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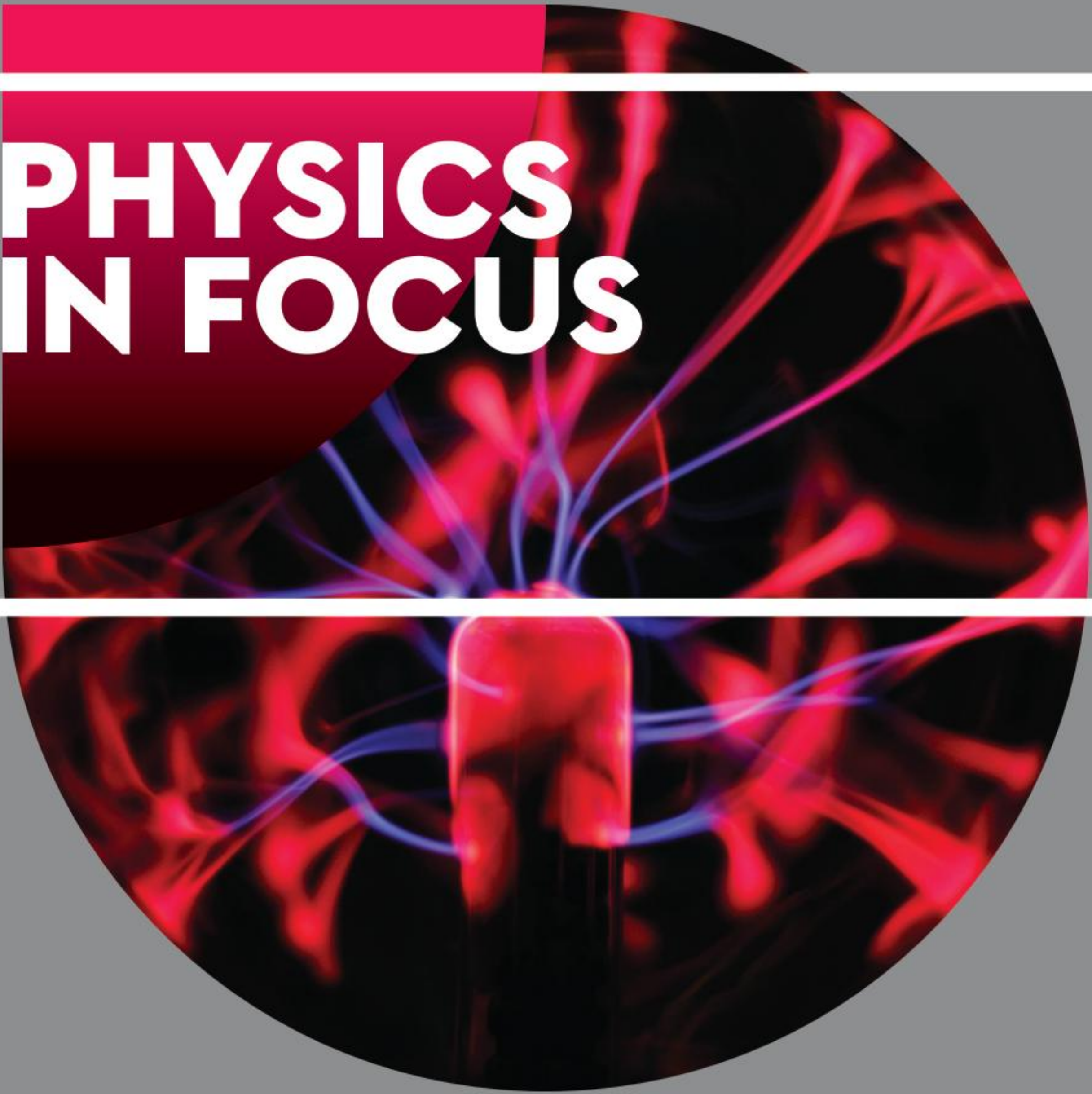
PHYSICS IN FOCUS



YEAR 11

Robert Farr
Kate Wilson
Philip Young
Darren Goossens

2ND EDITION

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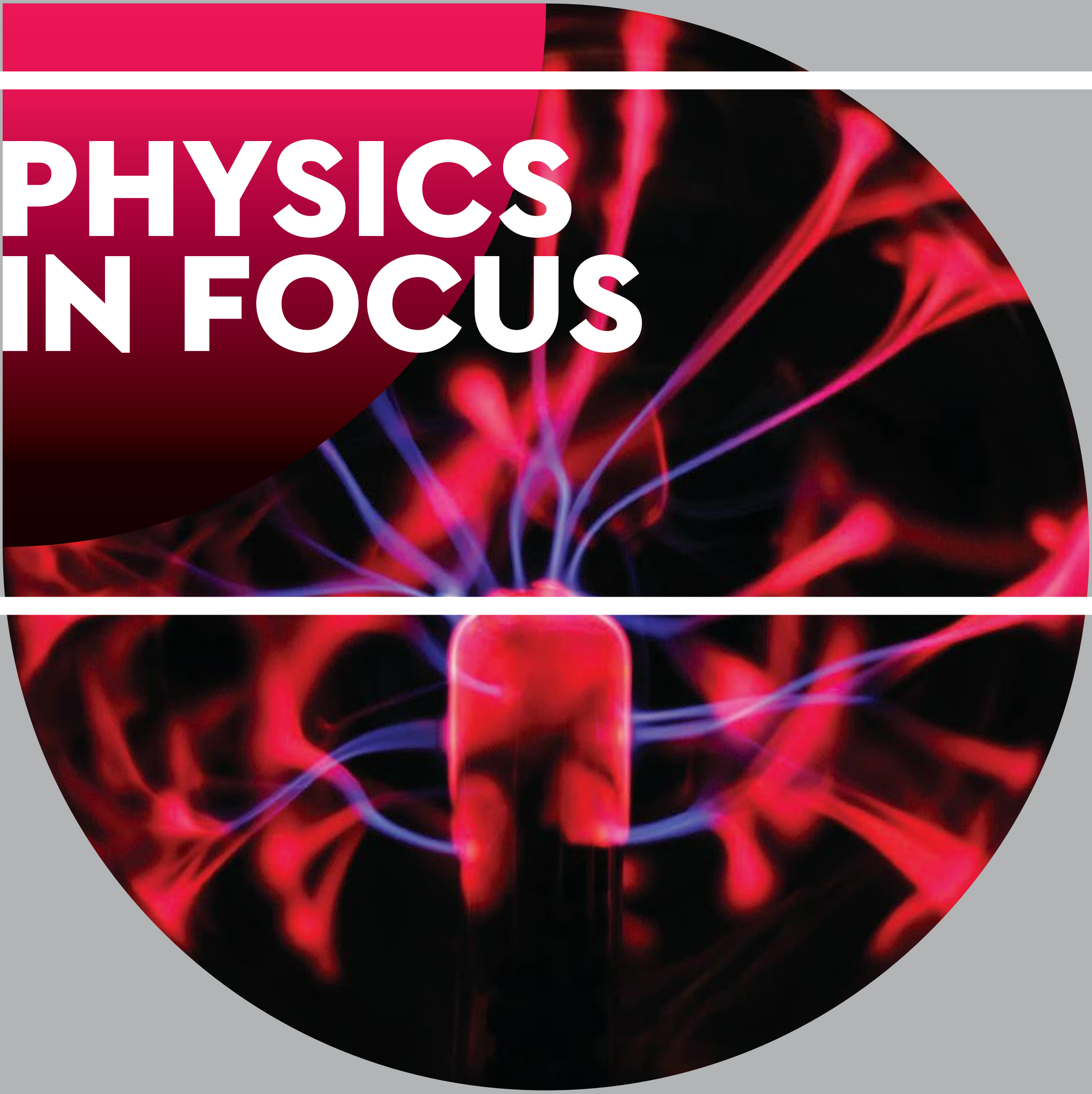

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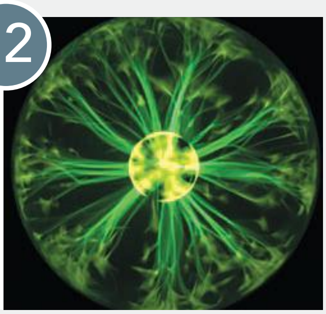

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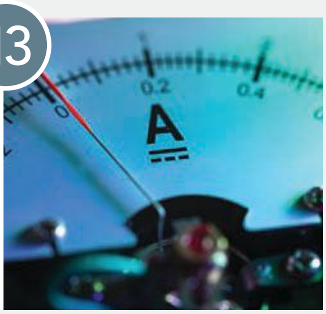

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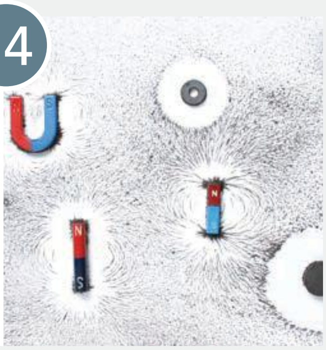

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INTRODUCTION

Physics in Focus Year 11 (2nd edition) has been written to meet the requirements of the NESA NSW Physics Stage 6 Syllabus (2017). The text has been written to enable students to meet the requirements of achieving a Band 6 in the Higher School Certificate. It also allows all students to maximise their learning and results.

Physics deals with the wonderfully interesting and sometimes strange Universe. Physicists investigate space and time (and space–time), from the incredibly small to the incredibly large, from nuclear atoms to the origin of the Universe. They look at important, challenging and fun puzzles and try to work out solutions.

Physicists deal with the physical world where energy is transferred and transformed, where things move, where electricity and magnetism affect each other, where light and matter interact. As a result, physics has been responsible for about 95% of the world's wealth – including electricity supply and distribution, heating and cooling systems, computers, diagnostic and therapeutic health machines, telecommunications and safe road transport.

Physicists are not just concerned with observing the Universe. They explain these observations, using models, laws and theories. Models are central to physics. Physicists use models to describe, explain, relate and predict phenomena. Models can be expressed in a range of ways – via words, images, mathematics (numerical, algebraic, geometric, graphical), or physical constructions. Models help physicists to frame physical laws and theories, and these laws and theories are also models of the world. Models are not static – as scientific understanding of concepts or physical data or phenomena evolves, so too do the models scientists use to describe, explain, relate and predict these. Thus, the text emphasises both the observations and quantitative data from which physicists develop the models they use to explain the data. Central to this is the rigorous use of mathematical representations as a key element of physics explanations.

Physics in Focus Year 11 (2nd edition) is written by academic and classroom teaching experts. They were chosen for their comprehensive knowledge of the physics discipline and best teaching practice in physics education at secondary and tertiary levels. They have written the text to make it accessible, readable and appealing to students. They have included numerous, current contexts to ensure students gain a wide perspective on the breadth and depth of physics. This mathematically rigorous and methodological approach is designed to ensure students can reach the highest possible standard. The intention is to ensure all students achieve the level of depth and interest necessary to pursue tertiary studies in physics, engineering, technology and other STEM related courses. Physics taken for the Higher School Certificate provides opportunities for students to arrive at a deeper understanding of their world whether they are intending to pursue STEM related careers or take a different pathway.

Each chapter of the *Physics in Focus* text follows a consistent pattern. Learning outcomes from the syllabus appear on the opening page. The text is then broken into manageable sections under headings and sub-headings. Question sets are found at the end of each section within the chapter. Relevant diagrams which are easy to interpret and illustrate important concepts support the text. New terms are bolded and defined in a glossary at the end of the book. Important concepts are summarised to assist students to take notes.

Worked examples, written to connect important ideas and solution strategies, are included throughout the text. Solutions are written in full, including algebraic transformations with substitution of values with units and significant figures. In order to consolidate learning, students are challenged to try similar questions on their own.

There is a comprehensive set of review questions at the end of each chapter which expand on the questions sets for further revision and practice. Questions have been set to accommodate the abilities of all students. Complete worked answers appear on the teacher website.

Investigations demonstrate the high level of importance the authors attach to understanding-by-doing physics. These activities introduce, reinforce and enable students to practise first hand investigation skills, especially experimental design, data collection, analysis and conclusions. Chapter 1 explores the concepts of reliability, validity and the nature of scientific investigation using the scientific method in detail and provides valuable information for performing and analysing investigations. Detailed information is provided that is designed to enhance students' experiences and to provide them with information that will maximise their marks in this fundamental area which is reinforced throughout the course.

Système Internationale d'Unités (SI) units and conventions, including accuracy, precision, uncertainty and error are also introduced in the first chapter. This invaluable chapter supports student learning through questions and investigations.

Physics in Focus Year 11 (2nd edition) provides students with a comprehensive study of modern physics that will fully prepare them for exams and any future studies in the area.

Robert Farr (lead author)

AUTHOR AND REVIEWER TEAMS

Author team

Rob Farr has taught Science for over 30 years, 20 of those as Head of Department. He has extensive experiencen as an HSC marker in Physics and Chemistry, and is a past Supervisor of Marking. Rob has co-authored the very successful *Physics in Focus* series and is a contributing author to the *iScience for NSW* series and the *Nelson Physics for the Australian Curriculum* books. He writes trial HSC examinations for Physics, used in over 120 schools across NSW, and leads workshops for the Broken Bay Diocese Science teachers to help improve their HSC results. Rob maintains his passion for Science teaching through active engagement with bodies such as the CSIRO and the STANSW, as well as sitting on the experienced teacher accreditation assessment panel for the NSW Association of Independent Schools (AIS). He is a BOSTES Board Curriculum Committee (BCC) member for the new Stage 6 Science syllabuses about to be introduced in NSW, representing the NSW AIS.

Dr Kate Wilson has a PhD in Physics from Monash University and a Grad. Dip. Ed. (Secondary Teaching) from the University of Canberra. She is currently a senior lecturer at UNSW Canberra (ADFA) in the School of Engineering and IT and the Learning and Teaching Group, where she teaches first year engineering mechanics and the Graduate Teaching Program. Previously Kate was first year coordinator in Physics at the ANU and Director of the Australian Science Olympiads Physics Program. She is a past member of the Sydney University Physics Education Research Group and has held an Innovative Teaching and Educational Technology Fellowship at UNSW (Kensington). Kate has published more than 30 research papers, including more than 20 in Physics education research. She is also an author of the first year university text *Physics* by Serway, Jewett, Wilson and Wilson, and *Nelson Physics Units 3&4 for the Australian Curriculum*.

Philip Young is a former director of the National Space Society in Washington DC, and former President of the National Space Society of Australia. He was Coordinator for the Australian Space Network run by the Australian Centre for Astrobiology, a NASA-affiliated organisation hosted at Macquarie University. For the last decade he has been teaching high school Science, specialising in Physics, and writing textbook materials at both senior and junior levels for the Australian Syllabus in several states. He holds a Bachelor of Science (Physics and Mathematics) from the University of Sydney.

Dr Darren Goossens has a PhD in Physics from Monash University and a Grad. Cert. in Professional Writing (Editing) from the University of Canberra. He is currently a freelance editor and writer, specialising in scientific writing. Darren has worked as a research scientist at the Australian Nuclear Science and Technology Organisation, and as an educator and researcher at the University of New South Wales and the Australian National University, where he won several awards for his teaching. He has published over 100 research papers, including work in education research. In 2012 he won the inaugural Sandy Mathieson Medal of the Society of Crystallographers in Australia and New Zealand for distinguished contributions to crystallography.

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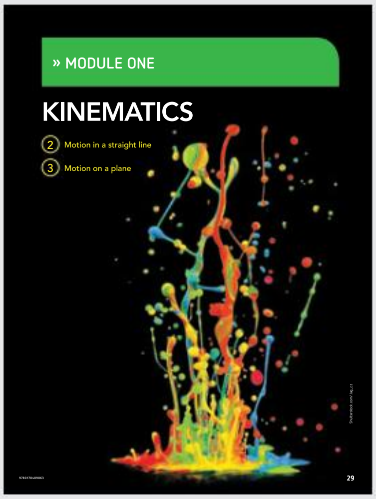
USING *PHYSICS IN FOCUS*

Physics in Focus has been purposely crafted to enable you, the student, to achieve maximum understanding and success in this subject. The text has been authored and reviewed by experienced Physics educators, academics and researchers to ensure up-to-date scientific accuracy for users. Each page has been carefully considered to provide you with all the information you need without appearing cluttered or overwhelming. You will find it easy to navigate through each chapter and see connections between chapters through the use of margin notes. Practical investigations have been integrated within the text so you can see the importance of the interconnectedness between the conceptual and practical aspects of Physics.



» MODULE ONE

KINEMATICS

2 Motion in a straight line

3 Motion on a plane

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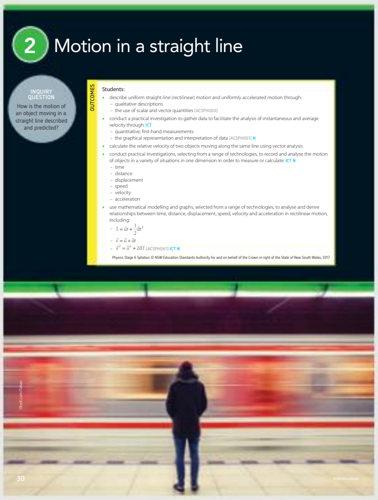

2 Motion in a straight line

INQUIRY QUESTION

How is the motion of an object moving in a straight line described and predicted?

OUTCOMES

Students:

- describe uniform straight-line (rectilinear) motion and uniformly accelerated motion through:
  - qualitative descriptions
  - the use of scalar and vector quantities (ACSPH060)
- conduct a practical investigation to gather data to facilitate the analysis of instantaneous and average velocity through: ICT
  - quantitative, first-hand measurements
  - the graphical representation and interpretation of data (ACSPH061) N
- calculate the relative velocity of two objects moving along the same line using vector analysis
- conduct practical investigations, selecting from a range of technologies, to record and analyse the motion of objects in a variety of situations in one dimension in order to measure or calculate: ICT N
  - time
  - distance
  - displacement
  - speed
  - velocity
  - acceleration
- use mathematical modelling and graphs, selected from a range of technologies, to analyse and derive relationships between time, distance, displacement, speed, velocity and acceleration in rectilinear motion, including:
  - $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
  - $\vec{v} = \vec{u} + \vec{a}t$
  - $\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$ (ACSPH061) ICT N

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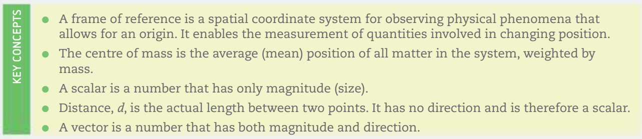
The content is organised under four modules as set out in the NESA Stage 6 Physics syllabus. Each module begins with a **Module opener**.

Each chapter begins with a **Chapter opener**. This presents the learning outcomes from the NESA Stage 6 Physics syllabus that will be covered in the chapter and also gives you the opportunity to monitor your own progress and learning.

To improve comprehension, a number of strategies have been applied to the preparation of our text to improve literacy and understanding. One of these is the use of shorter sentences and paragraphs. This is coupled with clear and concise explanations and real-world examples. New terms are bolded as they are introduced and are consolidated in an end-of-book glossary.

Throughout the text, important ideas, concepts and theories are summarised in **Concept boxes**. This provides repetition and summary for improved assimilation of new ideas.



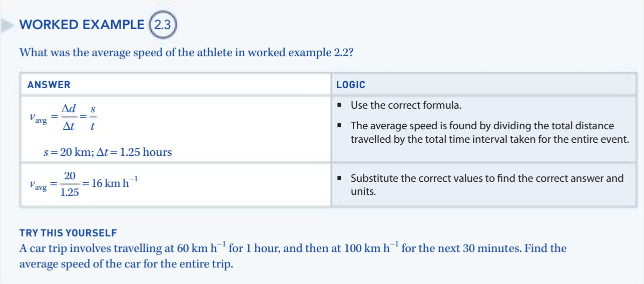
KEY CONCEPTS

- A frame of reference is a spatial coordinate system for observing physical phenomena that allows for an origin. It enables the measurement of quantities involved in changing position.
- The centre of mass is the average (mean) position of all matter in the system, weighted by mass.
- A scalar is a number that has only magnitude (size).
- Distance, *d*, is the actual length between two points. It has no direction and is therefore a scalar.
- A vector is a number that has both magnitude and direction.

Learning across the curriculum content has been identified by NESA as important learning for all students. This content provides you with the opportunity to develop general capabilities beyond the Physics course, as well as links into areas that are important to Australia and beyond. This content has been identified by a margin icon.

Critical and creative thinking

Mathematical relationships are presented in context. Step-by-step instructions on how to perform mathematical calculations are shown in the **Worked examples**. The logic behind each step is explained and you can practice these steps by attempting the related problems presented at the end of the worked example.



WORKED EXAMPLE (2.3)

What was the average speed of the athlete in worked example 2.2?

ANSWER	LOGIC
$v_{avg} = \frac{\Delta d}{\Delta t} = \frac{s}{t}$ $s = 20$ km; $\Delta t = 1.25$ hours	▪ Use the correct formula. ▪ The average speed is found by dividing the total distance travelled by the total time interval taken for the entire event.
$v_{avg} = \frac{20}{1.25} = 16\ \text{km h}^{-1}$	▪ Substitute the correct values to find the correct answer and units.

TRY THIS YOURSELF

A car trip involves travelling at 60 km h^{-1} for 1 hour, and then at 100 km h^{-1} for the next 30 minutes. Find the average speed of the car for the entire trip.

Physics is a science and you need to be given the opportunity to explore and discover the physical world through practical investigations. **Investigations** introduce and reinforce the Working scientifically skills listed in the NESA Stage 6 Physics syllabus. In some cases, the investigations are open-ended. These provide you with the opportunity to design and carry out your own scientific investigation, either individually or in a group. At times you are prompted to consider ideas for improvement to illustrate that science is constantly undergoing review and improvement. At other times investigations are secondary-sourced, meaning that you need to research the subject using data and information gained by other people. Further information on how to conduct a scientific investigation can be found in the **Working scientifically** and depth study chapter on page 1.

INVESTIGATION 2.1

The speeds of common objects

We can make objects move at different speeds. Some first-class cricketers can bowl a ball at speeds approaching 45 m s^{-1}. The mechanical advantage conferred by a bat or racquet can increase or decrease ball speeds.

Numeracy

Information and communication technology capability

AIM

To measure the speeds of some human-propelled objects

MATERIALS

- Stopwatch
- Measuring tape
- Various bats, racquets and balls
- Optional: video camera or motion data-logger

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
A ball hit with a bat could hit a person or break a window.	Perform the experiment in an open space, such as a school oval, and keep bystanders well back.

RISK ASSESSMENT

What other risks are associated with your investigation, and how can you manage them?

METHOD

1 Measure out an appropriate length (e.g. 20 m) between two lines on the school oval or in a clear area.
2 By either throwing or hitting a ball with a bat or racquet, reproduce the actions of several different ball sports (e.g. cricket, tennis, hockey, golf) that propel a ball from one line past the other.
3 Measure the time it takes for the ball to travel the designated distance. For this, use a stopwatch or you may be able to video the motion and use the clock on the video. You might also have access to a motion data-logger that is able to measure speed directly.
4 Repeat step 3 for the same sport several times.
5 Repeat steps 3 and 4 for a different sport.

RESULTS

Record the results of your timing measurements for each sport in a table.

ANALYSIS OF RESULTS

1 Find the average speed of the ball for each sport. Include an estimate of the uncertainty in each value.
2 Convert the results from m s^{-1} to km h^{-1}.

DISCUSSION

Discuss the difficulties encountered during this experiment and suggest ways in which the data collection could be made to be more accurate.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

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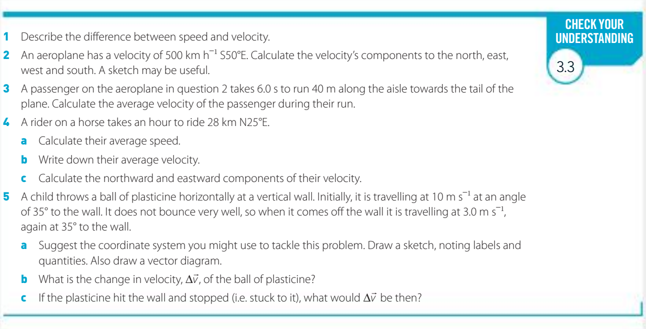
The **Risk assessment** table occurs within the investigations. The table highlights the risks of the investigation and provides suggestions on how to minimise these risks – they are not to be considered comprehensive. Teachers are expected to amend this table in the case of substitutions or in the case of any additional risks. This may mean obtaining and following Safety Data Sheets (SDS) for certain chemicals. All teachers are required to follow the safety guidelines of their specific school and associated government legislation when students are in their care.

RISK ASSESSMENT

Full understanding of a concept is often constructed from many pieces of information. Due to the sequential nature of a book, this information cannot always be presented together as it is best placed in other chapters. Links between concepts that occur on other pages and chapters are indicated using the **Margin notes**.

You will learn more about fluid transport in Chapter 6.

Regular opportunities to recall new terms and review recent concepts are provided as short **Check your understanding** question sets throughout each chapter.



CHECK YOUR UNDERSTANDING 3.3

1 Describe the difference between speed and velocity.
2 An aeroplane has a velocity of 500 km h^{-1} S50°E. Calculate the velocity's components to the north, east, west and south. A sketch may be useful.
3 A passenger on the aeroplane in question 2 takes 6.0 s to run 40 m along the aisle towards the tail of the plane. Calculate the average velocity of the passenger during their run.
4 A rider on a horse takes an hour to ride 28 km N25°E.
  a Calculate their average speed.
  b Write down their average velocity.
  c Calculate the northward and eastward components of their velocity.
5 A child throws a ball of plasticine horizontally at a vertical wall. Initially, it is travelling at 10 m s^{-1} at an angle of 35° to the wall. It does not bounce very well, so when it comes off the wall it is travelling at 3.0 m s^{-1}, again at 35° to the wall.
  a Suggest the coordinate system you might use to tackle this problem. Draw a sketch, noting labels and quantities. Also draw a vector diagram.
  b What is the change in velocity, $\Delta\vec{v}$, of the ball of plasticine?
  c If the plasticine hit the wall and stopped (i.e. stuck to it), what would $\Delta\vec{v}$ be then?

The end-of-chapter review provides:

- a **Summary** of the important concepts that have been covered in the chapter. This will be a valuable tool when you are revising for tests and exams

2 CHAPTER SUMMARY

- A frame of reference is a spatial coordinate system for observing physical phenomena that allows for an origin. It enables the measurement of quantities involved in changing position.
- The centre of mass is the average (mean) position of all matter in the system, weighted by mass.
- A scalar is a number that has only magnitude (size).
- Distance, *d*, is the actual length between two points. It has no direction and is therefore a scalar.
- A vector is a number that has both magnitude and direction.
- Displacement, $\vec{s}$, represents a change of position with respect to the starting point. It has both magnitude (the distance) and direction, so it is a vector.
- Movement is the change in position as time changes.
- Any time interval can be shown as Δt, where $\Delta t = t_2 - t_1$ (Unit: s).
- Speed, *v*, relates to the distance covered in a time interval.
- Velocity, $\vec{v}$, specifically relates to the change in displacement during a time interval.
- Speed is the magnitude of the velocity. Velocity also includes direction.
- Change in distance, called the distance interval, is given the symbol *s*, where $s = d_2 - d_1$ (Unit: m).
- Speed is measured as distance travelled over time (Unit: m s^{-1}).
- Average speed is the one single speed that would enable an object to cover a specified distance in a given time interval.
- Instantaneous speed is the rate at which distance is covered over a time interval that is so brief as to be negligible.
- For constant speed, the gradient on a distance–time graph is the same at all points. The graph is a straight line.
- The area under the curve on a speed–time graph shows the distance travelled.
- A graph of *v* versus *t* shows that the area under the line equals *s*, which is the distance travelled.
- Relative velocity depends on the frame of reference.
- Relative velocity is given by $\vec{v}_o - \vec{v}_d$.
- Using vector addition, the resultant vector from the point of view of one object is with respect to the other object, not the fixed external frame of reference.
- Acceleration is defined as the change in velocity divided by the time interval.
- Acceleration in a straight line can be positive or negative, depending on whether the object is speeding up or slowing down.
- On a speed–time graph, the average acceleration is the gradient of the line drawn covering the time interval Δt.
- The instantaneous acceleration is found as the time interval becomes small enough to be negligible.
- On a speed–time graph, the area under the line drawn covering the time interval Δt represents the distance travelled during that interval.
- A graph of *a* versus *t* shows that the area under the line equals the change in speed, Δv.
- For uniformly accelerated motion, $\vec{a} = \frac{\vec{v}}{t}$, implying $\vec{v} = \vec{a}t$.
- If we include an initial velocity $\vec{u}$, then $\vec{v} = \vec{u} + \vec{a}t$.
- If we know the initial velocity, acceleration and time of travel, then the distance covered is given by $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$.
- If we do not have the time interval, the relationship between initial velocity, final velocity, acceleration and distance travelled is given by $\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$.

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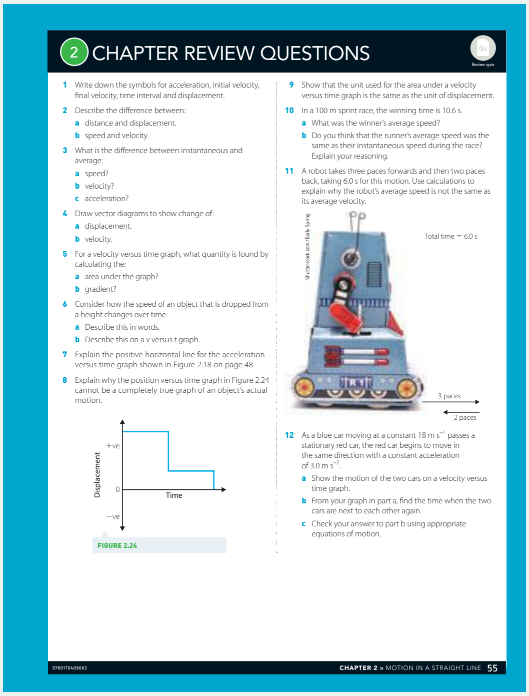
- **Chapter review questions** that review understanding and provide opportunities for application and analysis of concepts and how they interrelate.



2 CHAPTER REVIEW QUESTIONS

Review quiz

1 Write down the symbols for acceleration, initial velocity, final velocity, time interval and displacement.

2 Describe the difference between:
  a distance and displacement.
  b speed and velocity.

3 What is the difference between instantaneous and average:
  a speed?
  b velocity?
  c acceleration?

4 Draw vector diagrams to show change of:
  a displacement.
  b velocity.

5 For a velocity versus time graph, what quantity is found by calculating the:
  a area under the graph?
  b gradient?

6 Consider how the speed of an object that is dropped from a height changes over time.
  a Describe this in words.
  b Describe this on a *v* versus *t* graph.

7 Explain the positive horizontal line for the acceleration versus time graph shown in Figure 2.18 on page 48.

8 Explain why the position versus time graph in Figure 2.24 cannot be a completely true graph of an object's actual motion.

FIGURE 2.24

9 Show that the unit used for the area under a velocity versus time graph is the same as the unit of displacement.

10 In a 100 m sprint race, the winning time is 10.6 s.
  a What was the winner's average speed?
  b Do you think that the runner's average speed was the same as their instantaneous speed during the race? Explain your reasoning.

11 A robot takes three paces forwards and then two paces back, taking 6.0 s for this motion. Use calculations to explain why the robot's average speed is not the same as its average velocity.

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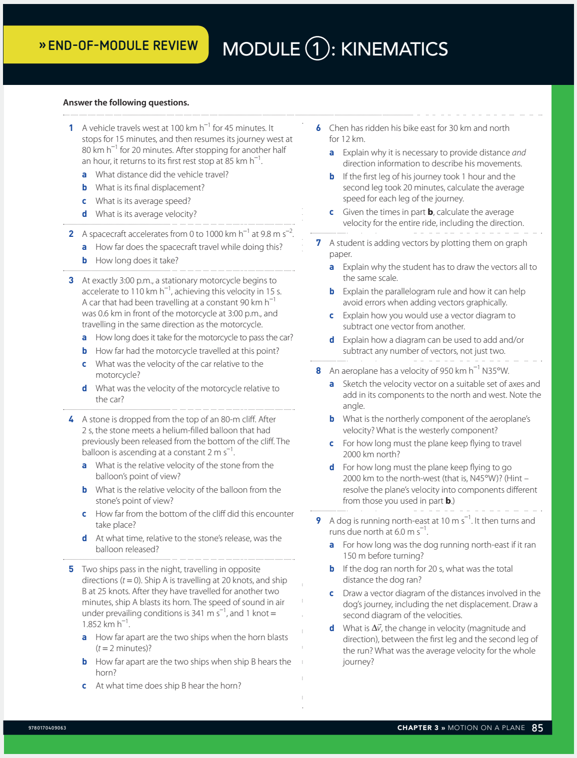
12 As a blue car moving at a constant 18 m s^{-1} passes a stationary red car, the red car begins to move in the same direction with a constant acceleration of 3.0 m s^{-2}.
  a Show the motion of the two cars on a velocity versus time graph.
  b From your graph in part a, find the time when the two cars are next to each other again.
  c Check your answer to part b using appropriate equations of motion.

9780170409063 CHAPTER 2 » MOTION IN A STRAIGHT LINE 55

Each module concludes with a **Module review**. This contains short-answer questions that provide you with the opportunity to assimilate content from across the chapters that fall within that module.



» END-OF-MODULE REVIEW MODULE ①: KINEMATICS

Answer the following questions.

1 A vehicle travels west at 100 km h^{-1} for 45 minutes. It stops for 15 minutes, and then resumes its journey west at 80 km h^{-1} for 20 minutes. After stopping for another half an hour, it returns to its first rest stop at 85 km h^{-1}.
  a What distance did the vehicle travel?
  b What is its final displacement?
  c What is its average speed?
  d What is its average velocity?

2 A spacecraft accelerates from 0 to 1000 km h^{-1} at 9.8 m s^{-2}.
  a How far does the spacecraft travel while doing this?
  b How long does it take?

3 At exactly 3:00 p.m., a stationary motorcycle begins to accelerate to 110 km h^{-1}, achieving this velocity in 15 s. A car that had been travelling at a constant 90 km h^{-1} was 0.6 km in front of the motorcycle at 3:00 p.m., and travelling in the same direction as the motorcycle.
  a How long does it take for the motorcycle to pass the car?
  b How far had the motorcycle travelled at this point?
  c What was the velocity of the car relative to the motorcycle?
  d What was the velocity of the motorcycle relative to the car?

4 A stone is dropped from the top of an 80-m cliff. After 2 s, the stone meets a helium-filled balloon that had previously been released from the bottom of the cliff. The balloon is ascending at a constant 2 m s^{-1}.
  a What is the relative velocity of the stone from the balloon's point of view?
  b What is the relative velocity of the balloon from the stone's point of view?
  c How far from the bottom of the cliff did this encounter take place?
  d At what time, relative to the stone's release, was the balloon released?

5 Two ships pass in the night, travelling in opposite directions ($t = 0$). Ship A is travelling at 20 knots, and ship B at 25 knots. After they have travelled for another two minutes, ship A blasts its horn. The speed of sound in air under prevailing conditions is 341 m s^{-1}, and 1 knot = 1.852 km h^{-1}.
  a How far apart are the two ships when the horn blasts ($t = 2$ minutes)?
  b How far apart are the two ships when ship B hears the horn?
  c At what time does ship B hear the horn?

6 Chen has ridden his bike east for 30 km and north for 12 km.
  a Explain why it is necessary to provide distance *and* direction information to describe his movements.
  b If the first leg of his journey took 1 hour and the second leg took 20 minutes, calculate the average speed for each leg of the journey.
  c Given the times in part **b**, calculate the average velocity for the entire ride, including the direction.

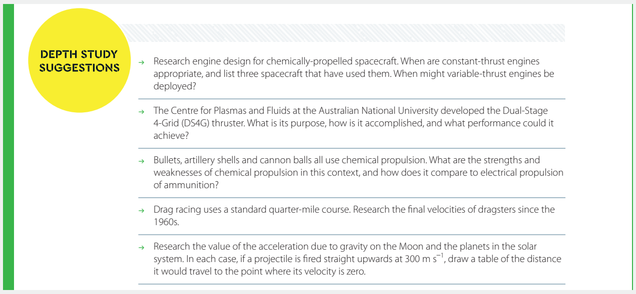
7 A student is adding vectors by plotting them on graph paper.
  a Explain why the student has to draw the vectors all to the same scale.
  b Explain the parallelogram rule and how it can help avoid errors when adding vectors graphically.
  c Explain how you would use a vector diagram to subtract one vector from another.
  d Explain how a diagram can be used to add and/or subtract any number of vectors, not just two.

8 An aeroplane has a velocity of 950 km h^{-1} N35°W.
  a Sketch the velocity vector on a suitable set of axes and add in its components to the north and west. Note the angle.
  b What is the northerly component of the aeroplane's velocity? What is the westerly component?
  c For how long must the plane keep flying to travel 2000 km north?
  d For how long must the plane keep flying to go 2000 km to the north-west (that is, N45°W)? (Hint – resolve the plane's velocity into components different from those you used in part **b**.)

9 A dog is running north-east at 10 m s^{-1}. It then turns and runs due north at 6.0 m s^{-1}.
  a For how long was the dog running north-east if it ran 150 m before turning?
  b If the dog ran north for 20 s, what was the total distance the dog ran?
  c Draw a vector diagram of the distances involved in the dog's journey, including the net displacement. Draw a second diagram of the velocities.
  d What is $\Delta\vec{v}$, the change in velocity (magnitude and direction), between the first leg and the second leg of the run? What was the average velocity for the whole journey?

9780170409063 CHAPTER 3 » MOTION ON A PLANE 85

The **Depth study** provides you with the opportunity to pursue a topic of interest from within the course. It enables you to study a topic in more depth and present your findings in a format of your choice. Advice and support to assist you in undertaking your depth study can be found in chapter 1, and there are suggestions for topics provided at the end of each module review. Refer to the NESA Stage 6 Physics syllabus for the full details on scoping and completion of your depth study.



DEPTH STUDY SUGGESTIONS

→ Research engine design for chemically-propelled spacecraft. When are constant-thrust engines appropriate, and list three spacecraft that have used them. When might variable-thrust engines be deployed?

→ The Centre for Plasmas and Fluids at the Australian National University developed the Dual-Stage 4-Grid (DS4G) thruster. What is its purpose, how is it accomplished, and what performance could it achieve?

→ Bullets, artillery shells and cannon balls all use chemical propulsion. What are the strengths and weaknesses of chemical propulsion in this context, and how does it compare to electrical propulsion of ammunition?

→ Drag racing uses a standard quarter-mile course. Research the final velocities of dragsters since the 1960s.

→ Research the value of the acceleration due to gravity on the Moon and the planets in the solar system. In each case, if a projectile is fired straight upwards at 300 m s^{-1}, draw a table of the distance it would travel to the point where its velocity is zero.

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OUTCOME GRID

Working Scientifically mapping

Content statements from the NESA Stage 6 Physics syllabus are shown in full on the chapter opening pages of the chapters where they are dealt with. A full mapping of chapters and content statements can be found on the NelsonNet Teacher website. Below is a mapping of the outcome statements for Working scientifically across all the chapters of *Physics in Focus Year 11*.

OUTCOME STATEMENTS STUDENTS:	CHAPTER													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
PH11/12-1 develops and evaluates questions and hypotheses for scientific investigation	✓		✓	✓	✓	✓	✓		✓			✓	✓	✓
PH11/12-2 designs and evaluates investigations in order to obtain primary and secondary data and information	✓		✓	✓	✓		✓		✓		✓			
PH11/12-3 conducts investigations to collect valid and reliable primary and secondary data and information	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
PH11/12-4 selects and processes appropriate qualitative and quantitative data and information using a range of appropriate media	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
PH11/12-5 analyses and evaluates primary and secondary data and information	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
PH11/12-6 solves scientific problems using primary and secondary data, critical thinking skills and scientific processes	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓
PH11/12-7 communicates scientific understanding using suitable language and terminology for a specific audience or purpose	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

1 Working scientifically and depth studies

OUTCOMES

Skills

A student:

- develops and evaluates questions and hypotheses for scientific investigation PH11-1
- designs and evaluates investigations in order to obtain primary and secondary data and information PH11-2
- conducts investigations to collect valid and reliable primary and secondary data and information PH11-3
- selects and processes appropriate qualitative and quantitative data and information using a range of appropriate media PH11-4
- analyses and evaluates primary and secondary data and information PH11-5
- solves scientific problems using primary and secondary data, critical thinking skills and scientific processes PH11-6
- communicates scientific understanding using suitable language and terminology for a specific audience or purpose PH11-7

Knowledge and understanding

A student:

- describes and analyses motion in terms of scalar and vector quantities in two dimensions and makes quantitative measurements and calculations for distance, displacement, speed velocity and acceleration PH11-8
- describes and explains events in terms of Newton's laws of motion, the law of conservation of momentum and the law of conservation of energy PH11-9
- explains and analyses waves and the transfer of energy by sound, light and thermodynamic principles PH11-10
- explains and quantitatively analyses electric fields, circuitry and magnetism PH11-11

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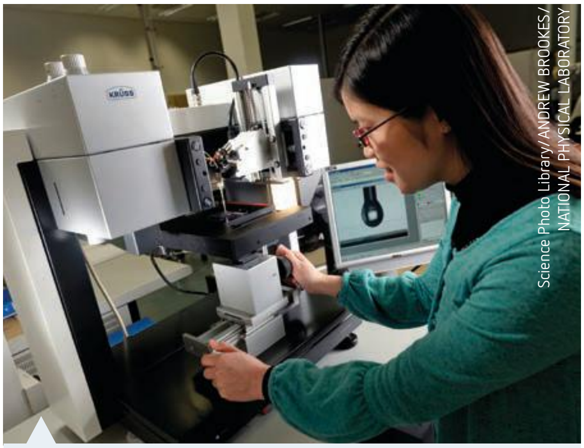

Science Photo Library/ANDREW BROOKES/ NATIONAL PHYSICAL LABORATORY

FIGURE 1.1 Science is characterised by a way of thinking, working and questioning.

Science is the systematic study, by observation and experiment, of the natural and physical world. Science is characterised by a way of thinking and working, and, most fundamentally, by questioning. The knowledge and understanding that arises from this questioning is not in itself science – it is the product of science, as is the technology that arises from this knowledge and understanding. Science is **empirical** – when scientists ask questions, they seek to answer them by using evidence, in particular observational and experimental evidence. Physics, the oldest and most fundamental of the sciences, is the study of how things work. Physics asks questions about matter and energy, and the interactions between matter and energy. Given that everything is composed of matter and energy, physics is a very broad science!

1.1 The nature of physics

Scientific knowledge and theories are testable and **falsifiable** (able to be disproved). This applies to all sciences, including physics. It means that for a theory to be considered scientific, it must be possible to test it and, most importantly, to show that it is not true. This sets science apart from disciplines in which there are theories that cannot be tested or disproved. Such theories are not scientific.

Theories cannot be proved, only ever disproved – no matter how much evidence you gather that agrees with a theory, it only takes one experiment that disagrees to disprove a theory. As Einstein said, '*No amount of experimentation can ever prove me right; a single experiment can prove me wrong.*' This is why scientists never talk about proving a theory, but rather about providing evidence to support a theory. When a large enough amount of evidence has been gathered that supports a theory, that theory is then accepted by the scientific community. Conservation laws are examples of theories that have so much evidence supporting them that they are generally accepted; for example, conservation of energy and momentum.

When a theory, model or idea has been accepted by the scientific community, it means that there is an overwhelming amount of evidence supporting it. Nonetheless, a scientist will still not describe that theory as proven – instead they will say it is well supported or accepted. Unfortunately, this sometimes means that people who don't know how science works think that all theories are equally valid. This is not the case. Scientific theories must have a great amount of evidence to support them before they are accepted. A scientific theory is not merely someone's opinion.

The scientific method

Critical and creative thinking

The **scientific method** is the process of systematically gathering data by observation and measurement, and using it to test and formulate hypotheses. A **hypothesis** is a tentative answer to a question. It is an idea or explanation that can be tested. For example, we might hypothesise that it takes more force to accelerate a heavy object than a lighter one by some amount. We could test this hypothesis by performing experiments where we measure the acceleration of different objects subject to the same force.

The scientific method is summarised in Figure 1.2. The basic process for the scientific method is to start with a question or questions (sometimes called your research question). Based on these questions, you formulate a hypothesis, which is a tentative answer to your question. Usually this involves reading the literature to see if anyone has already answered your question or investigated a similar question. The hypothesis is then used to make a prediction of what will happen in particular circumstances. An experiment is designed and performed to test the prediction, and the results are analysed. If the results of the experiment agree with the prediction, then the hypothesis is supported. Note that it is *not proved*, only supported. There may be other explanations that would also be supported by the results. If the results do not agree with the prediction, then the hypothesis is not supported and you need to come up with another explanation.

Reproducibility and peer review are important aspects of science. If an experiment cannot be repeated to give the same results, then there is a good chance that a mistake was made. Experiments are considered **valid** when they can be repeated to give the same results, and when the uncertainties in the measurements allow the hypothesis to be clearly disproved.

Scientists communicate their work to each other to share new ideas and information. If the outcomes of an experiment are not shared, then they cannot contribute to the ongoing development of science. Scientists usually communicate new findings to each other by writing articles for scientific journals. When you conduct an experiment and write a report on it, the report is very much like a scientific paper.

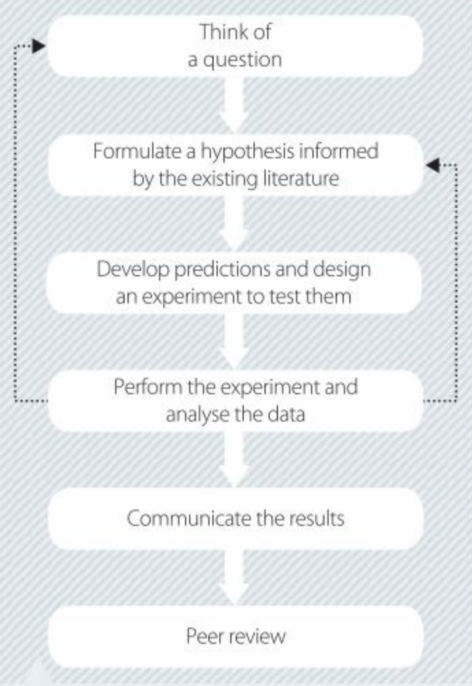


FIGURE 1.2 The scientific method

Before a scientific paper is published, it is reviewed by other scientists (experts in the particular area), who evaluate it. They try to determine whether the experiments conducted were appropriate, whether the conclusions drawn were valid, and whether the hypothesis is clearly supported or not. If the paper is considered to make a useful contribution to science, and the experiments and analysis are valid, then it will be published. Other scientists can then read the scientific paper and use it to inform their own work. Scientists also communicate their work in other ways to students and the public.

This description of the scientific method is somewhat idealised. Sometimes scientists only have questions, and no hypothesis to answer them. Experiments are conducted or observations are made to try to form a hypothesis that can then be tested. Sometimes, while trying to answer one question, a whole new and more interesting question arises so a scientist will change their experiments to work on that instead. However, even when a new and exciting discovery is made by accident, the scientific method will still be used to formulate and test hypotheses that arise to explain it.

The scientific method

Read this article about the scientific method and come up with your own explanation of the difference between science and pseudoscience.

Physics

Disciplines within science can be characterised by the sorts of questions that they ask. Physics asks questions about how the universe works, why things happen, and why things are the way they are. Physicists have found that these questions can generally be answered by looking at the way matter and energy interact via forces.

The more we find out, the more questions are generated. There are many questions we haven't answered yet. As current and future physicists answer these questions, yet more questions will arise that no one has thought of yet. Since answering one question often leads to further questions, it is unlikely that we will ever have a complete understanding of how the universe works. However, scientists will continue to work towards developing a deeper understanding of how the universe works. Science is an adventure, and will continue to be for a long time before all the possible questions about how the universe works are answered.

Many generations of scientists have asked questions and sought answers to these questions. From their answers, we have constructed models of how our universe works. These models are always changing as we get better answers to existing questions, or ask new questions. **Models** are representations of physical reality – they are not the physical reality itself any more than a model aeroplane is a real aeroplane. Models can be physical models, or mathematical models made up of equations and data, or conceptual models consisting of principles, laws and theories. Physicists use all sorts of models, combine models, and switch between models, as they ask and try to answer questions.

Models in physics have two important purposes – to explain how things work, and to predict what will happen. A model that does not accurately predict the results of an experiment will generally be revised or replaced.

The model that we choose to represent a situation depends on the situation. For example, classical mechanics and quantum mechanics can both be used to describe and analyse the behaviour of moving objects. Although the two models give the same results for large objects, such as cars, the quantum mechanical model is much more complicated so we generally choose to use classical mechanics when we analyse the behaviour of cars. When we analyse the behaviour of electrons, the two models give quite different results – quantum mechanics gives results that match experiments involving electrons, while classical mechanics does not. So when analysing the behaviour of electrons, we use quantum rather than classical mechanics. This doesn't mean that either model is 'right' or 'true', just that quantum mechanics is a better model than classical mechanics in this situation.

Choosing the right model for a situation is an important skill in solving problems in physics.

Physics knowledge and understanding

As you progress through your Physics course, you will learn a lot of useful skills and practise working scientifically by performing investigations and depth studies. You will also gain some knowledge and develop a deeper understanding of physics.

The knowledge that has arisen from answering the questions asked by physicists can be broadly categorised into five areas.

1. **Mechanics** describes the motion and interaction of objects, and uses the ideas of force and energy to explain phenomena. We use mechanics to describe and predict the behaviour of small numbers of macroscopic (bigger than atomic-sized) objects.
2. **Waves** are how we model the organised movement of energy and information when there is no overall movement of objects. Information is transmitted via waves, including light and sound. We use the same ideas as in mechanics, with the addition of superposition.
3. **Thermodynamics** is the study of how energy moves in a system, and is transformed from one form to another. It describes the behaviour of very large numbers of microscopic particles, such as the atoms in a material. It uses the same ideas as mechanics and applies them to find average values of quantities, such as energy per particle, and relate them to measurable quantities, such as temperature. Thermodynamics gave rise to most modern transport via the invention of engines.
4. **Electromagnetism** describes the electric and magnetic properties and interactions of matter and energy. It is the foundation of electronics. Electromagnetism uses the same core ideas as mechanics and waves, with the addition of the concepts of charge and electromagnetic field. Fields are a way of describing how forces act at a distance.
5. **Quantum physics** describes the behaviour and interactions of waves, atoms and subatomic particles, so it combines ideas from mechanics and waves. The idea of quantisation comes from standing waves, such as those created on musical instruments. Quantum physics is described in *Physics in Focus Year 12.*

These five areas are not separate – they overlap, and we typically need to draw on ideas from more than one of them to understand any phenomenon. For example, light is an electromagnetic wave. To understand how light interacts, we need to use ideas from waves and electromagnetism, and sometimes from quantum mechanics too. To understand how a car works, we need ideas from thermodynamics and mechanics.

There are core concepts and principles in physics that underlie all of these areas. These are energy, forces, and conservation principles. Figure 1.3 shows a concept map for mechanics outlining just some of the ideas you will meet in chapters 2–6, and how they are related.

Figure 1.4 shows a similar concept map for waves (chapters 7–10). Note that the same core ideas are present in both maps, with the addition of superposition for waves. Neither of these maps show everything that you will study in these topics, but they do show the core ideas and some of the relationships between them.

You will only study a very small part of thermodynamics, so the concept map for thermodynamics shown in Figure 1.5 is sparse compared with those for mechanics and waves. The concepts shown in grey are not covered in your Physics course, but you may meet them in your other studies, such as Chemistry.

Electromagnetism is sometimes seen by students as quite a different topic, with a new set of ideas and equations to learn. In fact, apart from the idea of charge and the electromagnetic field, we use all the same ideas again, as shown in Figure 1.6. The idea of a field is introduced in electricity and magnetism, but it is also used in mechanics to understand gravity, which will be described in *Physics in Focus Year 12.*

One of the most powerful aspects of physics is that the same ideas apply to many different situations and systems. A good understanding of any one part of physics will help you analyse any unfamiliar situation. You will find that you always draw upon the core ideas of force and energy and use conservation principles to help you frame and answer questions in physics.

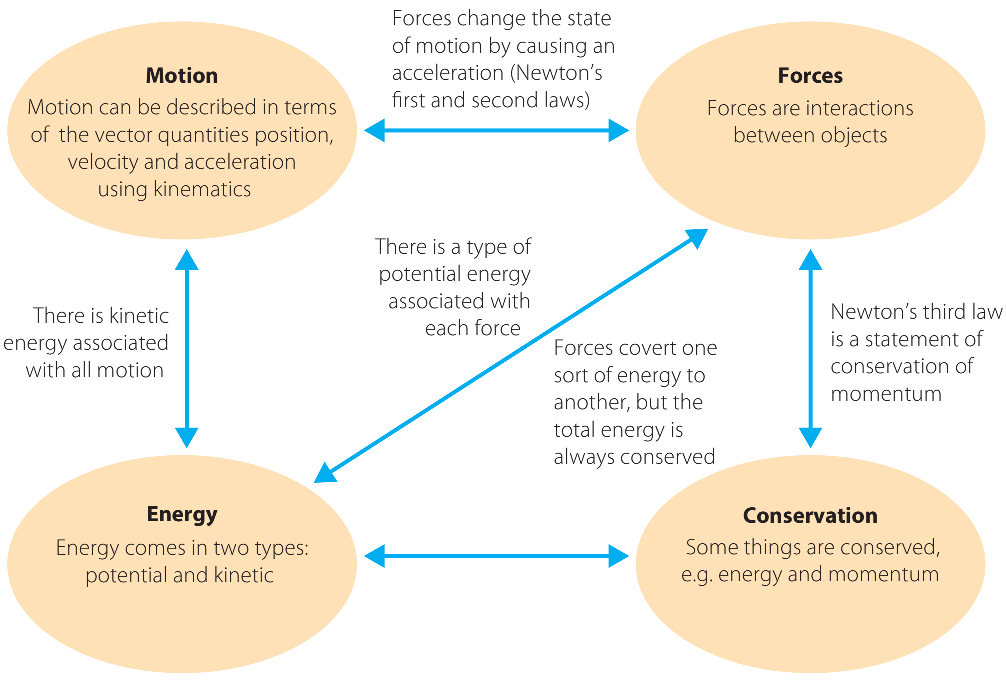


FIGURE 1.3 Concept map for mechanics (chapters 2–6)

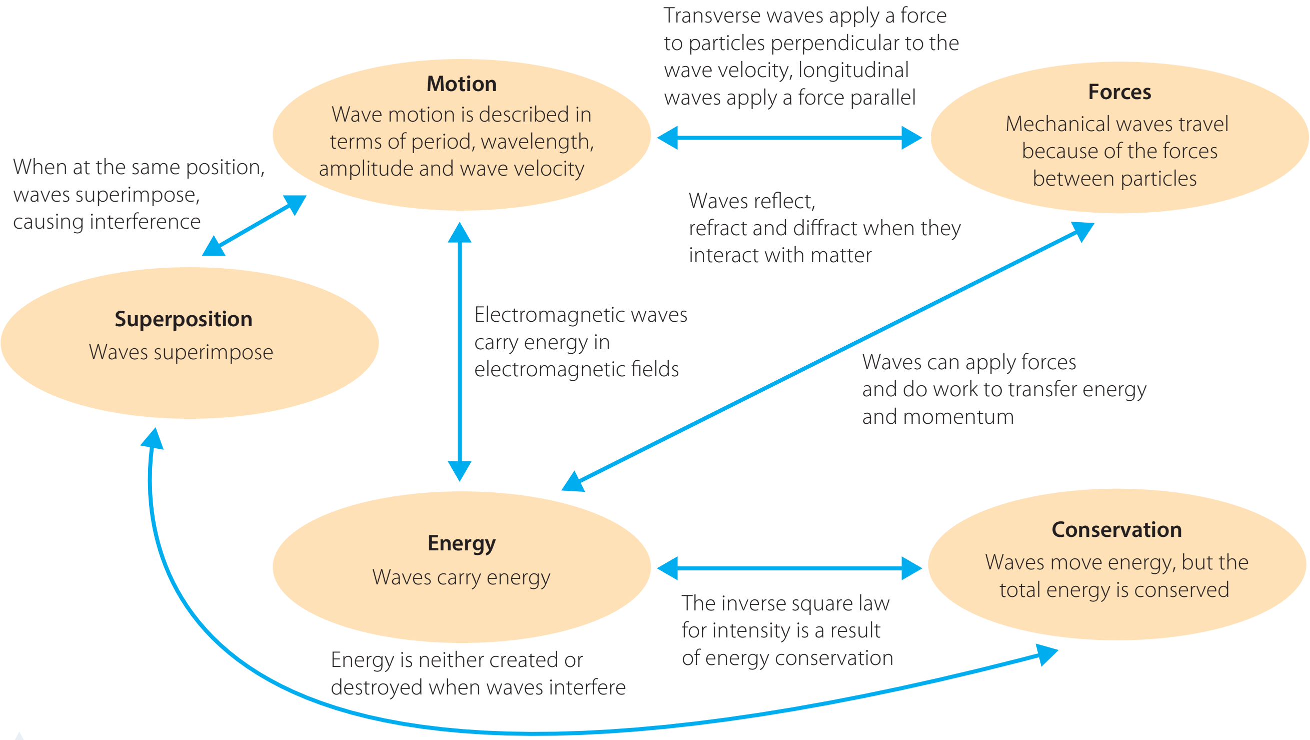


FIGURE 1.4 Concept map for waves (chapters 7–10)

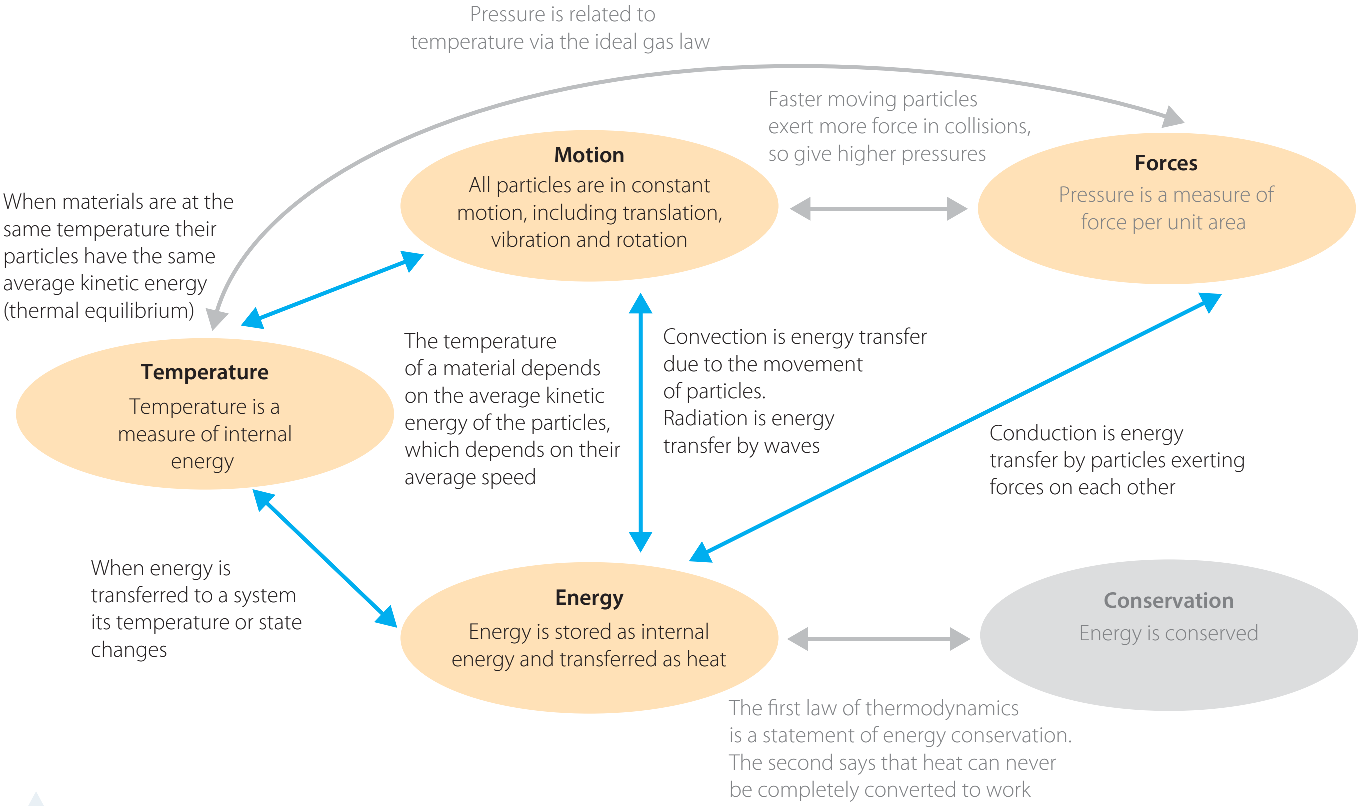


FIGURE 1.5 Concept map for thermodynamics (chapter 11). The concepts shown in grey are not part of this Physics course, but are important ideas that you may meet in your Chemistry course or later studies of physics.

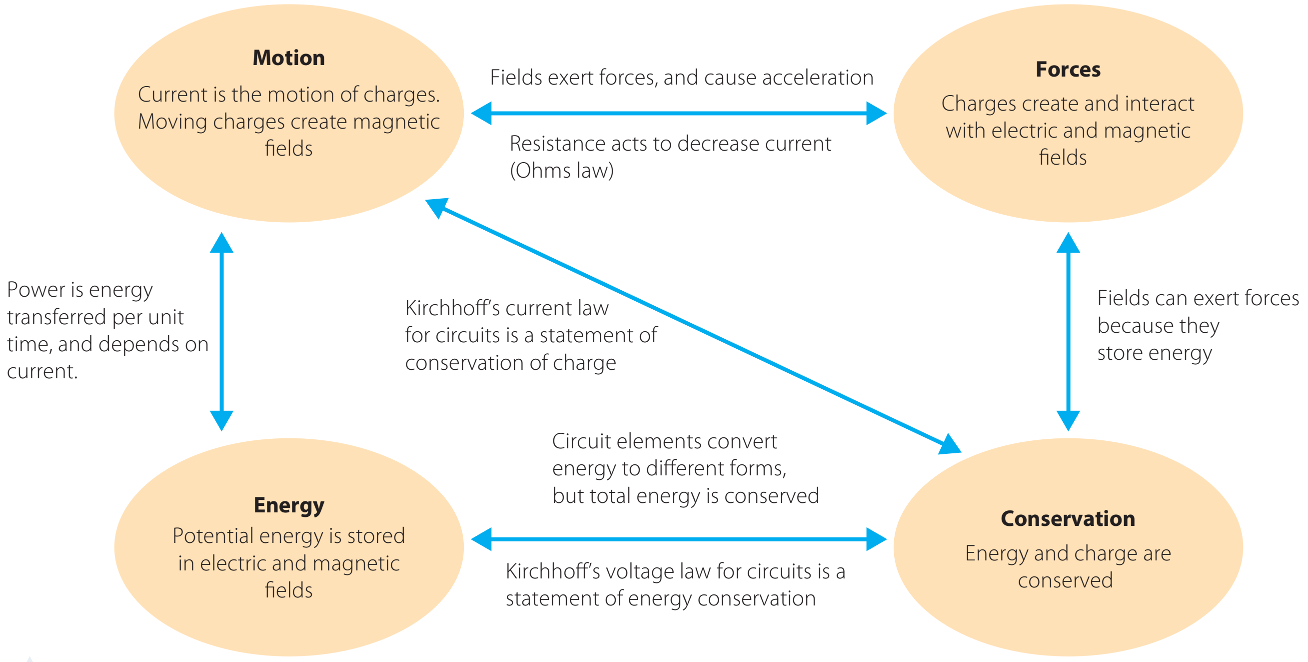


FIGURE 1.6 Concept map for electricity and magnetism (chapters 12–14)

As you learn more of the content knowledge of physics, you need to create your own mental models to help you understand it. Concept maps are a useful way of representing your mental models. They help to remind you that physics is not simply a collection of facts and formulae. Every idea in physics is connected to other ideas, and always to one of the fundamental ideas of force, energy and conservation. All of the theories, laws and equations of physics are fundamentally interconnected; none of them stands alone.

These concept maps (Figures 1.3–1.6) summarise just some of our knowledge and understanding of physics. This knowledge and understanding was arrived at by physicists who asked questions and then tried to answer those questions by working scientifically (Figure 1.2). Working scientifically is more characteristic of, and more important to, the study of physics than any particular collection of content knowledge. You will practise working scientifically (working like a physicist) when you undertake investigations and depth studies.

KEY CONCEPTS

- Scientific theories are falsifiable – they can be disproved, but they cannot be proved. For a theory to be accepted, it must be supported by a great amount of evidence.
- The scientific method consists of questioning, formulating hypotheses, making measurements to test the hypotheses, analysing the results, and communicating them for peer review. It is the process by which science proceeds.
- Physics uses models (physical, mathematical, conceptual, etc.) to describe the world and to make predictions. Models are constantly being refined as we learn more.
- Physics is not a collection of facts and formulae. All of the theories, laws and equations of physics are fundamentally interconnected. Force, energy and conservation are the central ideas in physics that we use to understand the interactions of matter and energy. All the knowledge that you learn in physics will be related to one or more of these ideas.

1.2 Solving scientific problems: depth studies

Depth studies are your opportunity to work scientifically and solve scientific problems. When performing a depth study, you will pose questions, develop hypotheses to answer your questions, and then seek evidence to support or disprove your hypotheses. The evidence may come from the existing scientific literature, or from your own experiments. You will need to analyse data to determine whether your hypotheses are supported. Analysing data usually requires you to represent it in some way, often mathematically or graphically. Finally, as scientists do, you need to communicate your findings to others. There are many ways that you can do this, and you need to choose the communication method most appropriate for your audience.

Depth studies provide you with an opportunity to:

- use the research methods that scientists use
- analyse works for scientific relevance and validity
- broaden your range of reading in a field of interest
- extend your depth of thinking and understanding
- ask questions and investigate areas that do not have definite answers
- investigate contentious issues and use critical thinking skills to consider the validity of views expressed in a variety of sources
- use inquiry-based learning to develop your creative thinking.

Depth studies can take different forms, and over the year you may undertake several different types of depth studies.

Types of depth studies

There are two broad types of depth studies. In first-hand practical investigations, you design and perform experiments or make observations to gather **primary data**. Investigations based on **secondary** sources require you to research and review information and data collected by other people.

First-hand investigations may be work undertaken in a laboratory, field work at home, school or elsewhere, or the creation and testing of a model or device.

Secondary-sourced depth studies may include undertaking a literature review, investigating emerging technologies, analysing a science-fiction movie or novel, or developing an evidence-based argument.

Depth studies may be presented in different forms, some of which include:

- written texts (reports, summaries, essays)
- visual presentations (diagrams, flow charts, posters, portfolios)
- multimedia presentations
- physical models
- a blend of the above.

All depth studies will involve the analysis of data, either from primary data that you collect or from analysing other people's research. Looking for patterns and trends in data will involve analysing and constructing graphs, tables, flow charts and diagrams.

Stages in a depth study

The summary below outlines four main stages of conducting a depth study, as well as the Working Scientifically skills that you will need to develop and apply at each stage.

Questioning and predicting
A student develops and evaluates questions and hypotheses for scientific investigation (PH11-1)
Planning investigations
A student designs and evaluates investigations in order to obtain primary and secondary data and information (PH11-2)
Conducting investigations
A student conducts investigations to collect valid and reliable primary and secondary data and information (PH11-3)
Processing data and information
A student selects and processes appropriate qualitative and quantitative data and information using a range of appropriate media (PH11-4)
Analysing data and information
A student analyses and evaluates primary and secondary data and information (PH11-5)
Problem solving
A student solves scientific problems using primary and secondary data, critical thinking skills and scientific processes (PH11-6)
Communicating
A student communicates scientific understanding using suitable language and terminology for a specific audience or purpose (PH11-7)

Developing and evaluating questions and hypotheses

The first step to beginning any investigation or depth study is deciding on a question.

Obviously, it is a good idea to investigate something that you find interesting. If you are working in a group, try to find something that is interesting to everyone.

A good way to start is by 'brainstorming' for ideas. This works whether you are working on your own or in a group. Write down as many ideas as you can think of. Don't be critical at this stage. Get everyone in the group to contribute and write every idea down.

After you have run out of ideas, it is time to start being critical. Decide which questions or ideas are the most interesting. Think about which ones are actually possible to investigate given the time and resources available. Make a shortlist of questions, but keep the long list too for the moment. Once you have your shortlist, it is time to start refining your ideas.

Alamy Stock Photo/Jim West

FIGURE 1.7 Brainstorm as many ideas as you can in your group.

Refining your question

Your depth study will be based on one of the areas described in Figures 1.3–1.6, which are described in the remaining chapters. However, the purpose of a depth study is to *extend* your knowledge while at the same time building your skills at working scientifically, so you will need to go beyond the basic syllabus content.

The next step is therefore to find out what is already known about the ideas on your list. You will need to conduct a literature review. If your depth study is a secondary-sourced investigation, then the literature review may be the investigation itself. A formal written literature review includes the information you have found, and complete references to the sources of information. It also includes interpretation and critique of what you have read. This is particularly important for a secondary investigation.

Literacy

Information and communication technology capability

Critical and creative thinking

Literature reviews are important to increase your breadth of knowledge, to learn from others and to stimulate new ideas. They are necessary to identify gaps in current knowledge that you may wish to research and to identify methods that you could use. You may also find there are a variety of views (sometimes opposing views) in an area of research.

Literature review

Literacy

A literature review is a search and evaluation of available literature in a particular subject area. It has a particular focus, which is defined by your research question or hypothesis.

The process of conducting a literature review involves researching, analysing and evaluating the literature. It is not merely a descriptive list of the information gathered on a topic, or a summary of one piece of literature after another. It outlines any opposing points of view in the research, and also expresses *your perspective* of the strengths and weaknesses of the research being reviewed. A literature review brings together results of different studies, pointing out areas where researchers or studies agree, where they disagree, and where major questions remain. By identifying gaps in research, literature reviews often indicate directions for future research.

Your literature review will give you an idea of past findings, and procedures, techniques and research designs that have already been used. This will help you to decide which methods are worth following, which need modifying, and which to avoid (those that have been inconclusive or invalid). You may plan your investigation to target a gap in research or try to replicate an investigation to test or validate it.

The length of your literature review will depend on its purpose. If the literature review is a depth study in itself, it will need to be more detailed and draw conclusions about the research. If the literature review is used as an introduction to inform your own research, it will be shorter and more focused.

To write a literature review, you first need to define the topic. It may help to formulate a literature review question, and then write a list of key words that will help you search for information.

To find articles, you can use library catalogues, databases and the internet. Refine your search technique by using specific words that narrow your search. Record search words that are successful and, if necessary, modify your search strategy.

When you write your literature review for your report, it should have an introduction that defines the topic and gives your specific focus. It may also explain the structure of the review for a lengthy, secondary-sourced depth study.

The main body of the review will then group the literature according to common themes and provide an explanation of the relationship between the research question and the literature reviewed. It should proceed from the general, wider view of the research to the specific area you are targeting. Include information about the usefulness, currency and major authors or sources of the literature.

The literature review concludes by summarising the major contributions of the literature, and explaining the link between your investigation and the literature reviewed. It may also point out major flaws or gaps in research if appropriate.

Literature reviews

More information about literature reviews and how to complete them.

Evaluating sources

Always be critical of what you read. Be wary of pseudoscience, and any material that has not been peer-reviewed. Apply the CRAAP (currency, relevance, authority, accuracy, purpose) test to websites that you find. The most reliable sites are from educational institutions (particularly universities), government and scientific organisations such as the CSIRO and NASA, and professional organisations such the Australian Institute of Physics and international equivalents. You can narrow your search to particular types of sites by including in your search terms 'site:edu' or 'site:gov' so that you only find sites from educational or government sources.

The CRAAP test

Apply the CRAAP tests to any websites that you find.

Make sure you keep a record of the information you find as well as the sources, so you can correctly reference them later. It is a good idea to start a **logbook** at this stage. You can write in references, or attach printouts to your logbook. This can save you a lot of time later on. Your logbook may be hardcopy or electronic.

Finally, talk to your teacher about your ideas. They may be able to suggest sources of information. They will also be able to tell you whether your ideas are likely to be possible given the equipment available. They may have had students with similar ideas in the past and will make helpful suggestions.

After you have researched your questions and ideas, you will hopefully be able to narrow the shortlist down to the one question you want to tackle. If none of the questions or ideas look possible (or still interesting), then you need to go back to the long list.

Shutterstock.com/Ermolaev Alexander

FIGURE 1.8 Start researching your topic, and make sure you keep a record of all your references. Good record-keeping is important in scientific research, and it begins at this stage of the investigation.

Proposing a research question or hypothesis

A research question is one that can be answered by performing experiments or making observations. A hypothesis is a prediction of the results of an experiment, which can be tested by performing experiments or making observations.

You need to frame a research question carefully. A good research question should define the investigation, set boundaries and provide some direction to the investigation. It needs to be specific enough that it guides the design of the investigation. A specific question rather than a vague one will make the design of your investigation much easier. Asking 'What volume of water gives the maximum height for a water rocket?' tells you what you will be varying and what you will be measuring. It also gives a criterion for judging whether you have answered the question.

Asking 'How can we make a water rocket fly the best?' is not a good question. This question does not say what will be varied, nor does it tell you when you have answered the question. 'Best' is a vague term. What you mean by 'best' may not be what someone else means.

A hypothesis is a tentative explanation or prediction, such as 'The height attained by a water rocket will increase with the amount of water contained in the rocket'. Your hypothesis should give a prediction that you can test, ideally quantitatively.

A hypothesis is usually based on some existing model or theory. It is a prediction of what will happen in a specific situation based on that model. For example, kinematics describes the trajectory of projectiles. A hypothesis based on the kinematics model predicts the range of a specific projectile launched at a given angle and speed.

A good research question or hypothesis identifies the variables that will be investigated. Usually you will have one dependent variable and one independent variable. For a depth study, you may have two or more independent variables that you control.

Alamy Stock Photo/Marmaduke St John

FIGURE 1.9 You need to frame your research question carefully. These students are investigating the launch angle at which their water rocket will achieve maximum range.

If your experiments agree with predictions based on your hypothesis, then you can claim that they support your hypothesis. This *increases your confidence* in your model, but *it does not prove that it is true.* Hence, an aim for an experiment should never start 'To prove …', as it is not possible to actually prove a hypothesis – only to disprove it.

If your experimental results disagree with your hypothesis, then you may have disproved it. This is *not* a bad thing! Often the most interesting discoveries in science start when a hypothesis based on an existing model is disproved, because this raises more questions.

Even if your question or hypothesis meets these criteria, do not be surprised if you change or modify it during the course of your investigation or depth study. In scientific research, the question you set out to answer is often only a starting point for more questions.

KEY CONCEPTS

- Investigations begin with a question, which is used to formulate a hypothesis. A good research question is specific, and can be answered by performing experiments and making measurements. A good hypothesis is a statement that predicts the results of an experiment and can be tested using measurements.
- A literature review helps you refine your question or hypothesis. It helps you to know what knowledge and ideas have been established on a topic, and the areas of strength and weakness in the research.

1.3 Planning your depth study

There are many things to consider when planning an investigation. You need to think about how much time you will have, what space and equipment you will need, and where you will go if you want to make measurements or observations outside. If you are conducting a secondary-sourced investigation or some other type of depth study (such as a creative work), you still need to plan ahead to make sure you have the resources you need.

You may be working in a group or on your own. Most scientists work in groups. If you can choose who you work with, think about it carefully. It is not always best to work with friends. Think about working with people who have skills that are different from your own.

Having a plan allows you to ensure you collect the data (whether primary- or secondary-sourced) that you need to test your hypothesis. The longer the investigation, the more important it is that you have a clear plan. There are several factors to consider, as shown in Table 1.1.

Critical and creative thinking

TABLE 1.1 Factors to consider when planning your depth study

PRIMARY-SOURCED INVESTIGATION	SECONDARY-SOURCED INVESTIGATION
What data will you need to collect?	What information will you need to gather?
What materials and equipment will you need?	What sources will you use?
When and where will you collect the data?	When and where will you gather the information?
If you are working in a group, what tasks are assigned to which people?	If you are working in a group, what tasks are assigned to which people?
Who will collect the data?	Who will collect what information?
Who will be responsible for record-keeping?	How will record-keeping be done to avoid plagiarism?
How will the data be analysed?	How will the information be analysed?
How will sources be referenced?	How will sources be referenced?

The most common problem that students have is time management. It is important to plan to have enough time to perform the experiments (including repeat measurements), and also to analyse the experiments *and* to report on them.

A good plan will help you keep on track. Your teacher may ask you to submit a plan for your depth study before you begin the implementation stage. Table 1.2 gives an idea of the areas you should consider.

TABLE 1.2 Depth study plan

1 INTRODUCTION	
Title What?	Choose a title for your depth study.
Rationale Why?	Explain why you have chosen this area of research. Describe what you are hoping to achieve through this investigation. Include any applications.
Type of depth study Which?	State the type of depth study you intend conducting (e.g. literature review, practical investigation etc.). Where applicable, describe any theoretical models (e.g. kinematics) that you will use.
2 TIMELINE	
Action and time frame – when?	Working Scientifically skills – how?
Initiating and planning When? (e.g. week 1 and 2)	**Questioning and predicting:** formulate questions and/or a hypothesis. **Planning:** wide reading to research background information, assess risks and ethical issues, plan methods, and design experiments.
Implementation and recording	**Conducting investigations:** safely carry out experiments, make observations and/or measurements, use appropriate technology, and use measuring instruments. **Process and record data and information:** collect, organise, record, and process information and/or data as you go.
Analysing and interpreting	**Analyse data and information:** begin looking for trends, patterns or mathematical relationships. **Problem-solve:** evaluate the adequacy of data (relevance, accuracy, validity and reliability) from primary and/or secondary sources, answer your research question, and draw and justify conclusions.
Communicating	**Present your depth study:** Write the report or other presentation using appropriate language, visualisations and technologies.
Final presentation	**Due date:** allow time for proofreading and editing.
3 DATA COLLECTION	
Variables What will you measure and what will you hold constant? Identify dependent and independent variables.	**Measurements and uncertainties** How will you make measurements? What equipment will you need? How will you minimise uncertainties?
4 DATA ANALYSIS AND PROBLEM SOLVING	
Data analysis What method(s) will you use to analyse the data and how will you represent the trends and patterns?	**Conclusions** How will you judge whether the experiment was valid? How will your data allow you to test your hypothesis or answer your question?

Keep a record of your planning. This should go in your logbook. Recording what you plan to do, and why, will help you stay focused. This is particularly important for a depth study. If you are working in a group, keep a record of what each person agrees to do. But remember, the plan may need to be adjusted as you go.

Designing your depth study

When designing your depth study or investigation, you should be aiming for reliable and valid measurements with good accuracy and precision. For a secondary-sourced investigation, you should be trying to find resources that have these characteristics.

Some good questions to ask to assess the reliability, validity, accuracy and precision of your design are outlined in Table 1.3.

TABLE 1.3 Assessing reliability, accuracy and validity in investigations

	PRIMARY INFORMATION AND DATA	SECONDARY INFORMATION AND DATA
Reliability	• Have I tested with repetition?	• How consistent is the information with other reputable sources? • Is the data presented based on repeatable processes?
Accuracy and precision	• Have I designed my experiments to minimise uncertainties? • Have I used repeat measurements to estimate random errors? • Have I used the best measuring equipment available, and used it correctly?	• Is this information similar to information presented in peer-reviewed scientific journals? • Is the data given with uncertainties, and are these uncertainties small compared with the measured values?
Validity	• Does my experiment actually test the hypothesis that I want it to? • Have all variables (apart from those being tested) been kept constant?	• Do the findings relate to the hypothesis or problem? • Are the findings accurate and the sources reliable?

Selecting equipment

A well-framed question or hypothesis will help you choose the equipment that you need. For example, if your hypothesis predicts a temperature change of 0.5°C, then you will need a thermometer that can measure to at least this precision. You also need to know how to use the equipment correctly. Always ask if you are unsure. The user manual will usually specify the precision of the device, and let you know of any potential safety risks.

You need to think about how you can minimise **uncertainties**. Minimising uncertainty is not just about using the most precise equipment you can find – it is also about clever experimental technique. Very precise measurements are possible using simple equipment. For example, in 1862, Léon Foucault measured the speed of light with an uncertainty of 0.2%, without a computer, data logger or even a digital stopwatch. Remember that it is a poor worker who blames their tools!

Minimising uncertainty

Find out how Foucault measured the speed of light so precisely.

Working safely: risk assessment

Ethical understanding

Personal and social capability

Work and enterprise

You may be required to complete a risk assessment before you begin your depth study. You need to think about three factors:

1. *What are the possible risks* to you, to other people, to the environment or property?
2. *How likely* is it that there will be an injury or damage?
3. *How serious are the consequences* likely to be if there is an injury or damage to property or environment?

A 'risk matrix', such as Table 1.4, can be used to assess the severity of a risk associated with an investigation. The consequences are listed across the top from negligible to catastrophic. 'Negligible' may be getting clothes dirty. 'Marginal' might be a bruise from falling off a bike, or a broken branch in a tree. 'Severe' could be a more substantial injury or a broken window. 'Catastrophic' would be a death or the release of a toxin into the environment. You need to ensure that your investigation is low risk.

TABLE 1.4 Risk matrix for assessing for severity of risk

CONSEQUENCES → LIKELIHOOD ↓	Negligible	Marginal	Severe	Catastrophic
Rare	Low risk	Low risk	Moderate risk	High risk
Unlikely	Low risk	Low risk	High risk	Extreme risk
Possible	Low risk	Moderate risk	Extreme risk	Extreme risk
Likely	Moderate risk	High risk	Extreme risk	Extreme risk
Certain	Moderate risk	High risk	Extreme risk	Extreme risk

Once you have considered what the possible risks are, you need to think about what you will do about them. What will you do to minimise the possible risks, and what will you do to deal with the consequences if something does happen? You can use a risk assessment table such as the one shown in Table 1.5.

TABLE 1.5 Sample risk assessment table

WHAT ARE THE RISKS IN DOING THIS EXPERIMENT?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Water from the rocket may be spilled and someone might slip.	Clean up all spills immediately.

RISK ASSESSMENT

Consider where you will perform your experiments or observations. Will you need to consider the convenience or safety of others? Talk to your teacher about what space is available.

In a secondary-sourced investigation, take precautions with cyber safety and remember to keep your personal information private.

Stay safe online!

Read the material on this site, and think about what you could do to keep yourself safe online.

KEY CONCEPTS

- In primary-sourced investigations you collect and analyse your own data. In secondary-sourced investigations you analyse someone else's data.
- Investigations need to be carefully planned so that they answer your research question. You also need to consider safety and possible environmental impacts of your investigation.

1.4 Conducting your investigation

Scientists keep a logbook for each project they work on. The logbook is the primary source of information when a scientist writes up their work for publication.

A logbook is a legal document for a working scientist. If the work is called into question, then the logbook acts as important evidence. Logbooks are sometimes provided as evidence in court cases; for example, in patent disputes. Every entry in a scientist's logbook is dated, records are kept in indelible form (i.e. in pen, *not pencil*), and entries may even be signed. Logbooks include details of experiments, ideas and analysis. They frequently include printouts of data, photocopies of relevant information, photos, and other items.

Literacy

FIGURE 1.10 Make sure you keep an accurate record of what you do as you do it. Keeping a logbook is important.

Your logbook records

It is a good idea to start keeping a logbook as soon as you begin planning your depth study. Your logbook may be paper or electronic. Either way, your logbook is a detailed record of *what you did* and *what you found out* during your investigation. Make an entry in the logbook every time you work on your depth study. At the start of each session, record the date and the names of all the people you are working with at the time. A logbook is particularly vital for primary-sourced investigations, but is also important for secondary-sourced investigations.

Always write down what you do as you do it. It is easy to forget what you did if you do not write it down immediately.

Record the results of *all* measurements *immediately and directly into your logbook, in pen if using hardcopy*. Never record data onto bits of scrap paper instead of your logbook. Results must be recorded in indelible form. Never write your results in pencil or use white-out. If you want to cross something out, just put a line through it and make a note explaining why it was crossed out. If you are using an electronic logbook, do not delete data or any working – instead, label it appropriately and keep it.

A good logbook contains:

- notes taken during the planning of your investigation
- a record of when, where and how you carried out each experiment
- diagrams showing the experimental set-ups, circuit diagrams, etc.
- all your raw results
- all your derived results, analysis and graphs
- all the ideas you had while planning, carrying out experiments and analysing data
- printouts, file names and locations of any data not recorded directly in the logbook.

It is not a neat record, but it is a *complete* record.

Literacy

Numeracy

Data collection

If you are conducting a secondary-sourced investigation, your literature review will be the basis of your investigation and your data will come from the existing literature. Remember that a literature review is not simply a summary of what you have read – you need to add meaning. This may come from comparing and contrasting competing models and constructing an argument, or by analysing and presenting secondary-sourced data. When using secondary sources, remember to make comparisons between data and claims in a number of reputable sources, including science texts, scientific journals and reputable internet sites, and to reference these appropriately.

If you are conducting a primary-sourced investigation, then you will be performing measurements to gather data yourself. You can collect data by performing experiments or making observations in the field. You will gain practice at making measurements if you do some of the investigations in the following chapters. These investigations can form a basis for a depth study.

Data collection for primary-sourced investigations

When conducting an experiment, you need to decide which variables you will measure and which variables you will control. In an experiment, typically we have an independent variable, which we control and vary, and a dependent variable, which is what we measure. We assume that the dependent variable is in some way dependent on the independent variable. There may also be controlled variables, which are kept constant so that they do not interfere with the results.

Whenever possible you should make repeat measurements. This allows you to check that your measurements are reliable. Your results are reliable if repeat measurements give the same results within experimental uncertainty. If a result is not reproducible, it is not a reliable result. If a result is not reproducible, then it may be that a variable other than the one you are controlling is affecting its value. If this is the case, you need to determine what this other variable is, and control it if possible.

You also need to consider how many data points to collect. In general, it is better to have more data than less. However, you will have limited time to collect your data, and you need to allow time for analysis and communicating your results. A minimum of six data points is usually required to establish a relationship between variables if the relationship is linear. A linear relationship is one where if you plot one variable against the other you get a straight line. If you think the relationship might not be linear, then take more data points and think carefully about how they will be spaced. You should try to collect more data in the range where you expect the dependent variable to be changing more quickly. For example, if you are measuring the temperature of a hot object as it cools, then you should collect more data early on when cooling is more rapid.

Draw a table to record data in. Label the columns in the table with the name and units of the variables. If you know that the uncertainty in all your measurements is the same, then you can record this at the top of the column as well. Otherwise, each data entry should have its uncertainty recorded in the cell with it.

It is a good idea to start your analysis while you are collecting your data. If you spot an outlier while you are still making measurements, then you have the opportunity to repeat that measurement. If you made a mistake, then put a line through the mistake, write in the new data, and make a comment in your logbook.

If you have not made a mistake, then plotting and analysing as you go allows you to spot something interesting early on. You then have a choice between revising your hypothesis or question to follow this new discovery, or continuing with your plan. Many investigations start with one question and end up answering a completely different one. These are often the most fun, because they involve something new and exciting.

Accuracy and precision of measurements

When making measurements, your aim is to be as precise and accurate as possible.

An accurate measurement result is one that represents the 'true value' of the measured quantity as closely as possible. When we take repeated measurements, we assume that the mean of the measurements will be close to the 'true value' of the variable. However, this may not always be the case. For example, if you have ever been a passenger in a car with an analogue speedometer and tried to read it, your reading will be consistently different from what the driver reads. This is due to parallax error. The needle sits above the scale, and when viewed from the side it does not correctly line up with the true speed. Beware of parallax error with any equipment using a needle. This is an example of a systematic error, in which measurements differ from the true value by a consistent amount. Note that often we do not know what the 'true value' is. Figure 1.11 compares accurate measurements with inaccurate measurements due to a systematic error.

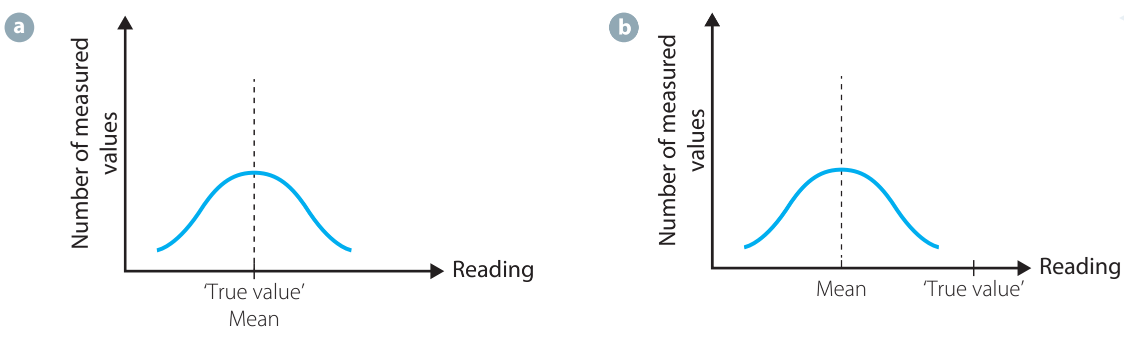


FIGURE 1.11 In a plot of number of measured values versus reading, results may: **a** be accurate and cluster close to the 'true value', or **b** cluster about some other value.

Precision is a measure of the variability of the measurements, so it affects the spread of the repeated measurements about the mean value. The smaller the spread, the greater the precision. Figure 1.12a shows precise measurements, and Figure 1.12b shows less precise measurements. Note that both data sets are centred about the same average, so they have the same accuracy.

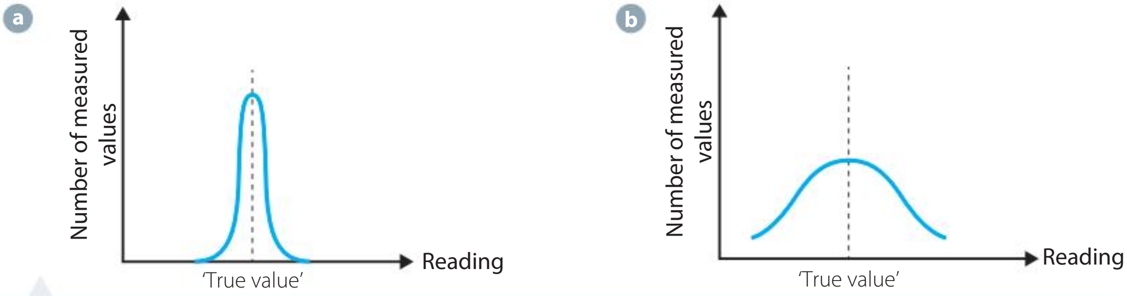
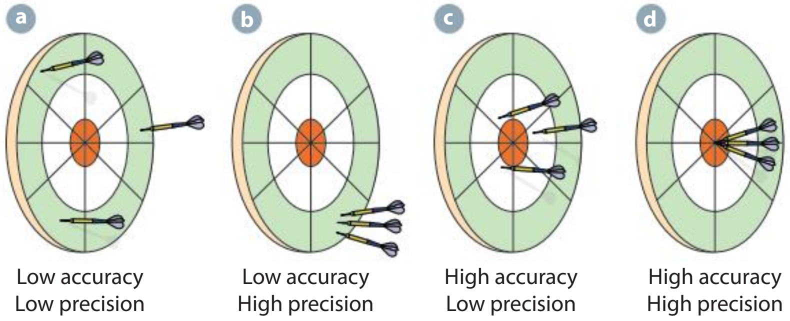


FIGURE 1.12 In a plot of number of measured values versus reading, results may: **a** be precise and have a small spread about the mean, or **b** be imprecise and have a large spread about the mean.

Figure 1.13 shows the difference between accuracy and precision for a game of darts. Accuracy is how close to the centre your darts hit, and precision is how closely the darts are grouped.

FIGURE 1.13 On a dart board, accuracy is determined by how close to the centre (bull's eye) your dart lands. Precision is how closely you can group your darts.



Low accuracy
Low precision

Low accuracy
High precision

High accuracy
Low precision

High accuracy
High precision

Estimating uncertainties

When you perform experiments, there are typically several sources of uncertainty in your data.

Sources of uncertainty that you need to consider are the:

- limit of reading of measuring devices
- precision of measuring devices
- variation of the measurand (the variable being measured).

For *all* devices there is an uncertainty due to the limit of reading of the device. The limit of reading is different for analogue and digital devices.

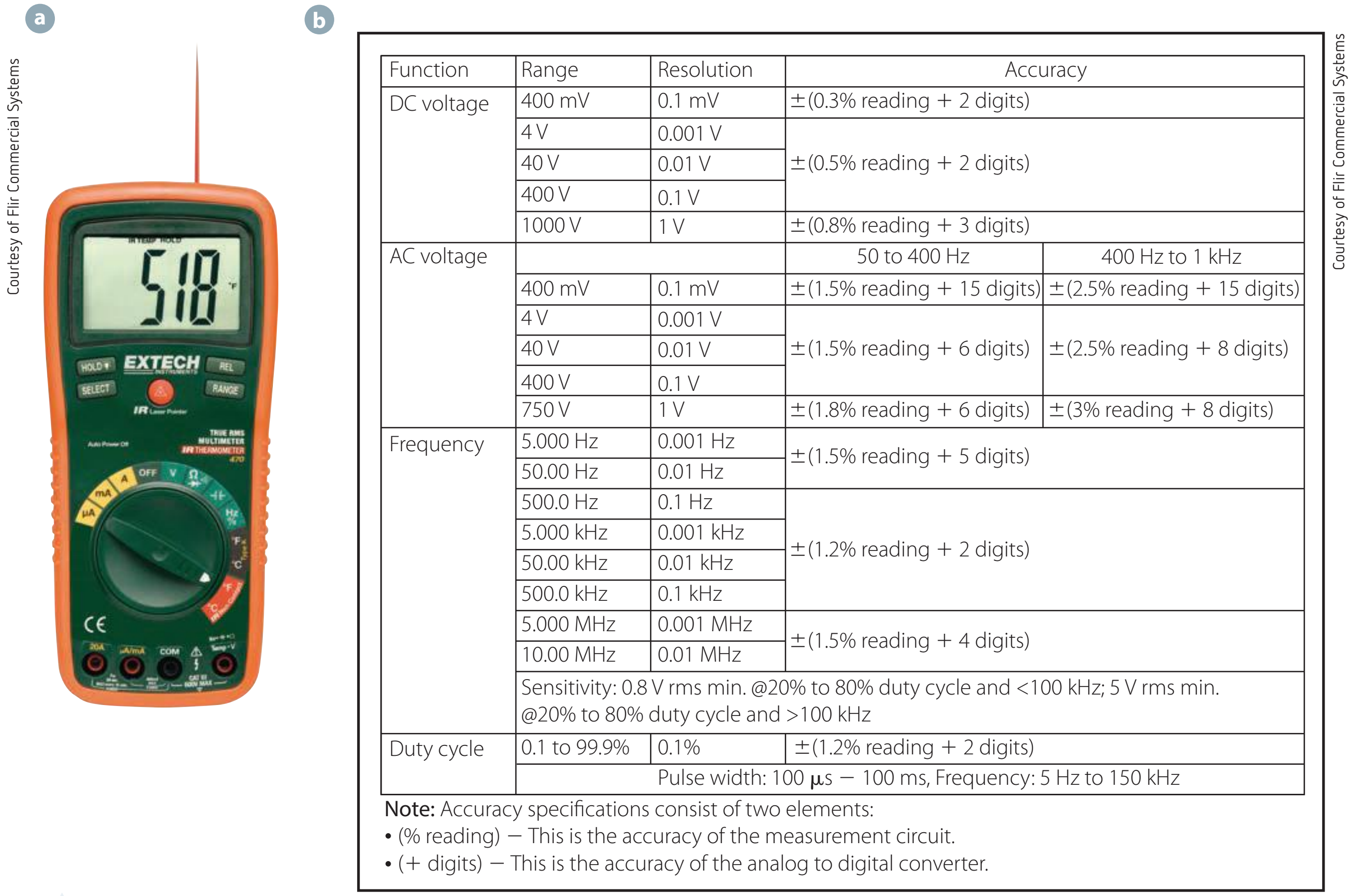
Analogue devices have continuous scales and include swinging-needle multimeters and liquid-in-glass thermometers. For an analogue device, the limit of reading (sometimes called the resolution) is half the smallest division on the scale. We take it as half the smallest division because you will generally be able to see which division mark the indicator (needle, fluid level, etc.) is closest to. So, for a liquid-in-glass thermometer with a scale marked in degrees Celsius, the limit of reading is 0.5°C.

Digital devices, such as digital multimeters and digital thermometers, have a scale that gives you a number. A digital device has a limit of reading uncertainty of a whole division. So, a digital thermometer that reads to whole degrees has an uncertainty of 1°C. For a digital device, the limit of reading is *always* a whole division (not a half), because you do not know whether it rounds up or down, or at what point it rounds.

The resolution or limit of reading is the *minimum* uncertainty in any measurement. Usually the uncertainty is greater than this minimum.

The measuring device used will have a precision, usually given in the user manual. For example, the multimeter shown in Figure 1.14 may have a precision of 1.5% on a voltage scale. This means if you measure a potential difference of 12.55 V on this scale, the uncertainty due to the precision of the meter is $0.015 \times 12.55\ \text{V} = 0.19\ \text{V}$. This is greater than the limit of reading uncertainty, which is 0.01 V in this case. The precision is a measure of repeatability. If you measured a potential difference of 12.55 V on this multimeter, you could expect the reading to vary by as much as 0.19 V, even if the potential difference stayed the same.

Many students think that digital devices are more precise than analogue devices. This is often not the case. A digital device may be easier for you to read, but *this does not mean it is more precise*. The uncertainty due to the limited precision of the device is generally greater than the limit of reading.



Courtesy of Flir Commercial Systems

Function	Range	Resolution	Accuracy	
DC voltage	400 mV	0.1 mV	±(0.3% reading + 2 digits)	
	4 V	0.001 V	±(0.5% reading + 2 digits)	
	40 V	0.01 V		
	400 V	0.1 V		
	1000 V	1 V	±(0.8% reading + 3 digits)	
AC voltage			50 to 400 Hz	400 Hz to 1 kHz
	400 mV	0.1 mV	±(1.5% reading + 15 digits)	±(2.5% reading + 15 digits)
	4 V	0.001 V	±(1.5% reading + 6 digits)	±(2.5% reading + 8 digits)
	40 V	0.01 V		
	400 V	0.1 V		
	750 V	1 V	±(1.8% reading + 6 digits)	±(3% reading + 8 digits)
Frequency	5.000 Hz	0.001 Hz	±(1.5% reading + 5 digits)	
	50.00 Hz	0.01 Hz		
	500.0 Hz	0.1 Hz	±(1.2% reading + 2 digits)	
	5.000 kHz	0.001 kHz		
	50.00 kHz	0.01 kHz		
	500.0 kHz	0.1 kHz		
	5.000 MHz	0.001 MHz	±(1.5% reading + 4 digits)	
	10.00 MHz	0.01 MHz		
	Sensitivity: 0.8 V rms min. @20% to 80% duty cycle and <100 kHz; 5 V rms min. @20% to 80% duty cycle and >100 kHz			
Duty cycle	0.1 to 99.9%	0.1%	±(1.2% reading + 2 digits)	
	Pulse width: 100 μs – 100 ms, Frequency: 5 Hz to 150 kHz			

Note: Accuracy specifications consist of two elements:

- (% reading) – This is the accuracy of the measurement circuit.
- (+ digits) – This is the accuracy of the analog to digital converter.

Courtesy of Flir Commercial Systems

FIGURE 1.14 **a** A typical small digital multimeter; **b** A page from the user manual giving the precision on various scales

Finally, the measurand itself may vary. For example, the flight of a water rocket is strongly dependent on initial conditions, wind and other factors. Even keeping launch conditions as close to identical as possible, it is unlikely that in repeat experiments you will be able to get a rocket to attain the same height within the limit of reading or equipment precision. Making repeat measurements allows you to estimate the size of the variation, using the maximum and minimum values.

Sometimes you will be able to see how the measurand varies during a measurement by watching a needle move or the readings change on a digital device. Watch and record the maximum and minimum values.

Accuracy, precision and resolution – what is the difference?

The difference between the maximum and minimum value is the range:

$$\text{Range} = \text{maximum value} - \text{minimum value}$$

The value of the measurand is the average value for repeated measurements, or the centre of the range for a single varying measurement:

$$\text{Measurand} = \text{minimum value} + \frac{1}{2}\text{(range)}$$

The uncertainty in the measurement is half the range:

$$\text{Uncertainty} = \frac{1}{2}\ \text{(range)} = \frac{1}{2}\ \text{(maximum value} - \text{minimum value)}$$

For example, if you are using a multimeter and you observe that the reading fluctuates between 12.2 V and 12.6 V, then your measurement should be recorded as (12.4 ± 0.2) V. Note that the measurement and uncertainty are together in the brackets, indicating that the unit applies to both the measurement and its uncertainty.

When you take repeat measurements, the best estimate of the measurand is the average value. If you have taken fewer than 10 measurements, then the best estimate of the uncertainty is half the range. If you have 10 or more measurements, the best estimate of the uncertainty is the standard deviation, given by:

$$\text{Standard deviation} = \left[\frac{\Sigma_i (x_i - \overline{x})^2}{n}\right]^{\frac{1}{2}}$$

where x_i is an individual value of the measurand, $\overline{x}$ is the average value of the measurand and n is the total number of measurements. The sum is over all values of x_i. Most calculators and spreadsheet software have built-in statistical functions such as standard deviation.

Remember that repeat measurements means repeating under the same conditions. It is not the same as collecting lots of data points under different conditions.

Uncertainties can be expressed in two ways: **absolute** and **fractional** (also called relative or proportional) uncertainty. The absolute uncertainty has the same units as the measurement, and is the uncertainty we have been describing above. The relative or fractional uncertainty is the absolute uncertainty as a fraction of the measurement, often expressed as a percentage. For example, if you measure your height to be 164 cm with an absolute uncertainty of 1 cm, then the fractional uncertainty is $\frac{1\ \text{cm}}{164\ \text{cm}} = 0.006$ or 0.6%.

Analysing data

Numeracy

Information and communication technology capability

The first step in analysing data (whether primary or secondary) is organising it. This will usually involve tabulating the data. Tables of data need to have headings with units for each column, and a caption telling you what the data means or how it was collected. Tables are used for recording raw data, and also for organising derived data.

Calculating derived data from raw data

Raw data is what you actually measured (with units and uncertainties). **Derived data** is data that you have calculated using your raw data. For example, your raw data may be time and distance measurements, from which you derive average speed and acceleration data.

When you record your data, write down the units for all your measurements. You may need to convert these to SI units; for example, centimetres to metres (see appendix 1). Include the units with all numbers as you perform your calculations to ensure you have the correct units on all derived data. It also allows you to check that any equations you are using are dimensionally correct. It is good practice in general, not just in investigations, to include units at each step in all your calculations.

Your raw data should be recorded with uncertainties. All your derived results should also have uncertainties.

Whenever you add or subtract raw data, you simply add the absolute uncertainties.

Whenever you multiply or divide raw data, you add the fractional uncertainties in all the values used. For example, if you are calculating a using the equation

$$a = \frac{1}{2} \times \frac{b^2}{c}$$

then the relative uncertainty in a will be the relative uncertainty in c plus the relative uncertainty in b plus the relative uncertainty in b again:

$$\frac{\Delta a}{a} = \frac{\Delta b}{b} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$$

We add the relative uncertainty of b twice because we multiply by b^2, which is the same as multiplying by b twice. Note that we do not include the relative uncertainty in $\frac{1}{2}$ because we assume it to be an exact number with no uncertainty.

If you have more complicated calculations, then you should refer to a guide such as the *Guide to Expression of Uncertainty in Measurement* or a book on experimental techniques.

GUM

Find out more about uncertainties in the GUM *(Guide to Expression of Uncertainty in Measurement).*

Drawing and using graphs

If you look at any physics journal, you will see that almost every article contains graphs. Graphs are not only a useful way of representing data, but they are also commonly used to analyse relationships between variables. You should have lots of graphs in your logbook as part of your exploration of the data. It is often useful to plot your data in different ways, especially if you are unsure what relationship to expect between your dependent and independent variables.

Graphs should be large and clear. The axes should be labelled with the names of the variables and their units. Choose a scale so that your data takes up most of the plot area. This will often mean that the **origin** is not shown in your graph. Usually there is no reason why it should be. Figure 1.15 shows examples of a poor graph and a good one.

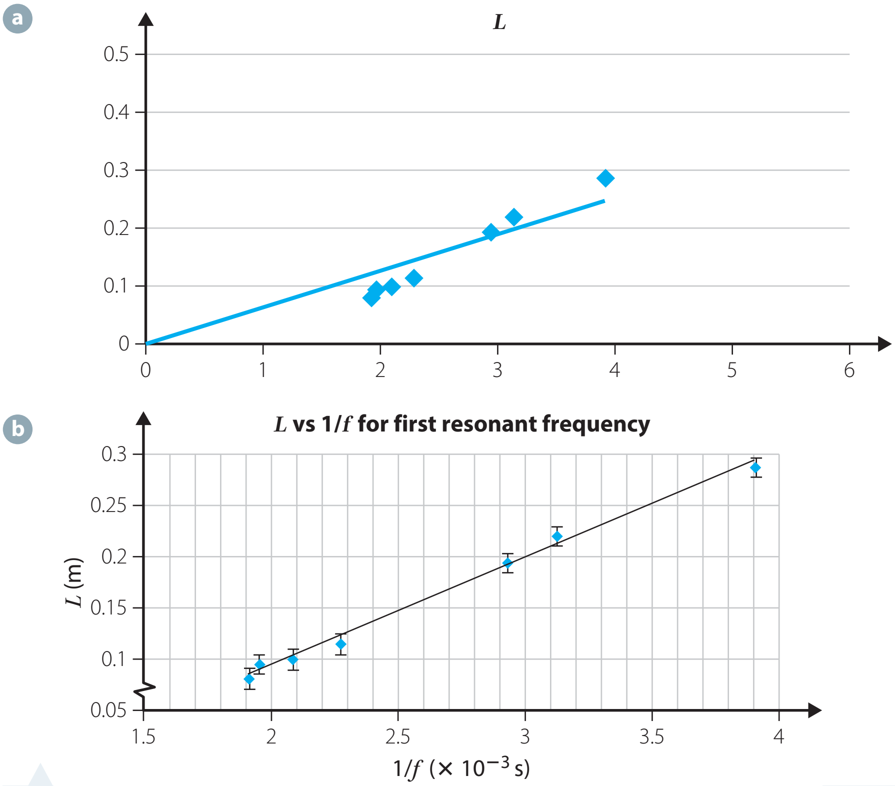
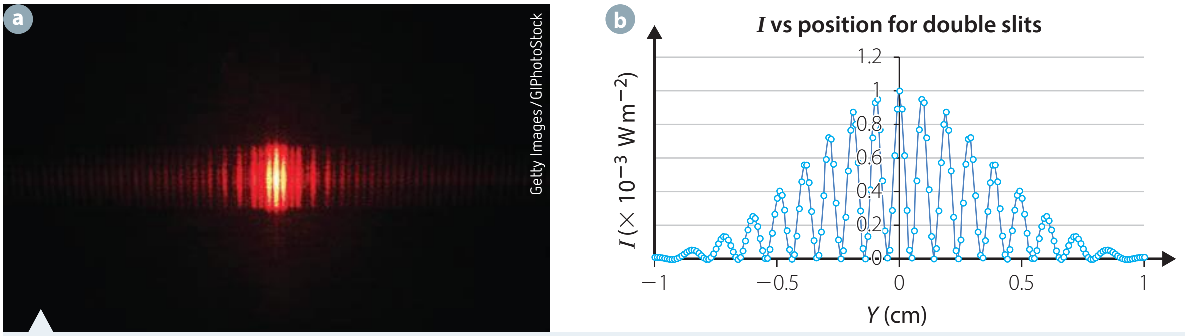


FIGURE 1.15 **a** A poor example of a graph; **b** A good example of a graph of the same data. How many problems can you identify on the graph in part **a**?

When you are looking for a relationship between variables, use a **scatter plot** (also called a scatter graph). This is a graph showing your data as points. *Do not join them up as in a dot-to-dot picture.* Usually the independent variable is plotted on the *x* axis and the dependent variable is on the *y* axis.

To determine a relationship, you need to have enough data points, and the range of your data points should be as large as possible. A minimum of six data points is generally considered adequate if the relationship is expected to be linear (a straight line), but always collect as many as you reasonably can in the available time.

For non-linear relationships, you need more data points than this. Try to collect more data in regions where you expect rapid variation. Imagine you are measuring an interference pattern from two slits, as shown in Figure 1.16. You may need more than a hundred data points to clearly see the sinusoidal pattern due to the two-slit interference.



Getty Images/GIPhotoStock

FIGURE 1.16 **a** Interference pattern from two slits; **b** Plot of intensity as a function of position for this experiment

Data points
Some helpful advice on deciding the number of data points.

A good graph to start with is simply a graph of the raw data. You will usually be able to tell by looking whether the graph is linear. If it is, then fit a straight line either by hand or using graphing software. Graphing software has a linear regression tool that calculates an R^2 number, which is a measure of 'goodness of fit'. The closer R^2 is to 1, the better the fit. If it is not very close to 1 (typically better than 0.95), then the relationship is probably not linear. Alternatively, you can calculate the uncertainty in the gradient by using lines of maximum and minimum gradient. If the uncertainty is large, then the relationship may not be linear.

If you have a hypothesised equation, then use it to generate a fit on a graph of your data. *Do not substitute your data into your hypothesised equation and try to show that it fits.* Note that a line of best fit is *not* the same as joining the dots. It is rarely useful or appropriate to join the dots, even though this is often the default setting in spreadsheet software.

If it is a linear relationship, then finding the equation for the line of best fit will be useful. Remember that a linear relationship is of the form $y = mx + c$, where y is the dependent variable plotted on the vertical axis, x is the independent variable on the horizontal axis, $m = \dfrac{\Delta y}{\Delta x}$ is the gradient of your graph, and c is the y intercept from your graph.

Never force a line of best fit through the origin. The intercept gives you useful information. It may even indicate a systematic error, such as a zero error in calibration of your equipment.

When you plot your raw data, you may find that one or two points are outliers. These are points that do not fit the pattern of the rest of the data. These points may be mistakes; for example, they may have been incorrectly recorded or the result of a mistake made during measurement. They may also be telling you something important. For example, if they occur at extreme values of the independent variable, then it might be that the behaviour of the system is linear in a certain range only. You may choose to ignore outliers when fitting a line to your data, but you should be able to justify why.

Non-linear data and linearising

Relationships between variables are often not linear. If you plot your raw data and it is a curve, then *do not draw a straight line through it*. In this case you need to think a little harder. If your hypothesis predicts the shape of the curve, then try fitting a theoretical curve to your data. If it fits well, then your hypothesis is supported.

If possible, you should linearise your data based on your hypothesis – that means to write it in the form $y = mx + c$. For example, if your hypothesis is that $h = \frac{1}{2}gt^2$, try plotting your data as a function of t^2. Here, h is the initial height of a falling object, g is the acceleration due to gravity, and t is the time taken for it to fall:

$$\begin{array}{ccccc} h = & \left(\frac{1}{2}g\right) & (t^2) & + & 0 \\ \uparrow & \uparrow & \uparrow & & \uparrow \\ y = & m & x & + & c \end{array}$$

Hence, a plot of h vs t^2 should be a straight line with gradient of $\frac{1}{2}g$ and a y intercept of zero. So, if you plot h vs t^2 and get a straight line of gradient $\frac{1}{2}g$ with a y intercept of zero, then your hypothesis is supported.

It is better to linearise your data rather than to try fitting a curve to non-linear data. Often a curve for an exponential relationship can look very much like a curve for a power law. Linearising your data allows you to distinguish between the two.

Interpreting your results

Once you have analysed your results, you need to interpret them. This means being able to either answer your research question or state whether your results support your hypothesis.

You need to consider the uncertainties in your results when you decide whether they support your hypothesis. For example, suppose you have hypothesised that the maximum range of a rocket occurs at a launch angle of 45°, but your results show that the maximum range occurs at an angle of 47°. To say whether the result agrees with the prediction, you need to consider the uncertainty. If the uncertainty is 1°, then the results disagree with the hypothesis. If the uncertainty is 2° or more, then the results do agree and the hypothesis is supported.

If your hypothesis is not supported, it is not enough to simply say 'our hypothesis is wrong'. If the hypothesis is wrong, *what* is wrong with it?

It may be that you have used a model that is too simple. For example, if you have based your hypothesis on the kinematics model and ignored the effect of air resistance, the range is likely to be shorter and the maximum height lower than you predicted. For many projectiles, including rockets, air resistance is not negligible. If you find this is the case, then you may conclude that your situation is better described by a model that includes air resistance.

However, before you decide that the model is at fault, it is a good idea to check carefully that you have not made any mistakes.

It is *never* good enough to conclude that 'the experiment didn't work'. Either a mistake was made or the model used was not appropriate for the situation. It is your job to work out which is the case.

Experiments that do not support predictions based on existing models are crucial in the progress of science. It is these experiments that tell us there is more to find out, and that inspire our curiosity as scientists.

KEY CONCEPTS

- Logbooks are important working documents for scientists. It is a good idea to record all your data in a logbook, along with all records of your investigations.
- Precision is a measure of the variability or spread in repeat measurements.
- Accuracy is a measure of the difference between the average value of repeat measurements and the ‘true value’ of the thing being measured. Often, we do not know the true value.
- The uncertainty in any measurement depends upon the limit of reading of the measuring device, the precision of the device and the variation of the measurand. The uncertainty in the measurand is whichever is the greatest of these.
- Uncertainties can be recorded as absolute uncertainties, which have the same units as the measurand, or fractional uncertainties, which have no units and are usually expressed as percentages.
- Data is usually recorded in tables. Graphs are used to represent and analyse data.
- Linear graphs are useful for analysing data.
- If you predict a particular mathematical relationship, you should linearise your data and then graph it to test your prediction.
- You must know the uncertainty in your results to be able to test your hypothesis.
- When your hypothesis is not supported, try to figure out why.

Literacy

Numeracy

Information and communication technology capability

1.5 Communicating your understanding

If research is not reported on, then no-one else can learn from it. An investigation is not complete until the results have been communicated. Most commonly a report is written. Scientists also use other means to communicate their research to each other such as posters and talks. School science fairs may also include posters and oral presentations. Science shows and demonstrations, websites, videos and blogs may also be used. You need to select the mode that best suits the *content* you wish to communicate and your intended *audience*.

Writing reports

A report is a formal and carefully structured account of your investigation or depth study. It is based on the data and analysis in your logbook. However, the report is a summary and contains only a small fraction of what appears in the logbook. Your logbook contains all your ideas, rough working and raw data. The report typically contains very little of this.

A report consists of several distinct sections, each with a particular purpose:

- Abstract
- Introduction
- Method
- Results and analysis
- Discussion
- Conclusion
- Acknowledgements
- References
- Appendices.

Reports are *always* written in the past tense because they describe what you have done.

The abstract is a very short summary of the entire report, typically 50–200 words. It appears at the start of the report, but is always the last section that you write. Try writing just one sentence to summarise each part of your report.

The introduction tells the reader why you did this investigation or depth study, and what your research question or hypothesis is. This is the place to explain why this research is interesting. The introduction also includes the literature review, which gives the background information needed to be able to understand the rest of the report. The introduction for secondary-sourced reports is similar to that for a primary-sourced investigation. In either case, it is important to reference all your sources correctly.

The method summarises what you did. It says what you measured and how you measured it. It is *not a recipe for someone else to follow*. It is *always* written in past tense; for example, 'We measured the temperature every 10 s', *not* 'Measure the temperature every 10 s'. It also briefly explains why you chose a particular method or technique.

For a primary-sourced investigation, the method describes how you carried out your experiments or observations in enough detail that someone with a similar knowledge level could repeat your experiments. It should include large, clear diagrams of equipment set-up, circuits etc. You should have diagrams in your logbook, but these are generally rough sketches. Diagrams should be redrawn neatly for a report, as in Figure 1.17.

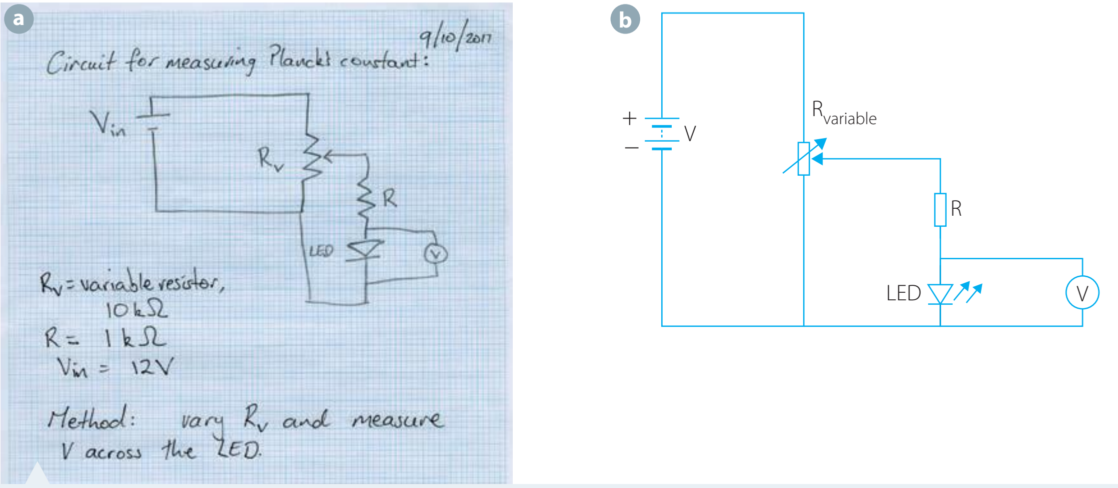


FIGURE 1.17 **a** A circuit diagram from a logbook; **b** The same circuit diagram redrawn in a formal report

The method section for a secondary-sourced investigation is generally shorter. If you are conducting a review of the current literature on a topic, then your method will say what literature searches you carried out and how you decided which sources to use.

The results section is a summary of your results. It is usually combined with the analysis section, although they may be kept separate.

Tables comparing the results of different experiments or secondary sources are useful, but avoid including long tables of raw data in your report. Wherever possible, use a graph instead of a table. If you need to include a lot of raw data, then put it in an appendix attached to the end of the report.

Think about what sort of graph is appropriate. If you want to show a relationship between two variables, then use a scatter plot. Display your data as points with uncertainty bars and clearly label any lines you have fitted to the data. Column and bar charts are used for comparing different data sets. *Do not* use a column or bar chart to try to show a mathematical relationship between variables. Examples of these two types of graphs are shown in Figure 1.18.

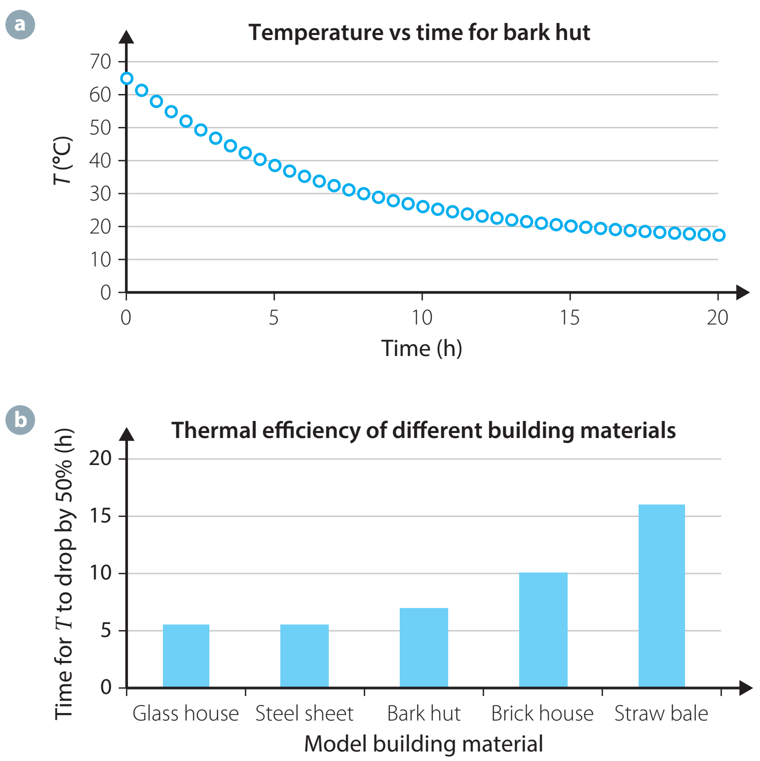


FIGURE 1.18
a A scatter plot demonstrating a mathematical relationship; **b** A column graph comparing results from different experiments

Any data and derived results should be given in correct SI units with their uncertainties. If you performed calculations, then show the equations you used. You might want to show one example calculation, but do not show more than one if the procedure used is repeated.

The discussion should summarise what your results mean. If you began with a research question, give the answer to the question here. If you began with a hypothesis, state whether or not your results supported your hypothesis. If not, explain why. If your investigation led you to more questions (as is often the case), say what further work could be done to answer those questions.

The conclusion is a *very* brief summary of the results and their implications. Say what you found out and what it means. A conclusion should only be a few sentences long.

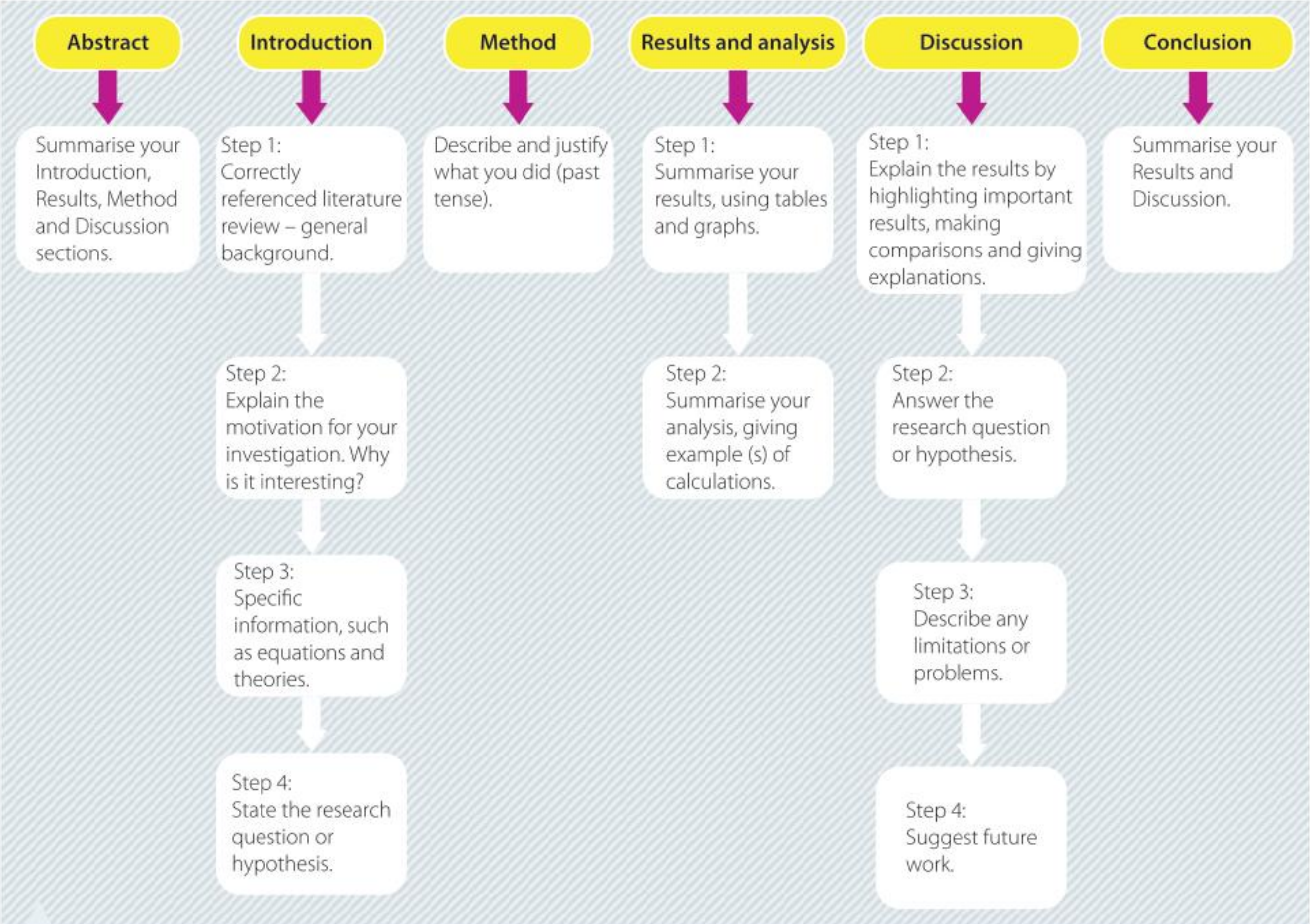
Scientific reports often include acknowledgements thanking people and organisations that helped with the investigation. This includes people who supplied equipment or funding, as well as people who gave you good ideas or helped with the analysis. In science, as in other aspects of your life, it is always polite to say thank you.

Referencing guide
This guide is designed to help you with referencing your sources.

The final section of a report is the reference list. It details the sources of all information that were actually used to write the report. This will generally be longer for secondary-sourced investigations. Wherever a piece of information or quotation is used in your report, it must be referenced *at that point*. This is typically done either by placing a number in brackets at the point; for example, [2], or the author and year of publication; for example, (Smith, 2014). The reference list is then either provided in a footnote at the end of each page, or a single complete list at the end of the report. There are different formats for referencing, so check with your teacher what format is preferred. There are several online guides to referencing.

Guide to referencing in different styles
This tutorial will help you understand referencing and show you how to avoid plagiarism.

Note that a reference list is *not* the same as a bibliography. A bibliography is a list of sources that are useful to understanding the research. They may or may not have actually been used in writing the report. You should have a bibliography in your logbook from the planning stage of your investigation. The references will be a subset of these sources. A primary-sourced investigation does not include a bibliography. A secondary-sourced investigation may include a bibliography, as well as references, to demonstrate the scope of your literature search. For some secondary-sourced investigations, such as an annotated bibliography, the bibliography itself may be a major section of the report.



Courtesy R A Gayani Ranawake

FIGURE 1.19 The steps in each of the major sections of a formal report

Figure 1.19 is a flowchart showing the steps in each of the major sections of a formal report. It shows the steps as they appear in the final report to give a logical sequence for your reader, but typically you would not write your report in this order. For example, students often find it easiest to write the method section first and the abstract last.

Other ways of communicating your work

You may want to present the results of your investigation in some other way. Look at examples of science investigations reported in places such as websites, newspapers, television etc. This will give you an idea of the different styles used in the different media. Think about the purpose – is it to inform, to persuade, or both? What sort of language is used?

Think about your audience and purpose, and use appropriate language and style. A poster is not usually as formal as a report. A video or webpage may be more or less formal, depending on your audience.

Posters and websites use a lot of images. Images are usually more appealing than words and numbers, but they need to be relevant. Make sure they communicate the information you want them to.

Consider accessibility if you are creating a website. Fonts need to be large enough and digital images should have tags. You can follow the weblink for more information on accessibility and website design.

Website accessibility

The Royal Society for the Blind has information on making websites accessible.

If you plan to make a video, consider who your audience is and what will appeal to them. Think about how you will balance content with entertainment.

A formal report uses referencing to show where you found information. Other means of communicating about your depth study or investigation also need to acknowledge the sources of information you used. You also need to be very careful about using copyright content – for example, you cannot copy images from other people's websites without permission unless the site gives that permission. Talk to your teacher about how they would like you to acknowledge your sources.

However you communicate your work, make sure you know what the message is and who the audience is. Once you have established that, you will be able to let other people know about the interesting things you have discovered in your investigation.

Ideas for depth studies

As you progress through this course, you will see investigations in each chapter. Some of these investigations are described in detail. These detailed investigations are designed to be useful as training exercises in learning how to perform primary investigations – how to set up equipment, make measurements and analyse data. Even if your depth study is secondary-sourced, it is important to gain some experience of conducting investigations as physics is an empirical science.

At the end of each end-of-module review there is a short section called depth study suggestions. Here you will find ideas for primary- and secondary-sourced investigations, which build on the content of the preceding chapters. These suggestions are sourced from experienced teachers and university academics, and the physics education literature. Your teacher will also have ideas and suggestions, and you can generate your own ideas by reading about topics you are interested in.

Consider what skills from other areas you might bring to a depth study, particularly if you are artistic or musical, or good at making things. Many physicists combine their love of science with other creative pursuits.

By carrying out depth studies, not only will you extend your knowledge and understanding in physics, but more importantly you will also learn how to work scientifically – you will learn how to 'do' physics.

KEY CONCEPTS

- A formal report has the same form as an article written by a scientist. It begins with an abstract briefly summarising the entire work. It includes an introduction section (with a literature review), and method, results and analysis, discussion and conclusion sections.
- All sources need to be referenced correctly.
- There are many ways of communicating your findings. Choose a method that is appropriate to your investigation and your intended audience.

KINEMATICS

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2 Motion in a straight line

INQUIRY QUESTION

How is the motion of an object moving in a straight line described and predicted?

OUTCOMES

Students:

- describe uniform straight-line (rectilinear) motion and uniformly accelerated motion through:
  - qualitative descriptions
  - the use of scalar and vector quantities (ACSPH060)
- conduct a practical investigation to gather data to facilitate the analysis of instantaneous and average velocity through: ICT
  - quantitative, first-hand measurements
  - the graphical representation and interpretation of data (ACSPH061) N
- calculate the relative velocity of two objects moving along the same line using vector analysis
- conduct practical investigations, selecting from a range of technologies, to record and analyse the motion of objects in a variety of situations in one dimension in order to measure or calculate: ICT N
  - time
  - distance
  - displacement
  - speed
  - velocity
  - acceleration
- use mathematical modelling and graphs, selected from a range of technologies, to analyse and derive relationships between time, distance, displacement, speed, velocity and acceleration in rectilinear motion, including:
  - $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
  - $\vec{v} = \vec{u} + \vec{a}t$
  - $\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$ (ACSPH061) ICT N

iStock.com/Cebas

It was once thought that moving objects would slow down and stop when they got tired – their motion was something that was natural to them. Aristotle (384–322 BCE) believed that heavier objects naturally fell faster than light objects. He said an 'efficient cause' made an arrow travel in a straight line. It moved off a straight line by reason of violent force. Aristotle developed a sophisticated view about motion that was used extensively for nearly 2000 years. However, not every observation could be explained using these ideas. As a result, Aristotle's ideas came under scrutiny. Galileo and Newton changed the way motion is explained by using the results of many observations and investigations to develop new ideas about motion. These ideas are still used today to explain all but the most unusual forms of motion. Einstein's theories are required to explain the behaviour of objects and particles that are moving at speeds close to the speed of light. These theories and laws are models for the behaviour of the motion of objects, which are themselves modelled as point masses. We use graphs and equations as equivalent and complementary representations of the motion of these model particles.

WS From Aristotle to Galileo to Newton


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FIGURE 2.1 Relative motion on a railway platform

2.1 Motion along a straight line

Motion can occur in straight lines, on a surface (two dimensions) or in three dimensions. The simplest type of motion of an object is along a straight line – forward or backward, to the left or right, or up or down. When we describe motion, we choose a sensible **frame of reference**. For example, stationary Earth is a sensible frame of reference for an aeroplane; for Venus, the Sun can be considered to be stationary. The chosen frame of reference can be used even if it is in motion, such as Earth moving around the Sun at about 30 km s^{-1}.

A car has many moving parts. However, in our studies we will model a car or other object as having its entire mass concentrated at one point – its **centre of mass**. This will simplify calculations about the motion of the car, even though a real car is not a single point. Such models are often used in physics.

Simplifications of real situations make it possible to analyse the motion of an object with length, such as a car, even though it is obviously not a single point. More detailed analysis of the different parts of the car, such as that undertaken by car designers, might be required to understand car safety and how vehicles deform in crashes.

Distance and displacement

In order to describe the motion of an object, information about the position of the object and the time are needed. Position cannot change instantaneously. An object cannot move from one place to another without taking time to do so. It takes time for the position of anything to change.

The position of an object is measured as the distance away from some reference point, such as the starting place, or origin, O. In Figure 2.2, point A is a distance of 25 cm from O while point B is 40 cm from O. A position can either be to the right or to the left of the origin O in this example. A position to the right is given a positive value while a position to the left is negative. In a method similar to a number line in mathematics, point A is said to be +25 cm from O while point B is –40 cm from O.

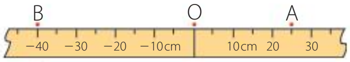


FIGURE 2.2 By defining a starting point or origin, opposite directions can be designated positive and negative.

Distance

Distance, *d*, is the actual length between two points. It has no direction. For example, the distance between points A and B in Figure 2.2 is simply stated as 65 cm.

Displacement

Displacement is the position of an object relative to the origin, or starting point. In straight-line motion, displacement must be given a positive or negative value to show which side of the origin the object is positioned.

In a marathon in which the finish line is the same as the starting line, a runner will have a final displacement of zero at the end of the race. The distance run will be 42.2 km, but the displacement measures how far you are from the start or origin. At times during the race, the runner's displacement may have been many kilometres, as this is the straight-line distance from the start to the runner. We will use *d* as the symbol for distance and $\vec{s}$ for displacement. Both displacement and distance have the same SI unit, the metre (m).

WORKED EXAMPLE 2.1

A snail starts at a position 20 cm from the origin, and then moves to a new position 40 cm further away before going back past the origin to a position 20 cm on the other side of the origin. It finally ends up at the origin. The positions are shown in Figure 2.3a.

Numeracy

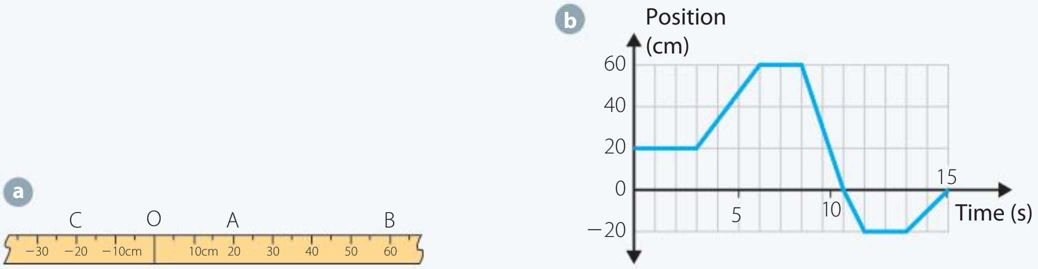


FIGURE 2.3 **a** A snail moves along a straight line from +20 cm to +60 cm, then to –20 cm, and then ends up at the origin, O. **b** Cartesian position–time graph of the motion of a snail along a straight line

The position–time graph of the motion is shown in Figure 2.3b.

1 What was the distance travelled by the snail's centre of mass on its trip?

2 What was the final displacement of the snail's centre of mass relative to the:

- **a** starting point?
- **b** origin?

ANSWERS	**LOGIC**
1 $d = 40\text{ cm} + (60\text{ cm} - (-20\text{ cm})) + (20\text{ cm} - 0\text{ cm})$ $= 40\text{ cm} + 80\text{ cm} + 20\text{ cm}$ $= 140\text{ cm}$	▪ Read data from graph correctly. ▪ Calculate the correct answer.
2 a $\vec{s} = d_f - d_i$ $= 0\text{ cm} - 20\text{ cm}$ $\vec{s} = -20\text{ cm}$ (i.e. 20 cm to the left of the starting point)	▪ Identify the appropriate formula. ▪ Substitute all known values into the formula. ▪ Calculate the correct answer.
b The snail is at the origin after the movement. This means its displacement is zero relative to the origin.	▪ Interpret the answer correctly.

TRY THESE YOURSELF

Tia lives on a long straight road 250 m from the shops and 400 m from her friend James, who lives in the opposite direction. Tia walks to the shops, and then goes to see James.

1 Before she goes out, what is Tia's position relative to:

- **a** the shops?
- **b** James?

2 After the walk:

- **a** what distance did Tia travel?
- **b** what was Tia's displacement relative to home?

A word about numbers

If we measure a quantity of something, such as marbles in a jar, the number just has a size (magnitude). An ordinary number such as this is called a **scalar**. Distance travelled is a scalar. We only care about how many metres were covered, not what direction we were going in.

With displacement, we care about the magnitude and we also care about the direction. On a number line, direction is positive or negative. Numbers that require both magnitude and direction are called **vectors**. We will represent vectors such as displacement by writing their pronumerals with a small arrow above them; for example, $\vec{s}$.

Time, speed and velocity

The position of an object is measured at a particular time, t. The SI unit for time measurements is the second, s.

Movement is the change in position as time changes. Any **time interval** can be shown as Δt, where:

$$\Delta t = t_2 - t_1 \text{ (Unit: s)}$$

Δt represents the amount of time that has passed from one measurement to the next. Δt is a single symbol. It is often replaced by t, but retains the meaning $\Delta t = t_2 - t_1$.

In everyday speech, the terms speed and velocity may be used to mean the same thing. In physics, speed relates to the distance covered in a time interval; velocity specifically relates to the change in displacement during a time interval.

The change in distance, called the **distance interval**, is given the symbol *s* (no arrow – it is not a vector).

$$s = d_2 - d_1 \text{ (Unit: m)}$$

TABLE 2.1 Scalars and vectors

SCALARS	VECTORS
Distance, s	Displacement, $\vec{s}$
Speed, v	Velocity, $\vec{v}$

Speed

Distance versus time graphs: designing a walk

Speed, *v*, is a measure of how fast something is travelling. It is the rate at which distance changes as time changes. Speed can be measured by observing the distance travelled and dividing by the observed time taken. Speed may be reported as instantaneous, such as the reading on the speedometer of a car, or it may be average, where a whole trip is measured. Whether it is instantaneous or average, speed is always measured by measuring distance intervals, and time intervals and performing the calculation:

$$v = \frac{s}{t}$$

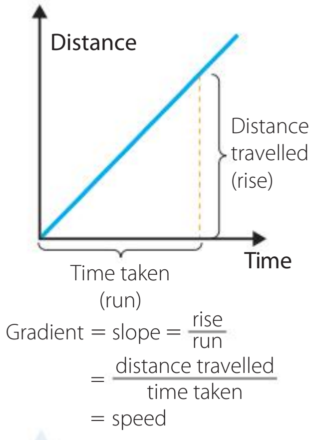


FIGURE 2.4 Speed is given by the gradient (rise/run) of the distance–time graph.

The SI unit of speed and velocity is the unit of $\frac{\text{distance travelled (m)}}{\text{time taken (s)}}$, that is, m/s or m s^{-1}.

In Figure 2.4, the gradient of the distance–time graph gives the speed. For constant speed, the gradient is the same at all points. The graph is a straight line. In this situation, the average speed would be the same as the **instantaneous speed** throughout the journey. If the speed were to vary, the gradient of the line would change too.

You cannot measure the speed of a car by taking a single photo. You can measure its speed over a very short time interval from a video of its motion, using the frame just before and just after the instant in time in question. Therefore, reference to instantaneous speed is really a reference to an average speed that is the same value as all the speeds in the time interval measured. This is why the time interval needs to be very small.

Velocity

The change in displacement, the **displacement interval**, is given the symbol $\vec{s}$. The arrow above the symbol is used to signify that this quantity has a direction associated with it.

Velocity is the rate of change of displacement as time changes:

$$\Delta\vec{v} = \frac{\vec{s}}{t} \text{ (Unit: m s}^{-1}\text{)}$$

Speed is the magnitude of the velocity. It can be found by dividing the straight-line difference between initial and final displacements by the time interval over which the change takes place. The direction of velocity is either positive or negative for straight-line motion. In general, the direction of the velocity is the same as the direction of the *change* of displacement.

INVESTIGATION 2.1

The speeds of common objects

We can make objects move at different speeds. Some first-class cricketers can bowl a ball at speeds approaching 45 m s^{-1}. The mechanical advantage conferred by a bat or racquet can increase or decrease ball speeds.

Numeracy

Information and communication technology capability

AIM

To measure the speeds of some human-propelled objects

MATERIALS

- Stopwatch
- Measuring tape
- Various bats, racquets and balls
- Optional: video camera or motion data-logger

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
A ball hit with a bat could hit a person or break a window.	Perform the experiment in an open space, such as a school oval, and keep bystanders well back.

RISK ASSESSMENT

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Measure out an appropriate length (e.g. 20 m) between two lines on the school oval or in a clear area.
2. By either throwing or hitting a ball with a bat or racquet, reproduce the actions of several different ball sports (e.g. cricket, tennis, hockey, golf) that propel a ball from one line past the other.
3. Measure the time it takes for the ball to travel the designated distance. For this, use a stopwatch or you may be able to video the motion and use the clock on the video. You might also have access to a motion data-logger that is able to measure speed directly.
4. Repeat step 3 for the same sport several times.
5. Repeat steps 3 and 4 for a different sport.

RESULTS

Record the results of your timing measurements for each sport in a table.

ANALYSIS OF RESULTS

1. Find the average speed of the ball for each sport. Include an estimate of the uncertainty in each value.
2. Convert the results from m s^{-1} to km h^{-1}.

DISCUSSION

Discuss the difficulties encountered during this experiment and suggest ways in which the data collection could be made to be more accurate.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

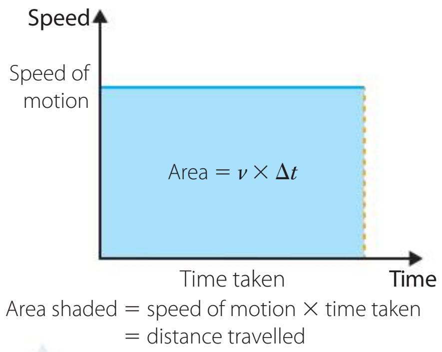


FIGURE 2.5 A speed versus time graph of an object moving at constant speed

Relationship between speed, distance and time

If you walk steadily taking two paces each second, you are walking at 2 paces per second (i.e. 2 paces s^{-1}). In 10 seconds, you would have moved 20 paces. Using metres as the unit of length allows any motion to be described in the same way anywhere. Calculations to find the distance moved can be made:

$$v = \frac{\Delta d}{\Delta t} = \frac{s}{t}$$

$$s = vt$$

The distance travelled can also be found by calculating the area under a speed–time graph. This shows that v times t, which equals s (the distance travelled), is the area under the line, as shown in Figure 2.5.

Motion when speed changes instantaneously

When an object changes speed in a very short amount of time, the change in speed is often regarded as being instantaneous. Truly instantaneous speed change is impossible for the same reasons that truly instantaneous distance change is impossible. In Worked example 2.2, we will consider that the runner changes her speed almost instantaneously (at 15 minutes in Figure 2.6) for practical purposes.

WORKED EXAMPLE 2.2

An athlete runs at 20 km h^{-1} for 15 minutes. She then gets a stitch and slows to 15 km h^{-1} for the next hour. The speed versus time graph is shown in Figure 2.6.

1 How far does the runner travel in the first 15 minutes?

2 How far does she run in total?

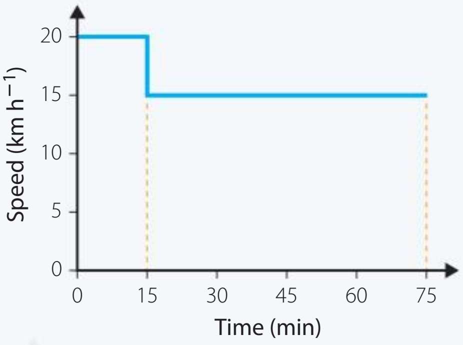


FIGURE 2.6

ANSWERS	LOGIC
1 Area = s_1 $= v_1 \Delta t_1$ $= 20\ \text{km h}^{-1} \times 0.25\ \text{h}$ $= 5\ \text{km}$	▪ Identify required area under the graph. ▪ Calculate the correct answer.
2 Area = $s_1 + s_2$ $= v_1 \Delta t_1 + v_2 \Delta t_2$ $= 5\ \text{km} + (15\ \text{km h}^{-1} \times 1\ \text{h})$ Area = 20 km	▪ Use area under the graph. ▪ Calculate the correct answer.

TRY THIS YOURSELF

If the runner continues at 15 km h^{-1} for another hour, how far will she have run in total?

Average speed, v_{avg}

When you are travelling through the city in a car, your speed changes all the time. If you have travelled 20 km in half an hour, you would say that your average speed, v_{avg}, was 40 km h^{-1} for that trip. It does not mean that you were always moving at 40 km h^{-1}; however, if you had been travelling at a constant 40 km h^{-1}, the same trip would have taken the same time. Average speed is the one single speed that would enable the car to cover the same distance in the same time interval:

$$v_{avg} = \frac{\Delta d}{\Delta t} = \frac{s}{t} \text{ (Units: m s}^{-1}\text{)}$$

WORKED EXAMPLE 2.3

What was the average speed of the athlete in worked example 2.2?

ANSWER	LOGIC
$v_{avg} = \frac{\Delta d}{\Delta t} = \frac{s}{t}$ $s = 20$ km; $\Delta t = 1.25$ hours	- Use the correct formula. - The average speed is found by dividing the total distance travelled by the total time interval taken for the entire event.
$v_{avg} = \frac{20}{1.25} = 16 \text{ km h}^{-1}$	- Substitute the correct values to find the correct answer and units.

TRY THIS YOURSELF

A car trip involves travelling at 60 km h^{-1} for 1 hour, and then at 100 km h^{-1} for the next 30 minutes. Find the average speed of the car for the entire trip.

Instantaneous speed, v_{inst}

Glancing down at the speedometer of a car will give information about the vehicle's speed at that moment. This is the car's instantaneous speed, v_{inst}. When observing an object in motion, it is often very difficult to measure its instantaneous speed. To find a speed, we need to measure the distance and the time intervals. This means that every measurement of speed is really a measurement of some average speed. In reality, all measurements are averages. However, if the time interval is very small, the average speed becomes very close to the instantaneous speed; $v_{inst} \approx v_{avg}$.

If the time interval is made smaller and smaller so that it approaches zero, the value of the v_{avg} being measured approaches the gradient of the *d* versus *t* graph.

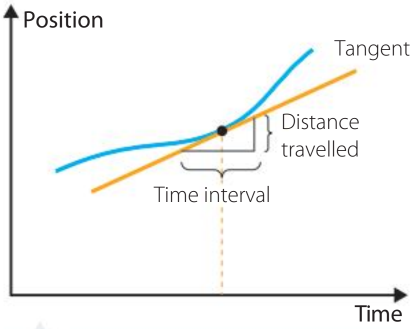


FIGURE 2.7 Instantaneous speed, v_{inst}, is the value of the gradient of the tangent to the position–time graph.

WORKED EXAMPLE 2.4

Refer to the speed versus time graph for an object shown in Figure 2.8.

1 Find the distance travelled:
   a in the first 10.0 s.
   b between 10.0 s and 30.0 s.
   c between 30.0 s and 35.0 s.

2 Find the average speed of the object for the trip.

3 Find the distance travelled in 15 s.

4 How long does the object take to travel 70 cm?

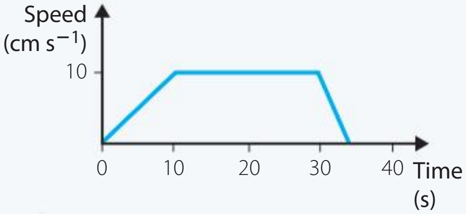


FIGURE 2.8

ANSWERS	LOGIC
1 a Distance is the area under the graph. $=\frac{1}{2}\times 10\text{ cm s}^{-1}\times 10\text{ s}$ $= 50\text{ cm}$	▪ Identify how to determine distance from the graph. ▪ Identify the formula required to calculate area under the graph. ▪ Substitute known values into the formula and calculate the area.
b Distance $= 10\text{ cm s}^{-1}\times 20\text{ s}$ $= 200\text{ cm}$	▪ Substitute known values into the formula and calculate the area.
c Distance $=\frac{1}{2}\times 10\text{ cm s}^{-1}\times 5\text{ s}$ $= 25\text{ cm}$	▪ Substitute known values into the formula and calculate the area.
2 Average speed $=\frac{\text{total distance}}{\text{time interval}}$ $=\frac{275\text{ cm}}{35\text{ s}}$ $= 7.9\text{ cm s}^{-1}$	▪ Identify the formula required to calculate the gradient. ▪ Substitute known values into the formula and calculate the gradient.
3 Distance $=\frac{1}{2}\times 10\text{ cm s}^{-1}\times 10\text{ s} + (10\text{ cm s}^{-1}\times 5\text{ s})$ $= 100\text{ cm}$	▪ Identify the formula required to calculate the distance. ▪ Substitute known values into the formula and calculate the distance.
4 Somewhere between 10 s and 15 s, the object has travelled 70 cm. Let t be the time at which the object has travelled 70 cm. The area under the graph after 10 s is $(t - 10\text{ s})\times v$ and $s = 20$ cm. $s = (t - 10\text{ s})\times v$ $t - 10\text{ s} = \frac{s}{v}$ $t = \frac{s}{v} + 10\text{ s}$ $= \frac{20\text{ cm}}{10\text{ cm s}^{-1}} + 10\text{ s}$ $= 12\text{ s}$	▪ Identify relevant information from the question and determine the appropriate formula. ▪ Substitute known values into the formula and calculate the answer.

TRY THESE YOURSELF

At the school sports, Elle runs the 100 m event in 17.25 s (see Figure 2.9).

1 Calculate the distance covered:

  a in the first 6.0 s.

  b from 6.0 s to 8.0 s.

2 Calculate the average speed in the first 8.0 s.

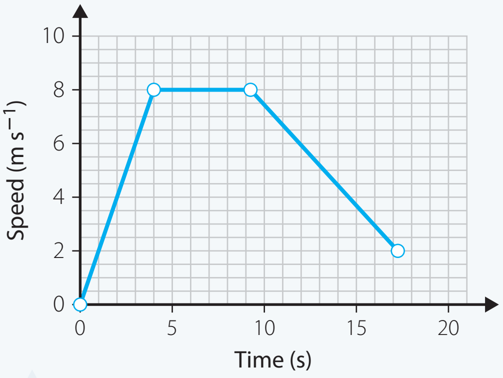


FIGURE 2.9

Relative velocity

Suppose you are standing beside a road, and a vehicle is approaching you from the left at 20 m s^{-1}. Looking right, another vehicle is approaching at 20 m s^{-1}. From your frame of reference, modelled as the origin on a number line, both vehicles have a speed of 20 m s^{-1}. However, from the point of view of either of the drivers, the other vehicle is approaching at 40 m s^{-1}.

Relative velocity depends on the frame of reference. It can be a very useful concept when considering the consequences of events such as collisions. A crash at 40 m s^{-1} does considerably more damage than a crash at 20 m s^{-1}.

To analyse these linear situations, choose one of the moving objects and call it the observer, with $\vec{v}_o$ as its velocity. Let $\vec{v}_d$ be the velocity of the other object, which we will call the distant object.

The relative velocity is given by:

$$\vec{v}_o - \vec{v}_d$$

Table 2.2 gives the relative velocity of the observer (relative to the distant object), assuming $\vec{v}_o$ is moving in the positive direction.

TABLE 2.2 Relative velocities and their meaning

SITUATION	RESULT	INTERPRETATION
$\vec{v}_o < \vec{v}_d$ same direction	A negative number	$\vec{v}_o$ is moving in the negative direction with respect to $\vec{v}_d$
$\vec{v}_o > \vec{v}_d$ same direction	A positive number	$\vec{v}_o$ is moving in the positive direction with respect to $\vec{v}_d$
Opposite directions	A positive number	$\vec{v}_o$ is moving in the positive direction with respect to $\vec{v}_d$

Note that if the objects are travelling in opposite directions, $\vec{v}_d$ is necessarily moving in a negative direction with respect to $\vec{v}_o$, and so $\vec{v}_o - \vec{v}_d$ will necessarily yield a positive number. Whether they are closing or separating will depend on their initial positions, but the relative velocity will have the same magnitude.

WORKED EXAMPLE 2.5

A car is travelling along a road at 22 m s^{-1}. A truck is travelling in the same direction 100 m ahead at 18 m s^{-1}.

1 How long will it take for the car to overtake the truck?

2 How far will the car have travelled in that time?

ANSWERS	LOGIC
1 Relative velocity $= \vec{v}_{car} - \vec{v}_{truck}$ $= 22\text{ m s}^{-1} - 18\text{ m s}^{-1}$ $= 4\text{ m s}^{-1}$ $\vec{v} = \frac{\vec{s}}{t}$, so $t = \frac{\vec{s}}{\vec{v}}$ $= \frac{100\text{ m}}{4\text{ m s}^{-1}}$ $= 25\text{ s}$	▪ Identify the appropriate formula to determine the difference in speed. ▪ Substitute known values into the formula and calculate relative velocity. ▪ Identify the correct formula to determine the time taken and rearrange to find the unknown. ▪ Substitute known values into the formula and calculate the time taken.
2 Absolute distance travelled: $\vec{s} = \vec{v}t$ $= 22 \times 25$ $= 550\text{ m}$	▪ Identify the correct formula to determine the absolute distance. ▪ Substitute known values into the formula and calculate the answer.

TRY THESE YOURSELF

1 Two vehicles are approaching each other on opposite sides of a motorway. The first is moving at 100 km h^{-1}, while the other is travelling at 85 km h^{-1}. If their initial separation is 2.0 km, how long will it take them to pass each other?

2 A dog is chasing a postman. The dog sprints at 8.5 m s^{-1} and starts 10 m behind the postman. The postman sprints at 6 m s^{-1}. The dog will stop 50 m from its starting point.

  a Will the dog reach the postman?

  b How long will the pursuit take?

KEY CONCEPTS

- A frame of reference is a spatial coordinate system for observing physical phenomena that allows for an origin. It enables the measurement of quantities involved in changing position.
- The centre of mass is the average (mean) position of all matter in the system, weighted by mass.
- A scalar is a number that has only magnitude (size).
- Distance, d, is the actual length between two points. It has no direction and is therefore a scalar.
- A vector is a number that has both magnitude and direction.
- Displacement, $\vec{s}$, represents a change of position with respect to the starting point. It has both magnitude (the distance) and direction, so it is a vector.
- Movement is the change in position as time changes.
- Any time interval can be shown as Δt, where: $\Delta t = t_2 - t_1$ (Unit: s).
- Speed, v, relates to the distance covered in a time interval.
- Velocity, $\vec{v}$, specifically relates to the change in displacement during a time interval.

- Speed is the magnitude of the velocity. Velocity also includes direction.
- Change in distance, called the distance interval, is given the symbol s, where $s = d_2 - d_1$ (Unit: m).
- Speed is measured as distance travelled over time (Unit: s^{-1}).
- Average speed is the one single speed that would enable an object to cover a specified distance in a given time interval.
- Instantaneous speed is the rate at which distance is covered over a time interval that is so brief as to be negligible.
- For constant speed, the gradient on a distance–time graph is the same at all points. The graph is a straight line.
- The area under the curve on a speed–time graph shows the distance travelled.
- A graph of v versus t shows that the area under the line equals s, which is the distance travelled.
- Relative velocity depends on the frame of reference.
- Relative velocity is given by: $\vec{v}_o - \vec{v}_d$.
- Using vector addition, the resultant vector from the point of view of one object is with respect to the other object, not the fixed external frame of reference.

CHECK YOUR UNDERSTANDING

2.1

1 What is the difference between:

  a distance and displacement?

  b speed and velocity?

  c average and instantaneous speed?

  d average and instantaneous velocity?

2 What does the value of the gradient of a distance versus time graph for an object represent?

3 Explain why instantaneous speed at a time during a journey can be quite different from the average speed for the whole journey.

4 For a speed versus time graph, show how to find the units of:

  a gradient.

  b area.

