Into the page
   B Out of the page
   C Up
   D Down

3. As the distance *r* from a current-carrying wire increases, the *B* field at *r*:
   A increases linearly.
   B increases exponentially.
   C decreases exponentially.
   D decreases inversely.

4. There is no interaction between a magnetic field and a:
   A stationary electric charge.
   B moving electric charge.
   C stationary magnet.
   D moving magnet.

5. The magnetic field lines around a long, straight current are:
   A straight lines parallel to the current.
   B straight lines that radiate from the current like spokes of a wheel.
   C concentric circles around the current.
   D concentric helixes around the current.

6. When a moving charged particle enters a uniform magnetic field in a direction parallel to the field lines, the particle's:
   A direction is changed.
   B velocity's magnitude is changed.
   C energy is changed.
   D motion in unaffected.

7. When a moving charged particle enters a uniform magnetic field in a direction perpendicular to the field lines, the particle's:
   A direction is changed.
   B velocity's magnitude is changed.
   C energy is changed.
   D motion in unaffected.

8. A long, straight wire carries a current of 1.2 A. What is the magnetic field 8.0 mm from the wire?
   A 3.0×10^{-8} T
   B 3.0×10^{-5} T
   C 3.6×10^{-5} T
   D 1.9×10^{-4} T

9. A 200-turn solenoid is 40 mm long. If the magnetic field inside it is to be 0.010 T, what current will it carry?
   A 1.6 A
   B 251 A
   C 1.6×10^{3} A
   D 4.0×10^{7} A

10. Two parallel wires 800 cm long are 5 cm apart and carry currents of 20 A each in the same direction. Each wire exerts a force on the other of:
   A 1.6×10^{-19} N, attractive.
   B 1.3×10^{-2} N, attractive.
   C 1.6×10^{-3} N, repulsive.
   D 1.3×10^{-2} N, repulsive.

SHORT RESPONSE

11. An electron having a charge of 1.6×10^{-19} C and velocity of 2.8×10^{3} m s^{-1} enters a magnetic field of strength 20 μT at an angle of 30° to the field lines. Calculate the force and acceleration experienced by the electron as a result of interaction with the field.

12. A current balance is set up in a solenoid with *B* field 0.20 T. The current through the wire in the current balance is 3.6 A and 3 cm of the wire is perpendicular to the field in the solenoid. The side length of the current balance is 12 cm long. What is the magnitude of force the current balance experiences in this magnetic field?

CROSS-CHAPTER QUESTION

13. A beam of electrons travelling perpendicularly to a *B* field of 4.5×10^{-3} T is caused to move in a circular path of radius 2.0 cm. What is the velocity of the electrons?

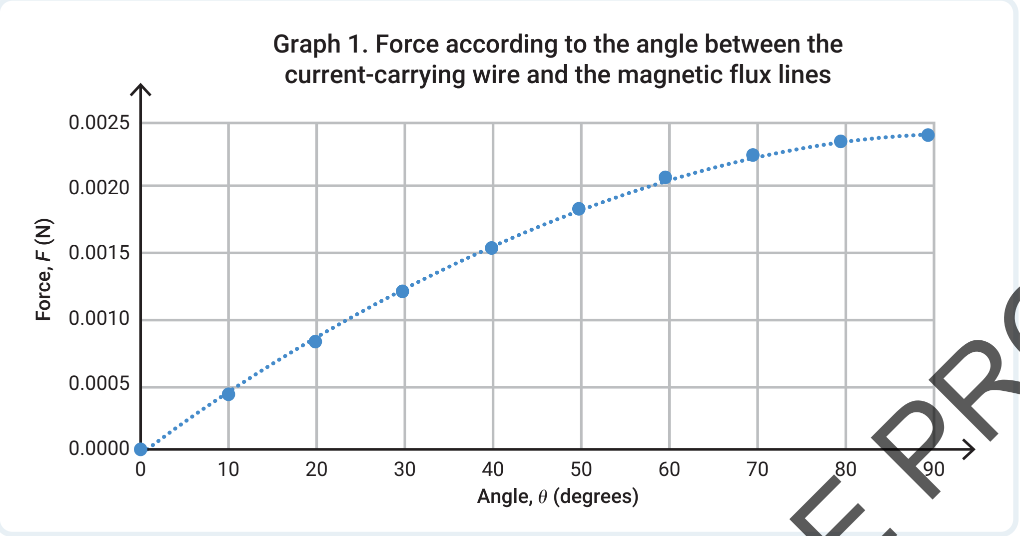
DATA ANALYSIS

14. Interpret evidence
    A student conducted an experiment to answer the following research question.

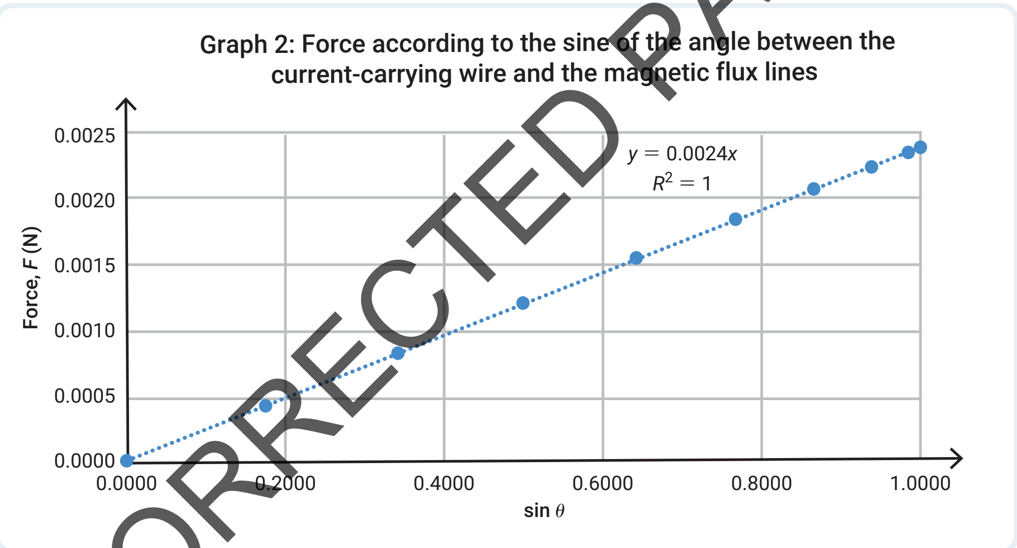
    What is the relationship between the force on a straight current-carrying wire and the angle of the wire relative to the magnetic flux lines it is immersed in?

    The student set up a current balance in which a 4 cm long current-carrying wire was immersed in a 0.03 T magnetic field. The angle of the wire relative to the flux lines within the magnetic field was changed and the resultant force on the wire was measured.

The raw data gathered is presented in Graph 1.



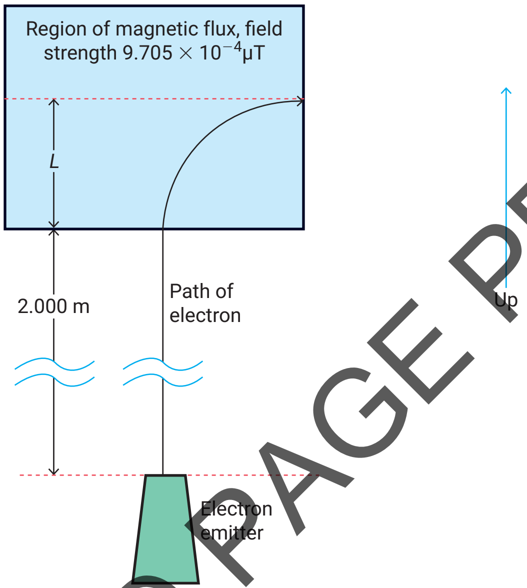
This data was then further processed to produce Graph 2.



a Describe and explain the shape of the trend line in Graph 1.
b Deduce why the student did not collect data for angles greater than 90°.
c Predict using a sketch what Graph 1 would look like if the data set was extended from 90° to 180°.
d Describe the mathematical relationship between force, F and $\sin\theta$ evident in Graph 2.
e Draw a conclusion about the magnitude of the current in the wire by using the data.
f Calculate the expected force on the wire in the apparatus if it is at an angle of 35° relative to the magnetic flux lines.
g Infer how accurate and precise the data collected is by referring to the line equation and R^2 value of Graph 2.

15. Analyse data

In a vacuum chamber, an electron was released vertically from an emitter with a velocity of $7.120\ \text{m s}^{-1}$. It then entered a region of uniform magnetic flux perpendicular to the path of the electron with a field strength of $9.705 \times 10^{-4}\ \mu\text{T}$. The apparatus and path of the electron is illustrated below.



a Deduce the direction of the perpendicular magnetic field. Is it into the page or out of the page?

b Sketch in the magnetic field using conventional icons

c Calculate the velocity of the electron as it enters the magnetic field.

d Draw a conclusion about the magnitude of L; will it be greater than or less than 2 cm?

 9780170483704

CHAPTER 9

Electromagnetic induction

VICTOR HABBICK VISIONS/SCIENCE PHOTO LIBRARY

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Describe the concepts of magnetic flux, magnetic flux density, electromagnetic induction, electromotive force (EMF), Faraday's Law and Lenz's Law.
- Solve problems involving the magnetic flux in an electric current-carrying loop using $\phi = BA\cos\theta$.
- Describe the process of inducing an EMF across a moving conductor in a magnetic field.
- Explain how Lenz's Law is consistent with the principle of conservation of energy.
- Explain how transformers work in terms of Faraday's Law and electromagnetic induction.
- Solve problems involving electromagnetic induction using $\text{emf} = -\frac{N\Delta(BA_{\perp})}{\Delta t}$, $\text{emf} = -N\frac{\Delta\phi}{\Delta t}$, $I_pV_p = I_sV_s$ and $\frac{V_p}{V_s} = \frac{N_p}{N_s}$.

Introduction

In Chapter 8, we discovered that there was an intrinsic link between electricity and magnetism in that an electric current in a conductor can create a magnetic field. It was a logical step for scientists to ask the question whether the opposite was true: can a magnetic field produce an electric current? The work of Joseph Henry and Michael Faraday independently showed in 1820 that this was indeed possible through the process of electromagnetic induction. This is an example of a principle of symmetry that many physicists regard as a fundamental property of nature. The impact of the discovery of electromagnetic induction cannot be understated because it resulted in the development, among other things, of the electric generator and motor.

Assessments
- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practicals
- Investigating the magnification of a microscrope

Worksheets
- Name
- Name
- Name

FPO

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ The concepts of current (I) and voltage (V) or EMF.
- ✓ Electric circuits can be depicted by circuit diagrams.
- ✓ The relationship between voltage, current and resistance can be calculated using Ohm's law, $V = I \times R$.
- ✓ Power (P) can be calculated by using $P = V \times I$.
- ✓ The characteristics of solenoids and magnets.
- ✓ A magnetic field is also called a B field and can be quantified using magnetic field strength in tesla (T).
- ✓ Magnetic fields can be represented visually by field lines.
- ✓ The SI unit for area (A) is the square metre (m^2)
- ✓ The normal is a direction or line perpendicular to a given surface or plane.
- ✓ Area can be calculated for a circle by using $A = \pi r^2$, and for a rectangle/square by using A = length × width.
- ✓ In geometry, a plane is a flat two dimensional surface, physical or imaginary.
- ✓ The frequency (f) of a cyclic or oscillating phenomena is the number of cycles or oscillations per second; units are hertz (Hz) or s^{-1}.
- ✓ The period (T) of a cyclic or oscillating phenomena is the time it takes for a single cycle or oscillation; units are seconds (s).
- ✓ A significant percentage of household electricity is produced by generators.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe the concepts of magnetic flux, magnetic flux density, electromagnetic induction, electromotive force (emf), Faraday's law and Lenz's law
- ✓ solve problems involving the magnetic flux in an electric current-carrying loop using $\phi = BA\cos\theta$
- ✓ describe the process of inducing an emf across a moving conductor in a magnetic field
- ✓ explain how Lenz's law is consistent with the principle of conservation of energy
- ✓ explain how transformers work in terms of Faraday's law and electromagnetic induction
- ✓ solve problems involving electromagnetic induction using $\text{emf} = -\frac{N\Delta(BA_{\perp})}{\Delta t}$, $\text{emf} = -N\frac{\Delta\phi}{\Delta t}$, $I_pV_p = I_sV_s$ and $\frac{V_p}{V_s} = \frac{N_p}{N_s}$
- ✓ predict the resultant induced current in a wire, wire loop or solenoid using Lenz's law and/or the right-hand rule.
- ✓ describe step-up and step-down transformers
- ✓ describe how a rotary generator works, using the principles of electromagnetic induction
- ✓ determine the output of a rotary generator using $\text{emf} = 2\pi fnBA\cos(2\pi ft)$ and $f = \frac{1}{T}$
- ✓ determine RMS voltage by using $V_{RMS} = \frac{1}{\sqrt{2}}V_{peak}$
- ✓ use Lenz's law to describe eddy currents and how magnetic braking works.

9.1 Electromagnetic induction

Electromagnetic induction is the production of an electric field by a changing magnetic field. An electric field in a region means that there is an **electromotive force (emf)** acting between points in that region. If this emf acts upon free charge carriers, then a current will be generated. This current is referred to as an **induced current**. This is the basis of electromagnetic induction used in electricity generation.

Faraday quantitatively investigated the size of the current produced by moving a magnet near a conducting wire using a device like that shown in Figure 9.1.1. During this investigation, Faraday discovered that not only did the current depend on the strength of the magnet used, but it also depended on the area enclosed by the loop and the angle at which the two are orientated to each other. These three factors are combined in the concept of magnetic flux.

To understand how the process works, it is important to investigate magnetic flux.

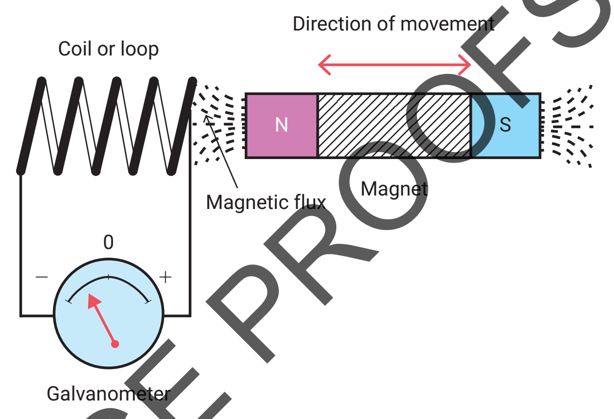


FIGURE 9.1.1 A current is induced when a magnet is moved relative to a coil of wire connected to a circuit.

Magnetic flux

Magnetic flux (Φ) is a measurement of the total magnetic field passing through a given area. It is directly proportional to the number of magnetic field lines passing through the defined area, which is referred to as the **magnetic flux density (*B*)**.

To derive a formula for magnetic flux, first consider a uniform magnetic field passing through an area, *A*, as shown in Figure 9.1.2. We choose the direction of the vector $\vec{A}$ to be perpendicular to the area, as this gives us a visually simplified way of representing the orientation of area. Note how this vector is aligned with the normal to the plane of the area. The magnitude of the vector $\vec{A}$ is the area *A*, in units of m^2. The magnetic flux density, $\vec{B}$, is the number of field lines crossing the area and depends on the directions of both $\vec{B}$ and $\vec{A}$. The magnetic flux, Φ, for a uniform magnetic field through a loop of area *A* is defined as:

$$\Phi = B_{\perp}A = BA\cos\theta$$

KEY FORMULA

Magnetic flux

$\Phi = B_{\perp}A = BA\cos\theta$

where:

Φ = magnetic flux (Wb)

B = magnetic field strength (T)

A = area of the surface (m^2)

θ = angle between the magnetic field lines and a normal to the plane of the area (°)

electromagnetic induction the production of an electromotive force (emf) or a voltage in an electrical conductor due to its dynamic interaction with a magnetic field

electromotive force (emf) a difference in potential that tends to give rise to an electric current

induced current a current that is produced due to the presence of an electromotive force

magnetic flux (Φ) a measurement of the total magnetic field that passes through a given area; unit is the weber (Wb)

magnetic flux density (*B*) the strength of a magnetic field per unit area otherwise known as magnetic field strength; unit is the tesla (T)

Weblink
What is magnetic flux?

The magnetic flux, Φ has units of T m^2 or the weber, Wb, after Wilhelm Weber (1804–91), who co-invented the first electromagnetic telegraph.

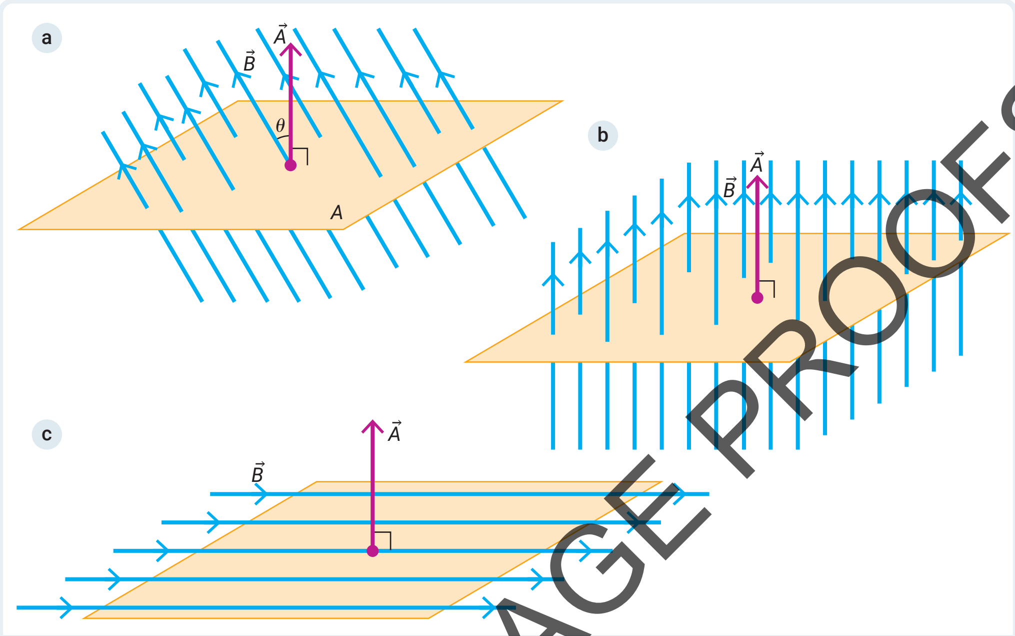


FIGURE 9.1.2 (a) The flux through the area A depends on the angle, θ, between the area vector $\vec{A}$ and the field, $\vec{B}$: $\Phi = BA \cos\theta$. (b) Flux is at a maximum when θ is zero and $\vec{B}$ and $\vec{A}$ are perpendicular. (c) Flux is at a minimum when $\theta = 90°$ and $\vec{B}$ and $\vec{A}$ are perpendicular.

The flux has maximum amplitude when the field is in the same direction as, or opposite to, the vector $\vec{A}$; that is, when $\vec{B}$ is perpendicular to the plane of the area. The flux is zero when $\vec{A}$ is perpendicular to $\vec{B}$; that is, when $\vec{B}$ is parallel to the plane of the area.

WORKED EXAMPLE 9.1.1

A loop of cross-sectional area 0.050 m^2 is in a uniform magnetic field of magnitude 0.24 T.

a Sketch diagrams showing the loop and field and identifying the angle, θ, between the area vector $\vec{A}$ and the field $\vec{B}$, when the flux is a:

- **i** minimum
- **ii** maximum.

b Determine the maximum and minimum values of the flux through the loop.

ANSWERS

a **i** Identify that flux has a minimum value of $\Phi = BA\cos\theta = 0$ when $\theta = 90°$ and that this is when the plane of loop is parallel to the field.

ii Identify that flux has a maximum value of $\Phi = BA$ when $\theta = 0°$ and that this is when the plane of loop is perpendicular to the field.

b **1** **Determine the formula.**

$\Phi = BA\cos\theta$

The maximum flux occurs when the field lines and the plane of the loop are perpendicular:

$\Phi_{max} = BA\cos 0° = BA$

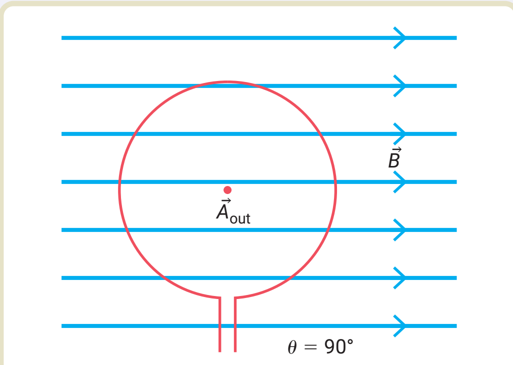


FIGURE 9.1.3 The loop is parallel to the magnetic field, thus the effective area is zero and the magnetic flux is zero.

2 **Substitute the known values.**

$\Phi_{max} = 0.24\ T \times 0.05\ m^2$

3 **Calculate the answer.**

$\Phi_{max} = 1.2 \times 10^{-2}\ Wb$

4 **The minimum flux of 0 occurs when the field lines and the plane of the loop are parallel.**

$\Phi_{max} = BA \cos 90° = 0$

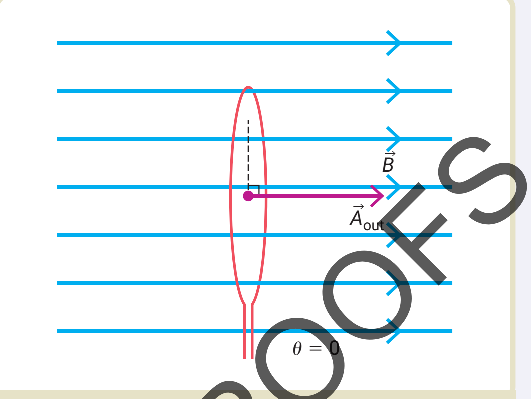


FIGURE 9.1.4 The loop is perpendicular to the field, thus the magnetic flux is at a maximum.

WORKED EXAMPLE 9.1.2

In a magnetic field experiment, the angle formed between the normal to a surface with an area of 0.55 m² and a uniform magnetic field of strength 0.15 T is measured to be 25°.

Determine the magnetic flux passing through the surface.

ANSWER

1 **Determine the formula.**

The magnetic flux is found using $\Phi = BA\cos\theta$.

2 **Substitute the known values.**

$\Phi = 0.15 \times 0.55 \times \cos 25°$

3 **Calculate the answer.**

$\Phi = 0.0748\ Wb$

LEARNING CHECK 9.1

DESCRIBING

1 **Recall** the units of magnetic flux.

2 **Define** 'electromagnetic induction'.

3 **Explain** why the magnetic flux through a loop of wire orientated perpendicularly to a uniform magnetic field will decrease as the wire is rotated to become parallel to the magnetic field.

4 **Compare** the concepts of magnetic field strength and magnetic flux and **explain** their differences.

APPLYING

5 **Calculate** the magnetic flux that passes through a loop of wire that has a diameter of 10.0 cm when it is placed perpendicularly within a uniform magnetic field of strength 0.050 T.

6 If the angle formed between the normal to a surface of area 0.25 m² and a uniform magnetic field of strength 0.10 T is equal to 35°, **determine** the magnitude of the magnetic flux passing through the surface.

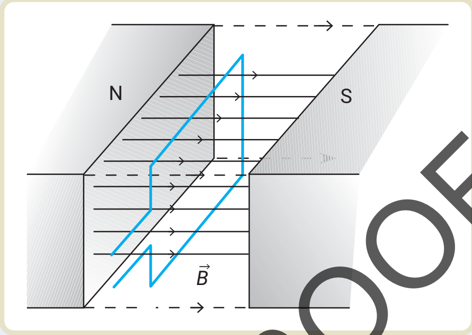
7 A solenoid produces a magnetic field of 0.25 T in its interior. The field is approximately uniform. **Determine** the radius of the coil, given that the flux through any loop of the solenoid is equal to 5.0 mWb.

ANALYSING

8 A loop of cross-sectional area 0.015 m^2 is in a uniform magnetic field of 0.030 T. Initially the loop is perpendicular to the field lines, as shown below. The loop is rotated about an axis parallel to its long sides at a uniform angular velocity of five revolutions per second.

a **Determine** the flux through the loop at $t = 0$ s.

b **Sketch** a graph of the flux through the loop as a function of time. Mark important features on your graph including the maximum flux and the period.



9.2 Faraday's law of induction

induced emf an emf created by a changing magnetic field

Weblinks
Faraday's Law
Magnetic Flux, Induction, and Faraday's Law

Faraday's experiments showed that when magnetic flux changes with time, an electric field is induced. If there is a loop within this field, an **induced emf** is produced within it.

When there is a change in the magnitude of the magnetic flux ($\Delta\Phi$) through a loop of wire over a small timeframe (Δt), the magnitude of the induced emf is given by Faraday's law.

KEY FORMULA

Faraday's law of induction

The induced emf in a loop of wire is equal to the negative of the change in magnetic flux ($\Delta\Phi$) divided by the change in time (Δt).

$$\text{emf} = \frac{-\Delta\Phi}{\Delta t}$$

$$= \frac{-(\Phi_f - \Phi_i)}{\Delta t}$$

$$= \frac{-(BA\cos\theta_f - BA\cos\theta_i)}{\Delta t}\ \text{V}$$

According to Faraday's law, the induced emf will have units of T m^2 s^{-1}, which is the same as the volt, V. The negative sign indicates that the induced emf *opposes* the change in flux.

Once an emf is induced, a current will flow if there are free charge carriers and a path for them to flow along. This is usually achieved by putting a metal coil in the field. This induced current is related to the emf by Ohm's law:

$$I = \frac{V}{R} \text{ or } I = \frac{\text{emf}}{R}$$

where R is the resistance of the path, I is the current, and emf is the voltage.

The term 'emf' is used here rather than potential difference for a reason. Because they often do the same thing and have the same unit, volt (V), they are often treated as if they are the same thing, but from the definitions given below, we can see that they are not.

- Potential difference is the unique difference in potential energy per unit charge *between any two points* in an electric field.

- emf is the energy per unit charge available to a charged particle. The induced emf between any two points in a changing magnetic field *does not* have a unique value but depends on the path between the two points. This is because it depends on the flux enclosed. Different closed paths between two points may contain different fluxes.

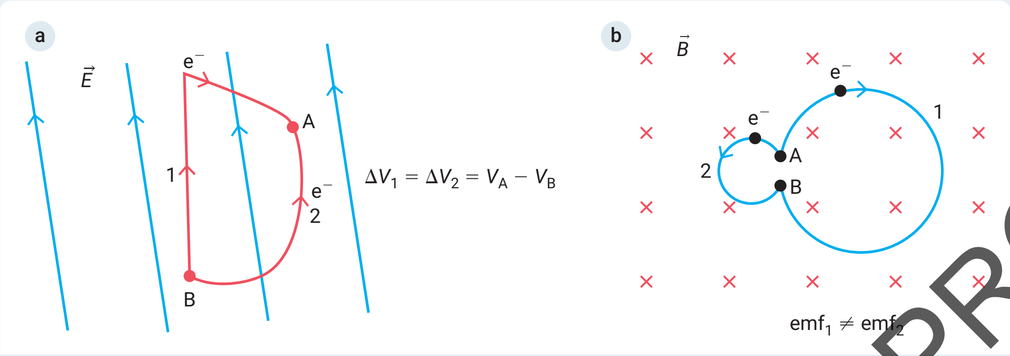


FIGURE 9.2.1 (**a**) Potential difference: an electron moves from point A to point B in an electric field. The change in potential energy of the electron is the same regardless of the path taken. (**b**) emf: an electron passes through two loops in a changing magnetic field. The emf measured across each loop is different because the loops contain different magnetic fluxes, even though they have the same beginning and end points.

WORKED EXAMPLE 9.2.1

Determine the emf generated in a loop of area 0.2 m² that lies perpendicular to a magnetic field that decreases from 0.4 T to 0.0 T over a period of 2 s.

ANSWER

1 Determine the formula.

The emf may be found through application of the formula:

$$\text{emf} = \frac{-\Delta\Phi}{\Delta t} = \frac{-(\Phi_f - \Phi_i)}{\Delta t}\ \text{V}$$

$$= \frac{-(BA\cos\theta_f - BA\cos\theta_i)}{\Delta t}\ \text{V}$$

2 Substitute the known values.

$$\text{emf} = \frac{-(BA\cos\theta_f - BA\cos\theta_i)}{\Delta t}$$

$$= \frac{-A\cos\theta(B_f - B_i)}{\Delta t}$$

$$= \frac{-0.2\text{m}^2\cos 0°(0.0 - 0.4)}{2}$$

3 Calculate the answer.

emf = 0.04 V

An induction cooktop (**Figure 9.2.2**) uses electromagnetic induction to transfer electrical energy to a ferromagnetic cooking vessel placed on it. The coil of the cooktop, which is mounted below its surface, carries an alternating current that, in turn, creates an alternating current that passes through the cooking surface and into the base of the vessel. This process creates a changing magnetic flux through the surface of the vessel and induces a current in its walls. This current flows through the vessel and ultimately produces heat, which is transferred to its contents.

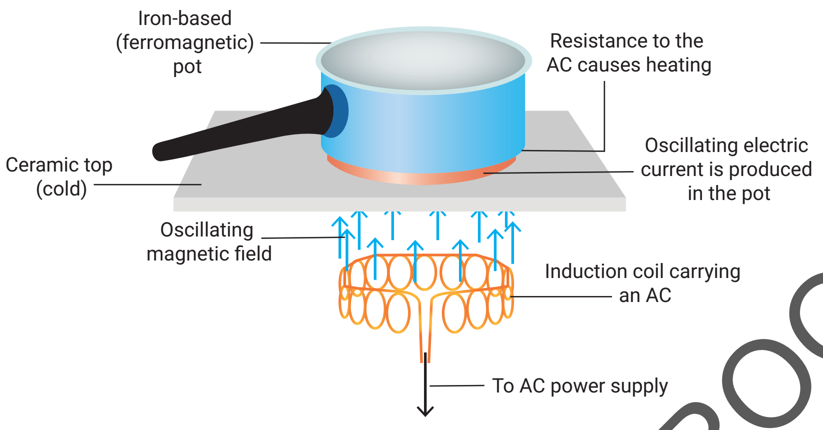


FIGURE 9.2.2 Induction cooking uses electromagnetic induction to transfer energy very effectively to the cooking vessel. This energy is manifested as heat.

Solving problems with Faraday's law

By, substituting the magnetic flux equation into Faraday's law, we can see that:

$$\text{emf} = \frac{-N\Delta\Phi}{\Delta t} = \frac{-N\Delta(BA\cos\theta)}{\Delta t} = \frac{-N(\Phi_f - \Phi_i)}{\Delta t}$$

KEY FORMULA

Faraday's law and the magnetic flux equation combined

$$\text{emf} = \frac{-N\Delta\Phi}{\Delta t}$$
$$= \frac{-N\Delta(BA\cos\theta)}{\Delta t}$$
$$= \frac{-N(\Phi_f - \Phi_i)}{\Delta t}$$

where:
emf = electromagnetic force induced (V)
B = magnetic field strength (T)
A = area of the surface (m^2)
θ = angle between the magnetic field lines and a normal to the surface (°)
Φ = magnetic flux (Wb)
t = time over which the change occurs (s)

From this equation, it can be seen that there are three ways to induce an emf:

- Change the magnetic flux density, B.
- Change the area, A.
- Change the angle, θ, between the area and the field.

In practice, it is usually either the magnetic field or the angle that is changed. For example, when a loop or coil of wire with area A is placed in a field, the flux through the loop can be varied by spinning the loop. This changes the angle and induces an emf in the loop. The same effect can be achieved by spinning a magnet near the loop. In both cases the flux varies in time. This is used in electric generators.

We can take any parameter kept constant out of the brackets. For example, if area and angle are kept constant while B is varied, we write:

$$\text{emf} = \frac{-A\cos\theta\Delta B}{\Delta t} = \frac{-A\cos\theta(B_f - B_i)}{\Delta t}$$

An emf is produced by a changing magnetic flux density.

WORKED EXAMPLE 9.2.1

A loop of conducting wire with an area of 0.500 m^2 is placed perpendicularly in a uniform 0.300 T magnetic field. Calculate the induced emf in the coil if it is removed from the magnetic field in 0.100 s.

ANSWER

1 Determine the formula/law.

$$\text{emf} = \frac{-\Delta\Phi}{\Delta t}$$

2 Insert the magnetic flux equation.

$$\text{emf} = \frac{-\Delta(BA\cos\theta)}{\Delta t}$$

3 Rearrange the equation to find the unknown.

$$\text{emf} = \frac{-A\cos\theta(B_f - B_i)}{\Delta t}$$

4 Substitute the known values.

$$\text{emf} = \frac{-0.500\,\text{m}^2 \times \cos 0^\circ \times (0\,\text{T} - 0.300\,\text{T})}{0.100\,\text{s}}$$

5 Calculate the answer.

emf = 15.0 V

If the area and field are held constant but the angle is changed, use the rearranged equation:

$$\text{emf} = \frac{-BA\Delta\cos\theta}{\Delta t} = \frac{-BA(\cos\theta_f - \cos\theta_i)}{\Delta t}$$

WORKED EXAMPLE 9.2.2

A loop of conducting wire with an area of 0.0250 m^2 is placed perpendicularly within a uniform 0.500 T magnetic field. Calculate the induced emf in the loop if the loop is rotated so that it becomes parallel to the magnetic field in 0.600 s.

ANSWER

1 Determine the formula/law.

$$\text{emf} = \frac{-\Delta\Phi}{\Delta t}$$

2 Insert the magnetic flux formula.

$$\text{emf} = \frac{-\Delta(BA\cos\theta)}{\Delta t}$$

3 Rearrange the formula to find the unknown.

$$\text{emf} = \frac{-BA(\cos\theta_f - \cos\theta_i)}{\Delta t}$$

4 Substitute the known values.

$$\text{emf} = \frac{-0.500\,\text{T} \times 0.0250\,\text{m}^2(\cos 90° - \cos 0°)}{0.600\,\text{s}}$$

5 Calculate the answer.

$\text{emf} = 2.08 \times 10^{-2}$ V

To generate a larger emf, a coil containing multiple loops of wire is used. Each loop will have an emf induced between its ends, so connecting *N* loops in series is like connecting *N* batteries in series. Simply add the emf in all loops. Therefore:

KEY FORMULA

Faraday's law for multiple loops

$$\text{emf} = \frac{-N\Delta\Phi}{\Delta t}$$

$$= \frac{-N\Delta(BA\cos\theta)}{\Delta t}$$

where:

emf = electromagnetic force induced (V)

N = number of loops of wire

B = magnetic field strength (T)

A = area of the surface (m^2)

θ = angle between the magnetic field lines and a normal to the surface (°)

Φ = magnetic flux (Wb)

t = time over which the change occurs (s)

WORKED EXAMPLE 9.2.3

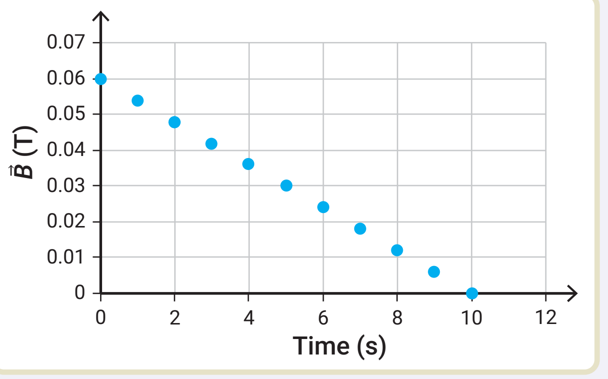
A wire loop of cross-sectional area 0.050 m^2 is in a magnetic field. The loop is perpendicular to the field. The field strength through the wire loop changes with time as shown.

a Sketch a graph of flux through the loop as a function of time.

b Sketch a graph of $\frac{\Delta\Phi}{\Delta t}$ as a function of time.

c Determine the emf induced between the ends of the wire.

d Determine the current induced in the loop when the loop has a resistance of 0.15 Ω.



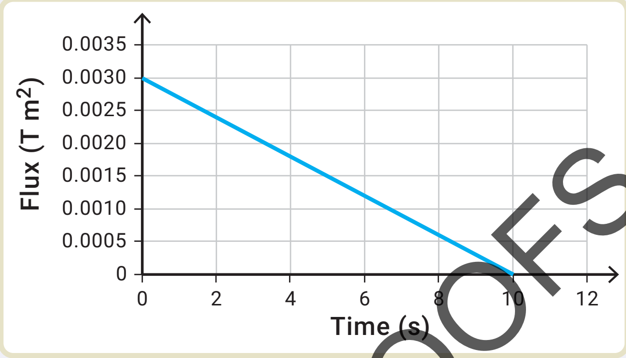
ANSWERS

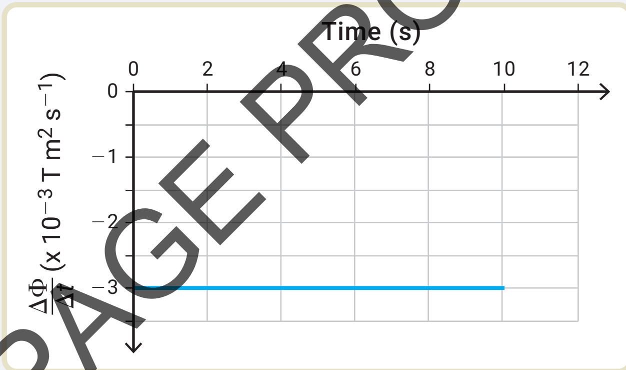
a **Use the values given in the question to sketch the graph.**

Use $\Phi = BA\cos\theta$, noting that $\theta = 0°$ and $A = 0.05\ m^2$.

b **Sketch the graph.**

$\frac{\Delta\Phi}{\Delta t}$ is the gradient of the flux against time graph, which is constant. We find this gradient by taking the rise over run for a section of the $\Phi(t)$ graph.





c **Find the emf.**
The emf is the negative of the gradient of our flux against time graph, which we can see is –0.3 mV; hence the emf is +0.3 mV.
emf = +0.3 mV

d **1** **Determine the formula/law.**

$$I = \frac{\text{emf}}{R}$$

2 **Substitute the known values.**

$$I = \frac{-3.0 \times 10^{-4}\ \text{V}}{0.15\ \Omega}$$

3 **Calculate the answer:**
$I = +0.002\ \text{A} = +0.2\ \text{mA}$ or $2 \times 10^{-3}\ \text{A}$

LEARNING CHECK 9.2

DESCRIBING

1 **Define** the concept of a change in magnetic flux.
2 **Explain** Faraday's law of electromagnetic induction.
3 List the three variables that can be altered to induce an emf in a loop of wire placed in a magnetic field.
4 Write an equation that can be used to calculate the magnitude of the emf induced in a conductive loop if the magnetic field strength is varied over time.
5 **Compare** the concept of potential difference with that of emf.
6 Reducing the value of one of these variables will result in a greater emf: *B*, *A* or *t*. **Identify** which one of these variables results in a larger emf.
7 A loop is placed in a uniform magnetic field and is moved in a straight line at a constant velocity through the field. **Explain** why no current is induced in the loop.
8 Figure 9.2.6 shows the flux through a loop of wire as a function of time.
   a State the time at which the emf produced will be at a minimum.

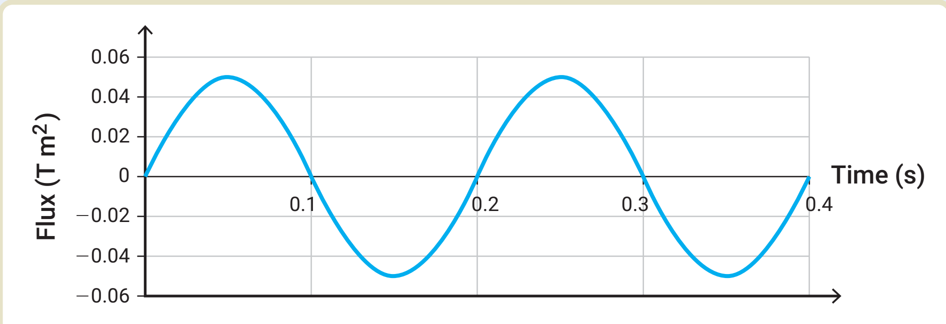


FIGURE 9.2.6 A graph of changing magnetic flux against time

b State the time at which the emf produced will be at a maximum.

c **Sketch** the induced emf across the loop as a function of time using the same time scale.

APPLYING

9 A circular loop of $r = 0.2$ m is placed in a uniform magnetic field perpendicular to the plane of the loop. The magnetic field is increased uniformly from 0.0 T to 0.5 T in 4 s. **Calculate** the emf in the loop.

10 Use a flow diagram to **explain** how a cast iron pot placed on an induction cooktop is able to heat its contents.

11 A circular loop of radius $r = 0.15$ m is placed in a uniform field perpendicular to its plane. The magnetic field decreases uniformly, inducing an emf of 0.02 V. If the magnetic field changes over $\Delta t = 5$ s, **determine** the change in magnetic field strength over this time.

12 A loop of conducting wire with an area of 0.45 m^2 is placed perpendicularly within a uniform 0.10 T magnetic field. **Calculate** the induced emf in the coil if the loop is removed from the magnetic field in 0.20 s.

13 A loop of conducting wire with an area of 0.250 m^2 is placed perpendicularly within a uniform 0.350 T magnetic field. **Calculate** the induced emf in the loop if the loop is rotated so that it becomes parallel to the magnetic field in 0.50 s.

14 **Calculate** the rate at which a uniform magnetic field passing perpendicularly through a loop of area 0.35 m^2 must change in order to produce an emf of 12.0 V.

9.3 Lenz's law

The negative sign in the equation for induced emf indicates the direction of the induced current.

Consider a loop in a magnetic field that is getting stronger with time (Figure 9.3.1). There are two possible directions in which the induced current can flow. The negative sign tells us that the current must flow so that the flux through the loop decreases. Why is this? The short answer is conservation of energy.

The potential energy of the changing magnetic field is transformed into electric potential energy. The result is an electric field that can do work by applying a force. Work is done on any free electrons by the induced electric field. The electrons then flow, giving the induced current. The induced current gets its energy from the changing magnetic flux (via the electric field) and so reduces the rate at which the flux changes.

Consider what would happen if the current acted to produce a further increase in the magnetic flux through the loop. The flux would increase more, giving a bigger induced current, giving a bigger flux and so on. The result would be a 'perpetual motion machine' that made more and more current without any source of energy. This would violate the law of conservation of energy and cannot happen. So the current *must* flow in the other direction and act to decrease the magnetic flux through the loop. Lenz's law is essentially a statement of *conservation of energy*.

KEY LAW

Lenz's law

An induced emf acts to produce an induced current. The induced current is in the direction that causes a change in magnetic flux that opposes the change in flux which induced the emf.

Weblink
Lenz's law

The current through a solenoid due to a changing magnetic field flows in a direction so that its own magnetic field opposes the motion that created it. This is known as Lenz's law. Lenz's law is a direct implication of the law of conservation of energy. In Physics circles it is also stated as 'there is no such thing as a free lunch!'

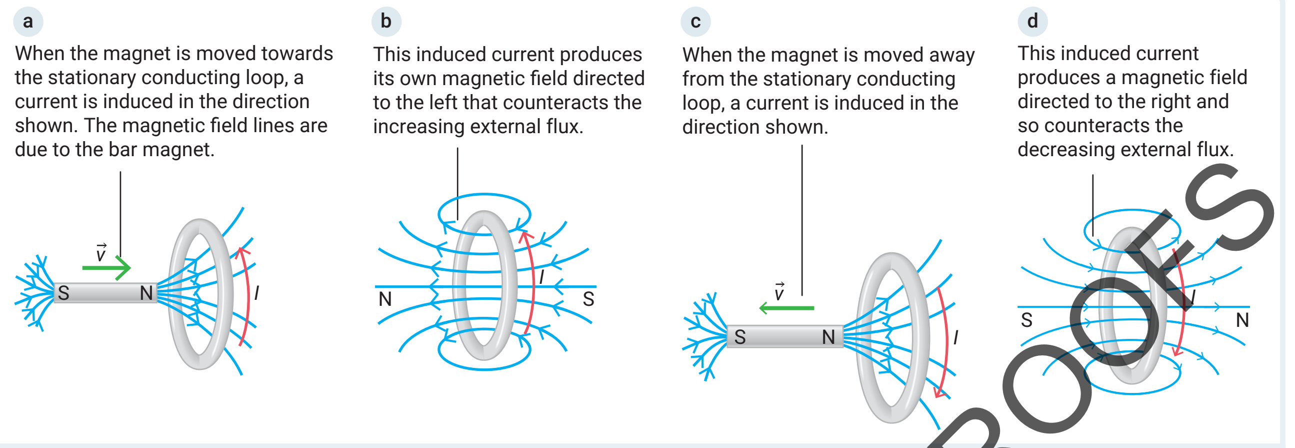


FIGURE 9.3.1 A moving bar magnet induces a current in a conducting loop. The direction of the current is determined by Lenz's law.

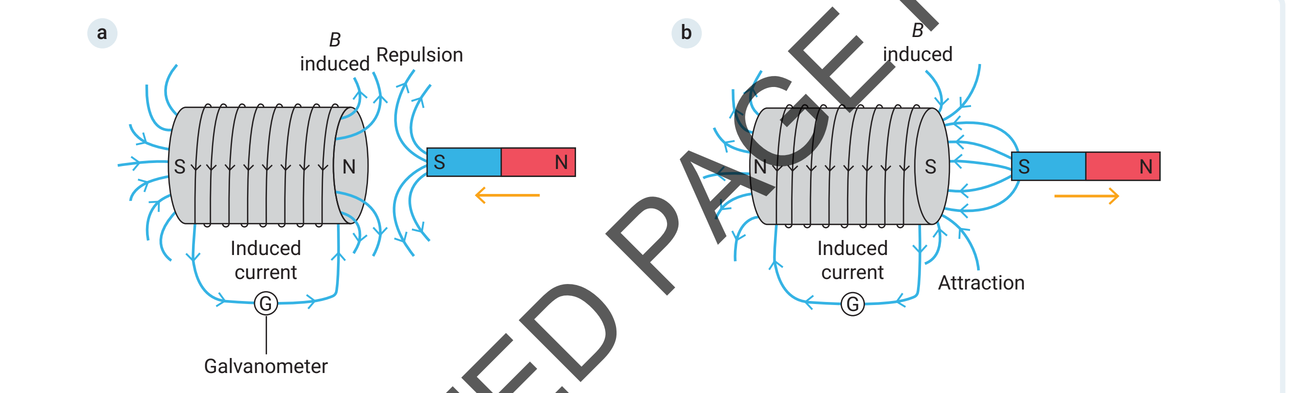


FIGURE 9.3.2 Introducing or removing a bar magnet induces a current in a conducting loop. The direction of the current is determined by Lenz's law. (a) Introducing a north pole is equivalent to withdrawing a south pole. (b) Withdrawing a north pole is equivalent to introducing a south pole.

The magnitude of the conventional current flowing through a solenoid may be readily determined by using the formulas described earlier. However, to determine the direction of current flow, you need to know how to apply Lenz's law and the right-hand grip rule.

Determining the direction of conventional current flow in a solenoid

For questions that involve determining the direction of conventional current in a solenoid, follow the steps below:

1. Recognise that Faraday's law will lead to an induced current due to a changing magnetic field. (This may be as a result of an introduced permanent magnet, turning on an electromagnet, or introducing a solenoid or coil to a magnetic field.)
2. Recognise that Lenz's law stipulates that any induced conventional current will circulate in a direction so that its own magnetic field will oppose the motion. (There is no such thing as a free lunch!)
3. Recognise the polarity (poles of N and of S) of the solenoid as a result. Label these and draw the magnetic field lines around and through the solenoid. (Magnetic field lines run north to south outside of the magnet.)
4. Apply the right-hand grip rule to determine the direction of the conventional current through the solenoid.

WORKED EXAMPLE 9.3.1

A permanent magnet is introduced to, or withdrawn from, a solenoid, inducing an electric current. Determine the direction of the field acting around the electromagnet (solenoid) as well as the direction of conventional current flow within the coil for each case from Figure 9.3.3.

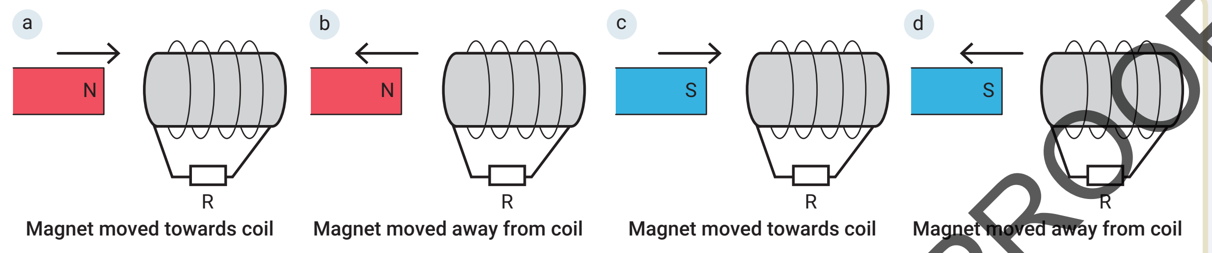


FIGURE 9.3.3 Introducing or removing a bar magnet induces a current in a conducting loop.

ANSWER

1. **Understand how Faraday's law affects current.**
   We recognise that Faraday's law will lead to an induced current due to a changing magnetic field with the introduction or removal of a magnet to the coil.
2. **Understand the impact of Lenz's law.**
   Lenz's law stipulates that any induced conventional current will circulate in a direction so that its own magnetic field will oppose the motion. Where a north pole is introduced, a north pole is created to oppose the introduction of the north pole.
3. **Label poles.**
   Label poles of the electromagnet (solenoid). Magnetic field lines run north to south outside the magnet.
4. **Apply the relevant rule.**
   Apply the right-hand grip rule to determine the direction of the conventional current through the solenoid.

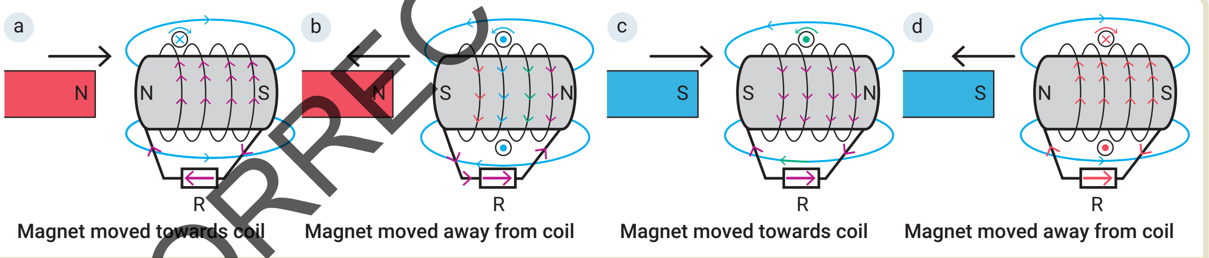


FIGURE 9.3.4 The direction of the induced conventional current may be determined, along with the magnetic field around the solenoid, when introducing or removing a bar magnet.

eddy current a circular current induced in a conductor due to a changing magnetic field

magnetic braking braking due to the interaction of eddy currents and an external magnetic field

Eddy currents

Induced currents are seen not only in wire loops, but in any material in which there are free charge carriers. If a magnet is moved around over a piece of metal, the changing magnetic field will induce **eddy currents** in the metal. The electrons move in circles in the region where the field is changing. They form loops and spirals of current, like eddies in a cup of tea when you stir it. These eddy currents create magnetic fields that oppose the changing flux from the moving magnet. They act to slow down or brake the magnet. This is called **magnetic braking**.

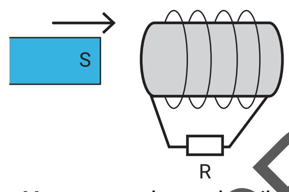
LEARNING CHECK 9.3

DESCRIBING

1 **Recall** Lenz's law.
2 **Define** 'eddy current'.
3 **Explain** how Lenz's law is consistent with the principle of the conservation of energy.
4 **Sketch** a flow diagram that summarises Lenz's law for a magnet that is being pushed into a coil of wire.

APPLYING

5 Use Lenz's law to **determine** the direction in which the induced current will flow in the solenoid as a south magnetic pole is introduced. Copy the following diagram and label the induced magnetic poles on the solenoid shown.



Magnet moved towards coil

9.4 Production and transmission of alternating current

The production and transmission of **alternating current (AC)** by electrical generators and transformers rely on the phenomenon of electromagnetic induction.

alternating current (AC) electrical current that alternates its direction of travel sinusoidally with time

Generators

Most electricity produced worldwide relies on the operation of an AC **generator**. At a fundamental level, a generator transforms kinetic energy, in the form of the relative movement of coils of wire and magnets, into electrical energy through the induction of an emf across the coils to generate a current. The energy required to produce the movement may come from any source. In Australia, it is mostly supplied by burning coal or gas. A small fraction comes from the gravitational potential energy of water (hydroelectric power stations) and the kinetic energy of air molecules (wind turbines). Many other nations use nuclear energy. All of these energy sources require generators to produce electricity.

generator a device used to produce electrical current by electromagnetic induction

Weblink
AC circuits: alternating current electricity

Figure 9.4.1 shows a very simple AC generator. An alternating current is one that varies between positive and negative values. Typically, AC varies sinusoidally. The coil is attached to an **armature** that rotates in the magnetic field between the poles of the two magnets. As the armature rotates, the flux through the loops of the coil varies, causing an emf across the ends of the coil. Each end of the coil is attached to a conducting slip ring that slides against a brush. The brushes are then connected to the external circuit that uses the emf generated.

armature the frame of the rotating part of a motor or generator, holding one or more coils

In Figure 9.4.2, the graph of magnetic flux against time is a sine curve because the original flux is zero.

The flux varies with the angle, θ, which varies in time so that:

$$\theta(t) = \left(\frac{2\pi}{T}\right)t = 2\pi ft$$

where T is the period of rotation.

The frequency is $f = \frac{1}{T}$. Hence, the flux as a function of time is given by:

$$\Phi = nBA\sin(2\pi ft)$$

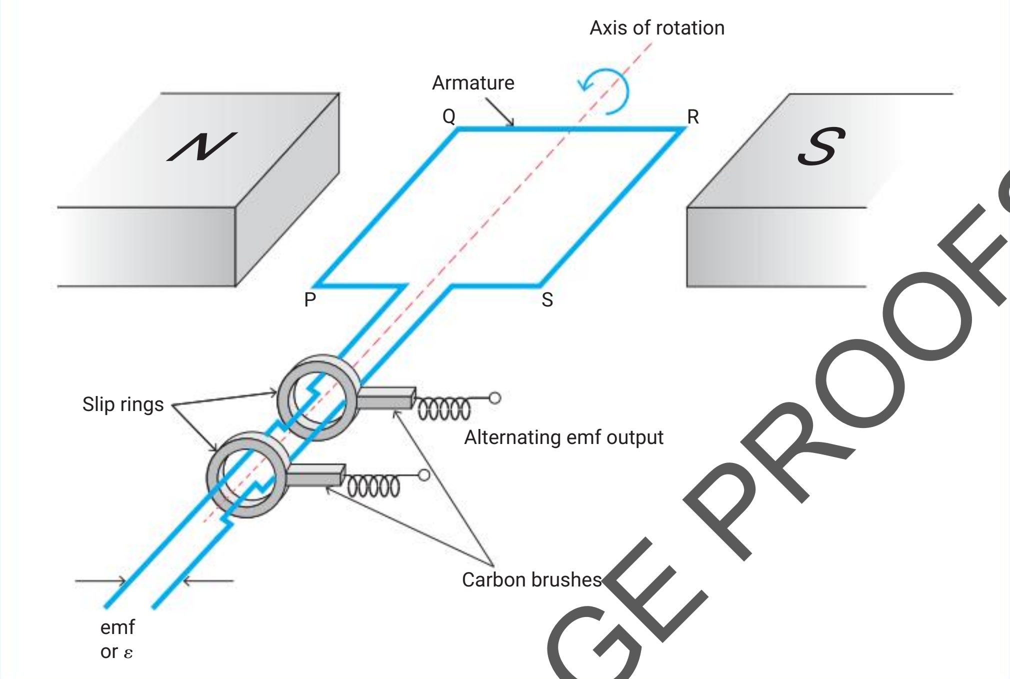


FIGURE 9.4.1 The rotation is circular, so the circular or periodic functions (sine and cosine) represent the loop's motion.

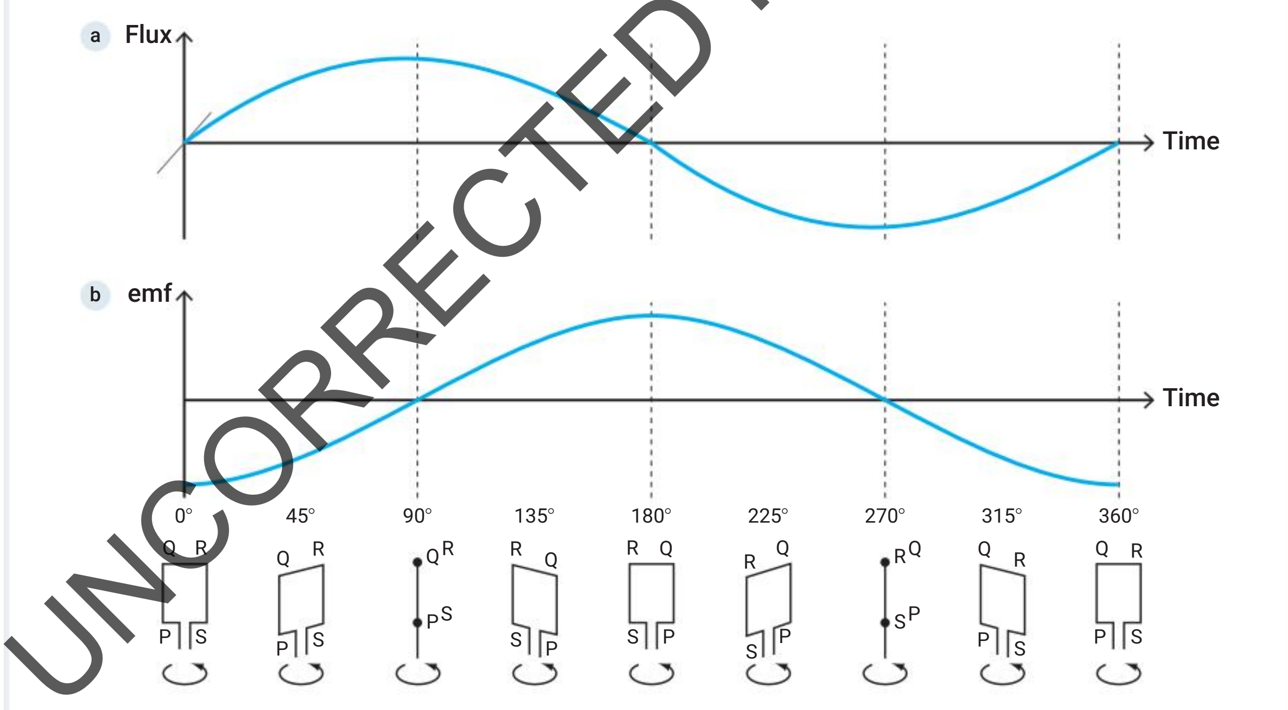


FIGURE 9.4.2 A loop rotates in a circle. (**a**) The flux through the loop describes a sine curve. (**b**) The rate of change of flux – the gradient – also describes a sinusoidal curve, but the emf is the negative of the rate of change of flux. The rotation of the coil is shown for each one-eighth of a turn.

KEY FORMULA

Flux through a rotating armature

The flux passing through a rotating armature in which a coil with n turns of area A rotates in a magnetic field of magnitude B

$\Phi = nBA \sin(2\pi ft)$

where:
Φ = magnetic flux (Wb)
n = number of coils of wire
B = magnetic field strength (T)
A = area of coil surface (m^2)
f = frequency of rotation (Hz)
t = time (s)

The emf is the negative of the gradient of the flux as a function of time.
So the emf is given by emf = $2\pi fnBA \cos(2\pi ft)$.

KEY FORMULA

emf produced by a rotating armature in a magnetic field

emf = $2\pi fnBA \cos(2\pi ft)$

where:
emf = electromagnetic force induced (V)
f = frequency of rotation (Hz)
B = magnetic field strength (T)
A = area of coil surface (m^2)
t = time (s)

This is shown in Figure 9.4.2. Note that when the flux is changing most rapidly, the emf has its maximum values. For a sine curve, the gradient is greatest at $t = 0$, $t = \frac{T}{2}$ and again at the end of each cycle. When the flux is at a peak, at $t = \frac{T}{4}$ and $t = \frac{3T}{4}$, the gradient is momentarily zero, so the emf is zero. The flux and emf have the same frequency.

The maximum emf occurs when $\cos(2\pi ft) = \pm 1$.

The maximum emf can be changed by changing f, n, B or A. If f is changed, the period changes as well as the emf.

Usually the armature has a large coil of wire, as the emf is proportional to the number of loops or turns in the coil. However, the bigger the coil, the heavier it is, so sometimes it is the magnets that are rotated instead of the coil.

KEY FORMULA

Maximum emf produced by a rotating armature in a magnetic field

$\text{emf}_{max} = 2\pi fnBA$

where:
f = frequency of rotation
n = number of turns
B = magnetic flux density
A = area of the armature

WORKED EXAMPLE 9.4.1

A square coil of side length 0.10 m is made up of 400 turns. It is rotated at 25 Hz in a magnetic field of magnitude 0.10 T.

a Determine the maximum emf induced.

b There is a maximum flux through the coil at $t = 0$. Sketch the emf as a function of time.

ANSWERS

a **1** **Determine the formula.**

Apply the maximum emf in an armature equation:

$emf_{max} = 2\pi fnBA$

2 **Substitute the known values.**

$emf_{max} = 2\pi \times 25\text{ Hz} \times 400 \times 0.10\text{ T} \times (0.10\text{ m})^2$

3 **Calculate the answer.**

$emf_{max} = 63\text{ V}$

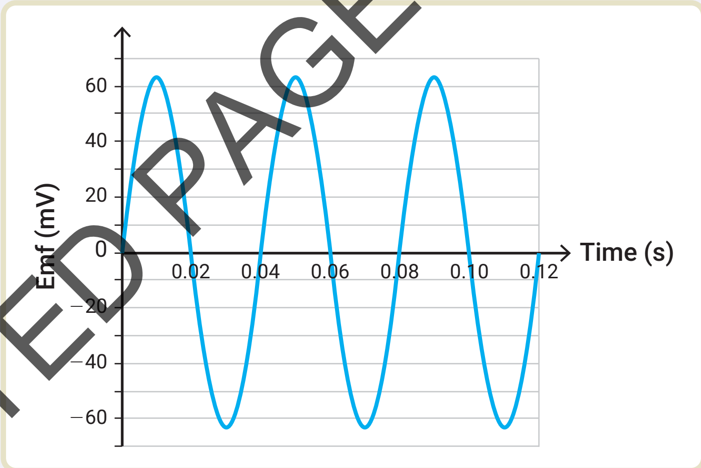
b **1** **Summarise the information known about emf and time.**

We know from part a that the emf varies between −63 V and +63 V. The period of its oscillations is the same as the period of rotation:

$$T = \frac{1}{f}$$

$$= \frac{1}{25\text{ Hz}}$$

$$= 0.04\text{ s}$$



2 **Sketch the graph.**

Note that as we do not know which way the coil is turning, a sketch showing emf starting from zero and decreasing first is also possible.

Alternating current

For a sinusoidal potential difference, the relevant quantities are peak potential difference, V_{peak}; peak-to-peak potential difference, $V_{peak-peak}$; period, T; and frequency, f. These are shown graphically in **Figure 9.4.3**.

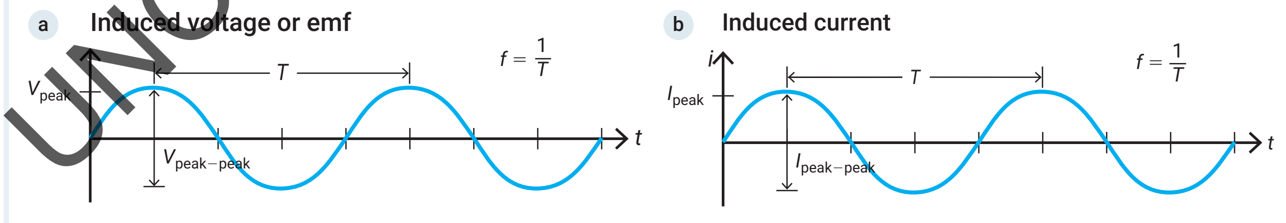


FIGURE 9.4.3 An induced emf gives rise to an induced current. The peak voltage and peak current are proportional to each other and they have the same frequency. (**a**) The induced emf or voltage has a peak value, V_{peak}, that is half the peak-to-peak value, $V_{peak-peak}$. It describes one cycle in one period of time, T. (**b**) The induced current, I, has a peak value, I_{peak}, that is half the peak-to-peak value $I_{peak-peak}$. It describes one cycle in one period of time, T.

When dealing with **direct current (DC)**, potential difference and current are constant, or at least constantly in the same direction. Values for AC vary between a peak positive value and a peak negative value, oscillating back and forth in each cycle. The average of the AC potential difference over one cycle is zero, yet the AC potential difference obviously delivers energy during that time; that is, it delivers power to a circuit. Power is proportional to the square of the potential difference. If we square the potential difference and find the average, we can get a value for the average power. To convert this to a single potential difference that would deliver the same power as the original AC potential difference, we take the square root of this average. The single value of potential difference that we get when we square, average and take the square root is called the **root mean square (rms)** value **(Figure 9.4.4)**.

direct current (DC) a current that flows in a single direction

root mean square (rms) the average AC potential difference or current that produces the same power in a load as a DC potential difference or current of the same magnitude

Weblink
What's the difference between AC and DC power?

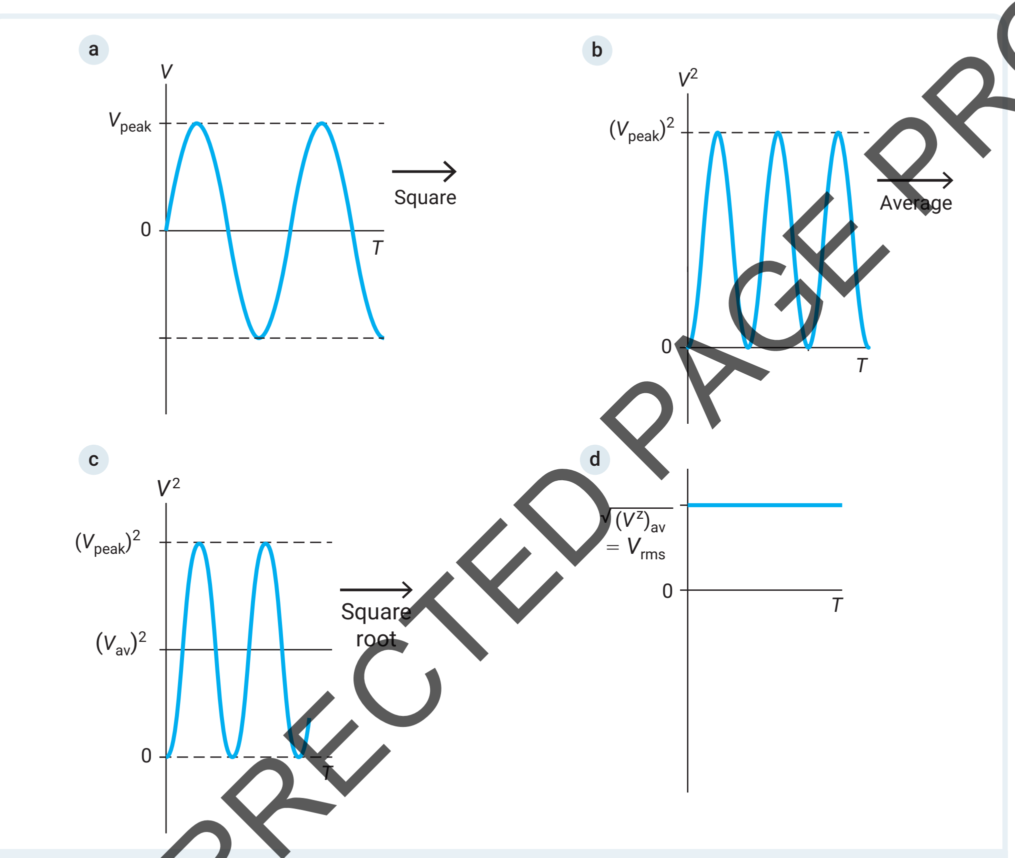


FIGURE 9.4.4 (**a**) A sinusoidal $V(t)$. (**b**) Square $V(t)$ to get $V^2(t)$. This has a peak value of $V_{peak}{}^2$. (**c**) The average value of $V^2(t)$, which has the value $\frac{1}{2}V_{peak}{}^2$. (**d**) Taking the square root of this value gives the rms value: $V_{rms} = \frac{1}{\sqrt{2}}V_{peak}$.

For an AC generator, the rms output emf is therefore calculated as:

$$\text{emf}_{rms} = \frac{\text{emf}_{max}}{\sqrt{2}} = \frac{2\pi fnBA}{\sqrt{2}} = \sqrt{2}\pi fnBA$$

The rms potential difference is an average AC potential difference that produces the same power in a resistive component as a constant DC potential difference of the same magnitude. AC systems are usually described using rms values.

Similarly, to calculate the root mean square current of an AC:

$$I_{rms} = \frac{I_{peak}}{\sqrt{2}} = 0.707\ I_{peak}$$

KEY FORMULA

Root mean square emf of an AC generator

$$\text{emf}_{rms} = \frac{\text{emf}_{max}}{\sqrt{2}} = \frac{2\pi fnBA}{\sqrt{2}} = \sqrt{2}\pi fnBA$$

KEY FORMULA

Root mean square current, I_{rms}, as a function of the peak current, I_{peak}

$$I_{rms} = \frac{I_{peak}}{\sqrt{2}}$$
$$= 0.707 I_{peak}$$

The rms current produced in a generator that is connected to a load of resistance R is calculated as:

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{\sqrt{2}\pi fnBA}{R}$$

KEY FORMULA

Root mean square current of an AC generator

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}\pi fnBA}{R}$$

The average power of an alternating current can be calculated as:

$$P_{av} = V_{rms} I_{rms} = \frac{V_{peak} I_{peak}}{2}$$

KEY FORMULA

Average power of AC as a function of rms voltage and current and of peak voltage and current

$$P_{av} = V_{rms} I_{rms}$$
$$= \frac{V_{peak} I_{peak}}{2}$$

WORKED EXAMPLE 9.4.2

If an AC generator that produces a peak voltage of 340 V is connected to an external load with a resistance of 55 Ω, calculate the:

a rms potential difference produced
b rms current produced
c average power of the alternating current.

