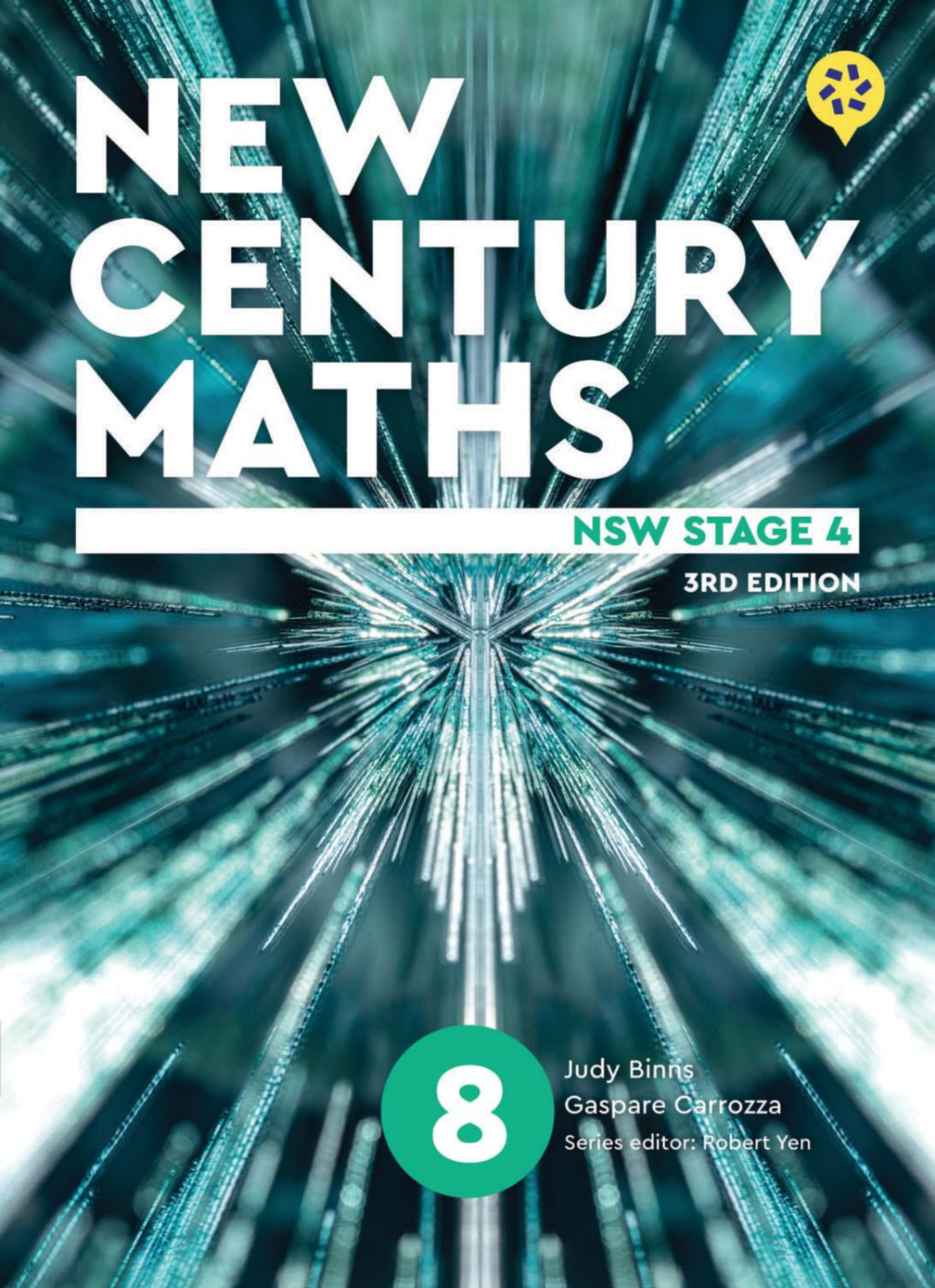
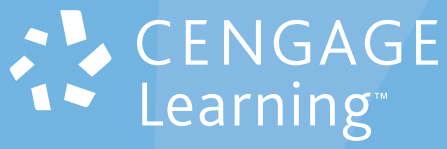

NEW CENTURY MATHS
NSW STAGE 4
3RD EDITION
8
Judy Binns
Gaspare Carrozza
Series editor: Robert Yen




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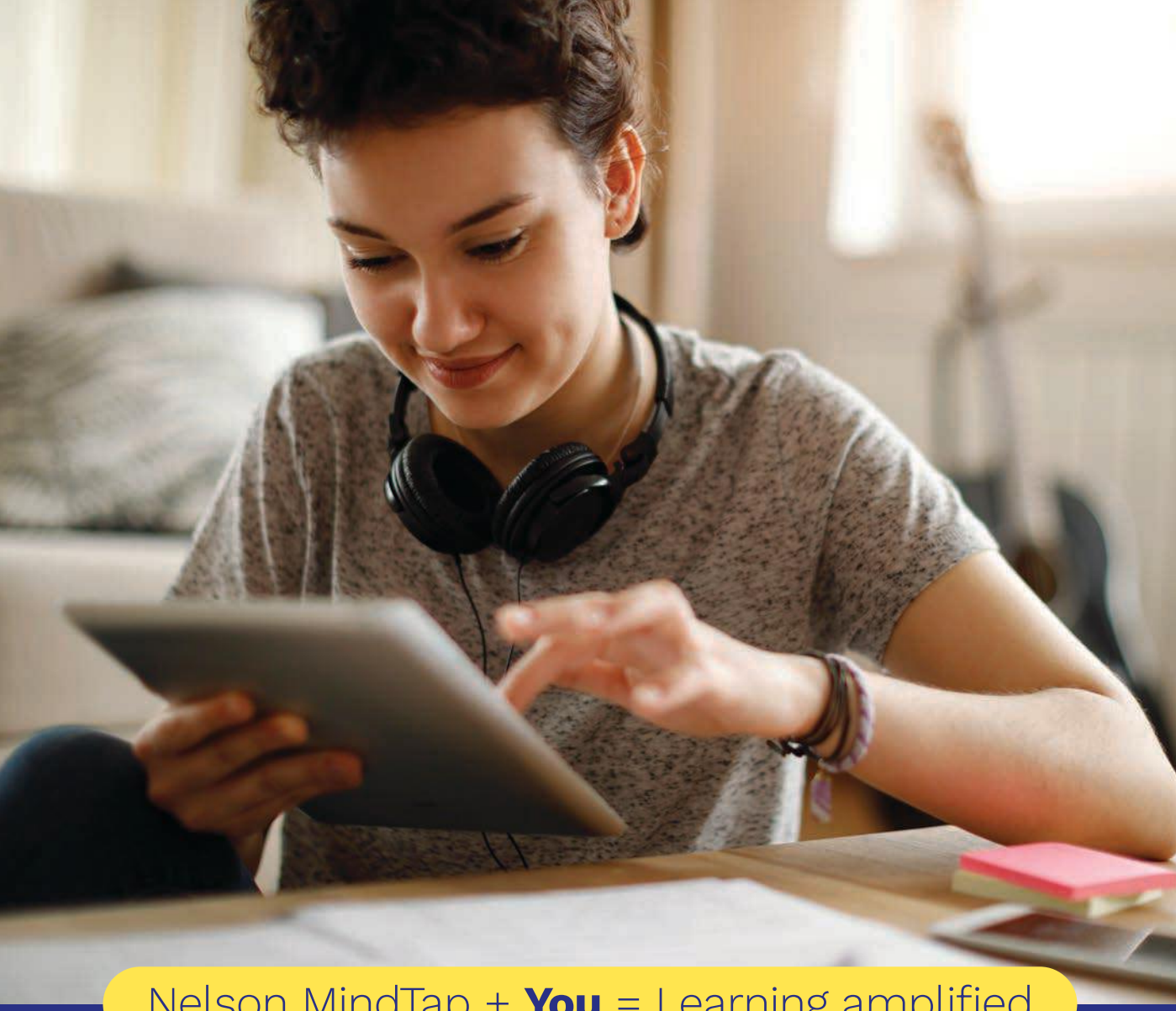
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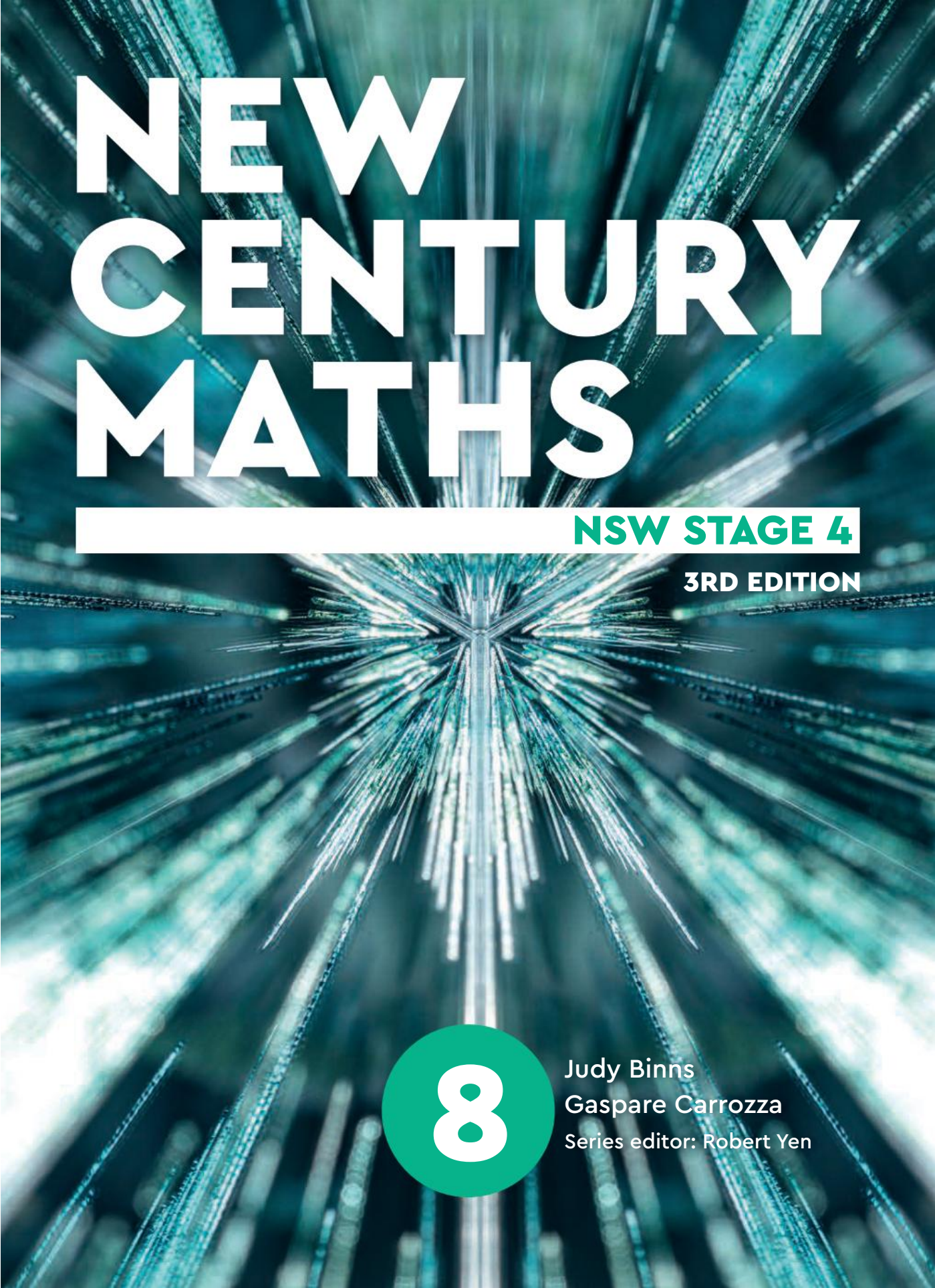


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— **Sarah,** Secondary School Teacher

Nelson MindTap


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**3rd Edition**
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# PREFACE

Teachers, it's time to start teaching maths again. You asked and we listened.

*New Century Maths 7-10* is an original series written by NSW teachers for NSW teachers, focusing on core skills to discourage 'teaching by syllabus dot points', featuring explicit grading and labelling of exercise questions, more 'flipped classroom' video tutorials, more applications and problem-solving questions, and worked solutions to every question.

In schools for 30 years, *New Century Maths* is carefully mapped to the NSW syllabus and built on solid pedagogical foundations that integrate into every chapter practical classroom activities, engaging investigations, problem-solving, reasoning, communicating, reflecting, summarising, extension, revision, mental calculation, technology, numeracy and literacy.

This book, *New Century Maths 8,* has been designed for Year 8 students progressing along the Stage 4 continuum, but includes Year 7 revision and Year 9 extension work.

The *Nelson MindTap* online learning platform contains an eBook, quizzes, interactives, 200+ worksheets, 100+ videos, topic tests, worked solutions and much more. We have provided an abundance of resources for teachers to plan and teach for a variety of pathways. *New Century Maths* is clear, concise, fresh and smart. We have designed this series to be user-friendly and uncomplicated so that teachers and students everywhere can pick it up and use straight away. When it comes to teaching in the spirit, breadth and depth of the new syllabus, we've got your back.

## About the authors

**Judy Binns** was head teacher of mathematics at Mulwaree High School in Goulburn and has taught at Homebush Boys High School. She has an interest in motivating students with learning difficulties and wide expereince in teaching senior practical mathematics courses. Judy also co-wrote *New Century Maths 11–12 Mathematics Standard 1.*

**Gaspare Carrozza** was head teacher of mathematics at Homebush Boys High School and South Sydney High School. He has conducted training workshops in both mathematics and computing. Gaspare has been involved in HSC marking and trial HSC exam writing, and also wrote resources for *Maths in Focus* 11–12.

Series editor **Robert Yen** taught at Hurlstone Agricultural High School. He co-wrote *New Century Maths 11–12 Mathematics Standard 2*, co-edited *Reflections,* the MANSW journal, and works for Cengage as the mathematics publisher.

**Contributing authors**

**Sarah Hamper** wrote the technology sections.

**Kuldip Khehra, Judy Binns, Gaspare Carrozza** and **Robert Yen** wrote many of the *NelsonNet* worksheets and topic tests.

**John Drake, Katie Jackson, Joanne Magner, Scott Smith, Rashmi Bhagwati, Deborah Da Cruz, Monique Ellement, Jade Lori** and **Robert Yen** created the video tutorials.

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# NSW SYLLABUS GRID

| Strand and substrand | *New Century Maths 7* chapter | | *New Century Maths 8* chapter | |
|---|---|---|---|---|
| **NUMBER AND ALGEBRA** | | | | |
| Computation with integers | 1 | Integers | 2 | Working with numbers |
| Fractions, decimals and percentages | 3 | Whole numbers | 2 | Working with numbers |
| | 4 | Fractions and percentages | 6 | Fractions and percentages |
| | 7 | Decimals | | |
| Ratios and rates | 12 | Ratios, rates and time | 10 | Ratios and rates |
| Algebraic techniques | 5 | Algebra and equations | 3 | Algebra |
| | | | 11 | Graphing linear equations |
| Indices | 3 | Whole numbers | 2 | Working with numbers |
| | | | 3 | Algebra |
| Equations | 5 | Algebra and equations | 9 | Equations |
| Linear relationships | 9 | The number plane | 11 | Graphing linear equations |
| **MEASUREMENT AND SPACE** | | | | |
| Length | 8 | Area and volume | 5 | Area and volume |
| Right-angled triangles (Pythagoras) | | | 1 | Pythagoras' theorem |
| Area | 8 | Area and volume | 5 | Area and volume |
| Volume | 8 | Area and volume | 5 | Area and volume |
| Angle relationships | 2 | Angles | 4 | Geometry |
| Properties of geometrical figures | 6 | Geometrical figures | 4 | Geometry |
| **STATISTICS AND PROBABILITY** | | | | |
| Data classification and visualisation | 10 | Analysing data | 7 | Investigating data |
| Data analysis | 10 | Analysing data | 7 | Investigating data |
| Probability | 11 | Probability | 8 | Probability |

# ABOUT THIS BOOK

## Coverage of the syllabus

- *New Century Maths 8* covers the NSW syllabus based on the Australian curriculum, as shown by the table of contents and syllabus grids on the previous pages.
- This book contains Stage 4 content. It also contains revision of some Stage 3 content, and some Year 9 extension work marked in **red** and by *.
- Each chapter begins with a **chapter outline** that includes the Working Mathematically processes covered in each section.

**Chapter outline**

| | Working mathematically | | | | |
|---|---|---|---|---|---|
| **9.01** One-step equations | U | F | | R | C |
| **9.02** Two-step equations | U | F | | R | C |
| **9.03** Equations with variables on both sides* | U | F | | R | C |
| **9.04** Equations with brackets* | U | F | | R | C |
| **9.05** Simple quadratic equations $ax^2 = c$ | U | F | | R | C |
| **9.06** Equation problems | U | F | PS | R | C |
| **9.07** Equations and formulas | U | F | PS | R | C |

**U = UNDERSTANDING**

**Understanding** is 'knowing and relating' maths. It is more than just learning facts. It's deep understanding, seeing how mathematical content is interconnected, knowing 'why' as well as 'how'.

**F = FLUENCY**

**Fluency** is 'applying' maths. It is being able to use mathematics competently and effectively. When you are fluent in a language, you have mastered it so that you can improvise and confidently use the correct word or phrase. Fluency in maths is choosing an appropriate skill, method or formula to use at the right place and time.

**PS = PROBLEM SOLVING**

**Problem solving** is 'modelling and investigating' with maths. It involves interpreting a rich, elaborate problem, selecting an appropriate strategy or model, solving the problem, then evaluating, communicating and justifying the solution.

**R = REASONING**

**Reasoning** is 'generalising and proving' with maths, using higher-order thinking to connect specific facts to general principles, using algebra, logic, proof and justification.

**C = COMMUNICATING**

**Communicating** is 'describing and explaining' maths, representing mathematical theory and solutions in words, algebraic symbols, special notations, diagrams, graphs and tables.

- **Understanding** and **Fluency** can be found in every exercise and activity, while **Problem solving**, **Reasoning** and **Communicating** are found in the **Investigations**, **Technology**, **Mental skills**, **Language of Maths** and **Topic summary** activities, and explicitly labelled in every exercise (see 'In each chapter' next page).

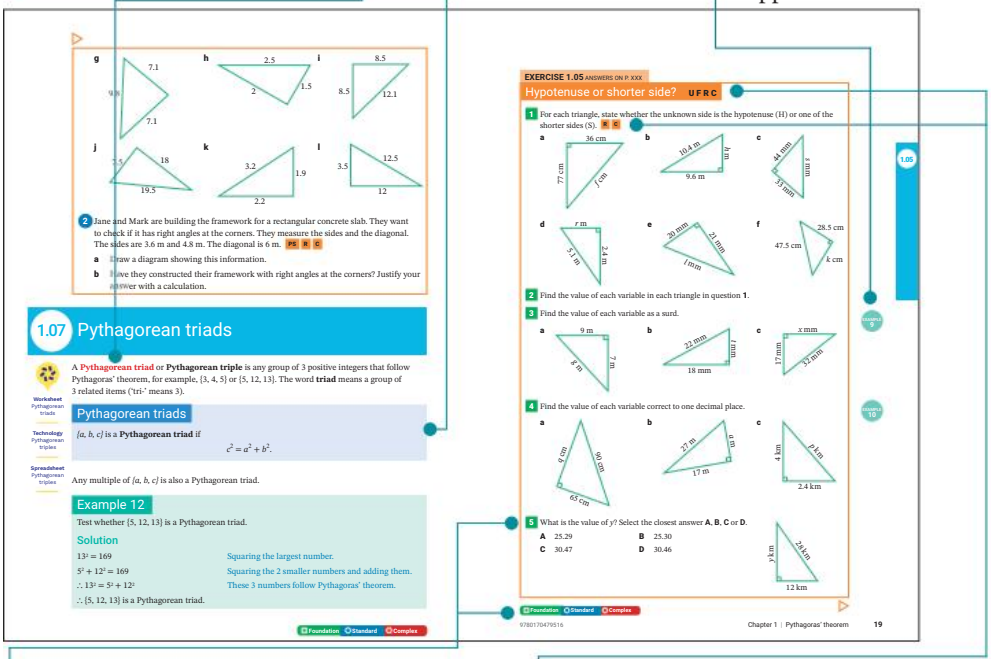
## At the beginning of each chapter

- A listing of *Nelson MindTap* **chapter resources**
- A **Chapter outline** and a **Wordbank** chapter glossary (a full glossary appears at the back of the book)
- **SkillCheck** reviews prerequisite skills and knowledge for the chapter



## In each chapter

- Important facts and formulas are highlighted in a shaded box.
- Glossary terms are printed in **red**.
- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications.



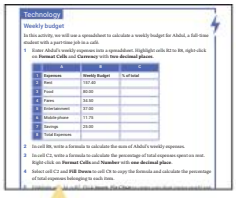
- The exercise questions are colour-coded for level of difficulty:

Foundation | Standard | Complex

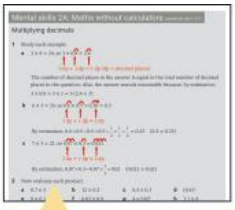
- Specific questions are also labelled **PS** for **Problem solving**, **R** for **Reasoning** and **C** for **Communicating**.




**Investigations** explore the syllabus in more detail, through group work, discovery and modelling activities.




**Technology** includes spreadsheets, dynamic geometry software and the Internet.




**Mental skills** reinforce mental calculation strategies ('calculator-free maths')




**Did you know?** contains interesting facts and applications of the mathematics learned in the chapter.

## At the end of each chapter

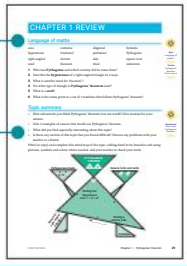
**Power plus** is an extension/challenge exercise.

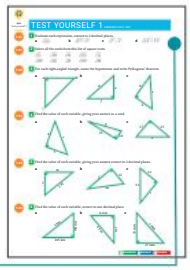
**Language of maths** has a chapter word list and literacy questions.

**Topic summary** has a mind map activity with downloadable solutions.

**Test Yourself** contains chapter revision linked to the relevant exercise set.

**Practice sets** after every 3 chapters.



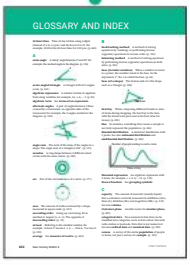

## At the end of the book

**General practice** exercise

**Answers** (worked solutions are on the teacher website).

**Glossary and index**



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**An online learning space that provides students with tailored learning experiences.**

- Access tools and content that make learning simpler yet smarter to help you achieve maths mastery.
- Includes an eText with integrated interactives and online assessment.
- Margin links in the student book signpost multimedia student resources found on MindTap.



**Video**
Equation problems

**For students:**

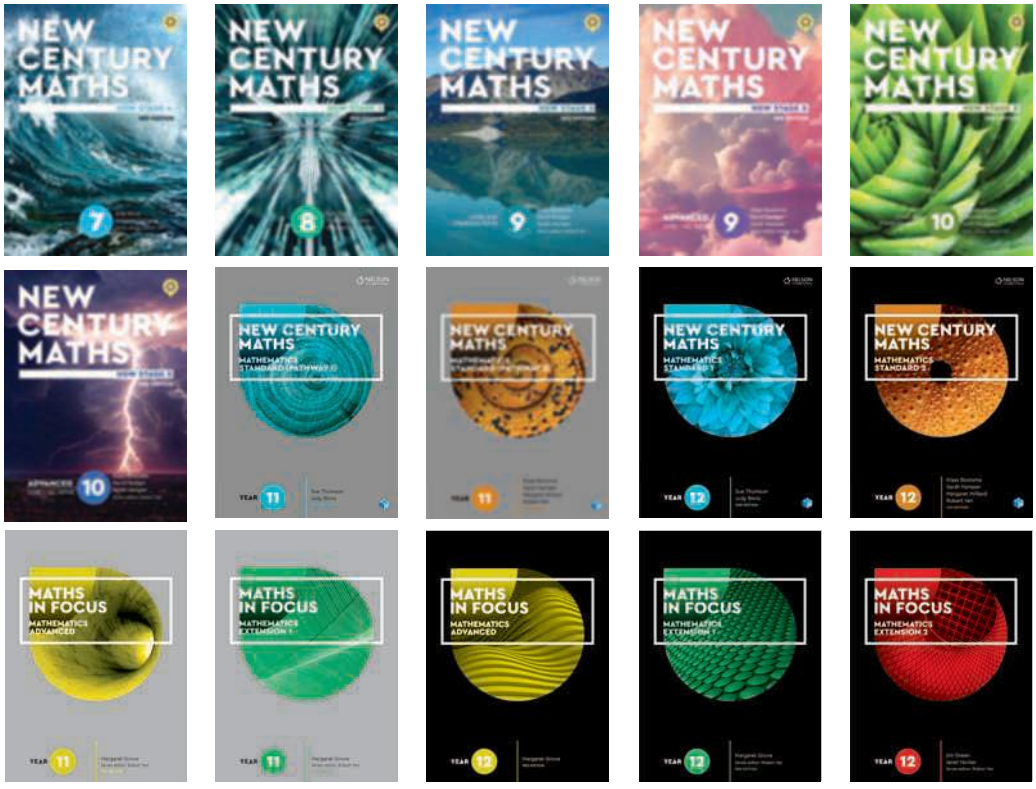
- **Watch** video tutorials featuring expert teacher advice to unpack new concepts and develop your understanding.
- **Revise** using quizzes, worksheets and skillsheets to practise your skills and build your confidence.
- **Navigate** your own path, accessing the content, analytics and support as you need it.
- ***Twig* mini-documentaries** showing the background or context of mathematics
- ***PhET* interactives:** online maths widgets

**For teachers*:**

- Tailor content to different learning needs – assign directly to the student, or the whole class.
- Monitor progress using assessment tools like Gradebook and Reports.
- Integrate content and assessment directly within your school's LMS for ease of access.
- Access topic tests, teaching programs and worked solutions to each exercise set.

* Complimentary access to these resources is only available to teachers who use this book as part of a class set, book hire or booklist. Contact your Cengage Education Consultant for information about access and conditions

## New Century Maths / Maths in Focus 7–12 series



# MATHEMATICAL VERBS

## A glossary of 'doing words' commonly found in mathematics problems

**analyse:** study in detail the parts of a situation

**bisect:** cut in half

**calculate:** *see* **evaluate**

**classify, identify:** state the type, category or feature of an item or situation

**comment:** express an observation or opinion about a result

**complete:** fill in detail to make a statement, diagram or table correct or finished

**compare:** show how 2 or more things are similar or different

**construct:** draw an accurate diagram

**convert:** change from one form to another, for example, from a fraction to a decimal, or from kilograms to grams

**decrease:** make smaller

**describe:** state the features of a situation

**estimate:** make an educated guess for a number, measurement or solution, to find roughly or approximately.

**evaluate, calculate:** find the value of a numerical expression, for example, $3 \times 8^2$ or $4x + 1$ when $x = 5$

**expand:** remove brackets in an algebraic expression by multiplying, for example, expanding $3(2y + 1)$ gives $6y + 3$

**explain:** describe why or how

**factorise:** take out the highest common factor (HCF) of an expression and insert brackets, for example, factorising $5x - 20$ gives $5(x - 4)$. The opposite of **expand**.

**give reasons:** show the rules or thinking used when solving a problem. See also **justify**.

**graph:** display on a number line, number plane or statistical graph.

**hence find/prove:** find an answer or prove a result using previous answers or information supplied

**identify:** *see* **classify**.

**increase:** make larger

**interpret:** find meaning in an answer or result

**justify:** give reasons or evidence to support your argument or conclusion. See also **give reasons**.

**measure:** use an instrument to find the size of something, for example, use a ruler to determine the length of a pen.

**prove that:** *see* **show that**

**recall:** remember and state.

**reduce (a fraction) to its lowest terms:** see **simplify (a fraction)**.

**round (a number):** find the nearest approximation of a number. For example, 4.3 rounded to the nearest whole number is 4, $12.9598 rounded to the nearest cent is $12.96, 0.166 66 rounded to 3 decimal places is 0.167.

**show that, prove that:** (in questions where the answer is given) use calculation, procedure or reasoning to prove that an answer or result is true

**show working:** show the steps you used to find the answer

**simplify:** give a result in its most basic, shortest, neatest form, for example, simplifying a ratio or algebraic expression

**simplify (a fraction):** reduce the numerator and denominator of a fraction by dividing by their highest common factor (HCF), for example, $\frac{16}{20}$ simplified is $\frac{4}{5}$.

**simplify (a ratio or rate):** reduce the terms or units of a ratio or rate by dividing by their highest common factor (HCF), for example, 10 : 4 simplified is 5 : 2.

**sketch:** draw a rough diagram that shows the general shape or idea, less accurate than **construct**

**solve:** find the value(s) of an unknown variable in an equation or inequality

**state:** *see* **write**.

**substitute:** replace a variable by a number and evaluate

**verify:** check that a solution or result is correct, usually by substituting back into the equation or referring back to the problem

**write correct to:** See **round (a number)**.

**write, state:** give the answer, formula or result without showing any working or explanation (this usually means that the answer can be found mentally, or in one step)

# SYMBOLS AND ABBREVIATIONS

| | | | |
|---|---|---|---|
| $=$ | is equal to | $\triangle ABC$ | triangle $ABC$ |
| $\neq$ | is not equal to | $\parallel$ | is parallel to |
| $\approx$ | is approximately equal to | $\perp$ | is perpendicular to |
| $<$ | is less than | $\equiv$ | is congruent to |
| $>$ | is greater than | $\therefore$ | therefore |
| $\leq$ | is less than or equal to | $x^2$ | $x$ squared, $x \times x$ |
| $\geq$ | is greater than or equal to | $x^3$ | $x$ cubed, $x \times x \times x$ |
| ( ) | parentheses, round brackets | $\sqrt{\ }$ | square root, radical sign |
| [ ] | (square) brackets | $\sqrt[3]{\ }$ | cube root |
| { } | braces | $P(E)$ | the probability of event $E$ |
| $\pm$ | plus or minus | LHS | left-hand side |
| $-3$ | negative 3 | RHS | right-hand side |
| $\pi$ | pi $= 3.14159...$ | % | percentage |
| $0.\dot{1}5\dot{2}$ | the recurring decimal 0.152152 ... | p.a. | per annum (per year) |
| $^\circ$ | degree | $\overline{x}$ | the mean (average) |
| $\angle A$ | angle $A$ | $\mu$ | micro-, mu |



1

MEASUREMENT AND SPACE

# PYTHAGORAS' THEOREM

Pythagoras' theorem expresses the relationship between the sides of a right-angled triangle. It is used in the building and construction industries to ensure right angles in building frames and to calculate the lengths of building materials. It is also used in Geography and GPS systems when calculating distances, and in checking for corrupted data sent electronically. Pythagoras had no idea how useful his theorem would be!




## Chapter outline

| | Working mathematically | | | | |
|---|---|---|---|---|---|
| **1.01** Square roots and surds | U | F | | | |
| **1.02** Discovering Pythagoras' theorem | U | F | | R | C |
| **1.03** Finding the hypotenuse | U | F | | | |
| **1.04** Finding a shorter side | U | F | | | |
| **1.05** Hypotenuse or shorter side? | U | F | | | |
| **1.06** Testing for right-angled triangles | U | F | PS | R | C |
| **1.07** Pythagorean triads | U | F | | | |
| **1.08** Pythagoras' theorem problems | U | F | PS | | |

## Wordbank

**converse** A rule or statement turned back-to-front; the reverse statement

**hypotenuse** The longest side of a right-angled triangle; the side opposite the right angle

**Pythagoras** An ancient Greek mathematician who discovered an important formula about the sides of a right-angled triangle

**surd** A square root (or other type of root) whose exact decimal value cannot be found

**theorem** A formal rule or a formula

**triad** A group of 3 related objects

**Quiz**
Wordbank 1

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

**Videos (5):**

**1.01** Surds and irrational numbers
**1.03, 1.04** Pythagoras' theorem 1 • Pythagoras' theorem 2
**1.06** Testing for right-angled triangles
**1.08** Pythagoras' theorem problems

***Twig* videos (2):**

**1.03** Irrational numbers: Pythagoras
**1.06** Building the pyramids

**Quizzes (5):**

- Wordbank 1
- SkillCheck 1
- Mental skills 1
- Language of maths 1
- Test yourself 1

**Skillsheet (1):**

**1.05** Pythagoras' theorem

**Worksheets (10):**

**1.01** Table of squares and square roots • Surds
**1.02** Pythagoras' discovery • A page of right-angled triangles • Proving Pythagoras' theorem
**1.05** Finding an unknown side
**1.06** Testing for right-angled triangles
**1.07** Pythagorean triads
**1.08** Applications of Pythagoras' theorem • Pythagoras' theorem in 2D and 3D • TV screens
**Mind map** Pythagoras' theorem

**Puzzles (3):**

**1.05** Pythagoras 1 • Pythagoras 2
**Language of maths** Pythagoras' theorem find-a-word

**Technology (3):**

**1.03, 1.04** Pythagoras' theorem
**1.06** Pythagoras' theorem tester
**1.07** Pythagorean triples

**Spreadsheets (2):**

**1.03, 1.04** Pythagoras' theorem
**1.07** Pythagorean triples

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To access resources above, visit **cengage.com.au/nelsonmindtap**

## In this chapter you will:

- investigate square roots, surds and irrational numbers
- investigate the relationship between the sides of a right-angled triangle
- solve problems involving Pythagoras' theorem, writing the answer as a decimal or as a surd
- test whether a triangle is right-angled by using the converse of Pythagoras' theorem
- investigate Pythagorean triads

Quiz
SkillCheck 1

## SkillCheck ANSWERS ON P. 570

**1** Evaluate each expression.

**a** $4^2$ **b** $12^2$ **c** $30^2$ **d** $2.1^2$
**e** $10.3^2$ **f** $3^2 + 4^2$ **g** $5^2 + 12^2$ **h** $5.4^2 - 3.2^2$

**2** Evaluate each square root.

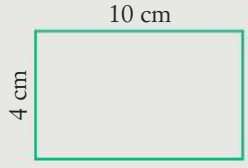
**a** $\sqrt{16}$ **b** $\sqrt{64}$ **c** $\sqrt{100}$ **d** $\sqrt{196}$
**e** $\sqrt{121}$ **f** $\sqrt{49}$ **g** $\sqrt{1.21}$ **h** $\sqrt{2.25}$

**3** Evaluate each expression correct to one decimal place.

**a** $\sqrt{3^2 + 7^2}$ **b** $\sqrt{5.6^2 + 9.2^2}$ **c** $\sqrt{15^2 - 8^2}$

**4** Find the area of each shape.

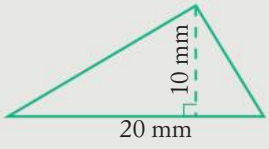
**a**




**b**




**c**






# Square roots and surds

1.01

The **square root** ($\sqrt{\ }$) of a number is the **positive** value which, if squared, will give that number. For example:

- $\sqrt{25} = 5$ because $5^2 = 25$ 'the square root of 25'
- $\sqrt{49} = 7$ because $7^2 = 49$ 'the square root of 49'

Most square roots do not give exact answers like the ones above. For example, $\sqrt{10} = 3.16227766\ldots \approx 3.2$. Such square roots are called **surds**. A surd is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any other type of root whose exact decimal or fraction value cannot be found. As a decimal, its digits run endlessly *without repeating*, so they are *neither* terminating nor recurring decimals. A surd cannot be written in fraction form $\frac{a}{b}$ so it is also called an **irrational** number.

**Worksheets**
Table of squares and square roots

Surds

## Example 1

Evaluate each expression correct to 2 decimal places.

**a** $\sqrt{7^2 + 9^2}$ **b** $\sqrt{12^2 - 10^2}$

### Solution

**a** $\sqrt{7^2 + 9^2} = 11.40175\ldots$
$\approx 11.40$

On a calculator, enter √ ( 7 $x^2$ + 9 $x^2$ ) =

OR:

$7^2 + 9^2 = 130$

On a calculator, enter 7 $x^2$ + 9 $x^2$ =

$\sqrt{130} = 11.40175\ldots$
$\approx 11.40$

On a calculator, enter √ Ans =

**b** $\sqrt{12^2 - 10^2} = 6.63324\ldots$
$\approx 6.63$

On a calculator, enter √ ( 12 $x^2$ − 10 $x^2$ ) =

OR:

$12^2 - 10^2 = 44$

On a calculator, enter 12 $x^2$ − 10 $x^2$ =

$\sqrt{44} = 6.63324\ldots$
$\approx 6.63$

On a calculator, enter √ Ans =

**Video**
Surds and irrational numbers

## Example 2

Select the surds from this list of square roots:

$\sqrt{72}$ $\sqrt{121}$ $\sqrt{64}$ $\sqrt{90}$ $\sqrt{28}$

### Solution

$\sqrt{72} = 8.4852...$ $\sqrt{121} = 11$ $\sqrt{64} = 8$

$\sqrt{90} = 9.4868...$ $\sqrt{28} = 5.2915...$

so the surds are $\sqrt{72}$, $\sqrt{90}$ and $\sqrt{28}$.

**EXERCISE 1.01** ANSWERS ON P. 570

## Square roots and surds U F

EXAMPLE 1

**1** Evaluate each square root.

**a** $\sqrt{196}$ **b** $\sqrt{900}$ **c** $\sqrt{64}$

**d** $\sqrt{625}$ **e** $\sqrt{121}$ **f** $\sqrt{361}$

**2** Evaluate each square root, correct to 2 decimal places.

**a** $\sqrt{12}$ **b** $\sqrt{45}$ **c** $\sqrt{1001}$ **d** $\sqrt{325}$

**e** $\sqrt{153}$ **f** $\sqrt{207}$ **g** $\sqrt{98}$ **h** $\sqrt{888}$

**i** $\sqrt{24}$ **j** $\sqrt{110}$ **k** $\sqrt{297}$ **l** $\sqrt{689}$

**3** Evaluate each expression, correct to 2 decimal places.

**a** $\sqrt{12^2 - 5^2}$ **b** $\sqrt{5^2 + 7^2}$ **c** $\sqrt{6^2 + 11^2}$

**d** $\sqrt{17^2 - 12^2}$ **e** $\sqrt{8^2 - 3^2}$ **f** $\sqrt{4^2 + 9^2}$

**g** $\sqrt{(1.5)^2 + (4.2)^2}$ **h** $\sqrt{(12.5)^2 - (7.1)^2}$ **i** $\sqrt{(25.7)^2 + (18.2)^2}$

EXAMPLE 2

**4** Which of the following is **not** a surd? Select the correct answer **A**, **B**, **C** or **D**.

**A** $\sqrt{10.24}$ **B** $\sqrt{24}$ **C** $\sqrt{1000}$ **D** $\sqrt{3}$

**5** Select all the surds from this list of square roots.

$\sqrt{98}$ $\sqrt{9}$ $\sqrt{225}$ $\sqrt{160}$ $\sqrt{36}$ $\sqrt{52}$

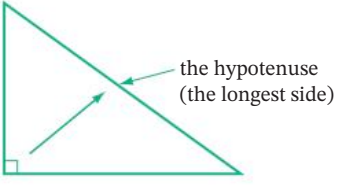
$\sqrt{144}$ $\sqrt{77}$ $\sqrt{18}$ $\sqrt{196}$ $\sqrt{200}$ $\sqrt{81}$

# 1.02 Discovering Pythagoras' theorem

**Worksheets**
Pythagoras' discovery
A page of right-angled triangles
Proving Pythagoras' theorem

A **right-angled triangle** has one right angle (90°) and 2 smaller angles. The side opposite the right angle is the *longest* side and is called the **hypotenuse**.

**Pythagoras' theorem** is a rule that describes the relationship between the hypotenuse and the other 2 (shorter) sides.




The hypotenuse is always opposite the right angle.

Foundation Standard Complex

## Example 3

**a** Construct a right-angled triangle with the 2 shorter sides being of length 3 cm and 4 cm.

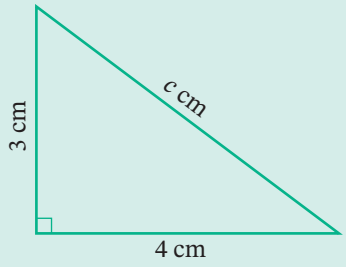
**b** Measure the length of the hypotenuse.

**c** Evaluate $3^2 + 4^2$.

**d** If $c$ is the length of the hypotenuse in centimetres, evaluate $c^2$.

### Solution

**a**




**b** Measuring with a ruler, the hypotenuse is 5 cm long.

**c** $3^2 + 4^2 = 25$

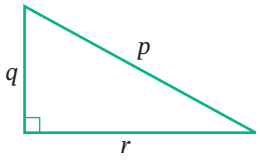
**d** $c^2 = 5^2 = 25$

**EXERCISE 1.02** ANSWERS ON P. 570

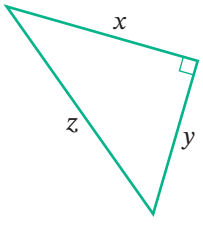
## Discovering Pythagoras' theorem UFRC

**1** Name the hypotenuse in each triangle. C

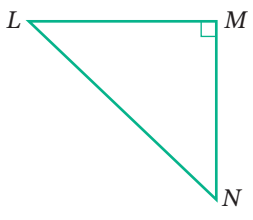
**a**




**b**




**c**




**2** **a** Draw a right-angled triangle with short sides measuring 5 cm and 12 cm.

**b** Measure the length of the hypotenuse.

**c** Evaluate $5^2 + 12^2$.

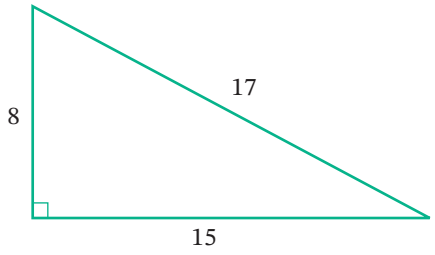
**d** If $c$ is the length of the hypotenuse in centimetres, evaluate $c^2$.

**e** Copy and complete each statement. C

The square of the hypotenuse = $____^2 = ____$

The sum of the squares of the 2 shorter sides = $____^2 + ____^2 = ____$

**3** Which equation is true for the right-angled triangle shown? Select the correct answer **A**, **B**, **C** or **D**. C




**A** $8 + 15 = 17$

**B** $8^2 + 15^2 = 17^2$

**C** $8^2 + 17^2 = 15^2$

**D** $17^2 + 15^2 = 8^2$

EXAMPLE 3

Foundation Standard Complex

**4** **a** Use dynamic geometry software (or pencil, ruler and 5 mm grid paper) to draw a right-angled triangle with the shorter sides of length 6 cm and 8 cm, then measure the length of the hypotenuse.

**b** Copy and complete this table by:

- constructing each right-angled triangle with the measurements given for the 2 shorter sides $a$ and $b$ (as you did in part **a**)
- measuring the length, $c$, of the hypotenuse
- evaluating the values of $a^2$, $b^2$, $a^2 + b^2$, $c^2$ C

| Shorter sides (cm) | | Hypotenuse (cm) | | | | |
|---|---|---|---|---|---|---|
| $a$ | $b$ | $c$ | $a^2$ | $b^2$ | $a^2 + b^2$ | $c^2$ |
| 6 | 8 | | | | | |
| 2.5 | 6 | | | | | |
| 1.5 | 2 | | | | | |
| 4 | 7.5 | | | | | |

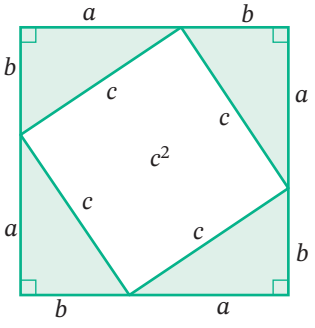
**5** What rule or relationship did you discover between the length of the hypotenuse and the lengths of the other 2 sides? R C

## Investigation

### Geometrical proof of Pythagoras' theorem

There is a proof of Pythagoras' theorem involving 4 identical right-angled triangles as shown in the diagram.

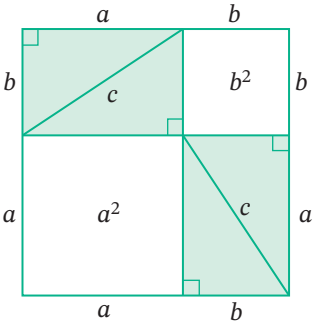
Each triangle has shorter sides of length $a$ and $b$ and hypotenuse of length $c$.




**1** Prove that the white space enclosed by the 4 triangles is a square with 4 right angles.

**2** What is the area of the white square?

**3** Copy the above diagram onto a sheet of paper, then cut out the 4 triangles and rearrange them into 2 identical rectangles and 2 different white squares as shown here.




**4** What is the total area of the 2 white squares?

**5** What does this prove?

**6** Search 'Geometrical proof of Pythagoras' theorem' on the Internet to find an animation or video of this proof.

**7** Research other geometrical proofs of Pythagoras' theorem.

# Finding the hypotenuse

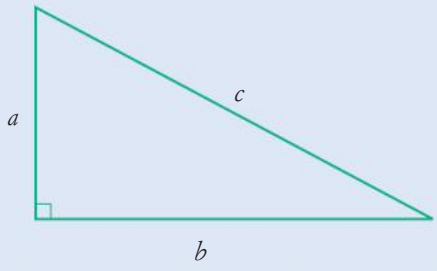
1.03

## Pythagoras' theorem

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other 2 sides.

If $c$ is the length of the hypotenuse, and $a$ and $b$ are the lengths of the other 2 sides, then:

$$c^2 = a^2 + b^2$$




A **theorem** is a formal rule or formula. **Pythagoras' theorem** was discovered by the ancient Greek mathematician Pythagoras. This rule is used to find the length of one side of a right-angled triangle when the other 2 sides are known.

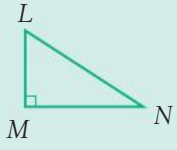


Alamy Stock Photo/Ilan Rosen

*This artwork of Pythagoras and his theorem is from the University of Coimbra in Portugal.*

## Example 4

Write Pythagoras' theorem for $\triangle LNM$.




### Solution

The hypotenuse is $LN$, the 2 shorter sides are $LM$ and $MN$.

So Pythagoras' theorem is $LN^2 = LM^2 + MN^2$

## Example 5

Find the value of $c$ in this triangle.




### Solution

$c$ cm is the length of the hypotenuse. Using Pythagoras' theorem:

$c^2 = a^2 + b^2$

$c^2 = 9^2 + 40^2$ Substitute the lengths of the 2 shorter sides.

$= 1681$

$c = \sqrt{1681}$ Use the square root to find $c$.

$= 41$

*Check*: A hypotenuse of 41 cm looks reasonable from the diagram: it is the longest side and a little longer than 40 cm.

**Technology**
Pythagoras' theorem

**Spreadsheet**
Pythagoras' theorem

**Videos**
Pythagoras' theorem 1

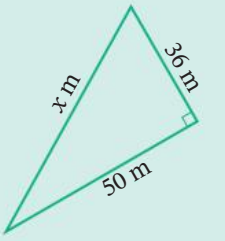
Pythagoras' theorem 2

## Example 6

Find the value of $x$ in this triangle:

**a** as a surd

**b** correct to 2 decimal places.




### Solution

**a** Write Pythagoras' theorem.

$c^2 = a^2 + b^2$

$x^2 = 50^2 + 36^2$ — The hypotenuse in this question is $x$.

$= 3796$ — Substitute the lengths of the 2 shorter sides.

$x = \sqrt{3796}$ — Because the question asks for a surd, leave the answer in square root ($\sqrt{}$) form.

**b** $x = \sqrt{3796}$ — From part **a**.

$= 61.6116\ldots$

$\approx 61.61$ — *Check*: From the diagram, this answer looks reasonable.

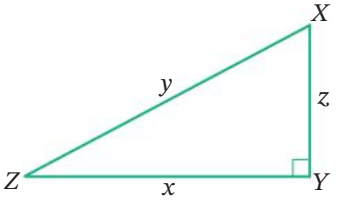
Note: The surd answer is more exact than the decimal answer because it is not rounded.

**EXERCISE 1.03** ANSWERS ON P. 570

## Finding the hypotenuse U F C

EXAMPLE 4

**1** What is Pythagoras' theorem for this triangle?
Select the correct answer **A**, **B**, **C** or **D**. C




**A** $z^2 = x^2 + y^2$

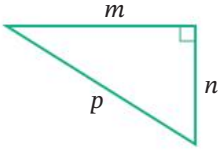
**B** $ZY^2 = ZX^2 + XY^2$

**C** $XY^2 = ZY^2 + ZX^2$

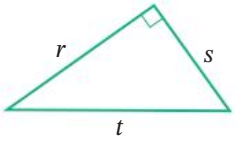
**D** $y^2 = x^2 + z^2$

**2** Write Pythagoras' theorem for each right-angled triangle. C

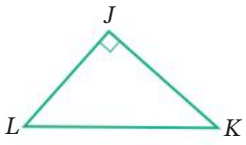
**a**



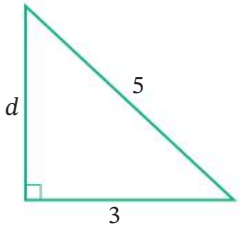

**b**



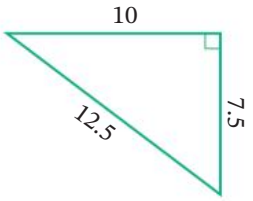

**c**



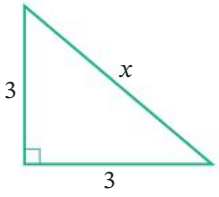

**d**




**e**




**f**



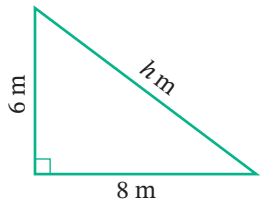

Foundation Standard Complex


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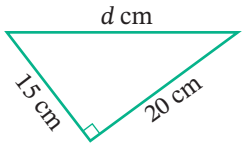
EXAMPLE 5

**3** Find the value of each variable.

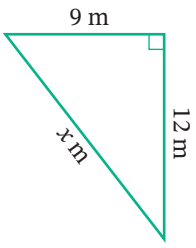
**a**




**b**



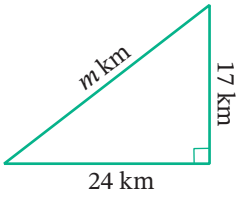

**c**



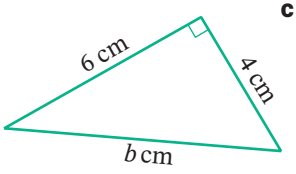

EXAMPLE 6

**4** Find the value of each variable as a surd.

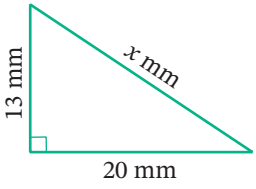
**a**



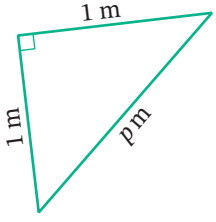

**b**



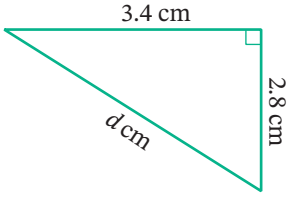

**c**




**d**




**e**

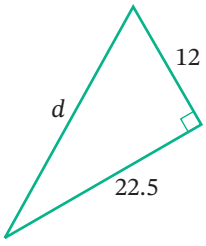



**5** For each triangle in question **4**, find correct to one decimal place the value of each variable.

**6** In this triangle, what is the value of $d$? Select **A**, **B**, **C** or **D**.

**A** 22.8 **B** 12.9

**C** 34.5 **D** 25.5

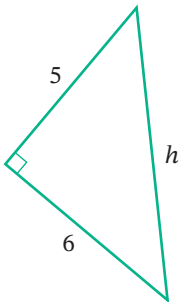



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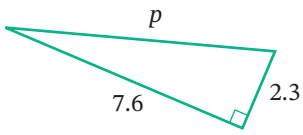
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**7** Find the value of each variable, correct to 2 decimal places.

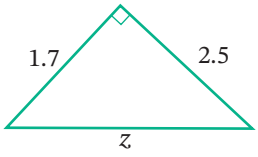
**a**




**b**




**c**




**8** Find the length of the hypotenuse of a right-angled triangle with shorter sides of length 7.5 cm and 10 cm.

**9** Copy and complete each equation. C

**a** $24^2 + 32^2 = ____^2$

**b** $36^2 + 15^2 = ____^2$

**c** $9^2 + 40^2 = ____^2$

**d** $21^2 + ____^2 = 35^2$

**e** $60^2 + ____^2 = 61^2$

**f** $____^2 + 15^2 = 17^2$

**g** $____^2 + 24^2 = 51^2$

**h** $____^2 + ____^2 = 10^2$

**i** $____^2 + ____^2 = 15^2$

Video
Irrational numbers: Pythagoras

## Did you know?

### Pythagoras

Pythagoras was a Greek philosopher and mathematician. He was born on the island of Samos, part of ancient Greece, and lived from 570 BCE to 490 BCE. He was the first person to set down a mathematical proof for his famous theorem, even though it was known and used long before his time.

The Pythagoreans were a group of men who followed Pythagoras' philosophical ideas and worked on mathematical problems. They formed a secret society known as the Pythagoreans. Apparently, they were so upset about the discovery of surds that they tried to keep it a secret. Hippasus, one of the Pythagoreans, was drowned for revealing the secret to outsiders.

**Which American president published a proof of Pythagoras' theorem? When did he do this?**



Shutterstock.com/OmoPhoto

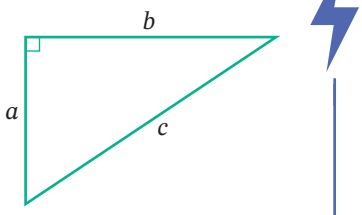
Statue of Pythagoras in modern Samos, Greece.

Foundation Standard Complex


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## Technology

### Finding the hypotenuse




In this activity, we will create a spreadsheet to calculate the length of the hypotenuse, $c$, of a right-angled triangle, given the lengths of the other two sides ($a$ and $b$).

**1** Enter the headings and values shown.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | a | b | a^2+b^2 | c | |
| 2 | 3 | 4 | | | |
| 3 | 5 | 12 | | | |
| 4 | 6 | 8 | | | |
| 5 | 7 | 24 | | | |
| 6 | | | | | |
| 7 | | | | | |

Remember: a^2 + b^2 means $a^2 + b^2$.

**2** Enter into cell C2 the formula **=A2^2+B2^2**, and use **Fill Down** to copy this formula into cells C3 to C5.

**3** To calculate the length of each hypotenuse in column D, we need to find the square root of the values in column C, so in cell D2, enter **=sqrt(C2)**. Then **Fill Down** to copy this formula into cells D3 to D5.

**4** Use your spreadsheet to check your answers to questions **3** to **6** of Exercise 1.03.



# Finding a shorter side

1.04

Pythagoras' theorem can also be used to find the length of one of the shorter sides in a right-angled triangle if the hypotenuse and the other side are given. In this case, we *subtract* the square of the length of the known shorter side from the square of the length of the hypotenuse.



Alamy Stock Photo/Myrleen Pearson

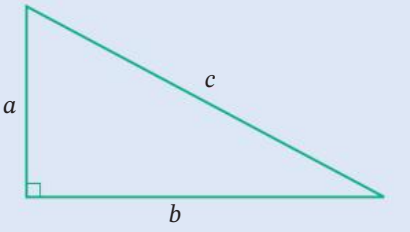
**Technology**
Pythagoras' theorem

**Spreadsheet**
Pythagoras' theorem

## Finding a shorter side

To find the length of a shorter side, $a$, in a right-angled triangle with hypotenuse $c$ and other shorter side $b$, use Pythagoras' theorem in the form:

$$a^2 = c^2 - b^2$$

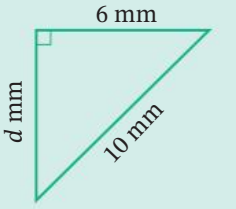



Remember the correct order for $c^2 - b^2$, the hypotenuse is squared first, otherwise we will get a negative number.

Video
Pythagoras' theorem 2

## Example 7

Find the value of $d$ in this triangle.




### Solution

$d$ is one of the shorter sides.

$a^2 = c^2 - b^2$

$d^2 = 10^2 - 6^2$ (Hypotenuse)$^2$ − (other side)$^2$

$= 64$

$d = \sqrt{64}$ Take the square root to find $d$.

$= 8$ *Check*: From the diagram, the answer $d = 8$ looks reasonable.

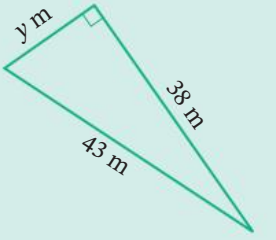
Video
Pythagoras' theorem 1

## Example 8

Find the value of $y$ in this triangle:

**a** as a surd

**b** correct to 2 decimal places.




### Solution

**a** $a^2 = c^2 - b^2$

$y^2 = 43^2 - 38^2$ (Hypotenuse)$^2$ − (other side)$^2$

$= 405$

$y = \sqrt{405}$ Surd answer

**b** $y = \sqrt{405}$ from part **a**

$= 20.1246\ldots$

$\approx 20.12$

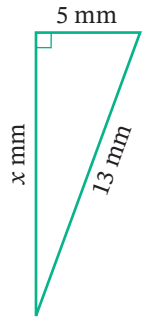
*Check*: From the diagram, this answer appears to be reasonable.

## Finding a shorter side UFR

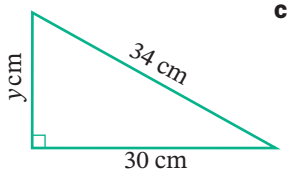
EXAMPLE 7

**1** Find the value of each variable.

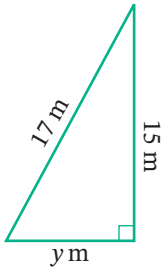
**a**



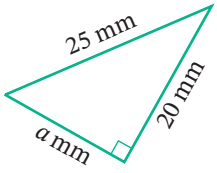

**b**



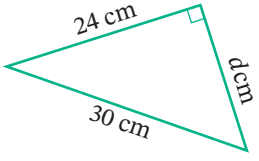

**c**



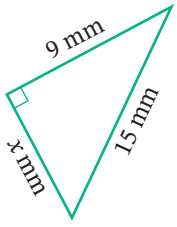

**d**




**e**



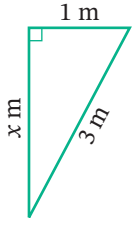

**f**



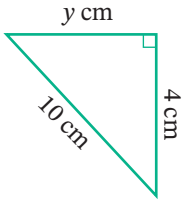

EXAMPLE 8

**2** Find the value of each variable, giving your answer in surd form.

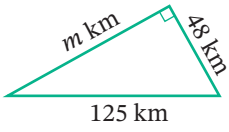
**a**



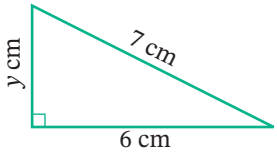

**b**



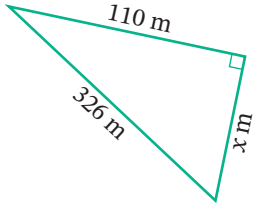

**c**



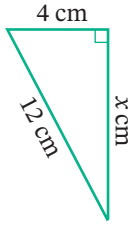

**d**




**e**




**f**

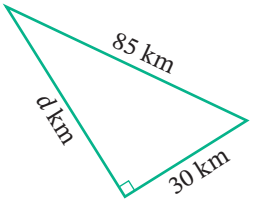



**3** Find the value of each variable in question **2**, correct to one decimal place.

**4** What is the value of $d$? Select the closest answer **A**, **B**, **C** or **D**.

**A** 74.16 **B** 79.53

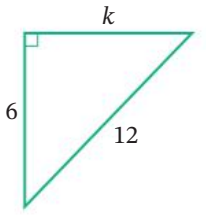
**C** 90.14 **D** 127.50



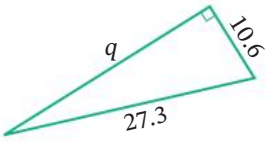

Foundation Standard Complex

**5** Find the value of each variable, correct to 2 decimal places.

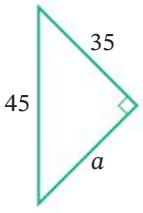
**a**




**b**




**c**




**6** Find the length of the unknown side of a right-angled triangle with a hypotenuse of length 32.5 cm and one of the short sides of length 12.5 cm.

**7** What could be the lengths of the other 2 sides of a right-angled triangle if: R

**a** one of the shorter sides is $\sqrt{7}$ cm long?

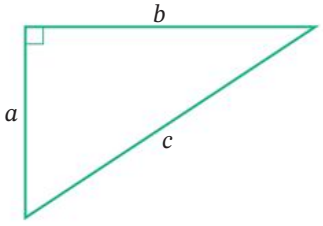
**b** one of the shorter sides is 7 cm long?

**c** the hypotenuse is $\sqrt{20}$ cm long?

**d** the hypotenuse is 20 cm long?

## Technology

### Finding a shorter side




In this activity, we will create a spreadsheet to calculate the length of a shorter side, $a$, of a right-angled triangle, given the lengths of the hypotenuse ($c$) and the other side ($b$).

**1** Enter the headings and values shown below.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | c | b | c^2 - b^2 | a |
| 2 | 25 | 20 | | |
| 3 | 37 | 12 | | |
| 4 | 11 | 7 | | |
| 5 | 29 | 21 | | |

**2** Enter a formula into cell C2 to calculate $c^2 - b^2$, which is 225, and use **Fill Down** to copy this formula into cells C3 to C5.

**3** To calculate the length of the shorter side ($a$) in column D, we need to find the square root of $c^2 - b^2$. Enter a formula into cell D2 to calculate the square root of cell C2, which is 15, and use **Fill Down** to copy this formula into cells D3 to D5.

**4** Use your spreadsheet to check your answers to Exercise 1.04.

# Mental skills 1: Maths without calculators ANSWERS ON P. 570

Quiz
Mental skills 1



## Squaring a number ending in 5 or 1

The square of a number ending in 5 always ends in 25.
For example, $35^2 = 1225$ and $105^2 = 11\,025$.
A mental calculation trick requires 3 easy steps:

- delete the 5 from the number
- multiply the remaining number by the next consecutive number
- write '25' at the end of the product.

**1** Study each example.

**a** To calculate $35^2$:

- deleting the 5 from 35 leaves 3
- multiply 3 by the next consecutive number: $3 \times 4 = 12$
- write '25' at the end: 1225

$35^2 = 1225$

**b** To calculate $105^2$:

- deleting the 5 from 105 leaves 10
- multiply 10 by the next consecutive number: $10 \times 11 = 110$
- write 25 at the end: 11 025

$105^2 = 11\,025$

**2** Now calculate each square number.

**a** $25^2$ **b** $55^2$ **c** $45^2$ **d** $85^2$ **e** $115^2$ **f** $7.5^2$
**g** $95^2$ **h** $195^2$ **i** $1.5^2$ **j** $65^2$ **k** $155^2$ **l** $245^2$

The square of a number ending in 1 always ends in 1.
For example, $41^2 = 1681$ and $71^2 = 5041$.
A mental calculation trick requires 3 steps:

- round the number down to the nearest 10 (by subtracting 1) to make a new number
- square the new number
- to your answer, add the new number and its next consecutive number

**3** Study each example.

**a** To calculate $41^2$:

- round 41 down to 40
- square 40: $40^2 = 1600$
- add 40 and 41: $1600 + 40 + 41 = 1681$

$41^2 = 1681$

**b** To calculate $71^2$:

- round 71 down to 70
- square 70: $70^2 = 4900$
- $4900 + 70 + 71 = 5041$

$71^2 = 5041$

**4** Now calculate each square number.

**a** $21^2$ **b** $101^2$ **c** $31^2$ **d** $91^2$ **e** $5.1^2$
**f** $81^2$ **g** $61^2$ **h** $201^2$ **i** $1.1^2$ **j** $4.1^2$

# 1.05 Hypotenuse or shorter side?

Skillsheet
Pythagoras' theorem

Worksheet
Finding an unknown side

Puzzles
Pythagoras 1

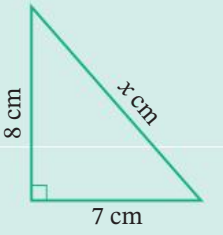
Pythagoras 2

Let's solve some mixed problems that require using Pythagoras' theorem. To find the length of any unknown side in a right-angled triangle, follow these steps.

1. Decide whether the unknown side is the **hypotenuse** or one of the **shorter sides**.
2. To find the hypotenuse, use $c^2 = a^2 + b^2$ and *add*.
   To find a shorter side, use $a^2 = c^2 - b^2$ and *subtract*.
3. If a **surd answer** is required, leave the answer in square root ($\sqrt{\ }$) form. Otherwise, calculate the value correct to the required number of decimal places.

## Example 9

Find the value of $x$ as a surd.




### Solution

$x$ is the hypotenuse.

$c^2 = a^2 + b^2$

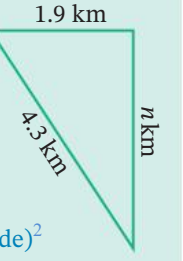
$x^2 = 7^2 + 8^2$
$= 113$

$x = \sqrt{113}$

As a surd.

## Example 10

Find the value of $n$, correct to one decimal place.




### Solution

$n$ is a shorter side.

$a^2 = c^2 - b^2$

$n^2 = 4.3^2 - 1.9^2$
$= 14.88$

(Hypotenuse)$^2$ – (other side)$^2$

$n = \sqrt{14.88}$
$= 3.85746\ldots$
$\approx 3.9$

Find the square root.

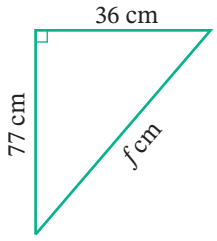

9780170479516

**EXERCISE 1.05** ANSWERS ON P. 570

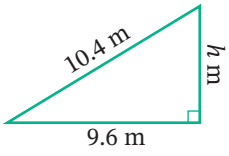
## Hypotenuse or shorter side? U F R C

**1** For each triangle, state whether the unknown side is the hypotenuse (H) or one of the shorter sides (S). R C

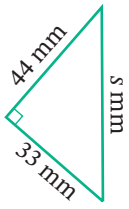
**a**



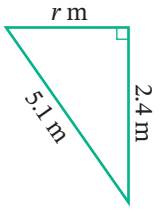

**b**



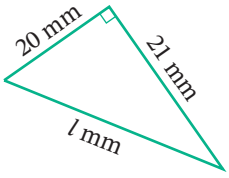

**c**



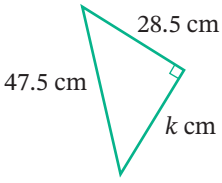

**d**




**e**




**f**

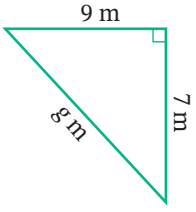



**2** Find the value of each variable in each triangle in question **1**.

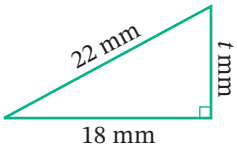
**3** Find the value of each variable as a surd.

EXAMPLE 9

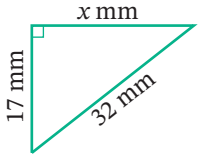
**a**




**b**



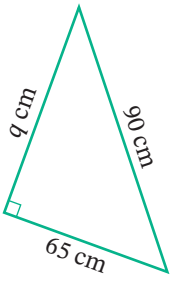

**c**



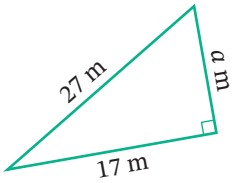

**4** Find the value of each variable correct to one decimal place.

EXAMPLE 10

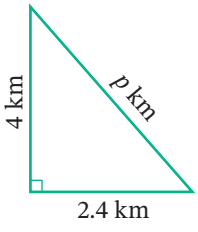
**a**




**b**



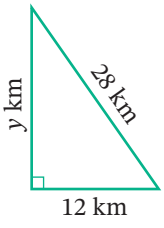

**c**




**5** What is the value of $y$? Select the closest answer **A**, **B**, **C** or **D**.

**A** 25.29 **B** 25.30

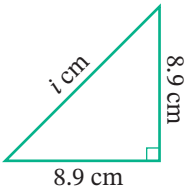
**C** 30.47 **D** 30.46



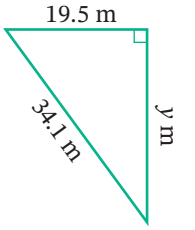

Foundation Standard Complex

**6** Find the value of each variable correct to 2 decimal places.

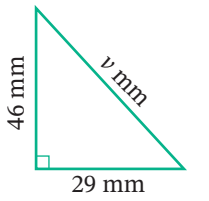
**a**




**b**

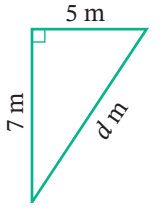



**c**

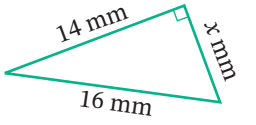



**7** Find the value of each variable as a surd.

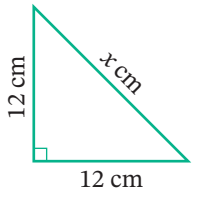
**a**



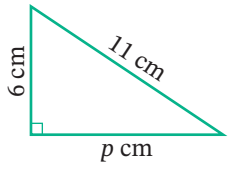

**b**



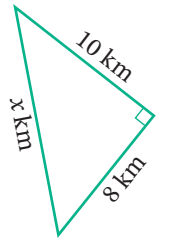

**c**



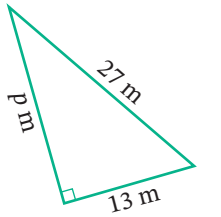

**d**



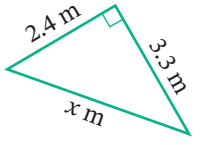

**e**



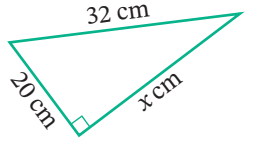

**f**



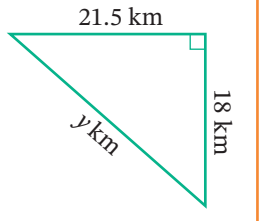

**g**




**h**




**i**




# 1.06 Testing for right-angled triangles

Pythagoras' theorem can be used to test whether a triangle is right-angled.

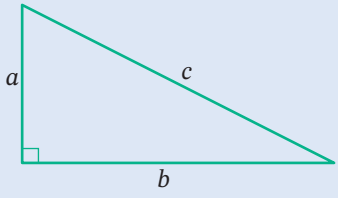
**Worksheet**
Testing for right-angled triangles

**Technology**
Pythagoras' theorem tester

## Testing for right-angled triangles

If the sides of a triangle have lengths $a$, $b$ and $c$, where $c$ is the largest, and they follow the formula $c^2 = a^2 + b^2$, then the triangle must be right-angled.

The right angle is opposite the longest side, $c$.



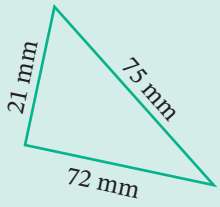

This is called the **converse** of Pythagoras' theorem. The word 'converse' means to 'turn around' or 'reverse'. So the converse of a theorem is the theorem written 'back-to-front'.

Foundation Standard Complex

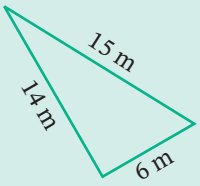
## Example 11

Test whether each triangle is right-angled.

**a**




**b**




### Solution

**a** $75^2 = 5625$ — Squaring the longest side.

$21^2 + 72^2 = 5625$ — Squaring the 2 shorter sides, and adding.

$\therefore 75^2 = 21^2 + 72^2$ — The 3 sides follow $c^2 = a^2 + b^2$.

$\therefore$ The triangle is right-angled (with the right angle opposite the 75 mm side).

$\therefore$ is the symbol for 'therefore'

**b** $15^2 = 225$ — Squaring the longest side.

$6^2 + 14^2 = 232$ — Squaring the 2 shorter sides, and adding.

$\therefore 15^2 \neq 6^2 + 14^2$ — The 3 sides do not follow $c^2 = a^2 + b^2$.

$\therefore$ The triangle is not right-angled.

**Videos**
Testing for right-angled triangles

Building the pyramids

1.06

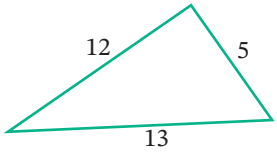
**EXERCISE 1.06** ANSWERS ON P. 570

## Testing for right-angled triangles U F PS R C

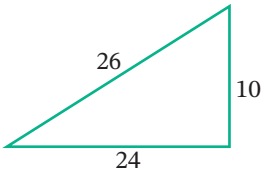
EXAMPLE 11

**1** Copy each triangle, test whether it is right-angled, then mark the right angle on the triangle. R

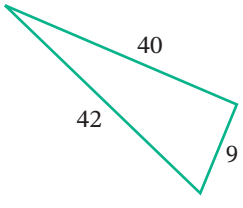
**a**



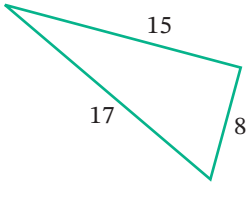

**b**



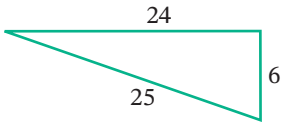

**c**



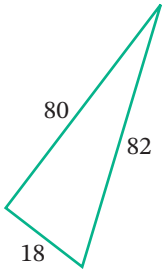

**d**




**e**

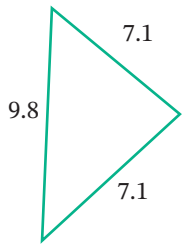



**f**

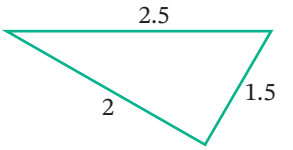



Foundation | Standard | Complex

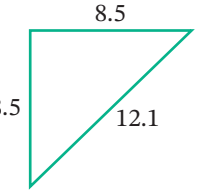
**g**



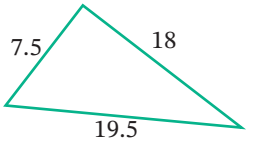

**h**



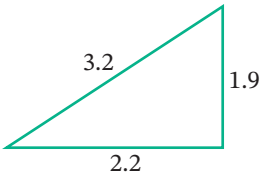

**i**



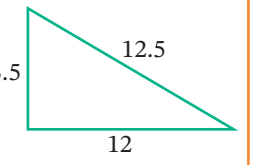

**j**




**k**




**l**




**2** Jane and Mark are building the framework for a rectangular concrete slab. They want to check if it has right angles at the corners. They measure the sides and the diagonal. The sides are 3.6 m and 4.8 m. The diagonal is 6 m. PS R C

**a** Draw a diagram showing this information.

**b** Have they constructed their framework with right angles at the corners? Justify your answer with a calculation.

# 1.07 Pythagorean triads

**Worksheet**
Pythagorean triads

**Technology**
Pythagorean triples

**Spreadsheet**
Pythagorean triples

A **Pythagorean triad** or **Pythagorean triple** is any group of 3 positive integers that follow Pythagoras' theorem, for example, {3, 4, 5} or {5, 12, 13}. The word **triad** means a group of 3 related items ('tri-' means 3).

## Pythagorean triads

$\{a, b, c\}$ is a **Pythagorean triad** if

$$c^2 = a^2 + b^2.$$

Any multiple of $\{a, b, c\}$ is also a Pythagorean triad.

## Example 12

Test whether {5, 12, 13} is a Pythagorean triad.

### Solution

$13^2 = 169$ — Squaring the largest number.

$5^2 + 12^2 = 169$ — Squaring the 2 smaller numbers and adding them.

$\therefore 13^2 = 5^2 + 12^2$ — These 3 numbers follow Pythagoras' theorem.

$\therefore$ {5, 12, 13} is a Pythagorean triad.

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## Example 13

{3, 4, 5} is a Pythagorean triad. Create other Pythagorean triads by multiplying {3, 4, 5} by:

**a** 2 **b** 9 **c** $\frac{1}{2}$

### Solution

**a** $2 \times \{3, 4, 5\} = \{6, 8, 10\}$

*Checking*: $10^2 = 100$

$6^2 + 8^2 = 100$

$\therefore 10^2 = 6^2 + 8^2$

$\therefore \{6, 8, 10\}$ is a Pythagorean triad.

**b** $9 \times \{3, 4, 5\} = \{27, 36, 45\}$

*Checking*: $45^2 = 2025$

$27^2 + 36^2 = 2025$

$\therefore 45^2 = 27^2 + 36^2$

$\therefore \{27, 36, 45\}$ is a Pythagorean triad.

**c** $\frac{1}{2} \times \{3, 4, 5\} = \{1.5, 2, 2.5\}$

*Checking*: $2.5^2 = 6.25$

$1.5^2 + 2^2 = 6.25$

$\therefore 2.5^2 = 1.5^2 + 2^2$

$\therefore \{1.5, 2, 2.5\}$ is a Pythagorean triad.

Strictly speaking, this is not a Pythagorean triad because they are not whole numbers.



**EXERCISE 1.07** ANSWERS ON P. 571

## Pythagorean triads U F R

**1** Test whether each triad is a Pythagorean triad. R

EXAMPLE 12

**a** {8, 15, 17} **b** {10, 24, 26} **c** {30, 40, 50}

**d** {5, 7, 9} **e** {9, 40, 41} **f** {4, 5, 9}

**g** {11, 60, 61} **h** {7, 24, 25} **i** {15, 114, 115}

**2** Which of the following is a Pythagorean triad? Select the correct answer **A**, **B**, **C** or **D**. R

**A** {4, 6, 8} **B** {5, 10, 12} **C** {6, 7, 10} **D** {20, 48, 52}

**3** If you used the technology worksheet **Pythagoras' theorem tester** on page 20 to create a spreadsheet, then use it to check your answers to questions **1** and **2**.

**4** For each Pythagorean triad, create another Pythagorean triad by multiplying each number in the triad by:

EXAMPLE 13

**i** a whole number **ii** a fraction **iii** a decimal.

**a** {5, 12, 13} **b** {8, 15, 17} **c** {30, 40, 50} **d** {7, 24, 25}

Check that each answer follows Pythagoras' theorem. R

**5** Pythagoras developed a formula for finding Pythagorean triads $\{a, b, c\}$.
If one number in the triad is $a$, the formulas for the other 2 numbers are $b = \frac{1}{2}(a^2 - 1)$ and $c = \frac{1}{2}(a^2 + 1)$. R

**a** If $a = 5$, use the formulas to find the values of $b$ and $c$.

**b** Hence show that $\{a, b, c\}$ is a Pythagorean triad.

**6** Use the formulas to find Pythagorean triads for each value of $a$.

**a** $a = 7$ **b** $a = 11$ **c** $a = 15$ **d** $a = 4$

**e** $a = 9$ **f** $a = 19$ **g** $a = 10$ **h** $a = 51$

**7** There are many other formulas for creating Pythagorean triads. Use the Internet to search for some of them and test that they are correct.



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# 1.08 Pythagoras' theorem problems

Pythagoras' theorem has many practical applications, from finding the diagonal length of a TV screen to calculating the distance a ship has sailed.

**Worksheets**
Applications of Pythagoras' theorem

Pythagoras' theorem in 2D and 3D

TV screens

## Example 14

The size of a TV screen is described by the length of its diagonal. If a flat-screen TV is 89 cm wide and 50 cm high, what is the size of its screen? Answer to the nearest centimetre.




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### Solution

Let the diagonal length be $d$ cm.

$d^2 = 89^2 + 50^2$ $d$ is the hypotenuse.

$= 10421$

$d = \sqrt{10421}$

$= 102.0832\ldots$

$\approx 102$

The size of the screen is 102 cm.

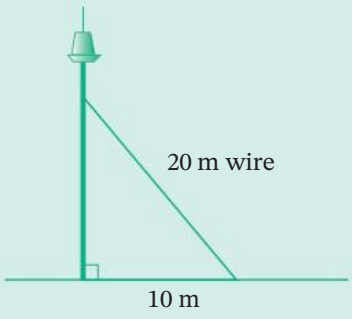
Foundation Standard Complex



9780170479516

## Example 15

A tower is supported by a wire that is 20 m long and attached to the ground 10 m from the base of the tower. How high does the wire reach up the tower? Answer correct to the nearest 0.1 m.




### Solution

Let $h$ m be how high the wire reaches up the tower.

$h^2 = 20^2 - 10^2$ — $h$ is a shorter side.
$= 300$

$h = \sqrt{300}$
$= 17.3205$
$\approx 17.3$

The wire reaches 17.3 m up the tower.

Video
Pythagoras' theorem problems

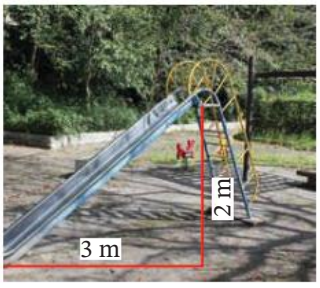
1.08

EXAMPLE 14

**EXERCISE 1.08** ANSWERS ON P. 571

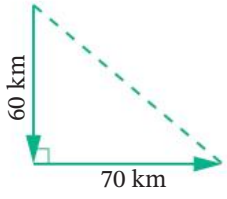
## Pythagoras' theorem problems U F PS R

**1** Find the length of this playground slide, correct to 2 decimal places.



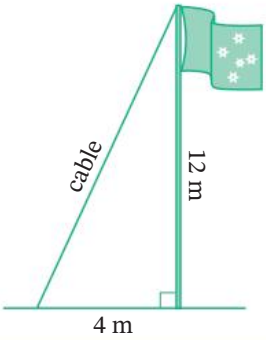
iStock.com/Masahito Ueno

**2** A ship sails 60 km south and then 70 km east. How far is it from its starting point, correct to one decimal place?

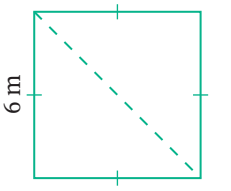



**3** What is the length of a cable used to stabilise a flagpole that is 12 m high, if the cable is secured to the ground 4 m from the base of the flagpole? Select the closest answer **A**, **B**, **C** or **D**.

**A** 11.3 m **B** 12.6 m
**C** 16 m **D** 80 m



**4** Find the length of the diagonal of a square with sides of length 6 m. Give your answer as a surd.




**5** A firefighter places a ladder on a window sill 9.5 m above the ground. If the foot of the ladder is 1.8 m from the wall, how long is the ladder, correct to one decimal place?



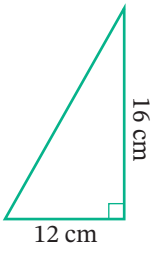

Alamy Stock Photo/Willows Photos UK

EXAMPLE 15

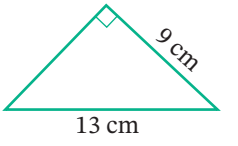
**6** Find the height of a TV screen with a 140 cm diagonal if its length is 122 cm.
Give your answer correct to one decimal place. PS

**7** By first using Pythagoras' theorem to find the length of the unknown side, find the perimeter of each shape below, correct to 2 decimal places where necessary.

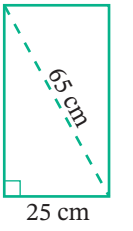
**a**



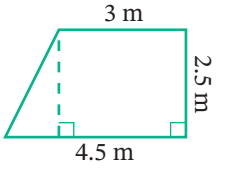

**b**



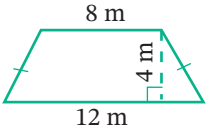

**c**



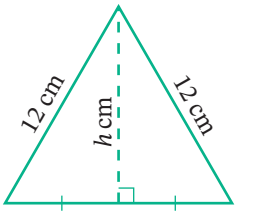

**d**




**e**

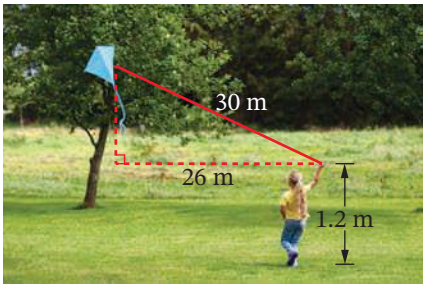



**8** An equilateral triangle has sides of length 12 cm. Find its perpendicular height, $h$, correct to 2 decimal places. PS R




**9** A rope is tied to the top of the 6 m wall of a tent and tied to a peg in the ground. The peg is 2 m from the bottom of the tent. How long is the rope, correct to 2 decimal places? PS R

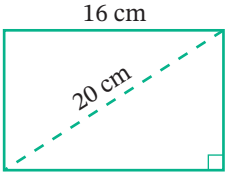
**10** Olga holds a kite string 1.2 m above the ground. How high is the kite above the ground, to the nearest metre? PS R



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**11** Find the area of each shape below, correct to one decimal place where necessary. R

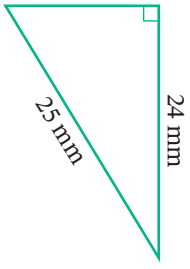
**a**



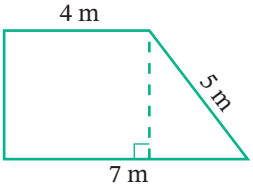

**b**



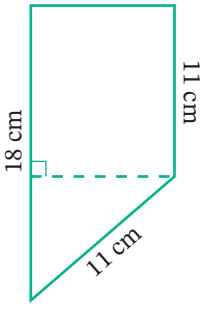

**c**



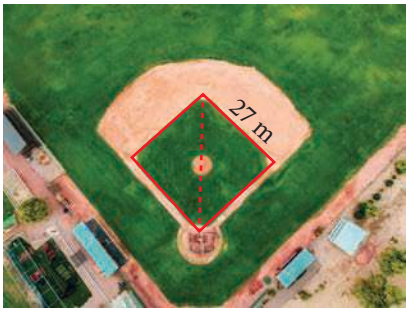

**d**




**e**

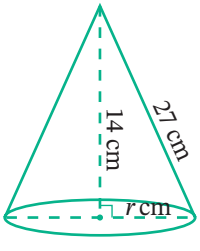



**12** This baseball diamond is a square shape of length 27 m. What is the distance from the home plate to second base, correct to the nearest metre?



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**13** The slant height of a cone is 27 cm and its vertical height is 14 cm. Find the radius of the base circle, correct to the nearest centimetre.



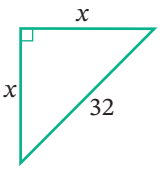

**14** A ladder 5 m long leans against a wall, with its base 2 m from the bottom of the wall. How far does the ladder reach up the wall, correct to nearest centimetre? PS R

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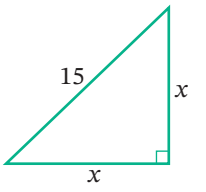
## Power plus ANSWERS ON P. 571

**1** Find the value of $x$ in each triangle, correct to one decimal place.

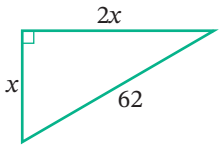
**a**



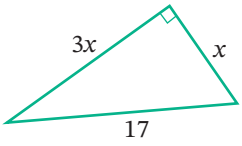

**b**



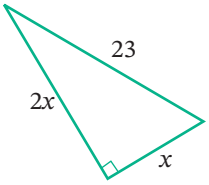

**c**



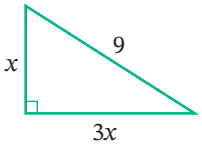

**d**




**e**



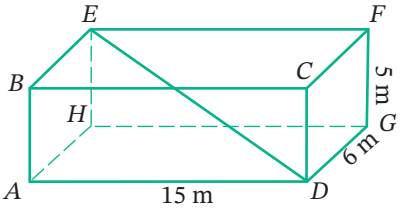

**f**




**2** Find, correct to one decimal place, the length of:

**a** $HD$

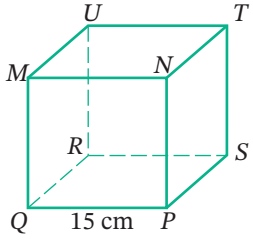
**b** $DE$



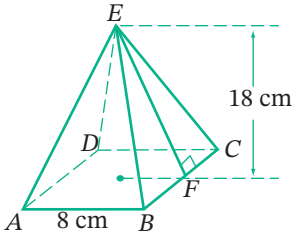

**3** For this cube of side length 15 cm, find, correct to one decimal place, the length of:

**a** $QS$, the diagonal of the base

**b** $QT$, the diagonal of the cube




**4** For this square pyramid, find the slant height $EF$ correct to one decimal place.




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# CHAPTER 1 REVIEW

## Language of maths

| | | | |
|---|---|---|---|
| area | converse | diagonal | formula |
| hypotenuse | irrational | perimeter | Pythagoras |
| right-angled | shorter | side | square root |
| surd | theorem | triad | unknown |

**1** Who was **Pythagoras** and which country did he come from?

**2** Describe the **hypotenuse** of a right-angled triangle in 2 ways.

**3** What is another word for 'theorem'?

**4** For what type of triangle is **Pythagoras' theorem** used?

**5** What is a **surd**?

**6** What is the name given to a set of 3 numbers that follows Pythagoras' theorem?

**Quiz**
Language of maths 1

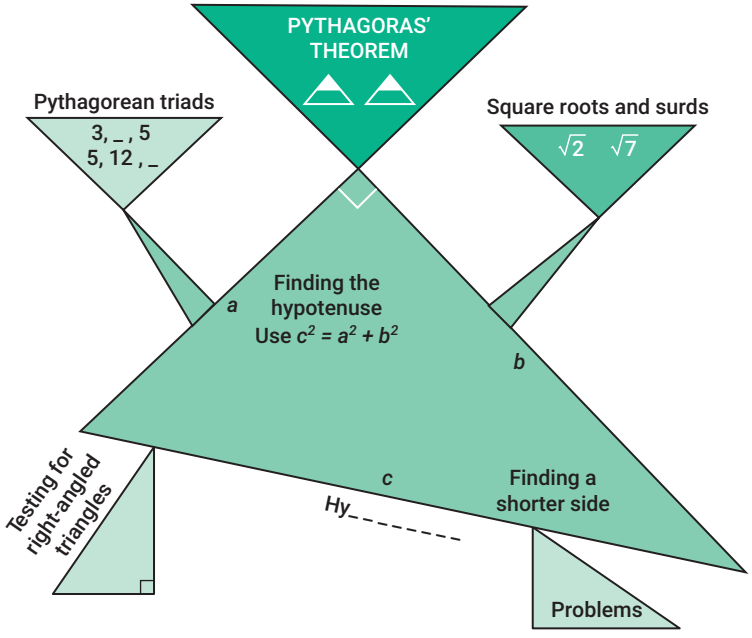
**Puzzle**
Pythagoras' theorem find-a-word

## Topic summary

- How relevant do you think Pythagoras' theorem is to our world? Give reasons for your answer.
- Give 3 examples of careers that would use Pythagoras' theorem.
- What did you find especially interesting about this topic?
- Is there any section of this topic that you found difficult? Discuss any problems with your teacher or a friend.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

**Worksheet**
Mind map: Pythagoras' theorem



Quiz
Test yourself 1

# TEST YOURSELF 1

ANSWERS ON P. 571

1.01 **1** Evaluate each expression, correct to 2 decimal places.

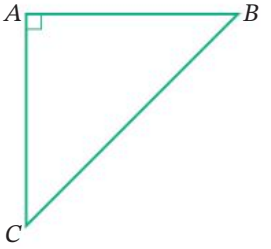
**a** $\sqrt{203}$ **b** $\sqrt{15^2 + 6^2}$ **c** $\sqrt{7^2 - 4^2}$ **d** $\sqrt{3.4^2 + 5.2^2}$

1.01 **2** Select all the surds from this list of square roots.

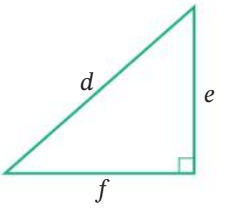
$\sqrt{25}$ $\sqrt{104}$ $\sqrt{96}$ $\sqrt{256}$ $\sqrt{12}$
$\sqrt{169}$ $\sqrt{121}$ $\sqrt{45}$ $\sqrt{729}$ $\sqrt{88}$

1.02 **3** For each right-angled triangle, name the hypotenuse and write Pythagoras' theorem.

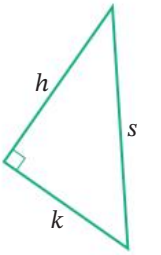
**a**




**b**

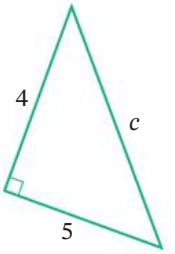



**c**

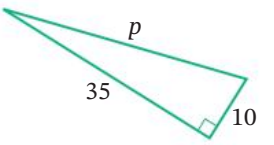



1.03 **4** Find the value of each variable, giving your answer as a surd.

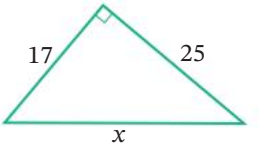
**a**




**b**

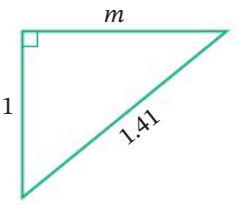



**c**

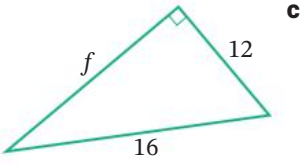



1.04 **5** Find the value of each variable, giving your answer correct to 2 decimal places.

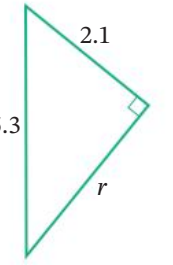
**a**




**b**

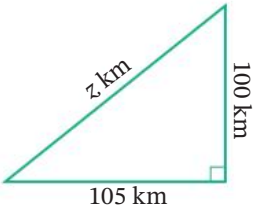



**c**

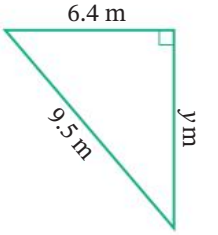



1.05 **6** Find the value of each variable, correct to one decimal place.

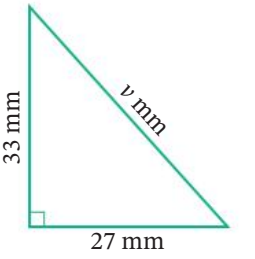
**a**




**b**




**c**

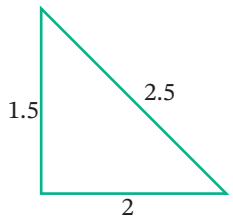



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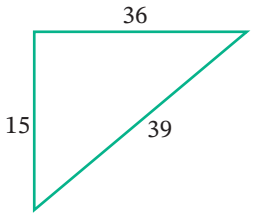

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**7** Test whether each triangle is right-angled. If the triangle is right-angled, sketch it showing the right angle. 1.06

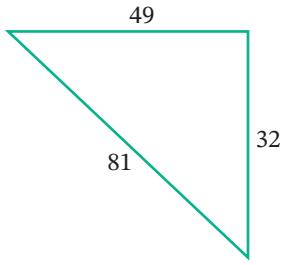
**a**




**b**




**c**




**8** Test whether each triad is a Pythagorean triad. 1.07

**a** {15, 20, 25} **b** {11, 14, 20} **c** {20, 21, 29} **d** {2.5, 6, 6.5}

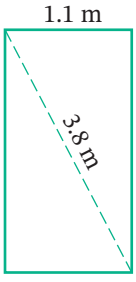
**9** Find the length of the longest pencil that can fit inside this pencil case. Answer to the nearest mm. 1.08



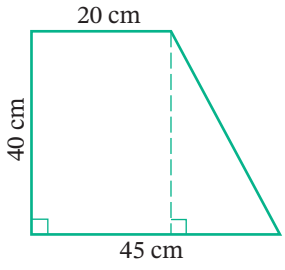

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**10** Find the perimeter of each shape, correct to one decimal place. 1.08

**a**




**b**




Foundation Standard Complex



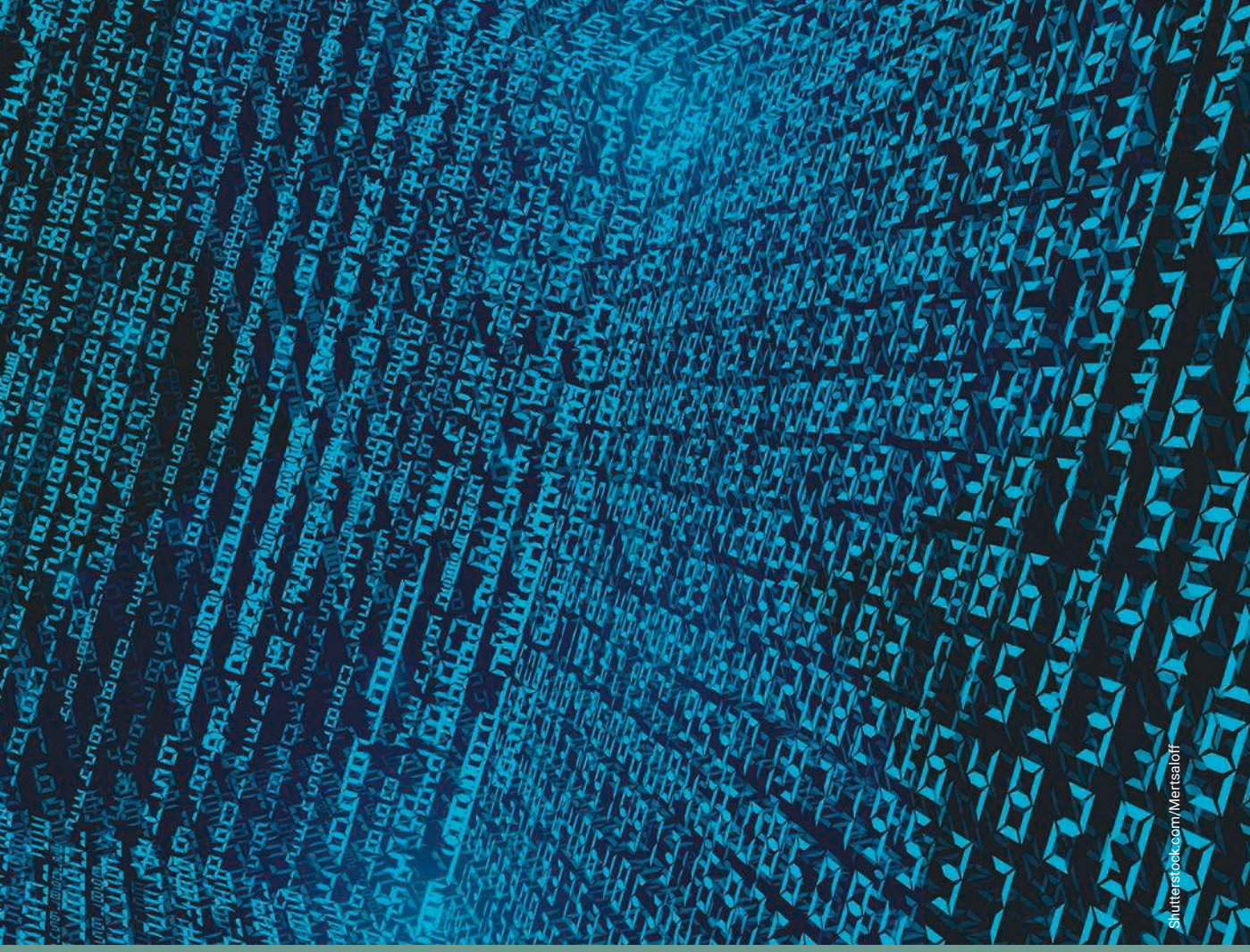
# 2

NUMBER AND ALGEBRA

# WORKING WITH NUMBERS

Numbers are interesting! You already know about odd and even numbers, prime and composite numbers and integers. There are also triangular and square numbers, Fibonacci numbers, perfect and amicable numbers, and complex numbers. Numbers can form special patterns such as in magic squares, Pythagorean triads and Pascal's triangle. These patterns can be found in biology, probability, digital technology and software design.

In this chapter, we will revise and extend our number skills and examine the patterns involved when calculating with powers, using mental calculation, pen-and-paper methods and calculators.




## Chapter outline

| | Working mathematically | | | | |
|---|---|---|---|---|---|
| **2.01** Mental calculation | U | F | PS | R | |
| **2.02** Adding and subtracting integers | U | F | PS | R | |
| **2.03** Multiplying and dividing integers | U | F | | R | C |
| **2.04** Order of operations | U | F | PS | R | C |
| **2.05** Decimals | U | F | PS | R | |
| **2.06** Multiplying and dividing decimals | U | F | PS | R | |
| **2.07** Terminating and recurring decimals | U | F | | R | C |
| **2.08** Powers and roots | U | F | | R | C |
| **2.09** Simplifying surds* | U | F | | R | C |
| **2.10** Factor trees | U | F | | R | C |
| **2.11** Index laws for multiplying and dividing | U | F | | R | C |
| **2.12** More index laws | U | F | | R | C |

*YEAR 9 EXTENSION

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

## Wordbank

**base (in index notation)** A number raised to a power, for example, in $3^5$, 3 is the base

**cube root** The value which, if cubed, will give the number required, for example $\sqrt[3]{8} = 2$ because $2^3 = 8$

**factor tree** A diagram that lists the prime factors of a number.

**index notation** A way of writing powers for the repeated multiplication of a number, for example, $3^5$.

**mental calculation** To operate with numbers 'in your head', without using a pen or calculator

**order of operations** The rules for calculating an expression involving mixed operations, such as $14 - 2 \times 4 + 1$

**recurring decimal** A decimal that has one or more digits that repeat endlessly

**terminating decimal** A decimal that is not recurring, but comes to an end

**Quiz**
Wordbank 2

## Videos (21):

**2.01** Mental multiplication • Dividing numbers • Multiplying by 8, 9, 11 and 12
**2.02** Adding integers • Subtracting integers
**2.03** Multiplying and dividing integers
**2.04** Order of operations 1 • Order of operations 2 • BODMAS
**2.05** Rounding decimals • Adding and subtracting decimals
**2.06** Multiplying decimals • Dividing decimals
**2.07** Recurring decimals • Converting fractions to decimals
**2.10** Factor trees • Highest common factor • HCF and factor trees • Using factor trees to find square roots
**2.11** Powers, indices and exponents • Index laws
**2.12** Index laws

## *Twig* videos (7):

**2.01** Number theory: Gauss • Chinese development of maths • The Fibonacci sequence
**2.02** India and negative numbers
**2.03** Chinese development of maths
**2.05** Rounding: Snails vs rockets
**2.08** Binary: the computer language
**2.10** The prime number code

## *PhET* interactives (5):

**SkillCheck** Number line: Integers
**2.01** Arithmetic
**2.02** Number line: Operations • Number line: Distance
**2.06** Area model decimals

## Quizzes (6):

- Wordbank 2
- SkillCheck 2
- Mental skills 2A
- Mental skills 2B
- Language of maths 2
- Test yourself 2

## Skillsheets (11):

**SkillCheck** Decimals • Fractions and decimals
**2.02** Integers • Integers using diagrams • Integers using coloured squares
**2.03** Integers
**2.04** Order of operations • Spreadsheets
**2.05** Decimals
**2.07** Fractions and decimals
**2.08** Index notation • Square roots and cube roots
**2.10** Factors and divisibility • Prime factors by repeated division

## Worksheets (22):

**SkillCheck** Calculation aids • Number grids
**2.02** A page of number lines
**2.03** What is the integer question? • Integer review • Integers writing activity
**2.05** Decimals wall • Dewey decimals • Decimals 1 • Decimals 10
**2.06** Decimal cards • Shopping and change • What's the point? • Decimals 7
**2.07** Fraction families
**2.08** Powers and roots • Big numbers
**2.09** Surds
**2.10** Perfect and amicable numbers
**2.12** What is the power question? • Power calculations
**Mind map** Working with numbers

## Puzzles (16):

**SkillCheck** Magic squares
**2.01** The accidental detective
**2.02** Integer Snap
**2.03** Multiplying integers game • Directed numbers • Integers group clues
**2.04** Directed numbers • Order of operations puzzle
**2.06** Operations with decimals • Which decimals? • Decimal number grids
**2.07** Decimal squaresaw 2
**2.08** Square root Snap
**2.09** Simplifying surds
**2.10** Crossnumber puzzles • Crossnumber challenges
**2.12** Crossnumber challenges
**Language of maths** Numbers crossword

## Presentations (4):

**2.02** Number lines • Adding directed numbers • Subtracting directed numbers
**2.10** Highest common factors

## Technology (3):

**2.04** Review of spreadsheets
**2.06** Movie night
**2.09** Simplifying surds quiz

## Spreadsheets (2):

**2.03** Integer quiz: Multiplication
**2.09** Simplifying surds quiz

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

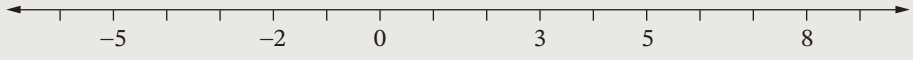
## In this chapter you will:

- apply number laws to help mental calculation
- calculate with integers, decimals and fractions, using mental and written strategies and technology
- apply the order of operations to evaluate mixed expressions
- investigate terminating and recurring decimals
- evaluate expressions involving powers, square root and cube root, after first estimating
- find square roots and cube roots of any non-square whole number using a calculator, after first estimating
- graph square roots (surds) on a number line using Pythagoras' theorem
- (EXTENSION, YEAR 9 PATH) simplify a surd
- explore the properties of squares and square roots of products: $(ab)^2$ and $\sqrt{ab}$
- use a factor tree to write a number as a product of its prime factors
- use factor trees to find the highest common factor and lowest common multiple of 2 or more numbers, and to find the square root or cube root of a square number or cube number respectively
- explore the index laws with positive and zero powers

## SkillCheck ANSWERS ON P. 571

**1** Copy and complete each number line.

**a**




**b**




**2** Write these integers in ascending order: −7, 6, −5, 11, −2, 1, 0, 7.

**3** Write these integers in descending order: 12, −3, 5, −1, 2, −9, 4, 8.

**4** In the decimal 2.718, name the digit in the:

**a** tenths place **b** thousandths place **c** hundredths place

**5** Convert each decimal to a fraction.

**a** 0.03 **b** 0.007 **c** 0.49 **d** 0.9

**6** How many decimal places has each number?

**a** 14.451 **b** 86.6 **c** 28.04 **d** 3.141 59

**7** **a** List the factors of 15.

**b** List the first 5 **multiples** of 15.

**8** State whether each number is prime or composite.

**a** 2 **b** 17 **c** 25 **d** 27



**Quiz**
SkillCheck 2

**Skillsheets**
Decimals

Fractions and Decimals

**Worksheets**
Calculation aids

Number grids

**Puzzle**
Magic squares

**Interactive**
Number line: Integers

# 2.01 Mental calculation

**Videos**
Mental multiplication
Dividing numbers
Number theory: Gauss
Chinese development of maths

**Puzzle**
The accidental detective

**Interactive**
Arithmetic

The word 'mental' means 'using the mind', and mental calculation is the skill of working with numbers 'in your head', without using a pen or calculator. In the 'Mental skills' sections of *New Century Maths 7*, you learnt many mental strategies for calculating with numbers.

## Example 1 Estimating answers by rounding numbers

**a** $15 + 37 + 18 + 45 + 22 \approx 20 + 40 + 20 + 40 + 20$
$= 140$ (exact answer = 137)

**b** $18 \times 12 \approx 20 \times 11$
$= 220$ (exact answer = 216)

**c** $504 \div 8 \approx 500 \div 10$
$= 50$ (exact answer = 63)

## Example 2 Adding and multiplying numbers in any order

The commutative laws: $a + b = b + a$ and $a \times b = b \times a$

The associative laws: $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$

**a** $15 + 37 + 18 + 45 + 22 = (15 + 45) + (18 + 22) + 37$
$= 60 + 40 + 37$
$= 137$
Pair numbers that add to multiples of 10.

**b** $7 \times 4 \times 5 = 7 \times (4 \times 5)$
$= 7 \times 20$
$= 140$
Pair numbers that multiply to multiples of 10.

## Example 3 Adding and subtracting 8 or 9

**a** $43 + 9 = 43 + 10 - 1$
$= 53 - 1$
$= 52$
Add 10, count back 1.

**b** $97 + 8 = 97 + 10 - 2$
$= 107 - 2$
$= 105$
Add 10, count back 2.

**c** $61 - 8 = 61 - 10 + 2$
$= 51 + 2$
$= 53$
Subtract 10, count forward 2.


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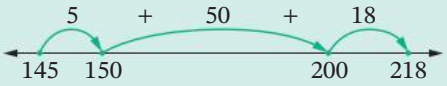
## Example 4 Calculating differences using a number line

$218 - 145$




$145 + ___ = 218?$

Think of this as the difference or 'gap' between 145 and 218 on a number line.




Make smaller jumps.

Add the jumps.

$5 + 50 + 18 = 73$

This is like counting out change with money.

So $218 - 145 = 73$

## Example 5 Multiplying and dividing by a multiple of 10

**a** $9 \times 60 = 9 \times 6 \times 10$
$= 54 \times 10$
$= 540$

**b** $2700 \div 30 = 270\cancel{0} \div 3\cancel{0}$
$= 270 \div 3$
$= 90$

## Example 6 Doubling and halving numbers, Multiplying and dividing by 4

**a** $47 \times 2 = 40 \times 2 + 7 \times 2$
$= 80 + 14$
$= 94$

**b** $\frac{1}{2} \times 68 = \frac{1}{2} \times 60 + \frac{1}{2} \times 8$
$= 30 + 4$
$= 34$

**c** $\frac{1}{2} \times 192 = \frac{1}{2} \times 180 + \frac{1}{2} \times 12$
$= 90 + 6$
$= 96$

**d** $29 \times 4 = 29 \times 2 \times 2$

Double twice

Double 29 = 58, double 58 = 116

$29 \times 4 = 116$

**e** $16 \times 8 = 16 \times 2 \times 2 \times 2$

Double 3 times

Double 16 = 32, double 32 = 64, double 64 = 128

$16 \times 8 = 128$

**f** $560 \div 4 = 560 \div 2 \div 2$

Halve twice

$\frac{1}{2} \times 560 = 280$, $\frac{1}{2} \times 280 = 140$

$560 \div 4 = 140$

## Example 7 Simplifying multiplication by factorising

**a**
$$\begin{aligned} 45\times16 &= 9\times5\times4\times4 \\ &= 5\times4\times4\times9 \\ &= 20\times36 \\ &= 720 \end{aligned}$$

**b**
$$\begin{aligned} 12\times18 &= 6\times2\times9\times2 \\ &= 6\times9\times2\times2 \\ &= 54\times4 \\ &= 216 \end{aligned}$$

## Example 8 Multiplying and dividing by 5, 15, 25, 50

**a**
$$\begin{aligned} 16\times15 &= 16\times\frac{1}{2}\times30 \quad \text{or} \quad 8\times2\times15 \\ &= 8\times30 \\ &= 240 \end{aligned}$$

**b**
$$\begin{aligned} 24\times25 &= 24\times\frac{1}{4}\times100 \quad \text{or} \quad 6\times4\times25 \\ &= 6\times100 \\ &= 600 \end{aligned}$$

**c**
$$\begin{aligned} 420\div5 &= 420\div10\times2 \\ &= 42\times2 \\ &= 84 \end{aligned}$$

**d**
$$\begin{aligned} 300\div25 &= 300\div100\times4 \\ &= 3\times4 \\ &= 12 \end{aligned}$$

Video
Multiplying by 8, 9, 11 and 12

## Example 9 Multiplying by 8, 9, 11, 12

The distributive laws:

$$a\times(b+c)=a\times b+a\times c \quad \text{and} \quad a\times(b-c)=a\times b-a\times c$$

**a**
$$\begin{aligned} 25\times9 &= 25\times(10-1) \\ &= 25\times10-25\times1 \\ &= 250-25 \\ &= 225 \end{aligned}$$

**b**
$$\begin{aligned} 18\times8 &= 18\times(10-2) \\ &= 18\times10-18\times2 \\ &= 180-36 \\ &= 144 \end{aligned}$$

**c**
$$\begin{aligned} 13\times11 &= 13\times(10+1) \\ &= 13\times10+13\times1 \\ &= 130+13 \\ &= 143 \end{aligned}$$

**d**
$$\begin{aligned} 27\times12 &= 27\times(10+2) \\ &= 27\times10+27\times2 \\ &= 270+54 \\ &= 324 \end{aligned}$$

## Short and long division

**Short division** is a technique for dividing by one-digit numbers (whole numbers less than 10).

**Long division** is a technique for dividing by numbers with 2 or more digits (whole numbers greater than 10).

## Example 10

Evaluate each quotient.

**a** $520 \div 8$ **b** $945 \div 21$

### Solution

**a** By short division:

$8\overline{)520}$ — 8 into 5 goes 0

$8\overline{)52^{4}0}$ — 8 into 52 goes 6, remainder 4

$8\overline{)52^{4}0}$ — 8 into 40 goes 5 (exactly)

$520 \div 8 = 65$ — Check by estimating: $520 \div 8 \approx 500 \div 10 = 50$ (65 is close to 50)

OR:

$520 \div 8 = 520 \div 2 \div 2 \div 2$ — Halve 3 times

$520 \div 2 = 260$, $260 \div 2 = 130$, $130 \div 2 = 65$

$520 \div 8 = 65$

**b** By long division:

```
   45
21)945
  -84↓
  105
 -105
    0
```

21 into 94 is 4, remainder 10.
21 into 105 is 5.

$945 \div 21 = 45$ — Check by estimating: $945 \div 21 \approx 900 \div 20 = 45$

OR:

Guessing with 'easy' multiples of 21.

| | |
|---|---|
| $21\overline{)945}$ | |
| $-420$ | 20 times |
| 525 | |
| $-420$ | 20 times |
| 105 | |
| $-84$ | 4 times |
| 21 | |
| $-21$ | 1 time |
| 0 | 45 times |

EXTENSION

**EXERCISE 2.01** ANSWERS ON P. 571

## Mental calculation U F PS R

EXAMPLE 1

**1** Estimate the value of each expression, then check your estimate with a calculator.

**a** $27 + 11 + 87 + 142 + 64$
**b** $55 + 34 - 22 - 46 + 136$
**c** $684 + 903$
**d** $35 + 81 + 110 + 22 + 7$
**e** $517 - 96$
**f** $766 - 353$
**g** $46 \times 8$
**h** $367 \times 2$
**i** $51 \times 12$
**j** $245 \div 7$
**k** $828 \div 3$
**l** $564 \div 12$

EXAMPLE 2

**2** Evaluate each sum. PS R

**a** $55 + 18 + 25 + 9 + 12$
**b** $23 + 42 + 16 + 24 + 7$
**c** $140 + 33 + 12 + 20 + 28$
**d** $37 + 11 + 29 + 5 + 13$
**e** $45 + 49 + 121 + 25 + 10$
**f** $74 + 99 + 21 + 32 + 16$

**3** What is the value of $720 + 1456$? Select the correct answer **A**, **B**, **C** or **D**.

**A** 2176 **B** 2206 **C** 2806 **D** 8656

EXAMPLE 3

**4** Evaluate each expression. R

**a** $27 + 9$
**b** $45 + 9$
**c** $16 + 8$
**d** $34 + 28$
**e** $17 + 11$
**f** $22 + 21$
**g** $69 + 12$
**h** $114 + 32$
**i** $45 - 9$
**j** $27 - 9$
**k** $76 - 8$
**l** $32 - 18$
**m** $55 - 12$
**n** $183 - 11$
**o** $67 - 41$
**p** $121 - 22$

EXAMPLE 4

**5** Evaluate each difference. PS R

**a** $560 - 327$
**b** $1614 - 239$
**c** $452 - 367$

**6** Write the answer to each multiplication.

**a** $3 \times 8$
**b** $4 \times 4$
**c** $7 \times 5$
**d** $6 \times 9$
**e** $4 \times 9$
**f** $8 \times 5$
**g** $8 \times 8$
**h** $7 \times 3$
**i** $4 \times 6$
**j** $6 \times 6$
**k** $9 \times 3$
**l** $7 \times 4$

**7** Write the answer to each division.

**a** $49 \div 7$
**b** $20 \div 4$
**c** $36 \div 9$
**d** $27 \div 3$
**e** $54 \div 6$
**f** $28 \div 7$
**g** $15 \div 5$
**h** $72 \div 8$
**i** $16 \div 4$
**j** $45 \div 5$
**k** $24 \div 8$
**l** $40 \div 8$

**8** Evaluate each product. PS R

**a** $8 \times 2 \times 50$
**b** $6 \times 5 \times 3$
**c** $25 \times 7 \times 4$
**d** $4 \times 8 \times 5$
**e** $5 \times 2 \times 10$
**f** $10 \times 4 \times 25$
**g** $15 \times 50 \times 2$
**h** $3 \times 4 \times 20$
**i** $6 \times 6 \times 5$
**j** $2 \times 5 \times 32$
**k** $20 \times 17 \times 5$
**l** $10 \times 7 \times 20$



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**9** Evaluate each expression. R

EXAMPLE 5

| | | | |
|---|---|---|---|
| **a** $8 \times 20$ | **b** $7 \times 500$ | **c** $2 \times 9000$ | **d** $15 \times 20$ |
| **e** $3 \times 70$ | **f** $6 \times 300$ | **g** $9 \times 4000$ | **h** $12 \times 200$ |
| **i** $900 \div 30$ | **j** $1200 \div 200$ | **k** $250 \div 50$ | **l** $320 \div 40$ |
| **m** $1800 \div 90$ | **n** $2000 \div 20$ | **o** $42\,000 \div 6000$ | **p** $240 \div 80$ |

**10** How many hours are there in one week? Select **A**, **B**, **C** or **D**.

**A** 84 **B** 151 **C** 168 **D** 240

**11** Evaluate each expression. R

EXAMPLE 6

| | | | |
|---|---|---|---|
| **a** $85 \times 2$ | **b** $39 \times 2$ | **c** $57 \times 4$ | **d** $28 \times 4$ |
| **e** $16 \times 8$ | **f** $33 \times 8$ | **g** $\frac{1}{2} \times 648$ | **h** $\frac{1}{2} \times 232$ |
| **i** $216 \div 4$ | **j** $488 \div 4$ | **k** $872 \div 8$ | **l** $208 \div 8$ |

**12** Evaluate each product by factorising. PS R

EXAMPLE 7

| | | | |
|---|---|---|---|
| **a** $7 \times 40$ | **b** $25 \times 16$ | **c** $22 \times 6$ | **d** $13 \times 20$ |
| **e** $35 \times 6$ | **f** $21 \times 3$ | **g** $16 \times 12$ | **h** $18 \times 18$ |

**13** Evaluate each expression. PS R

EXAMPLE 8

| | | | |
|---|---|---|---|
| **a** $22 \times 5$ | **b** $14 \times 5$ | **c** $28 \times 50$ | **d** $36 \times 15$ |
| **e** $44 \times 25$ | **f** $52 \times 50$ | **g** $800 \div 50$ | **h** $90 \div 5$ |
| **i** $120 \div 15$ | **j** $1000 \div 25$ | **k** $410 \div 5$ | **l** $700 \div 50$ |

**14** Nathan can type 76 words per minute. How many words can he type in 15 minutes?

**15** Evaluate each product by multiplying by 10 first, then adding or subtracting. R

EXAMPLE 9

| | | | |
|---|---|---|---|
| **a** $34 \times 9$ | **b** $51 \times 9$ | **c** $27 \times 8$ | **d** $72 \times 8$ |
| **e** $36 \times 11$ | **f** $41 \times 11$ | **g** $64 \times 12$ | **h** $22 \times 12$ |

EXTENSION

**16** Evaluate each quotient.

EXAMPLE 10

| | | | |
|---|---|---|---|
| **a** $2640 \div 8$ | **b** $1638 \div 9$ | **c** $616 \div 7$ | **d** $1584 \div 4$ |
| **e** $675 \div 15$ | **f** $756 \div 14$ | **g** $806 \div 31$ | **h** $729 \div 27$ |

**17** There are 135 students in Year 8. If they are divided evenly into 5 classes, how many students will be in each class? Select **A**, **B**, **C** or **D**.

**A** 21 **B** 25 **C** 27 **D** 29

**18** Divide a restaurant bill of $204 evenly among 6 people.

**Video**
The Fibonacci sequence

## Did you know?

### Fibonacci numbers

Fibonacci numbers are a number pattern 1, 1, 2, 3, 5, 8, ... named after the Italian mathematician Fibonnacci (pronounced 'fibbon-archie'), also known as Leonardo of Pisa. He introduced this number pattern to Europe in 1202, although it was known earlier in India.

The Fibonacci sequence of numbers is calculated as follows:

Start with 1 and 1

| | |
|---|---|
| $1 + 1 = 2$ | 1, 1, 2 |
| $1 + 2 = 3$ | 1, 1, 2, 3 |
| $2 + 3 = 5$ | 1, 1, 2, 3, 5 |
| $3 + 5 = 8$ | 1, 1, 2, 3, 5, 8 |

Each number is the sum of the previous 2 numbers.

Surprisingly, Fibonacci numbers have proved useful in computer algorithms, the growth of plants, software development and optics, even though they were first calculated hundreds of years ago.

**Calculate the next 5 Fibonacci numbers.**



iStock.com/Manakin

19th century statue of Fibonacci today in Pisa, Italy.

# 2.02 Adding and subtracting integers

**Skillsheets**
Integers

Integers using diagrams

Integers using coloured squares

**Presentation**
Number lines

**Integers** are all the positive and negative whole numbers and zero. Integers can be added and subtracted on a number line. A negative number can be entered into a calculator using the (−) or +/− key.

## Adding and subtracting integers

**Adding a negative number** is the same as subtracting its opposite: $7 + (-4) = 7 - 4 = 3$

**Subtracting a negative number** is the same as adding its opposite: $7 - (-4) = 7 + 4 = 11$

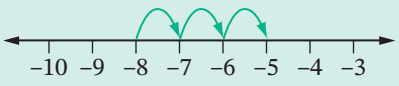
## Example 11

Evaluate each expression.

**a** $-8 + 3$ **b** $-8 - 3$ **c** $2 + (-6)$ **d** $2 - (-6)$

### Solution

**a** $-8 + 3 = -5$

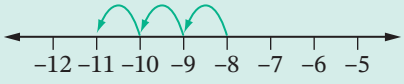



Start at −8, go forward 3

On a calculator, enter:

(–) 8 + 3 =.

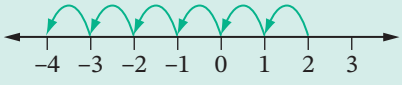
**b** $-8 - 3 = -11$




Start at −8, go back 3.

(–) 8 – 3 =.

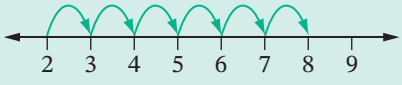
**c** $2 + (-6) = 2 - 6 = -4$




Start at 2, go back 6.

2 + (–) 6 =.

**d** $2 - (-6) = 2 + 6 = 8$




Start at 2, go forward 6.

2 – (–) 6 =.

## EXERCISE 2.02 ANSWERS ON P. 572

### Adding and subtracting integers U F PS R

A **sum** is the answer to an addition

**1** Evaluate each sum and check your answer using a calculator.

| | | | |
|---|---|---|---|
| **a** $-3 + 1$ | **b** $-5 + 10$ | **c** $-2 + 4$ | **d** $-1 + 7$ |
| **e** $-10 + 14$ | **f** $-9 + 6$ | **g** $-4 + 8$ | **h** $-5 + 5$ |
| **i** $9 + (-3)$ | **j** $4 + (-1)$ | **k** $6 + (-4)$ | **l** $2 + (-2)$ |
| **m** $3 + (-8)$ | **n** $7 + (-10)$ | **o** $5 + (-6)$ | **p** $1 + (-9)$ |

A **difference** is the answer to a subtraction

**2** Evaluate each difference and check your answer using a calculator.

| | | | |
|---|---|---|---|
| **a** $7 - 9$ | **b** $5 - 11$ | **c** $3 - 10$ | **d** $2 - 6$ |
| **e** $-4 - 4$ | **f** $-8 - 2$ | **g** $-1 - 5$ | **h** $-3 - 9$ |
| **i** $6 - (-6)$ | **j** $4 - (-1)$ | **k** $7 - (-5)$ | **l** $10 - (-8)$ |
| **m** $-5 - (-4)$ | **n** $-6 - (-9)$ | **o** $-3 - (-3)$ | **p** $-1 - (-2)$ |

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**3** At the ski resort, the temperature rose from −2°C to 7°C. What was the increase in temperature? R

**4** Rhys' bank account balance was −$44, meaning he was $44 in debt. He deposited $120 into his account. What is his balance now? Select the correct answer **A**, **B**, **C** or **D**. PS R

**A** $64 **B** $74 **C** $76 **D** $164

**5** Evaluate each expression.

**a** $12 - 3 - 11$ **b** $6 - 10 + 4$ **c** $-18 + 10 - 3$

**d** $-7 + 3 + 8$ **e** $8 - 15 + (-6)$ **f** $-2 - 12 + 20$

**g** $3 + (-5) + 1$ **h** $12 - 14 - (-4)$ **i** $-9 + 6 - (-1)$

**j** $7 - 7 + (-7)$ **k** $-5 + (-5) + (-5)$ **l** $8 - 10 + (-3)$

**6** The temperature changed from 6°C to −8°C overnight. What was the change in temperature? Select **A**, **B**, **C** or **D**. R

**A** 2°C decrease **B** 14°C decrease **C** 14°C increase **D** 2°C increase

**7** Copy and fill in the blank for each equation. Use a number line to help you if you need. PS R

**a** $5 + ___ = -4$ **b** $-2 + ___ = 1$ **c** $-4 + ___ = -9$

**d** $___ + (-7) = -11$ **e** $3 - ___ = -3$ **f** $-6 - ___ = -7$

**g** $-2 - ___ = 5$ **h** $___ - 2 = -6$ **i** $___ - (-8) = 0$

**8** The table shows the temperature in 4 towns at 2 different times of the morning. Which town had the smallest change in temperature? PS R

| Town | 5 a.m. | 8 a.m. |
|---|---|---|
| Jackson Creek | −6°C | −3°C |
| Smith Valley | −2°C | 3°C |
| Lake Drake | 2°C | 6°C |
| Mount Magner | −3°C | 4°C |

**9** A bird dives into a river from a height of 450 cm above the water, travelling at an average speed of 120 cm per second. How far below the water will it be after 4 seconds? PS

**10** Copy and complete the blanks for each equation, where at least one of the numbers must be negative. R

**a** $___ + ___ = 4$ **b** $___ + ___ = -1$ **c** $___ + ___ = 0$

**d** $___ + ___ = -9$ **e** $___ + ___ = 2$ **f** $___ - ___ = 4$

**g** $___ - ___ = -8$ **h** $___ - ___ = 9$ **i** $___ - ___ = -1$

**11** If $a$ and $b$ are integers, test whether each equation is true or false. R

**a** $a + b = b + a$ **b** $a - b = b - a$

**12** Simplify each algebraic expression by using integers to test them. R

**a** $a + (-a)$ **b** $a - (-a)$ **c** $a + a$ **d** $a - a$

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9780170479516

**13** Investigate whether each sum or difference is always positive (+), always negative (−), or could be either positive or negative (E). R

- **a** A larger integer minus a smaller integer
- **b** A positive integer minus a positive integer
- **c** A positive integer plus a positive integer
- **d** A smaller integer minus a larger integer
- **e** A positive integer plus a negative integer
- **f** A negative integer plus a negative integer
- **g** A positive integer minus a negative integer
- **h** A negative integer minus a negative integer



# Multiplying and dividing integers 2.03

## Multiplying integers

positive × positive = positive
positive × negative = negative
negative × positive = negative
negative × negative = positive

| × | + | − |
|---|---|---|
| + | + | − |
| − | − | + |

or

If both numbers have the **same** sign, the answer is **positive**.
If both numbers have **different** signs, the answer is **negative**.

Because division is the opposite of multiplication, the rules for dividing integers are the same.

## Dividing integers

positive ÷ positive = positive
positive ÷ negative = negative
negative ÷ positive = negative
negative ÷ negative = positive

| ÷ | + | − |
|---|---|---|
| + | + | − |
| − | − | + |

**Skillsheet**
Integers

**Worksheets**
What is the integer question?
Integer review
Integers writing activity

**Puzzles**
Mutiplying integers game
Directed numbers
Integers group clues

**Spreadsheet**
Integer quiz: Multiplication

Foundation Standard Complex

Video
Multiplying and dividing integers

## Example 12

Evaluate each product. A **product** is the answer to a multiplication

**a** $-3 \times 5$ **b** $-6 \times (-9)$ **c** $(-7)^2$

### Solution

**a** $-3 \times 5 = -15$

$(-) \times (+) = (-)$

On a calculator, enter: (–) 3 × 5 =

**b** $-6 \times (-9) = 54$

$(-) \times (-) = (+)$

(–) 6 × (–) 9 =

**c** $(-7)^2 = (-7) \times (-7) = 49$

$(-) \times (-) = (+)$

( (–) 7 ) $x^2$ =

## Example 13

Simplify each quotient. A **quotient** is the answer to a division

**a** $16 \div (-2)$ **b** $-20 \div (-10)$ **c** $\frac{-27}{9}$

### Solution

**a** $16 \div (-2) = -8$

$(+) \div (-) = (-)$

On a calculator, enter: 16 ÷ (–) 2 =.

**b** $-20 \div (-10) = 2$

$(-) \div (-) = (+)$

(–) 20 ÷ (–) 10 =

**c** $\frac{-27}{9} = -27 \div 9 = -3$

$(-) \div (+) = (-)$

(–) 27 ÷ 9 =.

## EXERCISE 2.03 ANSWERS ON P. 572

### Multiplying and dividing integers UFRC

EXAMPLE 12

**1** Evaluate each product and check your answer using a calculator.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $-5 \times 4$ | **b** | $3 \times (-6)$ | **c** | $-4 \times (-8)$ | **d** | $-9 \times 5$ |
| **e** | $10 \times (-7)$ | **f** | $-15 \times (-2)$ | **g** | $(-3)^2$ | **h** | $6 \times (-4)$ |
| **i** | $-7 \times 3$ | **j** | $(-10)^2$ | **k** | $9 \times (-4)$ | **l** | $-8 \times (-5)$ |
| **m** | $4 \times (-11)$ | **n** | $-20 \times 3$ | **o** | $(-6)^2$ | **p** | $-7 \times (-9)$ |
| **q** | $-11 \times 5$ | **r** | $(-9)^2$ | **s** | $10 \times (-10)$ | **t** | $-6 \times (-7)$ |

EXAMPLE 13

**2** For each product, select the correct answer **A**, **B**, **C** or **D**.

**a** $12 \times (-3)$

**A** 15 **B** $-15$ **C** $-36$ **D** 36

**b** $-3 \times 6 \times 2$

**A** 0 **B** $-16$ **C** $-18$ **D** $-36$

**c** $5 \times 3 \times (-2)$

**A** $-15$ **B** $-30$ **C** 15 **D** 30

**3** Evaluate each quotient and check your answer using a calculator.

**a** $36 \div (-4)$ **b** $-15 \div (-3)$ **c** $-14 \div 2$ **d** $60 \div (-10)$

**e** $-25 \div (-5)$ **f** $-28 \div 4$ **g** $45 \div (-9)$ **h** $-32 \div 16$

**i** $-36 \div (-6)$ **j** $56 \div (-7)$ **k** $-24 \div 3$ **l** $-42 \div (-6)$

**m** $40 \div (-8)$ **n** $-20 \div (-1)$ **o** $-81 \div 9$ **p** $24 \div (-4)$

**4** Evaluate each quotient. Select **A**, **B**, **C** or **D**.

**a** $-45 \div (-5)$

**A** 9 **B** $-9$ **C** $-50$ **D** $-40$

**b** $\frac{42}{-6}$

**A** 36 **B** $-7$ **C** 7 **D** $-48$

**5** Evaluate each quotient.

**a** $\frac{-18}{2}$ **b** $\frac{16}{-4}$ **c** $\frac{-30}{-5}$ **d** $\frac{21}{-3}$

**e** $\frac{-32}{-8}$ **f** $\frac{-100}{25}$ **g** $\frac{-81}{-9}$ **h** $\frac{56}{-8}$

**6** Copy and complete each equation. R

**a** $7 \times ___ = -14$ **b** $-3 \times ___ = 12$ **c** $-5 \times ___ = -25$

**d** $____ \times 3 = -21$ **e** $_____ \times (-4) = 28$ **f** $____ \times 9 = -63$

**7** Evaluate each product and check your answer using a calculator.

**a** $-2 \times (-2) \times 7$ **b** $6 \times 5 \times (-3)$ **c** $4 \times (-1) \times 10$

**d** $3 \times (-5)^2$ **e** $-5 \times 2 \times (-2)$ **f** $(-2)^3$

**8** Evaluate $-2 + 7 \times (-3) + 12$. Select **A**, **B**, **C** or **D**.

**A** $-3$ **B** $-11$ **C** 31 **D** 61

**9** Evaluate each expression and check your answer using a calculator.

**a** $-28 \div 2 \div 7$ **b** $45 \div (-3) \div (-5)$ **c** $36 \div (-9) \div 2$

**d** $-90 \div 3 \div (-1)$ **e** $100 \div 2 \div (-10)$ **f** $-24 \div (-6) \div (-4)$

**g** $8 \times (-5) \div 2$ **h** $-4 \times 7 \div 14$ **i** $35 \div (-5) \times 10$

**10** Copy and complete each equation. R

**a** $-12 \div ___ = 2$ **b** $15 \div ___ = -3$ **c** $-9 \div ___ = -1$

**d** $____ \div (-4) = -8$ **e** $____ \div 12 = -2$ **f** $____ \div (-10) = -10$

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**11** Evaluate $4 - 24 \div (-6) + 7$. Select **A**, **B**, **C** or **D.**

**A** $-20$ **B** $7$ **C** $10$ **D** $15$

**12** If $a$ and $b$ are integers, test whether each equation is true or false. R

**a** $a \times b = b \times a$ **b** $a \div b = b \div a$

**13** What happens when you multiply an integer by its opposite? R C

**14** Investigate whether each product is always positive (+), always negative (−), or could be either positive or negative (E). R

**a** The product of 2 positive integers

**b** The product of 2 negative integers

**c** The product of a positive integer and a negative integer

**d** A positive integer squared

**e** A negative integer squared

**15** Copy and complete for each equation, where at least one of the numbers must be negative. R

**a** ___ × ___ = 20 **b** ___ × ___ = −16 **c** ___ × ___ = 36

**d** ___ × ___ = −5 **e** ___ × ___ = 9 **f** ___ × ___ = −22

**16** What happens when you divide an integer: R C

**a** by itself? **b** by its opposite?

**17** Simplify each algebraic expression by using integers to test them. R

**a** $a \times a$ **b** $(-a) \times (-a)$ **c** $a \div a$ **d** $(-a) \div a$

**18** Copy and complete each equation, where at least one of the numbers must be negative. R

**a** ___ ÷ ___ = 9 **b** ___ ÷ ___ = −5 **c** ___ ÷ ___ = 3

**d** ___ ÷ ___ = −4 **e** ___ ÷ ___ = 10 **f** ___ ÷ ___ = −7

**Video**
Chinese development of maths

## Did you know?

### Pascal's triangle

Pascal's triangle is a number pattern named after the French mathematician Blaise Pascal (1623–1662) but it was known about much earlier.

Each number is found by adding the 2 numbers just to the left and right of it in the previous line.

There are many patterns in this triangle:

