Essential Insight Exam Guide

Mathematics Applications

Year 12 WACE

Western Australian Curriculum

2025 Edition

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Essential Insight Exam Guide
Mathematics Applications
Year 12 WACE

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Contents

Unit 3

Unit 3.1 – Bivariate data analysis

Section 1

2023 Section 1 Question 1

Bivariate data analysis

A survey was conducted by a film studio executive of filmgoers' favourite genres. The categories chosen were action, drama and comedy. The information is displayed in the table below.

		Genre			
		Action	Drama	Comedy	Total
Age (years)	1–10	40	45		125
	11–20	17		43	86
	21–30		25		125
	Total		96	113	336

(a) Complete the two-way table above. (3 marks)

(b) Identify the response variable for these data. (1 mark)

(c) The incomplete two-way percentaged table is shown below.

		Genre			
		Action	Drama	Comedy	Total
Age (years)	1–10		36		
	11–20		30		
	21–30	56		24	
	Total				

(i) Complete the table above by using either row percentages or column percentages, as appropriate. (3 marks)

(ii) State an association that can be observed from the two-way percentaged table. (1 mark)

2022 Section 1 Question 3

Bivariate data analysis

An analysis was done on hospital patients based on their age and injuries received from various outdoor activities. The information is displayed below.

		Activity			
		Trampoline	Bike	Skateboard	Total
Age (years)	0–8	150	120	30	300
	9–16	180		90	450
	17–24	120	80		400
	25–32	20			250
	Total	470		400	1400

(a) Complete the two-way table above. (2 marks)

(b) Identify the explanatory variable for these data. (1 mark)

(c) The incomplete two-way percentaged table is shown below.

		Activity			
		Trampoline	Bike	Skateboard	Total
Age (years)	0–8	50			
	9–16			20	
	17–24	30			
	25–32		60		
	Total				

(i) Show how the value of 20% was calculated. (1 mark)

(ii) Complete the table by using either row percentages **or** column percentages, as appropriate. (3 marks)

(d) State an association that can be observed from the two-way percentaged table. (1 mark)

2021 Section 1 Question 4

Bivariate data analysis

A public opinion survey was conducted on the statement 'our overwhelming dependence on computers is a good thing', with partial results being shown in the table below.

		Opinion			
		Agree	**Disagree**	**Undecided**	**Total**
Age	**20–39 years**	40	28		80
	40–59 years	38		20	100
	60–79 years	20		18	
	Total				230

(a) Complete the table above. (3 marks)

(b) Identify the response variable. (1 mark)

(c) Use the template below to construct a percentaged two-way frequency table showing either column or row percentages as appropriate, to investigate if there is an association between age and opinion. (4 marks)

		Opinion			
		Agree	**Disagree**	**Undecided**	
Age	**20–39 years**				
	40–59 years				
	60–79 years				

(d) State an association that can be observed from the percentaged two-way frequency table. (1 mark)

2021 Section 1 Question 7

Bivariate data analysis

The ages in years, and salaries in thousands of dollars ($'000), of eight employees at a company are shown below. The equation of the least-squares line for these data is $y = 0.2x + 38$.

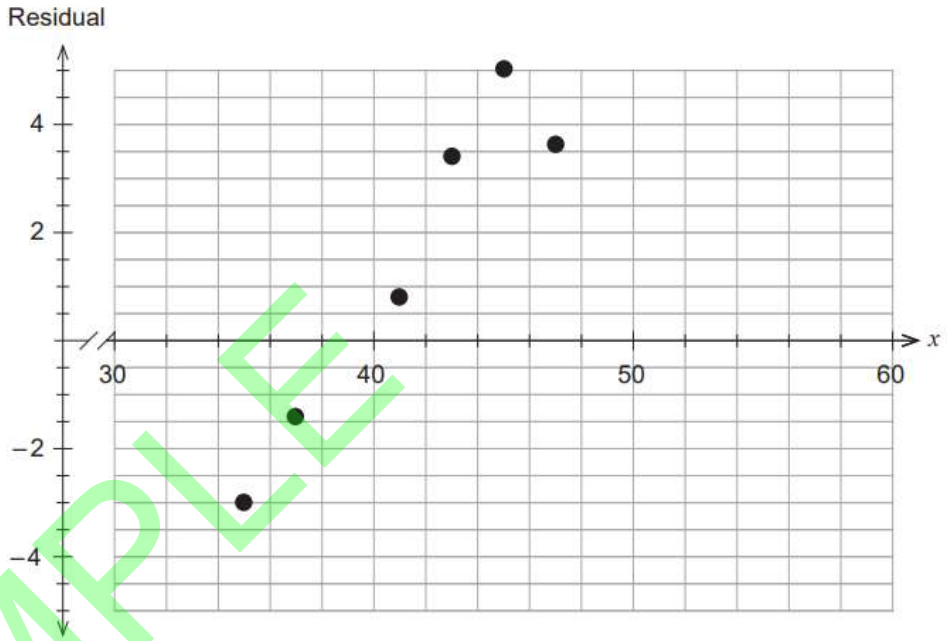
Age (x)	35	37	41	43	45	47	53	55
Salary (y)	42	44	47	50	52	51	49	45

The table below shows the predicted y-values, obtained from the equation of the least-squares line, and the corresponding residuals.

x	y	Predicted y-value	Residual
35	42	45.0	−3.0
37	44	45.4	−1.4
41	47	46.2	0.8
43	50	46.6	3.4
45	52	47.0	5.0
47	51	47.4	3.6
53	49	48.6	0.4
55	45	**A**	**B**

(a) Determine the value of **A** and **B**. (2 marks)

(b) Plot the last two residuals on the graph below. (2 marks)



(c) Justify, using the residual plot in part (b), whether the least-squares line is a good model for these data. (2 marks)

The calculated correlation coefficient for these data is 0.42.

(d) Describe how this supports your response in part (c). (1 mark)

2020 Section 1 Question 1

Bivariate data analysis

The owner of a bicycle shop recorded the type of repairs he made to bicycles with different purchase prices.

		Purchase price			
		Less than $500	From $500 to $1000	Greater than $1000	Total
Type of repair	Wheels and tyres	36	6	18	60
	Gears and brakes	20	12	8	40
	Frame and suspension	15	2	3	20

(a) Identify the explanatory variable for the table above. (1 mark)

The percentages in each row of the following table show the proportion of bicycles with different purchase prices requiring that type of repair.

(b) Complete the table. (2 marks)

		Purchase price			
		Less than $500	From $500 to $1000	Greater than $1000	Total
Type of repair	Wheels and tyres		10		100
	Gears and brakes		30	20	100
	Frame and suspension	75			100

(c) Using the information from the table in part (b), describe **one** association between these variables. (2 marks)

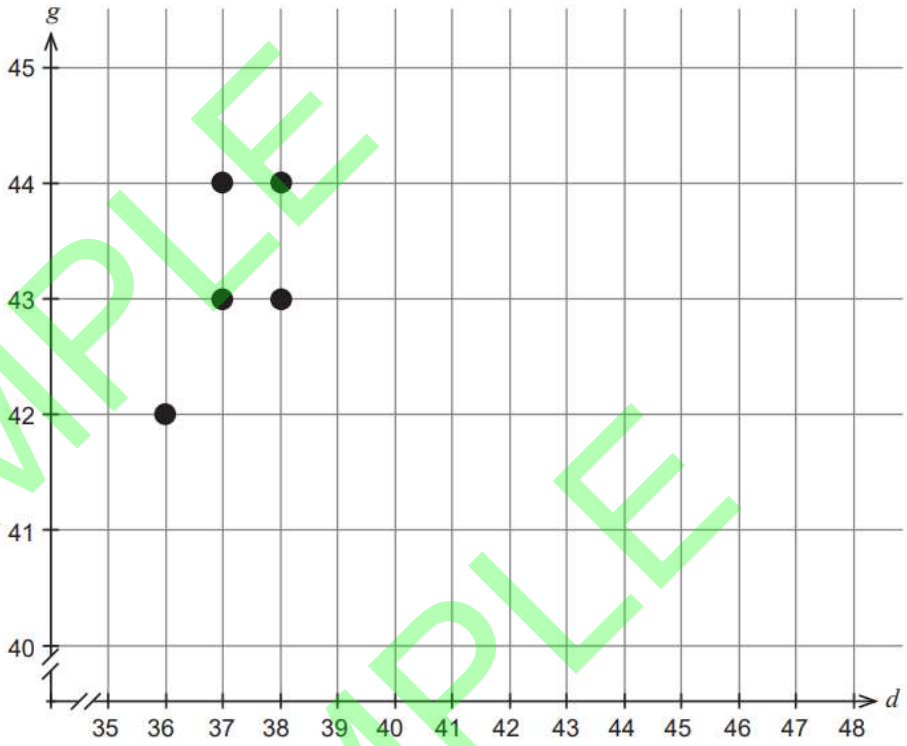
2020 Section 1 Question 4

Bivariate data analysis

The table shows data comparing the gestation period (in days) with the birth weight (in grams) for ten Tasmanian possums.

Gestation period in days (d)	36	37	37	38	38	42	43	44	44	45
Birth weight in grams (g)	42	43	44	43	44	41	42	43	41	42

(a) Plot the last five data points on the axes below. (2 marks)



The correlation coefficient for these observations is approximately –0.6 and the least-squares line is $g = -0.17d + 49$.

(b) Describe what this correlation suggests about the general pattern of association between gestation period and birth weight. (2 marks)

(c) Determine the coefficient of determination for these data. (1 mark)

(d) State the meaning of the coefficient of determination in the context of the question. (1 mark)

(e) Use the least-squares line to predict the birth weight of a possum after 40 days gestation. (1 mark)

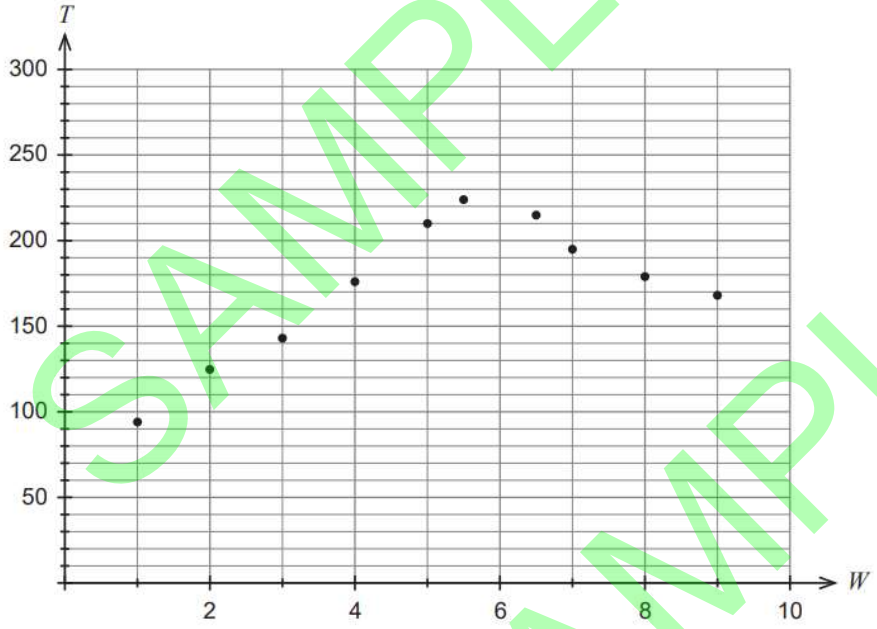
(f) Comment on the validity of this prediction. (2 marks)

(g) Is there any statistical evidence to support the research view that a higher birth weight will cause a shorter gestation period? Justify your answer. (2 marks)

2019 Section 1 Question 2

Bivariate data analysis

Katie is a hobby farmer who has been experimenting with a species of tomato plant growing under the same soil and climatic conditions. She varied the amount of water (W), in millimetres, used during each week and recorded the total number of tomatoes (T) produced by each plant. The scatterplot showing her results is drawn below.



Katie determined the following summary information:

- $r_{WT} = 0.66$
- the equation of the least-squares line is $T = 10.55W + 119.11$

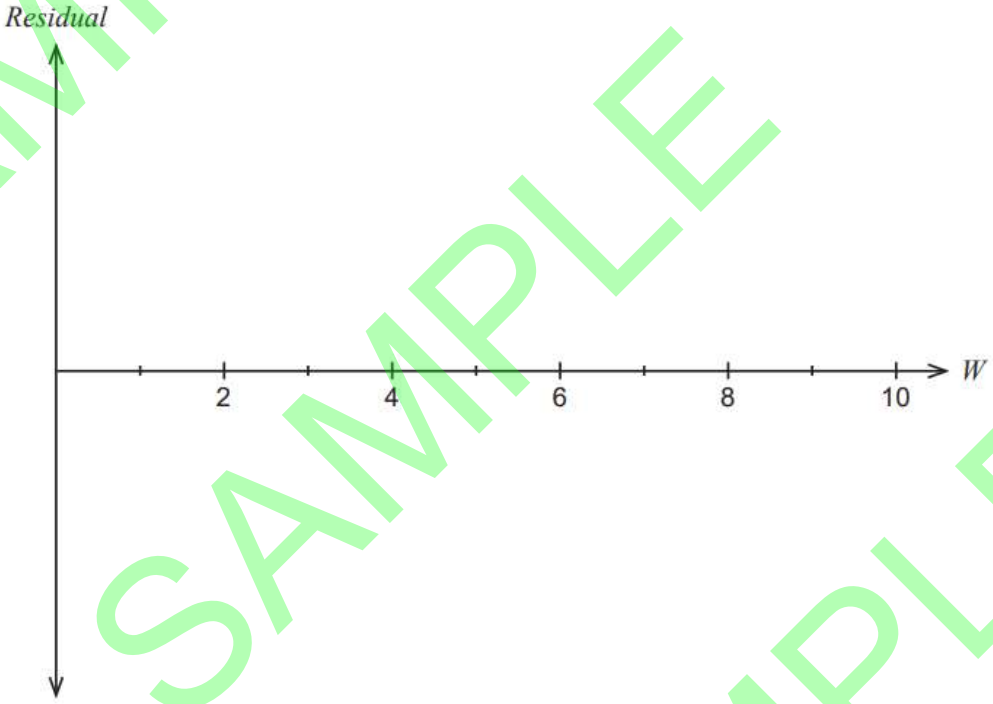
(a) Identify the response variable. (1 mark)

(b) Use the equation of the least-squares line to predict the total number of tomatoes produced when 10 millimetres of water are given to a plant during each week. (2 marks)

(c) Fit the least-squares line to the scatterplot. (2 marks)

Katie decided to draw a residual plot to gather more information about her results.

(d) (i) Sketch a residual plot she would have likely drawn for the given data. Note: you do not have to calculate actual values. (2 marks)



(ii) Use your residual plot to discuss the appropriateness of fitting a linear model to the data. (2 marks)

Section 2

2023 Section 2 Question 7

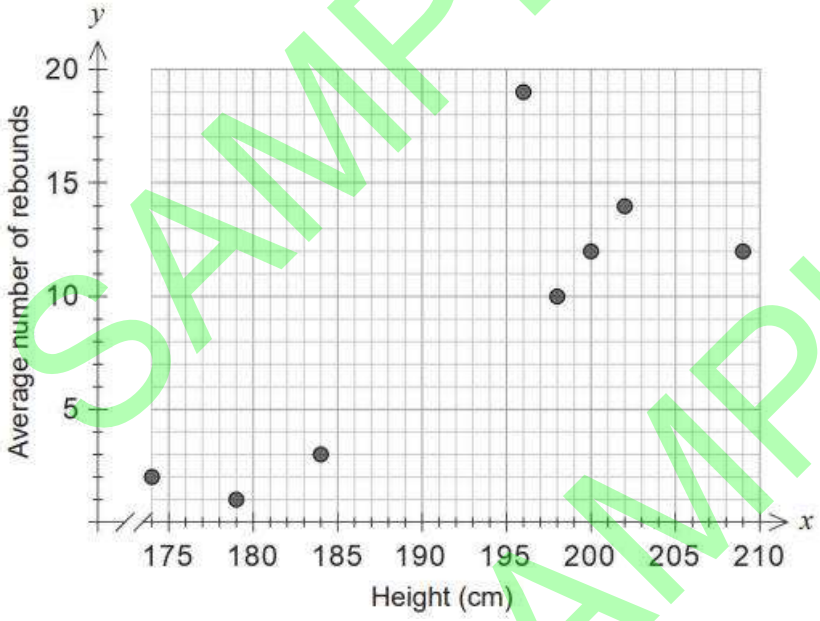
Bivariate data analysis

The heights of players and the average number of rebounds per game were recorded for a basketball team over the course of a 30 game season. The data collected is shown in the table below.