5 Jane averages 80 km h^{-1} for a 120 km journey. For the first 60 km, she averages 60 km h^{-1}. What must her average speed have been for the remainder of the journey?

6 A battery-operated car travels 6.0 m north in 2.4 s and then 6.0 m south in 1.8 s.

  a What is the displacement of the car?

  b What is the average velocity of the car?

  c What is the distance travelled by the car?

  d What is the average speed of the car?

7 A cyclist travels at 30 km h^{-1} for 0.5 hours, and then at 50 km h^{-1} for the next hour until the destination is reached.

  a How fast would a second cyclist, travelling at a constant speed, need to ride to arrive at the destination in the same time?

  b Explain why the answer to part **a** above is not simply 40 km h^{-1}.

8 The world record for the 100 m sprint is approximately 10 s. At this average speed, in what time would the record-holder run 1500 m?

9 The Blue Orchid and the Yellow Devil taxi services pick up passengers at an airport at midday to drive to the same destination. The driver of the Yellow Devil taxi averages 100 km h^{-1} for 4 h, while the driver of the Blue Orchid taxi travels more sedately for 3 h at an average speed of 80 km h^{-1}.

At what speed must the Blue Orchid taxi driver travel during the next hour so that the two taxis arrive at the same place at 4 p.m.?

2.2 Acceleration along a straight line

Kinematics: analysis of data

The speed of sprinters in a 100 m race increases very quickly in the first few seconds of the race. Once they cross the finish line, their speeds gradually decrease until they stop. In everyday language, this would be described as **acceleration** and **deceleration**. Both involve a change in speed over a time interval, so they are both examples of acceleration – the first is positive acceleration and the second is negative acceleration. When speed in the positive direction is increasing, it is positive acceleration. When the speed in the positive direction is decreasing, it is negative acceleration.

Acceleration can therefore be positive or negative. A car becoming faster in the negative (reverse) direction would be undergoing negative acceleration, or acceleration in the negative direction. Acceleration occurs over a time interval, and is defined as the change in velocity divided by the time interval:

Reaction times investigation

$$\vec{a}_{\text{avg}} = \frac{\Delta v\ (\text{m s}^{-1})}{\Delta t\ (\text{s})} = \frac{v}{t}\ (\text{Units: m s}^{-2})$$

Instantaneous acceleration can be found using the same method as instantaneous speed. As Δt approaches zero, the value of the acceleration is the value of the gradient of the velocity versus time graph at that time. For constant acceleration, $\vec{a}_{\text{avg}} = \vec{a}_{\text{inst}} = \vec{a}$.

As velocity is speed with direction, acceleration will be occurring when there is a change in the direction of motion, even if the speed is constant. This affects our analysis of projectile motion and circular motion. However, for the time being, we will concentrate only on motion in a straight line.

WORKED EXAMPLE 2.6

The speed of a car increases from 5.0 m s^{-1} to 15.0 m s^{-1} in 4.0 seconds. What was the car's average acceleration?

ANSWER	LOGIC
$v_f = 15.0\ \text{m s}^{-1}$; $v_i = 5.0\ \text{m s}^{-1}$; $\Delta t = 4.0\ \text{s}$	- Extract relevant data from the question.
$a_{\text{avg}} = \frac{\Delta v\ (\text{m s}^{-1})}{\Delta t\ (\text{s})}$	- Identify the appropriate formula.
$\frac{\Delta v}{\Delta t} = \frac{v_f - v_i\ \text{m s}^{-1}}{\Delta t\ \text{s}}$ $= \frac{15\ \text{m s}^{-1} - 5\ \text{m s}^{-1}}{4.0\ \text{s}}$	- Substitute the known values, with units, into the formula.
$= 2.5\ \text{m s}^{-2}$	- Calculate the correct answer, and express it with correct significant figures and units.

TRY THESE YOURSELF

Find the average acceleration, in m s^{-2}, for a car when its speed:

1 increases from 16 m s^{-1} to 40 m s^{-1} in 3.0 s.

2 decreases from 75 km h^{-1} to 40 km h^{-1} in 5.0 s.

The area under an acceleration versus time graph has the units of speed, m s^{-1}. Finding the change in the speed between two values of time will involve finding the rectangular area under the acceleration versus time graph. This applies to objects with constant acceleration, as shown in Figure 2.10a.

For non-constant acceleration, the change in speed is still found using the area under the acceleration versus time graph, as shown in Figure 2.10b.

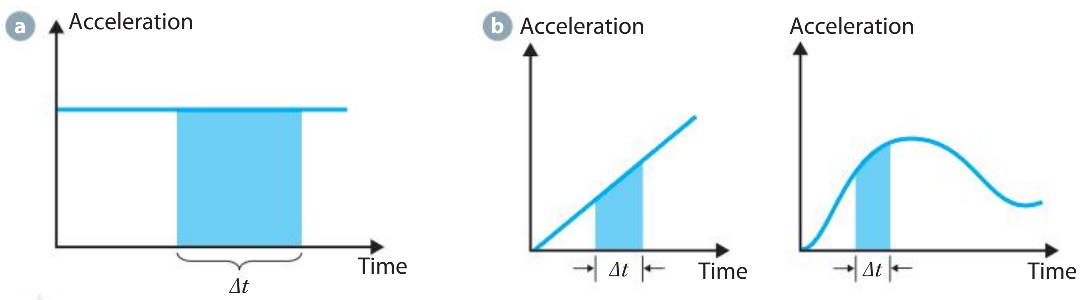


FIGURE 2.10 **a** The area under the acceleration–time graph gives the change in speed. **b** The area under the acceleration–time graph is the change of speed, even for non-constant accelerations.

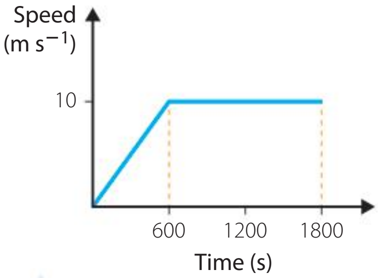
WORKED EXAMPLE 2.7

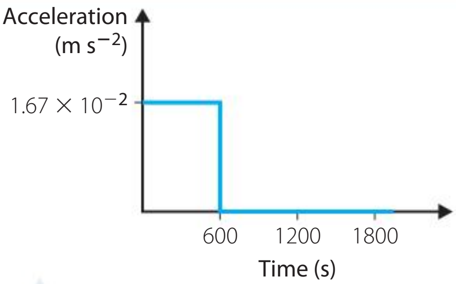
A cruise ship accelerates at a constant rate for 10.0 minutes until it reaches a speed of 10 m s^{-1}. It then continues to travel in a straight line for 20.0 minutes at 10 m s^{-1}.

1 Sketch a speed (m s^{-1}) versus time (s) graph for the ship for the 30 minutes. Note: convert minutes to seconds so that the time axis goes from 0 to 1800 s.

2 What was the ship's acceleration, in m s^{-2}, for the first 10.0 minutes?

3 Sketch an acceleration versus time graph for the ship for the 30 minutes.

ANSWERS	**LOGIC**
1  **FIGURE 2.11**	▪ Draw the axes and label correctly. ▪ Plot the line using the information provided.
2 Acceleration (m s^{-2}) $a = \frac{\Delta v}{\Delta t}$ $= \frac{10 \text{ m s}^{-1} - 0 \text{ m s}^{-1}}{10 \text{ min} \times 60 \text{ s per min}} \text{ m s}^{-1}$ $= 1.67 \times 10^{-2} \text{ m s}^{-2}$	▪ Identify the appropriate formula. ▪ Substitute the known values into the formula, calculate the answer and express it with correct significant figures and units

ANSWERS	LOGIC
3  **FIGURE 2.12**	- Draw the axes and label correctly. - Plot the line using the information provided.

TRY THESE YOURSELF

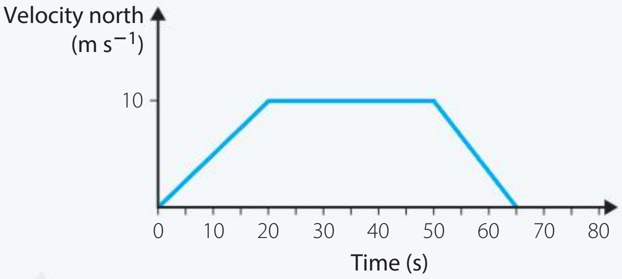


FIGURE 2.13

The velocity versus time graph for a bus travelling along a straight road is shown in Figure 2.13.

1. What distance is travelled by the bus while it is accelerating in a positive direction?
2. What is the distance between the two bus stops?
3. What is the acceleration of the bus in the first 10 s of motion?

WORKED EXAMPLE 2.8

Figure 2.14 shows the velocity–time graph for an object as it moves along a straight line.

1. Find the displacement after 20 s.
2. Find the acceleration at 17.0 s.
3. What is the acceleration at $t = 35.0$ s?

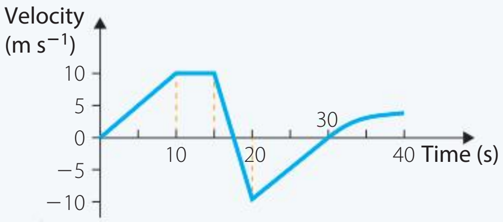


FIGURE 2.14

ANSWERS	LOGIC
1 Displacement = area under graph $\vec{s} = \frac{1}{2}\left(10\text{ m s}^{-1} \times 10\text{ s}\right) + \left(10\text{ m s}^{-1} \times 5\text{ s}\right)$ $+\frac{1}{2}\left(10\text{ m s}^{-1} \times 2.5\text{ s}\right) - \frac{1}{2}\left(10\text{ m s}^{-1} \times 2.5\text{ s}\right)$ $= 50\text{ m} + 50\text{ m} + 12.5\text{ m} - 12.5\text{ m}$ $= 100\text{ m}$	■ Identify the relevant information for the required area, determine the appropriate formula and substitute the known values. ■ Calculate the correct answer.
2 Acceleration $= \dfrac{\text{change in velocity}}{\text{time}}$ $\vec{a} = \dfrac{\left(-10\text{ m s}^{-1}\right) - \left(+10\text{ m s}^{-1}\right)}{20\text{ s} - 15\text{ s}}$ $= -4.0\text{ m s}^{-2}$	■ Identify the correct formula to determine the acceleration. ■ Substitute the known values and calculate the correct answer.
3 Finding the gradient of the tangent to the graph at $t = 35.0$ s gives $\vec{a} = \dfrac{5\text{ m s}^{-1} - 0\text{ m s}^{-1}}{40\text{ s} - 25\text{ s}}$ $= \dfrac{5\text{ m s}^{-1}}{15\text{ s}}$ $\vec{a} = 0.3\text{ m s}^{-2}$	■ Identify the relevant data from the graph and the appropriate formula to determine the average acceleration over the segment, then substitute the known values into the formula. ■ Calculate the correct answer.

TRY THIS YOURSELF

A motorbike, initially at rest, accelerates constantly at 5 m s^{-2} until it reaches a speed of 20 m s^{-1}. Sketch a speed versus time graph for the period before it reaches 20 m s^{-1}. Clearly show the time when this final speed is reached.

KEY CONCEPTS

- Acceleration is defined as the change in velocity divided by the time interval.
- Acceleration in a straight line can be positive or negative, depending on whether the object is speeding up or slowing down.
- On a speed–time graph, the average acceleration is the gradient of the line drawn covering the time interval Δt.
- The instantaneous acceleration is found as the time interval becomes small enough to be negligible.

CHECK YOUR UNDERSTANDING

2.2

1 For the motion of the two cars, A and B, in Figure 2.15, explain how it can be deduced that they never travel at the same speed.

2 The velocity versus time graph for the motion of a sprinter is shown in Figure 2.16.

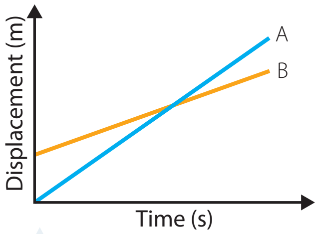


FIGURE 2.15

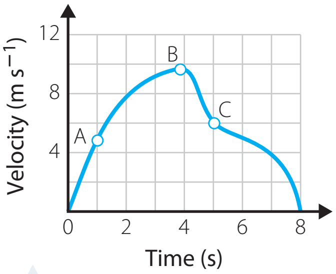


FIGURE 2.16

a From the graph, find the acceleration of the athlete at the following points.

  i A

  ii B

  iii C

b Estimate the distance travelled by the athlete in the first 4.0 s.

c At what time is the acceleration of the athlete the greatest?

d What is the maximum velocity reached by the athlete?

3 A car that is accelerating uniformly from rest travels 22.5 m in the eighth second of its motion.

a Calculate the acceleration of the car.

b Calculate how far the car travels in the first 6.0 s.

4 A car accelerates uniformly from rest to 25 m s^{-1} in 10 s.

a Sketch the graphs for:

  i displacement versus time.

  ii velocity versus time.

  iii acceleration versus time.

b From the velocity versus time graph, state the car's:

  i acceleration.

  ii speed at 5.0 s.

  iii displacement at 10 s.

5 A driver travelling at 75 km h^{-1} has a reaction time of 0.31 s. When a hazard occurs 50 m ahead, the driver slams on the brakes and comes to a stop just a few millimetres from the hazard.

Calculate and show the following on a carefully drawn sketch graph.

a how far the driver travels before the brakes are applied (m)

b how far the driver travels after the brakes are applied (m)

c braking time

d acceleration when braking

2.3

Equations for straight-line motion with constant acceleration

Graphs and algebraic formulae can both be used to represent motion. They are equivalent representations or models of motion. There is no reason why one representation of motion should be taken to be more important than another. For motion at constant acceleration, the speed versus time graph is a straight line. The gradient represents the acceleration and the area under the graph represents the distance covered. From the graph, a number of simple equations can be derived. These equations represent the same motion as the graph and are, therefore, exactly equivalent to the graphs.

So far we have observed that, for uniformly accelerated motion, $\vec{a} = \frac{\vec{v}}{t}$.

It follows then that $\vec{v} = \vec{a}t$.

This assumes that there is no initial, non-accelerated velocity. To account for this possibility, assume there is an initial velocity (which could still be zero). Let this initial velocity have the symbol $\vec{u}$. The final velocity will be the sum of this plus the uniformly accelerated portion, and we can write:

- Equation 1 $\vec{v} = \vec{u} + \vec{a}t$

If we plot this on a velocity–time graph, the distance travelled is given by the area under the curve (see Figure 2.17).

Speed is, by definition, the distance travelled divided by the time interval: $v = \frac{s}{t}$, which implies $s = vt$; that is, distance is speed multiplied by time. On a speed–time graph, the axes are respectively v and t, so the height times the base of the rectangle is $v \times t$, which represents distance for constant speed. Adding the uniformly accelerated portion, we see there is a triangle on top of the original rectangle, with an area given by $\frac{1}{2} v \times t$.

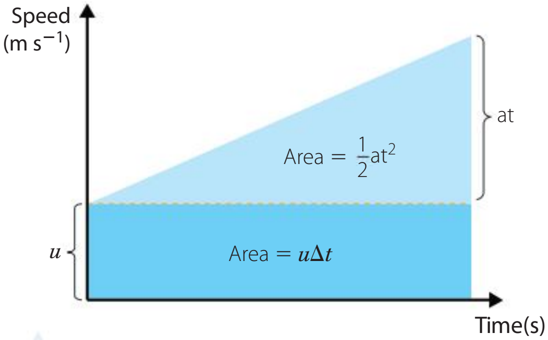


FIGURE 2.17

By definition, $\vec{a} = \frac{\vec{v}}{t}$, which implies $\vec{v} = \vec{a}t$. Therefore, the area of the triangle is:

$$\text{Area} = \frac{1}{2}\vec{v} \times t = \frac{1}{2}\vec{a}t \times t = \frac{1}{2}\vec{a}t^2$$

The total distance travelled is the sum of the rectangle plus the triangle:

- Equation 2 $\quad \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

Equation 2 can be used to algebraically find distance or displacement in uniformly accelerated situations, provided we know starting velocity, elapsed time and acceleration.

There is a third equation we can use if we do not know the time interval. By making t the subject of equation 1 and substituting into equation 2, it can be shown that:

- Equation 3 $\quad \vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$

This allows us to relate initial and final velocities with acceleration and displacement in a manner that is independent of time.

Each of these equations involves four variables. When solving problems algebraically, you will need to know the values of three of the five variables: v, u, a, t or s. The fourth can be found by simple substitution in the appropriate equation. It is then possible to use another equation to find the fifth variable.

Motion summary

Graphical analysis is often simpler and more obvious than algebraic analysis. Acceleration (gradient) and distance (area) can often be computed easily once a v versus t graph is sketched and relevant data points are identified.

Both methods yield the same answers because they are both models of the same motion.

WORKED EXAMPLE (2.9)

A car is travelling along a straight road at 20 m s^{-1}. It accelerates at a uniform 2 m s^{-2} for 10 seconds.

1 What is the car's final velocity?

2 What is the total distance travelled by the car during its acceleration phase?

ANSWERS	LOGIC
1 $\vec{u} = 20\ \text{m s}^{-1}$; $\vec{a} = 2\ \text{m s}^{-2}$; $t = 10\ \text{s}$	▪ Identify the relevant data from the question.
$\vec{v} = \vec{u} + \vec{a}t$	▪ Identify the appropriate formula.
$= 20\ \text{m s}^{-1} + 2\ \text{m s}^{-2} \times 10\ \text{s}$	▪ Substitute the known values, with units, into the formula.
$= 40\ \text{m s}^{-1}$	▪ Calculate the answer and express with correct significant figures and units.

ANSWERS	LOGIC
2 $\vec{u} = 20\ \text{m s}^{-1};\ \vec{a} = 2\ \text{m s}^{-2};\ t = 10\ \text{s}$ $s = \vec{u}t + \frac{1}{2}\vec{a}t^2$ $= 20\ \text{m s}^{-1} \times 10\ \text{s} + \frac{1}{2} \times 2\ \text{m s}^{-2} \times 10^2\ \text{s}^2$ $= 200\ \text{m} + 1\ \text{m s}^{-2} \times 100\ \text{s}^2$ $= 300\ \text{m}$	▪ Identify the relevant data from the question. ▪ Identify the appropriate formula. ▪ Substitute the known values, with units, into the formula. ▪ Calculate the answer and express with correct significant figures and units.

TRY THESE YOURSELF

1 A motorist driving at 80 km h^{-1} decelerates to 60 km h^{-1} in 5 seconds. What was the motorist's acceleration?

2 A lead ball is dropped 30 feet from a church tower. If the acceleration due to gravity is 9.8 m s^{-2}, how long does it take the ball to fall to the ground?

3 A bus accelerates from 40 km h^{-1} to 80 km h^{-1} at a rate of 2.4 m s^{-2}. How much distance did it cover in that time?

WORKED EXAMPLE 2.10

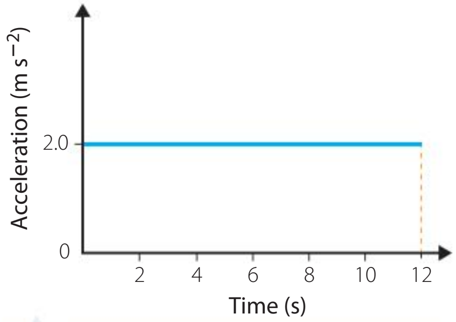
A car initially travelling at a speed of 4.0 m s^{-1} accelerates at 2.0 m s^{-2} for 12 s.

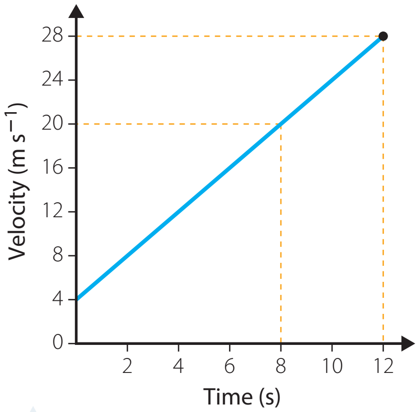
1 Sketch the acceleration versus time graph for the car.

2 Find the velocity, $\vec{v}$, of the car after 8.0 s.

3 Sketch the $\vec{v}$ versus t graph.

4 Find the distance moved by the car in the 8.0 s.

ANSWERS	LOGIC
1  **FIGURE 2.18**	▪ Correctly sketch graph and label points using the data provided in the question.
2 Change in velocity = area under the *a* versus *t* graph $\Delta\vec{v} = 2.0\ \text{m s}^{-2} \times 8.0\ \text{s}$ $= 16\ \text{m s}^{-1}$	▪ Identify velocity as the area under the curve. ▪ Calculate the car's initial velocity.

$\vec{v} = 4.0\ \text{m s}^{-1} + 16\ \text{m s}^{-1}$ $= 20\ \text{m s}^{-1}$	▪ Calculate the answer. ▪ Express the final answer with correct significant figures and units.
3  **FIGURE 2.19**	▪ Correctly sketch the graph using the data provided in the question.
4 Distance = area under the *v* versus *t* graph using the trapezium $s = \frac{1}{2}(4.0\ \text{m s}^{-1} + 20\ \text{m s}^{-1}) \times 8.0\ \text{s}$ $= 96\ \text{m}$	▪ Identify distance as the area under the curve. ▪ Substitute known values and correctly calculate the final answer.

TRY THESE YOURSELF

A train accelerates uniformly from rest to 8.0 m s^{-1} in 25 s. It then travels for 50 s at 8.0 m s^{-1} before coming uniformly to a stop in 12 s (see Figure 2.20).

1 What is the distance travelled by the train in the first 20 s?

2 What is the distance between the two stops?

3 What is the acceleration of the train in:

   a the first 10 s of the train's motion?

   b the last 10 s of the train's motion?

4 Sketch the acceleration versus time graph for the motion of the train.

5 After what time interval had the train travelled 300 m?

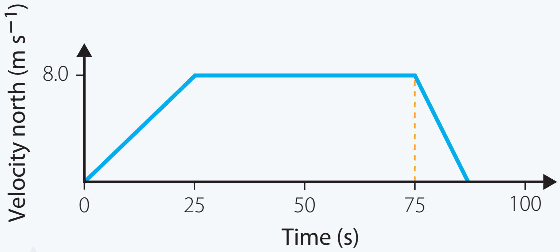


FIGURE 2.20

Linear acceleration under gravity

Once an object is released or thrown, the only force acting upon it is the gravitational force. Gravitational force is the force applied by one mass on another mass. Gravity is the common name given to the force applied by Earth's rather large mass of 6.0×10^{24} kg on ordinarily rather small masses on and near Earth. Near Earth's surface, gravity will cause a force on an object that results in that object accelerating vertically downwards with an acceleration of 9.8 m s^{-2}. This is known as gravitational acceleration, and is given the symbol *g*. Although the value of *g* varies slightly around the world, using a value of 9.8 m s^{-2} gives sufficiently accurate answers for our purposes.


Science Photo Library/NASA

FIGURE 2.21 Apollo 15 Commander David Scott dropped a feather and a hammer on the Moon in 1971.

Hammer versus feather

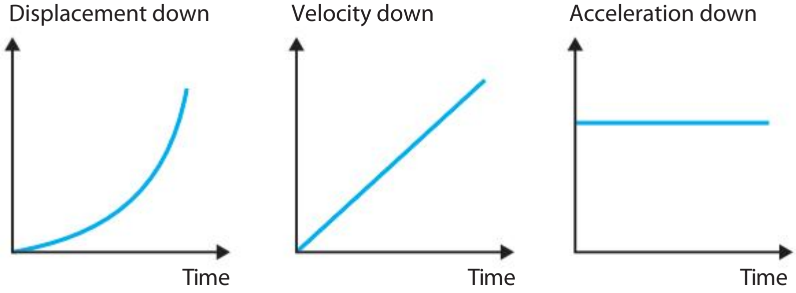
Why does a hammer fall faster than a feather when both are released together? Mostly this is because air resistance has a greater effect on lighter, fluffier objects than it does on denser, smoother objects. There is also a buoyancy force that is more significant for lighter objects. An experiment to show this was conducted by Apollo 15 astronaut Commander David Scott. He dropped a hammer and a feather on the Moon, which has no atmosphere. With no air resistance, the objects fell at the same rate.

For our purposes, we can ignore air resistance and buoyancy effects. The only significant acceleration for any projectile motion will be 9.8 m s^{-2} vertically downwards. Graphs of the motion of an object that is dropped are shown in Figure 2.22 for displacement, velocity and acceleration versus time. Notice that the vertical axis is for downwards values

Commander Dave Scott dropping a feather and a hammer

Use the information in the video to justify the theory that different masses fall at the same rate.

FIGURE 2.22 Graphs showing the motion of a falling object



The consequence of *g*

The gravitational acceleration value of 9.8 m s^{-2} means that anything dropped from a height will reach a speed of 9.8 m s^{-1} after the first second. This is about 35 km h^{-1}. After the next second, it will be falling with a speed of 9.8 + 9.8 = 19.6 m s^{-1}, or about 70 km h^{-1}. For every second it falls, the speed of the object will increase by 9.8 m s^{-1}. With no air friction, a brick would be falling at 98 m s^{-1} 10 seconds after being dropped. This is why we say 9.8 m s^{-2} is '9.8 metres per second per second'. It means that the speed is changing by 9.8 metres per second every second.

WS Revision

Objects falling directly downwards

When analysing the motion of a falling object, we can make the origin the point at which the object starts to move. If we let the downward direction be positive, variables *s*, *u*, *v*, *a* and *t* will all be positive, so that $a = g = +9.8$ m s^{-2}.

Worked example 2.11 uses the same equations of motion as used previously.

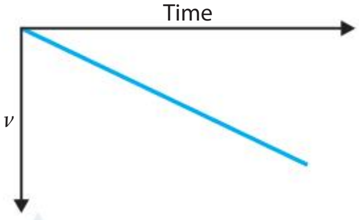
WORKED EXAMPLE 2.11

A watch falls from a Sydney Harbour Bridge climber's wrist. The watch falls for 2.5 s before hitting a car below.

1 Sketch a velocity versus time graph of this motion.

2 With what velocity does the watch hit the car?

3 How far did the watch fall?

ANSWERS	LOGIC
1  FIGURE 2.23	▪ Correctly sketch the graph using the data provided in the question.
2 $\vec{u} = 0;\ \vec{a} = -9.8\ \text{m s}^{-2};\ t = 2.5\ \text{s};\ \vec{v} = ?$ $\vec{v} = \vec{u} + \vec{a}t$ $\vec{v} = -9.8\ \text{m s}^{-1} \times 2.5\ \text{s}$ $= -24.5\ \text{m s}^{-1}$ vertically down	▪ Identify the relevant data from the question and identify the variable required. ▪ Identify the appropriate formula. ▪ Substitute the known values, with units, into the formula and calculate the answer, expressed with the correct significant figures, units and direction.
3 $\vec{u} = 0;\ \vec{a} = 9.8\ \text{m s}^{-2};\ t = 2.5\ \text{s};\ \vec{s} = ?$ $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$ $= 0 + \frac{1}{2} \times 9.8\ \text{m s}^{-2} \times (2.5\ \text{s})^2$ $= 31$ m down (correct to two significant figures)	▪ Identify the relevant data from the question and identify the variable required. ▪ Identify the appropriate formula. ▪ Substitute the known values, with units, into the formula and calculate the answer, expressed with the correct significant figures and units.

TRY THESE YOURSELF

A rock dropped from a cliff hits the ocean with a speed of 44.1 m s^{-1}.

1 Sketch the velocity versus time graph for this motion.

2 For how long did the rock fall?

3 How high is the cliff?

INVESTIGATION 2.2

Gravitational acceleration

For a falling object not affected significantly by air resistance, the value of the gravitational acceleration, g, can be found by collecting first-hand information.

Numeracy

Information and communication technology capability

AIM

a To find the value of the gravitational acceleration, g

b To investigate average and instantaneous velocities

MATERIALS

- Ruler
- Ball bearing
- Electronic timer or timing photogate

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU SAFELY MANAGE THESE RISKS?
The ball bearing may cause injury if thrown, dropped or stood on.	Never throw ball bearings. Manage the use of the ball bearing carefully. Never leave the ball bearing lying on the ground.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Set up the electronic timing apparatus.
2. Carefully measure the vertical distance, s, that the ball bearing will fall.
3. Using the timing apparatus, measure the time, t, taken for the ball bearing to fall through the known vertical height when released from rest from the upper position.
4. Repeat this several times.
5. Change the height of the fall and repeat the procedure.
6. Record sufficient data to plot a graph.

RESULTS

- Record all raw and derived data in a correctly constructed data table.
- Plot the data as it is collected.
- Estimate and record uncertainties in the data.

ANALYSIS OF RESULTS

1. Plot s versus t_{avg}, showing uncertainty bars.
2. Draw a line of best fit.
3. From the line of best fit, construct a data table of data points, (t_{avg}, s). Add an extra column for $(t_{avg})^2$ and a further column for v_{avg}.
4. Plot s versus $(t_{avg})^2$.
5. Draw a straight line of best fit and calculate the gradient.
6. Show that the equation $s = ut + \frac{1}{2}at^2$ can be used to find the acceleration from the gradient of the s versus $(t_{avg})^2$ graph.
7. Justify the best estimate of the value of the acceleration due to gravity, g, found in this experiment.
8. Use the least and greatest possible values of the gradient of the s versus $(t_{avg})^2$ graph to estimate the uncertainty in the experimental value of g. (Do not use the regression equation from your calculator!)
9. For each (s, t_{avg}) pair, calculate the average speed, v_{avg}.
10. Plot v_{avg} verus s.
11. Draw a line of best fit.
12. Describe the trend you observe in the data.
13. From the graph, interpolate the instantaneous velocities midway between the first and second, and second and third, data points.

DISCUSSION

1 Suggest ways in which this experiment could be made more accurate.

2 Evaluate the reliability of this procedure by analysing the variation in the separate measurements of time taken by the ball bearing before the average was found.

3 Suggest why a ball bearing was used rather than a tennis ball or other similar object.

4 Provide a precise value for g (best estimate with uncertainty limits).

5 Comment on any difficulties you had when undertaking this experiment. What changes could be made to overcome these difficulties?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aims of this investigation.

KEY CONCEPTS

- On a speed–time graph, the area under the line drawn covering the time interval Δt represents the distance travelled during that interval.
- A graph of a versus t shows that the area under the line equals the change in speed, Δv.
- For uniformly accelerated motion, $\vec{a} = \frac{\vec{v}}{t}$, implying $\vec{v} = \vec{a}t$.
- If we include an initial velocity $\vec{u}$, then $\vec{v} = \vec{u} + \vec{a}t$.
- If we know the initial velocity, acceleration and time of travel, then the distance covered is given by $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$.
- If we do not have the time interval, the relationship between initial velocity, final velocity, acceleration and distance travelled is given by $\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$.
- Graphically,

  $\vec{v} = \frac{\vec{s}}{t}$ = gradient of the $\vec{s}$ versus t graph

  $\vec{s}$ = area under the $\vec{v}$ versus t graph

  $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$ = gradient of the $\vec{v}$ versus t graph.

CHECK YOUR UNDERSTANDING

2.3

1 An object is dropped from a tower 128 m high.

  a What is the speed of the object that falls freely from rest for a distance of 128 m?

  b How long does it take for the object to reach the ground?

  c What is the speed of the object 2.0 s after being released?

  d When is the speed of the object 35 m s^{-1}?

2 Two objects, A and B, are released from a tower 125 m high. Object A is thrown downwards with an initial speed of 15.0 m^{-1}, while object B is allowed to fall from rest at the same instant.

  a Calculate the speed of each object on reaching the ground.

  b What is the difference in the time taken for the two objects to reach the ground?

  c How far apart are the two objects after 2.0 s?

3 A parachutist is falling vertically downwards with a constant speed of 4.8 m s^{-1}. When 120 m above the ground, the parachutist drops a small parcel. What is the time difference between the parcel and the parachutist reaching the ground? (Ignore air resistance on the parcel.)

4 Pelicans tuck in their wings to fall freely when diving for fish. A fish near the surface of the water needs 0.10 s to take evasive action. A pelican 25.0 m above the water starts its dive. The fish first notices the pelican when the pelican is 5.0 m above the water.

  Does the pelican go hungry, or does it catch its prey? Use graphs and calculations to support your answer.

2 CHAPTER SUMMARY

- A frame of reference is a spatial coordinate system for observing physical phenomena that allows for an origin. It enables the measurement of quantities involved in changing position.
- The centre of mass is the average (mean) position of all matter in the system, weighted by mass.
- A scalar is a number that has only magnitude (size).
- Distance, *d*, is the actual length between two points. It has no direction and is therefore a scalar.
- A vector is a number that has both magnitude and direction.
- Displacement, $\vec{s}$, represents a change of position with respect to the starting point. It has both magnitude (the distance) and direction, so it is a vector.
- Movement is the change in position as time changes.
- Any time interval can be shown as Δt, where $\Delta t = t_2 - t_1$ (Unit: s).
- Speed, *v*, relates to the distance covered in a time interval.
- Velocity, $\vec{v}$, specifically relates to the change in displacement during a time interval.
- Speed is the magnitude of the velocity. Velocity also includes direction.
- Change in distance, called the distance interval, is given the symbol *s*, where $s = d_2 - d_1$ (Unit: m).
- Speed is measured as distance travelled over time (Unit: m s^{-1}).
- Average speed is the one single speed that would enable an object to cover a specified distance in a given time interval.
- Instantaneous speed is the rate at which distance is covered over a time interval that is so brief as to be negligible.
- For constant speed, the gradient on a distance–time graph is the same at all points. The graph is a straight line.
- The area under the curve on a speed–time graph shows the distance travelled.
- A graph of *v* versus *t* shows that the area under the line equals *s*, which is the distance travelled.
- Relative velocity depends on the frame of reference.
- Relative velocity is given by $\vec{v}_o - \vec{v}_d$.
- Using vector addition, the resultant vector from the point of view of one object is with respect to the other object, not the fixed external frame of reference.
- Acceleration is defined as the change in velocity divided by the time interval.
- Acceleration in a straight line can be positive or negative, depending on whether the object is speeding up or slowing down.
- On a speed–time graph, the average acceleration is the gradient of the line drawn covering the time interval Δt.
- The instantaneous acceleration is found as the time interval becomes small enough to be negligible.
- On a speed–time graph, the area under the line drawn covering the time interval Δt represents the distance travelled during that interval.
- A graph of *a* versus *t* shows that the area under the line equals the change in speed, Δv.
- For uniformly accelerated motion, $\vec{a} = \frac{\vec{v}}{t}$, implying $\vec{v} = \vec{a}t$.
- If we include an initial velocity $\vec{u}$, then $\vec{v} = \vec{u} + \vec{a}t$.
- If we know the initial velocity, acceleration and time of travel, then the distance covered is given by $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$.
- If we do not have the time interval, the relationship between initial velocity, final velocity, acceleration and distance travelled is given by $\vec{v}^2 = \vec{u}^2 + 2\vec{a}\vec{s}$.

2 CHAPTER REVIEW QUESTIONS

Review quiz

1 Write down the symbols for acceleration, initial velocity, final velocity, time interval and displacement.

2 Describe the difference between:

a distance and displacement.

b speed and velocity.

3 What is the difference between instantaneous and average:

a speed?

b velocity?

c acceleration?

4 Draw vector diagrams to show change of:

a displacement.

b velocity.

5 For a velocity versus time graph, what quantity is found by calculating the:

a area under the graph?

b gradient?

6 Consider how the speed of an object that is dropped from a height changes over time.

a Describe this in words.

b Describe this on a *v* versus *t* graph.

7 Explain the positive horizontal line for the acceleration versus time graph shown in Figure 2.18 on page 48.

8 Explain why the position versus time graph in Figure 2.24 cannot be a completely true graph of an object's actual motion.

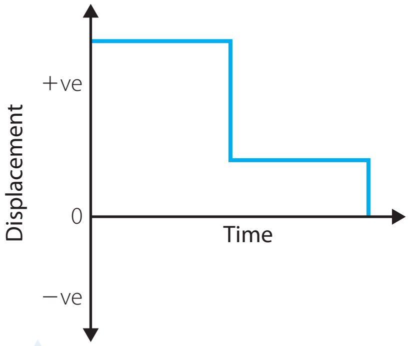


FIGURE 2.24

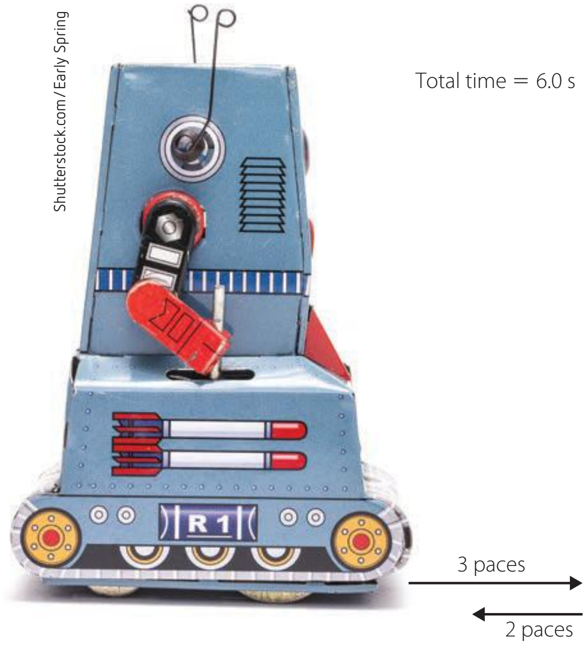
9 Show that the unit used for the area under a velocity versus time graph is the same as the unit of displacement.

10 In a 100 m sprint race, the winning time is 10.6 s.

a What was the winner's average speed?

b Do you think that the runner's average speed was the same as their instantaneous speed during the race? Explain your reasoning.

11 A robot takes three paces forwards and then two paces back, taking 6.0 s for this motion. Use calculations to explain why the robot's average speed is not the same as its average velocity.



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12 As a blue car moving at a constant 18 m s^{-1} passes a stationary red car, the red car begins to move in the same direction with a constant acceleration of 3.0 m s^{-2}.

a Show the motion of the two cars on a velocity versus time graph.

b From your graph in part a, find the time when the two cars are next to each other again.

c Check your answer to part b using appropriate equations of motion.

13 A builder drops a hammer from scaffolding 25 m above the footpath.

  a How fast will the hammer be moving just before it hits the ground?

  b How long will it take the hammer to fall?

14 Discuss why using the ground around us is a useful frame of reference for analysing motion, even though we know that Earth is rotating on its axis and revolving around the Sun.

15 Sketch a velocity versus time graph that shows the motion of a car that starts from rest and reaches a speed of 15 m s^{-1} after the first 5.0 s of motion. It then maintains this speed for 10 s before the brakes are applied and the car stops in 2.0 s. Add appropriate scales to the axes.

16 From a standing start, an XY Ford Falcon GTHO Phase 3 could cover a $\frac{1}{4}$ mile (400 metres) in 14.4 seconds. What is its acceleration, assuming it is constant?

17 The acceleration due to gravity on the Moon is 1.6 m s^{-2}, and the acceleration due to gravity on Earth is 9.8 m s^{-2}.

  a How long would it take the following items to fall from a 200 metre cliff on the Moon?

    i a hammer

    ii a feather

  b How long would it take the following items to fall from a 200 metre cliff on Earth?

    i a hammer

    ii a feather

18 A diver jumps from the 10-metre platform with a vertically upwards speed of 3.0 m s^{-1}, rising for 0.31 s before slowing to a stop. The diver then falls back down for 0.31 s and will be moving at 3 m s^{-1} by the time they are back level with the board.

  a Sketch the diver's velocity versus time graph.

  b Use the graph to find the:

    i distance travelled by the diver for the entire dive.

    ii diver's displacement.

    iii time the diver has to execute their manoeuvres before hitting the water.

3 Motion on a plane

INQUIRY QUESTION

How is the motion of an object that changes its direction of movement on a plane described?

OUTCOMES

Students:

- analyse vectors in one and two dimensions to:
  - resolve a vector into two perpendicular components
  - add two perpendicular vector components to obtain a single vector (ACSPH061) N
- represent the distance and displacement of objects moving on a horizontal plane using:
  - vector addition
  - resolution of components of vectors (ACSPH060) ICT N
- describe and analyse algebraically, graphically and with vector diagrams, the ways in which the motion of objects changes, including: ICT
  - velocity
  - displacement (ACSPH060, ACSPH061) N
- describe and analyse, using vector analysis, the relative positions and motions of one object relative to another object on a plane (ACSPH061)
- analyse the relative motion of objects in two dimensions in a variety of situations, for example:
  - a boat on a flowing river relative to the bank
  - two moving cars
  - an aeroplane in a crosswind relative to the ground (ACSPH060, ACSPH132) ICT N

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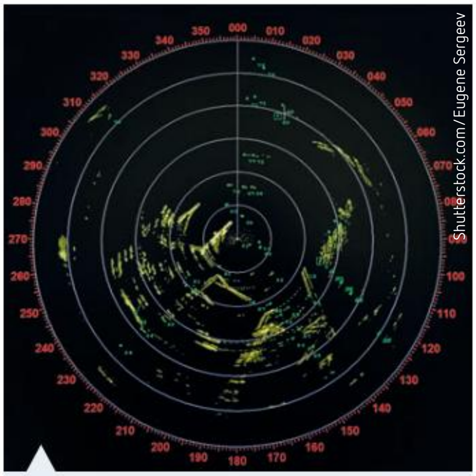

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FIGURE 3.1. A radar plot shows how far away something is *and* the direction it is in. Vectors are used to describe quantities with magnitude and direction.

The previous chapter explored motion in a straight line. It also introduced the idea of a vector, which is a quantity with a magnitude (a size or length) *and* a direction. This modern idea of a vector was developed independently by Oliver Heaviside (1850–1925) and J. Willard Gibbs (1839–1903) late in the 19th century. They also showed how to work with vectors mathematically. Gibbs in particular developed much of the notation we use today. Although the use of vectors may at times seem complicated and difficult, their invention was an improvement over earlier approaches. Today, vectors are used in most branches of physics, and are common in engineering, computer science and elsewhere.

Many quantities can be represented as vectors, where if $\vec{A}$ is a vector then A is its magnitude. Displacement, velocity and acceleration were presented in chapter 2. Later chapters will explore concepts such as force (chapters 4 and 5), momentum (chapter 6), electric field (chapter 12) and magnetic field (chapter 14). All these are vector quantities. In fact, vector mathematics was first invented to deal with electromagnetism. Because they are so widespread, being able to manipulate vectors is a useful skill.

Willard Gibbs
Find out what other contributions Gibbs made to physics and mathematics.

3.1 Analysing vectors in one and two dimensions

Some useful background

Quite often, motion is not restricted to a single direction. If a car is driven 100 km north and then 100 km west, it is easy to see that it has gone a total of 200 km. For one thing, the odometer will read 200 km more than it did at the start. But how far from the starting point is it? It might be 200 km by road, but how far is that 'as the crow flies'? Simple addition will not work. We need to add up the two legs of the journey *allowing for direction*. The way to do that is to treat the two legs as vectors. Vectors can be added to form new vectors, or subtracted to obtain the difference between two vectors. A single vector can be resolved into its **components**.

Vectors can also be multiplied (or divided) by a scalar. In chapter 2, a velocity was calculated from a displacement. One vector, $\vec{s}$, was divided by a scalar, t, to obtain another vector, $\vec{v}$. The vector $\vec{s}$ is the displacement, t is the time over which the displacement occurred and $\vec{v}$ is the average velocity during the displacement. The scalar, t, has units (seconds), which means that $\vec{v}$ has different units from $\vec{s}$. However, because t is a scalar, the division does not change the *direction* of the vector, so $\vec{v}$ is parallel to $\vec{s}$.

Resolving a vector into perpendicular components

It is possible to take a vector that lies in a two-dimensional plane and break it up into two perpendicular vectors. This is called resolving the vector into components.

When a vector is to be resolved into components, it is important to think about what coordinates should be used. If the motion is along the ground, such as the car, it may make sense to use the compass directions (north, south, east and west). Other motions, such as a ball being thrown, may take place in a vertical plane. In that case, the two directions would be horizontal and vertical. Perpendicular axes can be represented on paper as an x axis whose positive arm points to the right, and a y axis whose positive arm points up the page. This is known as the **x–y-plane**.

We try to choose axes that will help us solve the problem most easily. Chapters 4 and 5 look at objects sliding down slopes. In that case, we choose one component parallel to the slope and another perpendicular.

In every case, the two axes are perpendicular. There are some useful tools for working with vectors at right angles: Pythagoras' theorem and trigonometry.

Say we have a vector, $\vec{s}$. It can be expressed as the sum of two vectors, one pointing along x and one along y, as shown in Figure 3.2.

$$\vec{s} = \vec{s}_x + \vec{s}_y$$

The magnitudes of these vectors are related. (Remember, when we take the arrow off the top of a vector we are talking about magnitudes.)

$$s_x = s\cos\theta$$

$$s_y = s\sin\theta$$

where s is the magnitude of $\vec{s}$ and s_x and s_y are the magnitudes of the component vectors. $\vec{s}_x$ is sometimes referred to as the projection of $\vec{s}$ onto the x axis; similarly for $\vec{s}_y$.

Figure 3.2 shows that s_y is opposite the angle, so sine is used to find its length. The side s_x is adjacent to the angle, and so cosine is used. This may vary, depending on how the angles and sides are defined. In this case, θ is measured relative to the x axis and positive angles are anticlockwise. This is the convention for x and y coordinates. The vector $\vec{s}_x$ is a vector of length s_x in the x direction. In Figure 3.2, $\vec{s}_x$ points along the positive x direction. With θ defined as shown, the equation for s_x given above will give a positive answer.

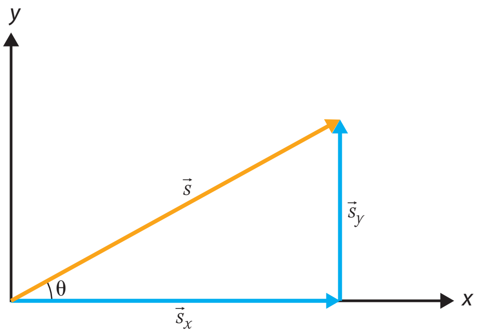


FIGURE 3.2 A vector, $\vec{s}$, and its components, $\vec{s}_x$ and $\vec{s}_y$. The angle is measured from the x axis. It is a positive angle because it is anticlockwise from the x axis.

It is always important to think about the signs of quantities. A vector of length −12 m in the +x direction is the same as one of +12 m in the −x direction.

Angles will not always be defined as anticlockwise from the x axis. In situations that use the compass points, angles *increase* as they go clockwise. An angle may be written like this: θ = N30°E. This means the angle is 30° east of north. An angle of N45°E is the same as north-east. Sometimes the important angles will be angles between vectors themselves. Because there is no one way to define an angle, it is important to draw a diagram of the situation. A diagram can help in working out which angles are relevant, which trigonometric functions to use, which quantities are positive and which negative.

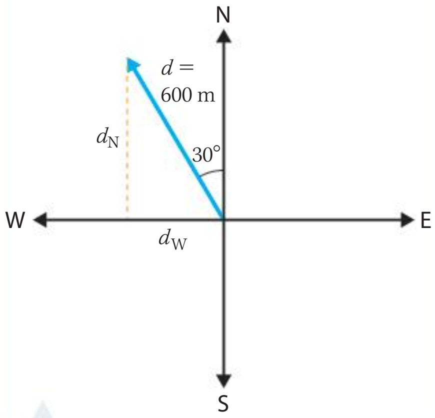
Once you have drawn a diagram, the components of a vector can be found using trigonometry or a careful scale drawing.

Vectors in two dimensions

Watch the video to help visual vectors in two dimensions.

WORKED EXAMPLE 3.1

An orienteer is at a position, *d*, which is 600 m N30°W from the origin.
Use trigonometry to find the magnitudes of the northward and westward components, d_N and d_W, of the orienteer's position. Take 600 m as having three significant figures.

ANSWER	**LOGIC**
 **FIGURE 3.3** The position of the orienteer, shown on a set of compass axes	- Draw a diagram showing the position as a vector.
$d_N = d\cos\theta$ $d_W = d\sin\theta$	- Use your diagram to work out which angle is sine, and which is cosine.
$d_N = (600\text{ m})\cos 30°$ $= 520\text{ m}$ $d_W = (600\text{ m})\sin 30°$ $= 300\text{ m}$	- Substitute the known values with units into the equations, calculate the answers and state them with correct units and appropriate significant figures.

TRY THIS YOURSELF

The orienteer now moves to a point 600 m N60°E from the origin. What are the north and east components of their position now? What are the north and west components of their position?

Vector components

Change a vector's magnitude and direction and see how the components vary.

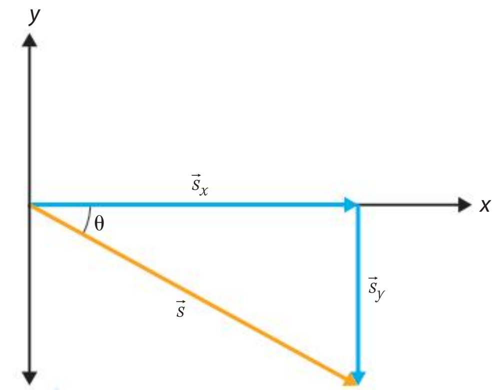


FIGURE 3.4 Adding components $\vec{s}_x$ and $\vec{s}_y$ to obtain the resultant vector, $\vec{s}$

Adding vector components

The reverse of resolving a vector into components is adding components to obtain a vector. Mathematically, $\vec{s} = \vec{s}_x + \vec{s}_y$. Assuming the components are perpendicular along *x* and *y*, then the length of $\vec{s}$, *s*, is obtained from Pythagoras' theorem,

$$s = \sqrt{s_x{}^2 + s_y{}^2}$$

where s_x and s_y are the magnitudes of the components.

However, $\vec{s}$ is a vector, so its direction must also be found. Figure 3.4 shows the component vectors, $\vec{s}_x$ and $\vec{s}_y$, as well as $\vec{s}$, and the angle $\vec{s}$ makes to the *x* axis, θ.

From Figure 3.4 it can be seen that

$$\tan\theta = \frac{s_y}{s_x}$$

So, by using Pythagoras' theorem and some trigonometry, it is possible to find the length and direction of the resultant vector.

WORKED EXAMPLE 3.2

Figure 3.5 represents the journey of a car. The journey begins at the bottom right-hand side of the picture, where the tails of the black and red arrows meet. The black arrows represent the actual path the car took. The red represents the resulting displacement away from the starting point. First the car drives 125 km north (up the page) then 125 km to the left (west).

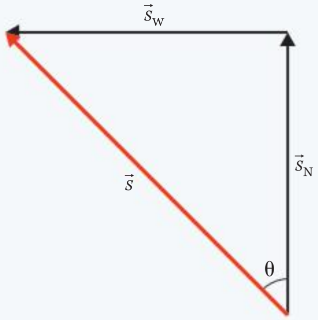


FIGURE 3.5 A car drives north and then west. The two perpendicular black vectors combine to give a third, the diagonal red vector. Conversely, it could be said that the black vectors are components of the red vector.

Find the magnitude and direction of the resultant vector, $\vec{s}$, in Figure 3.5.

ANSWER	LOGIC
$s_N = 125$ km; $s_W = 125$ km	- Identify the relevant data in the question.
The two paths are at 90° to each other.	- Recognise that the paths are perpendicular.
$s = \sqrt{{s_N}^2 + {s_W}^2}$ $= \sqrt{(125\text{ km})^2 + (125\text{ km})^2}$ $= 177$ km	- Apply Pythagoras' theorem to find the distance from the origin. - Substitute the known values with units into the equation. - Calculate the answer. State the final answer with correct units and appropriate significant figures.
That gives s, the length of the vector $\vec{s}$. Next, its direction must be established. $s_N = 125$ km; $s_W = 125$ km	- Identify the relevant data in the question.
$\tan\theta = \frac{s_W}{s_N}$ $\tan\theta = \frac{125\text{ km}}{125\text{ km}} = 1$ $\theta = 45°$ The total (or net) result is the same as if the car had driven 177 km north-west.	- Identify the appropriate formula. In this case s_W is opposite θ, s_N is adjacent. - Substitute the known values with units. - Calculate the answer. State the final answer with correct units and appropriate significant figures.

TRY THESE YOURSELF

1 Repeat the calculation, but with a northward component of 200 km.

2 Copy Figure 3.5 onto a sheet of graph paper at a scale of 1 cm = 10 km. Measure the resultant vector with a ruler. Does it agree with the calculation? Compare the precision of the methods.

Note that in Worked example 3.2, the hypotenuse was found in the first part of the question. It could have been used to find the angle. By using tan, we avoided relying on a quantity that we had to calculate ourselves. That removed one possible source of error from the angle. When working out the angle, the units (km) cancel out. This is a useful test, because it does not make sense to put a number with units into a trigonometric function. Lastly, a diagram is very useful when working out where the vector is actually pointing. The maths gives a 45° angle, but the diagram shows what that means.

INVESTIGATION 3.1

Displacement vectors

Critical and creative thinking

Numeracy

AIM

To investigate how displacement vectors can be decomposed into components, and to practise calculating experimental uncertainties

Write an appropriate inquiry question or hypothesis for this investigation.

MATERIALS

- 20-m tape measure
- Marker pegs
- Coin
- 2 dice
- Set square
- Open space, such as the school oval

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Excess sun exposure is dangerous.	Wear a hat, and any other appropriate sun protection.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Place a marker peg in the starting position.
2. Stand at the start position and throw the dice. Take as many steps forward in a straight line as the total shown on the dice. Take the dice with you.
3. Place a marker peg at your position.
4. Toss the coin. If it comes up heads, turn left 90°. If it comes up tails, turn right 90°. Use the set square to help you turn through the correct angle.
5. Throw the dice. Take as many steps forward in a straight line as the total shown on the dice. Place a marker peg at your final position.
6. The pegs should make a right-angled triangle, if you have been careful. Measure the distances (side lengths) between the pegs.
7. Repeat steps 2–6, either with the same person or different people.

RESULTS

- Record the distances measured and the number of steps each time.
- Record the uncertainty in each measurement.

»

ANALYSIS OF RESULTS

1 Draw a diagram for each set of results.

2 Based on the two perpendicular sides, calculate the length of the hypotenuse.
Compare the calculated length to the measured length.
The uncertainty in the calculated length can be found using the range method:

$$\Delta d = \frac{1}{2}(d_{\text{max}} - d_{\text{min}})$$

where

$$d_{\text{max}} = \sqrt{(x_{\text{max}})^2 + (y_{\text{max}})^2} \text{ and } d_{\text{min}} = \sqrt{(x_{\text{min}})^2 + (y_{\text{min}})^2}$$

x_{max} is the maximum possible value of the first distance walked allowing for the measurement uncertainty, x_{min} is the minimum possible value of the first distance walked allowing for the uncertainty. y_{max} and y_{min} are the corresponding values for the second distance walked.

3 Was there a consistent relationship between number of steps taken and distance travelled for an individual? What about between different people?

DISCUSSION

1 Did the calculated and measured values of the displacement agree, within the uncertainties? If not, can you explain why? Was there a source of uncertainty that you did not take into account?

2 Give the answer to your inquiry question or state whether your hypothesis was supported.

3 How could you improve or extend this experiment?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- A vector is a quantity that has both magnitude and direction.
- Vectors can be resolved into perpendicular components. These are aligned with the axes on a sketch or the cardinal directions on a compass.
- Trigonometry and Pythagoras' theorem are useful tools for resolving vectors into components and for combining components into vectors.
- If $\vec{s}$ is a vector in a plane, it is possible to write $\vec{s} = \vec{s}_x + \vec{s}_y$ where $\vec{s}_x$ is a vector parallel with the x axis and $\vec{s}_y$ is a vector parallel with the y axis.
- If s is the magnitude of $\vec{s}$, and s_x and s_y are the magnitudes of $\vec{s}_x$ and $\vec{s}_y$, then $s_x = s\cos\theta$, $s_y = s\sin\theta$, $s = \sqrt{{s_x}^2 + {s_y}^2}$ and $\tan\theta = \frac{s_y}{s_x}$.
- Compass directions (north, south, east and west) can be used as axes when resolving vectors.
- Vectors can be represented as arrows on scale diagrams. This is very useful in interpreting the results of calculations.

CHECK YOUR UNDERSTANDING

3.1

1 Describe the difference between a scalar and a vector.

2 Identify examples of scalars and vectors.

3 A rider on a horse rides 28 km N25°E. Calculate the perpendicular components of the journey – one component pointing east, and one pointing north.

4 Calculate the components of the journey in question 3, except now one component points north-east, and one points north-west.

5 For the journey in question 3, sketch the two different sets of components on a single drawing.

6 Gayani takes a piece of chalk and a tape measure, and goes onto the outdoor basketball court. She faces south and draws a straight line 3.5 m long. She turns east and draws another line, 4.5 m long. Calculate Gayani's net displacement, including direction.

7 Two children push a shopping trolley. Marcus pulls it 4 m along the aisle. Laurence pushes it sideways by 2 m.

- **a** On graph paper, accurately draw the two components to the same scale. Assume 'along the aisle' means up the page and 'across' means to the right. Use a ruler to obtain the magnitude, and a protractor to determine the angle relative to the aisle.
- **b** Calculate the net displacement of the trolley and the direction relative to the aisle. Compare with your result from part **a**.

3.2 Distance and displacement in a plane

Two important quantities are distance and displacement, as described in chapter 2. Displacement means how far the object is from where it started *and* in what direction. It may or may not have started on the origin of the coordinate system.

In two dimensions, it is important to remember that displacement is a vector. The displacement, $\vec{s}$, is the difference between initial and final positions, $\vec{d}_\mathrm{i}$ and $\vec{d}_\mathrm{f}$. Therefore, $\vec{s} = \vec{d}_\mathrm{f} - \vec{d}_\mathrm{i}$. The path used does not matter.

A displacement can begin anywhere, but a position is measured relative to the origin. It is an important difference.

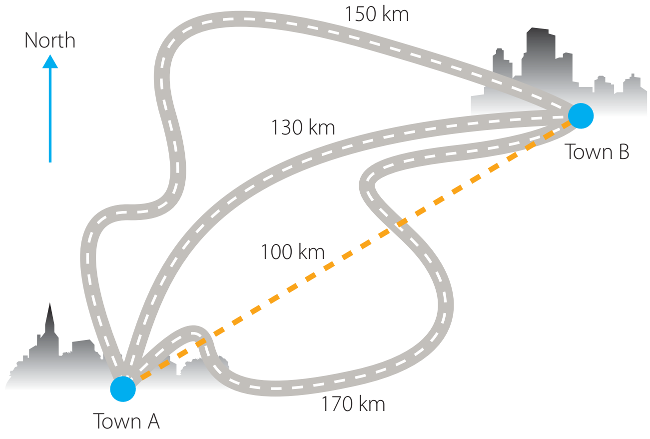


FIGURE 3.6 The road distances and displacement (orange dotted straight line) between town A and town B

Cars travelling from Town A to Town B in Figure 3.6 would, when they get to B, have the same displacements (100 km in an approximately north-easterly direction). But they would have travelled different distances. Someone who drove from A to B and back again might have driven a distance of over 300 km, but their displacement would be zero. When they were at B, their displacement was 100 km NE, but when they got home it was zero. So, the displacement depends on when the measurement is made.

Chapter 2 introduced the idea that displacement can depend on time. The constant acceleration kinematics equations, such as $s = ut + \frac{1}{2}at^2$, allow calculation of the magnitude of the displacement, s, as a function of time. This chapter examines displacements in a two-dimensional plane. We have to be aware of their vector nature as well as their time dependence.

Adding displacements using vectors

In section 3.1, two components were combined to make a single vector. It is a logical extension of that to put two vectors together to make a third one. In fact, the components of a vector are vectors themselves. They are just vectors that point along a convenient set of perpendicular axes. When adding the vectors together, we are effectively putting them head-to-tail and seeing where they point to. When using a graphical method like this, it is important that all the vectors are drawn to the same scale and the angles are correct.

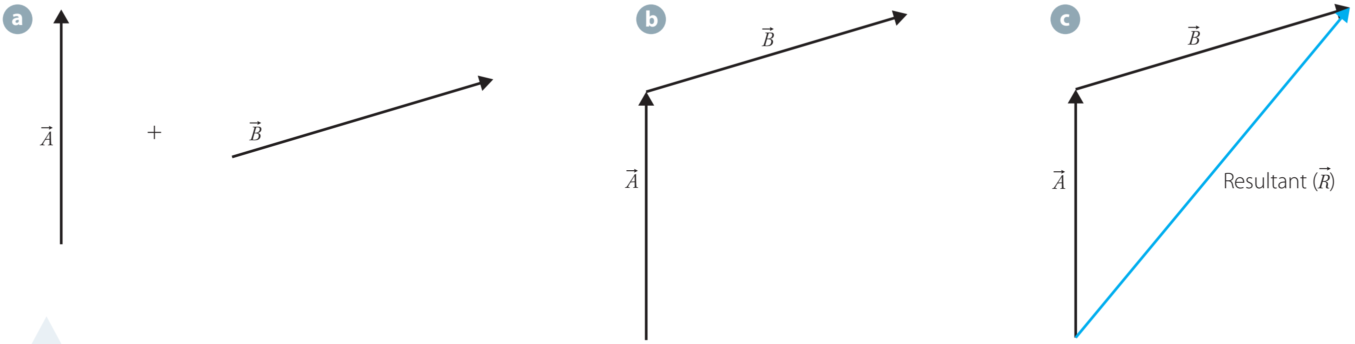


FIGURE 3.7 **a** Vector $\vec{A}$ and vector $\vec{B}$ are to be added. **b** Vector $\vec{B}$ is moved so that its tail is made to connect with the head of vector $\vec{A}$. **c** The resultant vector, $\vec{R}$, is found as the arrow running from the tail of $\vec{A}$ to the head of $\vec{B}$.

Figure 3.7 shows the process of adding two vectors to find the resultant vector. The same idea works for any number of vectors. If all vectors to be summed are drawn head-to-tail, then the resultant is found by drawing an arrow from the tail of the first vector to the tip of the last. The result of vector addition does not depend on whether $\vec{B}$ is added to $\vec{A}$, or $\vec{A}$ to $\vec{B}$. This is known as the parallelogram rule, and is illustrated in Figure 3.8.

Subtracting vectors can be thought of as adding the negative of one vector to another, because $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$. This is illustrated in Figure 3.9 (page 66). The negative of a vector is obtained by swapping the head and the tail. The parallelogram rule still works, as long as the negative of the second vector is taken *before* the construction is drawn. In mathematical terms,

$$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})=(-\vec{B})+\vec{A}$$

but

$$\vec{A}-\vec{B}\neq-\vec{A}+\vec{B}$$

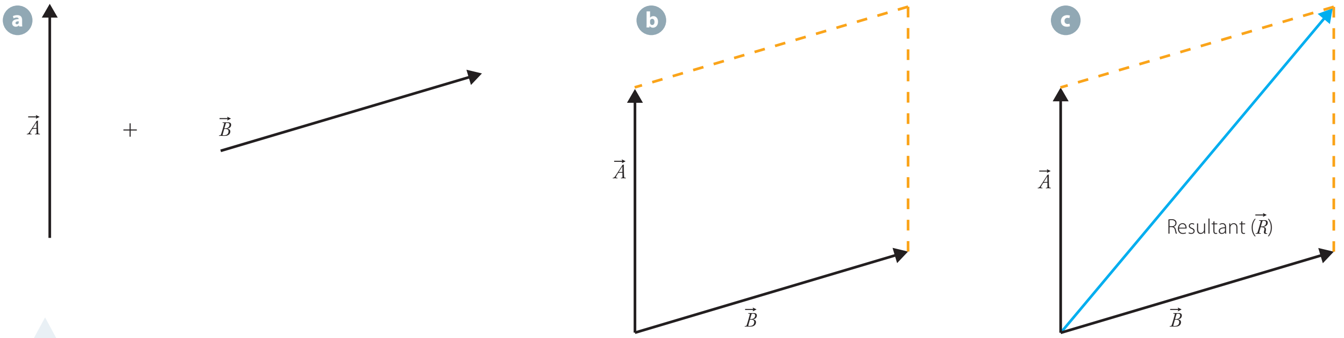


FIGURE 3.8 **a** Vector $\vec{A}$ and vector $\vec{B}$ are to be added. **b** Vector $\vec{B}$ is moved so that its tail is made to contact the tail of vector $\vec{A}$, and a copy is put on the head of $\vec{A}$. A copy of vector $\vec{A}$ is added so that it connects the heads of the copies of vector $\vec{B}$ (these copies are shown by the dashed orange lines). This draws a parallelogram. **c** The resultant vector, $\vec{R}$, is the diagonal of the parallelogram. It runs from the tails of the two vectors to the opposite corner.

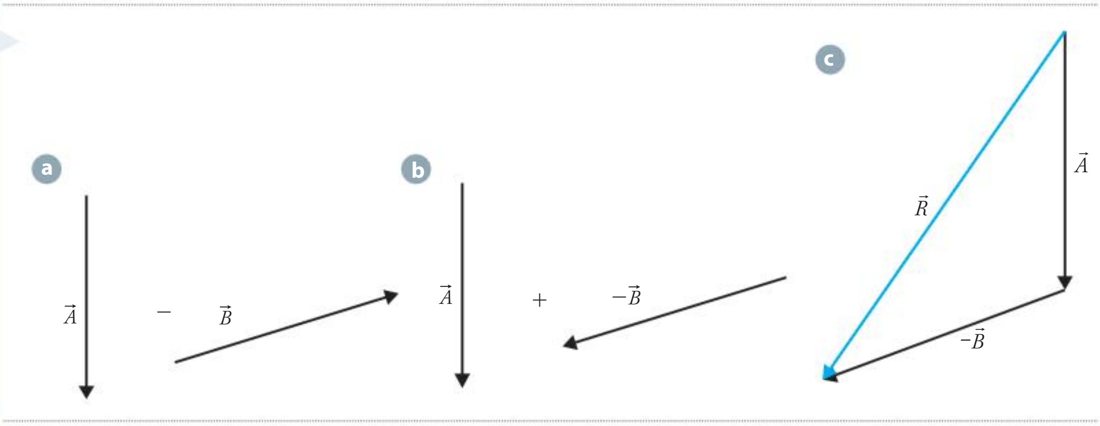


FIGURE 3.9 **a** Vector $\vec{B}$ is to be subtracted from vector $\vec{A}$. **b** Vector $\vec{B}$ is changed to '$-\vec{B}$' by flipping the arrow to point in the opposite direction. **c** Vector $\vec{A}$ is then added to vector $-\vec{B}$ using the head-to-tail method.

Vector subtraction is very useful. Velocity is the change in displacement divided by the change in time. Displacement is a vector, so the change in displacement is the difference between two vectors. We obtain it by subtracting the initial displacement from the final displacement. If we divide that by the time, we get the average velocity for the journey.

Resolving and adding vectors

The graph paper game

The previous section demonstrated vector addition and subtraction using diagrams. Here, a more mathematical approach is described.

First, each vector is resolved into its components. The same set of axes must be used for both vectors. The parallel components can then be added. This gives the components of the resultant vector. These can be combined to obtain the final vector. Similar to the discussion of Figure 3.2, we might say that if $\vec{s}$ is the vector sum of $\vec{s}_1$ and $\vec{s}_2$, then

$$\vec{s} = \vec{s}_1 + \vec{s}_2$$

where each of the three vectors can be resolved into x and y components:

$$\vec{s} = \vec{s}_x + \vec{s}_y$$

$$\vec{s}_1 = \vec{s}_{1,x} + \vec{s}_{1,y}$$

$$\vec{s}_2 = \vec{s}_{2,x} + \vec{s}_{2,y}$$

The x and y terms can be collected to make two equations,

$$\vec{s}_x = \vec{s}_{1,x} + \vec{s}_{2,x} \text{ and } \vec{s}_y = \vec{s}_{1,y} + \vec{s}_{2,y}$$

and these can be combined to give the final vector:

$$\vec{s} = (\vec{s}_{1,x} + \vec{s}_{2,x}) + (\vec{s}_{1,y} + \vec{s}_{2,y})$$

This process would work equally well when adding more than two vectors.

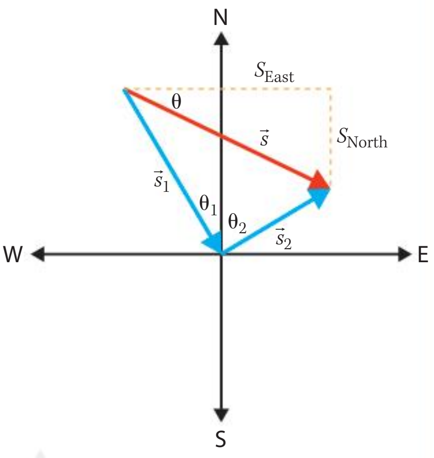
Pythagoras' theorem can be used to obtain the magnitude, s, of $\vec{s}$:

$$s = \sqrt{(s_{1,x} + s_{2,x})^2 + (s_{1,y} + s_{2,y})^2}$$

and trigonometry gives the direction, since $\vec{s}$, $\vec{s}_x$ and $\vec{s}_y$ make a right-angled triangle. The process is best illustrated with an example. In Worked example 3.3, it may help to think of east as the x component and north as the y component.

WORKED EXAMPLE 3.3

A number of orienteers run two legs of a course. First, they run 600 m S30°E, back to the origin. This first displacement can be referred to as $\vec{s}_1$. Then they run 400 m N60°E. That is $\vec{s}_2$. Their final position is at the tip of $\vec{s}_2$.
What is their final displacement, $\vec{s}$, relative to where they started? Take 400 m and 600 m as having three significant figures. Note that the runner is *not* starting on the origin.

ANSWER	LOGIC
 **FIGURE 3.10**	- Draw a diagram using the information in the question. - Resolve vectors into components, add them, and determine the resultant.
$\vec{s}_1 = 600\text{ m S30°E}$ $\vec{s}_2 = 400\text{ m N60°E}$ $\vec{s} = \text{resultant}$ $s_1 = 600\text{ m};\ \theta_1 = 30°$ $s_2 = 400\text{ m};\ \theta_2 = 60°$	- Identify relevant data in the question.
East is positive horizontal; north is positive vertical	- Define coordinates.
$\vec{s} = \vec{s}_1 + \vec{s}_2$	- Write an expression for the total displacement.
$\vec{s}_{1\text{East}} = s_1 \sin\theta_1$ $\vec{s}_{2\text{East}} = s_2 \sin\theta_2$	- Write the expressions for the east–west components (magnitudes).
$\vec{s}_{1\text{North}} = s_1 \cos\theta_1$ $\vec{s}_{2\text{North}} = -s_2 \cos\theta_2$	- Write the expressions for the north–south components (magnitudes).
$\vec{s}_{1\text{East}} = (600\text{ m})\sin 30° = 300\text{ m E}$ $\vec{s}_{2\text{East}} = (400\text{ m})\sin 60° = 346\text{ m E}$ $\vec{s}_{1\text{North}} = (-600\text{ m})\cos 30° = -520\text{ m N}$ $\vec{s}_{2\text{North}} = (400\text{ m})\cos 60° = 200\text{ m N}$	- Substitute the known values with units (written as vectors, with units and directions included).

ANSWER	**LOGIC**
$\vec{s}_{\text{East}} = \vec{s}_{1\text{East}} + \vec{s}_{2\text{East}}$ $= 300\text{ m E} + 346\text{ m E}$ $= 646\text{ m E}$ $\vec{s}_{\text{North}} = \vec{s}_{1\text{North}} + \vec{s}_{2\text{North}}$ $= -520\text{ m N} + 200\text{ m N}$ $= -320\text{ m N}$	■ Add components. Remember that north and south are negative of each other. ■ We can see from the vector resolution of $\vec{s}_2$ that the runner's final position (compared to the origin) is 346 m E, 200 m N. Relative to where they started, their final position is 646 m E, 320 m S. A pair of components like that is a valid way of expressing the answer. The direction is implicit in the relative lengths of the components. However, the more standard form is to give a direction and a length.
$s = \sqrt{(s_{\text{North}})^2 + (s_{\text{East}})^2}$ $s = \sqrt{(-320)^2 + (646)^2} = 721\text{ m}$	■ Apply Pythagoras' theorem to the magnitudes to find the net length. ■ Substitute the known values with units and calculate the answer.
$\tan\theta = \dfrac{s_{\text{North}}}{s_{\text{East}}}$ $\tan\theta = \dfrac{-320\text{ m}}{646\text{ m}} = -0.495$ $\theta = -26°$ $\vec{s} = 721\text{ m E}26°\text{S}$ $= 721\text{ m S}64°\text{E}.$	■ Write the relationship between the angle and the distances. ■ Substitute the known values with units. ■ Calculate the answer. ■ State the final answer with correct units and significant figures.

TRY THESE YOURSELF

1 If the orienteer is to have the maximum possible displacement, at what angle should the second leg of their run be if we keep the length fixed but let the angle vary?

2 If the orienteer runs at the angles in the example above, but both legs have a length of 600 m, what is their final displacement from their starting position?

INVESTIGATION 3.2

Adding vectors

Explore how the resultant of adding two vectors changes as you change the vectors.

Critical and creative thinking

Numeracy

Information and communication technology capability

Adding vectors

AIM

To investigate the addition of more than two vectors

Write an inquiry question for this investigation.

MATERIALS

- Computer with internet access

METHOD

1 Open the weblink 'Vector addition simulator'.

2 Add three vectors (not necessarily of the same length) to the simulation to create a resultant vector of length 20 at an angle of 30°. There are many possible ways of doing this – compare your method with other people's. Sketch your own arrangement, and at least two others.

»

3 Clear the simulation. Select three vectors of the same length, l. Arrange them to give the resultant vector of maximum possible length. What is this maximum possible length in terms of l? Now arrange the vectors to give the minimum possible length. What is this length? Sketch a vector diagram for each arrangement.

4 Clear the simulation. Select three vectors of different lengths. Arrange them to give the resultant vector of maximum possible length. Now arrange the vectors to give the minimum possible length. Sketch a vector diagram for each arrangement.

5 Clear the simulation and add three vectors of equal length to create a resultant of length 0. Sketch a vector diagram for your arrangement. Can you find more than one way of getting a resultant of zero?

6 Repeat step 4 with four, five, six and seven vectors of equal length. Sketch a vector diagram for each case, and note whether there is more than one way of making a resultant of zero length.

Vector addition simulator

This simulation of vector additions will be used for your investigation.

RESULTS AND ANALYSIS

Record your vector diagrams with the length of the resultant in each case.

DISCUSSION

- Describe a general method for adding vectors to give the resultant vector of maximum and minimum possible lengths.
- Describe a general method for adding vectors to give a resultant vector of zero.

CONCLUSION

Summarise your results and answer your inquiry question for this investigation.

KEY CONCEPTS

- Displacement, $\vec{s}$, represents a change of position ($\vec{d}$) with respect to the starting point, which may or may not be the origin. It has both magnitude (the distance) and direction, so it is a vector.
- The displacement is the difference between the final and initial positions, $\vec{s} = \vec{d}_f - \vec{d}_i$.
- We can add displacements by resolving them into components and adding the components: $\vec{s} = (\vec{s}_{1,x} + \vec{s}_{2,x}) + (\vec{s}_{1,y} + \vec{s}_{2,y})$
- We can add displacements graphically by drawing vectors head-to-tail. Subtraction is performed by reversing the second vector, and then adding them. $\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = (-\vec{B}) + \vec{A}$
- We can work using compass directions, x and y coordinates, or any other sensible set of axes.

CHECK YOUR UNDERSTANDING 3.2

1 **a** On paper (preferably graph paper), construct the vector triangles needed to add the pairs of vectors shown in Figure 3.11. Label the resultant $\vec{R}$ in each case.

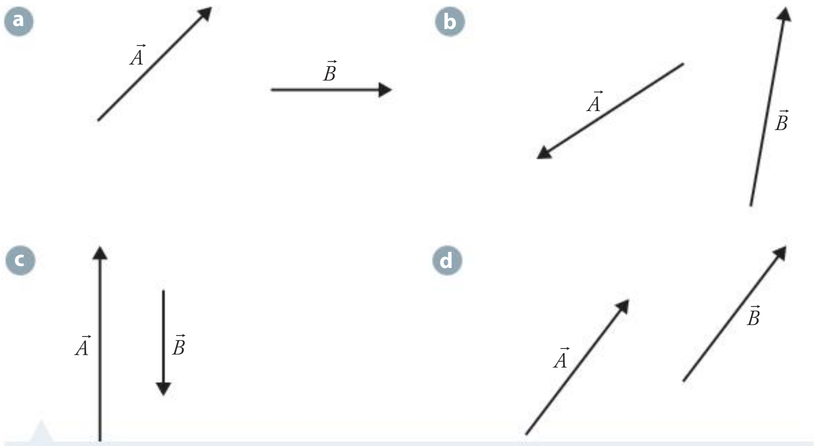


FIGURE 3.11 Pairs of vectors to be graphically added and subtracted

- **b** Repeat part **a**, but this time subtract the second vector from the first in each pair. Again, label the resultant in each case.

2 On the screen of a GPS map, each grid square is 10 km wide (east–west) and 10 km tall (north–south). Rosa follows a road north for seven grid squares and west for four grid squares.

- **a** Calculate Rosa's displacement.

Rosa now drives north-west for 50 km.

- **b** Calculate Rosa's total displacement.

3 When going from one to two dimensions, describe what extra factors need to be considered when working with positions, distances and displacements.

4 Describe the axes of a coordinate system (perpendicular axes) that would be suitable for analysing the motion of a bowling ball along a lane.

5 A boat is tacking into a wind that is blowing from north to south. First, the boat sails for 5.2 km N40°E. Then it sails 4.8 km N45°W. Calculate the boat's displacement.

6 A triathlete swims 5.5 km due south, runs 25.0 km S30°E, and cycles 30 km N25°W.

- **a** Find the distance they have run after 1, 2 and 3 legs of the triathlon.
- **b** Calculate their displacement after 1, 2 and 3 legs of the triathlon.
- **c** If the triathlete took a total of 2 hours, calculate their average speed and average velocity.

3.3 Describing motion using vectors

In chapter 2, we saw that the velocity vector is found by dividing the displacement vector by the time over which the displacement occurred. This remains true when the motion is in a two-dimensional plane.

Velocity is a vector

When an object moves, at any moment it is going in a particular direction with a particular speed. To describe the motion, we need to specify the magnitude (size) of the speed and its direction. For example, a car may be going south at 60 km h^{-1} but the car might be speeding up (accelerating) or slowing down. Just like the displacement, the velocity, $\vec{v}$, sometimes changes with time. When it does so, there is an acceleration.

Just like displacement, a velocity vector can be resolved into components. It may have a component along x and a component along y, or a component to the east and one to the north. If a car is heading north-west at a speed of 60 km h^{-1}, it has a velocity component to the north and one to the west. Trigonometry can be used to resolve a velocity into components. Components can be combined to obtain a resultant velocity.

Figure 3.12 shows a velocity vector, $\vec{v}$, and its components (in this case along the x and y directions). In vector notation, we write

$$\vec{v} = \vec{v}_x + \vec{v}_y$$

where the magnitudes are related by Pythagoras' theorem:

$$v = \sqrt{v_x{}^2 + v_y{}^2}$$

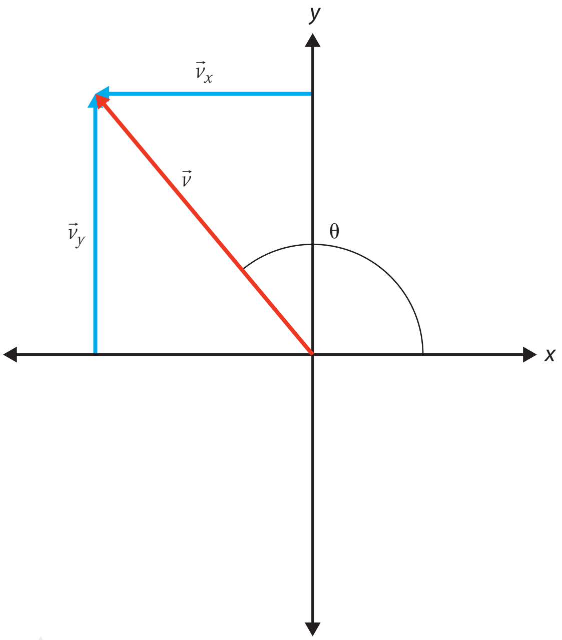


FIGURE 3.12 Using trigonometry, velocities can be resolved into components along perpendicular axes.

As previously for displacements, the angle is obtained from trigonometry:

$$\tan\theta = \frac{v_y}{v_x}$$

When drawing displacement vectors on graph paper, it is necessary to specify a scale, such as '1 cm represents 100 m'. Velocities can be illustrated in exactly the same way. The difference is the scale might now read '1 cm represents 100 m s^{-1}'. As long as the units are correct, all the results pertaining to vectors still apply. Because the units are different, velocities and displacements cannot be added together.

Velocity components
Watch how velocities are broken into components

Adding velocities on a wheel

Adding and subtracting velocities

It is possible to think of a situation where a motion has two perpendicular components; for example, people moving around on a train or bus. The net (total) velocity of a passenger is the vector sum of the passenger's movement within the vehicle and the movement of the vehicle itself.

WORKED EXAMPLE 3.4

Mario is on a train that is going east along a straight track at 60.0 km h^{-1}. The train is nearly empty. He decides to change seats, from a left window to a right window. He moves from one side of the carriage to the other in 1.0 s. The carriage is 3.0 m wide. What is Mario's net velocity while he is moving?

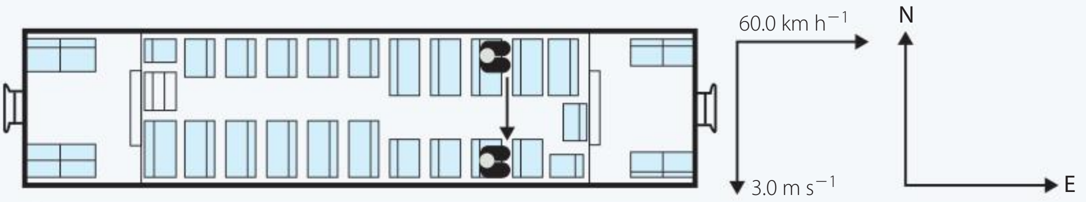


FIGURE 3.13 The train carriage is moving east at 60.0 km h^{-1} and Mario is walking across the carriage at 3.0 m s^{-1}. What is Mario's total velocity relative to the train tracks? Arrows show directions of velocities, but not magnitudes.

ANSWERS	LOGIC
$\vec{v}_{\text{East}} = \vec{v}_{\text{Train}} = 60.0 \text{ km h}^{-1} \text{east}$ $\vec{v}_{\text{South}} = \vec{v}_{\text{Mario to train}} = 3.0 \text{ m s}^{-1} \text{ south}$	▪ Identify relevant data and define variables. $\vec{v}_{\text{Mario to train}}$ is Mario's velocity relative to his seat.
$\vec{v}_{\text{Mario, total}} = \vec{v}_{\text{East}} + \vec{v}_{\text{South}}$	▪ Write the relationship between the net velocity and the components.
$v_{\text{East}} = 60.0 \text{ km h}^{-1} \times \dfrac{1000 \text{ m km}^{-1}}{3600 \text{ s h}^{-1}}$ $v_{\text{East}} = 16.7 \text{ m s}^{-1}$	▪ Convert to SI units.
$v_{\text{Mario, total}} = \sqrt{(v_{\text{East}})^2 + (v_{\text{South}})^2}$ $= \sqrt{(16.7 \text{ m s}^{-1})^2 + (3.0 \text{ m s}^{-1})^2}$ $= 17.0 \text{ m s}^{-1}$	▪ Recognise that the velocities are perpendicular, so we can apply Pythagoras' theorem to obtain the net magnitude. ▪ Substitute the known values with units. ▪ Calculate the answer.

ANSWERS	LOGIC
$\tan\theta = \frac{v_{\text{South}}}{v_{\text{East}}} = \frac{3.0\ \text{m s}^{-1}}{16.7\ \text{m s}^{-1}} = 0.18$	- Find the expression for the angle. Substitute the known values with units. - Since v_{South} ($\vec{v}_{\text{Mario to train}}$) was the 'opposite' side of the triangle we used to obtain the angle, θ is measured away from the direction of the train (east).
$\theta = 10.2°$	- Calculate the answer.
$\vec{v}_{\text{Mario, total}} = 17.0\ \text{m s}^{-1}\ \text{E}10°\text{S}$ $= 7.0\ \text{m s}^{-1}\ \text{S}80°\text{E}$	- State the final answer with correct units and appropriate significant figures.

TRY THIS YOURSELF

Mario now runs back to his original seat at a speed of 6 m s^{-1}. What is his net velocity as he does this? Give the magnitude and direction.

When two velocities are perpendicular, one can be equated with $\vec{v}_x$ and the other with $\vec{v}_y$. This allows the total to be found from Pythagoras' theorem. When they are not perpendicular (the more general case), we resolve the vectors and add the components. This adds steps to the calculation. If $\vec{v}$ is the vector sum of $\vec{v}_1$ and $\vec{v}_2$, then

$$\vec{v} = \vec{v}_1 + \vec{v}_2$$

where the velocities can be resolved into their components:

$$\vec{v}_1 = \vec{v}_{1,x} + \vec{v}_{1,y} \text{ and } \vec{v}_2 = \vec{v}_{2,x} + \vec{v}_{2,y}$$

$\vec{v}$ can also be resolved into its components:

$$\vec{v} = \vec{v}_x + \vec{v}_y$$

We then collect the x components into one equation and the y into another:

$$\vec{v}_x = \vec{v}_{1,x} + \vec{v}_{2,x} \text{ and } \vec{v}_y = \vec{v}_{1,y} + \vec{v}_{2,y}$$

Pythagoras' theorem can be used to obtain the magnitude, v, of $\vec{v}$:

$$v = \sqrt{(v_{1,x} + v_{2,x})^2 + (v_{1,y} + v_{2,y})^2}$$

By definition, $\vec{v}$, $\vec{v}_x$ and $\vec{v}_y$ make a right-angled triangle. That means trigonometry can be used to obtain the direction of $\vec{v}$.

With that in mind, what if Mario from Worked example 3.4 is still fidgeting about on the train, and has seen a seat somewhere in front of him that he likes? See Worked example 3.5.

WORKED EXAMPLE 3.5

Continuing on from Worked example 3.4, Mario moves from the left to the right of the train carriage, but also moves two seats forward. He moves a total of 3.6 m in 1.0 s, in a direction 56° south of east, relative to the carriage (see Figure 3.14). What was Mario's resultant velocity relative to the tracks while he was moving? Give the magnitude and direction. The train is still moving at 60.0 km h^{-1}.

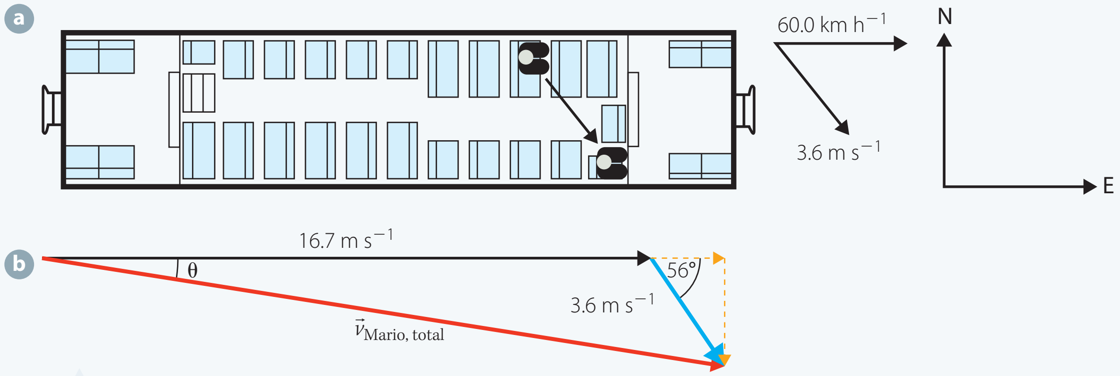


FIGURE 3.14 **a** A schematic diagram of the situation. The vectors are not to scale, but it shows how Mario is moving in the carriage. **b** A vector diagram with vectors to scale. It illustrates the train's velocity relative to the tracks (16.7 m s^{-1} to the east) and Mario's velocity relative to the train (3.6 m s^{-1} 56° south of east).

ANSWER	LOGIC
$\vec{v}_{\text{Train}} = 16.7\text{ m s}^{-1}$ to the east $\vec{v}_{\text{Mario on train}} = 3.6\text{ m s}^{-1}$ 56° south of east	▪ Identify the relevant data in the question.
$\vec{v}_{\text{Mario, total}} = \vec{v}_{\text{Train}} + \vec{v}_{\text{Mario to train}}$ $\vec{v}_{\text{Mario total}} = \vec{v}_{\text{East}} + \vec{v}_{\text{South}}$	▪ Define variables, and use them in the vector equations.
$v_{\text{East}} = v_{\text{Train, east}} + v_{\text{Mario to train, east}}$ $v_{\text{South}} = v_{\text{Train, south}} + v_{\text{Mario to train, south}}$	▪ Write an appropriate equation for each component. (We are now working with magnitudes.)
$v_{\text{Mario to train, east}} = (v_{\text{Mario to train}})\cos\theta$ $v_{\text{Mario to train, south}} = (v_{\text{Mario to train}})\sin\theta$	▪ Break $\vec{v}_{\text{Mario to train}}$ up into its components.
$v_{\text{Train, east}} = v_{\text{Train}}$ $v_{\text{Train, south}} = 0$	▪ Break $\vec{v}_{\text{Train}}$ up into its components. (It is going east, so the other component is zero.)
$v_{\text{East}} = 16.7\text{ m s}^{-1} + 3.6\text{ m s}^{-1}\cos 56°$ $= 18.7\text{ m s}^{-1}$ east $v_{\text{South}} = 0\text{ m s}^{-1} + 3.6\text{ m s}^{-1}\sin 56°$ $= 3.0\text{ m s}^{-1}$ south	▪ Substitute the known values with units into the equations for each component and calculate the answers.
$v_{\text{Mario, total}} = \sqrt{(v_{\text{East}})^2 + (v_{\text{South}})^2}$ $= \sqrt{(18.7\text{ m s}^{-1}\text{ east})^2 + (3.0\text{ m s}^{-1}\text{ south})^2}$ $= 18.9\text{ m s}^{-1}$	▪ Apply Pythagoras' theorem to find the magnitude of the total velocity. ▪ Substitute the known values and calculate the answer.
$\tan\theta = \frac{v_{\text{South}}}{v_{\text{East}}} = \frac{3.0\text{ m s}^{-1}}{18.7\text{ m s}^{-1}} = 0.16$ $\theta = 9.1°$ $\vec{v}_{\text{Mario, total}} = 18.9\text{ m s}^{-1}$ S81°E	▪ Find the expression for the angle. Substitute the known values with units. ▪ Calculate the answer. ▪ State the final answer with correct units and appropriate significant figures.

TRY THIS YOURSELF

What if Mario went across the carriage, but two seats backwards instead? Without doing any maths, would you expect $v_{\text{Mario, total}}$ to become bigger or smaller, or stay the same? What about $v_{\text{Mario on train, east}}$ and $v_{\text{Mario on train, south}}$? What about θ?

How fast are you moving now?

Earth is spinning and going around the Sun. Find out just how fast you're really moving.

Change in velocity

We have looked at adding velocity vectors to obtain the resultant velocity. The change in a vector is found by subtracting 'before' from 'after' (or 'initial' from 'final'). The change in velocity of a body may be referred to as $\Delta\vec{v}$ ('delta-vee'). It is worked out by subtracting the initial velocity from the final:

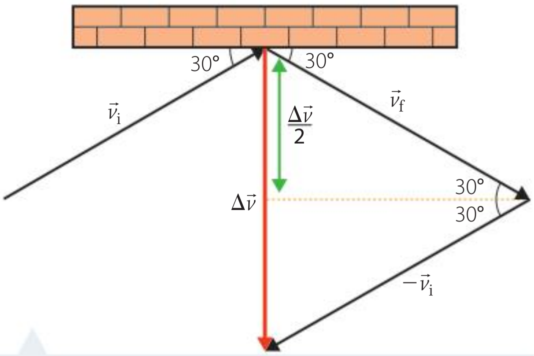
$$\Delta\vec{v} = \vec{v}_\text{f} - \vec{v}_\text{i}$$

If $\Delta\vec{v}$ is then divided by the time taken for the change in velocity, the result is the average acceleration. This can be positive or negative, and since it is a vector subtraction, it need not be in the direction of the initial or final velocity. The magnitude of $\Delta\vec{v}$ is written as Δv.

We will look at two kinds of velocity subtraction. The first is the case of a single object before and after some event (see Worked example 3.6). The second looks at two bodies moving relative to each other (see the next section).

WORKED EXAMPLE 3.6

A tennis ball strikes the wall at an angle of 30° to the wall, with a velocity of 6.0 m s^{-1}, and bounces off at the same speed and angle. Determine the change in the velocity of this tennis ball.

ANSWER	LOGIC
 **FIGURE 3.15** A ball bouncing off a wall. The red arrow gives the change in velocity, $\Delta\vec{v}$. Half that length is $\frac{\Delta\vec{v}}{2}$, as shown in green.	■ Draw a diagram based on the information given in the question. ■ The first step is to take the negative of the initial vector ($\vec{v}_\text{i}$) and then add it to the final, $\vec{v}_\text{f}$, and draw in the resultant, $\Delta\vec{v}$. This is shown in Figure 3.15. The angles are marked in carefully. We can see from the drawing that the components of $-\vec{v}_\text{i}$ and $\vec{v}_\text{f}$ that are parallel to the wall will cancel out. This is because the vectors have the same magnitude and the angles have the same magnitudes. The components of $-\vec{v}_\text{i}$ and $\vec{v}_\text{f}$ perpendicular to the wall are the same.
$\Delta\vec{v} = \vec{v}_\text{f} + (-\vec{v}_\text{i})$	■ Write the vector equation for subtraction.
$\frac{\Delta v}{2} = v_\text{f} \sin 30°$	■ Identify a right-angled triangle and use trigonometry to relate the magnitudes.
$\frac{\Delta v}{2} = 6.0\ \text{m s}^{-1} \sin 30° = 3.0\ \text{m s}^{-1}$	■ Substitute the known values with units and calculate the answer

ANSWER	LOGIC
$\Delta v = 6.0 \text{ m s}^{-1}$ Since the component of velocity parallel to the wall does not change, the change in velocity is 6.0 m s^{-1} perpendicular to the wall and away from the wall.	▪ State the final answer with correct units and appropriate significant figures.

TRY THIS YOURSELF

This time, imagine the ball is a bit squishy. When bounced off the wall, the ball came off with half the speed but in the same direction. What is $\Delta\vec{v}$ in this case?

In Worked example 3.6, the initial and final velocities were different in direction but the same in magnitude. In other worlds, the ball's *speed* did not change, but its *velocity* did.

If the time for the ball to collide with the wall and bounce off is known, the acceleration could be determined from $\vec{a} = \dfrac{\Delta\vec{v}}{t}$.

INVESTIGATION 3.3

Measuring velocity vectors and components

Critical and creative thinking

Numeracy

Information and communication technology capability

In this experiment, you will roll a ball at various speeds to derive a relationship between the components of the velocity, the total velocity, and the angle at which the ball is rolled.

AIM

Write a hypothesis for this investigation.

MATERIALS

- Tape measure
- Chalk
- Large protractor
- Stopwatch
- Ball
- Basketball court or tennis court

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Excess sun exposure is dangerous.	Wear a hat, and any other appropriate sun protection.

RISK ASSESSMENT

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Measure the width and length of the court – these are d_x and d_y. Use the chalk and protractor to measure the angle between the long side and the diagonal.
2. One person stays at a corner of the court with a ball. A second person stands at the opposite corner from them with the stopwatch, and measures times. The experimental arrangement is shown in Figure 3.16.

»

3. The first person slowly rolls the ball to the second person. The second person measures the time taken for the ball to reach them.
4. Repeat step 3 at least 10 times, rolling the ball a bit faster each time.

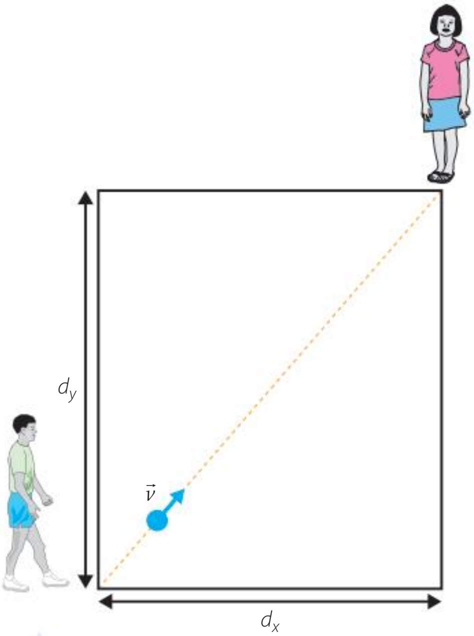


FIGURE 3.16 Experimental arrangement showing distances to be measured

RESULTS

- Record the distances d_x and d_y.
- Record your time measurements in a table similar to the one shown.

TRIAL	TIME (S)	v_x (m s^{-1})	v_y (m s^{-1})	v_{total} (m s^{-1})
1				
2				
…				

ANALYSIS OF RESULTS

1. Complete the table by:

calculating v_x for each time using $v_x = \frac{d_x}{t}$

calculating v_y for each time using $v_y = \frac{d_y}{t}$

calculating v_{total} for each time using $v_t = \sqrt{v_x{}^2 + v_y{}^2}$.

2. Use a spreadsheet to plot a graph of v_x vs v_{total}. Display a line of best fit and the equation for the line on your graph. Record the gradient.
3. Use a spreadsheet to plot a graph of v_y vs v_{total}. Display a line of best fit and the equation for the line on your graph. Record the gradient.

DISCUSSION

1. Did your calculated gradient agree with what you would expect from your measurements of the court? If not, can you explain why?
2. State whether your hypothesis was supported.
3. How could you improve or extend this experiment?

CONCLUSION

Summarise your results and write a statement of whether your hypothesis was supported or disproved by your results.

KEY CONCEPTS

- Velocities are vectors. Speeds are scalar.
- A velocity, $\vec{v}$, can be resolved into perpendicular components, $\vec{v} = \vec{v}_x + \vec{v}_y$.
- If v is the magnitude of $\vec{v}$, and v_x and v_y are the magnitudes of $\vec{v}_x$ and $\vec{v}_y$, then $v_x = v\cos\theta$, $v_y = v\sin\theta$, $v = \sqrt{v_x^2 + v_y^2}$ and $\tan\theta = \frac{v_y}{v_x}$.
- We can add and subtract velocities by resolving them into components.
- As with displacements, we can add velocities graphically by drawing vectors head-to-tail.
- Change in velocity, Δv, is found by subtraction of initial velocity from final velocity.

CHECK YOUR UNDERSTANDING

3.3

1 Describe the difference between speed and velocity.

2 An aeroplane has a velocity of 500 km h^{-1} S50°E. Calculate the velocity's components to the north, east, west and south. A sketch may be useful.

3 A passenger on the aeroplane in question 2 takes 6.0 s to run 40 m along the aisle towards the tail of the plane. Calculate the average velocity of the passenger during their run.

4 A rider on a horse takes an hour to ride 28 km N25°E.

   a Calculate their average speed.

   b Write down their average velocity.

   c Calculate the northward and eastward components of their velocity.

5 A child throws a ball of plasticine horizontally at a vertical wall. Initially, it is travelling at 10 m s^{-1} at an angle of 35° to the wall. It does not bounce very well, so when it comes off the wall it is travelling at 3.0 m s^{-1}, again at 35° to the wall.

   a Suggest the coordinate system you might use to tackle this problem. Draw a sketch, noting labels and quantities. Also draw a vector diagram.

   b What is the change in velocity, $\Delta\vec{v}$, of the ball of plasticine?

   c If the plasticine hit the wall and stopped (i.e. stuck to it), what would $\Delta\vec{v}$ be then?

3.4 Relative motion in a plane

The relative position of an object depends on the position of the observer. If an object is at position $\vec{d}_1$ and an observer is at $\vec{d}_2$, then

$$\vec{s}_{1 \text{ relative to } 2} = \vec{d}_1 - \vec{d}_2$$

For example, if a dog is at $\vec{d}_1$, 3 m from a wall, and a cat is at $\vec{d}_2$, 6 m from the wall in the same direction, then the dog is (3 – 6) = –3 m from the cat. The minus sign comes in because the distance away from the wall is positive. From the cat's point of view, the dog is towards the wall, which is the negative direction.

$\vec{s}_{1 \text{ relative to } 2}$ is also the displacement that needs to be applied to the cat at $\vec{d}_2$ to bring it to $\vec{d}_1$. This is a one-dimensional example. We have already looked at subtraction of displacement vectors (for example, Worked example 3.3), and this is another application. Sometimes, $\vec{s}_{1 \text{ relative to } 2}$ may be written as $\vec{s}_{1,2}$ or $\vec{d}_{1,2}$, but this notation does not make it very clear what is relative to what. It is generally preferable to be as clear as possible, even if it means using more words.

The relative motion of an object depends on the motion of the observer. It might be said that it depends on what the object's velocity is being measured against. In Worked example 3.4, when Mario was sitting down, his velocity relative to the carriage was zero. Relative to the tracks, his velocity was 60 km h^{-1}. A passenger in the same carriage (the 'carriage frame of reference') saw Mario as stationary. A passenger standing beside the tracks ('track frame of reference') would see Mario moving. When Mario changed seats, the passenger in the carriage saw his movement relative to the carriage – they saw only one aspect of his motion.

In Figure 3.17 (page 78), car A is moving 2 m s^{-1} faster than car B. To an observer standing still on the roadside (looking into the page), both cars are moving to the right. If the observer was seated *in* car B, still looking into the page, car A would seem to be moving to the right at 2 m s^{-1}. Car B would be stationary relative to the observer. If the observer was a passenger in car A, car B would seem to be moving to the left. This shows that the situation looks different for different frames of reference. The cars are in motion relative to each other and relative to the road. They have positions, velocities and accelerations relative to each other and relative to the road. For now, we are looking at velocities.

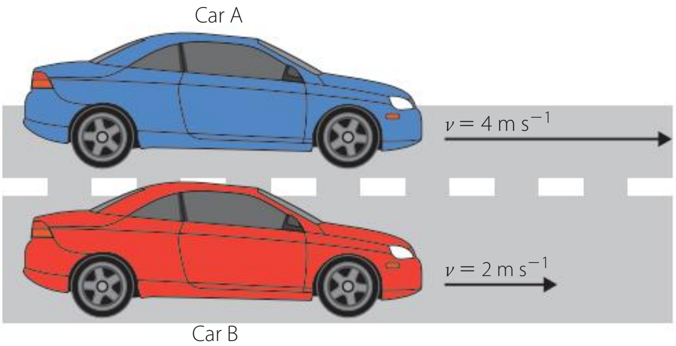


FIGURE 3.17 Two cars with different velocities relative to the road, but in the same direction

For this one-dimensional example,

$$v_{1 \text{ relative to } 2} = v_1 - v_2$$

where $v_{1 \text{ relative to } 2}$ is the velocity of object 1 relative to object 2. In this case, v_1 is the velocity of object 1 relative to the ground, and v_2 is that of object 2 relative to the ground. In the more general case, when the velocities are not parallel,

$$\vec{v}_{1 \text{ relative to } 2} = \vec{v}_1 - \vec{v}_2$$

Change in velocity and relative motion

where the velocities are now vectors. The vectors could be parallel, perpendicular or somewhere in between.

WORKED EXAMPLE 3.7

Aditya (A) and Belinda (B) are riding motorbikes. Aditya is riding north at 15 m s^{-1}. Belinda is riding east at 20 m s^{-1}, as shown in Figure 3.18. What is Aditya's velocity relative to Belinda? Take north as positive *y* and east as positive *x*.

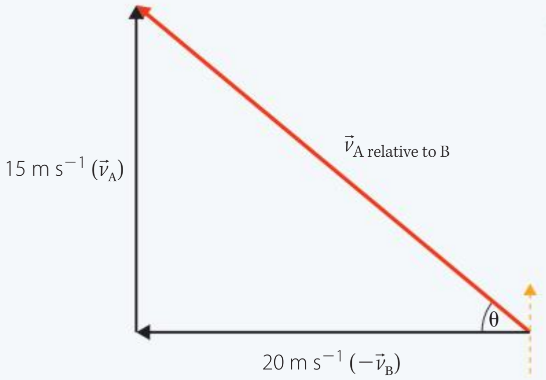


FIGURE 3.18 Vector diagram for Belinda's and Aditya's relative velocities. The horizontal vector has been reversed for subtraction.

ANSWER	LOGIC
$v_A = 15 \text{ m s}^{-1}$ north $v_B = 20 \text{ m s}^{-1}$ east	▪ Identify relevant data from the question.
$\vec{v}_{\text{A relative to B}} = \vec{v}_A - \vec{v}_B$	▪ Identify the appropriate vector equation.

ANSWER	LOGIC
$\vec{v}_{\text{A relative to B}} = \vec{v}_{\text{A}} + (-\vec{v}_{\text{B}})$ $v_{\text{A relative to B}} = \sqrt{(v_{\text{A}})^2 + (-v_{\text{B}})^2}$ $= \sqrt{(15\ \text{m s}^{-1})^2 + (-20\ \text{m s}^{-1})^2}$ $= 25\ \text{m s}^{-1}$	▪ Subtract by adding the negative. ▪ Velocities are perpendicular, so apply Pythagoras' theorem to find the magnitude of the resultant. ▪ Substitute the known values with units and calculate the answer.
$\tan\theta = \frac{v_{\text{A}}}{v_{\text{B}}}$ $\tan\theta = \frac{15\ \text{m s}^{-1}}{20\ \text{m s}^{-1}} = 0.75$ $\theta = 37°$ $\vec{v}_{\text{A relative to B}} = 25\ \text{m s}^{-1}\ \text{N}53°\text{W}$	▪ Use trigonometry to determine the angle. ▪ Substitute the known values with units. ▪ Calculate the answer. ▪ State the final answer with correct units and appropriate significant figures.

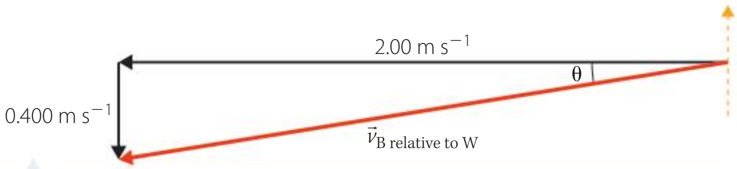
TRY THIS YOURSELF

What is Belinda's velocity relative to Aditya? Is there a really quick way of working this out from your first answer?

In Worked example 3.7, the velocity of one motorbike relative to another was calculated. It may be desirable to work the other way around. If a particular relative velocity is required, what components need to go into it? This is a common situation. An aeroplane travels through the air, which is moving. But what is important to the passengers is its speed relative to the ground. Similarly, a swimmer or a boat moves through flowing water, but ultimately it is their movement relative to the land that is important.

WORKED EXAMPLE 3.8

Imagine a boat (B) on a river. The river flows from south to north at 0.400 m s^{-1} (relative to the shore). What must the boat's velocity be relative to the water (W) if it is to go due west at 2.00 m s^{-1} relative to the shore?

ANSWER	LOGIC
$v_{\text{B}} = 2.00\ \text{m s}^{-1}$ west $v_{\text{W}} = 0.400\ \text{m s}^{-1}$ north	▪ Identify the relevant data from the question.
 **FIGURE 3.19** Vector diagram for the boat crossing the river. The boat must have a velocity component to the south of 0.400 m s^{-1}. The remaining velocity component must be westerly at 2.00 m s^{-1}.	▪ Draw the vector diagram based on the relevant information.

ANSWER	LOGIC
$\vec{v}_{\text{B relative to W}} = \vec{v}_\text{B} - \vec{v}_\text{W}$	▪ Identify the appropriate vector equation.
$\vec{v}_{\text{B relative to W}} = \vec{v}_\text{B} + (-\vec{v}_\text{W})$	▪ Subtract by adding the negative.
$v_{\text{B relative to W}} = \sqrt{(v_\text{B})^2 + (-v_\text{W})^2}$ $= \sqrt{(2.00\text{ m s}^{-1})^2 + (-0.400\text{ m s}^{-1})^2}$ $= 2.04\text{ m s}^{-1}$	▪ Velocities are perpendicular, so apply Pythagoras' theorem to find the magnitude of the resultant. ▪ Substitute the known values with units and calculate answer.
$\tan\theta = \frac{v_\text{W}}{v_\text{B}}$ $\tan\theta = \frac{0.400\text{ m s}^{-1}}{2.00\text{ m s}^{-1}} = 0.2$ $\theta = 11.3°$	▪ Use trigonometry to determine the angle. ▪ Substitute the known values with units. ▪ Calculate the answer.
$v_{\text{B relative to W}} = 2.04\text{ m s}^{-1}$ S79°W So the boat has to aim at an angle of 11° south of west to end up travelling perpendicularly across.	▪ State the final answer with correct units and appropriate significant figures.

TRY THESE YOURSELF

1 As the river flows faster, what happens to the magnitude and angle of $\vec{v}_{\text{B relative to W}}$?

2 Calculate the velocity of the boat relative to the water to still go due west at 2.00 m s^{-1} if the water flows at 1.00 m s^{-1}.

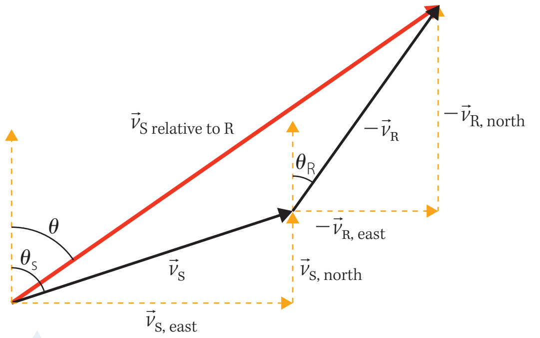
The example of an aeroplane in a crosswind may be treated much like the boat in Worked example 3.8. However, in real life, the aeroplane will not be dragged sideways at the same speed as the crosswind, but at some lesser velocity. When it comes to aeroplanes, the assumption that the air and the plane have the same crosswise velocity is an approximation.

Worked examples 3.7 and 3.8 both look at velocities that are perpendicular. This may not always be so. The vector equation $\vec{v}_{1\text{ relative to }2} = \vec{v}_1 - \vec{v}_2$ still works if the velocities are not perpendicular, but it is necessary to resolve the vectors into components. Worked example 3.9 brings together vector subtraction, resolving vectors into components, and the Red Baron.

WORKED EXAMPLE 3.9

The Red Baron (R) is flying at 100 km h^{-1} S35°W. He spots a Sopwith Camel (S). It is at the same height, but flying at 120 km h^{-1} N72°E. What is the velocity of the Sopwith relative to the Red Baron's Fokker triplane?

ANSWERS	LOGIC
$v_\text{R} = 100\text{ km h}^{-1}$ S35°W $v_\text{S} = 120\text{ km h}^{-1}$ N72°E	▪ Identify the relevant data; remember one vector has to be reversed when drawing the diagram.

ANSWERS	**LOGIC**
 **FIGURE 3.20** Vector diagram for the relative velocities, $\vec{v}_{\text{S}}$, $\vec{v}_{\text{R}}$ and $\vec{v}_{\text{S relative to R}}$. Note that $\vec{v}_{\text{R}}$ has been reversed to do the subtraction.	- Draw a labelled diagram, showing angles and vector lengths.
$\vec{v}_{\text{S relative to R}} = \vec{v}_{\text{S}} - \vec{v}_{\text{R}} = \vec{v}_{\text{S}} + (-\vec{v}_{\text{R}})$	- Identify the appropriate vector equation.
$v_{\text{S, east}} = v_{\text{S}} \sin \theta_{\text{S}}$ $v_{\text{S, north}} = v_{\text{S}} \cos \theta_{\text{S}}$ $-v_{\text{R, east}} = v_{\text{R}} \sin \theta_{\text{R}}$ $-v_{\text{R, north}} = v_{\text{R}} \cos \theta_{\text{R}}$	- Write an expression for each component. Note that signs are important.
$v_{\text{S relative to R, east}} = v_{\text{S, east}} + (-v_{\text{R, east}})$ $v_{\text{S relative to R, north}} = v_{\text{S, north}} + (-v_{\text{R, north}})$	- Write the expressions for the vector components of the solution.
$v_{\text{S relative to R, east}} = v_{\text{S}} \sin \theta_{\text{S}} + v_{\text{R}} \sin \theta_{\text{R}}$ $v_{\text{S relative to R, north}} = v_{\text{S}} \cos \theta_{\text{S}} + v_{\text{R}} \cos \theta_{\text{R}}$	- From these derive expressions for the scalar components of the solution.
$v_{\text{S relative to R, east}} = 120\text{ km h}^{-1} \sin 72° + 100\text{ km h}^{-1} \sin 35°$ $= 171.5\text{ km h}^{-1}$ $v_{\text{S relative to R, north}} = 120\text{ km h}^{-1} \cos 72° + 100\text{ km h}^{-1} \cos 35°$ $= 119.0\text{ km h}^{-1}$ $v_{\text{S relative to R}} = \sqrt{(171.5\text{ km h}^{-1})^2 + (119.0\text{ km h}^{-1})^2}$ $v_{\text{S relative to R}} = 209\text{ km h}^{-1}$	- Substitute the known values with units and calculate the answers. - Apply Pythagoras' theorem to find the net magnitude. - Calculate the answer.
$\tan\theta = \dfrac{v_{\text{S relative to R, east}}}{v_{\text{S relative to R, north}}}$ $\tan\theta = \dfrac{171.5\text{ km h}^{-1}}{119.0\text{ km h}^{-1}} = 1.44$ $\theta = 55.2°$	- Use trigonometry to determine the angle. - Substitute the known values with units. - Calculate the answer.

ANSWERS	**LOGIC**
$\vec{v}_{\text{S relative to R}} = 209\ \text{km h}^{-1}$ N55°E Relative to the Red Baron the Sopwith is moving at 209 km h^{-1} and appears to be travelling in a direction 55° east of north. Because the angles were measured from north, cosine is used to obtain the northerly components.	- State the final answer with correct units and appropriate significant figures.

TRY THIS YOURSELF

If we let the Sopwith's direction (but not the speed) vary, what is the maximum possible magnitude of $v_{\text{S relative to R}}$? What is the minimum? Thinking about aspects such as these can help you to check your mathematical answers.

Relative velocities

Watch these animations and work through the examples.

KEY CONCEPTS

- The relative position of an object depends on the position of the observer.
- The displacement of object 1 relative to object 2 is found from subtraction of position vectors, $\vec{d}_{1\ \text{relative to}\ 2} = \vec{d}_1 - \vec{d}_2$.
- The relative motion of an object depends on the motion of the observer.
- The velocity of object 1 relative to object 2 is found from subtraction of velocity vectors, $\vec{v}_{1\ \text{relative to}\ 2} = \vec{v}_1 - \vec{v}_2$.

CHECK YOUR UNDERSTANDING

3.4

1 Describe what is meant by 'frame of reference'.

2 An ant is on a chessboard. It is in the centre of the fourth square up in the third column from the left. A beetle is in the centre of the second square up in the seventh column from the left. Calculate the displacement of the beetle relative to the ant, in units of squares. (Hint: drawing a diagram may be useful.)

3 A passenger on a bus walks from the back to the front. It takes them 2.0 s to walk 10 m. The bus is moving south at 30 km h^{-1} relative to the road.

  a Calculate the passenger's velocity:

    i relative to the bus.

    ii relative to the road.

  b What is the bus's velocity relative to the passenger?

4 A spider is near the top of a vertical water spout. If rain is washing down the spout at 2.0 m s^{-1} relative to the spout, and the spider is climbing up at 5.0 cm s^{-1} relative to the water, what is the spider's velocity relative to the spout? How long will it take for the spider to get washed out if it is 2.2 m above the bottom of the spout?

5 An aeroplane is flying north at 200 km h^{-1} relative to the ground. There is a crosswind blowing from west to east at 55 km h^{-1} relative to the ground.

  a Calculate the velocity of the aeroplane relative to the wind. Include a sketch in your working.

  The wind now swings around and blows towards the north-east at 55 km h^{-1} relative to the ground.

  b Calculate the velocity of the aeroplane relative to the wind, assuming its velocity remains unchanged.

3 CHAPTER SUMMARY

- Motion can occur in one, two or three dimensions.
- When motion occurs in more than one dimension, it is useful to express displacements and velocities as vectors.
- Vectors can be represented as arrows, where the length of the arrow gives the magnitude and the angle of the arrow relative to some axis gives the direction.
- We use a variable with an arrow on top to represent a vector. If $\vec{A}$ is a vector, then A (no arrow) is its magnitude.
- A vector can be resolved into components. These are vectors along useful directions (perpendicular axes) that add to give the original vector.
- The vector $\vec{s}$ can be written as the sum of components in the x and y directions: $\vec{s} = \vec{s}_x + \vec{s}_y$.
- If the direction of the vector $\vec{s}$ is given by an angle to the x axis, θ, then the magnitudes of $\vec{s}_x$ and $\vec{s}_y$ are given by $s_x = s\cos\theta$ and $s_y = s\sin\theta$.
- If constructing a vector from its components, the angle can be found from $\tan\theta = \frac{s_y}{s_x}$ and the magnitude from $s = \sqrt{s_x{}^2 + s_y{}^2}$.
- Vectors can be added graphically by putting vector arrows head-to-tail.
- Vectors can be subtracted by reversing the second vector (exchanging head and tail) and then adding: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.
- Displacement is the vector, $\vec{s}$, from an initial position, $\vec{d}_\text{i}$, to a final position, $\vec{d}_\text{f}$. This means $\vec{d}_\text{f} = \vec{d}_\text{i} + \vec{s}$ or $\vec{s} = \vec{d}_\text{f} - \vec{d}_\text{i}$.
- When calculating displacement, the path followed does not matter.
- Velocity is a vector ($\vec{v}$). Its magnitude is the speed.
- Velocity can be resolved into components: $\vec{v} = \vec{v}_x + \vec{v}_y$, where $v_x = v\cos\theta$ and $v_y = v\sin\theta$.
- Vector magnitude and direction can be calculated from the magnitudes of the components: $v = \sqrt{v_x{}^2 + v_y{}^2}$ and $\tan\theta = \frac{v_y}{v_x}$.
- Two velocities can be added or subtracted by resolving them into components. Just like displacements, they can also be added and subtracted using diagrams.
- The change in an object's velocity after an event, such as a collision or acceleration, is denoted $\Delta\vec{v}$ ('delta-vee'). It is calculated from the vector equation $\Delta\vec{v} = \vec{v}_\text{f} - \vec{v}_\text{i}$.
- If $\Delta\vec{v}$ is divided by the time taken for the change in velocity, the result is the average acceleration: $\vec{a} = \frac{\Delta\vec{v}}{t}$.
- The velocity of object 1 relative to that of object 2 is given by the vector equation $\vec{v}_{1\text{ relative to }2} = \vec{v}_1 - \vec{v}_2$.

3 CHAPTER REVIEW QUESTIONS

Review quiz

1 Name four areas in physics where vectors are used.

2 Explain why it is useful to represent some quantities as vectors.

3 Explain what happens to a vector when it is multiplied or divided by a scalar quantity with units.

4 The second hand of a wall clock is 11 cm long. If the x axis points from the centre to the '3' and the y axis points from the centre to the '12', then:

- **a** calculate the angle to the x axis if the second hand is pointing to the '2'.
- **b** calculate the x and y components of the position of the tip of the second hand.

5 A dog runs away from its kennel 332 m S28°W. Calculate how far south and how far west it is of the kennel.

6 A basketball court is 28 m long and 15 m wide. Calculate the length of the diagonal and the angle it makes to the long side.

7 Draw vector diagrams to show two-dimensional changes in:

- **a** displacement.
- **b** velocity.

8 Find the change in displacement and average velocity for the following movements. For each case, give your answer in both m and km for the displacement and m s^{-1} and km h^{-1} for the average velocity.

- **a** 100 km, N to 240 km, W in 2.0 h
- **b** 250 km, N45°E to 550 km, N45°W in 88 min
- **c** 350 m, S35°E to 475 m, S20°E in 17.5 min

9 Draw the following to scale.

- **a** The change of displacement of a body that moves from 10 m N25°W to 20 m N25°E
- **b** The change of velocity of a body that has an initial velocity of 20 m s^{-1} S40°W, and a final velocity of 20 m s^{-1} S65°W

10 Use vector diagrams to add the pairs of vectors shown in Figure 3.21. Label the resultant $\vec{R}$.

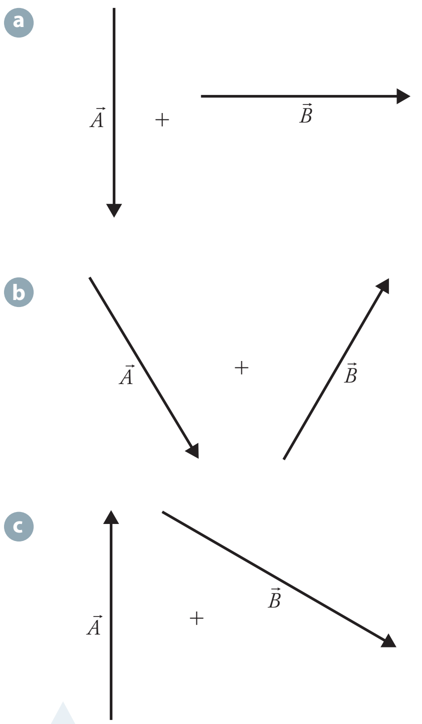


FIGURE 3.21 Add the vectors $\vec{A}$ and $\vec{B}$. Label the resulting vector $\vec{R}$.

11 Use vector diagrams to subtract the pairs of vectors shown in Figure 3.22. Label the resultant $\vec{R}$.

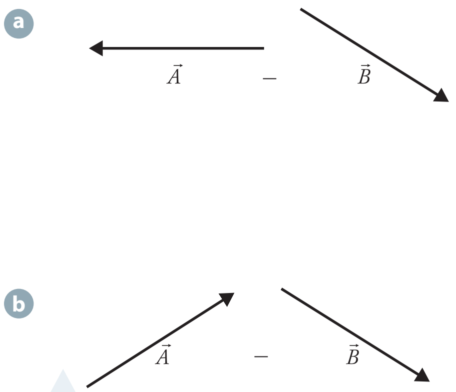


FIGURE 3.22 Subtract $\vec{B}$ from $\vec{A}$. Label the resultant vector $\vec{R}$.

12 In an orienteering event, a runner moves from a checkpoint at 200 m N30°E from the start to another checkpoint at 400 m S60°E. What is the change of displacement of the runner?

13 An ant walks 34 cm north-west, and then 52 cm east. Calculate its total displacement.

14 After a walk consisting of two segments, an ant has a displacement from its starting point of 1.25 m N25°E. The first part of its journey was a walk 2.00 m north-west. Calculate the length and direction of the second part of the walk.

15 A car is heading north-west at a speed of 60 km h^{-1}. Calculate the northerly and westerly components of its velocity.

16 A car is heading north-west at a speed of 60 km h^{-1}. Calculate the north-easterly component of its velocity. Explain your logic.

17 A car is heading S35°E at a speed of 75 km h^{-1}. Draw a scale vector diagram representing the car's velocity. Indicate the scale and label the axes. Work out the southerly and westerly velocity components using your diagram.

18 A toy car rolls the length of a corridor. The corridor is 2.0 m wide and 22.0 m long. The car goes from one end to the other, and also goes sideways by 1.2 m. If the toy car's speed is 10 m s^{-1}, what are the velocities parallel and perpendicular to the length of the corridor?

19 A girl is riding her bike north at 35 km h^{-1}. At the same time, she throws a tennis ball east at 20 m s^{-1}. Calculate the velocity of the tennis ball just after it leaves her hand.

20 A bird flies at 25 km h^{-1} in a north-easterly direction. It sees a cat and turns and flies at 56 km h^{-1} S35°W. Calculate $\Delta\vec{v}$ for the bird.

21 A girl (G) is sitting in a train travelling north at 35 km h^{-1}. She throws a tennis ball (B) to a friend across the aisle (east) at 20 m s^{-1}. Calculate $\vec{v}_{\text{B relative to G}}$.

» END-OF-MODULE REVIEW

MODULE ①: KINEMATICS

Answer the following questions.

1 A vehicle travels west at 100 km h^{-1} for 45 minutes. It stops for 15 minutes, and then resumes its journey west at 80 km h^{-1} for 20 minutes. After stopping for another half an hour, it returns to its first rest stop at 85 km h^{-1}.

- **a** What distance did the vehicle travel?
- **b** What is its final displacement?
- **c** What is its average speed?
- **d** What is its average velocity?

2 A spacecraft accelerates from 0 to 1000 km h^{-1} at 9.8 m s^{-2}.

- **a** How far does the spacecraft travel while doing this?
- **b** How long does it take?

3 At exactly 3:00 p.m., a stationary motorcycle begins to accelerate to 110 km h^{-1}, achieving this velocity in 15 s. A car that had been travelling at a constant 90 km h^{-1} was 0.6 km in front of the motorcycle at 3:00 p.m., and travelling in the same direction as the motorcycle.

- **a** How long does it take for the motorcycle to pass the car?
- **b** How far had the motorcycle travelled at this point?
- **c** What was the velocity of the car relative to the motorcycle?
- **d** What was the velocity of the motorcycle relative to the car?

4 A stone is dropped from the top of an 80-m cliff. After 2 s, the stone meets a helium-filled balloon that had previously been released from the bottom of the cliff. The balloon is ascending at a constant 2 m s^{-1}.

- **a** What is the relative velocity of the stone from the balloon's point of view?
- **b** What is the relative velocity of the balloon from the stone's point of view?
- **c** How far from the bottom of the cliff did this encounter take place?
- **d** At what time, relative to the stone's release, was the balloon released?

5 Two ships pass in the night, travelling in opposite directions ($t = 0$). Ship A is travelling at 20 knots, and ship B at 25 knots. After they have travelled for another two minutes, ship A blasts its horn. The speed of sound in air under prevailing conditions is 341 m s^{-1}, and 1 knot = 1.852 km h^{-1}.

- **a** How far apart are the two ships when the horn blasts ($t = 2$ minutes)?
- **b** How far apart are the two ships when ship B hears the horn?
- **c** At what time does ship B hear the horn?

6 Chen has ridden his bike east for 30 km and north for 12 km.

- **a** Explain why it is necessary to provide distance *and* direction information to describe his movements.
- **b** If the first leg of his journey took 1 hour and the second leg took 20 minutes, calculate the average speed for each leg of the journey.
- **c** Given the times in part **b**, calculate the average velocity for the entire ride, including the direction.

7 A student is adding vectors by plotting them on graph paper.

- **a** Explain why the student has to draw the vectors all to the same scale.
- **b** Explain the parallelogram rule and how it can help avoid errors when adding vectors graphically.
- **c** Explain how you would use a vector diagram to subtract one vector from another.
- **d** Explain how a diagram can be used to add and/or subtract any number of vectors, not just two.

8 An aeroplane has a velocity of 950 km h^{-1} N35°W.

- **a** Sketch the velocity vector on a suitable set of axes and add in its components to the north and west. Note the angle.
- **b** What is the northerly component of the aeroplane's velocity? What is the westerly component?
- **c** For how long must the plane keep flying to travel 2000 km north?
- **d** For how long must the plane keep flying to go 2000 km to the north-west (that is, N45°W)? (Hint – resolve the plane's velocity into components different from those you used in part **b**.)

9 A dog is running north-east at 10 m s^{-1}. It then turns and runs due north at 6.0 m s^{-1}.

- **a** For how long was the dog running north-east if it ran 150 m before turning?
- **b** If the dog ran north for 20 s, what was the total distance the dog ran?
- **c** Draw a vector diagram of the distances involved in the dog's journey, including the net displacement. Draw a second diagram of the velocities.
- **d** What is $\Delta\vec{v}$, the change in velocity (magnitude and direction), between the first leg and the second leg of the run? What was the average velocity for the whole journey?

10 Two girls are misbehaving on a train (train 1) and throwing a basketball back and forth along the aisle of a carriage. While they do this, the carriage passes under a bridge and another train (train 2) passes them, going the same direction but faster.

a List four frames of reference that might be used to describe this situation.

b An observer in train 2 looks across at train 1, and for a moment sees the basketball as stationary. What does this tell us about the velocities of train 1, train 2 and the basketball?

c From the frame of reference of the bridge, are the two trains moving in the same or different directions?

d From the frame of reference of train 1, are the two trains moving in the same or different directions? Are they moving at all?

DEPTH STUDY SUGGESTIONS

→ Research engine design for chemically-propelled spacecraft. When are constant-thrust engines appropriate, and list three spacecraft that have used them. When might variable-thrust engines be deployed?

→ The Centre for Plasmas and Fluids at the Australian National University developed the Dual-Stage 4-Grid (DS4G) thruster. What is its purpose, how is it accomplished, and what performance could it achieve?

→ Bullets, artillery shells and cannon balls all use chemical propulsion. What are the strengths and weaknesses of chemical propulsion in this context, and how does it compare to electrical propulsion of ammunition?

→ Drag racing uses a standard quarter-mile course. Research the final velocities of dragsters since the 1960s.

→ Research the value of the acceleration due to gravity on the Moon and the planets in the solar system. In each case, if a projectile is fired straight upwards at 300 m s^{-1}, draw a table of the distance it would travel to the point where its velocity is zero.

→ Investigate vectors in three dimensions. Vector decomposition becomes a bit trickier, but show some examples of how it can be done.

→ Using GPS, maps, and actual journeys, compare the distance you actually travel to the net displacement for some journeys you commonly take, such as the trip to your school or the shops.

→ Trace the use of vectors back through the history of science, looking at other attempts to tackle quantities with magnitude and direction.

→ Assemble a spreadsheet that can plot two vectors and their sum, or can resolve a vector into components.

→ Using skateboards, tennis balls tossed back and forth, and an open space such as a basketball court, explore some aspects of relative motion.

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» MODULE TWO

DYNAMICS

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4 Forces

INQUIRY QUESTION

How are forces produced between objects and what effects do forces produce?

OUTCOMES

Students:

- using Newton's Laws of Motion, describe static and dynamic interactions between two or more objects and the changes that result from:
  - a contact force
  - a force mediated by fields
- explore the concept of net force and equilibrium in one-dimensional and simple two-dimensional contexts using: (ACSPH050) ICT N
  - algebraic addition
  - vector addition
  - vector addition by resolution into components
- solve problems or make quantitative predictions about resultant and component forces by applying the following relationships: ICT N
  - $\vec{F}_{AB} = -\vec{F}_{BA}$
  - $\vec{F}_x = \vec{F} \cos \theta$, $\vec{F}_y = \vec{F} \sin \theta$
- conduct a practical investigation to explain and predict the motion of objects on inclined planes (ACSPH098) CCT ICT

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In the previous two chapters, we described the motion of objects and explored what happens when an object accelerates. In this chapter, we will look at the cause of acceleration, which is force. Forces make things move, stop and change direction. Whenever the state of motion of an object is changing, there must be a force acting on it. This means every time you move, a force is acting on you, and every time you move something, you exert a force on it. All your interactions with the world, from tapping a screen to making the air vibrate when you speak, involve forces.

Our current understanding of forces was largely developed by Sir Isaac Newton (1642–1727). Newton's three laws of motion describe how objects interact via forces, and what happens when an object experiences one or more forces. The unit of force, the newton, N, is named in honour of Newton. We will describe how forces are produced between objects and use Newton's laws to describe and predict the effects that forces produce.


Getty Images/Bettmann

FIGURE 4.1 Sir Isaac Newton

Sir Isaac Newton

Find out what other contributions Newton made to physics and mathematics.

4.1 Forces are interactions

General knowledge

When objects interact, they do so by exerting forces on each other. Forces are not 'contained' in an object, so we do not say that an object 'has' a force. Forces are an interaction, and they are exerted *on* one object *by* another object. When we write the symbol for a force, it is a good idea to use subscripts that say which object is exerting the force, and which object the force is exerted on. For example, if an object A is applying a force to object B, this is written as $\vec{F}_{\text{A on B}}$.

Forces can be categorised into two types: **contact forces** and **field forces**. Contact forces act when objects are touching each other (in contact). Field forces act whenever objects interact without touching. When you hold a book in your hand, you exert a contact force on the book with the skin of your hand. The book also experiences a **gravitational force** due to Earth, even though it is not touching it. This is a field force.

Contact forces

Everyday pushes and pulls are examples of contact forces. When you pick something up with your hand, or push on something, you are exerting a contact force on it. When you walk, you are exerting a contact force on the ground.

When you exert a contact force on a surface, such as pushing your foot against the ground to walk (as shown in Figure 4.2), we consider that force as having two components: the **normal force** and the **friction force**. You saw in chapter 3 how to resolve vectors into perpendicular components.

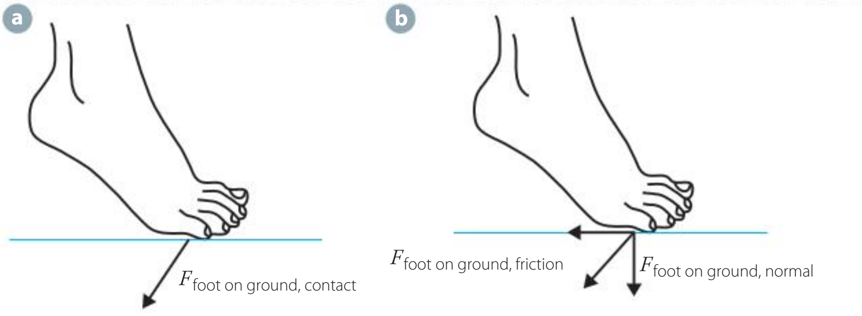


FIGURE 4.2 The contact force can be considered as the sum of a normal force and a friction force. **a** The total contact force; **b** The contact force broken up into normal force (perpendicular to the surface) and friction force (parallel to the surface)

The normal force acts *perpendicular to the surface* (normal means perpendicular in mathematics), and prevents your foot from moving into the ground. The normal force is due to interactions between atoms on the two surfaces – the sole of your foot and the surface of the ground. The normal force acts to prevent solid surfaces from moving into each other.

The friction force acts *parallel to the two surfaces*, and prevents your foot from sliding along the ground. The friction force is also due to interactions between atoms on the two surfaces. The friction force acts to prevent *relative* movement, or slipping, of two surfaces which are in contact. Note that friction doesn't prevent movement. Without friction, you couldn't walk or drive a car. Friction opposes the sliding of one surface against another.

Friction is described in detail in chapter 5.

Static friction acts to prevent sliding of one surface against another. **Kinetic friction** occurs when two surfaces are already sliding relative to one another, and it slows down the sliding.

Note that there is a limit to how large a normal or friction force a surface is capable of applying. If the limit of the normal force is exceeded, then one surface *will* move into the other. For example, a sharp stone may pierce your foot. If the limit of the static friction force is exceeded, then one surface will slide relative to the other. For example, if you try to run on wet slippery grass, you may skid and slide.

Terminal velocity

Find out what terminal velocity is, and how cats can survive falls from high windows.

Contact forces are also exerted by liquids and gases. When you swim, you exert a contact force on the water. Air resistance (drag) is the friction force of air on an object moving through the air.

Often we make the assumption that air resistance is very small, and ignore it. However, in many everyday situations (such as driving a car), air resistance is important. Knowing when to make approximations, such as ignoring friction or air resistance, *and knowing when not to do this*, is an important skill in physics.

Field forces

Contact and field forces

Objects that are not in contact can still interact and exert forces on each other. They do so by forces that are mediated by fields. Three fields will be described: the **gravitational field**, the **electric field** and the **magnetic field**.

If you hold a pencil and let it go, it will accelerate towards Earth. It does so because of the gravitational force that Earth exerts on it, as shown in Figure 4.3. This force is mediated by the gravitational field of Earth. The pencil is not in contact with Earth, but it is within the gravitational field of Earth. Earth creates the field, and exerts a force via this field. Hence, we say the force is *mediated* by the field. When you talk to someone on your phone, your communication is mediated by the phone – you are doing the talking, but the communication is enabled by the phone.

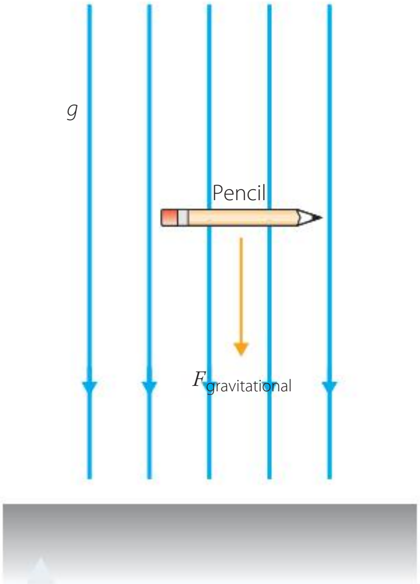


FIGURE 4.3 A pencil released above the ground will accelerate downwards due to the gravitational force. The gravitational force is mediated by the gravitational field.

Measuring Earth's gravitational field

The GRACE project is measuring variations in Earth's gravitational field. How do the two satellites do this?

All objects with mass have a gravitational field – the bigger the mass is, the stronger the field. You have already met the acceleration, $\vec{g}$, due to the gravitational field in the previous chapter. In chapter 3, we expressed $\vec{g}$ in units of m s^{-2}. We can also write $\vec{g}$ in units of N kg^{-1}, which is a useful way to remember that $\vec{g}$ is also the gravitational field strength close to the surface of Earth. The gravitational force, $\vec{F}_{\text{Earth on object}}$, on an object close to the surface of Earth, is:

$$\vec{F}_{\text{gravitational}} = \vec{F}_{\text{Earth on object}} = m\vec{g}$$

where m is the mass of the object.

The units of $\vec{g}$ are N kg^{-1} and the units of mass are kg, so this gives the correct units for force. N N kg^{-1} is the same as m s^{-2}. It is very useful to be able to convert between units when you are solving problems.

If you rub a balloon on your hair, charged particles are transferred from your hair to the balloon. The hair and balloon both end up being charged. Charged objects have an electric field. This electric field exerts a force on any charged object. Hence, the electric field of the balloon exerts a force on your hair. When you hold the balloon close to your hair, you can feel (and see, if your hair is long enough) the hair pulling towards the balloon. You can see this happening in Figure 4.4. This is because the hair and balloon have opposite charges and exert an attractive force on each other via the electric field. Like charges exert a repulsive force, also via a field.

The electric field and electrostatic forces are discussed in detail in chapter 12.

Moving charged objects and magnetic materials have a magnetic field. Magnetic fields exert force on moving charged objects and on magnetic materials. If you bring the north and south poles of a magnet close together, they will pull together. If you try to bring two north poles or two south poles together, they will push apart.

Getty Images/Digital Light Source

FIGURE 4.4 The person's hair is attracted to the balloon because the hair and balloon are both charged. This force is mediated by the electric field.

Newton's laws of motion

The effects of forces can be described and predicted using Newton's laws of motion. Newton's laws apply in both **static** and **dynamic** situations. Statics is the study of systems where the objects are not moving. Dynamics refers to systems where one or more components are moving, and often accelerations are involved.

Newton's first law

Newton's first law states that, in the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity. In other words, when no force acts on an object, the acceleration of the object is zero.

The magnetic field is described in detail in chapter 14.

This statement was enormously important for two reasons.

1. Newton's first law tells us that if there is no force acting, there will be no acceleration. It explicitly links the idea of force to acceleration. This leads us to the definition of force as something that causes acceleration. Hence, Newton's first law is fundamental to our understanding of what forces are.
2. Newton's first law contradicted the writings of Aristotle. This lead to a revolution in our understanding of the laws of nature.

For nearly 2000 years, the writings of the Greek philosopher Aristotle (384–322 BCE) were taught in schools and universities. Their significance was such that they came to be considered fundamental to the understanding of the natural world. The Aristotelian view of motion was that an object 'contained' force or 'impetus', and it moved until the 'impetus' ran out.

Newton and others, including Galileo, disagreed with the Aristotelian view. Newton's first law contradicts this Aristotelian view, and was a direct response to it.

One of the reasons the Aristotelian view was considered correct for so long was that it *appears* to match our experience. We observe that moving objects on Earth do not keep moving forever – they slow down and stop. In the Aristotelian model, they 'run out of impetus' and so stop moving. In the Newtonian model, if an object is slowing down, it is accelerating, so there is a force acting on it to cause this acceleration. Usually, this force is friction.

Note that Newton's first law deals with the case of a single object with no forces acting on it. Newton's second law, which we will discuss next, tells us what happens when one or more forces are acting.

Newton's laws and road safety

Explain how seatbelts and airbags work in terms of Newton's first law.

Newton's second law

Newton's second law tells us what happens when one or more forces act on an object. It quantifies the relationship between force and acceleration that was introduced in the first law.

Newton's second law tells us that, when a force acts on an object, the acceleration of the object as a result of that force is:

$$\vec{a}_{\mathrm{A}} = \frac{\vec{F}_{\mathrm{on\ A}}}{m_{\mathrm{A}}}$$

where m_{A} is the mass of the object. Or, as it is often written:

$$\vec{F}_{\mathrm{on\ A}} = m_{\mathrm{A}}\vec{a}_{\mathrm{A}}$$

Note that both force and acceleration are vectors, and the direction of the acceleration is the same as that of the force.

Newton's second law also tells us about the units of force, N. For the equation $\vec{F}_{\mathrm{on\ A}} = m_{\mathrm{A}}\vec{a}_{\mathrm{A}}$ to be dimensionally correct, then the units on each side must be the same:

$$[\mathrm{N}] = [\mathrm{kg}][\mathrm{m\ s}^{-2}]$$

Measuring mass

What is inertial mass? Is it different to gravitational mass? Find out what it is, and how it is measured.

So, in fundamental units, $1\ \mathrm{N} = 1\ \mathrm{kg\ m\ s}^{-2}$.

We will apply Newton's second law to solve a range of problems involving equilibrium situations (including static equilibrium) in the next two sections, and to solve problems in dynamic situations in the following chapter.

Newton's third law

Newton's third law

Newton's third law reminds us that forces are interactions between objects. *Both objects* are affected by an interaction. If you push the tips of your two index fingers together, you will see the skin compress on both fingers and you will feel the force on both fingers.

Newton's third law states that, whenever an object A exerts a force on object B, then object B exerts an equal and opposite force on object A. We write this mathematically as:

$$\vec{F}_{\mathrm{A\ on\ B}} = -\vec{F}_{\mathrm{B\ on\ A}}$$

or sometimes more simply as

$$\vec{F}_{\mathrm{AB}} = -\vec{F}_{\mathrm{BA}}$$

These two forces, $\vec{F}_{\mathrm{A\ on\ B}}$ and $\vec{F}_{\mathrm{B\ on\ A}}$, are called a Newton's third law force pair. Also they are sometimes referred to as an action–reaction pair. As the two forces are part of a single interaction, either one could be the action or the reaction. When you push your two index fingers together, both push and both are pushed.

There are some important things to remember about Newton's third law force pairs.

1 The forces act on different objects. If a pair of forces are acting on the same object, then they *cannot* be a Newton's third law force pair. Always writing forces with subscripts will help you remember this, and help you identify whether two forces are a Newton's third law force pair or not.

2 The forces are of the same type, because they are the two parts of a single interaction. The Newton's third law force pair to a gravitational force is *always* a gravitational force. The Newton's third law force pair to a contact force is *always* a contact force. A contact force can *never* be the Newton's third law force pair to a gravitational force.

3 As already stated, the two forces are equal in magnitude but act in opposite directions.

Newton car

Build your own 'Newton car' and use it to investigate Newton's third law.

Figure 4.5 shows two Newton's third law force pairs. The satellite and Earth exert equal and opposite gravitational forces on each other. The gravitational force is due to the gravitational fields created by the masses of Earth and the satellite. Even though the masses of the two objects are very different, both experience a force of the same magnitude. Similarly, the man pushing on the wall experiences an equal and opposite push by the wall. The man pushes the wall to the right, and the wall pushes the man to the left. In this case, both forces are contact forces – they are experienced by the two different objects, and act in opposite directions.

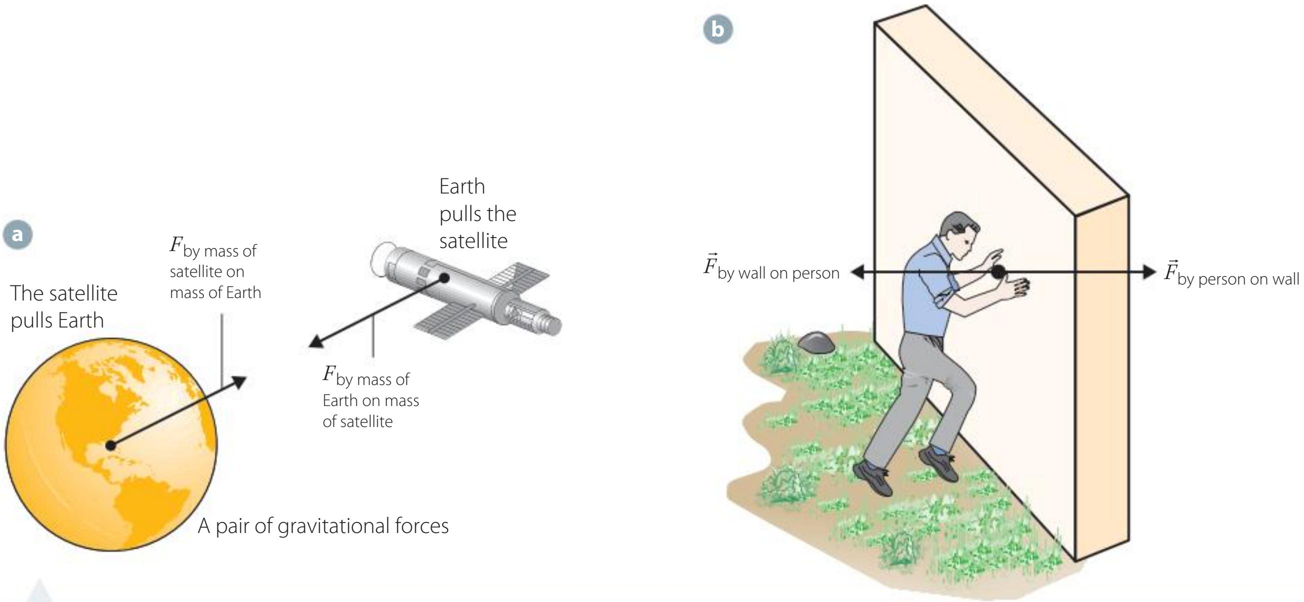


FIGURE 4.5 Two Newton's third law force pairs. **a** A pair of field forces: the gravitational force of Earth on a satellite, $F_{\text{by mass of Earth on mass of satellite}}$, and its Newton's third law force pair, the gravitational force of the satellite on Earth, $F_{\text{by mass of satellite on mass of Earth}}$; **b** A pair of contact forces: the person pushes on the wall, $F_{\text{by person on wall}}$, and the wall pushes on the person, $F_{\text{by wall on person}}$

WORKED EXAMPLE 4.1

Figure 4.6 shows a cup sitting on a table. The forces acting on the cup are the gravitational force of Earth pulling downwards on the cup, and the normal force of the table pushing upwards on the cup. Identify the Newton's third law force pairs to each of these forces.

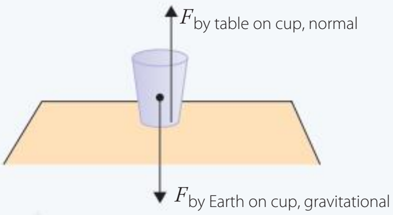


FIGURE 4.6 The forces acting on a cup sitting at rest on a table

ANSWER	**LOGIC**
$\vec{F}_{\text{gravitational}} = \vec{F}_{\text{Earth on cup}}$	▪ Write the gravitational force in the form $\vec{F}_{\text{A on B}}$.
$\vec{F}_{\text{Earth on cup}} = -\vec{F}_{\text{cup on Earth}}$	▪ Write Newton's third law using the forces involved in the question.
$\vec{F}_{\text{cup on Earth}}$ is the Newton's third law force pair to the gravitational force of Earth on the cup. This must be a gravitational force.	▪ Identify the Newton's third law pair.
$\vec{F}_{\text{normal}} = \vec{F}_{\text{table on cup}}$	▪ Write the normal force in the form $\vec{F}_{\text{A on B}}$.
$\vec{F}_{\text{table on cup}} = -\vec{F}_{\text{cup on table}}$	▪ Write Newton's third law using the forces involved in the question.

ANSWER	LOGIC
$\vec{F}_{\text{cup on table}}$ is the Newton's third law force pair to the normal force of the table on the cup. This must be a normal (contact) force.	▪ Identify the Newton's third law pair.
The gravitational force $\vec{F}_{\text{cup on Earth}}$ is the Newton's third law force pair to the gravitational force of Earth on the cup, and the contact force $\vec{F}_{\text{cup on table}}$ is the Newton's third law force pair to the normal force of the table on the cup.	▪ State your final answer.

TRY THESE YOURSELF

1 The cup described above is pushed along the table. It experiences two horizontal forces: the push from someone's hand, and friction from the surface of the table. Identify the Newton's third law force pair to each of these forces.

2 Explain why the normal force and the gravitational force on the cup cannot be a Newton's third law force pair.

KEY CONCEPTS

- Objects interact via forces.
- Forces can be contact forces (objects are touching) or field forces (objects are not touching).
- Newton's first law says that if no force acts on an object, it will not accelerate. It will move in a straight line with constant speed, or remain at rest.
- Newton's first law relates force to acceleration, and so provides a definition of force.
- Newton's second law quantifies the relationship between force and acceleration: $\vec{F} = m\vec{a}$. The direction of force and acceleration are the same.
- Newton's third law says that when object A exerts a force on object B, then object B exerts an equal and opposite force on object A: $\vec{F}_{AB} = -\vec{F}_{BA}$
- Newton's third law force pairs act on different objects, and are always the same type of force.

CHECK YOUR UNDERSTANDING

4.1

1 Describe the difference between field forces and contact forces.

2 Identify three different field forces.

3 Identify the two components of the contact force that one solid surface exerts on another. Draw a diagram to explain your answer.

4 A rolling ball gradually slows down and comes to a stop. How would Aristotle have explained this observation? How would Newton have explained it?

5 Is a force caused by an acceleration? Explain your answer with reference to Newton's second law.

6 Explain the following with reference to Newton's first law.
 a An object at rest does not spontaneously move.
 b A person standing on a bus stumbles forwards when the bus stops suddenly.
 c A seated person not wearing a seatbelt is not 'thrown forwards' towards the windscreen in a car crash, but they do hit the windscreen.

7 Apply Newton's second law to explain why it takes longer for a car towing a trailer full of gravel to come to a stop than the same car without the trailer.

8 Which applies the greater gravitational force: Earth on you, or you on Earth? Explain your answer.

9 Identify the Newton's third law force pairs to the following forces, and write the force pairs in the form $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$.
 a A car's tyres push backwards against the road.
 b A charged balloon is repelled by a second charged balloon.
 c The tip of a compass needle is attracted towards the north pole of Earth.

10 When you take a step forward, you push your foot down and backwards against the ground. Identify the reaction force to the contact force that you exert against the ground. In what direction does it act?

4.2 Net force in one and two dimensions

In this section, we will look at how to solve problems and make quantitative predictions about resultant and component forces. To do this we will need to be able to add forces in one and two dimensions. In one dimension, we can add forces algebraically. In two or more dimensions, we must add forces vectorially.

Newton's second law was introduced in the previous section as $\vec{F} = m\vec{a}$, which can be rearranged as $\vec{a} = \frac{\vec{F}}{m}$. This is the case when only a single force acts on an object. However, in almost all situations, there are at least two forces acting, and often more. In these situations, we need to consider the **net force** (total force) that acts on an object.

The more general form of Newton's second law states that the acceleration of an object depends upon the net force that acts on that object:

$$\vec{a}_{\mathrm{A}} = \frac{\vec{F}_{\text{net on A}}}{m_{\mathrm{A}}} = \frac{\Sigma \vec{F}_{\text{on A}}}{m_{\mathrm{A}}}$$

Adding forces to find the net force

The net force that acts on an object is the *vector sum* of all the forces that are acting on that object:

$$\vec{F}_{\text{net, A}} = \Sigma \vec{F}_{\mathrm{A}}$$

Consider the case of the block shown in Figure 4.7. This block is subject to two forces, both acting towards the right. If we call the right the positive direction, then the net force acting on the block is $\vec{F}_{\text{net}} = (+4.0\text{ N}) + (+3.0\text{ N}) = 4.0\text{ N} + 3.0\text{ N} = +7.0\text{ N}$. Usually we drop the + sign, and just write this as 7.0 N. This block will accelerate to the right.

FIGURE 4.7 A block subject to two forces acting in the same direction

Now consider the block shown in Figure 4.8. This block is also subject to two forces, one acting towards the right and the other to the left. If right is the positive direction, then the 3.0 N force that acts left must be negative. The net force now acting on the block is $\vec{F}_{\text{net}} = (+4.0\text{ N}) + (-3.0\text{ N}) = 4.0\text{ N} - 3.0\text{ N} = +1.0\text{ N}$. This block will also accelerate to the right, but at a much lower rate than the block shown in Figure 4.7.

FIGURE 4.8 A block subject to two forces acting in opposite directions

Imagine that the block is again subjected to the same two forces, but now the forces are acting at right angles to each other, as in Figure 4.9.

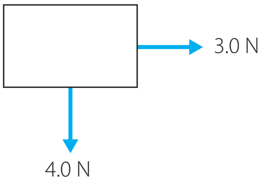


FIGURE 4.9 A block (seen from above) subject to two forces, acting at right angles to each other

We cannot simply add the forces algebraically now – we need to treat them as vectors. However, because these forces are at right angles, we can treat them as the two perpendicular components of the net force, as shown in Figure 4.10. The net force (the sum of the forces) can be found using Pythagoras's theorem or using trigonometry.

Using Pythagoras's theorem:

$$\vec{F}_{\text{net}} = \sqrt{(3.0\text{ N})^2 + (4.0\text{ N})^2} = 5.0\text{ N}$$

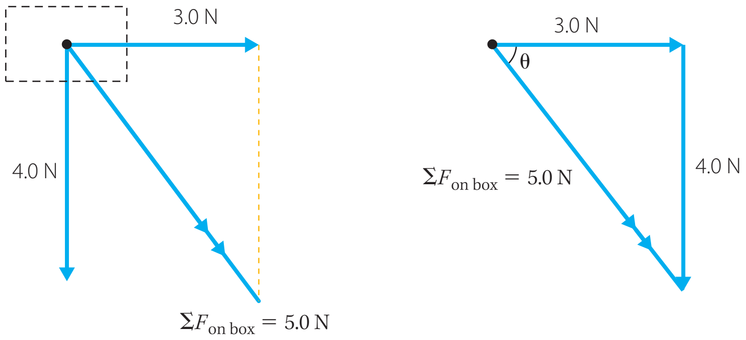


FIGURE 4.10 Vector addition of forces acting at right angles

Note that the net force now is less than when the two forces were acting in the same direction, but greater than when they acted in opposite directions.

Using trigonometry, we can find the angle, θ, at which the force acts:

$$\tan\theta = \frac{4.0\ \text{N}}{3.0\ \text{N}}$$

so

$$\theta = \tan^{-1}\left(\frac{4.0\ \text{N}}{3.0\ \text{N}}\right) = 53°$$

WORKED EXAMPLE 4.2

Two tugboats apply forces on a barge through ropes, as shown in Figure 4.11. What is the resultant force due to the two barges?