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
    1   6   15  20  15  6   1
  1   7   21  35  35  21  7   1
```

- The sum of each row are the powers of 2
- The Fibonacci numbers are found by summing minor diagonals
- The counting numbers and triangular numbers are found along major diagonals

The triangle has these simple patterns but also provides much more complex patterns, such as binomial expansions in algebra and probability.

**Use the Internet to research the pattern in the triangle that gives the powers of 11. Find 2 other patterns in Pascal's triangle.**

Foundation Standard Complex


9780170479516

# Mental skills 2A: Maths without calculators ANSWERS ON P. 572



Quiz
Mental skills 2A



## Multiplying decimals

**1** Study each example.

**a** $3 \times 8 = 24$, so $3 \times 0.8 = 2.4$

0 dp + 1 dp = 1 dp (dp = decimal places)

The number of decimal places in the answer is equal to the total number of decimal places in the question. Also, the answer sounds reasonable because, by estimation:

$3 \times 0.8 \approx 3 \times 1 = 3$ $(2.4 \approx 3)$

**b** $6 \times 5 = 30$, so $0.6 \times 0.5 = 0.30 = 0.3$

1 dp + 1 dp = 2 dp

By estimation, $0.6 \times 0.5 \approx 0.5 \times 0.5 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 0.25$ $(0.3 \approx 0.25)$

**c** $7 \times 3 = 21$, so $0.07 \times 0.3 = 0.021$

2 dp + 1 dp = 3 dp

By estimation, $0.07 \times 0.3 \approx 0.07 \times \frac{1}{3} \approx 0.02$ $(0.021 \approx 0.02)$

**2** Now evaluate each product.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $0.7 \times 5$ | **b** | $12 \times 0.2$ | **c** | $0.4 \times 0.3$ | **d** | $(0.6)^2$ |
| **e** | $8 \times 0.1$ | **f** | $0.03 \times 0.9$ | **g** | $4 \times 0.05$ | **h** | $1.1 \times 8$ |
| **i** | $0.3 \times 0.8$ | **j** | $0.2 \times 0.06$ | **k** | $9 \times 0.2$ | **l** | $0.07 \times 0.4$ |

**3** Study each example.

Given that $15 \times 23 = 345$, evaluate each product.

**a** $1.5 \times 2.3 = 3.45$

1 dp + 1 dp = 2 dp (*Estimate*: $1.5 \times 2.3 \approx 2 \times 2 = 4$)

**b** $150 \times 0.23 = 15 \times 10 \times 0.23 = 15 \times 0.23 \times 10 = 3.45 \times 10 = 34.5$

0 dp + 2 dp = 2 dp

(*Estimate*: $150 \times 0.23 \approx 150 \times 0.2 = 150 \times \frac{1}{5} = 30$)

**c** $0.15 \times 2300 = 0.15 \times 23 \times 100 = 3.45 \times 100 = 345$

2 dp + 0 dp = 2 dp

(*Estimate*: $0.15 \times 2300 \approx 0.2 \times 2300 = \frac{1}{5} \times 2300 = 460$)

**4** Now given that $39 \times 17 = 663$, evaluate each product.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $3.9 \times 17$ | **b** | $39 \times 170$ | **c** | $39 \times 0.17$ | **d** | $0.39 \times 1.7$ |
| **e** | $3.9 \times 1.7$ | **f** | $390 \times 1.7$ | **g** | $3.9 \times 0.17$ | **h** | $3.9 \times 170$ |
| **i** | $3900 \times 1.7$ | **j** | $39 \times 1.7$ | **k** | $39 \times 0.017$ | **l** | $0.39 \times 0.17$ |

# 2.04 Order of operations

When evaluating a mixed expression such as $18 \div (-2 + 1) \times 2$, there is a specific order in which the different operations are performed.

**Skillsheet**
Order of operations

**Puzzles**
Directed numbers

Order of operations puzzle

## Order of operations

1. Evaluate any expression inside **grouping symbols** (brackets): start with the innermost brackets first, or in the numerator (top) and denominator (bottom) of a fraction.
2. Evaluate any multiplication ($\times$) and division ($\div$), from left to right.
3. Evaluate any addition ($+$) and subtraction ($-$), from left to right.

Beware of cheap calculators that do not follow the 'order of operations' rules!

**Videos**
Order of operations 1

Order of operations 2

BODMAS

## Example 14

Evaluate each expression.

**a** $5 \times 2 + 3 \times (-9)$ **b** $2 \times [(25 - 4) \div 3]$

### Solution

**a** $5 \times 2 + 3 \times (-9) = 10 + (-27)$
$= -17$

Multiply first: $5 \times 2 = 10$, and $3 \times (-9) = -27$
On a calculator, enter:
5 × 2 + 3 × (–) 9 =.

**b** $2 \times [(25 - 4) \div 3] = 2 \times [21 \div 3]$
$= 2 \times 7$
$= 14$

Innermost brackets first: $25 - 4 = 21$
Next brackets: $21 \div 3 = 7$
2 × ( ( 25 − 4 ) ÷ 3 ) =.

## Example 15

Simplify each fraction.

**a** $\dfrac{8 + 16}{40 \div 10}$ **b** $\dfrac{10 - 30}{6^2}$

Imagine 'invisible brackets' in the top and the bottom of the fraction.

### Solution

To simplify each fraction, evaluate the numerator and denominator separately, then divide the numerator by the denominator.

**a** $\frac{8+16}{40\div10}=\frac{24}{4}$

$=6$

On a calculator, enter: ( 8 + 16 ) ▤ ( 40 ÷ 10 ) =

OR if using MATH Mode, press ▤ and enter 8 + 16 and 40 ÷ 10 separately into the blanks.

**b** $\frac{10-30}{6^2}=\frac{-20}{36}$

$=-\frac{5}{9}$

Simplifying the fraction.



## EXERCISE 2.04 ANSWERS ON P. 572

### Order of operations U F PS R C

**1** Evaluate each expression and check your answer on a calculator.

EXAMPLE 14

**a** $15\div(7-4)$ **b** $[14+(-9)]\times6$ **c** $20-(5-7)$

**d** $8\times(-3+5-6)$ **e** $-2\times(10-9)+28$ **f** $(16+8)\div2-4$

**g** $18\div(5-2)\times(-2)$ **h** $[4-(-7)]\times3-10$ **i** $-26\div(14+12)$

**j** $(7-10)\times10\div6$ **k** $5\times[(22-10)\div(-3)]+1$ **l** $30+[7\times(2-6)]-2$

**2** Evaluate each expression and check your answer on a calculator.

**a** $8+5\times(-2)$ **b** $7-2\times3$ **c** $6\times5-(-1)$

**d** $12+(-6)\div3$ **e** $3\times6+2\times5$ **f** $9-11+15\div3$

**g** $-5\times10+16\div2$ **h** $-3\times6-(-2)\times5$ **i** $42\div6+9$

**j** $17+8-3\times2$ **k** $4\times3-(-7)+5$ **l** $16-3\times(-4)\div2$

**3** Simplify each fraction.

EXAMPLE 15

**a** $\frac{5\times2}{16+4}$ **b** $\frac{19+5}{18-6}$ **c** $\frac{9\times3}{40-10}$ **d** $\frac{58+8}{-7-4}$

**e** $\frac{4^2}{16\div(-2)}$ **f** $\frac{5\times3-1}{16+10\times4}$ **g** $\frac{28-5\times3}{(56-30)\div2}$ **h** $\frac{25\div5+13}{\sqrt{36}}$

**4** Which expression is equal to 4? Select the correct answer **A**, **B**, **C** or **D**.

**A** $20\div4+6\times2$ **B** $20\div(4+6)\times2$

**C** $20\div[(4+6)\times2]$ **D** $(20\div4+6)\times2$

**5** Ms Ferme, the Mathematics Head Teacher, has a parent complaining about the marking of his daughter's maths exam. He claims his daughter's correct answer was marked wrong.

This is the question: $-9+13\times4$

His daughter's answer was 16. R C

**a** Why was the daughter's answer wrong?

**b** How could Ms Ferme explain why the daughter's answer is wrong?

**c** Put brackets in $-9+13\times4$ to make his daughter's answer correct.

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**6** Copy and complete each equation. R

**a** $-8 \times 3 - ___ = -32$ **b** $___ \times (5 - 9) = -20$ **c** $(8 - ___) \times 7 = 35$

**d** $9 \times ___ + (-2) = -20$ **e** $24 \div (-3) \times ___ = 8$ **f** $___ + 5 \times 3 = -5$

**g** $(-2 - ___) \times (-4) = -16$ **h** $(-3 - 9) \div __ = -2$ **i** $___ \div (-4) + 6 = 1$

**7** Copy each equation and insert grouping symbols in the correct places to make the equation true. R

**a** $8 - 3 + 7 = -2$ **b** $40 - 10 \times 5 = 150$ **c** $27 \div 9 \div 3 = 9$

**d** $8 + 4 - 3 \times 2 = 10$ **e** $8 + 4 - 3 \times 2 = 18$ **f** $6 + 4 \times (-1) - 1 = -11$

**g** $13 + 3 \div 4 - 6 = -8$ **h** $100 \div 10 + 10 + 5 = 10$ **i** $100 \div 10 + 10 + 5 = 4$

**8** Use all the integers 5, −2 and 10 and grouping symbols to complete each equation. PS R

**a** _____ = 30 **b** _____ = −1 **c** _____ = −60

Skillsheet
Spreadsheets

Technology
Review of spreadsheets

## Technology

### Order of operations

**1** Enter these 7 values in column A of a new spreadsheet: 36, 7, 8.5, 18, 50, 2, −3.

**2** In column B, write an appropriate formula to evaluate each expression in the given cell. Try to predict the answer before you enter each formula. The first one has been done for you.

B1: $36 + 7 - 2$ In cell B1, enter **=A1+A2−A6**

B2: $(7 + 8.5) \div 2$ B3: $7 + 8.5 \div 2$ B4: $50 - 2 \times (-3)$

B5: $18 \times 7 + 36 \times 8.5$ B6: $18 \times (7 + 36) \times 8.5$ B7: $\frac{50}{2+18}$

B8: $50 + 8.5 - 2 - (-3)$ B9: $7 \times 2 + 50 \div 2$ B10: $\frac{50 \times (2+18+8.5)}{-3}$

# 2.05 Decimals

Skillsheet
Decimals

Worksheets
Decimals wall

Dewey decimals

## Ordering decimals

- List them in a column with the decimal points in line so that the place values can be compared
- Fill any gaps at the end with 0s
- **'Ascending order'** means going up, from smallest to largest
- **'Descending order'** means going down, from largest to smallest.

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## Example 16

Write these decimals in ascending order: 3.5, 3.751, 3.15, 3.157.

### Solution

3.500 — List the numbers in a column with the decimal points aligned.

3.751 — Fill any gaps at the end with zeros.

3.150 — Order the decimals from smallest to largest by comparing place values.

3.157 — The smallest is 3.150, next is 3.157, next is 3.500, the largest is 3.751.

In ascending order, the decimals are 3.15, 3.157, 3.5, 3.751.

**Worksheets**
Decimals 1

Decimals 10

2.05

## Rounding decimals

To **round a decimal**, cut it at the required decimal place and look at the digit in the next place:

- if that digit is less than 5 (that is, 0, 1, 2, 3 or 4), **round down**
- if the digit is 5 or more (that is, 5, 6,7, 8 or 9), **round up**.

**Videos**
Rounding decimals

Rounding: Snails vs rockets

## Example 17

**a** Round 5.261 to the nearest tenth.

**b** Write 14.8239 correct to 2 decimal places.

### Solution

**a** 'nearest tenth' = to one decimal place

5.2 | 61

cut — the next digit is 6 (greater than 5), so round **up** to 5.3

$5.261 \approx 5.3$

**b** 14.82 | 39

cut — the next digit is 3 (less than 5), so round **down** to 14.82

$14.8239 \approx 14.82$

## Adding and subtracting decimals

When **adding and subtracting decimals**, keep decimal points below one another.

Fill any gaps with 0s.

Video
Adding and subtracting decimals

## Example 18

Evaluate each expression.

**a** $2.7 + 44.3 + 13.25$ **b** $23.8 - 5.65$

### Solution

**a**

$$\begin{array}{r} {\scriptstyle 1\ \ 1} \\ 2.70 \\ 44.30 \\ +13.25 \\ \hline 60.25 \end{array}$$

Line up decimal points in the same column.
Fill any gap with 0s.

Check by estimating: $2.7 + 44.3 + 13.25 \approx 3 + 44 + 10 = 57$ (60.25 is close to 57)

**b**

$$\begin{array}{r} {\scriptstyle 1\ 13\ 7\ 10} \\ 2\not{3}.\not{8}\not{0} \\ -5.65 \\ \hline 18.15 \end{array}$$

Line up decimal points in the same column.
Fill any gap with 0s.

Check by estimating:
$23.8 - 5.65 \approx 24 - 6$
$= 18$ (18.15 is close to 18)

**EXERCISE 2.05** ANSWERS ON P. 572

## Decimals U F PS R

EXAMPLE 16

**1** Which decimal is the largest? Select the correct answer **A**, **B**, **C** or **D**.

**A** 6.408 **B** 6.48 **C** 6.048 **D** 6.4

**2** Write each set of decimals in ascending order.

**a** 4.35, 4.3, 4.05, 4.035 **b** 17.2103, 17.12, 12.173, 17.231

**c** 14.8, 14.08, 14.801, 14.81 **d** 0.3, 0.295, 0.032, 0.2

**3** Write each set of decimals in descending order.

**a** 1.612, 1.621, 1.61, 1.16 **b** 3.05, 3.053, 3.035, 3.305

**4** How many decimal places has each decimal?

**a** 3.824 **b** 1.09 **c** 51.6 **d** 22

**e** 12.573 **f** 450.304 **g** 788.5 **h** 0.45

EXAMPLE 17

**5** Write each decimal correct to one decimal place.

**a** 3.851 **b** 4.0736 **c** 0.3333 **d** 7.34

**e** 15.0801 **f** 2.976 **g** 2.048 **h** 16.1919

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9780170479516

**6** What is 12.3752 rounded to the nearest hundredth? Select **A**, **B**, **C** or **D**.

**A** 12.8 **B** 12.37 **C** 12.38 **D** 12.375

**7** Round each decimal to the nearest hundredth.

**a** 68.9109 **b** 107.0594 **c** 3.5963 **d** 4.7077

**e** 3.198 **f** 32.999 **g** 19.7291 **h** 11.254

**8** Write each decimal correct to 3 decimal places.

**a** 9.7043 **b** 13.167 54 **c** 0.08281 **d** 53.094 23

**9** Round $460.39512 to the nearest:

**a** cent **b** dollar **c** tenth **d** 5 cents **e** thousandth

EXAMPLE 18

**10** Evaluate each expression, then check your answer with an estimate.

**a** $28.51 + 136.4$ **b** $7.2 + 18.16$

**c** $34.7 + 29.4 + 8.5$ **d** $127.81 - 36.2$

**e** $46.5 - 30.8$ **f** $100.87 - 23.6$

**g** $20.03 - 1.59$ **h** $12.56 - 9.88$

**i** $71.5 - 4.82$ **j** $65.21 - 13.6$

**k** $9 - 4.36$ **l** $3.671 - 1.28$



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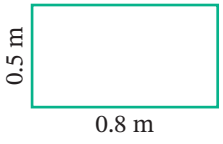
**11** Evaluate $0.52 + 14.025$. Select **A**, **B**, **C** or **D**.

**A** 65.025 **B** 19.225

**C** 14.545 **D** 14.077

**12** Yesterday, the temperature dropped from 24.1°C to 16.5°C. What was the difference between the 2 temperatures?

**13** Monique built a wooden frame with dimensions 0.8 m by 0.5 m. How much wood will be left from a 3.4 m length of timber? PS




**14** Tahir bought the following items: an exercise book for $2.70, 2 pens for $1.60 each, a drink for $1.50 and a packet of chips for $2.65. PS

**a** How much did Tahir spend in total?

**b** If he paid with a $20 note, how much change did he receive?

**15** Copy and complete each equation with the correct decimals. R

**a** $2.8 + _____ = 5.84$ **b** $_____ - 7.6 = 5.14$

**c** $18.75 - ____ = 13.3$ **d** $_____ + 0.83 = 1.719$

**e** $_____ - 1.07 = 3.256$ **f** $_____ - _____ = 4.89$

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## Investigation

### Rounding up or down?

Sometimes, the decision on whether to round an answer up or down depends on the situation. In groups of 2 to 4, discuss each of the following situations and decide whether it is more appropriate to round up or to round down. You must give a reason for your choice.

**1** 7 friends have dinner at a restaurant and the total bill is $206. They decide to share the bill evenly: $206 ÷ 7 = $29.42857...

How much should each friend pay, to the nearest dollar: $29 or $30?



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**2** You have a budget of $65 for buying drinks for a party. One can of drink costs $1.74 at the supermarket: $65 ÷ $1.74 = 37.35632...

How many whole cans of drink can you buy: 37 or 38?

**3** You need to find the average number of people living in each home in your street. You survey 48 homes and count a total of 166 people: 166 ÷ 48 = 3.45833...

What is the average whole number of people living in each house: 3 or 4?

**4** You need to paint the walls of a house with a total surface area of 334 $m^2$. One tin of paint covers 64 $m^2$: 334 ÷ 64 = 5.21875

How many whole tins of paint do you need for the job: 5 or 6?

**5** Jodie has $2098 in her bank account and the bank pays her 4.71% in interest:

4.71% × $2098 = $98.8158...

How much interest will the bank pay Jodie: $98.81 or $98.82?

## Technology

### The FIX mode on a calculator

Most scientific calculators have a FIX mode that rounds a displayed answer to a given number of decimal places. However, the exact answer is still stored in the calculator's memory to make sure that any further calculations involving this answer are accurate.
Find out how to use the FIX mode on your calculator.
The FIX mode has been used to set this calculator to display answers to 5 decimal places.






# Multiplying and dividing decimals 2.06

Note: Multiplying and dividing decimals (including by whole numbers) using written methods is no longer in the syllabus. Teachers can use judgment to allow calculator use instead.

## Multiplying decimals

When multiplying decimals, the number of decimal places in the answer equals the *total* number of decimal places in the question.

## Example 19

Evaluate each product.

**a** $0.8 \times 0.3$ **b** $7.2 \times 5$ **c** $5.31 \times 1.3$

### Solution

**a** $8 \times 3 = 24$ — Multiply without decimal points first.

Count the number of decimal places in the question. — 0.8 has one decimal place, 0.3 has one decimal place.

$0.8 \times 0.3 = 0.24$ — Write the answer with $1 + 1 = 2$ decimal places.

Check by estimating: $0.8 \times 0.3 \approx 1 \times 0.3 = 0.3$ $(0.24 \approx 0.3)$

**b** $72 \times 5 = 360$ — Multiply without decimal points first.

7.2 has one decimal place, 5 has no decimal places.

$7.2 \times 5 = 36.0$
$= 36$ — Write the answer with $1 + 0 = 1$ decimal place.

Check by estimating: $7.2 \times 5 \approx 7 \times 5 = 35$ $(36 \approx 35)$

**Video** Multiplying decimals

**Interactive** Area model decimals

**Worksheets** Decimal cards

Shopping and change

Where's the point?

Decimals 7

**Puzzles** Operations with decimals

Which decimals?

Decimal number grids

**Technology**
Movie night

**c**
$$\begin{array}{r} 531 \\ \times\ 13 \\ \hline 1593 \\ 5310 \\ \hline 6903 \end{array}$$

Multiply without decimal points first.

5.31 has 2 decimal places, and 1.3 has 1 decimal place.

$5.31 \times 1.3 = 6.903$

Write the answer with $2 + 1 = 3$ decimal places.

Check by estimating: $5.31 \times 1.3 \approx 5 \times 1 = 5$

$(6.903 \approx 5)$

## Dividing a decimal by a whole number

- Rewrite the question in 'short division' form
- Make the decimal point in the answer line up with the decimal point in the question
- Add 0s to the end of the decimal being divided, if needed

**Video**
Dividing decimals

## Dividing a decimal by another decimal

- Make the second decimal a whole number by moving the decimal point the required number of places to the right
- Move the point in the first decimal the same number of places to the right
- Divide the new first number by the whole number

This works because we are multiplying both decimals by the same power of 10 before dividing.

## Example 20

Evaluate each quotient.

**a** $6.28 \div 5$ **b** $12.4 \div 0.04$ **c** $4.98 \div 1.2$

### Solution

**a**
$$5\overline{)6.^{1}2^{2}8^{3}0}\quad 1.256$$

Write $6.28 \div 5$ as a short division.

- 5 into 6 goes 1, remainder 1
- Make the decimal point in the answer line up with the decimal point in the question
- 5 into 12 goes 2, remainder 2.
- 5 into 28 goes 5, remainder 3.
- Add a zero to 6.28 so that you can complete the division.
- 5 into 30 goes 6 exactly.

$6.28 \div 5 = 1.256$.

Check by estimating: $6.28 \div 5 \approx 6 \div 5 \approx 1$

$(1.256 \approx 1)$

**b** $12.4 \div 0.04 = 1240 \div 0\,04$

$= 1240 \div 4$

$= 310$

Move both decimal points 2 places to the right so that 0.04 becomes a whole number.

We added a 0 to the end of 12.4 so that we can move the point 2 places to the right.

Check by estimating: $1240 \div 4 \approx 1200 \div 4 \approx 300$ ($310 \approx 300$)

**c** $4.98 \div 1.2 = 49.8 \div 12$

Move both decimal points one place to the right so that 1.2 becomes a whole number.

$$\begin{array}{r} 4.15 \\ 12\overline{)49.80} \\ -48\downarrow \\ \hline 18 \\ -12 \\ \hline 60 \\ -60 \\ \hline 0 \end{array}$$

Write $49.8 \div 12$ as a long division:

12 into 49 is 4, remainder 1

12 into 18 is 1, remainder 6

12 into 60 is 5 exactly

$4.98 \div 1.2 = 4.15$

Check by estimating: $4.98 \div 1.2 \approx 5 \div 1 = 5$ ($4.15 \approx 5$)

## EXERCISE 2.06 ANSWERS ON P. 573

### Multiplying and dividing decimals U F PS R

**1** Evaluate each product, then check your answer with an estimate. EXAMPLE 19

| | | | |
|---|---|---|---|
| **a** $6.5 \times 4$ | **b** $0.45 \times 9$ | **c** $2.3 \times 5$ | **d** $7 \times 3.9$ |
| **e** $0.7 \times 0.3$ | **f** $0.8 \times 0.11$ | **g** $0.05 \times 0.4$ | **h** $4.2 \times 0.3$ |
| **i** $6.3 \times 4.5$ | **j** $28.7 \times 3.1$ | **k** $1.3 \times 0.62$ | **l** $0.72 \times 0.51$ |

**2** Phillip was building a fence. He needed 9.4 metres of wire at $5.80 per metre. What was the cost of the wire? Select the correct answer **A**, **B**, **C** or **D**. PS

**A** $15.20 **B** $54.52 **C** $152.00 **D** $545.20

**3** Sureti makes dolls to sell at a local market. She uses buttons for the eyes and sometimes for clothing. She buys 110 buttons in a batch. How much will she pay for a batch if each button costs 4.5 cents and she pays $9.90 for postage? PS

**4** Evaluate each quotient, then check your answer with an estimate. EXAMPLE 20

| | | |
|---|---|---|
| **a** $24.8 \div 4$ | **b** $114.1 \div 7$ | **c** $7.83 \div 3$ |
| **d** $0.695 \div 5$ | **e** $63.3 \div 0.2$ | **f** $4.173 \div 0.06$ |
| **g** $18.5 \div 0.05$ | **h** $23.22 \div 0.4$ | **i** $14.5 \div 25$ |
| **j** $6.24 \div 1.2$ | **k** $238 \div 1.4$ | **l** $0.252 \div 0.21$ |

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**5** What is the value of $12.92 \div 0.04$? Select **A**, **B**, **C** or **D**.

**A** 32.3 **B** 323 **C** 3.23 **D** 3230

**6** Brad uses 7.24 m of wood to make a garden bed fence. PS R



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**a** How much wood does he need to make 6 garden bed fences?

**b** If one of the garden beds is square-shaped, how long is each side?

**7** Evaluate each expression.

**a** $0.3 \times 0.7 + 5.7$ **b** $2.8 \div (7.4 - 7.2)$ **c** $3.62 + 4.08 \div 2$

**8** Jordan's car holds 45 litres of petrol. If the price of petrol is 189.9 cents per litre, how much will Jordan need to pay to fill the tank? Give your answer to the nearest 5 cents. PS

**9** Copy and complete each equation with the correct decimals. R

**a** $0.3 \times _____ = 2.4$ **b** $_____ \times 0.6 = 1.2$ **c** $_____ \times 0.7 = 0.028$

**d** $3.1 \times ____ = 2.79$ **e** $83.4 \div _____ = 6$ **f** $____ \div 8.1 = 3.2$

**10** A car travels 145.8 kilometres on 9 litres of petrol. How many kilometres could it travel on one litre of petrol? PS

**11** Anja earned $295.20 for working a 12-hour shift. How much was earned each hour? PS

**12** Henry has a faulty calculator that does not show the decimal point. For each calculation in the table shown, write the correct answer. PS R

| | Calculation | Answer with missing decimal point |
|---|---|---|
| **a** | 3.42 × 12 | 4104 |
| **b** | 4.145 × 0.2 | 829 |
| **c** | 37.3 × 8.8 | 32824 |
| **d** | 0.03 × 157.64 | 47292 |
| **e** | 8.3902 × 0.3 | 251706 |

# Terminating and recurring decimals 2.07

When converting a fraction to a decimal, or dividing 2 numbers, the decimal answer can be **terminating** or **recurring**.

**Terminating decimals**, such as 0.625, have a definite number of decimal places, while **recurring decimals**, such as 0.272727..., written as $0.\dot{2}\dot{7}$ or $0.\overline{27}$, have one or more digits that repeat endlessly. We use dots or lines to mark the repeating section. For example, 0.259 259 259... $= 0.\dot{2}5\dot{9}$ or $0.\overline{259}$.

'Terminate' means 'to stop', while 'recurring' means 'repeating'.

## Example 21

Convert each fraction to a recurring decimal.

**a** $\frac{5}{6}$ **b** $\frac{2}{11}$

### Solution

**a** $\frac{5}{6}$ means $5 \div 6$

$$6\overline{)5.0000\ldots}\quad 0.8333\ldots$$

$\frac{5}{6} = 0.8333... = 0.8\dot{3}$ or $0.8\overline{3}$

**b** $\frac{2}{11}$ means $2 \div 11$

$$11\overline{)2.00000\ldots}\quad 0.18181\ldots$$

$\frac{2}{11} = 0.181\,818... = 0.\dot{1}\dot{8}$ or $0.\overline{18}$

## Example 22

Evaluate each quotient.

**a** $2.8 \div 9$ **b** $0.7 \div 0.12$

### Solution

**a** $2.8 \div 9$

$$9\overline{)2.8000\ldots}\quad 0.3111\ldots$$

$2.8 \div 9 = 0.3111... = 0.3\dot{1}$ or $0.3\overline{1}$

**b** $0.7 \div 0.12 = 70 \div 12$

$$12\overline{)70.0000\ldots}\quad 5.8333\ldots$$

$0.7 \div 0.12 = 5.83333... = 5.8\dot{3}$ or $5.8\overline{3}$

**Skillsheet** Fractions and decimals

**Worksheet** Fraction families

**Puzzle** Decimals squaresaw 2

**Videos** Recurring decimals

Converting fractions to decimals

## EXERCISE 2.07 ANSWERS ON P. 573

# Terminating and recurring decimals U F R C

**1** Convert each fraction to a terminating decimal.

**a** $\frac{2}{5}$ **b** $\frac{3}{8}$ **c** $\frac{3}{4}$ **d** $\frac{2}{2}$

**e** $\frac{2}{4}$ **f** $\frac{6}{8}$ **g** $\frac{3}{5}$ **h** $\frac{2}{8}$

**i** $\frac{5}{8}$ **j** $\frac{7}{8}$ **k** $\frac{4}{8}$ **l** $\frac{5}{5}$

**2** Explain why some of the fractions in question **1** have the same decimal value. R C

**3** Rewrite each recurring decimal using dot notation. C

**a** 0.6666... **b** 4.2727... **c** 0.2828... **d** 3.8333...

**e** 9.607607... **f** 0.15191519... **g** 0.05252... **h** 12.23444...

**4** Write each recurring decimal showing the repeated pattern. R C

**a** $0.858\dot{3}$ **b** $0.1\dot{6}\dot{5}$ **c** $0.\dot{2}\dot{7}$ **d** $0.\dot{4}61538\dot{4}$

EXAMPLE 21

**5** Convert each fraction to a recurring decimal. C

**a** $\frac{1}{9}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$ **d** $\frac{1}{7}$ **e** $\frac{2}{3}$

**f** $\frac{2}{7}$ **g** $\frac{2}{9}$ **h** $\frac{3}{7}$ **i** $\frac{4}{7}$ **j** $\frac{4}{9}$

**k** $\frac{4}{6}$ **l** $\frac{5}{9}$ **m** $\frac{6}{7}$ **n** $\frac{7}{9}$ **o** $\frac{5}{7}$

**6** Copy and complete the table and note the pattern.

| Fraction | $\frac{1}{7}$ | $\frac{2}{7}$ | $\frac{3}{7}$ | $\frac{4}{7}$ | $\frac{5}{7}$ | $\frac{6}{7}$ |
|---|---|---|---|---|---|---|
| Decimal | | | | | | |

EXAMPLE 22

**7** Evaluate each expression as a recurring decimal.

**a** $11 \div 3$ **b** $15.4 \div 9$ **c** $58.43 \div 0.11$ **d** $1.96 \div 0.6$

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# Mental skills 2B: Maths without calculators ANSWERS ON P. 573

Quiz
Mental skills 2B



## Dividing decimals

To divide one decimal by another, first move the decimal points in **both** decimals the same number of places to the right so that the second decimal is a **whole number**.

**1** Study each example.

**a** $0.24 \div 0.06 = 24 \div 6 = 4$

**b** $0.45 \div 0.5 = 4.5 \div 5 = 0.9$

**c** $0.006 \div 0.3 = 0.06 \div 3 = 0.02$

**d** $27 \div 0.9 = 270 \div 9 = 30$

**e** $1.6 \div 0.4 = 16 \div 4 = 4$

**f** $5.6 \div 0.07 = 560 \div 7 = 80$

**2** Now evaluate each quotient.

**a** $0.25 \div 0.5$

**b** $63 \div 0.7$

**c** $3.2 \div 0.4$

**d** $0.18 \div 0.2$

**e** $2.7 \div 0.03$

**f** $0.042 \div 0.06$

**g** $4 \div 0.5$

**h** $1.2 \div 0.04$

**i** $0.072 \div 0.9$

**j** $0.35 \div 0.1$

**k** $0.28 \div 0.07$

**l** $0.033 \div 0.11$

**3** Study each example.

Given that $112 \div 14 = 8$, evaluate each expression.

**a** $112 \div 1.4 = 112.0 \div 1.4$
$= 1120 \div 14$
$= 112 \times 10 \div 14$
$= 112 \div 14 \times 10$
$= 8 \times 10$
$= 80$
Estimate: $112 \div 1.4 \approx 112 \div 1 = 112$

**b** $0.112 \div 0.14 = 0.112 \div 1.4$
$= 11.2 \div 14$
$= 112 \div 10 \div 14$
$= 112 \div 14 \div 10$
$= 8 \div 10$
$= 0.8$
Estimate: $0.112 \div 0.14 \approx 0.1 \div 0.1 = 1$

**c** $1120 \div 1.4 = 11200 \div 14$
$= 112 \times 100 \div 14$
$= 112 \div 14 \times 100$
$= 8 \times 100$
$= 800$
Estimate: $1120 \div 1.4 \approx 1120 \div 1 = 1120$

**d** $1.12 \div 14 = 112 \div 100 \div 14$
$= 112 \div 14 \div 100$
$= 8 \div 100$
$= 0.08$
Estimate: $1.12 \div 14 \approx 1.12 \div 10 = 0.112$

**4** Now given that $368 \div 23 = 16$, evaluate each quotient.

**a** $36.8 \div 2.3$

**b** $368 \div 2.3$

**c** $3.68 \div 2.3$

**d** $0.368 \div 0.23$

**e** $36.8 \div 23$

**f** $3.68 \div 0.23$

**g** $36.8 \div 0.23$

**h** $0.368 \div 2.3$

**i** $0.368 \div 23$

**j** $3.68 \div 0.023$

**k** $3.68 \div 23$

**l** $0.368 \div 230$

# 2.08 Powers and roots

**Skillsheets**
Index notation

Square roots and cube roots

**Worksheets**
Powers and roots

Big numbers

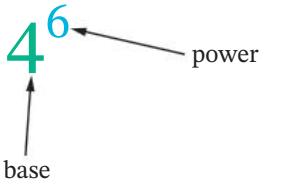
**Puzzle**
Square root Snap

**Video**
Binary: the computer language

Powers and the use of **index notation** allow us to write repeated multiplication in a shorter way.

$4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$ '4 to the power of 6'

In $4^6$, the number 4 is called the **base** and is the number that is repeated in the multiplication. The small raised number 6 is called the **power** or **index**.




## The power keys on a calculator

- The square of a number is found using the $x^2$ key
- The cube of a number is found using the $x^3$ key
- Any power of a number can be found using the $x^{\blacksquare}$ or $y^x$ key.

## Example 23

Evaluate each power.

**a** $11^3$ **b** $(-8)^4$

### Solution

**a** $11^3 = 1331$ On a calculator, enter: 11 $x^3$ =

**b** $(-8)^4 = 4096$ ( (–) 8 ) $x^{\blacksquare}$ 4 =

## Square root and cube root

- The **square root** ($\sqrt{\ }$) of a number is the **positive** value which, if squared, will give that number.
- The **cube root** ($\sqrt[3]{\ }$) of a given number is the positive or negative value which, if cubed, will give that number.

For example:

| | | |
|---|---|---|
| $\sqrt{36} = 6$ | because $6^2 = 36$ | 'the square root of 36' |
| $\sqrt[3]{125} = 5$ | because $5^3 = 125$ | 'the cube root of 125' |
| $\sqrt[3]{-8} = -2$ | because $(-2)^3 = -8$ | 'the cube root of − 8' |

Most roots do not give exact answers like the ones above, and are called **surds**. For example, $\sqrt{7} = 2.645751311... \approx 2.6$. A surd is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any other type of root whose exact decimal or fraction value cannot be found. As a decimal, its digits run endlessly *without repeating*, so they are *neither* terminating nor recurring decimals. A surd cannot be written in fraction form $\frac{a}{b}$, so it is also called an **irrational** number.

## Example 24

Evaluate each cube root, correct to 2 decimal places where necessary.

**a** $\sqrt[3]{729}$ **b** $\sqrt[3]{100}$

### Solution

**a** $\sqrt[3]{729} = 9$ On a calculator, enter: $\sqrt[3]{\square}$ 729 =

**b** $\sqrt[3]{100} = 4.6415 \ldots \approx 4.64$ $\sqrt[3]{\square}$ 100 =

## Example 25

Estimate the value of $\sqrt{50}$, correct to one decimal place.

### Solution

There is no *exact* answer for the square root of 50, because there isn't a number which, if squared, equals 50 exactly. However, we can find a decimal whose square is close to 50.

Noting that:

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

we can tell that $\sqrt{50}$ must lie somewhere between 7 and 8. Also, it is much closer to 7 because 50 is just over 49.

So we can estimate that $\sqrt{50} \approx 7.1$.

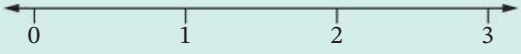
(In fact, $7.1^2 = 50.41$)

## Example 26

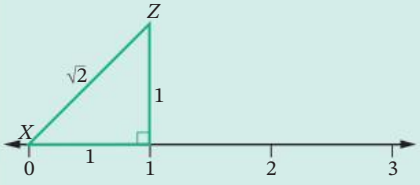
Use a pair of compasses and Pythagoras' theorem to estimate the value of $\sqrt{2}$ on a number line.

### Solution

Draw a number line using a large scale, such as 1 unit = 2 cm.




Draw a vertical line at 1 of height 1 unit to create a right-angled triangle.

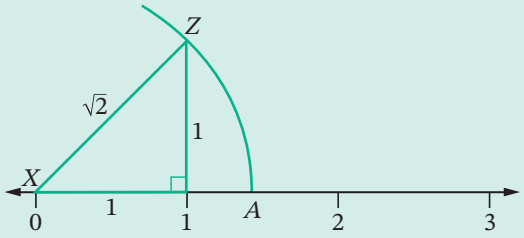



This is an isosceles right-angled triangle with base length 1 and perpendicular height 1.

The length of the hypotenuse $XZ$ of the triangle can be calculated using Pythagoras' theorem:

$$XZ^2 = 1^2 + 1^2$$
$$= 1 + 1$$
$$= 2$$
$$XZ = \sqrt{2} \text{ units}$$

To rotate this length onto the number line, use compasses to draw an arc from $Z$ to $A$ on the number line, using $X$ as the centre. The point $A$ on the number line represents the value of $\sqrt{2}$. Check this using a calculator.




Using the scale of the number line, $\sqrt{2} \approx 1.4$

## EXERCISE 2.08 ANSWERS ON P. 573

### Powers and roots U F R C

**1** Write each expression using index notation. C

**a** $7 \times 7 \times 7$

**b** $4 \times 4 \times 4 \times 4 \times 4$

**c** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

**d** $-8 \times (-8) \times (-8) \times (-8) \times (-8) \times (-8) \times (-8)$

EXAMPLE 23

**2** Evaluate each expression.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $(-8)^2$ | **b** | $2^3$ | **c** | $(-6)^3$ | **d** | $3^4$ |
| **e** | $7^1$ | **f** | $(-2)^5$ | **g** | $10^3$ | **h** | $4^4$ |
| **i** | $(-11)^6$ | **j** | $3^9 - 9^3$ | **k** | $4 \times 3^5$ | **l** | $7^4 + 2 \times 8^2$ |
| **m** | $16^4 \div 2^{10} - 5^2$ | **n** | $(-4)^6 - 18 \div 3^2$ | | | | |

**3** Find the value of $\square$ to make each equation true. R

| | | | | | |
|---|---|---|---|---|---|
| **a** | $2^{\square} = 8$ | **b** | $3^{\square} = 27$ | **c** | $(-10)^{\square} = 100$ |
| **d** | $4^{\square} = 4096$ | **e** | $5^{\square} = 125$ | **f** | $(-3)^{\square} = -243$ |

**4** What is the value of $(1.3)^2$? Select the correct answer **A**, **B**, **C** or **D**.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **A** | 1.9 | **B** | 1.69 | **C** | 2.6 | **D** | 1.09 |

**5** Which is smaller: $100^3$ or $3^{100}$?

**6** Which is larger: $10^2$ or $2^{10}$?

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**7** **a** Evaluate $(2 \times 3)^2$ without using a calculator.

**b** Evaluate $2^2 \times 3^2$ without using a calculator.

**c** Does $(2 \times 3)^2 = 2^2 \times 3^2$? Explain your answer. C

**8** **a** Evaluate $(4 \times 5)^2$.

**b** Evaluate $4^2 \times 5^2$.

**c** Does $(4 \times 5)^2 = 4^2 \times 5^2$? Explain your answer. C

**9** Use the pattern you found in questions **7** and **8** to complete each equation. R C

**a** $(3 \times 8)^2 =$ _____ $\times$ _____ **b** $(a \times b)^2 =$ _____ $\times$ _____

**10** Copy and complete each pattern. R C

**a** $18^2 = (6 \times 3)^2 = 6^2 \times$_____ = _____

**b** $22^2 = (2 \times 11)^2 =$ ____ $\times$ ____ = ____

**c** $16^2 = (2 \times$ ___$)^2 =$ ___ $\times$ ____ = _____

**d** $15^2 =$ (___ $\times 5)^2 =$ ____ $\times$____ = ____

**e** $30^2 = (10 \times$ ___$)^2 =$ ___ $\times$ ____ = _____

**f** $28^2 =$ (___ $\times$ ___$)^2 =$ ____ $\times$____ = ____

EXAMPLE 24

**11** Evaluate each root.

**a** $\sqrt{784}$ **b** $\sqrt{256}$ **c** $\sqrt{289}$ **d** $\sqrt{1089}$

**e** $\sqrt[3]{8}$ **f** $\sqrt[3]{343}$ **g** $\sqrt[3]{2197}$ **h** $\sqrt[3]{216}$

**12** Evaluate each root, correct to one decimal place.

**a** $\sqrt{37}$ **b** $\sqrt[3]{900}$ **c** $\sqrt{502}$ **d** $\sqrt[3]{6.5}$

**e** $\sqrt[3]{-495}$ **f** $\sqrt{2000}$ **g** $\sqrt{1.1}$ **h** $\sqrt[3]{1103}$

**13** Copy and complete this table.

| Number, $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number squared, $x^2$ | | | | 16 | | | 49 | | | | | |
| Number cubed, $x^3$ | | | | | | | | 512 | | 1000 | | |

Use the table from question **13** to answer questions **14** to **17**.

EXAMPLE 25

**14** Between which 2 consecutive whole numbers does $\sqrt{80}$ lie? Select **A**, **B**, **C** or **D**.

**A** 40 and 41 **B** 9 and 10 **C** 79 and 81 **D** 8 and 9

**15** Between which 2 consecutive whole numbers does $\sqrt[3]{130}$ lie? Select **A**, **B**, **C** or **D**.

**A** 4 and 5 **B** 10 and 11 **C** 11 and 12 **D** 5 and 6

**16** Between which 2 consecutive whole numbers does $\sqrt[3]{31}$ lie?

**17** Estimate, correct to one decimal place, the value of each square root, then check your estimate using a calculator.

**a** $\sqrt{33}$ **b** $\sqrt{105}$ **c** $\sqrt{68}$

**18** **a** Evaluate $\sqrt{4 \times 25}$.

**b** Evaluate $\sqrt{4} \times \sqrt{25}$.

**c** What do you notice about your answers to parts **a** and **b**? C

**19** **a** Evaluate $\sqrt{27 \times 3}$.

**b** Evaluate $\sqrt{27} \times \sqrt{3}$.

**c** What do you notice about your answers to parts **a** and **b**? C

**20** Use the pattern you found in questions **19** and **20** to complete each equation. R C

**a** $\sqrt{16 \times 9} =$ _____ $\times$ _____ **b** $\sqrt{a \times b} =$ _____ $\times$ _____

**21** Copy and complete each pattern. R C

**a** $\sqrt{64} = \sqrt{4} \times \sqrt{16} =$ _____ $\times$ _____ $=$ _____

**b** $\sqrt{441} = \sqrt{9} \times \sqrt{\phantom{00}} =$ _____ $\times$ _____ $=$ _____

**c** $\sqrt{144} = \sqrt{36} \times \sqrt{\phantom{00}} =$ _____ $\times$ _____ $=$ _____

**d** $\sqrt{2025} = \sqrt{81} \times \sqrt{\phantom{00}} =$ _____ $\times$ _____ $=$ _____

EXAMPLE 26

**22** **a** Use the method of Example 26 to estimate the value of $\sqrt{2}$ on a number line. R

**b** Draw a vertical line of height 1 unit at $\sqrt{2}$ on a number line to create another right-angled triangle. Calculate the length of the hypotenuse as a surd. Using compasses, draw an arc to rotate this length onto the number line. Check your approximation using a calculator.

**c** Repeat the above procedure to approximate the values of $\sqrt{4}$, $\sqrt{5}$ and $\sqrt{6}$ on the number line. Check your approximations using a calculator.

## Technology

### Powers and roots

1 Start a new spreadsheet and enter the label **Powers** in cell A1.

2 In cell A2, enter **=2^3** to evaluate $2^3$. The caret symbol, ^ (shift 6), is the symbol for power on a spreadsheet.

3 In cell A3, enter **=3^5** to evaluate $3^5$.

4 In cell A4, enter **=4^2** to evaluate $4^2$.

5 Write an appropriate formula to evaluate each expression in the given cell.

A5: $6.1^2$ A6: $4.25^2$ A7: $5^2 \times 3^2$ A8: $(5 \times 3)^2$

6 Does your answer in cell A7 equal your answer in cell A8?

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**7** Write an appropriate formula to evaluate each expression in the given cell.

A9: $2.9^3$ A10: $15^4$ A11: $8.3^5$ A12: $(-1)^3 + (-2)^4$ A13: $-1^3 + (-2^4)$

**8** Does your answer in cell A12 equal your answer in cell A13?

**9** In cell B1, enter **Square roots**.

**10** In cell B2, enter **=sqrt(49)** to evaluate $\sqrt{49}$ (**=sqrt** stands for 'square root')

**11** Write an appropriate formula to evaluate each square root in the given cell.

B3: $\sqrt{81}$ B4: $\sqrt{625}$ B5: $\sqrt{10000}$

B6: $\sqrt{71289}$ B7: $\sqrt{0.064}$ B8: $\sqrt{2.89}$

B9: $\sqrt{16.9}$ (Use **Format Cells** to round to 2 decimal places)

B10: $\sqrt{\frac{81}{25}}$

## Investigation

### Products of surds

**1** **a** Predict the answer to $\sqrt{2} \times \sqrt{2}$ and check it using a calculator.

**b** Do the same for $\sqrt{17} \times \sqrt{17}$.

**c** Write 3 more equations of this type.

**d** Write a general rule for this property algebraically.

**2** **a** Predict the answer to $\sqrt{4^2}$ and check it using a calculator.

**b** Do the same for $\sqrt{13^2}$.

**c** Write 3 more equations of this type.

**d** Write a general rule for this property algebraically.

**3** Using your calculator, check to see if these are true.

**a** $\sqrt{20} = \sqrt{4} \times \sqrt{5}$ **b** $\sqrt{20} = \sqrt{2} \times \sqrt{10}$ **c** $\sqrt{48} = \sqrt{6} \times \sqrt{8}$

**d** $\sqrt{48} = \sqrt{2} \times \sqrt{24}$ **e** $\sqrt{48} = \sqrt{3} \times \sqrt{16}$ **f** $\sqrt{48} = \sqrt{4} \times \sqrt{12}$

**4** **a** Write 3 more equations of the type shown in Question **3**.

**b** Write a general rule for this property algebraically.

# 2.09 Simplifying surds

YEAR 9 PATH

Worksheet
Surds

Puzzle
Simplifying surds

Technology
Simplifying surds quiz

Spreadsheet
Simplifying surds quiz

Videos
Simplifying surds 1

Simplifying surds 2

The square of any number is always positive (except for $0^2 = 0$), so it is not possible to give the square root of a *negative* number.

## The square root of $x$

For $x < 0$ (negative), $\sqrt{x}$ is undefined.

For $x = 0$, $\sqrt{x} = 0$.

For $x > 0$ (positive), $\sqrt{x}$ is the positive square root of $x$.

For $x \geq 0$, $(\sqrt{x})^2 = \sqrt{x} \times \sqrt{x} = x$ and $\sqrt{x^2} = x$.

## Example 27

Simplify each expression.

**a** $\left(\sqrt{5}\right)^2$ **b** $\sqrt{11^2}$ **c** $\left(7\sqrt{3}\right)^2$

### Solution

**a** $\left(\sqrt{5}\right)^2 = \sqrt{5} \times \sqrt{5}$

$= 5$

**b** $\sqrt{11^2} = 11$

**c** $\left(7\sqrt{3}\right)^2 = 7\sqrt{3} \times 7\sqrt{3}$

$= 7 \times 7 \times \sqrt{3} \times \sqrt{3}$

$= 49 \times 3$

$= 147$

## The square root of a product

For $x > 0$ and $y > 0$, $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$.

A surd $\sqrt{n}$ can be simplified if $n$ can be divided into 2 factors, where one of them is a square number such as 4, 9, 16, 25, 36, 49, ... This table of square numbers can be useful.

| Number | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| square | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 |

## Example 28

Simplify each surd.

**a** $\sqrt{48}$ **b** $\sqrt{175}$ **c** $4\sqrt{27}$

### Solution

Look for a square number that divides evenly into the number under the square root sign and factorise the number using the square number as one of the factors.

**Method 1**

**a** $\sqrt{48} = \sqrt{4} \times \sqrt{12}$ 4 is a square number
$= 2 \times \sqrt{4} \times \sqrt{3}$
$= 2 \times 2 \times \sqrt{3}$
$= 4 \times \sqrt{3}$
$= 4\sqrt{3}$

**Method 2**

$\sqrt{48} = \sqrt{16} \times \sqrt{3}$ 16 is a square number
$= 4 \times \sqrt{3}$
$= 4\sqrt{3}$

Method 1 involves simplifying surds *twice* ($\sqrt{48}$ and $\sqrt{12}$). Method 2 shows that when simplifying surds, it is more efficient to first look for the **highest square factor** possible.

**b** $\sqrt{175} = \sqrt{25} \times \sqrt{7}$
$= 5 \times \sqrt{7}$
$= 5\sqrt{7}$

**c** $4\sqrt{27} = 4 \times \sqrt{9} \times \sqrt{3}$
$= 4 \times 3 \times \sqrt{3}$
$= 12 \times \sqrt{3}$
$= 12\sqrt{3}$

## EXERCISE 2.09 ANSWERS ON P. 574

### Simplifying surds U F R C

**1** Simplify each expression. EXAMPLE 27

**a** $\left(\sqrt{8}\right)^2$ **b** $\sqrt{15^2}$ **c** $\left(\sqrt{7}\right)^2$ **d** $\sqrt{2^2}$

**e** $\sqrt{31^2}$ **f** $\left(\sqrt{83}\right)^2$ **g** $\sqrt{16^2}$ **h** $\left(\sqrt{25}\right)^2$

**i** $\left(2\sqrt{5}\right)^2$ **j** $\left(3\sqrt{2}\right)^2$ **k** $\left(6\sqrt{5}\right)^2$ **l** $\left(10\sqrt{10}\right)^2$

**2** Simplify each surd. EXAMPLE 28

**a** $\sqrt{20}$ **b** $\sqrt{18}$ **c** $\sqrt{50}$ **d** $\sqrt{12}$

**e** $\sqrt{90}$ **f** $\sqrt{32}$ **g** $\sqrt{8}$ **h** $\sqrt{45}$

**I** $\sqrt{108}$ **j** $\sqrt{98}$ **k** $\sqrt{200}$ **l** $\sqrt{162}$

**3** Simplify each surd.

**a** $5\sqrt{12}$ **b** $3\sqrt{24}$ **c** $4\sqrt{300}$ **d** $4\sqrt{18}$

**e** $4\sqrt{63}$ **f** $2\sqrt{75}$ **g** $2\sqrt{96}$ **h** $10\sqrt{27}$

**i** $3\sqrt{72}$ **j** $4\sqrt{125}$ **k** $5\sqrt{98}$ **l** $9\sqrt{108}$

**4** Convert $4\sqrt{3}$ into an 'entire surd' of the form $\sqrt{x}$. R C

**5** Convert each surd to an entire surd $\sqrt{x}$. R C

**a** $2\sqrt{3}$ **b** $4\sqrt{2}$ **c** $5\sqrt{6}$ **d** $3\sqrt{6}$

**e** $10\sqrt{2}$ **f** $9\sqrt{3}$ **g** $6\sqrt{5}$ **h** $8\sqrt{7}$

**i** $7\sqrt{10}$ **j** $4\sqrt{15}$ **k** $5\sqrt{3}$ **l** $12\sqrt{2}$

# 2.10 Factor trees

Every number can be written as a product of its prime factors. The prime factors can be found by using a **factor tree**.

**Videos**
Factor trees
The prime number code

**Skillsheets**
Factors and divisibility
Prime factors by repeated division

**Worksheet**
Perfect and amicable numbers

**Puzzles**
Crossnumber puzzles
Crossnumber challenges

## Example 29

Write 60 as a product of its prime factors.

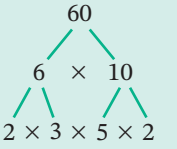
### Solution

Draw a factor tree for 60.

Stop when all of the factors are prime.

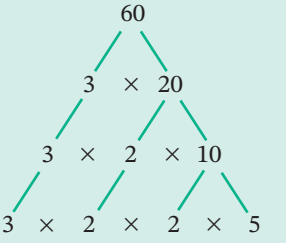
So 60 as a product of its prime factors is:

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$




Note: It is possible to draw different factor trees using different factors for the same number, but the final list of prime factors should still be the same. Here is another factor tree for 60:

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$




**Presentation**
Highest common factors

## Using prime factors to find the highest common factor

### Highest common factor

The **highest common factor** (HCF) or **greatest common divisor** (GCD) of 2 (or more) numbers is the largest number that is a factor of **both (or all)** of these numbers.

**To find the HCF of 2 numbers using their prime factors:**

**1** circle common prime factors

**2** multiply them together.

**Divisor** is just another name for **factor**.

**Video**
Highest common factor


9780170479516

## Example 30

Find the highest common factor of 36 and 45:

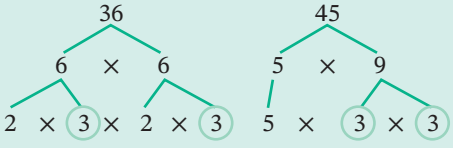
**a** by listing factors **b** using factor trees.

### Solution

**a** Factors of 36 = **1**, 2, **3**, 4, 6, **9**, 12, 18, 36.
Factors of 45 = **1**, **3**, 5, **9**, 15, 30, 45.
The common factors are 1, 3, 9.
The highest common factor is **9**.

**b**
- Draw factor trees for 36 and 45
- Circle common prime factors: 3 and 3
- Multiply 3 and 3 to calculate the HCF

HCF = $3 \times 3 = \mathbf{9}$.




Video
HCF and factor trees

2.10

EXTENSION

# Using prime factors to find the lowest common multiple

## Lowest common multiple

The **lowest common multiple** (LCM) of 2 (or more) numbers is the smallest number that is a multiple of **both (or all)** of these numbers.

**To find the LCM of 2 numbers using their prime factors:**

**1** circle common prime factors

**2** cross out **one** of each pair of common factors

**3** multiply the remaining factors.

also called **least** common multiple

## Example 31

Find the lowest common multiple of 36 and 45:

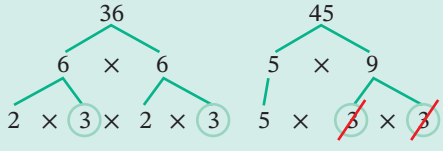
**a** by listing multiples **b** using factor trees.

### Solution

**a** Multiples of 36 = 36, 72, 108, 144, **180**, ...
Multiples of 45 = 45, 90, 105, **180**, 225, ...
The lowest common multiple is **180**.

**b**
- Draw factor trees for 36 and 45
- Circle common prime factors: 3 and 3
- Cross out the 2 3s in the second factor tree
- Multiply the remaining factors to calculate the LCM

LCM = $(2 \times 3 \times 2 \times 3) \times 5 = 36 \times 5 = \mathbf{180}$



# Finding square and cube roots using prime factors

**Video**
Using factor trees to find square roots

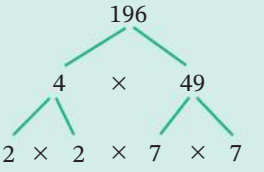
## Example 32

Use a factor tree to find the value of:

**a** $\sqrt{196}$ **b** $\sqrt[3]{216}$

### Solution

**a** Draw a factor tree for 196.



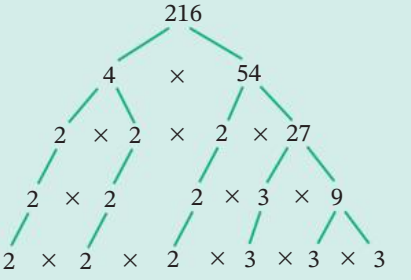

So $196 = 2 \times 2 \times 7 \times 7$

$\therefore \sqrt{196} = \sqrt{2 \times 2 \times 7 \times 7}$ $\sqrt{2 \times 2} = 2,\ \sqrt{7 \times 7} = 7$

$= 2 \times 7$

$= 14$

**b** Draw a factor tree for 216.




So $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$\therefore \sqrt[3]{216} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$ $\sqrt[3]{2 \times 2 \times 2} = 2,\ \sqrt[3]{3 \times 3 \times 3} = 3$

$= 2 \times 3$

$= 6$

## EXERCISE 2.10 ANSWERS ON P. 574

### Factor trees U F R C

EXAMPLE 29

**1** Use a factor tree to write each number as a product of its prime factors in index notation. R C

**a** 27 **b** 108 **c** 588 **d** 84

**e** 600 **f** 51 **g** 125 **h** 432

**2** What is 42 expressed as a product of its prime factors? Select the correct answer **A**, **B**, **C** or **D**.

**A** $2 \times 21$ **B** $3 \times 3 \times 7$ **C** $2 \times 3 \times 7$ **D** $3 \times 14$

Foundation Standard Complex


9780170479516

**3** Draw 2 different factor trees for 80 and show that both give the same prime factors. R C

**4** Find the highest common factor of each pair of numbers. R

EXAMPLE 30

| | | | |
|---|---|---|---|
| **a** 8, 28 | **b** 14, 35 | **c** 20, 5 | **d** 45, 60 |
| **e** 24, 90 | **f** 72, 24 | **g** 16, 48 | **h** 30, 18 |

**5** Use factor trees to find the highest common factor of each pair of numbers. R C

**a** 45, 60 (check against answer to Question **4d**)

**b** 208, 78 **c** 33, 176

2.10

**6** Find the lowest common multiple of each pair of numbers. R

YEAR 9 EXTENSION

EXAMPLE 31

| | | | |
|---|---|---|---|
| **a** 9, 8 | **b** 4, 6 | **c** 10, 22 | **d** 7, 4 |
| **e** 6, 10 | **f** 12, 8 | **g** 9, 45 | **h** 11, 10 |

**7** Use factor trees to find the lowest common multiple of each pair of numbers. R C

**a** 90, 15 **b** 28, 36 **c** 16, 20

**8** Use a factor tree to evaluate each root. R C

EXAMPLE 32

| | | |
|---|---|---|
| **a** $\sqrt{2025}$ | **b** $\sqrt{441}$ | **c** $\sqrt{256}$ |
| **d** $\sqrt[3]{4096}$ | **e** $\sqrt[3]{729}$ | **f** $\sqrt[3]{5832}$ |

**9** Explain why the 'factor trees' method for finding the LCM works, using 28 and 36 from question **7b** as an example. R C

**10** **a** Find the HCF of 28 and 76.

**b** Use the formula (rule)

$$\text{LCM of } a \text{ and } b = \frac{a \times b}{\text{HCF of } a \text{ and } b}.$$

to find the LCM of 28 and 76.

## Investigation

### Index laws for multiplying and dividing

The terms $3^6$ and $3^2$ have the same base, 3.

$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$ and $3^2 = 3 \times 3$.

What happens when we multiply them?

$3^6 \times 3^2 = (3 \times 3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3) = 3^8 = 6561$.

What happens when we divide $3^6$ by $3^2$?

$3^6 \div 3^2 = \frac{3 \times 3 \times 3 \times 3 \times \cancel{3^1} \times \cancel{3^1}}{\cancel{3^1} \times \cancel{3^1}} = 3 \times 3 \times 3 \times 3 = 3^4 = 81$.

Foundation Standard Complex

**1** Copy and complete each pattern.

**a** $2^4 \times 2^3 = (2 \times __ \times __ \times __) \times (2 \times __ \times __) = 2^{\square} = ____$

**b** $7^2 \times 7^2 = (7 \times __ ) \times (__ \times __) = 7^{\square} = ____$

**c** $10^2 \times 10^3 = (__ \times __ ) \times (__ \times __ \times __) = 10^{\square} = ____$

**d** $4^3 \times 4^5 = (_____) \times (_____) = 4^{\square} = ____$

**e** $5^6 \times 5 = (_____) \times ____ = 5^{\square} = ____$

**2** Copy and complete each pattern.

**a** $3^3 \times 3^7 = 3^{\square}$

**b** $2^5 \times 2^3 = 2^{\square}$

**c** $9^6 \times 9^4 = 9^{\square}$

**d** $6^3 \times 6^3 = 6^{\square}$

**e** $8^8 \times 8 = 8^{\square}$

**3** Copy and complete each statement.

**a** $a^4 \times a^5 = _____$

**b** $a^m \times a^n = _____$

**c** When multiplying terms with the same base, _____ the powers.

**4** Copy and complete each pattern.

**a** $2^5 \div 2^3 = \dfrac{2 \times 2 \times 2^{\cancel{1}} \times 2^{\cancel{1}} \times 2^{\cancel{1}}}{2^{\cancel{1}} \times 2^{\cancel{1}} \times 2^{\cancel{1}}} = ___ \times ___ = 2^{\square} = ____$

**b** $7^8 \div 7^4 = \dfrac{7 \times 7 \times 7 \times __ \times __ \times __ \times __ \times __}{7 \times 7 \times 7 \times __} = ___ \times ___ \times ___ \times ___ = 7^{\square} = ____$

**c** $4^6 \div 4^3 = \dfrac{__ \times __ \times __ \times __ \times __ \times __}{__ \times __ \times __} = ___ \times ___ \times ___ = 4^{\square} = ____$

**d** $3^6 \div 3 = _____ = _____ = 3^{\square} = ____$

**e** $10^7 \div 10^2 = _____ = _____ = 10^{\square} = ____$

**5** Copy and complete each pattern.

**a** $3^5 \div 3^2 = 3^{\square}$

**b** $10^9 \div 10^5 = 10^{\square}$

**c** $8^6 \div 8^4 = 8^{\square}$

**d** $6^7 \div 6 = 6^{\square}$

**e** $2^6 \div 2^5 = 2^{\square}$

**6** Copy and complete each statement.

**a** $a^5 \div a^3 = _____$

**b** $a^m \div a^n = _____$

**c** When dividing terms with the same base, _____ the powers.

# Index laws for multiplying and dividing 2.11

## Index laws

When **multiplying terms with the same base**, *add* the powers.

$$a^m \times a^n = a^{m+n}$$

When **dividing terms with the same base**, *subtract* the powers.

$$a^m \div a^n = a^{m-n}$$

These are called **index laws** because **index** is another name for 'power'.

Video
Powers, indices and exponents

Video
Index laws



## Example 33

Use index notation to simplify each product.

**a** $5^3 \times 5^4$ **b** $6^4 \times 6^5$ **c** $2 \times 2^3 \times 2^4$

### Solution

**a** $5^3 \times 5^4 = 5^{3+4} = 5^7$ Adding the powers

**b** $6^4 \times 6^5 = 6^{4+5} = 6^9$ Note: Do **not** multiply the bases

**c** $2 \times 2^3 \times 2^4 = 2^{1+3+4} = 2^8$ Note: $2 = 2^1$

## Example 34

Use index notation to simplify each quotient.

**a** $5^8 \div 5^3$ **b** $\frac{7^5}{7^2}$ **c** $6^{11} \div 6^{10}$

### Solution

**a** $5^8 \div 5^3 = 5^{8-3} = 5^5$ Subtracting the powers

**b** $7^5 \div 7^2 = 7^{5-2} = 7^3$ Note: Do **not** divide the bases

**c** $6^{11} \div 6^{10} = 6^{11-10} = 6^1 = 6$

Video
Index laws

## EXERCISE 2.11 ANSWERS ON P. 574

# Index laws for multiplying and dividing U F R C

EXAMPLE 33

**1** Use index notation to simplify each product. R C

**a** $6^3 \times 6^2$ **b** $4^3 \times 4^4$ **c** $2^5 \times 2^6$ **d** $8 \times 8^9$

**e** $11^4 \times 11^4$ **f** $7^3 \times 7^7$ **g** $5 \times 5^6$ **h** $6^6 \times 6^{10}$

**i** $2^3 \times 2^2 \times 2^4$ **j** $5^3 \times 5^4 \times 5^2$ **k** $2^4 \times 2^4 \times 2^6$ **l** $3^2 \times 3^9 \times 3^3 \times 3^2$

**2** Which term is equal to $2^3 \times 2^3 \times 2$? Select the correct answer **A**, **B**, **C** or **D**.

**A** $2^6$ **B** $2^9$ **C** $8^7$ **D** $2^7$

**3** Simplify each product, then evaluate it as a whole number.

**a** $5^2 \times 5^4$ **b** $7^6 \times 7^2$ **c** $3^4 \times 3^9$ **d** $4^3 \times 4^3$

**e** $9^2 \times 9^5$ **f** $6^5 \times 6^4$ **g** $8^4 \times 8^6$ **h** $7 \times 7^3$

EXAMPLE 34

**4** Use index notation to simplify each quotient. C

**a** $2^6 \div 2^2$ **b** $3^9 \div 3^7$ **c** $7^8 \div 7^2$ **d** $\frac{2^9}{2^4}$

**e** $5^{11} \div 5^9$ **f** $10^{11} \div 10^7$ **g** $11^{12} \div 11^5$ **h** $6^{18} \div 6^{11}$

**i** $2^5 \div 2^4$ **j** $\frac{16^{13}}{16^7}$ **k** $35^{12} \div 35$ **l** $\frac{20^7}{20^6}$

**5** Which term is equal to $5^8 \div 5^2$? Select **A**, **B**, **C** or **D**.

**A** $1^6$ **B** $5^4$ **C** $1^5$ **D** $5^6$

**6** Simplify each quotient, then evaluate it as a whole number.

**a** $2^6 \div 2^2$ **b** $5^9 \div 5^7$ **c** $4^5 \div 4^2$ **d** $10^9 \div 10^4$

**e** $\frac{8^{11}}{8^8}$ **f** $6^{10} \div 6^2$ **g** $11^4 \div 11^3$ **h** $\frac{3^7}{3}$

**7** Copy and complete this table of powers.

| Number, $n$ | $n^2$ | $n^3$ | $n^4$ | $n^5$ | $n^6$ |
|---|---|---|---|---|---|
| 2 | 4 | 8 | | | |
| 3 | | 27 | | 243 | |
| 4 | 16 | | 256 | | |
| 5 | | 125 | | | 15 625 |

**8** Use your answers from question **7** to evaluate each expression.

**a** $2^3 \times 2^2$ **b** $3^2 \times 3^2$ **c** $5^2 \times 5$ **d** $5^3 \times 5^2$

**e** $4^4 \times 4^2$ **f** $2^2 \times 3^2 \times 3^3 \times 2$ **g** $4^5 \div 4^2$ **h** $5^6 \div 5$

**i** $\frac{3^6}{3^4}$ **j** $2^5 \div 2^4$ **k** $\frac{4^4}{4^3}$ **l** $3^3 \div 3^3$

**9** Copy and complete each equation. R

**a** $5^2 \times$ ____ $= 5^6$ **b** ____ $\div 3^4 = 3^3$ **c** $2^7 \div$ ____ $= 2^4$

**10** Simplify each algebraic expression. R C

**a** $a^3 \times a^7$ **b** $x^2 \times x^4$ **c** $b^4 \times b$ **d** $y^3 \times y^2$

**e** $d^6 \div d^2$ **f** $x^5 \div x^3$ **g** $\frac{a^8}{a^7}$ **h** $\frac{r^5}{r}$

Foundation Standard Complex

## Investigation

### More index laws

What happens when we raise $2^3$ to a power of 4? How do we simplify $(2^3)^4$ using index notation?

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{3+3+3+3} = 2^{12}$

**1** Copy and complete each pattern.

**a** $(3^2)^5 = __ \times __ \times __ \times __ \times __ = 3$ —— $= 3$–

**b** $(7^4)^2 = __ \times __ = 7$ — $= 7$–

**c** $(16^3)^3 = ____ \times ____ \times ____ = 16$ —— $= 16$–

**d** $(2^6)^4 = __ \times __ \times __ \times __ = 2$ —— $= 2$–

**e** $(5^2)^3 = __ \times __ \times __ = 5$—— $= 5$–

**2** Copy and complete each statement.

**a** $(a^5)^2 =$ _____

**b** $(a^m)^n =$ _____

**c** When a term with a power is raised to another power, _____ the powers.

**3** We know that $3^2 = 3 \times 3$ and $3^1 = 3$, but what is the value of $3^0$ (3 to the power of 0)?

**a** Copy and complete this pattern.

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$3^3 = 3 \times ___ \times ___ = ___$

$3^2 =$ _____ $= ___$

$3^1 =$ _____ $= ___$

**b** As the powers of 3 decrease by 1, what happens to its calculated value?

**c** According to the pattern, what should be the value of $3^0$?

**4** **a** Use index notation to simplify $2^5 \div 2^5$.

**b** But $2^5 \div 2^5 = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2}$. What is the answer to this?

**c** What is any number divided by itself equal to?

**d** What is the value of $2^0$?

**5** **a** Use index notation to simplify $5^2 \times 5^0$. What do you notice about the answer?

**b** If a number multiplied by $5^0$ equals itself, what must be the value of $5^0$?

**c** Use index notation to simplify $10^6 \div 10^0$. What do you notice about the answer?

**d** If a number divided by $10^0$ equals itself, what must be the value of $10^0$?

**6** Copy and complete each statement.

**a** $4^0 =$ _____

**b** $a^0 =$ _____

**c** When any number is raised to a power of zero, the answer is always _____.

# 2.12 More index laws

**Worksheets**
What is the power question?

Power calculations

## More index laws

When a **term with a power** is raised to **another power**, **multiply** the powers.

$$(a^m)^n = a^{m \times n}$$

Any number raised to the **power of zero** is equal to **1**.

$$a^0 = 1$$

**Video**
Index laws

## Example 35

Use index notation to simplify each expression.

**a** $(6^2)^5$ **b** $(7^3)^3$

### Solution

**a** $(6^2)^5 = 6^{2 \times 5} = 6^{10}$ Multiply the powers

**b** $(7^3)^3 = 7^{3 \times 3} = 7^9$

**Video**
Index laws

## Example 36

Evaluate each expression.

**a** $6^0$ **b** $(3 \times 4)^0$ **c** $3 \times 4^0$

### Solution

**a** $6^0 = 1$ **b** $(3 \times 4)^0 = 12^0 = 1$ **c** $3 \times 4^0 = 3 \times 1 = 3$

**EXERCISE 2.12** ANSWERS ON P. 575

## More index laws U F R C

EXAMPLE 35

**1** Use index notation to simplify each expression. R C

**a** $(2^9)^2$ **b** $(5^2)^3$ **c** $(7^4)^5$ **d** $(8^2)^7$

**e** $(3^8)^3$ **f** $(9^6)^4$ **g** $(6^7)^2$ **h** $(11^5)^5$

**i** $(8^{10})^3$ **j** $(4^9)^5$ **k** $[(-2)^4]^7$ **l** $[(-7)^3]^6$

**2** Which term is equal to $(3^4)^2$? Select the correct answer **A**, **B**, **C** or **D**.

**A** $9^6$ **B** $6^4$ **C** $3^6$ **D** $3^8$

Foundation Standard Complex


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**3** Simplify each expression, then evaluate it as a whole number.

**a** $(5^2)^4$ **b** $(2^5)^3$ **c** $(3^9)^2$ **d** $(6^3)^4$

**e** $(7^2)^3$ **f** $(4^6)^2$ **g** $(8^0)^5$ **h** $(4^3)^3$

EXAMPLE 36

2.12

**4** Evaluate each expression.

**a** $9^0$ **b** $3^0$ **c** $10^0$ **d** $16^0$

**e** $(-2)^0$ **f** $6^0$ **g** $(7-3)^0$ **h** $7-3^0$

**i** $(-5)^0$ **j** $7\times3^0$ **k** $(7\times3)^0$ **l** $2+3^0$

**m** $(6+6)^0$ **n** $6^0+6^0$ **o** $6\div6^0$ **p** $10-10^0$

**5** Evaluate $6^2\div2^0$. Select **A**, **B**, **C** or **D**.

**A** 3 **B** 36 **C** 18 **D** 6

**6** Copy and complete this table of powers of 2.

| $2^0$ | $2^1$ | $2^2$ | $2^3$ | $2^4$ | $2^5$ | $2^6$ | $2^7$ | $2^8$ | $2^9$ | $2^{10}$ | $2^{11}$ | $2^{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 4 | | | 32 | | | | | 1024 | | |

**7** Use your answers from question **6** to evaluate each expression. R

**a** $(2^2)^3$ **b** $(2^6)^2$ **c** $(2^2)^2$ **d** $(2^3)^3$

**e** $(2^0)^4$ **f** $(2^2)^4$ **g** $(2^{10})^0$ **h** $(2^7)^1$

**8** Copy and complete each equation. R C

**a** $(6^2)^{-}=6^8$ **b** $(5^{-})^3=5^9$ **c** $(10^{-})^{-}=10^{18}$

**9** Simplify each algebraic expression. R C

**a** $(a^4)^2$ **b** $(x^6)^6$ **c** $(d^2)^5$ **d** $(n^3)^6$

**e** $p^0$ **f** $4\times y^0$ **g** $(4\times y)^0$ **h** $a^0-b^0$

**10** What happens when we have a product of 2 numbers all squared? How do we simplify $(3\times4)^2$ using index notation?

$(3\times4)^2=(3\times4)\times(3\times4)$

$=3\times4\times3\times4$

$=3^2\times4^2$

Use your calculator to check this is true.

We did some examples like this in Exercise 2.08 on page 67.

**11** Copy and complete this rule: C

$(ab)^2=$ _____

**12** Use index notation to simplify each expression, then evaluate it as a whole number. C

**a** $(10\times2)^2$ **b** $(4\times3)^2$ **c** $(3\times10)^2$ **d** $(12\times5)^2$

**e** $(2\times12)^2$ **f** $(8\times4)^2$ **g** $15^2$ **h** $21^2$

**i** $18^2$ **j** $22^2$ **k** $27^2$ **l** $36^2$

Foundation Standard Complex



**Puzzle**
Crossnumber challenges

## Power plus ANSWERS ON P. 575

**1** Simplify each surd.

| | | | |
|---|---|---|---|
| **a** $\sqrt{81}$ | **b** $\sqrt{8}$ | **c** $\sqrt{27}$ | **d** $\sqrt{50}$ |
| **e** $\sqrt{72}$ | **f** $\sqrt{45}$ | **g** $\sqrt{180}$ | **h** $\sqrt{128}$ |
| **i** $\sqrt{28}$ | **j** $\sqrt{200}$ | **k** $\sqrt{48}$ | **l** $\sqrt{245}$ |

**2** **a** Use index notation to simplify $2^4 \div 2^5$.

**b** But $2^4 \div 2^5 = \frac{2\times2\times2\times2}{2\times2\times2\times2\times2}$. What is the answer to this?

**c** What is the value of $2^{-1}$?

**d** What is the value of $2^{-2}$?

**3** **a** Does $(-2)^5 = -2^5$?

**b** Does $(-3)^4 = -3^4$?

**c** For what values of $n$ does $(-1)^n = -1$?

**d** For what value of $x$ is $x^0$ not defined?

**4** Scientific notation is a special way of writing very large or very small numbers using powers of 10. For example:

- 25 600 can be written as $2.56 \times 10 \times 10 \times 10 \times 10 = 2.56 \times 10^4$ because $2.56 \times 10^4 = 2.56 \times 10 \times 10 \times 10 \times 10$
- 0.001678 can be written as $1.678 \div 10 \div 10 \div 10 = 1.678 \times 10^{-3}$ because $1.678 \times 10^{-3} = 1.678 \div 10 \div 10 \div 10$

Note that the first number is a decimal between 1 and 10 and the second number is a power of 10.

Convert each number written in scientific notation to an ordinary number.

| | | |
|---|---|---|
| **a** $2.4 \times 10^4$ | **b** $4.55 \times 10^5$ | **c** $9.33 \times 10^{-2}$ |
| **d** $6.38 \times 10^{-2}$ | **e** $8.7 \times 10^6$ | **f** $5.82 \times 10^{-3}$ |
| **g** $1.26 \times 10^{-1}$ | **h** $2.69 \times 10^3$ | **i** $3.14 \times 10^8$ |

Can you see a quick way of writing the answer each time?

**5** Write each number in scientific notation.

| | | |
|---|---|---|
| **a** 12 000 | **b** 345 000 000 | **c** 0.007 |
| **d** 4000 | **e** 0.0005 | **f** 0.00041 |
| **g** 1 920 000 | **h** 0.000 361 | **i** 0.000 000 063 7 |

**6** **a** What is the largest number that can be displayed on your calculator?

**b** What is the smallest?

# CHAPTER 2 REVIEW



## Language of maths

| | | | |
|---|---|---|---|
| base | consecutive | cube root | decimal places |
| estimate | evaluate | factor tree | highest common factor |
| grouping symbol | index laws | index notation | integer |
| long division | lowest common multiple | mental calculation | order of operations |
| prime factors | product | quotient | recurring decimal |
| round | square root | surd | terminating decimal |

**1** If you round a decimal to the nearest hundredth, how many decimal places is this?

**2** Write a numerical expression that requires the use of:

**a** 'order of operations' rules **b** long division

**3** Write 'the cube root of $-64$' using mathematical symbols, then evaluate it.

**4** In $8^5$, what are the names given to the '8' and the '5'?

**5** What is another name for:

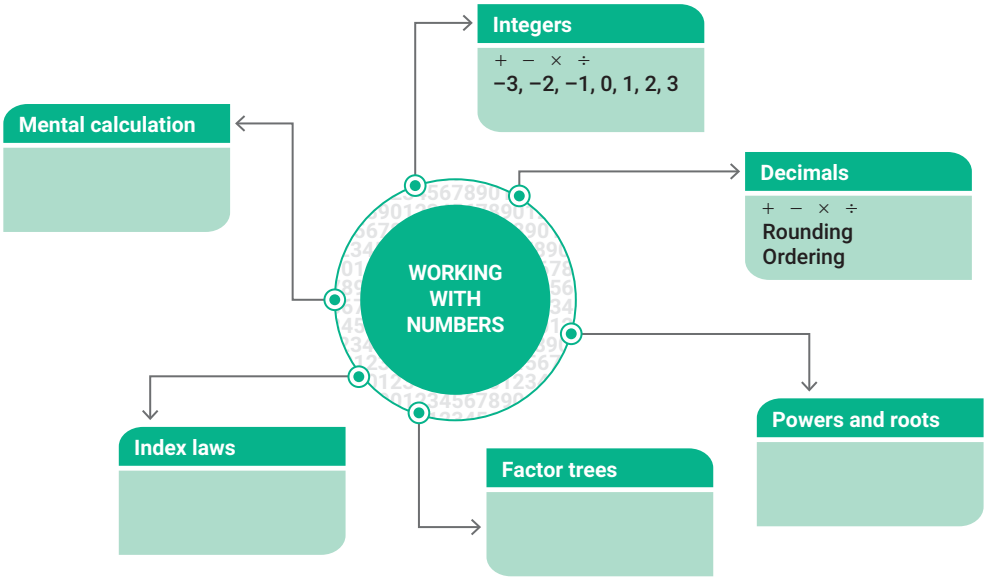
**a** grouping symbols? **b** index?

**6** What diagram is used to write a number as a product of its prime factors?

## Topic summary

- What parts of this chapter do you remember from last year?
- Are there any parts of this chapter that you do not understand?
- Discuss any problems with your teacher or a friend.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.




Quiz
Language of maths 2

Puzzle
Numbers crossword

Worksheet
Mind map: Working with numbers

Quiz
Test yourself 2

# TEST YOURSELF 2

ANSWERS ON P. 575

2.01 **1** Use mental calculation strategies to evaluate each expression.

**a** $7 \times 200$ **b** $14 \times 15$ **c** $8000 \div 50$
**d** $\frac{1}{2} \times 262$ **e** $68 + 19$ **f** $35 \times 11$
**g** $65 \times 4$ **h** $8 \times 60$ **i** $26 + 61 + 156 + 19 + 14$
**j** $8 \times 25 \times 4$ **k** $23 \times 12$ **l** $168 \div 4$
**m** $2700 \div 300$ **n** $40 \times 70$ **o** $37 \times 9$
**p** $18 \times 5$ **q** $384 \div 8$ **r** $970 - 245$
**s** $4 \times 8 \times 5$

2.01 **2** Evaluate each quotient.

**a** $812 \div 7$ **b** $846 \div 9$ **c** $396 \div 18$ **d** $2139 \div 23$

2.02 **3** Evaluate each expression.

**a** $-4 + 6$ **b** $5 - (-2)$ **c** $-3 - 8$ **d** $-2 - (-2)$

2.02 **4** Death Valley in the USA is 397 m below sea level and Mt Everest is 8840 m above sea level. What is the vertical distance between the bottom of Death Valley and the top of Mt Everest?

2.02 **5** On 17 July, the maximum temperature in Armidale was 17°C and the minimum temperature was − 6°C. By how much had the temperature risen during the day?

2.03 **6** Evaluate each product.

**a** $-2 \times 7$ **b** $-3 \times (-8)$ **c** $(-7)^2$ **d** $16 \times (-5)$

2.03 **7** Evaluate each quotient.

**a** $36 \div (-4)$ **b** $-28 \div 7$ **c** $24 \div (-2)$ **d** $-15 \div (-5)$

2.04 **8** Evaluate each expression.

**a** $2 + (13 - 8) \times 11$ **b** $14 - 5 \times (-3)$ **c** $15 - 18 \div 3 \times 2$
**d** $80 - [(7 - 10) \times (-6)]$ **e** $\frac{8-3}{100 \div 4}$ **f** $\frac{15 \times (-6+8)}{24 \div 2^2}$

2.05 **9** Write these decimals in descending order: 0.417, 0.47, 0.147, 0.471.

2.05 **10** Write each number correct to the number of decimal places shown in the brackets.

**a** 0.473 [1] **b** 13.1051 [2] **c** 98.4273 [3]

2.05 **11** To complete the wiring of a house, Joe the electrician needed these lengths of cables: 2.3 m, 1.9 m, 4.2 m and 3.8 m. How much cable did he need altogether?

2.06 **12** Evaluate each expression.

**a** $37.4 - 6.9$ **b** $13.3 + 0.82 + 5.6$ **c** $2.6 \times 4$ **d** $5.5 \times 0.6$
**e** $0.71 \times 1.3$ **f** $(2.5)^2$ **g** $9.57 \div 4$ **h** $8.12 \div 0.7$

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**13** A drink bottle holds 0.5 L. How many drink bottles can be filled from a container that holds 7.5 L? 2.06

**14** Convert each fraction to a decimal. 2.07

**a** $\frac{2}{9}$ **b** $\frac{7}{8}$ **c** $\frac{1}{6}$

**15** Evaluate each expression. 2.08

**a** $7^3$ **b** $11^5$ **c** $(-5)^4$

**d** $\sqrt{400}$ **e** $\sqrt[3]{27}$ **f** $\sqrt[3]{-125}$

**16** Without using a calculator, estimate $\sqrt{31}$, correct to one decimal place. 2.08

**17** Copy and complete: 2.08

**a** $20^2 = (_ \times 10)^2$
$= _^2 \times 10^2$
$= _ \times _$
$= _$

**b** $\sqrt{196} = \sqrt{\ } \times \sqrt{49}$
$= _ \times _$
$= _$

**18** Simplify each expression. YEAR 9 PATH 2.09

**a** $\left(\sqrt{3}\right)^2$ **b** $\sqrt{17^2}$ **c** $\left(3\sqrt{8}\right)^2$

**d** $\sqrt{40}$ **e** $\sqrt{27}$ **f** $2\sqrt{75}$

**19** **a** Find the highest common factor of 36 and 42. 2.10

**b** Find the lowest common multiple of 20 and 14.

**20** Use factor trees to write 132 and 88 as products of their prime factors, then use them to find the highest common factor of 132 and 88. 2.10

**21** Use factor trees to write 12 and 45 as products of their prime factors, then use them to find the lowest common multiple of 12 and 45. 2.10

**22** Use a factor tree to write 784 as a product of its prime factors, then use it to evaluate $\sqrt{784}$. 2.10

**23** Simplify each expression using index notation. 2.11

**a** $4^2 \times 4^5$ **b** $6^4 \times 6^7$ **c** $12^4 \times 12^4$ **d** $9^3 \times 9$

**e** $2^7 \div 2^3$ **f** $5^9 \div 5^6$ **g** $3^8 \div 3^7$ **h** $6^{11} \div 6^4 \times 6^5$

**24** Simplify each expression using index notation. 2.12

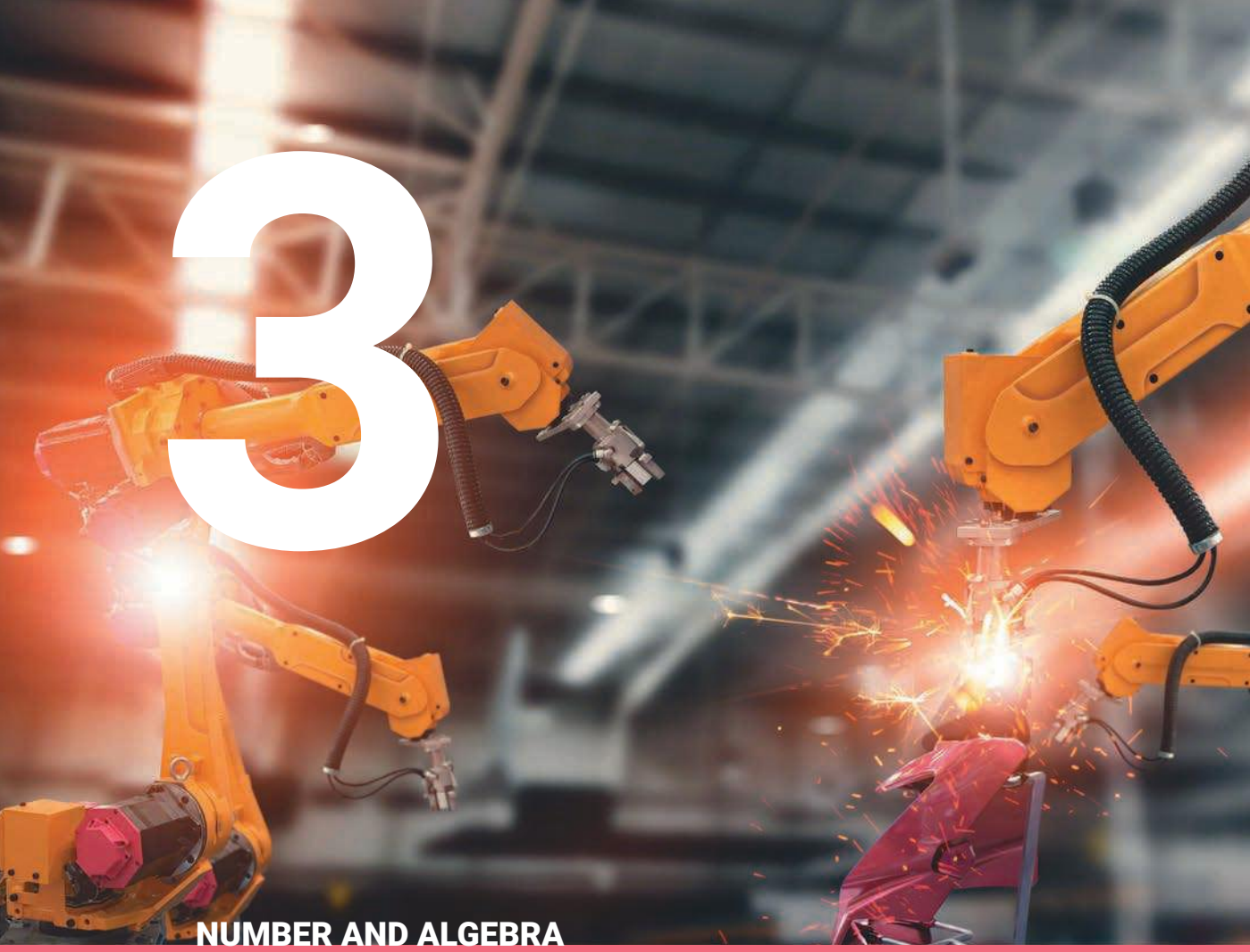
**a** $(3^4)^2$ **b** $(7^2)^5$ **c** $(6^3)^3$ **d** $(3^5)^3$

**25** Evaluate each expression. 2.12

**a** $3^0$ **b** $6^0$ **c** $4 \times 4^0$ **d** $9 - 9^0$

**e** $10^0$ **f** $(12 \times 3)^0$ **g** $(5 - 3)^0$ **h** $5^0 - 3^0$

Foundation Standard Complex



3

NUMBER AND ALGEBRA

# ALGEBRA

Algebra is the study of mathematical patterns and rules. The subject can be traced back over 4000 years ago to the ancient Babylonians. Today, all modern technology relies on algebra – the internet, satellites, computers, TVs and all other digital devices rely on the application of algebra.




## Chapter outline

| | Working mathematically | | | | |
|---|---|---|---|---|---|
| **3.01** Variables | U | F | | R | C |
| **3.02** Creating algebraic expressions | U | F | | R | C |
| **3.03** Substitution | U | F | PS | | |
| **3.04** Adding and subtracting terms | U | F | | R | C |
| **3.05** Multiplying and dividing terms | U | F | | R | C |
| **3.06** The index laws* | U | F | | | |
| **3.07** Expanding expressions | U | F | | R | C |
| **3.08** Factorising algebraic terms | U | F | | R | |
| **3.09** Factorising expressions | U | F | | R | C |
| **3.10** Factorising with negative terms | U | F | | R | C |
| **3.11** Expanding binomial products* | U | F | | R | C |
| **3.12** Perfect squares* | U | F | | R | |
| **3.13** Difference of 2 squares* | U | F | | R | C |

*YEAR 9 EXTENSION

## Wordbank

**algebraic expression** An expression that describes a quantity by using variables and numerals, such as $3x + 4y - 8$

**algebraic term** A part of an algebraic expression, such as $3x$ in $4x^2 + 3x - 11$

**evaluate** To find the value of an expression

**expand** To rewrite an expression such as $4(2k + 4)$ without brackets

**factorise** The opposite of expand; to rewrite an expression, such as $8k + 16$, with brackets

**like terms** Terms that have exactly the same variables, for example, $5p$ and $2p$

**substitute** To replace a variable with a number

**variable** (or **pronumeral**) A quantity that can take on different values, represented by a symbol such as a letter of the alphabet

**Quiz**
Wordbank 3

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

## Videos (19):

**3.01** Algebraic expressions • Simplifying algebraic expressions 2
**3.02** From words to algebraic expressions
**3.03** Substitution 2 • Substitution 1
**3.04** Adding and subtracting terms
**3.05** Multiplying terms • Dividing terms
**3.06** Powers, indices and exponents • Simplifying with the index laws • Index laws 1 • Index laws 2
**3.07** Expanding expressions • Expanding expressions 2
**3.09, 3.10** Factorising expressions
**3.11** Expanding binomial products 1 • Expanding binomial products 2
**3.12** Special binomial products
**3.13** Difference of 2 squares • Special binomial products

## *Twig* videos (2):

**3.03** The heartbeat formula • The heptathlon

## *PhET* interactives (2):

**3.04** Expression exchange
**3.07** Area model Algebra

## Quizzes (6):

- Wordbank 3
- SkillCheck 3
- Mental skills 3A
- Mental skills 3B
- Language of maths 3
- Test yourself 3

## Skillsheets (4):

**3.01** Order of operations
**3.02** Algebraic expressions
**3.04, 3.07** Algebra using diagrams
**3.08, 3.09, 3.10** Algebra using diagrams • Factorising using diagrams

## Worksheets (14):

**SkillCheck** What does the symbol mean?
**3.02** Writing algebraic expressions • What does the symbol mean?
**3.03** Substitution
**3.04** Collecting like terms
**3.05** Simplifying algebraic expressions • Perimeter and area • Why aren't they the same?
**3.07** Expanding brackets
**3.10** Algebra review • Algebraic expressions review
**3.11** Area diagrams • Binomial products • Special products
**3.12, 3.13** Special products
**Mind map** Algebra

## Puzzles (19):

**3.01** Algebra bingo game
**3.02** Generalised arithmetic • What's the expression?
**3.03** Formula 1 game • Substitution game
**3.04** Algebra bingo game • Adding and subtracting like terms
**3.06** Indices puzzle
**3.07** Expandominoes • Expanding • Expand the brackets • Expand negative brackets
**3.09** Factorising
**3.10** Expandominoes • Factorising puzzle • Factorising with negative terms
**3.11** Expanding brackets • Trinomials
**3.12** Factorominoes
**3.13** Enough time
**Language of maths** Algebra find-a-word

## Presentation (1):

**3.04** Collecting like terms

## Technology (1):

**3.11** Expanding binomials

## Spreadsheet (1):

**3.11** Expanding binomials

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

## In this chapter you will:

- learn about variables as a way of representing numbers using letters
- generalise number laws and properties to algebraic terms and expressions
- convert between word and algebraic expressions
- evaluate algebraic expressions by substituting values for each variable
- simplify algebraic expressions using the 4 arithmetic operations
- (EXTENSION, YEAR 9 CORE) apply index laws to algebraic expressions with whole number and zero powers
- use the distributive law to expand algebraic expressions with grouping symbols (brackets)
- factorise algebraic terms and expressions
- (EXTENSION, YEAR 9 PATH) expand binomial products, including perfect squares and difference of 2 squares



## SkillCheck ANSWERS ON P. 575



Quiz
SkillCheck 3

**1** Evaluate each expression.

| | | | | | |
|---|---|---|---|---|---|
| **a** | $3 - 4$ | **b** | $-5 + 9$ | **c** | $-8 - 2$ |
| **d** | $3 - (-2)$ | **e** | $-3 \times 6$ | **f** | $-12 \div (-4)$ |
| **g** | $-12 \div 4 - 10$ | **h** | $-4 \times (-8)$ | **i** | $(-6)^2$ |
| **j** | $2^3$ | **k** | $(-1)^3$ | **l** | $2 \times 3^2$ |

**2** Simplify each algebraic expression.

| | | | | | |
|---|---|---|---|---|---|
| **a** | $7 \times p$ | **b** | $a \times b$ | **c** | $k \times k$ |
| **d** | $n \times n \times n$ | **e** | $2 \times p \times m$ | **f** | $p \times k \times d$ |
| **g** | $y \times 6$ | **h** | $t \times 3 \times q$ | **i** | $6d - 7d$ |
| **j** | $q + q + q$ | **k** | $a + a + a + a$ | **l** | $1x$ |

**3** Choose which operation ($+$, $-$, $\times$ or $\div$) is related to each word.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | sum | **b** | product | **c** | difference | **d** | quotient |
| **e** | decrease | **f** | increase | **g** | more than | **h** | less than |

**4** If $x = 8$ and $y = 2$, evaluate each expression.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $x + y$ | **b** | $3x$ | **c** | $x - y$ | **d** | $5y$ |
| **e** | $xy$ | **f** | $9 - y$ | **g** | $x + 17$ | **h** | $2x + 4$ |
| **i** | $3y - 2$ | **j** | $x \div y$ | **k** | $5x \div y$ | **l** | $4x + 5y$ |

**5** Find:

| | | | |
|---|---|---|---|
| **a** | the sum of 8 and 5 | **b** | double 9 |
| **c** | 6 increased by 10 | **d** | the difference between 14 and 11 |
| **e** | the product of 3 and 7 | **f** | how many times 5 divides into 40 |



Worksheet
What does the symbol mean?

# 3.01 Variables

**Video**
Algebraic expressions

**Puzzle**
Algebra bingo game

In algebra, a **variable** or **pronumeral** is a symbol, usually a letter of the alphabet, that stands for a number. It is called a variable because its value can vary (change).

When writing **algebraic expressions**, we use the following abbreviations.

- $3 \times k = 3k$ for multiplication, we leave out the '×' symbol
- $m \div 4 = \frac{m}{4}$ for division, we can write in fraction form
- $r \times r = r^2$ for powers
- $1x = x$ for multiplying by 1
- $c \times a \times 6 = 6ac$ write the number first, then the variables in alphabetical order

**Video**
Simplifying algebraic expressions 2

## Example 1

Simplify each expression.

**a** $p + p + p + p$ **b** $d \times 5$ **c** $r \div 7$

**d** $m \times n \times 6$ **e** $4v + 2v$ **f** $5 \times k \times 2 \times h$

**g** $y \times y \times y$ **h** $5a - 4a$ **i** $b \times b \times 15$

### Solution

**a** $p + p + p + p = 4p$ — 4 lots of $p$.

**b** $d \times 5 = 5d$ — Write the number first.

**c** $r \div 7 = \frac{r}{7}$

**d** $m \times n \times 6 = 6mn$ — Number first, then variables in alphabetical order.

**e** $4v + 2v = 6v$ — $(v + v + v + v) + (v + v) = 6$ lots of $v$

**f** $5 \times k \times 2 \times h = 5 \times 2 \times k \times h$
$= 10hk$

**g** $y \times y \times y = y^3$

**h** $5a - 4a = 1a$ — $(a + a + a + a + a) - (a + a + a + a) =$ one $a$ left
$= a$ — Just $a$: no need to write the '1'.

**i** $b \times b \times 15 = 15b^2$ — The '2' (squared) belongs to the $b$ only.


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## Example 2

Use order of operations to simplify each expression.

**a** $2 \times m - 4$ **b** $15 - k \div 3$ **c** $13 \div (d \times d) + 4$

### Solution

**a** $2 \times m - 4 = 2m - 4$ $\times$ first, then $-$

**b** $15 - k \div 3 = 15 - (k \div 3)$ $\div$ first, then $-$

$= 15 - \frac{k}{3}$

**c** $13 \div (d \times d) + 4 = 13 \div d^2 + 4$ ( ) first, then $\div$, then $+$

$= \frac{13}{d^2} + 4$

Skillsheet
Order of operations

3.01

## Example 3

Write each expression in expanded form.

Note that expanding is the opposite of simplifying.

**a** $5bc$ **b** $-4kr^2$ **c** $x^2 - \frac{t}{11}$

### Solution

**a** $5bc = 5 \times b \times c$ **b** $-4kr^2 = -4 \times k \times r \times r$ **c** $x^2 - \frac{t}{11} = x \times x - t \div 11$

## EXERCISE 3.01 ANSWERS ON P. 575

### Variables U F R C

**1** Simplify each expression. R C

EXAMPLE 1

**a** $w \times 5$ **b** $m \times m$ **c** $3 \times c \times b \times a$

**d** $k + k + k + k + k + k$ **e** $m \div 8$ **f** $f + f + f$

**g** $12m + 4m$ **h** $7 \times w \times 3 \times x$ **i** $11p - 2p$

**j** $4 \times t \times t$ **k** $6d - 5d$ **l** $b \times c \times d \times 16$

**m** $26 \div n$ **n** $h \times 9 \times h$ **o** $9x + 11x$

**p** $a + a - a$ **q** $12q - 4q$ **r** $2 \times c \times 6 \times n$

**s** $3 \times r \times d \times (-2) \times d$ **t** $a + a + a + c + c$ **u** $a \times a \times a \times x \times x$

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EXAMPLE 2

**2** Use order of operations to simplify each expression. R C

**a** $13 \div (a \times 2)$ **b** $9 + 4 \times m$ **c** $(e - 6) \div 5$

**d** $4 \times t - 8$ **e** $y \times y + z \times z$ **f** $k \div 9 - n$

**g** $p \div (9 + n)$ **h** $22 - e \div 2$ **i** $11 + u \times u \times 11$

**3** Which expression is equal to $m \times m \times 5 + 2$? Select the correct answer **A**, **B**, **C** or **D**. R C

**A** $5m^2 + 2$ **B** $5m^3 + 2$ **C** $5m + 2$ **D** $m^2 + 7$

**4** Explain the meaning of: C

**a** $pq$ **b** $\frac{3y}{x}$ **c** $1x = x$

**5** Which expression is **not** equal to $4p$? Select **A**, **B**, **C** or **D**. R C

**A** $5p - p$ **B** $p^4$ **C** $p + p + p + p$ **D** $p \times 4$

EXAMPLE 3

**6** Write each expression in expanded form. C

**a** $8st$ **b** $-2y^2$ **c** $\frac{8}{f}$ **d** $r^2 + t^2$

**e** $4b^2 d$ **f** $5xy - 2a$ **g** $\frac{x-2}{3}$ **h** $q^3$

**i** $16 - 2d^2$ **j** $\frac{-4d}{3}$ **k** $h^2 i^2$ **l** $4j - \frac{9}{d}$

**7** What is $7e^3$ in expanded form? Select **A**, **B**, **C** or **D**. C

**A** $7e \times 7e \times 7e$ **B** $7e + 7e + 7e$ **C** $7 \times e + e + e$ **D** $7 \times e \times e \times e$

## Investigation

### The laws of arithmetic

We can use algebraic symbols to describe general laws about arithmetic.

For example, if we add zero to any number, the answer is still that number.

If we let $N$ stand for any number: $N + 0 = N$

In groups of 2 to 4, answer the following questions.

**1** Describe in words what each number rule below means.

**a** $N \times 1 = N$ **b** $N \times 0 = 0$

**c** $a - b \neq b - a$ **d** $a + b + c = b + a + c$

**e** $N - 0 = N$ **f** $ab = ba$

**2** Write each rule algebraically using variables.

**a** Any number divided by 1 equals itself.

**b** Multiplying a number by 8 is the same as doubling it 3 times.

**c** Any 3 numbers can be multiplied together in any order.

**d** Any number added to itself is the same as multiplying that number by 2.

**e** Any number subtracted from itself equals 0.

**f** Any number multiplied by its reciprocal equals 1.

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**3** Is each equation true or false? Test your answer by substituting a number for each variable and checking.

**a** $a \div b = b \div a$ **b** $N \div N = 1$

**c** $4a - a = 4$ **d** $a$ is a factor of $a$

**e** If $N$ is even, then $N + 3$ is odd **f** $\frac{1}{2}N = N - 2$

**g** $a + (-a) = 2a$ **h** $0 \div N = 0$

**i** $N \div 0 = 0$ **j** $N \div 1 = N$

**4** If $k$ is an odd number, what is an expression for:

**a** the previous odd number? **b** the next even number?



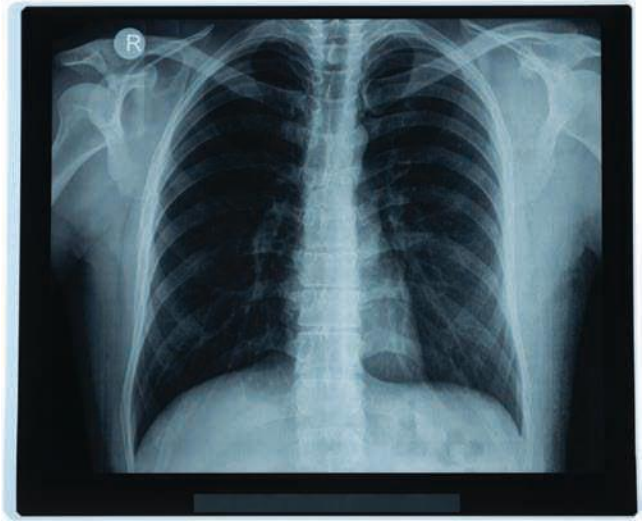
## Did you know?

### The X factor

In algebra the letter '***x***' is used quite frequently as a variable. During the 9th century, the Arabic word ***al-jabr*** ('algebra') described the process of moving variables from one side of an equation to the other to find the value of an unknown. The word for 'thing' or 'object' (the unknown number) in Arabic was ***shei***, translated into Greek as ***xei***, and shortened to ***x***. ***Xenos*** is also the Greek word for unknown, stranger or foreigner, which could explain why early European mathematicians used the letter ***x*** in algebra.

One further explanation is that ancient scholars could only write using feathers dipped in ink. The two easiest letters to write were ***x*** and ***y***. Mathematicians needed variables they could write quickly without mixing them up. As a result, $x$ and $y$ are still the two most commonly used variables in algebra.

**Find examples of X-words that involve something unknown, such as X-ray, Brand X or X factor.**




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# 3.02 Creating algebraic expressions

**Video**
From words to algebraic expressions

**Skillsheet**
Algebraic expressions

**Worksheets**
Writing algebraic expressions

What does the symbol mean?

**Puzzles**
Generalised arithmetic

What's the expression?

## Example 4

If $x$ is a variable representing any number, write an algebraic expression for:

**a** 4 times the number

**b** the number divided by 6

**c** the previous consecutive number

**d** one-third of the number

**consecutive** means 'following in order': for example, 7, 8, 9 are consecutive numbers

### Solution

**a** $4 \times x = 4x$

**b** $x \div 6$ or $\frac{x}{6}$

**c** $x - 1$ The number before $x$ is $x - 1$ (for example, the number before 4 is $4 - 1 = 3$).

**d** $\frac{1}{3}x = \frac{x}{3}$

## Example 5

Think of a number, $y$, multiply it by 3 and subtract 7.
Write an algebraic expression for the answer.

### Solution

First, we multiply the number $y$ by 3 to get $y \times 3 = 3y$.

Then, we subtract 7 from our result to get the answer, $3y - 7$.

## Example 6

Describe the algebraic expression $\frac{a+b}{2}$ in words.

### Solution

$\frac{a+b}{2}$ is the sum of $a$ and $b$, divided by 2, or the average of $a$ and $b$.

**EXERCISE 3.02** ANSWERS ON P. 576

## Creating algebraic expressions U F R C

EXAMPLE 4

3.02

**1** If $n$ represents any number, write an algebraic expression for: R C

**a** 5 times the number
**b** the number plus 6
**c** the number divided by 10
**d** the next consecutive number
**e** half of the number
**f** 20 less than the number
**g** the number multiplied by itself
**h** 20 minus the number
**i** the previous consecutive number
**j** double the number
**k** 6 divided by the number
**l** the difference between the number and 8
**m** the product of the number and $(-4)$
**n** the square root of the number

**2** If $a$, $b$ and $c$ represent any 3 numbers, write an algebraic expression for: R C

**a** the sum of $a$, $b$ and $c$
**b** the product of $a$ and $b$
**c** the difference between $b$ and $c$, where $b$ is greater than $c$
**d** the product of $a$ and $c$, plus $b$
**e** 3 times $b$, plus $a$
**f** $c$ divided by $b$
**g** how much $a$ is greater than $b$
**h** 5 more than ($b$ times $c$)
**i** $c$ less than $b$
**j** $a$ times $b$ divided by $c$
**k** the square root of the sum of $a$ and $c$
**l** the product of $a$ and $b$, decreased by 7 times $c$

**3** A cricketer scored a total of $x$ runs in 6 innings. What was her average score? Select the correct answer **A, B, C** or **D**. R C

**A** $6x$
**B** $6 \div x$
**C** $x^6$
**D** $x \div 6$



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EXAMPLE 5

**4** Write an algebraic expression for the answer to each instruction. C

**a** Think of a number ($y$), multiply it by 3 and add 8

**b** Think of a number ($p$), divide it by 3 and then subtract 6

**c** Think of a number ($M$), multiply it by 4 and then add 27

**d** Think of a number ($k$), multiply it by 5 and then divide it by 2

**e** Think of a number ($x$), divide it by 6 and add 13

**f** Think of a number ($B$), halve it and then subtract 2

**g** Think of a number ($R$), divide it by 9 and then add 10

**h** Think of a number ($n$), multiply it by 7 and then multiply it by 3

EXAMPLE 6

**5** Describe each algebraic expression in words. C

**a** $x + y - z$ **b** $5K + L$ **c** $xy - 3$ **d** $\frac{W}{5}$

**e** $3x - 1$ **f** $\frac{y - x}{10}$ **g** $2N + 21$ **h** $7 - e$

**6** Write an expression for: R C

**a** the number of girls in a class if there are $b$ boys and a total of $t$ students

**b** the number of bottles of drink needed for a party if each bottle can be shared by 5 guests and there are $n$ guests

**c** the amount of change in dollars that Sandra receives if she paid for 3 hamburgers costing \$$p$ each with \$$y$ in cash

**d** the perimeter of an isosceles triangle with equal sides of length $r$ and an uneven side of length 9

**e** the cost of one egg if a carton of a dozen eggs costs \$$a$

**f** the perimeter of a square of length $x$

**g** the number of rows of chairs needed to place 160 chairs in the hall if each row contains $r$ chairs



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**h** the total cost of attending the pool for $x$ adults and $y$ children, where admission is \$5 per adult and \$3 per child

**i** the side length of an equilateral triangle with perimeter $5m - 1$

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# Substitution 3.03

The word **substitute** means replacing one thing with another. In algebra, substitution means to replace a variable with a value to evaluate an algebraic expression.

## Example 7

If $p = 3$ and $q = 14$, evaluate each expression.

**a** $2p + q$ **b** $\frac{pq+2}{4}$ **c** $5q - p^2$

### Solution

**a** $2p+q=2\times3+14$
$=20$

**b** $\frac{pq+2}{4}=\frac{3\times14+2}{4}$
$=\frac{44}{4}$
$=11$

**c** $5q-p^2=5\times14-3^2$
$=61$

Videos
Substitution 2
Substitution 1
The heartbeat formula
The heptathlon

Puzzles
Formula 1 game
Substitution game

Worksheet
Substitution

3.03

## Substituting into formulas

A **formula** is a general rule that is written as an algebraic equation and shows the relationship between variables. Solving mathematical problems often involves substituting into formulas.

## Example 8

The formula for the perimeter of a rectangle is $P = 2l + 2w$, where $P$ is the perimeter, $l$ is the length and $w$ is the width. Use this formula to calculate the perimeter of a rectangle with length 12 m and width 9.5 m.

### Solution

When length $l = 12$ and width $w = 9.5$,

$P=2l+2w$
$=2\times12+2\times9.5$
$=43$ m

**EXERCISE 3.03** ANSWERS ON P. 576

### Substitution U F PS

**1** If $x = 5$ and $y = 6$, what is the value of $2x - y$? Select the correct answer **A**, **B**, **C** or **D**.

**A** 19 **B** −2 **C** 4 **D** 16

EXAMPLE 7

Foundation Standard Complex

**2** If $a = 3$, $b = 5$ and $c = 2$, evaluate each expression.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $a + b$ | **b** | $4a$ | **c** | $c^2$ | **d** | $a + b + c$ |
| **e** | $abc$ | **f** | $3b - a$ | **g** | $c + a - b$ | **h** | $4a - 2c$ |
| **i** | $10 - b$ | **j** | $ab + c^2$ | **k** | $c - b$ | **l** | $\frac{2a}{c}$ |

**3** If $d = 10$, $e = -3$ and $f = 5$, evaluate each expression.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $def$ | **b** | $f^2$ | **c** | $e^3$ | **d** | $d \div f$ |
| **e** | $de \div 2$ | **f** | $d^2 - f^2$ | **g** | $7e$ | **h** | $3e + 2d - f$ |
| **i** | $f + d - e$ | **j** | $\frac{d}{2} - 3f$ | **k** | $6ef$ | **l** | $\frac{e+f}{4}$ |

**4** If $p = -3$, what is the value of $2p^2$? Select **A**, **B**, **C** or **D**.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **A** | 36 | **B** | 18 | **C** | $-18$ | **D** | $-36$ |

**5** If $y = -1$ and $z = 4$, evaluate each expression.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $3 - 4z$ | **b** | $-3y^2$ | **c** | $5(z - 2)$ | **d** | $y(z + 1)$ |
| **e** | $\frac{y}{z}$ | **f** | $\frac{z}{4}$ | **g** | $\frac{y+z}{9}$ | **h** | $\frac{10}{3+y}$ |

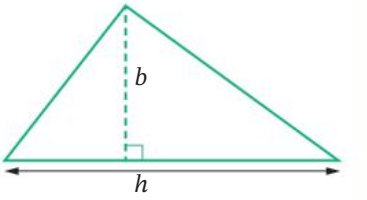
**6** If $n = 6$, which of the following expressions is equal to 22? Select **A**, **B**, **C** or **D**.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **A** | $\frac{n+4}{2}$ | **B** | $\frac{2n+4}{2}$ | **C** | $\frac{n^2}{2}+4$ | **D** | $\frac{n^2+4}{2}$ |

EXAMPLE 8

**7** The formula for the area of a triangle is $A = \frac{1}{2}bh$, where $A$ is the area of the triangle, $b$ is the length of the base and $h$ is the height.

Find the area of a triangle with base length 12 m and height 7 m.




**8** A plumber charges according to the formula $C = 42h + 65$, where $C$ is the charge in dollars and $h$ is the number of hours worked. How much does the plumber charge for working for $3\frac{1}{2}$ hours?



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**9** The formula for converting Celsius temperatures to Fahrenheit temperatures is $F = \frac{9}{5}C + 32$, where $F$ is the temperature in Fahrenheit and $C$ is the temperature in Celsius. Use the formula to convert 26°C to Fahrenheit. PS

Foundation Standard Complex


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**10** The number of hours of sleep recommended for children has the formula $H = 17 - \frac{A}{2}$, where $H$ is the number of hours and $A$ is the age of the child.

Find the number of hours of sleep recommended for a 6-year-old. PS

**11** The formula for the cost of a party is $C = 45n + 500$, where $C$ is the cost in dollars and $n$ is the number of guests. Use this formula to calculate the cost of a party for 120 guests. PS

**12** The time ($t$ minutes) taken to travel to school depends on how far the student walks and how far the student travels on the bus. The formula for the time taken is $t = 11w + 4b$, where $w$ is the walking distance and $b$ is the distance travelled by bus.

**a** Find the time taken for each student to travel to school: PS

**i** Michael, who walks 1 km and goes 5 km on the bus

**ii** Vittoria, who walks 4 km and goes 3 km on the bus

**b** If school starts at 9 a.m., what is the latest time Wendy can leave home if she needs to walk 2 km and go on the bus for 7 km and still reach school on time?

## Technology

### Substitution

**1** Enter the variables and values shown into a new spreadsheet.

**2** In cell D1, enter the formula **=B4-B9** to evaluate $p - u$. You should get 9 because $4 - (-5) = 9$.

**3** In cell D2, enter the formula **=B2+B3** to evaluate $m + n$. You should get 22 because $10 + 12 = 22$.

**4** For each cell below, enter a formula to evaluate the given expression.

| | A | B |
|---|---|---|
| 1 | variable | value |
| 2 | m | 10 |
| 3 | n | 12 |
| 4 | p | 4 |
| 5 | q | 0 |
| 6 | r | 2.7 |
| 7 | s | -8 |
| 8 | t | 25 |
| 9 | u | -5 |

This means $m$ = **10.**

D3: $pr$

D4: $\frac{t}{u}$

D5: $n + t - p$

D6: $n - \frac{m}{u}$

D7: $\frac{n-m}{u}$

D8: $\frac{s}{p} \times r$

D9: $p^3$

D10: $(s + n) \times (p + r)$

D11: $t + pq + s$

D12: $\sqrt{t}$

D13: $mnpq$

D14: the average of $r$, $s$, $t$ and $u$

D15: $\frac{6m-n}{p+s}$

D16: $\sqrt{n+p}$

D17: $ts^2$

D18: $n^3 - r^2$

# 3.04 Adding and subtracting terms

**Skillsheet**
Algebra using diagrams

**Worksheet**
Collecting like terms

**Puzzles**
Algebra bingo game

Adding and subtracting like terms

**Interactive**
Expression exchange

Suppose an unknown number $p$ is represented by a cup  and another unknown number $m$ is represented by an envelope  . Now consider these 2 sums.

**a**



$p + m + p + m = 2p + 2m$

**b**



$m + p + m + p + m = 3m + 2p$

Note that we can add all the $p$ **terms** together, and we can add all the $m$ terms together, but we cannot add the $p$s and $m$s together because $p$ and $m$ represent different numbers. We cannot simplify the answers $2p + 2m$ and $3m + 2p$ because $p$ and $m$ are different.

These are examples of collecting **like terms**, terms that contain exactly the same variable(s), such as $4h$, $5h$ and $h$. Here are some more examples.

$3x$ and $-5x$ $\quad$ $4mw$ and $2mw$ $\quad$ $12m$ and $m$

$-6d$ and $10d$ $\quad$ $5ab$ and $2ba$ $\quad$ $xyz$ and $2yzx$

Note: $ab = ba$

Here are some examples of **unlike terms**, terms that have different variables.

$3x$ and $5m$ $\quad$ $4mw$ and $2mn$ $\quad$ $-6d$ and $10d^2$

## Adding and subtracting terms

When **adding and subtracting terms**, only **like terms** can be collected.

Like terms have exactly the same variables.

## Example 9

Select the like terms from each list below.

**a** $5x, -x, x^2, xy, \frac{1}{2}x, -8x, \frac{4x}{3}$ $\quad$ **b** $3ab, 5, a, ba, -2b, 4ab^2, -ab$

### Solution

**a** $5x, -x, \frac{1}{2}x, -8x, \frac{4x}{3}$ are like terms because they each have a single $x$ in them.

**b** $3ab$, $ba$ and $-ab$ are like terms because each contains $ab$.


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## Example 10

Simplify each expression by collecting like terms.

**a** $7d + 3e + 3d + e$

**b** $6m - 8 + 3m + 6$

**c** $6xy + 7x^2 - 3yx - 12x^2$

**d** $14 - 4t - 9t + 2$

### Solution

**a** $7d + 3e + 3d + e = 7d + 3d + 3e + e$

$= 10d + 4e$

Grouping like terms

**b** $6m - 8 + 3m + 6 = 6m + 3m - 8 + 6$

$= 9m - 2$

The + or − sign goes with the term after it.

**c** $6xy + 7x^2 - 3yx - 12x^2 = 6xy - 3xy + 7x^2 - 12x^2$

$= 3xy - 5x^2$

$3yx = 3xy$

**d** $14 - 4t - 9t + 2 = 14 + 2 - 4t - 9t$

$= 16 - 13t$

Video
Adding and subtracting terms

Presentation
Collecting like terms

3.04

## EXERCISE 3.04 ANSWERS ON P. 576

### Adding and subtracting terms U F R C

EXAMPLE 9

**1** Select the like terms from each list. C

**a** $4p, 2y, 2p, 5z, p$

**b** $m^2, m, n, 3m^2, mn$

**c** $5ac, 4a, 7ca, 5$

**d** $vw, 5v, 9v, w, v^2, 2wv$

**e** $p, 6pq, 2qp, 3pq, 7q$

**f** $2d, 3d^3, 7d^2, 9, 3d$

**g** $4mn, 3m, 4, 2nm, mn, 2n$

**h** $x^2y, 2x, 3y, 4x^2y$

**i** $7, 2a, 4b, 5ba, ab$

**2** What are the like terms in $3ab, 2a^2, 2b^2a, 2p^2, 2ba$?
Select the correct answer **A**, **B**, **C** or **D**. C

**A** $2a^2, 2p^2$

**B** $2a^2, 2b^2a, 2p^2, 2ba$

**C** $3ab, 2ba$

**D** $2b^2a, 2ba$

**3** Simplify each expression by collecting like terms.

**a** $4k + 7k$

**b** $5mn + 2nm$

**c** $xy + xy$

**d** $3abc + 4abc + 2bac$

**e** $3x^2 + 2x^2 + x^2$

**f** $ef^2 + 4ef^2$

**g** $8d - 3d$

**h** $12mk - 7mk$

**i** $12xy - 7yx - xy$

**j** $8de - 12de$

**k** $6k^2 - 3k^2$

**l** $4w^2 - w^2$

EXAMPLE 10

**4** Which expression is $2m + 4p - 5m - 2p$ simplified? Select **A**, **B**, **C** or **D**.

**A** $-mp$

**B** $7m + 6p$

**C** $-3m + 2p$

**D** $-3p$

Foundation | Standard | Complex

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**5** Simplify each expression.

| | | | | | |
|---|---|---|---|---|---|
| **a** | $5k - 2j + 6j$ | **b** | $10ab + 2 - ab$ | **c** | $12m^2 - 5m^2 + 2m$ |
| **d** | $12s^2 - 7st - 2s^2$ | **e** | $4mn + 7mn - mn$ | **f** | $14k + 3 - 5k$ |
| **g** | $y^2 + 2y^2 - y^2$ | **h** | $9d - 4e + 3d$ | **i** | $7 - 2x - 3$ |
| **j** | $15gb - 8gb + gb$ | **k** | $4 - 2q + 5$ | **l** | $5a + 6c - 4c$ |

**6** Simplify each expression.

| | | | | | |
|---|---|---|---|---|---|
| **a** | $4x - y - x - 2y$ | **b** | $7ab + 2bc - 3ab + bc$ | **c** | $4k^2 + 3 + 5k^2 + 8$ |
| **d** | $4bc - 8a + 2bc - a$ | **e** | $x^2 - 6x - x + 7$ | **f** | $p^2 + q^2 + 3p^2 - q^2$ |
| **g** | $3 + 4y + 8y + 11$ | **h** | $11r - 3s - 4r + 6s$ | **i** | $7g + 8b - b - g$ |
| **j** | $6x^2 - 4x - 9x + 6$ | **k** | $20a + 11m + 32a - 11m$ | **l** | $3q - 7 - 8q + 12q$ |
| **m** | $5 - 2y - 5 - 4y$ | **n** | $s^2 + 4s^2 + 12 - 5s^2$ | | |

**7** Which expression is $a^5 + a^5$ simplified? Select **A**, **B**, **C** or **D**.

**A** $a^{10}$ **B** $a^{25}$ **C** $2a^5$ **D** $2a^{10}$

**8** Write an algebraic expression involving a sum that simplifies to: R C

| | | | | | |
|---|---|---|---|---|---|
| **a** | $2x + 6$ | **b** | $6m + 12p$ | **c** | $x^2 - x$ |

**9** Write an algebraic expression involving a difference that simplifies to: R C

| | | | | | |
|---|---|---|---|---|---|
| **a** | $2x + 6$ | **b** | $6m + 12p$ | **c** | $x^2 - x$ |

# 3.05 Multiplying and dividing terms

**Worksheets**
Why aren't they the same?

Simplifying algebraic expressions

Perimeter and area

The associative law of multiplication tells us that to calculate the product $6 \times 4 \times 5 \times 3$ mentally, we can rearrange the numbers to pair convenient factors like this:

$$\begin{aligned} 6 \times 4 \times 5 \times 3 &= (4 \times 5) \times (6 \times 3) \\ &= 20 \times 18 \\ &= 360 \end{aligned}$$

We can do the same algebraically. As numbers can be multiplied in any order, we can multiply like *and* unlike terms.

**Video**
Multiplying terms

## Example 11

Simplify each product.

| | | | | | |
|---|---|---|---|---|---|
| **a** | $7b \times 3a$ | **b** | $8x \times (-2y)$ | **c** | $5d \times 6d \times 4$ |
| **d** | $9m^2 \times 4p$ | **e** | $4a \times 3b \times (-2a)$ | | |

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### Solution

**a** $7b \times 3a = 7 \times b \times 3 \times a$

$= 7 \times 3 \times a \times b$ Group numbers and variables separately

$= 21ab$

**b** $8x \times (-2y) = 8 \times (-2) \times x \times y$

$= -16xy$

**c** $5d \times 6d \times 4 = 5 \times 6 \times 4 \times d \times d$

$= 120d^2$

**d** $9m^2 \times 4p = 9 \times 4 \times m^2 \times p$

$= 36m^2p$

**e** $4a \times 3b \times (-2a) = 4 \times 3 \times (-2) \times a \times a \times b$

$= -24a^2b$

3.05

## Multiplying terms

When **multiplying algebraic terms**, multiply the numbers first, then multiply the variables. Write all variables in alphabetical order.

## Example 12

Use index notation to simplify each expression.

**a** $k \times k \times k \times k$

**b** $j \times j \times j \times j \times j \times j$

Video
Multiplying terms

### Solution

**a** $k \times k \times k \times k = k^4$

**b** $j \times j \times j \times j \times j \times j = j^6$

## Dividing terms

When **dividing algebraic terms**, divide the numbers first, then divide the variables. Write all variables in alphabetical order.

## Example 13

Simplify each quotient.

**a** $20p \div 5$

**b** $\frac{24y}{8y}$

**c** $\frac{-2m}{4mn}$

**d** $12a^2b \div 3a$

Video
Dividing terms

### Solution

**a** $20p \div 5 = \left(\frac{20}{5}\right)p$

$= 4p$

**b** $\frac{24y}{8y} = \left(\frac{24}{8}\right)\frac{y}{y}$

$= 3$

**c** $\frac{-2m}{4mn} = \left(\frac{-2}{4}\right)\frac{m}{mn}$

$= \left(-\frac{1}{2}\right)\frac{1}{n}$

$= -\frac{1}{2n}$

**d** $12a^2b \div 3a = \left(\frac{12}{3}\right)\frac{ab}{a}$

$= 4ab$

**EXERCISE 3.05** ANSWERS ON P. 576

## Multiplying and dividing terms UFRC

EXAMPLE 11

**1** Simplify $2p \times 5pq$. Select the correct answer **A**, **B**, **C** or **D**.

**A** $10pq$ **B** $7p^2q$ **C** $7pq$ **D** $10p^2q$

**2** Simplify each expression.

**a** $5 \times 3y$ **b** $8 \times 2m$ **c** $4k \times m$ **d** $4t \times 5$

**e** $2 \times f \times 6$ **f** $2x \times 8y$ **g** $10p \times 3m$ **h** $3b \times 6d$

**i** $3d \times 4d$ **j** $5 \times 4m \times 6n$ **k** $2y \times 3y \times 8$ **l** $3r \times 3 \times 3r$

**m** $2 \times 4a \times 9b$ **n** $x \times xy \times y$ **o** $2j \times 3k \times 7$ **p** $5rst \times 2rs$

**q** $3ac \times 6cd$ **r** $hjk \times hj \times jk$ **s** $4q^2 \times 6p^2$ **t** $2m \times 3n \times mn$

**3** Simplify each expression.

**a** $-4b \times 2d$ **b** $6m \times (-4m)$ **c** $-2 \times 5k \times 9l$ **d** $20 \times (-3p)$

**e** $9k \times (-7)$ **f** $-15b \times (-5b)$ **g** $-3m \times mn \times 4n$ **h** $6abc \times (-2ab)$

**i** $5r \times (-2r) \times (-4)$ **j** $9d \times (-4e) \times 6d$ **k** $-8x \times 9x \times (-2)$ **l** $5m \times 4 \times (-2m)$

**4** Which expression simplifies to $14a^2n^2$? Select **A**, **B**, **C** or **D**.

**A** $2a \times an \times 7$ **B** $2a \times 7n \times an$ **C** $7a^2 \times 2n$ **D** $7a^2n^2 \times 2a^2n^2$

EXAMPLE 12

**5** Simplify each expression.

**a** $y \times y \times y \times y$ **b** $a \times a \times a$ **c** $2 \times c \times c$

**d** $t \times t \times t \times t \times t$ **e** $q \times q \times q \times 7$ **f** $k \times k \times k \times l \times l$

**g** $m \times m \times m \times n \times n$ **h** $-5 \times p \times p \times p \times p \times k \times k \times k$

**i** $-3 \times y \times y \times y$ **j** $d \times d \times (-1) \times e \times e \times f$ **k** $q \times q \times r \times r \times r \times 4$

EXAMPLE 13

**6** Simplify $\frac{55m^2n}{5m}$. Select **A**, **B**, **C** or **D**.

**A** $11mn$ **B** $11m^2n$ **C** $50mn$ **D** $11n$

**7** Simplify each quotient.

**a** $12m \div 3$ **b** $36e \div 4$ **c** $21p \div (-3)$ **d** $\frac{81x}{9}$

**e** $\frac{60w}{15}$ **f** $\frac{8m}{m}$ **g** $-25k \div 5k$ **h** $30y \div 40y$

**i** $28pq \div 14q$ **j** $\frac{63a}{-9a}$ **k** $\frac{4pq}{20q}$ **l** $\frac{-56mn}{8m^2}$

**8** Simplify each quotient.

**a** $14m^2 \div 7$ **b** $6e^2 \div (-2e)$ **c** $-5p^2 \div p$ **d** $\frac{14x^2}{2x}$

**e** $\frac{6wm}{-3m}$ **f** $\frac{8m^2n}{2mn}$ **g** $32k^2y \div (-4k)$ **h** $9jk^2 \div 12jk^2$

**i** $\frac{15d^2e^2}{3de}$ **j** $\frac{3a^2b^2c}{3ab^2c}$ **k** $\frac{-24acb}{3bc}$ **l** $\frac{-10m^2n}{5mn}$

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**9** Simplify $12x \div (-3) \times 2x$. Select **A**, **B**, **C** or **D**.

**A** $-2x$ **B** $-2$ **C** $-8x$ **D** $-8x^2$

**10** Simplify each expression.

**a** $3p + 2 \times 4p$ **b** $7a^2 - 2a \times 3a$ **c** $6r \times 2y - y \times 10r$

**d** $5p - 4p \times 8$ **e** $8x \times 9x \div 12$ **f** $48m^2 \div 12m \times 2n$

**g** $6x + 4x \div 2$ **h** $24p - \frac{18p^2}{3p}$ **i** $6w^2 - \frac{10w^5}{5w^3}$

**j** $x^4 - \frac{x^7}{x^3}$ **k** $\frac{15mn^2p}{3m^2n^2}$ **l** $\frac{-4x^2y \times 3y}{2xy \times (-6xy)}$

**11** Write an algebraic expression involving a product that simplifies to: R C

**a** $18de$ **b** $-20a^2y$ **c** $5pr^2$

**12** Write an algebraic expression involving a quotient that simplifies to: R C

**a** $7m$ **b** $-3ab$ **c** $5pr^2$

3.05

Quiz
Mental skills 3A

## Mental skills 3A: Maths without calculators ANSWERS ON P. 577

### Simplifying multiplication by factorising

Sometimes, one big multiplication may be made into many simpler little multiplications if we break one or both terms into 2 'easier' factors. Then we use the property that numbers can be multiplied in any order.

**1** Study each example.

**a** $24 \times 8 = 6 \times 4 \times 2 \times 4$
$= 4 \times 4 \times 6 \times 2$
$= 16 \times 6 \times 2$
$= 96 \times 2$
$= 192$

Look for pairs of numbers that you can multiply easily and rearrange

**b** $15 \times 6 = 3 \times 5 \times 6$
$= 3 \times 30$
$= 90$

**c** $14 \times 36 = 7 \times 2 \times 6 \times 6$
$= 6 \times 7 \times 6 \times 2$
$= 42 \times 6 \times 2$
$= 252 \times 2$
$= 504$

**d** $30 \times 22 = 10 \times 3 \times 22$
$= 10 \times 66$
$= 660$

*Note*: For each question, there are many different ways of arriving at the correct result.

**2** Now evaluate each product by factorising first.

**a** $30 \times 24$ **b** $18 \times 27$ **c** $25 \times 33$ **d** $16 \times 8$

**e** $28 \times 20$ **f** $12 \times 18$ **g** $21 \times 9$ **h** $36 \times 12$

**i** $16 \times 35$ **j** $22 \times 28$ **k** $9 \times 15$ **l** $27 \times 25$

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## Investigation

### Equivalent expressions

State whether each statement below is true or false, then check by substituting a number for the variable and testing whether the left-hand side equals the right-hand side.

**a** $6 + 4m + 4 = 4m + 10$ **b** $5x + 3x = 8x^2$ **c** $4k - 7k = -3k$

**d** $7g + 8h - h - g = 7h + 6g$ **e** $3 + 7n + 13 + 2n = 25n$ **f** $3p \times 7q = 21pq$

**g** $21m \div 3m = 7m$ **h** $5y - 4y = y$ **i** $\dfrac{-16a}{4} = -4a$

# 3.06 Extension: The index laws

YEAR 9 CORE

**Puzzle** Indices puzzle

**Videos** Powers, indices and exponents

Simplifying with the index laws

Index laws 1

Index laws 2

In the previous chapter, *Working with numbers*, we discovered properties about powers, called **index laws**.

Here are some examples:

- $2^3 \times 2^4 = 2^{3+4} = 2^7$
- $7^5 \times 7 = 7^{5+1} = 7^6$
- $4^6 \div 4^2 = 4^{6-2} = 4^4$
- $\dfrac{10^3}{10^2} = 10^{3-2} = 10^1$
- $(3^4)^2 = 3^{4\times 2} = 3^8$
- $(8^5)^3 = 8^{5\times 3} = 8^{15}$
- $5^0 = 1$
- $8^0 = 1$

Now we can write these index laws algebraically and use them to simplify algebraic expressions.

## Index laws

When **multiplying terms with powers** of the same base, **add** the powers.

$$a^m \times a^n = a^{m+n}$$

When **dividing terms with powers** of the same base, **subtract** the powers.

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

When **raising a term with a power** to another power, **multiply** the powers.

$$(a^m)^n = a^{m\times n}$$

Any number raised to the **power of zero** equals **1**.

$$a^0 = 1$$

Now we can apply these index laws algebraically to variables.

## Example 14

Simplify each expression.

**a** $p^6 \times p^2$ **b** $3m^2n \times 7m^3n^5$ **c** $a^{12} \div a^4$ **d** $\dfrac{42y^6}{7y^2}$

**e** $(d^4)^5$ **f** $(4m^2)^3$ **g** $2k^0$ **h** $(2k)^0$

### Solution

**a** $p^6 \times p^2 = p^{6+2}$ Adding the powers
$= p^8$

**b** $3m^2n \times 7m^3n^5 = 3 \times 7 \times m^{2+3} \times n^{1+5}$
$= 21m^5n^6$

**c** $a^{12} \div a^4 = a^{12-4}$ Subtracting the powers
$= a^8$

**d** $\dfrac{42y^6}{7y^2} = \dfrac{42}{7} \times y^{6-2}$
$= 6y^4$

**e** $(d^4)^5 = d^{4\times5} = d^{20}$ Multiplying the powers

**f** $(4m^2)^3 = 4^3 \times (m^2)^3$ because $(4m^2)^3 = 4m^2 \times 4m^2 \times 4m^2$
$= 64m^{2\times3}$
$= 64m^6$

**g** $2k^0 = 2 \times k^0$ The zero power belongs to $k$ only
$= 2 \times 1$
$= 2$

**h** $(2k)^0 = 1$ The zero power belongs to $(2k)$

**EXERCISE 3.06** ANSWERS ON P. 577

## The index laws U F

**1** Simplify $a^2b \times a^4b^3$. Select the correct answer **A**, **B**, **C** or **D**.

**A** $a^8b^3$ **B** $a^8b^4$ **C** $a^6b^3$ **D** $a^6b^4$

EXAMPLE 14

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YEAR 9 CORE

**2** Simplify each product.

**a** $m^5 \times m^2$ **b** $k^8 \times k$ **c** $y^4 \times y^6$ **d** $x^7 \times x$

**e** $n^4 \times n^4$ **f** $q^2 \times q^8$ **g** $b^4 \times b^{10} \times b^2$ **h** $s^3 \times s^5 \times s^7$

**i** $6k^3 \times 7k^3$ **j** $5y^3 \times 4y$ **k** $d^4 \times 3d^5$ **l** $11e^3 \times 5e$

**m** $h^4n^2 \times h^6n^5$ **n** $10c^3d^4 \times (-3c^3d)$ **o** $f^5g^4 \times f^3g^7$ **p** $-5p^3q \times 2p^4q$

**3** Simplify $\frac{36d^6e^4}{12d^3e^4}$. Select **A**, **B**, **C** or **D**.

**A** $3d^3$ **B** $3d^3e$ **C** $3d^2e$ **D** $3d^2$

**4** Simplify each quotient.

**a** $y^6 \div y^5$ **b** $p^4 \div p$ **c** $\frac{q^5}{q^2}$ **d** $n^8 \div n^3$

**e** $\frac{k^{11}}{k^9}$ **f** $x^7 \div x^5$ **g** $r^8 \div r^8$ **h** $m^{12} \div m^5 \div m^2$

**i** $q^5 \times q^3 \div q^7$ **j** $45g^6 \div 5g^2$ **k** $18p^7 \div 6p^3$ **l** $a^{12} \div a^6 \div a$

**m** $66x^4y^2 \div 6x^3y$ **n** $\frac{x^4 \times x^2}{x^3}$ **o** $\frac{70k^5l^6}{10k^2l}$ **p** $\frac{15e^3f}{21ef^2}$

**5** Simplify $(a^4b^2)^3$. Select **A**, **B**, **C** or **D**.

**A** $a^7b^5$ **B** $a^{12}b^6$ **C** $a^{12}b^5$ **D** $a^7b^6$

**6** Simplify each expression.

**a** $(y^8)^2$ **b** $(x^2)^5$ **c** $(y^3)^3$ **d** $(m^2)^9$

**e** $(k^7)^3$ **f** $(t^{20})^2$ **g** $(3m)^2$ **h** $(2m^4)^3$

**i** $(xy)^3$ **j** $(p^3q)^4$ **k** $(3h^4)^4$ **l** $(k^3z)^5$

**m** $(2kj)^4$ **n** $(4c^3d)^2$ **o** $(5x^2y^8)^3$ **p** $(2m^4p^2)^6$

**7** Simplify each expression.

**a** $y^0$ **b** $(xy)^0$ **c** $(2r - r)^0$ **d** $5u^0$

**e** $\left(\frac{x}{2}\right)^0$ **f** $(2k^7)^0$ **g** $m^8 \div m^8$ **h** $h^2 \div (h^0)^3$

**i** $(b^3)^2 \div 2b^0$ **j** $(3x)^0 \times (4x)^2$ **k** $(d^0)^5 \times (d^8)^0$ **l** $26x^4y^3 \div 2x^3y^3$

**m** $4m^0 + 3m - 3k^0$ **n** $\frac{27b^3d}{9bd}$ **o** $(8h)^2 \times 2h^0 \div 4h^2$ **p** $\frac{6ef^4}{12ef^4}$

**8** Simplify each expression.

**a** $\frac{3pq}{9p^2q^2}$ **b** $\frac{2m^3n^3}{m^2n^2}$ **c** $\frac{8xy}{12xy^2}$

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# Expanding expressions 3.07

The **distributive law** says that you can multiply by a number by splitting it into the sum or difference of 2 other numbers. Look at these 2 examples.

**a** $23 \times 12 = 23 \times (10 + 2)$

$= (10 + 2) + (10 + 2) + (10 + 2) + \ldots$ 23 times

$= 10 + 10 + 10 + \ldots + 2 + 2 + 2 + \ldots$ 23 times each

$= (23 \times 10) + (23 \times 2)$

$= 230 + 46$

$= 276$

so $23 \times (10 + 2) = 23 \times 10 + 23 \times 2$

**b** $35 \times 9 = 35 \times (10 - 1)$

$= (10 - 1) + (10 - 1) + (10 - 1) + \ldots$ 35 times

$= 10 + 10 + 10 + \ldots - 1 - 1 - 1 -$ 35 times each

$= (35 \times 10) - (35 \times 1)$

$= 350 - 35$

$= 315$

so $35 \times (10 - 1) = 35 \times 10 - 35 \times 1$


**Skillsheet** Algebra using diagrams

3.07

**Worksheet** Expanding brackets

**Puzzles** Expandominoes

Expanding

Expand the brackets

Expand negative brackets

**Interactive** Area model Algebra


## The distributive law for expanding algebraic expressions

If $a$, $b$ and $c$ stand for numbers, then: $a(b + c)$ means $a \times (b + c)$

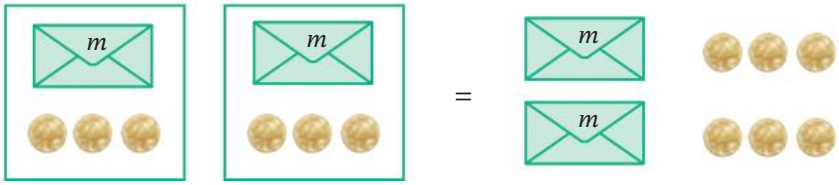
$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

We can apply the distributive law to algebraic expressions. Suppose an unknown number $m$ is represented by an envelope 

that contains $m$ coins.

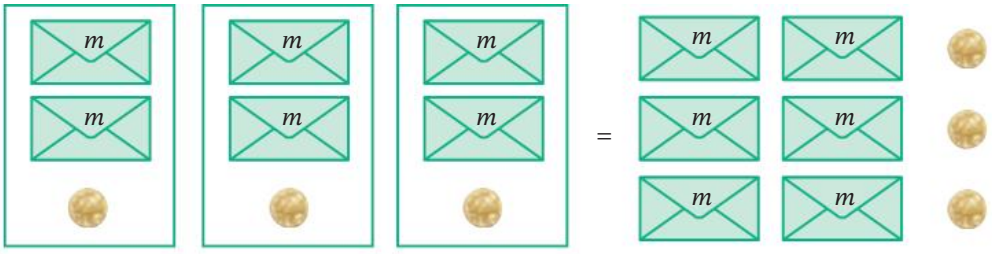
Then $2(m + 3)$ or '2 lots of $(m + 3)$', can be represented by this diagram.




It can be seen that $2(m + 3) = 2 \times m + 2 \times 3 = 2m + 6$.

The number outside the brackets is *multiplied* by each term inside.

Here is another example for $3(2m + 1)$.




It can be seen that $3(2m + 1) = 3 \times 2m + 3 \times 1 = 6m + 3$.

Removing the brackets in an algebraic expression by multiplying each term inside by the term outside is called **expanding** the expression.

Video
Expanding expressions

## Example 15

Expand each expression.

**a** $7(p + 4)$ **b** $3(x - 5)$ **c** $5(2m - 7)$

**d** $-2(a + 3)$ **e** $-(4d - 2)$ **f** $w(9w - 1)$

### Solution

**a** $7(p + 4) = 7 \times p + 7 \times 4$
$= 7p + 28$

7 is multiplied by each term inside the brackets separately.

**b** $3(x - 5) = 3 \times x - 3 \times 5$
$= 3x - 15$

**c** $5(2m - 7) = 5 \times 2m - 5 \times 7$
$= 10m - 35$

**d** $-2(a + 3) = -2 \times a + (-2) \times 3$
$= -2a + (-6)$
$= -2a - 6$

$-2$ is multiplied by each term inside the brackets separately.

**e** $-(4d - 2) = -1(4d - 2)$
$= -1 \times 4d - (-1) \times 2$
$= -4d + 2$

$-(4d - 2)$ is the same as $-1 \times (4d - 2)$.

**f** $w(9w - 1) = w \times 9w - w \times 1$
$= 9w^2 - w$


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## Example 16

Expand and simplify each expression.

**a** $2(x + 5) + 6(x - 4)$ **b** $9k - 4(k - 3)$ **c** $3p - (p + 6)$

### Solution

**a** $2(x+5)+6(x-4) = 2\times x+2\times 5+6\times x+6\times(-4)$ Expanding

$= 2x+10+6x-24$

$= 8x - 14$ Collecting like terms

**b** $9k-4(k-3) = 9k-4\times k-4\times(-3)$

$= 9k-4k+12$

$= 5k+12$

**c** $3p-(p+6) = 3p-1(p+6)$

$= 3p-1\times p-1\times 6$

$= 3p-p-6$

$= 2p-6$

Video
Expanding expressions 2

3.07

EXAMPLE 15

**EXERCISE 3.07** ANSWERS ON P. 577

## Expanding expressions U F R C

**1** Expand each expression.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $5(m + 3)$ | **b** | $8(m - 2)$ | **c** | $2(j + 10)$ | **d** | $3(a + 2)$ |
| **e** | $4(h - 7)$ | **f** | $6(x + y)$ | **g** | $9(4 - b)$ | **h** | $11(y - 6)$ |
| **i** | $4(2k + 5)$ | **j** | $3(5m - 2)$ | **k** | $6(3x + 4)$ | **l** | $10(2a + 5b)$ |

**2** Expand each expression.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $-3(n + 4)$ | **b** | $-4(v - 5)$ | **c** | $-7(d + 3)$ | **d** | $-10(k - 2)$ |
| **e** | $-2(d + e)$ | **f** | $-5(5 - z)$ | **g** | $-6(h + 1)$ | **h** | $-(m - 1)$ |
| **i** | $-9(5 + 2x)$ | **j** | $-(2f + 3)$ | **k** | $-3(u - v)$ | **l** | $-8(4 - k)$ |

**3** Expand $-3(a - 2b)$. Select the correct answer **A**, **B**, **C** or **D**.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **A** | $-3a - 2b$ | **B** | $-3a + 6b$ | **C** | $-3a - 6b$ | **D** | $-3a + 5b$ |

**4** Expand each expression.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $x(y + 6)$ | **b** | $p(q - 2)$ | **c** | $a(a + 4)$ | **d** | $b(3b - 1)$ |
| **e** | $d(d + 3e)$ | **f** | $2(4m + 5n)$ | **g** | $5k(3k - 2)$ | **h** | $7j(3j + ik)$ |
| **i** | $x(4x + y)$ | **j** | $-2(4p - 3q)$ | **k** | $k(k - 12)$ | **l** | $-m(4 - m)$ |
| **m** | $4n(2n + 5m)$ | **n** | $2x(1 - 7x)$ | **o** | $-f(2f + 4)$ | **p** | $-(5j - 7k)$ |

**5** Expand $3y(y - 4)$. Select **A**, **B**, **C** or **D**.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **A** | $3y + 12$ | **B** | $3y^2 - 7y$ | **C** | $3y^2 - 12y$ | **D** | $3y + 7$ |

**6** Explain the difference between $2a + 1$ and $2(a + 1)$. R C

**7** Scott says that, to find the perimeter of a rectangle, you double its length, double its width, and add the answers together. Thomas says that you add the length and width first, then double the answer. Who is correct? Can you write each of their answers algebraically? R C

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EXAMPLE 16

**8** Expand $12 - 8(x + 2)$. Select **A**, **B**, **C** or **D**.

**A** $4(x + 2)$ **B** $4x + 8$ **C** $-8x + 14$ **D** $-8x - 4$

**9** Expand and simplify each expression.

**a** $6(a + 2) + 14$ **b** $4k + 2(k + 3)$ **c** $3(q - 3m) + 5m$

**d** $8 + 6(p - 5)$ **e** $18 - (2y + 14)$ **f** $31 - 7(t + 2)$

**g** $6(2h - 1) + 11$ **h** $4x - 4(3x - 2)$ **i** $2(9n - 2) + 8$

**10** Check that $8(2b - 3) = 16b - 24$ by substituting any value for $b$ and showing that the answers to both sides are equal. R

**11** Expand and simplify each expression.

**a** $3(a + 4) + 2(a + 1)$ **b** $2(x + 5) + 6(x - 1)$ **c** $7(4 + k) + 5(2k - 2)$

**d** $5(2t + 1) + 3(4t + 2)$ **e** $3(n + 5) - 2(n + 4)$ **f** $10(d + 2) - 6(d + 3)$

**g** $8(4y + 1) - 5(2y - 3)$ **h** $3(p - 4) - 5(p - 1)$ **i** $m(m + 1) + 3(m + 1)$

**j** $4(d + e) + 3(d - e)$ **k** $2f(3f + 4) - 4(2f - 1)$ **l** $5e(1 - 7e) - e(3e + 4)$

**12** Copy and complete each equation. R

**a** $2(_____) = 2x + 10$ **b** $4(_____) = 4r - 12$ **c** $3(_____) = 24k - 9$

**d** $5(_____) = 10a + 35$ **e** $-2(_____) = -2y - 18$ **f** $-3(_____) = -6t + 21$

# 3.08 Factorising algebraic terms

**Skillsheets**
Algebra using diagrams

Factorising using diagrams

The **factors** of a number are those numbers that divide into it evenly.
For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.

Algebraic terms also have factors. The **factors** of an algebraic term are those numbers or terms that divide into it evenly. For example, $10xy = 2 \times 5 \times x \times y$, so *some* of the factors of $10xy$ are 2, 5, $x$, $y$, 10, $2y$ and $10x$. Often there are too many to list them all easily.

## Example 17

List 4 factors of $40a^2b$.

### Solution

There are many ways of multiplying terms to get $40a^2b$, such as:

$8 \times 5 \times a \times a \times b$

$4 \times 10 \times a^2 \times b$

$40 \times a^2 \times b$

So 4 factors of $40a^2b$ are: 4, 5, $a$ and $a^2b$. (*Note*: It is impractical to list them all.)

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# The highest common factor of algebraic terms

The **highest common factor (HCF)** (or **greatest common divisor (GCD)**) of 2 numbers is the largest number that divides into both numbers evenly. For example, the HCF of 18 and 30 is 6.

The **highest common factor** of 2 algebraic terms is the largest term that divides into both terms evenly. For example, the HCF of $18ab$ and $30b$ is $6b$.

## Highest common factor

To find the **highest common factor (HCF)** of 2 or more algebraic terms:

- find the HCF of the numbers
- find the HCF of the variables
- multiply them together to make the highest common algebraic factor.

## Example 18

Find the highest common factor of each pair of terms.

**a** $16x$ and $40xy$ **b** $12r$ and $28$

### Solution

**a** First, find the HCF of the numbers.

The factors of 16 are **1**, **2**, **4**, **8**, 16.

The factors of 40 are **1**, **2**, **4**, 5, **8**, 10, 20, 40.

So the HCF of 16 and 40 is **8**.

Then find the HCF of the variables:

The HCF of $x$ and $xy$ is $x$.

$\therefore$ The HCF of $16x$ and $40xy$ is $8 \times x = 8x$.

**b** First, find the HCF of the numbers:

The factors of 12 are **1**, **2**, 3, **4**, 6, 12.

The factors of 28 are **1**, **2**, **4**, 7, 14, 28.

So the HCF of 12 and 28 is **4**.

Then find the HCF of the variables:

The second term (28) has no variable so the HCF of the variables is 1.

$\therefore$ The HCF of $12r$ and 28 is $4 \times 1 = 4$.

**EXERCISE 3.08** ANSWERS ON P. 577

## Factorising algebraic terms U F R

**1** List the factors of each number.

**a** 15 **b** 8 **c** 17

**d** 25 **e** 55 **f** 42

**2** Find the highest common factor of each pair of numbers. R

**a** 9 and 6 **b** 6 and 14 **c** 18 and 24

**d** 16 and 12 **e** 33 and 22 **f** 15 and 25

**g** 21 and 9 **h** 45 and 30 **i** 16 and 48

EXAMPLE 17

**3** List 4 factors of each term.

**a** $5m$ **b** $7pq$ **c** $10x$ **d** $18d$

**e** $4k^2$ **f** $21bc$ **g** $32x^2y$ **h** $15ab$

**4** Which term is **NOT** a factor of $36m^2n$? Select the correct answer **A**, **B**, **C** or **D**.

**A** $4m$ **B** $9n$ **C** $8mn$ **D** $12m^2n$

EXAMPLE 18

**5** Find the highest common factor of each pair of terms. R

**a** $3xy$ and $12x$ **b** $4j$ and $24j$ **c** $6x$ and $4ax$

**d** $21kj$ and $14k$ **e** 5 and $10p$ **f** $16m$ and $12m^2$

**g** $48cd$ and $32bc$ **h** $de$ and $de^2$ **i** 9 and $27x^2$

**j** $3m$ and $6n$ **k** $22m$ and $4mn$ **l** $16y$ and $4y$

**m** $a^2x$ and $ay$ **n** $64j$ and 32 **o** $15w^2$ and $20wy$

**p** $22x^2y$ and $36xy^2$ **q** $cd$ and $ad$ **r** $36t$ and $12ty^2$

**6** Which term is the highest common factor of $8x^2$ and $12xy$? Select **A**, **B**, **C** or **D**.

**A** 4 **B** $2x$ **C** $4x$ **D** $24x^2y$

# 3.09 Factorising expressions

**Skillsheets**
Algebra using diagrams

Factorising using diagrams

Expanding an algebraic expression means removing its brackets by multiplying terms.

**Factorising** is the opposite of expanding. To factorise an algebraic expression, find the HCF of all terms and insert brackets.

expanding

$3(x + 6) = 3x + 18$

factorising

## Factorising algebraic expressions

**Puzzle**
Factorising

- Find the HCF of all terms and write it outside the brackets
- Divide each term by the HCF and write them inside the brackets

$ab + ac = a(b + c)$

$ab - ac = a(b - c)$

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To check that the answer is correct, expand it.

## Example 19

Factorise each expression.

**a** $2x + 6$ **b** $7 - 7y$ **c** $3ab - 5b$ **d** $r^2 + 7rs$

### Solution

**a** $2x + 6 = 2 \times x + 2 \times 3$
$= 2(x + 3)$

HCF of $2x$ and 6 is 2.
Think: $2 \times ___ = 2x$, $2 \times ___ = 6$
Check by expanding: $2(x + 3) = 2x + 6$

**b** $7 - 7y = 7 \times 1 - 7 \times y$
$= 7(1 - y)$

HCF of 7 and $7y$ is 7.
Think: $7 \times ___ = 7$, $7 \times ___ = 7y$
Check by expanding: $7(1 - y) = 7 - 7y$

**c** $3ab - 5b = b \times 3a - b \times 5$
$= b(3a - 5)$

HCF of $3ab$ and $5b$ is $b$.
Think: $b \times ___ = 3ab$, $b \times ___ = 5b$
Check by expanding: $b(3a - 5) = 3ab - 5b$

**d** $r^2 + 7rs = r \times r + r \times 7s$
$= r(r + 7s)$

HCF of $r^2$ and $7rs$ is $r$.
Think: $r \times ___ = r^2$, $r \times ___ = 7rs$
Check by expanding: $r(r + 7s) = r^2 + 7rs$

Video
Factorising expressions

3.09

**EXERCISE 3.09** ANSWERS ON P. 577

## Factorising expressions U F R C

**1** Find the highest common factor of the terms in each expression. R C

**a** $10a + 2b$ **b** $4 - 12x$ **c** $ax + xy$ **d** $y^2 - y$

**2** Copy and complete each factorisation. R C

**a** $2x + 4 = 2(_____)$
**b** $4p - 16 = 4(_____)$
**c** $12k - 15 = 3(_____)$
**d** $5j - 20k = 5(_____)$
**e** $3x^2 + 3y^2 = 3(_____)$
**f** $5x - 15 = _____(x - 3)$
**g** $12x - 6 = _____(2x - 1)$
**h** $2a + 8b = _____(a + 4b)$
**i** $4pq + 10 = _____(2pq + 5)$
**j** $5 + 15x = 5(_____)$
**k** $xy + xz = x(_____)$
**l** $ab - ac = a(_____)$
**m** $pq - p = p(_____)$
**n** $cd + bc = c(_____)$
**o** $a^2 + 2a = a(_____)$
**p** $5hk - 8h = _____(5k - 8)$
**q** $24mn + 11n = _____(24m + 11)$
**r** $xy + y^2 = _____(x + y)$
**s** $3k^2 + 4jk = _____(3k + 4j)$
**t** $ef - 2f^2 = _____(e - 2f)$

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EXAMPLE 19

**3** Factorise each expression. R C

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $4x + 8$ | **b** | $3m + 6$ | **c** | $5a - 10$ | **d** | $15m - 10a$ |
| **e** | $8x - 4y$ | **f** | $12m - 16w$ | **g** | $hk - h$ | **h** | $2xy - xm$ |
| **i** | $12p + 5pr$ | **j** | $x^2 + 5x$ | **k** | $4m - m^2$ | **l** | $y^2 - 11y$ |

**4** Factorise $16k^2 + 18k$. Select the correct answer **A**, **B**, **C** or **D**.

**A** $2(8k^2 + 9k)$ **B** $k(16k + 18)$ **C** $2k(8k + 9)$ **D** $4k(4k + 9)$

**5** Copy and complete each factorisation. R C

| | | | |
|---|---|---|---|
| **a** | $2am + 2an = 2a($_____$)$ | **b** | $6xy - 3x = 3x($_____$)$ |
| **c** | $4ap + 2a = 2a($_____$)$ | **d** | $5mn - 5mp = 5m($_____$)$ |
| **e** | $3xy - 12x = 3x($_____$)$ | **f** | $2pq + 2pr = 2p($_____$)$ |
| **g** | $14x - 2xy = 2x($_____$)$ | **h** | $20kj + 5j = 5j($_____$)$ |
| **i** | $9xy - 12yz = 3y($_____$)$ | | |

**6** Factorise $3d^2 + 21d$. Select **A**, **B**, **C** or **D**.

**A** $3(d^2 + 7d)$ **B** $d(3d + 21)$ **C** $d(3d + 7)$ **D** $3d(d + 7)$

**7** Factorise each expression. R C

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $7ab + 14a$ | **b** | $2x - 6xy$ | **c** | $3m - 9mw$ | **d** | $10xy - 25xw$ |
| **e** | $8am - 12mp$ | **f** | $9xy + 6my$ | **g** | $3x + 9xy$ | **h** | $ab + abc$ |
| **i** | $xyz + xya$ | **j** | $18m - 24mn$ | **k** | $2ac + 3abc$ | **l** | $pqr + mpq$ |
| **m** | $rs + 7r$ | **n** | $4s^2 + 16$ | **o** | $p^2 - 2p$ | **p** | $6k^3 - 2k$ |
| **q** | $3j + j^2$ | **r** | $7n - n^2$ | **s** | $7e + 21e^2$ | **t** | $6q - 12q^2$ |
| **u** | $a^4 + a^2$ | **v** | $x^2y + xy^2$ | **w** | $3m^2n + 9m$ | **x** | $12a^2b^2c^2 + 3abc$ |

# 3.10 Factorising with negative terms

**Skillsheets**
Algebra using diagrams

Factorising using diagrams

**Worksheets**
Algebra review

Algebraic expressions review

## Factorising with negative terms

When factorising an algebraic expression whose first term is *negative*, include the negative sign in the factor.

$$-ab - ac = -a(b + c)$$
$$-ab + ac = -a(b - c)$$

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## Example 20

Factorise each expression.

**a** $-5x - 10$ **b** $-9n + 21$ **c** $-gh - h$ **d** $-mn + n^2$

### Solution

**a** $-5x - 10 = -5 \times x + (-5) \times 2$
$= -5(x + 2)$

HCF of $-5x$ and $-10$ is $-5$.
Think: $-5 \times ___ = -5x$, $-5 \times ___ = -10$
Check by expanding: $-5(x + 2) = -5x - 10$

**b** $-9n + 21 = -3 \times 3n + (-3) \times (-7)$
$= -3(3n + (-7))$
$= -3(3n - 7)$

HCF of $-9n$ and $21$ is $-3$.
Think: $-3 \times ___ = -9n$, $-3 \times ___ = 21$
Check by expanding: $-3(3n - 7) = -9n + 21$

**c** $-gh - h = -h \times g + (-h) \times 1$
$= -h(g + 1)$

HCF of $-gh$ and $-h$ is $-h$.
Think: $-h \times ___ = -gh$, $-h \times ___ = -h$
Check by expanding: $-h(g + 1) = -gh - h$

**d** $-mn + n^2 = -n \times m + (-n) \times (-n)$
$= -n(m + (-n))$
$= -n(m - n)$

HCF of $-mn$ and $n^2$ is $-n$.
Think: $-n \times ___ = -mn$, $-n \times ___ = n^2$
Check by expanding: $-n(m - n) = -mn + n^2$



Video
Factorising expressions

Puzzles
Expandominoes
Factorising puzzle
Factorising with negative terms

**EXERCISE 3.10** ANSWERS ON P. 577

## Factorising with negative terms U F R C

**1** Copy and complete each factorisation. R C

**a** $-6k - 6 = -6(_____)$ **b** $-mn - mp = -m(_____)$
**c** $-5r + 10 = -5(_____)$ **d** $-3k + 27 = -3(_____)$
**e** $-ac + ad = _____(c - d)$ **f** $-y^2 - 7y = _____(y + 7)$
**g** $-12y - 30 = _____(2y + 5)$ **h** $-4f + f^2 = _____(4 - f)$

**2** Factorise each expression. R C

EXAMPLE 20

**a** $-4x + 12$ **b** $-3m - 9$ **c** $-10x + 20$ **d** $-15x + 10m$
**e** $-6x - 8m$ **f** $-5x - 30$ **g** $-a - ab$ **h** $-4xyz + 8abc$
**i** $-20 + 4x$ **j** $-dx - dy$ **k** $-2m + 5mp$ **l** $-24x - 16y$

**3** Factorise $-14wy - 28wz$. Select the correct answer **A**, **B**, **C** or **D**.

**A** $-2(7wy + 14wz)$ **B** $-14w(y + 2z)$ **C** $14w(-y - 2z)$ **D** $-2w(7y + 14z)$

**4** Factorise each expression. R C

**a** $-16xy - 24xk$ **b** $-20x + 12xy$ **c** $-8mw + 12m$
**d** $5e - 15ef$ **e** $-ab - abc$ **f** $xya + xyw$
**g** $-12c - 18cw$ **h** $-2rt + 7rst$ **i** $20fh + 5h$
**j** $-19bw - 38w$ **k** $-abc + abcd$ **l** $-abc - abcd$

Foundation Standard Complex

**5** **a** Substitute $w = 2$ into the expressions $-w^2 + 7w$ and $-w(w - 7)$.

**b** Is $-w^2 + 7w = -w(w - 7)$ for any value of $w$? R C

**6** Factorise $-ab + a^2$. Select **A**, **B**, **C** or **D**.

**A** $-a(b - a)$ **B** $a(-b - a)$ **C** $-a(b + a)$ **D** $-a(b + a^2)$

**7** Factorise each expression. R C

**a** $a^4bc - a^3b^2c$ **b** $2\pi r^2 + 2\pi rh$

Quiz
Mental skills 3B

## Mental skills 3B: Maths without calculators ANSWERS ON P. 578

### Multiplying by 9, 11, 99 or 101

We can use expanding when multiplying by a number near 10 or near 100.

**1** Study each example.

**a**
$$\begin{aligned} 25\times11 &= 25\times(10+1)\\ &= 25\times10+25\times1\\ &= 250+25\\ &= 275 \end{aligned}$$

**b**
$$\begin{aligned} 14\times9 &= 14\times(10-1)\\ &= 14\times10-14\times1\\ &= 140-14\\ &= 126 \end{aligned}$$

**c**
$$\begin{aligned} 32\times12 &= 32\times(10+2)\\ &= 32\times10+32\times2\\ &= 320+64\\ &= 384 \end{aligned}$$

**d**
$$\begin{aligned} 7\times99 &= 7\times(100-1)\\ &= 7\times100-7\times1\\ &= 700-7\\ &= 693 \end{aligned}$$

**e**
$$\begin{aligned} 27\times101 &= 27\times(100+1)\\ &= 27\times100+27\times1\\ &= 2700+27\\ &= 2727 \end{aligned}$$

**f**
$$\begin{aligned} 18\times8 &= 18\times(10-2)\\ &= 18\times10-18\times2\\ &= 180-36\\ &= 144 \end{aligned}$$

**2** Now evaluate each product.

**a** $16 \times 11$ **b** $33 \times 11$ **c** $29 \times 9$ **d** $45 \times 9$

**e** $62 \times 11$ **f** $7 \times 101$ **g** $18 \times 101$ **h** $36 \times 99$

**i** $19 \times 8$ **j** $45 \times 12$ **k** $21 \times 102$ **l** $6 \times 98$

# 3.11 Extension: Expanding binomial products

YEAR 9 CORE

$(x + 4)$ and $(x - 7)$ are called **binomial expressions** because they each have exactly 2 terms.

$(x + 3)(x - 1)$ is called a **binomial product** because it is a product of 2 binomial expressions.

binomial = '2 terms'

Foundation Standard Complex

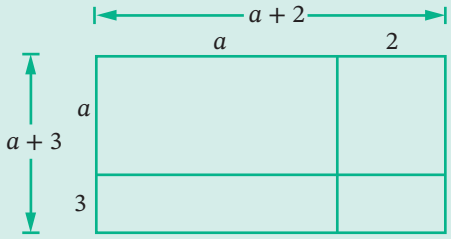

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## Example 21

Expand $(a + 2)(a + 3)$ using an area diagram.

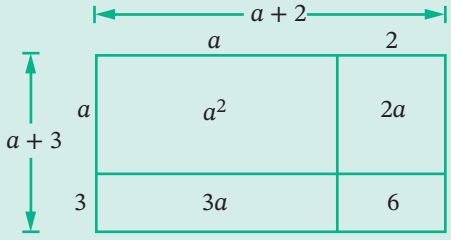
### Solution

Draw an area diagram (rectangle) with length $(a + 2)$ and width $(a + 3)$ and divide the diagram into 4 smaller rectangles.




Find the area of each smaller rectangle.

Expand $(a + 2)(a + 3)$ by adding the areas of the 4 rectangles.




$(a + 2)(a + 3) = a^2 + 2a + 3a + 6$ Expanding

$= a^2 + 5a + 6$ Simplifying by collecting like terms.

Another way of expanding binomial products is to use the distributive law. Each term in the first binomial is multiplied by each term in the second binomial to give **4 terms**, which are collected and simplified.

## Example 22

Expand each binomial product.

**a** $(x + 5)(x + 9)$ **b** $(k + 3)(k - 7)$ **c** $(x - 6)(4x + 2)$

**d** $(3t - 1)(2t - 5)$ **e** $(a - 6)^2$

### Solution

**a** $(x + 5)(x + 9) = x(x + 9) + 5(x + 9)$

$= x^2 + 9x + 5x + 45$ Expanding to make 4 terms.

$= x^2 + 14x + 45$ Simplifying.

**b** $(k + 3)(k - 7) = k(k - 7) + 3(k - 7)$

$= k^2 - 7k + 3k - 21$

$= k^2 - 4k - 21$

**c** $(x - 6)(4x + 2) = x(4x + 2) - 6(4x + 2)$

$= 4x^2 + 2x - 24x - 12$

$= 4x^2 - 22x - 12$

**d** $(3t - 1)(2t - 5) = 3t(2t - 5) - 1(2t - 5)$

$= 6t^2 - 15t - 2t + 5$

$= 6t^2 - 17t + 5$

**e** $(a - 6)^2 = (a - 6)(a - 6)$

$= a(a - 6) - 6(a - 6)$

$= a^2 - 6a - 6a + 36$

$= a^2 - 12a + 36$

YEAR 9 CORE

3.11

**Worksheets**
Area diagrams
Binomial products
Special products

**Puzzles**
Expanding brackets
Trinomials

**Technology**
Expanding binomials

**Spreadsheet**
Expanding binomials

**Videos**
Expanding binomial products 1
Expanding binomial products 2

YEAR 9 CORE

## EXERCISE 3.11 ANSWERS ON P. 578

# Expanding binomial products U F R C

EXAMPLE 21

**1** Expand each binomial product by copying and completing the area diagram.

**a** $(x + 2)(x + 7)$

| | $x$ | $2$ |
|---|---|---|
| $x$ | --- | --- |
| $7$ | --- | --- |

**b** $(x + 5)(x + 1)$

| | $x$ | $5$ |
|---|---|---|
| $x$ | --- | --- |
| $1$ | --- | --- |

**2** Expand $(p + 6)(p + 2)$. Select the correct answer **A**, **B**, **C** or **D**.

**A** $p^2 + 12$ **B** $p^2 + 6p + 12$ **C** $p^2 + 8p + 8$ **D** $p^2 + 8p + 12$

EXAMPLE 22

**3** Use the distributive law to expand each binomial product algebraically. R

**a** $(x + 1)(x + 4)$ **b** $(x + 6)(x + 3)$ **c** $(x + 5)(x + 4)$ **d** $(x + 2)(x + 2)$

**e** $(x + 2)(x + 6)$ **f** $(x + 10)(x + 3)$ **g** $(x + 8)(x + 1)$ **h** $(x + 7)(x + 7)$

**i** $(x - 1)(x + 4)$ **j** $(x - 3)(x + 2)$ **k** $(x + 1)(x - 4)$ **l** $(x - 5)(x + 6)$

**m** $(x - 4)(x - 5)$ **n** $(x + 9)(x - 3)$ **o** $(x - 5)(x - 3)$ **p** $(x - 7)(x - 7)$

**4** **a** Expand each binomial product.

**i** $(x + 3)(x - 3)$ **ii** $(x + 4)(x - 4)$ **iii** $(x - 2)(x + 2)$ **iv** $(x - 7)(x + 7)$

**b** Can you see a simpler way of expanding these products? Explain how you would expand $(x + 8)(x - 8)$. R C

**5** Expand each binomial product.

**a** $(x + 1)(2x + 4)$ **b** $(3x - 1)(x + 2)$ **c** $(2x + 3)(x + 1)$ **d** $(x - 3)(5x + 2)$

**e** $(4x - 3)(x - 2)$ **f** $(5x + 2)(2x - 3)$ **g** $(x - 3)(3x - 1)$ **h** $(4x + 7)(x - 3)$

**6** Expand each binomial square.

**a** $(a + 5)^2$ **b** $(x + 2)^2$ **c** $(y + 7)^2$

**d** $(x - 3)^2$ **e** $(2a + 1)^2$ **f** $(3a - 5)^2$

**7** Expand:

**a** $(a + b)^2$ **b** $(a + b)(a - b)$

**8** Expand $\left(4x + \frac{1}{2}\right)^2$. Select **A**, **B**, **C** or **D**.

**A** $16x^2 + 4x + \frac{1}{4}$ **B** $16x^2 + \frac{1}{4}$ **C** $4x^2 + 2x + \frac{1}{4}$ **D** $16x^2 + 2x + \frac{1}{4}$

Foundation Standard Complex


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# Extension: Perfect squares

3.12

Another name for **square number** is **perfect square**.

9 is a **perfect square** because it is $3^2$.

36 is another **perfect square** because it is $6^2$.

There is a special pattern when you expand a binomial multiplied by itself, for example, $(x + 5)(x + 5)$.

$(x + 5)(x + 5)$ is a perfect square because it is $(x + 5)^2$.

$(x + 5)^2$ is also called **the square of a binomial**, or a **perfect square**.

YEAR 9 PATH

**Puzzle**
Factorominoes

**Worksheet**
Special products



## Example 23

Expand each perfect square.

**a** $(x + 5)^2$ **b** $(3p + 2)^2$

### Solution

**a** $(x + 5)^2 = (x + 5)(x + 5)$

$= x(x + 5) + 5(x + 5)$

$= x^2 + 5x + 5x + 25$

$= x^2 + 10x + 25$

**b** $(3p + 2)^2 = (3p + 2)(3p + 2)$

$= 3p(3p + 2) + 2(3p + 2)$

$= 9p^2 + 6p + 6p + 4$

$= 9p^2 + 12p + 4$

Do you notice the pattern between the question and the answer for these perfect squares?

## Perfect squares

The formulas for expanding the perfect square of a binomial are:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

**Proof 1:** Using an area diagram

| | $a$ | $b$ |
|---|---|---|
| $a$ | $a^2$ | $ab$ |
| $b$ | $ab$ | $b^2$ |

$(a + b)^2 = a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

**Proof 2:** Using the distributive law

$(a + b)^2 = (a + b)(a + b)$

$= a(a + b) + b(a + b)$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

YEAR 9 PATH

Video
Special binomial products

## Example 24

Expand each perfect square.

**a** $(k + 4)^2$ **b** $(3y - 8)^2$

### Solution

**a** $(k + 4)^2 = k^2 + 2 \times k \times 4 + 4^2$
$= k^2 + 8k + 16$

**b** $(3y - 8)^2 = (3y)^2 - 2 \times (3y) \times 8 + 8^2$
$= 9y^2 - 48y + 64$

**EXERCISE 3.12** ANSWERS ON P. 578

## Perfect squares U F R

EXAMPLE 23

**1** Find the missing number in each expansion.

**a** $(x + 6)^2 = x^2 + 12x + \square$ **b** $(x - 3)^2 = x^2 - \square x + 9$

**c** $(m - 4)^2 = m^2 - \square m + 16$

**2** Expand $(x - 9)^2$. Select the correct answer **A**, **B**, **C** or **D**.

**A** $x^2 - 9x + 81$ **B** $x^2 - 18x - 81$ **C** $x^2 + 18x + 81$ **D** $x^2 - 18x + 81$

EXAMPLE 24

**3** Expand each perfect square.

**a** $(y + 6)^2$ **b** $(x - 1)^2$ **c** $(h + 3)^2$ **d** $(n - 8)^2$

**e** $(a - 7)^2$ **f** $(x + 9)^2$ **g** $(y - 11)^2$ **h** $(d + 10)^2$

**4** Expand each perfect square.

**a** $(2a + 3)^2$ **b** $(3x + 5)^2$ **c** $(2a - 1)^2$ **d** $(4x + 3)^2$

**e** $(5y - 1)^2$ **f** $(2p + 7)^2$ **g** $(3y - 5)^2$ **h** $(3 - 3a)^2$

**5** Use the expansion of a perfect square to evaluate each square number without using a calculator. R

**a** $11^2 = (10 + 1)^2$ **b** $19^2 = (20 - 1)^2$ **c** $51^2 = (50 + 1)^2$

**d** $35^2 = (30 + 5)^2$ **e** $65^2 = (60 + 5)^2$ **f** $82^2 = (80 + 2)^2$

**6** Expand and simplify each expression.

**a** $(a + 3)^2 + 4$ **b** $(x - 5)^2 - 3x$ **c** $2 + (a - 1)^2$ **d** $(x - 3)^2 - 3(x - 3)$

Foundation Standard Complex


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# Extension: Difference of 2 squares

3.13

There is also a special pattern when you multiply the sum of 2 terms by the difference of those 2 terms, for example, $(x + 7)(x - 7)$.

$$\begin{aligned}(x + 3)(x - 3) &= x(x - 3) + 3(x - 3)\\ &= x^2 - 3x + 3x - 9\\ &= x^2 - 9\end{aligned}$$

Notice that the 2 middle terms cancel each other out and you are left with the 1st term squared minus the 2nd term squared. The answer is called the **difference of 2 squares**.

## Difference of 2 squares

$$(a + b)(a - b) = a^2 - b^2$$

## Example 25

Expand each expression.

**a** $(d + 3)(d - 3)$ **b** $(7x + 2)(7x - 2)$

### Solution

**a** $(d + 3)(d - 3) = d^2 - 3^2$
$= d^2 - 9$

**b** $(7x + 2)(7x - 2) = (7x)^2 - 2^2$
$= 49x^2 - 4$

**EXERCISE 3.13** ANSWERS ON P. 578

## Difference of 2 squares U F R C

**1** Expand each expression.

**a** $(x + 3)(x - 3)$ **b** $(y - 4)(y + 4)$ **c** $(x - 6)(x + 6)$ **d** $(d + 2)(d - 2)$
**e** $(x - 1)(x + 1)$ **f** $(n - 8)(n + 8)$ **g** $(k + 11)(k - 11)$ **h** $(a - 12)(a + 12)$

**2** Expand each expression.

**a** $(2x + 1)(2x - 1)$ **b** $(3y - 5)(3y + 5)$ **c** $(5x - 6)(5x + 6)$ **d** $(3d + 5)(3d - 5)$
**e** $(4x - 1)(4x + 1)$ **f** $(6n - 8)(6n + 8)$ **g** $(2k + 7)(2k - 7)$ **h** $(6a - 2)(6a + 2)$

YEAR 9 PATH

3.13

Worksheet
Special products

Puzzle
Enough time

Videos
Difference of 2 squares

Special binomial products

EXAMPLE 25

Foundation Standard Complex

YEAR 9 PATH

**3** Expand each expression.

**a** $(1 + x)(1 - x)$ **b** $(3 - m)(3 + m)$ **c** $(5 - 6a)(5 + 6a)$ **d** $(2d + 3)(2d - 3)$

**e** $(3x - m)(3x + m)$ **f** $(2n - 5k)(2n + 5k)$ **g** $(k + 7d)(k - 7d)$ **h** $(3a - 5c)(3a + 5c)$

**4** By expressing $101 \times 99$ as $(100 + 1)(100 - 1)$, use the difference of 2 squares to find the value of $101 \times 99$. R

**5** Use the method of question 4 to evaluate each product. R C

**a** $51 \times 49$ **b** $81 \times 79$ **c** $29 \times 31$ **d** $98 \times 102$

## Power plus ANSWERS ON P. 578

**1** Simplify each expression.

**a** $-7p - 8q + 3p - 5q + 4q$ **b** $-m - 3n - 5m - 7mn$

**c** $3k - 2j - 5 - 4k + 6j + 9$ **d** $a^2 + 2b^2 + 3b^2 - a^2$

**e** $4x^2 + 6x - x^2 - 12x$ **f** $xy + yz + xz + 4xy + 3xz + yz$

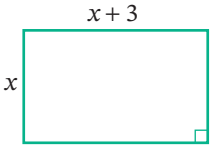
**g** $-d^2 + 7 + 6e^2 - 3d^2 - 2e^2 + 9$ **h** $5f^2 + 6f + 2 + 8f^2 - 1$

**2** For each shape, write an algebraic expression for:

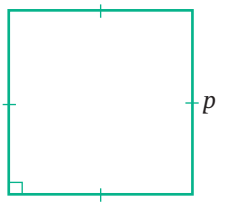
**i** its perimeter **ii** its area.

(Give your answer in simplest form.)

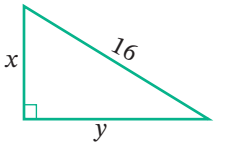
**a**



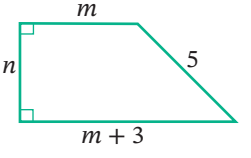

**b**



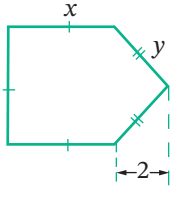

**c**



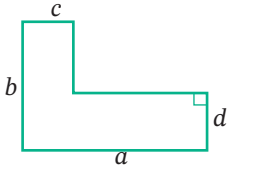

**d**




**e**




**f**




**3** Expand and simplify each expression.

**a** $x(x - 3) + 5(x - 3)$ **b** $m(a + 4) + m(a - 7)$

**c** $-3(y - 4) + 7(y - 3)$ **d** $4(2a + b + 3c) - 2(a + 3b - c)$

**e** $p(p - 3q + 2r) + 2p(4r - 3p - 2q)$ **f** $7k^2 + 3(k^2 - 3k - 1) - (6k^2 - 4k + 2)$

**g** $12 - 5(4f - 6g - 3)$ **h** $5(2x^2 - 5x + 3) - 4(3x^2 - 8x - 25)$

**i** $(x + 3)(x + 5)$ **j** $(p - 1)(p + 10)$

Foundation Standard Complex


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**4** Evaluate each expression if $p = 4$, $q = -6$, $r = 3$ and $t = -1$.

**a** $p + q$ **b** $pq$ **c** $r^2 + t^2$ **d** $q^2 - 6p + rt$

**e** $\frac{pq}{rt}$ **f** $6 - 2r + p$ **g** $2p + 3q + 4r - 5t$ **h** $\frac{p+4}{2}$

**i** $\frac{q+r}{p+t}$ **j** $\frac{r+7}{5} - \frac{q-t}{5}$ **k** $\frac{p+3}{10} + \frac{r-2}{2}$ **l** $q + pr - qt$

**5** Factorise each expression.

**a** $9pq + 5p - 2p^2$ **b** $44k - 33l - 11m$ **c** $2x - 7x^2 + 3xy$

**d** $x(y + 2) + 3(y + 2)$ **e** $a(b - 3) - 7(b - 3)$ **f** $4(k + 1) - j(k + 1)$

**6** Simplify each expression.

**a** $\frac{m}{2} + \frac{m}{2}$ **b** $\frac{7n}{10} - \frac{3n}{10}$ **c** $\frac{2k}{3} + \frac{k}{3}$ **d** $\frac{3d}{5} - \frac{d}{5}$

**e** $\frac{5f}{8} + \frac{7f}{8}$ **f** $\frac{3c}{10} + \frac{4c}{10}$ **g** $\frac{4}{x} + \frac{2}{x}$ **h** $\frac{7}{a} + \frac{5}{b}$

# CHAPTER 3 REVIEW

**Quiz**
Language of maths 3

**Puzzle**
Algebra find-a-word

## Language of maths

| | | | |
|---|---|---|---|
| base | binomial | brackets | collecting |
| consecutive | distributive law | evaluate | expand |
| expanded form | expression | factor | factorise |
| formula | highest common factor (HCF) | index laws | indices |
| like terms | power | pronumeral | substitute |
| term | unknown | unlike terms | variable |

**1** Why does the word 'variable' have that name? Look up its meaning in the dictionary.

**2** What are **unlike terms**?

**3** How do you **expand** an algebraic expression?

**4** What is the name given to the largest algebraic term that divides evenly into 2 or more given terms?

**5** What is the opposite of **expanding** an algebraic expression?

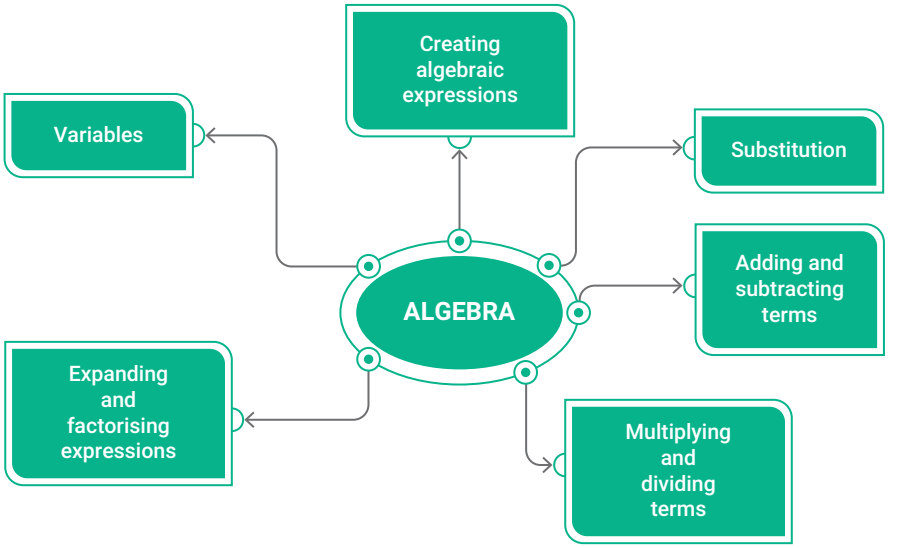
**6** Look up some non-mathematical meanings of the word 'factor'. Use 'factor' in a sentence to show one of these meanings.

**Worksheet**
Mind map: Algebra

## Topic summary

- What parts of algebra did you remember from Year 7? What new rules have you learnt?
- Are there parts of this chapter that you still do not understand? Talk to your teacher.
- Give 2 examples of jobs where algebra is needed.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.





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# TEST YOURSELF 3 ANSWERS ON P. 578

Quiz
Test yourself 3