Player	Height (x) (cm)	Average number of rebounds (y)
1	198	10
2	174	2
3	200	12
4	182	2
5	184	3
6	209	12
7	196	19
8	185	8
9	202	14
10	179	1

(a) Identify the explanatory variable. (1 mark)

(b) Complete the scatter graph below by plotting the missing data from the table. (2 marks)



The data has a correlation coefficient of 0.814, and the equation of the least-squares line is y = 0.43x – 74.23.

(c) Draw the least-squares line on the graph above. (2 marks)

(d) Describe the association between players' heights and average number of rebounds in terms of direction and strength. (2 marks)

(e) Determine the coefficient of determination and state its meaning in the context of the question. (2 marks)

2023 Section 2 Question 10

Bivariate data analysis

Data concerning rental properties have been collected from 10 suburbs of a city. The data is for median property value (p) ($'000), median weekly rent ($\$w$) and percentage vacancy rate (v%) within each suburb.

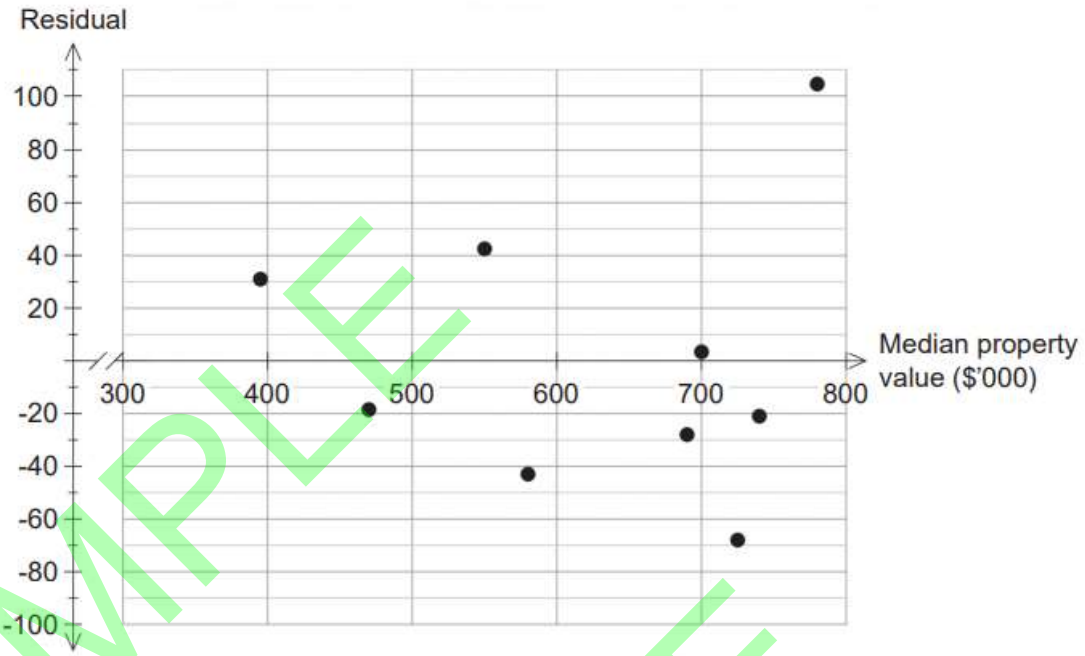
The data in the table below show the median property value and the median weekly rent for the 10 suburbs.

Median property value (p) ($'000)	395	470	550	725	580	780	700	740	690	585
Median weekly rent (w) ($)	445	460	590	630	530	850	680	690	640	575

(a) Calculate the correlation coefficient and the equation of the least-squares line for these data. (3 marks)

(b) In the context of this question, interpret the gradient of the least-squares line determined in part (a). (2 marks)

(c) The graph below shows the residual plot for the first nine suburbs as given in the table. Determine the residual for the 10th suburb and plot this value on the graph. (2 marks)



(d) State a conclusion that can be drawn from the residual plot. (1 mark)

(e) The predicted weekly rent of a property was calculated to be $612. What property value was this based on? (2 marks)

(f) If the data point (780, 850) was removed from all calculations, would the gradient of the least-squares line determined in part (a) increase, decrease or stay the same? (1 mark)

Bivariate data analysis between percentage vacancy rate and median weekly rent produced the following: $r^2_{vw} = 0.85$ and $w = -82.64v + 940.64$.

(g) Explain why $r_{vw} = -0.92$. (2 marks)

(h) A property has a vacancy rate of 4.1% and a median property value of $605 000. Predict the median weekly rent using the most reliable predictor. Justify which predictor is used. (2 marks)

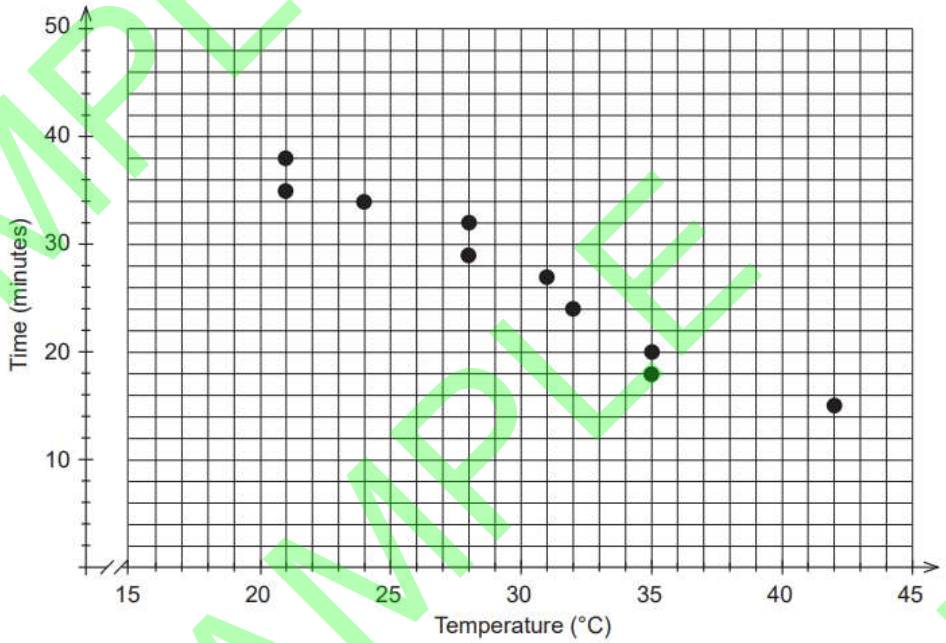
(i) Calculate the expected change in the weekly rent if the percentage vacancy rate increases by 0.4%. (1 mark)

(j) Comment on the statement 'it is clear both property price and vacancy rate will cause changes to the median weekly rent'. Justify your answer. (2 marks)

2022 Section 2 Question 11

Bivariate data analysis

Nullah wanted to see if there was a relationship between outside temperature and the time taken to dry his laundry. The following data was collected over a 10 day period.



Temperature (°C)	28	35	42	31	24	21	21	35	32	28
Time (minutes)	29	20	15	27	34	38	35	18		

(a) Complete the table by locating the data in the graph. (2 marks)

(b) Determine the equation of the least-squares line and state the correlation coefficient. (2 marks)

(c) Draw the least-squares line onto the graph above. (2 marks)

(d) Describe the association between the two variables in terms of direction and strength. (2 marks)

(e) What percentage of the variation in drying time can be explained by the variation in outside temperature? (1 mark)

(f) Identify at least one other factor that could explain the variation in drying time. (1 mark)

(g) The temperature on Day 11 is predicted to be 17 °C.

(i) Use the equation for the least-squares line from part (b) to predict the time Nullah should expect his laundry to dry on this day. (1 mark)

(ii) Is this prediction reliable? Justify your answer. (2 marks)

2022 Section 2 Question 13

Bivariate data analysis

Data have been collected for nine suburbs within a city about the number of mobile phone towers and the number of births in the last 12 months for each suburb.

	Suburb								
	1	2	3	4	5	6	7	8	9
Number of mobile phone towers (n)	4	6	7	8	6	10	5	8	7
Number of births in the last 12 months (b)	25	29	35	45	38	54	22	38	39

The data has a correlation coefficient of 0.92, and the equation of the least-squares line is $b = 5.13n + 1.31$.

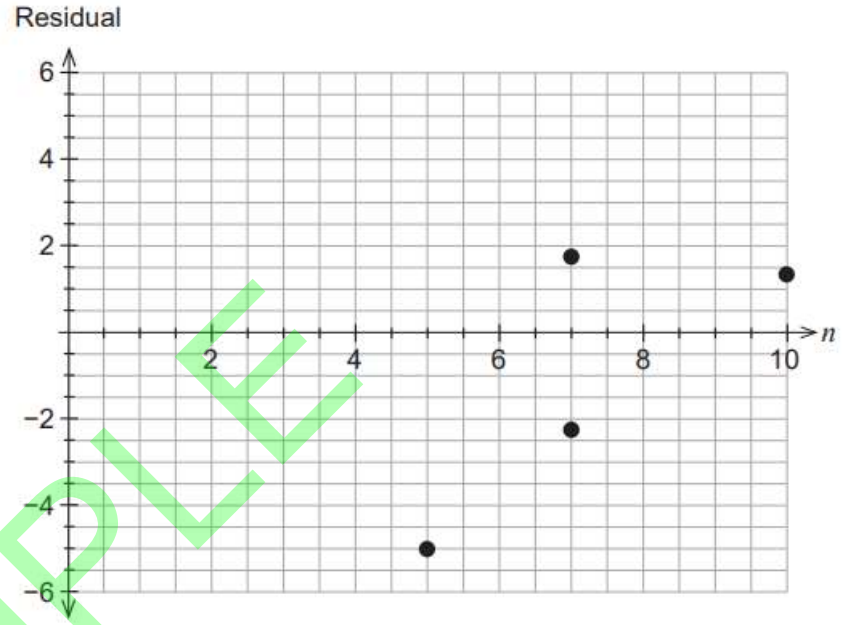
(a) Interpret the gradient of the least-squares line in the context of the question. (2 marks)

(b) Explain the significance of the correlation coefficient in the context of the question. (2 marks)

(c) (i) Predict the number of births for a suburb in this city that has nine mobile phone towers. (1 mark)

(ii) Comment on the validity of the prediction in part (c)(i). Justify your response. (2 marks)

(d) Complete the residual plot below.



(e) Based on the residual plot, comment on whether the least-squares line is a suitable model for these data. (2 marks)

(f) A 10th suburb has a data point (5,12) which has been verified as correct. State a practical explanation of how this could be a correct data point. (1 mark)

(g) A journalist has followed the mathematics involved in working with bivariate data and is writing a report for a newspaper. What is a valid statement that could be made about the observed association between the number of mobile phone towers and the number of births in the last 12 months for suburbs within the city? (2 marks)

2021 Section 2 Question 9

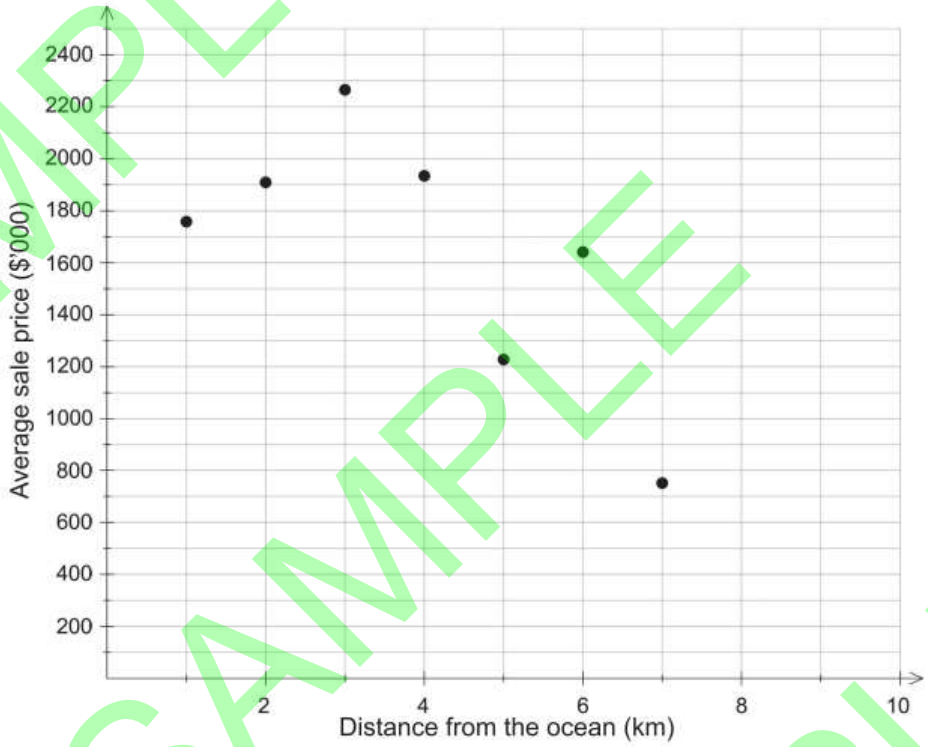
Bivariate data analysis

A real estate agent is analysing data on the sale of houses over the last six months. The table shows the average sale price of houses, in thousands of dollars ($'000), and their distance from the ocean, to the nearest kilometre.

Distance from the ocean (km)	1	2	3	4	5	6	7	8	9
Average sale price ($'000)	1758	1909	2265	1934	1228	1641	751	967	676

(a) State the explanatory variable. (1 mark)

(b) On the scatterplot below, plot the last two data points from the table. (1 mark)



(c) Determine the equation of the least-squares line for these data. (1 mark)

(d) Interpret the slope of the least-squares line from part (c) in the context of this question. (2 marks)

(e) (i) State the value of the correlation coefficient for these data. (1 mark)

(ii) What does the correlation coefficient measure? (1 mark)

(iii) Describe the association between the variables in terms of direction and strength. (2 marks)

(f) What percentage of the variation in average sale price can be explained by the variation in the distance from the ocean? (1 mark)

(g) In six months time, a homebuyer will have saved enough money for a deposit on a house. He would like to live about four kilometres from the ocean.

(i) Use the equation of the least-squares line from part (c) to predict the average sale price of houses four kilometres from the ocean. (1 mark)

(ii) Explain why your prediction is different from the average sale price given in the table. (1 mark)

(h) Give a reason why extrapolation in the context of this question would not make sense. (1 mark)

(i) The real estate agent was talking to some potential buyers and was heard to make the statement, “Having property closer to the ocean causes higher selling prices”. Comment on this statement. (2 marks)

2020 Section 2 Question 10

Bivariate data analysis

A football club records body measurements for all of their players. Shown below are the waistline measurements (cm) and percentage body fat for eleven players.

Player	1	2	3	4	5	6	7	8	9	10	11
Waistline measurement (w)	89	100	87	96	94	83	81	83	84	97	98
Percentage body fat (p)	14	17	11	19	17	12	9	10	8	14	19

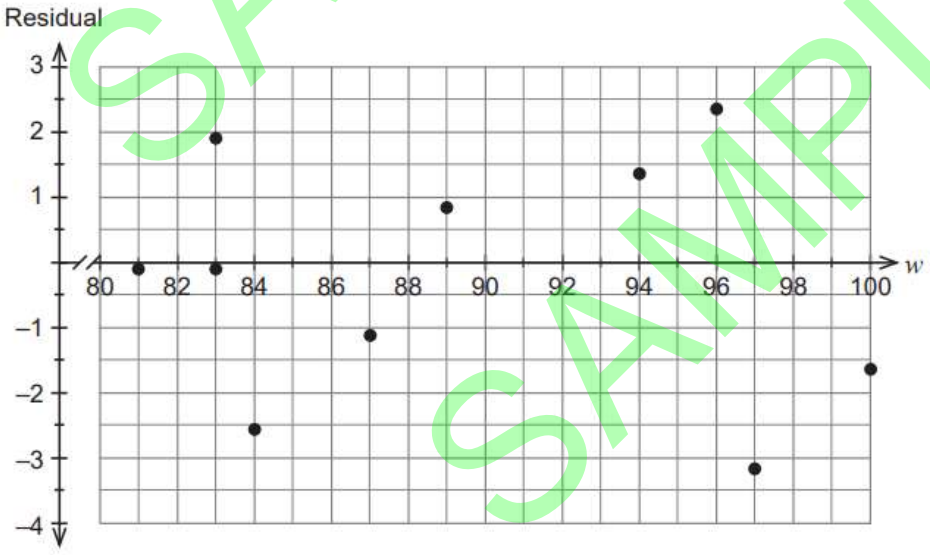
Research has shown that estimates for percentage body fat can be determined by using waistline measurements.

(a) Calculate the correlation coefficient r_{wp} for these data. (1 mark)

(b) Determine the equation of the least-squares line for these data. (1 mark)

(c) In the context of this question, interpret the slope of the line found in part (b). (2 marks)

(d) The residual plot shown below is for the first 10 players' data. Calculate the residual for player number 11 and plot this point on the graph. (2 marks)



(e) Comment on the appropriateness of fitting a linear model to the data. Justify your answer. (2 marks)

(f) What percentage of the variation in the percentage body fat measurements is **unexplained** by the variation in the waistline measurements? (2 marks)

(g) Wayne is player number 12 and has a waistline measurement of 105 cm.