ANSWERS

a **1** **Determine the formula.**

The rms voltage equation:

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

2 **Substitute the known values.**

$$V_{rms} = \frac{340\,V}{\sqrt{2}}$$

3 **Calculate the answer.**

$V_{rms} = 240$ V

b **1** **Determine the formula/law.**

$V_{rms} = I_{rms}R$

2 **Rearrange to find the unknown.**

$$I_{rms} = \frac{V_{rms}}{R}$$

3 **Substitute the known values.**

$$I_{rms} = \frac{240\,V}{55\,\Omega}$$

4 **Calculate the answer.**

$I_{rms} = 4.37$ A

c **1** **Determine the formula.**

$P_{av} = V_{rms}I_{rms}$

2 **Substitute the known values.**

$P_{av} = 240\text{ V} \times 4.37\text{ A}$

3 **Calculate the answer.**

$P_{av} = 1.05$ kW

Transformers

Most appliances use a transformer to convert the 240 V mains power (at 50 Hz) to a lower potential difference. Some also convert the AC potential difference to direct current. The transformer may be inside the device, or it may be a separate adaptor or 'brick'. There are also transformers at electricity substations and in suburban streets **(Figure 9.4.5)**. These drop the potential difference from the thousands of volts at which electricity is transmitted to the 240 V that is supplied to homes and businesses.

FIGURE 9.4.5 Various transformers

A transformer consists of two solenoids or coils of wire placed near each other so that an alternating current in the primary coil can induce a current in the secondary coil. The link between input and output is by electromagnetic induction; there is no electrical connection.

Solenoids are used because they produce a large and approximately uniform magnetic field inside the coil. The field in the primary solenoid varies sinusoidally with the alternating current in the coil. The coils need to be coupled so that the changing magnetic field in the primary coil causes a changing magnetic flux in the secondary solenoid. There are two ways of doing this. First, the coils can share the same space by placing one within the other. This is sometimes used in cordless appliances such as kettles. The second, and more usual, way is to link the coils using a ferromagnetic core.

The primary coil is wound around one side of an iron core. The current in the primary coil magnetises the whole core, not just the part within the primary coil. The time-varying current in the primary coil causes a time-varying magnetic field inside the secondary core. This creates a time-varying electric field, hence an emf and current in the secondary coil. Figure 9.4.6 shows how this works.

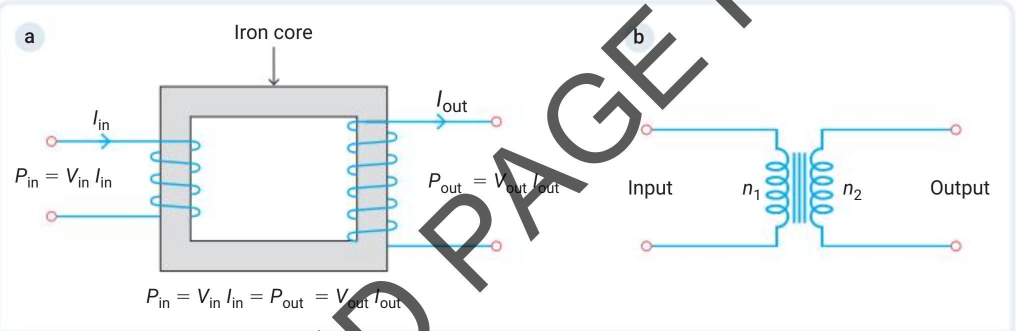


FIGURE 9.4.6 (a) A schematic diagram of a transformer and (b) its circuit symbol

The flux through any loop is the same for both coils. If the primary coil has n_p turns then:

$$V_p = -n_p \frac{\Delta\Phi}{\Delta t}$$

KEY FORMULA

$$V_p = -n_p \frac{\Delta\Phi}{\Delta t}$$

The alternating potential difference in the primary coil of a transformer (V_p) creates an alternating magnetic field in the iron core, which is equal to the negative of the number of loops in the coil multiplied by the rate of change of the magnetic flux $-n_p \frac{\Delta\Phi}{\Delta t}$.

KEY FORMULA

$$V_s = -n_s \frac{\Delta\Phi}{\Delta t}$$

An alternating magnetic flux through the secondary coil in a transformer $-n_s \frac{\Delta\Phi}{\Delta t}$, produces a potential difference in the coil (V_s).

As $\frac{\Delta\Phi}{\Delta t}$ is the same for both coils:

$$\frac{V_p}{V_s} = \frac{n_p}{n_s}$$

KEY FORMULA

$$\frac{V_p}{V_s} = \frac{n_p}{n_s}$$

The ratio of the voltages in the two arms of a transformer $\frac{V_p}{V_s}$ is equal to the ratio of the number of coils of each arm $\frac{n_p}{n_s}$.

Assuming that the transformer is 100% efficient, power out equals power in or $P_{out} = P_{in}$. We can apply Ohm's law to show that $I_s V_s = I_p V_p$. This can be included in the previous equation to give the full transformer equation:

$$\frac{V_p}{V_s} = \frac{n_p}{n_s} = \frac{I_p}{I_s}$$

KEY FORMULA

Transformer equation

$$\frac{V_p}{V_s} = \frac{n_p}{n_s} = \frac{I_p}{I_s}$$

The transformer equation: The ratio of the secondary voltage (V_s) to the primary voltage (V_p) is equal to the ratio of the primary current (I_p) to the secondary current (I_s) and is also equal to the ratio of the secondary number of coils (n_s) to the primary number of coils (n_p).

In reality, transformers are not 100% efficient, and a small amount of energy is lost as heat through resistance and eddy currents, although this is usually less than 1% of the total energy transformed.

It is important to realise that transformers only work when an alternating current is passing through the primary coil and would not work if direct current were passed through the coil. This is due to the need for a time-changing magnetic field to produce an emf in the secondary coil. This is one of the primary reasons that AC is widely used today – it can easily be transformed.

A **step-up transformer** ($n_s > n_p$) has a higher emf and lower current on the secondary side. A **step-down transformer** ($n_s < n_p$) has a lower emf and higher current on the secondary side.

step-down transformer
a transformer with an output potential difference that is lower than the input potential difference

step-up transformer
a transformer with an output potential difference that is higher than the input potential difference

Applications of transformers in power transmission

Electrical transformers are essential components in modern power systems, enabling the efficient transmission and safe delivery of electricity. Power plants generate electricity at a relatively low voltage, which is not suitable for long-distance transmission because of energy losses caused by resistance in power lines. (Recall that power loss is proportional to the square of the current $P_L = I^2 \times R$). To minimise these losses, transformers are used to step up the voltage to high levels, allowing electricity to travel more efficiently over vast distances. (Recall that $P = V \times I$, so a high voltage has a low current for the same power, but therefore has less power loss).

When the electricity reaches its destination, it needs to be made suitable for local use. This is achieved by step-down transformers, which reduce the high transmission voltage to lower, safer levels for distribution to homes, businesses and industries. This dual role of transformers ensures reliable, cost-effective and safe delivery of electricity across the power grid.

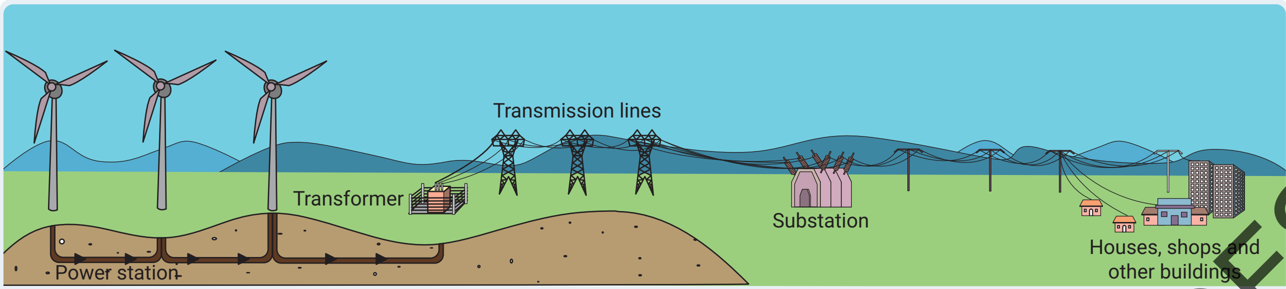


FIGURE 9.4.7 Step-up and step-down transformers are used to vary voltages and currents. This can be used to reduce power loss in transmission and to deliver energy safely to devices in homes and businesses.

WORKED EXAMPLE 9.5.3

A 120 W, 24 V AC supply is connected to the input terminals of a transformer. The primary coil is wound with 240 turns. The output emf is 72 V. Assume there is no power loss in the transformer.

a Determine the number of turns on the secondary coil.
b Deduce if this is a step-up or step-down transformer.
c Calculate the output current.

ANSWERS

a **1** **Determine the formula.**

$$\frac{V_p}{V_s} = \frac{n_p}{n_s}$$

2 **Rearrange to find the unknown.**

$$n_s = \frac{n_p \times V_s}{V_p}$$

3 **Substitute the known values.**

$$n_s = \frac{240 \times 72}{24}$$

4 **Calculate the answer.**

$n_s = 720$

b **Determine the relationship between n_p and n_s.**

Step-up transformer because $n_s > n_p$

c **1** **Determine the formula.**

Apply the conservation of energy:

$P_i = P_{out}$

2 **Apply Ohm's law.**

$P_{in} = I_s V_s$

3 **Rearrange to find the unknown.**

$$I_s = \frac{P_{in}}{V_s}$$

4 **Substitute the known values.**

$$I_{rms} = \frac{V_{rms}}{R}$$

5 **Calculate the answer.**

$I_s = 1.7$ A

LEARNING CHECK 9.4

DESCRIBING

1 **Recall** which two physical laws apply when an electrical generator makes electric current.

2 **Explain** alternating current.

3 **Describe** the purpose of a step-up transformer before transmitting energy over large distances.

4 **Describe** the difference between a step-up transformer and a step-down transformer.

5 **Explain** how a rotating coil in a magnetic field produces an alternating emf.

6 **Explain** why transformers can only operate with alternating current.

APPLYING

7 A rectangular coil of 30 turns and area 100 cm^2 rotates at 1200 revolutions per minute in a uniform magnetic field of flux density 0.50 T.

- a **Determine** the frequency of the generated emf.
- b **Determine** the maximum emf.
- c **Calculate** the rms emf.
- d Write the equation that gives the emf at any instant.

8 The armature of an AC generator is rotating at a constant speed of 35 revolutions per second in a horizontal field of flux density 1.0 T. The diameter of the cylindrical armature is 24 cm and its length is 40 cm. The generator is connected to a load of resistance 10.0 Ω. **Determine** the:

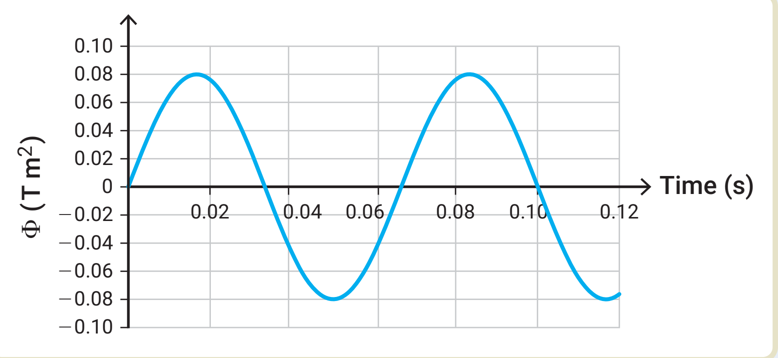
- a maximum emf induced in the armature if it has 30 turns
- b rms emf produced by this generator
- c rms current produced by the generator
- d average power output of the generator.

9 A step-up transformer is connected to an AC generator that delivers 8.0 A at 120 V. The ratio of the number of turns in the secondary coil to the number of turns in the primary is 500. **Determine** the:

- a emf in the secondary coil
- b average power input
- c maximum power output
- d maximum current in the secondary coil.

ANALYSING

10 The following graph shows the magnetic flux as a function of time through each loop of a 30-turn coil in a generator.



- a **Determine** the maximum and minimum emf and the period of oscillation of the emf.
- b **Derive** an equation that describes the emf produced as a function of time.
- c **Sketch** a graph showing the emf produced as a function of time.
- d Include the rms emf on the graph.

CHAPTER SUMMARY

Electromagnetic induction

- Electromagnetic induction is the production of an electromotive force (emf) across a current-carrying conductor due to its dynamic interaction with a magnetic field.
- emf is a difference in potential, or a voltage, which tends to give rise to an electric current (induced current).

Magnetic flux

- Magnetic flux, ϕ, is a measure of the total magnetic field passing through a given area. This is also indicated by the number of magnetic field lines passing through a given area.

  $\Phi = BA = BA\cos\theta$

  where:

  Φ = magnetic flux (Wb)

  B = magnetic field strength (T)

  A = area of the surface (m^2)

  θ = angle between the magnetic field lines and a normal to the surface (°)
- Magnetic flux density (B) is the strength of a magnetic field or the number of magnetic field lines per unit area.

Faraday's law

- Faraday's law explores the relationship between different factors and the magnitude of the voltage (emf) produced.

$$\text{emf} = \frac{-\Delta\Phi}{\Delta t} = \frac{-(\Phi_f - \Phi_i)}{\Delta t} = \frac{-(BA\cos\theta_f - BA\cos\theta_i)}{\Delta t}\,\text{V}$$

Lenz's law

- Lenz's law states the direction of a current, induced in a conductor by changing magnetic field, is such that the magnetic field created by the induced current opposes changes in the magnetic field that produces it.
- To determine the direction of the induced current, the right-hand rule can be used. The fingers point in the direction of the magnetic field, the thumb points in the direction of the conventional current and the palm points in the direction of the magnetic force. Think about an individual positive charge and where it would be forced.
- By opposing the change in magnetic flux, the induced current ensures no energy is created or destroyed, only transformed. For example, when a magnet moves towards a coil, the induced current creates a magnetic field that opposes the magnet's motion, the kinetic energy is transformed into electrical energy as the induced current in the coil, conserving the energy.

Inducing a current

- When a conductor moves through a magnetic field, it cuts across magnetic field lines, causing the magnetic flux to change.
- According to Faraday's law, this change in magnetic flux induces an emf in the conductor.
- The direction of the induced emf is such that it opposes the change in magnetic flux and can be determined using right-hand rule.

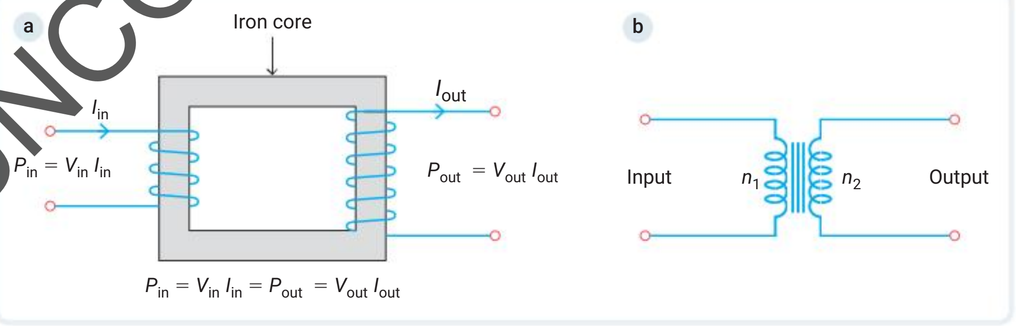
Alternating current in transformers

- Alternating current is electrical current that changes direction sinusoidally with time.
- Transformers are electrical devices that require alternating current to transform the voltage levels of electrical power by either stepping it up (low voltage to high voltage) or stepping it down (high voltage to low voltage).

  $$\frac{V_p}{V_s} = \frac{n_p}{n_s} = \frac{I_p}{I_s}$$

  where:

  I_p = current in primary coil (A)
  V_p = voltage in primary coil (V)
  I_s = current in secondary coil (A)
  V_s = voltage in secondary coil (V)
  n_p = number of turns in primary coil
  n_s = number of turns in secondary coil
- Step-up transformers increase the voltage from the primary coil to the secondary coil.
- Step-down transformers decrease the voltage from the primary coil to the secondary coil.
- Transformers consist of two solenoids placed near each other with a common core.
- An alternating current flows through the primary coil, which creates a changing magnetic field around it. The changing magnetic field is carried through the common core to the secondary coil, where it induces an alternating current in the secondary coil, according to Faraday's law.
- Faraday's law states the voltage in the secondary coil can be higher or lower than the voltage in the primary coil, depending on the number of turns of wire in each coil.



CHAPTER EXAM

MULTIPLE CHOICE

1. Which of the following does not affect the magnitude of the magnetic flux passing through a surface?

A The composition of the surface
B The magnitude of the magnetic field
C The area of the surface
D The orientation of the surface to the magnetic field

2. The unit used of magnetic flux is the weber (Wb). What is another unit that can be used?

A T m
B T m^{-1}
C T m^2
D T m^{-2}

3. On a graph of the magnetic flux passing through a surface as a function of time, the feature that gives an indication of the size of the emf as a function of time is the:

A *y* intercept.
B *x* intercept.
C area under the line.
D gradient of the line.

4. The cause of the alternating current in the secondary coil of a transformer is an emf produced by the:

A voltage applied to the primary coil.
B varying electric field of the primary coil.
C varying magnetic field of the primary coil.
D varying magnetic field of the secondary coil.

5. The horizontal steel cargo boom of a freighter travelling at 10 m s^{-1} is 7.0 m long and is at an angle of 75° relative to the direction of the ship's motion. The magnetic field of Earth in that region has a vertical component of 4.0×10^{-5} T. What is the potential difference between the ends of the boom?

A 0.06 mV
B 0.72 mV
C 2.7 mV
D 2.8 mV

6. A wire loop that encloses an area of 15 cm^2 is perpendicular to a magnetic field of 0.10 T. If the field drops to 0.04 T in 0.2 s, what is the average emf induced in the loop?

A 0.3 mV
B 0.45 mV
C 4.0 mV
D 4.5 mV

7. A 200-turn coil whose resistance is 4 Ω encloses an area of 20 cm^2. A changing magnetic field parallel to the coil axis induces a current of 1.2 A in the coil. How rapidly is the magnetic field changing?

A 0.75 T s^{-1}
B 12 T s^{-1}
C 14.4 T s^{-1}
D 30 T s^{-1}

Questions 8 and 9 relate to the following information.
A transformer has 300 turns in its primary coil and 75 turns in its secondary coil.

8. When the current in the secondary coil is 20 A, what is the current in the primary coil?

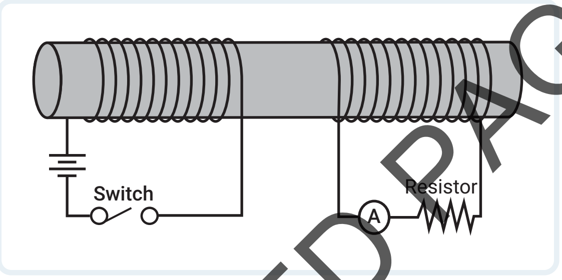
A 5 A
B 25 A
C 80 A
D 6.4 kA

9. If the power input to the transformer is 40 W, what is the power output?

A 2.5 W
B 10 W
C 40 W
D 160 W

10. The diagram displays two coils that share a common iron core.

When the switch on the left is closed (turned on), the current through the ammeter and resistor will:



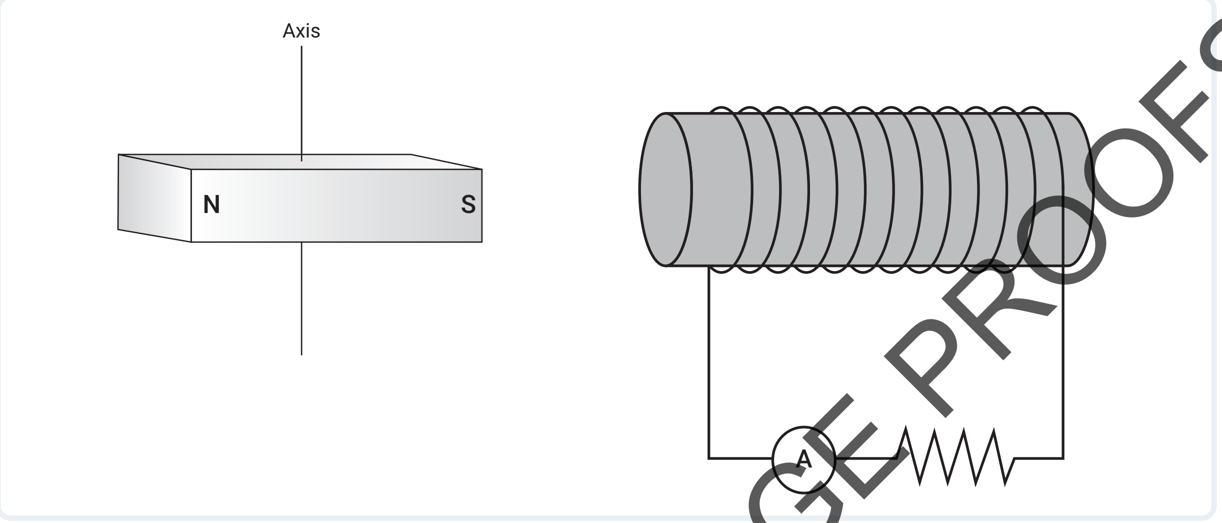
A flow from left to right.
B flow from right to left.
C alternate back and forth.
D not current flow.

SHORT RESPONSE

11. A loop of area 0.025 m^2 that is initially placed perpendicularly to a uniform magnetic field of 5.0×10^{-2} T is quickly removed from the magnetic field over a period of 0.05 s. **Calculate** the emf that will be produced in the loop.

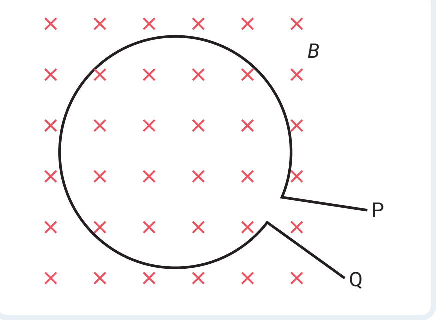
12. **Calculate** the velocity a wire 20 cm long should have through a 0.05 T magnetic field to induce an emf of 1.00 V.

13. **Describe** the direction of the current that flows in the resistor when the following changes are made to the position/orientation of the magnet near the iron core of the coil shown below.



- **a** The south pole of the magnet is moved closer to the coil.
- **b** The south pole of the magnet is then moved further from the coil.
- **c** The magnet is now rotated 90º clockwise (as seen from above) about the axis shown.
- **d** The magnet is rotated another 90º clockwise (as seen from above) about the axis shown until the north pole faces the coil.
- **e** The north pole is moved further from the coil.
- **f** The north pole is moved closer to the coil.

14. A flat, horizontal loop of wire is in a region in which the magnetic field is into the page. **Describe** which end of the wire (P or Q) will acquire a positive charge due to the induced emf (if any) resulting from the following changes:



- **a** The magnitude of the magnetic flux density *B* increases.
- **b** The loop is moved down into the page.
- **c** The loop is moved to the left along the page.
- **d** The radius of the loop is increased.
- **e** The coil is moved up the page.
- **f** The radius of the loop is reduced.
- **g** The magnitude of the magnetic flux density B decreases.

DATA ANALYSIS

15. Apply understanding

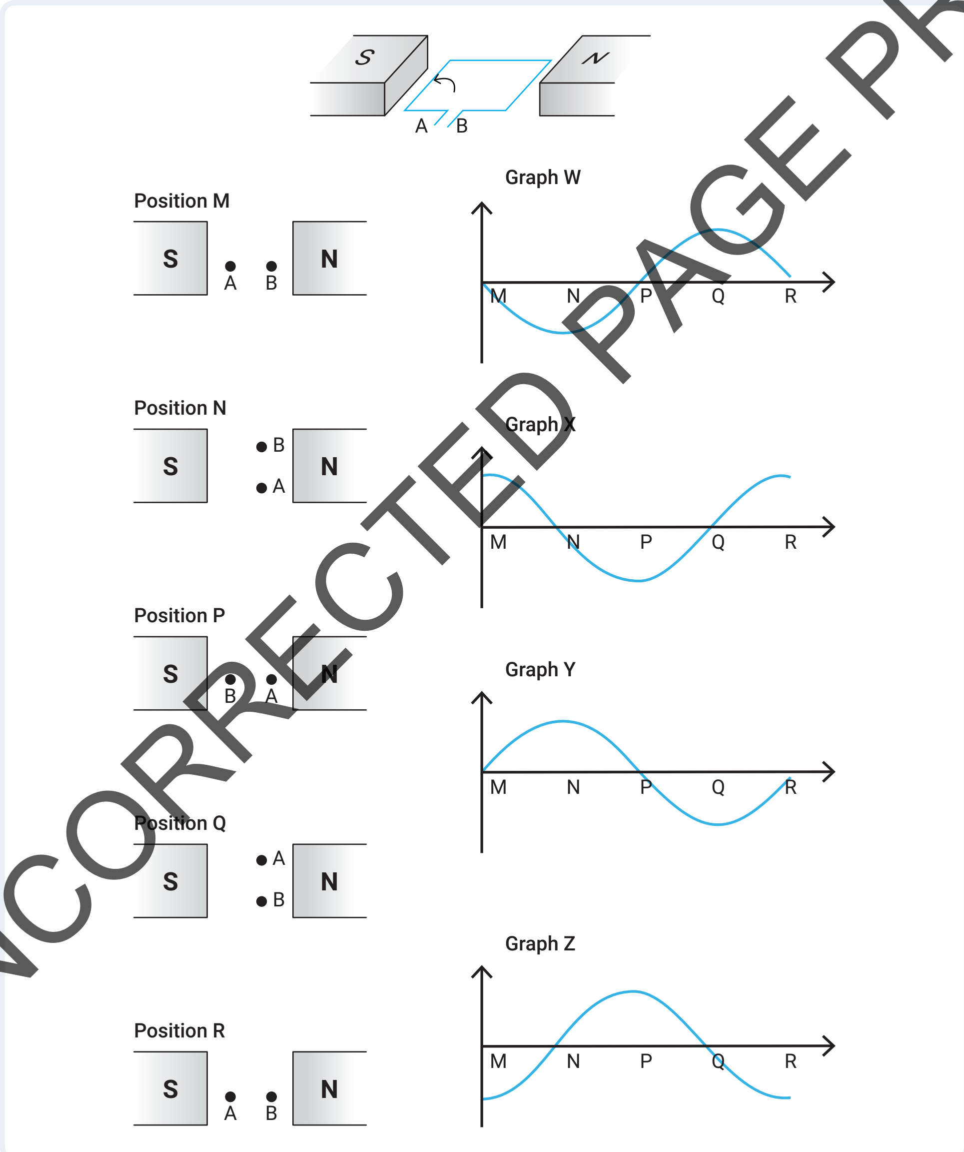
A loop of wire is rotating in an anticlockwise direction (as shown below) in a region between two magnetic poles. The ends of the loop are identified as A and B. The positions of A and B are noted as the loop rotates and these positions are labelled M, N, P, Q and R.

a In which position(s) does A have a maximum positive electric potential?

b In which position(s) does A have a zero electric potential?

c Which graph (W, X, Y or Z) corresponds to the variation of flux through the coil versus position?

d Which graph (W, X, Y or Z) corresponds to the variation of electric potential at A versus position?



CHAPTER 10

Electromagnetic radiation

Getty Images/Comezora

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Describe the concept of an electromagnetic wave.
- Explain the relationship between oscillating electric charges and electromagnetic waves.

SCIENCE AS A HUMAN ENDEAVOUR

- Appreciate the significant contributions of scientists such as Charles-Augustin de Coulomb, Michael Faraday, Emil Lenz, Mary Somerville and James Clerk Maxwell who furthered our understanding of electromagnetism.

Physics 2025 v1.2 General Senior Syllabus © QCAA 2024

Introduction

The culmination of electromagnetic theory came in the 19th century with the work of theoretical physicist James Clark Maxwell. Maxwell unified all the known phenomena of electricity and magnetism with the field theory of Michael Faraday into four equations that completely altered the way we view light. By analysing these four equations, Maxwell could describe many of the properties of light, but, most particularly, he was able to predict that light was an electromagnetic wave. This chapter will investigate the implications of this prediction.

Assessments

- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practicals

- Investigating the magnification of a microscrope

Worksheets

- Name
- Name
- Name

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ The frequency (f) of a cyclic or oscillating phenomena is the number of cycles or oscillations per second; units are hertz (Hz) or s^{-1}.
- ✓ The period (T) of a cyclic or oscillating phenomena is the time it takes for a single cycle or oscillation; units are seconds (s).
- ✓ The relationship between frequency and period can be stated as $f = \frac{1}{T}$.
- ✓ The speed of light in a vacuum is a constant: $c = 3 \times 10^8$ m s^{-1}.
- ✓ The wave equation is $v = f\lambda$.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe the concept of an electromagnetic wave and how the work of Maxwell contributed to our understanding
- ✓ explain the relationship between oscillating electric charges and electromagnetic waves
- ✓ interpret visual and graphical representations of an electromagnetic wave
- ✓ use the wave equation to quantify aspects of an electromagnetic wave
- ✓ categorise an electromagnetic wave within the electromagnetic spectrum
- ✓ describe how electromagnetic waves can be produced and detected
- ✓ analyse graphical data related to frequency, wavelength, wavenumber and energy.

10.1 Electromagnetic waves

Physicist James Clark Maxwell's (1831–79) most important contribution to physics was to take the equations and experimental results of Faraday and others and unify them into a single theory of electromagnetism. This theory is summed up in four **differential equations**.

differential equation an equation that relates the rate of change of displacement in space to the rate of change of displacement in time

Maxwell's four equations describe all the known classical phenomena involving electricity and magnetism. They may be summarised as:

- Gauss's law: Electric fields are created by charges.
- Gauss's law in magnetism: There are no isolated magnetic poles (monopoles).
- Faraday's law: Electric fields are created by changing magnetic fields.
- Ampere–Maxwell law: Magnetic fields are created by moving charges (currents) and changing electric fields.

Weblinks
Electromagnetic waves
James Clerk Maxwell: The greatest physicist you've never heard of

Electromagnetic waves

When Maxwell combined the second pair of his equations, he made a fundamental discovery. When a changing magnetic field produces an electric field, this electric field must also be changing. This changing electric field will then produce a changing magnetic field that would go on to produce another changing electric field, and so on.

Armed with this discovery, Maxwell could show that by manipulating the equations, the result of the interaction of these changing electric and magnetic fields was a wave of coupled, self-sustaining oscillating electric and magnetic fields that travel as transverse waves and can propagate through empty space. The equations went on to show that these **electromagnetic waves** explained all the known phenomena of electricity and magnetism.

electromagnetic wave a wave produced by an oscillating charge resulting in mutually perpendicular electric and magnetic fields

9780170483704

Electromagnetic waves can travel through a vacuum

Electromagnetic (EM) waves propagate as transverse waves consisting of electric and magnetic fields oscillating both at right angles to each other and to the direction of travel **(Figure 10.1.1)**. Because the oscillations of EM waves occur in fields rather than a medium (as is required by mechanical waves), they do not require a medium to travel through and can therefore propagate through empty space.

Even though light had been shown to behave like a wave some 60 years before Maxwell's discovery, there was much debate about the medium through which it travelled. Maxwell's description of EM waves could put to rest the need for the luminous aether – the hypothesised substance that pervaded the universe and provided a medium through which light waves propagated.

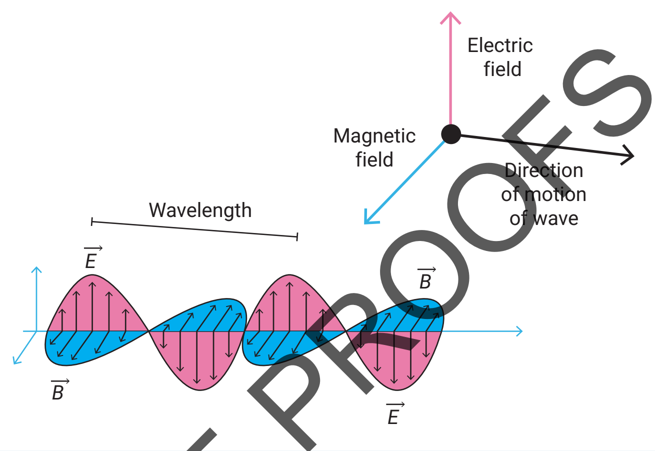


FIGURE 10.1.1 Electromagnetic waves are transverse waves consisting of coupled electric and magnetic field oscillations.

The speed of light

From his equations, Maxwell could predict that the speed of light in a vacuum (c) was inversely proportional to the square root of the product of the magnetic permeability (μ_0) and the electrical permittivity (ε_0) of the vacuum.

This value agreed, within uncertainty, with the experimentally measured value of the speed of light. In addition, the speed depends only on the constants μ_0 and ε_0, which are properties of empty space. This agreement between theory and experiment provided strong support for Maxwell's theories.

KEY FORMULA

Speed of light in a vacuum

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8 \text{ m s}^{-1}$$

The speed of light in a vacuum (c), is dependent upon the magnetic permeability (μ_0) and the electrical permittivity (ε_0) of the vacuum.

For light in a vacuum, $v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8 \text{ m s}^{-1}$, but for light in any other medium, the speed is lower and depends on the permittivity and permeability of the medium.

KEY FORMULA

Speed of light in any medium

The speed of light in any medium (v) is dependent upon the magnetic permeability (μ) and the electrical permittivity (μ) of that substance.

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Weblink
What is the speed of light?

universal wave equation a fundamental relationship that describes the propagation of waves in any medium, given by $v = f\lambda$; applies to mechanical waves (e.g. sound, water) and electromagnetic waves (e.g. light, radio).

The **universal wave equation**, $v = f\lambda$, gives us a relationship between the speed, frequency and wavelength for electromagnetic waves, just as it does for mechanical waves.

KEY FORMULA

Wave velocity

The wave velocity (v) is directly proportional to the frequency (f) and the wavelength (λ) of the electromagnetic wave:

$v = f\lambda$

where:

v = velocity (m s^{-1})

f = frequency (Hz)

λ = wavelength (m)

WORKED EXAMPLE 10.1.1

An antenna for a radio station transmits a signal with frequency of 106.3 MHz. Calculate the wavelength of this electromagnetic wave.

ANSWERS

1 **Determine the formula.**
$v = f\lambda$

2 **Rearrange to find the unknown.**
$\lambda = \frac{v}{f}$

3 **Identify the relationship between v and c.**
$v = c$ for electromagnetic waves

4 **Substitute the known values.**
$\lambda = \frac{3.0 \times 10^8 \text{ ms}^{-1}}{106.3 \times 10^6 \text{ Hz}}$

5 **Calculate the answer.**
$\lambda = 2.82$ m

WORKED EXAMPLE 10.1.2

Use the universal wave equation, $v = f\lambda$, to determine the missing values of frequency and wavelength of electromagnetic waves listed in Table 10.1.1.

TABLE 10.1.1 Wavelength and frequency data for a range of electromagnetic waves

Wave region	Wavelength, λ (m)	Wavelength, λ (nm)	Frequency, f (Hz)
Radio wave	2.0×10^3		
Visible light (red)			4.3×10^{14}
X-ray	1.0×10^{-10}		
Gamma ray			5.0×10^{20}

ANSWER

Wave region	Wavelength, λ (m)	Wavelength, λ (nm)	Frequency, *f* (Hz)
Radio wave	2.0×10^{3}	2.0×10^{12}	1.5×10^{5}
Visible light (red)	6.98×10^{-7}	698	4.3×10^{14}
X-ray	1.0×10^{-10}	0.1	3.0×10^{18}
Gamma ray	6.0×10^{-13}	6.0×10^{-4}	5.0×10^{20}

WORKED EXAMPLE 10.1.3

A light wave in a vacuum has a wavelength of 500 nm. If the speed of light is 3×10^{8} m s^{-1}, calculate the frequency of the light.

ANSWER

1 **Determine the formula.**

$v = f\lambda$

2 **Rearrange to find the unknown.**

$f = \frac{v}{\lambda}$

3 **Identify the relationship between *v* and *c*.**

$v = c$ for electromagnetic waves

4 **Substitute known values.**

$f = \frac{3 \times 10^{-8}}{500 \times 10^{-9}}$

5 **Calculate the answer.**

$f = 6.0 \times 10^{14}$ Hz

The electromagnetic spectrum

Visible light is only one kind of electromagnetic wave. Maxwell also predicted that a large range of frequencies was possible for electromagnetic waves, well beyond the visible spectrum. Figure 10.1.2 shows an alternative display of the **electromagnetic spectrum**, which shows the full range of EM waves that have been produced or detected. They form a continuous spectrum of energies from long-wavelength, low-frequency radio waves through to short-wavelength and high-frequency gamma rays.

electromagnetic spectrum the family of electromagnetic radiations – radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays – all of which travel at 3.0×10^{8} m s^{-1} in a vacuum

Wave behaviour of light

Maxwell provided definitive proof that light and all other forms of **electromagnetic radiation** travels as a wave. This agreed with the fact that all EM waves are known to exhibit wave behaviour including reflection, refraction, diffraction and the principle of superposition, as discussed in Chapter 15 of *Nelson QCE Physics Units 1 & 2*.

Syllabus link
Chapter 1 of *Nelson QCE Physics Units 1 & 2* discusses that heat can be transferred as infrared radiation.

electromagnetic radiation radiant energy consisting of synchronised oscillations of electric and magnetic fields, or electromagnetic waves, propagated at the speed of light in a vacuum

Weblink
The electromagnetic spectrum

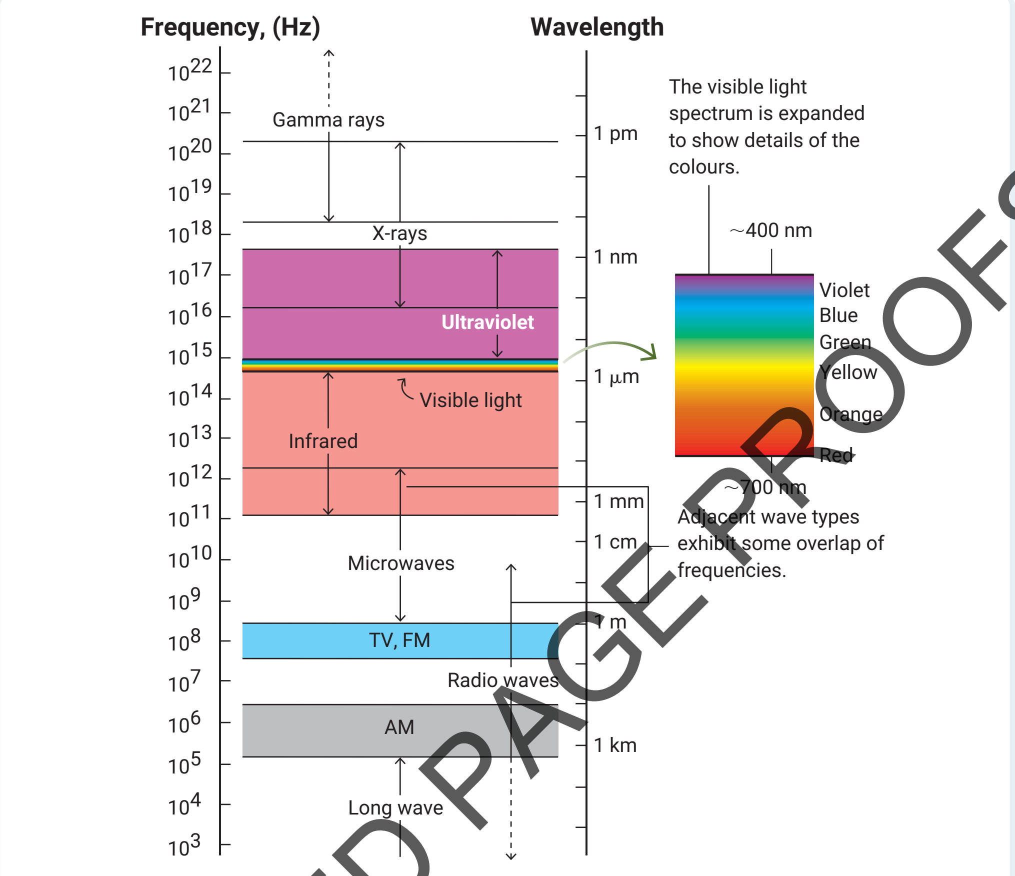


FIGURE 10.1.2 The electromagnetic spectrum showing all EM waves that have been produced or detected ordered according to their wavelengths and frequencies

Making and detecting electromagnetic waves

According to Maxwell's equations, EM waves consist of oscillating electric and magnetic fields that self-propagate, but for this to occur they require an initial change in one of these fields. What is the source of these initial oscillations?

All charged particles and magnetised materials produce fields in the space around them. However, these are insufficient to produce EM waves, as the fields are static and do not change with time. The same is true of steady electric currents.

However, if the current in a wire changes with time, as is the case in alternating current (AC), the wire emits electromagnetic radiation. This can be generalised to the important statement *that whenever a charged particle accelerates, it radiates energy in the form of electromagnetic waves.*

An important application of the fact that accelerating charges produce EM radiation is that of the antenna, which can be used to transmit or receive EM waves.

A transmitting antenna works by being connected to an AC. This current causes charges to oscillate back and forth along the length of the antenna, which creates a magnetic field. The magnetic field is perpendicular to the axis of the antenna and forms loops around the antenna.

The current oscillates with frequency *f*. The magnetic field also oscillates with this frequency. This time-varying magnetic field creates a time-varying electric field. This electric field also oscillates with frequency *f*, at right angles to the magnetic field.

These two oscillating fields are the EM wave. The EM wave travels in a direction perpendicular to both the fields that make up the wave, which are themselves mutually perpendicular. This is shown in Figure 10.1.2 and **Figure 10.1.3**.

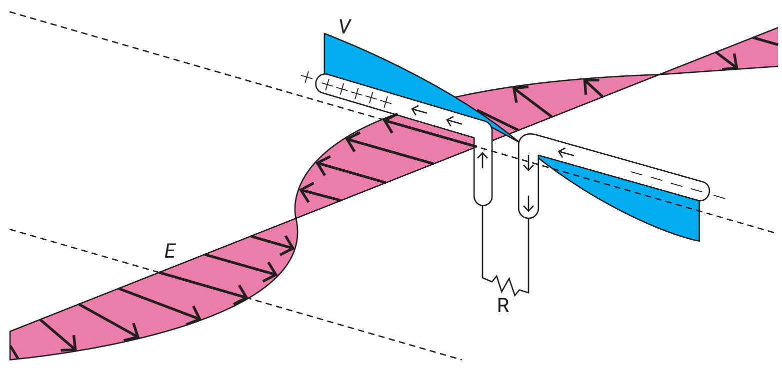


FIGURE 10.1.3 An antenna can transmit EM waves as a result of an AC travelling through it.

The frequency of the wave is f, the same as the frequency of the current, and it travels at speed c in a vacuum or very close to this in air. In this way, an EM wave of any frequency can be produced by changing the frequency of the oscillations in the current.

When this travelling electromagnetic wave is incident on free charges, such as electrons in a receiving antenna, it will make them oscillate. The electric field applies a force to the electrons, and an AC with the same frequency, f, is produced. This current can be picked up and converted into another form. For example, a transducer attached to the antenna can convert the current into sound waves. Hence, an antenna can either transmit electromagnetic waves if it is connected to an AC, or receive them and produce an AC. This is how radio, TV and mobile phone antennae work. The length of the antenna depends on the frequency of the waves it must transmit or receive and the current it must carry. Longer antennae are used for lower frequency electromagnetic waves.

Syllabus links
Chapter 13 discusses how the movement of electrons within an atom produces most of EM waves that exist in the universe.

Chapters 7 and 8 discuss charged particles and magnetised materials.

Maxwell's legacy

Maxwell's equations were startling, largely because of their ability to describe known phenomena, but also because of their predictive power. However, in many ways the largest impact of Maxwell's work was the scientific process that he followed. He combined existing theories into a new model that made predictions that could be investigated. This was in stark contrast to the usual method of making observations and then attempting to formulate a theory that agreed with these observations. Many theoretical physicists have since followed the same process, including, most notably, Albert Einstein, whose famous 'thought experiments' made incredible predictions that are still being shown to be experimentally correct today more than 100 years later.

Wavelength versus wavenumber

Wavelength (λ) is the physical distance between two consecutive points in a wave that are in phase, such as two peaks or two troughs. Wavelength is measured in metres (m), although wavelengths are often written in nanometres (nm) for convenience. Wavelength is related to the frequency through the universal wave equation, $v = f\lambda$.

Wavenumber (k or $\frac{1}{\lambda}$) is the number of wavelengths per unit distance, effectively describing the spatial frequency of a wave. It is the reciprocal of wavelength. The wavenumber is measured per metre (m^{-1}).

WORKED EXAMPLE 10.1.4

A light wave has a wavelength of 450 nm. To which part of the electromagnetic spectrum does this wave belong? Refer to Figure 10.1.3.

ANSWER

A wavelength of 450 nm is represented in the visible light spectrum (400–700 nm) and is more specifically within the blue-violet colour of the spectrum (see Figure 10.1.3).

WORKED EXAMPLE 10.1.5

If a beam of light has a frequency of 5×10^{14} Hz. Determine its:

a wavelength

b wavenumber.

ANSWERS

a **1** **Determine the formula.**

$$\lambda = \frac{c}{f}$$

2 **Substitute the known values.**

$$\lambda = \frac{3.0 \times 10^{8}}{5 \times 10^{14}}$$

3 **Calculate the answer.**

$\lambda = 6.0 \times 10^{-7}$ m or 600 nm

b **1** **Determine the formula.**

$$\frac{1}{\lambda} = \frac{f}{c}$$

2 **Substitute the known values.**

$$\frac{1}{\lambda} = \frac{5 \times 10^{14}}{3.0 \times 10^{8}}$$

3 **Calculate the answer.**

$$\frac{1}{\lambda} = 1.67 \times 10^{6}\ \text{m}^{-1}$$

Weblink
The SKA project in Australia

LEARNING CHECK 10.1

DESCRIBING

1. **Identify** the scientist who unified all the information about electromagnetic phenomena into a single theory, described by four equations.
2. State the range of wavelengths for visible light.
3. A beam of light has a frequency of 2.5×10^{16} Hz. **Determine** its:

   a wavelength

   b wavenumber.

4 When light (or other electromagnetic radiation) travels across a given region, what:

   a oscillates? **b** is transported?

5 What does a radio wave do to the charges in the receiving antenna to provide a signal for your car radio?

APPLYING

6 The red light emitted by a helium–neon laser has a wavelength of 633 nm.

   a What is the frequency of the light waves?

   b How long would it take for a signal from this laser to travel from Earth to the Moon and back, a distance of 768 000 km?

ANALYSING

7 What experiments may be done to show that light is a wave? Provide two examples and state the property of waves that they exhibit.

8 The human eye is most sensitive to light with a frequency of 5.45×10^{14} Hz, which is in the green-yellow region of the visible electromagnetic spectrum.

   a What is the wavelength of this light?

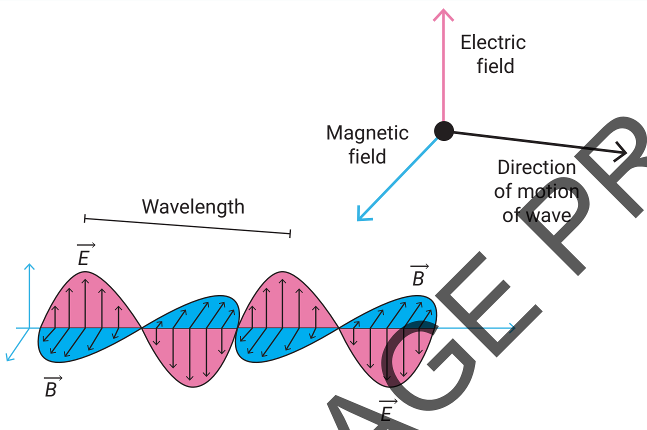
   b What energy does a single photon of this light contain?

9 Draw an electromagnetic spectrum, labelling at least three wavelength regions that you have used today.

CHAPTER SUMMARY

Electromagnetic waves

- Maxwell combined his equations to discover that when a changing magnetic field produces an electric field, this electric field must also be changing, which will then produce a changing magnetic field which will produce another changing electric field, and so on.
- Electromagnetic waves propagate through space as oscillating electric and magnetic fields that are perpendicular to each other and to the direction of travel.



- Mechanical waves need a material substance to carry their vibrations; however, electromagnetic waves are self-sustaining; the changing electric field generates a magnetic field, and so on, allowing the wave to move forward independently, without a medium.
- The speed of light in a vacuum (c), depends on the magnetic permeability (μ_0) and the electrical permittivity (ε_0) of the vacuum:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8 \text{ m s}^{-1}$$

- The speed of light in any medium (v) depends on the magnetic permeability (μ) and the electrical permittivity (ε) of that substance.

$$v = \frac{1}{\sqrt{\mu \varepsilon}}$$

The electromagnetic spectrum

- The electromagnetic spectrum is a continuous spectrum of radiations that encompasses a range of wavelengths and frequencies, from radio waves to gamma rays.
- The universal wave equation, $v = f\lambda$, relates the velocity to the frequency to the wavelength of the wave.

Making and detecting electromagnetic waves

- Accelerating charges, such as those in an alternating current, produce electromagnetic waves due to oscillating electric and magnetic fields.
- Antennas transmit electromagnetic waves by creating oscillating currents and receive them by converting the changing electric field into an alternating current.
- The frequency of the oscillating current determines the frequency of the electromagnetic wave, with longer antennas required for lower frequency waves.

MULTIPLE CHOICE

1. The orientation of oscillations in the electric and magnetic fields in an electromagnetic wave is:
   A perpendicular.
   B congruent.
   C parallel.
   D antiparallel.

2. An electromagnetic wave is a:
   A transverse wave.
   B surface wave.
   C longitudinal wave.
   D sound wave.

3. Which of the following waves has a wavelength that lies in the range of 1 cm to 1 m?
   A Gamma rays
   B X-rays
   C Microwaves
   D Visible light

4. Which of the following waves has the highest frequency?
   A Radio waves
   B Microwaves
   C Infrared waves
   D Visible light

5. The electromagnetic spectrum consists of several regions. Which option shows electromagnetic regions in order of increasing energy?
   A Ultraviolet, infrared, gamma rays and X-rays
   B Infrared, gamma rays, X-rays and ultraviolet
   C Infrared, ultraviolet, gamma rays and X-rays
   D Infrared, ultraviolet, X-rays and gamma rays

6. If a beam of light has a frequency of 5×10^{14} Hz, what is its wavelength?
   A 1.5×10^{-7} m
   B 3×10^{-7} m
   C 6×10^{-7} m
   D 1×10^{-6} m

7. A light wave has a frequency of 6.0×10^{14} Hz. Given that the speed of light is 3.0×10^{8} m s^{-1}, what is the wavenumber?
   A 2.0×10^{6} m^{-1}
   B 1.5×10^{6} m^{-1}
   C 2.0×10^{7} m^{-1}
   D 1.5×10^{7} m^{-1}

8. A light wave has a frequency of 3.0×10^{10} Hz. To which part of the electromagnetic spectrum does this wave belong to?
   A Infrared
   B Microwaves
   C Radio waves
   D Ultraviolet

9. A light wave has a wavelength of 2.0 nm. To which part of the electromagnetic spectrum does this wave belong?
   A Gamma rays
   B Infrared
   C Ultraviolet
   D X-rays

10. How long would it take for light from a laser to travel from Earth to a reflector on the Moon and back again, a distance of twice 384 400 km?
   A 0.74 s
   B 1.28 s
   C 2.56 s
   D 5.12 s

SHORT RESPONSE

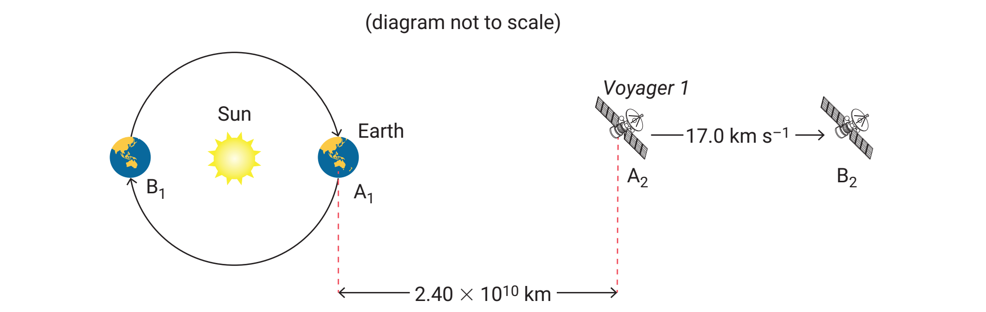
11. Show that if the electrical permittivity of a vacuum is $8.85 \times 10^{-12}\ C^2\,N^{-1}\,m^{-2}$ and its magnetic permeability is $1.257 \times 10^{-6}\ N\,s^2\,C^{-2}$, then the speed of light in a vacuum is $3.0 \times 10^8\ m\ s^{-1}$.

12. If light is travelling at a velocity of $2.54 \times 10^8\ m\ s^{-1}$ through a medium that has an electrical permittivity of $7.86 \times 10^{-12}\ C^2\,N^{-1}\,m^{-2}$, **calculate** the magnetic permeability of the substance.

13. A mobile phone transmits a signal with a frequency of 1800 MHz.
   a Name the part of the electromagnetic spectrum this wave belongs to.
   b **Determine** the wavelength of this signal.
   c **Calculate** how long it takes this wave to travel from Perth to Sydney, a distance of about 3300 km.
   d **Explain** why it actually takes longer than this for a signal to travel from a mobile phone in Perth to a mobile phone in Sydney.

CROSS-CHAPTER QUESTION

14. *Voyager 1* is the furthest human-made object from Earth and one of the fastest human-made objects ever. It is currently more than 24 billion kilometres away and has left the solar system, reaching interstellar space. Despite being launched about 50 years ago, physicists on Earth are still in contact with the remotely operated spacecraft via radio signals. Because of the vast distance involved, it takes a significant period of time for a radio signal travelling at the speed of light, c, from Earth to reach *Voyager 1*. This required time is continuously changing according to where Earth is in its orbit of the Sun and because of the probe's outward-bound velocity, which is an amazing $17.0\ km\ s^{-1}$.



a Determine the time is takes in hours for a radio signal from Earth to reach *Voyager 1* when at a particular point in time Earth and *Voyager 1* are at positions A_1 and A_2 respectively.

b 183 days later, Earth and *Voyager 1* are at new positions, B_1 and B_2 respectively, due to Earth orbiting around to the other side of the Sun and *Voyager*'s velocity. Considering the radius of Earth's orbit around the Sun is 1.5×10^8 km and *Voyager 1*'s velocity is 17.0 km s^{-1}, determine the time in hours it takes for a radio signal from Earth to reach *Voyager 1* at this second point in time.

DATA ANALYSIS

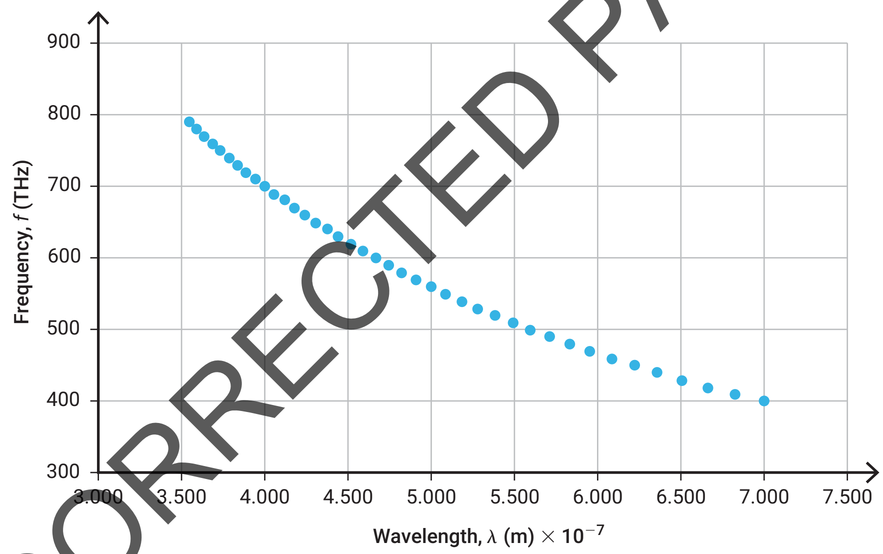
15. Interpret evidence

A physicist performed an experiment to answer the following research question:

'Can the value of *c* be accurately determined by analysing the visible spectrum using the combination of equipment at hand?'

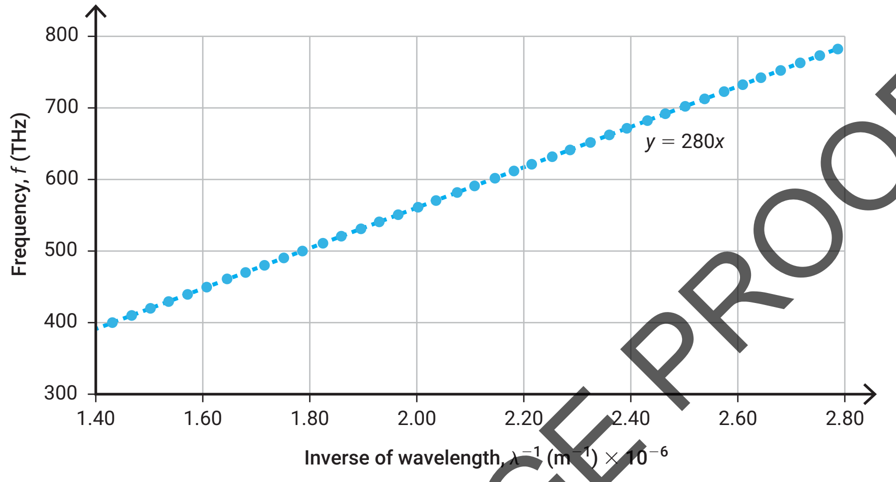
Two specialised pieces of equipment were available in the laboratory: one could measure the frequency of light, *f*, and the other could measure the wavelength of light, λ. The physicist used a prism to disperse white light and two different pieces of equipment to analyse the resultant spectrum. The raw data gathered is presented in Graph 1.

Graph 1. Frequency according to wavelength from data collected by two different pieces of equipment



The data was then processed to produce Graph 2.

Graph 2: Frequency according to the inverse of wavelength using data collected from two different pieces of equipment



a **Describe** and **explain** the trend illustrated in Graph 1.

b **Deduce** why the range of the x-axis in Graph 1 is 3.000 to 7.500×10^{-7} m.

c **Deduce** why the range of the y-axis in Graph 1 is 300 to 900 THz.

d **Infer** what the colour of light was that, when analysed, produced the last (far right) data point in Graph 1. You may need to refer to Figure 10.1.3.

e **Describe** the mathematical relationship between frequency, f, and the inverse of wavelength, λ^{-1}, evident in Graph 2.

f **Calculate** an experimentally determined value for the speed of light, using the line equation in Graph 2.

g **Calculate** the percentage error of the experimentally determined value of the speed of light.

h Make a statement about the accuracy of the experimental value.

SCIENCE AS A HUMAN ENDEAVOUR

Syllabus dot point

- Consider the scientific evidence concerning the risks of electromagnetic phenomena and associated technologies (e.g. wi-fi and mobile phones) as reported in the media.

Physics 2025 v1.0 General Senior Syllabus © QCAA 2024

The physics of wireless technologies and public health

The rapid growth of wireless technologies such as wi-fi and mobile phones has revolutionised the way society communicates, shares information and stays connected. However, these advancements come with a growing concern about the potential health risks associated with electromagnetic phenomena. In particular, the radiofrequency (RF) radiation emitted by these devices has raised questions about their long-term effects on human health. We will examine the science behind RF radiation, what research says about its safety, and how the scientific method continues to address public concerns.

Understanding electromagnetic radiation

Wireless devices such as mobile phones and wi-fi routers rely on electromagnetic radiation to transmit data. This radiation belongs to the category of radiofrequency waves, a type of non-ionising radiation that is much lower in energy than harmful **ionising radiation** such as X-rays and gamma rays. The key difference between **non-ionising radiation** and ionising radiation is their ability to remove tightly bound electrons from atoms, which can cause direct damage to biological tissues.

The scientific community agrees that non-ionising RF radiation does not have the same biological effects as ionising radiation, but concerns about prolonged exposure to RF waves, particularly regarding cancer or neurological effects, have spurred numerous studies to assess potential risks.

ionising radiation
high-energy radiation that has enough energy to ionise atoms and molecules, potentially leading to cancer or other harmful health effects (e.g. ultraviolet light, X-rays and gamma rays)

non-ionising radiation
radiofrequency radiation from wireless devices that lacks the energy to ionise atoms, so it cannot break chemical bonds or directly damage DNA

The role of scientific research

The widespread use of wireless technology makes understanding the effects of RF radiation a key public health issue. To address concerns, organisations such as the World Health Organization (WHO), national health agencies, and independent researchers have conducted extensive investigations. Epidemiological, animal and laboratory studies into the impact of RF radiation on the body have all contributed to our current understanding of the risks of using this technology. Most studies focus on whether exposure to RF radiation from devices such as mobile phones, which operate at frequencies between 700 MHz and 2.7 GHz, could lead to serious health outcomes.

FIGURE 1 The rapid growth of wireless technologies such as wi-fi and mobile phones has revolutionised how we communicate, share information and stay connected.

The scientific consensus emerging from this research is that the RF radiation from everyday devices such as mobile phones and wi-fi routers is safe when used within regulated guidelines.

Key physics principles of electromagnetic radiation

To understand the health effects of RF radiation, it is important to review the key physics principles that govern electromagnetic phenomena:

- **Electromagnetic spectrum:** RF waves are at the lower-energy end of the electromagnetic spectrum, between 3 kHz and 300 GHz. They are used for wireless communication because they can carry signals over long distances without causing ionisation.
- **Inverse-square law:** The intensity of RF radiation decreases rapidly with distance from the source. For example, holding a mobile phone close to your ear results in higher RF exposure than using a hands-free device or placing the phone on speaker mode. Doubling the distance reduces the intensity by a factor of four.
- **Specific absorption rate (SAR):** SAR is a measure of the rate at which the body absorbs RF energy, expressed in watts per kilogram. Mobile phone manufacturers are required to adhere to SAR limits, ensuring that devices remain within safe exposure levels set by regulatory agencies.

Distinguishing between evidence and sensationalism

Despite the overwhelming evidence supporting the safety of RF radiation, public debate persists. Media outlets sometimes report isolated studies that suggest possible health risks, often without considering the broader body of scientific evidence. These reports can lead to public fear, even when the studies in question are inconclusive or have been contradicted by further research.

- **Importance of peer review:** In science, the peer-review process ensures that research is thoroughly evaluated by experts in the field before it is accepted as valid. Claims of harm from RF radiation that have not undergone this process should be treated with caution.
- **Sensationalism versus science:** Headlines that exaggerate potential risks can fuel unnecessary anxiety. It is important to distinguish between well-conducted, repeatable studies by reputable and authoritative bodies and sensationalised reports that may not reflect the true state of scientific knowledge.

Ongoing research and safety guidelines

Although current scientific evidence does not support the idea that RF radiation from mobile phones or wi-fi routers poses a significant health risk, research is ongoing. The International Agency for Research on Cancer has classified RF radiation as 'possibly carcinogenic' to humans, based largely on limited evidence from animal studies. This classification highlights the need for continued vigilance, especially as technology evolves and new devices enter the market.

The rapid advancement of wireless technologies represents a triumph of physics and engineering. By understanding the electromagnetic phenomena at the heart of these technologies, scientists have been able to design systems that enable global communication, data sharing and connectivity. Extensive research supports the safety of these technologies, showing that RF radiation from everyday devices poses no significant health risks when used according to established guidelines. As science continues to explore this area, it highlights the importance of distinguishing between well-founded scientific evidence and media sensationalism, ensuring that the benefits of these technologies are enjoyed while public health is maintained.

UNIT

4

Revolutions in modern physics

Unit opener

Topic 1: Special relativity
CHAPTERS RELATED TO THIS TOPIC AREA: 11

Topic 2: Quantum theory
CHAPTERS RELATED TO THIS TOPIC AREA: 12–13

Topic 3: The standard model
CHAPTERS RELATED TO THIS TOPIC AREA: 14–15

9780170483704

Revolutions in modern physics provides a basis for you to examine relative motion, light and matter, including the limitations of classical physics theories that led to the development of the special theory of relativity and the quantum theory of light and matter. The development of the quantum theory of the atom and the derivation of the standard model of particle physics are examined, while technologies such as GPS navigation, lasers, modern electric lighting, medical imaging, quantum computers and particle accelerators are also investigated. Your inquiry and analytic skills are developed through experimentation and investigation of a range of phenomena, such as atomic emission and absorption spectra and the photoelectric effect.

UNIT OBJECTIVES

By the end of this unit, students should be able to:

1. Describe ideas and findings about special relativity, quantum theory and the Standard Model.
2. Apply understanding of special relativity, quantum theory and the Standard Model.
3. Analyse data about special relativity, quantum theory and the Standard Model.
4. Interpret evidence about special relativity, quantum theory and the Standard Model.
5. Evaluate processes, claims and conclusions about special relativity, quantum theory and the Standard Model.
6. Investigate phenomena associated with special relativity, quantum theory and the Standard Model.

Physics 2025 v1.2 General Senior Syllabus © QCAA 2024

CHAPTER 11

Special relativity

Sueddeutsche Zeitung Photo / Alamy Stock Photo

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Describe observations of natural phenomena that cannot be explained by classical physics, e.g. the presence of muons in the atmosphere and the momentum of high speed particles in particle accelerators.
- Describe the concepts of frame of reference and inertial frame of reference.
- State the two postulates of special relativity.
- Explain how motion can only be measured relative to an observer.
- Explain the concept of simultaneity.
- Describe the consequences of the constant speed of light in a vacuum, e.g. time dilation and length contraction.
- Describe the concept of time dilation, proper time interval, relativistic time interval, length contraction, proper length, relativistic length, rest mass and relativistic momentum.
- Describe the phenomena of time dilation and length contraction, including examples of experimental evidence of the phenomena.
- State the mass-energy equivalence relationship.

- Solve problems involving time dilation, length contraction and relativistic momentum using $t = \frac{t_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$, $L = L_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}$, $p_v = \frac{m_0 v}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$ and $E = mc^2$.
- Explain the implications of relativistic momentum of objects increasing as they approach the speed of light.
- Explain paradoxical scenarios that may arise as a result of special relativity including the twins' paradox, flashlights on a train, and the ladder in the barn paradox.

SCIENCE AS A HUMAN ENDEAVOUR

- Appreciate the significant contributions of scientists such as Albert Einstein and Amalie 'Emmy' Noether who furthered our understanding of relativity.
- Explore how special relativity built upon the work of previous scientists and led to the development of relativistic theories of gravitation, mass–energy equivalence and quantum field theory.
- Explore how special relativity leads to the idea of mass–energy equivalence, which has subsequently been applied in nuclear fission reactors.

Introduction

Towards the end of the 19th century, most scientists were quite satisfied with the development of mathematical models to explain the results of experiments and observations of the natural world. Newton's earlier work on motion had been thoroughly tested and found to be valid in all circumstances. However, within the space of 30 years, there would be a total transformation of these ideas. Small, but significant, discrepancies were beginning to be observed between predictions based on Newtonian physics and experimental results. Refinements to the theory of electromagnetism by James Maxwell were a trigger for this change. Albert Einstein then used some powerful ideas and mathematical modelling to convince others that we needed a new way to look at the nature of time, space, matter and energy. Enter special relativity.

Assessments

- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practicals

- Investigating the magnification of a microscrope

Worksheets

- Name
- Name
- Name

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ The speed of light in a vacuum is a constant: $c = 3 \times 10^8$ m s^{-1}.
- ✓ The SI unit for time (t) is the second (s).
- ✓ The SI unit for energy is the joule (J).
- ✓ The SI unit for mass is the kilogram (kg).
- ✓ The SI unit for velocity (v) is metres per second (m s^{-1}) and can be calculated using $v = \frac{s}{t}$.
- ✓ The SI unit for length (L) is the metre (m).
- ✓ Momentum can be calculated using $p = mv$.
- ✓ A light-year (ly) is the distance that light travels in one year in a vacuum.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe observations of natural phenomena that cannot be explained by classical physics (e.g. the presence of muons in the atmosphere and the momentum of high-speed particles in particle accelerators) and identify them as relativistic effects
- ✓ describe the concepts of frame of reference and inertial frame of reference
- ✓ state the two postulates of special relativity
- ✓ describe relative motion and calculate relative velocity
- ✓ describe how special relativity was developed by Einstein using previous work done by Maxwell and Lorentz
- ✓ explain how motion can only be measured relative to an observer
- ✓ explain the concept of simultaneity
- ✓ describe the consequences of the constant speed of light in a vacuum (e.g. time dilation and length contraction) using kinematic and geometric principles
- ✓ describe the concepts of time dilation, proper time interval, relativistic time interval, length contraction, proper length, relativistic length, rest mass and relativistic momentum
- ✓ describe the phenomena of time dilation and length contraction, including examples of experimental evidence of the phenomena
- ✓ state the mass–energy equivalence relationship and describe how it can be derived
- ✓ describe rest energy and relativistic kinetic energy
- ✓ describe time dilation, length contraction and relativistic momentum
- ✓ solve problems involving time dilations, length contraction and relativistic momentum using:

  $t = \frac{t_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$, $L = L_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}$, $p_v = \frac{m_0 v}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$ and $E = mc^2$
- ✓ explain the implications of relativistic momentum of objects increasing as they approach the speed of light
- ✓ describe examples of experimental evidence for time dilation and length contraction
- ✓ describe the subjectivity of simultaneity
- ✓ explain paradoxical scenarios that may arise as a result of special relativity, including the twins paradox, flashlights on a train, and the ladder in the barn paradox
- ✓ contrast classical mechanics with special relativity.

11.1 Newtonian physics

Newton's three laws of motion are the foundations of **classical mechanics**. The way macroscopic objects act when coming into contact with each other are predictable based on the known properties of inertia, momentum, force and action–reaction pairs. These descriptions of motion, although applicable to all macroscopic bodies, cannot describe what happens at the microscopic level. After many experiments, it was determined that there must be a new model of physics to describe all the unknown phenomena. This is known as **quantum physics**, and this new model accounts for all the discrepancies that classical mechanics cannot describe.

Muons in the atmosphere

At this stage, you are probably sceptical about an example that is not explained by classical mechanics. Consider **muons**, which are particles similar to electrons, but are 200 times greater in mass.

The upper atmosphere is constantly being bombarded by energy from cosmic rays. When these cosmic rays collide with molecules, muons are created from this interaction. These muons are accelerated towards Earth at very high speeds, close to the speed of light. Muons have a half-life of only 2.2 μs, and hence typically decay before reaching the surface of Earth, even at these extremely high speeds.