**1** Simplify each expression. 3.01

**a** $p \times 6$ **b** $r \times (-1)$ **c** $4 \times d \times g$

**d** $t + t + t + t + t$ **e** $14m + 6m$ **f** $11k - 6k$

**g** $6 \times y \times y$ **h** $16 \div a$ **i** $5 \times c \times 4 \times d$

**2** Write each expression in expanded form. 3.01

**a** $cd$ **b** $3y^2$ **c** $\frac{w}{6}$ **d** $c^2 + y$

**3** Write each statement as an algebraic expression. 3.02

**a** 4 more than $y$ **b** 6 times $b$ minus 3 **c** $p$ divided by $q$

**4** Evaluate $2n - p$ when $n = 11$ and $p = -3$. Select the correct answer **A**, **B**, **C** or **D**. 3.03

**A** 19 **B** 25 **C** 208 **D** 214

**5** If $x = 5$ and $y = -2$, evaluate each expression. 3.03

**a** $x + y$ **b** $xy$ **c** $3x - 2y$

**d** $6y^2$ **e** $6x \div y$ **f** $x^2 + y^2$

**6** Simplify each expression. 3.04

**a** $5k + 7k$ **b** $29d - 12d$

**c** $4m + 18m - 5m$ **d** $13m - 3m + 8m$

**e** $4ac + 4ac - 5ca$ **f** $11cd + 7cd - 9cd$

**g** $7x + 3y + 3x + 7y$ **h** $13r + 4s + 11s + 2r$

**i** $33h + 6g - 5h + g$

**7** Simplify each expression. 3.05

**a** $5 \times 2x$ **b** $2b \times 7d$ **c** $11m \times 6$

**d** $3p \times 8q$ **e** $4 \times 2w \times 4w$ **f** $15j \times 5$

**g** $3m \times 6m \times 2$ **h** $4p \times 3p \times 5p$

**8** Simplify each expression. 3.05

**a** $50x \div 2$ **b** $28b \div 7d$ **c** $66m \div 6$

**d** $\frac{32rs}{4}$ **e** $\frac{15p}{3p}$ **f** $14d^2 \div 2$

**g** $14e^2 \div 2e$ **h** $13b^2 \div b$ **i** $\frac{18x^2}{9x}$

**j** $\frac{44m^2n^2}{11m}$ **k** $\frac{16e^2g}{4eg}$ **l** $\frac{35a^2e^2}{7ae}$

Foundation | Standard | Complex

9780170479516

3.07 **9** Expand and simplify each expression.

**a** $6(n + 2)$ **b** $-5(2b + 7)$

**c** $j(9j - 3)$ **d** $10 + 2(n + 4)$

**e** $22v - 3(v + 5)$ **f** $5(y + 2) + 4(y - 7)$

3.07 **10** Which expression is $5(c - d) - 2c + 3d$ simplified? Select **A**, **B**, **C** or **D**.

**A** $3c - 2d$ **B** $3c + 8d$ **C** $3c - 8d$ **D** $5c - 5d - 2c + 3d$

3.08 **11** Find the highest common factor of each pair of terms.

**a** $6a$ and 12 **b** $3p^2$ and $18p$ **c** $7t$ and 14

3.09 **12** Factorise each expression.

**a** $3m + 9$ **b** $5h - 35$ **c** $6a + 9ab$

**d** $10h - 12hm$ **e** $xy + xyz$ **f** $cd - de$

**g** $a^2 - 6a$ **h** $8kj + 24j^2$

3.10 **13** Factorise each expression.

**a** $-2 - 4q$ **b** $-4f - 16$ **c** $-12 + 15d$

**d** $-ab + b$ **e** $-16b - 24bd$ **f** $-4k^2 - k$

**g** $-x^2 + 7x$ **h** $-h^2i - hi^2$

YEAR 9 CORE

3.06 **14** Simplify each expression.

**a** $x^2 \times x^4$ **b** $m^6 \div m^2$ **c** $5k^2 \times 6k$ **d** $-2hy^7 \times 3h^2y^4$

**e** $n^7 \div n^3$ **f** $8a^4 \div 2a^3$ **g** $q^7 \div q^3 \div q$ **h** $\frac{9x^5y}{3x^2}$

**i** $\frac{28m^3n}{6mn^2}$ **j** $\left(w^3\right)^4$ **k** $\left(3a^4\right)^2$ **l** $\left(2d^3e^2\right)^5$

**m** $p^0$ **n** $6a^0$ **o** $5h^3 \div h^3$ **p** $(4x)^0 \times (3x)^2 \div 9x$

3.11 **15** Expand each binomial product.

**a** $(m + 9)(m + 2)$ **b** $(5h + 3)(h - 1)$ **c** $(y + 6)(y - 5)$ **d** $(2a - 3)(2a - 7)$

YEAR 9 PATH

**16** Expand each binomial product.

3.12 **a** $(m + 9)^2$ **b** $(h - 1)^2$ **c** $(2y - 5)^2$ **d** $(3a - 2)^2$

3.13 **17** Expand each binomial product.

**a** $(m + 9)(m - 9)$ **b** $(h - 3)(h + 3)$ **c** $(5y - 2)(5y + 2)$ **d** $(2a + 3c)(2a - 3c)$

Foundation Standard Complex



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# PRACTICE SET 1

ANSWERS ON P. 579

**1** Evaluate each expression. 2.08

**a** $7^3$ **b** $(-35)^2$ **c** $4^4$

**d** $\sqrt{400}$ **e** $\sqrt[3]{2744}$ **f** $\sqrt{2.25}$

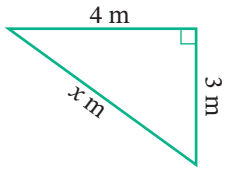
**2** Evaluate each expression without using a calculator. 2.01

**a** $28 \times 5$ **b** $15 \times 9$ **c** $8 \times 3 \times 5$

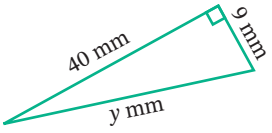
**d** $16 \times 30$ **e** $720 \div 4$ **f** $300 \div 20$

**3** Find the value of each variable, correct to one decimal place where necessary. 1.05

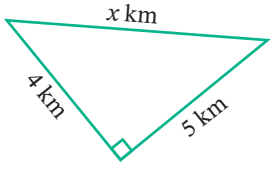
**a**



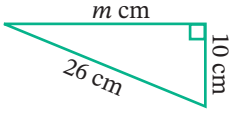

**b**



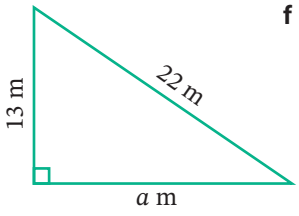

**c**



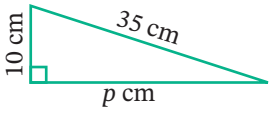

**d**




**e**




**f**




**4** Evaluate each expression. 2.02 2.03

**a** $-3 + 11$ **b** $6 \times (-5)$ **c** $-11 - 9$

**d** $3 - (-2)$ **e** $-7 \times (-4)$ **f** $\frac{42}{-7}$

**g** $7 - 9$ **h** $-4 - (-6)$ **i** $-6 - 10$

**j** $-4 \times 7$ **k** $-54 \div (-9)$ **l** $75 \div (-5)$

**5** Evaluate each expression. 2.04

**a** $4 + 7 \times 2$ **b** $16 - 28 \div (-4)$

**c** $(-32 + 11) \div 7$ **d** $32 \div (-8) - 7 \times 1$

**e** $(3 + 7 \times 5) \div 2$ **f** $(17 - 6) \times (3 - 12)$

**g** $\frac{12 + 7}{4}$ **h** $\frac{8.8}{6.5 - 5.4}$

**i** $\frac{64 - 12}{15 + 11}$

**6** If $a = 3$ and $b = 5$, then evaluate each expression. 3.03

**a** $2b + a$ **b** $a^2$ **c** $5a - b$ **d** $ab - b$

Foundation Standard Complex

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3.04

**7** Simplify each expression.

**a** $3k + 2j - k + 2j$ **b** $5p - p$

**c** $4p + 3q + 7p + 2q$ **d** $10x + 7y - 2x - y$

2.05

**8** Evaluate each expression.

**a** $1.35 + 0.2 - 5.78$ **b** $35.3 - 12.91$

2.06

**9** Evaluate each expression.

**a** $0.8 \times 9$ **b** $1.5 \times 0.3$ **c** $2.1 \div 5$ **d** $5.52 \div 0.3$

3.02

**10** Write each statement as an algebraic expression.

**a** 5 more than $k$ **b** 20 times $k$

**c** half of $k$ **d** 3 lots of $k$ increased by 10

**e** the difference between $k$ and 12 where $k$ is larger **f** $k$ multiplied by itself

3.05

**11** Simplify each expression.

**a** $5 \times 2p$ **b** $12m \div 2m$ **c** $-4x \times 3xy$

**d** $3a \times 2b \times 4$ **e** $\frac{14xy}{7xy}$ **f** $\frac{5a \times 4a}{10a}$

2.10

**12** Use a factor tree to write each number as a product of its prime factors, in index notation.

**a** 180 **b** 875

2.10

**13** Use factor trees to find the highest common factor of each pair of numbers.

**a** 28 and 72 **b** 36 and 90

2.11, 2.12

**14** Use index notation to simplify each expression.

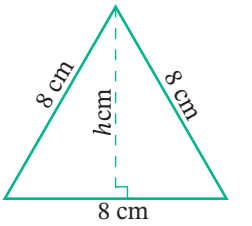
**a** $2^4 \times 2^3$ **b** $7^6 \times 7^5$ **c** $5^8 \div 5^3$ **d** $8^5 \div 8^4$

**e** $(3^4)^2$ **f** $(11^3)^5$ **g** $7^0$ **h** $(8 - 5)^0$

1.08

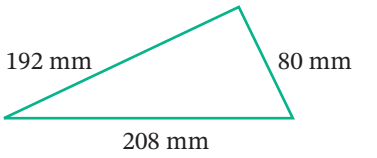
**15** Find the height, $h$ cm, of an equilateral triangle with sides 8 cm long. Write your answer:

**a** as a surd **b** correct to 2 decimal places.




1.06

**16** Test whether this triangle is right-angled. If it is, copy the triangle and draw the right angle.




Foundation Standard Complex


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**17** Use factor trees to find the lowest common multiple of each pair of numbers. 2.10

**a** 12 and 18 **b** 18 and 27

**18** Expand and simplify each expression. 3.07

**a** $4(x + 2)$ **b** $6(m - 4)$ **c** $-7(2k + 1)$

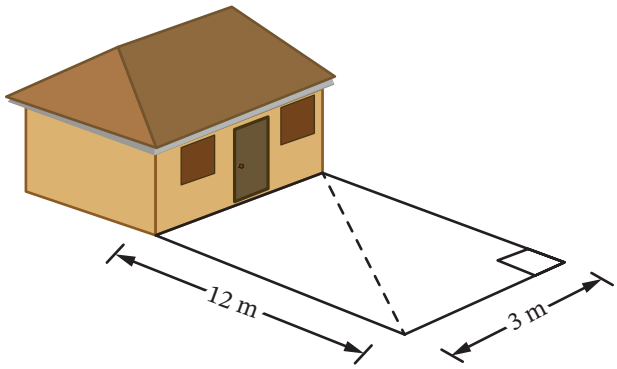
**d** $-(3p + q)$ **e** $2(m + 2) + 4m$ **f** $8 + 4(p - 2)$

**19** Factorise each expression. 3.08

**a** $2m + 4$ **b** $d^2 - d$ **c** $6p + 12q$

**d** $-12x - 24$ **e** $2ab + 3a$ **f** $-2x^2 + 6x$

**20** Rohan is laying a concrete slab 12 m by 3m in front of his shed. He needs to check that the corners of the slab are right angles. How long should the diagonal be? Answer correct to 2 decimal places. 1.07





# 4

MEASUREMENT AND SPACE

# GEOMETRY

Geometry is all around us. Designs and patterns involving angles, parallel lines, triangles and quadrilaterals can be found everywhere, in our homes, on transport, in construction, art and nature. This scene from the National Museum of Australia in Canberra shows the importance of angles, lines and shapes in architecture and design.




## Chapter outline

| | Working mathematically | | | | |
|---|---|---|---|---|---|
| **4.01** Angle geometry | U | F | | R | C |
| **4.02** Angles on parallel lines | U | F | | R | C |
| **4.03** Classifying triangles | U | F | | R | C |
| **4.04** Classifying quadrilaterals | U | F | | R | C |
| **4.05** Properties of quadrilaterals | U | F | | R | C |
| **4.06** Angle sums of triangles and quadrilaterals | U | F | | R | C |
| **4.07** Angle sum of a polygon* | U | F | | R | C |
| **4.08** Networks* | U | F | | | C |

*YEAR 9 EXTENSION

## Wordbank

**angle sum** The total of the sizes of the angles in a shape, such as a triangle

**bisect** To cut in half

**convex quadrilateral** A quadrilateral whose vertices all point outwards.

**diagonal** An interval joining 2 non-adjacent vertices of a shape

**exterior angle of a triangle** An 'outside' angle of a triangle formed after extending one of the sides of the triangle

**scalene triangle** A triangle with no equal sides

**supplementary angles** 2 angles that add to 180°

**Quiz**
Wordbank 4

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

## Videos (12):

**4.01** Angles at a point and vertically opposite angles • Angle geometry

**4.02** Angles on parallel lines • Angle relationships • Angle geometry

**4.03** Properties of triangles • Classifying triangles

**4.04, 4.05** Properties of quadrilaterals • Classifying quadrilaterals

**4.06** Angle sums of triangles and quadrilaterals • Angles in polygons • Exterior angle of a triangle

**4.08** Networks

## *Twig* videos (2)

**4.08** The seven bridges of Konigsberg • Networks: Labyrinths and mazes

## Quizzes (6):

- Wordbank 4
- SkillCheck 4
- Mental skills 4A
- Mental skills 4B
- Language of maths 4
- Test yourself 4

## Skillsheets (4):

**SkillCheck** Types of angles • Line and rotational symmetry

**4.02** Angles and parallel lines

**4.03** Naming shapes

## Worksheets (23):

**SkillCheck** A page of angles • Symmetry

**4.01** Straight angles, right angles and revolutions

**4.02** Find the unknown angle 1 • Investigating angles on parallel lines • Angles in parallel lines • What is the diagram? • Matching angles

**4.03** Properties of triangles • Triangle geometry

**4.04** Classifying quadrilaterals • Properties of quadrilaterals • Always, sometimes, never true • Classifying triangles and quadrilaterals

**4.05** Properties of quadrilaterals • Diagonal properties of quadrilaterals • Always, sometimes, never true

**4.06** Angles in triangles • Find the unknown angle 2 • Deductive geometry • Shapes and angles review

**4.07** Angle sum of a polygon • Angles in regular polygons • Equal angles

**Mind map**: Geometry

## Puzzles (12):

**SkillCheck** Angle cards

**4.01** Angles: A dog day • Angles

**4.02** Angles in parallel lines

**4.03** Classifying triangles

**4.04** What shape am I? • Singing in the car

**4.06** Mixed angles • Angles in triangles • Triangles and quadrilaterals

**4.07** Shape puzzle 1

**Language of maths** Geometry crossword

## Presentation (1):

**4.05** Angles and shapes



Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

## In this chapter you will:

- solve geometrical problems involving angles on a straight line, angles at a point and vertically opposite angles
- solve geometrical problems involving corresponding, alternate and co-interior angles on parallel lines crossed by a transversal
- classify triangles according to their side and angle properties
- identify convex and non-convex quadrilaterals
- classify squares, rectangles, rhombuses, parallelograms, kites and trapeziums

- solve geometrical problems involving the angle sum of a triangle and quadrilateral and the exterior angle of a triangle
- (EXTENSION, YEAR 9 PATH) solve problems involving the angle sum of a polygon
- (EXTENSION, YEAR 9 PATH) identify the parts of a network and investigate Euler's rule

## SkillCheck ANSWERS ON P. 579

**1** Draw 2 different examples of each type of angle.

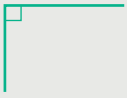
**a** acute angle **b** right angle **c** obtuse angle **d** reflex angle

**2** Classify each angle size as being acute, obtuse, right, reflex, straight or a revolution.

**a** 25° **b** 100° **c** 300° **d** 128°

**e** 90° **f** 360° **g** 286° **h** 180°

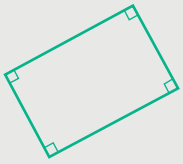
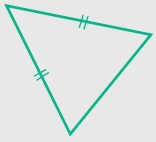
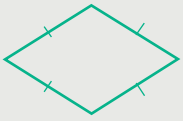
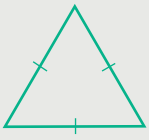
**3** Name each type of angle(s) marked.

**a**  **b**  **c** 

**4** Find the value of $x$ in each equation.

**a** $x + 30 = 90$ **b** $x + 57 = 180$ **c** $x + 121 + 77 = 360$

**5** Copy each shape, name it and draw all axes of **symmetry**.

**a**  **b**  **c**  **d** 

**6** Write the order of **rotational symmetry** of each shape in question **5**.

**7** Which quadrilateral has all 4 sides equal and all 4 angles equal?
Select the correct answer **A**, **B**, **C** or **D**.

**A** parallelogram **B** rhombus **C** rectangle **D** square

**8** Draw a scalene triangle.

**9** **a** Draw a parallelogram and its diagonals.

**b** Are the lengths of the diagonals of a parallelogram equal?

4

**Quiz**
SkillCheck 4

**Skillsheets**
Types of angles

Line and rotational symmetry

**Worksheets**
A page of angles

Symmetry

**Puzzle**
Angle cards

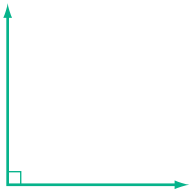
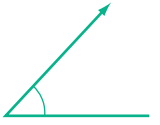
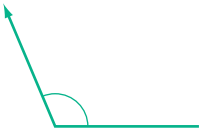
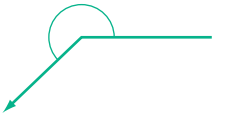
# 4.01 Angle geometry

## Classifying angles

**Worksheet**
Straight angles, right angles and revolutions

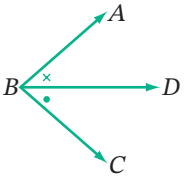
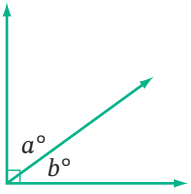
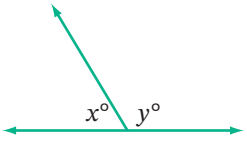
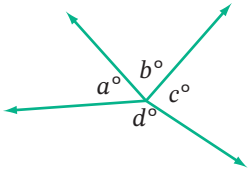
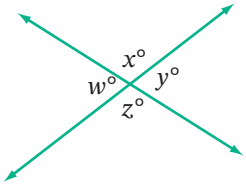
**Puzzles**
Angles: A dog day

Angles

| Right angle | Straight angle | Revolution |
|---|---|---|
| 90° (quarter-turn)  | 180° (half-turn)  | 360° (complete turn)  |
| **Acute angle** | **Obtuse angle** | **Reflex angle** |
| Less than 90°  | Between 90° and 180°  | Between 180° and 360°  |

## Angle facts

**Video**
Angles at a point and vertically opposite angles

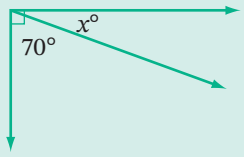
| Adjacent angles | Complementary angles | Angles in a right angle |
|---|---|---|
| Angles next to each other, sharing a common arm.<br>$\angle ABD$ and $\angle DBC$ are adjacent.   | 2 angles that have a sum of 90°, for example, 35° and 55°. | Are complementary.<br>$a + b = 90$   |
| | **Supplementary angles** | |
| | 2 angles that have a sum of 180°, for example, 140° and 40°. | |
| **Angles on a straight line** | **Angles at a point** | **Vertically opposite angles** |
| Are supplementary.<br>$x + y = 180$   | (In a revolution)<br>Add up to 360°.<br>$a + b + c + d = 360$   | Are equal.<br>$w = y$ and $x = z$   |


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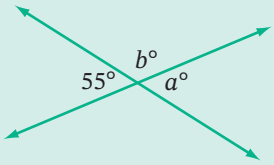
## Example 1

Find the value of each variable, giving reasons.

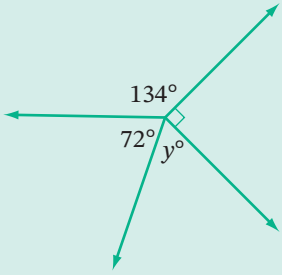
**a**




**b**




**c**




### Solution

**a** $x + 70 = 90$ (Angles in a right angle)

$x = 90 - 70$

$= 20$

The reason is written inside brackets.

**b** $a = 55$ (Vertically opposite angles.)

$b + 55 = 180$ (Angles in a straight line)

$b = 180 - 55$

$= 125$

**c** $y + 72 + 134 + 90 = 360$ (Angles at a point)

$y + 296 = 360$

$y = 360 - 296$

$= 64$

Video
Angle geometry

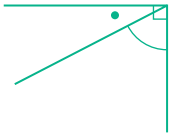
4.01

**EXERCISE 4.01** ANSWERS ON P. 580

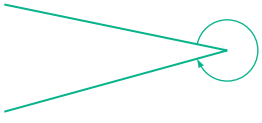
## Angle geometry U F R C

**1** Classify each type of angle(s). C

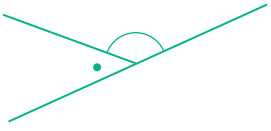
**a**



**b**



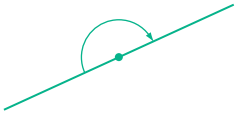
**c**



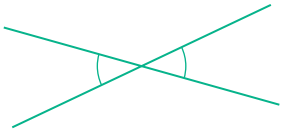
**d**



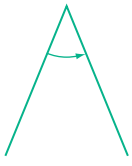
**e**



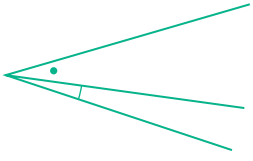
**f**



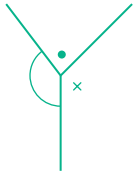
**g**



**h**



**i**



Foundation | Standard | Complex

**2** For each angle size, find the complementary angle.

**a** 16° **b** 33° **c** 71° **d** 2°

**3** For each angle size, find the supplementary angle.

**a** 25° **b** 149° **c** 107° **d** 48°

**4** What is the sum of the angles at a point? Select the correct answer **A**, **B**, **C** or **D**.

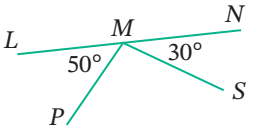
**A** 90° **B** 180° **C** 270° **D** 360°

EXAMPLE 1

**5** Find size of $\angle SMP$. Select **A**, **B**, **C** or **D**.

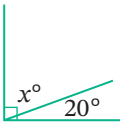
**A** 80° **B** 280°

**C** 100° **D** 10°

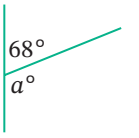



**6** Find the value of each variable, giving reasons. R C

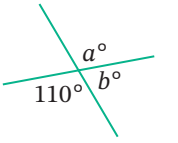
**a**



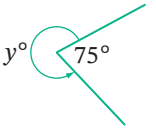

**b**



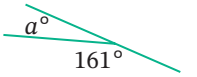

**c**



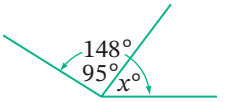

**d**



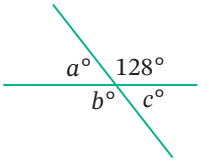

**e**



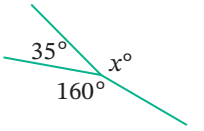

**f**



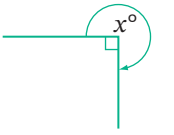

**g**



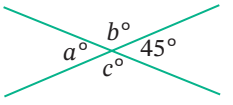

**h**



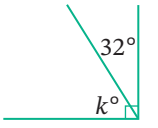

**i**



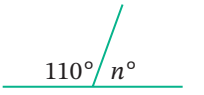

**j**



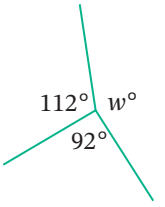

**k**



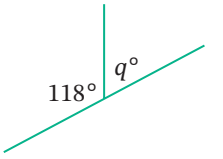

**l**



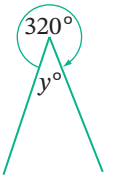

**m**



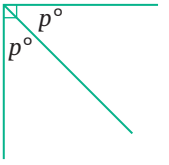

**n**



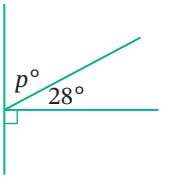

**o**



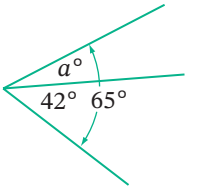

**p**




**q**




**r**

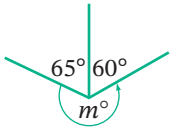



Foundation Standard Complex

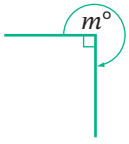

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**7** Find the value of each variable, giving reasons. R C

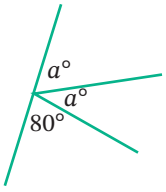
**a**



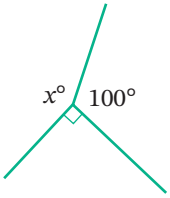

**b**



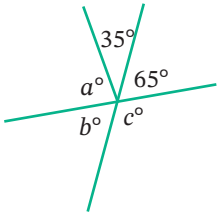

**c**



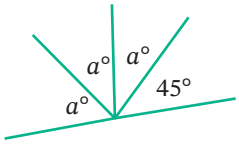

**d**



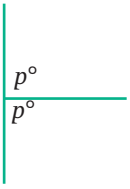

**e**



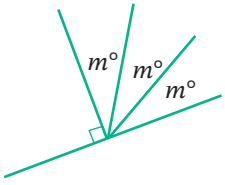

**f**




**g**

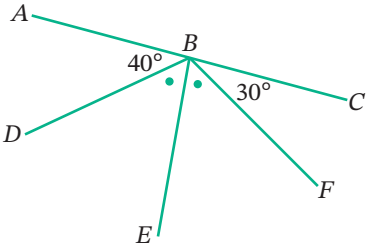


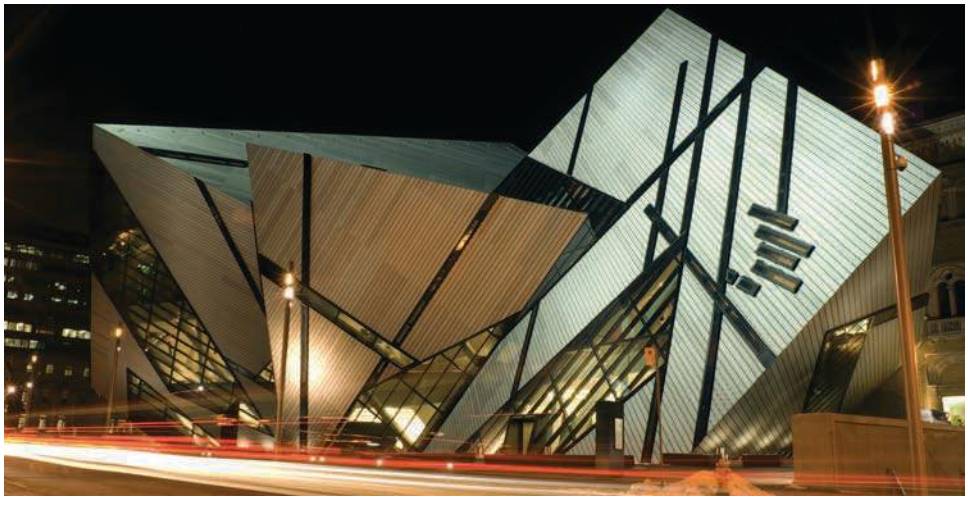

**h**




**8** *ABC* is a straight line and *EB* bisects ∠*DBF*. Find the size of ∠*EBC*.
Select **A**, **B**, **C** or **D**. R

**A** 95°
**B** 110°
**C** 85°
**D** 140°



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Foundation | Standard | Complex

# 4.02 Angles on parallel lines

**Skillsheet**
Angles and parallel lines

**Worksheets**
Find the unknown angle 1

Investigating angles on parallel lines

Angles in parallel lines

What is the diagram?

Matching angles

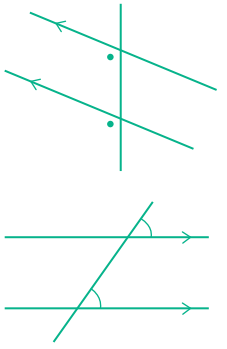
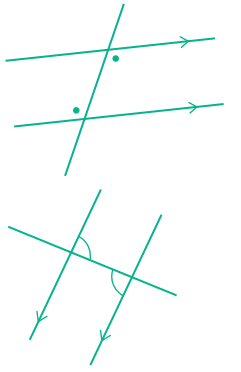
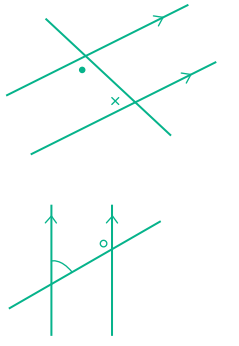
**Puzzle**
Angles in parallel lines

**Videos**
Angles on parallel lines

Angle relationships

**Video**
Angle geometry

When **parallel lines** are crossed by another line (called a **transversal**), special pairs of angles are formed.

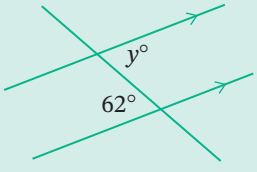
| Corresponding angles | Alternate angles | Co-interior angles |
|---|---|---|
| Corresponding angles on parallel lines are equal. | Alternate angles on parallel lines are equal. | Co-interior angles on parallel lines are supplementary (add to 180°). |
|  |  |  |

- **Corresponding angles** are in 'matching' positions on the *same* side of the transversal: 'corresponding' means 'matching'
- **Alternate angles** are between the parallel lines on *opposite* sides of the transversal: 'alternate' means 'going back and forth in turns'
- **Co-interior angles** are between the parallel lines on the *same* side of the transversal: 'co-interior' means 'together inside'

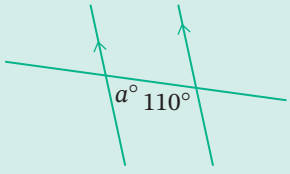
## Example 2

Find the value of each variable, giving reasons.

**a**




**b**




### Solution

**a** $y = 62$ (Alternate angles on parallel lines)

**b** $a + 110 = 180$ (Co-interior angles on parallel lines)

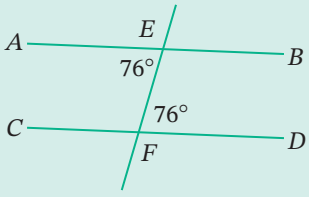
$a = 180 - 110$

$= 70$


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## Example 3

Prove that the lines $AB$ and $CD$ are parallel.




### Solution

$\angle AEF$ and $\angle EFD$ are alternate angles.

$\angle AEF = \angle EFD = 76°$

$\therefore AB \parallel CD$ (Alternate angles are equal)

'$\therefore AB \parallel CD$' means 'Therefore line $AB$ is parallel to line $CD$'

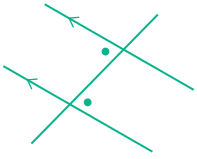


**EXERCISE 4.02** ANSWERS ON P. 580

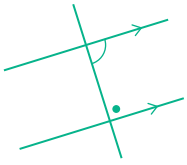
## Angles on parallel lines UFRC

**1** Is each marked pair of angles corresponding, alternate or co-interior?

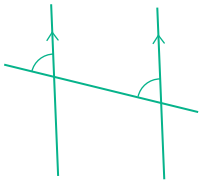
**a**



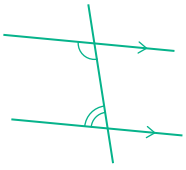
**b**



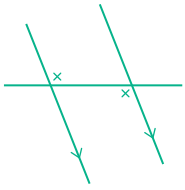
**c**



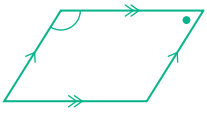
**d**



**e**

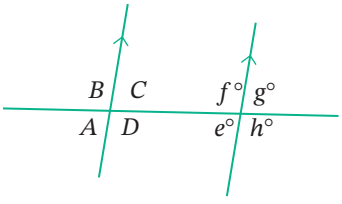


**f**



**2** Which angle is corresponding to the angle marked $g°$?
Select $A$, $B$, $C$ or $D$.

**3** For the diagram in question **2**, name an angle that is:




**a** co-interior to $D$

**b** alternate to the angle marked $e°$

**c** equal to $C$

**d** corresponding to $B$

**e** supplementary to the angle marked $e°$.

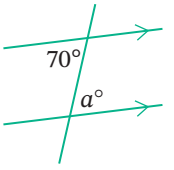
Foundation Standard Complex

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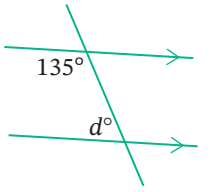
EXAMPLE 2

**4** Find the value of the variable in each diagram, giving reasons for your answers. R C

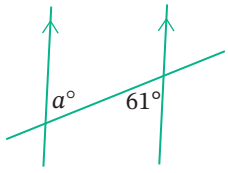
**a**



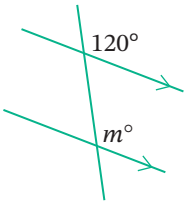

**b**



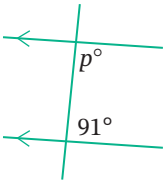

**c**



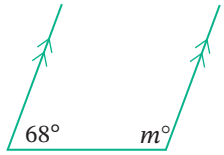

**d**




**e**

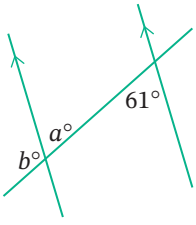



**f**

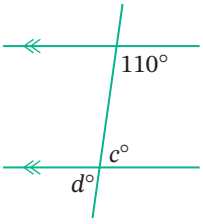



**5** Find the value of each variable, giving reasons. R C

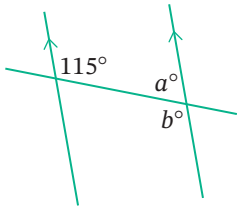
**a**



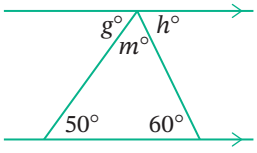

**b**



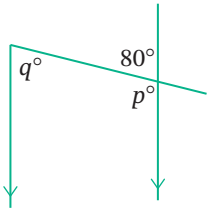

**c**



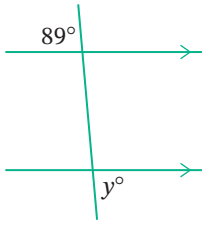

**d**



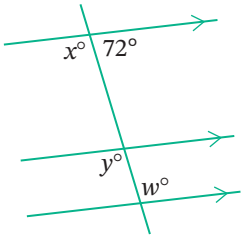

**e**



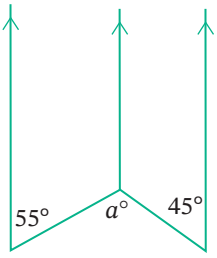

**f**



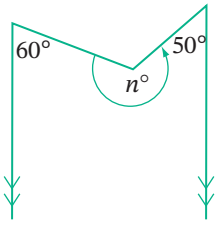

**g**




**h**



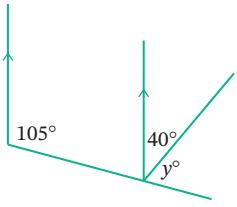

**i**




**6** What is the value of $y$ in this diagram? Select the correct answer **A**, **B**, **C** or **D**. R

**A** 85 **B** 40

**C** 35 **D** 65



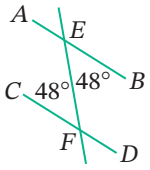

Foundation Standard Complex


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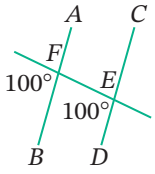
**7** For each diagram, decide whether $AB$ is parallel to $CD$. If it is, then prove it. R C

EXAMPLE 3

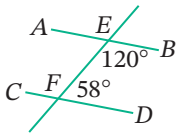
**a**



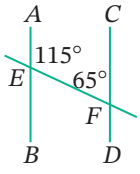

**b**




**c**




**d**

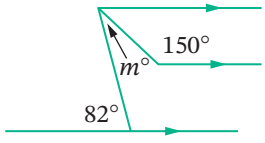



**8** Find the value of $m$ in each diagram. R

**a**




**b**

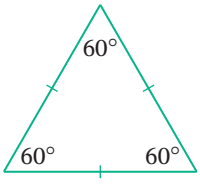
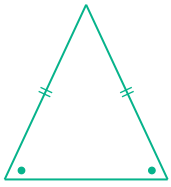
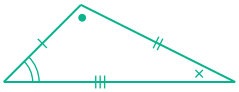


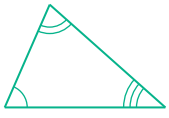
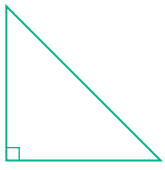

4.03

# Classifying triangles

4.03

Triangles can be classified in 2 ways: by their sides or by their angles.

| Classifying by sides | | |
|---|---|---|
| **Equilateral triangle** | **Isosceles triangle** | **Scalene triangle** |
|   |  |  |
| 3 equal sides (Also 3 equal angles, each 60°) | 2 equal sides (Also 2 equal angles, opposite the equal sides) | No equal sides (Also no equal angles) |

| Classifying by angles | | |
|---|---|---|
| **Acute-angled triangle** | **Obtuse-angled triangle** | **Right-angled triangle** |
|  |  |  |
| 3 acute angles (less than 90°) | One obtuse angle (between 90° and 180°) | One right angle (90°) |

**Skillsheet** Naming shapes

**Worksheets** Properties of triangles

Triangle geometry

**Puzzle** Classifying triangles

**Videos** Properties of triangles

Classifying triangles

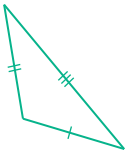
Foundation Standard Complex

**EXERCISE 4.03** ANSWERS ON P. 580

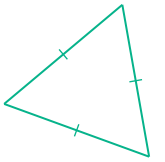
## Classifying triangles U F R C

**1** Classify each triangle according to its sides and angles. R C

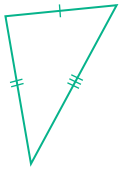
**a**



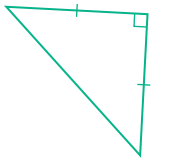
**b**



**c**



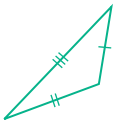
**d**



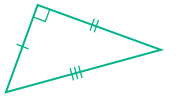
**e**



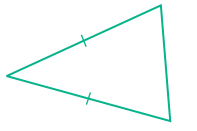
**f**



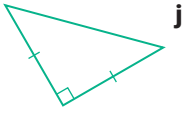
**g**



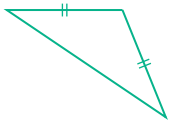
**h**



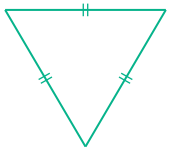
**i**



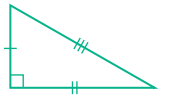
**j**



**k**



**l**



**2** Sketch a triangle that is: R C

**a** right-angled and isosceles

**b** equilateral

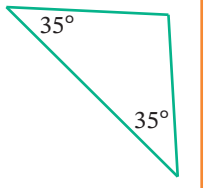
**c** scalene and obtuse-angled

**d** acute-angled and scalene

**e** right-angled and scalene

**f** acute-angled and isosceles

**3** Which type of triangle is this? Select the correct answer **A**, **B**, **C** or **D**. R C




**A** isosceles and obtuse-angled

**B** isosceles and acute-angled

**C** scalene and obtuse-angled

**D** scalene and acute-angled

**4** Is it possible to draw an obtuse-angled equilateral triangle? Justify your answer. R C

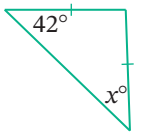
**5** Which triangles in question **1** have:

**a** line symmetry?

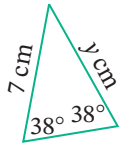
**b** rotational symmetry?

**6** Find the value of each variable, giving a reason. R C

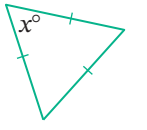
**a**




**b**



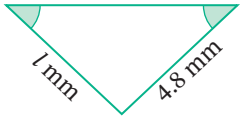

**c**



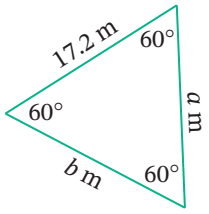

Foundation Standard Complex


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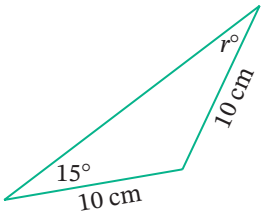
**d**




**e**




**f**




**7** Is it possible to draw a triangle with 2 obtuse angles? Why? R C

**8** Which triangle is both obtuse-angled and scalene? Select **A**, **B**, **C** or **D**. R C

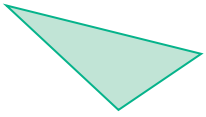
**A**



**B**



**C**



**D**



**9** Copy and complete this table. R C

| Triangle | Number of axes of symmetry | Order of rotational symmetry |
|---|---|---|
| Equilateral triangle | | |
| Isosceles triangle | | No rotational symmetry |
| Scalene triangle | | |

## Did you know?

### It's all Greek or Latin to me!

Many of our words in geometry come from Greek or Latin. Latin was the language of the ancient Roman Empire.

| Word | Origin | Meaning |
|---|---|---|
| Equilateral | Latin: aequus latus | Equal sides |
| Equiangular | Latin: aequus angulus | Equal corners |
| Isosceles | Greek: isos skelos | Equal legs |
| Scalene | Greek: skalenos | Uneven leg |
| Acute | Latin: acutus | Sharp |
| Obtuse | Latin: obtusus | Dull or blunt |
| Reflex | Latin: reflexus | Bent back |
| Triangle | Latin: tri angulus | 3 corners |
| Rectangle | Latin: rectus angulus | Right corners |
| Quadrilateral | Latin: quadri latus | 4 sides |
| Polygon | Greek: poly gonon | Many angles |
| Diagonal | Greek: dia gonios | From angle to angle |
| Trapezium/Trapezoid | Latin/Greek: trapeza | Small table |

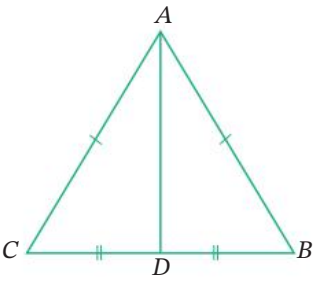
**Explain what this sentence means, and illustrate with a diagram:**
**'A rhombus is equilateral but not equiangular'.**

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## Investigation

### The perpendicular bisector in an isosceles triangle

$\triangle ABC$ is an isosceles triangle with $AC = AB$. It has one axis of symmetry, $AD$.




1. Why is $CD = DB$?
2. Why is $\angle ADC = \angle ADB$?
3. What is the size of $\angle ADC$ and $\angle ADB$?
4. $AD$ bisects side $CB$. What does 'bisect' mean?
5. $AD \perp CB$. What does '$\perp$' mean?
6. 'In an isosceles triangle, the axis of symmetry is the perpendicular bisector of the uneven side.' Explain what this means in your own words.

Quiz
Mental skills 4A

## Mental skills 4A: Maths without calculators ANSWERS ON P. 581

### Converting fractions and decimals to percentages

To convert a fraction or decimal into a percentage, multiply it by 100%.

**1** Study each example.

**a** $\frac{2}{5} = \frac{2}{5} \times 100\% = \frac{2}{\cancel{5}_1} \times \cancel{100}^{20}\% = 2 \times 20\% = 40\%$.

**b** $\frac{24}{40} = \frac{24}{40} \times 100\% = \frac{\cancel{24}^{3}}{\cancel{40}_{5}} \times 100\% = \frac{3}{\cancel{5}_1} \times \cancel{100}^{20}\% = 3 \times 20\% = 60\%$

**2** Now convert each fraction to a percentage.

| | | | | |
|---|---|---|---|---|
| **a** $\frac{7}{10}$ | **b** $\frac{33}{50}$ | **c** $\frac{27}{60}$ | **d** $\frac{22}{25}$ | **e** $\frac{24}{32}$ |
| **f** $\frac{30}{40}$ | **g** $\frac{60}{75}$ | **h** $\frac{4}{5}$ | **i** $\frac{11}{20}$ | **j** $\frac{28}{80}$ |
| **k** $\frac{15}{50}$ | **l** $\frac{16}{20}$ | **m** $\frac{54}{60}$ | **n** $\frac{18}{40}$ | **o** $\frac{13}{25}$ |

**3** Study each example.

**a** $0.41 = 0.41 \times 100\%$
$= 0.41$
$= 41\%$

**b** $0.08 = 0.08 \times 100\%$
$= 0.08$
$= 8\%$

**c** $0.9 = 0.9 \times 100\%$
$= 0.90$
$= 90\%$

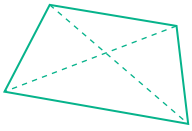
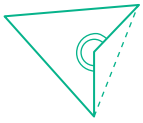
**d** $0.375 = 0.375 \times 100\%$
$= 0.375$
$= 37.5\%$

**4** Now convert each decimal to a percentage.

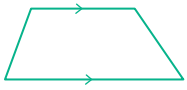
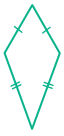
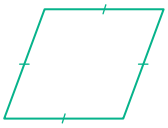
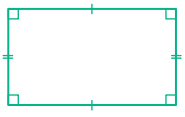
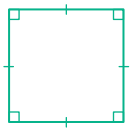
| | | | | |
|---|---|---|---|---|
| **a** 0.25 | **b** 0.68 | **c** 0.17 | **d** 0.6 | **e** 0.1 |
| **f** 0.333 | **g** 0.59 | **h** 0.702 | **i** 0.84 | **j** 0.7 |
| **k** 0.428 | **l** 0.055 | **m** 0.91 | **n** 0.7825 | **o** 0.314 |

# Classifying quadrilaterals 4.04

A **quadrilateral** is any shape with 4 straight sides. A quadrilateral may be **convex** or non-convex.

| Convex quadrilateral | Non-convex quadrilateral |
|---|---|
|  |  |
| • All vertices (corners) point outwards.<br>• All diagonals lie within the shape.<br>• All angles are less than 180°. | • One vertex points inwards.<br>• One diagonal lies outside the shape.<br>• One angle is more than 180° (reflex angle). |

There are 6 special types of quadrilaterals.

| Trapezium | Kite | Parallelogram |
|---|---|---|
|  <br>One pair of parallel sides |  <br>2 pairs of equal adjacent sides<br>'Adjacent' means 'next to each other'. |  <br>2 pairs of parallel sides |
| **Rhombus** | **Rectangle** | **Square** |
|  <br>4 equal sides |  <br>4 right angles |  <br>4 equal sides and 4 right angles |

**Worksheets**
Classifying quadrilaterals
Properties of quadrilaterals
Always, sometimes, never true
Classifying triangles and quadrilaterals

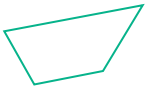
**Puzzles**
What shape am I?
Singing in the car

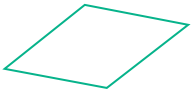
**Videos**
Properties of quadrilaterals
Classifying quadrilaterals

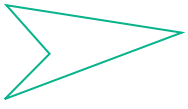
## EXERCISE 4.04 ANSWERS ON P. 581

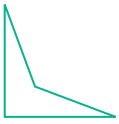
### Classifying quadrilaterals U F R C

**1** Is each quadrilateral convex or non-convex?

**a** 

**b** 

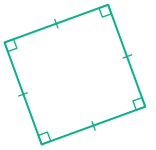
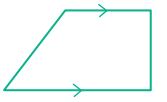
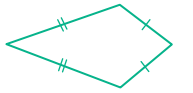
**c** 

**d** 

**e** 

**f** 

**2** Name each quadrilateral. C

**a**  **b**  **c**  **d** 

**3** Name all the quadrilaterals that have: R C

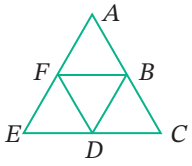
**a** 4 right angles
**b** exactly one pair of parallel sides
**c** 4 equal sides
**d** opposite sides equal
**e** opposite sides parallel
**f** 2 pairs of equal adjacent sides

**4** Which quadrilateral is also called a 'diamond'? R C

**5** Copy and complete this table.

| Quadrilateral | Number of axes of symmetry | Order of rotational symmetry |
|---|---|---|
| Rectangle | | |
| Parallelogram | | |
| Trapezium | | No rotational symmetry |
| Rhombus | | |
| Square | | |
| Kite | | |

**6** For this diagram, what type of quadrilateral is: C




**a** *ACDF*? **b** *FBCD*?

**7** A parallelogram is any quadrilateral with both pairs of opposite sides parallel. Which of the following is **not** a special type of parallelogram? Select **A**, **B**, **C** or **D**. R C

**A** square **B** kite **C** rectangle **D** rhombus

**8** Sketch each quadrilateral, showing its main features. C

**a** rectangle **b** trapezium **c** rhombus **d** kite

**9** True or false? (Explain your answers). R C

**a** A rhombus is a square.
**b** A square is a rhombus.
**c** A rectangle is a parallelogram.
**d** A parallelogram is a quadrilateral with its opposite sides parallel and equal.
**e** The diagonals of a parallelogram meet at right angles.
**f** A square is a rectangle.
**g** A rectangle is a square.

## Technology

### Properties of quadrilaterals

In this activity, you will use your dynamic geometry software to construct quadrilaterals.

**Square**

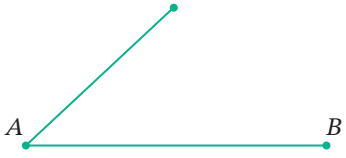
1. Draw a square of length 7.5 cm.
2. Construct the 2 diagonals for the square. Measure the length of each diagonal in the square. What do you notice?
3. Now measure the **size** of each angle of the square. What do you notice?
4. Draw another square with side length 9 cm. Repeat steps **2** and **3**. List 2 properties of the square.

**Rectangle**

1. Draw a rectangle with sides 6 cm by 3 cm.
2. Measure the length of the diagonals of the rectangle.
3. Now measure the **size** of each angle to check your accuracy. What should the size of each angle be?
4. Draw another rectangle with length 5 cm and width 8.4 cm. Measure the length of each diagonal and the size of each angle.
5. Copy and complete this property: The _____ in a rectangle are _____.
6. In one rectangle, measure the distance from each vertex to the point of intersection of the 2 diagonals. Repeat for the second rectangle. What do you notice?
7. Copy and complete this property: The _____ of a rectangle _____ each other.

**Parallelogram**

1. Draw an interval 6 cm long. Label it $AB$.
2. From $A$, draw the interval $AC$. Make the interval 4 cm.
3. Complete the parallelogram by drawing an interval from $C$ to $D$ and from $B$ to $D$.




4. Copy and complete: The _____ sides of a parallelogram are _____.
5. Now measure the size of $\angle CAB$ and $\angle CDB$. Repeat for $\angle ACD$ and $\angle ABD$. What do you notice?
6. Copy and complete: The _____ angles of a parallelogram are _____.
7. Now draw the diagonals of the parallelogram. Measure the length of each diagonal. What do you notice?
8. Copy and complete: The diagonals of a parallelogram are _____.

**9** Do the diagonals of a parallelogram bisect each other? Repeat step **6** from 'Rectangle' to help you.

**10** Copy and complete: The diagonals of a parallelogram _____ bisect each other.

**11** Drag any vertices of the parallelogram that you can. Is it possible to draw other parallelograms with the same dimensions, 6 cm by 4 cm? What do you notice?

**12** Use your dynamic geometry software to **accurately** construct other quadrilaterals such as a rhombus, kite or trapezium.



Shutterstock.com/JoeyPhoto

Do you see rhombuses or cubes?

# 4.05 Properties of quadrilaterals

**Worksheets**
Properties of quadrilaterals

Diagonal properties of quadrilaterals

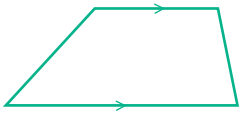
Always, sometimes, never true

**Presentation**
Angles and shapes

**EXERCISE 4.05** ANSWERS ON P. 581

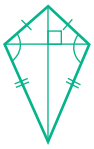
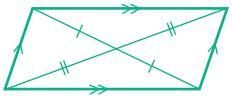
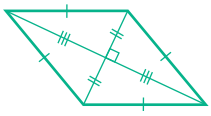
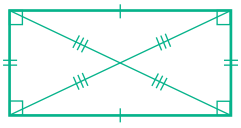
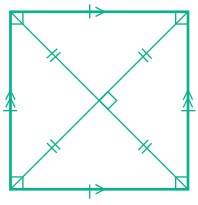
## Properties of quadrilaterals U F R C

**1** Copy the table below or use the link to print one out. Accurately draw each of the 6 special quadrilaterals below on a sheet of paper, then cut them out. Use your shapes to help you complete the table of quadrilateral properties. R C

| | | |
|---|---|---|
| **a** | Trapezium  | • One pair of opposite sides are _____ |

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9780170479516

| | | |
|---|---|---|
| **b** | Kite  | • 2 pairs of adjacent sides are _____<br>• One pair of opposite _____ are equal.<br>• Diagonals intersect at _____ |
| **c** | Parallelogram  | • _____ sides are equal.<br>• Opposite _____ are parallel.<br>• Opposite angles are _____<br>• Diagonals _____ each other. |
| **d** | Rhombus  | • All _____ are equal.<br>• _____ sides are _____<br>• _____ angles are _____<br>• Diagonals bisect each other at _____ angles.<br>• Diagonals _____ the angles of the rhombus. |
| **e** | Rectangle  | • Opposite sides are _____<br>• Opposite sides are also _____<br>• All angles are _____<br>• Diagonals are _____<br>• Diagonals _____ each other. |
| **f** | Square  | • All sides are _____<br>• All angles are _____<br>• Opposite _____ are parallel.<br>• Diagonals are _____<br>• Diagonals bisect each other at _____ |

**2** Name all special quadrilaterals that have each property. R C

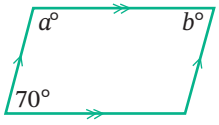
**a** Opposite sides are equal
**b** Diagonals cross at right angles
**c** Opposite angles are equal
**d** One pair of opposite sides are parallel
**e** Diagonals bisect each other
**f** Opposite sides are parallel
**g** Adjacent sides are of different lengths
**h** 2 equal diagonals
**i** All angles are equal
**j** All sides are equal

**3** Copy and complete the working to find the values of $a$ and $b$. R C

$a + 70 = 180$ (_____ angles on _____ lines)

$a =$ _____

$b =$ _____ (opposite _____ of a parallelogram)



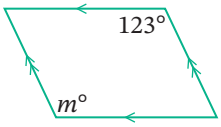

**Videos**
Properties of quadrilaterals
Classifying quadrilaterals

4.05

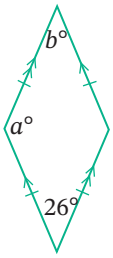
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**4** Find the value of each variable, giving reasons. R C

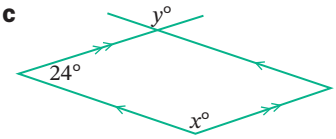
**a**



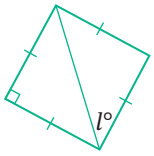

**b**



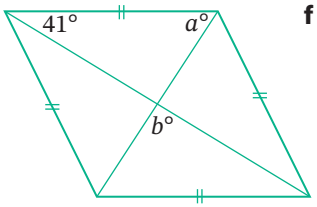

**c**



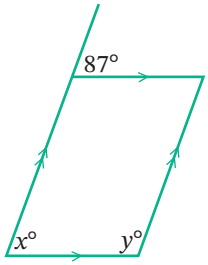

**d**



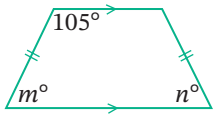

**e**



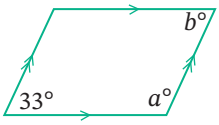

**f**



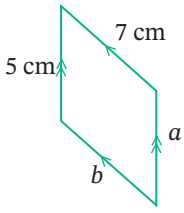

**g**



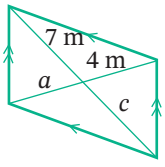

**h**



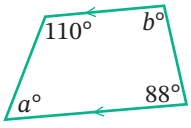

**i**




**j**




**k**




**5** I am a quadrilateral with opposite sides equal and parallel.

My diagonals are equal and I have 2 axes of symmetry.

Which quadrilateral am I? Select the correct answer **A**, **B**, **C** or **D**. R C

**A** parallelogram **B** rectangle **C** square **D** rhombus

**6** I am a quadrilateral with opposite sides equal.

My diagonals bisect each other and meet at right angles.

Which quadrilateral am I? Select **A**, **B**, **C** or **D**. R C

**A** parallelogram **B** trapezium **C** rectangle **D** rhombus

**7** A rectangle is a quadrilateral with 4 right angles.

Which one of the following is a special type of rectangle? Select **A**, **B**, **C** or **D**. R C

**A** square **B** kite **C** parallelogram **D** rhombus

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**8** A rhombus is a quadrilateral with all sides equal.

Which one of the following is a special type of rhombus? Select **A**, **B**, **C** or **D**. R C

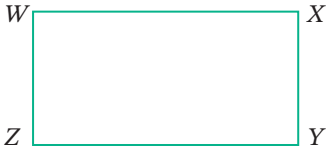
**A** square **B** kite **C** parallelogram **D** rectangle

**9** The quadrilateral $WXYZ$ is shown. R C

**a** If $WX = YZ$ and $WZ = XY$, must $WXYZ$ be a rectangle? Why?

**b** If $\angle WZY = 90°$, must $WXYZ$ be a rectangle? Why?

**c** If the information in parts **a** and **b** are both true, must $WXYZ$ be a rectangle? Why?

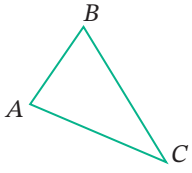



## Technology

### Exterior angle of a triangle

In this activity, you will use dynamic geometry software to discover an important property about the interior and exterior angles of any triangle.

**1** Draw a triangle and label each vertex, $A$, $B$ and $C$, as shown.

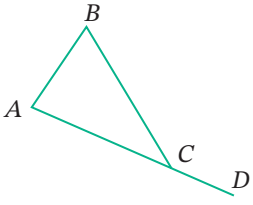



**2** Draw a ray from $A$ through $C$ to point $D$.

**3** Find the size of $\angle ACB$, $\angle BAC$ and $\angle BCD$.

**4** Calculate $\angle ACB + \angle BAC$.

**5** Compare your answer from question **4** with the size of $\angle BCD$. What do you notice?




**6** Repeat steps **1** to **5** for 3 other triangles.

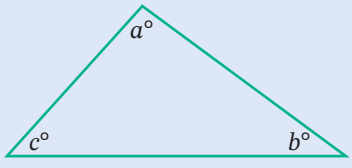
**7** Copy and complete: The _____ angle of a triangle is _____ to the sum of the _____ opposite _____ angles.

# Angle sums of triangles and quadrilaterals 4.06

## Angle sum of a triangle

The angle sum of a triangle is 180°.

$$a + b + c = 180$$




**Videos**
Angle sums of triangles and quadrilaterals

Angle in polygons

Foundation Standard Complex

**Worksheets**
Angles in triangles

Find the unknown angle 2

Deductive geometry

Shapes and angles review

**Puzzles**
Mixed angles

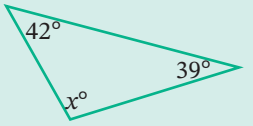
Angles in triangles

Triangles and quadrilaterals

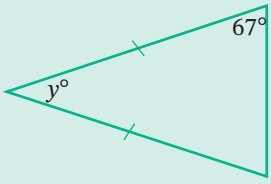
## Example 4

Find the value of each variable, giving reasons.

**a**




**b**




### Solution

**a** $x + 42 + 39 = 180$ (angle sum of a triangle)

$x = 180 - 42 - 39$

$= 99$

**b** $y + 67 + 67 = 180$ (angle sum of an isosceles triangle)

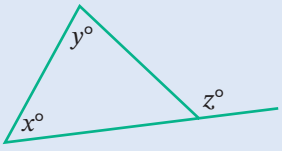
$y = 180 - 134$

$= 46$

## Exterior angle of a triangle

The exterior angle of a triangle is equal to the sum of the 2 interior opposite angles.

$$z = x + y$$

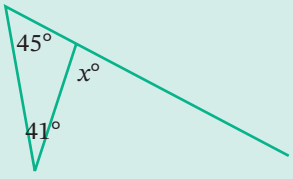



**Video**
Exterior angle of a triangle

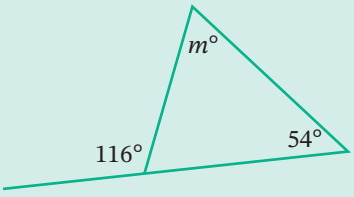
## Example 5

Find the value of each variable, giving reasons.

**a**




**b**




### Solution

**a** $x = 45 + 41$ (exterior angle of a triangle)

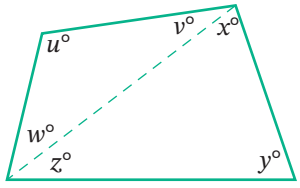
$= 86$

**b** $m + 54 = 116$ (exterior angle of a triangle)

$m = 116 - 54$

$= 62$

To prove the angle sum of a quadrilateral, we can divide a quadrilateral into 2 triangles along one of its diagonals. Because the angles in each triangle add to 180°, the angles in both triangles add to $2 \times 180° = 360°$.




$u° + v° + w° = 180°$ and $x° + y° + z° = 180°$

$\therefore$ Angle sum of quadrilateral $= 180° + 180°$

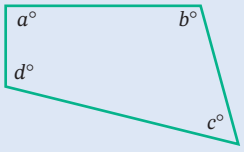
$= 360°$

## Angle sum of a quadrilateral

The angle sum of a quadrilateral is 360°.

$$a + b + c + d = 360$$

This property is true for both convex and non-convex quadrilaterals.




4.06

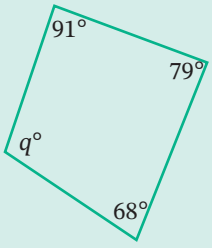


**Video**
Angle sums of triangles and quadrilaterals

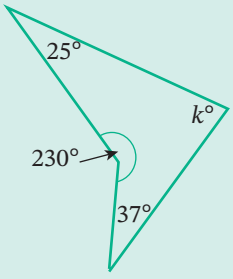
## Example 6

Find the value of each variable, giving reasons.

**a**




**b**




### Solution

**a** $q + 91 + 79 + 68 = 360$ (angle sum of a quadrilateral)

$q + 238 = 360$

$q = 360 - 238$

$= 122$

**b** $k + 25 + 37 + 230 = 360$ (angle sum of a quadrilateral)

$k + 292 = 360$

$k = 360 - 292$

$= 68$

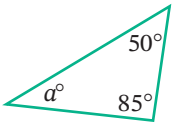
**EXERCISE 4.06** ANSWERS ON P. 582

## Angle sums of triangles and quadrilaterals UFRC

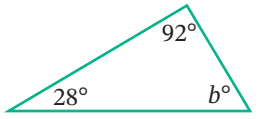
EXAMPLE 4

**1** Find the value of each variable. R

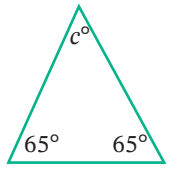
**a**



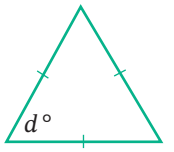

**b**



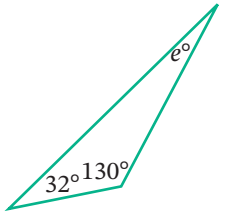

**c**



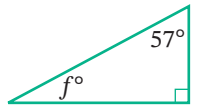

**d**



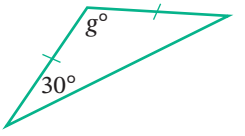

**e**



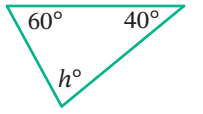

**f**



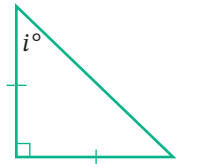

**g**




**h**



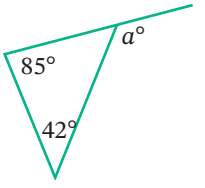

**i**



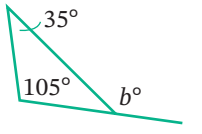

EXAMPLE 5

**2** Find the value of each variable. R

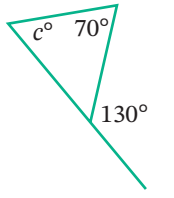
**a**



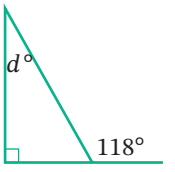

**b**



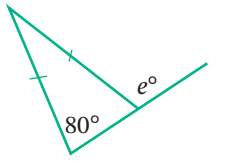

**c**



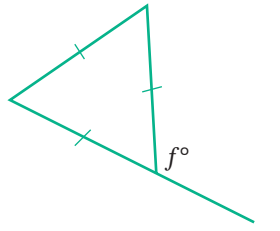

**d**



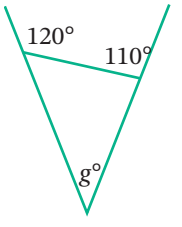

**e**



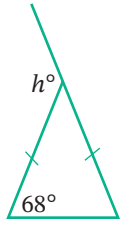

**f**



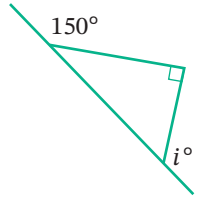

**g**




**h**




**i**




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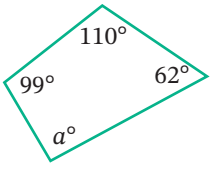

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EXAMPLE 6

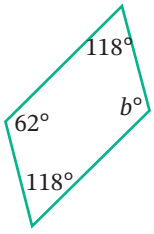
4.06

**3** Find the value of each variable. R

**a**




**b**



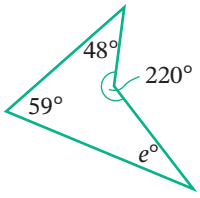

**c**



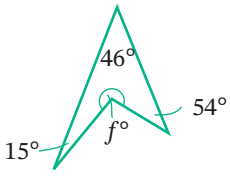

**d**



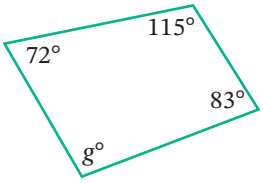

**e**



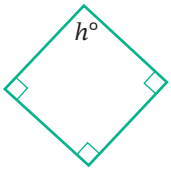

**f**



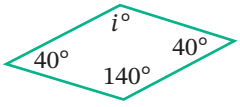

**g**




**h**

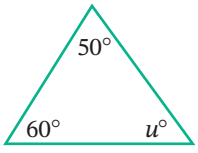



**i**




**4** Find the value of $u$. Select the correct answer **A**, **B**, **C** or **D**.

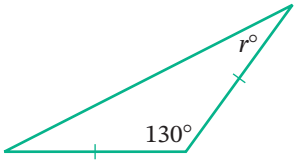
**A** 50 **B** 60 **C** 70 **D** 80




**5** Find the value of $r$. Select **A**, **B**, **C** or **D**.

**A** 25 **B** 35

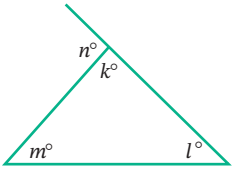
**C** 50 **D** 100




**6** Which equation is correct for this triangle? Select **A**, **B** or **C**. R

**A** $n = l + m$ **B** $n = k + m$

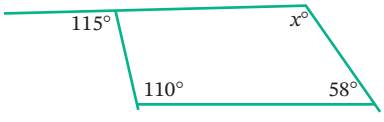
**C** $n = l + k$



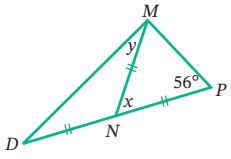

**7** Find the value of $x$. Select **A**, **B**, **C** or **D**.

**A** 115 **B** 65

**C** 127 **D** 122



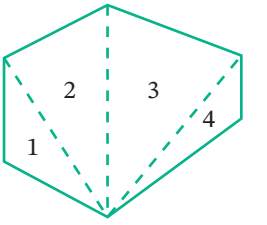

**8** Find the size of angles $x$ and $y$, giving reasons. R C




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## Investigation

### Angle sum of a polygon



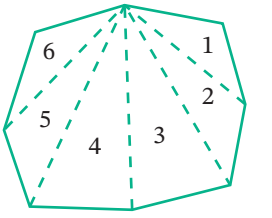

We know that the angle sum of a triangle is 180° and that the angle sum of a quadrilateral is 360°, but how do we find the angle sum of other convex polygons?

A hexagon can be divided into 4 triangles by drawing the diagonals from one vertex.

The sum of the angles in a hexagon = (angle sum of a triangle) × 4

$= 180° \times 4$

$= 720°$




An octagon can be divided into 6 triangles by drawing the diagonals from one vertex.

The total of the angles in an octagon = $180° \times 6 = 1080°$.

**1** How is the number of sides related to the number of triangles formed in a shape?

**2** Copy and complete these sentences:

The angle sum of a polygon with $n$ sides is

$A = 180 \times$ (number of sides $-$ _____)°

or: $A = 180 \times (n -$ _____)°

**3** Find the angle sum of each polygon.

**a** 11-sided polygon **b** 20-sided polygon **c** 14-sided polygon

# 4.07 Extension: Angle sum of a polygon

YEAR 9 PATH

A **convex polygon** has all vertices pointing outwards.

A **regular polygon** has all sides the same length and all angles the same size.

**Worksheets**
Angle sum of a polygon

Angles in regular polygons

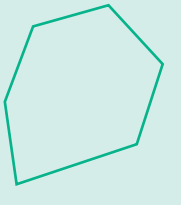
## Example 7

These shapes are all hexagons (6-sided).

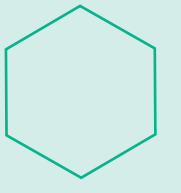
**A**



**B**



**C**



**a** Which hexagons are convex?

**b** Which hexagon is regular?

### Solution

**a** Hexagons **B** and **C** are convex because all of their vertices point outwards.

**b** Hexagon **C** is regular, because all its sides are equal and all its angles are equal.

## Angle sum of a polygon

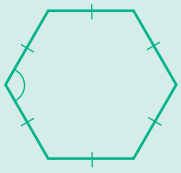
The angle sum of a polygon with $n$ sides is given by the formula:

$$A = 180(n - 2)^\circ$$

This property applies to both convex and non-convex polygons.

## Example 8

Find the size of one angle in a regular hexagon.



### Solution

A hexagon has 6 sides ($n = 6$).

Angle sum of a hexagon $= 180(6 - 2)^\circ$

$= 180 \times 4^\circ$

$= 720^\circ$

For a regular hexagon:

One angle $= 720^\circ \div 6$

$= 120^\circ$

$\therefore$ Each angle in a regular hexagon is 120°.

**EXERCISE 4.07** ANSWERS ON P. 582

## Angle sum of a polygon U F R C

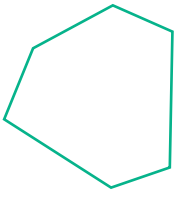
**1** Copy each polygon and write its correct name from this list.

| | | | |
|---|---|---|---|
| hexagon | nonagon | heptagon | decagon |
| octagon | triangle | quadrilateral | pentagon |

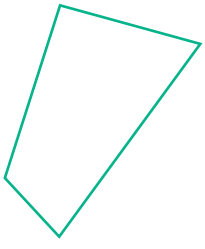
Also, state if each polygon is **regular** or **irregular**, **convex** or **non-convex**. C

EXAMPLE 7

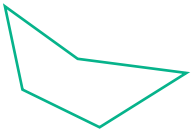
**a**



**b**



**c**



Foundation Standard Complex

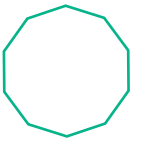
9780170479516

YEAR 9 PATH

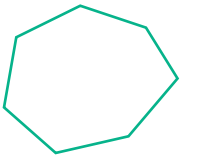
**d**



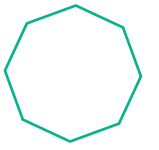
**e**



**f**



**g**



**h**



**2** Copy and complete this table. C

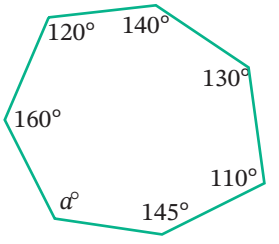
| Polygon | Number of sides | Sum of angles inside polygon |
|---|---|---|
| hexagon | | |
| heptagon | | |
| octagon | | |
| nonagon | | |
| decagon | | |

**3** Find the sum of the interior angles in:

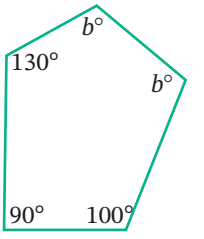
**a** a 15-agon **b** a 20-agon **c** a 25-agon **d** a 100-agon

**4** Find the value of each variable. R

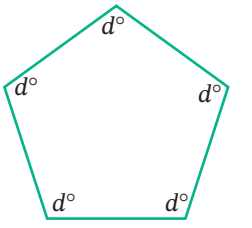
**a**



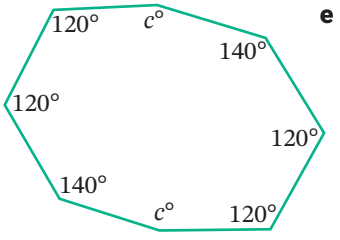

**b**



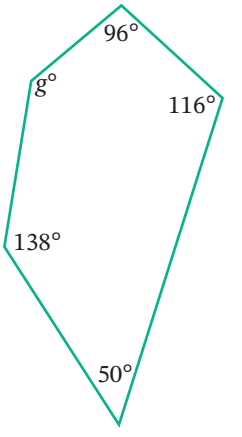

**c**



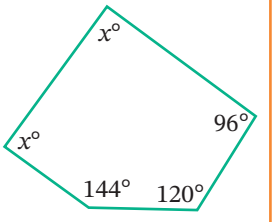

**d**



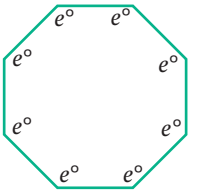

**e**




**f**




**g**




Foundation Standard Complex


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**5** Find the size of one interior angle in each regular polygon.

**a** square **b** equilateral triangle **c** regular hexagon
**d** regular octagon **e** regular decagon **f** regular pentagon
**g** regular dodecagon

**6** Find the number of sides of the polygon that has an angle sum of: R

**a** 2160° **b** 5760° **c** 4320° **d** 9180° **e** 22 140°

**7** Is it possible for a regular polygon to have each interior angle:

**a** 144°? **b** 130°?

If so, find the number of sides. R

YEAR 9 PATH

EXAMPLE 8

4.08

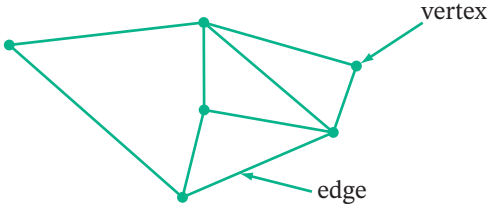
# Extension: Networks

4.08

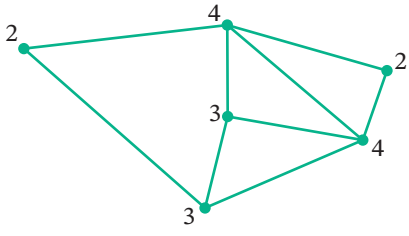
YEAR 9 PATH

A **network** is a collection of objects connected to each other in some way or system. Examples of networks include train networks, electrical circuits, the road traffic network of a city, and social friendship networks such as Instagram.

A **graph** is a diagram of a network where objects are represented by points called **vertices** (or **nodes**), which are joined by lines called **edges**. This graph has 6 vertices and 9 edges.




The **degree** of a vertex is the number of edges connected to it. The degree of each vertex of the graph is shown below.




An **even** vertex has an even number of edges connected to it.

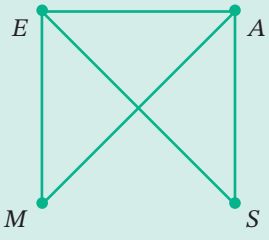
An **odd** vertex has an odd number of edges connected to it.

The graph above has 4 even vertices and 2 odd vertices.

Videos
Networks
The seven bridges of Konigsberg
Networks: Labyrinths and mazes

## Example 9

This network shows the friendships between 4 people, Emma ($E$), Alessio ($A$), Spyros ($S$) and Marie ($M$).




**a** Find the degree of each vertex in this network.

**b** How many even vertices are there?

**c** Show that the sum of the degrees is equal to double the number of edges.

Foundation Standard Complex

YEAR 9 PATH

## Solution

**a**

| Vertex | E | A | S | M |
|---|---|---|---|---|
| Degree | 3 | 3 | 2 | 2 |

**b** There are 2 even vertices ($S$ and $M$).

**c** Sum of degrees $= 3 + 3 + 2 + 2 = 10$

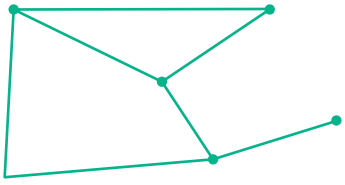
Number of edges $= 5$

Sum of degrees $= 2 \times 5$

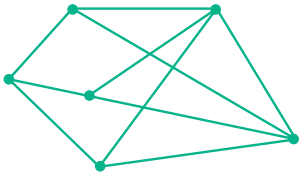
= double number of edges.

# Planar graphs

A **planar graph** is a graph hat can be drawn so that no edges cross over. 'Planar' means 2D, on a flat plane, and a planar graph can be drawn on a flat plane so that its edges don't overlap.



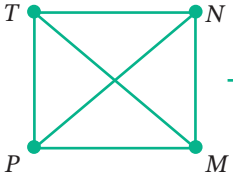
**Planar graph**



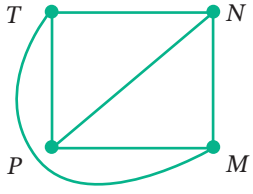
**A non-planar graph**

Some graphs may have edges crossing but they are still planar graphs because they can be redrawn without any of the edges crossing. For example, Graph **A** below is planar because it can be redrawn as graph **B** or **C**.

**A**

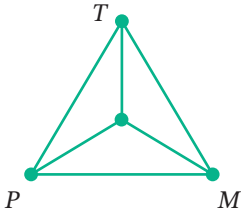



**B**

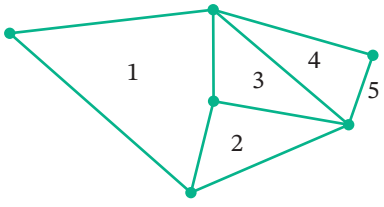



**or**

**C**




The **faces** of a network are the regions bounded by edges. When counting the faces of a network, the 'outside' region of the network is also considered a face, called the **infinite face**. This network has 5 faces.




# Euler's formula for planar graphs

In 1758, the Swiss mathematician, Leonhard Euler (pronounced 'oiler'), discovered a relationship between the vertices, edges and faces of a planar graph.

Euler is considered the 'father' of network theory.

## Euler's formula for planar graphs

For a planar graph,

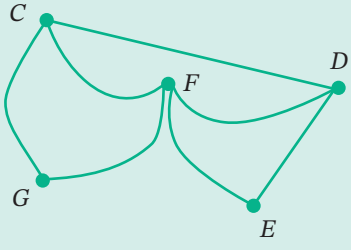
$$v - e + f = 2$$

where $v$ = number of vertices, $e$ = number of edges, $f$ = number of faces

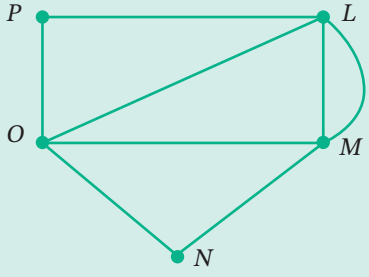
## Example 10

Show that each planar graph follows Euler's formula.

**a**




**b**




### Solution

**a** $v = 5, e = 7, f = 4$

So $v - e + f = 5 - 7 + 4$

$= 2$

So Euler's formula is true.

**b** $v = 5, e = 8, f = 5$

So $v - e + f = 5 - 8 + 5$

$= 2$

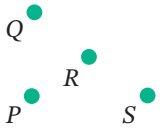
So Euler's formula is true.

**EXERCISE 4.08** ANSWERS ON P. 582

## Networks U F C

**1** Mark, Sue and Alyssa work as a team at a company and email messages to each other each day. Draw a network to represent the movement of the messages. C

**2** Towns *P*, *Q*, *R* and *S* are to be joined with roads so that there is a road directly connecting each town to every other town. Copy and complete this road network if the roads must not cross each other. C



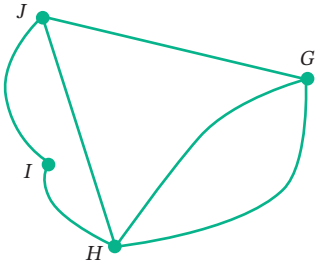

**3** For each network:

**i** find the degree of each vertex

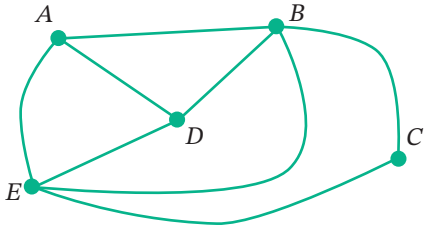
**ii** state which vertices are even and which are odd

**iii** show that the sum of the degrees is equal to double the number of edges

**a**




**b**




EXAMPLE 9

Foundation | Standard | Complex

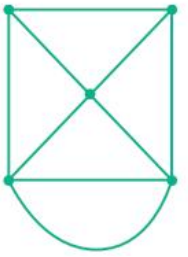
YEAR 9 PATH

EXAMPLE 10

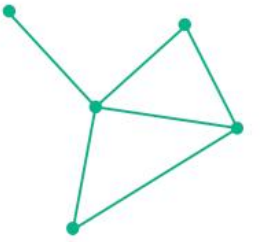
**4** Explain why the sum of the degrees of a network is always double the number of edges.

**5** For each planar graph, count the number of vertices, edges and faces and check that Euler's formula is true. For graphs **f** and **g**, first redraw one of the edges so that it doesn't cross another edge.

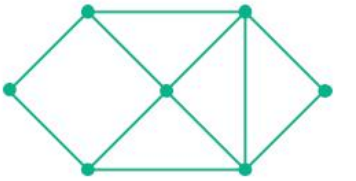
**a**



**b**



**c**



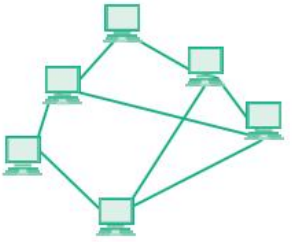
**d**



**e**



**f**



**g**



Quiz
Mental skills 4B

## Mental skills 4B: Maths without calculators ANSWERS ON P. 582

### Converting decimals and percentages to fractions

**1** Consider each example.

**a** $0.35 = \frac{35^{7}}{100_{20}} = \frac{7}{20}$

(2 decimal places, two 0s in the denominator)

**b** $0.8 = \frac{8^{4}}{10_{5}} = \frac{4}{5}$

(one decimal place, one 0 in the denominator)

**c** $0.64 = \frac{64^{16}}{100_{25}} = \frac{16}{25}$

**d** $0.22 = \frac{22^{11}}{100_{50}} = \frac{11}{50}$

**2** Now convert each decimal to a fraction.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | 0.75 | **b** | 0.28 | **c** | 0.3 | **d** | 0.14 |
| **e** | 0.06 | **f** | 0.85 | **g** | 0.32 | **h** | 0.49 |
| **i** | 0.56 | **j** | 0.9 | **k** | 0.72 | **l** | 0.65 |
| **m** | 0.2 | **n** | 0.24 | **o** | 0.53 | | |

Foundation | Standard | Complex


9780170479516

**3** Consider each example.

**a** $26\% = \frac{26^{13}}{100_{50}} = \frac{13}{50}$

**b** $40\% = \frac{40^{2}}{100_{5}} = \frac{2}{5}$

**c** $8\% = \frac{8^{2}}{100_{25}} = \frac{2}{25}$

**d** $95\% = \frac{95^{19}}{100_{20}} = \frac{19}{20}$

**4** Now convert each percentage to a fraction.

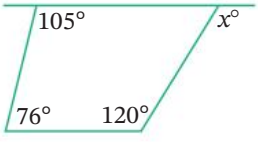
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | 76% | **b** | 10% | **c** | 80% | **d** | 45% |
| **e** | 88% | **f** | 56% | **g** | 75% | **h** | 31% |
| **i** | 68% | **j** | 5% | **k** | 60% | **l** | 54% |
| **m** | 6% | **n** | 49% | **o** | 82% | | |

## Power plus ANSWERS ON P. 582

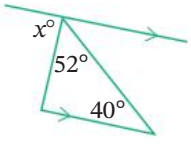
As questions become more complex, it may not be possible to find the answer in one step. It may be necessary to find another angle first.

Find the value of $x$ in each diagram. (Give reasons for all steps.)

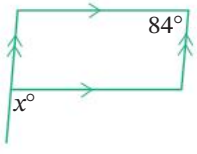
**a**



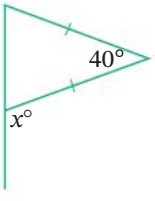

**b**



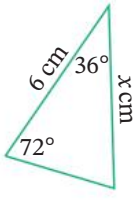

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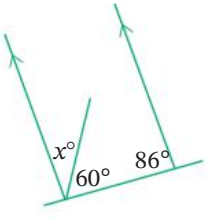

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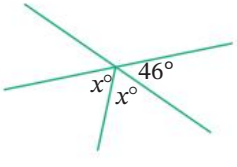

**e**



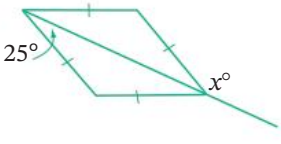

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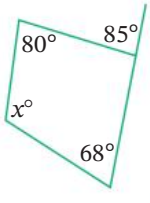

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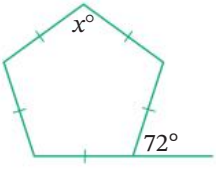

**h**



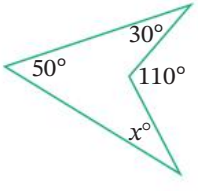

**i**



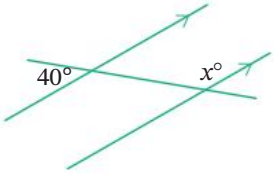

**j**




**k**




**l**






**Puzzle** Shapes puzzle 1

**Worksheet** Equal angles

# CHAPTER 4 REVIEW

**Quiz**
Language of maths 4

**Puzzle**
Geometry crossword

## Language of maths

| | | | |
|---|---|---|---|
| acute | alternate | angle sum | axis/axes |
| co-interior | complementary | convex | corresponding |
| diagonal | equilateral | exterior angle | interior angle |
| isosceles | kite | obtuse | parallelogram |
| quadrilateral | rectangle | reflex | rhombus |
| scalene | supplementary | trapezium | vertically opposite |

**1** What are **supplementary angles**?

**2** Name the types of angles associated with parallel lines cut by a transversal.

**3** What type of triangle has one angle that is greater than 90°?

**4** What is the **angle sum** of a quadrilateral?

**5** Illustrate the difference between an **interior angle** and an **exterior angle** of a triangle.

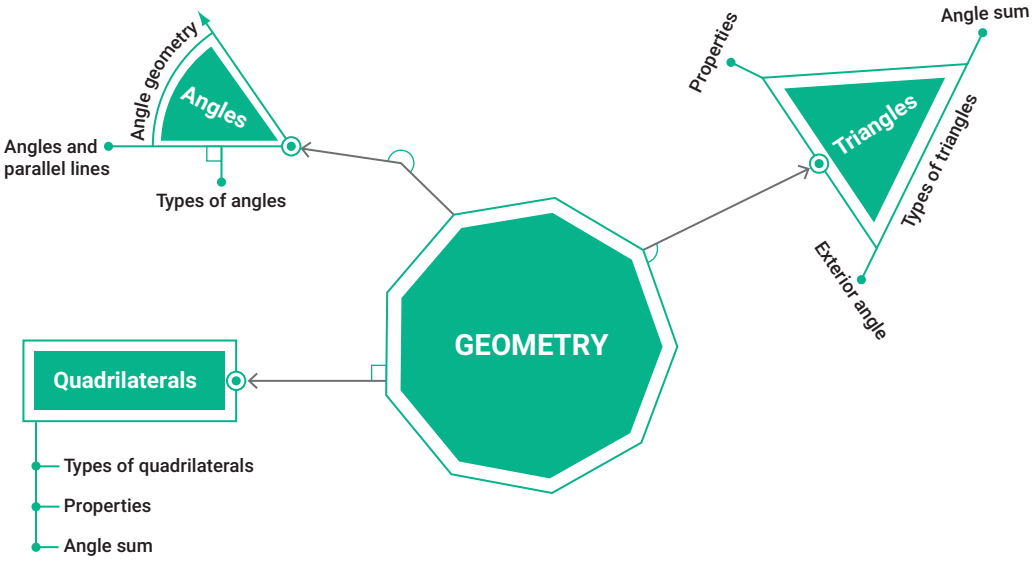
**6** What does **equilateral** mean? What is the common name for an 'equilateral parallelogram'?

## Topic summary

**Worksheet**
Mind map: Geometry

- Do you think this chapter is useful? Why? What did you learn in this chapter?
- How confident do you feel with geometry?
- List anything in this chapter that you did not understand. Show your teacher.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.





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# TEST YOURSELF 4

ANSWERS ON P. 582

Quiz
Test yourself 4

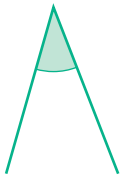


**1** Name each type of angle. 4.01

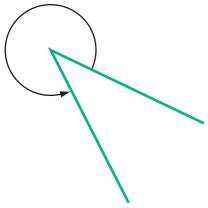
**a**



**b**

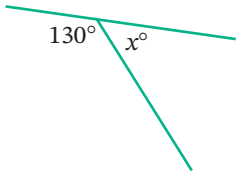


**c**

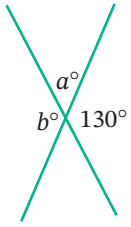


**2** Find the value of each variable, giving reasons. 4.01

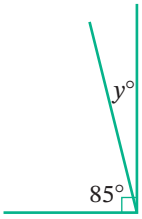
**a**



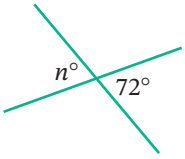

**b**



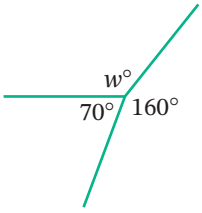

**c**




**d**

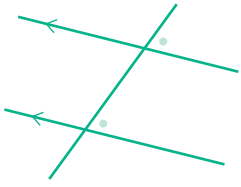



**e**

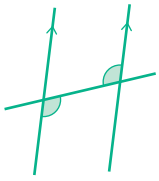



**3** Name each type of angle pair. 4.02

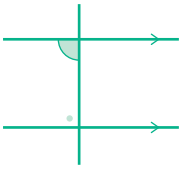
**a**



**b**

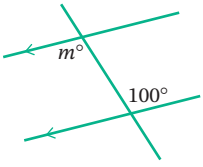


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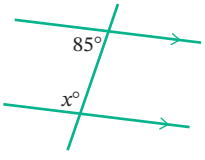


**4** Find the value of each variable, giving reasons. 4.02

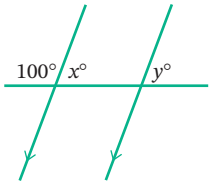
**a**



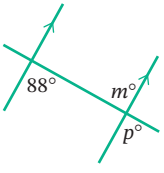

**b**




**c**




**d**




Foundation | Standard | Complex

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4.03

**5** Classify each triangle by its sides and angles.

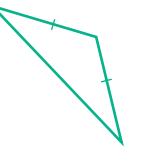
**a**



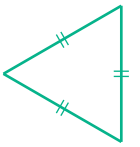
**b**



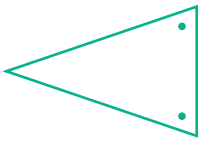
**c**



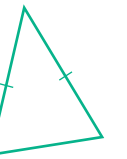
**d**



**e**



**f**



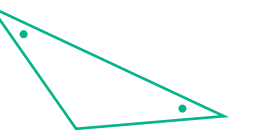
**g**



**h**



**i**



4.04

**6** Sketch each quadrilateral.

**a** parallelogram **b** kite **c** trapezium

4.05

**7** Name all the quadrilaterals that have each property.

**a** All sides equal in length

**b** No parallel sides

**c** All angles right angles

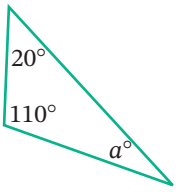
**d** 2 pairs of opposite angles equal

**e** Diagonals bisect each other

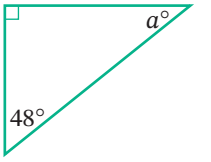
4.07

**8** Find the value of each variable.

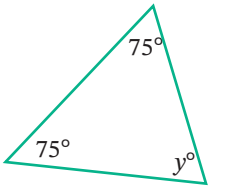
**a**



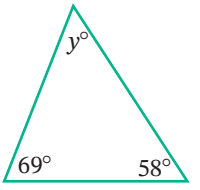

**b**



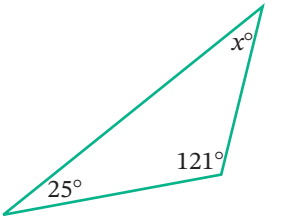

**c**



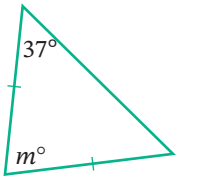

**d**




**e**




**f**



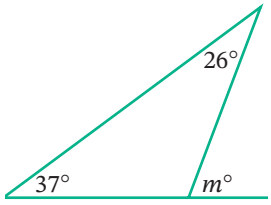

Foundation Standard Complex



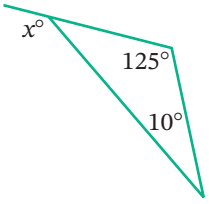
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**9** Find the value of each variable. 4.06

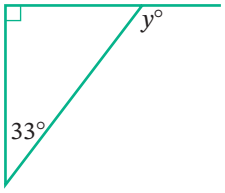
**a**



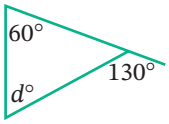

**b**



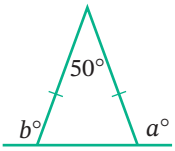

**c**



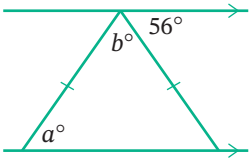

**d**



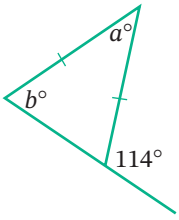

**e**




**f**

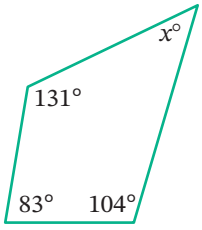



**g**

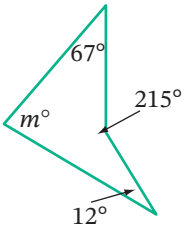



**10** Find the value of each variable. 4.06

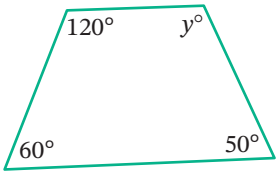
**a**




**b**



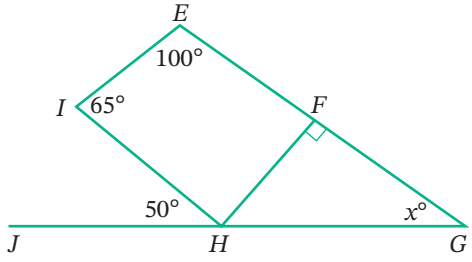

**c**




**11** Find the value of $x$. Select **A**, **B**, **C** or **D**. 4.06

**A** 65 **B** 25

**C** 55 **D** 80




YEAR 9 PATH

**12** **a** Find the angle sum of a nonagon. 4.07

**b** Find the size of one angle in a regular 14-sided polygon, correct to the nearest degree.

**c** Find the number of sides of the polygon with an angle sum of 3240°.

**13** For this network, find: 4.08

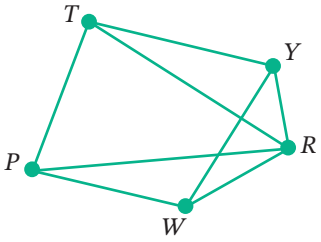
**a** the number of vertices

**b** the number of edges

**c** the degree of $T$

**d** the degree of $R$

**e** the number of odd vertices




Foundation | Standard | Complex

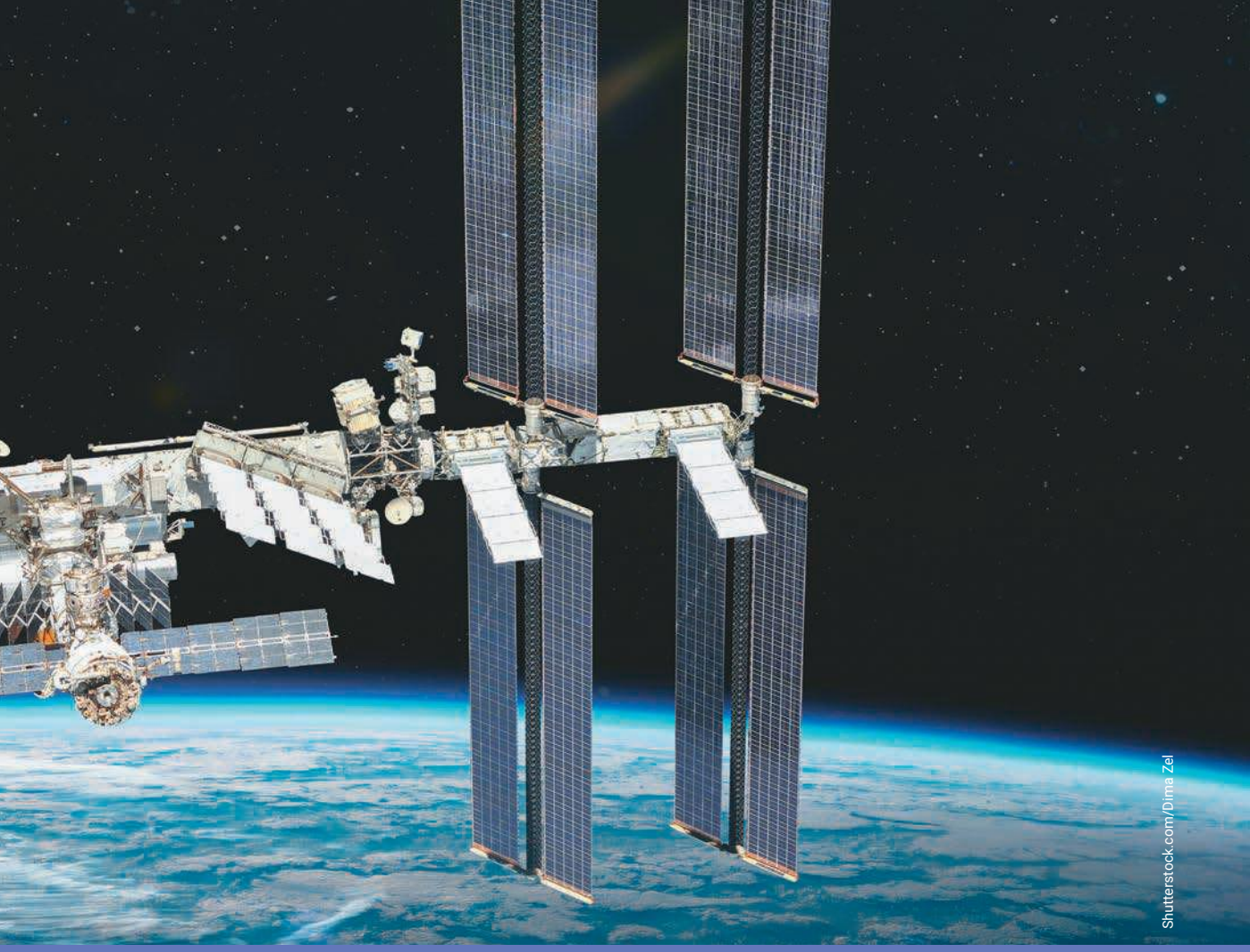
# 5



MEASUREMENT AND SPACE

# AREA AND VOLUME

The International Space Station (ISS) is the largest man-made object in space.
The construction of the ISS began in 1998. It circles the Earth every 90 minutes at a height of approximately 400 km above the Earth. ISS has a pressurised volume of 915 $m^3$ and it is 109 metres long, the same length as a Boeing 707. The solar panels of the station cover an area of 2500 $m^2$ and produce enough electricity to power 10 average-sized homes.




## Chapter outline

| | Working mathematically | | | | |
|---|---|---|---|---|---|
| **5.01** Perimeter | U | F | PS | R | |
| **5.02** Parts of a circle | U | F | | R | C |
| **5.03** Circumference of a circle | U | F | PS | R | |
| **5.04** Metric units of area | U | F | | R | C |
| **5.05** Areas of rectangles, triangles and parallelograms | U | F | PS | R | |
| **5.06** Areas of composite shapes | U | F | PS | R | |
| **5.07** Area of a circle | U | F | PS | R | |
| **5.08** Perimeter and area of a sector | U | F | PS | R | |
| **5.09** Area of a trapezium | U | F | PS | R | |
| **5.10** Areas of kites and rhombuses | U | F | | | |
| **5.11** Metric units of volume | U | F | | R | C |
| **5.12** Volume of a prism | U | F | PS | R | C |
| **5.13** Volume of a cylinder | U | F | PS | R | |
| **5.14** Volume and capacity | U | F | PS | R | C |

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

## Wordbank

**capacity** The amount of fluid (liquid or gas) in a container

**circumference** The perimeter of a circle; a circle's outer boundary

**cross-section** A 'slice' of a solid, taken across the solid rather than along it

**cubic metre** The volume of a cube that measures 1 m by 1 m by 1 m

**perpendicular height** The height of a shape taken at right angles to its base

**pi (π)** A special irrational number, approximately 3.1416, used in calculating circular measurements

**radius** The distance from the centre of a circle to the circle's edge

**sector** A 'pizza-slice' of a circle, cut along 2 radii

**Quiz**
Wordbank 5

## Videos (12):

**5.01** Perimeter of composite shapes
**5.03** Circumference of a circle
**5.04** Metric units of area
**5.05** Area formulas for triangles and quadrilaterals
**5.06** Areas of composite shapes
**5.07** Area of a circle
**5.08** Perimeter and area of a sector
**5.09** Area formulas for triangles and quadrilaterals • Area of a trapezium
**5.10** Area formulas for triangles and quadrilaterals
**5.11** Metric units of volume
**5.12** Volume of a prism
**5.13** Volumes of prisms and cylinders
**5.14** Volume and capacity

## *Twig* videos (2):

**5.03** Discovering pi: Archimedes
**5.07** Beating the U-boats

## *PhET* interactive (1):

**5.04** Area builder

## Quizzes (6):

- Wordbank 5
- SkillCheck 5
- Mental skills 5A
- Mental skills 5B
- Language of maths 5
- Test yourself 5

## Skillsheets (4):

**SkillCheck** Multiplying by 10, 100, 1000
**5.04, 5.05** What is area?
**5.11** What is volume?
**5.12** Solid shapes • What is volume?

## Worksheets (23):

**5.01** A page of composite shapes
**5.02** Parts of a circle • Discovering pi
**5.03** Discovering pi • A page of circles • Circumference and area
**5.04** Australian areas
**5.05** Rectangle and triangle areas
**5.06** A page of composite shapes • Odd areas • Composite areas • Approximating the area of a circle
**5.07** A page of circles • Circumference and area • Area ID • Area and perimeter investigations
**5.08** A page of circular shapes • Applications of area • Back-to-front problems • Area and perimeter investigations
**5.10** Quadrilateral and triangle areas
**5.11** Volume
**5.12** Volume • Measuring shapes review
**5.14** Volume and capacity • Volume and capacity cards • Capacity

**Mind map** Area and volume

## Puzzles (9):

**5.03** Circle crossword
**5.05** Area group clues
**5.06** Areas 1 • Area puzzles
**5.07** Circle areas
**5.08** Carpet talk
**5.10** Areas of quadrilaterals and triangles
**5.14** Capacity puzzle

**Language of maths** Area and volume find-a-word

## Presentation (1):

**5.01** Calculating the perimeter

Nelson MindTap

To access resources above, visit
**cengage.com.au/nelsonmindtap**

## In this chapter you will:

- convert between units of area
- calculate perimeters and areas of triangles, quadrilaterals and composite shapes
- learn the terminology for parts of a circle
- investigate the line and rotational symmetries of circles and circular shapes
- calculate perimeters and areas of circles, sectors and circular shapes, giving answers in terms of $\pi$ and as decimals
- convert between units of volume and capacity
- calculate volumes of prisms and cylinders



Quiz
SkillCheck 5

Skillsheet
Multiplying by 10, 100, 1000

## SkillCheck ANSWERS ON P. 583

**1** Copy and complete each conversion.

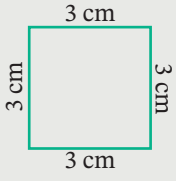
**a** 20 cm = _____ mm **b** 350 cm = _____ m **c** 2500 m = _____ km

**d** 46 mm = _____ cm **e** 4 km = _____ m **f** 5200 m = _____ km

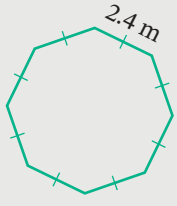
**g** 600 m = _____ km **h** 8200 mm = _____ m **i** 0.4 m = _____ mm

**2** Find the perimeter of each shape.

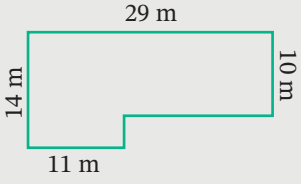
**a**



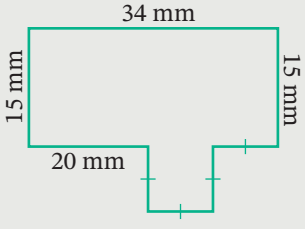

**b**



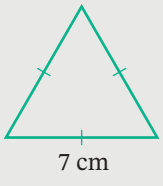

**c**



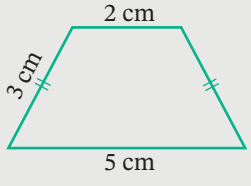

**d**




**e**

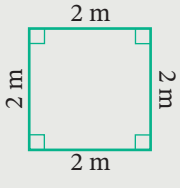



**f**

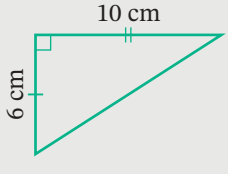



**3** Find the area of each shape.

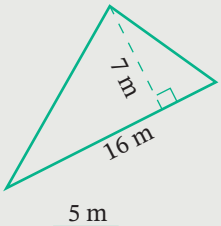
**a**



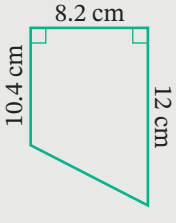

**b**



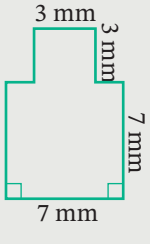

**c**



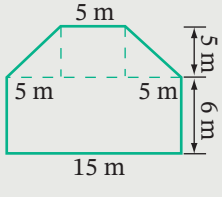

**d**




**e**




**f**



# 5.01 Perimeter

**Worksheet**
A page of composite shapes

**Presentation**
Calculating the perimeter

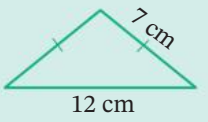
## Perimeter

The **perimeter** of a shape is the distance around the shape.
It is the sum of the lengths of the sides of the shape.

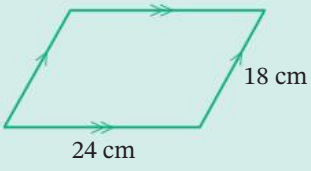
## Example 1

Find the perimeter of each shape.

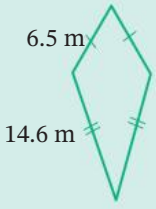
**a**




**b**




**c**




### Solution

**a** Perimeter $= 7 + 7 + 12$
$= 26$ cm

Adding the 3 sides of the isosceles triangle.

**b** Perimeter $= 24 + 18 + 24 + 18$
$= 84$ cm

In a parallelogram, opposite sides are equal.

**c** Perimeter $= 6.5 + 6.5 + 14.6 + 14.6$
$= 42.2$ m

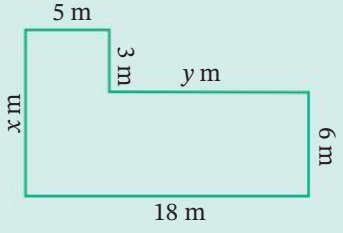
In a kite, adjacent sides are equal.

**Video**
Perimeter of composite shapes

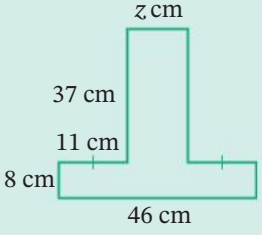
## Example 2

Find the perimeter of each composite shape.

**a**




**b**




### Solution

**a** Find the unknown sides first.
$x = 3 + 6 = 9$
$y = 18 - 5 = 13$

Perimeter $= 9 + 5 + 3 + 13 + 6 + 18$
$= 54$ m

**b** Find the unknown sides first.
$z = 46 - 11 - 11 = 24$

Perimeter $= 8 + 11 + 37 + 24 + 37 + 11 + 8 + 46$
$= 182$ cm


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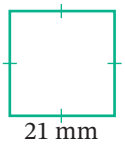
**EXERCISE 5.01** ANSWERS ON P. 583

## Perimeter U F PS R

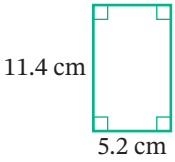
EXAMPLE 1

**1** Find the perimeter of each shape.

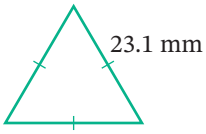
**a**



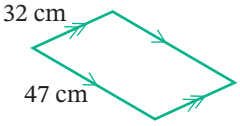

**b**



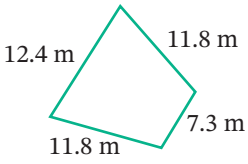

**c**



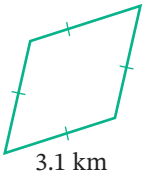

**d**




**e**



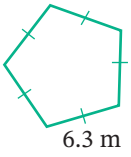

**f**



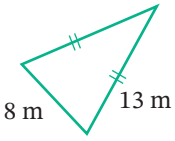

EXAMPLE 2

**2** Find the perimeter of each shape.

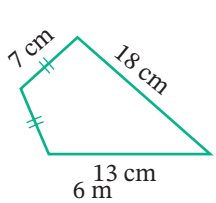
**a**



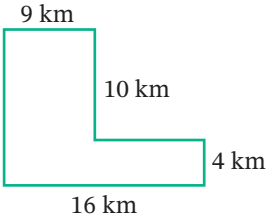

**b**



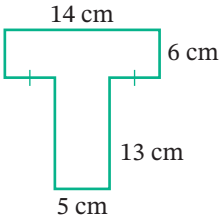

**c**



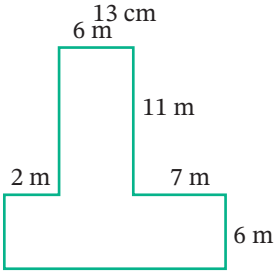

**d**



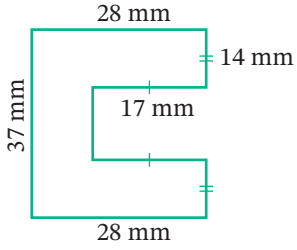

**e**



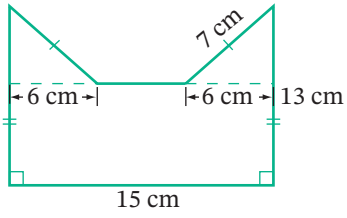

**f**



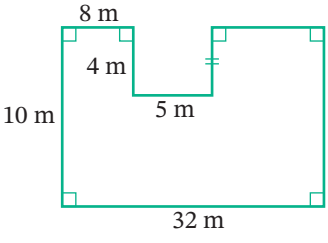

**g**




**h**




**i**




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**3** Find the perimeter of:

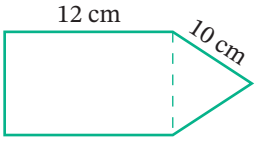
**a** a square of side length 3 cm

**b** a parallelogram with adjacent sides of length 7 m and 10 m

**c** an equilateral triangle of side length 5 cm

**d** an isosceles triangle with equal sides of 9 m and a side of 4 m

**4** This composite shape is made up of a rectangle and an equilateral triangle. Find its perimeter.
Select the correct answer **A**, **B**, **C** or **D**.




**A** 22 cm **B** 44 cm

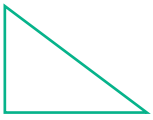
**C** 54 cm **D** 64 cm

**5** Which of the following lengths is the closest to the perimeter of your classroom?
Select **A**, **B**, **C** or **D**. R

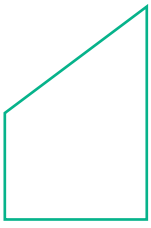
**A** 250 m **B** 2.5 km **C** 250 cm **D** 25 m

**6** Measure the sides of each shape in millimetres and find its perimeter.

**a**



**b**



**c**



**d**



**7** A square has a perimeter of 24 cm. What is the length of one side?
Select **A**, **B**, **C** or **D**. R

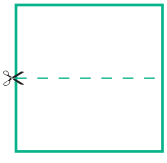
**A** 6 cm **B** 8 cm **C** 12 cm **D** 96 cm

**8** A rectangle has a perimeter of 40 m. If its length is 13 m, find its width. R

**9** A rhombus has a perimeter of 18 m. Find the length of one side. R

**10** A kite has a perimeter of 26 cm. If one of its longer sides is 9 cm, find the length of a shorter side. R

**11** A square is cut in half as shown. The 2 rectangles have a combined perimeter of 24 cm. Find the perimeter of the original square. PS R



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**12** Jade uses a piece of wire to build a square with a side of 12 cm. If, from the same wire, she builds a rectangle with a width of 6 cm, what is the length of the rectangle? PS R

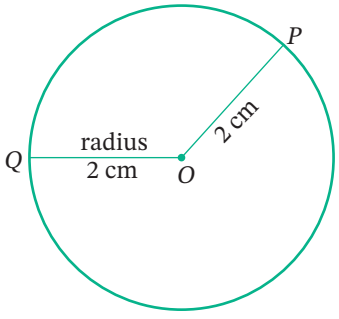
**13** Farouk's rectangular garden is 14 m long and 10 m wide. A hose is buried inside the garden $1\frac{1}{2}$ m from each side. How long is the hose? PS R

# Parts of a circle

5.02



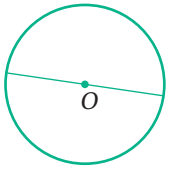
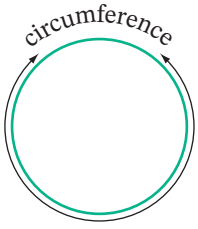
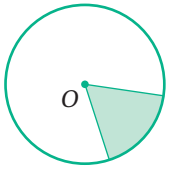
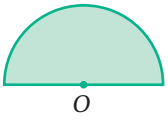
The circle is a completely round shape. Every point on a circle is the same distance from its centre. The circle shown has centre $O$, and points $P$ and $Q$ are exactly 2 cm from $O$. This distance, from the centre to the edge of the circle, is called the **radius** of the circle (*plural*: radii).



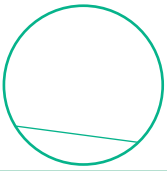
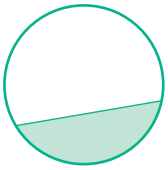
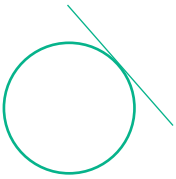

Worksheet
Parts of a circle

## Circle words

There are special words that describe the parts of a circle or the related lines.

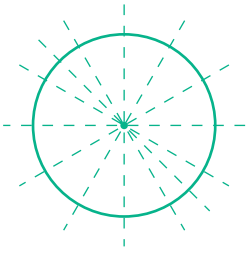
| Diameter | Circumference | Arc |
|---|---|---|
| An interval running from one side of a circle to the other side and through the circle's centre   | The perimeter of a circle   | Part of the circumference  |
| **Sector** | **Quadrant** | **Semicircle** |
| A 'pizza-slice' of a circle cut along 2 radii   | A sector that is a quarter of a circle, with angle 90°   | Half a circle   |

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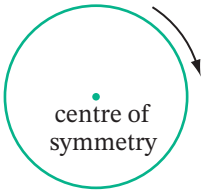
| Chord | Segment | Tangent |
|---|---|---|
| An interval joining any 2 points on a circle | A region cut off by a chord | A line outside a circle that touches it at exactly one point |
|  |  |  |

## Circle symmetry

A circle can be folded in half in an infinite number of ways, along any diameter, so it has an *infinite* number of axes of symmetry.



A circle can also be rotated through any angle size and still map onto itself, so it has an *infinite* order of rotational symmetry. The centre of symmetry is the centre of the circle.



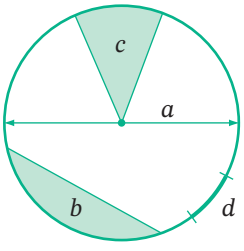

**EXERCISE 5.02** ANSWERS ON P. 583

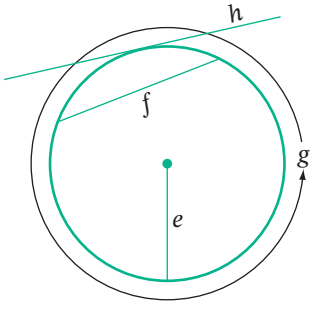
### Parts of a circle **U F R C**

**1** Use a ruler and compasses to construct a circle of:

**a** radius 4 cm **b** radius 2.5 cm **c** radius 56 mm

**2** Name each part of a circle marked by letters in these diagrams. **C**




**3** Match each description to the correct word from this list. **C**

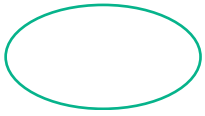
| | | |
|---|---|---|
| circumference | radius | sector |
| tangent | arc | quadrant |
| segment | diameter | chord |

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**a** The distance from the centre of a circle to its side
**b** A quarter of a circle
**c** A line that touches the outside of a circle once
**d** The distance from one side of a circle to the other side, through the circle's centre
**e** A line segment from one side of the circle to the other side, not through the centre
**f** Part of the circumference of a circle
**g** The area inside a circle formed by 2 radii and an arc
**h** The distance around a circle
**i** The area inside a circle formed by a chord and an arc

**4** This ellipse (oval) is a 'flattened' circle. How many axes of symmetry has an ellipse? Select the correct answer **A**, **B**, **C** or **D**. R C



**A** 1 **B** 2
**C** 4 **D** an infinite number

**5** What is the order of rotational symmetry of an ellipse? Select **A**, **B**, **C** or **D**. R C

**A** 1 **B** 2 **C** 4 **D** infinite number

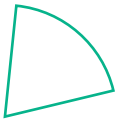
**6** Sketch a diagram of each term. C

**a** diameter **b** circumference **c** chord
**d** sector **e** tangent **f** segment
**g** quadrant **h** arc **i** radius

**7** Copy each shape. Then state: R C

**i** the number of axes of symmetry in the shape and draw them
**ii** the order of rotational symmetry of the shape if it has rotational symmetry.

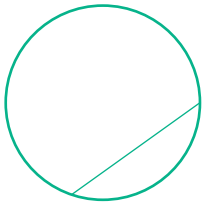
**a**



**b**



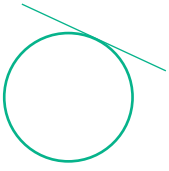
**c**



**d**



**e**



**f**



**8** Using a radius of 4 cm, draw:

**a** a semicircle
**b** a quadrant
**c** a sector whose angle at the centre of the circle is 60°
**d** an arc that is longer than half the circumference of the circle
**e** a circle with a chord of length 2.5 cm
**f** a circle with a tangent of length 4 cm.

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## Technology

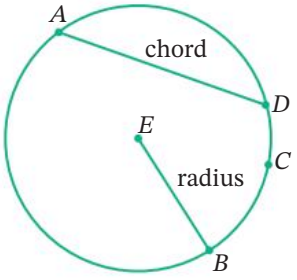
### Circle parts

In this activity you will use your dynamic geometry software to construct a circle and display all the circle parts.

**1** Construct a large circle, radius 4 cm.

**2** Draw a radius and a chord in your circle.

**3** Label the chord and the radius as shown.




**4** Construct each circle part below on your page and label each one.

| | | | | |
|---|---|---|---|---|
| centre | diameter | circumference | sector | arc |
| segment | semicircle | tangent | quadrant | |

**5** What are **concentric circles**? Find out and construct a diagram showing 3 or more concentric circles. Remember to clearly label the centre of the circle.

## Investigation

### Measuring the circumference of a circle

Worksheet
Discovering pi

This is a measuring activity to be done in groups of 3.

*You will need*: A measuring tape, or some string and a ruler; graph paper; 6 round objects such as cans, round cake tins, pipes, coins, drums, bottles, and so on.

**1** Copy this table.

| Object | Diameter, $d$ (mm) | Circumference, $C$ (mm) | The ratio $\frac{C}{d}$ |
|---|---|---|---|
| a | | | |
| b | | | |
| c | | | |
| d | | | |
| e | | | |
| f | | | |

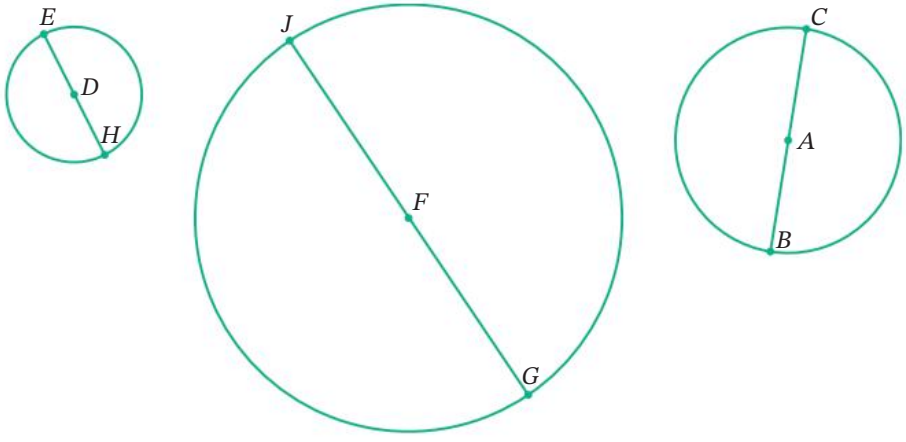
**2** Measure (in millimetres) to find the diameter and the circumference of 6 different objects. Record your results in the table.

**3** Calculate the ratio $\frac{C}{d}$ for each object. Round your answers to 2 decimal places.

## Technology

### The circumference and diameter of a circle

**1** Use your dynamic geometry software to construct 3 different circles. Construct the diameter of each circle.




**2** Calculate each circle's circumference and diameter.

**3** For each circle, use your calculator to find $\frac{\text{length of circumference}}{\text{length of diameter}}$

**4** What do you notice? Discuss this with another student.



# Circumference of a circle 5.03

The **circumference** of a circle can be found by multiplying its **diameter** by a special number called **pi** (pronounced 'pie'), represented by the Greek letter $\pi$. For any circle, $\frac{\text{circumference}}{\text{diameter}} = \pi = 3.14\ldots$

Pi is often estimated as 3.14, but a more accurate value can be found on your calculator when you press the $\pi$ key (you may need to press SHIFT first). As a decimal, the digits of $\pi$ continue endlessly without any repeats or patterns:

$\pi = 3.141\ 592\ 653\ 589\ 793\ \ldots$

so, like $\sqrt{2}$, it is called an **irrational number**.

This number was named 'pi' by the Swiss mathematician Leonhard Euler in 1737.

The formula for the circumference of a circle is $C = \pi \times \text{diameter} = \pi d$.

Because the diameter of a circle is double its radius, the circumference, $C$, of a circle with radius $r$ is $C = \pi \times 2 \times \text{radius} = 2\pi r$.

**Worksheets**
Discovering pi

A page of circles

Circumference and area

**Puzzle**
Circle crossword

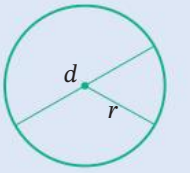
**Video**
Discovering pi: Archimedes

## Circumference of a circle

The **circumference (perimeter) of a circle** is:

$C = \pi \times \text{diameter}$ or $C = 2 \times \pi \times \text{radius}$

$C = \pi d$ or $C = 2\pi r$




Video
Circumference of a circle

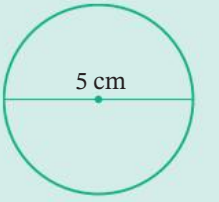
## Example 3

**a** Estimate the circumference of a circle with a diameter of 5 cm.

**b** Calculate the circumference of the circle:

**i** correct to 2 decimal places

**ii** in terms of $\pi$




### Solution

**a** $C = \pi d$

$= \pi \times 5$

$\approx 3 \times 5$ $\pi$ is approximately 3.

$= 15$ cm

**b** **i** $C = \pi d$ On your calculator enter π × 5 =.

$= \pi \times 5$

$= 15.707\,963\ldots$

$\approx 15.71$cm

**ii** $C = \pi d$

$= \pi \times 5$

$= 5\pi$ cm

Like writing an answer as a surd in Pythagoras' theorem problems, writing an answer in terms of $\pi$ is more exact as there is no rounding involved.

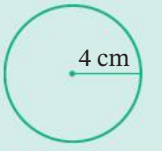
Video
Circumference of a circle

## Example 4

Calculate the circumference of the circle:

**a** correct to one decimal place

**b** in terms of $\pi$




### Solution

**a** $C = 2\pi r$

$= 2 \times \pi \times 4$

$= 25.13274\ldots$

$\approx 25.1$ cm

**b** $C = 2\pi r$

$= 2 \times \pi \times 4$

$= 8\pi$ cm


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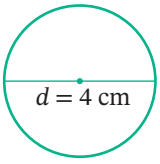
**EXERCISE 5.03** ANSWERS ON P. 583

## Circumference of a circle U F PS R

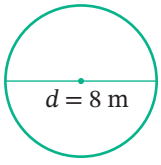
EXAMPLE 3

**1** Estimate the circumference of each circle.

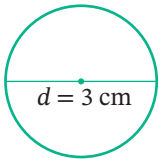
**a**



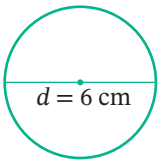

**b**




**c**




**d**



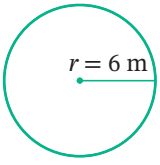

**2** Calculate, correct to 2 decimal places, the circumference of each circle in question **1**.

**3** Calculate the circumference of each circle in question **1** in terms of $\pi$.

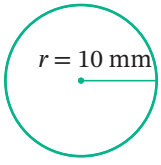
EXAMPLE 4

**4** Estimate the circumference of each circle.

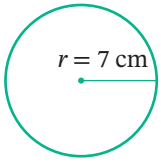
**a**



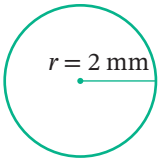

**b**




**c**




**d**




**5** Calculate, correct to one decimal place, the circumference of each circle in question **4**.

**6** Calculate the circumference of each circle in question **4** in terms of $\pi$.

**7** A child's inflatable swimming pool has a diameter of 1.4 m. Find its circumference correct to 2 decimal places.

**8** Tina's bicycle has wheels with a diameter of 60 cm. PS

**a** How far does the bicycle move when a wheel turns around once?

**b** If Tina cycles 900 m to school, how many complete turns does the bicycle wheel make?

**9** The Earth has a radius of 6370 km. Find the distance around the Equator.

**10** A 20-cent coin has a radius of 14 mm. Calculate its circumference.

**11** This tin of tomatoes has a diameter of 75 mm. If the label wraps around the tin completely, how long is the label? Answer correct to the nearest millimetre. PS R




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**12** Calculate, correct to 2 decimal places, the circumference of a circle with:

**a** a diameter of 2 cm

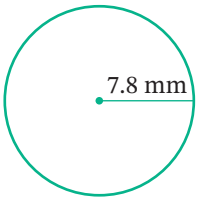
**b** a diameter of 7 cm

**c** a radius of 3 cm

**d** a radius of 5 cm

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**13** Which of the following intervals best shows the circumference of this circle? Select **A**, **B**, **C** or **D**. R




**A** _____

**B** _____

**C** _____

**D** _____

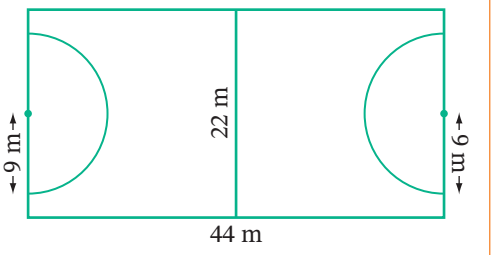
**14** The International Space Station completes one circular orbit of the Earth at a height of 400 km. If the radius of the Earth is approximately 6400 km, find the distance travelled by the station. Select the closest answer **A**, **B**, **C** or **D**.

**A** 2513 km **B** 37 699 km **C** 40 212 km **D** 42 726 km

**15** Tape has been placed on all the lines of this indoor hockey pitch. How much tape was used? Express the answer: PS R

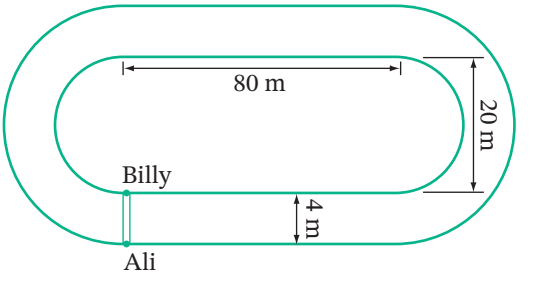
**a** correct to one decimal place

**b** in terms of $\pi$.




**16** A circle has a circumference of 50.265 cm. Find its diameter, correct to the nearest centimetre. R

**17** Ali and Billy raced each other around this athletic track. Ali ran along the outside perimeter while Billy ran along the inside perimeter. After one lap of the track, who ran the longer distance, and by how much? Answer correct to the nearest metre. PS R






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## Investigation

### Belt around the Equator



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Imagine that we wrapped a giant belt tightly around the Earth, along the Equator. This belt would touch the Earth at all points on its circumference, assuming the Earth was a perfect sphere or ball (with no mountains or valleys).

Now suppose we added one extra metre to the length of the belt. Then it would become loose and not touch the Earth any more. There would be a small gap between the Earth and the belt. How wide is this gap?

1 In a group of 2 to 4 people, guess whether you could:
   - **a** slip your hand between the belt and the ground
   - **b** crawl under the belt
   - **c** sit under the belt
   - **d** stand under the belt.

2 If the diameter of the Earth is 12 755 m, calculate to 2 decimal places, the length of the tight belt.

3 Add 1 m to the length of the tight belt.

4 Calculate the diameter of the longer, looser belt.

5 So what is the length of the gap between the Earth and the loose belt?

6 Check whether your guesses in question **1** were correct.

5.04

# Metric units of area

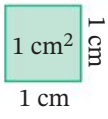
5.04

The **area** of a shape is the amount of surface covered by the shape. Area is measured in **square units**.

**square centimetre ($cm^2$)**

1 cm = 10 mm

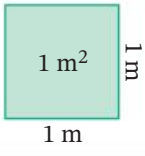
$1\ cm^2 = 1\ cm \times 1\ cm$
$= 10\ mm \times 10\ mm$
$= 100\ mm^2$




**square metre ($m^2$)**

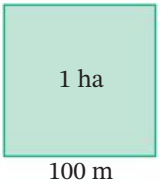
1 m = 100 cm

$1\ m^2 = 1\ m \times 1\ m$
$= 100\ cm \times 100\ cm$
$= 10\ 000\ cm^2$




**hectare (ha)**

$1\ ha = 100\ m \times 100\ m$
$= 10\ 000\ m^2$

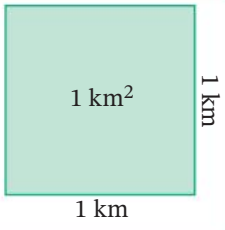



**square kilometre ($km^2$)**

1 km = 1000 m

$1\ km^2 = 1\ km \times 1\ km$
$= 1000\ m \times 1000\ m$
$= 1\ 000\ 000\ m^2$

$1\ km^2 = 100\ ha$




**Skillsheet**
What is area?

**Worksheet**
Australian areas

**Interactive**
Area builder

A **square centimetre** is about the area of a fingernail.

A **square metre** is approximately the area of the floor of a large shower recess.

A **hectare** is about the area of Sydney (Allianz) Football Stadium, or the grassed area inside an athletics (400 m) track.

A **square kilometre** is about the area of a theme park, like Dreamworld on the Gold Coast.

Note that, while 1 cm = 10 mm, 1 $cm^2$ = 100 $mm^2$ (double the number of 0s), and while 1 m = 100 cm, 1 $m^2$ = 10 000 $cm^2$ (double the number of 0s).

## Metric units of area

1 $cm^2$ = 100 $mm^2$

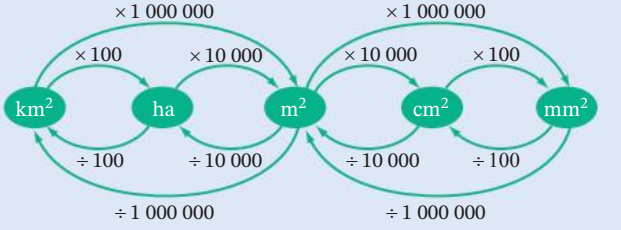
1 $m^2$ = 10 000 $cm^2$

1 $m^2$ = 1 000 000 $mm^2$

1 ha = 10 000 $m^2$

1 $km^2$ = 1 000 000 $m^2$

1 $km^2$ = 100 ha




Video
Metric units of area

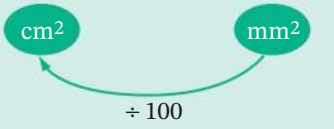
## Example 5

Convert:

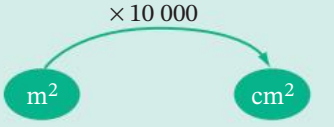
**a** 800 $mm^2$ to $cm^2$ **b** 4.6 $m^2$ to $cm^2$ **c** 250 000 $m^2$ to ha

### Solution

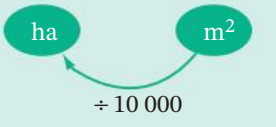
**a** 800 $mm^2$ = 800 ÷ 100 $cm^2$
= 8 $cm^2$




**b** 4.6 $m^2$ = 4.6 × 10 000 $cm^2$
= 46 000 $cm^2$




**c** 250 000 $m^2$ = 250 000 ÷ 10 000 $cm^2$
= 25 ha




**EXERCISE 5.04** ANSWERS ON P. 584

## Metric units of area U F R C

**1** What units of area would you use when measuring the area of: R C

**a** your home? **b** your pencil case? **c** a football field?

**d** the state of NSW? **e** your body? **f** a desk top?

Foundation Standard Complex


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**2** How many square centimetres are there in 3.57 $m^2$?
Select the correct answer **A**, **B**, **C** or **D**. R

**A** 35 700 **B** 357 **C** 0.357 **D** 0.0357

EXAMPLE 5

**3** Copy and complete each conversion. R

**a** 100 $mm^2$ = _____ $cm^2$ **b** 750 $mm^2$ = _____ $cm^2$
**c** 4500 $cm^2$ = _____ $m^2$ **d** 34 000 $m^2$ = _____ $km^2$
**e** 60 000 $mm^2$ = _____ $m^2$ **f** 432 000 $cm^2$ = _____ $m^2$
**g** 950 $mm^2$ = _____ $m^2$ **h** 7600 $m^2$ = _____ ha
**i** 4.8 $mm^2$ = _____ $cm^2$ **j** 30 $m^2$ = _____ $cm^2$
**k** 850 $cm^2$ = _____ $mm^2$ **l** 6.9 $km^2$ = _____ $m^2$
**m** 54 $cm^2$ = _____ $mm^2$ **n** 700 ha = _____ $m^2$
**o** 0.48 $m^2$ = _____ $cm^2$ **p** 0.45 $km^2$ = _____ $m^2$

**4** How many square metres are there in one square kilometre? Select **A**, **B**, **C** or **D**. R

**A** 1000 **B** 10 000 **C** 100 000 **D** 1 000 000

**5** The solar panels of the International Space Station cover an area of 2500 $m^2$.
What is this area in $km^2$? R C

**6** A large cattle station in South Australia has an area of 30 028 $km^2$.
What is this in hectares? R C

**7** The Cradle Mountain–St Clair National Park in Tasmania has an area of 134 805 ha.
What is this in $km^2$? R C

**8** The centre of Sydney is approximately a rectangle bounded by George Street, Circular Quay, Macquarie and College Streets and Liverpool Street. This rectangle is about 1.75 km by 0.5 km. What is its area in: R C

**a** $km^2$? **b** hectares?

**9** Copy and complete each conversion. R

**a** 7600 $mm^2$ = _____ $m^2$ **b** 0.7 ha = _____ $cm^2$
**c** 8 $m^2$ = _____ $mm^2$ **d** 850 $mm^2$ = _____ $cm^2$
**e** 6.90 ha = _____ $m^2$ **f** 15.2 $m^2$ = _____ $cm^2$
**g** 8500 $m^2$ = _____ ha **h** 49 000 000 $m^2$ = _____ $km^2$

**10** The area of Australia is 7 682 300 $km^2$. Western Australia has an area of 2 526 000 $km^2$. Calculate, correct to one decimal place, the area of Western Australia as a percentage of the area of Australia.

**11** Sydney Airport has an area of 881 hectares. Sydney Harbour has an area of about 5500 hectares. R C

**a** What is the area of Sydney Airport in square kilometres?

**b** How many 'Sydney Airports' would fit onto Sydney Harbour? Answer correct to one decimal place.

# 5.05 Areas of rectangles, triangles and parallelograms

**Skillsheet**
What is area?

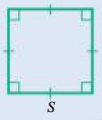
**Worksheet**
Rectangle and triangle areas

**Video**
Area formulas for triangles and quadrilaterals

**Puzzle**
Area group clues

## Area formulas

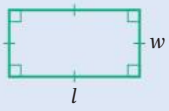
Square




$A = (\text{side})^2$

$A = s^2$

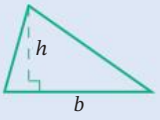
Rectangle




$A = \text{length} \times \text{width}$

$A = lw$

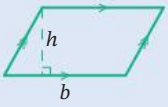
Triangle




$A = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$

$A = \frac{1}{2}bh$

Parallelogram



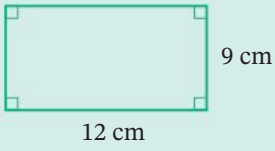

$A = \text{base} \times \text{perpendicular height}$

$A = bh$

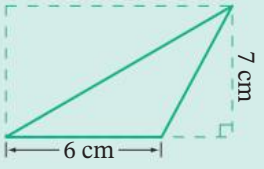
## Example 6

Find the area of each shape.

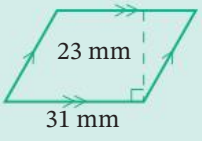
**a**




**b**




**c**




### Solution

**a** $A = lw$

$A = 12 \times 9$

$= 108 \text{ cm}^2$

Area of a rectangle = length × width

**b** $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6 \times 7$

$= 21 \text{ cm}^2$

Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

**c** $A = bh$

$= 31 \times 23$

$= 713 \text{ m}^2$

Area of a parallelogram = base × height


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**EXERCISE 5.05** ANSWERS ON P. 584

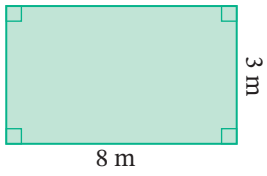
## Areas of rectangles, triangles and parallelograms U F PS R

EXAMPLE 6

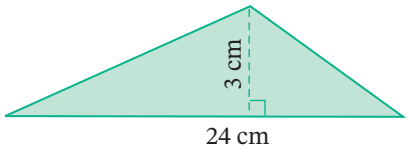
5.05

**1** Find the area of each shape.

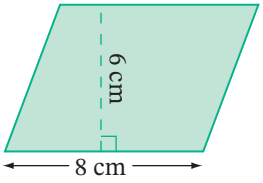
**a**



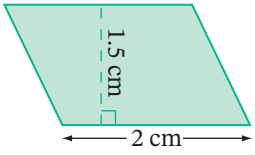

**b**



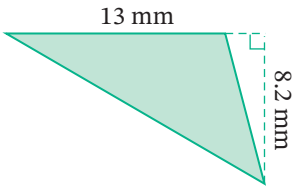

**c**



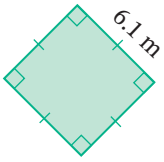

**d**



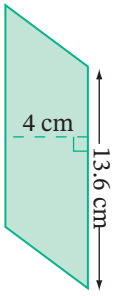

**e**



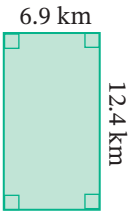

**f**



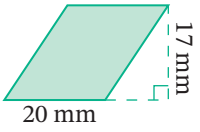

**g**



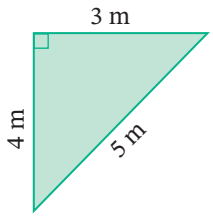

**h**



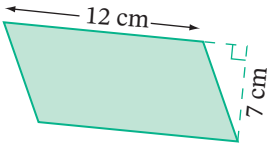

**i**



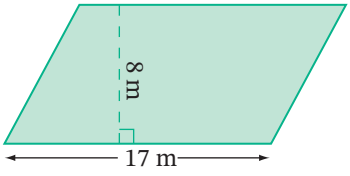

**j**




**k**



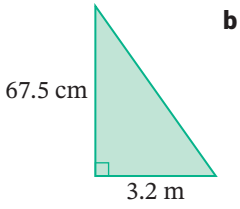

**l**



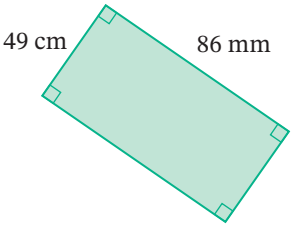

Foundation | Standard | Complex

**2** Find the area of each shape. Make sure that you change all lengths to the same units first.

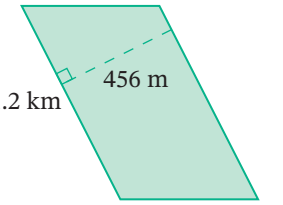
**a**



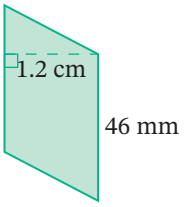

**b**



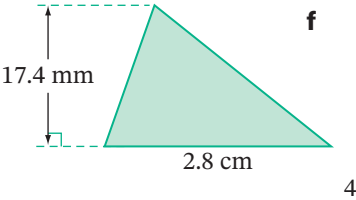

**c**



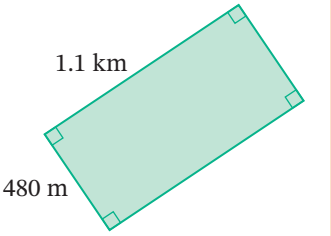

**d**




**e**



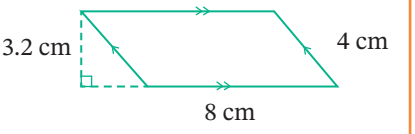

**f**




**3** One hectare of land is subdivided into blocks. If the average block measures 20 m by 25 m, how many blocks would there be?
Select the correct answer **A**, **B**, **C** or **D**. PS R

**A** 50 **B** 25 **C** 40 **D** 20

**4** Find the area of this shape. Select **A**, **B**, **C** or **D**.




**A** 12.8 cm$^2$ **B** 24 cm$^2$
**C** 25.6 cm$^2$ **D** 32 cm$^2$

**5** A square has an area of 25 cm$^2$. Find the length of one side. R

**6** A rectangle has an area of 27 m$^2$. If its length is 9 m, find its width. R

**7** A triangle has an area of 35 cm$^2$. If its base length is 10 cm, what is its perpendicular height? R

**8** A rectangle has an area of 72 mm$^2$. What could its length and width be? R

**9** The area of a square is 900 m$^2$. If the area of the square quadrupled, what would be the length of the new square? PS R

**10** The area of a triangle can be found using **Heron's formula** if you know the lengths of all 3 sides $a$, $b$ and $c$:

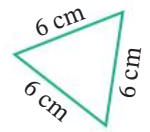
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \qquad \text{where } s = \text{half the perimeter} = \frac{a+b+c}{2}$$

Find the area of each triangle using Heron's formula, correct to one decimal place.

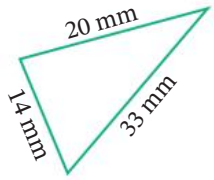
Foundation Standard Complex


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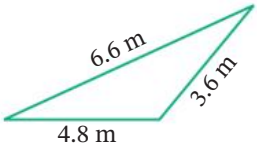
**a**



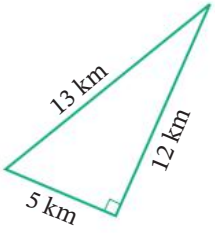

**b**




**c**




**d**




5.05

Quiz
Mental skills 5A

## Mental skills 5A: Maths without calculators ANSWERS ON P. 584

### Converting fractions and percentages to decimals

To convert a fraction into a decimal, change the denominator to a power of 10. This may require simplifying the fraction first.

**1** Study each example.

**a** $\frac{31}{50} = \frac{31 \times 2}{50 \times 2} = \frac{62}{100} = 0.62$

**b** $\frac{18}{40} = \frac{18 \div 2}{40 \div 2} = \frac{9}{20} = \frac{9 \times 5}{20 \times 5} = \frac{45}{100} = 0.45.$

**2** Now convert each fraction into a decimal.

**a** $\frac{7}{10}$ **b** $\frac{15}{25}$ **c** $\frac{14}{200}$

**d** $\frac{9}{20}$ **e** $\frac{49}{50}$ **f** $\frac{12}{30}$

To convert a percentage into a decimal, divide by 100 by moving the decimal point 2 places to the left.

**3** Study each example.

**a** $18\% = 0.18$ **b** $70\% = 0.70 = 0.7$ **c** $11.4\% = 0.114$ **d** $2.9\% = 0.029$

**4** Now convert each percentage to a decimal.

**a** 61% **b** 4% **c** 90%

**d** 38% **e** 27.1% **f** 0.7%

# 5.06 Areas of composite shapes

Video
Areas of composite shapes

Worksheets
A page of composite shapes
Odd areas
Composite areas

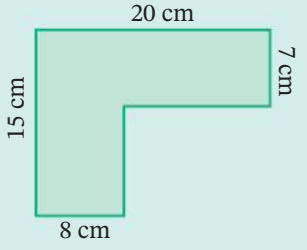
Puzzles
Areas 1
Area puzzles

Shapes made by combining other simpler shapes are called **composite shapes.**

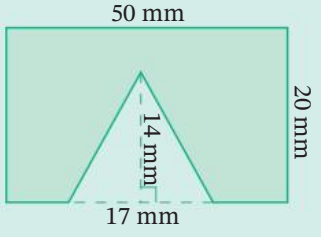
## Example 7

Find the area of each composite shape.

**a**



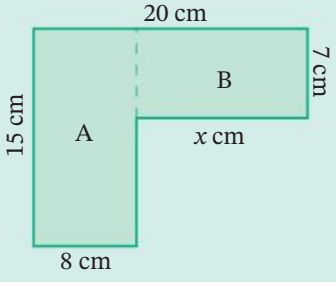

**b**




### Solution

**a** **Method 1**

Divide the shape into 2 rectangles, A and B, and find any unknown lengths.




$x = 20 - 8 = 12$

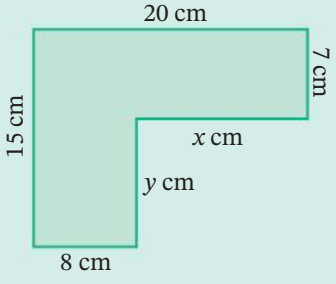
Area of shape = area of A + area of B

$= 8 \times 15 + 12 \times 7$

$= 204 \text{ cm}^2$

**Method 2**

The area can also be found by subtracting areas.




$x = 20 - 8 = 12$

$y = 15 - 7 = 8$

Area = large rectangle − small rectangle

$= 20 \times 15 - 12 \times 8$

$= 204 \text{ cm}^2$

**b** Area = area of rectangle − area of triangle

$= 50 \times 20 - \frac{1}{2} \times 17 \times 14$

$= 1000 - 119$

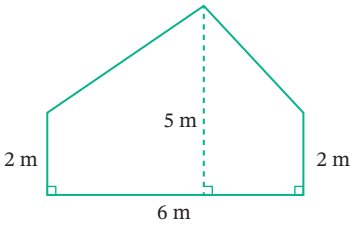
$= 881 \text{ mm}^2$


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## EXERCISE 5.06 ANSWERS ON P. 584

# Areas of composite shapes U F PS R

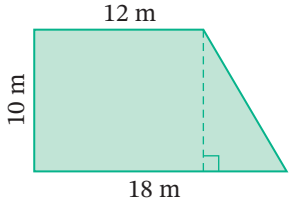
**1** What is the area of this shape? Select the correct answer **A**, **B**, **C** or **D**. R



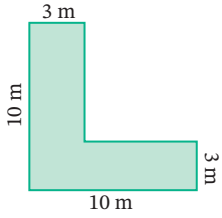

**A** 21 m²  **B** 27 m²
**C** 30 m²  **D** 42 m²

**2** Find the area of each composite shape. PS R

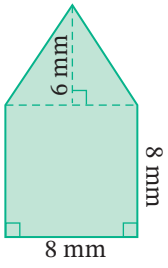
**a**



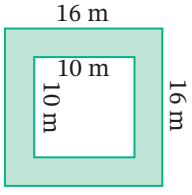

**b**



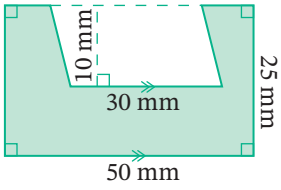

**c**



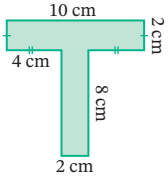

**d**



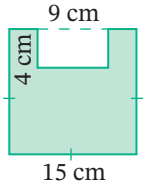

**e**



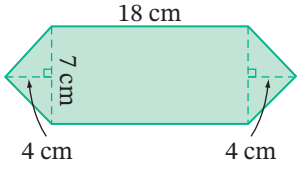

**f**



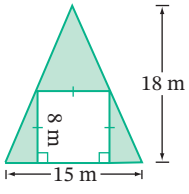

**g**



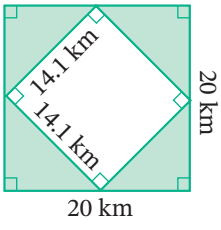

**h**



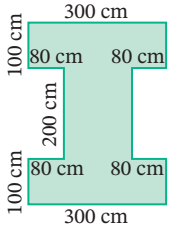

**i**



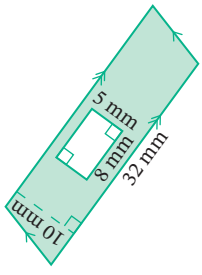

**j**



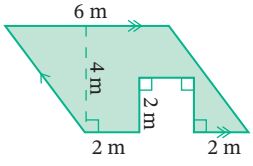

**k**




**l**

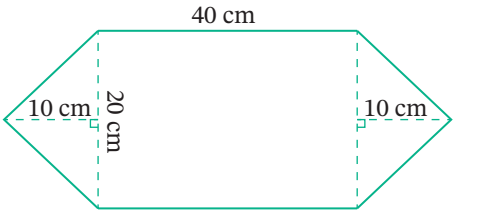



**m**




Foundation Standard Complex

**3** What is the area of this shape?
Select **A**, **B**, **C** or **D**.



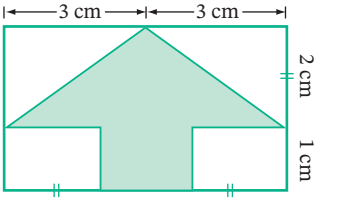

**A** 820 cm²

**B** 900 cm²

**C** 1000 cm²

**D** 1200 cm²

**4** What is the area of the shaded region?
Select **A**, **B**, **C** or **D**. PS R



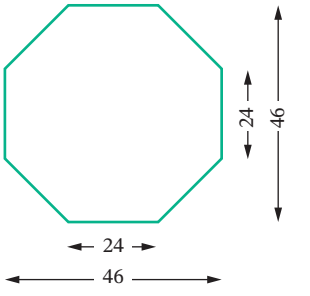

**A** 14 cm²

**B** 8 cm²

**C** 9 cm²

**D** 12 cm²

**5** Find the area of this octagon. All dimensions in centimetres. PS R



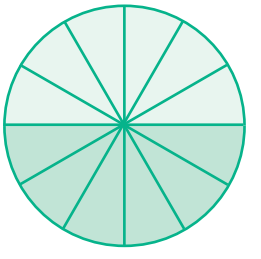

Worksheet
Approximating the area of a circle

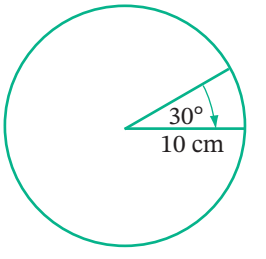
## Investigation

### Area by cutting out sectors

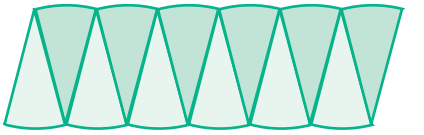
*You will need*: compasses, a ruler, a protractor and a pair of scissors.

**1** Draw a circle of radius 10 cm. Use your protractor to divide the circle into 12 sectors, each with angle size 30°.






**2** Cut out all the sectors and arrange them into a shape like a parallelogram.



Foundation Standard Complex

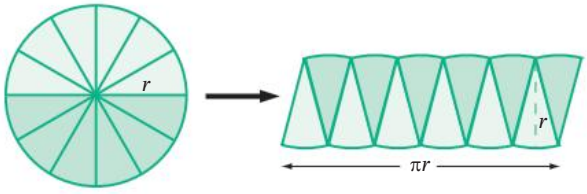

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3 Use a ruler to measure the *base* and the *height* of your 'parallelogram'.

a Now calculate the area of the 'parallelogram'.

5 Which formula gives the better approximation for the area of this circle: $A = 3 \times r^2$, or $A = 4 \times r^2$?

6 We will now find the actual formula for the area of a circle. A circle of radius $r$ is cut up into many sectors and rearranged into a 'parallelogram'.




What is the formula for the circumference of this circle?

7 Explain why the length of the 'parallelogram' is $\pi r$.

8 What is the formula for the area of this 'parallelogram'?

9 Explain why the area of the circle is $\pi r^2$.

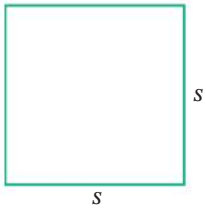


# Area of a circle

5.07

The area of a square is calculated by squaring its length.
$A = s \times s = s^2$

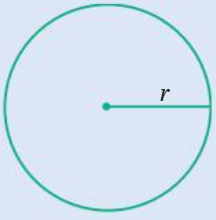
In a similar way, the area of a circle is calculated by squaring its radius and multiplying by $\pi$.




## Area of a circle

$A = \pi \times (\text{radius})^2$

$A = \pi r^2$




**Worksheets**
A page of circles

Circumference and area

Area ID

Area and perimeter investigations

**Puzzle**
Circle areas

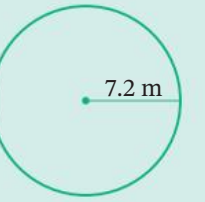
Video
Beating the U-boats

## Example 8

Calculate the area of this circle:

**a** correct to 2 decimal places

**b** in terms of $\pi$




### Solution

**a** $A = \pi r^2$

$= \pi \times 7.2^2$

$= 162.8601\ldots$

$\approx 162.86\ \text{m}^2$

On a calculator enter: π × 7.2 $x^2$ =.

Area is measured in square units, such as $\text{m}^2$

**b** $A = \pi r^2$

$= \pi \times 7.2^2$

$= 51.84\pi\ \text{m}^2$

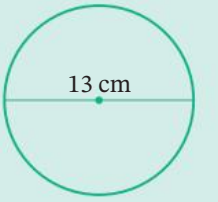
Video
Area of a circle

## Example 9

Find the area of this circle:

**a** correct to one decimal place

**b** in terms of $\pi$




### Solution

**a** $r = \frac{1}{2} \times 13 = 6.5\ \text{cm}$

$A = \pi r^2$

$= \pi \times 6.5^2$

$= 132.7322\ldots$

$\approx 132.7\text{cm}^2$

The radius is half of the diameter

**b** From **a**,

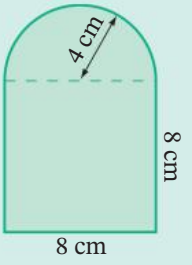
$A = \pi \times 6.5^2$

$= 42.25\pi\ \text{cm}^2$

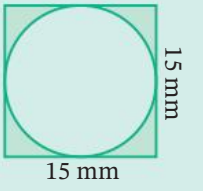
## Example 10

Find, correct to 2 decimal places, the area of each shaded region.

**a**




**b**




### Solution

**a** Area = area of square + area of semicircle

$= s^2 + \frac{1}{2} \times \pi r^2$

$= 8^2 + \frac{1}{2} \times \pi \times 4^2$

$= 89.132741\ldots$

$\approx 89.13\ \text{cm}^2$

Radius: $r = \frac{1}{2} 15 = 7.5\text{mm}$

**b** Shaded area = area of square − area of circle

$= s^2 - \pi r^2$

$= 15^2 - \pi \times 7.5^2$

$= 48.285413\ldots$

$\approx 48.29\ \text{mm}^2$

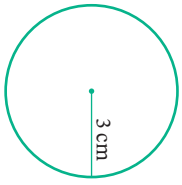
**EXERCISE 5.07** ANSWERS ON P. 584

## Area of a circle **U F PS R**

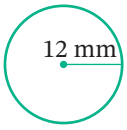
EXAMPLE 8

**1** Calculate, correct to one decimal place, the area of each circle.

**a**



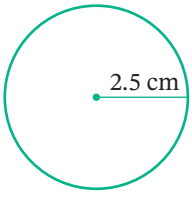

**b**



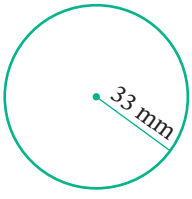

**c**




**d**




**e**

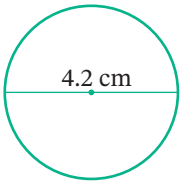



**2** Calculate the area of each circle in question **1** in terms of $\pi$.

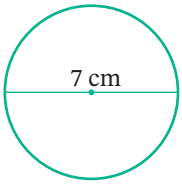
EXAMPLE 9

**3** Calculate, correct to 2 decimal places, the area of each circle.

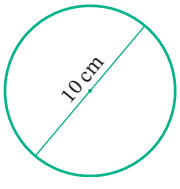
**a**




**b**




**c**



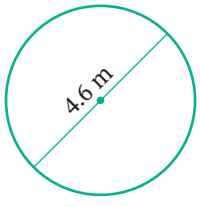

**4** Calculate the area of each circle in question **3** in terms of $\pi$.

**5** A sprinkler on the school playing field sprays water in a circular pattern of radius 13.1 m. Calculate the area being sprayed, to the nearest square metre.

**6** Kevlar is a very strong light plastic. What area of it is needed to make a solid bicycle wheel of diameter 685.5 mm? Give your answer in square centimetres, correct to 2 decimal places.

**7** A dinner plate has a radius of 14 cm. Calculate its area in terms of $\pi$.

**8** What is the area of this circle, correct to one decimal place?
Select the correct answer **A**, **B**, **C** or **D**.




**A** 14.5 m$^2$ **B** 66.5 m$^2$

**C** 7.2 m$^2$ **D** 16.6 m$^2$

5.07

Foundation Standard Complex

**9** The circular floor of a fishpond is to be covered in plastic. Find, correct to one decimal place, the area of plastic needed if the diameter of the pond is 2.8 m.

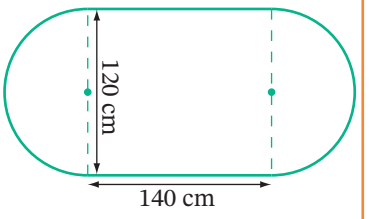
**10** Find the approximate radius of a circular tree trunk whose cross-sectional area is 454.40 cm$^2$. Give your answer correct to 2 decimal places. R

**11** The area of a DVD is 113 cm$^2$. Which of the following is closest to its diameter? Select **A**, **B**, **C** or **D**. R

**A** 12 cm **B** 13 cm **C** 12.5 cm **D** 13.1 cm

EXAMPLE 10

**12** What is the area of this shape closest to? Select **A**, **B**, **C** or **D**. PS R

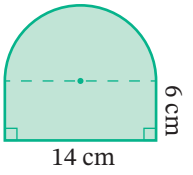



**A** 28 110 cm$^2$ **B** 22 455 cm$^2$

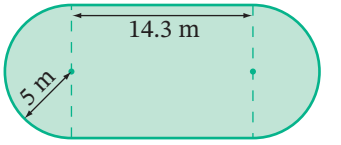
**C** 62 039 cm$^2$ **D** 11 570 cm$^2$

**13** Find the area of each shaded region, correct to 2 decimal places. PS R

**a**

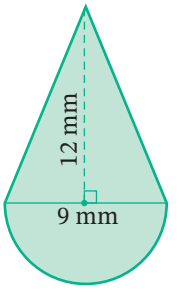



**b**

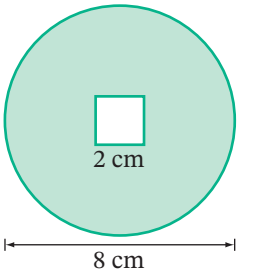



*Hint:* Combine 2 semicircles to make a whole circle.

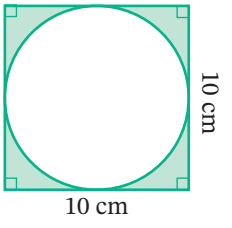
**c**



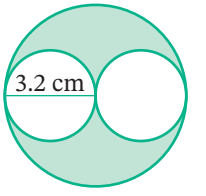

**d**




**e**




**f**

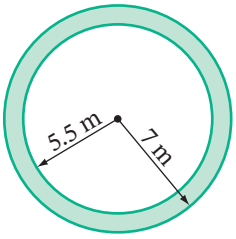



Foundation Standard Complex

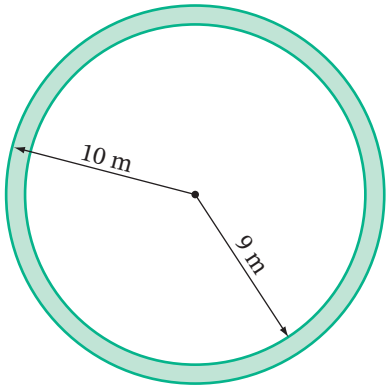

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**14** Each ring-shape or donut-shape shown below is called an **annulus**, made from 2 circles that are concentric (with the same centre) but of different size.

**a**




**b**



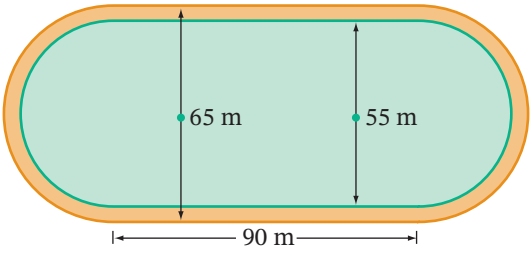

For each annulus, find, correct to one decimal place, the area of: R

**i** the larger circle

**ii** the smaller circle

**iii** the (shaded) annulus

**15** The dimensions of a running track are shown. The ends are circular. PS R




**a** Find the combined area of the (orange) track and the (green) grassed central area.

**b** Find the area of the grassed area.

**c** Find the area of the track.

**d** It is decided to cover the track with synthetic grass that costs $24.25 per square metre. How much will the track surface cost?

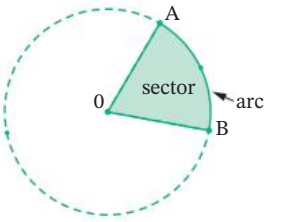
# 5.08 Perimeter and area of a sector

**Video**
Perimeter and area of a sector

A sector is a fraction of a circle.

An arc is a fraction of a circle's circumference.

As there are 360° in a circle, the fraction is $\frac{\text{angle at the centre}}{360°}$.




**Worksheets**
A page of circular shapes

Applications of area

Back-to-front problems

Area and perimeter investigations

**Puzzle**
Carpet talk

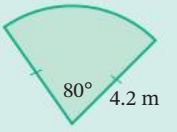
## Example 11

Find, correct to 2 decimal places, the perimeter and area of each shape.

**a**




**b**




### Solution

**a** This is a quadrant as the angle at the centre is 90°.

Fraction of circle is $\frac{90°}{360°} = \frac{1}{4}$

Perimeter = length of arc + radius + radius

Length of arc = $\frac{1}{4} \times$ circumference

$= \frac{1}{4} \times 2\pi r$

$= \frac{1}{4} \times 2 \times \pi \times 10$

$= 15.70796\ldots$

Perimeter of sector = 15.70796... + 10 + 10

= 35.70796...

≈ 35.71 mm

Area of sector = $\frac{1}{4} \times$ area of circle

$= \frac{1}{4} \times \pi \times 10^2$

$= 78.5398\ldots$

$\approx 78.54\ \text{m}^2$

**b** Angle at the centre is 80°.

Fraction of circle is $\frac{80°}{360°} = \frac{2}{9}$

Perimeter = length of arc + radius + radius

Length of arc = $\frac{2}{9} \times$ circumference

$= \frac{2}{9} \times 2\pi r$

$= \frac{2}{9} \times 2 \times \pi \times 4.2$

$= 5.8643\ldots$

Perimeter of sector = 5.8643... + 4.2 + 4.2

= 14.2643...

≈ 14.26 m

Area of sector = $\frac{2}{9} \times$ area of circle

$= \frac{2}{9} \times \pi \times 4.2^2$

$= 12.3150\ldots$

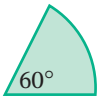
$\approx 12.32\ \text{m}^2$

**EXERCISE 5.08** ANSWERS ON P. 584

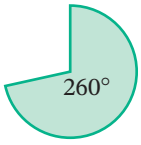
## Perimeter and area of a sector U F PS R

**1** What fraction of a whole circle is each sector?

**a** 

**b** 

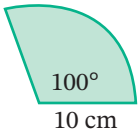

**c** 

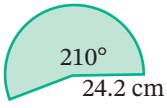

**d** 

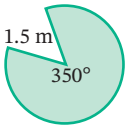

**2** Find the perimeter of each sector, correct to one decimal place.

EXAMPLE 11

**a** 


**b** 

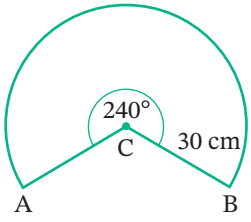

**c** 


**d** 


**3** Find the area of this shape, correct to the nearest square centimetre. Select the correct answer **A**, **B**, **C** or **D**.

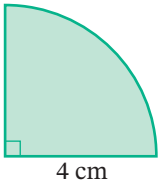
**A** 40 cm²  **B** 63 cm²

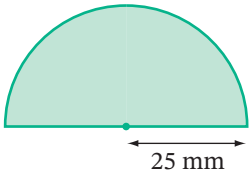
**C** 1200 cm²  **D** 1885 cm²

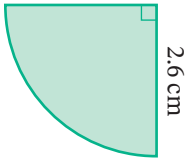


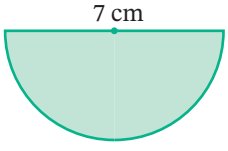

**4** Find the area of each sector in question **2** correct to 2 decimal places.

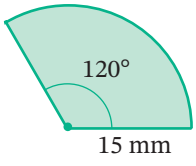
**5** Find, correct to one decimal place, the perimeter and area of each shape.

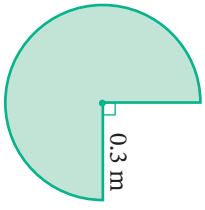
**a** 

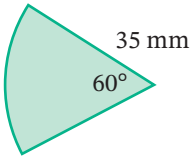

**b** 

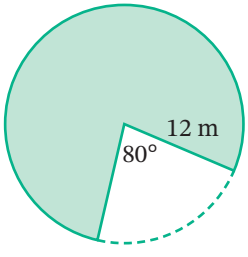

**c** 

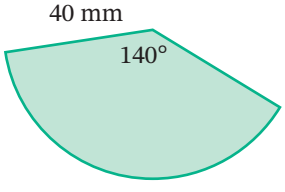

**d** 


**e** 


**f** 

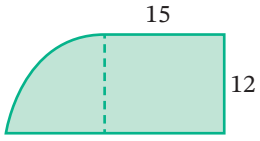

**g** 


**h** 


**i** 


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**6** Find the perimeter of this shape, correct to the nearest whole square unit. Select the correct answer **A**, **B**, **C** or **D**.




**A** 64 **B** 63

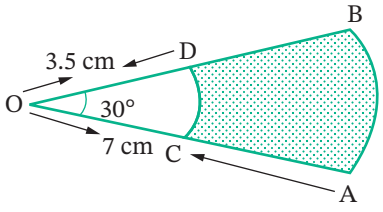
**C** 61 **D** 73

**7** The length of the minute hand on a clock is 14 cm. PS R

**a** How many degrees does the minute hand rotate through in one minute?

**b** Find, correct to 2 decimal places, the area swept by the minute hand in one minute.

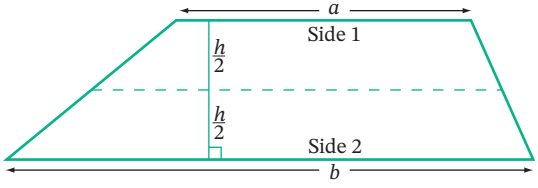
**8** Find the area of the shaded section, corrrect to 2 decimal places. R




## Investigation

### Area of a trapezium

**1** Draw 2 copies of this trapezium onto a sheet of paper. Draw a dotted line, parallel to the sides labelled $a$ and $b$, halfway down the height of each trapezium and label the sides as shown.




**2** Cut out one trapezium and then cut along the dotted line to make 2 smaller trapeziums. Join the pieces to make a long parallelogram.

**3** By measuring the height and base of the parallelogram to the nearest millimetre, find its area.

**4** By comparing the measurements of the parallelogram to the measurements of the original trapezium, suggest a general formula for finding the area of any trapezium. Check your answer with your teacher.

**5** Cut out the original trapezium and paste it and the parallelogram into your book.

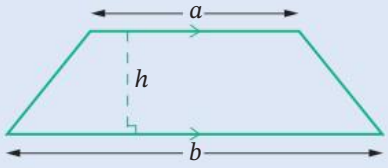
# Area of a trapezium

5.09

## Area of a trapezium

$A = \frac{1}{2} \times$ sum of lengths of parallel sides $\times$ perpendicular height

$A = \frac{1}{2}(a+b)h$




Video
Area formulas for triangles and quadrilaterals

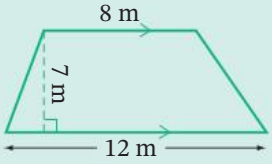


## Example 12

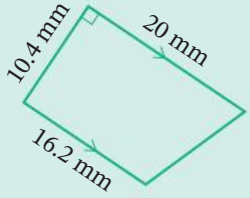
Video
Area of a trapezium

Find the area of each trapezium.

**a**




**b**




### Solution

**a**
$A = \frac{1}{2}(a+b)h$
$= \frac{1}{2}(8+12)\times 7$
$= 70\text{ m}^2$

**b**
$A = \frac{1}{2}(a+b)h$
$= \frac{1}{2}(20+16.2)\times 10.4$
$= 188.24\text{ mm}^2$



Alamy Stock Photo/lowefoto

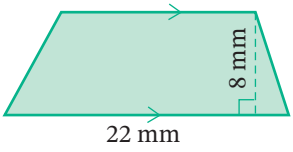
**EXERCISE 5.09** ANSWERS ON P. 584

## Area of a trapezium U F PS R

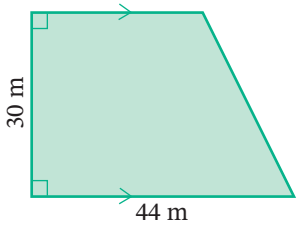
EXAMPLE 12

**1** Find the area of each trapezium.

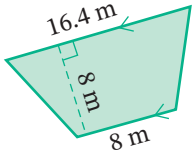
**a**



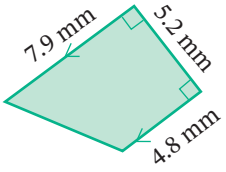

**b**



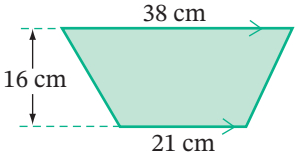

**c**



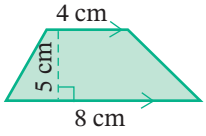

**d**



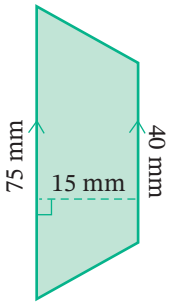

**e**



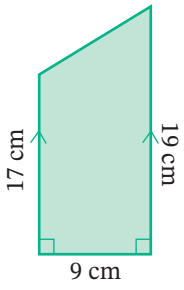

**f**



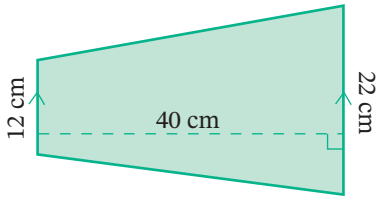

**g**




**h**

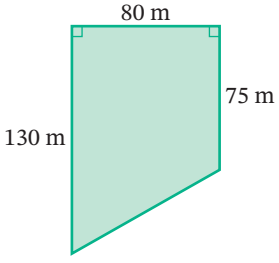



**i**

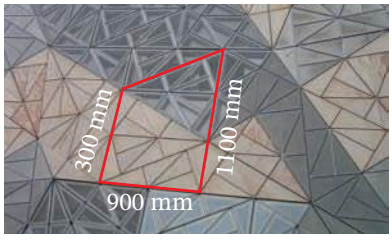



**2** **a** Find the area of the block of land shown.

**b** Council regulations state that $\frac{1}{5}$ of the land must be reserved for gardens. How much land is available for building?




**3** Find the area of the trapezoidal shape in the photo of the exterior of Federation Square, Melbourne.



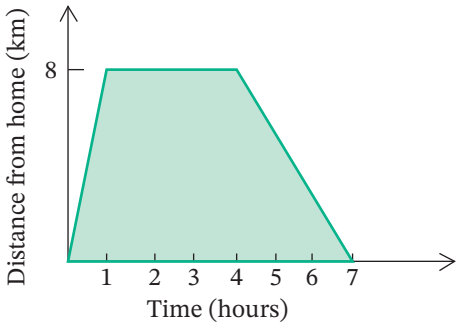

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**4** The travel graph shows a trip made by Katerina. What is the shaded area in the graph? Select **A**, **B**, **C** or **D**.

**A** 44 units$^2$ **B** 48 units$^2$

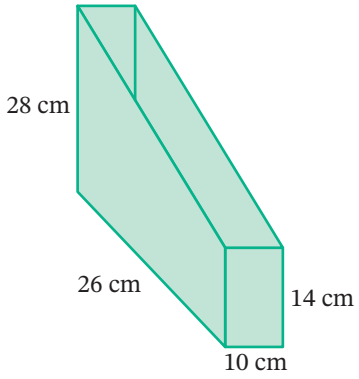
**C** 42 units$^2$ **D** 40 units$^2$




**5** A magazine holder has trapezoidal sides as shown. PS R

**a** Find the area of one of the trapezoidal sides.

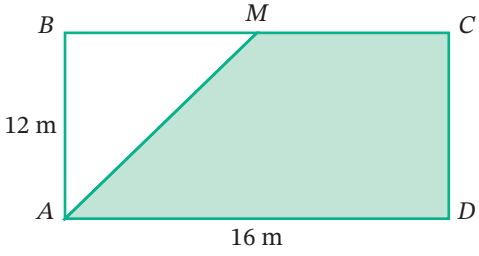
**b** Find the total area of cardboard needed to build this holder.




**6** In the diagram, $M$ is halfway between $B$ and $C$. Calculate the shaded area. Select **A**, **B**, **C** or **D**. R

**A** 96 m$^2$ **B** 132 m$^2$

**C** 224 m$^2$ **D** 144 m$^2$




**7** One side of the stage where Greta's drama club performs is shaped like a trapezium with an area of 144 m$^2$. The trapezium's parallel sides are 30 m and 42 m. What is the height of the trapezium? PS R



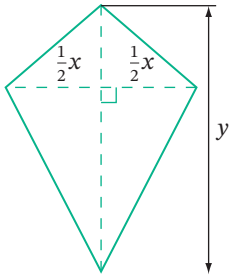
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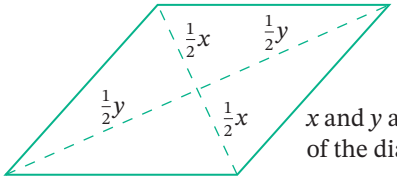
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## Investigation

### Area of a kite and a rhombus

**1** Draw 2 copies of the kite and rhombus shown below on a sheet of paper. Draw the diagonals of length $x$ and $y$ and label them as shown. Notice that they cross at right angles. For the kite, one diagonal ($x$) is bisected by the other ($y$), while for the rhombus, *both* diagonals bisect each other.




$x$ and $y$ are the lengths of the diagonals

**2** Cut out one of your kites and rhombuses and cut along their diagonals to make 4 triangles each. Join each set of triangles to make a rectangle.

**3** By comparing the measurements of the kite to the measurements of the rectangle formed from it, suggest a general formula for finding the area of any kite. Check your answer with your teacher.

**4** **a** What is the definition of a kite?

**b** Is a rhombus a special type of kite?

**5** By comparing the measurements of the rhombus to the measurements of the rectangle formed from it, suggest a general formula for finding the area of any rhombus. Check your answer with your teacher.

**6** Cut out the original kite and rhombus and paste all shapes into your book.



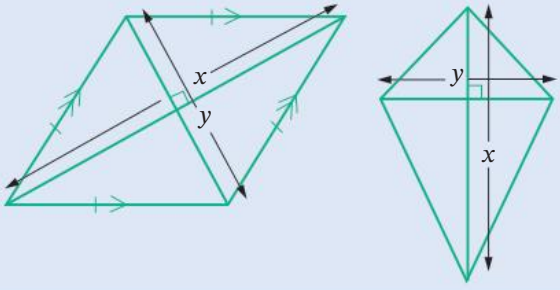
Alamy Stock Photo/Gabor Havasi

# Areas of kites and rhombuses 5.10

## Area of a kite or a rhombus

$$A = \frac{1}{2} \times (\text{diagonal}) \times (\text{other diagonal})$$

$$A = \frac{1}{2}xy$$




**Video**
Area formulas for triangles and quadrilaterals

**Worksheet**
Quadrilateral and triangle areas

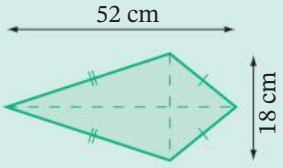
**Puzzle**
Areas of quadrilaterals and triangles

5.10

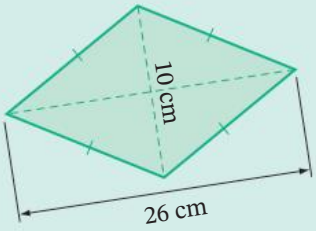
## Example 13

Find the area of each shape.

**a**




**b**




### Solution

**a**
$$A = \frac{1}{2}xy$$
$$= \frac{1}{2} \times 18 \times 52$$
$$= 468 \text{ cm}^2$$

**b**
$$A = \frac{1}{2}xy$$
$$= \frac{1}{2} \times 10 \times 26$$
$$= 130 \text{ cm}^2$$

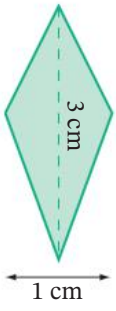
## EXERCISE 5.10 ANSWERS ON P. 584

## Areas of kites and rhombuses U F

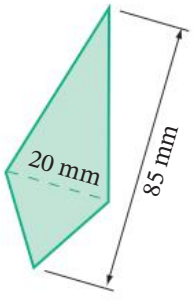
**1** Find the area of each kite.

EXAMPLE 13

**a**

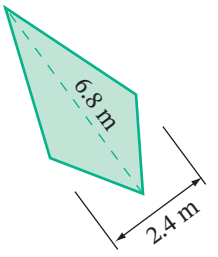



**b**

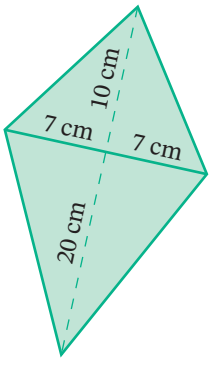



Foundation | Standard | Complex

**c**

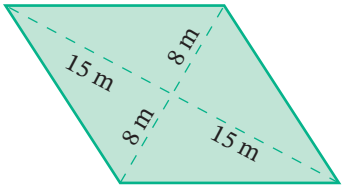



**d**

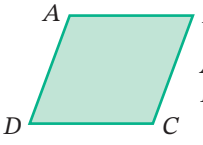



**2** Find the area of each rhombus.

**a**

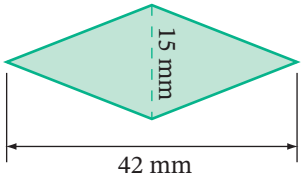



**b**

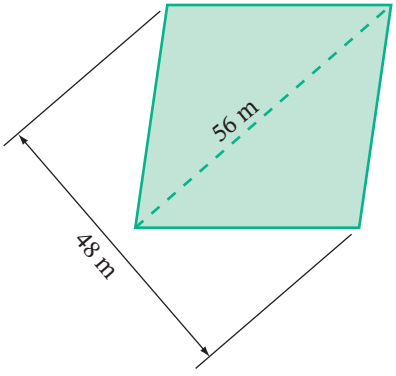



$AC = 10$ cm
$BD = 16$ cm

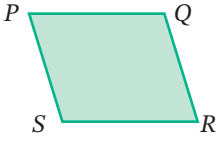
**c**




**d**




**e**




$PR = 2.4$ m
$QS = 1.5$ m

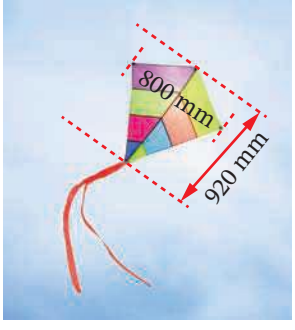
**3** What area of material is needed to build this kite?
Select **A**, **B**, **C** or **D**.

**A** 36.8 cm$^2$

**B** 3680 cm$^2$

**C** 73.6 cm$^2$

**D** 7360 cm$^2$



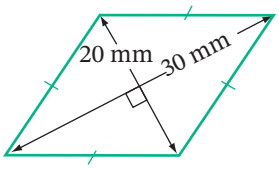

Shutterstock.com/benemale

Foundation Standard Complex

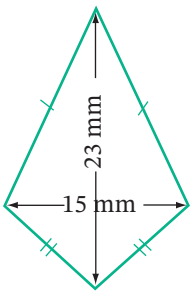

9780170479516

**4** Find the area of each shape.

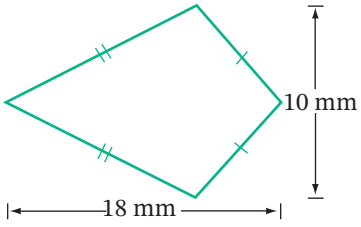
**a**



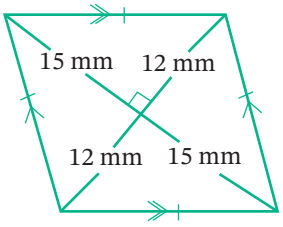

**b**



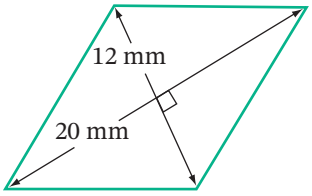

**c**



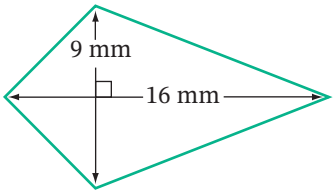

**d**



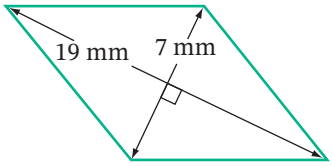

**e**



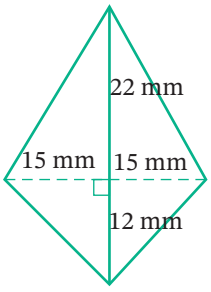

**f**



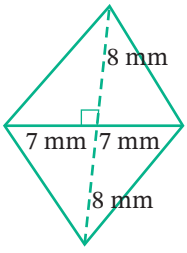

**g**




**h**




**i**

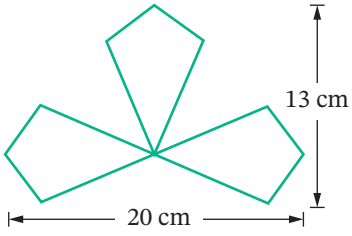



**5** Amanda used 3 identical kites to draw this logo.

**a** What are the lengths of the diagonals for each kite?

**b** Find the area of one kite.

**c** Find the area of the entire logo.




Foundation | Standard | Complex

Quiz
Mental skills 5B

# Mental skills 5B: Maths without calculators ANSWERS ON P. 585

## Comparing fractions, decimals and percentages

To compare or order fractions, express them with a common denominator first. To compare or order decimals, express them with the same number of decimal places first.

**1** Study each example.

**a** Which fraction is smaller: $\frac{4}{10}$ or $\frac{3}{8}$?

Using a common denominator of 80 ($8 \times 10$)

$\frac{4}{10} = \frac{4 \times 8}{10 \times 8} = \frac{32}{80}$

$\frac{3}{8} = \frac{3 \times 10}{8 \times 10} = \frac{30}{80}$

By comparing numerators: $\frac{30}{80} < \frac{32}{80}$

$\therefore \frac{3}{8}$ is smaller.

Using a common denominator of 40 (the LCM of 10 and 8)

$\frac{4}{10} = \frac{4 \times 4}{10 \times 4} = \frac{16}{40}$

$\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$

By comparing numerators: $\frac{15}{40} < \frac{16}{40}$

$\therefore \frac{3}{8}$ is smaller.

**b** Arrange these decimals in ascending order: 0.407, 0.47, 0.047, 0.4.

Express all decimals with 3 decimal places by inserting zeros at the end where necessary.

0.407 0.470 0.047 0.400

By comparing digits in the same decimal place, the decimals from smallest to largest are: 0.047 0.400 0.407 0.470

That is, 0.047, 0.4, 0.407, 0.47

**2** Now find the smaller number in each pair.

**a** $\frac{2}{3}$ and $\frac{3}{5}$ **b** $\frac{1}{4}$ and $\frac{1}{3}$ **c** 0.15 and 0.105

**d** 3.826 and 3.68 **e** $\frac{3}{7}$ and $\frac{4}{10}$ **f** $\frac{5}{6}$ and $\frac{7}{8}$

**g** 2.87 and 2.817 **h** 0.5301 and 0.503 **i** $\frac{1}{5}$ and $\frac{2}{9}$

**3** Arrange each set of numbers in ascending order.

**a** 0.81, 0.082, 0.821 **b** 3.5, 3.51, 3.55, 3.513 **c** $\frac{2}{3}, \frac{1}{6}, \frac{2}{5}$

**d** 0.007, 0.07, 0.7, 0.707 **e** $\frac{3}{8}, \frac{2}{10}, \frac{1}{2}$ **f** $\frac{1}{4}, \frac{4}{10}, \frac{3}{5}$

**4** Study this example.

Arrange these numbers in descending order: 68%, $\frac{13}{20}$, $0.\dot{6}$, $\frac{3}{5}$.

To order fractions, decimals and percentages, express them all as percentages first.

As percentages: 68% = 68%, $\frac{13}{20}$ = 65%, $0.\dot{6} = 66.66...\%$, $\frac{3}{5}$ = 60%.

Largest to smallest, this is 68%, 66.66%, 65%, 60%, that is, 68%, $0.\dot{6}$, $\frac{13}{20}$, $\frac{3}{5}$

**5** Now arrange each set of numbers in descending order.

**a** 0.25, $\frac{1}{6}$, 16%, $\frac{1}{5}$ **b** 27%, $\frac{1}{3}$, 0.4, 0.28 **c** 0.05, 50%, 6%, $\frac{1}{8}$

**d** $\frac{3}{4}$, 0.639, 55%, $\frac{2}{5}$ **e** 69%, 0.609, $\frac{2}{3}$, 0.6 **f** $\frac{2}{9}$, 0.105, 17%, 22.5%



# Metric units of volume 5.11

The **volume** of a solid is the amount of space it takes up. It is measured in **cubic units**.

| cubic centimetre ($cm^3$) | cubic metre ($m^3$) |
|---|---|
| 1 cm = 10 mm<br>$1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm$<br>$= 10\ mm \times 10\ mm \times 10\ mm$<br>$= 1000\ mm^3$<br>  | 1 m = 100 cm<br>$1\ m^3 = 1\ m \times 1\ m \times 1\ m$<br>$= 100\ cm \times 100\ cm \times 100\ cm$<br>$= 1000\ 000\ cm^3$<br>  |

Skillsheet What is volume?

Worksheet Volume

A **cubic centimetre** is about the volume of a person's tooth or a medical pill.

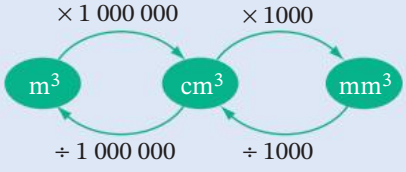
A **cubic metre** is about the volume of 2 washing machines.

Note that, while 1 cm = 10 mm, $1\ cm^3 = 1\ 000\ mm^3$ (triple the number of 0s), and while 1 m = 100 cm, $1\ m^3 = 1\ 000\ 000\ cm^3$ (triple the number of 0s).

## Metric units of volume

$1\ cm^3 = 1000\ mm^3$

$1\ m^3 = 1\ 000\ 000\ cm^3$



Video
Metric units of volume

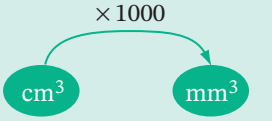
## Example 14

Convert:

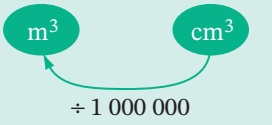
**a** 36 $cm^3$ to $mm^3$ **b** 84 000 000 $cm^3$ to $m^3$

### Solution

**a** 36 $cm^3 = 36 \times 1000\ mm^3$
$= 36\,000\ mm^3$




**b** 84 000 000 $cm^3 = 84\,000\,000 \div 1\,000\,000\ m^3$
$= 84\ m^3$




**EXERCISE 5.11** ANSWERS ON P. 585

## Metric units of volume U F R C

EXAMPLE 14

**1** What unit of volume should you use to measure the volume of: R C

**a** a bedroom? **b** a backpack?
**c** a mobile phone? **d** a concert hall?
**e** your body? **f** an aeroplane?

**2** A swimming pool has a volume of 38 $m^3$. How many cubic centimetres is this pool? Select the correct answer **A**, **B**, **C** or **D**. R C

**A** 380 000 **B** 3 800 000 **C** 38 000 000 **D** 380 000 000

**3** Copy and complete each conversion. R C

**a** 5000 $cm^3$ = _____ $mm^3$ **b** 1.6 $m^3$ = _____ $cm^3$
**c** 2 $cm^3$ = _____ $mm^3$ **d** 4 $m^3$ = _____ $mm^3$
**e** 6000 $cm^3$ = _____ $m^3$ **f** 8.2 $m^3$ = _____ $cm^3$
**g** 0.007 $m^3$ = _____ $mm^3$ **h** 9 600 000 $mm^3$ = _____ $m^3$
**i** 4000 $mm^3$ = _____ $cm^3$ **j** 160 000 $cm^3$ = _____ $m^3$
**k** 250 $mm^3$ = _____ $cm^3$ **l** 12 $cm^3$ =_____ $mm^3$
**m** 0.18 $m^3$ = _____ $cm^3$ **n** 200 000 $cm^3$ =_____ $m^3$

**4** The volume of a chest of drawers is 272 000 $cm^3$. Convert this to cubic metres. R C

**5** What is the approximate volume of a house brick? Select **A**, **B**, **C** or **D** R

**A** 1000 $cm^3$ **B** 20 $cm^3$ **C** 800 $cm^3$ **D** 2100 $cm^3$

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**6** How many cubic millimetres are there in 2.3 $m^3$? Select **A**, **B**, **C** or **D**. R C

**A** 2 300 000 000 **B** 2300 **C** 23 000 **D** 0.0023

**7** Match the correct volume (**A** to **G**) with each item (**a** to **g**) listed. R



Shutterstock.com/Monkey Business Images

| | | | |
|---|---|---|---|
| **a** | Mobile phone | **A** | 200 $m^3$ |
| **b** | Box of tissues | **B** | 3890 $m^3$ |
| **c** | Glass of water | **C** | 1250 $cm^3$ |
| **d** | Bottle of soft drink | **D** | 5000 $cm^3$ |
| **e** | Classroom | **E** | 80 $cm^3$ |
| **f** | School hall | **F** | 250 $cm^3$ |
| **g** | Box of cereal | **G** | 2200 $cm^3$ |

**8** The volume of a lunchbox is 2460 $cm^3$. Convert this to cubic millimetres. R C

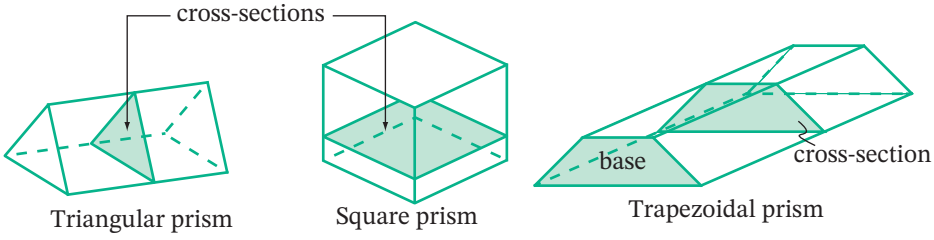


# Volume of a prism

5.12

A **cross-section** of a solid is a 'slice' of the solid, cut *across* it, parallel to its end faces, rather than along it.

If a solid has the same (uniform) cross-section along its length, and each cross-section is a **polygon** (with straight sides, not rounded), then the solid is called a **prism**. Here are some examples of prisms:




Triangular prism Square prism Trapezoidal prism

**Skillsheets**
Solid shapes

What is volume?

**Worksheets**
Volume

Measuring shapes review

Foundation Standard Complex

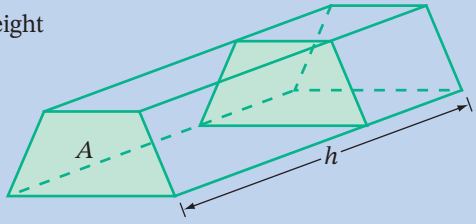
Prisms take their names from their cross-section. For example, the prism shown above is called a **trapezoidal prism** because its cross-sections are all trapeziums.

Because a prism is made up of identical cross-section 'slices' along its length, its volume can be calculated by multiplying the **area of the cross-section** by its **perpendicular height** (the length or depth of the prism).

## Volume of a prism

Volume = area of cross-section × perpendicular height

$V = Ah$




## Example 15

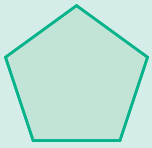
Convert:

**a** What is the name of this polygon?

**b** Draw a prism with this polygon as a cross-section.

**c** What shape are the side faces?

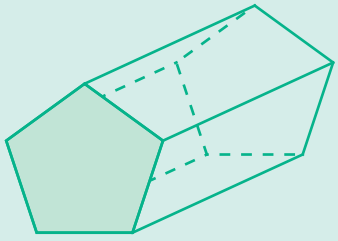
**d** What is the name of the prism?



### Solution

**a** Pentagon

**b**



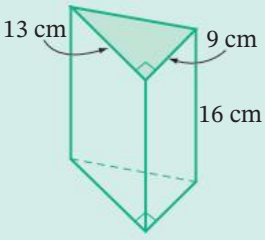
**c** The side faces are all rectangles.

**d** This is a pentagonal prism.

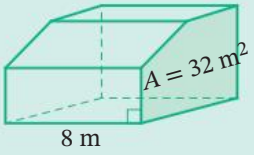
## Example 16

Find the volume of each prism.

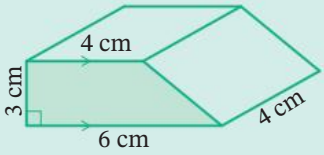
**a**




**b**




**c**




### Solution

**a** $A = \frac{1}{2} \times 9 \times 13$ — Area of shaded triangular cross-section

$= 58.5 \text{ cm}^2$

$V = 58.5 \times 16$ — $V = Ah$, where height $h = 16$ cm

$= 963 \text{ cm}^3$

**b** $A = 32 \text{ m}^2$ — Area of shaded cross-section

$V = 32 \times 8$ — $V = Ah$, where height $h = 8$

$= 256 \text{ m}^3$

**c** $A = \frac{1}{2} \times (4+6) \times 3$ — Cross-section is a trapezium

$= 15 \text{ cm}^2$

$V = 15 \times 4$ — $V = Ah$, where height $h = 4$

$= 60 \text{ cm}^3$

Video
Volume of a prism

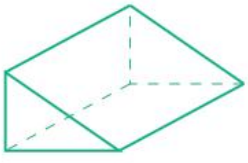
5.12

## EXERCISE 5.12 ANSWERS ON P. 585

### Volume of a prism U F PS R C

**1** State whether each solid is a prism or not. C

**a**



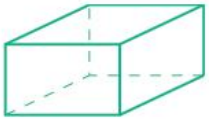
**b**



**c**



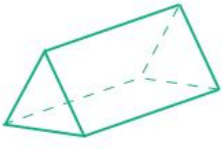
**d**



**e**



**f**



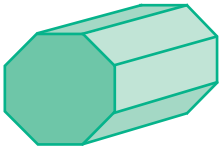
Foundation | Standard | Complex

EXAMPLE 15

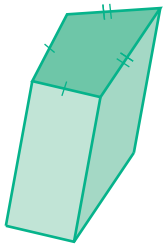
**2** For each prism:

**i** draw its cross-section **ii** write the name of the prism

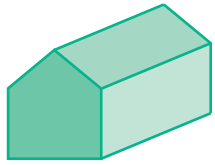
**a**



**b**



**c**



**3** Draw each prism and shade its base.

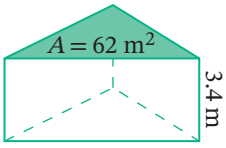
**a** square prism **b** isosceles triangular prism

**c** trapezoidal prism **d** hexagonal prism

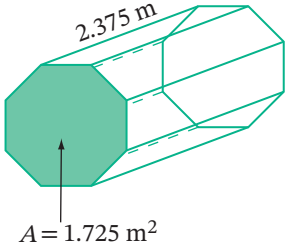
EXAMPLE 16

**4** Find the volume of each prism.

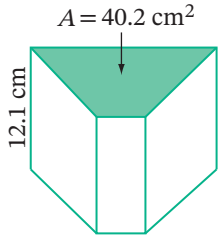
**a**




**b**

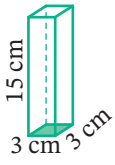



**c**

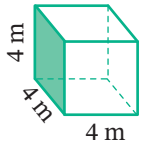



**5** The volume of a rectangular prism can be found using the formula $V = lwh$.
Use this formula to find the volume of each prism below.

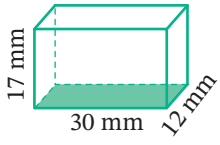
**a**



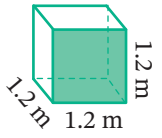

**b**



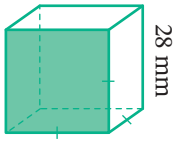

**c**



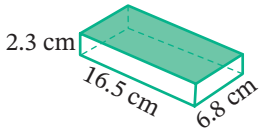

**d**




**e**




**f**

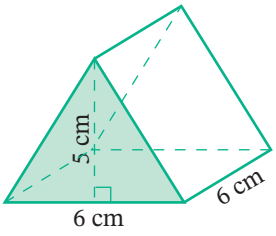



Foundation Standard Complex

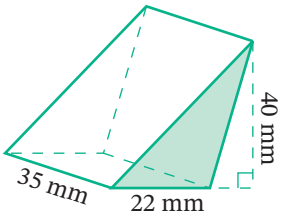

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**6** Find the volume of each triangular prism.

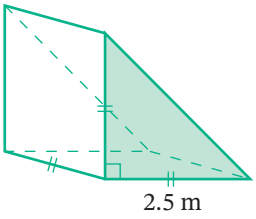
**a**



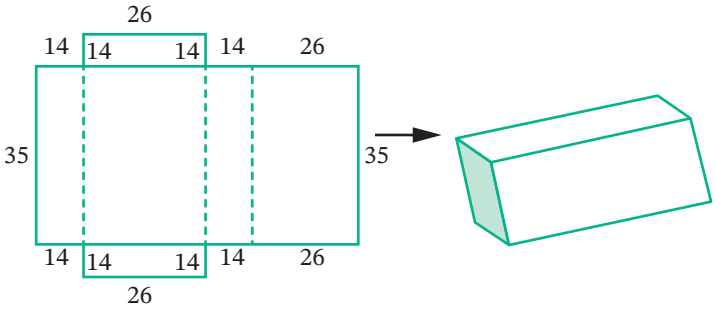

**b**




**c**



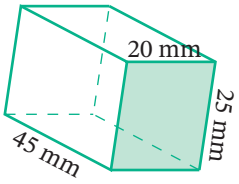

**7** This shape is folded along the dotted lines to form a rectangular prism.
Find the volume of the box in cubic units. Select the correct answer **A**, **B**, **C** or **D**. R



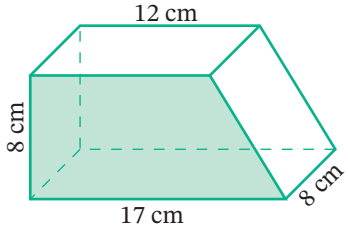

**A** 23 660 **B** 12 740 **C** 5096 **D** 258

**8** Find the volume of each prism.

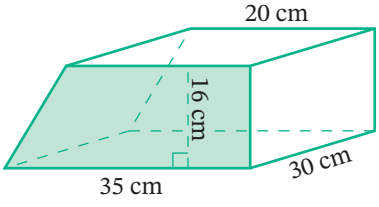
**a**



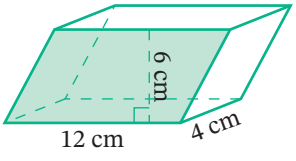

**b**




**c**

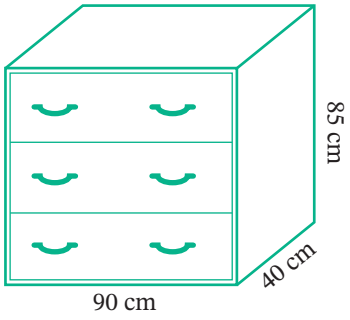



**d**




Foundation Standard Complex

**9** Find the volume of this chest of drawers. Give the answer in $m^3$: R C

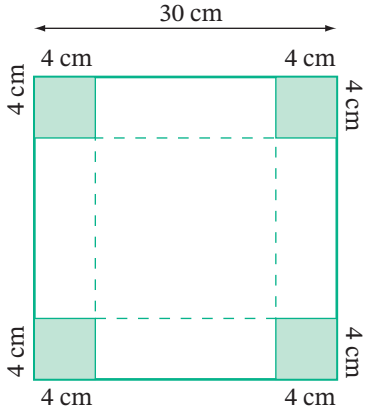



**a** by calculating the volume in $cm^3$, then converting to $m^3$

**b** by converting each length to metres first, then calculating the volume.

Which method is easier?

**10** A square sheet of metal has a side length of 30 cm. 4 identical squares of length 4 cm are cut away from the corners, as shown. Find the volume of the container formed when the remaining shape is folded along the dotted lines. Select **A**, **B**, **C** or **D**. PS R




**A** 1936 $cm^2$

**B** 2704 $cm^2$

**C** 64 $cm^2$

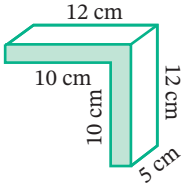
**D** 3600 $cm^2$

**11** A triangular prism has base length 6 m, height 10 m and volume 150 $m^3$. What is its length? R

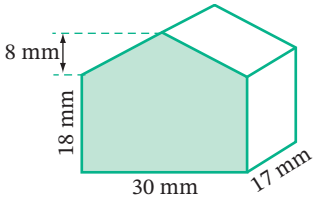
**12** A cube has a volume of 343 $m^3$. What is its side length? R

**13** Calculate the volume of each prism PS R

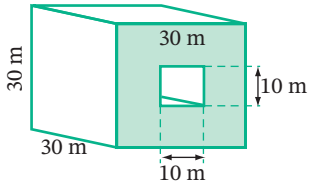
**a**



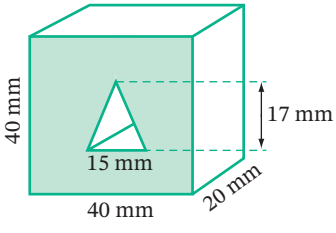

**b**



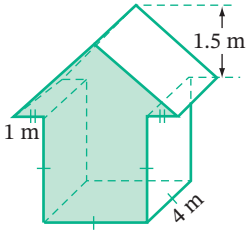

**c**




**d**




**e**




Foundation Standard Complex


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**14** A rectangular garden is 12 m long and 4 m wide. It is filled with soil to a depth of 15 cm. Calculate the volume of the soil. Select **A**, **B**, **C** or **D**. PS R

**A** 0.72 $m^3$ **B** 7.2 $m^3$ **C** 72 $m^3$ **D** 720 $m^3$

**15** A triangular prism has a volume of 36 $cm^3$. What could its length, perpendicular height and base length be? R



# Volume of a cylinder 5.13

A **cylinder** is like a 'circular prism' because its cross-sections are all identical circles. Because of this, we can also apply the prism formula $V = Ah$ to calculate the volume of a cylinder.

For a circle, $A = \pi r^2$, so:

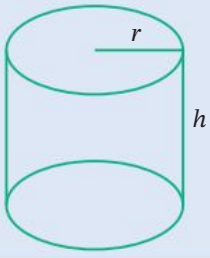
Volume $= Ah = \pi r^2 \times h = \pi r^2 h$

Video
Volumes of prisms and cylinders

## Volume of a cylinder

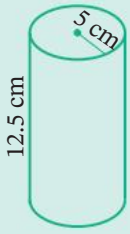
$V = \pi \times (\text{radius})^2 \times \text{perpendicular height}$

$V = \pi r^2 h$




## Example 17

Find the volume of this cylinder, correct to one decimal place.




### Solution

$V = \pi r^2 h$

$V = \pi \times 5^2 \times 12.5$

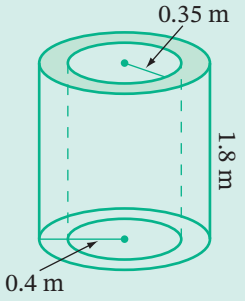
$= 981.7477\ldots$

$\approx 981.7\ \text{cm}^3$

Foundation Standard Complex

## Example 18

Calculate the volume of concrete needed to make this pipe, correct to 2 decimal places.




### Solution

Volume of concrete = volume of big cylinder – volume of small cylinder (hole)

$= \pi \times 0.4^2 \times 1.8 - \pi \times 0.35^2 \times 1.8$

$= 0.212057\ldots$

$\approx 0.21\ \text{m}^3$

**EXERCISE 5.13** ANSWERS ON P. 585

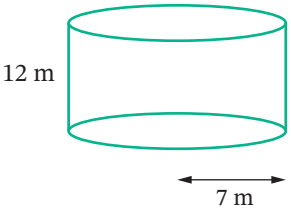
## Volume of a cylinder U F PS R

EXAMPLE 17

**1** A cylinder has a radius of 7 m and a height of 12 m. Find correct to 2 decimal places:

**a** the area of its circular base

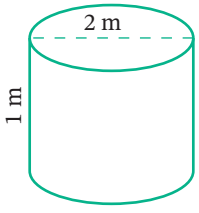
**b** the volume of the cylinder.




**2** What is the volume of this cylinder? Select the closest answer **A**, **B**, **C** or **D**.

**A** $2\ \text{m}^3$ **B** $12\ \text{m}^3$

**C** $3\ \text{m}^3$ **D** $6\ \text{m}^3$

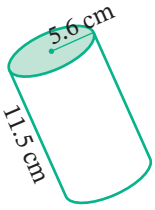



Foundation Standard Complex

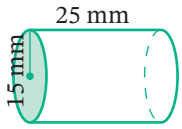

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**3** Find, correct to one decimal place, the volume of each cylinder.

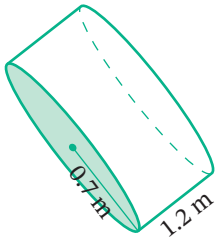
**a**



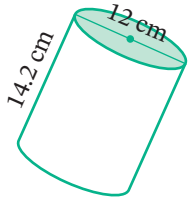

**b**



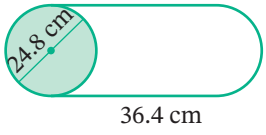

**c**



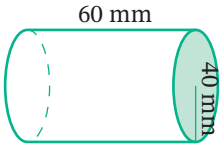

**d**



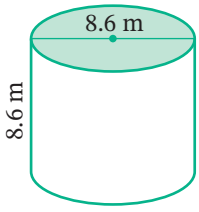

**e**



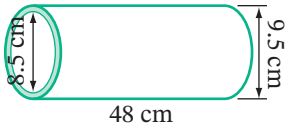

**f**




**g**




**4** This metal pipe has an inner diameter of 8.5 cm and an outer diameter of 9.5 cm. PS R




EXAMPLE 18

**a** What is the inner radius?

**b** What is the outer radius?

**c** Calculate, correct to 2 decimal places, the volume of metal needed to make the pipe.

**5** A manufacturer is experimenting with a new size of can. Which of these options provides the larger volume? PS R

**A** A can of radius 3.95 cm and height 11.8 cm

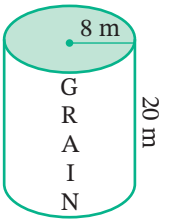
**B** A can of diameter 8.1 cm and height 15 cm

**6** A cylinder has a radius of 5 cm and a volume of 628.32 $cm^3$. Find its height, correct to the nearest centimetre. R

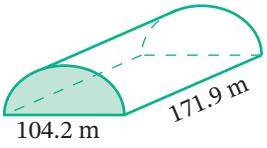
Foundation Standard Complex

**7** Find the volume of each solid, correct to one decimal place. PS R

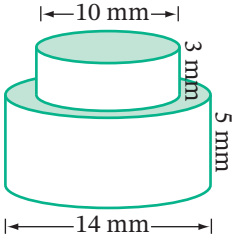
**a**



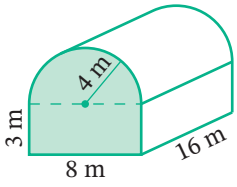

**b**



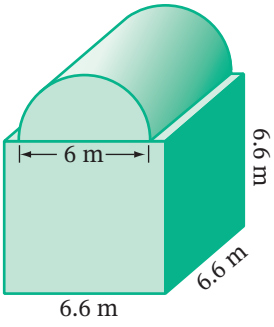

**c**



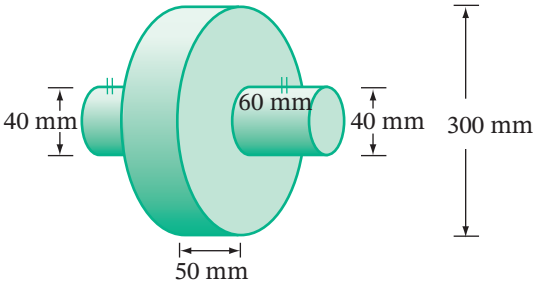

**d**




**e**




**f**




**8** A cylinder has a volume of 552.92 $m^3$ and a height of 11 m. Find its radius, correct to the nearest metre. R

# 5.14 Volume and capacity

**Worksheets**
Volume and capacity

Volume and capacity cards

Capacity

**Puzzle**
Capacity puzzle

**Capacity** is the amount of fluid (liquid or gas) in a container, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML).

- A large drop of water is about 1 mL
- A teaspoon holds about 5 mL
- A tall standard carton of milk holds 1 L
- A small rainwater tank holds about 1 kL



A can of drink holds 375 mL.

Shutterstock.com/Oleksiy Mark



A can of paint holds 4 L.

Shutterstock.com/Hintau Aliaksei

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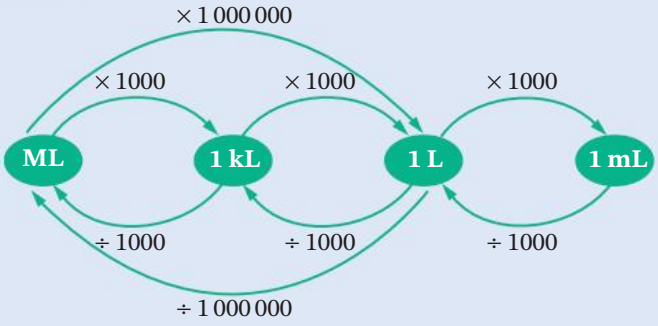

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## Metric units of capacity

1 L = 1000 mL

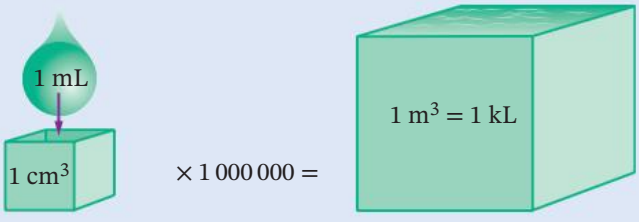
1 kL = 1000 L

1 ML = 1000 kL = 1 000 000 L




1 $cm^3$ contains 1 mL

1 $m^3$ contains 1000 L or 1 kL




Video
Volume and capacity

5.14

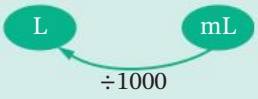
## Example 19

Convert:

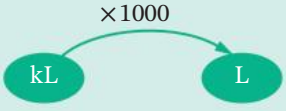
**a** 3400 mL to L **b** 2.9 $m^3$ to L

### Solution

**a** 3400 mL = 3400 ÷ 1000 L
= 3.4 L

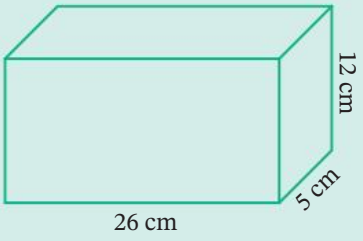



**b** 2.9 $m^3$ = 2.9 kL
= 2.9 × 1000 L
= 2900 L



## Example 20

Find the capacity of this container in litres.




### Solution

$V = 32 \times 8 \times 15$ $\quad V = lwh$

$= 3840\ \text{cm}^3$ $\quad 1\ \text{cm}^3 = 1\ \text{mL}$

Capacity $= 3840\ \text{mL}$

$= 3840 \div 1000\ \text{L}$

$= 3.84\ \text{L}$

**EXERCISE 5.14** ANSWERS ON P. 585

## Volume and capacity U F PS R C

**1** State what unit of capacity you would use when measuring: R C

**a** a glass of water **b** a dam **c** a car's petrol tank

**d** a bottle of medicine **e** an office water cooler **f** Olympic swimming pool

EXAMPLE 19

**2** Copy and complete each conversion. R C

**a** 5000 mL = _____ L **b** 3.4 kL = _____ L **c** 1.6 L = _____ mL

**d** 4000 $\text{cm}^3$ = _____ L **e** 2980 kL = _____ ML **f** 7.1 ML = _____ L

**g** 875 L = _____ kL **h** 8.2 $\text{m}^3$ = _____ L **i** 0.8 ML = _____ kL

**j** 1850 mL = _____ L **k** 5.4 kL = _____ L **l** 900 000 L = _____ $\text{m}^3$

**m** 6 kL = _____ L **n** 3500 $\text{mm}^3$ = _____ mL **o** 1.2 $\text{m}^3$ = _____ mL

**3** What is the capacity of a regular bottle of cough medicine? Select the closest answer **A**, **B**, **C** or **D**. R

**A** 200 mL **B** 500 mL **C** 1500 mL **D** 2000 mL

EXAMPLE 20

**4** The internal dimensions of a refrigerator are 150 cm (height), 60 cm (width) and 40 cm (depth). What is the capacity of the refrigerator? Select the closest answer **A**, **B**, **C** or **D**. R C

**A** 350 L **B** 400 L **C** 250 L **D** 300L

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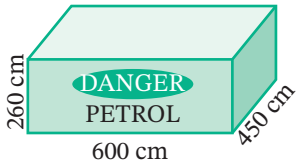
**5** A rectangular fish tank is 90 cm long and 30 cm wide, and is filled to a depth of 45 cm. Find the volume of water in: PS R C

**a** $cm^3$ **b** litres

**6** A can of drink has a radius of 3 cm and a height of 15 cm. Find its volume, correct to the nearest millilitre. PS R C

**7** A swimming pool is the shape of a rectangular prism 50 m × 18 m × 2 m. Find its volume in litres. PS R C

**8** A tank in the shape of a rectangular prism has dimensions as shown. PS R C




**a** Find the volume of the tank in $cm^3$.

**b** How many litres of petrol would fit in the tank?

**9** Samantha is inviting 18 friends to a party. She calculates that each person will drink 1300 mL of soft drink. PS R

**a** How many litres of soft drink must she buy?

**b** If Samantha intends to buy large 2.5 L bottles of drink, how many bottles must she buy?

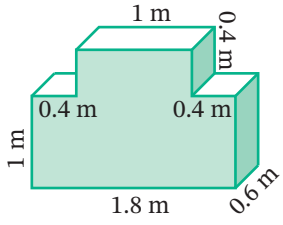
**10** A gardener orders a cubic metre of topsoil. She wants to spread it in her garden to a depth of 10 cm. If her garden is rectangular, what might be its dimensions? PS R C

**11** How many litres of rainwater were collected by a tank when 40 mm of rain fell on a roof 12 m long and 3.6 m wide? PS R C

**12** A plastic road block barrier is shown. It is filled with water to weigh it down. PS R C

**a** Find the volume of the barrier.

**b** How many litres of water would it take to fill the barrier?




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**13** The tank on a fuel tanker is in the shape of a cylinder 6 m long, with a diameter of 2 m. PS R C

**a** Find, correct to 3 decimal places, the volume of the tank in cubic metres.

**b** How many litres of fuel can this tank hold?




Shutterstock.com/Krivosheev Vitaly

**14** A tap leaks 10 mL of water every 50 seconds. How many litres of water will be lost in: PS R

**a** 1 hour? **b** 1 day?

**15** A cylindrical can holds 2 L of pesticide. What could its radius and height be? PS R

## Did you know?

### Water is everywhere

Water is the most valuable resource in our planet. Here are some interesting facts about water.

- 68.7% of the fresh water on Earth is trapped in glaciers
- A jellyfish and a cucumber are made of 95% water
- 70% of the human brain is water
- A human can live about one month without food, but only about a week without water
- About 25 700 L of water are needed to grow a day's food for a family of 4
- About 80% of a newborn baby's mass is water

**How many megalitres of water are needed to feed a family of 4 for a year?**

**If a baby weighs 3.8 kg at birth, how much of this is water?**



Shutterstock.com/Christina Fink

Wallaman Falls in Queensland, the tallest waterfall in Australia.

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## Investigation

### Drinking the swimming pool dry



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Charlotte's mum told her that it was healthy to drink 6 full glasses of water each day. Assume that Charlotte uses a cylindrical glass with a height of 9 cm and a diameter of 7 cm.

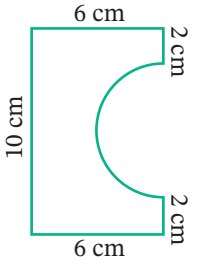
1. Calculate the volume of water Charlotte drinks each day, in millilitres, correct to 4 decimal places.
2. How much water will Charlotte drink in a year? Answer in litres, correct to 4 decimal places.
3. The family's swimming pool is in the shape of a cylinder with a diameter of 8.3 m, and is filled to a constant depth of 1.2 m. What is the amount of water in the pool? Answer in litres, correct to 4 decimal places.
4. How long would it take Charlotte to drink the equivalent of the swimming pool? Select the closest answer **A**, **B**, **C** or **D**.

**A** 6 years **B** 26 years **C** 56 years **D** 86 years

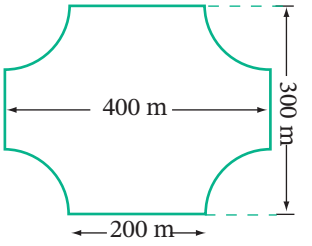
## Power plus ANSWERS ON P. 586

**1** Find, correct to one decimal place, the perimeter of each shape.

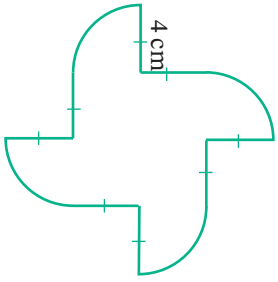
**a**



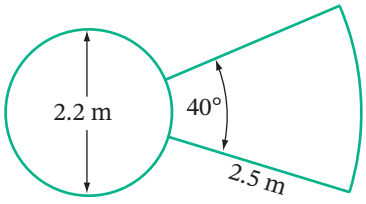

**b**




**c**

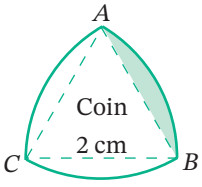


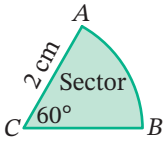

**2** This diagram shows how a shotput field is marked out for an athletics event. Calculate, correct to one decimal place:




- **a** the area of the field
- **b** the total length of the lines used.

**3** Mathsland has introduced a new 30-cent coin, as shown in the first diagram.
$\triangle ABC$ is an equilateral triangle of length 2 cm.

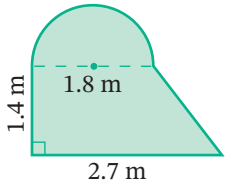


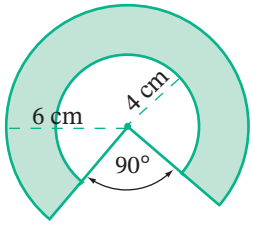

- **a** Find, correct to 3 decimal places, the area of the sector in the second diagram.
- **b** Use Pythagoras' theorem to prove that the perpendicular height of $\triangle ABC$ is $\sqrt{3}$ cm.
- **c** Calculate the area of $\triangle ABC$, correct to 3 decimal places.
- **d** Calculate the area of the shaded segment in the first diagram.
- **e** Hence calculate the area of the 30-cent coin, correct to 2 decimal places.

**4** Find, correct to one decimal place, the area of each shape.

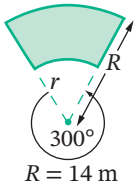
**a**




**b**




**c**




$R = 14$ m
$r = 11$ m


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# CHAPTER 5 REVIEW

## Language of maths

| | | | |
|---|---|---|---|
| arc | base | capacity | chord |
| circumference | cross-section | cubic metre | cylinder |
| diameter | hectare | kite | perimeter |
| perpendicular height | pi ($\pi$) | prism | quadrant |
| radius/radii | rhombus | sector | segment |
| semicircle | square metre | tangent | trapezium |

**1** Why are there 2 formulas for the circumference of a circle?

**2** What is meant by the **perpendicular height** of a shape?

**3** What is the correct name for a 'pizza slice' of a circle bounded by 2 radii and an arc?

**4** In the formula $A = \frac{1}{2}xy$ for the area of a rhombus, what do $x$ and $y$ stand for?

**5** What name is given to a 'slice' of a solid taken across the solid, rather than along it?

**6** What shape could be called a 'circular prism'?

**Quiz**
Language of maths 5

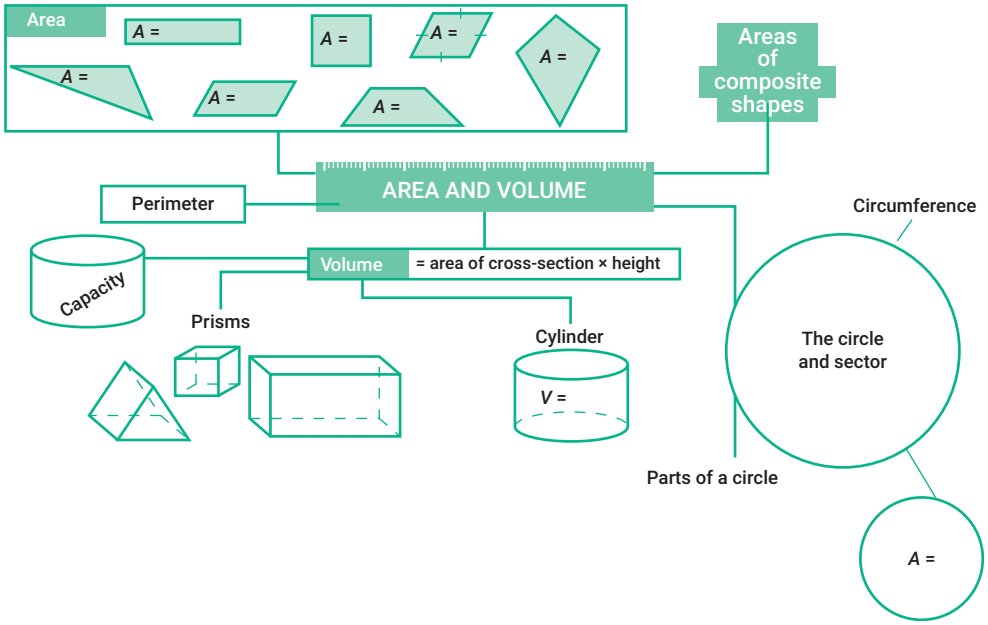
**Puzzle**
Area and volume find-a-word

## Topic summary

- What new things did you learn in this chapter?
- How often do you measure items? Give examples of the types of objects measured in your home.
- List 4 occupations that rely on being able to measure accurately.
- Did you have problems with any of the questions in this topic? If you did, discuss them with your teacher or a friend.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

**Worksheet**
Mind map: Area and volume



# TEST YOURSELF 5

ANSWERS ON P. 586

5.01

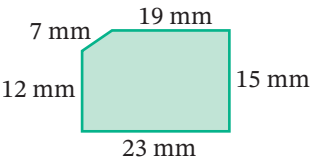
**1** What is the perimeter of a rectangle with length 8 m and width 4 m?
Select the correct answer **A**, **B**, **C** or **D**.

**A** 12 m **B** 16 m **C** 24 m **D** 48 m

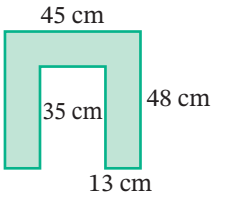
5.01

**2** Find the perimeter of each shape.

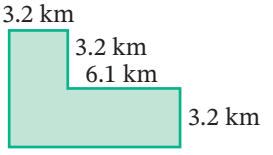
**a**




**b**

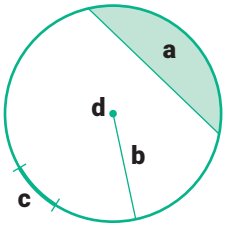


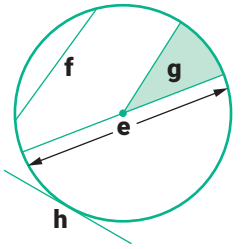