(i) Determine his predicted percentage of body fat. (1 mark)

(ii) Comment on the validity of the prediction and give a justification for your answer. (2 marks)

(h) Player number 13 has a residual of –2.6. What information does this provide about the percentage body fat for this player? (2 marks)

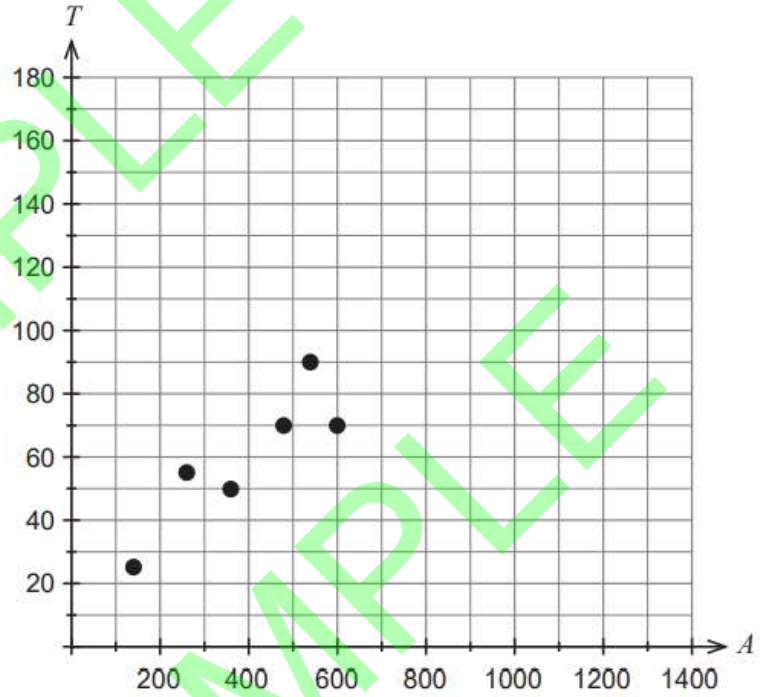
2019 Section 2 Question 8

Bivariate data analysis

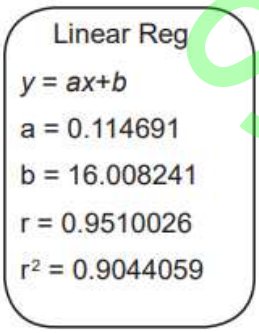
Abdul has a lawnmowing business and is investigating if there is a relationship between the size of a lawn and the length of time it takes to cut the lawn. He takes a random sample of eight customers and measures the areas of their lawns and notes the times, in minutes, it takes to mow their lawns. The results are in the table below, where A is the area of the lawn in square metres and T is the time in minutes. (Note: some values are missing.)

Customer	A	B	C	D	E	F	G	H
A (m^2)		260		480	540	600	860	1180
T (min)	25	55	50	70	90	70	135	140

(a) Complete the scatterplot below. (1 mark)



(b) From the information below, determine the equation of the least-squares line in terms of A and T and state the coefficient of determination for these data. (2 marks)



(c) Interpret the value of the gradient of the least-squares line in the context of the question. (2 marks)

(d) Given that Abdul charges $30 per hour, estimate the charge for mowing a customer's lawn with an area of 500 m2 . (2 marks)

(e) Explain whether the estimate determined in part (d) would be valid. (2 marks)

(f) Using the least-squares line correct to three decimal places

(i) calculate the residuals for Customers B and D. (2 marks)

(ii) explain the significance of the sign and the size of these residuals in reference to the least-squares line. (2 marks)

2019 Section 2 Question 14

Bivariate data analysis

The table below contains data provided by the Australian Bureau of Statistics. It shows the number of households with and without internet access from 2014–2017. All values are in thousands of households.

Internet access

State/territory	2014–15			2016–17		
	Households with internet access '000	Households without internet access '000	Total '000	Households with internet access '000	Households without internet access '000	Total '000
New South Wales	2407.9	414.5	2822.4	2439.9	421.8	2861.7
Victoria	1934.2	305.1	**A**	2008.2	305.8	2314.0
Queensland	1552.4	248.5	1800.9	1591.9	249.8	1841.7
South Australia	565.1	121.4	686.5	575.5	**B**	696.6
Western Australia	843.6	113.0	956.6	859.7	112.6	972.3
Tasmania	172.0	38.7	210.7	177.7	36.2	213.9
Northern Territory	58.1	6.3	64.4	57.6	7.3	64.9
Australian Capital Territory	137.2	9.0	146.2	140.1	9.7	149.8
Total	7670.5	1256.5	8927.0	7850.6	1264.3	9114.9

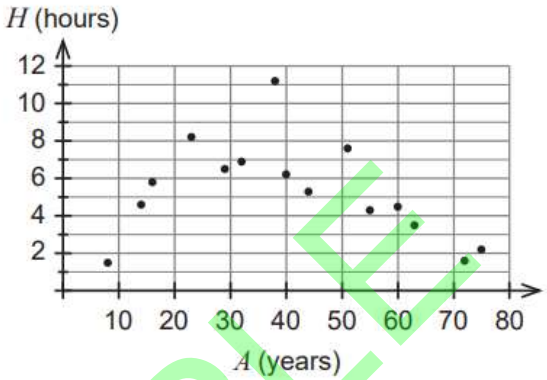
(a) (i) Determine the value of **A** and **B** in the table above. (2 marks)

(ii) Compare the percentages, correct to two decimal places, of households with internet access in New South Wales between 2014–15 and 2016–17. Comment on your results. (3 marks)

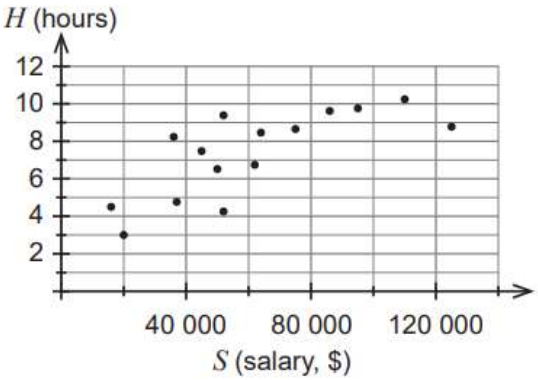
(iii) What is the difference in the data for households with internet access for the Northern Territory over the time period shown, compared to other States and Territories? (1 mark)

An internet service provider from Tasmania wanted to determine whether a person's age, *A*, or salary, *S*, affected the number of hours, *H*, of internet usage per day. The graphs below each show the recorded data for people surveyed.

Daily Internet Usage According to Age



Daily Internet Usage According to Salary



(b) (i) Describe the association between a person's salary and the number of hours of internet usage per day, in terms of direction and form. (2 marks)

(ii) The internet service provider calculated the correlation coefficient for the data contained in each graph. The values they calculated are contained in the following list.

–1.25, –0.95, –0.75, –0.3, 0.1, 0.3, 0.75, 0.95, 1.25

Choose the best estimate from the list for each of the graphs shown above. (2 marks)

Graph	Correlation coefficient
Daily internet usage according to age	
Daily internet usage according to salary	

2019 Section 2 Question 16

Bivariate data analysis

The table below records the altitude (metres above sea level), latitude (° S) and mean maximum temperature (°C) during January for eight cities in the southern hemisphere.

Altitude (A)	Latitude (L)	Mean maximum temperature (T)
15	31.95	25
20	43.53	20
24	42.88	18
314	45.03	16
8	6.18	28
154	12.05	26
37	12.46	29
8	34.60	25

Comparing altitude and the mean maximum temperature, it was determined that the least-squares line for these data was $T = -0.022A + 24.97$ and $r_{AT} = -0.50$.

(a) Determine the coefficient of determination for altitude and the mean maximum temperature and interpret this value. (2 marks)

(b) Determine the equation of the least-squares line for comparing latitude and the mean maximum temperature and state the correlation coefficient. (2 marks)

Rio de Janeiro has a latitude of 22.93° S and an altitude of 9 metres.

(c) Use the two least-squares lines above to predict the mean maximum temperature in January for Rio de Janeiro. Which prediction is more valid? Justify your choice. (3 marks)

2023 Section 1 Question 1

Bivariate data analysis

A survey was conducted by a film studio executive of filmgoers' favourite genres. The categories chosen were action, drama and comedy. The information is displayed in the table below.

		Genre			
		Action	Drama	Comedy	Total
Age (years)	1–10	40	45		125
	11–20	17		43	86
	21–30		25		125
	Total		96	113	336

(a) Complete the two-way table above. (3 marks)

		Genre			
		Action	Drama	Comedy	Total
Age (years)	1–10	40	45	**40**	125
	11–20	17	**26**	43	86
	21–30	**70**	25	**30**	125
	Total	**127**	96	113	336

Solution
see table above
Specific behaviours
✓ determines three correct values ✓ determines four correct values ✓ determines all correct values

(b) Identify the response variable for these data. (1 mark)

Solution
genre
Specific behaviours
✓ correct response variable

(c) The incomplete two-way percentaged table is shown below.

		Genre			
		Action	**Drama**	**Comedy**	**Total**
Age (years)	**1–10**		36		
	11–20		30		
	21–30	56		24	
	Total				

(i) Complete the table above by using either row percentages or column percentages, as appropriate. (3 marks)

		Genre			
		Action	**Drama**	**Comedy**	**Total**
Age (years)	**1–10**	**32**	36	**32**	**100**
	11–20	**20**	30	**50**	**100**
	21–30	56	**20**	24	**100**
	Total				

Solution
see table above
Specific behaviours
✓ determines row percentages are required ✓ determines three correct values ✓ determines all correct values

(ii) State an association that can be observed from the two-way percentaged table. (1 mark)

Solution
as age increases, the percentage of the drama genre decreases
Specific behaviours
✓ correctly states association

2022 Section 1 Question 3

Bivariate data analysis

An analysis was done on hospital patients based on their age and injuries received from various outdoor activities. The information is displayed below.

		Activity			
		Trampoline	**Bike**	**Skateboard**	**Total**
Age (years)	**0–8**	150	120	30	300
	9–16	180	**180**	90	450
	17–24	120	80	**200**	400
	25–32	20	**150**	**80**	250
	Total	470	**530**	400	1400

(a) Complete the two-way table above. (2 marks)

Solution
See table above
Specific behaviours
✓ correctly completes three or more entries ✓ correctly completes all entries

(b) Identify the explanatory variable for these data. (1 mark)

Solution
Age
Specific behaviours
✓ correctly identifies age

(c) The incomplete two-way percentaged table is shown below.

		Activity			
		Trampoline	**Bike**	**Skateboard**	**Total**
Age (years)	**0–8**	50	**40**	**10**	**100**
	9–16	**40**	**40**	20	**100**
	17–24	30	**20**	**50**	**100**
	25–32	**8**	60	**32**	**100**
	Total				

(i) Show how the value of 20% was calculated. (1 mark)

Solution
$\frac{90}{450} = \frac{1}{5} = 20\%$
Specific behaviours
✓ shows correct calculation

(ii) Complete the table by using either row percentages **or** column percentages, as appropriate. (3 marks)

Solution
See above table
Specific behaviours
✓ correctly identifies row percentages required and row totals equal 100 ✓ correctly calculates values for age 25–32 ✓ correctly calculates remaining values

(d) State an association that can be observed from the two-way percentaged table. (1 mark)

Solution
As a patient's age increases, injuries received due to trampoline decreases.
Specific behaviours
✓ correctly states association
Accept other relevant answers.

2021 Section 1 Question 4

Bivariate data analysis

A public opinion survey was conducted on the statement 'our overwhelming dependence on computers is a good thing', with partial results being shown in the table below.

		Opinion			
		Agree	**Disagree**	**Undecided**	**Total**
Age	**20–39 years**	40	28		80
	40–59 years	38		20	100
	60–79 years	20		18	
	Total				230

(a) Complete the table above. (3 marks)

Solution
See table above
Specific behaviours
✓ determines at least 3 correct entries ✓ determines at least 5 correct entries ✓ determines correctly all entries

(b) Identify the response variable. (1 mark)

Solution
Opinion
Specific behaviours
✓ correct answer

(c) Use the template below to construct a percentaged two-way frequency table showing either column or row percentages as appropriate, to investigate if there is an association between age and opinion. (4 marks)

		Opinion			
		Agree	Disagree	Undecided	Total
Age	20–39 years	50	35	15	100
	40–59 years	38	42	20	100
	60–79 years	40	24	36	100

Solution
See table above
Specific behaviours
✓ determines row percentages are required ✓ determines at least 3 correct entries ✓ determines at least 6 correct entries ✓ determines correctly all entries

(d) State an association that can be observed from the percentaged two-way frequency table. (1 mark)

Solution
As age increases, the percentage undecided increases.
Specific behaviours
✓ gives correct association

2021 Section 1 Question 7

Bivariate data analysis

The ages in years, and salaries in thousands of dollars ($'000), of eight employees at a company are shown below. The equation of the least-squares line for these data is $y = 0.2x + 38$.

Age (x)	35	37	41	43	45	47	53	55
Salary (y)	42	44	47	50	52	51	49	45

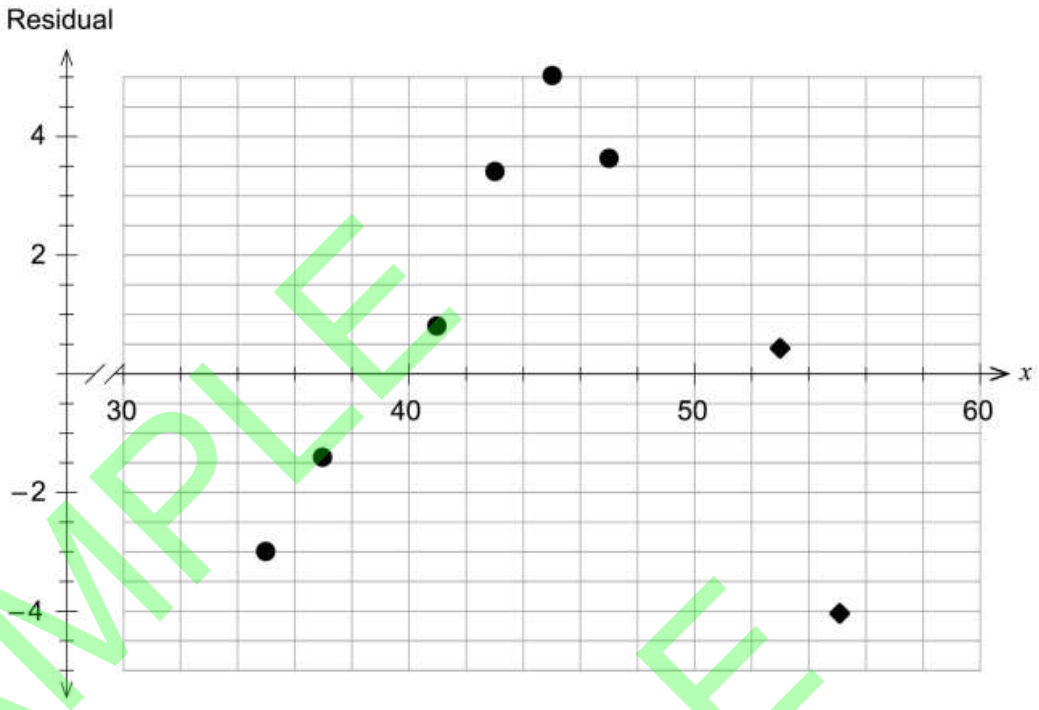
The table below shows the predicted y-values, obtained from the equation of the least-squares line, and the corresponding residuals.

x	y	Predicted y-value	Residual
35	42	45.0	−3.0
37	44	45.4	−1.4
41	47	46.2	0.8
43	50	46.6	3.4
45	52	47.0	5.0
47	51	47.4	3.6
53	49	48.6	0.4
55	45	**A**	**B**

(a) Determine the value of **A** and **B**. (2 marks)

Solution
$y = 0.2(55) + 38$ $= 11 + 38$ $= 49$ Therefore, **A** = 49 **B** = 45 – 49 = –4.0
Specific behaviours
✓ correctly calculates the value of **A** ✓ correctly calculates the value of **B**

(b) Plot the last two residuals on the graph below. (2 marks)

Solution

Specific behaviours
✓ correctly plots residuals at (53, 0.4) ✓ correctly plots residual at (55, **B**) i.e. (55,−4.0)

(c) Justify, using the residual plot in part (b), whether the least-squares line is a good model for these data. (2 marks)

Solution
The residual plot shows a pattern, therefore, this suggests that the least-squares line is unsuitable
Specific behaviours
✓ correctly states the model is unsuitable ✓ correctly states the residual plot shows a pattern

The calculated correlation coefficient for these data is 0.42.