An experiment carried out on this phenomenon in the early 1960s by David Frisch (1918–91) and James Smith collected data that could not be explained by classical mechanics. Frisch and Smith's data showed that of the muons detected in the upper atmosphere, a large majority of them were detected close to Earth's surface, approximately 6.5 km further down. Considering the muon was travelling at almost the speed of light, and only exists for 2.2 μs, the muons should only be able to travel less than 1 km before decaying. This observation was evidence against classical mechanics and hence falls into the realm of quantum physics. Specifically, this experimental result suggested that the muon has undergone **relativistic effects** – muons do not obey the time and space constraints of Newtonian physics.

LEARNING CHECK 11.1

DESCRIBING

1 **Describe** a relativistic effect.
2 State the differences between classical mechanics and quantum physics.
3 **Infer** why the principles of relativity and quantum physics were postulated more recently than the laws of classical mechanics.
4 **Explain** how muons in the atmosphere provide evidence for relativistic effects.

classical mechanics the study of motion in accordance with Newton's laws; also known as Newtonian physics

quantum physics the science of atoms and subatomic particles, for which classical mechanics fails to explain interactions observed; also known as quantum mechanics

muon a type of elementary particle belonging to the lepton family; an unstable particle formed by cosmic rays in the upper atmosphere

Weblink
Particles and waves: the central mystery of quantum mechanics

relativistic effect an effect where time and space are measured differently when objects travel at speeds nearing the speed of light (e.g. include time dilation and length contraction)

Syllabus links
Chapter 14 of *Nelson QCE Physics Units 1 & 2* discusses Newton's laws of motion.

Chapter 17 of *Nelson QCE Physics Units 1 & 2* discusses the speed of light and properties of light.

11.2 Frames of reference

Galileo was one of the earliest to comment on events observed in different **frames of reference**. In his book *Dialogues Concerning the Two Chief World Systems*, Galileo described a thought experiment in which a sailor drops an object from the mast of a sailing ship moving at a steady velocity. He asked the question: 'Where would the object land relative to the deck of the ship?'

frame of reference a framework in which motion of an object is described according to a coordinate system; is observational and can be inertial or non-inertial (accelerating)

In his frame of reference, the sailor would see the object fall straight down parallel to the mast (Figure 11.2.1a); however, a nearby observer on land (a different frame of reference) would see the object follow a parabolic path (Figure 11.2.1b).

Weblink
Frames of reference

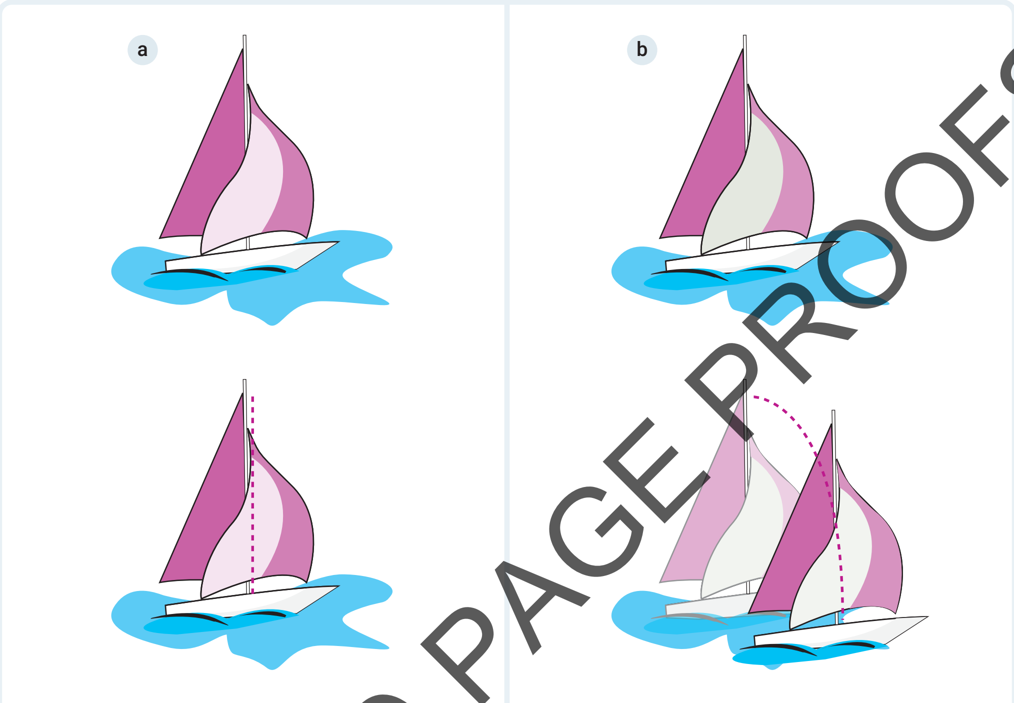


FIGURE 11.2.1 The path of a falling object in the reference frame of (a) a sailor on the ship and (b) an observer on land

Later, Newton would agree with Galileo. He described frames of reference that were stationary or moving at constant velocity as **inertial frames of reference**. Inertial frames of reference do not accelerate. An inertial frame of reference is an ideal situation in which objects at rest remain at rest and objects travelling at a constant velocity remain travelling at a constant velocity. Examples are a spaceship, a table and a cruising aeroplane. Non-inertial frames of reference include merry-go-rounds and aeroplanes taking off or landing (changing velocity).

inertial frame of reference a frame of reference in which Newton's first law applies to a very good approximation, and there has no acceleration

Galilean transformations

Classical physics, the physics of Galileo and Newton, relies on the 'sensible' idea that inertial coordinate systems are equivalent. That is, there is a set of translation rules to connect measurements in one frame of reference (or coordinate system) to any other reference frame.

Consider Figure 11.2.2. This represents two frames of reference in two dimensions. The *privileged* frame of reference, P, has coordinates (x, y). P is stationary with respect to a moving reference frame, P′, with coordinates (x', y'). The two coordinates can be made to coincide at the same time, but then P′ moves further and further away from P. This motion depends on the relative speed, v, of P′ with respect to P and the time elapsed, Δt, after the two frames coincided.

The time interval in each frame of reference is the same. That is, for classical physics, time intervals are measured in the same way and occur at the same rate. Clocks in all frames of reference are identical in their time keeping. Time is **invariant**. Thus, Δt in P and $\Delta t'$ in P′ are equal – the time interval is denoted as Δt.

invariant does not vary; is the same in all reference frames

Suppose an observer located at (0, 0) in frame P sees a rocket some distance y away. It is travelling parallel to the x-axis at speed v (Figure 11.2.2). Initially (at $t = 0$ s), the rocket's coordinates in P′ coincide with those in P. But as the rocket travels in P′, its x' coordinates increase

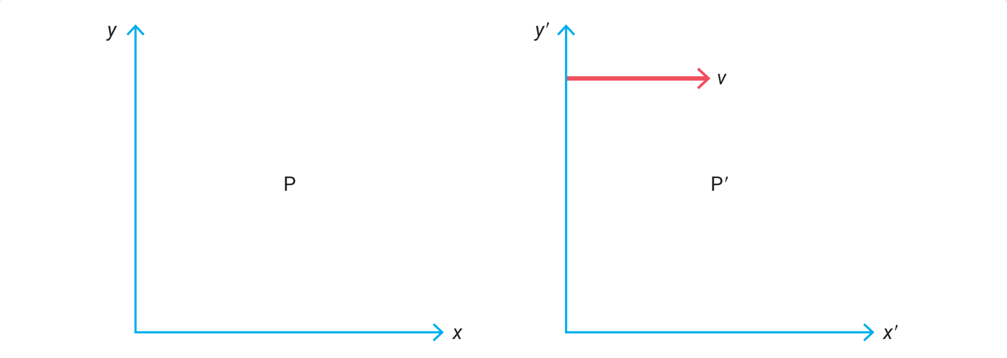


FIGURE 11.2.2 Two inertial frames that are moving relative to each other. P is regarded as the stationary frame.

in the same time interval Δt, by $v\Delta t$. This is just the distance between the two coordinate systems. Thus, the x coordinates in P relate to the x' coordinates in P′ by the **Galilean transformation**:

$$x = x' + v\Delta t$$

Galilean transformation for a two-dimensional inertial frame of reference, P′, moving at constant speed in the *x*-direction with respect to another inertial frame, P: $x = x' + v\Delta t$ and $y = y'$

Similar arguments can be used to show that if the rocket is travelling at speed v parallel to the y and y' axes:

$$y = y'$$

The transformation equations rely on the relative speeds of the two frames of reference. If P′ had been chosen as the stationary frame, then P would be travelling at negative speed with respect to P′. This needs to be taken into account in the Galilean transformation equations. Finally, if the rocket was travelling with vector velocity $\vec{v}$, the transformation equations in the x and y directions require the use of the x and y components of the velocity respectively.

WORKED EXAMPLE 11.2.1

A car travels at 12 m s^{-1} through an intersection. After 10 s, what is the position of the car with respect to the coordinates based on the frame of reference of the:

a intersection?

b car?

ANSWERS

a 1 **Determine the formula.**

$x = x' + v\Delta t$

2 **Substitute the known values.**

Consider the intersection as (0, 0). The intersection is the privileged frame for this question.

$x = 0 \text{ m} + 12 \times 10$

3 **Calculate the answer.**

$x = 120$ m

The car has travelled 120 m with respect to the intersection.

b **Determine the privileged frame.**

Consider the car as the privileged frame. The car has not moved with respect to itself.

$x' = 0$ m

LEARNING CHECK 11.2

DESCRIBING

1 **Define** 'inertial frame of reference'.
2 Write the Galilean transformation for an object moving at a constant speed in the *x*-direction.
3 The path followed by an object dropped from the mast of a moving ship can appear to be both a straight line and a parabola. **Explain** why.
4 **Explain** the term 'frame of reference'.

APPLYING

5 A bus is travelling at 12 m s^{-1} past a stationary pedestrian. After 5.0 s, what is the position of the:
   a bus relative to the observer?
   b observer relative to the bus?
6 A train passes through a station at 20 m s^{-1}. A person on the train walks towards the front of the train at 2 m s^{-1}. After 5.0 s, the person has moved a distance from the common origin on the train and station. What are the coordinates of the person with respect to the:
   a station?
   b train?
   Remember to choose the stationary frame according to the question.

11.3 The two postulates of special relativity

As well as considering a sailor dropping an object from the mast of a sailing ship, Galileo discussed the situation of a person walking within the cabin of a ship. If the sailing ship moves forwards at a velocity of 5 m s^{-1} relative to Earth, a person moving forwards at a velocity of 1 m s^{-1} relative to the cabin will be moving forwards at a velocity of 6 m s^{-1} relative to Earth. The position and velocity of the person is different in each reference frame.

According to Galileo and Newton, acceleration of a body will be the same in each frame of reference, providing they are inertial frames. For example, in the sailing ship's cabin, the person may have accelerated from 0 to 1 m s^{-1} in 0.5 s in the cabin, which is an acceleration of 2 m s^{-2}. From the perspective of a nearby land-based observer, the person will have accelerated from 5 to 6 m s^{-1} in the same time frame, which is again an acceleration of 2 m s^{-2}.

Principles of classical relativity

relativity principle the laws of physics are the same in all inertial frames of reference; the first postulate of special relativity

The **relativity principle** states that the laws of physics are the same in all inertial frames of reference. It can be shown that the laws of motion, including those for the conservation of energy and momentum, are the same in all inertial reference frames. Observers in different inertial frames would record different velocities and therefore determine different values of energy and momentum. Nevertheless, they would agree that there had been no net change of either energy or momentum. Consequently, they would agree on conservation of energy and conservation of momentum.

This suggests that there is no privileged inertial frame. No inertial reference frame is better than any other. If you are on a train travelling at a steady velocity of 80 km h^{-1} west across the Nullarbor

Plain in southern Australia, it is quite valid for you to argue that, from your reference frame, you are stationary and Earth is moving at 80 km h^{-1} east. Providing your ride is smooth and at steady velocity, there is no experiment you can perform to test whether you are moving or stationary.

Galileo wrote about this in his book, in which he discussed the example of a person observing a range of movements in the cabin of a sailing ship that was sailing steadily at constant velocity. He argued that the person would not be able to tell if the ship was moving or not. He believed that there is no absolute frame of reference against which the velocities of all other frames can be measured. In this he differed from Newton, who believed that the Earth–Sun system could be considered the absolute frame of reference.

The following, can be agreed on.

- The laws of motion are the same in all inertial frames of reference.
- The laws of conservation of energy and conservation of momentum apply in all inertial frames of reference.
- All inertial frames are equivalent. All are equally valid.

These are represented within the postulates of special relativity.

KEY CONCEPT

The two postulates of special relativity

Einstein's theory was based on the following two clear propositions, known as the two postulates of special relativity.

1. First postulate: The laws of physics are the same in all inertial frames of reference – the principle of special relativity.
2. Second postulate: The speed of light has the same value, *c*, in all inertial frames. It does not depend on the speed of either the source or the observer.

Special relativity

Albert Einstein (1879–1955) reflected long and hard about what an electromagnetic wave (travelling at the speed of light) would look like if you travelled along with it. He concluded that you would see no change in time; the wave would be stuck in time, but it would continue to oscillate in space. How could something change, yet no time pass? To resolve this problem, Einstein decided to investigate the nature of space and time. The result was the theory of **special relativity**. This was a bold step but a well-trodden path to discovery: questioning taken-for-granted assumptions leads to fresh, more powerful ways of understanding the world.

special relativity the physics theory about the relationship between space and time, which is not explained by Newtonian or Galilean relativity

Einstein's theory was based on the following two clear propositions.

1. *First postulate of special relativity*: The laws of physics are the same in all inertial frames of reference – the principle of special relativity.
2. *Second postulate of special relativity:* The speed of light has the same value, *c*, in all inertial frames. It does not depend on the speed of either the source or the observer.

These postulates contradict a few of the ideas of Newtonian and Galilean relativity presented earlier. The first postulate of special relativity excludes the possibility of there being a privileged reference frame. The second postulate contradicts the idea that, according to different reference frames, different objects would have different velocities (such as the sailor dropping an object on the boat). To elaborate on the second postulate of relativity, light will travel at 3×10^8 m s^{-1} in a vacuum, regardless of the speed of the observer.

Weblink
Einstein's theory of special relativity

LEARNING CHECK 11.3

DESCRIBING

1 State the two postulates of special relativity.
2 How did Einstein's postulates differ from the agreed-upon rules of relativity deduced by Galileo and Newton?
3 **Compare** how the speed of light in a moving inertial reference frame changes with that in a stationary inertial reference frame.

11.4 Measuring motion

To measure how motion is considered in different reference frames, you need to consider the velocities of objects relative to each other. It is straightforward to consider the velocity of an object when there is a stationary observer. For example, if a car travelling at 20 m s^{-1} east passes a person sitting on a bench, they will observe the car travelling at 20 m s^{-1} east. But if the person is walking at 2 m s^{-1} east, then the **relative motion** of the car will be different. The relative motion of the moving car and moving person depend on whose reference frame is being considered at the time.

relative motion the motion of a moving object according to a moving observer; when evaluating relative motion, one reference frame must always be considered stationary

Relative velocities

It is possible to calculate the velocity of an object in one inertial frame of reference relative to another inertial frame of reference, providing the object is travelling along the same axes of direction. If a person is moving at 2 m s^{-1} east, and a car passes them at 20 m s^{-1} east, the person would observe the car as travelling at 18 m s^{-1} east. To consider this in two or three dimensions, this becomes much more complex, and we need to consider vectors and how they compare to the relative motion of an object in different inertial frames of reference.

Take the example of an aeroplane flying from Sydney to Perth. At the cruising altitude of a passenger aircraft, about 10 km, there is nearly always a strong westerly wind that varies from 50 km h^{-1} to more than 300 km h^{-1}. The direction of the aircraft's movement is almost exactly parallel to the direction of the wind. The westbound flight is scheduled at 4 h 25 min (Figure 11.4.1a), while the eastbound flight is 3 h 50 min (Figure 11.4.1b). The aeroplane's velocity, relative to the ground, is higher on the way to Sydney than on the way to Perth.

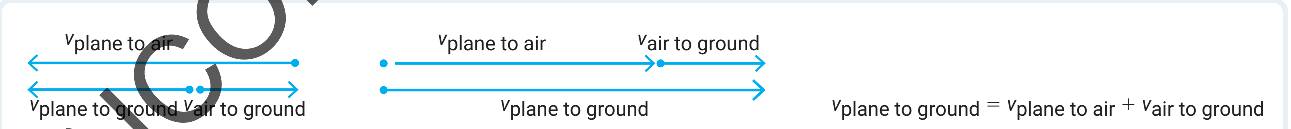


FIGURE 11.4.1 Vector addition showing the velocity of the plane relative to the ground is (**a**) smaller when the plane is travelling against the wind, known as a *head wind* and (**b**) greater when the plane is travelling in the same direction as the wind, known as a *tail wind*.

The velocity of the plane relative to the ground is the velocity of the plane relative to the air *plus* the velocity of the air relative to the ground. More generally, the velocity of A relative to B is the velocity of A relative to C *plus* the velocity of C relative to B. This holds true for vectors in two dimensions.

$$v_{AB} = v_{AC} + v_{CB}$$

KEY FORMULA

$v_{AB} = v_{AC} + v_{CB}$

where:

v_{AB} = velocity and direction of object A relative to B (m s^{-1})
v_{AC} = velocity and direction of object A relative to C (m s^{-1})
v_{CB} = velocity and direction of object C relative to B (m s^{-1})
Note that v_{AC} or v_{CB} may be negative relative to the positive reference direction.

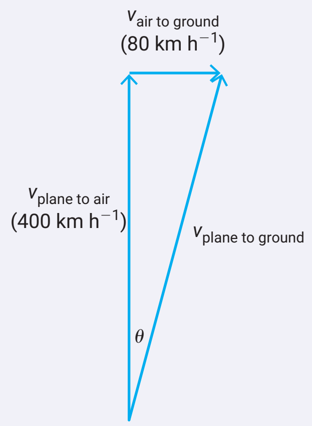
WORKED EXAMPLE 11.4.1

An aeroplane is headed due north at a speed of 400 km h^{-1}. There is a westerly wind (i.e. coming from the west) of 80 km h^{-1}.

a Draw a vector diagram to show the velocity of the plane relative to the ground.
b What is the speed of the plane relative to the ground?
c What is the resultant velocity of the plane relative to the ground?

ANSWERS

a **Draw a vector diagram showing the movement of an aeroplane and the impact of wind.**
The vector diagram shows a plane travelling north, through a westerly wind. This results as a north-easterly direction of the plane.



b 1 **Determine the formula.**
From the diagram and Pythagoras's theorem:
$v^2 = v_{\text{plane to air}}{}^2 + v_{\text{plane to ground}}{}^2$

2 **Substitute the known values.**
$v^2 = 400^2 + 80^2$

3 **Calculate the answer.**
$v^2 = \sqrt{(400^2 + 80^2)}$
$= 408 \text{ km h}^{-1}$

c 1 Determine the formula.
We now know the speed, so we must find the direction.
Finding θ can be done with tangent:

$\tan\theta = \frac{O}{A}$

2 **Substitute the known values.**

$\tan\theta = \frac{80}{400}$

3 **Calculate the answer.**
$\theta = \tan^{-1}(0.2)$
$= 11.3°$

LEARNING CHECK 11.4

DESCRIBING

1 State the formula for calculating relative velocity.

2 **Explain** whether the velocity of a moving train is the same in all reference frames.

APPLYING

3 A taxi is travelling at 10 m s^{-1} when a person nearby starts to walk towards it at 2 m s^{-1}. A stationary person on the other side of the street also observes the situation. What is the speed of the person nearby relative to the:

a taxi?

b observer on the other side of the road?

4 The tide is running south at 3.0 m s^{-1}. At the same time, a yacht is heading east at 4.0 m s^{-1} directly towards a buoy. What is the velocity of the yacht relative to the shore? Include a vector diagram in your answer.

5 A person rows at 1.0 m s^{-1} through the water of a river that is flowing at 0.5 m s^{-1} north. The rower keeps the boat moving perpendicular to the bank in an easterly direction.

a Draw a vector diagram to show the velocity of the rower from the reference frame of a person on the bank.

b Using your diagram, specify completely the velocity of the boat relative to the bank.

SYNTHESISING

6 Consider the relative motion of cars on a highway. A police car is travelling south and wants to know if any of the north-bound traffic is speeding. **Explain** how they would do this. **Compare** this method with a police officer standing on the side of the road with a speed camera.

11.5 Simultaneity

When two events happen simultaneously, they are said to happen at the same time. However, what may be considered a simultaneous event in one inertial reference frame may not be considered simultaneous in another reference frame. Measurement of time in different reference frames produces interesting results, not the least being that for simultaneous events. The idea that simultaneous events happen at different times in different reference frames argues against the existence of a universal time frame.

Consider a light positioned in the centre of a train carriage (**Figure 11.5.1**). At each end of the carriage is a sensor or camera that can detect when the light is on or off. From the perspective of someone inside the carriage, when the light turns on, both sensors will recognise this at the same time; that is, *simultaneously*. Even if the train carriage is moving at a constant speed, an observer inside the carriage will notice that both sensors detect the light turning on at the same time.

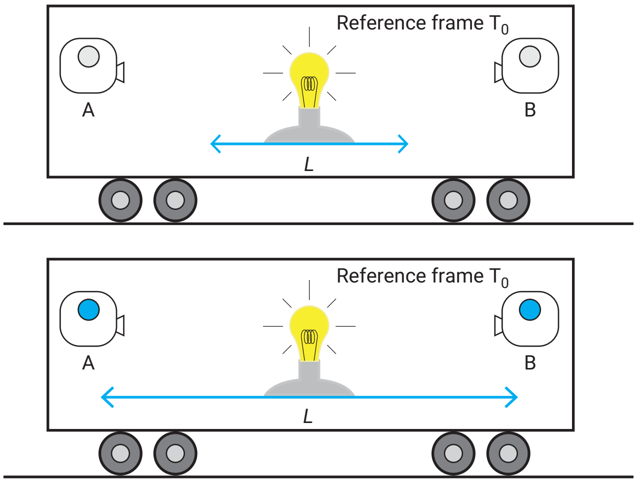


FIGURE 11.5.1 Reference frame T_0, inside a moving train. Light from the source reaches the two sensors A and B simultaneously when considered from the train (reference frame T_0).

Now consider a situation in which the train carriage *is* moving at a very high constant velocity, *v*, relative to the platform. An observer on the platform (reference frame T) watches the train go past and can see the light globe and each sensor (Figure 11.5.2).

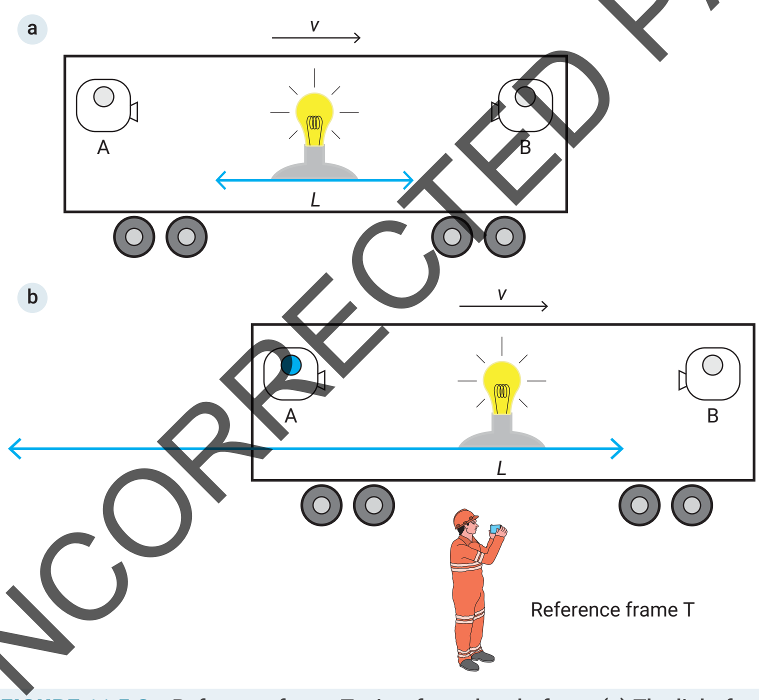


FIGURE 11.5.2 Reference frame T: view from the platform. (a) The light from the source at time *t* has not reached either sensor A or sensor B. (b) The light from the source by time *t′* has reached sensor A, but has not yet reached sensor B. The external observer does not see the events as simultaneous.

As the train carriage moves past, the observer in reference frame T notices that the globe, and sensor A are both on, but sensor B is off. In this reference frame, the observer sees sensor A moving forwards to meet the light waves moving towards the globe, so sensor A is triggered first. Sensor B is moving away from the light waves and hence takes longer to detect the light. The simultaneous events in reference frame T_0 are not simultaneous in reference frame T. It is important to note that the speed of light in both reference frames is the same.

simultaneity when two events occur simultaneously in one reference frame but do not occur simultaneously in another reference frame; occurs if the reference frames are moving close to the speed of light relative to each other

These events occur because **simultaneity** depends on the agreement of time measurements, and time is relative. If Einstein's second postulate were not true, then simultaneity could be agreed on across different reference frames. This seems to be against common sense or experience. Because the speed of light is so much greater than ordinary speeds, we do not see any evidence of timing problems in everyday life.

LEARNING CHECK 11.5

DESCRIBING

1 **Define** 'simultaneity'.

2 **Explain** why simultaneous events cannot occur at the same time in different frames of reference when one of the frames of reference is travelling at close to the speed of light.

3 **Describe** how, in the light on a train carriage scenario, the events can be seen as simultaneous and non-simultaneous.

APPLYING

4 Consider two trees, tree A and tree B, which are separated by a distance *d*. An observer is positioned at the centre between these two trees as a storm starts brewing. Each tree is hit by a bolt of lightning simultaneously from the reference frame of the observer. At this moment, a spacecraft travels past the trees from A to B at a velocity close to the speed of light. From the external spacecraft observer's reference frame, which tree does the lightning strike first?

11.6 Consequences of a constant speed

The theory of special relativity asks us to give up our Newtonian view of space and time and accept some very strange and puzzling ideas. To illustrate this, we will use a technique that Einstein used himself: simple thought experiments (from the German *gendanken*) that are based on the two postulates of special relativity. The example in the previous section of the light on a train carriage was an example of a thought experiment: one that we are unable to carry out, but which obeys the laws of special relativity.

Time

Assume two rockets, each travelling at 0.25*c* but in opposite directions, pass each other at the instant they receive a light pulse from a distant pulsar (Figure 11.6.1). Observers on each rocket attempt to measure the speed of this pulse of light by using sensors on the outside of the rocket (A_1 and B_1, and A_2 and B_2 respectively).

Each rocket uses a timer and the distance between the sensors to measure the speed of light relative to its own reference frame. Both rockets measure the speed of light at 3×10^8 m s^{-1} relative to their respective reference frames.

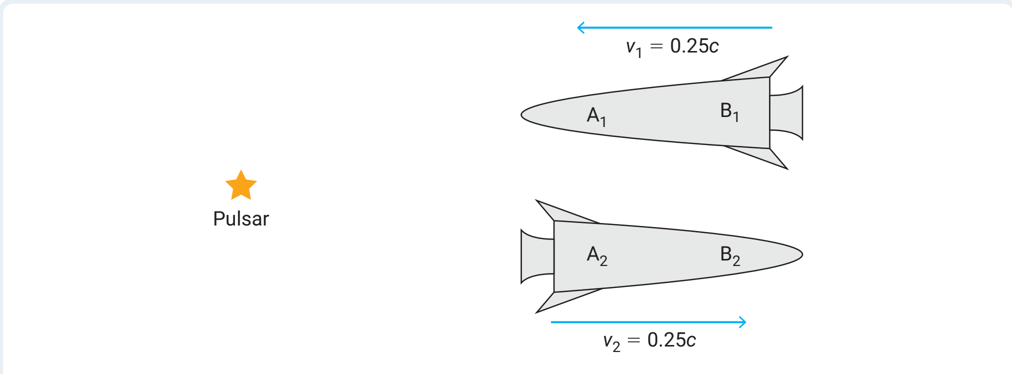


FIGURE 11.6.1 Passing rockets both view light from a pulsar, but with different relative velocities.

Galilean relativity would have argued that the rocket heading towards the pulsar would have registered a light speed of 1.25*c*, and the rocket moving away from the pulsar would have measured a light speed of 0.75*c*. Of course, in practice this would be a difficult experiment to carry out. However, we can conceive of such a scenario and use logic and Einstein's postulates to relate what each observer would see or measure. In this case, the time taken for the light to travel between sensors A and B on each ship is the same, and each would measure the speed of light as *c*.

Consider another experiment using a train. The train (a rail cart in this case) is running on a smooth track at high velocity v relative to the ground. On one carriage, a pair of mirrors m_1 and m_2 is set up so that a series of light pulses can bounce back and forth between them. Observer A is standing on the carriage with the mirrors, and observer B is standing on the ground nearby, watching the train pass (Figure 11.6.2). Both observer A and observer B have identical, very accurate watches that are capable of measuring very small time increments. They both agree that the distance between the two mirrors is width, w.

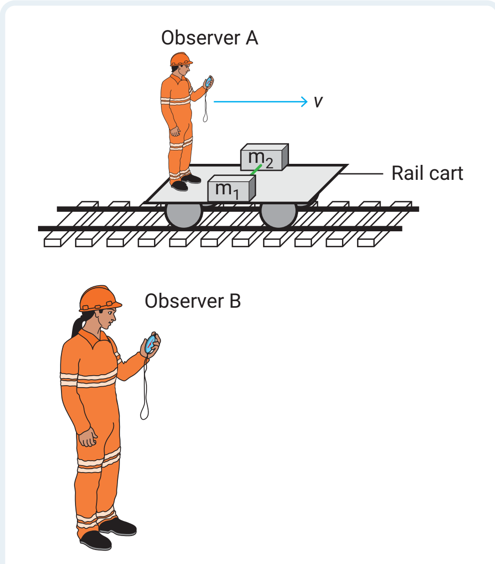


FIGURE 11.6.2 A passing high-speed rail cart

Observers A and B both measure the time it takes for the light to travel between the mirrors. Observer A sees the situation as a simple path of light between the two mirrors (Figure 11.6.3a), and hence measures the time it takes for the light to travel from m_1 to m_2 and back again as $t_0 = \frac{2w}{c}$ (from $t = \frac{s}{v}$). As the train is moving very fast, observer B sees the situation quite differently. From their viewpoint, the velocity of the mirrors results in the light pulse forming a triangle (Figure 11.6.3b). Because both mirrors are moving with velocity v to the right relative to observer B, she measures the time for one pulse to move from m_1 to m_2 and back again as time t. One half of the journey forms a right-angled triangle with sides of length w, $\frac{vt}{2}$ and $\frac{ct}{2}$, as c is constant in all reference frames.

Using Pythagoras' theorem, we can relate the time interval that observer B records for this light pulse to happen to the time observer A records as follows:

$$\left(\frac{ct}{2}\right)^2 = \left(\frac{vt}{2}\right)^2 + w^2$$

$$\left(\frac{ct}{2}\right)^2 - \left(\frac{vt}{2}\right)^2 = w^2$$

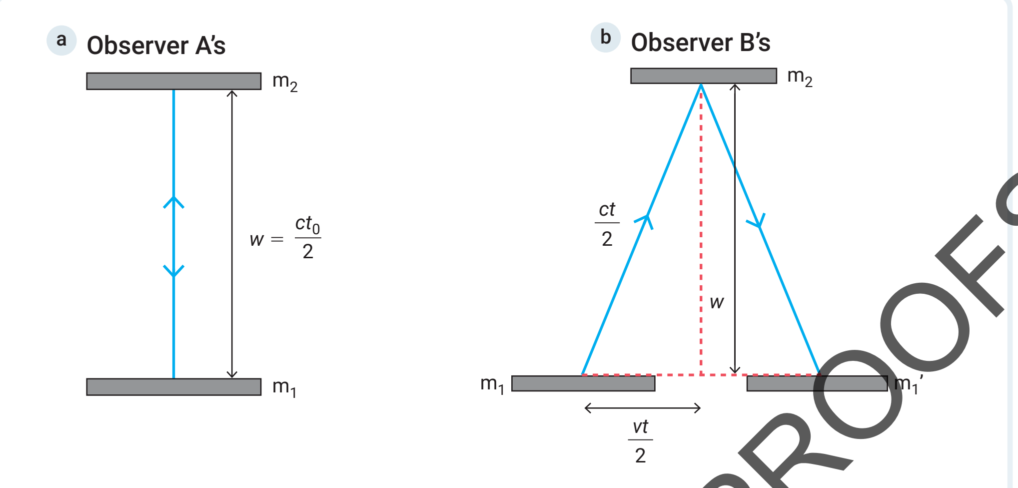


FIGURE 11.6.3 An event (light reflecting between two mirrors) as viewed from (**a**) observer A's reference frame and (**b**) observer B's reference frame

$$\frac{t^2}{4}(c^2 - v^2) = w^2$$

$$\frac{t^2}{4}\left(1 - \frac{v^2}{c^2}\right) = \frac{w^2}{c^2}$$

$$\frac{t}{2}\sqrt{1 - \frac{v^2}{c^2}} = \frac{w}{c}$$

$$t\sqrt{1 - \frac{v^2}{c^2}} = \frac{2w}{c}$$

$$t\sqrt{1 - \frac{v^2}{c^2}} = t_0$$

$$\therefore t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

proper time the time interval between two events occurring at the same place in an inertial reference frame, as measured by an observer in that inertial frame (t_0)

time dilation a longer time measured by an observer outside the reference frame in which the event occurs (t)

Observer A's measurement t_0 is called the **proper time** because the measurement from observer A was taken in the frame of reference in which the event was occurring. Observer B, who is observing the carriage moving past them, observes that the time t they recorded as *longer* than the time t_0 that observer A recorded. Observer B's time measurement has been dilated. Einstein argued that the postulates of special relativity led to the understanding that observers in different inertial reference frames would not agree on time measurements. **Time dilation** is only significant when inertial reference frames are moving relative to each other at speeds close to the speed of light. Time dilation is not encountered day-to-day, as the velocities that are observed daily, v, are much smaller than c, meaning that $\frac{v^2}{c^2}$ tends to zero, and $t = t_0$. Time dilation is a consequence of events occurring at speeds close to the speed of light, including those travelling at the speed of light.

KEY FORMULA

Relative time

The relative time relates to the proper time through the Lorentz factor, $\gamma = \frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$, thus, $t = \frac{t_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$. The Lorentz factor increases as v approaches c.

Length

If time measurements are different relative to different inertial frames, what happens to the measurements of length? If time dilation occurs, then so too does length contraction. Moving objects contract along their direction of motion.

This means that an observer will measure the length of an object moving relative to them as shorter than when it is at rest. The length of an object at rest is called its **proper length**. **Length contraction** is only observed when v is close to c. At ordinary speeds on Earth, $\frac{v}{c}$ becomes zero; therefore, $L = L_0$. The **relative length** is also known as the contracted length.

proper length length measured in an inertial frame of reference in which the object is stationary (L_0)

length contraction length measurements are shorter in a reference frame that is moving relative to an inertial frame

relative length length measured in an inertial frame of reference in which the object is moving relative to the direction of the length (L)

KEY FORMULA

Length contraction

Length contraction relates to the proper length through the Lorentz factor, $\gamma = \frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$, thus $L = L_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}$. Length contraction is experienced in the direction of motion only.

LEARNING CHECK 11.6

DESCRIBING

1 Does time dilation cause a time interval to increase or decrease when an observer is viewing an event moving close to the speed of light?

2 Does length contraction cause the length of an object to appear longer or shorter when an observer is viewing an object moving close to the speed of light?

3 **Define** 'proper time'.

4 **Define** 'proper length'.

5 **Explain** why time dilation and length contraction are not observed in our day to daily interactions.

6 Rearrange the time dilation formula, $t = \frac{t_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$, to make the proper time the subject of the equation.

7 Rearrange the length contraction formula, $L = L_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}$, to make the proper length the subject of the equation.

11.7 Time dilation, length contraction and relativistic momentum

To gain a clear understanding of special relativity, a few terms need to be defined in order to find their quantities mathematically. So far, we have understood that there is a time dilation and length contraction associated with reference frames moving at close to the speed of light. To understand these, we have had to define proper time and proper length, quantities that can only be observed when measured in the frame in which the event is occurring.

Time dilation

Weblink
Time dilation – Einstein's theory of relativity explained

Time dilation is only significant when inertial reference frames are moving relative to each other at speeds close to the speed of light. As previously derived, time dilation can be expressed mathematically as:

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Time measurements depend on the frame of reference in which they are made. A clock in an inertial frame that is moving relative to a clock in a second inertial frame will be regarded as running slow. This means that the time interval for the moving clock is greater than the time interval for the stationary clock.

Lorentz factor the factor by which both time dilation and length contraction are affected when v approaches c

In the time dilation equation, the reciprocal of the term $\sqrt{1 - \frac{v^2}{c^2}}$ is called the **Lorentz factor**, γ. It is also common to represent the factor $\frac{v}{c}$ as β, such that the Lorentz factor can be simplified to $\frac{1}{\sqrt{1-\beta^2}}$.

KEY FORMULA

Time dilation

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where:

t = dilated time, measured in the frame of a moving observer (s)

t_0 = proper time, measured by an observer in the inertial reference frame (s)

v = velocity of the inertial reference frame (m s^{-1})

c = speed of light (3×10^8 m s^{-1})

KEY FORMULA

Lorentz factor

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where:

γ = Lorentz factor

This allows for time dilation to be expressed as $t = \gamma t_0$. Table 11.7.1 demonstrates how γ, and hence time dilation, changes as v approaches c.

TABLE 11.7.1 Approximate values of the Lorentz factor, γ, for various values for β, $\frac{v}{c}$

$\beta = \frac{v}{c}$	Lorentz factor, γ
0	1
0.0010	1.000 0005
0.010	1.000 05
0.10	1.005
0.20	1.021
0.50	1.155
0.80	1.667
0.90	2.294
0.94	2.931
0.99	7.089
0.999	22.37

Proper time interval

Proper time is the time interval between two events occurring at the same place in an inertial frame, as measured by an observer in that inertial frame. Refer to observer B and observer A from section 11.6. The time measured by observer B is not the proper time, because observer B is not travelling with the mirrors. Now consider what would happen if the mirrors were on the platform with observer B and the experiment was repeated. Observer A could argue that their train was stationary and that observer B was moving at $-v$ with respect to them. This means he would find that observer B's clock is slow compared to his. How do we reconcile these two observations?

Time dilation is about measurement in *different* inertial frames; it is a result of relative movement between the frames. The clocks do not physically change. Time dilation is about what an observer in one frame measures about an event in another, and it must be reciprocal because there is no absolute frame of reference.

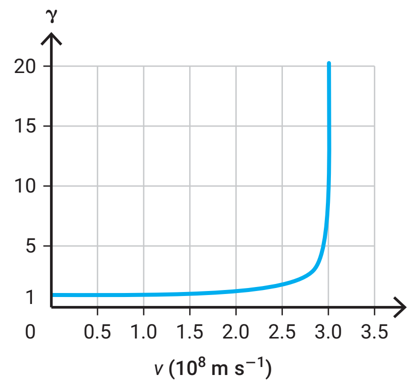


FIGURE 11.7.1 A graph of γ against v. Note the variation of the Lorentz factor as the velocity approaches the speed of light.

WORKED EXAMPLE 11.7.1

A pilot in a rocket travelling with a velocity of 0.250*c* presses a button to flash a light for period of 5.00 s at a space station as the rocket passes. How long is the flash seen by an observer on the space station?

ANSWER

1 Determine the formula.

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

2 Substitute the known values.

$$t = \frac{5.00\ \text{s}}{\sqrt{\left(1 - \frac{(0.250c)^2}{c^2}\right)}}$$

3 Calculate the answer.

$$t = \frac{5.00\ \text{s}}{\sqrt{0.9375}}$$
$$= 5.16\ \text{s}$$

This is **relativistic time** due to the reference frames moving so fast relative to each other. The observer in the space station views the rocket travelling towards it at a velocity of 0.250*c*. From this viewpoint, the clock on the rocket will be seen to run slowly compared to the one in the space station. Any observer regards a clock that is moving relative to their frame of reference as running slow.

relativistic time time dilation observed due to objects moving at very high speeds relative to each other (t)

Length contraction and proper length

All observers will measure an object moving at relativistic speeds as being shorter or contracted in the direction of relative motion than when the object is at rest. This phenomenon is known as length contraction and is represented as follows: $L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$, or simply $L = \frac{L_0}{\gamma}$.

KEY FORMULA

Length contraction

$$L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

where:

L = length of the object measured by an observer who is moving at constant velocity relative to the object's inertial frame (m)

L_0 = proper length, the length of the object at rest (m)

v = velocity of the object relative to the observer (m s^{-1})

c = speed of light (3×10^8 m s^{-1})

relativistic length length contraction due to objects moving at very high speeds relative to each other (L)

Proper length of an object is best measured at rest, or else in the frame of an observer moving with the object being measured. The length contraction or **relativistic length** is only observed when reference frames are travelling at very high speeds relative to each other.

WORKED EXAMPLE 11.7.2

a An observer on the Moon notices a spaceship travelling past at a speed of 2.08×10^8 m s^{-1}. The spaceship has a proper length of 120 m. What length will the observer on the Moon measure the spaceship to be?

b A crewed mission is to be sent to a newly discovered exoplanet 8 light-years away. Their spacecraft will travel at a velocity of 0.5c to get there.

i According to the mission crew, how far away from Earth is the exoplanet?

ii According to the mission crew, how long will the journey take?

iii According to the mission command on Earth, how long will the journey take?

ANSWERS

a 1 **Determine the formula.**

$$L = L_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}$$

2 **Substitute the known values.**

$$L = 120\sqrt{\left(1-\frac{(2.08\times10^8)^2}{(3.00\times10^8)^2}\right)}$$

3 **Calculate the answer.**

$$L = 120\times0.72$$
$$= 86.5 \text{ m}$$

b i 1 **Determine the formula.**

$$L = L_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}$$

2 **Substitute the known values.**

$$L = 8\sqrt{\left(1-\frac{(0.5c)^2}{c^2}\right)}$$

3 **Calculate the answer.**

$$L = 8 \text{ ly}\times0.866$$
$$= 6.9 \text{ ly}$$

ii 1 **Determine the formula.**

$$t_{crew} = \frac{L}{v_{crew}}$$

2 **Substitute the known values.**

$$t_{crew} = \frac{6 \text{ ly}}{0.5c}$$

3 **Calculate the answer.**

$$t_{crew} = 14 \text{ years}$$

iii 1 **Determine the formula.**

$$t_{Earth} = \frac{L_0}{v}$$

2 **Substitute the known values.**

$$t_{Earth} = \frac{8 \text{ ly}}{0.5c}$$

3 **Calculate the answer.**

$$t_{Earth} = 16 \text{ years}$$

Rest mass

Length and time have been shown to have values that depend on the relative motion of the observers. The third fundamental physical quantity is mass. From energy and momentum analyses, Einstein deduced that the mass of an object will be relative, and will depend on its relative speed as follows:

$$m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

rest mass the mass as measured when the mass is stationary in an inertial reference frame (m_0); also known as proper mass and never changes

relativistic mass the greater the relative velocity, the greater the relativistic mass: $m = \gamma m_0$; also known as relativistically corrected mass

where: m_0 = **rest mass**, as measured when the object is stationary in an inertial reference frame
m = measurement of its mass in a reference frame moving in relation to a stationary frame and is known as the **relativistic mass**.

KEY FORMULA

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where:

m = rest mass (kg)

m_0 = relativistic mass (kg)

v = speed of the moving inertial frame of reference (m s^{-1})

c = speed of light (3×10^8 m s^{-1})

This means that as the velocity of an object approaches the speed of light, the mass of the object will greatly increase. There is an ultimate velocity. As v approaches c in size, γ approaches zero, and m will be very large in size. If v was greater than c, then the term would result in the square root of a negative number from $\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$, which is an invalid result. This implies that the speed of an object with non-zero rest mass cannot be equal to, or exceed, the speed of light. It appears that the speed of light is the ultimate speed limit in the physical world.

WORKED EXAMPLE 11.7.3

a What is the relativistically corrected mass of an electron whose speed is measured to be 1.8×10^8 m s^{-1}? Rest mass of an electron is 9.109×10^{-31} kg.

b At what speed is a particle moving if its relativistic mass is five times larger than its rest mass?

ANSWERS

a 1 Determine the formula.

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

2 Substitute the known values.

$$m = \frac{9.109 \times 10^{-31}}{\sqrt{\left(1 - \frac{(1.8 \times 10^8)^2}{(3 \times 10^8)^2}\right)}}$$

3 Calculate the answer.

$m = 1.1 \times 10^{-30}$ kg

b 1 Determine the formula.

$m = \gamma m_0$

2 Rearrange to find the unknown.

$\gamma = \frac{m}{m_0}$

3 Substitute the known values.

Since $\gamma = 5$

$5 = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$

4 Calculate the answer.

$1 - \frac{v^2}{c^2} = 0.04$

$\frac{v^2}{c^2} = 0.96$

$\frac{v}{c} = \sqrt{0.96}$

$\frac{v}{c} = 0.9798$

$v = 2.4 \times 10^8 \text{ m s}^{-1}$

Relativistic momentum

Now that we have a definition for relativistic mass, what does this mean about momentum? Special relativity uses the classical equation $p = mv$. However, m is now the relativistic mass whose magnitude depends on the velocity of the mass. This means that the magnitude of the **relativistic momentum** of an object is given by:

$$p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{p_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

relativistic momentum the momentum of a particle due to the relativistic motion at high relative speeds (p_v)

KEY FORMULA

Relativistic momentum

$$p_v = \frac{p_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where:

p_v = relativistically corrected momentum (N s)

p_0 = momentum calculated from the rest mass in (N s)

v = velocity of the object relative to the observer (m s^{-1})

c = speed of light (3×10^8 m s^{-1})

Syllabus link
Chapter 14 of *Nelson QCE Physics Units 1 & 2* discusses momentum and impulse.

WORKED EXAMPLE 11.7.4.

What is the relativistic momentum of an electron whose speed is measured to be 2.0×10^8 m s^{-1}? Rest mass of an electron is 9.109×10^{-31} kg.

ANSWER

1 Determine the formula.

$$p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

2 Substitute the known values.

$$p_v = \frac{9.109 \times 10^{-31} \times 2.0 \times 10^8}{\sqrt{\left(1 - \frac{(2.0 \times 10^8)^2}{(3.0 \times 10^8)^2}\right)}}$$

3 Calculate the answer.

$$p_v = \frac{1.822 \times 10^{-22}}{0.745}$$

$$= 2.446 \times 10^{-22} \text{ N s}$$

LEARNING CHECK 11.7

DESCRIBING

1 Write the equations for time dilation and length contraction.

2 Write the equation for determining relativistic momentum.

APPLYING

3 A spacecraft has a proper length of 50 m. It travels past an observer at 0.6*c*. What is its length in the observer's frame of reference?

4 A train carriage travels at 0.75*c*. To an observer outside the carriage, it fits exactly between two markers, P and Q. To an observer on the train, does the carriage appear to fit exactly between P and Q? If not, what can you conclude about events in the different frames of reference?

5 What is the relativistic mass of a proton travelling at 0.65*c*, if its rest mass is 1.673×10^{-27} kg?

6 A spaceship travels at an average velocity of 0.4*c* to an exoplanet 4.4 light-years away.

  a What is the distance from Earth to the exoplanet in the frame of reference of the spacecraft?

  b What is the difference between the times taken from Earth's perspective and the perspective of the spacecraft?

7 What is the relativistic momentum of a proton whose speed is 0.75*c*?

8 A neutron has a relativistic mass that is 2.5 times greater than its rest mass. What is the speed of the neutron?

9 What is the relativistic momentum of a proton whose speed is measured to be 0.1*c*?

ANALYSING

10 A pion has a mean lifetime of 26 ns in Earth's inertial frame of reference. What is its mean lifetime in the pion's frame of reference if it has velocity $v = 0.75c$?

11.8 Experimental evidence for time dilation and length contraction

Experimental evidence for time dilation

Recall that muons have a mean lifetime of 2.2 μs and travel at close to the speed of light through Earth's atmosphere. This means that the muons are expected to only travel approximately 656 m during their lifetime.

Frisch and Smith measured an average of 563 muons per hour in the upper atmosphere. Assuming that other muons were passing the detector at the same rate, and assuming no muons are created between detectors, the number of muons at the second detector a few kilometres down should be almost zero due to their short lifetimes. In fact, they measured 412 muons per hour – far too many to be ignored and close to their predicted count rate based on the relativistic analysis. Their count could be explained when applying time dilation and length contraction.

This time dilation and length contraction is caused by relativistic effects. This is summarised in **Figure 11.8.1**. An observer on a moving muon (the muon's frame of reference) and an observer on Earth (Earth's frame of reference) will agree on the following about the event:

- relative speed, 0.995*c*
- number of physical decays of muons
- number of elapsed mean lifetimes.

They will disagree on:

- mean lifetime
- distance between the collectors.

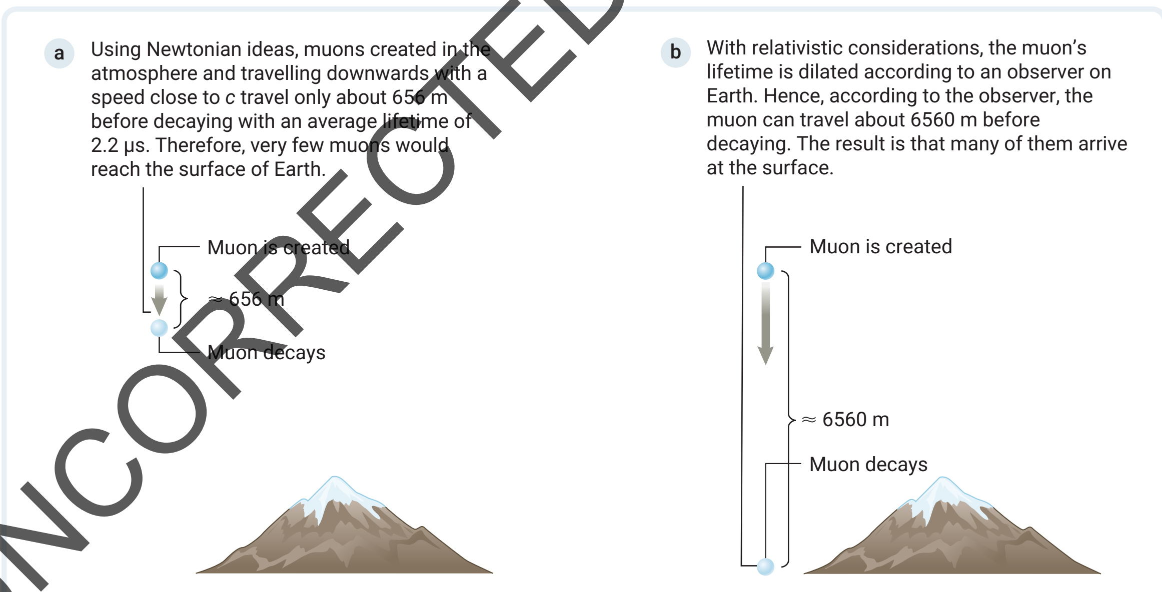


FIGURE 11.8.1 Travel of muons according to an observer (**a**) in the muon's frame of reference and (**b**) as an external observer on Earth

In regard to the time dilation observed of the muon:

$$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$= \frac{2.2}{\sqrt{\left(1 - \frac{(0.995c)^2}{c^2}\right)}}$$

$$= 22.0\ \mu s$$

suggesting that according to an observer on Earth, the muon will not decay until after 22.0 µs and can travel much further, as its half-life has increased by an order of magnitude, $\times 10^1$.

WORKED EXAMPLE 11.8.1

Muons with a mean lifetime of 2.2×10^{-6} s and travelling at 0.999*c* were observed at a height of 10 000 m above Earth (in Earth's reference frame).

a Use Newtonian ideas to calculate the distance travelled by a muon in one lifetime.

b Use Newtonian ideas to calculate the number of mean lifetimes that would elapse before the muon reached the ground.

c Justify why it is unlikely that the muon will reach the ground.

ANSWER

a **1 Determine the formula.**

$s = vt$

2 Substitute the known values.

$s = 0.999 \times 3 \times 10^8\ \text{m s}^{-1} \times 2.2 \times 10^6\ \text{s}$

3 Calculate the answer.

$s = 659$ m

b **1 Determine the formula.**

$$n = \frac{h}{d}$$

2 Substitute the known values.

$$n = \frac{10000\ \text{m}}{659\ \text{m}}$$

3 Calculate the answer.

$n = 15.2$

c One mean lifetime is the time in which the detected muons would be expected to have decayed. Using Newtonian ideas, it is unlikely that the muon would last for 15.2 lifetimes.

Experimental evidence for length contraction

Consider the muon decay phenomenon from a position of length contraction. The time dilation is observed from Earth's reference frame. However, length contraction is observed from the reference frame of the muon. Neither reference frame sees the relativistic effects of both time dilation and length contraction. In fact, they represent an alternative explanation from each frame of reference – one person measures relativistic time dilation and proper length; the other measures relativistic length contraction and a proper time.

The proper length in this scenario is measured from Earth's reference frame. This means that, in the muon's frame of reference, length is contracted due to the muon measuring the proper time. The muon sees the length contracted to the original 659 m in this circumstance.

Applying special relativity formulas

When solving problems in special relativity, it is sometimes difficult to determine the proper time, proper length and rest mass to, in turn, calculate the time dilation, length contraction and relativistic momentum. Proper time and proper length are *not* defined in the same frame of reference. As a general rule, the proper time is defined in the frame of reference of the object moving, and proper length is defined by a stationary observer.

Consider again a moving train, whose length is measured from the front of the carriage, F, to the rear of the carriage, R. The train moves between points A and B. The proper time is the time it takes for the train to get from A to B, as measured by an observer travelling on the train. The proper length of the train is measured by an observer not on the train, as the distance between F and R.

KEY CONCEPT

Proper time is defined in the frame of reference of the object moving.

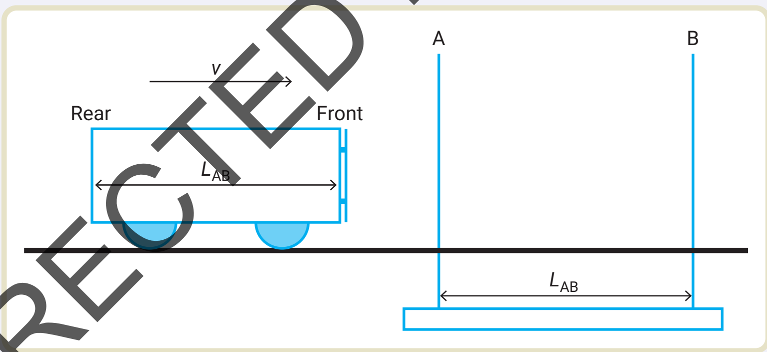
Proper length is defined by a stationary observer.

Rest mass is an intrinsic property of an object. The rest mass does not vary under effects of time dilation and length contraction and can only be measured by an observer in the same frame of reference as the object whose rest mass is being measured. In the case of the moving train, the rest mass would be the mass as measured by the observer on the train.

Once proper time, proper length and rest mass are known, calculating the relativistic effect is simple based on the formulas derived earlier.

WORKED EXAMPLE 11.8.2

A rapidly moving rail cart travelling at speed *v* approaches a pair of markers A and B, as shown.



An observer on the side of the track measures the cart to be L_{AB} in length. As the cart speeds past the markers, the land-based observer notices that the front of the cart coincides with marker B and the rear of the cart coincides exactly with marker A. This means the observer sees these events as simultaneous. In this reference frame, the cart took 1.2 s for the front of the car to move from A to B.

For this scenario, define the proper length and proper time, and state which reference frame will observe time dilation, length contraction and relativistic momentum.

ANSWER

There are two frames of reference to consider: the cart-based observer and the ground-based observer. The cart-based observer will measure the proper time and observe length contraction (as the length L_{AB} measured is moving relative to them). As the cart-based observer is stationary relative to the cart, this is the reference frame in which rest mass can be measured.

The ground-based observer measures the proper length L_{AB}, but the 1.2 s they observe is a dilated (longer) time (as a moving clock runs slowly). As rest mass is observed in the other reference frame, the ground-based observer will also perceive the cart as having a larger, relative momentum.

WORKED EXAMPLE 11.8.3

A spacecraft travels at 0.87*c* and is measured externally to have a momentum of 9.65×10^{10} N s. Determine the momentum of the spacecraft as measured in its reference frame.

ANSWER

The external measurement of the momentum of the spacecraft is 9.65×10^{10} N s. This momentum is relativistic, as the rest mass is measured from the spacecraft's reference frame. Therefore, the spacecraft itself will measure the proper momentum as follows:

1 **Determine the formula.**

$$p = \frac{p_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

2 **Rearrange to find the unknown.**

$$p_0 = p\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

3 **Substitute the known values.**

$$p_0 = 9.65 \times 10^{10}\sqrt{\left(1 - \frac{0.87^2 \times c^2}{c^2}\right)}$$

4 **Calculate the momentum.**

$p_0 = 9.65 \times 10^{10}$ N s $\times 0.493$

$= 4.76 \times 10^{10}$ N s

5 **Determine the formula for rest mass.**

$$m_0 = \frac{p_0}{v}$$

6 **Substitute the known values.**

$$m_0 = \frac{4.76 \times 10^{10}\ \text{Ns}}{0.87 \times 3 \times 10^8\ \text{ms}^{-1}}$$

7 **Calculate the answer.**

$m_0 = 182.28$ kg

LEARNING CHECK 11.8

DESCRIBING

1 Can one reference frame measure both the proper time and proper length?

2 **Explain** the experiment used to verify time dilation in muon decay.

APPLYING

3 Muons are formed 3.0 km above Earth. They travel at 0.996*c* and have a mean lifetime in the muon's rest frame of 2.2 μs. For an observer on a muon, how long does it take for the muon to travel to the ground?

4 Electrons in an electron gun are measured to be about 0.5% more massive than electrons at rest. What is the velocity of the electrons in the electron gun?

5 Two identical clocks are synchronised. One clock is sent off in a spaceship travelling with a speed of 0.7*c*. **Calculate** the time on the spaceship, as observed from Earth after 49 years has elapsed on the Earth clock.

6 A pion, travelling at 0.5*c*, has a relativistically corrected mean lifetime of 26 ns in Earth's frame of reference. **Calculate** the mean lifetime as measured in the pion's reference frame.

7 The pilot of a non-accelerating spacecraft, moving away from Earth at great speed, celebrates the passing of six birthdays. Earth-bound observers measure this elapsed time to be 10 years. Relative to Earth, **calculate**:

a the speed of the spacecraft

b how far the spaceship travels over these 6 years.

11.9 The mass–energy equivalence relationship

To explore the relationship between matter and energy, consider a thought experiment involving the momentum of a photon, $p = mv$, that we know to be $\frac{E}{c}$. Imagine a stationary spacecraft in distant space.

The pilot of the spacecraft is practising aiming with a laser gun that can fire a single photon or a high-energy beam. The single photon is fired towards the rear of the spacecraft (Figure 11.9.1). The photon has momentum $\frac{E}{c}$ (where E is the energy of the photon). Given that

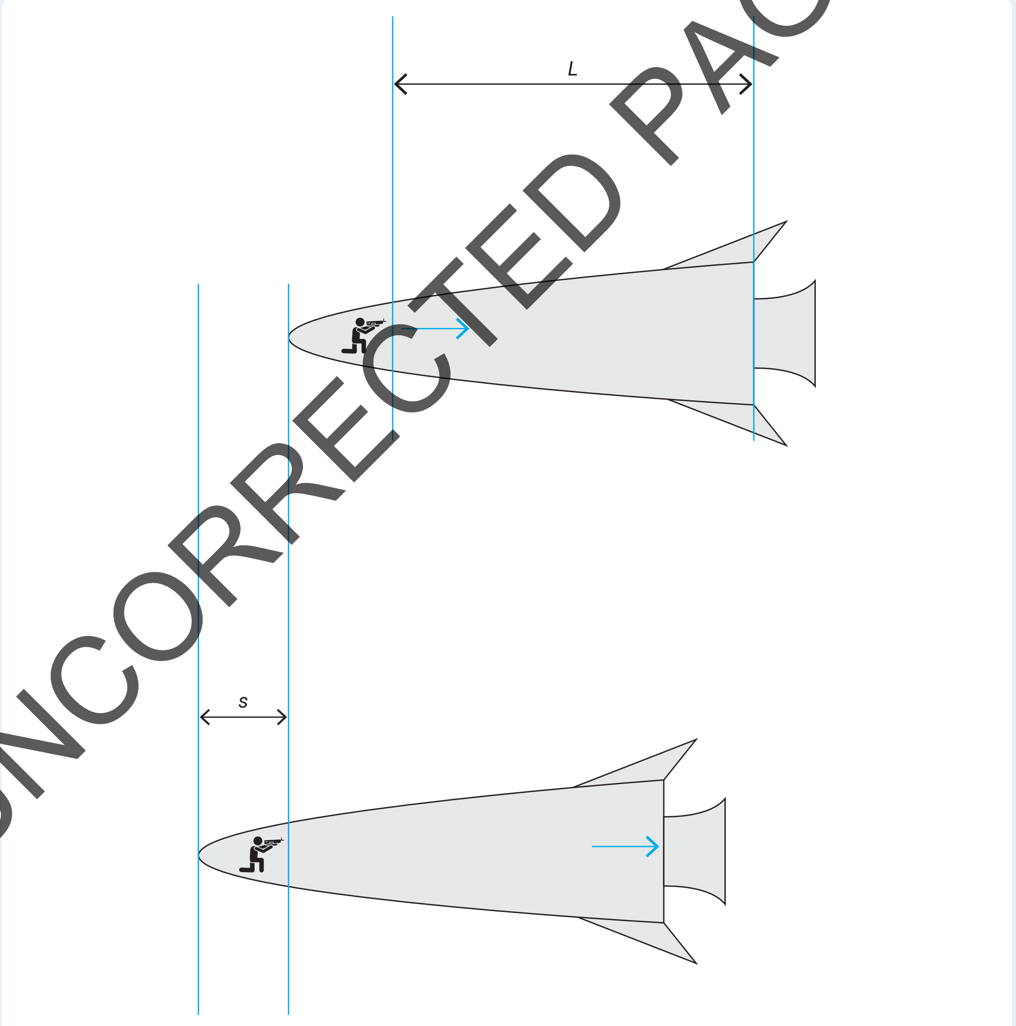


FIGURE 11.9.1 A photon fired towards the rear of a spacecraft. *L* denotes the distance the photon travels; *s* denotes the distance the spacecraft recoils due to conservation of momentum.

momentum is conserved in this isolated system, the pilot and, in turn, the spacecraft (because the pilot is seated in the spacecraft) will recoil with equal but opposite momentum. The distance the spacecraft moves, s, will be equal to vt, the velocity of the craft multiplied by the time the photon takes to reach the rear of the spacecraft.

The magnitude of the momentum the spacecraft receives will be mv, and M is the mass of the spacecraft. Of course, the pilot will probably not notice this movement because of the small size of the momentum of the photon. However, as small as it is, the movement will occur, just as it does when you jump into the air on Earth and Earth recoils with the same momentum. The time it takes for the photon to reach the rear of the craft, t, will be equal to $\frac{L}{c}$, where L is the length of the spacecraft cabin. From this, we can deduce that:

$$Mv = \frac{E}{c}$$

$$v = \frac{E}{mc}$$

where v is the velocity the spacecraft recoils with. As $s = vt$, we can express the distance the spacecraft travels as follows:

$$\begin{aligned} s &= vt \\ &= \frac{E}{Mc} \times t \\ &= \frac{E}{Mc} \times \frac{L}{c} \\ &= \frac{EL}{Mc^2} \end{aligned}$$

As the photon reaches the rear of the spacecraft, the momentum will be transferred, causing the spacecraft to stop. There is no net external force applied to the system; the total momentum of the system has not changed, yet the system has moved. It is the photon that has changed position and caused this redistribution. In this interaction with the photon, the spacecraft has behaved exactly as would be expected if there had been a redistribution of mass (Einstein's 'mass equivalent' of energy hypothesis).

If we suppose the photon to have a relativistically corrected mass, m, then its momentum would be $p = mc$. Thus, the speed of the photon as it travels to the rear of the spacecraft is $c = \frac{L}{t}$. Therefore, we could equate the momentum as follows:

$$\frac{Ms}{t} = \frac{mL}{t}$$

$$s = \frac{mL}{M}$$

KEY FORMULA

Mass–energy equivalency

$E = mc^2$

where:

E = change in energy equivalent to the change in mass (J)

m = change in mass (kg)

c = speed of light (3×10^8 m s^{-1})

We now have two equations for s, and equating these we obtain:

$$\frac{EL}{Mc^2} = \frac{mL}{M}$$
$$\Rightarrow E = mc^2$$

Of course, this thought experiment does not necessarily prove anything. However, the above equation has been derived in a variety of situations and, more importantly, has been successfully tested by experimentation.

The energy associated with mass at rest is called the **rest energy** of the mass and is given more specifically by the equation $E = m_0c^2$. Mass and energy are equivalent, as mass is a manifestation of energy.

rest energy defined from the rest mass by $E = m_0c^2$

The relativistic total energy of an object that is moving is defined by:

total energy = rest energy + relativistic kinetic energy

where the **relativistic kinetic energy** (specifically the change in the relativistic kinetic energy) is related to a change in mass as follows:

relativistic kinetic energy defined from the rest mass by $\Delta E_k = (\gamma - 1)m_0c^2$

$$\begin{aligned}\Delta E_k &= \Delta mc^2 \\ &= (m - m_0)c^2 \\ &= mc^2 - m_0c^2 \\ &= \gamma m_0c^2 - m_0c^2 \\ &= (\gamma - 1)m_0c^2\end{aligned}$$

where γ is the reciprocal of the Lorentz factor $\frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$.

WORKED EXAMPLE 11.9.1

An electron is accelerated from rest by a potential difference of 500 kV.

a What speed does the electron obtain?

b What speed would the electron attain if non-relativistic mechanics were used?

ANSWERS

a 1 **Determine the formula.**

Equate $\Delta E = qV$ to $\Delta E_k = (\gamma - 1)m_0c^2$

$qV = (\gamma - 1)m_0c^2$

2 **Rearrange to find the unknown.**

$$\gamma = \frac{qV}{m_0c^2} + 1$$

3 **Substitute the known values.**

$$\gamma = \frac{1.6\times10^{-19}\ \text{C}\times 500\,000\ \text{V}}{(9.1\times10^{-31}\ \text{kg})(3\times10^{8}\ \text{m s}^{-1})^2} + 1$$

4 **Calculate the value for γ.**

$\gamma = 1.9784$

5 Determine the relationship between γ and v.

Now, $\gamma = \dfrac{1}{\sqrt{\left(1-\dfrac{v^2}{c^2}\right)}}$, and v can be found as follows:

$$\gamma = \frac{1}{\sqrt{\left(1-\dfrac{v^2}{c^2}\right)}}$$

$$\gamma^2 \times \left(1-\frac{v^2}{c^2}\right) = 1$$

$$\gamma^2 c^2 - c^2 = v^2\gamma^2$$

$$v = \sqrt{\frac{\gamma^2 c^2 - c^2}{\gamma^2}}$$

6 Substitute the known values.

$$v = \sqrt{\frac{1.9784^2(3\times10^{-8})^2 - (3\times10^{-8})^2}{1.9784^2}}$$

7 Calculate the answer.

$v = 2.59 \times 10^8$ m s^{-1}

b 1 Determine the formula.

Non-relativistically:

$$qV = \frac{1}{2}mv^2$$

2 Rearrange to find the unknown.

$$v = \sqrt{\frac{2qV}{m}}$$

3 Substitute the known values.

$$v = \sqrt{\frac{2\times(1.6\times10^{-19}\,c)\times 500000\,\text{V}}{9.109\times10^{-31}\,\text{kg}}}$$

4 Calculate the answer.

$v = 4.19\times10^8$ m s^{-1}

ACTIVITY 11.9.1

THE AUSTRALIAN SYNCHROTRON

Introduction

The storage ring at the Australian Synchrotron has a radius of about 355 m but it should fit on a kitchen table, from the viewpoint of the particles inside it. The reason has to do with Einstein's relativity. Electrons travel in the storage ring **(Figure 11.9.2)** at very near the speed of light. For the 3 GeV Australian Synchrotron, electrons travel at 99.999 995% the speed of light. This means that their masses must be relativistically corrected. Relativistic corrections become necessary when the speed of the electron is about 0.1c.

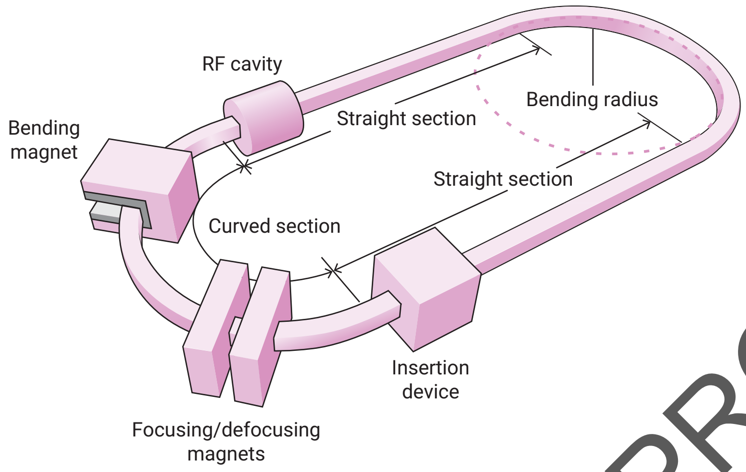


FIGURE 11.9.2 A synchrotron storage ring comprises straight and curved sections.

Electrons do not follow a perfect circular path. They travel along straight sections and are then subjected to curved sections. When a 3 GeV electron is affected by bending magnets that produce a 1.3 T field, it follows a circular path. The radius of the path is larger than that of a kitchen table. In the straight sections, it is affected by insertion devices called wigglers and undulators. These produce radiation in the nanometre range. This is a result of the combined effect of the relativistically corrected Doppler shift and length contraction in the electron's frame of reference.

Questions

1 For the Australian Synchrotron what is:
  - **a** the maximum energy available?
  - **b** its overall radius?
  - **c** the speed of the electrons?
  - **d** the magnitude of the magnetic field produced by the bending magnets?
  - **e** the value of the Lorentz factor when the electron is travelling at 0.1c and maximum speed? (Hint: You will need to use a value for c that has enough significant figures.)

2 For the electrons in the storage ring at the Australian Synchrotron, if $\beta = \frac{v}{c}$, what is the value of:
  - **a** β?
  - **b** v?

3 Copy and complete Table 11.9.1 to compare the kinetic energy and momentum for an electron travelling at 0.999 according to both classical and relativistic physics.

TABLE 11.9.1 Comparisons of classical and relativistic energy and momentum

	Kinetic energy (J)	Momentum (N s)
Classical		
Relativistic		

4 On a single set of axes, plot kinetic energy versus β for kinetic energies up to 1000 keV for both classical and relativistic energies. Use a spreadsheet to produce values of v from values of β; hence, find γ and E_k. Compare and contrast these two graphs.