**c**




5.02

**3** Name each part of the circle labelled.




5.02

**4** For each shape, state:

**i** the number of axes of symmetry

**ii** the order of rotational symmetry if it has rotational symmetry

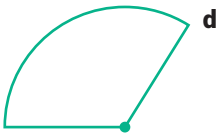
**a**



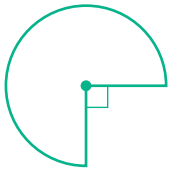
**b**



**c**



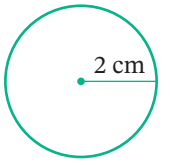
**d**



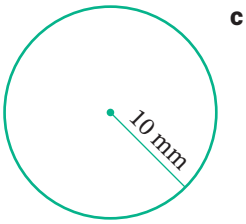
5.03

**5** Calculate, correct to one decimal place, the circumference of each circle.

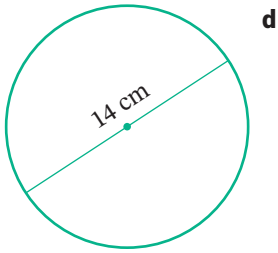
**a**



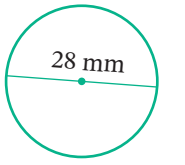

**b**




**c**




**d**




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**6** Calculate the area of each circle in Question 5: 5.07

**i** in terms of $\pi$ **ii** correct to one decimal place

**7** Copy and complete each conversion. 5.04

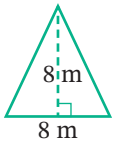
**a** 200 $mm^2$ = _____ $cm^2$ **b** 7 $m^2$ = _____ $cm^2$ **c** 8000 $cm^2$ = _____ $m^2$

**d** 30 $km^2$ = _____ $m^2$ **e** 70 000 $cm^2$ = _____ $m^2$ **f** 770 $mm^2$ = _____ $cm^2$

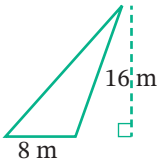
**g** 700 $mm^2$ = _____ $m^2$ **h** 7.2 ha = _____ $m^2$ **i** 5.8 $mm^2$ = _____ $cm^2$

**8** Which triangle has an area of 64 $m^2$? 5.05

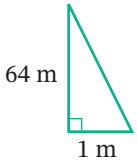
**A**



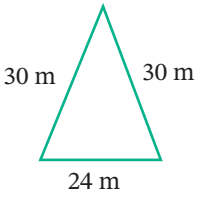

**B**




**C**

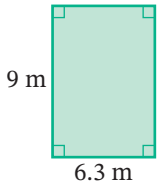



**D**

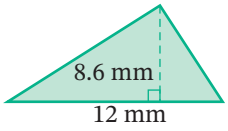



**9** Find the area of each shape. 5.05

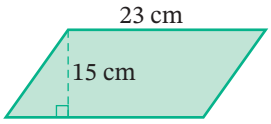
**a**



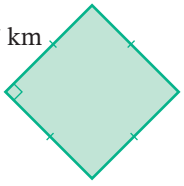

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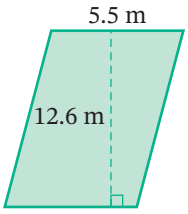

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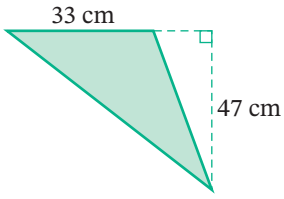

**d**




**e**

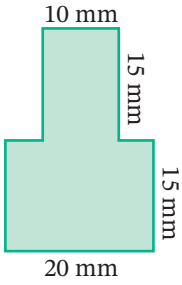



**f**

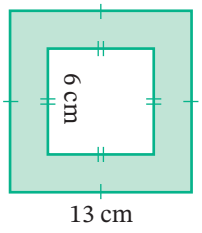



**10** Find the area of each shape. 5.06

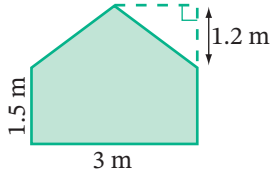
**a**



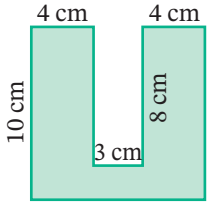

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**c**



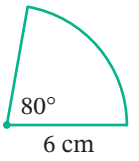

**d**



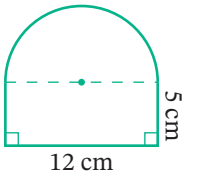

Foundation | Standard | Complex

5.08 **11** Calculate the perimeter of each shape, correct to 2 decimal places.

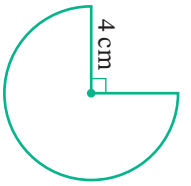
**a**



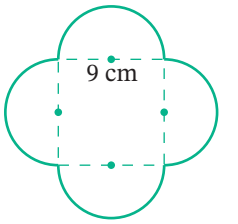

**b**




**c**



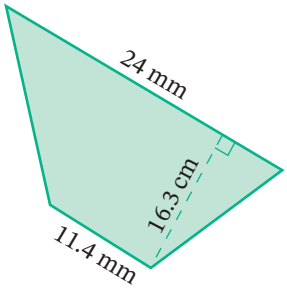

**d**



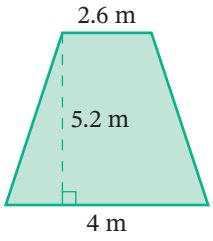

5.08 **12** Calculate, correct to 2 decimal places, the area of each shape in Question 11.

5.09 **13** Find the area of each shape.

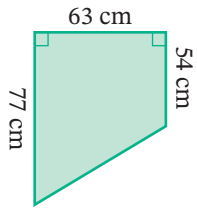
**a**




**b**

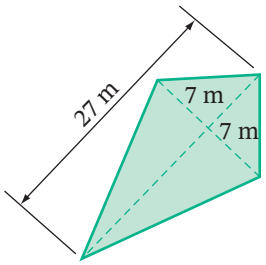



**c**

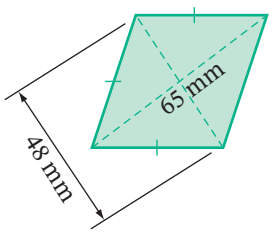



5.10 **14** Find the area of each shape.

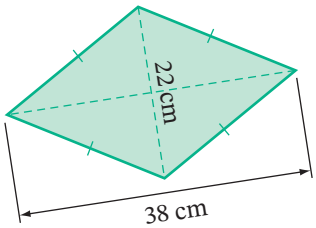
**a**




**b**




**c**




5.11 **15** Copy and complete each conversion.

**a** $8.2\ m^3 =$ _____ $cm^3$

**b** $3.4\ cm^3 =$ _____ $mm^3$

**c** $2\ 000\ 000\ cm^3 =$ _____ $m^3$

**d** $1\ km^3 =$ _____ $m^3$

**e** $8\ m^3 =$ _____ $mm^3$

**f** $45\ 000\ mm^3 =$ _____ $cm^3$

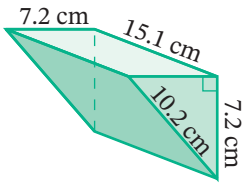
Foundation Standard Complex



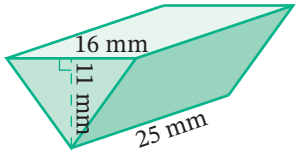
9780170479516

**16** Find the volume of each prism. 5.12

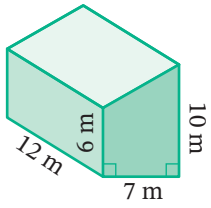
**a**



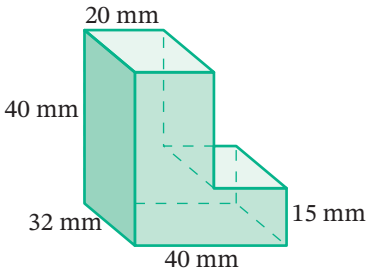

**b**



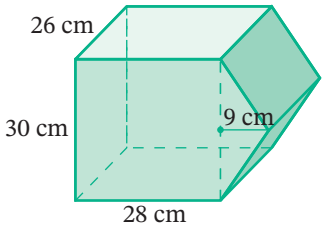

**c**




**d**

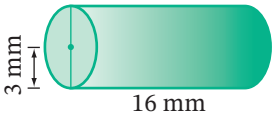



**e**

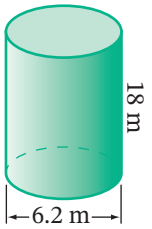



**17** Find the volume of each solid, correct to 2 decimal places. 5.13

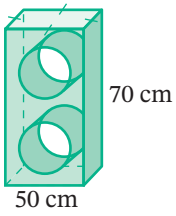
**a**




**b**




**c**




Diameter of circles = 16 cm

**18** Copy and complete each conversion. 5.14

**a** 3 L = _____mL

**b** 2500 mL = _____L

**c** 6.5 kL = _____L

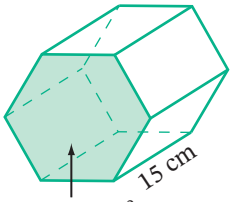
**d** 7.2 $mm^3$ = _____mL

**e** 120 mL = _____L

**f** 35 $cm^3$ = _____L

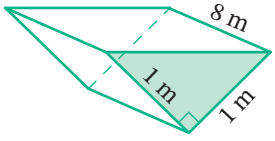
**19** Find the capacity of each solid in litres (correct to one decimal place for **c**). 5.14

**a**

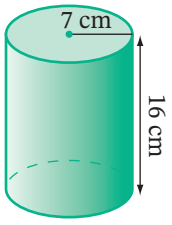



A = 380 $cm^2$

**b**




**c**

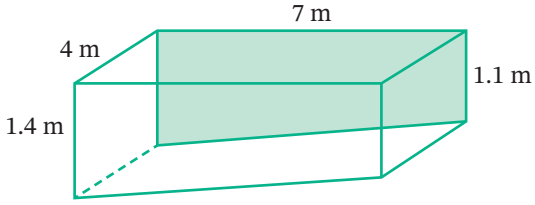



**20** Shreya's pool has the shape of a trapezoidal prism. 5.14
The shaded area is a trapezium.

**a** Find the shaded area.

**b** Find the volume of the pool.

**c** Find the capacity of the pool in litres.




Foundation | Standard | Complex




6

NUMBER AND ALGEBRA

# FRACTIONS AND PERCENTAGES

Football players, tennis players and elite athletes often say “I gave it 110%” after a match or event. What do they mean? Giving 100% is giving everything or the whole effort. They mean they have given extra effort or tried even harder than usual. When someone says “I support them 110%”, it is a similar thing. In both situations, the importance of the event or action is being emphasised.




Shutterstock.com/FrankHH

## Chapter outline

| | Working mathematically | | | | |
|---|---|---|---|---|---|
| **6.01** Fractions | U | F | | R | C |
| **6.02** Adding and subtracting fractions | U | F | PS | R | |
| **6.03** Multiplying and dividing fractions | U | F | PS | R | C |
| **6.04** Percentages, fractions and decimals | U | F | | R | C |
| **6.05** Fraction and percentage of a quantity | U | F | | R | |
| **6.06** Expressing amounts as fractions and percentages | U | F | PS | R | |
| **6.07** Percentage increase and decrease | U | F | PS | R | C |
| **6.08** Percentages without calculators | U | F | PS | R | |
| **6.09** The unitary method | U | F | PS | R | |
| **6.10** Profit, loss and GST | U | F | PS | R | |
| **6.11** Simple interest | U | F | PS | | |
| **6.12** Percentage problems | U | F | PS | R | |

## Wordbank

**Quiz**
Wordbank 6

**cost price** The price an item costs the retailer

**discount** The saving made between the original price of an item and the reduced price

**improper fraction** A fraction such as $\frac{7}{3}$, in which the numerator is larger than or equal to the denominator.

**loss** The amount lost when selling an item at a lower price

**GST** Goods and services tax, a 10% tax added to the original price of an item or service

**profit** The amount made when selling an item at a higher price

**selling price** The price at which an item is sold by the retailer

**unitary method** A method for finding the whole amount when a percentage of that amount is known, by first finding the size of 1%

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

## Videos (19):

**SkillCheck** Equivalent fractions • Using fractions

**6.01** Improper fractions and mixed numerals • Simplifying fractions

**6.02** Adding and subtracting fractions • Working out fractions

**6.03** Multiplying and dividing fractions • Dividing fractions • Working out fractions • Multiplying fractions

**6.04** Converting percentages to fractions and decimals • Converting fractions and decimals to percentages

**6.05** Fraction and percentage of a quantity • Fraction of a quantity

**6.06** Expressing quantities as fractions and percentages

**6.07** Percentage increase and decrease

**6.09** The unitary method

**6.10** Business maths • Profit and loss • GST

**6.11** Simple interest

## *Twig* videos (3):

**6.05** Percentages: Tax breaks

**6.07** Hyperinflation: 1920s Germany • The most populous country

## *PhET* interactives (5):

**6.02** Fractions: Intro • Fraction matcher • Build a fraction • Fractions: Mixed numbers • Fractions: Equality

## Quizzes (6):

- Wordbank 6
- SkillCheck 6
- Mental skills 6A
- Mental skills 6B
- Language of maths 6
- Test yourself 6

## Skillsheets (6):

**SkillCheck** Fractions

**6.01** Fractions • Equivalent fractions • Improper fractions and mixed numerals • Simplifying fractions

**6.04** Multiplying and dividing by 10, 100, 1000

**6.08** Mental percentages

## Worksheets (30):

**SkillCheck** Fraction diagrams • Classifying fractions

**6.01** Fractions wall • Pop stick calculator • Fraction strips

**6.02** Fraction arithmagons • Pop stick calculator • Magic squares

**6.03** Multiplying fractions using rectangles • Multiplying and dividing fractions number grids • What's the fraction question? • Fractions and decimals review

**6.04** Fractions wall • Decimals wall • Percentages wall • Fractions, decimals and percentages • Percentage shapes

**6.05** Percentages of an amount

**6.06** Calculating percentages • Money and percentages review • Percentages

**6.07** Percentage shortcuts • Discounts and special offers • Discounts

**6.08** Percentages without calculators

**6.09** The unitary method • Working with percentages

**6.10** Profit and loss

**6.11** Simple interest

**6.12** Percentage problems

**Mind map** Fractions and percentages

## Puzzles (15):

**6.01** Mixed numerals to improper fractions

**6.02** Adding and subtracting fractions

**6.04** Converting percentages to fractions • Converting fractions to percentages • Converting decimals to percentages • Percentages to decimals • Percentominoes

**6.05** Percentage crossnumber • Percentages • Fractions of words • Percentages of an amount

**6.06** Calculating percentages

**6.07** Discounts

**6.10** Profit and loss

**Language of maths** Fractions and percentages find-a-word

## Technology (3):

**6.06** Online discounts • Coffee shop sales

**6.10** Profit and loss

## Presentations (3):

**6.02** Rational numbers • Improper fractions

**6.04** Percentages, fractions and decimals

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

## In this chapter you will:

- order, add, subtract, multiply and divide fractions
- convert between fractions, decimals and percentages
- find a fraction or percentage of a quantity
- express one quantity as a fraction or percentage of another
- increase or decrease a quantity by a percentage
- given a percentage of an amount, use the unitary method to find the whole amount
- solve problems involving percentages, including profit, loss, GST and simple interest
- (EXTENSION, YEAR 9 CORE) solve problems using the simple interest formula

## SkillCheck ANSWERS ON P. 586

**1** Copy and complete each pair of **equivalent** fractions.

**a** $\frac{1}{2}=\frac{_}{6}$ **b** $\frac{2}{3}=\frac{_}{12}$ **c** $\frac{4}{5}=\frac{16}{_}$ **d** $\frac{7}{28}=\frac{1}{_}$

**e** $\frac{3}{4}=\frac{_}{8}$ **f** $\frac{15}{20}=\frac{_}{4}$ **g** $\frac{6}{10}=\frac{24}{_}$ **h** $\frac{3}{_}=\frac{9}{_}$

**2** Write each percentage as a fraction.

**a** 17% **b** 3% **c** 51% **d** 63% **e** 89%

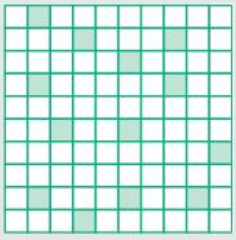
**3** Evaluate each difference.

**a** 100% − 30% **b** 100% − 95% **c** 100% − 39%

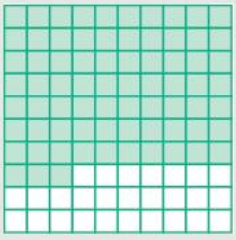
**d** 100% − 71% **e** 100% − 12% **f** 100% − 7%

**4** Write the percentage that is shaded in each diagram.

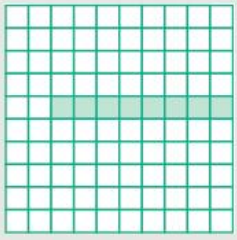
**a**



**b**



**c**



**5** Write the percentage that is unshaded in each diagram in question **4**.

**6** On a plane, 10% of the seats are first-class and 25% are business class. The rest are economy class. What percentage of the seating is economy class?

**7** Write all of the factors of 100.

**8** Evaluate each expression.

**a** 50% of $80 **b** 50% of 12 kg **c** 25% of $20

**d** 25% of 40 kg **e** 10% of $30 **f** 10% of 20 kg

**Quiz** SkillCheck 6

**Videos** Equivalent fractions

Using fractions

**Worksheets** Fractions diagrams

Classifying fractions

**Skillsheet** Fractions

# 6.01 Fractions

**Videos**
Improper fractions and mixed numerals
Simplifying fractions

**Skillsheets**
Fractions
Equivalent fractions
Improper fractions and mixed numerals
Simplifying fractions

**Worksheet**
Fractions wall

**Presentations**
Rational numbers
Improper fractions

**Puzzle**
Mixed numbers to improper fractions

**Worksheets**
Pop stick calculator
Fraction strips

**Interactives**
Fractions: Intro
Fraction matcher
Build a fraction
Fractions: Mixed numbers
Fractions: Equality

Scientific calculators have a fraction key for entering fractions: $\square\!\!\square$ or $a^{b/c}$

Casio calculators have 2 modes for entering and displaying fractions. MATH mode shows fractions more realistically as it allows you to enter the numerator and denominator into 2 blank spaces on the calculator's screen, while LINE mode shows the fraction in a line (for example, 1 ⌟ 3) and makes the fraction key act like a vinculum (fraction bar). Press SHIFT MODE to change between MATH and LINE modes. When MATH mode is selected, 'Math' appears on the calculator display. To ensure that mixed numeral answers are not converted to improper fractions, press SHIFT MODE and the down arrow to choose 'ab/c'.

## Example 1

Convert $\frac{27}{4}$ into a mixed numeral.

### Solution

$\frac{27}{4} = 27 \div 4$

$= 6$ remainder $3$

$= 6\frac{3}{4}$

To find the number of 'wholes' in 27 quarters, divide 27 by 4.

Write the remainder in the numerator of a fraction.

Or on a calculator, enter: 27 $\square\!\!\square$ 4 = or 27 $a^{b/c}$ 4 =.

For Casio MATH mode, press the $\square\!\!\square$ key first and use arrow keys to move the cursor to the 2 blank spaces to enter the values. Then press =.

If the display shows $\frac{27}{4}$, press SHIFT S⇔D or 2nd F $a^{b/c}$ to convert it to a mixed numeral.

SHIFT S⇔D or 2nd F $a^{b/c}$ converts an answer between **mixed numeral** and **improper fraction** forms.

## Example 2

Convert $4\frac{2}{5}$ to an improper fraction.

### Solution

$+$ $4\frac{2}{5} = \frac{5 \times 4 + 2}{5} = \frac{22}{5}$
$\times$

This works because $4\frac{2}{5} = 4 + \frac{2}{5} = \frac{5 \times 4}{5} + \frac{2}{5}$.

Or on a calculator, enter: 4 $\square\!\!\square$ 2 $\square\!\!\square$ 5 = SHIFT S⇔D or 4 $a^{b/c}$ 2 $a^{b/c}$ 5 = 2nd F $a^{b/c}$.

For entering mixed numerals in Casio MATH mode, press SHIFT $\square\!\!\square$ and move the cursor to the 3 blank spaces to enter the values of the mixed numeral. Then press =.


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## Example 3

Which fraction is larger, $\frac{4}{10}$ or $\frac{3}{8}$?

### Solution

**Method 1**

Convert both fractions so that they share a common denominator of $10 \times 8 = 80$.

$\frac{4}{10} = \frac{4 \times 8}{10 \times 8} = \frac{32}{80}$ and $\frac{3}{8} = \frac{3 \times 10}{8 \times 10} = \frac{30}{80}$

Note that we multiply the numerator and denominator of each fraction by the denominator of the **other** fraction.

By comparing numerators, $\frac{32}{80} > \frac{30}{80}$

$\therefore \frac{4}{10}$ is larger.

**Method 2**

Another suitable denominator is the lowest common denominator (LCD) of both denominators.

The LCD of 10 and 8 is 40.

The LCD is the lowest common multiple (LCM) of both denominators.

Convert both fractions.

$\frac{4}{10} = \frac{4 \times 4}{10 \times 4} = \frac{16}{40}$ and $\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$

By comparing numerators, $\frac{16}{40} > \frac{15}{40}$.

$\therefore \frac{4}{10}$ is larger.

## Example 4

Simplify each fraction.

**a** $\frac{10}{15}$ **b** $\frac{36}{60}$

### Solution

**a** $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$

Or on a calculator, enter: 10 ▭ 15 = or 10 $a^{b/c}$ 15 =.

**b** $\frac{36}{60} = \frac{36 \div 6}{60 \div 6} = \frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$

or $\frac{36}{60} = \frac{36 \div 12}{60 \div 12} = \frac{3}{5}$

We can simplify in one step if we divide by 12, the HCF of 36 and 60.

Or on a calculator, enter: 36 ▭ 60 = or 36 $a^{b/c}$ 60 =.

**EXERCISE 6.01** ANSWERS ON P. 586

## Fractions U F R C

EXAMPLE 1

**1** Convert each improper fraction into a mixed numeral or whole number. C

**a** $\frac{3}{2}$ **b** $\frac{11}{3}$ **c** $\frac{9}{4}$ **d** $\frac{12}{5}$ **e** $\frac{20}{4}$ **f** $\frac{47}{11}$

EXAMPLE 2

**2** Convert each mixed numeral into an improper fraction. C

**a** $3\frac{1}{2}$ **b** $4\frac{1}{3}$ **c** $5\frac{3}{4}$ **d** $7\frac{2}{3}$

**e** $7\frac{4}{5}$ **f** $7\frac{2}{9}$ **g** $10\frac{1}{7}$ **h** $9\frac{3}{10}$

**3** Write an improper fraction that can be converted into a whole number. R C

**4** Which is larger: a proper fraction or a mixed numeral? Explain your answer. R C

EXAMPLE 3

**5** For each pair of fractions, find the larger fraction.

**a** $\frac{1}{2}, \frac{1}{3}$ **b** $\frac{2}{3}, \frac{3}{4}$ **c** $\frac{3}{8}, \frac{1}{3}$

**d** $2\frac{3}{5}, 2\frac{7}{10}$ **e** $\frac{11}{15}, \frac{3}{5}$ **f** $\frac{5}{12}, \frac{2}{5}$

EXAMPLE 4

**6** Simplify each fraction.

**a** $\frac{5}{10}$ **b** $\frac{4}{12}$ **c** $\frac{12}{26}$ **d** $\frac{18}{24}$ **e** $\frac{15}{25}$ **f** $\frac{16}{28}$

**g** $\frac{32}{48}$ **h** $\frac{60}{100}$ **i** $\frac{44}{77}$ **j** $\frac{20}{35}$ **k** $\frac{21}{35}$ **l** $\frac{72}{80}$

**7** Which one of these fractions is **NOT** equivalent to $\frac{10}{25}$? Select **A**, **B**, **C**, or **D**.

**A** $\frac{1}{5}$ **B** $\frac{2}{5}$ **C** $\frac{20}{50}$ **D** $\frac{40}{100}$

**8** Convert each improper fraction into a simplified mixed numeral.

**a** $\frac{9}{6}$ **b** $\frac{18}{6}$ **c** $\frac{45}{10}$ **d** $\frac{36}{20}$

# 6.02 Adding and subtracting fractions

**Videos**
Adding and subtracting fractions
Working out fractions

## Adding and subtracting fractions

- To **add or subtract fractions**, convert them (if needed) so that they have the same denominator, then simply add or subtract just the numerators
- To **add or subtract mixed numerals**, add or subtract the whole numbers and fractions separately.

Foundation Standard Complex


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## Example 5

Evaluate each expression.

**a** $\frac{1}{3}+\frac{5}{6}$ **b** $\frac{5}{7}-\frac{2}{3}$ **c** $3-\frac{3}{4}$ **d** $4\frac{1}{5}-1\frac{2}{3}$

### Solution

**a** Common denominator $= 3 \times 6 = 18$

$$\frac{1}{3}=\frac{1\times6}{3\times6}=\frac{6}{18}$$
$$\frac{5}{6}=\frac{5\times3}{6\times3}=\frac{15}{18}$$
$$\frac{1}{3}+\frac{5}{6}=\frac{6}{18}+\frac{15}{18}$$
$$=\frac{21}{18}$$
$$=\frac{7}{6}$$
$$=1\frac{1}{6}$$

Note that we multiply the numerator and denominator of each fraction by the denominator of the *other* fraction.

OR Using the lowest common denominator (LCD) of 3 and 6 = 6:

$$\frac{1}{3}=\frac{1\times2}{3\times2}=\frac{2}{6}$$
$$\frac{5}{6}=\frac{5}{6}$$
$$\frac{1}{3}+\frac{5}{6}=\frac{2}{6}+\frac{5}{6}$$
$$=\frac{7}{6}$$
$$=1\frac{1}{6}$$

OR on a calculator,
enter: 1 $\square\!\!\square$ 3 + 5 $\square\!\!\square$ 6 =. Note: $\square\!\!\square$ or $a^{b/c}$.

**b** Common denominator $= 7 \times 3 = 21$:

$$\frac{5}{7}=\frac{5\times3}{7\times3}=\frac{15}{21}$$
$$\frac{2}{3}=\frac{2\times7}{3\times7}=\frac{14}{21}$$
$$\frac{5}{7}-\frac{2}{3}=\frac{15}{21}-\frac{14}{21}$$
$$=\frac{1}{21}$$

OR 5 $\square\!\!\square$ 7 − 2 $\square\!\!\square$ 3 =.

**c**
$$3-\frac{3}{4}=2+1-\frac{3}{4}$$
$$=2+\frac{4}{4}-\frac{3}{4}$$
$$=2\frac{1}{4}$$

OR 3 − 3 $\square\!\!\square$ 4 =.

**d**
$$4\frac{1}{5}-1\frac{2}{3}=4-1+\frac{1}{5}-\frac{2}{3}$$
$$=3+\frac{3}{15}-\frac{10}{15}$$
$$=3-\frac{7}{15}$$
$$=2+\frac{15}{15}-\frac{7}{15}$$
$$=2\frac{8}{15}$$

OR 4 $\square\!\!\square$ 1 $\square\!\!\square$ 5 − 1 $\square\!\!\square$ 2 $\square\!\!\square$ 3 =.

For MATH mode, press SHIFT $\square\!\!\square$ first.

## EXERCISE 6.02 ANSWERS ON P. 586

### Adding and subtracting fractions U F PS R

**1** Evaluate each expression.

**a** $\frac{1}{5}+\frac{3}{5}$ **b** $\frac{3}{4}-\frac{1}{4}$ **c** $\frac{3}{8}+\frac{2}{8}$ **d** $\frac{7}{8}+\frac{4}{8}$

EXAMPLE 5

**2** Evaluate each expression.

**a** $\frac{2}{5}+\frac{3}{10}$ **b** $\frac{2}{3}-\frac{3}{7}$ **c** $\frac{3}{5}+\frac{1}{4}$ **d** $\frac{1}{2}-\frac{1}{4}$

**e** $\frac{1}{6}+\frac{3}{8}$ **f** $\frac{7}{10}-\frac{2}{3}$ **g** $\frac{4}{9}+\frac{5}{6}$ **h** $\frac{7}{8}-\frac{5}{12}$

**i** $\frac{5}{8}+\frac{1}{2}$ **j** $1-\frac{6}{7}$ **k** $4-\frac{4}{9}$ **l** $3-\frac{2}{5}$

**3** Joe bought a length of timber to build fences for his garden beds. He used $\frac{1}{5}$ of the wood for the first garden and $\frac{3}{8}$ of the wood for the second garden. What fraction of the wood remains for the third garden? R

**4** What fraction goes in the blank? $\frac{7}{8}-____=\frac{1}{4}$. Select the correct answer **A**, **B**, **C**, or **D**. PS R

**A** $\frac{6}{4}$ **B** $\frac{8}{12}$ **C** $\frac{3}{8}$ **D** $\frac{5}{8}$

**5** Copy and complete each equation with the correct fraction. PS R

**a** $\frac{1}{4}+___=\frac{2}{3}$ **b** $___-\frac{2}{5}=\frac{1}{6}$

**6** A bottle is $\frac{3}{4}$ full. $\frac{1}{3}$ of the liquid is then poured out. What fraction of the bottle is filled? PS R

**7** Danielle used half a sheet of adhesive plastic to cover her books, while Christina used $\frac{2}{5}$ of the same sheet. What fraction of the original sheet remains? Select **A**, **B**, **C**, or **D**.

PS R

**A** $\frac{3}{5}$ **B** $\frac{4}{5}$ **C** $\frac{1}{10}$ **D** $\frac{9}{10}$

**8** Evaluate $2\frac{7}{8}+1\frac{3}{8}$. Select **A**, **B**, **C**, or **D**.

**A** $4\frac{1}{4}$ **B** $3\frac{4}{5}$ **C** $3\frac{1}{2}$ **D** $13\frac{1}{4}$

**9** Evaluate each expression.

**a** $1\frac{1}{2}+\frac{2}{5}$ **b** $2\frac{1}{4}+1\frac{3}{8}$ **c** $7\frac{3}{8}+9\frac{1}{3}$ **d** $3\frac{4}{5}+1\frac{2}{9}$

**e** $6\frac{2}{5}-4\frac{7}{8}$ **f** $4\frac{3}{4}-2\frac{4}{5}$ **g** $2\frac{1}{3}-1\frac{1}{2}$ **h** $4\frac{3}{5}-2\frac{9}{10}$

Foundation Standard Complex


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**10** In a magic square, each row, column and diagonal must add to the same total. Complete each magic square with appropriate fractions. PS R

**a**

| | | |
|---|---|---|
| 4 | $1\frac{1}{2}$ | |
| $\frac{1}{2}$ | | $4\frac{1}{2}$ |
| 3 | | |

**b**

| | | |
|---|---|---|
| $\frac{1}{6}$ | | |
| | $\frac{2}{3}$ | |
| | $\frac{1}{3}$ | $1\frac{1}{6}$ |



# Multiplying and dividing fractions 6.03

## Multiplying and dividing fractions

- To **multiply fractions**, multiply the numerators and denominators separately, then simplify if possible (sometimes, it is easier to simplify the fractions first).
- To **divide by a fraction**, $\frac{a}{b}$, multiply by its reciprocal $\frac{b}{a}$.
- To **multiply or divide mixed numerals**, convert them to improper fractions first.

## Example 6

Evaluate each expression.

**a** $\frac{3}{5}\times\frac{2}{7}$ **b** $\frac{2}{3}\times\frac{3}{8}$ **c** $\frac{4}{5}\div\frac{2}{3}$ **d** $1\frac{1}{2}\times2\frac{2}{3}$

### Solution

**a** $\frac{3}{5}\times\frac{2}{7}=\frac{3\times2}{5\times7}$

$=\frac{6}{35}$

OR on a calculator, enter:
3 $\square\!\square$ 5 × 2 $\square\!\square$ 7 =
Note: $\square\!\square$ or $a^{b/c}$

**b** $\frac{2}{3}\times\frac{3}{8}=\frac{2\times3}{3\times8}$

$=\frac{6}{24}$

$=\frac{1}{4}$

OR by simplifying first before multiplying:

$\frac{2}{3}\times\frac{3}{8}=\frac{2^{1}}{3^{1}}\times\frac{3^{1}}{8^{4}}$

$=\frac{1}{4}$

OR on a calculator, enter:
2 $\square\!\square$ 3 × 3 $\square\!\square$ 8 =

**Worksheets**
Multiplying fractions using rectangles
Multiplying and dividing fractions number grids
What's the fraction question?
Fractions and decimals review

**Videos**
Multiplying and dividing fractions
Dividing fractions
Working out fractions
Multiplying fractions

Foundation Standard Complex

**c** $\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2}$

$= \frac{12}{10}$

$= \frac{6}{5}$

$= 1\frac{1}{5}$

OR by simplifying first before multiplying:

$\frac{4}{5} \div \frac{2}{3} = \frac{\cancel{4}^{2}}{5} \times \frac{3}{\cancel{2}_{1}} = \frac{6}{5} = 1\frac{1}{5}$

OR on a calculator, enter:

4 [a b/c] 5 [÷] 2 [a b/c] 3 [=]

**d** $1\frac{1}{2} \times 2\frac{2}{3} = \frac{3}{2} \times \frac{8}{3}$

$= \frac{\cancel{3}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{8}^{4}}{\cancel{3}_{1}}$

$= \frac{4}{1}$

$= 4$

OR on a calculator, enter:

1 [a b/c] 1 [a b/c] 2 [×] 2 [a b/c] 2 [a b/c] 3 [=]

For MATH mode, press [SHIFT] [a b/c] first.

## EXERCISE 6.03 ANSWERS ON P. 587

### Multiplying and dividing fractions U F PS R C

EXAMPLE 6

**1** Evaluate each product.

**a** $\frac{1}{2} \times \frac{2}{3}$ **b** $\frac{2}{3} \times \frac{5}{7}$ **c** $\left(\frac{1}{8}\right)^2$ **d** $\frac{9}{10} \times \frac{5}{6}$

**e** $\frac{3}{5} \times \frac{3}{4}$ **f** $1\frac{1}{2} \times 3$ **g** $3\frac{1}{4} \times 3\frac{4}{5}$ **h** $1\frac{1}{3} \times 2\frac{1}{3}$

**2** What is the value of $\frac{7}{10} \div \frac{1}{5}$? Select the correct answer **A**, **B**, **C**, or **D**.

**A** $\frac{7}{50}$ **B** $1\frac{1}{5}$ **C** $1\frac{2}{5}$ **D** $3\frac{1}{2}$

**3** Evaluate each quotient.

**a** $\frac{1}{7} \div \frac{2}{3}$ **b** $\frac{2}{5} \div \frac{3}{4}$ **c** $\frac{5}{9} \div \frac{5}{6}$ **d** $\frac{9}{10} \div \frac{3}{5}$

**e** $4 \div \frac{1}{4}$ **f** $6 \div \frac{2}{3}$ **g** $\frac{5}{8} \div 5$ **h** $\frac{2}{5} \div 6$

**i** $\frac{7}{8} \div \frac{1}{4}$ **j** $10 \div 3\frac{1}{7}$ **k** $3\frac{1}{2} \div 1\frac{1}{4}$ **l** $3\frac{3}{4} \div 1\frac{1}{10}$

**4** From a box of chocolates, Lindy takes $\frac{3}{8}$ of the chocolates and shares them equally among her 4 children. What fraction of the box of lollies does each child receive? PS R

**5** **a** What number's reciprocal is itself?

**b** What number does not have a reciprocal?

**c** What is the product of a number and its reciprocal?

**d** What is the reciprocal of the reciprocal of $\frac{1}{3}$? R C

**6** Copy and complete each equation. R

**a** $\frac{3}{4} \times __ = \frac{21}{32}$ **b** $__ \times \frac{1}{5} = \frac{2}{15}$

**c** $__ \div \frac{2}{3} = \frac{1}{7}$ **d** $\frac{4}{15} \div __ = \frac{8}{21}$

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**7** **a** When a number is divided by a proper fraction, does the number increase or decrease? Give reasons for your answer.

**b** When a number is multiplied by an improper fraction, does the number increase or decrease? Give reasons for your answer. R C

# Percentages, fractions and decimals 6.04



A **percentage** is a fraction whose denominator is 100.

## Example 7

Convert each percentage into a simplified fraction.

**a** 55% **b** 130% **c** $37\frac{1}{2}\%$

### Solution

**a** $55\% = \frac{55}{100}$
$= \frac{11}{20}$

**b** $130\% = \frac{130}{100}$
$= \frac{13}{10}$
$= 1\frac{3}{10}$

**c** $37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100}$
$= \frac{37\frac{1}{2} \times 2}{100 \times 2}$
$= \frac{75}{200}$
$= \frac{3}{8}$

## Example 8

Convert each percentage into a decimal.

**a** 8% **b** 43.6% **c** $18\frac{1}{2}\%$

### Solution

**a** $8\% = \frac{8}{100}$
$= 8 \div 100$
$= 0.08$

**b** $43.6\% = \frac{43.6}{100}$
$= 43.6 \div 100$
$= 0.436$

**c** $18\frac{1}{2}\% = \frac{18\frac{1}{2}}{100}$
$= 18.5 \div 100$
$= 0.185$

To convert a percentage into a decimal mentally, move the decimal point 2 places to the left

**Skillsheet**
Multiplying and dividing by 10, 100, 1000

**Video**
Converting percentages to fractions and decimals

**Worksheets**
Fractions wall
Decimals wall
Percentages wall
Fractions, decimals and percentages
Percentage shapes

**Presentation**
Percentages, fractions and decimals

**Puzzle**
Converting percentages to fractions

Foundation Standard Complex

**Puzzles**
Converting fractions to percentages

Converting decimals to percentages

**Video**
Converting fractions and decimals to percentages

## Converting to a percentage

To convert a fraction or a decimal into a percentage, multiply it by 100%

### Example 9

Convert each fraction into a percentage.

**a** $\frac{1}{8}$ **b** $\frac{34}{40}$

#### Solution

**a** $\frac{1}{8} = \frac{1}{8} \times 100\%$

$= 12\frac{1}{2}\%$

**b** $\frac{34}{40} = \frac{34}{40} \times 100\%$

$= 85\%$

### Example 10

Convert each decimal into a percentage.

**a** 0.147 **b** 0.6

#### Solution

**a** $0.147 = 0.147 \times 100\%$

$= 14.7\%$

**b** $0.6 = 0.6 \times 100\%$

$= 60\%$

To mentally convert a decimal into a percentage, just move the decimal point 2 places to the right

**Puzzles**
Percentages to decimals

Percentominoes

**Video**
Converting fractions and decimals to percentages

## Ordering fractions, decimals and percentages

To order fractions, percentages and decimals, convert each into a percentage first.

### Example 11

Arrange in ascending order: 0.667, 66%, $\frac{7}{11}$.

#### Solution

Convert all numbers into percentages.

$0.667 \times 100\% = 66.7\%$

$66\% = 66\%$

$\frac{7}{11} \times 100\% = 63\frac{7}{11}\%$

Arrange the percentages from smallest to largest: $63\frac{7}{11}\%$, 66% and 66.7%

So, in ascending order, the numbers are $\frac{7}{11}$, 66% and 0.667.

## EXERCISE 6.04 ANSWERS ON P. 587

### Percentages, fractions and decimals U F R C

**1** Convert each percentage into a simplified fraction. C

EXAMPLE 7

| | | | |
|---|---|---|---|
| **a** 60% | **b** 75% | **c** 31% | **d** 8% |
| **e** 30% | **f** 85% | **g** 99% | **h** 3% |
| **i** 160% | **j** 135% | **k** 25% | **l** 250% |

**2** Which decimal is equal to $62\frac{1}{2}\%$? Select the correct answer **A**, **B**, **C**, or **D**. C

EXAMPLE 8

| | | | |
|---|---|---|---|
| **A** 62.12 | **B** 62.5 | **C** 0.625 | **D** 0.0625 |

**3** Convert each percentage to a decimal. C

| | | | |
|---|---|---|---|
| **a** 18% | **b** 82% | **c** 2% | **d** 50% |
| **e** 120% | **f** 51.1% | **g** 79% | **h** $12\frac{1}{2}\%$ |
| **i** 16.3% | **j** 4% | **k** 18.7% | **l** $5\frac{1}{4}\%$ |

**4** Convert each fraction into a percentage. C

EXAMPLE 9

| | | | |
|---|---|---|---|
| **a** $\frac{17}{100}$ | **b** $\frac{7}{10}$ | **c** $\frac{13}{50}$ | **d** $\frac{11}{20}$ |
| **e** $\frac{5}{8}$ | **f** $\frac{24}{25}$ | **g** $\frac{2}{3}$ | **h** $\frac{5}{4}$ |
| **i** $1\frac{2}{5}$ | **j** $\frac{27}{40}$ | **k** $\frac{1}{16}$ | **l** $\frac{4}{9}$ |

**5** Convert each decimal into a percentage. C

EXAMPLE 10

| | | | |
|---|---|---|---|
| **a** 0.38 | **b** 0.55 | **c** 0.96 | **d** 0.625 |
| **e** 0.08 | **f** 0.054 | **g** 0.6 | **h** 0.003 |
| **i** 1.9 | **j** 0.405 | **k** 1.26 | **l** 0.114 |

**6** Copy and complete this table. C

| | Fraction | Decimal | Percentage |
|---|---|---|---|
| **a** | | 0.65 | |
| **b** | | 0.6 | |
| **c** | | | 20% |
| **d** | | | 84% |
| **e** | $\frac{1}{2}$ | | |
| **f** | $\frac{1}{8}$ | | |
| **g** | | | 36% |
| **h** | $\frac{5}{8}$ | | |
| **i** | | 0.73 | |
| **j** | $\frac{1}{3}$ | | |
| **k** | | | $66\frac{2}{3}\%$ |

EXAMPLE 11

**7** For each pair of numbers, determine which is larger. R

**a** 75% and 0.73 **b** $\frac{4}{25}$ and 14% **c** $\frac{2}{3}$ and 0.64

**d** 18% and $\frac{1}{7}$ **e** 0.22 and $\frac{1}{4}$ **f** 55% and $\frac{6}{11}$

**g** 0.93 and $\frac{19}{20}$ **h** 60% and $\frac{13}{25}$ **i** $\frac{1}{6}$ and 0.08

**8** Arrange each set of numbers in ascending order. R

**a** $\frac{4}{5}$, 78%, 0.75, $\frac{9}{11}$ **b** 22%, $\frac{1}{4}$, 0.29, $\frac{7}{20}$ **c** 0.62, $\frac{3}{5}$, 57%, 0.605

**9** Which list of numbers is arranged in descending order? Select **A**, **B**, **C**, or **D**. R

**A** $\frac{4}{11}$, $\frac{2}{5}$, 0.41, 43% **B** $\frac{2}{5}$, 0.41, $\frac{4}{11}$, 43%

**C** 43%, 0.41, $\frac{2}{5}$, $\frac{4}{11}$ **D** 0.41, $\frac{4}{11}$, 43%, $\frac{2}{5}$

**10** Arrange each set of numbers in descending order. R

**a** $\frac{9}{20}$, 0.47, $\frac{2}{5}$, 43% **b** 0.08, 86%, $\frac{21}{25}$, 0.88 **c** 0.905, $\frac{19}{20}$, 91%, $\frac{9}{10}$

Quiz Mental skills 6A

## Mental skills 6A: Maths without calculators ANSWERS ON P. 587

### Finding 10%, 20% and 5%

To find 10% or $\frac{1}{10}$ of a number, simply divide the number by 10 by moving the decimal point one place to the left.

**1** Study each example.

**a** $10\% \times 150 = 15\overset{\curvearrowleft}{0}. = 15$

**b** $10\% \times \$1256.80 = \$125\overset{\curvearrowleft}{6}.8 = \$125.68$

**c** $10\% \times 4917 = 491\overset{\curvearrowleft}{7}. = 491.7$

**d** $10\% \times \$48.55 = \$\overset{\curvearrowleft}{4}8.55 = \$4.885$

**2** Now find 10% of each amount.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | 190 | **b** | $75 | **c** | 875 | **d** | $202 |
| **e** | $37.60 | **f** | 400 | **g** | $9.25 | **h** | 896 |
| **i** | $2700 | **j** | $3.80 | **k** | $1527.60 | **l** | $72.50 |
| **m** | 3154 | **n** | $10.70 | **o** | 426 | **p** | $24 317.60 |

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20% is 10% doubled so to find 20% of a number, first find 10% then double it.

**3** Study each example.

**a** $20\% \times 700$

$10\% \times 700 = 70$

$\therefore 20\% \times 700 = 70 \times 2 = 140$

**b** $20\% \times \$876$

$10\% \times \$876 = \$87.60$

$\therefore 20\% \times \$876 = \$87.60 \times 2 = \$175.20$

**c** $20\% \times 325$

$10\% \times 325 = 32.5$

$\therefore 20\% \times 325 = 32.5 \times 2 = 65$

**d** $20\% \times \$38.50$

$10\% \times \$38.50 = \$3.85$

$\therefore 20\% \times \$38.50 = \$3.85 \times 2 = \$7.70$

**4** Now find 20% of each amount.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | 50 | **b** | 620 | **c** | \$2450 | **d** | \$8.60 |
| **e** | 72 | **f** | \$12 700 | **g** | 390 | **h** | \$5.80 |
| **i** | \$45 | **j** | \$84 | **k** | \$4600 | **l** | 320 |

5% is half of 10%, so to find 5% of a number, first find 10% then divide it by 2.

**5** Study each example.

**a** $5\% \times 180$

$10\% \times 180 = 18$

$\therefore 5\% \times 180 = 18 \div 2 = 9$

**b** $5\% \times \$76$

$10\% \times \$76 = \$7.60$

$\therefore 5\% \times \$76 = \$7.60 \div 2 = \$3.80$

**c** $5\% \times 120$

$10\% \times 120 = 12$

$\therefore 5\% \times 12 = 12 \div 2 = 6$

**d** $5\% \times \$142.20$

$10\% \times \$142.20 = \$14.22$

$\therefore 5\% \times \$142.20 = \$14.22 \div 2 = \$7.11$

**6** Now find 5% of each amount.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | 2000 | **b** | \$12 | **c** | 50 | **d** | \$27 |
| **e** | \$36.80 | **f** | \$84 | **g** | 800 | **h** | 130 |
| **i** | \$9.60 | **j** | \$138 | **k** | \$72 | **l** | 840 |



# Fraction and percentage of a quantity 6.05

## Example 12

Find:

**a** $\frac{1}{5} \times 45$

**b** $\frac{7}{8} \times \$32$

**c** $\frac{1}{3}$ of one year

**d** $\frac{4}{5}$ of 2 kg (in grams)

### Solution

**a** $\frac{1}{5} \times 45 = 45 \div 5$

$= 9$

**b** $\frac{7}{8} \times \$32 = \left(\frac{1}{8} \times \$32\right) \times 7$

$= \$4 \times 7$

$= \$28$

**Videos**
Fraction and percentage of a quantity
Fraction of a quantity
Percentages: Tax breaks

**Puzzles**
Percentage crossnumber

Percentages

Fractions of words

Percentages of an amount

**Worksheet**
Percentages of an amount

**c** $\frac{1}{3}$ of 1 year $= \frac{1}{3} \times 12$ months
$= 4$ months
Convert 1 year to 12 months.

**d** $\frac{4}{5}$ of 2 kg $= \frac{4}{5} \times 2000$ g
$= \left(\frac{1}{5} \times 2000 \text{ g}\right) \times 4$
$= 400 \text{ g} \times 4$
$= 1600$ g
Convert 2 kg to 2000 g.

## Percentage of a quantity

To find a **percentage of a quantity**, calculate:

$$\frac{\text{percentage}}{100} \times \text{quantity} \quad \text{or} \quad \text{percentage} \div 100 \times \text{quantity}$$

**Video**
Fraction and percentage of a quantity

## Example 13

Find:

**a** 8% of \$400

**b** 12.5% of 1 hour (in minutes)

**c** 20% of 3 m (in centimetres).

### Solution

**a** 8% of \$400 $= \frac{8}{100} \times$ \$400
$=$ \$32
or $8 \div 100 \times$ \$400 or $0.08 \times$ \$400

**b** 12.5% of 1 h $= 12.5\% \times 60$ min
$= \frac{12.5}{100} \times 60$ min
$= 7.5$ min
Convert 1 h to 60 min
or $12.5 \div 100 \times 60$ or $0.125 \times 60$

**c** 20% of 3 m $= 20\% \times 300$ cm
$= \frac{20}{100} \times 300$ cm
$= 60$ cm
Convert 3 m to 300 cm
or $20 \div 100 \times 300$ or $0.2 \times 300$

**EXERCISE 6.05** ANSWERS ON P. 587

## Fraction and percentage of a quantity UFR

EXAMPLE 12

**1** Find:

**a** $\frac{3}{5} \times 40$ **b** $\frac{1}{4} \times 28$ **c** $\frac{1}{6} \times 24$

**d** $\frac{2}{3} \times 15$ **e** $\frac{7}{10} \times 60$ **f** $\frac{5}{8} \times 16$

**g** $\frac{3}{4}$ of 1 km (in metres) **h** $\frac{1}{3}$ of 1 day (in hours) **i** $\frac{2}{5}$ of 1 L (in mL)

**j** $\frac{1}{8}$ of 1 t (in kg) **k** $\frac{5}{6}$ of 1 year (in months) **l** $\frac{7}{12}$ of 1 hour (in minutes)

**2** What is 35% of $75? Select the correct answer **A**, **B**, **C**, or **D**.

**A** $40 **B** $110 **C** $26.25 **D** $48.75

EXAMPLE 13

**3** Find:

**a** 11% of $500 **b** 2% of 250 kg **c** 13% of 150 L

**d** 21% of 600 cm **e** 15% of $450 **f** 5% of 5700 g

**g** 42% of 1128 m **h** 112% of 256 km **i** 19.4% of 785 mL

**j** 7.1% of $220 **k** 23.6% of $380 **l** 11.3% of 403 kg

**m** 110% of 95 **n** 150% of 302 **o** 130% of $2010

**p** 105% of 120 m **q** 170% of 350 g **r** 115% of $400

**4** Taffy the cat had a mass of 2.7 kg when she was found. If her mass increased by $\frac{1}{4}$, how much did she gain?



Shutterstock.com/Galushko Sergey

**5** A **discount** is a saving made between the original price of an item and the cheaper price.

**a** How much do you *save* if you get a 25% discount on a $420 games system?

**b** What would be the discount price of the games system?

**6** An Airbus can seat 351 passengers. If the plane flew with $\frac{2}{3}$ of the seats occupied, how many passengers were on the flight?

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**7** Calculate each amount (convert to a smaller unit first).

**a** 12% of 8 m **b** 35% of 1 t **c** $\frac{1}{5}$ of 12 hours

**d** $2\frac{1}{2}$% of 3 L **e** $\frac{3}{4}$ of 15 kg **f** $7\frac{1}{2}$% of 12 km

**g** 35% of 10 days **h** $\frac{1}{8}$ of 4 hours **i** 67.5% of 40 g

**8** 12% of the 525 swimmers at the swimming carnival wore black caps. How many swimmers wore black caps?

**9** Haroula earns $17.20 an hour in her part-time job. If she is given a 5% pay increase, by how much does her hourly rate increase?

**10** If 42% of the 650 road deaths occurred on country roads, how many road deaths happened in the country?

**11** In a city of 3 million people, 1% of the population are doctors. How many doctors are there? Select **A**, **B**, **C** or **D**.

**A** 3 **B** 3000 **C** 300 **D** 30 000

**12** $\frac{1}{20}$ of the cars produced in a factory were found to have steering defects. If 340 cars were produced, how many had steering defects?

**13** How many children are there in a crowd of 40 530 if $\frac{3}{5}$ are children?

**14** If 70% of the seats at a rock concert must be sold to make a profit, how many must be sold in a 2000-seat theatre?

**15** Copy and complete each equation. R

**a** 20% × _____ = $18 **b** 15% × _____ = $75

## Technology

### Discounts

In this activity, you will calculate the discount and sale price of items in a department store given their original price.

**1** Enter the following data into a spreadsheet. Make the headings in row 1 **bold**.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Item | Original Price | % Discount | Discount | Sale Price |
| 2 | Backpack | $69.95 | 10% | =B2*C2 | |
| 3 | Headphones | $38.00 | 5% | | |
| 4 | Beach towel | $20 | 30% | | |
| 5 | Book | $29.95 | 15% | | |
| 6 | Swimwear | $60 | 20% | | |
| 7 | Media player | $145 | 10% | | |

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2 Cell D2 shows the formula for calculating the discount on the backpack. Right-click on D2 and **Fill Down** to copy the formula down to cell D7. This will calculate the discount that applies to each item.

3 In cell E2, enter the formula **=B2-D2** to calculate the sale price of the backpack. Use **Fill Down** to calculate the sale prices of the remaining items.

# Expressing amounts as fractions and percentages

6.06



## Expressing amounts as fractions and percentages

To write an amount as a fraction of a whole amount:

- write the amount in the numerator of the fraction
- write the whole amount (total) in the denominator

$$\text{Fraction} = \frac{\text{amount}}{\text{whole amount}}$$

To write an amount as a percentage of a whole amount:

- write the amount in the numerator of a fraction
- write the whole amount (total) in the denominator
- multiply by 100%

$$\text{Percentage} = \frac{\text{amount}}{\text{whole amount}} \times 100\%$$

## Example 14

There were 250 people at the school fete, and 160 of them were children.

**a** What fraction of the people at the fete were children?

**b** What percentage of the people at the fete were children?

### Solution

**a** $$\text{Fraction} = \frac{\text{Number of children}}{\text{Total number of people}}$$
$$= \frac{160}{250}$$
$$= \frac{16}{25}$$

**b** $$\text{Percentage} = \frac{160}{250} \times 100\%$$
$$= 64\%$$

## Example 15

Express 36 minutes as:

**a** a fraction of an hour

**b** a percentage of an hour

### Solution

Quantities need to be expressed in the same units, so change 1 hour to 60 minutes.

**a** $$\text{Fraction} = \frac{36}{60} = \frac{3}{5}$$

**b** $$\text{Percentage} = \frac{36}{60} \times 100\%$$
$$= 60\%$$

**Worksheets**
Calculating percentages
Money and percentages review
Percentages

**Puzzle**
Calculating percentages

**Video**
Expressing quantities as fractions and percentages

**Technology**
Online discounts
Coffee shop sales

**EXERCISE 6.06** ANSWERS ON P. 587

## Expressing amounts as fractions and percentages U F PS R

EXAMPLE 14

**1** Convert each test mark into a simplified fraction.

**a** 50 out of 100 **b** 38 out of 50 **c** 87 out of 100

**d** 8 out of 12 **e** 5 out of 20 **f** 45 out of 120

**2** Convert each test mark in question **1** into a percentage.

**3** A hockey team scored 8 goals. If one player scored 5 of them, what is this as a percentage of the team score?



Shutterstock.com/Alexander Kalina

**4** In a class of 25 students, 6 ride bikes, 10 walk to school and the rest catch the bus.

**a** What fraction walk to school?

**b** What percentage catch the bus?

**5** Wakeel answered 21 questions correctly out of 24 in his driving test. If the pass mark is 95%, did he pass? PS R

**6** The World Cup cricket final attracted 86 000 people to the game, but 94 500 tickets were sold before the match.

**a** What fraction of the sold tickets were used?

**b** What percentage (correct to the nearest whole number) of the sold tickets were not used?

**7** Tania earns $1340 a week. She pays $428.80 in tax and saves $180 a week.

**a** What percentage of Tania's earnings is paid in tax?

**b** What fraction of her earnings does Tania save?

**c** What percentage (correct to one decimal place) of her earnings does Tania save?

**8** Louise sells a house for $458 000. If the real estate agent is paid a commission of $22 900, what percentage of the sale price does the agent receive?

**9** The Great Gals are having a sale on microwave ovens. The sale price of each oven is listed below, along with the discount. PS R

**i**



Shutterstock.com/Markus Gann

$149
Save $20

**ii**



Shutterstock.com/Maxx-Studio

$179
Save $26

**iii**



Dreamstime.com/Pioneer111

$219
Save $30

**a** Calculate the original price of each microwave oven.

**b** Calculate, to one decimal place, the percentage discount on each oven.

**c** Which oven has the greatest percentage discount?

**10** Jamal bought a pair of roller skates for $180 and sold them at a profit of $45. What is the profit as a percentage? Select the correct answer **A**, **B**, **C**, or **D**. PS

**A** 25% **B** 4% **C** 45% **D** 20%

**11** Express each measurement as a simplified fraction.

EXAMPLE 15

**a** 5 min of 1 h **b** 250 mL of 1 L **c** 700 m of 1 km **d** 230 kg of 1 t

**e** 75c of $6 **f** 40 min of 4 h **g** 300 g of 2 kg **h** $3.80 of $14

**i** 12 h of 4 days **j** 75 mm of 20 cm **k** 800 m of 1.5 km **l** 400 mL of 3.5 L

**12** Express each measurement in question **11** as a percentage.

**13** What is 40 minutes as a percentage of 2 hours? Select **A**, **B**, **C**, or **D**.

**A** 20% **B** 5% **C** $33\frac{1}{3}\%$ **D** 0.33%

**14** A football team is scoring well in its matches if its points percentage is over 100, according to the formula:

$$\text{Points percentage} = \frac{\text{Points for}}{\text{Points against}} \times 100$$

where 'points for' are the total number of points the team has scored and 'points against' are the total number of points the other teams have scored when playing against that team.

**a** Calculate, correct to one decimal place, the points percentage for each team listed in the table. PS R

| Team | Points for | Points against | Team | Points for | Points against |
|---|---|---|---|---|---|
| Broncos | 391 | 313 | Raiders | 405 | 466 |
| Bulldogs | 410 | 366 | Roosters | 296 | 479 |
| Cowboys | 383 | 452 | Sea Eagles | 434 | 235 |
| Dragons | 321 | 395 | Sharks | 328 | 279 |
| Eels | 407 | 352 | Storm | 478 | 207 |
| Knights | 332 | 382 | Tigers | 498 | 642 |
| Panthers | 357 | 469 | Titans | 319 | 399 |
| Rabbitohs | 275 | 283 | Warriors | 688 | 454 |

**b** What are the top 4 teams based on these results?

## Did you know?

### Where are the percentages?

Percentages are all around us.

- Financial institutions advertise interest rates they pay on investments and charge on loans as percentages
- Companies report profits or losses as percentages. They also report dividends paid as percentages
- When the value of a product increases or the cost to the consumer of different items increases, it is reported as a percentage.
- Many statistics about the population are given as percentages.

An understanding of percentages is vital to understanding the world we live in.

**Find 5 other situations where percentages are used.**

## Investigation

### Success rates in netball and basketball



Alamy Stock Photo/Action Plus Sports Images

Percentages are a good way of comparing sporting performances. Commentators often give statistics in the form of percentages so that we can compare the success of teams or players.

**1** In netball, goal shooters rarely have the same number of shooting chances. To work out their success rates, we must use percentages to compare the number of shooting chances with the number of successful shots.

The following statistics were recorded in a match between Australia and New Zealand.

| | Australia | | New Zealand | |
|---|---|---|---|---|
| | Shooting chances | Goals | Shooting chances | Goals |
| 1st quarter | 15 | 11 | 9 | 9 |
| 2nd quarter | 25 | 17 | 14 | 13 |
| 3rd quarter | 18 | 14 | 23 | 20 |
| 4th quarter | 27 | 20 | 18 | 17 |

**a** Calculate as a percentage correct to one decimal place:

- **i** each team's success rate for each quarter
- **ii** the total success rate for each team.

**b** Comment on the relationship between the success rates and the final scores. What can you say about each team?

**2** At any stage during the season in the National Basketball League (NBL), the teams will have played different numbers of games. The highest position on the ladder is awarded to the team with the highest **wins percentage**, which is calculated as follows:

$$\text{Wins percentage} = \frac{\text{Number of games won}}{\text{Number of games played}} \times 100\%$$



Alamy Stock Photo/ZUMA Press

At one stage of a season the teams had won and lost the number of games shown below.

| Team | Won | Lost |
|---|---|---|
| Adelaide 36ers | 6 | 6 |
| Cairns Taipans | 8 | 7 |
| Brisbane Bullets | 7 | 4 |
| Melbourne United | 6 | 9 |
| New Zealand Breakers | 9 | 4 |
| Perth Wildcats | 3 | 11 |
| Sydney Kings | 11 | 3 |
| Tasmanian Jackjumpers | 8 | 4 |
| Illawarra Hawks | 2 | 12 |

Calculate the wins percentage for each team and construct the 'NBL Ladder' showing the teams in the correct order.

# 6.07 Percentage increase and decrease

- Percentage increase means to increase (make bigger) a quantity by a percentage
- Percentage decrease means to decrease (make smaller) a quantity by a percentage

**Videos**
Percentage increase and decrease
Hyperinflation: 1920s Germany
The most populous country

**Worksheets**
Percentage shortcuts
Discounts and special offers
Discounts

**Puzzle**
Discounts

**Video**
Percentage increase and decrease

## Example 16

Increase $200 by 7%.

### Solution

**Method 1**

$$7\% \text{ of } \$200 = \frac{7}{100} \times \$200$$
$$= \$14$$
$$\$200 + \$14 = \$214$$

or $7 \div 100 \times \$200$ or $0.07 \times \$200$

**Method 2**

$$(100\% + 7\%) \text{ of } \$200 = 107\% \text{ of } \$200$$
$$= \frac{107}{100} \times \$200$$
$$= \$214$$

Increasing by 7% is the same as calculating 107%

or $107 \div 100 \times \$200$ or $1.07 \times \$200$

## Example 17

Decrease $150 by 12%.

### Solution

**Method 1**

$$12\% \text{ of } \$150 = \frac{12}{100} \times \$150$$
$$= \$18$$
$$\$150 - \$18 = \$132$$

or $12 \div 100 \times \$150$ or $0.12 \times \$150$

**Method 2**

$$(100\% - 12\%) \text{ of } \$150 = 88\% \text{ of } \$150$$
$$= \frac{88}{100} \times \$150$$
$$= \$132$$

Decreasing by 12% is the same as calculating 88%

or $88 \div 100 \times \$150$ or $0.88 \times \$150$

## Example 18

The price of a watch increases by 15%. If its original price was $35, find its new price.

### Solution

**Method 1**

$$15\% \text{ of } \$35 = \$5.25$$
$$\$35 + \$5.25 = \$40.25$$

**Method 2**

$$(100\% + 15\%) \text{ of } \$35 = 115\% \text{ of } \$35$$
$$= \$40.25$$

## Example 19

Find the price of a computer game system, originally priced at $420, after a 9% discount.



Shutterstock.com/Saikorn

### Solution

**Method 1**

9% of $420 = $37.80

$420 − $37.80 = $382.20

**Method 2**

(100% − 9%) of $420 = 91% of $420

= $382.20

**EXERCISE 6.07** ANSWERS ON P. 588

## Percentage increase and decrease U F PS R C

**1** Increase:

**a** $150 by 5% **b** 400 by 20% **c** 60 km by 22%

**d** $2500 by 6% **e** 95 kg by 60% **f** 10 L by 33%

EXAMPLE 16

**2** Decrease:

**a** $440 by 60% **b** 120 by 15% **c** 110 kg by 8%

**d** $325 by 25% **e** 2000 L by 38% **f** $1570 by 3%

EXAMPLE 17

**3** Wooden chairs cost $172 to make. Calculate the selling price if the chairs are marked up by 35% when sold. PS

EXAMPLE 18

**4** Julie buys a bike for $2700 and sells it a year later, making a 15% profit.

**a** How much profit did Julie make?

**b** What was the selling price of the bike?

**5** Geeva's weekly pay of $980 increased by 4.5%. What is his new pay?

**6** Yumi's height of 168 cm increased by 2%. What is her new height?

**7** A department store has a mark-up of 200% on clothing. If the store buys a vest for $12, what will be its selling price after the mark-up? Select the correct answer **A**, **B**, **C**, or **D**.

**A** $32 **B** $14.40 **C** $24 **D** $36

**8** A car dealer offers a 15% discount on all new car purchases. What would you pay for a car marked at $21 990?

EXAMPLE 19

**9** Calculate the sale price of a pair of sports shoes marked at $165 after a discount of 12%.

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**10** After speed cameras were installed, the road toll of 840 deaths decreased by 5%. What is the new road toll?



iStock.com/Michele Jackson

**11** Aaron bought a house for $464 000 and sold it 10 years later, making a 150% profit. Calculate the selling price of the house.

**12** What is the new price of a pair of jeans worth $75 if they are discounted by 25%? Select **A**, **B**, **C**, or **D**.

**A** $50 **B** $56.25 **C** $18.75 **D** $55

**13** A netball sells for $45, but you receive a 11% discount if you buy 2. What is the discounted price of 2 netballs? PS

**14** The owner of a store buys a tablet device for $470. She adds a mark-up of $70 but, at sale time, offers a 30% discount. PS

**a** Find the selling price after the mark-up.

**b** Find the discounted price at sale time.

**c** How much loss did the store owner make on the tablet device?

**d** Calculate the percentage loss correct to one decimal place.

**15** Find the number which, if increased by 18%, gives 767. R

**16** Winter coats priced at $830 were reduced by 15% at the end-of-season sale. On the last weekend of the sale, they were reduced by a further 40% off the discounted price. What was the final price of a coat? PS R

**17** Find the number which, if decreased by 40%, gives 306. R

**18** A portable air conditioner with an original price of $680 has 10% GST added to it. It is then sold at an end-of-year sale for '10% off'. Is the sale price of the air conditioner more than, less than, or equal to its original price? Justify your answer by calculation. PS R C

# Percentages without calculators

6.08

If we know the equivalent fractions for common percentages, then we can find percentages mentally.

| Fraction | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{3}{5}$ | $\frac{1}{10}$ | $\frac{1}{8}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Percentage | 25% | 50% | 75% | $33\frac{1}{3}\%$ | $66\frac{2}{3}\%$ | 20% | 40% | 60% | 10% | $12\frac{1}{2}\%$ |

**Skillsheet** Mental percentages

**Worksheet** Percentages without calculators



## Example 20

Calculate each percentage mentally.

**a** $33\frac{1}{3}\%$ of \$1800

**b** $12\frac{1}{2}\%$ of 72 kg

### Solution

**a** $33\frac{1}{3}\%$ of \$1800 $= \frac{1}{3}$ of \$1800
$= \$600$

**b** $12\frac{1}{2}\%$ of 72 kg $= \frac{1}{8}$ of 72 kg
$= 9$ kg

## Example 21

**a** Increase \$470 by 10%.

**b** Decrease \$75 by 20%.

### Solution

**a** 10% of \$470 $= \frac{1}{10} \times \$470$
$= \$47$
\$470 + \$47 = \$517

**b** 20% of \$75 $= \frac{1}{5} \times \$75$ (or $10\% \times \$75 \times 2$)
$= \$15$
\$75 − \$15 = \$60

Note: More examples and exercises on increasing and decreasing an amount by a percentage mentally can be found in Mental skills 6B: Percentage increase and decrease on p. 263.




Shutterstock.com/Lin Xiu Xiu

**EXERCISE 6.08** ANSWERS ON P. 588

## Percentages without calculators U F PS R

Complete this exercise without using a calculator.

EXAMPLE 20

**1** Find 25% of each amount.

**a** 32 kg **b** 180 mm **c** 24 L **d** \$1000

**2** Find 40% of \$60. Select the correct answer **A**, **B**, **C**, or **D**.

**A** \$6 **B** \$12 **C** \$24 **D** \$48

**3** Find each amount.

**a** 50% of 36 m **b** 75% of 12 g **c** $12\frac{1}{2}\%$ of 24 h

**d** 20% of 5 L **e** $66\frac{2}{3}\%$ of \$30 **f** 60% of 20 km

**g** 25% of 20 mL **h** $33\frac{1}{3}\%$ of 18 days **i** 75% of 100 kg

**4** Find 10% of:

**a** 60 t **b** 40 hours **c** 300 sheep **d** \$90

**5** Find each amount.

**a** 10% of 70 **b** 50% of \$80 **c** 40% of 120 minutes

**d** 30% of \$210 **e** 60% of 250 cm **f** 20% of 400 mL

**6** What is 5% of 80 days? Select **A**, **B**, **C**, or **D**.

**A** 8 days **B** 4 days **C** 40 days **D** 80 days

**7** Find $12\frac{1}{2}\%$ of each amount.

**a** \$72 **b** 40 kg **c** 160 kL

EXAMPLE 21

**8** Increase:

**a** \$80 by 25% **b** 140 by 10% **c** \$150 by $33\frac{1}{3}\%$

**d** 28 by 50% **e** \$240 by $12\frac{1}{2}\%$ **f** 45 by 20%

**9** Decrease:

**a** \$310 by 10% **b** 120 by 20% **c** \$80 by $12\frac{1}{2}\%$

**d** 60 by 5% **e** \$900 by $66\frac{2}{3}\%$ **f** 2000 by 75%

**10** A class of 28 students sat for a History test out of 40. After 25 minutes, Leonie had already answered 60% of the test correctly. PS R

**a** How many marks has Leonie already scored?

**b** If Leonie's goal is to obtain 85% or more in the test, how many more marks will she need?

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# Mental skills 6B: Maths without calculators ANSWERS ON P. 588

Quiz
Mental skills 6B



## Percentage increase and decrease

The fraction equivalents of commonly-used percentages can help us when we need to increase or decrease a number by a percentage.

| Percentage | 10% | $12\frac{1}{2}\%$ | 20% | 25% | $33\frac{1}{3}\%$ | 50% | $66\frac{2}{3}\%$ | 75% |
|---|---|---|---|---|---|---|---|---|
| Fraction | $\frac{1}{10}$ | $\frac{1}{8}$ | $\frac{1}{5}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{3}{4}$ |

**1** Consider each example.

**a** Increase 80 by 20%

$20\% \text{ of } 80 = \frac{1}{5} \times 80$
$= 80 \div 5$
$= 16$
$80 + 16 = 96$

or

$10\% \text{ of } 80 = 8$
$20\% \text{ of } 80 = 8 \times 2$
$= 16$

**b** Increase \$36 by $66\frac{2}{3}\%$

$66\frac{2}{3}\% \text{ of } 36 = \frac{2}{3} \times 36$
$= \$36 \div 3 \times 2$
$= \$12 \times 2$
$= \$24$
$\$36 + \$24 = \$60$

**2** Now increase:

**a** \$280 by 10%
**b** 45 by $33\frac{1}{3}\%$
**c** 25 by 20%
**d** \$400 by $12\frac{1}{2}\%$
**e** 64 by 50%
**f** \$72 by 25%
**g** \$55 by 10%
**h** 90 by $66\frac{2}{3}\%$
**i** 120 by 75%
**j** \$80 by 5%
**k** \$250 by 20%
**l** 70 by 40%

**3** Consider each example.

**a** Decrease 225 by $33\frac{1}{3}\%$

$33\frac{1}{3}\% \text{ of } 225 = \frac{1}{3} \times 225$
$= 225 \div 3$
$= 75$
$225 - 75 = 150$

**b** Decrease \$70 by 15%

$10\% \text{ of } \$70 = \frac{1}{10} \times \$70 = \$7$

$\therefore 5\% \text{ of } \$70 = \frac{1}{2} \times \$7 = \$3.50$

$\therefore 15\% \text{ of } \$70 = (10\% \times \$70) + (5\% \times \$70)$
$= \$7 + \$3.50$
$= \$10.50$
$\$70 - \$10.50 = \$59.50$

**4** Now decrease:

**a** \$480 by 25%
**b** 60 by $33\frac{1}{3}\%$
**c** 110 by 20%
**d** \$25 by 10%
**e** 900 by 50%
**f** \$72 by $12\frac{1}{2}\%$
**g** \$320 by 75%
**h** 150 by $66\frac{2}{3}\%$
**i** \$63 by 20%
**j** \$100 by $12\frac{1}{2}\%$
**k** 250 by 10%
**l** \$150 by 30%

# 6.09 The unitary method

Worksheets
The unitary method
Working with percentages

Video
The unitary method

If we know the percentage of an amount, but not the actual amount, we can use the **unitary method** to find the whole amount. The word **unit** means 'one', and with the unitary method we find 1% first.

## Example 22

If 15% of an amount is \$75, what is the amount?

### Solution

15% of amount = \$75

1% of amount = \$75 ÷ 15
= \$5 — Find 1% first by dividing by 15.

100% of amount = \$5 × 100
= \$500 — Then find the whole amount by multiplying by 100.

The amount is \$500. — Check: 15% × \$500 = \$75

## The unitary method

To find an amount given a **percentage** of the amount:

- first find 1% of the amount by dividing by the known percentage
- then multiply by 100 to find the whole amount (100%).

## Example 23

Farmer Kate lost 30% of her sheep in a flood. If she had 560 sheep left, how many sheep did she have before the flood?

### Solution

Because 30% of the total were lost,
100% − 30% = 70% of the total were left.

So: 70% of the total = 560 sheep

1% of the total = 560 ÷ 70
= 8 sheep
100% of the total = 8 × 100
= 800 sheep

Kate had 800 sheep before the flood. — Check: 70% × 800 = 560

**EXERCISE 6.09** ANSWERS ON P. 588

## The unitary method U F PS R

EXAMPLE 22

**1** What is the whole amount if:

| | | | | | |
|---|---|---|---|---|---|
| **a** | 10% of it is $70? | **b** | 25% of it is $140? | **c** | 17% of it is $782? |
| **d** | 8.5% of it is $161.50? | **e** | 16% of it is 64 cm? | **f** | 20% of it is 56 kg? |
| **g** | 42% of it is 1.26 m? | **h** | 30% of it is 45 minutes? | **i** | 70% of it is $280? |
| **j** | 110% of it is $396? | **k** | 45% of it is 180 kg? | **l** | 120% of it is 132 minutes? |

**2** The deposit of 15% on a kitchen stove is $180. What is the price of the stove? PS R

**3** There were 51 870 men in the crowd at the football match. If this was 65% of the crowd, what was the total attendance? PS R

**4** Last year Joanna paid $23 009.28 in income tax. If this represents 28% of her income, calculate her income. PS R

EXAMPLE 23

**5** Jude sold his car for $12 200, making a loss of 20% on the original price. What was the original price? PS R

**6** Pip makes 18% profit on cakes she decorates. If she sells a cake for $330.40, what was the original cost of the cake? Select the closest answer **A**, **B**, **C** or **D**. PS R

| | | | |
|---|---|---|---|
| **A** | $184 | **B** | $238 |
| **C** | $271 | **D** | $280 |



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**7** Binnsville's population increased by 5%, or 1200 people. What was the old population? PS R

**8** The interest earned on an investment is $168. If the interest rate is 5.6%, how much was invested? PS R

**9** Venus buys a dress that has been discounted by 25%. If the sale price is $225, what was the original price of the dress? PS R

**10** Matthew buys a tent for $348 at a discount of 20%. What was the original price of the tent? PS R

# 6.10 Profit, loss and GST

## Profit and loss

Worksheet
Profit and loss

Technology
Profit and loss

Retailers (store owners) buy goods and sell them to people.

- **Cost price** is how much they buy the item for
- **Selling price** is how much they sell them for
- If they sell the item for more than what they paid for it, they make a **profit**
- If they sell the item for less than what they paid for it, they make a **loss**

The **percentage profit** or **percentage loss** is usually calculated as a percentage of the cost price.

Videos
Business maths
Profit and loss

### Example 24

*The Book Bin* buys books for \$18 each and sells them for \$22.50. For each book, calculate:

**a** the profit **b** the percentage profit.

#### Solution

**a** Cost price = \$18
Selling price = \$22.50

Profit = \$22.50 − \$18
= \$4.50

selling price − cost price

**b** Percentage profit $= \frac{\$4.50}{\$18} \times 100\%$
$= 25\%$

$\frac{\text{profit}}{\text{cost price}} \times 100\%$

Puzzle
Profit and loss

Video
Profit and loss

### Example 25

Renae buys a car for \$17 500 and sells it 2 years later for \$15 000. Find the percentage loss, correct to one decimal place.

#### Solution

Cost price = \$17 500
Selling price = \$15 000

Loss = \$17 500 − \$15 000
= \$2500

cost price − selling price

Percentage loss $= \frac{\$2500}{\$17\,500} \times 100\%$
$= 14.2857\ldots\%$
$\approx 14.3\%$

$\frac{\text{loss}}{\text{cost price}} \times 100\%$

## Example 26

Cooper buys a mountain bike for $850 and sells it a year later at 13% profit. Find the selling price.

### Solution

$\text{Profit} = 13\% \times \$850$
$= \$110.50$

$\text{Selling price} = \$850 + \$110.50$
$= \$960.50$

OR $\text{Selling price} = (100\% + 13\%) \times \$850$ Increasing $850 by 13%
$= 113\% \times \$850$
$= \$960.50$



# GST

**Goods and Services Tax (GST)** is a tax paid to the government on most goods (items) and services that we purchase. In Australia, GST is charged at 10% of the original price and is generally included in the marked price of the good or service.

## Example 27


Video GST

The selling price of a TV is $722 + 10% GST. Calculate:

**a** the GST payable

**b** the selling price

### Solution

**a** $\text{GST} = 10\% \text{ of } \$722$
$= \$72.20$

**b** $\text{Selling price} = \$722 + \$72.20$
$= \$794.20$

OR $\text{Selling price} = 110\% \times \$722$
$= \$794.20$

## Example 28


Video GST

The selling price of a lounge suite is $2695 with GST included.
How much of this price is the GST?

### Solution

Selling price + GST = 100% + 10% = 110%

110% of the selling price = $2695

$10\% \text{ GST} = \$2695 \div 11$
$= \$245$

**EXERCISE 6.10** ANSWERS ON P. 588

## Profit, loss and GST U F PS R

**1** In each situation, state:

**i** the cost price

**ii** whether a profit or loss was made.

**a** A car was bought for $18 700 and sold for $15 000.

**b** A house was bought for $543 000 and sold for $674 000.

**c** A video camera was sold for $350 when it originally cost $799.

**d** An antique desk sold for $8000 when it was purchased for $6800.

EXAMPLE 24

**2** For each situation, find:

**i** the profit

**ii** the percentage profit, correct to one decimal place where necessary.

**a** cost price $85 selling price $102

**b** cost price $415 selling price $600

**c** cost price $2.75 selling price $4.20

EXAMPLE 25

**3** For each situation, find:

**i** the loss

**ii** the percentage loss, correct to one decimal place where necessary.

**a** cost price $19 selling price $14

**b** cost price $2700 selling price $1450

**c** cost price $79 selling price $58

**4** Anthony bought a computer for $519 and sold it 3 years later for $220. Calculate his percentage loss, correct to 1 decimal place. Select the closest answer **A**, **B**, **C** or **D**.

**A** 26.2% **B** 42.4% **C** 57.6% **D** 172.8%

**5** A 3D printer costing $1250 is sold for $1400. Find:

**a** the profit **b** the percentage profit.

**6** A car costing $17 000 is sold for $15 200. Find:

**a** the loss **b** the percentage loss, correct to one decimal place

EXAMPLE 26

**7** Calculate the selling price for each situation.

**a** cost price $200 profit 17% **b** cost price $42 profit 70%

**c** cost price $720 loss 35% **d** cost price $1400 loss 9.5%

**e** cost price $2.50 profit 150% **f** cost price $27 loss 40%

**8** A block of land is purchased by Erin and Mark for $133 000. Six years later, they sell it for $164 000. Calculate their percentage profit, correct to 2 decimal places. PS

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**9** Rose buys a bed for $2700. She later sells it for $1500. Calculate her percentage loss, correct to one decimal place. PS

**10** For each item, calculate the GST payable and the final price.

EXAMPLE 27

**a** a car priced at $20 900
**b** a home entertainment unit priced at $1810
**c** an ice cream priced at $3.00
**d** a theatre ticket priced at $145
**e** plumber's fees of $180
**f** a desk lamp priced at $27

6.10

**11** Given that the final price of each item has GST included, calculate the GST and the original price, correct to the nearest cent. PS R

EXAMPLE 28

**a** accountant's fees: $792
**b** piano lessons: $198 per term
**c** refrigerator: $924
**d** box of chocolates: $18.70
**e** travel guide: $34.65
**f** diary: $16.70

**12** A manufacturer sells TVs to stores for $899 each. *Electric City* store adds on 60% profit for its selling price. It then must add on 10% GST. PS R

**a** Calculate the store's selling price of one TV before GST is added. Round your answer to the nearest $10.
**b** Calculate the final selling price of each television after GST is added.
**c** In the January sales all goods are sold at 40% off the marked price. Calculate the discounted price of a TV.
**d** Find how much of this discounted price will be paid in GST by dividing by 11.
**e** After paying GST to the government, does the store make a profit or a loss on each TV? How much profit or loss?
**f** Calculate the percentage profit or loss. Round your answer to one decimal place.

## Investigation

### Uses of percentages

1 Prepare a one-page percentage collage for class display. Look through newspapers, magazines and brochures for examples to cut out. Find photographs or draw situations where percentages are used.

2 Select 2 examples of situations where percentages are used. Write a set of 10 questions for each example, using the rules about percentages you have learnt. Calculate the answers.

3 Swap the questions you wrote in question **2** with others in the class. Each person should answer at least 3 sets of questions. Ask the student who prepared the questions to mark your answers. If there are any disagreements, check with your teacher.

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# 6.11 Simple interest

Worksheet
Simple interest

Banks, credit unions and other financial institutions reward you by paying you interest on your savings or investments. However, they also charge you interest if you borrow money from them.

- The original amount of money invested or borrowed is called the **principal**.
- **Interest** is calculated as a percentage of the principal.
- This percentage is called the **interest rate**, usually written as a rate **per annum** ('per year'), abbreviated 'p.a.'
- **Simple interest** (or **flat rate interest**) is interest calculated simply on the original principal.

## Example 29

Find the simple interest paid on $7500 borrowed at 8% p.a. for:

**a** 1 year **b** 3 years

### Solution

**a** Interest = 8% of $7500
= $600

**b** Interest = 8% of $7500 × 3
= $1800

YEAR 9 CORE

## The simple interest formula

$I = Prn$, where:

$I$ is the simple interest

$P$ is the principal

$r$ is the interest rate per year, expressed as a decimal

$n$ is the number of years

Applying this to the above example,

$P = \$7500$, $r = 8\% = 0.08$, $n = 3$ years

$I = Prn$

$= \$7500 \times 0.08 \times 3$

$= \$1800$

## Example 30

Find the simple interest earned on $8620 invested at 2.4% p.a. for 7 months.

Video
Simple interest

### Solution

$P = \$8620$, $r = 2.4\% = 0.024$,
$n = 7$ months $= \frac{7}{12}$ years

Divide by 12 to convert 7 months into years

$I = Prn$

$= \$8620 \times 0.024 \times \frac{7}{12}$

$= \$120.68$


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## EXERCISE 6.11 ANSWERS ON P. 589

### Simple interest U F PS

*Round answers to the nearest cent if needed.*

EXAMPLE 29

**1** Which of the following is the simple interest earned on $600 at 5% p.a. for 4 years? Select the correct answer **A**, **B**, **C**, or **D**.

**A** $20 **B** $30 **C** $80 **D** $120

**2** Find the simple interest paid on a loan of:

**a** $2400 at 5% p.a. for 1 year
**b** $2400 at 5% p.a. for 6 years
**c** $820 at 8% p.a. for 4 years
**d** $75 at 4% p.a. for 2 years
**e** $16 250 at 3.5% p.a. for 3 years
**f** $6570 at 11.5% p.a. for 5 years
**g** $532 at 9.25% p.a. for 8 years
**h** $4200 at 7% p.a. for 2 years
**i** $985 at 4.3% p.a. for 1 year
**j** $14 990 at 5.4% p.a. for 3 years

**3** Find the interest earned if you invest:

**a** $5000 at 9% p.a. for 3 years
**b** $7500 at 6% p.a. for 5 years
**c** $20 000 at 8% p.a. for 7 years
**d** $45 000 at 12% p.a. for 20 years
**e** $875 at 5.8% p.a. for 2 years
**f** $9000 at 6.95% p.a. for 10 years
**g** $11 452 at 7.25% p.a. for 6 years
**h** $4500 at 4% p.a. for 3 years
**i** $737 at 5% p.a. for 1 year
**j** $2340 at 8% p.a. for 2 years

YEAR 9 CORE

EXAMPLE 30

**4** Find the simple interest earned on:

**a** $1080 at 5% p.a. for 6 months
**b** $700 at 6.5% p.a. for 3 months
**c** $2500 at 9% p.a. for 1 month
**d** $40 000 at 3.4% p.a. for 9 months
**e** $475 at 2% p.a. for 5 months
**f** $8250 at 4.1% p.a. for 8 months
**g** $15 400 at 7% p.a. for 11 months
**h** $4700 at 2.8% p.a. for 17 months
**i** $350 at 3% p.a. for 14 months
**j** $12 900 at 4.7% p.a. for 27 months

**5** Xander borrowed $7000 from New Century Bank for 3 years at 3.5% p.a. interest. PS

**a** How much interest was he charged?
**b** How much, including interest, did Xander have to repay the bank?

**6** Goran borrowed $30 000 from Nelson Finance to set up a catering business. He is charged 7% p.a. simple interest over 5 years. PS

**a** How much interest will Goran have to pay?
**b** Calculate the total amount he must repay.
**c** Goran agrees to repay the loan in monthly payments over 5 years. How much must he repay each month?

**7** When Jesinta borrowed $18 000 from a credit union, she was charged 9% p.a. simple interest. PS

**a** How much interest would she pay in one year?
**b** Her total interest bill was $6480. For how long did Jesinta borrow the money?

# 6.12 Percentage problems

Worksheet
Percentage problems

## EXERCISE 6.12 ANSWERS ON P. 589

### Percentage problems U F PS R

**1** In a box of 180 oranges, 15% are not ripe. How many oranges are not ripe?

**2** A test result is 65 out of 80. What percentage is this?

**3** Southside High's hockey team wins 12 games out of 15. What percentage is this?

**4** A shirt costing $55 is reduced by 25%. What is the new selling price?

**5** In some mixed paint, 25% of the mixture is blue, 15% is yellow and the remainder is white. What percentage is white?

**6** Westvale High's basketball team scored 1065 points in a season. Claire shot 22% of the total points. How many points did Claire score?

**7** At Upper Darling High, 60% of students are girls. How many boys are there in this school of 870 students? PS R

**8** What percentage of 9 carat gold is pure gold, if pure gold is 24 carat? PS R

**9** When a floor was being tiled, 830 green tiles were used out of a total of 2075 tiles. What percentage of the floor is made of green tiles? PS R

**10** In a Year 1 class of 24 students, there were 3 absent students. Find the percentage of students absent from the class. PS R

**11** A real estate agent earns $2\frac{1}{2}$% commission on a sale of $345 000. How much does the agent earn?

**12** A worker's pay rate of $27.30/hour is increased to $30.40/hour. Find the increase as a percentage of the original pay rate, correct to one decimal place. PS R

**13** Find how much rent is paid if it is 30% of a weekly income of $875. PS R

**14** Anton receives a 7% pay rise. If his old salary was $67 000 per year, what is his new salary? Select the correct answer **A**, **B**, **C**, or **D**. PS R

**A** $67 007 **B** $67 469 **C** $71 690 **D** $113 900

**15** Maxine is a car salesperson. She is paid a retainer of $620 per week to cover her living expenses, plus 5% of all sales she makes. Maxine's sales in the first week of December total $74 999. Calculate her pay for that week. PS R

**16** Robert writes novels with a recommended retail price of $34.95. He is paid 10% royalty on the first 4000 copies sold and 12.5% royalty on the remaining sales. Calculate the royalties Robert will receive if his latest novel sells 11 000 copies. PS R

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# Technology

## Weekly budget

In this activity, we will use a spreadsheet to calculate a weekly budget for Abdul, a full-time student with a part-time job in a café.

**1** Enter Abdul's weekly expenses into a spreadsheet. Highlight cells B2 to B8, right-click on **Format Cells** and **Currency** with **two decimal places**.

| | A | B | C |
|---|---|---|---|
| 1 | **Expenses** | **Weekly Budget** | **% of total** |
| 2 | Rent | 157.40 | |
| 3 | Food | 80.00 | |
| 4 | Fares | 34.50 | |
| 5 | Entertainment | 37.00 | |
| 6 | Mobile phone | 11.75 | |
| 7 | Savings | 25.00 | |
| 8 | Total Expenses | | |

**2** In cell B8, write a formula to calculate the sum of Abdul's weekly expenses.

**3** In cell C2, write a formula to calculate the percentage of total expenses spent on rent. Right-click on **Format Cells** and **Number** with **one decimal place**.

**4** Select cell C2 and **Fill Down** to cell C8 to copy the formula and calculate the percentage of total expenses belonging to each item.

**5** Highlight cells A1 to B7. Click **Insert**, **Pie Chart** to create a pie chart (sector graph) and give your pie chart an appropriate title.

**6** Point your mouse (do **not** click) in each sector of the pie chart to read off the percentage breakdown (in brackets) of Abdul's expenses.



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## Power plus ANSWERS ON P. 589

**1** A retailer pays $55 for a chair, then marks it up 60% for the selling price, then adds 10% of this price for GST. Calculate the final selling price.

**2** Some items increase in value over time, such as jewellery, antiques and real estate. This is called **appreciation**. A gold chain costs $1500 and appreciates by 8.5% per year. Find:

**a** the value of the chain after one year

**b** the value of the chain after 2 years.

**3** The Blewes family buys a home for $480 000, and it appreciates at 4% per year. Calculate the home's value:

**a** after one year **b** after 2 years **c** after 5 years

**4** **a** A number is increased by 25%, then the result is decreased by 25%. Is the final answer more than or less than the original number?

**b** If a number is increased by 25%, then by what percentage must the result be decreased so that the answer is equal to the original number?

**5** Most items lose value over time. This is called **depreciation**. A Ferrari car costs $420 000 and depreciates at 15% per year. Calculate how much the Ferrari is worth (correct to the nearest $100) after:

**a** 1 year **b** 2 years **c** 5 years.

**6** **a** A number is decreased by $33\frac{1}{3}\%$, then the result is increased by $33\frac{1}{3}\%$. Is the final answer more than or less than the original number?

**b** If a number is decreased by $33\frac{1}{3}\%$, then by what percentage must the result be increased so that the answer is equal to the original number?

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# CHAPTER 6 REVIEW



Quiz
Language of maths 6

Puzzle
Fractions and percentages find-a-word

## Language of maths

| | | | |
|---|---|---|---|
| commission | cost price | decimal | decrease |
| denominator | discount | fraction | goods and services tax (GST) |
| improper fraction | increase | loss | mark-up |
| mixed numeral | numerator | percentage | profit |
| proper fraction | quantity | reciprocal | reduction |
| selling price | simple interest | simplify | unitary method |

**1** Which words in the list above refer to something getting:

**a** bigger? **b** smaller?

**2** What is the product of a number and its reciprocal?

**3** Name one good and one service that would have a **GST**.

**4** The 'cent' in percentage means 100. Find other 'cent' words that are related to 100.

**5** What do we mean when we say that someone gives '110%' effort or support to something?

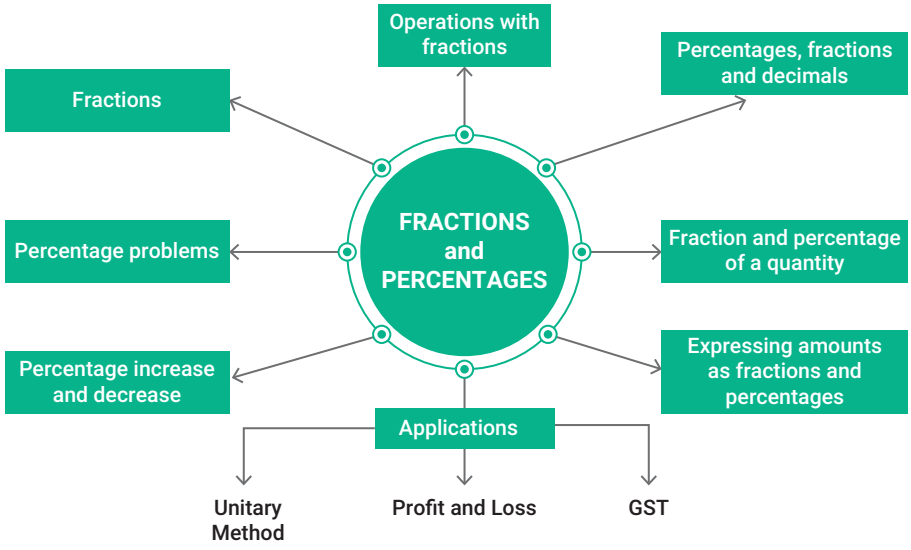
**6** Copy and complete: "A **loss** is made by a retailer if the **cost price** is _____ than the **selling price**."

## Topic summary

Worksheet
Mind map: Fractions and percentages

- List any rules you remember from your work on fractions and percentages.
- Name at least 3 careers where percentages would be used.
- Is there anything you still do not understand about fractions or percentages? See your teacher.
- Give examples of where you may use fractions or percentages in the future.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Quiz
Test yourself 6

# TEST YOURSELF 6

ANSWERS ON P. 589

6.01 **1** Simplify each fraction.

**a** $\frac{6}{8}$ **b** $\frac{28}{48}$ **c** $\frac{35}{45}$

6.01 **2** Convert each improper fraction into a mixed numeral.

**a** $\frac{15}{4}$ **b** $\frac{22}{5}$ **c** $\frac{26}{6}$

6.01 **3** Convert each mixed numeral into an improper fraction.

**a** $4\frac{1}{2}$ **b** $3\frac{2}{3}$ **c** $6\frac{4}{5}$

6.01 **4** Arrange these fractions in descending order: $\frac{5}{6}, \frac{7}{12}, \frac{1}{2}, \frac{3}{4}$

6.02 **5** Evaluate each expression.

**a** $\frac{1}{3}+\frac{2}{5}$ **b** $\frac{4}{7}-\frac{1}{2}$ **c** $\frac{3}{4}+\frac{2}{3}$

**d** $2\frac{1}{5}+4\frac{1}{2}$ **e** $7-\frac{5}{6}$ **f** $5\frac{3}{8}-2\frac{1}{2}$

6.02 **6** Santi buys 16 m of timber to fix her back deck. She uses $\frac{5}{8}$ of it to fix the deck. What fraction of the timber is left?

6.03 **7** Evaluate each expression.

**a** $\frac{7}{8}\times\frac{4}{9}$ **b** $\frac{4}{5}\times\frac{2}{3}$ **c** $\frac{6}{7}\times\frac{5}{8}$

**d** $2\frac{1}{4}\times\frac{3}{10}$ **e** $\frac{1}{6}\div\frac{5}{12}$ **f** $5\frac{3}{8}\div1\frac{1}{2}$

6.03 **8** In Carrozza Park's population of 27 500, $\frac{3}{5}$ of the people are adults. How many people in Carrozza Park are adults?

6.04 **9** Convert each percentage into a simplified fraction.

**a** 74% **b** 29% **c** 65% **d** 157%

6.04 **10** Convert each percentage into a decimal.

**a** 12% **b** 16.2% **c** 2% **d** 5.4%

6.04 **11** Convert each fraction into a percentage.

**a** $\frac{7}{8}$ **b** $\frac{12}{50}$ **c** $\frac{2}{9}$ **d** $\frac{33}{40}$

6.04 **12** Convert each decimal into a percentage.

**a** 0.56 **b** 0.613 **c** 0.7 **d** 0.048

6.04 **13** Arrange these numbers in ascending order: $\frac{3}{4}$, 0.725, 77%, 0.7

6.05 **14** Evaluate each expression.

**a** 15% of 40 **b** $\frac{1}{4}$ of 64 **c** 27.5% of \$880

**d** $\frac{2}{3}$ of 1 year (in months) **e** 120% of 90 L **f** 35% of 6 m (in cm)

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**15** Asam decided to go on a weight loss program. His starting weight was 187 kg. In the first 6 weeks he lost 13% of his body weight. 6.05

**a** How many kilograms did Asam lose in the first 6 weeks?

**b** What is his body weight now?

**16** Write each test mark as: 6.06

**i** a simplified fraction **ii** a percentage

**a** 32 out of 40 **b** 45 out of 60 **c** 11 out of 20

**17** Gianna scored 68 out of 80 in a test for her driving licence. Write her score as: 6.06

**a** a simplified fraction **b** a percentage

**18** Westvale Secondary College has 375 boys and 405 girls. What percentage (correct to one decimal place) of the students are boys? 6.06

**19** A refrigerator with a marked price of $545 is discounted by 40%. Calculate its sale price. 6.07

**20** Bella is to receive a salary increase of 3.5%. If she currently earns $65 000, what will her new salary be? 6.07

**21** Evaluate each expression without using a calculator. 6.08

**a** 20% of 55 t **b** $12\frac{1}{2}$% of 96 cm **c** 75% of 12 minutes

**22** A deposit of 18% paid on a car is $26 480. What is the cost of the car? 6.09

**23** James bought a guitar for $750, then sold it for $600. Calculate his percentage loss on the cost price. 6.10

**24** The price of a mobile phone is $315.70, including GST. Calculate: 6.10

**a** the amount of GST included in this price

**b** the price of the phone before GST was added

**25** Ishween sold $850 worth of books last week. If she receives 5% commission on all sales, calculate her commission. 6.12

**26** The cost of Evelyn's new glasses is $425. Her health fund pays her a benefit of $245. What percentage is her benefit of the cost of the glasses? Answer correct to one decimal place. 6.12

# PRACTICE SET 2 ANSWERS ON P. 589

6.01

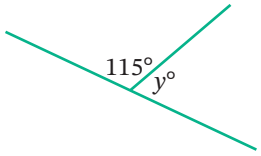
**1** Simplify each fraction.

**a** $\frac{12}{24}$ **b** $\frac{7}{21}$ **c** $\frac{15}{25}$

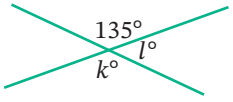
4.01

**2** Find the value of each variable, giving reasons.

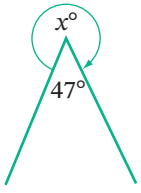
**a**



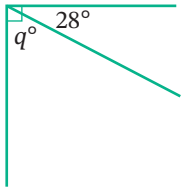

**b**




**c**



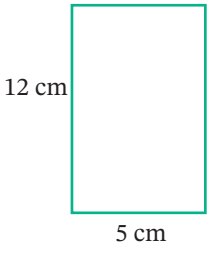

**d**



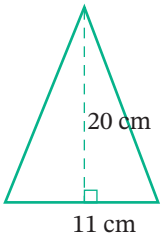

5.05

**3** Find the area of each shape.

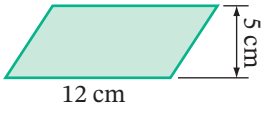
**a**



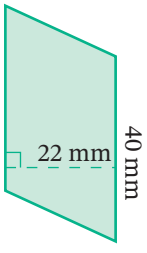

**b**




**c**




**d**




5.02

**4** Draw each part of a circle.

**a** chord **b** segment **c** arc **d** quadrant

Foundation Standard Complex

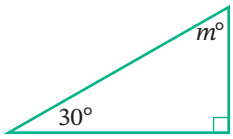


9780170479516

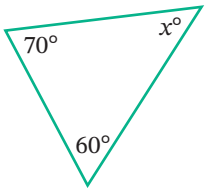
**5** Classify each triangle: 4.03

**i** by sides **ii** by angles.

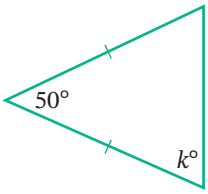
**a**



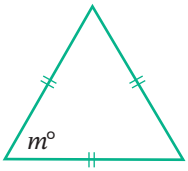

**b**




**c**




**d**




**6** Find the value of each variable in question **5** above. 4.06

**7** Convert each mixed numeral into an improper fraction. 6.01

**a** $2\frac{1}{2}$ **b** $1\frac{3}{4}$ **c** $4\frac{2}{5}$

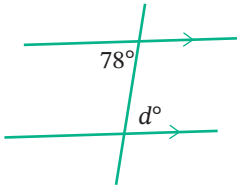
**8** Convert each percentage into: 6.04

**i** a simple fraction **ii** a decimal.

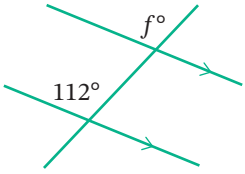
**a** 25% **b** 40% **c** 85% **d** $12\frac{1}{4}\%$

**9** Find the value of each variable, giving reasons. 4.02

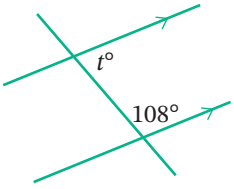
**a**




**b**




**c**




**10** Convert each number into a percentage. 6.04

**a** 0.34 **b** $\frac{3}{4}$ **c** 0.215 **d** $\frac{5}{8}$

**11** Name the (most general) quadrilateral with: 4.05

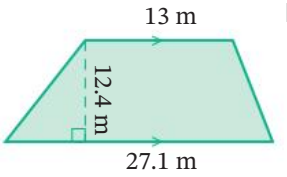
**a** opposite sides parallel **b** 4 right angles

**c** 4 equal sides and 4 equal angles **d** 2 pairs of equal adjacent sides.

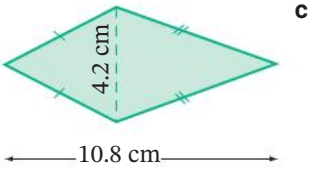
5.09
5.10

**12** Find the area of each shape.

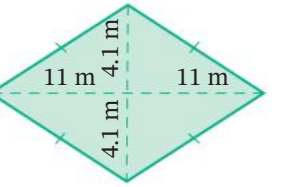
**a**




**b**




**c**




6.01

**13** Convert each improper fraction into a mixed numeral.

**a** $\frac{11}{2}$ **b** $\frac{19}{3}$ **c** $\frac{14}{4}$

6.02
6.03

**14** Evaluate each expression.

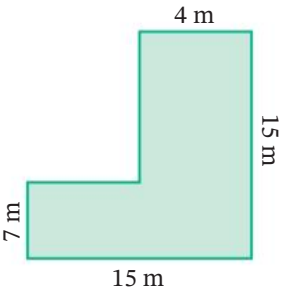
**a** $\frac{2}{5} + \frac{4}{5}$ **b** $\frac{1}{3} + \frac{2}{5}$ **c** $\frac{3}{5} - \frac{1}{4}$ **d** $1\frac{3}{5} + 2\frac{1}{2}$

**e** $2 - \frac{2}{3}$ **f** $\frac{1}{2} \times \frac{3}{5}$ **g** $\frac{3}{4} \times \frac{4}{5}$ **h** $\frac{4}{5} \div \frac{1}{2}$

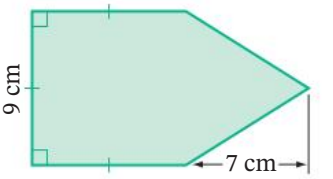
5.06

**15** Find the area of each shape.

**a**




**b**




5.03

**16** Find, correct to 2 decimal places, the circumference of a circle with:

**a** radius 5 cm **b** diameter 7 cm

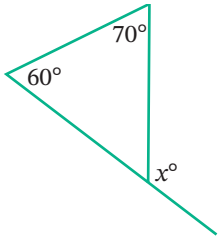
5.07

**17** Find, correct to 2 decimal places, the area of a circle with:

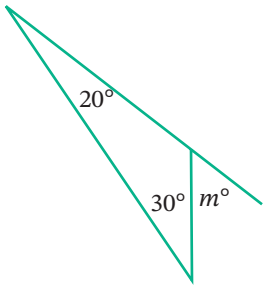
**a** radius 5 cm **b** diameter 7 cm

**18** Find the value of each variable, giving reasons. 4.06

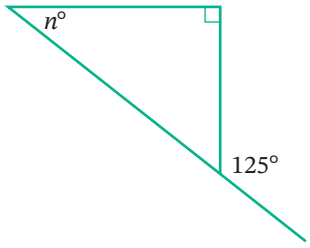
**a**



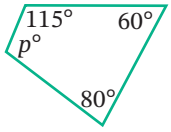

**b**



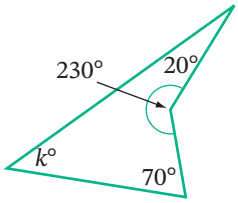

**c**



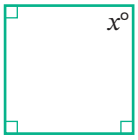

**d**




**e**




**f**

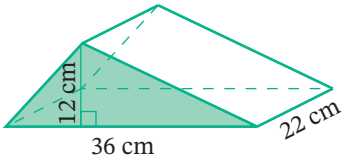



**19** School canteen prices are to increase by 15%. Find the new price for a $4 sandwich. 6.07

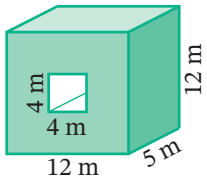
**20** What is the capacity, in litres, of a box measuring 48 cm by 33 cm by 17 cm? 5.14

**21** Find the volume of each prism. 5.12

**a**




**b**




**22** Convert: 5.04

**a** 3.6 $m^2$ to $cm^2$ **b** 875 $mm^2$ to $cm^2$

**23** Arrange these numbers in ascending order: 63%, $\frac{3}{5}$, 0.67, $\frac{6}{7}$. 6.04

**24** True or false? 4.05

**a** The sides of a rhombus are equal.

**b** The diagonals of a rectangle bisect each other at right angles.

**c** The opposite sides of a parallelogram are equal.

**d** The opposite sides of a trapezium are parallel.

**25** Find: 6.05

**a** 2% of $15 **b** $\frac{2}{5}$ of 1320 kg

**c** $\frac{3}{4}$ of $6.40 **d** 25% of 1 hour (in minutes)

Foundation Standard Complex

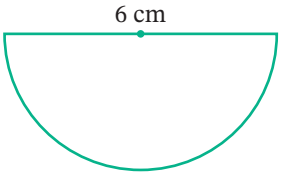
6.06 **26** What percentage is:

**a** 40 cm of 1 m? **b** 350 mL of 1 litre? **c** 45 minutes of $1\frac{1}{2}$ hours?

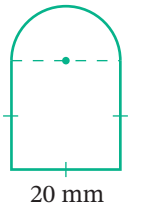
6.07 **27** The population of Banksia decreased by 3.5%, from 3400 people. Find the new population.

5.08 **28** Find, correct to 2 decimal places, the perimeter of each shape.

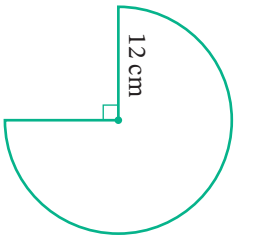
**a**




**b**



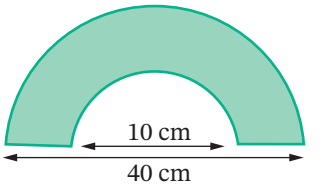

**c**




5.08 **29** Find, correct to 2 decimal places, the area of each shape above.

5.08 **30** Find the perimeter of this shape. Select the closest answer **A**, **B**, **C** or **D**.

**A** 109 cm **B** 77 cm **C** 737 cm **D** 129 cm




5.08 **31** Find the area of the shape above, correct to one decimal place.

6.09 **32** Mark saves 15% of his weekly income. If he saves $204, what is his weekly income?

6.10 **33** A computer has a cost price of $1100 and a selling price of $1599.

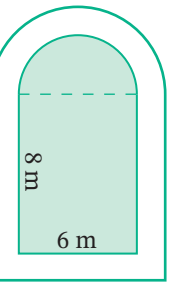
**a** Calculate the profit made.

**b** Calculate, correct to one decimal place, the percentage profit on the cost price.

5.08 **34** Mr Moneybags wants to put a new garden with a path around it in the grounds of his country estate. The plan for this garden is shown on the right. Find, correct to one decimal place:

**a** the area of the garden.

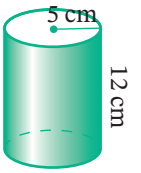
**b** the area of the path.



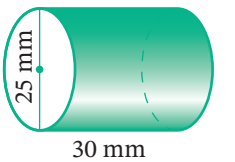

Width of path = 1.5 m

5.13 **35** Find, correct to one decimal place, the volume of each cylinder.

**a**




**b**




Foundation Standard Complex



9780170479516




7

STATISTICS AND PROBABILITY

# INVESTIGATING DATA

The Bureau of Meteorology collects weather data across Australia, including rainfall and temperatures. Polling organisations like Roy Morgan and Galaxy Research collect data on people's opinions on a wide range of issues. In NSW, the Bureau of Crime Statistics and Research (BOCSAR) collects data on crimes in different areas of the state. Health departments collect data on medical issues and health care. In all cases the data is analysed to evaluate decisions already made and to inform decision making in the future.




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## Chapter outline

| | Working mathematically | | | | |
|---|---|---|---|---|---|
| **7.01** Column, sector and divided bar graphs | U | F | | R | C |
| **7.02** Line graphs, pictograms and infographics | U | F | | R | C |
| **7.03** Types of data | U | F | | R | C |
| **7.04** Census vs sample | U | F | | R | C |
| **7.05** The mean and mode | U | F | | R | C |
| **7.06** The median and range | U | F | PS | R | C |
| **7.07** Analysing frequency tables | U | F | | | |
| **7.08** Dot plots and stem-and-leaf plots | U | F | PS | R | C |
| **7.09** Frequency histograms and polygons | U | F | PS | R | |
| **7.10** Choosing the best graph | U | F | | R | C |
| **7.11** The shape of data | U | F | | R | C |
| **7.12** Comparing data | U | F | PS | R | C |

## Wordbank

**bias** Something unwanted that causes a sample to not truly represent the population

**categorical data** Non-numerical data that can be classified into categories (for example, people's hair colour, or their favourite football team)

**frequency** How often a value appears in a data set

**frequency histogram** A special column graph that shows the frequencies of numerical data

**frequency polygon** A special line graph that shows the frequencies of numerical data

**infographic** A combination of graphs, images and words to describe information in a clear and interesting way

**measure of centre** An average, middle or typical value of a data set; a general name for the mean, median or mode

**numerical data** Data that can be measured and counted using values (for example people's height, or the number of devices they own)

**skewed** Twisted to one side, the shape of data that is not symmetrical

**Quiz**
Wordbank 7

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

## Videos (12):

**SkillCheck** Statistics

**7.03** Types of data

**7.06** The mean, mode, median and range
• The mode, median and mean

**7.07** Analysing frequency tables
• Statistics from a frequency table

**7.08** Analysing dot plots • Dot plots
• Stem-and-leaf plots 1
• Stem-and-leaf plots 2
• Back-to-back stem-and-leaf plots

**7.11** The shape of a frequency distribution

## *Twig* videos (8):

**7.01** Distorted graphs: Heat wave
• Nightingale's diagram

**7.04** Counting crowds • Can fish oil make you smarter? • Can you trust your IQ? • The wrong guy won

**7.05** Most popular pet

**7.06** Average Joe

## *PhET* interactive (1):

**7.06** Centre and variability

## Quizzes (5):

- Wordbank 7
- SkillCheck 7
- Mental skills 7
- Language of maths 7
- Test yourself 7

## Skillsheets (3):

**7.01** Displaying data

**7.05** Statistical measures

**7.06** Statistical measures
• Reading linear scales

## Worksheets (10):

**7.01** Every picture tells a story
• Frequency tables

**7.06** Mean, median, mode

**7.07** The mean and *fx* tables

**7.08** Stem-and-leaf plots • What data?

**7.09** Analysing data

**7.12** Analysing data • Comparing word lengths • Investigating young drivers

**Mind map** Investigating data

## Puzzles (8):

**7.01** Where all the cars are red

**7.03** Data match-up

**7.05** Looking for gold

**7.06** Ranges and averages • Data puzzles
• Mode, median and mean

**7.09** Statistical match-up • Highway elephants

**7.11** Statistical match-up

**Language of maths** Data crossword

## Technology (6):

**7.01** Creating a graph

**7.06** Students' marks • Finding the mean, median, mode and range
• Statistical measures calculator

**7.12** Population polygons
• Daily rainfall

## Presentation (1):

**7.08** Stem-and-leaf plots

## Spreadsheet (1):

**7.06** Statistical measures calculator

Nelson MindTap

To access resources above, visit
**cengage.com.au/nelsonmindtap**

## In this chapter you will:

- interpret and draw column graphs, sector graphs, divided bar graphs, line graphs, pictograms, infographics, dot plots and stem-and-leaf plots
- organise data into a frequency table
- identify categorical and numerical data, sort categorical data into ordinal and nominal data, and sort numerical data into discrete and continuous data
- identify ways of collecting data, including census, surveys, sampling and observation

- calculate and interpret the mean, median, mode and range of a set of data, including those represented on a graph or plot
- interpret and draw frequency histograms and polygons
- investigate the effect of individual data values, including outliers, on the mean and median
- identify the shape of a set of data

## SkillCheck ANSWERS ON P. 590





Quiz
SkillCheck 7



Video
Statistics

**1** A group of school students visiting Japan have the following ages.

14 13 12 15 13 14 12 14 15 14 14 13 13

Show this information on a dot plot.

**2** The masses (in kilograms) of some hospital patients are:

68 59 63 68 54 48 49 64 47 48 59 68 30

**a** Show this data on an ordered stem-and-leaf plot.

**b** What is the outlier?

**3** A group of people was surveyed about the number of phone calls they made on a Saturday night. The results are shown below.

6 4 0 3 1 2 2 3 5 4 3 3

4 1 0 6 6 3 4 3 1 5 4 3

**a** Draw a column graph for this data.

**b** How many people were surveyed?

**c** What was the most common number of phone calls?

**d** What was the least common number of phone calls?

**e** What was the highest number of phone calls?

**4** This back-to-back stem-and-leaf plot shows the points scored for and against the Koala Rugby team in each match, at one stage in the season.

| For | | Against |
|---|---|---|
| 4 | 0 | 6 |
| 8 4 | 1 | 4 6 6 |
| 2 2 0 0 | 2 | 2 9 9 |
| | 3 | 0 1 |
| 8 1 | 4 | 7 |
| 3 | 5 | |

**a** What is the highest number of points scored by the Koalas in one game?

**b** How many matches have the Koalas played so far?

**c** How many points have been scored against the Koalas?

# 7.01 Column, sector and divided bar graphs

**Skillsheet**
Displaying data

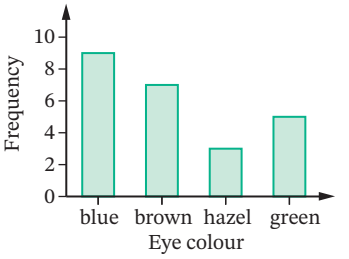
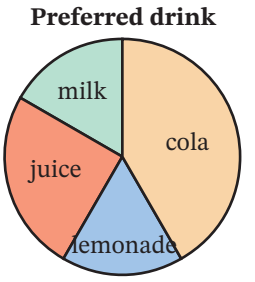
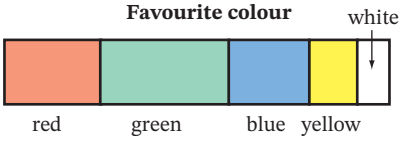
**Worksheet**
Every picture tells a story

**Puzzle**
Where all the cars are red

**Videos**
Distorted graphs: Heat wave

Nightingale's diagram

**Data** is a collection of facts or raw information. Data can be organised and displayed in a variety of ways so that we can **analyse** (study) it in more detail. This table shows 3 examples of statistical graphs.

| Graph | Description |
|---|---|
| Column graph (or bar chart)<br>  | • Mostly used for data that are in categories<br>• Uses columns to show the quantity or frequency of each category<br>• The horizontal axis (going across) shows the categories<br>• The vertical axis (going up) shows the frequency |
| Sector graph (or pie chart)<br>  | • Shows parts of a whole in circle form<br>• Uses sectors of a circle to show sizes of parts |
| Divided bar graph<br>  | • Shows parts of a whole in rectangular form<br>• Uses sections of a rectangle to show sizes of parts |

## Example 1

A sample of 80 people was surveyed on their favourite TV sport, and the results are shown.

| TV sport | Australian football | Cricket | Motor racing | Rugby league | Rugby union | Football |
|---|---|---|---|---|---|---|
| Frequency | 13 | 16 | 8 | 19 | 10 | 14 |

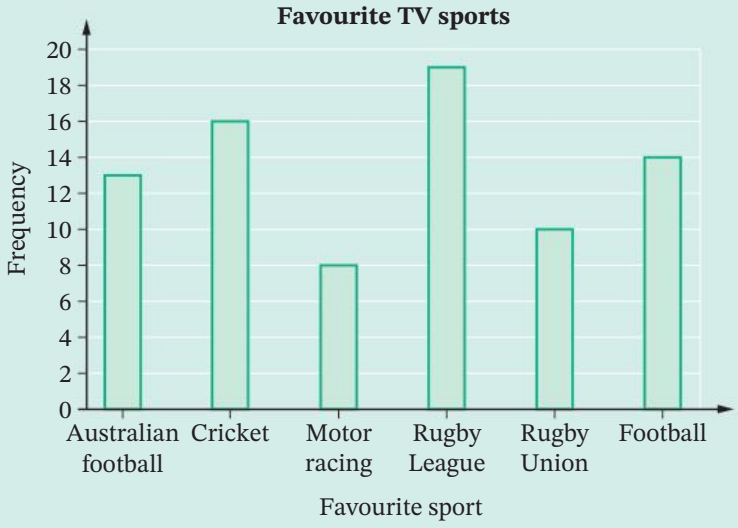
Display this data on a:

**a** column graph **b** sector graph **c** divided bar graph


9780170479516

## Solution

**a** The highest frequency is 19 (for rugby league), so use a scale of 1 unit = 2 on the 'Frequency' axis. Label both axes, draw a column for each category, then label the graph.




**b** Draw a large circle, then calculate the angle sizes for each sector using fractions of 360° (because there are 360° in a revolution).

Total number of people in sample = 80.

Sector angles:

Australian football: $\frac{13}{80} \times 360° = 58.5°$

Cricket: $\frac{16}{80} \times 360° = 72°$

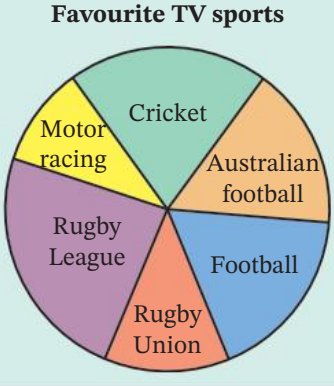
Motor racing: $\frac{8}{80} \times 360° = 36°$

Rugby League: $\frac{19}{80} \times 360° = 85.5°$

Rugby Union: $\frac{10}{80} \times 360° = 45°$

Football: $\frac{14}{80} \times 360° = 63°$

Construct the sector graph with the calculated angles, then label the sectors and the graph.



**c** Draw a rectangle whose length will represent 80, the total frequency (the whole), for example, 12 cm. Then calculate the length of each section using fractions of 12 cm.

Section sizes:

Australian football: $\frac{13}{80} \times 12\text{ cm} = 1.95\text{ cm}$ Cricket: $\frac{16}{80} \times 12\text{ cm} = 2.4\text{ cm}$

Motor racing: $\frac{8}{80} \times 12\text{ cm} = 1.2\text{ cm}$ Rugby League: $\frac{19}{80} \times 12\text{ cm} = 2.85\text{ cm}$

Rugby Union: $\frac{10}{80} \times 12\text{ cm} = 1.5\text{ cm}$ Football: $\frac{14}{80} \times 12\text{ cm} = 2.1\text{ cm}$

Construct the divided bar graph with the calculated section lengths, then label the sections and the graph.

**Favourite TV sports**




Worksheet
Frequency tables

# Frequency tables

A **frequency table** (or **frequency distribution table**) can be used to efficiently organise and count a large set of numerical data values. The **frequency** of a value is the number of times that value appears in the set of data.

## Example 2

20 people were surveyed about the number of cars in their household.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 1 | 2 | 0 | 3 | 2 | 1 | 1 | 4 |
| 1 | 1 | 1 | 2 | 2 | 0 | 3 | 2 | 1 | 2 |

Complete a frequency table for the data.

### Solution

Draw a table with columns for Score, Tally and Frequency, as shown next page.
The lowest score is 0 and the highest is 4, so list the numbers 0 to 4 in the Score column.

For each value in the data set, draw a tally mark in the tally column. Tally means 'to count', and we are counting how often each value occurs.

The first value is a 2, so mark a '|' in the 'Tally' column beside 2. The second value is another 2, so mark another '|' there. Continue until you have counted all of the values.

| Score | Tally | Frequency |
|---|---|---|
| 0 | II | |
| 1 | 𝍸 II | 𝍸 means a tally of 5. |
| 2 | 𝍸 III | |
| 3 | II | |
| 4 | I | |
| Total | | |

For each value, count up the tally marks and write the answer in the 'Frequency' column. Then write the total in the final row and check that it is equal to the total number of values.

| Score | Tally | Frequency |
|---|---|---|
| 0 | II | 2 |
| 1 | IIII II | 7 |
| 2 | 𝍸 III | 8 |
| 3 | II | 2 |
| 4 | I | 1 |
| Total | | 20 |



## EXERCISE 7.01 ANSWERS ON P. 590

## Column, sector and divided bar graphs U F R C

**1** A class of 30 PE students was surveyed about the day of the week that each student trained for their chosen sport.

EXAMPLE 1

| Day | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| Number of students | 3 | 13 | 4 | 7 | 3 |

Display this data in a sector graph. Label the graph. C

**2** The colours of the jelly beans in a jar were counted.

| Colour | Red | Black | Purple | Pink | Orange | White |
|---|---|---|---|---|---|---|
| Frequency | 11 | 5 | 2 | 4 | 6 | 8 |

Display this data as a column graph, using an appropriate scale. Label the graph and both axes. C



Shutterstock.com/Hayati Kayhan

**3** A sample of men were surveyed on the number of hours they slept the previous night.

| Number of hours | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|
| Frequency | 3 | 9 | 11 | 6 | 5 | 2 |

Display this data as a divided bar graph, 9 cm long. Label the graph. C

**4** This table shows the number of kilolitres of water used by a household in different areas over a year.

| Area | Garden | Shower | Toilet | Kitchen |
|---|---|---|---|---|
| Water usage (kL) | 90 | 520 | 140 | 50 |

If this information is shown in a divided bar graph of length 10 cm, which of the following is the length of the section representing 'Shower'? Select the correct answer **A**, **B**, **C** or **D**. R

**A** 1.5 cm **B** 1.9 cm **C** 5.2 cm **D** 6.5 cm

**5** The results of a spelling quiz (out of 10) for a class of students are as shown. C

EXAMPLE 2

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 7 | 8 | 9 | 7 | 5 | 4 | 8 | 9 |
| 9 | 8 | 10 | 8 | 7 | 5 | 5 | 4 | 1 |
| 8 | 8 | 9 | 7 | 7 | 4 | 8 | 3 | 1 |

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**a** Complete a frequency table for this data.

**b** What was the highest value?

**c** How many students scored 4 marks in the quiz?

**d** What was the most frequent value?

**e** Draw a column graph of this data.

**6** 30 customers were asked to name their favourite car colour. B = black, S = silver, W = white, U = blue, R = red. C

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| B | B | W | R | W | R | B | S | U |
| W | R | B | S | B | U | W | B | W |
| S | B | B | R | R | U | W | B | B |

**a** Complete a frequency table for this data.

**b** Draw a divided bar graph for this data.

**c** Which is the most popular colour for this group of customers?

**7** A sample of women were asked how many siblings (brothers and sisters) they had. C

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 0 | 2 | 3 | 2 | 4 | 3 | 5 | 4 | 3 |
| 0 | 0 | 1 | 2 | 3 | 2 | 2 | 1 | 4 | 3 |
| 1 | 1 | 1 | 5 | 2 | 5 | 0 | 1 | 2 | 1 |
| 0 | 1 | 3 | 2 | 3 | 4 | 1 | 1 | 3 | 3 |

**a** Complete a frequency table for this data.

**b** What was the highest number of siblings?

**c** What was the mode (the most frequent value)?

**d** Illustrate this data on a sector graph.

Technology
Creating a graph

# Technology

## Graphing life expectancy

Life expectancy describes the age to which a person is expected to live, based on statistical data covering different factors.

**1** Enter the data below into a new spreadsheet. It compares the life expectancy at birth for males and females in 10 countries.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Life expectancy at birth | | | |
| 2 | | | Years | |
| 3 | Country | Male | Female | Total |
| 4 | Australia | 80.5 | 85.0 | 82.7 |
| 5 | Bangladesh | 72.0 | 76.5 | 74.2 |
| 6 | Brazil | 71.2 | 78.4 | 74.7 |
| 7 | Canada | 81.1 | 85.9 | 83.4 |
| 8 | Chad | 56.5 | 60.1 | 79.4 |
| 9 | China | 74.0 | 78.4 | 76.1 |
| 10 | India | 68.4 | 71.2 | 69.7 |
| 11 | Spain | 79.0 | 85.2 | 82.0 |
| 12 | Sweden | 80.4 | 84.5 | 82.4 |
| 13 | USA | 78.0 | 82.5 | 80.3 |

Source: www.cia.gov

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2 Highlight all 4 columns, including headings. Click **Insert** and **2-D Column Graph**. Give the graph an appropriate title and label the axes.

3 Which country has:

- **a** the lowest total life expectancy?
- **b** the highest total life expectancy?
- **c** the lowest male life expectancy?
- **d** the lowest female life expectancy?
- **e** the highest male life expectancy?
- **f** the highest female life expectancy?

4 Suggest reasons why some countries would have higher life expectancies than others.

5 In column E, calculate the difference between the male and female life expectancies for each country.

6 Generally, do males or females live longer?

7 Which country has the greatest difference between its male and female life expectancies?

8 Which country has the least difference?



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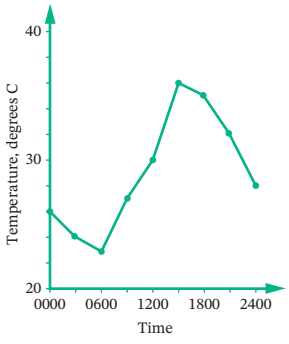


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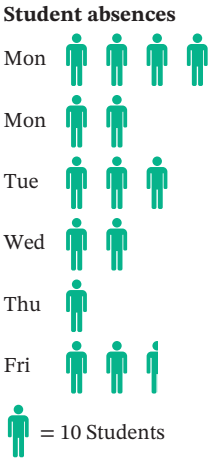


# Line graphs, pictograms and infographics 7.02

There are many different ways to display data. This table shows 3 more examples.

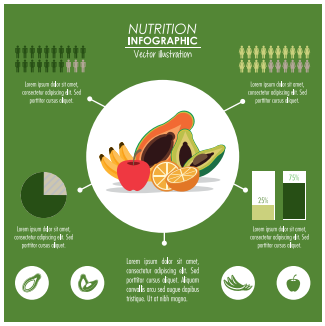
| Graph | Description |
|---|---|
| **Line graph**<br>  | • Used to show changes over time<br>• Lines connect points that describe the values of a numerical quantity<br>• The horizontal axis shows the time<br>• The vertical axis shows the quantity |

**Pictogram (or picture graph)**




- Use a picture or symbol to represent data
- Each picture represents a quantity, as shown in the key
- Mostly used for data that are in categories
- Easy to understand but not very accurate

**Infographic**




Alamy Stock Photo/Jemastock

- Uses images, graphs, and a small amount of text to illustrate data, statistics and information
- Often illustrates multiple facts
- Easy to understand

## Example 3

Display this data from Example 1 on a pictogram.

| TV sport | Australian football | Cricket | Motor racing | Rugby league | Rugby union | Football |
|---|---|---|---|---|---|---|
| Frequency | 13 | 16 | 8 | 19 | 10 | 14 |

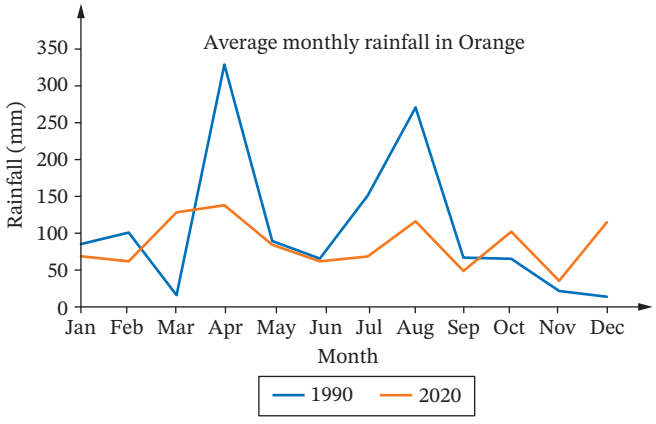
### Solution



## EXERCISE 7.02 ANSWERS ON P. 591

# Line graphs, pictograms and infographics U F R C

**1** This line graph shows the average monthly rainfall for Orange in the years 1990 and 2020. R C




Source: Bureau of Meteorology. Creative Commons License

**a** Which month in which year had the highest rainfall?

**b** Which month in which year had the sharpest drop in rainfall?

**c** In which 2 months was the rainfall approximately the same in both 1990 and 2020?

**d** In which 2 consecutive months in 1990 was the rainfall approximately the same?

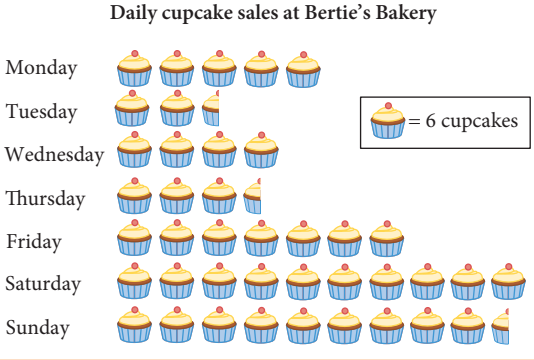
**e** Which year do you think had the higher rainfall overall: 1990 or 2020? How can you tell?

**f** You are planning a holiday in Orange. Based on these graphs, what month of the year would you choose in the hope of having least rain? Justify your answer.

**2** Draw a line graph for the data below, showing the monthly mean minimum temperatures (in °C) in Griffith in 2022. C

| Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 17 | 16 | 12 | 8 | 5 | 3 | 5 | 6 | 10 | 10 | 13 |

**3** This pictogram shows the daily sales figures for cupcakes at Bertie's Bakery for one week. R C




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**a** How many cupcakes were sold on Saturday?

**b** On which day were 42 cupcakes sold?

**c** True or false: More cupcakes were sold on the weekend than over the rest of the week combined.

**d** On which day were the least number of cupcakes sold? How many cupcakes were sold on this day?

**e** What is one disadvantage of using a pictogram to present data? Consider how you would represent 31 cupcakes being sold.

EXAMPLE 3

**4** Draw a pictogram for this data from Exercise 7.01, Question 1, about the day of the week on which students trained for their chosen sport. C

| Day | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| Number of students | 3 | 13 | 4 | 7 | 3 |

**5** This is a pictogram and an infographic. C

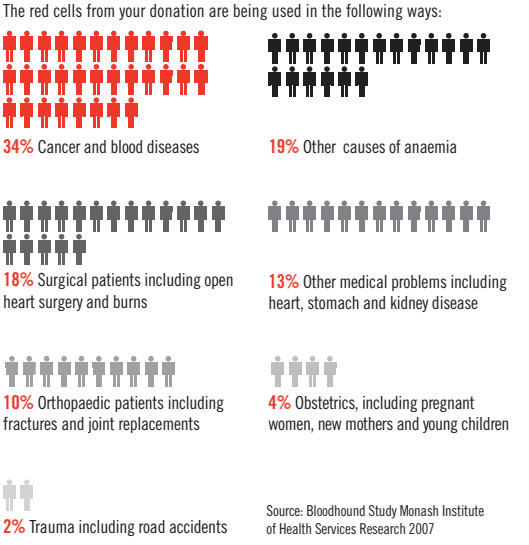



**a** What is one advantage of this type of pictogram?

**b** Design a similar pictogram and infographic to illustrate the statistic:

'In 2007, 4 in every 10 people in Australia were not connected to the internet.'

**6** This graph was published by the Australian Red Cross Blood Service. It shows where donated blood is used. R C




Source: Bloodhound Study Monash Institute of Health Services Research 2007. With permission from Australian Red Cross Blood Service.

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9780170479516

**a** What does each person symbol stand for?

**b** True or false?

**i** The most popular category is more than double the size of the 2nd most popular category.

**ii** The 2 most popular categories make up more than half of the use of donated blood.

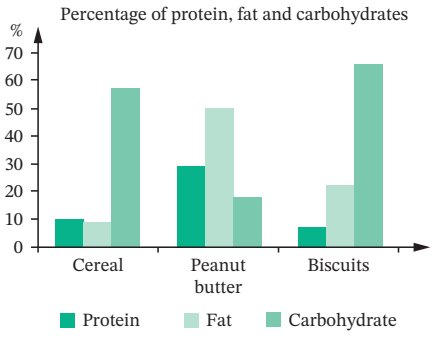
**c** What percentage of donations goes towards helping people with bone issues?

**d** Is there any information in this graph that surprises you? Explain your answer.

**e** Does this graph make you more likely to donate blood?

**f** If 27 000 L of blood are collected in one week, how much of it goes to help patients with cancer and blood diseases?

**7** We use a clustered column graph for categories we want to compare. This graph shows the nutritional information for different foods. R C




**a** Which food has the highest percentage of protein?

**b** Which food has the lowest percentage of fat?

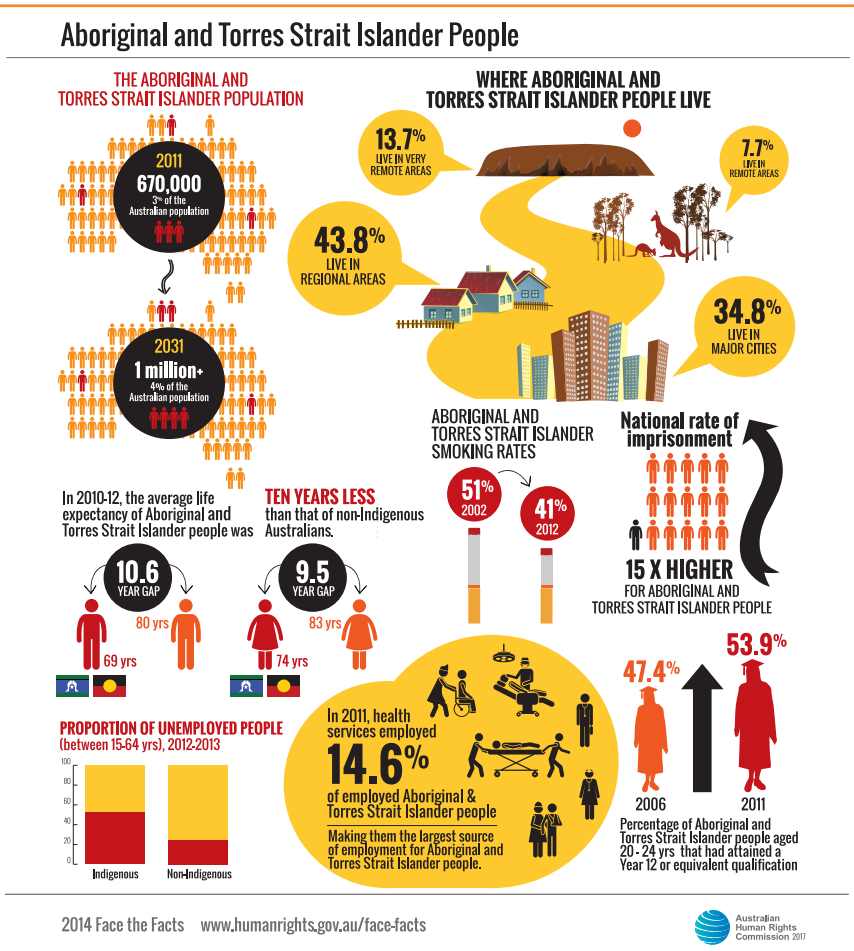
**c** If you were on a low carbohydrate diet, which food could you include in your diet?

**d** If you were on a low protein diet, which foods could be included in your diet?

**e** Jack had a 140 g serving of cereal for breakfast. Approximately how many grams of protein did the serving of cereal contain?

**f** Lizzie has to limit her carbohydrate intake. What foods should she avoid?

**8** This is an infographic about Aboriginal and Torres Strait Islander people published by the Australian Human Rights Commission. Read through it and notice the way it presents information in a clear and colourful way. R C




© Australian Human Rights Commission 2017 CC BY 4.0 (https://creativecommons.org/licenses/by/4.0/legalcode)

**a** Which piece of information did you read first? Why?

**b** List 3 facts that you have learned from this infographic.

**c** What type of message is this infographic trying to convey?

**d** Where do most Aboriginal and Torres Strait Islander people live?

**e** What was the life expectancy of a non-Indigenous woman in 2010–12?

**f** There's a graphic that shows the population of Aboriginal and Torres Strait Islander people in the year 2031, in the future. How is it possible to have that data?

**g** Which industry was in 2011 the most popular place for Aboriginal and Torres Strait Islander people to work in?

**h** How does this infographic display data in an interesting way, as opposed to just text or just graphs?

**i** What are the features of a good infographic?

**j** Design an infographic about the students who make up your maths class.

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**9** Here is some data about Nusentry High School.

| | | | |
|---|---|---|---|
| Male students | 484 | Female teachers | 25 |
| Female students | 492 | Total teachers | 45 |
| Total students | 976 | Year 7 students | 203 |
| Students of First Nations background | 230 | Year 8 students | 194 |
| Students of Asian background | 310 | Year 9 students | 190 |
| Students who live in town | 780 | Year 10 students | 182 |
| Students who live out of town | 196 | Year 11 students | 111 |
| Male teachers | 20 | Year 12 students | 96 |

Create an infographic describing some of this data. R C

**10** According to the 2021 Australian Census, there were 167 Aboriginal and Torres Strait Islander languages spoken at home by 76 978 people. The most widely used language groups are listed below.

| Language group | Percentage of all ATSI languages |
|---|---|
| Arnhem Land and Daly River Region | 14.5 |
| Western Desert | 10.9 |
| Torres Strait Island | 12.0 |
| Yolngu Matha | 8.5 |
| Arandic | 7.4 |

Source: Australian Bureau of Statistics

Aboriginal and Torres Strait Islander people: Census 2021. CC BY 4.0 License (https://creativecommons.org/licenses/by/4.0/)

Create an infographic using this data. R C

**11** Infographics are used to convey many different types of information. Search the Internet for different types of infographics. There are also different programs which can help you make an infographic. Find the names of these.



# Types of data 7.03

Statistical data may be in the form of categories (such as hair colour or favourite sport), or in the form of numbers (such as soccer score or number of cars owned). There are 2 types of data.

**Categorical data** can be grouped into categories, such as a type of seafood or the suburb where you live.

**Numerical data** (or **quantitative data**) can be counted or measured, such as number of children or a person's height.

Puzzle
Data match-up

Video
Types of data

## Example 4

Classify each of the following types of data as categorical or numerical.

**a** make of car

**b** weekly pay

**c** weight of athlete

**d** favourite radio station

### Solution

**a** Make of car is categorical data. Involves words or categories

**b** Weekly pay is numerical data. Involves numbers

**c** Weight of athlete is numerical data. Involves numbers

**d** Favourite radio station is categorical data. Involves words or categories

## Types of categorical data

Categorical data can be further divided into **ordinal data** and **nominal data**.

**Ordinal data** can be ordered in some way, for example:

school report grades – extensive, thorough, sound, basic, elementary

pain severity – no pain, moderate, severe, very severe

**Nominal data** cannot be ordered, for example, hair colour, the street where you live.

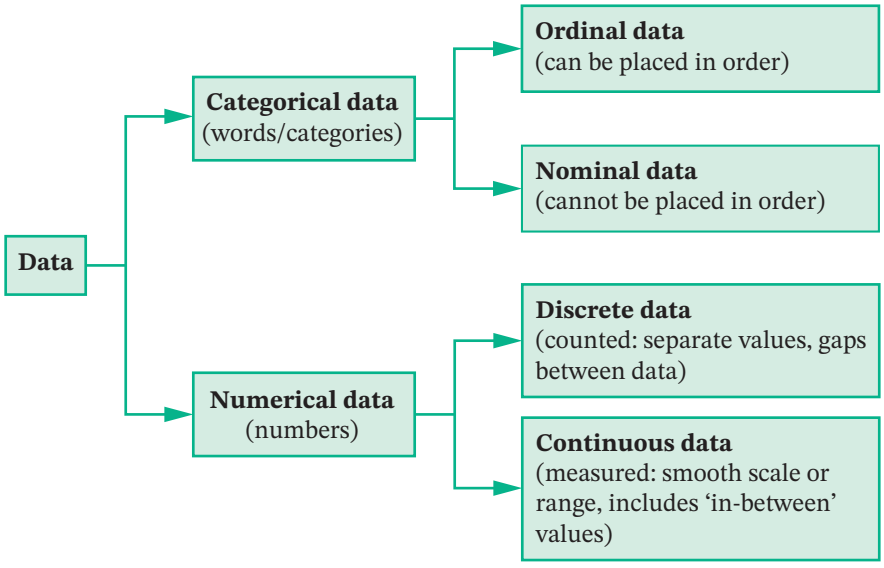
Ordinal means 'order', nominal means 'name'.

## Types of numerical data

Numerical data can be further divided into **discrete data** and **continuous data**.

**Discrete data** are counted or measured and can only take on separate, distinct values, with 'gaps' or 'jumps'. 'In-between' values are not allowed. For example, the number of children in a family is discrete because it could be 0, 1, 2, etc., but never 2.3 or $1\frac{2}{5}$.

**Continuous data** are measured on a 'smooth' scale without any gaps, and can take on a full range of values. 'In-between' values are allowed. For example, a town's temperature is continuous because it could be 28°C, 29°C, 29.5°C, 29.8°C, etc.






## Example 5

Classify each type of categorical data as nominal or ordinal:

**a** make of car (for example, Ford, BMW, Toyota)

**b** colour of Olympic medal

**c** sex of newborn baby

**d** finishing position in race

### Solution

**a** Make of car is nominal data. They cannot be placed in order.

**b** Colour of Olympic medal is ordinal data. Gold, silver, bronze have an order.

**c** Sex of newborn baby is nominal data. They cannot be placed in order.

**d** Position in race is ordinal data. 1st, 2nd, 3rd, etc. have an order.

Video
Types of data

## Example 6

Classify each type of numerical data as discrete or continuous:

**a** number of rooms in a home
**b** loudness of aeroplane
**c** shoe size
**d** weight of whale



Shutterstock.com/Paul S. Wolf

### Solution

**a** Number of rooms is discrete data. — Takes on separate values
**b** Loudness of aeroplane is continuous data. — Takes on a smooth scale of values
**c** Shoe size is discrete data. — Takes on separate values
**d** Weight of whale is continuous data. — Takes on a smooth scale of values

**EXERCISE 7.03** ANSWERS ON P. 591

## Types of data U F R C

EXAMPLE 4

**1** Classify each type of data as categorical (C) or numerical (N). R C

**a** mass of student
**b** brand of computers
**c** price of ice cream
**d** number of TV sets owned by family
**e** amount of rainfall
**f** cause of tooth decay
**g** sets won in a tennis match
**h** speed of truck

EXAMPLE 5

**2** Classify each type of categorical data as ordinal (O) or nominal (N). R C

**a** size of soft drink cup (for example, large)
**b** type of road vehicle
**c** ranking of a football team on the competition ladder
**d** type of seafood
**e** political party
**f** level of agreement with an opinion (for example, strongly disagree)

EXAMPLE 6

**3** Classify each type of numerical data as discrete (D) or continuous (C). R C

**a** number of homes with solar power
**b** patients' body temperature
**c** sizes of the crowd at a rock concert
**d** star ratings of hotel
**e** times taken for athlete to run 100 m
**f** school population
**g** amount of cough medicine taken by child
**h** number of road accidents in Grafton

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**4** Which of the following is **not** discrete data? Select the correct answer **A**, **B**, **C** or **D**. C

**A** The number of shoes you own

**B** The number of blue-eyed students at your school

**C** The number of options in a multiple-choice test

**D** The maximum daily temperature

**5** Classify each type of data as categorical ordinal (CO), categorical nominal (CN), numerical discrete (ND) or numerical continuous (NC). R C

**a** hair colour

**b** customer rating of a restaurant's level of service (poor, satisfactory, good, very good)

**c** number of points a player scored in a netball match

**d** film classification (G, PG, M, MA15+, R)

**e** the age you turn this year

**f** height of bridge

**g** number of staff employed

**h** country where cars were manufactured



# Census vs sample

7.04

In statistics, the **population** means all of the items being studied, whether they be people, cars, businesses or houses. Statistical data is collected in a variety of ways, such as surveys, Internet polls, interviews and questionnaires. To collect information, statisticians usually survey a large representative group called a **sample**. For example, to investigate opinions about school uniform, we can survey a selection of students, teachers and parents. The bigger the sample, the more representative and accurate it will be.

To collect information about a whole population, **all** people or items must be surveyed. This is called taking a **census**. For example, to plan the locations of childcare centres, the government may need to count the number of babies in different areas.

**Videos**
Counting crowds
Can fish oil make you smarter?
Can you trust your IQ?
The wrong guy won

| Sample | Census |
|---|---|
| Surveys a selected representative group of people or items. | Surveys **all** people or items (in the population). |
| Only gives **approximate** information about the population. Helps us make predictions about the population. | Gives **exact** information about the population. |
| Simple and inexpensive to survey. | Complex and expensive to survey. |
| Can be done quickly. | Takes a lot of time to collect and process the information. |

In Australia, a national census is conducted every 5 years to count the Australian population and obtain important information about every person. It takes place in a year ending in 1 or 6, such as 2021 and 2026.

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## Example 7

Should a sample or a census be used to investigate each fact?

**a** The most popular car colour.

**b** Whether energy-saving light globes really use less electricity.

**c** The number of retired people living on the Gold Coast.

**d** People's views on which nation will win the Rugby World Cup.

### Solution

**a** Sample, because it is easier and faster than asking all car-owners in Australia.

**b** Sample, because it is easier and faster than testing every light globe.

**c** Census, because the exact number is required.

**d** Sample, because it is very difficult to ask all people

# Random sampling and bias

Remember that in probability, a **random experiment** is a situation where every possible outcome has the same chance of occurring. When taking a sample, it is important that each person or item in the population has an equal chance of being chosen to be surveyed. This is called **random sampling**. To find a sample of students to survey about school uniform, we could place students' names in a box, mix them up, and then choose 100 names without looking. Alternatively, we could use a computer or calculator to generate random numbers to select students.

A sample that is not random is called a **biased sample**, and is not truly representative of the population. For example, if you survey the first 100 students to arrive at school today, or the first 100 students on the school roll, then not all students in the school have an equal chance of being selected. Your sample would not represent students who arrive at school later or whose names begin with letters at the end of the alphabet.

## Example 8

A random sample of students is to be selected for their views on a new school uniform. Will each of the following sampling methods provide a sample that is representative or biased? If biased, explain why.

**a** Randomly selecting one student from each class.

**b** Selecting 20 students from your mathematics class.

**c** Selecting the students from the school's netball teams.

**d** Randomly selecting students from the school roll by rolling a die and counting down the roll.

### Solution

**a** Representative. Every student in the school has an equal chance.

**b** Biased, because your mathematics class does not represent the other Year groups (or even the other Year 8 classes) at your school.

**c** Biased, because the netball teams do not represent the whole school and will have mainly female students.

**d** Representative. Every student in the school has an equal chance.

**EXERCISE 7.04** ANSWERS ON P. 592

## Census vs sample U F R C

EXAMPLE 7

**1** Is a sample or a census more appropriate for each situation? R C

- **a** Testing coffee for taste.
- **b** Finding the number of migrants from New Zealand.
- **c** Investigating people's views on road safety.
- **d** Finding out people's opinions on whether Australia should become a republic.
- **e** Testing a flu vaccine for effectiveness.
- **f** Counting the number of Spanish-speaking children in Australian schools.
- **g** Finding the population of Gundagai.
- **h** Finding the number of people in Australia over 40, to plan for health care.

**2** For each situation, state whether a sample or a census would be more appropriate and how the data could be collected. R C

- **a** A car dealer wants to find the level of customer satisfaction with his business.
- **b** To find the most popular TV program on Friday nights.
- **c** To find the busiest airport in Australia.
- **d** A principal wishes to find the least-liked subject in the school.
- **e** The number of people who support whaling.
- **f** The average height of students in your school.
- **g** The amount of recyclable garbage each household produces in a week.
- **h** The number of times the students in your class visit a restaurant in a month.

EXAMPLE 8

**3** The parent association for Nelson Vale High School want to know what people think of a new design for the school uniform. Which of the following samples are NOT biased? Select the correct answer **A**, **B**, **C** or **D**. R

- **A** Students who visit the school office during the day
- **B** Parents as they are picking up their children after school
- **C** Randomly-selected families from an alphabetical list of school families
- **D** Families watching the school sporting teams

**4** For each survey, describe the total population, give a suitable way of choosing a representative sample, write any problems that may be encountered and explain how bias could be minimised. R C

- **a** The attitudes of people to smoking in public.
- **b** Whether girls are better than boys at mathematics.
- **c** The choice of music at a school disco.
- **d** The political parties people will vote for at the next state election.

**5** In each survey, state whether the sample is random. If not, explain why you think it is biased. R C

**a** To find participants for a survey of students' study habits, a teacher wrote a message in the daily notices asking for volunteers.

**b** To ensure that items being made in a factory were of good quality, 2 items coming off the assembly line were tested at a randomly selected time each hour.

**c** In a poll to find voters' preferences in the next election, every 100th person listed in the White pages phone database was called. If there was no answer, the next person listed was called, and so on.

**d** In a parent survey about school uniform, the school council posted letters to all parents of students at the school asking for their opinion on the issue.

**e** In a survey about the best song of the decade, listeners of a rock radio station were asked to ring and nominate a song.

**6** The student council wants to survey students on their attitude to mobile phone usage at school. 15 students from each Year group were selected to take part in the survey. R C

**a** Do you believe the results of this survey would be reliable? Explain your answer.

**b** What factors should have been considered in selecting the sample?

**c** How would you have selected the students to be surveyed?

## Technology

### Random numbers

Random numbers can be used to select people or items for a sample. A calculator or spreadsheet can be set up to generate a random number from 1 to 100, for example, and that number can be matched to the name of a person or item from a list. For a scientific calculator:

| Casio | Sharp |
|---|---|
| Enter the formula RanInt#(1,100) as shown: ALPHA . ( 1 SHIFT ) 100 ) = | 2nd F 7 (RANDOM), select 3 R-INT, + 1 = |

This generates a random number from 1 to 100. Press = more times to generate more random numbers.

Alternatively, on a **spreadsheet,** type **=INT(RAND()*100+1)** into a cell and press the ENTER key, then the F9 key repeatedly for more random numbers.

**1** Use your calculator or spreadsheet to generate 12 random numbers from 1 to 100.

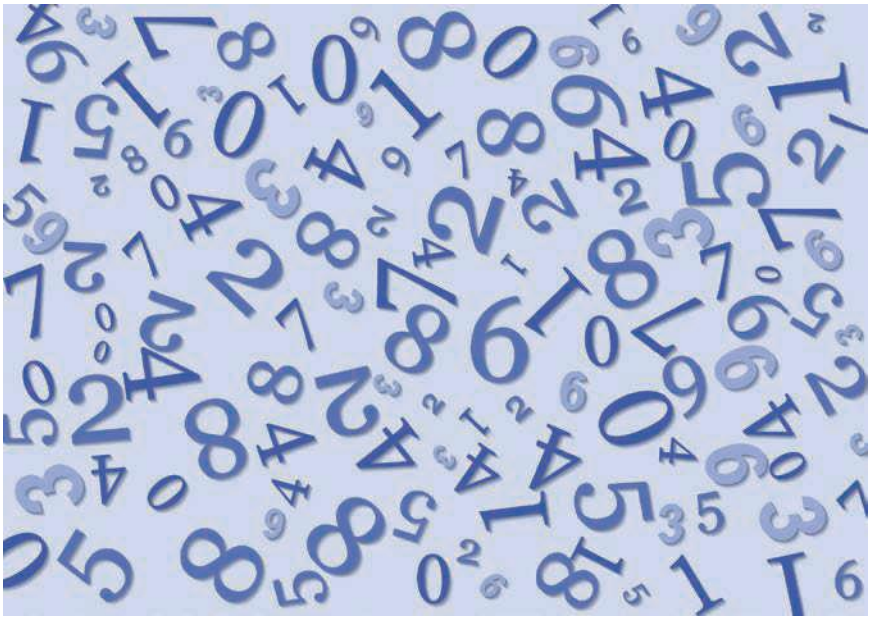
**2** Use your calculator or spreadsheet to generate 12 random numbers from 1 to the number of students in your class.

**3** Use a class roll to number the students in your class from 1 onwards. Suppose you need to choose a random sample of 9 students from your class for a survey on homework. Use your calculator or spreadsheet to generate 9 random numbers to select the 9 students for your survey.

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**4** Choose 4 students from each Year 8 maths class to answer some questions.
Devise several different methods to ensure that your selection is random. Prepare a short class talk about your methods.



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## Investigation

### Statistics in the media

We live in a world of 24-hour news: from TV, newspapers, websites, Facebook, Twitter, blogs, etc. To detect bias in news, we need to consider where the news item comes from and what types of samples the statistics are based on. Is the information supplied by a reporter, the police, a company executive, a government official or an opinionated blogger? Are the statistics based on a small sample, large sample, unrepresentative sample, or a phone or Internet poll of volunteers? Each may have a particular bias that may influence how the story is reported.

Find examples of news items or surveys reported in a newspaper, magazine or the Internet and investigate the following questions.

**a** Where did it come from?

**b** Who wrote the story?

**c** Does it show any bias?

**d** Can it be supported by other news providers?

**e** What type of sampling was used? Was it representative?

**f** How many people were questioned?

**g** When was the survey conducted?




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## Did you know?

### The Australian Census

The first population counts in Australia were known as musters, where people assembled at a location to be counted. The first census when people were recorded at their dwellings was in NSW in 1828. The first simultaneous census across all states was in 1881 and the population was counted as 2.25 million.

Following Federation in 1901, the first national census was developed and undertaken by the Commonwealth in 1911. From 1961, the Census has been held every 5 years in the years ending in 1 and 6.

The current census asks questions relating to the family, where you live, income, religion, level of education and other aspects of modern life.



Newpix.com/Andy Tyndall

**Find 5 questions that were asked in the most recent census.**

# 7.05 The mean and mode

**Skillsheet**
Statistical measures

**Puzzle**
Looking for gold

**Video**
Most popular pet

The mean, mode and median are known as **measures of centre** (averages) or **measures of central tendency**: they are typical or central values that represent all of the data in the set.

The range is a **measure of spread**: it describes how spread out the values are.

The **mean** is found by adding all the data values and dividing by the number of values.

## The mean

The symbol for the mean is $\bar{x}$.

$$\bar{x} = \frac{\text{sum of values}}{\text{number of values}}$$

- The mean is a single value that represents all of the data.
- *All* of the data values are used in the calculation of the mean.
- The mean is *not* necessarily one of the data values.

The **mode** is the most common or frequent value (or values). A set of data may have more than one mode, or no mode at all.

## The mode

The **mode** is the data value (or values) that occurs the most often: the value with highest frequency.

*Think:* mode = '**m**ost **o**ften'

- The mode can be one or more values.
- The mode does not depend on all of the values in the data set.
- The mode is one (or more) of the data values.

## Example 9

The number of goals scored by Courtney's soccer team in each match over a season were:

3 2 1 4 0 2 3 1 3 1 1 0

**a** Find the mean. **b** Find the mode.

### Solution

**a** Mean $\bar{x} = \dfrac{\text{sum of data values}}{\text{number of values}}$

$= \dfrac{3+2+1+4+0+2+3+1+3+1+1+0}{12}$

$= \dfrac{21}{12}$

$= 1.75$

Note that the value of the mean is at the centre of the data.

**b** The mode is 1. It is the most common value (occurs 4 times).

# The mean from a calculator's statistics mode

Scientific and graphics calculators have a statistics mode (SD or STAT). Follow the instructions in the table below to calculate the mean of the values from Example **5** using your calculator's statistics mode.

| Operation | Casio scientific | Sharp scientific |
|---|---|---|
| Start statistics mode. | MODE STAT 1-VAR | MODE STAT = |
| Clear the statistical memory. | SHIFT 1 Edit, Del-A | 2nd F DEL |
| Enter data. | SHIFT 1 Data to get table 3 = 2 =, etc. to enter in column. Press AC to leave table. | 3 M+ 2 M+, etc. |
| Calculate the mean. $(\bar{x} = 1.75)$ | SHIFT 1 Var $\bar{x}$ = | RCL $\bar{x}$ |
| Check the number of values. ($n$ = 12) | SHIFT 1 Var $n$ = | RCL $n$ |
| Return to normal (COMP) mode. | MODE COMP | MODE 0 |

## EXERCISE 7.05 ANSWERS ON P. 592

## The mean and mode U F R C

EXAMPLE 9

**1** The residents in a street were surveyed about the number of children living in each household. The results were:

2 2 1 2 0 3 2 1 1 4 1 0

**a** Find the mean, correct to one decimal place. **b** Find the mode.

**2** For each set of data, find:

**i** the mean (rounded to 2 decimal places, if needed) **ii** the mode(s).

**a** 6 8 7 6

**b** 7 9 5 2 2 6 2

**c** 11 7 7 6 1 2 4 1

**d** 96 53 96 45 64 76 54 54 96

**e** 47.31 47.20 47.24 47.21 47.28 47.26

**3** Use your calculator's statistics mode to calculate the mean of each data set in question **2**.

**4** Find the mode of each set of categorical data.

**a** Holden, Ford, Nissan, Holden, Hyundai, Ford, Hyundai, Nissan, Hyundai, Toyota, Toyota

**b** heart, spade, spade, diamond, spade, club, heart, heart, diamond, spade

**c** Chinese, Korean, Japanese, Korean, Chinese, Japanese, Japanese, Korean, Korean

**5** Why is it not possible to find the **mean** of each data set in question **4**? R C

**6** In a diving competition, the judges awarded the following scores.

6.2 7.0 8.2 8.8 7.0 9.1 7.0 8.1 8.0

**a** Find the mean of these scores, correct to 2 decimal places.

**b** Find the mode of these scores.

**c** Which measure of centre represents this set of scores better? Give a reason for your answer. R C

**7** What is the difference between the mean and the mode of the following set of scores?

20 60 20 50 40

Select the correct answer **A**, **B**, **C** or **D**.

**A** 20 **B** 12 **C** 18 **D** 2

**8** The ages of the members of the Carrozza family are:

1 3 11 25 26 30 31 52 58

The ages of the members of the Binns family are: R C

5 8 16 21 25 29 40 63 93

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**a** Calculate the mean age (correct to one decimal place) of each family.

**b** What is the main difference between these 2 sets of ages?

**c** What effect does this difference have on the means?

**9** 2 Year 8 classes scored the following marks for their PE assignments. R C

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **8Y:** | 5 | 5 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 8 |
| **8Z:** | 3 | 3 | 4 | 5 | 5 | 6 | 6 | 7 | 9 | 9 | 10 |

**a** Calculate the mean mark (correct to one decimal place) for each class.

**b** Describe the differences between the way each class has its marks spread out.

**c** Class 8X scored the following marks, but Hannah was away.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **8X:** | 4 | 5 | 5 | 6 | 6 | 6 | 7 | 7 | 7 | 10 |

What mark would Hannah have to score so that her class has the same mean as the other 2 classes?

**10** The mean points score per game by a basketball team over a season of 30 games was 98. What was the total number of points scored over the season? R

**11** The takings at the school canteen were: R

Monday $1600 Tuesday $2400 Wednesday $1500 Thursday $1750

**a** Find the average takings for these 4 days.

**b** How much should the canteen earn on Friday to make the average daily takings $2000?

**12 a** Find 4 values that have a mean of 10 and a mode of 11. R

**b** Find 7 values that have a mean of 13 and a mode of 5.



# The median and range

7.06

The **median** is the middle value of a set of data or the average of the 2 middle values.

## The median

When the values of a data set are ordered, the **median** is:

- the middle value if there is an odd number of values
- the average of the 2 middle values if there is an even number of values.

*Think:* 'Median' sounds like 'medium', which is halfway between small and large.

A 'median' strip is the strip of grass or concrete down the *middle* of a highway.

- The median is a single value that represents all of the data.
- The median does not depend on all of the data values, so it is not affected by outliers (extremely high or low values).
- The median is often one of the data values.

The **range** is a measure of the spread of a data set, and is simply the difference between the highest and lowest values.

**Skillsheet** Statistical measures

**Worksheet** Mean, median, mode

**Puzzles** Ranges and averages

Data puzzles

Mode, median and mean

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**Videos**
The mean, mode, median and range

The mode, median and mean

Average Joe

**Interactive**
Centre and variability

## The range

Range = highest value − lowest value

## Example 10

The ages of the members of the Carrozza family are:

3 1 25 58 26 52 30 31 11

Find:

**a** the median **b** the range

### Solution

**a** First, rewrite the ages in order.




Note that the value of the median is at the centre of the data.

There are 9 values, an odd number, so there is one middle value.

$\frac{1}{2} \times 9 = 4\frac{1}{2}$, so round up to 5 and count to the 5th value (circled above) to find the median.

Median = 26

**b** Range = 58 − 1
= 57

Highest value − lowest value

## Example 11

The lap times (in seconds) of 6 cyclists were:

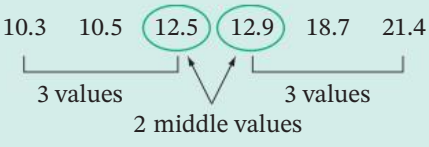
12.5 21.4 10.5 18.7 12.9 10.3

Find:

**a** the median **b** the range

### Solution

**a** Rewrite the values in order:




There are 6 values, an even number, so there are 2 middle values.

$\frac{1}{2} \times 6 = 3$, so count to the 3rd and 4th values (circled above) and find their average.

$$\text{Median} = \frac{12.5 + 12.9}{2}$$
$$= 12.7$$

12.7 is halfway between 12.5 and 12.9

**b** $\text{Range} = 21.4 - 10.3$
$= 11.1$

Highest value − lowest value



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7.06

## EXERCISE 7.06 ANSWERS ON P. 592

### The median and range U F PS R C

EXAMPLE 10

**1** What is the median of this set of data?

10 3 5 9 4 3 10

Select the correct answer **A**, **B**, **C** or **D**.

**A** 3 **B** 5 **C** 7 **D** 10

**2** What is the range of this set of data?

17.1 18.3 14.5 11.8 21.9 14.5

**A** 16.4 **B** 15.8 **C** 14.5 **D** 10.1

EXAMPLE 11

**3** For each set of data, find:

**i** the median **ii** the range.

**a** 11 9 7 4

**b** 46 50 48 47 50 40

**c** 12 13 11 14 10

**d** 84 50 0 28 14 99 51 66 51

**e** 57 57 57 57 57 57 57

**f** 64.2 99 99.2 71.2 93.5 61.8

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**4** Manish likes to gamble on horse races. His winnings or losses (in dollars) each day are as follows (losses are shown as negative values). R C

| | | | | | | |
|---|---|---|---|---|---|---|
| 8 | 21 | −16 | −25 | 21 | −25 | 16 |
| 11 | −12 | 12 | −25 | 17 | −25 | 14 |

**a** Calculate the mean, correct to the nearest cent.

**b** Find the median and the mode.

**c** As Manish wins more often than he loses, should he continue to gamble on horse races? Give a reason for your answer.

**5** 11 houses were sold in Jacaranda Avenue. The selling prices are listed here. R C

| | | | | | |
|---|---|---|---|---|---|
| $580 000 | $485 000 | $660 000 | $758 000 | $644 000 | $1 670 000 |
| $815 000 | $686 000 | $666 500 | $720 800 | $805 000 | |

**a** Find the mode.

**b** Find the median.

**c** Find the mean.

**d** Which measure of central tendency best describes this set of house prices? Give a reason for your answer.

**6** 2 ten-pin bowlers recorded the following scores over 10 games. PS R C

| | | | | | |
|---|---|---|---|---|---|
| *Aldo*: | 175 | 175 | 160 | 165 | 180 |
| | 145 | 150 | 177 | 177 | 177 |
| *Peter*: | 173 | 173 | 174 | 175 | 175 |
| | 176 | 176 | 177 | 179 | 179 |


Shutterstock.com/Mindscape studio

**a** Calculate the mean for each bowler.

**b** Calculate the range for each bowler.

**c** Who would you choose, and why, if you were asked to present an award for:

**i** the more talented bowler?

**ii** the more consistent bowler?

**d** A third player, Rob, recorded the following scores over 9 games.

| | | | | | |
|---|---|---|---|---|---|
| *Rob*: | 134 | 175 | 175 | 144 | 176 |
| | 150 | 187 | 166 | 180 | |

What score would Rob need to achieve in his tenth game to have the same mean score as Aldo?

**7** The favourite party food for a group of 3-year-old children was recorded. R

| | | | | |
|---|---|---|---|---|
| popcorn | fruit | chocolate | fruit | popcorn |
| chocolate | chocolate | fruit | fruit | fruit |

Which is the only statistical measure that can be found for this data?

**A** mean **B** median **C** mode **D** range

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**8** Samir is the coach of the Panthers athletics team. He needs to pick his best runner for the 200 m race. He has recorded the times (in seconds) for the last few races of Daniel and Cristian. R C

| | | | | | | |
|---|---|---|---|---|---|---|
| **Daniel** | 29.4 | 31 | 27.5 | 32.5 | 28 | |
| **Cristian** | 28.5 | 27 | 30.5 | 28.5 | 31.5 | 28.5 |

**a** Calculate the mean for each runner.

**b** Calculate the range for each runner.

**c** Who do you think Samir should choose as the runner for the next event? Give a reason for your answer.

**9** A group of 8 friends counted the number of letters in their surnames. The results were:

4 6 7 5 4 6 9 ☐