(d) Describe how this supports your response in part (c). (1 mark)

Solution
As the correlation coefficient is low, this supports the answer to part (c), that the least squares lines is unsuitable
Specific behaviours
✓ states a low correlation coefficient supports unsuitability

2020 Section 1 Question 1

Bivariate data analysis

The owner of a bicycle shop recorded the type of repairs he made to bicycles with different purchase prices.

		Purchase price			
		Less than $500	From $500 to $1000	Greater than $1000	Total
Type of repair	Wheels and tyres	36	6	18	60
	Gears and brakes	20	12	8	40
	Frame and suspension	15	2	3	20

(a) Identify the explanatory variable for the table above. (1 mark)

Solution
Purchase price
Specific behaviours
✓ identifies correct explanatory variable

The percentages in each row of the following table show the proportion of bicycles with different purchase prices requiring that type of repair.

(b) Complete the table. (2 marks)

		Purchase price			
		Less than $500	From $500 to $1000	Greater than $1000	Total
Type of repair	Wheels and tyres	**60**	10	**30**	100
	Gears and brakes	**50**	30	20	100
	Frame and suspension	75	**10**	**15**	100

Solution
See table above
Specific behaviours
✓ calculates 3 correct values ✓ calculates all correct values

(c) Using the information from the table in part (b), describe **one** association between these variables. (2 marks)

Solution
The higher the purchase price, the lower the percentage of gears and brakes repairs that are needed.
Specific behaviours
✓ identifies an association between gears and brakes repairs and purchase price ✓ describes association in terms of differences in the percentages from the information in part (b)

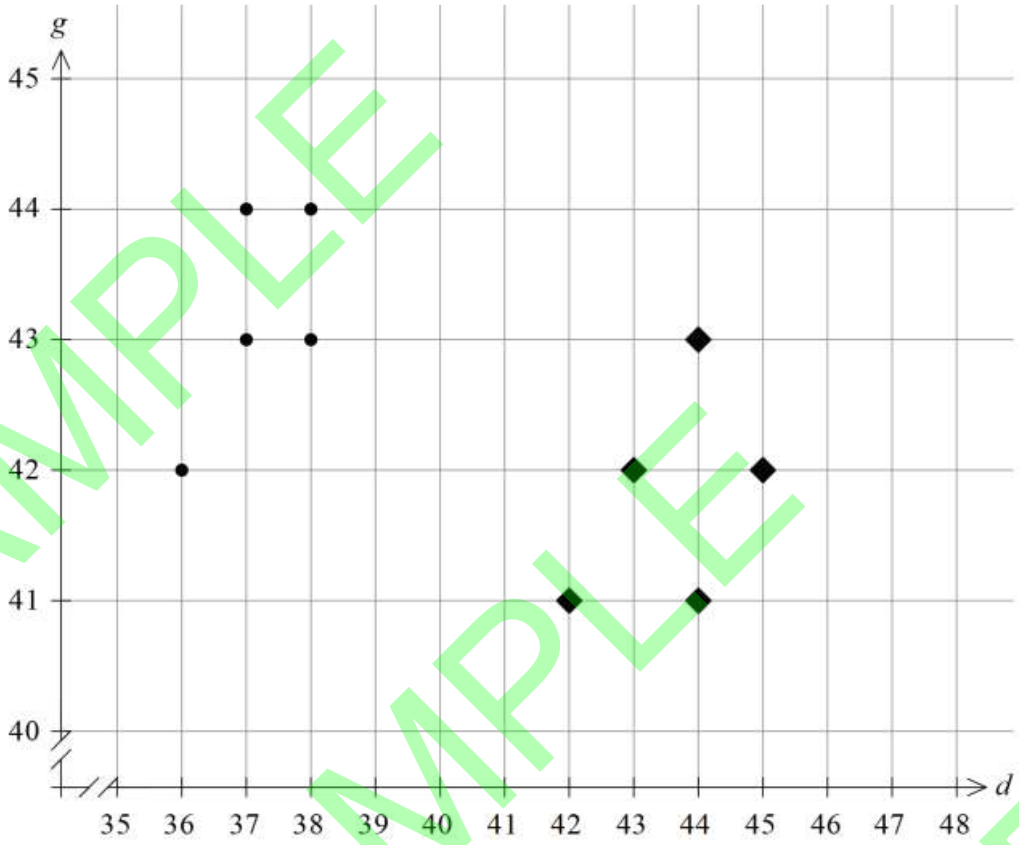
2020 Section 1 Question 4

Bivariate data analysis

The table shows data comparing the gestation period (in days) with the birth weight (in grams) for ten Tasmanian possums.

Gestation period in days (d)	36	37	37	38	38	42	43	44	44	45
Birth weight in grams (g)	42	43	44	43	44	41	42	43	41	42

(a) Plot the last five data points on the axes below. (2 marks)



Solution
See graph above
Specific behaviours
✓ correctly plots at least three points
✓ correctly plots all points

The correlation coefficient for these observations is approximately –0.6 and the least-squares line is $g = -0.17d + 49$.

(b) Describe what this correlation suggests about the general pattern of association between gestation period and birth weight. (2 marks)

Solution
Moderate negative linear relationship
Specific behaviours
✓ correctly mentions negative linear relationship
✓ correctly mentions moderate relationship

(c) Determine the coefficient of determination for these data. (1 mark)

Solution
36% (0.36)
Specific behaviours
✓ correctly calculates the coefficient of determination

(d) State the meaning of the coefficient of determination in the context of the question. (1 mark)

Solution
36% of the variation in their birth weight can be explained by the variation in the gestation period
Specific behaviours
✓ correctly states the meaning in the context of the question

(e) Use the least-squares line to predict the birth weight of a possum after 40 days gestation. (1 mark)

Solution
$g = -0.17 \times 40 + 49$ $g = 42.2$ grams
Specific behaviours
✓ correctly calculates birth weight

(f) Comment on the validity of this prediction. (2 marks)

Solution
It may not be valid (even though it is interpolation) as the correlation is moderate.
Specific behaviours
✓ correctly states prediction may not be valid ✓ correctly states correlation is moderate

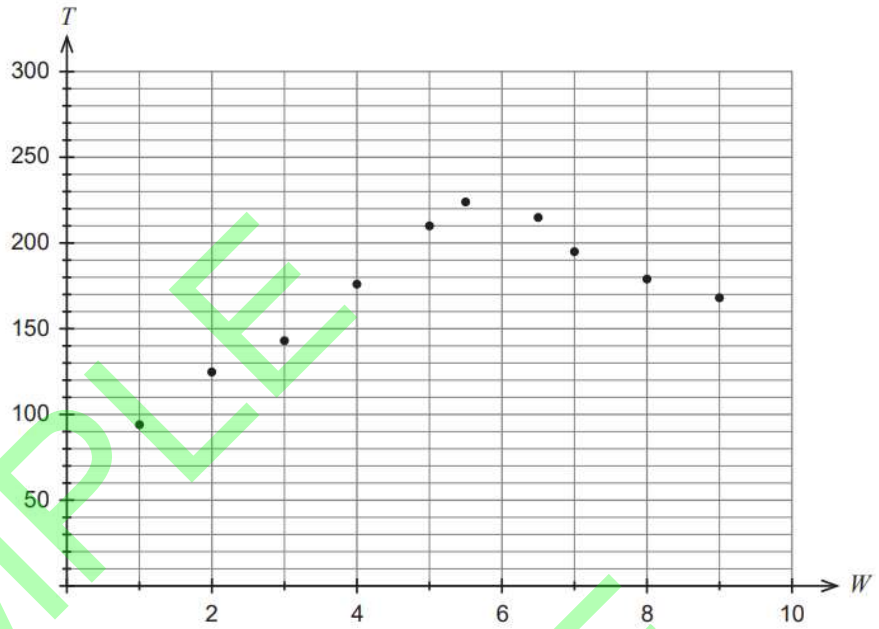
(g) Is there any statistical evidence to support the research view that a higher birth weight will cause a shorter gestation period? Justify your answer. (2 marks)

Solution
There is not enough evidence to say there is a causal relationship between birth weight and a shorter gestation period, there may be other factors involved.
Specific behaviours
✓ correctly states there is not enough supporting evidence ✓ states there may be other factors involved

2019 Section 1 Question 2

Bivariate data analysis

Katie is a hobby farmer who has been experimenting with a species of tomato plant growing under the same soil and climatic conditions. She varied the amount of water (W), in millimetres, used during each week and recorded the total number of tomatoes (T) produced by each plant. The scatterplot showing her results is drawn below.



Katie determined the following summary information:

- $r_{WT} = 0.66$
- the equation of the least-squares line is $T = 10.55W + 119.11$

(a) Identify the response variable. (1 mark)

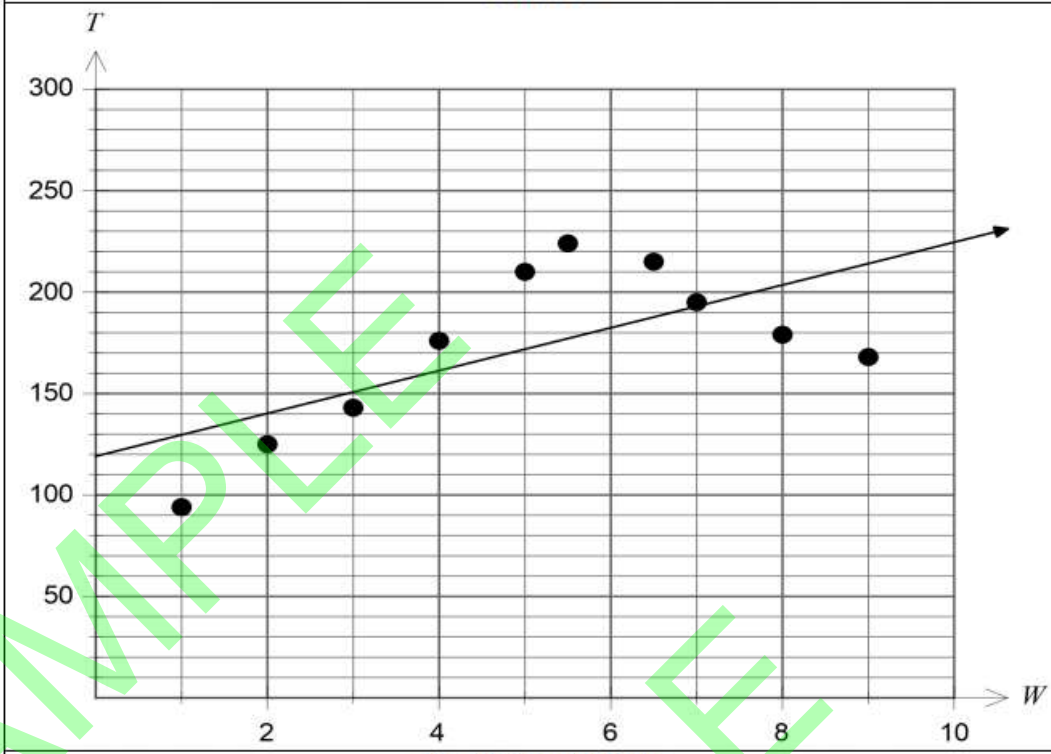
Solution
Number of tomatoes
Specific behaviours
✓ identifies correct variable

(b) Use the equation of the least-squares line to predict the total number of tomatoes produced when 10 millimetres of water are given to a plant during each week. (2 marks)

Solution
$T = 10.55 \times 10 + 119.11$ $T = 105.5 + 119.11$ $T = 224.61 \approx 224 / 225$
Specific behaviours
✓ correctly substitutes 10 into least-squares line ✓ rounds correctly to a whole number of tomatoes

(c) Fit the least-squares line to the scatterplot. (2 marks)

Solution

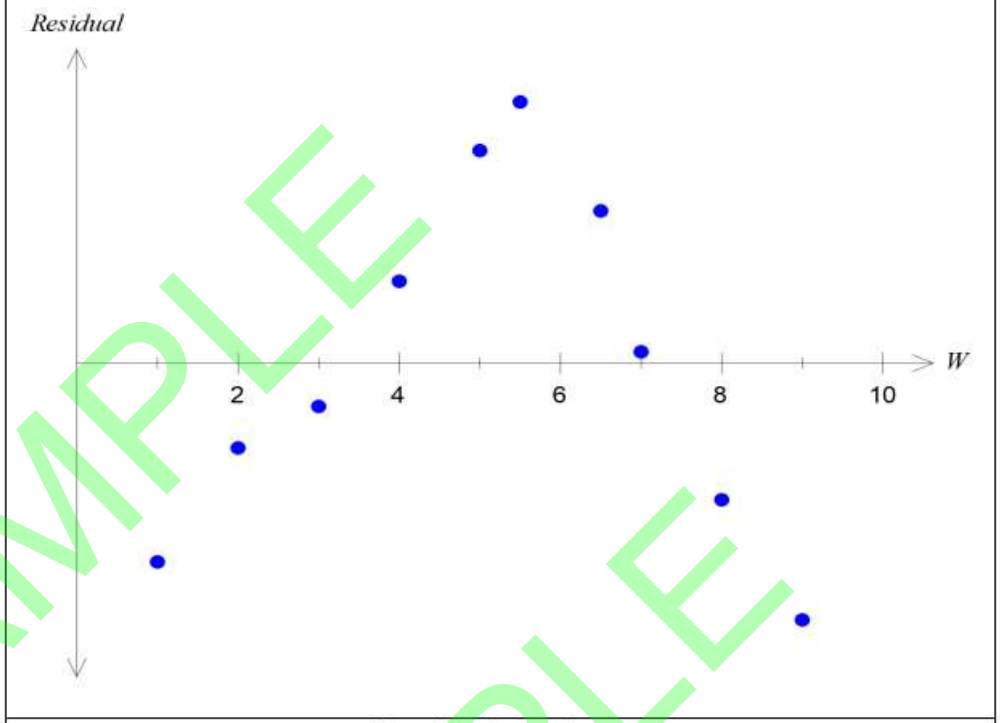


Specific behaviours

✓ plots one correct point
✓ plots a second correct point and draws least-squares line

Katie decided to draw a residual plot to gather more information about her results.

(d) (i) Sketch a residual plot she would have likely drawn for the given data. Note: you do not have to calculate actual values. (2 marks)

Solution

Specific behaviours
✓ correctly shows shape of residual plot ✓ plots all points on correct side of horizontal axis

(ii) Use your residual plot to discuss the appropriateness of fitting a linear model to the data. (2 marks)

Solution
A linear model is not appropriate as a pattern is evident in the residual plot.
Specific behaviours
✓ states linear model is not appropriate ✓ states a valid reason

2023 Section 2 Question 7

Bivariate data analysis

The heights of players and the average number of rebounds per game were recorded for a basketball team over the course of a 30 game season. The data collected is shown in the table below.