Weblink
The Australian Synchotron

The ultimate speed limit

Only light, massless photons with high energy can travel at the speed of light, *c*. Although these photons are massless, they still have intrinsic momentum capable of having effects on matter with mass. In the scenarios we have dealt with so far, all particles have been travelling at close to the speed of light, but not at the speed of light. This raises the question: Can particles with mass travel at this ultimate speed limit?

Consider a particle with mass travelling through a vacuum. There is no friction. For this particle to increase its speed, it must accelerate. For order acceleration to occur, an unbalanced force must be applied to the particle, and in for a force to be exerted on the particle, work must be done. In other words, for a particle to move with higher velocity, more energy must be applied. Additionally, the heavier the particle is, the more energy will be required to accelerate it.

Now consider the particle's rest mass m_0. As the particle begins to travel at velocities closer and closer to the speed of light, the mass of the particle becomes relativistically corrected. An outside observer who is applying energy to the particle to make it go faster now observes the particle getting heavier. No matter how hard they try, as they apply energy to cause the particle to travel faster and faster, the particle will become heavier and heavier in proportion to this increased energy. This leads to an impasse: applying more energy to increase the velocity of an object that is already travelling at close to *c* will increase its relative mass, causing the energy input for acceleration to be negated. Quite simply, for an object with mass to travel at the speed of light, it would have infinite mass. Due to the mass–energy equivalency, to move infinite mass would require infinite energy. An impractical reality!

LEARNING CHECK 11.9

DESCRIBING

1 State the mass–energy equivalency equation.

2 **Explain** why we need to understand the mass–energy equivalency to solve problems of special relativity.

APPLYING

3 An electron gains 600 keV of energy in an electron gun. What speed does the electron attain?

4 A positron is the antiparticle of an electron. Both have a rest mass of 9.109×10^{-31} kg. When an electron and a positron collide, they annihilate each other and produce energy in the form of electromagnetic radiation. How much energy is released by a collision of a positron and electron?

ANALYSING

5 The ultimate speed is *c*, but there does not appear to be an ultimate kinetic energy, $E = mc^2$. **Explain** this apparent contradiction.

6 **Explain** with reference to relativistic effects why no object can travel at the speed of light in a vacuum.

11.10 Paradoxical scenarios

A paradox is a statement that negates itself. In other words, it presents a scenario that is self-contradictory in that it is both true and cannot be true at the same time. A famous example is the grandfather paradox. In this scenario, a person travels back in time and kills their grandfather before the conception of their father or mother. This presents a contradiction in that if the grandfather were dead, the time traveller would never have been born to begin with and so could not have gone back in time to kill their grandfather. From this, several questions are presented, and many are debated.

Not unlike the grandfather paradox scenario, special relativity presents its own paradoxes to do with time and space. Both time dilation and length contraction result in realities that are seemingly contradictory but can be explained by relativistic effects. These include thought experiments known as the twins paradox, the flashlights on a train paradox, and the ladder in the barn paradox.

Twins paradox

Identical twins look the same, their DNA is the same (they are from a single, split, fertilised egg) and they age with time in the same way, assuming all factors are constant. Consider identical twins Hannah and Sophie. Hannah makes a journey into outer space in a high-speed spacecraft, while Sophie remains on Earth. When Hannah returns to Earth, she observes that Sophie has aged more.

This poses a problem: each twin sees the other moving relative to their own reference frame and, so paradoxically, each twin should have found the other to have aged less. For example, it can be argued that the clocks on Earth are running slow, or the clocks on the spacecraft are running slow, depending on where you measure proper time. It depends on whose perspective you take for the scenario, Hannah's or Sophie's.

Solution to the twins paradox

The easiest way to think about the scenario is to first consider the journey of the travelling twin. Hannah's spacecraft needs to change direction at some point in order to arrive back on Earth. This means there are two inertial frames of reference for Hannah, as we cannot consider the whole journey as one reference frame because of the change in velocity (acceleration). So we now consider that while Sophie is at rest in the same inertial frame throughout the entire journey, Hannah's frame switches from being at rest to a velocity away from Earth, to being at rest and again with a velocity towards Earth.

Consider the first half of the journey in which Hannah is headed outbound on the spacecraft. The twins align their clocks, and Hannah heads off at a constant velocity close to the speed of light, say $0.8c$. Hannah travels to a planet that is measured to be 10 light-years away from Earth. In Hannah's reference frame, length is contracted, and she measures only 6 light-years between Earth and the planet. This means Hannah measures the trip to the planet to take 7.5 years $\left(\frac{6 \text{ light-years}}{0.8c}\right)$, whereas Sophie would measure the time taken for this part of the journey to be 12.5 years $\left(\frac{10 \text{ light-years}}{0.8c}\right)$.

This has taken the time measurement from each perspective based on length contraction but has not considered how each twin reads the other's clock. When she reaches the planet, Hannah's clock reads that 7.5 years have gone by, but when this event is observed by Sophie,

Sophie's clock does not read that 12.5 years have gone by. Rather, from Sophie's perspective, it takes 12.5 years for Hannah to get to the star, plus an additional 10 years for the light to travel back to Earth for Sophie to observe the event. Hence Sophie's clock will indicate that 22.5 years have gone by since Hannah left Earth.

When Hannah reaches the star, she sees Sophie's clock as it was 10 years ago, and hence Sophie's clock appears to be the one running slow from Hannah's reference frame: it reads 12.5 – 10 = 2.5 years. So Hannah and Sophie are each observing the other's clock as running slow, from their reference frame.

This is where the paradox comes in, but it is resolved when considering the return journey. When Hannah travels back to Earth, Sophie views Hannah's clock as changing from 7.5 years to 15 years in just 2.5 years' time. This is because Sophie's clock goes from reading 22.5 years to 25 years from the time it took to observe Hannah reaching the planet to when Hannah arrived back on Earth. From Hannah's perspective, the return journey took 15 years (7.5 years each way), and it was not until after returning to Earth that Sophie's clock could be observed.

Resolution

The conclusion is that Sophie, the Earth-bound twin, will have a faster clock, and hence she will have aged more. As the twins are reunited, and the length contraction and time dilation are observed from both frames of reference, less time will have elapsed on Hannah's (the travelling twin) clock. This solution only works when considering the travelling twin as moving at speeds close to c, to a very long distance away, and then returning. The travelling twin undergoes acceleration and deceleration. She thus has periods of not being in an inertial reference frame. The apparent paradox is that the travelling twin may make the same claims as the Earth-bound twin, expecting the Earth twin to age less. This opposes both the predictions and experimentation. If there was no return, there would be no way to determine whose clock was in fact 'correct'. Of course, no one has ever attempted this with people; however, it has been shown with atomic clocks, and less time has been shown to pass for the travelling clock than the stationary clock on Earth.

Flashlight on a train paradox

Consider a moving train. There are two people, one at each end of a carriage, and an observer on the platform. The train has a length L and moves between markers A and B with velocity v, from west to east. The observer on the platform is standing aligned with marker B. If the person at the rear of the carriage shines a flashlight at the person standing at the front of the carriage, an observer on the carriage sees the light travelling at $v = c$. The person on the ground observes the velocity of the light as travelling at $v + c$. This is paradoxical, as nothing can travel faster than c – all observers measure the speed of light as c.

Resolution

Einstein's second postulate states that the speed of light has the same value, c, in all inertial frames. It does not depend on the speed of either the source or the observer. Therefore, the flashlight on a train scenario is not a simple relative motion problem, as the speed of light is invariant. This problem is solved by considering the laws of special relativity, not the laws of Newtonian mechanics or Galilean relativity. To the stationary observer on the platform, events on the train occur in a frame of reference where time runs more slowly (time dilation), and the trains length is contracted. These relativistic effects ensure that both observers measure the speed of light as c, thus there is no paradox.

Ladder in the barn paradox

Consider a ladder with length L_L and a barn with length L_B with big doors on opposite ends (Figure 11.10.1). When at rest $L_L > L_B$; that is, the ladder does not fit in the length of the barn. Consider the ladder moving towards the barn at very high, relativistic speed through the open barn doors. From length contraction, the ladder will be able to fit inside the barn and there will be a brief moment when both doors could be shut and the ladder can be contained within, from an observer in the barns frame of reference. However, if the reference frame of the ladder is considered, it is the barn that is moving, and the barn that is contracting in length to an even smaller size. This paradox arises from a misunderstanding of simultaneity.

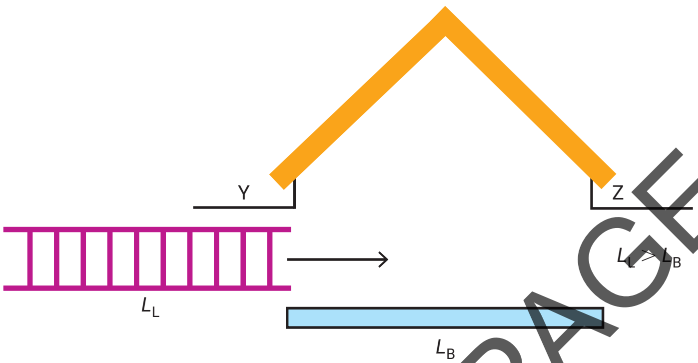


FIGURE 11.10.1 The ladder in the barn paradox, where $L_L > L_B$ when both the barn and ladder are at rest.

Resolution

This paradox is resolved by understanding the relativity of simultaneity. According to special relativity, events cannot occur simultaneously in both reference frames when the frames of reference have very high relative speeds. If the scenario is considered from the perspective of the ladder moving and the barn stationary, there will be a length contraction of the ladder, and for a moment the ladder will be able to fit in the barn and both the front and the rear of the ladder are inside the barn simultaneously. The barn doors can close momentarily at both ends and trap the ladder inside the barn. However, if the situation is considered in which the barn is moving and the ladder is stationary (the ladder's reference frame), the barn will undergo a length contraction, and even less of the ladder will be able to fit into the barn than previously. The ladder is not contracted in its own rest frame, so it cannot fit inside the shorter barn.

Now, consider the doors Y and Z on either end of the barn are closed for a brief period, in the frame in which the ladder is moving (contracted). This is the event as observed by a stationary observer in the barn (where the ladder is moving and the barn is not). Consider what would happen from the ladder's perspective. As the ladder approaches door Z (Figure 11.10.1), it closes but opens in time for the front of the ladder to head out. Later, the back of the ladder passes through door Y, which closes then opens again (still in the reference frame of the barn moving). At no point from this reference frame did the ladder need to fit inside the barn, but each door could still close while the ladder moved through, as simultaneity is relative.

In both reference frames, the ladder can fit inside the barn so that its doors can close, but this does not happen simultaneously in both frames of reference. The doors, from the frame of reference of the ladder, do not open and close at the same time. However, they do close at the same time according to the outside observer. Hence, there is no paradox because both perspectives are correct within their respective frames of reference.

LEARNING CHECK 11.10

DESCRIBING

1 **Explain** each of the following paradoxical scenarios.

  a Flashlights on a train
  b Ladder in the barn
  c Twins paradox

2 The effect of relativistic impacts may be seen in the Lorentz factor, γ.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

  a Complete the following table with correct Lorentz factor values.

Velocity (fraction of *c*)	0.01	0.10	0.20	0.50	0.90	0.99
Gamma (γ)						

  b State what happens to γ as v approaches c.

APPLYING

3 The twins paradox illustrates the effects of time dilation. Alex and Sam are twins. Alex takes an extraterrestrial journey that takes him to a distant stellar system and then returns. The average speed of Alex's spacecraft is 0.6*c*. Sam remains on Earth. When Alex returns, Sam is 20 years older. How much has Alex aged according to his space watch?

4 Abana, Zuri and Imani are triplets. Abana pilots a spacecraft away from Earth towards the centre of the Milky Way for a distance of 7 light-years relative to Earth. Imani pilots a similar spacecraft in the opposite direction, away from the centre of the Milky Way. Her craft also travels outwards for a distance of 7 light-years. Their average speed is 0.61*c*. Zuri remains on Earth. Both Imani and Abana return at the same time.

  a Will Abana and Imani appear to be the same age on their return?
  b How much older than the travellers will Lily be when the travellers return?

CHAPTER SUMMARY

Newtonian physics

- Newtonian physics (also known as classical mechanics) describes the motion of objects and the forces acting on them using principles such as Newton's three laws of motion and the law of universal gravitation.
- These principles can be effectively applied to the motion and forces of objects at relatively low speeds but are inadequate to accurately describe phenomena at speeds close to the speed of light.

Special relativity

- When objects travel at speeds close to the speed of light, they experience relativistic effects where time and space are measured differently.
- A frame of reference is a framework in which the motion of an object can be described according to a coordinate system.
- An inertial frame of reference is either at rest or moving with a constant velocity.
- Motion is always described relative to a specific frame of reference, meaning the observed event/movement depends on the observer's viewpoint.

Postulates of special relativity

- Relativity principle – the laws of physics are the same in all inertial frames of reference.
- The speed of light has the same value, m s^{-1}, in all inertial frames, and does not depend on the speed of the source or observer.

Simultaneity

- What can be considered simultaneous (two events occurring simultaneously) in one inertial frame of reference, may not be considered simultaneous in another inertial reference frame.
- Imagine two observers: one standing on a train platform and another inside a train moving past the platform at relativistic speeds. A light is switched on at the centre of the train carriage. The observer on the train sees the light hit the front and back of the train carriage simultaneously. However, the platform observer sees the light hit the back of the train carriage first, then the front. This difference arises because, with the train in motion and the speed of light constant, the distance between the light and the back of the carriage is shortened while the length between the light and front of the carriage is extended.

Consequences of speed of light in a vacuum

- Time dilation is the phenomenon where time appears to pass more slowly for an object in motion relative to an observer at rest.
- Proper time interval (t_0) is the time measured by an observer at rest relative to the event being timed, i.e. in the frame of reference travelling at speed.
- The relativistic time interval (t) is the time measured by an observer in a different frame of reference.
- Length contraction is the phenomenon where an object in motion appears shorter along the direction of motion when observed from a stationary frame of reference.

- Proper length (L_0) is the length of an object measured by an observer who is at rest relative to the object.
- Relativistic length (L) is the length measured by an observer in a different frame of reference.

Time dilation

- $$t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

  where:

  t = dilated time, measured in the frame of a moving observer (s)

  t_0 = proper time, measured by an observer in the inertial reference frame (s)

  v = velocity of the inertial reference frame (m s^{-1})

  c = speed of light (3×10^8 m s^{-1})
- Muons have a mean lifetime of 2.2 μs and travel at close to the speed of light through Earth's atmosphere.
- According to Newtonian physics, muons are therefore only expected to travel 656 m during their lifetime.
- However, the number of muons lower down in the atmosphere, compared to the number in the upper atmosphere, is much higher than expected.
- Time dilation is observed from Earth's reference frame which has caused their mean lifetime to increase to around 22 μs, providing reason for the high abundance of muons in the lower atmosphere.

Length contraction

- $$L = L_0\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

  where:

  L = length of the object measured by an observer who is moving at constant velocity relative to the object's inertial frame (m)

  L_0 = proper length, the length of the object at rest (m)

  v = velocity of the object relative to the observer (m s^{-1})

  c = speed of light (3×10^8 m s^{-1})
- In the muon decay phenomenon, length contraction is observed from the reference frame of the muon, allowing it to travel further into the atmosphere.

Rest mass

- The mass of an object is relative and depends on its speed.
- Rest mass (m_0) is the mass measured when the object is stationary in an inertial reference frame.
- Relativistic mass is the measurement of its mass in a reference frame moving in relation to a stationary frame.

- As the velocity of an object approaches the speed of light, the mass of the object will greatly increase.

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where:

m = rest mass (kg)

m_0 = relativistic mass (kg)

v =speed of the moving inertial frame of reference (m s^{-1})

c = speed of light (3×10^8 m s^{-1})

- If v is greater than c, the equation would result in the square root of a negative number, which is invalid. This implies the speed of an object with non-zero rest mass cannot be equal to, or exceed, the speed of light.

Relativistic momentum

- The momentum of a particle due to relativistic motion.

$$p_v = \frac{p_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where:

p_v = relativistically corrected momentum (N s)

p_0 = momentum calculated from the rest mass in (N s)

v = velocity of the object relative to the observer (m s^{-1})

c = speed of light (3×10^8 m s^{-1})

- As approaches, the object appears to 'gain mass' from the perspective of an external observer, meaning it requires more energy to continue increasing its speed. Further, when approaches, the denominator of the relativistic momentum equation approaches 0, making the momentum approach infinity. So, infinite energy is required to accelerate the particle to reach or exceed the speed of light.

Mass–energy equivalency

- Mass can be converted into energy and vice versa.
- This explains why objects with mass require infinite energy to reach the speed of light as increasing speed means increasing energy and, consequently, mass.

$E = mc^2$

where:

E = change in energy equivalent to the change in mass (J)

m = change in mass (kg)

c = speed of light (3×10^8 m s^{-1})

The twins paradox

- Imagine a scenario where there are two twins: one twin stays on Earth while the other travels to a distant star, at relativistic speeds, then returns to Earth.
- At first glance, the situation seems symmetrical – each twin should see the other twin as travelling, so, each should expect the other twin to age more slowly.
- However, the travelling twin ages less.
- This is due to the turning point of the travelling twin's journey, which involves acceleration, impacting the symmetry of the scenario.

The flashlight on a train paradox

- Imagine a train moving at relativistic speeds when a flashlight is turned on at the centre of the train, emitting light towards both the front of the train, and towards the back.
- An observer on the train sees the light hitting both ends of the carriage simultaneously while an observer on a platform next to the train tracks sees the light hitting the back of the train first and then the front.
- Special relativity resolves this by acknowledging simultaneity is relative to the observer's reference frame.
- As the speed of light is constant, the observer on the train sees the light hitting both ends of the train simultaneously, as the distance is equal. However, due to the train's motion, the light appears to travel a shorter distance to the back of the train and a longer distance to the front for the observer on the platform.

Ladder in a barn paradox

- Imagine a ladder moving at relativistic speeds towards a barn. When the ladder is inside the barn, the doors are closed momentarily. The ladder is longer than the barn at rest.
- From the barn's perspective, the ladder moves at a high speed, causing it to contract in length. So, the ladder appears shorter and can momentarily fit within the barn when both doors are closed simultaneously.
- From the ladder's perspective, the barn moves at a high speed, causing the barn to contract in length. So, the ladder is still longer than the barn and at no point does the entire ladder fit inside the barn simultaneously. The ladder sees the front and back doors close one by one, then open again, allowing the ladder to not touch the barn doors, but travel through the barn.
- The paradox is resolved by understanding the relativity of simultaneity. Both perspectives are valid within their own frames of reference, showing how events perceived as simultaneous in one frame may not be in another.

CHAPTER EXAM

MULTIPLE CHOICE

1. Which of the following is one of the postulates of special relativity?

A Particles can move at the speed of light.
B Proper length and proper time are measured in the same frame of reference.
C The speed of light is the same in all frames of reference.
D The speed of light changes depending on the frame of reference.

2. Ike observes a clock 3 light-seconds away. Ike is at rest relative to the clock. Which statement is true about the time Ike observes?

A Ike observes the clock as showing the same time as the watch on his wrist.
B Ike observes the time on the clock as different but by an uncertain amount.
C Ike observes the time as 3 s faster than his watch.
D Ike observes the time as 3 s slower than his watch.

3. A particle is travelling at close to *c* in space, and a floating, stationary observer watches it go by. Which of the following must be true?

A The particle measures the proper time in its reference frame.
B The particle measures the proper length in its reference frame.
C The particle measures the rest mass in its reference frame.
D The particle is massless.

4. A train is moving east at 6 km h^{-1} and passes a train heading west at 2 km h^{-1}. According to the eastbound train, what is the oncoming train travelling at?

A 2 km h^{-1}
B 4 km h^{-1}
C 6 km h^{-1}
D 8 km h^{-1}

5. What observation suggests classical physics cannot fully explain the presence of muons in the atmosphere?

A Muons are created only in deep underground labs.
B Muons decay too quickly to reach Earth's surface based on classical predictions.
C Muons have infinite rest mass.
D Muons travel faster than light in the atmosphere.

6. What distinguishes an inertial frame of reference from a non-inertial frame?

A Only inertial frames are in motion.
B Non-inertial frames experience acceleration or deceleration.
C Inertial frames can only exist in space.
D Non-inertial frames always have a velocity of zero.

7. An example of length contraction is:

A a car shrinking in size when observed from a stationary position.
B a spaceship appearing shorter to an external observer when it moves at relativistic speeds.
C a stationary ruler appearing shorter when observed from another frame.
D light travelling shorter distances in space.

8. Why does relativistic momentum increase as objects approach *c*?

A Mass decreases at high speeds.
B Time slows down.
C Energy increases exponentially with velocity.
D Space contracts in the observer's frame.

9. A spaceship travelling at $0.8c$ measures its proper length as 100 m. What is its length as observed from Earth?
   A 100 m
   B 80 m
   C 60 m
   D 36 m

10. If a clock in motion relative to an observer runs at 0.5 times the observer's clock, what is the relative velocity of the moving clock?
   A $0.6c$
   B $0.8c$
   C $0.9c$
   D $1.0c$

SHORT RESPONSE

11. A muon travels close to the speed of light, at $0.99c$, down towards Earth. An observer on the muon notices that it decays after 3 μs.
   a How long does it take for the muon to decay for an observer on Earth?
   b According to the muon, it travels a distance of 700 m before it decays. How far does the observer on Earth see the muon travel before it decays?

12. Barnard's Star in the Milky Way is 6 light-years away from Earth, as measured by a person on Earth. A rocket leaves for Barnard's Star at a speed of $v = 0.65c$ relative to Earth. Assume that Earth and Barnard's Star are stationary relative to each other. According to the frame of reference of the rocket, what is the distance between Earth and Barnard's Star?

13. In a famous experiment to verify the principle of relativity, a US B52 aircraft carrying an atomic clock left Florida and travelled at 600 m s^{-1} with respect to Earth around the equator (refuelling in the air). An identical atomic clock (synchronised with the first) was left at the airport in Florida. How long must the airborne clock travel at this speed before the two clocks will be out of synchronisation by 2 m s?

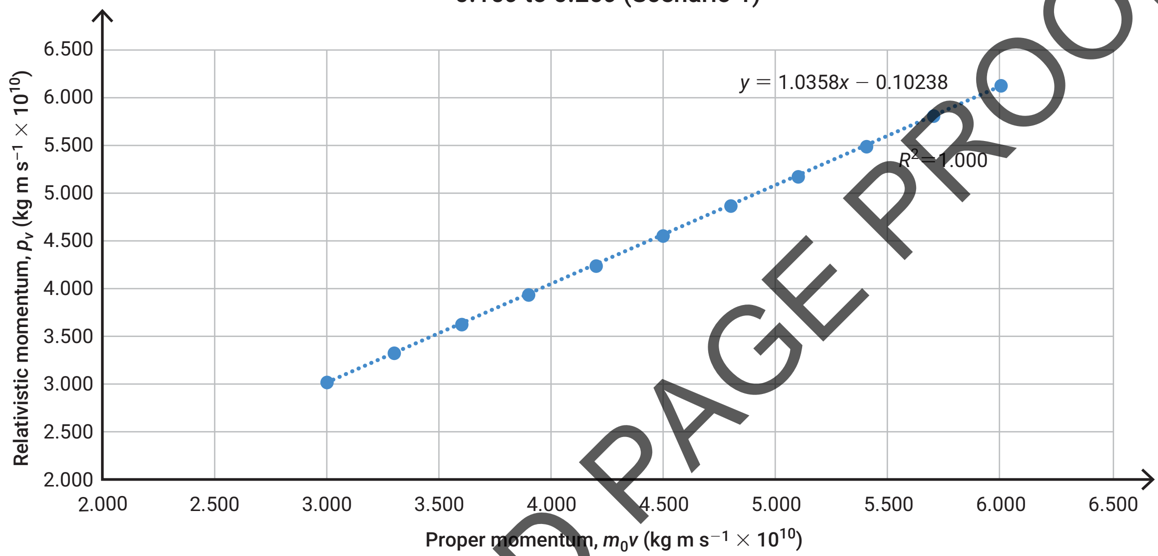
14. Many people think that the Twin Paradox arises because relativity suggests that one twin can become a lot older than the other. Is this the real paradox or is there more to it?

DATA ANALYSIS

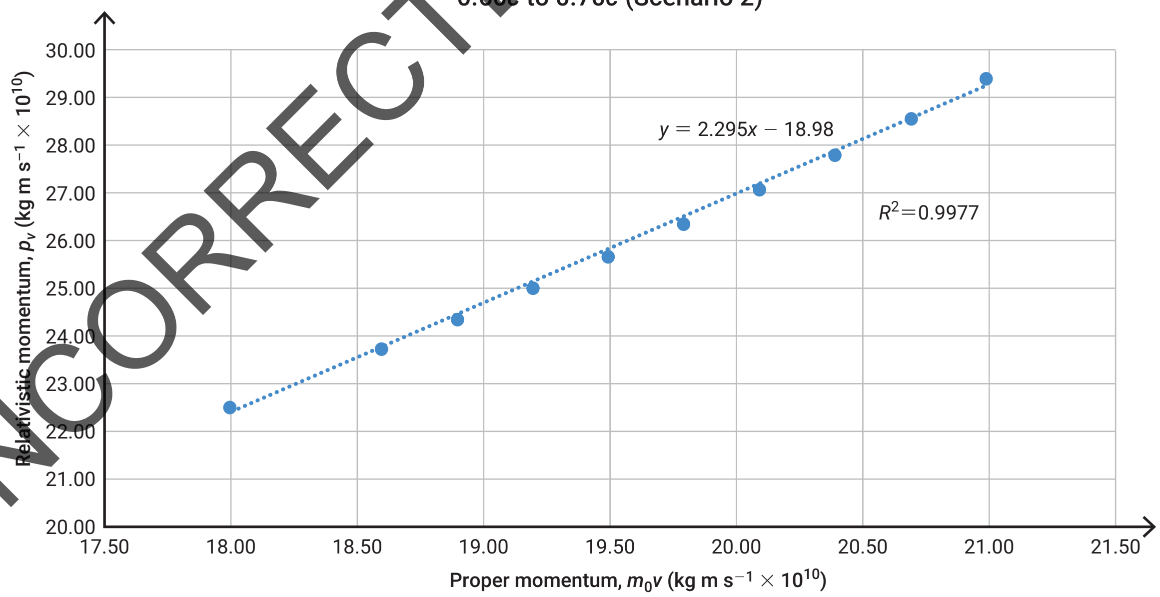
15. **Analyse data**
   A physicist decided to mathematically model the relativistic effects on the momentum of a hypothetical 1000 kg spacecraft that can attain velocities that are a significant fraction of the speed of light. They chose two scenarios:
   - Scenario 1: The spacecraft can attain velocities of $0.10c$ to $0.20c$.
   - Scenario 2: the spacecraft can attain velocities of $0.60c$ to $0.70c$.

Using $p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$ and a plotting program, the physicist constructed the following graphs:

Graph 1. Relativistic momentum according to proper momentum for velocities 0.10*c* to 0.20*c* (Scenario 1)



Graph 2. Relativistic momentum according to proper momentum for velocities 0.60*c* to 0.70*c* (Scenario 2)



Note: Lorentz factor = $\dfrac{1}{\sqrt{\left(1-\dfrac{v^2}{c^2}\right)}}$

a **Determine** the average value of the Lorentz factor for Scenario 1.

b **Determine** the average value of the Lorentz factor for Scenario 2.

c **Explain** why they are different, using your knowledge of special relativity and relativistic momentum

d The plotting program used by the physicist determined that both graphs have a linear line of best fit. **Explain** why the trends appear linear, even though theoretically they should not be.

e **Sketch** what the trend line would look like if a relativistic momentum versus proper momentum graph included a range from low velocity all the way up to 0.99*c*.

SCIENCE AS A HUMAN ENDEAVOUR

Syllabus dot point

- Explore how technologies such as satellites have dramatically increased the size, accuracy, and geographic and temporal scope of datasets with which scientists work.

The physics of satellite technologies in scientific endeavour

Satellite technologies have transformed the way scientists gather and analyse data, enabling a deeper understanding of Earth's complex systems. Here, we will discuss how advancements in satellite technology have expanded the geographic and temporal scope of data collection, and how these innovations are crucial for studying phenomena such as climate change, weather patterns and natural disasters.

Role of satellites in scientific research

Satellites orbiting Earth provide a continuous stream of data on atmospheric conditions, ocean currents, land use, and much more (**Figure 1**). This data is invaluable for studying environmental changes on a global scale, and offers insights that were previously inaccessible. For example:

- **Climate monitoring:** Satellites allow scientists to track long-term climate trends by observing changes in Earth's atmosphere, oceans and ice caps. By monitoring key indicators such as sea surface temperatures, atmospheric composition and polar ice melt, satellites contribute to our understanding of global warming and its effects on ecosystems.
- **Weather prediction:** Geostationary satellites positioned above the same point on Earth's surface provide real-time weather data, making it possible to predict storms, hurricanes and other extreme weather events with increasing accuracy.
- **Natural disasters assessment:** Satellites are essential in disaster response and mitigation. They provide rapid assessments of regions affected by earthquakes, tsunamis, floods or fires, enabling faster and more effective relief efforts.

FIGURE 1 Data obtained by satellites can help monitor Earth's health.

The vast datasets generated by satellites have revolutionised Earth science, allowing researchers to monitor regions that were once difficult or impossible to study.

Physics principles behind satellite orbits and data collection

The orbit of satellites is based on gravitational force, orbital velocity and Kepler's laws. In terms of data collection, electromagnetic waves allow satellites to communicate with ground stations by using radio waves, a form of electromagnetic radiation. By using specific frequencies for communication, satellites transmit data about weather, land use and environmental changes back to Earth without interference.

These principles enable satellites to gather data on vast geographic regions and ensure that the information reaches scientists in real time.

Expanding the scope of data collection

Satellites have dramatically increased the size and scope of datasets available to scientists, enabling global monitoring with unprecedented accuracy. Here are some key ways that satellite technology has expanded our capacity to collect and analyse data.

- **Global coverage:** Satellites provide continuous coverage of the entire Earth, allowing scientists to collect data from even the most remote and inaccessible areas, such as the polar regions and deep oceans.
- **Temporal resolution:** Satellites collect data over extended periods, allowing scientists to track changes over time. The ability to observe long-term trends is essential for understanding processes such as deforestation, glacier retreat and shifts in weather patterns.
- **High-resolution imaging:** Modern satellites can capture incredibly detailed images of Earth's surface, to a resolution of a few metres. This allows for precise monitoring of changes in land use, urban development and even vegetation health.

For example, the Landsat program has been collecting satellite imagery of Earth for more than four decades, and provides one of the longest continuous records of Earth's surface. This enables scientists to study trends in deforestation, urban sprawl and agricultural practices, contributing to more informed environmental policies.

Weblink
Landsat science

The human endeavour behind satellite technologies

The advancements in satellite technology exemplify how scientific progress leads to transformative tools that enhance our ability to study and interact with the world. These technologies, which stem from decades of research in physics, engineering and computer science, are a testament to human ingenuity.

- **Interdisciplinary collaboration:** Satellite missions often involve collaboration between physicists, engineers, environmental scientists and computer scientists. Each discipline contributes to designing, building and operating satellites, as well as to analysing the data they provide.
- **Practical applications:** Beyond scientific research, satellites have practical applications that benefit society. For example, they improve resource management by monitoring water availability, soil health and crop growth, helping to optimise agricultural practices. They also play a critical role in national security, providing information on global military movements and enabling better-informed diplomatic decisions.
- **Impact on society:** Satellites have a direct impact on our daily lives. The weather forecasts we check every morning, the GPS systems we use for navigation, and the internet services we rely on all depend on satellite technologies.

The use of satellite data for climate modelling, environmental protection, disaster response and resource management highlights how scientific advancements serve the broader public good. They are not just tools for scientific exploration; they are also critical resources for decision makers worldwide.

CHAPTER 12 Quantum theory I – the nature of light

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Explain how the double slit experiment provides evidence for the wave model of light.
- Describe light as an electromagnetic wave.
- Explain the concept of black-body radiation and the significance of the evidence it provides.
- Describe wave-particle duality of light by identifying evidence that supports the wave characteristics of light and evidence that supports the particle characteristics of light.

SCIENCE AS A HUMAN ENDEAVOUR

- Consider how theories are contested, refined or replaced when new evidence challenges them, or when a new model or theory has greater explanatory power.

Introduction

Quantum theory was developed when classical models, such as the wave model of light, were unable to explain experimental results.

In this chapter, the nature of light is explored through the study of such experiments and phenomena as Young's double-slit experiment and black-body radiation to identify evidence supporting the dual nature of light as both a wave and as a particle.

Assessments

- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practicals

- The wave nature of [illegible] e-on[illegible] resource)
- Black-body radiatio[illegible] online-only resource)

Worksheets

- Name
- [illegible]ame
- [illegible]ame

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ The frequency (f) of a cyclic or oscillating phenomenon is the number of cycles or oscillations per second; units are hertz (Hz) or s^{-1}.
- ✓ The wavelength (λ) of a wave is the distance over which its shape repeats; units are metres (m).
- ✓ The period (T) of a cyclic or oscillating phenomena is the time it takes for a single cycle or oscillation; units are seconds (s).
- ✓ The relationship between frequency and period is $f = \frac{1}{T}$.
- ✓ The speed of light in a vacuum is a constant: $c = 3 \times 10^8$ m s^{-1}.
- ✓ The wave equation is $v = f\lambda$.
- ✓ Light is an electromagnetic wave consisting of a magnetic field and electric field oscillating perpendicular to one another and to the direction of propagation.
- ✓ The electromagnetic spectrum is a continuum describing all electromagnetic radiation from high energy to low energy.
- ✓ Atoms consist of electrons surrounding a central nucleus made up of protons and neutrons.
- ✓ Elements are distinguished according to their atomic number and hence the number of protons and electrons each atom has.
- ✓ The unit of electric charge is the coulomb (C).

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe the development of quantum theory and the contributions of Roemer, Bradley, Fizeau, Foucault, Michelson, Young, Maxwell, Hertz, Schrödinger, Kirchhoff, Heisenberg and Bohr
- ✓ explain how the double-slit experiment provides evidence for the wave model of light
- ✓ interpret data from the double-slit experiment using $\Delta y = \frac{n\lambda L}{d}$, $\delta = n\lambda$ and $\delta = \left(n - \frac{1}{2}\right)\lambda$.
- ✓ describe light as an electromagnetic wave
- ✓ describe light as consisting of photons
- ✓ describe the wave–particle duality of light by identifying evidence that supports the wave characteristics of light and evidence that supports the particle characteristics of light
- ✓ describe the probabilistic and deterministic aspects of quantum mechanics
- ✓ explain the concept of black-body radiation and the significance of the evidence it provides
- ✓ describe the 'ultraviolet catastrophe' scenario and how it supports quantum theory
- ✓ describe the quantisation of energy according to quantum theory
- ✓ solve problems involving black-body radiation and the photoelectric effect using $\lambda_{max} = \frac{b}{T}$, $E = hf = \frac{hc}{\lambda}$, $E_k = hf - W$, $W = hf_0$ and $\lambda = \frac{h}{p}$.

12.1 The nature of light

Many early scientists conjectured that light had a finite speed. This was first demonstrated by Ole Roemer (1644–1710) at the end of the 17th century and confirmed by astronomer James Bradley (1693–1762) in 1728. Bradley measured the difference between where a star was expected to be seen and where it was actually observed. This **stellar aberration** was due to the finite speed of light and the speed of Earth combining to affect the position from which the starlight appeared to come.

He determined the speed of light to be about 298 000 km s^{-1} (in modern units). This value was later refined in land-based experiments by Fizeau (1849), Foucault (1852) and Michelson (1879, 1883 and 1926). The standard constant speed of light is now accepted as 299 792 458 m s^{-1}; however, for the purpose of our calculations, we will use 3.00×10^8 m s^{-1}.

In 1801, Thomas Young (1773–1829) demonstrated the effects of interference for light. This indicated that light had wave-like properties. Young's method was used to determine the wavelengths of visible light. Later, in the same century, James Clerk Maxwell (1831–79) used calculus-based mathematics to bring all the concepts of electricity and magnetism neatly together into four simple relationships. As well as being a triumph of theoretical physics, it also began the transformation of our understanding of light and matter. Maxwell was a giant of physics. Born in Scotland, he was educated at the University of Edinburgh and Trinity College, Cambridge. At Cambridge, he began his most important work. In statistical thermodynamics, he developed the Maxwell distribution, which predicted the range of molecular speeds in a gas. In optics, he produced the first colour photograph, and he improved our understanding of colour perception and its link to colour deficiency. In astronomy, Maxwell used mathematical modelling to show that the rings of Saturn were most likely made of small rock particles. Maxwell's greatest work was the unification of electric and magnetic theory into electromagnetism. Einstein's reflections on Maxwell's work led to the theory of relativity.

Maxwell's equations were seen as a great triumph of theoretical physics because they unified the two fields of electricity and magnetism. They are as important to electromagnetism as Newton's laws of motion and the law of universal gravitation are to mechanics. Maxwell realised that his latter two equations meant that an electric field produced by a varying magnetic field would itself be varying. Therefore, it would produce a varying magnetic field, and so on.

Maxwell deduced that these waves could move through space with a fixed velocity, *v*. The velocity of the electromagnetic wave only depended on the constants of **magnetic permeability (μ_0)** and **electrical permittivity (ε_0)**. Using modern values, this turns out to be 2.99×10^8 m s^{-1}.

The speed of electromagnetic radiation in a vacuum was the same as that of light. Maxwell argued that this suggested two things.

1. Visible light must be like an electromagnetic wave.
2. The speed of light depends only on the medium.

Initially, scientists were sceptical, but it was not long before Maxwell's view was accepted because of the strength of his mathematical arguments and the accumulation of evidence.

The first experiment to confirm the presence of these electromagnetic waves was conducted by Heinrich Hertz (1857–94) in 1886. Hertz used a high-voltage spark gap to produce some electromagnetic waves that were detected a few metres away. He later showed that these waves could be reflected and refracted and had a speed of 3.0×10^8 m s^{-1}. This was experimental confirmation of predictions based on Maxwell's equations.

We now know that, as well as visible light, the electromagnetic wave spectrum consists of radio waves, microwaves, infrared light, ultraviolet light, X-rays and gamma radiation (Figure 12.1.1). They are all produced by the acceleration of charged particles, as predicted by Maxwell's equations.

stellar aberration the variation in the visible and actual position of a star due to the relative motion of Earth

Weblink
James Clerk Maxwell

Syllabus link
Chapter 10 discusses Maxwell's equations.

magnetic permeability (μ_0) physical property of a medium associated with magnetism; $\mu_0 = 4 \times 10^{-7}$ T mA^{-1}

electrical permittivity (ε_0) a physical property of a medium associated with electricity; $\varepsilon_0 = 8.85 \times 10^{-12}$ N^{-1} m^{-2} C^2 or F m^{-1}

KEY FORMULA

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$= \frac{1}{\sqrt{(8.85 \times 10^{-12})(4\pi \times 10^{-7})}}$$

$$= 2.99 \times 10^8 \text{ ms}^{-1}$$

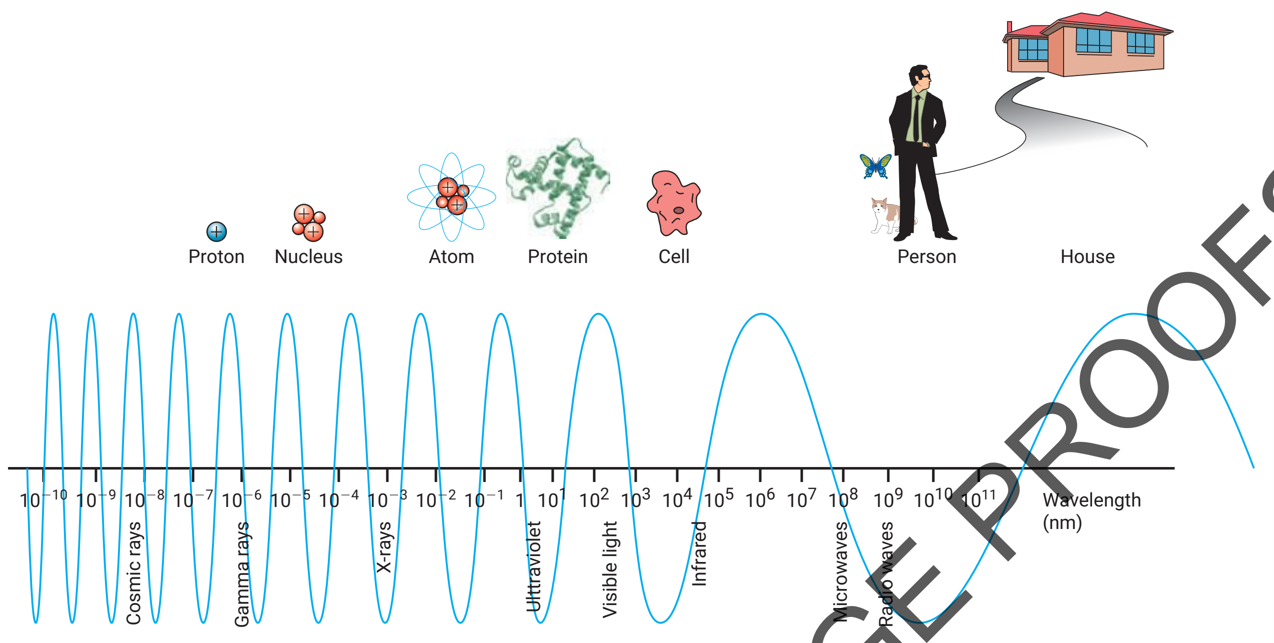


FIGURE 12.1.1 The electromagnetic spectrum. As wavelength increases, energy and frequency decrease. All light in the electromagnetic spectrum travels at c, $3.00 \times 10^8\ \mathrm{m\ s^{-1}}$.

Light as an electromagnetic wave

A scientific model is generally considered successful when it has both explanatory and predictive power. Newtonian mechanics, the study of forces and motion, is a very powerful model that helps us to understand and predict how objects will behave when forces are exerted upon them. The classical wave model, coupled with the electromagnetic field model, explains many of the behaviours of light. Light is an electromagnetic wave and behaves just like other waves – it reflects, refracts, and demonstrates diffraction and interference, just as sound and other mechanical waves do **(Figure 12.1.2)**.

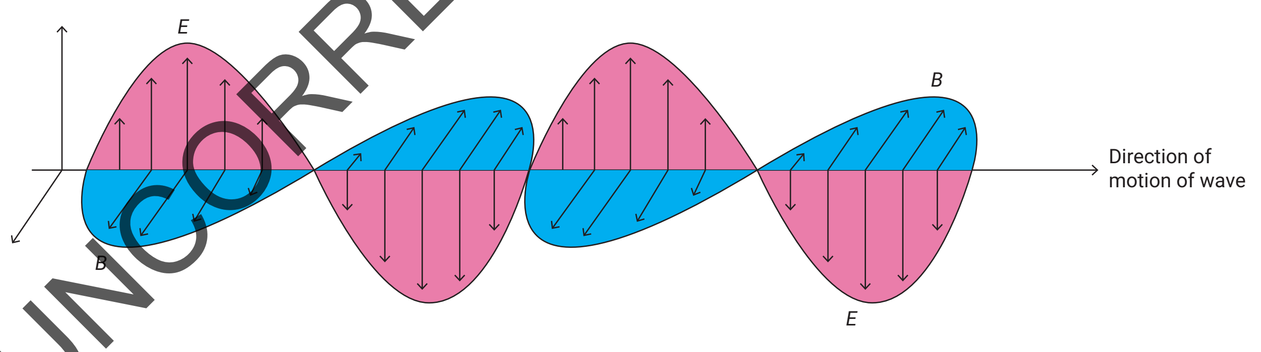


FIGURE 12.1.2 Light travels as a wave with two perpendicular waveforms – the electric field, *E*, and the magnetic field, *B*, which are at 90° to each other. Hence, light is referred to as an electromagnetic wave.

Each scientific model replaces earlier, less successful models. Newtonian mechanics replaced Aristotelian mechanics, and the electromagnetic wave model replaced Newton's particle model of light. For hundreds of years it seemed as if these models could explain the behaviour of all physical systems. However, experiments conducted during the scientific revolution of the 17th century started to provide results that could *not* be explained by these models.

You may wonder why it took so long for these experiments to be conducted. Often in science new developments occur or findings are determined following the development of new instruments. Consequently, new or adapted theories need to be constructed, as such improvements in technology allow for more precise, or different sorts, measurements. Thus, there is a strong relationship between science and technology. If the results of these experiments disagree with the existing models and theories, then new models and theories need to be developed.

Experiments from the late 19th and early 20th century showed results that did not agree with the classical models of light. Gustav Kirchhoff introduced the term **black-body radiation** in 1859, referring to the infrared, visible or higher frequency light emitted by an object due to its thermal energy. The **emission spectrum** released by a black body could not be explained in terms of the classical models of waves. The classical wave model was also unable to explain the **photoelectric effect** or the phenomenon of **atomic spectral lines**. These mismatches between theory and experiment suggested that physicists needed to rethink the very nature of light itself. This was the beginning of **quantum** mechanics.

The development of quantum theory was a major revolution in science. It changed the way in which we understand the behaviour of matter and the nature of the universe. It caused divisions among scientists, and vigorous arguments and debate. There is still debate among physicists as to the interpretation of aspects of quantum theory. However, it has been a spectacularly successful theory, arguably having a greater impact on our daily lives than any other theory in science. The quantum theory led to the development of semiconductors and semiconductor devices, including diodes and transistors. All of our modern information and communication technology is based on these developments, and you use them constantly. Every time you use a computer, smartphone or television, you are using an application of quantum mechanics. It has been estimated that 90 per cent or more of the wealth of the world today is directly related to quantum mechanics!

The oscillating electric and magnetic fields that make up light waves are coupled. As the electric field varies, it creates a varying magnetic field, which in turn creates a varying electric field, and so on, and thus the wave can propagate through empty space.

When a light wave meets a medium other than a vacuum, the electric and magnetic fields interact with the atoms and electrons in the material, slowing the light wave and causing its path to refract and bend. The electromagnetic wave model of light also explains polarisation. Polarisation is a result of the electric field component of light only being allowed to oscillate in one particular plane.

Maxwell's electromagnetic wave model of light successfully predicted and explained all of these behaviours of light and correctly predicted the speed of light. Hence, the wave model of light became the accepted model. It was a very successful model for more than a 100 years, and is still very useful; however, this electromagnetic wave model did not always correctly predict the outcome of experiments. Two experiments in particular showed that a new model was needed. These were experiments with black-body radiation and the photoelectric effect that we will explore later in this chapter.

black-body radiation the electromagnetic radiation emitted by a black body, with a spectrum characteristic of the temperature of the body

emission spectrum the spectrum of radiation emitted by an object (e.g. black-body radiation or atomic spectra from a discharge tube)

photoelectric effect the ejection of electrons from a surface by incident photons of sufficient energy

atomic spectral lines an emission or absorption spectrum consisting of discrete lines, characteristic of the energy levels of a particular atom or molecule; also called a line spectrum

quantum a discrete unit or amount of some physical property, such as energy, charge, mass or angular momentum

LEARNING CHECK 12.1

DESCRIBING

1 **Recall** where you have seen magnetic permeability, μ_0, and electrical permittivity, ε_0, before.
2 **Explain** what the speed of electromagnetic radiation depends on.
3 **Explain** why the wave model of light was questioned.
4 **Identify** two phenomena that the wave–particle model of light can explain.
5 **Sketch** a representation of an electromagnetic wave and describe how the fields interrelate.

APPLYING

6 **Describe** Hertz's experiment and **explain** why it was so important.

ANALYSING

7 Refer to Figure 12.1.1 and **compare** the properties of radio waves and X-rays. How do their frequencies and wavelengths affect their energy?

REFLECTING

8 **Infer** why historically physicists have encountered more difficulties explaining light compared to many other physical phenomena.

9 **Construct** a table that lists two examples to support the wave nature and the particle nature of light.

12.2 Young's double-slit experiment

Weblink
Thomas Young's double-slit experiment

In Newton's time (the 17th century) there were two competing models for light – the 'undulatory' or wave model and the 'corpuscular' or particle model. Newton was a proponent of the particle model while at around the same time Christian Huygens was working on his wave model. In the early 19th century, experiments such as Young's double-slit experiment provided convincing evidence that light acts like a wave.

Young's double-slit experiment is one of many experiments that may be explained by the wave model of light. Light waves refract in a similar way to mechanical waves. Unlike mechanical waves, such as sound and water waves, light waves do not require a medium through which to propagate.

The wave model also correctly predicts the diffraction of light by small apertures and around small objects. In the wave model, light propagates as a wave and, hence, is spread out over a region of space, just as a water wave is spread out over a surface. It is this spreading out or delocalisation that allows diffraction and interference to occur. When more than one light wave exists in a region of space, superposition means the waves can add to give points of constructive and destructive interference.

In Young's double-slit experiment, a coherent light source is shone through a pair of narrow, closely spaced slits. The resulting interference pattern can be observed on a distant screen. The interference pattern of constructive interference (bright bands) and destructive interference (dark bands) provides a pattern of bright and dark fringes **(Figure 12.2.1a)**. This is due to the path difference between the light waves coming from the two different slits.

The path difference is calculated as shown in **Figure 12.2.1b**.

KEY FORMULA

Distance between any two adjacent maxima

$$\Delta y = \frac{n\lambda L}{d}$$

where:
L = distance from the slits (m)
d = slit separation (m)
$L >> d$
λ = wavelength (m)
y = distance between any two adjacent maxima (m)

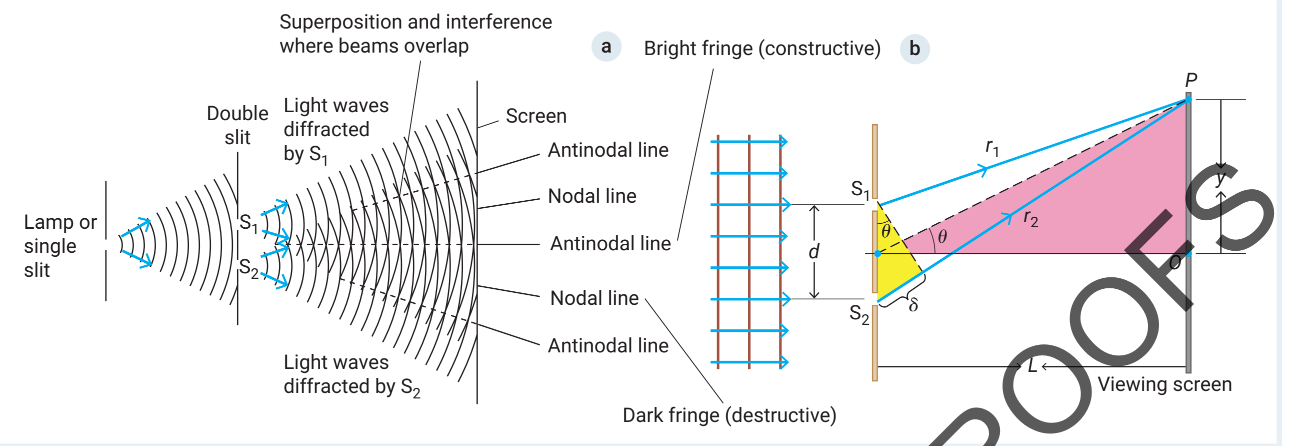


FIGURE 12.2.1 Young's double-slit experiment. (**a**) Bright fringes (constructive interference) occur along antinodal lines, as crests interact with crests, while dark fringes (destructive interference) occur along nodal lines, as crests interact with troughs. (**b**) Bright fringes occur when the path difference, δ, is equal to a multiple of the wavelength of light, i.e. $\delta = n\lambda$. Dark fringes occur when the path difference, δ, differs by exactly one-half of the wavelength and the waves are precisely out of phase, i.e. $\delta = \left(n - \frac{1}{2}\right)\lambda$.

When the screen is a long way from the slits, the two rays, r_1 and r_2, are approximately parallel. The interference pattern produced at the screen is due to the two rays travelling different distances to reach a given point, P, on the screen. The difference in distance travelled from each source slit is called the path difference. **Constructive interference** occurs when crests intersect with crests and troughs intersect with troughs. At these points, there is an antinode or bright fringe in the pattern. This occurs whenever the path difference is equal to a whole number of wavelengths, $\delta = n\lambda$. At the points where a crest intersects with a trough, **destructive interference** occurs, as the waves are always half a cycle out of phase. This results in a node where the path difference, δ, is equal to a multiple of one-half of the wavelength, i.e. $\delta = \left(n - \frac{1}{2}\right)\lambda$.

KEY FORMULA

Path difference for constructive and destructive interference

Path difference for constructive interference (points on an antinodal line):

$$\delta = n\lambda$$

Path difference for destructive interference (points on a nodal line):

$$\delta = \left(n - \frac{1}{2}\right)\lambda$$

where:
δ = path difference (m)
n = number of adjacent maxima from the central maximum (an integer)
λ = wavelength (m)

antinodal lines a region where waves from two sources arrive in phase with crests meeting crests, producing constructive interference, and combining to create a bright fringe

nodal lines a region where waves arrive out of phase with crests meeting troughs, producing destructive interference, essentially cancelling each other out and creating a dark fringe

constructive interference the superposition of waves where crests intersect with crests and troughs intersect with troughs; characterised by antinodes or bright fringes and occurs whenever the path difference is equal to a whole number of wavelengths, $\delta = n\lambda$

destructive interference the superposition of waves where a crest intersects with a trough, due to incoherent wave sources or sources being half a cycle out of phase; results in a node where the path difference, δ, is equal to a multiple of one-half of the wavelength, $\delta = \left(n - \frac{1}{2}\right)\lambda$

WORKED EXAMPLE 12.2.1

A coherent light source of wavelength 450 nm is shone through a pair of slits onto a screen. Determine the path difference from the slits to the third bright maximum.

ANSWER

1 **Determine the formula.**
$\delta = n\lambda$

2 **Substitute the known values.**
A bright fringe indicates a maximum, i.e. a point of constructive interference (an antinodal line). As it is the third bright fringe, then $n = 3$.
$\delta = 3 \times 450 \times 10^{-9}$ m

3 **Calculate the answer.**
$\delta = 1.35 \times 10^{-6}$ m

WORKED EXAMPLE 12.2.2

When light of wavelength 528 nm is incident upon a pair of slits 0.05 mm apart, there is an interference pattern produced on a screen 2.4 m away.

a Determine the distance to the third-order bright fringe from the central maximum.
b Infer how this distance would differ if the wavelength of light was increased.

ANSWERS

a 1 **Determine the formula.**

$$\Delta y = \frac{n\lambda L}{d}$$

2 **Substitute the known values.**

$$\Delta y = \frac{3 \times 528 \times 10^{-9} \times 2.4}{0.05 \times 10^{-3}}$$

3 **Calculate the answer.**
$\Delta y = 0.076$ m

b If the wavelength was increased, the numerator of the equation would increase; hence, the distance between the central maximum and the third-order bright fringe would increase.

LEARNING CHECK 12.2

DESCRIBING

1 **Recall** what result (bright fringe or dark fringe) occurs when two wave crests meet on a screen.
2 **Contrast** a nodal point (dark fringe) with an antinodal point (bright fringe).
3 **Explain** how the wave nature of light allows an interference pattern to form in the double-slit experiment.
4 In Young's double-slit experiment, **explain** what happens to the spacing of the light and dark fringes (increases, decreases or stays the same) if the:
   a wavelength of light is decreased
   b screen is moved closer to the slits
   c space between the slits is decreased.

APPLYING

5 In a double-slit experiment, light of wavelength 630 nm is incident on a pair of slits spaced a distance of 1.5 mm apart. The screen is a distance 2.0 m from the slits. **Describe** the positions of the first three bright spots.

6 In a measurement to find the wavelength of a light source, a viewing screen is placed a distance 4.8 m from a pair of slits with a separation a distance of 0.030 mm. The first dark fringe is a distance of 4.5 cm from the centre line on the screen. **Determine** the:

a wavelength of the light source

b distance between any two adjacent bright spots.

ANALYSING

7 A pair of slits spaced 0.015 mm apart is illuminated with light of two wavelengths at the same time: $\lambda_1 = 630$ nm and $\lambda_2 = 420$ nm. The viewing screen is a distance 3.0 m from the slits. Draw a conclusion about the position on the screen other than $y = 0$ where the maxima from the two interference patterns would first coincide.

12.3 Wave–particle duality of light

wave–particle duality the dual nature of matter and energy, requiring both the wave and the particle model to completely explain all observed behaviour of matter (particles) and energy (light)

Waves and particles

We have seen that if light is shone through two slits, as in a Young's double-slit experiment, an interference pattern is seen. This is shown in **Figure 12.3.1a**. In this experiment, light clearly acts like a wave and produces a pattern of high and low intensity just like a mechanical water wave, such as that shown in **Figure 12.3.1b**.

Experiments such as this successfully supported Maxwell's electromagnetic wave model of light; however, this wave model did not always correctly predict the outcome of experiments, such as with black-body radiation and the photoelectric effect for which light was found to behave as a particle.

So, is light a particle or a wave? The answer is that it acts like both, and the behaviour you see depends on the experiment you conduct. On its own, neither the wave nor the particle model completely explains the behaviour of light. They are *complementary models*; both are needed, and which one is used depends on the situation. We describe this need for both models as the **wave–particle duality**.

The question then arises: What about things that we know to be particles? If light is a wave and a particle, what about an electron? Or a proton? Could they also have a dual wave–particle nature?

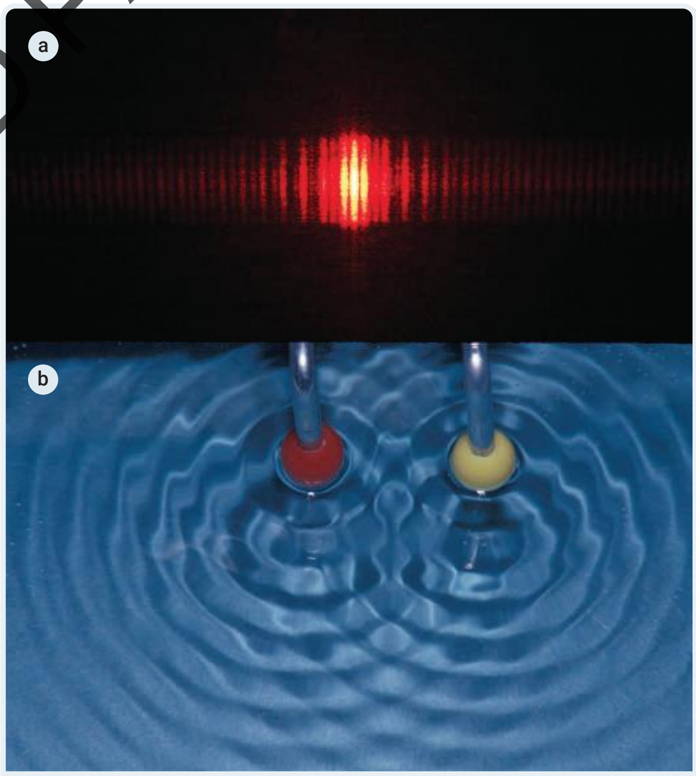


FIGURE 12.3.1 The characteristic interference patterns from (**a**) a double-slit experiment using a laser and (**b**) two coherent wave sources in water

KEY FORMULA

De Broglie wavelength

$$\lambda = \frac{h}{mv} \text{ or } \lambda = \frac{h}{p}$$

where:
h = Planck constant, 6.626×10^{-34} J s
$p = m \times v$ = the momentum of the particle (kg m s^{-1})

The de Broglie wavelength for particles

Weblink
Wave–particle duality

In 1924, Louis de Broglie (1892–1987) introduced the idea (in his doctoral thesis) that any moving particle has an associated wavelength. The idea was revolutionary in physics, and the implications enormously important; however, many physicists were sceptical at the time.

De Broglie claimed that a particle of mass m, moving at a velocity v, would have an associated wavelength. The wavelength λ, is known as the de Broglie wavelength.

WORKED EXAMPLE 12.3.1

a Determine the de Broglie wavelength of an electron travelling at 25 m s^{-1}

b Calculate the velocity that a cricket ball with a mass of 160 g would require if it had the same de Broglie wavelength as the electron in part a.

ANSWERS

a 1 Determine the formula.

$$\lambda = \frac{h}{mv}$$

2 Substitute the known values.

$m = 9.11 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ J s

$$\lambda = \frac{6.63 \times 10^{-34} \text{ J s}}{9.11 \times 10^{-31} \text{kg} \times 25 \text{ m s}^{-1}}$$

3 Calculate the answer.

$\lambda = 2.9 \times 10^{-5}$ m

b 1 Determine the formula.

$$\lambda = \frac{h}{mv}$$

2 Rearrange to find the unknown.

$$v = \frac{h}{m\lambda}$$

3 Substitute the known values.

$$v = \frac{6.63 \times 10^{-34} \text{ J s}}{0.16 \text{ kg} \times 2.9 \times 10^{-5} \text{m}}$$

4 Calculate the answer.

$v = 1.4 \times 10^{-28}$ m s^{-1}

As the momentum of a photon is $p = \frac{E}{c}$, and $E = hf = \frac{hc}{\lambda}$, the momentum of a photon may also be written as $p = \frac{h}{\lambda}$. Hence, de Broglie's equation also allows us to calculate the momentum of a photon: $p = \frac{h}{\lambda}$.

In any interaction between objects, including collisions, both energy and momentum must be conserved. We will see the results of this when we explore phenomena such as the photoelectric effect.

KEY FORMULA

Momentum of a photon

$$p = \frac{h}{\lambda}$$

where:
h = Planck constant, 6.626×10^{-34} J s
p = momentum of the photon (kg m s^{-1})
λ = de Broglie wavelength of the photon (m)

De Broglie waves and the Bohr model

When the idea of the de Broglie wavelength was incorporated into the Bohr model of the atom, it gave a justification for the quantisation of energies. The explanation treats the electrons as standing waves; hence, there is an integer number of wavelengths that fit the orbit of the electron around the nucleus. The electrons act like standing waves on a string the exact length of the orbit.

The condition for a stable orbit is then $n\lambda = 2\pi r$.

KEY FORMULA

De Broglie wavelength for an electron

$$n\lambda = 2\pi r$$

where:
n = an integer
λ = de Broglie wavelength of the electron (m)
r = radius of the orbit (m)

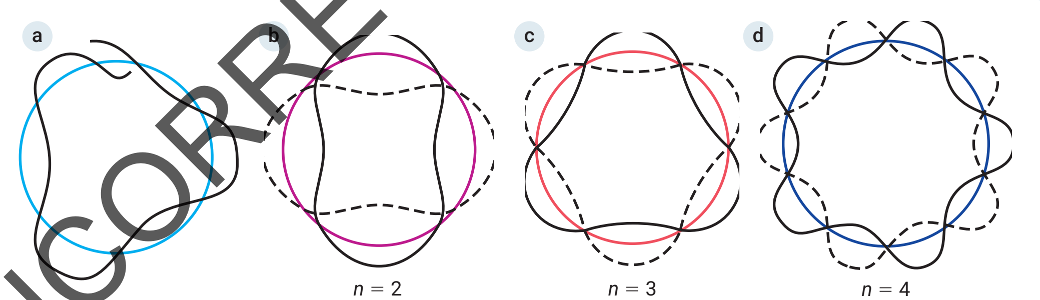


FIGURE 12.3.2 Electrons as standing waves, showing how energy levels correspond to different standing wave modes. (**a**) is not a standing wave and hence not a stable orbit; (**b**), (**c**) and (**d**) are stable orbits of circumference 2λ, 3λ and 4λ.

WORKED EXAMPLE 12.3.2

Calculate the longest three wavelengths of an electron in an orbit of radius 5.3 nm.

ANSWER

1 Determine the formula.

$n\lambda = 2\pi r$

2 Rearrange to find the unknown.

$\lambda = \frac{2\pi r}{n}$

where $\lambda_1 = \frac{2\pi r}{1}$, $\lambda_2 = \frac{2\pi r}{2}$, $\lambda_3 = \frac{2\pi r}{3}$

3 Substitute the known values.

$\lambda_1 = \frac{2\pi(5.3 \times 10^{-9})}{3}$

4 Calculate the answer.

$\lambda_1 = \frac{2\pi(5.3 \times 10^{-9})}{3} = 3.3 \times 10^{-8}\,\text{m}$

5 Repeat steps 3 and 4 for λ_3 and λ_4.

$\lambda_2 = \frac{2\pi(5.3 \times 10^{-9})}{3} = 1.7 \times 10^{-8}\,\text{m}$

$\lambda_3 = \frac{2\pi(5.3 \times 10^{-9})}{3} = 1.1 \times 10^{-8}\,\text{m}$

So, what does it mean for a particle or object to have a wavelength? It does *not* mean that it follows a wiggly path, undulating up and down as it travels. What it does mean is that in some sense the particle is delocalised, or spread out, in space, just as a wave is.

Although the idea of electrons as waves gave some physical explanation for the quantisation of energy levels in the Bohr model, it did not address the other failings of the model. As the wave nature of electrons became better understood, better models of the atom were developed. However, the Bohr model was of great historical importance and can still be used as a useful model of simple atomic systems.

Electron microscopes

In a scanning electron microscope (SEM) **(Figure 12.3.3a)**, the electrons interact with atoms at the surface of the sample. Some electrons are reflected, and some are ejected from atoms at the surface. These are used to create images such as that in **Figure 12.3.3b**. SEMs can be used to 'see' objects as small as 1 nm. In a transmission electron microscope (TEM) **(Figure 12.3.3c)**, the electrons are passed through a very thin sample and collected by a detector on the other side. The image can be formed by the sample simply absorbing some electrons and allowing others to pass through, like a shadow. The electrons can also produce a diffraction pattern on the other side. TEMs can be used to 'see' objects as small as 0.1 nm.

Quantum tunnelling

Quantum tunnelling is another phenomenon that cannot be explained by classical physics. Particles 'tunnel' through barriers they do not have enough energy to 'jump'. The Tonomura

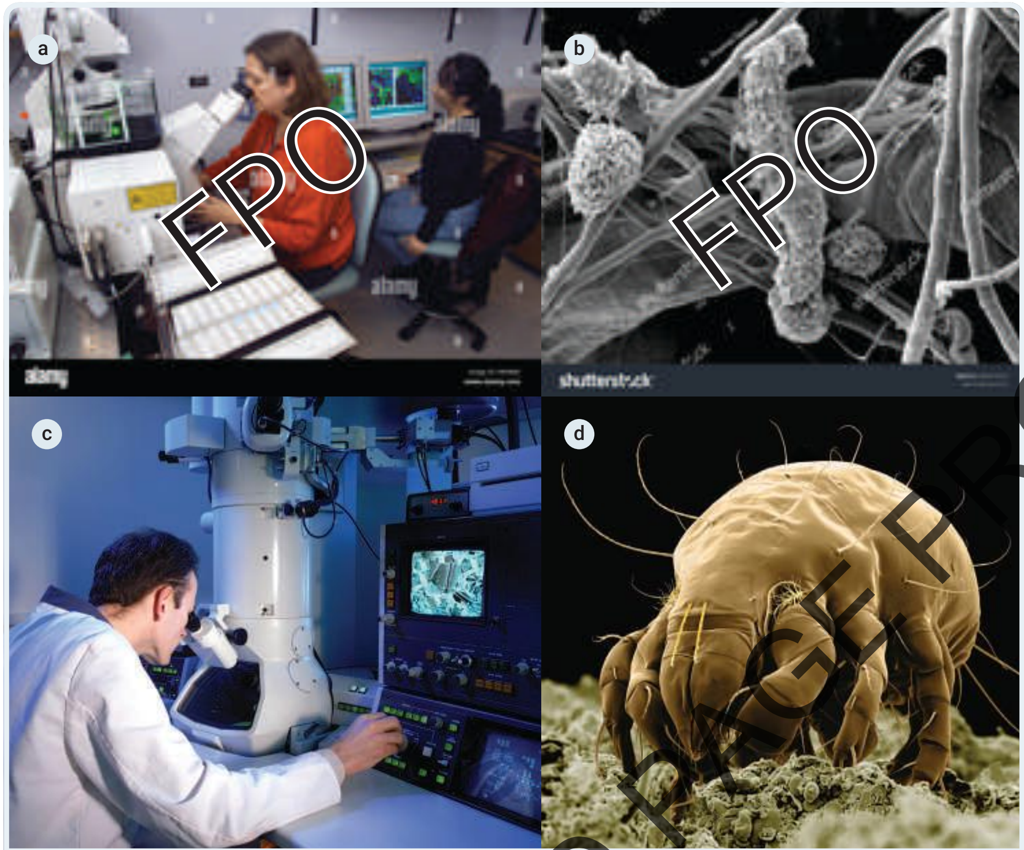


FIGURE 12.3.3 (a) A scanning electron microscope; (b) a photo taken with a scanning electron microscope of human connective tissue and red blood cells. (c) A transmission electron microscope; (d) a photo of a mite taken with a transmission electron microscope.

experiment, named after Japanese physicist Akira Tonomura (1942–2012), was voted by physicists to be the most beautiful experiment ever performed. In this experiment, the wave property of electron beams was found to create a vector potential to act on a beam of charged particles.

From the 1920s, what is now known as modern quantum mechanics was developed by physicists such as Niels Bohr (1885–1962), Louis de Broglie (1892–1987), Werner Heisenberg (1901–76) and Erwin Schrödinger (1887–1961). The term 'modern' is used to distinguish it from earlier quantum mechanics, such as the Bohr model.

The new model included the idea of uncertainty. In any experiment, there are uncertainties due to various sources, including equipment limitations. Heisenberg proposed that there is also an intrinsic uncertainty, and that the behaviour of particles is **probabilistic**. In other words, it cannot be predicted with certainty, no matter how much you know about the particles. In classical mechanics, the behaviour of all objects, including subatomic particles, is **deterministic** and completely predictable, once you have enough information. This is a fundamental difference between quantum mechanics and classical mechanics.

In the double-slit experiment, a probability wave associated with each particle passes through both slits. These probability waves interfere on the other side. When we measure which slit the wave passes through, we no longer have the probability wave passing through both slits, so we no longer have an interference pattern. The modern probabilistic model of quantum mechanics correctly predicts the results of these types of experiments.

probabilistic not deterministic, cannot be predicted regardless of how much information is known

deterministic predictable, can be determined if enough information is available

LEARNING CHECK 12.3

DESCRIBING

1 **Identify** the key features of the Bohr model of the atom.

2 A photon, an electron and a neutron all have the same wavelength. **Sequence** the velocities, from fastest to slowest, of the three particles with reference to the de Broglie wavelength formula and their relative masses.

3 **Explain** why we would not notice the wave nature of a cricket ball moving at 25 m s^{-1}.

APPLYING

4 A bullet of mass 50 g travels at 1.2×10^3 m s^{-1}. **Determine** its de Broglie wavelength.

5 A beam of electrons with de Broglie wavelength 1.5×10^{-10} m is incident on a double slit with a separation of 4.5×10^{-9} m. If the detectors are arranged on a screen a distance 20 cm from the slits, **determine** the location of the:

a first interference maximum

b first interference minimum

c second interference maximum.

