



# Apex Exam Guide

## General Mathematics

Year 12 QCE

Queensland Curriculum

2025 Edition

Edward Nyugen



# Apex Exam Guide

## General Mathematics

### Year 12 QCE

**Resource Hub Link:** <https://www.apexbooks.com.au/>

- Subscribe for future updates.
- Contact details for any queries or feedback.
- Relevant QCE subject information.
- University scholarship information.
- Other useful information.



#### Key Features of the Apex Exam Guides

- Contains the past four years of QCE exam questions from 2023 to 2020, categorised based on the most up to date QCAA syllabus reviews.
- Significantly reduced overall cost compared to alternative book options.
- Future editions will be released with the most up to date QCE exams.
- Students can review chapters, specific questions or solutions at any given time throughout the course.
- Students can refer to relevant chapters to complete during class or for private study to prepare for topic tests or exams.
- Teachers can use these eBooks as a great reference point for lesson planning, developing internal assessments or when preparing class activities to ensure alignment with exam level questions.
- Mark allocation included to guide student time management.
- Fully worked answers from the QCE marking guide are included for feedback.
- Increase student confidence and enable high achievement.

#### How to Use Apex Exam Guides

Purchase of these eBook PDFs allows your students access to previous QCE exam questions, arranged by syllabus topics and answers from the marking guide. Revision for QCE assessments or exams becomes much more straightforward. Students can review chapters, specific questions or solutions at any given time throughout the course. Students can be referred to relevant chapters to complete during class or privately to prepare for topic tests or exams. These textbooks also act as a great reference for teachers regarding lesson planning, developing internal assessments or when preparing class activities to ensure alignment with exam level questions.

#### Books available in this series

Mathematics	Sciences	English
<ul style="list-style-type: none"> <li>• Essential Mathematics</li> <li>• General Mathematics</li> <li>• Mathematical Methods</li> <li>• Specialist Mathematics</li> </ul>	<ul style="list-style-type: none"> <li>• Biology</li> <li>• Chemistry</li> <li>• Physics</li> <li>• Psychology</li> </ul>	<ul style="list-style-type: none"> <li>• English</li> <li>• Essential English</li> <li>• Literature</li> </ul>
Humanities and Social Sciences	Health and Physical Education	Technologies
<ul style="list-style-type: none"> <li>• Accounting</li> <li>• Ancient History</li> <li>• Business</li> <li>• Economics</li> </ul>	<ul style="list-style-type: none"> <li>• Health</li> <li>• Physical Education</li> </ul>	<ul style="list-style-type: none"> <li>• Design</li> <li>• Digital Solutions</li> <li>• Engineering</li> </ul>

#### Acknowledgements

External Assessment Resources © State of Queensland (QCAA) 2024.