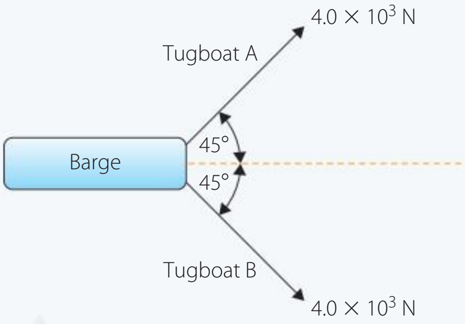


FIGURE 4.11 Two tugboats pull on a barge

ANSWER	LOGIC
$\vec{F}_A = 4.0 \times 10^3\ \text{N};\ \vec{F}_B = 4.0 \times 10^3\ \text{N}$	- Identify the relevant data in the question
The two forces are acting at 90° to each other.	- Recognise that the forces are perpendicular.
$F_{\text{net}} = \sqrt{F_A^2 + F_B^2}$ $= \sqrt{(4.0 \times 10^3\ \text{N})^2 + (4.0 \times 10^3\ \text{N})^2}$ $= 5.66 \times 10^3\ \text{N}$ $F_{\text{net}} = 5.7 \times 10^3\ \text{N}$	- Apply Pythagoras's theorem to find the net force and write an appropriate equation. - Substitute the known values with units and calculate the answer. - Calculate the answer. - State the final answer with correct units and appropriate significant figures.

TRY THIS YOURSELF

What would the magnitude and direction of the net force be if tugboat B doubled the force with which it pulled?

Force diagrams

Force diagrams, such as those in Figures 4.7 and 4.8, are very useful for helping us to understand the forces acting on an object. This is particularly true in two dimensions, as shown in Figures 4.9 and 4.10.

When you draw a force diagram, a force is drawn so the tail is at the point at which the force acts. The length of the arrow is proportional to the magnitude of the force. The direction of the arrow is in the direction of the force. Look again at Figure 4.9 to see how this is done.

Only the forces acting on the object of interest are shown. Forces exerted *by* the object on other things are not shown. This means you cannot show a Newton's third law force pair on the force diagram for a single object. In Figure 4.11, the forces of the tugboats acting on the barge are shown, but *not* the forces exerted by the barge on the tugboats.

The net force is not an actual force (like gravity or friction) – it is a sum of forces. The net force is not generally drawn on a force diagram. If you do include it, make sure it is a different sort of arrow so it is not confused with the actual forces acting. Look again at Figure 4.10 and see how the net force arrow is a different style to the arrows representing actual forces. Note that this net force arrow is *not* shown on the force diagram for the block in Figure 4.9.

Vector decomposition and addition

Resolving forces into components

When we are adding forces that are not perpendicular, we generally need to resolve them into perpendicular components. Recall that you have already done this for velocities when you studied kinematics.

Sometimes we break forces into vertical and horizontal components. Sometimes we break forces into components parallel and perpendicular to a surface. Which we choose will depend on the problem we are trying to solve.

Resolving forces into components

Use this physics applet to investigate what happens to the net force when you vary the angle between the two forces being added.

Figure 4.12 shows a person pulling a suitcase. The person exerts a force of magnitude F at an angle θ to the horizontal. Using trigonometry, we can decompose this force into horizontal and vertical components. The horizontal component, F_x, is given by:

$$F_x = F\cos\theta$$

and the vertical component by:

$$F_y = F\sin\theta$$

The two components add to give the total force, $\vec{F}$, which has magnitude F. Using Pythagoras's theorem, we can find the magnitude of the force from the magnitudes of the components:

$$F = \sqrt{F_x^2 + F_y^2}$$

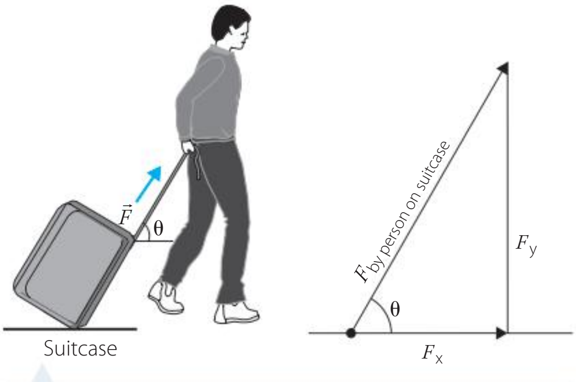


FIGURE 4.12 The force exerted by a person on a suitcase, decomposed into components

WORKED EXAMPLE 4.3

Phil is racing to get to the airport luggage check-in before his flight closes. He is dragging his heavy suitcase, as shown in Figure 4.12, and is wishing he'd just packed carry-on luggage instead. He exerts a force of 30 N at an angle of 60° to the horizontal on the suitcase. Calculate the horizontal and vertical components of this force.

ANSWER	LOGIC
$\theta = 60°$; $F = 30\ \text{N}$	▪ Identify the relevant data in the question.
$F_x = F\cos\theta$ $= 30\ \text{N}\cos 60°$ $= 15\ \text{N}$	▪ Write the expression for the horizontal component of the force. ▪ Substitute the known values with units. ▪ Calculate the answer.
$F_y = F\sin\theta$ $= 30\ \text{N}\sin 60°$ $= 26\ \text{N}$	▪ Write the expression for the vertical component of the force. ▪ Substitute the known values with units. ▪ Calculate the answer.
$F_x = 15\ \text{N}$; $F_y = 26\ \text{N}$	▪ State the final answer with correct units and appropriate significant figures.

TRY THESE YOURSELF

1 Calculate the horizontal and vertical components of the force Phil exerts if the magnitude of the force is 50 N and it acts at an angle of 30° to the horizontal.

2 Describe what happens to the two components of the force if Phil gradually increases the angle at which he pulls until he is pulling directly upwards.

Adding forces using perpendicular components

Once we have decomposed the forces into components, we can add them to find the net force. All the x components are added to find the total x component, and all the y components are added to find the total y component of the net force.

We first break each force into its component parts, as described above:

$$F_{1x} = F_1 \cos\theta_1, F_{2x} = F_2 \cos\theta_2 \ldots$$

$$F_{1y} = F_1 \sin\theta_1, F_{2y} = F_2 \sin\theta_2 \ldots$$

Then we add all the x components to get the x component of the net force:

$$F_{\text{net},x} = F_{1x} + F_{2x} + \ldots$$

and add all the y components to get the y component of the net force:

$$F_{\text{net},y} = F_{1y} + F_{2y} + \ldots$$

We can then find the magnitude of the total force from:

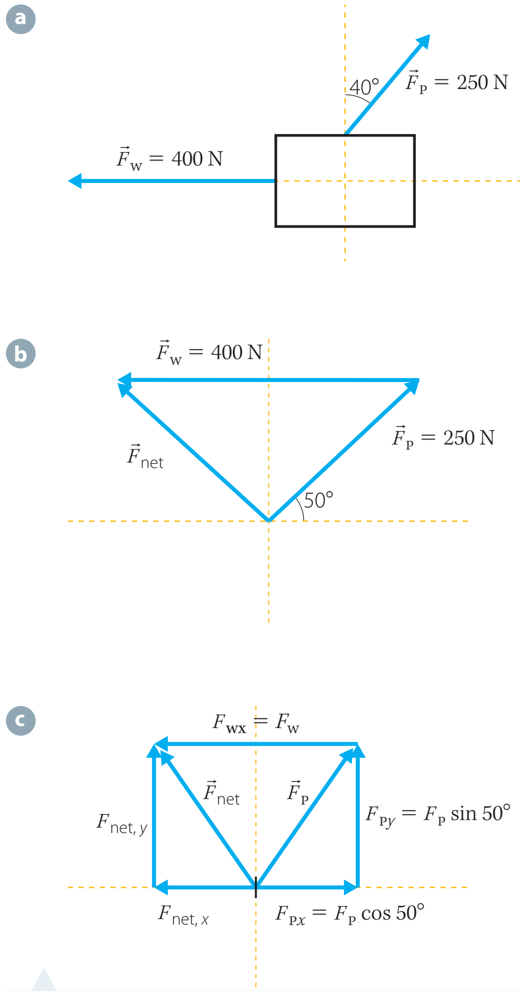
$$F_{\text{net}} = \sqrt{(F_{\text{net},x})^2 + (F_{\text{net},y})^2}$$

and find the angle at which it acts using trigonometry:

$$\tan\theta_{\text{net}} = \frac{F_{\text{net},y}}{F_{\text{net},x}}$$

WORKED EXAMPLE (4.4)

Eleanor is taking her boat out on the river to do some fishing. Due to the river current, she has to steer the boat at an angle to the direction she actually wants to go. The propeller exerts a force, $\vec{F}_P$, on the boat, with magnitude 250 N pointing N40°E. The water exerts a force, $\vec{F}_W$, with magnitude 400 N pointing directly west. What is the magnitude of the force exerted on the boat?

ANSWER	**LOGIC**
 **FIGURE 4.13** **a** A force diagram showing the two forces; **b** Vector addition of the two forces; **c** Breaking the forces into components and adding the components	- Draw force diagrams to visualise the situation.
North is y direction and east is x direction.	- Choose coordinate axes.
$F_{Px} = F_P \cos\theta = 250\text{ N} \cos 50° = 161\text{ N}$ $F_{Py} = F_P \sin\theta = 250\text{ N} \sin 50° = 192\text{ N}$ $F_{Wx} = F_W \cos\theta = 400\text{ N} \cos 180° = -400\text{ N}$ $F_{Wy} = F_W \sin\theta = 400\text{ N} \sin 180° = 0\text{ N}$	- Decompose the two forces into x and y components.
$F_{net,x} = F_{Px} + F_{Wx} = 161\text{ N} - 400\text{ N} = -239\text{ N}$ $F_{net,y} = F_{Py} + F_{Wy} = 192\text{ N} + 0\text{ N} = 192\text{ N}$	- Add the components to find the x and y components of the net force.
$F_{net} = \sqrt{\left(F_{net,x}\right)^2 + \left(F_{net,y}\right)^2}$ $= \sqrt{(-239\text{ N})^2 + (192\text{ N})^2}$ $= 307\text{ N}$ $F_{net} = 310\text{ N}$	- Use Pythagoras's theorem to find the magnitude of the net force. - Substitute known values with units. - Calculate the final answer. - State the final answer with correct units and appropriate significant figures.

TRY THESE YOURSELF

1. For the situation in Worked example 4.4, calculate the angle at which the net force acts.
2. A wedge-tailed eagle is swooping. It experiences a gravitational force of 45 N directly downwards, and a force due to the air of 25 N at an angle of 45° above the horizontal. Calculate the net force acting on the eagle, and the angle at which it acts.

Applying Newton's third law

Remember that forces are interactions, so forces come in pairs. Whenever an object exerts a force on something, it experiences an equal and opposite force.

Newton's third law explains how a propeller can make a boat move. In Worked example 4.4, the propeller is turned by the engine. The turning blades of the propeller push on the water, pushing it backwards away from the boat. According to Newton's third law, the water exerts an equal and opposite force on the propeller, pushing it forwards. As the propeller is attached to the boat, this pushes the boat forwards. The propeller doesn't work unless it has something to push against.

Newton's third law also helps us understand why we need friction between our feet and the ground to walk. When you push backwards against the ground with your foot, you exert a friction force against the ground. From Newton's third law, the ground exerts an equal and opposite friction force forwards on you. It is this friction force that pushes you forwards when you walk. Without friction between your feet and the ground, you wouldn't be able to walk.

WORKED EXAMPLE 4.5

The foot in Figure 4.14 exerts a normal force of 600 N downwards on the ground and a friction force of 300 N to the left on the ground. Calculate the total contact force exerted by the ground on the foot, assuming the foot does not slip.

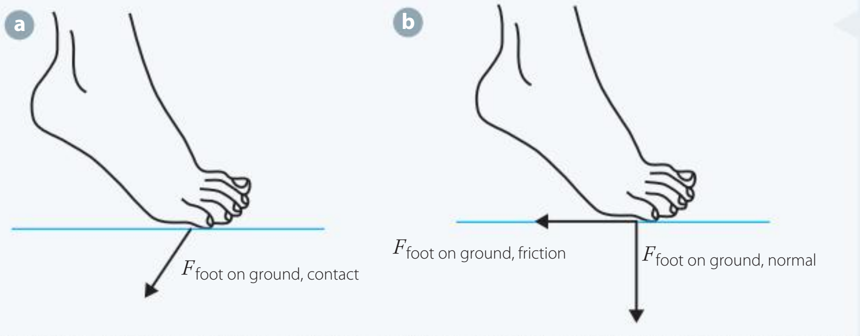


FIGURE 4.14 The contact force can be considered as the sum of a normal force and a friction force. **a** The total contact force; **b** The contact force broken up into normal force (perpendicular to the surface) and friction force (parallel to the surface)

ANSWER	LOGIC
We take the positive *x* direction to be right and the positive *y* direction to be up.	▪ Choose coordinate directions.
$\vec{F}_{\text{foot on ground, friction}} = F_x = -300\text{ N}$ $\vec{F}_{\text{foot on ground, normal}} = F_y = -600\text{ N}$	▪ Identify the relevant data in the question.
$\vec{F}_{\text{ground on foot}} = -\vec{F}_{\text{foot on ground}}$	▪ Write Newton's third law for the situation in the question.

ANSWER	**LOGIC**
$\vec{F}_{\text{ground on foot, friction}} = -F_x = +300\text{ N}$ $\vec{F}_{\text{ground on foot, normal}} = -F_y = +600\text{ N}$	- Apply Newton's third law.
$F_{\text{contact}} = \sqrt{F_x^2 + F_y^2}$	- Write the expression for the total contact force.
$= \sqrt{(300\text{ N})^2 + (600\text{ N})^2}$	- Substitute the known values with units.
$= 670.8\text{ N}$	- Calculate the answer.
$F_{\text{contact}} = 671\text{ N}$	- State the final answer with correct units and appropriate significant figures.

TRY THESE YOURSELF

1 For the situation in Worked example 4.5, calculate the angle at which the contact force exerted by the ground on the foot acts.

2 Repeat Worked example 4.5 for a normal force of 450 N and a friction force of 450 N.

KEY CONCEPTS

- When more than one force acts on an object, its acceleration is determined by the net force, $\vec{F}_{\text{net}}$.
- The net force on an object is the vector sum of all forces acting on that object: $\vec{F}_{\text{net on A}} = \Sigma \vec{F}_{\text{on A}}$.
- Newton's second law states that $\vec{a}_{\text{A}} = \dfrac{\vec{F}_{\text{net on A}}}{m_{\text{A}}}$.
- Forces are vectors with magnitude and direction.
- Forces can be broken into components: $F_x = F\cos\theta$ and $F_y = F\sin\theta$.
- Forces can be added by adding them up component-wise: $F_{\text{net},\,x} = F_{1x} + F_{2x} + \ldots$ and $F_{\text{net},\,y} = F_{1y} + F_{2y} + \ldots$
- The magnitude of the net force is found using Pythagoras's theorem: $F_{\text{net}} = \sqrt{\left(F_{\text{net},\,x}\right)^2 + \left(F_{\text{net},\,y}\right)^2}$.
- The net force acts at an angle $\tan\theta_{\text{net}} = \dfrac{F_{\text{net},\,y}}{F_{\text{net},\,x}}$.
- Newton's third law says if object A exerts a force on object B, then object B exerts an equal and opposite force on object A: $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$.
- Force diagrams are a useful way of representing forces on an object. They show all the forces, and only the forces, acting *on* the object.

CHECK YOUR UNDERSTANDING

4.2

1 In a tug-of-war competition, team A exerts a force of 200 N to the left on the rope, and team B exerts a force of 300 N to the right. Calculate the net force acting on the rope, and give its direction.

2 A force of 30 N acts in a direction N30°E on an object with mass 100 kg at rest on a smooth horizontal surface.

- **a** Calculate the northerly component of the force.
- **b** Calculate the easterly component of the force.

3 A naughty dog is trying to pull a sock off a washing line. The dog exerts a force with a horizontal component of magnitude 100 N and vertical component of 50 N.

- **a** Draw a diagram showing the force and its components.
- **b** Calculate the magnitude of the force.
- **c** Calculate the angle to the horizontal at which the force acts.

4 When vacuuming the living room, you need to clean under the couch. You exert a force on the couch of 500 N, at an angle of 65° above the horizontal. Describe the force that the couch exerts on you. Give its magnitude and direction.

5 Two forces, of magnitudes 5 N and 15 N, act on an object.

- **a** What is maximum possible net force on the object?
- **b** What is the minimum possible net force?
- **c** Draw a diagram for each case.

6 Two children push a shopping trolley. Marcus pushes forwards with a force of 100 N, and Laurence pushes to the right with a force of 150 N. Calculate the net force acting on the trolley.

7 When an aeroplane flies, its engines provide a forwards force called thrust, and its wings provide an upwards force called lift. The lift force acts perpendicular to the wings. It is also subject to drag from the air when flying, which acts in the opposite direction to the motion of the plane.

Draw force diagrams for an aeroplane in each of the following situations.

- **a** Sitting on the tarmac as passengers board
- **b** Ascending at an angle θ at constant velocity
- **c** Cruising at constant speed at a fixed altitude
- **d** Descending at an angle θ at constant velocity

Remember that the lengths of the force arrows should indicate relative sizes of forces.

8 Three dogs are fighting over a frisbee. One dog is pulling in a northerly direction with a force of 100 N, the second dog is pulling in an easterly direction with a force of 250 N, and the third dog is pulling south-east with a force of 150 N. Calculate the net force acting on the frisbee and give its direction.

9 Determine the net force acting on the object in each of the situations shown in Figure 4.15.

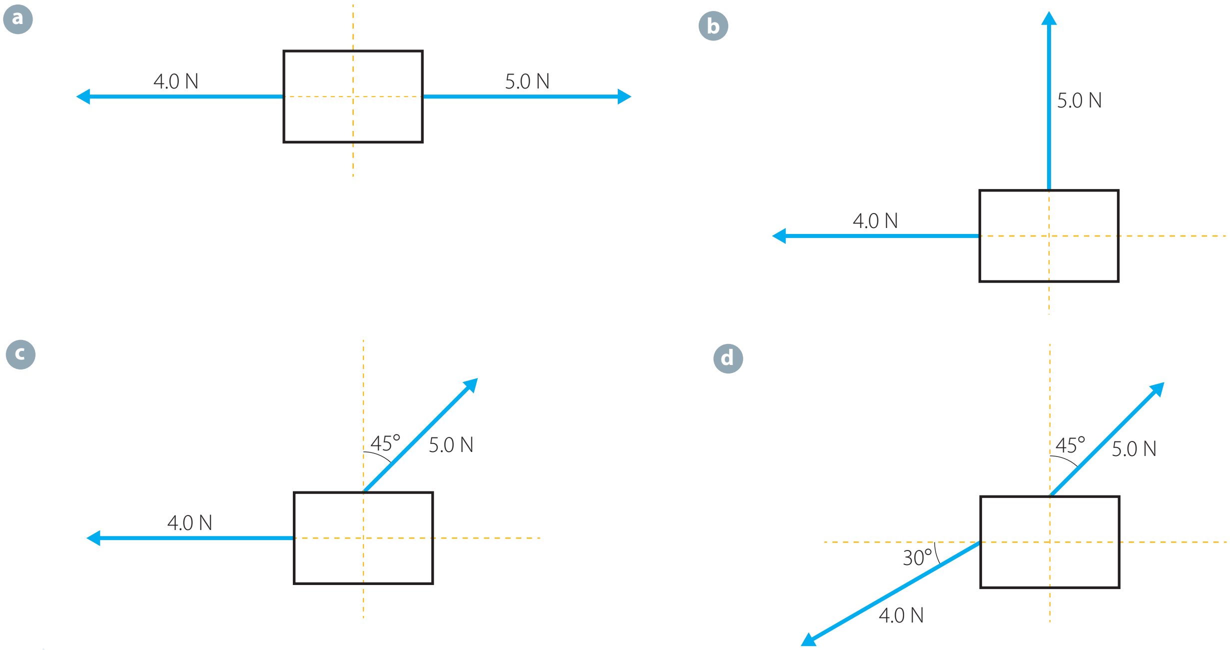


FIGURE 4.15 What is the net force on the object in each of these situations?

4.3 Zero net force: equilibrium in one and two dimensions

Once we have found the net force acting on an object, we can calculate its acceleration using Newton's second law. When all the forces acting add to zero, the acceleration is zero. This is called **equilibrium**.

When an object is at rest or moving with constant velocity, it has no acceleration. If the acceleration of an object is zero, then the net force acting on the object must also be zero. We call this condition equilibrium. Note that an object in equilibrium is not necessarily static.

An object can be in equilibrium if no forces at all are acting, but usually it is because all the forces acting on it balance to produce a zero net force.

Consider a cup sitting on a table, as shown in Figure 4.16a. The cup is at rest, so the forces acting it must balance:

$$\vec{F}_{\text{table on cup}} + \vec{F}_{\text{Earth on cup}} = 0$$

or

$$\vec{F}_{\text{table on cup}} = -\vec{F}_{\text{Earth on cup}}$$

The two forces, the normal force and the gravitational force, are equal in this case and act in opposite directions, but they are *not* a Newton's third law force pair. We can see this immediately from the subscripts – both forces act on the same object, the cup. Furthermore, the forces are of different types – one is a gravitational force (a field force), and the other is a contact force.

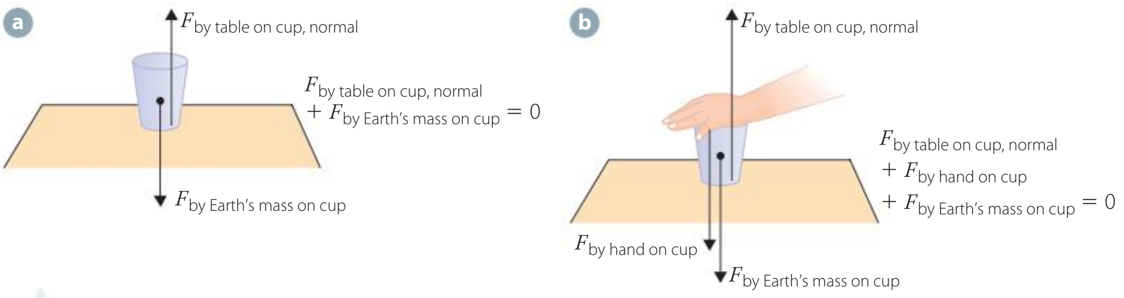


FIGURE 4.16 The cup is at rest, and so it is in equilibrium. The net force acting on the cup is zero. **a** The gravitational force and normal force balance each other. **b** The normal force increases so that it balances the gravitational force *and* the push by the hand.

Now consider what happens if you push down on the cup, as shown in Figure 4.16b. The total downwards force has increased. The gravitational force has not changed, but there is an additional force due to the hand. Now:

$$\vec{F}_{\text{table on cup}} + \vec{F}_{\text{Earth on cup}} + \vec{F}_{\text{hand on cup}} = 0$$

or

$$\vec{F}_{\text{table on cup}} = -\left(\vec{F}_{\text{Earth on cup}} + \vec{F}_{\text{hand on cup}}\right)$$

We can see that the normal force has increased, as it must now balance the combination of the gravitational force on the cup and the push downwards by the hand.

An object that is moving at constant velocity is also in equilibrium. Consider the person in Figure 4.17 (page 104) who is dragging a rock at constant speed along the ground. The rock is subject to three forces, as shown in Figure 4.17b. In order for the forces to add to zero, their vertical (y) components must add to zero, and their horizontal (x) components must also add to zero. The gravitational force due to Earth and the normal force due to the surface of the ground both act vertically. The friction force acts horizontally, and opposes the sliding of the rock relative to the ground. The force by the person acts at an angle θ to the ground. The vertical and horizontal components of each force are shown in Figure 4.17c.

In the horizontal direction:

$$F_{\text{net}} = F_{\text{person on rock}, x} - F_{\text{ground on rock, friction}} = F_{\text{person on rock}} \cos\theta - F_{\text{ground on rock, friction}} = 0$$

In the vertical direction:

$$\begin{aligned} F_{\text{net}, y} &= F_{\text{person on rock}, y} + F_{\text{ground on rock, normal}} - F_{\text{Earth on rock}} \\ &= F_{\text{person on rock}} \sin\theta + F_{\text{ground on rock, normal}} - F_{\text{Earth on rock}} = 0 \end{aligned}$$

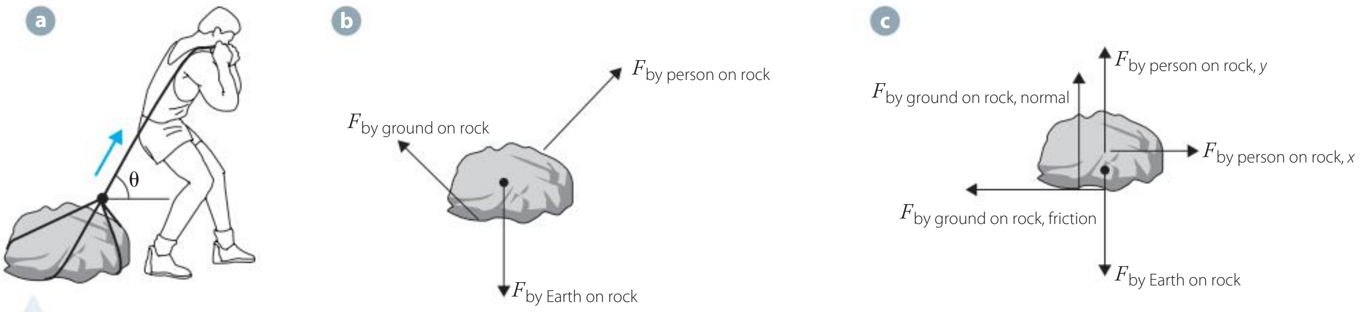


FIGURE 4.17 **a** A person drags a rock at constant speed; **b** Force diagram showing forces acting on the rock; **c** Forces acting on the rock broken into horizontal (x) and vertical (y) components

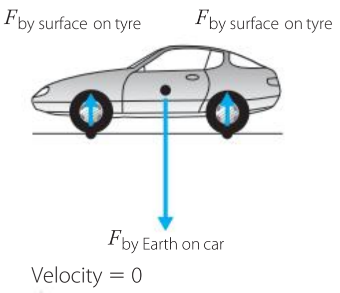
WORKED EXAMPLE 4.6

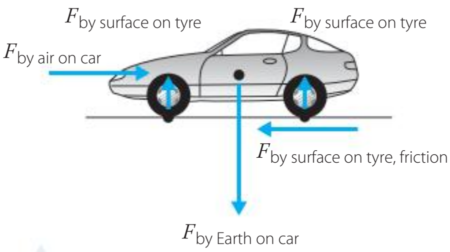
Draw force diagrams to show the forces acting on a car when it is:

1 stationary.

2 moving along a straight stretch of road at a constant 60 km h^{-1}.

Remember that the lengths of the arrows represent approximately the magnitudes of the forces and show whether a force is equal to, larger than, or smaller than other forces. Label the forces in the form $\vec{F}_{\text{A on B}}$.

ANSWERS	LOGIC
1 When a car is stationary, the forces acting on the car are: • the normal force of the ground surface on each tyre • the gravitational force of Earth on the car.	▪ Identify the forces acting from the information in the question.
 **FIGURE 4.18** Forces acting on a stationary car.	▪ Draw a diagram. ▪ The contact forces of the ground on the tyres should sum to be the same length as the gravitational force downwards.

ANSWERS	**LOGIC**
2 When a car is moving at constant velocity, the forces acting on the car are: • the normal force of the ground surface on each tyre • the friction force of the surface on the car (forwards) • air resistance (backwards) • the gravitational force of Earth on the car	▪ Identify the forces acting from the information in the question.
 **FIGURE 4.19** Forces acting on a car moving at constant velocity.	▪ Draw a diagram. ▪ The contact forces of the ground on the tyres should sum to be the same length as the gravitational force downwards. ▪ The friction force by the ground must be the same length as the force due to air resistance backwards.

TRY THESE YOURSELF

1 Draw a force diagram for a car parked on a hillside.

2 Draw a force diagram for the cup shown in Figure 4.16b, but with the hand instead applying an upwards force to the cup, where that force is less than the gravitational force on the cup.

Note that in Worked example 4.6, we have put the direction of the friction force of the road surface on the car pointing forwards. Remember that a car drives forwards by pushing backwards against the road. According to Newton's third law, if the car pushes backwards against the road, the road must push forwards against the car. Therefore, it is the friction force of the road on the tyres that pushes the car forwards. Opposing this force is the force of air resistance, which can be very large, and increases as the speed of the car increases. At equilibrium, when the car is moving at constant velocity, these two forces are equal.

WORKED EXAMPLE 4.7

Rob is dragging a rock through his garden to make it the centrepiece of his new water feature. The rock has a mass of 75 kg. Rob exerts a force of 240 N at an angle of 60° to the horizontal. The rock moves at a constant speed. Calculate the magnitudes of the two components of the contact force that the ground exerts on the rock. Figure 4.17 will help you.

ANSWER	**LOGIC**
$F_{\text{Rob on rock}} = 240\text{ N}; \theta = 60°$ $m_{\text{rock}} = 75\text{ kg}$	▪ Identify the relevant data from the question.
$F_{\text{net}} = 0$ because constant speed	▪ Recognise that the rock is in equilibrium.
$F_{\text{net},x} = F_{\text{Rob on rock},x} - F_{\text{ground on rock, friction}}$ $= F_{\text{Rob on rock}} \cos\theta - F_{\text{ground on rock, friction}}$ $= 0$	▪ Write the equation for equilibrium in the horizontal (*x*) direction ($F_{\text{net},x} = 0$) (refer to Figure 4.17c).

ANSWER	LOGIC
$F_{\text{ground on rock, friction}} = -F_{\text{Rob on rock}} \cos\theta$ $= -240\text{ N} \cos 60^\circ$ $= -120\text{ N}$	▪ Rearrange the equation to find the unknown force. ▪ Substitute known values with units. ▪ Calculate the answer.
$F_{\text{net},\,y} = F_{\text{Rob on rock},\,y} + F_{\text{ground on rock, normal}} - F_{\text{Earth on rock}}$ $= F_{\text{Rob on rock}} \sin\theta + F_{\text{ground on rock, normal}} - F_{\text{Earth on rock}}$ $= 0$	▪ Write the equation for equilibrium in the vertical (*y*) direction ($F_{\text{net},\,y} = 0$) (refer to Figure 4.17c).
$F_{\text{ground on rock, normal}} = F_{\text{Earth on rock}} - F_{\text{Rob on rock}} \sin\theta$ $F_{\text{Earth on rock}} = m_{\text{rock}}g$ $= (75\text{ kg})(9.8\text{ N kg}^{-1})$ $= 735\text{ N (down)}$ $F_{\text{ground on rock, normal}} = 735\text{ N} - 240\text{ N} \sin 60^\circ$ $= 530\text{ N}$ (two significant figures)	▪ Rearrange the equation to find the unknown force. ▪ Write an expression for the gravitational force acting on the rock. ▪ Substitute known values with units. ▪ Calculate the answer. ▪ Substitute values into the expression for $F_{\text{ground on rock, normal}}$. ▪ Calculate the final answer.
$F_{\text{ground on rock, friction}} = -120$ N in the opposite direction to the motion $F_{\text{ground on rock, normal}} = 530\text{ N}$	▪ State the final answer with correct units and significant figures.

TRY THESE YOURSELF

1 Explain what will happen to the force exerted by the ground on the rock if Rob increases the force he applies.

2 Refer back to Figure 4.11, showing two tugboats pulling on a barge. If the barge moves at constant velocity, what is the drag force due to the water acting on the barge?

INVESTIGATION 4.1

Static equilibrium

Critical and creative thinking

Numeracy

When an object has zero net force acing on it, it is in equilibrium. When the object is also at rest, we call it static equilibrium.

AIM

To show that the net force on a stationary, non-accelerating object is zero, within uncertainty limits

Our hypothesis is that the net force on the stationary ring is zero.

MATERIALS

- Wooden board and 3 nails
- Hammer
- String
- Protractor
- 3 spring balances
- Small rigid ring
- Paper
- Pencil
- Tape

»

9780170409063

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
The spring balances may flick back or flick an object into a person's eye.	Wear safety glasses when working with springs.

RISK ASSESSMENT

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Tape the piece of paper to the middle of the board and place the ring on top.
2. Hammer the three nails into the board, each near an edge of the board but well separated from each other (see Figure 4.20).
3. Hook one spring balance over each nail.
4. Use a piece of string to tie each spring balance to the hook. Do not double-knot it until you have adjusted the length!
5. Adjust the lengths of each piece of string so that the ring is over the piece of paper, and each balance is reading within its limits.

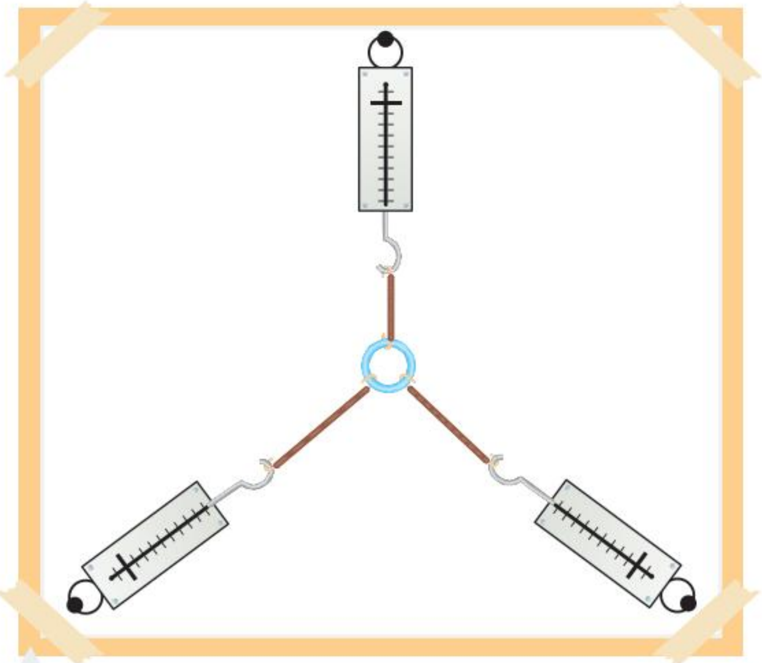


FIGURE 4.20 Experimental set-up

RESULTS

1. Trace the ring and the direction of each string on the paper.
2. Record the force measured by each balance against the tracing of the correct string.
3. Estimate the uncertainty in each spring balance reading.

When you have recorded your results, you can remove the ring and spring balances, and make measurements on your paper.

4. Draw a set of axes on your paper with the origin at the position where the ring was.
5. Measure and record the angle of each force (string line) to one or both axes.

You now have a force diagram for your ring.

ANALYSIS OF RESULTS

1. Calculate the net force acting on the ring.
2. Calculate the uncertainty in the net force.

DISCUSSION

Was your hypothesis supported, within the uncertainty in your net force? Explain any discrepancies.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- When the net force acting on an object is zero, the object is said to be in equilibrium.
- An object in equilibrium has zero acceleration.
- An object in equilibrium has constant velocity, which may or may not be zero velocity.
- An object at rest with zero net force acting on it is said to be in static equilibrium.

CHECK YOUR UNDERSTANDING

4.3

1. Is it possible for an object in equilibrium to be moving? Explain your answer.
2. What is the magnitude of the net force acting on an aeroplane flying horizontally at a constant speed of 400 km h^{-1}? Explain your answer.
3. A child is pulling a toy truck behind her over level ground at constant speed. How do you know that all the forces acting on the toy truck are balanced?
4. For each of the situations in Figure 4.21, what additional force would need to be added for the object to be in equilibrium? Calculate the magnitude and direction.

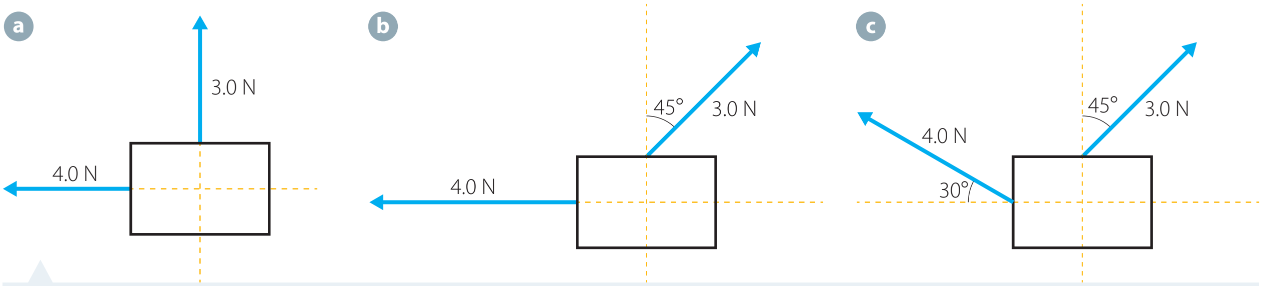


FIGURE 4.21

5. A heavy, 10.0 kg painting is suspended from two cables in a gallery. Each cable applies the same force to the painting, at an angle of 30° above the horizontal (Figure 4.22). What is the force applied by each cable on the painting?

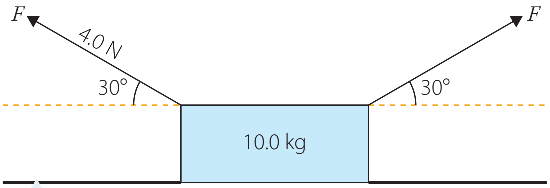


FIGURE 4.22 A heavy painting suspended by two cables

4.4 Investigating the motion of objects on inclined planes

An object on a slope is always affected by (at least) three forces – the gravitational force, $\vec{F}_{\text{Earth on object}}$, acts vertically downwards; the normal force, N, acts perpendicular to the surface of the slope; and the friction force, $\vec{F}_{\text{surfact on object, friction}}$, acts parallel to the slope. These forces are shown in Figure 4.23a.

In previous examples, we have found it convenient to decompose vectors into vertical and horizontal components. When dealing with inclined planes, it is usually more convenient to use axes parallel and perpendicular to the slope.

Consider an object on a slope, as in Figure 4.23a. We assume the object can only accelerate parallel to the slope – that is, it will not sink into the slope, or lift off above it. If acceleration can only occur parallel to the slope, then the net force perpendicular to the slope must be zero. Acceleration is possible parallel to the slope, so the net force parallel may not be zero. Hence it makes sense to choose our x axis as parallel to the slope and our y axis as perpendicular to the slope. When we do this, two of the three forces are already acting along one axis or the other. The friction force acts along the x axis (parallel to slope), and the normal force acts along the y axis (perpendicular to slope). The gravitational force can be decomposed into a component along each axis, as shown in Figure 4.23b.

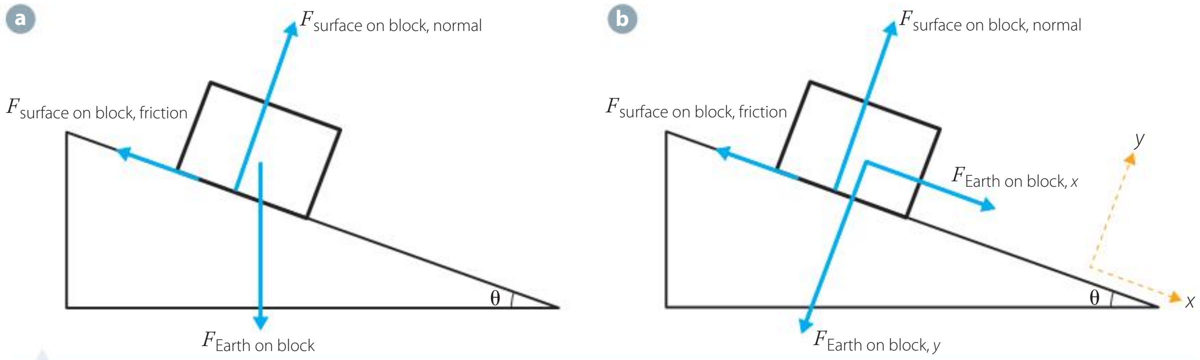


FIGURE 4.23 **a** The forces acting on an object on an inclined plane; **b** The forces decomposed into components parallel (x) and perpendicular (y) to the slope

We can then use Newton's second law in component form to analyse the behaviour of the object. In the y direction (perpendicular), where the net force is zero:

$$F_{\text{net, }y} = \Sigma F_y = F_{\text{surface on object,normal}} - F_{\text{Earth on object, }y} = F_{\text{surface on object,normal}} - F_{\text{Earth on object}} \cos\theta = 0$$

In the x direction, where the net force may be non-zero:

$$F_{\text{net, }x} = \Sigma F_x = F_{\text{surface on object, friction}} - F_{\text{Earth on object, }x} = F_{\text{surface on object, friction}} - F_{\text{(Earth on object)}} \sin\theta = m_{\text{object}}\, a$$

There are two things that can happen when we place an object on an inclined plane such as this – it can stay in place, or it can slide down. If it stays in place, then the object is in equilibrium, and $F_{\text{net, }x} = \Sigma F_x = 0$. In this case,

$$F_{\text{surface on object, friction}} = -F_{\text{Earth on object}} \sin\theta = m_{\text{object}}\, g \sin\theta$$

This occurs if the maximum static friction force is greater than or equal to the component of the gravitational force acting down the slope.

WORKED EXAMPLE 4.8

Harriet sits on a slide so that her brother Laurence can't use it. Harriet has her bare feet against the slide, and there is enough friction to prevent her moving. If Harriet has a mass of 21 kg and the slide makes an angle of 35° to the horizontal, calculate the magnitude of each force acting on her.

ANSWER	LOGIC
$m = 21$ kg; $\theta = 35°$ $F_{\text{net}} = 0$, because not moving Figure 4.23 shows the forces acting, but in this case they act on Harriet, rather than a block. The three forces acting on Harriet are the gravitational force of Earth, and the normal and frictional forces due to the surface.	▪ Identify the relevant data, and recognise that Harriet is in equilibrium.
$F_{\text{Earth on Harriet}} = mg$ $= (21\text{ kg})\,(9.8\text{ N kg}^{-1})$ $= 205.8$ N downwards	▪ Write the expression for the gravitational force. ▪ Substitute known values with units. ▪ Calculate the value (do not round at this stage).

ANSWER	LOGIC
$F_{\text{net},\,x} = F_{\text{Earth on Harriet, gravitational},\,x} - F_{\text{slide on Harriet, friction}} = 0$	■ Write the equation for equilibrium in the parallel (*x*) direction ($F_{\text{net},\,x} = 0$) (refer to Figure 4.23b).
$F_{\text{slide on Harriet, friction}} = F_{\text{Earth on Harriet},\,x}$	■ Rearrange to find the unknown force.
$= F_{\text{Earth on Harriet}} \sin\theta$	■ Expand the expression.
$= (205.8\text{ N}) \sin 35°$	■ Substitute known values with units.
$= 118\text{ N}$	■ Calculate the value.
$F_{\text{net},\,y} = F_{\text{Earth on Harriet, gravitational},\,y} - F_{\text{slide on Harriet, normal}} = 0$	■ Write the equation for equilibrium in the perpendicular (*y*) direction ($F_{\text{net},\,y} = 0$) (refer to Figure 4.23b).
$F_{\text{slide on Harriet, normal}} = F_{\text{Earth on Harriet},\,y}$	■ Rearrange to find the unknown force.
$= F_{\text{Earth on Harriet}} \cos\theta$	■ Expand the expression.
$= (205.8\text{ N}) \cos 35°$	■ Substitute known values with units.
$= 169\text{ N}$	■ Calculate the value.
$F_{\text{slide on Harriet, normal}} = 170\text{ N}$ $F_{\text{slide on Harriet, friction}} = 120\text{ N}$ $F_{\text{Earth on Harriet}} = 210\text{ N}$	■ State the final answer with correct units and significant figures.

TRY THESE YOURSELF

Continuing on from Worked example 4.8, Laurence tries to get Harriet out of his way by pushing her down parallel to the slide with a constant force of 150 N.

1 If the friction force acting on Harriet is unchanged, what is the net force acting on her? Describe her motion while this force is exerted.

2 If this extra force causes Harriet to slide down at a constant speed, calculate the magnitude of each force acting on Harriet now. What must have happened to the friction force? What is the net force in this case?

Chapter 5 section 5.2 gives more details and includes worked examples showing how to do this.

Forces on a box on an inclined plane

Watch the video, and work along with it to decompose the gravitational force.

IgNobel award: the physics of sheep-dragging

Read this article about Australian research on sheep-dragging. Draw a force diagram for a sheep being dragged up a rough ramp.

If the object sitting on an inclined plane slides down, this tells us that the maximum static friction force was less than the component of the gravitational force acting down the slope. So, in the *x* direction we have:

$$F_{\text{net},\,x} = \sum F_x = F_{\text{surface on object, friction}} - F_{\text{Earth on object}} \sin\theta = m_{\text{object}}\, a$$

and the object accelerates down the slope at a rate of:

$$a = \frac{F_{\text{surface on object, friction}} - F_{\text{Earth on object, gravity}} \sin\theta}{m_{\text{object}}}$$

$$= \frac{F_{\text{surface on object, friction}} - m_{\text{object}}\, g \sin\theta}{m_{\text{object}}}$$

Hence, we can measure acceleration of an object sliding on an inclined plane to calculate the kinetic friction force acting between the object and the surface of the plane.

If we vary the angle between the plane and the horizontal, we can find the maximum static friction force between the surfaces. If the angle at which the object just starts to slide is θ_c, which is called the critical angle, then

$$F_{\text{static friction max}} = m_{\text{object}}\, g \sin\theta_c$$

Note that here we have discussed the case of an object at rest subject to the static friction force, or an object sliding and subject to the kinetic friction force. The case of an object rolling, such as a ball rolling down a slide, is more complicated. The simple analysis described here for objects at rest or sliding is *not* a good model for a rolling ball.

INVESTIGATION 4.2

Motion of objects on inclined planes

The measurement of the acceleration of objects on inclined planes can be used to deduce the frictional force that acts between the plane and an object sliding on it. The frictional force will be different for different surfaces and for different angles. You can also use an inclined plane that can be adjusted to different angles to measure the maximum static frictional force between the plane and the object. Reading ahead to the next chapter where friction is discussed in more detail, and dynamics is presented, may help you with this investigation.

Critical and creative thinking

Numeracy

Information and communication technology capability

Skiing the super_G: the ulitmate inclined plane

AIM

- Write an aim for an investigation using objects sliding on inclined planes.
- Write an inquiry question or hypothesis.

MATERIALS

- An inclined plane
- One or more objects to slide down the inclined place

What else will you need? Write your own equipment list. For example, you might need timing equipment, motion-sensors, a range of different surface materials, a protractor, and various weights. What you need will depend on your inquiry question or hypothesis. Remember to talk to your teacher about what equipment is available.

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?

RISK ASSESSMENT

What risks are associated with your investigation, and how can you manage them?

METHOD

- Write a detailed method for your investigation. Check it with your teacher before you proceed.
- Remember to include what will be measured, and think about how you can minimise uncertainties.

RESULTS

Record your results as you measure them. A table is often a good way of organising your results. Remember to include units and uncertainties!

An investigation involving motion on inclined planes could be extended to a depth study by using it to study friction in detail, or comparing rolling with sliding motion using objects of different shapes and surface materials.

ANALYSIS OF RESULTS

How will you analyse your results so that you can answer your inquiry question or test your hypothesis?

DISCUSSION

- Have you answered your inquiry question? Did your results support your hypothesis?
- If other groups were working on similar investigations, did their results agree with yours?
- Are there ways you could improve your investigation?

CONCLUSION

Write a conclusion summarising the outcomes of your investigation.

Important theory

KEY CONCEPTS

- An object on an inclined plane is subject to the contact force due to the plane (normal and friction) and the gravitational force.
- It is usually convenient to decompose the forces acting on an object on an inclined plane into components parallel and perpendicular to the plane.
- Objects on inclined planes are generally in equilibrium (zero net force) perpendicular to the plane, but may be accelerating (non-zero net force) parallel to the plane.
- The kinetic friction force that the surface of a plane exerts on an object can be found by measuring the acceleration of the object sliding down the plane.
- The maximum static friction force that the surface of a plane exerts on an object can be found by measuring the angle at which the object begins to slide.

CHECK YOUR UNDERSTANDING

4.4

1 Why is it usually more convenient to decompose forces into components parallel and perpendicular to the plane for objects on inclined planes, than into horizontal and vertical components? Explain your answer.

2 A 1200 kg car is parked on a slope at an angle of 15° to the horizontal.
  a Draw a force diagram showing the forces acting on the car.
  b Calculate the magnitude of each force.

3 A 1200 kg car is travelling at constant speed up a slope at an angle of 15° to the horizontal.
  a Draw a force diagram showing the forces acting on the car.
  b Calculate the magnitude of each force.

4 In an experiment to measure the static friction force, students are using a plate sliding down an inclined plane. If the plate has a mass of 1.0 kg and the maximum static friction force that the inclined plane can exert on it is 4.2 N, calculate the minimum angle of the plane such that the plate will begin to slide.

5 In an experiment to measure the kinetic friction force, students are using a 1.0 kg plate sliding down an inclined plane. The plane is at an angle of 60° to the horizontal. The time taken for the plate to slide a distance of 30 cm, starting from rest, is measured to be 0.32 s.
  a Calculate the acceleration of the plate.
  b Calculate the net force acting on the plate.
  c Calculate the kinetic friction force that the plane exerts on the plate. You may wish to review kinematics in chapter 3.

6 Describe how you could calculate the uncertainties in the experiment described in question 5. What information would you need?

4 CHAPTER SUMMARY

- Objects interact via forces.
- Forces can be contact forces (objects touching) or field forces (objects not touching).
- Newton's first law says that if no force acts on an object, it will not accelerate. It will move in a straight line with constant speed, or remain at rest.
- Newton's first law relates force to acceleration, and so provides a definition of force.
- Newton's second law quantifies the relationship between force and acceleration: $\vec{a}_A = \frac{\vec{F}_{\text{on A}}}{m_A}$ or $\vec{F} = m\vec{a}$. The direction of force and acceleration are the same.
- Newton's third law tells us that when an object A exerts a force on object B, then object B exerts an equal and opposite force on object A: $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$.
- Newton's third law pairs act on different objects, and are always of the same type of force.
- When more than one force acts on an object, its acceleration is determined by the net force, $\vec{F}_{\text{net}}$.
- The net force on an object is the vector sum of all forces acting on that object: $\vec{F}_{\text{net on A}} = \Sigma\vec{F}_{\text{on A}}$.
- Forces can be broken into perpendicular components: $F_x = F\cos\theta$ and $F_y = F\sin\theta$
- Forces can be added by adding them up component-wise: $F_{\text{net},x} = F_{1x} + F_{2x} + \ldots$ and $F_{\text{net},y} = F_{1y} + F_{2y} + \ldots$
- The magnitude of the net force is found using Pythagoras's theorem: $F_{\text{net}} = \sqrt{(F_{\text{net},x})^2 + (F_{\text{net},y})^2}$.
- The net force acts at an angle given by $\tan\theta_{\text{net}} = \frac{F_{\text{net},y}}{F_{\text{net},x}}$.
- Force diagrams are a useful way of representing forces on an object. They show all the forces, and only the forces, acting *on* an object.
- When the net force acting on an object is zero, the object is said to be in equilibrium.
- An object in equilibrium has zero acceleration and constant velocity.
- An object at rest with zero net force acting on it is in static equilibrium.
- An object on an inclined plane is subject to the contact force due to the plane (normal and friction) and the gravitational force.
- We decompose the forces acting on an object on an inclined plane into components parallel and perpendicular to the plane.
- Objects on inclined planes are generally in equilibrium (zero net force) perpendicular to the plane, but may be accelerating (non-zero net force) parallel to the plane.

4 CHAPTER REVIEW QUESTIONS

Review quiz

1 State:

a Newton's first law.

b Newton's second law.

c Newton's third law.

2 Explain why Newton's third law force pairs can never be shown on a single force diagram.

3 When adding up the forces acting on an object, should you include the Newton's third law force pairs to the forces acting on the object? Explain your answer.

4 An elephant pulls on a rope with a force of 500 N. With what force does the rope pull on the elephant? Explain your answer.

5 Draw a diagram to show:

a the vertical and horizontal components of a force vector directed at an angle θ to the horizontal.

b the parallel and perpendicular components of the gravitational force on a mass sliding on a frictionless surface inclined at an angle θ to the horizontal.

6 Two people have a tug-of-war competition that results in a tie. Sketch a diagram to show all the forces acting on:

a each person.

b the rope.

7 Draw a force diagram showing the forces acting on a ball in each of the following situations.

- **a** When you hold it at rest in your hand.
- **b** As you throw the ball upwards.
- **c** After the ball has left your hand and is moving up.
- **d** When the ball reaches the top of its path.
- **e** When the ball is on the way back down.

8 A physics textbook is resting on the rear parcel shelf of a car moving at 50 km h^{-1}. The book becomes a dangerous missile when the car comes to a sudden stop. Explain why.

9 A book is sitting at rest on a table. Marcus puts his hand on the book and pushes downwards, with slowly increasing force. Describe what happens to:

- **a** the gravitational force acting on the book.
- **b** the normal force of the table acting on the book as Marcus increases the force with which he pushes.

10 A ball sits at rest on the ground. In the form of $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$, identify the Newton's third law force pair to:

- **a** the gravitational force acting on the ball.
- **b** the normal force of the ground acting on the ball.

11 A ball has been thrown and it is flying through the air. In the form of $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$, identify the Newton's third law force pair to:

- **a** the gravitational force acting on the ball.
- **b** the drag (friction) of the air acting on the ball.

12 Calculate the net force acting on a boat when the force by the wind on the sails is 500 N to the west, and there is an additional force due to a motor of 400 N to the north.

13 Calculate the horizontal and vertical components of the following force vectors.

- **a** 25 N at 30° above the horizontal, pointing right.
- **b** 25 N at 30° below the horizontal, pointing right.
- **c** 25 N at 30° above the horizontal, pointing left.
- **d** 25 N at 30° below the horizontal, pointing left.

14 A 40 kg block is on a frictionless plane inclined at 42°.

- **a** Calculate the component of the gravitational force that is:
  - **i** parallel to the surface.
  - **ii** perpendicular to the surface.
- **b** **i** What is the resultant force on the block?
  - **ii** What is the normal force on the block?

15 Two children run in and kick a soccer ball at exactly the same moment. Anna kicks the ball with a force of 150 N west, and Belinda kicks it with a force of 220 N north. Calculate the net force on the ball at this moment.

16 In a static equilibrium experiment, a ring is connected to three strings, each of which exerts a force on the ring as shown in Figure 4.24. Calculate the magnitude of the unknown force, *F*, and the angle, θ, at which it acts.

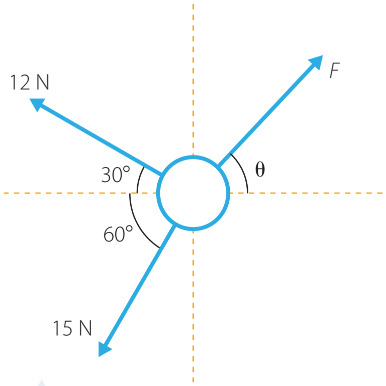


FIGURE 4.24 Three forces act on a stationary ring

17 Rob is pushing a shopping trolley with wonky wheels. He exerts a force of 90 N in a direction N45°E on the trolley, and at the same time the ground exerts a friction force of 30 N N10°W on the trolley. Calculate the magnitude and direction of the net force on the trolley.

18 A toboggan and child of combined mass 67 kg slide down a frictionless snowfield inclined at 15° to the horizontal.

Calculate the magnitude of:

- **a** the component of the weight force perpendicular to the snowfield.
- **b** the force down the slope of the snowfield on the toboggan and child.

19 A toboggan and child of combined mass 67 kg slide down a grassy slope inclined at 15° to the horizontal. The force of friction exerted by the slope on the toboggan is 50 N.

Calculate the magnitude of:

- **a** the component of the weight force perpendicular to the snowfield.
- **b** the net force down the slope on the toboggan and child.

20 A crowded school bus enters a left-hand corner while maintaining a constant speed. The younger pupils standing say that they were pushed to the right of the bus. Explain why they are incorrect, and give a proper explanation for their apparent motion from within the bus.

21 It is often observed that a force is required to maintain the motion of an object, such as a car along a straight, level road. Does this mean that Aristotle was correct after all? Justify your opinion.

22 A rocket launch involves engines expelling large quantities of gas downwards with high speed. Explain how this results in the rocket moving upwards.

23 Three books sit in a pile on a student's desk: a maths book on top, a chemistry book in the middle, and a physics book on the bottom (Figure 4.25).

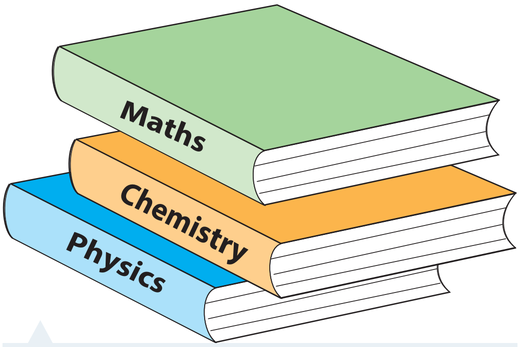


FIGURE 4.25 A pile of books on a student's desk

- **a** Identify the forces acting on each book and draw a separate force diagram for each.
- **b** Apply Newton's third law to identify equal and opposite forces in your diagrams (remember that a Newton's third law force pair cannot be shown on a single force diagram).
- **c** Apply Newton's second law to write a relationship between the forces acting on the chemistry book.

24 The anchor chain of a large ship exerts a force on the ship of 6.0×10^3 N in a direction N30°E at an angle of 60° to the horizontal.

- **a** Sketch a diagram to show this situation.
- **b** For this force, calculate the magnitude of the component in each of the following directions.
  - **i** Horizontally
  - **ii** Vertically up
  - **iii** North
  - **iv** East

5

Forces, acceleration and energy

INQUIRY QUESTION

How can the motion of objects be explained and analysed?

OUTCOMES

Students:

- apply Newton's first two laws of motion to a variety of everyday situations, including both static and dynamic examples, and include the role played by friction (friction = $\mu \vec{F}_N$) (ACSPH063) CCT
- investigate, describe and analyse the acceleration of a single object subjected to a constant net force and relate the motion of the object to Newton's second law of motion through the use of: (ACSPH062, ACSPH063)
  - qualitative descriptions CCT
  - graphs and vectors ICT N
  - deriving relationships from graphical representations including $\vec{F} = m\vec{a}$ and relationships of uniformly accelerated motion ICT N
- apply the special case of conservation of mechanical energy to the quantitative analysis of motion involving: ICT N
  - work done and change in kinetic energy of an object undergoing accelerated rectilinear motion in one dimension ($W = \vec{F}_{\text{net}}\vec{s}$)
  - changes in gravitational potential energy of an object in a uniform field ($\Delta U = m\vec{g}\Delta\vec{h}$)
- conduct investigations over a range of mechanical processes to analyse qualitatively and quantitatively the concept of average power $\left(P = \frac{\Delta E}{t}, P = \vec{F}\vec{v}\right)$, including but not limited to: ICT N
  - uniformly accelerated rectilinear motion
  - objects raised against the force of gravity
  - work done against air resistance, rolling resistance and friction

iStock.com/DaveAlan

In the previous chapter, we began our investigation of forces. Newton's laws were described, and we applied Newton's second and third laws to objects in equilibrium. In this chapter, we will apply our understanding of forces to objects that are *not* in equilibrium. We will see how the movement of cars, trains, aeroplanes and any other accelerating object can be explained and analysed using Newton's laws.


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FIGURE 5.1 A train accelerates away from the platform. It is the friction force between the train's wheels and the rails that causes the acceleration.

This chapter also introduces another of the central concepts in physics: energy. Energy can change forms, and be transferred from one object to another, but the total energy of the universe is constant. Energy is what allows us to apply forces and do work. Fuels such as petrol provide energy to power our cars, and most of our electrical energy in Australia comes from coal. Energy security is an important issue in the modern world. Here we shall see how the law of conservation of energy may be applied to analyse motion, and how force is related to energy.

5.1 Newton's laws and friction

Newton's first and second laws revisited

Recall that Newton's first law says that in the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity. This second part of the statement, that *an object in motion continues in motion with a constant velocity* seems to contradict our everyday experience.

If you slide a book along the table (Figure 5.2), it will not continue with constant velocity – instead it will slow down and stop. To apply Newton's first law to this situation, we first need to recognise that there is an acceleration. The book is moving forwards, but it is slowing down, therefore the book has an acceleration backwards. This acceleration, from Newton's first law, means that there must be a force acting on the book.

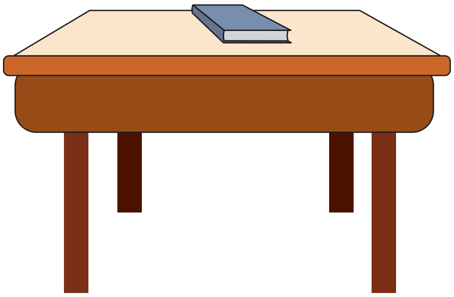


FIGURE 5.2 A book slides across the table, slowing down as it moves

Remember that forces are interactions. To identify the force causing the acceleration, we look to see what other objects the book is interacting with. The book is in the gravitational field of Earth, so it is subject to the gravitational force. The gravitational field of Earth exerts a force directly downwards, but this is not the direction of acceleration of the book, so it is not the gravitational force that is causing the acceleration of the book. The only object the book is in contact with is the table. The contact force of the table on the book has components parallel and perpendicular to the surface. The perpendicular component (normal force) is vertical, so it cannot be causing the acceleration. The parallel component of the contact force acts horizontally, so we conclude that it is this force that is acting to slow down the book. Recall that the parallel component of the contact force is called **friction**.

Force and friction

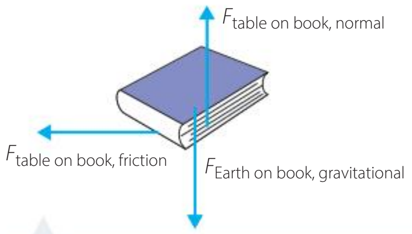
Note that we have ignored any force due to the air that the book moves in. The air will also be exerting forces both horizontally and vertically on the book – the horizontal force is drag or air resistance, and the vertical force is the buoyant force that acts upwards. Both forces are very small compared with the force exerted by the table, so we choose to ignore them.

Whenever we want to analyse the motion of an accelerating object, we use this process to identify the forces acting:

- First, consider any fields that may be exerting a force on the object. If the object has mass and is in a gravitational field, it will experience a gravitational force. If the object has charge and is in an electric field, it will experience an electrostatic force (see chapter 12). If the object has charge and is moving, or is magnetic, it will experience a force in a magnetic field (see chapter 14). Ask yourself which fields need to be included in the analysis and which can be ignored.
- Second, consider everything that the object is in contact with. All surfaces exert a contact force, as do air, water and other fluids. Ask yourself which contact forces need to be included. Sometimes friction or air resistance is very small and can be ignored, but often it is significant and should be included in your analysis.
- Once you have identified all the forces that are acting, draw a force diagram for the object showing all the forces acting on it. The object's acceleration will tell you the direction of the net force. This will help you determine the direction of the forces acting on the object and their relative sizes.

WORKED EXAMPLE 5.1

Identify the forces acting, and draw a force diagram, for a book sliding across a table, slowing down as it moves to the right.

ANSWER	LOGIC
The book is subject to the gravitational force of Earth downwards, the normal force of the table upwards and the friction force of the table horizontally.	▪ Identify all the forces acting and their directions.
Assume the forces due to the air are negligible, so can be ignored.	▪ State any assumptions or approximations.
The net force is horizontal and to the left.	▪ Identify the direction of the net force.
The net force is due to the friction force. The friction force is to the left.	▪ Relate the net force to the acting forces.
 **FIGURE 5.3**	▪ Draw the diagram. ▪ Remember that the force arrows on a force diagram should have lengths that indicate their relative sizes. As there is no vertical acceleration, we know that there is no net force in the vertical direction. Hence, the normal force must be equal to the gravitational force, so their arrows are the same length.

TRY THESE YOURSELF

Identify the forces acting on, and draw a force diagram for:

1 a book being pushed across a table at constant velocity.

2 a book being pushed across a table at increasing velocity.

Once we have identified all the forces acting, we can use Newton's second law to relate the acceleration to the net force. Recall that Newton's second law tells us that the acceleration is proportional to the net force, and is in the same direction as the net force:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

WORKED EXAMPLE 5.2

Ranji asks Phil if she can borrow his maths textbook. Phil slides the textbook along the table towards Ranji with an initial speed of 1.0 m s^{-1}. The book, with a mass of 1.5 kg, slides along and comes to rest in 2.0 s. Calculate the frictional force acting on the book.

ANSWER	LOGIC
$m = 1.5$ kg; $u = 1.0$ m s^{-1}; $v = 0$; $t = 2.0$ s	▪ Identify the relevant data.
$F = ma$	▪ Write Newton's second law.
$v = u + at$	▪ Relate acceleration to given data.
$a = \frac{(v-u)}{t}$	▪ Rearrange the equation for acceleration.
$F = \frac{m(v-u)}{t}$ $= \frac{1.5\text{ kg}\,(0 - 1.0\text{ m s}^{-1})}{2.0\text{ s}}$ $= -0.75$ kg m s^{-2} $F = -0.75$ N	▪ Substitute the expression for *a* into Newton's second law. ▪ Substitute known values with correct units. ▪ Calculate the final value. ▪ State the final answer with correct units, sign and significant figures.

TRY THESE YOURSELF

1 If the friction force acting on the book was only 0.25 N, calculate how long it would take the book to stop.

2 Calculate the friction force acting on the sliding textbook if it has an initial speed of 1.0 m s^{-1} and slides a distance of 1.5 m.

Static and kinetic friction

Friction is a significant force in almost all everyday situations. When you slide a book along a table to a friend, it slows and stops because of friction. When you ride a bicycle or drive a car, the wheels rotate and grip the road, accelerating you forwards because of friction. Every step you take, the friction force is what enables you to walk.

The friction force is the component of the contact force that prevents one surface from sliding over another surface. It always acts parallel to a surface. The friction force acting on an object may be in the direction of motion of the object, or it may be in the opposite direction. Friction, acting in the direction of motion, allows you to walk. Friction, acting in the direction opposite to motion, causes sliding to be reduced; for example, slowing down the sliding book. The important thing to remember is that friction always opposes the *relative* motion of one surface against another.

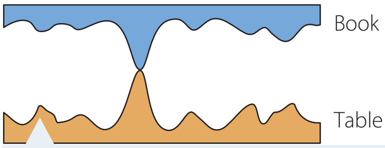


FIGURE 5.4 A very close-up view of the surfaces of a book and a table

Recall that friction is one component of the contact force that one surface exerts on another. The contact force is due to interactions of atoms at the surfaces of the two materials as their electrons get closer, and is electrostatic in nature. If you could look very, very closely at any surface, you would see that it is bumpy, as shown in Figure 5.4.

Where there are bumps on the two surfaces, they can catch as one surface tries to slide along the other. They can also bond at those points. The more one surface pushes into another, so that more atoms are interacting, the greater the friction force is.

Imagine you are going to slide a book along a table. You must apply a force to get the book moving. Try pushing very, very gently on a book. If you start by applying a very small force, the book does not move. It does not move because the **static friction** force is acting. If the book is not moving, then the friction force must be equal to the applied force to give a zero net force, as shown in Figure 5.5a. If you slowly increase the applied force (Figure 5.5b), the friction force will also increase, always remaining equal to the applied force, up to some maximum value. This maximum value is called the **maximum static friction** force. It is the maximum friction force that the two surfaces can exert on each other – an applied force greater than this breaks the bonds between the surfaces and pushes the bumps past each other. If you do the experiment very carefully, you will feel when this happens and you will notice that suddenly the force that the book exerts back on you decreases a little bit. This is because, once the book starts to slide, the force of friction acting on it decreases, as shown in Figure 5.5c. When surfaces are sliding, we call the friction force acting the **kinetic friction** force. It is the same sort of force as the static friction force.

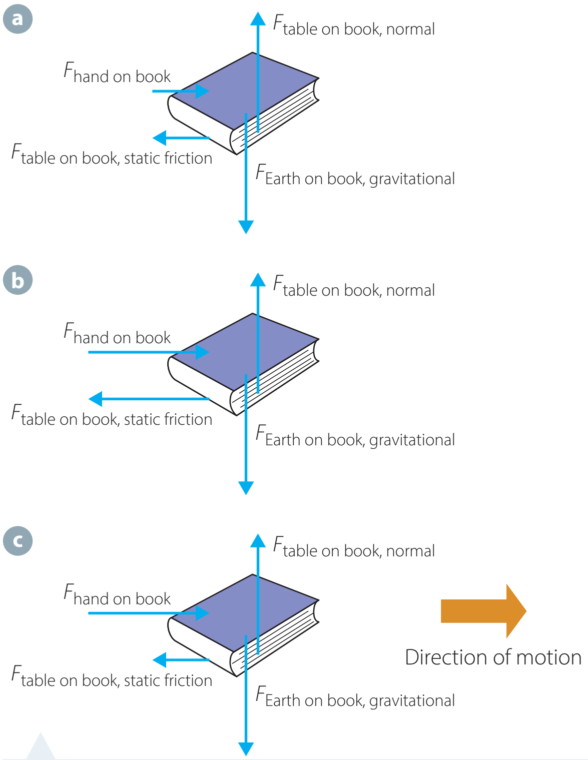


FIGURE 5.5 **a** When a small force, $F_{\text{hand on book}}$, is applied to the book, the static friction force is equal to the applied force and prevents the book from sliding. **b** As the applied force increases, so does the static friction force. **c** When the applied force exceeds the maximum static friction force, the book begins to slide. It is now the kinetic friction force that acts on the book.

The kinetic friction force is independent of other forces applied parallel to the surfaces. Figure 5.6 shows a graph of friction force as a function of applied force. You can see that the two are equal for small values of applied force, where the object is not yet sliding. This is the static friction region. In the kinetic friction region, the surfaces are sliding against each other, and the friction force is constant and less than the maximum static friction.

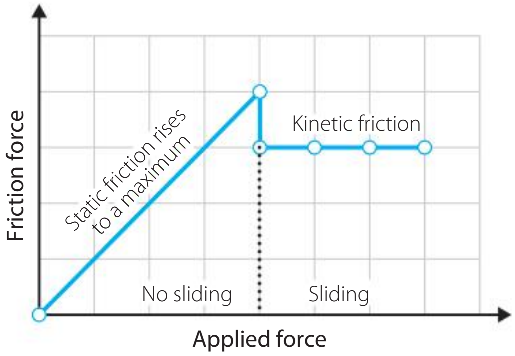


FIGURE 5.6 Static friction increases up to a maximum, staying equal to the applied force, until sliding begins. The kinetic friction is less than the maximum static friction and is constant.

The more that surface A pushes into surface B, the greater the kinetic friction $F_{\text{B on A, friction}}$. To a good approximation, kinetic friction is proportional to the normal force, since the normal force is a measure of how hard A is pushing into B. Remember that from Newton's third law, whatever force surface A exerts on surface B, surface B will exert an equal and opposite force on surface A. Hence, applying a perpendicular force (such as pushing down on a sliding book) changes the friction force.

Rollercoaster design

We can model the kinetic friction force mathematically as:

$$F_{\text{A on B, kinetic friction}} = \mu_k N_{\text{A on B}}$$

where N is the normal force exerted by surface A on surface B, and μ_k is a constant called the **coefficient of kinetic friction**. Note that as both F and N have units of N, the coefficient μ_k must be dimensionless and have no units. The value of μ_k is usually between 0 and 1, but can be greater than 1.

Remember that the static friction force varies, and can take any value up to some maximum. This maximum also depends on the normal force and is given by:

$$F_{\text{A on B, maximum static friction}} = \mu_s N_{\text{A on B}}$$

where μ_s is the **coefficient of static friction**, and, like μ_k, typically varies between 0 and 1, but can be greater than 1.

More generally, the magnitude of the static friction force is:

$$F_{\text{A on B, static friction}} \leq \mu_s N_{\text{A on B}}$$

and will take whatever value is necessary to prevent the surfaces sliding against each other, up to the maximum possible. The values of μ_k and μ_s depend on the details of the surfaces. Smoother surfaces have lower coefficients of friction, and rougher surfaces have higher coefficients of friction.

TABLE 5.1 Some examples of coefficients of friction. Note that these values are approximate, and will vary depending on the details of the surfaces.

SURFACES	μ_S	μ_K
Rubber on concrete	1.0	0.8
Glass on glass	0.94	0.4
Wood on wood	0.25–0.5	0.2
Steel on steel, unlubricated	0.74	0.57
Steel on steel, lubricated	0.15	0.06
Ice on ice	0.1	0.03

Car tyres and friction

Explore this presentation on car tyres and friction. What sort of tyres are best for the road conditions where you live?

WORKED EXAMPLE 5.3

A 1.5 kg textbook sits on a table. The coefficient of static friction for the book and table is 0.50 and the coefficient of kinetic friction is 0.45. Calculate the friction force acting on the book when Ranji pushes it with a horizontal force of:

1. 2.5 N
2. 5.0 N
3. 7.5 N

ANSWER	LOGIC
1 $m = 1.5\ \text{kg};\ \mu_s = 0.50,\ \mu_k = 0.45$	▪ Identify the relevant data.
$F_{\text{table on book, } maximum \text{ static friction}} = \mu_s N_{\text{table on book}}$ Check whether the applied force exceeds the maximum friction force, and then write an expression for the maximum friction force.	▪ Write the expression for maximum static friction force.
$N_{\text{table on book}} = m_{\text{book}}\, g$	▪ Recognise that as no other vertical forces are acting, the normal force is equal to the gravitational force on the book (Newton's second law).
$F_{\text{table on book, maximum static friction}} = \mu_s m_{\text{book}}\, g$ $= 0.5\ (1.5\ \text{kg})\ (9.8\ \text{N kg}^{-1})$ $= 7.35\ \text{N}$	▪ Write an expression for the maximum friction force in terms of known quantities. ▪ Substitute known values with correct units. ▪ Calculate the final value.
This maximum static friction force is greater than the applied force of 2.5 N, so the static friction force will take the value required to prevent the book sliding. This is the magnitude of the applied force. $F_{\text{table on book, static friction}} = 2.5\ \text{N}$	▪ State the final answer with correct units and significant figures.
2 $F_{\text{table on book, static friction}} = 5.0\ \text{N}$	▪ The maximum static friction force is the same as calculated in part 1. The applied force of 5.0 N is less than this, so the static friction force again takes the value of the applied force. ▪ State the final answer with correct units and significant figures.
3 $F_{\text{table on book, kinetic friction}} = \mu_k N_{\text{table on book}}$	▪ The applied force is now greater than the maximum static friction force, so the book will slide. The book is now subject to the kinetic friction force rather than the static friction force. ▪ Write an expression for the kinetic friction force
$N_{\text{table on book}} = m_{\text{book}}\, g$	▪ Recognise that as no other vertical forces are acting, the normal force is equal to the gravitational force on the book (Newton's second law).
$F_{\text{table on book, kinetic friction}} = \mu_k m_{\text{book}}\, g$ $= 0.45\ (1.5\ \text{kg})\ (9.8\ \text{N kg}^{-1})$ $= 6.615\ \text{N}$ $F_{\text{table on book, kinetic friction}} = 6.6\ \text{N}$	▪ Write an expression for the kinetic friction force in terms of known quantities. ▪ Substitute known values with correct units. ▪ Calculate the final value. ▪ State the final answer with correct units and significant figures

TRY THIS YOURSELF

Repeat Worked example 5.3, but with a second book of mass 1.0 kg sitting on top of the textbook. Assume that the books do not slide relative to each other.

When we need to slide one surface against another, we need to overcome the friction force. For most movement, such as any motion involving rolling, we rely on the static friction force. In a car race, it is the engine power that determines how fast the wheels can spin, but it is the friction force between the tyres and road that determines the maximum possible acceleration. When the engine revs and the wheels start to turn, the tyres push backwards against the road. By Newton's third law, the road pushes forwards against the tyres. It is this force that accelerates the car – the static friction force that the road exerts on the tyres. If this force is not great enough, the wheels will spin and the car goes nowhere.

WORKED EXAMPLE 5.4

The coefficient of static friction of a racing car's tyres against a road surface is 1.1. The car has a mass of 750 kg. Calculate the maximum acceleration possible for this car on a flat road.

ANSWER	**LOGIC**
$m = 750\ \text{kg};\ \mu_s = 1.1$	▪ Identify the relevant data.
$a_{\text{max}} = \dfrac{F_{\text{net, max}}}{m_{\text{car}}}$	▪ Relate acceleration to force using Newton's second law.
$F_{\text{net, max}} = F_{\text{road on tyres, maximum static friction}}$	▪ Recognise that the only horizontal force is the static friction force of the road on the tyres. The maximum net force is therefore the maximum static friction force.
$F_{\text{road on tyres, maximum static friction}} = \mu_s N_{\text{road on tyres}}$	▪ Write an expression for the maximum friction force.
$N_{\text{road on tyres}} = m_{\text{car}}\, g$	▪ Recognise that as no other vertical forces are acting, the normal force is equal to the gravitational force on the car (Newton's second law).
$F_{\text{road on tyres, maximum static friction}} = \mu_s\, m_{\text{car}}\, g$	▪ Write an expression for the force.
$a_{\text{max}} = \dfrac{F_{\text{net, max}}}{m_{\text{car}}} = \dfrac{\mu_s m_{\text{car}}\, g}{m_{\text{car}}} = m_s g$ $= 1.1\ (9.8\ \text{m s}^{-2})$ $= 10.78\ \text{m s}^{-2}$ $a_{\text{max}} = 11\ \text{m s}^{-2}$	▪ Substitute the expression for force into Newton's second law and simplify. ▪ Substitute known values with correct units. ▪ Calculate the final value. ▪ State the final answer with correct units and significant figures.

TRY THESE YOURSELF

1 As the car race in Worked example 5.4 proceeds, the tyres become worn and the maximum acceleration decreases to 8.5 m s^{-2}. Calculate the coefficient of static friction for the tyres against the road at this stage.

2 Would it make any difference to the answer if the car had a larger or smaller mass? Explain your answer.

INVESTIGATION 5.1

The static and kinetic friction forces

Critical and creative thinking

Numeracy

Information and communication technology capability

AIMS

- To determine how the static and kinetic friction forces that a surface exerts on an object depend on the object's mass
- To measure the coefficient of kinetic friction between two surfaces

Write a hypothesis and an inquiry question for this investigation.

MATERIALS

- Adjustable ramp
- Box
- Set of weights
- Protractor
- Stopwatch or data logger
- Weighing scales

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
The box with weights may slide off the end of the ramp and hit someone.	Keep the area at the end of the ramp clear.

What other risks are associated with your investigation, and how can you manage them?

METHOD

Part 1 (for the static friction force)

1. Measure the mass of the box with only a single weight in it.
2. Beginning with the ramp horizontal, place the box with the weight on the ramp.
3. Slowly raise the ramp until the box just begins to slide. Record the angle at which this happens. Make repeated measurements.
4. Add a weight to the box and measure the total mass of box and weights.
5. Repeat steps 2–4 until you have run out of weights or have at least six data points.

Part 2 (for the kinetic friction force)

1. Set the ramp at an angle that is large enough to overcome the maximum static friction force.
2. Measure the mass of the box with only a single weight in it.
3. Place the box with weight on the ramp and allow it to slide down.
4. Measure the time taken for the box to slide from rest to some known distance along the ramp. Make repeat measurements.
5. Add a weight to the box and measure the total mass of box and weights.
6. Repeat steps 3–5 until you have run out of weights or have at least six data points.

RESULTS

Record your results as you measure them.

Part 1

Make a table of data using your measurements from part 1, with mass in one column and angle in a second column. Add one more column for analysis. Include units and uncertainties. Uncertainties can be calculated from the spread in your repeat measurements.

»

Part 2

Make a table of data using your measurements from part 2, with mass in one column and time in a second column. Add two more columns for analysis. Include units and uncertainties. Uncertainties can be calculated from the spread in your repeat measurements.

ANALYSIS OF RESULTS

Part 1

1. Draw a force diagram for the box on the ramp.
2. Identify the forces acting parallel to the ramp surface.
3. Write an expression for the static friction force in terms of the measured angle and mass.
4. Calculate the maximum static friction force for each mass.
5. Plot a graph of maximum static friction force against mass using appropriate software. Add a trend-line (line of best fit) to your graph, and display the equation for the line on your graph.

Part 2

1. Calculate the acceleration of the box for each mass using kinematics equations. Record the acceleration in your table of data.
2. Calculate the net force acting on the box using Newton's second law and add it to your table.
3. Plot a graph of net force against mass using appropriate software. Add a trend-line to your graph, and display the equation for the line on your graph.
4. Find an expression for the gradient of your graph in terms of θ, μ_k and g. Use the value for the gradient of the trend-line to calculate μ_k.

DISCUSSION

- Did the two graphs have the shape that you expected? Did the line of best fit pass through the origin?
- Give the answer to your inquiry question and state whether your hypothesis was supported or not.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Fluids (liquids and gases) also exert frictional forces on objects that are moving relative to the fluid. We usually refer to these forces as **drag** or resistive forces; for example, **air resistance**. These forces can be very large. The mechanism of these friction forces is different from that between solid surfaces. For friction between solid surfaces, the friction force is approximately independent of the relative speed of the two surfaces. This is not the case for drag forces. Air resistance has been found to increase with the square of the speed of the object relative to the air: $F_{\text{air drag}} \propto v^2$. This is why it is more fuel efficient to drive at 90 km h^{-1} than at 110 km h^{-1}. Drag in liquids is generally directly proportional to the speed of the object: $F_{\text{liquid drag}} \propto v$.

Drag on aeroplanes

Find out more about air resistance and aircraft flight.

KEY CONCEPTS

- If there is an acceleration, then there must be a force acting. This includes when objects are slowing down.
- To identify the forces causing an object to accelerate, consider all surfaces the object is in contact with and all fields it is in.
- Draw force diagrams to help you analyse the effects of forces.
- Friction forces act to oppose the sliding of one surface against another. Friction is due to the interaction of atoms on the two surfaces.
- The static friction force acts to *prevent* sliding, and can take any value up a maximum given by $F_{\text{maximun static friction}} = \mu_s F_N$. In general, $F_{\text{static friction}} \leq \mu_s F_N$. The static friction force is what allows wheels to roll and enables us to walk.

- The kinetic friction force acts when surfaces are sliding against each other, and opposes the *relative* motion. The kinetic friction force is given by $F_{\text{kinetic friction}} = \mu_k F_N$.
- Air resistance is small at low speeds, but increases rapidly with speed. Air resistance cannot be ignored on objects moving quickly.

CHECK YOUR UNDERSTANDING

5.1

1 Compare and contrast the static and kinetic friction forces.

2 Identify the direction of the friction force acting on a car by the road in each of the following cases, and state whether it is the static or kinetic friction force that is acting.

- **a** A car accelerates forwards
- **b** A car makes a left turn
- **c** A car brakes to a stop

3 Identify the direction of the friction force acting on the box by the ute tray in each of the following cases, and state whether it is the static or kinetic friction force that is acting.

- **a** The box slides forwards as it is loaded on to the back of the ute
- **b** The box sits, without sliding, on the tray of the ute as the ute accelerates forwards
- **c** The box sits, without sliding, on the tray of the ute as the ute brakes

4 Harriet is pushing on a couch. She begins by pushing gently, gradually increasing the force she applies. Suddenly the couch start to move, and accelerates rapidly. Explain why this happened.

5 Harriet pushes on a couch of mass 40 kg with a force of 100 N, but it doesn't move.

- **a** What is the friction force acting on the couch?
- **b** Harriet increases the force she applies. The couch begins to move when she applies a force of 180 N. Calculate μ_s for the floor on the couch.

6 Marcus sits on the couch described in the previous question while Harriet is trying to push it. Explain why she now needs to apply a greater force to make it start moving.

7 Harriet finds that she needs to exert a force of 150 N for the couch described in the previous question to continue moving.

- **a** Calculate μ_k for the floor on the couch.
- **b** Describe the motion of the couch if Harriet pushes with a constant force of 160 N after the couch has begun to slide.

5.2 Acceleration of an object subject to a constant net force

Newton's second law tells us what happens when one or more forces act on an object. When an object is subjected to forces that do not add to zero, then the object will accelerate. From Newton's second law:

$$\vec{a}_A = \frac{\vec{F}_{\text{net, A}}}{m_A} = \frac{\sum \vec{F}_A}{m_A}$$

The acceleration is directly proportional to the net force, and inversely proportional to the mass. If the net force is constant, then the acceleration will also be constant.

To analyse the motion of an object subject to a net force, we first apply Newton's second law to find the acceleration. We can then apply the kinematics equations for constant acceleration:

$$s = ut + \frac{1}{2}at^2 = ut + \frac{1}{2}\frac{F_{\text{net}}}{m}t^2$$

$$v = u + at = u + \frac{F_{\text{net}}}{m}t$$

$$v^2 = u^2 + 2as = u^2 + 2\frac{F_{\text{net}}}{m}s$$

to describe and predict the motion.

Figure 5.7 shows what happens to the velocity and position of an object that is subject to a constant net force.

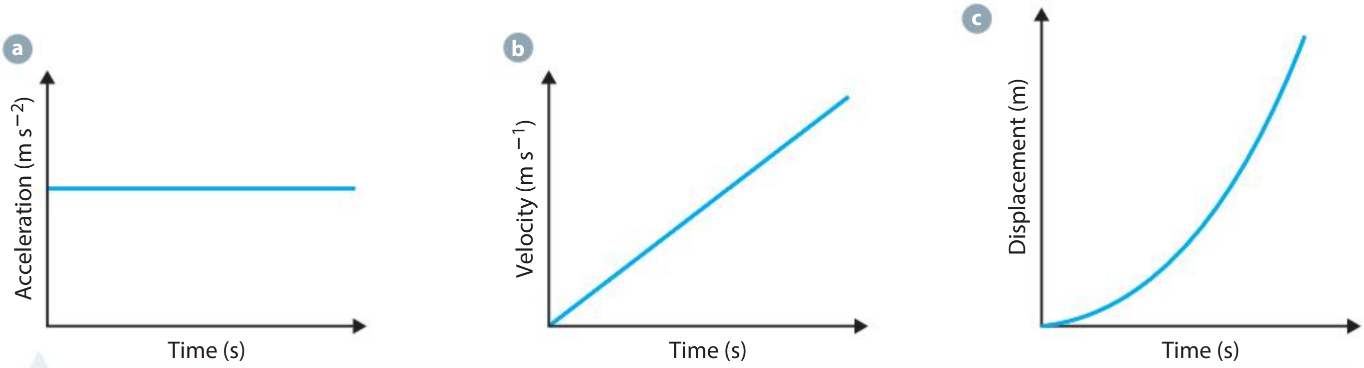


FIGURE 5.7 The **a** acceleration, **b** velocity, and **c** displacement of an object starting at rest and subject to a constant net force

Remember that forces are vectors, and need to be added using vector addition. The net force is also a vector. The direction of the net force gives the direction of the acceleration.

Alternatively, if we know the direction of an object's acceleration, this tells us the direction of the net force acting on it. This in turn tells us about the relative magnitudes of the forces acting on the object.

WORKED EXAMPLE 5.5

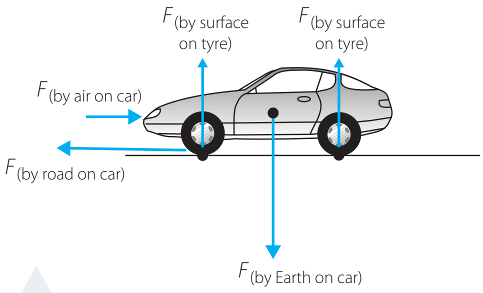
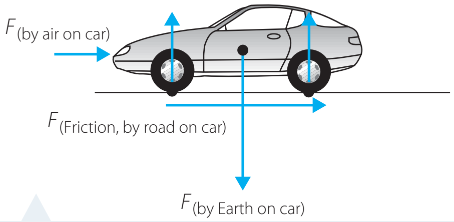
Draw force diagrams to show the forces acting on a car when it is:

1 speeding up.

2 slowing down by braking.

Remember that the lengths of the arrows represent the relative magnitudes of the forces. Label the forces in the form $\vec{F}_{\text{B on A}}$.

ANSWER	LOGIC
1 When a car is speeding up, the acceleration is forwards so the net force is also forwards.	▪ Relate the change in motion to $\vec{a}$ and $\vec{F}_{\text{net}}$.
When a car is speeding up, the forces acting on the car are: • the normal force of the ground surface on each tyre (up) • the friction force of the road surface on the car (forwards) • air resistance (backwards) • the gravitational force of Earth on the car (down).	▪ Identify the forces acting.

ANSWER	**LOGIC**
The normal and gravitational forces are equal. Friction force of the road surface must be greater than the air resistance	■ Identify the relationships between the forces.
 **FIGURE 5.8**	■ Draw a diagram. ■ The contact forces of the ground on the tyres should sum to be the same length as the gravitational force downwards. The force by the road on the car is greater than the force by the air on the car.
2 When the car is braking, the acceleration is backwards so the net force is also backwards.	■ Relate change in motion to $\vec{a}$ and $\vec{F}_{net}$.
When a car is braking, the forces acting on the car are: • the normal force of the road surface on each tyre (up) • the friction force of the road surface on the tyres (backwards) • air resistance (backwards) • the gravitational force of Earth on the car (down).	■ Identify the forces acting.
The normal and gravitational forces are equal. Friction force of the road surface is now in the opposite direction, and may be greater or smaller than the air resistance.	■ Identify the relationships between the forces.
 **FIGURE 5.9**	■ Draw the diagram. ■ The contact forces of the ground on the tyres should sum to be the same length as the gravitational force downwards. The force by the road on the car is now in the opposite direction.

TRY THIS YOURSELF

Draw a force diagram for a car that is rolling on a flat road in neutral, with no braking being applied by the driver.

If we know the net force that acts on an object and we know the object's mass, then we can calculate the acceleration of the object. If we know the acceleration of an object, we can use the kinematics equations from pages 46–7 in chapter 2 to find the change in the object's velocity and position. Solving problems in physics often involves combining ideas.

WORKED EXAMPLE 5.6

Megan is driving her Mercedes, with a mass of 1000 kg, at a speed of 20 m s^{-1}. Calculate the average net force applied to the car if it is to stop in a distance of 25 m to avoid running over a cat.

ANSWER	LOGIC
$s = 25$ m; $u = 20$ m s^{-1}; $v = 0$; $m = 1000$ kg We want to find the force.	▪ Identify the relevant data in the question.
Use kinematics to find the acceleration, then use Newton's second law to find the force.	▪ Plan how to solve the problem.
$v^2 = u^2 + 2as$ $a = \dfrac{v^2 - u^2}{2s}$ $= \dfrac{(0\ \text{m s}^{-1})^2 - (20\ \text{m s}^{-1})^2}{2(25\ \text{m})}$ $= -8.0\ \text{m s}^{-2}$	▪ Relate acceleration to given data. ▪ Rearrange for acceleration. ▪ Substitute known values with units. ▪ Calculate the answer.
$\sum F = F_{\text{net}} = ma$ $F_{\text{net}} = (1000\ \text{kg})(-8.0\ \text{m s}^{-2})$ $= -8000\ \text{kg m s}^{-2}$ $= -8000$ N $F_{\text{net}} = -8000$ N	▪ Write Newton's second law. ▪ Substitute known values with units. ▪ Calculate the answer. ▪ State the answer with correct units, significant figures and direction.

TRY THESE YOURSELF

1 An aeroplane with a mass of 8.0×10^4 kg accelerates from rest to take-off speed of 75 m s^{-1} in 25 s. What is the average net force on the aeroplane during this time?

2 A car with a mass of 1.2×10^3 kg accelerates from rest to 25 m s^{-1} in 5.0 s. Sketch a v versus t graph to find acceleration as the gradient. If the same car has a load of 400 kg added to it, how long, under the same net force, would it take to accelerate from rest to 25 m s^{-1}?

Newton's second law

Watch the video and follow along with the worked examples.

INVESTIGATION 5.2

Acceleration due to a constant net force

Critical and creative thinking

Numeracy

Information and communication technology capability

In this investigation, we will use the gravitational field to exert a constant force on a falling weight. We attach the weight to a string, which passes over a pulley and is attached to a toy car. The forces acting on the weight are the gravitational force and the tension in the string. These act in the vertical direction. The purpose of the pulley is to change the direction of the tension in the string without changing the magnitude of the tension. This is an approximation, called the 'ideal pulley' approximation. In the horizontal direction, there is the tension in the string pulling the car forwards and friction forces opposing this motion. These will give an approximately constant horizontal net force.

AIM

Write a hypothesis describing how you expect the toy car to behave under the influence of a constant net force.

»

MATERIALS

- Pulley attached to table edge
- String
- Masses and mass holder
- Tape measure
- Tape
- Toy car
- Stopwatch or motion-sensor with data-logger

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?

What risks are associated with your investigation, and how can you manage them?

METHOD

1 Set up your equipment as shown in Figure 5.10. Line up the toy car with the pulley so it is pulled directly towards it. Set up your data-logging equipment if you are using this system.

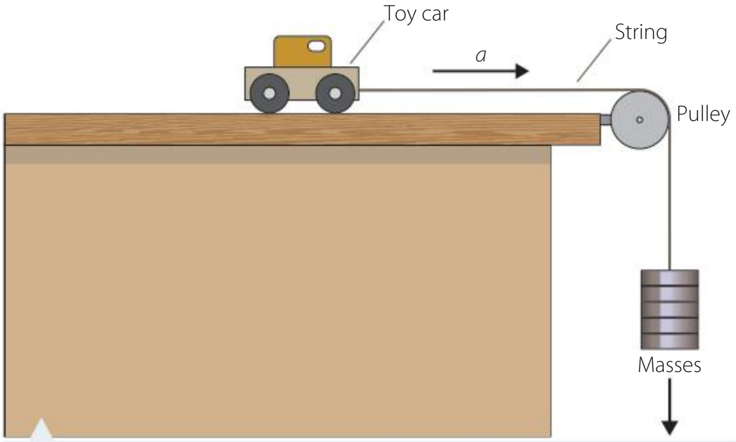


FIGURE 5.10 Experimental set-up for measuring acceleration due to a constant force

2 You will need to make a start and finish line on the table (you can do this using tape).

3 Place the car at the start line. Start the data-logger.

4 Allow the weights to fall freely and time how long it takes the car to reach the finish line.

5 Repeat steps 3 and 4 at least twice to get a measure of the uncertainty in the time taken.

6 Repeat steps 3–5 with a range of different weights added to the mass holder.

RESULTS

1 Record the distance travelled by the car.

2 Record the times taken for each run, and the weight used. Use a table to organise your results.

ANALYSIS OF RESULTS

1 Draw a force diagram for the car. Show the horizontal and vertical forces acting.

2 Calculate the average time taken for each value of the falling weights used, and the uncertainty in that value (use the range method as described in chapter 1). Include this information in your table, as shown.

»

MASS OF WEIGHTS USED (kg)	TIMES TAKEN (s)	AVERAGE TIME ± UNCERTAINTY (s)	ACCELERATION ± UNCERTAINTY (m s^{-2})

3 Use the kinematics equations to calculate the average acceleration of the toy car and write these in your table.

4 Plot a graph of the acceleration of the car versus mass of the falling weights used, using appropriate graphing software.

5 Fit a trend-line to your graph and display the equation. Record the gradient.

6 Derive an expression for the gradient of your graph. You will need to use $F = ma$. Be very careful to distinguish between the mass of the car and the mass of the falling weights.

DISCUSSION

1 Comment on the shape of your graph. Is it what you expect based on your hypothesis? Does the gradient agree with what you would expect from Newton's second law?

2 What other information can you obtain from your graph? For example, can it give you some idea of the friction forces acting?

CONCLUSION

Write a conclusion that links your findings to your hypothesis. Account for any discrepancies.

KEY CONCEPTS

- Newton's second law quantifies the relationship between net force and acceleration: $\vec{F}_{\text{net}} = m\vec{a}$.
- The direction of the acceleration is given by the direction of the net force.
- If the net force is constant, then the acceleration is also constant: $\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$
- The kinematics equations for constant acceleration are used to analyse the motion of an object that is subject to a constant net force.

CHECK YOUR UNDERSTANDING

5.2

1 An ice skater is travelling in a straight line at constant speed. She turns a skate slightly so that she now experiences a constant friction force in the direction opposite to her velocity.

  a Qualitatively describe her motion.

  b From before she started braking until she stops, sketch graphs of:

    i her acceleration as a function of time.

    ii her speed as a function of time.

    iii her position as a function of time.

2 A positron (which has same mass as an electron) has been ejected from a nucleus with a speed of 6.5×10^5 m s^{-1}. It passes into an electric field and experiences a force of 1.5×10^{-16} N in the direction opposite to its motion.

  a Calculate its acceleration.

  b How far does it travel before coming to a stop?

3 Kate is driving her car at 80 km h^{-1} when she sees a kangaroo on the road in front of her. The mass of the car is 1900 kg (assume the mass of Kate is negligible compared with the car).

  a If the maximum frictional force between the road and tyres is 1.4×10^4 N, what is the minimum stopping distance when Kate applies the brakes?

  b If the car skids, so that kinetic friction rather than static friction is acting on the tyres, will the stopping distance be greater or smaller? Explain your answer.

4 A force of 30 N acts in a direction N30°E on a mass of 100 kg at rest on a smooth horizontal surface.

- **a** Calculate the northerly component of the force.
- **b** Calculate the easterly component of the force.
- **c** What is the magnitude and the direction of the acceleration of the mass?

5 Miriam is playing with a toy car. Figure 5.11 shows the velocity–time graph for the toy car, which has a mass 0.5 kg and is travelling in a straight line to the right of the origin.

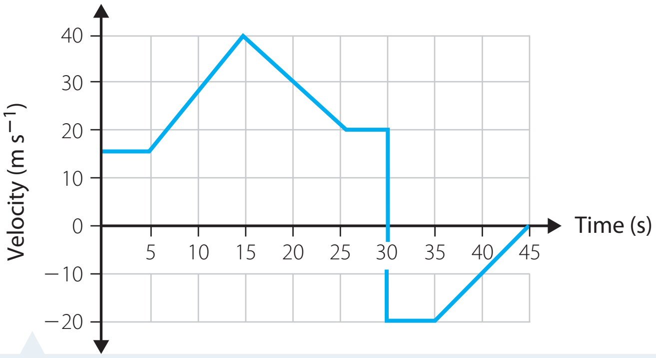


FIGURE 5.11 Velocity versus time graph for a toy car

a Copy the following table and use the graph to complete it.

TIME INTERVAL (s)	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45
ACCELERATION (m s^{-2})									
FORCE (N)									

b At a time of 10.0 s, the toy car experiences combined frictional forces of 0.2 N. What must be the forwards force to the right on the car when it is travelling at this time?

6 Two masses, A and B, are accelerated together along a frictionless horizontal surface by a force of 30 N, as shown in Figure 5.12.

The mass of A is 1.5 kg and the mass of B is 3.0 kg.

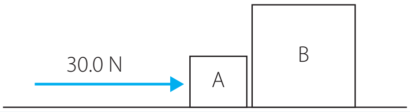


FIGURE 5.12 Two blocks on a frictionless surface are pushed to the right by a force of 30.0 N

- **a** What is the acceleration of the two masses?
- **b** What is the magnitude of the force exerted by B on A?
- **c** The two masses are now accelerated together along the same smooth surface, in the opposite direction by a force of 30 N. What is the force exerted by B on A?

5.3 Energy

Energy is one of the central concepts of physics. If you look again at the concept maps in chapter 1 on pages 5–7, you will see that energy is one of the most important ideas and is connected to our understanding of forces.

There are two forms of energy: **kinetic energy** is the energy possessed by objects due to their motion, and **potential energy** is due to the forces acting on objects in a system.

Kinetic energy

In this chapter, we will consider the kinetic energy of single, macroscopic objects. In chapter 7, we will use the idea of kinetic energy of microscopic particles when we describe waves. In a wave, there is no net movement of material, but all the particles in the medium have kinetic energy as the wave passes through the material. In chapter 11, we will look at the kinetic energy of the disorganised motion of particles in materials. We call this thermal energy, but it is really just the kinetic energy of many particles, which we measure as temperature. Figure 5.13 shows these different examples of kinetic energy.

For a single object, the kinetic energy is given by:

$$E_k = \frac{1}{2}mv^2$$

where m is the mass of the object and v is its velocity.

Looking at this equation, we can see that the units of kinetic energy are kg $\text{m}^2\ \text{s}^{-2}$. We give this the name joule, 1 J = 1 kg $\text{m}^2\ \text{s}^{-2}$, in honour of James Prescott Joule.

Unlike force, which is a vector, energy is a scalar and does not have a direction. Energy is often an easier quantity than force to work with when solving problems. Kinetic energy is always positive.

James Prescott Joule

Find out more about James Prescott Joule – physicist and brewer.

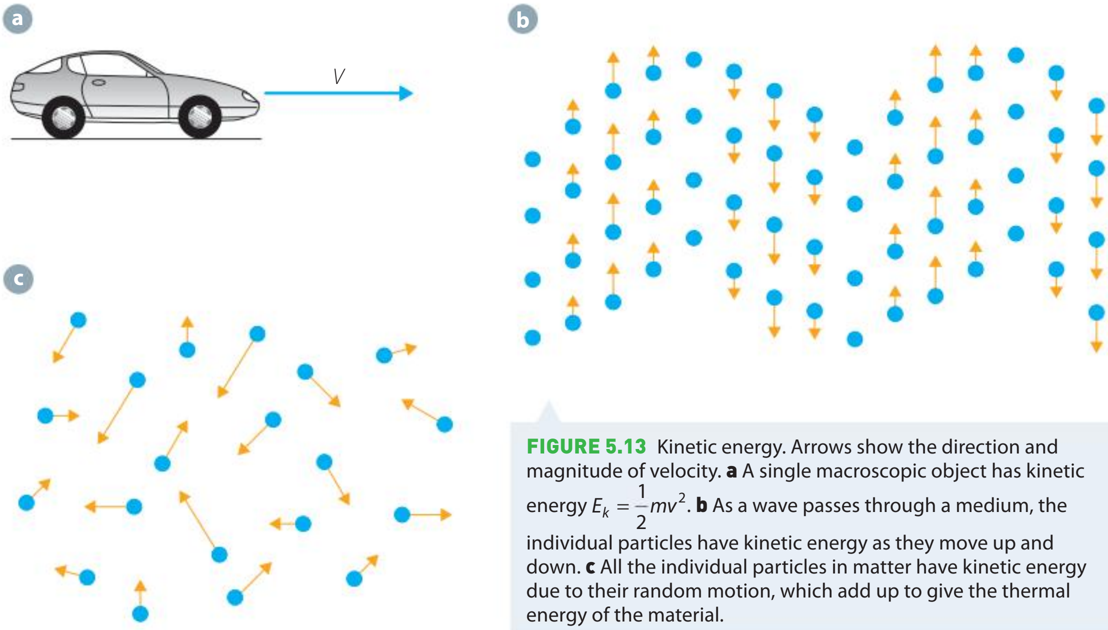


FIGURE 5.13 Kinetic energy. Arrows show the direction and magnitude of velocity. **a** A single macroscopic object has kinetic energy $E_k = \frac{1}{2}mv^2$. **b** As a wave passes through a medium, the individual particles have kinetic energy as they move up and down. **c** All the individual particles in matter have kinetic energy due to their random motion, which add up to give the thermal energy of the material.

WORKED EXAMPLE 5.7

Bill is driving his car (mass 1900 kg) through a small town at 50 km h^{-1}. Calculate the kinetic energy of the car.

ANSWER	LOGIC
$m = 1900$ kg; $v = 50$ km h^{-1}	▪ Identify the relevant data.
$v = \frac{50\ \text{km}}{1\ \text{h}} \times \frac{1\ \text{h}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \text{km}} = 13.9\ \text{m s}^{-1}$	▪ Convert to SI units.

ANSWER	LOGIC
$E_k = \frac{1}{2}mv^2$	- Write the expression for kinetic energy.
$= \frac{1}{2}(1900\ \text{kg})(13.9\ \text{m s}^{-1})^2$	- Substitute known values with correct units.
$= 183\ 549.5\ \text{kg m}^2\ \text{s}^{-2}$	- Calculate the answer.
$E_k = 180\ \text{kJ}$	- State the final answer with correct units and significant figures.

Minimum stopping distance of a car

Calculate the minimum stopping distance of a car using energy considerations.

TRY THESE YOURSELF

1 Continuing on from Worked example 5.7, Bill passes through the town and speeds up to 100 km h^{-1}. Calculate the kinetic energy of the car now.

2 Calculate the speed of the car if its kinetic energy is doubled, compared to its value when travelling at 50 km h^{-1}.

Potential energy

Potential energy is stored energy, ready to do work. Whenever a force acts on an object, there is a potential energy associated with that force.

If you hold an object up, you must do so against the gravitational force of Earth. Potential energy is stored in the Earth–object system. If you let go of the object, the gravitational field of Earth accelerates the object towards the ground. As it accelerates, potential energy is converted to kinetic energy.

As we will see in chapter 12, an electric field also stores potential energy. A charged particle experiences a force in an electric field, so it will accelerate. Magnetic fields (see chapter 14) also store potential energy because they exert forces.

A compressed spring has stored energy. If released, the spring is able to do work by applying a force through a distance. This is called elastic potential energy. Elastic potential energy is really another form of electromagnetic potential energy, because the force that a spring exerts is due to its atoms being pushed closer together or pulled further apart than their normal distance. Atoms interact via their electrons and the electromagnetic force.

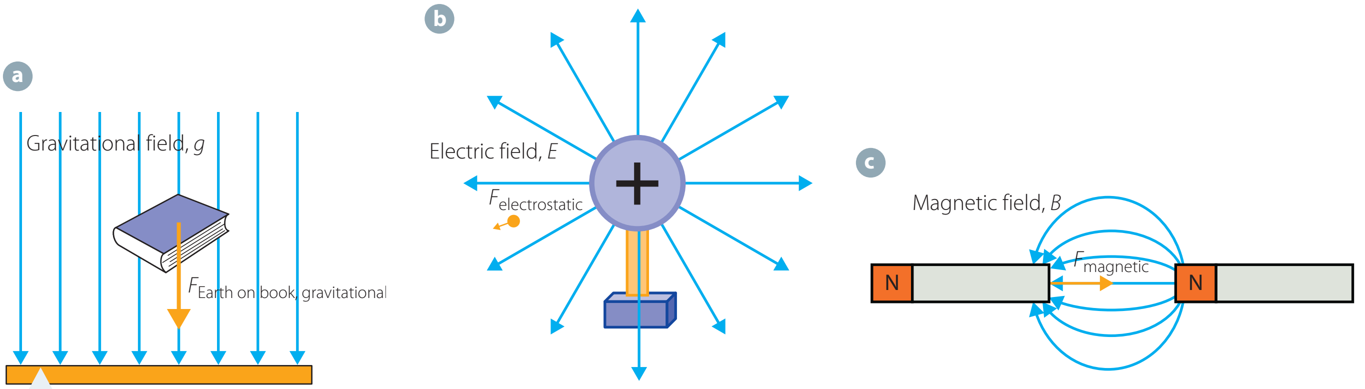


FIGURE 5.14 Whenever there is a force, there is potential energy. Potential energy is stored in fields. **a** Gravitational; **b** electrostatic; **c** magnetic

Mass can also be seen as a form of potential energy, as mass can be converted into energy in nuclear reactions. The forces involved in such reactions are the strong and weak forces that act between subatomic particles, including quarks. You will learn more about this in *Physics in Focus Year 12*.

Potential energy belongs to the system, *not to a single object*. A book held up above Earth's surface would not experience a gravitational force if Earth wasn't there. So the potential energy really belongs to the combination of the book and Earth, or the book–Earth system. We can model the potential energy as being stored in the field, just as we model the force as being exerted by the field.

Potential energy can be positive or negative. We define the zero of potential energy using some convenient or easily measurable configuration of the system. Once we define this configuration, potential energy is given by the force acting on an object multiplied by the object's distance from the reference position. The distance is measured in the direction of the force. For example, for the gravitational force, if we define the zero of potential energy for the book–Earth system as the book resting on the surface of Earth, then

$$U_g = F_{\text{gravitational}}\, h = m_{\text{object}}\, g\, h$$

where U_g is the gravitational potential energy, m is the mass of the object in Earth's gravitational field, g is the gravitational field strength (the acceleration due to gravity) and h is the height of the object above Earth's surface. Note that if the object falls into a hole, its potential energy becomes negative.

For the case of the gravitational potential energy at or close to Earth's surface, we often refer to the energy as belonging to the object, even though strictly it belongs to the system.

In summary, potential energy is due to the forces that objects exert on each other, and is calculated from the positions of the objects in the system.

WORKED EXAMPLE 5.8

Louise throws a ball to Rob, who misses the catch. The ball, with a mass of 150 g, rolls down a drain.

1 Calculate the gravitational potential energy of the ball when it is at a height of 2.5 m above the ground.

2 Calculate the gravitational potential energy of the ball when it is at the bottom of the drain, 1.5 m below the ground.

ANSWERS	LOGIC
1 $m = 150\ \text{g}; h = 2.5\ \text{m}$	▪ Identify the relevant data.
$m = 0.15\ \text{kg}$	▪ Convert to SI units.
$U_g = mgh$ $= (0.15\ \text{kg})\,(9.8\ \text{m s}^{-2})\,(2.5\ \text{m})$ $= 3.675\ \text{kg m}^2\ \text{s}^{-2}$ $U_g = 3.7\ \text{J}$	▪ Write the expression for gravitational potential energy. ▪ Substitute known values with correct units. ▪ Calculate the answer. ▪ State the final answer with correct units and significant figures.
2 $m = 150\ \text{g}; h = -1.5\ \text{m}$	▪ Identify the relevant data, noting that the height is now negative.
$m = 0.15\ \text{kg}$	▪ Convert to SI units.
$U_g = mgh$ $= (0.15\ \text{kg})\,(9.8\ \text{m s}^{-2})\,(-1.5\ \text{m})$ $= -2.205\ \text{kg m}^2\ \text{s}^{-2}$ $U_g = -2.2\ \text{J}$	▪ Write the expression for gravitational potential energy. ▪ Substitute known values with correct units. ▪ Calculate the answer. ▪ State the final answer with correct units and significant figures. Note that we have taken ground level to be $h = 0$ in this example, but this is an arbitrary choice.

TRY THIS YOURSELF

For the same situation, calculate at what height the ball would need to be to have a potential energy of 10 J.

Conservation of energy

Energy is a conserved quantity. The law of conservation of energy says that energy can neither be created nor destroyed. It can change forms, but the total amount of energy in the universe remains constant.

When we want to solve problems using the law of conservation of energy, we need to carefully define the system we are interested in.

An **isolated system** is one that cannot have energy (or mass) transferred into or out of it. Hence, the total energy of an isolated system remains constant:

$$E_{total} = \Sigma U + \Sigma E_k = \text{constant}$$

For a system that is not isolated, the total energy can change, but it only changes by the amount that is added or removed from the system. Remember that the total energy of the universe is constant, so any energy added to a system must be taken away from somewhere else in the universe.

Work done by a constant force

Forces act to transfer energy, and to change the form of energy. If a force acts on an object, causing it to accelerate so that its kinetic energy increases, then there must be an equal decrease in potential energy. If an object is slowed down by a force, then potential energy or some other kinetic energy, such as thermal energy, increases.

We call the amount of energy transferred by a force the **work** done by that force. The amount of energy transferred to an object when a force acts on it is given by:

$$W = \vec{F}\vec{s} = Fs\cos\theta$$

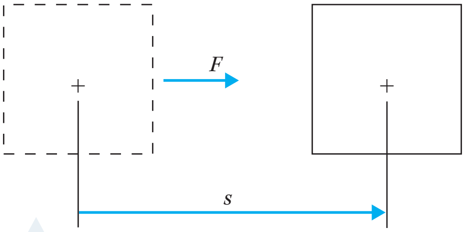
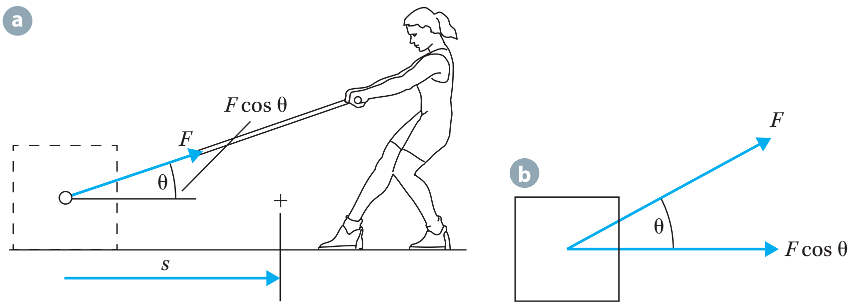


FIGURE 5.15 When the force is applied in the direction of motion, $W = \vec{F}\vec{s}$.

where $\vec{F}$ is the force applied to the object and $\vec{s}$ is the displacement of the object *in the direction of the force*. The angle, θ, is the angle between the applied force and the displacement, as shown in Figure 5.16.

Note that directions are important as we are dealing with forces. If an object moves in the opposite direction to the force, then the work done is negative. If the displacement, *s*, is in the direction of the force, then the work done is positive. In cases where the applied force and the motion are not in the same direction (Figure 5.16), we only consider the component of the force that is in the direction of motion.

FIGURE 5.16 **a** The force is applied at an angle to the direction of the displacement. **b** The component of the force parallel to the displacement is $F\cos\theta$, so the work done is $W = Fs\cos\theta$.



This equation for work also tells us that the unit J can be written as N m, which makes sense as $1\text{ N} = 1\text{ kg m s}^{-2}$, so $1\text{ N m} = 1\text{ kg m}^2\text{ s}^{-2} = 1\text{ J}$.

The work done by a force can be found from a graph of the force versus position. The work done is the area under the curve, as shown in Figure 5.17. If the curve is an irregular shape, as in Figure 5.17b, then you may need to find the area by adding up the areas of many small segments.

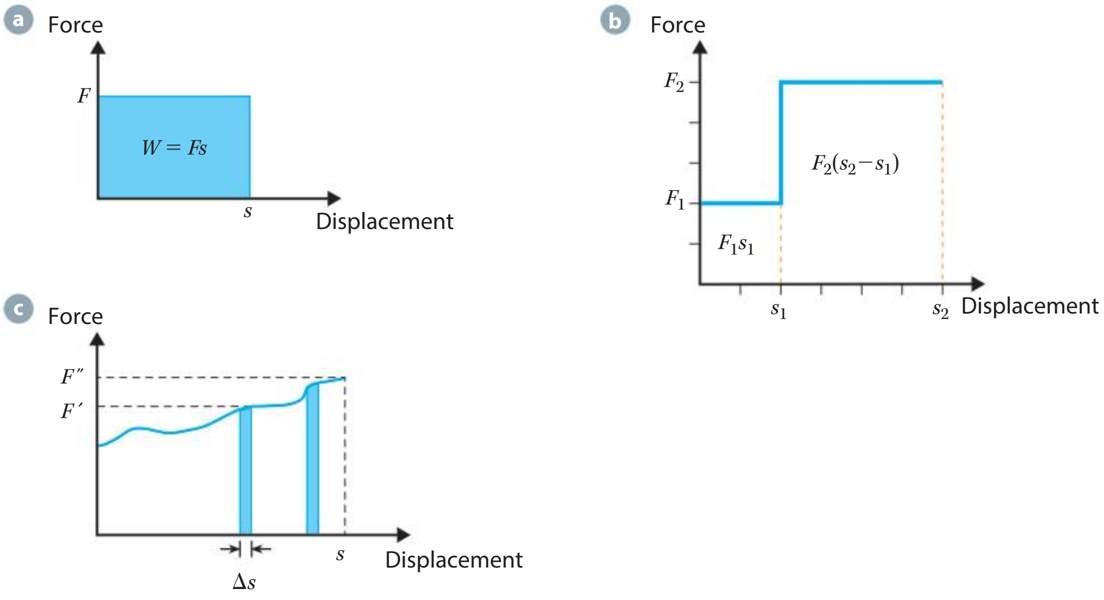


FIGURE 5.17 **a** The work done by a constant force. **b** When the force varies, divide the area under the curve into sections and add all the areas. **c** The work done is equal to the area under the curve, which is the sum of all the small areas (such as the segments shown in blue).

WORKED EXAMPLE 5.9

Phil pushes a textbook across a desk towards Ranji, applying a constant force of 60 N for a distance of 5 cm. The book then slides an additional 20 cm, while subject to a friction force of 10 N, before Ranji stops it.

1 How much work does Phil do on the book?

2 How much work does the desk do on the book?

ANSWER	LOGIC
1 $F = 60$ N; $s = 5$ cm	■ Identify the relevant data.
$s = 0.05$ m	■ Convert to SI units.
$W = Fs$ $= (60\text{ N})\,(0.05\text{ m})$ $= 3\text{ N m}$ $W = 3\text{ J}$	■ Write the expression for work done. ■ Substitute known values with correct units. ■ Calculate the answer. ■ State the final answer with correct units and significant figures.
2 $F = -10$ N; $s = 20$ cm	■ Identify the relevant data, noting that *F* is negative because the friction force is in the opposite direction to the direction of motion.
$s = 0.20$ m	■ Convert to SI units.
$W = Fs$ $= (-10\text{ N})\,(0.20\text{ m})$ $= -2\text{ N m}$ $W = -2\text{ J}$	■ Write the expression for work done. ■ Substitute known values with correct units. ■ Calculate the answer. ■ State the final answer with correct units and significant figures.

TRY THIS YOURSELF

The friction force of the road on the tyres of an accelerating car is 8.5 kN. The car accelerates for a distance of 10 m. Calculate the work done by the friction force on the car. Identify whether the work is positive or negative.

When the work done on an object is positive, so that the force is in the direction of motion, the kinetic energy of the object increases. When the force is in the opposite direction to the motion, the object slows down and its kinetic energy decreases.

The total work done on an object is equal to the sum of the work done by all forces acting:

$$W_{\text{net}} = \Sigma(\vec{F}\vec{s}) = \vec{F}_{\text{net}}\vec{s}$$

We can say that $\Sigma(\vec{F}\vec{s}) = \vec{F}_{\text{net}}\vec{s}$ because, while there may be many forces acting on an object, an object only has one displacement.

Using Newton's second law, we can say that:

$$W_{\text{net}} = \vec{F}_{\text{net}}\vec{s} = m\vec{a}\vec{s}$$

which, using the kinematic equation, $v^2 = u^2 + 2as$ we can write as:

$$W_{\text{net}} = \vec{F}_{\text{net}}\vec{s} = m\frac{1}{2}(v^2 - u^2) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Remembering that kinetic energy is

$$E_k = \frac{1}{2}mv^2$$

we can see that:

$$W_{\text{net}} = \vec{F}_{\text{net}}\vec{s} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \Delta E_{\text{k}}$$

where the Δ (delta) symbol indicates the change in a quantity.

WORKED EXAMPLE 5.10

Phil pushes a textbook from rest across a desk towards Ranji, applying a constant force of 60 N for a distance of 5 cm. At the same time, the book is subject to a friction force of 10 N.

1 Calculate the change in kinetic energy of the book.

2 Calculate the final speed of the book.

ANSWER	LOGIC
1 $F_{\text{Phil}} = 60$ N; $F_{\text{friction}} = -10$ N; $s = 5$ cm	- Identify the relevant data, noting that the forces have opposite signs as they act in opposite directions.
$s = 0.05$ m	- Convert to SI units.
$W_{\text{net}} = F_{\text{net}}s = \Delta E_{\text{k}}$	- Relate forces to change in kinetic energy.
$\Delta E_{\text{k}} = F_{\text{net}}s = (F_{\text{Phil}} + F_{\text{friction}})s$ $= (60\text{ N} - 10\text{ N})(0.05\text{ m})$ $= 2.5\text{ N m}$ $\Delta E_{\text{k}} = 2.5\text{ J}$	- Rearrange for change in kinetic energy and expand the expression for net force. - Substitute known values with correct units. - Calculate the answer. - State the final answer with correct units and significant figures.
2 $\Delta E_{\text{k}} = 2.5$ J; $E_{\text{k, initial}} = 0$; $m = 1.5$ kg	- Identify the relevant data.
$\Delta E_{\text{k}} = E_{\text{k, final}} - E_{\text{k, initial}}$	- Write the expression for change in kinetic energy.
$\Delta E_{\text{k}} = E_{\text{k, final}}$	- Recognise that the initial E_{k} is zero.

ANSWER	**LOGIC**
$E_{\text{k, final}} = \frac{1}{2}mv^2$	▪ Relate kinetic energy to velocity.
$v = \sqrt{\frac{2E_\text{k}}{m}}$	▪ Rearrange for velocity.
$= \sqrt{\frac{2(2.5\ \text{J})}{1.5\ \text{kg}}}$	▪ Substitute known values with correct units.
$= 1.83\ (\text{J kg}^{-1})^{\frac{1}{2}}$	▪ Calculate the answer.
$v = 1.8\ \text{m s}^{-1}$	▪ State the final answer with correct units and significant figures noting that $1\ \text{J} = 1\ \text{kg m}^2\ \text{s}^{-2}$.

TRY THIS YOURSELF

In the same situation, calculate the distance over which the friction force must act to bring the book to a stop, once Phil has stopped pushing.

Energy and the gravitational field

The force that the gravitational field of Earth exerts on an object is given by $\vec{F}_{\text{gravitational}} = m\vec{g}$, where m is the mass of the object and $\vec{g}$ is the field strength, which is the same as the acceleration due to gravity.

Remember that potential energy of an object is related to the force exerted on the object and its position. The potential energy of an object in Earth's gravitational field is given by:

$$U_\text{g} = F_{\text{gravitational}}\, h = m_{\text{object}}\, g\, h$$

where we usually take h to be the height above Earth's surface at the location of interest. Note that this assumes that g is constant, which is only a valid approximation close to Earth's surface.

When you drop an object, it falls (Figure 5.18a, page 140), and accelerates as it falls due to the gravitational force. The kinetic energy of the object increases because the gravitational force is doing work on it. If we make the approximation that air resistance is negligible, then the only force acting is the gravitational force. No other forces act, so the Earth–object system can be modelled as isolated. In this case:

$$W_{\text{gravity}} = F_{\text{gravitational}}\, s = m_{\text{object}}\, g\, \Delta h$$

if the object falls through a height Δh, so that $s = \Delta h$.

Look again at the expression for the potential energy. We can see that the magnitude of the work done by the gravitational field is equal to the change in gravitational potential energy:

$$W_{\text{gravity}} = m_{\text{object}}\, g\, \Delta h = -\Delta U_\text{g}$$

Note the negative sign here, as the force is in the opposite direction to the change in height (measuring height upwards from the ground), so the work done by gravity acts to *decrease* the potential energy of the object.

The work done is also equal to the change in kinetic energy of the object:

$$W_{\text{gravity}} = \Delta E_\text{k} = -\Delta U_\text{g}$$

This allows us to write:

$$mg\Delta h = \frac{1}{2}m(v^2 - u^2)$$

for an object falling freely in Earth's gravitational field.

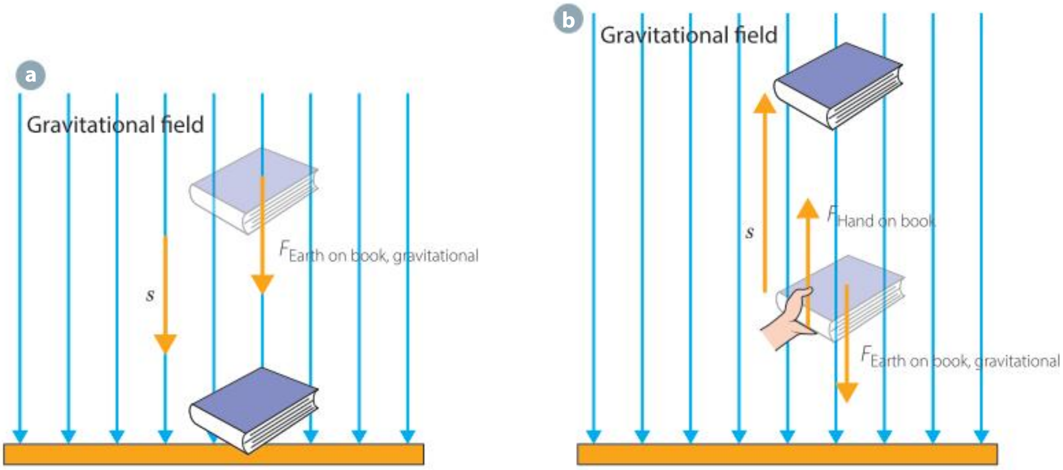


FIGURE 5.18 **a** Force and displacement are in the same direction when an object falls through some height. Work is positive and the object gains kinetic energy. **b** When an object is lifted against the gravitational field, the work done by gravity is negative and the work done by the applied force is positive. The change in kinetic energy is equal to the net work done.

The equation $W_{\text{gravity}} = \Delta E_k = -\Delta U_g$ is really a statement of conservation of energy, as it can be rearranged to say

$$\Delta E_k + \Delta U_g = 0$$

WORKED EXAMPLE 5.11

Phil pushes a textbook across a table towards Ranji, but it falls off the edge of the table. Consider motion in the vertical direction only and ignore air resistance. Calculate the speed at which the book hits the floor if the table is 75 cm high.

ANSWER	LOGIC
$\Delta h = 75\text{ cm}; u = 0$	▪ Identify the relevant data.
$\Delta h = 0.75\text{ m}$	▪ Convert data to SI units.
$\Delta E_k = -\Delta U_g$	▪ Apply conservation of energy.
$\frac{1}{2}mv^2 = mg\Delta h$	▪ Substitute the expressions for ΔE_k and ΔU_g, and recognise that $u = 0$.
$v = \sqrt{2g\Delta h}$ $= \sqrt{2(9.8\text{ m s}^{-2})(0.75\text{ m})}$ $= 3.83\text{ m s}^{-1}$ $v = 3.8\text{ m s}^{-1}$	▪ Rearrange for velocity. ▪ Substitute known values with correct units. ▪ Calculate the answer. ▪ State the final answer with correct units and significant figures.

TRY THIS YOURSELF

Calculate the height a ball must be dropped from if it is to hit the ground at a speed of 5.0 m s^{-1}.

If you lift an object, you must exert a force to do so, and do work on the object (Figure 5.18b). At the same time that you are applying a force upwards, the gravitational field is applying a force downwards. If the object is moving upwards, then you are doing positive work on it and the gravitational field is doing negative work.

WORKED EXAMPLE 5.12

Ranji picks up the fallen book with mass 1.5 kg and places it back on the table 75 cm above the floor.

1 How much work must Ranji do?

2 How much work is done by the gravitational field?

ANSWER	LOGIC
1 $\Delta h = 75$ cm; $u = 0$; $v = 0$; $m = 1.5$ kg	▪ Identify the relevant data.
$\Delta h = 0.75$ m	▪ Convert data to SI units.
$\Delta E_k = 0$	▪ Recognise that the change in kinetic energy of the book is zero if it starts at rest and ends at rest.
$\Delta U_g = mg\Delta h = W_{\text{Ranji}}$ $W_{\text{Ranji}} = (1.5\text{ kg})\,(9.8\text{ m s}^{-2})\,(0.75\text{ m})$ $= 11.025\text{ kg m}^2\text{ s}^{-2}$ $W_{\text{Ranji}} = 11\text{ J}$ Note: this is the work done by Ranji to increase the potential energy of the book by lifting it against the gravitational field to the table. She also has to do a small amount of work to accelerate the book from its initial zero velocity to the velocity at which she raises it. How much work this is depends on how fast she lifts it.	▪ Write an expression for the change in potential energy. This is the energy that Ranji must add to the system by doing work in lifting the book. ▪ Substitute known values with correct units. ▪ Calculate the answer. ▪ State the final answer with correct units and significant figures.
2 $m = 1.5$ kg; $\Delta h = 0.75$ m	▪ Identify the relevant data.
$W = Fs = F_{\text{gravitational}}\,\Delta h$	▪ Write the expression for the work done by the gravitational force.
$W = -mg\Delta h$ $W = (1.5\text{ kg})\,(-9.8\text{ m s}^{-2})\,(0.75\text{ m})$ $U_g = -11.025\text{ kg m}^2\text{ s}^{-2}$ $U_g = -11\text{ J}$ Note: this answer is the same as the work done by Ranji in lifting the book, but negative. The change in kinetic energy of the book is zero. But work, equal to the change in potential energy, has been done on the book–Earth system by Ranji.	▪ Substitute the expression for the gravitational force, noting that the force is negative as it acts down and the displacement is upwards. ▪ Substitute known values with correct units. ▪ Calculate the answer. ▪ State the final answer with correct units and significant figures.

TRY THIS YOURSELF

Continuing on from the example, Ranji throws Phil’s textbook directly up into the air and then catches it at the same height that she threw it from. How much work has the gravitational force done on the book over the entire trajectory of the book? Explain your answer and state any assumptions that you are making.

INVESTIGATION 5.3

Energy changes of falling objects

Numeracy

Critical and creative thinking

Information and communication technology capability

AIM

To investigate the transformation of gravitational potential energy to kinetic energy as an object is accelerated by a gravitational field.

Write a hypothesis or inquiry question for this investigation.

MATERIAL

- Objects to drop
- Data-logger equipment for measuring motion, with computer and software

What other equipment will you need?

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?

What risks are associated with your investigation, and how can you manage them?

METHOD

- Design a method to answer your inquiry question or test your hypothesis.
- When using a data-logger, you need to read the instructions and make sure that you can use it and the associated software.
- You will need to be able to record the motion of a falling object and display graphs of position, velocity and acceleration as a function of time and/or distance.

RESULTS

Record your results as you measure them. Make sure you save each set of graphs with a name that tells you what the data is for.

ANALYSIS OF RESULTS

- How will you analyse your data?
- A graph of velocity as a function of time will allow you to calculate kinetic energy as a function of time.
- A graph of position as a function of time will allow you to calculate gravitational potential energy as a function of time.
- A graph of acceleration as a function of time will allow you to calculate net force as a function of time.

DISCUSSION

1. Did your graphs have the shapes that you expected?
2. How much gravitational potential energy was converted to kinetic energy? Was air resistance significant? If you used a range of different objects, did the size, shape or mass of the objects make any difference?
3. Give the answer to your inquiry question and state whether your hypothesis was supported or not.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Mechanical energy

The kinetic energy of macroscopic objects and gravitational potential energy are sometimes grouped under the term **mechanical energy**. In the absence of friction and other resistive forces, such as air resistance, mechanical energy is conserved. Sometimes potential energy stored in springs is also considered a form of mechanical energy.

Often we make the approximation that mechanical energy is conserved. This makes calculations simpler. When objects are moving slowly so air resistance is small, and there are no solid surfaces sliding against each other, this is a reasonable approximation.

However, remember that it *is* an approximation. In reality, mechanical energy is almost never conserved. Real systems usually have friction and air resistance acting. When the kinetic friction force acts to slow down an object, the kinetic energy lost by the object is converted into the random kinetic energy of the atoms on the surfaces – the surfaces get warmer. This random kinetic energy of microscopic particles is not considered a form of mechanical energy.

KEY CONCEPTS

- There are two types of energy: kinetic and potential. Kinetic energy is associated with motion and potential energy with forces.
- Kinetic energy is given by $E_k = \frac{1}{2}mv^2$.
- Potential energy belongs to a system, and is stored because forces act between objects in the system. Potential energy is calculated from the positions of objects.
- The gravitational potential energy of an object close to Earth is $U_g = mgh$.
- Energy is conserved. The total amount of energy in the universe is constant.
- An isolated system is one that energy cannot be transferred into or out of. For an isolated system, $E_{total} = \sum U + \sum E_k = \text{constant}$.
- When a force acts on an object and there is a displacement in the direction of the force, the force does work, $W = Fs$, on the object. Work can be positive or negative.
- Work can be found from the area under a F versus s graph.
- When the force and motion are in the same direction, positive work is done and the object speeds up, gaining kinetic energy.
- When the force is in the opposite direction to the motion, negative work is done and the object slows down, losing kinetic energy.
- Kinetic energy and gravitational potential energy are types of mechanical energy. Mechanical energy is conserved only in the absence of friction.

CHECK YOUR UNDERSTANDING

5.3

1 Define 'work' and give its units.

2 When you are holding a book above your head, explain why you are not doing any work on the book.

3 Identify whether the work done by the force is positive or negative in each of the following situations.

- **a** the force you exert on a ball with your hand when you throw it
- **b** the gravitational force on the ball as it rises
- **c** the gravitational force on the ball as it falls
- **d** the force of your hand on the ball as you catch it

4 A car has a mass of 1500 kg.

- **a** Calculate the kinetic energy of the car at different speeds to complete the table below.
- **b** Plot a graph of kinetic energy as a function of speed. Comment on the shape of your graph.

SPEED (M S^{-1})	0	5	10	15	20	25	30	35
E_k (J)								

5 A cockatoo is chewing pinecones from a pine tree. A pinecone falls from a height of 4.5 m. Calculate the speed with which it hits the ground.

6 A toolbox of mass 15 kg sits on the back of a ute without slipping as the ute accelerates forwards. The ute takes 10 s to accelerate to a speed of 80 km h^{-1}. Calculate the work done by the static friction force on the toolbox.

5.4 Power

A crane that can lift a 10 tonne block of concrete to the top of a building in 1 minute is more powerful than a crane that can do the same task but takes 2 minutes. Both cranes do the same amount of work on the load, but they take different times to do so.

The rate at which work is done is called **power**, P. As work is the energy transferred by a force, power is the rate at which energy is transferred, or transformed from one form into another.

$$P = \frac{\Delta W}{t} = \frac{\Delta E}{t}$$

The unit of power is the watt (W); 1 W = 1 J s^{-1} = 1 N s^{-1}. A 100 W light globe transforms 100 J of electrical energy into light and heat every second. A 1 kW crane motor increases the gravitational potential energy of a load by 1 kJ every second (assuming 100% efficiency).

The amount of energy transformed or transferred in a process is the power multiplied by the time interval:

$$\Delta E = Pt$$

If you plot the energy being transferred to an object as a function of time (Figure 5.19a), then the gradient of this graph at any moment is the power. If you plot a graph of power as a function of time (Figure 5.19b), then the area under the line is the total energy transferred.

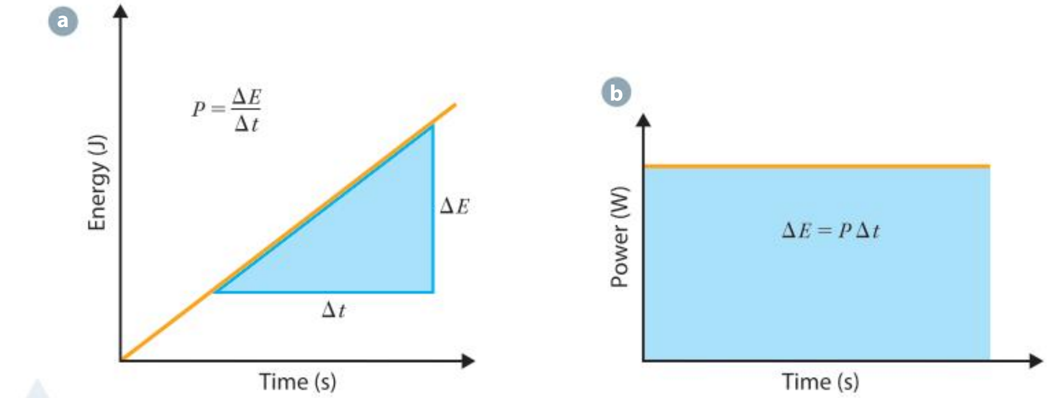


FIGURE 5.19 **a** Power is the gradient of an energy–time graph. **b** Energy transferred is the area under the line of a power–time graph.

Saturn V F-1 engines

Find out about the most powerful engines ever built.

We can relate power to force using the equation for work, $W = \vec{F}\vec{s}$:

$$P = \frac{\Delta E}{t} = \frac{W}{t} = \frac{\Delta(Fs)}{t} = F\frac{\Delta s}{t} = Fv$$

This means that the power, or rate at which energy is being transferred by a force, is equal to the force applied multiplied by the speed at which the object is moving.

WORKED EXAMPLE 5.13

A crane takes 1 minute to lift a 1000 kg block of concrete through a height of 10 m at constant speed. Calculate the power used by the crane to lift the block.

ANSWER	**LOGIC**
$m = 1000\text{ kg}; s = 10\text{ m}, t = 1\text{ min}$	- Identify the relevant data.
$t = 60\text{ s}$	- Convert to SI units.
$P = \frac{W}{t} = Fv$	- Identify the appropriate formula to determine the power.
$F_{\text{net}} = 0 \text{ so } F = F_{\text{gravitational}}$	- As speed is constant, net force = 0 (Newton's second law), so the force applied by the crane must be equal to the gravitational force.
$F = mg$	- Identify the appropriate formula to determine the force.
$v = \frac{\Delta s}{t}$	- Identify the appropriate formula to determine the velocity.
$P = Fv = \frac{mg\Delta s}{t}$	- Substitute these expressions into the equation for *P*.
$= \frac{(1000\text{ kg})(9.8\text{ m s}^{-2})(10\text{ m})}{60\text{ s}}$	- Substitute known values with correct units.
$= 1633\text{ kg m}^2\text{ s}^{-3}$	- Calculate the answer.
$P = 1600\text{ W or }1.6\text{ kW}$	- State the final answer with correct units and significant figures.

TRY THIS YOURSELF

Calculate the power of the crane if the crane takes 2 minutes to raise the 1000 kg block.

One of the factors that determines the top speed of a car is the power of its engine. Air resistance on the car increases significantly as the speed of the car increases. The work done on the car by this air resistance decreases the energy of the car. In order to keep the car travelling at constant speed, the engine must provide this amount of energy by converting the energy stored in the fuel, so fuel consumption increases with speed.

WS The power of modern aircraft

Other frictional forces also oppose the motion of a car. Rolling friction acts on the tyres. Ideally, when a wheel rolls, there is no slipping of the wheel against the ground. In reality, there may be slight slipping as well as deformation of the tyre and the road surface. These combine to give rolling friction, which must also be overcome to keep the car at constant speed.

Internal friction forces also affect a car's efficiency as they reduce the power that goes from the engine to the wheels. There is friction at all points along the drive train of a car. This reduces the amount of force with which the tyres can push against the ground.

INVESTIGATION 5.4

Power

Critical and creative thinking

Numeracy

Information and communication technology capability

In this investigation, you will explore the way energy is transformed from gravitational potential energy to kinetic energy. Falling weights do work on a toy car or block. The rate at which the work is done is the power that is transferred to the car/block.

AIM

Write an inquiry question for this investigation.

MATERIALS

- Pulley attached to table edge
- String
- Masses and mass holder
- Tape measure
- Tape
- Toy cars, blocks with different surfaces (smooth, rough)
- Stopwatch or motion-sensor with data-logger

RISK ASSESSMENT

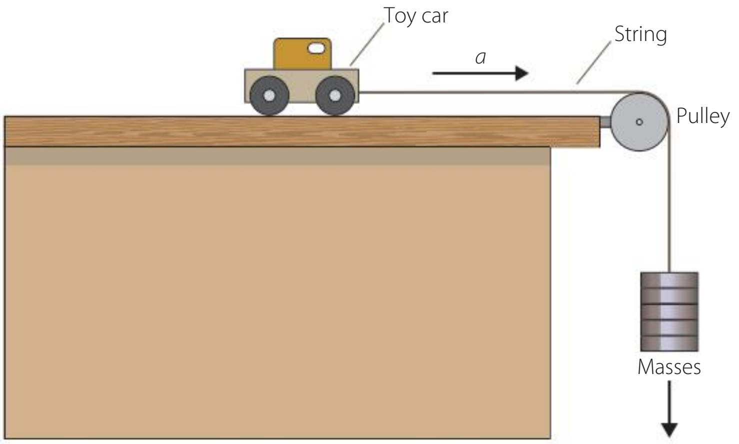
WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Falling weights could land on someone.	Keep area beneath weights clear.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Measure and record the masses of the toy car, blocks and falling weights.
2. Set up your equipment as shown in Figure 5.20. Line up the toy car with the pulley so it is pulled directly towards it.
3. You will need to make a start and finish line on the table (you can do this using tape).
4. Set up your data-logging equipment to record the motion of the car. If you don't have data-logging equipment, you can use a stopwatch to record the time taken for the car to travel a given distance.

FIGURE 5.20 Experimental set-up for investigating power transferred



5 Place the car at the start line. Allow the weights to fall freely through some measured distance. Record the motion of the car or time how long it takes the car to reach the finish line.
6 Lock the wheels of the car so they cannot rotate (you can do this using tape). Repeat step 5.
7 Repeat step 5 with blocks with various surfaces.

RESULTS

- Using the data-logger, record the motion of the car and blocks. You should get a set of data including speed as a function of time, or a graphical display including a plot of speed as a function of time.
- You will need to know the mass of the falling weights, the height through which they fell, the speed of the car when the weights reached the end of the fall and the time taken for the fall. Record this data in a table as shown.

OBJECT BEING PULLED	TIME FOR FALL (s)	FINAL SPEED (m s^{-1})	CHANGE IN E_k (J)	MASS OF FALLING WEIGHTS (kg)	DISTANCE FALLEN (m)	CHANGE IN U_g (J)	CHANGE IN E_k (J)
CAR – ROLLING							
CAR – SLIDING							
ROUGH BLOCK – SLIDING							
SMOOTH BLOCK – SLIDING							

ANALYSIS OF RESULTS

You will need to perform calculations for each measurement to complete your table.

1 Calculate the final kinetic energy of the car/block using $E_k = \frac{1}{2}m_{car}v^2$.

2 Calculate the power being transferred to the car from $P_{to\ car} = \frac{\Delta E_k}{t}$.

3 Calculate the change in gravitational potential energy of the falling weights using $U_g = m_{weight}g\Delta h$.

4 Calculate the final kinetic energy of the weights using $E_k = \frac{1}{2}m_{weights}v^2$. As the weights are attached to the car via the string, they must move at the same speed. So, the final speed of the car is the same as the final speed of the weights.

5 Calculate the power being transformed into kinetic energy of the weights: $P_{to\ weights} = \frac{\Delta E_k}{t}$.

6 Calculate the power being transferred from the weights from $P_{from\ weights} = \frac{\Delta U_g}{t}$.

7 Calculate the difference between the power transformed from potential energy and the combined kinetic energy of the weights and car. This difference is the rate at which energy is lost as thermal energy.

DISCUSSION

1 For each of the rolling and sliding situations you investigated, comment on how much of the power transferred from the falling weights went into increasing the kinetic energy of the car/block.
2 Was there a difference between sliding and rolling? Was there significant rolling resistance for your car? What effect did kinetic friction have for sliding objects?
3 Was mechanical energy conserved for any of these situations?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion that answers the inquiry question of this investigation.

General knowledge

KEY CONCEPTS

- Power is the rate at which energy is transferred, or transformed from one form into another: $P = \frac{\Delta E}{t} = \frac{\Delta W}{t}$.
- When a force acts on a moving object, the rate at which energy is transferred by the force is $P = \vec{F}\vec{v}$.

CHECK YOUR UNDERSTANDING

5.4

1. Explain how power, work and time are related.
2. Give the units of power in fundamental units.
3. The kinetic energy of a 200 kg satellite is raised by 4.0×10^7 J in a 10-minute rocket burn. Calculate the average power of the rocket motors.
4. A crane is lifting a 900 kg elephant upwards.
   - **a** Calculate the work done by the crane if the elephant is lifted through a vertical displacement of 16 m.
   - **b** The crane is lifting the elephant at a speed of 1.5 m s^{-1}. Calculate the power at which the crane is operating.
5. Calculate the constant vertical speed with which a 2.5 kW winch motor could lift an injured hiker into a helicopter if the hiker and stretcher have a combined mass of 100 kg.
6. A sprinter can run 100 m in 10 s, accelerating the entire time. Calculate the average power required for this if the sprinter has a mass of 75 kg.

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5 CHAPTER SUMMARY

- If there is an acceleration, there must be a force acting. To identify the forces causing an object to accelerate, consider all surfaces the object is in contact with and all fields it is in.
- Draw force diagrams to help you analyse the effects of forces.
- Friction forces act to oppose the sliding of one surface against another.
- The static friction force acts to prevent sliding, and can take any value up to a maximum given by $F_{\text{maximun static friction}} = \mu_s F_N$. In general, $F_{\text{static friction}} \leq \mu_s F_N$. The static friction force is what allows wheels to roll and enables us to walk.
- The kinetic friction force acts when surfaces are sliding against each other, and opposes the relative motion. The kinetic friction force is given by $F_{\text{kinetic friction}} = \mu_k F_N$.
- Air resistance is small at low speeds, but increases rapidly with speed. Air resistance cannot be ignored on objects moving quickly.
- Acceleration can be calculated from Newton's second law: $\vec{F}_{\text{net}} = m\vec{a}$.
- If the net force is constant, then the acceleration is also constant: $\vec{a} = \dfrac{\vec{F}_{\text{net}}}{m}$.
- There are two types of energy: kinetic and potential. Kinetic energy is associated with motion and potential energy with forces.
- Kinetic energy is given by $E_k = \frac{1}{2}mv^2$.
- Potential energy belongs to a system, and is stored because forces act between objects in the system. The gravitational potential energy of an object close to Earth is $U_g = mgh$.
- Energy is conserved. The total amount of energy in the universe is constant.
- An isolated system is one that energy cannot be transferred into or out of. For an isolated system, $E_{\text{total}} = \sum U + \sum E_k = \text{constant}$.
- When a force acts on an object and there is a displacement in the direction of the force, the force does work on the object. Work can be positive or negative.
- When the force and motion are in the same direction, positive work is done and the object speeds up, gaining kinetic energy. When force and motion are in opposite directions, negative work is done and the object slows down.
- Kinetic energy of macroscopic objects and gravitational potential energy are types of mechanical energy. Mechanical energy is conserved only in the absence of friction.
- Power is the rate at which energy is transferred, or transformed from one form into another: $P = \dfrac{\Delta E}{t} = \dfrac{\Delta W}{t}$.
- When a force acts on a moving object, the rate at which energy is transferred by the force is $P = \vec{F}\vec{v}$.

5 CHAPTER REVIEW QUESTIONS

Review quiz

1 Identify the quantity calculated by finding the area under a F versus s graph.

2 Explain how you could tell if work has been done on an object.

3 How does a falling object gain its kinetic energy, given that energy cannot be created or destroyed?

4 A 1000 kg car and a 2.00×10^5 kg aeroplane accelerate at the same rate of 3.5 m s^{-2}. Calculate the ratio $\dfrac{F_{\text{net on aeroplane}}}{F_{\text{net on car}}}$.

5 Does friction always oppose motion? Justify your answer.

6 Kate is driving when a kangaroo jumps in front of her car. She slams on the brakes and the car skids. Explain why it takes longer for a skidding car to come to a stop than when the wheels are still gripping the road and rolling.

7 A car accelerates at a rate of 2.0 m s^{-2} when a force of 3.0 kN is applied to it by the road. Calculate the mass of the car.

8 A student draws a force diagram for a car and includes a force of the engine on the car. The student says that the engine is what makes the car go, so it must exert a force on the car. Explain what is wrong with this diagram and explanation.

9 Define 'isolated system'. Apply the ideas of both force and energy to explain what an isolated system is.

10 Two children are fighting over a toy of mass 500 g. Marcus pulls it to the left with a force of 25 N. Laurence pulls it to the right with a force of 35 N.

  a Calculate the acceleration of the toy.

  b If Marcus suddenly lets go, what is the acceleration of the toy now?

11 An object with an initial speed u is subject to a constant force in the direction opposite to its motion. Sketch graphs, as a function of time, of the object's:

- **a** acceleration.
- **b** speed.
- **c** kinetic energy.

12 A toboggan and child of combined mass 67 kg slide down a frictionless snowfield inclined at 15° to the horizontal. Calculate the magnitude of:

- **a** the component of the gravitational force perpendicular to the snowfield.
- **b** the force down the slope of the snowfield on the toboggan and child.
- **c** the acceleration.

13 A snowboard and rider with combined mass of 95 kg are sliding down a snow-covered slope inclined at 38° to the horizontal. The coefficient of kinetic friction for the snow on the board is 0.08. Calculate the magnitude of:

- **a** the net force on the snowboard and rider.
- **b** the force down the slope on the snowboard and rider.
- **c** their acceleration.

14 When an aeroplane is flying, it is subject to four forces: gravity, lift (upwards), thrust (forwards) and air resistance (backwards). For each of these forces, identify whether it does positive, negative or no work when the plane is:

- **a** climbing after take-off.
- **b** cruising at constant speed.
- **c** descending to land.

15 The force that must be applied to stretch a spring varies with the distance it is stretched. The graph in Figure 5.21 shows the force exerted on the end of a particular spring versus the distance it is stretched. Use the graph to find:

- **a** the work that must be done to stretch the spring from no extension to 10 cm.
- **b** the work that must be done to then stretch the spring by an additional 5 cm.

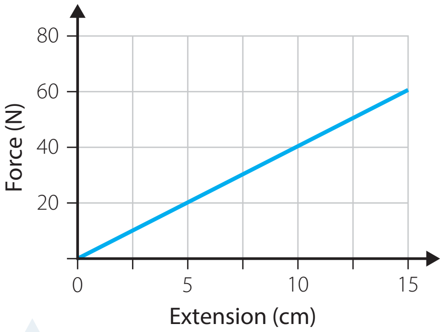

FIGURE 5.21 Force required to stretch a spring versus distance stretched

16 A student carries a heavy backpack to school along a level street. The student says, 'that was hard work'. Evaluate the student's statement.

17 A box is subject to two forces, as shown in Figure 5.22. The box has a mass of 2 kg and force F_1 has a magnitude of 10 N.

Calculate

- **a** the magnitude of the acceleration, a.
- **b** the magnitude of force F_2.

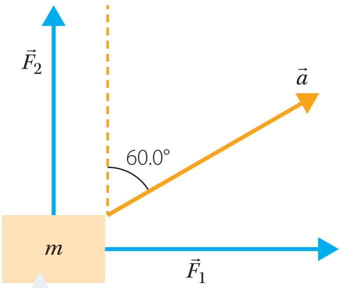

FIGURE 5.22

18 Darren's car is stuck in slippery mud and he is trying to push it out.

- **a** Draw a force diagram showing the forces acting on the car.
- **b** Draw a force diagram showing the forces acting on Darren.
- **c** Apply Newton's second and third laws to explain what will happen if Darren pushes with more force on the car than the maximum static friction force that the mud exerts on him.

19 Two objects are connected via a pulley, as shown in Figure 5.23. Assume the table is very smooth so friction can be ignored.

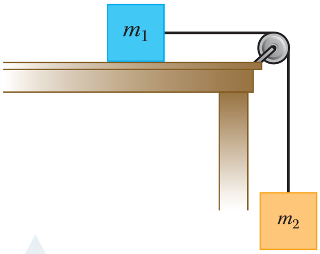

FIGURE 5.23

- **a** Draw a force diagram for each object.
- **b** Calculate the acceleration of the objects if $m_1 = 5$ kg and $m_2 = 10$ kg.
- **c** Now assume the coefficient of kinetic friction between the table and object 1 is 0.6. Calculate the acceleration of the objects now.

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20 A meteor enters the atmosphere and slows down quickly while burning up and vaporising. Describe how the law of conservation of energy applies to this event.

21 The maximum static friction force between a 1200 kg car's tyres and the road is 9.5×10^3 N.

- **a** Calculate the minimum stopping distance if the car is travelling at 60 km h^{-1}.
- **b** Calculate the minimum stopping distance if the car is travelling at 40 km h^{-1}.
- **c** Compare your answers to parts **a** and **b**, and use them to justify the 40 km h^{-1} speed limit in school zones.

22 Using the ideas of work and energy, calculate the speed with which a 10 kg rock released from a height of 30 m hits the ground.

23 A 10 kg rock is dropped from a height of 30 m. Assume air resistance is negligible during the fall.

- **a** Apply kinematics to find the time taken for the rock to fall to the ground.
- **b** Calculate the kinetic energy of the rock just before it hits the ground.
- **c** Calculate the average power being transferred to the rock as it falls.

24 A 10 kg rock is dropped from a height of 30 m. Assume air resistance is negligible during the fall.

- **a** Draw a graph of kinetic energy as a function of time for the rock.
- **b** Describe how you could find the power being transferred at any moment from your graph.
- **c** Describe how the power varies as the rock falls.

25 A toboggan and child of combined mass 67 kg slide down a snowfield inclined at 15° to the horizontal. They start at rest and travel a distance of 10 m down the slope. The coefficient of kinetic friction between the snow and the toboggan is 0.10. Calculate the magnitude of:

- **a** the net force down the slope on the toboggan and child.
- **b** the acceleration of the toboggan and child.
- **c** the work done on the child and toboggan.
- **d** the final velocity of the child and toboggan.
- **e** the average power being transferred to the child and toboggan as they slide.

6

Momentum, energy and simple systems

INQUIRY QUESTION

How is the motion of objects in a simple system dependent on the interaction between the objects?

OUTCOMES

Students:

- conduct an investigation to describe and analyse one-dimensional (collinear) and two-dimensional interactions of objects in simple closed systems (ACSPH064) CCT
- analyse quantitatively and predict, using the law of conservation of momentum $\left(\Sigma m\vec{v}_{\text{before}} = \Sigma m\vec{v}_{\text{after}}\right)$ and, where appropriate, conservation of kinetic energy $\left(\Sigma \frac{1}{2} m\vec{v}^2_{\text{before}} = \Sigma \frac{1}{2} m\vec{v}^2_{\text{after}}\right)$, the results of interactions in elastic collisions (ACSPH066) ICT N
- investigate the relationship and analyse information obtained from graphical representations of force as a function of time
- evaluate the effects of forces involved in collisions and other interactions, and analyse quantitatively the interactions using the concept of impulse $\left(\Delta\vec{p} = \vec{F}\Delta t\right)$ ICT N
- analyse and compare the momentum and kinetic energy of elastic and inelastic collisions (ACSPH066) ICT N

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In this chapter, we will explore how the movements of objects in a simple system are dependent on the interactions between the objects.

In a car crash, there are large and damaging forces exerted on the vehicles and the people inside them due to the interaction. Energy is transformed during a collision as the state of motion of the vehicles and people inside changes. In chapter 5, we looked at how forces cause accelerations. The first of several conservation principles was also introduced: conservation of energy. In this chapter, a second conservation law is introduced: the law of conservation of momentum. We will see how this law helps us to understand the effects of interactions between objects such as colliding vehicles.

Getty Images/ Bill Pugliano

FIGURE 6.1 The concept of momentum is important in analysing car crashes, and in designing cars to be safer.

6.1 Momentum

Momentum is a very useful concept in physics. Imagine being hit by a tomato thrown at 10 m s^{-1}. Now imagine being hit by a watermelon thrown at 10 m s^{-1}. The experience is quite different, and the concept of momentum allows us to quantify the difference.

Momentum, $\vec{p}$, is defined as the product of an object's mass, m, and its velocity, $\vec{v}$:

$$\vec{p} = m\vec{v}$$

Momentum has units of kg m s^{-1}. It is a vector quantity and has a direction the same as that of the object's velocity.

Momentum and Newton's laws

Sir Isaac Newton used the idea of momentum when he developed his three laws of motion. Newton actually wrote his second law (see chapter 4, page 92) in terms of momentum rather than acceleration. In terms of momentum, Newton's second law is:

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

If we substitute the definition of momentum into this equation, and assume that mass is constant, we can see that:

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a}$$

WORKED EXAMPLE 6.1

During a food fight, a child is hit by a 150 g tomato travelling at 10 m s^{-1}. Calculate the momentum of the tomato.

ANSWER	LOGIC
$m = 150$ g; $v = 10$ m s^{-1} $m = 0.15$ kg	- Identify the relevant data. - Convert data to SI units.
$p = mv$ $p = (0.15 \text{ kg}) \times (10 \text{ m s}^{-1})$ $p = 1.5 \text{ kg m s}^{-1}$	- Identify the appropriate formula. - Substitute in known values with correct units. - Calculate the final value and state the final answer with correct units and significant figures.

TRY THESE YOURSELF

1 Calculate the momentum of a watermelon with mass 5.0 kg moving at 10 m s^{-1}.

2 Calculate the speed that a 150 g tomato would need to move at to have the same momentum as a 5.0 kg watermelon moving at 10 m s^{-1}.

The law of conservation of momentum

Recall that for a conserved quantity, the total amount of that quantity in the universe, or in any isolated system, remains constant.