where ☐ represents a missing value.

Find a possible value of ☐ if the median is 6 and the range is 5. PS R

**10** Find 8 values that have a median of 8 and a range of 8.

**b** Find 7 values from 3 to 9 inclusive that have a mode of 4 and a median of 6. PS R

## Technology

### Mean, mode, median and range

**1** Enter the data below into a new spreadsheet, listing the daily maximum temperatures in Camden for one week in January.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Day | Temperature (deg C) | | | |
| 2 | Saturday | 31.5 | | Mean | |
| 3 | Sunday | 32.2 | | Mode | |
| 4 | Monday | 33 | | Median | |
| 5 | Tuesday | 36 | | Maximum | |
| 6 | Wednesday | 36.8 | | Minimum | |
| 7 | Thursday | 35.9 | | Range | |
| 8 | Friday | 26.7 | | | |

**2** Enter each formula into the given cell to calculate each statistic.

Cell E2: **=average(B2:B8)**

Cell E3:**=mode(B2:B8)**

Cell E4:**=median(B2:B8)**

Cell E5:**=max(B2:B8)**

Cell E6: **=min(B2:B8)**

Cell E7: **=E5-E6**

7.06

**Technology**
Students' marks

Finding the mean, median, mode and range

Statistical measures calculator

**Spreadsheet**
Statistical measures calculator

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Quiz
Mental skills 7

Skillsheet
Reading linear scales

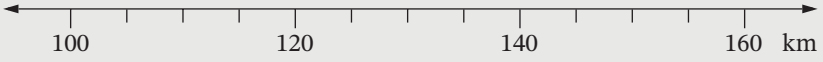
# Mental skills 7: Maths without calculators ANSWERS ON P. 592

## Reading linear scales

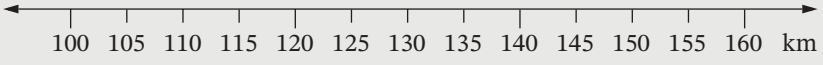
Understanding and reading the scale on a measuring instrument, on a number line or on the axis of a graph is an important mathematical skill.

**1** Study each example.

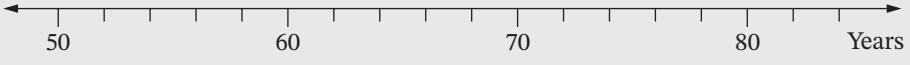
**a** Complete the missing values on this scale.



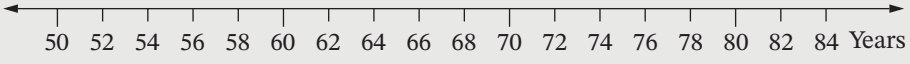

- First, choose 2 values on the scale, say 100 and 120.
- Count the number of intervals ('spaces') between the 2 values. There are 4 intervals between 100 and 120.
- To find the size of each interval, divide the difference between the 2 values by the number of intervals.
- Difference = 120 – 100 = 20 km
- Number of intervals = 4
- Size of an interval = 20 ÷ 4 = 5 km
- Use the calculated size of an interval to complete the missing values.




**b** Complete the values on this scale.




- Choose 50 and 60 on the scale.
- Number of intervals (between 50 and 60) = 5
- Difference (between 50 and 60) = 60 – 50 = 10 years
- Size of an interval = 10 ÷ 5 = 2 years.

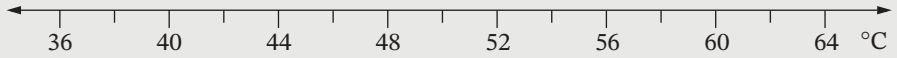




9780170479516

**2** Now copy and complete the following scales.

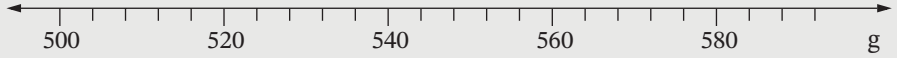
**a**



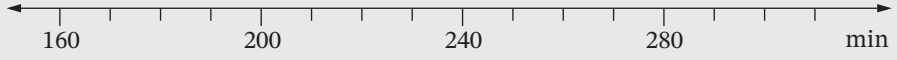

**b**



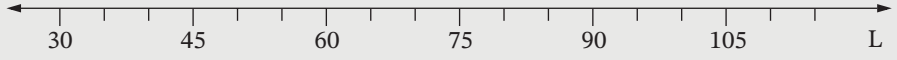

**c**



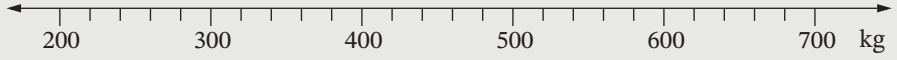

**d**



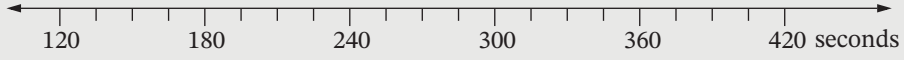

**e**




**f**




**g**




**h**




## Investigation

### Gym friends

7 friends joined a gym to lose weight.

| Name | Sex | Age | Weight loss |
|---|---|---|---|
| Brenda | F | 36 | 4 kg |
| Peter | M | 34 | 28 kg |
| Mark | M | 28 | 21 kg |
| Fiona | F | 33 | 2 kg |
| Manjeet | F | 30 | 6 kg |
| Mia | F | 34 | 3 kg |
| Natisa | F | 29 | 6 kg |



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In the following questions, round all mean answers to 2 decimal places.

**1** What is the mean age?

**2** Find the mean age of the males.

3 What is the median age of the group? Who has this age?

4 What is the median age of the males? Who has this age?

5 If the men were not counted, would the mean age or the median age be affected more?

6 Predict what would happen to the mean age if Imraan (aged 21) joined the group. Test your prediction.

7 What was the mean age 2 years ago? How does this compare to the group's mean age now?

8 Compare the mean weight loss with the median weight loss of the whole group. Who has the median weight loss?

9 Compare the mean weight loss and the median weight loss of the females only. Who has the median weight loss now?

10 When Diego joined the group, the mean weight loss of the group became 20 kg. How much weight did Diego lose?

## Investigation

### Finding the mean of a large set of data

How do we find the mean of a large amount of data? Is there a simpler way than adding all the values, then dividing the total by the number of values?

A group of students was surveyed about the number of hours they watched TV on Saturday:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 4 | 0 | 3 | 1 | 2 | 2 | 3 | 5 | 4 | 3 | 3 |
| 4 | 1 | 0 | 6 | 6 | 3 | 4 | 3 | 1 | 5 | 4 | 3 |

We can calculate the mean of this set of data using 3 different methods.

**Method 1: Adding all the values and dividing by the number of values**

1 Find the sum of all the data values.

2 How many values are there in the data set?

3 Hence show that the mean is approximately 3.17 hours.

**Method 2: Arranging the values in order first to simplify the addition**

1 Sort all of the values in ascending order and group the equal values:

0 0 1 1 1 2 2 3 3 3 3 3 3 4 4 4 4 4 5 5 6 6 6

2 To find the sum of all the values, notice that we can multiply each value by its frequency and add those amounts together.

Evaluate $(0 \times 2) + (1 \times 3) + (2 \times 2) + (3 \times 7) + (4 \times 5) + (5 \times 2) + (6 \times 3)$.

3 Hence show that the mean is approximately 3.17 hours.

**Method 3: Using a frequency table with an extra column**

| Score | Frequency | Frequency × Score |
|---|---|---|
| 0 | 2 | 0 |
| 1 | 3 | 3 |
| 2 | 2 | 4 |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| **Total** | | |

This is a more organised version of Method **2**.

**1** Copy and complete the frequency table.

**2** What is the total number of values?

**3** What is the sum of all the values?

**4** Hence show that the mean is approximately 3.17 hours.



# Analysing frequency tables 7.07

## Example 12

The number of minutes each train was late at a railway station in one afternoon were:

2 4 1 0 0 3 2 3
3 2 0 3 2 1 3

Copy and complete this frequency table for the data and use it to find:

**a** the range
**b** the mode
**c** the median
**d** the mean, correct to one decimal place.

| Score, $x$ | Frequency, $f$ | $fx$ |
|---|---|---|
| 0 | | |
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| Total | | |

**Worksheet**
The mean and *fx* tables

**Videos**
Analysing frequency tables

Statistics from a frequency table

## Solution

$fx$ means $f \times x$, so in the $fx$ column, multiply each score ($x$) by its frequency ($f$):

$3 \times 0 = 0$

$2 \times 1 = 2$

$4 \times 2 = 8$

| Score, $x$ | Frequency, $f$ | $fx$ |
|---|---|---|
| 0 | 3 | 0 |
| 1 | 2 | 2 |
| 2 | 4 | 8 |
| 3 | 5 | 15 |
| 4 | 1 | 4 |
| Total | 15 | 29 |

**a** $\text{Range} = 4 - 0$
$= 4$

Highest value − lowest value

**b** Mode = 3

The score with the highest frequency (5)

**c** There are 15 data values (an odd number), so the median (middle) is the **8th** value. According to the frequency table, the values in order are:

0, 0, 0  1, 1  2,2,(2),2  3,3,3,3,3  4

Half of 15 is $7\frac{1}{2}$, so round up to 8 and count the 8th value.

The first 3 values are 0, the next 2 values are 1, the next 4 values are 2, and so on.

$\therefore$ The 8th value (circled) is 2.

Median = 2.

**d** $\text{Mean } \bar{x} = \dfrac{\text{sum of values}}{\text{number of values}}$.

The $fx$ column multiplies each score by its frequency, so its total, 29, is the sum of all the data values. If we write all the values, we can see that the $fx$ column groups and adds the values.

$\underbrace{0,0,0}_{\text{Sum } 0}\ \underbrace{1,1}_{2}\ \underbrace{2,2,2,2}_{8}\ \underbrace{3,3,3,3,3}_{15}\ \underbrace{4}_{4}$

The $f$ column shows the frequency of each value so its total, 15, is the number of values.

$$\text{Mean} = \frac{\text{sum of } fx}{\text{sum of } f}$$
$$= \frac{29}{15}$$
$$= 1.9333\ldots$$
$$= 1.9$$

## Statistics for data in a frequency table

- the **mean** is $\bar{x} = \dfrac{\text{sum of } fx}{\text{sum of } f}$
- the **mode** is the data value(s) with the highest frequency, $f$
- the **range** is the difference between the last and first values in the score ($x$) column

## Example 13

The ages of children at a play centre are shown in the frequency table. Copy and complete the table and use it to find:

| Score, $x$ | Frequency, $f$ | $fx$ |
|---|---|---|
| 5 | 3 | |
| 6 | 4 | |
| 7 | 5 | |
| 8 | 2 | |
| 9 | 2 | |
| Total | | |



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**a** the mode

**b** the mean

**c** the median

**d** the range.

### Solution

| Score, $x$ | Frequency, $f$ | $fx$ |
|---|---|---|
| 5 | 3 | 15 |
| 6 | 4 | 24 |
| 7 | 5 | 35 |
| 8 | 2 | 16 |
| 9 | 2 | 8 |
| Total | 16 | 108 |

**a** Mode = 7

Has the highest frequency, 5.

**b** Mean:

$$\bar{x} = \frac{\text{sum of } fx}{\text{sum of } f} = \frac{108}{16} = 6.75$$

**c** There are 16 values (an even number), so there are 2 middle values.

5 5 5 6 6 6 6 (7 7) 7 7 7 8 8 9 9

The 2 middle values

Half of 16 is 8, so count the 8th and 9th values (circled) and find the average of them.

$$\text{Median} = \frac{7+7}{2} = 7$$

Note that the mean and median are close to each other and at the centre of the values.

**d** Range $= 9 - 5 = 4$

# The mean of data in a frequency table using a calculator's statistics mode

Data from a frequency table can be entered into a scientific or graphics calculator using the statistics mode. Follow the instructions in the table below to calculate the mean of the children's ages from the previous page using your calculator's statistics mode.

| Operation | Casio scientific | Sharp scientific |
|---|---|---|
| Start statistics mode. | MODE STAT 1-VAR<br>SHIFT MODE scroll down to STAT Frequency? ON | MODE STAT = |
| Clear the statistical memory. | SHIFT 1 Edit, Del-A | 2nd F DEL |
| Enter data. | SHIFT 1 Data to get table<br>5 = 6 =, etc. to enter in x column<br>3 = 4 =, etc. to enter in FREQ column<br>AC to leave table | 5 2nd F STO 3 M+ 6 2nd F STO 4 M+, etc. |
| Calculate the mean. $(\bar{x} = 6.75)$ | SHIFT 1 Var $\bar{x}$ = | RCL $\bar{x}$ |
| Check the number of values. ($n$ = 16) | SHIFT 1 Var $n$ = | RCL $n$ |
| Return to normal (COMP) mode. | MODE COMP | MODE 0 |

## EXERCISE 7.07 ANSWERS ON P. 593

## Analysing frequency tables U F

In this exercise, round all means to one decimal place where necessary.

EXAMPLES 12, 13

**1** This table shows how many times a sample of children visited a doctor over 3 months. Copy and complete the table, and find:

**a** the number of people surveyed

**b** the median

**c** the mean

**d** the mode

**e** the range

| No. of visits, $x$ | Frequency, $f$ | $fx$ |
|---|---|---|
| 0 | 5 | |
| 1 | 6 | |
| 2 | 14 | |
| 3 | 3 | |
| 4 | 5 | |
| 8 | 1 | |
| Total | | |

**2** A sample of hospital patients were surveyed on the number of hours they slept the previous night:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 7 | 9 | 8 | 10 | 11 | 9 | 10 | 8 | 9 | 7 | 6 |
| 11 | 9 | 8 | 8 | 8 | 10 | 10 | 9 | 7 | 8 | 9 | 10 |

**a** How many people were surveyed?

**b** What was the highest number of hours people slept?

**c** Use a frequency table with an *fx* column to find the mean.

**d** Find the range.

**e** Find the median.

**3** The judge's scores for students' speeches (out of 10) for an English class are as shown.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 7 | 8 | 9 | 7 | 5 | 4 | 8 | 9 |
| 9 | 8 | 10 | 8 | 7 | 5 | 5 | 4 | 1 |
| 8 | 8 | 9 | 7 | 7 | 4 | 8 | 3 | 1 |

**a** Complete a frequency table for this data.

**b** What is the mode?

**c** What is the mean?

**d** What is the median?

**e** What is the range?

**4** A class of students was asked how many children there were in their families.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 5 | 1 | 2 | 3 | 1 | 3 | 2 | 4 | 2 | 4 |
| 2 | 4 | 2 | 3 | 1 | 5 | 6 | 2 | 2 | 1 | 3 | 3 |

Complete a frequency table for this data, including an *fx* column, and use it to find:

**a** the range **b** the mode **c** the median **d** the mean

**5** This frequency table shows the heights in metres of a sample of students.

| Height (m) | Frequency |
|---|---|
| 1.42 | 4 |
| 1.44 | 5 |
| 1.46 | 8 |
| 1.48 | 11 |
| 1.5 | 6 |
| 1.52 | 3 |

Find:

**a** the number of students in the sample

**b** the mean correct to 2 decimal places, using your calculator's statistics mode

**c** the range

**d** the median

**e** the mode

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**6** Carmelina recorded the number of books bought by the people entering her shop in one hour.

| No. of books | Frequency |
|---|---|
| 0 | 7 |
| 1 | 5 |
| 2 | 13 |
| 3 | 4 |
| 4 | 1 |
| 5 | 2 |
| 6 | 1 |


Shutterstock.com/SpeedKingz

What is the median number of books bought? Select the correct answer **A**, **B**, **C** or **D**.

**A** 2 **B** 3

**C** 13 **D** 4

**7** 30 people were asked how often they went to a cinema in the previous month.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4 | 5 | 3 | 1 | 3 | 7 | 1 | 1 | 4 | 0 |
| 0 | 4 | 3 | 2 | 4 | 2 | 4 | 7 | 6 | 6 |
| 1 | 3 | 5 | 5 | 1 | 3 | 6 | 7 | 0 | 3 |

Construct a frequency table and use it to find:

**a** the range **b** the mode **c** the median **d** the mean.

**8** A pigeon breeders' club has 25 members. This frequency table shows the number of pigeons each member has.

| Score, *x* | Frequency, *f* |
|---|---|
| 8 | 3 |
| 10 | 4 |
| 14 | 3 |
| 15 | 6 |
| 17 | 2 |
| 18 | 4 |
| 20 | 3 |

ÆÆ


iStock.com/CaoChunhai

**a** What is the range?

**b** What is the median?

**c** Use your calculator's statistics mode to find the mean.

## Technology

### Daily temperatures

1 Enter the following data into a spreadsheet: a frequency table showing the maximum daily temperatures in Coffs Harbour in July.

2 How many temperatures are in this data set? In cell B11, enter an appropriate formula for the sum of the Frequency column.

3 To calculate the mean, first complete the *fx* column. In cell C2, enter the formula for multiplying the values in the A and B columns, then **Fill Down** to copy the formula into cells C3 to C10.

4 In cell C11, enter an appropriate formula for the sum of the *fx* column.

| | A | B | C |
|---|---|---|---|
| 1 | Temperatures (x) | Frequency (f) | *fx* |
| 2 | 15 | 1 | |
| 3 | 16 | 3 | |
| 4 | 17 | 3 | |
| 5 | 18 | 7 | |
| 6 | 19 | 7 | |
| 7 | 20 | 2 | |
| 8 | 21 | 7 | |
| 9 | 22 | 0 | |
| 10 | 23 | 1 | |
| 11 | Total: | | |
| 12 | Mean: | | |
| 13 | Range: | | |

5 In cell B12, calculate the mean by entering the formula for dividing the sum of the *fx* column by the sum of the *f* column.

6 In cell B13, calculate the range by entering the formula for subtracting the lowest temperature from the highest temperature.



# Dot plots and stem-and-leaf plots 7.08

## Dot plots

A **dot plot** is a graph that uses dots to show the frequency of each data value. A dot plot shows:

- any gaps in the data
- any **clusters**, where values are grouped or bunched together
- any **outliers**: extremely high or low values that are very different from the rest of the data
- how the values are spread out.

An **outlier** 'lies outside' the other values in the data set. Outlier is pronounced 'out-ly-er'

**Worksheets**
Stem-and-leaf plots
What data?

**Presentation**
Stem-and-leaf plots

Videos
Analysing dot plots
Dot plots

## Example 14

An online music store recorded the daily sales figure of a particular album while it was popular, and displayed the results in a dot plot.

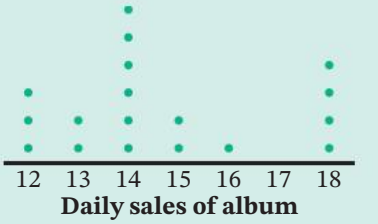
**a** On how many days were there exactly 13 sales?

**b** What is the range?

**c** Find the mode.

**d** Find the median.

**e** Calculate the mean, correct to one decimal place.



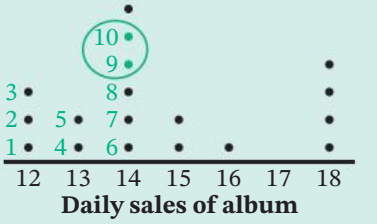

### Solution

**a** 13 sales were made on 2 days — 2 dots above 13

**b** Range $= 18 - 12 = 6$ — Highest value − lowest value

**c** Mode = 14 — The value with the most dots.

**d** There are 18 values (18 dots). This is an even number, so there are 2 middle values (the 9th and 10th values). By counting the dots, or by crossing out pairs of dots at each end, we can see that the middle values (circled) are 14 and 14.




$$\text{Median} = \frac{14+14}{2} = 14$$

**e** $\text{Mean: } \bar{x} = \dfrac{\text{sum of values}}{\text{number of values}}$ $\left(\text{or } \dfrac{\text{sum of } fx}{\text{sum of } f}\right)$

Number of values = 18 — 18 dots

$$\bar{x} = \frac{3\times12+2\times13+6\times14+2\times15+16+4\times18}{18}$$
$$= 14.6666\ldots$$
$$= 14.7$$

Note that the mean and the median are close to each other and at the centre of the data.

*Note*: This answer can also be found using a frequency table and/or your calculator's statistics mode.

Videos
Stem-and-leaf plots 1
Stem-and-leaf plots 2

## Stem-and-leaf plots

A **stem-and-leaf plot** (or **stemplot**) is like a sideways column graph, but one that lists the actual values on the horizontal columns. An **ordered** stem-and-leaf plot shows:

- all the data values, listed from smallest to largest
- any modes (the most common values)
- any clusters or outliers
- how the data are spread out.

## Example 15

The heights (in centimetres) of a sample group of Year 8 boys and girls were measured.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *Girls*: | 148 | 155 | 153 | 137 | 155 | 137 | 146 | 165 | 157 | 168 |
| *Boys*: | 140 | 152 | 143 | 139 | 166 | 144 | 136 | 151 | 130 | 138 |

**a** Show this data on a back-to-back stem-and-leaf plot.

**b** Find the median for each set of data.

**c** Find the range for each set of data.

**d** Comment on the differences between the data for girls and boys.

### Solution

**a** Heights of Year 8 students in centimetres

| Girls | | Boys |
|---|---|---|
| 7 7 | 13 | 0 6 8 9 |
| 8 6 | 14 | 0 3 4 |
| 7 5 5 3 | 15 | 1 2 |
| 8 5 | 16 | 6 |

**b** Median: There are 10 values in each set.

The median is the average of the 5th and 6th values (circled in red on the plot).

Median: Girls $= \frac{153+155}{2} = 154$

Boys $= \frac{140+143}{2} = 141.5$

**c** Range: Girls = 168 – 137 = 31

Boys = 166 – 130 = 36

**d** The girls in the sample are generally taller (higher median), but the boys' heights are more spread out (higher range).

Video
Back-to-back stem-and-leaf plots

7.08

**EXERCISE 7.08** ANSWERS ON P. 593

## Dot plots and stem-and-leaf plots U F PS R C

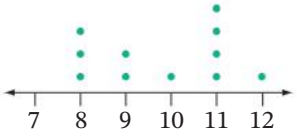
In this exercise, round all mean answers to one decimal place where necessary.

EXAMPLE 14

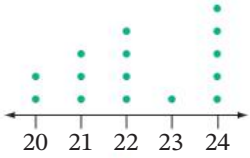
**1** For each dot plot, find:

**i** the range **ii** the mode **iii** the median **iv** the mean.

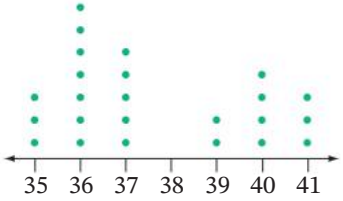
**a**



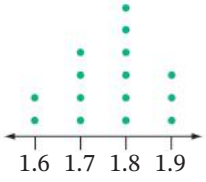

**b**




**c**




**d**




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**2** The number of goals a football player kicked for each match last season were:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 7 | 3 | 2 | 4 | 0 | 3 | 2 | 4 | 3 | 5 |
| 3 | 1 | 1 | 2 | 4 | 2 | 5 | 3 | 0 | 1 | 0 |

**a** Draw a dot plot to represent this data.

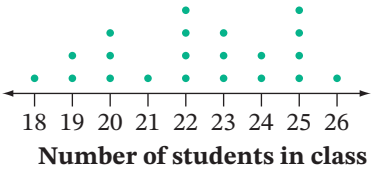
**b** How many matches were there last season?

**c** Which value is an outlier?

**d** Calculate the median.

**e** Calculate the mean.

**3** This dot plot shows the number of students in each class at Northvale Primary School.




**a** How many students are in the smallest class?

**b** Find the range.

**c** Find the mode.

**d** Calculate the mean.

**e** What percentage (to the nearest whole number) of classes have over 20 students?

**f** Which of the following values is the median? Select the correct answer **A**, **B**, **C**, or **D**.

**A** 11 **B** 22 **C** 22.5 **D** 23

**4** The number of songs Udani downloaded each day was: R

1, 2, 0, 2, 3, 3, 3, 1, 1, 1, 6, 2, 3, 0

**a** Draw a dot plot for this data.

**b** How many days are represented by this dot plot?

**c** What is the outlier?

**d** Find the median.

**e** Calculate the mean.

**f** If this pattern continues in the following weeks, how many songs would she expect to download on average in a week?

**5** The quiz marks out of 10 for 2 Year 8 classes are shown. R C

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *8 Hamper:* | 3 | 2 | 0 | 1 | 5 | 8 | 6 | 7 | 6 | 3 | |
| | 5 | 4 | 5 | 6 | 7 | 9 | 2 | 5 | 7 | | |
| *8 Yen:* | 7 | 6 | 3 | 7 | 8 | 1 | 9 | 4 | 6 | 7 | 2 |
| | 7 | 2 | 8 | 10 | 9 | 9 | 5 | 7 | 8 | 9 | 10 |

**a** Draw a dot plot of the marks for each class.

**b** What is the mode of the marks for class 8 Yen?

**c** What is the median of the marks for class 8 Hamper?

**d** What is the range of the marks for class 8 Yen?

**e** Draw a frequency table of the data for class 8 Hamper and use it to calculate the mean.

**f** Which do you think is the 'better' class? Give a reason for your answer.

EXAMPLE 15

7.08

**6** For each stem-and-leaf plot below, find:

**i** the range **ii** the mode **iii** the median **iv** the mean.

**a**

| Stem | Leaf |
|---|---|
| 1 | 0 2 3 |
| 2 | 1 4 4 5 6 |
| 3 | 3 3 3 7 |
| 4 | 1 2 3 5 9 |

**b**

| Stem | Leaf |
|---|---|
| 7 | 3 4 5 7 7 |
| 8 | 1 2 2 3 8 9 |
| 9 | 0 4 4 4 4 6 7 9 |

**c**

| Stem | Leaf |
|---|---|
| 10 | 0 1 1 2 |
| 11 | 7 8 8 8 9 |
| 12 | 3 6 6 7 |
| 13 | 1 1 1 1 1 8 |
| 14 | 0 |

**d**

| Stem | Leaf |
|---|---|
| 0 | 5 5 6 |
| 1 | 1 4 4 7 7 7 7 |
| 2 | 0 3 8 8 |
| 3 | |
| 4 | 9 9 9 |

**7** A survey shows that the number of students served at the school canteen daily over a 3-week period is:

105 76 97 108 114 106 124 101
112 98 95 105 117 101 92

**a** Show this information on a stem-and-leaf plot.

**b** What is the outlier?

**c** Find the mode.

**d** What was the highest value?

**e** On what stems were the values clustered?

**f** On what percentage (to one decimal place) of days were more than 100 students served?

**8** The heights in centimetres of the students in a Science class are:

156 154 158 167 164 163 155 166 159 158 137 163
171 160 164 169 157 161 160 165 145 175 167 167

**a** Make a stem-and-leaf plot for this data.

**b** On what stem are the data clustered?

**c** How many students were measured?

**d** How many students were shorter than 150 cm?

**e** Calculate the range.

**f** Find the median.

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**9** The times taken, in minutes, to drive between 2 towns along 2 different roads are shown on this back-to-back stem-and-leaf plot. PS R C

| Fuller's Ridge Rd | | Great Southern Drive |
|---|---|---|
| 7 5 3 2 | 2 | 6 8 |
| 9 8 8 5 1 1 | 3 | 0 2 4 7 8 |
| 3 3 2 0 0 | 4 | 0 1 2 6 9 9 |
| 2 | 5 | 0 1 3 7 |
| 1 0 | 6 | 2 |
| | 7 | 1 |

**a** How many trips were made along the Great Southern Drive?

**b** How many trips made along Fuller's Ridge Road took over an hour?

**c** Which road is the faster one to take to drive between the 2 towns? Explain your answer.

**d** Which statement is true for the data sets above? Select **A**, **B**, **C** or **D**.

**A** The range is the same for both data sets

**B** There is a difference of 5 minutes in the median travel times

**C** The mean travel time for Fuller's Ridge Road is 36 minutes

**D** A travel time of 71 minutes along the Great Southern Drive is an outlier

**10** This back-to-back stem-and-leaf plot shows the number of penalties that 2 football teams received for each match last season. PS R C

| Wee Waa Reds | | Narrabri Lions |
|---|---|---|
| 9 8 4 4 2 | 0 | 7 9 9 |
| 8 6 5 5 2 2 2 | 1 | 2 3 3 6 8 |
| 3 1 1 | 2 | 5 6 8 9 |
| 0 | 3 | 0 0 1 3 |

**a** How many matches did these teams play in one season?

**b** How many times did the Narrabri Lions receive fewer than 20 penalties in a match?

**c** Find the highest number of penalties received by the Wee Waa Reds in a game.

**d** Find the mode for the Narrabri Lions.

**e** Which value was an outlier for the Wee Waa Reds?

**f** Which team generally had a higher number of penalties?

**g** Which team had a greater range in their number of penalties? Give a reason for your answer.

## Investigation

### Measures of centre

**1** Create 6 values in a data set where the mean equals the median.

**2** Create 5 values in a data set where the median equals the mode.

**3** Create 4 values where the mean, the median and mode all differ.

**4** Create 5 values where the mean, median and mode are the same.

**5** 7 judges score a gymnast out of a possible 10. Their values are:

6 7 8 6 8 9 6

The scoring system requires that the lowest and highest scores are rejected and that the mean of the remaining 5 gives the final score. What is the final score?



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**6** In 4 games Dan scores 23, 11, 17 and 26 points. What score must Dan achieve in his next game so that he can average 20 on the 5 games? (Show the working or reasoning that you use.)

**7** 11 film students rate a movie out of a possible 10. Their ratings are:

7 9 9 4 6 4 8 9 7 8 5

The group cannot agree on a group rating for the movie. In one sentence, comment on each of these suggestions.

**a** Kim thinks that the rating should be 9 (the mode) because that was the rating given the most often.

**b** Vlado thinks that the rating should be 7 (the median) because it is not swayed by extremely high or extremely low ratings.

**c** Thanh thinks that the rating should be 6.9 (the mean) because it considers the ratings of all the students.

What overall rating would you give the film based on these opinions?

**8** Find 3 numbers for which the median is 15, the mean is 13 and the range is 12.

**9** Find 5 numbers for which the median is 3, the mean is 4, and the mode is 7.

# 7.09 Frequency histograms and polygons

Numerical data can be graphed on a **frequency histogram**, a special type of column graph, or a **frequency polygon**, a special type of line graph.

**Puzzles**
Statistical match-up
Highway elephants

**Worksheet**
Analysing data

## Example 16

20 families were surveyed about how many phones they owned.

Graph this data as:

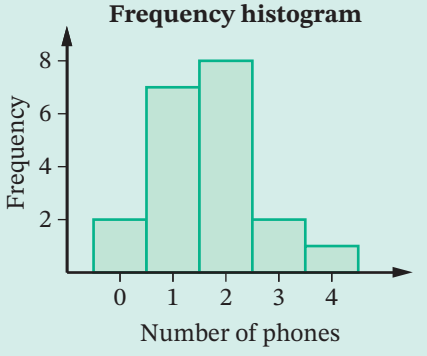
**a** a frequency histogram

**b** a frequency polygon.

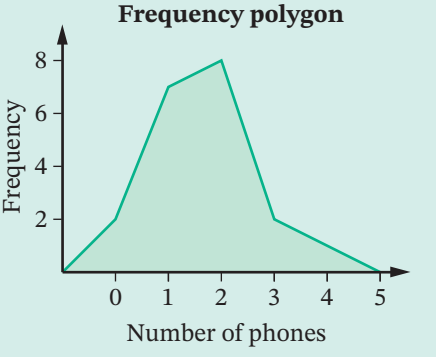
| No. of phones | Frequency |
|---|---|
| 0 | 2 |
| 1 | 7 |
| 2 | 8 |
| 3 | 2 |
| 4 | 1 |
| **Total** | 20 |

### Solution

**a**



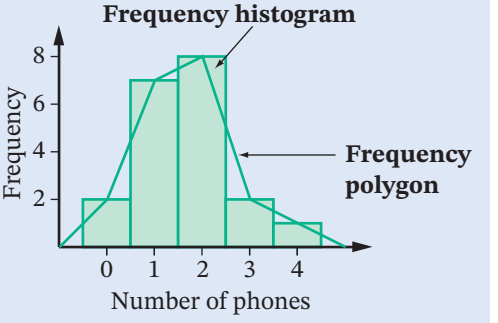

**b**




*Note:* A frequency polygon can be constructed from a frequency histogram by joining the midpoints of the tops of the columns.

## Frequency histogram

- A column graph that looks like a row of office buildings
- Frequency is shown on the vertical axis
- The values are shown on the horizontal axis
- The columns stand together without gaps between them
- The columns are centred on the values
- There is a gap of half a column width on the left (between the frequency axis and the first column)





9780170479516

## Frequency polygon

- A line graph that looks like a mountain
- Frequency is shown on the vertical axis
- The values are shown on the horizontal axis
- Begins and ends on the horizontal axis
- Called a 'polygon' because the graph and the horizontal axis together make a shape with straight sides



**EXERCISE 7.09** ANSWERS ON P. 594

## Frequency histograms and polygons **U F PS R**

**1** The students in a class were surveyed about how many pets they owned.

| No. of pets | Frequency |
|---|---|
| 0 | 7 |
| 1 | 11 |
| 2 | 9 |
| 3 | 3 |

EXAMPLE 16

**a** Graph this data as a frequency polygon.

**b** How many students were surveyed?

**c** How many students had no pets?

**d** What percentage of students had 2 pets?

**e** Find the median.

**f** Calculate the mean, correct to one decimal place.
Select the correct answer **A**, **B**, **C** and **D**.

**A** 1.0 **B** 1.3 **C** 1.5 **D** 3



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**2** Hadieya counted the number of phone calls she made each day for 30 days.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 6 | 2 | 4 | 3 | 3 | 5 | 4 | 7 |
| 5 | 1 | 6 | 3 | 2 | 3 | 5 | 6 | 1 | 6 |
| 2 | 4 | 3 | 1 | 5 | 4 | 2 | 7 | 3 | 2 |

**a** Complete a frequency table for this data.

**b** Draw a combined frequency histogram and polygon of the data.

**c** On how many days did Hadieya make 3 calls?

**d** How many phone calls were made over the 30 days?

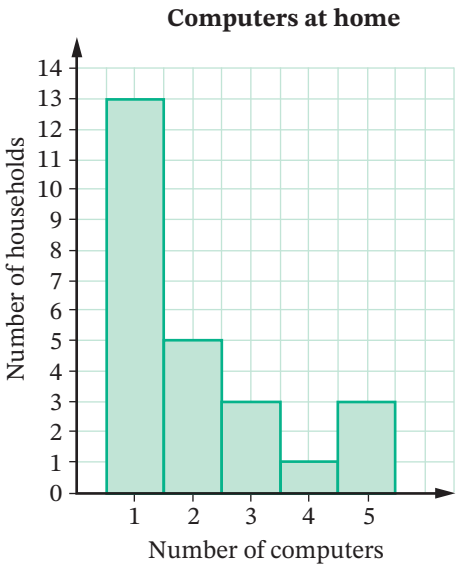
**e** On how many days did Hadieya make fewer than 3 calls?

**f** What was the mode?

**g** What was the mean (correct to one decimal place)?

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**3** This histogram shows the number of computers found in a sample of homes.




**a** How many households were surveyed?

**b** Where are most of the values clustered?

**c** Find the mode.

**d** Find the range.

**e** Complete a frequency table, including an *fx* column, for the data and use it to calculate the mean.

**f** Find the median.

**4** The numbers of goals scored by a soccer team in each match last season were:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 1 | 2 | 3 | 4 | 3 | 1 | 2 | 0 |
| 2 | 1 | 4 | 3 | 2 | 5 | 1 | 2 | 0 | 3 |

**a** Complete a frequency table for this data.

**b** Draw a frequency histogram and frequency polygon of the data on the same axes.

**c** How many matches were played by the team?

**d** What was the highest number of goals scored?

**e** What was the mode?

**f** What was the mean?

**g** In how many matches were more than 2 goals scored?

**h** In what percentage of matches were no goals scored?



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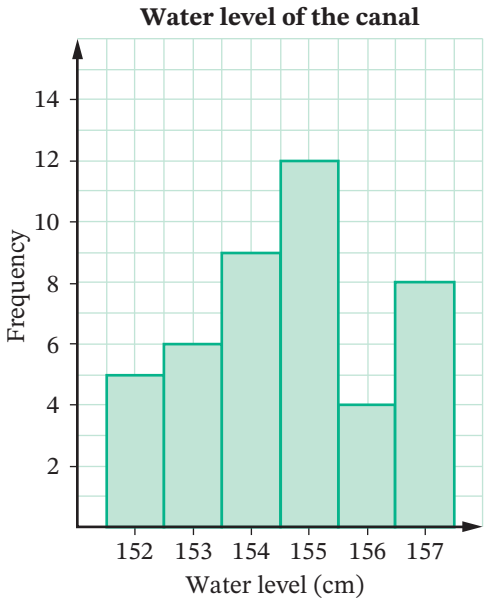
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**5** The water level of a canal was recorded over 44 days and graphed on this histogram. PS R

**a** Use the histogram to copy and complete the frequency table.

| Water level (cm) | Frequency |
|---|---|
| 152 | |
| 153 | |
| 154 | |
| 155 | |
| 156 | |
| 157 | |
| Total | |




**b** On how many days was the water level 156 cm high?

**c** What is the range?

**d** What is the mode?

**e** What is the mean, correct to 2 decimal places?

**f** On how many days was the water level lower than 154 cm?

**g** What is the median?

**h** There needs to be at least 155 cm of water in the canal for a boat to travel on it safely. On how many days would the boat be able to use the canal?

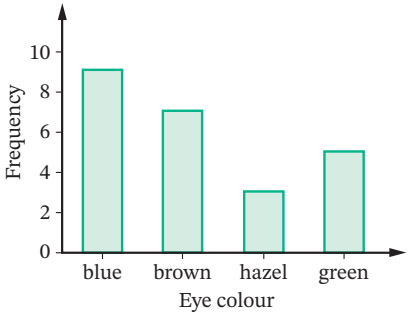
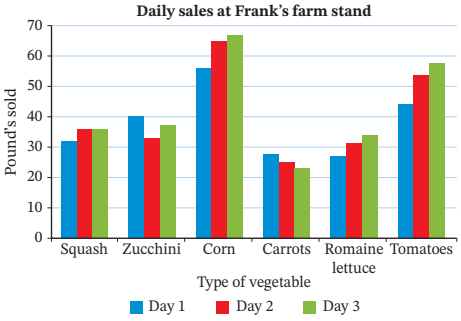
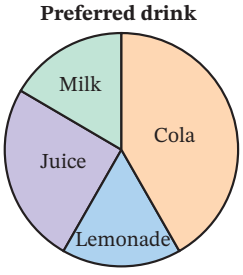
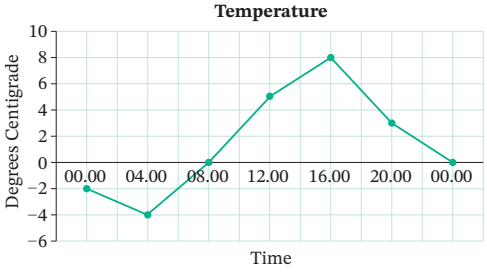
7.10

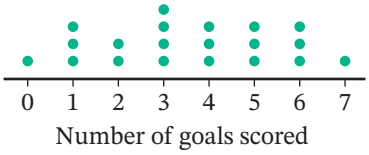
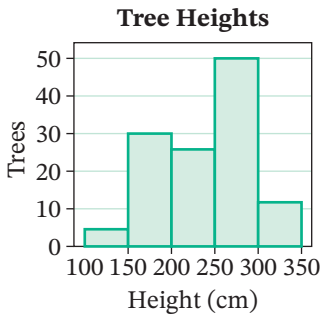
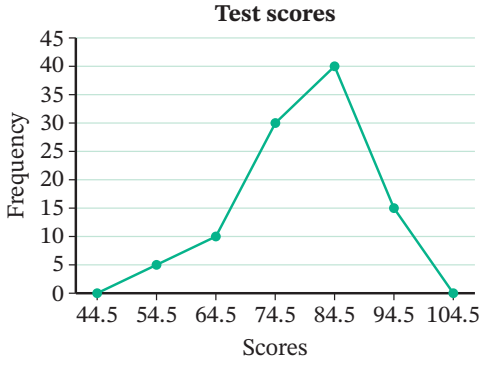
# Choosing the best graph 7.10

The best graph to use to represent a data set depends on the type of data being analysed. Here are some examples of the most appropriate graphs to use.

| Graph | Appropriate for | Description |
|---|---|---|
| **Pictogram**<br>Mode of transport: Bus — ☺☺☺☺☺☺☺<br>Car — ☺☺☺☺<br>Walking — ☺☺☺☺☺☺<br>Bicycle — ☺☺☺<br>**Key:** ☺ Represents 4 children | Simple data that doesn't require a lot of detail<br>Categorical data | • Uses pictures or symbols to represent frequencies<br>• Has a key to show what one picture represents<br>• Not precise |

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| Graph | Appropriate for | Description |
|---|---|---|
| **Column graph (or bar chart)**<br>  | Categorical data | • Uses columns to show frequency, gaps between columns<br>• Frequency shown on the vertical axis |
| **Clustered column graph**<br>  | Comparisons to be made between 2 or more data sets<br>Categorical data | • A key must be included to identify what the different colour bars represent |
| **Sector graph (or pie chart)**<br>  | Showing parts of a whole<br>Categorical data<br>Discrete numerical data | • Shows parts of a whole in circle form<br>• Uses sectors of a circle to show sizes of parts |
| **Divided bar graph**<br>  | Showing parts of a whole<br>Categorical data<br>Discrete numerical data | • Shows parts of a whole in rectangular form<br>• Uses sections of a rectangle to show sizes of parts |
| **Line graph**<br>  | Changes in a quantity over time<br>Numerical data | • Time on the horizontal axis, quantity on the vertical axis |

| Graph | Appropriate for | Description |
|---|---|---|
| **Dot plot**<br>  | Small data sets with low range and frequencies<br>Discrete numerical data | • A simplified column graph |
| **Stem-and-leaf plot**<br>**Stem** \| **Leaf**<br>0 \| 1 1 2 2 3 4 4 4 5 8<br>1 \| 0 0 0 1 1 3 7 9<br>2 \| 5 5 7 7 8 8 9 9<br>3 \| 0 1 1 1 2 2 2 4 5<br>4 \| 0 4 8 9<br>5 \| 1 6 7 7 8<br>6 \| 3 6<br>key: 6\|3 = 63 years old | Data sets with low frequencies<br>Discrete numerical data | • Lists every value in the data set |
| **Frequency histogram**<br>  | Large data sets with high frequencies<br>Numerical data | • No gaps between columns<br>• A space at the beginning and end of the graph<br>• Frequency is shown on the vertical axis |
| **Frequency polygon**<br>  | Large data sets with high frequencies<br>Numerical data | • Frequency is shown on the vertical axis<br>• Graph starts and finishes on the horizontal axis |

## Example 17

Identify what type of graph is appropriate to represent each data set.

**a** The number of coffees sold by a café on each day of the week

**b** The hourly temperature of a suburb over a day

### Solution

**a** Pictogram or column graph.

Days of week is categorical data and number of coffees sold will be frequency. A pictogram or column graph would easily show the differences in sales between the days.

**b** Line graph.

This is best to show a changing quantity over time.

**EXERCISE 7.10** ANSWERS ON P. 594

## Choosing the best graph U F R C

EXAMPLE 17

**1** What is the best graph to use for each data set? Justify your answer. R C

**a** Baby's length each month for the first 12 months

**b** Science test marks for all Year 8 students

**c** Minimum temperature each day for a fortnight in Coffs Harbour

**d** How students in your class travel to school

**e** Favourite sport watched on TV

**f** Heights of all students in your class

**g** Costs of manufacturing a car as a percentage of the car's price

**h** Number of households that own dogs in different streets

**i** Number of bedrooms per home in Mary Street

**2** A group of Year 8 students were surveyed on the make of car owned by their families. The results are shown in the table. R C

| Make of car | Frequency |
|---|---|
| Holden | 12 |
| Ford | 10 |
| Mitsubishi | 7 |
| Subaru | 5 |
| Toyota | 8 |
| Other | 3 |

**a** What is the best graph to use for this data?

**b** Draw the graph you have chosen.

**3** The Cairns Tourist Board wants to increase its tourist numbers in winter. It records the number of rainy days in June for a number of cities. It wants a graph that is simple, easy to read and attractive. R C

| City | Rainy days in June |
|---|---|
| Cairns | 10 |
| Melbourne | 18 |
| Adelaide | 24 |
| Sydney | 15 |
| Hobart | 17 |
| Perth | 16 |
| Brisbane | 15 |

**a** What is the best graph for this data?

**b** Draw the graph you have chosen.

**4** A survey was conducted about the colour of people's cars. The results are shown in the table below: R C

| Colour | Red | Blue | White | Green | Silver | Black |
|---|---|---|---|---|---|---|
| Frequency | 42 | 16 | 53 | 5 | 45 | 32 |

**a** What type of graph would you use to represent the results of this survey?

**b** Why is this an appropriate graph to use?

**c** Draw the graph you chose in part **a**.

**d** Is there a different type of graph that can be used to represent the same data set? Explain.

**5** The table below shows the sale of fruit (in kilograms) from a fruit shop over two days. R C

| Fruit | Monday | Tuesday |
|---|---|---|
| Apples | 10 | 15 |
| Bananas | 32.5 | 22.5 |
| Mangos | 17.5 | 25 |
| Oranges | 27.5 | 35 |
| Strawberries | 15 | 20 |

**a** What display would you use to represent the results of this survey?

**b** Why is this an appropriate display to use?

**6** This table shows the temperatures over 24 hours in Alice Springs for a November day. R C

| Time | Noon | 3 p.m. | 6 p.m. | 9 p.m. | Midnight | 3 a.m. | 6 a.m. | 9 a.m. | Noon |
|---|---|---|---|---|---|---|---|---|---|
| Temperature | 32°C | 35°C | 33°C | 29°C | 26°C | 24°C | 23°C | 29°C | 34°C |

**a** What is the best graph for this data?

**b** Draw the graph you have chosen.

# 7.11 The shape of data

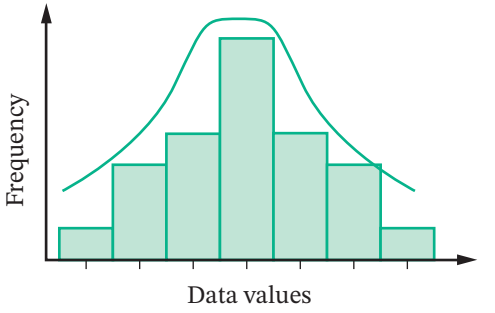
**Puzzle**
Statistical match-up

**Video**
The shape of a frequency distribution

A **statistical distribution** is the way the values of a data set are arranged, especially when graphed. When looking at histograms, dot plots and stem-and-leaf plots, an overall pattern can be seen from the shape of the display.

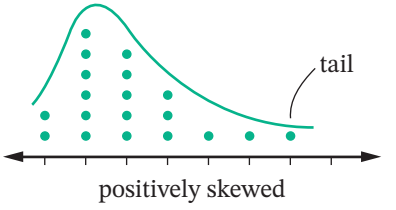
The shape of a statistical distribution shows how the data is spread and can be seen by drawing a curve around the graph or display.

A distribution is **symmetrical** if the data is evenly spread or balanced about the centre. For example, the frequency histogram and stem-and-leaf plot shown below are both symmetrical in shape. The most frequent values are the middle ones, and the least frequent values are the lower and higher ones.

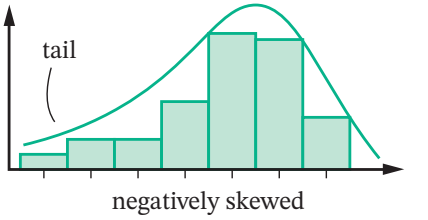



| Stem | Leaf |
|---|---|
| 2 | 6 |
| 3 | 3 7 9 |
| 4 | 5 8 |
| 5 | 0 2 3 3 7 8 |
| 6 | 0 4 |
| 7 | 6 7 8 |
| 8 | 9 |

A distribution is **skewed** if most of the data is bunched or clustered at one 'end' of the distribution and the other end has a 'tail', as shown on this dot plot and frequency histogram. The most frequent values are the lower ones for positively skewed and the higher ones for negatively skewed, and the least frequent values are at the opposite end, at the tail.




A distribution is **positively skewed** if its tail points to the right (the positive direction) and the data is bunched to the left. This could represent house prices in a country town, with mostly moderate prices and a few high prices for bigger houses and farms.

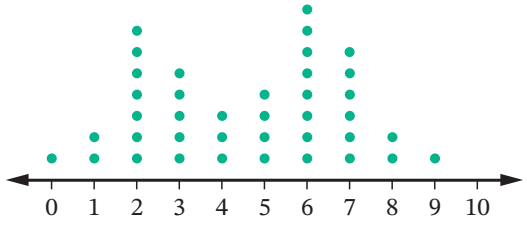



A distribution is **negatively skewed** if its tail points to the left (the negative direction) and the data is bunched to the right. This could represent the marks scored by students on an easy test, with many high marks and a few low marks.

A distribution is **unimodal** if it has one peak. All the above distributions are unimodal, and the peak indicates the mode, the data value with the highest frequency.

A distribution is **bimodal** if it has 2 peaks. The higher peak is the mode, while the other peak indicates another data value that has a high frequency.

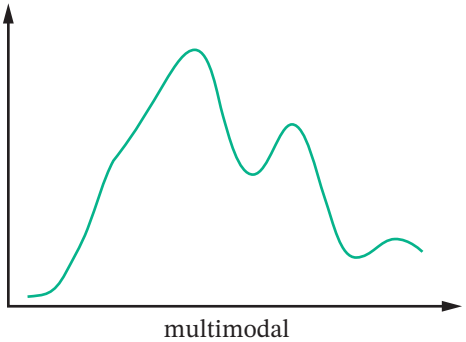
For example, this dot plot has peaks at 2 and 6 so it is bimodal. The mode, however, is 6.



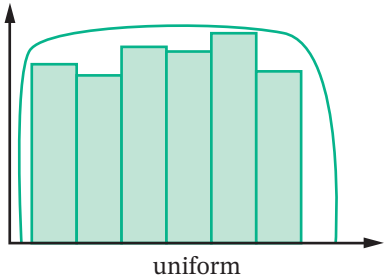

bimodal

A distribution is **multimodal** if it has 2 or more peaks (including a bimodal distribution).

A distribution is **uniform** if all the data values have roughly the same frequency. There are no peaks, and its shape is rectangular.



multimodal



uniform

The **mode** of a statistical distribution is data value that has the highest peak of the distribution.

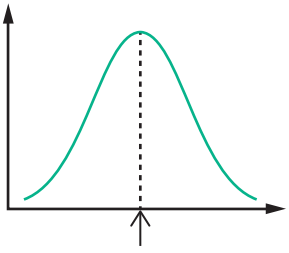
The **median** of a distribution is the data value that divides the area under the curve into half.

The **mean** of a distribution is most affected by outliers.

For a **symmetrical distribution**, the mean, median and mode are the same value.

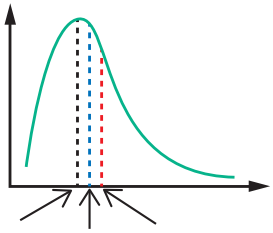
For a **skewed distribution**, the mean and median are pulled to the left or right so that the mean is further from the centre.

**symmetrical**



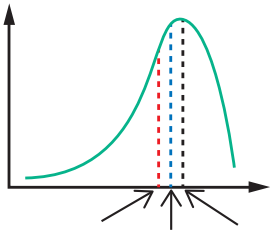
mean
median
mode

positively skewed



mode median mean

negatively skewed

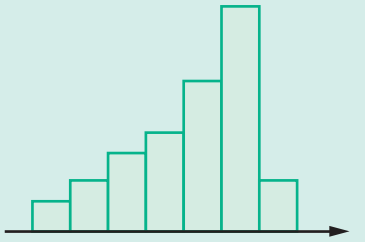


mean median mode

## Example 18

For each statistical distribution, describe the shape as symmetrical, positively skewed or negatively skewed.

**a**



**b**




**c**

| Stem | Leaf |
|---|---|
| 3 | 0 1 2 |
| 4 | 1 3 4 4 5 6 |
| 5 | 0 4 5 7 8 |
| 6 | 3 7 8 |
| 7 | 0 1 |
| 8 | 4 |
| 9 | 8 |

### Solution

**a** The shape is negatively skewed, tail points to the left.

**b** The shape is positively skewed, tail points to the right.

**c** The shape is positively skewed, tail points towards the higher values (down)

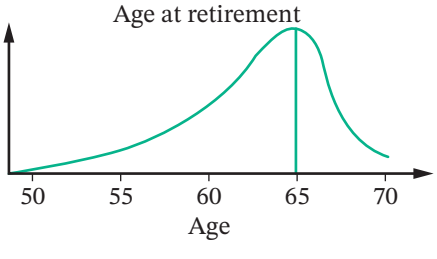
**EXERCISE 7.11** ANSWERS ON P. 595

## The shape of data UFRC

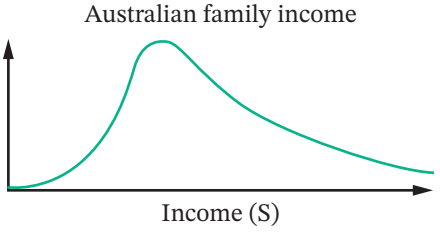
EXAMPLE 18

**1** What type of distribution is shown in each graph? Choose 2 from symmetrical, positively skewed, negatively skewed, unimodal and bimodal. **c**

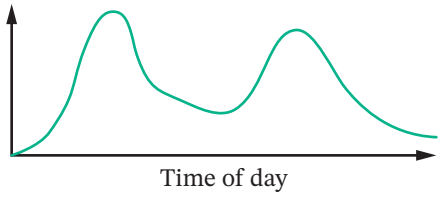
**a**



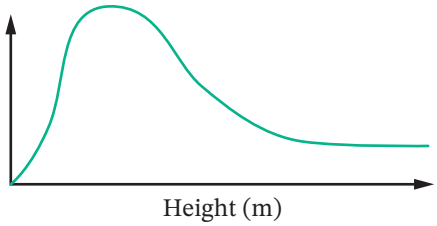

**b**



**c** Number of people waiting to catch a train




**d** Height of trees in a forest




**2** Which statement is true about this data set? Select the correct answer **A**, **B**, **C** or **D**.

**A** positively skewed and unimodal

**B** positively skewed and bimodal

**C** negatively skewed and unimodal

**D** negatively skewed and bimodal

| | Maths test results |
|---|---|
| 9 | 7 6 6 4 3 3 2 |
| 8 | 9 9 8 8 5 4 2 0 |
| 7 | 8 6 4 4 1 |
| 6 | 8 4 3 |
| 5 | 6 |

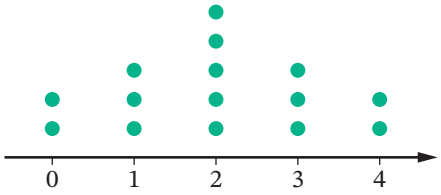
**3** The dot plot shown below shows the number of pets in each household in Lismore Ave. R C

**a** How many households were surveyed?

**b** Describe the shape of the data.

**c** What is the mode?

**d** Are the mean and median equal to the mode?




**4** The pulse rate (in beats/min) of the players of the Canberra football team is shown. R C

**a** Describe the shape of the data set.

**b** How many players are there in the team?

**c** Find the mean pulse rate for the team.

**d** Find the median pulse rate for the team.

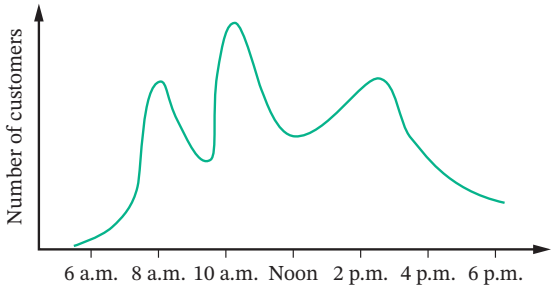
**e** Is the mean or median further away from the mode?

| Stem | Leaf |
|---|---|
| 5 | 0 8 |
| 6 | 4 8 |
| 7 | 0 4 6 |
| 8 | 2 2 2 6 6 6 |
| 9 | 0 0 0 0 0 0 2 4 |

**5** This 'trimodal' distribution represents the number of customers at a city business at different times of the day. R C

**a** Why is it called a trimodal distribution?

**b** What type of business do you think it could be? Give a reason for your answer.




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**6** This stem-and-leaf plot shows the exam marks of a group of History students. R C

| Stem | Leaf |
|---|---|
| 5 | 3 6 7 8 9 |
| 6 | 2 5 4 |
| 7 | 1 2 |
| 8 | 2 |
| 9 | 0 |

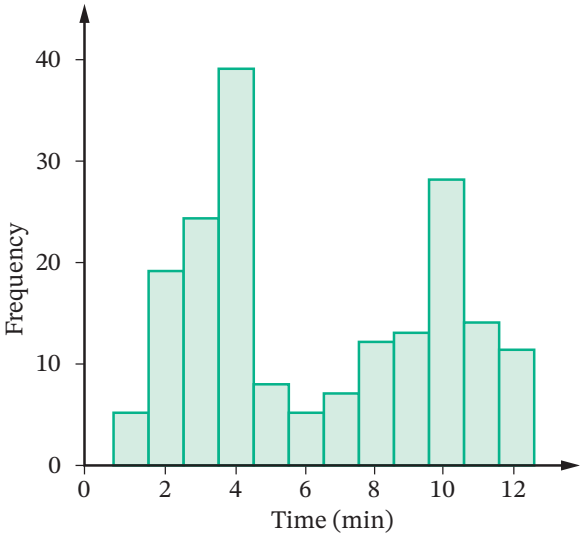
**a** Describe the shape of the distribution.

**b** Change 4 of the exam marks to make the distribution more like a symmetrical distribution. For example, you could change the 59 to 79.

**7** This histogram shows the amount of time customers at an appliance store had to wait for items to be brought in from the warehouse. R C

**a** Describe the shape of the distribution.

**b** Suggest a possible reason for the shape of the distribution.




**8** Sketch a graph to represent each distribution described. Remember to label the axes. R C

**a** The age at which a person dies is negatively skewed, with a mode of 80 years.

**b** The percentage marks students scored on a difficult exam has a bimodal, positively skewed distribution. A small group scored 90% but the majority scored around 40%.

**c** A bus carrying passengers to a concert includes a large group of dancers and a small group of rugby players. The distribution of the body massed (weights) of the passengers is bimodal.

**d** The heights of 2000 randomly-selected girls aged 17 are distributed symmetrically.

**e** When a die is rolled 100 times, the numbers that come up have a uniform distribution.

**f** The amount of time visitors spend at a museum exhibit that includes a 5-minute video is bimodal. Most visitors spend less than 1 minute looking at the exhibit, but some also watch the entire video and the exhibit. Very few people look at the display from 1 to 5 minutes.

# Comparing data 7.12

This table describes the 4 statistical measures and when it is more appropriate to use each one.

| Statistical measure | Features | When is it appropriate to use? |
|---|---|---|
| **Mean** | | |
| $\bar{x} = \frac{\text{sum of values}}{\text{number of values}}$<br>For a frequency table:<br>$\bar{x} = \frac{\text{sum of } fx}{\text{sum of } f}$ | • For numerical data<br>• Depends on all the values in the data set<br>• Affected by outliers<br>• Can be a decimal that is not one of the actual values | When the data set does not have many outliers. |
| **Mode** | | |
| Most common value(s) | • For numerical and categorical data<br>• There may be more than one mode, or no mode at all<br>• Not affected by outliers | When the most common value or category is needed. |
| **Median** | | |
| Middle value, or average of the 2 middle values, when values are arranged in order | • For numerical data<br>• Can be one of the values<br>• Not affected by outliers | When the data set has many outliers. |
| **Range** | | |
| Highest value – lowest value | • For numerical data<br>• Depends on the highest and lowest values only | When a measure of spread is needed. |

## Example 19

A group of 8 children was surveyed about the amount of pocket money (in dollars) they received each week.

20 32 32 40 18 32 18 48

**a** Find:

**i** the mean **ii** the mode **iii** the median **iv** the range

**b** If the last student's pocket money increased from \$48 to \$75, describe how this would affect:

**i** the mean **ii** the mode **iii** the median **iv** the range

**c** Which measure of centre best represents the new set of data?



**Worksheets**
Analysing data
Comparing word lengths
Investigating young drivers

**Technologys**
Population polygons
Daily rainfall

## Solution

**a** **i** Mean: $\bar{x} = \frac{\text{sum of values}}{\text{number of values}}$

$= \frac{20+32+32+40+18+32+18+48}{8}$

$= \frac{240}{8}$

$= \$30$

**ii** Mode = \$32

**iii** Rewriting the 8 values in order:

18 18 20 (32) (32) 32 40 48

There are 2 middle values, 32 and 32 (circled above).

$\text{Median} = \frac{32+32}{2}$

$= \$32$

**iv** Range $= 48 - 18$

$= \$30$

**b** If the last student's pocket money increased to \$75, this value would be a high outlier compared to the others.

**i** The mean should increase.

Mean: $\bar{x} = \frac{267}{8}$

$= \$33.375$

**ii** The mode will not change because \$32 is still the most common value.

**iii** The median will not change because the order of the values has not been affected.

18 18 20 (32) (32) 32 40 75

The 2 middle values are still 32 and 32 (circled above).

**iv** The range will increase because the highest value is now even higher.

Range $= 75 - 18$

$= \$57$

**c** The median is the best measure of centre because it is not affected by the outlier of \$75.

Note:

- The mode (\$32) is a good measure, but it only represents 3 of the values.
- The mean (\$33.375) is the least appropriate because it has been affected by the outlier and is now higher than most of the values.

**EXERCISE 7.12** ANSWERS ON P. 595

## Comparing data U F PS R C

In this exercise, round all mean answers to one decimal place where necessary.

EXAMPLE 19

7.12

**1** The distances (in kilometres) that a salesperson travelled each day for 10 days were:

49 77 63 56 49 40 56 40 74 49

**a** Find:

**i** the range **ii** the mode **iii** the median **iv** the mean.

**b** Which measure of centre best represents the data?

**2** The ages of the mathematics teachers at Westvale Secondary College are: R C

49 32 37 32 25 41 39

**a** Find:

**i** the range **ii** the mode **iii** the median **iv** the mean

**b** The 39-year-old teacher was replaced by a 22-year-old teacher. Without recalculating, describe how this will affect:

**i** the range **ii** the mode **iii** the median **iv** the mean

**3** In a swimming competition, each swimmer was graded A, B, C, D or E. One squad achieved the results shown. R C

| Grade | A | B | C | D | E |
|---|---|---|---|---|---|
| No. of swimmers | 4 | 9 | 8 | 6 | 3 |

**a** How many swimmers were in the squad?

**b** What is the mode?

**c** Why isn't it possible to find the mean or median of these grades?

**4** The annual incomes (× $1000) of 12 people are: R C

84 57 121 75 81 134 66 484 210 93 125 90

**a** Find (in dollars):

**i** the range **ii** the median **iii** the mean

**b** Why isn't the mean or mode an appropriate measure of centre for the incomes?

**c** Remove the outlier from the set of data and recalculate the mean and median. Are they closer together now?



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**5** The heights (in centimetres) of a team of basketball players are: R C

204 196 181 185 190 198 193

**a** Find:

**i** the mean **ii** the median **iii** the mode **iv** the range

**b** The 190 cm player is replaced by a 181 cm player. Without recalculating, describe how this will affect:

**i** the mean **ii** the median **iii** the mode **iv** the range

**6** The cricket batting scores for Sunjeet and Chris over six innings are shown. PS R C

Sunjeet: 29, 35, 40, 26, 28, 39 Chris: 2, 2, 3, 2, 188, 5

**a** What are the median scores for each player? Select the correct answer **A**, **B**, **C**, or **D**.

**A** Sunjeet 32, Chris 2 **B** Sunjeet 40, Chris 2.5

**C** Sunjeet 33, Chris 2.5 **D** Sunjeet 32, Chris 2.5

**b** Find the mean score for each player.

**c** Who is the better player if:

**i** the median is used? **ii** the mean is used?

**d** Who do you think is the better player? Explain your answer.

**7** Hardly Normal Discounts recorded the weekly number of sales of GPS navigators at 2 stores over a 20-week period. PS R C

*Forbes*: 44, 30, 54, 39, 63, 69, 75, 32, 80, 76, 90, 80, 55, 58, 71, 91, 36, 61, 65, 84

*Cobar*: 37, 36, 79, 77, 65, 47, 33, 49, 51, 60, 72, 49, 51, 73, 88, 31, 41, 97, 58, 30

**a** Display this information on a back-to-back stem-and-leaf plot.

**b** Calculate the mean, mode, median and range for each store.

**c** Which store performed better? Explain your answer.

**d** Which sales figure from the Cobar store is an outlier? Which of the measures of centre calculated in part **b** are most affected by this outlier?

**8** A netball coach kept a record of the number of goals scored by Christina and Mary during the first half of the season to compare their performances. PS R C

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Christina | 12 | 23 | 25 | 19 | 10 | 35 | 20 | 32 | 19 | 25 | 4 | 27 |
| Mary | 13 | 30 | 20 | 24 | 17 | 25 | 17 | 34 | 20 | 29 | 19 | 15 |

**a** Calculate the mean, mode, median and range for each player.

**b** Who do you think is the more consistent player? Explain.

**c** Who is the higher scorer?

**d** Who do you think is the better player? Why?

**e** What is the outlier value?

**f** If the outlier is removed, does it change:

**i** who the more consistent player is? **ii** who the higher scorer is?

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**9** 2 judges rated 8 contestants in a singing contest. R C

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Delta** | 8 | 5 | 6 | 8 | 5 | 9 | 2 | 6 |
| **Guy** | 7 | 8 | 5 | 4 | 9 | 5 | 5 | 6 |

**a** Draw a dot plot for each judge's marks.

**b** Calculate the range and median for each judge.

**c** Describe differences in the way each judge allocates marks.



## Power plus ANSWERS ON P. 595

**1** A small class achieved these marks in an exam:

66 68 74 76 82 79

**a** Find the mean of these marks, correct to one decimal place.

**b** The teacher realised there was an error in the marking and added 3 to each mark. Find the mean of the new marks.

**c** What effect did the extra 3 marks have on the mean?

**d** What effect does adding or subtracting the same number for all of the data have on the mean?

**2** Yelena sat 5 exams. Her average mark was 74%. What mark should Yelena obtain in her next exam if she wishes her average mark for the 6 exams to be 77%?

**3** If the mean of a set of values is 20, must 20 be one of the values? Give an example to illustrate your answer.

**4** 5 items of data were collected, but the figures were lost. The mean of the data was known to be 8 and the median was 9.

**a** What was the total of the data?

**b** What could the data have been?

**c** If you are told that the range is 7, what could the set of data have been?

**5** 4 sisters work at the same bank. One earns $500 per week, another earns $800 per week. The mean weekly wage of the 4 sisters is $2000. Is this possible? How?

# CHAPTER 7 REVIEW

## Language of maths

**Quiz**
Language of maths 7

**Puzzle**
Data crossword

| | | | |
|---|---|---|---|
| bias | categorical | census | cluster |
| continuous | discrete | dot plot | frequency histogram |
| frequency polygon | infographic | mean | measure of centre |
| median | mode | negatively skewed | nominal |
| numerical | ordinal | outlier | population |
| positively skewed | sample | stem-and-leaf plot | symmetrical |

**1** Name the **measures of centre**.

**2** What word means how often a value is listed in a set of data?

**3** What type of data is described by words rather than numbers?

**4** Is a **frequency polygon** a column graph or a line graph?

**5** What is a survey of the whole population called?

**6** What word describes a sample that misrepresents the population?

**7** List all the words that are types of graphs.

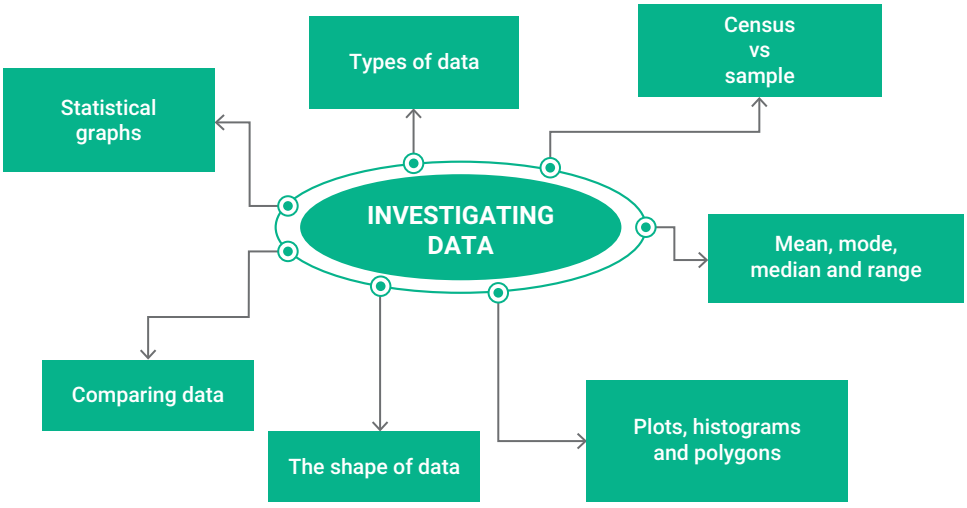
**8** List all the words that describe the shape of data.

## Topic summary

**Worksheet**
Mind map: Investigating data

- What did you find easy in this chapter? What don't you understand? Talk to your teacher.
- Give some examples of situations where statistics and measures of centre are used. Explain, where possible, which measure of centre is being used.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.





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# TEST YOURSELF 7

ANSWERS ON P. 595

Quiz
Test yourself 7



In this exercise, round all mean values to 2 decimal places where necessary.

**1** 20 families were asked how many computers they owned. 7.01

3 1 2 3 4 3 2 1 4 3
3 2 4 6 3 5 4 2 3 4

**a** Complete a frequency table for this data.
**b** What was the most frequent value?
**c** How many people owned 4 computers?
**d** What percentage of people owned one computer?
**e** Draw a sector graph of this data.

**2** Draw a pictogram for the average number of sunny days per month in Newcastle, shown below. 7.02

| J | F | M | A | M | J | J | A | S | O | N | D |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 12 | 13 | 13 | 17 | 13 | 16 | 18 | 18 | 16 | 13 | 15 |

**3** Classify each type of data below as categorical ordinal (CO), categorical nominal (CN), numerical discrete (ND) or numerical continuous (NC). 7.03

**a** life of a battery
**b** type of milk sold each day
**c** number of rooms in home
**d** volume of water in pools
**e** person's gender
**f** month of birthday

**4** Determine whether a sample or a census would be more appropriate for each survey below. 7.04

**a** Electing a Year 8 representative for student council.
**b** Investigating audience reaction to a new film.
**c** Testing the safety of a new car.
**d** Finding how many people live in Australia.

**5** A school of 1200 students wants to survey a sample of students to determine how often they use mobile phones. 7.04

**a** State a suitable way of choosing a representative sample of 50 students.
**b** Write one problem that may be encountered in conducting the survey.
**c** Explain how bias could be minimised.

**6** For each set of data, find: 7.05

**i** the mean **ii** the mode **iii** the median **iv** the range

**a** 6 12 11 12 10 6 6 10 6 7.06
**b** 8 4 1 1 4 1 3 6
**c** 20 21 23 24 25

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7.07

**7** The number of goals scored in each soccer match by the Southwest Strikers is shown in this frequency table.

| Score, $x$ | Frequency, $x$ | $fx$ |
|---|---|---|
| 0 | 3 | |
| 1 | 7 | |
| 2 | 6 | |
| 3 | 4 | |
| 4 | 2 | |
| 5 | 1 | |
| 6 | 1 | |
| Total | | |

**a** Copy and complete the frequency table.

**b** How many matches did the Strikers play?

**c** Find the range.

**d** Find the mode.

**e** Find the median.

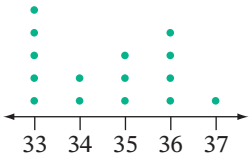
**f** Find the mean.

7.08

**8** For each plot below, find:

**i** the range **ii** the median **iii** the mode **iv** the mean

**a**




**b**

| Stem | Leaf |
|---|---|
| 12 | 3 5 6 |
| 13 | 1 1 |
| 14 | 0 8 8 8 8 |
| 15 | 7 8 |

7.09

**9** The daily number of people visiting a house that was for sale was:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 5 | 6 | 5 | 4 | 5 | 4 | 3 | 5 |
| 3 | 5 | 6 | 6 | 5 | 4 | 3 | 3 | 5 | 4 |
| 1 | 2 | 2 | 5 | 4 | 3 | 6 | 6 | 2 | 5 |

**a** Arrange this information into a frequency table.

**b** Draw a frequency histogram and frequency polygon.

**c** On how many days did the house receive more than 4 visitors?

**d** Find the mode.

**e** Find the median.

7.10

**10** **a** What type of graph is represented by parallel rectangles separated by gaps?

**b** Name the 2 types of graph used to illustrate parts of a whole.

**c** What type of graph uses an icon or symbol to represent a numerical value?

**d** What type of graph usually has Time on the horizontal axis?

**e** What type of data is represented on a column graph?

**f** What type of plot lists the actual values of a data set?

**g** What type of plot is useful for small sets of numerical data?

**h** What type of graph is represented by rectangles not separated by gaps?

**i** Name one type of graph that easily shows the shape of a distribution.

**j** What type of graph uses joined lines to show the frequencies of numerical data?

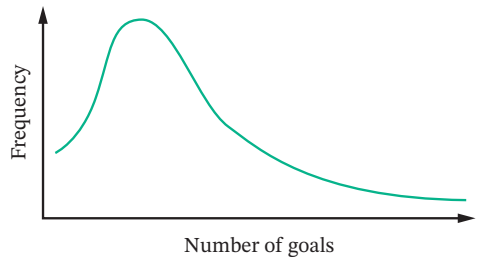
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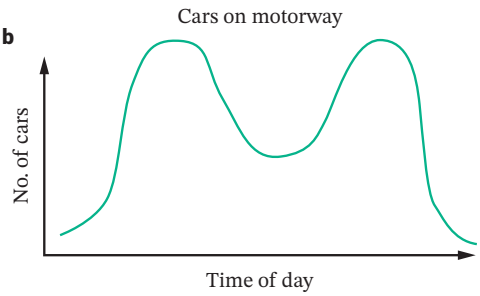
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**11** What shape of distribution is shown in each graph? Choose from symmetrical, positively skewed, negatively skewed, uniform and bimodal. 7.11

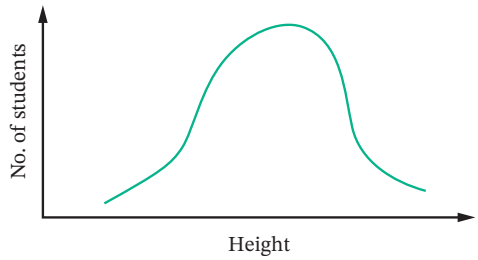
**a**




**b**




**c**




**12** Sketch a graph for each distribution described. Remember to label the axes. 7.11

**a** The percentage marks students scored on an easy test – the results are negatively skewed with most scoring between 80% and 90%.

**b** The masses of all 14-year-old girls in Australia are distributed symmetrically.

**c** When a playing die is rolled 500 times, the observed frequency of each value (1 to 6) has a uniform distribution.

**13** 21 Year 8 students were asked to find out the ages of their mothers. The results were: 7.12

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 30 | 35 | 35 | 35 | 36 | 36 | 36 | 36 | 36 | 37 | 37 |
| 37 | 37 | 38 | 39 | 39 | 41 | 43 | 43 | 45 | 45 | |

**a** Find the median.

**b** Find the mode.

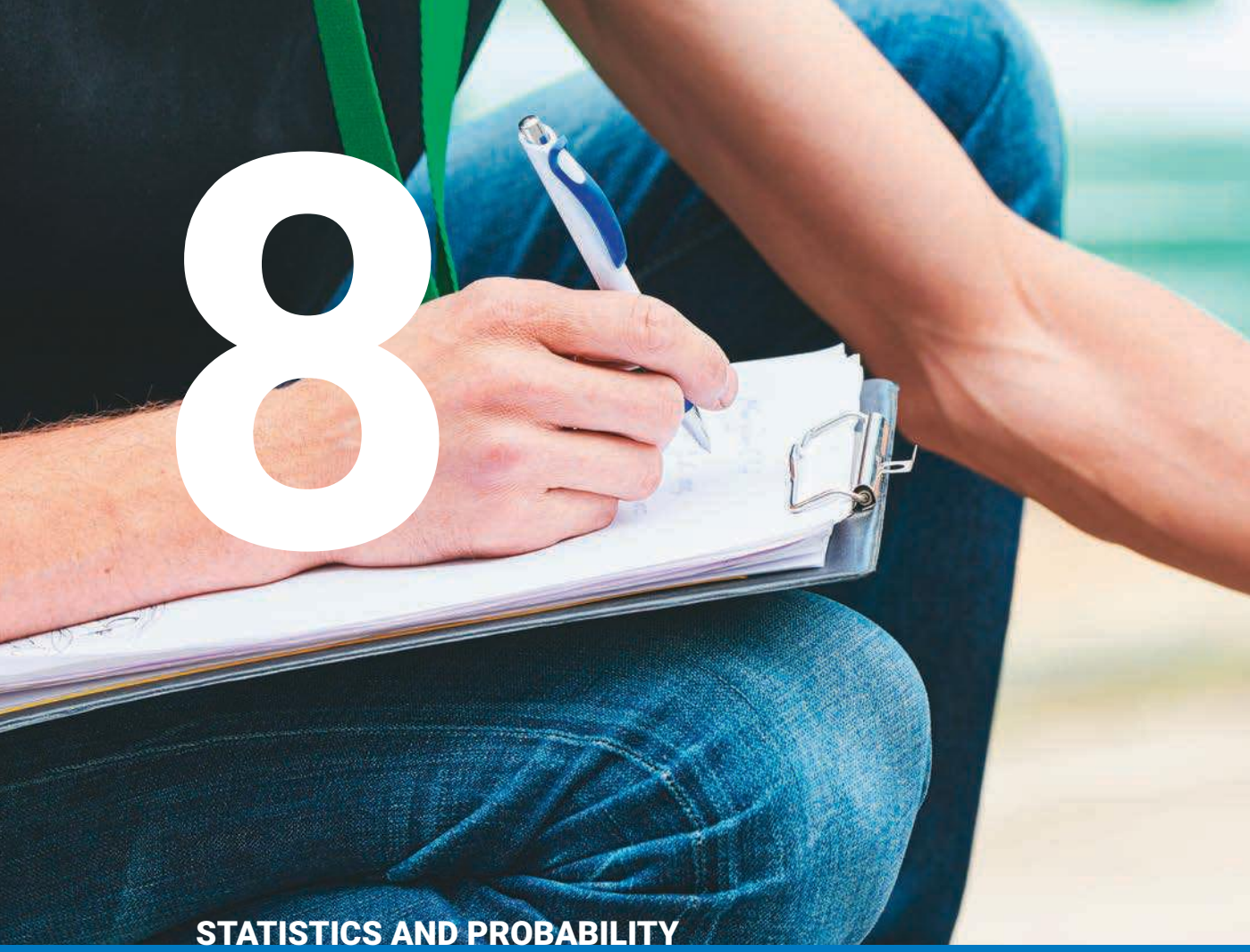
**c** Find the mean.

**d** Which measure of centre is the most appropriate for representing this data?

**e** Predict how many 39-year-old mothers there would be in a Year 8 group of 120 students.

**f** Suppose one of the ages is incorrect: one of the 39s should actually be 40. Without recalculating, describe how this will affect:

**i** the mean **ii** the median **iii** the mode **iv** the range



8

STATISTICS AND PROBABILITY

# PROBABILITY

Probability plays an important part in our daily lives. The study of chance has applications in weather forecasting, predicting disease, political campaigns, insurance and gambling. Insurance companies use probability to assess the risk of an event occuring. Statistics show that the age and sex of a driver affects the chances of a car accident, and the more distance a person drives, the higher the risk of an accident. These statistics determine how much drivers pay to insure their cars.




## Chapter outline

| | Working mathematically | | | | |
|---|---|---|---|---|---|
| **8.01** Probability | U | F | | R | C |
| **8.02** Complementary events | U | F | PS | R | C |
| **8.03** Venn diagrams* | U | F | PS | R | C |
| **8.04** Two-way tables* | U | F | | R | C |
| **8.05** Probability problems | U | F | PS | R | C |
| **8.06** Observed and expected frequencies | U | F | | R | C |

*YEAR 9 EXTENSION

## Wordbank

**at least** Referring to the smallest number, for example, 'at least 2' means 2, 3, 4, ..., that is, '2 or more'

**complementary event** The 'opposite' event (for example, the complementary event to selecting an Ace from a deck of cards is *not* selecting an Ace)

**Quiz**
Wordbank 8

**expected frequency** The expected number of times an event will occur over repeated trials

**mutually exclusive events** Events or categories that have no items in common

**observed frequency** The actual number of times an event occurred over repeated trials

**trial** One go or run of a repeated probability experiment, for example, one roll of a die

**Venn diagram** A diagram that uses circles (usually overlapping) to group items into categories

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

## Videos (8):

**SkillCheck** Probability
**8.01** Probability of an event
**8.02** Complementary events
**8.03** Venn diagrams 1 • Venn diagrams 2
**8.04** Two-way tables
**8.06** Experimental probability • Observed and expected frequencies

## *Twig* videos (2):

**Skillcheck** Probability: Irrational fears
**8.01** Venn diagrams: Global habitats

## *PhET* interactive (1):

**8.06** Plinko probability

## Quizzes (5):

- Wordbank 8
- SkillCheck 8
- Mental skills 8
- Language of maths 8
- Test yourself 8

## Skillsheets (2):

**SkillCheck** The language of chance
**8.01** Sample space

## Worksheets (19):

**SkillCheck** Chance cards • Describing probability
**8.01** Basic probability • Games of chance • Probability problems
**8.02** Theoretical probabilities
**8.03** Venn diagrams
**8.04** Two-way tables • Two-way probability tables
**8.05** Probability review
**8.06** Relative frequencies • Coins probability • Dice probability • Spinning chance • A page of spinners • Experimental probabilities • Probability review • Coin toss experiment

**Mind map** Probability

## Puzzles (12):

**8.01** Greedy Pig game • Spinner game • Basic probability • Theoretical probabilities
**8.02** Complementary events
**8.03** Venn diagrams group clues • Venn diagrams • Venn diagrams matching activity
**8.04** Two-way probability tables
**8.05** Matching probabilities
**8.06** Experimental probability • Duelling dice

**Language of maths** Probability crossword

## Technology (1):

**8.06** Simulating a spinner

## Presentations (3):

**8.03** Venn diagrams 1 • Venn diagrams 2
**8.06** Theoretical probability

## Spreadsheets (3):

**8.06** Coin-tossing simulator • Die-rolling simulator • Spinner simulator

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

## In this chapter you will:

- use the sample space to calculate the probability of an event
- solve problems involving complementary events
- (EXTENSION, YEAR 9 PATH) use Venn diagrams and two-way tables to represent sample spaces and events to solve probability problems
- (EXTENSION, YEAR 9 PATH) identify 2 or more events that are mutually exclusive or overlapping
- (EXTENSION, YEAR 9 PATH) describe events using words such as ‘and’, exclusive ‘or’, inclusive ‘or’ and ‘at least’
- compare the observed frequency of an event with its expected frequency

## SkillCheck ANSWERS ON P. 596

**1** Convert each number into a decimal.

**a** $\frac{17}{20}$ **b** $\frac{3}{8}$ **c** 2% **d** 30%

**2** List all the possible outcomes for each situation.

**a** tossing a coin **b** rolling a die

**c** the gender of a baby **d** the colour of traffic lights

**e** the result of a football game **f** the result of a driving test

**3** Convert each number into a percentage.

**a** 0.7 **b** 0.56 **c** $\frac{2}{3}$ **d** $\frac{6}{25}$

**4** Evaluate each expression.

**a** $1-\frac{1}{3}$ **b** $1-\frac{2}{5}$ **c** $1-\frac{3}{8}$

**5** Which term best describes the chance that the next baby born in Australia is a girl? Select the correct answer **A**, **B**, **C** or **D**.

**A** certain **B** definite **C** even chance **D** probable

**6** Convert each number into a simplified fraction.

**a** 0.28 **b** 0.02 **c** 64% **d** 80%

**7** Is the chance of each event greater than or less than $\frac{1}{2}$?

**a** You being home by 4 p.m. this afternoon

**b** You winning Lotto one day

**c** You listening to the radio today

**d** You learning to drive next year

**e** You going interstate this year

**f** You sending a text message on your phone today

Quiz
SkillCheck 8

Videos
Probability

Probability: Irrational fears

Skillsheet
The language of chance

Worksheets
Chance cards

Describing probability



# Probability 8.01

In probability (chance) situations, the set of all possible outcomes is called the **sample space**. For example, if a coin is tossed, the sample space is {heads, tails}. If each outcome has an equal chance, then we say that each outcome is **equally likely**.

An **event** consists of one or more outcomes. For example, the event that a baby is born in a summer month consists of the outcomes {December, January, February}. An event has the symbol $E$, and we can calculate its probability as a fraction.

Skillsheet
Sample space

Video
Probability of an event

## Probability of an event

$P(E)$ means 'the probability of an event, $E$ (occurring)'.
If all possible outcomes are **equally likely**, then:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

$$\text{or } P(E) = \frac{\text{number of outcomes matching } E}{\text{number of outcomes in the sample space}}$$

A **favourable outcome** is one of the outcomes in the event that you want, whose probability you are calculating.

The **probability** of an event can be written as a fraction, decimal or percentage. For example, the probability of tossing tails on a coin can be written as $\frac{1}{2}$, 0.5 or 50%.

Worksheets
Basic probability
Games of chance
Probability problems

Puzzles
Greedy Pig game
Spinner game
Basic probability
Theoretical probabilities

## Example 1

**a** Write the sample space for this spinner.

**b** Is each outcome equally likely?

**c** Find the probability that the spinner lands on red.

**d** Find the probability that the spinner lands on a 'traffic light' colour.



### Solution

**a** The sample space is {red, yellow, green, blue}.

**b** Each coloured region is equal in size $\left(\frac{1}{4} \text{ of the circle}\right)$, so each outcome is equally likely.

**c** $P(\text{red}) = \frac{1}{4}$ — One chance in 4

**d** $P(\text{traffic light colour}) = P(\text{red or yellow or green})$ — 3 favourable outcomes out of 4

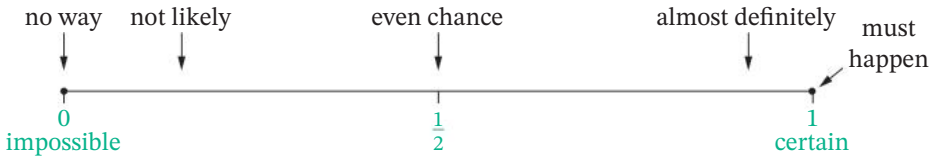
$$= \frac{3}{4}$$

## The language of probability

| Probability term | In Example 1 |
|---|---|
| An **experiment** (or **chance experiment**) is a situation involving chance that leads to results called outcomes. | Spinning a spinner |
| A **trial** is one go or run of the experiment. | One spin of the spinner |
| An **outcome** is the result of an experiment. | The arrow landing on red |
| The **sample space** is the set of all possible outcomes. | {red, yellow, green, blue} |
| An **event** is one or more outcomes of an experiment. | The arrow landing on a 'traffic light' colour: red, yellow or green |
| In a **random** experiment, every possible outcome has the same chance of occurring. | All spins on this spinner are random because every colour has the same chance. |

# The range of probability

Because probability is a fraction, its value must range from 0 to 1, or as a percentage, from 0% to 100%.




8.01

## Example 2

A die is rolled. Find the probability that the number rolled is:

**a** divisible by 3 **b** a factor of 6

**c** less than 7 (answer as a percentage) **d** **at least** 4 (answer as a decimal)

### Solution

There are 6 possible outcomes when a die is rolled: {1, 2, 3, 4, 5, 6}

**a** $P(\text{divisible by } 3) = \frac{2}{6} = \frac{1}{3}$ — 2 numbers {3 and 6}

**b** $P(\text{a factor of } 6) = \frac{4}{6} = \frac{2}{3}$ — 4 numbers {1, 2, 3, 6}

**c** $P(\text{less than } 7) = \frac{6}{6} = 100\%$ — All outcomes are less than 7: a certain event

**d** $P(\text{at least } 4) = \frac{3}{6} = \frac{1}{2} = 0.5$ — 3 numbers {4, 5, 6}.

'At least 4' means the smallest number is 4; that is, 4 or more.

**EXERCISE 8.01** ANSWERS ON P. 596

## Probability U F R C

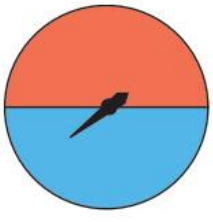
EXAMPLE 1

**1** For each spinner:

**i** write down the sample space C

**ii** for one spin, calculate the probability that the spinner lands on red.

**a**



**b**



**c**



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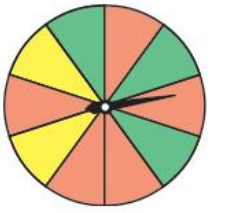
**d**



**e**



**f**



**2** For each experiment, count the number of possible outcomes and state whether each outcome is equally likely. C

- **a** tossing a coin
- **b** the result of a rugby match when Australia plays New Zealand
- **c** the first letter of a person's name
- **d** the gender of a baby
- **e** the last digit of a car number plate
- **f** the result of a driving test

**3** List the outcomes in each event. C

- **a** rolling an odd number on a die
- **b** selecting a vowel from the letters of the alphabet
- **c** having a house number greater than 4 but less than 10
- **d** having a birthday in a month beginning with A
- **e** living in a state of Australia
- **f** being in a high school grade

EXAMPLE 2

**4** A money box contains 4 $2 coins, 3 $1 coins, 2 50c coins, 6 20c coins and 5 10c coins. It is shaken and one coin falls out at random. Calculate:

- **a** $P$(50c coin)
- **b** $P$($1 coin) (answer as a decimal)
- **c** $P$(10c or 20c coin), as a percentage
- **d** $P$(not a 10c coin)
- **e** $P$(gold coin), as a decimal
- **f** $P$(a coin under $1), as a percentage

**5** A jar of lollies contains 5 red, 3 green, 6 yellow and 2 blue lollies. Taylor selects one lolly from the jar at random. Calculate each probability.



- **a** $P$(green)
- **b** $P$(yellow or blue)
- **c** $P$(traffic light colour), as a decimal
- **d** $P$(not yellow), as a percentage

**6** What is the probability that a person randomly chosen has a birthday in a month beginning with the letter J? Select the correct answer **A**, **B**, **C** or **D**. R

**A** $\frac{1}{12}$ **B** $\frac{1}{6}$ **C** $\frac{1}{4}$ **D** $\frac{1}{3}$

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**7** NSW is playing Queensland in a rugby league match. R C

**a** List the sample space for the outcomes of the match.

**b** Are all outcomes equally likely? Explain your answer.

**8** A packet of jelly beans has 4 yellow, 3 red, 6 green and 3 black jelly beans remaining. You tip the packet and one jelly bean rolls out at random. What is the probability that it is not a red jelly bean? Select **A**, **B**, **C** or **D**.

**A** $\frac{4}{12}$ **B** $\frac{3}{16}$ **C** $\frac{3}{13}$ **D** $\frac{13}{16}$

**9** Malik is holding 12 playing cards, as shown. His friend Lara picks a card without looking and calls out the number on the card. R C



**a** How many 5s are there in the 12 cards?

**b** List all the possible numbers that Lara could pick.

**c** Copy and complete this table.

| Outcome | 5 | 6 | 7 | 8 |
|---|---|---|---|---|
| Probability as a fraction | $\frac{1}{4}$ | | | |
| Probability as a decimal | 0.25 | | | |
| Probability as a percentage | 25% | | | |

**10** Elena has 2 $5 notes, 4 $10 notes and 3 $20 notes in her wallet. If she selects one note at random, what is the probability that it is a $20 note? Select **A**, **B**, **C** or **D**.

**A** $\frac{3}{35}$ **B** $\frac{60}{110}$ **C** $\frac{1}{3}$ **D** $\frac{20}{35}$

**11** Match each probability value to its correct description. C

**a** $\frac{1}{2}$ **b** 0 **c** 90% **d** 1

**e** $\frac{3}{4}$ **f** 0.1 **g** 0.6 **h** 2%

**A** cannot happen **B** better than average chance **C** even chance

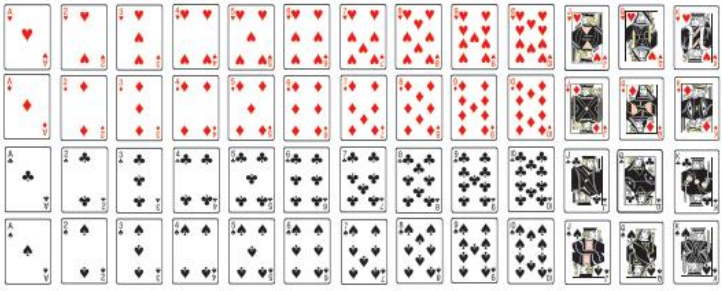
**D** good chance **E** very likely **F** almost impossible

**G** slim chance **H** must happen

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**12** The 52 cards in a standard deck of playing cards are shown below, divided evenly into 4 suits: hearts, diamonds, clubs, and spades.

Hearts:
Diamonds:
Clubs:
Spades:



The cards are shuffled and one is taken out at random.

**a** How many outcomes are in the sample space?

**b** Is each outcome equally likely?

**c** How many Aces are in the deck?

**d** How many hearts cards are there?

**e** How many red 7s are there?

**f** Find the probability of selecting:

**i** a red card **ii** a clubs card **iii** the King of spades

**iv** a Queen **v** a card with an even number **vi** a black picture card

**13** The weather forecaster says that the probability of a rainy day in April is 30%.

**a** Is this a high probability or a low probability?

**b** About how many rainy days are expected in April?

**14** Your maths teacher calls out a name randomly from your class roll.
What is the probability that it is:

**a** your name? **b** a girl's name?

**c** someone aged 14? **d** someone with brown hair?

**15** An Esky contains 8 cans of lemonade, 5 cans of orange drink and 2 cans of lime drink.
How many cans of cola must be added so that the probability of randomly selecting: R

**a** a lemonade is 50%? **b** an orange drink is 0.25?

**c** a lime drink is $\frac{1}{12}$? **d** a cola can is $\frac{2}{5}$?

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## Investigation

### Complementary events

In everyday life, we use the word 'complementary' to describe things that go together and 'complete the picture' when they are together. For example, when dressing for an occasion:

- a shirt and a matching tie complement each other
- a dress and a matching pair of shoes complement each other

Remember also that 'complementary' angles add to 90°.

In probability, **complementary events** are events that together make up all the possible outcomes.

| Event | Complementary event |
|---|---|
| Tossing a tail on a coin | Tossing a head on a coin |
| Rolling a 6 on a die | Rolling any of the other numbers, from 1 to 5, on a die |
| Raining | Not raining |
| Being born on a Monday | Being born on a day other than Monday |

**1** Suppose that there is an equal chance of being born on any day of the week: Monday to Sunday.

**a** What is $P(\text{Tues})$, the probability of being born on a Tuesday?

**b** What is $P(\text{not Tues})$, the probability of being born on a day other than Tuesday?

**c** What do you notice about $P(\text{Tues}) + P(\text{not Tues})$?

**2** A fruit bowl contains 7 apples, 4 oranges and 9 bananas. One piece of fruit is selected at random from the bowl.

**a** Find $P(\text{orange})$.

**b** Find $P(\text{not orange})$.

**c** What do you notice about $P(\text{orange}) + P(\text{not orange})$?

**3** A baby is selected at random from the maternity section of a large hospital. There is an equal chance of the baby being a boy or a girl.

**a** Find $P(\text{boy})$.

**b** Find $P(\text{girl})$.

**c** What do you notice about $P(\text{boy}) + P(\text{girl})$?

**4** Copy and complete the following sentence.

The probability of an event _____ the probability of its complementary event must always equal _____.

## Did you know?

### Two-up

Two-up is an Australian coin-tossing game. The game was very popular with soldiers during World War I and it is only legal to play it on Anzac Day (25 April). Two-up involves the tossing of 2 coins and betting on 'heads' or 'tails' on both coins.



AAP Photo/AP/Rick Rycroft

Heads: both coins land heads up.

Tails: both coins land tails up.

Odds: one head and one tail.

There is no winner if 'odds' are thrown and the coins are tossed again.

**Write down the sample space for this game.**

**What is the probability of throwing 'odds'?**

# 8.02 Complementary events

**Puzzle**
Complementary events

**Worksheets**
Theoretical probabilities

In any situation, the probabilities of all possible outcomes must add to 1.

**Complementary events** are 2 events that together make up all the possible outcomes, such as a **head** and a **tail** when tossing a coin. The **complement** of an event $E$, are all those outcomes that are *not* $E$, or that are the 'opposite' of $E$.

Because an event and its complement cover all possible outcomes, the sum of their probabilities must equal 1.

## Complementary events

$P(E) + P(\text{not } E) = 1$

or $P(\text{not } E) = 1 - P(E)$

or $P(\text{complementary event}) = 1 - P(\text{event})$

or $P(\text{event not occurring}) = 1 - P(\text{event occurring})$

If the probability is written in percentage form, then $P(\text{not } E) = 100\% - P(E)$

**Video**
Complementary events

## Example 3

A pack of 20 cards contains 10 red, 6 yellow and 4 green cards. One card is drawn from the pack at random. Find the probability that this card is:

**a** yellow **b** not yellow **c** not green

## Solution

**a** $P(\text{yellow}) = \frac{6}{20} = \frac{3}{10}$

**b** $P(\text{not yellow}) = 1 - P(\text{yellow})$

$= 1 - \frac{3}{10}$

$= \frac{7}{10}$

The complement of 'yellow' is 'not yellow', which is 'red or green'

Note that $P(\text{yellow}) + P(\text{not yellow}) = \frac{3}{10} + \frac{7}{10} = 1$, which covers all possible outcomes.

**c** $P(\text{green}) = \frac{4}{20} = \frac{1}{5}$

The complement of 'green' is 'not green', which is 'red or white'

$P(\text{not green}) = 1 - P(\text{green})$

$= 1 - \frac{1}{5}$

$= \frac{4}{5}$

Note that $P(\text{green}) + P(\text{not green}) = \frac{1}{5} + \frac{4}{5} = 1$, which covers all possible outcomes.



## EXERCISE 8.02 ANSWERS ON P. 597

### Complementary events U F PS R C

**1** Write the complement of each event. C

**a** Tossing tails on a coin
**b** Cloudy day tomorrow
**c** Selecting a white chocolate from a box containing white and brown chocolates
**d** Rolling a 6 on a die
**e** Winning a game of hockey
**f** Selecting a hearts card from a deck of cards
**g** Being born in winter
**h** Being under 15 years old

EXAMPLE 3

**2** A pile of 11 cards is placed on a desk. There are 6 red cards and the rest are blue. If one card is selected at random, what is the probability that it is not red? Select the correct answer **A**, **B**, **C** or **D**.

**A** $\frac{5}{11}$ **B** $\frac{6}{11}$ **C** $\frac{6}{5}$ **D** $\frac{5}{6}$

**3** A die is rolled. What is the probability that the result is:

**a** a 3?
**b** not a 3?
**c** not even?
**d** not prime?
**e** at least 2?
**f** at most 4?

**4** There are 6 books on a shelf: 3 Mathematics, 2 History and 1 Sport. If one book is chosen at random, what is the probability of selecting:

**a** a History book?
**b** a book that is not History?
**c** a book that is not Sport?
**d** a book that is not Science?

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**5** A car park has 450 cars, 100 motor bikes and 10 buses. One of them is selected at random.

**a** What is the probability that it is a car?

**b** What is the probability that it is not a car? Select **A**, **B**, **C** or **D**.

**A** $\frac{45}{56}$ **B** $\frac{110}{450}$ **C** $\frac{10}{560}$ **D** $\frac{11}{56}$

**6** What is the probability that your maths teacher was born in a month:

**a** beginning with the letter M?

**b** that does not begin with the letter M?

**7** What is the decimal probability that a mobile phone number selected at random does not end in 0 or 1?

**8** Tahnee buys 5 tickets in a raffle in which 1000 tickets are sold and there is only one prize. What is the probability of Tahnee not winning the prize?

**9** A jar contains 40 red, 25 blue, 50 black and 15 white jelly beans. One jelly bean is selected at random. What is the probability that it is:

**a** white? **b** not white? **c** not yellow?

**d** not blue or black? **e** not red? **f** not black or white?

**10** Write the probability of the event that is **complementary** to each event. C

**a** The probability of choosing a Jack from a pack of cards is $\frac{1}{13}$.

**b** The chance of shooting a basketball hoop is 61%.

**c** The probability of winning a prize is 0.15.

**11** In a bag of toy cars there are only 3 colours: red, blue and white. If you take out a car at random, the chance of it being red is 0.4, and the chance of it being white is 0.25. R

**a** What is the chance of selecting 'red or white'?

**b** What is the chance of the car you select being blue?

**c** If the bag holds 40 cars, how many of each colour would you expect to find?

**d** What is the chance of the car you select being pink?

**12** 4 students, Sue, Liam, Emily and Matt, write their names on cards and place the cards in a bag. A card is chosen, without looking, to select the class captain.

**a** Find the probability that Emily was not chosen.

**b** Find the probability that the captain is a boy.

**c** What is the chance that the captain is not a boy?

**d** What is the chance that the captain is the teacher?

**13** The probability that it will rain this weekend is 85%. What is the probability that it won't rain?

**14** Which of the following is the complementary event to 'winning a race'?
Select **A**, **B**, **C** or **D**. R C

**A** coming last **B** coming second or third

**C** not winning the race **D** coming second

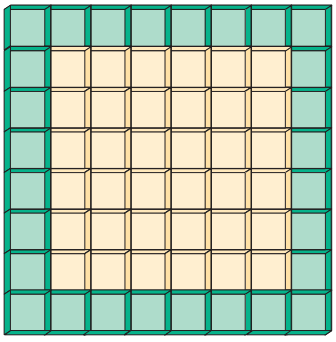
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**15** A game involves throwing a coin into a grid of squares. For a player to win, the coin must land in a yellow square. If the coin lands outside the grid or between squares, the throw is not counted and the player has another try.

For each throw, what is the probability of:

**a** not winning?

**b** winning?



**16** The letters of the word PROBABILITY are written on separate cards and one is selected randomly. What is the probability that a letter drawn out is:

**a** not P? **b** not a vowel? **c** not I? **d** not A or B?

**17** In a family there are Mum and Dad, 2 daughters and a son. Each day, they take it in turns to wash the dishes after dinner. If you visit the family, what is the chance that the person washing up is:

**a** male? **b** not a parent? **c** not Mum? **d** not male?

**18** A random 4-digit number is to be formed from the digits 1, 3, 5 and 8. What is the probability that the number will: PS R

**a** start with the digit 5? **b** be an odd number? **c** be greater than 5000?



# Extension: Venn diagrams

8.03

A **Venn diagram** is a diagram that uses circles (usually overlapping) to group items into categories. A rectangle represents the whole group while the circles represent categories. Items common to 2 or more categories are placed in the intersection (overlapping region) of the circles. The Venn diagram was invented in 1880 by English mathematician and priest, John Venn (1834–1923).



Alamy Stock Photo/GI Archive

YEAR 9 PATH

**Videos**
Venn diagrams 1

Venn diagrams: Global habitats

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YEAR 9 PATH

**Worksheet**
Venn diagrams

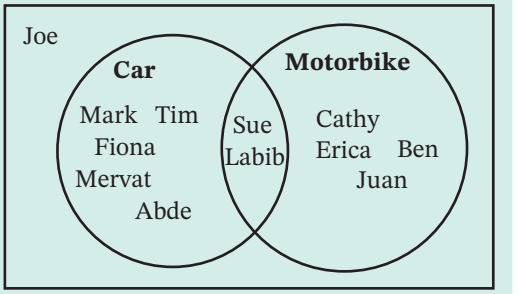
**Puzzles**
Venn diagrams group clues

Venn diagrams

Venn diagrams matching activity

## Example 4

A group of people were surveyed on the type of vehicle they drove, and the results are shown on the Venn diagram.




**a** How many people were surveyed?

**b** How many people drive a car?

**c** How many people drive a car or ride a motorbike?

**d** Who drives a car and rides a motorbike?

**e** Who only rides a motorbike?

**f** Why is Joe outside of the circles?

### Solution

**a** 12 people were surveyed.

**b** 7 people drive a car. The people within the Car circle.

**c** 11 people drive a car or ride a motorbike. All the people within the circles.

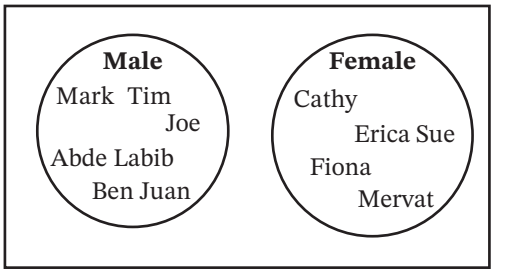
**d** Sue and Labib can drive a car and ride a bike. The people in the intersection of the circles.

**e** Cathy, Erica, Ben and Juan can only ride a bike.

**f** Joe does not drive a car or ride a bike.

## Mutually exclusive vs overlapping events

Sometimes, 2 categories must be represented on a Venn diagram as 2 separate circles because it is not possible for them to overlap. Here is an example:




In this case, it is not possible to be male *and* female. This is an example of a **mutually exclusive event**. Mutual means 'shared feature' and exclusive means 'belonging to one group only' (such as an exclusive party or an exclusive news story).

However, most Venn diagrams (as in Examples **4** and **5**) describe **overlapping events**, or **non-mutually exclusive events**.

## 'And' vs 'or'

For 2 categories or events A and B, the phrase **'A and B'** means to have both of them occurring together. For example, 'to drive a car' **and** 'to ride a motorbike' in Example **4** means to do both things.

If A and B are **overlapping**, the phrase **'A or B'** means to have A or B or both. For example, 'to drive a car' **or** 'to ride a motorbike' in Example **4** means to drive a car only, or to ride a motorbike only, or to do both. In this case, 'A or B' actually **includes** 'A and B' so this is an example of an **inclusive** 'or'.

If A and B are **mutually exclusive**, the phrase **'A or B'** means to have A only or B only (but not both). For example, 'male' **or** 'female' means to be male, or female, but not both. In this case, 'A or B' **excludes** 'A and B' so this is an example of an **exclusive** 'or'.

YEAR 9 PATH

8.03

Video
Venn diagrams 2

Presentations
Venn diagrams 1

Venn diagrams 2

## Example 5

30 students in a class were surveyed on how they relaxed after school.

- 18 play on their computer
- 15 play sport
- 6 play sports and play on their computer

**a** Show this information on a Venn diagram.

**b** If one student is selected at random from this class, find the probability that the student:

**i** plays sport, but not on their computer

**ii** plays sport or on their computer, but not both

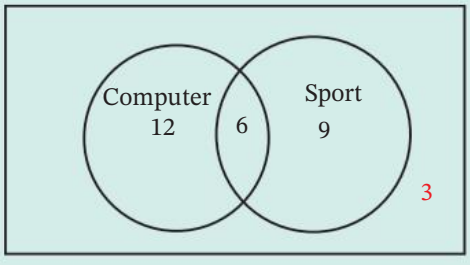


iStock.com/skynesher

### Solution

**a** When drawing Venn diagrams always begin with the intersection.

- 6 students belong to both groups
- 18 play on the computer but 6 have already been counted, so $18 - 6 = 12$ play on the computer only
- 15 play sport but 6 have already been counted, so $15 - 6 = 9$ play sport only
- $12 + 6 + 9 = 27$, but there are 30 students in the class, which means that 3 students do not belong to either group




**b** **i** $P(\text{plays sport but not computer}) = \frac{9}{30} = \frac{3}{10}$

**ii** $P(\text{plays sport or computer but not both}) = \frac{12+9}{30} = \frac{21}{30} = \frac{7}{10}$

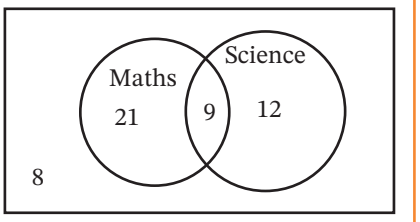
YEAR 9 PATH

## EXERCISE 8.03 ANSWERS ON P. 597

## Venn diagrams U F PS R C

EXAMPLE 4

**1** A group of students were asked whether they liked Maths or Science at school. The Venn diagram shows the results of this survey. How many students: R C




**a** were surveyed?

**b** liked Maths but not Science?

**c** liked Maths or Science?

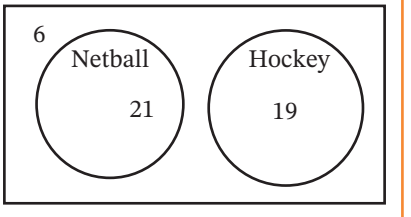
**d** liked Maths or Science but not both?

**e** did not like either subject?

**f** liked Maths and Science? Select the correct answer **A**, **B**, **C** or **D**.

**A** 9 **B** 21 **C** 33 **D** 42

**2** This Venn diagram shows the sports played by a number of female students. R C




**a** Are netball and hockey mutually exclusive or not?

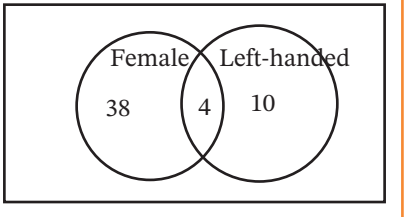
**b** How many students play neither netball nor hockey?

**c** How many students play netball or hockey?

**d** Find the total number of students surveyed.

**e** How many students play netball and hockey?

**3** This Venn diagram compares the categories Female and Left-handed. R C




**a** Is Female and Left-handed mutually exclusive or not?

**b** Find the total number of people represented in the diagram.

**c** How many left-handed females are there?

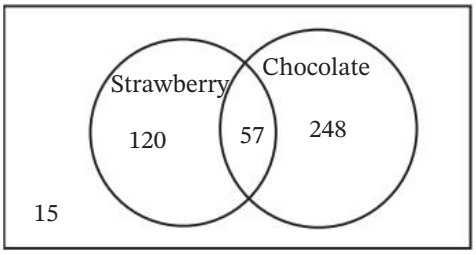
**d** What is the decimal probability (correct to 3 decimal places) that a person randomly chosen from this group is:

**i** female? **ii** female but not left-handed?

**iii** female or left-handed? **iv** a left-handed male?

**e** Describe the type of person that would be represented outside the 2 circles on the Venn diagram.

**4** A survey was carried out by an ice-cream shop to decide whether chocolate or strawberry was the more popular flavour. R C






iStock.com/Barcin

**a** How many people were surveyed?

**b** How many people liked strawberry or chocolate, but not both?

**c** How many people liked neither strawberry nor chocolate?

**d** If a person is randomly chosen from the survey, what is the probability that the person likes both chocolate and strawberry? Select **A**, **B**, **C** or **D**.

**A** $\frac{57}{440}$ **B** $\frac{368}{440}$ **C** $\frac{425}{440}$ **D** $\frac{57}{425}$

**5** The Tourism Council surveyed 130 people to find whether they prefer New South Wales or Queensland as a holiday destination. R C

EXAMPLE 5

NSW 44 Queensland 66 NSW and Queensland 20

**a** Construct a Venn diagram to represent the results.

**b** How many people like NSW or Queensland?

**c** How many people like NSW or Queensland, but not both?

**d** What is the probability that a person likes NSW exclusively?

**e** What is the probability that a person does not like NSW or Queensland?

**6** **a** Draw a Venn diagram representing the eye colours of this group of students. R C

| | |
|---|---|
| Blue eyes | 32 |
| Brown eyes | 38 |
| Neither blue nor brown eyes | 10 |

**b** Are these groups mutually exclusive or not? Why?

**c** How many students are in the group?

**d** How many students have blue or brown eyes?

**e** How many students have blue and brown eyes?

**f** What is the percentage probability that a student chosen at random from this group does not have brown eyes?



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YEAR 9 PATH

**7** A retailer surveyed the first 30 customers to see what they bought. PS R C

20 bought milk  13 bought bread  1 bought neither milk nor bread

**a** Show this information on a Venn diagram.

**b** How many customers bought milk and bread?

**c** How many customers bought milk only?

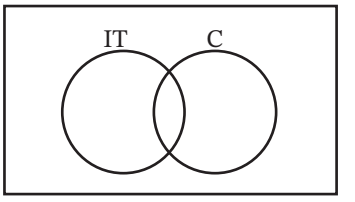
**d** What is the probability that a customer randomly chosen:

**i** bought bread only?  **ii** did not buy milk?

**iii** bought milk or bread?  **iii** bought milk or bread, but not both?

**8** There is a class of 30 students, all of whom study Information Technology or Chemistry. 7 of them study both subjects, and 20 study Information Technology. PS R C

**a** Copy and complete the Venn diagram to find how many study Chemistry.




**b** If one student is chosen at random, what is the probability that the student studies Chemistry?

# 8.04 Two-way tables

YEAR 9 PATH

Worksheets
Two-way tables

Two-way probability tables

Puzzle
Two-way probability tables

Video
Two-way tables

A **two-way table** is another way of grouping items into overlapping categories, especially when there are many overlaps that cannot be represented by Venn diagrams easily.

## Example 6

50 students were surveyed on whether they preferred dogs or cats more as pets. The results were sorted into this two-way table.

| | Preferred pet | |
|---|---|---|
| | Dog | Cat |
| Boys | 12 | 4 |
| Girls | 11 | 15 |

**a** How many students do not like dogs or cats?

**b** How many students like cats?

**c** How many boys like cats?

**d** How many students:

**i** like cats or are boys?

**ii** like cats or are boys, but not both?

**e** What is the probability that a student selected at random from the survey likes dogs?

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## Solution

**a** $12 + 4 + 11 + 15 = 42$ Adding the values in the table.

Students who do not like dogs or cats $= 50 - 42$ 50 students in survey.

$= 8$

**b** Students who like cats $= 4 + 15$

$= 19$

**c** Boys who like cats $= 4$

**d** **i** Students who like cats or who are boys $= 12 + 4 + 15$

$= 31$

**ii** Students who like cats or who are boys but not both $= 12 + 15$

$= 27$

**e** Students who like dogs $= 12 + 11$

$= 23$

Probability of selecting a student who likes dogs $= \frac{23}{50}$

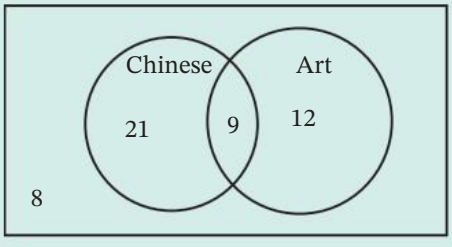


Shutterstock.com/Eric Isselee

## Example 7

A group of 50 Year 9 students were grouped in a Venn diagram according to whether they took Chinese or Art as an elective subject.

Represent this information in a two-way table.




## Solution

From the Venn diagram:

- 9 students take Chinese and Art
- 21 students take Chinese, but not Art
- 12 students take Art, but not Chinese
- 8 students do not take Chinese or Art

| | Chinese | Not Chinese | |
|---|---|---|---|
| Art | 9 | 12 | 21 |
| Not Art | 21 | 8 | 29 |
| | 30 | 20 | 50 |

YEAR 9 PATH

## EXERCISE 8.04 ANSWERS ON P. 597

# Two-way tables U F R C

EXAMPLE 6

**1** A primary school class was surveyed on whether its students could swim. The results are shown below. R C

| | Can swim | Cannot swim |
|---|---|---|
| Boys | 13 | 2 |
| Girls | 9 | 3 |

**a** How many students are in the class?

**b** How many students are boys or cannot swim?

**c** How many students are boys and cannot swim?

**d** What is the probability that a student randomly selected from this class is a girl?

**e** What is the probability that a student selected at random is:

**i** a non-swimmer? **ii** a girl who can swim?

**2** The players of a soccer club were divided into groups according to their age and weight. R C

| | Heavy | Light |
|---|---|---|
| Junior | 64 | 96 |
| Senior | 144 | 32 |

**a** How many players does the club have?

**b** How many players are juniors or light?

**c** How many players are juniors or light, but not both?

**d** What is the probability that a player selected at random is:

**i** a senior? **ii** a junior and heavy? **iii** a senior or light?

**3** This incomplete table describes the audience watching a movie at a cinema. R C

| | Under 18 | Over 18 | Total |
|---|---|---|---|
| Female | | 142 | 198 |
| Male | 45 | | |
| Total | | | 344 |

**a** Copy and complete the table.

**b** How many males were in the audience?

**c** How many under 18 females were there?

**d** If a person is selected at random from the audience, what is the probability that the person:

**i** is male and over 18? **ii** is male or over 18?

**iii** is male or over 18, but not both? **iv** is over 18?

**4** A group of children were asked whether they liked carrots. R C

| | Likes carrots | Dislikes carrots | Total |
|---|---|---|---|
| Boys | 75 | | 200 |
| Girls | | 40 | |
| Total | 145 | | |

**a** Copy and complete the table.

**b** How many children dislike carrots?



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**c** What is the probability that a child randomly selected is:

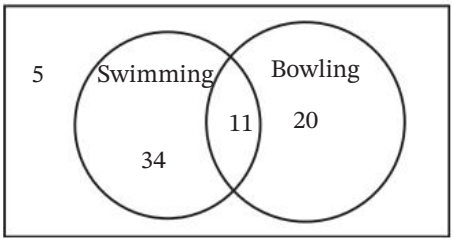
**i** a girl? **ii** a boy or dislikes carrots? **iii** a boy and dislikes carrots?

**d** What is the chance that a child randomly selected is a girl and likes carrots? Select **A**, **B**, **C** or **D**.

**A** $\frac{29}{62}$ **B** $\frac{11}{31}$ **C** 1 **D** $\frac{7}{31}$

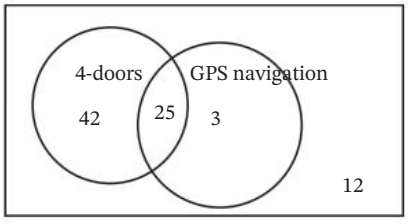


**5** The Venn diagram below shows the number of students who participate in swimming or bowling regularly. Copy and complete the two-way table for this data. R C




| | Swimming | Not swimming | Total |
|---|---|---|---|
| Bowling | | | |
| Not bowling | | | |
| Total | | | |

**6** The Venn diagram compares 2 features of the cars on display at Rusty Motors. R C




**a** Copy and complete the two-way table.

| | GPS navigation | No GPS navigation | Total |
|---|---|---|---|
| 4-doors | | | |
| Not 4-doors | | | |
| Total | | | |

**b** A car is chosen at random from the car yard. What is the probability that it has:

**i** GPS navigation? **ii** 4 doors?

**iii** 4 doors and GPS navigation? **iv** no GPS navigation?

**7** A group of university students were tested to see if they needed glasses. The table shows the results. R C

| | Needs glasses | Does not need glasses | Total |
|---|---|---|---|
| Male | 22 | | 82 |
| Female | | 85 | 98 |
| Total | | 145 | |

**a** Copy and complete the table.

**b** If a student is chosen randomly from this group, what is the probability that the student:

**i** is female? **ii** needs glasses?

**iii** is female and needs glasses? **iv** is male or does not need glasses?

**v** is female or needs glasses?

**vi** is male or does not need glasses, but not both?

# Technology

## Rolling a die

In this activity, you need a scientific calculator or spreadsheet to simulate the rolling of a die.

**1** Copy this table.

| Outcome | Frequency |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |



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**2** Random numbers from 1 to 6 can be generated on a calculator or spreadsheet using the RanInt, RANDOM or =RAND() functions.

| Casio scientific calculator | Sharp scientific calculator |
|---|---|
| **Enter** the formula RanInt#(1,6) as shown:<br>ALPHA · 1 SHIFT ) 6 ) = | 2nd F 7 (RANDOM), select 1 R-DICE, = |

Press = more times to generate more random integers from 1 to 6.

On a spreadsheet, type **=INT(RAND()*6+1)** into a cell and press the Enter key, then the F9 key repeatedly for more random numbers.

**3** Repeat the simulation 20 times and record the results in your table.

**4** Are certain numbers more likely to be rolled than others? For example, is a 2 more likely to be rolled than a 5? Do your results reflect this?

**5** Compare your results with the simulated results of other members in your class. Are they similar or different? Are they what you and your classmates expected? Discuss.

# Mental skills 8: Maths without calculators ANSWERS ON P. 598

Quiz
Mental skills 8



## Multiplying or dividing by a multiple of 10

**1** Study each example.

**a** $4 \times 700 = 4 \times 7 \times 100 = 28 \times 100 = 2800$

**b** $5 \times 60 = 5 \times 6 \times 10 = 30 \times 10 = 300$

**c** $12 \times 40 = 12 \times 4 \times 10 = 48 \times 10 = 480$

**d** $3.2 \times 30 = 3.2 \times 3 \times 10 = 9.6 \times 10 = 96$ (by estimation, $3 \times 30 = 90 \approx 96$)

**e** $4.5 \times 50 = 4.5 \times 5 \times 10 = 22.5 \times 10 = 225$ (by estimation, $5 \times 50 = 250 \approx 225$)

**f** $9.4 \times 200 = 9.4 \times 2 \times 100 = 18.8 \times 100 = 1880$
(by estimation, $9 \times 200 = 1800 \approx 1880$)

**2** Now evaluate each product.

| | | | | | |
|---|---|---|---|---|---|
| **a** | $8 \times 2000$ | **b** | $3 \times 70$ | **c** | $11 \times 900$ |
| **d** | $2 \times 300$ | **e** | $4 \times 4000$ | **f** | $5 \times 80$ |
| **g** | $7 \times 70$ | **h** | $1.3 \times 40$ | **i** | $2.5 \times 600$ |
| **j** | $6.8 \times 200$ | **k** | $3.9 \times 50$ | **l** | $4.4 \times 4000$ |

**3** Study each example.

**a** $8000 \div 400 = 80\cancel{00} \div 1\cancel{00} \div 4 = 80 \div 4 = 20$

**b** $200 \div 50 = 20\cancel{0} \div 1\cancel{0} \div 5 = 20 \div 5 = 4$

**c** $6000 \div 20 = 600\cancel{0} \div 1\cancel{0} \div 2 = 600 \div 2 = 300$

**d** $282 \div 30 = 282 \div 10 \div 3 = 28.2 \div 3 = 9.4$

**e** $3520 \div 40 = 352\cancel{0} \div 1\cancel{0} \div 4 = 352 \div 4 = 88$

**f** $8940 \div 2\cancel{00} = 8940 \div 1\cancel{00} \div 2 = 89.4 \div 2 = 44.7$

**4** Now evaluate each quotient.

| | | | | | |
|---|---|---|---|---|---|
| **a** | $560 \div 70$ | **b** | $2500 \div 50$ | **c** | $3200 \div 400$ |
| **d** | $440 \div 20$ | **e** | $160 \div 40$ | **f** | $1500 \div 30$ |
| **g** | $450 \div 50$ | **h** | $744 \div 80$ | **i** | $2550 \div 300$ |
| **j** | $846 \div 200$ | **k** | $576 \div 60$ | **l** | $2040 \div 50$ |

# 8.05 Probability problems

Worksheet
Probability review

Puzzle
Matching probabilities

The following exercise involves all the probability ideas that have been covered so far.

**EXERCISE 8.05** ANSWERS ON P. 598

## Probability problems U F PS R C

**1** There were 28 students on the school trip to Japan: 17 were born in Australia, 3 in Italy, 4 in Vietnam, 1 in Japan and 3 in China. One student was chosen at random.
Find the probability that the student was:

**a** born in Australia **b** born in Asia **c** not born in Vietnam.

**2** The 7 letters of the word SUCCESS are written on separate cards. A card is selected at random and the letter noted. R C

**a** List the sample space.

**b** Is each letter equally likely to be selected? Explain.

**c** Find the chance that the letter chosen:

**i** is an S **ii** is a vowel

**iii** is not a C **iv** is also a letter of the word FAIL.

**3** Alex selects one sock at random from a bag containing 2 black, 2 blue and 2 red socks. R C

**a** List the sample space.

**b** Find the probability that Alex selects:

**i** a blue sock **ii** a black sock **iii** a pink sock

**c** What is the complementary event to choosing a red sock? What is the probability of this event?

**4** A spinner is evenly divided into 5 sections numbered 1, 2, 3, 4 and 5. For one spin, find the probability, as a percentage, that it lands on:

**a** 2 **b** an even number

**c** a number less than 5 **d** a number at most 5

**5** Sophia bought 4 tickets in a lottery in which there were 100 000 tickets. What is the probability that she won first prize? Select the correct answer **A**, **B**, **C** or **D**.

**A** 0.000 0025 **B** 0.000 25 **C** 0.04 **D** 0.000 04

**6** One ball is selected at random from a barrel of balls numbered 1 to 100.
Find the probability that the number shown on the ball is:

**a** 12 **b** greater than 40 **c** even **d** at most 85

**7** Stathis flipped a coin 7 times and a tail showed each time. What is the chance of a tail showing on the next toss? R

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**8** A computer selected a random number from 1 to 15 inclusive. Find the probability that the number is:

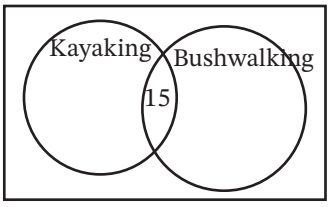
**a** odd
**b** not prime
**c** a multiple of 3 or 5
**d** a factor of 12

**9** The probability that a carton of eggs contains any broken eggs is 0.1. Find the probability that a carton contains no broken eggs.

**10** In a tennis tournament there are 60 players. Of these, 34 are from NSW, 20 are from Victoria and 6 are from Queensland. Find the probability that a player selected at random from the tournament is: R C

**a** from NSW
**b** from Queensland
**c** not from Victoria
**d** from Queensland or Victoria.

**11** 40 students at a school camp can select kayaking or bushwalking as an activity, or both. A total of 35 students chose kayaking, 20 chose bushwalking, including some who chose both. All students chose an activity. PS R C




**a** Copy and complete the Venn diagram representing these students.

**b** What is the probability that a student, chosen at random:

**i** chose both activities?
**ii** did not choose bushwalking?

**12** A sample of students from a sports high school were surveyed on whether they participated in hockey or judo.
The results are shown in this two-way table. R C

| | Judo | Not Judo | Total |
|---|---|---|---|
| Hockey | 128 | | 360 |
| Not hockey | 100 | 40 | |
| Total | | | |

**a** Copy and complete the table.

**b** What is the probability that a student chosen randomly:

**i** does judo?
**ii** doesn't do hockey?
**iii** does judo, but not hockey?
**iv** does hockey and judo?
**v** does hockey or judo?
**vi** does hockey or judo, but not both?

**13** In a bag there are lollies of 3 different colours: red, green and purple. The chance of taking out a red lolly at random is 0.5, the chance of it being green is 0.2. PS R C

**a** What is the chance of the lolly being purple?

**b** If the bag holds 10 lollies, how many of each colour would you expect to find?

**c** If a second bag had 100 lollies, how many of each would you expect to find?

**d** Which bag would give you a better chance of picking a red lolly? Explain your answer.

# 8.06 Observed and expected frequencies

**Worksheets**
Relative frequencies
Coins probability
Dice probability
Spinning chance
A page of spinners
Experimental probabilities
Probability review

**Interactive**
Plinko probability

**Puzzles**
Experimental probability
Duelling dice

**Presentation**
Theoretical probability

**Videos**
Experimental probability
Observed and expected frequencies

Probability calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

is more specifically called **theoretical probability** or **expected probability**.

We can also determine the probability of an event based on the results of an experiment or trial that has been repeated many times, such as the safety testing of different cars, or rely on past statistics, such as the number of rainy days in April. This type of probability is called **observed probability** or **experimental probability**, which is based on **relative frequency**, the number of times an event occurred as a fraction of the total frequency of outcomes.

## Relative frequency

$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

or $$P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

As in statistics, 'frequency' means the number of times something happens.

## Observed and expected frequencies

The expected number of times an event will occur over repeated trials is called the **expected frequency**.

Expected frequency = theoretical probability × number of trials

The actual number of times an event occurs over repeated trials in an experiment is called the **observed frequency**.

## Example 8

Declan rolled a die 60 times and recorded the results in a table.

| Outcome | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Frequency | 10 | 11 | 13 | 7 | 8 | 11 |

**a** What is the theoretical probability of rolling a 5 or 6 on a die?

**b** For 60 rolls, what is the expected frequency of rolling a 5 or 6? How does this compare with the observed frequency?

**c** What is the observed probability of rolling a 5 or 6?

## Solution

**a** $P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$

**b** Expected frequency of 5s or 6s $= \frac{1}{3} \times 60$ Probability × number of trials
$= 20$

From the table, the observed frequency of 5s and 6s = 8 + 11 = 19, which is close to the expected frequency, 20.

**c** Observed $P(5 \text{ or } 6) = \frac{8+11}{60} = \frac{19}{60}$

**EXERCISE 8.06** ANSWERS ON P. 598

## Observed and expected frequencies U F R C

EXAMPLE 8

**1** Harper rolled a biased die 60 times and the number 2 came up 15 times.

**a** What is the observed probability of rolling a 2?

**b** If the same die was rolled 500 times, what is the expected frequency of rolling a 2?

**2** **a** Copy this table.

| Outcome | Tally | Frequency |
|---|---|---|
| Head | | |
| Tail | | |
| | Total | |

**b** Flip a coin 50 times and record the results in the table.

**c** Find, as a decimal, the observed probability of flipping:

**i** a head **ii** a tail

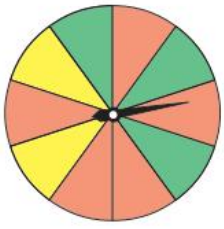
**d** Find, as a decimal, the theoretical probability of flipping:

**i** a head **ii** a tail

**e** How do the observed probabilities compare with the theoretical probabilities? C

**3** Vinh spun this spinner 80 times and found the following results:

| Outcome | Red | Green | Yellow |
|---|---|---|---|
| Frequency | 44 | 25 | 11 |



**a** What is the theoretical probability of spinning red?

**b** For 80 spins, what is the expected frequency of spinning red? How does this compare with the observed frequency? C

**c** What is the observed probability of spinning red?

**d** What is the theoretical probability of spinning yellow?

**e** Find the expected frequency of spinning yellow over 80 spins and compare this with the observed frequency. C

**f** What is the observed probability of spinning yellow?

Foundation Standard Complex

**4** The matches in a sample of matchboxes were counted. The number of matches in each box are recorded below. R

| Number of matches | 48 | 49 | 50 | 51 | 52 | 53 |
|---|---|---|---|---|---|---|
| Number of matchboxes | 3 | 6 | 20 | 16 | 4 | 1 |

**a** How many matchboxes were tested?

**b** What is the observed probability of finding 51 matches in a box?

**c** What is the observed probability of finding fewer than 50 matches in a box?

**d** In 1000 matchboxes, how many matchboxes would you expect to contain:

**i** exactly 50 matches? **ii** at least 50 matches?

**5** **a** Copy this table. R C

| Outcome | Tally | Frequency |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| | Total | |

**b** Roll a die 72 times and record the results in the table.

**c** Find the expected probability of rolling:

**i** 4 **ii** a number greater than 2

**iii** an even number

**d** When rolling a die 72 times, what is the expected frequency of rolling:

**i** 4? **ii** a number greater than 2? **iii** an even number?

**e** How do your expected frequencies compare with the observed frequencies from the table?

**f** Find the observed probability of rolling:

**i** 4 **ii** a number greater than 2 **iii** an even number

**g** If a die was rolled 1000 times, how many times should 4 come up:

**i** based on the expected probability?

**ii** based on the observed probability?

**6** Sefina tossed a coin many times and got 135 heads and 115 tails.
Calculate, as a percentage, the observed probability of tails with this coin.

**7** A die was rolled 80 times, with the results shown below. R C

| Outcome | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Frequency | 11 | 13 | 9 | 13 | 12 | 22 |

**a** Is each outcome equally likely?

**b** Do you think this die is biased (unfair)? Give a reason for your answer.

**c** Write, as a percentage, the observed probability of rolling a 1 on this die.

**d** If this die was rolled 100 times, what is the expected frequency of 3 coming up?

**8** A pair of dice was rolled 50 times and their sum calculated each time.

| Sum | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency | 0 | 2 | 4 | 6 | 5 | 5 | 9 | 6 | 8 | 3 | 2 |

**a** Find, as a decimal, the observed probability of rolling a sum:

**i** of 11

**ii** of 5 or 6

**iii** that is odd

**iv** of 6 or more

**v** of at most 6

**vi** that is a composite number

**b** Which sum(s):

**i** was most likely?

**ii** had a probability of $\frac{3}{25}$?

**iii** was least likely?

**iv** had a probability of 16%?

**v** was second-most likely?

**vi** had a probability of $\frac{1}{10}$?

## Technology

### Tossing a coin

In this activity, a spreadsheet is used to simulate the tossing of a coin. The computer can quickly generate either a 1 (representing heads) or a 2 (tails). We will use the command RAND to generate these random numbers.

1 In cell A1 enter the label 'Tossing a coin'.

2 In cell A2, enter the formula **=INT(RAND()*2+1)**. Press Enter and either 1 or 2 should appear randomly in the cell.

3 Select cell A2 and **Fill Down** to cell A11 to generate random numbers (1 or 2), to simulate the tossing of a coin 10 times.

4 Your results will probably be different from those of other students in the class. Compare.

5 Copy the table below and in the first blank row record your results for Trials 1–10 (the numbers of heads and tails in your first 10 'tosses').

| Trial | Number of heads | Number of tails |
|---|---|---|
| 1–10 | | |
| 11–20 | | |
| 21–30 | | |
| 31–40 | | |
| 41–50 | | |
| 51–60 | | |
| 61–70 | | |
| 71–80 | | |
| 81–90 | | |
| 91–100 | | |
| Total | | |

8.06

**Worksheet** Coin toss experiment

**Spreadsheets** Coin tossing simulator

Die rolling simulator

Spinner simulator

**Technology** Simulating a spinner

**Interactive** Plinko probability

Foundation | Standard | Complex

6 Simulate another 10 tosses of the coin by making the spreadsheet generate another set of random numbers. Do this by placing the cursor between the tops of columns A and B (so that it turns into a 'double arrow') and clicking.

7 Enter the results for Trials 11–20 in your table.

8 Repeat 8 more times so that you have 100 tosses of the coin recorded in the table.

9 **a** For 100 tosses of a coin, what is the expected frequency of heads based on the theoretical probability?

**b** Compare this with the observed frequency.

10 Calculate, as a decimal, the theoretical and observed probabilities of tossing heads.

11 Compare your results with those of other students in your class. Briefly explain any differences and why they may have occurred.

12 Combine the results of students in your class to calculate the observed probability of tossing heads, as a decimal. How does this compare with the theoretical probability?

## Investigation

### Family with 4 children

A **simulation** is an experiment that imitates a real event, used to estimate the probability of the event. For example, the sex of a newborn baby can be simulated by computer/calculator/spreadsheet, tossing a coin, rolling a die or selecting a card at random from a deck of cards.

What is the probability that for a family with 4 children, all 4 children are girls?

1 Determine a method of simulating the genders of the 4 children in a family, such as tossing 4 coins together or using a spreadsheet to generate 4 random numbers.

2 Record the outcome of each trial, for example, BBGB.

3 Run 50 trials of this simulation, then count how many instances of GGGG there were.

4 Calculate the observed probability that all 4 children are girls.

5 Research what the theoretical probability that all 4 children are girls, and therefore what the expected frequency for 50 trials is.

6 Run another 50 trials of this simulation to achieve a total of 100 trials, and calculate the observed probability that all 4 children are girls.

7 Is this observed probability closer to the theoretical probability? Is the observed frequency closer to the expected frequency?

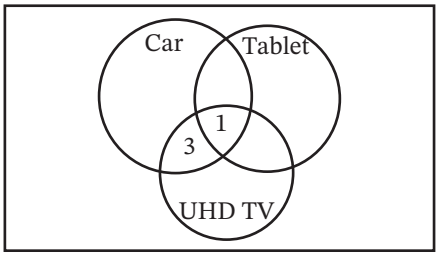
## Power plus ANSWERS ON P. 598



**1** A market research survey of 50 people shows that 42 people own a car, 20 own a tablet device and 8 own a UHD TV. Of these:

- 4 own a car and a UHD TV
- 3 own a UHD TV and a tablet
- 14 own a car and a tablet
- 1 person has a car, a tablet and a UHD TV

Copy and complete the following Venn diagram to represent the survey results.




**2** 2 dice are rolled and the numbers are **multiplied** together to arrive at a score.

**a** Copy and complete this table to show all possible products.

| | | 1st die | | | | | |
|---|---|---|---|---|---|---|---|
| | × | 1 | 2 | 3 | 4 | 5 | 6 |
| 2nd die | 1 | 1 | 2 | 3 | | | |
| | 2 | 2 | 4 | | | | |
| | 3 | 3 | 6 | | | | |
| | 4 | | | 12 | | | |
| | 5 | | | | | | |
| | 6 | | | | | | |

**b** How many different products are possible?

**c** Why isn't each product equally likely?

**d** Which product is most likely?

**e** Which product is least likely?

**f** What is the probability of a product of:

**i** 6? **ii** 20? **iii** at least 20?

**g** Which product has a probability of $\frac{1}{12}$?

**3** 3 friends decide to play a game with 2 dice. Danielle wins if the sum of the numbers is 3 or 5, and Vanessa wins if the total is 6 or 8. Any other total means that Karla wins. Is the game fair? Explain your answer.

**4** A committee of 2 people is to be selected from 2 boys (Paul and Sumeet) and 2 girls (Tash and Nadine).

**a** List all the committees you can form.

**b** If you were to choose a committee at random, what is the probability that it would include Tash?


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# CHAPTER 8 REVIEW

## Language of maths

| | | | |
|---|---|---|---|
| at least | certain | complementary | event |
| expected frequency | impossible | improbable | inclusive |
| likely | mutually exclusive | observed frequency | observed probability |
| outcome | overlapping | probability | probable |
| random | relative frequency | sample space | simulation |
| theoretical probability | trial | two-way table | unlikely |
| Venn diagram | | | |

**1** What is the meaning of **sample space**?

**2** What is the **complementary event** to winning a soccer match?

**3** What does it mean when a person is selected '**at random**' for a survey?

**4** What is the difference between **theoretical probability** and **observed probability?**

**5** What term means the number of times an event should occur over repeated trials?

**6** Express 'even chance' as a decimal probability value.

**Quiz**
Language of maths 8

**Puzzle**
Probability crossword

## Topic summary

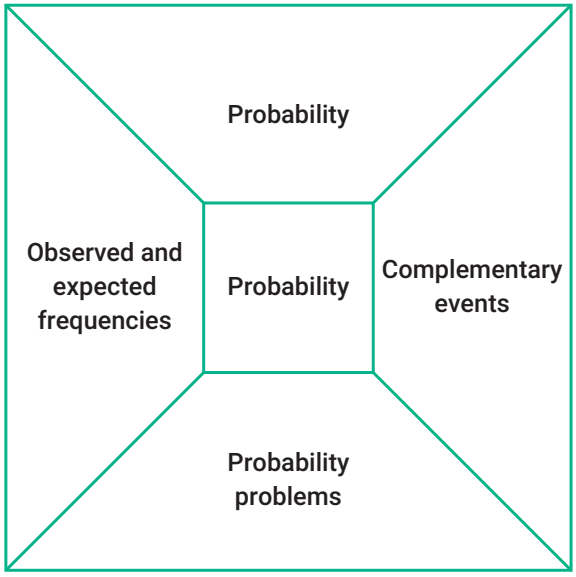
- Write in your own words what you have learnt in this chapter.
- Write which parts of the chapter were new to you.
- Copy and complete:

  The things I understand about probability are...

  The things I am still not confident in doing in this chapter are...

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

**Worksheet**
Mind map: Probability



Quiz
Test yourself 8

# TEST YOURSELF 8

ANSWERS ON P. 599

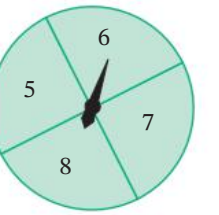
8.01 **1** There are 6 red, 3 yellow and 4 blue hair ties in a drawer. One hair tie is selected at random. What is the probability that it is red? Select the correct answer **A**, **B**, **C**, or **D**.

**A** $\frac{1}{13}$ **B** $\frac{6}{13}$ **C** $\frac{6}{7}$ **D** $\frac{13}{6}$

8.01 **2** A die is rolled. Find the probability that the number that comes up is:

**a** 1 **b** more than 2 **c** odd **d** composite

8.01 **3** **a** Write the sample space for this spinner.




**b** Find, as a percentage, the probability that the number spun is:

**i** 5 **ii** at least 5

**iii** 5 or less **iv** less than 5

8.02 **4** Write the complement of each event and its probability.

**a** Choosing an Ace from a standard deck of cards

**b** Rolling a factor of 6 on a die

**c** Buying the winning ticket out of 1000 tickets sold

8.02 **5** A golfer has a probability of 74% of sinking a putt. What is the probability that he will miss a putt?

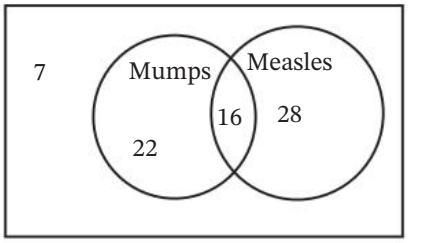
8.02 **6** A truck carries 240 boxes of hand sanitiser, 305 boxes of soap, 335 boxes of paper and 120 boxes of pens. The driver chooses one box at random. What is the probability that it is:

**a** a box of hand sanitiser? **b** a box of pens or soap?

**c** a box of paper? **d** not a box of paper?

YEAR 9 PATH

8.03 **7** The medical history of a group of children is shown in the Venn diagram.




**a** How many children are in the group?

**b** How many children have had the mumps and the measles?

**c** How many children have had the mumps or the measles, but not both?

**d** What is the probability that one child selected at random from this group has had:

**i** the measles?

**ii** the mumps or the measles?

**iii** neither the mumps nor the measles?

Foundation Standard Complex



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**8** A group of people were surveyed on whether they watched cricket or soccer on TV. The results were sorted into a two-way table. 8.04

| | TV sport | |
|---|---|---|
| | Cricket | Not cricket |
| Soccer | 28 | 15 |
| Not soccer | 20 | 7 |

**a** How many people were surveyed?

**b** How many people watch soccer?

**c** How many people watch soccer or cricket?

**d** What is the probability that a person selected at random from the survey:

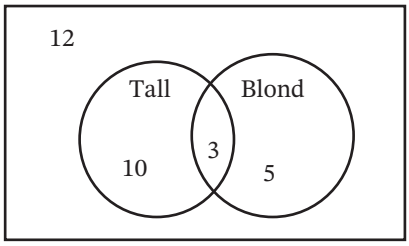
**i** watches neither soccer nor cricket?

**ii** watches soccer or cricket, but not both?

**iii** does not watch cricket?

**iv** does not watch soccer?

**9** Represent the data in the Venn diagram on the two-way table. 8.04




| | Tall | Not tall |
|---|---|---|
| Blond | | |
| Not blond | | |

**10** An 8-sided die has 2 red, 3 white, 1 blue and 2 yellow faces. If the die is rolled, find the decimal probability that the face that comes up is: 8.05

**a** blue **b** yellow **c** not red **d** white or yellow

**11** A die was rolled 80 times and the numbers 1 or 6 came up 25 times. 8.06

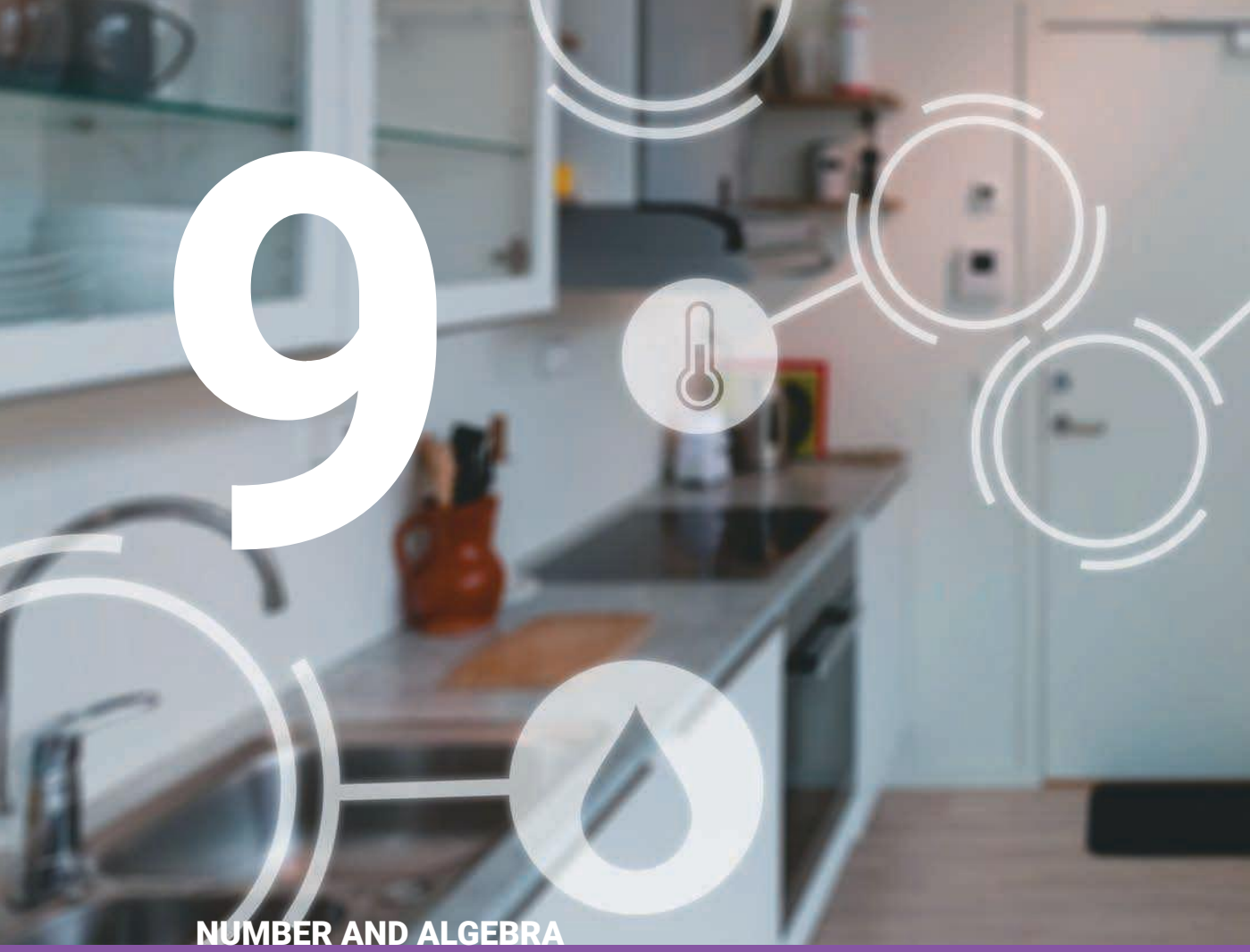
**a** What is the observed probability of rolling 1 or 6, as a percentage?

**b** What is the theoretical probability of rolling 1 or 6, as a percentage?

**c** Calculate the expected frequency of rolling 1 or 6 over 80 trials.



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9

NUMBER AND ALGEBRA

# EQUATIONS

Equations are extremely important for solving the same problem many times with different values. In animation and film-making, programmers use complex equations to make action figures move like real people. Most modern appliances are now smart devices with responses that rely on the use of equations.




Shutterstock.com/NicoElNino

## Chapter outline

| | Working mathematically | | | | |
|---|---|---|---|---|---|
| **9.01** One-step equations | U | F | | R | C |
| **9.02** Two-step equations | U | F | | R | C |
| **9.03** Equations with variables on both sides* | U | F | | R | C |
| **9.04** Equations with brackets* | U | F | | R | C |
| **9.05** Simple quadratic equations $ax^2 = c$ | U | F | | R | C |
| **9.06** Equation problems | U | F | PS | R | C |
| **9.07** Equations and formulas | U | F | PS | R | C |
| **9.08** Changing the subject of a formula* | U | F | | R | C |

*YEAR 9 EXTENSION

## Wordbank

**backtracking method** A method of solving equations by 'undoing', or performing inverse (opposite) operations in reverse order

**balancing method** A method of solving equations by using the same operations on both sides of the equation

**equation** A mathematical statement that 2 quantities are equal, involving algebraic expressions and an equals sign (=)

**inverse operation** An opposite used in solving an equation, for example, the inverse operation of multiplying is dividing

**quadratic equation** An equation involving a variable squared, such as $3x^2 = 45$.

**solve (an equation)** To find the value of an unknown variable in an equation

**solution** The answer to an equation or problem, the correct value(s) of a variable that makes an equation true

**substitute** To replace a variable with a number

**variable** A quantity that can take on different values, represented by a symbol such as a letter of the alphabet

**Quiz**
Wordbank 9

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

## Videos (10):

**9.01** One-step equations

**9.02** Two-step equations • Solving equations

**9.03** Equations with variables on both sides • Solving equations

**9.04** Equations with brackets • Solving equations

**9.05** Simple quadratic equations $ax^2 = c$

**9.06** Equation problems • Solving equations

**9.07** Equations and formulas 1 • Equations and formulas 2

**9.08** Changing the subject of a formula

## *Twig* video (1):

**9.01** The Arabic science of balancing

## *PhET* interactives (2):

**9.01** Equality explorer: Basics • Equality explorer

## Quizzes (5):

- Wordbank 9
- SkillCheck 9
- Mental skills 9
- Language of maths 9
- Test yourself 9

## Skillsheets (3):

**9.02** Solving equations by balancing • Solving equations by backtracking • Solving equations using diagrams

## Worksheets (16):

**9.01** Equations 1 • One-step equations • Guess-and-check

**9.02** Backtracking • Equations writing activity

**9.03** Equations with unknowns on both sides

**9.04** Equations 2 • Writing equations • Equations 3 (Extension) • Checking solutions

**9.06** Equation problems • Solving equations review • Angle problems with algebra

**9.07** Working with formulas

**9.08** Changing the subject of a formula

**Language of maths** Writing equations

**Mind map** Equations and inequalities

## Puzzles (10):

**9.01** Equations dominoes • One-step equations

**9.02** Solving equations

**9.03** Equations with unknowns on both sides

**9.04** Equations match • Equations • Equations order activity

**9.06** Writing and solving equations

**9.08** Formulas squaresaw

**Language of maths** Equations crossword

## Presentations (3):

**9.03** Formally solving equations

**9.06** Writing equations • Applying linear equations

## Technology (1):

**9.02** Solving linear equations

## Spreadsheet (1):

**9.02** Linear equation solver

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

## In this chapter you will:

- solve one- and two-step equations
- (EXTENSION, YEAR 9 CORE) solve equations involving variables on both sides, and brackets
- solve problems by using equations
- solve simple quadratic equations of the form $ax^2 = c$
- solve equations involving formulas to solve problems
- (EXTENSION, YEAR 9 PATH) change the subject of a formula

## SkillCheck ANSWERS ON P. 599

**1** Find the number represented by $\square$ to make each equation true.

| | | |
|---|---|---|
| **a** $\square + 3 = 10$ | **b** $4 \times \square = 28$ | **c** $12 - \square = 4$ |
| **d** $\square \div 3 = 9$ | **e** $9 + \square = 15$ | **f** $\square - 7 = 16$ |
| **g** $\square \times 6 = 30$ | **h** $24 \div \square = 8$ | **i** $10 + \square = 20$ |
| **j** $\square - 5 = 0$ | **k** $\frac{1}{2} \times \square = 4$ | **l** $\square \div 5 = 4$ |

**2** Evaluate each expression.

| | | |
|---|---|---|
| **a** $5 - 8$ | **b** $-8 + 3$ | **c** $3 \times (-4)$ |
| **d** $-6 \div 3$ | **e** $-2 + 5$ | **f** $-2 - 9$ |
| **g** $-4 \times (-9)$ | **h** $-12 \div (-4)$ | **i** $-3 - 7$ |
| **j** $80 \div (-10)$ | **k** $6 - 15$ | **l** $-8 \times 9$ |

**3** If $d = 7$, evaluate:

**a** $2d + 3$ **b** $3d - 4$ **c** $\frac{d+1}{2}$ **d** $18 - 2d$

**4** If $x = -3$, evaluate:

**a** $5x + 9$ **b** $4 - x$ **c** $\frac{x-3}{-1}$ **d** $2x - 8$

**5** What is the opposite operation to:

**a** adding? **b** dividing? **c** multiplying? **d** subtracting?

Quiz
SkillCheck 9



# One-step equations 9.01

An **equation** is a statement involving a variable (such as $x$), numbers and an equals (=) sign, for example, $\mathbf{2x - 5 = 11}$. In Year 7 we learnt how to **solve** equations, that is, to find the value of the variable that makes the equation true. This value is called the **solution** to the equation.

There are 2 algebraic methods for solving equations.

The **balancing method** involves representing both sides of an equation as balance scales, and solving the equation by performing the same operation on both sides to keep it 'balanced'.

The **backtracking method** involves using a flow chart to show what operations are performed on the variable (say, $x$) to create the equation, then using a reverse flow chart to undo each operation by performing the inverse (opposite) operation in reverse order.

Worksheets
Equations 1
One-step equations
Guess-and-check

Puzzles
Equations dominoes
One-step equations

## Solving equations

To solve an equation, aim to have the variable (such as $x$) on one side of the equation and a number on the other side, in the form:

$$x = ____$$

Check your solution by substituting it back into the equation.

**Videos**
One-step equations

The Arabic science of balancing

**Interactives**
Equality explorer: Basics

Equality explorer

## Example 1

Solve each equation.

**a** $m + 4 = 12$ **b** $3y = 18$ **c** $k - 3 = 5$ **d** $\frac{t}{8} = 6$

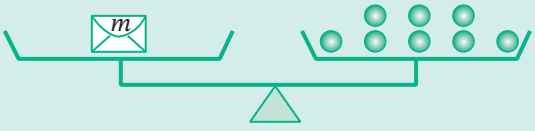
## Solution

**a** **Method 1: The balancing method**

$m + 4 = 12$ This equation can be represented on a balance scale:



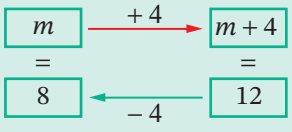

$m + 4 - 4 = 12 - 4$ Subtracting 4 from both sides.




$m = 8$ Check: $8 + 4 = 12$.

**Method 2: The backtracking method**

$m + 4 = 12$ This equation can be represented on a flowchart, starting at $m$ and ending at 12:



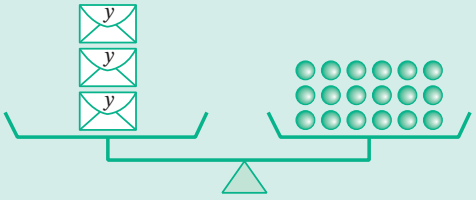

$m = 12 - 4$ Undo '+ 4' by subtracting 4.

Starting at 12 on the flowchart and ending at 8.

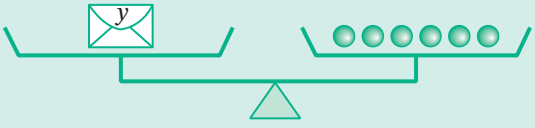
$m = 8$ Check: $8 + 4 = 12$.

**b** **Method 1: The balancing method**

$3y = 18$



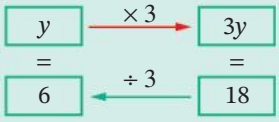

$\frac{3y}{3} = \frac{18}{3}$ Dividing both sides by 3.




$y = 6$ Check: $3 \times 6 = 18$

**Method 2: The backtracking method**

$3y = 18$




$y = \frac{18}{3}$ Undo '× 3' in $3y$ by dividing by 3.

$y = 6$ Check: $3 \times 6 = 18$

**c** **Method 1: The balancing method**

$k - 3 = 5$

$k - 3 + 3 = 5 + 3$ Adding 3 to both sides.

$k = 8$ Check: $8 - 3 = 5$

**Method 2: The backtracking method**

$k - 3 = 5$

$k = 5 + 3$ Undo '− 3' by adding 3.

$k = 8$ Check: $8 - 3 = 5$

**d** **Method 1: The balancing method**

$\frac{t}{8} = 6$

$\frac{t}{8} \times 8 = 6 \times 8$ Multiplying both sides by 8.

$t = 48$ Check: $\frac{48}{8} = 6$

**Method 2: The backtracking method**

$\frac{t}{8} = 6$

$t = 6 \times 8$ Undo '÷ 8' by multiplying by 8.

$t = 48$ Check: $\frac{48}{8} = 6$

These are all 'one-step' equations because they require only one step to solve.

## Inverse operations

| Operation | Inverse operation |
|---|---|
| + | − |
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## EXERCISE 9.01 ANSWERS ON P. 599

### One-step equations U F R C

EXAMPLE 1

**1** Solve each equation, showing the working.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $x + 5 = 12$ | **b** | $y + 13 = 36$ | **c** | $m + 9 = 21$ | **d** | $k + 27 = 54$ |
| **e** | $k + 6 = 2$ | **f** | $s + 3 = 1$ | **g** | $n + 9 = -4$ | **h** | $p + 17 = -3$ |
| **i** | $m + 8 = 0$ | **j** | $g + 4.5 = 20$ | **k** | $a + 1\frac{3}{4} = 12$ | **l** | $k + 2.7 = -5$ |

**2** Solve each equation, showing the working.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $t - 11 = 40$ | **b** | $w - 7 = 4$ | **c** | $a - 5 = 2$ | **d** | $j - 23 = 51$ |
| **e** | $q - 12 = 17$ | **f** | $f - 42 = 68$ | **g** | $x - 9 = 24$ | **h** | $g - 10 = 1$ |
| **i** | $j - 3 = -5$ | **j** | $w - 9 = -2$ | **k** | $m - 12 = -27$ | **l** | $d - 1 = -1$ |
| **m** | $y - 17 = 3.9$ | **n** | $n - 2.1 = 5.9$ | **o** | $b - \frac{4}{5} = \frac{1}{4}$ | **p** | $s - 2\frac{1}{3} = 4\frac{1}{2}$ |

**3** Solve each equation, showing the working.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $6x = 24$ | **b** | $4g = 16$ | **c** | $10y = 90$ | **d** | $5b = 45$ |
| **e** | $12d = 108$ | **f** | $9c = 81$ | **g** | $23p = 115$ | **h** | $2m = 26$ |
| **i** | $5x = -25$ | **j** | $7q = -42$ | **k** | $-3c = 54$ | **l** | $-15x = -120$ |
| **m** | $4p = 37$ | **n** | $8r = 18$ | **o** | $3q = -10$ | **p** | $9w = -12$ |

**4** Solve each equation, showing the working.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $\frac{s}{3} = 5$ | **b** | $\frac{m}{7} = 6$ | **c** | $\frac{d}{10} = 12$ | **d** | $\frac{w}{2} = 29$ |
| **e** | $\frac{a}{3} = -4$ | **f** | $\frac{f}{-2} = 9$ | **g** | $\frac{r}{5} = -3$ | **h** | $\frac{m}{-7} = 1$ |
| **i** | $\frac{x}{-8} = -6$ | **j** | $\frac{t}{2} = \frac{3}{4}$ | **k** | $\frac{n}{-4} = -4$ | **l** | $\frac{x}{3} = -\frac{1}{6}$ |

**5** Solve $3x = -12$. Select the correct answer **A**, **B**, **C** or **D**.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **A** | $x = -9$ | **B** | $x = -36$ | **C** | $x = -4$ | **D** | $x = -15$ |

**6** Write a one-step equation whose solution is: R C

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $m = 4$ | **b** | $x = 0$ | **c** | $r = 7$ | **d** | $d = -5$ |

Foundation Standard Complex


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# Two-step equations

9.02

**Two-step equations** require 2 steps or 2 inverse operations to solve.

## Example 2

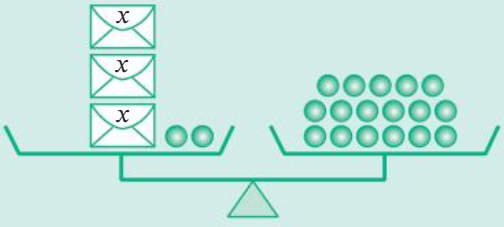
Solve each equation.

**a** $3x + 2 = 17$ **b** $\frac{k}{5} + 4 = 7$

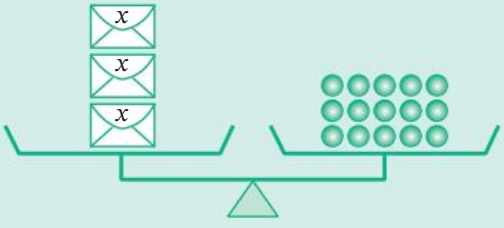
### Solution

**a** **Method 1: The balancing method**

$3x + 2 = 17$




$3x + 2 - 2 = 17 - 2$ *Step 1*: Subtracting 2 from both sides.




$3x = 15$

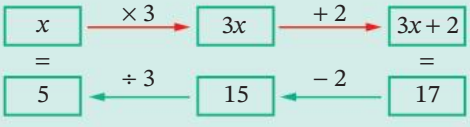
$\frac{3x}{3} = \frac{15}{3}$ *Step 2*: Dividing both sides by 3.




$x = 5$ Check: $3 \times 5 + 2 = 17$

**Method 2: The backtracking method**

$3x + 2 = 17$




**Videos**
Two-step equations
Solving equations

**Worksheets**
Backtracking
Equations writing activity

**Skillsheets**
Solving equations by balancing
Solving equations by backtracking
Solving equations using diagrams

**Technology**
Solving linear equations

**Spreadsheet**
Linear equation solver

**Puzzle**
Solving equations

$3x = 17 - 2$ *Step 1*: Undo '+ 2' by subtracting 2.

$x = \frac{15}{3}$ *Step 2:* Undo '× 3' in 3x by dividing by 3.

$x = 5$ Check: $3 \times 5 + 2 = 17$

**b** **Method 1: The balancing method**

$\frac{k}{5} + 4 = 7$

$\frac{k}{5} + 4 - 4 = 7 - 4$ *Step 1*: Subtracting 4 from both sides.

$\frac{k}{5} = 3$

$\frac{k}{5} \times 5 = 3 \times 5$ *Step 2*: Multiplying both sides by 5.

$k = 15$ Check: $\frac{15}{5} + 4 = 3 + 4 = 7$

**Method 2: The backtracking method**

$\frac{k}{5} + 4 = 7$

$\frac{k}{5} = 7 - 4$ *Step 1*: Undo '+ 4' by subtracting 4.

$\frac{k}{5} = 3$

$k = 3 \times 5$ *Step 2*: Undo '÷ 5' by multiplying by 5.

$k = 15$ Check: $\frac{15}{5} + 4 = 3 + 4 = 7$

**Video**
Two-step equations

## Example 3

Solve each equation.

**a** $\frac{2h}{3} = 4$ **b** $\frac{x+1}{2} = 7$

### Solution

**a** **Method 1: The balancing method**

$\frac{2h}{3} = 4$

$\frac{2h}{3} \times 3 = 4 \times 3$ *Step 1*: Multiplying both sides by 3.

$2h = 12$

$\frac{2h}{2} = \frac{12}{2}$ *Step 2*: Dividing both sides by 2.

$h = 6$ Check: $\frac{2 \times 6}{3} = 4$

**Method 2: The backtracking method**

$\frac{2h}{3} = 4$

$2h = 4 \times 3$ *Step 1*: Undo '÷ 3' by multiplying by 3.

$2h = 12$

$h = \frac{12}{2}$ *Step 2*: Undo '× 2' in 2h by dividing by 2.

$h = 6$ Check: $\frac{2 \times 6}{3} = 4$

**b** **Method 1: The balancing method**

$\frac{x+1}{2} = 7$

$\frac{x+1}{2} \times 2 = 7 \times 2$ *Step 1*: Multiplying both sides by 2.

$x + 1 = 14$

$x + 1 - 1 = 14 - 1$ *Step 2*: Subtracting 1 from both sides.

$x = 13$ Check: $\frac{13+1}{2} = 7$

**Method 2: The backtracking method**

$\frac{x+1}{2} = 7$

$x + 1 = 7 \times 2$ *Step 1*: Undo '÷ 2' by multiplying by 2.

$x + 1 = 14$

$x = 14 - 1$ *Step 2*: Undo '+ 1' by subtracting 1.

$x = 13$ Check: $\frac{13+1}{2} = 7$

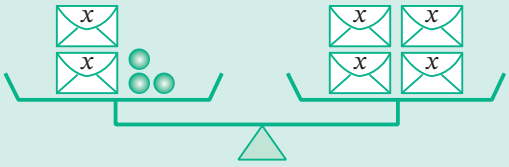
## Example 4

Solve $2x + 3 = 4x$.

### Solution

This equation has the variable on **both sides** of the equation, so we need to use the **balancing method** to 'collect like terms' and simplify the equation.

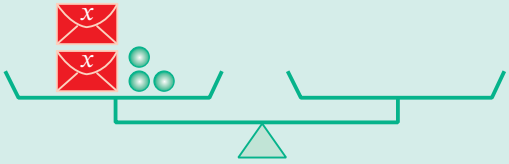
$2x + 3 = 4x$




$2x + 3 - 4x = 4x - 4x$

$-2x + 3 = 0$

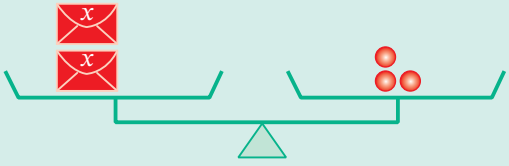
Subtracting $4x$ from both sides to remove it from the RHS (right-hand side).




$-2x + 3 - 3 = 0 - 3$

$-2x = -3$

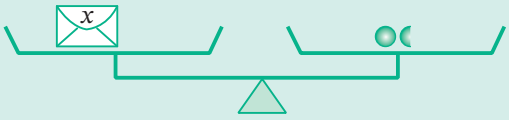
Subtracting 3 from both sides to remove it from the LHS (left-hand side).




$\frac{-2x}{-2} = \frac{-3}{-2}$

$x = 1\frac{1}{2}$

Divide both sides by $(-2)$.




Check:

LHS $= 2\left(1\frac{1}{2}\right) + 3 = 6$

RHS $= 4\left(1\frac{1}{2}\right) = 6$ LHS = RHS


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We can solve this equation in 2 steps if we move the variables to the RHS first by subtracting $2x$ from both sides.

$2x + 3 = 4x$

$3 = 2x$

$\frac{3}{2} = x$

$x = 1\frac{1}{2}$



**EXERCISE 9.02** ANSWERS ON P. 599

## Two-step equations U F R C

**1** Check which equation has $p = -2$ as its solution. Select the correct answer **A**, **B**, **C** or **D**.

**A** $2 + 3p = 8$ **B** $5p - 2 = 8$ **C** $-4p + 16 = 8$ **D** $-3p + 2 = 8$

**2** Solve each equation, showing all steps. Remember to check your answers.

EXAMPLE 2

**a** $2m + 7 = 19$ **b** $3x - 5 = 13$ **c** $5k + 12 = 52$

**d** $6w - 17 = 19$ **e** $4h + 21 = 9$ **f** $8d - 5 = -27$

**g** $-2x + 18 = 4$ **h** $-3a - 9 = -6$ **i** $14c + 2 = 37$

**j** $12p - 10 = 86$ **k** $9y + 11 = 101$ **l** $2r - 15 = 16$

**3** Solve each equation.

**a** $\frac{m}{2} + 4 = 13$ **b** $\frac{c}{5} + 9 = 12$ **c** $\frac{d}{7} + 2 = 8$

**d** $\frac{k}{3} + 15 = 6$ **e** $\frac{h}{2} + 7 = 17$ **f** $\frac{x}{4} + 2 = -4$

**g** $\frac{n}{5} - 9 = 4$ **h** $\frac{t}{3} - 6 = -2$ **i** $\frac{a}{8} - 2 = -6$

**j** $\frac{x}{6} - 16 = -13$ **k** $\frac{v}{15} - 2 = 4$ **l** $\frac{b}{3} - 4 = 3$

**4** This is Liam's incorrect solution for $7x + 5 = 13$.

$7x + 5 = 13$

$7x = 13 - 5$ **A**

$7x = 8$ **B**

$x = \frac{8}{7}$ **C**

$x = 1\frac{1}{8}$ **D**

In which line was the error made? Select **A**, **B**, **C** or **D**. R

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EXAMPLE 3

**5** Solve each equation.

**a** $\frac{3x}{2} = 9$ **b** $\frac{15x}{9} = 15$ **c** $\frac{2x}{3} = 8$ **d** $\frac{3N}{5} = 3$

**e** $\frac{6N}{7} = 18$ **f** $\frac{5B}{2} = -10$ **g** $\frac{3x}{4} = -3$ **h** $\frac{2x}{5} = -4$

**i** $\frac{5m}{2} = 11$ **j** $\frac{-x}{3} = 8$ **k** $\frac{-4x}{5} = 1$ **l** $\frac{-2x}{3} = -10$

**6** Check which value of $k$ is the solution to $\frac{k-12}{9} = 6$. Select **A**, **B**, **C** or **D**.

**A** $k = 27$ **B** $k = 42$ **C** $k = 66$ **D** $k = 162$

**7** Solve each equation.

**a** $\frac{x+1}{3} = 2$ **b** $\frac{x-3}{2} = 3$ **c** $\frac{N+2}{5} = 1$ **d** $\frac{N-3}{4} = 5$

**e** $\frac{N+8}{5} = 6$ **f** $\frac{x+1}{2} = -2$ **g** $\frac{k-5}{2} = -5$ **h** $\frac{m+2}{3} = 11$

EXAMPLE 4

**8** Solve each equation.

**a** $7x - 15 = 2x$ **b** $10p + 6 = 7p$ **c** $4r + 20 = r$ **d** $3u - 7 = 4u$

**e** $5d + 36 = 14d$ **f** $2y = y - 5$ **g** $9f = 4f + 11$ **h** $16 - 3a = a$

**9** Write 4 equations, one of each type shown in questions **2**, **3**, **5**, **7** and **8**, that have the solution $p = 2$. R C

# 9.03 Extension: Equations with variables on both sides

YEAR 9 CORE

Worksheet Equations with unknowns on both sides

For equations with variables on both sides, such as $3x + 4 = 2x + 7$, we can only use the **balancing method**, not the backtracking method.

## Equations with variables on both sides

Perform operations on both sides to move:

- all the variables onto one side of the equation
- all the numbers onto the other side of the equation.

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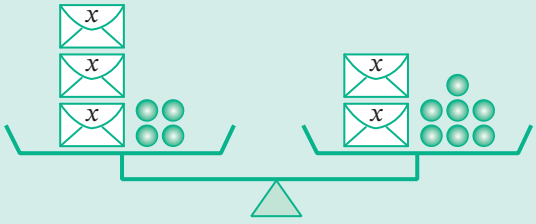
## Example 5

Solve each equation.

**a** $3x + 4 = 2x + 7$ **b** $5n - 3 = 2n - 15$ **c** $2d - 8 = -5d - 71$

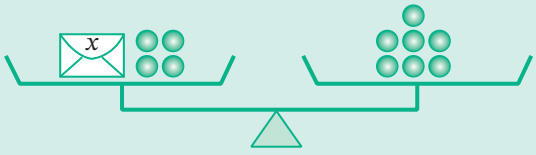
### Solution

**a** $3x + 4 = 2x + 7$



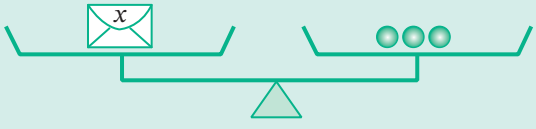

$3x + 4 - 2x = 2x + 7 - 2x$ Subtracting $2x$ from both sides to remove it from the RHS (right-hand side).

$x + 4 = 7$




$x + 4 - 4 = 7 - 4$ Subtracting 4 from both sides to remove it from the LHS (left-hand side).

$x = 3$




Check: LHS $= 3 \times 3 + 4 = 13$

RHS $= 2 \times 3 + 7 = 13$

LHS = RHS.

**b** $5n - 3 = 2n - 15$

$5n - 3 - 2n = 2n - 15 - 2n$ Subtracting $2n$ from both sides to remove it from the RHS.

$3n - 3 = -15$ Now this is a two-step equation.

$3n - 3 + 3 = -15 + 3$ Adding 3 to both sides to remove it from the LHS.