# Contents

<b>Unit 3 Bivariate data, sequences and change, and Earth geometry .....</b>	<b>4</b>
<b>Unit 3 – Topic 1: Bivariate data analysis .....</b>	<b>4</b>
Paper 1 Section 1 .....	4
Paper 1 Section 2 .....	10
Paper 2 Section 1 .....	19
Marking Guide – Paper 1 Section 1 .....	25
Marking Guide – Paper 1 Section 2 .....	31
Marking Guide – Paper 2 Section 1 .....	37
<b>Unit 3 – Topic 2: Time series analysis .....</b>	<b>44</b>
Paper 1 Section 1 .....	44
Paper 1 Section 2 .....	47
Paper 2 Section 1 .....	50
Marking Guide – Paper 1 Section 1 .....	53
Marking Guide – Paper 1 Section 2 .....	56
Marking Guide – Paper 2 Section 1 .....	59
<b>Unit 3 – Topic 3: Growth and decay in sequences .....</b>	<b>61</b>
Paper 1 Section 1 .....	61
Paper 1 Section 2 .....	63
Paper 2 Section 1 .....	66
Marking Guide – Paper 1 Section 1 .....	69
Marking Guide – Paper 1 Section 2 .....	71
Marking Guide – Paper 2 Section 1 .....	73
<b>Unit 3 – Topic 4: Earth geometry and time zones .....</b>	<b>76</b>
Paper 1 Section 1 .....	76
Paper 1 Section 2 .....	78
Paper 2 Section 1 .....	85
Marking Guide – Paper 1 Section 1 .....	87
Marking Guide – Paper 1 Section 2 .....	88
Marking Guide – Paper 2 Section 1 .....	95
<b>Unit 4: Investing and Networking .....</b>	<b>97</b>
<b>Unit 4 – Topic 1: Loans, investments and annuities .....</b>	<b>97</b>
Paper 1 Section 1 .....	97
Paper 1 Section 2 .....	99
Paper 2 Section 1 .....	105
Marking Guide – Paper 1 Section 1 .....	110
Marking Guide – Paper 1 Section 2 .....	112
Marking Guide – Paper 2 Section 1 .....	116
<b>Unit 4 – Topic 2: Graphs and networks .....</b>	<b>121</b>
Paper 1 Section 1 .....	121
Paper 1 Section 2 .....	125
Paper 2 Section 1 .....	128
Marking Guide – Paper 1 Section 1 .....	130
Marking Guide – Paper 1 Section 2 .....	134
Marking Guide – Paper 2 Section 1 .....	137
<b>Unit 4 – Topic 3: Networks and decision mathematics .....</b>	<b>139</b>
Paper 1 Section 1 .....	139
Paper 1 Section 2 .....	142
Paper 2 Section 1 .....	149
Marking Guide – Paper 1 Section 1 .....	158
Marking Guide – Paper 1 Section 2 .....	161
Marking Guide – Paper 2 Section 1 .....	168

## Unit 3 Bivariate data, sequences and change, and Earth geometry

### Unit 3 – Topic 1: Bivariate data analysis

#### Paper 1 Section 1

<b>2023 Paper 1 Section 1 Question 1  Bivariate data analysis</b>	<p>A linear association with a correlation coefficient of 0.23 is best described as</p> <p>(A) weak positive. (B) weak negative. (C) strong positive. (D) strong negative.</p>
---	--

<b>2023 Paper 1 Section 1 Question 4  Bivariate data analysis</b>	<p>Athletes were surveyed about their preferred shoe brand: X, Y or Z. The results are shown in the frequency table.</p> <table border="1"><thead><tr><th></th><th>X</th><th>Y</th><th>Z</th><th>Total</th></tr></thead><tbody><tr><th>Field athletes</th><td>26</td><td>12</td><td>2</td><td>40</td></tr><tr><th>Track athletes</th><td>14</td><td>18</td><td>8</td><td>40</td></tr><tr><th>Total</th><td>40</td><td>30</td><td>10</td><td>80</td></tr></tbody></table> <p>The percentage of field athletes who prefer brand Y is</p> <p>(A) 12% (B) 15% (C) 30% (D) 40%</p>		X	Y	Z	Total	Field athletes	26	12	2	40	Track athletes	14	18	8	40	Total	40	30	10	80
	X	Y	Z	Total																	
Field athletes	26	12	2	40																	
Track athletes	14	18	8	40																	
Total	40	30	10	80																	

<b>2023 Paper 1 Section 1 Question 5  Bivariate data analysis</b>	<p>A scatterplot is created to identify the nature of the relationship between two variables: vehicle age and distance travelled.</p> <p>Which statement is correct?</p> <p>(A) The vertical axis should show vehicle age as the response variable. (B) The horizontal axis should show vehicle age as the explanatory variable. (C) The horizontal axis should show distance travelled as the response variable. (D) The vertical axis should show distance travelled as the explanatory variable.</p>
---	---

<b>2023 Paper 1 Section 1 Question 7  Bivariate data analysis</b>	<p>Which statement is always true for a causal relationship between an explanatory variable and a response variable?</p> <p>(A) One of the variables is a confounding variable. (B) The relationship is explained by a third variable. (C) There is a positive association between the variables. (D) The response variable is dependent on the explanatory variable.</p>
---	---