Momentum is a conserved quantity. The sum of the momentum of all the objects in an isolated system remains constant:

$$\vec{p}_{\text{net}} = \Sigma \vec{p} = \text{constant}$$

or

$$\Sigma \vec{p}_{\text{initial}} = \Sigma \vec{p}_{\text{final}}$$

When we apply the law of conservation of momentum, we need to remember that it is a vector quantity. We must add momenta (the plural of momentum) as vectors. In one dimension, this means we must be careful to define positive and negative directions and make sure all quantities have the correct sign. In two dimensions, we must add momenta as vectors, using vector decomposition and addition by components. Vector decomposition and addition of forces was described in chapter 4. The same process is used for momenta.

Newton's third law was introduced in chapter 4 as $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$. It tells us that in any interaction between two objects, both experience a force, and these forces are equal in magnitude and opposite in direction. We can write Newton's second law as $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$, and substitute this into Newton's third law:

$$\vec{F}_{\text{A on B}} = \frac{\Delta\vec{p}_{\text{B}}}{\Delta t} = -\frac{\Delta\vec{p}_{\text{A}}}{\Delta t} = -\vec{F}_{\text{B on A}}$$

This tells us that in any interaction between two objects, A and B, the change in momentum of A is equal but opposite to the change in momentum of B. As the time during which the interaction takes place must be the same for both objects,

$$\Delta\vec{p}_{\text{A}} = -\Delta\vec{p}_{\text{B}}$$

or

$$\Delta\vec{p}_{\text{A}} + \Delta\vec{p}_{\text{B}} = 0$$

which is simply saying that in any interaction between two objects, the total change in momentum is zero. Hence, Newton's third law is really a statement of conservation of momentum.

This is true for any number of interacting objects, as we can consider the interactions pair-wise. For example, an interaction between objects A, B and C can be considered to be an interaction between A and B, an interaction between B and C and an interaction between A and C.

In any interaction, the total momentum is conserved. Hence, in an isolated system, the total momentum is constant.

The Chicxulub crater and the extinction of dinosaurs

What evidence is there that the last of the dinosaurs were wiped out by a giant meteorite?

WORKED EXAMPLE 6.2

A 1.5 kg maths textbook falls through a height of 2.0 m when it is dropped out a window. The book falls as a result of its interaction with Earth (mediated by Earth's gravitational field). Assume the book starts at rest and ignore air resistance.

1. What is the change in momentum of the book due to the fall?
2. What is the change in momentum of Earth?
3. What is the change in speed of Earth?

ANSWERS	**LOGIC**
1 m_b= 1.5 kg; $\Delta h = 2.0$ m; $u_b = 0.0$ m s^{-1}; v_b = ? We can use either kinematics or conservation of energy to solve this part. First we find v_b, then we multiply it by m_b to get Δp_b.	▪ Identify the relevant data from the question.
$\sum \vec{E}_{\text{initial}} = \sum \vec{E}_{\text{final}}$	▪ Write the equation for conservation of energy.
$m_b g\Delta h = \frac{1}{2} m_b v_b^2$ $v_b = \sqrt{2g\Delta h}$	▪ Expand the equation, noting that $u_b = 0$. ▪ Rearrange for v_b.
$\Delta p_b = m_b v_b = m_b \sqrt{2g\Delta h}$ $= 1.5\text{ kg}\sqrt{2\left(9.8\text{ m s}^{-2}\right)(2.0\text{ m})}$ $= 9.39\text{ kg m s}^{-1}$ $\Delta p_b = 9.4\text{ kg m s}^{-1}$	▪ Write the expression for change in momentum, noting that $u_b = 0.0$ m s^{-1}. ▪ Substitute in known values with units. ▪ Calculate the final value. ▪ State the final answer with correct units and significant figures.
2 Momentum is conserved in any interaction. $\Delta\vec{p}_b = -\Delta\vec{p}_{\text{Earth}}$ $\Delta p_{\text{Earth}} = -9.4\text{ kg m s}^{-1}$	▪ Write the expression for conservation of momentum.
3 $\Delta p_{\text{Earth}} = m_{\text{Earth}}\Delta v_{\text{Earth}}$	▪ Relate the momentum change to the change in speed.
$\Delta v_{\text{Earth}} = \frac{\Delta p_{\text{Earth}}}{m_{\text{Earth}}}$	▪ Rearrange for Δv_{Earth}.
$m_{\text{Earth}} = 5.97 \times 10^{24}\text{ kg}$	▪ Look up unknown data.
$\Delta v_{\text{Earth}} = \frac{-9.4\text{ kg m s}^{-1}}{5.97 \times 10^{24}\text{ kg}}$ $= -1.57 \times 10^{-24}\text{ m s}^{-1}$	▪ Substitute in known values with units. ▪ Calculate the final value.

ANSWERS	LOGIC
$\Delta v_{\text{Earth}} = -1.6 \times 10^{-24}$ m s^{-1} The negative sign indicates that this change in velocity is opposite to that of the book. This value is so small that we cannot observe changes in Earth's velocity due to its interactions with everyday things.	▪ State the final answer with correct units and significant figures.

TRY THIS YOURSELF

The meteor that hit Earth, leading to the extinction of the dinosaurs, probably had a mass of around 10^{15} kg. Calculate the change in Earth's velocity due to the collision with the meteor if it was moving at 10 m s^{-1} when it hit. Take Earth's initial velocity as zero.

INVESTIGATION 6.1

Conservation of momentum in one dimension

In this investigation, you will analyse the interaction of two objects in one dimension using the concept of momentum.

Critical and creative thinking

Numeracy

AIM

Formulate a hypothesis that you can test and use it to write an aim for your investigation.

MATERIALS

- 2 dynamics trolley cars
- Weighing scale
- Motion-sensor and data-logger
- Spring with loops at each end
- String
- Scissors

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
The spring may flick back or flick an object into a person's eye.	Wear safety glasses when working with springs.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Weigh the two dynamics trolleys cars.
2. Tie the two trolleys together using string with a compressed spring between them, as shown in Figure 6.2. Position the motion-sensors so you can make velocity measurements for each car.
3. The cars should start at rest. Release the cars by cutting the string holding them together.
4. Measure and record the velocities of each trolley car after the cutting of the string. Repeat this at least three times using the same spring and compression, Δx, each time.

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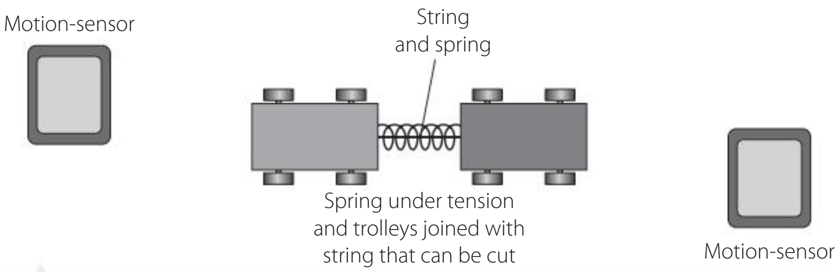


FIGURE 6.2 The set-up for the experiment

RESULTS

Record the mass of each car.
Record your measured velocities in a table as shown. Include the uncertainty in each measurement.

	v OF CAR A (m s^{-1})	p OF CAR A (kg m s^{-1})	v OF CAR B (m s^{-1})	p OF CAR B (kg m s^{-1})	Δp OF THE SYSTEM (kg m s^{-1})
TRIAL 1					
TRIAL 2					
…					
AVERAGE					

ANALYSIS OF RESULTS

1. Find the momentum of each of the two trolley cars after the string is cut for each trial. Make sure you clearly define a positive and negative direction for v and p. Remember to calculate the uncertainty in the momentum.
2. If the cars started at rest, their initial momentum was zero. Find the change in the momentum of the system for each trial.
3. Calculate the average value for the momentum of each car and the change in momentum of the car–spring system. You can use the range of measurements to find the uncertainty in the average value.

DISCUSSION

1. Was your hypothesis supported? If not, was there an approximation that you made that wasn't valid?
2. What was the main source of uncertainty in your investigation – was it the precision of the equipment, or your ability to have the same initial conditions (e.g. string length) each time?

CONCLUSION

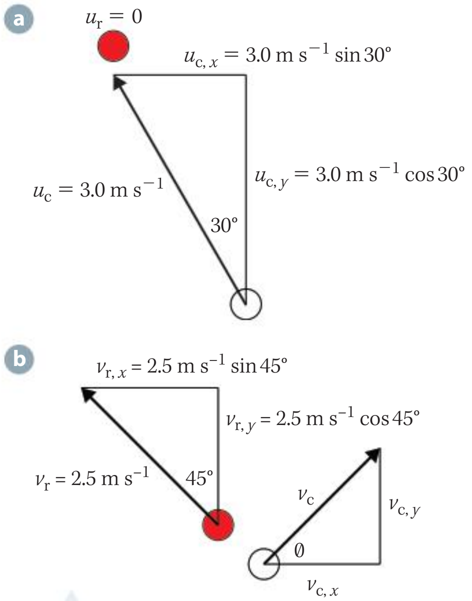
With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

In more than one dimension, we can apply the law of conservation of momentum in each dimension separately:

$$\sum p_{\text{initial},x} = \sum p_{\text{final},x} \text{ and } \sum p_{\text{initial},y} = \sum p_{\text{final},y}$$

WORKED EXAMPLE (6.3)

In a game of pool, the cue ball, moving with initial velocity 3.0 m s^{-1} N30°W, hits the stationary red ball. The red ball moves off with speed 2.5 m s^{-1} N45°W. Calculate the speed and direction of the cue ball's recoil. The balls have the same mass.

ANSWER	LOGIC
$m_c = m_r$ Before collision: $u_c = 3.0$ m s^{-1} N30°W; $u_r = 0.0$ m s^{-1} After collision: $v_r = 2.5$ m s^{-1} N45°W; $v_c = ?$	▪ Identify the relevant data from the question.
 **FIGURE 6.3** **a** Before the collision, showing the velocity of the cue ball decomposed into components; **b** After the collision, showing the velocity of the cue ball and the red ball decomposed into components	▪ Draw a diagram showing the system **a** before and **b** after.
$\sum p_{\text{initial},x} = \sum p_{\text{final},x}$ $p_{\text{c, initial},x} + p_{\text{r, initial},x} = p_{\text{c, final},x} + p_{\text{r, final},x}$ $m_c\, u_{c,x} + m_r\, u_{r,x} = m_c\, v_{c,x} + m_r v_{r,x}$	▪ Write the equation for conservation of momentum in the *x* direction. ▪ Write the equation for this interaction.
$u_{c,x} = v_{c\,x} + v_{r,x}$	▪ Simplify the equation, noting that $m_c = m_r$ and $u_r = 0$.
$v_{c,x} = u_{c,x} - v_{r,x}$	▪ Rearrange for $v_{c,x}$
$v_{c,x} = (-3.0\text{ m s}^{-1}\sin 30°) - (-2.5\text{ m s}^{-1}\sin 45°)$	▪ Substitute in known values with units, noting that west is the negative *x* direction.
$v_{c,x} = 0.268$ m s^{-1}	▪ Calculate the answer.
$p_{\text{c, initial},y} + p_{\text{r, initial},y} = p_{\text{c, final},y} + p_{\text{r, final},y}$ $m_c\, u_{c,y} + m_r\, u_{r,y} = m_c\, v_{c,y} + m_r v_{r,y}$	▪ Write the equation for this interaction in the *y* direction.
$u_{c,y} = v_{c,y} + v_{r,y}$	▪ Simplify the equation, noting that $m_c = m_r$ and $u_r = 0$.
$v_{c,y} = u_{c,y} - v_{r,y}$	▪ Rearrange for $v_{c,y}$.

ANSWER	LOGIC
$v_{c,y} = (3.0\ \mathrm{m\ s^{-1}}\cos 30°) - (2.5\ \mathrm{m\ s^{-1}}\cos 45°)$	- Substitute in known values with units, noting that north is in the positive *y* direction.
$v_{c,y} = 0.830\ \mathrm{m\ s^{-1}}$	- Calculate the answer.
$v_c = \sqrt{(v_{c,x})^2 + (v_{c,y})^2}$	- Write the expression for the total velocity.
$v_c = \sqrt{(0.268\ \mathrm{m\ s^{-1}})^2 + (0.830\ \mathrm{m\ s^{-1}})^2}$	- Substitute in known values with units.
$v_c = 0.87\ \mathrm{m\ s^{-1}}$	- Calculate the answer.
$\tan\theta = \dfrac{v_{c,y}}{v_{c,x}}$	- Write the expression for the angle.
$\tan\theta = \dfrac{0.830\ \mathrm{m\ s^{-1}}}{0.268\ \mathrm{m\ s^{-1}}}$	- Substitute in known values with units.
$\theta = 72°$	- Calculate the answer.
$v_c = 0.87\ \mathrm{m\ s^{-1}}$, N18°E	- State the final answer with correct units, direction and significant figures.

TRY THIS YOURSELF

Continuing on from Worked example 6.3, imagine that someone has stuck a lump of blu-tack to the red ball so that when the cue ball hits it, they move off stuck together. Calculate the velocity of the two balls immediately after the collision in this case.

INVESTIGATION 6.2

Collisions in two dimensions

Critical and creative thinking

Numeracy

Information and communication technology capability

AIM

To describe and analyse interactions in two dimensions

Write an inquiry question for your investigation.

MATERIALS

- 2 billiard balls of different colours
- Large sheet of paper (butcher's paper)
- Tape
- Pencil or marker
- Webcam with stand (e.g. retort stand), or phone with selfie-stick
- Stopwatch
- Weighing scale

»

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Billiard balls can hurt your foot if they fall on it.	Wear enclosed shoes.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1 Weigh the two billiard balls.
2 Draw a grid on your butcher's paper, with lines spaced 1 cm apart. Make every fifth line a different colour to make it easier to read.
3 Tape the paper to the bench. Make a mark approximately in the middle of the paper. This will be the starting position for the first ball.
4 Place the stopwatch face up on the paper.
5 Mount the camera looking down at the paper so it has as large a field of view as possible. The view through the camera should look like Figure 6.4. Note that you need to be able to read the stopwatch in the recording.
6 Place the first ball on the mark.
7 Start the stopwatch and start the camera recording.
8 Gently roll the second ball towards the first so that it collides.
9 Repeat steps 6–8. Try to roll the second ball with the same path and initial speed. Note that this is very difficult!

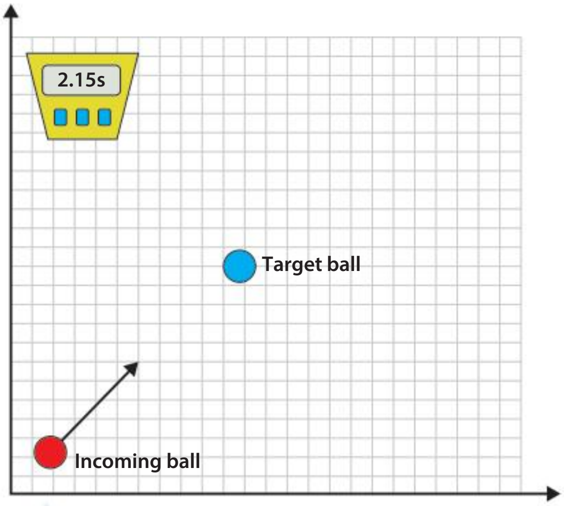


FIGURE 6.4 The view of the experiment, as seen by the webcam

RESULTS

You should have a video of the collision. The stopwatch reading on the video will allow you to calculate the time differences between selected frames.

ANALYSIS OF RESULTS

1 Choose two frames before the collision. Use the grid markings to calculate the change in position, Δx and Δy, of the second (incoming) ball. Use the stopwatch reading to calculate the time difference, Δt.
2 Choose two frames after the collision. Use the grid markings to calculate the change in position, Δx and Δy, of both balls. Use the stopwatch reading to calculate the time difference, Δt.
3 Estimate the uncertainty in your data.
4 Record your data in a table.

	BALL 2 (INCOMING)	BALL 2 (OUTGOING)	BALL 1 (TARGET)
Δx (m)			
v_x (m s^{-1})			
p_x (kg m s^{-1})			
Δy (m)			
v_y (m s^{-1})			
p_y (kg m s^{-1})			

» 5 Use your measurements of time, position and mass to calculate velocities and momenta to complete the table.

DISCUSSION

1 Was momentum conserved in this collision, within the bounds of your uncertainties? Is it a valid approximation to treat the balls as an isolated system?

2 Give the answer to your inquiry question.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

This investigation could be extended to a depth study by using a variety of different types of balls to model elastic and inelastic collisions.

KEY CONCEPTS

- Momentum is the product of mass and velocity: $\vec{p} = m\vec{v}$.
- Momentum is a vector quantity.
- Momentum is conserved in any isolated system: $\sum \vec{p} = \text{constant}$.
- Newton's third law is a statement of conservation of momentum. It says that in any interaction between two objects, $\vec{F}_{\text{A on B}} = \frac{\Delta\vec{p}_{\text{B}}}{\Delta t} = -\frac{\Delta\vec{p}_{\text{A}}}{\Delta t} = -\vec{F}_{\text{B on A}}$, so $\Delta\vec{p}_{\text{A}} = -\Delta\vec{p}_{\text{B}}$.

CHECK YOUR UNDERSTANDING

6.1

1 Define 'momentum'. Distinguish between the everyday usage of the word momentum and the physics usage.

2 Your PE teacher throws a tennis ball to you and you catch it. Next, the teacher is going to throw a medicine ball to you. The teacher gives you the choice of having it thrown with the same speed as the tennis ball, or the same momentum. Which would you choose? Justify your answer.

3 A 58 g tennis ball has a momentum of 0.87 kg m s^{-1}. Calculate the speed of the ball.

4 Imagine you are standing on an icy surface, so there is no friction between your feet and the ground. You throw a 58 g tennis ball with a speed of 12 m s^{-1}. Calculate the speed with which you recoil if you have a mass of 55 kg.

5 A four-wheel drive (mass 2500 kg) travelling north at 15 m s^{-1} collides with a car (mass 1500 kg) travelling east at 20 m s^{-1}, as shown in Figure 6.5. The vehicles stick together. Calculate the magnitude and direction of the velocity of the wreckage immediately after the crash.

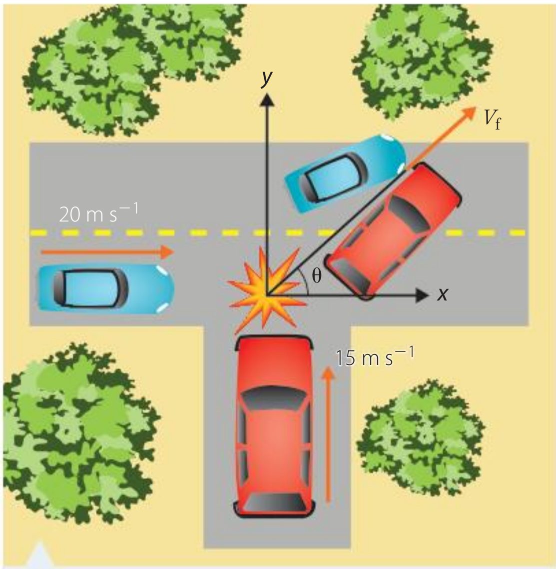


FIGURE 6.5

6 A slow-moving car and a fast-moving cyclist have the same momentum. Which has the greater kinetic energy? Explain your answer.

6.2 Momentum and energy in elastic collisions

Sling-shot manoeuvre

Find out how the Voyager space craft gained momentum from elastic 'collisions'.

Momentum is conserved in any interaction, as is energy. But unlike momentum, energy can change forms. When there is a collision between objects, the total momentum of the objects before the collision must be equal to the total momentum afterwards. However, the total kinetic energy afterwards may not be equal to the total kinetic energy before.

When the total kinetic energy *is* the same before and after the collision, we call it an **elastic collision**. For an elastic collision:

$$\Sigma E_{\text{k, before}} = \Sigma E_{\text{k, after}} \text{ or } \Sigma\frac{1}{2}mv^2_{\text{before}} = \Sigma\frac{1}{2}mv^2_{\text{after}}$$

and

$$\Sigma \vec{p}_{\text{before}} = \Sigma \vec{p}_{\text{after}} \text{ or } \Sigma m\vec{v}_{\text{before}} = \Sigma m\vec{v}_{\text{after}}$$

FIGURE 6.6 A Newton's cradle toy

Very few real collisions are elastic. In almost all collisions, some energy is transformed to other forms including thermal energy and sound. However, we can model a collision as elastic if the amount of energy transformed from kinetic to other forms is small compared with the total kinetic energy.

A Newton's cradle toy demonstrates (approximately) elastic collisions between steel balls, as shown in Figure 6.6. When you lift one ball to the side and release it, the ball swings down and collides with the next ball. Momentum and energy are transferred along the line of balls to the last one, which swings upwards. When that ball swings back down and collides with the previous ball, momentum and energy are transferred back in the other direction again.

A perfectly elastic collision can occur only when no friction is involved, and no energy is lost as heat or sound. This is possible when the interaction takes place via field forces rather than contact forces. A gravitational sling-shot manoeuvre is an example of this.

WORKED EXAMPLE 6.4

In a particle accelerator, a proton collides with a second proton in a head-on elastic collision. The first proton has a speed of 1.3×10^6 m s^{-1} to the right, and the second proton is also moving right but with a speed of 2.5×10^5 m s^{-1}. Calculate the speeds of the two protons after the collision.

ANSWER	LOGIC
$m_{p1} = m_{p2} = m_p$; $u_1 = 1.3 \times 10^6$ m s^{-1}; $u_2 = 2.5 \times 10^5$ m s^{-1} We want to find v_1 and v_2.	▪ Identify the relevant data from the question.
$\Sigma \vec{p}_{\text{before}} = \Sigma \vec{p}_{\text{after}}$	▪ Identify the equation for conservation of momentum.
$m_p u_1 + m_p u_2 = m_p v_1 + m_p v_2$	▪ Expand the equation.
$u_1 + u_2 = v_1 + v_2$	▪ Simplify the equation.

ANSWER	**LOGIC**
$u_1 - v_1 = v_2 - u_2$	- Group terms by particle. - We have one equation but two unknowns, so we need more information.
$\sum E_{\text{k, before}} = \sum E_{\text{k, after}}$	- Write the equation for kinetic energy.
$\frac{1}{2}m_p u_1^2 + \frac{1}{2}m_p u_2^2 = \frac{1}{2}m_p v_1^2 + \frac{1}{2}m_p v_2^2$	- Expand the equation.
$u_1^2 + u_2^2 = v_1^2 + v_2^2$	- Simplify the equation.
$u_1^2 - v_1^2 = v_2^2 - u_2^2$	- Group terms by particle.
$(u_1 - v_1)(u_1 + v_1) = (v_2 - u_2)(v_2 + u_2)$	- Factorise each side.
$\frac{(u_1 - v_1)(u_1 + v_1)}{(u_1 - v_1)} = \frac{(v_2 - u_2)(v_2 + u_2)}{(v_2 - u_2)}$	- Now we combine this equation with the conservation of momentum equation. - Divide the energy equation by the momentum equation.
$u_1 + v_1 = v_2 + u_2$	- Simplify the equation.
$v_1 = v_2 + u_2 - u_1$	- Rearrange for v_1
$u_1 + u_2 = v_1 + v_2 = v_2 + u_2 - u_1 + v_2$	- Substitute into momentum equation.
$u_1 = v_2$	- Simplify the equation.
$u_1 + v_1 = v_2 + u_2 = u_1 + u_2$	- Substitute $u_1 = v_2$ back into $u_1 + v_1 = v_2 + u_2$
$v_1 = u_2$	- Simplify.
The two protons have swapped velocities: $v_1 = u_2 = 2.5 \times 10^5$ m s^{-1} and $v_2 = u_1 = 1.3 \times 10^6$ m s^{-1}	- Write the final answer with correct units and significant figures.

TRY THIS YOURSELF

Repeat the calculation in Worked example 6.4 but with the second proton replaced by a helium nucleus, which has a mass equal to that of four protons.

From Worked example 6.4 we can derive the two very useful equations:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$u_1 + v_1 = v_2 + u_2$$

These two equations can be used when analysing elastic collisions.

The first equation is a statement of conservation of momentum for the two-object collision, so applies to any collision.

The second equation *only applies to elastic collisions*, because we assumed that kinetic energy was conserved to derive it. Note that while we assumed equal masses, this equation is also true if the masses are not equal. You can show this by doing the 'try this yourself' question.

A useful approximation can be made when one object is very much larger than the other, such that $m_2 >> m_1$. An example is when an object such as a ball collides with the surface of Earth. In this case, the momentum of Earth before and after the collision is not measurably different, so we can say that $u_2 = v_2$ and $v_1 = v_2 + u_2 - u_1 = 2u_2 - u_1$. If we take the velocity of Earth's surface to be zero, which is appropriate for a person standing on Earth's surface and bouncing a ball, then $v_1 = -u_1$.

Elastic collisions in 2D

So, for a completely elastic collision with a very much larger object, the small object reverses direction and bounces off with the same speed. The large object is unaffected.

We can also use the equation $v_1 = 2u_2 - u_1$ to understand how a gravitational sling-shot manoeuvre works. The spacecraft is object 1, with initial speed u_1. It approaches a planet moving at u_2 and swings close to it through its gravitational field (without coming into contact and crashing). It swings through the gravitational field and away again at speed $v_1 = 2u_2 - u_1$. Planets move very fast in their orbits (for example, the orbital speed of Earth is about 30 km s^{-1}, or 3×10^5 m s^{-1}), so a sling-shot manoeuvre about Earth could change the speed of a spacecraft by 6×10^5 m s^{-1}.

INVESTIGATION 6.3

Elastic collisions

Critical and creative thinking

Numeracy

Information and communication technology capability

AIM

To investigate approximately elastic collisions in one dimension

Write a hypothesis for your investigation.

MATERIALS

- Steel ball bearings
- Weighing scales
- Marker
- Containers for ball bearings
- Motion-sensors with data-loggers OR 3 stopwatches OR webcam and a stopwatch
- Flat track (for the ball bearings to run along)

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Ball bearings are hard and can cause damage.	Do not roll the ball bearings at high speeds.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Select some ball bearings and weigh them. If the weights are different, make sure you mark them or put them in labelled containers.
2. Set up your equipment so you can measure the speed of the ball bearings before and after the collision, or the times taken for each ball to move a measured distance.
3. Place the target ball bearing at its position.
4. Begin recording, or have everyone ready to use their stopwatches.
5. Roll the incoming ball bearing towards the target.
6. Record times/speeds for the incoming ball bearing before and after the collision, and for the target after the collision.
7. Repeat steps 3–6.

RESULTS

1. You should have either speeds for each ball bearing before and after the collision, or times taken to travel known distances.
2. Estimate the uncertainty in your data.
3. Record your data in a table.

»

	INCOMING BALL (BEFORE)	INCOMING BALL (AFTER)	TARGET BALL (AFTER)
Δx (m)			
Δt (s)			
v (m s^{-1})			
p_x (kg m s^{-1})			
E_k (kg m^2 s^{-2})			

ANALYSIS OF RESULTS

- Use your measurements of time, position and mass to complete the table.
- Calculate the total momentum of the system of ball bearings before and after the collision. Calculate the uncertainty in this value.
- Calculate the total kinetic energy of the system of ball bearings before and after the collision. Calculate the uncertainty in this value.

DISCUSSION

1. Was momentum conserved in this collision, within the bounds of your uncertainties?
2. Was kinetic energy conserved in this collision, within the bounds of your uncertainties?
3. Is the elastic-collisions model an appropriate one for the collision of steel ball bearings?
4. Was your hypothesis supported?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- In all collisions, momentum is conserved.
- In an elastic collision, no kinetic energy is transformed to other forms.
- For an elastic collision: $\sum E_{\text{k, before}} = \sum E_{\text{k, after}}$ or $\sum \frac{1}{2} mv^2_{\text{before}} = \sum \frac{1}{2} mv^2_{\text{after}}$, and $\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$ or $\sum m\vec{v}_{\text{before}} = \sum m\vec{v}_{\text{after}}$.
- These two equations can be used when analysing elastic collisions: $m_1u_1 + m_2u_2 = m_1u_1 + m_2u_2$ and $u_1 + v_1 = v_2 + u_2$.

CHECK YOUR UNDERSTANDING

6.2

1. Define 'elastic collision' and give an example of a collision that is elastic and one that is not.
2. A blue and a red billiard ball of equal mass roll directly towards each other at 2.0 m s^{-1}, as shown in Figure 6.7. Describe the motion of the balls after the collision, including their speeds. Assume that the collision is elastic.

FIGURE 6.7 Two billiard balls involved in an elastic collision

3 A neutron makes an elastic collision with a carbon nucleus, which has 12 times its mass. The carbon nucleus is initially at rest and the neutron is travelling at 1.5×10^7 m s^{-1}.

  a Assuming a one-dimensional collision, calculate the speed at which each particle is moving after the collision.

  b Calculate the fraction of its kinetic energy that the neutron has transferred to the carbon nucleus.

4 Marcus throws a bouncy superball directly downwards. He throws it so it has an initial speed of 5.0 m s^{-2} from a height of 1.5 m.

  a Calculate the speed with which the ball hits the ground.

  b Calculate the speed at which the ball rebounds.

  c Calculate the maximum height the ball reaches.

  d State all the assumptions you have made.

5 A spacecraft of mass 500 kg is using a gravity-assist (sling-shot) manoeuvre to gain kinetic energy. The spacecraft approaches Mercury with a speed of 3.5 km s^{-1}, and moves away after the interaction with a speed of 110 km s^{-1}. Modelling the interaction of the spacecraft with Mercury as a one-dimensional elastic collision:

  a calculate the orbital speed of Mercury.

  b calculate the kinetic energy of the spacecraft before and after the sling-shot manoeuvre.

  c explain where the change in kinetic energy came from.

6 A Newton's cradle toy has five identical steel balls. Demonstrate that it is not possible to have one ball swing in to collide, and two balls swing out at the other end as a result. Assume the collisions are elastic and show that if two balls swing out such that momentum is conserved, kinetic energy cannot also be conserved

6.3 Impulse

Imagine hitting a golf ball with a golf club. The ball starts at rest, and after the collision with the golf club it is moving – it has gained kinetic energy. This energy has been transferred to the ball by the club, because the club applies a force through a distance, and hence does work on the ball.

In chapter 5, you saw that a graph of force as a function of distance gave the quantity work. Recall that when a force, F, acts on an object through some distance, Δs, then an amount of energy $W = \Delta E = F\Delta s$ is transferred to the object. This amount of energy is the area under the curve on an F vs s graph (see Figure 5.17 on page 137).


iStock.com/ArtBoyMB

FIGURE 6.8 The golf club transfers kinetic energy to the golf ball

Consider Newton's second law, written in terms of momentum as in section 6.1:

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

We can rearrange this equation to find the change in momentum of an object when a force acts on it:

$$\Delta\vec{p} = \vec{F}\Delta t$$

We call this quantity **impulse** and give it the symbol I.

$$\vec{I} = \Delta\vec{p} = \vec{F}\Delta t$$

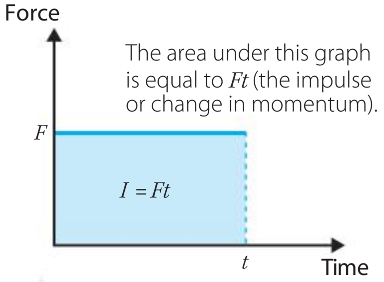
Impulse is a vector quantity and has the same units as momentum, kg m s^{-1}.

Graphical representations of force as a function of time

Impulse can be found from a force versus time graph in the same way that work can be found from a force versus distance graph. As shown in Figure 6.9, the impulse is the area under the F vs t graph.

When a constant force acts on an object for a time interval, the graph of force versus time is a straight line, as shown in Figure 6.9. The impulse, I, is the area under the F vs t graph, which in this case is the rectangle with area $I = Ft$.

When the force varies, we need to break up the area into small sections. We make the approximation that each small section is a rectangle of size $F\Delta t$. We then add up all the individual sections to find the total area, and hence the total impulse. This process is shown in Figure 6.10.



The area under this graph is equal to Ft (the impulse or change in momentum).

FIGURE 6.9 Impulse is the area under the F vs t graph.

Isaac Newton and the invention of calculus

The process of breaking the area under a curve into lots of tiny sections is the basis of integration, one of the two main processes of calculus. Find out how this process works.

Force against time

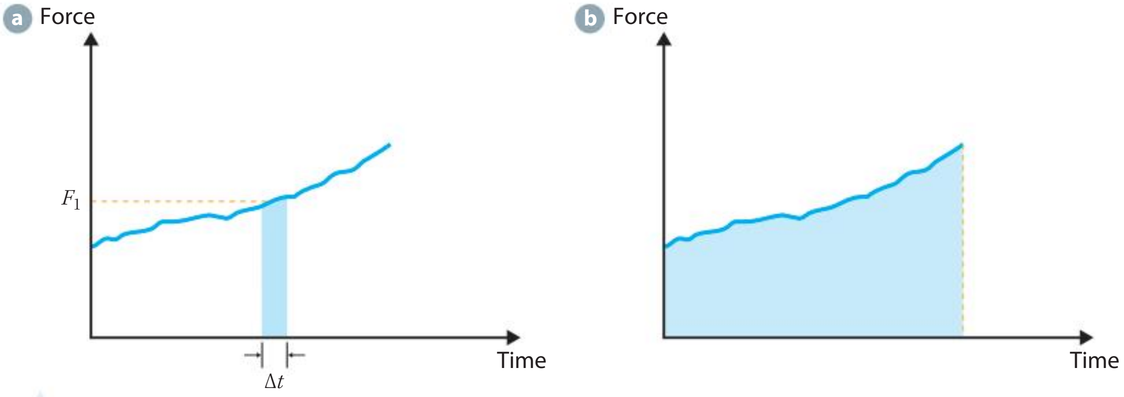


FIGURE 6.10 **a** Break the area into small sections of width Δt, each with area approximately $F\Delta t$. **b** The total impulse (change in momentum) is given by the sum of these small areas, which is the area under the curve.

WORKED EXAMPLE 6.5

A golf ball is hit by a golf club with a long 'follow-through'. The force applied to the ball as a function of time during the collision can be approximated by the curve shown in Figure 6.11. Calculate the impulse transferred to the ball.

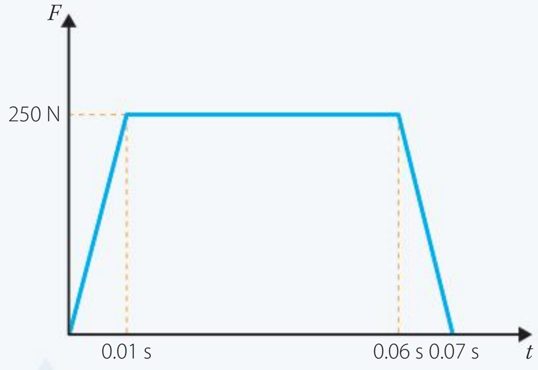
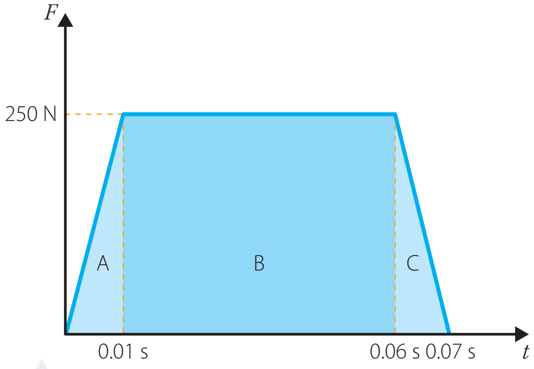


FIGURE 6.11 Force acting on a golf ball during a hit with a long follow-through.

ANSWER	**LOGIC**
 **FIGURE 6.12** Break the area into three sections: A, B and C	- The impulse is the area under the *F* vs *t* curve. Calculate this area by breaking the curve into three sections, as shown in Figure 6.12. - Find the area of each section and sum them to find the total area.
$F_{max} = 250\text{ N}; \Delta t_A = 0.01\text{s}$ $I_A = \text{area} = \frac{1}{2}F_{max}\Delta t_A$ $I_A = \frac{1}{2}(250\text{ N})(0.01\text{ s})$ $I_A = 1.25\text{ N s}$	- Find the area for section A. - Identify the relevant data from the graph. - Write the equation for impulse. - Substitute in known values with correct units. - Calculate the value of *I* for area A.
$F_{max} = 250\text{ N}; \Delta t_B = (0.06\text{ s} - 0.01\text{ s}) = 0.05\text{ s}$	- Find the area for section B. - Identify the relevant data from the graph.
$I_B = \text{area} = F_{max}\Delta t_B$ $I_B = (250\text{ N})(0.05\text{ s})$ $I_B = 12.5\text{ N s}$	- Write the equation for impulse. - Substitute in known values with correct units. - Calculate the value of *I* for area B.
$F_{max} = 250\text{ N}; \Delta t_C = (0.07\text{ s} - 0.06\text{ s}) = 0.01\text{ s}$	- Find the area for section C. - Identify the relevant data from the graph.
$I_C = \text{area} = \frac{1}{2}F_{max}\Delta t_C$ $I_C = \frac{1}{2}(250\text{ N})(0.01\text{ s})$ $I_C = 1.25\text{ N s}$	- Write the equation for impulse. - Substitute in known values with correct units. - Calculate the value of *I* for area C.
$I_{total} = I_A + I_B + I_C$ $= 1.25\text{ N s} + 12.5\text{ N s} + 1.25\text{ N s}$ $= 15\text{ N s}$ $I_{total} = 15\text{ kg m s}^{-1}$	- Write the expression for the total impulse. - Substitute in known values with correct units. - Calculate the value. - State the final answer with correct units and significant figures. - Note that either N s or kg m s^{-1} are appropriate units for impulse.

TRY THESE YOURSELF

1 Calculate the speed of the golf ball immediately after it is hit if it starts at rest and has a mass of 45 g.

2 Imagine there was no follow-through in this swing, such that the middle rectangular section was missing from the curve. Calculate the impulse and final speed for this case.

The forces involved in collisions and other interactions

We can use the concept of impulse to analyse the forces that act during collisions and other interactions.

The equation:

$$\vec{I} = \Delta\vec{p} = \vec{F}\,\Delta t$$

tells us that the change in momentum of an object depends on the force that is exerted on it, and how long the force is exerted for. In Worked example 6.5, the follow-through of the club is important in achieving a high speed of the golf ball. Keeping the club in contact with the ball, so that the force is exerted for longer, results in a larger transfer of momentum.

WORKED EXAMPLE 6.6

The coefficient of static friction between the tyres of Kate's car and the road is 0.75. The car has a mass of 1950 kg. Calculate the minimum 0–100 km h^{-1} acceleration time for Kate's car, assuming it is limited only by the maximum friction force.

ANSWER	LOGIC
$\mu_s = 0.75$; $m = 1950$ kg; $u = 0$ km h^{-1}; $v = 100$ km h^{-1}. We want to find Δt.	▪ Identify the relevant data from the question.
$u = 0 \text{ km h}^{-1} = 0 \text{ m s}^{-1}$ $v = 100 \text{ km h}^{-1} \times \dfrac{1000 \text{ m km}^{-1}}{3600 \text{ s h}^{-1}} = 27.8 \text{ m s}^{-1}$	▪ Convert data to SI units.
$I = \Delta\vec{p} = \vec{F}\,\Delta t$	▪ Write the equation for impulse.
$\Delta t = \dfrac{\Delta p}{F}$	▪ Rearrange for time taken.
$F = F_{\text{friction, max}} = \mu_s N = \mu_s mg$	▪ Write the expression for *F*.
$\Delta t = \dfrac{\Delta p}{F} = \dfrac{\Delta p}{\mu_s mg}$	▪ Substitute the expression for force into the equation for Δ*t*.
$\Delta p = mv - mu = mv$	▪ Write the expression for Δ*p*, and simplify, recognising that *u* = 0.
$\Delta t = \dfrac{\Delta p}{\mu_s mg} = \dfrac{mv}{\mu_s mg}$	▪ Substitute the expression for Δ*p* into the equation for Δ*t*.
$\Delta t = \dfrac{v}{\mu_s g}$ $\Delta t = \dfrac{27.8 \text{ m s}^{-1}}{0.75 \times 9.8 \text{ m s}^{-2}}$ $\Delta t = 3.78$ s $\Delta t = 3.8$ s	▪ Simplify the equation. ▪ Substitute in known values with units. ▪ Calculate the answer. ▪ State the final answer with correct units and significant figures.

TRY THIS YOURSELF

In reality, Kate's car does not have a powerful enough engine to achieve this acceleration. If the 0–100 km h^{-1} time is actually 8.5 s, what average force do the tyres exert against the road?

Forces and impulse in car crashes

In 2016, 1298 people were killed on Australian roads compared with 3403 in 1980. This decrease is even more significant when you consider the increase in the number of cars on the road. In 1980, there were 43 fatalities for every 100 000 cars on the road. In 2016, there were around 4.5 for every 100 000 cars – almost a factor of 10 decrease.

One reason for the decrease is the change in road rules and driver behaviour, including reducing speed limits and reducing alcohol consumption of drivers. Another important reason is better car design. Cars now have safety features including seatbelts, airbags and crumple zones.

When a car collides with another large object and comes to a sudden stop, a rapid deceleration occurs. Newton's first law tells us that the occupants of the car will continue with the same velocity unless a force acts on them. A seatbelt provides this force, slowing the occupants with the vehicle. When a car crash occurs and a passenger is not wearing a seat belt, they can be thrown from the car through the windscreen, or hit something or someone else inside the car. This is much more likely to result in death or serious injuries. Wearing a seatbelt also means that the force is applied to the occupant across the strongest parts of the body – the chest and hips. Seatbelts are designed to stretch a small amount, which also assists in decreasing the maximum force applied to the wearer while the impulse is transferred during the collision. The three-point seatbelt was invented by engineers at Volvo in 1959, and is now used in all road cars.

Why should you wear a seat belt?

Watch the video. Wear your seat belt for every trip – no matter how short, and how low the speed.

Crumple zones

Read more about how crumple zones work

A more recently introduced safety feature is the crumple zone. Figure 6.13 shows how the crumple zone at the front of a car crumples on impact with a rigid object.


Alamy Stock Photo/CTK

FIGURE 6.13 The crumple zone at the front of the car increases the time taken for the collision, thus reducing the force acting on the occupants.

When a car comes to a sudden stop during a crash, the impulse transferred to the passengers is their change in momentum. This is given by $I = \Delta p = F\Delta t$. Given that Δp is fixed, if the time, Δt, is increased, then the force exerted is decreased. By increasing the time that the collision takes, we decrease the force exerted on the passengers. This is what a crumple zone does. A crumple zone is a specially designed area surrounding the passenger compartment that is designed to crumple in a collision. Allowing the front of the car to crumple increases the time taken for the car to stop. This decreases the forces applied to the occupants.

WORKED EXAMPLE 6.7

A car travelling at 90 km h^{-1} collides with a rock wall and comes to a stop in 0.08 s. Assuming the car decelerates at a constant rate, calculate the force applied to a 60 kg occupant who does not move relative to the vehicle.

ANSWER	LOGIC
$m = 60$ kg; $u = 90$ km h^{-1}; $v = 0$; $\Delta t = 0.08$ s We want to find F.	- Identify the relevant data from the question.
$v = 0 \text{ m s}^{-1}$ $u = 90 \text{ km h}^{-1} \times \dfrac{1000 \text{ m km}^{-1}}{3600 \text{ s h}^{-1}} = 25 \text{ m s}^{-1}$	- Convert data to SI units.

ANSWER	**LOGIC**
$I = \Delta\vec{p} = \vec{F}\,\Delta t$	- Write the equation for impulse.
$F = \dfrac{\Delta p}{\Delta t}$	- Rearrange for the force.
$\Delta p = mv - mu = -mu$	- Write the expression for Δ*p*, and simplify, recognising that $v = 0$.
$F = \dfrac{\Delta p}{\Delta t} = \dfrac{-mu}{\Delta t}$ $F = \dfrac{(60\text{ kg})(-25\text{ m s}^{-1})}{0.08\text{ s}}$ $F = -18750\text{ N}$ $F = -19\text{ kN}$ The negative sign tells us that this force is acting in the opposite direction to the motion. The magnitude of the force is more than 30 times the gravitational force experienced by the person.	- Substitute the expression for Δ*p* into the equation for Δ*t*. - Substitute in known values with units. - Calculate the answer. - State the final answer with correct units and significant figures.

TRY THIS YOURSELF

Continuing on from the example, calculate the average force exerted on this passenger if the car had a better crumple zone that increased the time taken for the collision to 0.2 s.

Airbags, or supplemental restraint systems (SRS), were introduced into cars in the 1980s. Front airbags, as shown in Figure 6.14, are designed to increase the time taken for an occupant to decelerate. They also spread the force applied to the passenger more evenly. Later developments, including side and curtain airbags, prevent an occupant's head making an impact with hard parts of the car during collisions other than front-on collisions. In every case, the time taken to effect a change in momentum (the impulse) is increased so that the maximum force applied to any part of an occupant is decreased.

Alamy Stock Photo/Photo Network/Tom Tracy

FIGURE 6.14 Front airbags spread the force applied to a passenger during a collision, as well as decreasing it.

Safety features of cars

Find out how much the various safety features of a car contribute to preventing injury during a crash.

INVESTIGATION 6.4

Crash safety

Critical and creative thinking

Literacy

Some of the safety features of modern cars have been described. Your job in this investigation is to research and design a capsule to protect an egg during a collision. The collision occurs when the egg, in its capsule, is dropped from a height to collide with the ground.

AIM

To design, build and crash-test an egg capsule

MATERIAL

- Eggs* (raw or boiled)
- Materials to make the egg capsule – for example foam, plastic containers, tape, string, balloons.
- Tools to manufacture the egg capsule
- Tape measure

*Note: results are more dramatic, but messier, when raw eggs are used. Check with your teacher what is best to use.

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?

What risks are associated with your investigation, and how can you manage them?

METHOD

1. Do some research to find out more about safety features in modern cars. You might also choose to look at the design of bicycle helmets.
2. Design your egg capsule. Note that it needs to be reusable – it must be possible to remove the egg to check for cracking without destroying the capsule. *No parachutes are allowed.*
3. Justify your design. Explain the purpose of each feature, and describe how you expect it to contribute to protecting your egg. Record this in a design summary document. Include diagrams, and refer to the concepts of impulse and force.
4. Build your egg capsule.
5. Place your egg in the capsule. Drop the egg and capsule from a height of 30 cm.
6. Remove the egg and check for cracking.
7. Replace the egg and drop it from 10 cm higher than the previous trial.
8. Repeat steps 6 and 7 until the egg is cracked. Dispose of the egg appropriately.

RESULTS

Record the drop height at which the egg cracked.

ANALYSIS OF RESULTS

1. Compare the results for your egg capsule to other students' capsules.
2. What are the features of the most successful designs?
3. What was missing from the least successful designs?
4. Make recommendations for future egg-capsule design based on your analysis.

CONCLUSIONS

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

WS

Seatbelts and airbags

KEY CONCEPTS

- Impulse is the change in momentum of an object when it is acted on by a force: $\vec{I} = \Delta\vec{p} = \vec{F}\Delta t$.
- Impulse is a vector quantity and has the same units as momentum, kg m s^{-1}.
- Impulse can be found from a force versus time graph in the same way that work can be found from a force versus distance graph. Impulse is the area under the curve of an F vs t graph.
- For a given force, the longer the force is applied, the greater the change in momentum.
- For a given change in momentum, the longer the interaction time, the smaller the force exerted on the object.
- Seatbelts protect you during a collision by ensuring you decelerate as the car does. Other safety features such as crumple zones decrease the force applied by increasing the collision time.

CHECK YOUR UNDERSTANDING

6.3

1 Define 'impulse'.

2 Explain why it is desirable to have a crumple zone in a vehicle.

3 A force of 500 N is applied for 4.0 s to a 100 kg satellite in space.

  a Calculate the impulse given to the satellite.

  b What is the change to the satellite's velocity?

4 A 50 kg ice skater exerts an average force of 25 N on the ice in the skater's direction of motion for 3.0 s. The skater's initial speed is 4.5 m s^{-1}.

  a Calculate the skater's final speed.

  b If the force the skater applied to the ice had been 75 N, calculate how long the force needs to be applied for the skater to reach the same speed.

5 A 58 g tennis ball travelling at 30 m s^{-1} to the right is struck by a tennis racquet and returned along the same direction (Figure 6.15a). The force applied by the tennis racquet to the ball is modelled by the graph in Figure 6.15b.

  a Calculate the impulse applied to the ball.

  b Calculate the speed of the ball immediately after the collision with the racquet.

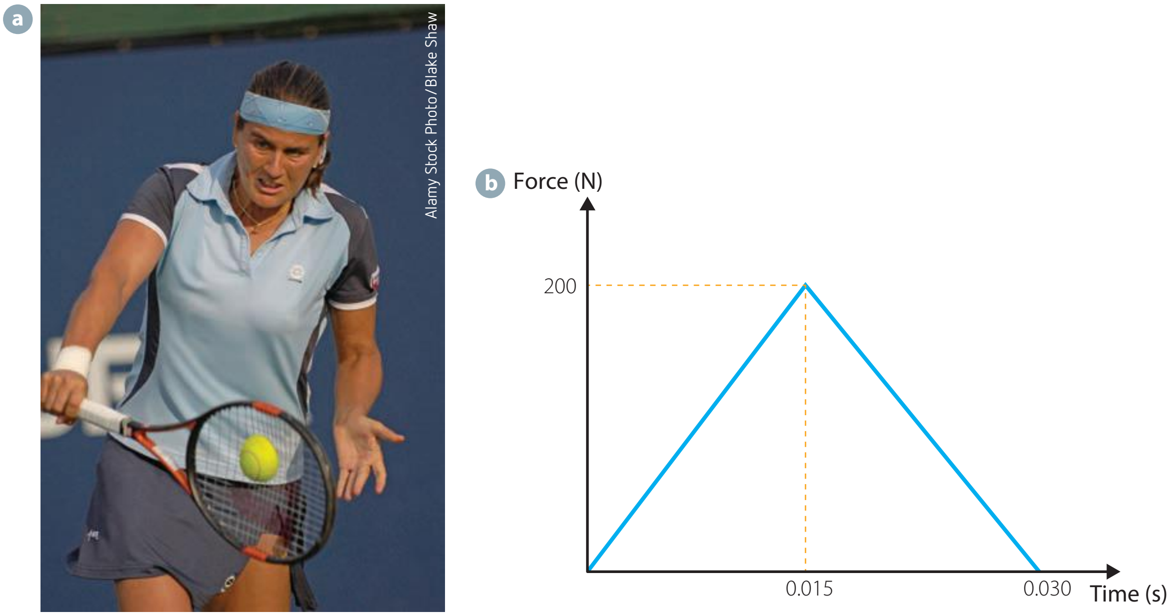


FIGURE 6.15 **a** A tennis ball is struck by a racquet and rebounds in the opposite direction; **b** The F vs t graph for the tennis ball being struck by the racquet

6.4 Momentum and energy in inelastic collisions

Momentum is conserved in all interactions, including all collisions. Energy is also a conserved quantity, so energy is conserved in all interactions. However, unlike momentum, energy can change forms. In most collisions, some kinetic energy is transformed into other forms including internal energy and sound.

In section 6.2, you learned about the elastic collision model. In a perfectly elastic collision, the total kinetic energy of the objects is the same before and after the collision. In reality, perfectly elastic collisions only happen when the collision is due to a field force and the objects don't actually come into contact, such as in interactions of subatomic particles. The elastic collision model is useful for collisions where friction forces are small.

Most real collisions are **inelastic collisions**. Inelastic collisions involve a loss of kinetic energy of the colliding objects. If sound or heat is produced, that energy must have come from somewhere – the decrease in the objects' kinetic energy. Energy can also be converted to potential energy in a collision; for example, due to deformation of the objects.

A second useful model is that of the **perfectly inelastic collision**. A perfectly inelastic collision is when the colliding objects stick together after the collision to form a single object.

WORKED EXAMPLE (6.8)

A 1.5×10^3 kg car is moving to the right at 20 m s^{-1} while a 5.0×10^3 kg truck is moving to the left at 10 m s^{-1}, as shown in Figure 6.16. The car and truck collide and move off as one mass, stuck together.

1. Calculate the velocity of the wreckage immediately after the collision.
2. Calculate the percentage of kinetic energy converted to other forms.

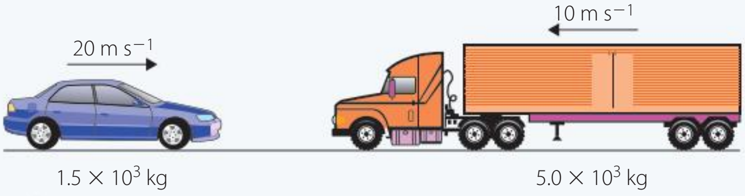


FIGURE 6.16

ANSWERS	**LOGIC**
1 $m_{\text{car}} = 1500$ kg; $u_{\text{car}} = +20$ m s^{-1}; $m_{\text{truck}} = 5000$ kg; $u_{\text{truck}} = -10$ m s^{-1} After the collision the car and truck form a single object with mass $m_{\text{car}} + m_{\text{truck}}$, moving with combined speed $v_{\text{car + truck}}$. We need to find $v_{\text{car + truck}}$.	▪ Identify the relevant data.
$\sum \vec{p}_{\text{initial}} = \sum \vec{p}_{\text{final}}$	▪ Write the general equation for conservation of momentum.
$\vec{p}_{\text{car}} + \vec{p}_{\text{truck}} = \vec{p}_{\text{car + truck}}$	▪ Write the equation for this interaction.
$m_{\text{car}}u_{\text{car}} + m_{\text{truck}}u_{\text{truck}} = m_{\text{car + truck}}v_{\text{car + truck}}$	▪ Expand the equation.
$v_{\text{car + truck}} = \frac{m_{\text{car}}u_{\text{car}} + m_{\text{truck}}u_{\text{truck}}}{m_{\text{car + truck}}}$ $v_{\text{car + truck}} = \frac{(1500 \text{ kg})(20 \text{ m s}^{-1}) + (5000 \text{ kg})(-10 \text{ m s}^{-1})}{(1500 \text{ kg} + 5000 \text{ kg})}$ $= -3.08$ m s^{-1} $= -3.1$ m s^{-1} Note that the negative sign means the combined wreckage is moving to the left, as we initially gave the velocity to the right a positive sign.	▪ Rearrange for $v_{\text{car + truck}}$ ▪ Substitute in known values with units. ▪ Calculate the answer. ▪ State the final answer with correct sign, units and significant figures.

ANSWERS	**LOGIC**
2 Before the collision: $m_{car} = 1500\text{ kg}$; $u_{car} = +20\text{ m s}^{-1}$; $m_{truck} = 5000\text{ kg}$; $u_{truck} = -10\text{ m s}^{-1}$ After collision: $m_{car+truck} = 6500\text{ kg}$; $v_{car+truck} = -3.08\text{ m s}^{-1}$ We need to find the fractional change in kinetic energy: $\frac{\Delta E_k}{E_{k,\,before}} = \frac{E_{k,\,before} - E_{k,\,after}}{E_{k,\,before}}$	▪ Identify the relevant data from the question.
$E_k = \frac{1}{2}mv^2$	▪ Write the equation for kinetic energy.
$E_{k,\,before} = \frac{1}{2}m_{car}u_{car}^2 + \frac{1}{2}m_{truck}\,u_{truck}^2$	▪ Write the expression for total kinetic energy before the collision.
$E_{k,\,before} = \frac{1}{2}(1500\text{ kg})(20\text{ m s}^{-1})^2 + \frac{1}{2}m(5000\text{ kg})(-10\text{ m s}^{-1})^2$	▪ Substitute in known values with units.
$= 5.5 \times 10^5\text{ kg m}^2\text{ s}^{-2} = 5.5 \times 10^5\text{ J}$	▪ Calculate the answer.
$E_{k,\,after} = \frac{1}{2}\;m_{car+truck}\,v_{car+truck}^2$	▪ Write the expression for total kinetic energy after the collision
$E_{k,\,after} = \frac{1}{2}\;(6500\text{ kg})\;(3.08\text{ m s}^{-1})^2$	▪ Substitute in known values with units.
$E_{k,\,after} = 3.08 \times 10^4\text{ kg m}^2\text{ s}^{-2} = 3.08 \times 10^4\text{ J}$	▪ Calculate the answer.
$\frac{\Delta E_k}{E_{k,\,before}} = \frac{E_{k,\,before} - E_{k,\,after}}{E_{k,\,before}}$	▪ Write the expression for fractional change in kinetic energy.
$\frac{\Delta E_k}{E_{k,\,before}} = \frac{5.5\times10^5\text{ J} - 3.08\times10^4\text{ J}}{5.5\times10^5\text{ J}}$	▪ Substitute in known values with units.
$\frac{\Delta E_k}{E_{k,\,before}} = 0.944$	▪ Calculate the answer.
94% of the kinetic energy was transformed to other forms.	▪ State the final answer with correct units and significant figures.

TRY THIS YOURSELF

Repeat Worked example 6.8, but replace the truck with a second car of mass 1500 kg travelling at 20 m s^{-1} to the left.

In Worked example 6.8, the question asks about *immediately* after the collision. This is because the car and truck cannot be modelled as an isolated system. The road exerts significant forces on them, particularly after the collision, changing the momentum of the car and truck system. However, during the very small time that the collision is actually taking place, the forces that the car and truck exert on each other are much, much greater than the forces due to anything else. Hence, just for those moments, we can reasonably make the approximation that the only significant interactions are between the car and truck.

Using approximations in rocket motion

In Worked example 6.8, you should also have noted that the fraction of kinetic energy converted to other forms was very large. This is typically the case with vehicle collisions where initial speeds can be quite high but the final speed is typically much lower. Vehicle collisions are always inelastic, and sometimes perfectly inelastic. Other examples of perfectly inelastic collisions include meteorite impacts with Earth and a person catching a ball.

Subatomic particles can have both perfectly elastic and perfectly inelastic collisions. For example, a neutron can be captured by a nucleus in a perfectly inelastic collision, or be scattered in an elastic collision.

WORKED EXAMPLE (6.9)

In the Open Pool Australian Light water reactor, OPAL (Figure 6.17), a neutron is emitted into the pool with a speed of 1.5×10^5 m s^{-1} where it collides with a hydrogen nucleus in the water. Assume the hydrogen nucleus is initially at rest.

1. Model the collision as perfectly elastic such that the neutron is scattered, and calculate the speeds of the two particles after the collision.
2. Model the collision as perfectly inelastic such that the two particles bind to form a deuterium nucleus, and calculate the speed of this deuterium nucleus.

Newspix / Alan Pryke

FIGURE 6.17 The Open Pool Australian Light water (OPAL) reactor in Sydney

ANSWERS	LOGIC
1 $u_n = 1.5 \times 10^5$ m s^{-1}, $u_p = 0$; we want to find v_n and v_p	- Identify the relevant data from the question.
A hydrogen nucleus is a single proton.	- Look up other data required.
Given that speed is only given to two significant figures, we will make the approximation that $m_p = m_n = 1.67 \times 10^{-27}$ kg	- Make sensible approximations.
$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$	- Write the equation for conservation of momentum.
$u_n = v_p + v_n$	- Simplify, given $m_p = m_n$ and $u_p = 0$.
$v_p = u_n - v_n$	- Rearrange for v_p.
$u_1 + v_1 = v_2 + u_2$	- Write the second equation for elastic collisions.
$u_n + \mathrm{v}_n = v_p$	- Simplify, given $u_p = 0$
$v_n = v_p - \mathrm{u}_n$	- Rearrange for v_n.
$v_n = v_p - \mathrm{u}_n = u_n - v_n - u_n$	- Substitute in the expression for v_p.
$v_n = 0$	- Solve for v_n.
$v_p = u_n - v_n = u_n - 0 = u_n$	- Substitute into the expression for v_p and simplify.
$v_n = 0,\ v_p = u_n = 1.5 \times 10^5$ m s^{-1}.	- State the final answer with correct units and significant figures.
2 $m_1 u_{1,} + m_2 u_2 = m_1 v_1 + m_2 v_2$	- Write the equation for conservation of momentum.
$\mathrm{m}_n \mathrm{u}_n = (\mathrm{m}_n + \mathrm{m}_p)\, v_{p+n} = 2m_n v_{p+n}$	- Simplify, given $m_p = m_n$, $u_p = 0$ and the collision is perfectly inelastic.
$v_{p+n} = \frac{1}{2} u_n$	- Rearrange for v_{p+n}.
$v_{p+n} = \frac{1}{2}\left(1.5 \times 10^5 \text{ m s}^{-1}\right) = 7.5 \times 10^4 \text{ m s}^{-1}$	- Substitute in known values and calculate answer.
The final speed of the deuterium nucleus is 7.5×10^4 m s^{-1}, half that of the captured neutron.	- State the final answer with correct units and significant figures.

TRY THIS YOURSELF

Repeat Worked example 6.9, but with the neutron colliding with a deuterium nucleus.

INVESTIGATION 6.5

Comparing elastic and inelastic collisions

Critical and creative thinking

Numeracy

Information and communication technology capability

AIM

To compare the collisions of different balls with Earth and identify which are closest to elastic or perfectly inelastic

Write an inquiry question for your investigation.

MATERIALS

- Tape measure or metre ruler
- Several different types of ball of approximately the same size, including super-ball and plasticine or blu-tack ball
- Scales
- Webcam or video camera

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?

What risks are associated with your investigation, and how can you manage them?

METHOD

1. Attach the ruler or tape measure to the wall so the zero is at the floor.
2. Set up the camera so it has a clear view of the ruler/tape measure, and you can read the scale on the recording (see Figure 6.18).
3. Weigh each ball and record its weight.
4. Start recording.
5. Hold a ball just in front of the 1 m mark on the ruler/tape measure. Drop the ball (don't throw it).
6. View the recording and note the maximum height to which the ball bounced. Estimate the uncertainty in your measurement.
7. Repeat steps 4–6 for each of the remaining balls.

RESULTS

Record your measurements of mass and maximum bounce height in a table.

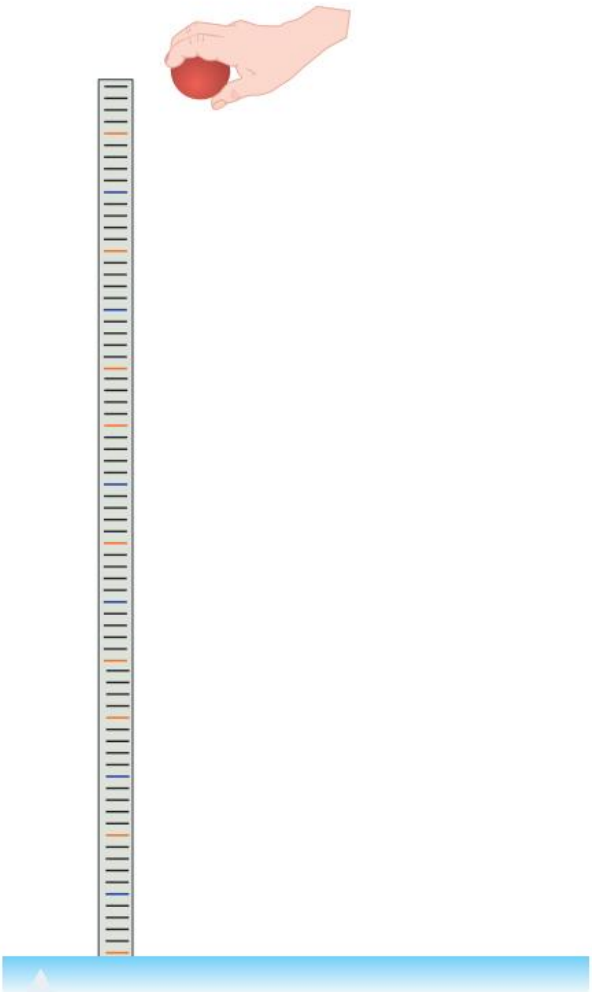


FIGURE 6.18 View by the webcam of the experimental set-up

»

BALL TYPE	m (kg)	h_{max} (m)	$\frac{\Delta E_k}{\Delta E_{k,\text{before}}}$	$\vec{p}_{before}$ (kg m s^{-1})	$\vec{p}_{after}$ (kg m s^{-1})	$\Delta\vec{p}$ (kg m s^{-1})

ANALYSIS OF RESULTS

1. Perform calculations to complete the table.
    - Make the approximation that the loss of energy due to air resistance is small compared with the loss due to friction forces during the collision.
    - Use the starting height and the maximum bounce height to calculate the fractional loss of kinetic energy during the collision.
    - In your write up, explain what assumptions you make and show that $\frac{\Delta E_k}{E_{k,\text{before}}} = \frac{1\text{ m} - h_{max}}{1\text{ m}}$.
2. Calculate the momentum of each ball just before and just after the collision. State what assumptions you have made. Remembering that momentum is a vector, calculate the change in momentum of each ball during the collision.

DISCUSSION

1. Which of the balls underwent a collision closest to elastic?
2. Which was closest to perfectly inelastic?
3. Were energy and momentum conserved in these collisions? What transfers of energy and momentum took place?

CONCLUSIONS

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Revision crossword

KEY CONCEPTS

- Most real collisions are inelastic collisions. Inelastic collisions involve a loss of kinetic energy of the colliding objects.
- In a perfectly inelastic collision, the colliding objects stick together after the collision to form a single object.
- Momentum is conserved in inelastic collisions.

CHECK YOUR UNDERSTANDING

6.4

1. Describe how an elastic collision is different from an inelastic collision.
2. Define 'perfectly inelastic collision'.
3. Marcus and Laurence are roller skating. Marcus stops to retie the laces on his skates and Laurence rolls into him. Marcus has a mass of 45 kg. Laurence has a mass of 39 kg, and is travelling at 4.5 m s^{-1} before the collision. If Laurence grabs Marcus as he hits him:
    - **a** at what speed do they move off together immediately after the collision?
    - **b** what fraction of Laurence's initial kinetic energy is converted into other forms of energy?
4. In another roller-skating incident, Marcus (45 kg) is travelling at 2.5 m s^{-1} north and Laurence (39 kg) is travelling at 3.0 m s^{-1} N45°E when they undergo an inelastic collision. What is their speed and direction immediately after the collision?

6 CHAPTER SUMMARY

- Momentum is the product of mass and velocity: $\vec{p} = m\vec{v}$.
- Momentum is a vector quantity.
- Momentum is conserved in any isolated system: $\Sigma\vec{p} = \text{constant}$.
- Newton's third law is a statement of conservation of momentum. In any interaction between two objects, $\vec{F}_{\text{A on B}} = \frac{\Delta\vec{p}_{\text{B}}}{\Delta t} = -\frac{\Delta\vec{p}_{\text{A}}}{\Delta t} = -\vec{F}_{\text{B on A}}$, so $\Delta\vec{p}_{\text{A}} = -\Delta\vec{p}_{\text{B}}$.
- Momentum is conserved in all collisions.
- In an elastic collision, no kinetic energy is transformed to other forms.
- For an elastic collision: $\Sigma E_{\text{k, before}} = \Sigma E_{\text{k, after}}$ or $\Sigma\frac{1}{2}mv^2_{\text{before}} = \Sigma\frac{1}{2}mv^2_{\text{after}}$, and $\Sigma\vec{p}_{\text{before}} = \Sigma\vec{p}_{\text{after}}$ or $\Sigma m\vec{v}_{\text{before}} = \Sigma m\vec{v}_{\text{after}}$.
- $m_1u_1 + m_2u_2 = mv_1 + m_2v_2$ is true for all collisions.
- $u_1 + v_1 = v_2 + u_2$ applies to elastic collisions only.
- Impulse is the change in momentum of an object when it is acted on by a force: $\vec{I} = \Delta\vec{p} = \vec{F}\Delta t$.
- Impulse is a vector quantity and has the same units as momentum, kg m s^{-1} or N s.
- Impulse can be found from a force versus time graph in the same way that work can be found from a force versus distance graph. Impulse is the area under the curve of an *F* vs *t* graph.
- For a given force, the longer the force is applied, the greater the change in momentum.
- For a given change in momentum, the longer the interaction time, the smaller the force exerted.
- Most real collisions are inelastic collisions. Inelastic collisions involve a loss of kinetic energy of the colliding objects.
- In a perfectly inelastic collision, the colliding objects stick together after the collision to form a single object.

6 CHAPTER REVIEW QUESTIONS

Review quiz

1 Write the equation for momentum and define all the symbols you use. Give the units of each quantity.

2 Classify the following quantities as conserved or not conserved: energy, force, momentum, mass.

3 Define 'impulse'.

4 Identify the quantity that is found from:

  a the area under an *F* vs *t* graph.

  b the area under an *F* vs *s* graph.

5 Demonstrate that the units for impulse can be written as N s.

6 A dog is running along next to a horse, keeping pace with it. Which has the greater momentum? Justify your answer.

7 What condition is necessary such that the total momentum of objects after a collision is zero? Does the same condition apply to elastic and inelastic collisions? Justify your answer.

8 A 1500 kg car is travelling at 20 m s^{-1}. Calculate its momentum.

9 A 20 kg dog has the same momentum as an 800 kg horse.

  a Calculate the ratio $\frac{v_{\text{dog}}}{v_{\text{horse}}}$.

  b Calculate the ratio $\frac{E_{\text{k, dog}}}{E_{\text{k, horse}}}$.

10 In an investigation of collisions, a student claims that momentum is not conserved when a ball bounces off the ground because only the ball has a change in momentum. Is momentum conserved in this case? Explain your answer.

11 The impulse given to the occupant of a car with a crumple zone is exactly the same as the impulse given to the occupant of a rigid vehicle. Explain why crumple zones are considered desirable.

12 Write Newton's second and third laws in terms of momentum. Explain how Newton's third law can be considered a statement of conservation of momentum.

13 During a crash test, a car is rolled into a wall with initial speed of 25 m s^{-1}. The car has a mass of 1450 kg and comes to rest in 0.20 s.

- **a** Calculate the impulse caused by the collision.
- **b** Calculate the average force applied to the car.

14 Figure 6.19 shows the net force acting on a 60 kg sprinter at the start of a race. Calculate the sprinter's speed 1.8 s after the start.

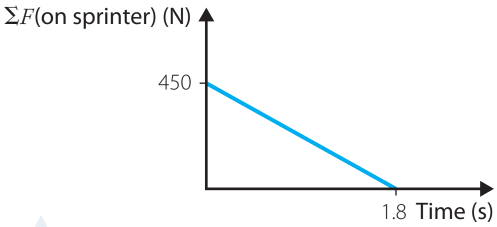


FIGURE 6.19

15 Chee and Matt are each sitting on their own sled. Matt has a mass larger than Chee's. Jenna pushes them each through the same distance, *s*, with the same force.

- **a** Which has greater kinetic energy after the push? Explain your answer.
- **b** Which has greater momentum? Explain your answer.
- **c** How would your answers to parts **a** and **b** change if instead Jenna pushed them for the same time?

16 An explosion can be modelled as a perfectly inelastic collision in reverse. Using what you know about momentum and collisions, explain why most fireworks produce a dome-shaped pattern of incandescent fragments.

17 A destroyer with a mass of 1200 tons (1.2×10^6 kg) fires an 8″ shell with a mass of 116 kg at a speed of 850 m s^{-1}.

- **a** Calculate the impulse given to the shell by the destroyer's gun.
- **b** If no other forces acted on the destroyer, at what speed would it recoil?
- **c** Explain why a ship tilts in the water when firing. Can the ship and shell be modelled as an isolated system?

18 In a particle accelerator, two particles with masses such that $m_1 = 3m_2$ collide. Particle 1 is initially moving at 1.5×10^7 m s^{-1} and particle 2 is moving in the same direction at 0.5×10^7 m s^{-1}.

- **a** Calculate the final speed of each particle assuming the collision is elastic.
- **b** Calculate the final speed of each particle assuming the collision is perfectly inelastic.

19 When a car brakes suddenly from 20 m s^{-1}, the head of a 90 kg occupant who is not wearing a seat belt comes to a stop on the windscreen. The skull is pushed inwards towards the brain by 3.5 cm. The mass of the head is 8% of the total body mass.

- **a** What is the mass of the occupant's head?
- **b** What is the kinetic energy of the head when it hits the windscreen?
- **c** With what force does the windscreen strike the head?
- **d** How long did it take for the head to be crushed by the windscreen?

20 Two cars make a perfectly inelastic collision at an intersection. Car 1 has a mass of 1200 kg, and car 2 has a mass of 1500 kg. Before the collision, car 1 is travelling at 25 m s^{-1} north and car 2 is travelling at 15 m s^{-1} west.

- **a** Calculate the speed and direction of the wreckage immediately after the collision.
- **b** Calculate the amount of kinetic energy that is transformed during the collision. What forms is this energy transformed into?

21 Construct a concept map showing all the ideas you have learned in your study of force, energy and momentum. Include all the equations you have used.

1 Sana puts on her seat belt, starts her car (mass 1200 kg), checks her mirrors and accelerates away on a straight horizontal road.

- **a** Calculate the force required to accelerate the car from 0 to 60 km h^{-1} in 9.0 s, assuming constant acceleration.
- **b** Sketch graphs of the position, speed and acceleration of the car as a function of time for this 9.0 s.
- **c** Draw a force diagram showing all the forces acting on the car as it accelerates. Do not ignore air resistance.
- **d** Describe how the forces shown in your diagram for part **c** vary with time.
- **e** Calculate the total work done on the car in this 9.0 s.

2 Sana is cruising at a constant speed of 60 km h^{-1} in her car (mass 1200 kg) on a straight horizontal road.

- **a** Draw a force diagram showing all the forces acting on the car.
- **b** Explain why Sana needs to keep her foot on the accelerator (she is not using cruise control) to maintain a constant speed.
- **c** If the road exerts a constant force of 400 N on the car, calculate how much work is done on the car per second by the road.
- **d** Where does the energy transferred to the car by the road come from? Explain your answer using Newton's third law.
- **e** Calculate the rate at which energy is lost (power) by the car due to air resistance.

3 A student states that 'friction always opposes motion'. Critique this statement, and give examples that support your critique.

4 In each of the following cases, is the net force acting on the car zero? Explain your answer for each case.

- **a** A car is travelling at constant speed on a flat, straight road.
- **b** A car is travelling at constant speed on a flat, curved road.
- **c** A car is travelling at constant speed up a hill on a straight road.
- **d** A car is travelling at constant speed down a hill on a straight road.
- **e** A car is travelling at increasing speed down a hill on a straight road.

5 Rafael is riding his motorbike (combined mass 260 kg) when he runs into a pigeon (mass 1 kg). Rafael is initially moving at 75 km h^{-1} and the pigeon is moving in the same direction at 25 km h^{-1} when it is surprised by Rafael running into it from behind.

- **a** If the collision is perfectly *elastic*, calculate the speed at which the pigeon is moving after the collision.
- **b** Calculate the work done by Rafael (and his bike) on the pigeon.
- **c** Compare this with the work done by the pigeon on Rafael and his bike.
- **d** Calculate the impulse transferred to the pigeon.
- **e** Compare this with the impulse transferred to Rafael and his bike.

6 Rafael is riding his motorbike (combined mass 260 kg) when he runs into a pigeon (mass 1 kg). Rafael is initially moving at 75 km h^{-1} and the pigeon is moving in the same direction at 25 km h^{-1} when it is surprised by Rafael running into it from behind.

- **a** If the collision is perfectly *inelastic*, calculate the speed at which Rafael and the pigeon are moving after the collision.
- **b** Calculate the work done by Rafael (and his bike) on the pigeon in this case.
- **c** Compare this with the work done by the pigeon on Rafael and his bike.
- **d** Calculate the impulse transferred to the pigeon.
- **e** Compare this with the impulse transferred to Rafael and his bike.
- **f** In which situation is the pigeon more likely to be injured; if it is hit by Rafael's rigid helmet or his padded leather jacket? Explain your answer in terms of force and momentum.

7 Mary picks up her 5.5 kg school bag, raising it through a total height of 1.5 m.

FIGURE 6.20

Dreamstime.com/Denise P. Lett

- **a** Calculate the force that Mary must apply if she is to lift her bag at constant speed, directly upwards.
- **b** Calculate how much work Mary does as she lifts the bag.
- **c** Calculate the work done by the gravitational field on the bag.
- **d** Is the Mary–bag system an isolated system? Explain your answer.

8 Kate is feeding her pig, Pigamajig. She puts the bucket on the ground and Pigamajig grabs hold of it. Pigamajig applies a horizontal force to the bucket, and Kate applies an equal and opposite force to the bucket so that it does not move.

- **a** Are the forces exerted by Kate and Pigamajig a Newton's third law action–reaction pair? Justify your answer.
- **b** Draw force diagrams for:

FIGURE 6.21

  - **i** the pig.
  - **ii** the bucket.
  - **iii** Kate.
- **c** Apply Newton's third law to identify forces that are equal and opposite in this situation, and are action–reaction pairs.
- **d** Apply Newton's second law to identify forces that are equal and opposite in this situation, but are not action–reaction pairs.

9 Kate is feeding her pig, Pigamajig, when the naughty pig grabs hold of the feed bucket before Kate has put it down. Pigamajig applies a force of 40 N to the bucket at an angle of 45° below the horizontal and to the left. Kate applies a force of 50 N at an angle of 60° above the horizontal and to the right.

- **a** Calculate the vertical and horizontal components of the force exerted by Pigamajig on the bucket.
- **b** Calculate the vertical and horizontal components of the force exerted by Kate on the bucket.
- **c** Calculate the net force acting on the bucket if it has a mass of 2.5 kg (including the food). State the magnitude and direction of the force.
- **d** Calculate the acceleration of the bucket.

10 An 80 kg sheep is being dragged up a ramp into a shearing shed. The coefficient of kinetic friction between the sheep and the boards of the ramp is 0.30, and the coefficient of static friction is 0.35. The angle of the ramp to the horizontal is 10°. The shearer pulling the sheep is applying a steadily increasing force.

- **a** Calculate the minimum force the shearer must exert to get the sheep to begin moving, assuming the sheep does not struggle.
- **b** Calculate the force required to drag the sheep at a constant speed, once it is moving.
- **c** Calculate the total work done by the shearer if he drags the sheep a distance of 2.5 m on the ramp at constant speed.

Newspix/Ian Currie

FIGURE 6.22

DEPTH STUDY SUGGESTIONS

- → Investigate friction forces (for example, by extending one of the investigations) or measure the static and kinetic coefficients of friction for different types of shoes and floors.
- → Car safety can be investigated using secondary sources, or by numerical modelling.
- → Investigate (first-hand and/or using secondary sources) how different sorts of flying machines work.
- → Measure drag forces in different types of fluids, or construct an anemometer.
- → Model projectile motion numerically, including air resistance. Compare your results with actual measurements.
- → Perform first-hand or secondary-sourced investigations of the forces involved in different sports.
- → Join an engineering challenge to make a static structure, such as a bridge or tower.
- → Investigate the causes of structural failures in buildings, dams and bridges.
- → Make your own Newton's cradle or ballistic pendulum to investigate momentum transfers.
- → Conduct a literature review and create a numerical model of a sling-shot manoeuvre.

» MODULE THREE

WAVES AND THERMODYNAMICS

Alamy Stock Photo/YAY Media AS

7 Wave characteristics

INQUIRY QUESTION

What are the properties of all waves and wave motion?

OUTCOMES

Students:

- conduct a practical investigation involving the creation of mechanical waves in a variety of situations in order to explain: CCT
  - the role of the medium in the propagation of mechanical waves
  - the transfer of energy involved in the propagation of mechanical waves (ACSPH067, ACSPH070)
- conduct practical investigations to explain and analyse the differences between: CCT
  - transverse and longitudinal waves (ACSPH068)
  - mechanical and electromagnetic waves (ACSPH070, ACSPH074)
- construct and/or interpret graphs of displacement as a function of time and as a function of position of transverse and longitudinal waves, and relate the features of those graphs to the following wave characteristics:
  - velocity
  - frequency
  - period
  - wavelength
  - wave number
  - displacement and amplitude (ACSPH069) ICT N
- solve problems and/or make predictions by modelling and applying the following relationships to a variety of situations: ICT N
  - $v = f\lambda$
  - $f = \frac{1}{T}$
  - $k = \frac{2\pi}{\lambda}$

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Water waves, **sound waves** and seismic waves are examples of mechanical waves. Mechanical waves are the result of an energy disturbance passing through the medium, such as water or air. The medium is not disturbed except briefly as the wave passes. The medium returns to exactly the same state as it was before the mechanical wave (energy disturbance) passed through the medium. There is no transfer of matter from point to point.


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FIGURE 7.1 Concentric circular waves formed by drops of water

As mechanical waves pass, the medium through which they are travelling (such as air or water) can be seen to vibrate but not travel with the wave itself. The medium stays where it is, undisturbed, after the wave passes. For example, a duck on a pond will bob up and down as a ripple passes, but it stays at its original position. It does not travel with the wave.

The waves described in the example of the duck on the pond are transverse waves. The motion of the medium as the wave passes through is perpendicular to the direction of propagation of the wave.

Mechanical waves come in two very distinct types: transverse and longitudinal waves. Sound waves are longitudinal waves, and are fundamentally different from transverse waves. When longitudinal waves (such as sound waves) travel through a medium (such as air), the medium is disturbed as the wave passes through. However, in this case, the vibration that is the origin of the sound (such as your voice) propagates through the medium by compression and rarefaction. The initial sound disturbance leads to a compression of the surrounding atmosphere. This leaves a following rarefaction of air, and the cycle continues.

Water waves have a circular or elliptical motion of the medium as the wave passes. However, a duck appears to move up and down as the wave passes rather than back and forth. A careful observer will notice that the duck moves a little back and then forwards with the passing of the wave, as well as up and down.

We are surrounded by sounds. The sound energy of the noise of the traffic on a busy road or the chirping of birds is carried to our ears by sound waves travelling through a medium. As the sound waves pass through the air, the air particles vibrate back and forth. The energy of the sound continues onwards, some continuing into our ear drums. The vibration of the air particles cause our ear drums to vibrate. This vibrating energy is converted into electrical impulses that our brain interprets as sound.

Waves generated by earthquakes, known as seismic waves, travel through Earth. These waves convey energy from earthquakes or from underground explosions. For sound waves travelling through air, the air is the medium. For seismic waves, Earth is the medium.

7.1 The mechanical wave model

Sound, water and **seismic waves** are mechanical waves. **Mechanical waves** transfer energy through a material called the **medium** without the medium itself being transported from one place to another. The particles which make up the medium are free to move from their normal positions but the whole medium does not move. Water waves are examples of **transverse waves**. When a water wave passes through the water, particles oscillate, or vibrate, at right angles to the direction of the motion of the wave.

When discussing water waves, it is important to note that once a wave has broken coming towards the beach, it is no longer regarded as being a wave but is now considered a movement of water.

Sound waves are **longitudinal waves**. Particles in the medium oscillate about their average positions in the same direction as the wave's propagation. Sound waves can travel through any medium such as air, water, rock or metal. As with all mechanical waves, sound cannot travel through a vacuum. Figure 7.2 shows the directions of motion of the medium for transverse waves and for longitudinal waves.

Mechanical transverse and longitudinal waves travel in a material medium made of particles that can affect other particles because they exert forces on each other. Energy is transferred through the medium, but none of the particles are. You may recall that all matter is made of particles. The arrangement of the particles differs between solids, liquids and gases.

Water waves

An in-depth description of the nature of water waves and the movement of the medium as they pass.

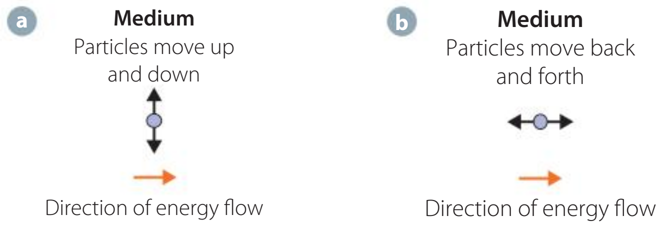


FIGURE 7.2 Mechanical waves cause particles to move around their average positions. **a** Transverse waves showing motion perpendicular to the direction of energy flow; **b** Longitudinal waves showing a back-and-forth motion in the same direction as energy flow

The role of the medium

The nature of the medium determines how fast the wave will travel through it. Table 7.1 shows the approximate speed of sound travelling through a variety of mediums.

TABLE 7.1 The speed of sound in a variety of mediums

MEDIUM	SPEED OF SOUND (m s^{-1})
Air	340
Water	1500
Sandstone	2000–6000
Steel	6100

There are a number of reasons for the large range of speeds of waves in different mediums. For air, the particles (which are mostly molecules of nitrogen or oxygen) are spaced far apart and are moving around with speeds of around 0.5 km s^{-1}. The wave motion must be transferred through the air by collisions between molecules, transferring the vibration of the air molecules to neighbouring air molecules. Such transfers occur when the air particles collide. The wave propagation travels through the air at 343 m s^{-1} at 20°C, slower than the speed of the particles in the medium themselves.

Wave characteristics – 20 questions

Table 7.1 shows the large range of the speed of sound in different mediums. In liquids and solids, the particles can influence the movement of neighbouring particles by exerting forces on each other. These forces can be pushes or pulls. In a gas, the forces between particles only occur when the particles collide. The speed of sound in steel is nearly 20 times faster than in air.

INVESTIGATION 7.1

The role of the medium in the propagation of mechanical waves

Critical and creative thinking

AIM

To investigate the role of the medium in the propagation of mechanical waves

MATERIALS

- 2 tin cans
- Small drill
- File (for sharp edges)
- 5 m length of string

»

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU SAFELY MANAGE THESE RISKS TO STAY SAFE?
Sharp edges on a tin can may cut you.	Take care handling the tins and file down any sharp edges.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1 Using the drill, make a small hole in the base of each tin can. File any sharp edges.

2 Pass an end of the string through the hole of one tin can. Make a knot in the string so that it cannot be pulled back out. Repeat with the other end of the string for the other can.

3 With the string pulled straight between the two cans, speak into one can and have a partner listen into the other. Observe how clearly the voice is transmitted through the string.

4 While staying the same distance from each other, speak directly to your partner in the same volume without using the string and cans. Compare how clearly you heard compared to when the cans and string were used.

5 Using a long bench in your laboratory, have your partner place their ear down at one end of the bench while you tap on the bench. Compare what is heard to when the partner's ear is not on the bench.

RESULTS

Record your observations in a suitable table.

DISCUSSION

1 Discuss your observations with regard to the role of the two different mediums for the transmission of the sound waves in this investigation by comparing:

  a whether the sound energy was being spread out as it travelled or if it was directed and concentrated in one direction.

  b the material that made up the medium – solid versus gas.

2 In this investigation, which medium was best at transmitting sound over a distance (i.e. clearest and loudest) and what reasons might there be for this?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Transfer of energy by mechanical waves

Mechanical waves can transfer energy from the source of the wave. The vibrations of the particles in the medium propagate through the medium, transferring energy as they travel. Seismic waves and ocean waves (tsunamis) caused by undersea earthquakes are examples of such energy transfer. These waves can transfer very large quantities of energy from the source of the earthquake (called the focus), causing devastation considerable distances away.

Seismic waves

When an earthquake occurs, energy radiates in waves from the focus through the rocks of Earth's crust. Such waves are known as seismic waves. The 2004 Indian Ocean earthquake that occurred off the Indonesian island of Sumatra radiated energy equivalent to around 32 000 Hiroshima atomic bombs, or 500 million tonnes of TNT. Figure 7.3 shows how the primary, or P waves, and the secondary, or S waves, travelled through the crust from the focus of the earthquake. P waves are longitudinal waves through the rocks while S waves are transverse waves.

9780170409063

The most destructive type of seismic waves are those that travel on the surface. There are two kinds of these waves: Love waves and **Rayleigh waves**. Both are quite complicated – Rayleigh waves result from a combination of many transverse waves and cause side-to-side shaking. Buildings are moved up and down and tilted as these waves pass, causing much damage. The energy in Rayleigh waves is quickly dispersed as they move away from the epicentre (the place on the surface immediately above the focus of the earthquake).

The difference between transverse and longitudinal waves will be discussed in the next section.

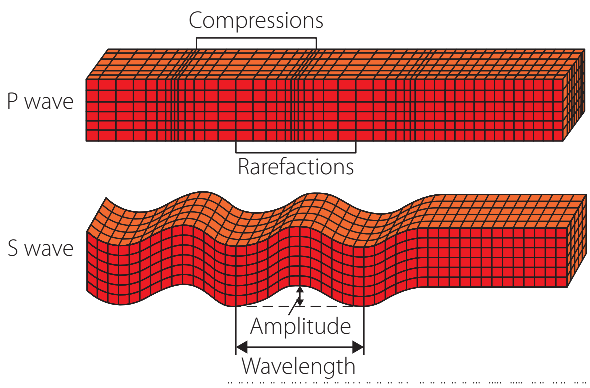
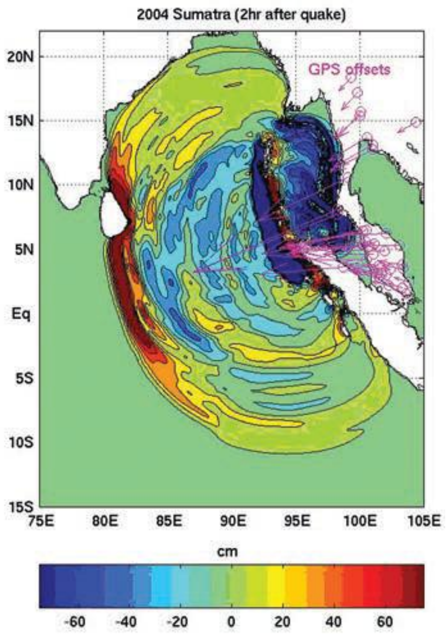


FIGURE 7.3 Propagation of longitudinal primary (P) waves and transverse Rayleigh waves through Earth away from an earthquake epicentre

The focus of the huge 2004 Indian Ocean earthquake was away from land, so there was little damage caused to buildings and structures due to the movement of the ground. The movement of huge slabs of the crust under the ocean resulted in another mechanical wave through the ocean – a tsunami. Reaching speeds of over 500 km h^{-1}, the tsunami carried energy from the earthquake to shores thousands of kilometres away. In some places, waves over 20 m in height caused the ocean to surge inland for 3 km.

Figure 7.4 is a computer-generated image showing the propagation of wave energy across the Indian Ocean 2 hours after the earthquake. As the wave spread out, its energy was also spread across a wider area, so its intensity decreased. However, closer to the event, the energy in the wave caused damage and devastation in many countries, with over 300 000 people killed.



Courtesy NASA/JPL-Caltech

FIGURE 7.4 Tsunami wave energy mapped spreading across the Indian Ocean 2 hours after the earthquake occurred. The computer-generated colours show the height of the ocean above or below normal, as indicated in the scale.

INVESTIGATION 7.2

Investigating the transfer of energy by a mechanical wave

AIM

To observe how energy can be transferred by a mechanical wave

MATERIALS

- Slinky springs
- Ribbons
- Chalk or non-permanent marker
- Shallow water tray
- Corks
- Optional: laptop with video editing software, a tablet or a phone

RISK ASSESSMENT

WHAT IS THE RISK IN DOING THIS INVESTIGATION?	HOW CAN YOU SAFELY MANAGE THESE RISKS TO STAY SAFE?
The spring may flick back and hit your eye.	Wear safety glasses when working with springs.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Place the stretched slinky spring on a floor with a smooth surface.
2. Tie two ribbons around a coil of the spring, each about 0.5 m from either end.
3. With a person on each end of the spring, have one person at a time move the spring quickly back and forth, and then from side to side.
4. Observe the motion within the spring and the movement of the ribbons in each case.
5. Using chalk or a non-permanent marker, measure out marks on the floor 1 cm apart, starting at each ribbon on the spring when it is at rest. These marks should continue either side of the ribbons both in the direction of the spring and perpendicular to the spring.
6. Repeat steps 3 and 4, this time taking care to record the distances each of the ribbons move.
7. Half fill a shallow tray with water.
8. Place a cork in the water at one end of the tray. At the other end, dip your finger in and out of the water.
9. Observe how the energy of the wave can cause the cork to move.

RESULTS

Draw a diagram of the motion of the ribbons in the spring. Record your observations of the cork movement in the water. You may wish to record the motion in the spring and in the tray on a suitable device such as a laptop with video editing software, a tablet or phone.

DISCUSSION

1. Using diagrams, describe how the energy in the two mediums travels from one place to another to cause motion without the medium itself travelling with the wave.
2. Compare the motion of the two ribbons in the spring. Which of the two ribbons moves further from its resting position? Suggest reasons why this might be happening.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- Mechanical waves travel through a medium.
- Energy is transported from one place to another by a mechanical wave.
- The particles in the medium oscillate as a mechanical wave passes.
- The medium itself does not travel with the wave.
- Examples of mechanical waves are sound, water and seismic waves.

CHECK YOUR UNDERSTANDING

7.1

1 Briefly define 'mechanical wave'.

2 Describe an example of when energy is conveyed by a wave from one place to another.

3 A student suggests that air travels with the sound wave. Using the example of sound travelling into a person's ear, describe why this could not be true.

4 An earthquake produces seismic waves in rocks. How can you tell that energy is being transported by the seismic wave?

5 Suggest why sound waves travel so much faster in steel than in air. Refer to the arrangement of the particles in these mediums.

7.2 Transverse and longitudinal waves

The way in which the medium moves as a wave passes determines whether the wave is classified as transverse or longitudinal. While these are not the only types of waves possible, they are the most commonly observed. In this section, both transverse and longitudinal wave motion will be discussed.

Transverse waves

Individual particles of a medium move up and down about their rest position. A series of wave **crests** and wave **troughs** move through the medium. The motions of different points in the medium along the wave are shown in Figure 7.5.

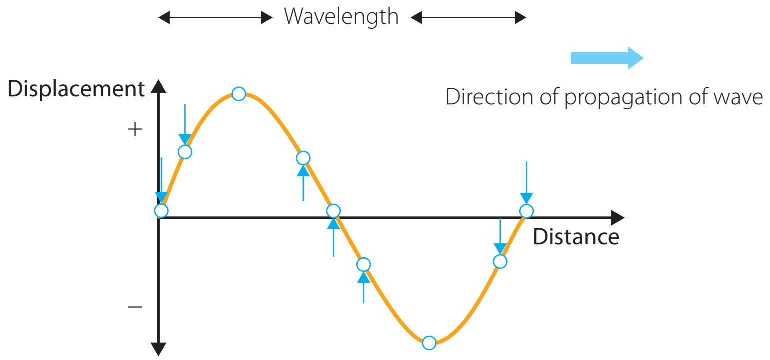


FIGURE 7.5 The motion of the medium in a transverse wave showing various points in the medium and the direction of motion as the wave moves

At a crest or a trough, the particles are momentarily stationary and are about to move back towards their rest position. Particles either side of a crest or a trough are moving perpendicular to the direction of the motion of the wave itself.

Reflection of waves will be studied further in chapter 8.

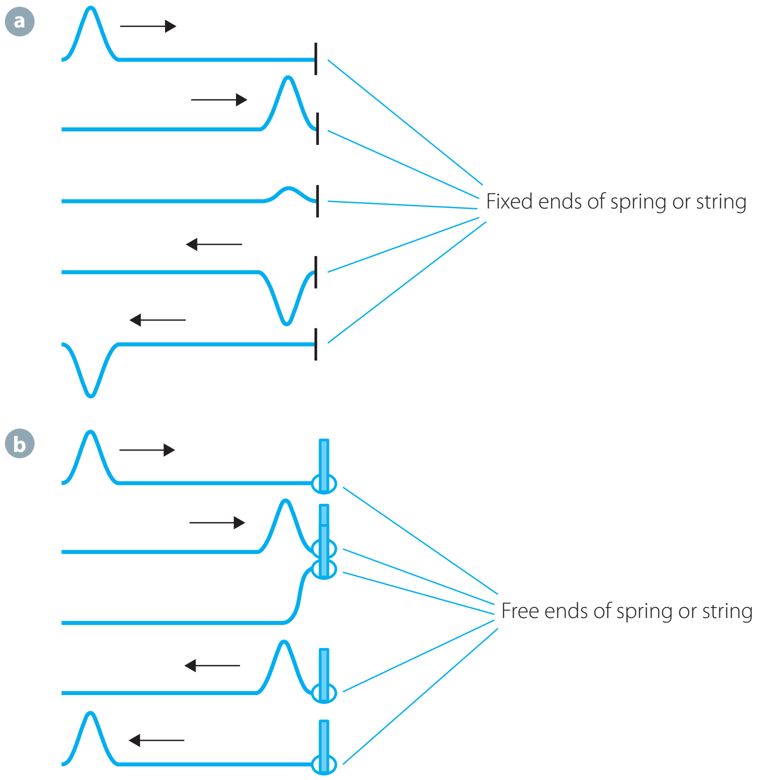
Reflection of transverse waves

When a transverse wave meets the end of the spring or string in which it is travelling, it is reflected. If the end of the spring or string is fixed, a crest is reflected as a trough – the wave is inverted or turned upside down when reflected. If the end of the spring or string is free to move up and down, the crest is reflected as a crest – the wave is reflected upright. Figure 7.6 shows these situations for a single transverse pulse. In both cases, the wave is reflected with the same shape and speed.

Wave on a string

This simulation can be used to visualise reflections from free and fixed ends of strings.

FIGURE 7.6 **a** A stretched spring or string fixed at one end reflects waves upside down. **b** A stretched spring or string free at one end reflects waves the same way up.



Longitudinal waves

In longitudinal waves, particles of the medium move back and forth in the same direction as the transfer of energy; that is, along the same direction as the wave is travelling.

When the particles around a point are all moving towards the point, there is a local **compression**. If they are all moving away from the point, there is a local **rarefaction**. A particular point in the medium through which the wave disturbance is travelling experiences a series of compressions and rarefactions (changes to the undisturbed pressure) as the energy passes through it. Figure 7.7 shows a snapshot at an instant in time of where the coils in the spring have been displaced from as the wave passes. The coils represent the particles in a medium such as air. Maximum pressure occurs at compressions; minimum pressure occurs at rarefactions.

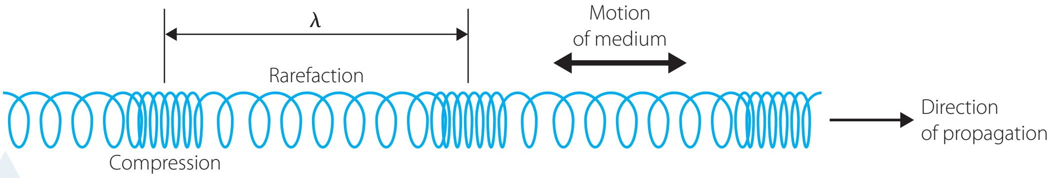


FIGURE 7.7 A snapshot in time showing rarefactions and compressions in a spring. This is an example of a longitudinal wave.

INVESTIGATION 7.3

Comparing transverse and longitudinal waves

Critical and creative thinking

AIM

To compare the motion of the medium in transverse and longitudinal waves

MATERIALS

- Slinky spring
- Ruler
- Motion-recording device (such as laptop, tablet or phone)

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
The spring may flick back into your eye if released.	Wear safety glasses when working with springs. Do not over stretch the spring.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Stretch the spring out on a smooth floor or desk so that there is a small amount of stretch in the spring. Measure the length of the spring.
2. Create a transverse pulse by having a partner hold one end of the spring stationary while you give the spring a quick sideways shake. Do this once to send a single pulse down the spring.
3. Observe the motion within the spring as the pulse travels down the spring. This motion can be recorded using a suitable device such as a laptop, tablet or phone.
4. Play the recording back at reduced speed and then draw a diagram to show the direction of motion of the coils of the spring.
5. Carefully observe the reflected pulse and draw a diagram to show two things you noticed about the nature and position of the reflected pulse.
6. Repeat the investigation with different sized pulses.
7. Stretch the spring a little further, taking care to hold it firmly. Measure the new length of the spring.
8. Create a longitudinal pulse in the spring by having a partner hold one end while you make a rapid back-and-forth movement in the spring in the same direction as the spring.
9. Again, record the motion of this pulse on a suitable device.
10. Play the recording back in slow motion and then draw a diagram to show the direction of motion of the coils in the spring as the pulse passes.

RESULTS

Draw your observations from the method as instructed.

ANALYSIS OF RESULTS

Use the recordings made of the motion in the spring to find the speed of each pulse in the spring. Remember, $\text{speed} = \frac{\text{distance}}{\text{time}}$. The time can be calculated from the recordings made.

DISCUSSION

1. Compare the motion of the coils in the spring in terms of the direction of motion of the coils between the transverse pulse and the longitudinal pulse.
2. Did the speeds of each type of wave change with a change in the amplitude of the waves?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Fill in the gaps

Investigation 7.3 compares the motion of the medium when transverse and longitudinal waves pass. Longitudinal waves are able to pass through gases and liquids. These substances cannot propagate transverse waves effectively as they cannot transmit sideways forces, which is necessary for transverse waves.

KEY CONCEPTS

- Longitudinal waves involve a back-and-forth motion of the particles in the medium as the wave passes.
- Transverse waves involve an up-and-down or side-to-side motion of the particles in the medium as the wave passes.
- Transverse waves reflect upright at a free end, and inverted at a fixed end.

CHECK YOUR UNDERSTANDING

7.2

1. Using a sketch, describe the key difference between longitudinal and transverse waves.
2. A buoy in the ocean is observed to move up and down as waves pass. Would this be an example of transverse or longitudinal wave motion? Explain your answer.
3. A girl on a swing is moving back and forth. Would such motion be regarded as a wave? Why or why not? Discuss this situation using your knowledge of waves.
4. Earthquake waves are generated by the movement of rocks under the surface. Explain why both longitudinal and transverse waves can be produced.
5. Classify the following examples of mechanical waves as transverse or longitudinal waves.
   - **a** A pulse transmitted along a string stretched at right angles to the direction of motion of the pulse.
   - **b** The wave produced by throwing a rock into a calm pond.
   - **c** Sound waves produced by a drum.
   - **d** Waves produced in the air by vibrating human vocal cords.
6. Recall why transverse waves cannot travel through water or air.

7.3 Electromagnetic waves

Electromagnetic radiation can be visualised as an electric and a magnetic field oscillating in unison perpendicular to each other, travelling through space, as shown in Figure 7.8.

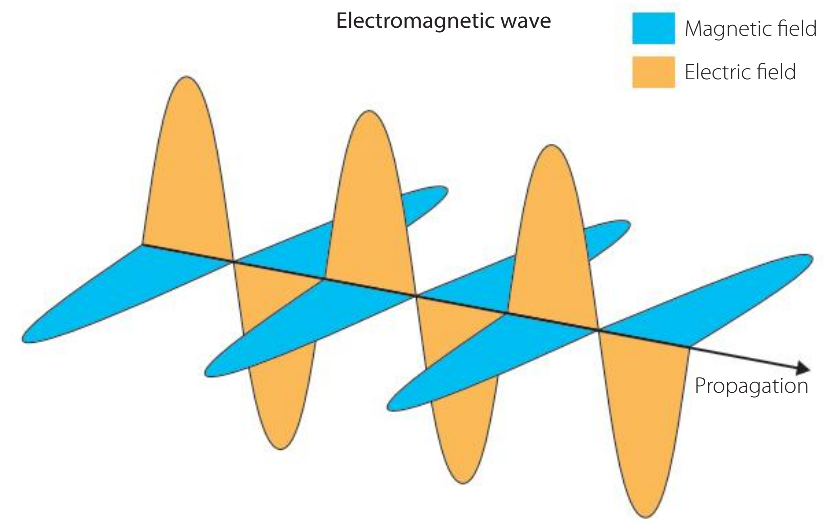


FIGURE 7.8 A representation of electromagnetic radiation as oscillating electric and magnetic fields perpendicular to each other, travelling through space.

The link between electricity and magnetism was first described by Hans Christian Ørsted in 1820 and subsequently studied by Michael Faraday and Humphry Davy. James Clerk Maxwell developed a mathematical description of electromagnetic radiation in the 1860s. The oscillation, or variation, of a magnetic field induces an electric field perpendicular to it. This changing electric field also induces a magnetic field perpendicular to it. The oscillations of the two fields are therefore able to self-propagate. For this reason, electromagnetic radiation is able to travel through the vacuum of space. When Maxwell's equations are solved to find the speed of propagation, the answer is the same regardless of the wave frequency, and is the same as the known speed of light. Maxwell hypothesised that light is an example of an electromagnetic wave, and that other similar waves with different frequencies might also exist. This turned out to be correct, with the discovery of X-rays, radio waves, infrared and ultraviolet, to name a few from what we now understand to be a continuous spectrum of electromagnetic radiation.

The speed of light in a vacuum, 2.9979×10^8 m s^{-1} or approximately 300 000 km s^{-1}, which came out of Maxwell's equations, is the theoretical universal 'speed limit' suggested in Einstein's special theory of relativity. This speed is often written as 3.0×10^8 m s^{-1}.

Maxwell's result, which showed the speed of light to be invariant, suggested that electromagnetic waves propagated through space without a medium being required for their propagation. The lack of the need for a medium for electromagnetic waves led to intense debate. Many experiments to detect the effect any such medium (referred to as the 'aether') would have on the speed of these waves were performed. The very existence of a medium for electromagnetic waves would imply that the medium itself becomes the universal frame of reference, a concept rejected by Einstein. Einstein, who realised that the invariant (constant) speed for the propagation of electromagnetic waves meant that there was no aether. To accommodate Maxwell's theories, Einstein showed by his 'thought experiments' that both time and space as experienced by local observers would vary with the location of the observer. This revolution in thought led to the special theory of relativity in 1905, which was influential in ushering in the modern physics we understand today.

The complete **electromagnetic spectrum** is shown in Figure 7.9. It includes wavelengths ranging from kilometres for long radio waves, down to 10 picometres (10^{-12} m) for gamma rays.

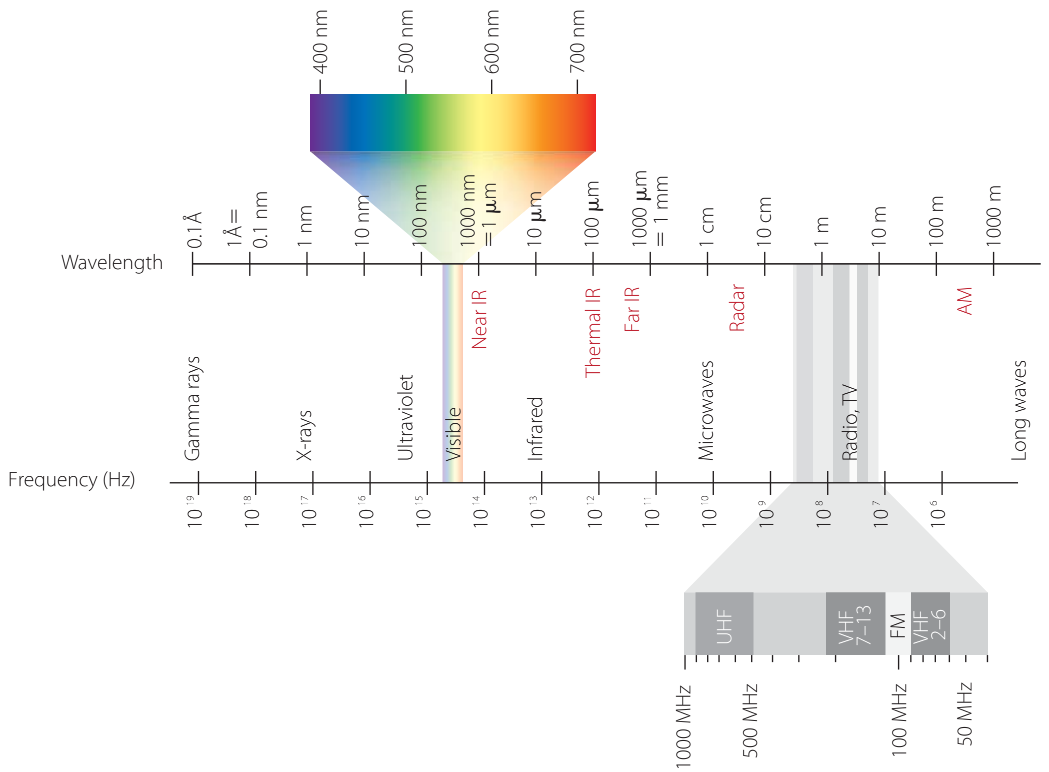


FIGURE 7.9 The electromagnetic spectrum includes radio, microwaves, infrared, light, ultraviolet, X-ray and gamma ray radiation.

Radio waves

Try this simulation to visualise travelling radio waves.

Unlike mechanical waves, electromagnetic waves do not require a medium. The radiation coming to us from the Sun, stars and pulsars travels through the emptiness of space at the speed of light. Electromagnetic waves are able to travel through certain mediums. For example, light can travel through the atmosphere, water, glass and diamond but not through rocks, wood or metals. Earth's atmosphere effectively absorbs or blocks most electromagnetic radiation with the exception of light, radio and microwaves.

With electromagnetic waves, as mentioned above, there is no physical vibration. It is invisible electric and magnetic fields (which do not involve matter) that are oscillating in space, transferring energy with it as the oscillating fields propagate.

INVESTIGATION 7.4

Differences between mechanical and electromagnetic waves

AIM

To observe differences between mechanical and electromagnetic waves

HYPOTHESIS

If the medium is removed, a mechanical wave will not pass but electromagnetic waves will.

MATERIAL

- Well-greased air-tight bell jar with air outlet
- Strong vacuum pump
- Battery-powered bell

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
The vacuum pump is very heavy and may cause injury if dropped.	Place the vacuum pump well away from the edge of the bench.
A strong vacuum can cause a glass container to implode and shatter, spreading shards of glass.	Only use a purpose built vacuum jar and wear safety glasses.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Place the battery-powered bell inside the bell jar, turn it on and seal the jar.
2. Connect the vacuum pump.
3. Observe the sound of the bell.
4. Start the vacuum pump and continue to observe the sound of the bell.
5. Once the vacuum has been attained, observe the sound from the bell and observe the bell itself.

RESULTS

Record your observations in a suitable format.

Arbor Scientific, with permission

FIGURE 7.10 A bell jar with bell inside suitable for use in this investigation

»

DISCUSSION

Explain why the sound from the bell changed the way it did, but the bell did not look any different. Which of the waves involved are mechanical and which are electromagnetic?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the hypothesis of this investigation.

It is difficult to conduct large-scale investigations in an environment where there is no medium. The bell jar vacuum shows that the sound from the bell is unable to travel from the bell directly to the walls of the jar. Some vibrations travel through the bell and its support inside the jar, which can be heard outside the jar. The light from the bell is not changed when there is a vacuum in the jar because light, a part of the electromagnetic spectrum, can travel through the vacuum.

KEY CONCEPTS

- Electromagnetic waves can be modelled as electric and magnetic fields oscillating perpendicular to each other.
- Electromagnetic waves do not need a medium.
- In a vacuum, all electromagnetic waves travel at the speed of light: 3.0×10^8 m s^{-1}.
- The range of electromagnetic waves make up the electromagnetic spectrum.

CHECK YOUR UNDERSTANDING

7.3

1 List three examples of electromagnetic waves travelling through a vacuum.

2 Discuss why modern space movies almost always portray sound as being able to travel through space.

3 If sound (a mechanical wave) could travel through the vacuum of space, what differences may there be for us here on Earth?

4 Explain why the oscillations involved in the production and propagation of electromagnetic waves cannot be observed directly.

5 Using the formula $\text{speed} = \frac{\text{distance}}{\text{time}}$, how long does it take light to reach Earth from the Sun, which is 150 million kilometres away?

7.4 Graphical representations of waves

It is important to understand the terminology used to describe waves. Plotting a graph of **displacement** versus time enables a visualisation of the period and the amplitude of a wave. Waves can also be compared easily if they are represented in graphical form.

The terminology used to describe waves

WS
Relationships between quantities

Period is the time it takes for a wave to repeat itself. **Amplitude** is the largest distance of the particle from the mean (or rest) position before returning. For transverse waves, the top of the wave is called a crest and the bottom is called a trough.

The **frequency** of a wave is the number of crests generated in a time interval of 1 second. For compression waves, this would be the number of compressions generated in 1 second. The unit used for frequency is hertz, Hz. For example, a sound wave with a frequency of 400 Hz has 400 compressions generated in 1 second.

Wavelength is the distance between two successive corresponding points on a wave; for example, between two crests or two troughs.

The wave model shows the relationship between amplitude (A), wavelength (λ), period (T), frequency (f) and speed (v) that can be used to describe the behaviour of continuous waves. Table 7.2 provides a summary of the definitions of these terms and the units in which they are measured.

TABLE 7.2 Wave terminology

TERM	DEFINITION	SYMBOL	UNIT
Displacement	distance away from the rest position of a particle	x	metre (m)
Amplitude	largest distance away from the rest position that a particle moves before returning	A	metre (m)
Frequency	number of crests (or compressions) generated in 1 second	f	hertz (Hz) or (s^{-1})
Wavelength	distance between successive crests	λ	metre (m)
Period	time it takes before a wave repeats itself	T	second (s)
Speed	distance travelled per second	v	metres per second ($m\ s^{-1}$)
Wavenumber	number of wavelengths or cycles of a wave per unit length	k	reciprocal of length (m^{-1})

Graphing waves

WS

Graphing a wave in Excel

Displacement versus time and displacement versus position graphs are useful in representing wave motion of the particles in the medium. A displacement versus time graph is useful in representing the period, T, of a wave, as shown in Figure 7.11.

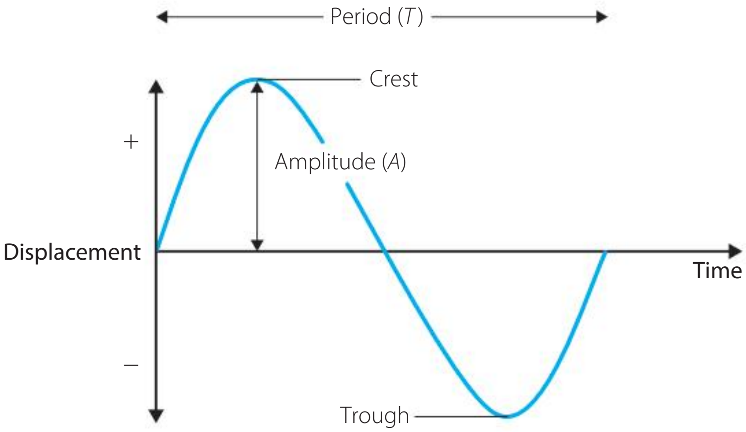


FIGURE 7.11 A displacement versus time graph of a mechanical wave represents the displacement of one particle of the medium as it experiences a wave disturbance over time.

Plotting a displacement versus distance graph enables the wavelength of the wave to be visualised, as shown in Figure 7.12. Both graphs show the amplitude of the wave, which is the displacement at a crest or the absolute value of displacement at a trough.

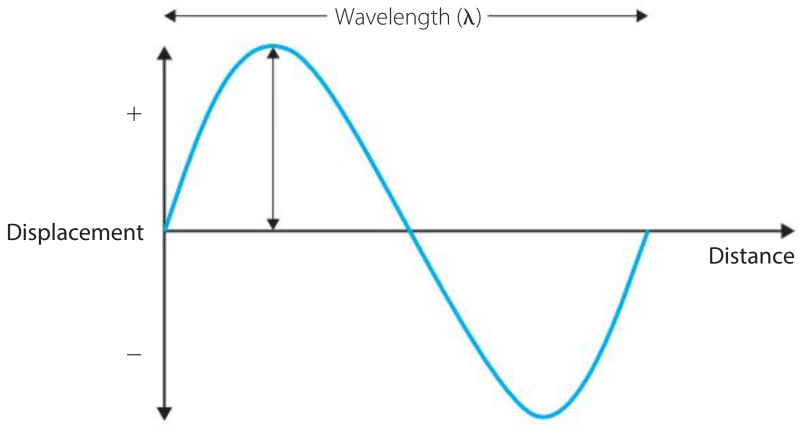


FIGURE 7.12 The displacement versus distance graph of a mechanical wave shows the displacement of all the particles of the medium at an instant in time.

Hyperphysics – transverse and longitudinal waves

Visit the Hyperphysics site to investigate transverse and longitudinal waves

Figure 7.11 and Figure 7.12 can also be used to represent the displacement versus time or displacement versus distance for longitudinal waves. The diagrams are not just images of how the wave looks, but rather plots of displacement of the medium at various times as a wave passes or plots of displacement along the length of a wave.

WORKED EXAMPLE 7.1

Numeracy

Sketch a displacement versus distance graph for a wave with a wavelength of 2.0 m and an amplitude of 40 cm. Each axis should have a label and a scale shown.

ANSWER

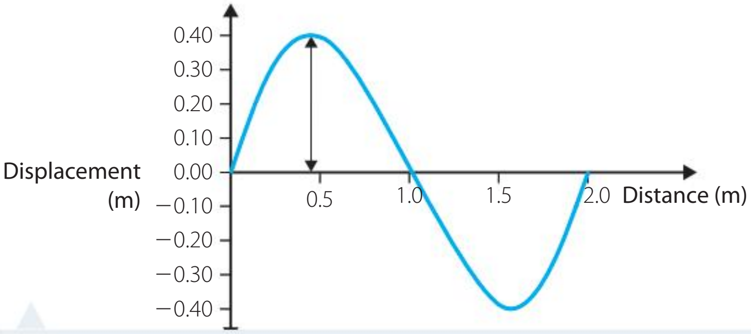


FIGURE 7.13 A displacement versus distance graph for a wave with wavelength 2.0 m and amplitude of 40 cm

LOGIC

- Draw and label axes and include an appropriate scale for each.
- Draw a smooth line that represents the information in the question.

TRY THESE YOURSELF

1 Choose appropriate axes to sketch a wave with an amplitude of 0.25 m and a period of 2.0 s.

2 Choose appropriate axes to sketch a wave with an amplitude of 10 cm and a wavelength of 50 cm.

Wavenumber

It is sometimes useful to use the **wavenumber** of a wave, which refers to the number of waves over a given unit of length, a metre.

By definition, wavenumber can be determined by:

The concept of wavenumber can be used in many other contexts. We will use it for our purposes as it appears here.

$$k = \frac{2\pi}{\lambda}$$

where the factor 2π is used as a mathematical constant.

The wavenumber can be used as a measure of the number of whole waves existing within the length of 1 m.

WORKED EXAMPLE 7.2

Find the wavenumber for a wave with a wavelength of 2.0 mm.

ANSWER	LOGIC
$\lambda = 2.0 \times 10^{-3}$ m	- Identify the relevant data in the question and convert to SI units.
$k = \frac{2\pi}{\lambda}$	- Identify the appropriate formula.
$= \frac{2\pi}{2.0 \times 10^{-3}\ \text{m}}$	- Substitute the known values, with units, into the formula.
$= 3.1 \times 10^{3}\ \text{m}^{-1}$	- Calculate the answer and express with correct significant figures and units.

TRY THESE YOURSELF

1 What is the wavenumber, k, for a wave with a wavelength of 40.0 cm?

2 Find the wavenumber of a light wave with wavelength of 650 nm.

Hyperphysics – period, frequency and amplitude

Investigate the relationships between period, frequency and amplitude.

KEY CONCEPTS

- All waves possess the characteristics of amplitude A, wavelength λ, frequency f, period T, speed v and wavenumber k.
- The displacement of a particle in the medium as a wave passes is its distance from its rest position.
- Waves can be represented using displacement versus time and displacement versus position graphs.
- Both types of graphs show the wave's amplitude.
- A displacement versus time graph shows the wave's period, T.
- A displacement versus position graph shows the wave's wavelength, λ.
- Wavenumber is the number of waves per unit length; $k = \frac{2\pi}{\lambda}$.

CHECK YOUR UNDERSTANDING 7.4

1 Outline the difference between displacement and amplitude.

2 Describe when the displacement can be equal to the amplitude of a wave.

3 Which graphical representation of a wave would you sketch if you intended to show the wavelength?

4 Is there any relationship between the wavelength of a wave and its amplitude?

5 Why is the horizontal axis of displacement versus time or displacement versus position graphs referred to as the 'rest position' of the medium?

6 State the equivalent SI unit for the unit 'hertz'.

7.5 Frequency, period, wavelength and velocity

These four characteristics of waves are related mathematically by simple but important ways that will be discussed in this section. They apply to every type of wave motion.

Relationship between frequency, *f*, and period, *T*

Frequency and period have an inverse relationship with each other for any wave. For example, if the frequency doubles, then the period halves. This relationship arises simply because of the definitions of frequency (f) = the number of whole waves per second; and period (T) = the time between each wave.

Consider a wave where two crests pass a point in 1 s. The frequency (f) of this wave is 2 Hz. The period (T) is therefore 0.5 s, as it takes 0.5 s for one whole wave to pass. Similarly, for a wave where 10 crests pass a point in 1 s, $f = 10$ Hz. As it takes 0.1 s for one wave to pass, $T = 0.1$ s.

This relationship can be written as:

$$f = \frac{1}{T}$$

WORKED EXAMPLE 7.3

Numeracy

A string is being moved up and down with a continuous vibration, taking 0.02 s to complete one full oscillation. What is the frequency of this vibration?

ANSWER	LOGIC
$T = 0.02\text{ s}$	▪ Identify the relevant data in the question.
$f = \frac{1}{T}$	▪ Identify the appropriate formula.
$= \frac{1}{0.02\text{ s}}$	▪ Substitute the known values, with units, into the formula.
$= 50\text{ Hz}$	▪ Calculate the answer with the correct significant figures and units.

TRY THESE YOURSELF

1 A wave in the ocean is observed to repeat every 20 s. Find its frequency, f.

2 The frequency of an electromagnetic wave is 5.0×10^{14} Hz. Find the time taken for one complete oscillation (the period, T) of this wave to be completed.

Relationship between velocity, frequency and wavelength

Consider a wave with a period T. In one period, the wave will move a distance of one wavelength, λ. As $v = \frac{d}{t}$ (seen in chapter 2), and as $d = \lambda$ and $t = T$, we have:

$$v = \frac{\lambda}{T}$$

As $f = \frac{1}{T}$, then:

$$T = \frac{1}{f}$$

and therefore:

$$v = \frac{\lambda}{\left(\frac{1}{f}\right)}$$

and we get the relationship:

$$v = f\lambda$$

Hyperphysics – Travelling wave relationships

Use this resource to practise examples using the wave relationships discussed.

Another example is a slinky spring that you are putting pulses into at a rate of 1 pulse every 0.5 s. This is the period of the wave (T). The distance between the tops of two adjacent pulses, the wavelength (λ), is found to be 0.6 m. It will take one period for a crest to travel a distance of one wavelength. One period is the time between identical 'snapshots'. This means that the wave in the spring is travelling 0.6 m every 0.5 s.

$$\begin{aligned} v &= \frac{\text{distance}}{\text{time}} \\ &= \frac{0.6\text{ m}}{0.5\text{ s}} \\ &= 1.2\text{ m s}^{-1} \end{aligned}$$

WORKED EXAMPLE 7.4

A sound wave with a frequency of 200 Hz is travelling through air at 340 m s^{-1}. What is its wavelength, λ?

ANSWER	LOGIC
$f = 200$ Hz; $v = 340$ m s^{-1}	▪ Identify the relevant data in the question.
$v = f\lambda$	▪ Identify the appropriate formula.
$\lambda = \frac{v}{f}$	▪ Rearrange formula.
$= \frac{340\text{ m s}^{-1}}{200\text{ Hz}}$	▪ Substitute the known values, with units, into the formula.
$= 1.70$ m	▪ Calculate the answer and express with the correct significant figures and units.

TRY THESE YOURSELF

1 A sound wave is travelling through a solid medium. The source of the sound is vibrating with a frequency of 800 Hz. The wavelength of the sound waves is 2.0 m. Find the velocity of sound in this solid medium.

2 The distance between successive crests of waves in the ocean is 50 m. A boat moves up and down once every 4.0 s. Find the velocity of these waves.

WORKED EXAMPLE (7.5)

A wave with a frequency of 440 Hz has a wavelength of 0.75 m. What is its speed?

ANSWER	LOGIC
$f = 440\ \text{Hz};\ \lambda = 0.75\ \text{m}$ $v = f\lambda$ $= 440\ \text{Hz} \times 0.75\ \text{m}$ $= 330\ \text{m s}^{-1}$	■ Identify the relevant data in the question. ■ Identify appropriate formula. ■ Substitute the known values, with units, into the formula. ■ Calculate the answer and express with the correct significant figures and units.

TRY THESE YOURSELF

1 The wavelength of a wave in the ocean is observed to be 250 m. The frequency of this wave is 0.050 Hz. Find this wave's speed.

2 What is the frequency of a sound wave with a wavelength of 1.5 m if the speed of sound is 340 m s^{-1}?

INVESTIGATION (7.5)

Modelling $v = f\lambda$ and $f = \dfrac{1}{T}$

Numeracy

Information and communication technology capability

AIM

To model the wave equations $v = f\lambda$ and $f = \frac{1}{T}$ using technology to visualise the relationships between the variables

MATERIALS

- Laptop or tablet with suitable spreadsheet software (Note: the instructions are for Microsoft Excel and may need to be modified for other spreadsheet software)

METHOD

1 Open a new spreadsheet and save it as 'Wave relationships'.

2 Enter data into worksheet 1 as shown in Table 7.3.

TABLE 7.3 Data entry for worksheet 1

	COLUMN	
ROW	A	B
1	Period, T (s)	Frequency, f (Hz)
2	0.1	=1/A2
3	0.2	
4	0.5	
5	1.0	
6	2.0	
7	5.0	
8	10.0	

»

3. Into cell B2, type the formula '=1/A2', and then copy and fill column B with this formula to cell B8.
4. Next, to produce a graph, highlight cells A1 to B8, and click on 'Insert' (on the ribbon or select from the menu).
5. Select the scatter graph with data points joined by a line. You may wish to add axis labels and a chart title.
6. Analyse the graph and describe it.
7. Copy the data from Table 7.4 into worksheet 2.

TABLE 7.4 Data entry for worksheet 2

	COLUMN	
ROW	A	B
1	Frequency, f (Hz)	Wavelength, λ (m)
2	100	=B9/A2
3	200	
4	400	
5	800	
6	1600	
7	3200	
8	6400	

8. Type a value for the speed of sound into cell B9. Use '340' to begin with, as this is the approximate speed of sound in air.
9. Copy and fill column B with the formula entered into cell B2.
10. Repeat steps 4 to 6 to insert a scatter graph as before.

RESULTS

Record your analysis and description beneath each graph on the spreadsheet.

ANALYSIS OF RESULTS

Analyse the graph and describe the relationship between f and λ for waves with the same speed. Identify the speed of the wave you have graphed.

DISCUSSION

1. Explain how the investigation can show how characteristics of waves are related.
2. Enter a new value in cell B9 (try '2000' to begin with). Describe the change observed in the graph produced from the data in the table. Is the shape of the graph similar? Why?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

When waves change velocity

The speed of a wave is a property of the medium. The frequency is determined by how quickly the source of the wave is oscillating or vibrating. The wavelength therefore will depend on both the velocity, v, and the frequency, f. If a wave changes speed because the nature of the medium, or the medium itself changes, how does the wave change?

In chapter 8, we will investigate what happens to a wave when its speed, v, changes. When this occurs, it is the wavelength, λ, that also changes while the frequency, f, remains constant.

KEY CONCEPTS

- A displacement versus time graph of a wave shows the period, T, of the wave.
- A displacement versus distance graph of a wave shows the wavelength, λ, of the wave.
- The frequency of a wave is the inverse of its period, i.e. $f = \frac{1}{T}$ and vice versa.
- The velocity of a wave is the product of its frequency and wavelength: $v = f\lambda$.

CHECK YOUR UNDERSTANDING

7.5

1 Using a sketch, show how the amplitude of a wave can be represented on a displacement versus position graph.

2 Sketch a displacement versus position graph of a wave with a wavelength of 8 cm and amplitude of 3 cm. Label this as wave A. On the same axes, sketch wave B with half the amplitude and the same wavelength as wave A.

3 **a** Explain why a displacement versus time graph does not show the wavelength of a wave.

b What extra information about the wave would you need to be able to calculate its wavelength?

4 With reference to the appropriate equation, show why the wavelength of a wave must decrease if the wave slows down but its frequency remains the same.

5 Calculate the speed of a wave that has a frequency of 2.00×10^3 Hz and a wavelength of 7.50×10^{-1} m. Give your answer with the correct number of significant figures and correct units.

6 Find the frequency of a wave that has a period of 2.00×10^{-2} s.

7 CHAPTER SUMMARY

- Mechanical waves travel through a medium.
- Energy is transported from one place to another by a mechanical wave.
- The particles in the medium oscillate as a mechanical wave passes.
- The medium itself does not travel with the wave.
- Examples of mechanical waves are sound, water and seismic waves.
- Longitudinal waves involve a back-and-forth motion of the particles in the medium as the wave passes.
- Transverse waves involve an up-and-down or side-to-side motion of the particles in the medium as the wave passes.
- Transverse waves reflect upright at a free end, and inverted at a fixed end.
- Electromagnetic waves can be modelled as electric and magnetic fields oscillating perpendicular to each other.
- Electromagnetic waves do not need a medium.
- In a vacuum, all electromagnetic waves travel at the speed of light: 3.0×10^8 m s^{-1}.
- The range of electromagnetic waves make up the electromagnetic spectrum.
- All waves possess the characteristics of amplitude A, wavelength λ, frequency f, period T, speed v and wavenumber k.
- The displacement of a particle in the medium as a wave passes is its distance from its rest position.
- Waves can be represented using displacement versus time and displacement versus position graphs.
- Both types of graphs show the wave's amplitude.
- A displacement versus time graph shows the wave's period, T.
- A displacement versus position graph shows the wave's wavelength, λ.
- Wavenumber is the number of waves per unit length; $k = \frac{2\pi}{\lambda}$.
- A displacement versus time graph of a wave shows the period, T, of the wave.
- A displacement versus distance graph of a wave shows the wavelength, λ, of the wave.
- The frequency of a wave is the inverse of its period, i.e. $f = \frac{1}{T}$ and vice versa.
- The velocity of a wave is the product of its frequency and wavelength: $v = f\lambda$.

7 CHAPTER REVIEW QUESTIONS

Review quiz

1 Give three examples of mechanical waves.

2 What do mechanical waves require in order for them to travel?

3 What do you call waves that travel through Earth's rock layers that are the result of earthquakes or explosions?

4 **a** Identify the type of mechanical wave that requires a solid medium to move through.

  b By referring to the type of motion within the medium, explain why these mechanical waves cannot travel through liquids or gases.

5 The velocity of a transverse wave in a string stretched between two points is 240 m s^{-1}. The end of the string is vibrating at a frequency of 480 Hz. What is the wavelength of the wave in the string?

6 The speed of sound in water is more than four times the speed of sound in air; water is much denser than air. Why does sound travel faster in water than in air?

7 Classify the following examples of waves or pulses as mechanical or electromagnetic and as transverse or longitudinal.

  a P wave

  b Rayleigh wave

  c Waves produced by the wind on the water

  d Visible light from the Sun

  e Waves produced by the vibrating air column of a trombone

8 Two waves travelling at the same speed are observed to have different wavelengths. Explain, using an appropriate equation, how it is also known that the frequencies of these two waves must also be different.

9 The unit of frequency, Hz, has the dimension of s^{-1}. Show that the formula $T = \frac{1}{f}$ gives the correct units for the period, T.

10 What is the wavenumber for a wave with a frequency of 550 Hz and a velocity of 1100 m s^{-1}?

11 In a diagram, show an example of a single pulse in a spring being reflected at:

a a free end.

b a fixed end.

12 As a pulse travels down a spring, its amplitude is observed to decrease. Does the speed of this pulse also decrease? Explain your answer.

13 The upper range of human hearing is 20 kHz. If sound travels at 340 m s^{-1}, what is the:

a wavelength of this sound?

b wavenumber of this sound?

14 A seismic wave travelling through rock at 4.00×10^3 m s^{-1} has a frequency of 0.2 Hz. What is its wavelength?

15 An electromagnetic wave is travelling through a vacuum.

a Is there any particle vibration involved in such a wave motion? Explain.

b Describe the nature of the oscillations involved in such a wave.

16 Using the example of sound waves travelling down a person's ear canal, describe how the medium does not travel with a wave but only vibrates as the wave passes.

17 In a vacuum and in air, light travels at 3.0×10^8 m s^{-1}. Humans can see ranges of wavelengths from about 400 nm to 700 nm. What is the frequency range of human sight?

18 Many science fiction movies show a spaceship in outer space moving past with a roar of sound. Explain why this is not correct.

19 How do you know that energy has been transferred when sound travels from a speaker to a person's ear?

20 Given the speed of light is 3.0×10^8 m s^{-1} and that the wavelength of red light is 620 nm, calculate the frequency of red light.

8 Wave behaviour

INQUIRY QUESTION

How do waves behave?

OUTCOMES

Students:

- explain the behaviour of waves in a variety of situations by investigating the phenomena of:
  - reflection
  - refraction
  - diffraction
  - wave superposition (ACSPH071, ACSPH072)
- conduct an investigation to distinguish between progressive and standing waves (ACSPH072)
- conduct an investigation to explore resonance in mechanical systems and the relationships between: CCT
  - driving frequency
  - natural frequency of the oscillating system
  - amplitude of motion
  - transfer/transformation of energy within the system (ACSPH073) ICT N

Shutterstock.com/Grezova Olga

Our understanding of the behaviour of waves is crucial to being able to manipulate them and to use them for the benefit of our technological society. Understanding how waves reflect, refract, diffract and superpose has allowed us to apply this knowledge in many ways. Some of these applications include better camera lenses, building telescopes that can receive signals from just after the Big Bang, and detecting toxic substances at extremely low concentrations.

In this chapter, wave behaviour will be investigated and the differences between progressive and stationary waves will be explored. Resonance and its applications will also be discussed, where the motion of systems as diverse as a child's swing to a collapsing road bridge will be explained.


Alamy Stock Photo/ILYA GENKIN

FIGURE 8.1 The Parkes radio telescope uses the behaviour of waves to probe and gather information from deep space.

8.1 Reflection of waves

We can represent the direction of travel of a series of waves by a single line, a ray, which is drawn perpendicular to the wavefronts. This simplification is used in the study of geometrical optics where we only consider the ray or rays. Chapter 10 deals with rays in detail.

There are many examples in everyday life of waves being reflected. We have all looked into a mirror, and similarly, we have all seen our reflected image. We can observe water waves 'bouncing' back off a wall in a pool or from a sea wall. Echoes are experienced when sound waves reflect back to the observer off a wall or cliff face. Air traffic controllers and ships use radar, the reflection of microwaves, for navigation and position finding. Even your microwave oven uses reflection as the microwaves bounce around off the metal walls until they are eventually absorbed by the food being heated.

Figure 8.2 represents how a wave, depicted here as a single line, or **ray**, behaves when it reflects off a surface. A construction line, the **normal**, perpendicular to the reflecting surface, is shown. All angles are measured from the ray to the normal, not to the reflecting surface. The **incident ray** has an **angle of incidence**, θ_i, while the reflected ray has an **angle of reflection**, θ_r.

Reflection also works when the direction of the wave is reversed.

The law of **reflection** can be stated as 'the angle of incidence, θ_i, equals the angle of reflection, θ_r'. That is,

$$\theta_i = \theta_r$$

When a wave is reflected, neither the speed of the wave nor its shape are altered. Curved mirrors may distort an image, but blue light is always reflected as blue and a 'coo-ee!' echoes back as the same sound.

The reflection of waves can be used for a variety of purposes.

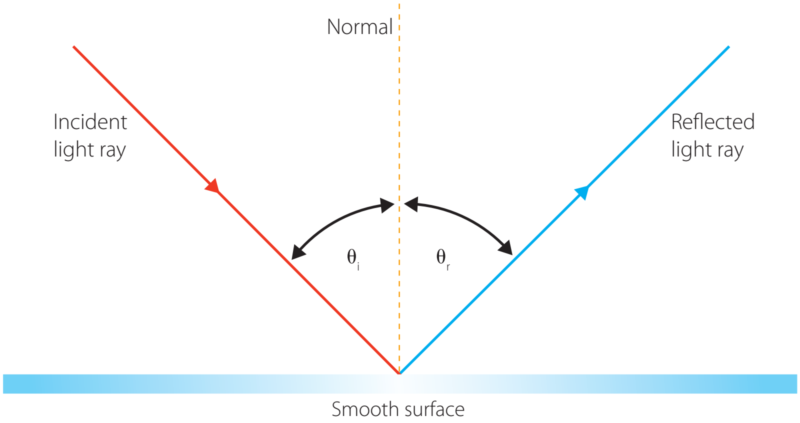


FIGURE 8.2 The law of reflection, showing angles of incidence and reflection, which are measured against a perpendicular construction line – the normal

Reflection off plane surfaces

Flat mirrors are an example of reflection off **plane surfaces**. The image is reflected without distortion. These mirrors are used in everyday applications such as bathrooms and in shops. At each point along the mirror, a normal will be

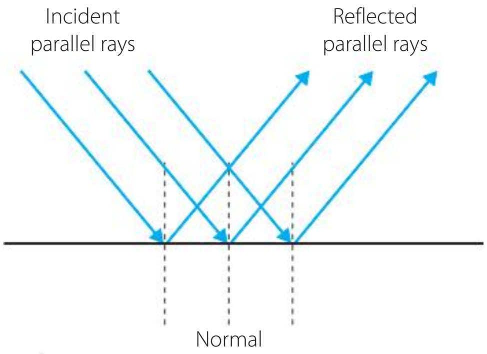


FIGURE 8.3 A plane mirror has normals parallel to each other.

parallel to any other normal, as shown in Figure 8.3. This results in a reflected image being seen in the mirror that has the same shape as the original object, as parallel waves are reflected parallel.

Reflection off concave surfaces

Figure 8.4 shows an application of reflection using a **concave surface**. Car headlights need to concentrate a narrow beam of light. The headlight globe is placed at the focal point facing a parabolic concave reflecting surface. The focal point is the place where incoming parallel rays of light will intersect when they are reflected. It is also the place where a source of light will produce rays that reflect off the mirror to form parallel rays of light, as shown in Figure 8.4. The light is then reflected ahead of the vehicle in a narrow beam.

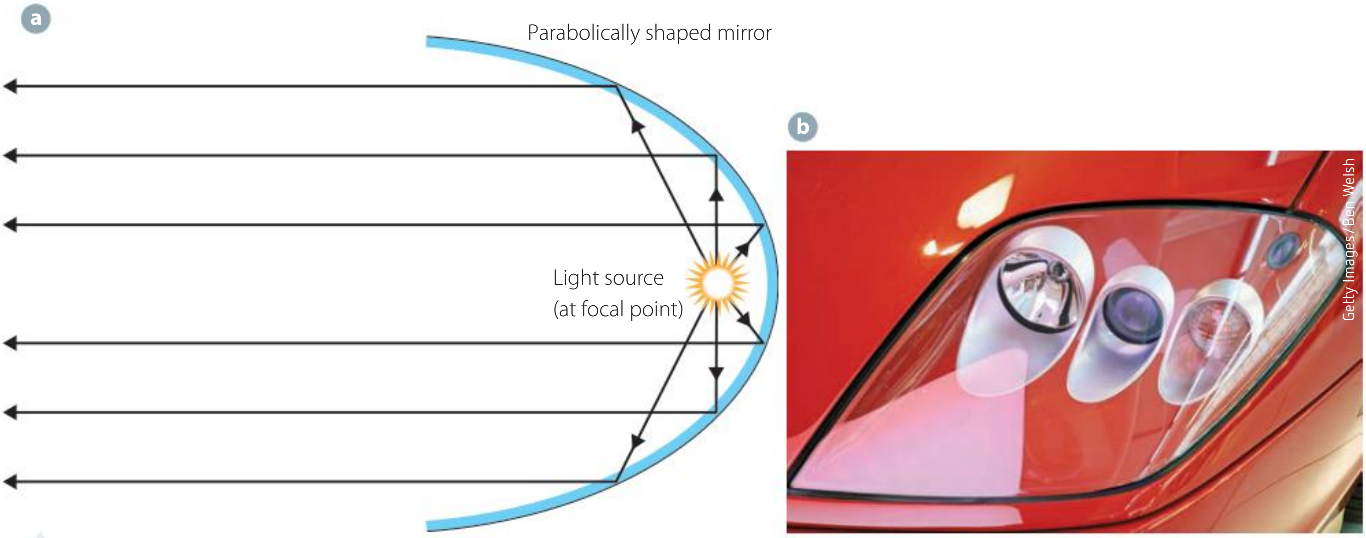


FIGURE 8.4 **a** A diagram showing that a headlight globe placed at the focal point of a parabolic mirror has the light reflected in a narrow beam ahead of the vehicle; **b** A car headlight constructed using the same geometry

FIGURE 8.5 A typical parabolic satellite dish with the antenna placed at the focal point

Satellite dishes, as shown in Figure 8.5, are designed to reflect the incident microwaves towards the focal point, where the antenna itself is placed. In this way, very weak signals can be collected over a large area. The dish is a parabolic curve. Curves that have a circular shape do not have a single focal point.

WS
Reflection of waves

INVESTIGATION 8.1

Applications of reflection

AIM

To investigate the application of different shapes used to reflect waves

MATERIALS

- Paper
- Pen
- Pencil
- Ruler
- Protractor

METHOD

1. On paper, trace a curve that is parabolic in shape and another curve that is an arc of a circle. Each curve should be about half a page in size. (A suitable parabolic curve can be found on the internet and then drawn by placing a piece of paper on your device's screen.)
2. Using a protractor, construct six normals to the surface of the curves at different locations.
3. Following the example shown in Figure 8.4, construct incident rays that are parallel – one ray for each normal you have drawn. The rays should meet the curves at a point where you have drawn a normal.
4. Obeying the law of reflection, draw the reflected ray for each of the incident rays. Remember to measure the incident and reflected angles from the ray to the normal.
5. Compare the two diagrams you have just drawn.

RESULTS

Write down your comparison beneath the diagrams.

DISCUSSION

1. Outline the differences you observed between the reflection from the parabola and that from the arc.
2. Discuss the suitability of each curved shape for reflecting waves.
3. Explain why parabolic shapes are preferred in many applications.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Reflection off convex surfaces

Mirrors that have a slightly **convex surface** are used to give a wider view. Applications include side mirrors on cars, which allow the driver to see what otherwise would be a blind spot in their vision. Some shops install convex mirrors at strategic locations so that the shopkeeper can see what is happening in all corners of the shop. Traffic mirrors are also useful in situations where drivers need to see a wider view, as shown in Figure 8.6.

iStock.com/Leadinglights

FIGURE 8.6 A convex mirror gives a wide field of view

INVESTIGATION 8.2

Reflection of light from a variety of surfaces

AIM

To investigate the reflection of light from plane, concave and convex surfaces

MATERIALS

- Ray box kits
- Power packs
- Protractor
- Pencil, ruler and paper

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Power packs use a 240 V mains power supply.	Devices plugged into 240 V mains power should be kept well away from water.
The laboratory may be darkened.	Trip hazards such as bags should be placed away carefully.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Set up a ray box so there are three or more parallel rays of light coming from the end of the ray box.
2. Trace the incident rays on each of the following mirror surfaces:
    - Plane mirror
    - Concave mirror
    - Convex mirror

   To do this, place the apparatus on paper and use a ruler and pencil to trace the rays. Draw arrows on the rays to show the direction of travel of the light.
3. At each point on the diagrams wherever an incident ray strikes the mirror, construct a normal to the surface of the mirror.
4. Using a protractor, measure the angle of incidence, $\theta_{i,}$ and the corresponding angle of reflection, $\theta_{r,}$ for each point. Verify that the law of reflection has been obeyed.

RESULTS

The results of this investigation should be kept or photographed for your files.

ANALYSIS OF RESULTS

Suggest ways in which the accuracy of this investigation could be improved.

DISCUSSION

1. In which cases were the reflected rays:
    a. parallel?
    b. diverging?
    c. converging?
2. Relate the way in which parallel rays of light are reflected to the applications of each of the shapes investigated.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- When waves reflect off a surface, they obey the law of reflection: $\theta_i = \theta_r$.
- Reflection can occur using plane, convex or concave surfaces.
- Applications of reflection include mirrors, satellite dishes, radio telescopes and traffic mirrors.

CHECK YOUR UNDERSTANDING

8.1

1 State the law of reflection.
2 What is the meaning of an 'incident' ray?
3 Identify one use of a parabolic mirror.
4 An echo is heard exactly 2.0 s after a sound is made. How far away is the surface where the sound was reflected?
5 'Reflection of light allows us to see objects that do not emit their own light.' Use a simple diagram to explain this statement.
6 Using a sketch showing incident and reflected rays of light and $\theta_i = \theta_r$, show why a mirror only needs to be half your height for you to see your whole body reflected in it.

8.2 Refraction of waves

When waves move from one medium into another, their speed may change. Light slows down when entering water after travelling through air. It slows down even further when travelling through glass. Waves can also change speed when the nature of the medium changes. Water waves travel more slowly in shallow water than in deep water. If the wave enters the new medium at an angle other than perpendicular, the change in its speed will cause it to change direction. This change of direction is called **refraction**.

The refraction of waves has many uses. Lenses used in glasses, cameras and telescopes, as well as the lens and cornea in the human eye, use refraction to bend light to form an image. A consequence of refraction, **total internal reflection**, allows fibre optics to carry information encoded in beams of light. The internet relies on such information transfer.

When a wave is refracted, the speed changes but the frequency remains constant. The frequency of a wave is determined by the vibrations of the source of the wave. Therefore, the wavelength of the wave must also change in the same ratio as the change in the wave's speed. When a wave changes speed when entering a new medium at an angle other than perpendicular to the boundary, the wave changes direction (as discussed in the next section).

Refraction of sound waves

Temperature differences in the same medium will also cause refraction because the density of the medium changes with temperature. For example, sound travels faster in warm air.

If a wave meets the interface at right angles, it will not change direction but its speed and wavelength will change. If it meets the interface at any other angle, its direction will change, as shown in Figure 8.7 (page 214). It is important to recall from chapter 7 that when a wave changes speed, the frequency of the wave does not change, only the wavelength.

The relationship $v = f\lambda$ can be used to calculate the change in the wavelength of the wave when its speed changes.

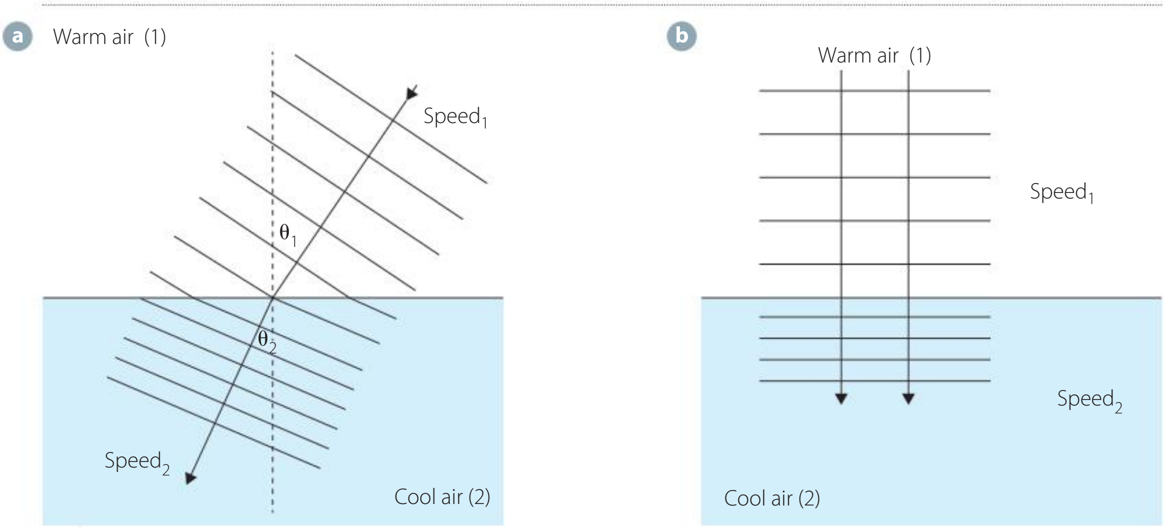


FIGURE 8.7 **a** A sound wave is refracted (changes direction) when it meets the boundary between two layers at an angle other than a right angle. **b** When a wave enters perpendicular to the surface, there is no change in direction. Speed and wavelength both change, but frequency remains constant.

WORKED EXAMPLE 8.1

Sound waves from a source vibrating at 680 Hz are travelling through air at 340 m s^{-1}.

1 What is the wavelength of these sound waves?

2 The sound waves enter cooler air and slow to 320 m s^{-1}. What is the wavelength of the sound waves now?

ANSWERS	LOGIC
1 $f = 680$ Hz; $v = 340$ m s^{-1}	▪ Identify the relevant data in the question.
$v = f\lambda$	▪ Identify the appropriate formula.
$\lambda = \frac{v}{f}$	▪ Rearrange the formula.
$= \frac{340 \text{ m s}^{-1}}{680 \text{ Hz}}$	▪ Substitute the known values, with units, into the formula.
$\lambda = 0.500$ m	▪ Express the answer with the correct significant figures and units.
2 $f = 680$ Hz; $v = 320$ m s^{-1}	▪ Identify the relevant data in the question (note: frequency remains constant).
$v = f\lambda$	▪ Identify the appropriate formula.
$\lambda = \frac{v}{f}$	▪ Rearrange the formula.
$= \frac{320 \text{ m s}^{-1}}{680 \text{ Hz}}$	▪ Substitute the known values, with units, into the formula.
$\lambda = 0.471$ m	▪ Express the answer with the correct significant figures and units.

TRY THESE YOURSELF

1. Water waves with a frequency of 0.20 Hz are travelling at 8.0 m s^{-1}. They enter deeper water and speed up to 10 m s^{-1}.
   - **a** What is the wavelength of the waves originally?
   - **b** What is the new wavelength of these water waves after they speed up?
2. Sound waves with a frequency of 915 Hz are travelling through air at 340 m s^{-1}. They then enter water and speed up to 1500 m s^{-1}. What are the original and new wavelengths for the sound waves?

Refraction of sound waves is evident in situations where the sound wave passes through a medium with gradually varying properties. This commonly occurs in the atmosphere where there is a gradual change in temperature with height. On occasions, the air higher up is warmer than air closer to the surface. This is known as a temperature inversion, as it is the opposite to the usual situation. Temperature inversions usually occur in winter at night when the ground cools quickly, cooling the air immediately above it.

During the day, the air close to the ground is warmed by the ground and the temperature decreases with height, creating a temperature gradient (Figure 8.8).

Sound travels faster in warmer air, so the edge of the wave in the warmer air travels faster than the edge of the wave in the cool air. This causes a gentle refraction (bending) of the wave away from the ground when the warm air is lower down. The reverse can happen at night when the ground is cooler than the air above it. The implication is that on nights when the ground is cooler than the air above it, you can clearly hear sounds coming from a distant source.

Demonstrations of the refraction of waves

Further information on the refraction of sound waves within a temperature gradient

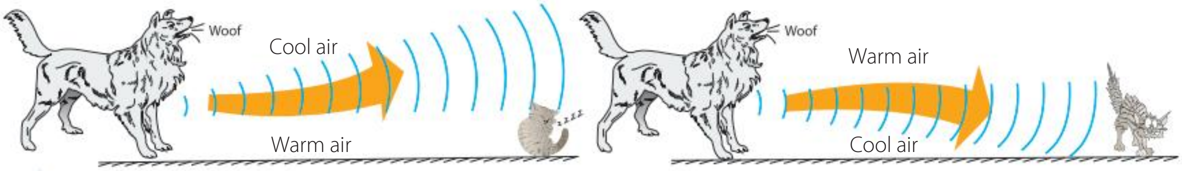


FIGURE 8.8 Refraction of sound in reverse temperature gradients

Wavefronts

When ocean waves approach a shoreline at an angle other than 90°, the **wavefronts** bend towards the shore. A wavefront is a line perpendicular to the direction of the propagation of the wave along which the wave is at the same part of its cycle, or oscillation. Wavefronts can be seen as lines of waves in the ocean, as shown in Figure 8.9. Here, wavefronts can be seen bending around a rocky island. The change in their direction is in part due to the waves' speed being slower in the shallower water closer to the island. This causes refraction, or bending, towards the island.

FIGURE 8.9 Ocean waves showing wavefronts bending around a rocky island

Refraction and wavefronts

INVESTIGATION 8.3

Observing everyday refraction

AIM

To observe the refraction of light in everyday examples

MATERIALS

- Reading glasses or magnifying lens
- Beaker of water
- Stick or rod
- Power pack
- Ray box kit with lenses

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Power packs have 240 V of mains electricity.	Devices plugged into 240 V mains power should be kept well away from water.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Place a stick in a beaker of water and observe from different angles how the stick appears to be bent.
2. Hold a pair of reading glasses or a magnifying lens at arm's length. Observe the way in which light is bent as it passes through the lens.
3. Trace rays of light coming from a ray box as they pass through a convex lens and a concave lens. Note how the rays tend to bend towards the normal where they strike the surface of the lens in the first instance.
4. Make a permanent record of your observations by taking photos of the investigation.

RESULTS

Record your results and observations. Traces of the light rays and lenses should be kept with your notes.

DISCUSSION

In each case observed, describe exactly where the light is being refracted.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- In different mediums, a wave can have different speeds.
- When a wave changes speed, its frequency remains constant and its wavelength changes because its speed has changed; for example, $v = f\lambda$ – the velocity of a wave equals the product of its frequency and wavelength.
- If a wave enters a different medium at an angle other than 90°, it will change direction and refract.

CHECK YOUR UNDERSTANDING

8.2

1. A swimmer opens their eyes underwater and sees a blurry image of a towel on the side of the pool. The colour of the towel is the same seen from underwater as it is from above the water. If light waves slow down in water, how can this be so?
2. Explain why a spoon standing in a glass of water appears to be bent.
3. Describe two examples of applications of refraction.
4. A hunter is hoping to spear a fish in a shallow pond, but does not aim directly at where the fish is observed by the hunter. Explain why this may still lead to success for the hunter.
5. A wave enters a different medium and slows down to half its original speed. Describe what effect this has on the wave's frequency and wavelength.

8.3 Diffraction of waves

Diffraction is the spreading of waves into a space beyond a gap or an obstacle. Figure 8.10 shows how a sound can still be heard even if the source is not in a direct line with your ears.

If you are sitting in a room reading, you can still be aware of a radio or TV in another room. In this situation, the sound will have both reflected and bent around corners and/or obstacles to reach your ears. The phenomenon of waves bending around corners or obstacles is known as diffraction. The waves spread out into the whole region as shown in Figure 8.11.

When you are listening to a loudspeaker, the higher pitched sounds (shorter wavelengths) will be best heard in front of the speaker. This is because higher pitched sounds have shorter wavelengths and are diffracted less than the longer wavelengths. The lower pitched sounds (longer wavelengths) will be heard in front and to the sides of the speaker.

The shorter wavelengths will be more directional – they will pass through openings with less bending or spreading than the longer wavelengths. The amount of diffraction of sound depends on the ratio of λ to w, where w is the slit width. This is shown in Figure 8.12. The smaller the opening for a given wavelength, the more diffraction there will be and the greater the angle of spread. Also, the longer the wavelength for a given opening, the more diffraction there will be and the greater the angle of spread.