$3n = -12$ Now this is a one-step equation.

$\frac{3n}{3} = \frac{-12}{3}$ Dividing both sides by 3.

$n = -4$ Check: LHS $= 5 \times (-4) - 3 = -23$

RHS $= 2 \times (-4) - 15 = -23$ LHS = RHS

YEAR 9 CORE

**Puzzle**
Equations with unknowns on both sides

**Presentation**
Formally solving equations

**Videos**
Equations with variables on both sides

Solving equations

9.03

YEAR 9 CORE

**c** $2d - 8 = -5d - 71$

$2d - 8 + 5d = -5d - 71 + 5d$ Adding $5d$ to both sides to remove it from the RHS.

$7d - 8 = -71$ Now this is a two-step equation.

$7d - 8 + 8 = -71 + 8$ Add 8 to both sides to remove it from the LHS.

$7d = -63$ Now this is a one-step equation.

$\frac{7d}{7} = \frac{-63}{7}$ Dividing both sides by 7.

$d = -9$ Check: LHS $= 2 \times (-9) - 8 = -26$

RHS $= -5 \times (-9) - 71 = -26$ LHS = RHS.

## EXERCISE 9.03 ANSWERS ON P. 600

## Equations with variables on both sides U F R C

EXAMPLE 5

**1** What is the first step in solving $3x - 4 = x + 5$? Select the correct answer **A**, **B**, **C** or **D**. R C

**A** subtract $x$ from both sides **B** add $x$ to both sides

**C** add $3x$ to both sides **D** subtract 4 from both sides

**2** Solve $10m - 2 = 6m$. Select **A**, **B**, **C** or **D**.

**A** $m = -\frac{1}{2}$ **B** $m = \frac{1}{8}$ **C** $m = \frac{1}{2}$ **D** $m = 2$

**3** Solve each equation, showing all steps. Remember to check your answers.

**a** $3a + 6 = a + 18$ **b** $2k + 4 = k + 8$ **c** $3x + 6 = 2x + 9$

**d** $6p + 4 = p + 19$ **e** $6n + 5 = 2n + 17$ **f** $5q + 6 = 4q + 12$

**g** $5y + 14 = 3y + 14$ **h** $4a + 7 = 2a + 21$ **i** $2r + 9 = r + 15$

**4** Solve $4x - 5 = 2x + 7$. Select **A**, **B**, **C** or **D**.

**A** $x = 1$ **B** $x = 2$ **C** $x = 4$ **D** $x = 6$

**5** Solve each equation.

**a** $5a - 3 = 2a + 6$ **b** $6x - 2 = 3x + 6$ **c** $3d - 6 = d - 4$

**d** $8p - 15 = 3p - 10$ **e** $3m + 5 = -m + 26$ **f** $4x + 6 = -6x + 56$

**g** $4x + 3 = -2x + 7$ **h** $2r + 18 = -5r + 11$ **i** $3y - 12 = -y + 6$

**6** Here is Bree's solution for $3x - 4 = -2x + 8$.

$3x - 4 = -2x + 8$

$3x + 2x - 4 = 8$ Line 1

$5x - 4 = 8$ Line 2

$5x = 8 - 4$ Line 3

$5x = 4$ Line 4

$x = \frac{5}{4}$ Line 5

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In which lines did Bree make mistakes? Select **A**, **B**, **C** or **D**. R

**A** Lines 1 and 3 **B** Lines 2 and 5 **C** Lines 1 and 4 **D** Lines 3 and 5

**7** Solve each equation.

**a** $3d + 4 = d$ **b** $10k = 12 - 8k$ **c** $5 - p = p + 9$

**d** $7 + x = 6x + 22$ **e** $6k - 11 = k$ **f** $3m + 20 = 7m - 2$

**g** $4t = 12 - 4t$ **h** $8j - 17 = 10j$ **i** $6 - 3q = 8 - q$

**8** Write an equation with $x$ on both sides that has the solution $x = 7$. R C

YEAR 9 CORE

9.04

# Extension: Equations with brackets 9.04

## Equations with brackets (grouping symbols)

Expand the expressions and then solve as usual.

### Example 6

Solve each equation.

**a** $3(x + 5) = 9$ **b** $2(3 - y) = 4y + 4$ **c** $5(r - 3) = 2(r + 6)$

### Solution

**a** $3(x + 5) = 9$

$3x + 15 = 9$ Expanding the expression to make it a two-step equation.

Can you think of another way to solve this equation without expanding?

$3x + 15 - 15 = 9 - 15$ Subtracting 15 from both sides.

$3x = -6$

$\frac{3x}{3} = \frac{-6}{3}$ Dividing both sides by 3.

$x = -2$ Check: $3(-2 + 5) = 3 \times 3 = 9$

**b** $2(3 - y) = 4y + 4$

$6 - 2y = 4y + 4$ Expanding the brackets.

$6 - 2y - 4y = 4y + 4 - 4y$ Subtracting $4y$ from both sides to remove it from the RHS.

$6 - 6y = 4$ Now this is a two-step equation.

$6 - 6y - 6 = 4 - 6$ Subtracting 6 from both sides to remove it from the LHS.

$-6y = -2$

YEAR 9 CORE

**Worksheets**
Equations 2
Writing equations
Equations 3 (Extension)
Checking solutions

**Puzzles**
Equations match
Equations
Equations order activity

**Videos**
Equations with brackets
Solving equations

Foundation Standard Complex

$\frac{-6y}{-6} = \frac{-2}{-6}$ Dividing both sides by $(-6)$.

$y = \frac{1}{3}$ Check: LHS $= 2 \times \left(3 - \frac{1}{3}\right) = 5\frac{1}{3}$

RHS $= 4 \times \frac{1}{3} + 4 = 5\frac{1}{3}$ LHS = RHS

**c** $5(r - 3) = 2(r + 6)$

$5r - 15 = 2r + 12$ Expanding both sides.

$5r - 15 - 2r = 2r + 12 - 2r$ Subtract $2r$ from both sides to remove it from the RHS.

$3r - 15 = 12$ Now this is a two-step equation.

$3r - 15 + 15 = 12 + 15$ Add 15 to both sides to remove it from the LHS.

$3r = 27$

$\frac{3r}{3} = \frac{27}{3}$ Dividing both sides by 3.

$r = 9$ Check: LHS $= 5 \times (9 - 3) = 5 \times 6 = 30$

RHS $= 2 \times (9 + 6) = 2 \times 15 = 30$ LHS = RHS

## EXERCISE 9.04 ANSWERS ON P. 600

### Equations with brackets UFRC

EXAMPLE 6

**1** Solve each equation, showing all steps. Remember to check your answers.

| | | | | | |
|---|---|---|---|---|---|
| **a** | $2(x + 3) = 8$ | **b** | $3(m + 2) = 18$ | **c** | $5(k + 1) = 35$ |
| **d** | $4(p + 4) = 32$ | **e** | $10(j + 2) = 50$ | **f** | $7(d + 4) = 63$ |
| **g** | $3(x - 4) = 15$ | **h** | $3(x - 4) = 12$ | **i** | $5(x - 1) = 10$ |
| **j** | $8(x - 5) = 24$ | **k** | $4(k - 5) = 28$ | **l** | $2(q - 7) = 26$ |

**2** Solve each equation.

| | | | | | |
|---|---|---|---|---|---|
| **a** | $2(6 - x) = 6$ | **b** | $3(4 - p) = 24$ | **c** | $2(1 - q) = 8$ |
| **d** | $3(16 - h) = 15$ | **e** | $2(4 + r) = -8$ | **f** | $3(e - 16) = -15$ |
| **g** | $5(z - 3) = -10$ | **h** | $-7(y + 2) = 35$ | **i** | $9(6 - a) = 0$ |
| **j** | $5(d + 3) = -20$ | **k** | $-2(3 - k) = 12$ | **l** | $4(6 - c) = -16$ |

**3** Solve $2(12 - 3x) = 30$. Select the correct answer **A**, **B**, **C** or **D**.

**A** $x = -2$ **B** $x = -1$ **C** $x = 1$ **D** $x = 2$

**4** Solve each equation.

| | | | | | |
|---|---|---|---|---|---|
| **a** | $6(x + 1) = 3x + 3$ | **b** | $2(r + 2) = r + 5$ | **c** | $3(p + 3) = 2p + 2$ |
| **d** | $2(x + 1) = x$ | **e** | $5(p - 2) = 3p$ | **f** | $4(e - 2) = 2e + 4$ |
| **g** | $5(y + 2) = 3y + 12$ | **h** | $3(a - 5) = 5 - 2a$ | **i** | $3(2w + 1) = 3w - 15$ |

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**5** Here is Ryan's solution for $7(h-4)=3h+20$.

$7(h-4)=3h+20$

$7h-4=3h+20$ **A**

$7h-4-3h=3h+20-3h$

$4h-4=20$ **B**

$4h-4+4=20+4$

$4h=24$ **C**

$\frac{4h}{4}=\frac{24}{4}$

$h=6$ **D**

In which line did Ryan make a mistake? Select **A**, **B**, **C** or **D**. R C

**6** Solve each equation.

**a** $5(x-1)=4(x+2)$ **b** $6(x+2)=4(x+6)$ **c** $4(k+3)=3(k-2)$

**d** $7(y+2)=4(y+5)$ **e** $3(v+2)=2(v+5)$ **f** $4(x-2)=2(x+7)$

**g** $3(p+1)=5(p-1)$ **h** $3(2s+1)=5(s+2)$ **i** $5(d+4)=2(2d-1)$

**7** Write an equation involving brackets whose solution is: R C

**a** $c=5$ **b** $k=-1$ **c** $n=2$ **d** $q=-4$

## Mental skills 9: Maths without calculators ANSWERS ON P. 600

Quiz
Mental skills 9

### Multiplying and dividing by 5, 15, 25 and 50

It is easier to multiply or divide a number by 10 than by 5. So whenever we multiply or divide a number by 5, we can double the 5 (to make 10) and then adjust the first number.

**1** Study each example.

**a** To multiply by 5, halve the number, then multiply by 10.

$18\times5=18\times\frac{1}{2}\times10$ (or $9\times2\times10$)

$=9\times10$

$=90$

**b** To multiply by 50, halve the number, then multiply by 100.

$26\times50=26\times\frac{1}{2}\times100$ (or $13\times2\times100$)

$=13\times100$

$=1300$

**c** To multiply by 25, quarter the number, then multiply by 100.

$44\times25=44\times\frac{1}{4}\times100$ (or $11\times4\times25$)

$=11\times100$

$=1100$

**d** To multiply by 15, halve the number, then multiply by 30.

$8\times15=8\times\frac{1}{2}\times30$ (or $4\times2\times15$)

$=4\times30$

$=120$

Foundation Standard Complex

**e** To divide by 5, divide by 10 and double the answer. We do this because there are two 5s in every 10.

$140 \div 5 = 140 \div 10 \times 2$
$= 14 \times 2$
$= 28$

**f** To divide by 50, divide by 100 and double the answer. This is because there are two 50s in every 100.

$400 \div 50 = 400 \div 100 \times 2$
$= 4 \times 2$
$= 8$

**g** To divide by 25, divide by 100 and multiply the answer by 4. This is because there are four 25s in every 100.

$600 \div 25 = 600 \div 100 \times 4$
$= 6 \times 4$
$= 24$

**h** To divide by 15, divide by 30 and double the answer. This is because there are two 15s in every 30.

$240 \div 15 = 240 \div 30 \times 2$
$= 8 \times 2$
$= 16$

**2** Now evaluate each expression.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $32 \times 5$ | **b** | $14 \times 5$ | **c** | $48 \times 5$ | **d** | $18 \times 50$ |
| **e** | $52 \times 50$ | **f** | $36 \times 25$ | **g** | $28 \times 5$ | **h** | $12 \times 25$ |
| **i** | $12 \times 15$ | **j** | $22 \times 35$ | **k** | $90 \div 5$ | **l** | $170 \div 5$ |
| **m** | $230 \div 5$ | **n** | $1300 \div 50$ | **o** | $900 \div 50$ | **p** | $300 \div 25$ |
| **q** | $1000 \div 25$ | **r** | $360 \div 45$ | **s** | $210 \div 15$ | **t** | $360 \div 15$ |

## Investigation

### Solving $x^2 = c$

**1** What is the inverse operation of 'squaring'?

**2** The equation $x^2 = 9$ has 2 solutions. What are the 2 numbers, $x$, which when squared give 9?

**3** What are the solutions for each equation? What is the pattern found in the answers?

**a** $x^2 = 25$ **b** $x^2 = 100$ **c** $x^2 = 1$

**4** Study this example:

$x^2 = 49$
$x = \pm\sqrt{49}$ which means $x = \sqrt{49}$ or $-\sqrt{49}$
$= \pm 7$ which means $x = 7$ or $x = -7$.

Check: When $x = 7$, $x^2 = 7^2 = 49$

When $x = -7$, $x^2 = (-7)^2 = 49$

Use the same method to solve each equation and check your answers:

**a** $x^2 = 81$ **b** $x^2 = 64$

# Simple quadratic equations $ax^2 = c$ 9.05

An equation involving a variable squared, such as $x^2 = 25$ or $3x^2 - 4 = 7$, is called a **quadratic equation**. In this section, we will solve simple quadratic equations of the type $ax^2 = c$, where $c$ is a number.



**Video**
Simple quadratic equations $ax^2 = c$

9.05

## Example 7

Solve each quadratic equation.

**a** $x^2 = 36$

**b** $x^2 = 121$

**c** $x^2 = 40$, writing the solution correct to one decimal place

**d** $x^2 = 83$, writing the solution as a surd

### Solution

**a**
$x^2 = 36$
$x = \pm\sqrt{36}$ Finding the square root of both sides.
$= \pm 6$ The equation has 2 solutions, $x = 6$ or $x = -6$.

**b**
$x^2 = 121$
$x = \pm\sqrt{121}$ Finding the square root of both sides.
$= \pm 11$ The equation has solutions $x = 11$ or $x = -11$.

**c**
$x^2 = 40$
$x = \pm\sqrt{40}$
$= \pm 6.3345\ldots$
$\approx \pm 6.3$ The equation has solutions $x \approx 6.3$ or $x \approx -6.3$.

**d**
$x^2 = 83$ Leaving the answer as a surd.
$x = \pm\sqrt{83}$ $x = \sqrt{83}$ or $-\sqrt{83}$

## Simple quadratic equations $x^2 = c$

The simple quadratic equation $x^2 = c$ (where $c$ is a positive number) has 2 solutions,

$$x = \pm\sqrt{c}$$

(which means $x = \sqrt{c}$ or $x = -\sqrt{c}$).

## Example 8

Solve $5x^2 = 125$.

### Solution

$5x^2 = 125$

$x^2 = 25$ Divide both sides by 5 first.

$x = \pm\sqrt{25}$

$= \pm 5$ The equation has solutions $x = 5$ or $x = -5$.

**EXERCISE 9.05** ANSWERS ON P. 600

## Simple quadratic equations $ax^2 = c$ U F R C

EXAMPLE 7

**1** Solve each quadratic equation.

**a** $x^2 = 81$ **b** $x^2 = 144$ **c** $x^2 = 1$

**d** $x^2 = 169$ **e** $m^2 = 5041$ **f** $u^2 = 1849$

**2** Solve each quadratic equation, writing the solution correct to 2 decimal places.

**a** $x^2 = 13$ **b** $x^2 = 54$ **c** $x^2 = 88$

**d** $t^2 = 129$ **e** $h^2 = 946$ **f** $z^2 = 527$

**3** Solve each quadratic equation, writing the solution as a surd.

**a** $x^2 = 41$ **b** $x^2 = 30$ **c** $x^2 = 48$

**d** $a^2 = 126$ **e** $p^2 = 75$ **f** $b^2 = 509$

**4** Many simple quadratic equations have 2 solutions (one positive, one negative), but there is one simple quadratic equation that has only **one** solution. What is this equation and what is its solution? R

**5** The equation $x^2 = -100$ has **no solutions**. Explain why. R C

**6** Write 2 more simple quadratic equations that have no solutions. R C

**7** Copy and complete: R C

**a** $x^2 = c$ has 2 solutions if $c$ is _____.

**b** $x^2 = c$ has no solutions if $c$ is _____.

**c** $x^2 = c$ has one solution if $c$ is _____.

EXAMPLE 8

**8** Solve each quadratic equation. R

**a** $3x^2 = 27$ **b** $2x^2 = 32$ **c** $3x^2 = 75$ **d** $5x^2 - 1 = 19$

**e** $5x^2 + 1 = 21$ **f** $7 + 3x^2 = 34$ **g** $6x^2 - 8 = 142$ **h** $7x^2 + 9 = 261$

## Did you know?

### Applications of quadratic equations

Quadratic equations can be more complex, for example, $3x^2 + 4x - 15 = 0$, with applications in many areas:

- the shape of satellite dishes and reflecting telescopes
- the path of a thrown object, such as a ball, shotput or javelin
- the orbits of planets as they move around the Sun.

**Research the formula that solves quadratic equations.**



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# Equation problems 9.06

## Solving word problems requiring an equation

- Choose your variable
- Translate the words into an equation
- Solve the equation
- Write a sentence that answers the problem

## Example 9

**a** When a number is doubled and 11 is subtracted, the result is 23. Find the number.

**b** 5 times a number is the same as 6 more than 3 times the number. What is the number?

### Solution

**a** Let the number be $x$.

$x \times 2 - 11 = 23$ Translating the words into an equation

$2x - 11 = 23$

Solving the equation:

$2x - 11 + 11 = 23 + 11$ Adding 11 to both sides

$2x = 34$

$\frac{2x}{2} = \frac{34}{2}$ Dividing both sides by 2

$x = 17$ Check: $2 \times 17 - 11 = 23$

The number is 17.

**Worksheets**
Equation problems
Solving equations review
Angle problems with algebra

**Puzzle**
Writing and solving equations

**Presentations**
Writing equations
Applying linear equations

**Videos**
Equation problems
Solving equations

**b** Let $n$ represent the number.

$$5n = 3n + 6$$

$5n - 3n = 3n + 6 - 3n$ Subtracting $3n$ from both sides

$$2n = 6$$

$\frac{2n}{2} = \frac{6}{2}$ Dividing both sides by 2

$$n = 3$$

The number is 3. Check: LHS $= 5 \times 3 = 15$

RHS $= 3 \times 3 + 6 = 15$ LHS = RHS

## Example 10

Cooper charges for fixing washing machines using the formula: $C = 45h + 70$, where $C$ is the charge in dollars and $h$ is the number of hours the job takes. Find:

**a** the charge for a job that takes 3 hours

**b** the number of hours Cooper worked if the charge is \$430.

### Solution

**a** Substitute $h = 3$ into the formula.

$$C = 45h + 70$$
$$= 45 \times 3 + 70$$
$$= 205$$

The charge is \$205.

**b** Substitute $C = 430$ into the formula and solve the equation.

$$430 = 45h + 70$$

$$45h + 70 = 430$$

$45h + 70 - 70 = 430 - 70$

$$45h = 360$$

$\frac{45h}{45} = \frac{360}{45}$

$$h = 8$$

The number of hours Cooper worked is 8.



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**EXERCISE 9.06** ANSWERS ON P. 600

## Equation problems U F PS R C

EXAMPLE 9

**1** Solve each problem by writing an equation and then solving it. You may use diagrams to help you think about the information. R C

**a** 4 family tickets for a theme park cost $352. How much does each ticket cost? (Let $t$ represent the price of a family ticket.)

**b** 20 cans of drink cost $36. How much does each can cost? (Let $c$ represent the cost of one can.)

**c** A number is tripled and the result is 99. What is the number? (Let $x$ represent the unknown number.)

**d** A number has 7 added to it and the result is 23. Find the number. (Let $n$ represent the unknown number.)

**2** Select the correct equation **A**, **B**, **C** or **D** for each problem. Then use the equation to solve the problem. R C

**a** If 23 is subtracted from a number, the answer is 79. What is the number?

**A** $79 + N = 23$ **B** $23N = 79$ **C** $23 - N = 79$ **D** $N - 23 = 79$

**b** The sum of a number and 18 is 67. What is the number?

**A** $N + 67 = 18$ **B** $N + 18 = 67$ **C** $18N = 67$ **D** $N - 18 = 67$

**c** When a number is multiplied by 23, the answer is 1288. What is the number?

**A** $1288N = 23$ **B** $N + 23 = 1288$ **C** $1288 - N = 23$ **D** $23N = 1288$

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**d** When a number is divided by 14, the answer is 42. What is the number?

**A** $N - 14 = 42$ **B** $\frac{N}{14} = 42$ **C** $14N = 42$ **D** $14 + N = 42$

**e** Marika buys a car that costs $13 499. She pays a deposit of $4525. How much does she owe?

**A** $4525N = 13\,499$ **B** $N - 4525 = 13\,499$

**C** $N + 4525 = 13\,499$ **D** $N = 13\,499 + 4525$

**3** For each problem, write an equation and find the unknown number. R C

**a** When an unknown number is divided by 5, the result is 15.

**b** When 12 is subtracted from a number, the answer is 37.

**c** When a number is subtracted from 52, the answer is 25.

**d** An unknown number is multiplied by 3, then 8 is subtracted. The result is 40.

**e** An unknown number is halved, then 7 is added. The result is 13.

**f** The sum of a number and 4 is doubled. The result is 44.

**g** The sum of a number and 6 is divided by 3 and the result is 7.

**h** The product of a number and 5 is decreased by 2. The result is 28.

**4** Translate each problem into an equation, then solve the equation to solve the problem. PS R C

**a** The student council is holding a disco to raise money. Each ticket bought by students raises $20, but the costs of running the disco total $1000. How many tickets must be sold to make a profit of $2000? (Let $n$ stand for the number of tickets sold.)

**b** In 8 years Kelly's age will be twice what it is now. How old is Kelly now? (Let $n$ stand for Kelly's age now.)

**c** 8 sheep have the same mass as 3 sheep and one cow. If the cow's mass is 500 kg, what is the mass of one sheep? (Let $s$ stand for the mass of one sheep.)

**d** If you multiply Liam's favourite number by 3 and add 1, you get the same answer as if you multiplied the number by 5 and took away 11. What is the number? (Let $x$ represent the number.)

**e** Mr Yen says, 'If you add 15 to my age and multiply by 7, the answer is 483.' How old is Mr Yen? (Let $a$ stand for his age.)

**f** The area of a rhombus is calculated by multiplying its diagonals and dividing by 2. If a rhombus has an area of 88 cm$^2$ and one diagonal is 11 cm, what is the length of the other diagonal? (Let $d$ represent the length of the diagonal.)

EXAMPLE 10

**5** A temperature in degrees Celsius (°C) can be converted to degrees Fahrenheit (°F) using the formula $F = \frac{9C}{5} + 32$. PS R

**a** Convert 25°C to °F.

**b** Convert 108°F to °C.

**6** The charge, $\$C$, for hiring a hall for an event is $C = 150 + 2N$, where $N$ stands for the number of people at the event. Find: PS R

**a** the charge when 225 people are at the event

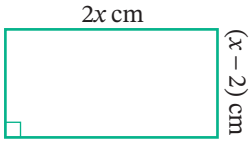
**b** the number of people at the event when the charge is \$394.

**7** The cost, $\$y$, of an online ad on a local news website is $y = 0.8w + 3.5$, where $w$ is the number of words in the ad. Find: PS R

**a** the cost of a 13-word ad

**b** the number of words in an ad costing \$25.90

**8** The perimeter of this rectangle is 47 cm. Find: PS R




**a** the value of $x$

**b** the length of the rectangle

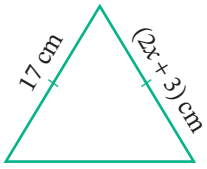
**c** the width of the rectangle.

**9** The profit, $\$P$, made by a hairdresser is given by $P = 18x - 900$, where $x$ represents the number of customers. Find: PS R

**a** the profit made with 195 customers

**b** the number of customers if the profit is \$3060.

**10** Find the value of $x$ in this isosceles triangle. R






# Equations and formulas 9.07

A **formula** is a rule written in algebraic form. It shows the relationship between variables.

For example:

- the formula $A = lw$ gives the area, $A$, of a rectangle with length $l$ and width $w$.
- the formula $D = ST$ gives the distance, $D$, travelled by a car at a speed $S$ for time $T$.

Solving mathematical problems often involves substituting values in formulas and solving equations.

**Videos**
Equations and formulas 1
Equations and formulas 2

**Worksheet**
Working with formulas

## Example 11

The formula for the perimeter, $P$, of a rectangle of length $l$ and width $w$ is given by $P = 2(l + w)$. Find a rectangle's:

**a** length if its perimeter is 40 cm and its width is 8 cm

**b** width if its perimeter is 100 cm and its length is 35 cm.

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## Solution

**a** Substitute $P = 40$, $w = 8$:

$$P = 2(l + w)$$
$$40 = 2(l + 8)$$
$$40 = 2l + 16$$
$$2l + 16 = 40$$
$$2l + 16 - 16 = 40 - 16$$
$$2l = 24$$
$$\frac{2l}{2} = \frac{24}{2}$$
$$l = 12$$

**b** Substitute $P = 100$, $l = 35$:

$$P = 2(l + w)$$
$$100 = 2(35 + w)$$
$$100 = 70 + 2w$$
$$70 + 2w = 100$$
$$70 + 2w - 70 = 100 - 70$$
$$2w = 30$$
$$\frac{2w}{2} = \frac{30}{2}$$
$$w = 15$$

**EXERCISE 9.07** ANSWERS ON P. 600

## Equations and formulas U F PS R C

EXAMPLE 11

**1** The volume of a rectangular prism with length, $l$, width, $w$, and height, $h$, is given by $V = lwh$. Use this formula to find the length of a rectangular prism with width 4 and height 5 if its volume is 340. Select the correct answer **A**, **B**, **C** or **D**.

**A** 17 **B** 68 **C** 85 **D** 425

**2** The number of toothpicks ($T$) needed to build a row of $N$ squares is $T = 3N + 1$.

**a** How many toothpicks are needed to build a row of 4 squares?

**b** How many squares can be built using 322 toothpicks?

**3** The number of hours ($H$) of sleep that children need depends on their age ($A$) in years and is given by the formula $H = 17 - \frac{A}{2}$. Find: PS R

**a** the hours of sleep needed when a child is 14 years old

**b** how old a child who needs 8 hours sleep is.

**4** Using the perimeter formula for a rectangle, $P = 2(l + w)$, find: PS R

**a** the perimeter when length = 13 and width = 4

**b** the length when perimeter = 36 and width = 7

**c** the width when perimeter = 58 and length = 16

**5** The area of a trapezium is given by $A = \frac{1}{2}(a+b)h$, where $a$ and $b$ are the lengths of its parallel sides and $h$ is the perpendicular height between them. Find the value of: PS R

**a** the area of a trapezium with parallel sides 20 and 13 and height 8

**b** the height of a trapezium with parallel sides 22 and 8 and area 90

**c** a parallel side of a trapezium with area 64, height 4 and other parallel side 17

**d** a parallel side of a trapezium with other parallel side 5, height 12 and area 120

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**6** The volume of a pyramid is given by $V = \frac{1}{3}Ah$, where $A$ is the area of the base and $h$ is the height of the pyramid. Find the value of: PS R

- **a** the volume of a pyramid with base area 25 and height 7
- **b** the base area of a pyramid with volume 100 and height 10
- **c** the height of a pyramid with volume 225 and base area 75

**7** The simple interest ($I$) earned on principal \$$P$ invested for $n$ years at an interest rate of $r$ p.a. (where $r$ is a decimal) is given by $I = Prn$. PS R

p.a. means 'per year'

- **a** Find the simple interest earned when \$1250 is invested at 2% p.a. for 5 years (convert 2% to a decimal first).
- **b** Find the principal invested if interest was \$60 and the interest rate was 5% p.a. for 3 years.
- **c** Find the interest rate (as a percentage) if \$700 invested for 3 years earned \$84 in interest.
- **d** Find the number of years that \$1650 was invested if it earned \$346.50 interest at 3% p.a.

**8** **a** 3 people meet and each person shakes hands with every other person. How many handshakes were there? R

- **b** In how many ways can 4 people shake hands?
- **c** The formula $H = \frac{P(P-1)}{2}$, where $H$ is the number of handshakes, and $P$ is the number of people, gives the number of shakes between people. In how many ways can 10 people shake hands?
- **d** How many people are needed for 105 handshakes?



# Extension: Changing the subject of a formula

9.08

The formula $C = 2\pi r$ is used to calculate the circumference of a circle if we know the value of $r$, the length of the circle's radius. The formula is used to find $C$, and so $C$ is called the **subject** of the formula. We could rewrite the formula so that $r$ was the subject and on the LHS while $C$ is on the RHS. This is called **changing the subject of the formula**. To change the subject of a formula, use the same rules as for solving an equation. The answer is not a number but an algebraic expression (another formula).

YEAR 9 PATH

**Worksheet**
Changing the subject of a formula

**Puzzle**
Formulas squaresaw

Foundation Standard Complex

YEAR 9 PATH

## Example 12

Rearrange the formula $C = 2\pi r$ to make $r$ the subject.

### Solution

To make $r$ the subject of the formula, solve it like an equation for $r$.

$C = 2\pi r$

$2\pi r = C$ Swapping sides so that $r$ appears on the LHS.

$r = \frac{C}{2\pi}$ Dividing both sides by $2\pi$.

## Example 13

Video
Changing the subject of a formula

Change the subject of the formula $v = u + at$ to $a$.

### Solution

$v = u + at$

$u + at = v$ Swapping sides so that $a$ appears on the LHS.

$at = v - u$ Subtracting $u$ from both sides.

$a = \frac{v-u}{t}$ Divide both sides by $t$.

**EXERCISE 9.08** ANSWERS ON P. 600

## Changing the subject of a formula U F R C

EXAMPLE 12

**1** Rearrange each formula to make the variable in brackets the subject. R C

If $a$ is the variable in brackets, the answer should be in the form '$a = \ldots$'.

**a** $F = ma\ (a)$ **b** $y = x + 3\ (x)$ **c** $A = \frac{d}{2}\ (d)$ **d** $f = \frac{120}{v}\ (v)$

**2** The formula $y = 2x + 1$ is rearranged to make $x$ the subject. What is the rearranged formula? Select the correct answer **A**, **B**, **C** or **D**. R C

**A** $x = y - 1 \div 2$ **B** $x = \frac{y}{-1}$ **C** $x = 2y - 1$ **D** $x = \frac{y-1}{2}$

EXAMPLE 13

**3** Change the subject of the formula $p = ax - 2$ to $x$. R C

**4** If $p = \frac{5q}{2}$, make $q$ the subject. R C

**5** The formula for converting temperatures from Celsius to Fahrenheit is $F = \frac{9C}{5} + 32$. Rearrange the formula to make $C$ the subject. R C

**6** The perimeter of a rectangle is $P = 2l + 2w$. Change the subject of the formula to $w$. R C

**7** Change the subject of the formula $a = \frac{v-u}{t}$ to $v$. R C

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YEAR 9 PATH

9.08

**8** The average speed $S$ km/h of a car travelling $D$ kilometres in $T$ hours is given by the formula $S = \frac{D}{T}$. R C

**a** Change the subject to $T$.

**b** Find the time taken for a car to travel 600 km at 75 km/h.

**9** Pythagoras' theorem is $c^2 = a^2 + b^2$. Change the subject of the formula to $a$. R C

**10** Make $d$ the subject of the formula $x = 2(d - 5)$. R C

**11** If the volume of a cylinder is $V = \pi r^2 h$, change the subject to the radius $r$. R C

**12** The area of a trapezium is $A = \frac{1}{2}(a + b)h$. Change the subject of the formula to $b$. R C

## Power plus ANSWERS ON P. 601

**1** Solve each equation.

**a** $2(x + 1) + 2(x - 1) = 12$

**b** $2(x + 4) - 3(x - 1) = 9$

**c** $4(2x - 1) - 5(x - 2) = 6$

**d** $2 - (3x + 5) = 4(x + 1)$

**e** $\frac{x+7}{4} = \frac{6(x-1)}{3}$

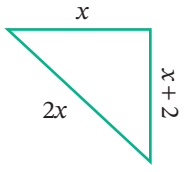
**f** $\frac{2(x+1)}{3} = \frac{5(x-2)}{2}$

**g** $\frac{2x}{3} - \frac{x}{6} = 10$

**h** $\frac{3x}{4} + \frac{9x}{10} = 44$

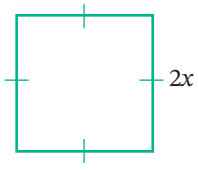
**2** For each diagram, write an equation for $x$ and solve it.

**a**



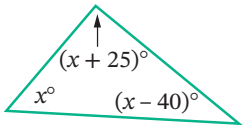

Perimeter = 30

**b**




Area = 100

**c**




**3** Solve each quadratic equation.

**a** $4x^2 = 100$

**b** $5x^2 - 7 = 73$

**c** $\frac{2x^2}{3} = 24$

**d** $3x^2 + 8 = 20$

**e** $(x + 4)^2 = 9$

**f** $x^2 + 2x = 15$

**4** Diophantus was a famous mathematician who was the first to abbreviate his mathematical thoughts using symbols. He is known as the Greek father of algebra and when he died, one of his admirers wrote the following riddle about his life:

Diophantus' youth lasted $\frac{1}{6}$ of his life. He grew a beard after $\frac{1}{12}$ more. After $\frac{1}{7}$ more of his life, Diophantus married; 5 years later he had a son. The son lived exactly $\frac{1}{2}$ as long as his father, and Diophantus died just 4 years after his son. All this adds to the years Diophantus lived.

Write this riddle as an equation and solve it to find how long Diophantus lived. (*Hint*: Let $x$ years equal his life.)

Foundation Standard Complex

# CHAPTER 9 REVIEW

## Language of maths

Quiz
Language of maths 9

Puzzle
Equations crossword

Worksheet
Writing equations

| | | | |
|---|---|---|---|
| backtracking | balancing | brackets | check |
| equation | expand | formula | inverse operation |
| LHS (left-hand side) | one-step equation | pronumeral | quadratic equation |
| RHS (right-hand side) | solution | solve | substitution |
| two-step equation | undoing | unknown | variable |

**1** What are the 2 algebraic methods for solving equations?

**2** Which method involves 'undoing' operations?

**3** What does the word 'solution' mean?

**4** Which word in the list means 'opposite'?

**5** Why is the **variable** in an equation sometimes called an **unknown**?

**6** What word means to rewrite an algebraic expression by removing the brackets?

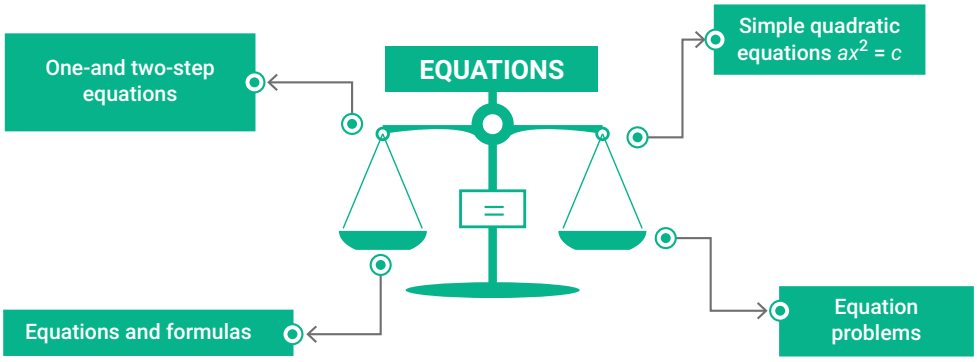
**7** What does **RHS** stand for?

## Topic summary

Worksheet
Mind map: Equations

- Which parts of this topic did you find easy? What did you already know?
- Give examples of some problems that might be solved using equations.
- Are there any parts of this topic that you still don't understand? Talk to your teacher about them.
- In what sort of careers would people use equations?

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.





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# TEST YOURSELF 9

ANSWERS ON P. 601

Quiz
Test yourself 9



**1** Solve each equation. 9.01

**a** $k + 6 = 13$ **b** $x - 3 = 8$ **c** $a + 3 = 17$

**d** $a - 12 = 21$ **e** $3x = 12$ **f** $10f = 120$

**g** $\frac{m}{4} = 7$ **h** $\frac{x}{3} = 8$ **i** $w + 9 = 3$

**j** $k - 5 = -7$ **k** $-2m = -16$ **l** $\frac{x}{3} = -12$

**2** Solve each equation. 9.02

**a** $4p + 3 = 23$ **b** $3m + 17 = 8$ **c** $2x - 12 = 18$

**d** $\frac{h}{7} + 5 = 16$ **e** $\frac{n}{2} - 8 = -12$ **f** $\frac{a}{5} - 8 = 4$

**g** $\frac{2y}{5} = 4$ **h** $\frac{x+2}{3} = 4$ **i** $\frac{n-5}{2} = 10$

**j** $\frac{5x}{2} = -15$ **k** $7k = 2k + 130$ **l** $10d - 58 = 12d$

YEAR 9 PATH

**3** Solve each equation. 9.03

**a** $3x + 4 = x + 6$ **b** $5u - 3 = 2u + 6$ **c** $12h - 8 = 8h + 4$

**d** $3v - 4 = 7v + 8$ **e** $2x + 9 = 7x - 4$ **f** $9 - 5t = 3t - 15$

**4** Solve each equation. 9.04

**a** $2(n + 3) = 18$ **b** $3(x + 1) = 15$ **c** $10(x - 3) = -10$

**d** $5(x - 2) = 3x + 4$ **e** $4(y + 1) = y + 18$ **f** $-2(d - 2) = 2d - 20$

**g** $3(r - 1) = 2(r + 9)$ **h** $7(a + 5) = 3(a + 9)$ **i** $2(2n - 4) = 2(5 - n)$

**5** Solve each equation: 9.05

**a** $x^2 = 64$ **b** $x^2 = 45$, correct to one decimal place **c** $5x^2 = 135$, as a surd

**6** **a** If a number is doubled and has 4 added to it, the answer is 14. What is the number? 9.06

**b** If a number has 4 added to it and is doubled, the answer is 14. What is the number?

**7** **a** Keilani is paid \$45 for each jumper she knits. If $n$ is the number of jumpers Keilani knits to earn a total of \$270, which equation can be used to find the value of $n$? Select the correct answer **A**, **B**, **C** or **D**. 9.06

**A** $\frac{n}{45} = 270$ **B** $45n = 270$ **C** $n + 45 = 270$ **D** $270 - n = 45$

**b** Solve the equation to find the value of $n$.

**8** 7 times a number is the same as 9 more than 4 times the same number. Use an equation to find the number. 9.06

**9** The sum ($S$) of the angles (in degrees) of a polygon is given by $S = 180(n - 2)$, where $n$ is the number of sides. Find: 9.07

**a** the sum of the interior angles when a polygon has 9 sides

**b** the number of sides in a polygon whose angle sum is 1080°.

YEAR 9 PATH

**10** Change the subject of each formula to the variable in the brackets. 9.08

**a** $V = IR$ $(R)$ **b** $p = 2q - 5$ $(q)$ **c** $x = 2(y - 3)$ $(y)$

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# PRACTICE SET 3

ANSWERS ON P. 601

9.02

**1** Solve each equation.

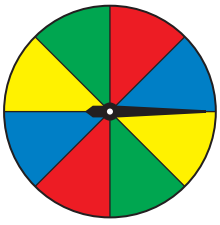
**a** $2a + 7 = 9$ **b** $3m - 2 = 7$ **c** $3x + 5 = -10$

**d** $\frac{t}{3} - 5 = 1$ **e** $\frac{k}{7} + 1 = 12$ **f** $8 - 2x = 4$

**g** $\frac{2x}{3} = 6$ **h** $\frac{5x}{4} = -10$ **i** $\frac{a+3}{2} = 8$

8.01

**2** If this spinner is spun, find the probability, as a percentage, that the arrow lands on:



**a** green

**b** a traffic light colour

**c** a colour that is not red

7.01

7.09

**3** Mrs Smart gave her class a quick quiz. The results are shown in the table.

**a** Copy and complete the table.

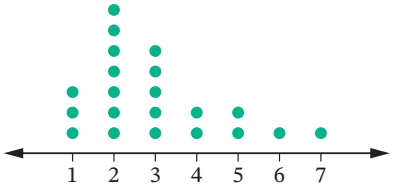
**b** How many students are there in Mrs Smart's class?

**c** Construct a frequency histogram and polygon for these results.

| Mark | Tally | Frequency |
|---|---|---|
| 6 | 𝍸 | |
| 7 | 𝍸 \|\| | |
| 8 | | 5 |
| 9 | 𝍸 𝍸 | |
| 10 | | 3 |

7.11

**4** Describe the shape of the data in this graph. Select the correct answer **A, B, C** or **D**.




**A** symmetrical

**B** bimodal

**C** negatively skewed

**D** positively skewed

7.01

**5** A die was rolled 40 times and the results are shown here.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 1 | 5 | 2 | 3 | 2 | 6 | 5 | 2 | 5 |
| 4 | 2 | 5 | 2 | 6 | 2 | 6 | 4 | 3 | 1 |
| 5 | 5 | 3 | 6 | 4 | 2 | 6 | 5 | 6 | 5 |
| 5 | 2 | 3 | 3 | 3 | 3 | 3 | 1 | 2 | 1 |

**a** Draw a frequency table for these results.

**b** Draw a divided bar graph to represent this information.

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**6** Is each type of data categorical or numerical? 7.03

- **a** number of cars owned
- **b** favourite drink
- **c** type of hairstyle

**7** Is each type of data discrete or continuous? 7.03

- **a** lap time of swimmer
- **b** distance from city
- **c** points scored by basketball players

**8** A die is rolled. Find the probability that the number that comes up is: 8.01

- **a** even
- **b** at least 2
- **c** a composite number

**9** Construct a dot plot for these Mathematics test marks. 7.08

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 22 | 16 | 10 | 8 | 12 | 11 | 15 | 13 | 12 | 13 | 11 | 16 |
| 11 | 12 | 10 | 10 | 12 | 12 | 15 | 11 | 13 | 15 | 17 | 13 |

- **a** Find the median.
- **b** Find the mode.
- **c** What is the outlier?
- **d** Where are the marks clustered?

**10** Is each type of data as nominal or ordinal? 7.03

- **a** World ranking of a professional tennis player.
- **b** Brand of laptop computer used by Year 8 students.
- **c** Size of pizza.

**11** For the data below, find: 7.05

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 90 | 89 | 95 | 92 | 90 | 92 | 97 | 93 | 90 | 97 |

- **a** the mean 7.06
- **b** the mode
- **c** the median
- **d** the range

**12** A pack of cards contains 10 blue, 6 purple and 4 pink cards. One card is drawn from the pack at random. Find the probability that this card is not purple. 8.02

Foundation Standard Complex

7.07

**13** The ages of people taking out a policy with an insurance company are shown.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 32 | 28 | 30 | 33 | 28 | 30 | 29 | 33 | 28 | 34 | 34 |
| 28 | 30 | 32 | 32 | 34 | 31 | 30 | 31 | 28 | 30 | 28 | 30 |

**a** Construct a frequency table of the ages, including an 'fx' column.

**b** What fraction of the people are under 30 years of age?

**c** For the above data, find:

**i** the mean (to one decimal place)

**ii** the median

**iii** the mode

**iv** the range.

9.05

**14** Solve $3x^2 = 48$. Select the correct answer **A**, **B**, **C** or **D**.

**A** $x = -2$ **B** $x = 2$ **C** $x = \pm 2$ **D** $x = \pm 4$

9.06

**15** 5 bricks have the same mass as one brick and 4 kg. What is the mass of one brick? Use an equation to solve this problem.

YEAR 9 PATH

8.04

**16** Year 8 students were surveyed on whether they participated in weekend sport.

**a** Copy and complete this 2-way table.

| | Plays weekend sport | Does not play weekend sport | Total |
|---|---|---|---|
| Male | 49 | | |
| Female | | 45 | 84 |
| Total | | 68 | |

**b** How many students are in Year 8?

**c** How many Year 8 students play weekend sport?

**d** If a student is chosen at random from Year 8, what is the probability that the student:

**i** is female?

**ii** is male or plays sport but not both?

7.12

**17** The marks scored by 10 students in Maths and Science exams are shown.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Maths | 35 | 58 | 87 | 43 | 52 | 96 | 23 | 35 | 48 | 58 |
| Science | 43 | 69 | 73 | 62 | 54 | 88 | 24 | 40 | 40 | 60 |

**a** Show these marks on a back-to-back stem-and-leaf plot.

**b** Find the median and the range for each exam.

**c** Based on these results, which exam do you think was more difficult? Justify your answer.

**18** The nationalities of players in a junior tennis tournament are shown below. 8.05

| Australia | USA | UK | Asia | Europe |
|---|---|---|---|---|
| 3 | 9 | 4 | 20 | 28 |

Find the probability that a player selected at random from this group:

**a** is Australian

**b** comes from Asia

**c** is not American

**d** is from Europe or America.

**19** Joanna wants to find the most popular TV program in Australia. 7.04

**a** Should she use a sample or a census? Give a reason for your answer.

**b** Suppose she decides to use a sample. How could she choose a representative sample?

**20** Using the formula for the area of a triangle $A = \frac{1}{2}bh$, find $h$, the height of the triangle with area 63 and base length 10.5. 9.07

**21** Emily is a park ranger. This table shows the data she recorded about kangaroo deaths in the park over one year. 8.06

| Cause of death | Number of deaths |
|---|---|
| Hit by a motor vehicle | 78 |
| Shot | 12 |
| Caught in a fence or trap | 11 |
| Old age | 25 |
| Starvation | 3 |
| Other | 7 |
| No known cause | 4 |
| Total | 140 |

**a** Calculate the observed probability that the death of a kangaroo was caused by a motor vehicle.

**b** Calculate as a percentage (to the nearest whole number) the observed probability that a kangaroo in the park will die from old age.

**c** In the next year, there were 90 kangaroo deaths. How many would you expect to have died from being shot?



# 10

NUMBER AND ALGEBRA

# RATIOS AND RATES

Ratios and rates are used in many everyday situations to compare things. Ratios are used when mixing hair colour, mixing paint to the chosen colour, paying rent in a share house and constructing buildings from plans. Rates are used when calculating the amount of paint required for a job, measuring blood alcohol content and preparing a holiday budget.




## Chapter outline

| | | Working mathematically | | | | |
|---|---|---|---|---|---|---|
| **10.01** | Ratios | U | F | | | C |
| **10.02** | Ratio problems | U | F | PS | R | |
| **10.03** | Scale maps and plans | U | F | PS | R | C |
| **10.04** | Dividing a quantity in a given ratio | U | F | PS | R | |
| **10.05** | Rates | U | F | | R | C |
| **10.06** | Best buys | U | F | PS | R | C |
| **10.07** | Rate problems | U | F | PS | R | C |
| **10.08** | Speed | U | F | PS | R | C |
| **10.09** | Converting rates | U | F | | R | C |
| **10.10** | Travel graphs | U | F | PS | R | C |

## Wordbank

**Quiz**
Wordbank 10

**best buy** When comparing different brands or sizes during shopping, this is the item with the lowest unit cost and is the best value for money

**per** (symbol '/') A word used in rates to mean 'for each'

**scaled length** A length on a map or plan that represents an actual length, usually much smaller, but in proportion to it

**speed** A rate that compares distance travelled with time taken

**travel graph** A line graph that describes a journey and shows distance travelled over time

**unit price** The price of one item or unit, such as 1 mL or 1 g

**unitary method** A method of finding a quantity by finding one part first

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

## Videos (17):

**SkillCheck** Why do we have ratios?

**10.01** Equivalent ratios • Simplifying ratios 1 • Simplifying ratios 2

**10.02** Ratio problems 1 • Ratio problems 2

**10.03** Map scales • Scale drawings • 24-hour time

**10.04** Dividing a quantity in a given ratio

**10.06** Unit pricing

**10.07** Rate problems 1 • Rate problems 2

**10.08** Speed

**10.09** Converting rates 1 • Converting rates 2

**10.10** Travel graphs

## *Twig* videos (4):

**SkillCheck** Ratios: Currency exchange • Ratios: The maths of baking

**10.01** The history of the golden ratio

**10.08** Speed of the Earth

## *PhET* interactives (3):

**10.01** Ratio and proportion • Proportion playground

**10.05** Unit rates

## Quizzes (5):

- Wordbank 10
- SkillCheck 10
- Mental skills 10
- Language of maths 10
- Test yourself 10

## Skillsheets (2):

**10.01** Ratios

**10.03** 24-hour time

## Worksheets (16):

**10.01** Simplifying ratios • Ratio calculations • Ratios match

**10.02** Ratio applications

**10.03** Map of Adelaide • Scale drawings

**10.05** Rates

**10.07** Rate problems • Ratios and rates review

**10.08** Speed • What's my speed?

**10.09** Converting rates • More rates

**10.10** Travel graph stories • The hare and the tortoise

**Mind map** Ratios and rates

## Puzzles (6):

**10.01** Simplifying ratios

**10.06** Best buys puzzle

**10.07** Rate problems

**10.08** Speed

**10.10** Jane's diary

**Language of maths** Ratios, rates and time crossword

## Technology (1)

**10.04** Applying ratios

## Presentations (2):

**10.04** Shares in ratios

**10.08** Rates: Triangle method

## Spreadsheet (1):

**10.04** Applying ratios

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

## In this chapter you will:

- simplify ratios and solve problems involving ratios
- interpret and calculate using scales on maps, plans and images
- divide a quantity in a given ratio
- simplify rates and solve problems involving rates, including speed
- calculate unit prices to determine 'best buys'
- convert between units for rates, for example, kilometres per hour to metres per second
- interpret and draw travel graphs, including the slope of the graph

## SkillCheck ANSWERS ON P. 602

**1** Copy and complete each conversion.

| | | | | | |
|---|---|---|---|---|---|
| **a** | 2 m = _____ cm | **b** | 3 h = _____ min | **c** | 3000 kg = _____ t |
| **d** | 2.5 L = _____ mL | **e** | 380 cm = _____ m | **f** | 180 mg = _____ g |
| **g** | 3 min = _____ s | **h** | 8.5 cm = _____ mm | **i** | 480 min = _____ h |
| **j** | 7500 mL = _____ L | **k** | 9.15 km = _____ m | **l** | 3840 mm = _____ cm |

**2** Find the highest common factor (HCF) of each pair of numbers.

**a** 12 and 18 **b** 35 and 21 **c** 16 and 40

**3** Complete each pair of equivalent fractions.

**a** $\frac{7}{10} = \frac{\square}{30}$ **b** $\frac{3}{5} = \frac{12}{\square}$ **c** $\frac{18}{45} = \frac{\square}{5}$

**4** Simplify each fraction.

**a** $\frac{18}{45}$ **b** $\frac{32}{64}$ **c** $\frac{80}{100}$ **d** $\frac{15}{36}$

**5** Evaluate each expression.

**a** $\frac{2}{5} \times 20$ **b** $\frac{1}{3} \times 15$ **c** $\frac{3}{4} \times 24$ **d** $\frac{5}{6} \times 36$

**e** $7.6 \times 100$ **f** $5.4 \times 10$ **g** $0.39 \times 100$ **h** $25 \times 10$

Quiz
SkillCheck 10

Videos
Why do we have ratios?
Ratios: Currency exchange
Ratios: The maths of baking

10.01

# Ratios 10.01

A **ratio** consists of 2 or more numbers that compare the parts or shares of things of the same type, in the same units. For example, if a cake recipe uses sugar to flour in a ratio of 1 to 2, written '1 : 2', it means that for every one part of sugar we need 2 parts of flour.

Each number in a ratio is called a **term** of the ratio.

**Equivalent ratios** are *equal* ratios and can be found in a similar way to that used for finding **equivalent fractions**.

Skillsheet
Ratios

Puzzle
Simplifying ratios

Worksheets
Simplifying ratios
Ratio calculations
Ratios match

Interactives
Ratio and proportion
Proportion playground

## Equivalent ratios

To find an **equivalent ratio**, multiply or divide each term by the same number.

## Example 1

Complete each pair of equivalent ratios.

**a** 6 : 4 = 18 : _____ **b** 3 : 4 = _____ : 24 **c** 12 : 8 = 3 : _____

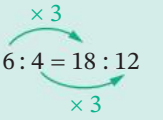
### Solution

**a** To find the missing term, look at the 2 known matching terms, 6 and 18.

6 is *multiplied* by 3 to give 18, so do the same thing to the 4.

$4 \times 3 = 12$

So 6 : 4 = 18 : 12

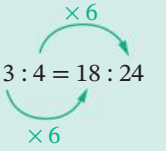



**b** The 2 known matching terms are 4 and 24.

4 is *multiplied* by 6 to give 24, so do the same thing to the 3.

$3 \times 6 = 18$

So 3 : 4 = 18 : 24

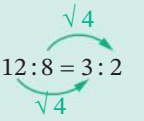



**c** The 2 known matching terms are 12 and 3.

12 is *divided* by 4 to give 3, so do the same thing to the 8.

$8 \div 4 = 2$

So 12 : 8 = 3 : 2




**Videos**
Equivalent ratios
The history of the golden ratio

## Simplifying ratios

- To simplify a ratio, keep dividing both terms by the same number, preferably a large number such as their highest common factor (HCF), until each term is as small as possible
- If all terms are even, divide by 2 or perhaps 4
- Otherwise, try dividing by the odd numbers 3, 5 or 7

## Example 2

Simplify each ratio.

**a** 25 : 40 **b** 24 : 16 **c** 18 : 30 : 15

### Solution

**a** $25 : 40 = \frac{25}{5} : \frac{40}{5}$

$= 5 : 8$

OR enter 25 : 40 as a fraction $\frac{25}{40}$ on the calculator:

25 $\boxed{\square\!\square}$ 40 $\boxed{=}$

Divide both terms by their HCF, 5.

$\boxed{a^{b/c}}$ is the fraction key on some calculators.

**Videos**
Simplifying ratios 1
Simplifying ratios 2

**b** $24:16 = \frac{24}{8} : \frac{16}{8}$

$= 3:2$

Divide both terms by their HCF, 8.

OR enter 24 : 16 as an improper fraction $\frac{24}{16}$ on the calculator: 24 ▭ 16 =

Then, to change the mixed numeral answer to an improper fraction, press:

SHIFT S⇔D or 2nd F $a^b/_c$

**c** $18:30:15 = \frac{18}{3} : \frac{30}{3} : \frac{15}{3}$

$= 6:10:5$

Divide all terms by their HCF, 3.

Because there are more than 2 terms in this ratio, the calculator cannot be used here.

## Example 3

Simplify each ratio.

**a** $\frac{3}{5} : \frac{1}{3}$ **b** 0.7 : 0.05

### Solution

If a ratio has terms that are fractions or decimals, it can be simplified by converting the terms to **whole numbers**.

**a** For fractions, multiply both terms by a common multiple, preferably the **lowest common multiple** (LCM) of the denominators.

$\frac{3}{5} : \frac{1}{3} = \left(\frac{3}{5} \times 15\right) : \left(\frac{1}{3} \times 15\right)$

$= 9:5$

The LCM of 5 and 3 is 15.

**b** For decimals, multiply both terms by the appropriate power of 10. In this case, multiply by 100 (move the decimal place 2 places to the right).

$0.7:0.05 = (0.7 \times 100) : (0.05 \times 100)$

$= 70:5$

$= 14:1$

Simplifying

## Example 4

Simplify the ratio of 2 hours to 1 day.

### Solution

First, change the values to the same units.

2 hours : 1 day = 2 hours : 24 hours

$= 2:24$

$= 1:12$

1 day = 24 hours

10.01

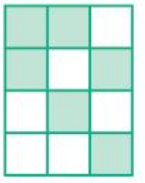
Video
Simplifying ratios 2

## EXERCISE 10.01 ANSWERS ON P. 602

# Ratios U F C

**1** For each shape, write the ratio of shaded to unshaded parts. C

**a**



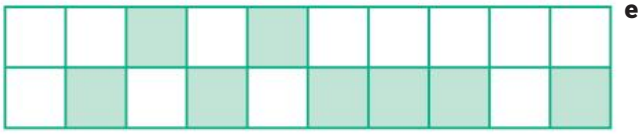
**b**



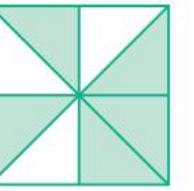
**c**



**d**



**e**



EXAMPLE 1

**2** Copy and complete each pair of equivalent ratios.

| | | | | | |
|---|---|---|---|---|---|
| **a** | 2 : 3 = 8 : _____ | **b** | 1 : 5 = 2 : _____ | **c** | 3 : 5 = ____ : 15 |
| **d** | 4 : 7 = _____ : 35 | **e** | 5 : 8 = 20 : _____ | **f** | 7 : 12 = 49 : _____ |
| **g** | 5 : 11 = _____ : 66 | **h** | 3 : 4 = _____ : 100 | **i** | 2 : 1 = 10 : _____ |
| **j** | _____ : 9 = 20 : 36 | **k** | 12 : _____ = 3 : 1 | **l** | 17 : 34 = _____ : 2 |
| **m** | _____ : 45 = 6 : 9 | **n** | 24 : 12 = 4 : _____ | **o** | 16 : _____ = 2 : 5 |
| **p** | _____ : 20 = 15 : 60 | **q** | 24 : 20 = 6 : _____ | **r** | 50 : 40 = ____ : 20 |

**3** Which of the following ratios is **not** equivalent to the ratio 32 : 48?
Select the correct answer **A**, **B**, **C** or **D**.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **A** | 16 : 24 | **B** | 4 : 6 | **C** | 2 : 3 | **D** | 6 : 8 |

EXAMPLE 2

**4** Simplify each ratio.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | 10 : 100 | **b** | 12 : 24 | **c** | 12 : 30 | **d** | 35 : 49 |
| **e** | 18 : 12 | **f** | 56 : 24 | **g** | 1000 : 100 | **h** | 45 : 99 |
| **i** | 87 : 87 | **j** | 123 : 321 | **k** | 51 : 17 | **l** | 3 : 48 |
| **m** | 8 : 12 : 20 | **n** | 15 : 20 : 30 | **o** | 27 : 9 : 36 | **p** | 14 : 35 : 21 : 49 |

EXAMPLE 3

**5** Simplify each ratio.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $\frac{1}{3} : \frac{2}{5}$ | **b** | $\frac{1}{4} : \frac{1}{3}$ | **c** | $\frac{3}{4} : \frac{2}{3}$ | **d** | $\frac{1}{2} : \frac{3}{8}$ |
| **e** | $\frac{2}{5} : \frac{3}{10}$ | **f** | $\frac{4}{5} : \frac{1}{2}$ | **g** | $\frac{5}{8} : \frac{1}{4}$ | **h** | $\frac{2}{3} : \frac{1}{2}$ |
| **i** | $\frac{3}{4} : \frac{7}{16}$ | **j** | $\frac{4}{5} : \frac{1}{2}$ | **k** | $\frac{5}{6} : \frac{2}{5}$ | **l** | $\frac{6}{5} : \frac{2}{3}$ |
| **m** | 0.4 : 0.7 | **n** | 1.3 : 0.8 | **o** | 0.5 : 0.3 | **p** | 0.9 : 1.8 |
| **q** | 0.6 : 0.8 | **r** | 3.6 : 2.4 | **s** | 0.05 : 0.2 | **t** | 0.25 : 0.5 |
| **u** | 0.375 : 0.25 | **v** | 12 : 8.4 | **w** | 2.4 : 1.2 : 3.6 | **x** | 4.5 : 1 : 0.9 |

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EXAMPLE 4

10.01

**6** Simplify each ratio.

| | | | | | |
|---|---|---|---|---|---|
| **a** | 50 cm to 2 m | **b** | 300 g to 1.2 kg | **c** | 5 days to 7 weeks |
| **d** | 30 min to 2 hours | **e** | 70 cents to $2.10 | **f** | 2 years to 6 months |
| **g** | 15 hours to 2 days | **h** | 20 mm to 1 m | **i** | 4 tonnes to 350 kg |
| **j** | 25 min to 3 hours | **k** | 18 m to 1 km | **l** | 8 months to 4 years |
| **m** | 2 days to 8 hours | **n** | 75 cents to $5 | **o** | $2.70 : $12 |

**7** On a farm there are 200 orange and mandarin trees in total. If there are 120 orange trees, what is the ratio of orange to mandarin trees? **C**

**8** In a class of 28 students, there are 16 girls. Find, in simplest form, the ratio of: **C**

**a** girls to boys **b** boys to girls **c** girls to students in the class

**9** A store has 30 gas heaters and 20 electric heaters in its warehouse. Find the ratio of: **C**

**a** gas heaters to electric heaters

**b** gas heaters to all heaters

**c** all heaters to electric heaters

**10** A man earns $75 000 a year and spends $63 000 a year. Find the ratio of his savings to earnings. Select **A**, **B**, **C** or **D**. **C**

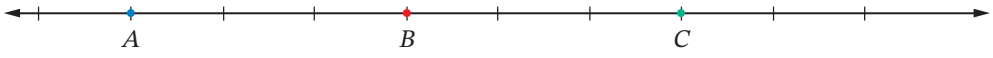
**A** 25 : 21 **B** 21 : 25 **C** 4 : 25 **D** 4: 21

**11** Zoe buys goods for $320 and sells them for $380. Find, in simplest form, the ratio of: **C**

**a** the cost price to the selling price **b** the selling price to the cost price

**c** the profit to the selling price **d** the selling price to the profit

**12** The line below is divided into units of length as shown.




Find each ratio of lengths. **C**

**a** $AB : BC$ **b** $AC : AB$ **c** $BC : AC$ **d** $AC : BC$

**13** A vending machine is filled with bottles and cans. The ratio of the number of bottles to the total number of contents is 3 : 8. **C**

**a** What fraction of the contents are cans?

**b** What is the ratio of the number of cans to the number of bottles?

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**14** 0.5 $m^3$ of cement is added to $\frac{3}{8}$ $m^3$ of metal to make a mixture. What is the ratio of cement to metal? Select **A**, **B**, **C** or **D**. C

**A** 4 : 3 **B** 7 : 8 **C** 3 : 4 **D** 3 : 16

**15** When comparing the rate at which babies are born in different countries (that have different population sizes), we use a base number of 1000. The annual birth rate in Australia is approximately 12 per 1000 people (a ratio of 12 : 1000). The rate in India is approximately 22 per 1000 people, or 22 : 1000. Express each ratio in simplest form. C

# 10.02 Ratio problems

**Worksheet**
Ratio applications

**Videos**
Ratio problems 1
Ratio problems 2

Problems involving ratios can be solved using **equivalent ratios** or the **unitary method**. With the unitary method, we find the size of **one part** first. We have used this method before when calculating with percentages.

## Example 5

The ratio of boys to girls in a class is 2 : 3. If there are 10 boys in the class, how many girls are there?

### Solution

**Method 1: Equivalent ratios**

Write the problem as a pair of equivalent ratios.

Boys : girls = 2 : 3 = 10 : ___ (10 boys; ×5 above, ×5 below)

Number of girls = 3 × 5 = 15
There are 15 girls in the class.

**Method 2: Unitary method**

Boys : girls = 2 : 3
2 parts (boys) = 10
1 part = 10 ÷ 2 = 5
3 parts (girls) = 3 × 5 = 15
There are 15 girls in the class.

'Unitary' means 'one'
Finding one part first.

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## Example 6

To make concrete, a builder mixes sand and cement in the ratio 5 : 4. If a mix of concrete contains 20 kg of cement, find:

**a** the amount of sand in the mix

**b** the total mass of the mix.



Shutterstock.com/franco lucato

### Solution

**a** **Method 1: Equivalent ratios**

Sand : cement = 5 : 4 = __ : 20 (×5 above, ×5 below)

20 kg of cement

Amount of sand = 5 × 5 = 25 kg

**Method 2: Unitary method**

Sand : cement = 5 : 4

4 parts (cement) = 20 kg

1 part = 20 ÷ 4 = 5 kg

5 parts (sand) = 5 × 5 = 25 kg

**b** Total mass = 20 kg + 25 kg = 45 kg

Cement and sand

So the total mixture was 45 kg.

## EXERCISE 10.02 ANSWERS ON P. 602

### Ratio problems U F PS R

**1** Alison and Elena buy a length of material and divide it between them in the ratio 2 : 3. If Alison has 5.4 m, what length of material does Elena have? Select the correct answer **A**, **B**, **C** or **D**. PS

EXAMPLE 5

**A** 2.7 m **B** 3.6 m **C** 8.1 m **D** 13.5 m

**2** A tiler uses 4 green tiles to every 3 white ones. How many white tiles are used if 100 green tiles are used? PS

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EXAMPLE 6

**3** When making concrete, sand and cement are mixed in the ratio 4 : 1. If 140 kg of cement has been delivered, what mass of sand is needed? PS

**4** 2 lengths of timber are in the ratio 4 : 7. The longer length is 56 cm. What is the shorter length?

**5** In a college, the ratio of teachers to students is 1 : 18. If the college has 80 teachers, how many students are there? PS

**6** The speeds of 2 boats are in the ratio 7 : 4. The speed of the slower boat is 10 km/h. Find the speed of the faster boat.

**7** In a rectangle, the ratio of the width to the length is 5 : 12. The length is 48 cm. PS R

**a** Find the width of the rectangle.

**b** Find the perimeter of the rectangle.

**8** An alloy contains copper and iron in the ratio 2 : 5. A quantity of alloy contains 20 kg of copper. What mass of iron does it contain?

**9** Toudi, Ash and Felicity share a Lotto prize in the ratio 11 : 8 : 6. Ash received $720. PS R

**a** How much did Toudi and Felicity each receive?

**b** What was the total prize money shared?

**10** In a triangle, the lengths of the sides are in the ratio 3 : 4 : 5. If the longest side is 45 cm long, find the perimeter of the triangle. Select **A**, **B**, **C** or **D**. PS R

**A** 57 cm **B** 72 cm **C** 81 cm **D** 108 cm

**11** The masses of 2 packets of detergent are in the ratio 3 : 10. PS R

**a** If the lighter packet has a mass of 1.5 kg, what is the mass of the larger one?

**b** If the heavier packet costs $12.50 and the lighter packet costs $3.90, which packet is the cheaper per kilogram and by how much?

**12** Ed's Farmers' Market buys fruit and vegetables in the ratio 7 : 9. The mass of vegetables ordered is 14.4 tonnes. What is the total mass of produce ordered? PS

**13** Samantha and James' heights are in the ratio 7 : 6. If Samantha is 1.75 m tall, how tall is James?

**14** In an outback mining town, the ratio of women to men is 2 : 5. If there are 240 women, how many men and women are there in the town altogether? Select **A**, **B**, **C** or **D**. PS R

**A** 840 **B** 600 **C** 960 **D** 1680

**15** To make Superglue, the contents of Tube A and Tube B are mixed in the ratio 3 : 1. PS

**a** If 21 mL of Tube A is used, how much of Tube B is needed?

**b** How much glue is made altogether if 21 mL of Tube B is used?

## Did you know?

### Loose change

All Australian coins are made by mixing copper with other elements.

The $2 and $1 coins were introduced in 1988 to replace notes. They are made of copper, aluminium and nickel in the ratio 46 : 3 : 1.

The 50c, 20c, 10c and 5c coins are made of copper and nickel in the ratio of 3 : 1.

**600 grams of aluminium are used to make some $2 coins. How much copper and nickel is used?**



Shutterstock.com/Alex Bogatyrev



# Scale maps and plans 10.03

Scale maps and plans are a special application of ratios in real life. Lengths and distances on scale diagrams are in the same ratio as the real lengths and distances.

**Worksheet**
Map of Adelaide

## Scale ratios

The scale ratio on a scale diagram is written in the form scaled length : real length, where **scaled length** is the length on the diagram.

For example, a scale ratio of 1 : 100 means that the real lengths are 100 times larger than the lengths on the diagram.

## Map scales

Map scales are often expressed in the same form as ratios. A scale of 1 cm : 1 km means that 1 cm on the map represents an actual distance of 1 km.

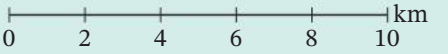
Video
Map scales

## Example 7

Simplify each map scale.

**a** 1 cm : 1 km

**b**




### Solution

**a** 1 cm : 1 km = 1 cm : 1000 m — 1 km = 1000 m

= 1 cm : 100 000 cm — 1 m = 100 cm

= 1 : 100 000

**b** The length of the scale from 0 to 10 on the diagram is 5 cm.

So 5 cm on the map represents 10 km of actual distance.

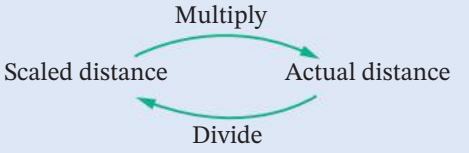
Scale = 5 cm : 10 km

= 5 cm : 10 × 1000 × 100 cm — Convert 10 km to cm.

= 5 cm : 1 000 000 cm

= 1 : 200 000 — Simplifying the ratio.

## Map scales




Video
Map scales

## Example 8

A map has a scale of 1 : 25 000.

**a** What is the actual distance if the scaled distance is 4 cm?

**b** What is the scaled distance if the actual distance is 3.5 km?

### Solution

**a** Scaled distance = 4 cm

Actual distance = 4 × 25 000 cm

= 100 000 cm

= 1000 m — 1 m = 100 cm

= 1 km — 1 km = 1000 m

**b** Actual distance = 3.5 km

= 3500 m — 1 km = 1000 m

= 350 000 cm — 1 m = 100 cm

Scaled distance = 350 000 cm ÷ 25 000

= 14 cm

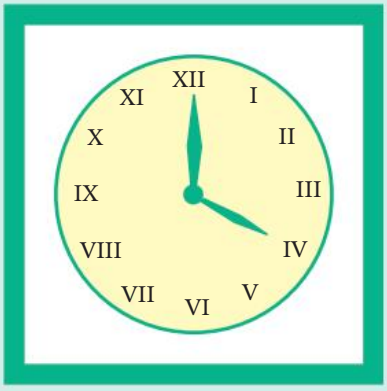

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# Scale drawings



## Example 9

This diagram of a clock is drawn to a scale of 1 : 6. Measure its length and calculate its actual length.




Scale 1 : 6

### Solution

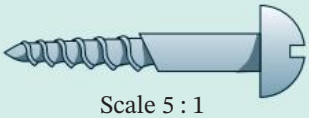
Scaled length = 5 cm by measurement

Actual length = 5 cm × 6

= 30 cm

## Example 10

This drawing of a screw is drawn to a scale of 5 : 1. Find its actual length.



Scale 5 : 1

### Solution

A scale of 5 : 1 means that the real screw is 5 times **smaller** than the one drawn.

Scaled length = 4 cm by measurement

Actual length = 4 cm ÷ 5

= 0.8 cm

**EXERCISE 10.03** ANSWERS ON P. 602

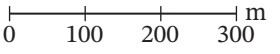
## Scale maps and plans U F PS R C

EXAMPLE 7

**1** Simplify each map scale. R C

**a** 1 cm : 5 km **b** 1 mm : 1 m **c** 1 cm : 500 km **d** 1 cm : 25 km

**e**




**f**



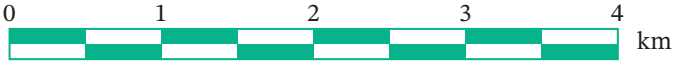

**g**




**h**




**i**




**j**




EXAMPLE 8

**2** A map has a scale of 1 : 50 000. What distance is represented by 64 mm on the map? Select the correct answer **A**, **B**, **C** or **D**. R C

**A** 0.32 km **B** 3.2 km **C** 32 km **D** 320 km

**3** A street map uses a scale of 1 cm : 200 m. PS R C

**a** Simplify this ratio.

**b** Find the actual distance, in kilometres, represented by each scaled distance.

**i** 7 cm **ii** 9.5 cm **iii** 12.4 cm

**c** Find the scaled distance, in centimetres, used to represent each actual distance.

**i** 18 km **ii** 1500 m **iii** 9.6 km

**4** On a map using a scale of 1 : 10 000 000, the world's longest river, the Nile in Egypt, is the length of an average shoe lace (66.7 cm). How many kilometres long is the River Nile? PS R C

**5** The town of Gilgandra is 66 km north of Dubbo. On a map with a scale of 1 : 100 000, what is the scaled distance between the 2 towns? PS R C

**6** Lord Howe Island is 2.8 km wide. How long would its scaled width be on a map with a scale of 1 : 50 000? PS R C

**7** This map of Nambucca Heads has a scale of 1 : 40 000. PS R C




**a** Find, in metres, the distance between:

**i** the Water Towers (C2) and the Catholic Church (F3)

**ii** the Anglican Church (G4) and the centre of Coronation Park (I4)

**iii** Rotary Lookout (H6) and Shelley Beach Lookout (J6)

**iv** the post office (F4) and the Foreshore Caravan Park (D6)

**b** Find the length of:

**i** West Street (E4)

**ii** Piggott Street (D5)

**c** How long is the lagoon (I7)?

**d** What are the dimensions of the cemetery (J2)?

**e** How long is the causeway leading to the Island Golf Course (B7)?

**f** To train for a fun run, Merridy decides to run 8 km 3 times a week. What distance will this be on the map in centimetres? Outline a possible course for her training run, starting and finishing at the Foreshore Caravan Park (D6).

**8** This is a scale plan of a bedroom. R C

By measurement and calculation, find in metres the actual:

**a** length of the bedroom

**b** width of the doorway (in cm)

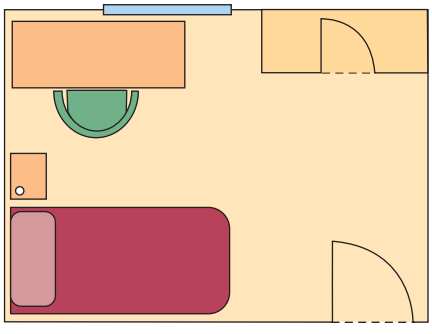
**c** length of the bed

**d** length of the window

**e** length of the table

**f** area of the bedroom

Scale: 1 : 70



EXAMPLE 9

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**9** Measure the length of each scaled-down image below, then use the scale ratio to calculate its actual length in centimetres (metres for the house). R C

**a** Fish 1 : 3




Shutterstock.com/Elena Blokhina

**b** House 1 : 300




iStock.com/pamspix

**c** Pen 1 : 3




Shutterstock.com/Pakhnyushchy

**d** Tennis racquet 1 : 16




Shutterstock.com/Gearstd

**10** This house plan is drawn to a scale of 1 : 140. PS

Measure and calculate to the nearest 0.1 m: R C

**a** the length of the main bedroom

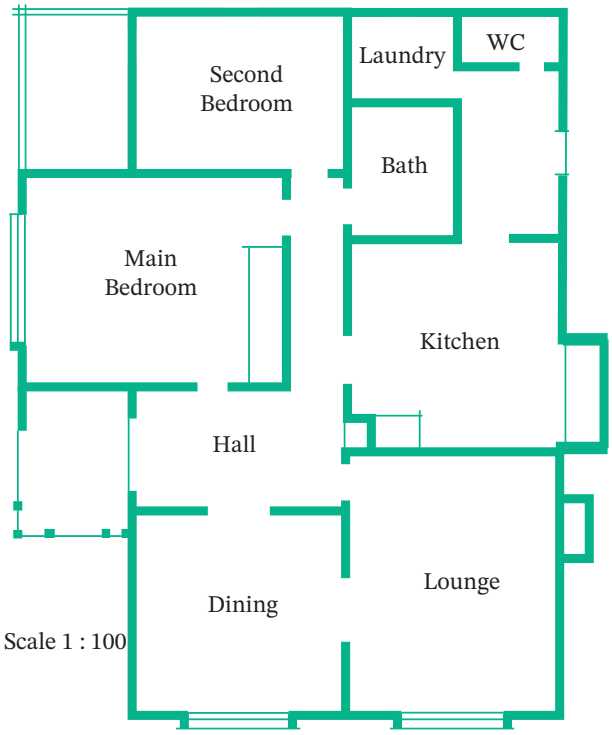
**b** the length of the window in that room

**c** the length of the laundry

**d** the area of the bathroom

**e** the longer side of the lounge room

**f** the area of the dining room




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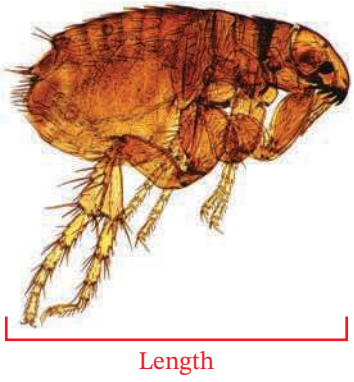
EXAMPLE 10

**11** An electronics engineer designs a mobile phone SIM card using a diagram with a scale of 100 : 1. If the scaled drawing is 80 cm long, what is its actual length in millimetres? Select **A**, **B**, **C** or **D**. R C

**A** 80 **B** 8 **C** 0.8 **D** 0.08

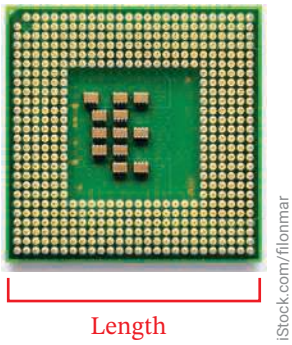
**12** Measure the length of each magnified image below, then use the scale ratio to calculate its actual length in millimetres. R C

**a** Flea 100 : 1



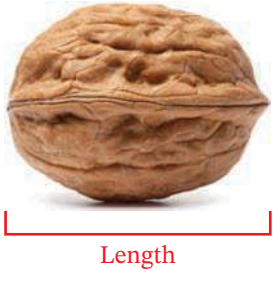

iStock.com/olikim

**b** Microchip 3 : 1




iStock.com/filonmar

**c** Nut 4 : 3




Shutterstock.com/Alexapicso

**d** Bacterium 100 : 1




Dreamstime.com/Puntasit Choksawatdikom

## Investigation

### Scale drawings

1. Measure the dimensions of your bedroom and the dimensions of the items of furniture in it.
2. Using a scale of 1 : 100, draw a scale diagram of your bedroom on graph paper.
3. Draw scale diagrams of each piece of furniture on another piece of graph paper, then cut them out.
4. Rearrange the furniture in your bedroom on the scale diagram to see which arrangement you prefer best.

Quiz
Mental skills 10

Skillsheet
24-hour time

Video
24-hour time

# Mental skills 10: Maths without calculators ANSWERS ON P. 603

## 24-hour time

| 24-hour time | 12-hour time | 24-hour time | 12-hour time |
|---|---|---|---|
| 00:00 | 12 a.m. (midnight) | 12:00 | 12 p.m. (midday) |
| 01:00 | 1 a.m. | 13:00 | 1 p.m. |
| 02:00 | 2 a.m. | 14:00 | 2 p.m. |
| 03:00 | 3 a.m. | 15:00 | 3 p.m. |
| 04:00 | 4 a.m. | 16:00 | 4 p.m. |
| 05:00 | 5 a.m. | 17:00 | 5 p.m. |
| 06:00 | 6 a.m. | 18:00 | 6 p.m. |
| 07:00 | 7 a.m. | 19:00 | 7 p.m. |
| 08:00 | 8 a.m. | 20:00 | 8 p.m. |
| 09:00 | 9 a.m. | 21:00 | 9 p.m. |
| 10:00 | 10 a.m. | 22:00 | 10 p.m. |
| 11:00 | 11 a.m. | 23:00 | 11 p.m. |

To convert from **24-hour time** to 12-hour time:

- if it begins with '00', then it is the 12 a.m. (midnight) hour
- if it begins with '12', then it is the 12 p.m. (midday) hour
- if it is less than 12:00, then it is a.m. (morning)
- if it is 13:00 or more, then it is p.m. (afternoon/evening) time: subtract 12 from the hour.

**1** Study each example.

**a** Convert 18:50 to 12-hour time.

18:00 > 13:00, so it is p.m. time, so subtract 12 from the hour.

$18 - 12 = 6$

18:50 = 6:50 p.m.

**b** Convert 04:30 to 12-hour time.

04:30 < 12:00, so it is a.m. time.

04:30 = 4:30 a.m.

**c** Convert 00:15 to 12-hour time.

00:15 begins with 00, so it is 12 a.m. time

00:15 = 12:15 a.m.

**2** Now convert each time to 12-hour time.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | 08:45 | **b** | 13:20 | **c** | 17:50 | **d** | 00:17 |
| **e** | 21:05 | **f** | 18:32 | **g** | 11:15 | **h** | 02:38 |
| **i** | 14:40 | **j** | 03:20 | **k** | 16:55 | **l** | 23:31 |
| **m** | 01:08 | **n** | 10:18 | **o** | 20:00 | **p** | 06:43 |

To convert from 12-hour time to 24-hour time:

- if it is the 12 a.m. (midnight) hour, begin with '00'
- if it is 'a.m.' time or the 12 p.m. (midday) hour, write as is but make sure the hour has 2 digits (for example, 02, 09).
- if it is 1 p.m. or later, then add 12 to the hour.

**3** Study each example.

**a** Convert 4:10 a.m. to 24-hour time.

It is 'a.m.' time, so write as a 4-digit number and rename 4 as 04.

4:10 a.m. = 04:10

**b** Convert 4:10 p.m. to 24-hour time.

It is after 1 p.m., so add 12 to the hour: 4 + 12 = 16

4:10 p.m. = 16:10

**c** Convert 12:47 a.m. to 24-hour time.

It is in the 12 a.m. (midnight) hour, so change the 12 to 00.

12:47 a.m. = 00:47

**4** Now convert each time to 24-hour time.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | 6:35 p.m. | **b** | 8:05 a.m. | **c** | 11:45 a.m. | **d** | 11:20 p.m. |
| **e** | 2:21 a.m. | **f** | 12:30 p.m. | **g** | 3:48 p.m. | **h** | 7:11 p.m. |
| **i** | 9:08 a.m. | **j** | 9:50 p.m. | **k** | 12:42 a.m. | **l** | 7:39 a.m. |
| **m** | 1:59 a.m. | **n** | 10:18 p.m. | **o** | 10:46 a.m. | **p** | 5:23 p.m. |



# Dividing a quantity in a given ratio 10.04

Elyse and Henry won a cash prize of $420 for winning an art competition, but instead of dividing the money evenly ($210 each), they decide to divide it in the ratio 3 : 4, so that Elyse receives 3 parts while Henry receives 4 parts. Henry receives more because he did more of the work and paid more when buying the materials for the artwork.

Problems involving dividing a quantity in a given ratio can be solved using the **unitary method** or **fraction method**.



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**Presentation**
Shares in ratios

**Technology**
Applying ratios

**Spreadsheet**
Applying ratios

**Video**
Dividing a quantity in a given ratio

## Example 11

Divide a cash prize of $420 between Elyse and Henry in the ratio 3 : 4.

### Solution

**Method 1: Unitary method**

Total number of parts = 3 + 4 = 7

7 parts = $420

1 part = $420 ÷ 7

= $60

Elyse share (3 parts) = 3 × $60

= $180

Henry's share (4 parts) = 4 × $60

= $240

So Elyse receives $180 and Henry receives $240.

Check: $180 + $240 = $420

**Method 2: Fraction method**

Total number of parts = 3 + 4 = 7

$$\text{Elyse share (3 parts)} = \frac{3}{7} \times \$420$$

$$= \$180$$

$$\text{Henry's share (4 parts)} = \frac{4}{7} \times \$420$$

$$= \$240$$

## Example 12

Nick, Hong and Rob agreed to divide the profits of their removalist business in the ratio 2 : 3 : 5. If their profit this year was $45 000, find the size of each person's share of the profits.

### Solution

**Method 1: Unitary method**

Total number of parts = 2 + 3 + 5 = 10

10 parts = $45 000

1 part = $45 000 ÷ 10

= $4500

Nick's share = 2 × $4500

= $9000

Hong's share = 3 × $4500

= $13 500

Rob's share = 5 × $4500

= $22 500

**Method 2: Fraction method**

Total number of parts = 2 + 3 + 5 = 10

$$\text{Nick's share} = \frac{2}{10} \times \$45\,000$$

$$= \$9000$$

$$\text{Hong's share} = \frac{3}{10} \times \$45\,000$$

$$= \$13\,500$$

$$\text{Rob's share} = \frac{5}{10} \times \$45\,000$$

$$= \$22\,500$$

∴ Nick, Hong and Rob receive $9000, $13 500 and $22 500 respectively.

Check: $9000 + $13 500 + $22 500 = $45 000

## EXERCISE 10.04 ANSWERS ON P. 603

### Dividing a quantity in a given ratio U F PS R

**1** Find the total number of parts if the ratio is:

**a** 2 : 7 **b** 4 : 1 **c** 3 : 4 **d** 2 : 5 : 6

EXAMPLE 11

**2** Divide $500 in the ratio:

**a** 4 : 1 **b** 7 : 3

**3** Phuong, Janelle and Ahmet bought a Lotto ticket for $12. They contributed $3, $3 and $6 respectively to the purchase price. They won $4000 and agree to split the winnings in the same ratio. PS R

**a** Simplify the ratio 3 : 3 : 6.

**b** How much prize money does each person get?

**4** George and Amal share the weekly rent of $390 in the ratio 7 : 6. What is Amal's share?

**5** Divide 450 kg in the ratio:

**a** 4 : 5 **b** 3 : 2

**6** Divide 720 cm in the ratio:

**a** 1 : 3 : 5 **b** 5 : 3 : 4

EXAMPLE 12

**7** Company directors Judy, Gas and Robert share the company profits in the ratio 5 : 3 : 3. Which of the following is the amount that Judy receives in a year when profits are $121 000? Select the correct answer **A**, **B**, **C** or **D**. PS R

**A** $11 000 **B** $24 000 **C** $33 000 **D** $55 000

**8** In Year 8, the ratio of boys to girls is 4 : 5. If there are 225 students in Year 8, find how many girls there are. PS

**9** In Year 9, the ratio of boys to girls is 3 : 2. If there are 125 students in Year 9, how many more boys than girls are there? PS R

**10** Adam needs to make 800 g of short-crust pastry. Flour and butter are needed in the ratio 3 : 1. How much flour is needed? PS R

**11** A company posts 1386 letters in a week. The ratio of local to overseas letters is 2 : 7. How many overseas letters are sent in a week?

**12** A truck carries fruit and vegetable boxes in the ratio 5 : 7. If it carries a total mass of 7.5 tonnes, what mass of vegetables does it carry? Select **A**, **B**, **C** or **D**. PS R

**A** 0.625 t **B** 3.125 t **C** 4.375 t **D** 6.873 t

**13** An alloy of mass 176 kg is made from copper and zinc in the ratio 5 : 6. Find the mass of copper in the alloy.

**14** At a school, the ratio of students who speak a second language to students who speak only English is 5 : 8. If there are 923 students at the school, how many students speak only English?

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15 A 20 m cable is cut into 3 sections in the ratio 2 : 3 : 5. Find the length of each section.

16 When making mortar, sand and concrete is mixed in the ratio 6 : 1. If we need 280 kg of mortar, how much sand will we need? PS R

17 In a study of 225 000 people, it was found that the ratio of right-handed people to left-handed people was 13 : 2. PS R

**a** How many left-handed people were there?

**b** How many more right-handed people than left-handed people were there?

18 Angus earns twice as much as Catriona. If the sum of their wages is $210 000, how much does each earn? R

# 10.05 Rates

Worksheet
Rates

Interactive
Unit rates

While a ratio compares 2 or more quantities measured in the same units, a **rate** compares 2 quantities measured in **different units**.

A rate shows how one quantity changes with another quantity. We write a rate using a '/' symbol in the form 'something *per* something else'. For example:

- the price of petrol is stated in cents per litre, or c/L
- your heart beats at a rate measured in beats per minute, or beats/min
- the speed of a car is measured in kilometres per hour, or km/h.

The word 'per' means 'for each' so we express a rate '*per* single unit'. For instance, travelling at a rate of 50 km/h means travelling 50 km in *each* hour.

## Example 13

Write each statement as a simplified rate.

**a** A factory produces 87 cars in 3 hours

**b** Ham costs $50 for 8 kg

### Solution

**a** The production rate would be expressed in cars per hour.

$$\text{Production rate} = \frac{87 \text{ cars}}{3 \text{ hours}}$$
$$= 29 \text{ cars/hour}$$

We can write a rate as a fraction: divide the number of cars by the number of hours

**b** The cost would be expressed in dollars per kilogram.

$$\text{Cost} = \frac{\$50}{8\text{kg}}$$
$$= \$6.25/\text{kg}$$

Divide the number of dollars by the number of kilograms

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## EXERCISE 10.05 ANSWERS ON P. 603

### Rates U F R C

**1** Write the units suitable for each rate below, in the form _____/_____. R C

**a** typing speed
**b** heart rate
**c** cost of a mobile phone call
**d** cost of bananas
**e** a person's wage
**f** a runner's speed
**g** population growth
**h** the cost of water
**i** population density of a country

**2** Write each statement as a simplified rate. C

**a** 51 sheep in 3 hours
**b** $10.75 for 2.5 kg
**c** 208 students for 8 teachers
**d** 136 points in 4 games
**e** 546 words in 6 minutes
**f** 34 articles in 4 hours
**g** 72 cars in 14 days
**h** 5040 boxes in 8 hours
**i** 259 m in 7 seconds
**j** 46 000 bottles in 50 hours
**k** 7944 revolutions in 6 minutes
**l** $175 for 5 hours
**m** 448 km in 8 hours
**n** $16.50 for 6 kg
**o** 114 runs in 24 overs
**p** 243 km using 30 litres
**q** $126 for 12 hours
**r** 2520 kg for 60 hectares

**3** The cost of sending a 5.5 kg parcel to Malaysia is $88. What is the postage rate? Select the correct answer **A**, **B**, **C** or **D**. C

**A** $0.34/kg **B** $16/kg **C** $0.07/kg **D** $484/kg

**4** In your own words, explain what is meant by each rate. C

**a** a speed of 100 km/hour
**b** a traffic flow of 150 cars/hour
**c** petrol consumption of 10.3 L/100 km
**d** a farmer keeping 60 sheep/ha

**5** A lift should carry no more than 1600 kg or 20 people. What is this weight allowance, in kg/person? C

**6** A pulp mill clears 12 600 hectares of forest in 7 years. At what rate in hectares/year does the mill clear the forest? C

**7** The cost of 53 L of petrol is $73.67. Express this cost in c/L. C

**8** A complaints hotline took 2190 calls in one year. Calculate the number of calls per month. C



EXAMPLE 13

# 10.06 Best buys

Video
Unit pricing

When shopping, it is important to compare the prices of different brands or sizes of items and calculate which is the **best buy** ('best value for money'). The biggest container does not always provide the best value. This can be done by comparing the **unit price** (cost of one item or unit) of each brand or size and choosing the cheapest one. A unit could be one gram or one millilitre.

Puzzle
Best buys puzzle

## Unit price

Unit price = cost ÷ number of items or units

Supermarkets are required by law to display unit prices to allow you to compare brands or sizes of items. If you look carefully at the price tags on their shelves, you will notice unit prices displayed in small print.

## Example 14

Which brand of baked beans is the better buy?



$4.60 $1.65

### Solution

Calculating the unit price (cost per gram) for each brand:

Bean There = $4.60 ÷ 500 = $0.0092 / g

Mr Beanz = $1.65 ÷ 200 = $0.00825 / g

Mr Beanz has the lower unit price, so it is the better buy.

**EXERCISE 10.06** ANSWERS ON P. 603

## Best buys U F PS R C

EXAMPLE 14

**1** Find the better buy for each pair of items. PS R C

**a** A 2 kg box of sultanas for $8.85 or a 1 kg box for $4.65

**b** 6 notepads for $15.50 or 8 notepads for $20

**c** 45 g chips for $2.10 or 150 g for $6.50

**d** 2 tubs of yoghurt for $1.24 or 7 tubs for $4.30

**e** 3 kg of corn flour for $5.85 or 500 g of corn flour for $1

**f** 600 mL bottle of fruit juice for $3.20 or a 2.25 L carton for $8.05

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**2** Michael purchased 400 g of ham for $3.59, while Kim purchased 300 g for $2.54. Who had the better buy? PS R C

**3** Which size of Fruit Bix cereal is the best buy? PS R C




**$4.75** **$3.49** **$2.51**

**4** Happy Tam cat food can be purchased in 4 different packages.

Pack 1: 24 × 250 g cans for $39.99 Pack 2: 12 × 300 g cans for $24.99

Pack 3: 9 × 400 g cans for $25.99 Pack 4: 6 × 1 kg cans for $38.50

Order these packs from best to worst buy. Select the correct answer **A**, **B**, **C** or **D**. PS R C

**A** 4, 2, 1, 3 **B** 3, 2, 1, 4 **C** 4, 1, 2, 3 **D** 2, 1, 3, 4

**5** For each item, find which size is the best value for money. PS R C

**a**




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Shutterstock.com/Bahadir Yeniceri

**b**




Shutterstock.com/Marek Tr

Shutterstock.com/Gts

**c**




Smart Media/Jalna

Smart Media/Jalna

**d**




Shutterstock.com/Lipowski Milan

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**6** For each item, which size gives the best value for money? PS R C

| | | Small | Medium | Large |
|---|---|---|---|---|
| **a** | Washing powder | 450 g for $3.99 | 600 g for $5.15 | 1 kg for $8.95 |
| **b** | Margarine | 350 g for $4.15 | 500 g for $5.99 | 750 g for $8.50 |
| **c** | Ice cream | 500 mL for $1.39 | 750 mL for $2.30 | 2 L for $5.99 |
| **d** | Peas | 400 g for $2.05 | 500 g for $2.49 | 600 g for $2.50 |

# 10.07 Rate problems

Problems involving rates can usually be solved by multiplying or dividing.

**Video**
Rate problems 1

**Worksheets**
Rate problems
Ratios and rates review

**Puzzle**
Rate problems

## Solving rate problems

- Write the units of the rate $x/y$ as a fraction: $\frac{x}{y}$
- To find the quantity in the numerator, $x$, **multiply** by the rate.
- To find the quantity in the denominator, $y$, **divide** by the rate.

## Example 15

Lucas types 55 words per minute. How many words can he type in 20 minutes?

### Solution

The units of the rate expressed as a fraction is $\frac{\text{words}}{\text{min}}$.

To find the number of words (the numerator), **multiply** by the rate.

$$\text{Number of words} = 20 \times 55$$
$$= 1100$$

No. of minutes × words typed per minute

**Video**
Rate problems 2

## Example 16

A factory makes pens at the rate of 50 pens per minute.

**a** How many hours and minutes will it take to produce 10 000 pens?

**b** How many pens are produced in an 8-hour day at the factory?

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## Solution

The units of the rate expressed as a fraction are $\frac{\text{pens}}{\text{min}}$.

**a** To find the number of minutes (the denominator), **divide** by the rate.

How many times 50 divides into 10 000.

$$\begin{aligned}\text{Time taken} &= 10\,000 \div 50 \\ &= 200 \text{ minutes} \\ &= \frac{200}{60} \text{ hours} \\ &= 3 \text{ h } 20 \text{ min}\end{aligned}$$

**b** To find the number of pens (the numerator), **multiply** by the rate.

$$\begin{aligned}\text{No. of pens in one hour} &= 60 \times 50 \\ &= 3000 \\ \text{No. of pens in 8 hours} &= 8 \times 3000 \\ &= 24\,000\end{aligned}$$

1 h = 60 min

No. of mins × pens per min



## EXERCISE 10.07 ANSWERS ON P. 603

### Rate problems U F PS R C

EXAMPLE 15

**1** Marisol is paid $24.20 per hour. How much will she earn if she works 38 hours in a week?

**2** A cricket batsman scores at a rate of 32 runs per innings. How many runs will he score in 5 innings?

EXAMPLE 16

**3** A car uses 14 L of petrol to travel 147 km. PS R

**a** Write this as a rate in km/L.

**b** How far can the car travel on 20 L of petrol?

**4** A farmer can graze 16 sheep per hectare. If he has 21 hectares set aside for sheep, how many sheep can he graze?

**5** A tap drips water at a rate of 18 mL/h. How much water would be wasted in one day? PS R

**6** An aeroplane can carry a total of 450 passengers per flight. How many flights would it take to carry 2700 passengers? PS R

**7** The Wong family needs to buy new carpet for the family home. The carpet costs $125/m and the carpet layers charge $480 to lay it. How much will it cost to carpet the house if the Wongs need 28 metres of carpet? PS R

**8** A farmer uses fertiliser at a rate of 28 kg/ha. How many hectares can she cover if she has 200 kg of fertiliser? PS R

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**9** Tatiana is paid $17.50/h to babysit. PS R

**a** How much does she earn for babysitting for 4 hours?

**b** How long did she babysit to earn $43.75?

**10** A plane travelled at 900 km/h. How far did it travel in 20 minutes? PS R

**11** **a** Petrol costs $2.03/L. If Rico had $30 in his pocket, how much petrol could he buy (correct to the nearest 0.1 L)?

**b** If Rico's car can travel 9 km on one litre of petrol, how far can he travel on the petrol he bought with $30? PS R

**12** Smokers lose approximately 3% of their lung capacity per year. If Ella with 100% lung capacity starts smoking at 20 years of age, at what age will she have 25% lung capacity? Select the correct answer **A**, **B**, **C** or **D**. PS R

**A** 75 **B** 28 **C** 45 **D** 33

**13** The temperature in Mittagong at 7 a.m. is 8°C. If the temperature rises at a rate of 3°C/h, find: PS R

**a** the temperature at 10 a.m. and at 2 p.m.

**b** the time at which the temperature will be 23°C.

**14** Keira and Xander take turns mowing a rectangular lawn 60 m long and 40 m wide. PS R

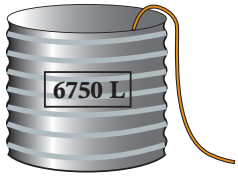
**a** Find the area of the lawn.

**b** When Keira cuts the grass, she does it in one hour using an old lawn mower. What is her mowing rate in $m^2$/min?

**c** Xander uses a new lawn mower and it takes him 40 minutes. What is his mowing rate in $m^2$/min?

**d** How long will it take them to mow the lawn if they mowed together?

**15** Jayden is filling this tank with a pipe that pumps water at 1.25 L per second.




How long will it take to fill the tank: PS R C

**a** in seconds? **b** in minutes? **c** in hours?

**16** Mikayla and Jesinta started their holiday on a full tank of petrol.

The reading on the odometer (in km) was

| 0 | 3 | 4 | 5 | 6 | 8 |
|---|---|---|---|---|---|

When the tank became empty, the reading was

| 0 | 3 | 5 | 2 | 1 | 8 |
|---|---|---|---|---|---|

Petrol cost $1.94/L and they needed to pay $93.12 to fill the tank. PS R

**a** How many kilometres did they travel on a full tank of petrol?

**b** How much petrol is there in a full tank?

**c** How far can their car travel on 1 L of petrol? Answer to the nearest 0.1 km.

# Speed 10.08

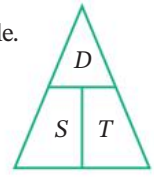
**Speed** is a rate that compares the distance travelled with the time taken. Average speed is calculated by dividing the distance travelled by the time taken.

## Speed

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

Speed can be measured in kilometres/hour (km/h) or metres/second (m/s).

The relationship between distance, speed and time can be remembered using this triangle.




If we cover the quantity we are looking for, the rest of the diagram tells us what to do.

- To find the speed, cover the $S$ and we are left with $\frac{D}{T}$, so speed = distance ÷ time
- To find the distance, cover the $D$ and we are left with $S \times T$, so distance = speed × time
- To find the time, cover the $T$ and we are left with $\frac{D}{S}$, so time = distance ÷ speed

## Example 17

A bullet train travels 320 km in 1 hour and 20 minutes.

**a** Calculate its average speed.

**b** How long will it take to travel 1000 km?

**c** How far will it travel in 5 hours?

### Solution

**a** $$\text{Average speed} = \frac{\text{distance}}{\text{time}}$$
$$= \frac{320 \text{ km}}{1 \text{ h } 20 \text{ min}}$$
$$= \frac{320 \text{ km}}{1\frac{1}{3} \text{ h}}$$
$$= 240 \text{ km/h}$$

$20 \text{ min} = \frac{20}{60} \text{ h} = \frac{1}{3} \text{ h}$

**b** $$\text{Time} = \frac{\text{distance}}{\text{speed}}$$
$$= \frac{1000}{240} \text{ h}$$
$$= 4\frac{1}{6} \text{ h}$$
$$= 4 \text{ h } 10 \text{ min}$$

By the speed triangle, or because we divide by the rate to find the h in $\frac{\text{km}}{\text{h}}$.

Press $\circ\,'\,''$ or DMS on the calculator.

Worksheets
Speed
What's my speed?

Puzzle
Speed

Presentation
Rates: Triangle method

Videos
Speed

Speed of the Earth

**c** Distance = speed × time
= 240 × 5
= 1200 km

By the speed triangle, or because we multiply by the rate to find the km in $\frac{\text{km}}{\text{h}}$.

## EXERCISE 10.08 ANSWERS ON P. 603

### Speed U F PS R C

EXAMPLE 17

**1** Find the average speed in km/h for each statement.

**a** A horse rider travels a distance of 15 km in 3 hours

**b** A jet travels 2400 km in 5 hours

**c** A bushwalker walks 25 km in 5 hours

**d** A cyclist rides 85 km in 5 hours

**e** A motorbike travels 250 km in 2.5 hours

**f** A car travels 280 km in 3.5 hours

**g** A truck travels 390 km in 3 hours and 15 minutes

**h** A car travels 90 km in 45 minutes

**i** A motorbike on the race track travels 250 km in 1 hour and 15 minutes

**j** A boat travels 15 km in 1 hour and 40 minutes

**k** An athlete runs 200 m in 20 seconds

**l** A kangaroo travels 1.5 km in 15 minutes

**2** Find the average speed in m/s for each statement.

**a** An athlete runs 200 m in 20 seconds

**b** A swimmer swims 100 m in 64 seconds

**c** A bird flies 348 m in 6 seconds

**d** A cyclist travels 960 m in 2 minutes

**e** A swimmer swims 1500 m in 16 minutes and 40 seconds

**3** Find the distance travelled for each statement.

**a** A car travels for 8 hours at an average speed of 90 km/h.

**b** A bushwalker travels for 7 hours and 30 minutes at an average speed of 4 km/h.

**c** A truck travels for 14 hours at an average speed of 95 km/h.

**4** Majid can walk at a speed of 6 km/h. How far can he walk in 30 minutes?

**5** A boat travels 24 km in 90 minutes. What is its average speed? Select the correct answer **A**, **B**, **C** or **D**.

**A** 12 km/h **B** 16 km/h **C** 18 km/h **D** 36 km/h

**6** Peter walked to school in 15 minutes, a distance of 1500 metres. Find his speed in: R C

**a** metres per minute **b** kilometres per hour

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**7** Mrs Hassam lives 20 km from the nearest railway station and it takes her 30 minutes to drive there in her car. Find her average speed in km/h.

**8** A racing car travels at an average speed of 210 km/h for 2 hours and 20 minutes. How far did it travel? Select **A**, **B**, **C** or **D**. PS R

**A** 90 km **B** 294 km **C** 462 km **D** 490 km

**9** A flywheel rotates at a rate of 2400 revolutions per minute. How many revolutions does it make in 30 seconds? R

**10** How long does it take a motor scooter to travel 60 km at a speed of 40 km/h? R

**11** There are other units for measuring speed, such as the **knot** and the **mach.** Find what these special terms mean and where they are used. C

**12** A bus travels at 75 km/h for 5 hours, and then travels a further distance of 200 km in 4 hours. Find the average speed for the whole journey, correct to one decimal place. PS R

**13** A car travels for 3 hours at 60 km/h and then travels a further distance of 100 km at 50 km/h. What is the average speed for the trip? PS R

**14** Kieran swims at a speed of 1.85 m/s. What would be his time (to 2 decimal places) for a 100 metre sprint? PS R

**15** Nia runs 200 metres in 24 seconds. How far will she run in one minute at the same rate? PS R

## Did you know?

### Running speeds

The current world records for some of the Olympics running events are shown below, along with the equivalent speed in km/h.

| Event | World record | Equivalent speed |
|---|---|---|
| **Mens** | | |
| 100 m | 9.58 s | 37.58 km/h |
| 200 m | 19.19 s | 37.52 km/h |
| 400 m | 43.03 s | 33.47 km/h |
| 800 m | 1 min 40.91 s | 28.54 km/h |
| 1500 m | 3 mins 26 s | 26.21 km/h |
| 4 × 100 m relay | 36.84 s | 39.09 km/h |
| **Womens** | | |
| 100 m | 10.49 s | 34.52 km/h |
| 200 m | 21.34 s | 33.74 km/h |
| 400 m | 47.6 s | 30.25 km/h |
| 800 m | 1 min 53.28 s | 25.42 km/h |
| 1500 m | 3 mins 50.07 s | 23.47 km/h |
| 4 × 100 m relay | 40.82 s | 35.28 km/h |



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**What do you notice about the speeds as the length of the race increases?**
**Why would the speeds for the 4 × 100 m relays be faster than the speeds for 400 m?**

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## Technology

### Travelling distances and times

Visit a map website such as Google Maps or WhereIs for this investigation to examine travelling distances and times between 2 locations.
Aidan lives near the Wyong Hill Reserve in NSW and is travelling to the Tuggerah shopping centre to see a movie.

1 Use the website to locate both Wyong Hill Reserve and Tuggerah shopping centre, then find the travelling distance and time for Aidan's trip if he is going by car.

2 Find the travelling distance and time for Aidan's trip if he is travelling on foot (walking).

3 How much further would you have to travel if you were going from the Wyong Hill Reserve to the shops by car instead of walking there?

4 Would you recommend that Aidan travel by car or on foot for the trip? Why?

5 **a** Calculate the average speed of the car for the trip.

**b** Why do you think this average speed is slower than the normal speed limit of 60 km/h?

6 Aidan is meeting Alinta at Tuggerah to see a movie. Alinta lives in Berkeley Vale. Find the travelling distance and time for Alinta's trip if she is travelling by car.

7 If Alinta's actual driving time was 5 minutes, what was her average speed?

8 If Aidan's actual speed was 80 km/h, how many minutes and seconds will his trip take?

9 So will Aidan or Alinta arrive at Tuggerah first? By how many minutes and seconds?

# 10.09 Converting rates

**Worksheets**
Converting rates

More rates

What is a speed of 90 km/h in m/s? Sometimes it is useful to be able to convert rates from one set of units to another. Doing this helps us to compare rates for different circumstances.

## Converting rates

To convert a rate from one pair of units to another:

- write the first rate as a fraction, including its units
- convert the unit on the numerator of the fraction to the new unit
- convert the unit on the denominator to the new unit
- simplify the rate

## Example 18

Convert 90 km/h to m/s.

### Solution

$90 \text{ km/h} = \dfrac{90 \text{ km}}{1 \text{ h}}$ — Writing the rate as a fraction.

$90 \text{ km} = 90 \times 1000 \text{ m}$
$= 90\,000 \text{ m}$ — Converting 90 km (the numerator) to metres.

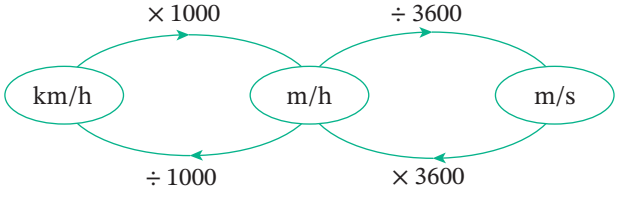
$1 \text{ h} = 60 \times 60 \text{ seconds}$
$= 3600 \text{ s}$ — Converting 1 hour (the denominator) to seconds.

$90 \text{ km/h} = \dfrac{90\,000 \text{ m}}{3600 \text{ s}}$ — Substituting these values back into the rate.

$= 25 \text{ m/s}$ — Dividing to simplify.

A car travelling at a speed of 90 km/h covers 25 metres in one second.

You may find this diagram useful for converting between km/h and m/s.




## Example 19

A farmer plants seed at a rate of 45 grams per square metre.

Convert 45 g/m$^2$ to kg/ha.

### Solution

$45 \text{ g/m}^2 = \dfrac{45 \text{ g}}{1 \text{ m}^2}$ — Writing the rate as a fraction.

$45 \text{ g} = 45 \div 1000 \text{ kg}$
$= 0.045 \text{ kg}$ — Converting 45 g to kg using 1 kg = 1000 g.

$1 \text{ m}^2 = 1 \div 10\,000 \text{ ha}$
$= 0.0001 \text{ ha}$ — Converting 1 m$^2$ to ha using 1 ha = 10 000 m$^2$.

$45 \text{ g/m}^2 = \dfrac{0.045 \text{ kg}}{0.0001 \text{ ha}}$ — Substituting these values back into the rate.

$= 450 \text{ kg/ha}$ — Dividing to simplify.

A seeding rate of 45 g/m$^2$ requires 450 kg of seed to cover one hectare.

**Videos**
Converting rates 1
Converting rates 2

10.09

## EXERCISE 10.09 ANSWERS ON P. 604

### Converting rates UFRC

In this exercise, round answers to one decimal place if needed. R C

**1** Convert:

**a** $92 424/year to $/month **b** 75 km/h to km/min **c** 5 L/h to L/day

**d** 3 m/s to m/min **e** 0.8 cm/min to cm/h **f** 78 words/min to words/s

**g** 8.2 L/100 km to L/km **h** 1750 mL/s to mL/min

EXAMPLE 18

**2** Convert each speed into m/s.

**a** 20 km/h **b** 50 km/h **c** 80 km/h **d** 110 km/h

**3** Convert:

**a** 24c/min to $/hour **b** 8 m/s to km/h **c** 15 mL/s to L/h

**d** $21.60/h to c/min **e** 60 km/h to m/s **f** 3 t/day to kg/min

**4** Asha is walking at a rate of 80 metres per minute. What is her speed in:

**a** km/h? **b** m/s?

**5** Convert each speed to km/h.

**a** 12.5 m/s **b** 2400 m/min **c** 3500 m/h

EXAMPLE 19

**6** Siobhan fertilises her land at the rate of 4 g/m$^2$. Express this rate in kg/ha.

**7** A tap leaks at a rate of 30 mL per minute. What is this rate in litres per hour?

**8** The steel mesh used in fencing weighs 1.2 kg/m. What is this rate in:

**a** g/mm? **b** tonnes/km?

**c** What do you notice about the answers to **a** and **b**? Can you explain why?

**d** What is the rate in g/cm?

**9** Jesse pumps water from his dam to irrigate his market gardens. The pump delivers water at the rate of 3.5 kL/h. What rate is this expressed as:

**a** L/h? **b** L/min? **c** L/s?

**10** A 5 L container of varnish costs $14.00.

**a** What is the cost in $/L?

**b** Express this as a cost in cents/mL.



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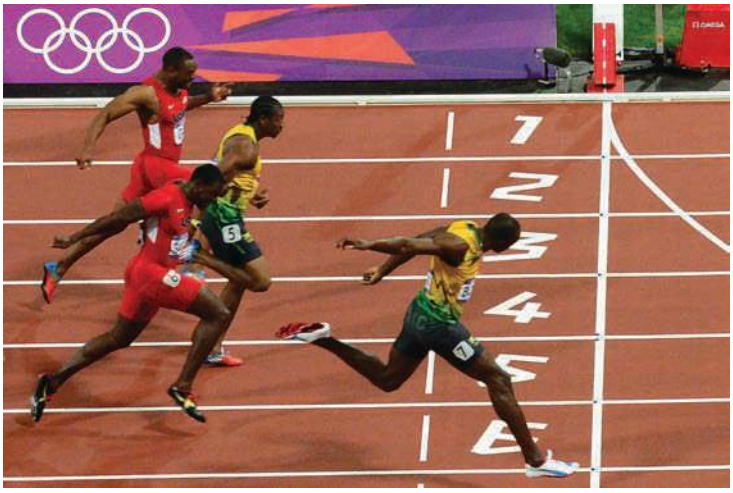


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**11** Jamaica's Usain Bolt set the world record for the 100 m sprint in 2009. He ran the distance in 9.58 seconds.

**a** Express this as a rate in m/s (to 2 decimal places)

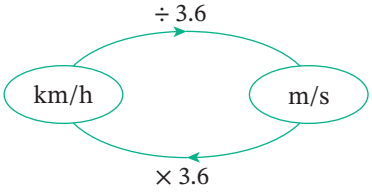
**b** Convert this speed to km/h (to the nearest km/h).



Getty Images/EMMANUEL DUNAND/AFP

**12** The world's fastest cricket bowler is Pakistan's Shoaib Akhtar. In 2003, he bowled a ball 20.12 m in 0.45 seconds. Calculate the speed of the ball he bowled.

**13** Explain why the method shown in this diagram works for converting between km/h and m/s. Test it on Questions **2**, **11** and **12**.



# 10.10 Travel graphs

A **travel graph** or **distance–time graph** is a line graph that describes a journey, by comparing distance (on the vertical axis) with time (on the horizontal axis). The slope or steepness of the graph indicates the speed.

**Worksheets**
Travel graph stories
The hare and the tortoise

**Puzzle**
Jane's diary

**Video**
Travel graphs

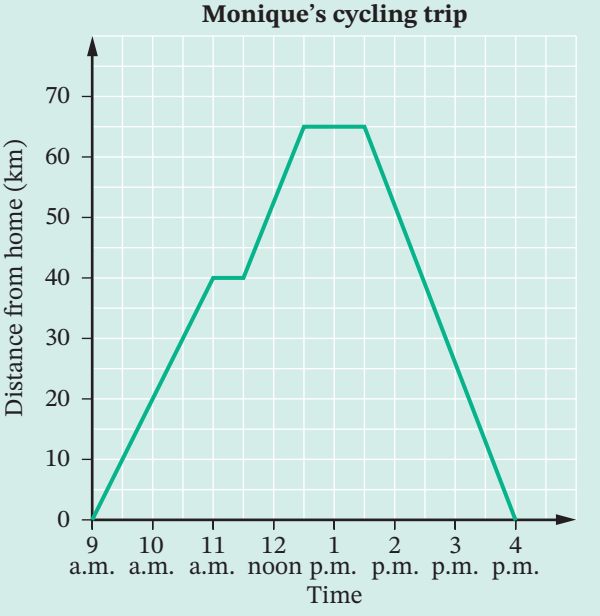
## Travel graphs

On a **travel graph**:

- a horizontal (flat) section on the graph indicates a stop (the traveller is stationary)
- the steeper the line, the greater the speed (more distance covered in less time)
- a section going down, towards the right, indicates a change in direction or that the traveller is returning towards the start.

## Example 20

This graph shows Monique's cycling trip.




**a** At what time did Monique leave home?

**b** When was Monique's first stop? How far from home was she?

**c** Find her average speed over the first 2 hours.

**d** What time was it when Monique began her journey home?

**e** How far did she travel all together?

**f** Find her average speed during the trip home.

**g** For how long did Monique stop altogether during the trip?

### Solution

**a** Monique left home at 9 a.m.

At the start of the graph, when distance = 0.

Where the graph is flat, the distance from home does not change, which means that Monique has stopped

**b** Monique first stopped at 11 a.m., 40 km from home. This is where the graph is flat.

**c** Distance = 40 km, time = 2 hours

$$\text{Average speed} = \frac{40 \text{ km}}{2 \text{ h}} = 20 \text{ km/h}$$

**d** Monique started returning home at 1:30 p.m. This is where the graph points downward.

**e** Monique travelled 65 km, then returned home.

Total distance = $2 \times 65$ km = 130 km

**f** Distance = 65 km, time = $2\frac{1}{2}$ hours

$$\text{Average speed} = \frac{65 \text{ km}}{2\frac{1}{2} \text{ h}} = 26 \text{ km / h}$$

**g** First stop: $\frac{1}{2}$ hour. Second stop: 1 hour

Total stopping time = $\frac{1}{2} + 1 = 1\frac{1}{2}$ hours

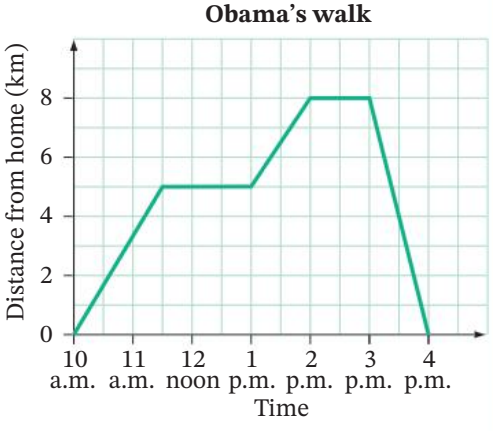


## EXERCISE 10.10 ANSWERS ON P. 604

### Travel graphs U F PS R C

EXAMPLE 20

**1** This travel graph shows Obama's return trip to his friend Joe's house. R C




**a** How long did it take Obama to walk to Joe's house?

**b** How far is Joe's house from Obama's house?

**c** Find Obama's average speed:

**i** before his first stop

**ii** after his first stop

**d** How is a higher speed shown on the graph?

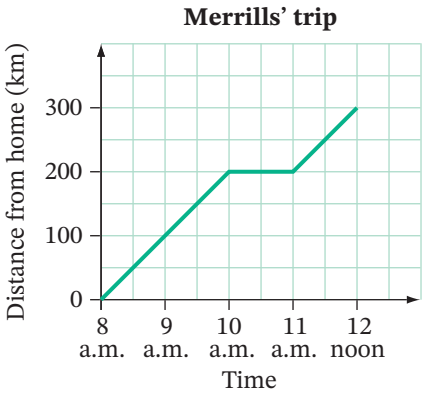
**e** What was the total distance that Obama travelled?

**f** Between what times does Obama stop on his walk?

**g** When did Obama start his trip home?

**h** How long did it take him to walk home?

**2** Merrill drives from Sydney to Canberra, stopping to visit friends in Goulburn. R C




**a** How far is Canberra from Sydney?

**b** How long does the trip take?

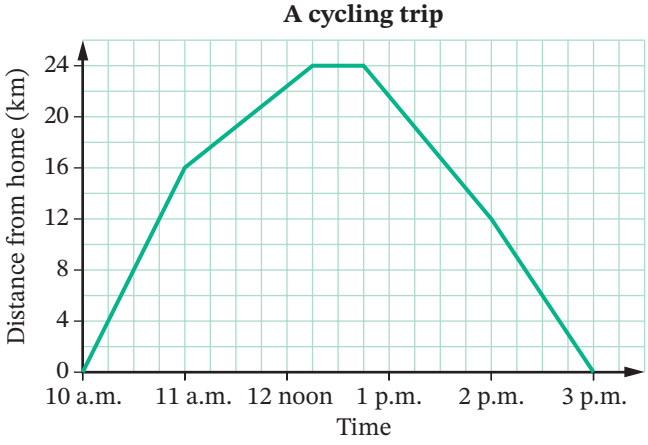
**c** For how long does Merrill stop at Goulburn?

**d** Find her average speed for the journey (excluding stops).

**e** How far is it from Goulburn to Canberra?

**f** When does Merrill travel faster: before or after Goulburn?

**3** This graph shows a cyclist's day trip. R C




**a** At what time did the speed of the cyclist

**i** increase? **ii** decrease?

**b** When did the cyclist start to return home?

**c** How far did the cyclist travel altogether on this day?

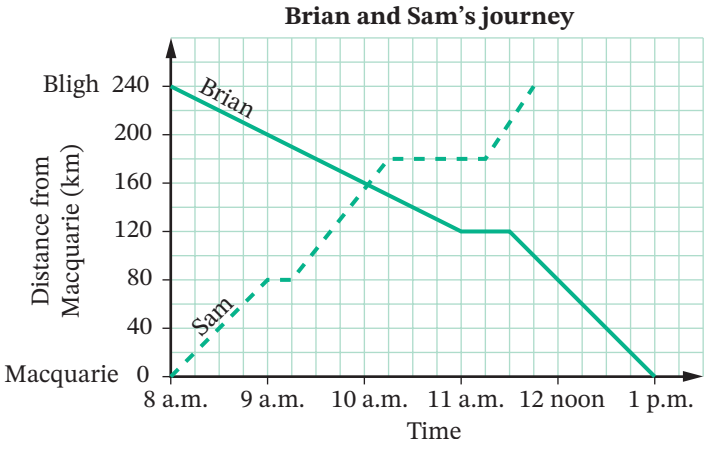
**d** How long did the cyclist spend 'on the road'?

**e** Find the cyclist's average speed for:

**i** the first hour **ii** 11 a.m. to 12:15 p.m.

**iii** 12:45 p.m. to 2 p.m. **iv** the entire day

**4** Brian travels from Bligh to Macquarie, while Sam travels from Macquarie to Bligh. R C




**a** How far is it between the 2 towns?

**b** Who is travelling faster? How can you tell?

**c** How far is Brian from Macquarie at 11 a.m.?

**d** How far is Sam from Macquarie at 11 a.m.?

**e** At what time do Brian and Sam pass each other? How far are they from Macquarie when they pass?

**f** Is Sam travelling faster before 9 a.m. or after 9:15 a.m.? How does the graph show this?

**g** Calculate Brian's average speed before he stops.

**h** For how long did Sam stop altogether on the trip? Select **A**, **B**, **C** or **D**.

**A** 60 minutes **B** 75 minutes **C** 5 minutes **D** 90 minutes

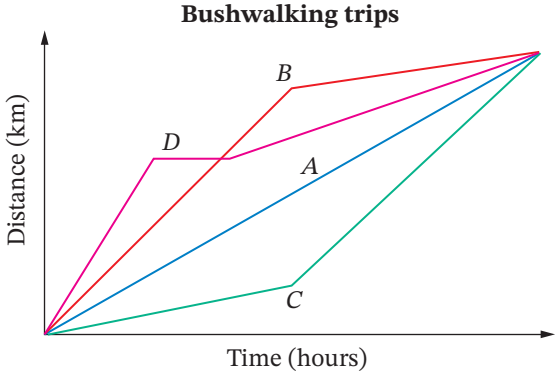
**5** Match each bushwalker described below with the correct travel graph. R C

**a** Santosh maintained the same speed all day.

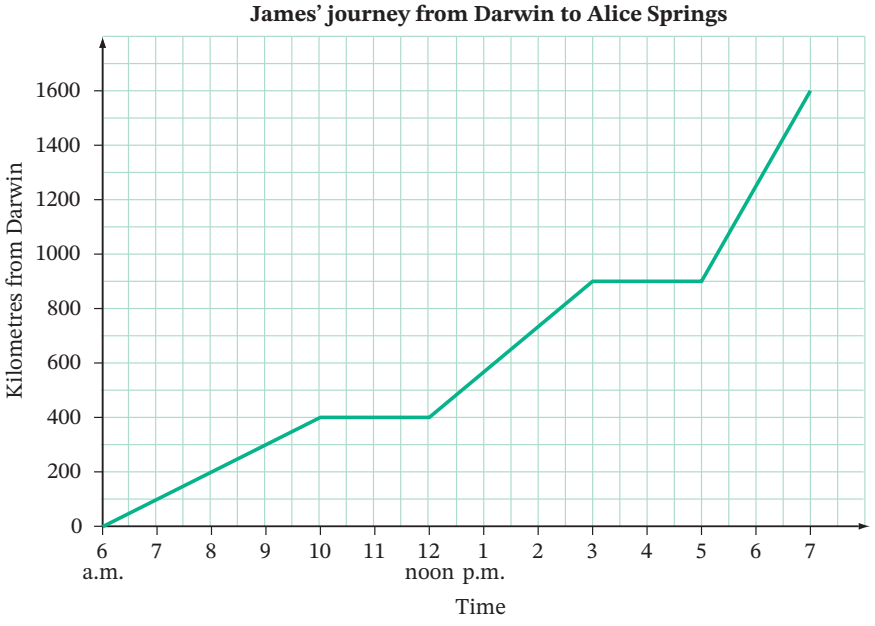
**b** Michelle was fast at first, but slowed down.

**c** Tahlia was the fastest, but then got slower after stopping for lunch.

**d** Jim was slow at first, but picked up speed.



**6** This graph illustrates James' trip from Darwin to Alice Springs. PS R C




**a** How far is Alice Springs from Darwin?

**b** When did James arrive in Alice Springs?

**c** What was James' average speed in the last 2 hours?

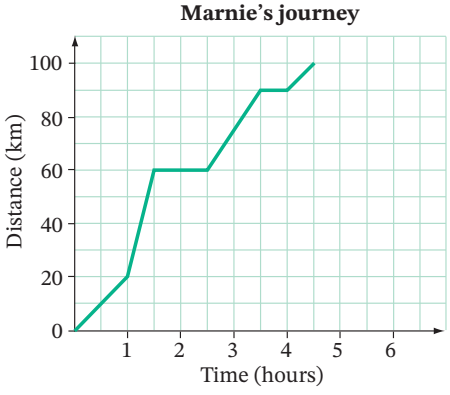
**d** Which of the following statements is false? Select **A**, **B**, **C** or **D**.

- **A** James stopped for 4 hours altogether.
- **B** James was travelling fastest in the last 2 hours.
- **C** James' speed for the first 4 hours was 100 km/h.
- **D** James' average speed for the whole trip was 120 km/h.

**e** Samantha leaves Darwin one hour after James and travels at a constant speed of 125 km/h towards Alice Springs. Copy the graph and add Samantha's journey to it.

**f** Find the approximate time when Samantha overtakes James.

**7** This travel graph shows Marnie's cycling journey. Write a story about her ride, based on the information in the graph. C

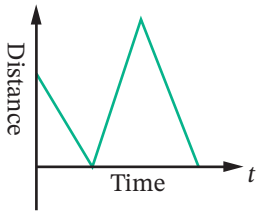



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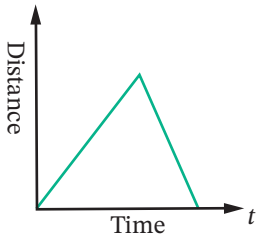

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**8** Which one of these graphs could **not** be a travel graph? Why not? R C

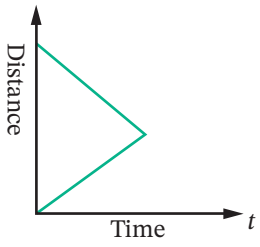
**a**



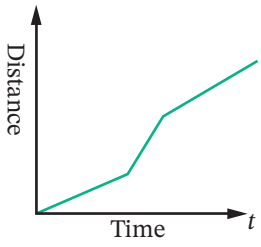

**b**




**c**




**d**



## Power plus ANSWERS ON P. 604

**1** Australia's annual birth rate in 2021 was approximately 12.1 per 1000. Given that its population then was 26 439 000, approximately how many babies were born in Australia in 2021?

**2** Singapore is a small island, but it has a population of 5 780 000, making it the 2nd most densely populated country in the world. Its population density is 8210 persons/km$^2$.

**a** What is the area of Singapore, to the nearest square km?

**b** Australia is the 3rd least-densely populated country in the world, at a rate of only 3.4 persons/km$^2$ and an area of 7 682 300 km$^2$. If Australia was as densely populated as Singapore, what would its population be?

**3** A spider moves at 1 cm/s. If the spider is in the back left-hand corner of your classroom, find how long (in minutes) it will take to reach:

**a** the nearest person **b** you **c** the teacher's desk

**4** Convert each speed to km/h. Round your answers to 2 decimal places.

**a** An African cheetah runs at 27 m/s.

**b** A German peregrine falcon dives at 97 m/s.

**c** A Tanzanian snake travels at 3.3 m/s.

**d** A racing cyclist rides at 23 m/s.

**5** Work out a formula for converting speed expressed as $x$ m/s to $y$ km/h.

# CHAPTER 10 REVIEW

## Language of maths

| | | | |
|---|---|---|---|
| best buy | distance | divide | equivalent |
| km/h | map | per (/) | plan |
| rate | ratio | scaled length | scale ratio |
| simplify | speed | stationary | term |
| travel graph | unitary method | unit price | |

**1** When comparing the prices and sizes of items to find the **best buy**, do we look for the highest or lowest **unit price**?

**2** What type of measurement compares quantities of different types, expressed using 2 units?

**3** What is meant by the **scale ratio** of a map or plan?

**4** What does an **average speed** of 65 km/h actually mean?

**5** How is a change in speed shown on a travel graph?

**6** What does it mean if a scale drawing has a scale of 20 : 1?

**Quiz**
Language of maths 10

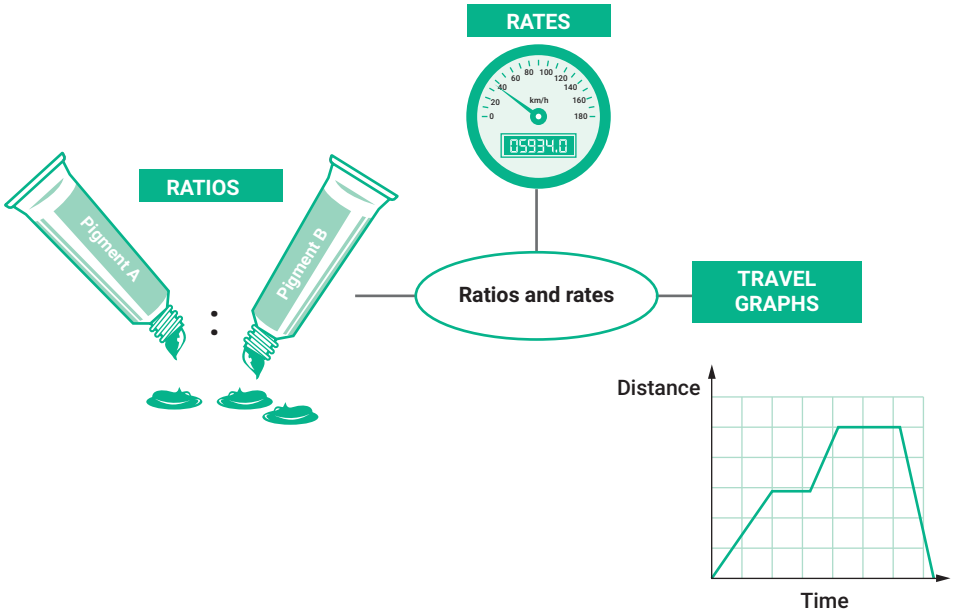
**Puzzle**
Ratios, rates and time crossword

## Topic summary

- Give examples of situations where ratios and rates are used.
- What did you learn in this chapter?
- What did you find the most difficult about this topic? Discuss any problems with your teacher or with a friend.

**Worksheet**
Mind map: Ratios and rates

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Quiz
Test yourself 10

# TEST YOURSELF 10

ANSWERS ON P. 604

10.01

**1** Copy and complete each pair of equivalent ratios.

**a** 1 : 3 = 4 : ____ **b** 2 : 5 = 6 : ____ **c** 3 : 7 = ____ : 21

**d** 4 : 9 = ____ : 45 **e** 0.5 : 0.8 = 2 : ____ **f** 3 : 4 : 5 = ____ : 20 : ____

10.01

**2** Simplify each ratio.

**a** 12 : 21 **b** 25 : 75 **c** 6 : 36

**d** 25 : 45 **e** 18 : 6 : 24 **f** 5 : 25 : 100

**g** $\frac{1}{3} : \frac{1}{2}$ **h** $\frac{3}{4} : \frac{9}{16}$ **i** $1\frac{1}{2} : 2\frac{1}{2}$

**j** 0.98 : 0.245 **k** 1.5 : 6 **l** 91 : 5.6

**m** 5 km : 200 m **n** 20 kg : 3600 g **o** $25 : $4.25

**p** 18 months: 4 years **q** 10 days: 5 weeks **r** 8 h : 3 days

10.02

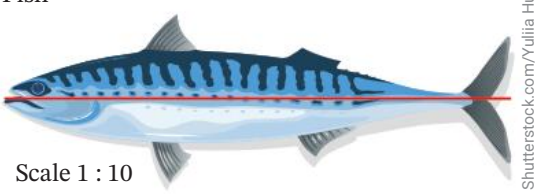
**3** **a** The ratio of girls to boys in Year 8 is 4 : 3. If there are 75 boys in Year 8, find how many girls there are.

**b** Adam and Eve invested in a business in the ratio 5 : 7. If Eve invested $63 000, how much did Adam invest?

10.03

**4** Measure the length of each scale drawing below, and then use the ratio to work out the actual length of the object correct to the nearest centimetre.

**a** Fish



Scale 1 : 10

Shutterstock.com/Yulia Husar

**b** Frog



Scale 1 : 4

Shutterstock.com/SGV_Arts

10.03

**5** On a tourist map of Sydney, the scale is given by 0 ——— 500 m.

**a** Write this scale as a simplified ratio.

**b** Find the actual distance between the following places given the scaled distance.

**i** Circular Quay station to the Opera House (2.5 cm)

**ii** Pyrmont Bridge to Parliament House (4.4 cm)

**c** Find the scaled distance between the following places given the actual distance.

**i** Art Gallery of NSW to Sydney Tower (875 m)

**ii** Circular Quay station to Central station (2.5 km)

10.04

**6** Ellen and Portia share the weekly rent of $420 on their apartment in the ratio 4 : 3. How much does Ellen pay? Select the correct answer **A**, **B**, **C** or **D**.

**A** $105 **B** $240 **C** $180 **D** $210

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**7** Write each statement as a rate in simplified form. 10.05

**a** $10.50 for 3 kg
**b** 220 km in 2 hours
**c** $56.40 for 4 hours
**d** 260 runs in 50 overs

**8** For each pair, determine which is the better buy. 10.06

**a** A 1 kg box of biscuits for $5.60 or a 700 g carton for $4.00.
**b** A 350 g can of soup for $4.20 or a 550 g can for $7.15.
**c** A 1.5 L bottle of lemonade for $2.45 or a 375 mL can for $0.70.
**d** 3 × 40 g chocolate bars for $2.36 or a 500 g chocolate bar for $9.

**9** **a** Mince is $9.99/kg. How much does 5 kg of mince cost? 10.07
**b** Tiwa earns $31.70/h. How much is she paid for 38 hours work?
**c** How many litres of petrol can you buy with $60 if petrol costs $1.95/L? Answer to the nearest 0.1 L.
**d** Fertiliser is used at 20 kg/ha. How many hectares can be covered with 144 kg of fertiliser?

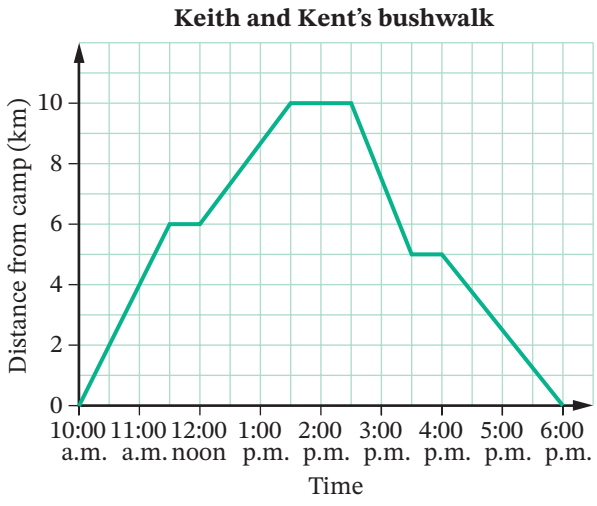
**10** Find the speed, in km/h, of a cyclist who travels a distance of 45 km in 2 hours and 15 minutes. 10.08

**11** Find the distance covered by a truck that travels for 5 hours and 30 minutes at an average speed of 45 km/h. 10.08

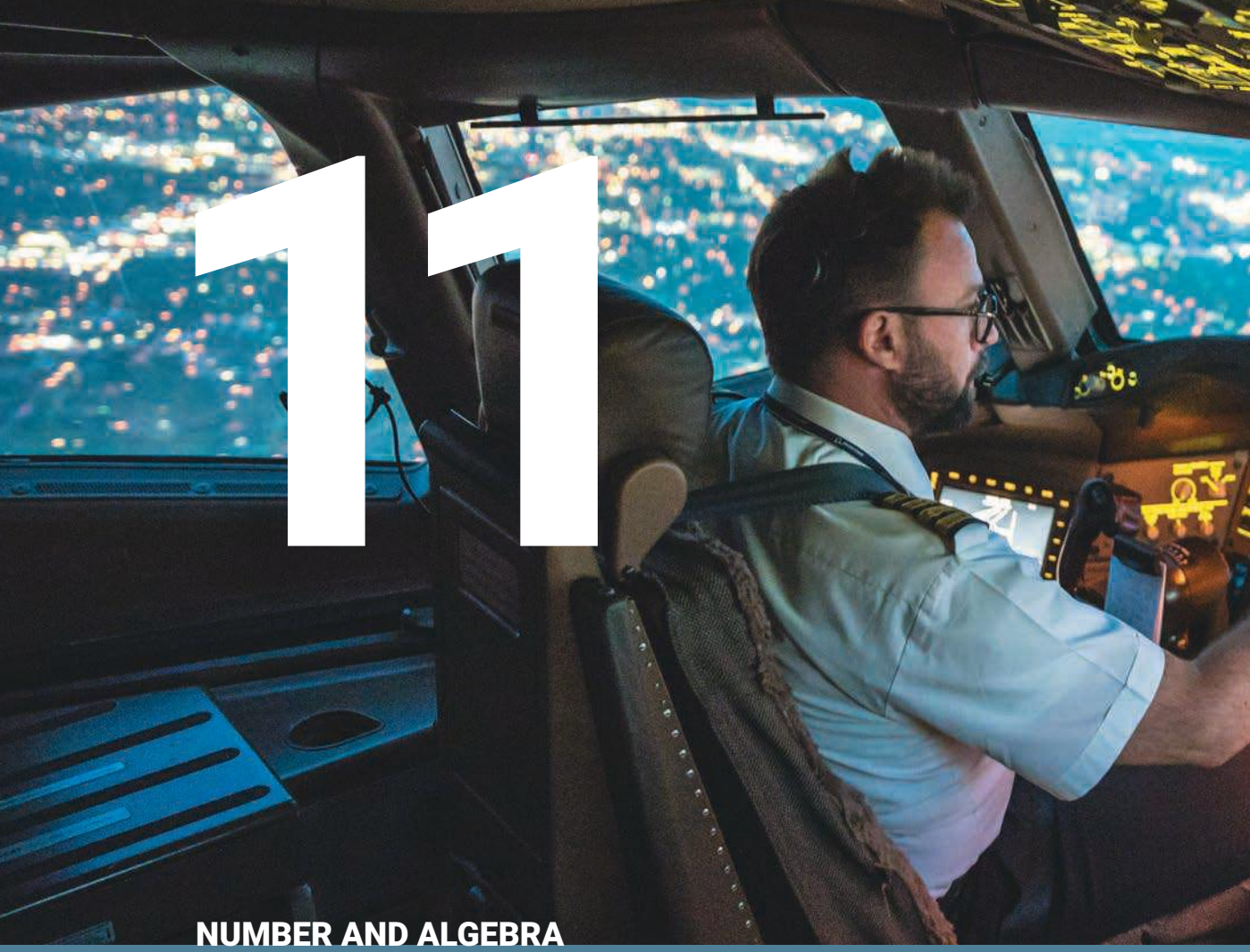
**12** **a** Convert a speed of 75 km/h into m/s, correct to one decimal place. 10.09
**b** A large vat is being filled at a rate of 35 mL/min. Convert this rate to a rate of L/day.

**13** Keith and Kent decided to go bushwalking. This travel graph shows their walk. 10.10

**a** How far did they travel from camp?
**b** How many stops did they make?
**c** Find their average speed between their first and second stops.
**d** How long did it take them to return home?
**e** Between what times were they walking the fastest?




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# 11

NUMBER AND ALGEBRA

# GRAPHING LINEAR EQUATIONS

The famous French philosopher and mathematician René Descartes (1596–1650) is famous for discovering the connections between algebra and geometry. His concept of coordinate geometry connected algebra and geometry using the graphs of straight lines and curves. In computer modelling, the use of coordinates allows us to locate things more precisely, especially in three-dimensional (3D) space. Number plane graphs are used to track changes over short and long periods of time, such as the height and speed of an aeroplane.




## Chapter outline

| | | Working mathematically | | | | |
|---|---|---|---|---|---|---|
| **11.01** | Tables of values | U | F | | | |
| **11.02** | Finding the rule | U | F | | R | C |
| **11.03** | Finding rules for number patterns | U | F | PS | R | C |
| **11.04** | The number plane | U | F | | R | C |
| **11.05** | Graphing number patterns | U | F | | R | C |
| **11.06** | Graphing linear equations | U | F | | R | C |
| **11.07** | Finding the equation of a line | U | F | | R | C |
| **11.08** | Comparing linear equations | U | F | | R | C |
| **11.09** | The gradient of a line* | U | F | | R | C |
| **11.10** | The linear equation $y = mx + c$* | U | F | | R | C |
| **11.11** | Applying linear equations | U | F | PS | R | C |
| **11.12** | Solving linear equations graphically | U | F | | | |
| **11.13** | Intersecting lines | U | F | | R | |

*YEAR 9 EXTENSION

## Wordbank

**consecutive numbers** Numbers that follow each other in order, for example, 2, 3, 4

**constant term** The term in an equation that is a number only and does not contain a variable, for example, the 5 in $y = 2x + 5$

**horizontal** Going across, sideways, flat

**linear** Involving a line

**linear equation** A formula whose graph is a straight line

**number plane** A coordinate grid system based on the $x$- and $y$-axes, also called a Cartesian plane

**table of values** A table of ordered pairs of numbers, usually following a formula and which can be graphed on a number plane

**vertical** Going up and down, at a right angle to the horizontal

**$y$-intercept** The $y$-value at which a line crosses the $y$-axis on the number plane

**Quiz**
Wordbank 11

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

## Videos (10):

**11.02** Finding the rule
**11.04** The number plane
**11.05** Graphing number patterns
**11.06** Linear functions • Graphing linear equations
**11.07** Finding the equation of a line • Horizontal and vertical lines
**11.08** Linear functions
**11.09** The gradient of a line • Gradient (slope) of a line
**11.10** Graphing $y = mx + c$

## *Twig* video (1):

**11.04** Coordinate geometry: Descartes

## *PhET* interactives (4):

**11.02** Function builder: Basics • Function builder
**11.06** Graphing lines
**11.08, 11.10** Graphing slope-intercept

## Skillsheet (1):

**11.04** The number plane

## Quizzes (5):

- Wordbank 11
- SkillCheck 11
- Mental skills 11
- Language of maths 11
- Test yourself 11

## Worksheets (17):

**11.01** Tables of values
**11.03** Patterns and rules
**11.04** The number plane • Plane grid 2 • Number plane review • Number plane grid paper • A page of number planes • Plotting points and lines • Number plane writing activity
**11.05** Number plane grid paper • A page of number planes • Graphing tables of values
**11.06** Number plane grid paper • A page of lines • A page of number planes
**11.07** Finding linear equations
**11.09** Drawing gradients • A page of lines • Gradient between two points
**11.09, 11.10, 11.12, 11.13** Number plane grid paper • A page of number planes
**11.11** Number plane grid paper • 1 cm grid paper
**11.13** Line intersections
**Mind map** Graphing linear equations

## Puzzles (6):

**11.04** Coordinates code puzzle
**11.06** Matching linear equations • Graphing functions
**11.07, 11.08** Matching linear equations
**11.09** Find the gradient 1
**11.10** Equation of a line
**Language of maths** Linear equations crossword

## Technology (2):

**11.04** Plotting points
**11.06** Graphing straight lines: Finding intercepts

## Presentation (1):

**11.04** Graph coordinates

## Spreadsheets (2):

**11.04** Points plotter
**11.07** Linear functions plotter

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

## In this chapter you will:

- complete a table of values to graph a linear equation on a number plane
- identify and describe a number pattern, then represent it algebraically and graphically on a number plane
- determine the formula for a table of values and set of points on a number plane
- compare and contrast the graphs of different linear equations
- (EXTENSION, YEAR 9 CORE) find the gradient of an interval or line
- (EXTENSION, YEAR 9 CORE) use the gradient-intercept equation of a straight line $y = mx + c$

- solve problems that can be modelled by linear equations
- solve linear equations graphically
- graph 2 intersecting lines and identify the point of intersection

## SkillCheck ANSWERS ON P. 605

Quiz SkillCheck 11



**1** For each equation, find the missing number.

**a** $2 \times 3 + ___ = 10$ **b** $2 \times (-1) + ___ = 3$ **c** $3 \times 1 - ___ = 1$

**d** $-2 \times 2 + ___ = -2$ **e** $4 \times 3 - ___ = 8$ **f** $-1 \times (-1) - ___ = -4$

**2** Evaluate each expression.

**a** $3x - 2$ if $x = 2$ **b** $\frac{1}{2}x + 3$ if $x = 6$ **c** $2x - 1$ if $x = -1$

**d** $-x + 1$ if $x = 0$ **e** $-2x - 1$ if $x = 1$ **f** $10 - x$ if $x = 3$

**3** Solve each equation.

**a** $3x - 2 = 7$ **b** $-2x - 1 = 3$ **c** $-x + 1 = 4$

**d** $2x - 1 = -5$ **e** $3x + 2 = x - 1$ **f** $x - 2 = 4 - x$

# Tables of values

11.01

In order to graph straight lines, we need points. In Year 7, you were given a **table of values** to plot on the number plane, or used a rule or formula to create a table of values.

Worksheet Tables of values

## Example 1

Complete this table of values using the formula $v = 2u + 1$.

| $u$ | −2 | −1 | 0 | 1 | 2 | 6 | 9 |
|---|---|---|---|---|---|---|---|
| $v$ | | | | | | | |

### Solution

Substitute each value of $u$ in the table into the formula $v = 2u + 1$ to find the value of $v$.

- When $u = -2$, $v = 2 \times (-2) + 1 = -3$
- When $u = -1$, $v = 2 \times (-1) + 1 = -1$
- When $u = 0$, $v = 2 \times 0 + 1 = 1$
- When $u = 1$, $v = 2 \times 1 + 1 = 3$
- When $u = 2$, $v = 2 \times 2 + 1 = 5$
- When $u = 6$, $v = 2 \times 6 + 1 = 13$
- When $u = 9$, $v = 2 \times 9 + 1 = 19$

Completing the table, we have:

| $u$ | −2 | −1 | 0 | 1 | 2 | 6 | 9 |
|---|---|---|---|---|---|---|---|
| $v$ | −3 | −1 | 1 | 3 | 5 | 13 | 19 |

**EXERCISE 11.01** ANSWERS ON P. 605

## Tables of values U F

EXAMPLE 1

**1** Complete each table of values given the formula.

**a** $y = x - 1$

| $x$ | 3 | 7 | −1 | −8 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$ | | | | | 3 | |

**b** $q = 3p$

| $p$ | 8 | −3 | 2 | 5 | 10 | −4 |
|---|---|---|---|---|---|---|
| $q$ | | | | 15 | | |

**c** $k = h \div 2$

| $h$ | 12 | 8 | −2 | −6 | 0 | 7 |
|---|---|---|---|---|---|---|
| $k$ | 6 | | | | | |

**d** $y = x + 7$

| $x$ | −2 | 11 | 7 | 0 | 13 | −9 |
|---|---|---|---|---|---|---|
| $y$ | | 18 | | | | |

**e** $b = 2a$

| $a$ | 8 | −1 | −5 | 0 | 4 | 2 |
|---|---|---|---|---|---|---|
| $b$ | | | −10 | | | |

**f** $y = x - 3$

| $x$ | 10 | −4 | 7 | 5 | −8 | 11 |
|---|---|---|---|---|---|---|
| $y$ | 7 | | | | | |

**2** Given the rule $y = -x - 4$, what is the value of $y$ when $x = -3$? Select **A**, **B**, **C** or **D**.

**A** $-1$ **B** $-7$ **C** 7 **D** 12

**3** Complete each table of values given the formula.

**a** $p = 5m - 1$

| $m$ | 3 | 10 | 1 | 4 | 8 | −6 |
|---|---|---|---|---|---|---|
| $p$ | | 49 | | | | |

**b** $q = 3p - 2$

| $p$ | 4 | −1 | 7 | 10 | 2 | −6 |
|---|---|---|---|---|---|---|
| $q$ | | | 19 | | | |

**c** $t = 4r + 6$

| $r$ | 2 | 0 | −3 | 9 | −5 | 4 |
|---|---|---|---|---|---|---|
| $t$ | 14 | | | | | |

**d** $z = 5y + 2$

| $y$ | 4 | 8 | 7 | −5 | 6 | −2 |
|---|---|---|---|---|---|---|
| $z$ | | 42 | | | | |

**e** $y = 2x - 3$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

**f** $b = \frac{1}{2}a - 2$

| $a$ | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $b$ | | | | | | |

**g** $k = 3h + 5$

| $h$ | −1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $k$ | | | | | | |

**h** $y = 1 - x$

| $x$ | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

**4** Which table of values matches the formula $y = 3x - 1$? Select **A**, **B**, **C** or **D**.

**A**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 2 | 3 | 4 | 5 | 6 |

**B**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 2 | 5 | 8 | 11 | 14 |

**C**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 4 | 7 | 10 | 13 | 16 |

**D**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 0 | 3 | 6 | 9 | 12 |

**5** Which table of values follows the rule $c = \frac{h+2}{2}$? Select **A**, **B**, **C** or **D**.

**A**

| $h$ | 0 | 2 | 4 |
|---|---|---|---|
| $c$ | 1 | 3 | 5 |

**B**

| $h$ | 0 | 2 | 4 |
|---|---|---|---|
| $c$ | 1 | 2 | 5 |

**C**

| $h$ | 0 | 2 | 4 |
|---|---|---|---|
| $c$ | 0 | 1 | 3 |

**D**

| $h$ | 0 | 2 | 4 |
|---|---|---|---|
| $c$ | 1 | 2 | 3 |



# Finding the rule

11.02

We can often find the formula for a table of values. If the values in the top row are **consecutive** (increase by 1 each time), look for a pattern in the values in the bottom row.

## Example 2

Find the formula for each table of values.

**a**

| $p$ | 2 | 3 | 4 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| $q$ | 1 | 2 | 3 | 5 | 6 | 7 |

**b**

| $m$ | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $n$ | −6 | −3 | 0 | 3 | 6 | 9 |

**c**

| $r$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $t$ | 1 | 3 | 5 | 7 | 9 | 11 |

**d**

| $d$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $e$ | 8 | 12 | 16 | 20 | 24 | 28 |

**Video**
Finding the rule

**Interactives**
Function builder: Basics

Function builder

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## Solution

**a** What has been done to each value in the top row to get the value in the bottom row?

| $p$ | 2 | 3 | 4 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| $q$ | 1 | 2 | 3 | 5 | 6 | 7 |

The pattern is:

$2 - 1 = 1$
$3 - 1 = 2$
$4 - 1 = 3$, and so on.
The formula is $q = p - 1$.

**b** The pattern is:

| $m$ | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $n$ | −6 | −3 | 0 | 3 | 6 | 9 |

$3 \times (-2) = -6$
$3 \times (-1) = -3$
$3 \times 0 = 0$, and so on.
The formula is $n = 3m$.

**c** The formula for this table of values is not so obvious. It involves **2** operations: a multiplication and either an addition or a subtraction.

| $r$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $t$ | 1 | 3 | 5 | 7 | 9 | 11 |
| | | 2 | 2 | 2 | 2 | 2 |

If the values in the top row are consecutive, the bottom row helps us find the **multiplier**.

The values in the bottom row go up by 2 each time, so the multiplier is 2. This means the formula must have $2 \times r$ in it.

$2 \times 1 - 1 = 1$
$2 \times 2 - 1 = 3$
$2 \times 3 - 1 = 5$, and so on.
The formula is $t = 2r - 1$.

**d** The values in the top row are **consecutive**, the values in the bottom row go up by 4, so the multiplier is 4, and the equation is of the form $e = 4d$ ____.

| $d$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $e$ | 8 | 12 | 16 | 20 | 24 | 28 |
| | | 4 | 4 | 4 | 4 | 4 |

Choose any ordered pair from the table to find the missing number, say (2, 12).

When $d = 2$, $e = 4 \times 2 + 4 = 12$, so the missing number is 4.

So the equation is $e = 4d + 4$.

(Checking that this is also true for another ordered pair (1, 8).
When $d = 1$, $e = 4 \times 1 + 4 = 8$)

## EXERCISE 11.02 ANSWERS ON P. 605

# Finding the rule UFRC

EXAMPLE 2

11.02

**1** Copy and complete the formula for each table of values. R C

**a** $y = ___ x$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 0 | 3 | 6 | 9 |

**b** $y = ___ x$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 0 | −2 | −4 | −6 |

**c** $y = x + _____$

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | 3 | 4 | 5 | 6 |

**d** $y = x - _____$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | −4 | −3 | −2 | −1 |

**e** $y = 10 - _____ x$

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | 11 | 10 | 9 | 8 |

**f** $y = x \div _____$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 0 | $\frac{1}{2}$ | 1 | $1\frac{1}{2}$ |

**g** $k = _____$

| $p$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $k$ | 5 | 6 | 7 | 8 |

**h** $y = _____$

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | −4 | −3 | −2 | −1 |

**i** $j = _____$

| $h$ | 4 | 5 | 9 | 11 |
|---|---|---|---|---|
| $j$ | 8 | 10 | 18 | 22 |

**j** $p = _____$

| $m$ | 0 | 5 | 30 | 45 |
|---|---|---|---|---|
| $p$ | 0 | 1 | 6 | 9 |

**2** Find the formula for each table, then complete the last 2 columns. R C

**a**

| $f$ | 1 | 2 | 3 | 4 | 7 | 8 |
|---|---|---|---|---|---|---|
| $h$ | 1 | 4 | 7 | 10 | | |

**b**

| $m$ | 1 | 2 | 3 | 4 | 6 | 9 |
|---|---|---|---|---|---|---|
| $p$ | 2 | 7 | 12 | 17 | | |

**c**

| $m$ | 0 | 1 | 2 | 3 | 5 | 8 |
|---|---|---|---|---|---|---|
| $b$ | 3 | 6 | 9 | 12 | | |

**d**

| $h$ | 3 | 4 | 5 | 6 | 9 | 11 |
|---|---|---|---|---|---|---|
| $k$ | 8 | 10 | 12 | 14 | | |

**e**

| $r$ | 0 | 1 | 2 | 3 | 4 | 7 |
|---|---|---|---|---|---|---|
| $s$ | 1 | 4 | 7 | 10 | | |

**f**

| $a$ | 2 | 3 | 4 | 5 | 10 | 11 |
|---|---|---|---|---|---|---|
| $b$ | 2 | 4 | 6 | 8 | | |

**g**

| $m$ | 0 | 1 | 2 | 3 | 5 | 6 |
|---|---|---|---|---|---|---|
| $n$ | 5 | 8 | 11 | 14 | | |

**h**

| $c$ | 3 | 4 | 5 | 6 | 9 | 10 |
|---|---|---|---|---|---|---|
| $d$ | 31 | 41 | 51 | 61 | | |

**i**

| $w$ | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| $x$ | 14 | 19 | 24 | 29 | | |

**j**

| $y$ | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|
| $z$ | 4 | 6 | 8 | 10 | | |

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**k**

| $a$ | 1 | 2 | 3 | 4 | 7 | 9 |
|---|---|---|---|---|---|---|
| $m$ | 5 | 9 | 13 | 17 | | |

**l**

| $z$ | 4 | 5 | 6 | 7 | 9 | 12 |
|---|---|---|---|---|---|---|
| $t$ | 1 | 3 | 5 | 7 | | |

**3** What is the formula for this table? Select the correct answer **A**, **B**, **C** or **D**.

| $p$ | 3 | 4 | 5 | 6 |
|---|---|---|---|---|
| $m$ | 8 | 11 | 14 | 17 |

**A** $m = 2p + 1$ **B** $m = 3p - 1$ **C** $m = 2p + 4$ **D** $m = 3p + 1$

**4** What is the formula for this table? Select **A**, **B**, **C** or **D**.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | −5 | −7 | −9 | −11 | −13 |

**A** $y = -2x - 5$ **B** $y = 2x - 5$ **C** $y = -2x - 3$ **D** $y = 2x - 3$

**5** Find the formula for each table, then complete the table. R C

**a**

| $x$ | 0 | 1 | 2 | 3 | 9 |
|---|---|---|---|---|---|
| $y$ | −1 | −3 | −5 | −7 | |

**b**

| $A$ | 0 | 1 | 2 | 3 | 10 |
|---|---|---|---|---|---|
| $C$ | 0 | 0.5 | 1 | 1.5 | |

**c**

| $s$ | −1 | 0 | 1 | 2 | 7 |
|---|---|---|---|---|---|
| $t$ | 18 | 13 | 8 | 3 | |

**d**

| $t$ | −1 | 0 | 1 | 2 | 5 |
|---|---|---|---|---|---|
| $v$ | 30 | 20 | 10 | | |

**e**

| $x$ | 0 | 1 | 2 | 3 | 6 |
|---|---|---|---|---|---|
| $D$ | −0.5 | −1 | −1.5 | | |

# 11.03 Finding rules for number patterns

**Worksheet**
Patterns and rules

## Example 3

For this number pattern: 2, 5, 8, 11, 14, ...

**a** find the 8th term

**b** find a formula for the $n$th term, $T$.

### Solution

**a** This number pattern is increasing by 3 each time. By continuing the pattern:

2, 5, 8, 11, 14, 17, 20, 23

8th term = 20 + 3 = 23

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**b** Write the number pattern as a table of values.

| Term of pattern, $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number, $T$ | 2 | 5 | 8 | 11 | 14 |

The number pattern goes up by 3 each time, so the multiplier for the formula is 3, and the equation is of the form $T = 3n$ ____.

Choose any ordered pair from the table to find the missing number, say (1, 2).

When $n = 1$, $T = 3 \times 1 - 1 = 2$, so the missing number is $-1$.

$3 \times 1 - 1 = 2$

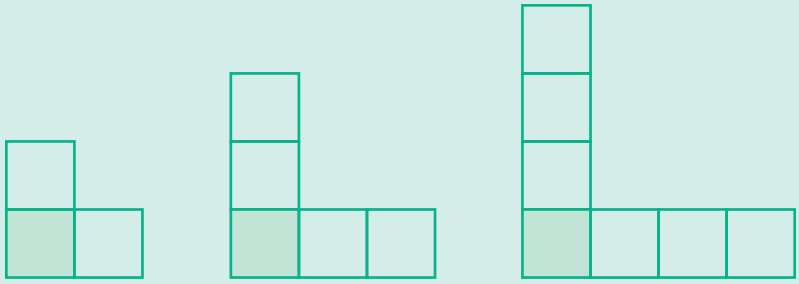
$3 \times 2 - 1 = 5$

$3 \times 3 - 1 = 8$, and so on

So the general formula for the $n$th term is $T = 3n - 1$.

## Example 4

Examine this geometrical pattern.



arm length = 1 arm length = 2 arm length = 3

**a** Copy and complete this table for the pattern.

| Arm length, $a$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of tiles, $t$ | | | | | | |

**b** Write the rule for this pattern in words.

**c** Write the rule as a formula.

**d** How many tiles would be needed to make a design with an arm length of:

**i** 18? **ii** 60?

### Solution

**a**

| Arm length, $a$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of tiles, $t$ | 3 | 5 | 7 | 9 | 11 | 13 |

**b** The number of tiles increases by 2 each time. This means that the multiplier is 2, and the equation is of the form $t = 2a$ ____.

Choose (1, 3) from the table to find the missing number.

When $a = 1$, $t = 2 \times 1 + 1 = 3$.

In words, the rule is: The number of tiles is 2 times the arm length plus 1.

**c** As a formula, the rule is $t = 2a + 1$.

**d** **i** When $a = 18$, $t = 2 \times 18 + 1 = 37$

37 tiles are needed for a design with an arm length of 18.

**ii** When $a = 60$, $t = 2 \times 60 + 1 = 121$

121 tiles are needed for a design with an arm length of 60.

## EXERCISE 11.03 ANSWERS ON P. 605

## Finding rules for number patterns U F PS R C

EXAMPLE 3

**1** Find the formula for the $n$th term, $T$, of the number pattern $-1, -3, -5, -7, \ldots$
Select the correct answer **A**, **B**, **C** or **D**. R C

**A** $T = n - 2$ **B** $T = 1 - 2n$ **C** $T = 2n - 1$ **D** $T = n - 3$

**2** For each number pattern: R C

**i** find the 8th term

**ii** find a formula for the $n$th term, $T$

**iii** use the formula to find the 8th term and the 20th term.

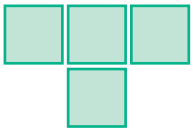
**a** 1, 3, 5, 7, 9, ... **b** 2, 4, 6, 8, 10, ... **c** 8, 10, 12, 14, 16, ...

**d** 7, 10, 13, 16, 19, ... **e** 7, 15, 23, 31, 39, ... **f** 2, 7, 12, 17, 22, ...

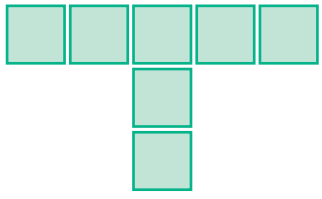
**g** 19, 17, 15, 13, 11, ... **h** 1, 4, 9, 16, 25, ...

EXAMPLE 4

**3** **a** Here are the first 2 T-shapes in a geometrical pattern. Draw the next 3 T-shapes. R C



arm length = 1



arm length = 2

**b** Copy and complete the table.

| Arm length, $a$ | 1 | 2 | 3 | 4 | 5 | 8 | 11 |
|---|---|---|---|---|---|---|---|
| Number of tiles, $t$ | 4 | | | | | | |

**c** Write the rule for the pattern in words.

**d** Write the rule as a formula.

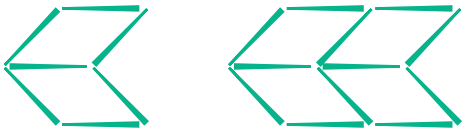
**e** How many tiles are needed to build a T-shape with arm length of:

**i** 15? **ii** 33?

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**4** **a** Here are the first 2 flight shapes, made from toothpicks. Draw the next 3. R C



flight shape 1 flight shape 2

**b** Copy and complete the table.

| Number of flight shape, $f$ | 1 | 2 | 3 | 4 | 6 | 9 | 10 |
|---|---|---|---|---|---|---|---|
| Number of toothpicks, $t$ | 7 | | | | | | |

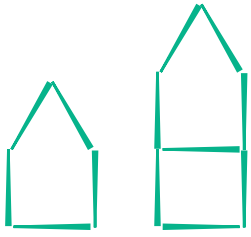
**c** Write the rule for the pattern in words.

**d** Write the rule as a formula.

**e** How many toothpicks are needed to build:

**i** flight shape 25? **ii** flight shape 40?

**5** **a** The first 2 rockets are shown below. Draw the next 3 rockets. R C



stage 1 stage 2

**b** Copy and complete the table.

| Number of stage, $s$ | 1 | 2 | 3 | 4 | 5 | 8 | 11 |
|---|---|---|---|---|---|---|---|
| Number of toothpicks, $t$ | | | | | | | |

**c** Write the rule for the pattern in words.

**d** Write the rule as a formula.

**e** How many toothpicks are needed to build:

**i** stage 12? **ii** stage 45?

**6** For each matchstick pattern on the next page: R C

**i** draw the next 3 terms

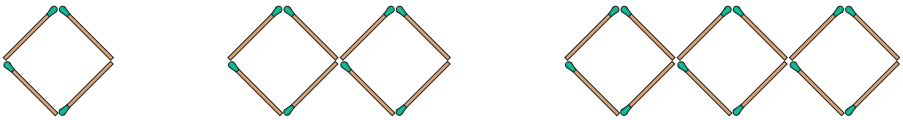
**ii** copy and complete this table

| Number of shapes ($n$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of matchsticks ($T$) | | | | | |

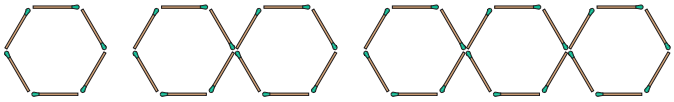
**iii** find the formula for the pattern

**iv** find the number of matchsticks required to make 12 shapes.

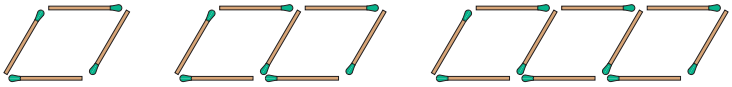
**a**



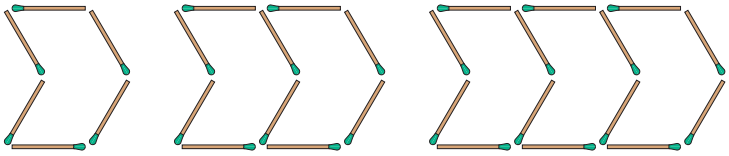
**b**



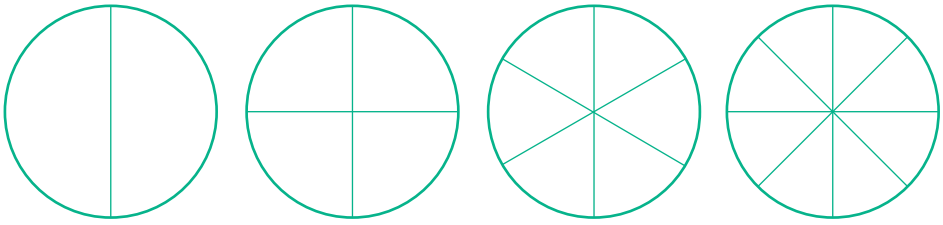
**c**



**d**



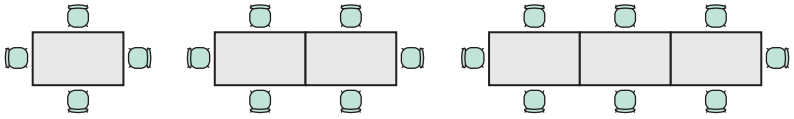
**7** The pattern below is formed by cutting a cake $c$ times to make $p$ pieces. Which of the following is the formula that best describes the pattern? Select **A**, **B**, **C** or **D**. R C



1 cut 2 cuts 3 cuts 4 cuts

**A** $p = 2c$ **B** $c = 2p$ **C** $p = c + 2$ **D** $c = p + 2$

**8** In a restaurant, chairs are placed around rectangular tables as shown below. PS R C



**a** Find the number of chairs required for:

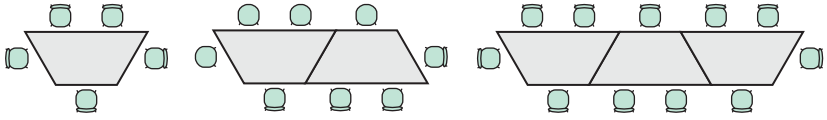
**i** 2 tables **ii** 3 tables **iii** 4 tables **iv** 12 tables

**b** Find a formula that relates the number of chairs, $C$, to the number of tables, $T$.

**c** How many chairs would be needed for a row of 50 tables?

**d** A booking is made at the restaurant for 30 people. How many tables are needed?

**9** Students arrange themselves around desks ready for group work in the classroom. The seating arrangement for $n$ students at $d$ desks is shown. PS R C



**a** Find the number of students who can be seated around a row of:

**i** 2 desks **ii** 3 desks **iii** 4 desks **iv** 10 desks

**b** Find a formula that relates the number of students, $n$, to the number of desks, $d$.

**c** Find the number of students that could be seated around 18 desks.

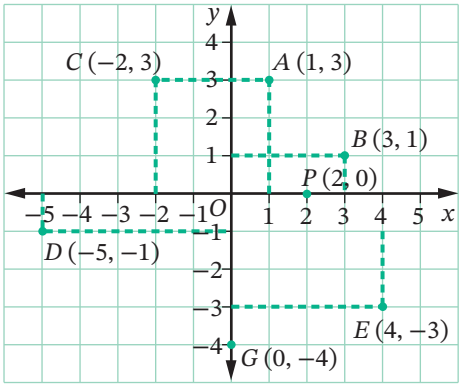
**d** Class 8Y has 29 students. How many desks are needed for them?



# The number plane 11.04

A **number plane** is a grid made from a horizontal number line called the ***x*-axis** and a vertical number line called the ***y*-axis**. The number plane is also called a **Cartesian plane**, named after René Descartes (pronounced 'Ren-ay Day-cart'), a French mathematician and philosopher who developed the idea.

Points on the number plane are located using a pair of **coordinates** or an **ordered pair**. Note that order is important with coordinates: $A(1, 3)$ is not the same as $B(3, 1)$.



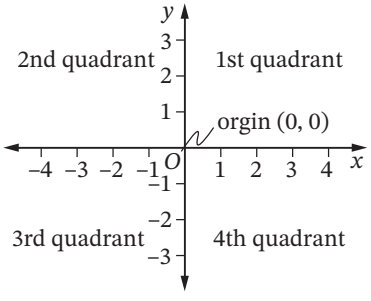

## Points on the number plane

In any ordered pair, the first number is called the ***x*-coordinate** and the second number is called the ***y*-coordinate**.

The point (0, 0) is called the **origin**.

To locate a point, always start from (0, 0): the $x$-coordinate tells you how far to move *across*, the $y$-coordinate tells you how far to move *up* or *down*.

The number plane is divided evenly into 4 regions called **quadrants**.




**Skillsheet**
The number plane

**Worksheets**
The number plane
Plane grid 2
Number plane review
Number plane grid paper
A page of number planes
Plotting points and lines'

**Videos**
The number plane
Coordinate geometry: Descartes

Foundation Standard Complex

**Puzzle**
Coordinates code puzzle

**Presentation**
Graph coordinates

**Technology**
Plotting points

**Spreadsheet**
Points plotter

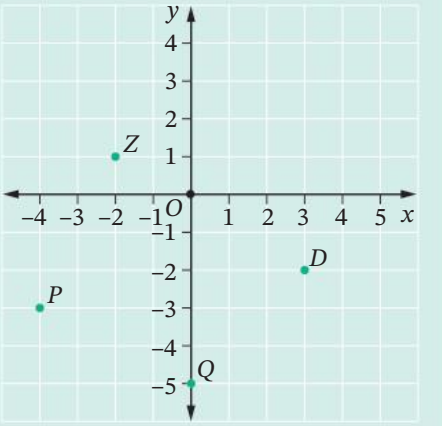
**Worksheet**
Number plane writing activity

## Example 5

**a** Write the coordinates of point $D$ and state what quadrant it is in.

**b** Write the point with coordinates $(-4, -3)$ and state what quadrant it is in.

**c** Write the coordinates of point $Q$.




### Solution

**a** $D(3, -2)$ is in the 4th quadrant.

**b** $(-4, -3)$ is the point $P$, which is in the 3rd quadrant

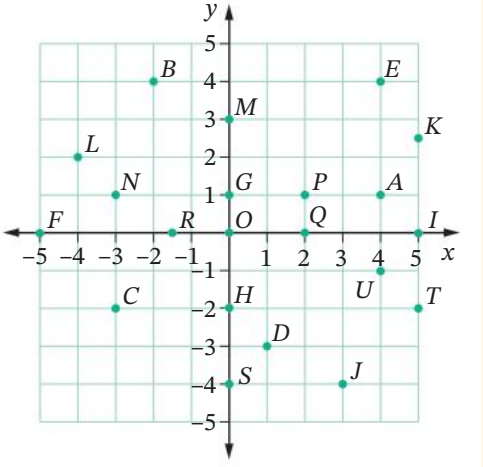
**c** Point $Q$ is 0 on the $x$-axis and $-5$ on the $y$-axis, so its coordinates are $(0, -5)$.

**EXERCISE 11.04** ANSWERS ON P. 606

## The number plane UFRC

EXAMPLE 5

Questions **1** to **5** refer to this diagram.




**1** Name the point with each pair of coordinates.

**a** $(2, 1)$ **b** $(-3, 1)$

**c** $(5, 2\frac{1}{2})$ **d** $(5, -2)$

**e** $(0, 3)$ **f** $(0, -4)$

**g** $(2, 0)$ **h** $(-1\frac{1}{2}, 0)$

**i** $(-4, 2)$ **j** $(4, -1)$

**2** Write the coordinates of each point. C

**a** $A$ **b** $B$ **c** $C$

**d** $D$ **e** $E$ **f** $F$

**g** $G$ **h** $H$ **i** $I$ **j** $J$

**3** In which quadrant does each point lie? C

**a** $N$ **b** $T$ **c** $E$ **d** $L$ **e** $C$

**4** List all of the points that are on the $x$-axis and write their coordinates. C

**5** List all of the points that are on the $y$-axis and write their coordinates. C

**6** If a point has a positive $x$-coordinate and a negative $y$-coordinate, which quadrant is it in? Select the correct answer **A**, **B**, **C** or **D**. R

**A** 1st quadrant **B** 2nd quadrant **C** 3rd quadrant **D** 4th quadrant

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**7** Determine the quadrant in which each point lies. R C

**a** $(3, -5)$ **b** $(-2, -4)$ **c** $(-8, 1)$ **d** $(5, 4)$

**8** **a** Draw a number plane, with both axes extending from −8 to 8, or use the link to print Plane grid 2.

**b** Plot and label these points.

$A(3, 0)$ $B(-2, 2)$ $C(4, 5)$ $D(3\frac{1}{2}, -5)$ $E(-1, 7)$

$F(0, -4)$ $G(-6, 0)$ $H(\frac{1}{2}, -7\frac{1}{2})$ $I(-2, 6)$ $J(7, 0)$

**9** **a** Draw a number plane, with both axes extending from −3 to 3, or use the link to print Number plane grid paper.

**b** Mark these points.

$A(0, 1)$, $B(2, -1)$, $C(-1, 2)$, $D(-2, 3)$, $E(3, -2)$ $F(1, 0)$.

**c** What do you notice about the points? Check this with your ruler. C

**d** Points that lie on the same straight line are said to be **collinear**. Write down the coordinates of any 4 points on a number plane that are collinear.

Worksheets
Plane grid 2

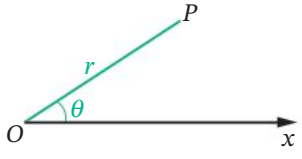
Number plane grid paper

11.04

## Did you know?

### Polar coordinates

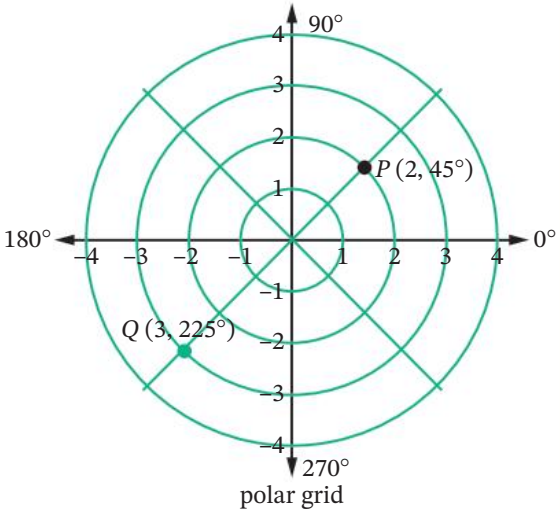
The number plane is not the only way to represent the position of a point. The **polar coordinates** of a point give its position in terms of a distance from the origin and angle measured from the $x$-axis.




We say that $(r, \theta)$ are the polar coordinates of the point $P$, where $r$ is the distance $P$ is from the origin $O$ and $\theta$ is the angle between $Ox$ and $OP$. A polar grid shows both distance and angle. For example, the grid below shows the coordinates of $P$ as $(2, 45°)$ and $Q$ as $(3, 225°)$.

Some of the real-life uses of polar coordinates include avoiding collisions between vessels and other ships, calculating the flow of ground water and guiding industrial robots.

**On polar grid paper, plot the points $L(2, 90°)$, $M(4, 135°)$ and $N(0, 270°)$**



polar grid

Foundation Standard Complex

Quiz
Mental skills 11

## Mental skills 11: Maths without calculators ANSWERS ON P. 606

### The unitary method with percentages

The unitary method is used when you are only given a *percentage* of an amount and you need to find the amount. It is called the unitary method because we find 1% of the amount first, then multiply that by 100 to find the whole (100%).

**1** Study each example.

**a** If 8% of a number is 24, what is the number?

8% of the number = 24

$\therefore$ 1% of the number = 24 ÷ 8 = 3

$\therefore$ 100% of the number = 3 × 100 = 300.

The number is 300. *Check*: 8% × 300 = 24

**b** If 15% of an amount is \$90, what is the whole amount?

15% of the amount = \$90

$\therefore$ 1% of the amount = \$90 ÷ 15 = \$6

$\therefore$ 100% of the amount = \$6 × 100 = \$600.

The amount is \$600. *Check*: 15% × \$600 = \$90

**2** Find the whole amount if:

**a** 5% of the amount is \$35

**b** 11% of the amount is \$88

**c** 20% of the amount is 80

**d** 6% of the amount is 42

**e** 90% of the amount is \$270

**f** 15% of the amount is \$60

**g** 40% of the amount is 100

**h** 120% of the amount is \$360

**i** 25% of the amount is \$75

**j** 8% of the amount is 40

# 11.05 Graphing number patterns

Video
Graphing number patterns

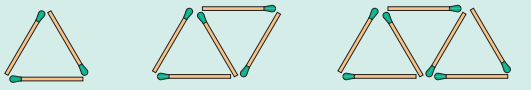
Worksheets
Number plane grid paper

A page of number planes

Graphing tables of values

## Example 6

Consider this pattern of matchsticks used to make triangles.



**a** Copy and complete the table of values for this pattern.

| No. of triangles, $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| No. of matchsticks, $y$ | | | | | |

**b** Find the formula for this table of values.

**c** Graph this table of values.

## Solution

**a**

| No. of triangles, $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| No. of matchsticks, $y$ | 3 | 5 | 7 | 9 | 11 |

**b** Bottom row of $y$-values increase by 2, so the multiplier in the formula is 2, and the equation is of the form $y = 2x$ ____.

Choose (2, 5) from the table to find the missing number.

When $x = 2$, $y = 2 \times 2 + 1 = 5$.

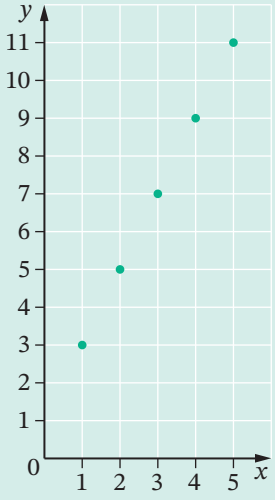
The formula is $y = 2x + 1$.

**c** Each column in the table forms an ordered pair.

| No. of triangle, $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| No. of matchsticks, $y$ | 3 | 5 | 7 | 9 | 11 |

(1, 3) (2, 5) (3, 7) (4, 9) (5, 11)

Graph each ordered pair on a number plane.




Note that the points are **collinear**, that is, they lie on a straight line. When this happens, we say that the relationship or pattern is **linear**.

- **linear** = involving a line
- **collinear points** = points that lie on a straight line

**EXERCISE 11.05** ANSWERS ON P. 606

## Graphing number patterns U F R C

**1** Find the formula for this table of values. Select the correct answer **A**, **B**, **C** or **D**. R C

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 2 | 6 | 10 | 14 | 18 |

**A** $y = 3x$ **B** $y = 2x + 2$ **C** $y = 4x - 2$ **D** $y = 4x$

**2** This table of values shows the height of a tree as it ages. Graph the values and determine whether there is a linear pattern between the age and height of the tree. R C

| Age, $x$ (years) | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|
| Height, $y$ (metres) | 10 | 12.5 | 17.5 | 21 | 26 |



EXAMPLE 6

Foundation Standard Complex

**3** For each pattern below: R C

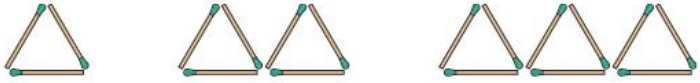
**i** copy and complete this table

| No. of shapes, $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| No. of matchsticks, $y$ | | | | | |

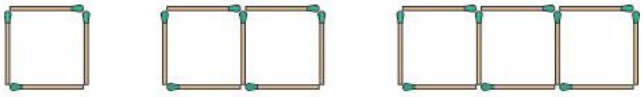
**ii** find the formula for the table of values

**iii** graph the table of values.

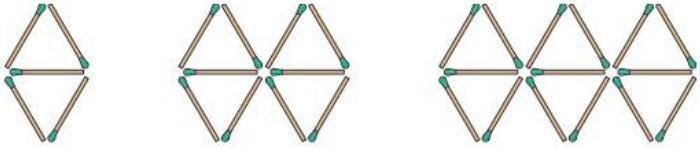
**a**



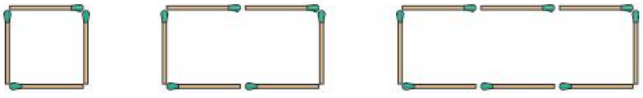
**b**



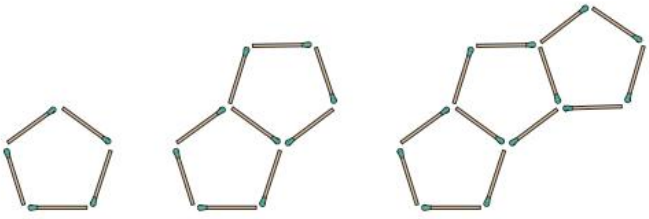
**c**



**d**



**e**



**4** Does each graph in question **3** show a linear relationship?

**5** Cans in supermarkets are often stacked neatly in triangles. Consider the relationship between the number of cans on the bottom row, $x$ and the total number of cans, $y$. R C



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**a** Copy and complete this table.

| Length of triangle, $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| No. of cans, $y$ | | | | | | | |

**b** Graph this table on a number plane.

**c** Is the relationship linear?

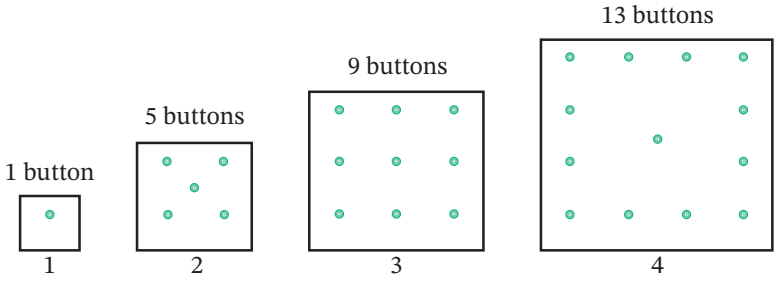
**d** Which formula matches the table of values? Select the correct answer **A**, **B** or **C**.

**A** $y = x(x-1)$ **B** $y = \frac{x+1}{2}$ **C** $y = \frac{x(x+1)}{2}$

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9780170479516

**6** The following pattern is made with buttons. R C




**a** Complete a table of values showing the relationship between the length of the square ($S$) and the number of buttons ($B$).

**b** Find a rule for $S$ in terms of $B$.

**c** Use your rule to find the number of buttons required to make a pattern for a square of length:

**i** 10 **ii** 24

**d** Kovo has 106 buttons. What is the length of the biggest complete square that she can build?



# Graphing linear equations 11.06

A **linear equation** is an equation or formula whose graph on a number plane is a **straight line**.