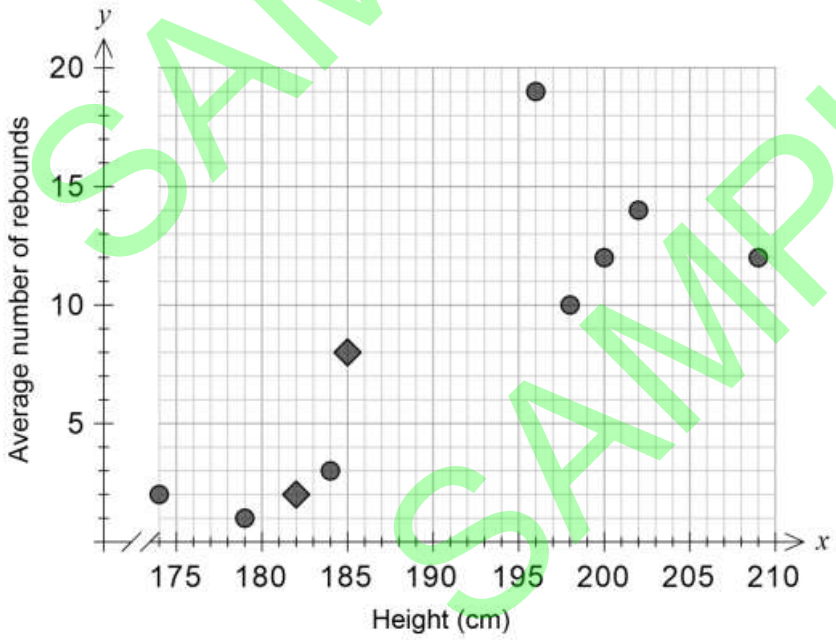
Player	Height (x) (cm)	Average number of rebounds (y)
1	198	10
2	174	2
3	200	12
4	182	2
5	184	3
6	209	12
7	196	19
8	185	8
9	202	14
10	179	1

(a) Identify the explanatory variable. (1 mark)

Solution
height
Specific behaviours
✓ identifies correct variable

(b) Complete the scatter graph below by plotting the missing data from the table. (2 marks)

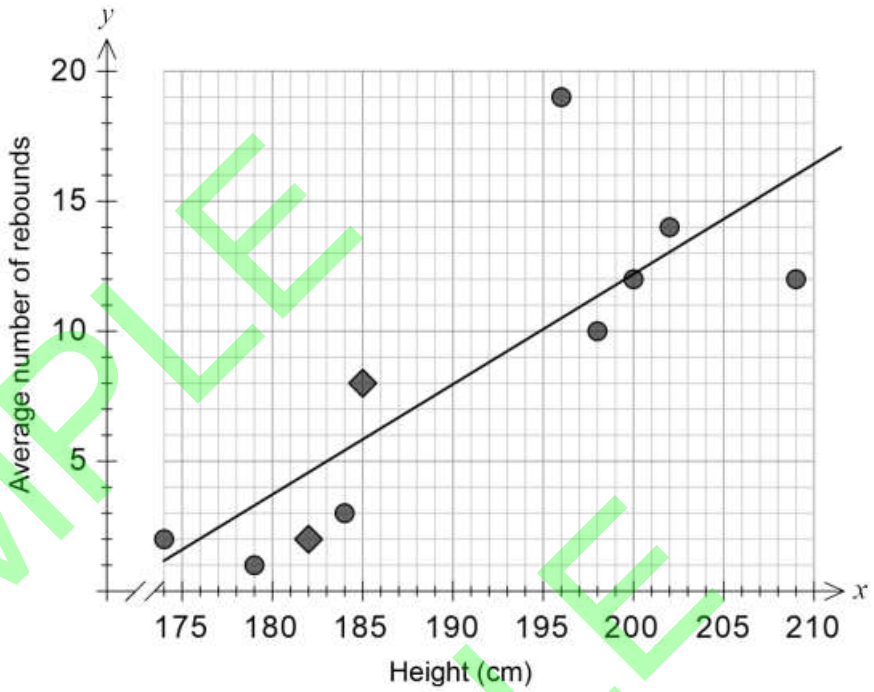
Solution



Specific behaviours
✓ plots one correct coordinate ✓ plots both correct coordinates

The data has a correlation coefficient of 0.814, and the equation of the least-squares line is y = 0.43x – 74.23.

(c) Draw the least-squares line on the graph above. (2 marks)

Solution

Specific behaviours
✓ shows correct slope ✓ shows that line passes approximately through (200, 12)

(d) Describe the association between players' heights and average number of rebounds in terms of direction and strength. (2 marks)

Solution
strong positive association
Specific behaviours
✓ correctly states strong ✓ correctly states positive association

(e) Determine the coefficient of determination and state its meaning in the context of the question. (2 marks)

Solution
$r^2 = 0.663$ Therefore 66.3% of the variation in average number of rebounds can be explained by the variation of the height of the basketball players.
Specific behaviours
✓ determines correct coefficient of determination ✓ determines correct meaning

2023 Section 2 Question 10

Bivariate data analysis

Data concerning rental properties have been collected from 10 suburbs of a city. The data is for median property value (p) ($'000), median weekly rent ($\$w$) and percentage vacancy rate (v%) within each suburb.

The data in the table below show the median property value and the median weekly rent for the 10 suburbs.

Median property value (p) ($'000)	395	470	550	725	580	780	700	740	690	585
Median weekly rent (w) ($)	445	460	590	630	530	850	680	690	640	575

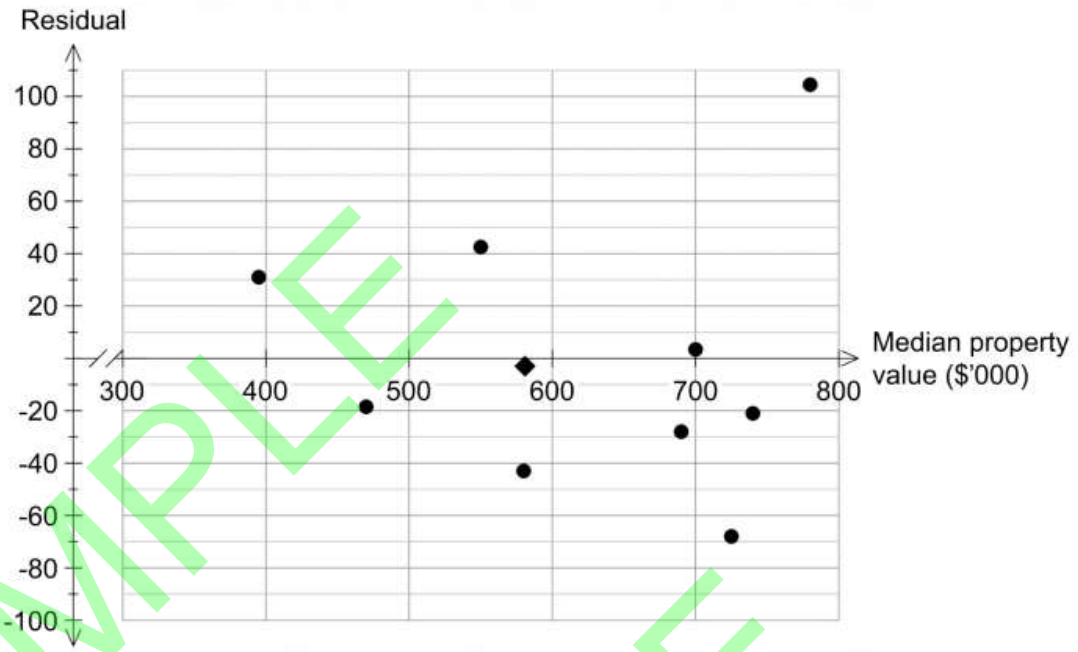
(a) Calculate the correlation coefficient and the equation of the least-squares line for these data. (3 marks)

Solution
$r = 0.91$ $\quad w = 0.861p + 73.84$
Specific behaviours
✓ determines correct correlation coefficient ✓ determines correct least squares line ✓ uses correct variables

(b) In the context of this question, interpret the gradient of the least-squares line determined in part (a). (2 marks)

Solution
For each $1000 increase in median property value, median weekly rent will increase by $0.86
Specific behaviours
✓ uses median property value and median weekly rent ✓ uses correct values

(c) The graph below shows the residual plot for the first nine suburbs as given in the table. Determine the residual for the 10th suburb and plot this value on the graph. (2 marks)



Solution
Residual: (585, –2.6) See graph above for plot
Specific behaviours
✓ determines correct point ✓ plots point correctly

(d) State a conclusion that can be drawn from the residual plot. (1 mark)

Solution
It is appropriate to use a linear model for these data as there is no clear pattern in the residual plot
Specific behaviours
✓ states correct conclusion

(e) The predicted weekly rent of a property was calculated to be $612. What property value was this based on? (2 marks)

Solution
$612 = 0.861p + 73.84 \quad \Rightarrow p = 625.04$ Property value $625 000
Specific behaviours
✓ solves for p ✓ expresses answer in thousands of dollars

(f) If the data point (780, 850) was removed from all calculations, would the gradient of the least-squares line determined in part (a) increase, decrease or stay the same? (1 mark)

Solution
decrease
Specific behaviours
✓ gives correct answer

Bivariate data analysis between percentage vacancy rate and median weekly rent produced the following: $r^2_{vw} = 0.85$ and $w = -82.64v + 940.64$.

(g) Explain why $r_{vw} = -0.92$. (2 marks)

Solution
$\sqrt{0.85} = 0.92$, but the gradient of least squares line is negative therefore r is negative.
Specific behaviours
✓ determines square root ✓ explains why negative

(h) A property has a vacancy rate of 4.1% and a median property value of $605 000. Predict the median weekly rent using the most reliable predictor. Justify which predictor is used. (2 marks)

Solution
the strength of $r_{vw} > r_{pw}$, therefore use vacancy rate predicted value $\approx \$602$
Specific behaviours
✓ explains why vacancy rate is the best predictor ✓ determines correct predicted value

(i) Calculate the expected change in the weekly rent if the percentage vacancy rate increases by 0.4%. (1 mark)

Solution
$-82.64 \times 0.4 = -\$33.06$ (A decrease of $33.06)
Specific behaviours
✓ determines correct change

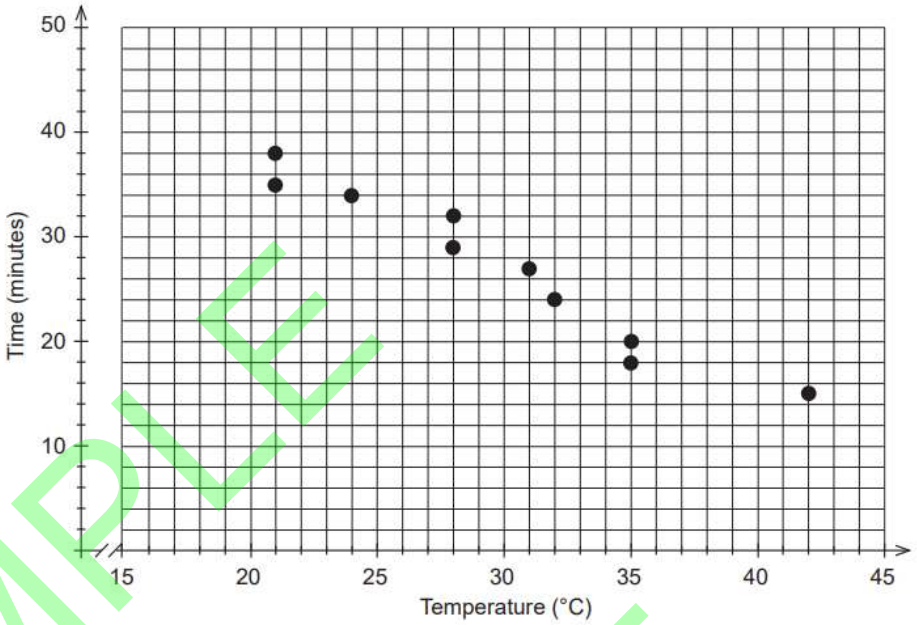
(j) Comment on the statement 'it is clear both property price and vacancy rate will cause changes to the median weekly rent'. Justify your answer. (2 marks)

Solution
statement incorrect, cause not established
Specific behaviours
✓ comments statement incorrect ✓ gives correct justification

2022 Section 2 Question 11

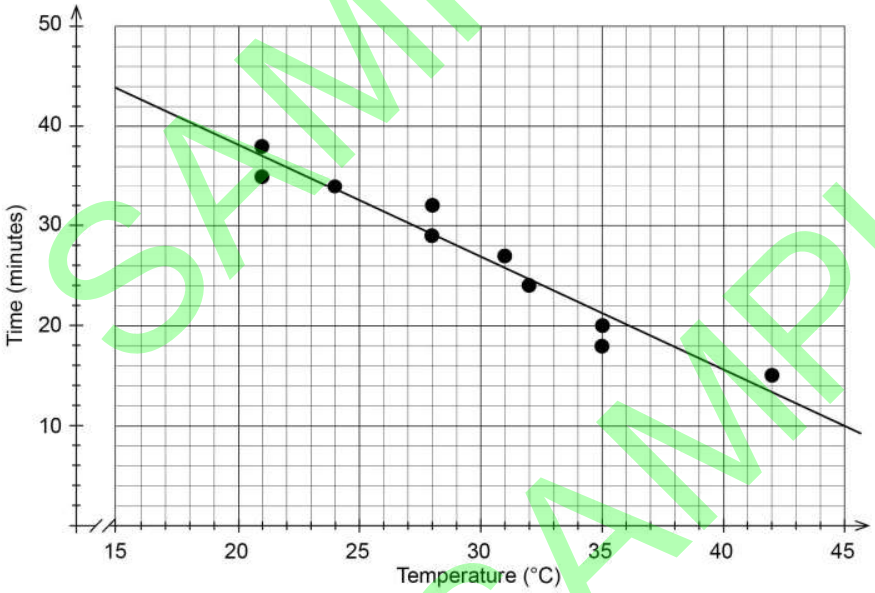
Bivariate data analysis

Nullah wanted to see if there was a relationship between outside temperature and the time taken to dry his laundry. The following data was collected over a 10 day period.



Temperature (°C)	28	35	42	31	24	21	21	35	32	28
Time (minutes)	29	20	15	27	34	38	35	18		

(a) Complete the table by locating the data in the graph. (2 marks)



Temperature (°C)	28	35	42	31	24	21	21	35	32	28
Time (minutes)	29	20	15	27	34	38	35	18	**24**	**32**

Solution
see table above
Specific behaviours
✓ one correct entry ✓ both correct entries

(b) Determine the equation of the least-squares line and state the correlation coefficient. (2 marks)

Solution
$y = -1.13x + 60.74, r = -0.97$
Specific behaviours
✓ states correct equation ✓ states correct correlation coefficient

(c) Draw the least-squares line onto the graph above. (2 marks)

Solution
see graph above
Specific behaviours
✓ line goes through approximately (28, 29) ✓ line has correct slope

(d) Describe the association between the two variables in terms of direction and strength. (2 marks)

Solution
The two variables have a strong, negative association
Specific behaviours
✓ correctly states strong association ✓ correctly states negative association

(e) What percentage of the variation in drying time can be explained by the variation in outside temperature? (1 mark)

Solution
$r^2 = 0.945 = 94.5\%$ ∴ 94.5% of the variation in the drying time can be explained by the variation in outside temperature
Specific behaviours
✓ states correct percentage

(f) Identify at least one other factor that could explain the variation in drying time. (1 mark)

Solution
Answers could include: • wind • type of laundry material • shade.
Specific behaviours
✓ identifies at least one valid factor
Accept other relevant answers.

(g) The temperature on Day 11 is predicted to be 17 °C.

(i) Use the equation for the least-squares line from part (b) to predict the time Nullah should expect his laundry to dry on this day. (1 mark)

Solution
$y = -1.13(17) + 60.74 = 41.53$ time = 41.53 minutes
Specific behaviours
✓ calculates correct time

(ii) Is this prediction reliable? Justify your answer. (2 marks)

Solution
No, it is unreliable since it is extrapolation
Specific behaviours
✓ correctly states it is unreliable ✓ correctly states extrapolation

2022 Section 2 Question 13

Bivariate data analysis

Data have been collected for nine suburbs within a city about the number of mobile phone towers and the number of births in the last 12 months for each suburb.

	Suburb								
	1	2	3	4	5	6	7	8	9
Number of mobile phone towers (n)	4	6	7	8	6	10	5	8	7
Number of births in the last 12 months (b)	25	29	35	45	38	54	22	38	39

The data has a correlation coefficient of 0.92, and the equation of the least-squares line is $b = 5.13n + 1.31$.

(a) Interpret the gradient of the least-squares line in the context of the question. (2 marks)

Solution
There is an average increase of approximately 5 births per year for each extra mobile phone tower
Specific behaviours
✓ correctly states increase ✓ correctly states 5 births per year

(b) Explain the significance of the correlation coefficient in the context of the question. (2 marks)

Solution
There is a strong, positive relationship between the number of mobile phone towers and the number of births in the last 12 months
Specific behaviours
✓ correctly mentions both variables ✓ correctly identifies strength and/or direction

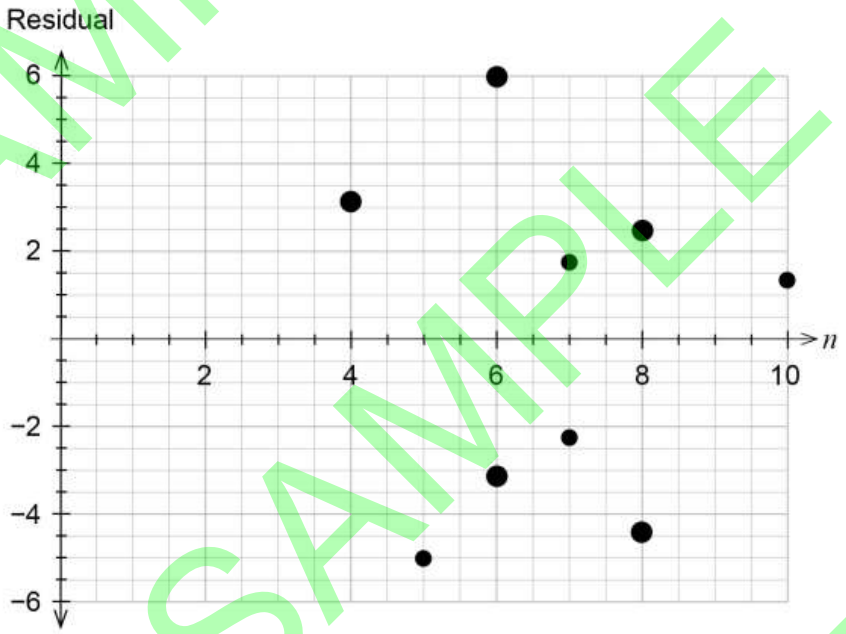
(c) (i) Predict the number of births for a suburb in this city that has nine mobile phone towers. (1 mark)

Solution
$b = 5.13(9) + 1.31 = 47.48 \approx 48$
Specific behaviours
✓ correct prediction

(ii) Comment on the validity of the prediction in part (c)(i). Justify your response. (2 marks)

Solution
Prediction is valid since r is strong and it is interpolation
Specific behaviours
✓ states prediction is valid ✓ states r is strong and it is interpolation

(d) Complete the residual plot below.