ANALYSING

6 A photon of energy 3.6×10^{-15} J collides with a stationary electron that is free to move.

a **Calculate** the magnitude of the momentum of the photon before the collision.

b After the collision, the photon returns along its original path and the electron moves forwards with a momentum of 2.1×10^{-23} kg m s^{-1}.

i **Determine** the de Broglie wavelength of the electron after the collision.

ii Draw a conclusion about whether the wavelength of the photon after the collision will be greater than, less than or the same as before the collision. **Justify** your conclusion.

iii **Calculate** the wavelength of the photon before and after the collision.

REFLECTING

7 **Recall** four or more devices that you have used today that rely on semiconductors and thus quantum physics.

8 **Reflect on** the implications of the de Broglie wavelength for macroscopic objects. Why does this wave–particle nature of particles become noticeable only at very small scales?

12.4 Black-body radiation

Weblink
What is black body radiation?

spectrum the distributed components of light or another wave arranged by frequency (or wavelength)

All objects continuously radiate energy in the form of electromagnetic waves. At any non-zero temperature, a body emits radiation of all wavelengths, but the distribution or **spectrum** of wavelengths depends on its temperature. If an object is very hot, you can see the light that is being emitted; for example, you can see the glowing coals in a fire or the filament of a light globe. At low temperatures, the wavelengths of the emitted radiation are mainly in the low-frequency, infrared region and cannot be seen, although you may still be able to feel the radiation as heat with your skin. Measurements show that hotter objects emit more electromagnetic radiation, and that greater amounts of this radiation is at shorter wavelengths (higher energies). Hence, if you heat a piece of metal slowly, it will glow a dim red at first, then bright yellow and eventually very bright white.

Measurement of the intensity of emitted radiation as a function of wavelength shows that it is a continuous distribution of wavelengths from the infrared, through the visible to the ultraviolet. This distribution is called a **continuous spectrum**. The shape of the spectrum depends only on the temperature of the object, and not on any of its other properties. A continuous spectrum is shown in **Figure 12.4.1**.

continuous spectrum a spectrum containing radiation of all wavelengths; for example, a rainbow is composed of the various wavelengths of the visible spectrum

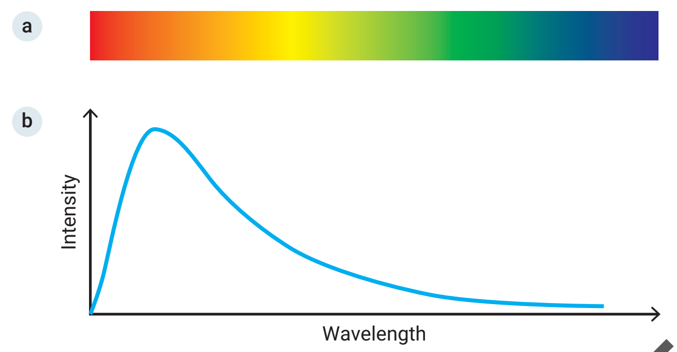


FIGURE 12.4.1 (a) A continuous emission spectrum. (b) The intensity of emitted radiation is a function of wavelength for a continuous black-body spectrum.

What is a black body?

A **black body** is an ideal surface that completely absorbs all wavelengths of electromagnetic radiation incident on it. Hence it is a *black* body. Such a surface will also be a perfect emitter of electromagnetic radiation at all wavelengths. The black-body radiation is characteristic of the temperature of the black body and is mainly in the infrared part of the electromagnetic spectrum when the body is at room temperature.

black body an object with a perfectly absorbing surface that emits radiation with a spectrum that is characteristic of the temperature of the object

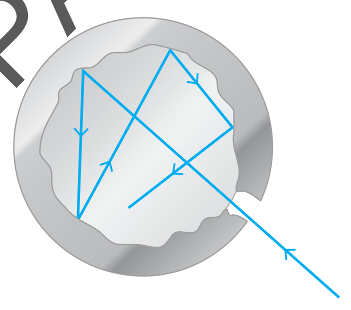


FIGURE 12.4.2 The opening to a cavity is a good approximation of an ideal black body. Note that it is the *opening* to the cavity that is the black body, not the entire hollow object. The hole acts as a perfect absorber.

Although a true black body is only a theoretical concept, it can be closely simulated in a laboratory. Consider a cavity (hollow space) that has the interior walls blackened and which is kept at a constant temperature (**Figure 12.4.2**). If a small hole is made in the wall of the cavity, it will act like a black-body radiator. Any radiation that falls on the hole from the outside will pass through it. After multiple reflections, the radiation will be absorbed by the interior surfaces. As the cavity is in thermal equilibrium with its surroundings, the interior surfaces will emit radiation at the same rate at which it is absorbed. The radiation that escapes depends only on the temperature of the cavity. It is not affected by the size of the cavity or the material of which it is made.

Remember that this is an idealised object, not a real one. In practice, materials that absorb most of the light incident on them are good approximations of a black body. This is where the term 'black body' comes from – black objects absorb most of the light incident on them, regardless of wavelength.

Black-body emission spectrum

The black-body model is useful because it allows us to determine the temperature of distant objects. For example, we can estimate the surface temperature of the Sun by measuring its electromagnetic spectrum.

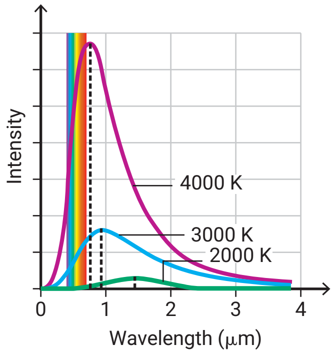


FIGURE 12.4.3 Intensity and distribution of wavelengths of radiation from a black body at different temperatures. Note that the peak intensity in the radiation curve gets larger and shifts to shorter wavelengths (higher frequency and higher energy) as temperature increases.

KEY FORMULA

$$\lambda_{max} = \frac{b}{T}$$

where:
λ_{max} = peak wavelength (m)
T = absolute temperature (K)
b = Wien's constant, 2.898×10^{-3} m K

In 1893, Wilhelm Wien (1864–1928) derived a relationship between the position of the peak wavelength at which radiation is emitted and the temperature of a black body. He used the idealised black-body cavity model to derive the relationship, now known as Wien's displacement law, or simply as Wien's law. The position of the peak wavelength is given by Wien's law, $\lambda_{max} = \frac{b}{T}$

WORKED EXAMPLE 12.4.1

The temperature of the surface of the Sun is approximately 5800 K. If we treat the Sun as a black body, determine the peak wavelength of the radiation emitted. Refer to an electromagnetic spectrum such as in Figure 12.1.1 to describe the part of the spectrum to which this wavelength belongs.

ANSWER

1 **Determine the formula.**

$$\lambda_{max} = \frac{b}{T}$$

2 **Substitute the known values**

$$\lambda_{max} = \frac{2.898 \times 10^{-3}\ \text{m K}}{5800\ \text{K}}$$

3 **Calculate the answer.**

$\lambda_{max} = 5.0 \times 10^{-7}$ m

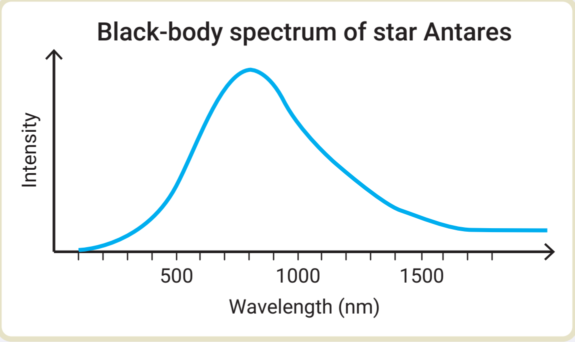
This is yellow (visible) light.

Wien's law was a successful model in that it accurately predicted the position of the peak wavelength. However, there were still two problems. First, there was no theory that explained the shape of the curve. Second, Wien's law was based on an idealised system – a cavity with a small hole. It is difficult to see how this theoretical model could represent the surface of a solid piece of material or of a star such as the Sun.

Classically, it was thought that the thermal radiation originated from oscillating charged particles near the surface of an object. Previously we saw that oscillating charges are a source of electromagnetic waves. This is how an antenna works; the oscillating charges in the antenna produce an electromagnetic wave of the same frequency as the oscillations.

WORKED EXAMPLE 12.4.2

The following black-body spectrum is for the star Antares. Identify the peak wavelength and determine the surface temperature of Antares,



ANSWER

1 Determine the formula.

$$\lambda_{max} = \frac{b}{T}$$

2 Rearrange to find the unknown.

$$T = \frac{\lambda_{max}}{b}$$

3 Substitute the known values.

$b = 2.898 \times 10^{-3}$ m K, $\lambda_{max} = 800 \times 10^{-9}$ m

$$T = \frac{2.898 \times 10^{-3}}{800 \times 10^{-9}}$$

4 Calculate the answer.

$T = 3622.5$ K

Recall that the temperature of a material is a measure of the average kinetic energy of the atoms of that material. In a gas or a liquid, the particles are free to move. The higher the temperature, the more kinetic energy the particles have and the faster they move. In a solid material, the atoms are not free to move, so this kinetic energy is observed as vibrations, and the higher the temperature, the higher the frequency of vibration. And as you know, atoms are made up of smaller particles including protons and electrons, which are charged. Therefore, this theory provided the oscillating charges needed to produce the electromagnetic radiation.

Now consider again the ideal model of the black-body cavity. If the atoms on the inside surface are acting as little antennas, we would see standing waves set up between the walls of the cavity. The waves produced by the vibrating atoms in the inside surface would reflect from the opposite surface. If the waves have the right wavelength, a standing wave is set up, just like a standing wave on a string. We call these standing waves **modes of vibration**.

modes of vibration characteristic patterns of oscillation, usually with a discrete set of allowed frequencies.

The 'ultraviolet catastrophe'

Classically, all the possible modes of vibration would be equally probable, and the total energy would be divided equally between them all. In any cavity, more short wavelength modes would be able to fit in the cavity. This means more short wavelength radiation should be emitted through the hole. As the temperature of the cavity increased, so should the total energy. As the energy increased, the energy associated with the short wavelengths (ultraviolet, X-rays and gamma rays) would approach infinity. According to this theory, even a regular heater should be emitting dangerous amounts of X-rays and gamma rays!

Figure 12.4.4 shows a comparison of a theoretical spectrum based on this model and a measured spectrum. This mismatch between theory and experiment was called the 'ultraviolet catastrophe'; however, it was only a catastrophe for the theory that predicted it. A new theory was needed to solve these problems. German physicists Max Planck (1858–1947) and Albert Einstein (1879–1955) solved the problem by postulating Planck's quanta, which we now know as photons.

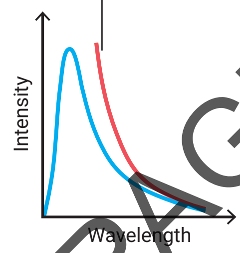


FIGURE 12.4.4 A comparison of the classically predicted and experimental black-body spectra

The comparison of the classically predicted and experimental black-body spectra led to further scientific exploration to resolve the contrary theoretical and experimental results. The ultraviolet catastrophe arose from classical physics' prediction of black-body radiation using the Rayleigh–Jeans law. According to classical theory, the intensity of radiation emitted by a black body increases indefinitely with frequency. This suggests that at high frequencies (ultraviolet and beyond), the energy emitted becomes infinite, which is a clear contradiction to experimental results, where the intensity peaks and then drops off with shorter wavelengths. Planck resolved this by rejecting the assumption that energy could vary continuously. Instead, he proposed that the energy of atoms emitting radiation is quantised. Planck's quantisation limited the number of high-energy modes available for radiation because higher frequencies require disproportionately higher quanta of energy, which are less likely to be excited at a given temperature.

LEARNING CHECK 12.4

DESCRIBING

1 **Define** 'continuous spectrum'.
2 **Recall** the name of the scientist who derived the relationship between the position of the peak wavelength emitted and the temperature of a black body.
3 **Describe** why a surface may be described as a 'black body'.
4 **Sketch** a typical graph of 'intensity versus wavelength' for a black body. Indicate the peak wavelength.
5 **Describe** what occurs to the peak wavelength and the intensity as the temperature of a black body increases.

APPLYING

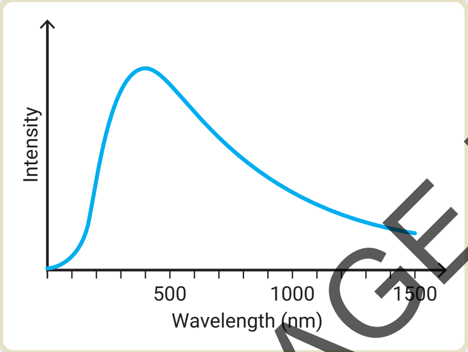
6 The surface of a star is measured to have a peak wavelength of 4.25×10^{-7} m (425 nm). **Determine** the surface temperature of the star. Use $b = 2.898 \times 10^{-3}$ m K.

7 A black body is known to have a surface temperature of 4000 K. Use the value of $b = 2.898 \times 10^{-3}$ m K to **determine** the peak wavelength.

ANALYSING

8 **Describe** the relationship between the peak wavelength emitted from a black body and its surface temperature.

9 The graph shows the black-body radiation spectrum for the star Vega.



a **Determine** the peak wavelength.

b **Calculate** the surface temperature of Vega.

10 The filament of an incandescent light globe can be modelled as a black body. A tungsten filament reaches a temperature of 2900 K.

a **Determine** its peak wavelength.

b **Explain** why such light globes emit more radiation in the infrared region than in the visible part of the electromagnetic spectrum.

12.5 Planck's quanta and photon characteristics

In 1900, Planck used 'lucky guesswork' (as he called it) to derive a formula that correctly matched the experimentally observed spectrum. Planck proposed that the atoms could only oscillate with **discrete** energies, given by the formula $E = nhf$, where h is now known as the **Planck constant**.

discrete able to take only specific values, not continuous (e.g. a line spectrum is a discrete spectrum)

Planck constant the constant of proportionality between energy and frequency for photons, $h = 6.626 \times 10^{-34}$ J s

Weblink
The Planck constant

KEY FORMULA

$$E = nhf$$

where:
n = an integer
f = frequency of oscillation (Hz)
h = Planck constant = 6.626×10^{-34} J s or 4.14×10^{-15} eV s
Note: Kinetic energy (E_k) and the Planck constant, h, must have consistent units. If E_k is in J, then h is in J s. If E_k is in eV, then h is in eV s.

quantised existing in discrete amounts, not able to be divided into arbitrarily smaller amounts

This was a radical proposition. It means that the energy of the oscillators is **quantised**; that is, it may only take the discrete values given by the equation, rather than any possible value within a continuous range.

Planck deduced that the oscillators could only emit and absorb electromagnetic radiation (i.e. light) in packets of specific energies. He called these packets of energy 'quanta'. The amount of energy emitted is equal to the amount of energy lost by an oscillator when it goes to a lower energy state. For example, if an oscillator goes from an energy of $E_3 = 3hf$ to $E_2 = hf$, the energy lost is $E_3 - E_2 = 3hf - 2hf = hf$.

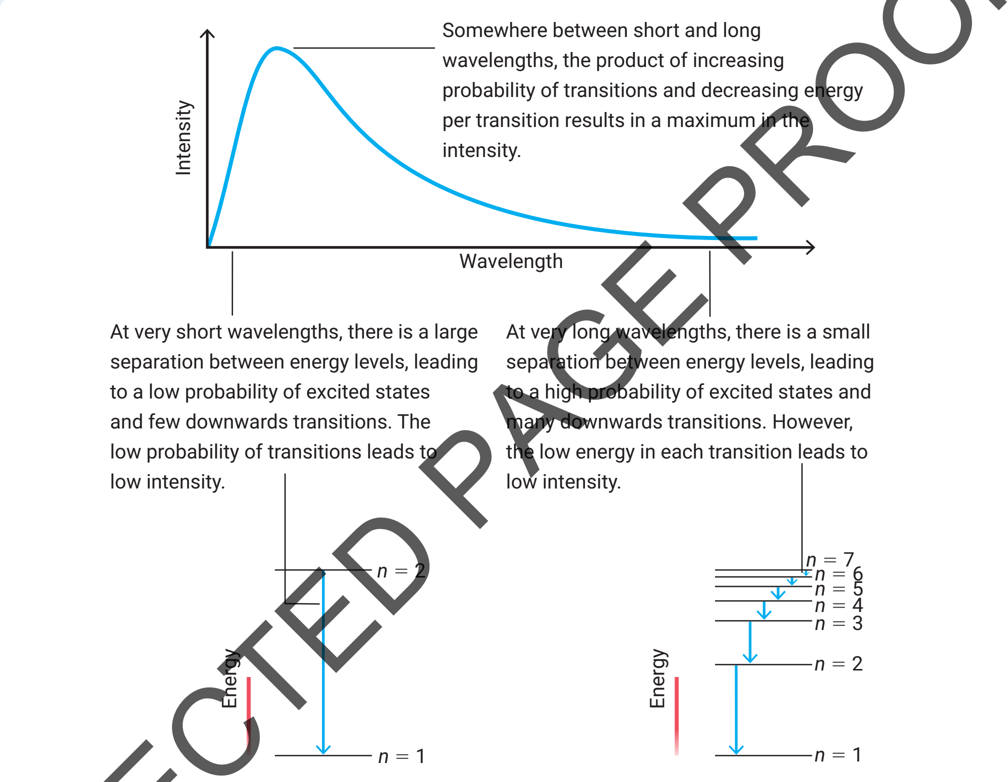


FIGURE 12.5.1 Planck's model for black-body radiation

KEY FORMULA

$$E = nhf = nh\frac{c}{\lambda}$$

where:
n = an integer
f = frequency of oscillation (Hz)
h = Planck's constant = 6.626×10^{-34} J s
c = 3.00×10^{8} m s^{-1}
λ = wavelength of light (m)

One quantum of light has energy $E = hf$. The universal wave equation $c = f \times \lambda$ may also be applied, using $f = \frac{c}{\lambda}$, to determine these discrete energies.

Planck combined this idea of quantisation with two ideas from classical statistical mechanics.

- First, the probability of an oscillator having a particular energy decreases as the energy increases. Hence, the probability of an atom being in a higher energy state (an 'excited state') is lower. This means that the intensity of radiation at high frequencies (short wavelengths) is small.
- Second, the probability of a *change* in energy decreases with the relative gap between energy levels. The relative gap is larger for lower energies, or long wavelengths, so intensity is again low at these wavelengths. In between these extremes, we see the peak intensity observed in the experimental spectra.

The quantisation of energy was such a revolutionary departure from the classical physics of Newtonian mechanics, electromagnetism and thermodynamics that even Planck was reluctant to accept his own idea. Although Planck had discovered a mathematical way of explaining the shape of the black-body spectrum, he was concerned that there was no physical model for how the energy could be in these discrete packets. It was Einstein who put physical meaning to Planck's quantum hypothesis.

WORKED EXAMPLE 12.5.1

A quantum of energy has a wavelength 5.8×10^{-5} m. Determine the:

a frequency of this quantum

b energy of this quantum.

ANSWERS

a

1. **Determine the formula.**
   $c = f\lambda$
2. **Rearrange to find the unknown.**
   $f = \frac{c}{\lambda}$
3. **Substitute the known values.**
   $f = \frac{3.00 \times 10^{8}\,\text{m s}^{-1}}{5.8 \times 10^{-5}\,\text{m}}$
4. **Calculate the answer.**
   $f = 5.2 \times 10^{12}$ Hz

b $E = hf$

$E = 6.626 \times 10^{-34}\,\text{J s} \times 5.2 \times 10^{12}\,\text{Hz}$

$= 3.45 \times 10^{-21}$ J

WORKED EXAMPLE 12.5.2

Determine the frequency of a quantum of energy with $E = 2.5$ keV.

ANSWER

1. **Determine the formula.**
   $E = hf$
2. **Rearrange to find the unknown.**
   $f = \frac{E}{h}$
3. **Substitute the known values.**
   $f = \frac{2.5 \times 10^{3} \times 1.60 \times 10^{-19}}{6.63 \times 10^{-34}}$
4. **Calculate the answer**
   $f = 6.03 \times 10^{17}$ Hz

WORKED EXAMPLE 12.5.3

A packet of energy 'quanta' of 7.96×10^{-19} J is emitted as a photon of light following an electron falling to a lower energy state, from $n = 3$ to $n = 1$.

a Determine the frequency of light emitted.

b Calculate the wavelength of the photon.

c Convert the magnitude of the energy from joules to electron volts.

ANSWERS

a **1** **Determine the formula.**

$E = nhf$

2 **Substitute the known values.**

$7.96 \times 10^{-19}\,\text{J} = 2 \times 6.626 \times 10^{-34}\,\text{J s} \times f$

3 **Calculate the answer**

$f = 6.00 \times 10^{14}\,\text{Hz}$

b **1** **Determine the formula.**

$c = f\lambda$

2 **Substitute the known values.**

$$\lambda = \frac{c}{f}$$

$$= \frac{3.00 \times 10^{8}\ \text{m s}^{-1}}{6.00 \times 10^{14}\ \text{Hz}}$$

3 **Calculate the answer.**

$\lambda = 5.00 \times 10^{-7}\,\text{m}$ or 500 nm

c **1** **Determine the formula.**

$$E = \frac{7.96 \times 10^{-19}\ \text{J}}{1.6 \times 10^{-19}\ \text{JeV}^{-1}}$$

2 **Calculate the answer.**

$E = 4.98\,\text{eV}$

LEARNING CHECK 12.5

REMEMBERING

1 **Describe** the relationship between the energy of a photon and its frequency

2 **Explain** what Planck meant by the term ‘quantised’.

UNDERSTANDING

3 **Describe** how the units for the Planck constant must be J s.

4 Vega is a blue star and Antares is a red star. Conclude which is hotter. **Justify** your answer with reference to black-body radiation and the electromagnetic spectrum.

APPLYING

5 An atomic oscillator has frequency $f = 6.1 \times 10^{12}\,\text{Hz}$, and is in the $n = 3$ state.

a **Determine** the energy of this oscillator.

b **Calculate** the frequency of light will be emitted if it transitions to the $n = 2$ state.

6 **Determine** the frequency and wavelength of a quantum of energy with $E = 1.7 \times 10^{-19}\,\text{J}$.

REFLECTING

7 For climate scientists, the evidence of anthropogenic global warming is clear. However, many people, including government and industry leaders, do not accept the evidence. Draw a conclusion about what cultural and economic factors might be important in decision and policy making about the climate. **Infer** why some politicians are reluctant to make policy decisions that require action to be taken to mitigate the risks associated with climate change.

CHAPTER SUMMARY

Light as electromagnetic waves

- The classical model of light introduces light as electromagnetic waves which travel with two perpendicular waveforms, the electric field and the magnetic field.
- Phenomena such as the photoelectric effect and atomic spectral lines cannot be fully explained by classical wave models, so quantum mechanics was developed.

Young's double-slit experiment

- Young's double-slit experiment provided evidence that light acts like a wave as light waves refract in a similar way to mechanical waves but do not require a medium through which to propagate.
- In the experiment, a coherent light source is shone through a pair of narrow, closely spaces slits and a resulting interference pattern can be observed on a distant screen.
- The bright and dark fringes observed on the screen align with the constructive (bright) and destructive (dark) interferences of waves, indicating that light behaves as a wave in this experiment.

Wave–particle duality

- A wave can behave as a wave or as a particle.
- De Broglie introduced the idea that any moving particle has an associated wavelength.

$$\lambda = \frac{h}{mv} \text{ or } \lambda = \frac{h}{p}$$

where:
h = Planck constant, 6.626×10^{-34} J s
$p = m \times v$ = the momentum of the particle (kg m s^{-1}).

- The de Broglie wavelength gave justification for the quantisation of energies. The condition for a stable orbit is $n\lambda = 2\pi r$.

Black-body radiation

- A black body is an object that completely absorbs and emits all wavelengths of electromagnetic radiation incident on it.
- Wien's law allows for the estimation of temperatures of distant objects by measuring electromagnetic spectrum. The law is a successful model to accurately predict the position of the peak wavelength of radiation.

$$\lambda_{max} = \frac{b}{T}$$

where:
λ_{max} = peak wavelength (m)
T = absolute temperature (K)
b = Wien's constant, 2.898×10^{-3} m K

- The ultraviolet catastrophe arose from classical physics' prediction of black-body radiation where the intensity of radiation emitted by a black-body increases indefinitely with frequency. This suggests the energy emitted becomes infinite at high frequencies.

- Planck resolved the catastrophe by proposing energy of atoms emitting radiation is quantised.
- Higher frequencies require significantly larger energy quanta, making high-energy states less likely to be excited at a given temperature, resulting in reduced intensity at shorter wavelengths. At the other end, large energy gaps at lower frequences limit transitions, also lowering intensity. Between these extremes, where transitions are most probable, the curve peaks, matching the observed experimental spectra.

$$E = nhf$$

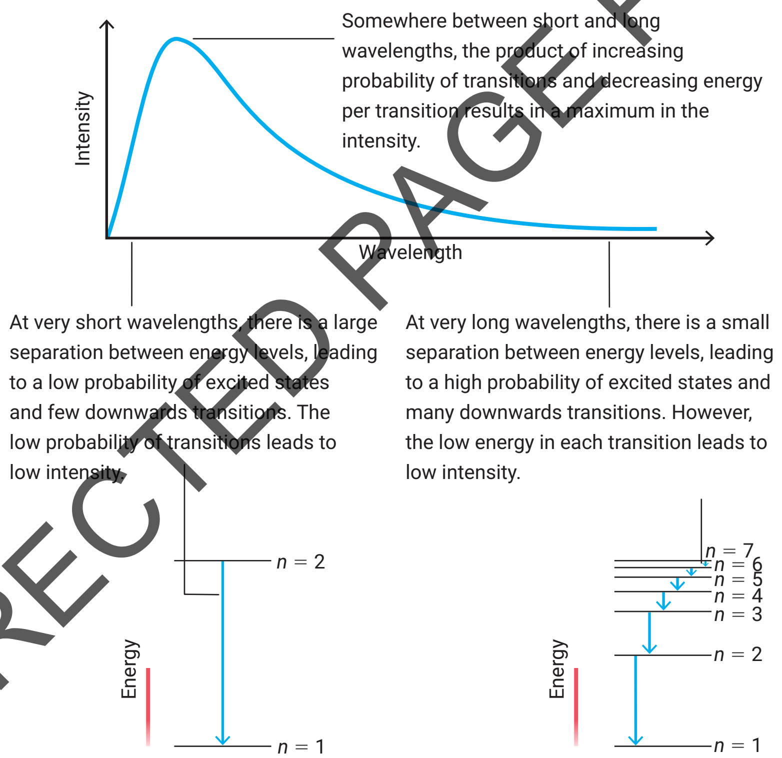
where:

n = an integer

f = frequency of oscillation (Hz)

h = Planck constant = 6.626×10^{-34} J s or 4.14×10^{-15} eV s

Planck's model for black-body radiation



CHAPTER EXAM

MULTIPLE CHOICE

1. Which statement regarding wave interference is correct?
   - **A** Constructive interference occurs along nodal lines.
   - **B** Destructive interference occurs when a crest meets with another crest.
   - **C** A crest meeting a trough is an example of destructive interference.
   - **D** Antinodal lines always result in a dark fringe.
2. The double-slit experiment with electrons supports wave-particle duality because it:
   - **A** confirms the existence of photons.
   - **B** reveals the existence of the nucleus.
   - **C** shows light behaves only as a wave.
   - **D** demonstrates interference patterns from particles.
3. Which observation from the double-slit experiment provides evidence for the wave model of light?
   - **A** Diffraction patterns of bright and dark fringes
   - **B** Creation of a single bright spot
   - **C** Continuous emission spectrum
   - **D** Photoelectric effect
4. What is the energy of a photon with a frequency of 6.0×10^{14} Hz?
   - **A** 4.0×10^{-14} J
   - **B** 1.2×10^{-18} J
   - **C** 4.0×10^{-19} J
   - **D** 3.98×10^{-20} J
5. Calculate the de Broglie wavelength of an electron travelling at 2.0×10^{6} ms^{-1}.
   - **A** 3.63×10^{-9} m
   - **B** 3.63×10^{-10} m
   - **C** 4.0×10^{-12} m
   - **D** 6.63×10^{-34} m
6. According to de Broglie, what determines the allowed orbits in the Bohr model?
   - **A** Energy levels are fixed.
   - **B** Frequency of light emitted.
   - **C** Electrons have particle properties only.
   - **D** Electron wavelength fits a whole number of times around the orbit.
7. What does the concept of wave–particle duality mean?
   - **A** Energy is always conserved.
   - **B** Particles exist in two places simultaneously.
   - **C** Matter consists of particles and waves separately.
   - **D** Light and matter exhibit both wave and particle properties.
8. What happens to the de Broglie wavelength of a particle as its momentum increases?
   - **A** It decreases.
   - **B** It increases.
   - **C** It remains constant.
   - **D** It becomes infinite.

9. According to Wien's displacement law, what happens to the wavelength of peak emission of a black body as its temperature increases?
   A The wavelength remains constant regardless of temperature.
   B The wavelength increases linearly with temperature.
   C The wavelength decreases as temperature increases.
   D The wavelength decreases and then increases.

10. How does the de Broglie wavelength of a particle change if its velocity is doubled?
   A Halved
   B Doubled
   C Unchanged
   D Reduced by a factor of four

SHORT RESPONSE

11. **a** The surface temperature of a star is 5000 K. **Calculate** the wavelength at which the star's radiation is the most intense.
   b Another star emits its maximum radiation at a frequency of 6.67×10^{14} Hz. **Calculate** the temperature of the star.

12. In a double-slit experiment, light with wavelength 426 nm is used to illuminate twin slits that are separated by 0.010 mm. The pattern produced is observed on a wall 2.00 m from the slits. **Determine** the position of the:
   a first interference maximum (bright spot)
   b first interference minimum (dark spot)
   c second interference maximum (bright spot).

CROSS-CHAPTER QUESTIONS

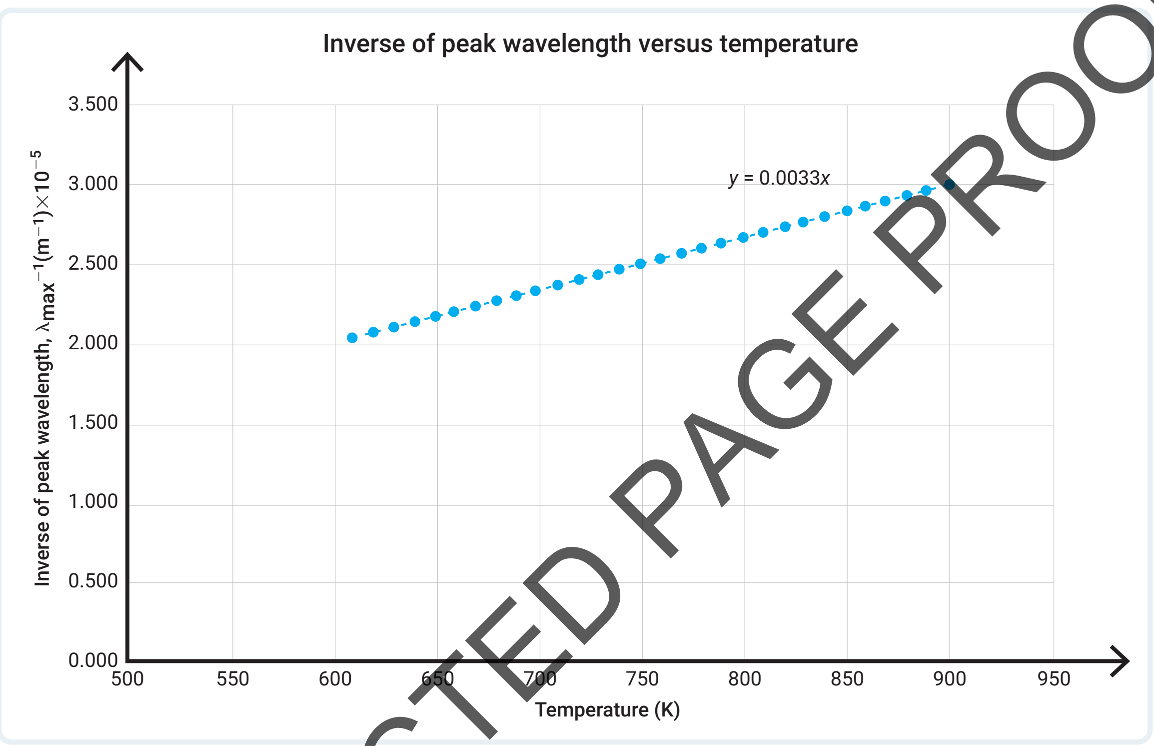
13. An electron is moving in a circular orbit around a nucleus with a radius of 0.053 nm.
   a **Calculate** the de Broglie wavelength of the electron if its kinetic energy is 13.6 eV.
   b Conclude whether the electron orbit is stable.

14. A high-energy proton is emitted from the surface of a star with the kinetic energy of 0.75 eV.
   a **Calculate** the initial velocity of the proton.
   b **Calculate** the relativistic momentum of the proton.
   c **Compare** the de Broglie wavelength of the proton in both classical and relativistic scenarios.

DATA ANALYSIS

15. Analyse data

A physicist conducted an experiment to study black-body radiation. They analysed the light emitted by a piece of metal to determine the peak wavelength over a range of temperatures from 610 to 900 K. The processed data from the experiment is presented in the following graph.



a **Describe** the mathematical relationship between peak wavelength λ_{max} and temperature *T*.

b Using the gradient of the trendline, **determine** the experimental value of Wein's displacement constant, *b*.

c **Calculate** the percentage error of the experimental value of *b*.

d **Sketch** the expected general shape of a graph of λ_{max} against *T*.

CHAPTER 13

Quantum theory II – the photoelectric effect and atomic spectra

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Describe the photoelectric effect in terms of the photon.
- Describe the concepts of threshold frequency and work function.
- Solve problems involving blackbody radiation and the photoelectric effect using $\lambda_{max} = \frac{b}{T}$, $E = hf = \frac{hc}{\lambda}$, $E_k = hf - W$, $W = hf_0$ and $\lambda = \frac{h}{p}$.
- Compare the different models of the atom proposed by Rutherford and Bohr.
- Explain how Bohr's model of the hydrogen atom integrates light quanta and atomic energy states to explain the specific wavelengths in the hydrogen line spectrum.
- Solve problems involving the line spectra of simple atoms using atomic energy states or atomic energy level diagrams using $n\lambda = 2\pi r$, $mvr = \frac{nh}{2\pi}$, $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ and $\lambda = \frac{h}{p}$.
- Interpret data related to the photoelectric effect.

SCIENCE AS A HUMAN ENDEAVOUR

- Appreciate the significant contributions of scientists such as Wilhelm Wien, Max Planck, Ernest Rutherford, Niels Bohr, Maria Goeppert-Mayer and Johannes Rydberg who furthered our understanding of quantum theory.
- Explore the historical development of the model of the atom in terms of traditional models.
- Explore how the approximation of Earth as a black body can be used to predict climate patterns.

SCIENCE INQUIRY

- Investigate variables related to the photoelectric effect such as work functions of surfaces.

Introduction

Quantum theory was developed when classical models, such as the wave model of light, were unable to explain experimental results.

In this chapter, the nature of light is explored through the study of such experiments and phenomena as atomic line spectra, the photoelectric effect and de Broglie wavelength to identify evidence supporting the dual nature of light as both a wave and as a particle.

Assessments

- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practicals

- Observing emission spectra (online-only resource)
- Measuring the Planck constant (online-only resource)
- THE PHOTOELECTRIC EFFECT (online-only resource)

Worksheets

- Name
- Name

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ The relationship between frequency and period is $f = \frac{1}{T}$.
- ✓ The speed of light in a vacuum is a constant: $c = 3 \times 10^{8}\ \text{m s}^{-1}$.
- ✓ The wave equation is $v = f\lambda$.
- ✓ The unit prefix nano (*n*) has a multiplication factor of 10^{-9}.
- ✓ Momentum can be calculated using $p = mv$.
- ✓ Atoms consist of electrons surrounding a central nucleus comprised of protons and neutrons.
- ✓ Elements are distinguished according to their atomic number and hence the number of protons and electrons each atom has.
- ✓ Kinetic energy can be calculated using $E_k = \frac{1}{2}mv^2$.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe the development of quantum theory and the contributions of Hertz, Schrödinger, Kirchhoff, Heisenberg and Bohr
- ✓ describe wave–particle duality of light by identifying evidence that supports the wave characteristics of light and evidence that supports the particle characteristics of light
- ✓ describe the de Broglie wavelength and how it contributes to the arrangement of electrons in an atom
- ✓ calculate the de Broglie wavelength
- ✓ describe the probabilistic and deterministic aspects of quantum mechanics
- ✓ describe the quantisation of energy according to quantum theory
- ✓ describe the photoelectric effect in terms of the photon, photocurrent, stopping voltage and photoelectrons
- ✓ describe the concepts of threshold frequency and work function
- ✓ interpret graphs constructed from photoelectric effect data
- ✓ interpret the behaviour and graphs of voltage against current of photocells, photoelectric tubes and photoelectric circuits
- ✓ solve problems involving black-body radiation and the photoelectric effect using $\lambda_{max} = \frac{b}{T}$, $E = hf = \frac{hc}{\lambda}$, $E_k = hf - W, W = hf_0$ and $\lambda = \frac{h}{p}$
- ✓ compare the different models of the atom proposed by Thomson, Rutherford and Bohr
- ✓ explain how Bohr's model of the hydrogen atom integrates light quanta and atomic energy states to explain the specific wavelengths in the hydrogen line spectrum
- ✓ describe how a spectroscope and emission and absorption line spectra are used to investigate the electron configuration of atoms
- ✓ describe the contributions of Balmer, Lyman, Paschen and Brackett to the analysis of the hydrogen emission spectrum
- ✓ describe how the Rydberg equation and constant were derived
- ✓ describe the ground state and excited state of electrons
- ✓ describe the phenomena of fluorescence
- ✓ describe the limitations of the Bohr model and the Zeeman effect
- ✓ solve problems involving the line spectra of simple atoms using atomic energy states or atomic energy level diagrams using $n\lambda = 2\pi r$, $mvr = \frac{nh}{2\pi}$ and $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$.

13.1 The photoelectric effect

Quantisation and the photoelectric effect

Max Planck (1858–1947) introduced the idea of quantised electromagnetic energy to explain the black-body spectrum. It was already known at the time that matter was quantised. Scientists accepted that matter came in discrete quanta, or atoms, and that atoms combined to form molecules, and so on. In 1897, J.J. Thomson (1856–1940) discovered electrons when he realised that cathode rays were made of tiny negatively charged subatomic particles. So, although it was known that atoms could be broken down into smaller components, these components were themselves quantised into discrete particles. Although the idea of quantisation of matter was already well established while the quantum model was being developed, the idea of quantisation of energy was completely new, as it contradicted the accepted model of light as a wave at the time. The photoelectric effect provided the evidence needed for quantisation of energy to be accepted.

Weblink
The photoelectric effect

Einstein's explanation of the photoelectric effect gave a physical meaning to the idea of quantisation of energy of electromagnetic radiation. It meant that, in some circumstances, light behaved like particles. The term '**photon**' was introduced in 1926 by the chemist Gilbert Lewis (1875–1946) to describe these particles. Further experiments by Arthur Holly Compton (1892–1962) provided evidence for the existence of photons. Compton scattered single photons from electrons and found that only particular energies were absorbed. Photons are now accepted as particles with zero rest mass, and with energy given by $E = hf$.

photon a particle or quantum of light, having energy $E = hf$

The photoelectric effect was first observed by Heinrich Hertz in 1887. He observed that when light is shone on a highly polished metal surface, electrons can be emitted from the surface. One of Hertz's assistants, Philipp Lenard (1862–1947), performed experiments to investigate the photoelectric effect in detail. Lenard developed much of the equipment needed to make quantitative measurements of the intensity and energy of the emitted 'cathode rays', as they were called at the time. Other physicists, including Robert Millikan (1868–1953), also investigated the effect.

Their data for the photoelectric effect showed that:

- no electrons were emitted unless the frequency of the light was above some minimum threshold (or critical) frequency, f_0, regardless of the intensity of the light
- the number of electrons (the current), if emitted, was proportional to the intensity. It did not vary with the frequency of the light (as long as the frequency was of the threshold frequency or greater).

In Figure 13.1.1b, when light is shone through the quartz window at the polished metal plate, X, **photoelectrons** are emitted. The photoelectrons are attracted to the positively charged metal plate, Y. The ammeter, A, measures the current of photoelectrons produced – the **photocurrent**.

photocurrent the current formed by electrons ejected from a surface by incident photons of frequency $f \geq f_0$.

photoelectron an electron ejected from a metal surface after absorption of a photon of sufficient energy

threshold frequency the minimum frequency of light needed to eject an electron from a metal surface (f_0)

Using this apparatus, experiments have shown that:

- a photocurrent is only produced when the frequency of the light is above some minimum value, termed the **threshold frequency**, (f_0). This implies a threshold wavelength also, $\lambda_0 = \frac{c}{f_0}$, above which no photocurrent is produced.
- The size of the current (the number of photoelectrons produced) depends on the intensity of the light but not on the frequency, as long as the frequency is above the threshold f_0.
- There is no time delay between light being incident on the metal and photoelectrons being emitted, regardless of intensity.
- Different metals have unique characteristic threshold frequencies.

The voltage divider is used to vary the potential difference between X and Y. When the potential difference is reversed, the maximum kinetic energy of the emitted photoelectrons can be measured. This is called a reverse bias voltage. In this case, plate Y is negative and repels the photoelectrons.

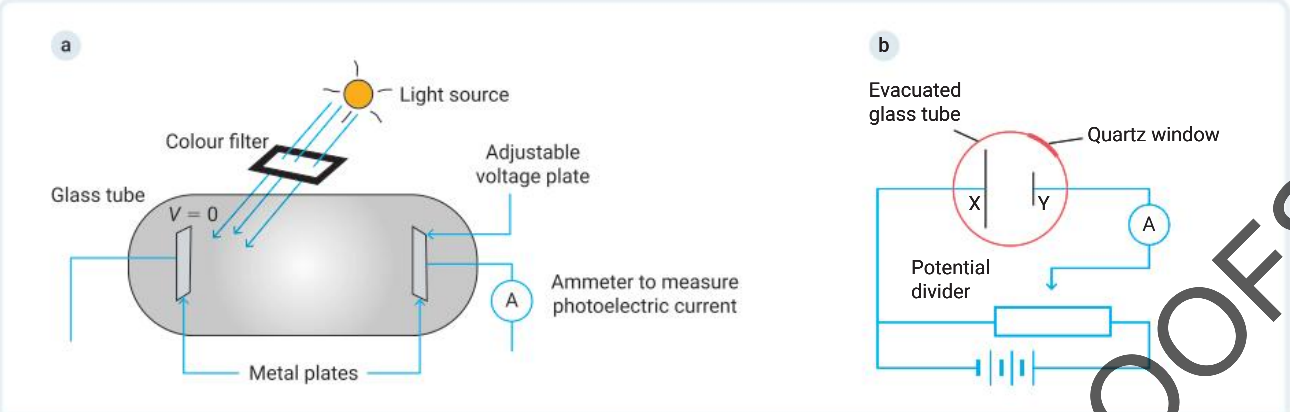


FIGURE 13.1.1 The photoelectric effect apparatus. (**a**) The external monochromatic incident light arrives on a target metal plate. (**b**) The photoelectrons ejected arrive at a cathode and make a photocurrent.

The reverse bias voltage between X and Y is slowly increased and the current observed until it drops to zero. At this point, the potential difference is equal to the maximum energy per unit charge of the electrons. This potential difference is called the **stopping voltage** (V_s). Hence, the product of the potential difference and the charge on the electron, qV_s, is equal to the maximum kinetic energy of the photoelectrons; that is, $E_{k(max)} = qV_s = eV_s$.

The photoelectric effect experiment provides the characteristic results that the maximum kinetic energy (measured using $E_k = qV_s$) depends on the frequency of light, but *not* on the intensity, as shown in Figure 13.1.2. Note that different metals have varied work functions, hence different characteristic threshold frequencies. The **work function** (W) is equivalent to the product of the Planck constant and the threshold frequency, $W = hf_0$.

KEY FORMULA

Kinetic energy of a photoelectron

$E_{k(max)} = qV_s = eV_s$

where:

E_k = kinetic energy (J)

q = charge (C)

V_s = stopping voltage (V)

e = charge on electron (1.60×10^{-19} C)

stopping voltage the reverse bias voltage required to stop the flow of photoelectrons in a photoelectric effect experiment (V_s)

work function the energy required to eject an electron from a metal surface; effectively, it is the ionisation energy for the bulk material (W)

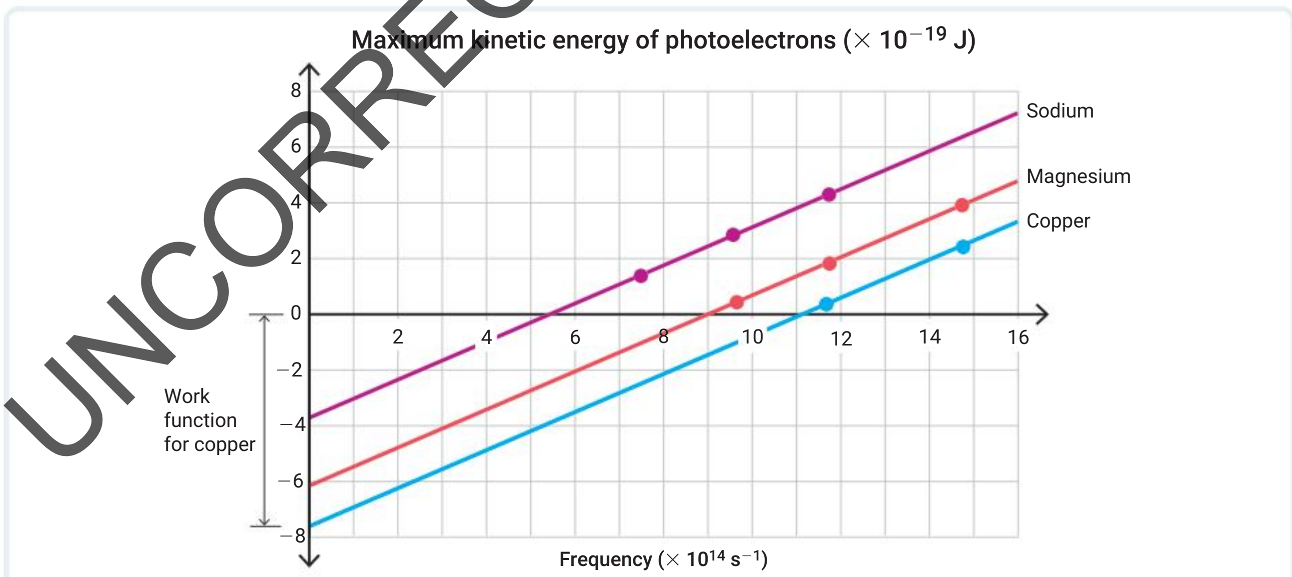


FIGURE 13.1.2 Experimental results for the photoelectric effect. Maximum kinetic energy as a function of frequency for three different metals. The data is extrapolated to determine the work functions for each metal. For this graph of kinetic energy versus frequency, the *y* intercept represents the work function, the *x* intercept represents the threshold frequency and the parallel gradients of the lines of best fit represent the Planck constant, *h*.

The electromagnetic wave model of light previously discussed could not explain the observations of the photoelectric effect. **Table 13.1.1** compares the results of these experiments with the predictions from classical electromagnetic wave theory.

TABLE 13.1.1 Comparison of the photoelectric experimental results with predictions of classical electromagnetic wave theory

Photoelectric experimental observation	Prediction of classical electromagnetic wave theory (unsupported)
Photocurrent only occurs for frequencies above f_0, and where f_0 is characteristic of the material.	Electrons should be emitted at any frequency, as long as the intensity is high enough.
The size of the current depends on intensity but not frequency.	The current should depend on both intensity and frequency.
There is no time delay between the absorption of light and the emission of a photoelectron at any intensity.	At low intensities, it takes time for enough energy to be absorbed by the atoms. Hence, there should be a delay between the light being turned on and electrons being emitted. The delay should be longer for lower intensities.
The maximum kinetic energy of the electrons depends on the frequency of light but *not* on the intensity.	The kinetic energy should only be related to the intensity, and not to the frequency.

Just as with black-body radiation, a new theory was needed to explain these results. Einstein came up with a new model in 1905. His explanation combined two ideas – the familiar conservation of energy and Planck's recently introduced idea of 'quantisation'. Note, it was for his explanation of the photoelectric effect that Einstein won his Nobel Prize in Physics (and not for the development of relativity, which was deemed too controversial at the time).

Conservation of energy, threshold frequency and work function

You are already familiar with the idea of conservation of energy. Einstein explained the photoelectric effect by saying that electromagnetic radiation, or light, is quantised. When it interacts with matter, such as the metal plate X in Figure 13.1.1, it can only give up its energy in discrete amounts. Each quantum of light has energy $E = hf$, a relationship between energy and frequency first introduced by Planck to explain black-body radiation.

When an electron in the metal plate X (Figure 13.1.1) absorbs a photon, it gains this energy. However, to leave the metal plate, a given amount of energy is required – effectively an ionisation energy. Hence, the threshold frequency, which is characteristic of the metal, is a measure of this ionisation energy. This energy is called the work function of the metal and is given by $W = hf_0$.

Putting this together with the conservation of energy, Einstein determined that $E_{k(max)} = hf - hf_0 = E - W$. The photoelectric equation relates the kinetic energy of the photoelectrons with the incident energy of the photon and the work function of the metal; that is, the maximum kinetic energy of photoelectrons equals the energy of the incident photon less the work function.

KEY FORMULA

$$E = hf = \frac{hc}{\lambda}$$

where:
E = energy (J)
h = Planck constant, $h = 6.626 \times 10^{-34}$ J s
f = frequency of incident light (Hz)
c = speed of light, 3.00×10^8 m s^{-1}
λ = wavelength (m)

KEY FORMULA

$$W = hf_0$$

where:
W = work function (J)
h = Planck constant (J s)
f_0 = threshold frequency (Hz)

> **KEY FORMULA**
>
> **Photoelectric equation**
>
> $E_{k(max)} = hf - hf_0 = E - W$

From Figure 13.1.2, we can see that the gradient of each line must be equal to the Planck constant, h. The extrapolated straight lines of best fit in Figure 13.1.2 cross the y-axis at the value of the work function; that is, the y intercept provides the value W. This graphical representation of the experimental data allows us to quickly find values for both the Planck constant, h, and the work function, W, of the metal used.

> **KEY FORMULA**
>
> The photoelectric effect equation may be written in a wide variety of forms to enable the different variables to be determined, including the velocity of the ejected photoelectrons, their electric stopping potential, the threshold frequency or threshold wavelength, the work function or the incident energy, frequency or wavelength. It is useful to be able to interchange each of these variables within the formula to apply it to find the value that you are looking for.
>
> $E_k = hf - W$
>
> $qV = hf - hf_0$
>
> $\frac{1}{2}mv^2 = h\frac{c}{\lambda} - h\frac{c}{\lambda_0}$

TABLE 13.1.2 Work functions of various metals

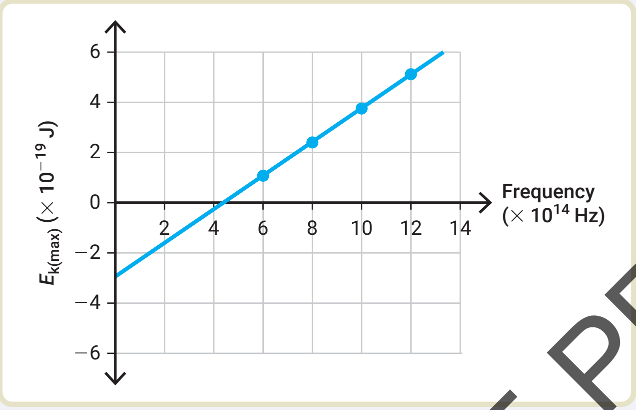
Metal	W (eV)	W (J)
Sodium (Na)	2.46	3.94×10^{-19}
Aluminium (Al)	4.08	6.53×10^{-19}
Iron (Fe)	4.50	7.20×10^{-19}
Copper (Cu)	4.70	7.52×10^{-19}
Zinc (Zn)	4.31	6.90×10^{-19}
Silver (Ag)	4.73	7.57×10^{-19}
Platinum (Pt)	6.35	1.02×10^{-18}
Lead (Pb)	4.14	6.62×10^{-19}

Note: These are typical values for these metals. Measured values depend on whether the metal is a single crystal or polycrystalline and which face of the crystal is illuminated.

> **KEY CONCEPT**
>
> **The photoelectric effect**
>
> The photoelectric effect is the ejection of electrons from a polished metal surface by incident light. The frequency of incident light must equal or exceed a minimum threshold frequency for this to occur, $f \geq f_0$.
>
> - The threshold frequency corresponds to a minimum energy, $E = hf_0$.
> - The minimum energy corresponds to the work function, W, of the metal.
> - The maximum kinetic energy of the photoelectrons is $E_{k(max)} = hf - W$. This is a statement of conservation of energy.
> - The photocurrent, which is proportional to the number of photoelectrons, depends on the intensity of the light. The intensity is a measure of the number of incident photons.

WORKED EXAMPLE 13.1.1

Using the following graph to find the value of the work function for caesium. Give your answer in eV and J.



ANSWER

1 Determine the formula.

$W = hf_0$

2 Substitute the known values.

Given that $f_0 = 4.4 \times 10^{14}$ Hz:

$W = 6.626 \times 10^{-34}$ J s $\times 4.4 \times 10^{14}$ Hz

3 Calculate the answer.

$W = 2.9 \times 10^{-19}$ J

To convert to eV:

$$W = \frac{2.9 \times 10^{-19}\ \text{J}}{1.6 \times 10^{-19}\ \text{J eV}^{-1}} = 1.8\ \text{eV}$$

You will have noticed that the formula refers to the *maximum* kinetic energy of the photoelectrons. The photoelectrons have all values of energy *up to* this maximum value. After absorbing the energy *hf* from the incident light, the electrons have energy *hf*. These are the valence electrons that are free to move through the metal and are not bound to any particular atom. It is these conduction electrons that can be ejected as photoelectrons. The electrons that are bound to the atoms cannot gain enough energy to be ejected in this way. Electrons that have absorbed $E = hf$ lose a minimum energy of $W = hf_0$, the work function, to escape the material. This may occur if they are at the very surface and have no interactions with other electrons or nuclei as they escape. However, most of the electrons will lose some of the absorbed energy in collisions, and many do not leave the metal at all. This results in an increase in the temperature of the metal. Hence, there is a continuous spectrum of electron energies from zero to the maximum value kinetic energy value of $E_{k(max)} = hf - W$.

KEY FORMULA

The photoelectric effect formula represents incident energy, work function and the remaining kinetic energy of photoelectrons using various related formulas, each of which are interchangeable, depending on the purpose and the variables provided.

$$E_{k(max)} = hf - hf_0 = E - W$$

$$\frac{1}{2}mv^2 = hf - hf_0$$

$$qV_s = h\frac{c}{\lambda} - h\frac{c}{\lambda_0}$$

WORKED EXAMPLE 13.1.2

Ultraviolet light of wavelength 200 nm is incident on a polished silver plate. The work function for silver is 4.73 eV.

a What is the incident energy of the photon, in J?

b What is the kinetic energy of the fastest moving electrons in J and in eV?

c Calculate the threshold frequency for silver.

d Calculate the threshold wavelength for silver.

ANSWERS

a 1 Determine the formula

$$E = hf = \frac{hc}{\lambda}$$

$$E = \frac{hc}{\lambda}$$

2 Substitute known values.

$$E = \frac{6.636 \times 10^{-34} \times 3.00 \times 10^{8}}{200 \times 10^{-9}}$$

3 Calculate the answer.

$E = 9.939 \times 10^{-18}$ J

b 1 Determine the formula.

$E_{k(max)} = hf - W$, and since $f = \frac{c}{\lambda}$ then:

$$E_{k(max)} = \frac{hc}{\lambda} - W$$

2 Substitute the known values.

$$E_{k(max)} = \frac{4.14 \times 10^{-15}\ \text{eV s} \times 3.0 \times 10^{8}\ \text{m s}^{-1}}{200 \times 10^{-9}\ \text{m}} - 4.73\ \text{eV}$$

3 Calculate the answer.

$E_{k(max)} = 1.5$ eV

c 1 Determine the formula.

$$E = \frac{hc}{\lambda} - W$$

2 Substitute the known values.

$$E = \frac{6.636 \times 10^{-34} \times 3.00 \times 10^{8}}{200 \times 10^{-9}} - 4.73\ \text{eV} \times 1.60 \times 10^{-19}\ \text{J eV}^{-1}$$

3 Calculate the answer.

$$E = 9.939 \times 10^{-18} - 7.568 \times 10^{-19}$$
$$= 9.182 \times 10^{-18}\ \text{J}$$

c 1 Determine the formula.

$W = hf_0$ where $f_0 = \frac{W}{h}$

2 Substitute the known values.

$$f_0 = \frac{4.73}{4.14 \times 10^{-15}}\ \text{(using eV)}$$

or

$$f_0 = \frac{4.73 \times 1.60 \times 10^{-19}}{6.626 \times 10^{-34}}\ \text{(using joules)}$$

3 **Calculate the answer.**

$f_0 = 1.14 \times 10^{15}$ Hz

d 1 **Determine the formula.**

$$\lambda_0 = \frac{c}{f_0}$$

2 **Substitute the known values.**

$$\lambda_0 = \frac{3.00 \times 10^{8}}{1.14 \times 10^{15}}$$

3 **Calculate the answer.**

$\lambda_0 = 2.63 \times 10^{-7}$ m or 263 nm

KEY FORMULA

Planck constant

The Planck constant, h, may be written in various units, most commonly:
$h = 6.626 \times 10^{-34}$ J s when working with joules
or
$h = 4.141 \times 10^{-15}$ eV s

WORKED EXAMPLE 13.1.3

Find the range of energies of photons in the visible spectrum, in eV. The visible spectrum ranges from blue light (wavelength approximately 400 nm) to red light (wavelength approximately 700 nm).

ANSWER

For 400 nm:

1 **Determine the formula.**

$E = hf$ given that $f = \frac{c}{\lambda}$:

$$E = \frac{hc}{\lambda}$$

$$E_{max} = \frac{hc}{\lambda_{min}}$$

2 **Substitute the known values.**

$$E_{max} = \frac{4.14 \times 10^{-15}\,\text{eV s} \times 3.0 \times 10^{8}\,\text{m s}^{-1}}{400 \times 10^{-9}\,\text{m}}$$

3 **Calculate the answer.**

$E_{max} = 3.1$ eV

For 700 nm:

1 **Determine the formula.**

$$E_{min} = \frac{hc}{\lambda_{max}}$$

2 **Substitute known values.**

$$E_{min} = \frac{4.14 \times 10^{-15}\,\text{eV s} \times 3.0 \times 10^{8}\,\text{m s}^{-1}}{700 \times 10^{-9}\,\text{m}}$$

3 **Calculate the answer.**

$E_{min} = 1.8$ eV

WORKED EXAMPLE 13.1.4

Light of frequency 6.20×10^{14} Hz is incident on a polished sodium surface. The work function for sodium is 3.94×10^{-19} J.

a Determine the maximum kinetic energy of any ejected photoelectrons.

b Calculate the threshold frequency for sodium.

ANSWER

a 1 **Determine the formula.**

$E_{k(max)} = hf - W$

2 **Substitute the known values.**

$E_{k(max)} = 6.626 \times 10^{-34}\,\text{J s} \times 6.20 \times 10^{14}\,\text{Hz} - 3.94 \times 10^{-19}\,\text{J}$

3 **Calculate the answer.**

$E_{k(max)} = 4.11 \times 10^{-19} - 3.94 \times 10^{-19}$ J

$E_{k(max)} = 1.71 \times 10^{-20}$ J

b 1 **Determine the formula.**

$W = hf_0$

2 **Rearrange to find the unknown.**

$$f_0 = \frac{W}{h}$$

3 **Substitute the known values.**

$$f_0 = \frac{3.94 \times 10^{-19}\,\text{J}}{6.63 \times 10^{-34}\,\text{J s}}$$

4 **Calculate the answer.**

$f_0 = 5.94 \times 10^{14}$ Hz

LEARNING CHECK 13.1

DESCRIBING

1 **Explain** what the work function of a metal is.

2 **Describe** how $E_{k(max)} = hf - W$ illustrates that the photoelectric effect obeys the law of conservation of energy.

3 **Describe** how a photoelectron differs from any other electron.

4 **Explain** how you can determine the Planck constant from a graph of frequency of incident light and kinetic energy, E_k.

5 **Explain** how the value of the Planck constant value may be determined by using the V_s and wavelength of incident light.

6 It requires more energy to remove an electron from the surface of a polished piece of copper than from a polished piece of lithium.

a Which metal has the larger work function?
b Which metal has the greater threshold frequency?
c Which metal has the greater threshold wavelength?

APPLYING

7 If a metal has a work function *W*, and is irradiated with incident light of frequency *f*, how is the possible energy of any emitted photoelectrons determined?

8 The graph in Figure 13.1.3 shows the results of a photoelectric experiment using magnesium metal.

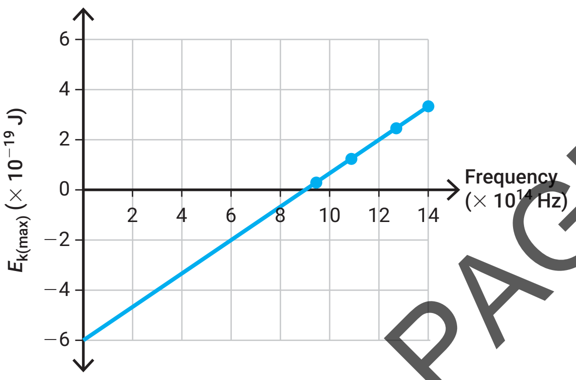


FIGURE 13.1.3 The results of a photoelectric effect for magnesium

a **Determine** a value for the Planck constant from this graph.
b **Determine** a value for the work function of magnesium.
c If silver (work function 4.73 eV) had been used in this experiment instead of magnesium, where on the graph would a line for silver lie relative to the line for magnesium?

ANALYSING

9 Figure 13.1.4 shows a photoelectric tube with light of frequency *f* and intensity *I* incident on a metal cathode. Electrons emitted from the cathode are collected at the anode. The potential difference (pd or V) between the anode and cathode is varied, and the resulting photocurrent is measured. Figure 13.1.5 shows the results of this experiment.

a Why is the photocurrent constant at positive values of potential difference?
b If the frequency of the light is varied, which of the graphs in Figure 13.1.6 represents the relationship between the stopping voltage, V_s, and *f*?

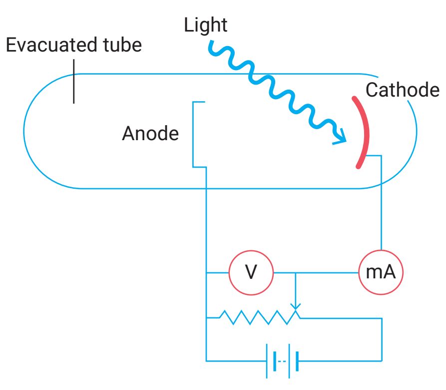


FIGURE 13.1.4 The apparatus for measuring the photoelectric effect

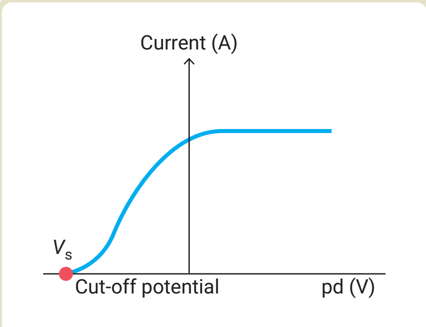


FIGURE 13.1.5 A graph of photocurrent versus applied potential difference

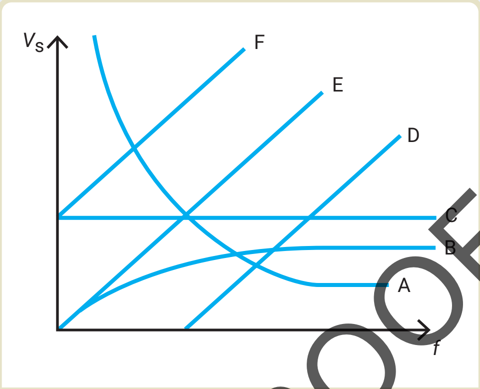


FIGURE 13.1.6

13.2 The photoelectric effect – graphical analysis

Graphing data to determine fundamental physical constants, such as the Planck constant or the work function, is a crucial technique in experimental physics because it allows for a more accurate and visual interpretation of the relationship between variables.

Clarifying relationships between variables

Graphing allows us to visualise the linear or non-linear relationships between variables in an experimental set-up. For instance, the Planck constant (h) is often determined using the photoelectric effect equation:

$$E_k = hf - W$$

where E_k is the kinetic energy of ejected electrons, h is the Planck constant, f is the frequency of incident light, and W is the work function of the material.

When plotted as E_k versus f, this relationship is linear, with the slope of the line providing h, and the y intercept giving W. This graphical method provides a representation of all data and quantitative results.

Reducing uncertainty through multiple data points

Using a graph also allows multiple experimental data points to be incorporated. A single measurement might be prone to random errors, but graphing a series of data points and fitting a line or curve of best fit minimises the impact of outliers. The slope and intercepts interpreted or extrapolated from the graph provide more reliable values for the Planck constant or the work function.

Choice of units for kinetic energy (joules vs electron-volts)

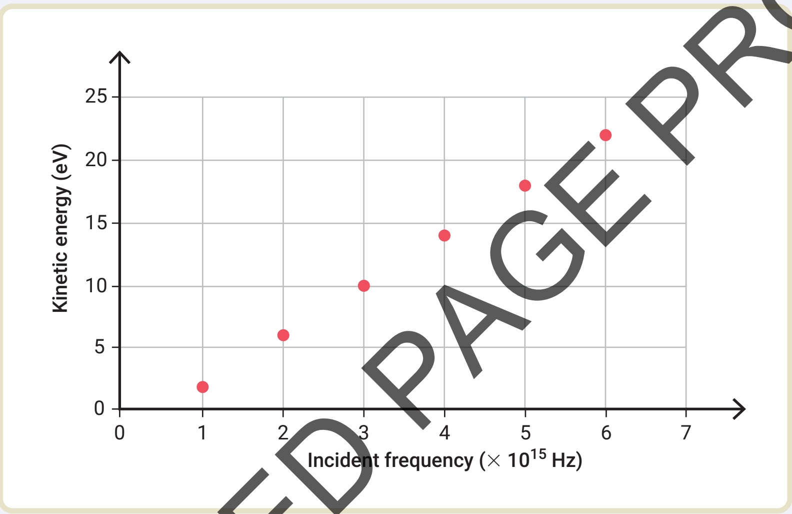
When working with atomic-scale energies, electron-volts (eV) are often more convenient because they align naturally with the scale of energy changes in quantum systems. They simplify interpretation and reduce large numerical values.

Joules (J) are used when maintaining consistency with SI units or when integrating the results into larger systems or classical contexts.

Graphing in either unit can highlight the same relationships, but the choice often depends on the experiment's context and audience; whether values are simply read from graphs or used in further photoelectric calculations.

WORKED EXAMPLE 13.2.1

A photoelectric effect experiment was conducted by shining light of various frequencies onto an unknown metal plate. The graph displays the kinetic energies (eV) of ejected photoelectrons with respect to the frequency of incident light (Hz).



Determine which metal is most likely to have ejected the photoelectrons in this experiment.

TABLE 13.2.1 Work functions of various metals

Metal	Work function (eV)
Potassium	2.30
Copper	4.70
Osmium	5.93

ANSWER

To determine the metal used in the experiment, the unique work function needs to be found and compared to the table of work functions of various metals. The work function, $W = hf_0$, can be calculated by using f_0 from the x intercept of the graph of E_k versus f.

1 Graph the data on your graphing calculator to determine *x* intercept, representing f_0, the threshold frequency.

FPO

2 Determine the formula.

Use the f_0 value to determine the work function using $W = hf_0$.

$f_0 = 0.55 \times 10^{15}$ Hz

3 **Calculate the work function.**

$W = hf_0 = 6.626 \times 10^{-34} \times 0.55 \times 10^{15}$

$W = 3.64 \times 10^{-19}$ J

Converting to eV (1.6×10^{-19} J per electron-volt):

$W = 2.28$ eV

4 **Compare the experimentally determined work function to the work functions of the various metals in the table.**

$W = 2.28$ eV is closest to that of potassium ($W = 2.39$ eV); hence, the metal is most likely potassium.

WORKED EXAMPLE 13.2.2

A photoelectric effect experiment was conducted by shining different wavelengths of light onto a sample of a known metal. The kinetic energy (J) of the ejected photoelectrons was measured against the incident light wavelength and the results are shown in Table 13.2.2.

a Use this data to graph the variables and to determine the relationship between E_k and λ.

b Use the relationship to find the threshold wavelength, λ_0.

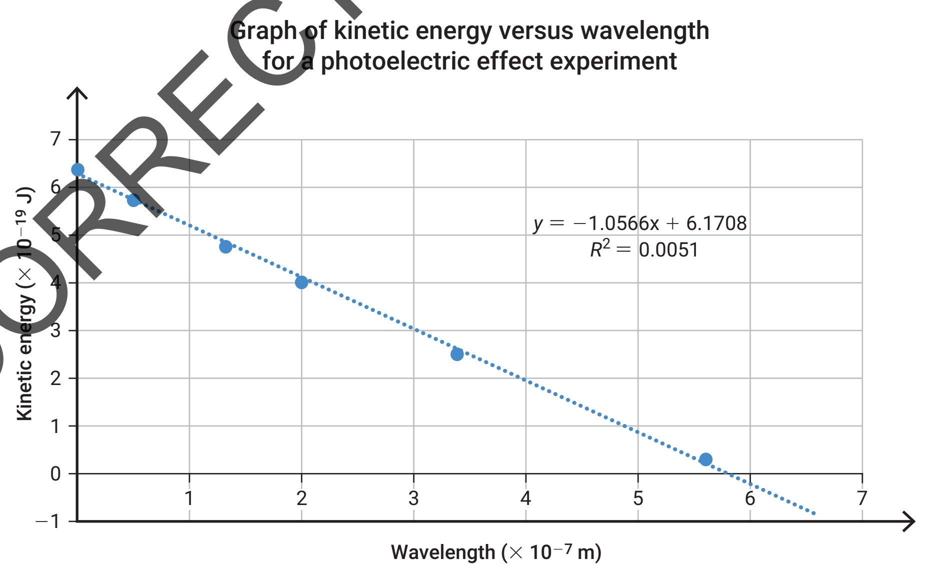
TABLE 13.2.2 The kinetic energy of ejected photoelectrons and wavelength of incident light in a photoelectric effect experiment

Kinetic energy ($\times 10^{-19}$ J)	Wavelength ($\times 10^{-7}$ m)
0.0	6.4
0.5	5.6
1.2	4.8
2.0	4.0
3.4	2.4
5.7	0.3

ANSWER

a 1 **Draw the graph to represent this information.**

The graph of kinetic energy (J) versus wavelength (m) is linear.