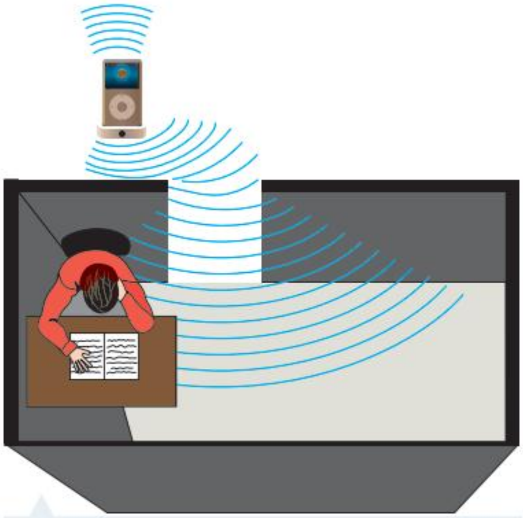


FIGURE 8.10 Simplified diagram of sound waves diffracted through an open door

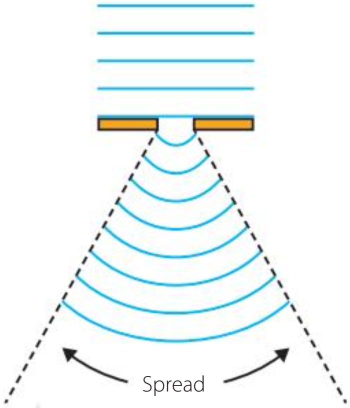


FIGURE 8.11 Waves spread out after passing through a gap. This is called diffraction.

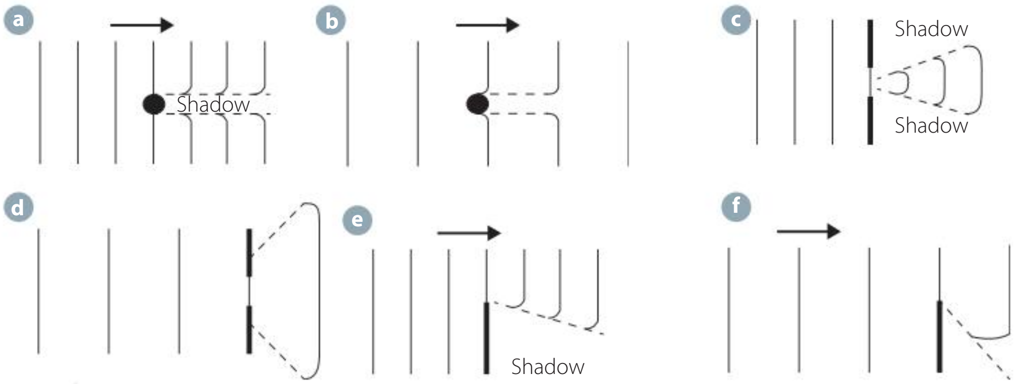


FIGURE 8.12 Diffraction of water waves: **a** Short wavelength around an object; **b** Long wavelength around an object; **c** Short wavelength through a gap; **d** Long wavelength through the same gap; **e** Short wavelength around the edge of a barrier; **f** Long wavelength around the edge of the same barrier

For substantial diffraction to occur, the wavelength must be similar to the size of the aperture (opening) through which it passes. Loud music on a car system always sounds much the same outside the car no matter what is being played because only the low frequency sound waves are diffracted effectively through a window. The wavelength of low frequency sound waves (for example, where $f = 100$ Hz) can be calculated using $v = f\lambda$, as done previously. Higher frequency sound waves with frequencies of 1000 Hz will have much shorter wavelengths and will therefore not undergo diffraction as much when they pass through a car window. When sound waves are diffracted, there is no change to the speed, wavelength or frequency.

WORKED EXAMPLE 8.2

1 Calculate the wavelengths for sound waves with frequencies of:

- **a** 100 Hz
- **b** 1000 Hz

(Use the speed of sound as $340\ \text{m s}^{-1}$.)

2 Compare these wavelengths to the width of a typical car window (0.40 m).

ANSWERS	LOGIC
1 **a** $v = 340\ \text{m s}^{-1}$; $f = 100\ \text{Hz}$	▪ Identify the relevant data in the question.
$v = f\lambda$	▪ Identify the appropriate formula.
$\lambda = \frac{v}{f}$	▪ Rearrange the formula.
$= \frac{340\ \text{m s}^{-1}}{100\ \text{Hz}}$	▪ Substitute the known values, with units, into the formula.
$\lambda = 3.40\ \text{m}$	▪ Calculate the answer and express with correct significant figures and units.
b $v = 340\ \text{m s}^{-1}$; $f = 1000\ \text{Hz}$	▪ Identify the relevant data in the question.
$v = f\lambda$	▪ Identify the appropriate formula.
$\lambda = \frac{v}{f}$	▪ Rearrange the formula.
$= \frac{340\ \text{m s}^{-1}}{1000\ \text{Hz}}$	▪ Substitute the known values, with units, into the formula.
$\lambda = 0.340\ \text{m}$	▪ Calculate the answer and express with correct significant figures and units.
2 Therefore, sound with a low frequency is diffracted through a car window more than a higher frequency sound as its wavelength is much longer than the size of the opening. The wavelength of the higher frequency sound is about the same length as the size of the opening.	

TRY THESE YOURSELF

1 A sound wave is passing through an opening with a diameter of 3.0 m. What is the frequency of the sound with the same wavelength as the width of the opening?

2 A sound has a frequency of 3.0 kHz. What is the width of an opening that has the same size as the wavelength of this sound?

Diffraction and echolocation

Echolocation is a technique used by some mammals that use sound echoes instead of light reflection to help them ‘see’ in the dark or in poor light. Bats use ultrasound to locate their prey. The frequencies used range from 20 kHz up to 200 kHz. Sound waves can only reflect from an object if its wavelength is similar to or less than the size of the object. If a sound wave has a long wavelength, it is completely diffracted around the object and there is no reflection. This puts a limit on the smallest size prey a bat can hunt.

WORKED EXAMPLE 8.3

If the echolocation frequency of a bat is 100 000 Hz, what is the smallest prey it can locate? (Assume the speed of sound in air is 340 m s^{-1}.)

ANSWER	**LOGIC**
$v = 340\ \text{m s}^{-1}; f = 100\ 000\ \text{Hz}$	▪ Identify the relevant data in the question.
$v = f\lambda$	▪ Identify the appropriate formula.
$\lambda = \frac{v}{f}$	▪ Rearrange the formula.
$= \frac{340\ \text{m s}^{-1}}{100\ 000\ \text{s}^{-1}}$	▪ Substitute the known values, with units, into the formula.
$\lambda = 0.003\,40\ \text{m}$	▪ Calculate the answer.
$\lambda = 3.40 \times 10^{-3}\ \text{m}$	▪ Express the answer with the correct significant figures and units.
That is 3.40 mm, a mosquito-sized meal.	

TRY THESE YOURSELF

1 Dolphins also use echolocation to catch their prey, using frequencies of about 20 000 Hz. Given that the speed of sound in salt water is 1531 m s^{-1}, what is the smallest-sized fish a dolphin could locate?

2 What frequency would be needed for an echolocation device that is designed to detect an object 1.0 mm in size?

KEY CONCEPTS

- Diffraction is the spreading of waves into a space beyond a gap or obstacle.
- The greater the wavelength compared to the object or aperture, the greater the spreading.
- When waves are diffracted, there is no change to the speed, wavelength or frequency of the wave.

CHECK YOUR UNDERSTANDING 8.3

1 In a game of hide and seek, a girl hiding behind a large tree can hear her friend approaching from the other side of the tree. Using a sketch, show how diffraction of sound is involved in this scenario.

2 A sound wave with a frequency of 30 kHz is sent out by a dolphin in sea water. What is the smallest object that the dolphin’s echolocation could detect?

3 The wavelength of visible light ranges from approximately 400×10^{-9} m to 800×10^{-9} m. Explain why the diffraction of light is not normally observed in everyday situations.

4 AM radio waves have frequencies of around 1000 kHz. FM radio is around 100 MHz, while digital TV transmissions have frequencies around 600 MHz. Of these three, AM radio is least affected by hills and buildings in the way. By calculating the wavelengths for each of the three transmissions, show why this is so by referring to the phenomenon of diffraction.

8.4 Wave superposition

Waves of the same nature (either longitudinal or transverse) and in the same medium can pass through each other. Their displacements add algebraically as they interact, then resume their original shape. This is **superposition**. Usually superposition results in a messy combination. At any point, **constructive interference** (larger amplitude) or **destructive interference** (reduced amplitude) may occur, depending on the two waves.

Figures 8.13 and 8.14 shows examples of how constructive and destructive interference can occur when two waves coincide and overlap. Two identical waves with their crests coinciding will add to form a resultant wave with twice the amplitude of the original waves. If the two waves arrive at the same place with a crest from one wave coinciding with a trough from the other, the waves' displacements will cancel completely. Such a situation is shown in Figure 8.14.

If two waves with different wavelengths coincide and overlap, the resultant wave has alternating regions of constructive and destructive interference, as shown in Figure 8.15.

FIGURE 8.13 Two waves constructively interfering, having crests coinciding with crests and troughs coinciding with troughs

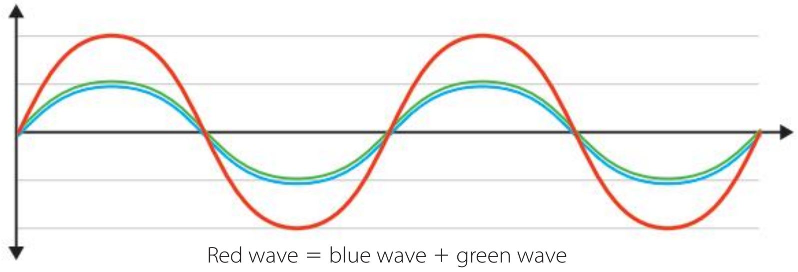
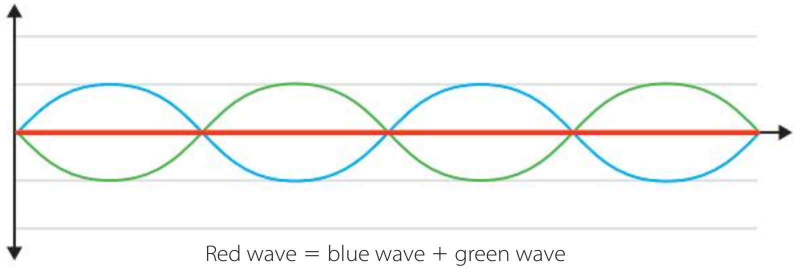
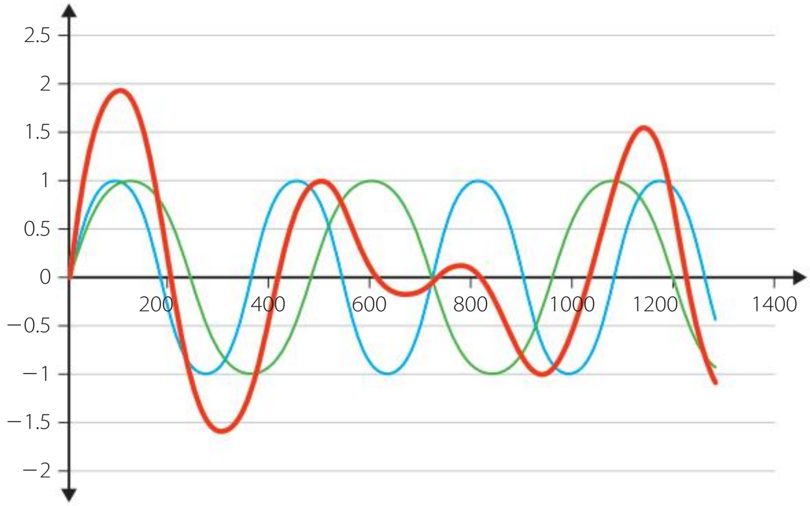


FIGURE 8.14 Two waves destructively interfering, having crests coinciding with troughs



Superposing waves in Excel

FIGURE 8.15 Two waves with different wavelengths coinciding to produce a resultant wave showing both constructive and destructive interference



Superposition and interference of waves

Animations by Pennsylvania State University showing superposition and interference of waves

Acoustics and the principle of superposition

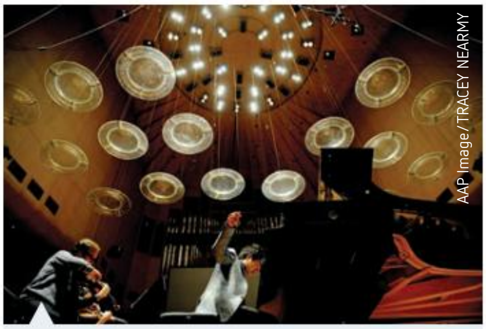

AAP Image/TRACEY NEARMY

FIGURE 8.16 The reflective rings above the stage of the Concert Hall in the Sydney Opera House

The field of acoustics and acoustic engineering predicts and analyses the behaviour of sound waves inside buildings. Buildings that are constructed for concerts, speeches or the theatre have appropriate sound qualities. Figure 8.16 shows the circular rings placed above the stage of the Concert Hall in the Sydney Opera House. These rings were added after the Concert Hall was opened in the 1970s and it was found that the acoustics (sound properties) of the building made it difficult to hear the orchestra playing on the stage. The rings reflect some of the sound back towards the audience. Without the rings, sound waves were undergoing destructive interference after reflecting off several different surfaces, making the orchestra sound muffled and without the clarity expected.

Nowadays, computer simulations used by acoustic engineers help prevent such mistakes from occurring. The superposition of sound waves forms the basis for this interesting field of study.

INVESTIGATION 8.4

Destructive and constructive interference in the classroom

AIM

To experience how destructive and constructive interference can occur with sound in a classroom

MATERIALS

- 2 signal generators (e.g. phones using apps to play the sounds through suitable speakers)
- Optional: smartphone app that measures sound level intensity

METHOD

1. Place the two speakers playing the same sine wave sound with exactly the same frequencies at least 3 m apart at the front of the room. The ideal frequency is around 600 Hz.
2. Commencing in a position equidistant from the two speakers, move slowly to one side of the room or the other, listening very carefully for changes to the sound from the speakers.
3. Measure how far sideways you need to move between two points that both have constructive interference occurring. A smartphone app that measures sound level intensity could be used to assist with these observations.

RESULTS

Record your results regarding the variation in the loudness of the sound heard and the distance moved in your notes. To do this, construct a scale map of the room and mark the points where constructive and destructive interference is observed along the line that you have moved.

ANALYSIS OF RESULTS

1. Calculate the wavelength, λ, of the sound played by the speakers (use $v_{\text{sound}} = 340\ \text{m s}^{-1}$).
2. Compare the wavelength of the sound waves with the distance you needed to move between two points with constructive interference.

»

Examples of reverberation

Use these examples to increase your understanding of reverberation.

DISCUSSION

1 Explain why destructive and constructive interference was occurring between the two sources of sound.

2 Use a diagram to explain your results obtained for the distance between two points with constructive interference, making references to interference of waves and the wavelength of the sound waves calculated above.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

INVESTIGATION 8.5

The principle of superposition

AIM

To demonstrate the principle of superposition

MATERIALS

- Cathode ray oscilloscope (CRO) with two inputs
- 2 microphones
- Signal generators or sound sources (such as guitars)
- Camera, mobile phone, laptop or tablet with videoing capability

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW WILL YOU MANAGE THESE RISKS TO STAY SAFE?
Electrical equipment is dangerous if water is present.	Devices plugged into 240 V mains power should be kept well away from water.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1 Connect the microphones to the CRO and have two separate sound sources (either signal generators or musical instruments such as guitars) generating sounds of slightly different frequencies being fed into the two microphones.

2 Adjust the CRO vertical scale so that each microphone's input has approximately the same vertical scale when displayed separately.

3 Now change the display mode to 'add' so that the signal from each microphone is added to form one waveform.

4 Observe the alternating periods of constructive and destructive interference both with your ears and on the CRO display.

RESULTS

Record your observations by videoing the changing display on the CRO screen or by taking snapshots of the screen at different times.

»

DISCUSSION

1. How did the waveform displayed on the CRO change over time?
2. How did the waveform on the CRO relate to the sound that you heard?
3. How is the principle of superposition involved with these observations?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Musical instruments and their characteristic sound

A single musical instrument's sound wave may be the result of many component sound waves that contribute to the recognisable sound that is heard. For example, a saxophone sounds different to a clarinet due to the different component sounds produced, even when they play the same note. This is despite both instruments using a vibrating reed (thin piece of bamboo) to generate the sound. The vibrations of the instruments themselves cause sound waves with different, unique shapes. As well as having different shapes, waves with different frequencies are generated that add, or superpose, to make the one resultant sound waves that we hear. The overall shape is the result of many superpositions and is unique to each instrument, making them recognisable.

A simple investigation can be performed that explains how superposition is the result of the addition of the component waves' displacements along the waves.

INVESTIGATION 8.6

Constructing a resultant wave using the principle of superposition

AIM

To apply the principle of superposition to produce a resultant waveform from two component waves

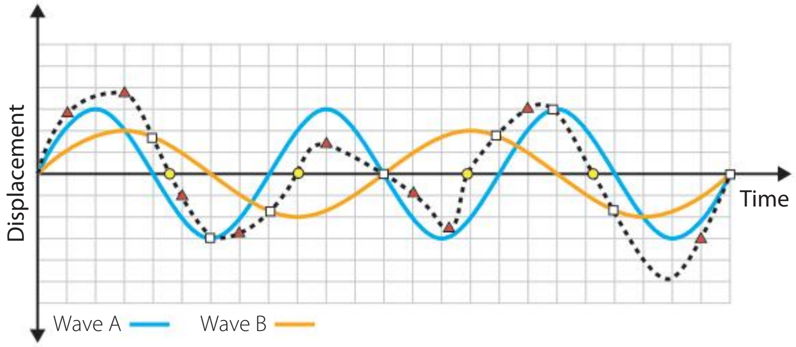
MATERIALS

- Grid paper (preferably 10 mm × 10 mm grids)
- Pencil
- Ruler

METHOD

1. On the grid paper, draw a graph of a wave commencing at the origin with wavelength $\lambda = 8$ cm and amplitude $A = 3$ cm. Label this wave A.
2. On the same axes, draw a graph of a wave commencing at the origin with $\lambda = 12$ cm and $A = 2$ cm. Label this wave B.
3. For each grid line, plot the sum of the two waves' displacements from the horizontal axis.
4. Join the dots you have plotted with a curved dashed line. An example is shown in Figure 8.17.

FIGURE 8.17 A dashed line representing the resultant of superposition of two component waves



RESULTS

Label the curved dashed line you have constructed as A + B. This is the resultant wave.

ANALYSIS OF RESULTS

1 Measure the amplitude of the resultant wave A + B.

2 Label regions of wave A + B where constructive and destructive interference have occurred between the original wave A and wave B.

DISCUSSION

1 Describe the shape of the resultant wave A + B compared with the original component wave A and wave B.

2 Compare the size of the resultant wave A + B in the regions where it was formed by constructive and by destructive interference of the original component wave A and wave B.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Further uses of superposition of sound waves will be discussed in chapter 9.

KEY CONCEPTS

- The principle of superposition states that when two or more waves coincide, their displacements add to form a resultant wave.
- When the crests and troughs from two component waves arrive at the same time, constructive interference results.
- When a crest from one wave coincides with a trough from the other wave, destructive interference results.

CHECK YOUR UNDERSTANDING

8.4

1 When two waves overlap, the resultant wave has a smaller amplitude than either component wave. What type of interference is this?

2 In order to find the resultant wave's displacement when two waves overlap, what calculation must be performed?

3 After two waves have overlapped and superposition has occurred, does the speed of the two component waves change?

4 Would it be possible for two sounds to be played simultaneously so that no sound would be heard?

8.5 Standing waves

A **standing wave** (often called a stationary wave) is one that does not appear to be travelling through the medium or along the length of the string or spring. The particles in the medium will move in their transverse or longitudinal vibration mode but the wave does not appear to go anywhere. If you shake waves into a string that is fixed at the other end, the forwards-moving waves and the reflected waves will interfere and superpose along the string. Usually the effect is messy, but at just the right frequency, a fixed pattern of maximum and minimum displacements will appear. A standing wave has been formed.

Standing waves can be created in stretched strings, springs and in air columns in pipes. This is used extensively in musical instruments to make resonant sounds.

A standing wave such as that shown in Figure 8.18 is created when two waves of the same frequency and amplitude, but travelling in opposite directions, exist in a medium. The length of the string, spring or air column must also have a relationship with the wavelength of the wave. Figure 8.18 shows the length of the string being the wavelength of the wave in the string.

The **nodes** are points where destructive interference always occurs. At these points, at any moment in time, the amplitudes of the two waves are always the same magnitude but in opposite directions. When a crest of one wave passes through this point, a trough of the same size from the reflected wave travelling in the opposite direction is also passing this point. When the displacement due to one wave is half the amplitude, the displacement due to the other wave is also half the amplitude, but in the opposite direction, and so on. Whatever the displacement due to one wave, the displacement due to the other is the same size but the opposite direction. Superposition means that these two displacements add to give a total displacement of zero at any time. This is why a particle at a node does not move, even though waves are travelling through the point.

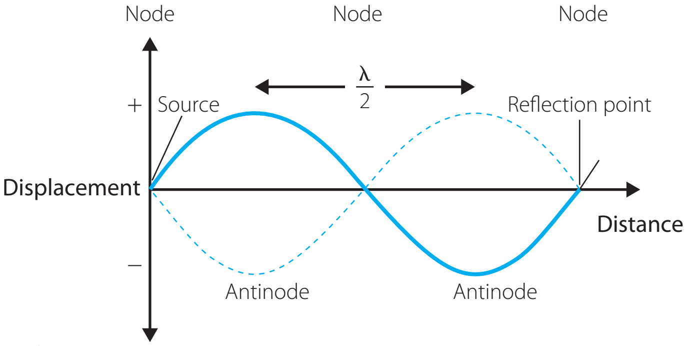


FIGURE 8.18 A standing wave in a stretched string fixed at both ends showing the nodes and antinodes. The solid line represents the string's displacement at an instant in time, and the dotted line the string's displacement half a period $\left(\frac{T}{2}\right)$ later.

At the **antinodes**, the particles move up and down repeatedly and reach the maximum displacement possible. The maximum displacement occurs when two crests (or two troughs) meet at this point to give a displacement equal to the sum of the amplitudes of the individual waves.

The frequency with which the particles in the medium move up and down is the same as the wave frequency.

In between nodes and antinodes, the particles oscillate up and down with the same frequency, but with smaller amplitudes, to produce the pattern shown in Figure 8.18.

Standing wave patterns are always characterised by an alternating pattern of nodes and antinodes. The distance between consecutive nodes or between consecutive antinodes along the string or spring is half a wavelength, $\frac{\lambda}{2}$.

Despite their appearance, it is important to keep in mind that when standing waves are observed, they are the result of two waves that are travelling in opposite directions. The standing wave is a result of the superposition of the two travelling waves.

INVESTIGATION 8.7

Using standing waves to calculate the speed of waves in a spring

AIM

To observe the effect of stretching a spring on the speed of waves in a spring by observing and measuring standing waves

HYPOTHESIS

Stretching a spring further increases the speed of the waves travelling in the spring.

MATERIALS

- Slinky springs
- Metre ruler
- Stopwatch
- Optional: digital camera or recording device (e.g. phone, tablet or laptop)

METHOD

1. Extend a slinky spring along the ground and fix or hold one end still. Have another person at the free end move the spring side to side and observe a travelling wave pass down the spring. Adjust the stretch of the spring to ensure that the wave reaches the end of the spring.
2. Again with the other end of the spring held fixed, make the side-to-side motion regularly and gradually increase the frequency until the waves in the spring appear to become stationary. You will observe nodes and antinodes appear along the spring that do not appear to be travelling along the spring.
3. Maintain the frequency with which the spring is being moved while a second person uses a stopwatch to time 10 full side-to-side motions. Record the result.
4. While step 3 is being performed, a third person measures the distance between two successive nodes in the spring using a metre ruler. Note that the fixed end of the spring is also a node. Record this measurement.
5. If possible, have another person use a digital camera or recording device to record this investigation.
6. Repeat the above method after stretching the spring further.

RESULTS

As stated in the method, record the time taken for 10 full side-to-side motions to be completed and record the distance between two successive nodes.

ANALYSIS OF RESULTS

1. Calculate the period, T, of the waves in the spring by dividing the time taken for 10 full side-to-side movements by 10.
2. Calculate the frequency of the waves using $f = \frac{1}{T}$.
3. Using the fact that the distance between two successive nodes is equal to $\frac{\lambda}{2}$ (see Figure 8.18), calculate the wavelength, λ, of the waves in the spring.
4. Finally, using $v = f\lambda$, calculate the speed of the waves in the spring.
5. Repeat these calculations and analysis for the results taken with the spring stretched further.

Standing waves in a string

This interactive simulation can be used to generate standing waves in the string.

DISCUSSION

1. Given that there is only one source of wave motion in the spring, explain how the reflection of waves plays a part in the production of standing waves in the spring.
2. Discuss why the amount of stretch in the spring affects the speed of waves in the spring.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the hypothesis of this investigation.

KEY CONCEPTS

- Travelling waves can be seen moving through the medium.
- Standing waves appear to be stationary (standing still); however, they are the result of the superposition of two waves – one being the transmitted wave and the other the reflected wave.

CHECK YOUR UNDERSTANDING 8.5

1 Give an example of a standing wave and describe what it looks like.

2 Could a standing wave be produced from just one wave travelling in a medium without reflection? Explain your answer.

3 Do reflected waves move with the same speed as the incident waves in a medium?

4 What is the name of the point on a standing wave where the displacement is:

  a a maximum?

  b always zero?

5 Outline a method that could be used to find the speed of a wave in a spring by observing standing waves in the spring.

6 Why must there be a node at a fixed end of a spring?

8.6 Resonance in mechanical systems

Resonance in bottles

If you blow across the mouth of a bottle, as shown in Figure 8.19, the air in the bottle is made to vibrate and you will hear a note. The frequency of this note is determined by the dimensions of the bottle. The sound results from the free vibrations of the air in the bottle.

If you purse your lips and blow through them, but don't whistle, you will hear the sound of the rushing air. This sound is made up of waves of many different frequencies. This sort of sound is called 'white noise' in analogy to 'white light', which is composed of many frequencies of light.

When you blow air across the top of a bottle, you are providing waves of many different frequencies to the air column inside the bottle. Most of these waves transfer energy inefficiently to the air column. But waves of one particular frequency, the natural or resonance frequency, transfer energy very efficiently and set up a standing wave in the bottle. The frequency of this standing wave is the frequency of the note you hear.

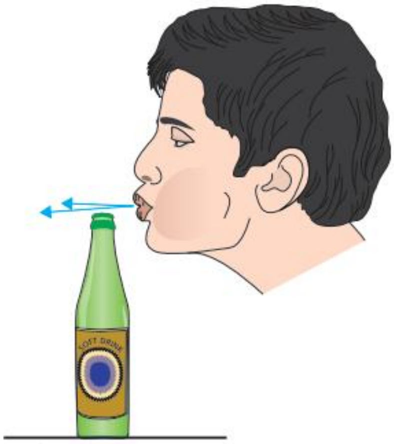


FIGURE 8.19 Blowing across an open bottle

Free vibrations

Free (natural) vibrations occur when an object is displaced from its equilibrium position and then left to vibrate by itself. Restoring forces will cause the vibrating objects to accelerate back towards their rest position.

When a tuning fork is struck, the prongs vibrate about their mean position. Elastic restoring forces strongly pull the prongs back and forth. The tuning fork vibrates at its natural frequency. This phenomenon can be observed in guitar strings, organ pipes, wind instruments, drums, pendulums and masses hanging on the end of springs (think bungee jumping!) – all these have natural frequencies.

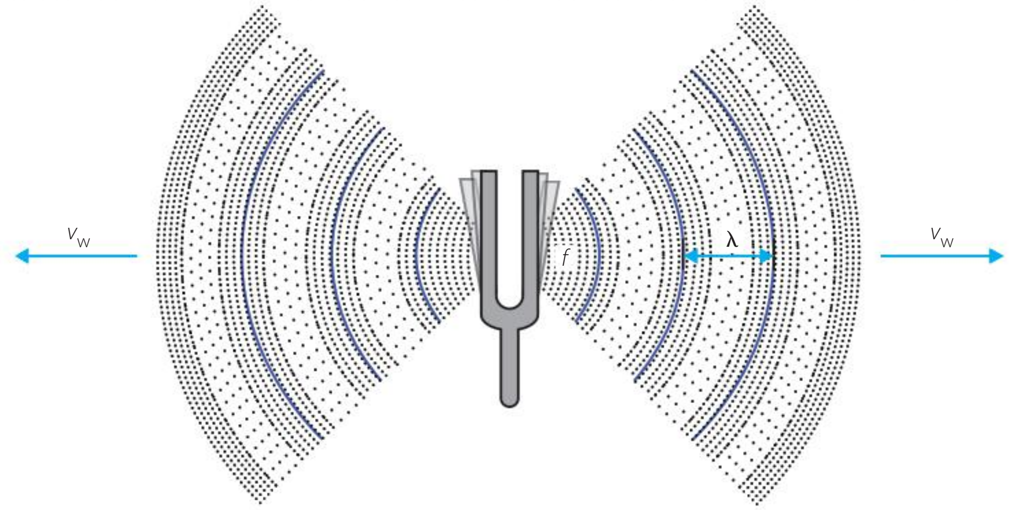


FIGURE 8.20 Sound waves produced from the freely vibrating tuning fork with a frequency f, speed v and wavelength λ

The frequency (and period) of vibration is determined by the properties of the vibrating object. For example, a plucked guitar string vibrates at different natural frequencies depending on its length, mass per unit length, and the tension in the string.

The only energy driving a free vibration is the initial energy. In time, these vibrations die away because of friction – energy transfers to the surroundings.

Forced vibrations

A **forced vibration** occurs when one vibrating object makes another object vibrate. If a vibrating tuning fork is struck on a rubber stopper, it emits a low-intensity sound that can only be heard with difficulty. However, if the same vibrating tuning fork is held with its shaft on a wooden bench or tabletop, the sound is heard throughout a classroom. Why is this?

The sound is louder when the fork is in contact with the bench because the fork causes the bench to vibrate with the same frequency. The benchtop has a larger vibrating area than the tuning fork. Consequently, these forced vibrations disturb a greater volume of air and produce a louder sound.

Resonance

Resonance occurs when the frequency of the forced vibration matches the natural (free) vibration frequency of the object. The source of the forced vibration enhances the free vibrations of the object so that they are amplified. The amplitude of resonant vibrations can become very large and, in certain circumstances, cause damage to the vibrating object.

When sound waves are projected towards a wine glass with a frequency that matches the resonant frequency of the wine glass, it is possible to shatter the glass without physically touching it.

Resonance of vibrating objects can be observed in the laboratory using a strobe light to 'freeze' the motion of the vibrating object.

Energy transfer within a system

When the frequency of the forced vibration coincides with the natural frequency of the system, energy is transferred with maximum efficiency. For these examples, a standing wave is produced. As we have seen, this phenomenon is called resonance. The energy from the speaker in front of the wine glass is transferred to the wine glass with maximum efficiency if the sound from the speaker has the same frequency as the natural resonance of the glass.

INVESTIGATION 8.8

Resonance

Critical and creative thinking

AIM

To observe objects vibrating at their resonant frequency

MATERIALS

- Wine glass
- Beaker of water
- Sonometer or guitar string
- Strobe light with adjustable frequency
- Frequency analyser (or suitable app on a smartphone)
- Digital camera or recording device (e.g. phone, tablet or laptop)

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
A strobe light may induce seizures in some susceptible people.	Ensure that no class member is susceptible to seizures or has a history of epilepsy.
Wine glasses are made from delicate glass that, if broken, will form very sharp pieces.	Ensure that all present are wearing enclosed shoes and that instructions on how to handle broken glass are given prior to this investigation.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Moisten your finger (using water from the beaker) and then rub your finger gently around the rim of the wine glass until the glass begins to 'sing' at its resonant frequency.
2. Use the frequency analyser or smartphone app to measure this resonant frequency.
3. Darken the laboratory. Set the strobe light to the same frequency as the resonant frequency of the wine glass.
4. Repeat step 1 with the strobe light illuminating the wine glass as it 'sings'. Adjust the frequency of the strobe light until the glass appears to move slowly. Observe carefully and record your observations on a phone, tablet or laptop.
5. Pluck a guitar or sonometer string and again use the frequency analyser to measure the frequency of the resonant vibration.
6. Set the frequency of the strobe light to this measured frequency.
7. In the darkened laboratory, illuminate the vibrating guitar or sonometer string and adjust the frequency of the strobe until the string is observed to be moving from side to side slowly. Again, record the motion on a suitable available device.
8. Repeat these steps for the glass and then for the string, but this time set the strobe light frequency to twice the measured resonant frequencies of the glass and string. Observe carefully.

RESULTS

1. Observe where nodes and antinodes occur within the glass and along the string when they vibrate at their resonant frequencies.
2. Discuss whether the vibrations observed are examples of travelling waves or standing waves.

»

»

DISCUSSION

1 Discuss why the vibrating objects appear to 'freeze' or move only slowly when the strobe light frequency is set to match the frequency of the sound produced.

2 Explain why doubling the strobe light frequency can also produce 'freezing' of the vibrating objects.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Energy transfer between oscillating systems

An animation of two connected vibrating oscillating systems transferring energy between them. Note that when both systems have the same natural (or free vibration) frequency, the transfer of energy is most efficient.

Role of resonance in the destruction of buildings in earthquakes

This site contains animations that explain the role of the resonant frequency of buildings and the damage they are subjected to during earthquakes.

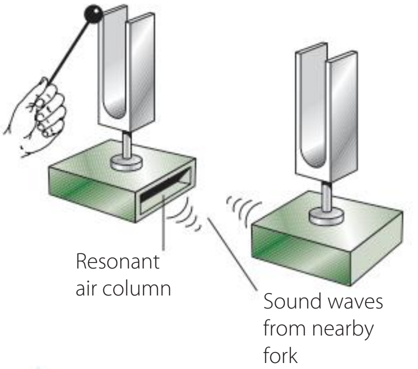


FIGURE 8.21 Energy is transferred from one tuning fork and causes the other to resonate

Figure 8.21 shows the transfer of energy from one tuning fork to another. The tuning forks are mounted on sounding boxes made so that they will vibrate with a resonant frequency equal to the tuning fork's resonant frequency. Energy is transferred from the struck tuning fork and its sounding box to the adjacent one, causing it to vibrate as well. This happens if both tuning forks have the same resonant frequency.

When the frequencies of the two tuning forks are different, the same effect is not observed. Resonance in the second tuning fork is not successful as the forced vibration is not the same as the free vibration frequency of the second tuning fork.

KEY CONCEPTS

- The amplitude of vibration of a resonating object will increase dramatically.
- Resonance will only occur when the frequency matches the natural frequency.
- When an object is resonating, energy is being transferred very efficiently from the object oscillating to the object being oscillated.

CHECK YOUR UNDERSTANDING 8.6

1 What is the difference between a free and a forced vibration?

2 Give two examples of:

   a free vibrations.

   b forced vibrations.

3 a Explain how blowing across the top of a bottle can produce a loud, clear note.

   b What is this phenomenon called?

4 What conditions are required for a standing wave to form?

5 The vibrating air column in the bottle in Figure 8.19 (page 227) is a standing wave. Why is it called a standing wave?

6 How could you use one tuning fork to force another tuning fork to vibrate at the second tuning fork's natural frequency?

7 When a wave is reflected off a beach and meets an incoming wave, a larger wave is formed as they pass through each other. Sketch diagrams to show the two waves:

   a before they interact.

   b in the middle of interacting.

   c after interacting.

INVESTIGATION 8.9

Energy transfer in resonating systems

Information and communication technology capability

Numeracy

AIM

To observe the transfer of energy from one resonating system to another

MATERIALS

- Several tuning forks mounted on sound boxes, two of which need to have the same frequency
- Ruler
- Audio signal generator or a signal generator app on a smartphone
- Speaker that can be wired into the signal generator or smartphone

METHOD

1. Use the ruler to measure the length of the sound box from the opening to the back of the box.
2. Place the two identical tuning forks on sounding boxes facing each other, as shown in Figure 8.21.
3. Strike one tuning fork and listen carefully to the other tuning fork. Touch the other fork gently to feel for vibrations. Record your observations.
4. Repeat with two tuning forks of different frequencies and record your observations after listening and touching carefully.
5. Set the frequency of the audio generator or smartphone app to the frequency of one of the tuning forks (usually stamped on the tuning fork itself).
6. With the speaker from the audio generator facing the opening of the sound box, play a sound of a single frequency into the sound box. Observe by listening and gently touching the tuning fork.
7. Vary the frequency of the source of the sound so that it is:
    - double,
    - triple, and then
    - five times the frequency of the tuning fork.

    Observe the strength of the vibration of the tuning fork at these frequencies.

RESULTS

Record your observations in a suitable table with headings 'Frequency of sound', 'Observations of tuning fork' and 'Wavelength of sound'.

ANALYSIS OF RESULTS

1. Using $v = f\lambda$, calculate the wavelength, λ, of the sound waves for each frequency used in the investigation. Assume that the speed of sound in air, $v_{\text{sound}} = 340 \text{ m s}^{-1}$.
2. Compare the wavelengths calculated with the physical length of the sound box (as measured from the opening to the back of the box).

DISCUSSION

1. In which cases did energy transfer occur most efficiently?
2. Is there a mathematical pattern observed that relates the wavelengths that cause resonance to the length of the sound box? If so, what is this relationship?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

8 CHAPTER SUMMARY

- Waves reflect off surfaces obeying the law of reflection. The angle of incidence equals the angle of reflection: $\theta_i = \theta_r$.
- Refraction of waves occurs when they change speed when they enter a different medium at an angle other than 90°.
- Diffraction occurs when a wave passes through a narrow opening and then spreads out.
- As the width of the opening decreases, diffraction becomes more pronounced.
- At any instant when waves are passing through each other, constructive or destructive interference can occur.
- Constructive interference occurs when two waves passing through each other result in a wave of greater amplitude.
- Destructive interference occurs when two waves passing through each other result in a wave of lesser amplitude.
- Standing (stationary) waves result when two continuous waves travelling in opposite directions and having the same frequency and amplitude superpose.
- Nodes are points on a standing wave where no displacement occurs.
- Antinodes are points on a standing wave where maximum displacement occurs.
- Objects will vibrate at their natural (free) frequency.
- Resonance occurs when an external driving vibration transfers energy to another system that has the same natural (free) frequency of vibration.

8 CHAPTER REVIEW QUESTIONS

Review quiz

1 What is a 'normal' to a surface?

2 State the law of reflection.

3 With the aid of a sketch, show why a parabolic mirror is used for satellite receiver dishes and radio telescopes.

4 Give two applications of refraction of waves.

5 What property of waves causes refraction when a wave changes medium?

6 In what medium does light travel the fastest?

7 The noise from a roadway can be heard from behind a very thick brick wall. What property of waves is likely to be responsible for this?

8 Using sketches to illustrate your answer, show how the width of a single opening compared with the wavelength of a wave determines the way in which the wave will diffract after it moves through the opening.

9 Describe what is meant by 'destructive interference' and 'constructive interference'.

10 Light is a wave, and waves can undergo destructive interference when they coincide.

  a Would it be possible for one light wave to completely cancel out another light wave?

  b What properties would need to be equal for the two light waves for this to happen?

11 In certain seats in a concert hall, the sound from the orchestra sounds softer than it should. Using sketches, show how reflection and destructive interference of the sound waves could cause this effect.

12 Is it possible to make a standing wave using one source of one wave? If so, identify what other wave behaviour must occur.

13 Using a child's swing as an example,

  a describe what is meant by:

    i driving frequency.

    ii natural frequency.

    iii the amplitude of the motion involved.

  b describe the most effective way to transfer energy from another system to the swing.

14 Figure 8.22 shows two sound waves represented as transverse waves. Use the principle of superposition to plot the shape of the resultant wave.

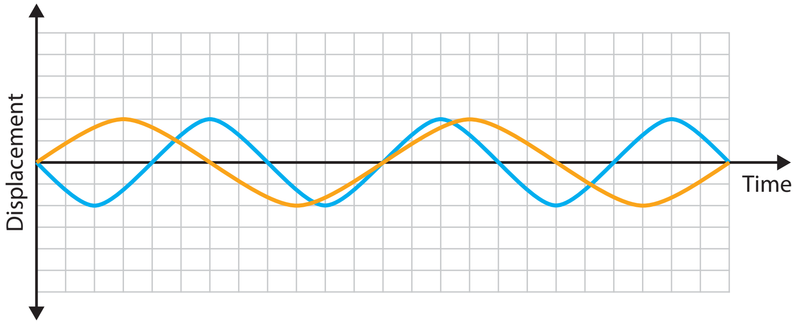


FIGURE 8.22

15 Surfers are familiar with the concept of 'sets' of waves – when several waves larger than normal appear. In between sets there is usually a period of comparative calm. Waves at the beach can be produced by more than one source from areas of wind hundreds of kilometres away in the ocean. Each source of waves may produce waves with slightly different frequencies. Explain the phenomenon of wave 'sets' using information learned in this chapter.

16 It is possible to make a wine glass 'sing' by rubbing a damp finger around the rim of the glass at just the right speed. Explain what is happening here to make the glass 'sing'.

17 It is possible for a wine glass to be broken by having someone loudly sing a certain note while holding the glass in front of their mouth. How could this happen?

18 Describe how energy can be transferred between two vibrating systems that have the same natural frequency.

19 **a** Copy and complete the diagrams shown in Figure 8.23 by tracing the rays after they have reflected from the concave surface of each shape shown.

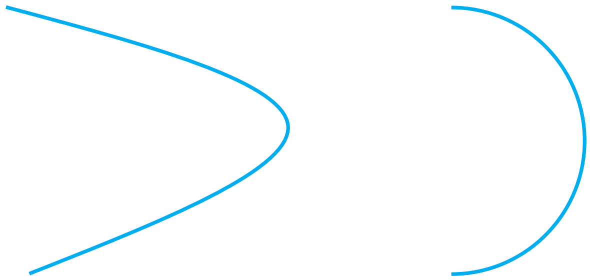


FIGURE 8.23

b Compare and contrast the nature of the reflected rays and the applications that each of the reflecting shapes could have.

20 Figure 8.24 shows parallel waves approaching a slit in a barrier. Copy the diagrams and show the shape of the wavefronts after they have passed through the slit. Summarise the differences in the two situations.

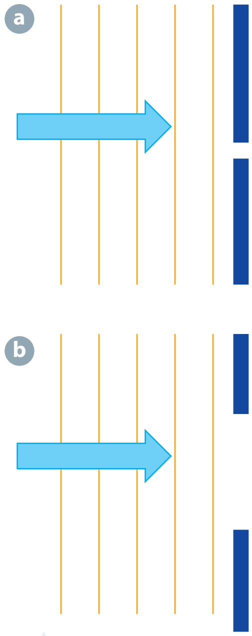


FIGURE 8.24

21 Discuss the application of acoustic design when a concert hall is being planned compared with an assembly hall.

22 'Musical instruments must be constructed with resonance in mind.' Evaluate this statement.

9 Sound waves

INQUIRY QUESTION

What evidence suggests that sound is a mechanical wave?

OUTCOMES

Students:

- conduct a practical investigation to relate the pitch and loudness of a sound to its wave characteristics
- model the behaviour of sound in air as a longitudinal wave
- relate the displacement of air molecules to variations in pressure (ACSPH070)
- investigate quantitatively the relationship between distance and intensity of sound
- conduct investigations to analyse the reflection, diffraction, resonance and superposition of sound waves (ACSPH071)
- investigate and model the behaviour of standing waves on strings and/or in pipes to relate quantitatively the fundamental and harmonic frequencies of the waves that are produced to the physical characteristics (e.g. length, mass, tension, wave velocity) of the medium (ACSPH072) ICT N
- analyse qualitatively and quantitatively the relationships of the wave nature of sound to explain: CCT
    - beats $\left(f_{\text{beat}} = \left|f_2 - f_1\right|\right)$
    - the Doppler effect $f' = f\dfrac{\left(v_{\text{wave}} + v_{\text{observer}}\right)}{\left(v_{\text{wave}} - v_{\text{source}}\right)}$

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9780170409063

The wave model is a powerful tool that can be used to explain observations of the behaviour of sound.

Sound is all around us. It is rarely that we can stop and hear no sounds. Sound waves are mechanical waves, with wave properties that have been explored in chapters 7 and 8. They require a medium to travel through and have speeds that vary considerably according to the arrangement of the particles in the medium and the density of the medium. Being waves, sound can reflect, refract, diffract and convey energy from one place to another. We perceive sounds with associated **pitch** (how high or low on the musical scale the sound is) and **loudness**. Human hearing has a rather limited range of frequencies that can be detected compared with some other mammals. We can, however, hear sounds over a great range of intensities.

Over the millennia, musical instruments have been produced that utilise the wave characteristics of sound. From a simple flute to the huge pipe organs found in cathedrals and concert halls around the world, the science of music is a fascinating study. This chapter presents some of the aspects of sound and uses the wave model to explain them.

iStock.com/Jokerbee12

FIGURE 9.1 A pipe organ, one of the largest musical instruments

9.1 Pitch and loudness

Prior knowledge

When we hear a sound, two characteristics are immediately obvious: the pitch of the sound, sometimes referred as being how high or low the 'note' sounds, and the loudness of the sound. It is useful to be able to visualise the sound waves producing the sounds we hear so that they can be analysed and measured.

An oscilloscope shows how electrical signals change with time on a screen. A microphone converts the energy of sound waves into electrical signals. These can be sent to an oscilloscope. The vertical axis on the oscilloscope represents the pressure variations in the sound waves, and the horizontal axis represents the time over which these changes take place.

We can use an audio frequency generator, a loudspeaker and an oscilloscope to investigate the waveforms of sound waves at the same time as they are being heard. This enables us to link what we hear to the characteristics of the sound waves.

To do this, a sound of known frequency is produced by the audio generator. This can be heard through the loudspeaker and, using a microphone, viewed on the oscilloscope. The audio generator vibrates the loudspeaker. Some of the same sound that causes our ears to vibrate also causes vibrations in the microphone. The resulting electrical signal is fed into the oscilloscope. The oscilloscope displays an apparently transverse wave. The vertical axis represents particle movement at a point, and the horizontal axis represents time. The oscilloscope displays a graph of amplitude versus time – it is not a snapshot of the wave at a particular time.

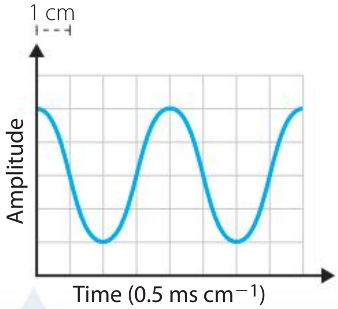


FIGURE 9.2 Sound wave as it appears on an oscilloscope

The **time-base scale** (horizontal axis) enables the period of the wave to be determined (Figure 9.2). The pitch of the note increases with frequency.

For the waveform in Figure 9.2, one complete wave on the oscilloscope is completed in 4 cm. If the time-base scale of the oscilloscope is set on 0.5 ms cm^{-1}, then each centimetre on the scale on the screen is equivalent to half a millisecond. The period (T) of the wave is 4 cm × 0.5 × 10^{-3} s cm^{-1}, and therefore equal to 2.0 × 10^{-3} s. As $f = \frac{1}{T}$, frequency $f = \frac{1}{2.0 \times 10^{-3}}$, which is 500 Hz.

Frequency can also be calculated as the number of waves passing per time taken:

$$f = \frac{n}{t}$$

Figure 9.3 depicts how changes in frequency and changes in amplitude would appear on the display as changes to the waveform. Increased frequency causes the waves to appear more 'bunched up'. An increase in the amplitude causes the waves to become taller.

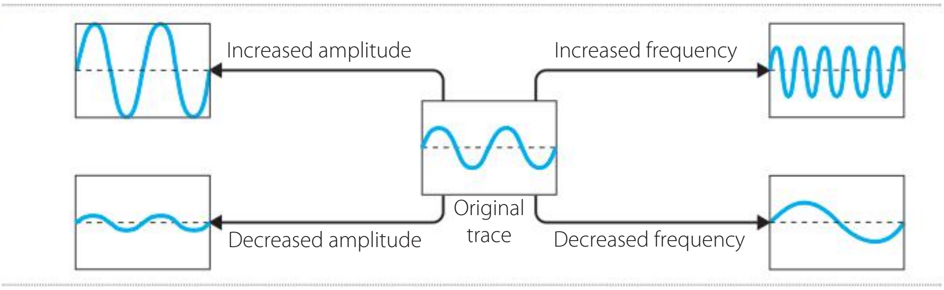


FIGURE 9.3 Effects of changes to frequency and amplitude of a wave on the waveform displayed on an oscilloscope

What does a change in the frequency of a sound wave sound like to us? What does changing the amplitude of a sound wave do to the sound we are listening to? The next investigation will explore these questions.

INVESTIGATION 9.1

Pitch, frequency, loudness and amplitude of sound waves

AIM

To investigate the relationship between pitch and frequency as well as loudness and amplitude of sound waves

Write an appropriate hypothesis for this investigation. Ideally it should read as an 'If … then …' statement (or statements).

MATERIALS

- Audio signal generator
- Oscilloscope or an equivalent app on a tablet or phone
- Loudspeaker (e.g. strong computer speakers)

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Devices plugged into 240 V mains power are a potential source of electric shock.	Devices plugged into 240 V mains power should be kept well away from water, and should have been tested and tagged.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Connect the apparatus as represented diagrammatically in Figure 9.4.
2. Practice using the features of the oscilloscope you have, as models vary considerably.
3. With the audio generator set to produce a 500 Hz sound wave, measure the horizontal axis wavelength of the waveform displayed to calculate the period, *T*, of the sound using the method shown previously. You will need to know the horizontal axis scale on the display.

»

4. Gradually increase the frequency on the audio generator while observing the change to the waveform displayed on the oscilloscope and the pitch of the sound being heard.
5. Next, vary the level of the output from the audio generator, again while observing the change to the shape of the waveform displayed and the change to the perceived loudness of the sound being heard.

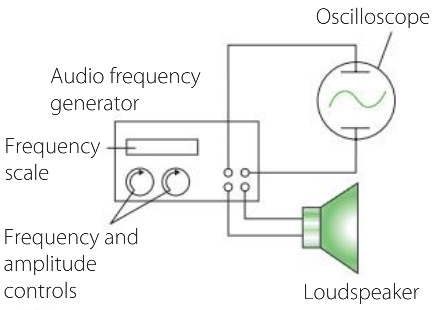


FIGURE 9.4 Apparatus used to investigate the waveform of sound waves. Note that the oscilloscope is connected directly to the audio frequency generator. It is therefore showing the signal going into the speaker.

RESULTS

1. Write down your observations of the change in the shape of the waveform displayed and the change in the perceived sound as the frequency of the audio generator was being increased.
2. Write down your observations of the change in the shape of the waveform displayed and the change in the perceived sound as the output level of the audio generator was being varied.

ANALYSIS OF RESULTS

Deduce the relationship between pitch and frequency, and also between loudness and amplitude, for the sound and the waves producing the sound. Summarise your findings.

DISCUSSION

Consider how these relationships could be extended to other types of waves such as light waves or earthquake waves.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the hypothesis of this investigation.

It is also possible to measure the frequency of a sound wave by displaying the waveform on an oscilloscope.

WORKED EXAMPLE 9.1

An oscilloscope has the time axis (horizontal axis) scale set to 5.0 ms cm^{-1}. A sound wave is received by a microphone connected to the oscilloscope's input. The distance between two successive crests on the screen is 3.0 cm. Find the period, T, and the frequency, f, of the sound wave.

ANSWER	LOGIC
5.0 ms cm^{-1} (on the screen) $\lambda = 3.0$ cm (on the screen)	▪ Identify relevant data in the question.
$T = 3.0 \text{ cm} \times 5.0 \text{ ms cm}^{-1}$ $= 15$ ms	▪ Identify the appropriate formula for period and substitute in known values. Calculate the answer.
$f = \dfrac{1}{T}$	▪ Identify the appropriate formula for frequency.
$= \dfrac{1}{15 \times 10^{-3} \text{ s}}$	▪ Substitute in the known values, with units, into the formula.
$= 67$ Hz	▪ Calculate the answer and express with correct significant figures and units.

TRY THESE YOURSELF

1. The horizontal axis scale is set to 0.50 ms cm^{-1} on an oscilloscope. Two successive crests on a waveform on a screen are 4.0 cm apart. Find the period and the frequency of the wave.
2. A sound wave with a frequency of 1200 Hz is displayed on an oscilloscope screen with a time axis scale set to 0.20 ms cm^{-1}. What is the distance between two successive crests on the waveform displayed?

INVESTIGATION 9.2

Measuring the frequency of a sound wave

AIM

To measure the frequency of a sound wave using an oscilloscope

MATERIALS

- Oscilloscope
- 3 tuning forks of different frequencies with sounding boards
- Microphone

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Devices plugged into the 240 V mains supply are a potential source of electric shock.	Devices plugged into 240 V mains power should be kept well away from water and should have been tested and tagged.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Set up the oscilloscope and adjust the display controls so that when a tuning fork is sounded near the microphone, a steady waveform is shown on the screen. Do this by adjusting the time-base (horizontal axis) input scale.
2. Measure the distance on the screen from one crest to the next, and convert this to the period.
3. Record your results in a data table as shown.
4. Repeat this procedure for the other tuning forks.
5. Calculate the frequency of the tuning forks and compare your results with the frequencies stamped on the forks.

TUNING FORK	TIME-BASE SETTING (ms cm^{-1})	CREST TO CREST DISTANCE (cm)	PERIOD (ms)	FREQUENCY (Hz)

ANALYSIS OF RESULTS

1. What did you observe about the frequencies of the tuning forks and the pitch heard?
2. Describe how the amplitude of the displayed wave changed as the loudness of the sound from the sounding boards changed.
3. Was there any relationship between frequency and amplitude observed?

»

» **DISCUSSION**

1 Explain why the tuning forks were mounted on sounding boards.

2 Are the sounding boards for different frequency tuning forks the same? Describe any differences observed.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- The pitch of a sound is related to the frequency of the sound wave.
- The loudness of a sound is related to the amplitude of the sound wave.
- An oscilloscope's time axis, the horizontal axis, can be used to measure the period and calculate the frequency of a sound wave.
- A sounding box can be used to amplify the sound caused by a vibrating tuning fork.

CHECK YOUR UNDERSTANDING 9.1

1 The distance between a crest of a wave and the next crest on an oscilloscope screen is 2.5 cm. The time-base axis scale is set to 2.0 ms cm^{-1}.
  a What is the period of this wave?
  b What is the frequency of this wave?

2 On a set of axes, sketch a sound wave represented as a transverse wave. Using the same axes, sketch a second wave that has a higher pitch and is louder than the wave you first sketched.

3 In music, the note known as concert pitch A has a frequency of 440 Hz. A note is played that has a higher pitch than concert pitch A. Use a mathematical expression that shows what the frequency of this note could be.

4 A wave with a period of 0.001 s is displayed on an oscilloscope with the time-base axis scale set to 0.5 ms cm^{-1}. What is the distance between successive crests on the screen?

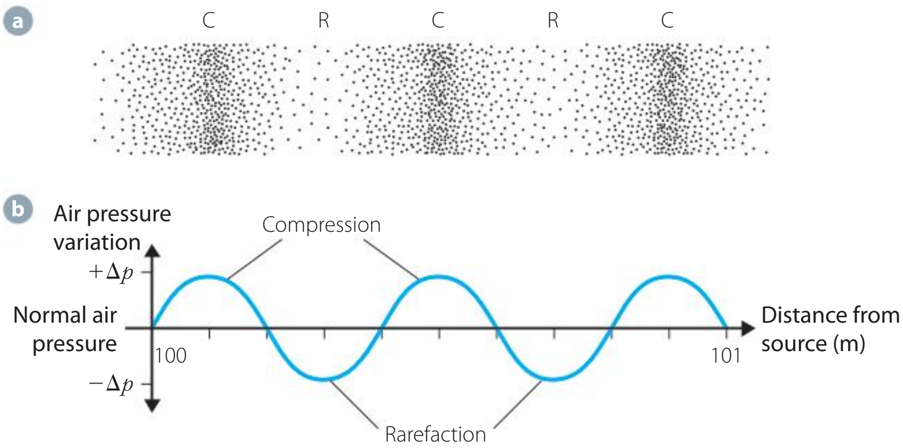
9.2 Sound in air is a longitudinal wave

In a sound wave in air, particle oscillations are always parallel to the direction of energy flow; therefore, sound waves are longitudinal waves. This sound wave model is not perfect. For example, if the particles of a gas are in constant random motion, how do they oscillate about a central mean (or rest) position? Models have their limitations and cannot exactly describe physical reality.

Sound transmission as a longitudinal pressure wave

When sound travels through a medium, the particles form a series of compressions and rarefactions as in Figure 9.5a (page 240). At compressions, the pressure is higher than the normal pressure. At rarefactions, the pressure is lower than normal. The graph of pressure variation, Δp, from the normal air pressure against the distance from source is shown in Figure 9.5b. Such graphs are often used to represent sound waves as they are easier to sketch than those such as Figure 9.5a.

FIGURE 9.5
a When sound travels through a medium, the particles form a series of compressions and rarefactions. **b** This graph shows the pressure variations, Δp, with distance in a sound wave.



University of Virginia
Animation of sound travelling through air as a longitudinal wave.

Further on in this chapter, the representation of pressure variation against distance, similar to Figure 9.5b, is used to explain sound waves in tubes or pipes. Attempting to do this using the representation used in Figure 9.5a would be difficult. If a microphone were being used to detect a sound wave as it passed, the compressions and rarefactions would be displayed on an oscilloscope screen as a function of time (rather than distance as shown in Figure 9.5).

WORKED EXAMPLE 9.2

As a sound wave passes a point, 30.0 compressions were detected in 0.100 s. What is the frequency of this sound wave?

ANSWER	LOGIC
$n = 30.0$; time $= 0.100$ s	- Identify relevant data in the question.
$f = \frac{n}{t}$	- Identify the appropriate formula (frequency is the number of waves passing each second).
$= \frac{30.0}{0.10\ \text{s}}$	- Substitute known values into formula.
$= 300$ Hz	- Calculate the answer and express with the correct significant figures and units.

TRY THESE YOURSELF

1 A sound wave results in 500 rarefactions passing a point in 0.20 s. Find the frequency of this wave.

2 How many compressions will a sound wave with a frequency of 600 Hz produce in 0.050 s?

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INVESTIGATION 9.3

Modelling sound as a longitudinal pressure wave

AIM

To use a slinky spring to model a sound wave as a longitudinal pressure wave

MATERIALS

- Slinky spring
- Ribbon
- Ruler
- Safety glasses
- Video recording device (preferably with slow-motion playback capabilities)

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
A stretched slinky spring can flick back and cause an injury.	Do not over-stretch the spring and do not let it go unexpectedly. Wear safety glasses.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Stretch a slinky spring on smooth ground with a student holding each end.
2. Tie a ribbon around one of the coils of the slinky spring at about the midpoint of the spring.
3. Place a clearly marked ruler next to spring with the centre of the ruler at the ribbon.
4. Have one student move the end of the spring rapidly and smoothly backwards and forwards in a line with the spring while the motion of the ribbon is videoed.

RESULTS

View the recording of the motion of the ribbon.

ANALYSIS OF RESULTS

1. If possible, view the motion of the ribbon in slow motion.
2. Describe the motion of the ribbon as a wave passes through the spring.

DISCUSSION

Describe how the spring's motion models the motion of air particles as a sound wave passes.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- Sound is a longitudinal wave with back-and-forth vibrations of air particles.
- These vibrations result in compressions and rarefactions in air.
- In air, compressions are regions of higher pressure and rarefactions are regions of lower pressure.

CHECK YOUR UNDERSTANDING

9.2

1 A sound with a frequency of 200 Hz was produced. How much time is there between each successive compression in air?

2 Describe the motion of air particles as a sound wave passes through the air.

3 How much time is there between a compression and a rarefaction in a sound wave with a frequency of 500 Hz?

4 What is the frequency of a sound wave if there is 0.025 s between each successive compression in the air?

9.3 Relationship between distance and intensity

The inverse square law

The energy produced by a source of sound waves will spread out into the surrounding air, covering an increasingly larger area. As the energy of the waves spreads out, the **intensity** of the sound will decrease in proportion to the inverse of the square of the distance from the source. Intensity is the amount of sound energy (measured in joules, J) passing through a unit area (a square metre, m^2) in 1 second. It has the unit of joules per second ($J\ s^{-1}$) per square metre. As 1 joule per second is the unit of power, watts (W), we get the unit of intensity being $W\ m^{-2}$.

We assume the source of sound is a **point source** and that there are no other objects in the vicinity to reflect or absorb the sound. Such a relationship between intensity and distance is known as the **inverse square law**:

$$I \propto \frac{1}{d^2}$$

where I is the intensity of sound, d is the distance from the sound source, and the symbol $\propto$ represents the relationship 'is proportional to'. In chapter 10, the inverse square law is investigated when applied to light. Other natural phenomena such as gravitational and electrostatic forces are related to distance by the inverse square law as well, but are not covered in this chapter.

It is useful to use the mathematical relationship between the ratio of the two intensities $\frac{I_1}{I_2}$ and the ratio of the two distances $\frac{d_1}{d_2}$ for the inverse square law:

$$\frac{I_1}{I_2} = \left(\frac{d_2}{d_1}\right)^2$$

Measuring the intensity of sound in decibels

The **decibel (dB)** scale is commonly used for measuring the intensity of sound. It is a logarithmic scale, due to the very large differences between the lowest intensity of sound that can be heard (the threshold of hearing) and the loudest common sounds.

The accepted sound intensity for the threshold of hearing is 10^{-12} W m^{-2}. This is assigned as zero decibels and is written as:

$$I_o = 10^{-12}\ W\ m^{-2}$$

A sound intensity of 10 dB has 10 times the intensity of 0 dB, or 10^{-11} W m^{-2}. This increase compounds so that a sound intensity of 20 dB has 10 times the intensity of a sound with 10 dB. Table 9.1 gives examples of the dB scale and sounds typically associated with these intensities.

TABLE 9.1 The decibel (dB) scale of sound intensity

SOUND	dB LEVEL	INTENSITY (W m^{-2})
Threshold of hearing – lowest intensity sound that can be heard	0	10^{-12}
Quiet breathing	10	10^{-11}
Whisper	20	10^{-10}
Leaves rustling	30	10^{-9}
Talking softly	40	10^{-8}
Conversation	50	10^{-7}
Noisy classroom	60	10^{-6}
Busy road at 15 m away	70	10^{-5}
Food processor	80	10^{-4}
Lawn mower	90	10^{-3}
Jackhammer	100	10^{-2}
Loud concert	110	10^{-1}
Jet take-off at 10 m away	120	1
Explosion causing hearing damage or loss	130+	10+

From the information in Table 9.1 it can be seen that the sound intensity at a loud concert is 10^{11} (100 billion) times the intensity of the threshold of hearing.

As almost all sound level meters and smartphone apps express sound intensity using the decibel scale, converting to the units of W m^{-2} is required. This can be done using the relationship:

$$I = 10^{\left(\frac{\text{dB}}{10} - 12\right)}$$

where I is the sound intensity in W m^{-2} and dB is the sound level in dB.

INVESTIGATING 9.4

Relationship between distance and intensity of sound

AIM

To investigate the relationship between the intensity of sound and the distance from the source

Write a hypothesis statement about the effect that the distance from the source of a sound is predicted to have on the measured intensity of the sound.

MATERIALS

- Constant sound source (e.g. signal generator and speaker, or an electric bell)
- Decibel meter (or suitable app on a smartphone)
- Measuring tape

METHOD

1. Select a suitable quiet area outdoors, preferably away from walls and other buildings that may reflect sound.
2. With the constant sound playing, measure the reading on the decibel meter or phone app at 10 different distances away from the source.
3. Record your results in a table as shown on page 244.

»

»

RESULTS

Record the readings on the meter and the distance from the source in the first two columns of the table.

DISTANCE *d* FROM SOURCE (m)	READING ON DECIBEL METER (dB)	SOUND INTENSITY (W m^{-2})	DISTANCE FROM SOURCE SQUARED, d^2 (m^2)	INVERSE OF DISTANCE SQUARED, $\frac{1}{d^2}$ (m^{-2})

ANALYSIS OF RESULTS

1. Calculate the values for the third, fourth and fifth columns of the table. For the third column, use the following formula to convert from decibels to intensity in W m^{-2}:

   $$I = 10^{\left(\frac{\mathrm{dB}}{10} - 12\right)}$$

   where dB is the sound level reading in decibels (dB).

2. On a set of axes, plot I (in W m^{-2}) versus $\frac{1}{d^2}$.
3. **a** Alternatively, copy the fifth and then the third columns of data into a spreadsheet. The fifth column of data from the table must be to the left of the third column or the axes will be transposed.

   b Highlight the columns in the spreadsheet, select 'Insert', and then select the scatter graph chart type. A graph will be inserted next to your data, which you can then label and give a title.
4. Draw a line of best fit to your plotted data points. This line should pass as close to the data points as possible.

DISCUSSION

1. Does your data support the inverse square law for sound? Explain your answer.
2. Discuss which other factors may have had an effect on your data.
3. Suggest ways in which this investigation could be improved.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

WORKED EXAMPLE (9.3)

The intensity of a sound wave is 4.00×10^{-4} W m^{-2} at a distance of 5.0 m from the source of the sound. What will the intensity be at a distance of 22.0 m?

ANSWER	LOGIC
$I_1 = 4.00 \times 10^{-4}$ W m^{-2}; $d_1 = 5.0$ m; $d_2 = 22.0$ m; $I_2 = ?$	- Identify relevant data in the question.
$\frac{I_1}{I_2} = \left(\frac{d_2}{d_1}\right)^2$	- Identify the appropriate formula.
$I_2 = \frac{I_1}{\left(\frac{d_2}{d_1}\right)^2}$	- Rearrange the formula.

ANSWER	**LOGIC**
$= \dfrac{4.00\times10^{-4}\ \text{W m}^{-2}}{\left(\dfrac{22.0\ \text{m}}{5.0\ \text{m}}\right)^2}$	▪ Substitute the known values, with units, into the formula.
$I_2 = 2.1\times10^{-5}\ \text{W m}^{-2}$	▪ Calculate the answer and express the final answer with the correct significant figures and units.

TRY THESE YOURSELF

1 How far away does a person need to move to reduce the intensity of sound from a jet engine to 1.00×10^{-3} W m^{-2} if, when 10.0 m away, the intensity is 2.50×10^{0} W m^{-2}?

2 By what factor does the intensity of sound from a single source increase if a person moves from a distance of 500.0 m to 25.0 m away?

KEY CONCEPTS

- Intensity of sound, I, is a measure of the amount of sound energy passing through a unit area per second.
- Intensity of sound is inversely proportional to the square of the distance from a point source of sound.
- The inverse square law is: $I \propto \dfrac{1}{d^2}$.
- Using the inverse square law, $\dfrac{I_1}{I_2} \propto \left(\dfrac{d_2}{d_1}\right)^2$.

CHECK YOUR UNDERSTANDING

9.3

1 State the inverse square law for the relationship between the intensity of sound and the distance from the source.

2 **a** In theory, how far from a source of sound do you need to be so that the intensity from the source is zero?

b Does this apply in practice? If not, why?

3 Explain why workers such as gardeners use earmuffs when using a mower, but it is considered safe to walk down the street when someone is mowing their lawn.

4 How many times further away would you need to go from a sound source to reduce the intensity of the sound to 1% of the original intensity experienced?

5 If the intensity of a sound source increased by a factor of 4, how much further away would you need to move from the source so that it sounded just as loud as before?

9.4 Reflection, diffraction and resonance of sound waves

The characteristics of waves in general were covered in chapter 8. As sound propagates as a wave, it can be reflected, diffracted, undergo superposition and cause resonance. The superposition of waves will be investigated in a later section in this chapter when beats are discussed in detail. The phenomenon of sound resonance will be examined in section 9.5 on standing waves in pipes and musical instruments.

Reflection of sound

Sound is reflected or absorbed when it meets a surface. Smooth, hard surfaces are better reflectors of sound than soft, rough surfaces. When a quick sharp sound reflects back to you quickly, it is perceived as being prolonged rather than as a separate, distinct echo. If the furthest reflecting surface is no more than 17 m away, the total distance covered by a reflected sound that you make is 34 m. If the sound is travelling through air at 340 m s^{-1}, this means that the time taken for the sound to be reflected back to you is no more than 0.1 s. We perceive such a reflection as a prolonging of the original sound rather than two distinct sounds. When in an empty room no larger than a typical classroom or laboratory, making a single sharp noise such as a clap of your hands allows you to experience this. Such a prolonging of the original sound is called **reverberation**.

In contrast to a reverberation, an **echo** is heard when sound is reflected back to you from a surface greater than 17 m away. The reflected sound wave arrives more than 0.1 s after the original sound, so it is perceived as a separate, distinct sound. Echoes are produced when the reflecting surface is hard, such as a rock wall, so that most of the sound energy is reflected rather than absorbed.

Simply cupping your hand over your ear results in more sound entering your ear canal. This is a result of the sound waves reflecting off your curved hand and continuing into your ear.

The law of reflection will be dealt with fully in chapter 10. Investigation 9.5 will consider the reflection of sound off a hard surface.

INVESTIGATION 9.5

Observing the reflection of sound waves

AIM

To observe cases of the reflection of sound waves

MATERIALS

- 2 cardboard or plastic tubes, approximately 30–40 cm in length and 5–8 cm in diameter
- Smooth, hard surface (e.g. a small portable whiteboard)
- Portable source of sound (e.g. a smartphone playing music)
- Protractor, paper and pencil

METHOD

1. Set up the apparatus as shown in Figure 9.6. The smooth hard surface is vertical while the tubes lie on a flat horizontal surface such as a table top.
2. Place the speaker of the sound source over the end of tube 1. Play a soft sound into the tube (music is suitable, or an app that plays the sound of a ticking clock could be used).
3. Move the tubes so that:
   - the angles that they make with a perpendicular line drawn from the hard surface are the same (as shown in Figure 9.6); and
   - the angles the tubes make with the perpendicular line are not equal.

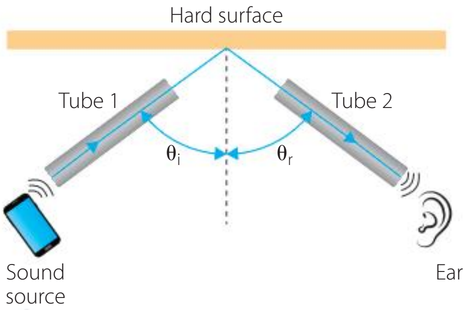


FIGURE 9.6 The arrangement of the apparatus. Two tubes placed on a horizontal surface face a smooth vertical surface. A soft sound such as music is played into the first tube, reflects off the smooth, hard surface, and travels back up the second tube where it is heard.

» **4** Compare how well the sound from tube 1 is heard through tube 2 for the different alignments of the tubes as described.

5 Repeat steps 3 and 4 for several different angles.

RESULTS

Record your results and observations in a table.

ANALYSIS OF RESULTS

Describe the alignment of the two tubes when the sound in tube 2 was heard the loudest.

DISCUSSION

As a result of your observations, discuss whether sound is reflected off a hard surface.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Diffraction of sound

The diffraction of waves in general was discussed in chapter 8. You should recall several important points about the diffraction of waves.

- Diffraction occurs as a wave spreads out after passing through an opening.
- Waves with longer wavelengths will spread out more than shorter wavelengths.

An investigation to observe the diffraction of sound can also show how the range of wavelengths of sound waves determines how much diffraction will occur. With music typically having a range of frequencies from 100 Hz to around 5 kHz, we can use $v = f\lambda$ to calculate the range of wavelengths, with the speed of sound in air as 340 m s^{-1}.

$f = 100$ Hz:

$$\begin{aligned} v &= f\lambda \\ \lambda &= \frac{v}{f} \\ &= \frac{340 \text{ m s}^{-1}}{100 \text{ Hz}} \\ &= 3.40 \text{ m} \end{aligned}$$

$f = 5$ kHz:

$$\begin{aligned} v &= f\lambda \\ \lambda &= \frac{v}{f} \\ &= \frac{340 \text{ m s}^{-1}}{5\times 10^{3} \text{ Hz}} \\ &= 0.068 \text{ m} \\ &= 68 \text{ mm} \end{aligned}$$

Given that a doorway has a width of approximately 1 m, we would expect low frequency sounds to diffract through a doorway more than high frequency sounds.

INVESTIGATION 9.6

Observing diffraction of sound waves

AIM

To observe the diffraction of sound and to relate the extent of diffraction to the wavelength of the sound
Write an appropriate hypothesis for this investigation.

MATERIALS

- Loudspeaker (or sound system) playing music
- A room with a doorway and a door that can close

METHOD

1. Set up the loudspeaker or sound system playing music inside the classroom or laboratory.
2. Move around inside the room and take notes about the quality of the sound you are hearing.
3. Now move outside the room, leaving the door open. Move around near the door so that you are:
    - directly in front of the doorway
    - to one side of the doorway
    - as far to one side of the doorway as you can be.
4. Record your observations about the quality of the sound you can hear in the positions described, with particular reference to the higher frequency sounds and the lower frequency sounds.

RESULTS

Which frequencies were more audible inside the room, and at the positions described outside the room?

ANALYSIS OF RESULTS

Relate your results and observations to the diffraction of sound and to the range of frequencies being heard.

DISCUSSION

1. If not for diffraction, would it still be possible to hear any of the sound from the loudspeaker when you are outside of the room?
2. Summarise the role diffraction plays in hearing music coming from inside a room when you are outside of the room.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the hypothesis of this investigation.

KEY CONCEPTS

- Sound is reflected from hard surfaces.
- Sound is diffracted when passing through an opening.
- Long wavelength sound waves diffract more than short wavelength sound waves.
- An echo is a distance reflection of a sound wave.
- Reverberation occurs when many reflections of sound are heard over a short period of time.

CHECK YOUR UNDERSTANDING

9.4

1 Identify three examples where the reflection of sound can be observed.

2 Outline the difference between the quality of sound heard when a person is speaking in:

  a the school gymnasium.

  b a cinema.

  Link these qualities to the material covering the walls and floor.

3 Why do megaphones and loud hailers have a cone-shaped structure surrounding the loudspeaker?

4 When a car with an open window drives past, the music we can hear is often just the bass (low frequency) component of what is being played inside the car. Explain why this happens.

9.5 Standing waves in strings

In chapter 8, the concept of standing, or stationary waves was explored. For strings and sound waves in air, standing waves can be produced in instruments with the right dimensions that will give rise to resonance. This is the basis of string and wind musical instruments.

Modes of vibrations in strings

Figure 9.7 shows possible vibration modes, or harmonics, for standing waves in a string or wire fixed at both ends. There is a node at each fixed end for all modes of vibrations.

The **fundamental** mode of vibration is also referred to as the **first harmonic**. The other modes of vibration are called the second harmonic, third harmonic, and so on. Wires and strings of musical instruments can be made to vibrate at frequencies other than their fundamental frequency.

Predicting the fundamental frequency

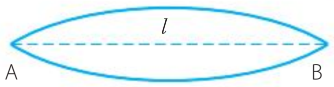
Vibration mode	Wave pattern	f and λ
Fundamental mode of vibration 1st harmonic		$\lambda_1 = 2l$ $f_1 = \frac{v}{2l}$
1st overtone 2nd harmonic		$\lambda_2 = l$ $f_2 = 2f_1$
2nd overtone 3rd harmonic		$\lambda_3 = \frac{2}{3}l$ $f_3 = 3f_1$
3rd overtone 4th harmonic		$\lambda_4 = \frac{2}{4}l$ $f_4 = 4f_1$
4th overtone 5th harmonic		$\lambda_5 = \frac{2}{5}l$ $f_5 = 5f_1$

FIGURE 9.7 The first five harmonics (four overtones) of a string fixed at both ends

Harmonics (other than the first harmonic) are also called **overtones**. Overtones are notes or tones of higher frequency than the fundamental or natural frequency (and are of smaller amplitude). Thus, the second harmonic is the first overtone; the third harmonic is the second overtone, and so on.

The frequency of each harmonic is its harmonic number times the fundamental frequency, $f_n = nf_1$. If the fundamental frequency of a stretched string is 40 Hz, the fourth harmonic has a frequency of $4 \times 40 = 160$ Hz.

The fundamental mode of vibration (the first harmonic) is generated when the stretched string is plucked in the middle. If you look at the fundamental vibration mode in Figure 9.7, the pattern represents half a wave, so wavelength λ_1 is twice the length of the string:

$$l = \frac{\lambda_1}{2}$$

The second harmonic (first overtone) is generated when the stretched string is plucked a quarter of the way along the string. The second harmonic mode in Figure 9.7 shows that the pattern represents a complete wave, so the wavelength is the length of the string:

$$l = \lambda$$

The third harmonic is generated when the stretched string is plucked a sixth of the way along the string. The third harmonic mode in Figure 9.7, the pattern represents one and a half complete waves so the wavelength is two-thirds the length of the string:

$$l = \frac{3\lambda_1}{2}$$

For strings attached at both ends to be resonating, the length of the string is related to the resonating wavelength by the relationship:

You will also see this relationship later for open pipes.

$$l = n\frac{\lambda_n}{2}$$

where l is the length of the string and λ_n is the wavelength of the nth harmonic ($n = 1, 2, 3$ ...).

Numeracy

Putting this together with $v = f\lambda$, the following relationships become apparent.

- The fundamental or first mode has frequency $f_1 = \frac{v}{\lambda_1} = \frac{v}{2l}$
- The second harmonic has frequency $f_2 = \frac{v}{\lambda_2} = \frac{2v}{2l} = 2f_1$.

To generalise, the nth harmonic has frequency:

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2l} = nf_1$$

WORKED EXAMPLE 9.4

The fundamental frequency of a string 2.4 m long, fixed at both ends, is 20 Hz.

1 What are the frequencies of the first three overtones?

2 Is it possible to produce standing waves of frequency 50 Hz in this string?

3 What is the speed of the waves in the string?

4 What is the wavelength of the first harmonic?

ANSWERS	LOGIC
1 The frequency of each harmonic is its harmonic number times the fundamental frequency: $f_n = nf_1$ 1st overtone = 2nd harmonic: $2 \times 20\ \text{Hz} = 40\ \text{Hz}$ 2nd overtone = 3rd harmonic: $3 \times 20\ \text{Hz} = 60\ \text{Hz}$ 3rd overtone = 4th harmonic: $4 \times 20\ \text{Hz} = 80\ \text{Hz}$	▪ The overtone number is one less than the harmonic number.
2 As 50 Hz is not the frequency of one of the harmonics of this string (a whole-number multiple of the fundamental frequency), it is not possible to produce standing waves of this frequency with the string under the same tension.	▪ Check possible harmonic frequencies.
3 $f_1 = 20\ \text{Hz};\ l = 2.4\ \text{m}$ $f_1 = \dfrac{v}{2l}$ $v = f_1 \times 2l$ $= 20\ \text{Hz} \times (2 \times 2.4\ \text{m})$ $v = 96\ \text{m s}^{-1}$	▪ Identify relevant data in the question. ▪ Identify the appropriate formula. ▪ Rearrange the formula. ▪ Calculate the answer. ▪ Express the final answer with the correct significant figures and units.
4 The first harmonic is the fundamental frequency; the wavelength is $2l$, so the wavelength is: $2 \times 2.4\ \text{m} = 4.8\ \text{m}$	▪ Identify the appropriate formula, and calculate the answer.

TRY THESE YOURSELF

A guitar string is 0.66 m in length. When plucked at its centre, it produces a fundamental note of 80 Hz.

1 What is the wavelength of the note?

2 A guitarist presses the string against the fret, shortening the string to produce notes of different frequencies. How long should the string be to produce a note with a fundamental frequency of 160 Hz?

Velocity of a wave in a string

The velocity of a wave in a stretched string depends on two factors: the tension, T, in the string and the mass per unit length, $m\,l^{-1}$, of the string. The mass per unit length is usually given as kilograms per metre, and the tension is given in newtons.

As anyone who has played a string musical instrument knows, increasing the tension in the string will increase the frequency of its pitch by causing the string to vibrate faster in its fundamental mode. Investigating the strings in a piano will reveal thicker wires for the lower notes and thinner wires for the higher notes in addition to variations in their lengths.

The speed of a wave travelling in a string is given by the relationship:

$$v = \sqrt{\frac{T}{m\,l^{-1}}}$$

where T is the tension in the string (expressed in newtons), $m\,l^{-1}$ is mass per unit length of the string (in kilograms per metre), and v is the speed of wave in the string (in metres per second).

Frequency of a vibrating string
Investigate further the factors affecting the frequency of vibration of a string.

Wave velocity in a string
Investigate the effects of the variables on the wave velocity in a stretched string.

Information and communication technology capability

The fundamental frequency of a vibrating string can be found using $v = f\lambda$ and the fact that the wavelength of this wave must be twice the length, l, of the string. Therefore,

$$\begin{aligned} f_1 &= \frac{v}{\lambda} \\ &= \frac{v}{2l} \\ &= \frac{\sqrt{\frac{T}{m\,l^{-1}}}}{2l} \\ &= \sqrt{\frac{T}{m\,l^{-1}}} \div 2l \\ &= \frac{1}{2}\sqrt{\frac{T}{m\,l^{-1}l^{2}}} \\ &= \frac{1}{2}\sqrt{\frac{T}{m\,l}} \end{aligned}$$

Sound waves

It is clear that the more tension there is in a string, the faster it will vibrate. More massive strings will vibrate slower and longer strings will also have a lower frequency. These characteristics are familiar to those who have ever played a string instrument.

WORKED EXAMPLE 9.5

A string in a piano is 80 cm long and has a mass of 20 g. When the tension in the string is 350 N, what is the fundamental frequency of the vibrating string?

ANSWER	LOGIC
$l = 0.80$ m; $m = 0.020$ kg; $T = 350$ N	▪ Identify relevant data in the question and convert to SI units.
$f_1 = \dfrac{\sqrt{\frac{T}{m\,l^{-1}}}}{2l}$	▪ Identify the appropriate formula.
$= \dfrac{\sqrt{\frac{350\text{ N}}{\frac{0.020\text{ kg}}{0.80\text{ m}}}}}{2 \times 0.80\text{ m}}$	▪ Substitute the known values, with units, into the formula.
$f_1 = 74$ Hz	▪ Calculate the answer with the correct significant figures and units.

TRY THESE YOURSELF

1 What is the fundamental frequency of a vibrating string with a length of 40 cm and mass of 15 g when the tension in the string is 300 N?

2 Without re-calculating the previous question, what would the fundamental frequency change to if the tension in the same string was increased to 1200 N, four times the previous value?

KEY CONCEPTS

- A vibrating string has a node at both ends.
- The fundamental mode of vibration in a string is also the first harmonic.
- The wavelength λ of the fundamental mode is $2l$, or twice the length of the string.
- For harmonics in a string, $l = n\frac{\lambda_n}{2}$ where $n = 1, 2, 3 \ldots$
- The speed, v, of a wave in a string is determined by the length of the string, l, the tension, T and the mass per unit length, $m\,l^{-1}$, according to the relationship: $v = \sqrt{\frac{T}{m\,l^{-1}}}$.
- The fundamental frequency of vibration in a string, f_1 is given by $f_1 = \frac{v}{2l}$.

CHECK YOUR UNDERSTANDING

9.5

1 What is the difference between the fundamental mode and the first harmonic mode of vibration in a string?

2 A standing wave in a spring results from the interference between an incident wave and its reflection. The two waves cancel at the nodes. Does this mean that energy is destroyed? Explain your answer.

3 When the two component waves producing a standing wave pattern, each has a wavelength of λ. What is the distance between:

- **a** adjacent nodes?
- **b** adjacent antinodes?
- **c** a node and the closest antinode?

4 Which pattern(s) in Figure 9.8 could represent a standing wave pattern on a string of length l, fixed at both ends?

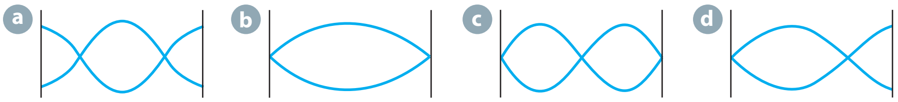


FIGURE 9.8

5 The apparatus used to investigate the vibrations of a stretched string or vibrating wire is called a sonometer (see Figure 9.9).

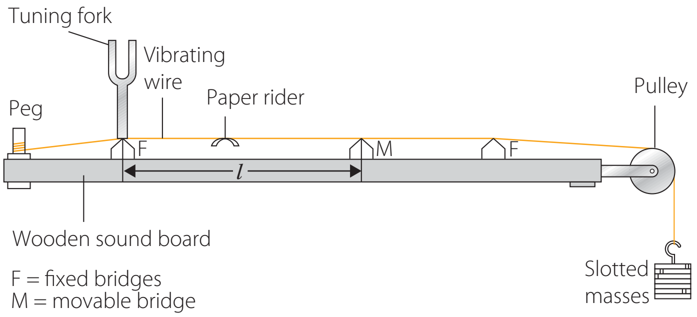


FIGURE 9.9 The sonometer. A paper rider is a small piece of paper taped to the stretched string. The bridges labelled F are fixed, while the one labelled M is movable so as to adjust the length of the wire, ℓ, that is allowed to vibrate. Here, ℓ is the distance from F to M as shown.

The stretched wire on the sonometer has a length l of 0.80 m, and it can vibrate.

- **a** What is the wavelength of the fundamental mode of vibration?
- **b** If the speed of the wave in the wire is 200 m s^{-1}, what is the fundamental frequency?
- **c** If the vibrating length of the wire is shortened, does the fundamental frequency increase or decrease? Give a reason for your answer.
- **d** If you added more slotted masses to the sonometer, the frequency of the note it produces will increase. Why is this?

6 What is the longest wavelength of a standing wave that can be created between fixed supports 12 cm apart?

7 Two successive overtones of a vibrating string are 300 Hz and 360 Hz. What is the fundamental frequency of the string?

9.6 Vibration in air columns

FIGURE 9.10 Trombones change the length of the resonator by sliding one tube through another.

Shutterstock.com/sirtravelalot

Longitudinal standing sound waves can be created in both open and closed pipes. Resonance occurs when sound waves match one of the harmonic wavelengths of the pipe. Open pipes are pipes that are open at both ends, and closed pipes are open at only one end. Resonance in air columns is related to the length of the pipe and the speed of sound in air, which is temperature dependent.

Reflection of sound waves in pipes

Waves confined in pipes travel as plane waves, the same as they do in the open air. As a compression travels along the pipe, it continually reflects off the walls of the pipe. This maintains the compression. When the compression reaches the end of an open pipe, it is no longer confined and rapidly expands into the air, leaving behind a rarefaction that travels back down the pipe. The compression has been reflected as a rarefaction. This is similar to a fixed-end reflection in strings and springs.

When a rarefaction travels along the pipe, it also continually reflects off the walls of the pipe. This maintains the rarefaction. When the rarefaction reaches the end of an open pipe, the higher pressure air outside the pipe rapidly expands into the rarefaction creating a compression that travels back down the pipe. The rarefaction has been reflected as a compression.

When a compression strikes the closed end of a pipe, it is reflected as a compression. This is similar to a free-end reflection in strings and springs.

Rarefactions are also reflected as rarefactions from the closed end of the pipe. The standing wave formed in the tube has its maximum air displacement (an antinode) at the open end. This means there will be a pressure node at the open end of the pipe and a pressure antinode at the closed end.

Stationary waves in open pipes

To indicate the stationary wave pattern in an air column, either the pressure variation, Δp, or the particle displacement, Δx, could be used. These were shown earlier in Figure 9.5 (page 240). Figure 9.11 represents the particle displacement, Δx, and pressure variations, Δp, in an open pipe. In all open pipes, the maximum air displacements (displacement antinode or a pressure node) occur at both ends of the tube, so that its natural frequencies are different from those of a tube closed at one end.

For pipes open at both ends to resonate, the length of the pipe must be related to the resonating wavelength by the relationship:

$$l = n\frac{\lambda_n}{2}$$

where l is the length of the pipe, λ_n is the wavelength of the resonance frequency, and n is a whole number relating to that harmonic at which it is resonating ($n = 1, 2, 3 \ldots$). In open pipes, all harmonics are possible. That is, every integer multiple of the fundamental frequency, f_1, is a possible overtone.

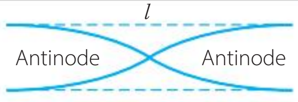
Vibration mode	**Particle displacement**	**Pressure variation**	***f* and λ**
Fundamental mode of vibration 1st harmonic			$\lambda_1 = 2l$ $f_1 = \frac{v}{2l}$
1st overtone 2nd harmonic			$\lambda_2 = l$ $f_2 = 2f_1$
2nd overtone 3rd harmonic			$\lambda_3 = \frac{2}{3}l$ $f_3 = 3f_1$
3rd overtone 4th harmonic			$\lambda_4 = \frac{2}{4}l$ $f_4 = 4f_1$
4th overtone 5th harmonic			$\lambda_5 = \frac{2}{5}l$ $f_5 = 5f_1$

FIGURE 9.11 The particle displacement, Δx, and pressure variations, Δp, of the resonant frequencies of a tube open at both ends. Compare this to Figure 9.7 for vibrations in strings.

WORKED EXAMPLE 9.6

Calculate the length of a pipe open at both ends whose fundamental frequency is 320 Hz, when the temperature is such that the speed of sound in air is 340 m s^{-1}.

ANSWER	**LOGIC**
$f_1 = 320\text{ Hz}; v = 340\text{ m s}^{-1}$	- Identify relevant data in the question.
$f_1 = \frac{v}{2l}$	- Identify the appropriate formula.
$l = \frac{v}{2f_1}$	- Rearrange the formula.
$= \frac{340\text{ m s}^{-1}}{2 \times 320\text{ Hz}}$	- Substitute the known values, with units, into the formula.
$l = 0.53\text{ m}$	- Calculate the answer and express with the correct significant figures and units.

TRY THESE YOURSELF

1 The fundamental harmonic of a pipe open at both ends is 640 Hz. What is its length?

2 A 2.2 m pipe open at both ends is resonating at its second harmonic frequency.

  a What is the wavelength of the stationary wave?

  b What is the frequency of the second harmonic?

Stationary waves in pipes closed at one end

Consider an air column that is in a tube closed at one end. When it is resonating, a pressure antinode (displacement node) occurs at the closed end. Figure 9.12 represents the particle displacement and pressure variations in a pipe closed at one end.

The fundamental resonance mode when $n = 1$:

$$l = \frac{\lambda}{4}$$

The resonance mode when $n = 2$ (third harmonic):

$$l = \frac{3\lambda}{4}$$

The resonance mode when $n = 3$ (fifth harmonic):

$$l = \frac{5\lambda}{4}$$

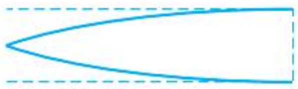
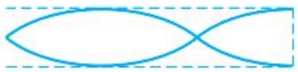
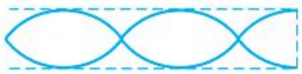
For a closed pipe to resonate, the length of the pipe must be related to the resonance wavelength by the relationship:

$$l = (2n - 1)\frac{\lambda}{4}$$

For any integer n, $(2n - 1)$ will be an odd value.

Only the odd harmonics are present in a closed pipe. This means that musical instruments with closed pipes can only play half the notes that those with open pipes can play (see Figure 9.11, page 255).

FIGURE 9.12 Particle displacement, Δx, and pressure variation, Δp, for pipes closed at one end. Only the odd harmonics are produced in these pipes.

Vibration mode	Particle displacement	Pressure variation	*f* and λ
Fundamental mode of vibration 1st harmonic	 Antinode Node	 Node Antinode	$\lambda_1 = 4l$ $f_1 = \frac{v}{4l}$
1st overtone 3rd harmonic			$\lambda_2 = \frac{4}{3}l$ $f_2 = 3f_1$
2nd overtone 5th harmonic			$\lambda_3 = \frac{4}{5}l$ $f_3 = 5f_1$
3rd overtone 7th harmonic			$\lambda_4 = \frac{4}{7}l$ $f_4 = 7f_1$
4th overtone 9th harmonic			$\lambda_5 = \frac{4}{9}l$ $f_5 = 9f_1$

WORKED EXAMPLE 9.7

When the air is at 0°C, the speed of sound in air is 331 m s^{-1}. What will be the fundamental frequency, f_1, and the frequency of the first two overtones, f_2 and f_3, for an organ pipe 2.4 m long if it is:

1 open at both ends?

2 closed at one end?

ANSWER	**LOGIC**
1 $v = 331\ \text{m s}^{-1}$; $l = 2.4\ \text{m}$ $f_1 = \frac{v}{2l}$ $= \frac{331\ \text{m s}^{-1}}{2 \times 2.4\ \text{m}}$ $f_1 = 69\ \text{Hz}$ For pipes open at both ends, the first two overtones are $2f_1$ and $3f_1$, i.e. 140 Hz and 210 Hz.	▪ Identify relevant data in the question. ▪ Identify the appropriate formula. ▪ Substitute the known values, with units, into the formula. ▪ Calculate the answer and express with the correct significant figures and units.
2 $f_1 = \frac{v}{4l}$ $= \frac{331\ \text{m s}^{-1}}{4 \times 2.4\ \text{m}}$ $f_1 = 34\ \text{Hz}$ For pipes closed at one end, the first two overtones are $3f_1$ and $5f_1$, i.e. 100 Hz and 170 Hz.	▪ Identify the appropriate formula. ▪ Substitute the known values, with units, into the formula. ▪ Calculate the answer and express with the correct significant figures and units.

TRY THESE YOURSELF

A girl blows across the mouth of a bottle and sets the air column inside it resonating at its third harmonic of 930 Hz. The speed of sound in the air is 340 m s^{-1}.

1 What is the length of the bottle?

2 What is the frequency of the next harmonic?

INVESTIGATION 9.7

Finding the speed of sound by air column resonance

The speed of sound in air varies with temperature, moisture content and the local atmospheric pressure.

AIM

To find the speed of sound in air in your classroom

MATERIALS

- Large graduated cylinder
- Glass tube of about 2–3 cm in diameter and about 30 cm long
- Tuning fork with a fundamental frequency of 440 Hz
- Ruler
- Marking pen that can write on wet glass

Animations and visualisations of standing sound waves in pipes

Use these visualisations to increase your understanding of standing sound waves in pipes.

»

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
The glass tube may break and produce sharp edges.	Take care handling the glass equipment.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Set up the equipment as shown in Figure 9.13.
2. Fill the graduated cylinder with water to just below the top.
3. Lower the glass tube until the top is just above the surface of the water.
4. Strike the tuning fork on a surface that will get it oscillating strongly and hold it near the open end of the tube.
5. Raise the tube slowly until it begins to resonate.
6. With the pen, mark the base of the tube where it contacts the water when the loudest resonance is detected.
8. Repeat the procedure twice and average the three values.
9. Analyse the results given that, for the fundamental frequency, the resonance length is $l = \frac{\lambda}{4}$.

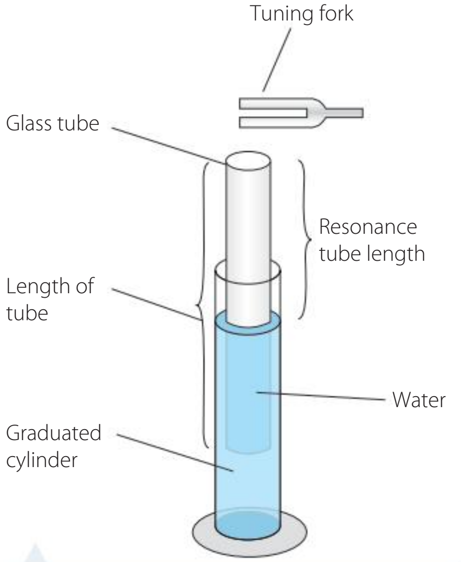


FIGURE 9.13 Experimental set-up. The glass tube fits inside the graduated cylinder (large measuring cylinder), and can be moved up and down to vary the length of the air column inside.

RESULTS

Record your results in a table.

DATA	TRIAL 1	TRIAL 2	TRIAL 3	AVERAGE
Frequency of the tuning fork				
First resonance length				

ANALYSIS OF RESULTS

1. Use the data to calculate your best estimate of the speed of sound. Include the uncertainty in this value.
2. A commonly accepted value for the speed of sound is 340 $\mathrm{m\ s^{-1}}$. Compare your best estimate with this accepted value. Do this by first calculating the percentage difference between the best estimate of the accepted value and your best estimate of the value.

DISCUSSION

1. What atmospheric conditions may have affected your result?
2. If you conducted the experiment on another day when the temperature in the room was hotter, how would your result change? Explain your answer.
3. What changes could you make to the experiment so the data gathered was more accurate?
4. If you replaced the air in the glass tube with carbon dioxide, would the value of the speed of sound change? Explain your answer.
5. Draw a diagram showing the pressure node and antinode in the closed tube when it was resonating at the tuning fork's fundamental frequency.
6. If you repeated the experiment with a tuning fork that had a higher fundamental frequency, would the first resonance occur in a longer or shorter air column? Justify your answer.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- Open pipes have an antinode at both ends.
- The fundamental wavelength is twice the length of the pipe.
- Open pipes have every harmonic; $f_n = nf_1$ where $n = 1, 2, 3, \ldots$
- A node exists at a closed end of a pipe.
- An antinode exists at the open end of a pipe.
- The fundamental mode of vibration is also the first harmonic.
- A closed pipe has only odd harmonics; $f_n = nf_1$ where $n = 1, 3, 5, \ldots$
- The length of a closed pipe is $l = (2n - 1)\frac{\lambda}{4}$.

CHECK YOUR UNDERSTANDING 9.6

Unless otherwise stated, take the velocity of sound in air at room temperature to be 340 m s^{-1}.

1 A tube of length *l* is open at both ends. A standing wave is set up in the tube. Only waves with certain frequencies will cause resonance within the tube. Which of the following gives the set of wavelengths that can exist in a tube open at both ends ($n = 1, 2, 3, \ldots$)?

A nl

B $\frac{l}{n}$

C $\frac{4l}{2n-1}$

D $\frac{2l}{n}$

2 Water is poured into a long metal tube closed at one end until the shortest resonant length is found for a fork of frequency 256 Hz. If the length of the air column in the tube is 31.0 cm, what is the velocity of sound in the air at the time?

3 A vertical pipe of length 1.40 m is filled with water, which is allowed to run out slowly from the lower end, while a vibrating tuning fork of frequency 512 Hz is held over its open end. How many positions of resonance will be obtained?

4 The human ear is most sensitive to sounds of a frequency of about 5000 Hz. The outer ear canal can be modelled as a tube closed at one end (Figure 9.14). Assuming that this frequency corresponds to the fundamental frequency, what is the length of the outer ear canal of a human?

5 The frequency of the maximum sensitivity for a domestic cat is different from that of humans. Assuming all other factors are the same, the frequency depends on the length of the ear canal. Will the frequency of maximum sensitivity for the cat be higher or lower than that for humans? Explain your answer.

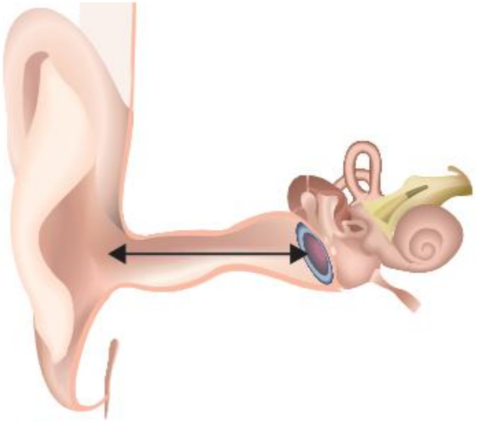


FIGURE 9.14 The human outer ear canal can be modelled as a tube closed at one end.

9.7 Beats and the Doppler effect

The wave nature of sound gives rise to phenomena such as beats and the Doppler effect.

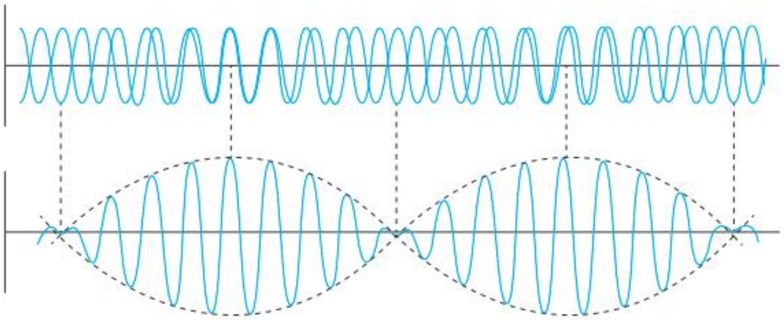
The superposition of sound producing beats

Superposition of waves

The superposition of waves was discussed in chapter 8. A very useful phenomenon arises due to superposition when two sounds are produced simultaneously.

If two sound sources have slightly different frequencies, the resultant wave will alternate between constructive and destructive interference, growing and shrinking with a third frequency known as the **beat frequency**. This beat frequency is equal to the difference between the two source frequencies. Experienced musicians and piano tuners use this phenomenon to tune their instruments. Figure 9.15 (page 260) shows the resultant sound wave produced when two component sound waves with slightly different frequencies superpose.

FIGURE 9.15 When two sound waves with slightly different frequencies superpose, the resultant sound wave grows and shrinks with time – its beat frequency. The dashed lines are the 'wave envelope' of the sound that is heard.


Two sound waves with slightly different frequencies coinciding

Resultant sound wave– the sound that is heard, with the wave envelope shown as a dashed line

Beat frequency
Various beat frequency animations can be heard here.

Mathematically, the beat frequency can be represented as:

$$f_{\text{beat}} = |f_2 - f_1|$$

where f_2 and f_1 are the two source frequencies.

Beats are sometimes heard from aircraft or boats with two engines operating at slightly different speeds. Many people would not notice this effect, but when you are listening for them, beats can be heard in many situations where two sound sources are involved.

WORKED EXAMPLE 9.8

A musical instrument is played with a frequency of 359 Hz. Another instrument is played at the same time with a frequency of 361 Hz. What beat frequency is heard?

ANSWER	LOGIC
$f_1 = 359\text{ Hz}; f_2 = 361\text{ Hz}$	Identify relevant data in the question.
$f_{\text{beat}} = \|f_2 - f_1\|$	Identify the appropriate formula.
$= 361\text{ Hz} - 359\text{ Hz}$	Substitute known values, with units, into the formula.
$f_{\text{beat}} = 2\text{ Hz}$	Calculate the answer and express with correct significant figures and units.

TRY THESE YOURSELF

1 Two sounds are played simultaneously. A beat frequency of 5 Hz is heard. One of the component frequencies is known to be 500 Hz. What could be the frequencies of the second sound?

2 What beat frequency will be heard if two sounds are played at the same time, one with a frequency of 348 Hz and the other 372 Hz?

INVESTIGATION 9.8

Critical and creative thinking

Beats

AIM

To observe beats in sound

MATERIALS

- 2 signal generators with speakers (or phone apps with external speakers)

»

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Electrical equipment is dangerous if water is present.	Devices plugged into 240 V mains power should be kept well away from water.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Set each signal generator to a frequency of 400 Hz. Listen to the resulting sound produced.
2. Now set the first signal generator to 399 Hz while keeping the second at 400 Hz. Listen and observe the sound created.
3. Change the frequency of the first signal generator to 398 Hz. Listen and observe again.
4. Set the signal generators' frequencies to different values a few Hz apart. Listen and observe the beating.
5. Measure the beat frequency heard by counting or tapping every peak in sound intensity over a period of 10 seconds, then divide by 10. This reduces the error in the measurement.
6. Change the signal generator's frequencies so that they are more than 20 Hz apart, and listen carefully.

RESULTS

Record your observations for each of the settings used.

ANALYSIS OF RESULTS

1. How did the beat frequency relate to the difference between the two source frequencies?
2. Describe what was heard as the difference between the two source frequencies was increased to beyond 20 Hz.

DISCUSSION

1. With the aid of a diagram, explain the cause of the phenomenon you have just observed.
2. Explain why beat frequencies are not heard when the difference between the source frequencies is greater than 20 Hz.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

The Doppler effect

When the distance between the observer and the source of a sound is changing, the time between successive compressions and rarefactions of sound waves reaching the observer also changes. If this distance is decreasing, successive compressions will be encountered sooner than if there was no relative motion. This results in the apparent (or observed) frequency of the sound being higher than the frequency of the source. If the distance is increasing, it takes longer for successive compressions to reach the observer, so the observed frequency is lower than the source.

When the source of a sound is moving closer to you, the soundwaves become closer together. The source of the sound waves 'chases' the waves it is emitting – the wavelength decreases. This results in the frequency you hear being increased. This increased frequency is heard as a higher pitched sound. Recall that $v = f\lambda$, and if the velocity of the sound in the air is constant, a decrease in the wavelength results in an increase in the frequency.

This phenomenon is called the **Doppler effect**. For sound, it can be noticed when a fast-moving noisy source approaches and passes an observer. An ambulance siren, racing motorbikes or aircraft doing a low-altitude flypast are typical examples of when the Doppler effect is observed.

Figure 9.16 shows sound waves from a siren moving from left to right. Observers to the right of the siren observe the siren approaching and hear a higher pitch. Observers to the left of the siren observe the siren moving further away and hear a lower pitch.

In the example shown in Figure 9.16, for a stationary observer to the right of the siren, the velocity of the siren, v_s, is towards the observer. The frequency observed, f', is given by:

$$f' = \frac{v}{(v - v_s)} f \text{, as } f' > f.$$

Table 9.2 shows the Doppler effect formula for the other possibilities involving the relative motion of the source and the observer.

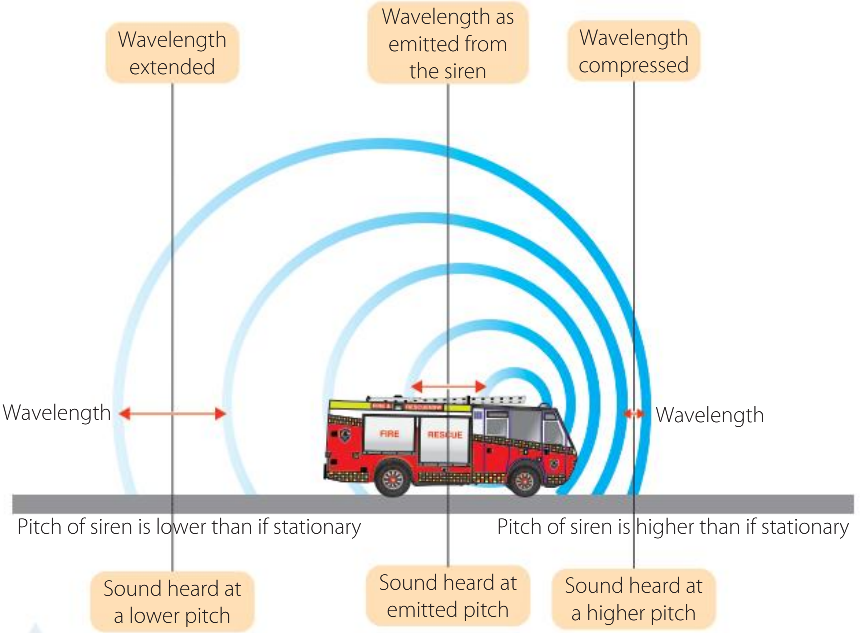


FIGURE 9.16 The Doppler effect for a siren moving from left to right. The siren 'chases' the waves it emits, causing a decrease in the wavelength of the sound waves measured by an observer to the right of the siren. The reverse occurs for an observer to the left of the siren as the siren moves away.

WS
Sound summary

TABLE 9.2 Motion of the source of sound and the observer and the relationship between the source frequency, *f*, and the observed frequency, *f′*

MOTION OF OBSERVER	MOTION OF SOURCE	f' COMPARED WITH f	FORMULA
stationary	towards observer	$f' > f$	$f' = \frac{v}{(v - v_s)} f$
stationary	away from observer	$f' < f$	$f' = \frac{v}{(v + v_s)} f$
towards source	stationary	$f' > f$	$f' = \frac{(v + v_o)}{v} f$
away from source	stationary	$f' < f$	$f' = \frac{(v - v_o)}{v} f$

Whenever the distance between the source and the observer is decreasing, $f' > f$. When this distance is increasing, $f' < f$.

If the speeds of the source, v_s, and the observer, v_o, are known, the observed frequency f' can be calculated using:

$$f' = f \frac{(v_{\text{wave}} + v_{\text{observer}})}{(v_{\text{wave}} - v_{\text{source}})}$$

where v is the speed of sound and f is the original source frequency. In our calculations, we will use the velocity of sound in air as $v = 340 \text{ m s}^{-1}$.

An interesting consequence of relative motion between source and observer is that if the source of the sound is moving faster than the speed of sound, no sound will be heard before the source passes the observer. The source 'chases' and overtakes the sound waves it is emitting. This can be seen as an animation in the weblink.

Doppler effect

Animations showing the Doppler effect in action as the source and/or the observer move relative to each other.

WORKED EXAMPLE 9.9

The siren from an approaching ambulance has a frequency of 800 Hz. The ambulance is travelling at 25.0 m s^{-1}. A car is moving towards the approaching ambulance at 15.0 m s^{-1}. What is the frequency of the sound heard by an occupant of the car?
Hint: refer to Table 9.2 and combine both of the appropriate cases into the one formula. Use common sense to determine the sign of both v_s and v_o.

ANSWER	LOGIC
$f = 800$ Hz; $v = 340$ m s^{-1}; $v_o = 15.0$ m s^{-1} $v_s = 25.0$ m s^{-1}	▪ Identify relevant data in the question.
$f' = \frac{(v + v_o)}{(v - v_s)} f$	▪ Identify the appropriate formula.
$= \frac{340 \text{ m s}^{-1} + 15.0 \text{ m s}^{-1}}{340 \text{ m s}^{-1} - (25.0 \text{ m s}^{-1})} \times 800 \text{ Hz}$	▪ Substitute the known values, with units, into the formula.
$f' = 902$ Hz	▪ Calculate the answer and express with correct significant figures and units.

TRY THESE YOURSELF

1 A racing car is moving towards a stationary observer with a speed of 60.0 m s^{-1}. Its engine is emitting a sound with a frequency of 1.80 kHz. What frequency is heard by this observer?

2 The same racing car is now moving away from the observer at 80.0 m s^{-1}, emitting the same frequency. What frequency is heard by the observer?

KEY CONCEPTS

- When two sounds waves with slightly different frequencies superpose, a third frequency, the beat frequency, is heard.
- The beat frequency, f_{beat}, is the difference between the two component waves' frequencies: $f_{beat} = |f_2 - f_1|$.
- The Doppler effect occurs when there is relative motion between the source of the sound and the observer.
- An increase in observed frequency occurs when the source and the observer are approaching each other.
- When there is relative motion between the source and the observer, the observed frequency is given by: $f' = f\frac{(v_{wave} + v_{observer})}{(v_{wave} - v_{source})}$.

Sonic booms

Information relating to the Doppler effect and sonic booms, showing how a source moving faster than the speed of sound overtakes the sound waves it is emitting.

CHECK YOUR UNDERSTANDING 9.7

1 By referring to the Doppler effect formula, explain why a source of sound needs to be moving quickly before the Doppler effect for sound is observed.

2 A sound source emitting a frequency of 500 Hz is moving towards a stationary observer at 200 m s^{-1}. What is the frequency of the observed sound?

3 Would the Doppler effect be observed if the source of the sound and the observer are both moving at the same speed in the same direction? Apply this situation to the Doppler effect formula to test your hypothesis.

4 With reference to the Doppler effect, explain why the pitch of an approaching ambulance siren decreases after it passes an observer.

5 If a source of sound is moving away from the observer faster than the speed of sound, will the observer hear anything? Check your answer by applying the Doppler effect formula.

9 CHAPTER SUMMARY

- The pitch of a sound is related to the frequency of the sound wave.
- The loudness of a sound is related to the amplitude of the sound wave.
- An oscilloscope's time-base axis, the horizontal axis, can be used to measure the period and calculate the frequency of a sound wave.
- A sounding box can be used to amplify the sound caused by a vibrating tuning fork.
- Sound is a longitudinal wave with back-and-forth vibrations of air particles. These vibrations result in compressions and rarefactions in air.
- In air, compressions are regions of higher pressure and rarefactions are regions of lower pressure.
- Intensity of sound, I, is a measure of the amount of sound energy passing through a unit area per second.
- Intensity of sound is inversely proportional to the square of the distance from a point source of sound.
- The inverse square law is $I \propto \frac{1}{d^2}$.
- Using the inverse square law, $\frac{I_1}{I_2} \propto \left(\frac{d_2}{d_1}\right)^2$.
- Sound is reflected from hard surfaces.
- Sound is diffracted when passing through an opening.
- Long wavelength sound waves diffract more than short wavelength sound waves.
- A vibrating string has a node at both ends.
- The fundamental mode of vibration in a string is also the first harmonic.
- The wavelength, λ, of the fundamental mode is $2l$, or twice the length of the string.
- For harmonics in a string, $l = n\frac{\lambda_n}{2}$ where $n = 1, 2, 3, \ldots$
- The speed, v, of a wave in a string is determined by the length of the string, l, the tension, T, and the mass per unit length, $m\,l^{-1}$, according to the relationship: $v = \sqrt{\frac{T}{m\,l^{-1}}}$.
- The fundamental frequency of vibration in a string, f_1, is given by $f_1 = \frac{v}{2l}$.
- A node exists at a closed end of a pipe.
- An antinode exists at the open end of a pipe.
- The fundamental mode of vibration is also the first harmonic.
- A closed pipe has only odd harmonics; $f_n = nf_1$ where $n = 1, 3, 5, \ldots$
- The length of a closed pipe is $l = (2n-1)\frac{\lambda}{4}$.
- When two sounds waves with slightly different frequencies superpose, a third frequency, the beat frequency, is heard.
- The beat frequency, f_{beat}, is the difference between the two component waves' frequencies: $f_{\text{beat}} = |f_2 - f_1|$.
- The Doppler effect occurs when there is relative motion between the source of the sound and the observer.
- An increase in observed frequency occurs when the source and the observer are approaching each other.
- When there is relative motion between the source and the observer, the observed frequency is given by:

$$f' = f\frac{(v_{\text{wave}} + v_{\text{observer}})}{(v_{\text{wave}} - v_{\text{source}})}.$$

9 CHAPTER REVIEW QUESTIONS

Review quiz

In all these review questions use 340 m s^{-1} as the value for the speed of sound.

1 For a soundwave, outline the relationship between:

a pitch and frequency.

b loudness and amplitude.

2 Describe a way in which a sound wave could be modelled in a school laboratory.

3 Outline the meaning of 'superposition'.

4 Using a sketch, explain what compressions and rarefactions are.

5 **a** Why are sound waves often represented as transverse waves in diagrams?

b What do the crests and troughs in the transverse wave diagrams represent in sound waves?

6 Using a sketch, explain the difference between a 'closed' air column and an 'open' air column.

7 What length of air column, closed at one end, will have a fundamental frequency of 256 Hz?

8 How does the slide on a trombone enable different notes to be played?

9 Pipe organs make use of both open and closed pipes. Calculate the frequencies of the first three harmonics of an organ pipe of length 0.50 m that is:

- **a** closed at one end.
- **b** open at both ends.

10 The length of a vibrating guitar string is 60 cm.

- **a** What is the wavelength of the fundamental mode of vibration?
- **b** If the speed of the wave in the guitar string is 360 m s^{-1}, what is its fundamental frequency?
- **c** If the vibrating length of the guitar string is increased, does the fundamental frequency increase or decrease? Give a reason for your answer.

11 The velocity of the transverse waves in a guitar string is 350 m s^{-1}. When the guitar string is plucked, it vibrates with a fundamental frequency of 330 Hz.

- **a** What is the wavelength of the standing waves in the plucked guitar string?
- **b** What is the length of the guitar string?
- **c** How far apart are the nodes in the standing waves in the string?

12 A clarinet acts as a tube closed at one end (the mouthpiece) and open at the other end. Vibration of the air column is produced in the mouthpiece. A particular clarinet has a fundamental frequency of 150 Hz.

- **a** What is the wavelength of this sound?
- **b** Which one of the graphs (A–F) of displacement amplitude plotted against distance from the open end, in Figure 9.17, best illustrates the amplitude of vibration of air particles at the fundamental frequency of 150 Hz? (On the distance axis, X is the open end of the clarinet and Y is the mouthpiece.)
- **c** What is the length of this model of clarinet?
- **d** What other frequencies are possible in this model?

13 Using a simple diagram, show how a fast-moving source of sound could overtake the sound waves that it is emitting.

14 Describe the reason for the formation of beat frequencies when two sounds close in frequency are played simultaneously.

15 A plastic tube open at both ends can produce a musical note if whirled in a horizontal circle.

- **a** The effective length of one such tube is 1.2 m. What would be the fundamental frequency of the note produced when the tube was whirled in a horizontal circle?
- **b** What is the frequency of the first two overtones?
- **c** Which one of the following best describes the displacement nodes for the fundamental mode of the vibration?
  - **A** There is only one displacement node in the tube, and this is at one end.
  - **B** There is only one displacement node in the tube, and this is in the centre.
  - **C** There are displacement nodes at each end, and none in between.
  - **D** There are two displacement nodes in the tube, at 0.40 m and 0.80 m from one end.
- **d** Explain why only a few frequencies are audible even though there will be a range of frequencies produced by the tube.
- **e** What difference would it make if a longer tube was used?

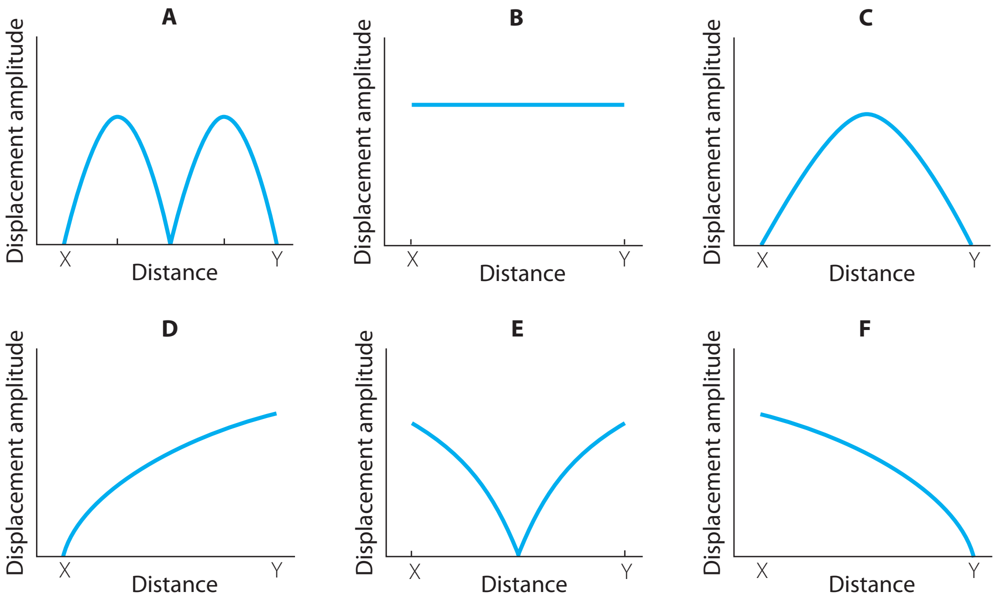


FIGURE 9.17

16 What is the beat frequency heard when sounds with frequencies of 344 Hz and 347 Hz are played simultaneously?

17 Estimate the length of a didgeridoo, and use this estimate in the following questions.

- **a** Calculate the fundamental wavelength of the sound waves produced by the didgeridoo.
- **b** What is the fundamental frequency of the didgeridoo?

18

- **a** Draw a diagram of a didgeridoo resonating at its fundamental frequency. On the diagram, show the pressure variation and displacement variation.
- **b** Explain why the air at the closed end is a pressure antinode and a displacement node.

19 When a loud clap is made in a room with large glass surfaces, the sound is heard to be prolonged, or reverberates. Explain how this is caused by the reflection of sound.

20 A student finds the speed of sound by measuring how long it takes the echo of a short clap to return from a wall 25.0 ± 0.1 m away. The time measured is 0.15 ± 0.02 s.

- **a** What is the speed of sound measured by the student?
- **b** What is the absolute uncertainty in the result?
- **c** What result should the student report?
- **d** Is the student's result consistent with the reported value for the speed of sound in air of (341 ± 5) m s^{-1}?

21 The Sydney Harbour Tunnel is 2.25 km in length.

- **a** What is the fundamental resonance frequency for sound in the tunnel? Assume the tunnel functions as a pipe open at both ends.
- **b** Did the designers of the tunnel need to take this resonance into consideration in their design? By referring to the range of frequencies that humans can hear (20 Hz–20 kHz), explain your reasons.

22 With the aid of a diagram, explain why the pitch of a sound increases when the source is approaching the observer.

23 What frequency is heard when a source emitting a 700 Hz sound is moving away from an observer with a relative speed of 120 m s^{-1}?

24 Explain why the Doppler effect is not noticed when a person talking to you is walking towards you.

25 Figure 9.18 shows wavefronts moving away from a point source of sound energy.

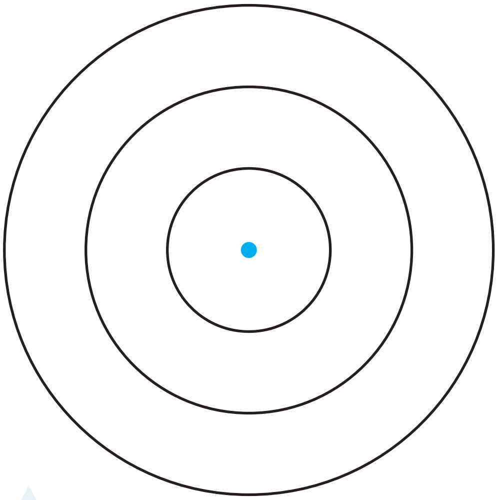


FIGURE 9.18

By measuring the radius (*r*) and the circumference (*c*) of each wavefront shown, show how the inverse square law applies to the intensity of the sound as its distance from the source varies.

26 A standing wave in a string 40 cm in length, as shown in Figure 9.19, produces a sound wave with a frequency of 500 Hz.

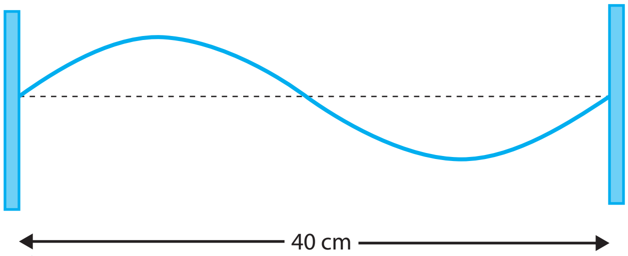


FIGURE 9.19

- **a** What is the wavelength of the standing wave?
- **b** Find the fundamental frequency for this string.
- **c** Calculate the frequency of the next overtone in this string.

10 Ray model of light

INQUIRY QUESTION

What properties can be demonstrated when using the ray model of light?

OUTCOMES

Students:

- conduct a practical investigation to analyse the formation of images in mirrors and lenses via reflection and refraction using the ray model of light (ACSPH075)
- conduct investigations to examine qualitatively and quantitatively the refraction and total internal reflection of light (ACSPH075, ACSPH076)
- predict quantitatively, using Snell's law, the refraction and total internal reflection of light in a variety of situations CCT
- conduct a practical investigation to demonstrate and explain the phenomenon of the dispersion of light CCT
- conduct an investigation to demonstrate the relationship between inverse square law, the intensity of light and the transfer of energy (ACSPH077)
- solve problems or make quantitative predictions in a variety of situations by applying the following relationships to: CCT N
  - $n_x = \frac{c}{v_x}$ for the refractive index of medium x, v_x is the speed of light in the medium
  - $n_1 \sin(i) = n_2 \sin(r)$ (Snell's law)
  - $\sin(i_c) = \frac{1}{n_x}$ for the critical angle i_c of medium x
  - $I_1 r_1^2 = I_2 r_2^2$ to compare the intensity of light at two points, r_1 and r_2

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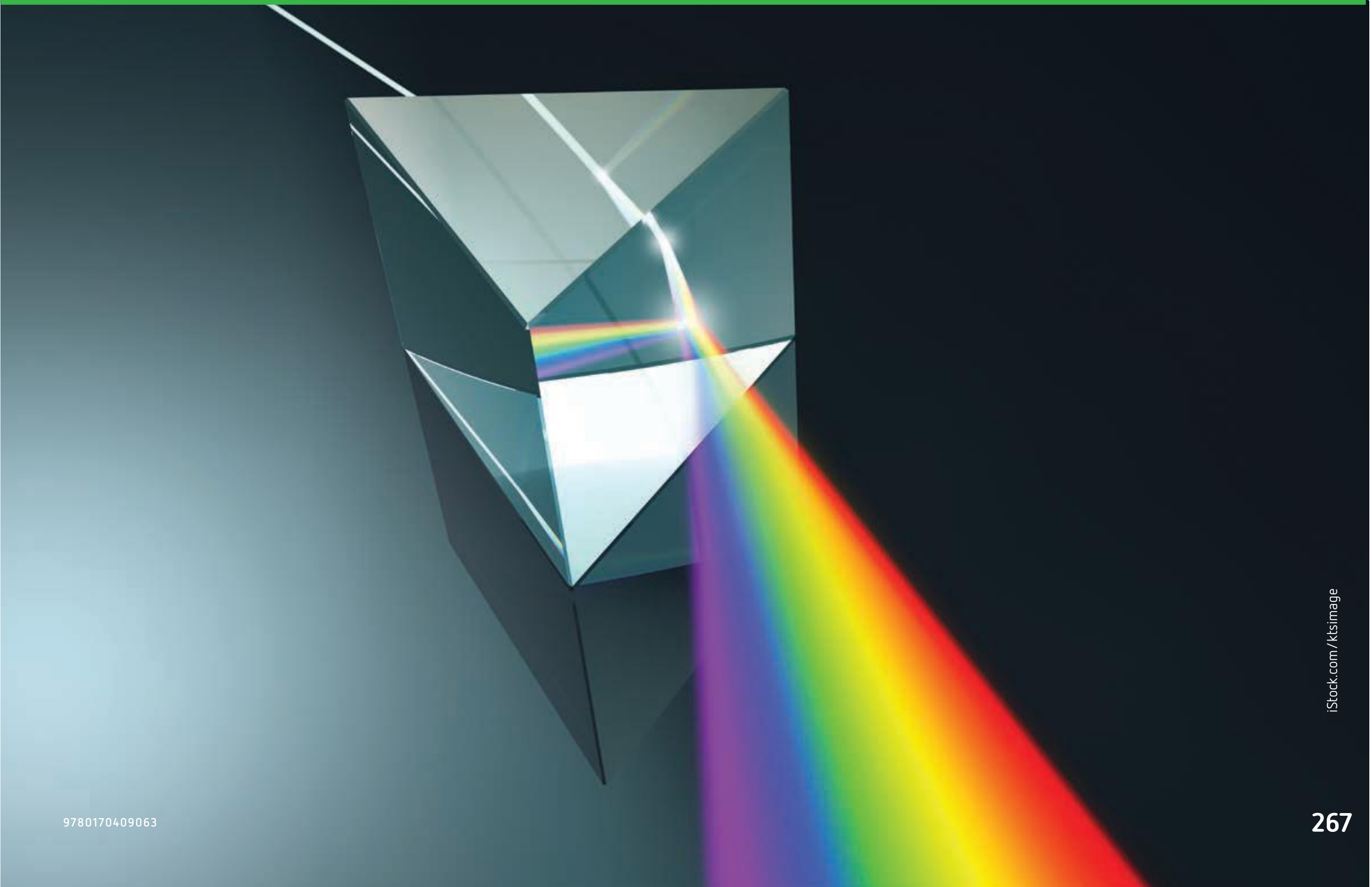

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Prior knowledge

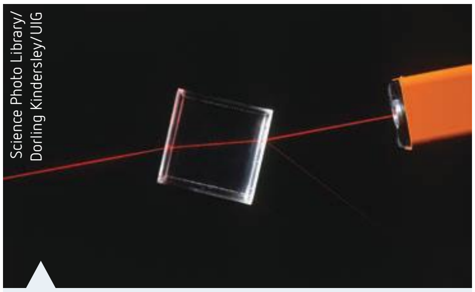

Science Photo Library/ Dorling Kindersley/UIG

FIGURE 10.1 A light ray travels in a straight line unless it is refracted by a prism.

The ray model of light states that light can be modelled as a series of rays (lines) that can be represented on a diagram. These rays are perpendicular to the wavefronts. Of course, light can still be a broad glow, such as that given off by a globe. The ray model largely ignores the wave nature of light, but it is useful in a number of ways. In explaining the formation of images using mirrors and lenses, the ray model allows for the use of simple line drawings showing the reflection and refraction that occurs as light is incident on (strikes) mirrors or as it passes through lenses.

The path of refracted light can be predicted using Snell's law. Total internal reflection is a phenomenon essential for modern fibre optic communications. This too can be explained and shown using the ray model of light.

10.1 Images in mirrors and lenses

For many centuries, mirrors and lenses have been used to manipulate light. In today's society, the manipulation of light to form images using mirrors and lenses is used in a huge variety of applications. Examples include rear-view mirrors in cars and sophisticated endoscopes used by surgeons. Reading glasses use the refraction of light to assist a person's eyes. Huge mirrors, metres in diameter, are used in telescopes to probe the depths of our universe. The ray model of light can be used to help analyse the formation of these images.

Reflection and the ray model

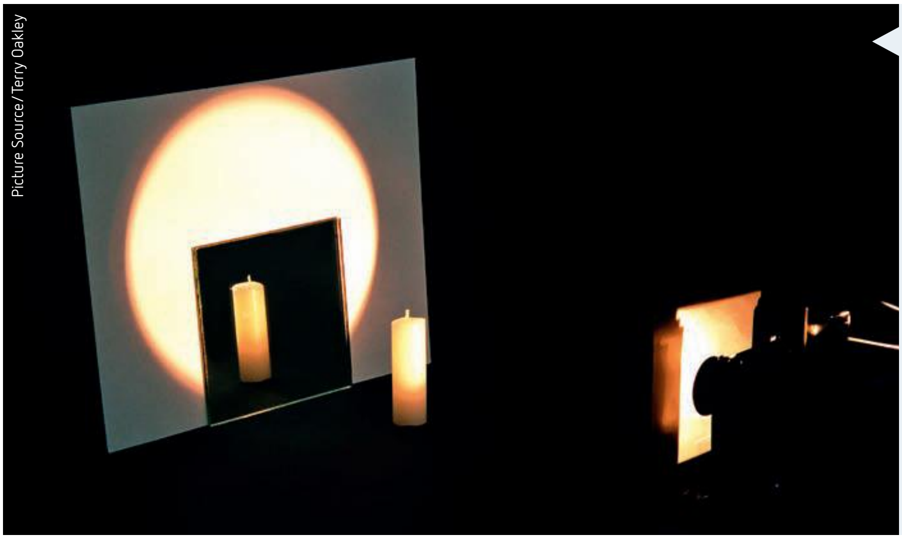
When a beam of light is incident on a smooth, polished surface, such as a plane mirror or a still water surface (such as shown in Figure 10.2), the rays of light forming the beam are reflected in a predictable and regular way. A clear image of the reflecting light from objects in the scene can be observed.

Most surfaces reflect incident light in all directions. This is known as **diffuse reflection**. For example, a sheet of paper or a painted wall appears smooth, but a microscopic examination will show that it is

FIGURE 10.2 An almost-perfect reflection in a still pool of water


Shutterstock.com/puttsk

rough when compared with the wavelength of the light. Parallel rays incident on a rough surface are scattered in all directions. This is a particularly important property as it allows objects to be visible from many different angles. Figure 10.3 illustrates both diffuse and regular reflection.



Picture Source/Terry Oakley

FIGURE 10.3 Diffuse and regular reflection from a mirror in front of a white piece of paper. The mirror is mainly dark because light is reflected, but not into the camera, while the paper reflects light in all directions, including towards the camera. The candle and its image are recorded by diffuse reflection to the camera.

The quantity of light reflected from a surface depends on the nature of the surface and the direction of the incident light. A good quality mirror, made by backing a sheet of glass with a thin layer of metal, reflects about 95% of the incident light. Optical fibres totally internally reflect more than 99% of the incident light.

Law of reflection

Reflection from surfaces always follows the law of reflection, as discussed in chapter 8. This is true for regular and diffuse reflection; however, it is easier to observe regular reflection, also called **specular reflection**.

1. The incident ray, the normal perpendicular to the surface and the reflected ray all lie in the same flat plane.
2. The angle of incidence equals the angle of reflection: $\theta_i = \theta_r$.
3. The law of reflection applies at each point on a surface.

Figure 10.4 compares reflection from a smooth surface and from a rough surface.

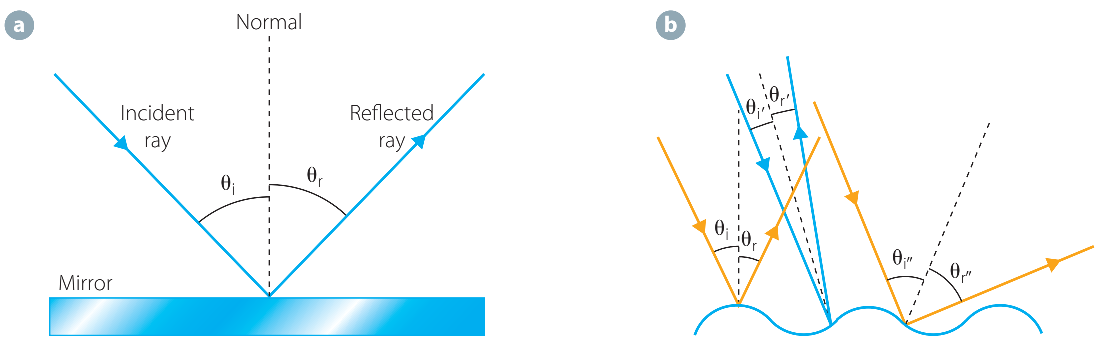


FIGURE 10.4 Reflection of light showing solid lines for rays of light and a dashed line representing the normal to the surface at the point of reflection. **a** Regular reflection. **b** Diffuse reflection, showing parallel incident rays being reflected at different angles. Angles θ_i, θ_i' and θ_i'' represent three different angles of incidence.

Reflection using the ray model

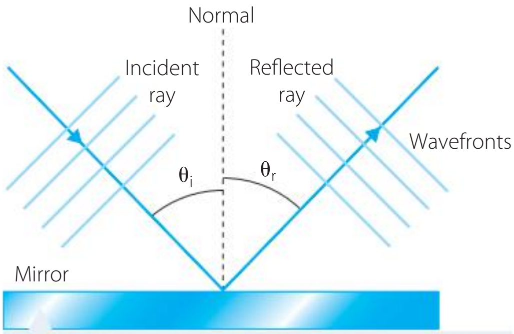


FIGURE 10.5 Using the wave model, the ray is perpendicular to the wavefronts. For clarity, the waves interacting at the surface are not shown.

If we measure the angle between the surface and the reflected ray, and the surface and the incident ray, we find that the angles are equal. This applies at any point on a surface, as shown in Figure 10.4. Figure 10.4a shows the simple case of a flat surface, and hence is regular reflection. Figure 10.4b shows the case of a rough surface, and hence shows diffuse reflection. In both cases, the angle between the surface and the incident ray, and the surface and the reflected ray, are equal at the point where reflection occurs.

The law of reflection states that the angle of reflection is always equal to the angle of incidence.

Figure 10.5 shows this law using light represented as a series of wavefronts. The directions of travel of the incoming and reflected wavefronts are shown by the rays drawn perpendicular to the wavefronts. Rays are drawn perpendicular to the wavefronts.

Images formed in plane mirrors

Light radiates from a point source in all directions. When the rays strike a plane mirror, they reflect ($\theta_i = \theta_r$). They appear to come from an image point, a virtual image, behind the mirror. The rays that enter our eyes affect our retinas. Reflected rays form a **real image** in our eyes. We perceive a virtual image of the object to be where it is not physically present.

A **virtual image** is an image that is formed when rays of light appear to diverge from a point where an object does not actually exist. Virtual images cannot be cast onto a screen in the way that real images can. Rays drawn on a ray diagram do not pass through a virtual image. For a real image, rays of light diverge from the actual object being observed. The virtual image appears at the same distance behind the mirror surface as the object is in front. The image is the same size as the object; that is, the magnification $M = 1$.

Magnification, M, is defined as being the ratio of the height of the image, h_i and the height of the object, h_o.

$$M = \frac{h_i}{h_o}$$

WORKED EXAMPLE 10.1

An object that is 5.0 cm high has an image formed that appears to be 15.0 cm high. Calculate the magnification.

ANSWER	LOGIC
$h_o = 5.0\text{ cm}; h_i = 15.0\text{ cm}$	- Identify the relevant data in the question.
$M = \frac{h_i}{h_o}$	- Identify the appropriate formula.
$= \frac{15.0\text{ cm}}{5.0\text{ cm}}$	- Substitute the known values, with units, into the formula.
$M = 3.0$	- Calculate and express the final answer with the correct significant figures. - Note that as magnification is a ratio, it does not have units.

TRY THESE YOURSELF

1 A virtual image of an object is 20 cm high. The magnification $M = 4$. What is the height of the object?

2 What magnification is needed so that an object 1.5×10^{-4} m in diameter appears to have a diameter of 3.0 mm?

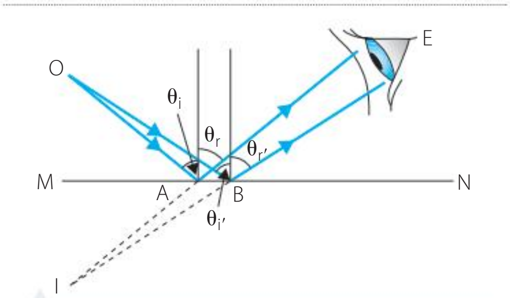


FIGURE 10.6 Reflected rays are perceived to be coming from behind the mirror. The image is virtual because the rays do not pass through the image. A real image is formed on the retina of the eye or in a camera.

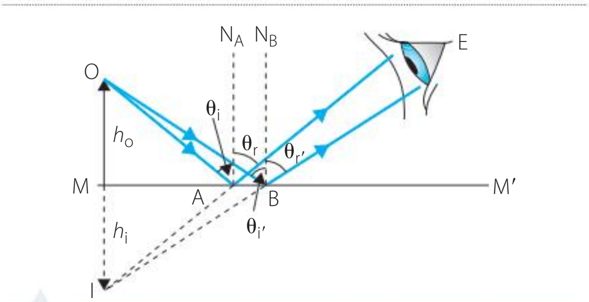


FIGURE 10.7 Geometric construction to show virtual image formation in a mirror. The line M–M′ is the mirror surface, h_o the height of the object, h_i the height of the image. N_A and N_B are the normals drawn from the surface of the mirror where the light rays are being reflected.

Figure 10.6 shows how an image is formed and seen by an observer. Rays of light from the object, O, travel to the mirror and reflect such that the angle of incidence is equal to the angle of reflection. Two rays are shown, which reflect at points A and B. When we look towards points A and B on the mirror, it appears that light is coming from these points. If we extend the rays behind the mirror, they intersect at point I behind the mirror. Point I is the position of the image.

Figure 10.7 shows a geometric construction using the law of reflection that allows us to find the magnification and position of the image. This sort of diagram is called a **ray diagram**. We draw our object as having some actual size, such as the arrow. We draw two rays coming from the top of the object and reflecting from the mirror. The rays must obey the law of reflection, as shown. We again extend the reflected rays behind the mirror to the point at which they intersect. This point corresponds to the top of the image, the arrowhead. Our object has a height equal to the distance between the mirror, M, and point O. The image has a height equal to the distance between the mirror, M, and point I. The ratio of these distances is the magnification, $M = \frac{h_i}{h_o}$. For a plane mirror, $M = \frac{h_i}{h_o} = 1$. For curved mirrors, the magnification may be greater or less than 1.

INVESTIGATION 10.1

Image formation in plane mirrors

AIM

To analyse the formation of an image in a plane mirror

MATERIALS

- Small plane mirror
- A4 paper
- Matchsticks (or small pencils)
- Adhesive such as blu-tack
- Pen, pencil and ruler
- Camera (or mobile phone, laptop or tablet)

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Mirrors may break if dropped, producing shards of sharp glass.	Take care handling glass mirrors and keep them away from the edges of desks.

RISK ASSESSMENT

What other risks are associated with your investigation, and how can you manage them?

»

»

METHOD

1. Place the mirror vertically in the middle of a sheet of paper. Use an adhesive to hold the mirror up if necessary.
2. Place the matchstick in a blob of adhesive so it stands vertically.
3. Move the matchstick so it is about 10 cm in front of the mirror.
4. Move your eyes down to the level of the desk and observe the image of the matchstick. Does it appear to be in the mirror or behind it?
5. Place a second matchstick behind the mirror so that the top of the matchstick that is visible above the mirror is aligned with the image and appears the same size.
6. Measure the distances from each matchstick to the mirror.
7. Photograph the apparatus as it is.
8. Compare the two distances found in step 6.
9. Repeats steps 3–7 using different positions for the matchstick.

ANALYSIS OF RESULTS

Construct a ray diagram of the apparatus showing the formation of the image.

DISCUSSION

1. Is the image formed virtual or real? Give reasons for your answer.
2. What is the magnification of the image formed in the mirror in this investigation?

CONCLUSION

By considering the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- The ray model of light models light as a series of rays, or lines, drawn perpendicular to the wavefronts.
- Reflection of light can be either diffuse or specular.
- Ray diagrams are construction diagrams showing the path of light as rays.
- The law of reflection states that $\theta_i = \theta_r$.
- All angles are measured away from the normal.
- Mirrors can be used to form virtual images.
- Magnification, M, is the ratio of the height of the image, h_i and the height of the object, h_o: $M = \frac{h_i}{h_o}$.
- The magnification for plane mirrors is $M = 1$.

CHECK YOUR UNDERSTANDING

10.1a

1. Write the statement for the law of reflection. Draw a diagram to show this law.
2. Use the ray model of light to illustrate diffuse reflection.
3. A ray from a single point strikes a plane mirror at an angle of incidence of 30°. Use a carefully measured diagram to show that the object and the image are equidistant on opposite sides of the mirror.
4. How do we see a virtual image in a plane mirror? Use a ray diagram to assist in your explanation.
5. Object O is placed in front of a plane mirror, as shown in Figure 10.8.
   a. Copy the diagram and locate the position of the image by constructing appropriate waves on the diagram.

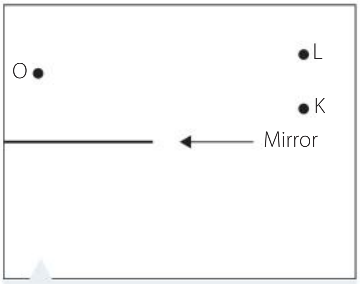


FIGURE 10.8

b Construct a ray model diagram to show that an observer at L can see the image of O.

c From the point of view of an observer moving from L to K, how does the image of O move?

6 The eyes of a 170 cm tall woman are 160 cm above the ground. She stands 0.60 m in front of a plane mirror that is mounted vertically and sees her entire image. What is the shortest mirror that can be used for such a purpose? Illustrate your answer with a diagram.

7 Prove that the image is exactly the same distance behind a flat mirror as the object is in front of the mirror.

8 State two ways that reflection affects your everyday experiences.

Images formed by refraction and reflection

Lenses are shaped, **transparent** objects. They may be convex, such as the cornea and lens in the eye, or concave. **Converging (convex) lenses** are thicker at the centre and thinner at the edges. **Diverging (concave) lenses** are thicker at the edges than at the centre. Lenses can produce real or virtual images by refraction. Figure 10.9 shows a variety of lens shapes and the names of these shaped lenses.

Curved mirrors can be concave or convex. They can produce real or virtual images by reflection. Figure 10.10 shows a variety of different mirror shapes. The black line represents the surface of the mirror.

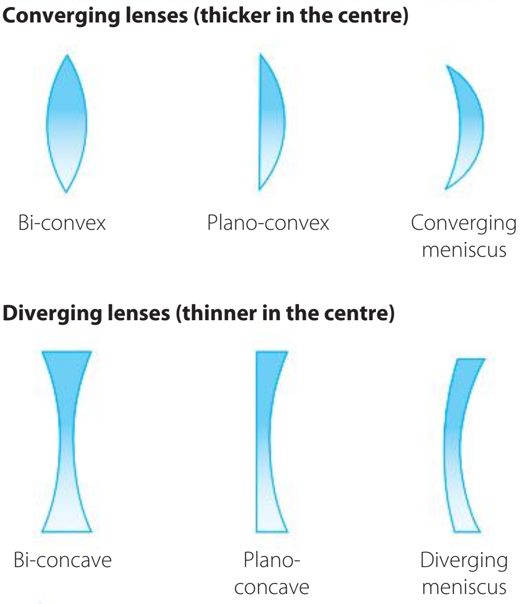


FIGURE 10.9 Different shapes of converging and diverging lenses, with names

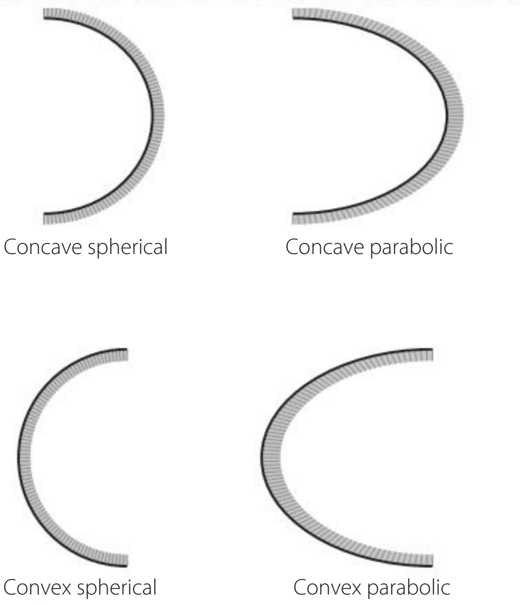


FIGURE 10.10 Different shapes of converging and diverging mirrors, with names

Images formed by refraction in lenses

Figure 10.11 (page 274) shows how rays of light are refracted in converging (convex) and diverging (concave) lenses. The **principal axis** is drawn perpendicular to the lens, passing through its centre. The converging lens refracts parallel incoming rays towards the principal axis. The rays converge and cross at the focal point (F). The focal point is on the opposite side of the lens to the source of the light. The converging lens forms a real image on the opposite side of the lens to the object. Remember, a real image is formed when the rays of light are actually coming from the point they appear to be coming from. A screen placed at this point will have an image on it, and a photodetector or camera placed at this point will detect light.

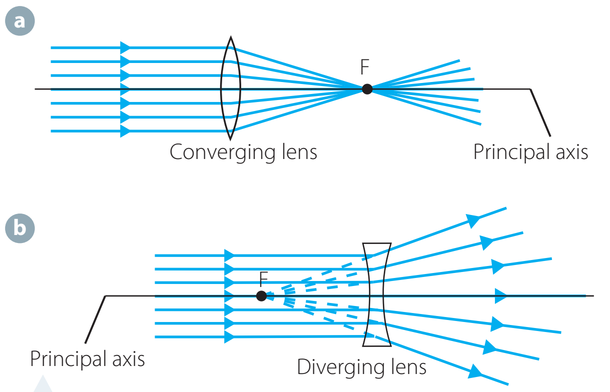


FIGURE 10.11 Rays that are parallel to the principal axis refract to: **a** a real focus (F) in a converging lens; **b** a virtual focus in a diverging lens.

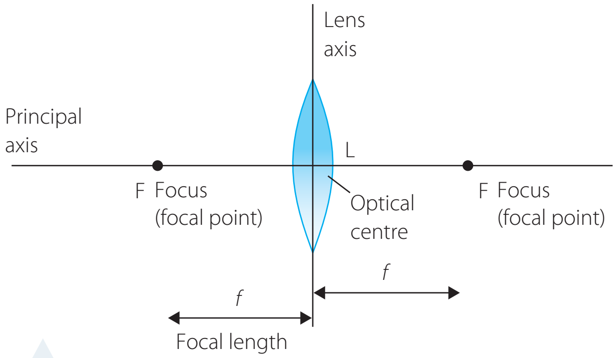


FIGURE 10.12 Geometry of the convex lens image-forming system showing lens axis, principal axis, optical centre and two foci

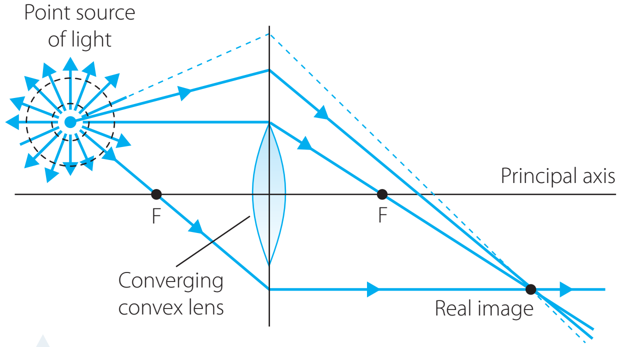


FIGURE 10.13 Some of the many rays that leave an object shown as a point source. These rays represent the waves emanating from the point in all directions.

The diverging lens (Figure 10.11b) refracts light so that the parallel rays diverge, and do not cross each other on the far side of the lens from the source. However, if we trace the rays backwards from the right-hand side of the lens, we see that they appear to originate from a focal point (F) on the same side of the lens as the object from which the parallel rays of light are originating. A diverging lens forms a virtual image, which is an image formed at a position where the light rays do not actually converge. A photodetector or camera placed at this point (labelled F in Figure 10.11b) will not detect light, nor will a screen show an image here. This is similar to the way a plane mirror forms a virtual image. The image still exists, and can be seen and photographed, it is just not due to light coming from the image position. Instead, the light making the image is being collected by the lens to form a real image in the camera.

Our eyes contain a convex lens that converges the rays of light incident on our eye to a focal point. If the lens and cornea of the eye forms an image in front of or behind the retina (rather than exactly on the light-sensitive cells of the retina), the result is blurry vision.

When the image is formed in front of the retina, this is usually because the lens and cornea are too strongly converging, or the eyeball is too long. The result is that the person can focus on objects up close, but not far away. This is called myopia (short- or near-sightedness). When the image is formed behind the retina, usually because the lens is not converging enough, the person can focus on objects far away but not up close. This is a called hyperopia (long-sightedness). When it is due to ageing of the eyes and the muscles that control the shape of the lens, it is called presbyopia (literally 'old-age vision').

Ray tracing biconvex lenses

Figure 10.12 shows the geometry of a biconvex lens system. The principal axis is a line that passes through the centre of the lens at a right angle to the plane in which the lens stands. The lens axis passes through the centre of the lens as shown. The **optical centre** is the point at which these two axes cross. The focal point (or focus) F nearest the object is at the **focal length**, *f*. The other focal point F is placed symmetrically on the opposite side of the lens.

Light leaves a point on an object, passes through the biconvex lens, and is recombined on the other side to form a real image, as shown for a point source of light in Figure 10.13.

The waves that leave the point all radiate uniformly into the surrounding space. Some parts of the wavefronts reach the lens and are refracted. The rays shown are representations of the direction of propagation of the wavefronts. There are, therefore, many rays that leave the point and reach the lens. All these rays are recombined beyond the lens to form a real image.

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Paraxial assumptions

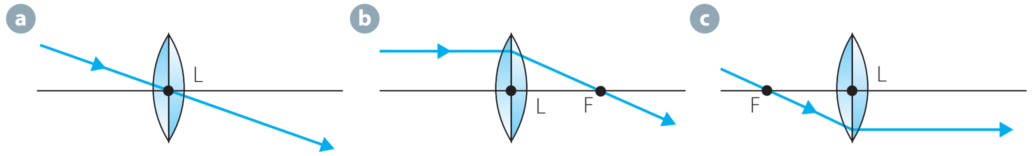
If an exact image is to be constructed by ray tracing, the exact lens surfaces need to be drawn and the rays traced exactly into, through and out the other side of the lens. It is possible to draw reasonably accurate (first approximation) ray tracing diagrams, but this requires certain assumptions to be made. These are called **paraxial assumptions**, and are as follows.

- The rays striking the lens or curved mirror are not too far away from the principal axis.
- The lens or curved mirror is small and thin so that it can be replaced in the diagram with a straight line. (However, we always draw a small lens or curved mirror around the centre to remind us of what we are doing!)
- When a ray strikes the straight line that represents a lens or curved mirror, it refracts or reflects respectively as though the line were the lens or curved mirror.

The paraxial assumptions only apply for thin lenses and curved mirrors. Highly accurate image reconstruction requires more sophisticated assumptions.

We can define any number of rays leaving a point on the object because the wavefront is continuous. Of these, three rays are useful to help trace the rays to the image in a convex lens:

1. A ray directed through the centre of the lens travels to the image unrefracted (Figure 10.14a)
2. A ray parallel to the principal axis refracts through the lens and passes through the focus on the other side (Figure 10.14b)
3. A ray through the focus nearer the object refracts at the lens and travels parallel to the principal axis (Figure 10.14c).



A ray through the optical centre L is undeviated.

A ray parallel to the principal axis is refracted (by the lens) to pass through F.

A ray arriving through F is refracted parallel to the principal axis.

FIGURE 10.14 Ray paths for three significant rays that can be used to locate the image in a convex lens: **a** the ray directed at the optical centre; **b** the ray parallel to the principal axis; **c** the ray through the focus nearer the object

Thin lens equation

An image may be inverted or upright with respect to the object, and it may be a different size from the object (enlarged or diminished). As described earlier, the magnification, M, is the ratio of image height, h_i to the object height, h_o.

$$M = \frac{h_i}{h_o}$$

Figure 10.15 enables us to deduce a mathematical relationship, the **thin lens equation**, which connects the position of the object, u, the position of the image, v, and the focal length, f.

The thin lens equation is deduced from similar triangles.

In ΔABL and ΔDEL:

$$\frac{h_i}{h_o} = \frac{v}{u}$$

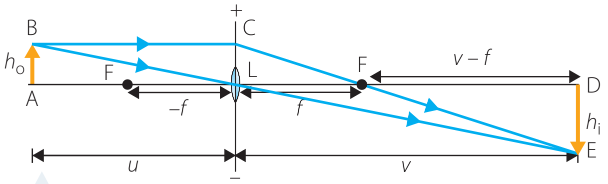


FIGURE 10.15 Ray tracing diagram to find an image in a biconvex lens, where u is the distance from the lens axis to the object, v is the distance of the image to the lens axis, f is the focal length of the lens, and F show the positions of the focal points.

In ΔEDF and ΔFLC:

$$\frac{h_i}{h_o} = \frac{v}{u}$$

$$\frac{h_i}{h_o} = \frac{v - f}{f} \text{ (CL} = h_o\text{, DF} = v - f\text{)}$$

$$\frac{v}{u} = \frac{v - f}{f}$$

$$vf = uv - uf$$

Divide by *uvf*:

$$\frac{vf}{uvf} = \frac{uv}{uvf} - \frac{uf}{uvf}$$

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Therefore, the thin lens equation is:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

The principal axis can be considered as a number line with distances to the left of the lens being negative, and distances to the right of the lens being positive. If the thin lens equation produces a negative value for v (the distance to the image), then the image is a virtual image produced on the same side of the lens as the object itself. A negative focal length means that the lens is a diverging lens, not converging.

For real images, M is negative (as real images are inverted). For virtual images, M is positive (as virtual images are upright).

$$M = \frac{-h_i}{+h_o} = -\frac{v}{u}$$

A small light forms a real image on a screen when it shines through a convex lens. The light must be further from the lens than the focal distance. The image may be enlarged, equal to, or diminished relative to the actual light. This depends on the distance of the object from the lens.

WORKED EXAMPLE (10.2)

An object 3.0 cm high is placed 16.0 cm in front of a converging lens of focal length 10.0 cm.