Solution
(4, 3.15), (6, –3.12), (6, 5.88), (8, 2.61), (8, –4.39)
Specific behaviours
✓ correctly plots one point ✓ correctly plots at least three points ✓ correctly plots all points

(e) Based on the residual plot, comment on whether the least-squares line is a suitable model for these data. (2 marks)

Solution
Least-squares line is suitable because the residual plot shows no pattern
Specific behaviours
✓ states line is suitable ✓ gives valid reason

(f) A 10th suburb has a data point (5,12) which has been verified as correct. State a practical explanation of how this could be a correct data point. (1 mark)

Solution
New suburb with mobile towers installed and not yet fully populated
Specific behaviours
✓ correct comment

(g) A journalist has followed the mathematics involved in working with bivariate data and is writing a report for a newspaper. What is a valid statement that could be made about the observed association between the number of mobile phone towers and the number of births in the last 12 months for suburbs within the city? (2 marks)

Solution
While the mathematics shows a strong association there is no evidence of a causal relationship. Other factors would need to be considered to establish a causal relationship.
Specific behaviours
✓ states strong mathematical relationship ✓ states causal relationship not evident

2021 Section 2 Question 9

Bivariate data analysis

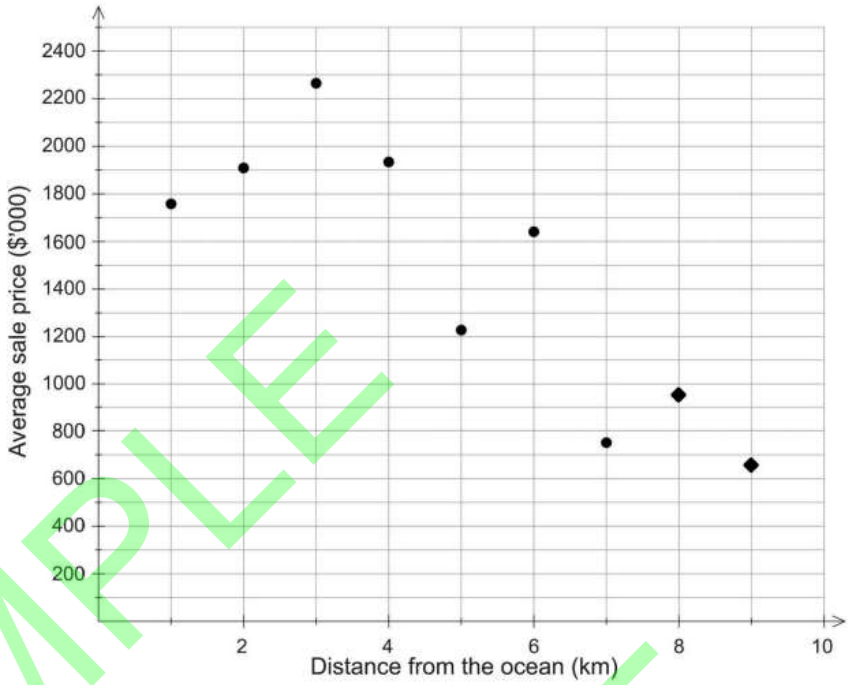
A real estate agent is analysing data on the sale of houses over the last six months. The table shows the average sale price of houses, in thousands of dollars ($'000), and their distance from the ocean, to the nearest kilometre.

Distance from the ocean (km)	1	2	3	4	5	6	7	8	9
Average sale price ($'000)	1758	1909	2265	1934	1228	1641	751	967	676

(a) State the explanatory variable. (1 mark)

Solution
Distance from the ocean
Specific behaviours
✓ states correct variable

(b) On the scatterplot below, plot the last two data points from the table. (1 mark)



Solution
see graph above
Specific behaviours
✓ both points are plotted correctly

(c) Determine the equation of the least-squares line for these data. (1 mark)

Solution
$p = -174.58d + 2331.7 \quad (y = -174.58x + 2331.7)$
Specific behaviours
✓ correctly determines equation

(d) Interpret the slope of the least-squares line from part (c) in the context of this question. (2 marks)

Solution
For each kilometre extra from the ocean, the selling price drops by an average of $174 580
Specific behaviours
✓ refers to correct variables ✓ correctly states average price drop of $174 580

(e) (i) State the value of the correlation coefficient for these data. (1 mark)

Solution
$r = -0.84$
Specific behaviours
✓ correct answer

(ii) What does the correlation coefficient measure? (1 mark)

Solution
Strength of the linear relationship
Specific behaviours
✓ correct answer

(iii) Describe the association between the variables in terms of direction and strength. (2 marks)

Solution
Strong, negative association
Specific behaviours
✓ correctly states direction ✓ correctly states strength

(f) What percentage of the variation in average sale price can be explained by the variation in the distance from the ocean? (1 mark)

Solution
$r^2 = 0.7007 \approx 70.1\%$
Specific behaviours
✓ calculates correct percentage

(g) In six months time, a homebuyer will have saved enough money for a deposit on a house. He would like to live about four kilometres from the ocean.

(i) Use the equation of the least-squares line from part (c) to predict the average sale price of houses four kilometres from the ocean. (1 mark)

Solution
$d = 4 \Rightarrow p = 1633.38 \quad \Rightarrow \text{price} = \$1\ 633\ 380$
Specific behaviours
✓ determines correct price estimate

(ii) Explain why your prediction is different from the average sale price given in the table. (1 mark)

Solution
The table value is the true average value, whereas the prediction comes from the least-squares line.
Specific behaviours
✓ gives a valid reason

(h) Give a reason why extrapolation in the context of this question would not make sense. (1 mark)

Solution
Following this model, the selling price would eventually become less than zero.
Specific behaviours
✓ gives a valid reason

(i) The real estate agent was talking to some potential buyers and was heard to make the statement, "Having property closer to the ocean causes higher selling prices". Comment on this statement. (2 marks)

Solution
Not a valid statement. Only **average** price increases the closer to the ocean. **or** Not a valid statement. One variable does not cause the other to occur (not causally related).
Specific behaviours
✓ states comment not valid ✓ gives an appropriate reason

2020 Section 2 Question 10

Bivariate data analysis

A football club records body measurements for all of their players. Shown below are the waistline measurements (cm) and percentage body fat for eleven players.

Player	1	2	3	4	5	6	7	8	9	10	11
Waistline measurement (w)	89	100	87	96	94	83	81	83	84	97	98
Percentage body fat (p)	14	17	11	19	17	12	9	10	8	14	19

Research has shown that estimates for percentage body fat can be determined by using waistline measurements.

(a) Calculate the correlation coefficient r_{wp} for these data. (1 mark)

Solution
$r = 0.88$
Specific behaviours
✓ correct answer

(b) Determine the equation of the least-squares line for these data. (1 mark)

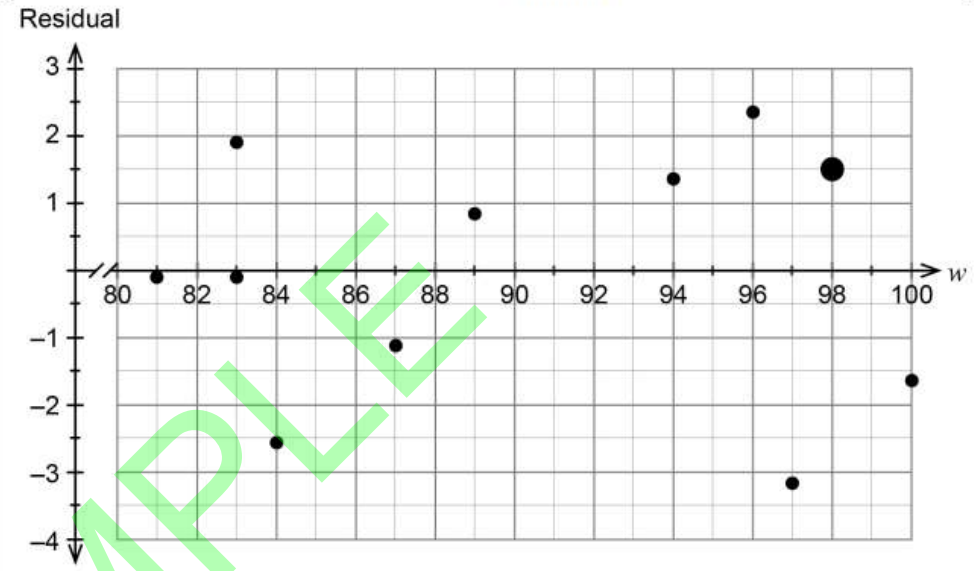
Solution
$p = 0.5w - 31.4$
Specific behaviours
✓ correct answer

(c) In the context of this question, interpret the slope of the line found in part (b). (2 marks)

Solution
For each 1 cm increase in waistline measurement, the percentage body fat increases by 0.5.
Specific behaviours
✓ matches waistline measurement to percentage body fat ✓ average 0.5 increase in percentage body fat for each 1 cm increase in waistline measurement

(d) The residual plot shown below is for the first 10 players' data. Calculate the residual for player number 11 and plot this point on the graph. (2 marks)

Solution



Residual = 1.5

Specific behaviours
✓ correct residual
✓ correctly plotted

(e) Comment on the appropriateness of fitting a linear model to the data. Justify your answer. (2 marks)

Solution
Appropriate as there is no clear pattern in the residuals
Specific behaviours
✓ correctly states that fitting a linear model is appropriate
✓ correctly states there is no clear pattern in residuals

(f) What percentage of the variation in the percentage body fat measurements is **unexplained** by the variation in the waistline measurements? (2 marks)

Solution
$r^2 = d = 0.77$ $\therefore$ 23% unexplained
Specific behaviours
✓ correctly determines coefficient of determination
✓ correctly gives unexplained variation percentage

(g) Wayne is player number 12 and has a waistline measurement of 105 cm.

(i) Determine his predicted percentage of body fat. (1 mark)

Solution
$p = 0.5 \times 105 - 31.4 = 21$
Specific behaviours
✓ correct answer

(ii) Comment on the validity of the prediction and give a justification for your answer. (2 marks)

Solution
Not valid, extrapolation
Specific behaviours
✓ correctly states prediction is not valid ✓ correctly states it is due to extrapolation

(h) Player number 13 has a residual of –2.6. What information does this provide about the percentage body fat for this player? (2 marks)

Solution
The percentage body fat for player 13 is 2.6 below their predicted percentage body fat.
Specific behaviours
✓correctly states that percentage body fat is below/under … ✓correctly relates actual percentage body fat measurement to the predicted percentage body fat (from the least-squares line)

2019 Section 2 Question 8

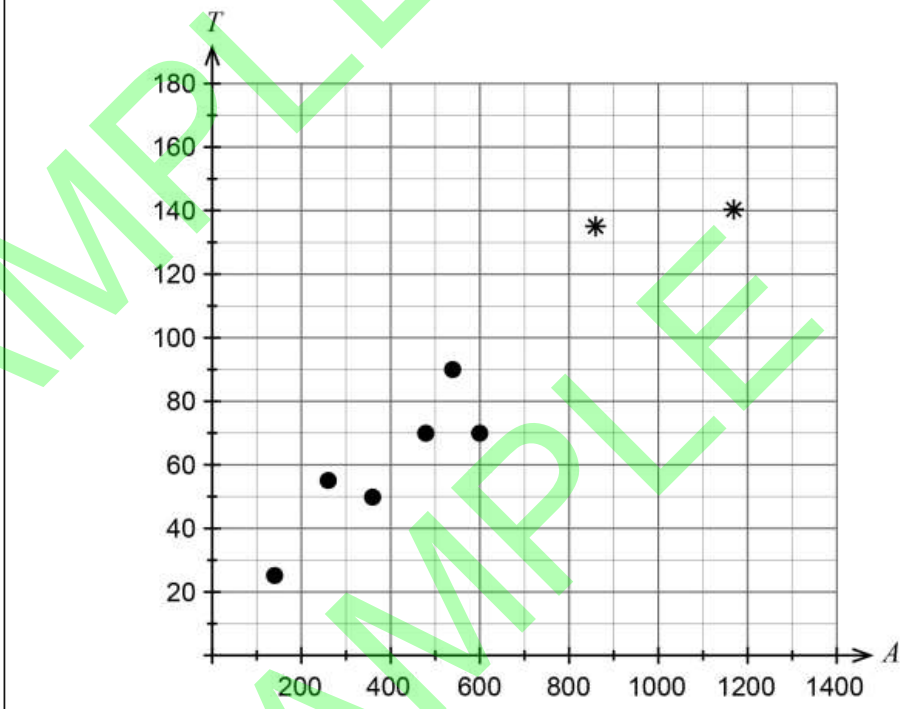
Bivariate data analysis

Abdul has a lawnmowing business and is investigating if there is a relationship between the size of a lawn and the length of time it takes to cut the lawn. He takes a random sample of eight customers and measures the areas of their lawns and notes the times, in minutes, it takes to mow their lawns. The results are in the table below, where A is the area of the lawn in square metres and T is the time in minutes. (Note: some values are missing.)

Customer	A	B	C	D	E	F	G	H
A (m^2)		260		480	540	600	860	1180
T (min)	25	55	50	70	90	70	135	140

(a) Complete the scatterplot below. (1 mark)

Solution



Specific behaviours

✓ plots both points correctly

(b) From the information below, determine the equation of the least-squares line in terms of A and T and state the coefficient of determination for these data. (2 marks)

Linear Reg
$y = ax+b$
a = 0.114691
b = 16.008241
r = 0.9510026
r^2 = 0.9044059

Solution
Equation of least-squares line is: $T = 0.115A + 16.008$ The coefficient of determination is 0.904
Specific behaviours
✓ states correct equation ✓ states correct coefficient of determination

(c) Interpret the value of the gradient of the least-squares line in the context of the question. (2 marks)

Solution
the time taken to mow the lawn increases by 0.115 minutes per square metre
Specific behaviours
✓ correctly defines an increase ✓ gives rate with correct units

(d) Given that Abdul charges $30 per hour, estimate the charge for mowing a customer's lawn with an area of 500 m2 . (2 marks)

Solution
16.008 + 0.115 × 500 = 73.508 Estimated charge = $\frac{73.508}{60} \times 30 = \$36.75$
Specific behaviours
✓ correctly calculates an estimate for T ✓ correctly calculates the charge

(e) Explain whether the estimate determined in part (d) would be valid. (2 marks)

Solution
Estimate would be valid since it is interpolation and the correlation coefficient is strong
Specific behaviours
✓ correctly explains validity ✓ explains validity with reference to either interpolation or correlation coefficient

(f) Using the least-squares line correct to three decimal places

(i) calculate the residuals for Customers B and D. (2 marks)

Solution
Residual for customer B is 9.092 Residual for customer D is –1.208
Specific behaviours
✓ correctly calculates residual for customer B ✓ correctly calculates residual for customer D

(ii) explain the significance of the sign and the size of these residuals in reference to the least-squares line. (2 marks)

Solution
The change in sign indicates the residuals are above and below the least-squares line The size indicates that the residual for D is closer to the line than the residual for customer B (or vice versa)
Specific behaviours
✓ states correct meaning of residual sign ✓ states correct meaning of residual size

2019 Section 2 Question 14

Bivariate data analysis

The table below contains data provided by the Australian Bureau of Statistics. It shows the number of households with and without internet access from 2014–2017. All values are in thousands of households.