2 Determine the relationship between the dependent and independent variables.

To determine the relationship between the variables, substitute the variables measured for y and x in the linear relationship. Remember to take into account the scientific notation attributed to each axis.

$$y = mx + c$$

$$= -1.0566 \times \frac{10^{-19}}{10^{-7}} x + 6.17 \times 10^{-19}$$

$$= -1.0566 \times 10^{-12} x + 6.17 \times 10^{-19}$$

where y represents E_k and x represents λ.

Hence: $E_k = -1.0566 \times 10^{-12} \lambda + 6.17 \times 10^{-19}$

3 Determine the threshold wavelength.

The threshold wavelength, λ_0, is found when $E_k = 0$ J.

$$E_k = -1.0566 \times 10^{-12} \lambda + 6.17 \times 10^{-19}$$

Let $E_k = 0$ J

$$0 = -1.0566 \times 10^{-12} \lambda + 6.17 \times 10^{-19}$$

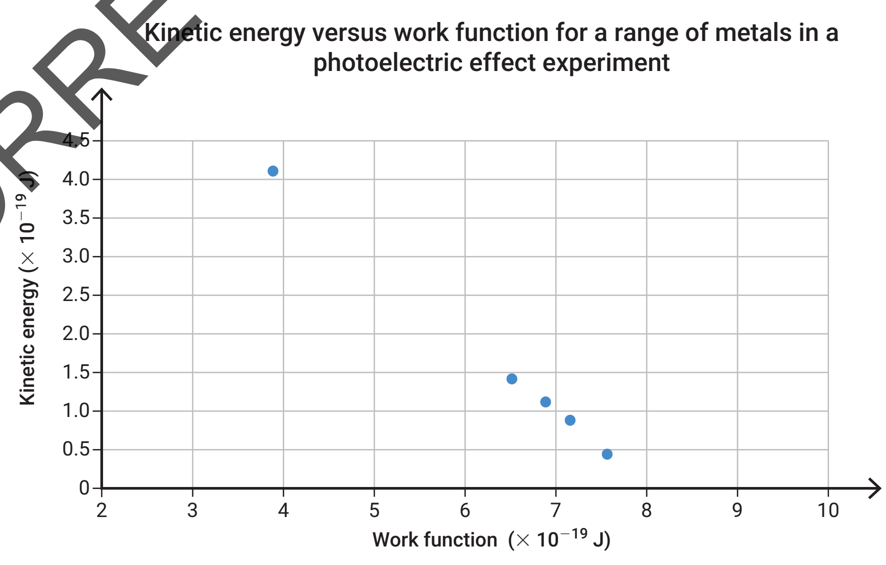
Therefore $1.0566 \times 10^{-12} \lambda = 6.17 \times 10^{-19}$

and $\lambda_0 = \frac{6.17 \times 10^{-19}}{1.0566 \times 10^{-12}}$ or 5.84×10^{-7} m

WORKED EXAMPLE 13.2.3

A photoelectric effect experiment using a range of metals of known work function was conducted with a single light source of unknown frequency. The following graph displays the kinetic energy, E_k, of the photoelectrons emitted (J) versus work function (J).

Use the graph of this data to determine the frequency of light used throughout the experiment.



ANSWER

1 Determine the formula.

To determine the frequency of the incident light, use the values of work function and kinetic energy and the formula $E_{k(max)} = hf - hf_0 = E - W$.

$E_k = E_{in} - W$

Therefore, $E_{in} = E_k + W$

2 Substitute the known values

This may be conducted for all five values and an average taken, as below.

$E_{in} = 4.05 \times 10^{-19} + 3.90 \times 10^{-19} = 7.95 \times 10^{-19}$

$E_{in} = 1.48 \times 10^{-19} + 6.50 \times 10^{-19} = 7.98 \times 10^{-19}$

$E_{in} = 1.15 \times 10^{-19} + 6.90 \times 10^{-19} = 8.05 \times 10^{-19}$

$E_{in} = 0.90 \times 10^{-19} + 7.20 \times 10^{-19} = 8.10 \times 10^{-19}$

$E_{in} = 0.45 \times 10^{-19} + 7.60 \times 10^{-19} = 8.05 \times 10^{-19}$

3 Calculate the average.

$$E_{in}(\text{average}) = \frac{(7.95 + 7.98 + 8.05 + 8.10 + 8.05) \times 10^{-19}}{5}$$

$$= 8.03 \times 10^{-19}\ \text{J}$$

4 Determine the formula to calculate frequency.

The incident light, E_{in}, has an energy of 8.03×10^{-19} J, and from $E = hf$, the frequency of light may be found.

$$f = \frac{E}{h}$$

5 Substitute the known values.

$$f = \frac{8.03 \times 10^{-19}}{6.626 \times 10^{-34}}$$

6 Calculate the answer.

$f = 1.21 \times 10^{15}$ Hz

Alternatively, the x intercept (about 8.10×10^{-19} J) of the linear relationship of the graph may be used.

The incident light, E_{in}, has an energy of 8.10×10^{-19} J, and from $E = hf$, the frequency of light may be found.

$$f = \frac{E}{h}$$

$$= \frac{8.10 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$$= 1.22 \times 10^{15}\ \text{Hz}$$

WORKED EXAMPLE 13.2.4

The following graph shows the results of a photoelectric effect experiment where electrons were emitted from a metal plate when irradiated by an incident light source. The stopping voltage of photoelectrons for a range of incident frequencies was tabulated. Use the data provided to determine an experimental value for the Planck constant, *h*.

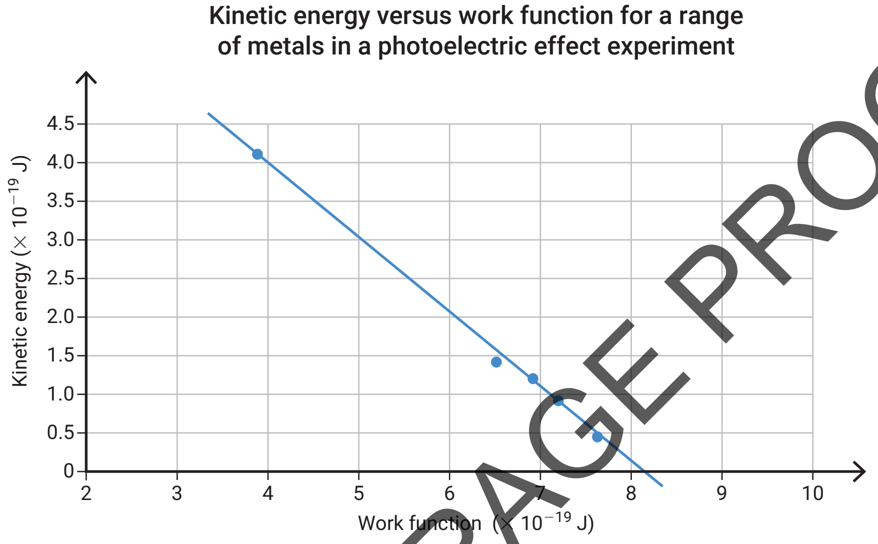


FIGURE 13.2.1 A graph of kinetic energy (J) versus work function (J) for a range of metals in a photoelectric effect experiment. The line of best fit may be used to extrapolate to the x-axis to determine the work function when $E_k = 0$ J.

TABLE 13.2.3 The stopping voltage, V_{stop}, for ejected photoelectrons in a photoelectric effect experiment

V_{stop} (V)	Frequency (× 10^{14} Hz)
0.13	12.0
1.00	14.0
1.38	15.0
1.75	16.0
2.25	17.0

ANSWER

1 **Determine the formula.**

To determine the experimental value for the Planck constant, calculate the gradient of the graph of V_{stop} versus frequency.

$E = qV_{stop} = hf$

$$\frac{V_{stop}}{f} = \frac{h}{q}$$

The gradient, *m* is: $m = \frac{\Delta y}{\Delta x} = \frac{V_{stop}}{f} = \frac{h}{q}$

So, the gradient x q (the charge on an electron) provides an experimental value for *h*.

$$m = \frac{\Delta y}{\Delta x}$$

2 **Substitute known values.**

$$m = \frac{2.25 - 0.13}{(17.0 - 12.0) \times 10^{14}}$$

3 **Calculate the gradient.**

$$m = \frac{2.12}{5.0 \times 10^{14}}$$

$$= 4.24 \times 10^{-13}\ \text{J s C}^{-1}$$

4 **Determine the formula to calculate *h*.**

$h = \text{gradient} \times q$

5 **Substitute the known values.**

$h_{exp} = 4.24 \times 10^{-13} \times 1.6 \times 10^{-19}$ J s

6 **Calculate the answer.**

$h_{exp} = 6.78 \times 10^{-34}$ J s

This experimental value of *h* is very close to the accepted theoretical value of $h = 6.626 \times 10^{-34}$ J s

LEARNING CHECK 13.2

DESCRIBING

1 Incident light with a frequency of 1.59×10^{15} Hz was shone onto a metal surface with a work function value of $W = 1.15 \times 10^{-18}$ J. **Determine** the kinetic energy of a photoelectron ejected from the metal surface in joules and in electron-volts.

2 A photoelectron with kinetic energy of 2.19×10^{-19} J is ejected when a photo of wavelength 2.273×10^{-7} m is incident on the metal surface. **Determine** the work function and the threshold frequency of the metal plate.

3 **Calculate** the frequency of incident light that would be required to eject a photoelectron with a velocity of 1.82×10^{6} m s^{-1} from a plate of metal with a work function of 4.52 eV.

APPLYING

4 When light of various frequencies was shone on a metal surface in an evacuated glass tube, photoelectrons were emitted and then decelerated by a retarding voltage (Table 13.2.4). Prepare a graph to plot this data to then **determine** the threshold frequency as well as a value for the Planck constant.

TABLE 13.2.4

Wavelength (nm)	V_{stop} (V)
582	0.361
534	0.565
488	0.713
468	0.803
426	0.978

5 When light of various frequencies shone on a metal surface in an evacuated glass tube, photoelectrons were emitted and then decelerated by a retarding voltage. Prepare a graph to plot this data (Table 13.2.5) to then determine the threshold frequency as well as a value for the Planck constant.

TABLE 13.2.5

Frequency ($\times 10^{14}$ Hz)	Kinetic energy (eV)	Kinetic energy ($\times 10^{-20}$ J)
4.08	0.70	
4.66	0.98	
5.10	1.08	
5.52	1.24	
6.01	1.43	

ANALYSING

6 When light of various frequencies was shone on a metal surface in an evacuated glass tube, photoelectrons were ejected and then decelerated by providing a retarding (stopping) voltage. The stopping potential, V_{stop}, was measured for each frequency of incident light. Data obtained from replication of this experiment is displayed in Table 13.2.6.

TABLE 13.2.6

Incident frequency ($\times 10^{14}$ Hz)	Stopping voltage (V)
4.75	0.43
5.80	0.91
6.70	1.23
7.20	1.41

a Graph the data and extrapolate to find the threshold frequency and hence **determine** the work function of the metal.

b **Predict** the maximum kinetic energy of the photoelectrons produced when ultraviolet light of frequency 1.93×10^{16} Hz is incident on the metal surface.

c If light of wavelength 821 nm is incident on the surface of the metal, **describe** what effect, if any, would be observed in the apparatus.

7 A photosensitive surface is illuminated with monochromatic light of different wavelengths. The potential differences applied to prevent electrons released by the light from crossing into the collector and registering as a current are shown in Table 13.2.7.

TABLE 13.2.7

Wavelength ($\times 10^{-7}$ m)	Potential difference (V)
3.66	1.48
4.05	1.15
4.36	0.93
4.92	0.62
5.46	0.36
5.79	0.24

Plot a suitable graph and hence **determine** the:

a threshold frequency

b work function of the metal surface

c experimental value of the Planck constant

d maximum kinetic energy (J) with which electrons are ejected from the surface by incident light of wavelength 3.66×10^{-7} m.

Weblink
The History of the Atom – Theories and Models

13.3 The model of the atom and atomic spectra

We saw earlier that observations of black-body radiation and the photoelectric effect led to the development of the photon (particle) model of light. While Planck, Einstein and others were developing this new quantum model of light, other physicists were investigating the nature of atoms.

Thomson model of the atom

The idea that matter consisted of atoms dates back to ancient Greece, and until the end of the 19th century it was believed that atoms were indivisible. Then in 1897, J.J. Thomson (1856–1940) experimentally observed electrons being emitted from atoms. This established that an atom was not a single homogeneous particle but consisted of matter – subatomic particles – with positive and negative charges. At the time, it was not known how the positive and negative parts were arranged.

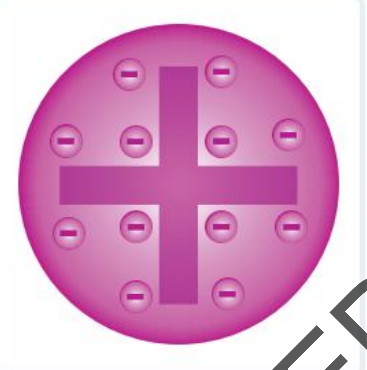


FIGURE 13.3.1 Thomson's plum-pudding model of the atom

Measurements of the mass of electrons showed that almost all the mass of an atom was associated with the positive part of the atom. On the basis of this evidence, in 1904 Thomson suggested a model in which the atom consisted of a sphere of positive charge with electrons embedded in it. This model was known as the 'plum-pudding' model of the atom (Figure 13.3.1), as the electrons were scattered through the positive matter, like raisins scattered through a plum pudding (or chocolate chips in a chocolate chip muffin).

Rutherford model of the atom

Then, in 1909 Hans Geiger (1882–1945) and Ernest Marsden (1889–1970), who were students working with Ernest Rutherford (1871–1937), performed the alpha-particle scattering experiment in which a thin gold foil was placed in a beam of alpha particles (helium nuclei). When they measured the paths of the scattered alpha particles, they found that most alpha particles passed through the metal foil and were only minimally deflected, if at all; however, about one in 20 000 was scattered backwards towards the source. This result implied the presence of a tiny, very dense and positively charged nucleus. Rutherford later said, 'It was as if you fired a 15-inch shell at a sheet of tissue paper and it came back to hit you'.

Rutherford went on to suggest a planetary model of the atom to explain these observations. This model involved electrons orbiting the nucleus in a circular path under the influence of the electrostatic force. This was analogous to the way planets orbit the Sun under the influence of the gravitational force.

Although Rutherford's model fitted the experimental data from the gold-foil scattering experiment, there were still problems. In this planetary model, the electrons (charged particles) were undergoing centripetal acceleration. This is not a problem for uncharged objects such as planets, but accelerating charged particles emit energy. The electrons should have been acting like those in an antenna and emitting electromagnetic waves. Because energy is always conserved, an electron that was emitting energy as electromagnetic radiation would lose kinetic and potential energy and spiral into the nucleus and the atom would collapse. This was a significant flaw in this model.

Atomic spectra

None of the classical models of the atom, such as those described above, fully explained or predicted the behaviour of atoms. New models and theories were needed. Experimental data showing that the energy in atoms is quantised would prove vital for developing the new quantum mechanical atomic theory. This data came from the measurements of atomic line spectra.

A **spectroscope** like the one shown in Figure 13.3.2 uses a prism or diffraction grating to separate, or disperse, light into its component colours. White light disperses into all the colours of the rainbow. We have already seen in Chapter 12 that black bodies produce a continuous spectrum. In contrast, when a gas is heated, it produces a spectrum consisting of discrete colours. This is observed through a spectroscope as a pattern of parallel coloured lines and hence is called a **line spectrum**. A heated gas produces an **emission spectrum**. When white light is passed through a cold gas, an **absorption spectrum** is produced. Absorption spectra have the same characteristic lines as emission spectra, but they are dark lines on a continuous coloured background.

FIGURE 13.3.2 In a spectroscope, a prism is used to disperse the light, allowing specific elements to be detected by their unique emission spectrum.

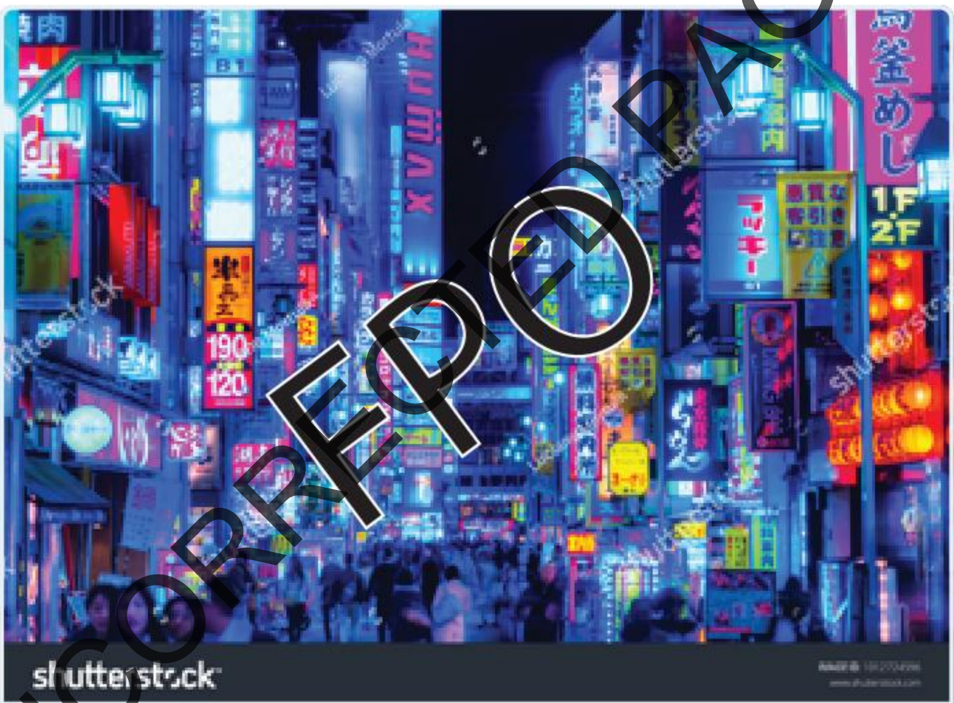


FIGURE 13.3.3 Neon lights (which are not necessarily neon, but other inert gases) are an example of gas discharge tubes.

spectroscope a device that disperses radiation by energy (or wavelength or frequency) so that a spectrum may be observed and measured

line spectrum an emission or absorption spectrum consisting of discrete lines that are characteristic of the energy levels of a particular atom or molecule

emission spectrum the spectrum of radiation emitted by an object (e.g. black-body radiation or atomic spectra from a discharge tube)

absorption spectrum the wavelengths (or frequencies or energies) of radiation absorbed by a material

Gustav Kirchhoff (1824–87) and Robert Bunsen (1811–99) had recognised as early as the 1860s that line spectra can be used to identify elements. Using spectroscopy, they discovered two new elements, caesium and rubidium, in 1861.

Although Kirchhoff, Bunsen and others had observed and used spectra, there was no theory that explained why the phenomena existed, although it was presumed that the characteristic spectra were related to the internal structure of the atom. To solve this puzzle, the simplest atom, hydrogen, was subject to intense theoretical and experimental investigation.

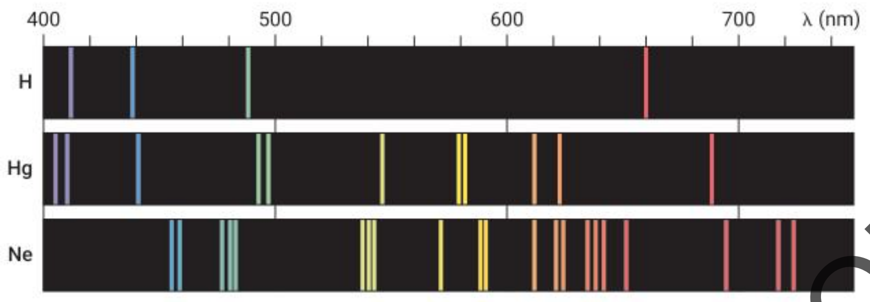


FIGURE 13.3.4 Emission spectra of hydrogen (H), mercury (Hg) and neon (Ne). Each element's emission spectrum is unique and reflects the energy differences between electron energy levels.

Hydrogen spectra

Hydrogen produces infrared, visible and ultraviolet emission spectra. The emission and absorption spectra of hydrogen are shown in **Figure 13.3.5**. Dark lines in the absorption spectrum of any element coincide with the bright lines in its emission spectrum. **Figure 13.3.6** shows the wavelengths on the emission spectra of hydrogen.

Weblink
Emission and absorption spectra

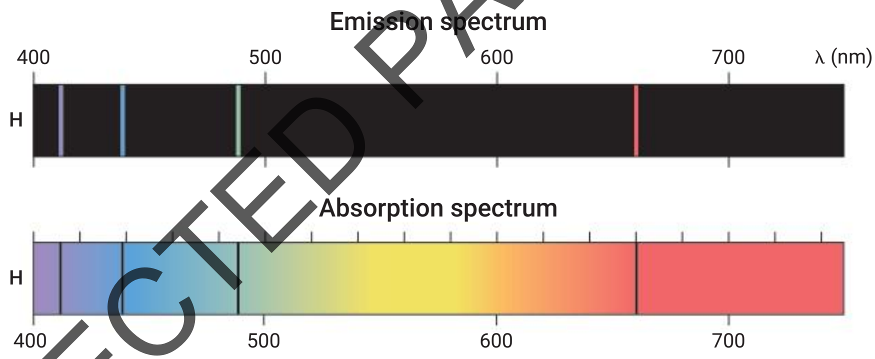


FIGURE 13.3.5 The emission and absorption spectra of hydrogen

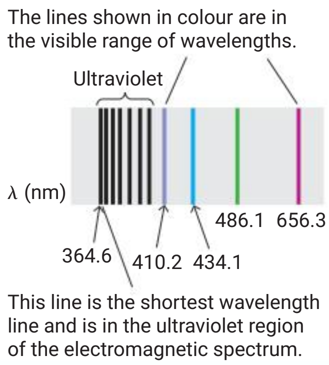


FIGURE 13.3.6 Wavelengths in the emission spectra of hydrogen, some of which are in the visible region

Balmer series of spectral lines for atomic hydrogen

In 1855, Johann Balmer (1825–1898) derived an empirical formula for the visible series that now bears his name.

Balmer showed that the observed wavelengths were proportional to $\frac{m^2}{(m^2 - n^2)}$ with $n = 2$ and m greater than n. In Balmer's calculations m represents the initial orbital, and n represents the final orbital, set at $n = 2$.

The other spectral series of hydrogen are named after those who discovered them. The Lyman series is in the ultraviolet region and the Paschen series is in the infrared region of the spectrum. Table 13.3.1 and shows the different wavelengths from the different spectral series of hydrogen. Other series of even longer wavelength are the Brackett series and the Pfund series. These lines have a similar pattern of separation, but with different values for n and m.

TABLE 13.3.1 Lines in the spectral series of hydrogen

Series name	Part of electromagnetic spectrum	Shortest wavelength (nm)	Longest wavelength (nm)
Lyman	Ultraviolet	91.1	121.6
Balmer	Visible	364.5	656.3
Paschen	Infrared	820.1	1870.0

In the 1880s, Johannes Rydberg (1854–1919) was working on finding a mathematical description of the line spectra of the alkali metals (e.g. lithium and sodium). He read Balmer's work on hydrogen and realised that his own mathematical model and Balmer's were equivalent. Rydberg expressed the relationship as:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where λ is the wavelength of the line, n_f and n_i are integers and R is a constant known as the Rydberg constant, $R = 1.097 \times 10^7\ m^{-1}$.

Rydberg arrived at his formula empirically; that is, by fitting an equation to the observed data. At the time there was no theoretical model of the atom that could predict the relationship between positions of spectral lines, or even the existence of spectral lines. His work was important in that it led to the development of the first quantum mechanical model of the atom – the Bohr model.

KEY FORMULA

R = Rydberg constant = $1.097 \times 10^7\ m^{-1}$

Note: The Rydberg constant is applicable only to the hydrogen spectrum.

WORKED EXAMPLE 13.3.1

For the Brackett series in the far infrared, $n_f = 4$, find the

a longest wavelength in the series

b shortest wavelength in the series.

ANSWER

a **1** **Determine the formula.**

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

2 **Substitute the known values.**

The longest wavelength corresponds to the smallest energy, which would occur between the $n = 5$ and $n = 4$ states:

$$\frac{1}{\lambda} = 1.097 \times 10^7\ \text{m}^{-1} \times \left(\frac{1}{4^2} - \frac{1}{5^2}\right)$$

3 **Calculate the answer.**

$$\frac{1}{\lambda} = 2.47 \times 10^5\ \text{m}^{-1}$$

$$\lambda = 4050\ \text{nm}$$

b **1** **Determine the formula.**

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$= R\left(\frac{1}{n_f^2} - \frac{1}{\infty^2}\right)$$

$$= R\left(\frac{1}{n_f^2}\right)$$

2 **Substitute the known values.**

The shortest wavelength is the largest energy, or when the electron comes from $n = \infty$ to $n = 4$:

$$\lambda = \frac{n_f^2}{R} = \frac{4^2}{1.097 \times 10^7\ \text{m}^{-1}}$$

3 **Calculate the answer.**

$$\lambda = 1460\ \text{nm}$$

Bohr model of the atom

In 1913, Niels Bohr (1885–1962) combined the concepts of Rutherford's planetary model of the atom and Einstein's photons to predict the observed spectra of hydrogen. To solve the problems of the planetary model, Bohr made several postulates.

KEY CONCEPT

Bohr's postulates

1 An electron in an atom moves in a circular orbit about the nucleus under the influence of the electrostatic attraction of the nucleus.
2 Only certain orbits are stable. Electrons in these orbits do not emit energy.
3 The greater the radius of the orbit, the greater its energy. Atoms emit radiation when an electron goes from one orbit to another orbit with lower energy. The energy released is:
$E = E_f - E_i = hf$
4 The orbits are characterised by quantised radii, given by:

$$r = \frac{nh}{2\pi m_e v}$$

where:
r = radius (m)
m_e = mass of electron (kg)
v = velocity (m s^{-1})
h = Planck constant
n = an integer

Weblink
Atomic flashback: a century of the Bohr model

The postulates were a mixture of classical physics (postulate 1), recently introduced quantum principles (postulate 3) and completely new ideas (postulates 2 and 4).

Postulate 1 was drawn directly from the earlier planetary model of the atom. Postulate 2 simply stated what had been observed – that atoms do not collapse. Postulates 3 and 4 distinguish his model as the first quantum model of the atom. Postulate 3 states that energies are quantised and may take only a discrete set of values. The relationship between radius and energy is given by classical electromagnetism. The greater the separation between a positive charge (the nucleus) and a negative charge (the electron), the greater the potential energy of the system. The different possible energies are called **energy levels**. Using classical electromagnetism, Bohr showed that the energy of any given level is proportional to $\frac{1}{n^2}$, where n is the integer in the equation for the radius in postulate 4. Hence, $E_n = \frac{k}{n^2}$, where k is a constant.

energy levels the allowed energies of a nucleus–electron system; often referred to as electron energy levels, even though they are characteristic of the atom, not of individual electrons

angular momentum the momentum associated with rotational or orbital motion, $L = mvr$

Postulate 4 was also based on the quantisation of a physical property, in this case the **angular momentum**, L, of the electron. Angular momentum is to circular motion what momentum is to linear motion. It is a conserved quantity and is given by $L = mvr$.

In postulate 3, Bohr was saying that only discrete values of r are possible; that is, that only discrete values of energy are allowed. We often refer to these energy levels as electron energies; however, we must remember that this energy belongs to the electron–nucleus system as they are separated charged particles. All isolated atoms of one element have the same set of energy levels, but specific elements have different sets of energy levels, due to their different nuclei and numbers of electrons.

KEY FORMULA

$L = mvr$

where:
L = angular momentum of object (kg m^2 s^{-1})
m = mass (kg)
v = velocity (m s^{-1})
r = orbital radius (m)
By saying that L may only have discrete values, then:

$$L = mvr = \frac{nh}{2\pi}$$

where:
h = Planck constant
n = an integer (the quantum number)

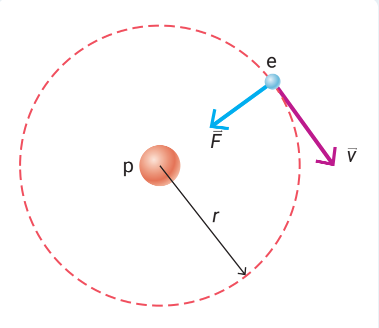


FIGURE 13.3.7 In Bohr's model of the hydrogen atom, the electron occupies discrete orbits.

An atom can make a transition from one level to a lower level by emitting a photon of energy equal to the difference between the levels. This occurs when an electron moves from an orbit further from the nucleus to one closer to the nucleus. In contrast, for an electron to move from a lower energy level to a higher energy level, it must absorb energy. The Bohr model was thus able to explain the existence of discrete line spectra, as emission spectra can be explained by electrons moving from higher to lower energy orbits. The energy gap between the energy levels is equal to the energy of the photon emitted when the transition occurs.

In fact, Bohr showed that the constant $\frac{k}{h}$ was equal to the Rydberg constant, $R = 1.097 \times 10^7\,\text{m}^{-1}$. Bohr's model now not only predicted the existence of line spectra but quantitatively predicted the positions of the lines for the hydrogen atom.

KEY FORMULA

$E_{gap} = E_f - E_i = hf_{photon}$

and recalling that $E_n = \frac{k}{n^2}$, we can write:

$hf_{photon} = E_f - E_i = k\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$, where k is a constant.

If we now use the universal wave equation, $f = \frac{c}{\lambda}$, we can see that:

$$\frac{1}{\lambda} = \frac{k}{h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

For hydrogen, this relationship is expressed as the Rydberg equation:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where R = Rydberg constant = $1.097 \times 10^7\,\text{m}^{-1}$

Hydrogen emission spectrum

ground level the lowest possible energy level of a nucleus–electron system

Bohr's model explained the observed line spectra as resulting from transitions between energy levels in atoms. The lowest wavelength (highest energy) line corresponds to the ionisation energy of electrons in the lowest possible energy level. This level is called the **ground level** of the atom and corresponds to the electron being in the orbit with the smallest radius. Ionisation is the removal of an electron from the atom to an infinite distance, or at least so far away that the electrostatic attraction is negligible.

Energy levels can be represented in two ways (Figure 13.3.8):

1. with the ionisation energy being taken as zero (all the energy states then have a negative potential energy)
2. with the ground state level being taken as zero.

The energy difference between levels is the same in both representations. We shall generally use the first representation because this corresponds to the usual convention for choosing the zero of potential energy, as in electromagnetism.

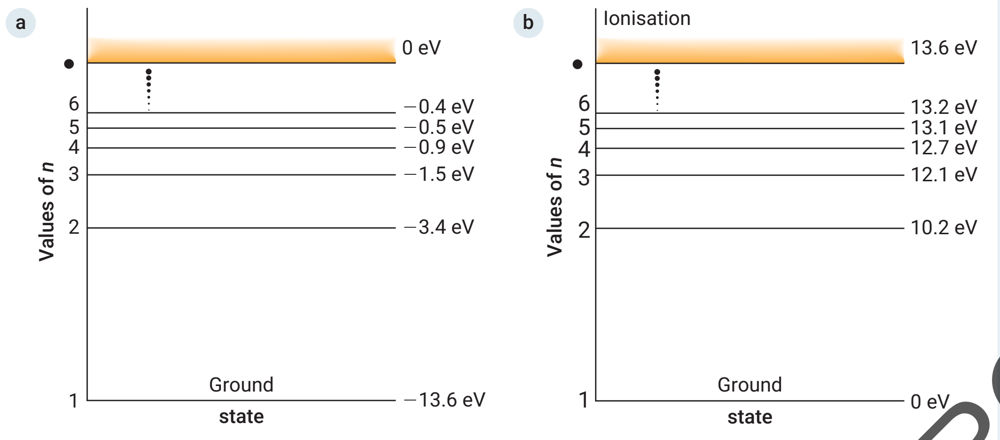


FIGURE 13.3.8 The energy levels of the hydrogen atom have the (**a**) ionisation energy as zero and (**b**) ground state as zero. Either form is acceptable although (a) is more common.

The energy of each level can be deduced from the wavelengths, and hence energies, of the lines in the emission spectrum. The highest energy lines for hydrogen are those in the Lyman series in the ultraviolet region. These lines correspond to transitions to the ground state from higher energy levels, hence for these lines $n_f = 1$ and $n_i = 2, 3, 4$ ... The Balmer series, in the visible region, corresponds to transitions to the $n = 2$ level, so $n_f = 2$ and $n_i = 3, 4, 5$ ...

Figure 13.3.9 shows the energy levels for hydrogen and the transitions corresponding to these two series of spectral lines.

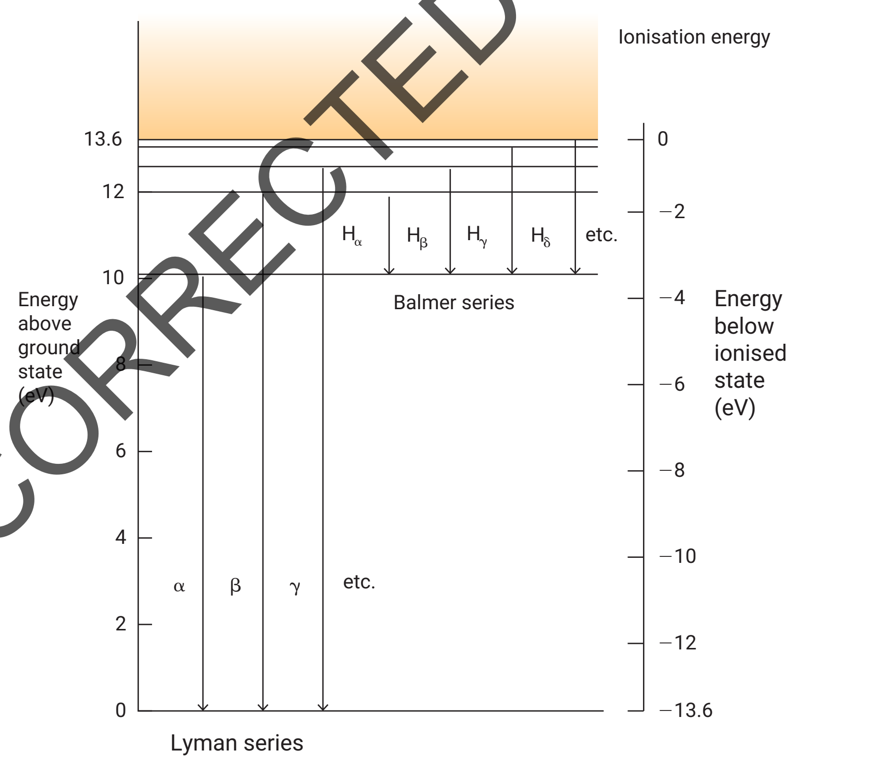


FIGURE 13.3.9 This energy level model for hydrogen shows the transitions corresponding to the Lyman and Balmer emission spectral series.

WORKED EXAMPLE 13.3.2

Refer to Figure 13.3.9 to calculate the:

a frequency of the lowest energy line in the Lyman series.

b wavelength corresponding to the highest energy line in the Balmer series.

ANSWER

a **1** **Determine the formula.**

$E_f - E_i = hf$

2 **Rearrange to find the unknown.**

$$f = \frac{E_f - E_i}{h}$$

3 **Substitute the known values.**

$$f = \frac{(10.2\,\text{eV} - 0\,\text{eV}) \times 1.6 \times 10^{-19}\,\text{J eV}^{-1}}{6.63 \times 10^{-34}\,\text{J s}}$$

4 **Calculate the answer.**

$$f = \frac{1.63 \times 10^{-18}\,\text{J}}{6.63 \times 10^{-34}\,\text{J s}}$$

$$= 2.46 \times 10^{15}\,\text{Hz}$$

Alternative method

1 **Determine the formula.**

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

2 **Substitute known values.**

R = the Rydberg constant = $1.097 \times 10^7\,\text{m}^{-1}$,
and the electron falls from $n = 2$ to $n = 1$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$= 8.23 \times 10^6$$

3 **Calculate λ.**

$\lambda = 1.22 \times 10^{-7}$ m or 122 nm

4 **Determine the formula to calculate *f*.**

Since $c = f\lambda$, then $f = \frac{c}{\lambda}$.

5 **Substitute the known values.**

$$f = \frac{3.00 \times 10^8}{1.22 \times 10^{-7}}$$

6 **Calculate the answer.**

$f = 2.46 \times 10^{15}$ Hz

b **1** **Determine the formula for *f*.**

$E_f - E_i = hf$

2 **Substitute the known values.**

E_f = 13.6 and 0 eV

$$f = \frac{(13.6\,\text{eV} - 0\,\text{eV}) \times 1.6 \times 10^{-19}\,\text{J eV}^{-1}}{h}$$

$$= \frac{13.6 \times 1.6 \times 10^{-19}\,\text{J}}{6.63 \times 10^{-34}\,\text{J s}}$$

3 **Calculate *f*.**
$f = 3.28 \times 10^{15}$ Hz

4 **Determine the formula to calculate λ.**
Since $c = f\lambda$, then $\lambda = \frac{c}{f}$.

5 **Substitute the known values.**
$$\lambda = \frac{3.00 \times 10^8}{3.28 \times 10^{15}}$$

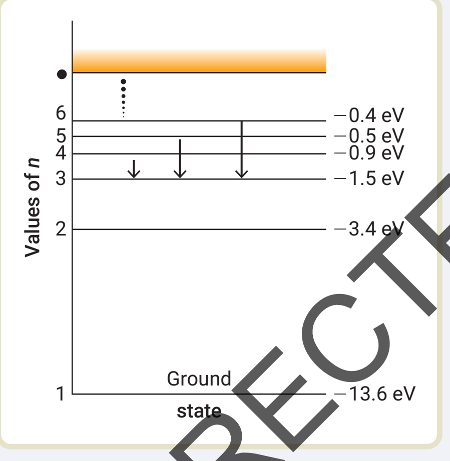
6 **Calculate the answer.**
$\lambda = 9.15 \times 10^{-8}$ m or 91.5 nm

WORKED EXAMPLE 13.3.3

a Construct an energy level diagram like that shown in Figure 13.3.9, showing the transitions corresponding to the Paschen series, for which $n_f = 3$ and $E_3 = -1.5$ eV.
b Calculate the frequency of a photon released in a transition from $n_i = 6$ to $n_f = 3$.

ANSWER

a



b 1 **Determine the formula.**
$E_f - E_i = hf$

2 **Rearrange to find the unknown.**
$$f = \frac{E_f - E_i}{h}$$

3 **Substitute the known values.**
Since $h = 6.626 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV
$$f = \frac{-0.4\,\text{eV} - (-1.5\,\text{eV})}{4.14 \times 10^{-15}\,\text{eV s}}$$

4 **Calculate the answer.**
$f = 2.66 \times 10^{14}$ Hz

Absorption spectra occur when a photon of exactly the right energy is incident on an atom. A photon with too little or too much energy cannot be absorbed; only a photon with energy corresponding exactly to a gap between energy levels can be absorbed. As a gas, most atoms will be in their ground state; that is, their electrons will be in their lowest possible energy levels. Hence, the most likely transitions will be from the ground state, $n = 1$, to higher levels. However, as long as the temperature is above absolute zero, there will always be some atoms in an excited state, with electrons in levels $n = 2$, $n = 3$ and so on. That is why in an absorption spectrum, we see lines corresponding to all the transitions of the emission spectrum.

As in the emission spectrum, the Balmer series in the absorption spectrum corresponds to electrons going *from* the $n = 2$ state to higher levels **(Figure 13.3.10)**.

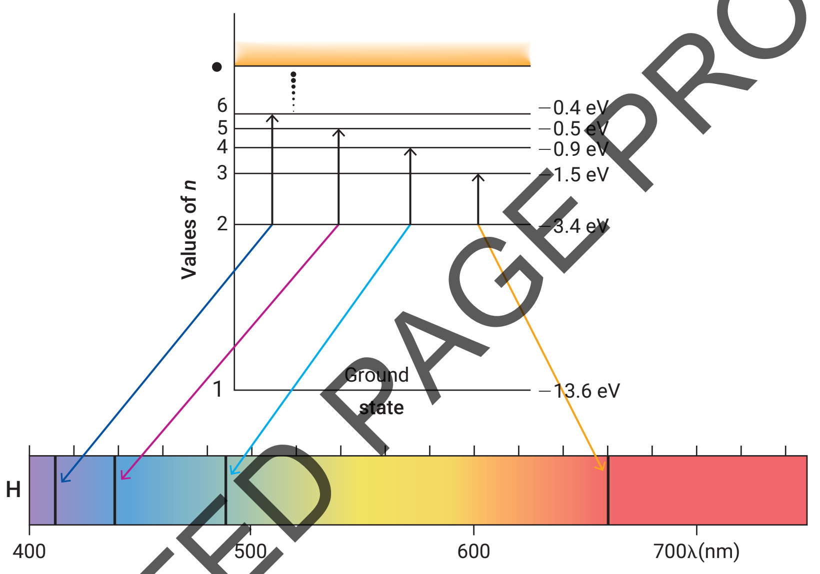


FIGURE 13.3.10 The absorption spectrum of hydrogen and the corresponding transitions are represented on this energy level diagram.

WORKED EXAMPLE 13.3.4

Consider photons of the following energies: 1.9, 10.2, 12.5 and 13.6 eV. Which of these could be absorbed by hydrogen gas? Explain your answer.

ANSWER

Referring to the hydrogen energy level diagram, Figure 13.3.10 and the possible energy differences using $E_f - E_i$, the:

- 1.9 eV photon could be absorbed (transition from $n = 2$ to $n = 3$)
- 10.2 eV photon could be absorbed (transition from $n = 1$ to $n = 2$)
- 12.5 eV photon could not be absorbed (there is no energy gap corresponding to 12.5 eV)
- 13.6 eV photon could be absorbed (transition from $n = 1$ to $n = \infty$, i.e. atom is ionised)

Spectra of other atomic species

Each type of atom produces a unique line spectrum. The energy levels are unique because each type of atom has a different number of protons and hence a different nuclear charge. Accordingly, the force exerted by the nucleus on the electrons differs, giving rise to different potential energies at different orbital radii, $r = \dfrac{nh}{2\pi m_e v}$. There is also a different number of electrons in different atoms. Electrons close to the nucleus 'shield' the outer electrons somewhat from the nuclear charge. This also acts to change the potential energy at the different allowed radii. These effects combine to give a unique fingerprint for each atom in the form of a unique line spectrum. This is extremely useful because it allows the presence of different types of atoms to be detected. For example, we know from the line spectrum of the Sun that there is a great deal of hydrogen and helium present, as well as larger atoms. Other stars have different characteristic spectra, indicating the presence of other atomic species.

Different molecules also have characteristic spectra. The spectrum of a molecule is not simply the sum of the spectra of the atoms that make up the molecule. This is because when atoms bind together to form a molecule, the energy levels change. Hence, it is possible to distinguish between ethanol and methanol by their spectra, even though both contain only carbon, hydrogen and oxygen.

Limitations of the Bohr model

Bohr was able to explain qualitatively and quantitatively the existence and positions of the spectral lines of a hydrogen atom. His estimate of the size of the largest stable radius also agreed closely with the measured size of the hydrogen atom. However, the Bohr model could not predict the spectra of multi-electron atoms, even one as simple as two-electron helium. It also could not explain the different intensities of lines or why some lines split into multiple, closely spaced lines – fine and hyperfine structure – or the magnetically induced Zeeman effect.

Finally, Bohr's model introduced the idea of quantised atomic energy levels, but it did not offer any explanation for why they should be quantised. Successful models have both predictive power and explanatory power. Bohr's model lacked explanatory power and had limited predictive power. It was superseded by a more comprehensive quantum mechanical model developed by Schrödinger (1887–1961) and Heisenberg (1901–76). This modern quantum model built on the ideas of de Broglie, described earlier in this chapter.

LEARNING CHECK 13.3

DESCRIBING

1 Name three physicists who contributed to the development of the model of the atom. Briefly describe their contributions.
2 What were the most important successes and limitations of the Bohr model?
3 State Bohr's four postulates.
4 **Identify** what was quantised in Bohr's model of the atom?
5 The spectral lines for transitions to the $n = 2$ state from higher levels for a particular atom are in the infrared region of the spectrum. Would you expect this atom to have any series of spectral lines in the visible range? If so, to what transitions would they correspond?

6 **Explain** the difference between an absorption spectrum and an emission spectrum. Why are there lines at the same frequencies and wavelengths for each element?

7 **Explain** why Bohr's model of the atom is considered the first quantum mechanical model.

APPLYING

8 How many different emission spectral lines would there be as a result of transitions from the −1.52 eV energy level in sodium gas? The sodium energy levels are shown in Figure 13.3.11.

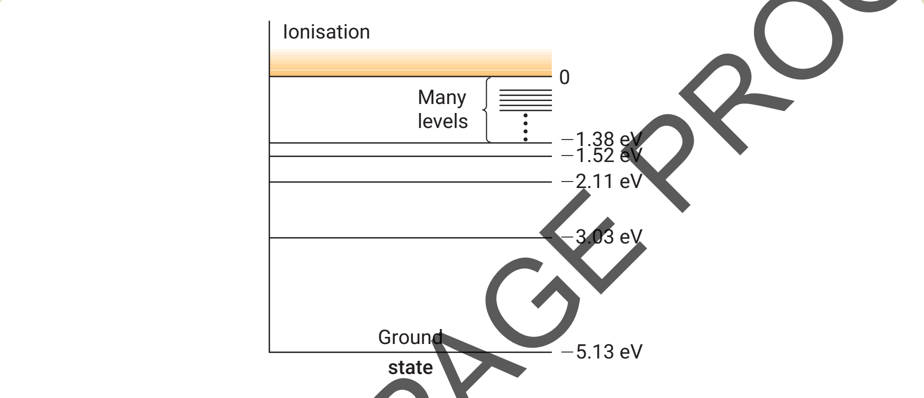


FIGURE 13.3.11 The energy level diagram for sodium

9 The ground-state energy level of the electron in a hydrogen atom is negative. What is the significance of the negative sign? What is the zero-energy level in this model?

APPLYING

10 Figure 13.3.11 shows an energy level diagram for sodium.

a How many possible emission spectral lines are there for transitions from the −2.11 eV level?

b **Determine** the energy released in each of these transitions.

c What are the wavelengths of the photons emitted in these transitions? If any are in the visible light region, identify their colour.

11 What is the shortest wavelength of photons that can be emitted from hydrogen atoms as they return from excited states to the ground state?

12 Two energy levels within a particular atom are 10 eV and *x* eV. When an atom of this element returns from the higher energy level to the lower energy level, radiation of wavelength 450 nm is emitted. What are the possible values of *x*?

13 What is the wavelength of X-ray photons of energy 3.6×10^4 eV?

14 An atom is excited to its third energy level above the ground state, $n = 4$.

a How many different spectral lines can it emit?

b Which energy level transition will produce the photon of greatest energy?

c Which energy level transition will produce a photon of the longest wavelength?

ANALYSING

15 Hydrogen atoms have only one electron, yet the spectrum of hydrogen contains many lines. **Explain** how this is possible.

CHAPTER SUMMARY

Photoelectric effect and experimental results

- When light is shone on a metallic surface, electrons can be emitted from the surface.
- No electrons are emitted unless the frequency of light is above a minimum threshold frequency, f_0, which is unique for each metal.
- The number of electrons emitted, if the frequency is above the threshold frequency, is proportional to the intensity of light.
- There is no time delay between incident light and ejected electrons.
- The kinetic energy of photons can be described by:

$$E_k = qV_s = \frac{1}{2}mv^2$$

Photoelectric effect and the particle nature of light

- The photoelectric effect experiment provides evidence of light acting as a particle because it only gives up its energy in discrete amounts.
- Each photon has energy defined by $E = hf$, where h is the Planck constant (6.626×10^{-34} J s).
- Electrons in the metal plate absorb all energy from the photon, which, if above work function ($W = hf_0$), allows the electron to be ejected with kinetic energy defined as $E_k = hf - W$.

Models of the atom

Model	Description
Thomson's model of the atom	• J.J. Thomson proposed the plum pudding model of the atom in 1904, which was a uniform, positively charged atom with negatively charged electrons embedded within it.
Rutherford's model of the atom	• Rutherford proposed that the atom consists of a small, dense, positively charged nucleus with electrons orbiting around. • However, the model does not explain why negatively charged electrons, which should lose energy and spiral into the positively charged nucleus, do not collapse into the nucleus.
Bohr's model of the atom	• Bohr proposed that electrons orbit the nucleus in specific, quantised energy levels, which correspond to certain fixed distances from the nucleus, without radiating energy. • Electrons can transition between these energy levels. When an electron jumps from a higher energy level to a lower one, it emits a photon with an energy equal to the distance between the two energy levels. • The energy of the emitted photon is defined by $E = hf$, which corresponds to a specific wavelength of light ($\lambda = \frac{c}{f}$, where c is the speed of light, f is the frequency of light, and λ is the wavelength of light). • This model explains why the hydrogen atom's line spectrum consists of discrete lines rather than a continuous range of wavelengths because only specific energy transitions are possible.

Rydberg's formula

- Rydberg expressed a relationship between the wavelength of light and the energy levels of electrons in an atom, which can be used to predict the wavelengths of spectral lines of many chemical elements:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

CHAPTER EXAM

MULTIPLE CHOICE

1. What series of the hydrogen emission spectrum emits light in the visible region?
   - **A** Lyman series
   - **B** Paschen series
   - **C** Balmer series
   - **D** Rydberg series

2. Threshold frequency is defined as the:
   - **A** frequency at which light is absorbed by an atom.
   - **B** frequency of light required for total internal reflection.
   - **C** maximum frequency a metal can absorb without emitting electrons.
   - **D** minimum frequency of light needed to eject an electron from a metal surface.

3. Rutherford's atomic model proposed that:
   - **A** atoms are indivisible.
   - **B** electrons exist in fixed energy levels.
   - **C** electrons orbit the nucleus in fixed paths.
   - **D** atoms have a dense, positively charged nucleus.

4. Bohr's model improved upon Rutherford's model by explaining:
   - **A** the distribution of protons in the atom.
   - **B** the quantised energy levels of electrons.
   - **C** how electrons are embedded within the nucleus.
   - **D** how atoms remain stable in the absence of a nucleus.

5. The term 'ground state' in an atom refers to the:
   - **A** highest energy level of an electron.
   - **B** lowest energy level of an electron.
   - **C** energy required to ionise an atom.
   - **D** state where all electrons are unbound.

6. An electron is emitted with a kinetic energy of 1.5 eV. The photon's energy was 4.0 eV. What is the work function of the metal?
   - **A** 1.5 eV
   - **B** 2.5 eV
   - **C** 3.0 eV
   - **D** 4.0 eV

7. What is the wavelength of light required to eject electrons from a metal with a work function of 3.0 eV?
   - **A** 207 nm
   - **B** 413 nm
   - **C** 515 nm
   - **D** 621 nm

8. If the threshold frequency of a metal is 5.5×10^{14} Hz, what is the work function of the metal?
   - **A** 1.83 eV
   - **B** 2.28 eV
   - **C** 3.65 eV
   - **D** 4.11 eV

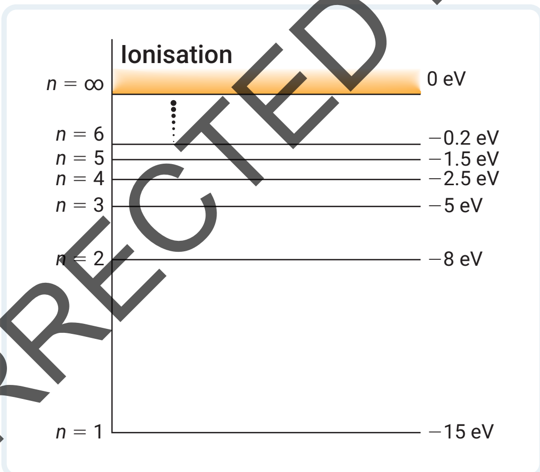
9. A photon with a wavelength of 400 nm strikes a metal with a work function of 2 eV. What is the maximum kinetic energy of the emitted electron?
A 1.24×10^{-19} J
B 2.98×10^{-19} J
C 4.97×10^{-19} J
D 7.45×10^{-19} J

10. A photon causes an electron to jump from $n = 2$ to $n = 4$ in a hydrogen atom. Calculate the energy of the photon.
A 2.56 eV
B 3.40 eV
C 4.09 eV
D 5.10 eV

SHORT RESPONSE

11. **a** **Determine** the energy (in J and eV) of a photon of light whose wavelength is:
i 600 nm
ii 450 nm.
b **Determine** the wavelength of a photon with energy of:
i 4.2 eV
ii 9.42×10^{-19} J .

12. Photons with 10 eV of energy were incident upon atoms with the following atomic energy levels.



a What energy levels could electrons move between?
b What photon energies (in eV) could be seen emitted from them?
c What wavelengths of light would be emitted?

CROSS-CHAPTER QUESTION

13. In a vacuum chamber, light with a frequency of 9.00×10^{14} Hz was incident on a piece of magnesium. The resultant photoelectrons entered a perpendicular magnetic field of 2 T once ejected. Given f_0 and W for magnesium are 8.84×10^{14} Hz and 3.66 eV, determine the radius of curvature of the path of the ejected photoelectrons.

DATA ANALYSIS

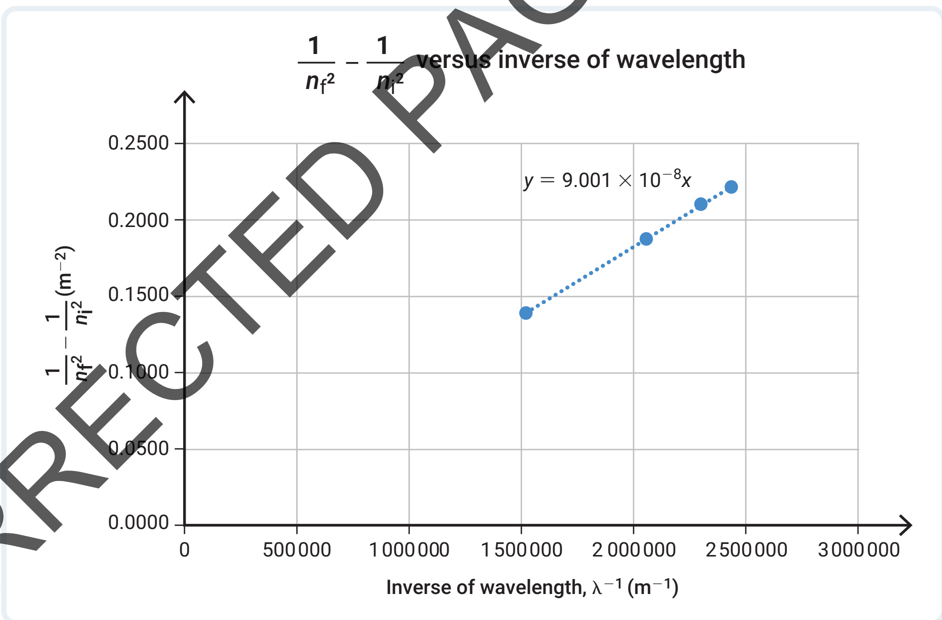
14. Analyse data

A physicist was studying the emission spectrum of hydrogen by measuring the wavelengths of a series of four adjacent spectral lines and matching them to known excited state energy levels. The raw data gathered is presented in Table 1 in which n_f is the integer value of the ground state energy level of the electrons and n_i is the integer value of the excited state energy level of the electrons:

TABLE 1

Measured wavelength, λ ($m \times 10^{-7}$)	n_i value	$\frac{1}{n_f^2} - \frac{1}{n_i^2}$
5.5101	3	0.1389
4.8402	4	0.1875
4.3199	5	0.2100
4.1203	6	0.2222

This data was processed to produce the following graph.



Refer to the data above and use the Rydberg equation:

$$\frac{1}{\lambda} = R \times \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

a **Determine** the experimental value for the Rydberg constant, R.

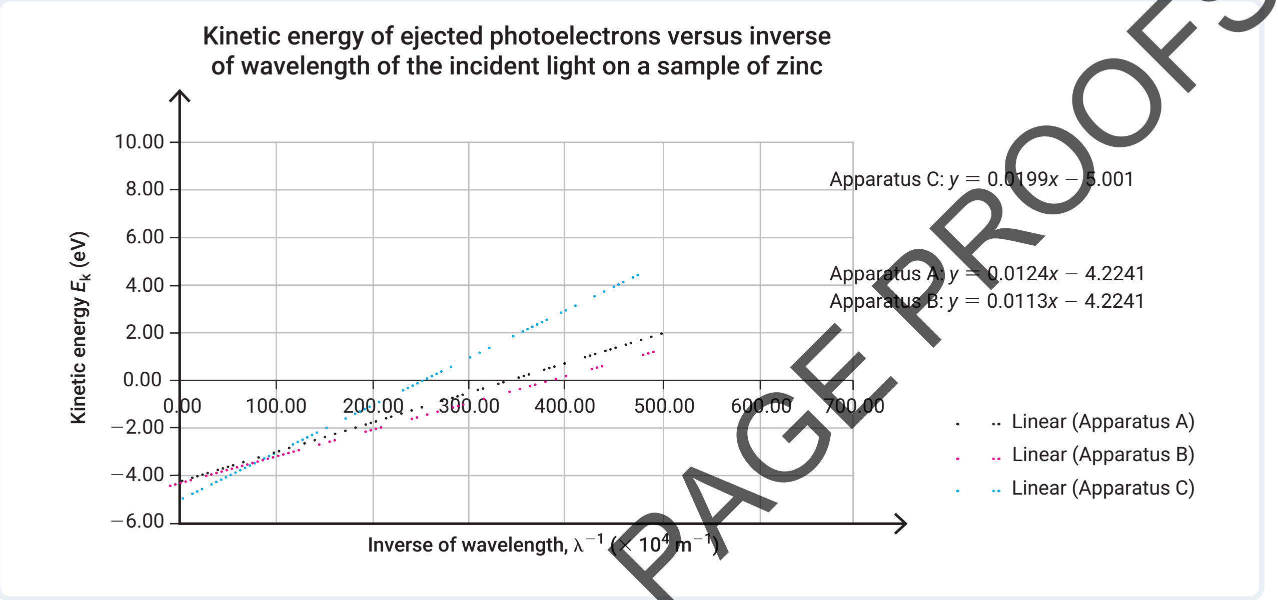
b **Calculate** the percentage error for the experimental value of R.

c **Determine** if this series is in the visible spectrum.

d Draw a conclusion about which series in the hydrogen emission spectrum the data represents – the Lyman, Balmer, Paschen or Brackett series.

15. Apply understanding

A student decided to analyse how a zinc sample with a known threshold frequency of 1.02×10^{15} Hz behaved due to the photoelectric effect using three different experimental set-ups: Apparatus A, Apparatus B and Apparatus C. The processed data is presented in the following graph.



a **Calculate** the three experimentally determined work functions (W) in joules from Apparatus A, Apparatus B and Apparatus C.

b **Calculate** the three experimentally determined values of the Planck constant, h, from Apparatus A, Apparatus B and Apparatus C.

c **Determine** the three experimentally obtained threshold frequencies (f_0) from Apparatus A, Apparatus B and Apparatus C by using the values for h calculated in part b and the work functions calculated in part a.

d Draw a conclusion about which apparatus (A, B or C) is the most reliable. Justify your conclusion by using your findings in parts a–c.

SCIENCE AS A HUMAN ENDEAVOUR

Syllabus dot point

- Explore how the approximation of Earth as a black body can be used to predict climate patterns.

The physics of Earth as a black body in climate science

Physics 2025 v1.0 General Senior Syllabus © QCAA 2024

The approximation of Earth as a black body is a powerful concept that helps climate scientists understand how energy flows through Earth's system and influences global climate patterns. A black body is an idealised object that absorbs all incident radiation and re-emits it perfectly; applying this model to Earth provides a simplified yet valuable framework for predicting how the planet's temperature responds to various factors such as solar radiation and greenhouse gases. Here, we will explore how the black-body approximation is used in climate science and how it contributes to our understanding of Earth's energy balance and climate change.

Role of black body approximation in climate modelling

In climate science, one of the key challenges is predicting how Earth's temperature changes in response to changes in solar energy and atmospheric conditions. By approximating Earth as a black body, scientists can use well-established physical principles to estimate its average temperature based on the balance between incoming and outgoing radiation.

- **Absorption and emission of radiation:** A black body absorbs all incoming electromagnetic radiation, regardless of wavelength, and re-emits the energy as thermal radiation. The temperature of the black body determines the intensity and wavelength of the emitted radiation. Although Earth is not a perfect black body, this model allows scientists to estimate how much solar energy Earth absorbs and how much heat it radiates back into space.
- **Energy balance:** Earth's climate is primarily determined by the balance between incoming solar radiation (shortwave radiation) and outgoing infrared radiation (longwave radiation). Solar radiation heats Earth's surface, and Earth radiates much, but not all, of this energy back into space as infrared radiation. The black-body approximation helps scientists model this energy exchange and predict the resulting average global temperature.

Stefan–Boltzmann law and Earth's temperature

A key equation that links the black-body model to Earth's climate is the Stefan–Boltzmann law, which states that the total energy radiated by a black body is proportional to the fourth power of its temperature:

$$E = \varepsilon\sigma T^4$$

where:

E = radiant energy per unit area (J m^{-2})

ε = emissivity of the surface emitting the radiation

σ = Stefan–Boltzmann constant (5.67×10^{-8} W m^{-2} K^{-4}), a constant of proportionality

T = temperature in kelvin (K).

This relationship allows scientists to calculate Earth's equilibrium temperature from the energy it receives from the Sun and how much of that energy it emits back into space.

The actual temperature of Earth is higher than what this simple black-body calculation predicts. This discrepancy is due to the greenhouse effect, where greenhouse gases such as carbon dioxide and methane trap some of the outgoing infrared radiation, warming the planet. By incorporating this effect into models, scientists can more accurately simulate Earth's climate and the role of greenhouse gases in global warming.

Impact of greenhouse gases on Earth's energy balance

One of the most significant uses of the black-body approximation in climate science is to understand how changes in atmospheric composition, particularly the concentration of greenhouse gases, affect Earth's energy balance.

- **Radiative forcing:** Climate scientists use the concept of radiative forcing to quantify how changes in factors such as greenhouse gas concentrations alter Earth's energy balance. Radiative forcing is measured in watts per square metre ($W\ m^{-2}$) and represents the change in net radiation (incoming radiation minus outgoing radiation) at the top of the atmosphere. A positive radiative forcing (due to increased greenhouse gases) leads to warming, while a negative forcing leads to cooling.
- **Greenhouse effect:** The greenhouse effect is the mechanism by which certain gases in the atmosphere absorb and re-emit infrared radiation, trapping heat that would otherwise escape into space. By modelling Earth as a black body, scientists can estimate how much heat is trapped by the atmosphere and how this affects surface temperatures. The higher the concentration of greenhouse gases in the atmosphere, the stronger the greenhouse effect, leading to higher global temperatures.

This approach is fundamental to climate modelling and helps predict how increasing levels of carbon dioxide and other greenhouse gases will affect future climate patterns.

Predicting climate patterns with black-body radiation models

The black-body approximation is crucial for predicting how various factors influence Earth's climate. By considering the planet as a system that absorbs and emits radiation, scientists can explore how changes in solar output, atmospheric composition and surface properties affect global temperatures.

- **Solar variability:** Changes in solar activity, such as sunspot cycles, affect how much solar energy Earth receives. Using the black-body model, scientists can estimate how variations in solar radiation influence global temperatures and climate patterns over time.
- **Albedo effect:** Earth's albedo refers to how much sunlight is reflected back into space by Earth's surface, such as ice, clouds and vegetation. A higher albedo means more solar energy is reflected, leading to a cooling effect. Scientists incorporate the albedo effect into black-body models to simulate how changes in surface properties, such as melting ice caps, affect Earth's climate.
- **Climate feedback mechanisms:** Feedback mechanisms, such as the melting of polar ice, amplify or dampen the effects of climate change. For instance, as ice melts, Earth's albedo decreases, meaning more solar energy is absorbed, leading to further warming. This is an example of a positive feedback cycle. Black-body models help scientists study these feedback loops and their implications for long-term climate predictions.

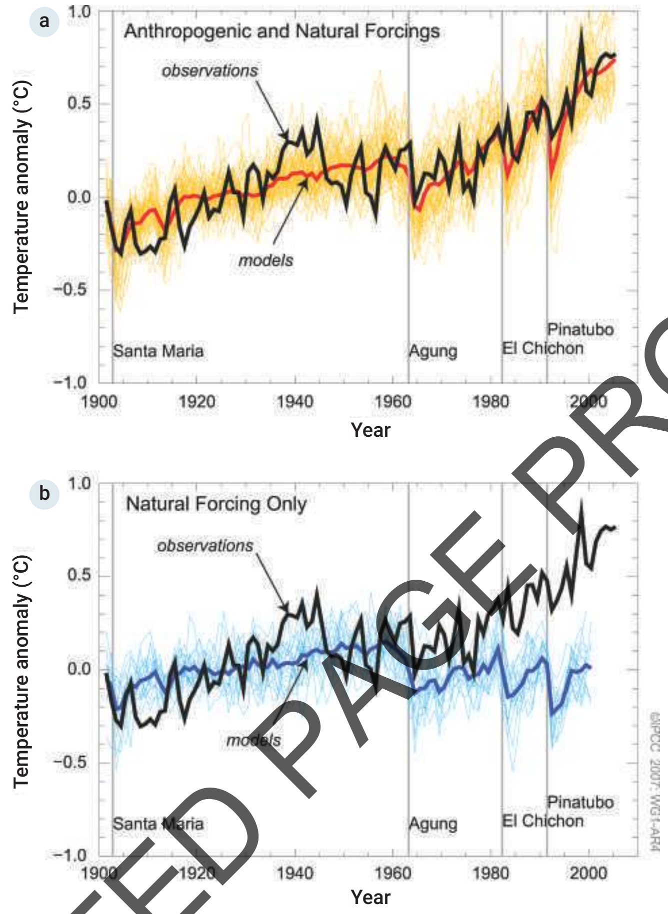


FIGURE 1 Models of climate change from two sets of models that (**a**) do include, and (**b**) do not include anthropogenic effects. The black lines are measured data; the coloured lines are simulations from models. The heavy coloured lines are averages from all the individual models. 'Temperature anomaly' is the difference in global temperature from the long-term mean. Grey vertical lines show when prominent volcanoes erupted.

Figure 1 shows that models do not reproduce the observed trends in warming unless the effects of greenhouse gases emitted as a result of human activities are included. Multiple sources of evidence, including results such as this, have convinced climate scientists that humans have significantly changed Earth's climate, and that even larger changes are to come.

Human endeavour in climate science and policy

The use of the black-body approximation in climate science highlights how theoretical models can provide valuable insights into complex systems. The ability to predict future climate conditions based on energy balance models has far-reaching implications for both scientific understanding and societal responses to climate change.

CHAPTER 14

The Standard Model

TK

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Describe the concepts of elementary particles and antiparticles.
- Identify the six types of quarks.
- Describe baryons and mesons.
- Identify the six types of leptons.
- Identify the four gauge bosons.
- Compare the strong nuclear, weak nuclear and electromagnetic forces in terms of the gauge bosons.
- Contrast the fundamental forces experienced by quarks and leptons.

SCIENCE AS A HUMAN ENDEAVOUR

- Appreciate the contribution of Australian scientists to the discovery of the Higgs boson.
- Explore the evidence relating to the Standard Model that supports the Big Bang theory.
- Appreciate the significant contributions of scientists such as Chien-Shiung Wu, Richard Feynman and Peter Higgs who furthered our understanding of particle physics and The Standard Model.
- Explore the history of particle physics models and theories through the development of particle accelerators and contributions from notable physicists.

Introduction

Over the past century, scientists and engineers working in particle physics have made a series of significant discoveries. These discoveries have led to new theories that describe and predict a plethora of elementary particles and their interactions. Physicists believed that there must be an underlying structure giving rise to the properties and behaviours of these particles. The Standard Model of particle physics ('the Standard Model') is our best current understanding of the universe on the smallest of scales. The model also provides answers to questions about the origins of the biggest of objects, the universe itself.

Assessments
- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practical
- Build your own cloud chamber and detect cosmic rays (online-only resource)

Worksheets
- Name
- Name

FPO

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

ASSUMED KNOWLEDGE

- ✓ Atoms consist of electrons surrounding a nucleus consisting of protons and neutrons.
- ✓ Matter is made up of particles.
- ✓ Subatomic particles can have a charge.
- ✓ Light consists of photons.
- ✓ The four fundamental forces of physics are the strong nuclear force, the electromagnetic force, the weak nuclear force and the gravitational force.

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe the concepts of elementary particles and antiparticles
- ✓ describe how evidence for the existence of antimatter was gathered by Carl Anderson by using cloud chambers and applying the Dirac and Schrödinger equations
- ✓ describe matter–antimatter annihilation
- ✓ describe examples of devices used by organisations such as CERN to investigate the Standard Model, such as particle accelerators and the Large Hadron Collider
- ✓ use conventional notation for representations in the Standard Model
- ✓ use MeV to quantify particle mass
- ✓ use lifetime and spin to characterise particles in the Standard Model
- ✓ describe the Pauli exclusion principle
- ✓ describe fermions and hadrons
- ✓ identify and describe the six types of quarks
- ✓ describe baryons and mesons
- ✓ identify and describe the six types of leptons
- ✓ identify the four gauge bosons
- ✓ describe the graviton as the hypothetical gauge boson of the gravitational force
- ✓ compare the strong nuclear, weak nuclear and electromagnetic forces in terms of the gauge bosons
- ✓ contrast the fundamental forces experienced by quarks and leptons
- ✓ describe the limitations of the current Standard Model and how it is being further developed in conjunction with the grand unified theory, dark energy and dark matter
- ✓ describe the discovery of the Higgs boson
- ✓ link the Standard Model to observed cosmological phenomena, the Big Bang theory and emerging cosmological theories.

14.1 Elementary particles and antiparticles

One of the characteristics of physics as a science is the belief that complex systems can be understood by examining and understanding the motion and interactions of simpler, component parts. Underlying this belief is another: that, as we break the system into smaller and smaller parts, we will eventually come to a point where there are no internal components. Then we will be dealing with the basic building blocks from which everything else is constructed.

The idea that there are fundamental building blocks of matter has persisted through the ages, although it has undergone many changes as new discoveries and observations have been made. The ancient Greeks thought that the universe was made up of four basic elements: earth, air, fire and water. Then Democritus introduced the concept of the atom. Later it was found that atoms were made of even smaller components, including electrons, protons and neutrons.