## Graphing linear equations

- complete a table of values
- graph the table of values on a number plane
- rule a line through the points and label the line with the equation.

## Example 7

Graph $y = x + 1$ on a number plane.

### Solution

Complete a table of values. Choose $x$-values close to 0 for easy calculation and graphing.

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −1 | 0 | 1 | 2 | 3 |

Graph the table of values on a number plane.

Rule a straight line through the points, place arrows at each end, and label the line with its equation.

**Worksheets**
Number plane grid paper
A page of lines

**Puzzles**
Matching linear equations
Graphing functions

**Interactive**
Graphing lines

**Videos**
Linear functions
Graphing linear equations

Foundation Standard Complex

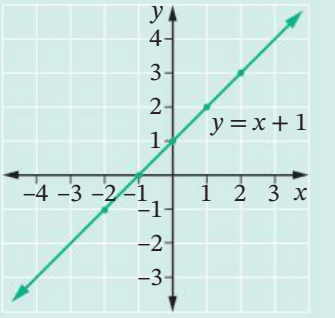
**Technology**
Graphing straight lines: Finding intercepts

**Spreadsheet**
Linear functions plotter

Note:

- Every pair of values that follows the linear equation $y = x + 1$ lies on this line, for example, (3, 4), $(-4, -3)$, $(1\frac{1}{2}, 2\frac{1}{2})$, (5, 6), (100, 101), $(-\frac{1}{2}, \frac{1}{2})$.
- Every point on the line follows the linear equation $y = x + 1$.
- There are an infinite number of points that satisfy the rule.

For these reasons, the line is infinite so we must draw arrows at each end.




**Video**
Graphing linear equations

## Example 8

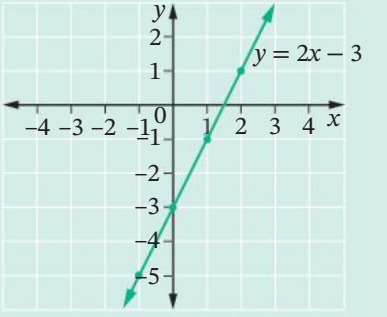
**a** Graph $y = 2x - 3$ on a number plane.

**b** Test whether the point (5, 8) lies on the line $y = 2x - 3$.

### Solution

**a**

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | −5 | −3 | −1 | 1 |




**b** We could extend the line $y = 2x - 3$ to see if (5, 8) lies on it, but a simpler way is to check whether (5, 8) follows or **satisfies** the formula $y = 2x - 3$. If it does, then when $x = 5$, $y = 8$.

When $x = 5$, $y = 2 \times 5 - 3 = 7 \neq 8$

$\therefore$ (5, 8) does not lie on the line. (In fact, (5, 7) does)



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9780170479516

**EXERCISE 11.06** ANSWERS ON P. 607

## Graphing linear equations U F R C

**1** For each linear equation, copy and complete the table of values and graph the equation on a number plane. C

**a** $y = x + 3$

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**b** $y = x - 2$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | | | | |

**c** $y = 2x$

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**d** $y = \frac{x}{2}$

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**e** $y = 6 - x$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | | | | |

**f** $y = x$

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**2** Graph each linear equation on a number plane.

**a** $y = 2x + 1$ **b** $y = 2x - 1$ **c** $y = 3x$

**d** $y = 3x - 2$ **e** $y = 2x + 4$ **f** $y = 3x - 5$

**3** Look at the graphs you drew in question **2**. Why are they called **increasing graphs**? C

**4** Graph each linear equation on a number plane.

**a** $y = -x$ **b** $y = -x + 5$ **c** $y = 4 - x$

**d** $y = -x - 3$ **e** $y = 1 - 3x$ **f** $y = -2x$

**5** Look at the graphs you drew in question **4**. Why are they called **decreasing graphs**? C

**6** By looking at each linear equation, predict whether its graph will be increasing or decreasing. R C

**a** $y = 5x - 4$ **b** $y = x + 9$ **c** $y = 3 - x$

**d** $y = 2x$ **e** $y = -x + 5$ **f** $y = -3x - 1$

**7** Test whether each point lies on the graph of the given linear equation by: R

**i** substituting its coordinates into the linear equation

**ii** examining the graphs of the lines you drew in questions **1** and **2**

**a** $(4, 7)\ y = x + 3$ **b** $(-1, -1)\ y = x - 2$

**c** $(5, 1)\ y = 6 - x$ **d** $(-3, -5)\ y = 2x + 1$

**e** $(4, 10)\ y = 3x - 2$ **f** $(-2, 1)\ y = 2x + 4$

**8** On which line does the point $(1, 3)$ lie? Select the correct answer **A**, **B**, **C** or **D**.

**A** $y = -x + 3$ **B** $y = x + 4$ **C** $y = 2 - 2x$ **D** $y = 2x + 1$

EXAMPLE 7

EXAMPLE 8

**Worksheets**
Number plane grid paper

A page of number planes

11.06

Foundation Standard Complex

## Technology

### Graphing linear equations

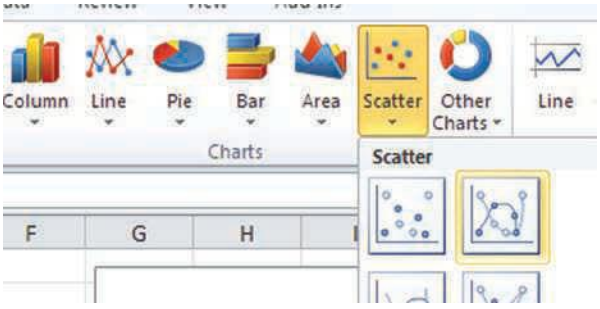
We are going to use a spreadsheet to graph $y = 2x - 3$.

**1** Enter these values into a spreadsheet.

| | A | B |
|---|---|---|
| 1 | x | y |
| 2 | -4 | |
| 3 | -3 | |
| 4 | -2 | |
| 5 | -1 | |
| 6 | 0 | |
| 7 | 1 | |
| 8 | 2 | |
| 9 | 3 | |
| 10 | 4 | |
| 11 | 5 | |
| 12 | | |

**2** In cell B2, enter the formula **=2*A2−3** to evaluate $2x - 3$ and use **Fill Down** to copy the formula down to cell B11.

**3** Highlight columns A and B, then click **Insert** and **Scatter** (with **Smooth Lines and Markers**).




**4** Click the bottom-right corner of the graph border to enlarge the graph.

**5** Use your graph to predict the value of $y$ when:

**a** $x = 5$ **b** $x = -6$ **c** $x = -1.5$

**6** Use your graph to predict the value of $x$ when:

**a** $y = 11$ **b** $y = -9$ **c** $y = 14$

## Investigation

### Special lines

**1** Draw a number plane with both axes extending from −4 to 4 and graph each table of values on it.

**a**

| $x$ | −1 | 0 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 2 | 2 | 2 | 2 |

**b**

| $x$ | −2 | 0 | 1 | 3 |
|---|---|---|---|---|
| $y$ | −1 | −1 | −1 | −1 |

**c**

| $x$ | 2 | −1 | 0 | −4 |
|---|---|---|---|---|
| $y$ | 1 | 1 | 1 | 1 |

**d**

| $x$ | −2 | 3 | 0 | 4 |
|---|---|---|---|---|
| $y$ | −3 | −3 | −3 | −3 |

**2** **a** What do all of the lines in question **1** have in common?

**b** What do all of the tables of values in question **1** have in common?

**c** The formula for the table of values and line in question **1 a** is $y = 2$. It does not have an $x$ in it because the value of $y$ does not depend on $x$, but is always equal to 2. What are the formulas for the other 3 lines in question **1**?

**3** Draw another number plane and graph each table of values on it.

**a**

| $x$ | 1 | 1 | 1 | 1 |
|---|---|---|---|---|
| $y$ | −3 | −2 | 0 | 2 |

**b**

| $x$ | −2 | −2 | −2 | −2 |
|---|---|---|---|---|
| $y$ | −2 | 0 | 1 | 3 |

**c**

| $x$ | 3 | 3 | 3 | 3 |
|---|---|---|---|---|
| $y$ | −3 | 2 | −2 | 0 |

**d**

| $x$ | −3 | −3 | −3 | −3 |
|---|---|---|---|---|
| $y$ | −4 | −1 | 2 | 4 |

**4** **a** What do all of the lines in question **3** have in common?

**b** What do all of the tables of values in question **3** have in common?

**c** The formula for the table of values and line in question **3 a** is $x = 1$. What are the formulas for the other 3 lines in question **3**?

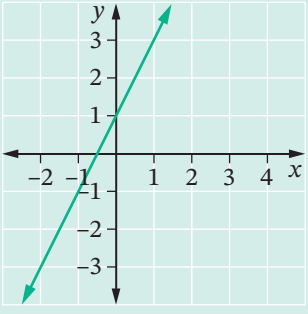


# Finding the equation of a line

11.07

## Example 9

Find the equation of this line.




### Solution

First draw a table of values using the coordinates of the points on the line, with consecutive $x$-values.

| $x$ | −2 | −1 | 0 | 1 |
|---|---|---|---|---|
| $y$ | −3 | −1 | 1 | 3 |

+2 +2 +2

The $y$-values go up by 2, so the multiplier is 2, and the equation is of the form $y = 2x$ ____.

Choose the point (1, 3) to find the missing number.

When $x = 1$, $y = 2 \times 1 + 1 = 3$.

So the equation is $y = 2x + 1$.

(Check that this is also true for (0, 1)).

**Worksheets**
Finding linear equations

**Puzzles**
Matching linear equations

**Video**
Finding the equation of a line

Video
Horizontal and vertical lines

# Horizontal and vertical lines

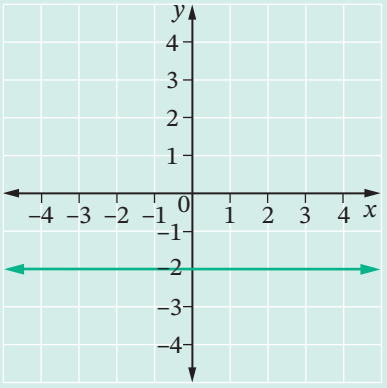
A **horizontal** line is flat and runs across (sideways). The word 'horizontal' comes from 'horizon,' the flat line that is the edge of the Earth as far as your eye can see.

A **vertical** line runs up and down, at right angles to a horizontal line. The word 'vertical' means upright. For example, vertical blinds are made up of columns of slats.

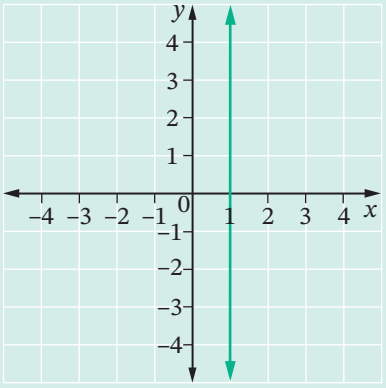
## Example 10

Find the equation of each line.

**a**




**b**




### Solution

**a** This is a **horizontal** line.

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | −2 | −2 | −2 | −2 |

$y$ is always equal to $-2$.

So the equation is $y = -2$.

**b** This is a **vertical** line.

| $x$ | 1 | 1 | 1 | 1 |
|---|---|---|---|---|
| $y$ | 0 | 1 | 2 | 3 |

$x$ is always equal to 1.

So the equation is $x = 1$.

## Horizontal and vertical lines

The **equation of a horizontal line** is of the form $y = c$ (where $c$ is a constant number).

The **equation of a vertical line** is of the form $x = c$ (where $c$ is a constant number).

**EXERCISE 11.07** ANSWERS ON P. 609

## Finding the equation of a line UFRC

EXAMPLE 9

**1** Find the equation that describes this table of values. Select the correct answer **A**, **B**, **C** or **D**.

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | 3 | 1 | −1 | −3 |

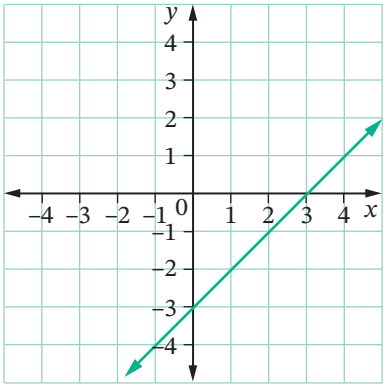
**A** $y = 2x + 1$ **B** $y = 2x - 1$ **C** $y = x - 2$ **D** $y = -2x + 1$

Foundation Standard Complex


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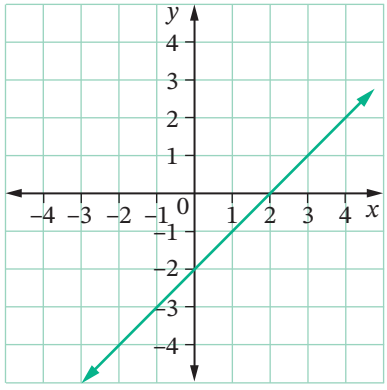
**2** For each graph, copy and complete the table of values and use it to find the equation of the line.

**a**



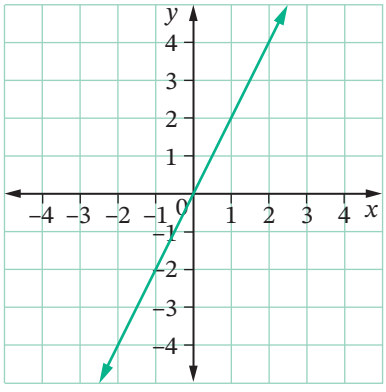
| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**b**



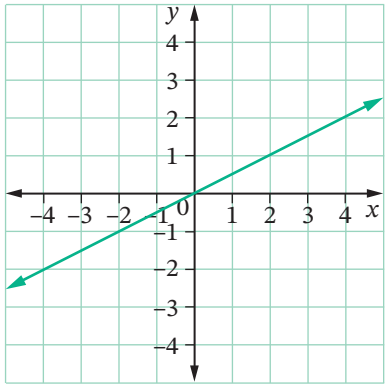
| $x$ | −2 | −1 | 0 | 1 |
|---|---|---|---|---|
| $y$ | | | | |

**c**



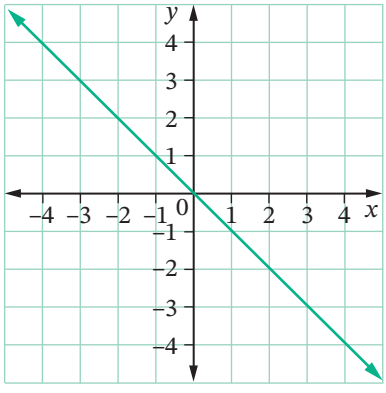
| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**d**



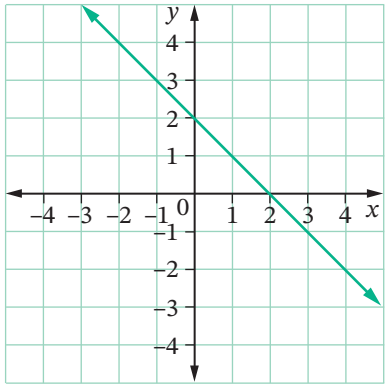
| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | | | | |

**e**



| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

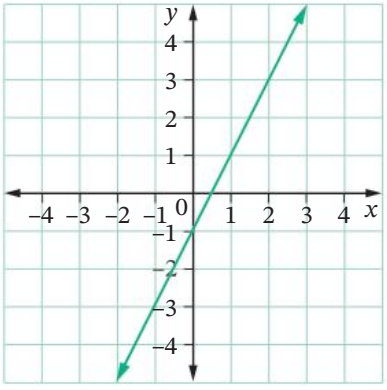
**f**



| $x$ | −3 | −2 | −1 | 0 |
|---|---|---|---|---|
| $y$ | | | | |

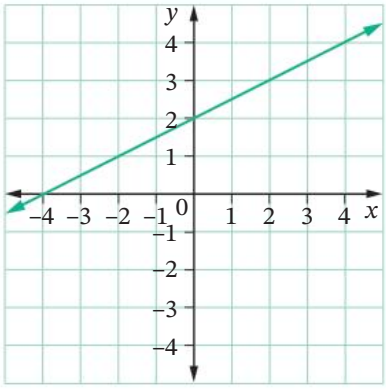
Foundation | Standard | Complex

**g**



| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**h**



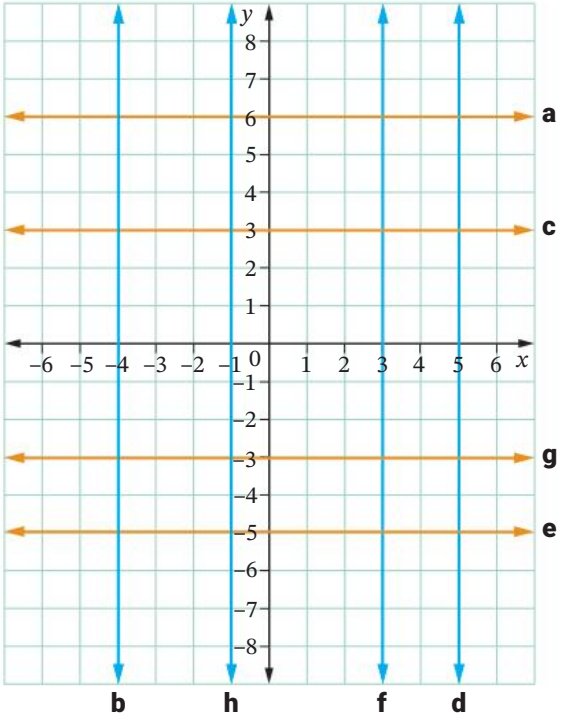
| $x$ | −2 | −1 | 0 | 1 |
|---|---|---|---|---|
| $y$ | | | | |

EXAMPLE 10

**3** **a** What do the lines represented by $x = 2$, $x = 5$, $x = -6$ and $x = \frac{1}{2}$ have in common? R C

**b** What do the lines represented by $y = 2$, $y = 5$, $y = -6$ and $y = \frac{1}{2}$ have in common?

**4** Write down the equation of each line in the diagram below.




**5** Graph each line on the same number plane.

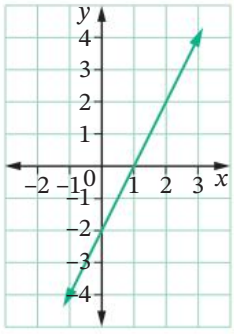
**a** $x = 4$ **b** $x = -2$ **c** $y = 5$ **d** $y = -1$

**e** $x = 1$ **f** $y = -3$ **g** $y = 4$ **h** $x = -3$

Foundation Standard Complex


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**6** What is the equation of this line? Select **A**, **B**, **C** or **D**.




**A** $y = -x + 2$ **B** $y = 2x - 2$

**C** $y = 2 - 2x$ **D** $y = x - 2$

**7** Harrison read the following points from a line that he drew.

$$(-4, 0), (-3, \tfrac{1}{2}), (-2, 1), (-1, 1\tfrac{1}{2})$$

Find the equation of the line that Harrison drew. R C

11.07

## Technology

### Comparing linear equations

In this activity, we will use a spreadsheet to compare the graphs of $y = x$, $y = 2x$ and $y = 3x$.

**1** Set up this spreadsheet.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | x | y = x | y = 2x | y = 3x |
| 2 | -2 | | | |
| 3 | -1 | | | |
| 4 | 0 | | | |
| 5 | 1 | | | |
| 6 | 2 | | | |

**2** In cell B2 enter **=A2** to calculate $y = x$, then use **Fill Down** to copy this formula down to B6.

**3** In cell C2 enter **=2*A2** to calculate $y = 2x$, then **Fill Down** to C6.

**4** In cell D2, write a formula to calculate $y = 3x$, then **Fill Down** to D6.

**5** Highlight all cells. Click **Insert** and **Scatter (Smooth Lines and Markers)** to graph all 3 lines on the same axes.

**6** Compare the 3 lines. What is the same and what is different? Describe any patterns and important features. Write your answers on your spreadsheet or in your workbook.

**7** Set up a similar spreadsheet for each set of 3 linear equations shown below.
Use appropriate formulas to complete each column and graph each set of 3 lines.
Compare the 3 lines in each set and describe the pattern and important features of each set.

**a**

| G | H | I | J |
|---|---|---|---|
| x | y=-x | y=-2x | y=-3x |
| -2 | | | |
| -1 | | | |
| 0 | | | |
| 1 | | | |
| 2 | | | |

**b**

| L | M | N | O |
|---|---|---|---|
| x | y=x+1 | y=x+2 | y=x-4 |
| -2 | | | |
| -1 | | | |
| 0 | | | |
| 1 | | | |
| 2 | | | |

**c**

| Q | R | S | T |
|---|---|---|---|
| x | y=-x+1 | y=-x+2 | y=-x-4 |
| -2 | | | |
| -1 | | | |
| 0 | | | |
| 1 | | | |
| 2 | | | |

Foundation Standard Complex

# 11.08 Comparing linear equations

**Puzzles**
Matching linear equations

**Interactive**
Graphing slope-intercept

**Video**
Linear functions

A **linear equation** such as $y = 2x + 1$ is made up of 2 terms:

- the **variable term**, or the term with the $x$
- the **constant term**, the number without the $x$.

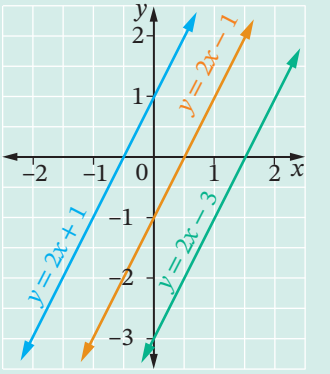
$y = 2x + 1$

The coefficient of $x$ is 2. The constant term is 1.

The number in front of the $x$ is called the **coefficient** of $x$.

## Example 11

The lines $y = 2x + 1$, $y = 2x - 1$ and $y = 2x - 3$ are graphed.




**a** How are the lines similar?

**b** How are the linear equations similar?

**c** The $y$-intercept of a line is the value at which the line crosses the $y$-axis. Find the $y$-intercept of each line in the diagram.

**d** How are the linear equations different?

### Solution

**a** The lines point in the same direction and have the same slope, they are parallel.

**b** The linear equations all have the same coefficient of $x$, namely, 2.

**c** $y = 2x + 1$: $y$-intercept $= 1$

$y = 2x - 1$: $y$-intercept $= -1$

$y = 2x - 3$: $y$-intercept $= -3$

**d** The linear equations have different constant terms: 1, −1, −3.

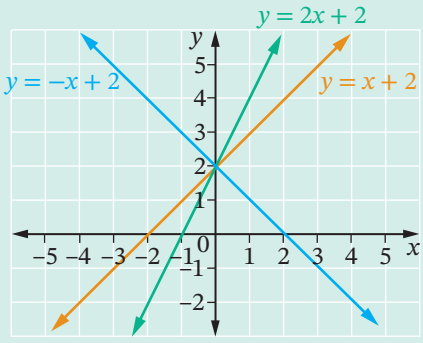


iStock.com/Ranimiro Lotufo Neto

The Octavio Frias de Oliveira bridge in São Paulo, Brazil.

## Example 12

The lines $y = x + 2$, $y = -x + 2$ and $y = 2x + 2$ are graphed.




**a** Find the $y$-intercept of each line.

**b** How are the linear equations similar?

**c** How are the lines different?

**d** How are the linear equations different?

**e** How does the coefficient of $x$ in a linear equation affect its graph?

**f** How does the constant term in a linear equation affect its graph?

### Solution

**a** $y = -x + 2$: $y$-intercept $= 2$

$y = 2x + 2$: $y$-intercept $= 2$

$y = x + 2$: $y$-intercept $= 2$

**b** The linear equations all have the same constant term, 2.

**c** The lines point in different directions: they are not parallel.

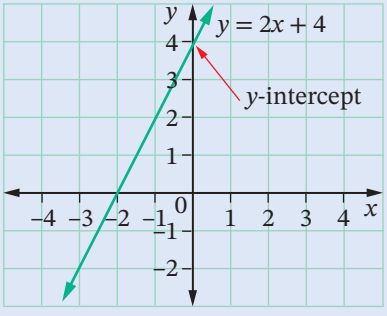
**d** The linear equations have different coefficients of $x$: $-1$, 2 and 1.

**e** The coefficient of $x$ affects the direction of the graph.

**f** The constant term gives the $y$-intercept of the graph.

## Features of a linear equation

- In a linear equation, the **coefficient of** $x$ gives the **direction** of the line.
- If 2 or more lines have the same coefficient of $x$ in their equations, then they are parallel.
- The **constant term** gives the $y$-**intercept** of the line.




**EXERCISE 11.08** ANSWERS ON P. 610

## Comparing linear equations U F R C

The exercise is best completed using dynamic geometry or graphing software.

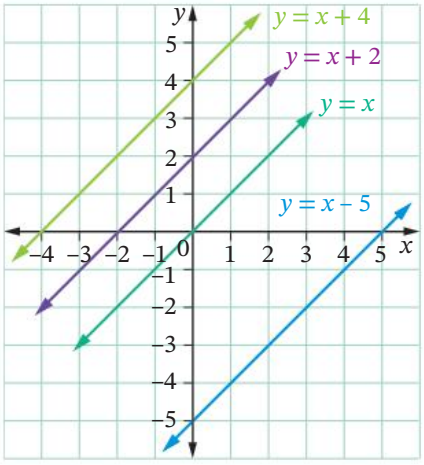
**1** For each linear equation, state the coefficient of $x$ and the constant term. C

**a** $y = 4x + 1$ **b** $y = \frac{1}{2}x - 3$ **c** $y = x - 5$ **d** $y = -x + 6$

EXAMPLE 11

Foundation Standard Complex

**2** The lines $y = x + 4$, $y = x + 2$, $y = x$ and $y = x - 5$ are graphed. R C




**a** How are the lines similar?

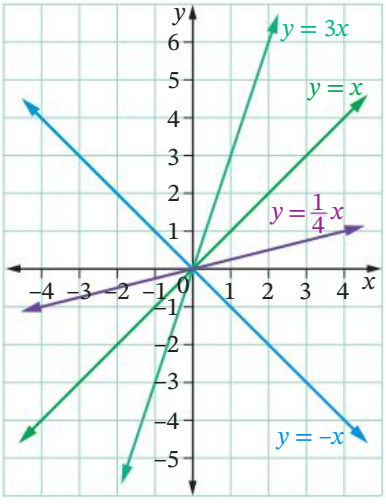
**b** How are the linear equations similar?

**c** How are the lines different?

**d** How are the linear equations different?

EXAMPLE 12

**3** The lines $y = \frac{1}{4}x$, $y = x$, $y = 3x$ and $y = -x$ are graphed. R C




**a** How are the lines similar?

**b** How are the linear equations similar?

**c** How are the lines different?

**d** How are the linear equations different?

**e** Which line is the steepest? What is the coefficient of $x$ in its equation?

**f** Which line is the least steep? What is the coefficient of $x$ in its equation?

**g** How does the size of the coefficient of $x$ in a linear equation affect the steepness of its line?

**4** Graph $y = 3x$, $y = 3x - 1$ and $y = 3x + 2$ on the same number plane. R C

**a** What is the same about all 3 lines?

**b** What is the same about all 3 linear equations?

**c** What is different about the lines?

**d** What is different about the linear equations?

**5** Graph $y = x + 1$, $y = 3x + 1$ and $y = \frac{1}{2}x + 1$ on the same number plane. R C

**a** What is the same about all 3 lines?

**b** What is the same about all 3 linear equations?

**c** What is different about the lines?

**d** What is different about the linear equations?

**e** What is the coefficient of $x$ in the equation with the steepest line?

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**6** Graph $y = -x - 2$, $y = -2x - 2$ and $y = -\frac{1}{2}x - 2$ on the same number plane. R C

**a** How are the lines similar?

**b** How are the linear equations similar?

**c** How are the lines different?

**d** How are the lines different to the lines in question **5**?

**e** What is the coefficient of $x$ in the equation with the least steepest line?

**7** The graphs in question **5** are all increasing, while the graphs in question **6** are all decreasing. What feature of their equations indicates this? R C

**8** Which equation below when graphed gives a decreasing line with a $y$-intercept of 5? Select the correct answer **A**, **B**, **C** or **D**. R C

**A** $y = 5x$ **B** $y = 5 - x$ **C** $y = 5x - 5$ **D** $y = -5x$

**9** Copy and complete each statement. R C

**a** The equations of parallel lines have the same _____.

**b** Linear equations with the same constant term have graphs that have the same _____.

**c** Linear equations with a negative coefficient of $x$ have graphs that are _____.

**d** The equations of lines with the same $y$-intercept have the same _____.

**e** The equations of lines that are decreasing have a _____ coefficient of $x$.

**10** Graph each **non-linear equation** after completing the table of values given. R C

**a** $y = x^2$

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**b** $y = \sqrt{x}$

| $x$ | 0 | 1 | 2 | 4 | 5 | 7 | 9 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**c** $y = x^3$

| $x$ | −2 | −1.5 | −1 | −0.5 | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | | |

**d** $y = \frac{1}{x}$

| $x$ | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | | | |

**11 a** Why do you think the equations graphed in question **10** are called non-linear equations? R C

**b** How are the non-linear equations algebraically different to the linear equations?

11.08

Foundation Standard Complex

# 11.09 Extension: The gradient of a line

YEAR 9 CORE

The steepness or slope of a line is called the **gradient** of the line, and is calculated as the ratio of the vertical **rise** to the horizontal **run** between any 2 points on the line.

The gradient is represented by the variable $m$ and is measured by counting how many units the line rises (up), for each unit across to the right.

**Worksheets**
Drawing gradients

A page of lines

Gradient between two points

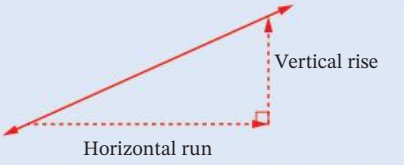
**Puzzle**
Find the gradient 1

**Videos**
The gradient of a line

Gradient (slope) of a line

## The gradient of a line

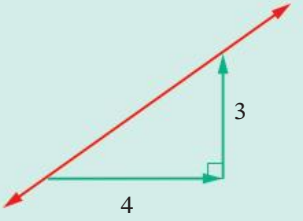
$$m = \frac{\text{vertical rise}}{\text{horizontal run}} = \frac{\text{rise}}{\text{run}}$$



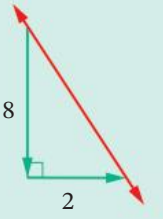

## Example 12

Calculate the gradient of each line.

**a**

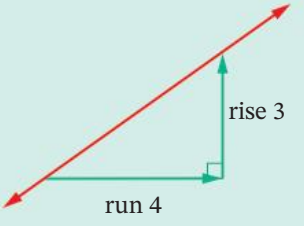



**b**



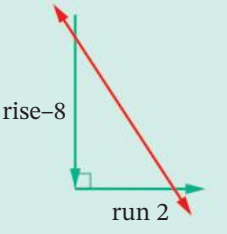

### Solution

**a** $m = \frac{\text{rise}}{\text{run}} = \frac{3}{4}$




Positive gradient,
line leans to the right.

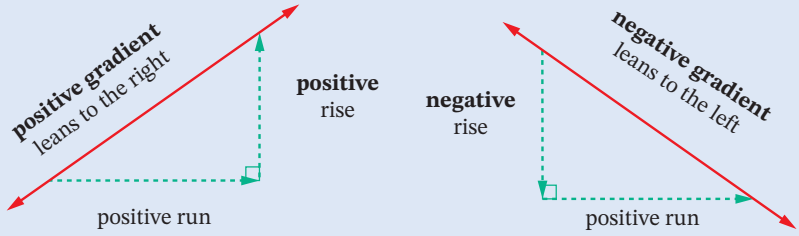
**b** $m = \frac{\text{rise}}{\text{run}} = \frac{-8}{2} = -4$




Negative gradient,
line leans to the left.
This line has a negative rise or a drop (↓).


9780170479516

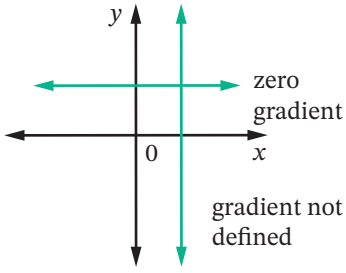
## Positive and negative gradients




Note: the run is always **positive**.

A **horizontal** (flat) line has **zero** gradient, as it has no steepness (also no 'rise').

A **vertical** line has infinite steepness and its gradient is **not defined** (also no 'run').

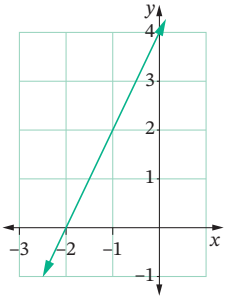



**EXERCISE 11.09** ANSWERS ON P. 611

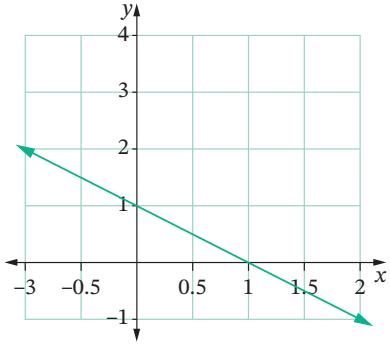
## The gradient of a line U F R C

**1** State whether the gradient of each line is positive or negative. R C

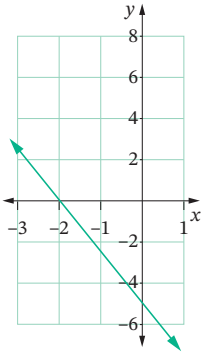
**a**




**b**




**c**




Foundation | Standard | Complex

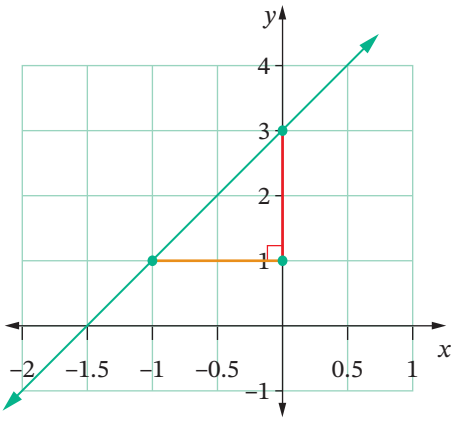
YEAR 9 CORE

EXAMPLE 12

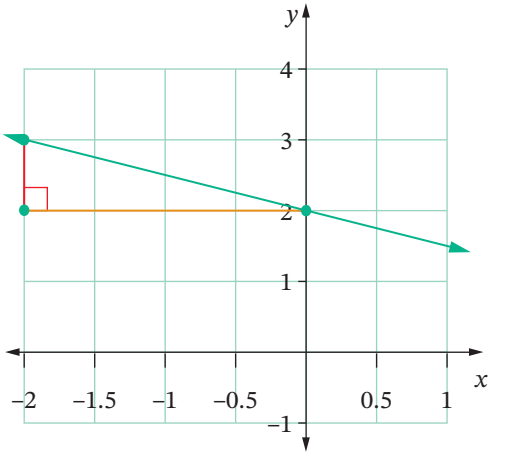
**2** Find the gradient of each line in question **1**.

**3** Using the given triangles, find the gradient of each line.

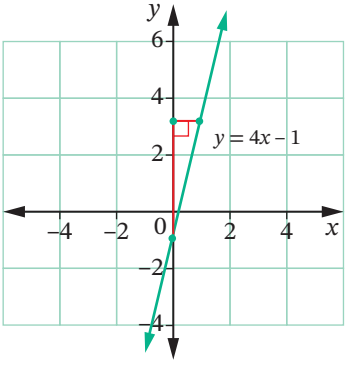
**a**




**b**




**c**




**Worksheets**
Number plane grid paper

A page of number planes

**4** Plot each pair of points on a separate number plane and find the gradient of the line that passes through both points.

**a** (2, 1), (3, 4) **b** (−2, 0), (2, 1) **c** (0, 0), (−4, 0)

**d** (3, 3), (6, 0) **e** (−3, 1), (−1, 3)

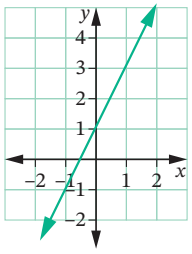
Foundation Standard Complex


9780170479516

## Investigation

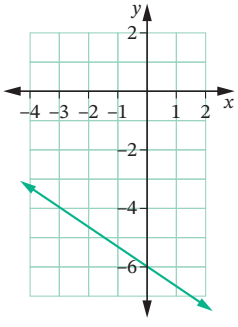
YEAR 9 CORE

### Comparing gradients and *y*-intercepts




**1** This is the graph of $y = 2x + 1$.

- **a** What is the gradient of the line?
- **b** What is the *y*-intercept?
- **c** How are the above values related to the linear equation $y = 2x + 1$?




**2** This is the graph of $y = -\frac{2}{3}x - 6$.

- **a** What is the gradient of the line?
- **b** What is the *y*-intercept?
- **c** Compare your answers with the linear equation $y = -\frac{2}{3}x - 6$.

**3** What is the gradient and *y*-intercept of the line with equation $y = 3x - 5$?

11.10

# Extension: The linear equation $y = mx + c$

11.10

YEAR 9 CORE

## The linear equation $y = mx + c$

The equation of a straight line is $y = mx + c$, where $m$ is the gradient and $c$ is the *y*-intercept.

$y = mx + c$ is called the **gradient-intercept form** of a linear equation.

## Example 13

Find the gradient and *y*-intercept of the line with equation:

**a** $y = 4x + 1$ **b** $y = -\frac{2}{3}x - 5$ **c** $y = 3 - \frac{1}{2}x$ **d** $x + y = 1$

### Solution

**a** $y = 4x + 1$ is in the form $y = mx + c$.

$\therefore$ Gradient $m = 4$ and *y*-intercept $c = 1$.

**b** For $y = -\frac{2}{3}x - 5$, gradient $m = -\frac{2}{3}$ and *y*-intercept $c = -5$.

**c** $y = 3 - \frac{1}{2}x$ can be rewritten as $y = -\frac{1}{2}x + 3$.

$\therefore$ Gradient $m = -\frac{1}{2}$ and *y*-intercept $c = 3$.

**d** $x + y = 1$ can be rewritten as $y = -x + 1$.

$\therefore$ Gradient $m = -1$ and *y*-intercept $c = 1$.

**Puzzle**
Equation of a line

**Interactive**
Graphing slope-intercept

**Video**
Graphing $y = mx + c$

YEAR 9 CORE

## Example 14

Graph each equation by finding its gradient and $y$-intercept.

**a** $y = 3x - 2$ **b** $y = -\frac{2}{3}x + 1$

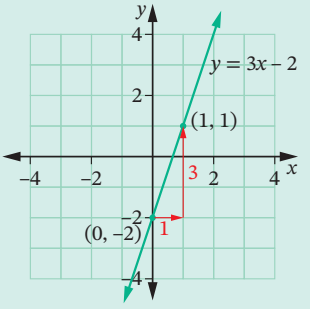
### Solution

**a** $y = 3x - 2$ has a gradient of 3 and a $y$-intercept of $-2$

Plot the $y$-intercept $-2$ on the $y$-axis.

Note that $3 = \frac{3}{1} = \frac{up}{right}$. Starting from the $y$-intercept, move 1 unit right and then 3 units up. This takes you to point $(1, 1)$.

Draw a line through $(1, 1)$ and the $y$-intercept.

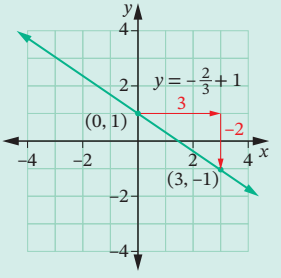



**b** $y = -\frac{2}{3}x + 1$ has a gradient of $-\frac{2}{3}$ and a $y$-intercept of 1.

Plot the $y$-intercept 1 on the $y$-axis.

Note that $-\frac{2}{3} = \frac{down}{right}$. Starting from the $y$-intercept, move 3 units right and then 2 units down. This takes you to point $(3, -1)$.

Draw a line through $(3, -1)$ and the $y$-intercept.




## EXERCISE 11.10 ANSWERS ON P. 611

## The linear equation $y = mx + c$ UFRC

EXAMPLE 13

**1** Write down the gradient and $y$-intercept of the line with equation:

**a** $y = 3x + 2$ **b** $y = -2x + 1$ **c** $y = -x + 6$ **d** $y = \frac{1}{3}x - 1$

**e** $y = -4x - 3$ **f** $y = \frac{2}{5}x + 3$ **g** $y = -\frac{4}{3}x - 8$ **h** $y = \frac{5}{6}x + \frac{1}{2}$

Foundation Standard Complex


9780170479516

**2** Which equation has a gradient of 4 and a $y$-intercept of 2? Select the correct answer **A**, **B**, **C** or **D**.

**A** $y = 2x + 4$ **B** $y = 4x - 2$ **C** $y = 2x - 4$ **D** $y = 4x + 2$

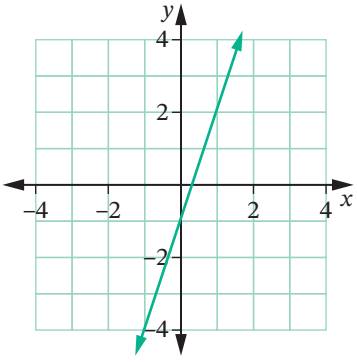
**3** Write the equation of the line with:

**a** a gradient of 3 and $y$-intercept of 1 **b** a gradient of $-2$ and $y$-intercept of 6

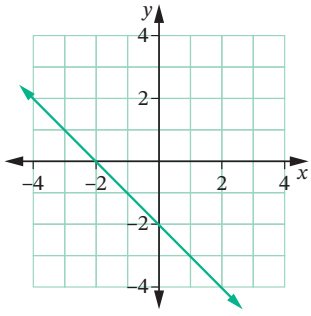
**c** a gradient of $-2$ and $y$-intercept of 11 **d** a gradient of $-\frac{1}{4}$ and $y$-intercept of $-5$

**4** Find the gradient and $y$-intercept of each line, then write its equation in the form $y = mx + c$.

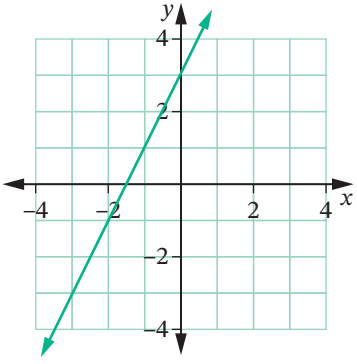
**a**



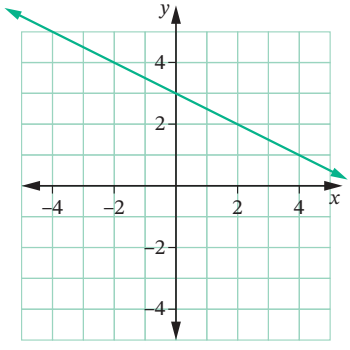

**b**




**c**




**d**




**5** Find the gradient and $y$-intercept of the line with equation:

**a** $y = 2 - 3x$ **b** $y = -4 - x$ **c** $2x + y = 6$ **d** $3x - y = -1$

**6** Graph each equation by finding its gradient and $y$-intercept.

**a** $y = 2x + 1$ **b** $y = -\frac{1}{2}x + 3$ **c** $y = -4x$

**Worksheets**
Number plane grid paper

A page of number planes

EXAMPLE 14

Foundation Standard Complex

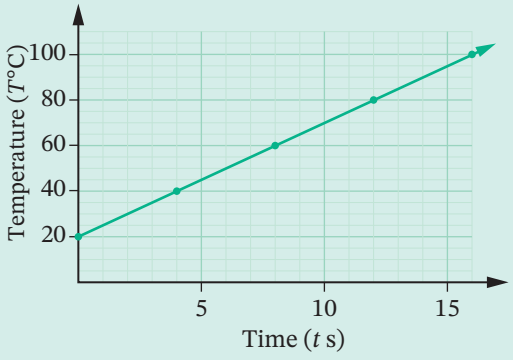
# 11.11 Applying linear equations

Linear equations can be used to model (represent) real-life situations where a quantity is increasing or decreasing at a constant rate.

## Example 15

This table and graph show the temperature, in degrees Celsius, of water being boiled in a kettle. It can be modelled by the linear equation $T = 5t + 20$.

| Time ($t$ seconds) | 0 | 4 | 8 | 12 | 16 |
|---|---|---|---|---|---|
| Temperature ($T$°C) | 20 | 40 | 60 | 80 | 100 |




**a** What is the temperature of the water after 10 seconds?

**b** Where does the line cross the $T$-axis? What does this represent?

**c** When is the temperature of the water 55°C?

**d** At what rate is the temperature of the water increasing each second?

### Solution

**a** From the graph or equation:

$T = 5 \times 10 + 20$

$= 70$°C

**b** 20, this represents the starting temperature of the water (20°C), the room temperature.

**c** From the graph or equation:

$55 = 5t + 20$

$35 = 5t$

$t = 7$ seconds

**d** From the graph or table, between 0 and 4 seconds, the temperature increased from 20°C to 40°C, an increase of 20°C.

So the temperature of the water is increasing at $\frac{20}{4} = 5$°C per second.

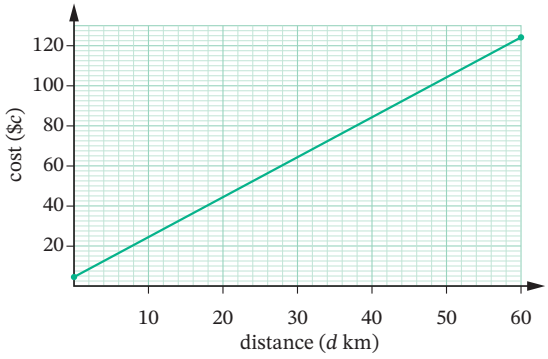
Note that this is also the coefficient of $t$ in the equation $T = 5t + 20$ and the gradient of the line.

**EXERCISE 11.11** ANSWERS ON P. 612

## Applying linear equations U F PS R C

For this exercise, graphs can be drawn on paper or using dynamic geometry software.

**1** The cost of a taxi fare can be represented by the linear equation $c = 2d + 4$ where $c$ is in dollars and $d$ is in kilometres. The graph of the equation is shown below. PS R C



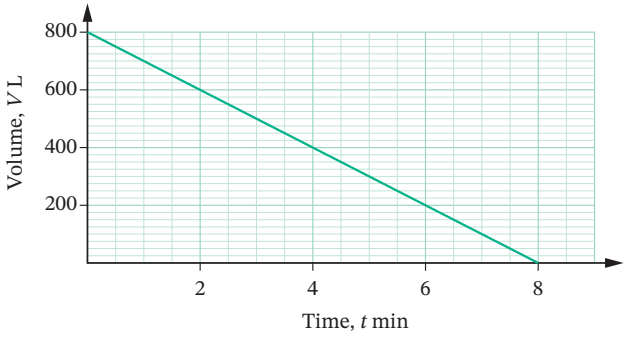

**a** Find the cost of a trip of 28 km. Use both the equation and the graph.

**b** If Juan had $100, how far could he travel on the taxi?

**c** What is the $c$-intercept of the line? What does this value represent?

**d** At what rate is the cost increasing per kilometre?

**2** This graph shows the volume of water in a 800 L tank being drained over a period of minutes. PS R C




**a** How long does it take for the tank to empty?

**b** Find the vertical intercept, $c$, of the line.

**c** Find the equation of the line in the form $V = mt + c$, where $m$ and $c$ are numbers.

**d** How much water was in the tank after 5 minutes?

**e** How much is the volume of water decreasing in one minute? Is this the same amount for each minute?

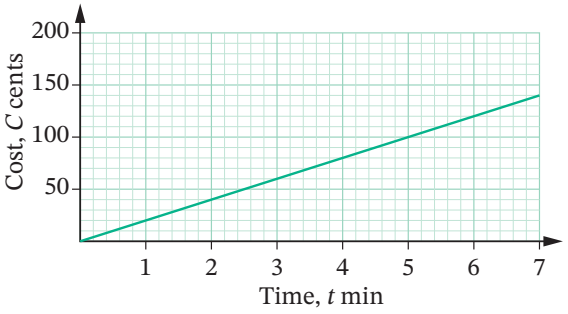
Worksheets
Number plane grid paper
1 cm grid paper

11.11

EXAMPLE 15

Foundation Standard Complex

**3** This graph shows the cost of an international phone call, where $C$ is the cost in cents and $t$ is the time in minutes. PS R C




**a** Find the cost of a phone call which lasts 5 minutes.

**b** At what rate is the cost increasing each minute?

**c** Find the vertical intercept of the line.

**d** Find the equation of the line.

**e** Alyssa has $1.20, for how long can she talk to her friend?

**4** A steel company makes square plates of different sizes. The cost of each plate is calculated using the formula $C = 1.5s + 5$, where $C$ is in dollars and $s$ is in centimetres. PS R C

**a** Copy and complete this table.

| Side length, $s$ cm | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| Cost, $C | | | | |

**b** Graph the line on a number plane.

**c** In the formula $C = 1.5s + 5$, what does the value of 1.5 represent?

**d** What does the value of 5 represent?

**e** Use your graph or equation to determine:

**i** the cost of manufacturing a plate with side 18 cm

**ii** the side length of a plate which cost $38

**5** Tony kicks a ball towards the goal 60 m away. The distance of the ball from the goal is given by the linear equation $d = 60 - 10t$, where $d$ is in metres and $t$ is in seconds. PS R C

**a** Copy and complete this table.

| Time, $t$ s | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| Distance, $d$ m | | | | |

**b** Graph the linear equation.

**c** What is the distance of the ball from goal after 5 seconds?

**d** How long does it take for the ball to reach 30 m from goal?

**e** How long does it take the ball to reach the goal?

**f** What is the vertical intercept of the line and what does it represent?

**g** What is the speed of the ball?

**6** Lara finds that the cost of running her car each week is determined by the equation $C = 0.5d + 20$, where $C$ is the cost in dollars and $d$ represents the distance travelled in kilometres. PS R C

**a** Copy and complete this table.

| Distance, $d$ km | 0 | 10 | 20 | 30 | 40 |
|---|---|---|---|---|---|
| Cost, \$$C$ | | | | | |

**b** Graph this equation on a number plane.

**c** What does the vertical intercept of the line represent?

**d** What does the value of 0.5 in the equation $C = 0.5d + 20$ represent?

**e** Use your graph or equation to determine:

**i** the cost of travelling 24 km

**ii** the distance that has a running cost of \$60



# Solving linear equations graphically 11.12

Equations can be solved **algebraically** using the balancing and backtracking methods. However, we can also solve an equation **graphically** by first graphing it on the number plane.

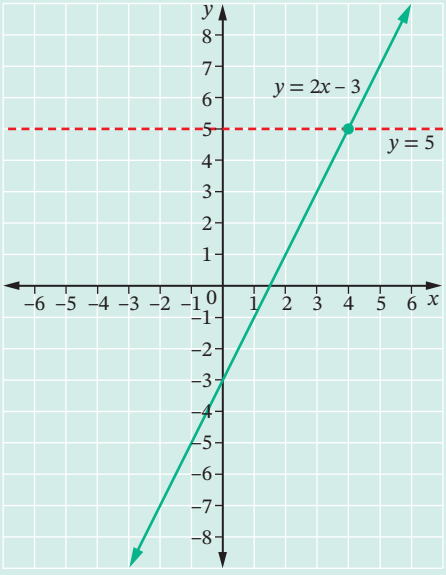
## Example 16

Solve the equation $2x - 3 = 5$ graphically.

### Solution

Graph $y = 2x - 3$ on a number plane (the LHS of the equation).

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | −5 | −3 | −1 | 1 |




For each point in the table and on the line, the $y$-coordinate is the value of $2x - 3$. So to solve $2x - 3 = 5$, we need to find the point whose $y$-coordinate is 5.

Draw a dotted horizontal line at $y = 5$ (shown in red on the diagram) and read off the coordinates for the point on $y = 2x - 3$ crossed by this line.

This point is (4, 5), which means that the solution to $2x - 3 = 5$ is $x = 4$.

Check: $2 \times 4 - 3 = 5$

Foundation Standard Complex

**EXERCISE 11.12** ANSWERS ON P. 612

## Solving linear equations graphically U F

This exercise is best completed using dynamic geometry software or a graphing website.

EXAMPLE 16

**Worksheets**
Number plane grid paper
A page of number planes

**1** Use the graph in Example **13** above to solve each equation below. Check your solutions.

**a** $2x - 3 = 3$ **b** $2x - 3 = -7$ **c** $2x - 3 = 0$

**2** Graph $y = 2x + 1$ and use it to solve each equation below graphically.

**a** $2x + 1 = 7$ **b** $2x + 1 = 10$ **c** $2x + 1 = -5$

**3** Solve each equation in question **2** algebraically.

**4** Graph $y = 2x - 1$ and use it to solve each equation below graphically.

**a** $2x - 1 = -5$ **b** $2x - 1 = -1$ **c** $2x - 1 = 4$

**5** Graph $y = 3x - 2$ and use it to solve each equation graphically.

**a** $3x - 2 = 7$ **b** $3x - 2 = -11$ **c** $3x - 2 = -2$

**6** Graph $y = -x + 1$ and use it to solve each equation graphically.

**a** $-x + 1 = -3$ **b** $-x + 1 = 4$ **c** $-x + 1 = -1$

**7** Graph $y = -2x - 1$ and use it to solve each equation graphically.

**a** $-2x - 1 = -9$ **b** $-2x - 1 = 0$ **c** $-2x - 1 = 3$

**8** Graph $y = \frac{1}{2}x + 3$ and use it to solve each equation graphically.

**a** $\frac{1}{2}x + 3 = 5$ **b** $\frac{1}{2}x + 3 = 1$ **c** $\frac{1}{2}x + 3 = 2$

## Technology

### Intersecting lines

We will graph the 2 straight lines $y = x + 5$ and $y = 3 - x$ using a spreadsheet to find their point of intersection.

| | A | B | C |
|---|---|---|---|
| 1 | x | y = x + 5 | y = 3 - x |
| 2 | -4 | | |
| 3 | -3 | | |
| 4 | -2 | | |
| 5 | -1 | | |
| 6 | 0 | | |
| 7 | 1 | | |
| 8 | 2 | | |
| 9 | 3 | | |
| 10 | 4 | | |

1. Set up this spreadsheet.
2. In cell B2, enter **=A2+5**, then **Fill Down** to cell B10.
3. In cell C2, enter **=3−A2**, then **Fill Down** to cell C10.
4. Highlight all 3 columns and click **Insert** and **Scatter (Smooth Lines and Markers)**.
5. From the graph, read the point of intersection of the 2 straight lines.

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**6** Find the point of intersection for each pair of straight lines below, by setting up a separate spreadsheet for each pair of lines and using **Insert** and **Scatter (Smooth Lines and Markers)** (as above) to graph each pair of lines on the same set of axes.

**a**

| | A | B | C |
|---|---|---|---|
| 1 | x | y = 2x - 2 | y = -x + 7 |
| 2 | -4 | | |
| 3 | -3 | | |
| 4 | -2 | | |
| 5 | -1 | | |
| 6 | 0 | | |
| 7 | 1 | | |
| 8 | 2 | | |
| 9 | 3 | | |
| 10 | 4 | | |

**b**

| | A | B | C |
|---|---|---|---|
| 1 | x | y = 3x - 6 | y = -4x - 6 |
| 2 | -4 | | |
| 3 | -3 | | |
| 4 | -2 | | |
| 5 | -1 | | |
| 6 | 0 | | |
| 7 | 1 | | |
| 8 | 2 | | |
| 9 | 3 | | |
| 10 | 4 | | |

**c**

| | A | B | C |
|---|---|---|---|
| 1 | x | y = 10 - 2x | y = 2x |
| 2 | -4 | | |
| 3 | -3 | | |
| 4 | -2 | | |
| 5 | -1 | | |
| 6 | 0 | | |
| 7 | 1 | | |
| 8 | 2 | | |
| 9 | 3 | | |
| 10 | 4 | | |



# Intersecting lines 11.13

**Worksheet**
Line intersections

## Example 17

Graph $y = 2x - 2$ and $y = x + 1$ on the same number plane and use the graphs to solve the equation $2x - 2 = x + 1$.

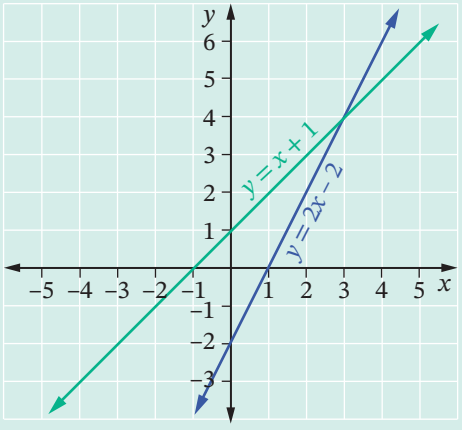
### Solution

$y = 2x - 2$

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | −4 | −2 | 0 | 2 |

$y = x + 1$

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | 0 | 1 | 2 | 3 |




To solve $2x - 2 = x + 1$, we need to find the point on both lines where the $y$-coordinate is the same.

This is the point where the 2 lines intersect.

This point is (3, 4), which means that the solution to $2x - 2 = x + 1$ is $x = 3$.

Check:

LHS $= 2 \times 3 - 2 = 4$

RHS $= 3 + 1 = 4$

LHS = RHS

**Worksheets**
Number plane grid paper

A page of number planes

EXAMPLE 17

## EXERCISE 11.13 ANSWERS ON P. 613

## Intersecting lines U F R

This exercise is best completed using dynamic geometry software.

**1** By substitution, find the point of intersection of the lines $y = 2x - 1$ and $y = 5 - x$.
Select the correct answer **A**, **B**, **C** or **D**. R

**A** $(2, -1)$ **B** $(2, 3)$ **C** $(5, -1)$ **D** $(\frac{4}{3}, \frac{5}{3})$

**2** Graph each pair of equations and find their point of intersection.

**a** $y = x$ and $y = -x + 2$ **b** $y = x + 1$ and $y = 2x$

**c** $y = -x - 1$ and $y = x + 1$ **d** $y = \frac{x}{2} - 2$ and $y = -x + 1$

**e** $y = 2x + 1$ and $y = x + 3$ **f** $y = -2x - 1$ and $y = x + 5$

**3** What is the point of intersection of $y = x + 1$ and $y = -x + 5$? Select **A**, **B**, **C** or **D**.

**A** $(3, 2)$ **B** $(2, 3)$ **C** $(4, 1)$ **D** $(4, 5)$

**4** Solve each equation graphically. R

**a** $2x - 1 = x + 2$ **b** $3x - 2 = 2x - 1$ **c** $x - 2 = 4 - x$

**d** $2x = x + 3$ **e** $3x + 2 = x - 1$ **f** $-2x + 3 = 2x + 1$

**5** Solve each of the equations above algebraically.

**6** These 3 linear equations form a triangle when graphed on the number plane.

$y = x + 2$ $y = \frac{x}{2} - 1$ $y = -3x + 6$

Find the coordinates of the 3 vertices of the triangle. R

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## Power plus ANSWERS ON P. 614

**1** **a** George receives twice as much as Bill in pocket money. This can be represented by $y = 2x$, where $y$ is George's pocket money and $x$ is Bill's amount of money. Draw the graph of this line on a number plane where the $x$-axis runs from 0 to 12 and the $y$-axis from 0 to 24.

**b** The total of George and Bill's pocket money is \$24. Write an equation that represents this.

**c** Draw the graph of your equation in part **b** on the same set of axes as part **a**.

**d** Use your graphs to find George's and Bill's pocket money.

**2** Graph each equation accurately and state whether it is linear or non-linear.

**a** $y = x^3 - 2$ **b** $x + y = 5$ **c** $y = x^2 - 1$

**d** $y = 4 - x^2$ **e** $x - y = 4$ **f** $y = \frac{4}{x}$

# CHAPTER 11 REVIEW

## Language of maths

**Quiz**
Language of maths 11

**Puzzle**
Linear equations crossword

| | | | |
|---|---|---|---|
| coefficient | collinear | consecutive | constant term |
| coordinates | decreasing | formula | graphically |
| horizontal | increasing | intersection | linear |
| number plane | origin | quadrant | satisfy |
| solution | table of values | term | variable |
| vertical | $x$-axis | $y$-axis | $y$-intercept |

**1** What is the difference between **linear** and **collinear**?

**2** In which quadrant of the number plane are the $x$- and $y$-coordinates of a point both negative?

**3** What are **consecutive** numbers?

**4** What is the 'answer' to an equation called?

**5** What does it mean if the coordinates of a point **satisfy** the equation of a line?

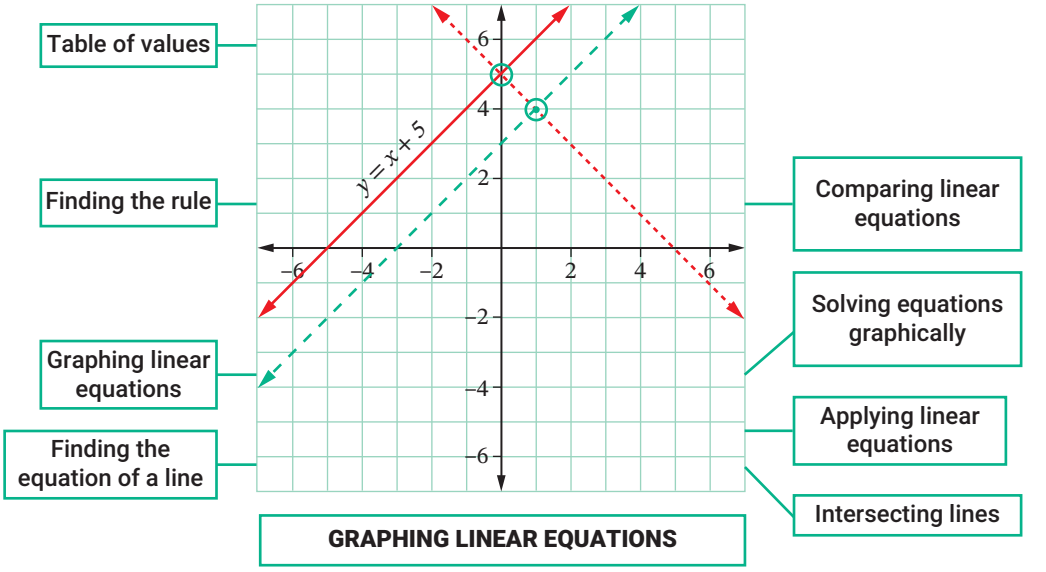
**6** For the linear equation $y = 4x + 1$, which is the **constant term**?

## Topic summary

**Worksheet**
Mind map: Graphing linear equations

- Write in your own words what you have learnt about linear equations and their graphs.
- What parts of this topic did you have difficulty with? Discuss them with a friend.
- Where might you use the skills acquired in this chapter?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.





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# TEST YOURSELF 11

ANSWERS ON P. 615

Quiz
Test yourself 11



**1** Copy and complete each table of values. 11.01

**a** $y = x + 3$

| $x$ | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y$ | | | | | |

**b** $q = 2p - 7$

| $p$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $q$ | | | | | |

**c** $d = 5 - c$

| $c$ | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $d$ | | | | | |

**2** Find the formula for each table of values. 11.02

**a**

| $x$ | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y$ | −3 | −2 | −1 | 0 | 1 |

**b**

| $m$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $p$ | 2 | 5 | 8 | 11 | 14 |

**c**

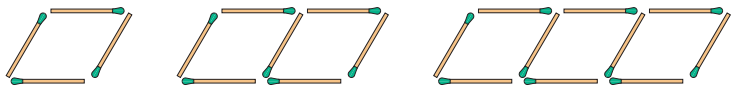
| $m$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $n$ | 10 | 8 | 6 | 4 | 2 |

**3** For this number pattern: 6, 10, 14, 18, 22, ... 11.03

**a** find the 7th term

**b** find a formula for the $n$th term, $T$.

**4** This pattern of rhombuses is made up of matchsticks. 11.03



**a** Copy and complete the table.

| No. of rhombuses, $x$ | 1 | 2 | 3 | 4 | 5 | 8 | 10 |
|---|---|---|---|---|---|---|---|
| No. of matchsticks, $y$ | 4 | | | | | | |

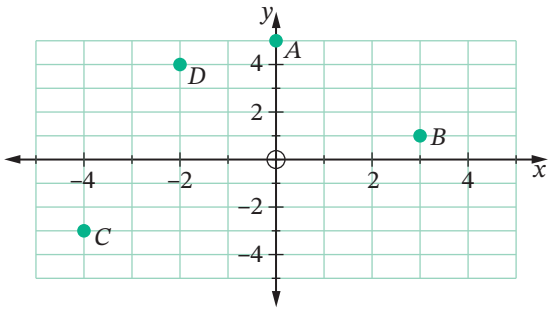
**b** Write the rule for the pattern in words.

**c** Write the rule as a formula.

**d** How many matchsticks are needed to build a pattern of:

**i** 12 rhombuses? **ii** 15 rhombuses?

**5** State the coordinates of the points $A$, $B$, $C$ and $D$. 11.04




**6** Plot each point on a number plane. 11.04

**a** $P(1, -4)$ **b** $Q(-3, -1)$ **c** $R(-1, 2)$

**d** $S(0, 4)$ **e** $T(2\frac{1}{2}, -2)$ **f** $U(-2, 0)$

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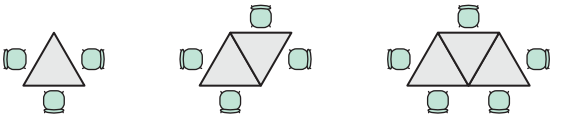
11.04 **7** Which point from question **6**:

**a** is on the $y$-axis? **b** is in the 2nd quadrant?

**c** is on the $x$-axis? **d** is in the 4th quadrant?

11.05 **8** **a** Copy and complete this table for the pattern of tables and chairs.

| Number of tables ($x$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of chairs ($y$) | | | | | |



**b** Find the formula for $y$.

**c** Graph the table of values.

11.06 **9** Graph each linear equation on a number plane after completing a table of values.

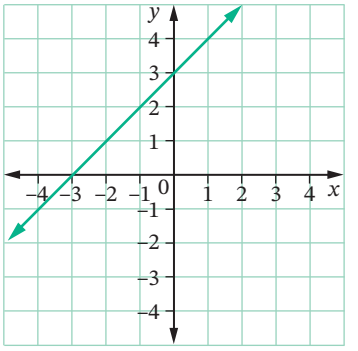
**a** $y = 2x - 1$ **b** $y = -x + 3$ **c** $y = \frac{1}{2}x - 2$

11.06 **10** Test whether each point lies on the line $y = 3x + 2$.

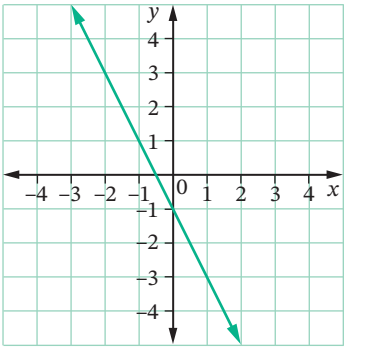
**a** $(1, 2)$ **b** $(-1, 4)$ **c** $(-2, -4)$ **d** $(0, 2)$

11.07 **11** Find the equation of each line.

**a**




**b**




11.07 **12** Graph each linear equation on the same number plane.

**a** $y = -3$ **b** $x = 4$

11.08 **13** **a** Graph $y = -x + 2$, $y = x + 2$ and $y = 2x + 2$ on the same number plane.

**b** What is the same about the 3 lines?

**c** What is the same about the 3 linear equations?

**d** What is different about the 3 lines?

**14** Find the gradient of each line shown in Question **11**.

**15** Draw a line through the 2 points (0, –3) and (2, 1) on a number plane and find its gradient.

**16** Write the equation of the line with:

**a** gradient –4 and $y$-intercept 8 **b** gradient $\frac{-3}{2}$ and $y$-intercept –6

**17** Graph $y = -2x - 1$ on a number plane by finding its gradient and $y$-intercept.

**18** Serge provides waiters for a catering company and charges according to the formula $C = 100 + 30w$, where $C$ is the cost in dollars and $w$ is the number of waiters.

**a** Copy and complete the table below.

| No. of waiters, $w$ | 1 | 2 | 4 | 6 |
|---|---|---|---|---|
| Cost, \$$C$ | | | | |

**b** Graph this line on the number plane.

**c** Find the cost of hiring 5 waiters.

**d** If Annalise paid \$310, how many waiters did she hire?

**e** What is the cost of each additional waiter?

**19** **a** Graph $y = 3x - 4$ and use it to solve $3x - 4 = 5$ graphically.

**b** Graph $y = 2x + 1$ and use it to solve $2x + 1 = -2$ graphically.

**20** Graph $y = 2x$ and $y = x - 2$ and find their point of intersection.

**21** Solve the equation $3x - 3 = x + 3$:

**a** graphically **b** algebraically



12

MEASUREMENT AND SPACE

# SIMILAR FIGURES AND SURFACE AREA

## OPTIONAL YEAR 9 CORE TOPIC

Figures which have the same shape but are different in size are said to be similar. For example, different-sized photographs of a person where one is an enlargement of the other show the same person, but the size is different. Similar figures are quite useful in calculating the size of objects which are difficult to measure directly, such as the height of a building, the height of a tree or the width of a river.




## Chapter outline

| | | Working mathematically | | | | |
|---|---|---|---|---|---|---|
| **12.01** | Similar figures* | U | F | | R | C |
| **12.02** | Properties of similar figures* | U | F | | R | C |
| **12.03** | Finding sides in similar figures* | U | F | PS | R | |
| **12.04** | Surface area of a prism* | U | F | PS | R | |
| **12.05** | Surface area of a cylinder* | U | F | | R | C |

*YEAR 9 EXTENSION

## Wordbank

**enlargement** An increase in the size of a shape

**image** A transformed shape after it has been enlarged or reduced

**matching angles** corresponding angles of similar figures in the same position

**matching sides** corresponding sides of similar figures in the same position

**scale factor** The amount by which a shape has been enlarged or reduced, equal to $\frac{\text{image length}}{\text{original length}}$

**similar** To have the same shape but not necessarily the same size, an enlargement or reduction (symbol: ~)

**surface area** The total area of all the faces of a solid shape

**Quiz**
Wordbank 12

**U** = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communicating

**Videos (4):**

**12.03** Finding an unknown side in similar figures

**12.04** Surface area of a prism • Surface area of prisms

**12.05** Surface area of a cylinder

***Twig* video (1):**

**12.04** Cubist art

**Quizzes (5):**

- Wordbank 12
- SkillCheck 12
- Mental skills 12
- Language of maths 12
- Test yourself 12

**Skillsheets (2):**

**12.03** Finding sides in similar triangles

**12.04** Solid shapes

**Worksheets (12):**

**12.01** A page of similar figures • A page of congruent and similar figures

**12.02** Enlargements and reductions • Enlarging a logo

**12.03** Finding sides in similar figures

**12.04** Surface area 1 • Nets of solids • Solid shapes

**12.05** Surface area 2 • A page of prisms and cylinders • Applications of area 3

**Mind map** Similar figures and surface area

**Puzzles (4):**

**12.02** Cartoon enlargement

**12.03** Similar triangles • Nested similar triangles

**12.05** Car song

**Technology (1):**

**12.04, 12.05** Surface area calculator

**Spreadsheets (1):**

**12.04, 12.05** Surface area calculator

**Presentation (1):**

**12.04** Surface area

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

## In this chapter you will:

(EXTENSION, YEAR 9 CORE)

- identify similar figures and investigate their properties
- use scale factors to find unknown sides in similar figures
- calculate the surface areas of rectangular and triangular prisms
- calculate the surface area of a cylinder

## SkillCheck ANSWERS ON P. 617

**1** Simplify each fraction.

**a** $\frac{4}{8}$ **b** $\frac{6}{4}$ **c** $\frac{15}{3}$ **d** $\frac{26}{14}$

**2** Evaluate each expression.

**a** $\frac{2}{3} \times 15$ **b** $\frac{1}{4} \times 32$ **c** $\frac{3}{2} \times 10$ **d** $\frac{5}{7} \times 84$

**3** Use a protractor to measure the size of each angle.

**a**



**b**



**c**



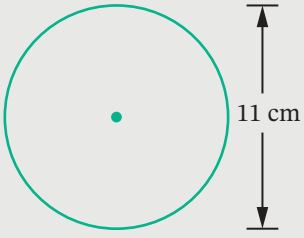
**4** Solve each equation.

**a** $\frac{x}{2} = \frac{10}{4}$ **b** $\frac{y}{40} = \frac{3}{5}$ **c** $\frac{p}{32} = \frac{4}{16}$ **d** $\frac{m}{2} = \frac{12}{18}$

**5** Find correct to 2 decimal places this circle's:

**a** circumference

**b** area

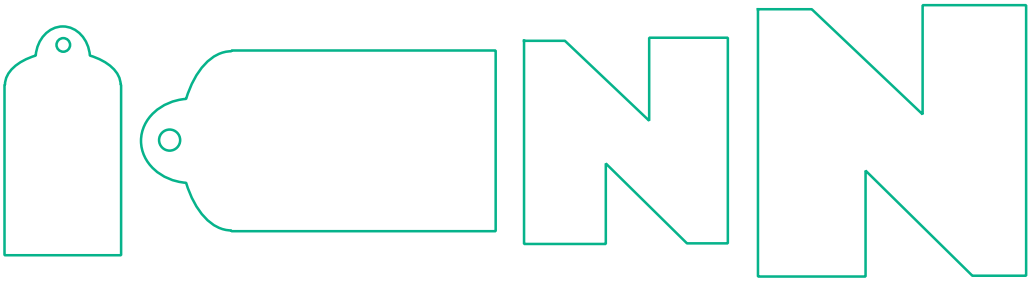



Quiz
SkillCheck 12

12.01

# Similar figures 12.01

**Similar figures** have the same shape but are not necessarily the same size. One figure is an enlargement or reduction of the other figure.



YEAR 9 CORE

**Worksheets**
A page of similar figures

A page of congruent and similar figures

YEAR 9 CORE

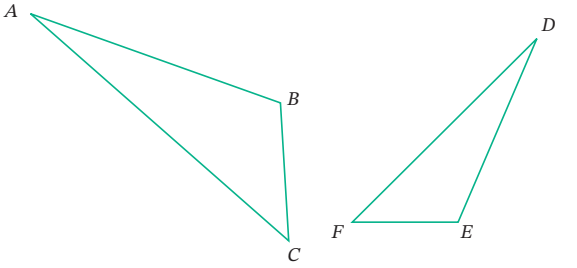
A photograph and its enlargement are examples of similar figures.





iStock.com/USO

Triangles $ABC$ and $DEF$ below are similar.




We can rotate $\triangle DEF$ anticlockwise to match up the angles so that the triangles look similar.

$\triangle DEF$ is a reduction of $\triangle ABC$.

The symbol for 'is similar to is '~'.

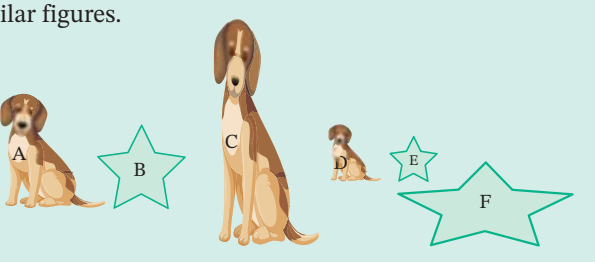
We can write '$\triangle ABC \sim \triangle DEF$', which is read 'Triangle $ABC$ is similar to triangle $DEF$'.

When using this notation, we must write the vertices (angles) of the similar figures in matching order:

- $\angle A$ matches $\angle D$
- $\angle B$ matches $\angle E$
- $\angle C$ matches $\angle F$

## Example 1

Select 2 pairs of similar figures.




### Solution

Figures A and D are similar.

Figures B and E are similar.

They are the same shape but different sizes.


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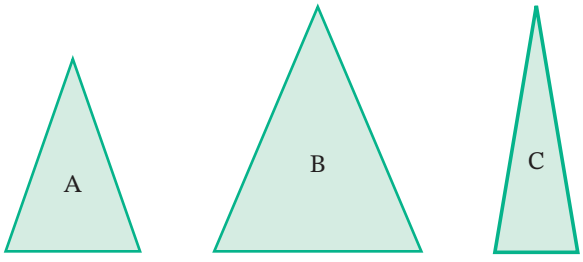
## Similar figures U F R C

EXAMPLE 1

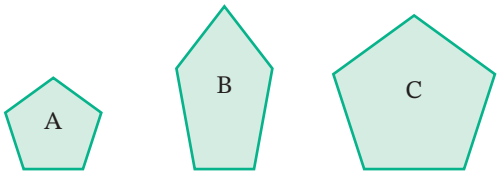
12.01

**1** Which pair of shapes are similar? R C

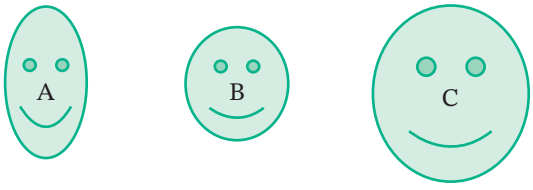
**a**



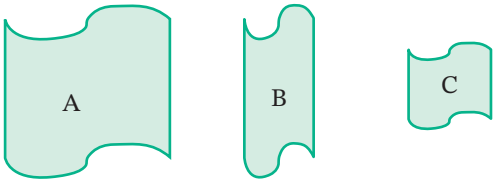

**b**



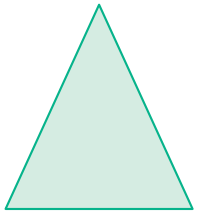

**c**




**d**



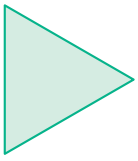

**2** Which triangle below is similar to this triangle?
Select **A**, **B**, **C** or **D**.



**A**



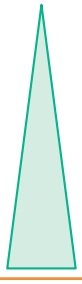
**B**



**C**



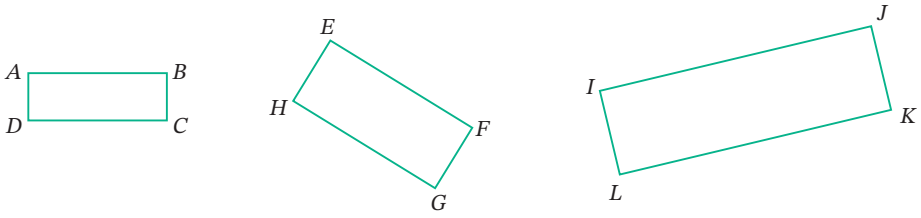
**D**



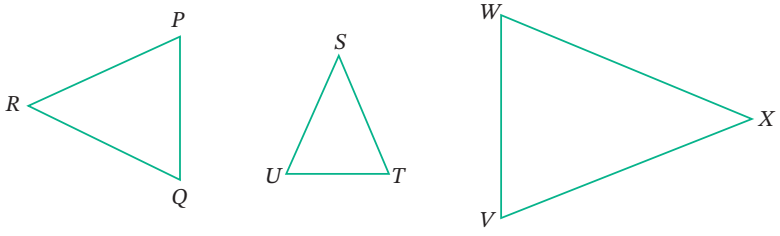
Foundation Standard Complex

**3** Find the pair of similar figures in each set of shapes below. Then write a similarity statement of the form '____ ~ ____' naming the vertices in the correct order. R C

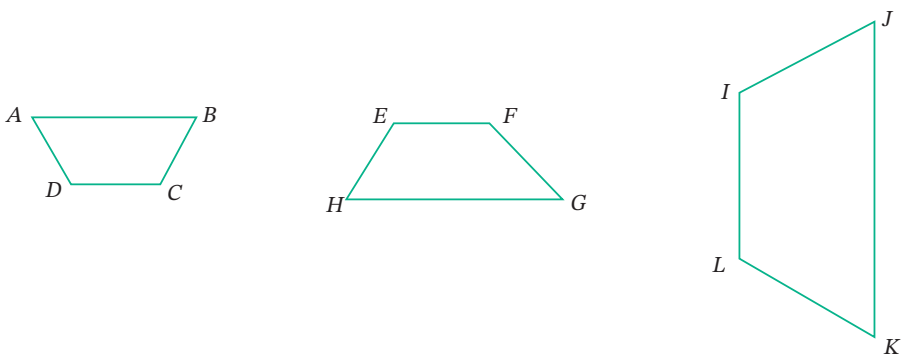
**a**



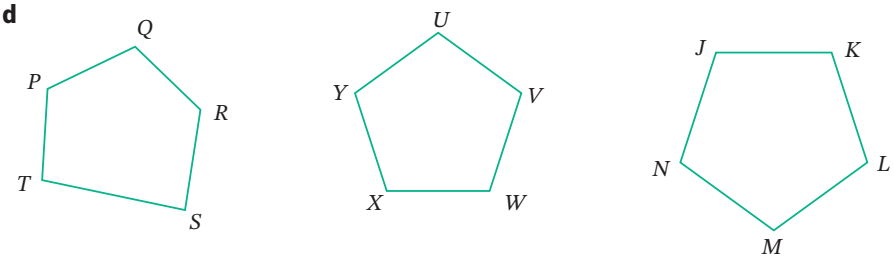

**b**



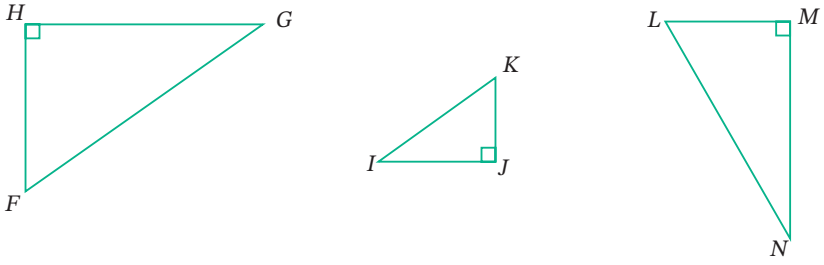

**c**




**d**




**e**

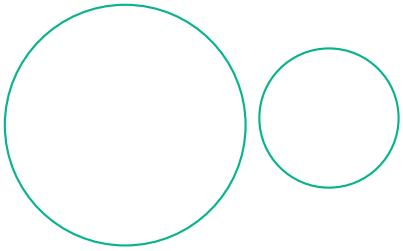



Foundation Standard Complex


9780170479516

**4** **a** State whether the figures in each pair below are similar. R C

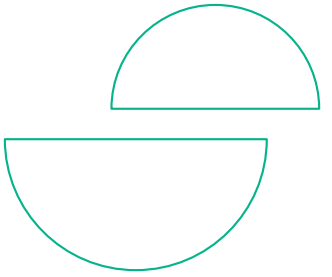
**i**



**ii**



**iii**



**b** How did you decide on your answer for part **a**?

**5** **a** Draw 2 squares that are similar.

**b** Draw 2 circles that are similar.

**c** Draw 2 rectangles that are similar.

**d** Draw 2 similar right-angled triangles.

**e** Make up your own shape and draw it. Draw another that is similar to it.

**6** Discuss these statements and decide whether they are true or false. R C

**a** All squares are similar.

**b** All rectangles are similar.

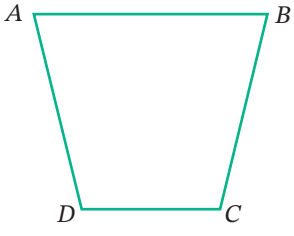
**c** All triangles are similar.

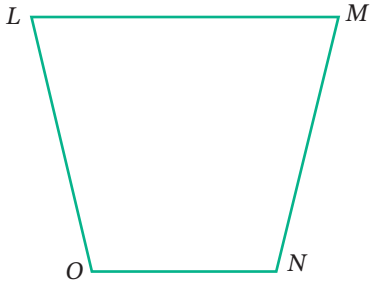
**d** All circles are similar.

## Investigation

### Properties of similar figures

You need a ruler and a protractor to complete this investigation. The similar trapeziums below have the same shape but are different in size.




Foundation Standard Complex

**1** **a** Use a ruler to measure the sides of $ABCD$ and the matching sides in $LMNO$ in millimetres. Copy and complete this table.

| *ABCD* | *LMNO* | Ratio |
|---|---|---|
| $AB =$ | $LM =$ | $\frac{LM}{AB} =$ |
| $BC =$ | $MN =$ | $\frac{MN}{BC} =$ |
| $CD =$ | $NO =$ | $\frac{NO}{CD} =$ |
| $DA =$ | $OL =$ | $\frac{OL}{DA} =$ |

**b** How much larger are all the sides in $LMNO$ than the matching sides in $ABCD$?

This number is called the **scale factor**. $ABCD$ has been enlarged by the scale factor to get $LMNO$.

**2** **a** Use a protractor to measure the angles in each trapezium. Copy and complete this table.

| *ABCD* | *LMNO* |
|---|---|
| $\angle A =$ | $\angle L =$ |
| $\angle B =$ | $\angle M =$ |
| $\angle C =$ | $\angle N =$ |
| $\angle D =$ | $\angle O =$ |

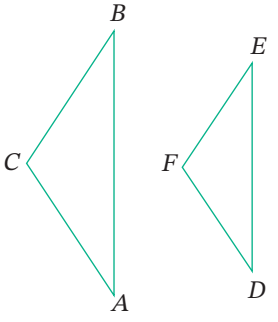
**b** What do you notice about the sizes of the matching angles?

**c** Copy and complete the following:

In similar figures

- Matching sides are in the same . .. .. . ... .. .
- Matching angles are .. . .. . .. . ...

**3** Look at these similar triangles.




**a** Measure the angles of each triangle as accurately as you can. Copy and complete this table.

| | |
|---|---|
| $\angle A =$ | $\angle E =$ |
| $\angle B =$ | $\angle F =$ |
| $\angle C =$ | $\angle G =$ |

**b** Measure the sides of each triangle as accurately as you can.
Copy and complete this table.

| Length of sides (mm) | | Ratio matching sides |
|---|---|---|
| $AB =$ | $DE =$ | $\frac{AB}{DE} =$ |
| $BC =$ | $EF =$ | $\frac{BC}{EF} =$ |
| $CA =$ | $FD =$ | $\frac{CA}{FD} =$ |



# Properties of similar figures 12.02

## Similar figures

Similar figures have the same shape. One figure is an enlargement of the other figure.

For similar figures:

- matching angles are equal
- the lengths of matching sides are in the same ratio

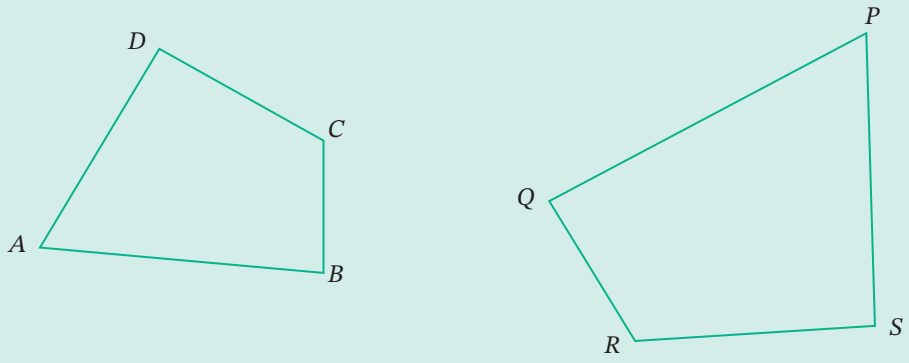
**Worksheets**
Enlargements and reductions

Enlarging a logo

**Puzzle**
Cartoon enlargement

## Example 2

**a** Use a protractor to measure the angles in these 2 quadrilaterals.




**b** Are the matching angles the same size?

**c** Are the quadrilaterals similar?

### Solution

**a**

| Angle | A | B | C | D | P | Q | R | S |
|---|---|---|---|---|---|---|---|---|
| Size | 80° | 90° | 115° | 75° | 80° | 90° | 115° | 75° |

**b** Matching angles are the same size.

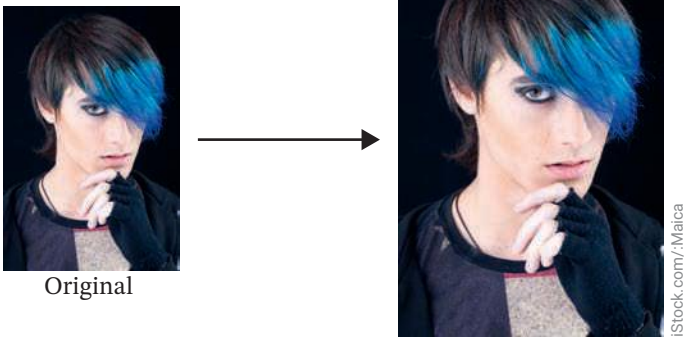
$\angle A = \angle P = 80^\circ$, $\angle B = \angle Q = 90^\circ$, $\angle C = \angle R = 115^\circ$, $\angle D = \angle S = 75^\circ$.

**c** The quadrilaterals are similar.

## The scale factor

A similar figure is an *enlargement* or a *reduction* of another figure. The figure we start with is called the **original**. The enlarged or reduced figure is called the **image**. The scale factor describes the change in size between the original and the image.

For example, this photograph has been enlarged to a bigger size to be placed in a frame, using a scale factor of 1.5. The image is 1.5 times the size of the original.



Original

Image

iStock.com/iMaica

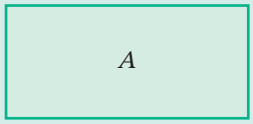
### Scale factor

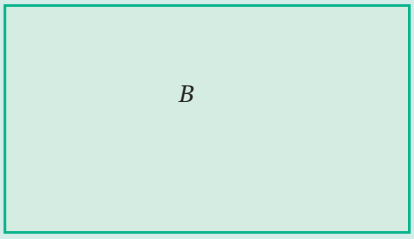
$$\text{Scale factor} = \frac{\text{image length}}{\text{object length}}$$

- A scale factor greater than 1 means that the original has been enlarged
- A scale factor between 0 and 1 means that the original has been reduced

## Example 3

These 2 rectangles are similar. By measuring the matching sides, find the scale factor.




In all questions, assume that the left figure is the original and the right figure is the image.

### Solution

Length of $A = 3$ cm

Width of $A = 2$ cm

$$\text{Scale factor} = \frac{\text{image length}}{\text{object length}}$$

$$= \frac{6}{3}$$

$$= 2$$

Scale factor is 2.

Length of $B = 6$ cm

Width of $B = 4$ cm

$$\text{Scale factor} = \frac{\text{image length}}{\text{object length}}$$

$$= \frac{4}{2}$$

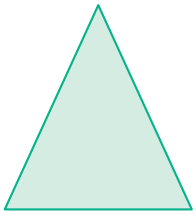
$$= 2$$

**EXERCISE 12.02** ANSWERS ON P. 617

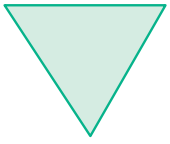
## Properties of similar figures U F R C

EXAMPLE 2

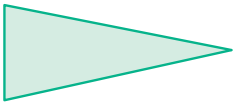
**1** By measuring the angles in each figure, determine which figure is similar to this triangle? Select the correct answer **A**, **B**, **C** or **D**.



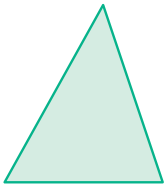
**A**



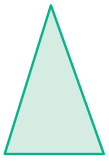
**B**



**C**

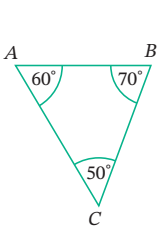


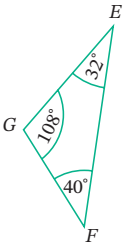
**D**

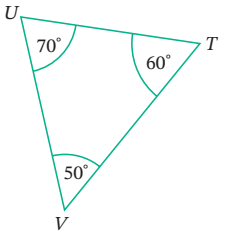


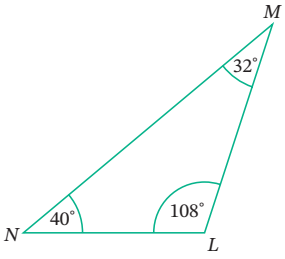
Foundation | Standard | Complex

**2** Name 2 pairs of similar triangles, writing a similarity statement of the form '____ ~ ____'. Make sure you write the vertices in matching order. R C



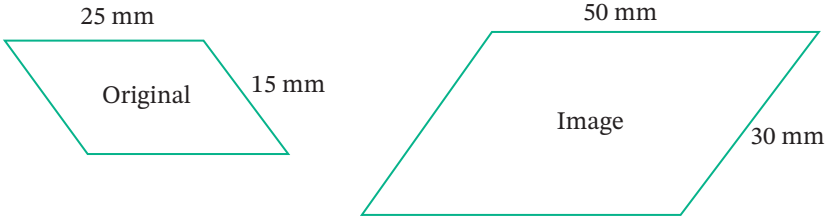



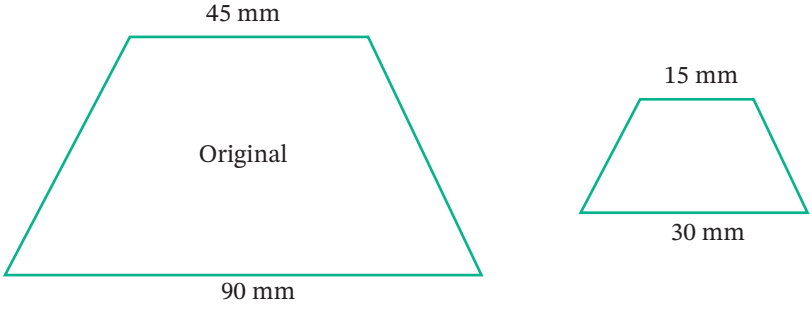

EXAMPLE 3

**3** Calculate the scale factor for each pair of similar quadrilaterals. R C

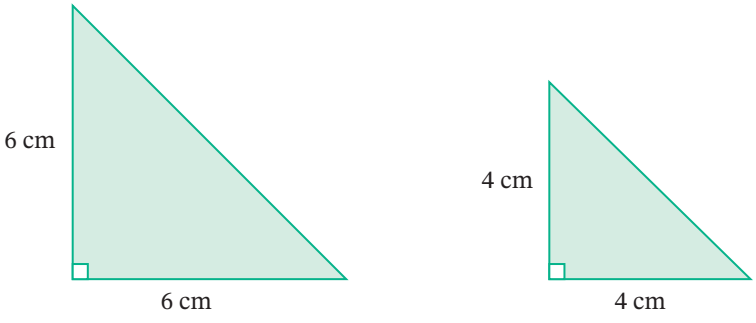
**a**




**b**




**4** These 2 triangles are similar. R




**a** Measure all the angles in both triangles. Are matching angles equal?

**b** Find the ratio of matching sides.

**c** What is the scale factor?

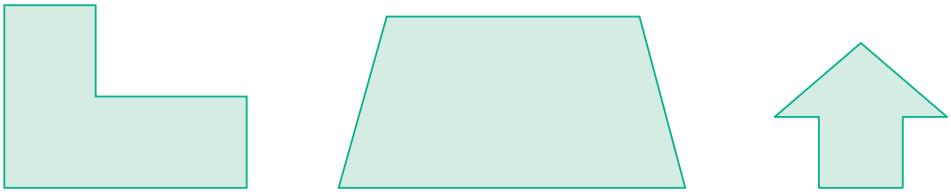
Foundation Standard Complex


9780170479516

**5** A photograph is 12 cm long and it is enlarged using a scale factor of 1.5. What is the length of the enlarged photograph? R

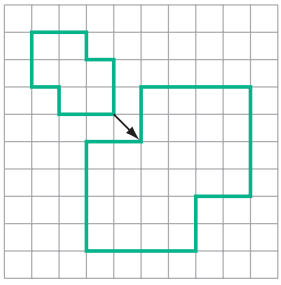
**6** Copy each shape and use the given scale factor to enlarge or reduce the figure.

**a** scale factor of 2 **b** scale factor of $\frac{1}{2}$ **c** scale factor of 1.5

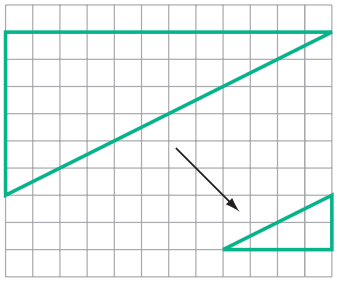


**7** By measuring with a ruler, find the scale factor for each pair of similar figures.

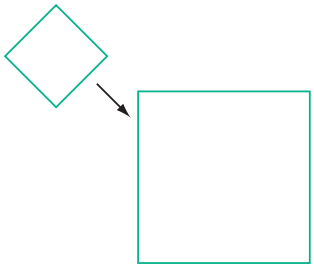
**a**



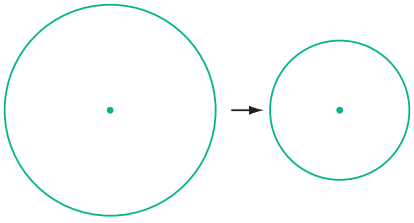
**b**



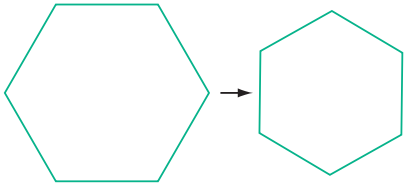
**c**



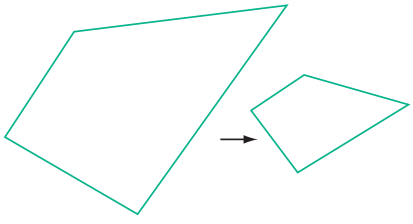
**d**



**e**



**f**

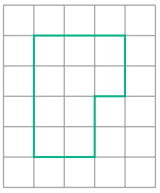


12.02

Foundation Standard Complex

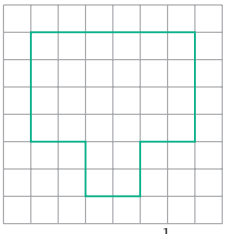
**8** Copy each figure and draw its image using the given scale factor.

**a**



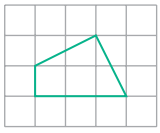
Scale factor 2

**b**



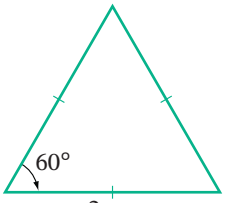
Scale factor $\frac{1}{2}$

**c**



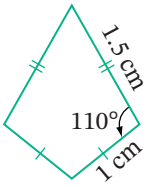
Scale factor 3

**d**



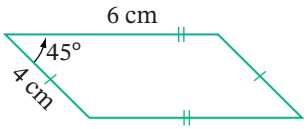

Scale factor 2

**e**




Scale factor 4

**f**




Scale factor $\frac{1}{4}$

# 12.03 Finding sides in similar figures

**Video**
Finding an unknown side in similar figures

**Skillsheet**
Finding sides in similar triangles'

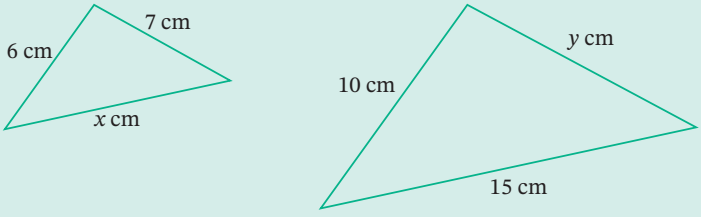
**Worksheet**
Finding sides in similar figures

Puzzles
Similar figures

Nested similar triangles

## Example 4

Find the value of $x$ and $y$ in this pair of similar triangles.




### Solution

The sides marked 6 cm and 10 cm are matching sides, and those marked $x$ cm and 15 cm are matching sides.
Since matching sides are in the same ratio:

$$\frac{x}{15} = \frac{6}{10}$$

$$x = \frac{6}{10} \times 15$$
$$= 9$$

**Alternative method**

Scale factor $= \frac{10}{6} = \frac{5}{3}$

So $x = 15 \div \frac{5}{3} = 9$

$y$ cm and 7 cm are matching sides.

Since matching sides are in the same ratio:

$$\frac{y}{7} = \frac{10}{6}$$

$$y = \frac{10}{6} \times 7$$
$$= 11\frac{2}{3}$$

So $y = 7 \times \frac{5}{3} = 11\frac{2}{3}$

Foundation Standard Complex


9780170479516

## EXERCISE 12.03 ANSWERS ON P. 617

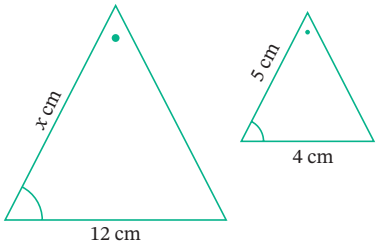
# Finding sides in similar figures U F PS R

EXAMPLE 4

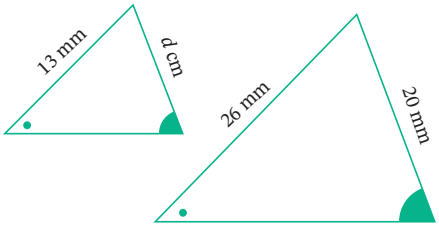
12.03

**1** Find the value of the variable in each pair of similar triangles.

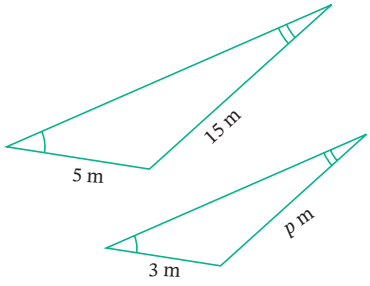
**a**



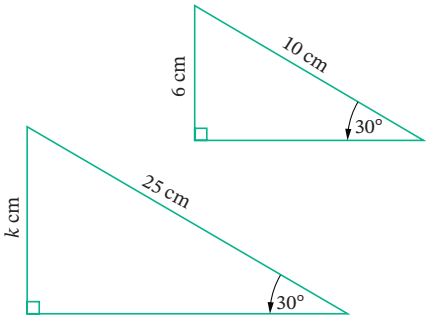

**b**



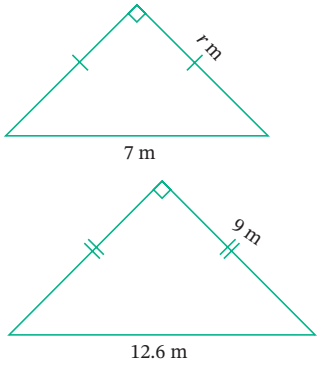

**c**



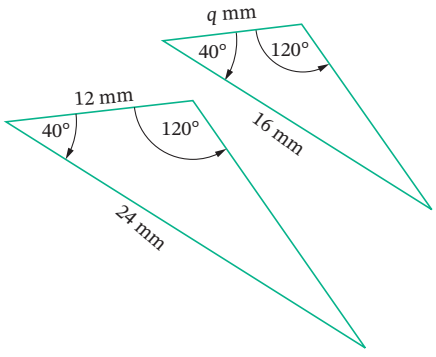

**d**



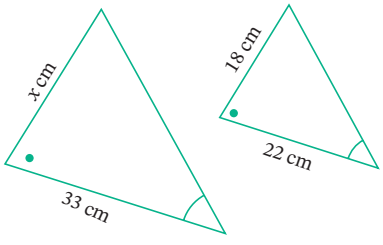

**e**



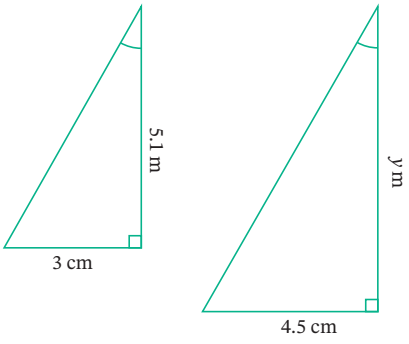

**f**




**g**

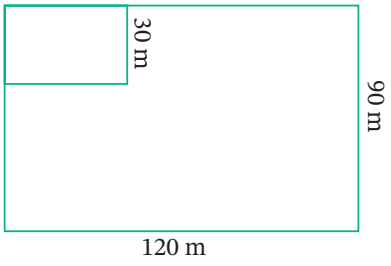



**h**




Foundation | Standard | Complex

**2** A large rectangular paddock, of length 120 m and width 90 m, has a small rectangular pen of width 30 m, in one corner. The paddock and the pen are similar figures.



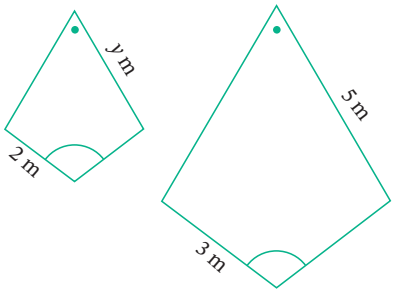

Which of the following is the length of the pen? Select the correct answer **A**, **B**, **C** or **D**.

PS R

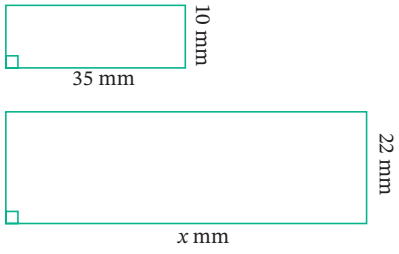
**A** 22.5 m **B** 37.5 m **C** 40 m **D** 60 m

**3** Find the value of the variable in each pair of similar figures.

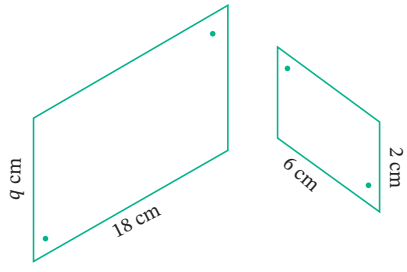
**a**



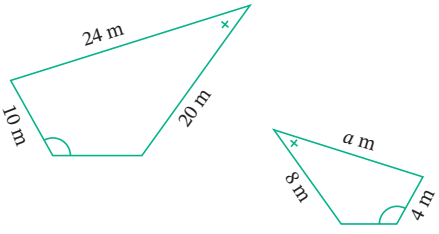

**b**



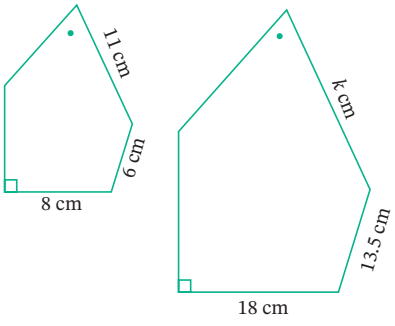

**c**



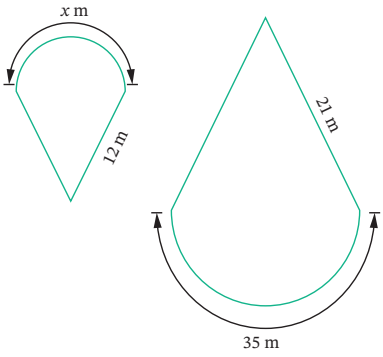

**d**




**e**




**f**

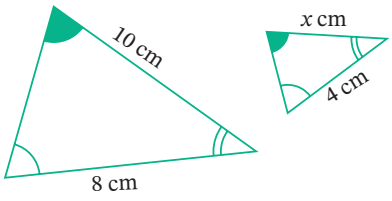



Foundation Standard Complex

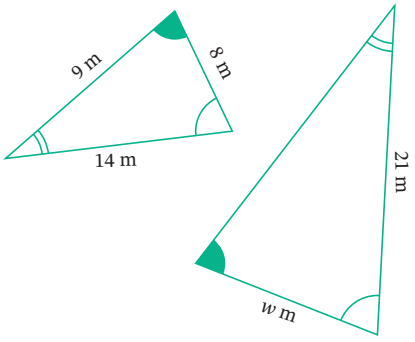

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**4** For each pair of similar triangles, find the value of the variable.

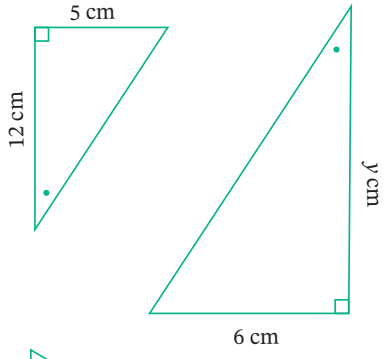
**a**



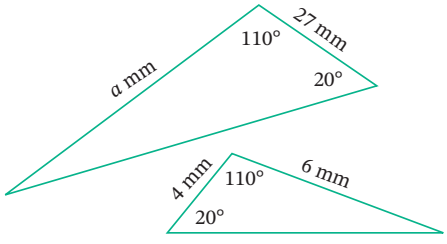

**b**



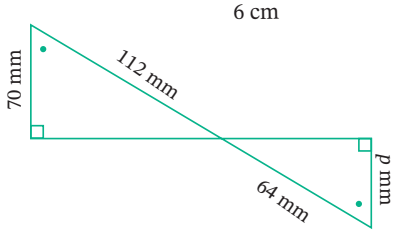

**c**



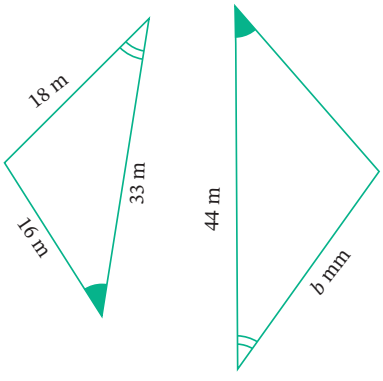

**d**



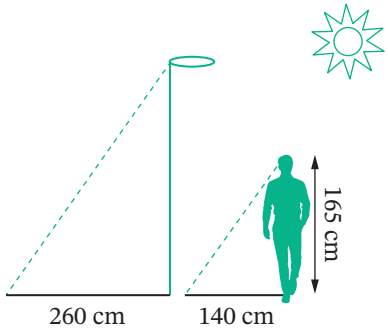

**e**




**f**




**5** Marcus stands next to a pole supporting a netball ring on a sunny day. He is 165 cm tall and casts a shadow of 140 cm. The pole casts a shadow of 260 cm. How high is the pole, correct to the nearest centimetre? **PS** **R**




Foundation Standard Complex

Quiz
Mental skills 12

# Mental skills 12: Maths without calculators ANSWERS ON P. 617

## Time differences

**1** Study each example.

**a** What is the time difference between 11:40 a.m. and 6:15 p.m.?

From 11:40 a.m. to 5:40 p.m. = 6 hours

*Count*: '11:40, 12:40, 1:40, 2:40, 3:40, 4:40, 5:40'

From 5:40 a.m. to 6:00 p.m. = 20 min

From 6:00 p.m. to 6:15 p.m. = 15 min

5 hours + 20 min + 15 min = 6 hours 35 min

OR:




**b** What is the time difference between 1645 and 2320?

From 1645 to 2245 = 6 hours (22 − 16 = 6)

From 2245 to 2300 = 15 min

From 2300 to 2320 = 20 min

6 hours + 15 minutes + 20 minutes = 6 hours 35 minutes

OR:




**2** Now find the time difference between:

**a** 11:10 a.m. and 7:40 p.m.

**b** 6:20 p.m. and 12:00 midnight

**c** 4:45 p.m. and 8:10 p.m.

**d** 2:35 a.m. and 10:50 a.m.

**e** 1:05 p.m. and 12:30 a.m.

**f** 9:35 a.m. and 11:15 a.m.

**g** 0425 and 0935

**h** 1440 and 2025

**i** 7:55 a.m. and 3:50 p.m.

**j** 2:40 p.m. and 10:20 p.m.

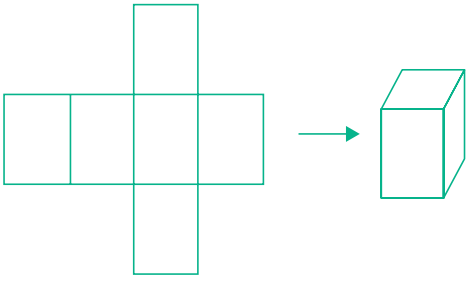

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# Surface area of a prism

12.04

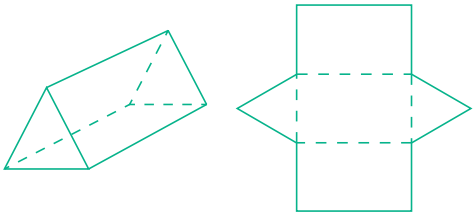
The **surface area** of a solid is the total area of all the faces of the solid.

The surface area of a solid can be easily calculated by first drawing the net of the solid, finding the area of each face, then adding the areas together.



A rectangular prism and its net.

The prism has 6 faces.



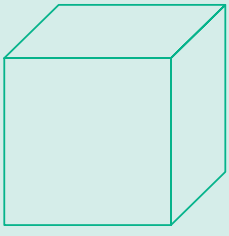
A triangular prism and its net.

The prism has 5 faces.

## Example 5

Find the surface area of each prism.

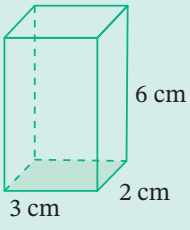
**a**



Cube

side length 5 cm

**b**




Rectangular prism

### Solution

**a** A cube has 6 identical square faces.

$$\text{Surface area} = 6 \times 5^2 \qquad 6 \times \text{area of a square}$$
$$= 150 \text{ cm}^2$$

Surface area is an area, so it is measured in square units such as $\text{cm}^2$ or $\text{m}^2$.


**Videos**
Surface area of a prism
Surface area of prisms
Cubist art

**Worksheets**
Surface area 1
Nets of solids
Solid shapes

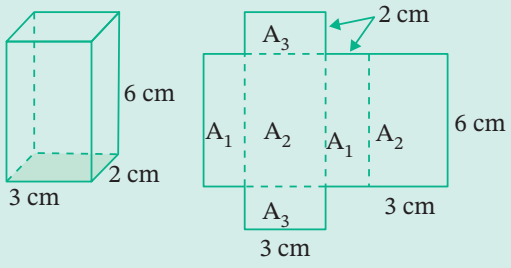
**Skillsheet**
Solid shapes

**Presentation**
Surface area

**Technology**
Surface area calculator

**Spreadsheet**
Surface area calculator

**b**




A rectangular prism has 6 rectangular faces. Opposite faces are the same size, such as the top and bottom, shown as $A_3$ in the diagram.

Surface area $= 2 \times A_1 + 2 \times A_2 + 2 \times A_3$ — 2 side faces + 2 front/back faces + 2 top/bottom faces

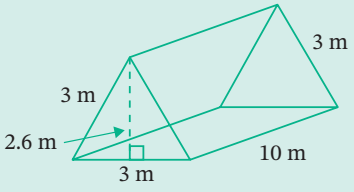
$= 2 \times (6 \times 2) + 2 \times (6 \times 3) + 2 \times (2 \times 3)$

$= 72 \text{ cm}^2$

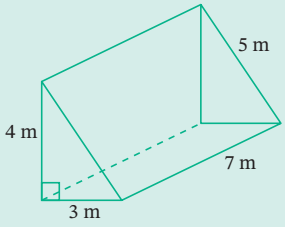
## Example 6

Find the surface area of each prism.

**a**

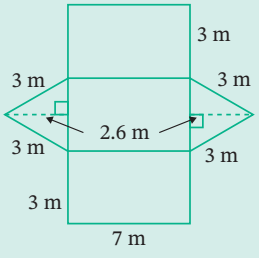



**b**




### Solution

**a** This triangular prism has 5 faces. Its base is an equilateral triangle with side length 3. Its faces are made of 2 identical triangles (front and back) and 3 equal rectangles (sides and bottom).




Surface area $= 2 \times \left(\frac{1}{2} \times 3 \times 2.6\right) + 3 \times (10 \times 3)$ — 2 front/back faces + 3 rectangular faces

$= 97.8 \text{ m}^2$

**b** This triangular prism has 5 faces: 2 identical triangles and 3 different rectangles of length 7 m.

Surface area $= 2 \times \left(\frac{1}{2} \times 3 \times 4\right) + (3 \times 7) + (4 \times 7) + (5 \times 7)$ — 2 front/back faces + bottom + left + right

$= 96 \text{ m}^2$

**EXERCISE 12.04** ANSWERS ON P. 617

## Surface area of a prism U F PS R

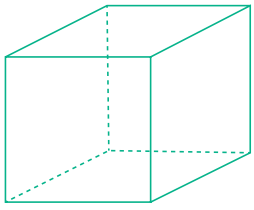
EXAMPLE 5

12.04

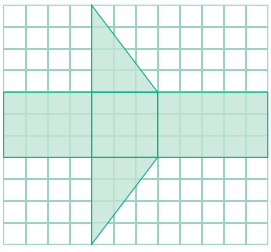
**1** Find the surface area of a cube with side length 6 cm.
Select the correct answer **A**, **B**, **C** or **D**.

**A** 24 cm$^2$ **B** 144 cm$^2$

**C** 36 cm$^2$ **D** 216 cm$^2$

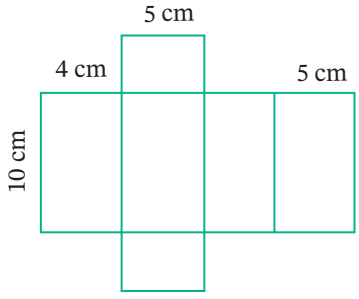



**2** Name and draw the solid that this net would make.

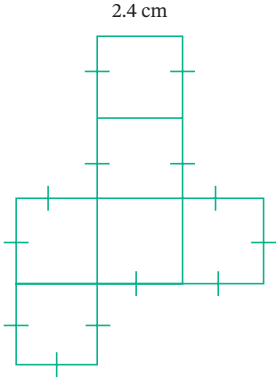


**3** Name the solid that can be made from each net and find its surface area.

**a**

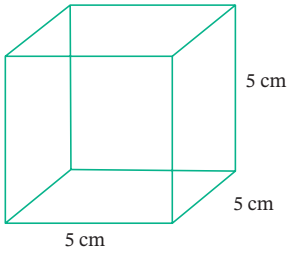



**b**

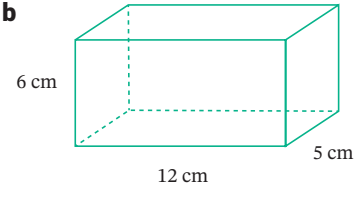



**4** Find the surface area of each prism.

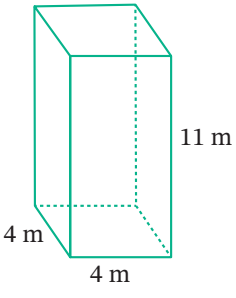
**a**




**b**




**c**

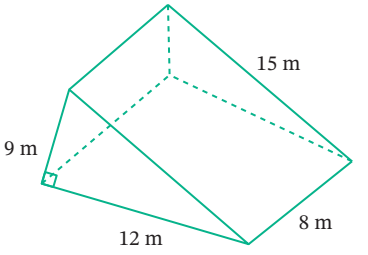



Foundation Standard Complex

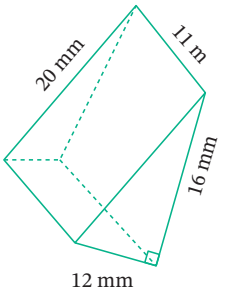
EXAMPLE 6

**5** Find the surface area of each triangular prism.

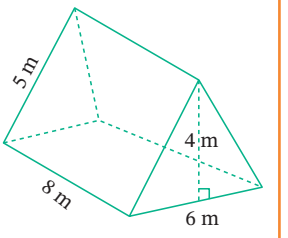
**a**



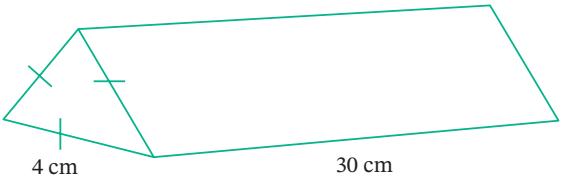

**b**




**c**




**6** Bodhi bought a ruler in the shape of a triangular prism as shown in the diagram. If the ruler is to be covered with aluminum foil, calculate the surface area to be covered. PS R




height of triangle = 3.5 cm

**7** Mary purchased a metal container and wishes to paint all the sides except the floor. PS R

**a** Calculate the surface area to be painted.

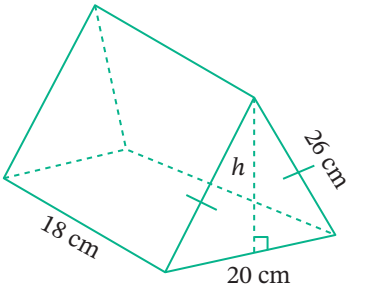
**b** If a 4-litre can of paint can cover 40 square metres, how many cans does Mary need to complete her task?




Shutterstock.com/Cathetus

**8** **a** Find the perpendicular height, $h$, of this triangular prism, correct to 2 decimal places.

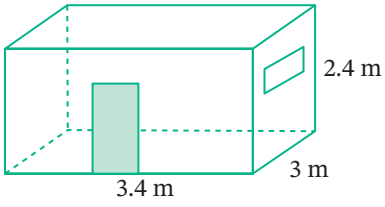
**b** Calculate the total surface area of the prism.




Foundation Standard Complex


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**9** Duc wishes to tile his new bathroom to a height of 1.2 m. The door is 0.9 m wide and the window is higher than 1.2 m. PS R



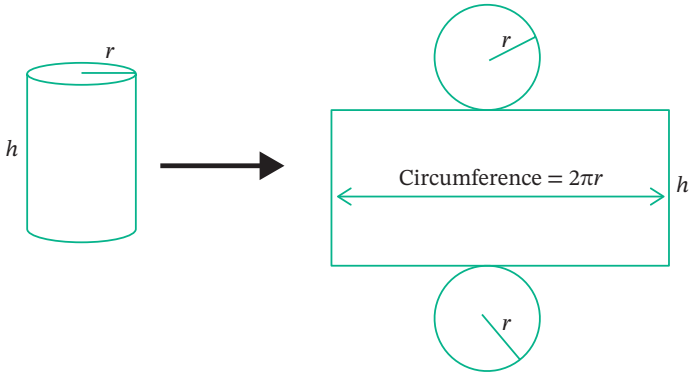

**a** Calculate how many square metres of tiles Duc needs? Answer to the nearest whole metre.

**b** If the tiles cost $65 per square metre and the tiler charges $60 per square metre, find the total cost for tiling the bathroom.



# Surface area of a cylinder 12.05

When a closed cylinder is opened out flat, its net is a rectangle and 2 identical circles:




The curved surface of the cylinder is the rectangle whose length is the circumference of the cylinder.

Surface area of a cylinder = area of 2 circles + area of rectangle

$$= 2 \times \pi r^2 + 2\pi r \times h$$

$$= 2\pi r^2 + 2\pi rh$$

## Surface area of a closed cylinder

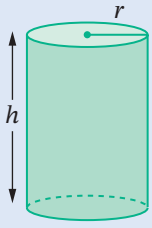
$SA = 2\pi r^2 + 2\pi rh$

where $r$ = radius of circular base

$h$ = perpendicular height

The area of the 2 circular ends = $2\pi r^2$

and the area of the curved surface = $2\pi rh$.




**Video**
Surface area of a cylinder

**Worksheets**
Surface area 2

A page of prisms and cylinders

Applications of area 3

**Puzzle**
Car song

**Technology**
Surface area calculator

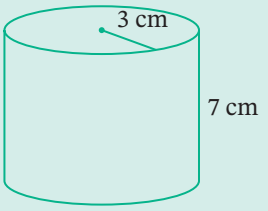
**Spreadsheet**
Surface area calculator

Foundation Standard Complex

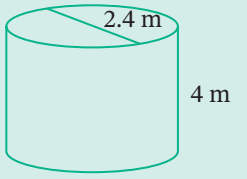
## Example 7

Find correct to one decimal the surface area of each cylinder.

**a**




**b** This water tank is open at the top.




### Solution

**a** $SA = 2 \times \pi \times 3^2 + 2 \times \pi \times 3 \times 7$

$= 188.4955 \cdots$

$\approx 188.5 \text{ cm}^2$

**b** Diameter $= 2.4$ m, so $r = \frac{1}{2} \times 2.4 = 1.2$ m

$SA =$ area of 1 circle + area of curved surface

$= \pi \times 1.2^2 + 2 \times \pi \times 1.2 \times 4$

$= 34.6831 \cdots$

$= 34.7 \text{ m}^2$

**EXERCISE 12.05** ANSWERS ON P. 618

## Surface area of a cylinder U F PS R C

EXAMPLE 7

**1** Find the surface area of a closed cylinder with radius 15 mm and height 20 mm, correct to 1 decimal place. Select the correct answer **A**, **B**, **C** or **D**.

**A** 3298.7 mm$^2$ **B** 1979.2 mm$^2$ **C** 2591.8 mm$^2$ **D** 28734.3 mm$^2$

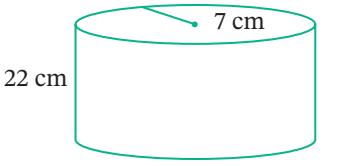
**2** For this cylinder, find correct to 2 decimal places:

**a** the circumference of the circular base

**b** the area of the circular base

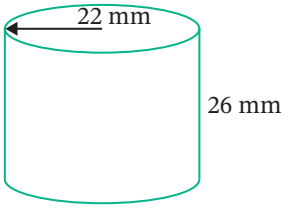
**c** the area of the curved surface

**d** the total surface area of the closed cylinder

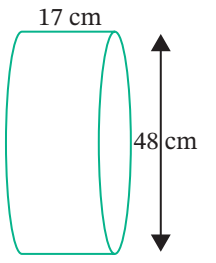



**3** Find the surface area of each cylinder, correct to one decimal place.

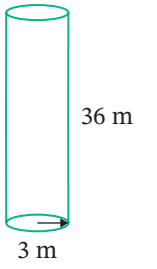
**a**




**b**




**c**

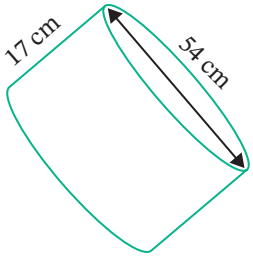



Foundation Standard Complex

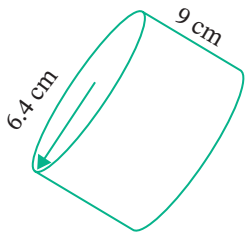

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**4** Find the surface area of each cylinder, which are open at the top. Answer to one decimal place. R C

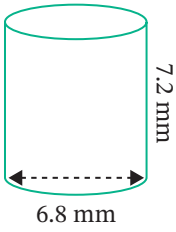
**a**




**b**

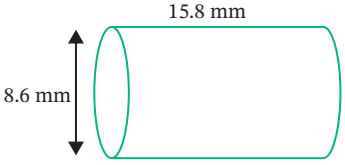



**c**

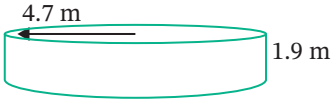



**5** Find the area of the curved surface of each cylinder correct to 2 decimal places.

**a**

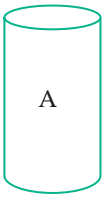



**b**



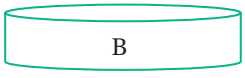

**6** Find correct to the nearest square centimetre the external (outside) surface area of an open-ended pipe 3 m long and with a 3 cm radius. R C

**7** Which of these cylinders has the greater surface area?




Radius 4 cm

Height 12 cm




Radius 12 cm

Height 4 cm

**8** An aeroplane hangar has the shape of half a cylinder with one open end.

The diameter of the hangar is 12 m.

The length of the hangar is 15 m.

Find its external surface area, correct to 2 decimal places.



Shutterstock.com/photographer DmitryStrong

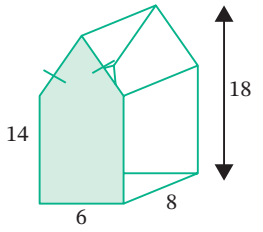
12.05

Foundation Standard Complex

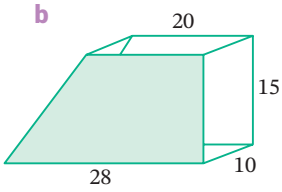
## Power plus ANSWERS ON P. 618

**1** Find the surface area of each prism. All measurements are in centimetres.

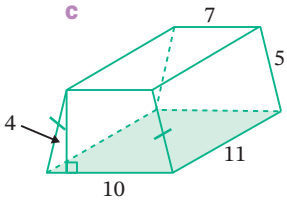
**a**



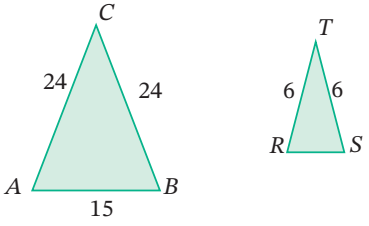

**b**



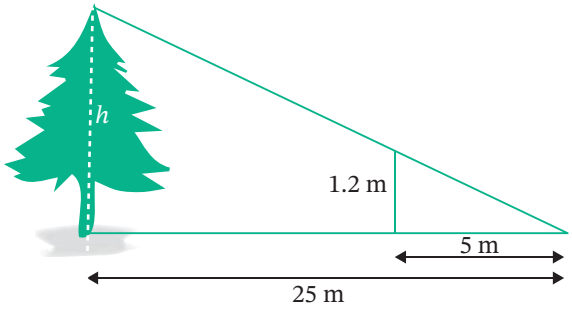

**c**




**2** $\triangle ABC$ and $\triangle RST$ are similar triangles. How many times is the area of $\triangle ABC$ the size of the area of $\triangle RST$?

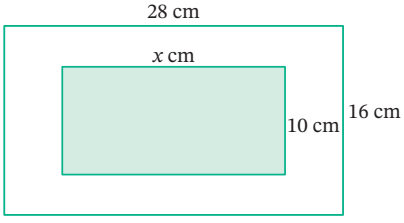



**3** A tree casts a shadow of 25 m long on the ground. At the same time a 1.2 m stick casts a shadow of 5 m.

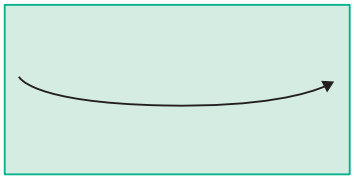



Find the height of the tree, correct to one decimal place.

**4** These 2 rectangles are similar. How many times is the area of the large rectangle as large as the area of the small rectangle?




**5** Toula has a metal sheet 30 cm long and 20 cm high. She curls it to form a tube. Find the diameter of the tube, correct to one decimal place.




9780170479516

# CHAPTER 12 REVIEW



Quiz
Language of maths 12

## Language of maths

| | | | |
|---|---|---|---|
| circumference | cube | curved surface | cylinder |
| enlargement | face | image | net |
| original | perpendicular height | pi ($\pi$) | prism |
| radius | ratio | rectangular prism | reduction |
| scale factor | similar (~) | surface area | triangular prism |

**1** Find 2 different meanings for the word 'scale'.

**2** How is the geometrical meaning of 'similar' different from its everyday meaning?

**3** What is $\frac{\text{image length}}{\text{object length}}$?

**4** What can you say about the value of the above fraction if a figure is reduced in size?

**5** How many faces has a triangular prism?

**6** What shape is the curved surface of a cylinder?

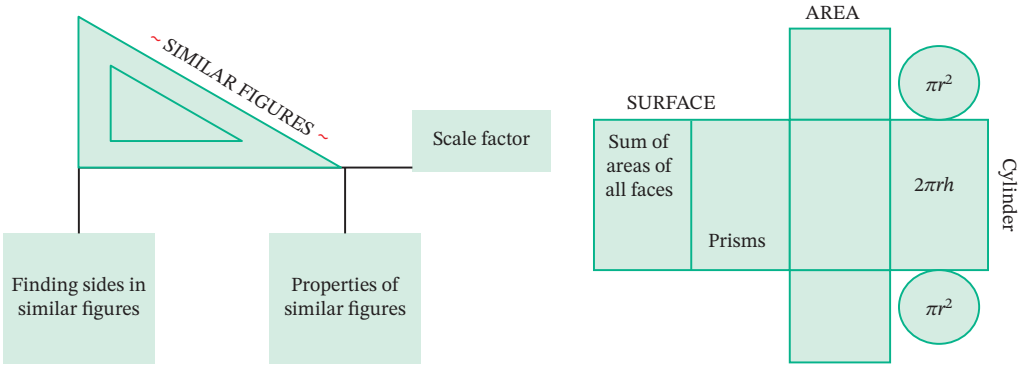
## Topic summary

Worksheet
Mind map: Similar figures and surface area

- Write in your own words what you have learnt about similar figures and surface area.
- What parts of this topic did you have difficulty with? Discuss them with a friend.
- Where might you use the skills acquired in this chapter?

Print (or copy) and complete these mind maps of the topics, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

**SIMILAR FIGURES AND SURFACE AREA**



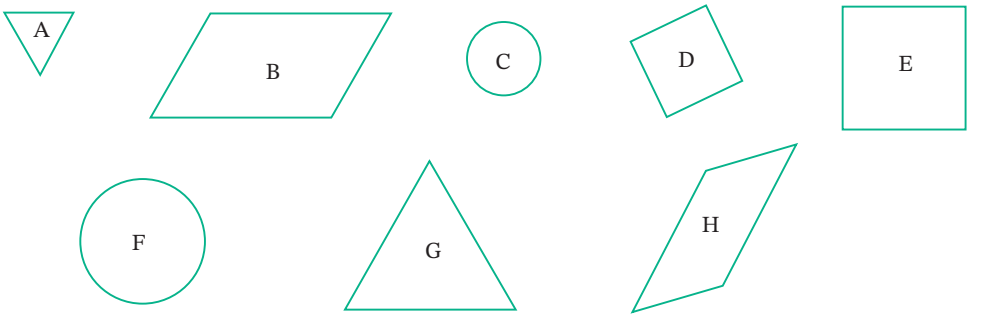
Quiz
Test yourself 12

# TEST YOURSELF 12

ANSWERS ON P. 618

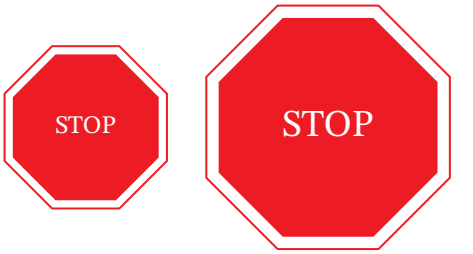
12.01

**1** Match pairs of similar figures.



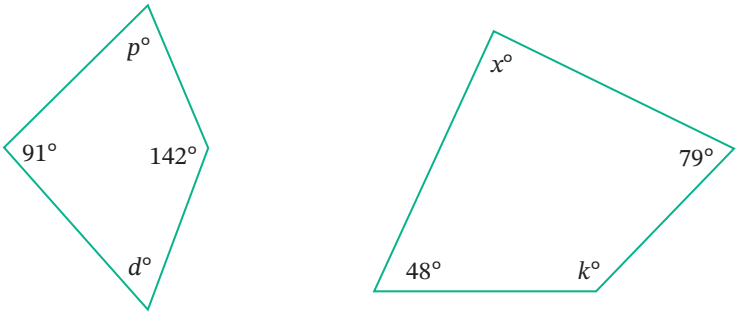

12.02

**2** By measuring the matching sides, find the scale factor of these similar figures.




12.02

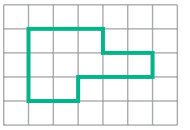
**3** Find the value of all variables in these 2 similar quadrilaterals.




12.02

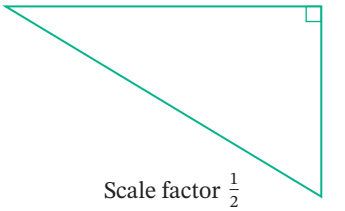
**4** Draw the image of each figure using the given scale factor.

**a**



Scale factor 2

**b**



Scale factor $\frac{1}{2}$

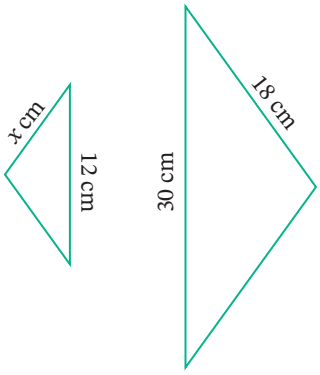
Foundation | Standard | Complex



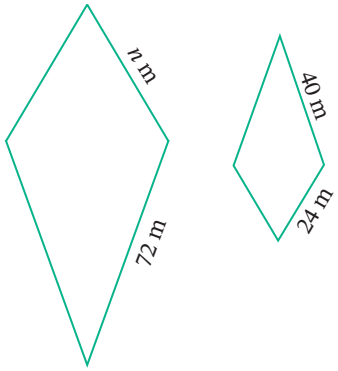
9780170479516

**5** Find the scale factor for each pair of similar figures. 12.03

**a**



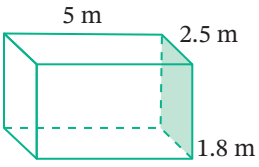

**b**



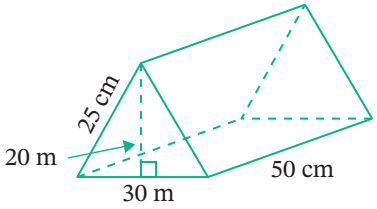

**6** Find the value of the variable in each pair of similar figures above. 12.03

**7** Find the surface area of each prism. 12.04

**a**

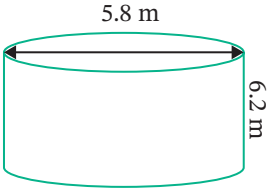



**b**

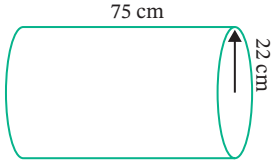



**8** Find the surface area of each cylinder, correct to one decimal place. 12.05

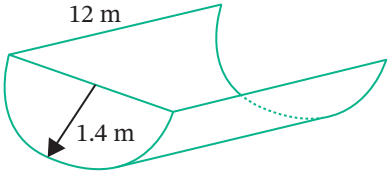
**a**




**b**




**9** Find the surface area of this half-cylinder that is open at one end. Answer correct to 2 decimal places. 12.05




Foundation | Standard | Complex

# PRACTICE SET 4 ANSWERS ON P. 618

10.01

**1** Simplify each ratio.

**a** 2 : 6

**b** 96 : 34

**c** 2 m to 15 cm

**d** 10 minutes to 2 hours

**e** $\frac{2}{5} : \frac{1}{3}$

**f** 0.7 : 2.8

10.06

**2** For each pair of sizes, find the better buy.

**a** Chips: 300 g for $5.20 or 400 g for $7.20

**b** Pasta: 1.5 kg for $3.30 or 750 g for $1.80

11.01

**3** Complete this table of values using the formula $y = 2x + 9$.

| $x$ | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y$ | | | | | |

10.02

**4** The ratio of boys to girls at a conference is 4 : 3. If there are 15 girls, find the number of boys.

10.03

**5** **a** If 5 cm on a map represents 1 km in actual distance, what length would be used to represent 5 km?

**b** Using a scale of 1 : 500, what actual length, in metres, would a scaled length of 4 cm represent?

10.08

**6** **a** A train travels 264 km in 8 hours. What is its average speed?

**b** A bus travels at 45 km/h for 5 hours. How far does it travel?

11.02

**7** Find the formula for this table of values.

| $x$ | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| $y$ | 6 | 9 | 12 | 15 | 18 | 21 |

11.03

**8** Here are the first 3 shapes in a pattern made of toothpicks.



**a** Copy and complete this table.

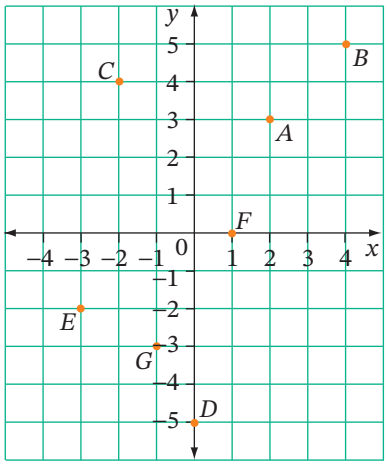
| Number of triangles $t$ | 1 | 2 | 3 | 4 | 5 | 8 |
|---|---|---|---|---|---|---|
| Number of toothpicks, $p$ | 3 | | | | | |

**b** Write the rule for the pattern in words.

**c** Write the rule as a formula.

**d** How many toothpicks are needed to build 20 triangles?

**9** Write the coordinates of each point marked on the number plane. 11.04




**10** Graph each linear equation on a number plane after copying and completing its table of values. 11.06

**a** $y = x + 3$

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**b** $y = 4x - 1$

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**11** A truck is loaded with fruit and vegetables in the ratio 5 : 9. The total mass is 4.2 tonnes. What is the mass of the fruit carried in the truck? 10.04

**12** In 20 hours, a train travels 800 km. How far would it travel in 11 hours at the same rate? 10.08

**13** Test whether the point (3, 7) lies on the line with the equation: 11.06

**a** $y = 2x + 4$

**b** $y = 3x - 2$

**14** Water flows into a tank at a rate of 15 L per hour. How long will it take to fill a tank that holds 750 L? 10.07

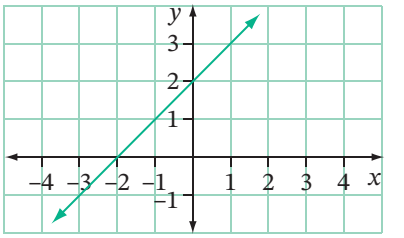
**15** Graph the line $y = 2$ on a number plane. 11.06

**16** Graph the line $y = 2x - 3$ and use it to solve the equation $2x - 3 = 5$ graphically. 11.12

Foundation Standard Complex

11.07

**17** Find the equation of this line.




10.09

**18** Convert

**a** 15 m/s to m/h

**b** 3 tonnes/day to kg/day

**c** 30 L/minute to mL/s

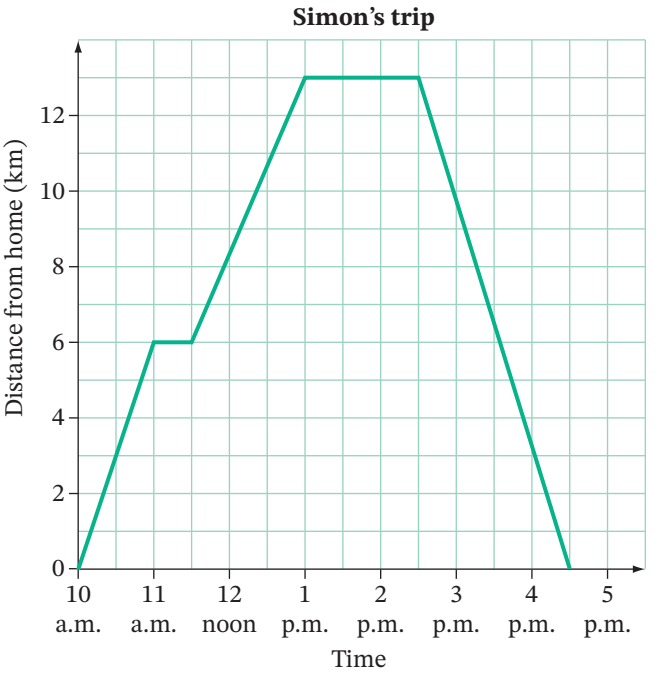
**d** 60 km/h to m/s

11.13

**19** Graph the lines $y = 2x$ and $y = x + 2$ on the same number plane and find their point of intersection.

10.10

**20** Simon's cycling trip is described by this travel graph.




**a** When, and for how long, did Simon stop for lunch?

**b** How far did he cycle in total?

**c** What was Simon's speed on the return journey home?

11.12

**21** Solve the equation $3x + 2 = x - 1$ graphically.

Foundation Standard Complex



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**22** Emilia planted a rose bush in her garden. The bush was 20 cm tall when it was first planted and grows 3 cm per month. 11.11

**a** Copy and complete this table, where $h$ is the height of rose bush in centimetres at $t$ months.

| Time, $t$ months | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| Height, $h$ cm | 20 | 26 | | | | | |

**b** Graph this table of values as a line on a number plane.

**c** Find the equation of the line.

**d** Find the height of the rose bush after 5 months.

**e** When will the height reach 74 cm?

**23** Leo and Chris share the cost of their internet bill in the ratio of 3 : 5. What fraction does Chris pay? Select the correct answer **A**, **B**, **C** or **D**. 10.02

**A** $\frac{2}{5}$ **B** $\frac{5}{8}$

**C** $\frac{5}{2}$ **D** $\frac{1}{5}$

**24** Dounya takes 24.5 seconds to walk 70 metres, while Anna walks 160 metres in 1 minute. Answer correct to one decimal place. 10.08

**a** What is the average speed of each person?

**b** If each person walks 2 km at their normal speed, who finishes first, and by how much?

# GENERAL PRACTICE

ANSWERS ON P. 619

CHAPTER 2

**1** Evaluate each expression.

**a** $\sqrt{169}$ **b** $\sqrt{256}$ **c** $\sqrt{289}$

**d** $\sqrt[3]{27}$ **e** $\sqrt[3]{125}$ **f** $\sqrt[3]{5832}$

CHAPTER 3

**2** Write an algebraic expression for:

**a** 2 times $x$, plus $y$

**b** $y$ times $z$ minus $x$

**c** the sum of $x$, $y$ and $z$

**d** the product of $x$ and $y$ decreased by 4 times $z$

CHAPTER 6

**3** Simplify each fraction.

**a** $\frac{3}{12}$ **b** $\frac{18}{24}$ **c** $\frac{35}{100}$

CHAPTER 6

**4** Convert each improper fraction into a simplified mixed numeral.

**a** $\frac{36}{5}$ **b** $\frac{67}{10}$ **c** $\frac{74}{12}$

CHAPTER 3

**5** If $m = 2$, $n = -1$ and $p = 3$, evaluate each expression.

**a** $mnp$ **b** $2p - m$ **c** $3m - n^2$

CHAPTER 3

**6** Simplify each expression.

**a** $3a \times 2r$ **b** $-7p - 2p + 6p$ **c** $7u \times 5u$

**d** $9x - 3 + 12 - 4x$ **e** $6p - 3q + 2p - 4q$ **f** $\frac{24ut}{4mt}$

CHAPTER 6

**7** Thay scores 32 points out of his team's score of 128. What percentage is this?

CHAPTER 10

**8** Write as a rate:

**a** 450 m in 15 seconds **b** 82 points in 5 games **c** 3.5 kg for $28

CHAPTER 7

**9** These are scores in a spelling test:

5 4 8 6 1 5 8 5 4 2 5

Find:

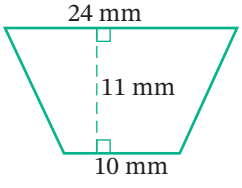
**a** the mean (to one decimal place) **b** the median

**c** the mode **d** the range

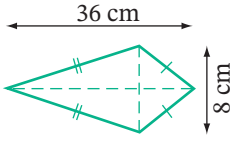
CHAPTER 5

**10** Find the area of each quadrilateral.

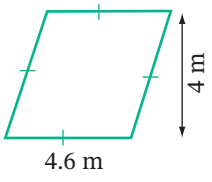
**a**




**b**




**c**




Foundation Standard Complex

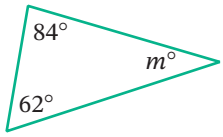


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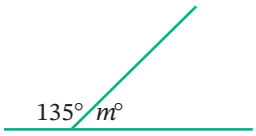
**11** Find the value of each variable, giving reasons.

CHAPTER 4

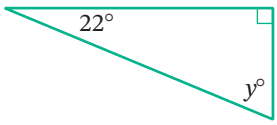
**a**




**b**



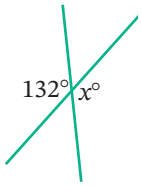

**c**



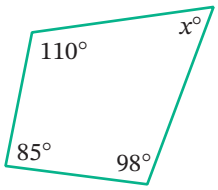

**d**



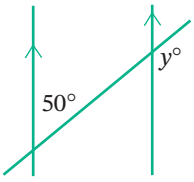

**e**



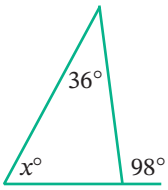

**f**



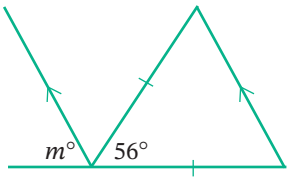

**g**




**h**




**i**




**12** Evaluate each expression.

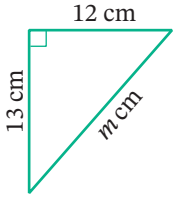
CHAPTER 2

**a** $10 + 4 \times (-3)$ **b** $15.35 \div 5$ **c** $8.3 - 1.986$ **d** $4 + 7 \times 2$

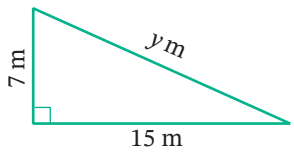
**13** Find the value of each variable, correct to one decimal place.

CHAPTER 1

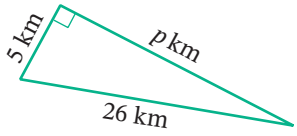
**a**



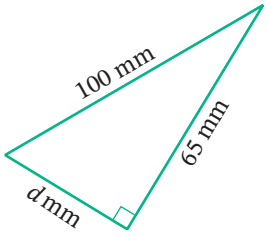

**b**




**c**




**d**




**14** Use index notation to simplify each expression.

CHAPTER 2

**a** $6^5 \div 6^3$ **b** $2^3 \times 2^4$ **c** $3^9 \div 3^7$ **d** $5 \times 5^2$

**e** $(2^3)^4$ **f** $(2^3 \times 5)^2$ **g** $3^2 \div 3^2$ **h** $6 \times 4^0$

**15** The probability of choosing an Ace from a pack of cards is $\frac{4}{52}$.

CHAPTER 8

**a** What is the complementary event to choosing an Ace?

**b** What is the probability of this complementary event?

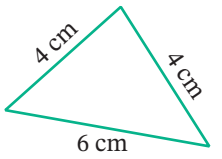
Foundation Standard Complex

CHAPTER 4

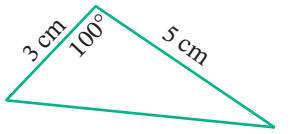
**16** Classify each triangle:

**i** by sides **ii** by angles

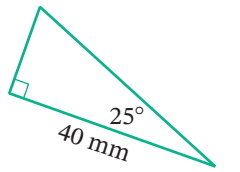
**a**




**b**




**c**




CHAPTER 2

**17** Use a factor tree to write 200 as a product of its prime factors.

CHAPTER 5

**18** Copy and complete each conversion.

**a** $8\ m^2 =$ ____ $cm^2$ **b** $4900\ m^2 =$ ____ ha **c** $650\ m^3 =$ ____ $cm^3$

CHAPTER 6

**19** Evaluate each expression.

**a** $\frac{2}{3}+\frac{3}{4}$ **b** $\frac{2}{3}-\frac{1}{5}$ **c** $\frac{7}{10}\times\frac{2}{3}$ **d** $\frac{4}{9}\div\frac{5}{6}$

**e** $1\frac{3}{8}+\frac{1}{4}$ **f** $4-\frac{5}{7}$ **g** $5\div\frac{4}{9}$ **h** $3\frac{2}{5}\times 1\frac{4}{5}$

CHAPTER 6

**20** Convert 15% into a simple fraction.

CHAPTER 9

**21** Solve each equation.

**a** $5a-13=12$ **b** $\frac{d}{9}+12=16$ **c** $\frac{-4r}{3}=28$

CHAPTER 7

**22** Classify each type of data as categorical ordinal (CO), categorical nominal (CN), numerical discrete (ND) or numerical continuous (NC).

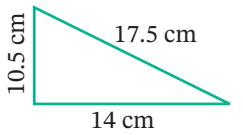
**a** number of taxis on the road **b** maturity rating of computer game

**c** weights of babies in a maternity ward **d** brands of sport shoes

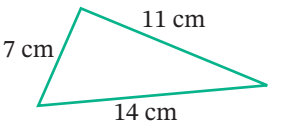
CHAPTER 1

**23** Test whether each triangle is right-angled.

**a**

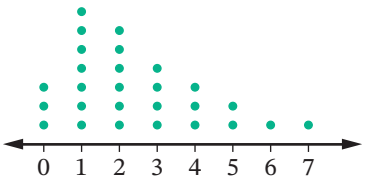



**b**




CHAPTER 7

**24** Describe the shape of this set of data.




CHAPTER 10

**25** Gaurish earns $22.40 per hour. How much will he earn if he works 42 hours in a week?

**26** Find the formula for each table of values.

**a**

| $x$ | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|
| $y$ | 16 | 19 | 22 | 25 | 28 |

**b**

| $x$ | 3 | 3 | 3 | 3 | 3 |
|---|---|---|---|---|---|
| $y$ | −1 | 2 | 4 | −2 | 0 |

CHAPTER 11

**27** When catering for events, *Dine with Us* allows 1.2 L of drink per person.

**a** If each glass contains 150 mL, how many glasses of drink is this per person?

**b** *Dine with Us* are catering at an exhibition where 500 guests are expected. How many litres of drink are required?

CHAPTER 5

**28** Graph each linear equation on the same number plane.

**a** $y = 3$ **b** $x = -1$

CHAPTER 11

**29** Expand and simplify each expression.

**a** $3(a + 5)$ **b** $4(5 - 2k)$ **c** $3(2p - 5) - 6p$

CHAPTER 3

**30** Simplify each ratio.

**a** 42 : 49 **b** 16 kg : 800 g **c** 10 mm : 1 m

CHAPTER 10

**31** A machine makes transistors, of which 2 in 35 are faulty. If the machine has made 16 faulty transistors, how many good ones would you expect?

CHAPTER 10

**32** Find the width of a TV screen if its diagonal is 125 cm and its height is 62 cm.

CHAPTER 1

**33** The simple interest formula is $I = Prn$, where $I$ is the simple interest, $P$ is the principal, $r$ is the interest rate per year expressed as a decimal and $n$ is the number of years that the money is invested. Find how much money was invested at a rate of 3.5% p.a. for 4 years to earn $63 in simple interest.

CHAPTER 9

**34** **a** Graph $y = 2x - 1$ on a number plane.

**b** Test whether (5, 9) lies on the line.

**c** Graph $y = x - 2$ on the same number plane.

**d** Solve the equation $2x - 1 = x - 2$ graphically.

CHAPTER 11

**35** A department store buys jeans for $60 and sells them for $96. Find:

**a** the store's profit

**b** the percentage profit

CHAPTER 6

**36** Name the shape that matches each description.

**a** A quadrilateral with equal diagonals.

**b** A triangle with 3 equal sides.

**c** A quadrilateral with opposite sides equal and 2 axes of symmetry.

**d** A triangle with 2 equal angles.

CHAPTER 4

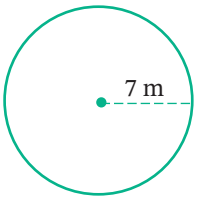
Foundation Standard Complex

CHAPTER 5

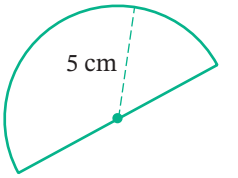
**37** For each shape, find correct to 2 decimal places:

**i** the perimeter **ii** the area.

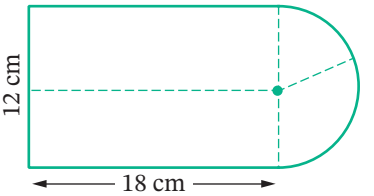
**a**



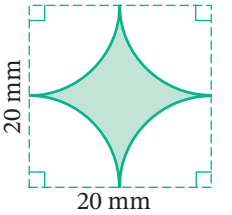

**b**




**c**




**d**




CHAPTER 3

**38** Factorise each expression.

**a** $2y + 8$ **b** $5a - 15ab$ **c** $-8ab + 2b$

CHAPTER 6

**39** A savings account pays 4% interest each year. If Haroon puts $5000 in his account, how much does he get in interest after one year?

CHAPTER 8

**40** A bag has 7 yellow badges, 3 red badges and 4 blue badges. If one badge is selected at random, what is the probability of selecting a:

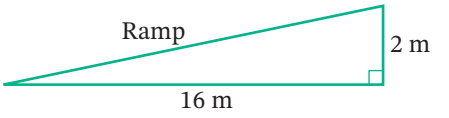
**a** yellow badge? **b** blue badge? **c** badge that is not red?

CHAPTER 10

**41** In an orchard, the ratio of orange trees to lemon trees is 7 : 4. If there are 2800 lemon trees, how many orange trees are there?

CHAPTER 1

**42** A ramp for a shopping centre is built as shown. Find the length of the ramp, correct to 2 decimal places.




CHAPTER 10

**43** Luis uses 8 litres of paint to cover 32 square metres of wall. What area can he cover with 20 litres of paint?

CHAPTER 7

**44** The ages of patients waiting at a doctor's surgery are shown on this stem-and-leaf plot.

| Stem | Leaf |
|---|---|
| 1 | 3 8 |
| 2 | 1 9 |
| 3 | 5 |
| 4 | 2 |
| 5 | 0 4 7 |
| 6 | |
| 7 | 3 5 |

Find:

**a** the range

**b** the median

**c** the mean, correct to one decimal place.

Foundation Standard Complex



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**45** A large vat is being filled at a rate of 35 mL/min. Convert this rate to L/day. CHAPTER 10

**46** Hawkes Landscaping Company supplies garden soil. The charge is $70 plus $30 per tonne to deliver the soil. This is calculated using the formula $C = 30n + 70$. CHAPTER 11

**a** Copy and complete this table of values.

| Number of tonnes, $n$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Cost of soil, $C | | | | | | |

**b** Graph the table of values as a line on the number plane.

**c** What is the vertical intercept of the line and what does it represent?

**d** Use your graph or equation to determine:

**i** the cost of purchasing 2.5 tonnes.

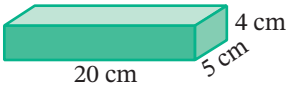
**ii** how many tonnes can be purchased for $205?

**47** Which is the best buy for strawberries? Select **A**, **B** or **C**. CHAPTER 10

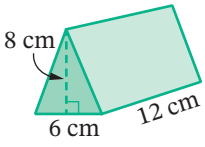
**A** 600 g for $8.60 **B** 500 g for $7.00 **C** 1 kg for $15

**48** Find the volume of each solid. (For part **f**, round your answer to one decimal place). CHAPTER 5

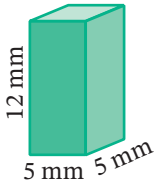
**a**



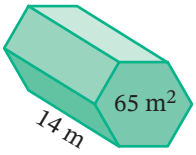

**b**



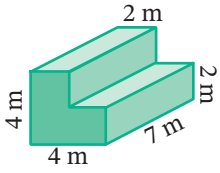

**c**



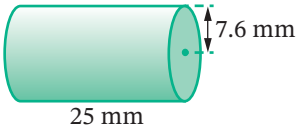

**d**




**e**




**f**




**49** The scale of a plan is 1 : 2000. What distance, in centimetres, on the plan would represent a real distance of 50 m? CHAPTER 10

**50** Anne earned $92 for delivering newspapers. If she was given a 20% pay rise, how much does she earn now? CHAPTER 6

**51** **a** Complete a frequency table for this data, including an '$fx$' column. CHAPTER 7

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 53 | 50 | 50 | 52 | 47 | 49 | 49 | 50 | 48 | 50 |
| 52 | 49 | 47 | 51 | 51 | 52 | 50 | 53 | 50 | 52 |
| 50 | 48 | 52 | 49 | 50 | 49 | 51 | 45 | 52 | 50 |
| 51 | 52 | 51 | 50 | 52 | 50 | 51 | 52 | 49 | 51 |

**b** Find the mean of this data.

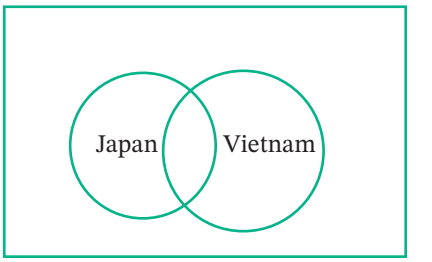
**c** Show this data in a frequency histogram and polygon.

YEAR 9 EXTENSION

CHAPTER 8

**52** A survey was held to find whether of a sample of 30 Australians had travelled to Japan or Vietnam. The results are shown below.

Japan only 4 Vietnam only 13 Japan and Vietnam 5




**a** Copy and complete the Venn diagram to show this information.

**b** How many people did not visit either country?

**c** How many people had visited Japan but not Vietnam?

**d** What is the probability that a person selected randomly from this sample has not been to Japan?

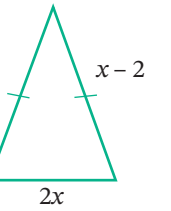
CHAPTER 5

**53** Adrian's fish tank is a rectangular prism 60 cm by 25 cm and filled to a depth of 13 cm. Find the volume of water in the tank in:

**a** $cm^3$ **b** litres

CHAPTER 9

**54** If the perimeter of this isosceles triangle is 24 cm, find the value of $x$.




CHAPTER 10

**55** If Dino can run 400 metres in 66 seconds, calculate his speed in m/s, correct to one decimal place.

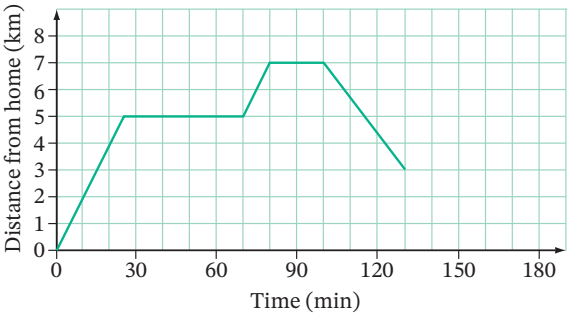
CHAPTER 6

**56** What is the whole amount if:

**a** 35% of it is $210? **b** 68% of it is 306 kg?

CHAPTER 10

**57** Mel went on a bike ride and her trip is shown in the travel graph.




**a** How far from home was Mel after 20 minutes?

**b** When and where was she furthest from home?

**c** For how long did Mel rest on her second stop?

**d** When Mel was travelling home, what was her average speed in km/h?

Foundation Standard Complex

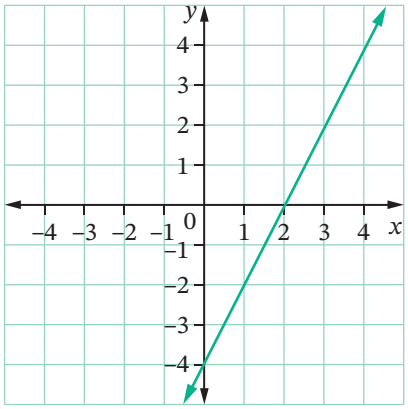

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**58** Solve the linear equation $4 - 2x = 6$ graphically.

CHAPTER 11

**59** Find the equation of this line.

CHAPTER 11




**60** Izak paid $88 for a shirt at a '20% off' sale. What was the original price of the shirt?

CHAPTER 6

Foundation Standard Complex

# CALCULATOR SKILLS

## Basic keys

| Key | Use | Key | Use |
|---|---|---|---|
| + , − , × , ÷ | Basic operations | (−) or +/− | Enters negative numbers |
| = | Equals sign, gives the answer | $\square\!\!\square$ or $a^{b/c}$ | Enters fractions |
| · | Decimal point | ( ) | Enters parentheses (brackets) |
| DEL | Deletes previous entry | $x^2$ | Squares a number |
| Ans | Retrieves previous answer | $\sqrt{}$ | Finds the square root of a number |
| ✣ | Moves cursor around the screen | $x^3$ | Cubes a number |
| MODE or SHIFT or 2ndF | Accesses other operations | $\sqrt[3]{}$ | Finds the cube root of a number |

## 1 Basic operations

### Example 1

#### Performing a calculation

| Question | Calculator steps | Answer |
|---|---|---|
| 34 + 6 − 16 | 34 + 6 − 16 = | 24 |
| 7.4 × 9.1 | 7.4 × 9.1 = | 67.34 |
| 234 ÷ 1.5 | 234 ÷ 1.5 = | 156 |
| 4569 ÷ (1.7 + 4.3) | 4569 ÷ ( 1.7 + 4.3 ) = | 761.5 |

### Example 2

#### Correcting a wrong entry

Calculating 34 + 6 − 16

**a** Wrong entry: 34 + 8

To delete the 8 and continue, enter DEL 6 − 16 =

**b** Wrong entry: 34 + 6 + 16 =

To correct this, press ← repeatedly to go back to the second +, press DEL and then −, then press =


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**EXERCISE 1** ANSWERS ON P. 621

## Basic operations

**1** Calculate each expression.

**a** $362 \times 14$
**b** $26.3 - 11.8$
**c** $810 \div 18$
**d** $5.3 \times 1.08$
**e** $51.1 \div 14.6$
**f** $19.3 + 8.8 - 1.2$
**g** $2.4 \div 0.2 + 11$
**h** $5.6 \times (0.1 + 0.2)$
**i** $(56 - 24) \div (1.2 - 1)$
**j** $12.6 \times 3.8 - 18.4$
**k** $(16.07 + 9.02) - (38.3 - 5.126)$
**l** $(24.8 + 3.06 + 9.4) \div 3$

**2** Bronte purchased a loaf of bread for $2.75, a packet of cornflakes for $4.55 and 3 kg of tomatoes at $3.95 per kg.

**a** Find the total cost.
**b** Find the change Bronte will receive from a $20 note.

**3** Mark started the month with $1650 in his bank account. He made deposits of $380 and $725.50 and withdrew $880. How much was left in Mark's account?

**4** An unloaded truck has a mass of 6550 kg. It is loaded with eight boxes, each weighing 124 kg. What is the total mass of the truck and the boxes?

**5** Salma needs to cut a 9.6 m length of rope into 4 equal pieces. Find the length of each piece.

**6** Hamish posted 25 Christmas cards to his friends. If the stamps were 75c each, how much did he spend on stamps?

**7** An orchard has 1288 trees arranged in 56 rows. How many trees are in each row?

**8** A train takes 15 hours to travel 1128 km. What is its average speed in km/h?

**9** Chloe bought 60 L of petrol at 192.4 c per litre. How much did she pay for the petrol?

**10** Nelson Anglican College has the following numbers of students in each Year level.

| Year 7 | Year 8 | Year 9 | Year 10 | Year 11 | Year 12 |
|---|---|---|---|---|---|
| 122 | 128 | 123 | 131 | 117 | 94 |

The whole school is going on an excursion. If each bus can carry 55 people, how many buses will be needed?

**11** The manager of a sports store purchased 32 tennis racquets for $79.60 each and sold them for $120.80 each.

**a** Find the profit made on each racquet.
**b** Find the total profit.

**12** A batsman scored the following runs over 5 cricket innings.

27 4 123 66 47

Find the batsman's average score.

## 2 Integers

### Example 3

| Question | Calculator steps | | Answer |
|---|---|---|---|
| | Casio | Sharp | |
| $-4 + 9$ | (–) 4 + 9 = | +/– 4 + 9 = | 5 |
| $17 \times (-2)$ | 17 × (–) 2 = | 17 × +/– 2 = | −34 |
| $18 - (-3) \times 4$ | 18 – (–) 3 × 4 = | 18 – +/– 3 × 4 = | 30 |

**EXERCISE 2** ANSWERS ON P. 621

### Integers

**1** Calculate each expression.

**a** $-5 + 12$ **b** $-6 - 3$ **c** $-3 \times 4$

**d** $16 \div (-2)$ **e** $12 - (-3)$ **f** $-19 + 4$

**g** $-25 \times (-3)$ **h** $-55 \div (-11)$ **i** $8 - 12 + (-1)$

**j** $-3 + 7 - 2 - (-5)$ **k** $6 \times (-2) \times 8$ **l** $-18 \div (-2) \times 5$

**m** $-3 \times 7 - 4$ **n** $(-3 + 9) \times 4$ **o** $80 - (-2) \times 8$

**p** $12 \times (-3) + 50$ **q** $64 \div (-8) \times 2$ **r** $27 - (-3) \times (-4)$

**s** $18 \times (-5 + 3)$ **t** $-14 - 3 \times (-8)$

## 3 Powers and roots

Look for these keys on your calculator: $x^2$ $x^3$ $\sqrt{\ }$ $\sqrt[3]{\ }$

You may need to use the SHIFT or 2ndF key.

### Example 4

| Question | Calculator steps | Answer |
|---|---|---|
| $16^2$ | 16 $x^2$ = | 256 |
| $11^3$ | 11 $x^3$ = | 1331 |
| $\sqrt{196}$ | $\sqrt{\ }$ 196 = | 14 |
| $\sqrt[3]{4913}$ | $\sqrt[3]{\ }$ 4913 = | 17 |

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**EXERCISE 3** ANSWERS ON P. 621

## Powers and roots

**1** Calculate each expression.

**a** $14^2$ **b** $23^2$ **c** $6^3$ **d** $1.2^3$

**e** $5.13^2$ **f** $101^2$ **g** $2.04^2$ **h** $19^3$

**i** $\sqrt{676}$ **j** $\sqrt{21\,904}$ **k** $\sqrt[3]{729}$ **l** $\sqrt{6^2 \times 7^2}$

**2** Evaluate each expression, correct to one decimal place.

**a** $\sqrt{24}$ **b** $\sqrt{101}$ **c** $\sqrt{11 \times 2}$ **d** $\sqrt[3]{100}$

**3** Calculate each expression.

**a** $5^2 - 3^2$ **b** $16^2 - 10^2$ **c** $\sqrt{6} \times \sqrt{6}$ **d** $\sqrt{25 \times 4}$

# 4 Fractions

Casio calculators have 2 ways of entering and displaying fractions: MATH mode or LINE mode. MATH mode shows fractions more realistically as it allows you to enter the numerator and denominator into 2 blank spaces on the calculator's screen, while LINE mode shows the fraction in a line (for example, 1 ⌟ 3) and makes the fraction key act like a vinculum (fraction bar). For MATH mode, use the arrow keys to move the cursor to the spaces to enter the numerator and denominator. Press SHIFT MODE to change between MATH I/O and LINE I/O (input/output). When MATH mode is selected, 'Math' appears on the calculator display. To ensure that mixed numeral answers are not converted to improper fractions, press SHIFT MODE and the down arrow to choose 'ab/c'.

## Example 5

### Converting fractions

| Question | Calculator steps | | Answer |
|---|---|---|---|
| | Casio LINE mode | Sharp | |
| Simplify $\frac{16}{20}$ | 16 ▭ 20 = | 16 $a^{b/c}$ 20 = | $\frac{4}{5}$ |
| Change $1\frac{3}{4}$ to an improper fraction | 1 ▭ 3 ▭ 4 = (SHIFT S⇔D) | 1 $a^{b/c}$ 3 $a^{b/c}$ 4 = 2ndF $a^{b/c}$ | $\frac{7}{4}$ |
| Change $\frac{19}{4}$ to a mixed numeral | 19 ▭ 4 = | 19 $a^{b/c}$ 4 = | $4\frac{3}{4}$ |
| Change $\frac{3}{4}$ to a decimal | 3 ▭ 4 = S⇔D or 3 ÷ 4 = | 3 $a^{b/c}$ 4 = $a^{b/c}$ or 3 ÷ 4 = | 0.75 |
| Change 1.7 to a fraction | 1.7 = ▭ | 1.7 = $a^{b/c}$ | $1\frac{7}{10}$ |

S⇔D converts an answer between standard fraction and decimal forms. SHIFT S⇔D or 2ndF $a^{b/c}$ converts an answer between mixed numeral and improper fraction forms.

## Example 6

### Operations with fractions

| Question | Calculator steps | | Answer |
|---|---|---|---|
| | Casio | Sharp | |
| $\frac{1}{2}+\frac{1}{3}$ | 1 $\square\!\!\square$ 2 + 1 $\square\!\!\square$ 3 = | 1 $a^{b/c}$ 2 + 1 $a^{b/c}$ 3 = | $\frac{5}{6}$ |
| $\frac{5}{7}-\frac{2}{3}$ | 5 $\square\!\!\square$ 7 − 2 $\square\!\!\square$ 3 = | 5 $a^{b/c}$ 7 − 2 $a^{b/c}$ 3 = | $\frac{1}{21}$ |
| $\frac{3}{5}\times\frac{2}{7}$ | 3 $\square\!\!\square$ 5 × 2 $\square\!\!\square$ 7 = | 3 $a^{b/c}$ 5 × 2 $a^{b/c}$ 7 = | $\frac{6}{35}$ |
| $\frac{4}{5}\div\frac{2}{3}$ | 4 $\square\!\!\square$ 5 ÷ 2 $\square\!\!\square$ 3 = | 4 $a^{b/c}$ 5 ÷ 2 $a^{b/c}$ 3 = | $1\frac{1}{5}$ |

## Example 7

### Mixed operations with fractions

| Question | Calculator steps | Answer |
|---|---|---|
| $\frac{13+7}{9\times2}$ | ( 13 + 7 ) ÷ ( 9 × 2 ) = | 1.1111... |
| | or | |
| | ( 13 + 7 ) $\square\!\!\square$ (or $a^{b/c}$) ( 9 × 2 ) = | $1\frac{1}{9}$ |

**EXERCISE 4** ANSWERS ON P. 621

## Fractions

**1** Simplify each fraction.

**a** $\frac{20}{32}$ **b** $\frac{40}{85}$ **c** $\frac{45}{72}$

**d** $\frac{112}{126}$ **e** $\frac{84}{100}$ **f** $\frac{48}{192}$

**2** Convert each mixed numeral into an improper fraction.

**a** $1\frac{7}{8}$ **b** $5\frac{2}{9}$ **c** $10\frac{3}{4}$

**3** Convert each improper fraction into a mixed numeral.

**a** $\frac{20}{9}$ **b** $\frac{44}{5}$ **c** $\frac{18}{8}$

**4** Convert each fraction into a decimal.

**a** $\frac{1}{2}$ **b** $\frac{4}{5}$ **c** $\frac{7}{8}$

**d** $\frac{9}{25}$ **e** $\frac{34}{200}$ **f** $\frac{5}{9}$

**5** Convert each decimal into a fraction in its simplest form.

**a** 0.35 **b** 2.04 **c** 0.001

**d** 0.365 **e** 1.06 **f** 0.325

**6** Simplify each expression.

**a** $\frac{3}{5}+\frac{1}{4}$ **b** $\frac{4}{7}+\frac{1}{2}$ **c** $\frac{2}{3}-\frac{3}{5}$ **d** $\frac{1}{3}\times\frac{4}{7}$

**e** $\frac{5}{7}\div\frac{15}{28}$ **f** $\frac{5}{6}-\frac{5}{8}$ **g** $\frac{9}{10}+\frac{2}{3}$ **h** $\frac{7}{8}\times\frac{5}{12}$

**i** $\frac{7}{8}\div\frac{1}{2}$ **j** $24\div1\frac{1}{4}$ **k** $5\frac{1}{4}+1\frac{2}{5}$ **l** $9\frac{7}{8}-1\frac{1}{3}$

**m** $5\frac{1}{4}\times2\frac{3}{7}$ **n** $4\frac{2}{3}\div\frac{2}{3}$ **o** $\frac{8}{9}\div6$ **p** $10\frac{1}{6}\times1\frac{9}{10}$

**7** Simplify each expression.

**a** $\dfrac{12-4\times2}{18+14}$

**b** $\dfrac{6^2}{100\div4+23}$

**c** $\dfrac{9\times9-2}{\sqrt{30-5}}$

**d** $\dfrac{20\div2+8}{3\times9\times4}$

**8** Mr Pettis is at his office for $7\frac{1}{2}$ hours each day. He takes $\frac{3}{4}$ of an hour for lunch each day. How long does he work:

**a** in one day?

**b** in 5 days?

# ANSWERS

## Chapter 1

### SkillCheck

**1** **a** 16 **b** 144 **c** 900 **d** 4.41
**e** 106.09 **f** 25 **g** 169 **h** 18.92

**2** **a** 4 **b** 8 **c** 10 **d** 14
**e** 11 **f** 7 **g** 1.1 **h** 1.5

**3** **a** 7.6 **b** 10.8 **c** 12.7

**4** **a** 40 cm² **b** 16 cm² **c** 100 mm²

### Exercise 1.01

**1** **a** 14 **b** 30 **c** 8
**d** 25 **e** 11 **f** 19

**2** **a** 3.46 **b** 6.71 **c** 31.64
**d** 18.03 **e** 12.37 **f** 14.39
**g** 9.90 **h** 29.80 **i** 4.90
**j** 10.49 **k** 17.23 **l** 26.25

**3** **a** 10.91 **b** 8.60 **c** 12.53
**d** 12.04 **e** 7.42 **f** 9.85
**g** 4.46 **h** 10.29 **i** 31.49

**4** A

**5** $\sqrt{98}$, $\sqrt{160}$, $\sqrt{52}$, $\sqrt{77}$, $\sqrt{18}$, $\sqrt{200}$

### Exercise 1.02

**1** **a** $p$ **b** $z$ **c** $LN$

**2** **b** 13 cm **c** 169 **d** 169
**e** 13, 169, 5, 12, 169

**3** B

**4** **b**

| | Shorter sides | | Hypote-nuse $c$ | $a^2$ | $b^2$ | $a^2 + b^2$ | $c^2$ |
|---|---|---|---|---|---|---|---|
| | $a$ | $b$ | | | | | |
| **a** | 6 | 8 | 10 | 36 | 64 | 36 + 64 | 100 |
| **b** | 2.5 | 6 | 6.5 | 6.25 | 36 | 6.25 + 36 | 42.25 |
| **c** | 1.5 | 2 | 2.5 | 2.25 | 4 | 2.25 + 4 | 6.25 |
| **d** | 4 | 7.5 | 8.5 | 16 | 56.25 | 16 + 56.25 | 72.25 |

**5** Teacher to check.

### Exercise 1.03

**1** D

**2** **a** $p^2 = m^2 + n^2$ **b** $t^2 = r^2 + s^2$
**c** $LK^2 = JL^2 + JK^2$ **d** $5^2 = d^2 + 3^2$
**e** $12.5^2 = 10^2 + 7.5^2$ **f** $x^2 = 3^2 + 3^2$

**3** **a** $h = 10$ **b** $d = 25$ **c** $x = 15$

**4** **a** $m = \sqrt{865}$ **b** $b = \sqrt{52}$ **c** $x = \sqrt{569}$
**d** $p = \sqrt{2}$ **e** $d = \sqrt{19.4}$

**5** **a** $m = 29.4$ **b** $b = 7.2$ **c** $x = 23.9$
**d** $p = 1.4$ **e** $d = 4.4$

**6** D

**7** **a** $h = 7.81$ **b** $p = 7.94$ **c** $z = 3.02$

**8** 12.5 cm

**9** **a** 40 **b** 39 **c** 41
**d** 28 **e** 11 **f** 8
**g** 45 **h** 6, 8 **i** 9, 12

### Exercise 1.04

**1** **a** $x = 12$ **b** $y = 16$ **c** $y = 8$
**d** $a = 15$ **e** $d = 18$ **f** $x = 12$

**2** **a** $x = \sqrt{8}$ **b** $y = \sqrt{84}$ **c** $m = \sqrt{13\,321}$
**d** $y = \sqrt{13}$ **e** $x = \sqrt{94\,176}$ **f** $x = \sqrt{128}$

**3** **a** $x = 2.8$ **b** $y = 9.2$ **c** $m = 115.4$
**d** $y = 3.6$ **e** $x = 306.9$ **f** $x = 11.3$

**4** B

**5** **a** $k = 10.39$ **b** $q = 25.16$ **c** $a = 28.28$

**6** 30 cm **7** Teacher to check

### Mental skills 1

**2** **a** 625 **b** 3025 **c** 2025 **d** 7225
**e** 13 225 **f** 56.25 **g** 9025 **h** 38 025
**i** 2.25 **j** 4225 **k** 24 025 **l** 60 025

**4** **a** 441 **b** 10 201 **c** 961 **d** 8281
**e** 26.01 **f** 6561 **g** 3721 **h** 40 401
**i** 1.21 **j** 16.81

### Exercise 1.05

**1** **a** H **b** S **c** H
**d** S **e** H **f** S

**2** **a** $f = 85$ **b** $h = 4$ **c** $s = 55$
**d** $r = 4.5$ **e** $l = 29$ **f** $k = 38$

**3** **a** $g = \sqrt{130}$ **b** $t = \sqrt{160}$ **c** $x = \sqrt{735}$

**4** **a** $q = 62.2$ **b** $a = 21.0$ **c** $p = 4.7$

**5** B

**6** **a** $i = 12.59$ **b** $y = 27.97$ **c** $v = 54.38$

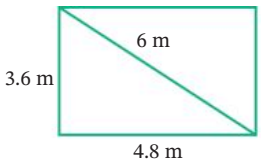
**7** **a** $d = \sqrt{74}$ **b** $x = \sqrt{60}$ **c** $x = \sqrt{288}$
**d** $p = \sqrt{85}$ **e** $x = \sqrt{164}$ **f** $p = \sqrt{560}$
**g** $x = \sqrt{16.65}$ **h** $x = \sqrt{624}$ **i** $y = \sqrt{786.25}$

### Exercise 1.06

**1** **a** right-angled **b** right-angled
**c** not right-angled **d** right-angled
**e** not right-angled **f** right-angled
**g** not right-angled **h** right-angled
**i** not right-angled **j** right-angled
**k** not right-angled **l** right-angled
Teacher to check diagrams.


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**2** **a**




**b** Yes, $6^2 = 36$ and $4.8^2 + 3.6^2 = 36$

## Exercise 1.07

**1** **a**, **b**, **c**, **e**, **g**, **h** are Pythagorean triads.

**2** D **3–4** Teacher to check.

**5** **a** 12, 13

**b** $a = 5, b = 12, c = 13$

$a^2 + b^2 = 5^2 + 12^2 = 25 + 144 = 169$

$c^2 = 13^2 = 169$

hence $a^2 + b^2 = c^2$, so it is a Pythagorean triad.

**6** **a** 7, 24, 25 **b** 11, 60, 61

**c** 15, 112, 113 **d** 4, 7.5, 8.5

**e** 9, 40, 41 **f** 19, 180, 181

**g** 10, 49.5, 50.5 **h** 51, 1300, 1301

## Exercise 1.08

**1** 3.61 m **2** 92.2 km **3** B

**4** $\sqrt{72}$ **5** 9.7 m **6** 68.7 cm

**7** **a** 48 cm **b** 31.38 cm **c** 170 cm

**d** 12.92 m **e** 28.94 m

**8** $h = 10.39$ cm **9** 6.32 m **10** 16 m

**11** **a** 192 cm$^2$ **b** 360 mm$^2$ **c** 84 mm$^2$

**d** 22 m$^2$ **e** 123.0 cm$^2$

**12** 38 m **13** 23 cm

**14** 4.58 m or 458 cm

## Power plus

**1** **a** 22.6 **b** 10.6 **c** 27.7

**d** 5.4 **e** 10.3 **f** 2.8

**2** **a** 16.2 m **b** 16.9 m

**3** **a** 21.2 cm **b** 26.0 cm

**4** 18.4 cm

## Test yourself 1

**1** **a** 14.25 **b** 16.16 **c** 5.74 **d** 6.21

**2** $\sqrt{104}, \sqrt{96}, \sqrt{12}, \sqrt{45}, \sqrt{88}$

**3** **a** $BC, BC^2 = AB^2 + AC^2$ **b** $d, d^2 = e^2 + f^2$

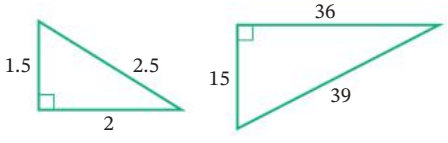
**c** $s, s^2 = h^2 + k^2$

**4** **a** $c = \sqrt{41}$ **b** $p = \sqrt{1325}$ **c** $x = \sqrt{914}$

**5** **a** $m = 0.99$ **b** $f = 10.58$ **c** $r = 4.87$

**6** **a** $z = 145.0$ **b** $y = 7.0$ **c** $v = 42.6$

**7** **a** right-angled **b** right-angled




**c** not right-angled

**8** **a**, **c**, **d** are Pythagorean triads.

**9** 206 mm

**10** **a** 9.5 m **b** 152.2 cm

# Chapter 2

## SkillCheck

**1** **a** Missing numbers are −6, −4, −3, −1, 1, 2, 4, 6, 7, 9

**b** Missing numbers are −35, −30, −25, −15, −10, 5

**2** −7, −5, −2, 0, 1, 6, 7, 11 **3** 12, 8, 5, 4, 2, −1, −3, −9

**4** **a** 7 **b** 8 **c** 1

**5** **a** $\frac{3}{100}$ **b** $\frac{7}{1000}$ **c** $\frac{49}{100}$ **d** $\frac{9}{10}$

**6** **a** 3 **b** 1 **c** 2 **d** 5

**7** **a** 1, 3, 5, 15 **b** 15, 30, 45, 60, 75

**8** **a** prime **b** prime

**c** composite **d** composite

## Exercise 2.01

**1** **a** 331 **b** 157 **c** 1587 **d** 255

**e** 421 **f** 413 **g** 368 **h** 734

**i** 612 **j** 35 **k** 276 **l** 47

**2** **a** 119 **b** 112 **c** 233

**d** 95 **e** 250 **f** 242

**3** A

**4** **a** 36 **b** 54 **c** 24 **d** 62

**e** 28 **f** 43 **g** 81 **h** 146

**i** 36 **j** 18 **k** 68 **l** 14

**m** 43 **n** 172 **o** 26 **p** 99

**5** **a** 233 **b** 1375 **c** 85

**6** **a** 24 **b** 16 **c** 35 **d** 54

**e** 36 **f** 40 **g** 64 **h** 21

**i** 24 **j** 36 **k** 27 **l** 28

**7** **a** 7 **b** 5 **c** 4 **d** 9

**e** 9 **f** 4 **g** 3 **h** 9

**i** 4 **j** 9 **k** 3 **l** 5

**8** **a** 800 **b** 90 **c** 700 **d** 160

**e** 100 **f** 1000 **g** 1500 **h** 240

**i** 180 **j** 320 **k** 1700 **l** 1400

**9** **a** 160 **b** 3500 **c** 18 000 **d** 300

**e** 210 **f** 1800 **g** 36 000 **h** 2400

**i** 30 **j** 6 **k** 5 **l** 8

**m** 20 **n** 100 **o** 7 **p** 3

**10** C

**11** **a** 170 **b** 78 **c** 228 **d** 112

**e** 128 **f** 264 **g** 324 **h** 116

**i** 54 **j** 122 **k** 109 **l** 26

**12** **a** 280 **b** 400 **c** 132 **d** 260

**e** 210 **f** 63 **g** 192 **h** 324

**13** **a** 110 **b** 70 **c** 1400 **d** 540

**e** 1100 **f** 2600 **g** 16 **h** 18

**i** 8 **j** 40 **k** 82 **l** 14

**14** 1140 words

**15** **a** 306 **b** 459 **c** 216 **d** 576

**e** 396 **f** 451 **g** 768 **h** 264

ANSWERS

**16** **a** 330 **b** 182 **c** 88 **d** 396
**e** 45 **f** 54 **g** 26 **h** 27
**17** C **18** $34

## Exercise 2.02

**1** **a** −2 **b** 5 **c** 2 **d** 6
**e** 4 **f** −3 **g** 4 **h** 0
**i** 6 **j** 3 **k** 2 **l** 0
**m** −5 **n** −3 **o** −1 **p** −8
**2** **a** −2 **b** −6 **c** −7 **d** −4
**e** −8 **f** −10 **g** −6 **h** −12
**i** 12 **j** 5 **k** 12 **l** 18
**m** −1 **n** 3 **o** 0 **p** 1
**3** 9°C **4** C
**5** **a** −2 **b** 0 **c** −11 **d** 4
**e** −13 **f** 6 **g** −1 **h** 2
**i** −2 **j** −7 **k** −15 **l** −5
**6** B
**7** **a** $5 + (-9) = -4$ **b** $-2 + 3 = 1$
**c** $-4 + (-5) = -9$ **d** $-4 + (-7) = -11$
**e** $3 - 6 = -3$ **f** $-6 - 1 = -7$
**g** $-2 - (-7) = 5$ **h** $-4 - 2 = -6$
**i** $-8 - (-8) = 0$
**8** Jackson Creek **9** 30 cm
**10** Teacher to check.
**11** **a** True **b** False
**12** **a** 0 **b** $2a$ **c** $2a$ **d** 0
**13** **a** + **b** E **c** + **d** −
**e** E **f** − **g** + **h** E

## Exercise 2.03

**1** **a** −20 **b** −18 **c** 32 **d** −45
**e** −70 **f** 30 **g** 9 **h** −24
**i** −21 **j** 100 **k** −36 **l** 40
**m** −44 **n** −60 **o** 36 **p** 63
**q** −55 **r** 81 **s** −100 **t** 42
**2** **a** C **b** D **c** B
**3** **a** −9 **b** 5 **c** −7 **d** −6
**e** 5 **f** −7 **g** −5 **h** −2
**i** 6 **j** −8 **k** −8 **l** 7
**m** −5 **n** 20 **o** −9 **p** −6
**4** **a** A **b** B
**5** **a** −9 **b** −4 **c** 6 **d** −7
**e** 4 **f** −4 **g** 9 **h** −7
**6** **a** (−2) **b** (−4) **c** 5
**d** −7 **e** −7 **f** −7
**7** **a** 28 **b** −90 **c** −40
**d** 75 **e** 20 **f** −8
**8** B
**9** **a** −2 **b** 3 **c** −2
**d** 30 **e** −5 **f** −1
**g** −20 **h** −2 **i** −70
**10** **a** (−6) **b** (−5) **c** 9
**d** 32 **e** −24 **f** 100
**11** D
**12** **a** True **b** False
**13** You get the negative of the integer squared.
**14** **a** + **b** + **c** − **d** + **e** +
**15** Teacher to check.
**16** **a** The answer is 1. **b** The answer is −1.
**17** **a** $a^2$ **b** $a^2$ **c** 1 **d** −1
**18** Teacher to check.

## Mental skills 2A

**2** **a** 3.5 **b** 2.4 **c** 0.12 **d** 0.36
**e** 0.8 **f** 0.027 **g** 0.2 **h** 8.8
**i** 0.24 **j** 0.012 **k** 1.8 **l** 0.028
**4** **a** 66.3 **b** 6630 **c** 6.63 **d** 0.663
**e** 6.63 **f** 663 **g** 0.663 **h** 663
**i** 6630 **j** 66.3 **k** 0.663 **l** 0.0663

## Exercise 2.04

**1** **a** 5 **b** 30 **c** 22 **d** −32
**e** 26 **f** 8 **g** −12 **h** 23
**i** −1 **j** −5 **k** −19 **l** 0
**2** **a** −2 **b** 1 **c** 31 **d** 10
**e** 28 **f** 3 **g** −42 **h** −8
**i** 16 **j** 19 **k** 24 **l** 22
**3** **a** $\frac{1}{2}$ **b** 2 **c** $\frac{9}{10}$ **d** −6
**e** −2 **f** $\frac{1}{4}$ **g** 1 **h** 3
**4** B
**5** **a** She has not followed the correct order of operations: need to multiply first, before adding
**b** Ms Ferme should explain the correct order of operations
**c** $(-9 + 13) \times 4$
**6** **a** 8 **b** 5 **c** 3
**d** (−2) **e** (−1) **f** −20
**g** (−6) **h** 6 **i** 20
**7** **a** $8 - (3 + 7) = -2$ **b** $(40 - 10) \times 5 = 150$
**c** $27 \div (9 \div 3) = 9$ **d** $8 + (4 - 3) \times 2 = 10$
**e** $[8 + (4 - 3)] \times 2 = 18$
**f** $(6 + 4) \times (-1) - 1 = -11$
**g** $(13 + 3) \div (4 - 6) = -8$
**h** $100 \div (10 + 10) + 5 = 10$
**i** $100 \div (10 + 10 + 5) = 4$
**8** Teacher to check.

## Exercise 2.05

**1** B
**2** **a** 4.035, 4.05, 4.3, 4.35
**b** 12.173, 17.12, 17.2103, 17.231
**c** 14.08, 14.8, 14.801, 14.81
**d** 0.032, 0.2, 0.295, 0.3
**3** **a** 1.621, 1.612, 1.61, 1.16 **b** 3.305, 3.053, 3.05, 3.035
**4** **a** 3 **b** 2 **c** 1 **d** 0
**e** 3 **f** 3 **g** 1 **h** 2
**5** **a** 3.9 **b** 4.1 **c** 0.3 **d** 7.3
**e** 15.1 **f** 3.0 **g** 2.0 **h** 16.2
**6** C

**7** **a** 68.91 **b** 107.06 **c** 3.60 **d** 4.71
**e** 3.20 **f** 33.00 **g** 19.73 **h** 11.25
**8** **a** 9.704 **b** 13.168 **c** 0.083 **d** 53.094
**9** **a** \$460.40 **b** \$460 **c** \$460.4
**d** \$460.40 **e** \$460.395
**10** **a** 164.91 **b** 25.36 **c** 72.6 **d** 91.61
**e** 15.7 **f** 77.27 **g** 18.44 **h** 2.68
**i** 66.68 **j** 51.61 **k** 4.64 **l** 2.391
**11** C **12** 7.6°C **13** 0.8 m
**13** **a** \$10.05 **b** \$9.95
**14** **a** 3.04 **b** 12.74 **c** 5.45 **d** 0.889
**e** 4.326 **f** Teacher to check.

## Exercise 2.06

**1** **a** 26 **b** 4.05 **c** 11.5 **d** 27.3
**e** 0.21 **f** 0.088 **g** 0.02 **h** 1.26
**i** 28.35 **j** 88.97 **k** 0.806 **l** 0.3672
**2** B **3** \$14.85
**4** **a** 6.2 **b** 16.3 **c** 2.61 **d** 0.139
**e** 316.5 **f** 69.55 **g** 370 **h** 58.05
**i** 0.58 **j** 5.2 **k** 170 **l** 1.2
**5** B
**6** **a** 43.44 m **b** 1.81 m
**7** **a** 5.91 **b** 14 **c** 5.66
**8** \$85.45
**9** **a** 8 **b** 2 **c** 0.04
**d** 0.9 **e** 13.9 **f** 25.92
**10** 16.2 km **11** \$24.60
**12** **a** 41.04 **b** 0.829 **c** 328.24
**d** 4.7292 **e** 2.51706

## Exercise 2.07

**1** **a** 0.4 **b** 0.375 **c** 0.75 **d** 1
**e** 0.5 **f** 0.75 **g** 0.6 **h** 0.25
**i** 0.625 **j** 0.875 **k** 0.5 **l** 1
**2** They are equivalent fractions.
**3** **a** $0.\dot{6}$ **b** $4.\dot{2}\dot{7}$ **c** $0.\dot{2}\dot{8}$ **d** $3.\dot{8}\dot{3}$
**e** $9.\dot{6}0\dot{7}$ **f** $0.\dot{1}5\dot{1}\dot{9}$ **g** $0.0\dot{5}\dot{2}$ **h** $12.2\dot{3}\dot{4}$
**4** **a** 0.858333... **b** 0.1656565...
**c** 0.272727... **d** 0.461538461538...
**5** **a** $0.\dot{1}$ **b** $0.1\dot{6}$ **c** $0.\dot{8}\dot{3}$
**d** $0.\dot{1}42857\dot{1}$ **e** $0.\dot{6}$ **f** $0.\dot{2}85714\dot{2}$
**g** $0.\dot{2}$ **h** $0.\dot{4}28571\dot{4}$ **i** $0.\dot{5}71428\dot{5}$
**j** $0.\dot{4}$ **k** $0.\dot{6}$ **l** $0.\dot{5}$
**m** $0.\dot{8}57142\dot{8}$ **n** $0.\dot{7}$ **o** $0.\dot{7}14285\dot{7}$
**6** 0.142857, 0.285714, 0.428571, 0.571428, 0.714285, 0.857142. It's the same digits, in order, but starting with a different digit each time.
**7** **a** $3.\dot{6}$ **b** $1.7\dot{1}$ **c** $531.1\dot{8}$ **d** $3.2\dot{6}$

## Mental skills 2B

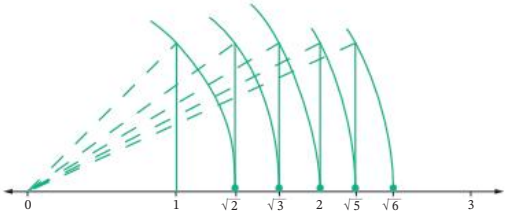
**2** **a** 0.5 **b** 90 **c** 8 **d** 0.9
**e** 90 **f** 0.7 **g** 8 **h** 30
**i** 0.08 **j** 3.5 **k** 4 **l** 0.3
**4** **a** 16 **b** 160 **c** 1.6 **d** 1.6
**e** 1.6 **f** 16 **g** 160 **h** 0.16
**i** 0.016 **j** 160 **k** 0.16 **l** 0.0016

## Exercise 2.08

**1** **a** $7^3$ **b** $4^5$ **c** $5^{10}$ **d** $(-8)^7$
**2** **a** 64 **b** 8 **c** −216 **d** 81
**e** 7 **f** −32 **g** 1000 **h** 256
**i** 1 771 561 **j** 18 954 **k** 972
**l** 2529 **m** 39 **n** 4094
**3** **a** 3 **b** 3 **c** 2
**d** 6 **e** 3 **f** 5
**4** B **5** $100^3$ **6** $2^{10}$
**7** **a** 36 **b** 36
**c** Yes, they both equal 36.
**8** **a** 400 **b** 400 **c** Yes
**9** **a** $3^2 \times 8^2$ **b** $a^2 \times b^2$
**10** **a** $6^2 \times 3^2 = 324$ **b** $2^2 \times 11^2 = 484$
**c** $(2 \times 8)^2 = 2^2 \times 8^2 = 256$
**d** $(3 \times 5)^2 = 3^2 \times 5^2 = 225$
**e** $(10 \times 3)^2 = 10^2 \times 3^2 = 900$
**f** $(4 \times 7)^2 = 4^2 \times 7^2 = 784$
**11** **a** 28 **b** 16 **c** 17 **d** 33
**e** 2 **f** 7 **g** 13 **h** 6
**12** **a** 6.1 **b** 9.7 **c** 22.4 **d** 1.9
**e** −7.9 **f** 44.7 **g** 1.0 **h** 10.3
**13**

| Number, $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Number squared, $x^2$ | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 | 121 | 144 |
| Number cubed , $x^3$ | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 | 1000 | 1331 | 1728 |

**14** D **15** D **16** 3 and 4
**17** **a** 5.7 **b** 10.2 **c** 8.2
**18** **a** 10 **b** 10 **c** The answer is the same.
**19** **a** 9 **b** 9
**c** The answer is the same.
**20** **a** $\sqrt{16} \times \sqrt{9}$ **b** $\sqrt{a} \times \sqrt{b}$
**21** **a** $2 \times 4 = 8$ **b** $\sqrt{9} \times \sqrt{49} = 3 \times 7 = 21$
**c** $\sqrt{36} \times \sqrt{4} = 6 \times 2 = 12$
**d** $\sqrt{81} \times \sqrt{25} = 9 \times 5 = 45$
**22**




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## Exercise 2.09

**1** **a** 8 **b** 15 **c** 7 **d** 2
**e** 31 **f** 83 **g** 16 **h** 25
**i** 20 **j** 18 **k** 180 **l** 1000

**2** **a** $2\sqrt{5}$ **b** $3\sqrt{2}$ **c** $5\sqrt{2}$ **d** $2\sqrt{3}$
**e** $3\sqrt{10}$ **f** $4\sqrt{2}$ **g** $2\sqrt{2}$ **h** $3\sqrt{5}$
**i** $6\sqrt{3}$ **j** $7\sqrt{2}$ **k** $10\sqrt{2}$ **l** $9\sqrt{2}$

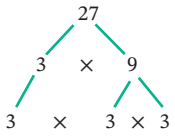
**3** **a** $10\sqrt{3}$ **b** $6\sqrt{6}$ **c** $40\sqrt{3}$ **d** $12\sqrt{2}$
**e** $12\sqrt{7}$ **f** $10\sqrt{3}$ **g** $8\sqrt{6}$ **h** $30\sqrt{3}$
**i** $18\sqrt{2}$ **j** $20\sqrt{5}$ **k** $35\sqrt{2}$ **l** $54\sqrt{3}$

**4** $\sqrt{48}$

**5** **a** $\sqrt{12}$ **b** $\sqrt{32}$ **c** $\sqrt{150}$ **d** $\sqrt{54}$
**e** $\sqrt{200}$ **f** $\sqrt{243}$ **g** $\sqrt{180}$ **h** $\sqrt{448}$
**i** $\sqrt{490}$ **j** $\sqrt{240}$ **k** $\sqrt{75}$ **l** $\sqrt{288}$

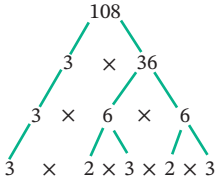
## Exercise 2.10

**1** **a**



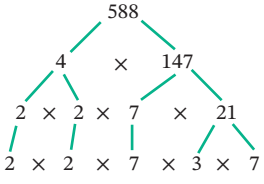

$27 = 3^3$

**b**



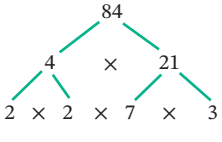

$108 = 2^2 \times 3^3$

**c**



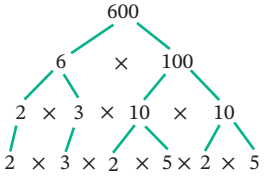

$588 = 2^2 \times 7^2 \times 3$

**d**




$84 = 2^2 \times 3 \times 7$

**e**



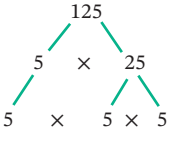

$600 = 2^3 \times 5^2 \times 3$

**f**



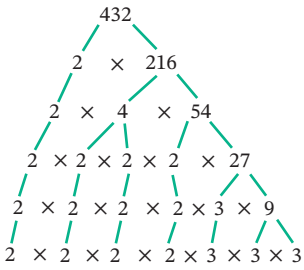

$51 = 3 \times 17$

**g**




$125 = 5^3$

**h**




$432 = 2^4 \times 3^3$

**2** C **3** Teacher to check.

**4** **a** 4 **b** 7 **c** 5 **d** 15
**e** 6 **f** 24 **g** 16 **h** 6

**5** **a** 15 **b** 26 **c** 11

**6** **a** 72 **b** 12 **c** 110 **d** 28
**e** 30 **f** 24 **g** 45 **h** 110

**7** **a** 90 **b** 252 **c** 80

**8** **a** 45 **b** 21 **c** 16
**d** 16 **e** 9 **f** 18

**9** Teacher to check.

**10** **a** 4 **b** 532

## Exercise 2.11

**1** **a** $6^5$ **b** $4^7$ **c** $2^{11}$ **d** $8^{10}$
**e** $11^8$ **f** $7^{10}$ **g** $5^7$ **h** $6^{16}$
**i** $2^9$ **j** $5^9$ **k** $2^{14}$ **l** $3^{16}$

**2** D

**3** **a** $5^6 = 15\,625$ **b** $7^8 = 5\,764\,801$
**c** $3^{13} = 1\,594\,323$ **d** $4^6 = 4096$
**e** $9^7 = 4\,782\,969$ **f** $6^9 = 10\,077\,696$
**g** $8^{10} = 1\,073\,741\,824$ **h** $7^4 = 2401$

**4** **a** $2^4$ **b** $3^2$ **c** $7^6$ **d** $2^5$
**e** $5^2$ **f** $10^4$ **g** $11^7$ **h** $6^7$
**i** 2 **j** $16^6$ **k** $35^{11}$ **l** 20

**5** D

**6** **a** $2^4 = 16$ **b** $5^2 = 25$ **c** $4^3 = 64$
**d** $10^5 = 100\,000$ **e** $8^3 = 512$
**f** $6^8 = 1\,679\,616$
**g** 11 **h** $3^6 = 729$

**7**

| Number, $n$ | $n^2$ | $n^3$ | $n^4$ | $n^5$ | $n^6$ |
|---|---|---|---|---|---|
| 2 | 4 | 8 | 16 | 32 | 64 |
| 3 | 9 | 27 | 81 | 243 | 729 |
| 4 | 16 | 64 | 256 | 1024 | 4096 |
| 5 | 25 | 125 | 625 | 3125 | 15 625 |

**8** **a** 32 **b** 81 **c** 125 **d** 3125
**e** 4096 **f** 1944 **g** 64 **h** 3125
**i** 9 **j** 2 **k** 4 **l** 1

**9** **a** $5^4$ **b** $3^7$ **c** $2^3$

**10** **a** $a^{10}$ **b** $x^6$ **c** $b^5$ **d** $y^5$
**e** $d^4$ **f** $x^2$ **g** $a$ **h** $r^4$


9780170479516

## Exercise 2.12

**1** **a** $2^{18}$ **b** $5^{6}$ **c** $7^{20}$ **d** $8^{14}$
**e** $3^{24}$ **f** $9^{24}$ **g** $6^{14}$ **h** $11^{25}$
**i** $8^{30}$ **j** $4^{45}$ **k** $(-2)^{28}$ **l** $(-7)^{18}$

**2** D

**3** **a** $5^8 = 390\,625$ **b** $2^{15} = 32\,768$
**c** $3^{18} = 387\,420\,489$ **d** $6^{12} = 2\,176\,782\,336$
**e** $7^6 = 117\,649$ **f** $4^{12} = 16\,777\,216$
**g** $8^0 = 1$ **h** $4^9 = 262\,144$

**4** **a** 1 **b** 1 **c** 1 **d** 1
**e** 1 **f** 1 **g** 1 **h** 6
**i** 1 **j** 7 **k** 1 **l** 3
**m** 1 **n** 2 **o** 6 **p** 9

**5** B

**6** 1, 2, 8, 16, 64, 128, 256, 512, 2048, 4096

**7** **a** 64 **b** 4096 **c** 16 **d** 512
**e** 1 **f** 256 **g** 1 **h** 128

**8** **a** $(6^2)^4 = 6^8$ **b** $(5^3)^3 = 5^9$
**c** $(10^6)^3 = 10^{18}$, other possible answers

**9** **a** $a^8$ **b** $x^{36}$ **c** $d^{10}$ **d** $n^{18}$
**e** 1 **f** 4 **g** 1 **h** 0

**10** Teacher to check.

**11** $a^2\,b^2$

**12** **a** 400 **b** 144 **c** 900 **d** 3600
**e** 576 **f** 1024 **g** 225 **h** 441
**i** 324 **j** 484 **k** 729 **l** 1296

## Power plus

**1** **a** 9 **b** $2\sqrt{2}$ **c** $3\sqrt{3}$ **d** $5\sqrt{2}$
**e** $6\sqrt{2}$ **f** $3\sqrt{5}$ **g** $6\sqrt{5}$ **h** $8\sqrt{2}$
**i** $2\sqrt{7}$ **j** $10\sqrt{2}$ **k** $4\sqrt{3}$ **l** $7\sqrt{5}$

**2** **a** $2^{-1}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2^2} = \frac{1}{4}$

**3** **a** Yes **b** No
**c** For odd values of $n$. **d** 0

**4** **a** 24 000 **b** 455 000 **c** 0.0933
**d** 0.0638 **e** 8 700 000 **f** 0.005 82
**g** 0.126 **h** 2690 **i** 314 000 000

**5** **a** $1.2 \times 10^4$ **b** $3.45 \times 10^8$ **c** $7 \times 10^{-3}$
**d** $4 \times 10^3$ **e** $5 \times 10^{-4}$ **f** $4.1 \times 10^{-4}$
**g** $1.92 \times 10^6$ **h** $3.61 \times 10^{-4}$ **i** $6.37 \times 10^{-8}$

**6** Teacher to check.

## Test yourself 2

**1** **a** 1400 **b** 210 **c** 160 **d** 131
**e** 87 **f** 385 **g** 260 **h** 480
**i** 276 **j** 800 **k** 276 **l** 42
**m** 9 **n** 2800 **o** 333 **p** 90
**q** 48 **r** 725 **s** 160

**2** **a** 116 **b** 94 **c** 22 **d** 93

**3** **a** 2 **b** 7 **c** −11 **d** 0

**4** 9237 m

**5** 23°C

**6** **a** −14 **b** 24 **c** 49 **d** −80

**7** **a** −9 **b** −4 **c** −12 **d** 3

**8** **a** 57 **b** 29 **c** 3
**d** 62 **e** $\frac{1}{5}$ **f** 5

**9** 0.471, 0.47, 0.417, 0.147

**10** **a** 0.5 **b** 13.11 **c** 98.427

**11** 12.2 m

**12** **a** 30.5 **b** 19.72 **c** 10.4 **d** 3.3
**e** 0.923 **f** 6.25 **g** 2.3925 **h** 11.6

**13** 15 bottles

**14** **a** $0.\dot{2}$ **b** 0.875 **c** $0.1\dot{6}$

**15** **a** 343 **b** 161 051 **c** 625
**d** 20 **e** 3 **f** −5

**16** 5.6

**17** **a** 2, 2, 4, 100, 400 **b** 4, 2, 7, 14

**18** **a** 3 **b** 17 **c** 72
**d** $2\sqrt{10}$ **e** $3\sqrt{3}$ **f** $10\sqrt{3}$

**19** **a** 6 **b** 140

**20** $132 = 2^2 \times 3 \times 11$, $88 = 2^3 \times 11$, HCF $= 2 \times 2 \times 11 = 44$

**21** $12 = 2^2 \times 3$, $45 = 3^2 \times 5$, LCM $= 2^2 \times 3 \times 3 \times 5 = 180$

**22** $784 = 2^4 \times 7^2$, $\sqrt{784} = 2^2 \times 7 = 28$

**23** **a** $4^7$ **b** $6^{11}$ **c** $12^8$ **d** $9^4$
**e** $2^4$ **f** $5^3$ **g** 3 **h** $6^{12}$

**24** **a** $3^8$ **b** $7^{10}$ **c** $6^9$ **d** $3^{15}$

**25** **a** 1 **b** 1 **c** 4 **d** 8
**e** 1 **f** 1 **g** 1 **h** 0

# Chapter 3

## SkillCheck

**1** **a** −1 **b** 4 **c** −10 **d** 5
**e** −18 **f** 3 **g** −13 **h** 32
**i** 36 **j** 8 **k** −1 **l** 18

**2** **a** $7p$ **b** $ab$ **c** $k^2$ **d** $n^3$
**e** $2mp$ **f** $dkp$ **g** $6y$ **h** $3qt$
**i** $-d$ **j** $3q$ **k** $4a$ **l** $x$

**3** **a** + **b** × **c** − **d** ÷
**e** − **f** + **g** + **h** −

**4** **a** 10 **b** 24 **c** 6 **d** 10
**e** 16 **f** 7 **g** 25 **h** 20
**i** 4 **j** 4 **k** 20 **l** 42

**5** **a** 13 **b** 18 **c** 16
**d** 3 **e** 21 **f** 8

## Exercise 3.01

**1** **a** $5w$ **b** $m^2$ **c** $3abc$ **d** $6k$
**e** $\frac{m}{8}$ **f** $3f$ **g** $16m$ **h** $21wx$
**i** $9p$ **j** $4t^2$ **k** $d$ **l** $16bcd$
**m** $\frac{26}{n}$ **n** $9h^2$ **o** $20x$ **p** $a$
**q** $8q$ **r** $12cn$ **s** $-6d^2r$ **t** $3a + 2c$
**u** $a^3x^2$

**2** **a** $\frac{13}{2a}$ **b** $9 + 4m$ **c** $\frac{e-6}{5}$
**d** $4t - 8$ **e** $y^2 + z^2$ **f** $\frac{k}{9} - n$
**g** $\frac{p}{n+9}$ **h** $22 - \frac{e}{2}$ **i** $11 + 11u^2$

**3** A **4** Teacher to check.

**5** B

**6** **a** $8 \times s \times t$ **b** $-2 \times y \times y$
**c** $8 \div f$ **d** $r \times r + t \times t$
**e** $4 \times b \times b \times d$ **f** $5 \times x \times y - 2 \times a$
**g** $(x - 2) \div 3$ **h** $q \times q \times q$
**i** $16 - 2 \times d \times d$ **j** $-4 \times d \div 3$
**k** $h \times h \times i \times i$ **l** $4 \times j - 9 \div d$

**7** D

## Exercise 3.02

**1** **a** $5n$ **b** $n + 6$ **c** $\frac{n}{10}$ **d** $n + 1$
**e** $\frac{n}{2}$ **f** $n - 20$ **g** $n^2$ **h** $20 - n$
**i** $n - 1$ **j** $2n$ **k** $\frac{6}{n}$ **l** $n - 8$
**m** $-4n$ **n** $\sqrt{n}$

**2** **a** $a + b + c$ **b** $ab$ **c** $b - c$
**d** $ac + b$ **e** $3b + a$ **f** $\frac{c}{b}$
**g** $a - b$ **h** $5 + bc$ **i** $b - c$
**j** $\frac{ab}{c}$ **k** $\sqrt{a+c}$ **l** $ab - 7c$

**3** D

**4** **a** $3y + 8$ **b** $\frac{p}{3} - 6$ **c** $4M + 27$
**d** $\frac{5k}{2}$ **e** $\frac{x}{6} + 13$ **f** $\frac{B}{2} - 2$
**g** $\frac{R}{9} + 10$ **h** $21n$

**5** **a** $x$ plus $y$ minus $z$ **b** 5 times $K$, plus $L$
**c** $x$ times $y$ minus 3 **d** $W$ divided by 5
**e** 3 times $x$, minus 1
**f** $y$ minus $x$, then divided by 10
**g** 2 times $N$, plus 21 **h** 7 minus $e$

**6** **a** $t - b$ **b** $\frac{n}{5}$ **c** $y - 3p$
**d** $2r + 9$ **e** $\frac{a}{12}$ **f** $4x$
**g** $\frac{160}{r}$ **h** $5x + 3y$ **i** $\frac{5m - 1}{3}$

## Exercise 3.03

**1** C

**2** **a** 8 **b** 12 **c** 4 **d** 10
**e** 30 **f** 12 **g** 0 **h** 8
**i** 5 **j** 19 **k** $-3$ **l** 3

**3** **a** $-150$ **b** 25 **c** $-27$ **d** 2
**e** $-15$ **f** 75 **g** $-21$ **h** 6
**i** 18 **j** $-10$ **k** $-90$ **l** $\frac{1}{2}$

**4** B

**5** **a** $-13$ **b** $-3$ **c** 10 **d** $-5$
**e** $-\frac{1}{4}$ **f** 1 **g** $\frac{1}{3}$ **h** 5

**6** C **7** $A = 42\ \text{m}^2$ **8** \$212
**9** 78.8°F **10** 14 hours **11** \$5900
**12** **a** **i** 31 minutes **ii** 56 minutes
**b** 8:10 am

## Exercise 3.04

**1** **a** $4p$, $2p$, $p$ **b** $m^2$, $3m^2$ **c** $5ac$, $7ca$
**d** $vw$, $2wv$ and $5v$, $9v$ **e** $6pq$, $2qp$, $3pq$
**f** $2d$, $3d$ **g** $4mn$, $2nm$, $mn$
**h** $x^2y$, $4x^2y$ **i** $5ba$, $ab$

**2** C

**3** **a** $11k$ **b** $7mn$ **c** $2xy$ **d** $9abc$
**e** $6x^2$ **f** $5ef^2$ **g** $5d$ **h** $5mk$
**i** $4xy$ **j** $-4de$ **k** $3k^2$ **l** $3w^2$

**4** C

**5** **a** $5k + 4j$ **b** $9ab + 2$ **c** $7m^2 + 2m$
**d** $10s^2 - 7st$ **e** $10mn$ **f** $9k + 3$
**g** $2y^2$ **h** $12d - 4e$ **i** $4 - 2x$
**j** $8bg$ **k** $9 - 2q$ **l** $5a + 2c$

**6** **a** $3x - 3y$ **b** $4ab + 3bc$ **c** $9k^2 + 11$
**d** $6bc - 9a$ **e** $x^2 - 7x + 7$ **f** $4p^2$
**g** $12y + 14$ **h** $7r + 3s$ **i** $7b + 6g$
**j** $6x^2 - 13x + 6$ **k** $52a$ **l** $7q - 7$
**m** $-6y$ **n** 12

**7** C

**8, 9** Teacher to check.

## Exercise 3.05

**1** D

**2** **a** $15y$ **b** $16m$ **c** $4km$ **d** $20t$
**e** $12f$ **f** $16xy$ **g** $30mp$ **h** $18bd$
**i** $12d^2$ **j** $120mn$ **k** $48y^2$ **l** $27r^2$
**m** $72ab$ **n** $x^2y^2$ **o** $42jk$ **p** $10r^2s^2t$
**q** $18ac^2d$ **r** $h^2j^3k^2$ **s** $24p^2q^2$ **t** $6m^2n^2$

**3** **a** $-8bd$ **b** $-24m^2$ **c** $-90kl$
**d** $-60p$ **e** $-63k$ **f** $75b^2$
**g** $-12m^2n^2$ **h** $-12a^2b^2c$ **i** $40r^2$
**j** $-216d^2e$ **k** $144x^2$ **l** $-40m^2$

**4** B

**5** **a** $y^4$ **b** $a^3$ **c** $2c^2$ **d** $t^5$
**e** $7q^3$ **f** $k^3l^2$ **g** $m^3n^2$ **h** $-5p^4k^3$
**i** $-3y^3$ **j** $-d^2e^2f$ **k** $4q^2r^4$

**6** A

**7** **a** $4m$ **b** $9e$ **c** $-7p$ **d** $9x$
**e** $4w$ **f** 8 **g** $-5$ **h** $\frac{3}{4}$
**i** $2p$ **j** $-7$ **k** $\frac{p}{5}$ **l** $-\frac{7n}{m}$

**8** **a** $2m^2$ **b** $-3e$ **c** $-5p$ **d** $7x$
**e** $-2w$ **f** $4m$ **g** $-8ky$ **h** $\frac{3}{4}$
**i** $5de$ **j** $a$ **k** $-8a$ **l** $-2m$

**9** D

**10** **a** $11p$ **b** $a^2$ **c** $2ry$ **d** $-27p$
**e** $6x^2$ **f** $8mn$ **g** $8x$ **h** $18p$
**i** $4w^2$ **j** 0 **k** $\frac{5p}{m}$ **l** 1

**11, 12** Teacher to check.

## Mental skills 3A

**2** **a** 720 **b** 486 **c** 825 **d** 128
**e** 560 **f** 216 **g** 189 **h** 432
**i** 560 **j** 616 **k** 135 **l** 675

## Exercise 3.06

**1** D

**2** **a** $m^7$ **b** $k^9$ **c** $y^{10}$ **d** $x^8$
**e** $n^8$ **f** $q^{10}$ **g** $b^{16}$ **h** $s^{15}$
**i** $42k^6$ **j** $20y^4$ **k** $3d^9$ **l** $55e^4$
**m** $h^{10}\,n^7$ **n** $-30c^6\,d^5$ **o** $f^8\,g^{11}$ **p** $-10p^7\,q^2$

**3** A

**4** **a** $y$ **b** $p^3$ **c** $q^3$ **d** $n^5$
**e** $k^2$ **f** $x^2$ **g** 1 **h** $m^5$
**i** $q$ **j** $9g^4$ **k** $3p^4$ **l** $a^5$
**m** $11xy$ **n** $x^3$ **o** $7k^3\,l^5$ **p** $\frac{5e^2}{7f}$

**5** B

**6** **a** $y^{16}$ **b** $x^{10}$ **c** $y^9$ **d** $m^{18}$
**e** $k^{21}$ **f** $t^{40}$ **g** $9m^2$ **h** $8m^{12}$
**i** $x^3\,y^3$ **j** $p^{12}\,q^4$ **k** $81h^{16}$ **l** $k^{15}\,z^5$
**m** $16k^4\,j^4$ **n** $16c^6\,d^2$ **o** $125x^6\,y^{24}$ **p** $64m^{24}\,p^{12}$

**7** **a** 1 **b** 1 **c** 1 **d** 5
**e** 1 **f** 1 **g** 1 **h** $h^2$
**i** $\frac{b^6}{2}$ **j** $16x^2$ **k** 1 **l** $13x$
**m** $3m + 1$ **n** $3b^2$ **o** 32 **p** $\frac{1}{2}$

**8** **a** $\frac{1}{3pq}$ **b** $2mn$ **c** $\frac{2}{3y}$

## Exercise 3.07

**1** **a** $5m + 15$ **b** $8m - 16$ **c** $2j + 20$
**d** $3a + 6$ **e** $4h - 28$ **f** $6x + 6y$
**g** $36 - 9b$ **h** $11y - 66$ **i** $8k + 20$
**j** $15m - 6$ **k** $18x + 24$ **l** $20a + 50b$

**2** **a** $-3n - 12$ **b** $-4v + 20$ **c** $-7d - 21$
**d** $-10k + 20$ **e** $-2d - 2e$ **f** $-25 + 5z$
**g** $-6h - 6$ **h** $-m + 1$ **i** $-45 - 18x$
**j** $-2f - 3$ **k** $-3u + 3v$ **l** $-32 + 8k$

**3** B

**4** **a** $xy + 6x$ **b** $pq - 2p$ **c** $a^2 + 4a$
**d** $3b^2 - b$ **e** $d^2 + 3de$ **f** $8m + 10n$
**g** $15k^2 - 10k$ **h** $21j^2 + 7ijk$ **i** $4x^2 + xy$
**j** $-8p + 6q$ **k** $k^2 - 12k$ **l** $-4m + m^2$
**m** $8n^2 + 20mn$ **n** $2x - 14x^2$ **o** $-2f^2 - 4f$
**p** $-5j + 7k$

**5** C **6, 7** Teacher to check. **8** D

**9** **a** $6a + 26$ **b** $6k + 6$ **c** $3q - 4m$
**d** $6p - 22$ **e** $4 - 2y$ **f** $17 - 7t$
**g** $12h + 5$ **h** $-8x + 8$ **i** $18n + 4$

**10** Teacher to check.

**11** **a** $5a + 14$ **b** $8x + 4$ **c** $17k + 18$
**d** $22t + 11$ **e** $n + 7$ **f** $4d + 2$
**g** $22y + 23$ **h** $-2p - 7$ **i** $m^2 + 4m + 3$
**j** $7d + e$ **k** $6f^2 + 4$ **l** $-38e^2 + e$

**12** **a** $x + 5$ **b** $r - 3$ **c** $8k - 3$
**d** $2a + 7$ **e** $y + 9$ **f** $2t - 7$

## Exercise 3.08

**1** **a** 1, 3, 5, 15 **b** 1, 2, 4, 8
**c** 1, 17 **d** 1, 5, 25
**e** 1, 5, 11, 55 **f** 1, 2, 3, 6, 7, 14, 21, 42

**2** **a** 3 **b** 2 **c** 6 **d** 4 **e** 11
**f** 5 **g** 3 **h** 15 **i** 16

**3** Teacher to check. **4** C

**5** **a** $3x$ **b** $4j$ **c** $2x$ **d** $7k$
**e** 5 **f** $4m$ **g** $16c$ **h** $de$
**i** 9 **j** 3 **k** $2m$ **l** $4y$
**m** $a$ **n** 32 **o** $5w$ **p** $2xy$
**q** $d$ **r** $12t$

**6** C

## Exercise 3.09

**1** **a** 2 **b** 4 **c** $x$ **d** $y$

**2** **a** $x + 2$ **b** $p - 4$ **c** $4k - 5$ **d** $j - 4k$
**e** $x^2 + y^2$ **f** 5 **g** 6 **h** 2
**i** 2 **j** $3x + 1$ **k** $y + z$ **l** $b - c$
**m** $q - 1$ **n** $b + d$ **o** $a + 2$ **p** $h$
**q** $n$ **r** $y$ **s** $k$ **t** $f$

**3** **a** $4(x + 2)$ **b** $3(m + 2)$ **c** $5(a - 2)$
**d** $5(3m - 2a)$ **e** $4(2x - y)$ **f** $4(3m - 4w)$
**g** $h(k - 1)$ **h** $x(2y - m)$ **i** $p(12 + 5r)$
**j** $x(x + 5)$ **k** $m(4 - m)$ **l** $y(y - 11)$

**4** C

**5** **a** $m + n$ **b** $2y - 1$ **c** $2p + 1$
**d** $n - p$ **e** $y - 4$ **f** $q + r$
**g** $7 - y$ **h** $4k + 1$ **i** $3x - 4z$

**6** D

**7** **a** $7a(b + 2)$ **b** $2x(1 - 3y)$ **c** $3m(1 - 3w)$
**d** $5x(2y - 5w)$ **e** $4m(2a - 3p)$ **f** $3y(3x + 2m)$
**g** $3x(1 + 3y)$ **h** $ab(1 + c)$ **i** $xy(z + a)$
**j** $6m(3 - 4n)$ **k** $ac(2 + 3b)$ **l** $pq(r + m)$
**m** $r(s + 7)$ **n** $4(s^2 + 4)$ **o** $p(p - 2)$
**p** $2k(3k^2 - 1)$ **q** $j(3 + j)$ **r** $n(7 - n)$
**s** $7e(1 + 3e)$ **t** $6q(1 - 2q)$ **u** $a^2(a^2 + 1)$
**v** $xy(x + y)$ **w** $3m(mn + 3)$
**x** $3abc(4abc + 1)$

## Exercise 3.10

**1** **a** $k + 1$ **b** $n + p$ **c** $r - 2$ **d** $k - 9$
**e** $-a$ **f** $-y$ **g** $-6$ **h** $-f$

**2** **a** $-4(x - 3)$ **b** $-3(m + 3)$ **c** $-10(x - 2)$
**d** $-5(3x - 2m)$ **e** $-2(3x + 4m)$ **f** $-5(x + 6)$
**g** $-a(1 + b)$ **h** $-4(xyz - 2abc)$ **i** $-4(5 - x)$
**j** $-d(x + y)$ **k** $-m(2 - 5p)$ **l** $-8(3x + 2y)$

**3** B

**4** **a** $-8x(2y + 3k)$ **b** $-4x(5 - 3y)$ **c** $-4m(2w - 3)$
**d** $5e(1 - 3f)$ **e** $-ab(1 + c)$ **f** $xy(a + w)$
**g** $-6c(2 + 3w)$ **h** $-rt(2 - 7s)$ **i** $5h(4f + 1)$
**j** $-19w(b + 2)$ **k** $-abc(1 - d)$ **l** $-abc(1 + d)$

**5** **a** 10, 10 **b** Yes

**6** A

**7** **a** $a^3bc(a - b)$ **b** $2\pi r(r + h)$

## Mental skills 3B

**2** **a** 176 **b** 363 **c** 261 **d** 405
**e** 682 **f** 707 **g** 1818 **h** 3564
**i** 152 **j** 540 **k** 2142 **l** 588

## Exercise 3.11

**1** **a**

| | $x$ | 2 |
|---|---|---|
| $x$ | $x^2$ | $2x$ |
| 7 | $7x$ | 14 |

$x^2 + 9x + 14$

**b**

| | $x$ | 5 |
|---|---|---|
| $x$ | $x^2$ | $5x$ |
| 1 | $x$ | 5 |

$x^2 + 6x + 5$

**2** D

**3** **a** $x^2 + 5x + 4$ **b** $x^2 + 9x + 18$
**c** $x^2 + 9x + 20$ **d** $x^2 + 4x + 4$
**e** $x^2 + 8x + 12$ **f** $x^2 + 13x + 30$
**g** $x^2 + 9x + 8$ **h** $x^2 + 14x + 49$
**i** $x^2 + 3x - 4$ **j** $x^2 - x - 6$
**k** $x^2 - 3x - 4$ **l** $x^2 + x - 30$
**m** $x^2 - 9x + 20$ **n** $x^2 + 6x - 27$
**o** $x^2 - 8x + 15$ **p** $x^2 - 14x + 49$

**4** **a** **i** $x^2 - 9$ **ii** $x^2 - 16$
**iii** $x^2 - 4$ **iv** $x^2 - 49$
**b** $x^2 - 8^2 = x^2 - 64$

**5** **a** $2x^2 + 6x + 4$ **b** $3x^2 + 5x - 2$
**c** $2x^2 + 5x + 3$ **d** $5x^2 - 13x - 6$
**e** $4x^2 - 11x + 6$ **f** $10x^2 - 11x - 6$
**g** $3x^2 - 10x + 3$ **h** $4x^2 - 5x - 21$

**6** **a** $a^2 + 10a + 25$ **b** $x^2 + 4x + 4$
**c** $y^2 + 14y + 49$ **d** $x^2 - 6x + 9$
**e** $4a^2 + 4a + 1$ **f** $9a^2 - 30a + 25$

**7** **a** $a^2 + 2ab + b^2$ **b** $a^2 - b^2$

**8** A

## Exercise 3.12

**1** **a** 36 **b** 6 **c** 8

**2** D

**3** **a** $y^2 + 12y + 36$ **b** $x^2 - 2x + 1$ **c** $h^2 + 6h + 9$
**d** $n^2 - 16n + 64$ **e** $a^2 - 14a + 49$ **f** $x^2 + 18x + 81$
**g** $y^2 - 22y + 121$ **h** $d^2 + 20d + 100$

**4** **a** $4a^2 + 12a + 9$ **b** $9x^2 + 30x + 25$
**c** $4a^2 - 4a + 1$ **d** $16x^2 + 24x + 9$
**e** $25y^2 - 10y + 1$ **f** $4p^2 + 28p + 49$
**g** $9y^2 - 30y + 25$ **h** $9 - 18a + 9a^2$

**5** **a** 121 **b** 361 **c** 2601
**d** 1225 **e** 4225 **f** 6724

**6** **a** $a^2 + 6a + 13$ **b** $x^2 - 13x + 25$
**c** $a^2 - 2a + 3$ **d** $x^2 - 9x + 18$

## Exercise 3.13

**1** **a** $x^2 - 9$ **b** $y^2 - 16$ **c** $x^2 - 36$ **d** $d^2 - 4$
**e** $x^2 - 1$ **f** $n^2 - 64$ **g** $k^2 - 121$ **h** $a^2 - 144$

**2** **a** $4x^2 - 1$ **b** $9y^2 - 25$ **c** $25x^2 - 36$
**d** $9d^2 - 25$ **e** $16x^2 - 1$ **f** $36n^2 - 64$
**g** $4k^2 - 49$ **h** $36a^2 - 4$

**3** **a** $1 - x^2$ **b** $9 - m^2$ **c** $25 - 36a^2$
**d** $4d^2 - 9$ **e** $9x^2 - m^2$ **f** $4n^2 - 25k^2$
**g** $k^2 - 49d^2$ **h** $9a^2 - 25c^2$

**4** 9999

**5** **a** 2499 **b** 6399 **c** 899 **d** 9996

## Power plus

**1** **a** $-4p - 9q$ **b** $-6m - 3n - 7mn$
**c** $4j - k + 4$ **d** $5b^2$
**e** $3x^2 - 6x$ **f** $5xy + 2yz + 4xz$
**g** $-4d^2 + 4e^2 + 16$ **h** $13f^2 + 6f + 1$

**2** **a** **i** $4x + 6$ **ii** $x^2 + 3x$
**b** **i** $4p$ **ii** $p^2$
**c** **i** $x + y + 16$ **ii** $\frac{xy}{2}$
**d** **i** $2m + n + 8$ **ii** $\frac{n}{2}(2m+3)$ or $mn + \frac{3m}{2}$
**e** **i** $3x + 2y$ **ii** $x^2 + x$
**f** **i** $2a + 2b$ **ii** $ad + bc - cd$

**3** **a** $x^2 + 2x - 15$ **b** $2am - 3m$
**c** $4y - 9$ **d** $6a - 2b + 14c$
**e** $-5p^2 - 7pq + 10pr$ **f** $4k^2 - 5k - 5$
**g** $27 - 20f + 30g$ **h** $-2x^2 + 7x + 115$
**i** $x^2 + 8x + 15$ **j** $p^2 + 9p - 10$

**4** **a** $-2$ **b** $-24$ **c** 10 **d** 9
**e** 8 **f** 4 **g** 7 **h** 4
**i** $-1$ **j** 3 **k** 1.2 **l** 0

**5** **a** $p(9q + 5 - 2p)$ **b** $11(4k - 3l - m)$
**c** $x(2 - 7x + 3y)$ **d** $(y + 2)(x + 3)$
**e** $(b - 3)(a - 7)$ **f** $(k + 1)(4 - j)$

**6** **a** $m$ **b** $\frac{2n}{5}$ **c** $k$ **d** $\frac{2d}{5}$
**e** $\frac{3f}{2}$ **f** $\frac{7c}{10}$ **g** $\frac{6}{x}$ **h** $\frac{7b+5a}{ab}$

## Test yourself 3

**1** **a** $6p$ **b** $-r$ **c** $4dg$
**d** $5t$ **e** $20m$ **f** $5k$
**g** $6y^2$ **h** $\frac{16}{a}$ **i** $20cd$

**2** **a** $c \times d$ **b** $3 \times y \times y$
**c** $w \div 6$ **d** $c \times c + y$

**3** **a** $y + 4$ **b** $6b - 3$ **c** $\frac{p}{q}$

**4** B

**5** **a** 3 **b** $-10$ **c** 19
**d** 24 **e** $-15$ **f** 29

**6** **a** $12k$ **b** $17d$ **c** $17m$
**d** $18m$ **e** $3ac$ **f** $9cd$
**g** $10x + 10y$ **h** $15r + 15s$ **i** $28h + 7g$

**7** **a** $10x$ **b** $14bd$ **c** $66m$ **d** $24pq$
**e** $32w^2$ **f** $75j$ **g** $36m^2$ **h** $60p^3$


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**8** **a** $25x$ **b** $\frac{4b}{d}$ **c** $11m$ **d** $8rs$
**e** 5 **f** $7d^2$ **g** $7e$ **h** $13b$
**i** $2x$ **j** $4mn^2$ **k** $4e$ **l** $5ae$

**9** **a** $6n + 12$ **b** $-10b - 35$ **c** $9j^2 - 3j$
**d** $2n + 18$ **e** $19v - 15$ **f** $9y - 18$

**10** A

**11** **a** 6 **b** $3p$ **c** 7

**12** **a** $3(m + 3)$ **b** $5(h - 7)$ **c** $3a(2 + 3b)$
**d** $2h(5 - 6m)$ **e** $xy(1 + z)$ **f** $d(c - e)$
**g** $a(a - 6)$ **h** $8j(k + 3j)$

**13** **a** $-2(1 + 2q)$ **b** $-4(f + 4)$ **c** $-3(4 - 5d)$
**d** $-b(a - 1)$ **e** $-8b(2 + 3d)$ **f** $-k(4k + 1)$
**g** $-x(x - 7)$ **h** $-hi(h + i)$

**14** **a** $x^6$ **b** $m^4$ **c** $30k^3$ **d** $-6h^3y^{11}$
**e** $n^4$ **f** $4a$ **g** $q^3$ **h** $3x^3y$
**i** $\frac{14m^2}{3n}$ **j** $w^{12}$ **k** $9a^8$ **l** $32d^{15}e^{10}$
**m** 1 **n** 6 **o** 5 **p** $x$

**15** **a** $m^2 + 11m + 18$ **b** $5h^2 - 2h - 3$
**c** $y^2 + y - 30$ **d** $4a^2 - 20a + 21$

**16** **a** $m^2 + 18m + 81$ **b** $h^2 - 2h + 1$
**c** $4y^2 - 20y + 25$ **d** $9a^2 - 12a + 4$

**17** **a** $m^2 - 81$ **b** $h^2 - 9$ **c** $25y^2 - 4$ **d** $4a^2 - 9c^2$

## Practice set 1

**1** **a** 343 **b** 1225 **c** 256
**d** 20 **e** 14 **f** 1.5

**2** **a** 140 **b** 135 **c** 120
**d** 480 **e** 180 **f** 15

**3** **a** $x = 5$ **b** $y = 41$ **c** $x = 6.4$
**d** $m = 24$ **e** $a = 17.7$ **f** $p = 33.5$

**4** **a** 8 **b** −30 **c** −20 **d** 5
**e** 28 **f** −6 **g** −2 **h** 2
**i** −16 **j** −28 **k** 6 **l** −15

**5** **a** 18 **b** 23 **c** −3
**d** −11 **e** 19
**f** −99 **g** $4\frac{3}{4}$ **h** 8 **i** 2

**6** **a** 13 **b** 9 **c** 10 **d** 10

**7** **a** $2k + 4j$ **b** $4p$ **c** $11p + 5q$ **d** $8x + 6y$

**8** **a** −4.23 **b** 22.39

**9** **a** 7.2 **b** 0.45 **c** 0.42 **d** 18.4

**10** **a** $k + 5$ **b** $20k$ **c** $\frac{k}{2}$
**d** $3k + 10$ **e** $k - 12$ **f** $k^2$

**11** **a** $10p$ **b** 6 **c** $-12x^2y$
**d** $24ab$ **e** 2 **f** $2a$

**12** **a** $180 = 2^2 \times 3^2 \times 5$ **b** $875 = 5^3 \times 7$

**13** **a** 4 **b** 18

**14** **a** $2^7$ **b** $7^{11}$ **c** $5^5$ **d** 8
**e** $3^8$ **f** $11^{15}$ **g** 1 **h** 1

**15** **a** $\sqrt{48}$ cm **b** 6.93 cm

**16** Right-angled.



**17** **a** 36 **b** 54

**18** **a** $4x + 8$ **b** $6m - 24$ **c** $-14k - 7$
**d** $-3p - q$ **e** $6m + 4$ **f** $4p$

**19** **a** $2(m + 2)$ **b** $d(d - 1)$ **c** $6(p + 2q)$
**d** $-12(x + 2)$ **e** $a(2b + 3)$ **f** $-2x(x - 3)$

**20** 12.37 m

# Chapter 4

## SkillCheck

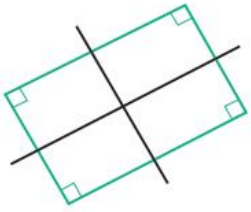
**1** Teacher to check.

**2** **a** acute **b** obtuse **c** reflex
**d** obtuse **e** right **f** revolution
**g** reflex **h** straight

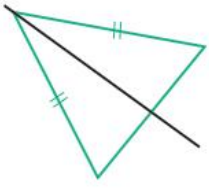
**3** **a** vertically opposite **b** right
**c** reflex

**4** **a** $x = 60$ **b** $x = 123$ **c** $x = 162$

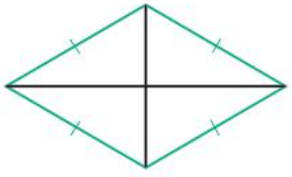
**5** **a** rectangle



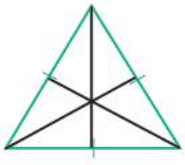
**b** isosceles triangle



**c** rhombus



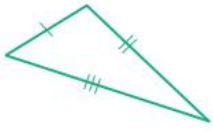