<b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 14</b>  <b>Bivariate data analysis</b>	<p>A calculator is used to determine the equation of the least-squares line for the plant growth data in the table.</p> <p>What is the correct equation?</p> <p>(A) <math>d = 0.6h + 5.7</math></p> <p>(B) <math>h = 0.6d + 5.7</math></p> <p>(C) <math>d = 5.7h + 0.6</math></p> <p>(D) <math>h = 5.7d + 0.6</math></p>
---	--

<b>2022</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 8</b>  <b>Bivariate data analysis</b>	<p>The scatterplot shows the annual number of visitors to the Great Barrier Reef Marine Park.</p> <p>For 2018, the annual number of visitors could best be</p> <p>(A) interpolated as 2.7 million.</p> <p>(B) extrapolated as 2.7 million.</p> <p>(C) interpolated as 3.2 million.</p> <p>(D) extrapolated as 3.2 million.</p>
--	--

<b>2022</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 10</b>  <b>Bivariate data analysis</b>	<p>Which example states an explanatory variable followed by a response variable?</p> <p>(A) car manufacturers and car colours</p> <p>(B) dog breeds and frequency of names</p> <p>(C) plant growth and amount of fertiliser used</p> <p>(D) daily temperatures and daily ice cream sales</p>
---	--

<b>2022</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 11</b>  <b>Bivariate data analysis</b>	<p>The equation of a fitted line for the number of free throws in basketball, <math>t</math>, and the number of hours in a training session, <math>h</math>, is <math>t = 26.781 + 12.974 h</math>.</p> <p>The predicted number of free throws for a 5-hour training session, when rounded to the nearest whole number, is</p> <p>(A) 64</p> <p>(B) 65</p> <p>(C) 91</p> <p>(D) 92</p>
---	--

**2022  
Paper 1  
Section 1  
Question 12**

**Bivariate data analysis**

Identify the scatterplot that best demonstrates a strong negative association.

**2022  
Paper 1  
Section 1  
Question 13**

**Bivariate data analysis**

The two-way table summarises the semester 1 results for students enrolled in two courses, Machinery and Electrical. Students achieved either satisfactory (S) or unsatisfactory (U).

		Machinery	
		S	U
Electrical	S	80%	10%
	U	20%	90%

The 10% cell in the table indicates that

(A) 10% of all students achieved satisfactory in Electrical.  
 (B) 10% of all students achieved unsatisfactory in Machinery.  
 (C) 10% of the students who achieved satisfactory in Electrical achieved unsatisfactory in Machinery.  
 (D) 10% of the students who achieved unsatisfactory in Machinery achieved satisfactory in Electrical.

**2021  
Paper 1  
Section 1  
Question 3**

**Bivariate data analysis**

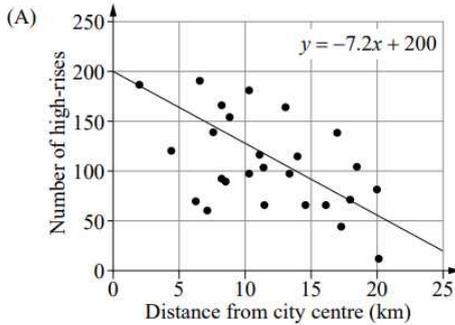
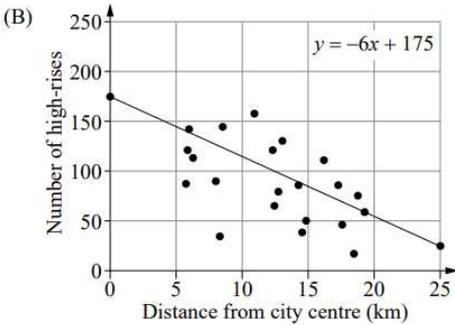
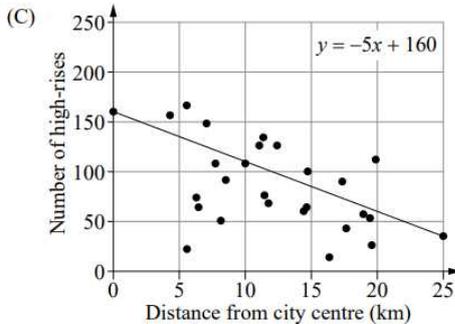
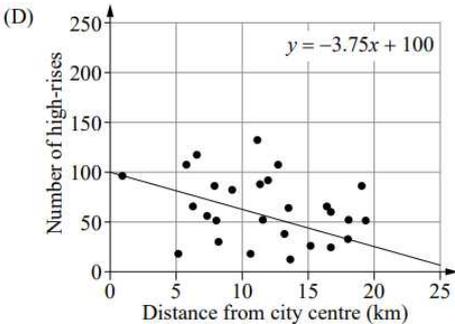
The table shows the results of a student survey about their preferred movie genre.