The Standard Model of particle physics makes the prediction that these components are themselves made of smaller components called **elementary particles**. Their existence has been supported by experimental evidence gathered from particle colliders.

elementary particle
a particle whose substructure is unknown

Weblinks
Fundamental particles
Positron

Elementary particles

Our contemporary quantum mechanical model of the atom was largely developed as of 1932. There is a tiny yet massive nucleus that contains positively charged protons and neutral neutrons (Figure 14.1.1). Surrounding this is a cloud of negatively charged electrons **(Figure 14.1.1)**.

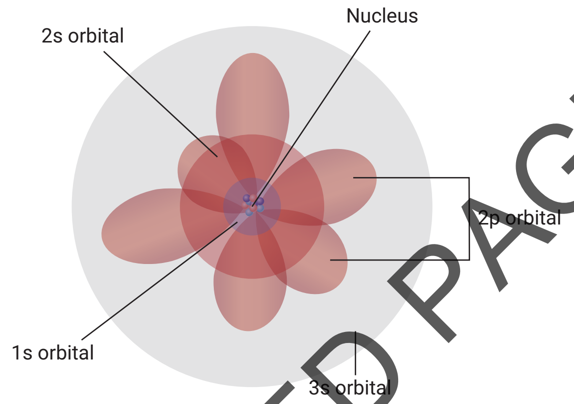


FIGURE 14.1.1 In the modern quantum mechanical model of the atom, the protons and neutrons are contained in the nucleus, surrounded by electron clouds.

A fourth particle, the photon, was also already known by the 1930s. Recall that the photon has no mass and no charge. It does have quantised energy and momentum, which depend on its frequency. The electron and the photon both appear to be elementary particles.

Under the Standard Model, there are three broad classes or families of particles. They are quarks, leptons and gauge bosons. The quarks and leptons represent the most fundamental particles, and they interact with each other by exchanging gauge bosons.

Recall that when we are considering the gravitational force between the Sun and Earth, we can treat Earth as if all its mass were concentrated at its centre and use Newton's law of gravitation.

Earth is not really a point mass because it has *internal structure*. When people are close to Earth's surface, the gravitational field no longer appears to be that due to a point mass; the internal structure becomes important. Similarly, a charged object can be treated as a point charge when it interacts with another charged object a long way away, but up close, we must treat it differently.

Syllabus link
Chapter 12 discusses the development of the model of the atom.

The electric field of an electron is exactly what is expected for a point charge, suggesting that it has no internal structure and strongly supporting the idea that it is an elementary particle.

However, experimental evidence suggests that the neutron is *not* an elementary particle. Although the neutron has no charge, it does have its own intrinsic magnetic field. A magnetic field is the result of moving charged particles or a changing electric field. This indicates that the neutron has some internal structure and contains charges that add to zero total charge. This suggested to physicists that the neutron is *not* an elementary particle. This suggestion was supported by experimental evidence gained from understanding radioactive decay.

Because the proton has a mass very close to that of a neutron and is otherwise a similar particle in its behaviour, the same question was also raised of the proton. Evidence that the proton is not an elementary particle also came from studies of radioactivity.

When a nucleus decays, different types of particles may be emitted. These include β^- and β^+ particles. It was observed that these β particles have the same mass as an electron, and either a positive or negative charge equal to the electron charge.

It was determined that the β^- particle is an electron. When a nucleus emits a β^- particle, its proton number increases by one and its neutron number decreases by one. Similarly, when a β^+ particle is emitted, a proton is converted to a neutron. Hence, it appeared that neutrons could be converted into protons, and protons into neutrons, by the emission of these negatively or positively charged electrons. This suggests that neither the neutron nor the proton is an elementary particle.

positron the antiparticle of an electron with charge $+e$ and mass m_e

antimatter matter composed of antiparticles, such as positrons, antiprotons and antineutrons

The positively charged electrons, called **positrons**, that are involved in β^+ decay were discovered in cosmic rays in 1932 by Carl Anderson (1905–91). Positrons were the first **antimatter** particles to be observed.

By 1932, there were five particles known:
- three ‘normal’ matter particles, the electron, proton and neutron
- one antimatter particle, the positron
- the photon.

Of these, it was already believed that the proton and neutron were not themselves elementary particles, and so the hunt for their component parts began.

Weblinks
Antimatter
The Schrödinger equation explained in 60 seconds

Antimatter

Schrödinger equation a differential equation that describes the wave-like properties of matter existing within a potential field

In the 1920s, Paul Dirac (1902–84) developed a relativistic version of the **Schrödinger equation** that could describe many properties of the electron.

However, there appeared to be an issue with this approach: the equation that Dirac had developed to describe the electron had two solutions. One of these solutions gave the wave functions for electrons; the other solution described a particle with the same mass but the opposite charge and magnetic moment. It also predicted that if these two particles should meet, they would both be destroyed, producing a burst of energy. This is called **annihilation**.

annihilation the destructive process resulting when a particle and its antiparticle meet

antiparticle a particle with the same mass and opposite charge and/or spin to a corresponding particle

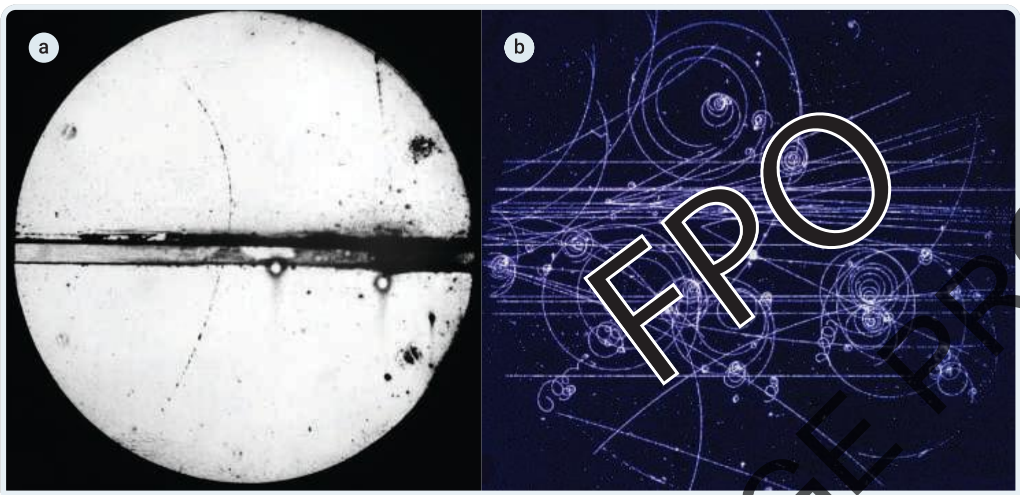
Hence, this second particle was the ‘anti-electron’. It was this particle, the **antiparticle** to the electron, that Anderson observed in 1932. We now refer to this particle as the positron, to indicate that it has a positive charge, opposite to the negative charge of an electron.

cloud chamber a chamber containing a supersaturated vapour through which high-energy particles pass, causing vapour trails to be formed and therefore allowing the path of the particles to be visualised

Anderson was using a **cloud chamber** to study cosmic rays. A cloud chamber is a particle detector that detects ionising radiation. It uses a supersaturated vapour, like a layer of fog, in a container or chamber. When a charged particle enters the chamber, it ionises the vapour. The resulting electrically charged vapour particles act as sites for condensation of the vapour. This produces a visible trail of condensation along the path of the particle.

Anderson placed his cloud chamber in a magnetic field. The magnetic field caused the moving charged particles to follow curved paths, just like the ions in a mass spectrometer. This allowed him to distinguish between positive and negative charges. Recall that the direction in which a particle’s path curves depends on its charge, and that the sharpness of the curve (radius of curvature) depends on the mass.

Anderson noted that some of the tracks in his cloud chamber had electron-like curvature but were deflected in a direction corresponding to a positively charged particle. Anderson had discovered the anti-electron, or positron (**Figure 14.1.2**). Anderson was awarded the Nobel Prize for Physics in 1936 for this discovery.



Science Photo Library/Emilio Segre Visual Archives

FIGURE 14.1.2 (**a**) This cloud chamber photograph shows the track of the first identified positron. (**b**) Modern cloud chamber images may be used to trace trajectories and determine both charge and mass of particles

Dirac's theory went further and suggested that *an antiparticle exists for every particle*. It has subsequently been verified that almost every known elementary particle has a distinct antiparticle.

The antiparticle of a known particle, such as the neutron (n) or electron (e^-), is represented either by placing a bar over the symbol (e.g. $\bar{n}$ for the antineutron) or by showing that the sign is reversed (e.g. e^+ for the positron).

LEARNING CHECK 14.1

DESCRIBING

1 **Define** 'elementary particle'.

2 State the differences between matter particles and their antimatter particles.

3 **List** the mass and charge of:

  a a positron

  b an antiproton

  c a neutron.

4 **Explain** how a cloud chamber can be used to detect whether an electron or a positron is passing through it. Consider both the direction and radius of curvature.

5 An electron enters a region of uniform magnetic field and curves to the right as shown in Figure 14.1.3. A second charged particle enters the same field but curves to the left. **Explain** how the particle could be identified as a positron or a proton.

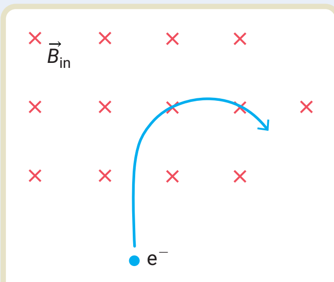
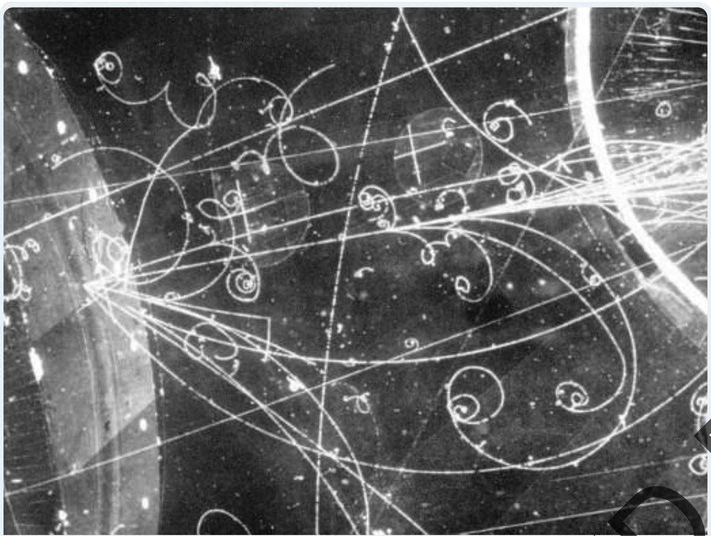


FIGURE 14.1.3 An electron's trajectory curves to the right when it enters the magnetic field.

14.2 Particle physics: the search for elementary particles

Weblink
How an accelerator work

Following on from the discovery of antiparticles, many other 'new' particles were discovered experimentally. Some of these were discovered by analysing the different tracks in cloud chamber experiments, others in cosmic ray experiments and others in nuclear decays or through particle accelerator experiments **(Figure 14.2.1)**.



U.S. Department of Energy, Fermi National Accelerator Laboratory

FIGURE 14.2.1 A cloud chamber, showing multiple tracks resulting from many particles. Some of the tracks seem to emerge from other tracks, indicating that a particle has decayed into a new particle.

Cosmic rays pass through Earth's atmosphere at random, and their energies vary widely. Hence, it is impossible to design well-controlled experiments that use cosmic rays to create new particles.

The energies of the particles emitted in some nuclear decays are well defined, but they are generally low. The equation $E = mc^2$ suggested that if higher energies could be used, more massive particles might be created. Thus, physicists looked for ways to produce beams in which there were more particles with high enough energies to produce new particles that they could study. The breakthrough was achieved with the invention of **particle accelerators**.

From the 1950s onwards, many more particles were discovered in experiments involving high-energy collisions between known particles in these accelerators. The more energy available in the collisions, the more new particles produced and the greater their mass.

particle accelerator a device in which electric and magnetic fields are used to accelerate beams of particles to high speeds

These new particles are characteristically very unstable and have very short half-lives that range between 10^{-6} and 10^{-23} s. Their decays produce lighter particles, some of which are also unstable. So, each collision between just two initial particles may result in many outgoing particles, which need to be detected and identified simultaneously. To enable this, physicists and engineers worked together to develop huge, complex apparatus to use at the new particle accelerators. This is an example of the role that technology plays in allowing new experiments, which in turn lead to the development and refinement of theory.

The largest, most powerful and possibly best-known particle accelerator in the world is the Large Hadron Collider at CERN (the European Organization for Nuclear Research) in Switzerland. The Large Hadron Collider consists of a 27 km long synchrotron ring built 100 m under the local mountains and uses superconducting electromagnets to accelerate beams of protons up to 7 TeV, approximately 0.999 999 991 times the speed of light **(Figure 14.2.2)**. Recall that the kinetic energy of an object is $E_k = \frac{1}{2}mv^2$. Even though the mass of the individual particles is very small, because of their very high velocity, the kinetic energy of the particles is very large.

When protons collide head-on at these colossal energies, a wide variety of particles are created. Seven detector stations are used at various points along the synchrotron ring and detect the properties such as the mass, charge and magnetic spin of the collision products.

The most well-publicised discovery at the Large Hadron Collider was the detection of the Higgs boson in 2012 and was the culmination of the work of many scientists and engineers.

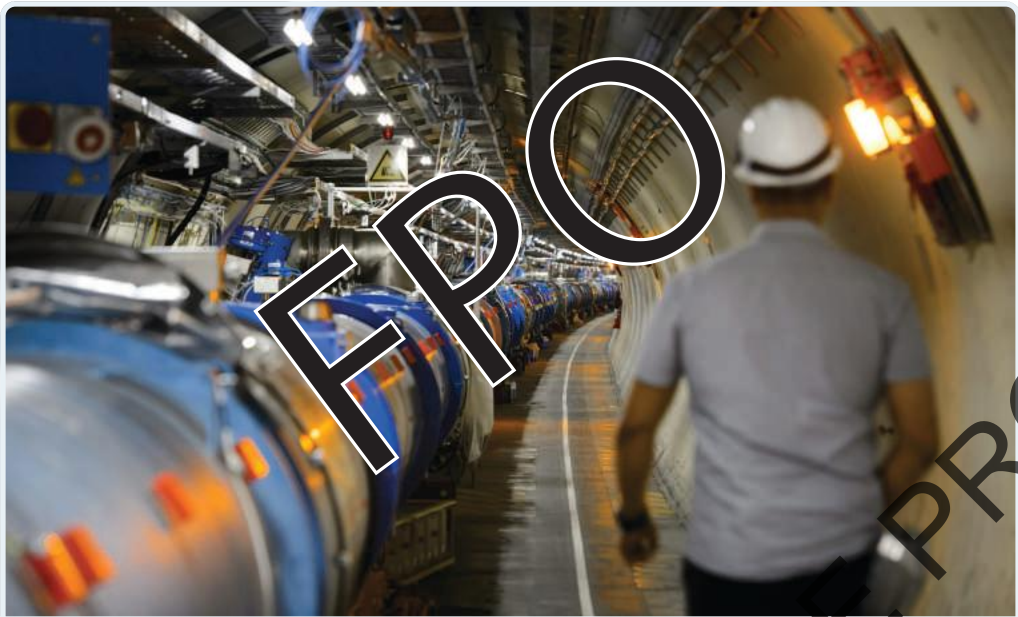


FIGURE 14.2.2 Inside the tunnel containing the Large Hadron Collider at CERN.

The particle zoo

So far, several hundred particles have been identified in particle accelerator experiments. The wide variety of properties and behaviours of these new particles led to the term the 'particle zoo'. These newly discovered particles included the antiproton, discovered by Emilio Segré and Owen Chamberlain in 1955, and the antineutron, discovered in 1956 by Bruce Cork.

Some of these particles and their properties are listed in Table 14.2.1. The various ways of classifying these particles will be explored in the following sections.

Weblinks
The Large Hadron Collider
The Higgs boson

Particle mass

The masses in Table 14.2.1 are given in units of MeV c^{-2}. Recall that mass and energy are related by Einstein's mass equivalence relationship: $E = mc^2$. In particle physics, it is common to give masses in terms of their energy equivalent, rather than in kilograms.

$$1 \text{ MeV } c^{-2} = 1.78 \times 10^{-30} \text{ kg}$$

Using $E = mc^2$

$$m = \frac{E}{c^2}$$

$$1.0 \text{ MeV } c^{-2} = \frac{1 \times 10^6 \times 1.6 \times 10^{-19}}{\left(3 \times 10^8\right)^2}$$

Therefore, $m = 1.78 \times 10^{-30}$ kg

Particle lifetime

The lifetimes of the particles given in Table 14.2.1 are the mean lifetimes, t_{mean}. Recall that unstable nuclei have a half-life that determines how likely they are to decay. In a large population of nuclei with a half-life $t_{\frac{1}{2}}$, half of the nuclei will decay in a time $t_{\frac{1}{2}}$.

The mean lifetime is related to the half-life by:

$$t_{\text{mean}} = \frac{t_{\frac{1}{2}}}{\ln 2} = 1.44\, t_{\frac{1}{2}}$$

Weblink
January 1925: Wolfgang Pauli announces the exclusion principle

TABLE 14.2.1 Some subatomic particles and their properties

Particle	Symbol	Antiparticle	Mass (MeV c^{-2})	$\underline{B}$	$\underline{L}_e$	$\underline{L}_m$	$\underline{L}_t$	Lifetime (s)	Spin
Leptons									
Electron	e^-	e^+	0.511	0	+1	0	0	Stable	$\frac{1}{2}$
Electron neutrino	ν_e	$\bar{\nu}_e$	<7 eV c^{-2}	0	+1	0	0	Stable	$\frac{1}{2}$
Muon	μ^-	μ^+	105.7	0	0	+1	0	2.20×10^{-6}	$\frac{1}{2}$
Muon neutrino	ν_μ	$\bar{\nu}_\mu$	<0.3	0	0	+1	0	Stable	$\frac{1}{2}$
Tau	τ	τ^+	1784	0	0	0	+1	$<4 \times 10^{-13}$	$\frac{1}{2}$
Tau neutrino	ν_τ	$\bar{\nu}_\tau$	<30	0	0	0	+1	Stable	$\frac{1}{2}$
Hadrons – mesons									
Pion	π^+	π^-	139.6	0	0	0	0	2.60×10^{-8}	0
	π^0	Self	135.0	0	0	0	0	0.83×10^{-16}	0
Kaon	K^+	K^-	493.7	0	0	0	0	1.24×10^{-8}	0
	$\bar{K}^0_S$	$\bar{K}^0_S$	497.7	0	0	0	0	0.89×10^{-10}	0
	$\bar{K}^0_L$	$\bar{K}^0_L$	497.7	0	0		0	5.2×10^{-8}	0
Eta	η	Self	548.8	0	0	0	0	$<10^{-8}$	0
	η'	Self	958	0	0	0	0	2.2×10^{-10}	0
Hadrons – baryons									
Proton	p	$\bar{p}$	938.3	+1	0	0	0	Stable	$\frac{1}{2}$
Neutron	n	$\bar{n}$	939.6	+1	0	0	0	614	$\frac{1}{2}$
Lambda	Λ^0	$\bar{\Lambda}^0$	1115.6	+1	0	0	0	2.6×10^{-10}	$\frac{1}{2}$
Sigma	Σ^+	$\bar{\Sigma}^-$	1189.4	+1	0	0	0	0.80×10^{-10}	$\frac{1}{2}$
	Σ^0	$\bar{\Sigma}^0$	1192.5	+1	0	0	0	63×10^{-20}	$\frac{1}{2}$
	Σ^-	$\bar{\Sigma}^+$	1197.3	+1	0	0	0	1.5×10^{-10}	$\frac{1}{2}$
Delta	Δ^{++}	$\bar{\Delta}^{--}$	1230	+1	0	0	0	6×10^{-24}	$\frac{3}{2}$
Xi	Ξ^0	$\bar{\Xi}^0$	1315	+1	0	0	0	2.9×10^{-10}	$\frac{1}{2}$
Omega	Ω^-	Ω^+	1672	+1	0	0	0	0.82×10^{-10}	$\frac{3}{2}$

We are using the symbol t_{mean} rather than τ to denote the mean lifetime, to avoid confusion with the τ (tau) particle. You may see lifetimes written as τ in other sources.

Particle spin

Spin is another important property of the particles listed in Table 14.2.1.

A particle's spin is a complex magnetic feature resulting from it having its own magnetic moment. Spin is a measure of its intrinsic magnetic field. Initially, this magnetic moment was thought to be due to the particle's rotational motion or spin; hence, the name. A modern understanding of quantum mechanics suggests that they are not in fact spinning, but the name remains.

Like other properties of particles, spin is quantised; that is, it can take only specific discrete values. Some particles have spins with integer values; others have half-integer values of spin. These values relate to the magnetic moment and, hence, magnetic fields of the particles. Particles with non-zero spin have a magnetic moment and, hence, their own magnetic field.

Particles can be classified according to their spin. Particles with half-integer spin, $s = \frac{1}{2}, \frac{3}{2}$, and so on, are called **fermions**. As can be seen in Table 14.2.1, **leptons** and **baryons** have half-integer spins and are classified as fermions.

Fermions obey the **Pauli exclusion principle**. The Pauli exclusion principle states that any two fermions in the same quantum system cannot have identical sets of quantum numbers. Electrons in an atom are an example of this – no two electrons in any given atom can have identical quantum numbers.

Particles with integer spin, $s = 0, 1, 2, \ldots$ are called **bosons**. **Mesons** and photons are bosons. Bosons do not obey the exclusion principle.

Fermions and bosons play different roles in the Standard Model, as we shall see in the following sections.

The other particle properties, such as baryon number (B) and lepton number (L), and their conservation are discussed in Chapter 15.

The search for order

You will note that the particles listed in Table 14.2.1 are classified into two different types: leptons and **hadrons**.

In fact, almost all particles can be divided into leptons and hadrons. Leptons, such as electrons, generally have small mass. Hadrons, such as protons and neutrons, generally have large mass and are themselves composed of subatomic particles called **quarks**.

The following sections will investigate the properties of these categories further, together with gauge bosons.

LEARNING CHECK 14.2

DESCRIBING

1 **Describe** how a particle accelerator can raise the energy of a beam of particles.

2 **Define** 'particle spin'.

3 State the two families to which particles can belong if they are being classified according to their spin. State the value of the spin for each family.

spin a quantum property of particles that results from them having their own magnetic moment and therefore magnetic field

fermions particles with half-integer spin ($s = \frac{1}{2}, \frac{3}{2}$ …) that obey the Pauli exclusion principle

leptons a family of elementary particles that include electrons, taus, muons, their neutrinos and all their antiparticles

baryons a family of heavy subatomic particles, such as neutrons and protons, which contain composite structures made up of three quarks

Pauli exclusion principle quantum mechanical principle that two fermions in the same quantum system cannot have identical sets of quantum numbers (e.g. no two electrons can be in the same shell or orbital around an atom and have the same energy)

bosons particles with integer spin ($s = 0, 1, 2, \ldots$); these particles do not obey the Pauli exclusion principle

mesons a family of heavy subatomic particles that contain composite structures made up of one quark and an antiquark

hadron a family of elementary particles with a large mass consisting of mesons and baryons; a subatomic particle made up of quarks and held together by the nuclear strong force (e.g. protons and neutrons; can be further categorised into baryons (protons and neutrons) made up of three quarks and mesons (consisting of one quark and one antiquark)

quark a fundamental particle of matter; they combine in specific ways to form hadrons (protons and neutrons) and experience all four fundamental forces; the six types of 'flavours' of quark are up, down, top, bottom, charm and strange

4 A synchrotron with a constant magnetic field can hold a particle in orbit with a constant speed and continuous acceleration. **Explain** how this is the case and use a diagram to assist.

5 **Explain** the impact that the Pauli exclusion principle has on the allowed states of fermions.

APPLYING

6 The mass of an electron is 9.109×10^{-31} kg. Convert this into units of MeV c^{-2}.

7 Consider a proton with an energy of 4 TeV in the Large Hadron Collider.
 a What is the energy of this particle in joules?
 b How fast would a mosquito, with a mass of 3 mg, need to fly to have the same amount of energy? Comment on your answer.

8 Use Table 14.2.1 to **describe** the following particles according to their category, symbol, antiparticle, mass, lifetime and spin.
 a Neutron **b** Proton **c** Electron
 d Tau **e** Tau neutrino

14.3 Gauge bosons and the fundamental forces of nature

gauge boson a fundamental force-carrying particle that mediates particle interactions through the four fundamental forces; there are four types, each associated with a specific fundamental force

The Standard Model of particle physics considers quarks and leptons to be the elementary particles that make up all *ordinary* matter. The distinction between the two is based on an understanding of how they interact with the forces of nature.

The photon is not listed in Table 14.2.1 The photon is known as a **gauge boson**. This is the third category of particles and they act as force carriers.

The four forces believed to be responsible for all interactions are (in order of relative strength) the strong nuclear force, the electromagnetic force, the weak nuclear force and the gravitational force. Each of these forces is mediated by a gauge boson.

- Photons (γ) mediate the electromagnetic force, which acts on charged particles
- W and Z bosons (W^-, W^+, Z^0) mediate the weak nuclear force and are responsible for beta decay.
- Gluons (g) mediate the strong nuclear force, which binds quarks together to form hadrons such as protons and neutrons.
- Gravitons (h) are hypothetical bosons, predicted to mediate the gravitational force in quantum field theory. This is not yet part of the Standard Model.

KEY CONCEPT

The gauge bosons mediate the fundamental forces of the Standard Model:

- Photons (γ) mediate the electromagnetic force, which acts on charged particles.
- W and Z bosons (W^-, W^+, Z^0) mediate the weak nuclear force and are responsible for beta decay.
- Gluons (g) mediate the strong nuclear force, which binds quarks together to form hadrons such as protons and neutrons.

In this particle exchange model of interactions, the basic process is that one particle emits a gauge boson that is subsequently absorbed by another elementary particle. This interaction will be further developed in Chapter 15.

Strong nuclear force

The discovery that atomic nuclei are composed of protons and neutrons led to the introduction of a new type of force. The protons in any nucleus should strongly repel one another due to their positive charges. So, there must be another force acting to stop the nucleus flying apart. We call the force that holds the nucleus together the strong nuclear force, or the nuclear force, because it must be strong to overcome the proton–proton repulsion and it acts between components of the nucleus.

The first theory attempting to explain the nature of the attractive force between nucleons was proposed in 1935 by Japanese physicist Hideki Yukawa. Yukawa used the idea of gauge bosons to explain the strong nuclear force through pi mesons (π), also known as the contracted name 'pions'. These particles were discovered in cosmic radiation in 1947.

The later development of quark theory led to the understanding that the strong nuclear force is in fact not fundamental but rather arises from interactions between quarks. The force that leads to these interactions is called the **strong force**.

strong force the attractive force that acts between quarks, holding them together; is mediated by gluons

The strong force is responsible for the attractive force that exists between quarks and has only a very short range of about 10^{-15} m (about the size of a nucleus). Its gauge boson is the **gluon**, so named because it 'glues' particles together.

gluon the gauge boson that mediates the strong nuclear force

Electromagnetic force

Recall that electrons and other charged particles interact through electric and magnetic fields. Recall also that the electromagnetic field consists of particles called photons (γ). Therefore, the electromagnetic force can be pictured in terms of the exchange of photons between electrically charged particles. The electromagnetic force is said to be mediated by photons.

The idea that electromagnetic interactions could be mediated by photon exchange led physicists to question whether other types of interaction might be modelled in the same way.

Weak nuclear force

The weak nuclear force is involved in nuclear decay. The exchange particles for the weak nuclear force, **W and Z bosons**, have since been detected.

W and Z bosons the particles that mediate the weak nuclear force; they include W^-, W^+, Z^0 and are responsible for beta decay

In 1979, Sheldon Glashow (1932–), Abdus Salam (1926–96) and Steven Weinberg (1933–2021) won the Nobel Prize in Physics for developing a theory that unifies the electromagnetic and weak interactions. This **electroweak theory** postulates that the weak and electromagnetic interactions have the same strength when the particles involved have very high energies. Because of the mass difference between photons and the W and Z bosons, the electromagnetic and weak forces are quite distinct at low energies, but become similar at very high energies, when the rest energy is negligible relative to the total energy. This behaviour, as one goes from high to low energies, is called *symmetry breaking* because the forces are similar, or symmetric, at high energies but are very different at low energies.

electroweak theory the theory that combines the electromagnetic and weak interactions

Gravitational gauge bosons

The gravitational force is a long-range force that has a strength of only about 10^{-39} times that of the strong force. This is the force that holds the planets, stars and galaxies together and is most recognised by humans in our daily lives. However, the gravitational effect on elementary particles is negligible. The gravitational force is thought to be mediated by field particles called **gravitons**. The gravitational force is not part of the Standard Model, and the graviton has not yet been detected;

graviton the hypothetical gauge boson of the gravitational force

Weblink
ARC Centre of Excellence for Gravitational Wave Discovery

however, several major international research facilities, including the Australian International Gravitational Observatory in Western Australia, are trying to detect gravity waves or gravitons.

The different forces and their gauge bosons are listed in Table 14.3.1.

TABLE 14.3.1 Forces and their gauge bosons. Note their relative strengths and ranges

Interactions	Relative strength	Range of force	Mediating field particle	Mass of field particle (GeV c^{-2})
Strong	1	Short (1 fm)	Gluon	0
Electromagnetic	10^{-2}	∞	Photon	0
Weak	10^{-5}	Short (10^{-3} fm)	W^-, W^+, Z^0 bosons	80.4, 80.4, 91.2
Gravitational	10^{-39}	∞	Graviton	0

LEARNING CHECK 14.3

DESCRIBING

1 Which forces are included in the Standard Model of particle physics?

2 Name the gauge bosons associated with the:

a electromagnetic force
b strong nuclear force
c weak nuclear force
d gravitational force.

3 **Describe** the electroweak interaction.

4 **Identify** what force the strong nuclear force that holds protons and neutrons together overcomes within the nucleus.

5 List the fundamental forces in order from greatest to least strength.

APPLYING

6 **Calculate** the gravitational and electrostatic forces acting between two protons that are 1 fm apart.

14.4 Leptons

As we have seen, in the 1950s a huge number of particles were known. Broadly, these could be classified as particles with mass – leptons and hadrons – and those without mass, which include exchange particles such as photons.

Weblink
Explainer: what are leptons?

Leptons are mostly particles with very small mass, such as the electron and electron neutrinos. We have already noted that electrons appear to be elementary particles because they show no signs of having internal structure. Because the heavier leptons and their antiparticles appear to be identical to electrons and positrons in all respects except for mass, it appears that they too are elementary particles. This leads physicists to believe that the whole family of leptons, including neutrinos, are elementary particles.

Leptons (from the Greek *leptos*, meaning small or light) are particles that do not interact by means of the strong nuclear force, but do interact via the gravitational, electromagnetic and weak forces. All leptons have spin $\frac{1}{2}$. Unlike hadrons, which have size and structure, leptons appear to be truly elementary, meaning that they have no structure and are point-like.

Unlike hadrons, the number of known leptons is small. Currently, scientists believe that there are only six leptons (plus their antiparticles): the electron, the muon and the tau, plus a neutrino associated with each:

- e, electron
- μ, muon
- τ, tau
- ν_e, electron neutrino
- ν_μ, muon neutrino
- ν_τ , tau neutrino.

The neutrino was first predicted in 1930 by Dirac. At this time, radioactive beta decay had been observed experimentally, but momentum and energy did not appear to be conserved in these decays. Hence, Dirac proposed the existence of a very light, uncharged particle that could carry the unaccounted-for energy and momentum.

Current experiments indicate that neutrinos have a small, but non-zero, mass. Direct experimental evidence for the neutrino associated with the tau was announced by the Fermi National Accelerator Laboratory (Fermilab) in 2000.

LEARNING CHECK 14.4

DESCRIBING

1 **Recall** the six types of leptons. Give the names and symbols of each.
2 **Recall** the forces that are felt by the six types of leptons.
3 **Explain** why leptons are considered to be elementary.
4 List the six leptons in order of increasing mass.
5 **Contrast** a quark with a lepton.

14.5 Hadrons: mesons, baryons and their quarks

Hadrons mostly have a large mass and are further divided into mesons and baryons based on their number of quarks, their mass and spin. Families of particles with similar mass but different electric charge are seen for hadrons but not leptons.

Hadrons interact with each other via all four of the fundamental forces of nature.

In addition to their high masses, many of the properties of hadrons raised questions with physicists about whether they were in fact fundamental particles or not. Some of these properties include the:

- existence of groups of hadrons with similar mass but different charge
- ways in which hadrons decay, often to other lighter hadrons
- ways particular hadrons are produced in particular reactions
- magnetic moments of uncharged particles such as the neutron.

Many experiments since their discovery have shown that hadrons are not elementary particles but rather are made up from constituent particles called quarks.

Weblink
Scientists gain new insights into how mass is distributed in hadrons

Quarks

In 1963, Murray Gell-Mann and George Zweig independently proposed a model for the substructure of hadrons. According to their model, all hadrons are composed of two or three elementary constituents called quarks. **Figure 14.5.1** is a pictorial representation of the quark compositions of several hadrons.

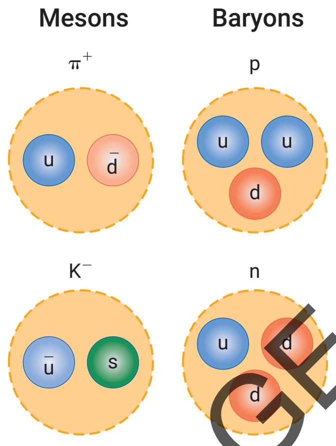


FIGURE 14.5.1 The quark composition of two mesons and two baryons. Mesons consist of a quark–antiquark pair whereas baryons consist of three quarks. Note that the antiquarks are indicated by a line above the quark (e.g. $\bar{u}$).

This early quark model had three types of quarks – up (u), down (d) and strange (s). The various types of quarks are called **flavours**.

flavours the six classifications of quark types: up, down, strange, charm, top and bottom

Each quark has an associated antiquark of opposite charge, baryon number, strangeness, charm, topness and bottomness.

The compositions of all hadrons known when Gell-Mann and Zweig presented their model can be completely specified by the following three simple rules.

- A meson consists of one quark and one antiquark.
- A baryon consists of three quarks.
- An antibaryon consists of three antiquarks.

WORKED EXAMPLE 14.5.1

Determine whether these quark combinations are possible mesons or baryons.

a $d\bar{d}$ **b** dd **c** $\bar{u}d\bar{d}$ **d** $\bar{u}\bar{d}\bar{d}$

ANSWERS

A meson consists of a quark–antiquark pair. A baryon consists of three quarks, and an antibaryon of three antiquarks.

a Possible (meson – it contains a quark–antiquark pair)
b Not possible
c Not possible
d Possible (antibaryon – it contains three antiquarks)

Table 14.5.1 lists the properties of quarks.

TABLE 14.5.1 Properties of the six quarks

Name	Symbol	Spin	Charge	Baryon number	Strangeness	Charm	Bottomness	Topness
Up	u	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	0	0	0
Down	d	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	0	0	0	0
Charm	c	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	+1	0	0
Strange	s	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	–1	0	0	0
Top	t	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0	0	0	+1
Bottom	b	$\frac{1}{2}$	$\frac{1}{3}e$	$\frac{1}{3}$	0	0	+1	0

Unlike any particles we have seen before, quarks carry a fractional electric charge. The u, d and s quarks have charges of $+\frac{2}{3}e$, $-\frac{1}{3}e$ and $-\frac{1}{3}e$ respectively, where e is the electron charge ($e = 1.60 \times 10^{-19}$ C). Quarks have spin $\frac{1}{2}$, which means that they are classified as fermions and have their own intrinsic magnetic moment and, therefore, magnetic field. When quarks combine to form a particle, the charge of the particle is the arithmetic sum of the charges of its quarks. We can show this for both protons and neutrons, by referring to Table 14.5.1 and the charge of each quark.

The proton contains the quarks u, u, d. The sum of their charge is $\frac{2}{3}+\frac{2}{3}+\frac{-1}{3}=\frac{3}{3}=1$.

The neutron contains the quarks u, d, d. The sum of their charge is $\frac{2}{3}+\frac{-1}{3}+\frac{-1}{3}=\frac{0}{3}=0$.

Similarly, the spin of the particle is the sum of the spins of its quarks. However, spin is a vector quantity, so we need to be careful how we add the spins. Both the magnitude and the direction of spin are quantised.

Although the original quark model was highly successful in classifying particles into families, some discrepancies occurred between its predictions and certain experimental decay rates. Consequently, several physicists proposed a fourth quark flavour in 1967. The fourth quark, designated c, was assigned a property called charm. A *charm* quark has charge $+\frac{2e}{3}$, just as the up quark does, but its charm distinguishes it from the other three quarks. This introduces a new quantum number C, representing charm. The new quark has charm $C = +1$, its antiquark has charm $C = -1$; all other quarks have $C = 0$.

Evidence that the charmed quark exists began to accumulate in the 1970s, when a series of heavy mesons with long lifetimes were discovered. The existence of these new mesons provided firm evidence for the fourth quark flavour.

Further developments in particle physics led to more elaborate quark models and the prediction of two new quarks, top (t) and bottom (b). To distinguish these quarks from the others, quantum numbers called *topness* and *bottomness* (with allowed values +1, 0, –1) were assigned to all quarks and antiquarks (**Table 14.5.1**).

Although no isolated quark has ever been observed experimentally, the quark model does describe the properties of mesons and baryons.

Mesons

The name *meson* is the Greek word for 'intermediate', or 'middle sized'. Several mesons have masses in the range between the masses of the electron and the proton, although mesons having masses greater than that of the proton have been found. Mesons all have zero or whole integer spin (0, 1, ...), and, therefore, they are also bosons (as opposed to fermions, with half-integer spin).

No stable meson has ever been observed. All mesons decay. Some decays produce lighter (but themselves unstable) mesons, but all meson decay chains ultimately produce electrons, positrons, neutrinos and photons.

As previously discussed, each meson is constructed from one quark and one antiquark. **Table 14.5.2** lists some representative mesons and their constituents.

TABLE 14.5.2 Quark compositions of a range of mesons

Quarks	Antiquarks									
	$\bar{b}$		$\bar{c}$		$\bar{s}$		$\bar{d}$		$\bar{u}$	
b	Y	$(\bar{b}b)$	B_c^-	$(\bar{c}b)$	$\bar{B}_S$	$(\bar{s}b)$	$\bar{B}_{D^0}$	$(\bar{d}b)$	B^-	$(\bar{u}b)$
c	B_c^{-1}	$(\bar{b}c)$	$\frac{J}{Y}$	$(\bar{c}c)$	D_S^+	$(\bar{s}c)$	D^+	$(\bar{d}c)$	D^0	$(\bar{u}c)$
s	B_S^0	$(\bar{b}s)$	D_S^-	$(\bar{c}s)$	ϕ	$(\bar{s}s)$	$\bar{K}_0$	$(\bar{d}s)$	K^-	$(\bar{u}s)$
d	B^+	$(\bar{b}u)$	$\bar{D}_0$	$(\bar{c}u)$	K^+	$(\bar{s}u)$	π^+	$(\bar{d}u)$	π^0	$(\bar{u}u)$
u	B_D^0	$(\bar{b}d)$	D^-	$(\bar{c}d)$	K^0	$(\bar{s}d)$	π^0	$(\bar{d}d)$	π^-	$(\bar{u}d)$

Baryons

The name *baryon* means 'heavy' in Greek. Baryons have masses equal to or greater than the proton mass. Their spin is always a half-integer value $\left(\frac{1}{2}, \frac{3}{2}, \ldots\right)$ and so they are fermions. Protons and neutrons are baryons. Protons are the only stable baryon. All others decay in such a way that the end products of the decay chain include a proton.

Baryons are composed of three quarks, and antibaryons of three antiquarks. **Table 14.5.3** lists the quark composition of some baryons.

Note that some baryons have the same quark composition; for example, the p and Δ^+ baryons have the same composition (uud), as do the n and Δ^0 baryons (udd). In these cases, the Δ particles are considered to be excited states of the proton and neutron.

TABLE 14.5.3 Quark compositions of a range of baryons

Particle	Symbol	Quark composition
Proton	p	uud
Neutron	n	udd
Lambda	Λ^0	uds
Sigma	Σ^+	uus
	Σ^0	uds
	Σ^-	dds

Particle	Symbol	Quark composition
Delta	Δ^{++}	uuu
	Δ^{+}	uud
	Δ^{0}	udd
	Δ^{-}	ddd
Xi	Ξ^{0}	uss
	Ξ^{-}	dss
Omega	Ω^{-}	sss

LEARNING CHECK 14.5

DESCRIBING

1 State the combination of quarks that is required to produce a meson.
2 State the combination of quarks that is required to produce a baryon.
3 List the fundamental forces experienced by quarks.
4 **Explain** why the name 'meson', meaning middle sized, is appropriate when compared to a proton.
5 **Explain** the evidence that supports hadrons not being elementary particles.

APPLYING

6 Which of the following quark combinations are possible mesons or baryons? **Explain** your answer.
a ud **b** udd **c** $\bar{u}dd$ **d** $\bar{u}\bar{u}\bar{d}$ **e** $ud\bar{d}$
7 Create a diagrammatic representation of the D^{-} meson. (Refer to Table 14.5.2.)
8 Create a diagrammatic representation of the Δ^{-} baryon. (Refer to Table 14.5.3.)

14.6 The Standard Model today

The Standard Model: a summary

Particle physicists now think that all matter is made up of two types of elementary particles – quarks and leptons – as well as the force carriers that are needed for them to interact. There appears to be a symmetry between the quarks and leptons, in that there are six types of each, along with their antiparticles.

Quarks combine to form mesons and baryons, which are both considered to be hadrons. Mesons are made up of two quarks, as shown in Table 14.5.2. Baryons are made up of three quarks each, as shown in Table 14.5.3.

In addition to these particles, there are also the exchange particles associated with the four fundamental forces, as shown in Table 14.3.1.

Although the details of the Standard Model are complex, its essential ingredients are summarised in Figure 14.6.1. This diagram shows that quarks participate in all the fundamental forces and that leptons participate in all except the strong force.

Weblink
The Standard Model

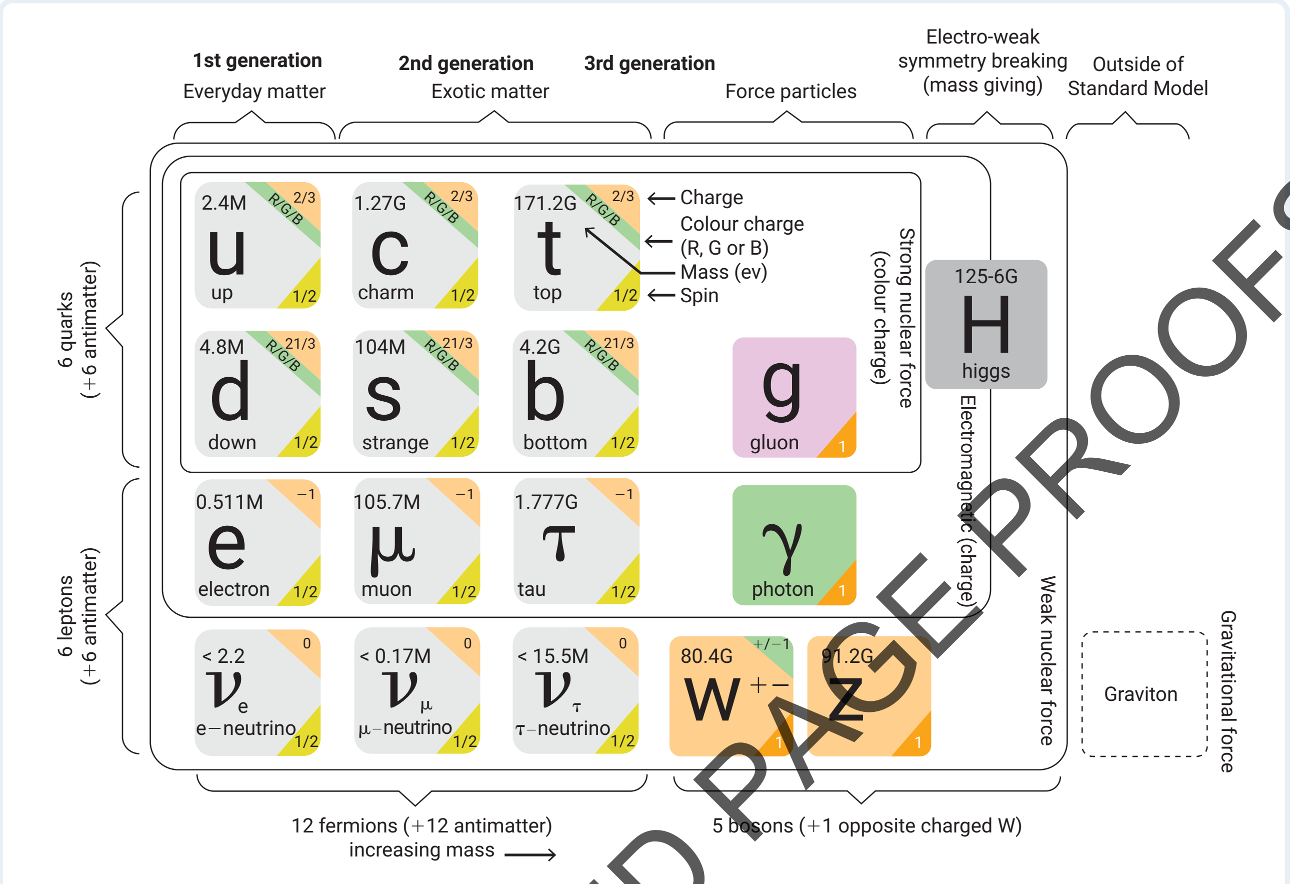


FIGURE 14.6.1 The 'periodic table' of the elementary particles of the Standard Model of particle physics

Note that the Standard Model *does not* include the gravitational force at present. However, gravity is included in Figure 14.6.1 because physicists hope to eventually incorporate this force into a **unified theory**.

unified theory any theory that demonstrates how fundamental forces can be united, and explains the mechanism by which they become distinct (e.g. the electroweak theory)

Limitations of the Standard Model

The Standard Model has helped us make sense of the huge number of particles. It has provided a sort of 'periodic table' to help us understand particle properties as arising from their underlying quark compositions.

The Standard Model has also been very successful in explaining the origin and nature of three of the four fundamental forces – the strong, weak and electromagnetic. However, the Standard Model does not include the gravitational force.

The inclusion of the Higgs boson in the Standard Model gives a mechanism by which particles have mass. Particles acquire mass because of interactions with Higgs bosons. As we have seen, mass is the property that causes the gravitational field; and therefore, the gravitational force. However, the Standard Model does not explain how the gravitational force is mediated. Many physicists believe that the gravitational field is mediated by massless exchange particles called gravitons. This is consistent with the way other fields are mediated by exchange particles (Table 14.3.1), but the Standard Model does not include the graviton; therefore, the Standard Model is incomplete.

Gravity is the force that is most significant in the large-scale structure and evolution of the universe. Theories such as the Big Bang theory base their predictions and explanations of cosmological phenomena largely on our current understanding of gravity as described by the general theory of relativity. For example, the predicted existence of **dark energy** and **dark matter** is based on observed gravitational effects. Therefore, the development of a more fundamental explanation for the gravitational field, or a **grand unified theory (GUT)** that explains all four forces, is of great importance not only to particle physicists but also to cosmologists.

dark energy energy that is predicted from the increasing rate of expansion of the universe but which is not identifiable as any currently known energy form

dark matter matter that is postulated to explain gravitational effects but which is not observable by the emission or reflection of light

grand unified theory (GUT) a theory that unites all four fundamental forces in a single model and explains the symmetry-breaking mechanism that caused them to separate into the four distinct forces we now know; there is as yet no widely accepted GUT

The structure and rate of expansion of the universe can be explained by the presence of dark matter and dark energy. Dark matter neither radiates nor reflects energy – hence, it is dark. Its presence is inferred from gravitational effects. Cosmologists think that dark matter and dark energy consist of subatomic particles of a type as yet undiscovered. The Standard Model does not predict such a particle – there is a mismatch between current theories in cosmology and the Standard Model of particle physics.

Finally, the Standard Model in its current form has been found to be consistent with experimental results over the last 50–60 years, with one exception. The Standard Model predicts that the neutrino should be massless.

In 1998, scientists at the Super-Kamiokande neutrino detector in Japan discovered that neutrinos can change from one type to another. This implies that they have mass.

Astronomical observations including 'galactic lensing' imply a neutrino mass of 0.2–1.5 eV c^{-2}. The masses of the various types of neutrinos are not yet known, other than that they are very, very small. However, there is general agreement that neutrinos are not massless. Various theories have been put forward modifying the Standard Model to allow for non-zero neutrino masses.

Mass and the Higgs boson

One of the questions raised by the Standard Model is why all the field particles except the W and Z bosons are massless. Or, put another way, why do the W and Z bosons have mass, and what is the origin of the mass of all the other massive elementary particles?

To resolve this problem, a hypothetical particle called the Higgs boson, which provides a mechanism for breaking the electroweak symmetry, was proposed. When particles interact via the Higgs field, they gain mass from this interaction. The field particle is the Higgs boson. The Standard Model modified to include the Higgs boson provides a logically consistent explanation of the massive nature of the W and Z bosons.

Weblink
How massive neutrinos broke the Standard Model

The Higgs boson was named for Peter Higgs (1929–2024), one of a group of physicists who proposed its existence in 1964. The existence of the Higgs boson was confirmed in March 2013 by scientists at CERN after a particle with properties corresponding to those predicted for the Higgs boson was observed in July 2012.

Discovery of the Higgs boson

As we have seen, the mass of the Z and W bosons that mediate the electroweak force is something of a mystery in the Standard Model. Related to this is the question of how particle masses arise and what determines the mass of a given particle.

The theories behind this are extremely complicated mathematically and well beyond the scope of this text. They deal with fundamental symmetries in nature and are referred to as *gauge theories*. They are based on quantum field theories, in which exchange particles are the mechanism by which particles interact and reactions occur.

In 1964, British physicist Higgs proposed the existence of a field, now called the Higgs field, with which particles with mass interact. It is this field that gives them the property of mass. The field is composed of particles, now called Higgs bosons.

In the same year, and published in the same journal (*Physical Review Letters*), were two other papers that also described a mechanism by which particles acquired mass via interactions with a field. The first paper was by Francois Englert and Robert Brout, who were working in Brussels, and the other was by Gerald Guralnik, Carl Hagen and Tom Kibble, a collaboration of British (Kibble) and American (Guralnik and Hagen) physicists working in London.

Although the popular literature generally only mentions Higgs, all three papers are considered of equal importance in the development of the relevant theory by particle physicists. Many physicists, including Higgs himself, have argued that the name 'Higgs boson' is inappropriate because it does not acknowledge the contribution made by the others. However, given the widespread and long-term use of the name, it is unlikely to change now.

Remember that for a model to be considered a good one, it must *explain existing phenomena* and *produce testable predictions*. The model developed by Englert, Brout, Higgs, Guralnik, Hagen and Kibble explained why the weak force has a short range, while the electromagnetic force has an infinite range. It also explained why particles have mass. The model predicted the existence of a gauge boson – the Higgs boson – as well as its approximate mass. The detection of this particle is an important test of the theory.

However, detecting a Higgs boson is not a simple task. It requires massive energies of colliding particles to produce a reaction in which isolated Higgs bosons can be detected. It also requires massive computing power to analyse the data generated in such collision events. One reason the Large Hadron Collider was constructed was to search for the Higgs boson.

In July 2012, almost 50 years after its prediction, the detection of a particle with the properties and approximate mass of the Higgs boson was announced by physicists at CERN. It is interesting to note that the announcement *did not* claim that the particle found was the Higgs boson. Such a finding is of enormous importance in confirming the Standard Model and in understanding the nature of matter, forces and energy – the basic concepts in physics. This explains why researchers were hesitant to make such a significant claim without further careful analysis and measurement. Nonetheless, the announcement caused enormous excitement.

In 2013, the Nobel Prize in Physics was awarded to Peter Higgs and Francois Englert (Robert Brout died in May 2011, and the Nobel Prize is not awarded posthumously). The citation by the Nobel Prize Foundation states that the award was given 'for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider'.

The Standard Model and cosmology

As you have seen, the Standard Model of particle physics deals with elementary particles, which are the smallest building blocks of matter. In contrast, cosmology deals with the large-scale structure of the universe, such as galaxies and nebulae. So, it is not immediately obvious how the two are connected.

To understand the current structure of the universe, and predict its future, we need to understand the evolution of the universe. The currently accepted theory of how the universe began, the Big Bang theory, states that the universe began as an infinitesimally small and dense singularity about 14 billion years ago. This singularity exploded, forming all the elementary particles described by the Standard Model of particle physics. The interaction of these particles, which led to the formation of other particles – atoms, molecules and eventually stars and galaxies – is governed by the fundamental forces.

Hence, to understand the evolution of the universe, we need to start with particle physics. But to understand where the particles came from in the first place, we need to use cosmology: the two are intimately connected.

The Big Bang theory is an expansion theory of the universe. It says that the universe was once much smaller and has been expanding since some time in the past. There are other expansion theories in cosmology, as well as steady state theories (which say that the universe is not expanding) and even oscillatory theories, which say that the universe is oscillating in size and happens to be expanding at the moment. However, the Big Bang theory is currently the most widely accepted cosmological theory.

There are two main experimental observations that support the Big Bang theory. The first is the **redshift** of light from distant galaxies, described in the next section. If we assume that this means that all points in space are moving away from us and extrapolate backwards in time, then the universe must once have been much smaller. The second important piece of evidence is the existence of the **cosmic microwave background radiation**, which is also discussed later.

redshift the observed shift to longer wavelength of spectral lines in distant stars

cosmic microwave background radiation the observed radiation coming from all points in space corresponding to radiation from a black body at 3 K; it is believed to come from an earlier, much hotter stage of the evolution of the universe

In addition to the redshift and cosmic microwave background radiation, the different characteristic spectra of distant stars and the structure of distant galaxies, compared to close ones, implies that the universe is changing with time. Therefore, it is not in a steady state. This argument is based on the idea that looking at distant galaxies is equivalent to looking backwards in time. This is because light takes a finite time to travel a given distance. If a star is 1000 light-years away, then light from that star that we observe today left the star 1000 years ago and what we are observing is what that star looked like 1000 years ago.

The expanding universe

In 1912, Vesto Melvin Slipher reported that most galaxies are receding from Earth at speeds of up to several million kilometres per hour. Slipher used Doppler shifts in spectral lines to measure the velocities of various galaxies.

Recall that at this time spectra had already been used to identify specific atomic species. Slipher observed spectra from distant galaxies that had the same line spacings as known species, but with the frequency of each line shifted by a fixed amount.

When the source of a wave is moving, the frequency of the wave detected by an observer is shifted. A higher frequency (blueshift) is detected if the source of the wave is travelling towards the observer. A lower frequency (redshift) is detected if the source is travelling away from the detector. You may have noticed this effect when an ambulance or police car goes past you with its siren on. Initially, as the vehicle approaches, you hear a higher frequency. As the vehicle passes you, the frequency decreases and you hear a lower frequency as it moves away.

The spectral lines from distant galaxies that Slipher observed were consistently shifted to lower frequencies. The shift varied for different galaxies, but in each case the shift was towards a lower frequency (or longer wavelength). This is called a redshift, because red is at the lower frequency end of the spectrum. From these shifts, Slipher deduced that the galaxies were moving away from us.

Subsequent observations by other astronomers consistently showed that the spectra of stars in all observed galaxies were redshifted. It appeared that everything was moving away from us!

Not only did observations show that all galaxies are moving away from us, but it also seems that the more distant the galaxy, the greater the redshift. This implies that the further away a galaxy is, the faster it is moving.

In the late 1920s, Edwin P. Hubble proposed the theory that the entire universe is expanding. Observations showed that the speeds at which galaxies are receding from Earth increase in direct proportion to their distance from us.

This is possible if the entirety of space–time is expanding, with all points getting further away from each other. So the universe must have started out much smaller.

The current theory in cosmology, the Big Bang theory, says that it started with an explosion of all matter from a single point, a singularity.

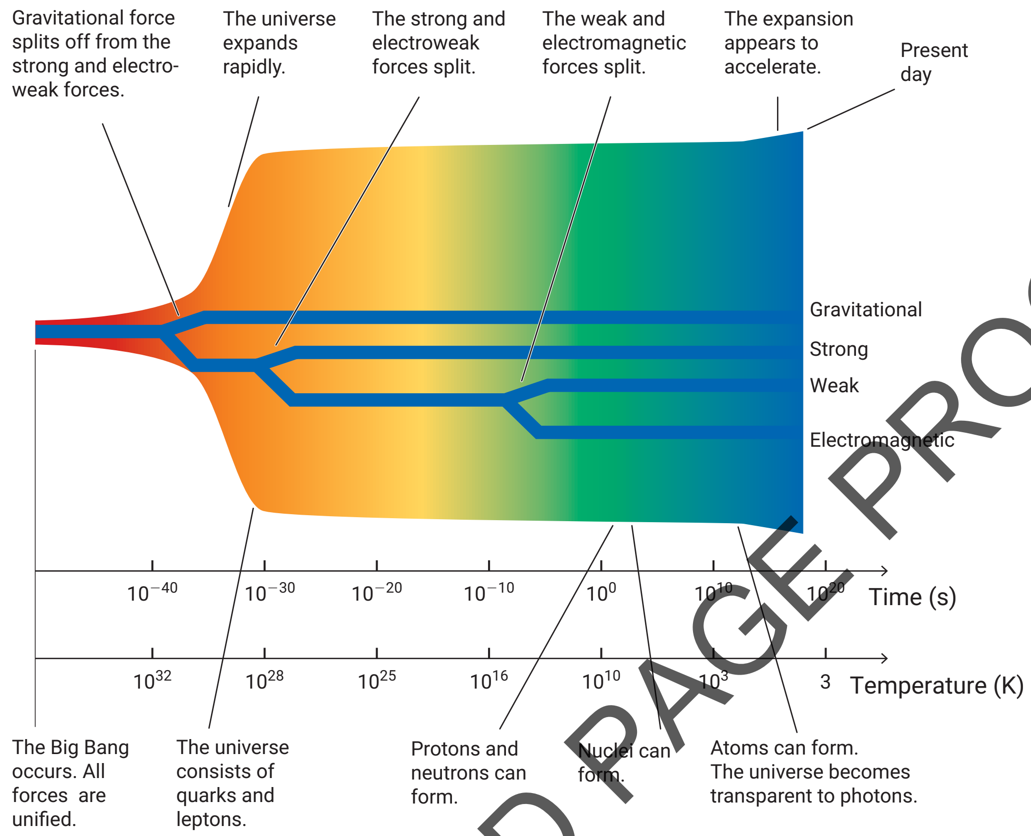


FIGURE 14.6.2 The Big Bang and the evolution of the fundamental forces from one original fundamental force

The Big Bang theory and the evolution of the universe

According to the Big Bang theory, the universe erupted from an infinitely dense singularity about 14 billion years ago.

For the first few moments after the Big Bang, the universe was at such extremely high energy (temperature) that all matter was contained in a quark–gluon plasma.

It is thought that in these first moments all four fundamental forces were unified; the strong, electroweak and gravitational forces were joined to form a single force.

The evolution of the four fundamental forces from the Big Bang to the present is shown in Figure 14.6.2.

Then, about 10^{-35} s after the Big Bang, when the temperature had dropped to about 10^{29} K, symmetry breaking occurred for gravity. This symmetry breaking meant that the properties of the gravitational force became distinct from those of the other forces. At this time, the strong and electroweak forces remained unified.

It was a period when particle energies were so great that very massive particles as well as quarks, leptons and their antiparticles existed. For some reason not yet understood, far more matter than antimatter particles formed. The amount of matter far exceeds the amount of antimatter in our universe to this day. This is not explained by the Standard Model of particle physics, or by the Big Bang theory.

Then the universe rapidly expanded and cooled, and the strong and electroweak forces became distinct.

The universe continued to cool and, at approximately 10^{-10} s after the Big Bang, the electroweak force split into the weak force and the electromagnetic force.

After a few minutes, protons and neutrons condensed out of the plasma. For half an hour, the universe underwent thermonuclear detonation, exploding as a hydrogen bomb and producing most of the helium nuclei that now exist. The universe continued to expand, and its temperature dropped.

Until about 700 000 years after the Big Bang, the universe was dominated by radiation. High-energy radiation prevented matter from forming neutral hydrogen atoms because collisions would instantly ionise any atoms formed. Photons were continuously scattered from the vast numbers of free electrons, resulting in a universe that was opaque to radiation.

This is an important point in the history of the universe. Astronomers claim that when you look through a telescope into space, you are looking into the past. This is because light takes a finite time to travel through space. Hence, the further away you look into space, the further back in time you are seeing, because the light has taken a long time to reach you.

No matter how far away you look with an optical telescope, you cannot see further back than the time when the universe was 700 000 years old. This is because the universe was opaque to light before then, so no light can reach us from that time. The development of more powerful radio telescopes, such as the Square Kilometre Array in Western Australia, that can detect much longer wavelengths will be able to analyse signals from beyond this time **(Figure 14.6.3)**.

FIGURE 14.6.3 An artist's impression of the Square Kilometre Array in Western Australia

The 700 000-year-old universe had expanded and cooled to approximately 3000 K and protons could now bind to electrons to form neutral hydrogen atoms. Now that atoms existed as the main state of matter, far more wavelengths of radiation were *not* absorbed by atoms than were absorbed, and the universe suddenly became transparent to photons.

Radiation no longer dominated the universe, and clumps of neutral matter steadily grew: first atoms, then molecules, gas clouds, stars and finally galaxies.

The universe has continued to expand and cool since the Big Bang.

One prediction of the Big Bang theory is that residual radiation from the Big Bang should currently be observable. This was predicted in 1948 by Ralph Alpher and Robert Herman, although their work was largely ignored at the time.

This radiation comes from the time when the universe was at a temperature of about 3000 K and photons were first able to pass through the matter in the universe. We would expect a radiation spectrum from the early universe to look like that of a 3000 K black-body curve.

The universe has since expanded and cooled, and the predicted radiation spectrum now corresponds to a 3 K black-body curve. This background radiation should have a peak intensity at a wavelength of a few millimetres, which is in the microwave region of the spectrum. Hence, this residual radiation is known as cosmic microwave background radiation **(Figure 14.6.4)**.

FIGURE 14.6.4 A map of cosmic microwave background radiation produced by NASA's Cosmic Microwave Background Explorer satellite

Cosmic microwave background radiation was first observed in 1965 by astronomers Arno Penzias and Robert Wilson of Bell Laboratories in the US. The measurement of the radiation was important in establishing the Big Bang theory as the accepted theory in cosmology.

In 1965, Penzias and Wilson were testing a sensitive microwave receiver for satellite communications. A faint background hiss was interfering with their experiments. They noticed that the hiss was the same regardless of the direction they pointed the antenna. They cooled the microwave detector and went outside to chase a flock of pigeons out of the horn-shaped antenna, but the signal remained.

In a casual conversation with colleagues they realised that what they had taken to be interference caused by pigeons was actually the residual radiation from the Big Bang.

Subsequent measurements confirmed that the radiation they measured corresponded to that of a black body at 2.7 K. In 1978, Penzias and Wilson were awarded the Nobel Prize for their discovery.

The Big Bang theory gives us a model for the beginning of the universe. But what will happen to the universe in the future? Will it continue to expand and cool forever, or at some time will it begin to contract again? What happens will depend on the amount of mass in the universe, and on the rate at which the universe is expanding.

If there is enough mass, then eventually the gravitational force will cause the expansion to slow down and stop and then reverse it. Gravity will pull all the galaxies, stars and planets back together into a 'big crunch'. If there is not enough mass, or if the rate of expansion is too great, then the universe will continue to expand and cool forever.

In 1998, observations of the apparent brightness and the redshift of supernovae were used to measure their distance and speed of recession from Earth. These observations led astronomers to conclude that the expansion rate is increasing.

To explain this acceleration, physicists proposed the existence of dark energy, which is energy possessed by the vacuum of space. The theory is that in the early universe, gravity dominated over the dark energy. As the universe expanded and the gravitational force between galaxies became smaller because of the great distances between them, the dark energy became comparatively more important **(Figure 14.6.5)**. The dark energy results in an effective repulsive force that causes the expansion rate to increase. This is a similar mechanism to that which causes hot gases to expand.

Other observations carried out between 2006 and 2011 at the Anglo-Australian Telescope at the Siding Spring Observatory in New South Wales have provided strong evidence that the expansion rate is indeed increasing. The new results imply that dark energy is likely to account for about 72% of the energy in the universe!

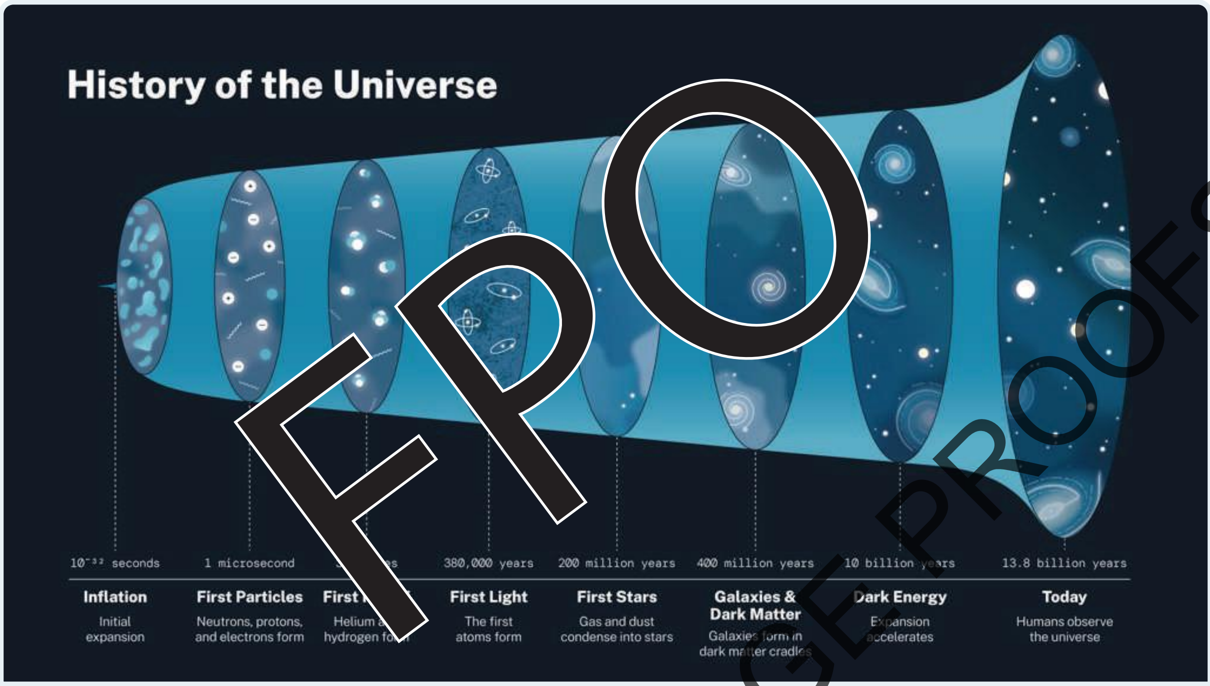


FIGURE 14.6.5 The history of the universe since the Big Bang to today, showing the discovery of dark energy

The dark energy may be acting to increase the rate of expansion of the universe. If the balance between dark energy and gravity favours dark energy, then the universe will continue to expand forever. Based on the observable mass of the universe, dark energy should win out. However, based on observable gravitational effects, there appears to more than just visible matter in the universe.

'Dark matter' was first postulated by Jan Oort in 1932. Recall from your study of gravity and orbital motion that the orbital velocity of a planet depends on the mass of the star about which it orbits. Similarly, the orbital velocity of stars about the galactic centre depends on the mass in the galaxy. Oort observed that the measured velocity of stars in the Milky Way did not correspond to that predicted by the observable mass in the Milky Way. The observable mass is that calculated from the objects that we can see – mainly stars.

Observations by other astronomers also provided evidence that the total mass in galaxies, including our own, is far greater than that due to visible matter. The 'missing matter' needed to explain these observations was given the name 'dark matter' because we cannot see it.

Cosmologists now theorised that most of the matter in the universe is this mysterious dark matter.

The nature of both dark energy and dark matter remains a mystery that many physicists hope to solve.

Limitations of the Big Bang theory

The Big Bang theory is the model of the evolution of the universe that is most widely accepted by cosmologists today. However, as with all the models and theories we have examined so far, it has limitations.

One of the predictions based on the combination of particle theory and the Big Bang theory is the creation of magnetic monopoles (isolated magnets with only one magnetic pole). However, no magnetic monopoles have ever been observed.

LEARNING CHECK 14.6

DESCRIBING

1 Name the three types of fundamental particles.
2 What causes redshift in observed matter?
3 What is dark matter and why is it given this name?
4 **Describe** a limitation of the Standard Model.
5 Why is the detection of the Higgs boson of such importance to the Standard Model?

APPLYING

6 The Higgs boson detected has a mass between 125 GeV c^{-2} and 126 GeV c^{-2}. **Calculate** this mass in kilograms.

REFLECTING

7 Why do you think a unified theory that incorporates all four forces is important to both particle physicists and cosmologists? How has your understanding of the way in which different areas of physics interact changed? Draw a concept map to illustrate your understanding.

CHAPTER SUMMARY

Elementary particles and antiparticles

- Elementary particles are particles whose substructure is unknown. These particles are classified into several categories: quarks, leptons, gauge bosons and the Higgs boson.
- For each elementary particle of matter, there is a corresponding antiparticle with the same mass but opposite charge. When an antiparticle collides with its corresponding particle, they annihilate each other, destroying both particles and releasing a significant burst of energy.
- An elementary particle's spin is a quantised, fundamental property of elementary particles, which is a measure of its intrinsic magnetic field.

Gauge bosons

- Gauge bosons are force-carrying particles that mediate particle interactions through the four fundamental forces.
  - The electromagnetic force involves the exchange of photons between electrically charged particles.
  - The strong nuclear force acts to hold the nucleus together, meaning it must be strong to overcome the proton–proton repulsion within the nucleus. Its gauge boson is the gluon.
  - The weak nuclear force is involved in nuclear decay, with the exchange particles being the W and Z bosons.
  - The gravitational force is the force which holds planets, stars and galaxies together, but its impact on elementary particles is negligible. It is thought to be mediated by the hypothetical gauge bosons called gravitons, which are not part of the Standard Model as the particles are yet to be detected.

Leptons

- Leptons are particles with very small masses that do not interact by means of the strong nuclear force, but do interact via the gravitational, electromagnetic and weak forces.
- All leptons have spin $\frac{1}{2}$.
- There are six leptons (plus their antiparticles): the electron, the muon and the tau, with a neutrino associated with each.
- In radioactive beta decay, leptons are responsible for the previously unaccounted-for energy and momentum.