1 Draw an accurate ray tracing diagram.

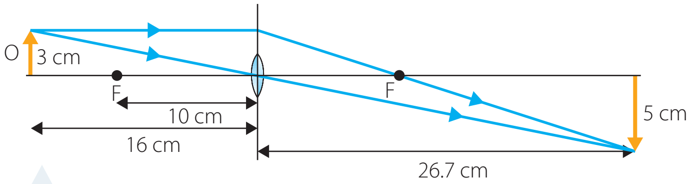
2 Use the diagram to find the following properties of the image.

   a Image position (upright or inverted)

   b Image nature (real or virtual)

   c Size of the image (enlarged or diminished)

   d Magnification

ANSWERS	**LOGIC**
1  **FIGURE 10.16**	▪ Draw the axes correctly, label the foci and mark in the object correctly. Use a consistent scale. ▪ Draw two useful rays to and from the mirror. ▪ Locate the image correctly. It must be located correctly, both horizontally and vertically.
2 From the accurately drawn ray diagram: **a** The image arrow is pointing down, so it is inverted.	▪ Measure the distance from the lens to the image on your diagram.
b The image is real.	▪ The rays pass through the image, so it is real.
c Size = –5.0 cm The size of the image is enlarged and the height is negative due to the image being inverted.	▪ Measure the height of the image on your diagram.
d $M = \frac{-h_i}{h_o}$ $M = \frac{-5\text{ cm}}{3\text{ cm}} = -\frac{26.7\text{ cm}}{16\text{ cm}} = -1.7$	▪ Identify the appropriate formula. ▪ Substitute known values from diagram into the formula and calculate the answer.

TRY THESE YOURSELF

1 An object 6.0 cm high is placed in front of a converging lens of focal length 5.0 cm. The object is at double the focal distance (i.e. 10.0 cm) from the lens. Use an accurately drawn ray tracing diagram to find the following properties of the image.

- **a** Image position (upright or inverted)
- **b** Image nature (real or virtual)
- **c** Size of the image (enlarged or diminished)
- **d** Magnification

2 An object 4.0 cm high is placed in front of a converging lens of focal length 6.0 cm. The object is at a position that is less than double the focal distance (i.e. less than 10.0 cm) from the lens but greater than the focal distance. Use an accurately drawn ray tracing diagram to find the following properties of the image.

- **a** Image position (upright or inverted)
- **b** Image nature (real or virtual)
- **c** Size of the image (enlarged or diminished)
- **d** Magnification

WORKED EXAMPLE 10.3

A 2.0 cm high object is 6.0 cm from a lens of focal length 10.0 cm. (The object is inside the focal length).

1 Draw an accurate ray tracing diagram.

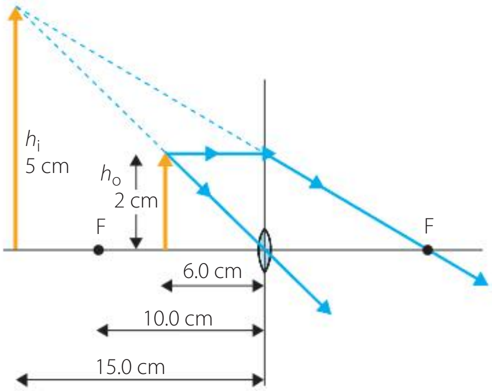
2 Use the thin lens equation to find the following properties of the image.

  a Image position (upright or inverted)

  b Image nature (real or virtual)

  c Magnification

  d Size of the image (enlarged or diminished)

ANSWERS	LOGIC
1  **FIGURE 10.17**	- Draw the axes correctly, label the foci and mark in the object correctly. Use a consistent scale. - Draw two useful rays to and from the mirror. - Locate the image correctly. It must be located correctly, both horizontally and vertically.
2 From the diagram, it can be seen that the image is magnified, upright and on the same side of the lens as the object, hence virtual. **a** $u = 6.0$ cm; $v = ?$; $f = 10.0$ cm By calculation: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$ $\frac{1}{v} = \frac{u - f}{fu}$ $v = \frac{fu}{u - f}$ $= \frac{10 \text{ cm} \times 6 \text{ cm}}{6 \text{ cm} - 10 \text{ cm}}$ $v = -15$ cm Image is 15 cm from the lens on the same side as the object, and it is upright.	- Identify the appropriate formula. - Rearrange the formula to make *v* the subject. - Substitute in the known values. - Calculate the answer. - State the numerical answer with an appropriate description to address the question.
b The image is virtual.	- A negative result in part **a** means a virtual image.

c $M = -\frac{v}{u}$ $= -\frac{-15\text{ cm}}{6\text{ cm}}$ $M = +2.5$	▪ Identify the appropriate formula. ▪ Substitute in the known values. ▪ Calculate the answer.
d $M = \frac{h_i}{h_o}$ $\frac{h_i}{h_o} = M$ $h_i = Mh_o$ $= 2.5 \times 2.0\text{ cm}$ $h_i = 5.0\text{ cm}$ The image is enlarged.	▪ Identify the appropriate formula. ▪ Substitute in the known values. ▪ Calculate the answer.

TRY THESE YOURSELF

1 An object 2.0 cm high is placed in front of a converging lens of focal length 10.0 cm. The object is placed at half the focal distance (i.e. 5.0 cm) from the lens. Use an accurately drawn ray tracing diagram to find the following properties of the image.

- **a** Image position
- **b** Image nature
- **c** Magnification
- **d** Size of the image

2 An object 2.0 cm high is placed in front of a converging lens of focal length 8.0 cm. The object is close to the focal distance away from the lens, at 6.0 cm from the lens. Use an accurately drawn ray tracing diagram to find the following properties of the image.

- **a** Image position
- **b** Image nature
- **c** Magnification
- **d** Size of the image

The thin lens equation applies equally to a biconcave lens, which is an example of a diverging lens. Figure 10.18 shows the three rays that can be used on a ray diagram to locate the virtual image formed. In this example, the object is placed more than two focal lengths away from the lens axis. Notice that it is a virtual image as it only appears to be where the rays of light have originated. The rest of the object's image can be constructed in the same way at any point on the object.

In Figure 10.18, the object is further than two focal lengths from the lens, creating a virtual, upright and diminished image. The magnification, M, is less than 1 and can be calculated by measuring the distance from the lens to the object, u, and the distance to the image, v.

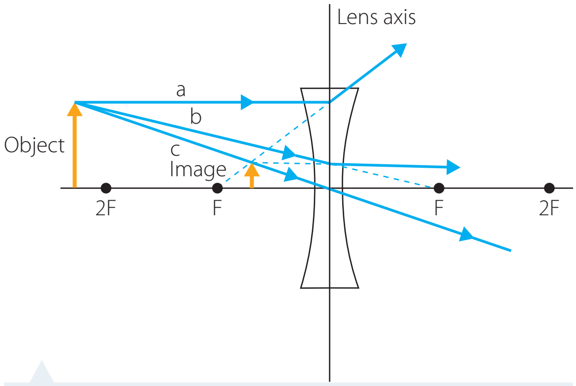


FIGURE 10.18 Three construction rays originating from the tip of the object. Ray a is parallel to the principal axis before being refracted in a direction as if it had originated from the focal point. Ray b travels towards the focal point on the other side of the lens and is refracted parallel to the principal axis. Ray c passes through the centre of the lens and does not change direction as a result of being refracted. The image point is located by extending the rays behind the lens to their point of intersection.

Image formation using mirrors and lenses

This interactive site includes activities with converging and diverging lenses and mirrors.

INVESTIGATION 10.2

Images in a convex lens

AIM

To apply the thin lens equation to data to find the focal length of a convex lens

MATERIALS

- Small convex lens and lens holder
- Vertical white screen
- Small bright light source (a single-filament globe is best) in a globe holder
- 9-volt DC battery pack
- Rubber band
- Metre rulers
- Tape
- Darkened space

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
While the room is darkened, trip hazards may not be visible.	Remove all bags and trip hazards out of the way.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Tape two rulers end to end along a table.
2. Put the lens in the lens holder and place it where the rulers join.
3. Adjust the height so the filament of the globe is at the same height as the centre of the lens.
4. Move the globe to one end of the rulers.
5. Place the screen on the opposite side of the lens to the globe.
6. With the globe as far from the lens as possible, move the screen so as to estimate the focal length of the lens. Record this length.
7. Move the lens and screen until a clear, focused image appears on the screen. Do this for at least three positions of the object (the filament of the globe).

RESULTS

In a properly constructed data table:

1. Record object and image distances from the lens.
2. Show the computed value for the focal length of the lens, to an appropriate number of significant figures, by applying the thin lens equation to the three sets of distances measured.

ANALYSIS OF RESULTS

1. What was the average focal length for the lens?
2. What is a reasonable estimate of the uncertainties in the distance measurements and in the derived focal length?

DISCUSSION

1. Compare the computed average focal length with your experimental estimate of the focal distance.
2. Explain the basis for the direct experimental estimate of the focal length.
3. Justify quantitative estimates of the uncertainty in the data.

CONCLUSION

By considering the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Images formed by reflection in curved mirrors

When you take the converging lens geometry and reflect it in the plane in which the lens stands, you produce the converging, concave mirror diagram. This diagram has all the same triangles as the convex lens diagram, only this time they are on the same side as the object (see Figure 10.19).

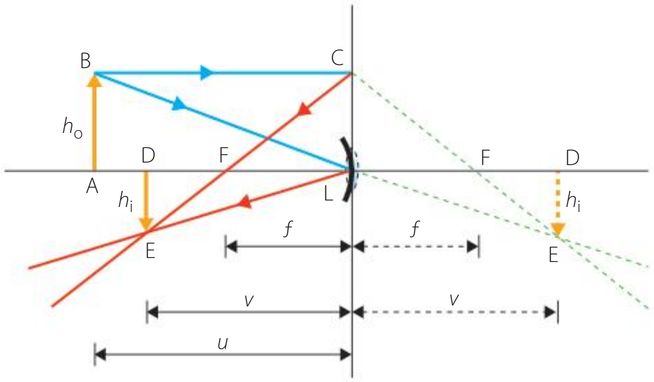


FIGURE 10.19 The geometry for a converging mirror (red) is the reflection of the geometry for the converging lens (green).

Curved mirrors simply reflect rays where lenses refract rays. The reflected geometry produces the same relationships between triangles. Hence the curved mirror formulae are the same as the thin lens formulae:

$$M = -\frac{v}{u}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

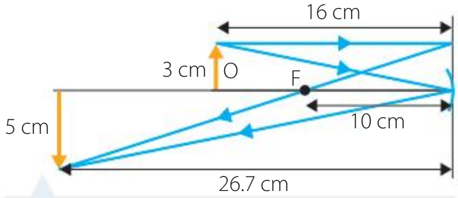
WORKED EXAMPLE (10.4)

An object 3.0 cm high is placed 16 cm in front of a concave mirror of focal length 10.0 cm.

1 Draw an accurate ray tracing diagram.

2 Use the diagram to find the following properties of the image.

- **a** Image position
- **b** Image nature
- **c** Size of the image
- **d** Magnification

ANSWERS	**LOGIC**
1  **FIGURE 10.20**	▪ Draw the axes correctly, label the foci and mark in the object correctly. Use a consistent scale. ▪ Draw two useful rays to and from the mirror. ▪ Locate the image correctly, both horizontally and vertically.

ANSWERS	**LOGIC**
2 From the accurately drawn ray diagram in Figure 10.20: **a** The image is 26.7 cm from mirror on the opposite side of the lens axis, and it is inverted.	▪ Measure the position of the image on your diagram.
b The image is real.	▪ Determine the location of the image relative to the object and the lens.
c Size of image is 5.0 cm.	▪ Measure the height of the image on your diagram.
d $M = \frac{h_i}{h_o}$ $M = \frac{-5 \text{ cm}}{3 \text{ cm}} = -1.7$	▪ Identify the appropriate formula. ▪ Substitute in the known values and calculate the answer.

TRY THESE YOURSELF

1 An object 6.0 cm high is placed 10 cm in front of a converging (concave) mirror. Use an accurately drawn ray tracing diagram to find the following properties of the image.

- **a** Image position
- **b** Image nature
- **c** Size of the image
- **d** Magnification

2 An object 4.0 cm high is placed in front of a concave mirror of focal length 6.0 cm. The object is at a position that is more than double the focal distance, at 15.0 cm from the mirror. Use an accurately drawn ray tracing diagram to find these properties of the image.

- **a** Image position
- **b** Image nature
- **c** Size of the image
- **d** Magnification

KEY CONCEPTS

- For curved mirrors as well as converging lenses, magnification is given by $M = \frac{-v}{u}$ and $M = \frac{h_i}{h_o}$.
- The thin lens equation is $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$.
- A virtual image has a negative value of v, the distance to the image.

CHECK YOUR UNDERSTANDING

10.1b

1 Draw diagrams to show the defining feature of:

- **a** convex lenses.
- **b** concave lenses.
- **c** convex mirrors.
- **d** concave mirrors.

2 a List the paraxial assumptions for the ray tracing representation of image formation in lenses and curved mirrors.
   b Illustrate these assumptions on a diagram.
3 a Write the equations used when calculating image positions and magnification.
   b Sketch and annotate a ray diagram showing real image formation in a convex lens.
4 An object 5.0 cm tall is placed 20 cm in front of a concave mirror of focal length 10 cm. Calculate the distance of the image from the mirror.
5 Using a scale drawing, determine the position, nature and size of the image of an object 5.0 cm tall placed 10.0 cm in front of a concave mirror of focal length 20.0 cm.
6 An object 15.0 cm high is placed 35.0 cm from a converging lens of focal length 10.0 cm. Using calculations, determine the position, nature and size of the image formed.

10.2 Refraction, Snell's law and total internal reflection

In chapter 8, the refraction of waves was introduced and discussed. We will now consider refraction quantitatively and show how light can be 'trapped' inside a medium so that it can be guided through narrow strands of glass called optical fibres.

Law of refraction

When a ray of light travels from one transparent medium into another, it changes direction. This phenomenon is called refraction. The amount of refraction is mainly related to differences in the electrical properties of each medium. The electromagnetic wave changes speed depending on how well the electromagnetic wave is permitted to move through the medium.

Refraction is responsible for many strange optical effects, such as the apparent bending of a straight pencil that is partly in water and partly in air, as shown in Figure 10.21.


Science Photo Library/ ANDREW LAMBERT PHOTOGRAPHY

FIGURE 10.21 A straight pencil apparently bends or breaks at the interface between air and water.

Measuring the speed of invisible electromagnetic radiation – Heinrich Hertz

Refractive index and Snell's law

Refraction refers to both the change of speed and the change in direction of the light. Refraction occurs whenever light passes from one medium into another. We can characterise any medium by its **refractive index**, n. A medium's refractive index is a relative measure of the speed of light in a vacuum to the speed of light in the medium.

The value of the refractive index of a vacuum is defined as the value 1.00. Relative to a vacuum, all other values are greater than 1.00 for visible light.

The expression for the value of n is:

$$n_x = \frac{c}{v_x}$$

where n_x is the refractive index of medium x, c is the speed of light in a vacuum, and v_x is the speed of light in medium x.

When light moves from one material to a second material with a similar refractive index there is very little refraction. This is the case when light moves from a vacuum to air, which has a refractive index close to 1.00. When light moves from one medium to a second medium with a very different refractive index, there is strong refraction. For example, diamond has a refractive index of 2.42 for visible light. Hence, light entering a diamond from air is slowed down a lot and bends a lot.

The values given for refractive indices are generally an average value for visible light. The refractive index is actually different for different wavelengths. This variation of refractive index with colour for a particular medium is important. It gives rainbows their colours and diamonds their sparkle. Diamonds sparkle because the refractive index for blue light in diamond is larger than for red light. Hence, blue light bends more than red light, and the different colours are separated. This phenomenon will be discussed further later in the chapter.

Most materials have refractive indices of between 1.00 and 1.60, but some materials have very high refractive indices. The refractive indices of some substances are given in Table 10.1. Diamond has a very high refractive index of 2.42, while pure silicon's refractive index is an even higher 4.01.

TABLE 10.1 Some common substances and their refractive indices

MEDIUM	REFRACTIVE INDEX
Vacuum	1 (exactly)
Air	1.00029
Water	1.33
Glass	1.50
Flint glass	1.65
Diamond	2.42
Silicon	4.01

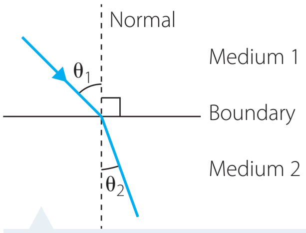


FIGURE 10.22 Angles are measured in a refraction experiment

When a light ray travels from one medium with a refractive index $n_{1,}$ and enters a second medium with a refractive index n_2, it makes an angle of incidence with the normal to the boundary in the first medium. As this is the first angle produced, it is labelled θ_1. The refracted ray makes an **angle of refraction** in the second medium, θ_2.

All experiments conducted along similar lines for refraction at a boundary demonstrate the two laws of refraction:

1. The incident ray, the normal and the refracted ray are coplanar (in the same plane).
2. Snell's law is obeyed.

Snell's law is the quantitative expression of the relationship between the incident and refracted rays:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

For simple cases where refraction occurs only once, this can also be written as:

$$n_1 \sin(i) = n_2 \sin(r)$$

where (i) is the angle of incidence and (r) is the angle of refraction.

In many investigations, light enters the second medium from air. Taking $n_{air} = 1.0$, the refractive index of the second medium, n_2 can be found by measuring θ_1 and θ_2 and rearranging Snell's law so that:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$n_2 = \frac{\sin\theta_1}{\sin\theta_2}$$

Snell's law interactive

This interactive site allows the user to explore Snell's law by refracting a ray of light.

INVESTIGATION 10.3

Investigating Snell's law

Critical and creative thinking

Numeracy

Information and communication technology capability

Refraction can occur when a light ray travels from one medium into another. The effect depends on the angle of incidence and the relative difference in the optical properties of the media.

AIM

To measure the angles of incidence and refraction, and hence find a relationship that links the two for a range of angles

Write a suitable hypothesis for this investigation.

MATERIALS

- Semi-circular glass or perspex block
- Ruler
- Protractor
- Graph paper
- Pencil
- Ray box
- Computer with suitable spreadsheet program

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
The globe in a ray box can get very hot.	Take care when handling the ray box and do not touch the globe.

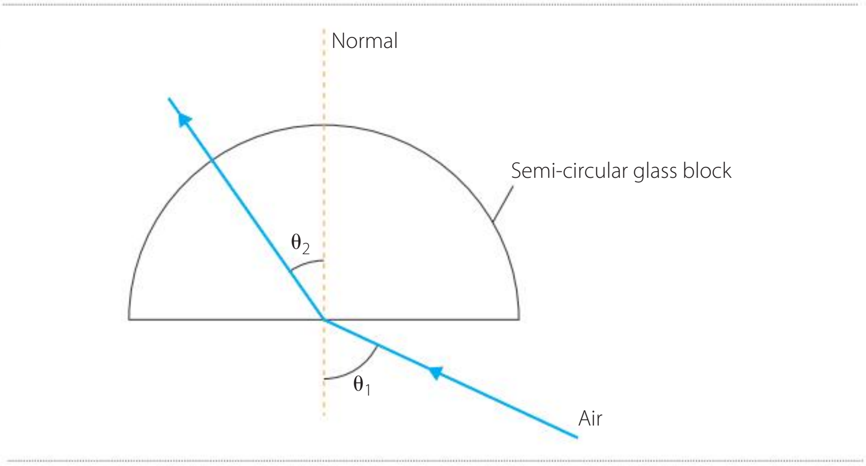
What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Trace the straight edge of the semi-circular glass block along the line on the graph paper.
2. Trace the outline of the block.
3. Construct the normal line perpendicular to the centre of the flat edge of the block, shown as a dashed line in Figure 10.23 (page 286).
4. Direct a single ray of light from the ray box to the centre of the flat edge of the block.
5. Trace this ray and the refracted ray (see the example shown in Figure 10.23).
6. Remove the block and carefully measure the angle of incidence, θ_1 and the angle of refraction, θ_2. Record these in a table.
7. Replace the block and repeat this for five different angles of incidence, from about 20° to 60°.

»

FIGURE 10.23 Arrangement for measuring angles θ_1 (incidence) and θ_2 (refraction) using a semi-circular glass or perspex block



RESULTS

Record the following data in a properly constructed data table.

1. Raw data:
    - Angle of incidence, θ_1
    - Angle of refraction, θ_2
2. Derived data:
    - $\sin\theta_1$
    - $\sin\theta_2$

ANALYSIS OF RESULTS

1. On a graph, plot data points $\sin\theta_1$ versus $\sin\theta_2$. This should be done on a spreadsheet (such as Excel).
2. Insert the trend-line for the plotted points and show the equation for this trend-line.

DISCUSSION

1. Is the ratio $\frac{\sin\theta_1}{\sin\theta_2}$ constant for all values of θ_1? You should consider the shape of the graph plotted in the analysis of results. Is it a straight line? If so, why?
2. Explain how you can derive the refractive index of glass from the graph of $\sin\theta_1$ versus $\sin\theta_2$ by considering the gradient of this graph.
3. Provide an estimate of the uncertainty in the value of the refractive index by first estimating the uncertainty in the measurement of your angles.
4. Suggest ways in which these measurements could be made more precise.

CONCLUSION

By considering the data obtained and its analysis, write a conclusion based on your hypothesis for this investigation.

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Refraction towards and away from the normal

Figure 10.24a shows a ray refracting towards the normal as it travels from air to glass. In this case, $n_2 > n_1$ and the ray is refracted towards the normal.

If the ray is reversed and travels from glass to air (Figure 10.24b), then $n_1 > n_2$ and refraction away from the normal occurs: $n_1 = 1.33$ (glass); $n_2 = 1.00$ (air).

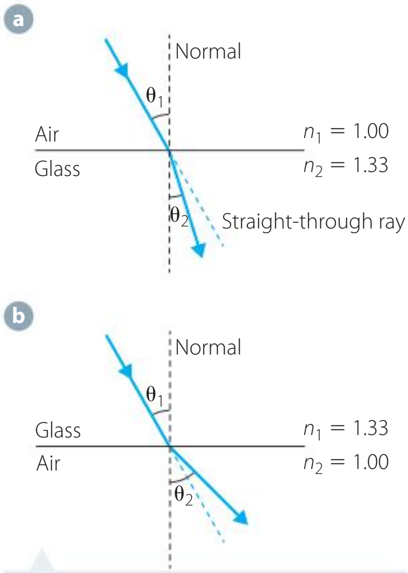


FIGURE 10.24 **a** Refraction at the air–glass boundary is towards the normal, $n_2 > n_1$. **b** Refraction is away from the normal when the ray is reversed (glass–air), $n_1 > n_2$.

Total internal reflection

At every boundary between media, reflection always occurs. Mostly, so does refraction. However, for refraction away from the normal (when $n_1 > n_2$), as the angle of incidence is increased, there comes an angle of incidence for which no refraction occurs. At angles of incidence greater than this angle, the ray is totally reflected back into the medium in which it was travelling when it reached the boundary. This is called **total internal reflection**. At this **critical angle** θ_c for the angle of incidence, the refracted angle θ_2 is 90°.

Thus:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

When $\theta_1 = \theta_c$ and $\theta_2 = 90°$,

$$n_1 \sin\theta_c = n_2 \sin 90°$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

This can also be written as

$$\sin(i_c) = \frac{1}{n_x}$$

where $n_2 = 1$, as it is for air or a vacuum.

As long as $n_2 < n_1$, a value of the critical angle, θ_c, can be calculated.

Total internal reflection interactive animation

This interactive animation can be used to show how total internal reflection occurs if the incident angle exceeds the critical angle

WORKED EXAMPLE 10.5

Numeracy

Light travelling in water ($n_1 = 1.33$) strikes the interface with flint glass ($n_2 = 1.65$) at an incident angle $\theta_1 = 36.0°$ to the normal.

1 What is the angle of refraction, θ_2, in the glass?

2 What is the critical angle, θ_c, for light travelling from flint glass into water?

ANSWERS	LOGIC
1 $n_1 = 1.33$; $n_2 = 1.65$; $\theta_1 = 36.0°$	▪ Identify the relevant data in the question.
$n_1 \sin\theta_1 = n_2 \sin\theta_2$	▪ Identify the appropriate formula.
$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$	▪ Rearrange the formula.
$\sin\theta_2 = \frac{1.33 \sin 36.0°}{1.65}$	▪ Substitute the known values, with units, into the formula.

ANSWERS	**LOGIC**
$\theta_2 = \sin^{-1}\left(\frac{1.33 \sin 36.0^\circ}{1.65}\right)$ $\theta_2 = 28^\circ$ (Note that the light is deviated by 7.7° towards the normal.)	- Calculate the answer. - Express the final answer with the correct significant figures and units.
2 The critical angle, θ_c occurs when the refracted angle $\theta_2 = 90^\circ$. $n_1 = 1.65$; $n_2 = 1.33$; $\theta_2 = 90^\circ$; $\theta_c = \theta_1$ $n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\sin\theta_1 = \frac{n_2 \sin\theta_2}{n_1}$ $\sin\theta_c = \frac{1.33 \sin 90^\circ}{1.65}$	- Extract data from the question. - Identify the relevant data in the question. - Identify the appropriate formula. - Substitute the known values, with units, into the formula.
$\theta_c = \sin^{-1}\left(\frac{1.33 \sin 90^\circ}{1.65}\right)$ $\theta_c = 53.7^\circ$	- Calculate the answer. - Express the final answer with the correct significant figures and units.

TRY THESE YOURSELF

1 A ray of light travelling in air enters a surface of a diamond (refractive index of 2.42) at an angle of 23.0°. What is the angle of refraction in the diamond?

2 A ray of light enters a medium of refractive index of 1.36 and refracts as it enters a new medium with $n = 1.29$. The angle of refraction (θ_2) is 25.0°. What is the angle of incidence (θ_1) for this ray?

3 What is the critical angle (θ_c) for light travelling out of diamond ($n = 2.42$) into water ($n = 1.33$)?

INVESTIGATION 10.4

Numeracy

Calculating the refractive index of a medium using total internal reflection

At the critical angle θ_c, the refracted angle $\theta_r = 90^\circ$ so that it is parallel to the boundary between the two mediums.

AIM

To calculate the refractive index of a medium by measuring the critical angle, θ_c

MATERIALS

- Ray box
- Semi-circular glass or perspex block
- Paper, pencil and ruler
- Protractor

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
The globe in the ray box gets very hot and may cause burns if touched.	Take care not to touch the globe in the ray box.

RISK ASSESSMENT

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Place the semi-circular block onto a flat piece of paper.
2. Arrange the ray box so that a single ray enters the curved surface perpendicular to the surface (i.e. parallel to the normal) at that point, as shown in Figure 10.25.
3. By moving the ray box, adjust the angle of the single ray so that it leaves the semi-circular block with an angle of refraction of 90°. This will show as the exiting ray travelling along the boundary between the flat side of the block and the air.
4. Trace the complete path of the ray and the semi-circular block on the paper.

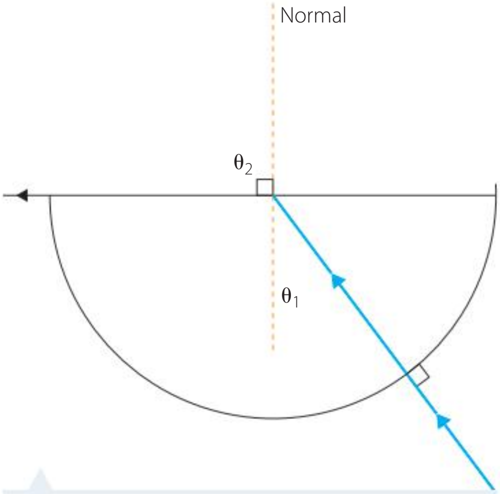


FIGURE 10.25 The ray of light enters the semi-circular block travelling towards the middle of the flat face of the block

RESULTS

Remove the block and measure the angle of incidence at the middle of the flat face inside the semi-circular block, as labelled θ_1 in Figure 10.25.

ANALYSIS OF RESULTS

Calculate the refractive index of the semi-circular block using Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, with the values:

- n_1 = refractive index of the medium that constitutes the semi-circular block
- θ_1 = angle of incidence
- $\theta_2 = 90°$ as it is the angle of refraction
- n_2 = refractive index of air (= 1.00).

DISCUSSION

1. Could this investigation be conducted with the glass block being in a medium other than air? Explain your answer.
2. Suggest changes that could be made to this investigation that would make the results more precise.

CONCLUSION

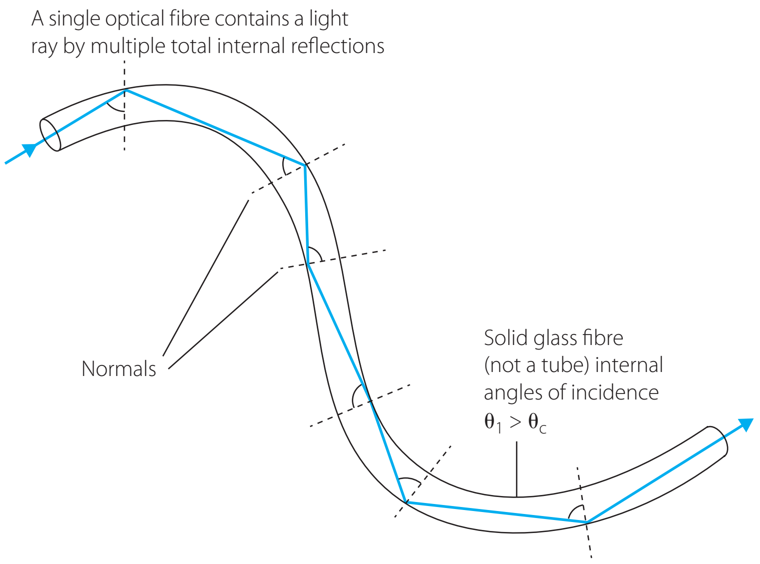
With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Optical fibres and other application of total internal reflection

An optical fibre is made of a glass core that has a refractive index slightly higher than the surrounding glass cladding. Today's society uses optical fibres to convey information around the world on the internet.

Light that is incident on the boundary between the core of the optical fibre and its cladding is mostly constrained to travel down the core by total internal reflection, as shown in Figure 10.26. The energy loss per reflection is about 500 times less than for a highly polished mirror surface. Optical fibres are highly flexible so that the light can be readily carried around corners. Every bend causes an increase in energy loss, but this is still much better than for ordinary mirror surfaces.

FIGURE 10.26 An optical fibre is made from core and cladding glass, and carries light around corners by total internal reflection.



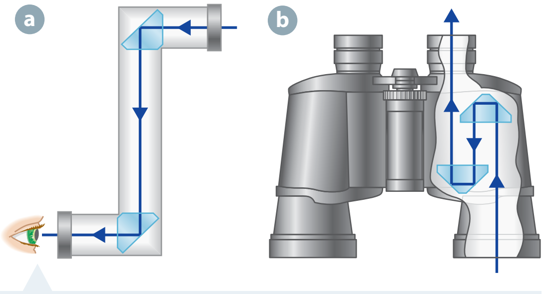


FIGURE 10.27 Rays of light undergoing total internal reflection within: **a** a periscope; **b** a pair of binoculars

The National Broadband Network (NBN), and telephone companies such as Telstra, Optus and Vodafone, use optical fibres. Optical fibres connect telephone exchanges and mobile base stations, and stretch around the world connecting us to the internet. Bundles of optical fibres are laid on the ocean floor to link the continents to the internet.

Total internal reflection can also be used in optical instruments such as periscopes or binoculars. Light is totally internally reflected from one glass–air surface using triangular prisms, as shown in Figure 10.27. The image formed has much better clarity than if a mirror were used for the same purpose.

KEY CONCEPTS

- A medium’s refractive index is a relative measure of the speed of light in a vacuum to the speed of light in the medium.
- The refractive index of medium *x* equals the speed of light in a vacuum divided by the speed of light in medium *x*: $n_x = \frac{c}{v_x}$.
- Refraction occurs when light enters a medium with a different refractive index at an angle other than 0° due to a change in the speed of light in the new medium.
- Snell’s law (the law of refraction) relates the angles of incidence and refraction to the refractive indices of the two mediums involved: $n_1 \sin i = n_2 \sin r$ with *i* and *r* being the angles of incidence and refraction respectively.
- Total internal reflection will occur if $\theta_1 > \theta_c$ when $n_2 < n_1$.
- $\sin\theta_c = \frac{n_2}{n_1}$ where $n_2 < n_1$. This is also written as $\sin(i_c) = \frac{1}{n_x}$ for $n_2 = 1$.
- Total internal reflection has applications in optical fibres and can be used in place of mirrors in optical instruments.

CHECK YOUR UNDERSTANDING

10.2

1 Draw a diagram to illustrate the:
   - a angle of incidence.
   - b angle of refraction.
   - c normal.
2 State Snell's law.
3 Define 'refractive index'. Why is it necessary to use a specific wavelength of light in the definition?
4 Use a diagram to show how Snell's law is obtained for waves in terms of their speed.
5 Refractive index is a ratio and therefore has no units. Explain.
6 Draw and label an optical fibre to show core, cladding, and total internal reflection at the core–cladding boundary.
7 Light travelling in air ($n_{air} = 1.00$) enters a glass block ($n_{glass} = 1.49$) at an angle of incidence of 30.0°.
   - a What is the angle of refraction in the glass?
   - b The glass block is now immersed in oil ($n_{oil} = 1.28$). Does the angle of refraction inside the glass block become larger or smaller when the angle of incidence is still 30.0°? Support your answer with calculations.

10.3 Dispersion of light

Different colours of light refract by different amounts. This effect is called **chromatic dispersion**. Red light refracts least, and violet light refracts most: $n_{red} < n_{blue}$.

Chromatic dispersion can be observed when white light is passed through a prism in the manner shown in Figure 10.28. The light is refracted twice – once upon entering the prism and again when leaving. The direction of refraction is the same in both events, exaggerating the difference between the angle of refraction for different colours. The component colours of white light can be observed as they diverge.

Rainbows are a result of colour dispersion. Colours disperse within every raindrop, as shown for a single raindrop in Figure 10.29 (page 292). Altogether, the raindrops produce different colours at slightly different angles.

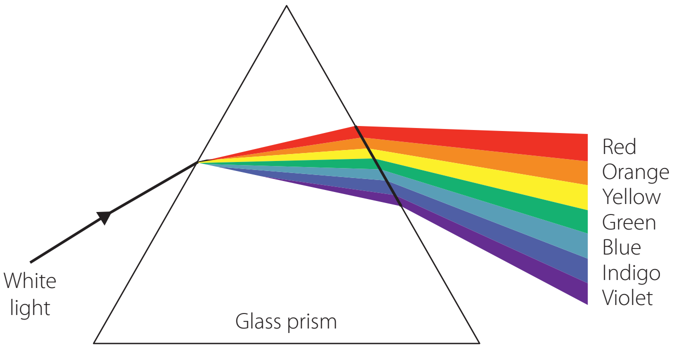


FIGURE 10.28 When white light is passed through a prism, red light is refracted the least while violet light is refracted the most, splitting white light into its component colours.

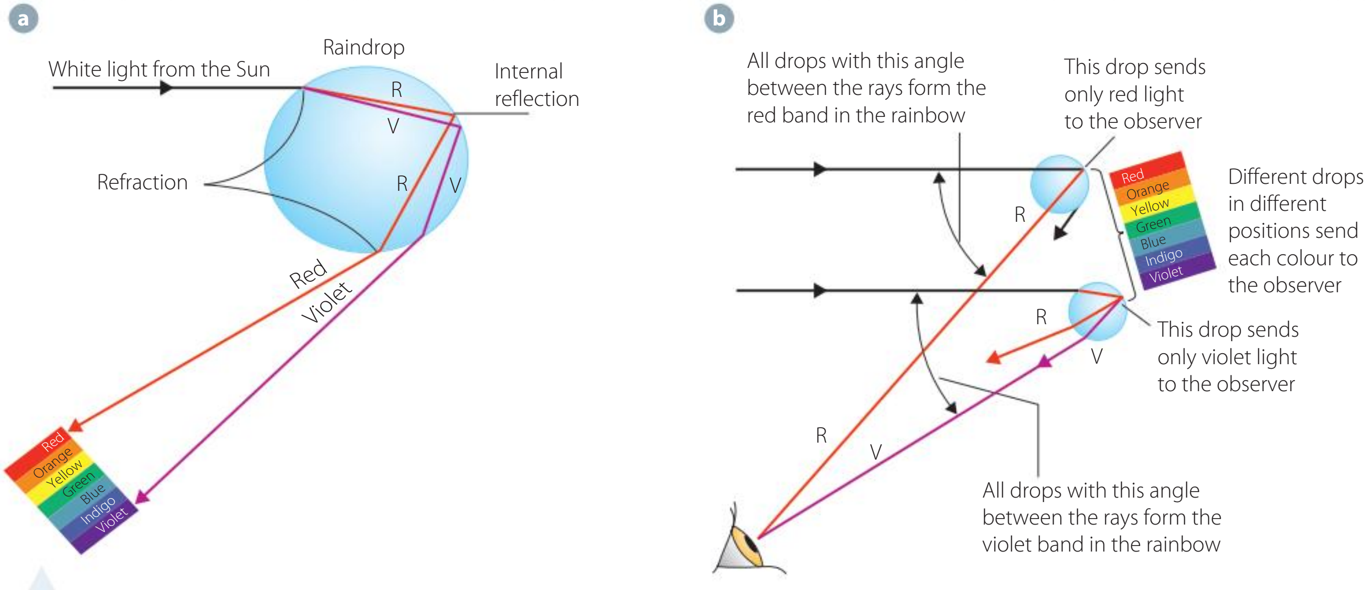


FIGURE 10.29 A rainbow is formed by the addition of the dispersed light in all the raindrops.

The composition of glass can be changed in order to change its sparkling qualities. The refractive index changes markedly between types of glass, and for different colours within a particular type of glass. Table 10.2 shows examples of such variations.

Dispersion of white light

This interactive allows the user to adjust the refractive indices of the mediums and the angle of incidence to observe dispersion.

TABLE 10.2 Refractive indices for different-coloured light in two types of glass

COLOUR	CROWN GLASS	FLINT GLASS
Red	1.514	1.638
Yellow	1.520	1.650
Blue	1.527	1.664
Violet	1.533	1.675

INVESTIGATION 10.5

Critical and creative thinking

Demonstrating dispersion of light

AIM

To demonstrate and explain the dispersion of light

MATERIALS

- Ray box
- Glass or perspex triangular prism
- Paper, ruler, pencil and protractor
- Optional: camera (or other recording device)

»

»

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
The globe in the ray box can get very hot and cause burns.	Take care handling the ray box to ensure you do not touch the globe.

RISK ASSESSMENT

What other risks are associated with your investigation, and how can you manage them?

METHOD

1 Set up the apparatus on a flat surface on a piece of paper so that a single ray of light is incident on the prism (as shown in Figure 10.28, page 291).

2 Trace the prism and the ray of incident white light onto the paper beneath the prism.

3 Trace the exiting rays of coloured light, taking care to be accurate. Label the colours.

4 Once complete, remove the paper. Using a protractor, measure the angle between the violet and the red light 'rays' that are leaving the prism.

RESULTS

Photograph your investigation and the ray diagram, or ensure that it is retained with your notes.

ANALYSIS OF RESULTS

Compare the overall change in direction of the red ray of light to the original white light ray. Repeat for the violet ray of light.

DISCUSSION

1 Explain why dispersion of light is not normally observed when single refraction occurs.

2 Summarise the necessary conditions for dispersion to occur.

3 What parts of the electromagnetic spectrum might be detected beyond the red light when white light is dispersed?

CONCLUSION

By considering the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- Dispersion occurs when different component colours in white light are refracted differently.
- Shorter wavelengths of light are refracted slightly more than longer wavelengths.
- Dispersion can be observed using a prism with refraction occurring twice.
- The production of a rainbow is by refraction and total internal reflection within raindrops.

WS Light questions

CHECK YOUR UNDERSTANDING

10.3

1 Describe why dispersion occurs when white light is refracted.

2 If dispersed white light was re-merged into a single beam of light, what colour would it be? Explain.

3 By analysing the geometry of Figure 10.29, explain why different colours in a rainbow are observed at slightly different angles to the sun.

4 Research ways in which the phenomenon of dispersion could be used in other applications.

10.4 Light intensity and the inverse square law

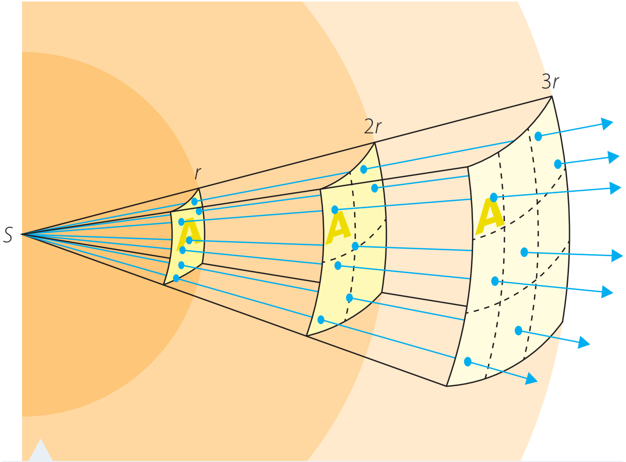


FIGURE 10.30 Light from a point source spreads uniformly into the surrounding space.

Light from a point source spreads uniformly into the surrounding space. The energy at the source becomes spread out over larger areas as the light travels away from the source.

The area of a sphere of radius r is:

$$A = 4\pi r^2$$

The source strength, S, is the energy per second being emitted. This has the units of joules per second, J s^{-1}, or watts, W. Area has the units of metres squared, m^2. The intensity, I, at distance, r, from the source, is the energy per second per area:

$$I = \frac{S}{4\pi r^2}$$

$$I = \frac{1}{4\pi}\frac{S}{r^2}$$

Thus, the intensity at any point is proportional to the source strength:

$$I \propto S$$

but inversely proportional to the square of the distance from the source:

$$I \propto \frac{1}{r^2}$$

This relationship is known as the **inverse square law**. You will encounter this law in other areas of physics such as gravitational fields, electrostatic forces and sound intensity.

The constant, $\frac{1}{4\pi}$, tells us that a sphere is involved in the calculations. As intensity is a measure of energy passing per second (power) per unit surface area, it can be measured using the units of watts per square metre, W m^{-2}.

If the distance away from the source of light is known, it is possible to compare the intensities I_1 and I_2 of the light at two points, r_1 and r_2 distant from the source.

Using the inverse square law:

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$$

$$I_1 r_1^2 = I_2 r_2^2$$

Inverse square law interactive

This interactive can be switched between light and paint to demonstrate the relationship between distance from the source and intensity.

WORKED EXAMPLE 10.6

3.0 m from a light source, the intensity is 2.40×10^2 W m^{-2}. What is the intensity at 1.5 m from the source?

ANSWER	LOGIC
$I_1 = 240$ W m^{-2}; $r_1 = 3.0$ m $r_2 = 1.5$ m; $I_2 = ?$	▪ Identify the relevant data in the question.
$I_1 r_1^2 = I_2 r_2^2$	▪ Identify the appropriate formula.
$I_2 = \frac{I_1 r_1^2}{r_2^2}$	▪ Rearrange the formula.
$= \frac{240 \text{ W m}^{-2} \times (3.0 \text{ m})^2}{(1.5 \text{ m})^2}$	▪ Substitute the known values, with units, into the formula.
$= 960 \text{ W m}^{-2}$	▪ Calculate the answer.
$I_2 = 9.6 \times 10^2 \text{ W m}^{-2}$	▪ Express the final answer with the correct significant figures and units.

TRY THESE YOURSELF

10.0 m from a light source, the intensity is 4.00×10^2 W m^{-2}. What is the intensity at:

1 40.0 m?

2 8.0 m?

WORKED EXAMPLE 10.7

Earth is 150 million km from the Sun. Mars is 228 million km from the Sun. What is the ratio of sunlight on Earth compared to on Mars?

ANSWER	LOGIC
$r_1 = 150$ million km (Earth's distance) $r_2 = 228$ million km (Mars' distance) $\frac{I_1}{I_2} =$ ratio of Earth : Mars sunlight intensity	▪ Identify the relevant data in the question.
$I_1 r_1^2 = I_2 r_2^2$	▪ Identify the appropriate formula.
$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$	▪ Rearrange the formula.
$= \frac{(228 \times 10^9 \text{ km})^2}{(150 \times 10^9 \text{ km})^2}$	▪ Substitute the known values, with units, into the formula.
$= 2.31$	▪ Calculate the answer.
Sunlight on Earth is 2.31 times more intense than on Mars.	▪ Express the final answer with the correct significant figures. ▪ Note that the answer is a ratio so it has no units.

TRY THESE YOURSELF

1. How many times more intense is the light from a candle viewed from 50.0 cm away than when the observer is 25.0 m away?
2. A lighthouse is observed from a ship from a distance of 14.0 km. How far from the lighthouse must the ship be so that the light from the lighthouse becomes exactly 9 times more intense?

INVESTIGATION 10.6

Information and communication technology capability

Numeracy

The inverse square law

AIM

To measure the variation in light intensity with the distance from the source

Write a suitable hypothesis for this investigation.

MATERIALS

- Point source of light (such as a light globe)
- Black curtain or fabric
- Measuring tape
- Light meter (or suitable app on a smartphone) to measure light intensity
- Graph paper (or computer with suitable spreadsheet program)

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
The room will be darkened, so items on the floor may present trip hazards.	Remove any potential trip hazards.
Staring at the light source may temporarily hurt your eyes.	Avoid staring directly at the light source.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Darken the room as much as possible. Place the source of light in front of a black curtain to prevent reflected light from reaching the light meter.
2. Over a range of distances from 0.20 m to 5.0 m, measure the intensity of light and record your results in a table. Approximately 10 different distances are needed. The actual units for the intensity of light are not important.

RESULTS

Your results should be recorded in a table with column headings as shown.

d (m)	I (UNITS)	$\frac{1}{d}$ (m^{-1})	$\frac{1}{d^2}$ (m^{-2})

»

9780170412483

» Complete the columns '$\frac{1}{d}$' and '$\frac{1}{d^2}$' by performing the appropriate calculations for each distance used in the investigation.

ANALYSIS OF RESULTS

1 On graph paper or by using appropriate software, plot the values of I versus $\frac{1}{d^2}$.

2 Compare the shape of your graph to the one expected if $I \propto \frac{1}{d^2}$.

DISCUSSION

1 Do the results of your investigation support the inverse square law for light? Justify your answer.

2 Comment on the positions of the data points obtained.

3 Suggest ways in which this investigation could be improved by reducing or eliminating sources of error in the measurements taken.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the hypothesis of this investigation.

KEY CONCEPTS

- The intensity of light varies with the inverse square of the distance from the source.
- The ratio of the intensities varies with the inverse square of the ratio of the distances.
- The inverse square law is $I_1r_1^2 = I_2r_2^2$.

Revision

CHECK YOUR UNDERSTANDING

10.4

1 Light spreads out uniformly from a 100 W point source. What is the intensity of the light at a distance of:
  - **a** 1.0 m?
  - **b** 2.0 m?
  - **c** 4.0 m?
  - **d** 5.2 m?

2 High-beam headlights can dazzle drivers at night when the oncoming car approaches. However, when several hundred metres away, this is not a problem. Explain this with reference to the inverse square law.

3 How many time brighter (more intense) will the light from a star be if the observer's distance is:
  - **a** doubled?
  - **b** halved?
  - **c** ten times the original distance?
  - **d** one-tenth of the original distance?

4 Other than for light, does the inverse square law for intensity versus distance hold for other forms of radiation? Briefly research to answer this question.

10 CHAPTER SUMMARY

- The interactions of light with matter can be modelled using the ray model of light where it is assumed that light interacts with matter along straight lines.
- For reflection (diffuse and regular):
  - the incident ray, the normal and the reflected ray are coplanar.
  - the two angles are equal: $\theta_i = \theta_r$.
- For refraction:
  - the incident ray, the normal and the refracted ray are coplanar.
  - Snell's law is $n_1 \sin\theta_1 = n_2 \sin\theta_2$, where θ_i is the angle of incidence; θ_2 is the angle of refraction; n_1 is the refractive index of the first medium; and n_2 is the refractive index of the second medium.
- The refractive index of a medium is $n_x = \frac{c}{v_x}$, where n_x is the refractive index of medium *x; c* is the speed of light in a vacuum, and v_x is the speed of light in medium *x*.
- Total internal reflection will occur if the angle of incidence θ_i exceeds the critical angle θ_c, and $n_1 > n_2$.
- Dispersion of white light occurs due to differences in refraction for different wavelengths of white light where shorter wavelengths are refracted more than longer wavelengths.
- For image formation in lenses and curved mirrors, $M = \frac{-h_i}{+h_o} = -\frac{v}{u}$ and $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$.
- Intensity *I* of light at a distance *r* from a source with power of *S* is $I = \frac{S}{4\pi r^2}$, which can also be written as $I = \frac{1}{4\pi}\frac{S}{r^2}$
- The inverse square law is $I_1 r_1^2 = I_2 r_2^2$.

10 CHAPTER REVIEW QUESTIONS

Review quiz

1 Draw ray model diagrams to show reflection of light from a painted wall that has a rough surface.

2 Write the equation for the way intensity of light from a point source changes with the distance from the source. Define the terms used.

3 **a** Write Snell's law.

b Annotate a diagram to define each quantity in Snell's law.

4 Sketch and annotate a ray diagram showing real image formation in a concave lens.

5 **a** 'Total internal reflection is a refraction phenomenon.' Discuss this statement.

b The critical angle for light passing from glycerine to air is 42.9°.

i What is the index of refraction of glycerine?

ii What is the angle of refraction for light passing from air into glycerine at an angle of incidence of 42.9°?

6 Why can light from a torch make a wall easier to see than a mirror?

7 Light rays from an object radiate in all directions from every point on an object.

a Identify which three rays are selected for analysing image information and for each of the rays show the rays' paths.

b For each of the three rays, show in a diagram the rays' paths as they pass through a concave lens.

c For the same three rays, describe their paths as they pass through a convex lens. Again, a diagram should be used.

8 A ray of blue light of wavelength 485 nm travels from air into a crown glass block at an angle of 40.0°. The speed of light in air = 3.00×10^8 m^{-1} and the refractive index for blue light in crown glass = 1.53.

a Calculate the angle of refraction as the light passes into the crown glass.

b For light transmitted into crown glass, find the speed of the light in the crown glass.

9 An object 4.0 cm tall is placed 12.0 cm in front of a concave mirror of focal length 8.0 cm. Use ray tracing to find:

a the height of the image.

b the distance of the image from the mirror.

10 An object 4.0 cm tall is placed 12.0 cm in front of a convex lens of focal length 8.0 cm. Determine these properties of the image.

a Position

b Nature

c Size

11 Jana is 160 cm tall and stands 2.0 m in front of a flat mirror mounted vertically on a wall. She is just able to see her entire image. Jana's friend, Rana, is 190 cm tall. The eyes of both girls are 10 cm below the top of their heads.

a What is the length of Jana's mirror? Illustrate your answer with a diagram.

b Rana cannot see herself fully in Jana's mirror, even if she stands back further from the mirror. Explain why.

c What length mirror is needed so that both Rana and Jana can see themselves fully?

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12 At 1.5 m from a light source, the intensity is 0.13 W m^{-2}. What is the intensity at:

a 4.5 m?

b 0.5 m?

13 How is it possible to 'see' a virtual image?

14 Two mirrors that meet at right angles are called a corner mirror. A ray of light from a point 2.5 cm from both mirrors is incident on one mirror at an angle of incidence of 20°. It reflects from both mirrors.

a What will be the subsequent path of the light ray?

b How many images will be formed in the two-mirror system?

c Show that an incoming narrow beam of parallel rays will be reflected from a corner mirror as a narrow beam that is parallel to the incoming beam.

15 At what minimum angle of incidence would a diver need to shine a laser towards the surface of the water ($n_{water} = 1.33$) so that the light is not transmitted into the air above?

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16 A ray of light is shone from air into an optical fibre. All the light transmits down the fibre by total internal reflection. What is the maximum angle of incidence at the air–core boundary for which this can occur, given that $n_{core} = 1.500$ and $n_{clad} = 1.490$?

17 The image of an object in a bi-convex mirror is magnified 2.5 times. Use a ray tracing diagram to scale to show how this is possible. Describe the image in terms of it being real or virtual, upright or inverted, enlarged or diminished.

18 Use a ray tracing diagram to show how a bi-concave lens produces an image of an object placed at a distance of twice the focal length from the lens. Describe the image formed.

19 Explain why a virtual image cannot be shown on a screen but a real image can. Use a ray tracing diagram to illustrate your answer.

11 Thermodynamics

INQUIRY QUESTION

How are temperature, thermal energy and particle motion related?

OUTCOMES

Students:

- explain the relationship between the temperature of an object and the kinetic energy of the particles within it (ACSPH018)
- explain the concept of thermal equilibrium (ACSPH022)
- analyse the relationship between the change in temperature of an object and its specific heat capacity through the equation $\Delta Q = mc\Delta T$ (ACSPH020)
- investigate energy transfer by the process of:
  - conduction
  - convection
  - radiation (ACSPH016)
- conduct an investigation to analyse qualitatively and quantitatively the latent heat involved in a change of state
- model and predict quantitatively energy transfer from hot objects by the process of thermal conductivity CCT
- apply the following relationships to solve problems and make quantitative predictions in a variety of situations: ICT N
  - $\Delta Q = mc\Delta T$, where c is the specific heat capacity of a substance
  - $\frac{Q}{t} = \frac{kA\Delta T}{d}$, where k is the thermal conductivity of a material

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Thermodynamics is the area of physics that studies the effects of heat, work and energy on a system. It began in the 19th century, when scientists and engineers were first trying to understand and build steam engines. It deals with system effects that are large enough for us to observe and measure. Thermodynamics has three laws, confusingly numbered the zeroth, first and second laws. These laws help us to explain system interactions involving heat, energy and work. The zeroth law defines and explains **thermal equilibrium**. The first (law of conservation of energy) and second laws help us to understand how this equilibrium comes about.

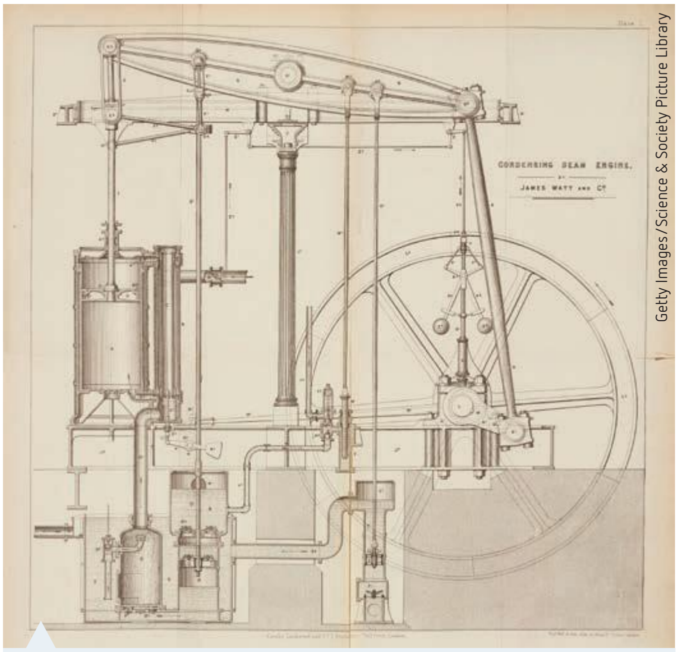

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FIGURE 11.1 Watt's condensing steam engine, from *Steam Engines Familiarly Explained*, 1836

11.1 Heat, work and energy

Energy is a central concept in many theories and models in physics. We use the **law of conservation of energy** to explain observations and predict what will happen in many situations. There are many forms of energy.

One of the key ideas we will be using in this chapter is the concept of heat or thermal energy. **Heat** is energy that is in the process of being transferred from one place to another due to a temperature difference.

We will also use the verb 'to heat' to mean the *process* of adding heat to something; for example, to heat food. In physics, we do not use the word heat to describe a quantity of stored energy – rather, it is energy that is being transferred.

This is similar to the way we define work. **Work** is energy that is being transferred due to the action of a force, but it is also not stored in an object or system. It took a long time for the scientific community to accept that heat was a form of energy. In the 17th century, Antoine Lavoisier (1743–94) developed the caloric theory. He argued that caloric was a kind of weightless fluid capable of flowing from hotter to colder bodies through tiny holes in materials.

Sir Benjamin Thompson (Count Rumford, 1753–1814) undertook many experiments on heat and mechanical work. He concluded they were manifestations of the same thing – energy. French physicist Sadi Carnot (1796–1832) developed these ideas further, especially for engines. James Prescott Joule (1818–89) conducted some extremely careful experiments on mechanical energy and heat. He used a known amount of mechanical energy to raise the temperature of a known quantity of water. His data demonstrated conclusively that heat and mechanical energy are quantitatively equivalent.

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FIGURE 11.2 Joule's apparatus

Joule's evidence led to the establishment of the law of conservation of energy (the first law of thermodynamics) – this is the foundation of all studies of heat. The importance of his work has been honoured by the SI unit for energy, joule, J. Joule worked with Lord Kelvin (William Thomson) to develop the absolute scale of temperature. He also discovered the relationship between the flow of current through a resistance and the amount of heat generated. This is now called Joule's law. Caloric theory and energy theory co-existed for a long time before energy became the preferred concept. In our discussions of heat transfer, there are still echoes of caloric theory in the way we discuss the flow of energy from hotter to colder objects. However, heat is not a substance.

A thermometer gives you an objective measurement (quantitative) of the temperature. Your hands give a subjective indication (qualitative) of the temperature. Heat-detecting nerves in your skin detect the rate at which energy (heat) is transferred to or from your skin. The rate of heat transfer depends upon temperature difference. We interpret the sensation as temperature.

INVESTIGATION 11.1

Sensing hot and cold

AIM

To explore how we sense hot and cold

MATERIALS

- 3 large bowls of water at different temperatures – cold (refrigerated but not frozen), warm (with an equal mixture of hot and cold water), and hot (as hot as your hands can stand)
- Thermometer

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Water that is too hot can burn.	Limit the temperature to 50°C.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Record the temperature of each bowl of water.
2. Place your left hand in the cold water and your right hand in the hot water for 2 minutes.
3. Now place both your hands in the warm-water mixture.

»

Mark Fergus Photography

Mark Fergus Photography

FIGURE 11.3 **a** First, place your hands in the hot and cold water as shown. **b** Second, place your hands in the warm water as shown.

RESULTS

Record descriptions of the sensation of 'hotness' or 'coldness' in each hand:

- At the start, when you first place your hands in the hot and cold water.
- After you've placed your hands in the warm water mixture.

DISCUSSION

Identify two or more advantages and disadvantages of subjective and objective measurements of heat.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Kinetic particle model of matter

According to the **kinetic particle model**, all matter is made up of small particles that are in constant motion. The interactions of particles are small in scale and are described by the kinetic particle model. Thermodynamics and **kinetic theory** complement each other. Some principles are more easily understood in one than the other. We use the kinetic particle model of matter to explain the states of matter, and changes between states.

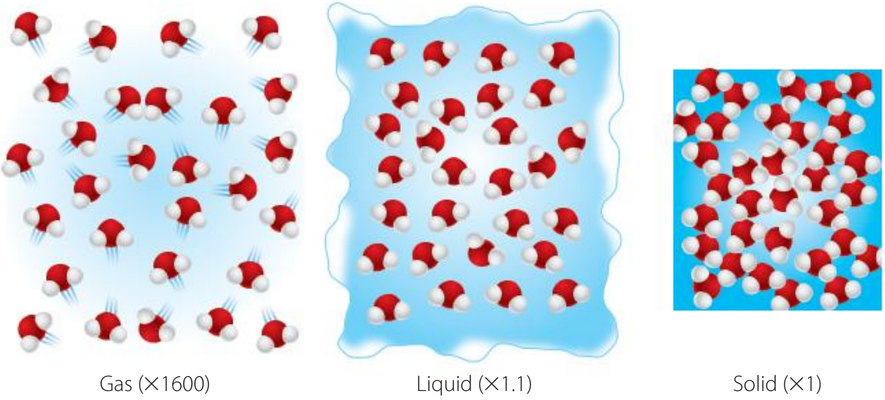
The kinetic particle model involves some assumptions.

- All matter is made up of small particles in constant motion; these particles have kinetic energy.
- Collisions between particles are perfectly elastic; the total kinetic energy before and after the collision is the same.
- Potential energy is stored in the 'springs' that connect the particles; potential energy depends on the distance between particles.

States of matter

Matter can exist in four different states: solid, liquid, gas and plasma. Solids have fixed shapes, fixed volumes and are mostly incompressible. Liquids have fixed shapes, fixed volumes and are more or less incompressible. Gases have no fixed shape or volume and are compressible. Plasmas are similar to gases, but are so hot that electrons have too much energy to be held by atomic nuclei. Plasmas are not relevant to this discussion.

FIGURE 11.4 The states of matter showing approximate volume changes (note the change in scale)



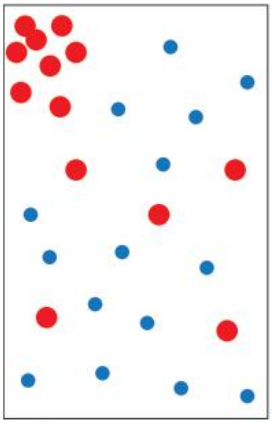


FIGURE 11.5 Particles of a perfume (red) diffuse through the air (blue) away from the source in the top left-hand corner (very small scale)

In a solid, the particles (atoms or molecules) are attached to each other by **intermolecular forces** (bonds) that behave a little like springs. There is an ideal length for any bond, but it is possible for the bond to be stretched and compressed like a spring. The solid material has potential energy because of these bonds. When people talk about chemical energy in food or fuels, it is this potential energy associated with the bonds between atoms to which they are referring. The atoms also have kinetic energy. Even in a solid, the atoms are constantly vibrating, even though there is no translational motion with respect to the atoms to which they are bonded. The material itself may not be going anywhere, but every atom is moving. Think of a large assembly of students all sitting in their own chairs, but each one fidgets and leans side to side to talk to their neighbours.

In a liquid, the particles are only loosely bound. There is still potential energy associated with interactions between the particles, but less than in a solid. However, the particles typically have much more kinetic energy.

In a gas, the bonds between molecules or atoms have broken and the particles are free to move. There is no longer potential energy associated with bonds between particles (although there is still energy associated with bonds within particles). We model the particles of a gas as being in constant motion. When they interact, they do so by colliding and undergoing elastic collisions. In an elastic collision, kinetic energy is conserved. Kinetic energy is transferred from one particle to another, but not converted into potential energy. This model of a gas is the kinetic particle model or ideal gas model. The pressure of a gas is due to the constant collisions between the gas particles.

Diffusion

Some things to think about

Smells waft to us from many places. This diffusion of gases is explained by the kinetic particle model. Particles move unseen from their source through a 'sea' of randomly moving air particles that are relatively far away from each other. Diffusion is very rapid in gases, slower in liquids, and can even occur between solids under pressure.

Energy model

Energy exists in many forms, including heat, light, mechanical, gravitational, electrical, magnetic, sound and chemical. Even mass is a form of energy. Regardless of the form, energy is still energy. The 'form' is often named by its origin (such as nuclear or solar). All forms of energy can be transformed from one form to another, and transferred from one place to another. For example, when you turn on an electrical bar heater, the electrical energy is transformed to radiant heat and light energy.

The SI unit of measurement of energy is the joule (J). It is approximately equivalent to the effort required to lift a 100 g apple from the ground to a height of 1 m.

The two major forms of energy are kinetic energy (energy associated with movement) and potential energy (energy ready to be used). All energy sources can ultimately be reduced to these two forms.

Kinetic energy

Kinetic energy is the energy a body possesses due to its motion. There is a number of forms of kinetic energy. For example, consider a moving train – it has bulk translational kinetic energy due to the straight-line motion of the whole train, bulk rotational kinetic energy in the rotating wheels and engine parts, and it has disorganised vibration kinetic energy due to the vibrations of the atoms and molecules in the solid materials from which it is made.

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FIGURE 11.6 A moving train possesses different forms of kinetic energy.

Potential energy

When you stretch an elastic band (sometimes called a rubber band), you do work on it and store energy in it. The elastic band now has the potential to do work. It has stored the energy. When the elastic band is released, it transforms this stored energy into kinetic energy and work is done. As stretching occurs, the atoms change position and the potential energy is increased. This extra potential energy can be recovered as work.

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FIGURE 11.7 Potential energy is stored in the stretched elastic bungee cord due to the change in position of the atoms from which it is made.

Internal energy

Even when un-stretched, there is potential energy stored in the way the particles are connected to each other in an elastic band. This keeps it solid, but it is not available to do extra work.

If a solid body is heated, its temperature increases. The particles will gain kinetic energy and (on average) vibrate faster. The particles move apart and the 'springs' also store more energy. The sum of kinetic energy and potential energy is the **internal energy**.

At **melting point** of a solid (the temperature at which it becomes a liquid), there is a **phase change**. The kinetic energy of the particles does not change any more until the phase change is completed. However, the intermolecular bonds are broken and the particles become further separated. At a phase change, the energy input is used to increase the distance between the particles, not to add to their kinetic energy.

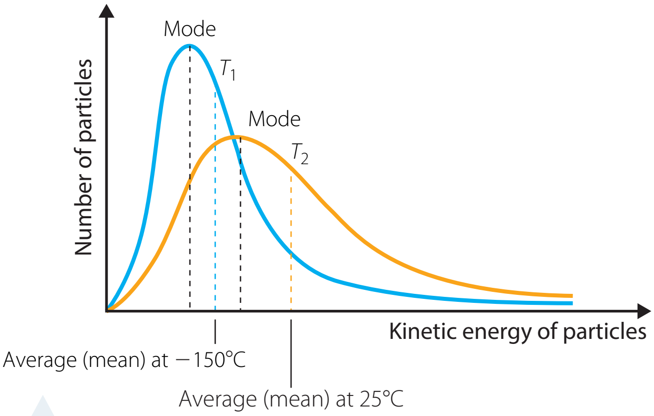


FIGURE 11.8 The graph shows the energy distribution of the particles in a sample of iron at two different temperatures, T_1 (−150°C) and T_2 (25°C)

Temperature explained

A cup of water takes a much shorter time to come to a boil (100°C) than a saucepan of water. The final temperature is the same for both, but the larger mass of water requires more heat to bring it to the same temperature. However, all the particles in the cup have (on average) the same kinetic energy as the particles in the saucepan. So, to a good first approximation, temperature is a measure of the average random kinetic energy of the particles making up a body.

Temperature variation and kinetic energy distribution

When you heat a material, the average kinetic energy of the particles increases. Figure 11.8 shows the wide range of kinetic energies of particles in the same mass of iron at two different temperatures. This distribution of energies is called a **Maxwell–Boltzmann distribution**. The peak of the curve is the most common kinetic energy among the particles. The average is to the right of the peak because the distribution is not symmetrical. This same distribution of particle energies also applies to liquids and gases. In a liquid, even if it is very cold, a few particles will still have enough energy to totally escape. This accounts for **evaporation**.

According to the particle model, it should be possible for all the particles in a substance to be entirely still. This condition is called **absolute zero**, and would be measured as 0 on the Kelvin scale, or −273.15°C. Kelvin and Celsius temperature measurements use the same scale except that the zero point of the Celsius scale is the freezing/melting point of water at standard atmospheric pressure.

When a material is heated and heat energy is added, the proportion of atoms vibrating faster increases. The average kinetic energy of particles, and therefore the temperature, increases.

KEY CONCEPTS

- Heat is energy that is in the process of being transferred from one place to another due to a temperature difference.
- Work is energy that is being transferred due to the action of a force.
- James Joule's careful experiments led to the law of conservation of energy.
- Normally when a substance is heated, its particles gain kinetic energy.
- During a phase change, the heat energy increases the distance between particles. The particles themselves do not gain kinetic energy.
- Internal energy is the sum of the kinetic energy and the potential energy of the particles in a substance.
- An object can have bulk (organised) kinetic energy (such as a train in motion), which is independent of its internal energy.
- Temperature is a measure of the average kinetic energy of particles in a substance.
- At any given temperature, some particles move slower than average and some move faster.

CHECK YOUR UNDERSTANDING

11.1

1. List the assumptions of the kinetic particle model.
2. What is the difference between energy transformation and energy transfer?
3. Explain why diffusion occurs faster in a gas than in a liquid.
4. Why are measurements using a thermometer considered to be objective and the sensations in your hands to be subjective?
5. Explain how kinetic energy, potential energy and internal energy are related.
6. Identify the types of kinetic energies the bodies in the following situations have.
   - **a** A free-falling, spinning ball.
   - **b** A stationary block of ice
7. Use the kinetic particle model to explain the increase in volume of steam compared with the same volume of liquid water.
8. Why is the mode (peak) of the graph in Figure 11.8 higher for $T_1 = -150$°C than for $T_2 = 25$°C?
9. Explain why the atmosphere contains between 1% and 4% water vapour in the lower atmosphere over much of Earth's surface.

11.2 Thermal equilibrium

The zeroth law of thermodynamics states that 'if two thermodynamic systems are each in thermal equilibrium with a third, then they are in thermal equilibrium with each other' (see Figure 11.9).

When two substances at different temperatures are mixed, the heat lost by one substance is equal to the heat gained by the other. If you place a hot stone in a container of cold water, heat energy is transferred from the hot stone to the cold water and container. The heat lost by the stone is equal to the heat gained by the water and the container. The stone gets cooler, and the water and its container get warmer. This transfer will continue until both reach the same temperature – when this occurs, they are said to be in thermal equilibrium (Figure 11.10). Collisions still occur between the water particles and the stone particles, but the amount of heat going from the water into the stone exactly balances the amount of heat leaving the stone and going into the water. There is no longer a net transfer of energy from one to the other.

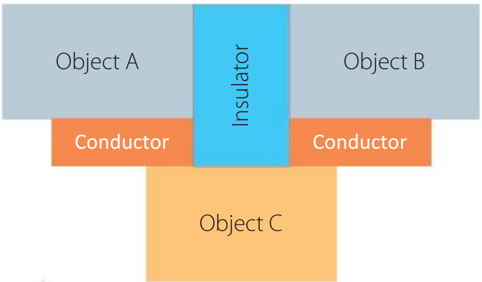


FIGURE 11.9 Objects A and B are not touching each other, but each is in thermal equilibrium with object C. According to the zeroth law of thermodynamics, they are therefore both in thermal equilibrium with each other – they are at the same temperature. Should they touch each other, no heat would flow between them.

Heating and cooling curves

When a pure substance is heated, its temperature is directly proportional to the energy transferred from a source provided that:

- it does not change phase
- the mass is constant
- the energy input rate is constant.

Heating and cooling curves always take the basic shape shown in Figure 11.11. The bottom left of the curve shows ice heating up to its melting point at 0°C. The curve becomes level because, as the ice melts, there is no change in temperature. The energy goes into the bonds between particles, not the kinetic energy of the particles.

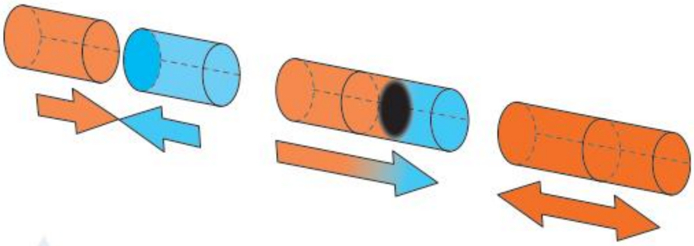


FIGURE 11.10 Hot and cold objects reach thermal equilibrium due to the transfer of energy by particle collisions. Eventually, the average kinetic energy of particles in both objects becomes the same.

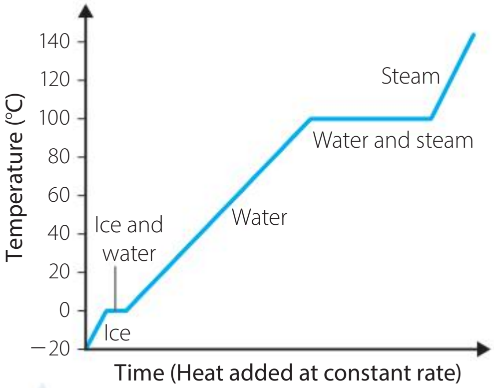


FIGURE 11.11 Heating curve for water. When all the ice has melted, the water continues to heat until it reaches 100°C. At 100°C, it starts to boil. The curve remains at that temperature until all the water turns to steam. The steam temperature then rises.

How temperature is measured

The zeroth law has practical applications. Many materials have properties that depend on their temperature; for example, the amount of expansion of a solid or liquid, or the electrical conductivity of a metal at a given temperature.

Consider a column of mercury. When no heat flows in or out of the column, its height remains steady. We can calibrate this column by placing it in contact with a physical system at a known temperature. Using water at freezing/melting point (0°C), the height of the column can be marked. The column can now be brought into thermal equilibrium with water at boiling point (100°C), and the height marked. By marking 100 even divisions along the column, we now have a Celsius thermometer.

There are other types of thermometers. Table 11.1 shows some thermometers that rely on changes to length, electrical resistance or colour. For example, a thermostat relies on the change of electrical resistance to measure temperature in an oven.

TABLE 11.1 Physical properties used by thermometers to indicate temperature change

TYPE OF THERMOMETER	PROPERTY UTILISED
Mercury in glass	Uses different coefficients of expansion between mercury and glass
Thermocouple	Uses different temperature-dependent electrical properties of different metals that are brought into contact
Thermostat	Uses variation in electrical resistivity of a material with temperature
Thermal paint	Uses colour change with temperature
Bimetallic strip	Uses variation in coefficients of expansion between two different metals to detect temperature changes
Infrared	Uses the electromagnetic radiation radiated from a surface to measure temperature on the absolute temperature scale
Digital	Uses the variation in resistivity of a material with temperature – the greater the resistance, the lower the current

INVESTIGATION 11.2

Bimetallic strips and thermometers

A bimetallic strip consists of two different metals that have been welded together, as shown in Figure 11.12. You are to investigate how a bimetallic strip behaves when heated, and then use this information to design and build a working thermometer. To do this, you will need to investigate the thermal expansion of various metals to understand how this works. Your design must be able to give quantitative temperature readings when placed in the oven. Your teacher will provide you with the equipment and materials you will need. You will then write a report to be submitted to your teacher.

FIGURE 11.12 Bimetallic strip

»

AIM

To build a working bimetallic strip thermometer

MATERIALS (SUGGESTED)

- Metals to be investigated
- Water baths (0°C, 50°C and 90°C)
- Metal clamps and tongs
- Thermometer
- Protective mats
- Protective gloves
- Calibrated background
- Access to oven and/or freezer

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Hot and cold metals, ovens and freezers all pose a burns risk.	Be extremely careful when manipulating the bimetallic bar, oven and freezer when in operation. Hot metal looks exactly the same as cold metal. Wear protective gloves.
Hot water can cause burns and scalds.	Be extremely careful. Wear safety glasses, lab coat and protective gloves. If spilt on skin, wash with plenty of cold water for 5 minutes. Apply ice pack.

RISK ASSESSMENT

What other risks are associated with your investigation, and how can you manage them?

METHOD

Write a procedure using point form for how you intend to carry out your investigation. Ask your teacher to check your method before proceeding.

RESULTS

Think about the types of results you will be collecting. Remember that your measurements will be quantitative and objective. Think about how you are going to display your results. Will you use a table or a graph?

DISCUSSION

What are your results telling you? The discussion section of your report should also include answers to the following questions:

1. How could you improve the temperature range of your thermometer?
2. How could you improve the accuracy of your scale?
3. Is your scale linear? Explain your answer.
4. Would your thermometer be as good as a glass–mercury bulb thermometer to measure daily temperature differences? Explain your reasons.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

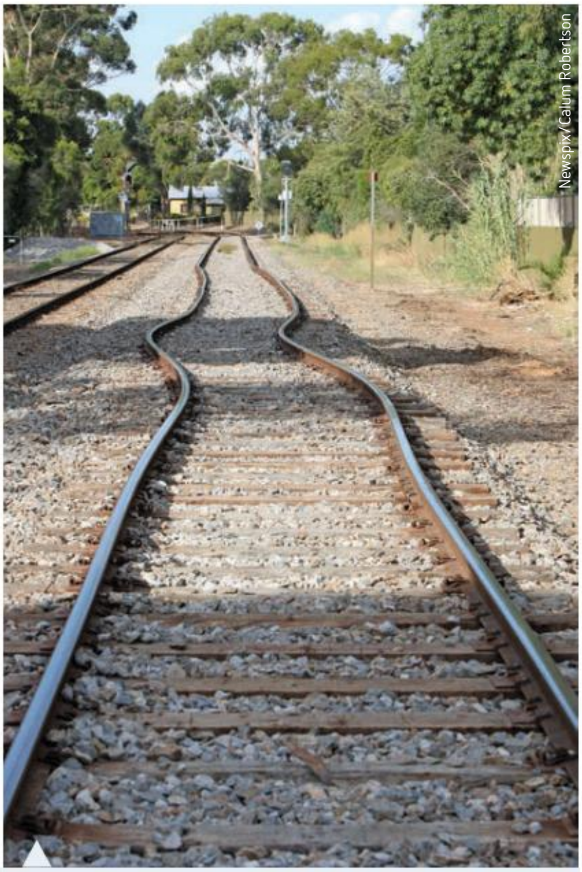

Newspix/Calum Robertson

FIGURE 11.13 Railway tracks buckle due to expansion during a heat wave. How is this related to this investigation?

KEY CONCEPTS

- Thermodynamics is the study of the effects of heat, work and energy on systems large enough to observe and measure.
- If two thermodynamic systems are each in thermal equilibrium with a third, then they are in thermal equilibrium with each other.
- No net heat flows between substances in thermal equilibrium.
- When a pure substance is heated, its temperature is directly proportional to the energy transferred from a source if it does not change phase, the mass is constant and the energy input rate is constant.
- During a change of phase, energy is still transferred but temperature does not change.
- Some materials change their properties in a regular way as their temperature changes.
- Such materials can be used to make thermometers.

CHECK YOUR UNDERSTANDING

11.2

1 Define 'temperature'.

2 List two different physical characteristics of materials that can be used to make thermometers.

3 What can we say about the scale of systems that thermodynamics examines?

4 Substance A is hotter than substance B, and the two come into contact.
  a What can we say about the energy lost from substance A compared to the energy gained by substance B?
  b Explain your reason for your answer to part **a**.

5 Why do most railway lines have gaps between each section of line?

6 How can we say two substances are in thermal equilibrium if they are not in contact with each other?

7 a What interrupts the trends shown in heating curves such as that in Figure 11.11 (page 308)?
  b Explain why this happens.

11.3 Specific heat capacity

To boil water, a 1000 W electric kettle transfers 1000 J of energy every second for quite a long time. It takes a large amount of energy to raise the temperature of water. The same mass of cooking oil at the same starting temperature would take about half the time to reach 100°C in the same kettle.

The **heat capacity** of a substance is the ratio of the amount of heat required for a given change in temperature. This is a physical property of the material and is related to its structure. It changes depending on how much of the substance is involved. A more useful quantity is the amount of energy required to raise the temperature of 1 kg of a substance by 1°C, without a change of phase. This is called the **specific heat capacity** (or just specific heat) of that substance. Water has a high specific heat capacity. Cooking oil has a much lower specific heat capacity. Oil heats up and cools down almost twice as quickly as water.

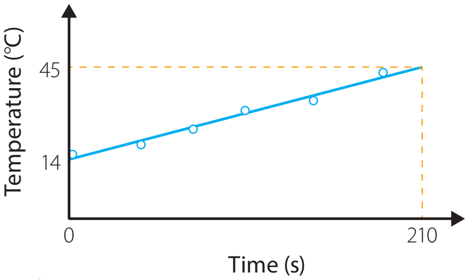


FIGURE 11.14 The graph shows that a change in temperature of the liquid is directly proportional to the amount of energy put in

Investigating specific heat

A student completed an investigation by heating 1 kg of an unknown liquid X by adding heat energy to it at a steady rate of 150 J s^{-1} for 210 s. The temperature was measured at regular intervals during the heating process and the data recorded was plotted as shown in Figure 11.14.

The graph shows that temperature T of a body (independent variable) increases in direct proportion to the amount of heat energy ΔQ (dependent variable) put into the liquid:

$$\Delta T \propto \Delta Q$$

The experiment was then repeated using only 0.5 kg of the unknown liquid. All other conditions were kept the same as the first experiment. The data were recorded and plotted as shown in Figure 11.15.

The second graph shows that for the same energy input, half the mass increases its temperature (independent variable) by twice as much. Therefore, the change in the temperature of the body is inversely proportional to the mass of the body *m* (a dependent variable):

$$\Delta T \propto \frac{1}{m}$$

Putting these two findings together gives us the relationship:

$$\Delta T \propto \frac{\Delta Q}{m}$$

There is always a constant, *c*, that makes a proportionality an equality, so by rearranging we get:

$$c\Delta T = \frac{\Delta Q}{m}$$

$$c = \frac{\Delta Q}{m\Delta T}$$

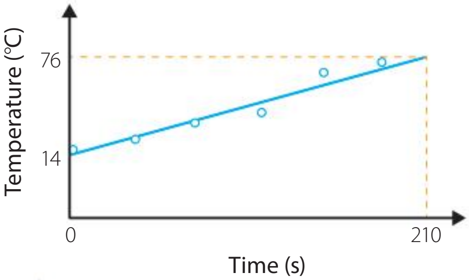


FIGURE 11.15 The graph shows that for the same energy input into half the mass, the temperature increase is doubled

The constant *c* is the specific heat capacity of the substance that is being heated; ΔQ is the quantity of energy supplied; *m* is the mass of the body being heated and ΔT is the change in temperature. The units of the specific heat capacity can be found by substitution of the units into the formula:

$$\text{Units of } c = \frac{\text{J}}{\text{kg K}} = \text{J kg}^{-1}\ \text{K}^{-1}$$

That is, joules per kilogram per kelvin. Since 1 K = 1°C, in practise we can use degrees Celsius instead of kelvin.

The relationships are then expressed in their simplest algebraic form as shown below:

$$\Delta Q = mc\Delta T$$

WS Steam-powered power stations

This is a good example of how careful experimentation provides useful data to find meaningful relationships (formulae). These relationships can then be used to predict what will happen under a different set of given conditions.

Specific heat capacity of water

Water plays an integral part in evolution and the sustaining of life. It has the highest specific heat capacity of the most commonly-occurring substances: 4200 J kg^{-1} K^{-1}. Compared with the same mass of most other substances, water:

- heats up more slowly
- cools down more slowly
- stores more internal energy.

Many cooling and heating systems (from hot water bottles and hydronic heaters to water-cooled engines) utilise water's high specific heat capacity. Large bodies of water (such as oceans, seas and lakes) absorb large amounts of energy with only small temperature changes. For the same amount of energy input, landmasses have much greater temperature changes than water. Thus, the temperatures inland are much hotter than on islands and in coastal regions. During the warmer months, when the sea temperature is less than the average air temperature, the sea acts as a heat sink – it stores energy. During the colder months, when sea temperature is warmer than the average air temperature, the sea releases the stored energy. This release of energy moderates the temperature of regions close to large bodies of water.

Specific heat capacities of metals

Just as land masses have specific heat capacities different to water, so do most other substances (Table 11.2), including metals. Experiments have demonstrated that a small metal block will take about 3–5 minutes to come to thermal equilibrium with boiling water. Once it has reached this temperature, it can be placed in a known mass of water (specific heat capacity = 4200 J kg^{-1}°C^{-1}) at a different temperature. Left for long enough in a calorimeter, the two will reach thermal equilibrium. From this and the mass of the metal block, the specific heat capacity of the metal can be calculated.

TABLE 11.2 Specific heat capacity of some common substances

SUBSTANCE	SPECIFIC HEAT CAPACITY (J Kg^{-1} K^{-1})
Water	4200
Ethylene glycol (antifreeze)	2400
Cooking oil	2800
Ice	2100
Steam	2000
Air	1000
Aluminium	900
Soil	800
Crown glass	670
Iron	450
Copper	380
Lead	130

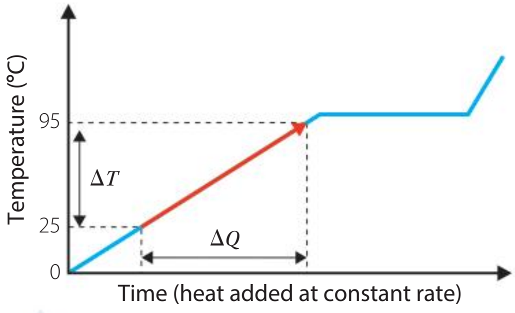
WORKED EXAMPLE 11.1

Numeracy

250 mL of pure water at 25°C is heated to 95°C.

1 Sketch a heating curve for the water from 0°C to 100°C. Show on the graph the section relevant to this question.

2 How much energy is needed to achieve this temperature change?

ANSWERS	LOGIC
1  **FIGURE 11.16**	- Identify the relevant data in the question and draw the curve. Highlight the relevant section of the graph.

ANSWERS	**LOGIC**
2 $m = 0.250\ \text{kg};\ \Delta T = 95°\text{C} - 25°\text{C} = 70°\text{C}$ $c = 4200\ \text{J kg}^{-1}\ °\text{C}^{-1}$	▪ Identify the relevant data in the question. ▪ Find *c* in list of specific heat constants.
$\Delta Q = mc\Delta T$ $\Delta Q = 0.250\ \text{kg} \times 4200\ \text{J kg}^{-1}\ °\text{C}^{-1} \times 70°\text{C}$ $= 7.35 \times 10^4\ \text{J}$ $\Delta Q = 7.4 \times 10^4\ \text{J}$	▪ Identify the appropriate formula. ▪ Substitute the known values, with units, into the formula. ▪ Calculate the answer. ▪ Express the final answer with the correct significant figures and units.

TRY THESE YOURSELF

1. A sample of an unknown substance of mass of 505 g is heated from 21.0°C to 56.0°C. The energy required was 4.90×10^4 J. Calculate the specific heat of the substance. Use Table 11.2 (on page 312) to identify the substance.
2. A pure iron nail of unknown mass requires 860 J of energy to change from 23.0°C to 305.0°C What is the mass of the nail?

WORKED EXAMPLE 11.2

A nurse prepares a bath that needs to be at 41°C for a patient. First, the nurse adds 53 L of water at 23°C from the cold tap. Next, the nurse will need to add water at 68°C from the hot tap so the bath is at the correct temperature.

1. What four assumptions must be made before starting to solve this problem?
2. How much water did the nurse need to add from the hot tap to achieve the required temperature of 41°C?

ANSWERS	**LOGIC**
1 Assumptions: • No energy is lost to the surroundings such as the taps, the air and the bath. • The mixing process does not add energy to the water. • The water is pure. • 1 L of water has a mass of 1 kg.	▪ Identify and state logical assumptions.
2 $T_{\text{cold}} = 23°\text{C};\ T_{\text{hot}} = 68°\text{C};\ T_{\text{final}} = 41°\text{C};\ m_{\text{cold}} = 53\ \text{kg}$	▪ Identify the relevant data in the question.
$\Delta Q_{\text{hot water lost}} = \Delta Q_{\text{cold water gained}}$ $-m_{\text{hot}}c\ \Delta T_{\text{hot}} = m_{\text{cold}}c\ \Delta T_{\text{cold}}$ $-m_{\text{hot}} = \frac{m_{\text{cold}}c\Delta T_{\text{cold}}}{c\Delta T_{\text{hot}}}$ $-m_{\text{hot}} = \frac{m_{\text{cold}}\Delta T}{\Delta T_{\text{hot}}} = \frac{53\ \text{kg} \times (41°\text{C} - 23°\text{C})}{41°\text{C} - 68°\text{C}}$ $m_{\text{hot}} = 35.3\ \text{kg}$ 35 L of hot water was added.	▪ Identify the appropriate formulae. Note that when heat is lost, Δ*Q* is negative. ▪ Rearrange the formula. ▪ Substitute the known values, with units, into the formula. ▪ Calculate the answer. ▪ Express the final answer with the correct significant figures and units

TRY THESE YOURSELF

A hot iron barbecue plate of mass 1.20 kg is placed in a tub that contains 22.2 L of cold water at 19.3°C. The water and the plate reach a final equilibrium temperature of 21.3°C.

1 What assumptions must be made before you start solving the problem?

2 What was the original temperature of the hot barbecue plate?

INVESTIGATION 11.3

Specific heat capacity of metals

AIM

To find the specific heat capacity of one or more metals

MATERIALS

- Calorimeter
- Thermometer or calibrated temperature probe
- Heating equipment
- 250 mL beaker
- Glass stirring rod
- Electronic scales
- Different metal cubes with dimensions about 2 cm^3
- Strong cotton thread
- Paper towel
- Protective gloves
- Safety glasses

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Hot metal blocks can cause burns.	Double check that the cotton thread is properly tied. Avoid touching the metal when transferring the block.
Heating equipment can cause burns.	Avoid touching the equipment. Wait for the equipment to cool before you put it away.
Hot water can cause burns and scalds.	Wear safety glasses and protective gloves. Lower the block gently into the water. Avoid splashing the water. If spilt on skin, wash it with plenty of cold water for 5 minutes. Apply ice pack.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1 Set up the equipment as shown in Figure 11.17.

2 Place 150 mL of water into the 250 mL beaker. Heat the water until it is boiling.

3 Determine the mass of the cube and securely tie the cotton thread around it.

4 Gently lower the metal cube into the boiling water and leave until it is at 100°C (3–5 minutes).

5 Meanwhile, measure and record the mass of the calorimeter.

»

»

6. Add approximately 150 mL of cold tap water to the calorimeter, and measure and record the mass of the calorimeter and water.
7. Suspend the thermometer (or temperature probe) in the cold water.
8. Gently stir the water with the stirring rod and wait for the temperature of water and thermometer (or temperature probe) to come to equilibrium. Record this temperature.
9. Carefully lift the hot metal cube out of the boiling water, quickly dry it, then lower it gently into the calorimeter water. Stir the water gently and frequently.
10. Record the temperature of the mixture of the metal block and the water when it reaches its maximum.
11. Repeat the experiment with a second trial.
12. If directed by your teacher, repeat the experiment with a different metal.

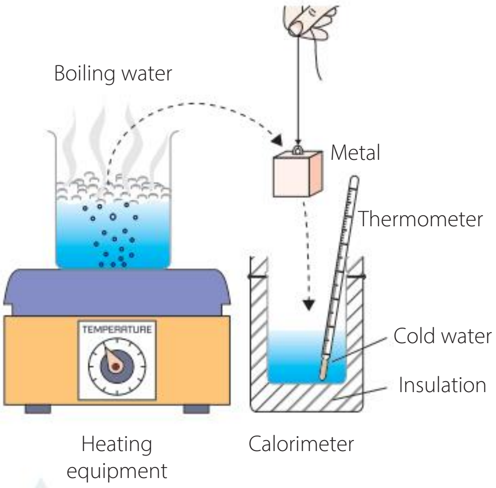


FIGURE 11.17 Experimental set-up for the transfer of the hot metal to the cold water

RESULTS

Record your results in a table. Include an estimate of the uncertainty in each measurement.

ANALYSIS OF RESULTS

1. Use the data to find the specific heat capacity of the metal for both trials.
2. Use the data to determine the measurement value and the estimate of the uncertainty.
3. Look up the accepted value of the specific heat capacity of the metal, including the uncertainty associated with this value. Decide whether the range of your measurement value overlaps the range of the accepted value.

DISCUSSION

1. Why were you instructed to dry the metal cube before placing it in the cold water?
2. Why is it desirable to start with the water temperature below room temperature and have a final temperature above room temperature?
3. Why were you asked to do two trials? Does this improve accuracy or precision?
4. Did your best estimate of the specific heat capacity of the metal differ from the accepted value? Explain.
5. Is it meaningful to calculate the percentage error in this experiment? Explain.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- The specific heat capacity of a substance, c, is the amount of energy required to raise the temperature of 1 kg of the substance by 1°C, without a change of phase.
- The temperature, T, of a body increase in direct proportion to the amount of heat energy, ΔQ.
- $\Delta Q = mc\Delta T$ where m is the mass of the substance and c is the specific heat capacity.
- Water has the highest specific heat capacity of the most commonly-occurring substances: 4200 J kg^{-1} $°\text{C}^{-1}$.
- The sea acts as a heat sink during the warmer months. During the colder months, it releases stored energy.

CHECK YOUR UNDERSTANDING

11.3

1. What does it mean when we say the specific heat capacity of iron is 450 J kg^{-1} K^{-1}?
2. Why do the oceans have a modifying effect on the temperature of the land near them?
3. Use the formula $\Delta Q = mc\Delta T$ to derive the units of specific heat capacity.
4. A 5.0 L tub contains 3.0 L of water at 23°C. The tub is then filled to the top with hot water. The final temperature of the water in the tub is 26°C. What was the temperature of the water added?
5. A 60 kg bushwalker is suffering from hypothermia, having an average body temperature of 33.5°C. When rescued, the bushwalker was wrapped in a blanket and given two 310 mL cups of warm tea (each at a temperature of 60°C). Calculate the maximum rise in the bushwalker's body temperature due to the heat of the tea. ($c_{\text{human body}} = 3.5 \times 10^3$ J kg^{-1} K^{-1})
6. A cook pours 0.80 kg of soup at 98.0°C into a 1.00 kg vacuum flask (specific heat capacity of 32.0 J kg^{-1} K^{-1}). When the soup and flask reach thermal equilibrium, the temperature of the flask has gone from 10.0°C to 97.0°C. Calculate the specific heat capacity of the soup.
7. Equal masses of lead and water are in contact and at thermal equilibrium. Equal amounts of heat are now transferred to the water and the lead. Will they be at thermal equilibrium immediately after the transfer of energy? Explain in detail.

11.4 Conservation of energy

In physics, a **closed system** is one that matter cannot enter or leave, but which energy can be transferred into or out of by work or heat. An **isolated system** is one that neither energy nor matter can enter or leave.

Within an isolated system, the total amount of energy remains constant. Energy can be converted from one form to another. Energy can be transferred from place to place. But the total energy within the isolated system remains constant.

Except for the universe, there is no such thing as a perfect isolated system. Very small losses of energy occur even in the most insulated systems (think about your calorimeter experiments). As long as the losses are negligible, we can model the system as isolated.

Energy transfer occurs when energy flows from one object or substance to another. Energy transformation occurs when energy changes from one form to another. If you turn on a torch, chemical energy is transformed into electrical energy, and the electrical energy is transformed into light and heat energy. In this case, the battery converts the stored chemical energy into electrical energy, and a globe or LED converts the electrical energy into light and heat energy.

As you will recall, the first law of thermodynamics (the law of conservation of energy) states that in an isolated system, energy can neither be created nor destroyed. Energy can be transferred or transformed but the total energy of an isolated system remains constant.

State changes

A pure solid starts to change state to a liquid at its melting point. A pure liquid starts to change state to a gas at its **boiling point**. Both processes, **melting** and **vaporisation** respectively, require energy input. Energy removal causes gases to undergo **condensation** and liquids, **solidification**.

Evaporation and vaporisation are often confused. Below the boiling point, evaporation from a liquid occurs at the surface. Some particles with high kinetic energy escape (see Figure 11.19).

Vaporisation occurs when the liquid changes to gas. At the boiling point, bubbles form below the surface. No temperature change occurs during vaporisation.

In the absence of sufficient external pressure, a solid can turn directly into a gas when heated, which is called **sublimation**. The reverse process, **deposition**, can also occur.

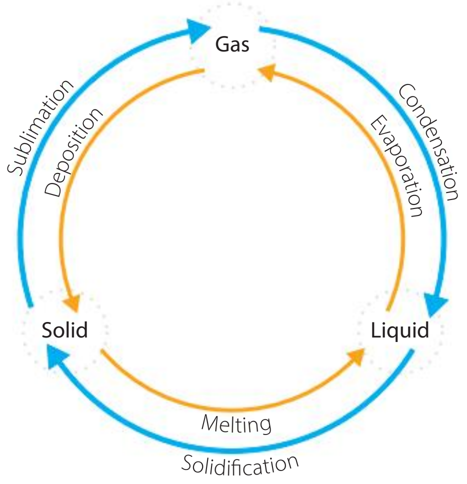


FIGURE 11.18 State change cycles

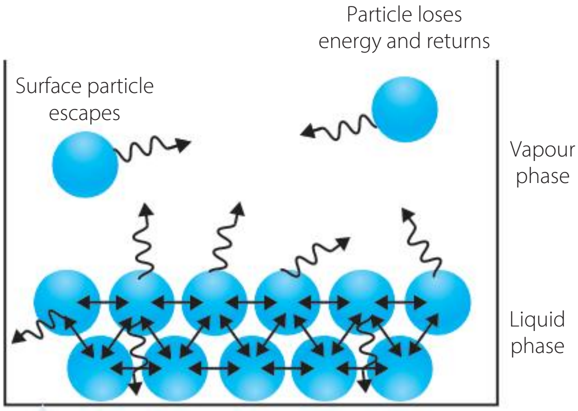


FIGURE 11.19 Evaporation occurs at the surface when water molecules that are less tightly bound and have relatively higher kinetic energy than those in the body of the water escape.

Latent heat

Measuring the latent heat of water

During a change of state, energy is added or removed. The energy added during a state change is called **latent heat**. The **specific latent heat of fusion** of a substance is the energy required to change the state of 1 kg of the substance from its solid state to its liquid state without any change in temperature. The specific latent heat of melting for water is 334 kJ kg^{-1}. In its solid state, the water particles are held tightly together. To pull them apart requires a large amount of energy. This energy to separate the particles is supplied externally. It does not increase the kinetic energy of the particles – instead, it is used to increase their average separation. As a result, only the internal energy increases. The change in internal energy cannot be measured by a thermometer because there is no increase in the average kinetic energy.

The **specific latent heat of vaporisation** of a substance is the heat required to change the state of 1 kg of the substance from its liquid to gaseous state. The specific latent heat of vaporisation of water is 2260 kJ kg^{-1}. You can see from the very large value that it requires a huge amount of energy to separate the particles from each other.

Both these processes are reversible. For example, a quantity of steam loses energy to its surroundings, cooling until it reaches its boiling point. It then remains at a constant temperature while the water molecules draw closer together. Energy is being released to the surroundings during the condensation process. This energy comes from a reduction of the internal energy of the molecules as they draw closer together to form liquid water. This is why steam at 100°C will cause more severe burns than the same mass of water at 100°C.

Condensation and heat exchange occurs in cloud formation. A pocket of moist air surrounded by dry air rises because it is less dense. It ascends into a cooler region, causing the water vapour to condense as clouds. The latent heat of vaporisation is released into the surrounding air, which becomes warmer. Warm air, being less dense than cooler air, continues to rise. Eventually, the moist air hits the 'roof' of the weather zone – the troposphere. Cloud formation then continues horizontally rather than vertically. Distinctive anvil-shaped clouds form at the tops of thunder clouds, especially in the tropics (Figure 11.20, page 318).

Latent heat

Use the information presented in the weblink to understand cyclical applications of latent heat such as refrigerators and heaters.

FIGURE 11.20 Distinctive anvil-shaped clouds are often seen at the tops of thunder clouds.

Alamy Stock Photo/PhotoStock-Israel

Investigating latent heat

Estimating the latent heat capacity of water

A student completed an investigation by heating different masses of ice at 0°C that had been dried to remove any water in liquid form. The heat energy was supplied at a steady rate of 1000 J s^{-1} until the ice had just melted. The time was recorded and precautions were taken to minimise any external heat gains or losses. The data was recorded in Table 11.3, and Figure 11.21 shows a graph of this data.

TABLE 11.3 Data from student investigation

MASS OF WATER PRODUCED (KG)	TIME (SEC)	ENERGY (KJ)
0.10	33	33
0.22	71	71
0.39	88	88
0.51	175	175
0.64	217	217
0.90	300	300

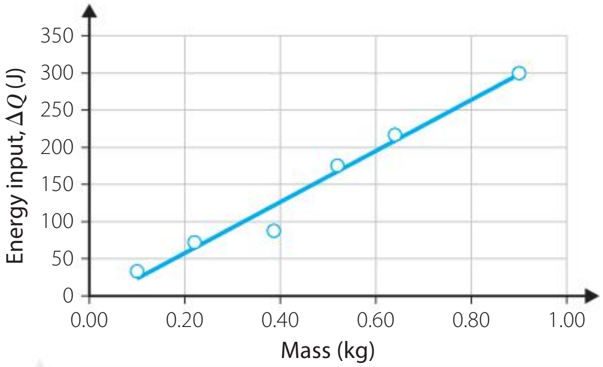


FIGURE 11.21 Finding the relationship between energy input and mass when ice melts. The equation of the line is $\Delta Q = m \times 334$.

The graph indicates there is a direct proportionality. In general:

$$\Delta Q \propto m$$

There is always a constant that makes a proportionality an equality; in this case L is used as the constant:

$$\Delta Q = Lm$$

Rearranging:

$$L = \frac{\Delta Q}{m}$$

where L, the latent heat, is the gradient of the graph. Units are clearly J kg^{-1}

This gives us the algebraic expression of the relationship between state changes and energy required to change the state:

$$\Delta Q = mL$$

WORKED EXAMPLE 11.3

Numeracy

Given that the latent heat of fusion of water is 334 kJ kg^{-1}, how much energy is required to melt 250 g of ice at 0°C to 250 mL of water at 0°C?

ANSWER	LOGIC
$L = 334\ \text{kJ kg}^{-1}$; $m = 250\ \text{g}$	- Identify the relevant data in the question.
$\Delta Q = mL$ $\Delta Q = 0.250\ \text{kg} \times 334\ \text{kJ kg}^{-1}$ $= 83.5\ \text{kJ}$ $\Delta Q = 83.5\ \text{kJ or } 8.35 \times 10^4\ \text{J}$	- Identify the appropriate formula. - Substitute the known values, with units, into the formula. - Calculate the answer. - Express the final answer with the correct significant figures and units

TRY THIS YOURSELF

A sample of lead (mass 505 g) is heated to its melting point of 327.5°C How much energy is needed to completely change its phase to liquid lead at its melting point temperature, given that its latent heat of fusion is 23 kJ kg^{-1}?

INVESTIGATION 11.4

Latent heat of fusion of solids

Modify Investigation 11.3 'Specific heat capacity of metals' (page 314) to find the latent heat of fusion of solids such as ice and other substances that melt at similar temperatures.

When crushed ice at 0°C is added to water, it will melt. The temperature change in the water can be used to find the latent heat of fusion of ice.

AIM

Consider the range of possible substances, then write your aim. The aim should be concise, indicate what quantity is to be measured, and provide information about the way the measurement is to be undertaken.

MATERIALS

Use the resources suggested in Investigation 11.3 'Specific heat capacity of metals'. What other resources will you need?

RISK ASSESSMENT

Construct a table similar to the one below. Identify specific risks involved in the investigation and ways that you will manage the risks to avoid injuries or damage to equipment.

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?

RISK ASSESSMENT

»

» METHOD

1. Consider the similarities and differences between Investigation 11.3 'Specific heat capacity of metals' and your investigation.
2. Construct a flowchart of the procedure you intend to follow.
3. Submit your flowchart and risk assessment to your teacher for approval before beginning the work.

RESULTS

Show on your flowchart the data that you intend to collect.

ANALYSIS OF RESULTS

Consider the value of graphs and equations in your analysis. Justify your analytic processes.

DISCUSSION

Provide a qualitative analysis of the latent heat involved in a change of state. Give quantitative values for all latent heat of fusions measured. Ensure the uncertainties claimed are justifiable. Consider whether the investigation met your aim, and include ideas for further improvements in reliability.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- A closed system is one that matter cannot enter or leave, but which energy can be transferred into or out of by work or heat.
- An isolated system is one that neither energy nor matter can enter or leave.
- The first law of thermodynamics (the law of conservation of energy) states that in an isolated system, energy can neither be created nor destroyed. Energy can be transferred or transformed, but the total energy of an isolated system remains constant.
- A pure solid starts to change state to a liquid at its melting point. A pure liquid starts to change state to a gas at its boiling point. Both processes, melting and vaporisation respectively, require energy input.
- Energy removal causes gases to undergo condensation, and causes liquids to undergo solidification.
- Below the boiling point, evaporation from a liquid occurs at the surface.
- At boiling point, bubbles form below the surface. No temperature change occurs during vaporisation.
- A solid can turn directly into a gas when heated (sublimation). The reverse process is deposition.
- The specific latent heat of fusion of a substance is the energy required to change the state of 1 kg of the substance from its solid state to its liquid state without any change in temperature.
- The specific latent heat of vaporisation of a substance is the heat required to change the state of 1 kg of the substance from its liquid state to its gaseous state.
- The energy required to change the state is: $\Delta Q = mL$.

CHECK YOUR UNDERSTANDING

11.4

1. What is the difference between a closed system and an isolated system?
2. Give an example of:
   a. a closed system.
   b. an isolated system.
3. Why is there no temperature rise when a substance is melting, even though energy is being input?

4 Why do clouds form?

5 **a** Why will a damp towel eventually dry out?

  b What factors would increase or decrease the rate of evaporation of a liquid?

6 What are two observable physical differences between boiling and evaporation?

7 Different substances have different melting points and boiling points, but the shapes of their heating curves are very similar. Solid iron is heated constantly from its solid state to its gaseous state in a furnace. The heating curve for the duration of this process is shown in Figure 11.22.

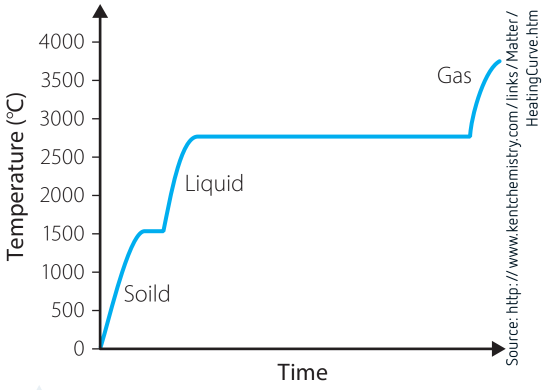
  Use Figure 11.22 to answer the following questions.

  a What is the melting point of iron?

  b What is the boiling point of iron?

  c Explain why two sections of the graph are parallel to the time axis.

  d What does the length of the longest horizontal section of the graph tell you about the latent heat of vaporisation of iron?



Source: http://www.kentchemistry.com/links/Matter/HeatingCurve.htm

FIGURE 11.22 Heating curve for iron

11.5 Energy transfer models

Heat energy always moves from a region of high temperature to a region of low temperature, using conduction, convection or radiation. The kinetic particle model is useful in describing heat transfer in materials, particularly concerning conduction and convection.

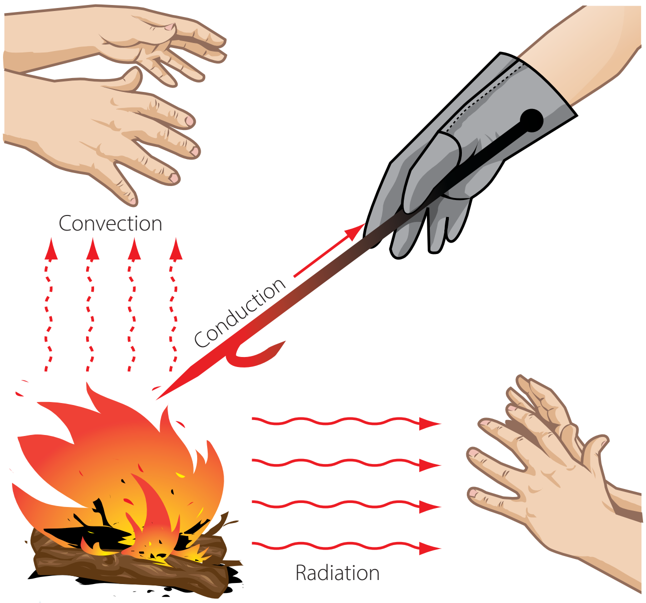


FIGURE 11.23 Heat can be transferred by conduction, convection or radiation.

Conduction

Conduction is the transfer of heat energy through a substance by particle collision. There is no net movement of particles. The particles in the hottest part of a material, such as a metal rod, vibrate further from their usual positions. Collisions with lower-energy particles transfer energy. Ultimately, the average kinetic energy of all the particles becomes the same – the substance reaches thermal equilibrium.

Thermal conductivity

Different materials have different conducting properties. **Thermal conductivity** measures how much energy per second flows through a unit area (1 square metre) that has a thickness of unity (1 metre in SI units) of a material per degree temperature difference between the two ends of the material. The unit of thermal conductivity is $\text{W m}^{-1}\text{ K}^{-1}$. Solids are better heat conductors than liquids or gases. Heat insulators are poor heat conductors.

Calculations in thermal conductivity can be done taking all the factors in the definition above into account, as follows.

$$\text{Energy change: } \Delta Q$$

Note that, since the temperature difference is always negative, ΔQ will be negative.

$$\text{Energy per second: } \frac{\Delta Q}{t}$$

In practice, we usually don't have exactly 1 square metre of material, so we need to include a variable, A, in our calculations to specify the actual area when working out heat flow through a material:

$$\text{Area: } A$$

Temperature difference is the final temperature (outgoing) less the initial temperature (incoming). Since the flow is from hot to cold, the final value (colder) less the initial value (hotter) will always yield a negative number:

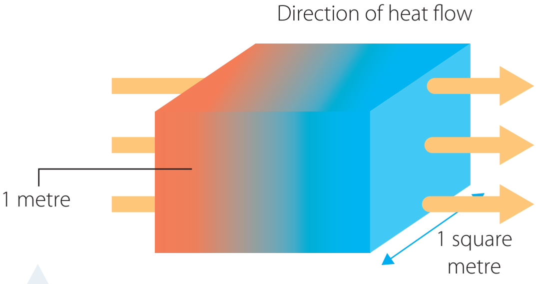


FIGURE 11.24 Heat flow in a solid

$$\text{Temperature difference: } T_{\text{final}} - T_{\text{initial}} = \Delta T$$

The distance travelled through the material is also generally not 1 m, so we need a variable to specify this:

$$\text{Distance: } d$$

Finally, since each substance has a different thermal conductivity because of its physical structure, we need to specify this.

$$\text{Thermal conductivity of material: } k$$

Putting it all together we arrive at:

$$\frac{\Delta Q}{t} = \frac{kA\Delta T}{d}$$

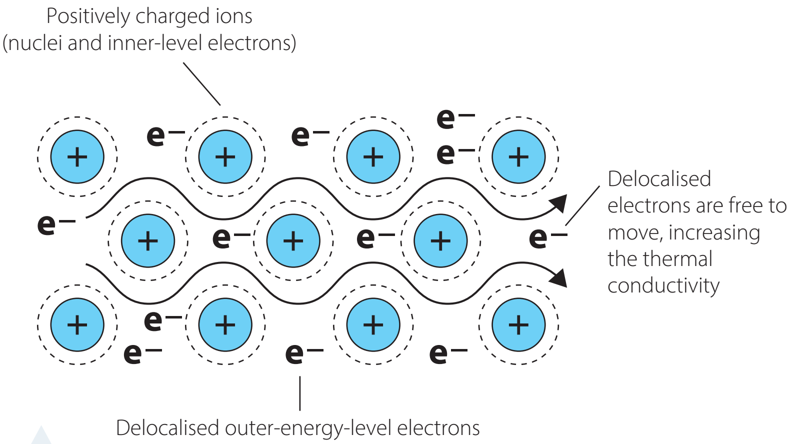


FIGURE 11.25 Delocalised electrons are free to move in metals and can conduct thermal energy quickly.

Metals are particularly good heat conductors – they have large thermal conductivities. The large numbers of relatively unattached electrons in metals, which are relatively free to move, transfer kinetic energy quickly. The **delocalised valence electrons** transfer energy to other electrons and atoms at a faster rate than electrons that are tightly bonded.

Good heat conductors, such as liquid sodium, are used in some nuclear reactors to transfer heat to water. Other good conductors are used in refrigerators, disc brakes, computer heat sinks and car engine radiators. The thermal conductivity of some metals is given in Table 11.4 below.

TABLE 11.4 Thermal conductivities of some metals

SUBSTANCE (AT 20°C)	THERMAL CONDUCTIVITY ($\text{W m}^{-1}\text{ K}^{-1}$)
Admiralty brass	111
Aluminium, pure	204
Copper, pure	386
Gold	315
Lead	35
Tungsten	170
Zirconium	23

Almost all non-metal materials, including gases, are insulators. Unlike metals, non-metals do not have free, delocalised electrons. Energy transfer occurs between particles that are relatively fixed (in solids) or widely spaced (in gases). When they are cold, birds and cats fluff their feathers and fur to trap air. Consequently, less heat is transferred from their bodies.

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FIGURE 11.26 **a** Birds fluff up their feathers to retain body heat when it is cold. **b** Fine down feathers found under the tougher exterior feathers trap air in their structures.

WORKED EXAMPLE 11.4

Numeracy

Given that the thermal conductivity of aluminium is 204 W $\text{m}^{-1}\ \text{K}^{-1}$, how much energy is transferred through a distance of 40 cm in a block of aluminium of square cross-section 0.25 m^2 with one side at 60°C and the other at 10°C, in 300 seconds?

ANSWER	**LOGIC**
$k = 204\ \text{W m}^{-1}\ \text{K}^{-1}$; $A = 0.25\ \text{m}^2$; $\Delta T = 10 - 60 = -50°\text{C}$; $d = 40\ \text{cm}$; $t = 300\ \text{s}$	▪ Identify the relevant data in the question.
$\frac{\Delta Q}{t} = \frac{kA\Delta T}{d}$	▪ Identify the appropriate formula.
$\Delta Q = \frac{tkA\Delta T}{d}$	▪ Rearrange the formula.
$\Delta Q = \frac{300\ \text{s} \times 204\ \text{W m}^{-1}\text{K}^{-1} \times 0.25\ \text{m}^2 \times -50\ \text{K}}{0.4\ \text{m}}$	▪ Substitute the known values, with units, into the formula.
$= -1\,912\,500\ \text{J}$	▪ Calculate the answer. The negative sign indicates energy transferred out, from one end to the other.
$\Delta Q = 1910\ \text{kJ}$ or $1.91 \times 10^6\ \text{J}$	▪ Express the final answer with the correct significant figures and units. The question asks for 'how much energy is transferred', so do not include the negative sign.

TRY THESE YOURSELF

1 A block of lead, with an area of 0.2 m^2 and length 10 cm, is heated at one end to 35°C while its other end is kept at 0°C. How much heat energy is transferred from one end to the other in 90 seconds?

2 A 30 cm copper wire has a diameter of 1 mm. If one end of the wire is at 65°C while the other is at 10°C, how long will it take to transfer 1.00 kJ of heat to the other end? (You will need to refer to Table 11.4.)

INVESTIGATION 11.5

Thermal conductivity

Metals have a wide range of thermal conductivities. Given the length and cross-sectional area, it is possible to determine the thermal conductivity of an unknown metal rod by examining heat transfer over a set amount of time, and so identify the metal.

AIM

To identify an unknown metal by determining its thermal conductivity

MATERIALS

- Hot plate (to maintain constant temperature for hot water bath)
- 250 mL beaker
- 150 mL beaker
- Ice cubes
- Water
- Measuring cylinder
- 2 thermometers
- Styrofoam sleeve or similar for insulation
- Styrofoam cup
- Dissecting probe
- Timer/stopwatch
- Putty or other sealant
- Metal rod (approximately 15 cm long, made of an unidentified metal)
- Retort stand, boss head and clamp (× 2)
- Safety glasses
- Protective gloves

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Hot water can cause burns and scalds.	Do not exceed the experimental temperature. Wear safety glasses and protective gloves. If spilt on skin, wash with plenty of cold water for 5 minutes. Apply ice pack.

What other risks are associated with your investigation, and how can you manage them?

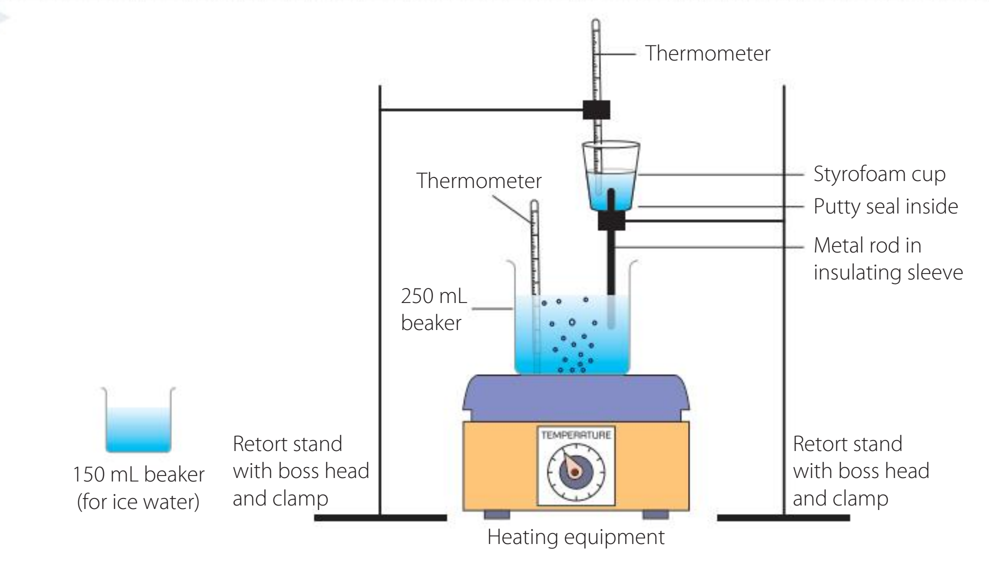

FIGURE 11.27 Experimental set-up for thermal conductivity

METHOD

1. Set up the equipment (as shown in Figure 11.27).
    - Place 100 mL of water in the 150 mL beaker. Fill with ice cubes and monitor the temperature of the water. When the temperature is 0°C, this will be your ice water.
    - Place 200 mL of water in the 250 mL beaker. Set on the hot plate and monitor the temperature of the water. This will be your hot water bath and it is ready when the temperature is 60°C.
    - Place the insulating sleeve around the middle of the metal rod, leaving about 2 cm exposed at each end.
    - Make a small hole in the middle of the base of the styrofoam cup with the dissecting probe. Insert the metal rod from below so that the 2 cm of exposed metal penetrates into the cup. The tighter the fit, the better. Seal with putty on the inside.
    - Clamp the styrofoam cup, with rod inserted, high on the retort stand.
    - Separately clamp the thermometer used to monitor the ice water temperature so it will now be able to measure the temperature of the styrofoam cup's contents.
2. Measure 100 mL of ice water (no ice cubes) into the styrofoam cup. Check that there are no leaks.
3. Lower the rod/cup assembly so that the bottom 2 cm of the rod is in the hot water bath.
4. Start the timer.
5. Record the elapsed time and the temperature of the water in the styrofoam cup at the beginning, and then every minute for 5 minutes.
6. Repeat for reliability.

RESULTS

Record your results in a table, and display the data as a graph.

ANALYSIS OF RESULTS

1. Determine the amount of energy transferred to the water in the styrofoam cup using the equation $\Delta Q = mc\Delta T$. Remember that the ΔT applies to the water in the styrofoam cup only.
2. If you haven't already done so, measure the length and diameter of the rod. Use the diameter to work out its cross-sectional area.
3. Now that we have ΔQ, we can determine the thermal conductivity, k, of the metal using the formula $\frac{\Delta Q}{t} = \frac{kA\Delta T}{d}$ (first, transpose the formula to make k the subject).
4. Refer to Table 11.4 (page 322) or consult an online table for thermal conductivities of metals to determine which metal the rod is made from.

DISCUSSION

1. Did your graph yield a straight line?
2. What does this mean?
3. Were the results of your second trial consistent with your first trial?
4. How would you assess the overall reliability of your investigation?
5. What improvements could you make to the investigation that would enhance its reliability?
6. How valid was the investigation? Did it meet the stated aim?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

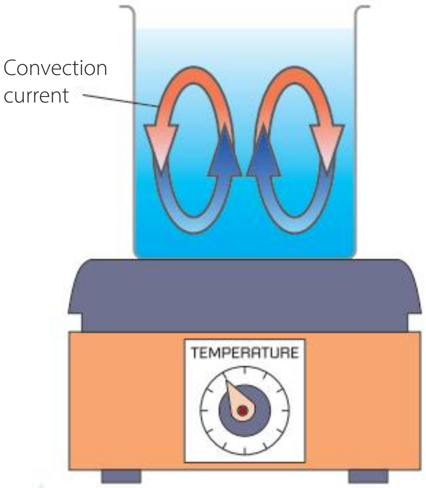

FIGURE 11.28 Convection currents transfer heat energy in water.

Convection

Birds soar gracefully on **convection currents** (also called **thermals**), which are caused by temperature differences between air masses.

Convection is the transfer of heat energy by bulk movement of particles. The flow of particles away from a warmer to a cooler region produces a convection current. Heated fluids expand, becoming less dense and more buoyant. Convection currents only occur in fluids (liquids and gases), which have relatively weakly connected particles, but more so in gases than liquids because the particles in a gas are less tightly connected.

Convection currents and **convection cells** occur where warm and cold fluid masses intersect; for example, in the atmosphere and oceans, and in hydronic home-heating systems.

Figure 11.28 shows convection currents – warm, less dense water at the bottom flows upwards, while more dense water at the top sinks. A convection cell is produced.

INVESTIGATION 11.6

Bottled convection currents

AIM

To explore convection in liquids

MATERIALS

- 4 empty identical transparent bottles with a mouth at least 8 cm across
- Warm and cold water
- Food colouring (yellow and blue)
- An old playing card (or card of similar dimensions and strength)
- Digital camera (optional)

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Water on the floor can cause slipping.	Perform the experiment in a sink or large tray to avoid spills. Clean up any spills immediately.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Fill the four bottles with water – two with warm tap water and two with cold water.
2. Add yellow food colouring to the bottles of warm water, and blue food colouring to the bottles of cold water.
3. Place the playing card over the mouth of one of the warm water bottles.
4. Over a sink or large tray, turn the bottle upside down and rest it on top of a cold water bottle. Make sure that they are exactly aligned mouth to mouth with the card separating the two liquids (as shown in Figure 11.29).

»

5. Bring the two liquids into direct contact by sliding the card out carefully.
6. Observe what happens to the coloured liquids.
7. Repeat steps 3–6, but this time place the cold water on top of the warm water.

ANALYSIS OF RESULTS

1. For each situation, describe any changes to the:
   - cold water.
   - warm water.
2. Support your descriptions with annotated diagrams, photos or video clips.
3. Use the kinetic particle model to explain what you observed.

DISCUSSION

Which of the two experiments could be used as a model to explain:

- ocean currents?
- the formation of thunder clouds?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

FIGURE 11.29 Experimental set-up

Radiation

Radiation is the transfer of energy that does not need a medium. Unlike conduction and convection, radiation does not involve particles of matter. Except at 0 K, all objects emit electromagnetic radiation.

Moving charged particles (electrons or ions) are the source of electromagnetic radiation. The faster they move, the hotter the substance is. This also means the radiation's frequency is higher and its wavelength is shorter. A graph of the intensity at each frequency is called a **Planck curve**, and is related to the Maxwell–Boltzmann distribution curve we saw in Figure 11.8 (on page 306).

In space, gas clouds (close to 0 K) emit radio waves, while stars (3000–30 000 K) emit ultraviolet and visible light. Warm bodies mostly emit infrared radiation. Planck curves are used to measure the temperature in furnaces and stars. At 800°C, objects glow dull red. Stars such as Spica, which mostly emit ultraviolet light, have temperatures of about 22 000 K.

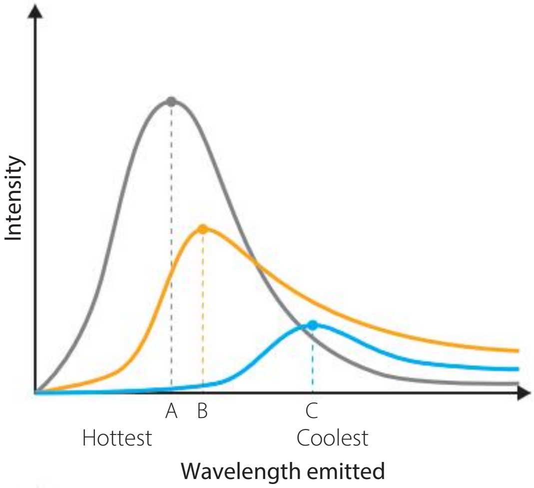

FIGURE 11.30 Planck curves showing peak intensities for three objects at different temperatures. Note that the temperature scale reads from right to left.

WS Hot water systems and heat energy loss

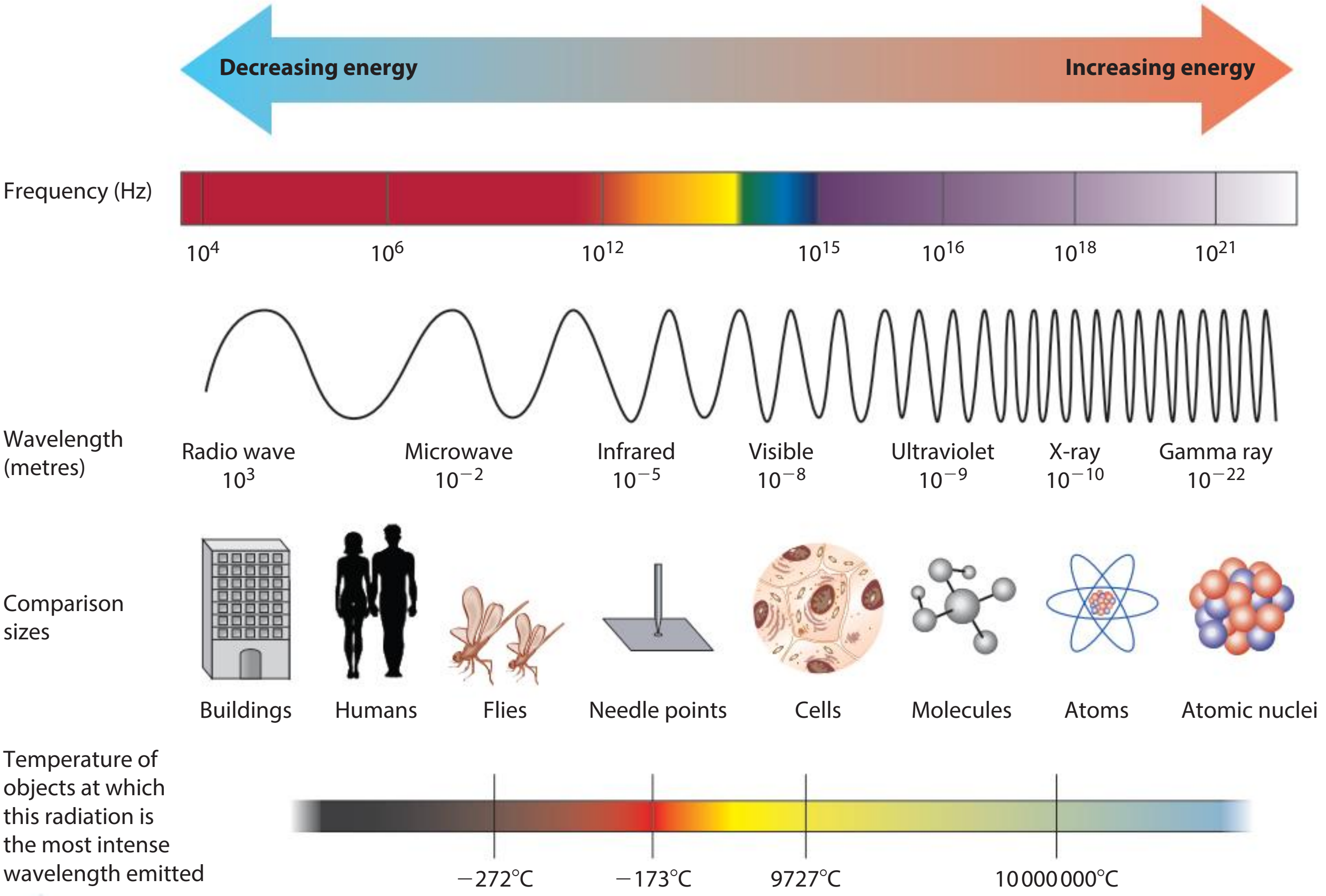

FIGURE 11.31 The electromagnetic spectrum

INVESTIGATION 11.7

Radiative heating

AIM

To investigate the heating properties of microwaves

MATERIALS

- Microwave oven (2.45 GHz)
- Oven mitts
- Baking paper
- 2 solid blocks of chocolate (length > 13 cm)
- Ruler
- Digital camera

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Hot molten chocolate can burn.	Only handle the molten chocolate with oven mitts. Do not eat it.

What other risks are associated with your investigation, and how can you manage them?

»

METHOD

1. Remove the rotating tray from inside the microwave unit
2. Cover the 'floor' of the microwave with baking paper, avoiding the rotating spindle.
3. Place one block of chocolate on baking paper, parallel to microwave door.
4. Cook for 30 seconds (assuming 1 kW microwave unit).
5. Remove paper with chocolate on it.
6. Repeat steps 2–5, this time with the chocolate block perpendicular to door.

RESULTS

1. Measure the distances between the hot spots and record in a table.
2. Take a digital image of your chocolate blocks.

DISCUSSION

Explain the existence of, and distance between, the hot spots.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- Heat can be transferred by conduction, convection or radiation.
- Conduction is the transfer of heat energy through a substance by particle collision.
- Thermal conductivity measures how much energy per second flows through a material per degree temperature difference between the two ends of the material.
- $\frac{\Delta Q}{t} = \frac{kA\Delta T}{d}$
- Metals are particularly good heat conductors.
- Almost all non-metal materials, including gases, are insulators.
- Convection is the transfer of heat energy by bulk movement of particles.
- With convection, particles flow from warmer to cooler regions.
- Convection currents only occur in fluids.
- Convection cells occur where warm and cold fluid masses intersect.
- Radiation involves the transfer of energy without the need for a medium.
- Moving charged particles generate electromagnetic radiation.
- In a substance at a certain temperature, particles have a range of energies with a particular peak. The radiation they generate has a corresponding range of frequencies.

CHECK YOUR UNDERSTANDING

11.5

1. What properties of the shape of an object affect the rate of heat conduction in it?
2. Why are metals generally good conductors of heat?
3. a Describe a convection cell.
   b Why does heat rise in a convection cell?
4. Why does water feel colder the deeper you dive?
5. What are two major differences between conduction and convection on the one hand, and radiation on the other?
6. Propose a method of measuring the temperature of an object remotely. Justify your choice.

11 CHAPTER SUMMARY

- Heat is energy that is in the process of being transferred from one place to another due to a temperature difference.
- Work is energy that is being transferred due to the action of a force.
- James Joule's careful experiments led to the law of conservation of energy.
- Collisions between particles are perfectly elastic; the total kinetic energy before and after the collision is the same.
- Potential energy is stored in the 'springs' that connect the particles; potential energy depends on the distance between particles.
- Normally when a substance is heated, its particles gain kinetic energy.
- During a phase change, the heat energy increases the distance between particles. The particles themselves do not gain kinetic energy.
- Internal energy is the sum of the kinetic energy and the potential energy of the particles in a substance.
- An object can have bulk (organised) kinetic energy (such as a train in motion), which is independent of its internal energy.
- Temperature is a measure of the average kinetic energy of particles in a substance.
- At any given temperature, some particles move slower than average and some move faster.
- Thermodynamics is the study of the effects of heat, work and energy on systems large enough to observe and measure.
- If two thermodynamic systems are each in thermal equilibrium with a third, then they are in thermal equilibrium with each other.
- No net heat flows between substances in thermal equilibrium.
- When a pure substance is heated, its temperature is directly proportional to the energy transferred from a source if it does not change phase, the mass is constant and the energy input rate is constant.
- During a change of phase, energy is still transferred but temperature does not change.
- Some materials change their properties in a regular way as their temperature changes.
- The specific heat capacity of a substance, *c*, is the amount of energy required to raise the temperature of 1 kg of the substance by 1°C, without a change of phase.
- The temperature, *T*, of a body goes up in direct proportion to the amount of heat energy, ΔQ.
- $\Delta Q = mc\Delta T$ where *m* is the mass of the substance and *c* is the specific heat capacity.
- Water has the highest specific heat capacity of most commonly occurring substances: 4200 J kg^{-1} $°\text{C}^{-1}$.
- A closed system is one that matter cannot enter or leave, but which energy can be transferred into or out of by work or heat.
- An isolated system is one that neither energy nor matter can enter or leave.
- The first law of thermodynamics (the law of conservation of energy) states that in an isolated system, energy can neither be created nor destroyed. Energy can be transferred or transformed but the total energy of an isolated system remains constant.
- A pure solid starts to change state to a liquid at its melting point. A pure liquid starts to change state to a gas at its boiling point. Both processes, melting and vaporisation respectively, require energy input.
- Energy removal causes gases to undergo condensation, and causes liquids to undergo solidification.
- Below the boiling point, evaporation from a liquid occurs at the surface.
- At boiling point, bubbles form below the surface. No temperature change occurs during vaporisation.
- A solid can turn directly into a gas when heated (sublimation). The reverse process is deposition.
- The specific latent heat of fusion of a substance is the energy required to change the state of 1 kg of the substance from its solid state to its liquid state without any change in temperature.
- The specific latent heat of vaporisation of a substance is the heat required to change the state of 1 kg of the substance from its liquid to gaseous state.
- The energy required to change the state is: $\Delta Q = mL$.
- Heat can be transferred by conduction, convection or radiation.

- Conduction is the transfer of heat energy through a substance by particle collision.
- Thermal conductivity measures how much energy per second flows through a material per degree temperature difference between the two ends of the material.
- $\frac{\Delta Q}{t} = \frac{kA\Delta T}{d}$
- Metals are particularly good heat conductors.
- Almost all non-metal materials, including gases, are insulators.
- Convection is the transfer of heat energy by bulk movement of particles.
- With convection, particles flow from warmer to cooler regions.
- Convection currents only occur in fluids.
- Convection cells occur where warm and cold fluid masses intersect.
- Radiation involves the transfer of energy without the need for a medium.
- Moving charged particles generate electromagnetic radiation.
- In a substance at a certain temperature, particles have a range of energies with a particular peak. The radiation they generate has a corresponding range of frequencies.

11 CHAPTER REVIEW QUESTIONS

Review quiz

1 Distinguish between temperature, kinetic energy and internal energy.

2 What happens to temperature during a phase change?

3 What are the 'fixed points' on a thermometer?

4 Define 'thermodynamics'.

5 Define 'conduction', 'convection' and 'radiation'.

6 What state of matter is the most efficient in the conduction of heat energy?

7 How do isolated and closed systems differ?

8 All the energy lost from Earth to space is radiant energy. Why is there no loss due to conduction and convection?

9 Why is it better to take an insulated hot water bottle to bed rather than an insulated hot brick?

10 Sydney recorded its highest temperature of 45.8°C on 18 January 2013. What is this temperature in kelvin?

11 A scalpel blade that needs to be sterilised is heated in a flame until it is glowing red-hot. It is then held above a glass of water in which it will be cooled.

- **a** Which has the highest average kinetic energy, the blade or the water?
- **b** The scalpel blade is dropped into the water. What happens to the average kinetic energy of the blade and of the water?

12 A hot cup of coffee is left to stand for a couple of hours. Explain on a particle level how it reaches thermal equilibrium with the surroundings.

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13 Why is it that the water going over a waterfall is warmer at the bottom of the fall than at the top?

14 In an espresso coffee machine, steam at 100°C is passed into milk to heat it.

- **a** Calculate the energy required to heat 150 g of milk from 20°C to 80°C ($c_{\text{milk}} = 4010$ J kg^{-1} K^{-1}).
- **b** Calculate the mass of steam condensed.

15 Why are bar heaters more effective than convection heaters in a room that has large doors that are frequently opened?

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16 Explain what role the air trapped between the two panes of glass in double-glazed windows plays in controlling heat flow.

17 To deaden pain in minor operations, a liquid that vaporises easily (vapocoolant) is sometimes sprayed onto the skin. Explain how vapocoolant cools the skin.

18 Explain why food cooks more quickly when it is steamed rather than boiled.

19 If you blow moist, warm air from your mouth onto a mirror on a cool day, the mirror fogs. Has this process cooled or warmed the mirror? Explain.

20 A lake can absorb huge amounts of radiation on a sunny summer's day, and yet the temperature of the water can remain virtually constant. Explain how this is possible.

21 Suppose two substances, lead and water, are in thermal equilibrium. Equal amounts of heat energy are now put into each. Assuming the mass of each substance is the same, will they still be at thermal equilibrium? Explain in detail.

22 Why is it important to keep the lid on a simmering pot of water?

23 A vacuum flask is designed to keep liquids hot or cold. A student seals 200 g of ice-cold water in a glass vacuum flask and finds that it warms up by 3.5 K per hour. Calculate the average rate of heat flow into the flask.

24 An electric hotplate is required to heat a 1.6 kg block of ice at 0°C to steam at 100°C. If it takes 10 minutes to boil the water away and 15% of the heat generated by the electric hot plate is lost to the surroundings, calculate the rate at which the electric hot plate supplies energy to convert the ice into steam.

25 To test the temperature of an oven, a 500 g aluminium tray was placed in the oven and allowed to reach thermal equilibrium. It was then dropped into a plastic bucket that contained 2 L of water at 0°C. The temperature of the water rose by 7°C.

a What was the temperature of the oven?

b Why was a plastic bucket used?

» END-OF-MODULE REVIEW

MODULE ③: WAVES AND THERMODYNAMICS

1 A student performs an investigation during which transverse waves are sent down a spring.

a Using a sketch, describe the motion of the particles in the spring as the wave passes.

b Outline a method the student could follow that would allow the student to calculate the velocity of the waves in the spring without using

$$\text{velocity} = \frac{\text{distance}}{\text{time}}.$$

c Construct and label a displacement versus time graph for such a wave with the following characteristics:

- velocity = $3.0\ \text{m s}^{-1}$
- frequency = 6.0 Hz
- amplitude = 4.0 cm

2 When earthquake waves pass through the ground on which a building stands, the building moves in different ways. If the first earthquake waves that reach the building are longitudinal waves, followed by a series of transverse waves through the ground, describe how an observer might see the building move relative to the source of the earthquake waves and the ground.

3 Using sketches to assist you, describe and outline one application of each of the following wave phenomena:

a reflection.

b refraction.

c diffraction.

4 An investigation was performed to explore resonance in a mechanical system.

a Outline what is meant by the term 'resonance'.

b Describe how the natural frequency of a mechanical system could be determined by varying the driving frequency until it matches the natural frequency.

5 With the aid of diagrams, compare the formation of overtones in air columns closed at one end and air columns open at both ends.

6 With the aid of diagrams and with reference to the relevant physics of waves, explain the formation of beats when two sources with different frequencies of sound are played simultaneously.

7 The total internal reflection of light has many applications.

a Explain the necessary conditions for total internal reflection.

b Find the critical angle for light when a ray is travelling in a medium with a refractive index $n_1 = 1.45$ when that medium is wrapped in a second medium with a refractive index $n_2 = 1.20$.

c Describe two applications of total internal reflection.

8 The intensity of the light from a lighthouse is measured over a range of distances.

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a On a set of axes, sketch the expected relationship between intensity and distance for the light from the lighthouse.

b When 2.50 km from the lighthouse, the intensity of the light is measured as being 35.0 units. Calculate the intensity of the light at a distance of 500 m.

9 Hydroelectric electricity production produces no greenhouse gases, with falling water turning the turbine blades of the generators. During periods of low electricity demand, water can be pumped back uphill so it is again available to generate electricity when the demand is higher. With reference to the Maxwell–Boltzmann distribution, explain a source of inefficiency in this energy storage scenario.

10 Explain thermal conduction in:

a solids.

b liquids.

c gases.

11 A pure aluminium rod of radius 2.5 mm and length 30 cm transfers 240 J in 60 seconds. What is the temperature difference between the two ends of the rod?

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DEPTH STUDY SUGGESTIONS

- → Investigate, research and make models of the Anzac Bridge structure in Sydney and the problems with resonance that this bridge had when first built.
- → Investigate, research and make models of antenna that use parabolic reflecting surfaces in light telescopes and in radio telescopes, e.g. Parkes radio telescope, Hubble Space Telescope or Square Kilometre Array (SKA).
- → Investigate, research and build a wave tank to show how 'rogue' waves in the ocean form and how they are responsible for the loss of large ships.
- → Investigate, research and model the role of resonance in speaker design and in building design.
- → Investigate the operation and application of the Doppler effect in modern ultrasound techniques used to detect blood flow through the heart and arteries.
- → Investigate and compare aircraft design for subsonic and supersonic aircraft.
- → Investigate methods that have been used to overcome the inverse square law when transmitting information by radio waves or microwaves through the enormous distances of space.
- → Investigate the applications of total internal reflection, the advantages and problems encountered, and how these problems have been overcome.
- → Investigate the applications of dispersion, particularly in spectrometers used in astronomy.
- → Investigate the relation between pressure, volume and gas in a contained gas. Research applications of these relationships in refrigerators and reverse-cycle air conditioners.
- → Investigate and compare the advantages and disadvantages of diesel engines and internal combustion engines that use spark plugs.
- → The steam engine revolutionised Western civilisation. Investigate the thermodynamic principles of the steam engine and the factors that make them more, or less, efficient.

» MODULE FOUR

ELECTRICITY AND MAGNETISM

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9780170409063

12 Electrostatics

INQUIRY QUESTION

How do charged objects interact with other charged objects and with neutral objects?

OUTCOMES

Students:

- conduct investigations to describe and analyse qualitatively and quantitatively: CCT ICT
  - processes by which objects become electrically charged (ACSPH002)
  - the forces produced by other objects as a result of their interactions with charged objects (ACSPH103)
  - variables that affect electrostatic forces between those objects (ACSPH103)
- using the electric field lines representation, model qualitatively the direction and strength of electric fields produced by:
  - simple point charges
  - pairs of charges
  - dipoles
  - parallel charged plates ICT
- apply the electric field model to account for and quantitatively analyse interactions between charged objects using: ICT N
  - $\vec{E} = \frac{\vec{F}}{q}$ (ACSPH103, ACSPH104)
  - $\vec{E} = -\frac{V}{\vec{d}}$
  - $\vec{F} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$ (ACSPH102)
- analyse the effects of a moving charge in an electric field, in order to relate potential energy, work and equipotential lines, by applying: (ACSPH105)
  - $V = \frac{\Delta U}{q}$ where U is potential energy and q is the charge

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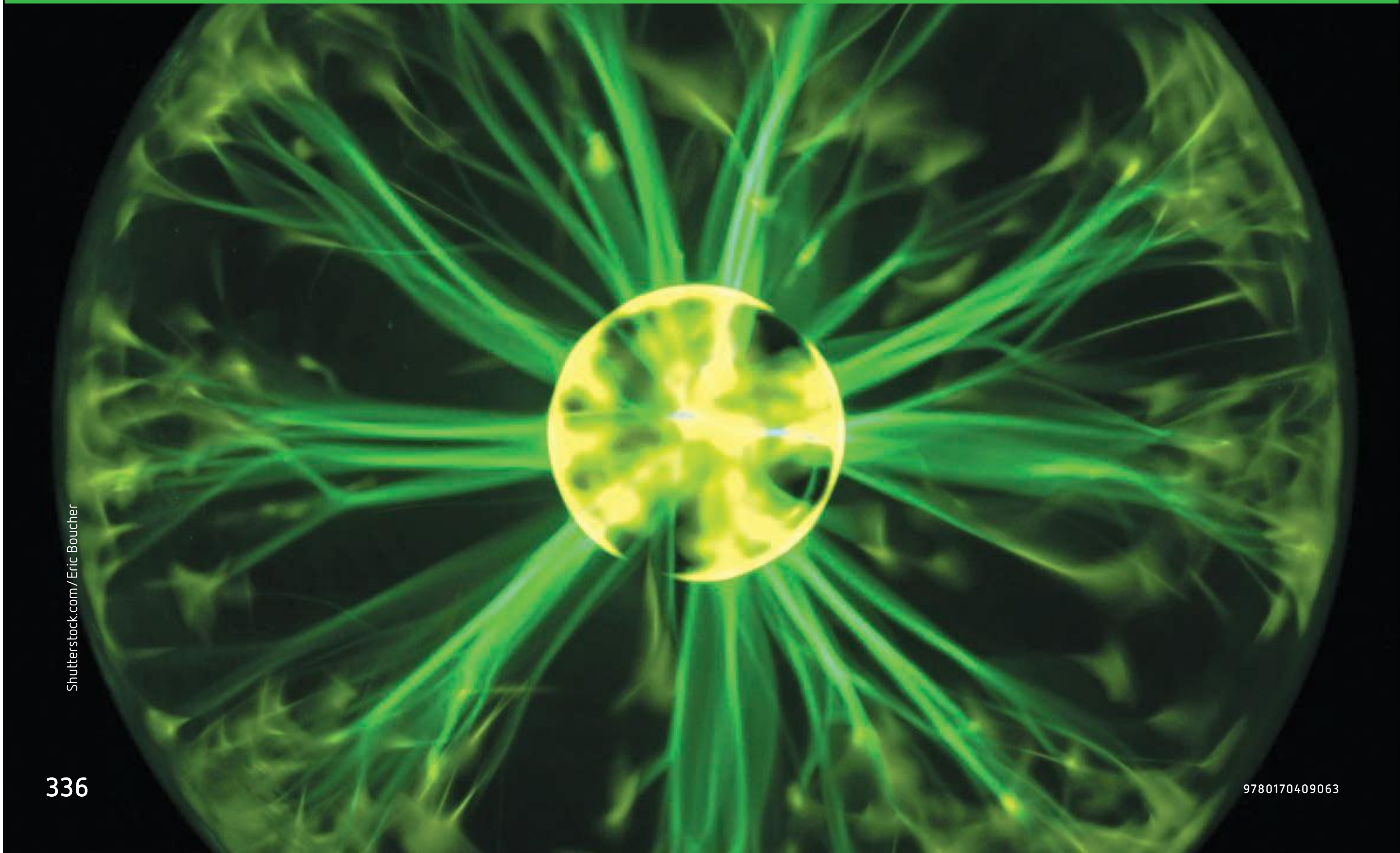

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In this chapter, we begin a new topic in physics: electromagnetism. Although new ideas, including charge and electric field, are introduced, you will find that many familiar ideas (particularly the ideas of force and energy) are also used. These ideas are central to this topic, just as they are central to the topics presented earlier. If you look back to Figure 1.6 on page 7, you will see that the same core ideas of forces, energy, motion and conservation principles are used in electromagnetism.

Matter interacts via forces, as described in chapter 4. There are only four fundamental forces. These are the electromagnetic force, gravity, and the strong and weak forces. This chapter will look at the electrostatic force, which is a part of the electromagnetic force. It is exerted on, and by, charged particles. In chapter 14, the magnetic part of the electromagnetic force will be described. The other three forces will be discussed in *Physics in Focus Year 12*.


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FIGURE 12.1 A lightning storm is a powerful and spectacular display of electrostatics in nature.

We will begin by exploring how charged objects interact with other charged objects and with neutral objects.

12.1 Electric charge

Matter is made of atoms. Figure 12.2 shows a diagram of an atom, based on a model by Rutherford and Bohr. Atoms are too small to be seen directly, so clever experiments had to be done to work out that atoms consist of a tiny nucleus that contains protons and neutrons, which is surrounded by orbiting electrons. In *Physics in Focus Year 12*, atomic structure is described in detail and a more modern model is introduced. For now, the Rutherford–Bohr model has all the essential features that we need.

You can see that in the nucleus of the atom shown in Figure 12.2 there are two types of particles. Those labelled with a + are protons (orange), and they have a positive **charge**. Those without a label (grey) are neutrons, and they are uncharged, or **neutral**. Outside the nucleus, the small particles labelled – are electrons (green). Electrons are negatively charged. Charge is a fundamental property of particles, like mass. We deduce the existence of this property from the forces that particles exert on one another. These forces can be attractive or repulsive, so there must be two types of charge.

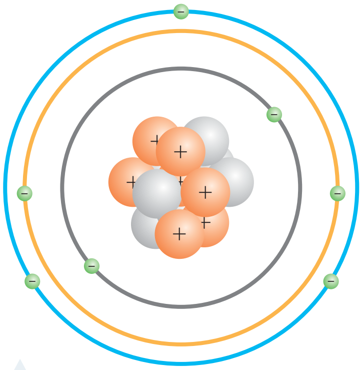

FIGURE 12.2 A model of an atom, with a small positively charged nucleus surrounded by orbiting electrons

Recall from chapter 4 that when an object accelerates, there must be a force acting on it. Gravity was identified as a force and described by Newton hundreds of years ago. The gravitational force is always attractive, and is due to the mass of objects. But there are also forces between objects that include repulsion, and are far too strong to be caused by gravity. Many of these interactions, particularly on a scale of atoms and larger, are explained by the existence of charge and the forces it exerts.

Charge is difficult to define, but you can think of it as an answer to the question 'how can we explain observed interactions that are not due to gravity?'. In *Physics in Focus Year 12*, you will see how other properties of particles are the answer to the question 'how can we explain observed interactions that are not due to gravity *or* electromagnetism?'. The electrostatic model, including the idea of charge, was developed to explain certain types of interactions.

Atomic structure of matter

Find out more about the atomic structure of matter and the basis of the periodic table.

How do objects become charged, and exert forces?

Atoms are generally neutral overall – this means they have as many positive protons in their nucleus as they have negative electrons orbiting it. The attractive force between protons and neutrons is what holds atoms together. Atoms can be ionised by adding or removing electrons. For example, it does not take a lot of energy to remove an electron from a sodium atom (Na) to make a sodium ion, Na^+. Chlorine, Cl, will readily accept an extra electron to form a chloride ion, Cl^-. This losing and gaining of electrons is the basis of ionic bonding, which you may have heard about in chemistry. Na and Cl bond to form NaCl (salt) by first transferring an electron from the Na to the Cl. This creates a positively charged ion and a negatively charged ion, which are attracted to each other – this attraction is the chemical bond. Figure 12.3 shows the process of ionic bonding for salt.

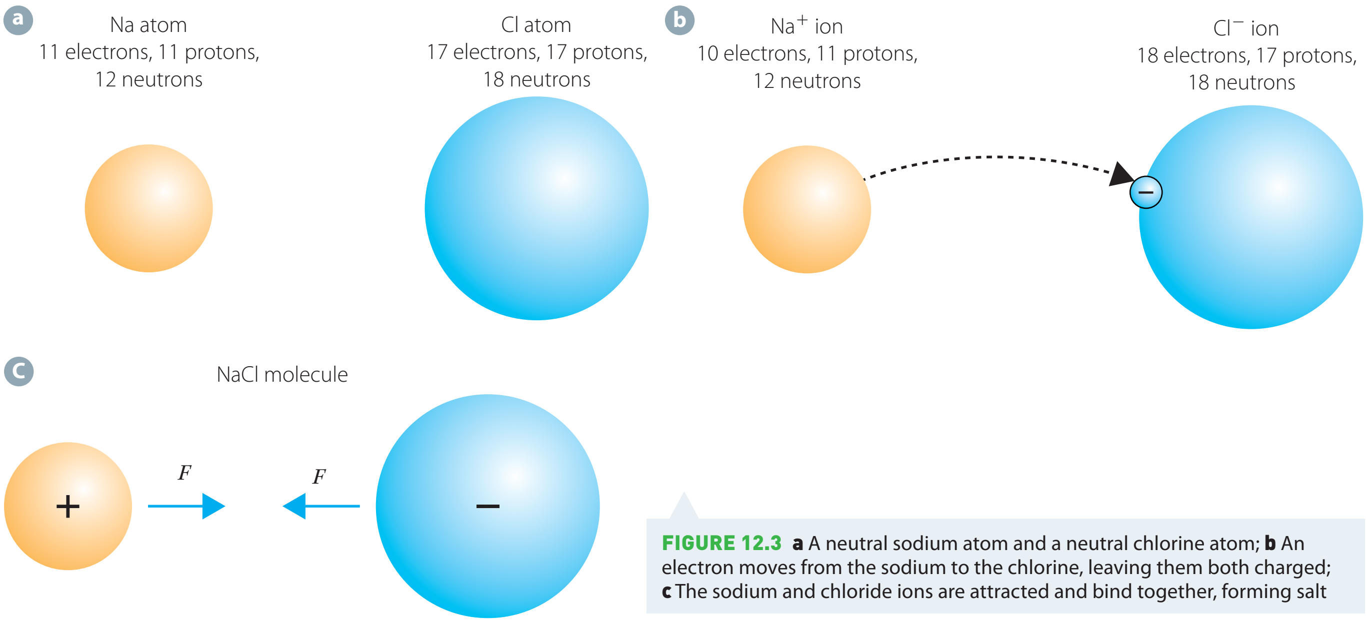

FIGURE 12.3 **a** A neutral sodium atom and a neutral chlorine atom; **b** An electron moves from the sodium to the chlorine, leaving them both charged; **c** The sodium and chloride ions are attracted and bind together, forming salt

We call the force that charged objects, including charged particles, exert on each other the **electrostatic force**. The 'static' in electrostatic means that this force acts when the charged objects are not moving. It also acts when the objects are moving, but as we will see in chapter 14, when charges move they also exert a magnetic force. Electrostatics deals with the simple case when we do not need to include the magnetic force.

The process of moving electrons can happen on a large scale as well as an atomic scale, leaving macroscopic objects charged. A Van de Graaff generator works by taking negative electrons from the metal dome on top, leaving it positively charged. The positively charged dome then exerts a force on other objects that are charged. Figure 12.4 shows a charged Van de Graaff generator dome.

It is not only charged objects that experience a force due to other charges. A neutral object will experience a force when close to a charged object. When you remember that neutral objects are made of atoms, and that these atoms are all made of charged particles, this makes sense. If you hold a piece of packing foam close to a charged Van de Graaff generator dome, it will fly off your hand to the dome. If you could look very closely at the atoms in the foam you would see that the atoms are changing shape very slightly. The electrons move towards one side more, making that side more negative, and leaving the other side more positive. The negative side is closer to the dome, so the attractive force it experiences is stronger than the repulsive force experienced by the positive side. The result is a net force, and an acceleration – the foam flies to the generator.

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FIGURE 12.4 A van de Graaff generator. The paper attached to the dome becomes charged as it loses electrons to the positively charged dome.

Electrons are easy to add or remove from conductors such as metals (as described in chapter 13), so conductors are normally charged by adding or removing electrons. However, electrons are not so easy to add or remove from other types of materials.

Electrostatics

One way of transferring electrons is by rubbing one material against another. Objects can also become charged by taking away pieces of molecules, leaving other pieces behind. If these pieces have a net charge, then the object will become charged. Again, this usually happens by rubbing. This is the process that occurs when you slide down a plastic slide and your hair stands up, or when you walk across a carpet and get a small shock when you touch a door handle.

We refer to any objects with a net charge, either positive or negative, as **charged**. Objects with zero net charge (as much positive as negative) are called neutral.

INVESTIGATION 12.1

Charge and charged objects

Critical and creative thinking

Numeracy

In this investigation, you will explore how charged objects interact with other charged objects and with neutral objects.

AIM

To investigate the interactions between charged and neutral objects.

Write one or more research questions for this investigation. Consider what factors might be important in the force that a charged object exerts on another object.

»

MATERIALS

- Paper torn into many tiny pieces (a few millimetres across)
- 2 plastic combs or perspex rods
- Different types of fabric (e.g. wool, silk, polyester)
- 2 or more balloons
- Sticky tape (some types work better than others, so experiment first)
- Scissors

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Paper scraps can easily end up all over the floor and can be a choking hazard if inhaled.	Keep the paper scraps in a bag until you need to use them.
Sticky tape stuck all over the desk is unsightly.	Clean up after yourself, and do not allow the entire length of tape to become stuck.

What other risks are associated with your investigation, and how can you manage them?

METHOD

A Charging using different materials

1. Place a few tiny pieces of paper on the desk.
2. Rub the plastic comb/rod up and down with a piece of fabric. Hold the plastic comb/rod approximately 2 cm from the paper. Observe what happens.
3. Remove any paper that is stuck to the comb/rod and throw it away.
4. Repeat step 2 with as many different types of fabric as you have. Note down what happens each time. Identify the material that most effectively charged the comb/rod.

B Charge and force

1. Rub the second (uncharged) comb or rod *once* up and down using the material you identified in part A.
2. Hold the plastic comb or rod approximately 2 cm from the paper. Observe what happens, and count the number of pieces of paper that stick to the comb/rod.
3. Remove any paper that is stuck to the comb/rod and throw it away.
4. Rub the comb/rod three times up and down with the fabric, and repeat steps 2 and 3. Repeat steps 2 and 3 again after rubbing the comb/rod 5, 10 and 20 times. Record your results.