Internet access

State/territory	2014–15			2016–17		
	Households with internet access '000	Households without internet access '000	Total '000	Households with internet access '000	Households without internet access '000	Total '000
New South Wales	2407.9	414.5	2822.4	2439.9	421.8	2861.7
Victoria	1934.2	305.1	**A**	2008.2	305.8	2314.0
Queensland	1552.4	248.5	1800.9	1591.9	249.8	1841.7
South Australia	565.1	121.4	686.5	575.5	**B**	696.6
Western Australia	843.6	113.0	956.6	859.7	112.6	972.3
Tasmania	172.0	38.7	210.7	177.7	36.2	213.9
Northern Territory	58.1	6.3	64.4	57.6	7.3	64.9
Australian Capital Territory	137.2	9.0	146.2	140.1	9.7	149.8
Total	7670.5	1256.5	8927.0	7850.6	1264.3	9114.9

(a) (i) Determine the value of **A** and **B** in the table above. (2 marks)

Solution
A = 1934.2 + 305.1 = 2239.3 **B** = 696.6 – 575.5 = 121.1
Specific behaviours
✓ correctly determines **A** ✓ correctly determines **B**

(ii) Compare the percentages, correct to two decimal places, of households with internet access in New South Wales between 2014–15 and 2016–17. Comment on your results. (3 marks)

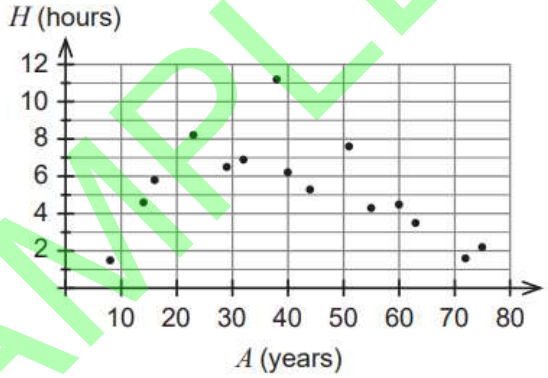
Solution
2014–15: $\frac{2407.9}{2822.4} = 85.31\%$, 2016–17: $\frac{2439.9}{2861.7} = 85.26\%$ There is a slight decrease in the percentage of households with internet access from 2014–15 to 2016–17.
Specific behaviours
✓ calculates the correct percentages ✓ correctly rounds percentages to two decimal places ✓ states there is a small drop in the percentage of households with internet access from 2014–15 to 2016–17

(iii) What is the difference in the data for households with internet access for the Northern Territory over the time period shown, compared to other States and Territories? (1 mark)

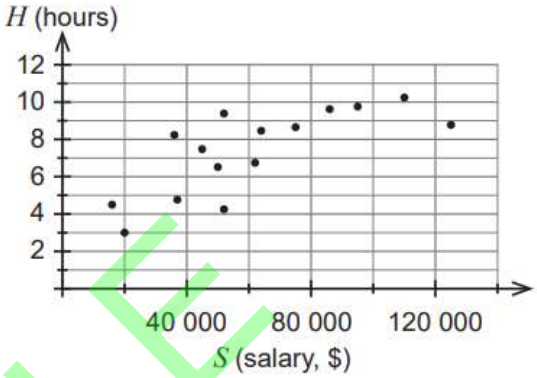
Solution
The number of households with internet access decreases from 2014–15 to 2016–17.
Specific behaviours
✓ correctly states how they are different

An internet service provider from Tasmania wanted to determine whether a person's age, *A*, or salary, *S*, affected the number of hours, *H*, of internet usage per day. The graphs below each show the recorded data for people surveyed.

Daily Internet Usage According to Age



Daily Internet Usage According to Salary



(b) (i) Describe the association between a person's salary and the number of hours of internet usage per day, in terms of direction and form. (2 marks)

Solution
Positive and linear
Specific behaviours
✓ states correct direction ✓ correctly identifies a linear form

(ii) The internet service provider calculated the correlation coefficient for the data contained in each graph. The values they calculated are contained in the following list.

–1.25, –0.95, –0.75, –0.3, 0.1, 0.3, 0.75, 0.95, 1.25

Choose the best estimate from the list for each of the graphs shown above. (2 marks)

Solution	
Graph	**Correlation coefficient**
Daily internet usage according to age	–0.3
Daily internet usage according to salary	0.75
Specific behaviours	
✓ chooses correct correlation coefficient for internet usage according to age ✓ chooses correct correlation coefficient for internet usage according to salary	

2019 Section 2 Question 16

Bivariate data analysis

The table below records the altitude (metres above sea level), latitude (° S) and mean maximum temperature (°C) during January for eight cities in the southern hemisphere.

Altitude (A)	Latitude (L)	Mean maximum temperature (T)
15	31.95	25
20	43.53	20
24	42.88	18
314	45.03	16
8	6.18	28
154	12.05	26
37	12.46	29
8	34.60	25

Comparing altitude and the mean maximum temperature, it was determined that the least-squares line for these data was $T = -0.022A + 24.97$ and $r_{AT} = -0.50$.

(a) Determine the coefficient of determination for altitude and the mean maximum temperature and interpret this value. (2 marks)

Solution
$r^2 = 0.25$ Approximately 25% of the variation in temperature can be explained by the variation in altitude.
Specific behaviours
✓ correctly determines the coefficient of determination ✓ gives correct description of its meaning

(b) Determine the equation of the least-squares line for comparing latitude and the mean maximum temperature and state the correlation coefficient. (2 marks)

Solution
$T = -0.264L + 30.94$ $r = -0.88$
Specific behaviours
✓ correctly states equation of least-squares line ✓ correctly states correlation coefficient

Rio de Janeiro has a latitude of 22.93° S and an altitude of 9 metres.

(c) Use the two least-squares lines above to predict the mean maximum temperature in January for Rio de Janeiro. Which prediction is more valid? Justify your choice. (3 marks)

Solution
$T = -0.264 \times 22.93 + 30.94 = 24.89$ $T = -0.022 \times 9 + 24.97 = 24.77$ The prediction using latitude is more valid as the correlation coefficient is much stronger.
Specific behaviours
✓ correctly determines a prediction using latitude and altitude ✓ correctly states that the prediction using latitude is more valid ✓ correctly explains that latitude has a stronger correlation coefficient

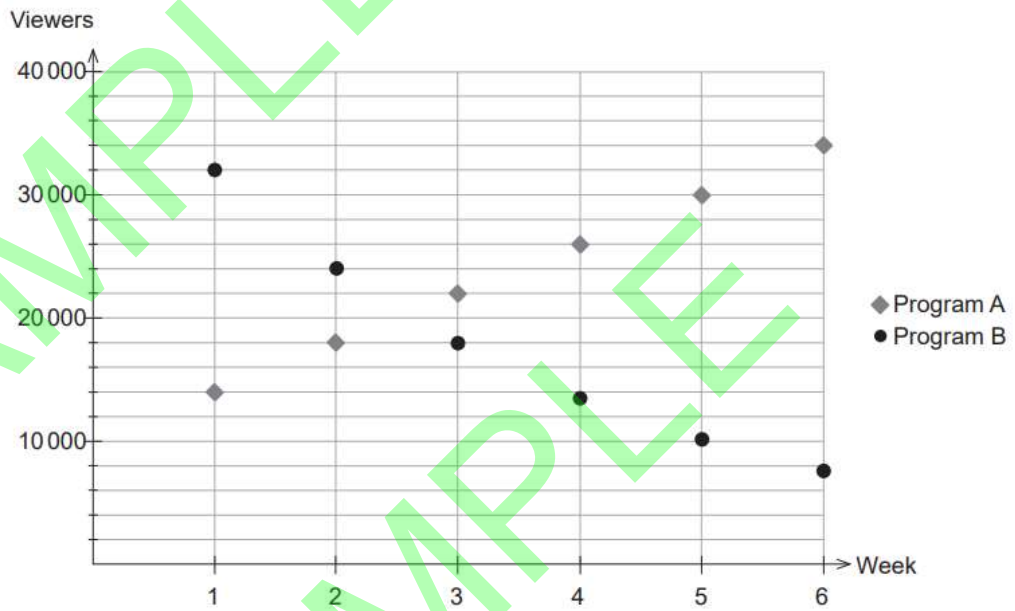
Unit 3.2 – Growth and decay in sequences

Section 1

2023 Section 1 Question 3 **Growth and decay in sequences**	From January 1, 2020, a company offered its employees an income package with a starting wage of $4000 per month, paid at the end of each month. Also, as an incentive to stay with the company, there was a monthly increase of $50 each month. (a) Determine a recursive rule for the monthly wage. (2 marks) (b) Deduce a simplified rule for the nth term of the monthly wage. (2 marks) (c) Determine the monthly wage for December 2020. (2 marks) The company has decided to make the monthly increase $60 from the end of December 2023. (d) Calculate the monthly wage for March 2024. (3 marks)

2021 Section 1 Question 1 **Growth and decay in sequences**	Hanai is a successful college basketball player. His coach has warned him that he will lose his scholarship if he scores 54% or below on a weekly assessment. On his first three weekly assessments he scored 84%, 81% and 78% respectively. Assume Hanai's weekly assessments continue to follow this pattern. (a) Deduce a rule for the n^{th} term of this sequence. (2 marks) (b) Determine Hanai's score on his sixth weekly assessment. (1 mark)

	(c) Predict when Hanai will lose his scholarship. (2 marks)

2021 Section 1 Question 6 **Growth and decay in sequences**	A television network programmer was analysing the number of viewers for two children's programs over a period of several weeks, to decide which program should be given the better time slot. The viewing numbers, displayed on the graph below, formed an arithmetic sequence and a geometric sequence.  (a) Write a recursive rule for the arithmetic sequence. (2 marks) (b) Using the first two data points, deduce a rule for the n^{th} term of the geometric sequence. (2 marks)

	(c) Explain which program should be given the better time slot. (2 marks) (d) Determine the number of viewers for the more successful program in Week 8. (1 mark)
2019 **Section 1** **Question 6** **Growth and decay in sequences**	The population of turtles in an artificial lake at a wildlife sanctuary is initially 32 and research has shown a natural decrease in population of 50% each year. Twenty extra turtles are introduced to the lake at the end of each year. (a) Determine a recursive rule for the turtle population. (2 marks) (b) Determine the long-term steady state of the turtle population. (2 marks)

	(c) If the wildlife sanctuary preferred a long-term steady state of 80 turtles, what yearly addition of turtles would be required to produce this steady state? Assume all other conditions remain the same. (2 marks)

Section 2

2022 Section 2 Question 9 **Growth and decay in sequences**	A study of a penguin colony on an island was conducted and it found the initial population size of 1200 was dropping by 14% each year due to the introduction of non-native predators. (a) Explain why the population after n weeks is $1200 \times (0.86)^n$ penguins. (2 marks) After eight weeks, the Parks and Wildlife Service set traps to reduce the predator numbers. This saw the penguin population increase weekly by 6%. (b) State the recursive formula that models the new population growth. (2 marks) (c) How many weeks will it take to get the population back up to the initial size? (1 mark)

Once the population returns to the initial size, it is further helped by the introduction of penguins from a breeding program at the zoo.

The new population growth model can be represented by

$P_{n+1} = -0.25P_n + 3000$, $P_0 = 1200$.

(d) Discuss the long-term behaviour of the penguin population, now that it is being supported by the breeding program. (2 marks)

2022 Section 2 Question 17 **Growth and decay in sequences**	Indie was in a line with 24 other people for a slide at a water park. She noticed that the approximate number of people (P) in the line for the slide increased by 1.5% every minute (m). (a) Write an exponential equation in the form $P = ar^m$ to represent this situation. (2 marks) (b) Determine the approximate number of people in the line after 2 hours. (2 marks) After 3 hours, the line started to decrease by 1% per minute. (c) Using this new information, calculate the approximate number of people in line, 5 hours after Indie initially lined up. (4 marks)

2021 Section 2 Question 11

Growth and decay in sequences

Judith monitors the water quality in her garden pond at the same time every day. She likes to maintain the concentration of algae at between 200 and 250 units per 100 litres (L). Her measurements show that the concentration increases daily according to the recursive rule $C_{n+1} = 1.025C_n$, where $C_1 = 200$ units per 100 L (the minimum concentration).

When the concentration gets above the 250 units per 100 L limit, she treats the water to bring the concentration back to the minimum 200 units per 100 L.

(a) If Judith treated the water on Sunday, 6 December 2020, determine

(i) the concentration on Wednesday, 9 December 2020. (2 marks)

(ii) the day and date when she next treated the water. (2 marks)

(b) During the first week of January 2021, Judith monitored the water and recorded the following readings.

Day	1	2	3	4	5	6	7
Concentration (C)	200	206	212.18	218.55	225.10	231.85	238.81

(i) Determine the revised recursive rule. (2 marks)

(ii) If she treated the water on 10 January and went on holiday until 20 January, when she next treated the water, calculate the concentration of the water on her return, assuming the recursive rule from part (b)(i) is used. (2 marks)

2020 Section 2 Question 7 **Growth and decay in sequences**	The world's tallest man was recorded as 60 cm long at birth. He grew 28 cm in his first year, 26 cm in his second year and so on, always 2 cm less than in the previous year until he stopped growing. (a) Calculate his annual growth (in cm) in his fourth and fifth years. (1 mark) (b) Deduce the rule for his annual growth in the n^{th} year, until he stopped growing. (2 marks) (c) In which year did he first not grow any taller? (1 mark) (d) Calculate his maximum height. (2 marks)

2020 Section 2 Question 8 **Growth and decay in sequences**	A farmer has a large lake on his farm and has started stocking it with fish of a variety that will flourish in the conditions in this lake. Monitoring has shown that the number of adult fish is increasing at a consistent rate of 9% per month and at the beginning of 2020 the lake holds 660 of the adult fish. (a) Write a recursive rule to give the number of adult fish in the lake at the end of each month from the beginning of 2020. (2 marks) (b) Deduce a rule for the nth term of this sequence. (2 marks)

	The farmer plans to allow the general public to pay to fish in the lake. This will commence at the beginning of the next month after the adult fish population first reaches 4000. (c) Determine how many months after the beginning of 2020 fishing will commence. (2 marks) (d) The farmer wishes to maintain a steady state in the adult fish population once fishing commences. Calculate how many adult fish can be taken from the lake each month. (3 marks)

2019 Section 2 Question 7 **Growth and decay in sequences**	A water tank is full. When a tap at the bottom of the tank is opened, 84 litres run out in the first minute, 78 litres in the second minute and 72 litres in the third minute. This pattern continues until the tank is empty. (a) Write a rule for the n^{th} term of a sequence in the form $T_n = A + Bn$, which will model this situation where T_n is the amount of water that runs out in the n^{th} minute. (2 marks) (b) How many litres run out in the seventh minute? (1 mark) (c) How many litres have run out after eight minutes? (1 mark) (d) What is the capacity of the tank? (2 marks)

2019 Section 2 Question 10 **Growth and decay in sequences**	Ruby Ducks Coffee shops commenced operations in 1992 and had 15 stores open by the end of the year. They have been so successful over the years that the number of stores worldwide has continued to grow exponentially since then. The number of shops operating, T, at the end of 2017 was 22 579 and at the end of 2018 was 30 256. The number of shops operating at the end of n years can be represented by the recursive rule Tn = 1.34T_n–1, T_1 = 15. (a) Show mathematically that the common ratio is approximately 1.34. (1 mark) (b) Write the rule for the n^{th} term of this sequence. (1 mark) (c) Determine the first year in which there is likely to be over 200 000 Ruby Ducks Coffee shops. (2 marks) Typically, each store has twelve employees working during the day across different shifts. Each employee earns, on average, $114.80 per day. (d) Calculate the total daily wages for all stores at the beginning of 2012. (3 marks)

2023 Section 1 Question 3

Growth and decay in sequences

From January 1, 2020, a company offered its employees an income package with a starting wage of $4000 per month, paid at the end of each month. Also, as an incentive to stay with the company, there was a monthly increase of $50 each month.

(a) Determine a recursive rule for the monthly wage. (2 marks)

Solution
$T_{n+1} = T_n + 50, T_1 = 4000$
Specific behaviours
✓ states correct recursive rule ✓ states correct first term

(b) Deduce a simplified rule for the nth term of the monthly wage. (2 marks)

Solution
$T_n = 4000 + (n-1)(50)$ $T_n = 3950 + 50n$
Specific behaviours
✓ uses correct arithmetic formula ✓ gives correct simplified rule for the n^{th} term

(c) Determine the monthly wage for December 2020. (2 marks)

Solution
$T_{12} = 3950 + 50(12)$ $= 4550$ Therefore, the monthly wage for December 2020 is $4550
Specific behaviours
✓ correctly identifies term 12 ✓ correctly calculates the $4550

The company has decided to make the monthly increase $60 from the end of December 2023.

(d) Calculate the monthly wage for March 2024. (3 marks)

Solution
$12 \times 4 = 48$ $T_{48} = 3950 + 50(48) = 6350$ Therefore, the monthly wage for March 2024 is 6350 + 60 + 60 + 60 = $6530
Specific behaviours
✓ correctly calculates T_{48} ✓ calculates correct term for March 2024 ✓ states correct solution for wage

2021 Section 1 Question 1

Growth and decay in sequences

Hanai is a successful college basketball player. His coach has warned him that he will lose his scholarship if he scores 54% or below on a weekly assessment. On his first three weekly assessments he scored 84%, 81% and 78% respectively.

Assume Hanai's weekly assessments continue to follow this pattern.