Quarks

- Quarks are the substructure of hadrons.
- There are six flavours of quarks: up, down, strange, charm, top and bottom.
- Each quark has an associated antiquark with opposite charge, baryon number, strangeness, charm, topness and bottomness.
- Quarks also carry a fractional electric charge.
- Quarks interact by the means of all four fundamental forces.

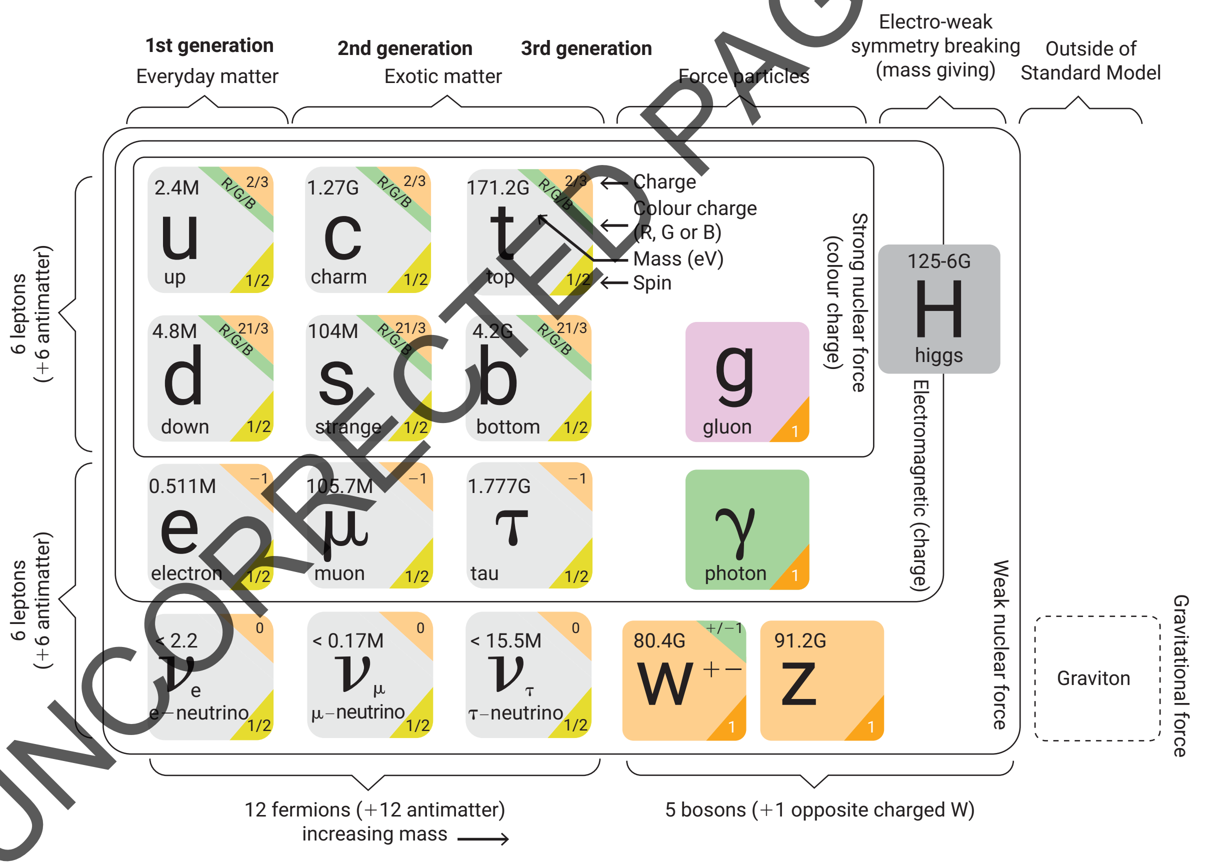
Hadrons

- Hadrons have a large mass and can be further divided into mesons and baryons.
  - A meson consists of one quark and one antiquark.
  - A baryon consists of three quarks.
  - An antibaryon consists of three antiquarks.

Higgs boson

- The Higgs boson helps to understand why particles have mass. It is not a gauge boson but is part of the Standard Model.
- The universe is permeated by an invisible field known as the Higgs field.
- When particles move through the Higgs field, mass is imparted to the particles.
- Some particles interact more strongly with the field, gaining more mass than others which interact weakly.

The 'periodic table' of the elementary particles of the Standard Model of particle physics



CHAPTER EXAM

MULTIPLE CHOICE

1. Which of the following particles is classified as a gauge boson?
- **A** Proton
- **B** Quark
- **C** Neutron
- **D** Photon

2. Which of the following particles is classified as a lepton?
- **A** Proton
- **B** Quark
- **C** Electron
- **D** Photon

3. Which of the following particles is classified as a hadron?
- **A** Proton
- **B** Quark
- **C** Electron
- **D** Photon

4. What is the correct symbol for a positron?
- **A** p^+
- **B** p^0
- **C** e
- **D** e^+

5. To which class of particles does the Pauli exclusion principle apply?
- **A** Leptons
- **B** Bosons
- **C** Mesons
- **D** All of the above

6. Which of the following forces is mediated by the Z^0 boson?
- **A** Strong
- **B** Weak
- **C** Electromagnetic
- **D** Gravitational

7. Which of the following forces is not experienced by leptons?
- **A** Strong
- **B** Weak
- **C** Electromagnetic
- **D** Gravitational

8. Which particle is responsible for giving other particles mass, according to the Standard Model?
   A Proton
   B Neutron
   C Higgs boson
   D Photon

9. How many types of quarks are there?
   A 2
   B 3
   C 6
   D 8

10. Which of the following is not yet confirmed as part of the Standard Model?
    A Electron
    B Graviton
    C Gluon
    D Photon

SHORT RESPONSE

11. **Calculate** the mass, in kg, of the following particles.
    a π
    b Ω

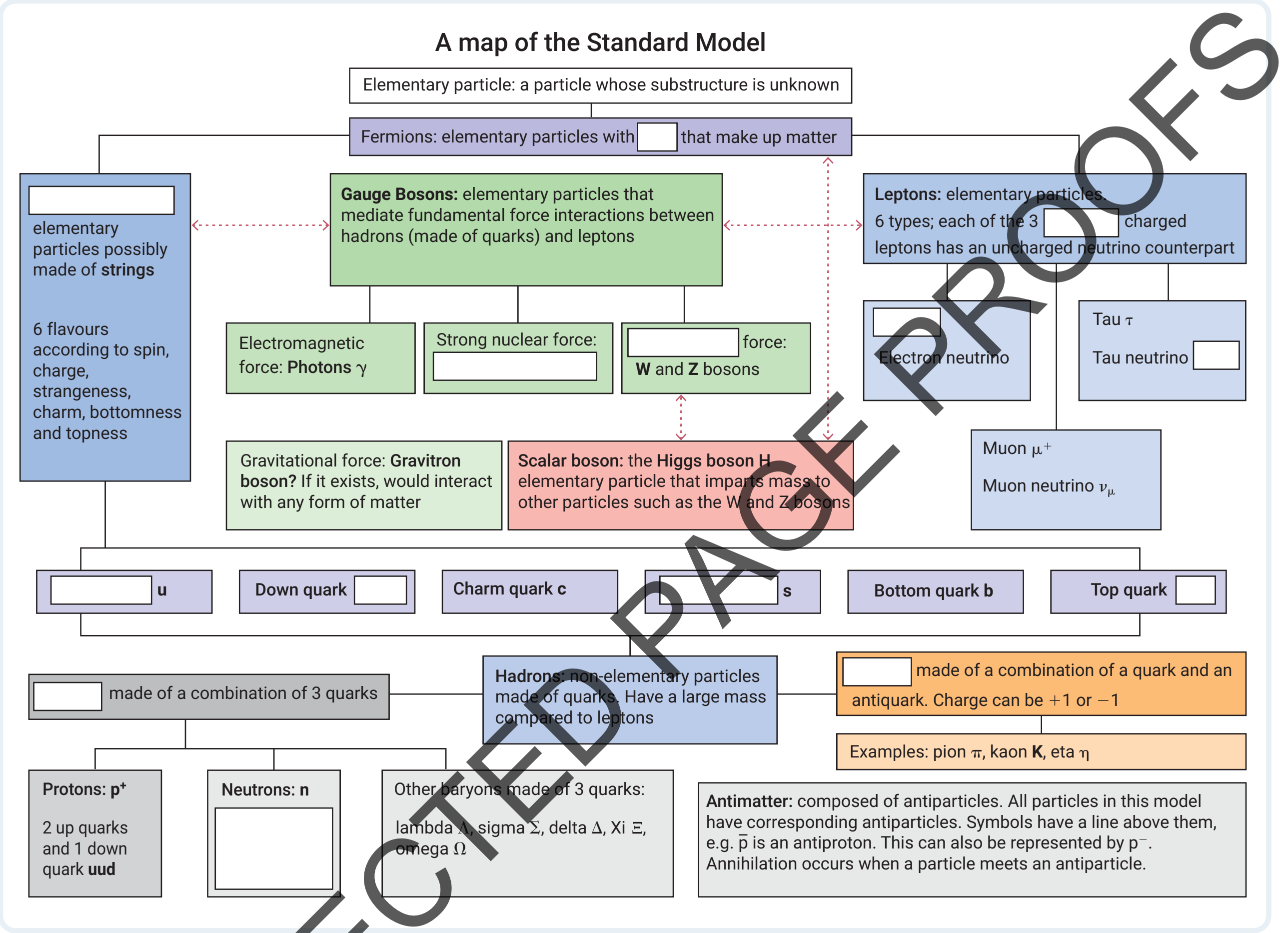
12. **Explain** the differences between fermions and bosons.

13. **Determine** whether these quark combinations are possible or not. For those that are possible, identify that particle.
    a cd
    b $\bar{u}ud$
    c $\bar{u}\bar{s}\bar{s}$
    d $c\bar{c}$

14. **Contrast** the muon and down quark in regard to the nature of the particles and their interactions.

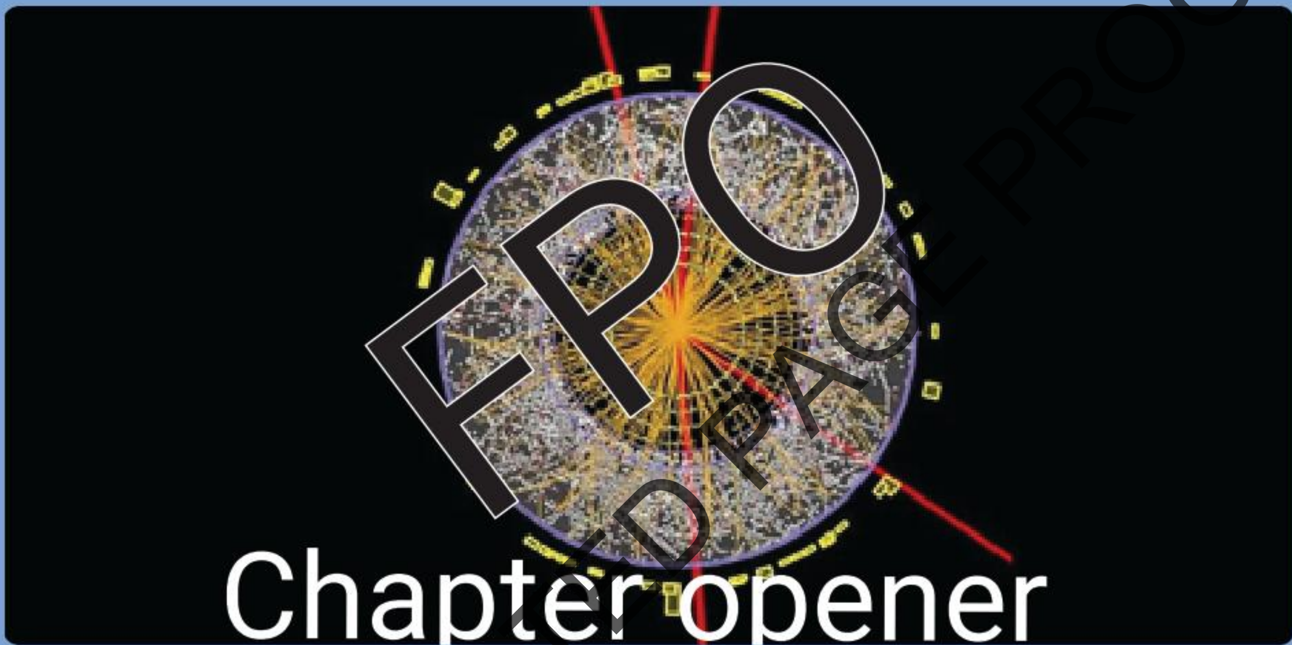
15. While studying, a Physics student constructed their own map of the Standard Model. However, some terms and symbols are missing from this map.

Copy and complete the student's map of the Standard Model by filling in the blank spaces.



CHAPTER 15

Particle interactions



TK

SYLLABUS DOT POINTS

SCIENCE UNDERSTANDING

- Describe the concepts of lepton number and baryon number.
- Solve problems relating to the conservation of lepton number and baryon number in particle interactions using $B = n_b - n_{\bar{b}}$, $B = \frac{1}{3}(n_q - n_{\bar{q}})$ and $L = n_l - n_{\bar{l}}$.
- Describe electron/electron, electron/positron and neutron decay interactions using particle interaction diagrams.
- Describe how symmetry in particle interactions occurs to maintain the principles of conservation.

SCIENCE INQUIRY

- Examine evidence supporting theories related to particle physics.

Physics 2025 v1.2 General Senior Syllabus © QCAA 2025

Introduction

The fundamental particles, also known as elementary particles, are the building blocks of the universe. In Chapter 14, we saw that these elementary particles are grouped according to specific properties. At this fundamental level, the interactions of these particles can only be represented theoretically because we are unable to see what is actually happening. The search for order in the universe is never ending, and at the particle level we need to consider which components of the building blocks are conserved.

Assessments

- Learning checks 5.1, 5.2, 5.3
- Chapter exam

Practical

- Pipe-Cleaner Reaction Diagrams (online-only resource)

Worksheets

- Name
- Name
- Name

FPO

ASSUMED KNOWLEDGE

- ✓ The Standard Model consists of quarks, leptons and gauge bosons.
- ✓ Electrons and positrons are antiparticles of each other.
- ✓ β^- decay involves the change from a proton to neutron, and the release of an electron antineutrino and an electron.
- ✓ Protons are made from two up quarks and one down quark (uud).
- ✓ Neutrons are made from one up quark and two down quarks (udd).

LEARNING OUTCOMES

By the end of this chapter, you should be able to:

- ✓ describe the concepts of lepton number and baryon number
- ✓ use conventional notation to represent particles, particle interactions and processes
- ✓ solve problems relating to the conservation of lepton number and baryon number in particle interactions using $B = n_b - n_{\bar{b}}$, $B = \frac{1}{3}(n_q - n_{\bar{q}})$ and $L = n_l - n_{\bar{l}}$.
- ✓ describe electron–electron, electron–positron and neutron decay interactions by using particle interaction diagrams and particle interaction equations
- ✓ describe Feynman's contribution to the development of particle interaction diagrams
- ✓ describe the role of exchange particles and exchange forces in particle interactions
- ✓ describe how time-reversal symmetry, charge-reversal symmetry and crossing symmetry in particle interactions occur to maintain the principles of conservation.

15.1 Lepton number and baryon number

Leptons and baryons are particles with unique characteristics. They are both categorised as fermions, which obey the Pauli exclusion principle.

Lepton number

lepton number a quantum number associated with each lepton, antilepton and non-leptonic particle

A lepton is a particle that interacts not by means of the strong nuclear force, but through the gravitational, electromagnetic and weak forces. There are three varieties of leptons: the electron (e), the muon (μ) and the tau (τ), each with an associated neutrino (ν). There are three quantum numbers called **lepton numbers**, L_e, L_μ and L_τ, associated with these particles. The electron and the electron neutrino (ν_e) have electron lepton number $L_e = +1$, and the antileptons e^+ and $\bar{\nu}_e$ have $L_e = -1$. All other particles have $L = 0$. Table 14.2.1 (page xxx) lists the lepton numbers of each type of lepton. L_ν and L_τ are also equal to 1, and the antileptons to these have lepton number –1. The lepton notation used depends on the particles in the reaction.

The lepton number is a quantum number associated with each lepton and its neutrino (+1), and each antilepton and antineutrino (–1), to help describe the absence of reactions that created new particles. The lepton numbers are conserved during a reaction and can be determined using the formula $L = n_l - n_{\bar{l}}$.

KEY FORMULA

Lepton number

$L = n_l - n_{\bar{l}}$

where:

L = lepton number

n_l = number of leptons

$n_{\bar{l}}$ = number of antileptons

WORKED EXAMPLE 15.1.1

Determine the lepton number of each of the following particles.

a Electron (e^-)
b Electron neutrino (ν_e)
c Positron (e^+)
d Antimuon ($\bar{\mu}^+$)
e Neutron (n)
f Antitau neutrino ($\bar{\nu}_\tau$)

ANSWERS

1 Understand lepton number.

Lepton number (L) is a conserved quantum number assigned to particles in the lepton family.
Particles in a specific lepton family (e, μ, τ) are assigned $L = +1$.
Their corresponding antiparticles are assigned $L = -1$.
Non-leptonic particles, such as protons and neutrons, are assigned $L = 0$.

2 Assign lepton numbers.

a Electron (e^-):
- Lepton in the electron family
- Lepton number: $L = +1$

b Electron neutrino (ν_e):
- Neutrino in the electron family
- Lepton number: $L = +1$

c Positron (e^+):
- Antiparticle of the electron
- Lepton number: $L = -1$

d Antimuon ($\bar{\mu}^+$):
- Antiparticle of the muon
- Lepton number: $L = -1$

e Neutron (n):
- Non-leptonic particle (baryon)
- Lepton number: $L = 0$

f Antitau neutrino ($\bar{\nu}_\tau$):
- Antiparticle of the tau neutrino
- Lepton number: $L = -1$

Baryon number

A baryon is a relatively heavy fermion with half-integer spin. Protons and neutrons are examples of baryons. Every particle can be assigned a **baryon number**: $B = +1$ for all baryons, $B = -1$ for all antibaryons; $B = 0$ for all other particles. Table 14.2.1 lists the baryon number for each type of particle. Baryon number is used to describe the annihilation and creation of particles at a very small scale. Experimental results show that whenever a baryon is created in a decay or nuclear reaction, an antibaryon is also created. Baryons and antibaryons can annihilate, just as electrons and positrons do. The baryon numbers are conserved during a reaction.

The baryon number is conserved in interactions and can be determined using the formula $B = n_b - n_{\bar{b}}$.

baryon number a quantum number associated with each baryon, antibaryon and non-baryonic particles

KEY FORMULA

Baryon number

$$B = n_b - n_{\bar{b}}$$

where:

B = baryon number

n_b = number of baryons

$n_{\bar{b}}$ = number of antibaryons

WORKED EXAMPLE 15.1.2

Determine the baryon number of each of the following particles.

a Proton (p)
b Neutron (n)
c Antiproton ($\bar{p}$)
d Pion (π^+)
e Lambda particle (Λ^0)
f Antineutron ($\bar{n}$)

ANSWERS

1 Understand baryon number.

The baryon number (*B*) is a conserved quantum number assigned to baryons (particles made of three quarks).

- Baryons (e.g. protons, neutrons) are assigned $B = +1$.
- Antibaryons (e.g. antiprotons, antineutrons) are assigned $B = -1$.
- Non-baryonic particles (e.g. mesons, leptons) are assigned $B = 0$.

2 Assign baryon numbers.

a Proton (p):
- A baryon made of three quarks (uud)
- Baryon number: $B = +1$

b Neutron (n):
- A baryon made of three quarks (udd)
- Baryon number: $B = +1$

c Antiproton ($\bar{p}$):
- An antibaryon made of three antiquarks ($\bar{u}\bar{u}\bar{d}$)
- Baryon number: $B = -1$

d Pion ($\pi+$):
- A meson made of a quark–antiquark pair ($u\bar{d}$)
- Baryon number: $B = 0$

e Lambda particle (Λ^0):
- A baryon made of three quarks (uds)
- Baryon number: $B = +1$

f Antineutron ($\bar{n}$):
- An antibaryon made of three antiquarks ($\bar{u}\bar{d}\bar{d}$)
- Baryon number: $B = -1$

KEY FORMULA

Baryon number of an individual particle

$$B = \frac{1}{3}(n_q - n_{\bar{q}})$$

where:

B = baryon number

n_q = number of quarks

$n_{\bar{q}}$ = number of antiquarks

The baryon number of an individual particle may be determined from the number of quarks and antiquarks in a particle. For example, the proton, which is made up of two up quarks and one down quark, has a baryon number of $B = \frac{1}{3}(3 - 0) = 1$.

9780170483704

WORKED EXAMPLE 15.1.3

A certain particle contains 2 up quarks, 1 down quark, and no antiquarks. Calculate the baryon number of the particle using the formula $B = \frac{1}{3}(n_q - n_{\bar{q}})$ where n_q is the number of quarks and $n_{\bar{q}}$ is the number of antiquarks. Based on your result, identify if the particle is a baryon, antibaryon, or neither.

ANSWER

1 **Determine the formula.**
The formula for baryon number is:
$$B = \frac{1}{3}(n_q - n_{\bar{q}})$$

2 **Substitute the known values.**
Here, the particle contains:
$n_q = 3$ (2 up quarks + 1 down quark)
$n_{\bar{q}} = 0$ (no antiquarks)
$$B = \frac{1}{3}(3 - 0)$$

3 **Calculate the answer.**
$$B = \frac{1}{3}(3)$$
$$= 1$$

4 **Interpret the result.**
The baryon number of the particle is $B = 1$.
A baryon is a particle with $B = 1$.
An antibaryon has $B = -1$.
Particles that are not baryons or antibaryons (like mesons) have $B = 0$.
Since $B = 1$, this particle is a baryon.
Therefore:
- baryon number is $B = 1$
- particle is a baryon.

LEARNING CHECK 15.1

DESCRIBING

1 **Recall** the lepton number for each lepton and antilepton.
2 **Explain** 'baryon number'.
3 **Determine** the baryon numbers for protons, neutrons, antiprotons and mesons.
4 **Explain** the difference in application of the lepton number and the baryon number.

APPLYING

5 A particle contains an up quark and one down antiquark. **Calculate** the baryon number of the particle using the formula $B = \frac{1}{3}(n_q - n_{\bar{q}})$ where n_q is the number of quarks and $n_{\bar{q}}$ is the number of antiquarks. **Identify** if the particle is a baryon, antibaryon or neither.
6 **Determine** the baryon number of each of the following particles.
a Electron (e^-)
b Neutron (n)
c Positron (e^+)
d Pion (π^+)
e Antiproton ($\bar{p}$)

law of conservation of lepton number each of the lepton numbers L_e, L_μ and L_τ is a conserved quantity

law of conservation of baryon number whenever a nuclear reaction or decay occurs, the sum of the baryon numbers before the process must equal the sum of the baryon numbers after the process

Weblink
Particle conservation laws

15.2 The conservation of lepton number and baryon number

The laws of conservation of energy, linear momentum, angular momentum, spin and electric charge provide us with a set of rules that all processes and interactions must obey. In addition, we are confined to a new set of conservation rules: the **law of conservation of lepton number** and the **law of conservation of baryon number** in any reaction or decay.

Lepton and baryon conservation

Consider the decay of a neutron:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Before the decay, the lepton electron number is $L_e = 0$; after the decay, it is $0 + 1 + (-1) = 0$. This shows that lepton electron number is conserved. The reason we use the L_e notation is that this decay is concerned with the electron.

Consider again the decay of the neutron. On the left-hand side, the neutron has baryon number $B = +1$. On the right-hand side, the proton has baryon number $B = +1$ and the electron and antineutrino have baryon number $B = 0$. As baryon number is conserved, this is an allowed process for neutrons.

Lack of conservation

neutrino oscillation a phenomenon in which a neutrino with a given lepton association (e, μ or τ) can later be measured to have switched to another neutrino type (e, μ or τ); lepton number is still conserved in this instance

In cases where the baryon number or the lepton number differs, the reaction may not be possible. The decay of a proton into a positron and a neutral pion satisfies the conservation of energy, momentum and charge. However, it does not satisfy the law of conservation of baryon number. Hence, this decay has never been observed. Similarly, if lepton number is not conserved in a reaction, it means that the process is not possible. It has been found that lepton number is conserved in all reactions between particles; however, neutrinos have been observed to change from one type to another. This is called **neutrino oscillation**.

WORKED EXAMPLE 15.2.1

Use the law of conservation of lepton number and baryon number to determine if the following reactions are possible.

a $p + n \rightarrow p + p + n + \bar{p}$

b $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

c $p + n \rightarrow p + p + \bar{p}$

ANSWERS

a All these particles are baryons, so baryon number will be conserved. (Lepton number on both sides is 0.)

1. **Determine the baryon number before the reaction.**
   $B = (+1) + (+1) = +2$
2. **Determine the baryon number after the reaction.**
   $B = (+1) + (+1) + (+1) + (-1) = +2$
3. **Compare the numbers.**
   Both baryon and lepton number are conserved; therefore, the reaction is allowed.

b These particles are all leptons, so lepton number will be conserved. (Baryon number on both sides is 0.)

1 **Determine the lepton number before the reaction.**

$L = +1$

2 **Determine the lepton number after the reaction.**

$L = +1 + (-1) + (+1) = +1$

3 **Compare the numbers.**

The lepton numbers are conserved, so the decay is possible.

c All these particles are baryons, so baryon number will be conserved. (Lepton number on both sides is 0.)

1 **Determine the baryon number before the reaction.**

$B = (+1) + (+1) = +2$

2 **Determine the baryon number after the reaction.**

$B = (+1) + (+1) + (-1) = +1$

3 **Compare the numbers.**

The total baryon number is reduced and not conserved. From this, we can conclude that the reaction cannot occur.

Particle interaction diagrams

A particle interaction diagram provides a pictorial representation of an interaction of particles before, at and after their interaction. These diagrams show the particles that exist before the reaction to the left and those that result from the reaction to the right. In particle interaction diagrams, the horizontal axis represents time, with the positive *x* direction pointing to the right.

Weblink
A beginner's guide to Feynman diagrams

Conventions in particle interaction diagrams

Convention 1: Space is vertical, time is horizontal

In particle interaction diagrams, space is represented on the vertical axis and time is represented on the horizontal axis (Figure 15.2.1). Particle interaction diagrams are drawn to be read from left to right.

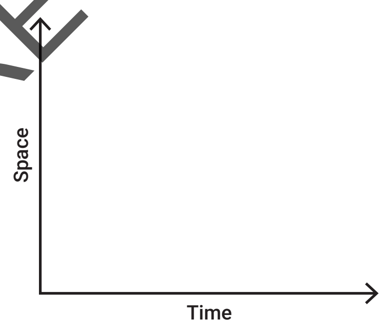


FIGURE 15.2.1 Particle interaction diagrams are drawn with space on the vertical axis and time on the horizontal axis.

Convention 2: Particle types are represented as straight lines with arrows

Particles are represented as straight lines with arrows pointing in the forward direction of time (Figure 15.2.2). They should be labelled with their appropriate symbol. Antiparticles are represented as straight lines also, but with arrows pointing in the backwards direction of time. They should also be labelled with their appropriate symbol.

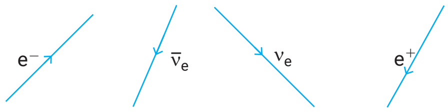


FIGURE 15.2.2 Particles in particle interaction diagrams are drawn with straight lines. Their arrows indicate the type of matter: particles are forward-pointing arrows with respect to the time axis; antiparticles are backward- pointing arrows with respect to the time axis.

Convention 3: Gauge bosons (force mediators) are represented with squiggly lines

In particle interaction diagrams, gauge bosons are represented as squiggly lines to indicate their nature as force carriers in particle interactions (Figure 15.2.3). Gauge bosons include the photon (γ), W^+, W^-, Z bosons and gluon, g. We typically see gauge bosons mediating (in the middle of) interactions.

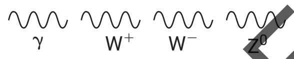


FIGURE 15.2.3 Gauge bosons are drawn with squiggly lines to indicate their nature as force carriers in particle interactions. Gauge bosons are typically seen mediating (in the middle of) interactions.

Summary of conventions

Follow these conventions when drawing particle interaction diagrams. They will ensure that the diagram correctly reflects the particles involved and the conservation laws applied.

1. Space is represented on the vertical axis; time is represented on the horizontal axis.
2. Particles are represented as straight lines with a forwards-pointing arrow. Antiparticles are represented as straight lines with backwards-pointing arrow. Label particles and antiparticles with their corresponding symbols.
3. Gauge bosons (the mediating particles) are represented as a squiggle. Label gauge bosons with their corresponding symbols.
4. A vertex, where multiple lines enter to indicate an interaction, is shown as a dot in the particle interaction diagram.

These conventions are a mathematical and graphical convenience. The use of particle interaction diagrams helps us to understand the interactions and the conservation laws applied.

KEY CONCEPT

Particle interaction diagram conventions

1. Space is on the vertical axis; time is on the horizontal axis.
2. A particle is represented as a straight line with a forwards-pointing arrow. An antiparticle is represented as a straight line with a backwards-pointing arrow. Label particles and antiparticles with their corresponding symbols.
3. Gauge bosons (the mediating particles) are represented as a squiggle. Label gauge bosons with their corresponding symbols.
4. A vertex, where multiple lines enter to indicate an interaction, is shown as a dot in the particle interaction diagram.

Figure 15.2.4 shows a particle interaction diagram for the decay of a neutron into a proton, an electron and an electron antineutrino. This is otherwise known as β^- (beta negative) decay.

Note that on the diagram each particle is represented by an arrow, and the direction of the arrows matches the direction of time. For antiparticles, the direction of the arrow is reversed. This allows us to distinguish between particles and antiparticles. The arrows on reaction diagrams *do not* represent trajectories; rather, they represent their nature as a particle or antiparticle. Straight lines represent matter and squiggly lines represent massless **exchange particles**, such as photons.

exchange particle a particle carrying force, which is responsible for behaviour during other particle interactions; sometimes exchange particles are the result of a particle interaction, such as in an electron–positron annihilation

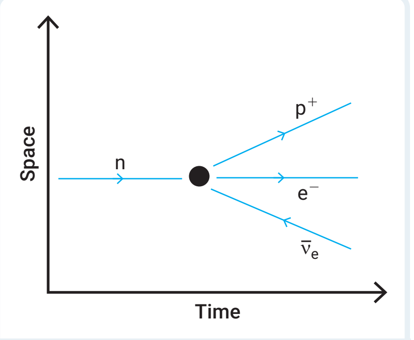


FIGURE 15.2.4 The β^- decay of a neutron into a proton, an electron and an electron antineutrino

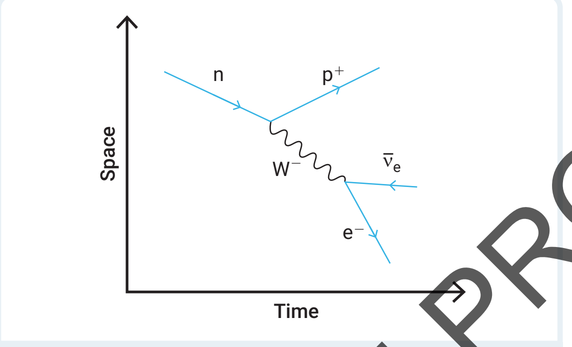
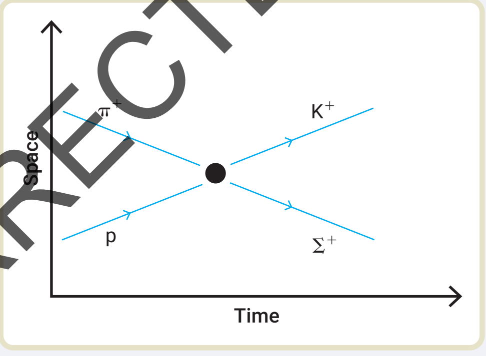


FIGURE 15.2.5 A particle interaction diagram showing a neutron decaying to a proton. The conservation of charge, mass and spin necessitates the generation of an electron and an electron antineutrino. This is otherwise known as a β^- decay.

WORKED EXAMPLE 15.2.2

Construct a particle interaction diagram for the process $\pi^+ + p \rightarrow K^+ + \Sigma^+$.

ANSWER



LEARNING CHECK 15.2

DESCRIBING

1 Energy, momentum and charge are conserved in reactions. **Recall** the other two quantities that must be conserved in all particle interaction reactions.

APPLYING

Refer to Table 14.2.1 (page 391) for lepton numbers and baryon numbers.

1 **Determine** if the decay $\pi^+ + p \rightarrow K^+ + \Sigma^+$ is allowed.

2 **Determine** if the reaction $p + e^- \rightarrow n + \nu_e$ is allowed.

3 **Describe** how the lepton number is conserved in the reaction $\pi^+ \rightarrow \mu^+ + \nu_e + \bar{\nu}_\mu$.

4 Show that baryon number is conserved in the reaction $p + p \rightarrow p + p + \pi^0$.

5 **Construct** the reaction diagram for a negatively charged tau changing to a tau neutrino, an electron and an antielectron neutrino.

6 **Construct** the reaction diagram for a proton and neutron changing to a neutron, an antiproton and two protons.

7 **Determine** the baryon number of the:

  a meson ($d\bar{u}$)

  b neutron (udd).

ANALYSING

9 The following reaction is forbidden. **Determine** which conservation law is violated.

$p + \bar{p} \rightarrow \mu^+ + e^-$

10 Consider the reaction $\pi^- + \_\_ \rightarrow K^0 + \Lambda^0$. **Deduce** what properties the missing particle must have for the reaction to be allowed. Provide an example of such a particle.

15.3 Particle interaction diagrams

particle interaction diagram a diagram that models exchange particles and exchange forces over time in space, when particles come into close proximity to each other

exchange force a strong, electromagnetic, weak or gravitational force associated with the exchange particles gluons, photons, Q and Z particles and gravitons respectively (e.g. an exchange of photons between electrons produces an electromagnetic force)

Particle interaction diagrams are used to show more detail of the behaviour of various particles and **exchange forces** in a given interaction. Particle interaction diagrams use specific rules to visually represent the behaviour of particles over time.

Recall our summary of conventions for drawing particle interaction diagrams.

1. Space is represented on the vertical axis; time is represented on the horizontal axis.
2. Particles are represented as straight lines with a forwards-pointing arrow. Antiparticles are represented as straight lines with backwards-pointing arrow. Label particles and antiparticles with their corresponding symbols.
3. Gauge bosons (the mediating particles) are represented as a squiggle. Label gauge bosons with their corresponding symbols.
4. A vertex, where multiple lines enter to indicate an interaction, is shown as a dot in the particle interaction diagram.

Consider the conservation laws applied.

1. Charge is always conserved during particle interactions.
2. Baryon number is conserved during particle interactions.
3. Lepton number is conserved during particle interactions.

It is important to recognise that in all particle interaction diagrams, left to right represents going forwards in time. The vertical axis represents distance in space. The direction of the arrows on particles indicates only what type of particle is interacting during each given scenario: arrows pointing left to right represent particles; arrows pointing right to left represent antiparticles.

Electron–electron interactions

When two electrons interact, there is an electrostatic repulsion between them. This repulsion is due to the exchange of a photon between the electrons. After this interaction, the electrons still exist as they did originally (Figure 15.3.1).

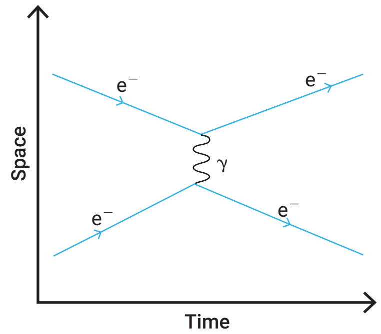


FIGURE 15.3.1 A particle interaction diagram for electron–electron scattering. Note that the exchange particle, gauge boson (the photon), is placed vertically. This indicates that the electrons do not actually touch in space. This exchange causes the electrostatic repulsion of the electrons and the electrons then scatter. Note: The arrows do not indicate a trajectory; they indicate that the electron is a particle moving in time

Electron–positron interactions

Electrons and positrons interact in either of two ways – scattering or annihilation.

Electron and positron scattering

Scattering is otherwise known as Bhabha scattering, named after the Indian physicist Homi J. Bhabha. An electron approaches a positron, but does not touch it. They exchange a virtual photon, γ, shown with as a vertical squiggly line, and scatter in different directions (**Figure 15.3.2**).

Electron and positron annihilation

When an electron and a positron annihilate, two high-energy photons are produced. During this process, mass is not conserved, but both lepton and baryon number are conserved. As a result, it is an allowed interaction. The interaction of the electron and positron is a particle interaction, denoted by the solid line in **Figure 15.3.3**.

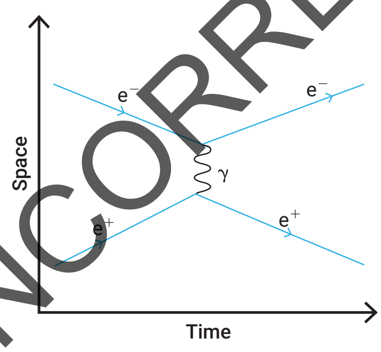


FIGURE 15.3.2 A particle interaction diagram for electron–positron scattering (also known as Bhabha scattering). Note the vertical gauge boson.

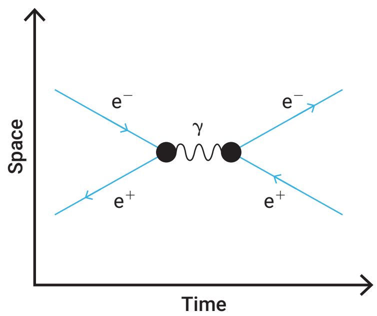


FIGURE 15.3.3 A particle interaction diagram for electron–positron annihilation. Note the mediating photon after the annihilation, before a new electron and positron are created.

Neutron decay (β^- decay)

Neutrons are generally stable within a nucleus. However, when a neutron does become unstable it decays into a proton, an electron and an electron antineutrino **(Figure 15.3.4)**. After this decay, there is a weak interaction between the new proton and the electron and antineutrino pair. The electron, because of its mass, can then leave with some velocity. This is what happens in radioactive nuclides during β^- decay. **Figure 15.3.5** shows the detail of the subatomic quarks for the neutron and proton

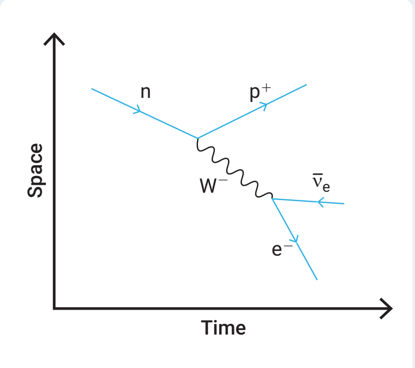


FIGURE 15.3.4 A particle interaction diagram for a neutron decay (β^- decay)

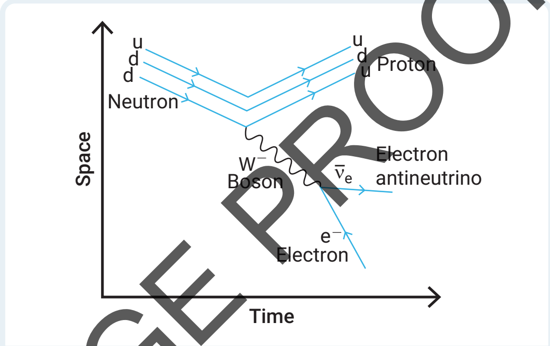


FIGURE 15.3.5 A particle interaction diagram for a neutron decay (β^- decay), showing the detail of the subatomic quarks for the neutron and proton.

Syllabus link
Chapter 7 of *Nelson QCE Physics Units 1 & 2* discusses neutrons decaying into a proton and an electron.

Proton decay (β^+ decay)

Protons are also generally stable within a nucleus. However, when a proton does become unstable, it decays into a neutron, a positron (an antielectron) and an electron neutrino **(Figure 15.3.6)**. After this decay, there is a weak interaction between the new neutron and the positron and neutrino pair. This is what happens in radioactive nuclides during β^+ decay. **Figure 15.3.7** shows the detail of the subatomic quarks in the proton decay.

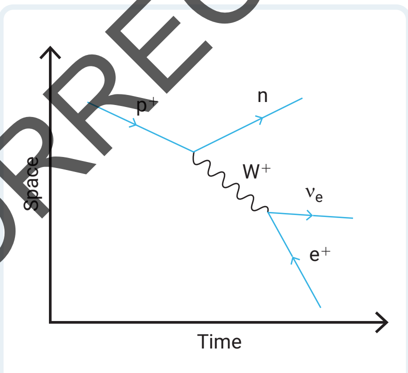


FIGURE 15.3.6 A particle interaction diagram for a proton decay (β^+ decay)

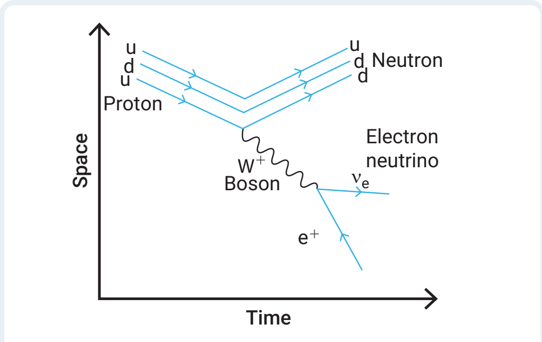


FIGURE 15.3.7 A particle interaction diagram for a proton decay (β^- decay), showing the detail of the subatomic quarks for the proton and neutron.

LEARNING CHECK 15.3

DESCRIBING

1 **Recall** what squiggly lines in a particle interaction diagram represent.

2 **Recall** the function of drawing arrows on a particle interaction diagram.

APPLYING

3 **Describe** the interaction taking place in Figure 15.3.8.

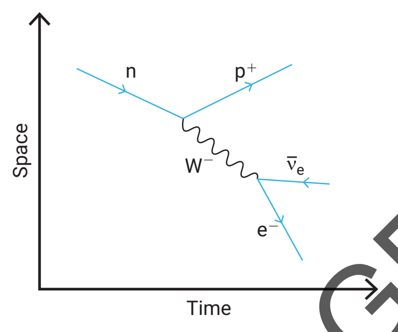


FIGURE 15.3.8 A particle interaction diagram. Can you explain the interaction including the different particles taking part?

4 **Construct** the particle interaction diagram of β^- decay.

5 **Construct** the particle interaction diagram of β^+ decay.

6 **Contrast** (identify the differences between processes) the particle interactions of electron–positron scattering and electron–positron annihilation.

15.4 Symmetry in particle interactions

The concept of **symmetry** in physics is an important one. Physicists believe that the laws of physics will correctly describe physical phenomena under various transformations – such as reversal of direction in space or time, or angular momentum (spin) or charge. These are called symmetries. Three symmetries give us the means of predicting possible particle interactions: time-reversal symmetry, charge-reversal symmetry and crossing symmetry.

If any of these symmetries is applied to an allowed reaction, then the resulting reaction is also allowed under the conservation laws. This does not mean that the reaction is likely to take place, just that it is allowed. In general, the probability of a new reaction occurring will be very different from the probability of the reaction from which it was derived. In some cases. the new reaction does not occur. The reason for this **symmetry breaking** is a matter of ongoing theoretical and experimental research.

Time-reversal symmetry

Recall that the horizontal direction on a reaction diagram represents time. If this diagram is then reflected, swapping left to right, the direction of time is reversed. This means that the reaction occurs in the reverse order. If all the conservation laws previously described were obeyed by the original reaction, then the new process will also obey all the conservation laws. This process is called **time reversal**.

symmetry the invariance of physical laws under transformations such as translation, reflection or rotation in time or space; symmetry in particle interactions reflect time-reversal, charge-reversal and crossing symmetry

symmetry breaking a change in the behaviour of a physical system or the laws of physics that govern its behaviour when a symmetry operation such as a translation, reflection or rotation in time or space takes place

time reversal when reactions are reversed in time

Figure 15.4.1 shows time reversal applied to electron capture. Figure 15.4.1a demonstrates the scenario in which an electron is captured by a proton, resulting in a neutron and an electron neutrino. This process occurs naturally in some nuclei. If time reversal is then applied to this process, the reaction becomes $n + \nu_e \rightarrow p + e^-$. This reaction still obeys all conservation laws and is hence an allowed reaction.

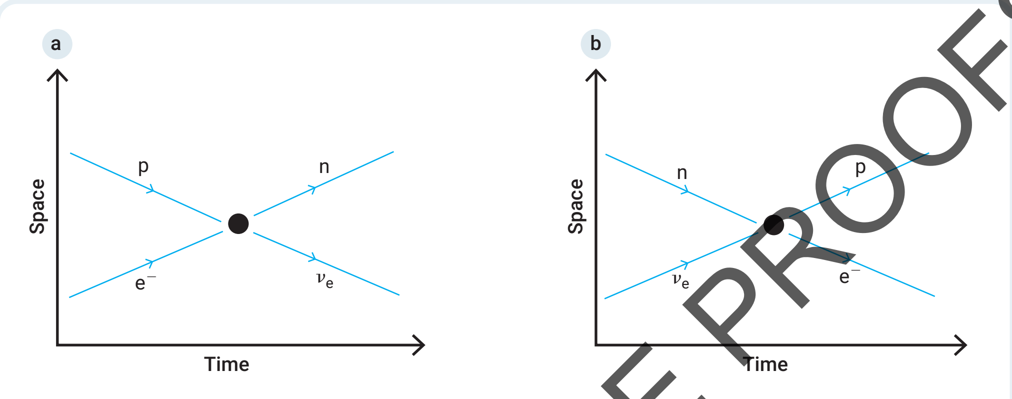


FIGURE 15.4.1 Time reversal applied to electron capture. (**a**) An electron is captured by a proton resulting in a neutron and electron antineutrino. (**b**) A neutron reacts with an electron neutrino to form a proton and electron.

time-reversal symmetry when an allowed reaction is written in the opposite direction in time; the new reaction is also allowed in that it does not break any of the known conservation laws

If **time-reversal symmetry** is applied to a known allowed reaction, then the new reaction generated is theoretically possible. However, this new reaction may not be experimentally probable.

Now consider the probability of particular reactions occurring under time reversal. An example is the decay reaction depicted in **Figure 15.4.2a** of a positive muon turning into a positron, an antimuon neutrino and an electron neutrino. This process has been experimentally observed. Applying time reversal to this reaction **(Figure 15.4.2b)** creates a reaction between a positron, antimuon neutrino and electron neutrino. Although this reaction is theoretically possible and it does not break any conservation laws, the probability of finding all three particles close enough together to react like this is negligible. This is particularly the case with short-lived exotic particles such as muons and neutrinos that only interact very weakly with matter. It is practically impossible to simultaneously collide three or more particles.

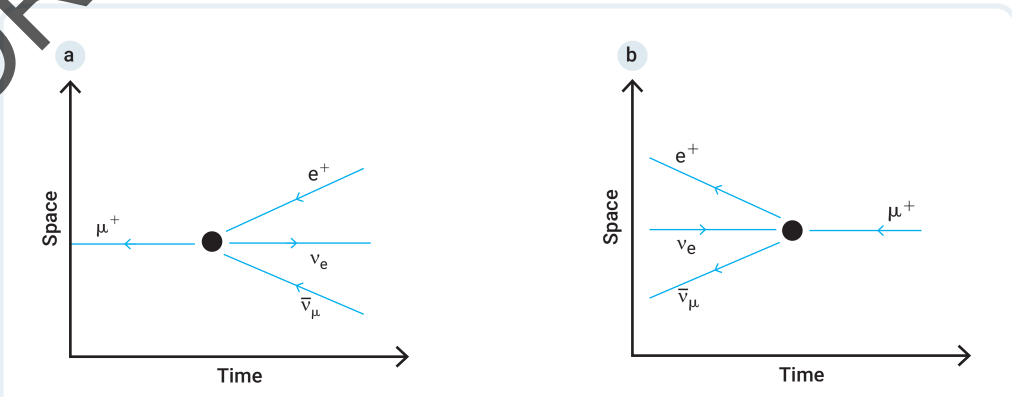


FIGURE 15.4.2 (**a**) A positive muon decays into a positron, antimuon neutrino and an electron neutrino. (**b**) Time reversal is applied to the process in part a. This reaction is not observed due to its extremely low probability, though is possible due to meeting conservation laws.

WORKED EXAMPLE 15.4.1

Predict the result of applying time reversal to the β^- decay of a neutron decaying into a proton. Conclude whether the time-reversal reaction is likely to occur.

ANSWER

1 Determine the β^- decay of a neutron.

$$n \rightarrow p + \beta^- + \bar{\nu}_e$$

2 Apply time reversal.

$$p + \beta^- + \bar{\nu}_e \rightarrow n$$

3 Determine the likelihood of the reaction.
This reaction is unlikely to occur because of the very low probability of having all three particles in such close proximity that they would collide to form a neutron.

Charge-reversal symmetry

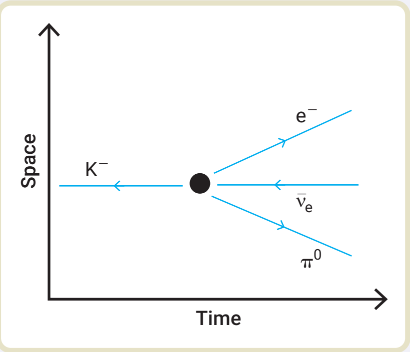
Charge-reversal symmetry says that if the charges on all particles in a reaction are reversed, then this new reaction is also possible in that it does not violate any conservation laws. Strictly speaking, charge reversal is used to refer to swapping all particles for their antiparticles, even those that are electrically neutral, such as neutrons and neutrinos. As with time-reversal symmetry, applying charge-reversal symmetry produces a reaction that does not violate conservation principles, but may also be unlikely.

Consider the decay of a muon: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$. Under charge reversal, this becomes $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$. This decay also occurs.

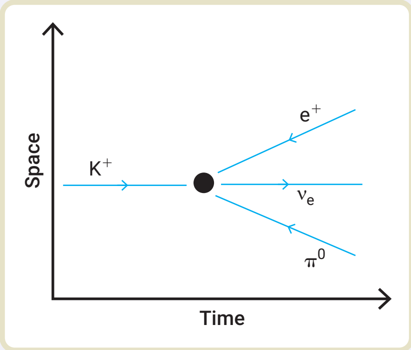
charge-reversal symmetry if all particles in an allowed reaction are replaced with their antiparticles (which have opposite charge), the new reaction is also allowed under known conservation laws

WORKED EXAMPLE 15.4.2

Apply charge-reversal symmetry to the decay of a negative kaon, shown here, to sketch a particle interaction diagram for the decay of a positive kaon.



ANSWER



crossing symmetry if a particle in an allowed reaction is crossed to the other side of the reaction and replaced with its antiparticle, the new reaction is also allowed under known conservation principles, provided enough energy is available

Crossing symmetry

In **crossing symmetry**, one particle is taken and 'crossed' to the other side of the reaction, and converted to its antiparticle. Crossing symmetry, as with time-reversal and charge-reversal symmetry, predicts reactions that do not violate conservation principles. Whether a reaction does occur, or the probability of its occurrence, depends on the energy available, the mass differences of the particles and the conservation of other properties, such as angular momentum. As with other symmetries, crossing-symmetry reactions may be theoretically possible, but have a low probability of occurring.

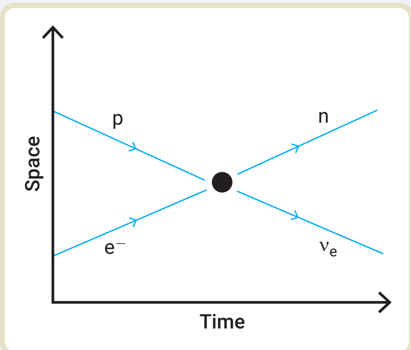
WORKED EXAMPLE 15.4.3

A proton captures an electron to form a neutron and an electron neutrino.

a Sketch the particle interaction diagram for this process.
b Apply crossing symmetry to the electron and sketch the resulting reaction diagram.

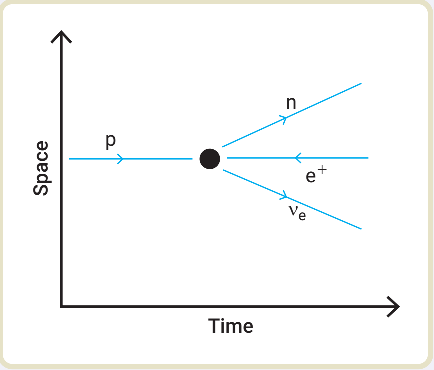
ANSWERS

a **Draw the reaction diagram for** $p + e^- \rightarrow n + \nu_e$.



b Draw the diagram for the potential new reaction with the application of crossing symmetry (crossing the electron).

Note how the electron, when moved to the right-hand side of the reaction, is represented as a positron. $p \rightarrow n + \nu_e + e^+$. This is an observed reaction



LEARNING CHECK 15.4

DESCRIBING

1 **Recall** the three types of symmetries that can be applied to generate potential new reactions.

2 When crossing symmetry is applied to move a particle from one side of a reaction to the other, **describe** what else must be done to ensure conservation.

UNDERSTANDING

3 **Explain** why many of the reactions predicted by a charge reversal are not observed, even though they are possible.

APPLYING

4 A negative sigma (Σ^-) commonly decays by the reaction $\Sigma^- \rightarrow n + \pi^-$. **Apply** a charge-reversal symmetry to **predict** the decay process for a positive sigma.

5 **Apply** time reversal to the reaction of electron–positron annihilation. Write the reaction and draw the reaction diagram. **Explain** why this reaction does not occur without other particles being involved.

6 **Apply** time reversal to the particle interaction shown in Figure 15.4.3. **Sketch** the time-reversal particle interaction diagram. Conclude whether the new reaction is likely to occur.

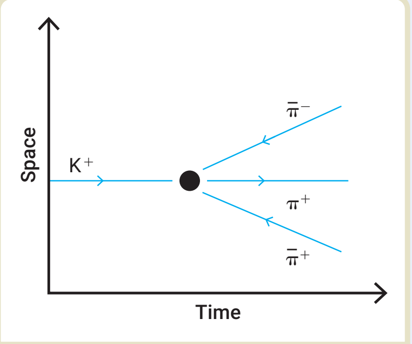


FIGURE 15.4.3 The particle interaction diagram for a positive kaon decay

ANALYSING

7 Consider the β^- decay of a neutron into a proton $n \rightarrow p + e^- + \bar{\nu}_e$. Apply crossing symmetry, time reversal and then a second symmetry operation to this reaction to **sketch** the process of β^+ decay of a proton to a neutron in the form of a particle interaction diagram.

8 **Deduce** one possible crossing symmetry reaction for the reaction $p \rightarrow n + e^+ + \nu_e$.

CHAPTER SUMMARY

Lepton number

- Lepton number is a conserved quantum number used to describe the number of leptons in a particle interaction.
- There are three quantum numbers, L_e, L_μ and L_τ, so each generation of leptons has its own lepton number.
- Leptons in the same family have a lepton number of +1, whereas their corresponding antileptons have a lepton number of –1. All other particles have a lepton number of 0.
- The total lepton number must be the same before and after any interaction or decay, so the lepton number must be conserved.
  $L = n_l - n_{\bar{l}}$

Baryon number

- Baryon number is a conserved quantum number used to describe the number of baryons in a particle interaction.
- There is a single quantum number, B, which applies to all baryons.
- Baryons (like protons and neutrons) have a baryon number of +1, whereas their corresponding antibaryons have a baryon number of –1. All other particles have a baryon number of 0.
- The total baryon number must be the same before and after any interaction or decay, so the baryon number must be conserved.
  $B = n_b - n_{\bar{b}}$
- When analysing an interaction, quarks can also be used to determine the initial and final baryon number.
  $B = \frac{1}{3}(n_q - n_{\bar{q}})$

Particle interaction diagrams

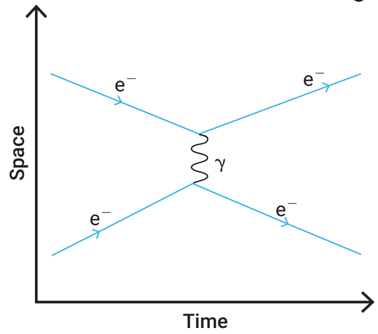
- Particle interaction diagrams model the exchange of particles and forces over time in space, when particles come into close proximity to each other.
- Space is measured in the positive y-axis and time is measured in the positive x-axis, so the diagram can be read left to right.
- Solid straight lines with arrows pointing in the forwards direction of time represent particles; solid straight lines with arrows pointing in the reverse direction of time represent antiparticles. All arrows should be marked with the appropriate symbol of the particle or antiparticle.
- Gauge bosons, which mediate interactions, are represented with squiggly lines. The lines should also be marked with the appropriate symbol of the gauge boson.
- A vertex, where multiple lines enter to indicate an interaction, is shown as a dot in a particle interaction diagram.

Electron–electron interactions

- When two electrons interact, there is an electrostatic repulsion between them, due to the exchange of a virtual photon.
- After the interaction, the electrons exist as they did originally.

 9780170483704

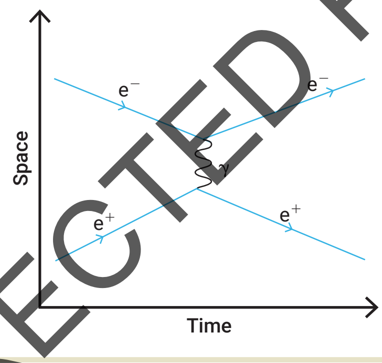
Particle interaction diagram for electron–electron scattering



Electron–positron interactions

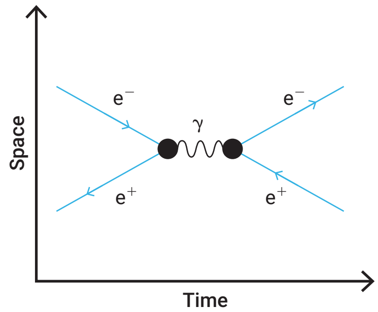
- When electrons and positrons interact, they may scatter or annihilate.
- Electron and positron scattering, or Bhabha scattering, involves an electron approaching a positron, but never touching. This interaction involves the exchange of a virtual photon.

Particle interaction diagram for electron–positron scattering



- Electron and positron annihilation occurs when an electron and positron annihilate, resulting in two high-energy photons.

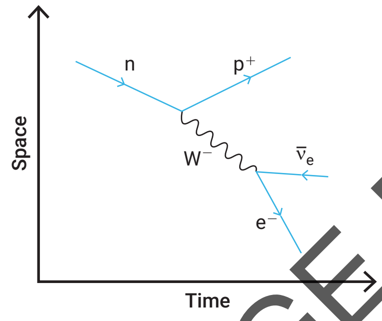
Particle interaction diagram for electron–positron annihilation



Neutron decay

- When a neutron becomes unstable, it decays into a proton, an electron and an electron neutrino.
- The W^- boson mediates this interaction.

Particle interaction diagram for a neutron decay (β- decay)



Symmetry in particle interactions

- Symmetry in particle interactions allows for the conservation of energy, momentum, angular momentum, and lepton and baryon numbers.
- There are several symmetries in particle interactions that, if applied to an allowed reaction, mean that the resulting reaction is also allowed.
- Resulting reactions are not always likely to take place, but are allowed.
- Time-reversal symmetry occurs when reactions are reversed in time.
- Charge-reversal symmetry is when all particles in an allowed reaction are replaced with their antiparticles of opposite charge. The new reaction is also allowed.
- Crossing symmetry occurs when a particle in an allowed reaction is crossed to the other side of the reaction and replaced with its antiparticle. The new reaction is also allowed.

MULTIPLE CHOICE

1. What is the lepton number of a positron?

A 0
B +1
C –1
D +2

2. How does the baryon number of an antiproton compare to that of a proton?

A It is the same.
B It is opposite in sign.
C It is twice as large.
D It is zero.

3. In beta decay ($n \rightarrow p + e^- + \bar{\nu}_e$), what is the lepton number of the products?

A 0
B +1
C –1
D +2

4. A neutrino interacts with an electron in a process that produces a new neutrino and positron. Which of the following correctly represents the interaction, considering the conservation of lepton number and charge?

A $\nu_e + e^- \rightarrow \nu_e + e^+$
B $\nu_e + e^- \rightarrow \nu_\mu + e^+$
C $\nu_e + e^- \rightarrow \nu_\mu + \mu^+$
D $\nu_\mu + e^- \rightarrow \nu_e + e^+$

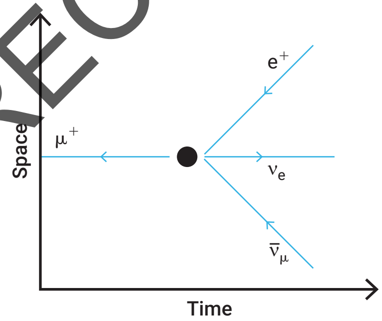
5. In a particle interaction diagram for $n \rightarrow p + e^- + \bar{\nu}_e$, which particle mediates this interaction?

A Photon
B W boson
C Z boson
D Gluon

6. In Feynman diagrams, antiparticles are represented by:

A straight lines with the arrows in the direction of time.
B straight lines with the arrows in the opposite direction of time.
C dashed lines.
D wavy lines.

7. Consider the following particle interaction diagram.



The particle interaction indicated by this diagram is:

A an electron, electron neutrino and antimuon antineutrino reacting to create a muon.
B a positron, an electron neutrino and an antimuon antineutrino reacting to create a muon.
C an antimuon decaying into an electron, an electron neutrino and a muon antineutrino.
D an antimuon decaying into a positron, an electron neutrino and amuon antineutrino.

8. If charge reversal is applied to an electron neutrino, the particle obtained is:
   - **A** a positron neutrino.
   - **B** an electron antineutrino.
   - **C** a positron antineutrino.
   - **D** an electron neutrino.

9. What is meant by symmetry in particle interactions?
   - **A** Identical particles behave identically in every reaction.
   - **C** Only particles with equal mass can interact.
   - **B** Physical laws remain invariant under transformations.
   - **D** Momentum is always conserved.

10. If a reaction violates lepton number conservation, this implies that it:
   - **A** cannot occur in nature.
   - **B** must involve a baryon.
   - **C** must release energy.
   - **D** must involve neutrinos.

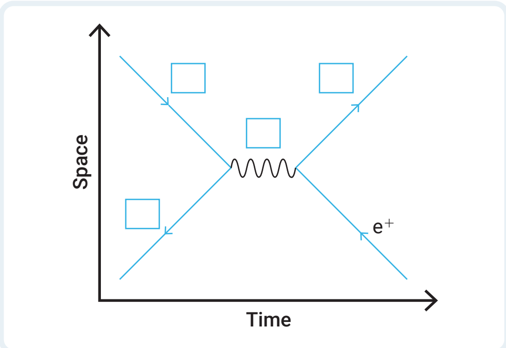
SHORT RESPONSE

11. Draw a conclusion about whether all reactions under all symmetries are theoretically possible. **Justify** your conclusion.

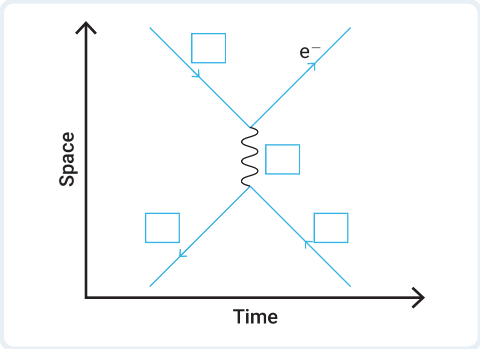
12. Apply crossing symmetry to the electron in the reaction $\tau^- \to e^- + \nu_\tau + \bar{\nu}_e$. **Sketch** a particle interaction diagram for the resulting reaction. **Infer** whether this reaction is likely to occur.

13. **Deduce** which of the following nuclear reactions are possible. For the reactions that are not possible, state which conservation law is broken.
   - **a** $p \to n + \pi^+$
   - **b** $\pi^+ \to e^+ + \nu_\mu$
   - **c** $\Xi^0 \to \Sigma^+ + \pi^-$
   - **d** $\Omega^- \to \pi^- + \Sigma^0 + \nu_\tau$
   - **e** $\Sigma^0 \to \Lambda^= + \gamma + \gamma$
   - **f** $p + \pi^- \to K^+ + \Sigma^-$
   - **g** $p + \pi^+ \to K^+ + \Sigma^+$
   - **h** $p + \pi^- \to K^0 + \Lambda^0 + \pi^0$

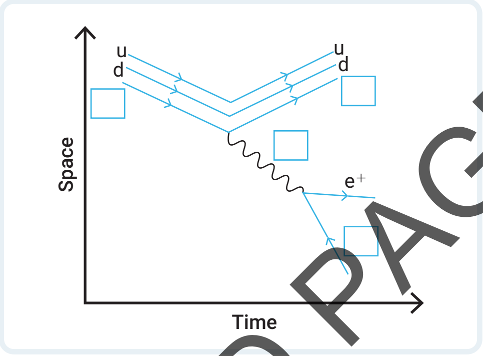
14. Copy the following particle interaction diagrams and fill in the blanks for the stated interactions.
   - **a** Electron–positron annihilation



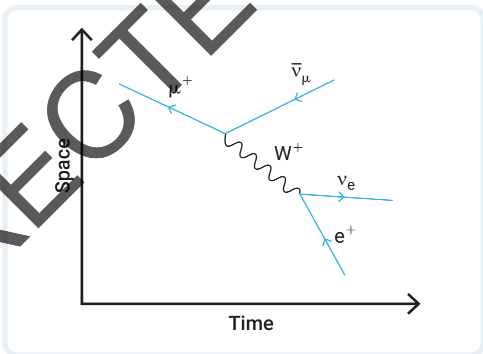
b Electron–positron scattering (Bhabha scattering)



c β^- decay



15. The following particle interaction diagram shows the decay of an antimuon.



a Write the equation for the decay shown in the above particle interaction diagram.

b **Determine** if charge and lepton number were conserved in this interaction.

c **Identify** the types of symmetry applied for the following interactions.

i $\bar{\nu}_\mu + e^+ + \nu_e \rightarrow \mu^+$

ii $\mu^- \rightarrow \bar{\nu}_e + \nu_\mu + e^-$

iii $\mu^+ + e^- \rightarrow \bar{\nu}_\mu + \nu_e$

iv $\mu^- + e^+ \rightarrow \bar{\nu}_e + \nu_\mu$

SCIENCE AS A HUMAN ENDEAVOUR

Syllabus dot point

- Explore the evidence relating to the Standard Model that supports the Big Bang theory.

Physics 2025 v1.2 General Senior Syllabus © QCAA 2024

Exploring the Standard Model and the Big Bang theory

Science is a dynamic and progressive field, constantly building on the discoveries of previous generations. It strives to expand our understanding of the natural world and drives technological advancements that improve society. One of the most profound examples of this ongoing endeavour is in the field of particle physics and cosmology, where the Standard Model and the Big Bang theory play pivotal roles in explaining the origins and evolution of the universe.

These two interconnected frameworks, supported by key discoveries such as cosmic microwave background radiation (CMBR) and the Higgs boson, illustrate this pursuit of knowledge.

The Standard Model of particle physics: understanding the universe's building blocks

As you learnt in Chapter 14, the Standard Model describes the fundamental particles – such as quarks, leptons and bosons – and the forces that govern their interactions.

1. **Fundamental particles and forces:** all matter is made up of elementary particles, such as quarks and electrons. These particles interact through fundamental forces: the electromagnetic, weak nuclear and strong nuclear forces, as well as gravity (which is not yet fully integrated into the Standard Model).
2. **A unified framework:** The Standard Model provides a unifying framework for understanding how these particles and forces behave at both the smallest and largest scales of the universe.

The Big Bang theory: tracing the origins of the universe

Chapter 14 also describes the Big Bang theory and how it helps to explain the origin and evolution of the universe.

3. **Cosmic microwave background radiation (CMBR):** One of the most significant pieces of evidence for the Big Bang theory is the discovery of CMBR. Predicted by theoretical models in the 1940s and confirmed by satellite observations in the 1960s, CMBR is a faint glow of radiation that permeates the universe. It is the afterglow of the Big Bang, left over from a time when the universe was just 380 000 years old. CMBR provides a snapshot of the universe in its early stages and has been crucial in confirming the Big Bang theory.
4. **Nucleosynthesis and the formation of light elements:** Another key piece of evidence for the Big Bang theory is nucleosynthesis, the process by which the first atomic nuclei – mainly hydrogen and helium – were formed in the minutes following the Big Bang. The abundance of these light elements observed throughout the universe today matches the predictions made by Big Bang nucleosynthesis models.

Discovery of the Higgs boson: confirming the Standard Model

In 2012, a major milestone in particle physics was achieved with the discovery of the Higgs boson at the Large Hadron Collider at CERN (the European Organization for

Nuclear Research). The Higgs boson is a fundamental particle that had been predicted by the Standard Model but had eluded detection for decades. Its discovery was not only a triumph for particle physics but also a vital piece of evidence supporting both the Standard Model and the Big Bang theory.

FIGURE 1 A section of the Large Hadron Collider

5. **The Higgs field and mass**: The Higgs boson is associated with the Higgs field, a theoretical field that permeates the universe and gives particles their mass. Without the Higgs field, particles would have no mass, and the universe as we know it could not exist. The discovery of the Higgs boson confirmed the existence of this field and provided crucial insights into the conditions of the early universe, just moments after the Big Bang.
6. **Impact on our understanding of the universe**: The discovery of the Higgs boson not only confirmed a key prediction of the Standard Model but also deepened our understanding of the universe's fundamental structure. It allowed scientists to better understand how particles acquired mass in the early universe, helping to explain why the universe evolved in the way it did after the Big Bang.

Interconnectedness of the Standard Model and Big Bang theory

Recall that the Standard Model and the Big Bang theory are deeply interconnected, with each framework offering crucial insights into the other. Together, they help scientists piece together a coherent narrative about the origins and evolution of the universe.

7. **Early universe conditions:** The Standard Model describes the fundamental particles and forces that governed the universe immediately after the Big Bang. In the first fractions of a second after the Big Bang, the universe was an incredibly hot and dense 'soup' of particles, governed by the forces described in the Standard Model. As the universe expanded and cooled, these particles coalesced into atoms, stars and galaxies, laying the groundwork for the universe we observe today.
8. **Cosmic evolution:** The Big Bang theory, supported by evidence such as CMBR and nucleosynthesis, explains how the universe has expanded and evolved over billions of years. The Standard Model provides the framework for understanding how the fundamental particles and forces have shaped this evolution, from the formation of the first atoms to the complex structures we see today.

The Standard Model and the Big Bang theory are two of the most profound achievements in modern physics, offering a unified explanation for the origins and evolution of the universe. Supported by key discoveries such as the CMBR and the Higgs boson, these frameworks illustrate the power of scientific inquiry and the ongoing human endeavour to uncover the mysteries of the cosmos.