C Charge, force and distance

1. Rub the comb/rod vigorously up and down with the fabric many times.
2. Hold the comb/rod 20 cm above the pieces of paper.
3. *Very* slowly move the comb/rod down towards the paper. Note at what distance the paper starts to be lifted, and whether the number of pieces being lifted increases, decreases or stays the same with distance.

D Forces between like and unlike charges

1. Stick a piece of sticky tape approximately 15 cm long to the desk, with 5 cm hanging off the edge unattached. Put a second similar piece close by but not overlapping.
2. One person is to peel off the first piece of tape, holding it carefully by the free end. A second person is to peel off the second piece of tape. Alternatively, peel off one piece with each hand. ***Do not*** allow them to get close together or touch.
3. Holding the pieces of tape so they hang vertically, ***slowly*** bring them close together. Record your observations. Discard the tape.
4. Repeat step 1, but this time place the second piece of tape carefully on top of the first piece, so it overlays it. Peel this piece off first, holding the piece underneath down so it does not come off. Then peel the underneath piece off.
5. Repeat steps 2 and 3.

If you find that the tape is curling up into a loop when you peel it off the desk, try peeling it off more slowly and gently, or using shorter pieces of tape.

» E Forces between charged objects and neutral objects.

1. Rub an inflated balloon vigorously on your hair (or someone else's, with their permission). The balloon should become charged. When it 'crackles' and lifts your hair when moved away, it is charged.
2. Gently place the charged balloon against the wall. Observe the behaviour of the balloon. Try placing charged balloons against different wall surfaces, such as windows, plaster, wood and metal. Record your observations.

RESULTS

- For part A, you should have noted which fabrics were most effective at charging the comb/rod.
- You should have quantitative data recorded for parts B and C. Record this in tables.
- For part B, show the number of pieces of paper attracted to the comb/rod as a function of how charged it was (how many times rubbed up and down).
- For part C, show the number of pieces of paper attracted to the comb/rod as a function of how far away it was (distance in cm).
- Part D and E have qualitative data. You should have noted whether objects were attracted or repelled in each case.

ANALYSIS OF RESULTS

A What sort of fabric most effectively charged the comb/rod?

B How did the force between the charged comb/rod and the paper vary with the amount of charge on the comb/rod? You could plot a graph of pieces of paper attracted versus amount of rubbing.

C How did the force between the charged comb/rod and the paper vary with the distance between them? You could plot a graph of pieces of paper versus distance.

D When the pieces of tape had the same sign charge (both transferred charge to the desk), did they attract or repel each other? When they had opposite sign charge (one transferred charge to the other), did they attract or repel?

E When the charged balloon was placed gently against the wall, was it attracted, repelled or neither? Did the wall surface make any difference?

DISCUSSION

- What general principles can you derive from your observations?
- Have you answered your research questions?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation. Give the answers to your research questions.

Experiments such as those in Investigation 12.1 have led to the following general observations about forces between charged and neutral objects.

1. Neutral objects are attracted to charged objects, whether they are positively or negatively charged. Recall from Newton's third law (see page 92) that forces are interactions experienced by the two interacting objects. So, charged objects are attracted to neutral objects, regardless of their charge. (Note that this breaks down at the level of fundamental particles, where charge cannot be separated.)
2. Like charges repel each other, and unlike charges attract each other.
3. The force that a charged object exerts on a neutral or charged object decreases with the distance between them. We will explore exactly how the force decreases in a later section.
4. The larger the charge, the greater the force that is exerted and experienced.

WORKED EXAMPLE 12.1

Four polystyrene beads (A, B, C and D) sit on a desk after an electrostatics experiment. When bead A and B are brought close together, they are repelled and move apart. When B and C are brought close together, they attract and move closer together. When C and D are brought close together, they do not move. If A has positive charge, what are the signs of the charge on B, C and D?

ANSWER	LOGIC
A has positive charge B is repelled by A C is attracted to B D experiences no force from C	- Identify the relevant data in the question.
B has positive charge C has either negative charge or is neutral C and D are neutral	- Like charges repel each other. - Unlike charges attract; charged and neutral objects attract. - No electrostatic force is exerted by neutral objects on other neutral objects.
A and B are positive, C and D are neutral	- State the final answer.

TRY THESE YOURSELF

1 A fifth bead, bead E, is attracted to all of beads A, B, C and D. Deduce the sign of the charge on E.

2 Three beads, X, Y and Z are all attracted to each other when put close together in pairs. What can you deduce about the charges on X, Y and Z?

KEY CONCEPTS

- Charge is a fundamental property of matter.
- Charge comes in two types: positive and negative.
- Charged objects exert an electrostatic force, and experience an electrostatic force.
- Atoms are made of positively charged protons, negatively charged electrons and uncharged (neutral) neutrons.
- Objects become charged when they gain or lose electrons.
- Objects with like charge (same sign) are repelled by each other.
- Objects with unlike charge (opposite sign) are attracted to each other.
- Neutral objects are attracted to both positively and negatively charged objects.
- The electrostatic force increases with increasing charge, and decreases with distance.

CHECK YOUR UNDERSTANDING 12.1

1 Name the two types of charge and the subatomic particles associated with each.

2 Which is easier to remove from an atom, the protons or the electrons? Why?

3 Kristy and Adam are discussing objects with neutral charge. Adam says that a neutral object doesn't have any charge. Kristy says that a neutral object has lots of positive and negative charges, there are just equal numbers of each. Who is correct and why?

4 Minh charges a balloon by rubbing it on her hair. She moves it away from her hair, then brings it close again, and her hair moves towards the balloon. Explain why her hair is attracted to the balloon.

5 Minh rubs a balloon (balloon A) on her head so that it has some amount of charge on it. Sam rubs a balloon (B) on his head so that it has twice as much charge as that on balloon A. Minh and Sam then hold the two balloons close together and observe that they repel each other.

   a What can you deduce about the signs of the charge on the two balloons?

b What is the ratio of the electrostatic forces $\frac{F_{\text{on balloon A}}}{F_{\text{on balloon B}}}$? Justify your answer.

6 Minh rubs a balloon (balloon A) on her head and holds it near another balloon (B). Balloon B is attracted to balloon A.

a What can you deduce about the net charge of balloon B?

b Minh holds balloon A near a third balloon (C) and they repel each other. What can you deduce about the net charge of balloon C?

12.2 Electric fields

Recall that, in chapter 4, we classified forces as either contact forces or field forces. Contact forces can only act when two surfaces are in contact. Field forces can act when the objects exerting and experiencing the force are *not* touching. Charged objects are able to exert forces on each other when they are not touching. We describe this force at a distance using the **electric field** model. This model was developed from Michael Faraday's idea of 'lines of force'. These lines of force enable an object to exert a force on a second object some distance away.

For the electric field model, we assume that the charges are not moving, and we ignore any quantum or relativistic effects. This model is extremely good at predicting the behaviour of most interacting charged objects. Later, when you study gravitation in Year 12, you will see that this model is very similar to Newton's model of universal gravitation.

You may have conducted experiments to show that a charged object exerts a force on another charged object some distance away. For example, if you rub a balloon on your hair and then hold it close to your head, you will see or feel your hair being pulled towards it. All objects with electric charge create an electric field around themselves. All objects with charge experience a force in an electric field. We define the electric field at a position in space as the force per unit charge that acts on a small positive charge at that position. In other words, the field is a measure of the force acting on a positively charged object. We always talk about 'at a particular point' because the electric field varies from place to place. The field depends only on the object that creates it.

You already know that like charges repel each other and unlike charges attract. Hence, a positive charge pushes another positive charge away. So, the field around a positive point charge (a very small charged object) points away from the charge. Remember that force is a vector – it has magnitude and direction. Field is a measure of force, so it is also a vector with magnitude and direction.

Electric field lines

Fields can be represented by field line diagrams. An **electric field line** diagram uses lines with arrows to show the direction of the force on a small positively charged particle.

To draw a field line diagram, start by considering the force on a test charge at various points. Let's start with a single positive charge and think about what happens when we put a small positive test charge close to it. We know that like charges repel, so the test charge will be pushed away from the positive charge, as shown in Figure 12.6a (page 344). The direction of the electric field is radially away from the charge. If we join up the arrows in Figure 12.6b we get field lines, as shown in Figure 12.6c.

Figure 12.6c shows a field line diagram for a single positive point charge. If the point charge were negative instead of positive, the field lines would all point inwards. A small positive test charge would be attracted to a negative charge (see Figure 12.7, page 344).

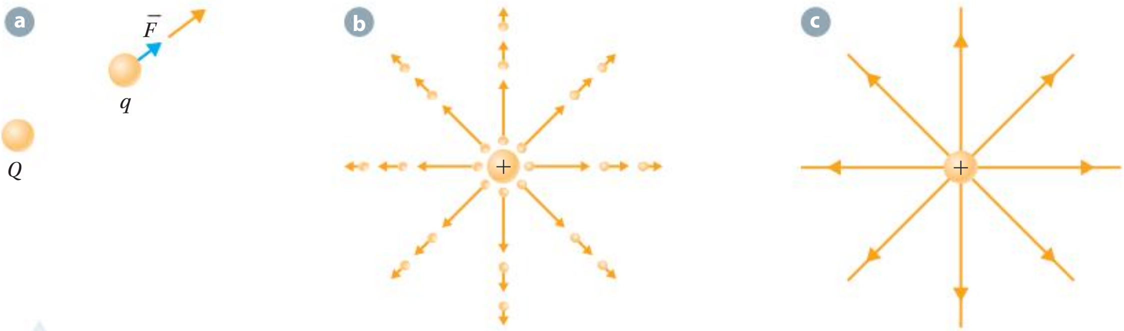


FIGURE 12.5 Constructing a field line diagram. Start by drawing arrows to show the force at various points, and then join them up to form field lines.

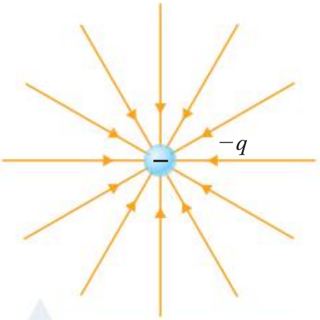


FIGURE 12.6 Field line diagram for a negative point charge

So, field lines come out of positive charges and go into negative charges. This is because positive charges are attracted to negative charges and repelled by positive charges.

Note that in Figure 12.5b, the force experienced by the small test charge gets smaller as it gets further away from the positive charge. The field lines also get further apart. In general, the further apart the field lines are, the weaker the field is, and the weaker the force is.

At a single point in space, the force can only have one direction. If you place a positive charge at a point in an electric field, it will experience a force and hence accelerate. It can only have one acceleration due to the field, so the force can only be in a single direction. The result is that field lines can never cross. If they did, this would imply that the force was simultaneously acting in two different directions.

We can summarise the characteristics of electric field lines:

- they point in the direction of the force acting on a positively charged particle due to the field
- they never cross
- they begin on positive charges and end on negative charges
- the field strength is proportional to the density of field lines.

Drawing field line diagrams

When you draw a field line diagram, there is an infinite number of possible field lines that you can draw. However, you only have finite time, so choose a sensible number! In general, make sure you draw enough lines so that you can see what the field looks like around the charge or charges. Typically, at least eight field lines are needed and they should be evenly spaced around a point charge. Make the ratio of field lines coming out from or entering any charge proportional to the magnitude of the charge. This will ensure your field lines have the properties previously listed.

WORKED EXAMPLE 12.2

Draw a field line diagram for the pair of equal positive charges shown in Figure 12.7.

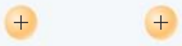
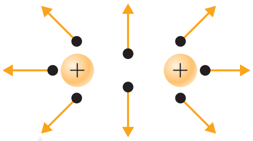
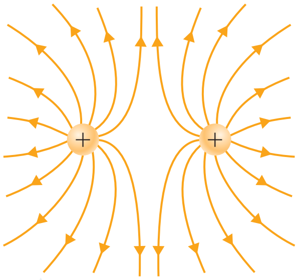


FIGURE 12.7
A pair of positively charged particles

ANSWER	**LOGIC**
 **FIGURE 12.8** The force acting on a small positive test charge at points around the charges	▪ Think about putting a small positive test charge at points close to the charges, as shown in Figure 12.8. ▪ Draw arrows showing the forces due to the two charges.
 **FIGURE 12.9** Field lines for a pair of positive charges	▪ Do this at lots of points and then join up your arrows to form field lines, as in Figure 12.9.

TRY THIS YOURSELF

Draw a field line diagram for a pair of equal negative charges.

A common configuration of charges is a **dipole**. This is a combination of a positively charged object and a negatively charged object close together. Figure 12.10 shows the electric field in the region around a dipole. The sodium and chlorine atoms in a single bound NaCl molecule are a dipole.

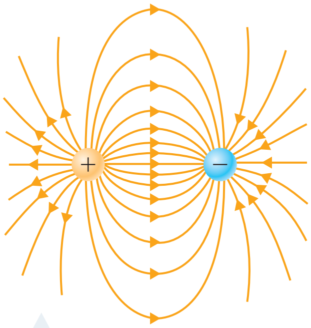


FIGURE 12.10 Electric field line diagram for a dipole

You may have met the idea of **polar** molecules in chemistry. Polar molecules or objects have charge separation, just like when a neutral object is attracted to a charged object. Water (H_2O) is a polar molecule because the hydrogens share their electrons with the oxygen, making the hydrogens positive and the oxygen negative. Figure 12.11 shows an electric field line diagram for water.

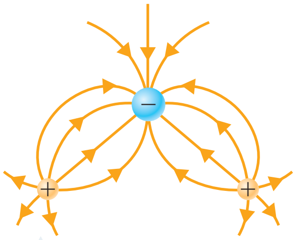


FIGURE 12.11 Electric field line diagram for a polar water molecule

All of the field line diagrams shown so far (Figures 12.5 to 12.11) have been for one or more point charges. For all these cases, the electric field varies with position, as we can see from the way the density of field lines varies.

Uniform electric fields

A **uniform electric field** is one that has the same magnitude and direction at all points. It is possible to create a uniform electric field using either a single, very large, uniformly charged plate or a pair of parallel charged plates.

Figure 12.12 shows a large, uniformly charged sheet. To draw a field line diagram, we start by considering the force that will act on a small positive test charge at different positions near the sheet. As shown in Figure 12.12b, the force at any position will be up, away from the sheet. Remember that field lines cannot cross, so when we draw our field lines they remain parallel (Figure 12.12c.). This tells us that as we get further above the sheet, the field remains constant, or uniform. You can see in Figure 12.12c that the field lines are parallel above most of the sheet, but spread apart at the edges. The field is weaker and not uniform near the edges of the sheet.

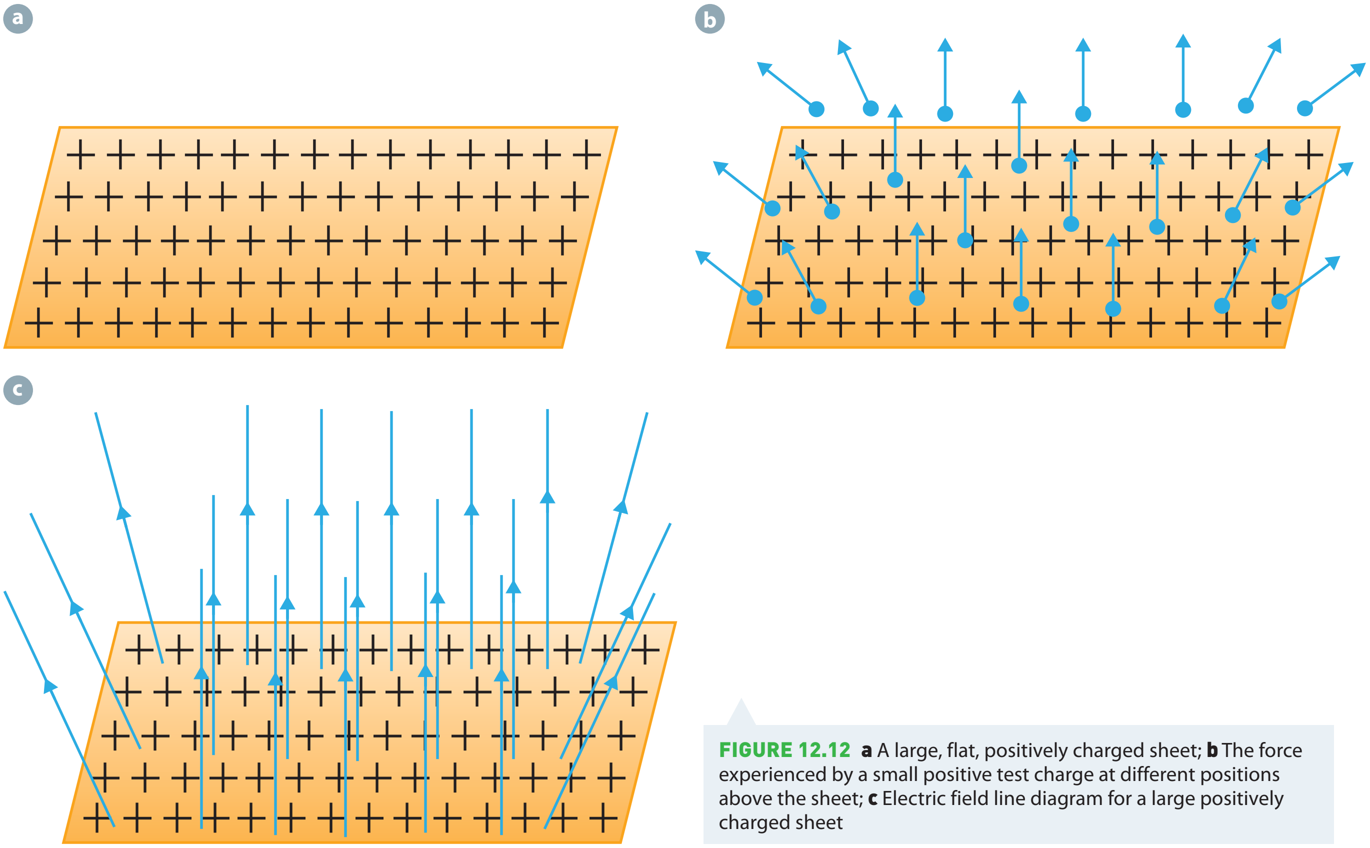
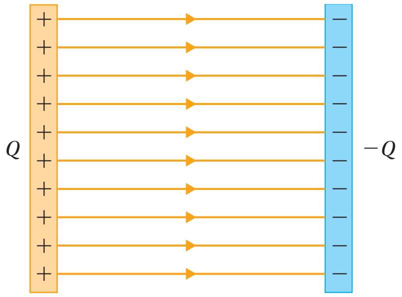


FIGURE 12.12 **a** A large, flat, positively charged sheet; **b** The force experienced by a small positive test charge at different positions above the sheet; **c** Electric field line diagram for a large positively charged sheet

The field for a single large sheet is only uniform close to the sheet. If we want to make a strong, uniform electric field, we can do so by using two parallel plates with opposite charges. The field is uniform between the plates, and only becomes non-uniform very close to the edges. Figure 12.13 shows the electric field created by a pair of parallel plates. Such an arrangement is called a capacitor, and is used in electric circuits for storing energy. The relationship between field and energy is discussed later in this chapter.

FIGURE 12.13 An electric field line diagram for a pair of oppositely charged, parallel plates



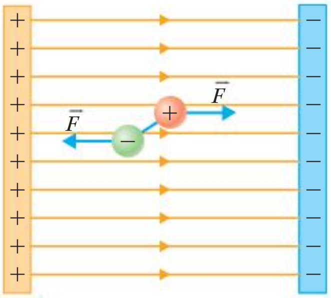
WORKED EXAMPLE (12.3)

In which direction will the following objects accelerate if placed between the charged parallel plates shown in Figure 12.13?

a A proton

b A dipole

Explain your answer.

ANSWERS	**LOGIC**
a The field points from left to right; the charge is positive (proton)	- Identify relevant data in the question.
Electric field lines point in the direction of force on a positive charge.	- Relate field lines to force on charges.
Acceleration is in the direction of force.	- Relate force to acceleration.
The acceleration is to the right.	- State the answer.
b The object is a dipole, so has equal positive and negative charge	- Identify relevant data in the question.
The negative charges experience a force to the left; the positive charge experiences an equal force to the right.	- Relate field lines to force on charges.
 **FIGURE 12.14** A dipole in a uniform electric field experiences zero net force.	- Draw a force diagram for the dipole.
The net force on the dipole is zero, so it does not accelerate.	- State the final answer.

TRY THIS YOURSELF

What would be the direction of acceleration of an electron in this field?

In Worked example 12.3, the dipole experiences zero net force in a uniform electric field. This does not mean that the dipole experiences no effect from the field. Note that in Figure 12.14, the forces are not along the same line. This will cause the dipole to rotate in the field and try to line up with the field lines.

When a charged object attracts a neutral object, it is because temporary dipoles are formed in the neutral object. The atoms and molecules distort slightly, becoming polar. For the dipoles to experience a net force and be attracted to the charged object, the field must not be uniform – one side of the dipole must experience a greater force. Most charged objects will produce a non-uniform field, which is why a charged comb can lift uncharged polystyrene beads (or tiny pieces of paper). However, a uniform electric field does not exert a net force on an uncharged object.

INVESTIGATION 12.2

Drawing electric field lines

Critical and creative thinking

Information and communication technology capability

AIM

To draw electric field line diagrams for various arrangements of charged particles

Write one or more research questions for this investigation.

MATERIALS

- Computer with internet access

METHOD

Charges and fields

Use this simulation for Investigation 12.2.

1. Open the simulation in the weblink 'Charges and fields'.
2. Use the simulation to draw an electric field line diagram for one positive charge. Make a copy (sketch or take a screen shot) of the result.
3. Use the simulation to draw an electric field line diagram for one negative charge. Make a copy.
4. Use the simulation to draw an electric field line diagram for a dipole. Make a copy.
5. Use the simulation to draw an electric field line diagram for two positive charges. Make a copy.
6. Place three positive charges, closely spaced together, in a line. Make a copy of the resultant electric field line diagram.
7. Add another positive charge to one end of the line.
8. Repeat step 7 until you have as large a line of closely spaced positive charges as possible. Make a copy of the final result and at least one intermediate result.
9. Create a line of closely spaced alternating positive and negative charges. Make a copy of the resultant electric field line diagram.

RESULTS

You should have a collection of field line diagrams.

ANALYSIS

1. Identify regions of large and small electric field in your diagrams.
2. Are there regions of uniform electric field in any of them?
3. Describe what happens to the field as you add charges in a line. Note that the simulation only shows you a two-dimensional slice through the field.
4. The final arrangement represents a neutral material that has been polarised. Describe the resulting field.

DISCUSSION

1. What general principles can you derive from your observations?
2. How does the electric field vary around a point charge, a dipole and a sheet of charge? What about a polarised neutral object?
3. Have you answered your inquiry questions?

CONCLUSION

Write a conclusion summarising your findings.

KEY CONCEPTS

- Electric field line diagrams are a way of representing electric fields.
- Electric field lines point in the direction of the force acting on a positively charged particle.
- Electric field lines never cross.
- Electric field lines begin on positive charges and end on negative charges.
- The field strength is proportional to the density of field lines.
- When drawing field line diagrams, choose a sensible number of lines to show how the field varies.
- The ratio of field lines coming out from or entering any charge should be proportional to the magnitude of the charge.
- Point charges create fields that decrease with distance from the charge.
- Large flat sheets of charge or parallel charged plates can be used to create a uniform field.

CHECK YOUR UNDERSTANDING 12.2

1 What does an electric field line represent?

2 Summarise the main characteristics of field lines.

3 When is an electric field directed towards a charge, and when is it directed away from a charge?

4 Draw a field line diagram for the electric field around:

  a two nearby positive charges.

  b two nearby negative charges.

  c a dipole.

5 An electron is between two charged parallel plates, one to the left and one to the right. The electron accelerates towards the left.

  a Draw the arrangement described.

  b Identify which plate is positive and which is negative.

  c Draw the electric field lines for the pair of plates.

6 An uncharged (neutral) polystyrene bead is attracted to a charged balloon. The same bead is not attracted to either plate when placed between charged parallel plates. Explain this behaviour.

12.3 The electrostatic force and the electric field

So far we have used the electric field line model to represent electric fields. This is a useful visual and conceptual model. Now we will use a mathematical model that allows us to quantify the electric field.

Electric field, force and charge

The electric field is defined as the force per unit charge that acts on a small positive test charge, at a given point in space. Mathematically, the electric field, $\vec{E}$, is given by

$$\vec{E} = \frac{\vec{F}}{q}$$

where both force, $\vec{F}$, and field are vectors. The charge, q, is the amount of positive charge on the small test charge.

Force was described and quantified in chapter 4, but we have not as yet quantified charge.

Charge is measured in units of **coulombs, C**. The unit coulomb was named to honour Charles-Augustin de Coulomb. Charge is a fundamental property of matter. Unlike most of the other units you have met so far, the coulomb cannot be reduced to a combination of length, mass and time.

Charles-Augustin de Coulomb

Find out more about the life of Charles-Augustin de Coulomb and his contributions to physics.

Particles such as electrons and protons have very small charges when measured in units of coulombs. An electron has charge -1.6×10^{-19} C, and a proton has charge $+1.6 \times 10^{-19}$ C. All stable particles have a charge equal to some whole-number multiple of 1.6×10^{-19} C. This charge is given the special name e, the electron charge, where $e = 1.6 \times 10^{-19}$ C. When you see an ion described with a charge, this refers to its net charge in multiples of e. For example, a Ca^{2+} ion has a charge $+2e = 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}$ C.

Looking back at our definition of electric field, we can now determine the units of the electric field. Force has units of newtons, N, and charge has units of coulombs, C, so electric field has units of newtons per coulomb, N C^{-1}. Some electric field strengths are shown in Table 12.1.

When a charged particle is placed in an electric field, it experiences a force. This force is proportional to the size of the field, and the charge on the particle. The definition of the field gives us the force:

$$\vec{F} = \vec{E}q$$

TABLE 12.1 Some electric field strengths

ELECTRIC FIELD DUE TO ...	APPROXIMATE FIELD STRENGTH, N C^{-1}
Hairdryer, 20 cm away	4
Earth's fair weather field	100 downwards
Earth's wet weather field	200–300 upwards
Thunderstorms	1000–10 000 or more (momentary spikes)
High voltage overhead power lines, 30 m away	10–1000
Old electric blanket, 10 cm away	2000

WORKED EXAMPLE 12.4

A battery is connected across a piece of copper wire giving an electric field of 3.0 N C^{-1} in the wire. Calculate the force on an electron in this wire.

ANSWER	LOGIC
$E = 3.0$ N C^{-1} This is the magnitude; we do not know the direction. $q = -e = -1.6 \times 10^{-19}$ C	▪ Identify the relevant data in the question.
$\vec{F} = \vec{E}q$ $F = (3.0\ \text{N C}^{-1})(-1.6 \times 10^{-19}\ \text{C})$ $F = -4.8 \times 10^{-19}$ N The negative sign tells us that the force is in the direction opposite that of the field.	▪ Identify the appropriate formula for the relationship between force, field and charge. ▪ Substitute known values, with units, into the formula. ▪ Calculate the final answer and state with correct significant figures and units.

TRY THIS YOURSELF

What magnitude electric field would be required to give the same force on a helium nucleus, which has a charge $+2e$?

To understand how a charged object behaves in a field, we need to use another model: Newton's laws. Newton's second law says that:

$$\vec{a} = \frac{\vec{F}}{m}$$

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Using the definition of electric field,

$$\vec{E} = \frac{\vec{F}}{q}$$

we can say:

$$\vec{a} = \frac{\vec{E}q}{m}$$

Electric field

In this simulation, position two or more charges in the electric field. Click 'go' to see how they accelerate.

WORKED EXAMPLE (12.5)

What is the acceleration of the electron in the 3.0 N C^{-1} field in the example given in Worked example 12.4? The mass of an electron is 9.1×10^{-31} kg.

ANSWER	LOGIC
$E = 3.0\ \mathrm{N\ C^{-1}}$; $q = -e = -1.6 \times 10^{-19}\ \mathrm{C}$	- Identify the relevant data in the question.
$m_e = 9.1 \times 10^{-31}\ \mathrm{kg}$	- Look up any additional data needed.
$\vec{a} = \frac{\vec{E}q}{m}$ $\vec{a} = \frac{(3.0\ \mathrm{N\ C^{-1}})(-1.6 \times 10^{-19}\ \mathrm{C})}{9.1 \times 10^{-31}\ \mathrm{kg}}$ $= -5.27 \times 10^{11}\ \mathrm{N\ kg^{-1}}$	- Identify the appropriate formula for the relationship between acceleration, field, charge and mass - Substitute known values, with units, into the formula. - Calculate the answer.
$a = -5.3 \times 10^{11}\ \mathrm{m\ s^{-2}}$ The negative sign tells us that the acceleration is in the direction opposite to that of the field.	- State the final answer with correct significant figures and units.

TRY THESE YOURSELF

1 What magnitude electric field is required to give an electron the same acceleration as that due to Earth's gravitational field, 9.8 m s^{-2}? Compare this to Earth's fair weather field, listed in Table 12.1.

2 A balloon has a mass of 250 g. After being rubbed on a student's head, the balloon has a charge of 1.2×10^{-6} C. Calculate the acceleration of this balloon due to Earth's fair weather field. Compare this to the acceleration due to gravity on Earth.

The field due to a charged particle

Any point-like (very, very small) object with charge creates an electric field that is symmetrical about the object. This means it looks the same from any direction. For this to be the case, the field lines must spread out evenly in all directions. A charged particle such as a proton or electron is an example of a point-like charge.

We know from experimentation that the bigger the charge, the bigger the force, and therefore the bigger the field. So, we expect field to be proportional to charge. We also expect the field to have spherical symmetry, as there is no preferred direction around a point. We can see this from our field line diagrams (Figures 12.5c and 12.6, page 344).

We also know that the field becomes weaker at greater distances from the charge. A charged balloon does not pull as hard at your hair if you move it further away. We can use electric field line diagrams to work out exactly how the field varies with distance.

The field lines become less dense the further away they are from a point charge (Figures 12.5 and 12.6). If you draw concentric circles around a point charge, the number of field lines crossing each circle is

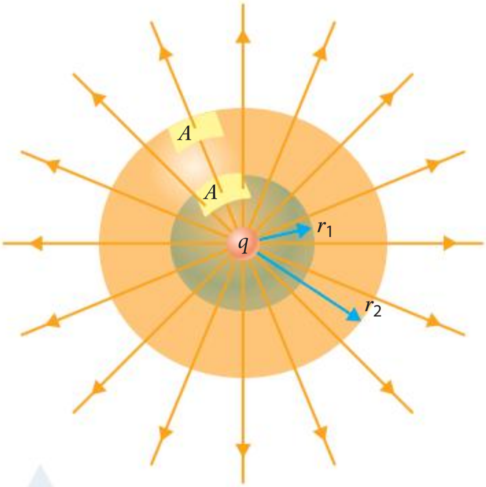


FIGURE 12.15 Field lines due to a point charge. The number of field lines passing through the sphere with radius r_1 is the same as that passing through the sphere with radius r_2. However, the number passing through any small area, A, which is a measure of the density of field lines and hence field strength, is greater for the smaller sphere.

constant. However, the density decreases with the radius of the circle. In three dimensions, the field lines radiate out in all directions. If you imagine concentric spheres around a point charge, as shown in Figure 12.15, the number, n, of field lines crossing each sphere is the same.

However, the density of lines decreases with the square of the radius because the surface area of each sphere is proportional to r^2:

$$\text{Density} = \frac{\text{number}}{\text{area}}$$

$$\text{Area} = 4\pi r^2$$

so

$$\text{Density} = \frac{n}{4\pi r^2}$$

Hence, the field strength, which is proportional to the field line density, decreases with distance squared from the source. This $\frac{1}{r^2}$ form is common to all point sources, whether they are sources of electric field, gravitational field, or light or sound waves. So we can say that

$$\vec{E} \propto \frac{q}{r^2}$$

We need only the constant of proportionality now to get the units and magnitude of the electric field. The constant of proportionality is $\frac{1}{4\pi\varepsilon_0}$ where ε_0 is called the **permittivity of free space**. It tells us how the field varies in a vacuum, as well as giving us the correct units for field. This is an important physical constant. The speed of light depends on this constant, as we shall see in Year 12.

$$\varepsilon_0 = 8.9 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$$

so

$$\frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$$

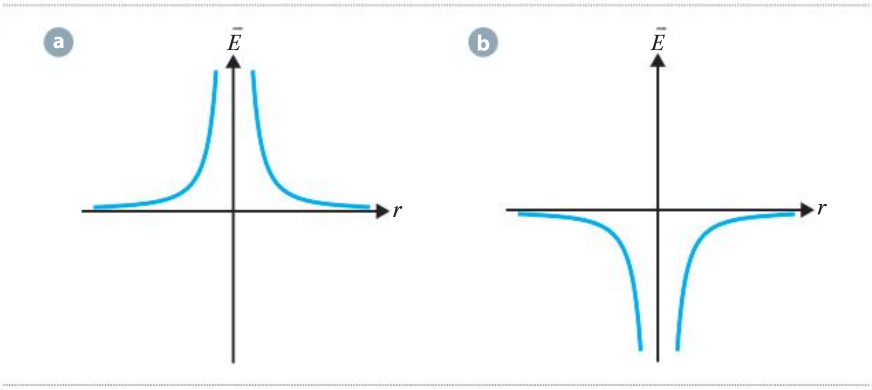
We now have an equation for the electric field due to a charged particle:

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}$$

This expression applies to any charged spherical object, as long as the radius we are calculating the field at is greater than the radius of the object. Hence, we can also apply it for objects such as the dome of a Van de Graaff generator.

Figure 12.16 shows plots of field strength as a function of distance for a positively and a negatively charged particle.

FIGURE 12.16 Plots of electric field as a function of distance, $E(r)$ for: **a** a positive point charge; **b** a negative point charge



WORKED EXAMPLE 12.6

Calculate the electric field due to a proton at a distance of 8.0×10^{-11} m. This is the average orbital radius of the electron in a hydrogen atom. This distance is four orders of magnitude larger than the size of the proton, which is about 10^{-15} m, so we can treat the proton as a point charge.

ANSWER	LOGIC
$q = +1e = 1.6 \times 10^{-19}\ \text{C};\ r = 8.0 \times 10^{-11}\ \text{m}$	▪ Identify the relevant data from the question.
$E = \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{q}{r^2}$ $E = 9.0 \times 10^9\ \text{N m}^2\ \text{C}^{-2} \times \dfrac{1.6 \times 10^{-19}\ \text{C}}{\left(8.0 \times 10^{-11}\ \text{m}\right)^2}$ $= 2.25 \times 10^{11}\ \text{N C}^{-1}$	▪ Identify the appropriate formula for the electric field due to a point charge. ▪ Substitute known values, with units, into the formula. ▪ Calculate the answer.
$E = 2.3 \times 10^{11}\ \text{N C}^{-1}$	▪ State the final answer with correct units and significant figures.

TRY THIS YOURSELF

Calculate the electric field due to a proton at distances of 1×10^{-10} m (a typical distance between atoms), 1 cm and 1 m.

Coulomb's law

Now that we can calculate the electric field due to a point charge, we can calculate the force that one point charge exerts on another.

Recall that the definition of electric field is

$$\vec{E} = \frac{\vec{F}}{q}$$

so the force exerted by the field is

$$\vec{F} = \vec{E}q$$

For a point charge the field is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1}{r^2}$$

where q_1 is the charge on the particle creating the field.

So, when we place a second charged particle, with charge q_2, within the field due to q_1, it experiences a force:

$$\vec{F} = \vec{E}q_2$$

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$$

This is known as Coulomb's law. It describes the force that a point charge q_1 exerts on a second point charge q_2 when they are separated by a distance r.

Remember that forces are vectors. The form of Coulomb's law given here only gives the magnitude of the force, as there is no vector on the right hand side. The force will be directed towards or away from charge q_1, in the direction on the radius vector, $\vec{r}$. You should always draw a diagram to show the direction of the force. Remember: like charges repel and unlike charges attract.

If we write an equation for the force that charge q_2 exerts on charge q_1, we get exactly the same expression. This should not come as a surprise. From Newton's third law, whatever force q_2 exerts on

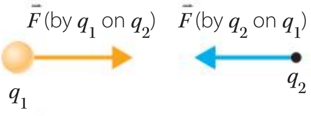


FIGURE 12.17 Two charges exert equal and opposite forces on each other.

q_1, q_1 must exert an equal and opposite force on q_2. They are a Newton's third law force pair; equal in magnitude, opposite in direction, have the same fundamental nature, and act on different objects. This is shown in Figure 12.17.

WORKED EXAMPLE (12.7)

Calculate the force exerted by a proton on an electron at a distance of 8.0×10^{-11} m.

ANSWER	LOGIC
$q_1 = +e = 1.6 \times 10^{-19}$ C; $q_2 = -e = -1.6 \times 10^{-19}$ C; $r = 8.0 \times 10^{-11}$ m	▪ Identify the relevant data in the question.
$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$ $F = 9.0 \times 10^9\ \text{N m}^2\ \text{C}^{-2} \times \frac{(1.6 \times 10^{-19}\ \text{C})(-1.6 \times 10^{-19}\ \text{C})}{(8.0 \times 10^{-11}\ \text{m})^2}$ $F = -3.6 \times 10^{-8}$ N In this case, the negative sign indicates that the force is towards the proton. We could also have done this by simply taking the answer to the previous example and multiplying it by the charge of an electron.	▪ Identify the appropriate formula to relate the force to the charges and distance (Coulomb's law) ▪ Substitute known values, with units, into the formula. ▪ Calculate the answer and express with correct significant figures and units.

TRY THESE YOURSELF

1 Calculate the force on an electron due to a helium nucleus (He^{2+}) at the same distance.

2 Calculate the force on a helium nucleus due to an electron. Compare it with the force on the electron due to the helium nucleus.

KEY CONCEPTS

- Charge is measured in units of coulombs, C.
- An electron has charge -1.6×10^{-19} C; a proton has charge $+1.6 \times 10^{-19}$ C.
- The electric field is the force per unit charge acting on a small positive test charge: $\vec{E} = \frac{\vec{F}}{q}$. Electric field has units newtons per coulomb, N C^{-1}.
- Force and field are vectors. They point in the same direction for a positive charge, q, and in opposite directions for a negative charge.
- Using Newton's second law, the acceleration of a charged particle in an electric field is given by $\vec{a} = \frac{\vec{E}q}{m}$.
- For a point charge or spherical charge distribution, the electric field is given by $E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}$.
- Coulomb's law is used to find the force that a point charge q_1 (such as a charged particle) exerts on a second point charge q_2: $\vec{F} = \vec{E}q_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$
- Both charged particles or objects experience the same magnitude force, but in opposite directions.

CHECK YOUR UNDERSTANDING

12.3

1 Write down the mathematical relationship between the electrostatic force and electric field.

2 Two charges, q_1 and q_2, are such that $q_1 = 3q_2$. What is the ratio of the force $F(q_1 \text{ on } q_2)$ to $F(q_2 \text{ on } q_1)$?

3 Calculate the electric field at a distance of 10 cm away from a point charge of +5.0 mC.

4 Calculate the acceleration of an electron in a constant electric field of 95 N C^{-1}, directed upwards. In which direction does the electron accelerate?

5 Calculate the force on a charge of +2.0 mC a distance 10 cm away from a point charge of +5.0 mC.

6 **a** At what distance from a proton does an electron experience a force of 2.3×10^{-19} N?

  b At what distance does the force have half this value?

  c What is the force if the distance is doubled?

12.4

Potential energy and work in an electric field

You already know that there are two types of energy: potential energy and kinetic energy (chapter 5). Objects that are moving have kinetic energy, and objects that are subject to a force have potential energy. For example, an object in the gravitational field of Earth has potential energy. If Earth was not there, so that there was no gravitational field, then the object would not have any potential energy. So, strictly, the energy belongs to the combination of the object and Earth.

You also know that energy is conserved. When you drop a pencil, gravitational potential energy is transformed into kinetic energy (Figure 12.19a). This transformation happens because of the gravitational force exerted by the gravitational field on the pencil.

Remember from chapter 4 that the gravitational force is given by $F = mg$. From chapter 5 you know that the work, W, done by a force, F, on an object as it is displaced a distance, s, in the direction of the force is

$$W = \vec{F}\vec{s}$$

Hence, when the pencil falls through a height h, the gravitational field does work on the pencil,

$$W = mgh$$

The same happens to a charged object in an electric field (Figure 12.18b, page 356). If the object starts at rest and is allowed to move, it will accelerate. It accelerates because the force (due to the field) does work on it – it exerts a force through some distance. The work done is equal to the change in the potential energy of the system,

$$W = \vec{F}\vec{s} = -\Delta U$$

Recall that the negative sign indicates a decrease in potential energy of the system when work is done by one part of the system on another.

Note that a small isolated charge cannot be said to have potential energy any more than an isolated pencil. It only has potential energy because of its interaction with an electric field, which is due to some other charged object. Again, the potential energy is due to the application of a force to the charge, which occurs via the field.

The electric field exerts a force on a charged particle or object given by:

$$\vec{F} = \vec{E}q$$

When a charged object is displaced by some distance d in the direction of the field, the work done on the object is:

$$W = Eqd$$

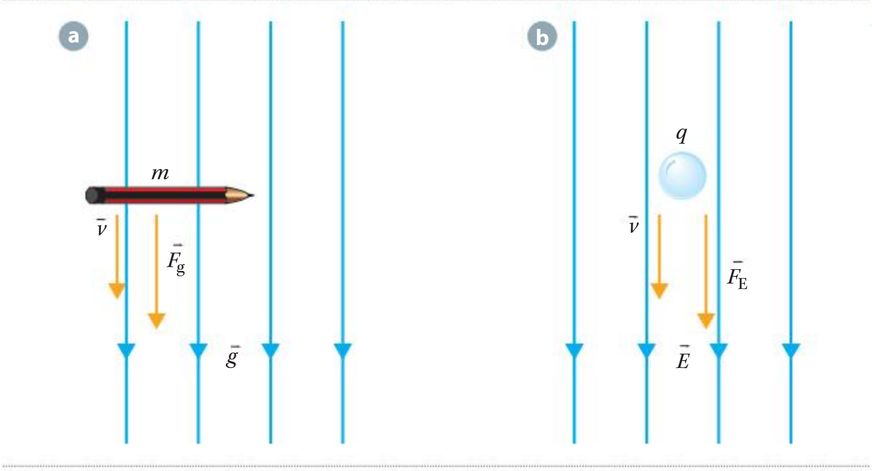


FIGURE 12.18
a A dropped pencil has gravitational potential energy that is transformed into kinetic energy.
b A charged object in an electric field has electric potential energy that is converted into kinetic energy.

The work done is equal to the change in potential energy of the object–field system.

$$W = Eqd = -\Delta U$$

The electric force is a conservative force, just like the gravitational force (and all other field forces), so the change in potential energy appears as a change in kinetic energy:

$$W = Eqd = -\Delta U = \Delta E_k$$

So, a particle moving in an electric field will gain kinetic energy if positive work is done on it, and will lose kinetic energy if negative work is done. Hence, fields are not only a way of exerting a force at a distance, *fields also store energy.* The gravitational field stores gravitational potential energy and the electric field stores **electric potential energy**.

Note the similarity of the expressions for work done by the gravitational field and work done by an electric field: $W = mgh$ and $W = Eqd$. Each expression is a product of the field strength (g is the gravitational field strength close to Earth's surface), the property that is affected by the field, and the distance moved through the field.

WORKED EXAMPLE 12.8

A charged polystyrene bead is placed between two charged parallel plates. The bead has a mass of 0.50 g and a charge of 25 nC. The field between the plates has a magnitude of 1.0 kN C^{-1}. Ignore any frictional forces acting on the bead and assume it starts from rest. Calculate the speed of the bead after it has moved a distance of 1.0 cm.

ANSWER	LOGIC
$q = 25\ \text{nC} = 2.5 \times 10^{-8}\ \text{C}; m = 0.50; g = 5.0 \times 10^{-4}\ \text{kg};$ $d = 1.0\ \text{cm} = 0.010\ \text{m}; E = 1.0\ \text{kN C}^{-1} = 1.0 \times 10^{3}\ \text{N C}^{-1}$	▪ Identify the relevant data in the question and convert to SI units.
$Eqd = \Delta E_k$	▪ Identify the appropriate formula to relate the kinetic energy to the field, charge and distance.
$\Delta E_k = \frac{1}{2}mv^2$	▪ Identify the appropriate formula to relate the kinetic energy to the speed (Chapter 5).
$Eqd = \frac{1}{2}mv^2$	▪ Combine the two formulae.

ANSWER	**LOGIC**
$v = \sqrt{\frac{2Eqd}{m}}$	- Rearrange for speed.
$\left[\mathrm{m\,s^{-1}}\right] = \sqrt{\frac{\left[\mathrm{N\,C^{-1}}\right][\mathrm{C}][\mathrm{m}]}{[\mathrm{kg}]}}$ $= \sqrt{\frac{[\mathrm{N}][\mathrm{m}]}{[\mathrm{kg}]}}$ $= \sqrt{\frac{\left[\mathrm{kg\,m\,s^{-2}}\right][\mathrm{m}]}{[\mathrm{kg}]}}$ $= \sqrt{\left[\mathrm{m^2\,s^{-2}}\right]}$ $= \left[\mathrm{m\,s^{-1}}\right]$	- With a complicated expression like this, it is worth checking that the units are correct before proceeding.
$v = \sqrt{\frac{2\left(1.0\times10^{3}\ \mathrm{N\,C^{-1}}\right)\left(2.5\times10^{-8}\ \mathrm{C}\right)(0.010\ \mathrm{m})}{5.0\times10^{-4}\ \mathrm{kg}}}$	- Substitute known values, with units, into the formula.
$= 0.0316\ \mathrm{m\,s^{-1}}$	- Calculate the answer.
$v = 0.032\ \mathrm{m\,s^{-1}}$ or $3.2\ \mathrm{cm\,s^{-1}}$	- State the final answer with correct significant figures and units.

TRY THESE YOURSELF

1 Calculate the force exerted on the bead by the field.

Do this two ways: first using kinematics and Newton's second law, and second using the field.

2 An electron is moving at $500\ \mathrm{m\,s^{-1}}$ when it enters an area of uniform electric field of magnitude $100\ \mathrm{N\,C^{-1}}$ pointing in the direction of its velocity. How far does the electron move before coming to a stop? What happens after it stops?

Does energy increase or decrease?

We need to pay close attention to signs. A positive charge moving in the direction of an electric field, such that q and E have the same sign, will have positive work done on it and gain kinetic energy. For a negative charge to gain kinetic energy, it must be moving in the opposite direction to the field.

This makes sense because the electric field points away from positive charges. Positive charges are repelled by positive charges, and accelerate away from them. Negative charges are attracted to positive charges, so accelerate towards them, against the direction of the field.

The 'zero' of electric potential energy

Note that in the discussion and equations above, we refer to *changes* in potential energy, ΔU, rather than an absolute potential energy, U. Potential energy is always measured relative to some zero point. For example, close to Earth's surface we usually choose ground level as the zero for gravitational potential energy. However, when we are considering other objects such as the Sun and planets, this choice of zero is not very useful.

There is no obvious choice for defining a zero of potential energy like this for electric potential energy. The convention is that we choose the system configuration that gives zero force. When the field, and hence force, is zero, it can do no work. This occurs when the object creating the field and the object experiencing the force due to it are so far apart they can no longer interact. So, we take the zero of electric potential energy for a system of charged objects as being when they are so far apart (effectively infinitely far apart) that they no longer interact.

Electric potential, energy and field

Electric potential is a very useful quantity when working with electric fields, and especially with electric circuits (as you will see in the next chapter).

Electric potential (or simply 'potential') has the same relationship to potential energy that field has to force. The field is defined as the force per unit charge:

$$\vec{E} = \frac{\vec{F}}{q}$$

The electric potential, V, is defined as the potential energy per unit charge at a point:

$$V = \frac{U}{q}$$

The units for V are joules per coulomb, J C^{-1}, also known as volts, V. Recall that force (an interaction) depends on both the object exerting it and the object experiencing it (as in Coulomb's law), but field depends only on the object creating the field. The same is true of potential. While potential energy depends on all objects in a system, potential depends only on the charged object creating the field.

We usually define the zero of potential as being infinitely far away from any charged objects. This is consistent with our choice for the zero reference point for electric potential energy. However, this is an arbitrary choice, and one that is not always practical to measure.

So instead of talking about absolute values of potential, we usually talk about **potential differences** rather than potentials. A potential difference is the difference in electric potential between two points:

$$\Delta V = V_{\text{final}} - V_{\text{initial}} = \frac{\Delta U}{q}$$

The unit for potential difference is the volt, V, the same unit as for potential. Sometimes just the symbol V is used for potential difference, but really the Δ should be included to indicate that it is a difference, not an absolute value. Sometimes potential difference is called **voltage**, from the unit volt, just as power is sometimes referred to as 'wattage'. The term 'potential difference' is more precise, but 'voltage' is very common.

When a charged particle moves through a potential difference $\Delta V = V_{\text{final}} - V_{\text{initial}}$, its potential energy changes by:

$$\Delta U = q\left(V_{\text{final}} - V_{\text{initial}}\right) = q\Delta V$$

The kinetic energy of the particle must change by the same amount (assuming no other forces are acting) but with the opposite sign, as energy is conserved.

WORKED EXAMPLE (12.9)

An alpha particle of charge $+3.2 \times 10^{-19}$ C passes through a potential difference of +100 V. What is the change in potential energy of this particle?

ANSWER	LOGIC
$q = +3.2 \times 10^{-19}$ C; $\Delta V = +100$ V	▪ Identify the relevant data in the question.
$\Delta U = q\Delta V$	▪ Identify the appropriate formula to relate the change in potential energy to the charge and potential difference.

ANSWER	LOGIC
$\Delta U = (+3.2 \times 10^{-19}\text{ C})(+100\text{ V})$	- Substitute known values, with units, into the formula.
$= +3.2 \times 10^{-17}\text{ C V}$	- Calculate the answer.
$\Delta U = +3.2 \times 10^{-17}\text{ J}$ The positive change in potential energy means that either kinetic energy was lost from the system, or the system was not isolated.	- State the final answer with correct significant figures and units.

TRY THIS YOURSELF

What is the change in potential energy of an electron moving through this potential difference?

We can use the relationship between force and energy to derive a relationship between field and potential. From chapter 5:

$$W = Fd = -\Delta U$$

For the electric field,

$$Eqd = -\Delta U$$

dividing both sides through by the charge, q, gives:

$$Ed = -\frac{\Delta U}{q} = -\Delta V$$

which can be rearranged to give:

$$E = -\frac{\Delta V}{d}$$

This equation tells us several things. First, it tells us that there is another way of expressing the units of field, V m^{-1} as well as N C^{-1}. Second, the negative sign tells us that the direction of the electric field is *opposite* to the direction in which the potential is increasing.

A positively charged object released from rest in an electric field will be accelerated in the direction of the field. The force exerted on the object by the field acts to increase its kinetic energy. From conservation of energy, this kinetic energy must come from somewhere. It comes from a decrease in potential energy. Hence, ΔV must be negative, and there is a drop in potential in the direction of a field line.

If a negatively charged object moves from higher to lower potential so that ΔV is negative, then the change in potential energy, ΔU, is positive. This can only happen if the charge has some initial kinetic energy, or if some external force is doing work on the system. This is shown in Figure 12.19. In a circuit, a battery provides this additional potential energy (chapter 13).

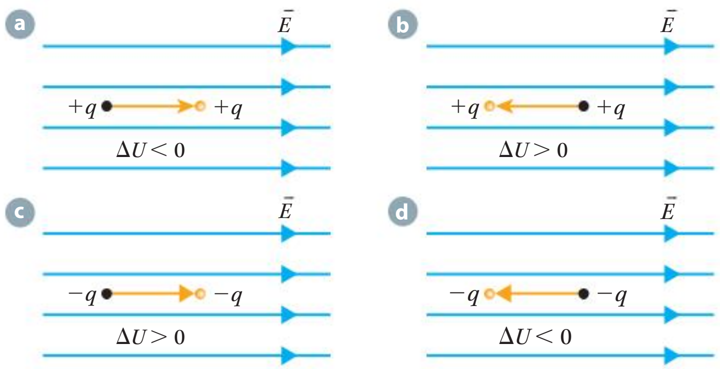


FIGURE 12.19 Energy changes when a charge moves in an electric field. **a** A positive charge moving in the direction of the field; **b** A positive charge moving against the direction of the field; **c** A negative charge moving in the direction of the field; **d** A negative charge moving against the direction of the field

Table 12.2 summarises the changes in potential and energy when a charge moves in a field. The field will do work on the charge when a positive charge moves with the field or a negative charge moves against the field. Work must be done on the system to make a positive charge move against the field or a negative charge move with the field.

Revision 1

TABLE 12.2 Changes in potential and energy for a charge moving in a field

CHARGE	MOVEMENT WITH OR AGAINST THE FIELD LINES	CHANGE IN POTENTIAL	CHANGE IN POTENTIAL ENERGY	WORK DONE BY OR ON THE FIELD
+	With	Negative (decrease)	Negative (decrease)	By
+	Against	Positive (increase)	Positive (increase)	On
–	With	Negative (decrease)	Positive (increase)	On
–	Against	Positive (increase)	Negative (decrease)	By

Charges, fields and potentials

Use this simulation to set up some charges, and then move the gauge around to measure the potential at different points. Try it with a single charge, and then with a dipole.

Electrons and other subatomic particles moving through potential differences are so common that a special unit is used to describe the change in energy when this happens; the **electron volt**. When an electron moves through a potential difference of 1 V, it has a change in energy of

$$\begin{aligned}\Delta U &= q\Delta V\\ &= e\Delta V\\ &= \left(1.6\times10^{-19}\ \mathrm{C}\right)(1\ \mathrm{V})\\ &= 1.6\times10^{-19}\ \mathrm{J}\\ &= 1\ \text{electron volt}\end{aligned}$$

The electron volt or eV is a good size for describing the energy of subatomic particles. The SI unit, the joule, is very large by comparison. You will see the unit eV often when you study nuclear or particle physics in Year 12. It is also used in chemistry to describe the energy of reactions. Remember that it is a unit of energy, not a unit of potential difference.

WORKED EXAMPLE 12.10

An electron in an X-ray machine is accelerated through a potential difference of 100 kV before colliding with a target and emitting X-rays. What is the energy of the electron just before it hits the target? Give your answer in electron volts and joules.

ANSWER	LOGIC
$q = -1e = -1.6\times10^{-19}$ C; $\Delta V = 100$ kV $= 1.00\times10^{5}$ V We want to find E_k in eV and J.	▪ Identify relevant data in the question and convert to SI units.

ANSWER	LOGIC
$E_k = -\Delta U$ $= -\Delta U = -q\Delta V$ $= -(-1.6 \times 10^{-19}\ \mathrm{C})(1.00 \times 10^5\ \mathrm{V})$ $= 1.6 \times 10^{-14}\ \mathrm{C\ V}$	▪ By conservation of energy, the kinetic energy is the decrease in potential energy. ▪ Identify the appropriate formula to relate the energy to the charge and potential difference. ▪ Substitute known values, with units, into the formula. ▪ Calculate the answer.
$E_k = 1.6 \times 10^{-14}\ \mathrm{J}$	▪ Convert to different units. State the answer with correct units and significant figures.
$E_k = 100\ \mathrm{keV}$	▪ An electron gains 1 eV for each V it passes through.

TRY THIS YOURSELF

Find the speed at which the electron will be moving after passing through this potential difference. Note that this speed is unrealistic, because at these energies we need to use relativity rather than classical mechanics.

Relativity is described in *Physics in Focus Year 12*, chapter 12.

Equipotentials

The Δ sign in the equation:

$$E = -\frac{\Delta V}{d}$$

is very important, because it reminds us that it is the *change* in potential that is important. Sometimes the equation is written as:

$$E = -\frac{V}{d}$$

but this is leaving out a very important aspect of the relationship between field and potential.

The ΔV tells us that whenever there is a *change* in potential, there is an electric field. It is not enough that the potential is non-zero, *it has to be changing*.

If there is no change in potential from one point to another, so that $\Delta V = 0$, then the electric field is zero in the direction in which ΔV is zero. But remember that field is a vector. The field may have non-zero components in other directions.

If we draw lines along which the potential is constant, then $\Delta V = 0$ from any point to any other point along which V is constant. We call these lines **equipotentials**. In two dimensions, we have equipotential lines; in three dimensions, we have equipotential surfaces.

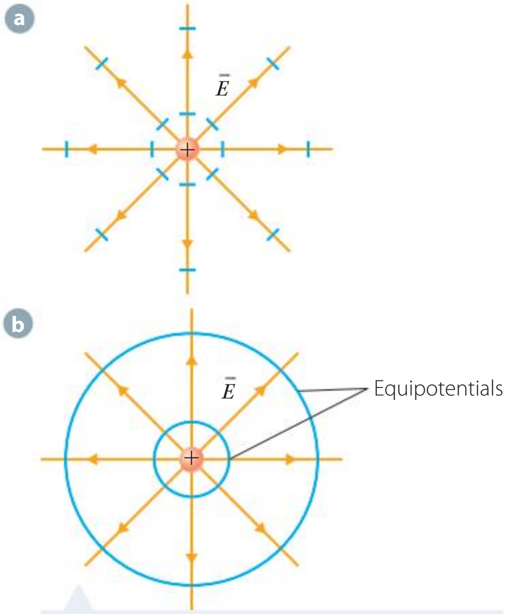
Equipotentials

Recall from chapter 3 that we can break any vector up into perpendicular components. If we break the electric field up into components parallel and perpendicular to an equipotential, then the parallel component must be zero. Only the perpendicular component (or components in three dimensions) are non-zero. This means that electric field lines are always perpendicular to equipotential lines.

WORKED EXAMPLE 12.11

Figure 12.5c (page 344) shows the electric field lines for a single positive charge. Draw the equipotential lines for this field.

ANSWER	LOGIC
The equipotential lines must be perpendicular to the field lines.	▪ Recognise that if field is perpendicular to equipotentials, then the reverse is true.

ANSWER	LOGIC
 **FIGURE 12.20** **a** Draw perpendicular lines to the field lines; **b** Join them to form equipotentials	▪ Redraw Figure 12.5 and mark the equipotentials on it.

TRY THIS YOURSELF

Repeat the worked example for a uniform field, such as that shown in Figure 12.14 (page 347).

INVESTIGATION 12.3

Critical and creative thinking

Numeracy

Information and communication technology capability

Mapping equipotential lines

AIM

To map equipotential lines for a dipole

Write a hypothesis for this investigation.

MATERIALS

- 12 V DC power supply with leads
- Conductive paper
- Voltmeter

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Power supplies can be dangerous if not used correctly.	Only connect the power supply as instructed.

What other risks are associated with your investigation, and how can you manage them?

9780170409063

METHOD

1. Attach the positive terminal of the power supply to a point near one end of your conductive paper and the negative terminal near the other end, as shown in Figure 12.21. You now have a dipole with a positive and negative electrode on your paper.

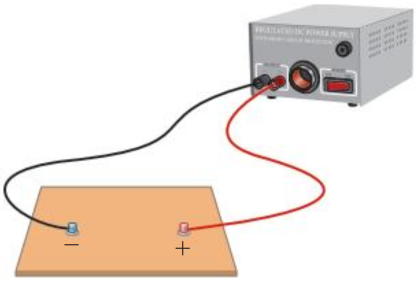


FIGURE 12.21 Experimental set-up for a dipole

2. Record the positions of the electrodes on your paper by tracing around them.
3. With one probe from your voltmeter touching the negative electrode, move the other probe around on the paper until you get a reading of 2 V (Figure 12.22). Mark this point.

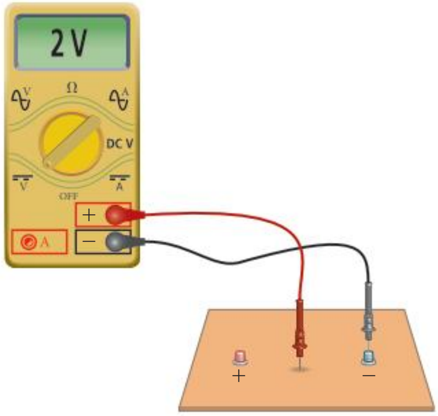


FIGURE 12.22 Experimental set-up for measuring potential difference

4. Carefully move the probe around and mark other points of 2 V potential on the paper. Join the dots – this is your first equipotential line. Label this line $\Delta V = 2$ V.
5. Repeat steps 3 and 4 to map out equipotential lines of 4 V, 6 V, 8 V and 10 V potential. Label these lines on the paper.

RESULTS

You should now have a piece of paper showing a set of equipotential lines.

ANALYSIS OF RESULTS

1. Use the equipotential lines to draw electric field lines for your arrangement of electrodes.
2. Plot graphs of potential as a function of distance from one of your electrodes. Do this for the line joining the two electrodes and at least one other line.

DISCUSSION

1. Are the equipotential lines equally spaced?
2. Do the field lines look the way you would expect from the field line drawing in Figure 12.10 (page 345)?

» **3** Describe the relationship between potential and distance from the electrodes. Write an equation to describe this relationship.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation. Was your hypothesis supported or disproved?

Revision 2

KEY CONCEPTS

- Electric fields do work on charged objects by exerting a force on them. This changes the potential energy of the field–object system, and the kinetic energy of the object: $W = Eqd = -\Delta U = \Delta E_\text{k}$.
- The zero of electric potential energy is usually defined as when the charged objects are infinitely far apart so they exert no forces on each other.
- Electric potential, V, is the potential energy per unit charge at a point: $V = \frac{U}{q}$. Potential has units volts, V. 1 V = 1 J C^{-1}.
- Usually we can only measure potential differences, ΔV, rather than absolute potentials.
- When a charged particle or object moves through a potential difference ΔV, its potential energy changes: $\Delta U = q\left(V_\text{final} - V_\text{initial}\right) = q\Delta V$. Energy is conserved, so the change in potential energy is equal to the change in kinetic energy, if no friction forces act.
- Electric field is related to electric potential difference by $E = -\frac{\Delta V}{d}$.
- The direction of the electric field is *opposite* to the direction in which the potential is increasing.
- Positive charges lose potential energy and gain kinetic energy when moving from higher to lower potential (in the direction of the field). Negative charges gain potential energy and lose kinetic energy when moving from higher to lower potential (in the direction of the field).
- Equipotential lines and surfaces have constant V, so ΔV between any two points on an equipotential line or surface is zero.
- Electric field lines are perpendicular to equipotential lines.

CHECK YOUR UNDERSTANDING

12.4

1 What is the relationship between electric potential and electric potential energy?

2 What are the units for electric potential, potential difference and potential energy?

3 A proton moves a distance 10 cm in a uniform electric field of 3.5 kN C^{-1}, in the direction of the field. Calculate the change in potential energy of the proton.

4 An alpha particle is ejected from a radioactive atom, leaving behind a negative ion. As the alpha particle moves away from the ion, is work done by or on the alpha particle by the field due to the negative ion?

5 What is the speed of a particle that has been accelerated from rest through a potential difference of 1000 V when the particle is:

- **a** an electron?
- **b** a proton?
- **c** an alpha particle?

6 An object with charge +1.0 μC moves a distance 15 cm in a uniform electric field of magnitude 150 V m^{-1} Calculate the change in electric potential energy of the object if it moves:

- **a** in the direction of the field.
- **b** in the direction opposite to the field.
- **c** perpendicular to the field.

12 CHAPTER SUMMARY

- Charge is a fundamental property of matter. It is measured in units of coulombs, C.
- Charged objects exert an electrostatic force, and experience an electrostatic force. They do this via the electric field.
- Objects become charged when they gain or lose electrons.
- Objects with like charge are repelled by each other, objects with opposite (unlike) charge are attracted to each other.
- Neutral objects are attracted to both positively and negatively charged objects.
- Electric field line diagrams are a way of representing electric fields.
- Electric field lines point in the direction of the force exerted by the field on a positively charged particle.
- Electric field lines begin on positive charges and end on negative charges. They never cross, and the density of field lines is an indication of field strength.
- Large flat sheets of charge or parallel charged plates can be used to create a uniform field.
- The electric field is the force per unit charge acting on a small positive test charge: $\vec{E} = \frac{\vec{F}}{q}$. Electric field has units newtons per coulomb, N C^{-1}, or V m^{-1}.
- For a point charge or spherical charge distribution, the electric field is given by $E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}$
- Coulomb's law is used to find the force that a point charge q_1 exerts on a second point charge q_2: $\vec{F} = \vec{E}q_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$.
- Electric fields do work on charged objects by exerting a force on them.
- Electric potential, V, is the potential energy per unit charge at a point: $V = \frac{U}{q}$. Potential has units of volts, V. 1 V = 1 J C^{-1}.
- When a charged particle or object moves through a potential difference ΔV, its potential energy changes: $\Delta U = q\left(V_{\text{final}} - V_{\text{initial}}\right) = q\Delta V$.
- Electric field is related to electric potential difference by: $E = -\frac{\Delta V}{d}$.
- The direction of the electric field is *opposite* to the direction in which the potential is increasing.
- Positive charges lose potential energy and gain kinetic energy when moving from higher to lower potential (in the direction of the field).
- Equipotential lines and surfaces have constant V, so ΔV between any two points on an equipotential line or surface is zero.
- Electric field lines are perpendicular to equipotential lines.

12 CHAPTER REVIEW QUESTIONS

Review quiz

1 How is electric field related to force?

2 How is electric potential related to electric potential energy?

3 Show that 1 V is equal to 1 kg m^2 s^{-3} A^{-1} in SI base units. Show that this is the same as 1 J C^{-1}.

4 Explain how it is possible for a neutral object to be attracted to a charged object. Is it possible for it to also be repelled?

5 Earth's fair weather electric field points downwards and has a magnitude of approximately 100 V m^{-1}. What is the potential difference (approximately) as measured from your feet to your head?

6 Two small balls with equal positive charges are hung from retort stands, as shown in Figure 12.23. Draw a diagram showing how the balls will hang if:

a the charge on the second ball is reduced but that on the first ball remains the same.

b the net charge on the second ball is removed, but that on the first ball remains the same.

For each diagram, clearly show how the angles change.

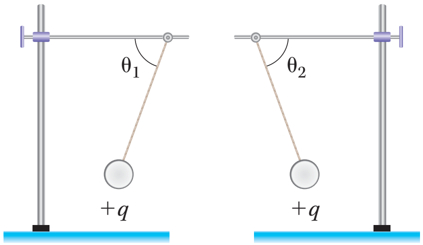


FIGURE 12.23 Two charged balls suspended close to each other

7 Draw a field line diagram for the pair of charged balls in Figure 12.23.

8 A polystyrene bead with mass 1.0 g and charge +1.0 nC is in an electric field of 50 N C^{-1}. Calculate the force on, and acceleration of, the bead due to the field.

9 Calculate the potential difference, ΔV, an electron must pass through to gain 1.0 J of potential energy. Give the magnitude and sign of ΔV.

10 Consider three charged particles, a helium nucleus and its two orbiting electrons, arranged as shown in Figure 12.24. Rank the magnitudes of the forces $F_{\text{a on b}}$, $F_{\text{a on c}}$, $F_{\text{b on a}}$, $F_{\text{b on c}}$, $F_{\text{c on a}}$ and $F_{\text{c on b}}$ from smallest to largest.

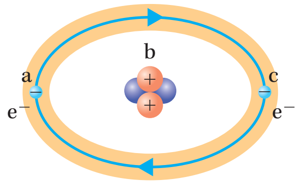


FIGURE 12.24 A helium atom

11 Two charged particles enter a uniform electric field, as shown in Figure 12.25. Both particles have the same magnitude charge.

- **a** Which of the two charged plates creating the field shown is the positively charged plate?
- **b** What can you say about the charges of the two particles?

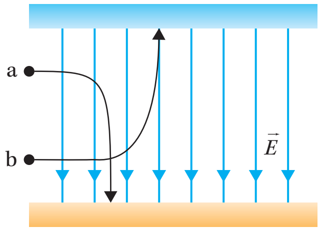


FIGURE 12.25 Two charged particles in a uniform electric field

12 Figure 12.26 shows an electric field line diagram for a pair of charged objects. q_1 has a charge of magnitude e. What is the charge, magnitude and sign of q_2?

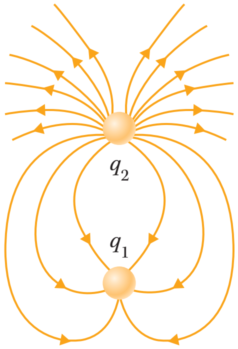


FIGURE 12.26 Field line diagram for a closely spaced pair of charged objects

13 Draw a diagram showing electric field lines and equipotential lines for:

- **a** a negative charge.
- **b** a pair of parallel plates.
- **c** a dipole.

14 Two protons in a molecule are 3.80×10^{-10} m apart.

- **a** Find the magnitude of the electric field due to one of the protons at this distance.
- **b** Find the magnitude of the electrostatic force exerted by one proton on the other.

15 The dome of a Van de Graaff generator is charged up such that there is an electric field in the region surrounding it. A small test charge with charge q is brought into this region and placed at a point P. If the test charge is replaced with a small test charge of $2q$, what happens to:

- **a** the electric field at point P?
- **b** the force on the test charge?
- **c** the electric potential at point P?
- **d** the potential energy of the dome and test charge system?

16 The field at a distance of 1 m from the dome of a Van de Graaff generator is 1000 N C^{-1}. Model the dome as a point charge at this distance.

- **a** By how much would the charge on the dome need to increase for the field at this point to double?
- **b** At what distance from the dome is the field now 1000 N C^{-1}?

17 A dust particle has a mass of 1×10^{-6} g. What charge must this dust particle have for the electrostatic force on it due to Earth's fair weather field ($100\ \text{V m}^{-1}$ down) to balance the gravitational force? Give the sign and magnitude of the charge.

18 An electron moving at $25\ \text{km s}^{-1}$ enters a uniform electric field of magnitude $550\ \text{V m}^{-1}$ in the direction of its velocity.

- **a** Calculate the force on the electron.
- **b** After what distance does the electron come to a stop?

19 An electron moves in an electric field from a point at which the potential is 1.5 V to a point at which the potential is 3.5 V.

- **a** By how much does the potential energy of the electron–field system change?
- **b** Is this change positive or negative?
- **c** Where does the energy come from or go to?

20 A proton is released from rest in a uniform electric field.

- **a** Describe the motion of the proton.
- **b** Describe the changes in the proton's kinetic energy and the potential energy of the field–proton system.
- **c** Does the field do work on the proton, or does the proton do work on the field?

21 An electron moving at $20\ \text{km s}^{-1}$ enters a uniform electric field of magnitude $500\ \text{V m}^{-1}$ in the direction of its velocity.

- **a** What is the kinetic energy of the electron?
- **b** Use an energy approach to calculate the distance it takes for the electron to come to a stop.
- **c** What is the potential difference between the initial point at which the electron entered the field and the point at which it comes to a stop?

13 Electric circuits

INQUIRY QUESTION

How do the processes of the transfer and the transformation of energy occur in electric circuits?

OUTCOMES

Students:

- investigate the flow of electric current in metals and apply models to represent current, including
  - $I = \frac{q}{t}$ (ACSHP038) CCT ICT N
- investigate quantitatively the current–voltage relationships in ohmic and non-ohmic resistors to explore the usefulness and limitations of Ohm's law using:
  - $V = \frac{W}{q}$
  - $R = \frac{V}{I}$ (ACSPH003, ACSPH041, ACSPH043) ICT N
- investigate quantitatively and analyse the rate of conversion of electrical energy in components of electric circuits, including the production of heat and light, by applying $P = VI$ and $E = Pt$ and variations that involve Ohm's law (ACSPH042) ICT N
- investigate qualitatively and quantitatively series and parallel circuits to relate the flow of current through the individual components, the potential differences across those components and the rate of energy conversion by the components to the laws of conservation of charge and energy, by deriving the following relationships: (ACSPH038, ACSPH039, ACSPH044) ICT N
  - $\sum I = 0$ (Kirchhoff's current law – conservation of charge)
  - $\sum V = 0$ (Kirchhoff's voltage law – conservation of energy)
  - $R_{\text{series}} = R_1 + R_2 + \ldots + R_n$
  - $\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$
- investigate quantitatively the application of the law of conservation of energy to the heating effects of electric currents, including the application of $P = VI$ and variations of this involving Ohm's law (ACSPH043) CCT N

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Energy is a core concept in physics. In previous chapters, we have used the concept of energy to help us understand how forces change the state of motion of objects. We have also used the idea of energy to define temperature and explore what happens when matter changes state, for example when ice melts.

In this chapter, we will explore how the processes of transfer and transformation of energy occur in electrical circuits. An electric circuit is one or more loops that current flows through. Every time you turn on a light, a computer, a TV, your phone or any other electrical or electronic device, energy is being transferred, and being transformed, within a circuit.

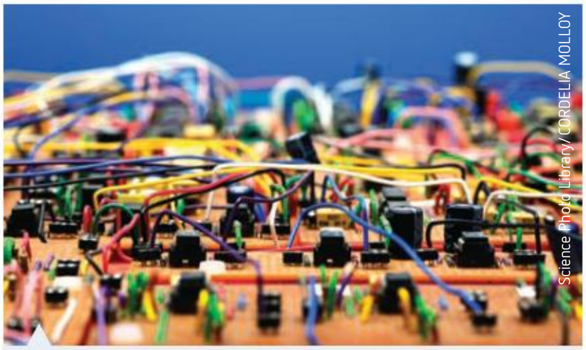

Science Photo Library/ CORDELIA MOLLOY

FIGURE 13.1 An electronic circuit board.

13.1 Electric current

Current is the flow of charged particles. In a circuit, this is usually the flow of electrons. In other systems, such as in your body, it is the flow of larger particles including calcium ions (Ca^{2+}) and potassium ions (K^+). We define current, I, as:

$$I = \frac{q}{t}$$

where q is the quantity of charge passing through a point in time, t. Remember that charge has units of coulombs, C, and time has units of seconds, s. So, the unit of current is coulombs per second, C s^{-1}, which we call amperes, A. The unit ampere is named for André-Marie Ampère, a French mathematician and physicist.

Ampère's contributions

Learn more about the life and work of André-Marie Ampère.

Useful information

When we define current as charge per unit time, it is a scalar. But it still has a sign because charges may be moving through the point in one direction or the other, and they may be positive or negative charges. It is often very difficult to tell whether the moving charges are positive or negative. In Figure 13.2, the right-hand side of the line between X and Y is becoming more positive by three units in three different possible ways: a flow of positive charges to the right, a flow of negative charges to the left, or a combination of these. Regardless of whether it is positively or negatively charged particles that are flowing, the result is the same.

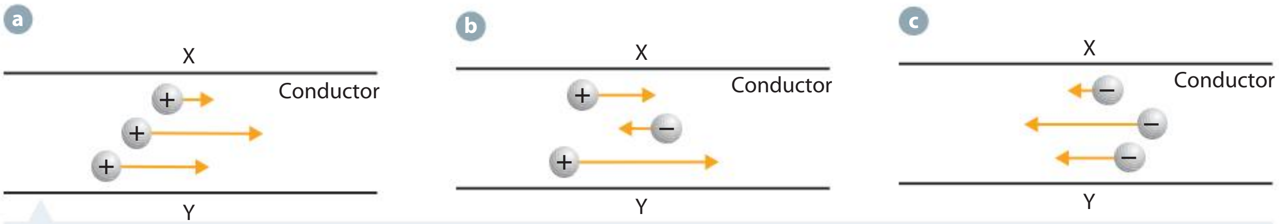


FIGURE 13.2 **a** Three positive charges moving to the right; **b** Two positive charges moving to the right and one negative charge moving to the left, which makes the same current; **c** Three negative charges moving to the left, which also makes the same current

Some convention is needed, so we define the direction of current as the direction that positive charges would be moving if it was positive charges creating the current. Hence, in your body, the direction of current matches the direction that calcium and sodium ions move. In an electric circuit, the direction of current is opposite to the direction that the electrons move. In Figure 13.2, the direction of the current is always to the right, regardless of the sign of the charge carriers.

WORKED EXAMPLE (13.1)

A cell membrane has 500 Ca^{2+} ions move across it in 1.0 s. What is the current flow through this membrane?

ANSWER	**LOGIC**
$t = 1.0$ s; q is the charge carried by 500 Ca^{2+} ions Each Ca^{2+} ion has charge $q = +2e$, where e is the electron charge.	- Identify the relevant data in the question.
$q = 500 \times 2e = 500 \times 2 \times 1.6 \times 10^{-19}$ C $q = 1.6 \times 10^{-16}$ C $I = \frac{q}{t}$ $I = \frac{1.6 \times 10^{-16}\ \text{C}}{1.0\ \text{s}}$ $= 1.6 \times 10^{-16}$ C s^{-1}	- Convert to SI units. - Identify the appropriate formula. - Substitute the known values, with units, into the formula. - Calculate the answer.
$I = 1.6 \times 10^{-16}$ A	- State the final answer with appropriate significant figures and units.

TRY THESE YOURSELF

1. If a current of 2.0×10^{-15} A flowed through a membrane in 1.0 s, calculate how many Na^+ ions this would require.
2. A current of 1.5 A flows through a wire out of a car battery terminal for 2.0 minutes. Calculate how much charge flows out of this battery terminal in this time.

There are three conditions required for a current to flow:

1. a path for the current to flow along
2. charge carriers (charged particles) that are free to move
3. a force applied to make them start moving.

A material that allows current to flow through it easily is called a **conductor**. Metals are good conductors because they have many free electrons.

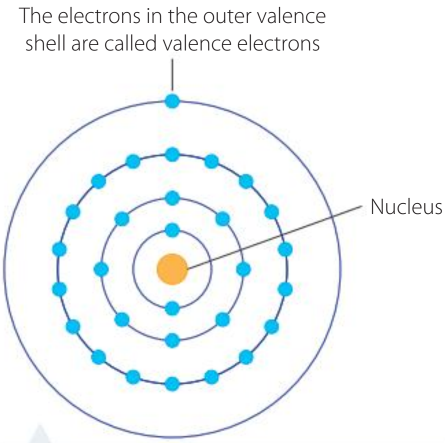


FIGURE 13.3 A single copper atom, with the electrons arranged in shells

Materials that do not allow current to flow through them are called **insulators**. Insulators do not have free electrons. There are also materials, called **semiconductors**, which have a small number of free electrons. These materials allow current to flow, but not easily.

Whether a material is a conductor, an insulator or a semiconductor depends on what sort of atoms it is made of and how those atoms are connected to each other.

In a metal, the atoms bond together in such a way that the outermost electrons are shared between a large number of atoms. Consider the single copper atom shown in Figure 13.3. There are many electrons surrounding the nucleus. When the copper atom is isolated, all the electrons are bound to the nucleus by the electrostatic force (see chapter 12).

However, when we bring a large number of copper atoms together, a small number of outer-shell (valence) electrons (one in copper, but one, two or three in other metals) become unbound or free of their original nuclei. These are the free electrons that move when a current flows. Hence, they are also referred to as conduction electrons. These free electrons also have an important role in holding the metal together – they act as a negative 'sea' or 'glue' of electrons surrounding the positive ions, which consist of the nuclei and their bound electrons. This is shown in Figure 13.4. This is what chemists mean when they talk about metallic bonding – it is a bond between many atoms due to these free electrons. So, a metal has free charge carriers and can provide a path for free electrons to move through.

In the next section, we will look at how a force can be applied to get the electrons moving and create a current.

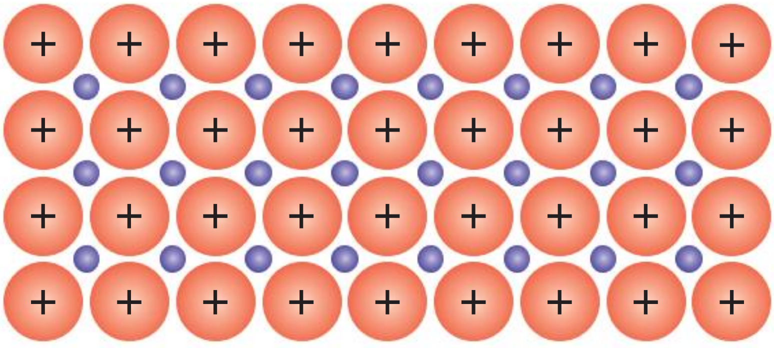
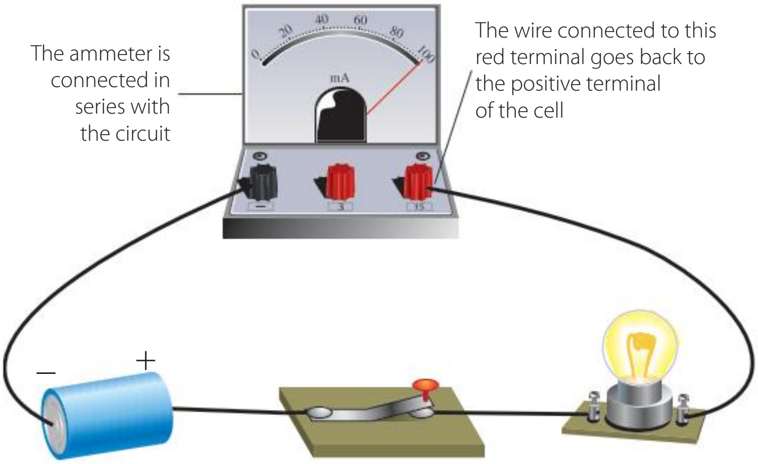


FIGURE 13.4 Many copper atoms bonded together by a sea of free electrons

Current is measured in a circuit using an ammeter. A multimeter has an ammeter setting for **direct current (DC)** and for **alternating current (AC)**. The DC setting is used when the current goes in the same direction all the time, such as when a battery is used in a circuit. The AC setting is used when the current is expected to oscillate in direction, such as the mains power supply. Because an ammeter measures current through a point, it must be placed at that point within the circuit. It is placed in **series** within the circuit, as shown in Figure 13.5.

Conductors, insulators and semiconductors

Find out why conductors, insulators and semiconductors behave the way they do.



The ammeter is connected in series with the circuit

The wire connected to this red terminal goes back to the positive terminal of the cell

FIGURE 13.5 An ammeter connected in series within a circuit. The ammeter measures how much current is flowing through itself.

INVESTIGATION 13.1

Current flow in metals

Critical and creative thinking

Numeracy

Information and communication technology capability

In this experiment, a resistor is used to ensure that the flow of current in the circuit is not too high. Resistors are described in detail in section 13.2.

AIM

To investigate the flow of current through metal wire

Write a research question for this investigation.

MATERIALS

- 1.5 V battery in holder
- 10 Ω resistor
- Current meter (ammeter or multimeter on current setting)
- Fine copper wire in various lengths of minimum 20 m (for example, 20 m, 40 m, 60 m, 80 m, 100 m) (alternatively, shorter lengths of nichrome wire can be used)
- Micrometer
- Crocodile clips for making connections

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Electricity can cause shocks.	Do not touch both terminals of the battery at once.
A length of wire across the battery can cause a 'short circuit', damaging the battery.	Always keep the resistor in the circuit.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Read the instructions for your multimeter or current meter, or ask your teacher to show you how to use it.
2. Connect the resistor to the positive terminal of the battery.
3. Connect one probe of your current meter to the resistor.
4. Connect the other probe of the current meter to one end of the wire.
5. Connect the other end of the length of wire to the negative terminal of the battery. Your circuit should look like Figure 13.6.
6. Record the current flowing through the wire, as shown on the current meter. Remember to include units.
7. Remove the wire from the circuit.
8. Repeat steps 4–7 for each different length of wire.
9. Use the micrometer to measure the diameter of the metal conductor within the wire.

   If you find that the current is not changing as you change lengths of wire, try using a smaller resistor (e.g. 5 Ω or 1 Ω).

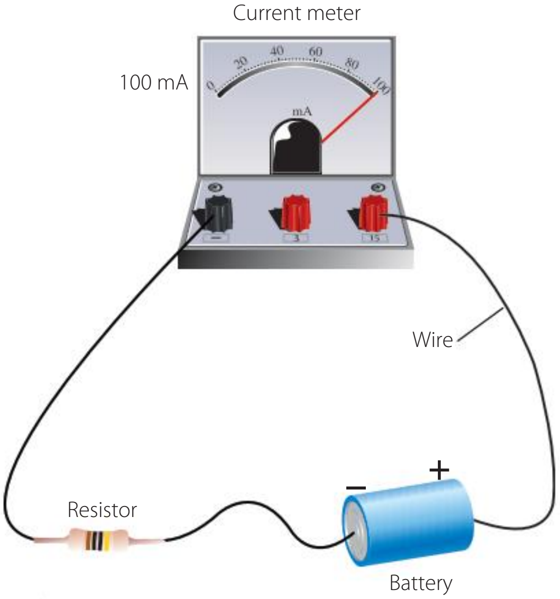


FIGURE 13.6 Experimental set-up

RESULTS

Record your data in a spreadsheet. Use columns headed 'length' and 'current'. Include units in the heading cells, or in the cell below.

ANALYSIS OF RESULTS

1. Use the spreadsheet to draw a scatter plot of your data, with the current on the vertical axis and the length on the horizontal axis. What shape is your graph? Does it look like a straight line?
2. Add another heading cell, $\frac{1}{\text{length}}$, to your spreadsheet. Include units in this cell, or in the cell below. Create a column of data below this by getting the spreadsheet to calculate $\frac{1}{\text{length}}$ for each length of wire used.

»

3 Plot a scatter graph of current as a function of $\frac{1}{\text{length}}$. What shape is this graph? If it is linear, add a line of best fit and display the equation on your graph.

4 Write an equation that describes the relationship between current and the length of the wire.

5 The current through a wire is expected to follow the relationship $I = \frac{VA}{\rho L}$, where A is the cross-sectional area of the wire and ρ is the resistivity of the wire. The resistivity is an important property of a metal because it determines how easily current can flow through the metal. Use the gradient of your graph and other measured data to calculate the resistivity of the metal in the wire.

Ohm's law and drift velocity in semiconductors

Use this Physclips animation to learn about resistance and conduction on a microscopic level.

DISCUSSION

1 Were you able to answer your research question?

2 What are the limitations of this experiment?

3 Can you suggest better ways of making the measurements, or further work?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation. Write an answer to your research question.

KEY CONCEPTS

- Current is the flow of charged particles, called charge carriers.
- Charge carriers can be positive or negative. In electric circuits, they are usually electrons, which have a negative charge.
- Current is the charge per unit time passing through a point: $I = \frac{q}{t}$, measured in units of A; $1\ \text{A} = 1\ \text{C s}^{-1}$.
- The direction of a current is the direction that positive charge carriers would be flowing if the current was carried by positive charge carriers.
- Metals are good conductors because they have many free electrons (charge carriers).
- Ammeters are used to measure current. Ammeters must be connected in series in a circuit.

CHECK YOUR UNDERSTANDING

13.1

1 A defibrillator takes 30 s to fully charge, and less than 1 s to discharge. In which phase is the current greater? Justify your answer.

2 Electrons are moving to the left through a wire. Explain why the direction of the current in the wire is to the right.

3 A cell membrane has a channel (hole) that allows sodium (Na^+) ions to pass into the cell.

  a What is the direction of the current in this case: into or out of the cell?

  b A different channel allows chloride (Cl^-) ions to enter the cell. What is the direction of the current for this channel: into or out of the cell? Explain your answer.

4 Copper has one valence electron and zinc has two. Predict which will be the better conductor. Justify your prediction.

5 It takes 30 s for 1.0 C to pass through the terminal of a battery. Calculate the current flow through this terminal.

6 A wire carries a current of 0.5 A.

  a Calculate the total charge that passes through a point in the wire in each minute.

  b Calculate how many electrons pass through this point per minute.

  c If the wire were replaced by one with the same diameter but twice the length, would you expect the current to increase or decrease?

13.2 Current–voltage relationships

In the previous section, we noted that three things are required to create a current. These are charged particles that can move (free charge carriers), a path for them to move along, and a force to start them moving. One way to provide this force is by an electric field, because fields store potential energy.

A charged particle in an electric field experiences a force, and if it moves in the field then work is done on the particle. In chapter 12, the idea of electric potential was introduced. We can relate the work done to the potential difference:

$$V = \frac{W}{q}$$

where V is the potential difference between two points, and q is the charge. The units of potential difference are volts, and so potential difference is often called '**voltage**'. Potential difference is more correct, but voltage is very commonly used. $1\ \text{V} = 1\ \text{J C}^{-1}$; it is the change in energy per unit charge of the charge carrier as it moves though the potential difference.

A battery or power supply provides energy to a circuit by doing work on charged particles (usually electrons). This potential energy is used to produce a current through the circuit.

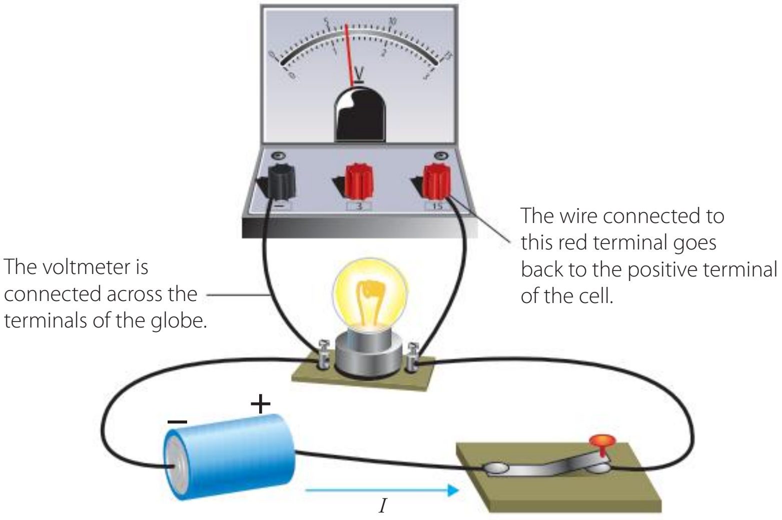
When a charge q moves between two points in a circuit, it will lose or gain potential energy. If it moves through an energy source, such as a battery, it gains energy. If it moves through other components, it will generally lose energy. The energy change, loss or gain, is equal to:

$$W = Vq$$

A positive potential difference means going from less positive (or more negative) to more positive (or less negative). When a positive charge goes through a positive potential difference, it gains potential energy because it moves against the direction of the field lines (see chapter 12). So, current in a circuit flows through a battery from negative to positive.

Potential difference is measured with a voltmeter. A voltmeter measures the difference in potential between two points. The voltmeter needs to be connected in **parallel** to two different parts of the circuit, as shown in Figure 13.7. It measures the potential difference between the two points to which it is connected.

FIGURE 13.7 The voltmeter is measuring the difference in potential between two points – in this case, the two sides of the light globe. It is placed in parallel across an element.



The voltmeter is connected across the terminals of the globe.

The wire connected to this red terminal goes back to the positive terminal of the cell.

WORKED EXAMPLE (13.2)

An electron moves through a potential difference of +12 V. How much work is done on the electron?

ANSWER	LOGIC
$V = +12\ \text{V};\ q = 1e = -1.6 \times 10^{-19}\ \text{C}$	- Identify the relevant data in the question.
$V = \frac{W}{q}$	- Identify the appropriate formula to relate voltage and work.
$W = Vq$	- Rearrange for work.
$W = (+12\text{V})\,(-1.6 \times 10^{-19}\ \text{C})$	- Substitute the known values, with units, into the formula.
$= -1.92 \times 10^{-18}\ \text{V C}$	- Calculate the answer.
$W = -1.9 \times 10^{-18}\ \text{J}$ The electron has lost -1.9×10^{-18} J.	- State the final answer with appropriate significant figures and units.

TRY THESE YOURSELF

1 Calculate how many electrons must move through a potential difference of −12 V for a total of 1.0 J of work to be done.

2 A current of 1.5 A flows from a 12 V car battery for 2.0 minutes. Calculate the energy used by the battery in this time.

Resistance and Ohm's law

In Investigation 13.1, you would have observed that when a battery is connected to a wire, current flows through the wire. The longer the wire, the less current flows. This is because even a very good conductor, such as a metal wire, has some **resistance**. (The only exceptions to this are superconductors, which need to be kept at very low temperatures.) Resistance is a measure of how much a circuit component opposes the flow of current through it.

Resistance, R, is defined as the ratio of potential difference *across* (V) to current *through* (I) a component:

$$R = \frac{V}{I}$$

The units of resistance are V A^{-1}, which are given the name ohm and symbol Ω (capital Greek letter omega). The unit Ohm is named for Georg Ohm, a German scientist.

This equation tells us that, for a given applied potential difference, a low current means a high resistance and vice versa. For an insulator, the resistance is very, very high; effectively infinite. So the current will be zero. For a good conductor, the resistance is very low, so the current flow will be very large.

All circuit components have a resistance. Components whose purpose is to provide a constant known resistance in a circuit are called **resistors**.

WORKED EXAMPLE 13.3

A student has connected a 1.5 V battery to a 100 Ω resistor. Calculate the current flow through the resistor.

ANSWER	LOGIC
$V = 1.5\text{ V}; R = 100\ \Omega$	Identify the relevant data in the question.
$R = \frac{V}{I}$	Identify the appropriate formula to relate resistance and voltage.
$I = \frac{V}{R}$	Rearrange for current.
$I = \frac{1.5\text{ V}}{100\ \Omega}$	Substitute the known values, with units, into the formula.
$= 0.015\text{ V }\Omega^{-1}$	Calculate the answer.
$I = 1.5 \times 10^{-2}\text{ A}$	State the final answer with appropriate significant figures and units.

TRY THIS YOURSELF

Calculate the current that would flow through the 100 Ω resistor if it were connected to a 12 V car battery.

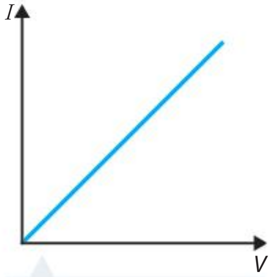


FIGURE 13.8 I–V graph for an ohmic device. In an ohmic device, the resistance is constant.

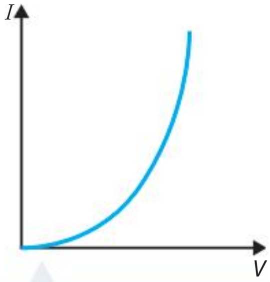


FIGURE 13.9 I–V graph for a non-ohmic device. In a non-ohmic device, the resistance changes as the voltage and current is varied.

Ohmic and non-ohmic circuit components

All circuit components act as resistors, in that they add resistance to a circuit. Components for which the resistance is constant for a wide range of applied voltage are called **ohmic**. For these components:

$$R = \frac{V}{I} = \text{constant}$$

This special case is called **Ohm's law**. Current through an ohmic component or device depends directly on the applied voltage (potential difference). If we plot a graph of current as a function of potential difference, we get a straight line, as shown in Figure 13.8. We can calculate the resistance of the component from the gradient of such a graph:

$$R = \frac{V}{I} = \frac{1}{\text{gradient}}$$

Current–voltage (I–V) graphs such as that shown in Figure 13.8 are commonly used to show the current–voltage characteristics of components. Different types of components have very different I–V characteristics, depending on their function. For example, diodes act as valves in a circuit so they have very low resistance above some minimum voltage, and very high resistance below this. Components that have resistance that varies with applied voltage are called **non-ohmic**. Figure 13.9 shows an example of non-ohmic I–V characteristics.

Non-ohmic components and devices cannot be characterised by a single constant resistance. They have a resistance that varies with applied voltage, so we need an I–V graph to determine how they will behave in a circuit. To find the resistance of a non-ohmic component, we look at the curve and read off the current for the voltage we are interested in. The resistance is then given by:

$$R = \frac{V}{I}$$

WORKED EXAMPLE 13.4

A group of students are conducting an investigation on non-ohmic components. Figure 13.10 shows the *I–V* characteristics that they have measured for a diode. Calculate the resistance of this diode for applied voltages of 0.5 V, 1 V and 1.5 V.

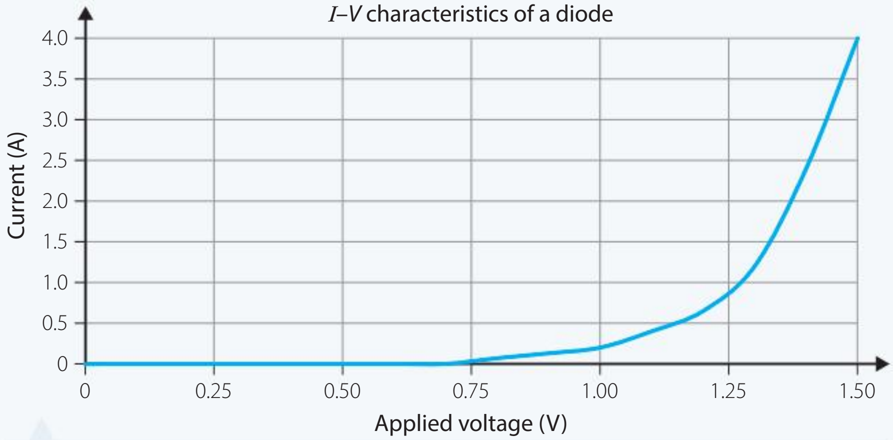


FIGURE 13.10 *I–V* characteristics of a diode

ANSWER	**LOGIC**
$R = \frac{V}{I}$	- Identify the appropriate formula to relate resistance, voltage and current.
At $V = 0.5$ V, $I = 0$ A At $V = 1$ V, $I = 0.2$ A At $V = 1.5$ V, $I = 4.0$ A	- Read the value of the current from the graph for each of the voltages given.
For $V = 0.5$ V, $R = \frac{V}{I} = \frac{0.5\text{ V}}{0\text{ A}} = \infty$ For $V = 1$ V, $R = \frac{V}{I} = \frac{1\text{ V}}{0.2\text{ A}} = 5\ \Omega$ For $V = 1.5$ V, $R = \frac{V}{I} = \frac{1.5\text{ V}}{4\text{ A}} = 0.375\ \Omega$	- Calculate the resistance for each value of *V*.
V (V) | *I* (A) | *R* (Ω) 0.5 | 0 | ∞ 1 | 0.2 | 5 1.5 | 4 | 0.4 When no current can flow at all, the resistance is effectively infinite. This condition is also called 'open circuit' because it is equivalent to opening up a circuit by disconnecting it, so that there is no path for the charge carriers (electrons) to flow along.	- State the final answer with appropriate significant figures and units. This can be done in a table or in words.

TRY THIS YOURSELF

The graph shown in Figure 13.10 is for a component whose resistance decreases with increasing voltage. Sketch a graph of the *I–V* characteristics for a component that has an increasing resistance with increased applied voltage.

INVESTIGATION 13.2

Ohmic and non-ohmic components

Ohm's law

Use these simulations to compare ohmic and non-ohmic components.

Critical and creative thinking

Numeracy

Information and communication technology capability

RISK ASSESSMENT

In this experiment, you will measure the I–V characteristics for a resistor and a light globe. By plotting I–V graphs, you will be able to determine whether they are ohmic or non-ohmic.

AIM

To investigate the I–V characteristics of a resistor and a light globe

Write a hypothesis for this investigation. Predict whether each component will be ohmic or non-ohmic. If you have predicted that a component will be non-ohmic, do you expect its resistance to decrease or increase with increasing voltage?

MATERIALS

- 12 V variable DC power supply
- 100 Ω resistor
- 12 V globe in holder
- Ammeter or multimeter on current setting
- Voltmeter or multimeter on volts setting
- Connectors (e.g. wires with crocodile clips)

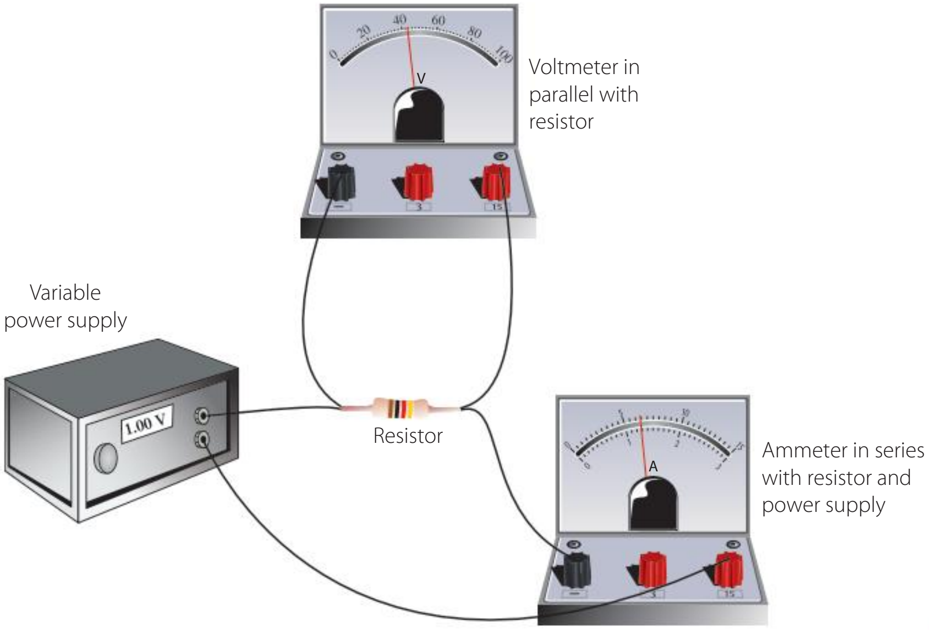
WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Electricity can cause shocks.	Keep the power supply turned off until your teacher has checked your circuit. Do not touch the terminals of the power supply when it is turned on.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Read the instructions for your meters, or ask your teacher to show you how to use them.
2. Connect a series circuit consisting of the power supply, ammeter and resistor. Do not turn the power supply on yet.
3. Connect the probes of the voltmeter across the resistor. Your circuit should look like Figure 13.11.

FIGURE 13.11
Experimental set-up



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» 4 Turn the power supply to its lowest voltage setting before turning it on.
5 Turn the power supply on and adjust it so that the voltmeter reads as close to 1 V as possible.
6 Record the current flowing through the resistor, as shown on the current meter. Remember to include units and an estimate of the uncertainty in the current.
7 Record the voltage across the resistor. Remember to include units and an estimate of the uncertainty in the current.
8 Increase the power supply output so that the voltage across the resistor is 1 V higher.
9 Repeat steps 6–8 until you have reached 12 V. Record your data in a table.
10 Turn off the power supply and remove the resistor from the circuit. Replace it with the light globe.
11 Repeat steps 4–9, measuring the current through and voltage across the light globe. Note that it is important to measure the current through the globe from low voltage to high, as the filament resistance changes as it heats up.

RESULTS

1 Record your data in a spreadsheet. You should have two tables of data: voltage and current for the resistor and for the globe.
2 Write down any other observations that you make; for example, how the brightness of the globe varies.

ANALYSIS OF RESULTS

1 Draw a scatter plot of your data for each component, with the current on the vertical axis and the voltage on the horizontal axis. What shape is each graph?
2 If either graph is linear, use the gradient to find the resistance of the component.
3 If either graph is not linear, calculate the resistance of the component for each voltage. Plot a graph of resistance as a function of applied voltage.

DISCUSSION

1 Were the components ohmic or non-ohmic? If either was non-ohmic, did its resistance increase or decrease with voltage?
2 Was your hypothesis supported or disproved?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- The potential difference, V, between two points is the work per unit charge that is done on a charged particle when it moves between those two points: $V = \frac{W}{q}$.
- Potential difference (voltage) has units volts; $1\ \text{V} = 1\ \text{J C}^{-1}$.
- A voltmeter measures the potential difference between two points. Voltmeters are connected in parallel with circuit components.
- Resistance, R, is defined as the ratio of potential difference *across* (V) to current *through* (I) a component: $R = \frac{V}{I}$ (Ohm's law).
- Components whose purpose is to provide a constant known resistance in a circuit are called resistors.
- Components for which the resistance is constant for a wide range of applied voltage are called ohmic. Components for which the resistance varies with applied voltage are called non-ohmic.
- I–V graphs are used to represent the current–voltage characteristics of a component or device, and allow the resistance to be calculated for any applied voltage.

CHECK YOUR UNDERSTANDING 13.2

1 Explain the difference between an ohmic and non-ohmic component.

2 Calculate the potential difference across a 1.8 kΩ resistor in which a current of 240 mA flows.

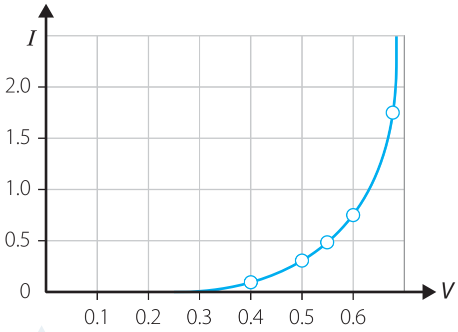


FIGURE 13.12 *I–V* characteristics of a diode

3 The *I–V* graph for a diode is shown in Figure 13.12. Calculate the resistance when there is a potential difference of 0.6 V across the diode.

4 Tuan measures the current through a light-emitting diode (LED) for five different potential differences. His measurements are shown in Table 13.1.

TABLE 13.1 Measurements of current through an LED for varying potential differences

POTENTIAL DIFFERENCE (V)	2.4	2.6	2.8	3	3.2
CURRENT (mA)	0	1	4	12	25

- **a** Plot an *I–V* graph for Tuan's data.
- **b** Is this an ohmic or non-ohmic component? Justify your answer.
- **c** Calculate the resistance of this LED when it has a potential difference (applied voltage) of 3 V across it.

5 When a potential difference of 16 V is applied across the ends of a wire, the current flowing in the wire is 2.4 A. Assume the wire is ohmic.

- **a** What is the resistance of the wire?
- **b** What potential difference is needed to make a current of 3.0 A flow through the wire?

6 Given that ammeters are always connected in series with a circuit, should ammeters have large or small resistance? Explain your reasoning, using Ohm's law to justify your answer.

13.3 Energy and power in electric circuits

A circuit typically consists of sources of potential energy, and components that convert that potential energy into different forms. A battery is a source of potential energy. As charged particles move through a battery, their potential energy increases. When charged particles flow through other components (such as light globes, resistors and motors), their potential energy decreases.

Recall from chapter 5 that energy is conserved. A battery stores potential energy, but as it increases the energy of the charged particles, its store of energy must decrease – it goes 'flat'.

When the potential energy of the charged particles decreases as they move through other components, that energy is converted into other forms. An incandescent light globe converts electrical potential energy into light and heat. An LED (light-emitting diode) converts electrical potential energy into light. A motor converts electrical potential energy into kinetic energy. In Investigation 13.2, you may have noticed that the resistor got warm. A resistor converts electrical potential energy into heat, which is conducted and radiated to the environment.

Power in electric circuits

The rate at which energy, *E*, is transformed is the power, *P*. From chapter 5, we know that:

$$P = \frac{E}{t}$$

so, the energy transferred in a time *t* is:

$$E = Pt$$

Potential difference is the change in energy per unit charge between two points. So, as current flows through a component:

$$V = \frac{E}{q}$$

The current is the charge per unit time (C s^{-1}):

$$I = \frac{q}{t}$$

If we multiply current by voltage, we get:

$$IV = \frac{q}{t} \times \frac{E}{q} = \frac{E}{t} = P$$

Hence, the rate at which energy is transformed by a component is given by:

$$P = VI$$

where I is the current through the component and V is the potential difference across it.

Power is measured in units of watts, W. $1\ \text{W} = 1\ \text{J s}^{-1} = 1\ \text{V A}$.

Power in electric circuits

Use this calculator to find the power transformed in a circuit. Follow the links to see how this relates to Ohm's law.

WORKED EXAMPLE 13.5

A light globe is rated at 15 W and is powered by the mains supply, which provides 240 V. What current flows through this globe when it is turned on?

ANSWER	LOGIC
$V = 240\ \text{V}$; $P = 15\ \text{W}$	■ Identify the relevant data in the question.
$P = VI$	■ Identify the appropriate formula to relate power, current and voltage.
$I = \frac{P}{V}$	■ Rearrange for current.
$I = \frac{15\ \text{W}}{240\ \text{V}}$	■ Substitute the known values, with units, into the formula.
$= 0.0625\ \text{W V}^{-1}$	■ Calculate the answer.
$I = 63\ \text{mA}$	■ State the final answer with appropriate significant figures and units.

TRY THESE YOURSELF

1 Calculate the energy that would be used (transformed into light and heat) by this globe in 1 hour.

2 Calculate the energy that would be used (transformed into heat) by a 240 V heater with a current of 2 A passing through it for 1 hour.

Power, energy and Ohm's law

Ohm's law gives us a relationship between resistance, current and voltage:

$$R = \frac{V}{I}$$

This can be rearranged to give:

$$V = IR \text{ and } I = \frac{V}{R}$$

If we substitute the first of these into the expression for power, $P = VI$, we get:

$$P = VI = I^2R$$

If we substitute the second of these into $P = VI$, we get:

$$P = VI = \frac{V^2}{R}$$

We now have three equivalent expressions for power:

$$P = VI = I^2 R = \frac{V^2}{R}$$

WORKED EXAMPLE 13.6

The cabin light of a car is powered by the 12 V car battery. When it is turned on, it has a resistance of 25 Ω. Calculate the rate at which it converts electrical potential energy to other forms.

ANSWER	LOGIC
$V = 12$ V; $R = 25\ \Omega$ We want to find P.	■ Identify the relevant data in the question.
$P = \frac{V^2}{R}$	■ Identify the appropriate formula to relate power, voltage and resistance.
$P = \frac{(12\ \text{V})^2}{25\ \Omega}$	■ Substitute the known values, with units, into the formula.
$= 5.76\ \text{V}^2\ \Omega^{-1}$	■ Calculate the answer.
$P = 5.8$ W	■ State the final answer with appropriate significant figures and units.

TRY THESE YOURSELF

1 Calculate the current flowing through this globe.

2 Calculate the energy used by this light in each minute.

INVESTIGATION 13.3

Critical and creative thinking

Numeracy

Information and communication technology capability

Power and energy conversions in a DC circuit

In this investigation, you will observe different energy transformations. You will also compare the power usage of various types of circuit elements.

AIM

To observe different energy transformations in an electric circuit

Write a research question or hypothesis for this investigation.

»

MATERIALS

- 3 V DC power supply (variable supply set on 3 V or two 1.5 V batteries in a holder)
- Wires and clips for making connections
- 3 V globe in holder
- 3 V DC buzzer
- Small DC motor
- Current meter (ammeter or multimeter on current setting)

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Electricity can cause shocks.	Keep the power supply turned off until your teacher has checked your circuit. Do not touch the terminals of the power supply when it is turned on.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Read the instructions for your ammeter, or ask your teacher to show you how to use it.
2. Connect a series circuit consisting of the power supply, ammeter and globe. Your circuit should look like Figure 13.13.

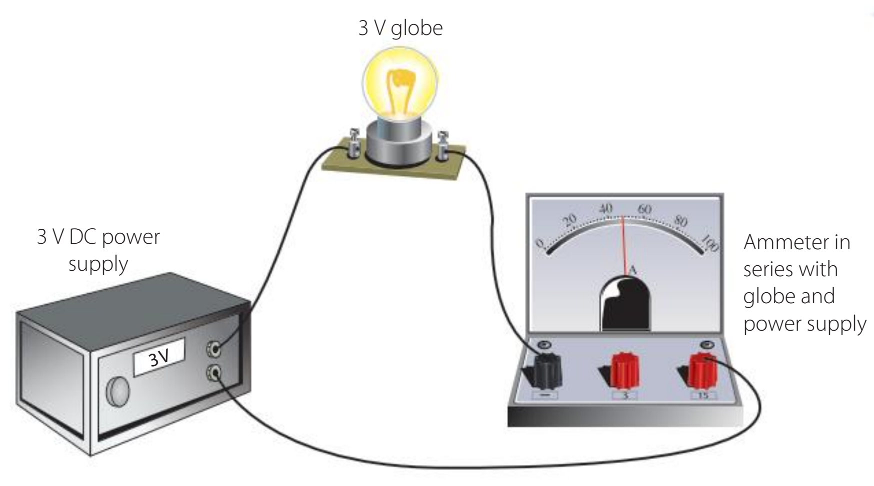


FIGURE 13.13 Experimental set-up

3. When your teacher has checked your circuit, turn on the power supply.
4. Record the reading on the ammeter. Note down any other observations.
5. Turn off the power supply and remove the globe from the circuit.
6. Repeat steps 2–5, replacing the globe with the buzzer. Repeat steps 2–5 again, replacing the buzzer with the motor.

RESULTS

Create a table for your results as shown:

COMPONENT	ELECTRIC POTENTIAL ENERGY TRANSFORMED TO:	CURRENT (mA)	POWER (W)	RESISTANCE (Ω)
Globe	light and heat			
Buzzer				
Motor				

Record your data for current and your observations of any energy transformations in the table.

ANALYSIS OF RESULTS

1 Calculate the power used by each component (globe, buzzer, motor) and write it in the table.
2 Calculate the resistance of each component and write it in the table.
3 Plot a bar graph comparing the power usage of the three components.

DISCUSSION

1 Describe the energy transformations that took place in each of your three circuits.
2 Compare the energy usage of the three components.
3 Give the answer to your research question, or state whether your hypothesis was supported.
4 Can you think of any further experiments you could do to extend this investigation?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

INVESTIGATION 13.4

Critical and creative thinking

Numeracy

Converting potential energy to heat

In this experiment, electric potential energy is converted to heat and used to increase the temperature of a known mass of water. You may wish to review chapter 11, particularly the sections on heat transfer and specific heat capacity. The principle of conservation of energy is applied to calculate the efficiency of a kettle.

AIM

To analyse the energy transformations and transfers taking place as a kettle is used to heat water

Write a research question for this investigation.

MATERIALS

- 240 V electric kettle
- 500 mL water
- Thermometer
- Stopwatch
- Wooden stirrer

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Hot water and steam can burn.	Avoid spills and do not heat the water to boiling. Use a wooden stirrer rather than a metal one.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Read the compliance label on the kettle, usually found on the base. Note down the operating voltage (230–240 V in Australia) and the power rating in W or kW.
2. Pour 500 mL (500 g) of water into the kettle. Wait a few minutes to allow the water and kettle to reach thermal equilibrium.
3. While you are waiting, use the power rating of the kettle to calculate the temperature change of the water if the kettle runs for 60 s. Assume that all energy transferred to the kettle is transformed to heat and used to heat the water (100% efficiency).
4. Measure the temperature of the water. Note this down, including the uncertainty in your measurement.
5. Turn the kettle on for 60 s, and then immediately turn it off again. Note that there will be an uncertainty in the time for which the kettle was run.
6. Quickly stir the water so it has uniform temperature, and measure the temperature of the water. Note this down along with its uncertainty.

RESULTS

1. You calculated a theoretical temperature change, assuming 100% efficiency of energy conversion and transfer.
2. You measured a temperature change for the same energy usage.

ANALYSIS OF RESULTS

1. Does your measured temperature change agree with the theoretical change, within experimental uncertainty?
2. Calculate the rate at which energy was actually transferred to the water by the kettle (the power).
3. Calculate the uncertainty in this value. To do so, remember that the uncertainty in the temperature difference is the sum of the absolute uncertainties in the two temperature measurements. The uncertainty in the power will be the sum of the *fractional* (relative) uncertainties in the temperature difference, time and mass of water. Remember that if any of these are very small compared to the others, they can be neglected.
4. Calculate the efficiency of the kettle. This is the fraction of energy used that was actually used to heat the water.
5. Calculate the current through the kettle while it was turned on, and the resistance of the kettle. Summarise your calculations in a table, giving all the characteristics and specifications of the kettle.

DISCUSSION

1. How does the experimental temperature change compare to your theoretical calculated change?
2. How can you explain any differences? Use the idea of energy conservation to explain your findings.
3. Provide an answer to your research question.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

KEY CONCEPTS

- A battery is a source of potential energy. As charged particles move through a battery, their potential energy increases.
- When charged particles flow through other components (such as light globes, resistors and motors), their potential energy decreases. The potential energy is transformed into other forms including light, heat and kinetic energy.
- The rate at which energy is transformed is given by $P = \frac{E}{t} = VI = I^2R = \frac{V^2}{R}$. Power has units of W; 1 W = 1 J s^{-1} = 1 V A.

CHECK YOUR UNDERSTANDING

13.3

1. A 240 V heater has two settings: high and low. The high setting produces more heating than the low setting. Apply Ohm's law to determine on which setting the heater has the higher resistance.
2. Define power. Give the units of power in at least three different ways, including fundamental units.
3. A 100 W light globe is left on for 1.0 hour. Calculate the energy used by the globe. Give your answer in joules.
4. **a** How much energy is transformed in a hair dryer if it has a potential difference of 240 V and 100 C of charge passes through it?
   b Describe all the energy transformations that take place when you use a hair dryer, starting with the potential energy from the grid and including the energy transferred to the air.
5. A current of 2.0 A flowing in a heater for an hour converts 1.7 MJ of electrical energy into heat.
   a Calculate the charge transferred through the heater.
   b Calculate the potential difference across the heater.

13.4 Circuits and circuit diagrams

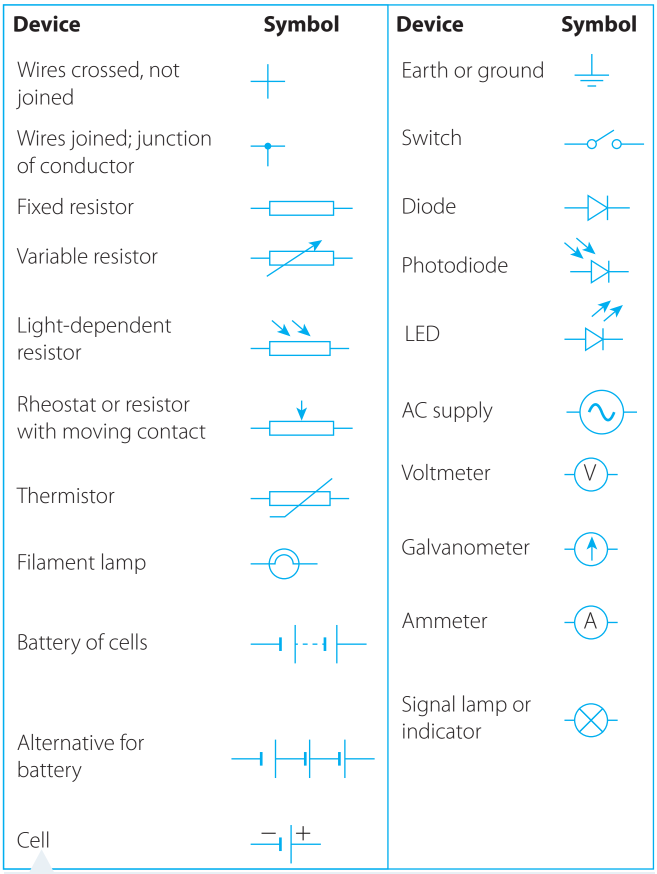


FIGURE 13.14 Symbols for commonly used circuit elements

Physicists use a lot of diagrams. In chapter 3 you learned how to draw vector diagrams, and in chapter 4 you learned how to draw force diagrams. Another type of diagram that is very useful is a **circuit diagram**.

Circuit diagrams are used to show how the various components in a circuit are connected. Each different type of component is represented by a different symbol. Figure 13.14 shows the most commonly used circuit symbols. Further information on these is given in appendix 3.

A circuit diagram is like a diagram for a transport system, such as that for a train network. It shows you how the components are related to each other – how they are connected, rather than their physical sizes and positions.

Figure 13.15a shows a picture of a circuit, and Figure 13.15b shows a circuit diagram for this same circuit. The direction of current is also shown. Note that current flows from the positive terminal of the battery around to the negative terminal.

Just as a train system diagram may not show physical distances, a circuit diagram doesn't indicate lengths of wires. We assume that the wires in a circuit have negligible resistance compared to any other component, regardless of their length.

Using electricity

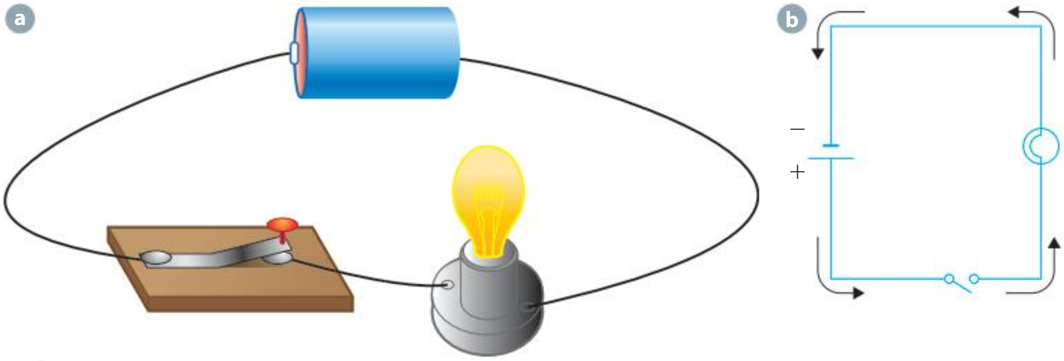
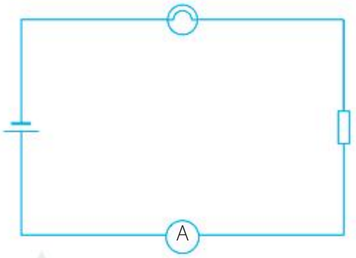


FIGURE 13.15 **a** A light globe, switch and a battery connected by wires; **b** A circuit diagram for the electric circuit shown in part **a**

WORKED EXAMPLE 13.7

A battery is connected in series with a light globe, a resistor and an ammeter. Draw a circuit diagram for this circuit.

ANSWER	**LOGIC**
The circuit consists of battery, globe (filament lamp), resistor and ammeter in series. In series means connected one after another.	- Identify the relevant data in the question. Look at Figure 13.14 to identify the symbol for each component.
 **FIGURE 13.16** Circuit diagram for a series circuit consisting of a battery, globe, resistor and ammeter.	- Draw the diagram with the components connected in a single loop.

TRY THESE YOURSELF

1. Draw a circuit diagram for the circuit shown in Figure 13.7 (on page 374). Note that the voltmeter is *not* in series with the other components in this circuit.
2. Identify each of the circuit components shown in Figure 13.17.

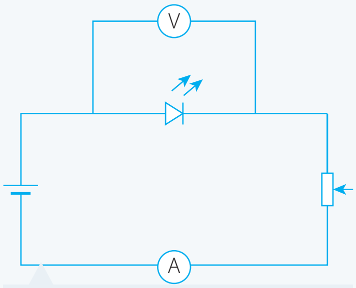


FIGURE 13.17 Identify each of the components shown in this circuit diagram.

Kirchhoff's laws and conservation principles

We can apply conservation principles to analyse the voltages and currents in a circuit. The two conservation principles that we use are conservation of energy and conservation of charge. German physicist Gustav Kirchhoff first proposed applying these conservation principles to circuits, so they are called Kirchhoff's laws in this context. But they are the same conservation principles that you met earlier in chapters 5 and 12.

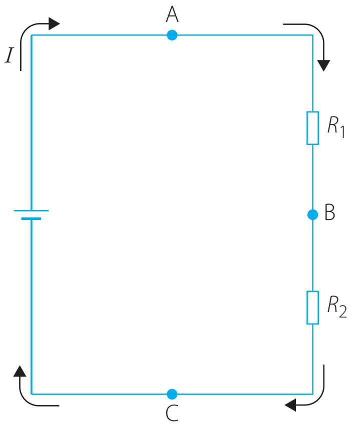


FIGURE 13.18 A simple series circuit containing a battery and two resistors

Kirchhoff's voltage law – conservation of energy

Figure 13.18 shows a simple single loop circuit. This sort of circuit is called a **series circuit** because the components are all connected in series, one after another.

In this circuit, current will flow from the positive terminal of the battery (the longer line), through the resistors and back to the battery through the negative (shorter) terminal. The current passes through points A, B and C in sequence. (Remember the convention that the direction of the current is the direction of flow of positive charges. Electrons will move in the opposite direction.)

Imagine a positively charged particle moving around the loop, starting at point A. As it passes through the first resistor between A and B, it gives up energy to the resistor, which is converted to heat.

The charged particles move at constant speed through the circuit – they do not get faster and slower. It is the potential energy of the particles that changes as they move around the loop. As the particle moves through the resistor, its potential energy decreases.

Recall that potential is potential energy per unit charge. As the particle moves through the resistor, its potential energy decreases – it has moved through a *potential difference*. From Ohm's law, this potential difference is $V = IR$. This potential difference is negative if we measure it from A to B.

As the charged particle continues from B to C, it passes through another resistor, again losing potential energy as it passes through a second potential difference.

From point C, the charged particle flows through the battery, gaining potential energy. As the change in potential energy is positive, so too is the potential difference across the battery from C to A.

When the charged particle is back at point A, it must have the same kinetic and potential energies as it had when it started. Hence, whatever decreases in potential energy it had from passing through the resistors must be equal to the potential energy gained by passing through the battery.

In circuits, we typically work in terms of potential differences (voltages) rather than potential energies. If the total change in potential energy around the loop is zero, then so is the total potential difference.

$$\Delta V_{\text{total}} = \sum V = V_{\text{battery}} - V_{R_1} - V_{R_2} = 0$$

Note that here we are using the symbol V to mean a potential difference. You may also see this written as ΔV sometimes.

This says that the sum over all the changes in potential difference around a loop must be equal to zero. This statement is generally true for any loop, and is known as Kirchhoff's loop law or **Kirchhoff's voltage law**. It is really just a statement of conservation of energy for the special case of an electric circuit.

The loop law

Use this animation to explore the loop law.

WORKED EXAMPLE 13.8

Consider the circuit shown in Figure 13.18. The battery supplies a potential difference of 12 V. A voltmeter is used to measure the potential difference across the first resistor (R_1), which is found to be 3 V. What is the potential difference across R_2?

ANSWER	LOGIC
$V_{\text{battery}} = 12\text{ V};\ V_{R_1} = 3\text{ V}$	▪ Identify the relevant data in the question.
$\sum V = 0$ $V_{\text{battery}} - V_{R_1} - V_{R_2} = 0$	▪ Write Kirchhoff's loop law to relate the voltages around the loop (from A to B to C to A).
$V_{R_2} = V_{\text{battery}} - V_{R_1}$	▪ Rearrange for V_{R_2}.
$= 12\text{ V} - (3\text{ V})$	▪ Substitute the known values, with units, into the formula.
$V_{R_2} = 9\text{ V}$	▪ Calculate the final answer and state with appropriate significant figures and units.

TRY THIS YOURSELF

Three identical resistors are connected in series to a 12 V battery. The same current flows through each resistor. Calculate the potential difference across each resistor.

Kirchhoff's current law – conservation of charge

Charge is a conserved quantity, and the total charge in a closed system is constant. As charge does not enter or leave a circuit, but only flows around within it, the total charge in a circuit is constant.

Figure 13.19a shows another circuit with two resistors, this time connected in parallel to a battery. This is called a **parallel circuit** or multi-loop circuit. Figure 13.19b shows the top section of this circuit, where the current flows from A to the junction between A, B and C.

A current I_A flows through point A and to the junction between A, B and C. At the junction, the current splits. Recall that current is charge per unit time, C s^{-1}. In any given second, an amount of charge equal to $1\text{ s} \times I_A$ enters the junction. Charge does not accumulate in junctions, nor is it used up or lost. However much charge enters, the same amount must flow out. Hence, the total current *into* a junction must be equal to the total current *out*. If we take current flowing in to have a positive sign, and current flowing out to have a negative sign, then:

$$\sum I = 0$$

This is called **Kirchhoff's current law** or junction law. It is really just a statement of conservation of charge.

In the case of the circuit in Figure 13.19, $I_A - I_B - I_C = 0$, or $I_A = I_B + I_C$.

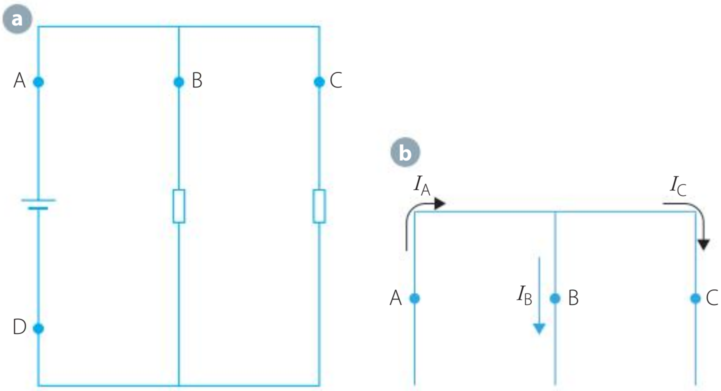


FIGURE 13.19 **a** A parallel (multi-loop) circuit. **b** The top section of the circuit, showing that the current splits into two different paths at the junction between A, B and C.

WORKED EXAMPLE 13.9

The current at point A in Figure 13.19 is 2.0 A. Three times as much current passes through B as through C. Calculate the currents at B, C and D.

ANSWER	**LOGIC**
$I_A = 2.0\text{ A}; I_B = 3I_C$	▪ Identify the relevant data in the question.
$\Sigma I = 0$ or $\Sigma I_{in} = \Sigma I_{out}$ $I_A = I_B + I_C$ $I_A = 3I_C + I_C = 4I_C$ $I_C = \frac{I_A}{4}$ $I_C = \frac{2.0\text{ A}}{4}$ $I_C = 0.5\text{ A}$	▪ Write Kirchhoff's current law to relate the currents in the different sections. ▪ Substitute the expression for I_B in terms of I_C. ▪ Rearrange for I_C. ▪ Substitute known values, with units, into the formula. ▪ Calculate the final answer and state with appropriate significant figures and units.
$I_B = 3I_C$ $I_B = 3(0.5\text{ A})$ $I_B = 1.5\text{ A}$ (At this point, we should quickly check that the sum of the currents out does equal the current in. If not, we have made a mistake and need to check our working!)	▪ Relate I_B to I_C. ▪ Substitute known values, with units, into the formula. ▪ Calculate the final answer and state with appropriate significant figures and units.
$I_D = I_B + I_C = I_A$ $I_D = 2.0\text{ A}$	▪ Recognise that there is another junction between B and C, leading to D, and apply Kirchhoff's current law. ▪ Substitute known values, with units, into the formula and calculate the answer.

TRY THIS YOURSELF

If the current at A was 1.0 A and the current through B was 0.7 A, what currents would be flowing through C and D?

Kirchhoff's current law is very useful for analysing circuits. Charge does not accumulate in batteries, resistors or wires, nor is it 'used up'. So Kirchhoff's current law applies not only to junctions but also to components and groups of components. For example, consider the series circuit shown in Figure 13.18 (page 388). The current at point A must be the same as the current at point B, because it only has one possible path and does not accumulate in a resistor. Similarly, the current at C must be the same as at B. Any point in a single loop circuit has the same current flowing through it. *The current is constant around the loop* – it is not 'used up' as it goes through components.

Series and parallel circuits

There are two main types of circuits. Series circuits have only one path that the charge can flow through. Parallel circuits have multiple paths that current can flow through.

Resistors in series

It is often useful to know the equivalent resistance to a group of components. This will allow us to calculate, for example, the current that will be drawn from a power supply.

So, for the series circuit in Figure 13.18, I is the same through all resistors. The total voltage across the two resistors is:

$$V_{\text{total}} = V_{R_1} + V_{R_2}$$

If we divide this through by the current, we get:

$$\frac{V_{\text{total}}}{I} = \frac{V_{R_1}}{I} + \frac{V_{R_2}}{I}$$

Using Ohm's law, $R = \frac{V_R}{I}$, we can write this as:

$$R_{\text{total}} = R_1 + R_2$$

R_{total} is the total resistance of the combination of the two resistors in series, and is the sum of the two resistors. We have derived this for the case of two resistors, but it applies to any number of resistors in series.

In general:

$$R_{\text{total}} = R_1 + R_2 + R_3 + \ldots + R_n$$

Once we know the total resistance, we can apply Ohm's law to the entire combination of resistors:

$$R_{\text{total}} = \frac{V_{\text{total}}}{I}$$

The more resistors there are added to a series circuit, the higher the total resistance is, and the lower the current.

WORKED EXAMPLE 13.10

Figure 13.20 shows a 6 V battery connected to two resistors in series, $R_1 = 8\ \Omega$; $R_2 = 4\ \Omega$.

1 Calculate the total resistance of the load (the two resistors).

2 Calculate the current through the circuit.

3 Calculate the potential difference across each resistor.

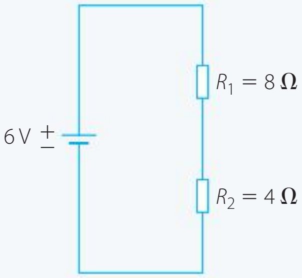


FIGURE 13.20 A load consisting of two resistors in series connected to a battery

ANSWER	LOGIC
$V_{\text{total}} = V_{\text{battery}} = 6\text{ V}; R_1 = 8\ \Omega; R_2 = 4\ \Omega$	▪ Identify the relevant data in the question.
1 $R_{\text{total}} = R_1 + R_2$	▪ Identify the appropriate formula to calculate the total resistance.
$R_{\text{total}} = 8\ \Omega + 4\ \Omega$	▪ Substitute the known values, with units, into the formula.
$R_{\text{total}} = 12\ \Omega$	▪ Calculate the final answer and state with appropriate significant figures and units.
2 $R_{\text{total}} = \frac{V_{\text{total}}}{I}$	▪ Write Ohm's law for the total load.
$I = \frac{V_{\text{total}}}{R_{\text{total}}}$	▪ Rearrange for current.
$I = \frac{6\text{ V}}{12\ \Omega}$	▪ Substitute the known values, with units, into the formula.
$= 0.5\text{ V }\Omega^{-1}$	▪ Calculate the answer.
$I = 0.5\text{ A}$	▪ State the final answer with appropriate significant figures and units.

3 $R_1 = \frac{V_1}{I}$ $V_1 = IR_1$ $V_1 = (0.5\ \text{A})(8\ \Omega)$ $V_1 = 4\ \text{A}\ \Omega$ $V_1 = 4\ \text{V}$	▪ Write Ohm's law for $R_{1.}$ ▪ Rearrange for potential difference across $R_{1.}$ ▪ Substitute the known values, with units, into the formula. ▪ Calculate the answer. ▪ State the final answer with appropriate significant figures and units.
We can either apply Ohm's law to the second resistor to find V_2 or we can apply Kirchhoff's voltage law. Applying Kirchhoff's voltage law: $V_{\text{battery}} = V_{R_1} + V_{R_2}$ $V_{R_2} = V_{\text{battery}} - V_{R_1}$ $V_{R_2} = 6\ \text{V} - 4\ \text{V}$ $V_{R_2} = 2\ \text{V}$	▪ Write Kirchhoff's voltage law for this loop ▪ Rearrange for V_{R_2}. ▪ Substitute the known values, with units, into the formula. ▪ Calculate the final answer and state with appropriate significant figures and units.

TRY THESE YOURSELF

1 Calculate V_{R_2} using Ohm's law instead, and check that you get the same answer.

2 a What voltage power supply would be needed to create a current flow of 1.0 A in this circuit?

b What would be the potential difference across each resistor in this case?

Resistors in parallel

When resistors are connected in parallel, as in Figure 13.19 (page 389), the voltage across each must be the same. Figure 13.19 can be thought of as two loops: one with the battery and the first resistor; the second with the battery and the second resistor. Figure 13.19 is redrawn in Figure 13.21, highlighting these two loops. If we apply Kirchhoff's voltage law to the first loop, we get:

$$V_{\text{battery}} = -V_{R_1}$$

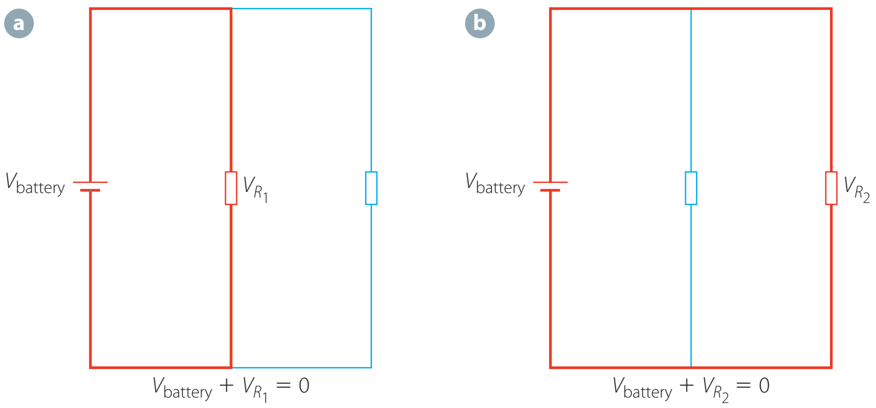
If we apply Kirchhoff's voltage law to the second loop, we get:

$$V_{\text{battery}} = -V_{R_2}$$

so,

$$V_{R_1} = V_{R_2} = -V_{\text{battery}}$$

FIGURE 13.21 The circuit shown in Figure 13.19 showing: **a** the first loop, with $V_{\text{battery}} = -V_{R_1}$; **b** the second loop, with $V_{\text{battery}} = -V_{R_2}$



In general, any parallel sections of a circuit (sections that connect across the same two points) must have the same potential difference across them. This is why we always connect voltmeters in parallel with the component we wish to measure the voltage across. The voltmeter then has the same potential across it as the component.

Figure 13.22 shows the same circuit as Figure 13.19 and Figure 13.21 again, but with voltmeters connected across each resistor and the currents in and out of the junction between the battery and the resistors.

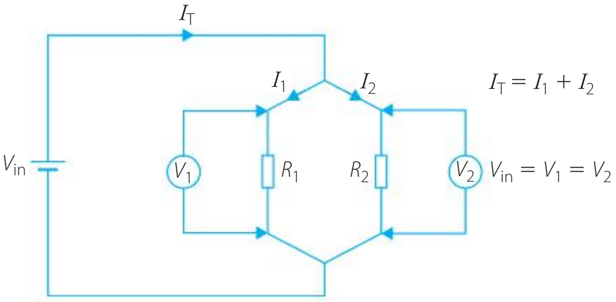


FIGURE 13.22 Parallel circuit with two resistors, as in Figure 13.19, showing voltages and currents

We know from Kirchhoff's laws that for this parallel circuit:

$$I_{total} = I_1 + I_2$$

and

$$V_{in} = V_1 = V_2$$

Using Ohm's law, the current through each resistor is given by:

$$I = \frac{V_R}{R}$$

Putting this into $I_{total} = I_1 + I_2$ gives:

$$\frac{V_{total}}{R_{total}} = \frac{V_{R_1}}{R_1} + \frac{V_{R_2}}{R_2}$$

where R_{total} is the total equivalent resistance of the parallel resistors, and V_{total} is the potential difference across the parallel resistors.

As $V_{in} = V_1 = V_2 = V_{total}$, we get

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Electricity multiple choice

As with resistors in series, we have derived this for the case of two resistors. However, it is generally true for any number of resistors in parallel:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$

WORKED EXAMPLE 13.11

For the circuit shown in Figure 13.23, calculate:

1. the total equivalent resistance for the load (two parallel resistors).
2. the current through the battery.
3. the current through each resistor.

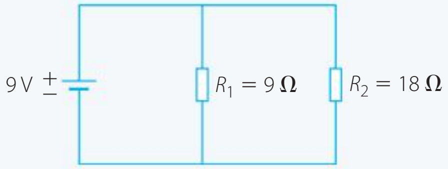


FIGURE 13.23 A circuit with two parallel resistors

ANSWER	LOGIC
$V_{\text{battery}} = 9\ \text{V}; R_1 = 9\ \Omega; R_2 = 18\ \Omega$	▪ Identify the relevant data in the question.
1 $\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2}$	▪ Write the expression for total resistance.
$\frac{1}{R_{\text{total}}} = \frac{1}{9\ \Omega} + \frac{1}{18\ \Omega}$	▪ Substitute the known values, with units, into the formula.
$= \frac{2}{18\ \Omega} + \frac{1}{18\ \Omega}$	▪ Rewrite the fractions with a common denominator.
$= \frac{3}{18\ \Omega}$	▪ Add the fractions.
$R_{\text{total}} = \frac{18\ \Omega}{3} = 6\ \Omega$	▪ Rearrange for $R_{\text{total.}}$
Note that in this case we have substituted values before rearranging. It is very rare that this is an appropriate thing to do, but in this case it makes the solution much faster.	
2 $R_{\text{total}} = \frac{V_{\text{total}}}{I} = \frac{V_{\text{battery}}}{I}$	▪ Write Ohm's law for the total load.
$I = \frac{V_{\text{battery}}}{R_{\text{total}}}$	▪ Rearrange for current.
$I = \frac{9\ \text{V}}{6\ \Omega}$	▪ Substitute the known values, with units, into the formula.
$I = 1.5\ \text{V}\ \Omega^{-1}$	▪ Calculate the answer.
$I = 1.5\ \text{A}$	▪ State the final answer with appropriate significant figures and units.
3 $R_1 = \frac{V_{\text{battery}}}{I_1}$	▪ Write Ohm's law for $R_{1.}$
$I_1 = \frac{V_{\text{battery}}}{R_1}$	▪ Rearrange for current through $R_{1.}$
$I_1 = \frac{9\ \text{V}}{9\ \Omega}$	▪ Substitute the known values, with units, into the formula.
$I_1 = 1\ \text{V}\ \Omega^{-1}$	▪ Calculate the answer.
$I_1 = 1\ \text{A}$	▪ State the final answer with appropriate significant figures and units.
We can either apply Ohm's law to the second resistor to find I_2 or we can apply Kirchhoff's current law. Applying Kirchhoff's current law: $I_T = I_1 + I_2$	▪ Write Kirchhoff's voltage law for this loop.
$I_2 = I_T - I_1$	▪ Rearrange for I_2.
$I_2 = 1.5\ \text{A} - 1\ \text{A}$	▪ Substitute the known values, with units, into the formula.
$I_2 = 0.5\ \text{A}$	▪ Calculate the final answer and state with appropriate significant figures and units.

TRY THIS YOURSELF

Repeat Worked example 13.11 with a third resistor in parallel with the first two. The third resistor has resistance 6 Ω. Begin by drawing a circuit diagram.

Notice that in Worked example 13.11, the total resistance of the parallel combination of resistors was less than either of the individual resistors. This is always the case, because the more paths there are for current to flow along, the more current there will be. The current will also divide in inverse proportion to the resistance of the paths. Current will flow in every path, but the path of least resistance will have the greatest current.

Series and parallel circuits

Find out more about series and parallel circuits, and follow the link to the interactive circuit builder activities

INVESTIGATION 13.5

Parallel and series circuits

Critical and creative thinking

Numeracy

Information and communication technology capability

In this investigation, you will perform two experiments. In the first, you make qualitative observations of light globes connected in series and parallel. In the second, you will make quantitative measurements for resistors in series and parallel.

AIM

To investigate light globes and resistors connected in series and parallel

Write a hypothesis for each part of the investigation. For the second part, your hypothesis should be quantitative.

MATERIALS

- 12 V DC power supply
- 3 × 12 V globes in holders
- 10 × 100 Ω resistors
- Ammeter
- Voltmeter
- Wires and connectors

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?

RISK ASSESSMENT

Identify the risks associated with your investigation, and state how you can manage them.

Part 1: globes in series and parallel

METHOD

Remember to write your hypothesis *before* you begin to make observations.

1 Connect the circuit shown in Figure 13.24.

2 Once your teacher has checked your circuit, turn on the power supply. Write down what you observe.

3 Connect the circuit shown in Figure 13.25.

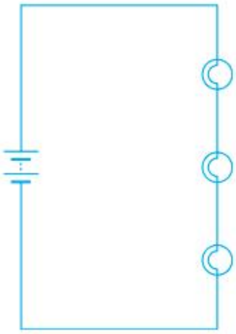


FIGURE 13.24 Globes connected in series

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4 Once your teacher has checked your circuit, turn on the power supply. Write down what you observe now.

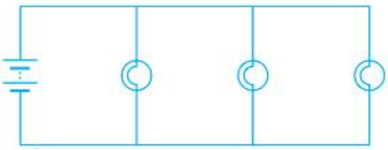


FIGURE 13.25 Globes connected in series

RESULTS

Record your observations, and include sketches of your circuits.

ANALYSIS OF RESULTS

1 For which circuit were the globes brighter?

2 In either circuit, were some globes brighter than others?

DISCUSSION

Did your results support or disprove your hypothesis? Explain why.

Part 2: resistors in series and parallel

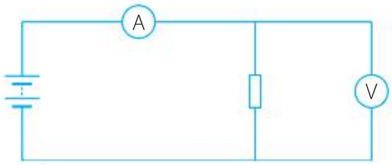


FIGURE 13.26 Starting circuit for part 2

METHOD

1 Connect the circuit shown in Figure 13.26. Ask your teacher to check your circuit before turning on the power supply.

2 Turn on the power supply and measure the current flowing from the power supply and the potential difference across the (first) resistor.

3 Turn off the power supply.

4 Add another resistor in series with the first. Leave the ammeter and voltmeter where they are so you continue to measure current from the power supply and voltage across the first resistor.

5 Repeat steps 2–4, each time recording the current through the circuit and the voltage across the first resistor.

6 When you have used all your resistors, take all but one out of the circuit and return your circuit to that shown in Figure 13.26.

7 Repeat the experiment above, but this time connect the additional resistors in parallel with the first.

RESULTS

Record your results in a table, either on paper or in a spreadsheet. Your tables should look like the following, but with more rows.

Resistors in series

NUMBER OF RESISTORS	CURRENT (mA)	V_{R_1} (V)	R_{TOTAL} (Ω)
1			
2			

Resistors in parallel

NUMBER OF RESISTORS	CURRENT (mA)	V_{R_1} (V)	R_{TOTAL} (Ω)
1			
2			

ANALYSIS OF RESULTS

1 Calculate the total resistance of your circuit using Ohm's law with the measured current and the power supply voltage (12 V) to complete the table.

2 Compare your calculated values for total resistance from your measured voltages and currents with what you expect from using the expressions for series and parallel resistors.

$$\frac{1}{R_{\text{parallel}}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\ldots \text{ and } R_{\text{series}}=R_1+R_2+R_3+\ldots$$

3 Use spreadsheet software to graph your data. Draw a scatter plot of current as a function of number of resistors for the resistors in series data. Is it a straight line? Is it the shape you expected it to be?

4 Plot a linear graph using your data. You will need to think about how you expect current to vary with number of resistors.

5 Draw a scatter plot of current as a function of number of resistors for the resistors in parallel data. Is this a straight line? If so, fit a line to your graph and display the equation on the graph. Does it match what you predicted?

CONCLUSION

Write a conclusion to summarise your findings. State whether your hypotheses were supported.

KEY CONCEPTS

- Circuit diagrams are used to show how the various components in a circuit are connected. Each different type of component is represented by a different symbol.
- Kirchhoff's voltage law says that the sum of potential differences around a loop is zero: $\Sigma V=0$. This is a result of conservation of energy.
- Kirchhoff's current law says that the total current entering a junction is zero: $\Sigma I=0$ or the total current in must equal the total current out. This is a result of conservation of charge.
- The current in a single-loop circuit must be the same at all points in the loop. Current is not 'used up' in resistors or other components.
- When resistors are connected in series (one after another, end to end), the total resistance is the sum of the resistances: $R_{\text{total}}=R_1+R_2+R_3+\ldots+R_n$.
- Resistors in series all have the same current through them.
- When resistors are connected in parallel (side by side), the total resistance is found from:
  $$\frac{1}{R_{\text{total}}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\ldots+\frac{1}{R_n}.$$
- Resistors in parallel all have the same potential difference across them.

Revision

CHECK YOUR UNDERSTANDING

13.4

1 Which has less equivalent resistance: two 5.0 Ω resistors in series or two 5.0 Ω resistors in parallel? Compare the total current when connected to a battery of voltage V.

2 Figure 13.27 shows three ammeters A, B and C in three wires as part of a continuous circuit. If ammeter A reads 1.40 A and ammeter B reads 0.50 A, what is the reading on ammeter C?

3 Figure 13.28 shows a series circuit with two resistors. If $V_1=4.2$ V what is the reading on V_2?

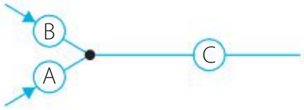


FIGURE 13.27 A junction in a circuit

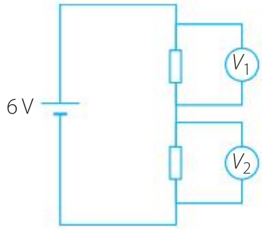


FIGURE 13.28 A series circuit with two resistors

4 Four different combinations of resistors are shown in Figure 13.29. Calculate the total or equivalent resistance of each combination.

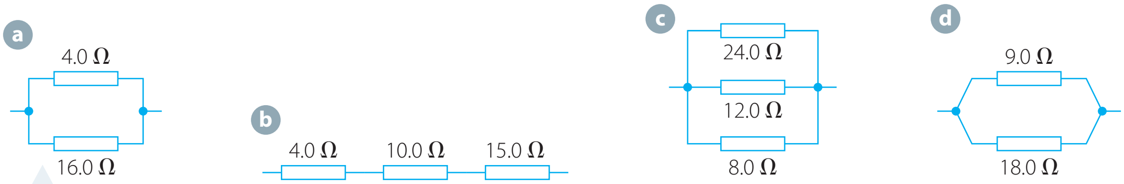


FIGURE 13.29 What is the total resistance of each of these combinations?

5 Two 12.0 V resistors and one 6.0 V resistor are connected in series across a 6.0 V battery.

- **a** Draw a circuit diagram to show this arrangement.
- **b** What is the total resistance of the circuit?
- **c** What current flows through the 6.0 Ω resistor?
- **d** What is the potential drop across each resistor?

6 Voltmeters are always connected in parallel. Measuring devices (ammeters, voltmeters etc.) should interfere with a circuit as little as possible. Explain why voltmeters always have very high resistance.

7 Figure 13.30 shows a voltage divider. If $V_{in} = 12$ V, $V_{out} = 9$ V and $R_1 = 10\ \Omega$, calculate

- **a** V_1
- **b** R_2
- **c** the current in the circuit.

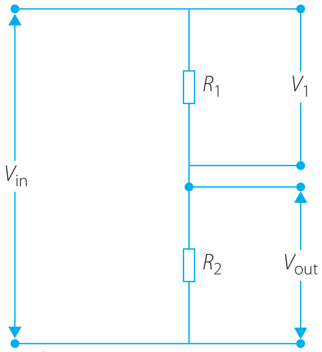


FIGURE 13.30 A voltage divider circuit

13 CHAPTER SUMMARY

- Current is the flow of charged particles, called charge carriers.
- Charge carriers can be positive or negative. In electric circuits, they are usually electrons, which have a negative charge.
- Current is the charge per unit time passing through a point: $I = \frac{q}{t}$, measured in units of A; 1 A = 1 C s^{-1}.
- The direction of a current is the direction that positive charge carriers would be flowing if the current was carried by positive charge carriers.
- Metals are good conductors because they have many free electrons (charge carriers).
- Ammeters are used to measure current. Ammeters must be connected in series in a circuit.
- The potential difference (also called voltage), *V*, between two points is the work per unit charge that is done on a charged particle when it moves between those two points. $V = \frac{W}{q}$ measured in units of volts; 1 V = 1 J C^{-1}.
- A voltmeter measures the potential difference between two points. Voltmeters are connected in parallel with circuit components.
- Resistance, *R*, is defined as the ratio of voltage across to current through a component: $R = \frac{V}{I}$ (Ohm's law).
- For ohmic components, the resistance is constant for a wide range of potential difference. For non-ohmic components the resistance varies with potential difference.
- *I*–*V* graphs are used to represent the current–voltage characteristics of a component, and allow the resistance to be calculated for any applied voltage.
- A battery is a source of potential energy. As charged particles move through a battery, their potential energy increases.
- When charged particles flow through other components (such as light globes, resistors and motors), their potential energy decreases. The potential energy is transformed into other forms including light, heat and kinetic energy.
- The rate at which energy is transformed is the power, given by $P = \frac{E}{t} = VI = I^2R = \frac{V^2}{R}$. Power has units of W; 1 W = 1 J s^{-1} = 1 V A.
- Circuit diagrams are used to show how the various components in a circuit are connected.
- Kirchhoff's voltage law (conservation of energy) says that the sum of potential differences around a loop is zero: $\sum V = 0$.
- Kirchhoff's current law (conservation of charge) says that the total current entering a junction is zero: $\sum I = 0$, or total current in = total current out.
- The current in a single-loop circuit must be the same at all points in the loop. Current is not 'used up' in resistors or other components.
- When resistors are connected in series (one after another, end to end): $R_{\text{total}} = R_1 + R_2 + R_3 + \ldots + R_n$.
- Resistors in series all have the same current through them.
- When resistors are connected in parallel (side by side): $\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$
- Resistors in parallel all have the same potential difference across them.

13 CHAPTER REVIEW QUESTIONS

Review quiz

1 List some of the forms of energy into which electric potential energy can be converted.

2 How much energy does a 12.0 V car battery supply to every coulomb of charge that passes through it?

3 How much energy does a 12.0 V car battery supply to every electron that passes through it?

4 What is the potential difference across a 220 mΩ resistor if it carries a current of 4.0 A?

5 A current of 0.50 A flows for 10 minutes in a conductor. Calculate the number of coulombs of charge that pass a given point in the conductor.

6 People often 'charge' their phones when their battery is running low. Are they adding more charge to their phones, or is another process taking place? Describe what is happening.

7 What is the difference between an ohmic and a non-ohmic resistor?

8 Calculate the potential difference of a battery that supplies 960 J of energy to every 80.0 C of charge that passes through the battery.

9 Calculate the potential difference of a battery that supplies 1.92×10^{-18} J of energy to every electron that passes through the battery.

10 How much charge passes through a load if a current of 4.0 A flows for 5.0 s?

11 When a potential difference of 16 V is applied across the ends of a wire, the current flowing in the wire is 2.4 A. Assume the wire is ohmic.

- **a** What is the resistance of the wire?
- **b** What potential difference is needed to make a current of 3.0 A flow through the wire?

12 What is the potential difference across a 1.8 kΩ resistor in which a current of 240 mA flows?

13 The reading of voltmeter V_1 in Figure 13.31 is 5 V. What is the reading of V_2?

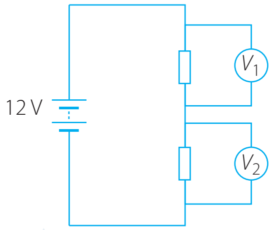


FIGURE 13.31 Voltmeters measuring potential difference across two resistors in series

14 Figure 13.32 shows a parallel circuit. The current through ammeter A_1 is 100 mA and the current through A_2 is 25 mA. What is the current through A_3?

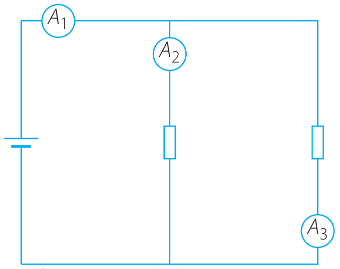


FIGURE 13.32 Measuring current in a parallel circuit

15 What is the size and direction of the current I in Figure 13.33?

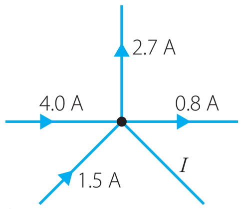


FIGURE 13.33 A junction in a circuit

16 A current of 2.0 A flows through a battery when a light globe is connected across its terminals. The potential difference across the terminals is 6.0 V.

- **a** What is the quantity of electrical charge that flows in the globe each second?
- **b** How much energy is given to each coulomb of charge that passes through the battery?
- **c** How long does it take the battery to supply 480 J of energy?
- **d** How many coulombs of charge will pass through the battery in this time?