**d** equilateral triangle



**6** **a** 2 **b** 1 **c** 2 **d** 3

**7** D

**8**



**9** **a**



**b** No

## Exercise 4.01

**1** **a** complementary **b** reflex
**c** supplementary **d** obtuse
**e** straight **f** vertically opposite
**g** acute **h** adjacent
**i** angles at a point

**2** **a** 74° **b** 57° **c** 19° **d** 88°

**3** **a** 155° **b** 31° **c** 73° **d** 132°

**4** D **5** C

**6** **a** $x = 70$ (angles in a right angle)
**b** $a = 112$ (angles on a straight line)
**c** $a = 110$ (vertically opposite angles), $b = 70$ (angles on a straight line)
**d** $y = 285$ (angles at a point)
**e** $a = 19$ (angles on a straight line)
**f** $x = 53$ (adjacent angles)
**g** $a = 52$ (angles on a straight line), $b = 128$ (vertically opposite angles), $c = 52$ (angles on a straight line)
**h** $x = 165$ (angles at a point)
**i** $x = 270$ (angles at a point)
**j** $a = 45$ (vertically opposite angles), $b = 135$ (angles on a straight line), $c = 135$ (vertically opposite angles)
**k** $k = 58$ (angles in a right angle)
**l** $n = 70$ (angles on a straight line)
**m** $w = 156$ (angles at a point)
**n** $q = 62$ (angles on a straight line)
**o** $y = 40$ (angles at a point)
**p** $p = 45$ (equal angles in a right angle)
**q** $p = 62$ (angles on a straight line)
**r** $a = 23$ (adjacent angles)

**7** **a** $m = 235$ (angles at a point)
**b** $m = 270$ (angles at a point)
**c** $a = 50$ (angles on a straight line)
**d** $x = 170$ (angles at a point)
**e** $a = 80$ (angles on a straight line), $b = 65$ (vertically opposite angles), $c = 115$ (angles on a straight line)
**f** $a = 45$ (angles on a straight line)
**g** $p = 90$ (angles on a straight line)
**h** $m = 30$ (angles on a straight line)

**8** C

## Exercise 4.02

**1** **a** alternate **b** co-interior
**c** corresponding **d** co-interior
**e** alternate **f** co-interior

**2** C

**3** **a** $e°$ **b** C **c** $e°$ or A or $g°$
**d** $f°$ **e** D or $h°$ or $f°$ or B

**4** **a** $a = 70$ (alternate angles on parallel lines)
**b** $d = 45$ (co-interior angles on parallel lines)
**c** $a = 61$ (alternate angles on parallel lines)
**d** $m = 120$ (corresponding angles on parallel lines)
**e** $p = 89$ (co-interior angles on parallel lines)
**f** $m = 112$ (co-interior angles on parallel lines)

**5** (Other reasons possible)
**a** $a = 61$ (alternate angles on parallel lines), $b = 119$ (angles on a straight line)
**b** $c = 70$ (co-interior angles on parallel lines), $d = 70$ (vertically opposite angles)
**c** $a = 65$ (co-interior angles on parallel lines, $b = 115$ (alternate angles on parallel lines
**d** $g = 50$ (alternate angles on parallel lines), $h = 60$ (alternate angles on parallel lines), $m = 70$ (angles on a straight line)
**e** $p = 100$ (angles on a straight line), $q = 80$ (alternate angles on parallel lines)
**f** $y = 89$ (vertically opposite angles and corresponding angles on parallel lines)
**g** $w = 108$ (co-interior angles on parallel lines), $x = 108$ (angles on a straight line), $y = 108$ (corresponding angles on parallel lines)
**h** $a = 100$ (alternate angles on parallel lines [by extending the middle parallel line])
**i** $n = 250$ (co-interior angles on parallel lines)

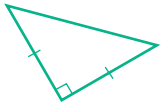
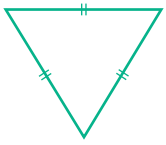
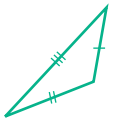
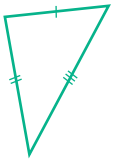
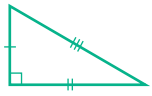
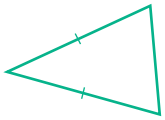
**6** D

**7** **a** Yes (alternate angles are equal)
**b** Yes (corresponding angles are equal)
**c** No (co-interior angles do not add to 180°)
**d** Yes (co-interior angles add to 180°)

**8** **a** 255 **b** 52

## Exercise 4.03

**1** **a** scalene, obtuse-angled
**b** equilateral, acute-angled
**c** scalene, acute-angled
**d** isosceles, right-angled
**e** isosceles, obtuse-angled
**f** scalene, obtuse-angled
**g** scalene, right-angled
**h** isosceles, acute-angled
**i** isosceles, right-angled
**j** isosceles, obtuse-angled
**k** equilateral, acute-angled
**l** scalene, right-angled

**2** **a**  **b** 
**c**  **d** 
**e**  **f** 

**3** A


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**4** No, all angles are 60° in equilateral triangles so they are not obtuse.

**5** **a** b, d, e, h, i, j, k **b** b, k

**6** **a** $x = 42$ (angles in an isosceles triangle)
**b** $y = 7$ (angles in an isosceles triangle)
**c** $x = 60$ (angles in an equilateral triangle)
**d** $l = 4.8$ (angles in an isosceles triangle)
**e** $a = 17.2$, $b = 17.2$ (angles in an equilateral triangle)
**f** $r = 15$ (angles in an isosceles triangle)

**7** No, the sum of all angles cannot be greater than 180°.

**8** C

**9**

| | | |
|---|---|---|
| **Equilateral triangle** | 3 | 3 |
| **Isosceles triangle** | 1 | No rotational symmetry |
| **Scalene triangle** | 0 | No rotational symmetry |

## Mental skills 4A

**2** **a** 70% **b** 66% **c** 45% **d** 88%
**e** 75% **f** 75% **g** 80% **h** 80%
**i** 55% **j** 35% **k** 30% **l** 80%
**m** 90% **n** 45% **o** 52%

**4** **a** 25% **b** 68% **c** 17% **d** 60%
**e** 10% **f** 33.3% **g** 59% **h** 70.2%
**i** 84% **j** 70% **k** 42.8% **l** 5.5%
**m** 91% **n** 78.25% **o** 31.4%

## Exercise 4.04

**1** **a** convex **b** convex **c** non-convex
**d** non-convex **e** convex **f** convex

**2** **a** square **b** trapezium **c** kite
**d** parallelogram

**3** **a** square, rectangle **b** trapezium
**c** square, rhombus
**d** parallelogram, rectangle, square, rhombus
**e** parallelogram, rectangle, square, rhombus
**f** kite, square, rhombus

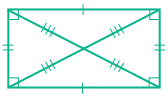
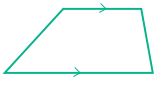
**4** rhombus

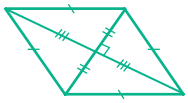
**5**

| Quadrilateral | Number of axes of symmetry | Order of rotational symmetry |
|---|---|---|
| Rectangle | 2 | 2 |
| Parallelogram | 0 | 2 |
| Trapezium | 0 | No rotational symmetry |
| Rhombus | 2 | 2 |
| Square | 4 | 4 |
| Kite | 1 | No rotational symmetry |

**6** **a** trapezium **b** rhombus

**7** B

**8** **a**  **b** 

**c**  **d** 

**9** **a** False **b** True **c** True **d** True
**e** False **f** True **g** False

## Exercise 4.05

**1** **a** parallel
**b** equal; angles; right angles
**c** opposite; sides; equal; bisect
**d** sides; opposite; parallel; opposite; equal; right; bisect
**e** equal; parallel; right angles; equal; bisect
**f** equal; right angles; sides; equal; right angles

**2** **a** Parallelogram, rectangle, rhombus, square
**b** Square, rhombus, kite
**c** Parallelogram, rectangle, rhombus, square
**d** Trapezium
**e** Parallelogram, rectangle, rhombus, square
**f** Parallelogram, rectangle, rhombus, square
**g** Parallelogram, rectangle, trapezium, kite
**h** Rectangle, square
**i** Rectangle, square
**j** Rhombus, square

**3** **a** $a + 70 = 180$ (co-interior angles on parallel lines); $a = 110$, $b = 70$ (opposite angles of a parallelogram)

**4** **a** $m = 123$ (opposite angles of a parallelogram)
**b** $a = 154$ (co-interior angles on parallel lines), $b = 26$ (opposite angles of a rhombus or co-interior angles on parallel lines)
**c** $x = 156$ (co-interior angles on parallel lines), $y = 156$ (opposite angles of a parallelogram and vertically opposite angles, or co-interior angles on parallel lines and vertically opposite angle)
**d** $l = 45$ (diagonals bisect the angles of a square)
**e** $a = 49$ (angle sum of a triangle), $b = 90$ (diagonals of a rhombus bisect each other at right angles)
**f** $x = 87$ (corresponding angles on parallel lines), $y = 93$ (co-interior angles on parallel lines)
**g** $m = 75$ (co-interior angles on parallel lines), $n = 75$ (equal angles of an isosceles trapezium, or by symmetry)
**h** $a = 147$ (co-interior angles on parallel lines), $b = 33$ (opposite angles of a parallelogram or co-interior angles on parallel lines)
**i** $a = 5$, $b = 7$ (opposite sides of a parallelogram)
**j** $a = 4$, $c = 7$ (diagonals of a parallelogram bisect each other)
**k** $a = 70$, $b = 92$ (co-interior angles on parallel lines)

**5** B **6** D **7** A **8** A

**9** **a** No, teacher to check **b** No, teacher to check
**c** Yes, teacher to check

ANSWERS

## Exercise 4.06

**1** **a** $a = 45$ **b** $b = 60$ **c** $c = 50$
**d** $d = 60$ **e** $e = 18$ **f** $f = 33$
**g** $g = 120$ **h** $h = 80$ **i** $i = 45$

**2** **a** $a = 127$ **b** $b = 140$ **c** $c = 60$
**d** $d = 28$ **e** $e = 100$ **f** $f = 120$
**g** $g = 50$ **h** $h = 136$ **i** $i = 120$

**3** **a** $a = 89$ **b** $b = 62$ **c** $c = 109$
**d** $d = 90$ **e** $e = 33$ **f** $f = 245$
**g** $g = 90$ **h** $h = 90$ **i** $i = 140$

**4** C **5** A **6** A **7** C

**8** $x = 68$ (angle sum of an isosceles triangle),
$y = 34$ (exterior angle of an isosceles triangle)

## Exercise 4.07

**1** **a** hexagon, irregular, convex
**b** quadrilateral, irregular, convex
**c** pentagon, irregular, non-convex
**d** triangle, irregular, convex
**e** decagon, regular, convex
**f** heptagon, irregular, convex
**g** octagon, regular, convex
**h** nonagon, irregular, non-convex

**2**

| Polygon | Number of sides | Sum of angles inside polygon |
|---|---|---|
| hexagon | 6 | 720° |
| heptagon | 7 | 900° |
| octagon | 8 | 1080° |
| nonagon | 9 | 1260° |
| decagon | 10 | 1440° |

**3** **a** 2340° **b** 3240° **c** 4140° **d** 17 640°

**4** **a** $a = 95$ **b** $b = 110$ **c** $d = 108$ **d** $c = 160$
**e** $g = 140$ **f** $x = 90$ **g** $e = 135$

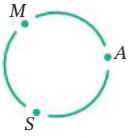
**5** **a** 90° **b** 60° **c** 120° **d** 135°
**e** 144° **f** 108° **g** 150°

**6** **a** 14 **b** 34 **c** 26 **d** 53 **e** 125

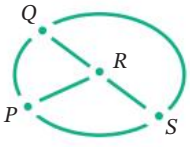
**7** **a** Yes, 10 **b** No

## Exercise 4.08

**1**




**2**




**3** **a** **i**

| Vertex | G | H | I | J |
|---|---|---|---|---|
| Degree | 3 | 4 | 2 | 3 |

**ii** even $H$, $I$; odd $G$, $J$

**iii** Sum of degrees = 12, number of edges = 6.

**b** **i**

| Vertex | A | B | C | D | E |
|---|---|---|---|---|---|
| Degree | 3 | 4 | 2 | 3 | 4 |

**ii** even $B$, $C$, $E$; odd $A$, $D$

**iii** Sum of degrees = 16, number of edges = 8.

**4** Teacher to check.

**5** **a** $v = 5, e = 9, f = 6, 5 - 9 + 6 = 2$
**b** $v = 5, e = 6, f = 3, 5 - 6 + 3 = 2$
**c** $v = 7, e = 11, f = 6, 7 - 11 + 6 = 2$
**d** $v = 5, e = 7, f = 4, 5 - 7 + 4 = 2$
**e** $v = 7, e = 9, f = 4, 7 - 9 + 4 = 2$
**f** $v = 6, e = 8, f = 4, 6 - 8 + 4 = 2$
**g** $v = 4, e = 6, f = 4, 4 - 6 + 4 = 2$

## Mental skills 4B

**2** **a** $\frac{3}{4}$ **b** $\frac{7}{25}$ **c** $\frac{3}{10}$ **d** $\frac{7}{50}$ **e** $\frac{3}{50}$
**f** $\frac{17}{20}$ **g** $\frac{8}{25}$ **h** $\frac{49}{100}$ **i** $\frac{14}{25}$ **j** $\frac{9}{10}$
**k** $\frac{18}{25}$ **l** $\frac{13}{20}$ **m** $\frac{1}{5}$ **n** $\frac{6}{25}$ **o** $\frac{53}{100}$

**4** **a** $\frac{19}{25}$ **b** $\frac{1}{10}$ **c** $\frac{4}{5}$ **d** $\frac{9}{20}$ **e** $\frac{22}{25}$
**f** $\frac{14}{25}$ **g** $\frac{3}{4}$ **h** $\frac{31}{100}$ **i** $\frac{17}{25}$ **j** $\frac{1}{20}$
**k** $\frac{3}{5}$ **l** $\frac{27}{50}$ **m** $\frac{3}{50}$ **n** $\frac{49}{100}$ **o** $\frac{41}{50}$

## Power plus

**1** **a** $x = 121$ **b** $x = 88$ **c** $x = 96$
**d** $x = 110$ **e** $x = 6$ **f** $x = 34$
**g** $x = 67$ **h** $x = 155$ **i** $x = 117$
**j** $x = 108$ **k** $x = 30$ **l** $x = 140$

## Test yourself 4

**1** **a** obtuse **b** acute **c** reflex

**2** **a** $x = 50$ (angles on a straight line)
**b** $a = 50$ (angles on a straight line), $b = 130$ (vertically opposite angles or angles on a straight line)
**c** $y = 5$ (angles in a right angle)
**d** $n = 72$ (vertically opposite angles)
**e** $w = 130$ (angles at a point)

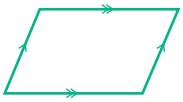
**3** **a** corresponding **b** alternate
**c** co-interior

**4** **a** $m = 100$ (alternate angles on parallel lines)
**b** $x = 95$ (co-interior angles on parallel lines)
**c** $x = 80$ (straight angle on parallel lines); $y = 80$ (corresponding angles on parallel lines)
**d** $m = 88$ (alternate angles on parallel lines); $p = 88$ (corresponding angles on parallel lines)


9780170479516

**5** **a** scalene, right-angled **b** scalene, obtuse-angled
**c** isosceles, obtuse-angled
**d** equilateral, acute-angled
**e** isosceles, acute-angled
**f** isosceles, acute-angled
**g** isosceles, right-angled
**h** scalene, obtuse-angled
**i** isosceles, obtuse-angled

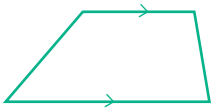
**6** **a**



**b**



**c**



**7** **a** rhombus, square **b** kite
**c** rectangle, square
**d** parallelogram, rhombus, rectangle, square
**e** parallelogram, rhombus, rectangle, square

**8** **a** $a = 50$ **b** $a = 42$ **c** $y = 30$
**d** $y = 53$ **e** $x = 34$ **f** $m = 106$

**9** **a** $m = 63$ **b** $x = 135$ **c** $y = 123$
**d** $d = 70$ **e** $a = b = 115$ **f** $a = 56, b = 68$
**g** $a = 48, b = 66$

**10** **a** $x = 42$ **b** $m = 66$ **c** $y = 130$

**11** A

**12** **a** 1260° **b** 154° **c** 20

**13** **a** 5 **b** 8 **c** 3 **d** 4 **e** 4

# Chapter 5

## SkillCheck

**1** **a** 200 **b** 3.5 **c** 2.5
**d** 4.6 **e** 4000 **f** 5.2
**g** 0.6 **h** 8.2 **i** 400

**2** **a** 12 cm **b** 19.2 m **c** 86 m
**d** 112 mm **e** 21 cm **f** 13 cm

**3** **a** 4 $m^2$ **b** 30 $cm^2$ **c** 56 $m^2$
**d** 91.84 $cm^2$ **e** 58 $mm^2$ **f** 140 $m^2$

## Exercise 5.01

**1** **a** 84 mm **b** 33.2 cm **c** 69.3 mm
**d** 158 cm **e** 43.3 m **f** 12.4 km

**2** **a** 31.5 m **b** 34 m **c** 45 cm
**d** 60 km **e** 66 cm **f** 64 m
**g** 164 mm **h** 58 cm **i** 92 m

**3** **a** 12 cm **b** 34 m
**c** 15 cm **d** 22 m

**4** C **5** D

**6** **a** 56 mm **b** 84 mm
**c** 92 mm **d** 80 mm

**7** A **8** 7 m **9** 4.5 m

**10** 4 cm **11** 16 cm

**12** 18 cm **13** 36 m

## Exercise 5.02

**1** Teacher to check.

**2** **a** diameter **b** segment **c** sector
**d** arc **e** radius **f** chord
**g** circumference **h** tangent

**3** **a** radius **b** quadrant **c** tangent
**d** diameter **e** chord **f** arc
**g** sector **h** circumference **i** segment

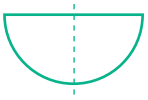
**4** B **5** B

**6** Teacher to check.

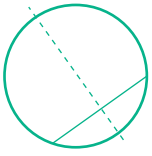
**7** **a** **i** 1 **ii** no rotational symmetry



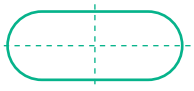
**b** **i** 1 **ii** no rotational symmetry



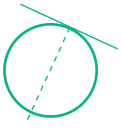
**c** **i** 1 **ii** no rotational symmetry



**d** **i** 2 **ii** 2



**e** **i** 1 **ii** no rotational symmetry



**f** **i** 1 **ii** no rotational symmetry



**8** Teacher to check.

## Exercise 5.03

**1** **a** 12 cm **b** 24 m **c** 9 cm **d** 18 cm

**2** **a** 12.57 cm **b** 25.13 m
**c** 9.42 cm **d** 18.85 cm

**3** **a** $4\pi$ cm **b** $8\pi$ m **c** $3\pi$ cm **d** $6\pi$ cm

**4** **a** 36 m **b** 60 mm **c** 42 cm **d** 12 mm

**5** **a** 37.7 m **b** 62.8 mm **c** 44.0 cm **d** 12.6 mm

**6** **a** $12\pi$ m **b** $20\pi$ mm **c** $14\pi$ cm **d** $4\pi$ mm

**7** 4.40 m
**8** **a** 188.5 cm **b** 477
**9** 40 023.9 km **10** 88.0 mm **11** 236 mm
**12** **a** 6.28 cm **b** 21.99 cm
**c** 18.85 cm **d** 31.42 cm
**13** C **14** D
**15** **a** 210.5 m **b** $(154 + 18\pi)$ m
**16** 16 cm **17** Ali, 25 m

## Exercise 5.04

**1** **a** $m^2$ **b** $cm^2$ **c** $m^2$
**d** $km^2$ **e** $cm^2$ **f** $cm^2$
**2** A
**3** **a** 1 **b** 7.5 **c** 0.45
**d** 0.034 **e** 0.06 **f** 43.2
**g** 0.000 95 **h** 0.76 **i** 0.048
**j** 300 000 **k** 85 000 **l** 6 900 000
**m** 5400 **n** 7 000 000 **o** 4800
**p** 450 000
**4** D **5** 0.0025 $km^2$
**6** 3 002 800 ha **7** 1348.05 $km^2$
**8** **a** 0.875 $km^2$ **b** 87.5 ha
**9** **a** 0.0076 **b** 70 000 000 **c** 8 000 000
**d** 8.5 **e** 69 000 **f** 152 000
**g** 0.85 **h** 49
**10** 32.9%
**11** **a** 8.81 $km^2$ **b** 6.2

## Exercise 5.05

**1** **a** 24 $m^2$ **b** 36 $cm^2$ **c** 48 $cm^2$
**d** 3 $cm^2$ **e** 53.3 $mm^2$ **f** 37.21 $m^2$
**g** 54.4 $cm^2$ **h** 85.56 $km^2$ **i** 340 $mm^2$
**j** 6 $m^2$ **k** 84 $cm^2$ **l** 136 $m^2$
**2** **a** 10 800 $cm^2$ **b** 42 140 $mm^2$ **c** 547 200 $m^2$
**d** 552 $mm^2$ **e** 243.6 $mm^2$ **f** 528 000 $m^2$
**3** D **4** C **5** 5 cm **6** 3 m
**7** 7 cm **8** Teacher to check. **9** 60 m
**10** **a** 15.6 $cm^2$ **b** 66.4 $mm^2$ **c** 8.4 $m^2$ **d** 30 $km^2$

## Mental skills 5A

**2** **a** 0.7 **b** 0.6 **c** 0.07
**d** 0.45 **e** 0.98 **f** 0.4
**4** **a** 0.61 **b** 0.04 **c** 0.9
**d** 0.38 **e** 0.271 **f** 0.007

## Exercise 5.06

**1** A
**2** **a** 150 $m^2$ **b** 51 $m^2$ **c** 88 $mm^2$
**d** 156 $m^2$ **e** 950 $mm^2$ **f** 36 $cm^2$
**g** 189 $cm^2$ **h** 154 $cm^2$ **i** 71 $m^2$
**j** 201.19 $km^2$ **k** 88 000 $cm^2$ **l** 280 $mm^2$
**m** 20 $m^2$
**3** C **4** B **5** 1874 $cm^2$

## Exercise 5.07

**1** **a** 28.3 $cm^2$ **b** 452.4 $mm^2$ **c** 0.2 $m^2$
**d** 19.6 $cm^2$ **e** 3421.2 $mm^2$
**2** **a** $9\pi$ $cm^2$ **b** $144\pi$ $mm^2$ **c** $0.0625\pi$ $m^2$
**d** $6.25\pi$ $cm^2$ **e** $1089\pi$ $mm^2$
**3** **a** 13.85 $cm^2$ **b** 38.48 $cm^2$ **c** 78.54 $cm^2$
**4** **a** $4.41\pi$ $cm^2$ **b** $12.25\pi$ $cm^2$ **c** $25\pi$ $cm^2$
**5** 539 $m^2$ **6** 3690.67 $cm^2$ **7** $196\pi$ $cm^2$
**8** D **9** 6.2 $m^2$ **10** 12.03 cm
**11** A **12** A
**13** **a** 160.97 $cm^2$ **b** 221.54 $m^2$ **c** 85.81 $mm^2$
**d** 46.27 $cm^2$ **e** 21.46 $cm^2$ **f** 16.08 $cm^2$
**14** **a** **i** 153.9 $m^2$ **ii** 95.0 $m^2$ **iii** 58.9 $m^2$
**b** **i** 314.2 $m^2$ **ii** 254.5 $m^2$ **iii** 59.7 $m^2$
**15** **a** 9168 $m^2$ **b** 7326 $m^2$
**c** 1842 $m^2$ **d** \$44 668.50

## Exercise 5.08

**1** **a** $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3}$ **d** $\frac{13}{18}$
**2** **a** 50.0 mm **b** 37.5 cm **c** 137.1 cm
**d** 12.2 m
**3** D
**4** **a** 153.94 $mm^2$ **b** 87.27 $cm^2$
**c** 1073.24 $cm^2$ **d** 6.87 $m^2$
**5** **a** $P = 14.3$ cm, $A = 12.6$ $cm^2$
**b** $P = 128.5$ mm, $A = 981.7$ $mm^2$
**c** $P = 9.3$ cm, $A = 5.3$ $cm^2$
**d** $P = 18.0$ cm, $A = 19.2$ $cm^2$
**e** $P = 61.4$ mm, $A = 235.6$ $mm^2$
**f** $P = 2.0$ m, $A = 0.2$ $m^2$
**g** $P = 106.7$ mm, $A = 641.4$ $mm^2$
**h** $P = 82.6$ m, $A = 351.9$ $m^2$
**i** $P = 177.7$ mm, $A = 1954.8$ $mm^2$
**6** D
**7** **a** 6° **b** 10.26 $cm^2$
**8** 9.62 $cm^2$

## Exercise 5.09

**1** **a** 148 $mm^2$ **b** 1080 $m^2$ **c** 97.6 $m^2$
**d** 33.02 $mm^2$ **e** 472 $cm^2$ **f** 30 $cm^2$
**g** 862.5 $mm^2$ **h** 162 $cm^2$ **i** 680 $cm^2$
**2** **a** 8200 $m^2$ **b** 6560 $m^2$
**3** 630 000 $mm^2$ **4** D
**5** **a** 546 $cm^2$ **b** 1772 $cm^2$
**6** D **7** 4 m

## Exercise 5.10

**1** **a** 1.5 $cm^2$ **b** 850 $mm^2$ **c** 8.16 $m^2$ **d** 210 $cm^2$
**2** **a** 240 $m^2$ **b** 80 $cm^2$ **c** 315 $mm^2$
**d** 1344 $m^2$ **e** 1.8 $m^2$
**3** B
**4** **a** 300 $mm^2$ **b** 172.5 $mm^2$ **c** 90 $mm^2$
**d** 360 $mm^2$ **e** 120 $mm^2$ **f** 72 $mm^2$
**g** 66.5 $mm^2$ **h** 510 $mm^2$ **i** 112 $mm^2$

**5** **a** 10 cm and 6 cm **b** 30 $cm^2$
**c** 90 $cm^2$

## Mental skills 5B

**2** **a** $\frac{3}{5}$ **b** $\frac{1}{4}$ **c** 0.105
**d** 3.68 **e** $\frac{4}{10}$ **f** $\frac{5}{6}$
**g** 2.817 **h** 0.503 **i** $\frac{1}{5}$

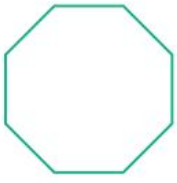
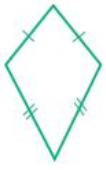
**3** **a** 0.082, 0.81, 0.821 **b** 3.5, 3.51, 3.513, 3.55
**c** $\frac{1}{6}, \frac{2}{5}, \frac{2}{3}$ **d** 0.007, 0.07, 0.7, 0.707
**e** $\frac{2}{10}, \frac{3}{8}, \frac{1}{2}$ **f** $\frac{1}{4}, \frac{4}{10}, \frac{3}{5}$

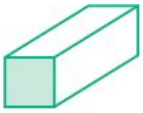
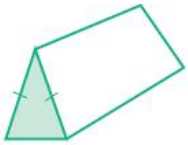
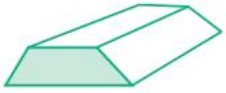
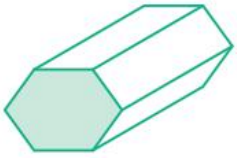
**5** **a** 0.25, $\frac{1}{5}$, $\frac{1}{6}$, 16% **b** 0.4, $\frac{1}{3}$, 0.28, 27%
**c** 50%, $\frac{1}{8}$, 6%, 0.05 **d** $\frac{3}{4}$, 0.639, 55%, $\frac{2}{5}$
**e** 69%, $\frac{2}{3}$, 0.609, 0.6 **f** 22.5%, $\frac{2}{9}$, 17%, 0.105

## Exercise 5.11

**1** **a** $m^3$ **b** $cm^3$ **c** $cm^3$
**d** $m^3$ **e** $cm^3$ **f** $m^3$
**2** C
**3** **a** 5 000 000 **b** 1 600 000 **c** 2000
**d** 4 000 000 000 **e** 0.006 **f** 8 200 000
**g** 7 000 000 **h** 0.0096 **i** 4
**j** 0.16 **k** 0.25 **l** 12 000
**m** 180 000 **n** 0.2
**4** 0.272 $m^3$ **5** D **6** A
**7** **a** E **b** G **c** F
**d** C **e** A **f** B
**g** D
**8** 2 460 000 $mm^3$

## Exercise 5.12

**1** **a** Yes **b** No **c** No
**d** Yes **e** No **f** Yes
**2** **a** **i**  **ii** octagonal prism
**b** **i**  **ii** kite prism
**c** **i**  **ii** pentagonal prism

**3** **a**  **b** 
**c** 
**d** 

**4** **a** 210.8 $m^3$ **b** 4.096 875 $m^3$ **c** 486.42 $cm^3$
**5** **a** 135 $cm^3$ **b** 64 $m^3$ **c** 6120 $mm^3$
**d** 1.728 $m^3$ **e** 21 952 $mm^3$ **f** 258.06 $cm^3$
**6** **a** 90 $cm^3$ **b** 15 400 $mm^3$ **c** 7.8125 $m^3$
**7** B
**8** **a** 22 500 $mm^3$ **b** 928 $cm^3$
**c** 13 200 $cm^3$ **d** 288 $cm^3$
**9** **a** 0.306 $m^3$ **b** 0.306 $m^3$ Teacher to check.
**10** A **11** 5 m **12** 7 m
**13** **a** 220 $cm^3$ **b** 11 220 $mm^3$ **c** 24 000 $m^3$
**d** 29 450 $mm^3$ **e** 82 $m^3$
**14** B **15** Teacher to check.

## Exercise 5.13

**1** **a** 153.93 $m^2$ **b** 1847.26 $m^3$
**2** C
**3** **a** 1133.0 $cm^3$ **b** 17 671.5 $mm^3$ **c** 1.8 $m^3$
**d** 1606.0 $cm^3$ **e** 17 583.1 $cm^3$
**f** 301 592.9 $mm^3$ **g** 499.6 $m^3$
**4** **a** 4.25 cm **b** 4.75 cm **c** 678.58 $cm^3$
**5** B **6** 8 cm
**7** **a** 4021.2 $m^3$ **b** 732 944.7 $m^3$
**c** 1005.3 $mm^3$ **d** 786.1 $m^3$
**e** 380.8 $m^3$ **f** 3 685 088.2 $mm^3$
**8** 4 m

## Exercise 5.14

**1** **a** mL **b** ML **c** L
**d** mL **e** L **f** kL
**2** **a** 5 **b** 3400 **c** 1600
**d** 4 **e** 2.98 **f** 7100000
**g** 0.875 **h** 8200 **i** 800
**j** 1.85 **k** 5400 **l** 900
**m** 6000 **n** 3.5 **o** 1 200 000
**3** A **4** A
**5** **a** 121 500 $cm^3$ **b** 121.5 L
**6** 424 mL **7** 1 800 000 L
**8** **a** 70 200 000 $cm^3$ **b** 70 200 L
**9** **a** 23.4 L **b** 10

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**10** Teacher to check.
**11** 1728 L
**12** **a** 1.32 $m^3$
**b** 1320 L
**13** **a** 18.850 $m^3$
**b** 18 849.6 L
**14** **a** 0.72 L
**b** 17.28 L
**15** Teacher to check.

## Power plus

**1** **a** 35.4 cm **b** 1228.3 m **c** 57.1 cm
**2** **a** 7.9 $m^2$ **b** 14.4 m
**3** **a** 2.09 $cm^2$ **b** Teacher to check
**c** 1.73 $cm^2$ **d** 0.36 $cm^2$ **e** 2.81 $cm^2$
**4** **a** 4.4 $m^2$ **b** 47.1 $cm^2$ **c** 39.3 $m^2$

## Test yourself 5

**1** C
**2** **a** 76 mm **b** 256 cm **c** 31.4 km
**3** **a** segment **b** radius **c** arc
**d** centre **e** diameter **f** chord
**g** sector **h** tangent
**4** **a** **i** 1 **ii** no rotational symmetry
**b** **i** 2 **ii** 2
**c** **i** 1 **ii** no rotational symmetry
**d** **i** 1 **ii** no rotational symmetry
**5** **a** 12.6 cm **b** 62.8 mm **c** 44.0 cm **d** 88.0 cm
**6** **a** **i** $4\pi$ $cm^2$ **ii** 12.6 $cm^2$
**b** **i** $100\pi$ $mm^2$ **ii** 314.2 $mm^2$
**c** **i** $49\pi$ $cm^2$ **ii** 153.9 $cm^2$
**d** **i** $196\pi$ $mm^2$ **ii** 615.8 $mm^2$
**7** **a** 2 **b** 70 000 **c** 0.8
**d** 30 000 000 **e** 7 **f** 7.7
**g** 0.0007 **h** 72 000 **i** 0.058
**8** B
**9** **a** 56.7 $m^2$ **b** 51.6 $mm^2$ **c** 345 $cm^2$
**d** 49 $km^2$ **e** 69.3 $m^2$ **f** 775.5 $cm^2$
**10** **a** 450 $mm^2$ **b** 133 $cm^2$
**c** 6.3 $m^2$ **d** 86 $cm^2$
**11** **a** 20.38 cm **b** 40.85 cm
**c** 26.85 cm **d** 56.55 cm
**12** **a** 25.13 $cm^2$ **b** 116.55 $cm^2$
**c** 37.70 $cm^2$ **d** 208.23 $cm^2$
**13** **a** 288.51 $mm^2$ **b** 17.16 $m^2$ **c** 4126.5 $cm^2$
**14** **a** 189 $m^2$ **b** 1560 $mm^2$ **c** 418 $cm^2$
**15** **a** 8 200 000 **b** 3400 **c** 2
**d** 1000000000 **e** 8 000000000 **f** 45
**16** **a** 391.392 $cm^3$ **b** 2200 $mm^3$ **c** 672 $m^3$
**d** 35 200 $mm^3$ **e** 25 350 $cm^3$
**17** **a** 452.39 $mm^3$ **b** 543.43 $m^3$ **c** 154 893.81 $cm^3$
**18** **a** 3000 **b** 2.5 **c** 6500
**d** 0.0072 **e** 0.12 **f** 0.035
**19** **a** 5.7 L **b** 4000 L **c** 2.5 L
**20** **a** 8.75 $m^2$ **b** 35 $m^3$ **c** 35 000 L

# Chapter 6

## SkillCheck

**1** **a** 3 **b** 8 **c** 20 **d** 4
**e** 6 **f** 3 **g** 40 **h** 15
**2** **a** $\frac{17}{100}$ **b** $\frac{3}{100}$ **c** $\frac{51}{100}$
**d** $\frac{63}{100}$ **e** $\frac{89}{100}$
**3** **a** 70% **b** 5% **c** 61%
**d** 29% **e** 88% **f** 93%
**4** **a** 13% **b** 73% **c** 8%
**5** **a** 87% **b** 27% **c** 92%
**6** 65%
**7** 1, 2, 4, 5, 10, 20, 25, 50, 100
**8** **a** \$40 **b** 6 kg **c** \$5
**d** 10 kg **e** \$3 **f** 2 kg

## Exercise 6.01

**1** **a** $1\frac{1}{2}$ **b** $3\frac{2}{3}$ **c** $2\frac{1}{4}$
**d** $2\frac{2}{5}$ **e** 5 **f** $4\frac{3}{11}$
**2** **a** $\frac{7}{2}$ **b** $\frac{13}{3}$ **c** $\frac{23}{4}$ **d** $\frac{23}{3}$
**e** $\frac{39}{5}$ **f** $\frac{65}{9}$ **g** $\frac{71}{7}$ **h** $\frac{93}{10}$
**3** Teacher to check.
**4** Mixed numeral, because it is greater than 1, while a proper fraction is less than 1.
**5** **a** $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{3}{8}$ **d** $2\frac{7}{10}$
**e** $\frac{11}{15}$ **f** $\frac{5}{12}$
**6** **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{6}{13}$ **d** $\frac{3}{4}$
**e** $\frac{3}{5}$ **f** $\frac{4}{7}$ **g** $\frac{2}{3}$ **h** $\frac{3}{5}$
**i** $\frac{4}{7}$ **j** $\frac{4}{7}$ **k** $\frac{3}{5}$ **l** $\frac{9}{10}$
**7** A
**8** **a** $1\frac{1}{2}$ **b** 3 **c** $4\frac{1}{2}$ **d** $1\frac{4}{5}$

## Exercise 6.02

**1** **a** $\frac{4}{5}$ **b** $\frac{1}{2}$ **c** $\frac{5}{8}$ **d** $1\frac{3}{8}$
**2** **a** $\frac{7}{10}$ **b** $\frac{5}{21}$ **c** $\frac{17}{20}$ **d** $\frac{1}{4}$
**e** $\frac{13}{24}$ **f** $\frac{1}{30}$ **g** $1\frac{5}{18}$ **h** $\frac{11}{24}$
**i** $1\frac{1}{8}$ **j** $\frac{1}{7}$ **k** $3\frac{5}{9}$ **l** $2\frac{3}{5}$
**3** $\frac{17}{40}$ **4** D
**5** **a** $\frac{5}{12}$ **b** $\frac{17}{30}$

**6** $\frac{1}{2}$ **7** C **8** A

**9** **a** $1\frac{9}{10}$ **b** $3\frac{5}{8}$ **c** $16\frac{17}{24}$ **d** $5\frac{1}{45}$

**e** $1\frac{21}{40}$ **f** $1\frac{19}{20}$ **g** $\frac{5}{6}$ **h** $1\frac{7}{10}$

**10** **a** $2, 2\frac{1}{2}, 3\frac{1}{2}, 1$ **b** $1, \frac{5}{6}, 1\frac{1}{3}, 0, \frac{1}{2}$

## Exercise 6.03

**1** **a** $\frac{1}{3}$ **b** $\frac{10}{21}$ **c** $\frac{1}{64}$ **d** $\frac{3}{4}$

**e** $\frac{9}{20}$ **f** $4\frac{1}{2}$ **g** $12\frac{7}{20}$ **h** $3\frac{1}{9}$

**2** D

**3** **a** $\frac{3}{14}$ **b** $\frac{8}{15}$ **c** $\frac{2}{3}$ **d** $1\frac{1}{2}$

**e** 16 **f** 9 **g** $\frac{1}{8}$ **h** $\frac{1}{15}$

**i** $3\frac{1}{2}$ **j** $3\frac{2}{11}$ **k** $2\frac{4}{5}$ **l** $3\frac{9}{22}$

**4** $\frac{3}{32}$

**5** **a** 1 (or −1) **b** 0 **c** 1 **d** $\frac{1}{3}$

**6** **a** $\frac{7}{8}$ **b** $\frac{2}{3}$ **c** $\frac{2}{21}$ **d** $\frac{7}{10}$

**7** **a** Increase: dividing by a proper fraction is the same as multiplying by its reciprocal, an improper fraction, whose value is greater than 1, so the number will increase.

**b** Increase: multiplying by an improper fraction, whose value is greater than 1, will make the number increase.

## Exercise 6.04

**1** **a** $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{31}{100}$ **d** $\frac{2}{25}$

**e** $\frac{3}{10}$ **f** $\frac{17}{20}$ **g** $\frac{99}{100}$ **h** $\frac{3}{100}$

**i** $1\frac{3}{5}$ **j** $1\frac{7}{20}$ **k** $\frac{1}{4}$ **l** $2\frac{1}{2}$

**2** C

**3** **a** 0.18 **b** 0.82 **c** 0.02 **d** 0.5

**e** 1.2 **f** 0.511 **g** 0.79 **h** 0.125

**i** 0.163 **j** 0.04 **k** 0.187 **l** 0.0525

**4** **a** 17% **b** 70% **c** 26%

**d** 55% **e** 62.5% **f** 96%

**g** $66\frac{2}{3}\%$ **h** 125% **i** 140%

**j** 67.5% **k** 6.25% **l** $44\frac{4}{9}\%$

**5** **a** 38% **b** 55% **c** 96% **d** 62.5%

**e** 8% **f** 5.4% **g** 60% **h** 0.3%

**i** 190% **j** 40.5% **k** 126% **l** 11.4%

**6** **a** $\frac{13}{20}$, 65% **b** $\frac{3}{5}$, 60% **c** $\frac{1}{5}$, 0.2

**d** $\frac{21}{25}$, 0.84 **e** 0.5, 50% **f** 0.125, 12.5%

**g** $\frac{9}{25}$, 0.36 **h** 0.625, 62.5% **i** $\frac{73}{100}$, 73%

**j** $0.\dot{3}, 33\frac{1}{3}\%$ **k** $\frac{2}{3}, 0.\dot{6}$

**7** **a** 75% **b** $\frac{4}{25}$ **c** $\frac{2}{3}$

**d** 18% **e** $\frac{1}{4}$ **f** 55%

**g** $\frac{19}{20}$ **h** 60% **i** $\frac{1}{6}$

**8** **a** 0.75, 78%, $\frac{4}{5}$, $\frac{9}{11}$ **b** 22%, $\frac{1}{4}$, 0.29, $\frac{7}{20}$.

**c** 57%, $\frac{3}{5}$, 0.605, 0.62

**9** C

**10** **a** 0.47, $\frac{9}{20}$, 43%, $\frac{2}{5}$ **b** 0.88, 86%, $\frac{21}{25}$, 0.08

**c** $\frac{19}{20}$, 91%, 0.905, $\frac{9}{10}$

## Mental skills 6A

**2** **a** 19 **b** \$7.50 **c** 87.5 **d** \$20.20

**e** \$3.76 **f** 40 **g** \$0.925 **h** 89.6

**i** \$270 **j** \$0.38 **k** \$152.76 **l** \$7.25

**m** 315.4 **n** \$1.07 **o** 42.6 **p** \$2431.76

**4** **a** 10 **b** 124 **c** \$490 **d** \$1.72

**e** 14.4 **f** \$2540 **g** 78 **h** \$1.16

**i** \$9 **j** \$16.80 **k** \$920 **l** 64

**6** **a** 100 **b** \$0.60 **c** 2.5 **d** \$1.35

**e** \$1.84 **f** \$4.20 **g** 40 **h** 6.5

**i** \$0.48 **j** \$6.90 **k** \$3.60 **l** 42

## Exercise 6.05

**1** **a** 24 **b** 7 **c** 4

**d** 10 **e** 42 **f** 10

**g** 750 m **h** 8 h **i** 400 mL

**j** 125 kg **k** 10 months **l** 35 min

**2** C

**3** **a** \$55 **b** 5 kg **c** 19.5 L

**d** 126 cm **e** \$67.50 **f** 285 g

**g** 473.76 m **h** 286.72 km **i** 152.29 mL

**j** \$15.62 **k** \$89.68 **l** 45.539 kg

**m** 104.5 **n** 453 **o** \$2613

**p** 126 m **q** 595 g **r** \$460

**4** 0.675 kg

**5** **a** \$105 **b** \$315

**6** 234

**7** **a** 96 cm **b** 350 kg

**c** 144 min **d** 75 mL

**e** 11 250 g **f** 900 m

**g** 84 h **h** 30 min

**i** 27 000 mg

**8** 63 **9** \$0.86 **10** 273

**11** D

**12** 17 **13** 24 318 **14** 1400

**15** **a** \$90 **b** \$500

## Exercise 6.06

**1** **a** $\frac{1}{2}$ **b** $\frac{19}{25}$ **c** $\frac{87}{100}$

**d** $\frac{2}{3}$ **e** $\frac{1}{4}$ **f** $\frac{3}{8}$

**2** **a** 50% **b** 76% **c** 87%
**d** $66\frac{2}{3}\%$ **e** 25% **f** $37\frac{1}{2}\%$

**3** 62.5%

**4** **a** $\frac{2}{5}$ **b** 36%

**5** No

**6** **a** $\frac{172}{189}$ **b** 9%

**7** **a** 32% **b** $\frac{9}{67}$ **c** 13.4%

**8** 5%

**9** **a** **i** $169 **ii** $205 **iii** $249
**b** **i** 11.8% **ii** 12.7% **iii** 12.0%
**c** **ii** $179 oven

**10** A

**11** **a** $\frac{1}{12}$ **b** $\frac{1}{4}$ **c** $\frac{7}{10}$ **d** $\frac{23}{100}$
**e** $\frac{1}{8}$ **f** $\frac{1}{6}$ **g** $\frac{3}{20}$ **h** $\frac{19}{70}$
**i** $\frac{1}{8}$ **j** $\frac{3}{8}$ **k** $\frac{8}{15}$ **l** $\frac{4}{35}$

**12** **a** $8\frac{1}{3}\%$ **b** 25% **c** 70% **d** 23%
**e** $12\frac{1}{2}\%$ **f** $16\frac{2}{3}\%$ **g** 15% **h** $27\frac{1}{7}\%$
**i** $12\frac{1}{2}\%$ **j** $37\frac{1}{2}\%$ **k** $53\frac{1}{3}\%$ **l** $11\frac{3}{7}$

**13** C

**14** **a** Broncos 124.9%, Bulldogs 112.0%, Cowboys 84.7%, Dragons 81.3%, Eels 115.6%, Knights 86.9%, Panthers 76.1%, Rabbitohs 97.2%, Raiders 86.9%, Roosters 61.8%, Sea Eagles 184.7%, Sharks 117.6%, Storm 230.9%, Tigers 77.6%, Titans 79.9%, Warriors 151.5%
**b** Storm, Sea Eagles, Warriors, Broncos

## Exercise 6.07

**1** **a** $157.50 **b** 480 **c** 73.2 km
**d** $2650 **e** 152 kg **f** 13.3 L

**2** **a** $176 **b** 102 **c** 101.2 kg
**d** $243.75 **e** 1240 L **f** $1522.90

**3** $232.20

**4** **a** $405 **b** $3105

**5** $1024.10 **6** 171.36 cm **7** D

**8** $18 691.50 **9** $145.20 **10** 798

**11** $1 160 000 **12** B **13** $80.10

**14** **a** $540 **b** $378 **c** $92 **d** 19.6%

**15** 650 **16** $423.30 **17** 510

**18** Less than the original price, $673.20

## Exercise 6.08

**1** **a** 8 kg **b** 45 mm **c** 6 L **d** $250

**2** C

**3** **a** 18 m **b** 9 g **c** 3 h
**d** 1 L **e** $20 **f** 12 km
**g** 5 mL **h** 6 days **i** 75 kg

**4** **a** 6 t **b** 4 h
**c** 30 sheep **d** $9

**5** **a** 7 **b** $40 **c** 48 min
**d** $63 **e** 150 cm **f** 80 mL

**6** B

**7** **a** $9 **b** 5 kg **c** 20 kL

**8** **a** $100 **b** 154 **c** $200
**d** 42 **e** $270 **f** 54

**9** **a** $279 **b** 96 **c** $70
**d** 57 **e** $300 **f** 500

**10** **a** 24 **b** 10

## Mental skills 6B

**2** **a** $308 **b** 60 **c** 30 **d** $450
**e** 96 **f** $90 **g** $60.50 **h** 150
**i** 210 **j** $84 **k** $300 **l** 98

**4** **a** $360 **b** 40 **c** 88 **d** $22.50
**e** 450 **f** $63 **g** $80 **h** 50
**i** $50.40 **j** $87.50 **k** 225 **l** $105

## Exercise 6.09

**1** **a** $700 **b** $560 **c** $4600
**d** $1900 **e** 400 cm **f** 280 kg
**g** 3 m **h** 150 min **i** $400
**j** $360 **k** 400 kg **l** 110 min

**2** $1200 **3** 79 800 **4** $82 176

**5** $15 250 **6** D **7** 24 000

**8** $3000 **9** $300 **10** $435

## Exercise 6.10

**1** **a** **i** $18 700 **ii** Loss
**b** **i** $543 000 **ii** Profit
**c** **i** $799 **ii** Loss
**d** **i** $6800 **ii** Profit

**2** **a** **i** $17 **ii** 20%
**b** **i** $185 **ii** 44.6%
**c** **i** $1.45 **ii** 52.7%

**3** **a** **i** $5 **ii** 26.3%
**b** **i** $1250 **ii** 46.3%
**c** **i** $21 **ii** 26.6%

**4** C

**5** **a** $150 **b** 12%

**6** **a** $1800 **b** 10.6%

**7** **a** $234 **b** $71.40 **c** $468
**d** $1267 **e** $6.25 **f** $16.20

**8** 23.31% **9** 44.4%

**10** **a** $2090, $22 990 **b** $181, $1991
**c** $0.30, $3.30 **d** $14.50, $159.50
**e** $18, $198 **f** $2.70, $29.70

**11** **a** $72, $720 **b** $18, $180 **c** $84, $840
**d** $1.70, $17 **e** $3.15, $31.50 **f** $1.52, $15.18

**12** **a** $1440 **b** $1584 **c** $950.40
**d** $86.40 **e** Loss of $35 **f** 4.0%


9780170479516

## Exercise 6.11

**1** D

**2** **a** \$120 **b** \$720 **c** \$262.40
**d** \$6 **e** \$1706.25 **f** \$3777.75
**g** \$393.68 **h** \$588 **i** \$42.36
**j** \$2428.38

**3** **a** \$1350 **b** \$2250 **c** \$11 200
**d** \$108 000 **e** \$101.50 **f** \$6255
**g** \$4981.62 **h** \$540 **i** \$36.85
**j** \$374.40

**4** **a** \$27 **b** \$11.38 **c** \$18.75
**d** \$1020 **e** \$3.96 **f** \$225.50
**g** \$988.17 **h** \$186.43 **i** \$12.25
**j** \$1364.18

**5** **a** \$735 **b** \$7735

**6** **a** \$10 500 **b** \$40 500 **c** \$675

**7** **a** \$1620 **b** 4 years

## Exercise 6.12

**1** 27 **2** 81.25% **3** 80%
**4** \$41.25 **5** 60% **6** 234 points
**7** 348 **8** 37.5% **9** 40%
**10** 12.5% **11** \$8625 **12** 11.4%
**13** \$262.50 **14** C **15** \$4369.95
**16** \$44 561.25

## Power plus

**1** \$96.80

**2** **a** \$1627.50 **b** \$1765.84

**3** **a** \$499 200 **b** \$519 168 **c** \$583 993.39

**4** **a** Less **b** 20%

**5** **a** \$357 000 **b** \$303 500 **c** \$186 400

**6** **a** Less **b** 50%

## Test yourself 6

**1** **a** $\frac{3}{4}$ **b** $\frac{7}{12}$ **c** $\frac{7}{9}$

**2** **a** $3\frac{3}{4}$ **b** $4\frac{2}{5}$ **c** $4\frac{1}{3}$

**3** **a** $\frac{9}{2}$ **b** $\frac{11}{3}$ **c** $\frac{34}{5}$

**4** $\frac{5}{6}, \frac{3}{4}, \frac{7}{12}, \frac{1}{2}$

**5** **a** $\frac{11}{15}$ **b** $\frac{1}{14}$ **c** $1\frac{5}{12}$
**d** $6\frac{7}{10}$ **e** $6\frac{1}{6}$ **f** $2\frac{7}{8}$

**6** $\frac{3}{8}$

**7** **a** $\frac{7}{18}$ **b** $\frac{8}{15}$ **c** $\frac{15}{28}$
**d** $\frac{27}{40}$ **e** $\frac{2}{5}$ **f** $3\frac{7}{12}$

**8** 16 500

**9** **a** $\frac{37}{50}$ **b** $\frac{29}{100}$ **c** $\frac{13}{20}$ **d** $1\frac{57}{100}$

**10** **a** 0.12 **b** 0.162 **c** 0.02 **d** 0.054

**11** **a** 87.5% **b** 24% **c** $22\frac{2}{9}\%$ **d** 82.5%

**12** **a** 56% **b** 61.3% **c** 70% **d** 4.8%

**13** $0.7, 0.725, \frac{3}{4}, 77\%$

**14** **a** 6 **b** 16 **c** \$242
**d** 8 months **e** 108 L **f** 210 cm

**15** **a** 24.31 kg **b** 162.69 kg

**16** **a** **i** $\frac{4}{5}$ **ii** 80%
**b** **i** $\frac{3}{4}$ **ii** 75%
**c** **i** $\frac{11}{20}$ **ii** 55%

**17** **a** $\frac{17}{20}$ **b** 85%

**18** 48.1% **19** \$327 **20** \$67 275

**21** **a** 11 t **b** 12 cm **c** 9 min

**22** \$147 111.11 **23** 20%

**24** **a** \$28.70 **b** \$287.00

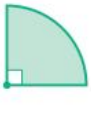
**25** \$42.50 **26** 57.6%

## Practice set 2

**1** **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{3}{5}$

**2** **a** $y = 65$ (angles on a straight line)
**b** $k = 135$ (vertically opposite angles), $l = 45$ (angles on a straight line)
**c** $x = 313$ (angles at a point)
**d** $q = 62$ (angles in a right angle)

**3** **a** 60 cm² **b** 110 cm² **c** 60 cm² **d** 880 mm²

**4** **a**  **b** 
**c**  **d** 


**5** **a** **i** scalene **ii** right-angled
**b** **i** scalene **ii** acute-angled
**c** **i** isosceles **ii** acute-angled
**d** **i** equilateral **ii** acute-angled

**6** **a** $m = 60$ **b** $x = 50$ **c** $k = 65$ **d** $m = 60$

**7** **a** $\frac{5}{2}$ **b** $\frac{7}{4}$ **c** $\frac{22}{5}$

**8** **a** **i** $\frac{1}{4}$ **ii** 0.25
**b** **i** $\frac{2}{5}$ **ii** 0.4
**c** **i** $\frac{17}{20}$ **ii** 0.85
**d** **i** $\frac{1}{8}$ **ii** 0.125

**9** **a** $d = 78$ (alternate angles on parallel lines)
**b** $f = 112$ (corresponding angles on parallel lines)
**c** $t = 72$ (co-interior angles between parallel lines)

**10** **a** 34% **b** 75% **c** 21.5% **d** 62.5%

**11** **a** parallelogram
**b** rectangle **c** square **d** kite

**12** **a** 248.62 m² **b** 22.68 cm² **c** 90.2 m²

**13** **a** $5\frac{1}{2}$ **b** $6\frac{1}{3}$ **c** $3\frac{1}{2}$

**14** **a** $1\frac{1}{5}$ **b** $\frac{11}{15}$ **c** $\frac{7}{20}$ **d** $4\frac{1}{10}$
**e** $1\frac{1}{3}$ **f** $\frac{3}{10}$ **g** $\frac{3}{5}$ **h** $1\frac{3}{5}$

**15** **a** 137 m² **b** 112.5 cm²

**16** **a** 31.42 cm **b** 21.99 cm

**17** **a** 78.54 cm² **b** 38.48 cm²

**18** **a** $x = 130$ (exterior angle of a triangle)
**b** $m = 50$ (exterior angle of a triangle)
**c** $n = 35$ (exterior angle of a triangle)
**d** $p = 105$ (angle sum of a quadrilateral)
**e** $k = 40$ (angle sum of a quadrilateral)
**f** $x = 90$ (angle sum of a quadrilateral)

**19** $4.60 **20** 26.928 L

**21** **a** 4752 cm³ **b** 640 m³

**22** **a** 36 000 cm² **b** 8.75 cm²

**23** $\frac{3}{5}$, 63%, 0.67, $\frac{6}{7}$

**24** **a** true **b** false **c** true **d** false

**25** **a** $0.30 **b** 528 kg **c** $4.80 **d** 15 min

**26** **a** 40% **b** 35% **c** 50%

**27** 3281

**28** **a** 15.42 cm **b** 91.42 mm **c** 80.55 cm

**29** **a** 14.14 cm² **b** 557.08 mm² **c** 339.29 cm²

**30** A **31** 589.0 cm² **32** $1360

**33** **a** $499 **b** 45.4%

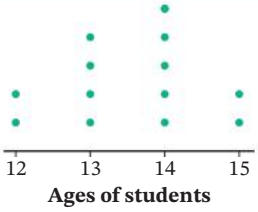
**34** **a** 62.1 m² **b** 55.2 m²

**35** **a** 942.5 cm³ **b** 14 726.2 mm³

# Chapter 7

## SkillCheck

**1**



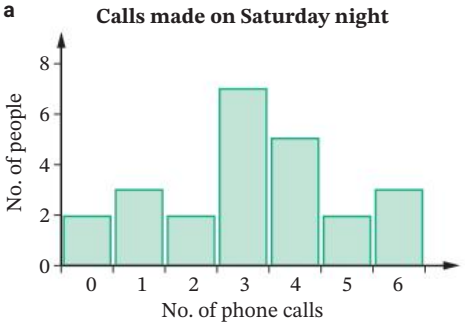

**Ages of students**

**2** **a**

| Stem | Leaf |
|---|---|
| 3 | 0 |
| 4 | 7 8 8 9 |
| 5 | 4 9 9 |
| 6 | 3 4 8 8 8 |

**b** 30

**3** **a**

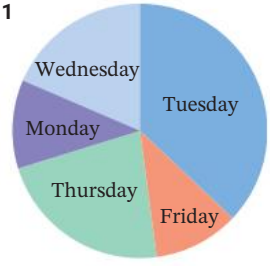



**b** 24 **c** 3 **d** 0, 2 and 5 **e** 6

**4** **a** 53 **b** 10 **c** 240

## Exercise 7.01

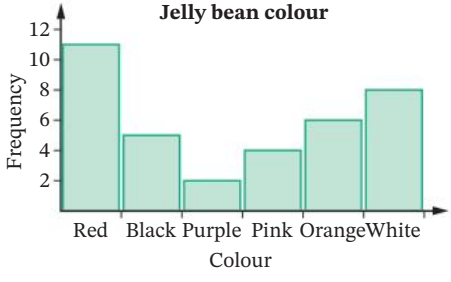
**1**




**Training day**

Day of sport training
Sector graph with angles:
Mon = 36°,
Tues = 156°,
Wed = 48°,
Thurs = 84°,
Fri = 36°

**2**




**3** Hours of sleep




6 = 0.75 cm, 7 = 2.25 cm , 8 = 2.75 cm,
9 = 1.5 cm, 10 = 1.25 cm, 11 = 0.5 cm

**4** D

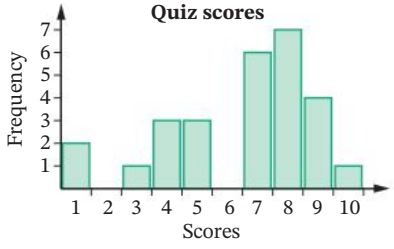

9780170479516

**5 a**

| Score | Tally | Frequency |
|---|---|---|
| 1 | ‖ | 2 |
| 2 | | 0 |
| 3 | \| | 1 |
| 4 | ‖\| | 3 |
| 5 | ‖\| | 3 |
| 6 | | 0 |
| 7 | 卌\| | 6 |
| 8 | 卌\| | 7 |
| 9 | ‖‖ | 4 |
| 10 | \| | 1 |
| | **Total** | 27 |

**b** 10 **c** 3 **d** 8

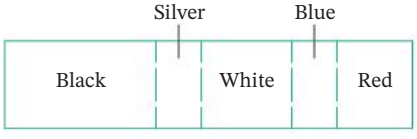
**e**




**6 a**

| Colour | Frequency |
|---|---|
| Black (B) | 10 |
| Silver (S) | 3 |
| White (W) | 6 |
| Blue (U) | 3 |
| Red (R) | 5 |
| **Total** | 27 |

**b**



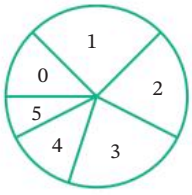

**c** Black

**7 a**

| No. of siblings | Tally | Frequency |
|---|---|---|
| 0 | 卌 | 5 |
| 1 | 卌 卌 | 10 |
| 2 | 卌 ‖ | 8 |
| 3 | 卌 ‖‖ | 9 |
| 4 | 卌 | 5 |
| 5 | ‖ | 3 |
| | **Total** | 40 |

**b** 5 **c** 1

**d** Sector graph with angles: 0 = 45°, 1 = 90°, 2 = 72°, 3 = 81°, 4 = 45°, 5 = 27°

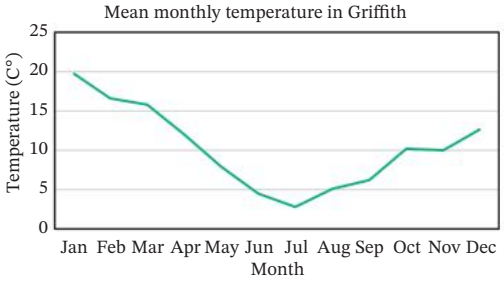



No. of siblings

## Exercise 7.02

**1 a** April 1990 **b** May 1990
**c** May, June **d** September, October
**e** 1990, 2 months with very high rainfall
**f** November, has low rainfall in both years

**2**




**3 a** 60 **b** Friday
**c** False **d** Tuesday, 15
**e** It is difficult to show exact or very large numbers.

**4** Teacher to check.

**5 a** easy to read, other answers possible
**b** Teacher to check

**6 a** 1% **b i** False **ii** True
**c** 10% **d, e** Teacher to check
**f** 9180 L

**7 a** Peanut butter **b** Cereal
**c** Peanut butter **d** Cereal, Biscuits
**e** 14 g of protein **f** Cereal, Biscuits

**8 a, b, c** Teacher to check **d** regional areas
**e** 74 years
**f** it's a prediction based on past statistics and modelling
**g** health services **h, i, j** Teacher to check.

**9, 10, 11** Teacher to check.

## Exercise 7.03

**1 a** N **b** C **c** N **d** N
**e** N **f** C **g** N **h** N

**2 a** O **b** N **c** O
**d** N **e** N **f** O

ANSWERS

**3** **a** D **b** C **c** D **d** D
**e** C **f** D **g** C **h** D
**4** D
**5** **a** CN **b** CO **c** ND **d** CO
**e** ND **f** NC **g** ND **h** CN

## Exercise 7.04

**1** **a** sample **b** census **c** sample **d** sample
**e** sample **f** census **g** census **h** census
**2** **a** sample, survey **b** sample, survey
**c** census, observation **d** sample, survey
**e** sample, survey **f** census, measurement
**g** sample, measurement **h** census, survey
**3** C **4** Teacher to check.
**5** **a** Not random, students volunteered themselves for the survey rather than be chosen.
**b** Random.
**c** Not random, many people do not have their phone numbers listed in the book.
**d** Random.
**e** Not random, station focuses on rock music only, and listeners call in to vote rather than be randomly selected.
**6** Teacher to check.

## Exercise 7.05

**1** **a** 1.6 **b** 1 and 2
**2** **a** 6.75, 6 **b** 4.71, 2 **c** 4.88, 1 and 7
**d** 70.44, 96 **e** 47.25, no mode
**3** See answers to question **2**.
**4** **a** Hyundai **b** spade **c** Korean
**5** Data are categorical, not numerical.
**6** **a** 7.71 **b** 7.0
**c** Mean, every value is involved in calculating it.
**7** C
**8** **a** Carrozza 26.3, Binns 33.3
**b** Binns family has higher values, outlier of 93.
**c** Binns family has higher mean.
**9** **a** 8Y 6.1; 8Z 6.1
**b** 8Z's marks are more spread out.
**c** 4
**10** 2940
**11** **a** \$1812.50 **b** \$2750
**12** Teacher to check.

## Exercise 7.06

**1** B **2** D
**3** **a** **i** 8 **ii** 7
**b** **i** 47.5 **ii** 10
**c** **i** 12 **ii** 4
**d** **i** 51 **ii** 99
**e** **i** 57 **ii** 0
**f** **i** 82.35 **ii** 37.4
**4** **a** −\$0.57
**b** median \$9.50, mode −\$25
**c** No. His winnings are small and his loss big. Not worth the effort.
**5** **a** no mode **b** \$686 000 **c** \$771 845.45
**d** Median, because it's not affected by the outlier of \$1 670 000.
**6** **a** Aldo 168.1, Peter 175.7
**b** Aldo 35, Peter 6
**c** **i** Aldo, higher mean.
**ii** Peter is more consistent, lower range.
**d** 194
**7** C
**8** **a** Daniel 29.68, Cristian 29.08
**b** Daniel 5, Cristian 4.5
**c** Cristian, he is consistent and fast.
**9** 6, 7, 8 or 9 **10** Teacher to check.

## Mental skills 7

**2** **a** 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 °C
**b** 200 210 220 230 240 250 260 270 280 290 300 310 320 330 340 350 360 370 mL
**c** 500 504 508 512 516 520 524 528 532 536 540 544 548 552 556 560 564 568 572 576 580 584 588 592 g
**d** 160 170 180 190 200 210 220 230 240 250 260 270 280 290 300 310 min
**e** 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 L
**f** 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640 660 680 700 kg
**g** 120 135 150 165 180 195 210 225 240 255 270 285 300 315 330 345 360 375 390 405 420 seconds
**h** 100 125 150 175 200 225 250 275 300 325 350 375 400 425 450 475 500 525 550 575 600 625 650 675 700 725 mL

## Exercise 7.07

**1** **a** 34 **b** 2 **c** 2.1 **d** 2 **e** 8

**2** **a** 24 **b** 11 **c** 8.6 **d** 5 **e** 9

**3** **a**

| Judge's score, $x$ | Frequency, $f$ | $fx$ |
|---|---|---|
| 1 | 2 | 2 |
| 3 | 1 | 3 |
| 4 | 3 | 12 |
| 5 | 3 | 15 |
| 7 | 6 | 42 |
| 8 | 7 | 56 |
| 9 | 4 | 36 |
| 10 | 1 | 10 |
| **Total** | 27 | 176 |

**b** 8 **c** 6.5 **d** 7 **e** 9

**4** **a** 5 **b** 2 **c** 2.5 **d** 2.8

**5** **a** 37 **b** 1.47 **c** 0.1 **d** 1.48 **e** 1.48

**6** A

**7** **a** 7 **b** 3 **c** 3 **d** 3.4

**8** **a** 12 **b** 15 **c** 14.5

## Exercise 7.08

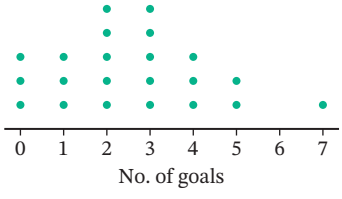
**1** **a** **i** 4 **ii** 11 **iii** 10 **iv** 9.8

**b** **i** 4 **ii** 24 **iii** 22 **iv** 22.3

**c** **i** 6 **ii** 36 **iii** 37 **iv** 37.6

**d** **i** 0.3 **ii** 1.8 **iii** 1.8 **iv** 1.8

**2** **a**

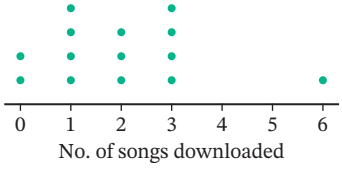



**b** 22 **c** 7 **d** 2.5 **e** 2.6

**3** **a** 18 **b** 8 **c** 22 and 25

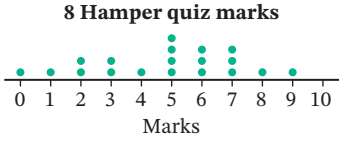
**d** 22.3 **e** 71% **f** B

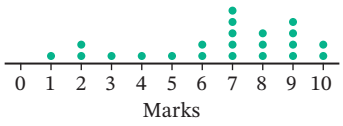
**4** **a**




**b** 14 **c** 6 **d** 2 **e** 2 **f** 14

**5** **a**




**b** 7 **c** 5 **d** 9

**e**

| $x$ | $f$ | $fx$ |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 2 | 2 | 4 |
| 3 | 2 | 6 |
| 4 | 1 | 4 |
| 5 | 4 | 20 |
| 6 | 3 | 18 |
| 7 | 3 | 21 |
| 8 | 1 | 8 |
| 9 | 1 | 9 |
| **Total** | 19 | 91 |

$\bar{x} = 4.8$

**f** 8 Yen, marks clustered around higher values 7 to 9.

**6** **a** **i** 39 **ii** 33 **iii** 33 **iv** 30.1

**b** **i** 26 **ii** 94 **iii** 88 **iv** 86.3

**c** **i** 40 **ii** 131 **iii** 124.5 **iv** 121.5

**d** **i** 44 **ii** 17 **iii** 17 **iv** 21.7

**7** **a**

| Stem | Leaf |
|---|---|
| 7 | 6 |
| 8 | |
| 9 | 2 5 7 8 |
| 10 | 1 1 5 5 6 8 |
| 11 | 2 4 7 |
| 12 | 4 |

**b** 76 **c** 101, 105 **d** 124

**e** 9, 10, 11 **f** 66.7%

**8** **a**

| Stem | Leaf |
|---|---|
| 13 | 7 |
| 14 | 5 |
| 15 | 4 5 6 7 8 8 9 |
| 16 | 0 0 1 3 3 4 4 5 6 7 7 7 9 |
| 17 | 15 |

**b** 16 **c** 24 **d** 2

**e** 38 **f** 162

**9** **a** 19 **b** 1

**c** Fuller's Ridge Rd, times are clustered around the lower values.

**d** D

**10** **a** 16 **b** 8 **c** 30

**d** 9, 13 and 30 **e** 30

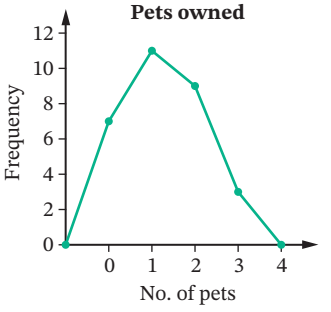
**f** Narrabri Lions

**g** Wee Waa Reds, it has a range of 28 compared to Narrabri Lion's 26.

ANSWERS

## Exercise 7.09

**1** **a**

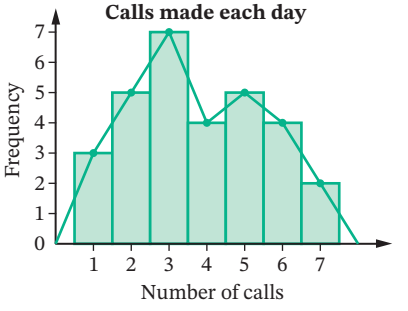



**b** 30 **c** 7 **d** 30%
**e** 1 **f** B

**2** **a**

| x | f |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |
| 4 | 4 |
| 5 | 5 |
| 6 | 4 |
| 7 | 2 |
| | 30 |

**b**




**c** 7 **d** 113 **e** 8 **f** 3 **g** 3.8

**3** **a** 25 **b** 1 **c** 1 **d** 4

**e**

| x | f | fx |
|---|---|---|
| 1 | 13 | 13 |
| 2 | 5 | 10 |
| 3 | 3 | 9 |
| 4 | 1 | 4 |
| 5 | 3 | 15 |
| | 25 | 51 |

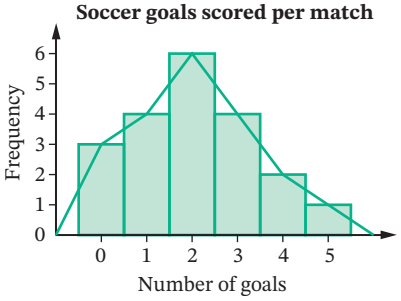
$\bar{x} = 2.04$

**f** 1

**4** **a**

| x | f |
|---|---|
| 0 | 3 |
| 1 | 4 |
| 2 | 6 |
| 3 | 4 |
| 4 | 2 |
| 5 | 1 |
| | 20 |

**b**




**c** 20 **d** 5 **e** 2
**f** 2.05 **g** 7 **h** 15%

**5** **a** 5, 6, 9, 12, 4, 8, 44 **b** 4
**c** 5 **d** 155 **e** 154.64
**f** 11 **g** 155 **h** 24

## Exercise 7.10

**1** Teacher to check reasons.
- **a** line graph
- **b** frequency histogram or polygon
- **c** dot plot, column graph or line graph
- **d** sector graph, divided bar graph or column graph
- **e** column graph, sector graph
- **f** stem-and-leaf plot, frequency histogram or polygon
- **g** sector graph, divided bar graph or column graph
- **h** pictogram or column graph
- **i** dot plot, column graph, sector graph or divided bar graph

**2** **a** column graph, sector graph or divided bar graph
**b** Teacher to check.

**3** **a** pictogram or column graph
**b** Teacher to check.

**4** **a** pictogram, column graph, sector graph or divided bar graph
**b, c, d** Teacher to check.

**5** **a** clustered column graph
**b** compares 2 data sets (Monday vs Tuesday sales)

**6** **a** line graph **b** Teacher to check.

## Exercise 7.11

1 **a** negatively skewed, unimodal
**b** positively skewed, unimodal
**c** bimodal, symmetrical
**d** positively skewed, unimodal

**2** A

**3** **a** 15 **b** symmetrical
**c** 2 **d** yes

**4** **a** negatively skewed **b** 21
**c** 80.5 **d** 86 **e** the mean

**5** **a** Has 3 peaks
**b** Many answers possible. For example, a café with peaks before work, morning tea and afternoon tea.

**6** **a** positively skewed
**b** Teacher to check.

**7** **a** bimodal **b** Teacher to check.

**8** Teacher to check

## Exercise 7.12

**1** **a** **i** 37 **ii** 49 **iii** 52.5 **iv** 55.3
**b** Mean, because there are no outliers.

**2** **a** **i** 24 **ii** 32 **iii** 37 **iv** 36.4
**b** **i** increase **ii** no change
**iii** decrease **iv** decrease

**3** **a** 30 **b** B
**c** Grades are categorical data, not quantitative (numerical).

**4** **a** **i** $427 000 **ii** $91 500 **iii** $135 000
**b** There is no mode, and the mean is affected by the outlier of $484 000.
**c** Mean $103 727, median $90 000, yes.

**5** **a** **i** 192.4 cm **ii** 193 cm
**iii** no mode **iv** 23
**b** **i** decrease **ii** no change
**iii** There will be a mode of 181
**iv** no change

**6** **a** D **b** Sunjeet 32.8, Chris 33.7
**c** **i** Sunjeet **ii** Chris
**d** Sunjeet: his scores are consistently higher.

**7** **a**

| Forbes | | Cobar |
|---|---|---|
| 9 6 2 0 | 3 | 0 1 3 6 7 |
| 4 | 4 | 1 7 9 9 |
| 8 5 4 | 5 | 1 1 8 |
| 9 5 3 1 | 6 | 0 5 |
| 6 5 1 | 7 | 2 3 7 9 |
| 4 0 0 | 8 | 8 |
| 1 0 | 9 | 7 |

**b**

| | Mean | Mode | Median | Range |
|---|---|---|---|---|
| Forbes | 62.7 | 80 | 64 | 61 |
| Cobar | 56.2 | 49, 51 | 51 | 67 |

**c** Forbes, it has a higher mean, mode and median.
**d** 97, mean

**8** **a**

| | Mean | Mode | Median | Range |
|---|---|---|---|---|
| Christina | 20.9 | 19, 25 | 21.5 | 31 |
| Mary | 21.9 | 17, 20 | 20 | 21 |

**b** Mary, lower range **c** Mary
**d** Mary, higher mean **e** 4
**f** **i** no **ii** yes

**9** **a**




**b** Delta: range 7, median 6; Guy: range 5, median 5.5
**c** Other than the outlier of 2, Delta allocates higher marks, while Guy's marks are spread out.

## Power plus

**1** **a** 74.2 **b** 77.2
**c** Increased by 3. **d** Does the same to the mean.

**2** 92%

**3** No, but the values will be centred around 20 (teacher to check).

**4** **a** 40 **b, c** Teacher to check.

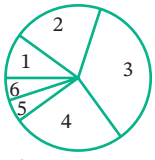
**5** Yes. The sum of the 4 sisters' wages is $8000, so each of the other 2 sisters earn much more (the sum of their wages is $6700).

## Test yourself 7

**1** **a**

| *x* | *f* |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 7 |
| 4 | 5 |
| 5 | 1 |
| 6 | 1 |
| | 20 |

**b** 3 **c** 5 **d** 10%
**e** Sector graph with angles: 1 = 36°, 2 = 72°, 3 = 126°, 4 = 90°, 5 = 18°, 6 = 18°




No. of computers owned

**2** Teacher to check

**3** **a** NC **b** CN **c** ND
**d** NC **e** CN **f** CO

**4** **a** census **b** sample **c** sample **d** census

**5** Teacher to check.

**6** **a** **i** 8.78 **ii** 6 **iii** 10 **iv** 6
**b** **i** 3.5 **ii** 1 **iii** 3.5 **iv** 7
**c** **i** 22.6 **ii** no mode **iii** 23 **iv** 5

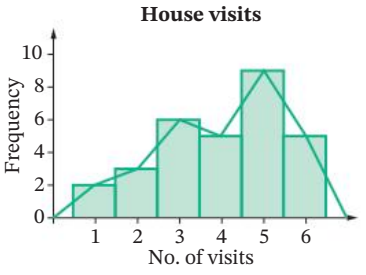
**7** **a** 0, 7, 12, 12, 8, 5, 6, totals 24, 50
**b** 24 **c** 6 **d** 1 **e** 2 **f** 2.08

**8** **a** **i** 4 **ii** 35 **iii** 33 **iv** 34.6
**b** **i** 35 **ii** 144 **iii** 148 **iv** 140.25

**9** **a**

| *x* | *f* |
|---|---|
| 1 | 2 |
| 2 | 3 |
| 3 | 6 |
| 4 | 5 |
| 5 | 9 |
| 6 | 5 |
| | 30 |

**b**




**c** 14 **d** 5 **e** 4

**10** **a** column graph
**b** sector graph, divided bar graph
**c** pictogram **d** line graph
**e** categorical **f** stem-and-leaf plot
**g** dot plot **h** frequency histogram
**i** frequency histogram, frequency polygon, dot plot or stem-and-leaf plot
**j** frequency polygon

**11** **a** positively skewed **b** bimodal, symmetrical
**c** symmetrical

**12** Teacher to check.

**13** **a** 37 **b** 36 **c** 37.9
**d** mean **e** 11
**f** **i** increase **ii** no change
**iii** no change **iv** no change

# Chapter 8

## SkillCheck

**1** **a** 0.85 **b** 0.375 **c** 0.02 **d** 0.3

**2** **a** {head, tail} **b** {1, 2, 3, 4, 5, 6}
**c** {female, male} **d** {red, amber, green}
**e** {win, lose, draw} **f** {pass, fail}

**3** **a** 70% **b** 56% **c** $66\frac{2}{3}\%$ **d** 24%

**4** **a** $\frac{2}{3}$ **b** $\frac{3}{5}$ **c** $\frac{5}{8}$

**5** C

**6** **a** $\frac{7}{25}$ **b** $\frac{1}{50}$ **c** $\frac{16}{25}$ **d** $\frac{4}{5}$

**7** Answers will vary between students.

## Exercise 8.01

**1** **a** **i** {red, blue} **ii** $\frac{1}{2}$
**b** **i** {red, blue, yellow, green} **ii** $\frac{1}{4}$
**c** **i** {red, blue} **ii** $\frac{1}{2}$
**d** **i** {red, white} **ii** $\frac{1}{5}$
**e** **i** {red, white, black, green} **ii** $\frac{1}{4}$
**f** **i** {red, green, yellow} **ii** $\frac{1}{2}$

**2** **a** 2, equally likely **b** 3, not equally likely
**c** 26, not equally likely **d** 2, equally likely
**e** 10, equally likely **f** 2, not equally likely

**3** **a** {1, 3, 5} **b** {A, E, I, O, U}
**c** {5, 6, 7, 8, 9} **d** {April, August}
**e** {NSW, Qld, SA, Tas., Vic., WA}
**f** {Years 7, 8, 9, 10, 11, 12}

**4** **a** $\frac{2}{20}=\frac{1}{10}$ **b** $\frac{3}{20}=0.15$ **c** $\frac{11}{20}=55\%$
**d** $\frac{15}{20}=\frac{3}{4}$ **e** $\frac{7}{20}=0.35$ **f** $\frac{13}{20}=65\%$

**5** **a** $\frac{3}{16}$ **b** $\frac{8}{16}=\frac{1}{2}$
**c** $\frac{14}{16}=0.875$ **d** $\frac{10}{16}=62.5\%$

**6** C

**7** **a** {win, lose, draw}
**b** No, teams have different abilities.

**8** D

**9** **a** 3 **b** {5, 6, 7, 8}
**c**

| Outcome | 5 | 6 | 7 | 8 |
|---|---|---|---|---|
| Probability as a fraction | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{6}$ | $\frac{1}{4}$ |
| Probability as a decimal | 0.25 | $0.\dot{3}$ | $0.1\dot{6}$ | 0.25 |
| Probability as a percentage | 25% | $33\frac{1}{3}\%$ | $16\frac{2}{3}\%$ | 25% |

**10** C

**11** **a** C **b** A **c** E **d** H
**e** D **f** G **g** B **h** F

**12** **a** 52 **b** Yes **c** 4
**d** 13 **e** 2
**f** **i** $\frac{1}{2}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{52}$
**iv** $\frac{1}{13}$ **v** $\frac{5}{13}$ **vi** $\frac{3}{26}$

**13** **a** low **b** 9

**14** Teacher to check.

**15** **a** 1 **b** 5 **c** 9 **d** 10

## Exercise 8.02

**1** **a** tossing heads

**b** sunny day tomorrow

**c** selecting a brown chocolate

**d** rolling 1, 2, 3, 4 or 5

**e** losing or having a draw (not winning)

**f** selecting a diamonds, clubs or spades card (not selecting hearts)

**g** born in spring, summer or autumn (not being born in winter)

**h** being 15 or over

**2** A

**3** **a** $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$

**e** $\frac{5}{6}$ **f** $\frac{2}{3}$

**4** **a** $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{5}{6}$ **d** 1

**5** **a** $\frac{45}{56}$ **b** D

**6** **a** $\frac{1}{6}$ **b** $\frac{5}{6}$

**7** 0.8 **8** $\frac{199}{200}$

**9** **a** $\frac{3}{26}$ **b** $\frac{23}{26}$ **c** 1

**d** $\frac{11}{26}$ **e** $\frac{9}{13}$ **f** $\frac{1}{2}$

**10** **a** $\frac{12}{13}$ **b** 39% **c** 0.85

**11** **a** 0.65 **b** 0.35

**c** Red 16, Blue 14, White 10

**d** 0

**12** **a** $\frac{3}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** 0

**13** 15% **14** C

**15** **a** $\frac{7}{16}$ **b** $\frac{9}{16}$

**16** **a** $\frac{10}{11}$ **b** $\frac{7}{11}$ **c** $\frac{9}{11}$ **d** $\frac{8}{11}$

**17** **a** $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{4}{5}$ **d** $\frac{3}{5}$

**18** **a** $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{1}{2}$

## Exercise 8.03

**1** **a** 50 **b** 21 **c** 42

**d** 33 **e** 8 **f** A

**2** **a** Yes, mutually exclusive **b** 6

**c** 40 **d** 46 **e** 0

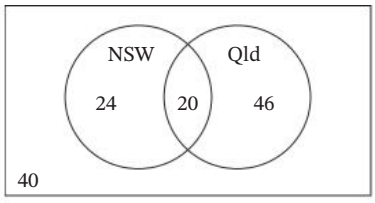
**3** **a** No **b** 52 **c** 4

**d** **i** 0.808 **ii** 0.731 **iii** 1 **iv** 0.192

**e** right-handed male

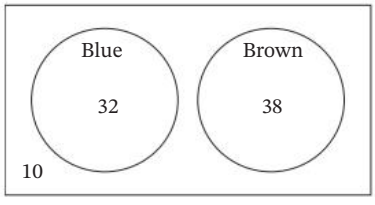
**4** **a** 440 **b** 368 **c** 15 **d** A

**5** **a**




**b** 90 **c** 70 **d** $\frac{12}{65}$ **e** $\frac{4}{13}$

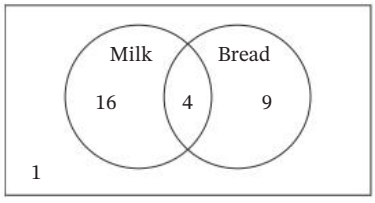
**6** **a**




**b** Yes, cannot be both blue-eyed and brown-eyed.

**c** 80 **d** 70 **e** 0 **f** 52.5%

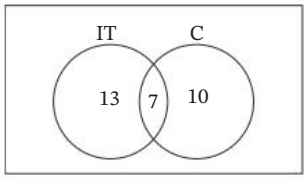
**7** **a**




**b** 4 **c** 16

**d** **i** $\frac{3}{10}$ **ii** $\frac{1}{3}$ **iii** $\frac{29}{30}$ **iv** $\frac{5}{6}$

**8** **a**




**b** $\frac{17}{30}$

## Exercise 8.04

**1** **a** 27 **b** 18 **c** 2 **d** $\frac{4}{9}$

**e** **i** $\frac{5}{27}$ **ii** $\frac{9}{27} = \frac{1}{3}$

**2** **a** 336 **b** 192 **c** 96

**d** **i** $\frac{11}{21}$ **ii** $\frac{4}{21}$ **iii** $\frac{17}{21}$

**3** **a** 56; 101, 146; 101, 243

**b** 146 **c** 56

**d** **i** $\frac{101}{344}$ **ii** $\frac{36}{43}$ **iii** $\frac{187}{344}$ **iv** $\frac{243}{344}$

**4** **a** 125; 70, 110; 165, 310 **b** 165

**c** **i** $\frac{11}{31}$ **ii** $\frac{24}{31}$ **iii** $\frac{25}{62}$

**d** D

ANSWERS

**5**

| | Swimming | Not swimming | |
|---|---|---|---|
| Bowling | 11 | 20 | 31 |
| Not bowling | 34 | 5 | 39 |
| | 45 | 25 | 70 |

**6 a**

| | GPS navigation | No GPS navigation | |
|---|---|---|---|
| 4-doors | 25 | 42 | 67 |
| Not 4-doors | 3 | 12 | 15 |
| | 28 | 54 | 82 |

**b i** $\frac{14}{41}$ **ii** $\frac{67}{82}$ **iii** $\frac{25}{82}$ **iv** $\frac{27}{41}$

**7 a**

| | Needs glasses | Does not need glasses | |
|---|---|---|---|
| Male | 22 | 60 | 82 |
| Female | 13 | 85 | 98 |
| | 35 | 145 | 180 |

**b i** $\frac{49}{90}$ **ii** $\frac{7}{36}$ **iii** $\frac{13}{180}$ **iv** $\frac{167}{180}$
**v** $\frac{2}{3}$ **vi** $\frac{107}{180}$

## Mental skills 8

**2 a** 16 000 **b** 210 **c** 9900 **d** 600
**e** 16 000 **f** 400 **g** 490 **h** 52
**i** 1500 **j** 1360 **k** 195 **l** 17 600

**4 a** 8 **b** 50 **c** 8 **d** 22
**e** 4 **f** 50 **g** 9 **h** 9.3
**i** 8.5 **j** 4.23 **k** 9.6 **l** 40.8

## Exercise 8.05

**1 a** $\frac{17}{28}$ **b** $\frac{2}{7}$ **c** $\frac{6}{7}$

**2 a** {U, E, C, S}
**b** No, some occur more than others.
**c i** $\frac{3}{7}$ **ii** $\frac{2}{7}$ **iii** $\frac{5}{7}$ **iv** 0

**3 a** {black, red, blue}
**b i** $\frac{1}{3}$ **ii** $\frac{1}{3}$ **iii** 0
**c** choosing a black or blue sock, $\frac{2}{3}$

**4 a** 20% **b** 40% **c** 80% **d** 100%

**5** D

**6 a** $\frac{1}{100}$ **b** $\frac{3}{5}$ **c** $\frac{1}{2}$ **d** $\frac{17}{20}$

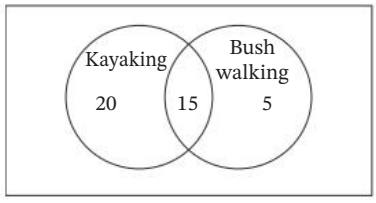
**7** $\frac{1}{2}$

**8 a** $\frac{8}{15}$ **b** $\frac{3}{5}$ **c** $\frac{7}{15}$ **d** $\frac{2}{5}$

**9** 0.9

**10 a** $\frac{17}{30}$ **b** $\frac{1}{10}$ **c** $\frac{2}{3}$ **d** $\frac{13}{30}$

**11 a**




**b i** $\frac{3}{8}$ **ii** $\frac{1}{2}$

**12 a** 232; 140; 228, 272, 500

**b i** $\frac{57}{125}$ **ii** $\frac{7}{25}$ **iii** $\frac{1}{5}$
**iv** $\frac{32}{125}$ **v** $\frac{23}{25}$ **vi** $\frac{83}{125}$

**13 a** 0.3 **b** 5 red, 2 green, 3 purple
**c** 50 red, 20 green, 30 purple
**d** No difference, the probabilities are the same.

## Exercise 8.06

**1 a** $\frac{1}{4}$ **b** 125

**2** Teacher to check.

**3 a** $\frac{1}{2}$ **b** 40, less than
**c** $\frac{11}{20}$ **d** $\frac{1}{5}$
**e** 16, the observed frequency was less than expected
**f** $\frac{11}{80}$

**4 a** 50 **b** $\frac{8}{25}$ **c** $\frac{9}{50}$
**d i** 400 **ii** 820

**5 c i** $\frac{1}{6}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{2}$
**d i** 12 **ii** 48 **iii** 36
**g i** 167

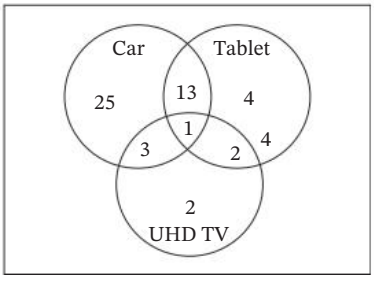
**6** 46%

**7 a** Yes
**b** Yes, 6 comes up much more often.
**c** $13\frac{3}{4}\%$ **d** 11

**8 a i** 0.06 **ii** 0.22 **iii** 0.44
**iv** 0.76 **v** 0.34 **vi** 0.68
**b i** 8 **ii** 5, 9 **iii** 2
**iv** 10 **v** 10 **vi** 6, 7

## Power plus

**1**





9780170479516

**2** **a**

| | | 1st die | | | | | |
|---|---|---|---|---|---|---|---|
| | × | 1 | 2 | 3 | 4 | 5 | 6 |
| 2nd die | 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| | 2 | 2 | 4 | 6 | 8 | 10 | 12 |
| | 3 | 3 | 6 | 9 | 12 | 15 | 18 |
| | 4 | 4 | 8 | 12 | 16 | 20 | 24 |
| | 5 | 5 | 10 | 15 | 20 | 25 | 30 |
| | 6 | 6 | 12 | 18 | 24 | 30 | 36 |

**b** 18 **c** They occur different numbers of times

**d** 6, 12 **e** 1, 9, 16, 25, 36

**f** **i** $\frac{1}{9}$ **ii** $\frac{1}{18}$ **iii** $\frac{2}{9}$

**g** 4

**3** No, the probabilities of the 3 events are not equal.

**4** **a** {Paul and Summet, Paul and Tash, Paul and Nadine, Sumeet and Tash, Sumeet and Nadine, Tash and Nadine}

**b** $\frac{1}{2}$

## Test yourself 8

**1** B

**2** **a** $\frac{1}{6}$ **b** $\frac{2}{3}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$

**3** **a** 5, 6, 7, 8

**b** **i** 25% **ii** 100% **iii** 25% **iv** 0%

**4** **a** Choosing a card that is not an Ace, $\frac{12}{13}$

**b** Rolling a number that is not a factor of 6, $\frac{1}{3}$

**c** Buying a ticket that does not win, $\frac{999}{1000}$

**5** 26%

**6** **a** $\frac{6}{25}$ **b** $\frac{17}{40}$ **c** $\frac{67}{200}$ **d** $\frac{133}{200}$

**7** **a** 73 **b** 16 **c** 50

**d** **i** $\frac{44}{73}$ **ii** $\frac{66}{73}$ **iii** $\frac{7}{73}$

**8** **a** 70 **b** 43 **c** 63

**d** **i** $\frac{1}{10}$ **ii** $\frac{1}{2}$ **iii** $\frac{11}{35}$ **iv** $\frac{27}{70}$

**9**

| | Tall | Not tall | |
|---|---|---|---|
| Blond | 3 | 5 | 8 |
| Not blond | 10 | 12 | 22 |
| | 13 | 17 | 30 |

**10** **a** 0.125 **b** 0.25 **c** 0.75 **d** 0.625

**11** **a** $31\frac{1}{4}\%$ **b** $33\frac{1}{3}\%$ **c** 27

# Chapter 9

## SkillCheck

**1** **a** 7 **b** 7 **c** 8 **d** 27
**e** 6 **f** 23 **g** 5 **h** 3
**i** 10 **j** 5 **k** 8 **l** 20

**2** **a** −3 **b** −5 **c** −12 **d** −2
**e** 3 **f** −11 **g** 36 **h** 3
**i** −10 **j** −8 **k** −9 **l** −72

**3** **a** 17 **b** 17 **c** 4 **d** 4

**4** **a** −6 **b** 7 **c** 6 **d** −14

**5** **a** subtracting **b** multiplying
**c** dividing **d** adding

## Exercise 9.01

**1** **a** $x = 7$ **b** $y = 23$ **c** $m = 12$
**d** $k = 27$ **e** $k = -4$ **f** $s = -2$
**g** $n = -13$ **h** $p = -20$ **i** $m = -8$
**j** $g = 15.5$ **k** $a = 10\frac{1}{4}$ **l** $k = -7.7$

**2** **a** $t = 51$ **b** $w = 11$ **c** $a = 7$
**d** $j = 74$ **e** $q = 29$ **f** $f = 110$
**g** $x = 33$ **h** $g = 11$ **i** $j = -2$
**j** $w = 7$ **k** $m = -15$ **l** $d = 0$
**m** $y = 20.9$ **n** $n = 8$ **o** $b = 1\frac{1}{20}$
**p** $s = 6\frac{5}{6}$

**3** **a** $x = 4$ **b** $g = 4$ **c** $y = 9$
**d** $b = 9$ **e** $d = 9$ **f** $c = 9$
**g** $p = 5$ **h** $m = 13$ **i** $x = -5$
**j** $q = -6$ **k** $c = -18$ **l** $x = 8$
**m** $p = 9\frac{1}{4}$ **n** $r = 2\frac{1}{4}$ **o** $q = -3\frac{1}{3}$
**p** $w = -1\frac{1}{3}$

**4** **a** $s = 15$ **b** $m = 42$ **c** $d = 120$
**d** $w = 58$ **e** $a = -12$ **f** $f = -18$
**g** $r = -15$ **h** $m = -7$ **i** $x = 48$
**j** $t = 1\frac{1}{2}$ **k** $n = 16$ **l** $x = -\frac{1}{2}$

**5** C **6** Teacher to check.

## Exercise 9.02

**1** D

**2** **a** $m = 6$ **b** $x = 6$ **c** $k = 8$
**d** $w = 6$ **e** $h = -3$ **f** $d = -2\frac{3}{4}$
**g** $x = 7$ **h** $a = -1$ **i** $c = 2\frac{1}{2}$
**j** $p = 8$ **k** $y = 10$ **l** $r = 15\frac{1}{2}$

ANSWERS

**3** **a** $m = 18$ **b** $c = 15$ **c** $d = 42$
**d** $k = -27$ **e** $h = 20$ **f** $x = -24$
**g** $n = 65$ **h** $t = 12$ **i** $a = -32$
**j** $x = 18$ **k** $v = 90$ **l** $b = 21$

**4** D

**5** **a** $x = 6$ **b** $x = 9$ **c** $x = 12$
**d** $N = 5$ **e** $N = 21$ **f** $B = -4$
**g** $x = -4$ **h** $x = -10$ **i** $m = 4\frac{2}{5}$
**j** $x = -24$ **k** $x = -1\frac{1}{4}$ **l** $x = 15$

**6** C

**7** **a** $x = 5$ **b** $x = 9$ **c** $N = 3$
**d** $N = 23$ **e** $N = 22$ **f** $x = -5$
**g** $k = -5$ **h** $m = 31$

**8** **a** $x = 3$ **b** $p = -2$ **c** $r = -6\frac{2}{3}$ **d** $u = -7$
**e** $d = 4$ **f** $y = -5$ **g** $f = 2\frac{1}{5}$ **h** $a = 4$

**9** Teacher to check.

## Exercise 9.03

**1** A **2** C

**3** **a** $a = 6$ **b** $k = 4$ **c** $x = 3$
**d** $p = 3$ **e** $n = 3$ **f** $q = 6$
**g** $y = 0$ **h** $a = 7$ **i** $r = 6$

**4** D

**5** **a** $a = 3$ **b** $x = 2\frac{2}{3}$ **c** $d = 1$
**d** $p = 1$ **e** $m = 5\frac{1}{4}$ **f** $x = 5$
**g** $x = \frac{2}{3}$ **h** $r = -1$ **i** $y = 4\frac{1}{2}$

**6** D

**7** **a** $d = -2$ **b** $k = \frac{2}{3}$ **c** $p = -2$
**d** $x = -3$ **e** $k = 2\frac{1}{5}$ **f** $m = 5\frac{1}{2}$
**g** $t = 1\frac{1}{2}$ **h** $j = -8\frac{1}{2}$ **i** $q = -1$

**8** Teacher to check.

## Exercise 9.04

**1** **a** $x = 1$ **b** $m = 4$ **c** $k = 6$
**d** $p = 4$ **e** $j = 3$ **f** $d = 5$
**g** $x = 9$ **h** $x = 8$ **i** $x = 3$
**j** $x = 8$ **k** $k = 12$ **l** $q = 20$

**2** **a** $x = 3$ **b** $p = -4$ **c** $q = -3$
**d** $h = 11$ **e** $r = -8$ **f** $e = 11$
**g** $z = 1$ **h** $y = -7$ **i** $a = 6$
**j** $d = -7$ **k** $k = 9$ **l** $c = 10$

**3** B

**4** **a** $x = -1$ **b** $r = 1$ **c** $p = -7$
**d** $x = -2$ **e** $p = 5$ **f** $e = 6$
**g** $y = 1$ **h** $a = 4$ **i** $w = -6$

**5** A

**6** **a** $x = 13$ **b** $x = 6$ **c** $k = -18$
**d** $y = 2$ **e** $v = 4$ **f** $x = 11$
**g** $p = 4$ **h** $s = 7$ **i** $d = -22$

**7** Teacher to check.

## Mental skills 9

**2** **a** 160 **b** 70 **c** 240 **d** 900
**e** 2600 **f** 900 **g** 140 **h** 300
**i** 180 **j** 770 **k** 18 **l** 34
**m** 46 **n** 26 **o** 18 **p** 12
**q** 40 **r** 8 **s** 14 **t** 24

## Exercise 9.05

**1** **a** $x = \pm 9$ **b** $x = \pm 12$ **c** $x = \pm 1$
**d** $x = \pm 13$ **e** $m = \pm 71$ **f** $u = \pm 43$

**2** **a** $x \approx \pm 3.61$ **b** $x \approx \pm 7.35$ **c** $x \approx \pm 9.38$
**d** $t \approx \pm 11.36$ **e** $h \approx \pm 30.76$ **f** $z \approx \pm 22.96$

**3** **a** $x = \pm\sqrt{41}$ **b** $x = \pm\sqrt{30}$
**c** $x = \pm\sqrt{48}$ or $\pm 4\sqrt{3}$ **d** $a = \pm\sqrt{126}$ or $\pm 3\sqrt{14}$
**e** $p = \pm\sqrt{75}$ or $\pm 5\sqrt{3}$ **f** $b = \pm\sqrt{509}$

**4** $x^2 = 0$, with solution $x = 0$

**5** It is not possible to find the square root of a negative number.

**6** For example, $x^2 = -1$ and $x^2 = -12$

**7** **a** positive **b** negative **c** zero

**8** **a** $x = \pm 3$ **b** $x = \pm 4$ **c** $x = \pm 5$ **d** $x = \pm 2$
**e** $x = \pm 2$ **f** $x = \pm 3$ **g** $x = \pm 5$ **h** $x = \pm 6$

## Exercise 9.06

**1** **a** \$88 **b** \$1.80 **c** 33 **d** 16

**2** **a** D, 102 **b** B, 49 **c** D, 56
**d** B, 588 **e** C, \$8974

**3** **a** 75 **b** 49 **c** 27 **d** 16
**e** 12 **f** 18 **g** 15 **h** 6

**4** **a** 150 **b** 8 **c** 100 kg
**d** 6 **e** 54 **f** 16 cm

**5** **a** 77°F **b** $42\frac{2}{9}$°C

**6** **a** \$600 **b** 122

**7** **a** \$13.90 **b** 28

**8** **a** $x = 8\frac{1}{2}$ **b** 17 cm **c** $6\frac{1}{2}$ cm

**9** **a** \$2610 **b** 220

**10** $x = 7$

## Exercise 9.07

**1** A

**2** **a** 13 **b** 107

**3** **a** 10 hours **b** 18 years old

**4** **a** 34 **b** 11 **c** 13

**5** **a** 132 **b** 6 **c** 15 **d** 15

**6** **a** $58\frac{1}{3}$ **b** 30 **c** 9

**7** **a** \$125 **b** \$400 **c** 4% p.a. **d** 7 years

**8** **a** 3 **b** 6 **c** 45 **d** 15

## Exercise 9.08

**1** **a** $a = \frac{F}{m}$ **b** $x = y - 3$ **c** $d = 2A$ **d** $V = \frac{120}{f}$

**2** D

**3** $x = \frac{p+2}{a}$

**4** $q = \frac{2p}{5}$

**5** $C = \frac{5}{9}(F-32)$ or $C = \frac{5F-160}{9}$

**6** $w = \frac{p-2l}{2}$ or $w = \frac{1}{2}(p-2l)$ or $w = \frac{p}{2} - l$

**7** $v = at + u$

**8** **a** $T = \frac{D}{S}$ **b** 8 hours

**9** $a = \sqrt{c^2 - b^2}$ (positive square root because the length, $a$, is positive)

**10** $d = \frac{x}{2} + 5$ or $d = \frac{x+10}{2}$

**11** $r = \sqrt{\frac{V}{\pi h}}$

**12** $d = \frac{2A}{h} - a$ or $b = \frac{2A - ha}{h}$

## Power plus

**1** **a** $x = 3$ **b** $x = 2$ **c** $x = 0$
**d** $x = -1$ **e** $x = 2\frac{1}{7}$ **f** $x = 3\frac{1}{11}$
**g** $x = 20$ **h** $x = 26\frac{2}{3}$

**2** **a** $x = 7$ **b** $x = 5$ **c** $x = 65$

**3** **a** $x = 5$ or $-5$ **b** $x = 4$ or $-4$ **c** $x = 6$ or $-6$
**d** $x = 2$ or $-2$ **e** $x = -7$ or $-1$ **f** $x = 3$ or $-5$

**4** $\frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + 4 = x$, 84 years

## Test yourself 9

**1** **a** $k = 7$ **b** $x = 11$ **c** $a = 14$
**d** $a = 33$ **e** $x = 4$ **f** $f = 12$
**g** $m = 28$ **h** $x = 24$ **i** $w = -6$
**j** $k = -2$ **k** $m = 8$ **l** $x = -36$

**2** **a** $p = 5$ **b** $m = -3$ **c** $x = 15$
**d** $h = 77$ **e** $n = -8$ **f** $a = 60$
**g** $y = 10$ **h** $x = 10$ **i** $n = 25$
**j** $x = -6$ **k** $k = 26$ **l** $d = -29$

**3** **a** $x = 1$ **b** $u = 3$ **c** $h = 3$
**d** $v = -3$ **e** $x = 2\frac{3}{5}$ **f** $t = 3$

**4** **a** $n = 6$ **b** $x = 4$ **c** $x = 2$
**d** $x = 7$ **e** $y = 4\frac{2}{3}$ **f** $d = 6$
**g** $r = 21$ **h** $a = -2$ **i** $n = 3$

**5** **a** $x = \pm 8$ **b** $x = \pm 6.7$ **c** $x = \pm\sqrt{27}$ (or $\pm 3\sqrt{3}$)

**6** **a** 5 **b** 3

**7** **a** B **b** $n = 6$

**8** $x = 3$

**9** **a** 1260° **b** 8

**10** **a** $R = \frac{V}{I}$ **b** $q = \frac{p+5}{2}$ **c** $y = \frac{x+6}{2}$ or $y = \frac{x}{2} + 3$

## Practice set 3

**1** **a** $a = 1$ **b** $m = 3$ **c** $x = -5$
**d** $t = 18$ **e** $k = 77$ **f** $x = 2$
**g** $x = 9$ **h** $x = -8$ **i** $a = 13$

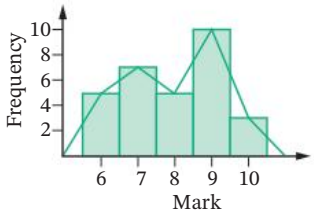
**2** **a** 25% **b** 75% **c** 75%

**3** **a**

| Mark | Tally | Frequency |
|---|---|---|
| 6 | 𝍸 | 5 |
| 7 | 𝍸 \|\| | 7 |
| 8 | 𝍸 | 5 |
| 9 | 𝍸 𝍸 | 10 |
| 10 | \|\|\| | 3 |

**b** 30

**c**

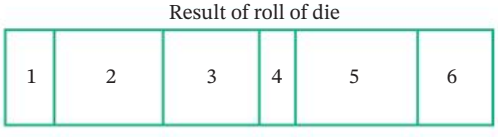



**4** D

**5** **a**

| Score | Frequency |
|---|---|
| 1 | 4 |
| 2 | 9 |
| 3 | 8 |
| 4 | 3 |
| 5 | 10 |
| 6 | 6 |
| Total | 40 |

**b**

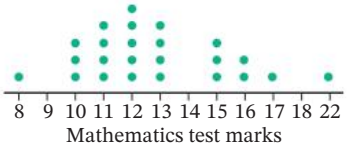



**6** **a** numerical **b** categorical **c** categorical

**7** **a** continuous **b** continuous **c** discrete

**8** **a** $\frac{1}{2}$ **b** $\frac{5}{6}$ **c** $\frac{1}{3}$

**9**




**a** 12 **b** 12 **c** 22 **d** 10–13

**10** **a** ordinal **b** nominal **c** ordinal

**11** **a** 92.5 **b** 90 **c** 92 **d** 8

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**12** $\frac{7}{10}$

**13 a**

| $x$ | $f$ | $fx$ |
|---|---|---|
| 28 | 6 | 168 |
| 29 | 1 | 29 |
| 30 | 6 | 180 |
| 31 | 2 | 62 |
| 32 | 3 | 96 |
| 33 | 2 | 66 |
| 34 | 4 | 136 |
| | 24 | 737 |

**b** $\frac{7}{24}$

**c i** 30.7 **ii** 30 **iii** 28, 30 **iv** 6

**14** D

**15** 1 kg

**16 a**

| | Plays weekend sport | Does not play weekend sport | Total |
|---|---|---|---|
| Male | 49 | 23 | 72 |
| Female | 39 | 45 | 84 |
| Total | 88 | 68 | 156 |

**b** 156 **c** 88

**d i** $\frac{7}{13}$ **ii** $\frac{31}{78}$

**17 a**

| Maths | | Science |
|---|---|---|
| 3 | 2 | 4 |
| 5 5 | 3 | |
| 8 3 | 4 | 0 0 3 |
| 8 8 2 | 5 | 4 |
| | 6 | 0 2 9 |
| | 7 | 3 |
| 7 | 8 | 8 |
| 6 | 9 | 9 |

**b**

| | Median | Range |
|---|---|---|
| Maths | 50 | 73 |
| Science | 57 | 64 |

**c** Maths, lower median.

**18 a** $\frac{3}{64}$ **b** $\frac{5}{16}$ **c** $\frac{55}{64}$ **d** $\frac{37}{64}$

**19 a** Sample – quick, simple, inexpensive

**b** Teacher to check

**20** $h = 12$

**21 a** $\frac{39}{70}$ **b** 18% **c** 8

# Chapter 10

## SkillCheck

**1 a** 200 **b** 180 **c** 3 **d** 2500
**e** 3.8 **f** 0.18 **g** 180 **h** 85
**i** 8 **j** 7.5 **k** 9150 **l** 384

**2 a** 6 **b** 7 **c** 8

**3 a** 21 **b** 20 **c** 2

**4 a** $\frac{2}{5}$ **b** $\frac{1}{2}$ **c** $\frac{4}{5}$ **d** $\frac{5}{12}$

**5 a** 8 **b** 5 **c** 18 **d** 30
**e** 760 **f** 54 **g** 39 **h** 250

## Exercise 10.01

**1 a** 6 : 6 or 1 : 1 **b** 2 : 6 or 1 : 3 **c** 6 : 10 or 3 : 5
**d** 8 : 12 or 2 : 3 **e** 5 : 3

**2 a** 12 **b** 10 **c** 9 **d** 20 **e** 32
**f** 84 **g** 30 **h** 75 **i** 5 **j** 5
**k** 4 **l** 1 **m** 30 **n** 2 **o** 40
**p** 5 **q** 5 **r** 25

**3** D

**4 a** 1 : 10 **b** 1 : 2 **c** 2 : 5
**d** 5 : 7 **e** 3 : 2 **f** 7 : 3
**g** 10 : 1 **h** 5 : 11 **i** 1 : 1
**j** 41 : 107 **k** 3 : 1 **l** 1 : 16
**m** 2 : 3 : 5 **n** 3 : 4 : 6 **o** 3 : 1 : 4
**p** 2 : 5 : 3 : 7

**5 a** 5 : 6 **b** 3 : 4 **c** 9 : 8 **d** 4 : 3
**e** 4 : 3 **f** 8 : 5 **g** 5 : 2 **h** 4 : 3
**i** 12 : 7 **j** 8 : 5 **k** 25 : 12 **l** 9 : 5
**m** 4 : 7 **n** 13 : 8 **o** 5 : 3 **p** 1 : 2
**q** 3 : 4 **r** 3 : 2 **s** 1 : 4 **t** 1 : 2
**u** 3 : 2 **v** 10 : 7 **w** 2 : 1 : 3 **x** 45 : 10 : 9

**6 a** 1 : 4 **b** 1 : 4 **c** 5 : 49 **d** 1 : 4
**e** 1 : 3 **f** 4 : 1 **g** 5 : 16 **h** 1 : 50
**i** 80 : 7 **j** 5 : 36 **k** 9 : 500 **l** 1 : 6
**m** 6 : 1 **n** 3 : 20 **o** 9 : 40

**7** 3 : 2

**8 a** 4 : 3 **b** 3 : 4 **c** 4 : 7

**9 a** 3 : 2 **b** 3 : 5 **c** 5 : 2

**10** C

**11 a** 16 : 19 **b** 19 : 16 **c** 3 : 19 **d** 19 : 3

**12 a** 1 : 1 **b** 2 : 1 **c** 1 : 2 **d** 2 : 1

**13 a** $\frac{5}{8}$ **b** 5 : 3

**14** A

**15** Australia 3 : 250, India 11 : 500

## Exercise 10.02

**1** C **2** 75 **3** 560 kg
**4** 32 cm **5** 1440 **6** 17.5 km/h
**7 a** 20 cm **b** 136 cm
**8** 50 kg
**9 a** Toudi $990, Felicity $540 **b** $2250
**10** D
**11 a** 5 kg **b** Heavier packet by 10 c/kg
**12** 25.6 t **13** 1.5 m **14** A
**15 a** 7 mL **b** 84 mL

## Exercise 10.03

**1 a** 1 : 500 000 **b** 1 : 1000 **c** 1 : 50 000 000
**d** 1 : 2 500 000 **e** 1 : 10 000 **f** 1 : 200 000
**g** 1 : 25 000 **h** 1 : 40 000 **i** 1 : 50 000
**j** 1 : 10 000

**2** B

**3** **a** 1 : 20 000
**b** **i** 1.4 km **ii** 1.9 km **iii** 2.48 km
**c** **i** 90 cm **ii** 7.5 cm **iii** 48 cm

**4** 6670 km **5** 66 cm **6** 5.6 cm

**7** **a** **i** 880 m **ii** 680 m
**iii** 440 m **iv** 960 m
**b** **i** 880 m **ii** 440 m
**c** 360 m **d** 120 m by 280 m
**e** 160 m **f** 20 cm, Teacher to check

**8** **a** 3.9 m **b** 77 cm **c** 2.0 m
**d** 1.2 m **e** 1.6 m
**f** 3.9 m × 2.9 m = 11.31 $m^2$

**9** **a** 12.6 cm **b** 7.2 m
**c** 13.5 cm **d** 67.2 cm

**10** **a** 5.0 m **b** 2.5 m
**c** 2.0 m **d** 2.0 m × 2.5 m = 5 $m^2$
**e** 5.0 m **f** 4.1 m × 3.8 m = 15.58 $m^2$

**11** B

**12** **a** 0.45 mm **b** 11 mm
**c** 26.25 mm **d** 0.37 mm

## Mental skills 10

**2** **a** 8:45 a.m. **b** 1:20 p.m. **c** 5:50 p.m.
**d** 12:17 a.m. **e** 9:05 p.m. **f** 6:32 p.m.
**g** 11:15 a.m. **h** 2:38 a.m. **i** 2:40 p.m.
**j** 3:20 a.m. **k** 4:55 p.m. **l** 11:31 p.m.
**m** 1:08 a.m. **n** 10:18 a.m. **o** 8 p.m.
**p** 6:43 a.m.

**4** **a** 18:35 **b** 08:05 **c** 11:45 **d** 23:20
**e** 02:21 **f** 12:30 **g** 15:48 **h** 19:11
**i** 09:08 **j** 21:50 **k** 00:42 **l** 07:39
**m** 01:59 **n** 22:18 **o** 10:46 **p** 17:23

## Exercise 10.04

**1** **a** 9 **b** 5 **c** 7 **d** 13

**2** **a** $400, $100 **b** $350, $150

**3** **a** 1 : 1 : 2
**b** Phuong $1000, Janelle $1000, Ahmet $2000

**4** $180

**5** **a** 200 kg, 250 kg **b** 270 kg, 180 kg

**6** **a** 80 cm, 240 cm, 400 cm
**b** 300 cm, 180 cm, 240 cm

**7** D **8** 125 **9** 25

**10** 600 g **11** 1078 **12** C

**13** 80 kg **14** 568

**15** 4 m, 6 m, 10 m

**16** 240 kg

**17** **a** 30 000 **b** 165 000

**18** $140 000, $70 000

## Exercise 10.05

**1** **a** words/min **b** beats/min **c** c/min
**d** $/kg **e** $/h **f** m/s or km/h
**g** persons/year **h** c/kL **i** persons/$km^2$

**2** **a** 17 sheep/h **b** $4.30/kg
**c** 26 students/teacher **d** 34 points/game
**e** 91 words/min **f** 8.5 articles/h
**g** 5.1 cars/day **h** 630 boxes/h
**i** 37 m/s **j** 920 bottles/h
**k** 1324 revolutions/min **l** $35/h
**m** 56 km/h **n** $2.75/kg
**o** 4.75 runs/over **p** 8.1 km/L
**q** $10.50/h **r** 42 kg/ha

**3** B

**4** **a** Can travel 100 km in one hour at this speed.
**b** 150 cars will pass in an hour at this rate.
**c** 10.3 L can be used over a distance of 100 km.
**d** For each hectare of land, there are 60 sheep.

**5** 80 kg/person **6** 1800 ha/year

**7** 139 c/L **8** 182.5

## Exercise 10.06

**1** **a** 2 kg box **b** 8 notepads **c** 150 g
**d** 7 tubs **e** 3 kg **f** 2.25 L carton

**2** Kim **3** 750 g box **4** C

**5** **a** 375 g **b** 2 L **c** 735 g **d** 115 g

**6** **a** medium **b** large **c** small **d** large

## Exercise 10.07

**1** $919.60 **2** 160 runs

**3** **a** 10.5 km/L **b** 210 km

**4** 336 **5** 432 mL **6** 6

**7** $3980 **8** $7\frac{1}{7}$ ha or 7.1 ha

**9** **a** $70 **b** 2.5 h

**10** 300 km

**11** **a** 14.8 L **b** 133.2 km

**12** C

**13** **a** 17°C, 29°C **b** 12 p.m.

**14** **a** 2400 $m^2$ **b** 40 $m^2$/min
**c** 60 $m^2$/min **d** 24 min (at 100 $m^2$/ min)

**15** **a** 5400 s **b** 90 min **c** 1.5 hr

**16** **a** 650 km **b** 48 L **c** 13.5 km

## Exercise 10.08

**1** **a** 5 km/h **b** 480 km/h **c** 5 km/h
**d** 17 km/h **e** 100 km/h **f** 80 km/h
**g** 120 km/h **h** 120 km/h **i** 200 km/h
**j** 9 km/h **k** 36 km/h **l** 6 km/h

**2** **a** 10 m/s **b** 1.5625 m/s **c** 58 m/s
**d** 8 m/s **e** 1.5 m/s

**3** **a** 720 km **b** 30 km **c** 1330 km

**4** 3 km **5** B

**6** **a** 100 m/min **b** 6 km/h

**7** 40 km/h **8** D **9** 1200

**10** 1.5 h **11** Teacher to check.

**12** 63.9 km/h **13** 56 km/h

**14** 54.05 s **15** 500 m

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## Exercise 10.09

**1** **a** $7702/month **b** 1.25 km/min
**c** 120 L/day **d** 180 m/min
**e** 48 cm/h **f** 1.3 words/s
**g** 0.1 L/km **h** 105 000 mL/min

**2** **a** 5.6 m/s **b** 13.9 m/s **c** 22.2 m/s **d** 30.6 m/s

**3** **a** $14.40/h **b** 28.8 km/h **c** 54 L/h
**d** 36 c/min **e** $16.7 \text{ or } 16\frac{2}{3}$ m/s **f** 2.1 kg/min

**4** **a** 4.8 km/h **b** 1.3 m/s

**5** **a** 45 km/h **b** 144 km/h **c** 3.5 km/h

**6** 40 kg/ha

**7** 1.8 L/h

**8** **a** 1.2 g/mm **b** 1.2 t/km
**c** the same, in both cases × 100 and ÷ 1000
**d** 12 g/cm

**9** **a** 3500 L/h **b** 58.3 L/min **c** 1.0 L/s

**10** **a** $2.80/L **b** 0.28 c/mL

**11** **a** 10.44 m/s **b** 38 km/h

**12** 161.0 km/h

**13** Teacher to check.

## Exercise 10.10

**1** **a** 4 h **b** 8 km
**c** **i** $3\frac{1}{3}$ km/h **ii** 3 km/h
**d** steeper graph **e** 16 km
**f** 11:30 a.m. to 1 p.m., 2 p.m. to 3 p.m.
**g** 3 p.m. **h** 1 h

**2** **a** 300 km **b** 4 h
**c** 1 h **d** 100 km/h
**e** 100 km **f** Neither: the same speed.

**3** **a** **i** 2 p.m. **ii** 11 a.m.
**b** 12:45 p.m. **c** 48 km **d** 4.5 h
**e** **i** 16 km/h **ii** 6.4 km/h
**iii** 9.6 km/h **iv** 9.6 km/h

**4** **a** 240 km
**b** Sam, steeper graph, ending journey earlier.
**c** 120 km **d** 180 km
**e** 10 a.m., 160 km
**f** After 9:15 a.m., graph is steeper.
**g** 40 km/h **h** B

**5** **a** A **b** B **c** D **d** C

**6** **a** 1600 km **b** 7 p.m.
**c** 350 km/h **d** D
**e** see below
**f** 10:10 a.m.

**7** Teacher to check.

**8** C, not possible to be at 2 different places/distances at the same time.

## Power plus

**1** 319 912

**2** **a** 704 km$^2$ **b** 63 071 683 000

**3** Teacher to check.

**4** **a** 97.2 km/h **b** 349.2 km/h
**c** 11.88 km/h **d** 82.8 km/h

**5** $y = 3.6x$

## Test yourself 10

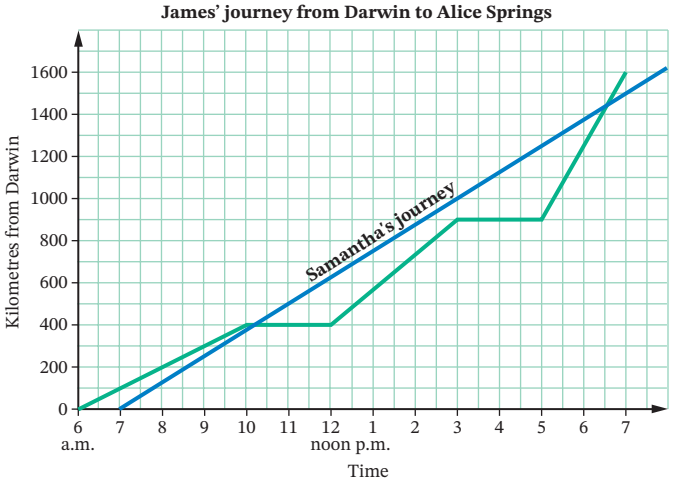
**1** **a** 4 : 12 **b** 6 : 15 **c** 9 : 21
**d** 20 : 45 **e** 2 : 3.2 **f** 15 : 20 : 25

**2** **a** 4 : 7 **b** 1 : 3 **c** 1 : 6 **d** 5 : 9
**e** 3 : 1 : 4 **f** 1 : 5 : 20 **g** 2 : 3 **h** 4 : 3
**i** 3 : 5 **j** 4 : 1 **k** 1 : 4 **l** 65 : 4
**m** 25 : 1 **n** 50 : 9 **o** 100 : 17 **p** 3 : 8
**q** 2 : 7 **r** 1 : 9

**3** **a** 100 **b** $45 000

**4** **a** 67 cm **b** 12 cm

**5** **a** 1 : 25 000
**b** **i** 625 m **ii** 1.1 km or 1100 m
**c** **i** 3.5 cm **ii** 10 cm





9780170479516

**6** B

**7** **a** \$3.50/kg **b** 110 km/h
**c** \$14.10/h **d** 5.2 runs/over

**8** **a** 1 kg box **b** 350 g can
**c** 1.5 L bottle **d** 500 g bar

**9** **a** \$49.95 **b** \$1204.60 **c** 30.8 L **d** 7.2 ha

**10** 20 km/h **11** 247.5 km

**12** **a** 20.8 m/s **b** 50.4 L/day

**13** **a** 10 km **b** 3 **c** $2\frac{2}{3}$ km/h
**d** $3\frac{1}{2}$ h **e** 2:30 p.m. to 3:30 p.m.

# Chapter 11

## SkillCheck

**1** **a** 4 **b** 5 **c** 2
**d** 2 **e** 4 **f** 5

**2** **a** 4 **b** 6 **c** −3
**d** 1 **e** −3 **f** 7

**3** **a** $x = 3$ **b** $x = -2$ **c** $x = -3$
**d** $x = -2$ **e** $x = -1\frac{1}{2}$ **f** $x = 3$

## Exercise 11.01

**1** **a** 2, 6, −2, −9, 4 **b** 24, −9, 6, 30, −12
**c** $4, -1, -3, 0, 3\frac{1}{2}$ **d** 5, 14, 7, 20, −2
**e** 16, −2, 0, 8, 4 **f** −7, 4, 2, −11, 8

**2** A

**3** **a** 14, 4, 19, 39, −31 **b** 10, −5, 28, 4, −20
**c** 6, −6, 42, −14, 22 **d** 22, 37, −23, 32, −8
**e** −3, −1, 1, 3, 5, 7 **f** $-3, -2\frac{1}{2}, -2, -1\frac{1}{2}, -1, -\frac{1}{2}$
**g** 2, 5, 8, 11, 14, 17 **h** 3, 2, 1, 0, −1, −2

**4** B **5** D

## Exercise 11.02

**1** **a** $y = 3x$ **b** $y = -2x$ **c** $y = x + 4$
**d** $y = x - 4$ **e** $y = 10 - x$ **f** $y = x \div 2$
**g** $k = p + 4$ **h** $y = x - 3$ **i** $j = 2h$
**j** $p = m \div 5$

**2** **a** $h = 3f - 2$, 19, 22 **b** $p = 5m - 3$, 27, 42
**c** $b = 3m + 3$, 18, 27 **d** $k = 2h + 2$, 20, 24
**e** $s = 3r + 1$, 13, 22 **f** $b = 2a - 2$, 18, 20
**g** $n = 3m + 5$, 20, 23 **h** $d = 10c + 1$, 91, 101
**i** $x = 5w - 1$, 34, 39 **j** $z = 2y - 6$, 12, 14
**k** $m = 4a + 1$, 29, 37 **l** $t = 2z - 7$, 11, 17

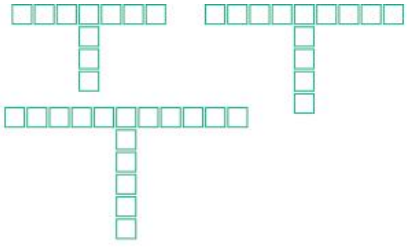
**3** B **4** C

**5** **a** $y = -2x - 1$, −19 **b** $c = \frac{1}{2}A$, 5
**c** $t = -5s + 13$, −22 **d** $v = -10t + 20$, 0, −30
**e** $D = -\frac{1}{2}x - \frac{1}{2}, -2, -3\frac{1}{2}$

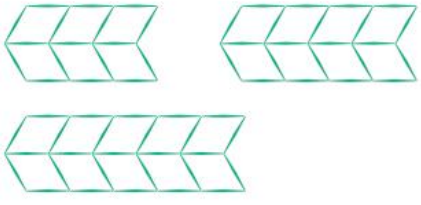
## Exercise 11.03

**1** B

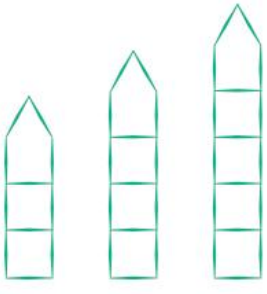
**2** **a** **i** 15 **ii** $T = 2n - 1$ **iii** 15, 39
**b** **i** 16 **ii** $T = 2n$ **iii** 16, 40
**c** **i** 22 **ii** $T = 2n + 6$ **iii** 22, 46
**d** **i** 28 **ii** $T = 3n + 4$ **iii** 28, 64
**e** **i** 63 **ii** $T = 8n - 1$ **iii** 63, 159
**f** **i** 37 **ii** $T = 5n - 3$ **iii** 37, 97
**g** **i** 5 **ii** $T = -2n + 21$ **iii** 5, −19
**h** **i** 64 **ii** $T = n^2$ **iii** 64, 400

**3** **a**


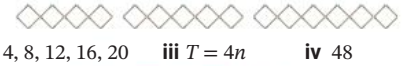
**b** 7, 10, 13, 16, 25, 34
**c** The number of tiles equals 3 times the arm length plus 1.
**d** $t = 3a + 1$
**e** **i** 46 **ii** 100

**4** **a**


**b** 12, 17, 22, 32, 47, 52
**c** The number of toothpicks equals 5 times the number of the flight shape plus 2.
**d** $t = 5f + 2$
**e** **i** 127 **ii** 202

**5** **a**


**b** 5, 8, 11, 14, 17, 26, 35
**c** The number of toothpicks equals 3 times the number of stages plus 2.
**d** $t = 3s + 2$
**e** **i** 38 **ii** 137

**6** **a** **i**


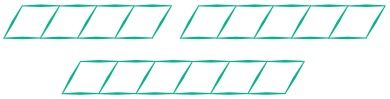
**ii** 4, 8, 12, 16, 20 **iii** $T = 4n$ **iv** 48

**b** **i**


**ii** 6, 12, 18, 24, 30 **iii** $T = 6n$ **iv** 72

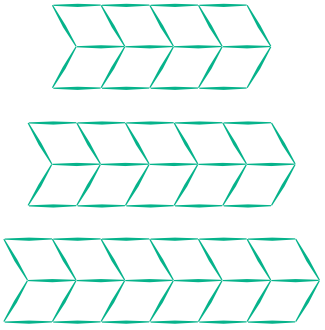
ANSWERS

**c i**



**ii** 4, 7, 10, 13, 16 **iii** $T = 3n + 1$ **iv** 37

**d i**



**ii** 6, 10, 14, 18, 22 **iii** $T = 4n + 2$ **iv** 50

**7** A

**8** **a i** 6 **ii** 8 **iii** 10 **iv** 26
**b** $C = 2T + 2$ **c** 102 **d** 14

**9** **a i** 8 **ii** 11 **iii** 14 **iv** 32
**b** $n = 3d + 2$ **c** 56 **d** 9

## Exercise 11.04

**1** **a** P **b** N **c** K **d** T **e** M
**f** S **g** Q **h** R **i** L **j** U

**2** **a** (4, 1) **b** (−2, 4) **c** (−3, −2) **d** (1, −3)
**e** (4, 4) **f** (−5, 0) **g** (0, 1) **h** (0, −2)
**i** (5, 0) **j** (3, −4)

**3** **a** 2nd **b** 4th **c** 1st
**d** 2nd **e** 3rd

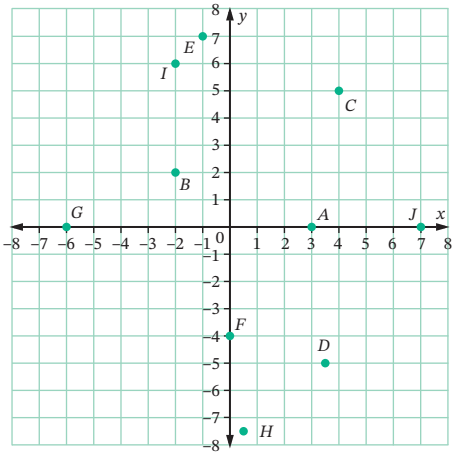
**4** $F(-5, 0)$, $R(-1\frac{1}{2}, 0)$, $O(0, 0)$, $Q(2, 0)$, $I(5, 0)$

**5** $M(0, 3)$, $G(0, 1)$, $O(0, 0)$, $H(0, -2)$, $S(0, -4)$

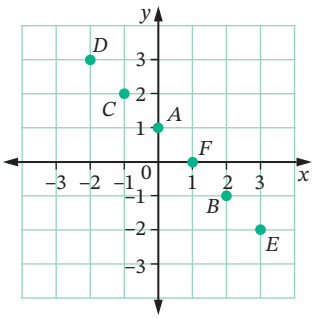
**6** D

**7** **a** 4th **b** 3rd **c** 2nd **d** 1st

**8**




**9** **b**




**c** They all lie on the same line.

**d** Teacher to check.

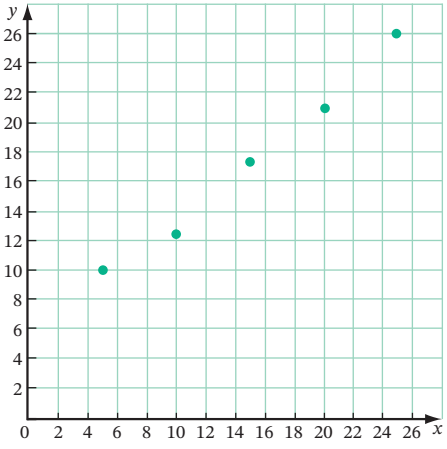
## Mental skills 11

**2** **a** \$700 **b** \$800 **c** 400 **d** 700
**e** \$300 **f** \$400 **g** 250 **h** \$300
**i** \$300 **j** 500

## Exercise 11.05

**1** C

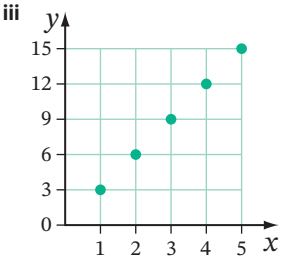
**2**




There is a rough linear pattern but it is not perfect.

**3** **a** **i** 3, 6, 9, 12, 15 **ii** $y = 3x$

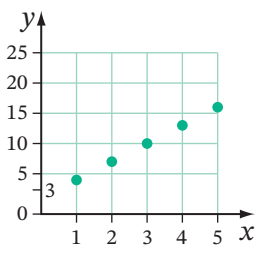
**iii**




**b** **i** 4, 7, 10, 13, 16 **ii** $y = 3x + 1$

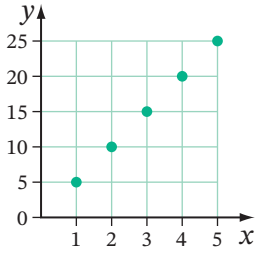

9780170479516

**iii**



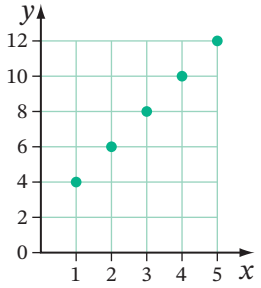

**c i** 5, 10, 15, 20, 25 **ii** $y = 5x$

**iii**



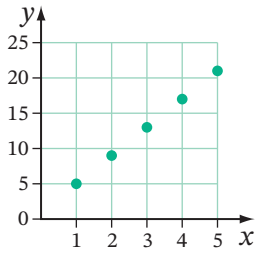

**d i** 4, 6, 8, 10, 12 **ii** $y = 2x + 2$

**iii**




**e i** 5, 9, 13, 17, 21 **ii** $y = 4x + 1$

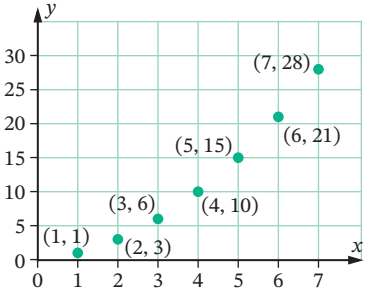
**iii**




**4** Yes

**5 a** 1, 3, 6, 10, 15, 21, 28

**b**




**c** No **d** C

**6 a**

| $S$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $B$ | 1 | 5 | 9 | 13 | 17 | 21 |

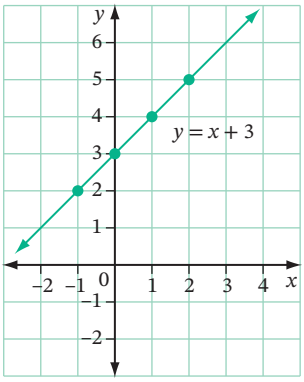
**b** $B = 4S - 3$

**c i** 37 **ii** 93

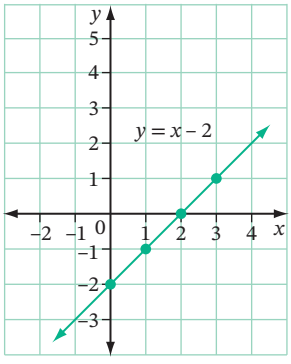
**d** 27

## Exercise 11.06

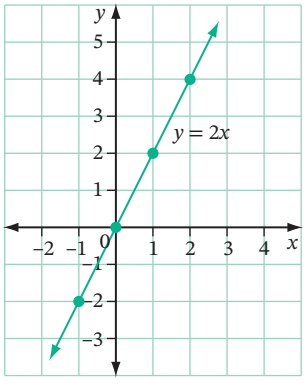
**1 a** 2, 3, 4, 5



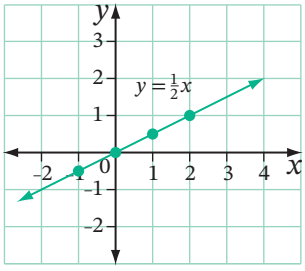

**b** −2, −1, 0, 1



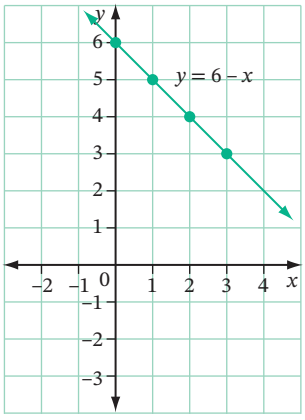

**c** −2, 0, 2, 4



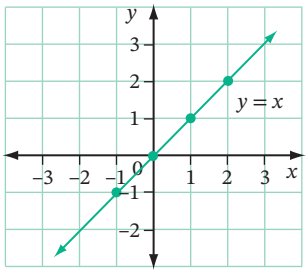
**d** $-\frac{1}{2}$, 0, $\frac{1}{2}$, 1



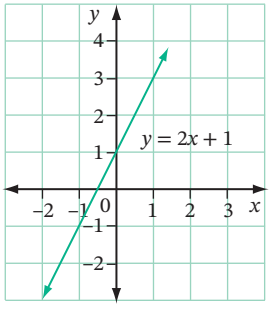

**e** 6, 5, 4, 3



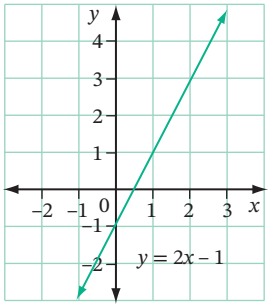

**f** −1, 0, 1, 2



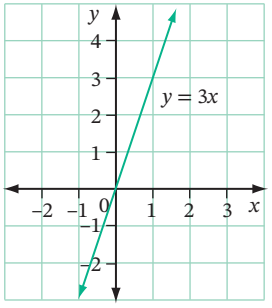

**2** **a**



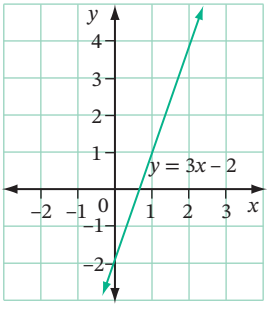

**b**



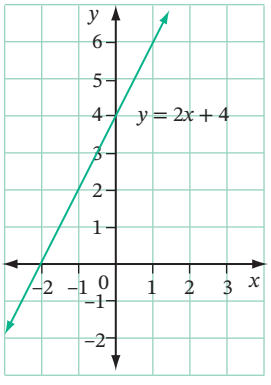

**c**




**d**



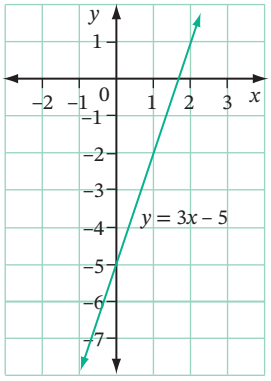

**e**





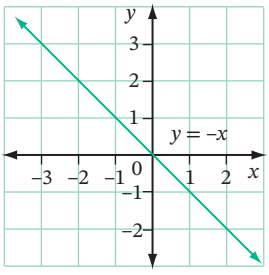
9780170479516

**f**

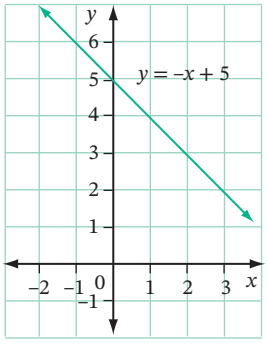



**3** From left to right, the line is rising, going up.

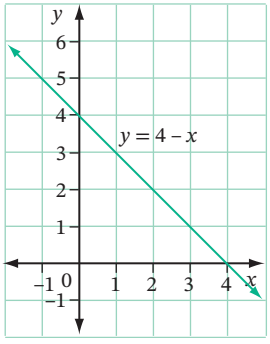
**4** **a**



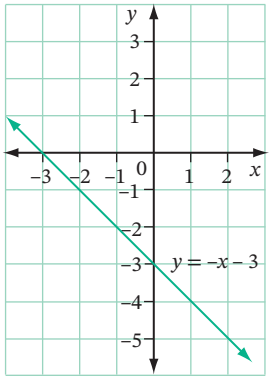

**b**



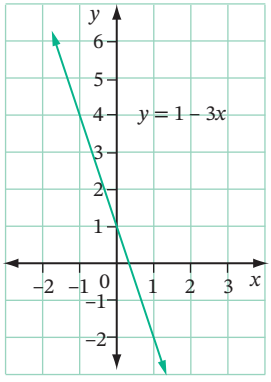

**c**



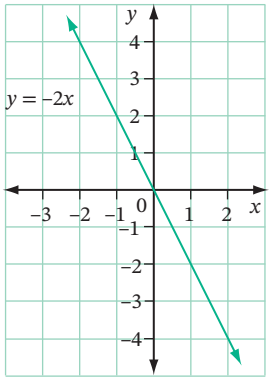

**d**




**e**




**f**




**5** From left to right, the line is falling, going down.

**6** **a** increasing **b** increasing **c** decreasing
**d** increasing **e** decreasing **f** decreasing

**7** **a** yes **b** no **c** yes
**d** yes **e** yes **f** no

**8** D

## Exercise 11.07

**1** D

**2** **a** $-4, -3, -2, -1, y = x - 3$
**b** $-4, -3, -2, -1, y = x - 2$
**c** $-2, 0, 2, 4, y = 2x$
**d** $0, \frac{1}{2}, 1, 1\frac{1}{2}, y = \frac{1}{2}x$
**e** $1, 0, -1, -2, y = -x$

**f** 5, 4, 3, 2, $y = 2 - x$

**g** $-3, -1, 1, 3, y = 2x - 1$

**h** $1, 1\frac{1}{2}, 2, 2\frac{1}{2}, y = \frac{1}{2}x + 2$

**3** **a** They are all vertical lines

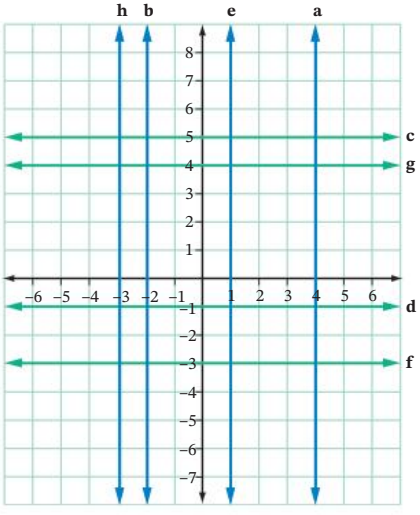
**b** They are all horizontal lines

**4** **a** $y = 6$ **b** $x = -4$ **c** $y = 3$

**d** $x = 5$ **e** $y = -5$ **f** $x = 3$

**g** $y = -3$ **h** $x = -1$

**5**




**6** B **7** $y = \frac{1}{2}x + 2$

## Exercise 11.08

**1** **a** 4, 1 **b** $\frac{1}{2}, -3$ **c** 1, −5 **d** −1, 6

**2** **a** point in the same direction, parallel

**b** coefficient of $x$ is 1

**c** different $y$-intercepts

**d** different constant terms

**3** **a** all pass through (0, 0), $y$-intercept of 0

**b** constant term of 0

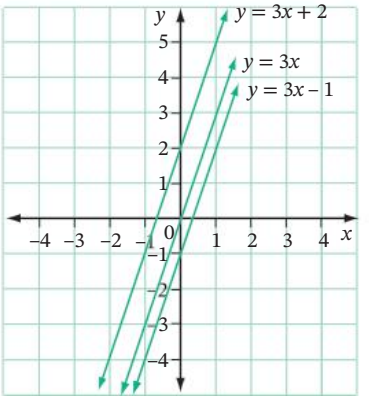
**c** point in different directions

**d** different coefficients of $x$

**e** $y = 3x$, 3 **f** $y = \frac{1}{4}x, \frac{1}{4}$

**g** The greater the size of the coefficient, the steeper the line.

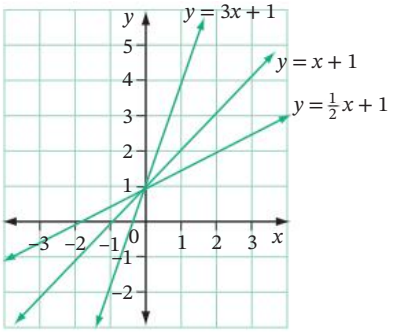
**4**




**a** parallel **b** coefficient of $x$ is 3

**c** $y$-intercepts **d** constant terms

**5**




**a** $y$-intercept of 1

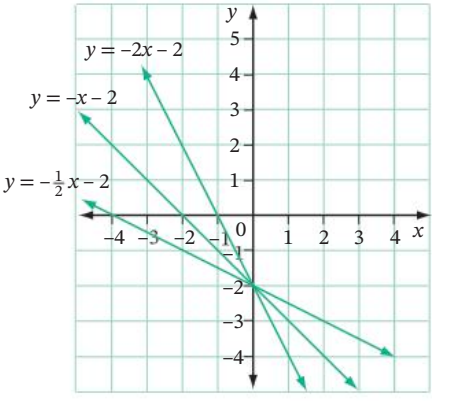
**b** constant term of 1

**c** different steepnesses, directions

**d** coefficients of $x$

**e** 3

**6**




**a** $y$-intercept of −2

**b** constant term of −2

**c** different steepnesses, directions

**d** decreasing lines, pointing downwards

**e** $-\frac{1}{2}$

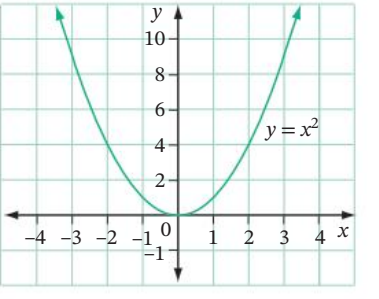
**7** the sign of the coefficients of $x$ **8** B

**9** **a** coefficient of $x$ **b** $y$-intercept

**c** decreasing **d** constant term

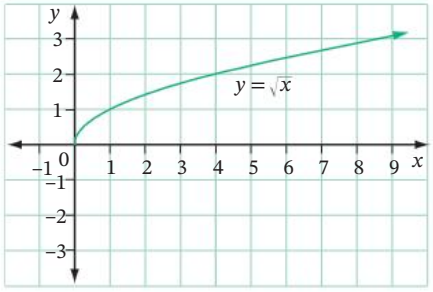
**e** negative

**10** **a** 9, 4, 1, 0, 1, 4, 9

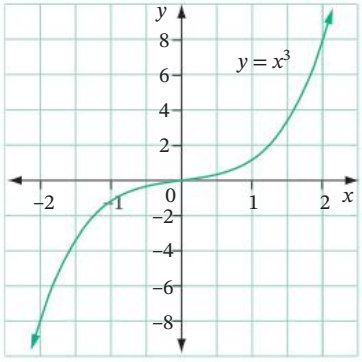


**b** 0, 1, 1.41, 2, 2.24, 2.65, 3

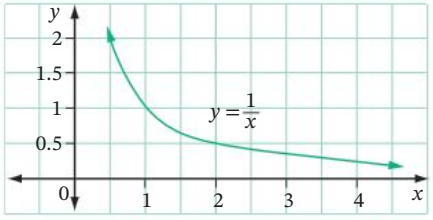
$y = \sqrt{x}$




**c** −8, −3.375, −1, −0.125, 0, 0.125, 1, 3.375, 8




**d** 2, 1, $0.\dot{6}$, 0.5, 0.4, $0.\dot{3}$, 0.29, 0.25, $0.\dot{2}$




**11** **a** Their graphs are not lines, but curves.

**b** $x$ is raised to a power, or square root of $x$, or appears in the denominator of a fraction.

## Exercise 11.09

**1** **a** positive **b** negative **c** negative

**2** **a** 2 **b** −1 **c** $-2\frac{1}{2}$

**3** **a** 2 **b** $-\frac{1}{2}$ **c** 4

4 **a** 3 **b** $\frac{1}{4}$ **c** 0

**d** −1 **e** 1

## Exercise 11.10

**1** **a** $m = 3$, $y$-intercept $= 2$

**b** $m = -2$, $y$-intercept $= 1$

**c** $m = -1$, $y$-intercept $= 6$

**d** $m = \frac{1}{3}$, $y$-intercept $= -1$

**e** $m = -4$, $y$-intercept $= -3$

**f** $m = \frac{2}{5}$, $y$-intercept $= 3$

**g** $m = -\frac{4}{3}$, $y$-intercept $= -8$

**h** $m = \frac{5}{6}$, $y$-intercept $= \frac{1}{2}$

**2** D

**3** **a** $y = 3x + 1$ **b** $y = -2x + 6$

**c** $y = -2x + 11$ **d** $y = -\frac{1}{4}x - 5$

**4** **a** $m = 3$, $y$-intercept $= -1$, $y = 3x - 1$

**b** $m = -1$, $y$-intercept $= -2$, $y = -x - 2$

**c** $m = 2$, $y$-intercept $= 3$, $y = 2x + 3$

**d** $m = -\frac{1}{2}$, $y$-intercept $= 3$, $y = -\frac{1}{2}x + 3$

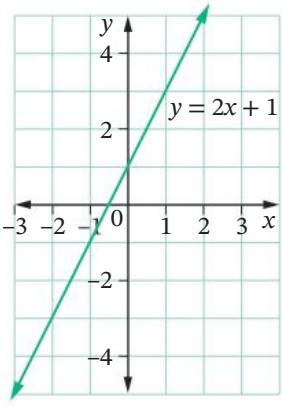
**5** **a** $m = -3$, $y$-intercept $= 2$

**b** $m = -1$, $y$-intercept $= -4$

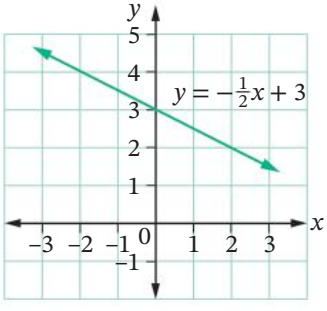
**c** $m = -2$, $y$-intercept $= 6$

**d** $m = 3$, $y$-intercept $= 1$

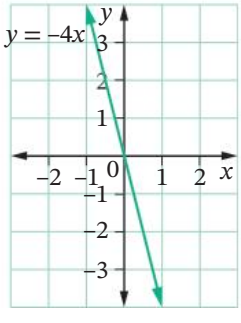
**6** **a**




**b**




**c**




ANSWERS

## Exercise 11.11

**1** **a** \$60 **b** 48 km
**c** 4, the starting cost of the taxi fare (or flagfall), \$4, before any distance is travelled
**d** \$2/km

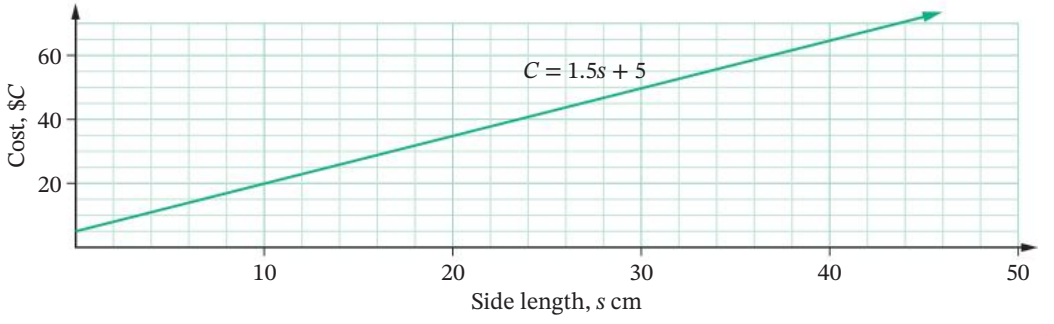
**2** **a** 8 minutes **b** 800 **c** $V = -100t + 800$
**d** 300 L **e** 100 L, yes

**3** **a** 100 cents = \$1.00 **b** 20 c/min
**c** 0 **d** $C = 20t$
**e** 6 minutes

**4** **a**

| Side length, $s$ cm | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| Cost, \$$C$ | 20 | 35 | 50 | 65 |

**b**

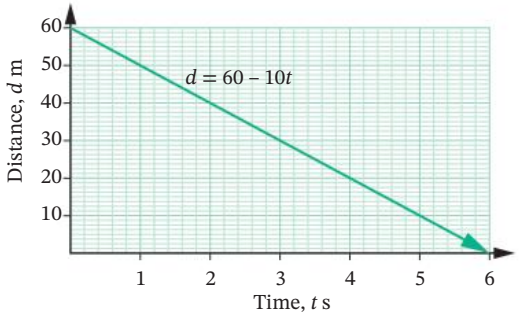



**c** The increase in cost, \$1.50, for each increase in centimetre of side length.
**d** \$5, the starting cost of making the plate regardless of side length.
**e** **i** \$32 **ii** 22 cm

**5** **a**

| Time, $t$ s | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| Distance, $d$ m | 60 | 40 | 20 | 0 |

**b**

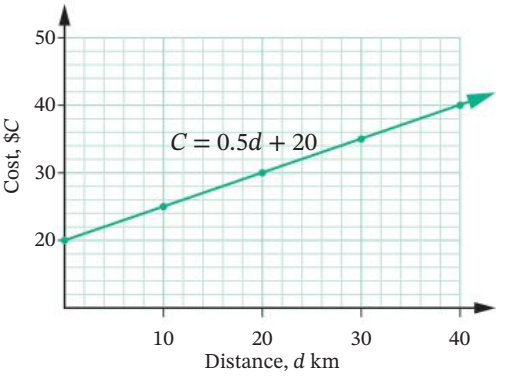



**c** 10 m **d** 3 s **e** 6 s
**f** 60, it represents the starting distance of 60 m
**g** 10 m/s

**6** **a**

| Distance, $d$ km | 0 | 10 | 20 | 30 | 40 |
|---|---|---|---|---|---|
| Cost, \$$C$ | 20 | 25 | 30 | 35 | 40 |

**b**




**c** The starting or fixed cost, \$20, before any distance is travelled.
**d** The additional running cost per km of distance travelled, \$0.50 (50 cents) per km.
**e** **i** \$32 **ii** 80 km

## Exercise 11.12

**1** **a** $x = 3$ **b** $x = -2$ **c** $x = 1\frac{1}{2}$

**2, 3** **a** $x = 3$ **b** $x = 4\frac{1}{2}$ **c** $x = -3$

**4** **a** $x = -2$ **b** $x = 0$ **c** $x = 2\frac{1}{2}$

**5** **a** $x = 3$ **b** $x = -3$ **c** $x = 0$

**6** **a** $x = 4$ **b** $x = -3$ **c** $x = 2$

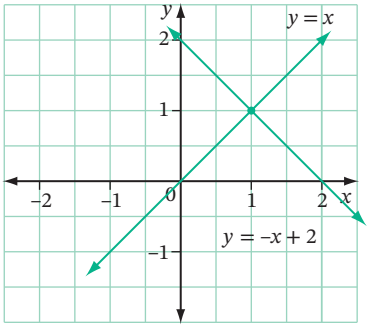
**7** **a** $x = 4$ **b** $x = -\frac{1}{2}$ **c** $x = -2$

**8** **a** $x = 4$ **b** $x = -4$ **c** $x = -2$

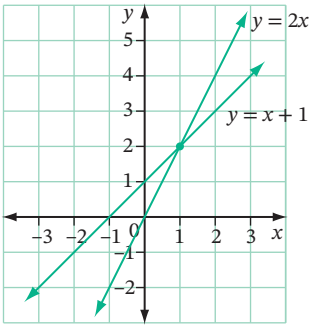
## Exercise 11.13

**1** B

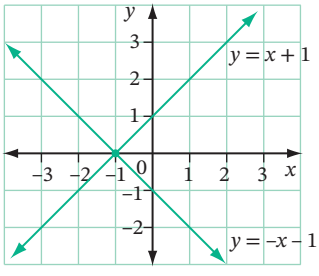
**2** **a** (1, 1)



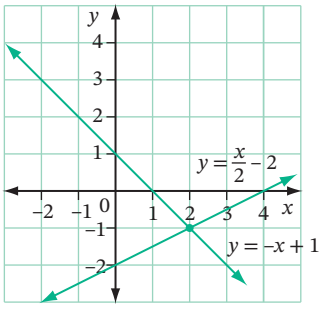

**b** (1, 2)



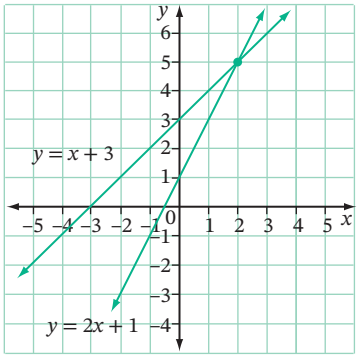

**c** (−1, 0)



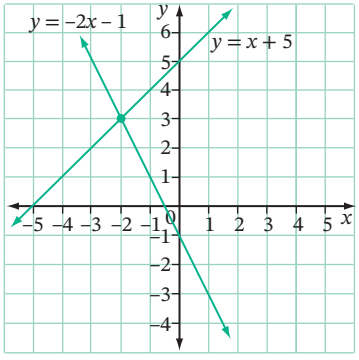

**d** (2, −1)




**e** (2, 5)

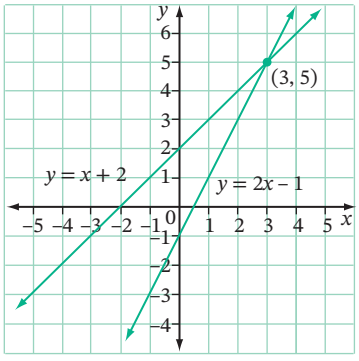



**f** (−2, 3)

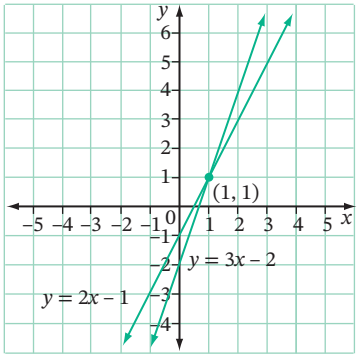



**3** B

**4, 5 a** $x = 3$

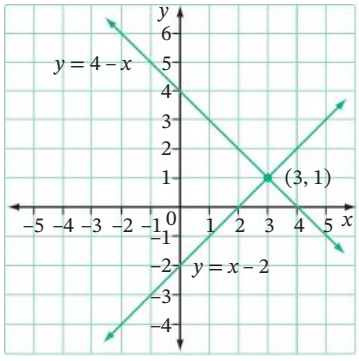



**b** $x = 1$

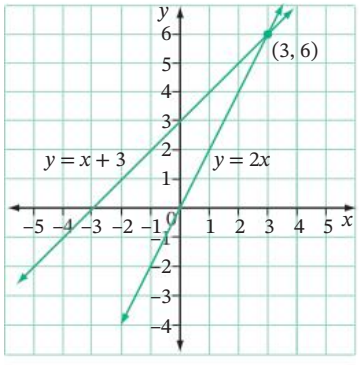



ANSWERS

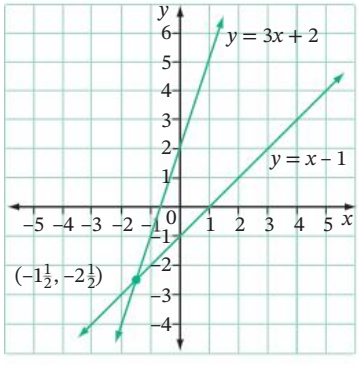
**c** $x = 3$



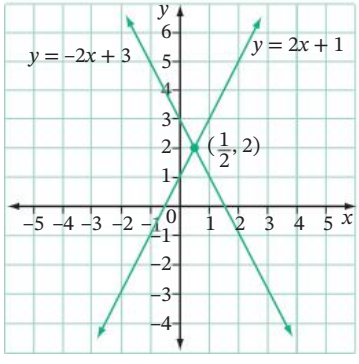

**d** $x = 3$



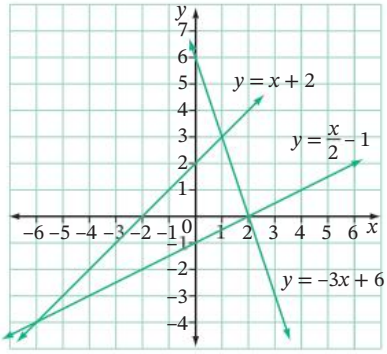

**e** $x = -1\frac{1}{2}$




**f** $x = \frac{1}{2}$

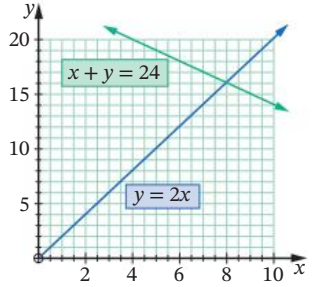



**6** (1, 3), (2, 0), (−6, −4)



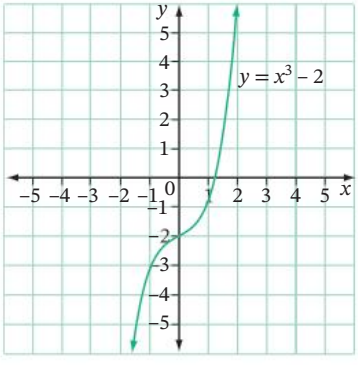

## Power plus

**1** **a, c**

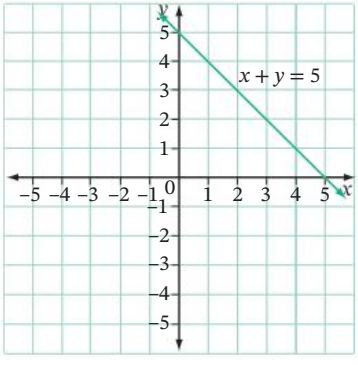



**b** $x + y = 24$ **d** George $16, Bill $8

**2** **a** non-linear



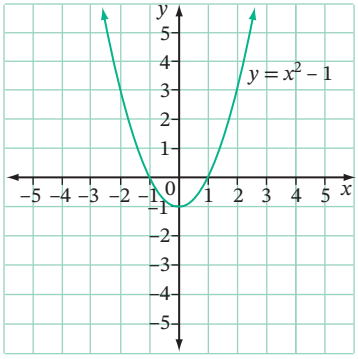

**b** linear



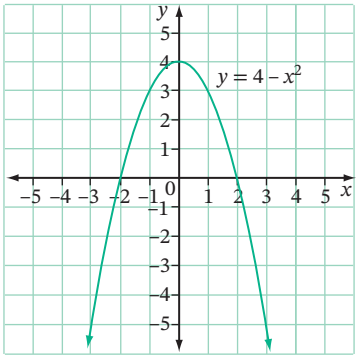


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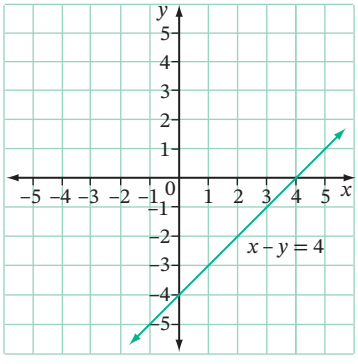
**c** non-linear



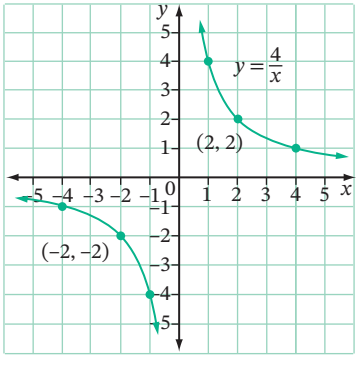

**d** non-linear




**e** linear




**f** non-linear




## Test yourself 11

**1** **a** 2, 3, 4, 5, 6 **b** −11, −9, −7, −5, −3
**c** 6, 5, 4, 3, 2

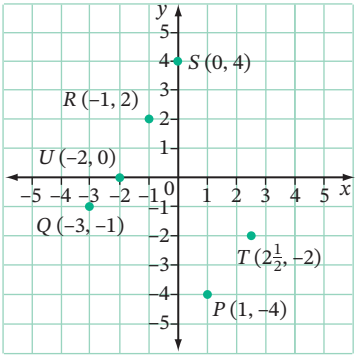
**2** **a** $y = x - 2$ **b** $p = 3m - 1$
**c** $n = -2m + 10$

**3** **a** 30 **b** $T = 4n + 2$

**4** **a** 4, 7, 10, 13, 16, 25, 31
**b** Number of matchsticks equals 3 times the number of rhombuses plus 1.
**c** $y = 3x + 1$
**d** **i** 37 **ii** 46

**5** $A(0, 5)$, $B(3, 1)$, $C(-4, -3)$, $D(-2, 4)$

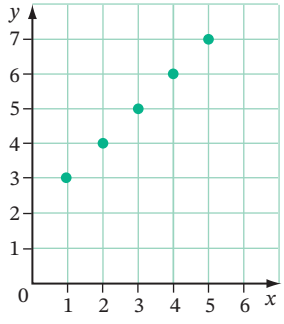
**6**



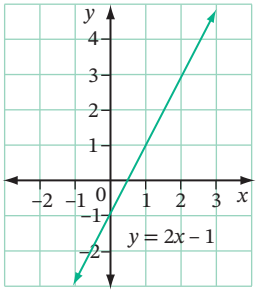

**7** **a** S **b** $R$ **c** $U$ **d** $T$ and $P$

**8** **a** 3, 4, 5, 6, 7 **b** $y = x + 2$

**c**

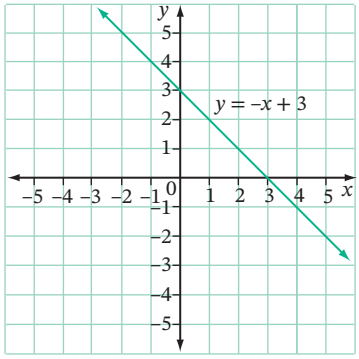



**9** **a**

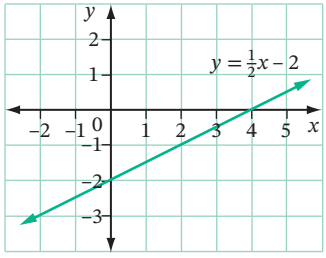



ANSWERS

**b**



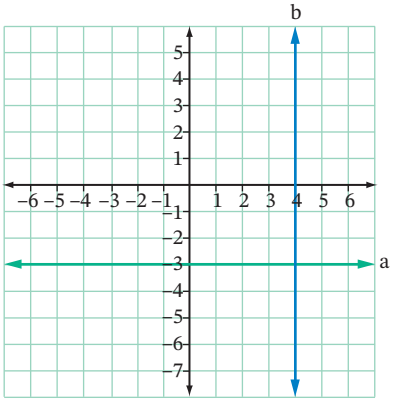

**c**



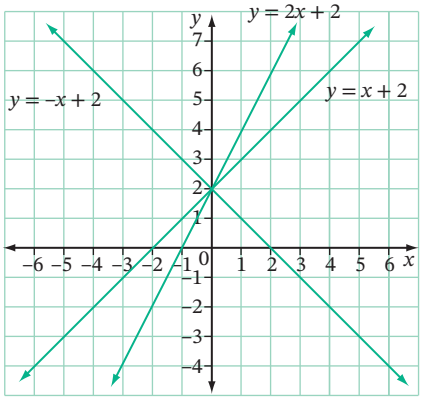

**10** **a** No **b** No **c** Yes **d** Yes

**11** **a** $y = x + 3$ **b** $y = -2x - 1$

**12**




**13** **a**




**b** $y$-intercept of 2

**c** constant term of 2

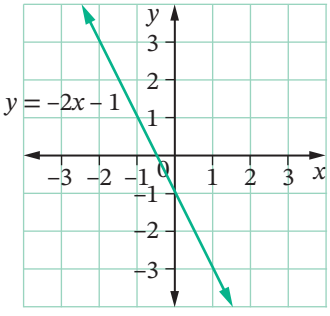
**d** different steepnesses, directions and coefficients of $x$

**14** **a** 1 **b** −2

**15** 2

**16** **a** $y = -4x + 8$ **b** $y = -\frac{3}{2}x - 6$

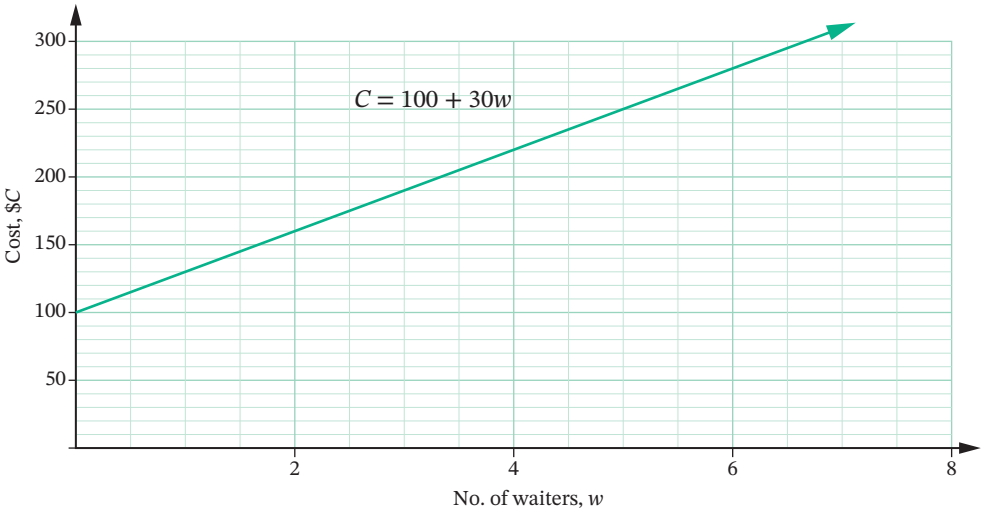
**17**




**18** **a**

| Waiters, $w$ | 1 | 2 | 4 | 6 |
|---|---|---|---|---|
| Cost, $C | 130 | 160 | 220 | 280 |

**b**

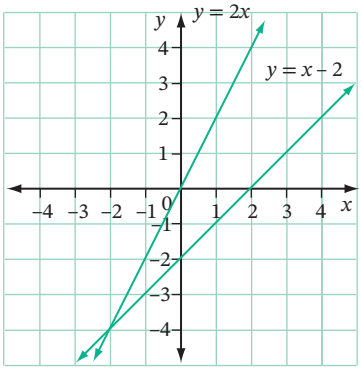



**c** $250 **d** 7 waiters **e** $30

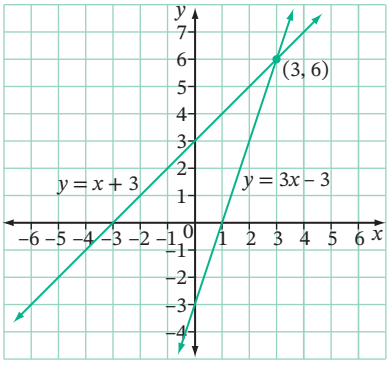

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**19** **a** $x = 3$ **b** $x = -1\frac{1}{2}$

**20** $(-2, -4)$




**21** $x = 3$




# Chapter 12

## SkillCheck

**1** **a** $\frac{1}{2}$ **b** $\frac{3}{2}$ **c** 5 **d** $\frac{13}{7}$

**2** **a** 10 **b** 8 **c** 15 **d** 60

**3** **a** 35° **b** 60° **c** 110°

**4** **a** $x = 5$ **b** $y = 24$ **c** $p = 8$ **d** $m = \frac{4}{3}$

**5** **a** 34.56 cm **b** 95.03 cm$^2$

## Exercise 12.01

**1** **a** A and B **b** A and C **c** B and C **d** A and C

**2** A

**3** **a** $ABCD \sim IJKL$ **b** $\triangle STU \sim \triangle XVW$
**c** $ABCD \sim JKLI$
**d** $UVWXY \sim MNJKL$ **e** $\triangle HGF \sim \triangle JIK$

**4** **a** **i** Yes **ii** No **iii** Yes
**b** **i** All circles are the same shape.
**ii** The ovals/ellipses are different-shaped.
**iii** All semi-circles are the same shape.

**5** Teacher to check.

**6** **a** True **b** False **c** False **d** True

## Exercise 12.02

**1** D

**2** $\triangle ABC \sim \triangle TUV$, $\triangle EGF \sim \triangle MLN$

**3** **a** 2 **b** $\frac{1}{3}$

**4** **a** 90°, 45°, 45°, matching angles are equal.
**b** $\frac{4}{6} = \frac{2}{3}$ **c** $\frac{2}{3}$

**5** 18 cm

**6** Teacher to check.

**7** **a** 2 **b** $\frac{1}{3}$ **c** 2.5 **d** $\frac{2}{3}$
**e** $\frac{4}{5}$ **f** $\frac{1}{5}$

**8** Teacher to check.

## Exercise 12.03

**1** **a** $x = 15$ **b** $d = 10$ **c** $p = 9$ **d** $k = 15$
**e** $r = 5$ **f** $q = 8$ **g** $x = 27$ **h** $y = 7.65$

**2** C

**3** **a** $y = 3\frac{1}{3}$ **b** $x = 77$ **c** $q = 6$
**d** $a = 9.6$ **e** $k = 24.75$ **f** $x = 20$

**4** **a** $x = 5$ **b** $w = 12$ **c** $y = 14.4$
**d** $a = 40.5$ **e** $p = 40$ **f** $b = 24$

**5** 306 cm

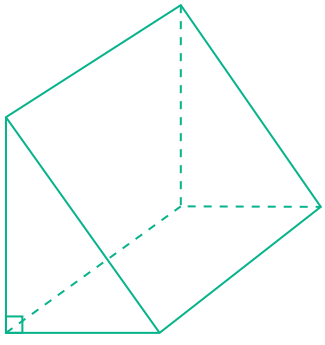
## Mental skills 12

**2** **a** 8 h 30 min **b** 5 h 40 min **c** 3 h 25 min
**d** 8 h 15 min **e** 11 h 25 min **f** 1 h 40 min
**g** 5 h 10 min **h** 5 h 45 min **i** 7 h 55 min
**j** 7 h 40 min

## Exercise 12.04

**1** D

**2** Triangular prism



**3** **a** rectangular prism, 220 cm$^2$
**b** cube, 34.56 cm$^2$

**4** **a** 150 cm$^2$ **b** 324 cm$^2$ **c** 208 m$^2$

**5** **a** 396 m$^2$ **b** 720 mm$^2$ **c** 152 m$^2$

**6** 374 cm$^2$

**7** **a** 58.84 m$^2$ **b** 2 cans

**8** **a** 24.00 m **b** 1776 m$^2$

**9** **a** 15 m$^2$ **b** $1875

## Exercise 12.05

**1** A

**2** **a** 43.98 cm **b** 307.88 cm$^2$
**c** 967.61 cm$^2$ **d** 1275.49 cm$^2$

**3** **a** 6635.0 mm$^2$ **b** 6182.7 cm$^2$ **c** 735.1 m$^2$

**4** **a** 5174.2 cm$^2$ **b** 490.6 cm$^2$ **c** 190.1 mm$^2$

**5** **a** 426.88 mm$^2$ **b** 56.11 m$^2$

**6** 5655 cm$^2$

**7** $B$, surface area of $A = 402.1$ cm$^2$, surface area of $B = 1206.4$ cm$^2$

**8** 339.29 m$^2$

## Power plus

**1** **a** 544 cm$^2$ **b** 1520 cm$^2$ **c** 365 cm$^2$

**2** 16 **3** 6 m **4** 2.56 **5** 9.55 cm

## Test yourself 12

**1** A and G, C and F, D and E

**2** 1.5

**3** $d = 48$, $k = 142$, $p = 79$, $x = 91$

**4** Teacher to check.
**a** Length 10 units **b** Length 80 mm

**5** **a** 2.5 or $\frac{5}{2}$ **b** $\frac{5}{9}$

**6** **a** $x = 7.2$ **b** $n = 43.2$

**7** **a** 52 m$^2$ **b** 4600 cm$^2$

**8** **a** 165.8 m$^2$ **b** 13 408.3 cm$^2$

**9** 55.86 m$^2$

## Practice set 4

**1** **a** 1 : 3 **b** 48 : 17 **c** 40 : 3
**d** 1 : 12 **e** 6 : 5 **f** 1 : 4

**2** **a** 300 g for $5.20 **b** 1.5 kg for $3.30

**3** 7, 9, 11, 13, 15

**4** 20

**5** **a** 25 cm **b** 20 m

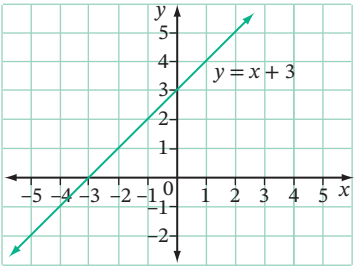
**6** **a** 33 km/h **b** 225 km

**7** $y = 3x$

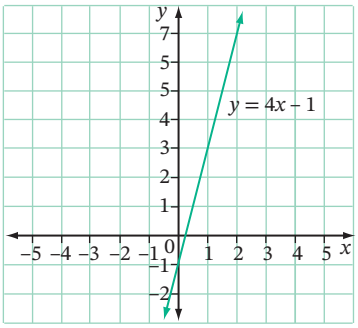
**8** **a** 5, 7, 9, 11, 17
**b** The number of toothpicks is equal to double the number of triangles plus 1.
**c** $p = 2t + 1$ **d** 41

**9** $A(2, 3)$, $B(4, 5)$, $C(-2, 4)$, $D(0, -5)$, $E(-3, -2)$, $F(1, 0)$, $G(-1, -3)$.

**10** **a** 2, 3, 4, 5




**b** −5, −1, 3, 7

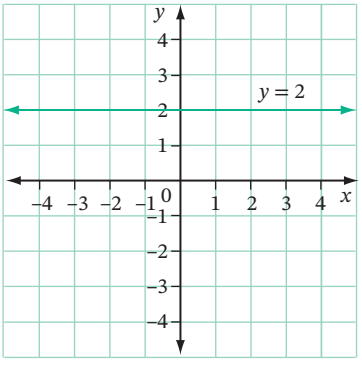



**11** 1.5 t **12** 440 km

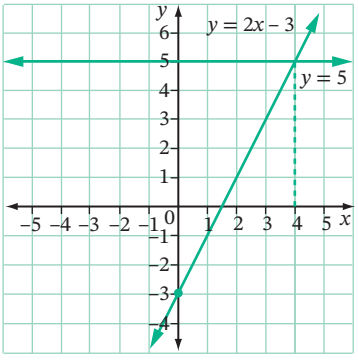
**13** **a** No **b** Yes

**14** 50 h

**15**



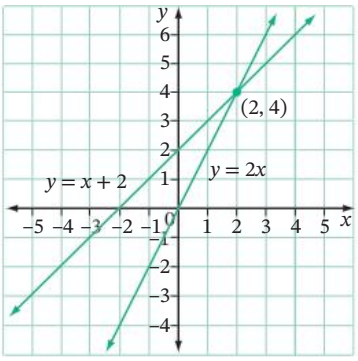

**16** $x = 4$




**17** $y = x + 2$

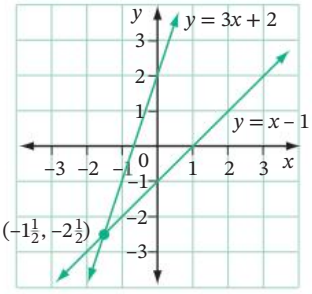
**18** **a** 54 000 m/h **b** 3000 kg/day
**c** 500 mL/s **d** 16.7 m/s

**19**




**20** **a** 1 p.m., for 1.5 hours **b** 26 km
**c** 6.5 km/h

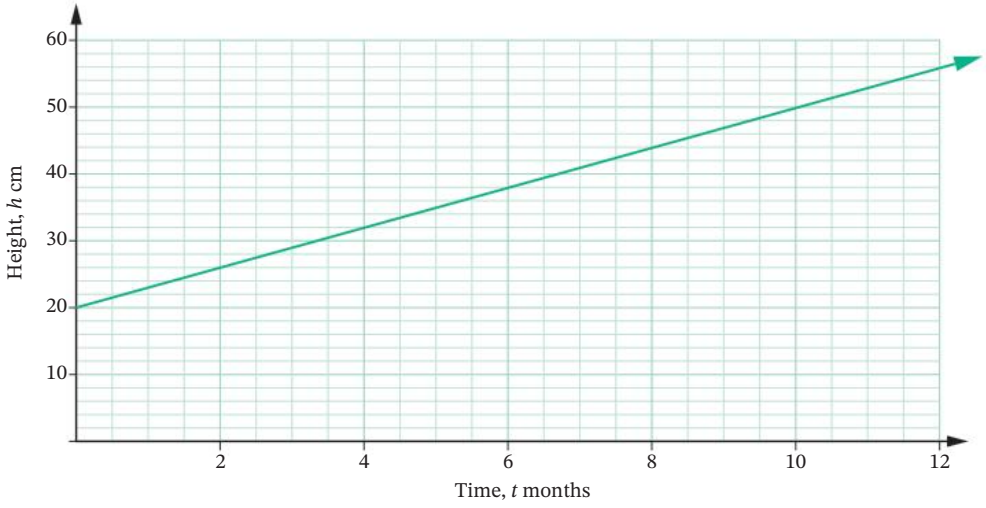
**21** $x = -1\frac{1}{2}$




**22** **a**

| Time, $t$ months | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| Height, $h$ cm | 20 | 26 | 32 | 38 | 44 | 50 | 56 |

**b**




**c** $h = 3t + 20$
**d** 35 cm
**e** 18 months

**23** B

**24** **a** Dounya 2.9 m/s, Anna 2.7 m/s
**b** Dounya by 51.1 seconds

## General practice

**1** **a** 13 **b** 16 **c** 17
**d** 3 **e** 5 **f** 18

**2** **a** $2x + y$ **b** $yz - x$
**c** $x + y + z$ **d** $xy - 4z$

**3** **a** $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{7}{20}$

**4** **a** $7\frac{1}{5}$ **b** $6\frac{7}{10}$ **c** $6\frac{1}{6}$

**5** **a** $-6$ **b** 4 **c** 5

**6** **a** $6ar$ **b** $-3p$ **c** $35u^2$
**d** $5x + 9$ **e** $8p - 7q$ **f** $\frac{6u}{m}$

**7** 25%

**8** **a** 30 m/s **b** 16.4 points/game
**c** \$8/kg

**9** **a** 4.8 **b** 5 **c** 5 **d** 7

**10** **a** 187 mm$^2$ **b** 144 cm$^2$ **c** 18.4 m$^2$

**11** **a** $m = 34$ (angle sum of a triangle)
**b** $m = 45$ (angles on a straight line)
**c** $y = 68$ (angle sum of a triangle)
**d** $m = 100$ (isosceles triangle, exterior angle of a triangle or angles on a straight line)
**e** $x = 132$ (vertically opposite angles)
**f** $x = 67$ (angle sum of a quadrilateral)
**g** $y = 130$ (different reasons possible: corresponding/alternate angles on parallel lines, angles on a straight line)
**h** $x = 62$ (exterior angle of a triangle)
**i** $m = 62$ (angle sum of an isosceles triangle, corresponding angles on parallel lines)

**12** **a** $-2$ **b** 3.07 **c** 6.314 **d** 18

**13** **a** $m = 17.7$ **b** $y = 16.6$
**c** $p = 25.5$ **d** $d = 76.0$

ANSWERS

**14** **a** $6^2$ **b** $2^7$ **c** $3^2$ **d** $5^3$
**e** $2^{12}$ **f** $2^6 \times 5^2$ **g** $3^0 = 1$ **h** 6

**15** **a** Not choosing an Ace **b** $\frac{48}{52} = \frac{12}{13}$

**16** **a** **i** isosceles **ii** acute-angled
**b** **i** scalene **ii** obtuse-angled
**c** **i** scalene **ii** right-angled

**17** $200 = 2^3 \times 5^2$

**18** **a** 80 000 **b** 0.49 **c** 650 000 000

**19** **a** $1\frac{5}{12}$ **b** $\frac{7}{15}$ **c** $\frac{7}{15}$ **d** $\frac{8}{15}$
**e** $1\frac{5}{8}$ **f** $3\frac{2}{7}$ **g** $11\frac{1}{4}$ **h** $6\frac{3}{25}$

**20** $\frac{3}{20}$

**21** **a** $a = 5$ **b** $d = 36$ **c** $r = -21$

**22** **a** ND **b** CO **c** NC **d** CN

**23** **a** Yes **b** No

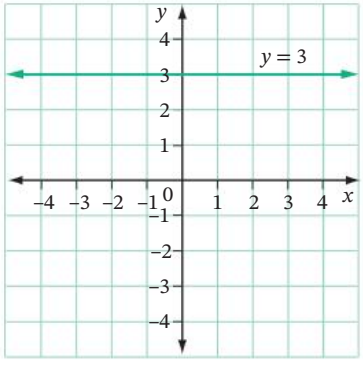
**24** positively skewed

**25** \$940.80

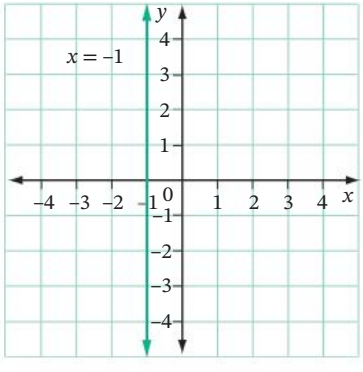
**26** **a** $y = 3x + 4$ **b** $x = 3$

**27** **a** 8 glasses **b** 600 L

**28** **a**




**b**



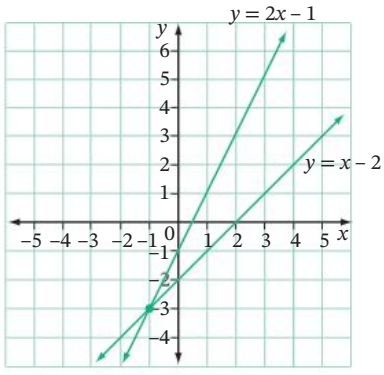

**29** **a** $3a + 15$ **b** $20 - 8k$ **c** $-15$

**30** **a** 6 : 7 **b** 20 : 1 **c** 1 : 100

**31** 280 **32** 109 cm

**33** \$450

**34** **a**, **c**




**b** yes **d** $x = -1$

**35** **a** \$36 **b** 60%

**36** **a** rectangle (or square)
**b** equilateral triangle
**c** rectangle (or rhombus)
**d** isosceles triangle

**37** **a** **i** 43.98 m **ii** 153.94 $m^2$
**b** **i** 25.71 cm **ii** 39.27 $cm^2$
**c** **i** 66.85 cm **ii** 272.55 $cm^2$
**d** **i** 62.83 mm **ii** 85.84 $mm^2$

**38** **a** $2(y + 4)$ **b** $5a(1 - 3b)$ **c** $-2b(4a - 1)$

**39** \$200

**40** **a** $\frac{1}{2}$ **b** $\frac{2}{7}$ **c** $\frac{11}{14}$

**41** 4900 **42** 16.12 m

**43** 80 $m^2$

**44** **a** 62 **b** 42 **c** 42.5

**45** 50.4 L/day

**46** **a** 100, 130, 160, 190, 210, 240
**b** Teacher to check **c** \$70, initial charge
**d** **i** \$145 **ii** 4.5 tonnes

**47** B

**48** **a** 400 $cm^3$ **b** 288 $cm^3$ **c** 300 $mm^3$
**d** 910 $m^3$ **e** 84 $m^3$ **f** 4536.5 $mm^3$

**49** 2.5 cm

**50** \$110.40

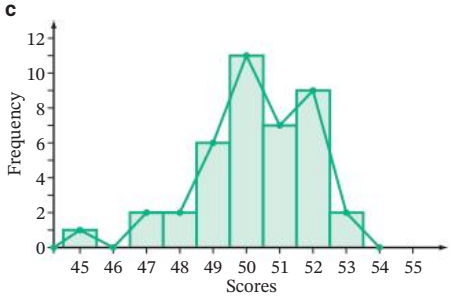
**51** **a**

| Score | Tally | *f* | *fx* |
|---|---|---|---|
| 45 | \| | 1 | 45 |
| 47 | \|\| | 2 | 94 |
| 48 | \|\| | 2 | 96 |
| 49 | 𝍸 \| | 6 | 294 |
| 50 | 𝍸 𝍸 \| | 11 | 550 |
| 51 | 𝍸 \|\| | 7 | 357 |
| 52 | 𝍸 𝍸 \|\| | 9 | 468 |
| 53 | \|\| | 2 | 106 |
| | Total | 40 | 2010 |

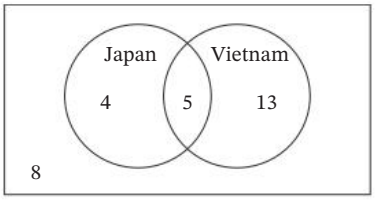
**b** 50.25


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**c**




**52 a**




**b** 8 **c** 4 **d** $\frac{7}{10}$

**53 a** 19 500 cm$^3$ **b** 19.5 L

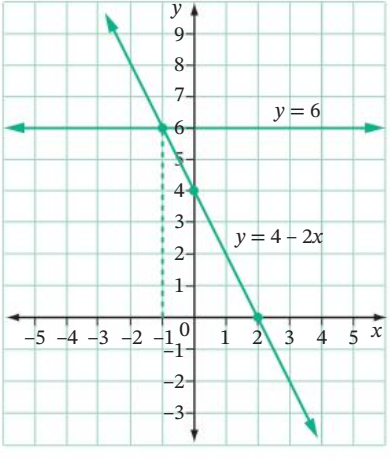
**54** $x = 7$ **55** 6.1 m/s

**56 a** \$600 **b** 450 kg

**57 a** 4 km **b** 80 min, 7 km

**c** 20 min **d** 8 km/h

**58** $x = -1$




**59** $y = 2x - 4$ **60** \$110

## Calculator skills

### Exercise 1

**1 a** 5068 **b** 14.5 **c** 45

**d** 5.724 **e** 3.5 **f** 26.9

**g** 23 **h** 1.68 **i** 160

**j** 29.48 **k** −8.084 **l** 12.42

**2 a** \$19.15 **b** 85 cents

**3** \$1875.50 **4** 7542 kg

**5** 2.4 m **6** \$18.75 **7** 23 trees

**8** 75.2 km/h **9** \$115.44 **10** 13 buses

**11 a** \$41.20 **b** \$1318.40

**12** 53.4

### Exercise 2

**1 a** 7 **b** −9 **c** −12 **d** −8

**e** 15 **f** −15 **g** 75 **h** 5

**i** −5 **j** 7 **k** −96 **l** 45

**m** −25 **n** 24 **o** 96 **p** 14

**q** −16 **r** 15 **s** −36 **t** 10

### Exercise 3

**1 a** 196 **b** 529 **c** 216 **d** 1.728

**e** 26.3169 **f** 10 201 **g** 4.1616 **h** 6859

**i** 26 **j** 148 **k** 9 **l** 42

**2 a** 4.9 **b** 10.0 **c** 4.7 **d** 4.6

**3 a** 16 **b** 156 **c** 6 **d** 10

### Exercise 4

**1 a** $\frac{5}{8}$ **b** $\frac{8}{17}$ **c** $\frac{5}{8}$

**d** $\frac{8}{9}$ **e** $\frac{21}{25}$ **f** $\frac{1}{4}$

**2 a** $\frac{15}{8}$ **b** $\frac{47}{9}$ **c** $\frac{43}{4}$

**3 a** $2\frac{2}{9}$ **b** $8\frac{4}{5}$ **c** $2\frac{1}{4}$

**4 a** 0.5 **b** 0.8 **c** 0.875

**d** 0.36 **e** 0.17 **f** $0.\dot{5}$

**5 a** $\frac{7}{20}$ **b** $2\frac{1}{25}$ **c** $\frac{1}{1000}$

**d** $\frac{73}{200}$ **e** $1\frac{3}{50}$ **f** $\frac{13}{40}$

**6 a** $\frac{17}{20}$ **b** $1\frac{1}{14}$ **c** $\frac{1}{15}$ **d** $\frac{4}{21}$

**e** $1\frac{1}{3}$ **f** $\frac{5}{24}$ **g** $1\frac{17}{30}$ **h** $\frac{35}{96}$

**i** $1\frac{3}{4}$ **j** $19\frac{1}{5}$ **k** $6\frac{13}{20}$ **l** $8\frac{13}{24}$

**m** $12\frac{3}{4}$ **n** 7 **o** $\frac{4}{27}$ **p** $19\frac{19}{60}$

**7 a** $\frac{1}{8}$ **b** $\frac{3}{4}$

**c** $15\frac{4}{5}$ or 15.8 **d** $\frac{1}{6}$ or $0.1\dot{6}$

**8 a** $6\frac{3}{4}$ hours **b** $33\frac{3}{4}$ hours

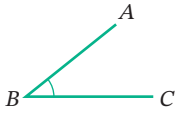
ANSWERS

# GLOSSARY AND INDEX

**24-hour time** Time of day written using 4 digits (instead of a.m. or p.m.) and the hours 0 to 23. For example, 18:20 is the 24-hour time for 6:20 p.m. (p. 444)

## A

**acute angle** A 'sharp' angle between 0° and 90°, for example, the marked angle in the diagram. (p. 136)

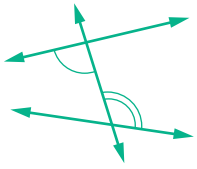



**acute-angled triangle** A triangle with all 3 angles acute. (p. 143)

**algebraic expression** A number written in algebraic form using variables, for example, $2xy + 4y - 5$. (p. 90)

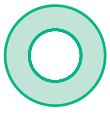
**algebraic term** See **terms of an expression**.

**alternate angles** A pair of angles between 2 lines crossed by a transversal, on opposite sides of the transversal; for example, the 2 angles marked in the diagram. (p. 140)



**angle sum** The total of the sizes of the angles in a shape. The angle sum of a triangle is 180°. (p. 153)

**annulus** A ring shape between 2 different-sized circles with the same centre. (p. 199)



**arc** Part of the circumference of a circle. (p. 177)




**area** The amount of surface enclosed by a shape, measured in square units. (p. 185)

**ascending order** Going up, increasing, from smallest to largest (1–2–3). The opposite of **descending order**. (p. 52)

**at least** Referring to the smallest number, for example, 'at least 2' means 2, 3, 4, ... , that is, '2 or more'. (p. 359)

**average** See **measure of centre**. (p. 308)

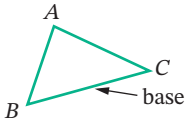
## B

**backtracking method** A method of solving equations by 'undoing', or performing inverse (opposite) operations in reverse order. (p. 393)

**balancing method** A method of solving equations by performing inverse (opposite) operations on both sides. (p. 393)

**base (in index notation)** When a number is raised to a power, the number raised is the base. In the expression $3^5$, the 3 is called the base. (p. 64)

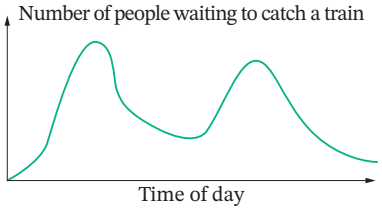
**base (of a shape)** The bottom side of a flat shape such as a triangle. (p. 188)




**best buy** When comparing different brands or sizes of items during shopping, the best buy is the item with the lowest unit price and is the best value for money. (p. 450)

**bias** In statistics, something that causes a sample to not truly represent the population. (p. 304)

**bimodal distribution** A statistical distribution with 2 peaks. See also **unimodal distribution** and **multimodal distribution**. (p. 340)




**binomial expression** An algebraic expression with 2 terms, for example, $x + 9$, $2y - 12$. (p. 118)

**braces/brackets** See **grouping symbols**.

## C

**capacity** The amount of material (usually liquid) that a container can hold, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML). (p. 222) See also **volume**.

**Cartesian plane** Another name for **number plane**. (p. 485)

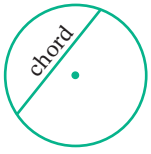
**categorical data** Non-numerical data that can be classified into categories, such as hair colour, favourite radio station or postcode. Data that is not numerical. See also **ordinal data** and **nominal data**. (p. 299)

**census** A survey of the entire **population** of people or items, not just a survey of a **sample**. (p. 303)

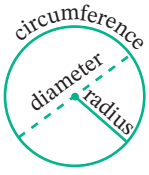

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**certain** Must happen; has a probability of 1 or 100%. (p. 359)

**chord** An interval joining 2 points on a circle. (p. 178)



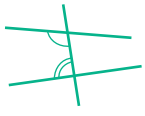

**circumference** The perimeter of a circle, or the length of that perimeter. $C = \pi d$ or $C = 2\pi r$, where $C$ is the circumference, $\pi$ is pi, $d$ is the diameter and $r$ is the radius. (p. 181)




**cluster** A group of data values that are bunched or close together. (p. 325)

**coefficient** The number 'in front of' a variable in an equation; a multiplier. For example, in $y = 3x - 7$, the coefficient of $x$ is 3. (p. 500)

**co-interior angles** A pair of angles between 2 lines crossed by a transversal, on the same side of the transversal (co-interior means 'together inside'), for example, the 2 angles marked in the diagram. (p. 140)

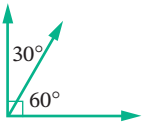


**collinear points** Points that lie in a straight line. (p. 489)

**compasses** Geometrical instrument for constructing circles and intervals of equal length.



**complementary angles** 2 angles whose sum is 90°. The angles 30° and 60° are complementary. (p. 136)




**complementary event** All the outcomes that are *not* the event; the 'opposite' event. For example, the complementary event to rolling 1 on a die is rolling a number that is not 1. (p. 364)

**composite shape** A shape made up of 2 or more basic shapes. (p. 192)

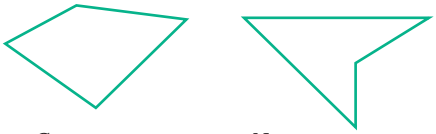
**consecutive numbers** Any series of integers that follow each other in order, for example, 8, 9 and 10. (p. 477)

**constant term** The term in an equation that is a number only and does not contain a variable. For example, in $y = 3x - 7$, the constant term is $-7$. (p. 500)

**continuous data** Numerical data that can be measured on a smooth scale without any gaps, that can take on a full range of values, such as the height of people. Continuous data is measured on a scale without 'gaps', unlike **discrete data**. (p. 300)

**converse** The 'reverse' of a rule, the rule written 'back-to-front' or 'turned around'. The converse of Pythagoras' theorem is that if the square of the longest side of a triangle is equal to the sum of the squares of the other 2 sides, then the triangle is right-angled. (p. 20)

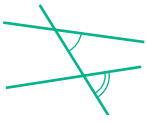
**convex quadrilateral** A quadrilateral whose vertices all point outwards. All diagonals lie within the shape, and all angles are less than 180°. (p. 147)



Convex Non-convex

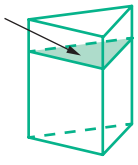
**coordinates** An ordered pair of numbers used to locate a point or position, for example, $A(2, 5)$ tells us that the point $A$ is located 2 units to the right of and 5 units up from the origin. (p. 485)

**corresponding angles** A pair of angles in matching positions when 2 lines are crossed by a transversal. They are on the same side of the transversal and the lines, for example, the 2 angles marked on the diagram. (p. 140)



**cost price** The price it costs a retailer (shop) to buy (to resell). The cost price is usually less than the **selling price**. (p. 266)

**cross-section** A 'slice' of a solid cut across it rather than along it. (p. 213)



**cube (of a number)** The number raised to the power of 3. For example, 7 cubed = $7^3 = 7 \times 7 \times 7 = 343$. (p. 64)

**cube root (of a number)** The value which, if cubed, gives the number. For example, $\sqrt[3]{8} = 2$ because $2^3 = 2 \times 2 \times 2 = 8$. (p. 64)

**cubic metre** A metric unit of volume, the volume of a 1 m cube. (p. 211)

**cylinder** A can-shaped solid with ends that are circles. (p. 219)

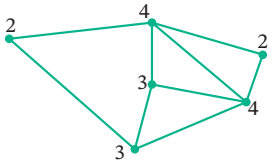


## D

**data** Information, a collection of facts. (p. 288)

**decimal places** The places after the decimal point in a number. For example, 3.1416 has 4 decimal places.

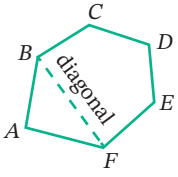
**degree (of a vertex)** The number of edges coming out of the vertex in a network. (p. 161)



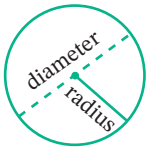

**denominator** The number below the line in a fraction. The denominator of $\frac{2}{3}$ is 3.

**descending order** Going down, decreasing, from largest to smallest (3–2–1). The opposite of **ascending order.** (p. 52)

**diagonal** An interval joining 2 non-adjacent vertices of a shape.




**diameter** An interval joining 2 points on the circumference and passing through the centre of a circle, or the length of that interval. The diameter is double the **radius**. (p. 181)




**difference** The result of a subtraction. The difference between 65 and 10 is 55. (p. 43)

**difference of 2 squares** An algebraic expression of the form $a^2 - b^2$, that can be factorised into $(a + b)(a - b)$. For example, $x^2 - 25 = (x + 5)(x - 5)$. (p. 123)

**discount** The saving made between the original price of an item and the reduced price. (p. 251)

**discrete data** Numerical data that are counted or measured, only taking on distinct, separate values, such as the number of children in a family (0, 1, 2, ...). Discrete data has a scale with 'gaps' or jumps, unlike **continuous data**. (p. 300)

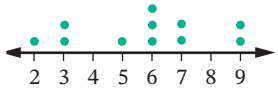
**distributive law** A law of arithmetic that says that you can multiply by a number by splitting it into the sum or difference of 2 other numbers. For example, $27 \times 12 = 27 \times (10 + 2) = 27 \times 10 + 27 \times 2$. More generally, $a \times (b + c) = a \times b + a \times c$ for any 3 numbers $a$, $b$ and $c$. (p. 109)

**divided bar graph** A rectangular graph that is divided into proportionately-sized sections to represent parts of a whole. (p. 288)



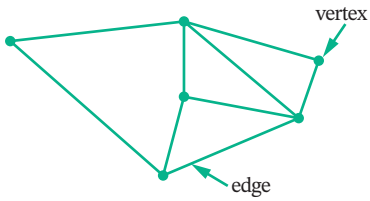
**divisor** See **factor**. (p. 112)

**dot plot** A graph that uses dots above a number line to show the frequencies of data values. (p. 325)




## E

**edge (of a network)** The connecting lines that join vertices together. (p. 161)




**equation** A mathematical statement that 2 quantities are equal. For example, $8 + 2 = 10$ or $3b - 7 = 5$. (p. 393)

**equilateral triangle** A triangle with all 3 sides equal. (p. 143)



**equivalent** Having the same value. For example, 0.75 and $\frac{3}{4}$ are equivalent. (pp. 237, 429)

**equivalent ratio** An equal ratio created by multiplying or dividing each term of a ratio by the same number. (p. 434)

**event** In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the 3 outcomes {2, 4, 6}. (pp. 357, 358)

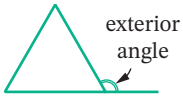
**expected frequency** The expected number of times an event will occur over repeated trials, calculated by multiplying the probability of the event by the number of trials. (p. 380)

**expected probability** See **theoretical probability**. (p. 380)

**experiment** A situation involving chance that leads to outcomes, for example, rolling a die. (p. 358)

**expression** A mathematical description of one or more operations. For example, $2 \times 4 + 5$ and $3b - 7$ are mathematical expressions (but $3b - 7 = 5$ is an equation). See also **algebraic expression**.

**exterior angle** An 'outside' angle of a shape created by extending one of the sides of the shape. (p. 154)

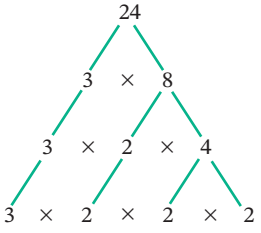



## F

**face (of a network)** A region bounded by the edges of the network, and the region outside the network (called the infinite face). (p. 62)

**factor (of a number)** A value that divides evenly into a given number. For example, the factors of 15 are 1, 3, 5 and 15. Also called **divisor**. (p. 112)

**factor tree** A diagram that lists the prime factors of a number. For example, the factor tree for 24 is shown. (p. 72)




**formula** (plural: **formulas** or **formulae**) A rule written as an algebraic equation, using variables.

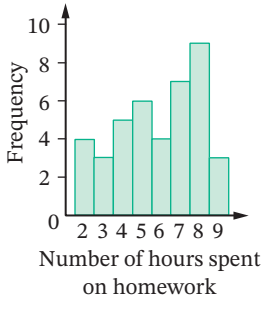
The formula for the area of a triangle is $A = \frac{1}{2}bh$. (pp. 97, 415)

**fraction** A number written in the form $\frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$. (p. 238)

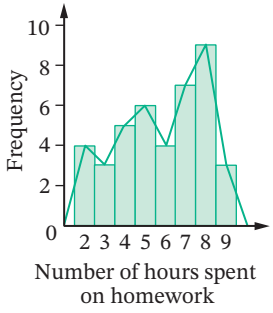
**frequency** The number of times an event occurs in repeated trials of a probability experiment, or the number of times a value appears in a set of data. (p. 290)

**frequency (distribution) table** A table listing the frequency of each value in a set of data, with columns for Score ($x$), Frequency ($f$) and sometimes Tally and $fx$. (p. 290)

**frequency histogram** A column graph that shows the frequencies of numerical data. There are no spaces between the columns, and the graph looks like a row of office buildings. (p. 332)



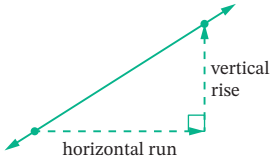

**frequency polygon** A line graph that shows the frequencies of numerical data. It can be made by joining the midpoints of the tops of the columns of a histogram. The graph looks like a mountain. (p. 332)




## G

**goods and services tax (GST)** A 10% tax added to the original price of an item or service. (p. 267)

**gradient** The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$. (p. 504)




**gradient–intercept form (of a linear equation)** The equation of a straight line $y = mx + c$, where $m$ is the gradient and $c$ is the $y$-intercept. (p. 507)

**graph** Another name for a network. The study of networks is called **graph theory**. (p. 161)

**greatest common divisor (GCD)** See **highest common factor**.

**grouping symbols** The collective name for parentheses ( ), (square) brackets [ ] and braces {}. (p. 50)

**GST** See **goods and services tax.**

## H

**hectare** A large metric unit of area, equivalent to 10 000 $m^2$ or to the area of a square 100 m × 100 m. (p. 186)

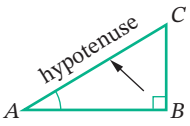
**height** How tall or high something is, the vertical distance between its top and its base. (p. 188)

**highest common factor (HCF)** Also called **greatest common divisor (GCD)** The largest factor shared by 2 or more numbers or algebraic terms. For example, the HCF of 36 and 8 is 4 and the HCF of $6xy$ and $12y^2$ is $6y$. (pp. 72, 113)

**horizontal** Going across, sideways, flat. (p. 496)




**hypotenuse** The longest side of a right-angled triangle, opposite the right angle. (p. 6)




## I

**image** A transformed shape after it has been enlarged or reduced. (p. 532)

**impossible** Cannot happen, no chance, has a probability of 0. (p. 359)

**improper fraction** A fraction whose numerator is greater than or equal to its denominator, such as $\frac{7}{4}$. (p. 238)

**index** (plural: **indices**, pronounced 'in-da-sees') See **power**. (p. 64)

**index notation** A way of writing repeated multiplication using indices (powers), in the form $a^n$. Index notation for $2 \times 2 \times 2 \times 2$ is $2^4$. (p. 64)

**infographic** A display that combines images, graphs and text to illustrate data, statistics and information. (p. 294)

**integer** A number that is a positive or negative whole number or zero. The numbers {..., −3, −2, −1, 0, 1, 2, 3, ...} are integers. (p. 42)

**intersect** To cross.

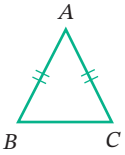
**interval** A section of a line with a definite length, such as $AB$ below. (p. 474)




**inverse operation** The opposite or reverse operation, used when solving equation. For example, the inverse operation to adding is subtracting, the inverse operation to dividing is multiplying. (p. 393)

**irrational number** A number such as $\pi$ or $\sqrt{2}$ that cannot be expressed as a fraction. In decimal form, its digits run endlessly without repeating. (pp. 64, 181)

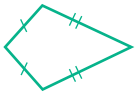
**isosceles triangle** A triangle with 2 equal sides. (p. 143)




## K

**kilo-** A prefix meaning one thousand (1000), represented by the symbol k. For example, one kilogram is 1000 grams.

**kite** A quadrilateral with 2 pairs of equal adjacent sides. (p. 147)



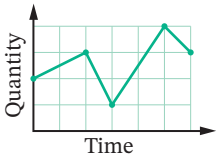
## L

**LHS** The left-hand side (of an equation). (p. 400)

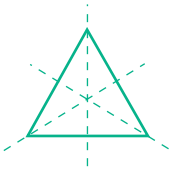
**like terms** Algebraic terms that have exactly the same variables. For example, $5xy$ and $2xy$ are like terms, $3xy$ and $4x^2$ are not like terms. (p. 100)

**likely** A high chance it will happen, probable. (p. 357)

**line graph** A graph made up of a line or several line intervals, often showing how a quantity is changing over time. (p. 293)




**line symmetry** A plane shape has line symmetry if it can be folded so that one half fits exactly on top of the other half; that is, one half is the mirror image of the other. The line of fold is called the **axis of symmetry** (plural: **axes**). An equilateral triangle has 3 axes of symmetry. (p. 178) See also **rotational symmetry**.



**linear** Involving a line. (p. 489)

**linear equation** A formula whose graph is a straight line, or an equation involving a variable that is not raised to a power, such as $2x + 9 = 17$. (p. 491)

**long division** A method for dividing by numbers with 2 or more digits, that is, a number greater than 10. (p. 38)

**loss** The money lost when selling an item at a lower price, when the selling price is less than the cost price. The opposite of **profit**. (p. 266)

**lowest common multiple (LCM)** The smallest multiple that is shared by 2 or more numbers. For example, the LCM of 4 and 10 is 20. (p. 73)

## M

**mean** The average of a set of data, represented by $\bar{x}$, calculated by dividing the sum of the data values by the number of values. (p. 308)

**measure of centre** or **measure of central tendency** An average, middle or typical value of a set of data. The 3 measures of centre are the **mean**, **median** and **mode**. (p. 308)

**median** The middle value when the values of a data set are arranged in order. If the number of values is even, then the median is the average of the 2 middle values. (p. 311)

**mega-** A prefix meaning one million (1 000 000), represented by the symbol M. A megalitre is one million litres.

**micro-** A prefix meaning one-millionth, represented by the Greek letter μ. One microsecond is one-millionth of a second.

**milli-** A prefix meaning one-thousandth $\left(\frac{1}{1000}\right)$, represented by the symbol m. One millimetre is one-thousandth of a metre.

**mixed numeral** A number written as a whole number and a fraction, for example $5\frac{3}{4}$. (p. 238)

**mode** The most common or frequent value(s) in a set of data. (p. 309)

**multimodal distribution** A statistical distribution with 2 or more peaks. See also **unimodal distribution** and **bimodal distribution**. (p. 340)

**multiple (of a number)** The product of the number and a whole number. For example, the multiples of 6 are 6, 12, 18, 24, ... (p. 35)

**mutually exclusive events** Events or categories that have no items in common. (p. 368)

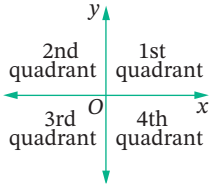
## N

**negative number** A number less than 0, written with a negative sign (−), for example, −3, −10 and −7.

**network (or graph)** A collection of points called vertices connected together by lines called edges. (p. 161)

**nominal data** Categorical data that are names and cannot be ordered, such as colour of eyes. (p. 300)

**number plane** A coordinate grid system based on 2 number lines that cross at right angles: a horizontal line called the $x$-axis and a vertical line called the $y$-axis. Also called **Cartesian plane**. (p. 485)




**numerator** The number above the line in a fraction. The numerator of $\frac{2}{3}$ is 2.

**numerical data** Data that can be measured or counted using values, such as a person's height or number of goals scored. Data that is not **categorical**. See also **discrete data** and **continuous data**. (p. 299)

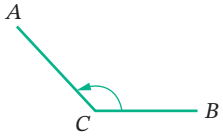
## O

**observed frequency** The actual number of times an event occurs over repeated trials in an experiment. (p. 380)

**observed probability** or **experimental probability** An estimate of theoretical probability; the **relative frequency** of an event in repeated trials of an experiment, found using the formula

$$P(E) = \frac{\text{frequency of } E}{\text{total frequency}}. \text{ (p. 380)}$$

**obtuse angle** A 'wide' angle greater than 90°, but less than 180°. (p. 136)



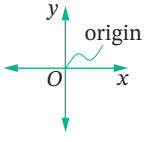

**obtuse-angled triangle** A triangle with one obtuse angle (between 90° and 180°). (p. 143)

**order of operations** The order in which a mixed expression such as $(17 - [3 + 1]) \times 2$ is evaluated. Work from left to right, first performing any operations in grouping symbols, then multiplication and division, and finally addition and subtraction. (p. 50)

**ordered pair** A pair of numbers $(x, y)$ that can be used as coordinates to plot a point on the number plane. (p. 485)

**ordinal data** Categorical data that can be ordered, such as size of coffee cup. (p. 300)

**origin** The point $O(0, 0)$ at the centre of the number plane, where the $x$-axis and $y$-axis cross. (p. 485)



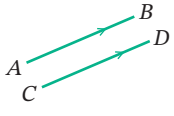

**outcome** In probability, the result of a situation or experiment. For example, when rolling a die, one possible outcome is rolling a 4. (p. 358)

**outlier** An extreme data value that is very different from the other values in a set. (p. 325)

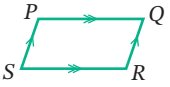
## P

**parallel lines** Lines that point in the same direction and do not intersect.

$AB \parallel CD$ means '$AB$ is parallel to $CD$.' (p. 140)



**parallelogram** A quadrilateral in which the opposite sides are parallel. (p. 147)




**parentheses** (Pronounced 'pa-ren-th-sees') See **grouping symbols**.

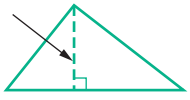
**per annum (p. a.)** Per year. (p. 270)

**percentage** A fraction with denominator 100 that is written in a special way. For example, 7% means $\frac{7}{100}$. (p. 245)

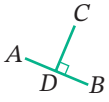
**perfect square** A square number or an algebraic expression that represents one, for example, 64, $(x + 9)^2$, $(a - b)^2$. (p. 121)

**perimeter** The distance around the outside of a shape. The sum of the lengths of its sides. (p. 174)

**perpendicular height** The height of a shape, measured at right angles to the base. (p. 188)

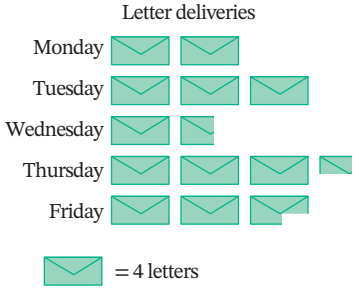


**perpendicular lines** Lines that intersect to form a right angle. $AB \perp CD$ means $AB$ is perpendicular to $CD$.



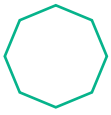

**pi** ($\pi$) A special number, approximately 3.1416, used in calculating circle measurements. Pi is a constant value found by dividing the circumference of any circle by the diameter. (p. 181)

**pictogram (or picture graph)** A type of graph where pictures are used to indicate quantities. (p. 294)




**planar graph** A connected network (graph) where the edges do not cross (overlap) (p. 162)

**polygon** Any flat shape made up of straight sides. (p. 158)



**population** In statistics, all of the items being studied, the entire group. (p. 303)

**positive number** A number greater than 0, for example, 11, 6 and 4.

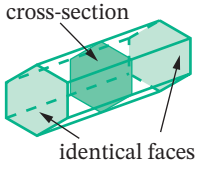
**power (or index)** The number of times a base is multiplied by itself. In $2^5$, the power is 5. Also called the **exponent**. (p. 64)

**power of 10** The numbers 10, 100, 1000 and so on, formed by multiplying 10 by itself repeatedly.

**prime number** A number that has only 2 factors, 1 and itself. For example, 2 and 7 are prime numbers, but 15 is not.

**principal** An amount of money invested or borrowed, on which interest is calculated. (p. 270)

**prism** A solid shape with identical cross-sections with straight sides, such as this hexagonal prism. (p. 213)




**probability** The chance of an event occurring, measured as a fraction, decimal or percentage between 0 and 1. (p. 358)

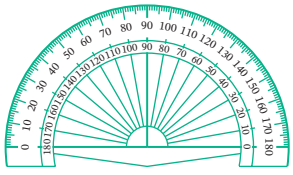
**product** The result of multiplication. The product of 7 and 3 is 21. (p. 102)

**profit** The amount made when selling an item at a higher price, when the selling price is more than the cost price. The opposite of **loss**. (p. 266)

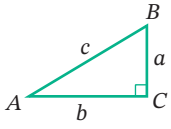
**pronumeral** Another name for **variable**. (p. 90)

**proper fraction** A fraction whose numerator is less than its denominator, such as $\frac{3}{8}$. (p. 240)

**protractor** A geometrical instrument that measures the size of an angle in degrees.



**Pythagoras' theorem** The formula $c^2 = a^2 + b^2$ for a right-angled triangle, where $c$ is the length of the hypotenuse and $a$ and $b$ are the lengths of the other 2 shorter sides. (p. 6)




**Pythagorean triad** A set of 3 positive integers that follow Pythagoras' theorem, such as 3, 4, 5. (p. 22)

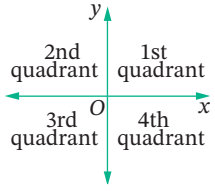

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## Q

**quadrant (of a circle)** A sector that is a quarter of a circle, containing a right angle. (p. 177)



**quadrant (of a number plane)** A quarter of the number plane created by the $x$-axis and $y$-axis crossing at right angles. (p. 485)




**quadratic equation** An equation in which the highest power of the variable is 2, that is, a variable squared, for example, $3x^2 - 6 = 69$. (p. 409)

**quadrilateral** Any polygon with 4 sides. (p. 147)



**quotient** The result of adivision. If 36 is divided by 3, the quotient is 12. (p. 485)

## R

**radius** (plural: **radii**) An interval joining the centre of a circle to the circumference, or the length of that interval. The radius is half of the **diameter**. (p. 177)




**random** In probability, describing a situation where every possible outcome has an equal chance, or is equally likely. (p. 358)

**random sampling** In statistics, selecting a sample in which every person or item in the population has an equal chance of being selected. A sample should be random to be truly representative of the population. (p. 304)

**range** In a set of data, the difference between the highest and lowest values. (p. 311)

**rate** A relationship between 2 quantities measured in different units. For example, a speed of 107 km/h compares distance travelled (in kilometres) with time (in hours). (p. 448)

**ratio** A relationship between quantities measured in the same units. For example, the ratio of 3 teachers to 40 students is 3: 40 (read '3 to 40'). (p. 429)

**reciprocal** The reciprocal of any number is found by first writing the number as a fraction and then swapping the numerator with the denominator. The reciprocal of 5 is $\frac{1}{5}$ and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. The product of any number and its reciprocal is 1. (p. 243)

**rectangle** A quadrilateral with 4 right angles. (p. 147)

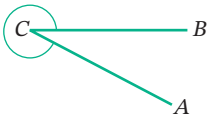


**recurring (or repeating) decimal** A decimal with one or more digits that repeat endlessly. For example 0.1666 ... is abbreviated as $0.1\dot{6}$. (p. 61)

**reflection** The process of 'flipping' a shape across a line to give a mirror image that is back-to-front.



**reflex angle** A 'bent-back' angle greater than 180° but less than 360°. (p. 136)




**regular polygon** A polygon that has all sides equal and all angles equal. For example, this regular pentagon has 5 equal sides and 5 equal angles. (p. 158)



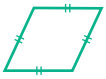
**relative frequency** The frequency of an event in repeated trials of a chance experiment, written as a fraction of the total frequency, found using the formula

$$P(E) = \frac{\text{frequency of } E}{\text{total frequency}}.$$ (p. 380)

**revolution** An angle of 360°. (p. 136)




**rhombus** A quadrilateral with 4 equal sides. (p. 147)



**RHS** The right-hand side (of an equation). (p. 400)

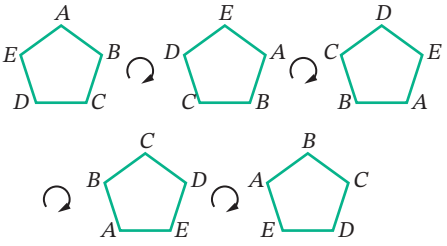
**right angle** A 90° angle, a 'square corner', a quarter-turn. (p. 136)




**right-angled triangle** A triangle with one 90° angle. (p. 6)

**rise** Short for 'vertical rise', the change in vertical position between 2 points on a line or interval, the number of units 'going up', used with the **run** to calculate the **gradient** of a line or interval. See **gradient** for diagram. (p. 504)

**rotational symmetry** A plane shape has rotational symmetry if it can be spun around its centre so that it fits onto itself again before making a complete revolution. The centre point is called the **centre of symmetry**. A regular pentagon has rotational symmetry, and because it can fit onto itself 5 times during one complete revolution, we say that it has *rotational symmetry of order* 5. (p. 135) See also **line symmetry**.




**run** Short for 'horizontal run', the change in horizontal position between 2 points on a line or interval, the number of units 'going right', used with the **rise** to calculate the **gradient** of a line or interval. See **gradient** for diagram. (p. 504)

## S

**sample** In statistics, a group of people or items selected from a population for study. (p. 303)

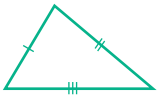
**sample space** In a probability situation, the set of all possible outcomes. (pp. 357, 358)

**satisfy an equation** When a variable (or variables) can be substituted into an equation to show that it is correct or true. For example, $x = 5, y = 8$ satisfies the equation $y = 3x - 7$. (p. 518)

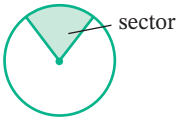
**scaled length** The length on a scale drawing, map or plan, not the actual length. (p. 437)

**scale factor** The amount by which a shape has been enlarged or reduced, equal to $\frac{\text{image length}}{\text{original length}}$. (p. 532)

**scalene triangle** A triangle with no equal sides. (p. 143)



**sector** A region of a circle cut off by 2 radii, shaped like a slice of pizza. (p. 177)




**sector graph** Also called a pie chart, a circular graph that is divided into proportionately-sized sectors to represent parts of a whole. (p. 288)



**segment** A region of a circle cut off by a chord. (p. 178)




**selling price** The price at which an item is sold by the retailer (shop). The shop buys the item for the **cost price**. (p. 266)

**semicircle** Half a circle. (p. 177)

**similar figures** Figures that have the same shape but not necessarily the same size, an enlargement or reduction. The symbol '~' means 'is similar to'. (p. 525)

**simple interest** Interest that is calculated as a percentage of the original principal. (p. 270)

**skewed distribution** A statistical distribution in which most of the data values are clustered at one end, creating a 'tail' at the other end. The tail determines whether the skew is positive or negative. See also **symmetrical distribution**. (p. 340)



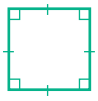
Negatively skewed Positively skewed

**solution** In algebra, the value that makes an equation true. (p. 393)

**speed** A rate that compares distance travelled with time taken. Usually measured in kilometres per hour (km/h) or metres per second (m/s). (p. 455)

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

**square** A quadrilateral with 4 equal sides and 4 right angles. (p. 147)



**square (of a number)** The number multiplied by itself. For example, $7^2 = 7 \times 7 = 49$. (p. 64)

**square root (of a number)** The positive value which, if squared, gives the number. For example, $\sqrt{25} = 5$ because $5^2 = 5 \times 5 = 25$. (pp. 5, 64)

**stationary** Not moving, stopped. (p. 462)

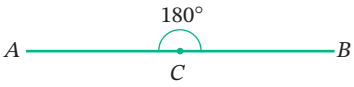
**stem-and-leaf plot** A 'number graph' that lists all the data values in groups. Each value is split into a 'stem' and a 'leaf'. This stem-and-leaf plot shows 12 test marks, from 42 to 82. (p. 326)

| Stem | Leaf |
|---|---|
| 4 | 2 5 |
| 5 | 0 2 8 |
| 6 | 6 7 |
| 7 | 3 5 7 7 |
| 8 | 2 |

Key, 5|8 stands for 58


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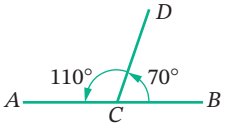
**straight angle** An angle of 180°. (p. 136)




**subject of a formula** The variable for which a formula is written, the variable on the left-hand-side of a formula. For example, the subject of the formula $A = \frac{1}{2}bh$ is $A$. (p. 417)

**sum** The result of an addition. The sum of 5 and 7 is 12. (p. 43)

**supplementary angles** 2 angles whose sum is 180°. The angles 70° and 110° are supplementary. (p. 136)

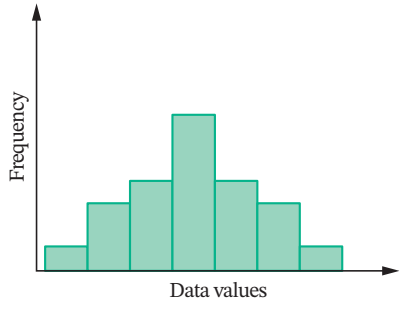



**surd** A square root (or other root) whose exact value cannot be found, such as $\sqrt{10}$ or $\sqrt[3]{7}$. (pp. 5, 64)

**surface area** The total area of all the faces of a solid shape. (p. 541)

**symmetry** See **line symmetry** and **rotational symmetry**.

**symmetrical distribution** A statistical distribution in which all data values are distributed equally on both sides of the centre, its shape having line symmetry. See also **skewed distribution**. (p. 340)

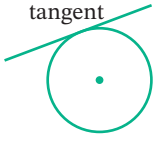



## T

**table of values** A table of ordered pairs of numbers, usually following a formula and which can be graphed on a number plane. The table of values below is for the formula and linear equation $y = 2x - 3$. (p. 475)

| $x$ | −1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | −5 | −3 | −1 | 1 |

**tangent** A line that touches a circle at one point. (A tangent cannot be inside a circle.) (p. 178)




**term (of a pattern)** A number or diagram in a pattern. (p. 480)

**term (of an expression)** A part of an algebraic expression. For example, $b^2 + 6b - 9$ has 3 terms: $b^2$, $6b$ and $-9$. (p. 100)

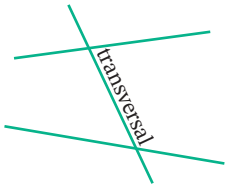
**term (of a ratio)** A number in a ratio, for example, in 14 : 9, the terms are 14 and 9. (p. 429)

**terminating decimal** A decimal whose digits do not repeat endlessly, for example, 0.125. (p. 61)

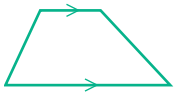
**theoretical probability** or **expected probability** Probability calculated using the formula

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}.$$ (p. 380)

**transversal** A straight line that intersects 2 or more lines at different points. (p. 147)




**trapezium** A quadrilateral with one pair of opposite sides parallel. (p. 147)



**travel graph** A line graph that describes a journey and shows the distance travelled over time. (p. 462)

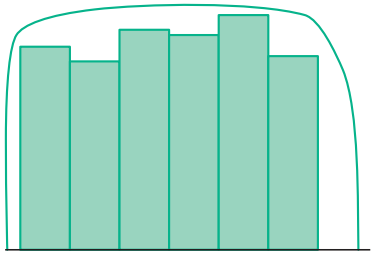
**trial** One go or run of a repeated probability experiment, for example, one roll of a die. (p. 358)

**two-way table** A table that shows the number of items belonging to overlapping categories. (p. 372)

| | Can swim | Cannot swim |
|---|---|---|
| Boys | 13 | 2 |
| Girls | 9 | 3 |

## U

**uniform distribution** The shape of a statistical distribution where each value has the same or similar frequency. Its graph is close to one big rectangle as the data values are evenly distributed. (p. 340)



**unimodal distribution** A statistical distribution with one mode or peak. See also **bimodal distribution** and **multimodal distribution**. (p. 340)

**unit price** The cost of one item or unit, found by dividing the cost of the item by the number of items or units. (p. 450)

**unitary method** A method for finding a quantity by finding the size of one part or 1% first. (pp. 264, 434)

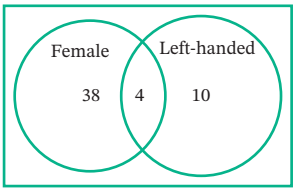
**unknown** Another name for **variable** because its value is usually not known, especially in an equation where the value can be found.

**unlikely** A low chance it will happen, improbable, probably won't happen. (p. 359)

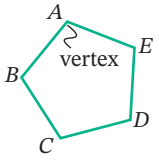
## V

**variable** A symbol, usually a letter of the alphabet, that stands for a number. Also called a **pronumeral** or **unknown**. (p. 90)

**Venn diagram** A diagram of circles (usually overlapping) for grouping items into categories. (p. 367)




**vertex** (plural: **vertices**) A corner of a shape or angle. (p. 147)




**vertex (or node, of a network, plural: vertices)** A point representing an item that is connected by lines called edges. See **edge** for diagram. (p. 161)

**vertical** Going up and down, at a right angle to the **horizontal**. (p. 496)



**vertically opposite angles** A pair of opposite and equal angles formed when 2 lines cross. (p. 136)

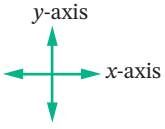
**volume** The amount of space taken up by a solid object, measured in cubic units. (p. 211)

## X

***x*-axis** The horizontal axis of a number plane (running across, see diagram below). (p. 485)

## Y

***y*-axis** The vertical axis of a number plane (running up and down). (p. 485)




***y*-intercept** The $y$-value at which a line cuts the $y$-axis. (p. 501)


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