(a) Deduce a rule for the n^{th} term of this sequence. (2 marks)

Solution
$T_n = 84 + (n-1)(-3)$ $= 87 - 3n$
Specific behaviours
✓ correctly identifies an arithmetic sequence ✓ correctly states the rule for the n^{th} term

(b) Determine Hanai's score on his sixth weekly assessment. (1 mark)

Solution
$T_6 = 87 - 3(6)$ $= 69$ Therefore, he gets 69% on his sixth assessment
Specific behaviours
✓ calculates the correct value

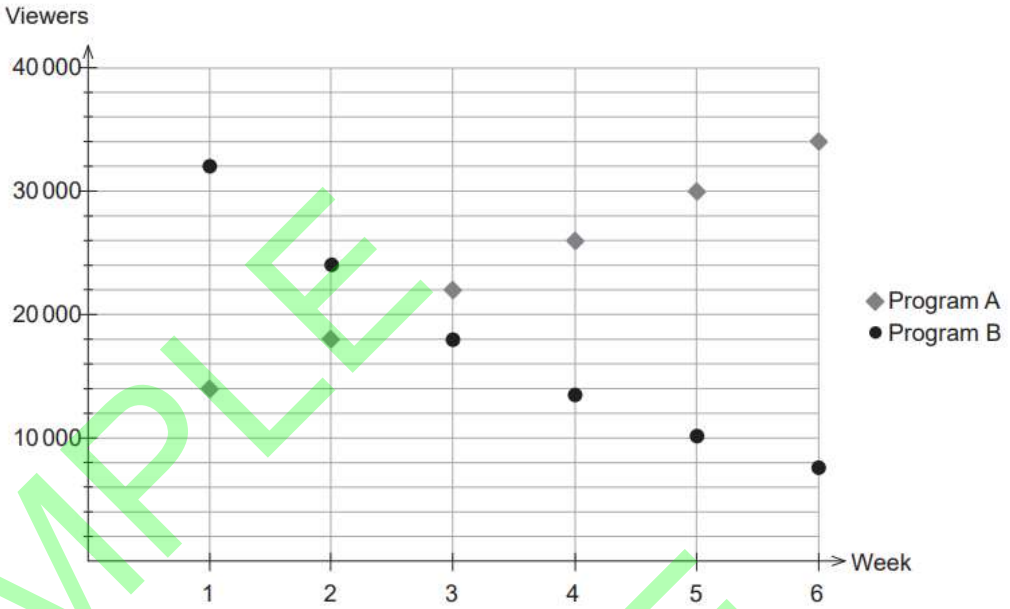
(c) Predict when Hanai will lose his scholarship. (2 marks)

Solution
$54 = 87 - 3n$ $3n = 33$ $n = 11$ Therefore, Hanai will lose his scholarship after the 11th weekly assessment
Specific behaviours
✓ substitutes 54 correctly ✓ identifies correct weekly assessment

2021 Section 1 Question 6

Growth and decay in sequences

A television network programmer was analysing the number of viewers for two children's programs over a period of several weeks, to decide which program should be given the better time slot. The viewing numbers, displayed on the graph below, formed an arithmetic sequence and a geometric sequence.



(a) Write a recursive rule for the arithmetic sequence. (2 marks)

Solution
$T_{n+1} = T_n + 4000, T_1 = 14\,000$
Specific behaviours
✓ states a correct recursive rule ✓ states correct first term

(b) Using the first two data points, deduce a rule for the n^{th} term of the geometric sequence. (2 marks)

Solution
Using 32 000 and 24 000 Ratio = $\frac{24\,000}{32\,000} = \frac{3}{4} = 0.75$ Therefore $T_n = 32\,000\left(\frac{3}{4}\right)^{n-1} = 32\,000(0.75)^{n-1}$
Specific behaviours
✓ determines correct ratio ✓ states correct n^{th} term

(c) Explain which program should be given the better time slot. (2 marks)

Solution
Program A because viewer numbers are increasing
Specific behaviours
✓ states correct program ✓ explains why Program A should be given the better time slot

(d) Determine the number of viewers for the more successful program in Week 8. (1 mark)

Solution
$T_8 = 34\,000 + 8\,000 = 42\,000$ Therefore, the number of viewers is 42 000
Specific behaviours
✓ correctly states number of viewers

2019 Section 1 Question 6

Growth and decay in sequences

The population of turtles in an artificial lake at a wildlife sanctuary is initially 32 and research has shown a natural decrease in population of 50% each year. Twenty extra turtles are introduced to the lake at the end of each year.

(a) Determine a recursive rule for the turtle population. (2 marks)

Solution
$T_{n+1} = 0.5T_n + 20, T_1 = 32$
Specific behaviours
✓ states correct rule ✓ states correct first term

(b) Determine the long-term steady state of the turtle population. (2 marks)

Solution 1
$T_2 = 0.5(32) + 20$ $= 36$ $T_3 = 0.5(36) + 20$ $= 38$ $T_4 = 0.5(38) + 20$ $= 39$ $T_5 = 0.5(39) + 20$ 39.5 Approaching a steady state of 40 turtles
Specific behaviours
✓ correctly calculates a further 4 terms ✓ correctly determines the long-term steady state

OR

Solution 2
$x = 0.5x + 20$ $0.5x = 20$ $x = 40$
Specific behaviours
✓ correctly writes steady state equation ✓ correctly determines the long-term steady state

(c) If the wildlife sanctuary preferred a long-term steady state of 80 turtles, what yearly addition of turtles would be required to produce this steady state? Assume all other conditions remain the same. (2 marks)

Solution 1
By trial and error. Example: if k = 30, $T_2 = 46, T_3 = 53, T_4 = 56.5$, not approaching 80 Example: if k = 40, $T_2 = 56, T_3 = 68, T_4 = 74, T_5 = 77$, approaching 80
Specific behaviours
✓ uses trial and error to correctly generate a sequence of at least 4 terms ✓ correctly determines the yearly addition of turtles

OR

Solution 2
$80 = 0.5 \times 80 + k$ $80 = 40 + k$ $k = 40$
Specific behaviours
✓ correctly writes steady state equation ✓ correctly determines the yearly addition of turtles

2022 Section 2 Question 9

Growth and decay in sequences

A study of a penguin colony on an island was conducted and it found the initial population size of 1200 was dropping by 14% each year due to the introduction of non-native predators.

(a) Explain why the population after n weeks is $1200 \times (0.86)^n$ penguins. (2 marks)

Solution
Assuming population size dropping by 14% per week: The population of penguins in each week is 86% of that of the previous week, so $r = 0.86$. The initial population (P_0) is 1200.
Specific behaviours
✓ identifies that 14% decrease is equivalent to 86% of previous value ✓ identifies 1200 is initial population size

Alternate solution
Assuming population size dropping by 14% per year: The population of penguins is not $1200\times(0.86)^n$ after n weeks. The given expression would be the population of penguins after n years, not weeks. For n years the expression would be $1200\times\left(1-\frac{0.14}{52}\right)^n = 1200\times 0.99731^n$.
Specific behaviours
✓ states given expression is not the population after n weeks ✓ states valid reason

After eight weeks, the Parks and Wildlife Service set traps to reduce the predator numbers. This saw the penguin population increase weekly by 6%.

(b) State the recursive formula that models the new population growth. (2 marks)

Solution
If used given expression from part (a): $P_8 = 1200\times 0.86^8 \approx 359$ penguins Therefore, the recursive formula is $P_{n+1} = 1.06P_n,\ P_0 = 359$ or If used alternate solution from part (a): $P_8 = 1200\times 0.99731^8 \approx 1174$ penguins Therefore, the recursive formula is $P_{n+1} = 1.06P_n,\ P_0 = 1174$
Specific behaviours
✓ states recursive formula ✓ includes correct P_0 value

(c) How many weeks will it take to get the population back up to the initial size? (1 mark)

Solution
If used given expression from part (a): It will take 21 weeks for population to return to 1200. or If used alternate solution from part (a): The population will return to 1200 within the first week (0.37 weeks)
Specific behaviours
✓ determines correct number of weeks

Once the population returns to the initial size, it is further helped by the introduction of penguins from a breeding program at the zoo.

The new population growth model can be represented by

$P_{n+1} = -0.25P_n + 3000$, $P_0 = 1200$.

(d) Discuss the long-term behaviour of the penguin population, now that it is being supported by the breeding program. (2 marks)

Solution
The long-term trend for the penguin population is to reach a steady state of 2400 penguins.
Specific behaviours
✓ identifies steady state solution ✓ states 2400 penguins

2022 Section 2 Question 17

Growth and decay in sequences

Indie was in a line with 24 other people for a slide at a water park. She noticed that the approximate number of people (P) in the line for the slide increased by 1.5% every minute (m).

(a) Write an exponential equation in the form $P = ar^m$ to represent this situation. (2 marks)

Solution
$P = 25 \times 1.015^m$
Specific behaviours
✓ states correct value of a ✓ states correct value of r

(b) Determine the approximate number of people in the line after 2 hours. (2 marks)

Solution
$P = 25 \times 1.015^{120}$ $P = 149.23$ ~149 people
Specific behaviours
✓ states correct value ✓ recognises integer value required

After 3 hours, the line started to decrease by 1% per minute.

(c) Using this new information, calculate the approximate number of people in line, 5 hours after Indie initially lined up. (4 marks)

Solution
$P = 25 \times 1.015^{180}$ $P = 364.61$ ~ 365 $P = 365 \times 0.99^{m}$ $P = 365 \times 0.99^{120}$ $P = 109.3$ ~ 109 people
Specific behaviours
✓ calculates P for $m = 180$ ✓ states new ratio of 0.99 ✓ identifies $m = 120$ ✓ uses equation to calculate $P \sim 109$

2021 Section 2 Question 11

Growth and decay in sequences

Judith monitors the water quality in her garden pond at the same time every day. She likes to maintain the concentration of algae at between 200 and 250 units per 100 litres (L). Her measurements show that the concentration increases daily according to the recursive rule $C_{n+1} = 1.025C_n$, where $C_1 = 200$ units per 100 L (the minimum concentration).

When the concentration gets above the 250 units per 100 L limit, she treats the water to bring the concentration back to the minimum 200 units per 100 L.

(a) If Judith treated the water on Sunday, 6 December 2020, determine

(i) the concentration on Wednesday, 9 December 2020. (2 marks)

Solution
$C_4 = 215.38$ units per 100 L
Specific behaviours
✓ correctly identifies Wednesday is C_4 ✓ correctly calculates the concentration

(ii) the day and date when she next treated the water. (2 marks)

Solution
$C_{10} = 249.77$, $C_{11} = 256.02$ i.e. Wednesday 16 December (11^{th} term)
Specific behaviours
✓ calculates correct day ✓ calculates correct date

(b) During the first week of January 2021, Judith monitored the water and recorded the following readings.

Day	1	2	3	4	5	6	7
Concentration (C)	200	206	212.18	218.55	225.10	231.85	238.81

(i) Determine the revised recursive rule. (2 marks)

Solution
$\frac{206}{200} = 1.03 \ \therefore C_{n+1} = 1.03C_n, C_1 = 200$
Specific behaviours
✓ calculates correct ratio ✓ states correct initial value

(ii) If she treated the water on 10 January and went on holiday until 20 January, when she next treated the water, calculate the concentration of the water on her return, assuming the recursive rule from part (b)(i) is used. (2 marks)

Solution
20 January is the eleventh term. $C_{11} = 268.78$ units per 100 L
Specific behaviours
✓ determines correct term ✓ calculates correct value

2020 Section 2 Question 7

Growth and decay in sequences

The world's tallest man was recorded as 60 cm long at birth. He grew 28 cm in his first year, 26 cm in his second year and so on, always 2 cm less than in the previous year until he stopped growing.

(a) Calculate his annual growth (in cm) in his fourth and fifth years. (1 mark)

Solution
Fourth year: 22 cm Fifth year: 20 cm
Specific behaviours
✓ calculates correct values

(b) Deduce the rule for his annual growth in the n^{th} year, until he stopped growing. (2 marks)

Solution
$T_n = 30 - 2n$
Specific behaviours
✓ determines the correct constant term ✓ determines the correct coefficient of n

(c) In which year did he first not grow any taller? (1 mark)

Solution
In the fifteenth year
Specific behaviours
✓ calculates correct value

(d) Calculate his maximum height. (2 marks)

Solution
$60 + (28 + 26 + 24 + \ldots + 2) = 270$ cm
Specific behaviours
✓ correctly sums terms from 28 to 2 ✓ correctly adds 60 to the sum

2020 Section 2 Question 8

Growth and decay in sequences

A farmer has a large lake on his farm and has started stocking it with fish of a variety that will flourish in the conditions in this lake. Monitoring has shown that the number of adult fish is increasing at a consistent rate of 9% per month and at the beginning of 2020 the lake holds 660 of the adult fish.

(a) Write a recursive rule to give the number of adult fish in the lake at the end of each month from the beginning of 2020. (2 marks)

Solution
$T_{n+1} = 1.09T_n \quad T_0 = 660$
Specific behaviours
✓ correctly states recursive rule ✓ correctly states T_0

(b) Deduce a rule for the nth term of this sequence. (2 marks)

Solution
$A_n = 660 \times 1.09^n$
Specific behaviours
✓ gives formula in exponential form ✓ states correct rule

The farmer plans to allow the general public to pay to fish in the lake. This will commence at the beginning of the next month after the adult fish population first reaches 4000.

(c) Determine how many months after the beginning of 2020 fishing will commence. (2 marks)

Solution
$4000 = 660 \times 1.09^n \quad n = 20.9$ Therefore fishing will commence 21 months after the beginning of 2020 **or** $T_{21} = 4031.8$ Therefore fishing will commence 21 months after the beginning of 2020
Specific behaviours
✓ correctly solves for n ✓ correctly states correct number of months after the beginning of 2020

(d) The farmer wishes to maintain a steady state in the adult fish population once fishing commences. Calculate how many adult fish can be taken from the lake each month. (3 marks)

Solution
$T_{21} = 4031$ $T_{n+1} = 1.09T_n - x$ $4031 = 1.09 \times 4031 - x \qquad x = 362.79$ 362 fish per month
Specific behaviours
✓ correctly determines $T_{21} = 4031$ ✓ correctly determines monthly increase ✓ correctly rounds down for number of fish

2019 Section 2 Question 7

Growth and decay in sequences

A water tank is full. When a tap at the bottom of the tank is opened, 84 litres run out in the first minute, 78 litres in the second minute and 72 litres in the third minute. This pattern continues until the tank is empty.

(a) Write a rule for the n^{th} term of a sequence in the form $T_n = A + Bn$, which will model this situation where T_n is the amount of water that runs out in the n^{th} minute. (2 marks)

Solution
$T_n = 84 + (n-1)(-6)$ $= 90 - 6n$
Specific behaviours
✓ states correct value for A ✓ states correct value for B

(b) How many litres run out in the seventh minute? (1 mark)

Solution
48 L
Specific behaviours
✓ states correct value

(c) How many litres have run out after eight minutes? (1 mark)

Solution
48 L
Specific behaviours
✓ states correct value

(d) What is the capacity of the tank? (2 marks)

Solution
$T_{15} = 0$, Sum of first 15 terms is 630 L
Specific behaviours
✓ states that the 15th term is zero ✓ states correct capacity

2019 Section 2 Question 10

Growth and decay in sequences

Ruby Ducks Coffee shops commenced operations in 1992 and had 15 stores open by the end of the year. They have been so successful over the years that the number of stores worldwide has continued to grow exponentially since then. The number of shops operating, T, at the end of 2017 was 22 579 and at the end of 2018 was 30 256.

The number of shops operating at the end of n years can be represented by the recursive rule $Tn = 1.34T_n–1$, $T_1 = 15$.

(a) Show mathematically that the common ratio is approximately 1.34. (1 mark)

Solution
$r = \frac{30\,256}{22\,579} \approx 1.34$
Specific behaviours
✓ shows correct calculation of the ratio

(b) Write the rule for the n^{th} term of this sequence. (1 mark)

Solution
$T_n = 15 \times 1.34^{n-1}$ or $T_n = 11.19 \times 1.34^n$
Specific behaviours
✓ states correct rule

(c) Determine the first year in which there is likely to be over 200 000 Ruby Ducks Coffee shops. (2 marks)

Solution
After 33 years there are approximately 234 719 shops Therefore, the first year is 2025
Specific behaviours
✓ determines correct number of years ✓ determines correct year

Typically, each store has twelve employees working during the day across different shifts. Each employee earns, on average, $114.80 per day.

(d) Calculate the total daily wages for all stores at the beginning of 2012. (3 marks)

Solution
Stores ≈ 3900 Employees $= 3900 \times 12 = 46\,800$ Daily wages $= 46\,800 \times 114.80 = \$5\,372\,640$
Specific behaviours
✓ correctly determines the number of stores ✓ correctly determines the number of employees ✓ correctly determines the total wages