17 Currents I_1 and I_2 respectively flow in resistances R_1 and R_2 placed in series across a potential difference V. Show that the potential difference is shared in the same ratio as the resistances: $\frac{V_1}{V_2} = \frac{R_1}{R_2}$.

18 Currents I_1 and I_2 respectively flow in resistances R_1 and R_2 placed in parallel across a potential difference V. Show that the current is shared in inverse ratio to the resistances: $\frac{I_1}{I_2} = \frac{R_2}{R_1}$

19 A student has three resistors: 50 Ω, 100 Ω and 150 Ω.

- **a** If the student arranges the resistors in series, what is the total resistance?
- **b** What is the equivalent resistance if the students arranges them in parallel?

20 Three 10 Ω resistors and one 20 Ω resistor are joined in series across a 10.0 V supply. What is the potential difference across each? Start by drawing a circuit diagram.

21 Three 10 Ω resistors and one 20 Ω resistor are joined in parallel across a 10.0 V supply. What is the current through each? Start by drawing a circuit diagram.

22 Two globes drawing 50 W each are connected in series across a 10.0 V supply. Each globe has a resistance of 0.5 Ω.

- **a** Calculate the potential difference across one globe.
- **b** Calculate the current through both globes.

23 Two 1 kΩ resistors are placed in series with a 9 V battery, an ammeter and an unknown resistor, *R*. The ammeter reads 1.5 mA.

- **a** What is the resistance of the resistor, *R*? Show your working, including a circuit diagram.
- **b** What would the ammeter read if it were placed in parallel with one of the 1 kΩ resistors? Give a detailed explanation.

24 Household globes, each 100 W, are connected to each other in parallel array, and then this arrangement is connected in series to a power supply with a 7.5 A fuse of negligible resistance. When 7.5 A or more flows in the fuse, it melts and breaks the circuit. The power supply can be considered to be 240 V DC.

- **a** What is the current in one 100 W globe?
- **b** If three 100 W globes are turned on, what is the current in the fuse?
- **c** How many globes can be turned on before the fuse melts?

25 A combination series and parallel circuit is shown in Figure 13.34.

- **a** Calculate the effective resistance of the circuit. Hint: start by calculating the equivalent resistance of the parallel section, and modelling it as a single resistor in series with the other components.
- **b** Find the current in each resistor.
- **c** Calculate the potential difference across each resistor.

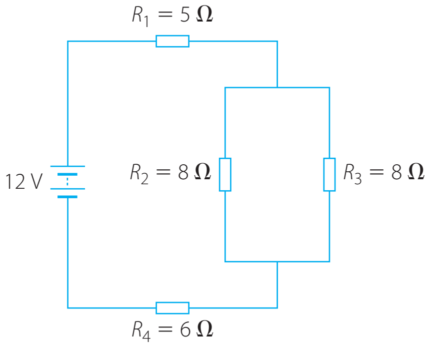


FIGURE 13.34 A combination series and parallel circuit

14 Magnetism

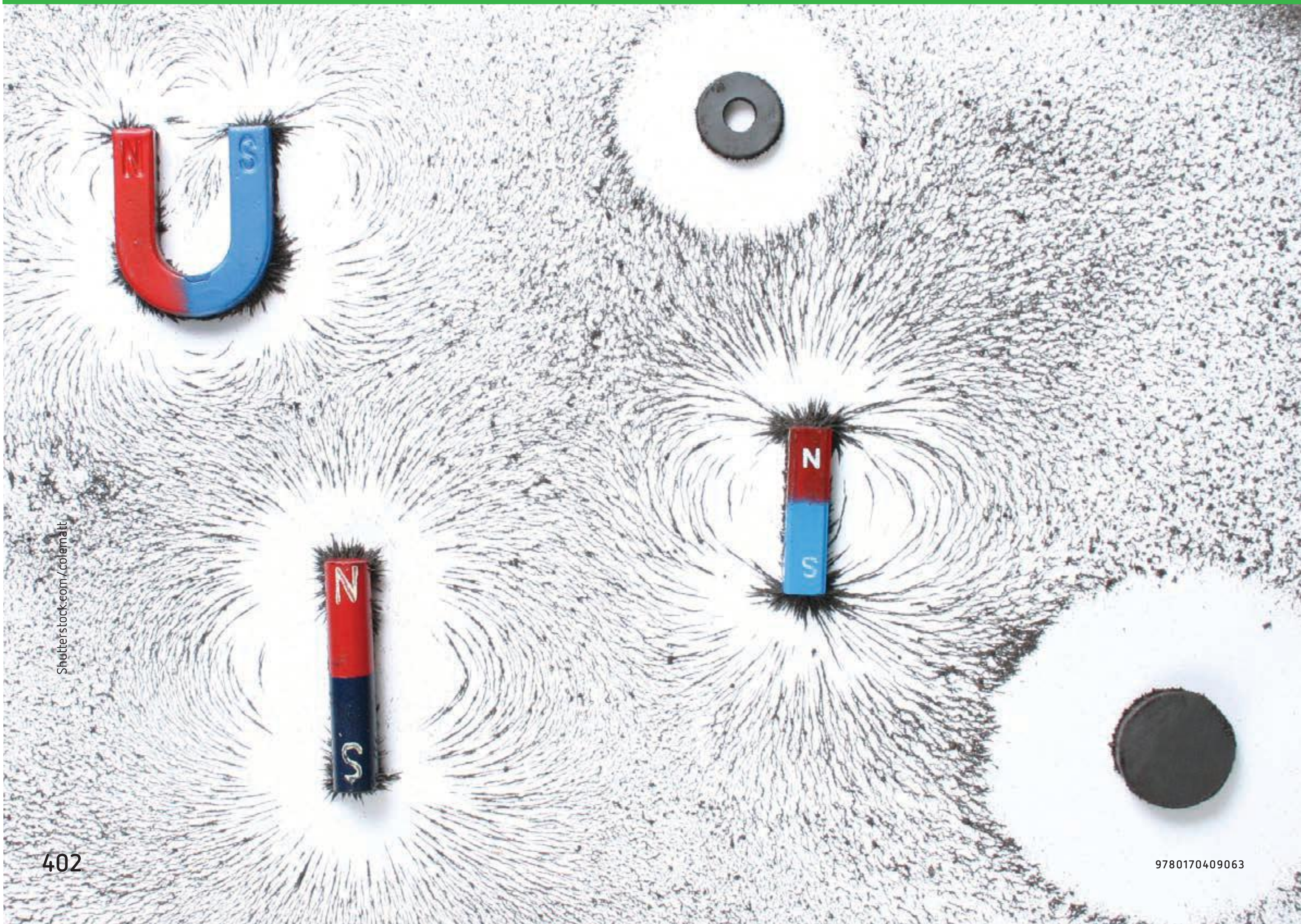
INQUIRY QUESTION

How do magnetised and magnetic objects interact?

OUTCOMES

Students:

- investigate and describe qualitatively the force produced between magnetised and magnetic materials in the context of ferromagnetic materials (ACSPH079)
- use magnetic field lines to model qualitatively the direction and strength of magnetic fields produced by magnets, current-carrying wires and solenoids and relate these fields to their effect on magnetic materials that are placed within them (ACSPH083) ICT
- conduct investigations into and describe quantitatively the magnetic fields produced by wires and solenoids, including: (ACSPH106, ACSPH107)
  - $B = \frac{\mu_0 I}{2\pi r}$ ICT N
  - $B = \frac{\mu_0 NI}{L}$ ICT N
- investigate and explain the process by which ferromagnetic materials become magnetised (ACSPH083)
- apply models to represent qualitatively and describe quantitatively features of magnetic fields ICT N



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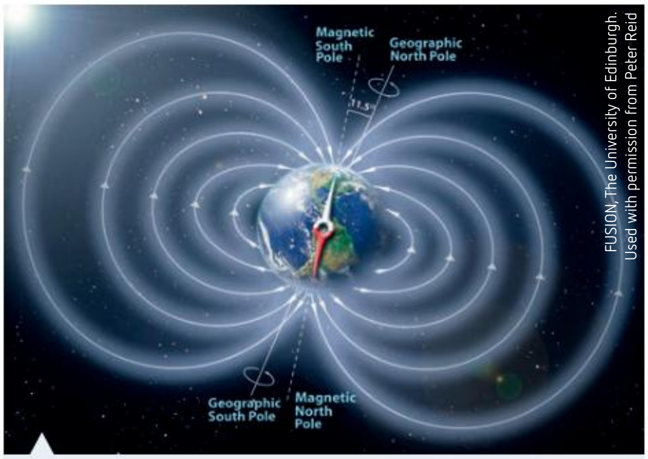
Gravitational fields are created by objects with mass (chapter 4), and electric fields (chapter 12) are created by objects with charge. Magnetic fields are created by moving charged particles, such as the movement of electrons in a current. Some materials also create magnetic fields due to the behaviour of their electrons. Recall that a field is a way for a force to act between objects that are not in contact.

Magnetic fields are important to us in many ways. Earth has a magnetic field that protects us from cosmic rays from the Sun (Figure 14.1). Magnetic materials are used in many devices, such as transformers and USB drives. In this chapter, we will describe the characteristics of magnetic fields and examine how magnetised and magnetic objects interact.



FUSION, The University of Edinburgh. Used with permission from Peter Reid

FIGURE 14.1 Earth's magnetosphere

14.1 Magnetism and magnetic materials

Magnetic fields are created by moving charges, and exert forces on moving charges. Magnets create magnetic fields and experience forces due to magnetic fields because of the moving charged particles (electrons) that they contain.

The magnetic field is given the symbol B in physics, and is measured in units of tesla, T. The tesla is a very large unit, and most everyday fields are measured in mT or μT. For example, the magnetic field due to Earth varies between about 30 μT and 60 μT on Earth's surface, depending upon where you are. Strong permanent magnets made of neodymium alloys can produce a field of around 1 T, and fields of nearly 100 T can be produced using high-power and high-tech equipment in laboratories.

The words magnet and magnetism come from the word 'magnetite', an iron ore (Fe_3O_4) that attracts iron. Magnetite was observed to attract iron by the ancient Greeks around 3000 years ago. One legend says that the material magnetite was named after the shepherd Magnes. The iron nails of his shoes reportedly stuck to pieces of magnetite as he pastured his flocks.

We call materials that can be magnetised **ferromagnetic**, where *ferro* means iron. Most magnetic materials in use today still contain iron. When a material is magnetised, it creates its own magnetic field. Most materials are not ferromagnetic and cannot be magnetised.

Ferromagnetic materials, when magnetised, are attracted into a magnetic field. You will have noticed this if you have ever played with a magnet and used it to pick up pins or paper clips. As described later, the presence of a magnetic field is what magnetises the material. Once magnetised, the material experiences a force due to the field.

Hyperphysics: ferromagnetism

Read more about ferromagnetism and follow the links to find out more.

WS

Ferromagnets

INVESTIGATION 14.1

Critical and creative thinking

Identifying magnetic materials

Ferromagnetic materials are attracted into magnetic fields. Can you predict what sort of materials in your classroom will be ferromagnetic?

AIM

To identify some ferromagnetic materials

MATERIALS

- Strong magnet
- Paper clips
- Coins
- Iron filings in a transparent container
- Other magnets including fridge magnets
- Samples of other materials of different types including plastics, copper, aluminium

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Magnets can become demagnetised by being dropped or banged together a lot.	Avoid dropping the magnets or allowing them to bang together.
A pair of strong magnets can pinch you.	Handle magnets one at a time.
Magnets can be toxic, and small magnets are a choking hazard.	Never put a magnet in your mouth and always wash your hands after any investigation.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Gather your sample materials and space them out on your desk.
2. Place the strong magnet close to the first sample.
3. Record how the sample responds.
4. Move the strong magnet away, reverse it, and place it close to the sample again. This time the opposite end of the magnet will be close to the sample.
5. Record how the sample responds.
6. Repeat steps 2–5 for each of your samples.

RESULTS

Make a table summarising your results as shown. Include your predictions. Write your prediction for each sample *before* you test it.

SAMPLE	PREDICTED BEHAVIOUR	OBSERVED BEHAVIOUR

ANALYSIS OF RESULTS

1. Can you find any patterns in your results?
2. Were all the metal samples attracted to the magnet?
3. Was it only metals that were attracted?
4. Did it matter which end of the magnet you used?

»

DISCUSSION

1. State if your results agree with what you would expect.
2. List two ways could you improve this experiment.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

In Investigation 14.1, you will have noticed that most materials are *not* ferromagnetic. It is the properties and behaviour of electrons within the material that give rise to ferromagnetism. You will know from chemistry that electrons orbit the nucleus in shells. This shell structure is important for understanding magnetic materials. Within shells, the electrons move in orbitals. If you are studying chemistry, you will be familiar with the names of these orbitals: 1s, 2s, 2p and so on. The shell structure changes when atoms bind together to form molecules.

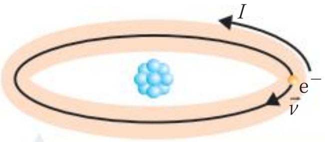


FIGURE 14.2 The electron orbiting the nucleus can be modelled as a current

Every electron in a material creates its own tiny magnetic field as it orbits a nucleus, because it is acting as a tiny circular current. This is shown in Figure 14.2. We shall see later in this chapter how currents create magnetic fields. When there are two electrons (paired electrons) in an orbital, they create opposite magnetic fields, which cancel out. Materials that only have paired electrons are not ferromagnetic.

Ferromagnetism arises from unpaired electrons in the shells.

Electrons also have a property known as **spin**. The electrons are probably not really spinning – this is just the name given to a property in quantum mechanics. However, they do have their own tiny magnetic field in addition to the field due to their orbital motion. A charged ball would have a magnetic field if it were spinning, so this property associated with a particle's magnetic field is called spin.

The spin of a particle is a measure of a particle's own magnetic field. Spin is a quantum mechanical property of particles, and is discussed in *Physics in Focus Year 12*.

Hence, electrons in materials have magnetic fields from two sources – their orbital motion and their spin. The total magnetic field from each electron is the sum of these two.

In materials where the electrons all have pairs, the magnetic fields due to each electron in a pair cancel out. Even in materials where there are unpaired electrons, their magnetic fields are randomly arranged and may end up all cancelling each other out.

In ferromagnetic materials, the fields due to the unpaired electrons tend to line up. Usually this happens only in small local regions. These small local regions of alignment are called magnetic domains. As the field in each domain may be different, the overall magnetic field is still zero or very small. However, if the local magnetic fields can be made to line up, the result is a material that has a large magnetic field – a magnet.

Creation of magnetic fields
Watch the animation and read about how orbiting electrons create magnetic fields.

Note that protons and neutrons in the nucleus also have spin, and their own magnetic fields, but the total magnetic field due to the nucleus is tiny compared to that due to electrons so it doesn't contribute to ferromagnetic behaviour. Nuclear magnetism is important in technologies such as nuclear magnetic resonance (NMR) imaging or magnetic resonance imaging (MRI).

When two magnets are brought close together, they exert a force on each other. This is how we know that the magnetic field must exist – there is a force between magnets that acts at a distance. Remember that the field model for forces, whether gravitational, electric or magnetic, is how we describe action at a distance.

Two magnets will either attract or repel each other. You may have noticed this in Investigation 14.1. This is because a magnet has two poles, which we call a north pole and south pole. Like poles repel, just as like charges do, so a north pole will repel another north pole. Unlike poles attract, so a north pole is attracted to a south pole and vice versa.

This naming convention arises from observations of the magnetic field of Earth (Figure 14.1, page 403). Compass needles can be observed to swing so that they point approximately towards the geographic North Pole of Earth. As it is the north pole of the compass needle that swings towards the geographic North Pole, this tells us that the geographic North Pole is a magnetic south pole. In fact, as you can see from Figure 14.1, the geographic and magnetic poles don't quite line up. The magnetic poles also move and occasionally reverse.

INVESTIGATION 14.2

Critical and creative thinking

Literacy

The force exerted between magnetised materials

AIM

To investigate the force between two magnets

MATERIALS

- 2 strong magnets with at least two parallel flat sides (e.g. 'button' or rectangular shaped magnets)
- Tape
- Sensitive kitchen scales (1 g precision)

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Magnets can be toxic and small magnets are a choking hazard.	Never put a magnet in your mouth and always wash your hands after any investigation.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Tape one of the magnets to the kitchen scale so that one pole is pointing upwards. You may need to observe the magnets first to determine which sides are the poles.
2. Record the reading on the scale.
3. Hold the second magnet at least 10 cm above the first, with one pole pointing directly down towards it (as in Figure 14.3).
4. Very slowly move the second magnet down and note how the scale reading changes – does it increase or decrease?
5. Note whether *you* can feel any force on the magnet in your hand. Does it appear to be pushing upwards or pulling downwards?
6. Record your observations.
7. Move the second magnet back up again, and reverse its direction.
8. Repeat steps 4–7.

FIGURE 14.3 Experimental set up for investigating the forces between two magnets

Refer to pages 96-7 to revise the rules on drawing force diagrams.

RESULTS

Record your observations. Draw a diagram showing the directions of the forces acting on each of the magnets.

»

ANALYSIS OF RESULTS

Draw a diagram showing the forces acting on the magnet taped to the scale when the second magnet was far away, and when the second magnet was close by. Make sure you include the normal force of the scale on the magnet.

DISCUSSION

By Newton's third law, the normal force exerted by the magnet on the scales is equal and opposite to the normal force the scales exert on the magnet. The normal force exerted on the scales is what it converts to a displayed mass reading. So, the apparent change in mass can be converted to a change in the normal force exerted on the magnet.

1 Apply Newton's second law to your diagram showing the forces on the magnet. Write an equation relating the forces acting, noting that the magnet is in equilibrium. (Hint: review Figure 4.16, page 103.)

2 Explain why the scale reading increased or decreased when the second magnet was moved close to it.

3 Identify the Newton's third-law force pair to the force exerted by the magnet in your hand on the magnet taped to the scale.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

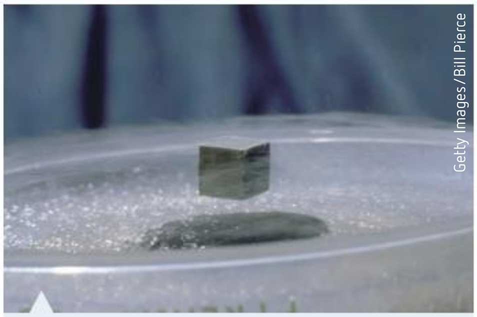

Getty Images/Bill Pierce

FIGURE 14.4 A magnet hovers over a disc of superconductor cooled with liquid nitrogen

Most materials are not ferromagnetic and cannot be magnetised to produce magnets. Most materials can be classified as either **diagmagnetic** or **paramagnetic**. Paramagnets are very, very weakly attracted into magnetic fields. Diamagnets are repelled by magnetic fields. Most diamagnets are only very, very weakly repelled. The behaviour of both paramagnetic and diamagnetic materials is hard to observe without specialised and highly sensitive equipment. An exception to this is a superconductor below its critical temperature. The critical temperature is the temperature at which it becomes superconducting. Below the critical temperature, a superconductor is a strong diamagnet and is repelled by a magnet. By Newton's third law, the magnet must also be repelled by the superconductor. This can be seen in Figure 14.4 where a magnet is levitating above a cooled superconductor. The downwards gravitational force is balanced by the upwards magnetic force.

The Meissner effect

A small magnet will levitate above a cooled superconductor because of the Meissner effect. Read about it and watch the video.

KEY CONCEPTS

- Ferromagnetic materials are materials that can be magnetised. Magnetised materials create magnetic fields.
- The magnetic fields are due to the motion of the electrons in the material.
- Most (but not all) ferromagnetic materials contain iron.

CHECK YOUR UNDERSTANDING

14.1

1 Define 'ferromagnetic'.

2 Distinguish between ferromagnetic, paramagnetic and diamagnetic materials.

3 a Explain how a fridge magnet can stick to a fridge when the fridge is not magnetised.
  b Draw a diagram showing all the forces acting on the magnet.
  c Identify the Newton's third-law reaction force to all the forces in your diagram.

4 Describe how you could make measurements to determine how the force that one magnet exerts on another varies with the distance between the magnets. What graph(s) would you plot?

14.2 Magnetic field lines

You learned in chapter 12 that field lines can be used to represent the strength and direction of an electric field. Recall that for an electric field, the direction of the field is the direction of the force exerted on a positive charge. The density of field lines is proportional to the strength of the field.

Magnetic field lines give the same sort of information for magnetic fields. Magnetic field lines show the direction of force acting on a magnetic north pole. The density of the magnetic field lines is proportional to the field strength. The field lines are a visual model of the field.

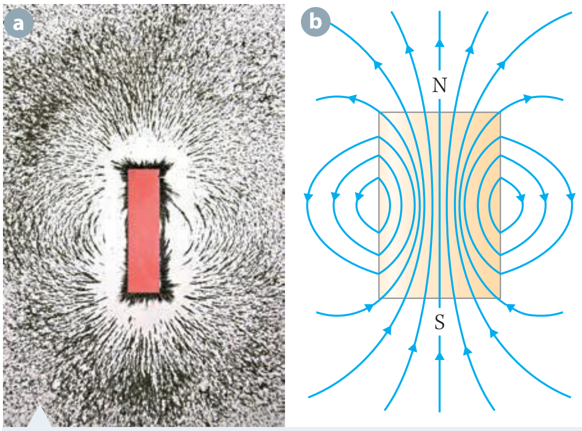


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FIGURE 14.5 **a** Iron filings align with the magnetic field around a bar magnet; **b** The magnetic field lines of the bar magnet

Remember what it feels like to push two magnets together with their north poles facing. The north pole of each is pushed away by the other – the force is away from the other magnet, so the magnetic field lines point out from a north pole. However, if you turn one magnet around, the north pole of one will be attracted to the south pole of the other. So, magnetic fields go into south poles, because this is the direction of the force on a north pole.

We can measure the direction of a magnetic field by placing a compass in it. Compass needles are made from magnetised iron or steel. A compass needle has a north and a south pole. The north pole is attracted to south poles, and repelled by other north poles. Hence, compass needles are useful for visualising magnetic fields.

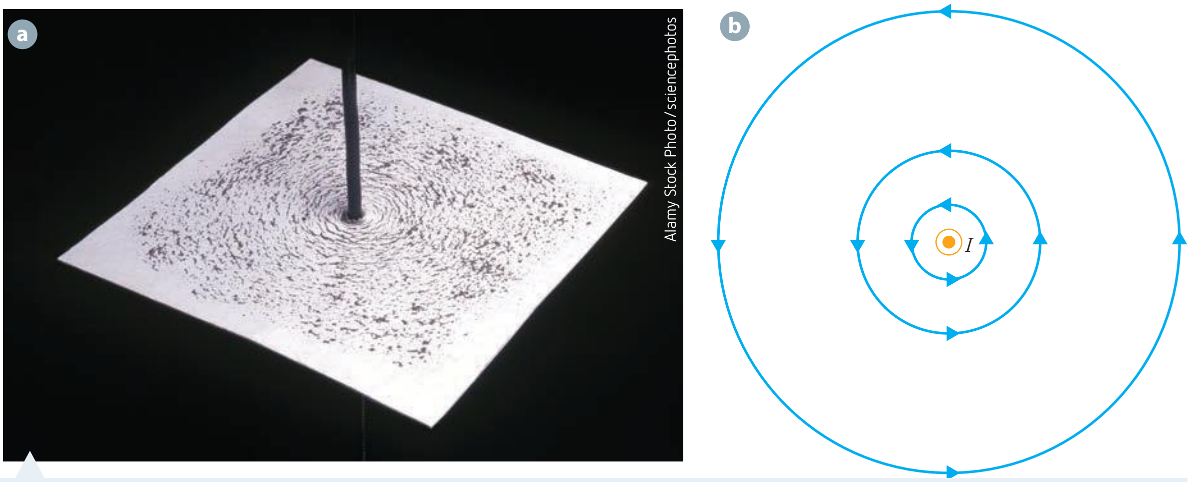
Magnetic field lines are also easy to visualise using iron filings, which act as tiny compass needles, as shown in Figure 14.5. Figure 14.5a shows a bar magnet with iron filings around it. The iron filings are ferromagnetic and become magnetised by the field of the bar magnet. Once magnetised, they align themselves with the magnetic field. Figure 14.5b shows a field line diagram for the bar magnet. Note that the field is stronger close to the poles of the magnet, and strongest inside the magnet.

There is a relationship between magnetism and electricity, which was discovered in 1819 by Hans Christian Øersted. During a lecture demonstration, he found that an electric current in a wire deflected a nearby compass needle. This showed that a magnetic field is created by the flow of charge, or current. This observation gave physicists the first clue to how ferromagnetic materials work, but it took more than another 200 years, and the development of quantum mechanics, to arrive at our current model.

When iron filings are placed around a current-carrying wire, as in Figure 14.6, they form loops. These loops show the field lines due to the current. The circular nature of these magnetic field lines was first

Magnetic field lines

Look at the field line diagrams. How does the field of a bar magnet differ from that of an electric charge?



Alamy Stock Photo/sciencephotos

FIGURE 14.6 **a** Iron filings showing the magnetic field lines around a current-carrying wire; **b** Magnetic field lines as seen from above for a current-carrying wire (current coming out of the page) observed and published by Michael Faraday. Careful measurements by Jean-Baptise Biot and Félix Savart, described in more detail in the next section, showed that the field lines get less dense further from the wire. This means that the field gets weaker as you get further from the wire.

Bending a current-carrying wire changes the shape of the magnetic field lines. If a current-carrying wire is formed into a loop (as shown in Figure 14.7a), it produces a field that is strong within the loop, and weaker outside the loop where the field lines spread out. This field is shown in Figure 14.7b. Note that the field also looks like that of a bar magnet, with a north and south pole.

Recall that in our discussion of ferromagnetic materials, we stated that an electron acts like a loop of current as it orbits the nucleus. Figure 14.7c shows the magnetic field lines due to an orbiting electron. It is these magnetic fields due to individual electrons, as well as the electron spin, that add up to give the magnetic field of an orbiting electron. Adding up the contributions from all the electrons in a material gives the magnetic field due to the material.

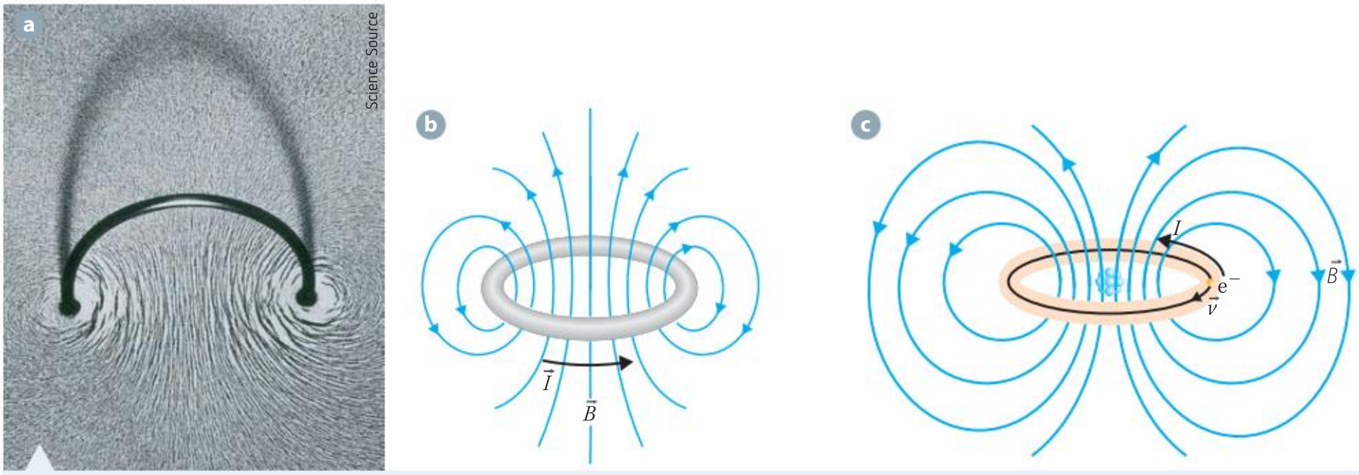


FIGURE 14.7 **a** Iron filings showing the magnetic field around a single current-carrying loop of wire; **b** Magnetic field lines for a current loop; **c** Magnetic field lines for an electron orbiting the nucleus

You can see from Figure 14.7b that the magnetic field lines within the current loop are more densely spaced than the lines outside the loop. Hence, the field is stronger in this region. By winding a wire into a coil, we get many current loops. This gives a much stronger field within the coil and a very weak field outside the coil. Such a device is called a **solenoid**. Figure 14.8 shows the magnetic field due to a solenoid. The loops or turns are shown loosely wound in Figure 14.8b so that you can see how the field lines due

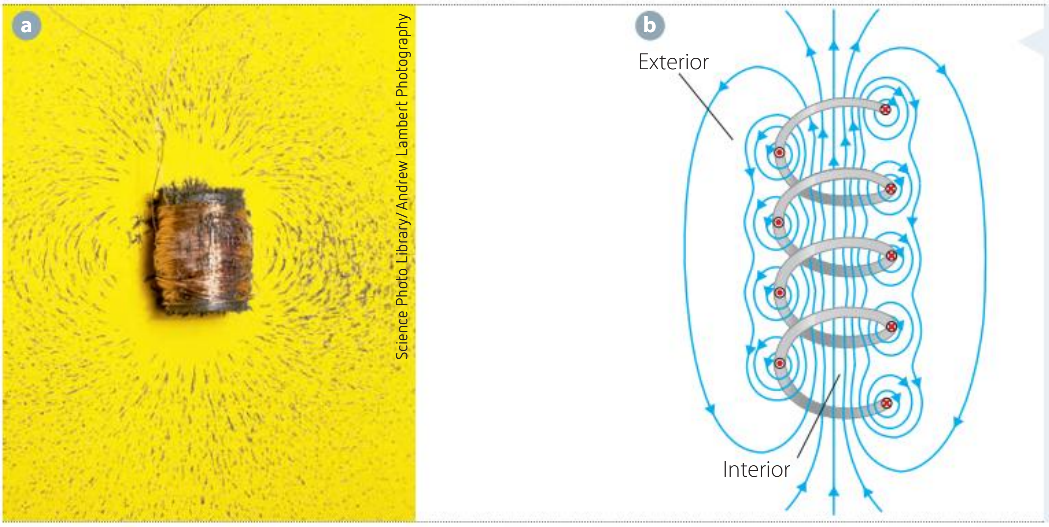


FIGURE 14.8 **a** Iron filings showing the magnetic field around a solenoid; **b** Magnetic field lines for a loosely wound solenoid

to the individual turns reinforce each other within the solenoid. Outside the solenoid, the fields from adjacent turns tend to cancel, giving a weak external field.

The more turns of wire, the greater the field. In a tightly wound solenoid, the internal field lines are straight and parallel. Outside the coil, the field lines spread out. Hence, there is a large uniform internal field and a very small external field. The result is an extremely useful device. Remember that energy is stored as potential energy in any field, so a solenoid is a way of storing energy in a magnetic field. Solenoids are used in inductors, transformers, electromagnets, magnetic switches and many other applications.

Look again at all the field line drawings in Figures 14.5 to 14.8. Except where the field lines extend beyond the edge of the diagram, you can see that they form loops. Magnetic field lines form closed loops around currents. Magnetic field lines from permanent magnets form closed loops passing out of the north pole, around the magnet and back into the south pole, and through the material back to the north pole. Unlike electric charges, which can be isolated, magnetic poles always come in pairs. Hence, the field lines always either form a closed loop or lead from a north to a south pole.

The diagrams shown in Figures 14.5 to 14.8 allow you to see a slice through the magnetic field in the plane containing the iron filings. The field line diagrams also show the field lines in a single plane, but remember that fields are actually three-dimensional. You can use iron filings suspended in oil to see magnetic field lines in three dimensions.

INVESTIGATION 14.3

Critical and creative thinking

Visualising magnetic field lines (1)

AIM

To observe the magnetic field lines of some magnets in three dimensions

MATERIALS

- Bottle of baby oil
- Iron filings (about 15 mL)
- A test tube that fits snugly into the neck of the bottle
- Small magnets that fit inside the test tube (e.g. small spherical magnets, bar magnets, strips of fridge magnets)
- Paper towel

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
The oil containing filings may be used by someone if not correctly disposed of, causing skin irritation and abrasions.	Dispose of the oil with filings properly, or label and keep in the laboratory stores.
Glassware may break and cut your hands.	Always be careful using glassware and clean up any breakages according to your teacher's instructions.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. Peel the label off the bottle. Tip a little of the oil out of the bottle so that the test tube fits in. Ask your teacher where to put this excess oil – *do not pour it down the sink*.
2. Add the iron filings to the oil in the bottle.
3. Insert the test tube into the neck of the bottle. It should fit snugly so oil does not seep out around it. *Clean up any spills immediately with paper towel.*
4. *Carefully* slide the first magnet into the tube. Do not drop it in, as the tube may break.

»

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5. Draw a diagram or take a photograph showing how the iron filings have arranged themselves in the oil. You may need more than one diagram or photo to show the field from different angles.
6. Carefully remove the first magnet.
7. Repeat steps 4–6 using each magnet in turn.

RESULTS

You should have a collection of diagrams showing the field lines for different magnets.

ANALYSIS OF RESULTS

1. Compare and contrast the fields for the different magnets.
2. Can you identify where the poles of the magnets are?
3. What indication is there of the way the field varies with distance from each magnet?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

It can be difficult to visualise magnetic fields directly, particularly for complicated arrangements of currents. Computer simulations are useful for visualising fields. You may have used one when you studied electric fields (chapter 12). Physicists often use simulations for modelling systems as well as for visualisation. However, remember that *a simulation is a model – it is not the real system*. A simulation has limitations, just like any other model.

INVESTIGATION 14.4

Visualising magnetic field lines (2)

Critical and creative thinking

Information and communication technology capability

AIM

To model the magnetic field lines of some current configurations using computer simulations

MATERIALS

- Computer
- Magnetic field line simulator (see weblink for examples)

Magnetic field line simulators

Use these simulations to model magnetic field lines. Try one or more of them.

METHOD

1. Follow the instructions in the magnetic field line simulator that you are using.
2. When you have the magnetic field lines displayed, make a copy of the diagram (for example by taking a screen shot). Save the image.
3. Vary the magnitude and direction of the current. Record how the field changes.
4. Try using different arrangements of currents (for example, a long straight wire, two parallel wires with parallel and antiparallel currents, a loop, a solenoid, etc.).
5. Keep records of each configuration that you examine and the resulting field lines.

RESULTS

You should have a collection of diagrams showing the field lines for different arrangements of currents.

»

ANALYSIS OF RESULTS

1 What do your magnetic field models (the field line diagrams) tell you about the force that would be exerted on a magnet at different positions?

2 What indication is there of the way the field varies with distance from each current?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Revision

KEY CONCEPTS

- Magnetic field lines are a visual model of the magnetic field.
- Magnetic field lines show the direction of force acting on a magnetic north pole at a point. The density of the magnetic field lines is proportional to the field strength.
- Magnetic field lines come out of north poles and go into south poles.
- Magnetic field lines for magnetic fields due to currents form closed loops about the current.

CHECK YOUR UNDERSTANDING

14.2

1 Copy Figure 14.5b (page 408) in the middle of a sheet of paper. Extrapolate the field lines that are cut off at the edges of Figure 14.5b to show their complete paths.

2 Imagine you have a source of magnetic field in an opaque box that you cannot open. Could you tell if it was a magnetic material or a current source inside the box? Justify your answer.

3 Figure 14.9 shows a horseshoe magnet.

  a Predict and draw the magnetic fields lines for this magnet.

  b Identify regions of strong magnetic field for this magnet.

  c Examine your field line drawing from part a. Do the magnetic field lines form loops in this case? Explain your answer. Modify your drawing if necessary.

4 Two current-carrying wires lie on a table in a cross. One is aligned east–west and carries current west. The other is aligned north–south and carries an equal current north.

  a Draw the magnetic field at the surface of the desk.

  b Identify any areas where the total magnetic field is zero. Hint: use Figure 14.6 (page 408) to help you decide on the direction of the field for each wire.

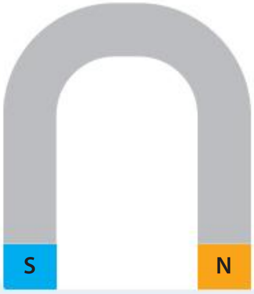


FIGURE 14.9 A horseshoe magnet

14.3

Magnetic fields produced by current-carrying wires and solenoids

Øersted's experiment

Find out about Øersted's experiment, and try it yourself.

As Øersted first observed in 1819, a current-carrying wire creates a magnetic field. This observation led to many discoveries in electromagnetism, and the development of motors, generators and many other useful devices.

Permanent magnets, made of magnetised ferromagnetic materials, are a convenient way of producing a small magnetic field. However, when a large field is needed, an electromagnet using a current-carrying coil of wire is usually used.

Different current configurations can be used to create magnetic fields of different magnitudes and different shapes.

INVESTIGATION 14.5

Investigating magnetic fields due to currents

Critical and creative thinking

AIM

Write an aim for your investigation. What is your inquiry question?

MATERIALS

- Magnetic field meter or compass (alternatively, a smart phone with compass and magnetic field meter (EMF meter) app installed)
- DC power supply
- Cardboard tube
- Light globe of same voltage as power supply (e.g. if you are using a 12 V supply, use a 12 V globe)
- Insulated wire (at least 2 m long)
- Connectors (such as alligator clips)
- Tape
- At least 1 m of clear bench space

Note: if you have a magnetic field meter, you will be able to make quantitative measurements of field strength. If not, you can use a compass to measure angles of deflection, which give an indication of field strength.

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
There is a danger of electric shock.	Be careful not to set the voltages and currents too high.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1. If you are using a magnetic field meter or an app on your smart phone, read the instructions before you begin. Make sure you know how to use it, and whether it needs to be aligned with the field to make measurements.
2. Measure the magnetic field strength or note the direction of the compass needle before connecting any equipment. Write down your result.
3. Connect the length of wire to the light globe at one end and the power supply at the other. Connect the other contact of the globe to the power supply. You now have a single loop circuit. Note: *never* connect the wire alone across the power supply as it does not have enough resistance and will short-circuit the power supply.
4. Stretch out the wire so you have some length that is well away from the power supply and the wire is straight. You may want to tape a section to the bench.
5. Turn on the power supply. The light should glow, indicating that you have current flow. If it does not, check your connections.
6. Measure the field around the straight length of wire, or note the compass deflection. Record how it varies above and to the sides of the wire. Note the direction of current. Observe what happens if the direction of the current is reversed.
7. Turn off the power supply and disconnect the wire from the power supply. Leave it connected to the light.

»

»

8. Wrap the wire around the cardboard tube to form a coil, holding it in place with tape as necessary. Make sure you always wrap in the same direction. Leave enough length free to reconnect to the power supply.
9. Connect the wire, now in a coil, to the power supply.
10. Repeat steps 6 and 7, making measurements inside the tube, at its ends, and to the side.
11. Lay the wire back and forth on the bench, taping down as needed (Figure 14.10).
12. Repeat steps 6 and 7.
    Other arrangements of the wire can also be investigated.

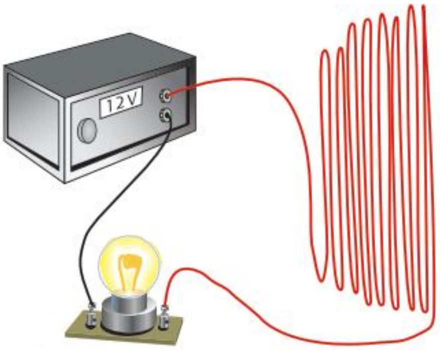


FIGURE 14.10 Experimental set-up for measuring magnetic field due to parallel current-carrying wires, with antiparallel currents

RESULTS

You should have a record of the magnetic field due to the current in a long straight length of wire, a coil, and a flat array of wires.

ANALYSIS OF RESULTS

1. Using your results, draw field line diagrams for the various current arrangements.
2. If you were able to make quantitative measurements, are there useful graphs you can use to represent your results?

CONCLUSIONS

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

Magnetic field due to a current-carrying wire

Jean-Baptiste Biot and Félix Savart performed many experiments with magnets and current-carrying wires. They found that for a point P some distance from a long straight current-carrying wire:

- the magnetic field is perpendicular to both the direction of the current and to a line between the wire and P
- the magnitude of the field is inversely proportional to the distance from the wire to P, as shown in Figure 14.11b
- the magnitude of the field is proportional to the current, as shown in Figure 14.11c.

These observations can be summarised mathematically as:

$$B = \frac{\mu_0 I}{2\pi r}$$

where B is the magnitude of the magnetic field at a distance r from a wire carrying current I. The constant μ_0 is called the **permeability of free space**.

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

The constant μ_0 has the same function for magnetic fields as the permittivity of free space, ε_0, has for electric fields. It tells us the strength of a field created by a given current in vacuum, and ensures the units of field are correct.

We can use this equation to deduce the units for magnetic field.

If:

$$B = \frac{\mu_0 I}{2\pi r}$$

then

$$[B] = \frac{[\mu_0][I]}{[r]} = \frac{(\text{T m A}^{-1})(\text{A})}{\text{m}} = \text{T}$$

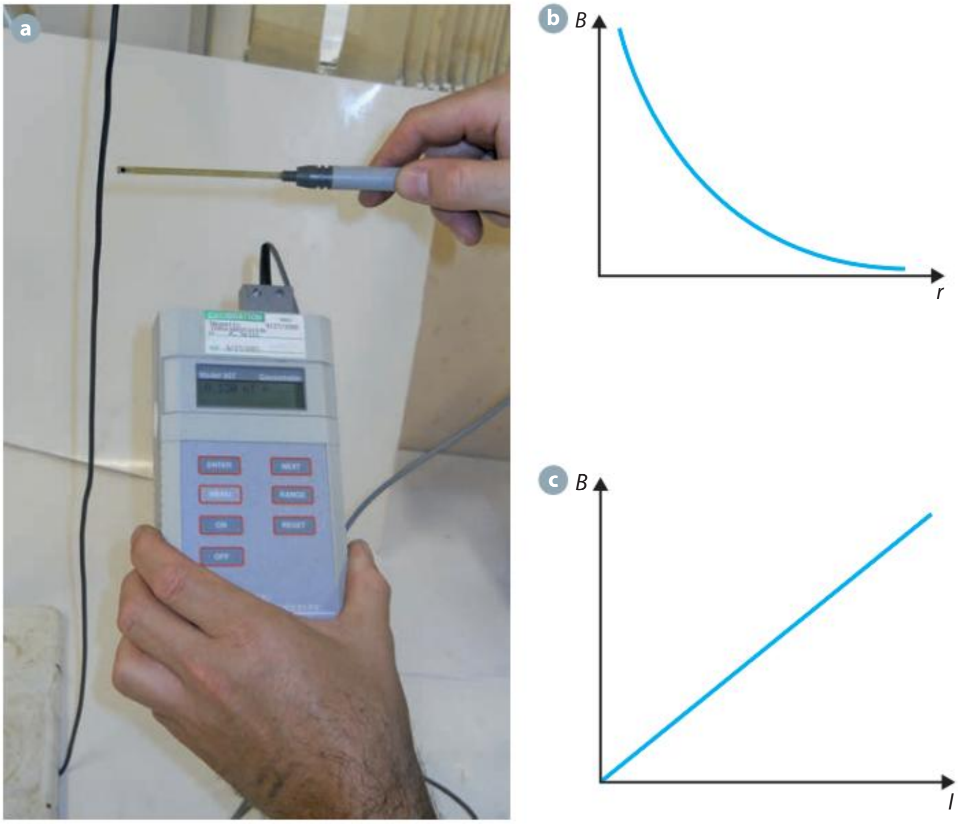


FIGURE 14.11 **a** Measuring field strength due to a current-carrying wire; **b** Field strength, *B*, as a function of distance, *r*, from a wire; **c** Field strength, *B*, as a function of current, *I*, carried by a wire

Hence, magnetic field is measured in units of T, where T is the symbol for tesla. The unit tesla was named for Nikola Tesla, who did important work in electricity and magnetism. The tesla can be reduced to fundamental units as $1\ \text{T} = 1\ \text{kg s}^{-2}\ \text{A}^{-1}$.

We have seen already in chapter 12 that point sources (such as a single positive or negative charge) create a field that varies with $\frac{1}{r^2}$. A current-carrying wire is not a point source – it is a line source, and so it creates a field that varies with $\frac{1}{r}$.

Nikola Tesla

Find out more about Tesla's interesting life and his contributions to science.

WORKED EXAMPLE (14.1)

Calculate the magnetic field strength at a distance of 10 cm from a long, straight wire carrying a current of 10 A.

ANSWER	LOGIC
$r = 10$ cm; $A = 10$ A $r = 0.10$ m	▪ Identify the relevant data in the question. ▪ Convert data to SI units.
$B = \frac{\mu_0 I}{2\pi r}$ $B = \frac{(4\pi \times 10^{-7}\ \text{T m A}^{-1})(10\ \text{A})}{2\pi \times 0.1\ \text{m}}$ $= 2 \times 10^{-5}\ \text{T}$	▪ Identify the appropriate formula to relate the field strength to current and distance. ▪ Substitute known values, with units, into the formula. ▪ Calculate the answer.
$B = 2.0 \times 10^{-5}\ \text{T}$	▪ State the final answer with appropriate significant figures and units.

TRY THESE YOURSELF

1 Find the field at distances of 20 cm, 40 cm, 60 cm, 80 cm and 1 m from the wire.

2 Plot a graph of field as a function of distance from the wire.

The right-hand rule

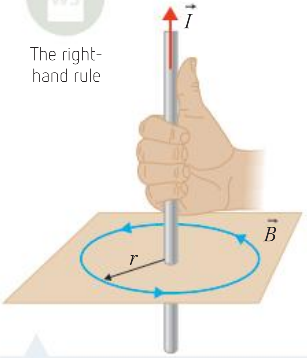


FIGURE 14.12 Quick 'rule of thumb' for finding the direction of magnetic field lines: point your right thumb in the direction of the current and your fingers curl in the direction of the field lines

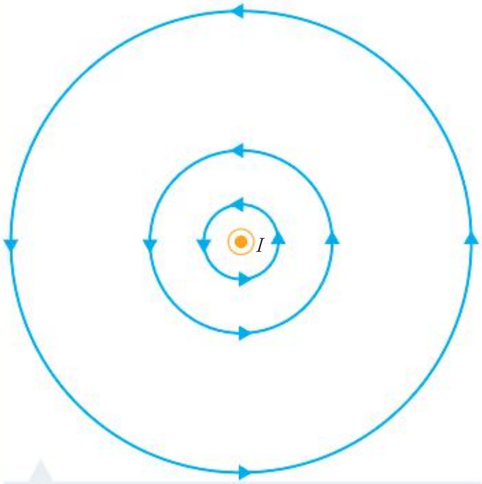
We have seen that current-carrying wires produce magnetic fields that form loops, as in Figure 14.6. We also know that field is a vector quantity, so the loops must have a direction. Placing compasses around a wire can tell us the direction of the field at any point. We find that if the current is coming towards you, the direction of the field lines is anticlockwise about the current.

A quick way to find the direction of a magnetic field due to a current is to use the 'rule of thumb' shown in Figure 14.12. Point your *right* thumb in the direction of the current. Your fingers then naturally curve in the direction of the field. *Remember to use your right hand.*

When drawing field line diagrams, remember there is an infinite number of possible magnetic field lines you can draw. Choose a sensible number, and space them so that the density of field lines is proportional to the field strength. This means that for a current-carrying wire, the distance between field lines should get bigger as you get further from the wire.

WORKED EXAMPLE 14.2

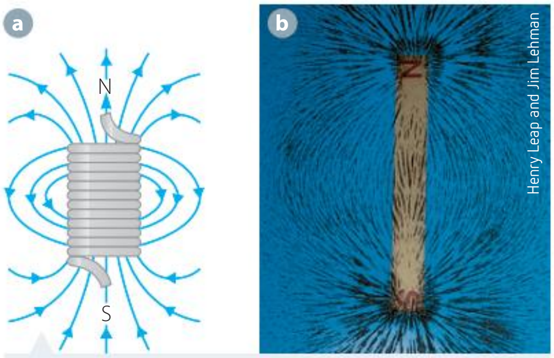
A long wire is carrying current directly upwards. Draw the magnetic field lines due to the current as seen from above.

ANSWER	LOGIC
I is upwards out of the page.  **FIGURE 14.13** Magnetic field lines due to a long, straight current-carrying wire, with current coming out of the page	▪ Recall that field lines form circular loops around a current. ▪ Use the 'rule of thumb' to find the direction of the field lines. ▪ Remember that the field decreases with $\frac{1}{r}$.

TRY THIS YOURSELF

Draw the magnetic field lines for a long straight wire carrying current to the left.

Magnetic field due to a current-carrying solenoid



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Henry Leap and Jim Lehman

FIGURE 14.14 Magnetic field lines of **a** a solenoid and **b** a bar magnet. The solenoid has a north pole at its top.

As we have seen, a current-carrying wire creates a magnetic field. If we want to create a large field, we use lots of wires. One way to do this is to coil a single wire into a solenoid, or coil.

Each loop of the wire creates a magnetic field, as shown in Figure 14.8 (page 409). Inside the coil these fields add up to give a large and approximately uniform magnetic field. The more loops or turns of wire, the greater the field, and the more uniform it is. In a tightly wound solenoid, the internal field lines are straight and parallel. Outside the coil, the field lines spread out. Hence, there is a large uniform internal field and a very small external field.

Figure 14.14 compares the magnetic field lines of a short, tightly wound coil to that of a bar magnet. Note the similarity in the field line arrangement. As the solenoid has almost the same magnetic field as the bar magnet, we can say that it too has a north and south pole. The poles are located at the ends of the coil.

The field inside a solenoid depends upon the current, I, that it carries and the number of loops of wire, N, and the length, L, of the solenoid:

$$B = \frac{\mu_0 N I}{L}$$

The more loops or turns for a given length, the greater the field.

WORKED EXAMPLE 14.3

As part of an investigation, a student winds 200 turns of wire onto a 10 cm long cardboard tube. If they want a magnetic field of 0.002 T inside the tube, what current must flow through the coil?

ANSWER	LOGIC
$N = 200$; $L = 10$ cm; $B = 0.002$ T; we want to find I.	▪ Identify the relevant data in the question.
$L = 0.10$ m	▪ Convert data to SI units.
$B = \frac{\mu_0 N I}{L}$	▪ Identify the appropriate formula to relate the field strength to the current, number of turns and coil length.
$I = \frac{BL}{\mu_0 N}$ $I = \frac{(0.002\ \text{T})(0.1\ \text{m})}{(4\pi \times 10^{-7}\ \text{T m A}^{-1})(200)}$ $I = 0.796$ A	▪ Rearrange for current. ▪ Substitute known values, with units, into the formula. ▪ Calculate the answer.
$I = 0.8$ A	▪ State the final answer with appropriate significant figures and units.

TRY THESE YOURSELF

What current would be needed to produce this field in the coil if:

1 the number of turns was doubled but the length kept the same?

2 the length of the coil was doubled but the number of turns stayed the same?

3 both the number of turns and the length was doubled?

KEY CONCEPTS

- Current-carrying wires produce magnetic fields.
- Magnetic field lines form circular loops around current-carrying wires.
- The direction of the field around a current-carrying wire can be found from the 'rule of thumb' (Figure 14.12).
- The magnitude of the field around a current-carrying wire is given by $B = \frac{\mu_0 I}{2\pi r}$.
- A solenoid is a coil of wire. A current-carrying solenoid has a large, approximately uniform magnetic field inside the coil, and a small magnetic field outside the coil.
- The magnitude of the field inside a solenoid is given by $B = \frac{\mu_0 NI}{L}$.

CHECK YOUR UNDERSTANDING

14.3

1 **a** Calculate the current necessary to produce a field of 0.15 T at a distance of 1.0 cm from a wire.

b What is the field at a distance of 2.0 cm from this wire?

2 The magnetic field is being measured at a distance of 1.0 cm from a long straight current-carrying wire.

a Calculate the magnetic field at this position to complete Table 14.1.

TABLE 14.1 Magnetic field due to a current-carrying wire

CURRENT (A)	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
FIELD (T)										

b Draw a graph to represent the data. Calculate the gradient of the graph *from the graph*. Explain the significance of the gradient.

3 Two current-carrying wires lie parallel to each other on a table.

a Draw a magnetic field line diagram to represent the field due to the wires:

i when they carry equal current in the same direction.

ii when they carry equal current in opposite directions.

b Describe the field at a point on the table equidistant between the two wires for each of the cases described in part **a**.

4 Describe the features of the magnetic field due to a current-carrying solenoid.

5 A student wishes to make a solenoid capable of producing a magnetic field of 0.01 T with a current of no more than 2 A. The student has a tube with a radius of 1.5 cm and a length of 15 cm. Calculate the length of wire needed if the student winds it along the entire length of the tube.

14.4 Magnetisation: making magnets

If you stroke a steel pin with a strong magnet, it will line up the magnetic fields in the iron in the steel. The pin then becomes magnetised and can be used as a compass needle, and can also be used to pick up other pins.

INVESTIGATION 14.6

Making magnets

Critical and creative thinking

AIM

To magnetise a nail by exposing it to the magnetic field of a strong magnet

Read the method below, and then write a hypothesis for this investigation.

MATERIALS

- Strong magnet
- Steel nail
- Small steel paper clips (or steel pins with a pincushion)
- Container for paper clips or pins

Note: not all metal nails or paper clips are ferromagnetic. You may need to try a few different sorts. Steel pins are generally ferromagnetic and lighter than paper clips, but are sharp and easily lost.

RISK ASSESSMENT

WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Pins are sharp and easily lost.	Use a pincushion to store pins, and always put them back immediately.

What other risks are associated with your investigation, and how can you manage them?

METHOD

Note: do not put your pins close to the magnet. You need to control their exposure to the magnetic field.

1 Before you begin magnetising your nail, check that the pins are not attracted to it.

2 Stroke the nail with the permanent magnet, once. Note which way the magnet was held, and which direction you moved it relative to the nail.

3 Hold the magnetised nail just above the container of un-magnetised pins. Count how many pins the magnetised nail picks up. Note whether all the pins picked up are directly touching the magnetised nail.

4 Remove the pins from your magnetised nail and set them aside.

5 Repeat steps 2–4, always stroking the nail in the same direction.

RESULTS

Make a table recording how many pins were picked up each time. Draw diagrams showing any interesting results, such as pins forming a chain attached to the nail.

ANALYSIS OF RESULTS

Can you identify a relationship between the number of times the nail was stroked and the number of pins picked up?

DISCUSSION

1 State whether your results agreed with your hypothesis.

2 Explain how it is possible for a chain of pins to form from the nail. Describe what is happening to the pins.

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation.

How are magnets made?

You can read about the magnet manufacturing process at these sites.

Many materials will be magnetised while they are within a magnetic field, but they lose their magnetisation when they are removed from the field.

A **permanent magnet** is a magnetic material that maintains its magnetic field after being magnetised. Permanent magnets are made by taking a ferromagnetic material and placing it in a large magnetic field. The material may be a metal alloy such as aluminium–nickel–cobalt (Alnico) or neodymium–iron–boron, or a ceramic such as barium ferrite or strontium ferrite. The magnet is usually made by compressing the powdered material into the desired shape, and then a solenoid, which creates a large magnetic field, magnetises the material.

A ferromagnet that has not been magnetised and is not in a magnetic field does not produce a magnetic field of its own. The tiny magnetic fields due to individual electrons line up in very tiny regions called domains, but the fields due to different domains do not line up. So, on average, the domain fields add up to give zero net field.

However, when you put a ferromagnet in a magnetic field, this applied field makes some of the electron fields line up with it. Domains where the internal field is aligned with the applied field get bigger. Domains where the internal field is not aligned get smaller as the electron fields rotate to line up with the external field. The larger the external field, the more domains line up with it and the bigger these domains get. So, the bigger the external field, the bigger the field that is produced by the material. This is shown in Figure 14.15.

If the applied field is large enough, then every unpaired electron will have its magnetic field aligned with the external field. This is called **saturation magnetisation**. At this point, the ferromagnet is producing its largest possible magnetic field.

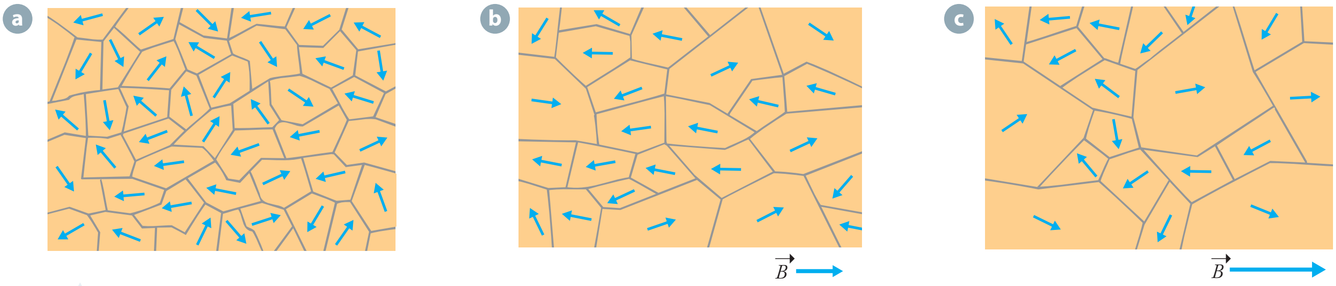

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FIGURE 14.15 **a** An un-magnetised ferromagnet will have small domains in which the magnetic fields due to individual atoms line up, but on average the fields all cancel out. **b** When subject to a magnetic field, the fields due to individual atoms start to line up, and the domains get bigger. The material starts to have its own magnetic field – it is magnetised. **c** A larger external field causes more domains to line up, and the material becomes more strongly magnetised.

Electromagnets

Follow the instructions to make your own electromagnet.

When the ferromagnet is removed from the applied field, then its magnetisation typically decreases as some domains becomes un-aligned. This happens because of internal energy. All the particles are vibrating and moving about (see chapter 11). The higher the temperature, the more internal energy, and the more the particles jiggle about. The more the particles jiggle about, the more random the arrangement of their magnetic fields becomes. The more random the arrangement of fields, the lower the total magnetic field, and the lower the magnetisation. However, for a ferromagnet at low temperature, the arrangement does not become completely random.

The lower the temperature is, the less the magnetisation is lost. The magnetisation does not return to its initial zero state – there is a **residual magnetisation**. This means the ferromagnet continues to produce its own magnetic field – it is magnetised, and hence is a magnet. How well the material remains magnetised depends on what it is made of and how large the magnetising field was.

Thin, flat fridge magnets, such as those often used for advertising, have domains running in strips along the length or width of the magnet. If you get two fridge magnets and put them with their magnetic sides together, and then slide them against each other, it will feel like their surfaces are rippled. However, when you feel the surfaces of the fridge magnets with your fingers, they feel smooth. The sensation of rippling comes from the north pole and south pole domains on the two magnets lining up and attracting, and then, as you slide them, the north pole domains line up with north pole domains and repel each other.

KEY CONCEPTS

- Ferromagnetic materials have their internal magnetic fields aligned within small domains.
- Ferromagnets can be magnetised by placing them in magnetic fields. Solenoids are used to create the large magnetic fields used for manufacturing magnets. The applied magnetic field pulls the magnetic fields of the individual domains into alignment. When the applied field is removed, some domains remain aligned and the material is magnetised.
- Saturation magnetisation is when all domains are fully aligned with the applied field. Residual magnetisation is the remaining magnetisation when the applied field is removed.
- A steel nail or pin can be magnetised by stroking it with a strong magnet.

CHECK YOUR UNDERSTANDING

14.4

1. Define 'saturation magnetisation' and 'residual magnetisation'.
2. Describe how you could make a compass using a magnet, a steel pin and a length of thread.
3. Which would be more useful for manufacturing magnets: a material with high residual magnetisation, or one with low residual magnetisation? Justify your answer.
4. Recall what you learned about thermodynamics in chapter 11. Explain how increasing the temperature of a magnet can decrease its magnetisation.

14.5 Modelling magnetic fields

We have seen that magnetic fields can be modelled in various ways.

Field lines are a way of modelling and visualising a magnetic field. The arrows on field lines represent the direction of the magnetic field at various points in space. The arrows show the direction of the force exerted on a north magnetic pole by the field. The spacing between the lines gives an indication of the strength of the field. Remember that field lines (electric, magnetic or gravitational) do not show the trajectory of a particle – they show the direction of the force at a point. When you draw field lines, you have a choice of how many lines you draw. Field lines do not physically exist – they are a way of modelling the field.

We have also used a conceptual model of the electron as a charged particle travelling in a circular path, and with its own magnetic field due to its spin. We used this model to help us understand magnetic materials, in particular, ferromagnets. This model gives us useful predictive and explanatory power, but again it is a model – not the physical reality. Like all models, it is limited. For example, it does not offer any explanation for why an electron should have its own magnetic field. You will meet more sophisticated models of electrons when you study quantum mechanics in *Physics in Focus Year 12*.

Another way of modelling magnetic fields is mathematically – using equations to describe the field. Two mathematical models have been introduced in this chapter.

First, we introduced a mathematical model for the magnetic field due to a long straight current-carrying wire. This model assumes that the wire is very long, so that the effects of the ends of the wire can be ignored. This works well when the point at which the field is to be calculated is very close to the wire compared to the length of the wire, and is not close to either end. However, for short lengths of wire, this model is not as accurate.

The second model introduced was for a current-carrying solenoid. Again, this model makes assumptions and approximations. We make the approximation that the field is constant inside the solenoid, which is based on the assumption that the solenoid is long compared to its radius, and that it is tightly wound.

Mathematical models are common in physics. They are particularly valuable because they allow us to make quantitative, testable predictions. In Investigation 14.7, you can compare the predictions for the mathematical model of the field inside a solenoid to your own measurements.

INVESTIGATION 14.7

Critical and creative thinking

Numeracy

Measuring the field inside a solenoid

AIM

Write a hypothesis for this investigation based on the mathematical model of the magnetic field inside a solenoid.

MATERIALS

- Hollow solenoid with a known (large) number of turns
- Variable DC power supply
- Ammeter
- Magnetic field meter with probe

RISK ASSESSMENT

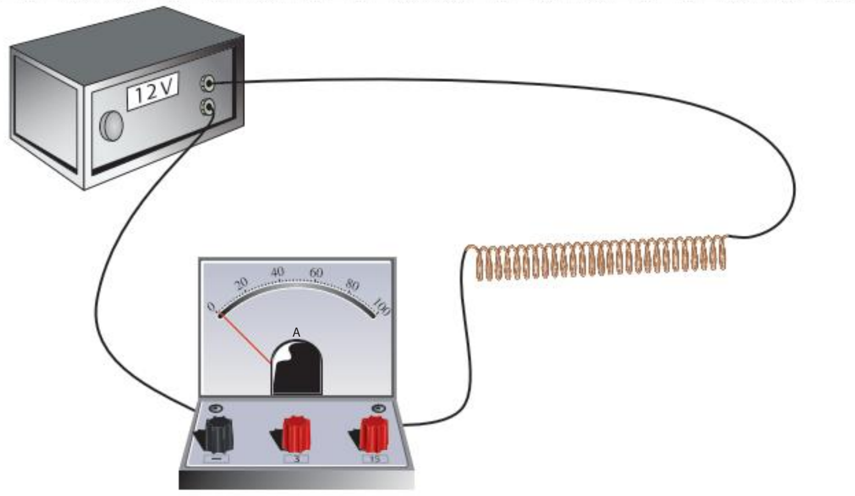
WHAT ARE THE RISKS IN DOING THIS INVESTIGATION?	HOW CAN YOU MANAGE THESE RISKS TO STAY SAFE?
Electricity can cause shocks.	Use only low voltages and currents.

What other risks are associated with your investigation, and how can you manage them?

METHOD

1 Connect the solenoid and ammeter to the power supply in a series circuit as shown in Figure 14.16.

FIGURE 14.16 Series circuit with ammeter and coil connected to power supply



2 Before you turn on the power supply, measure the magnetic field inside the solenoid. Record your measurement. Note the position of the meter probe – you will need to keep the meter at the same position for consistent results.

3 Turn on the power supply to a low voltage.

4 Record the magnetic field inside the solenoid. Record the current through the solenoid (the ammeter reading).

5 Increase the voltage from the power supply.

6 Repeat steps 4 and 5 until you have at least six data points.

7 With the current constant, slowly move the magnetic field meter probe around inside the solenoid. Note whether the reading changes.

»

RESULTS

1. Construct a table containing your measured data for current and field. Remember to include units and uncertainties. You may need to consult the manuals for the magnetic field meter and the ammeter to find their precision.
2. Draw a large diagram of the solenoid, and show on it how the field varied with position inside the coil, particularly near the ends.

ANALYSIS OF RESULTS

1. Plot a graph of magnetic field as a function of current. Draw a line of best fit on your graph and find the vertical axis intercept and the gradient.
2. Does your data match the predictions of the mathematical model $B = \frac{\mu_0 NI}{L}$ for the intercept and gradient?
3. How close to constant was the magnetic field inside the solenoid?

CONCLUSION

With reference to the data obtained and its analysis, write a conclusion based on the aim of this investigation including an evaluation of the mathematical model of the magnetic field inside a solenoid.

Mathematical models use equations that can be solved analytically or numerically. When you solve an equation analytically, you perform mathematical operations to rearrange it until you have an expression for the variable you are interested in. You may then substitute in values for a particular case to calculate the parameter you are interested in.

However, some equations are very difficult, or time consuming, or even impossible to solve. Sometimes we want to calculate so many values that doing it manually would take too long. In these cases, we use numerical methods and create numerical models. As we generally use a computer for this, they are often referred to as computational models or computational simulations. Physicists often use numerical methods and create computational simulations of complex systems. Sometimes these are written in specialised mathematical software languages, but common spreadsheet software can be used for simple simulations.

INVESTIGATION 14.8

Modelling the magnetic field due to a current-carrying wire

Critical and creative thinking

Numeracy

Information and communication technology capability

AIM

To model the magnetic field of a long current-carrying wire using the equation $B = \frac{\mu_0 I}{2\pi r}$ and spreadsheet software

MATERIALS

- Computer with suitable spreadsheet software (Note: the instructions given here are for Microsoft Excel and may need to be modified for other spreadsheet software)

METHOD

1 Open a worksheet in your spreadsheet software, and add a title at the top. Save the spreadsheet with a filename that you will remember.

2 Enter the data that will remain constant in cells near the top. Enter labels in the cells above so you know what each piece of data refers to (see Figure 14.17).

3 Create a set of data for the distance, r:

- Write the title, r, in a cell and put the first value you want to use in the cell below.
- Create a column of distance data as long as you want by entering in an appropriate formula into the cell below, and then copying it into a number of cells below. For example, in Figure 14.17, the first data point for r is in cell A7. Cell A8 has the formula '=A7+0.0005'. A8 was then copied into 100 cells in the column below A8.

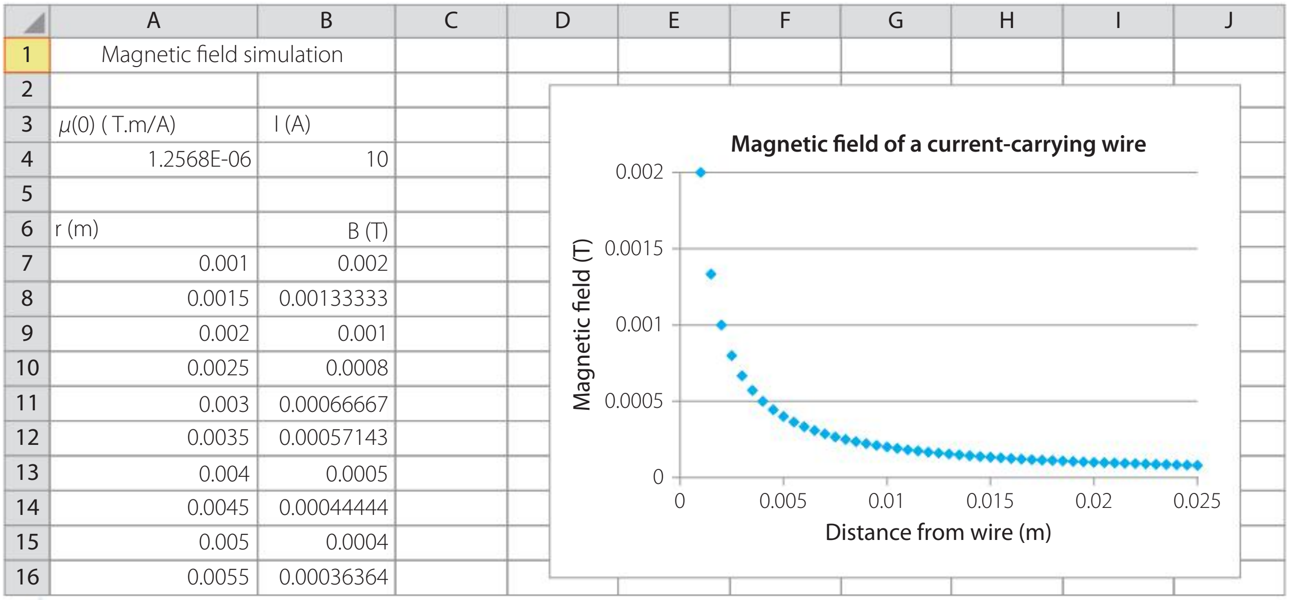
4 Calculate the magnetic field data:

- Write the title 'B' in the cell next to the 'r' title cell.
- In the cell below, enter an appropriate formula to calculate the magnetic field. This formula will need to refer to the cells that contain the values for μ_o and I, as well as the data for r. For the example shown in Figure 14.17, the formula used is '=(A4*B4)/(2*3.142*A7)' in cell B7. Note the use of $ signs in front of the cell references for the constant data. These $ signs tell the software not to change these references if you copy this cell.
- Copy the formula into the cells below, to the same number of cells as you have for r.

Other mathematical equations can also be modelled in this way, such as the equation for magnetic field inside a solenoid, $B = \frac{\mu_0 NI}{L}$.

5 Visualise your simulation results:

- Create a scatter plot of the magnetic field as a function of distance.
- Adjust the axes to give appropriate scales.
- Give your plot a descriptive title and label the axes.



	A	B	C	D	E	F	G	H	I	J
1	Magnetic field simulation									
2										
3	μ(0) (T.m/A)	I (A)								
4	1.2568E-06	10								
5										
6	r (m)	B (T)								
7	0.001	0.002								
8	0.0015	0.00133333								
9	0.002	0.001								
10	0.0025	0.0008								
11	0.003	0.00066667								
12	0.0035	0.00057143								
13	0.004	0.0005								
14	0.0045	0.00044444								
15	0.005	0.0004								
16	0.0055	0.00036364								

FIGURE 14.17 Spreadsheet simulation of magnetic field due to a current-carrying wire

6 Exploring your simulation:

- Change the value for the current. Try increasing it and decreasing it. Keep copies of the different graphs (for example, by making a copy of the sheet within the same file before making each change).
- Change the starting value of the distance, and the amount by which it increases as you go down your r data column.

RESULTS

Your results are the data you have created and the graphs.

ANALYSIS OF RESULTS

1 Does your graph show what you expect based on the equation?

2 What happens when you change the different variables?

3 How could you extend this investigation?

CONCLUSION

Write a short conclusion summarising your findings.

Excel spreadsheet for magnetic fields

The spreadsheet for this investigation, with extension sheets, can be found here.

KEY CONCEPTS

- Physicists use a range of models including conceptual, mathematical and numerical models. Numerical models are based on mathematical models.
- Magnetic fields can be described using all these types of models. Field lines are a conceptual model. The equations for magnetic field due to a current-carrying wire and inside a current-carrying solenoid are mathematical models. Computational simulations (numerical models) can be created using these mathematical models and spreadsheet software.

CHECK YOUR UNDERSTANDING

14.5

1 Distinguish between conceptual, mathematical and numerical models.

2 Apply your conceptual model for magnetic fields to the field line model to explain why field lines never cross.

3 Describe three models you have previously used in your physics studies. Classify them as conceptual or mathematical.

4 Describe one or more approximations typically made when solving problems involving motion in physics. When are they reasonable? When should they not be made?

5 The mathematical model for the magnetic field due to a current-carrying wire says that $B = \frac{\mu_0 I}{2\pi r}$.

   a What does this model predict the field to be inside the wire? Is this physically reasonable?

   b For what values of r is this model appropriate?

14 CHAPTER SUMMARY

- Magnetic fields are created by moving charges and current-carrying wires.
- Ferromagnetic materials are materials that can be magnetised. Magnetised materials create magnetic fields. The magnetic fields are due to the motion of the electrons in the material.
- Magnetic field lines are a visual model of the magnetic field. They show the direction of force acting on a magnetic north pole. The density of the magnetic field lines is proportional to the field strength.
- Magnetic field lines come out of north poles and go into south poles. Magnetic field lines form circular loops around current-carrying wires.
- The direction of the field around a wire can be found from the 'rule of thumb' (using your right hand, point your thumb in the direction of the current then your fingers will curl in the direction of the field). The magnitude of the field is given by $B = \frac{\mu_0 I}{2\pi r}$.
- A current-carrying solenoid has a large, approximately uniform magnetic field inside the coil, and a small magnetic field outside the coil. The magnitude of the field inside the coil is given by $B = \frac{\mu_0 NI}{L}$.
- Ferromagnets can be magnetised by placing them in magnetic fields. Solenoids are used to create the large magnetic fields used for manufacturing magnets. The applied magnetic field pulls the magnetic fields of the individual domains into alignment. When the applied field is removed, some domains remain aligned and the material is magnetised.
- Physicists use a range of models including conceptual, mathematical and numerical models. Numerical models are based on mathematical models.
- Magnetic fields can be described using all these types of models. Field lines are a conceptual model. The equations for magnetic field due to a current-carrying wire and inside a current-carrying solenoid are mathematical models. Computer simulations (numerical models) can be created using these mathematical models and spreadsheet software.

14 CHAPTER REVIEW QUESTIONS

Review quiz

1 List the two sources of magnetic fields.

2 Define 'magnetic domain'. Draw a diagram to help explain your answer.

3 Describe how you could test whether a material was ferromagnetic.

4 Describe how you could test whether a ferromagnetic material was magnetised.

5 Draw the magnetic field of a bar magnet, showing the north and south poles of the magnet.

6 Figure 14.18 shows two bar magnets placed with their north ends together.

  a Draw the magnetic field lines associated with this arrangement.

FIGURE 14.18

  b Are there any points in Figure 14.18 where the magnetic field is zero? If so, where?

7 Describe how magnetic field strength varies with distance from a long straight current-carrying wire.

8 What are the units of the constant μ_0? Write these units in fundamental units.

9 Explain why most materials are not magnetic. In what ways are ferromagnets different?

10 Draw the magnetic field lines due to a wire carrying current directly downwards, as seen from above.

11 A current-carrying wire lies in a north–south line. A small magnetic compass needle is placed beneath the wire.

  a Predict what will happen to the compass needle when the current in the wire flows:

    i from north to south.

    ii from south to north.

  b If the compass were placed above the wire, what difference would it make to the direction in which the compass needle pointed?

12 A small magnetic compass needle is placed at the centre of a single loop of wire that carries an electric current of 2 A, as shown in Figure 14.19. The loop has a radius of 2.0 cm. The plane of the coil is vertical and east–west. The magnitude of the magnetic field of the loop is much greater than Earth's magnetic field at this location.

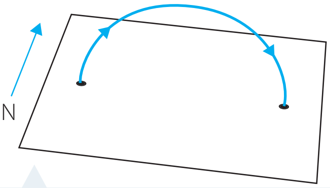


FIGURE 14.19

- **a** In which direction will the north pole of the compass needle point?
- **b** If the direction of the current in the wire is reversed, in which direction will the compass needle now point?

13 Calculate the magnetic field a distance 5.0 cm from a wire carrying a current of 0.5 A.

14 Calculate the distance from a wire carrying a current of 15 A at which the field is 1.0 mT.

15 Calculate the current necessary in a long straight wire to produce a magnetic field of 50 μT, approximately that due to Earth, at a distance of 1 cm.

16 Figure 14.20 shows a solenoid that carries a current in the direction as shown in the diagram.

- **a** Copy the diagram and carefully draw the magnetic field of the solenoid.
- **b** Predict the effect on the magnetic field within the solenoid of:
  - **i** increasing the current in the solenoid.
  - **ii** reversing the direction of the current in the solenoid.
  - **iii** increasing the number of turns of wire in the solenoid without changing the length of the solenoid.

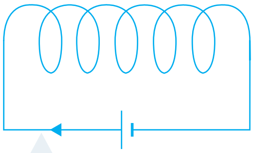


FIGURE 14.20

17 Calculate the magnetic field inside a 20 cm long solenoid with 500 turns when it carries a current of 2.0 A.

18 A 15 cm long solenoid produces a magnetic field of 0.015 T when carrying a current of 1.0 A. Calculate the number of turns, N, for this solenoid.

19 A solenoid has an internal magnetic field of 0.10 T. Predict the effect on the field when:

- **a** the current through the solenoid is doubled.
- **b** the radius of the solenoid is doubled without changing the number of turns.
- **c** the length of the solenoid is doubled without changing the number of turns.

20 Compare and contrast electric and magnetic fields.

» END-OF-MODULE REVIEW

MODULE ④: ELECTRICITY AND MAGNETISM

1 Chen is conducting an experiment on electric charge. He rubs a comb with a woollen cloth, and finds that the comb is then able to pick up polystyrene beads. Beads that come into contact with the comb become charged.

- **a** Explain how the comb becomes charged by rubbing with the cloth.
- **b** Explain how the charged comb is able to pick up neutral beads. Draw a diagram.
- **c** After the experiment, two beads (A and B) are found to repel each other, but each will attract a third bead (C). What can you deduce about the sign of charge on the three beads?

2 A Van de Graaff generator has been charged so that its 25 cm radius dome has a charge of 0.10 C. The dome is spherical, so it produces a field the same as that of a point charge located at the centre of the dome.

- **a** Draw a field line diagram for the dome.
- **b** Calculate the electric field at a distance of 10 cm from the surface of the dome.
- **c** Calculate the force experienced by a small object with charge +1.0 nC at this position.
- **d** Sketch the force acting on this object as a function of distance from the dome.

3 A Van de Graaff generator has been charged so that its dome has a positive charge. A positively charged polystyrene bead is placed close to the dome.

- **a** If allowed to move, will the bead accelerate towards or away from the dome?
- **b** Does the potential energy of the dome–bead system increase or decrease as it accelerates?
- **c** Does the bead move in the direction of increasing or decreasing potential?
- **d** Repeat parts **a**–**c** for a negatively charged bead.

4 A pair of parallel plates is used to create a uniform electric field. The potential difference between the plates is 12 V, and the plates are spaced 1.0 cm apart.

- **a** Draw a field line diagram for this arrangement.
- **b** Draw equipotential lines for this arrangement.
- **c** Calculate the electric field between the plates.

5 A 1.5 V rechargeable battery is rated at 2000 mAh.

- **a** How much potential energy does each electron gain as it flows through the battery?
- **b** How many electrons in total can flow through the battery before it is 'flat'?
- **c** For how long could this battery produce a current of 0.5 A?
- **d** A length of wire is connected across the terminals of the battery. Describe how changing each of the following characteristics of the wire would affect the length of time before the battery became flat.
  - **i** the length of the wire
  - **ii** the diameter of the wire

6 A circuit is shown in Figure 14.21. The battery supplies a potential difference of 3 V. Model the globes as identical resistors with resistance 5 Ω each when they are turned on.

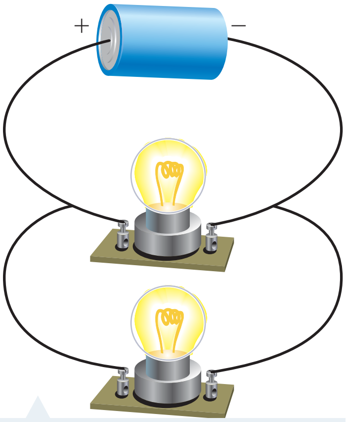


FIGURE 14.21 A circuit

- **a** Draw a circuit diagram for the circuit shown in Figure 14.21.
- **b** Calculate the voltage across and current through each globe.
- **c** Calculate the power used by each globe.
- **d** The globes are disconnected and then reconnected in series. Repeat parts **a**–**c** for this case.
- **e** In which case would the battery be drained faster? Justify your answer.

7 A student measures the voltage–current characteristics for a component, as shown in Table 14.2.

- **a** Draw a graph showing the I–V characteristics of this component.
- **b** Is this component ohmic or non-ohmic? Explain your answer.
- **c** Calculate the resistance of this component.
- **d** Explain the difference between a positive and a negative current.

TABLE 14.2

V (V)	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5	6
I (mA)	–12	–10	–8	–6	–4	–2	0	2	4	6	8	10	12

8 Materials can be classified by how they respond to magnetic fields.

- **a** Define 'ferromagnetic'.
- **b** Describe how a ferromagnetic material behaves in a magnetic field. Include a diagram showing magnetic domains in your answer.
- **c** Describe how a ferromagnetic material can be used to make a permanent magnet.

9 Magnetic field lines are used to visualise magnetic fields.

- **a** In what ways are magnetic field lines similar to electric field lines, and in what ways are they different?
- **b** Draw a magnetic field line diagram for a bar magnet.
- **c** Draw a magnetic field line diagram for a length of current-carrying wire. Show the direction of the current and the field lines.
- **d** Draw a magnetic field line diagram for a single loop of current. Show the direction of the current and the field lines.

10 A 50 m length of wire is initially laid out straight. It carries a current of 1.0 A.

- **a** Calculate the magnetic field at a distance of 2.0 cm from the wire.
- **b** Sketch a graph of the magnetic field as a function of distance from the wire.

The wire is now coiled into a solenoid (coil) of length 20 cm and radius 2.0 cm.

- **c** Calculate the number of loops in the coil.
- **d** Calculate the magnetic field in the centre of the coil.
- **e** Sketch a graph of the magnetic field as a function of radial position within the coil.

DEPTH STUDY SUGGESTIONS

- → Make a fluid model of a circuit using a pump and pipes.
- → Conduct a literature review and find out how semiconductor materials work.
- → Measure *I*–*V* characteristics for various components, such as resistors, globes, diodes, LEDs and motors.
- → Extend Investigation 13.1 (pages 371–3) to look at variation of current with area, or with type of metal.
- → Investigate more complex resistive circuits, including combinations of series and parallel resistors.
- → Build a circuit that performs a useful task.
- → Research Earth's electric and magnetic fields.
- → Research animals that are sensitive to electric or magnetic fields.
- → Conduct an energy audit of your home or school, or find out more about how the electricity is produced.
- → Measure the magnetic field in different places, and around different devices.

Appendix 1: SI and non-SI units

International System of Units (SI)

The international body that decides the appropriate units to be used for the various physical quantities is the Conférence Générale des Poids et Mesures (CGPM). The system of units approved by the CGPM and now widely used by the scientific community throughout the world is known as Système International d'Unités (abbreviated SI).

In your experimental work you should use SI units (or their multiples or submultiples).

The SI consists of seven base units and two supplementary units. All other derived units are based on these nine fundamental units.

The base and supplementary units, together with the derived units with special names that might be relevant to your experimental work, are listed in Tables A1.1 and A1.2.

TABLE A1.1 SI base units

PHYSICAL QUANTITY	NAME OF UNIT	ABBREVIATION
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

As you become familiar with each new unit you should make a practice of correctly using its abbreviated form.

The internationally recognised prefixes for the SI units together with their abbreviations are given in Table A1.3.

TABLE A1.2 SI supplementary units derived from SI base units

NAME	SYMBOL	QUANTITY	EQUIVALENTS	SI BASE UNIT EQUIVALENTS
hertz	Hz	frequency	1/s	s^{-1}
radian	rad	angle	m/m	dimensionless
steradian	sr	solid angle	m^2/m^2	dimensionless
newton	N	force, weight	$kg\ m/s^2$	$kg\ m\ s^{-2}$
pascal	Pa	pressure, stress	N/m^2	$kg\ m^{-1}\ s^{-2}$
joule	J	energy, work, heat	N m	$kg\ m^2\ s^{-2}$
watt	W	power, radiant flux	J/s	$kg\ m^2\ s^{-3}$
coulomb	C	quantity of electric charge	A s	A s

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NAME	SYMBOL	QUANTITY	EQUIVALENTS	SI BASE UNIT EQUIVALENTS
volt	V	electromotive force, electrical potential difference, electric potential voltage	J/C	kg m^{2} s^{-3} A^{-1}
farad	F	electrical capacitance	C/V s/V	kg^{-1} m^{-2} s^{4} A^{2}
ohm	V	electrical resistance, impedance, reactance	V/A	kg m^{2} s^{-3} A^{-2}
siemens	S	electrical conductance	1/V A/V	kg^{-1} m^{-2} s^{3} A^{2}
weber	Wb	magnetic flux	J/A	kg m^{2} s^{-2} A^{-1}
tesla	T	magnetic field strength, magnetic flux density	V s/m^{2} Wb/m^{2} N/(A m)	kg s^{-2} A^{-1}
degree Celsius	°C	temperature relative to 273.15 K	K – 273.15	K – 273.15
lumen	lm	luminous flux	cd sr	cd
lux	lx	illuminance	lm/m^{2}	m^{-2} cd
becquerel	Bq	radioactivity (decays per unit time)	1/s	s^{-1}
gray	Gy	absorbed dose (of ionising radiation)	J/kg	m^{2} s^{-2}
sievert	Sv	equivalent dose (of ionising radiation)	J/kg	m^{2} s^{-2}

TABLE A1.3 Prefixes for SI units

PREFIX	ABBREVIATION	VALUE
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

Non-SI units

A number of non-SI units are still in use in scientific literature for a variety of reasons. Some of these are being phased out, but others are likely to remain in use. The more common non-SI units that you might come across are listed in Table A1.4.

TABLE A1.4 Non-SI units

PHYSICAL QUANTITY	UNIT	ABBREVIATION	CONVERSION TO SI UNITS
time	minute hour day year	min h d y	60 s 3.6×10^3 s 8.64×10^4 s 3.156×10^7 s
mass	unified mass unit tonne	u t	1.661×10^{-27} kg 1000 kg
angle	degree	°	dimensionless
energy	electron-volt kilowatt hour	eV kW h	1.602×10^{-19} J 3.60×10^3 J
pressure	millimetre of mercury	mmHg	133.3 Pa
charge	elementary or electronic charge	e	1.602×10^{-19} C
source activity	curie	Ci	3.7×10^{10} Bq
radiation absorbed dose equivalent dose	rad rem	rad rem	0.01 Gy 0.01 Sv

Using SI units

There are certain conventions now adopted widely in scientific literature when SI units are being used. Some of the more important ones are given below.

1. When recording a measurement, write the unit in full or use the recommended abbreviation (e.g. 25 metre or 25 m). Using abbreviations save space and time. Notice the space between the numeral and the unit.
2. SI units named after scientists:
   - **a** If the full word is used, it starts with a lower case letter (e.g. 10 newton, 7 joule, 105 pascal, 50 hertz)
   - **b** If the abbreviation is used, it is (or at least commences with) a capital letter (e.g. 10 N, 7 J, 105 Pa, 50 Hz).
   - **c** Measurements are written as products. '3 kg' means 'the product of 3 and the mass known as a kilogram', just as '3*x*' in maths means the product of 3 and *x*. Therefore 's' is not added to units (e.g. 5 kg not 5 kgs).
   - **d** A full-stop is not placed after the abbreviation of a unit, unless it is at the end of a sentence.
   - **e** When units are combined as a quotient (e.g. metre per second), a solidus (/) or negative index may be used. So m/s or m s^{-1} are both acceptable, though the latter is used more widely. Never use more than one solidus in a unit as in m/s/s for acceleration, which should be m/s^2 or m s^{-2}. It is ambiguous, just as writing 36/6/3 in maths is ambiguous. (This could mean 2 or 18.)

Converting between units

Converting between units

Learn more about converting between units.

Treat the unit as a multiplier. Use the prefixes in Table A1.3 also as multipliers. For example, 4 kg is the same as:

$4 \times k \times g = 4 \times 1000 \times g = 4000$ g

There are 4000 g in 4 kg.

1500 cm is the same as $1500 \times (10^{-2}$ m$) = 15$ m.

Appendix 2: Some important physical quantities

From time to time you will need to find a value of a physical property from a reputable source. These might include finding:

- the value of a physical constant, such as Newton's universal gravitational constant or the electric constant.
- a physical property, such as boiling point or refractive index, which is characteristic of a particular material.
- a conversion factor such as micrometres to metres, electron-volt to joule, unified mass unit to kilogram.

All physical quantities, including physical constants, are measured to very precise levels of accuracy.

Some important physical quantities, including some physical constants, are listed alphabetically in Table A2.1. They are given to four significant figures. The uncertainty in most of these figures is better than six-figure accuracy. They are taken from sources such as the National Institute of Science and Technology (NIST). NIST is a specialist organisation dedicated to metrology (study of measurement).

NIST physical reference data

The National Institute of Standards and Technology (NIST) provides a wide range of data, including Standard reference data (SRF). For example, click on 'Other NIST Data' to enter the NIST Gateway.

NIST physical element laboratory

This website provides atomic and nuclear data for every element.

TABLE A2.1 Physical constants, physical measures and conversion factors

PHYSICAL CONSTANTS	
Avogadro constant, N_A	6.022×10^{23} mol^{-1}
Coulomb law constant, $\frac{1}{4\pi\omega_0}$	8.988×10^{9} N m^{2} C^{-2}
Universal gravitation constant, G	6.674×10^{-11} m^{3} kg^{-1} s^{-2}
Permittivity of free space electric constant, ε_0	8.854×10^{-12} F m^{-1}
Permeability of free space magnetic constant, μ_0	$4\pi \times 10^{-7}$ H m^{-1} = 12.57×10^{-7} H m^{-1}
Planck constant, h	6.626×10^{-34} J s = 4.136×10^{-15} eV s
Speed of electromagnetic radiation in free space, c	2.998×10^{8} m s^{-1}
PHYSICAL MEASURES	
Mass of electron	9.109×10^{-31} kg = 5.486×10^{4} u
Mass of proton	1.6726×10^{-27} kg
Mass of neutron	1.6749×10^{-27} kg
Rydberg constant (for hydrogen), R_H	1.097×10^{7} m^{-1}
Gravitational field strength at Earth's surface), g	(9.80 +/– 0.3) N kg^{-1}
Acceleration due to gravity at Earth's surface, g	(9.80 +/– 0.3) m s^{-2}
Mass of Earth	5.976×10^{24} kg
Mass of Moon	7.348×10^{22} kg
Mass of Sun	1.989×10^{30} kg
Period of rotation of Earth	8.616×10^{4} s
Radius of Earth (equatorial)	6.378×10^{6} m
Radius of Earth (mean)	6.371×10^{6} m
Radius of Earth's orbit about Sun (mean)	1.496×10^{11} m
Radius of Moon's orbit around Earth (mean)	3.844×10^{8} m
Radius of Sun	6.960×10^{8} m
Solar constant (mean)	1.370×10^{3} W m^{-2}

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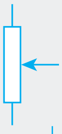
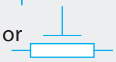
PHYSICAL MEASURES	
Density of water (pressure and temperature dependent)	9.982×10^3 kg m^{-3}
Air density (pressure and temperature dependent)	1.292×10^3 kg m^{-3}
Air pressure (temperature dependent)	1.013×10^5 Pa
Speed of sound in air at 0°C	331.4 m s^{-1}
CONVERSION FACTORS	
Absolute zero, 0 K	−273.15°C
Unified mass unit, u	1.661×10^{-27} kg = 931.5 MeV
Electron-volt, eV	1.602×10^{-19} J
Elementary electron charge, *e*	1.602×10^{-19} C
Coulomb	6.242×10^{18} elementary charges

Appendix 3: Electric and electronic symbols

TABLE A3.1 Electric and electronic symbols

COMPONENT GROUP	COMPONENT	SYMBOL
Sources of emf	Cell	
	Battery, DC power supply	
	Variable DC power supply	
	AC power supply	
Resistance	Resistor	
	Rheostat, resistor with sliding contact, potentiometer, voltage divider	 or 
	Variable resistor	
	Light-dependent resistor (LDR)	
	Filament globe	
	Thermistor	
	Fuse	 or 
Capacitance	(Non-polarised) capacitor	
	Variable capacitor	

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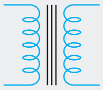
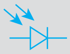
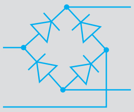
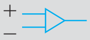
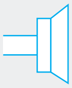
COMPONENT GROUP	COMPONENT	SYMBOL
	Polarised capacitor, electrolytic capacitor	
Transformer	Iron-cored transformer (one secondary winding)	
Diodes	Junction diode	
	Zener diode	
	Photodiode	
	Light-emitting diode (LED)	
	Four-diode bridge	
Meters	Ammeter	
	Voltmeter	
	Galvanometer	

COMPONENT GROUP	COMPONENT	SYMBOL
	Cathode ray oscilloscope (CRO)	
Amplifiers	Voltage amplifier	
	Operational amplifier (op amp)	
Transducers	Motor	
	Microphone	
	Loudspeaker	
External connections	Earth	
Circuit connections	Non-connected leads	
	Connected leads	 or 

Appendix 4: Resistance codes

A resistor is a physical object or circuit element that has resistance. The resistance of an actual, physical resistor is indicated in one of two ways:

- a colour code: a set of stripes on the resistor to indicate the resistance and the tolerance or uncertainty in the value
- a number–letter code: numbers with a letter to mark the decimal point.

Resistance colour code

The resistor is marked with three or four colour bands painted on the resistor near one end as shown in Figures A4.1 and A4.2.

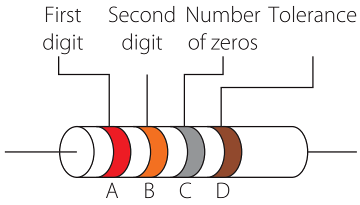


FIGURE A4.1 A resistor marked with four coloured bands

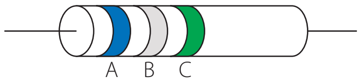


FIGURE A4.2 A resistor marked with three coloured bands

The colour of band A nearest the end is the first digit in the resistance value. The colour of band B represents the second digit. The colour of band C gives the number of zeros to follow these two digits (Table A4.1).

TABLE A4.1 Colour code for digits

DIGIT	COLOUR
0	Black
1	Brown
2	Red
3	Orange
4	Yellow
5	Green
6	Blue
7	Violet
8	Grey
9	White

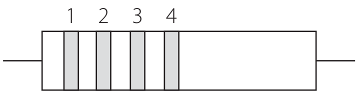
The fourth band from the end indicates the tolerance or percentage uncertainty in the resistance value (Table A4.2).

TABLE A4.2 Colour code for tolerances of resistors

TOLERANCE (%)	COLOUR
1	Brown
2	Red
5	Gold
10	Silver
20	No band D shown

If you hold the resistor so the stripes are on the left, you may find it easier to work out the resistance and the tolerance. If there are only three stripes, that is there is no tolerance band, the percentage uncertainty is 20%.

Figure A4.3 summarises the colour code details.



Colour	Value	Value	Multiply by	Tolerance
Black	0	0	1	Red = +/–2%
Brown	1	1	10	Gold = +/–5%
Red	2	2	100	Silver = +/–10%
Orange	3	3	1000	No band = +/–20%
Yellow	4	4	10 000	This gives the maximum error in the value of the resistor.
Green	5	5	100 000	
Blue	6	6	1 000 000	
Violet	7	7	not used	
Grey	8	8		
White	9	9		

FIGURE A4.3 Summary of resistance colour code system

Examples

Stripe (reading from the stripe nearest to the end):

- A–B–C: yellow (4), violet (7), orange (1000), gold (±5) ⇒ (47 000 ± 5%) Ω% or (47 ± 3) kΩ
- A–B–C–D: orange (3), white (9), brown (10), silver (±10%) ⇒ (390 ± 10%) Ω or (390 ± 39) Ω
- A–B–C: brown (1), green (5), black (1) and no fourth colour ⇒ (15 ± 20%) Ω or (15 ± 3) Ω

Resistance number–letter codes

In this system, which is often used on circuit diagrams, the numeral may have a letter in front of, behind or between the digits. The resistance is given to two significant figures. The letters R, K and M are used as multipliers: R for '1', K for '$\times 10^3$' or M for '$\times 10^6$'. The letters R, K or M are used to show where the decimal point goes.

Tolerances are given letter codes at the end (Table A4.3).

TABLE A4.3 Letter code for tolerances of resistors

TOLERANCE (%)	LETTER
1	F
2	G
5	J
10	K
20	M

Examples

- 2R5J $\Rightarrow$ 2.5 Ω (2.5 $\pm$ 5%) Ω or (2.5 $\pm$ 0.1) Ω
- 47KM $\Rightarrow$ 47 kΩ (47 $\pm$ 20%) kΩ or (47 $\pm$ 10) Ω
- M22K $\Rightarrow$ (0.22 $\pm$ 10%) MΩ or (220 $\pm$ 22) kΩ

Appendix 5: Scientific notation and significant figures

When we measure quantities on instruments, the last figure in the measurement is usually uncertain. This is because of the in-built uncertainty in the instrument itself, even if we have avoided errors such as parallax error. Thus, if our electronic balance gives a reading of, say, 10.514 g, then we need to be aware that the last figure (4) will be uncertain. It is likely that the true mass is somewhere between 10.512 g and 10.516 g.

Significant figures show how many digits in the reading are meaningful. The last figure is always deemed to be uncertain. By keeping track of the number of significant figures in all the instrumental measurements used to calculate a quantity, we can determine the extent to which our answer correctly represents the accuracy of our instruments. To reflect this accuracy, we always give our answer to the same number of significant figures as the least accurate data used. (If we use an even lower number of significant figures than this, then we might as well use less accurate instruments!)

The rules

1. Every non-zero digit is significant. For example, 3.78 and 294 both have three significant figures.
2. Every zero in the middle of a reading is significant. For example, the mass reading of 10.514 g has five significant figures.
3. Every zero to the right of a reading is significant. For example, 31.20 has four significant figures. The exception to this is a number with no decimal point and a trail of zeros, such as 500 kg. This volume may have one, two or three significant figures. To avoid this ambiguity, we must be given more information, stated in standard form. For example, if the volume is provided as 5.00×10^2 kg, then we know that it has three significant figures. If this is not clarified, we assume that it has the maximum number of significant figures.
4. Every zero before a number is not significant, and only shows the place value. For example, 0.005 only has one significant figure and 0.0090 has two significant figures. Again, rewriting these numbers in standard form clarifies this. (These numbers would be written as 5×10^{-3} and 9.0×10^{-3} respectively.)

Calculations

Rounding off

For rounding off an answer to a given number of significant figures, we examine the next figure on the right only. If it is 5 or more, then we round up.

Example

If we need to round off 10.9847 to:

- four significant figures, then we write 10.98 (8 is the fourth significant figure. The next figure on the right of 8 is 4, so we do not round up.)
- three significant figures, then we write 11.0. (9 is the third significant figure. The next figure on the right of 9 is 8, so we round up. Adding 1 to the 9 causes 10.9 to become 11.0.)

Adding or subtracting numbers

The answer cannot have more significant figures after the decimal point (i.e. decimal places) than the least accurate data.

Example

73.251 ← three decimal places
+11.4 ← one decimal place, therefore, this is the least accurate data
84.651

This answer cannot have more than one decimal place, so must be rounded off to 84.7.

Multiplying or dividing numbers

Again, the answer cannot have more significant figures than the least accurate data.

Example

$7.53 \times 6.0958 + 45.9$ (to three significant figures)

The least accurate figure (7.53) has three significant figures, so we round off the answer to three significant figures.

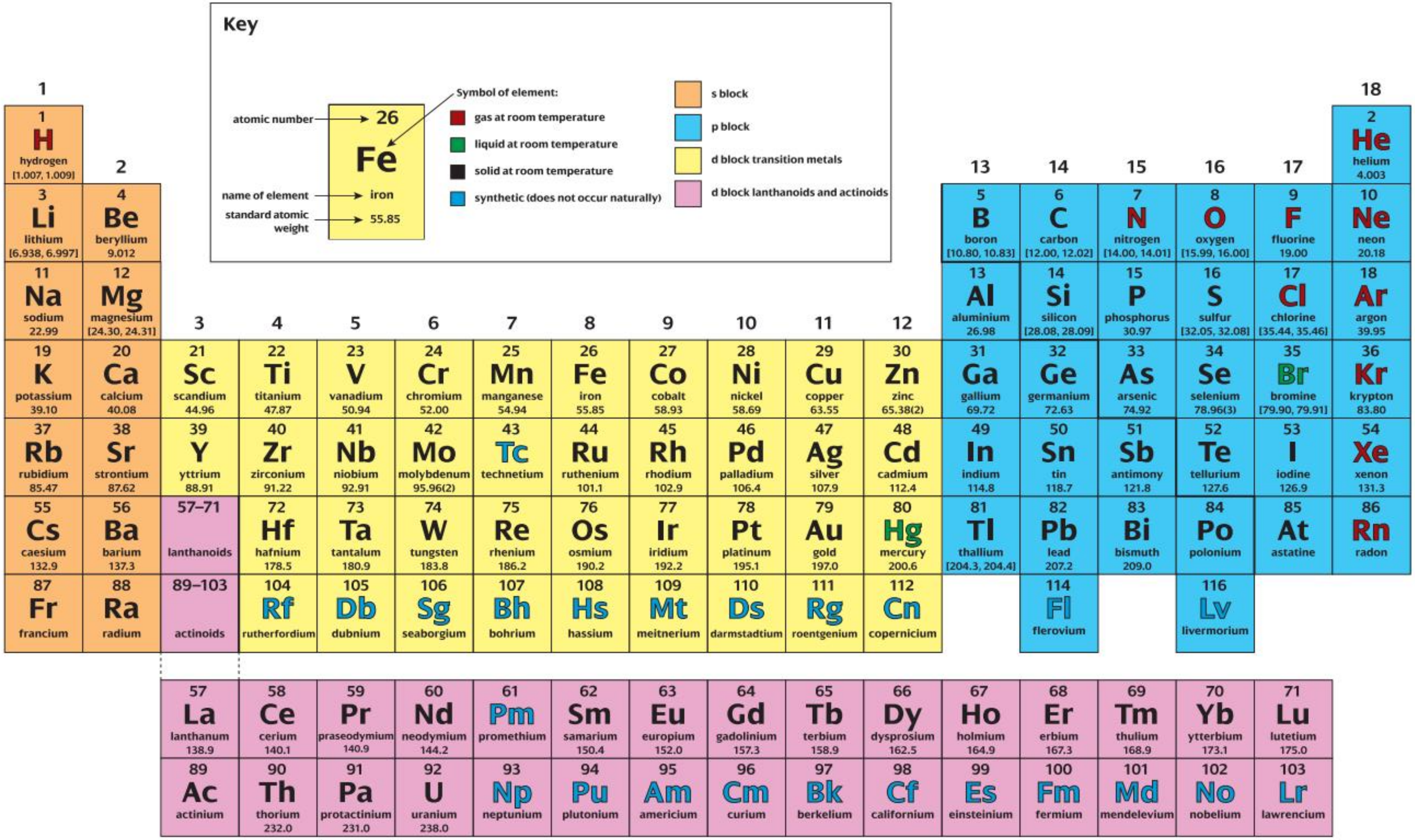
Use of data such as the universal gravitational constant, *G*

Ideally, physical data such as the universal gravitational constant should be quoted to at least the same number of significant figures as the experimental data. However, physical data is not taken into account when determining the number of significant figures in the answer – only experimental data such as masses and volumes is considered.

Advice

Data provided in a question or in a test should not be rounded off prior to its use in a calculation. For example, if G is given as 6.674×10^{-11} m^3 kg^{-1} s^{-2}, then in a calculation we write 6.674×10^{-11}, not 6.7×10^{-11}. Similarly, if a volume is given as 20.00 mL, we write 20.00 in the calculation, not 20 (even though on our calculator we would only enter the number as 20). This makes the accuracy of the reading clear for the purposes of determining the number of significant figures we provide in the answer.

Appendix 6: Periodic table of elements



Key

atomic number → 26

Fe

name of element → iron

standard atomic weight → 55.85

Symbol of element:

- 🟥 gas at room temperature
- 🟩 liquid at room temperature
- ■ solid at room temperature
- 🟦 synthetic (does not occur naturally)

- 🟧 s block
- 🟦 p block
- 🟨 d block transition metals
- 🟪 d block lanthanoids and actinoids

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H hydrogen [1.007, 1.009]																	2 He helium 4.003
3 Li lithium [6.938, 6.997]	4 Be beryllium 9.012											5 B boron [10.80, 10.83]	6 C carbon [12.00, 12.02]	7 N nitrogen [14.00, 14.01]	8 O oxygen [15.99, 16.00]	9 F fluorine 19.00	10 Ne neon 20.18
11 Na sodium 22.99	12 Mg magnesium [24.30, 24.31]											13 Al aluminium 26.98	14 Si silicon [28.08, 28.09]	15 P phosphorus 30.97	16 S sulfur [32.05, 32.08]	17 Cl chlorine [35.44, 35.46]	18 Ar argon 39.95
19 K potassium 39.10	20 Ca calcium 40.08	21 Sc scandium 44.96	22 Ti titanium 47.87	23 V vanadium 50.94	24 Cr chromium 52.00	25 Mn manganese 54.94	26 Fe iron 55.85	27 Co cobalt 58.93	28 Ni nickel 58.69	29 Cu copper 63.55	30 Zn zinc 65.38(2)	31 Ga gallium 69.72	32 Ge germanium 72.63	33 As arsenic 74.92	34 Se selenium 78.96(3)	35 Br bromine [79.90, 79.91]	36 Kr krypton 83.80
37 Rb rubidium 85.47	38 Sr strontium 87.62	39 Y yttrium 88.91	40 Zr zirconium 91.22	41 Nb niobium 92.91	42 Mo molybdenum 95.96(2)	43 Tc technetium	44 Ru ruthenium 101.1	45 Rh rhodium 102.9	46 Pd palladium 106.4	47 Ag silver 107.9	48 Cd cadmium 112.4	49 In indium 114.8	50 Sn tin 118.7	51 Sb antimony 121.8	52 Te tellurium 127.6	53 I iodine 126.9	54 Xe xenon 131.3
55 Cs caesium 132.9	56 Ba barium 137.3	57–71 lanthanoids	72 Hf hafnium 178.5	73 Ta tantalum 180.9	74 W tungsten 183.8	75 Re rhenium 186.2	76 Os osmium 190.2	77 Ir iridium 192.2	78 Pt platinum 195.1	79 Au gold 197.0	80 Hg mercury 200.6	81 Tl thallium [204.3, 204.4]	82 Pb lead 207.2	83 Bi bismuth 209.0	84 Po polonium	85 At astatine	86 Rn radon
87 Fr francium	88 Ra radium	89–103 actinoids	104 Rf rutherfordium	105 Db dubnium	106 Sg seaborgium	107 Bh bohrium	108 Hs hassium	109 Mt meitnerium	110 Ds darmstadtium	111 Rg roentgenium	112 Cn copernicium		114 Fl flerovium		116 Lv livermorium		
		57 La lanthanum 138.9	58 Ce cerium 140.1	59 Pr praseodymium 140.9	60 Nd neodymium 144.2	61 Pm promethium	62 Sm samarium 150.4	63 Eu europium 152.0	64 Gd gadolinium 157.3	65 Tb terbium 158.9	66 Dy dysprosium 162.5	67 Ho holmium 164.9	68 Er erbium 167.3	69 Tm thulium 168.9	70 Yb ytterbium 173.1	71 Lu lutetium 175.0	
		89 Ac actinium	90 Th thorium 232.0	91 Pa protactinium 231.0	92 U uranium 238.0	93 Np neptunium	94 Pu plutonium	95 Am americium	96 Cm curium	97 Bk berkelium	98 Cf californium	99 Es einsteinium	100 Fm fermium	101 Md mendelevium	102 No nobelium	103 Lr lawrencium	

NUMERICAL ANSWERS

CHAPTER 2: MOTION IN A STRAIGHT LINE

WORKED EXAMPLE 2.1

1 a 250 m
b 400 m
2 a 900 m
b 400 m

WORKED EXAMPLE 2.2

35 km

WORKED EXAMPLE 2.3

73 km h^{-1} to two significant figures

WORKED EXAMPLE 2.4

1 a 32 m
b 16 m
2 6 m s^{-1}

WORKED EXAMPLE 2.5

1 39 s
2 a Yes
b 4 s

CHECK YOUR UNDERSTANDING 2.1

5 120 km h^{-1}
6 a 0 m
b 0 m s^{-1}
c 12.0 m
d 2.9 m s^{-1}
7 a 43 km h^{-1}
8 2 m 30 s
9 160 km h^{-1}

WORKED EXAMPLE 2.6

1 8.0 m s^{-2}
2 1.9 m s^{-2}

WORKED EXAMPLE 2.7

1 100 m
2 475 m
3 0.50 m s^{-2}

CHECK YOUR UNDERSTANDING 2.2

2 a i At A, 5 m s^{-2}
ii At B, 0 m s^{-2}
iii At C,-2 m s^{-2}
b 26 m
d just under 10 m s^{-1}
3 a 3.0 m s^{-2}
b 54 m
4 b i 2.5 m s^{-2}
ii 12.5 m s^{-1}
iii 125 m
5 a a distance of about 6.5 m
b 43.5 m
c 4.2 s
d -5.0 m s^{-2} to two significant figures

WORKED EXAMPLE 2.9

1 -4 m s^{-2}
2 1.4 s
3 77 m

WORKED EXAMPLE 2.10

1 64 m
2 548 m
3 a 0.32 m s^{-2}
b -0.67 m s^{-2}
5 50 s

WORKED EXAMPLE 2.11

2 4.5 s
3 99 m

CHECK YOUR UNDERSTANDING 2.3

1 a 50 m s^{-1}
b 5.1 s
c 19.6 m s^{-1}
d 3.6 s
2 a 51.75 m s^{-1}
b 1.3 s
c 30 m apart.
3 20.5 s
4 Pelican goes hungry

END-OF-CHAPTER REVIEW

5 a distance travelled
b acceleration
10 a 9.4 m s^{-1}
12 b 12 s
13 a 22.1 m s^{-2}
b 2.26 s
16 3.86 m s^{-2}
17 a 15.8 s for both objects
b 6.4 s for hammer

18 b i 0.46 m
  ii 10.92 m
  iii 10.0 s

CHAPTER 3: MOTION ON A PLANE

WORKED EXAMPLE 3.1

300 m north, 520 m east

300 m north, –520 m west

WORKED EXAMPLE 3.2

1 236 km 32° west of north

CHECK YOUR UNDERSTANDING 3.1

3 $d_N = 25.4$ km
  $d_E = 11.8$ km
4 $d_{NE} = 26.3$ km
  $d_{NW} = 9.6$ km
6 37.9° south of east (or 52.1° east of south)
7 b $\theta = 26.6°$

WORKED EXAMPLE 3.3

1 S30°E
2 849 m S75°E

CHECK YOUR UNDERSTANDING 3.2

2 a 81 km at about 30° west of north
  b 130 km at about 36° west of north
5 7.4 km N0.4°W
6 a (1) 5.5 km (2) 30.5 km (3) 60.5 km
  b (1) 5.5 km S (2) 27.2 km S, 12.5 km E (3) 0.2 km W
  c Average speed is 30.25 km h^{-1}. Average velocity is 0.1 km h^{-1}.

WORKED EXAMPLE 3.4

17.7 m s^{-1} N70.2°E

WORKED EXAMPLE 3.6

8.2 m s^{-1} away from the wall, at an angle of 11° to the perpendicular to the wall

CHECK YOUR UNDERSTANDING 3.3

2 –321 km h^{-1} north, 321 km h^{-1} south, –383 km h^{-1} west, 383 km h^{-1} east
3 476 km h^{-1}
4 a 28 km h^{-1}
  b 28 km h^{-1} N25°E
  c $v_N = 25$ km h^{-1}
    $v_E = 12$ km h^{-1}

WORKED EXAMPLE 3.7

–25 m s^{-1} N53°W

WORKED EXAMPLE 3.8

1 increases
2 2.24 m s^{-1}

WORKED EXAMPLE 3.9

maximum = 220 km h^{-1}

minimum = 20 km h^{-1}

CHECK YOUR UNDERSTANDING 3.4

2 4 squares to the right and –2 squares above (2 squares below)
3 a i 5.0 m s^{-1} south
    ii 13.3 m s^{-1} relative to the road
  b 5.0 m s^{-1} south or 5.0 m s^{-1} north
4 Spider's net velocity relative to the pipe is 1.95 m s^{-1} down; 2.2 m journey will take 1.13 s
5 a 207 km h^{-1} at about 15° west of north
  b 166 km h^{-1} at about 14° west of north

END-OF-CHAPTER REVIEW

4 a 30°
  b $x = \dfrac{\sqrt{3}}{2}l$
    $y = l/2$
5 293 m south, 156 m west
6 31.76 m; 28°
8 a 260 km (2.60×10^3 m) 23° south of west; 130 km h^{-1} (36 m s^{-1})
  b 604 km (6.04×10^5 m) 21° north of west; 412 km h^{-1} (114 m s^{-1})
  c (163 m) S13°W; 0.56 km h^{-1} (0.155 m s^{-1})
12 33.4°
13 38 cm E39°N
14 1.96 m E8.3°S
15 42 km h^{-1}
16 0
18 9.98 m s^{-1} parallel; 0.05 m s^{-1} perp.
19 40 km h^{-1} E60°N
20 81 km h^{-1} 52° south of east
21 20 m s^{-1} E

MODULE 1 REVIEW

1 a 128.33 km
  b 75 km west
  c 60 km h^{-1}
  d 35 km h^{-1}
2 a 3.9 km
  b 28 s
3 a 149 s (about 2.5 min)
  b 4.3 km
  c –5.56 m s^{-1} (–20 km h^{-1})
  d 5.56 m s^{-1} (20 km h^{-1})

4 a 19.6 m s^{-1} (down)
b 21.6 m s^{-1}
c 19.6 m
d 28.2 s
5 a 2.8 km
b 2980 m, or 3.0 km
c 8.74 s
6 a Direction without distance does not tell us how far he went.
b Leg 1: 30 km h^{-1}, Leg 2: 36 km h^{-1}
c $\theta = 22°$
8 b 545 km h^{-1} west and 778 km h^{-1} north
c 2.6 h
d 2.1 h
9 a 15 s
b 270 m
d 7.1 m s^{-1}

CHAPTER 4: DYNAMICS

CHECK YOUR UNDERSTANDING 4.1

9 a $F_{\text{tyre on road}} = -F_{\text{road on tyre}}$; frictional force (contact)
b $F_{\text{balloon 1 on balloon 2}} = -F_{\text{balloon 2 on balloon 1}}$; electrostatic force (field)
c $F_{\text{Earth on needle}} = -F_{\text{needle on Earth}}$; magnetic force (field)

WORKED EXAMPLE 4.2

8.9×10^3 N at an angle of 18.4° to the right of the direction the barge is pointing

WORKED EXAMPLE 4.3

1 $F_x = 43$ N
$F_y = 25$ N

WORKED EXAMPLE 4.4

1 39° north of west, or N51°W
2 34° below the horizontal

WORKED EXAMPLE 4.5

1 63° above the horizontal, to the right
2 636 N

CHECK YOUR UNDERSTANDING 4.2

1 100 N (right)
2 a 26 N
b 15 N
3 b 118 N
c 27° below the horizontal
4 500 N at an angle of 65° below the horizontal
5 a 20 N
b 10 N
6 180 N at 56° right of the forwards direction
8 356 N E1°S
9 a 1.0 N in the x direction
b 6.4 N 129° from the $+x$ axis
c 3.5 N 98° from the $+x$ axis.
d 1.5 N straight up

WORKED EXAMPLE 4.7

2 5.7×10^3 N backwards

CHECK YOUR UNDERSTANDING 4.3

1 Yes
4 a 5.0 N 37° down and to the right
b 2.8 N 48° below the horizontal and to the right
c 4.3 N 72° below the horizontal and to the right
5 98 N

WORKED EXAMPLE 4.8

1 150 N down the slide

CHECK YOUR UNDERSTANDING 4.4

2 b $F_N = 1.1 \times 10^4$ N (perpendicular to the slope, upwards)
$F_f = 3.0 \times 10^3$ N (parallel to the slope, upwards)
3 Same as question 2
4 infinitesimally greater than 25°
5 a 5.9 m s^{-2}
b 5.859 N
c 2.6 N up the slope

CHAPTER REVIEW

3 No
4 500 N
10 a $F_{\text{ball on Earth}} = -F_{\text{Earth on ball}}$
b $F_{\text{ball on ground}} = -F_{\text{ground on ball}}$
11 a $F_{\text{ball on Earth}} = -F_{\text{Earth on ball}}$
b $F_{\text{ball on air}} = -F_{\text{air on ball}}$
12 640 N E39°N, or 640 N N51°E
13 a $F_x = 22$ N, $F_y = 13$ N
b $F_x = 22$ N, $F_y = -13$ N
c $F_x = -22$ N, $F_y = 13$ N
d $F_x = -22$ N, $F_y = -13$ N
14 a i 262 N (down the slope)
ii 291 N ('into' the plane, perpendicular to it)
b i 262 N down the slope
ii 291 N perpendicular to the surface and upward
15 266 N N34°W
16 19 N; 21°
17 110 N N32°E

18 a 634 N
b 170 N
19 a T634 N
b –50 N
24 b i 3.0×10^3 N, away from ship
ii 5.2×10^3 N, downwards
iii 2.6×10^3 N
iv 1.5×10^3 N
v -2.6×10^3 N

CHAPTER 5: FORCES, ACCELERATION AND ENERGY

WORKED EXAMPLE 5.2

1 6.0 s
2 –0.50 N

WORKED EXAMPLE 5.3

1 2.5 N
2 5.0 N
3 7.5 N

WORKED EXAMPLE 5.4

1 0.87
2 No difference

CHECK YOUR UNDERSTANDING 5.1

2 a Static
b Static
c Static
3 a Kinetic
b Static
c Static
5 a 100 N
b 0.46
7 a 0.38
b $0.25\,\mathrm{m\,s^{-2}}$ in the direction Harriet is pushing

WORKED EXAMPLE 5.6

1 2.4×10^5 N
2 7.1 s

CHECK YOUR UNDERSTANDING 5.2

2 a $1.65 \times 10^{14}\,\mathrm{m\,s^{-2}}$
b 1.3 mm
3 a 33.5 m
4 a 26 N
b 15 N
c $0.30\,\mathrm{m\,s^{-2}}$ N 30°E
5 b 1.45 N

6 a $6.67\,\mathrm{m\,s^{-1}}$
b 20 N
c 10 N

WORKED EXAMPLE 5.7

1 730 kJ
2 $71\,\mathrm{km\,h^{-1}}$

WORKED EXAMPLE 5.8

6.8 m

WORKED EXAMPLE 5.9

85 kJ, positive

WORKED EXAMPLE 5.10

25 cm

WORKED EXAMPLE 5.11

1.3 m

CHECK YOUR UNDERSTANDING 5.3

3 a Positive
b Negative
c Positive
d Negative
4 b Parabolic
5 $9.4\,\mathrm{m\,s^{-1}}$
6 3.7 kJ

WORKED EXAMPLE 5.13

0.82 kW

CHECK YOUR UNDERSTANDING 5.4

3 67 kW
4 a 140 kJ
b 13 kW
5 $2.6\,\mathrm{m\,s^{-1}}$
6 1500 W

CHAPTER REVIEW

4 200
5 No
7 1500 kg
10 a $20\,\mathrm{m\,s^{-2}}$ to the right
b $70\,\mathrm{m\,s^{-2}}$
12 a 630 N
c $2.5\,\mathrm{m\,s^{-2}}$ (down the slope)
13 a 510 N
b Same as a.
c $5.4\,\mathrm{m\,s^{-2}}$

15 a 2 J
   b 0.5 J
17 a $5.8\,m\,s^{-2}$
   b 5.8 N
19 b $6.5\,m\,s^{-2}$
   c $4.6\,m\,s^{-2}$
21 a 18 m
   b 7.8 m
22 $24.2\,m\,s^{-1}$
23 a 2.5 s
   b 2.9 kJ
   c 1.2 kW
25 a 110 N
   b $1.6\,m\,s^{-2}$
   c 1.1 kJ
   d $5.7\,m\,s^{-1}$
   e 301 W

CHAPTER 6: MOMENTUM, ENERGY AND SIMPLE SYSTEMS

WORKED EXAMPLE 6.1

1 $50\,kg\,m\,s^{-1}$
2 $3.3 \times 10^{2}\,m\,s^{-1}$

WORKED EXAMPLE 6.2

$1.7 \times 10^{-9}\,m\,s^{-1}$

WORKED EXAMPLE 6.3

$1.5\,m\,s^{-1}$ N30°W

CHECK YOUR UNDERSTANDING 6.1

3 $15\,m\,s^{-1}$
4 $-0.013\,m\,s^{-1}$
5 $12\,m\,s^{-1}$ N39°E

WORKED EXAMPLE 6.4

1 $v_1 = -3.8 \times 10^{5}\,m\,s^{-1}$
  $v_2 = 6.7 \times 10^{5}\,m\,s^{-1}$

CHECK YOUR UNDERSTANDING 6.2

3 a $2.3 \times 10^{6}\,m\,s^{-1}$
  b 28%
4 a $7.4\,m\,s^{-1}$
  b $-7.4\,m\,s^{-1}$
  c 2.8 m
5 a $53\,km\,s^{-1}$
  b 3000 GJ

WORKED EXAMPLE 6.5

1 $3.3 \times 10^{2}\,m\,s^{-1}$
2 impulse = 2.50 N s
  final speed = $56\,m\,s^{-1}$

WORKED EXAMPLE 6.6

$6.4 \times 10^{3}\,N$

WORKED EXAMPLE 6.7

−7.5 kN

CHECK YOUR UNDERSTANDING 6.3

3 a 2000 N s
  b $20\,m\,s^{-1}$
4 a $6.0\,m\,s^{-1}$
  b 1.0 s
5 a 3.00 N s
  b $22\,m\,s^{-1}$

WORKED EXAMPLE 6.8

1 $0\,m\,s^{-1}$
2 100% is converted to other forms.

WORKED EXAMPLE 6.9

1 $1.0 \times 10^{5}\,m\,s^{-1}$
2 $5.0 \times 10^{5}\,m\,s^{-1}$

CHECK YOUR UNDERSTANDING 6.4

3 a $2.1\,m\,s^{-1}$
  b 78%
4 $2.5\,m\,s^{-1}$; N 23° W

CHAPTER REVIEW

4 a Impulse
  b Work
6 horse
8 $3.0 \times 10^{4}\,kg\,m\,s^{-1}$
9 a 40
  b 40
13 a $-3.6 \times 10^{4}\,kg\,m\,s^{-1}$
   b $-1.8 \times 10^{5}\,N$
14 $6.75\,m\,s^{-1}$
17 a $9.86 \times 10^{4}\,kg\,m\,s^{-1}$
   b $-0.082\,m\,s^{-1}$ (−0.16 knots)
18 a $v_1 = 1.0 \times 10^{7}\,m\,s^{-1}$
     $v_2 = 2.0 \times 10^{7}\,m\,s^{-1}$
   b $2.0 \times 10^{7}\,m\,s^{-1}$

19 a 7.2 kg

b 1.44×10^3 J

c 4.1×10^4 N

d 3.5×10^{-3} s

20 a 14 m s^{-1} at N37°W

b -2.8×10^5 J

MODULE 2 REVIEW

1 a 2.2 kN

e 167 kJ

2 c 6.7 kJ

e 6.7 kJ s^{-1}, or 6.7 kW

5 a 124.6 km h^{-1}

b 575 kJ

d 99.6 kg m s^{-1}

6 a 74.8 km h^{-1}

b 192 kJ

d 14 kg m s^{-1}

7 a 54 N

b 81 J

c −81 J

9 a 28 N (parallel to ground); 28 N (perpendicular to ground, down)

b 25 N (parallel to ground); 43 N (perpendicular to ground, up)

c 10 N (below the horizontal and towards the pig); 72°

d 4.0 m s^{-2}

10 a 406 N (up the slope)

b 368 N (up the slope)

c 919 J

CHAPTER 7: WAVE CHARACTERISTICS

CHECK YOUR UNDERSTANDING 7.2

2 Transverse

5 a Transverse

b Transverse

c Longitudinal

d Longitudinal

CHECK YOUR UNDERSTANDING 7.3

5 approx. 8.5 min

WORKED EXAMPLE 7.2

1 15.7 m^{-1}

2 9.67×10^6 m^{-1}

CHECK YOUR UNDERSTANDING 7.4

4 No.

WORKED EXAMPLE 7.3

1 0.05 Hz

2 2.0×10^{-15} s

WORKED EXAMPLE 7.4

1 1.6×10^3 m s^{-1}

2 12.5 m s^{-1}

WORKED EXAMPLE 7.5

1 12.5 m s^{-1}

2 226.7 s^{-1}

CHECK YOUR UNDERSTANDING 7.5

5 50×10^3 m s^{-1}

6 50.0 s^{-1}

CHAPTER REVIEW

2 A medium

3 Seismic

4 a Transverse

5 0.500 m

10 3.14 m^{-1}

12 No

13 a 1.7×10^{-2} m

b 370 m^{-1}

14 20 km

15 a No

17 4.3×10^{14} s^{-1}

20 4.8×10^{14} s^{-1}

CHAPTER 8: WAVE BEHAVIOUR

CHECK YOUR UNDERSTANDING 8.1

4 340 m

WORKED EXAMPLE 8.1

1 a 40 m

b 50 m

2 Original: 0.372 m

New: 1.64 m

WORKED EXAMPLE 8.2

1 1.1×10^2 s^{-1}

2 11 cm

WORKED EXAMPLE 8.3

1 7.6 cm

2 1.5×10^6 s^{-1}

CHECK YOUR UNDERSTANDING 8.3

2 5.1 cm

CHECK YOUR UNDERSTANDING 8.4

1 Destructive

3 No

4 Yes

CHECK YOUR UNDERSTANDING 8.5

2 No

3 Yes

4 a Antinode

  b Node

CHECK YOUR UNDERSTANDING 8.6

3 b Resonance

CHAPTER REVIEW

6 Vacuum

7 Diffraction

10 a Yes

12 Yes

16 Resonance

CHAPTER 9: SOUND WAVES

WORKED EXAMPLE 9.1

1 500 Hz

2 4.2 cm

CHECK YOUR UNDERSTANDING 9.1

1 a 5.0 ms

  b 200 Hz

3 $f > 440$ Hz

4 2 cm

WORKED EXAMPLE 9.2

1 2.5 kHz

2 30

CHECK YOUR UNDERSTANDING 9.2

1 5.00 ms

3 1.00 ms

4 40 Hz

WORKED EXAMPLE 9.3

1 500 m

2 400

CHECK YOUR UNDERSTANDING 9.3

4 10 times

5 double

WORKED EXAMPLE 9.4

1 1.32 m

2 0.33 m

WORKED EXAMPLE 9.5

1 74 Hz

2 double

CHECK YOUR UNDERSTANDING 9.5

1 No difference

3 a $\frac{\lambda}{2}$

  b $\frac{\lambda}{2}$

  c $\frac{\lambda}{4}$

4 B and C

5 a 1.6 m

  b 125 Hz

  c increases

6 24 cm

7 60 Hz

WORKED EXAMPLE 9.6

1 0.27 m

2 a 2.2 m

  b 155 Hz

WORKED EXAMPLE 9.7

1 0.27 m

2 1550 Hz

CHECK YOUR UNDERSTANDING 9.6

1 D

2 3.2×10^2 m s^{-1}

3 4 resonances

4 1.7 cm

5 Higher

WORKED EXAMPLE 9.8

1 495 Hz or 505 Hz

2 24 Hz

WORKED EXAMPLE 9.9

1 2.19 kHz

2 1.46 kHz

CHECK YOUR UNDERSTANDING 9.7

2 1.21 kHz

3 No

5 Yes

CHAPTER REVIEW

7 0.33 m

9 a 170 Hz, 510 Hz, 850 Hz

  b 340 Hz, 680 Hz, 1020 Hz

10 a 1.20 m

b 300 Hz

11 a 106 m

b 0.53 m (53 cm)

c 53 cm

12 a 2.27 m

b F

c 0.57 m

d 150 Hz, 450 Hz, 750 Hz, 1050 Hz, …

15 a 142 Hz. 140 Hz to two significant figures

b 283 Hz; 425 Hz

c B

16 3 Hz

17 estimate 1.5 m

a 6 m

b 20 Hz

20 a 333.33 m s^{-1}

b 45 m s^{-1} rounded to 50 m s^{-1}

c $(3.3 \pm 0.5) \times 102$ m s^{-1}.

d Yes

21 a $f = 0.076$ Hz, or about 4 vibrations per minute

23 517 Hz

26 a 40 cm

b 250 Hz

c 750 Hz

CHAPTER 10: RAY MODEL OF LIGHT

WORKED EXAMPLE 10.1

1 5 cm

2 20

WORKED EXAMPLE 10.2

1 a inverted

b real

c −6.0 cm

d −1

2 a inverted

b real

c larger than 4 cm

d $M \geq 1$

WORKED EXAMPLE 10.3

1 a upright

b virtual

c 2.0

d 4.0 cm

2 a upright

b virtual

c 4.0

d 8.0 cm

WORKED EXAMPLE 10.4

1 a inverted

b real

c greater than 6.0 cm

d $M > 1.0$

2 a inverted

b real

c 2.7 cm

d 0.67

CHECK YOUR UNDERSTANDING 10.1B

4 20 cm

5 20 cm behind the mirror, virtual, upright and 10 cm high.

6 14 cm from the lens on opposite side of lens, real and diminished.

WORKED EXAMPLE 10.5

1 9.29°

2 23.6°

3 33.3°

CHECK YOUR UNDERSTANDING 10.2

7 a 19.6°

b larger

WORKED EXAMPLE 10.6

1 25.0 W m^{-2} (three significant figures)

2 625.0 W m^{-2} (three significant figures)

WORKED EXAMPLE 10.7

1 2500

2 4.67 km

CHECK YOUR UNDERSTANDING 10.4

1 a 7.96 rounds to 8.0 W m^{-2}

b 1.99 rounds to 2.0 W m^{-2}

c 0.50 W m^{-2}

d 0.29 W m^{-2}

3 a one quarter

b four times

c one hundredth

d one hundred times

END-OF-CHAPTER REVIEW

5 b i 1.47

ii 27.6°

8 a 24.8°

b 24

9 a 8.0 cm
b 24 cm, on the same side as the object (virtual)
10 a 24 cm
b 8.0 cm tall
11 a 80 cm high
b 110 cm
12 a 0.014 W m^{-2}
b 1.2 W m^{-2}
14 b Three images
15 at least 48.8° to the normal
16 83.4° from the normal
17 inverted and real

CHAPTER 11: THERMODYNAMICS

WORKED EXAMPLE 11.1

1 $c = 2.77 \times 10^3$ J kg C^{-1}; the substance is cooking oil
2 2.77×10^3 J kg

WORKED EXAMPLE 11.2

2 367°C (three significant figures)

CHECK YOUR UNDERSTANDING 11.3

4 30.5°C
5 0.3°C
6 3480 J kg^{-1}°C^{-1}

WORKED EXAMPLE 11.3

1 12 kJ to two significant figures

CHECK YOUR UNDERSTANDING 11.4

7 a 1500°C (actual melting point 1538°C)
b 2800°C (actual boiling point 2862°C)

WORKED EXAMPLE 11.4

1 W–2.2×10^5 J
2 5.0 hours

CHAPTER REVIEW

6 Solid
10 318.95 K
14 a 36 kJ
b 16 g (to two significant figures)
23 2.9 kJ hr^{-1} (to two significant figures)
24 9.5 kW
25 a 138° C

MODULE 3 REVIEW

7 b 55.9°
8 b 875 units
11 300 K

CHAPTER 12: ELECTRICITY AND MAGNETISM

WORKED EXAMPLE 12.1

1 negative

CHECK YOUR UNDERSTANDING 12.1

1 Positive protons and negative electrons
2 Electrons
3 Kristy is correct
5 a Same
b –1

WORKED EXAMPLE 12.3

to the left

WORKED EXAMPLE 12.4

1.5 N C^{-1}

WORKED EXAMPLE S 12.5

1 -5.6×10^{-11} N C^{-1}
2 4.8×10^{-4} m s^{-2} (down)

WORKED EXAMPLE 12.6

1.44×10^{-9} N C^{-1}

WORKED EXAMPLE 12.7

1 -7.2×10^{-8} N
2 -7.2×10^{-8} N

CHECK YOUR UNDERSTANDING 12.3

1 $\vec{F} = q\vec{E}$
3 4.5×10^9 N C^{-1}
4 -1.6710^{13} m s^{-2}
5 $9.0\ 10^6$ N
6 a 3.2×10^{-5} m
b 4.5×10^{-5} m
c 0.58×10^{-19} N

WORKED EXAMPLE 12.8

1 2.5×10^{-5} N
2 –7.1 nm

WORKED EXAMPLE 12.10

1.9×10^8 m s^{-1}

CHECK YOUR UNDERSTANDING 12.4

3 $-5.6\ 10^{-19}$ J
5 a Electron: 1.9×10^7 m s^{-1}
b Proton: 4.3×10^5 m s^{-1}
c Alpha: $3.1\ 10^5$ m s^{-1}
6 a 2.3×10^{-2} J
b Opposite of a
c No change

END-OF-CHAPTER REVIEW

8 5.0×10^{-8} N; 5.0×10^{-5} m s^{-2}
9 -6.25×10^{-18} V
10 $|F_{\text{a on b}}| = |F_{\text{b on a}}| = |F_{\text{b on c}}| = |F_{\text{c on b}}| > |F_{\text{a on c}}| = |F_{\text{c on a}}|$
11 a The upper plate
12 $+3e$
14 a 1.0×10^{10} N C^{-1}
b $1.6\ 10^{-9}$ N
16 a double the charge
b $\sqrt{2}$ m
17 -9.8×10^{-11} C
18 a -8.8×10^{-17} N
b 3.2×10^{-6} m
19 a -3.2×10^{-19} J
b negative
21 a 1.8×10^{-22} J
b -2.275×10^{-6} m
c 1.1 mV

CHAPTER 13: ELECTRIC CIRCUITS

WORKED EXAMPLE 13.1

1 1.3×10^{4} ions
2 180 C

CHECK YOUR UNDERSTANDING 13.1

3 a into the cell
b outwards
5 0.033 A
6 a 30 C
b 1.9×10^{20} electrons per minute
c decrease

WORKED EXAMPLE 13.2

1 5.2×10^{17} electrons
2 2160 J

WORKED EXAMPLE 13.3

0.12 A

CHECK YOUR UNDERSTANDING 13.2

2 432 V
3 0.8 Ω
4 b non-ohmic
c 250 Ω
5 a 6.7 Ω
b 20 V

WORKED EXAMPLE 13.5

1 54 kJ
2 1.7 MJ (to two significant figures)

WORKED EXAMPLE 13.6

1 0.48 A
2 3.5×10^{2} J (two significant figures)

CHECK YOUR UNDERSTANDING 13.3

3 3.6×10^{5} J
4 a 24 kJ
5 a 7.2 kC
b 236 V

WORKED EXAMPLE 13.8

$V_1 = V_2 = V_3 = -4$ V

WORKED EXAMPLE 13.9

$I_C = 0.3$ A and $I_D = 1.0$ A

WORKED EXAMPLE 13.10

1 $V_{R2} = IR_2 = 0.5\ \text{A} \times 4\ \Omega = 2\ \text{V}$
2 a 12 V
b Double what they were before, 8 V and 4 V

WORKED EXAMPLE 13.11

1 3 Ω
2 3 A
c $I_1 = 1$ A; $I_2 = 0.5$ A; $I_3 = 1.5$ A

CHECK YOUR UNDERSTANDING 13.4

2 1.90 A
3 1.8 V
4 a 3.2 Ω
b 20.0 Ω
c 4.0 Ω
d 6.0 Ω
5 b 30.0 Ω
c 0.20 A
d −2.4 V
7 a 3 V
b 30 Ω
c 0.3 A

CHAPTER REVIEW

2 12.0 J
3 1.92×10^{-19} J
4 0.88 V
5 300 C

8 12 V

9 12.0 V

10 20 C

11 a 6.7 Ω

b 20 V

12 432 V

13 –7 V

14 75 mA

15 2.0 A

16 a 2.0 C every second

b 6.0 J to every coulomb of charge

c 40 s

d 80 C

19 a 300 Ω

b 27 Ω

21 10 Ω resistor, 1.0 A and for 20 Ω, 0.5 A.

22 a 5.0 V each

b 10.0 A

23 a 4 kΩ

b 5 kΩ:

24 a 0.417 A (0.42 A)

b 1.25 A

c 17

25 a 15.0 Ω

b R_2 and R_3 – 0.4 A each; R_1 and R_4 – 0.8 A each

c $V_1 = 4.0$ V; $V_2 = 3.2$ V; $V_3 = 3.2$ V; $V_4 = 4.8$ V

CHAPTER 14: MAGNETISM

WORKED EXAMPLE 14.1

1 10×10^{-6} T; 5.0×10^{-6} T; 3.3×10^{-6} T; 2.5×10^{-6} T; 2.0×10^{-6} T

WORKED EXAMPLE 14.3

1 0.4 A

2 1.6 A

3 0.4 A

CHECK YOUR UNDERSTANDING 14.3

1 a 7.5 kA

b 0.075 T

5 57 m

CHAPTER REVIEW

13 2×10^{-6}

14 3.0 m

15 2.5 A

17 6.3 T

18 1790 turns

19 a Field will double.

b Field is unchanged.

c Field is halved.

MODULE 4 REVIEW

2 b 7.3×10^{9} N C^{-1}

c 7.3 N

4 c 1.2 kV m^{-1}

5 a 2.4×10^{-19} J

b 4.5×10^{22} electrons

c 0.5 A for 4 hours

6 b 0.6 A

c 1.8 W

d ii 0.3 A in both globes and 1.5 V across each

iii 0.45 W

7 c 500 Ω

10 a 1.0×10^{-5} T

c 398 turns

d 2.5 mT

GLOSSARY

A

absolute (uncertainty) uncertainty expressed in the units of the measured quantity

absolute zero the hypothetical condition where all particles in a substance have no kinetic energy

acceleration change in velocity over time

accurate the degree to which a measurement result approaches the 'true value'

air resistance the drag force exerted by air

alternating current (AC) current that changes direction, oscillating in direction

amplitude the largest distance of the particle from the mean, or rest, position before returning

angle of incidence the angle made between the incident ray and the normal drawn at the point of incidence

angle of reflection the angle made between the reflected ray and the normal drawn at the point of reflection

angle of refraction the angle from the ray to the normal after refraction has occurred

antinodes points along a standing wave where constructive interference always occurs

average the mean value

B

beat frequency the difference between two sound sources frequencies, the frequency at which the resultant sound oscillates between constructive and destructive interference

boiling point the temperature at which the vapour pressure from particles escaping the liquid equals the pressure on the liquid from its surrounding environment (such as the atmosphere). Characterised by bubbles forming below the surface.

C

centre of mass also known as the centre of gravity, this is the average (mean) position of all matter in the system, weighted by mass

charge a fundamental property of matter, which creates and is affected by electric fields

charged having a non-zero net charge

chromatic dispersion where different colours of light refract by slightly different amounts

circuit diagram a diagram showing the connections between components in a circuit, using circuit symbols to represent the components

closed system a system where matter cannot enter or leave, but which energy can be transferred into or out of by work or heat

coefficient of kinetic friction, μ_k dimensionless constant of proportionality between the friction force and normal force between two surfaces when the surfaces are sliding relative to one another: $F_{\text{friction, kinetic}} = \mu_k N$

coefficient of static friction, μ_s dimensionless constant of proportionality between the *maximum* friction force and normal force between two surfaces when the surfaces are stationary relative to one another: $F_{\text{friction, static}} = \mu_s N$

component (vector) the projection of a vector quantity along an axis

compression an area where the medium has become more dense as a wave passes

concave surface a surface curved inwards

condensation gas forming a liquid as its particles lose energy through cooling

conduction the transfer of heat energy through a substance by particle collision

conductor a material that allows current to flow through it easily, for example a metal. A material with a low resistivity.

constructive interference when superposition gives a resultant wave with a greater amplitude than the constituent waves

contact force the force that one surface exerts on another that it is in contact with. The contact force is the sum of the normal force and the friction force.

controlled variable variables that are kept constant so that they do not interfere with the outcome of the experiment

convection the transfer of heat energy by bulk movement of particles

convection cell the circulation of a fluid that is heated below and cooled from the top, under the influence of gravity

convection current the transfer of heat through the mass movement of a fluid to a region of lower temperature

converging (convex) lens a lens which causes parallel rays of light to converge

convex surface a surface curved outwards

coulomb, C unit of electric charge

crest the top of a wave

critical angle the angle of incidence that will give an angle of refraction of 90°

current the flow of charged particles, $I = q/t$. In a circuit the charged particles are most often electrons.

D

deceleration negative acceleration

decibel (dB) a logarithmic scale used for the intensity of sound, with zero decibels being the threshold of hearing

delocalised valence electrons electrons that are so loosely held by their atoms that they are free to move through a metallic lattice. Responsible for both good electrical and heat conductivity.

dependent variable the variable being measured in an experiment

deposition a cooled gas directly forming a solid, without an intermediate liquid phase

depth study an investigation or an activity completed by a student or students to explore more deeply a topic from the Year 11 Physics course that they find interesting

derived data data that is deduced from raw data by mathematical manipulation, such as graphs, algebraic equations or geometric constructions

destructive interference when superposition gives a resultant wave with a smaller amplitude than the constituent waves

diamagnetic not able to be permanently magnetised; diamagnets are repelled by magnetic fields. The internal magnetic fields of a diamagnet align in the opposite direction to an external field.

diffraction the bending of a wave around a corner as it passes an obstruction

diffuse reflection where light is reflected in many directions from a rough surface

dipole equal and opposite charges close together

direct current (DC) current that flows in the same direction at all times

displacement (kinematics) the position of an object relative to the origin. It is a vector and includes direction.

displacement (waves) the perpendicular distance a particle in the medium has moved from its rest or mean position

displacement interval the change in displacement

distance the actual length between two points. It is a scalar and has no direction.

distance interval the magnitude of the change in position between ending and beginning points. A scalar.

diverging (concave) lens a lens which causes parallel rays of light to diverge

Doppler effect the shift in the frequency and wavelength of waves that results from the relative motion between source and observer

drag the friction force that a fluid exerts on a solid surface when the surface moves relative to the fluid

dynamic a state in which the object or system is moving, and in which there may be a net force and hence an acceleration

E

echo a reflection of sound heard after the original sound

elastic collision a collision in which the total kinetic energy of the colliding objects is unchanged by the collision

electric field the force field created by charged objects, which exerts a force on other charged objects. The force per unit charge acting on a charged object.

electric field line a line with an arrow on a field diagram showing the direction of the force on a positive charge

electric potential the potential energy per unit charge at a point in space, measured in units of volts, V

electric potential energy potential energy arising from the interaction of charged objects; the potential energy stored in an electric field

electromagnetic radiation oscillating perpendicular magnetic and electric fields

electromagnetic spectrum the range of all wavelengths or frequencies of electromagnetic radiation

electromagnetism an area of science relating to the electric and magnetic properties and interactions of matter and energy

electron volt unit of energy (not potential difference), $1\ \mathrm{eV} = 1\ e \times 1\ \mathrm{V} = 1.6 \times 10^{-19}\ \mathrm{J}$

electrostatic force the force exerted by charged objects on other charged objects, when the objects are stationary

empirical able to be verified by gaining evidence through observation and experimentation

equilibrium (of an object) having no net force acting, so the acceleration of the object is zero and it has constant acceleration

equipotential line or surface on which potential is constant

evaporation particles on the surface of a liquid escaping to become gaseous, even though the liquid is below boiling point

F

falsifiable able to be disproved

ferromagnetic able to be permanently magnetised, having magnetic properties like that of iron. The internal magnetic fields of a ferromagnet align with an external field and ferromagnets are attracted into a magnetic field.

field force a force which is exerted via a field, so does not need direct contact. The four field forces are the gravitational, electromagnetic, strong and weak forces.

first harmonic another term for the fundamental, the mode of vibration with the lowest frequency

focal length the distance from the optical centre to the point where parallel rays passing through the lens will intersect

focal point the point at which rays or waves meet after they have been reflected or refracted

forced vibration vibrations that occur in an object when it is forced to vibrate by another object

fractional (uncertainty) absolute uncertainty expressed as a fraction of the measurement, often expressed as a percentage

frame of reference a framework for observing physical phenomena that allows for an origin. It enables the measurement of quantities involved in changing position.

free (natural) vibrations vibrations occurring when a body vibrates by itself

frequency the number of crests or whole waves generated in a time interval

friction the contact force that one surface exerts on another, parallel to the surfaces. Friction acts to resist the sliding of one surface against another.

friction force the component of the contact force that one surface exerts on another surface that is parallel to the surfaces. It acts to prevent the sliding of one surface against the other.

fundamental (mode of vibration) the lowest frequency resonant note or mode of vibration of an object

G

gravitational field the force per unit mass acting on an object with mass due to other objects with mass. The force field surrounding any object with mass.

gravitational force the force exerted on any object with mass by any other object with mass. It surrounds us and penetrates us, and it binds the galaxy together.

H

heat energy that is being transferred due to a temperature difference

heat capacity the ratio of the amount of heat added to or subtracted from a material, to the temperature change produced. Units are Joules/Kelvin (J/K).

hypothesis a tentative prediction, usually based on an existing model or theory; also a tentative explanation of an observation based on an existing model or theory

I

impulse, I the change in momentum of an object due to the action of a force

$\vec{I} = \vec{F}\Delta t = \Delta\vec{p}$

incident ray a ray striking a surface

independent variable the variable that is controlled or manipulated by the experimenter

inelastic collision any collision in which the total kinetic energy of the colliding objects is decreased by the collision

instantaneous (speed) the rate at which distance is covered over a time interval that is so brief as to be negligible

insulator a material that does not allow current to flow through it. A material with an effectively infinite resistivity.

intensity the amount of sound energy (measured in joules, J) passing through a unit area (a square metre, m^2) in one second

intermolecular forces relatively weak forces that hold molecules together, not to be confused with covalent or ionic bonding

internal energy the sum of the potential energy in bonds and the kinetic energy of particles in a substance

inverse square law describes how the intensity of light is inversely proportional to the distance from the source

Isolated system a system where neither energy nor matter can enter or leave

K

kinetic energy energy due to the motion of objects

kinetic friction the component of the contact force acting parallel to the surfaces in contact, when there is slipping of the surfaces against each other

kinetic particle model describes what happens when particles physically interact

kinetic theory theory explaining the different states of matter

Kirchhoff's current law a statement of conservation of charge saying that the total current in to a junction must be equal to the total current out of a junction. $\Sigma I = 0$. Also known as Kirchhoff's junction law.

Kirchhoff's voltage law a statement of conservation of energy for a circuit saying that the sum off all potential differences around a loop must be zero: $\Sigma V = 0$. Also known as Kirchhoff's loop law.

L

latent heat energy added during a change of state while the temperature remains constant

law of conservation of energy the energy of an isolated system is constant. Energy can neither be created nor destroyed. It may be transformed from one form to another.

limit of reading the smallest unit of measurement on a measuring instrument

literature review a report and evaluation of information from secondary sources on a topic of interest

logbook the record of an experiment or investigation kept by the scientist performing the experiments; it is a legal record of the experiments and their results

longitudinal waves waves which cause a back-and-forth motion of the medium along the same direction as that of the propagation of the wave

loudness a subjective quality of perception of the amount of sound energy arriving at a person's ear

M

magnetic field the force per unit current element that acts on moving charges or magnetic materials due to other moving charges or magnetic materials. The force field surrounding any moving charge or magnetic material.

magnification ratio of the size of an image and the object itself

maximum static friction the maximum friction force that one surface can exert on another, $F_{\text{friction, maximum static}} = \mu_s N$

Maxwell–Boltzmann distribution the distribution of energies of particles in a substance of temperature T

measurand quantity being measured

mechanical energy the total bulk kinetic energy and potential energy of an object or system, does not include thermal or other internal energies

mechanical waves waves which cause a vibration in a physical medium

mechanics an area of science relating to the motion and interaction of objects and that uses the ideas of force and energy to explain phenomena

medium the substance through which a wave is passing

melting a solid being changed to a liquid through heating

melting point the temperature at which melting occurs. At this same temperature, a liquid being cooled will solidify, so this is also the freezing point for that substance.

model a representation of a system or phenomenon that explains the system or phenomenon. A model may be mathematical equations, a computer simulation, a physical object, words or other form.

momentum, p the product of mass and velocity, $\vec{p} = m\vec{v}$. Momentum is a vector quantity with direction the same as that of the velocity and units kg m s^{-1}.

N

net force the total force acting on an object, the vector sum of all forces acting on the object

neutral uncharged, or having zero net charge

nodes points along a standing wave where destructive interference always occurs

non-ohmic describes a component for which resistance, R, depends on potential difference, so R is not constant

normal a line drawn perpendicular to the boundary between two substances

normal force the component of the contact force that one surface exerts on another surface that is perpendicular to the surfaces. It acts to prevent one surface moving into the other.

O

Ohm's law resistance is a constant, and is the ratio of potential difference to current: $R = \dfrac{V}{I}$

Ohmic describes a component for which resistance, R, does not depend on potential difference; $R = V/I = \text{constant}$

optical centre the point on the optical axis of a lens where the rays pass through and remain unrefracted

origin the point from which all other positions in a frame of reference are measured

outlier a data point that is distant from the other data points in the sample

overtone harmonics other than the first harmonic

P

parallel (circuits) arranged side by side, with common connections at each end

parallel circuit multi-loop circuit, with components arranged in parallel

paramagnetic not able to be permanently magnetised, paramagnets are weakly attracted by magnetic fields. The internal magnetic fields of a paramagnet align with an external field.

paraxial assumptions three assumptions made about lenses and refracting light when ray diagrams are constructed

perfectly inelastic collision a collision in which the colliding objects become stuck together and move as a single object after the collision

period the time it takes before a wave repeats itself

permanent magnet a material which stays magnetised after being exposed to a magnetic field and then removed from the field

permeability of free space, μ_0 constant of proportionality for magnetic fields, gives the magnetic field strength in vacuum due an electric current. It has units T m A^{-1}.

permittivity of free space the physical constant that determines how large an electric field is produced by a charge in vacuum. It has the value 8.9×10^{-12} C^2 N^{-1} m^{-2}.

phase change a physical change from one state of matter to another without changing chemical composition

pitch the perceived characteristic of a sound related to its frequency

Planck curve a graph of intensity versus wavelength for an object's electromagnetic radiation

plane surface a flat surface

point source (of light) a single point from where light radiates

point source (of sound) a source of sound that, for the purposes of measuring its location, is treated as being a single point in space

polar having an equal amount of positive and negative charge, but arranged so that one area is more positive and another more negative

potential difference the difference in potential between two points, also called voltage

potential energy energy contained in a system due to the forces exerted by objects on each other within the system, hence the energy due to the positions of objects in a system

power rate of change of energy; rate of energy transfer or transformation

precise the degree to which individual measurements cluster around the mean

primary data data that you have measured or collected yourself

principal axis a line that passes through the centre of the lens at right angles to the lens

Q

quantum physics an area of science relating to the behaviour and interactions of waves, atoms and subatomic particles

R

radiation the direct emission of energy as electromagnetic waves

rarefaction an area where the medium has become less dense as a wave passes

raw data original data taken directly from a measurement system

ray a representation of light as a line with an arrow

ray diagram a construction diagram tracing rays of light

Rayleigh wave a seismic wave that travels along the ground

real image an image which can be focused on a screen

reflection the return of a wave or particle from a surface or a boundary

refraction a change of direction of a wave upon entering a different medium at an angle other than 0° caused by a change of speed of the wave

refractive index a relative measure of how much refraction will occur when light enters a medium

reliable when repeat measurements give the same results within experimental uncertainty

reproducibility giving the same result, within uncertainty, when repeated measurements are made

reproducible giving the same result, within uncertainty, when repeated measurements are made

research question the specific questions that a particular experiment or investigation is designed to answer

residual magnetisation the magnetic field that remains in a ferromagnetic after it has been magnetised and the external field removed

resistance ratio of potential difference to current in a component, $R = V/I$. The larger the resistance, the smaller the current.

resistor an electrical device that resists the flow of electric current in a circuit

resonance the oscillation of a physical system at its natural frequency

resultant (vector) the sum of appropriate component vectors

reverberation the prolonging of an original sound without it being perceived as a distinct echo

S

saturation magnetisation the maximum magnetisation possible for a material, when all internal magnetic fields are completely aligned

scalar a number that has only magnitude (size)

scatter plot/graph a graphical representation of the relationship between the individual data points of two variables

scientific method a systematic process of observation, experimentation, measurement and analysis to either support or disprove a hypothesis

secondary data data or information that has been collected by someone else

seismic waves disturbances travelling as waves through Earth's crust or rocks

semiconductor material which allows current to flow, but not easily. A material with a high resistivity.

series arranged one after another, end to end

series circuit single loop circuit with all components in series

solenoid a coil of wire which can carry a current. A current-carrying solenoid has a large internal magnetic field and small external field. Also called an inductor when used in a circuit.

solidification the formation of a solid as liquid particles lose energy. Also called freezing.

sound waves waves which can be heard when transmitted to the human ear

specific heat capacity heat capacity per unit mass of a substance

specific latent heat of fusion the energy required to change the state of 1 kg of a substance from its solid to liquid without any change in temperature

specific latent heat of vaporisation the heat required to change 1 kg of a substance from its liquid to gaseous state

specular reflection the reflection of light which occurs from a smooth surface

speed the distance covered in a time interval

spin a quantum mechanical property of particles, a measure of the particle's intrinsic magnetic field

standing or stationary wave a wave that does not appear to be travelling

static a state in which there is no net force and the object or system is not moving

static friction the component of the contact force acting parallel to the surfaces in contact, when there is no slipping of the surfaces

sublimation the process where a heated solid turns directly into a gas

superposition the algebraic addition of the displacement of two or more waves interacting at the same point

systematic error an error that results in a consistent, predictable offset from the 'true value'; for example, a zero error

T

thermal fluid circulation generated by pressure gradients caused by differential heating. Concerning flight, it generally refers to the updraught. See **convection current**

thermal conductivity the flow of energy per second through 1 metre of a material per degree temperature difference between the two ends of the material

thermal equilibrium two substances in thermal equilibrium have the same temperature. They do not exchange heat energy.

thermals convection currents in the atmosphere where warm air is rising

thermodynamics an area of science relating to the transfer and transformation of energy

thin lens equation an equation used to find the focal length of a lens

time interval the difference between a finishing time and a starting time

time–base scale usually the x axis on an oscilloscope display with units of time

total internal reflection the internal reflection of a ray of light when the angle of incidence exceeds the critical angle, i_c

transparent allowing the transmission of light through, it without distortion

transverse waves waves which cause a transverse or perpendicular motion of the medium to the direction of propagation of the wave

trough the bottom of a wave

U

uncertainties estimate of the range of values within which the 'true value' of a measurement or derived quantity lies

uniform electric field electric field which does not vary with position in some region

V

valid results that are affected by a single independent variable and hence are reproducible

vaporisation particles escaping a liquid to form a gas

vector a mathematical quantity that has both magnitude (size) and direction

velocity the change in displacement during a time interval

virtual image a perceived image of an object where it is not physically present

voltage potential difference

W

wavefront a representation of a wave as a line perpendicular to the direction of propagation

wavelength distance between two successive corresponding points on a wave, e.g. between two crests or two troughs

wavenumber the number of complete waves per unit distance

waves an area of science relating to the movement of energy and information without the overall movement of objects

work energy transferred to an object when a force acts on it

X

***x–y*-plane** a coordinate system used to describe motion in two dimensions. In a diagram, the x axis points to the right and the y axis points up the page.

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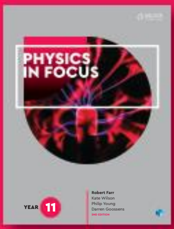

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