Year level	Genre		
	Comedy	Action	Science fiction
7–8	20	25	21
9–10	24	53	21
11–12	36	28	12

Of the students who preferred comedy, what percentage were in Year 9 or higher?

(A) 25%  
 (B) 30%  
 (C) 60%  
 (D) 75%

<p><b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 4</b></p> <p><b>Bivariate data analysis</b></p>	<p>A confounding variable is a variable that</p> <p>(A) can only take on a certain number of values.          (B) remains constant throughout a statistical investigation.          (C) is used to predict a difference in the response variable.          (D) other than the explanatory variable, influences the response variable.</p>
--	---

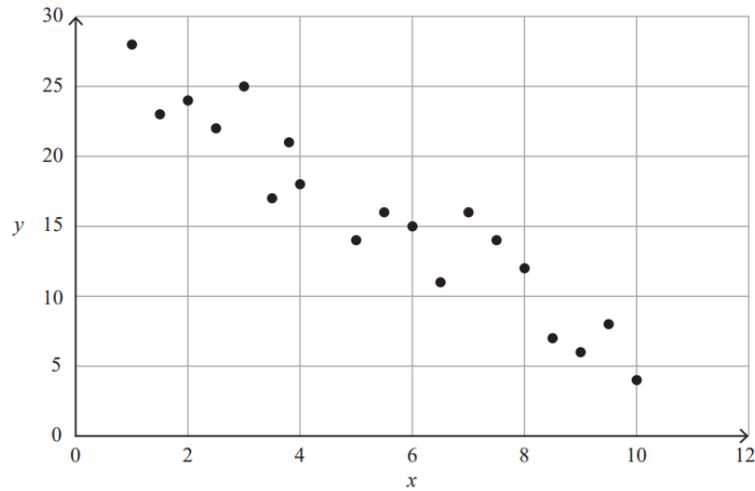
<p><b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 6</b></p> <p><b>Bivariate data analysis</b></p>	<p>These scatterplots show the number of high-rises in a city and their distance from the city centre.</p> <p>Which scatterplot was used to extrapolate that 25 km from the city centre there were 20 high-rises?</p> <div style="display: flex; flex-wrap: wrap;"> <div style="width: 50%;"> <p>(A) </p> </div> <div style="width: 50%;"> <p>(B) </p> </div> <div style="width: 50%;"> <p>(C) </p> </div> <div style="width: 50%;"> <p>(D) </p> </div> </div>
--	--

<p><b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 11</b></p> <p><b>Bivariate data analysis</b></p>	<p>Which option is an example of bivariate data?</p> <p>(A) The rating given to a brand of meat pies as poor, fair or good.          (B) The number of people in a household and amount of water used.          (C) The number of cars passing through a particular set of traffic lights.          (D) The time a person spends using a mobile phone on a Friday evening.</p>
---	--

**2020  
Paper 1  
Section 1  
Question 8**

**Bivariate  
data analysis**

The following scatterplot shows a linear association between two numerical variables.



Choose the best description for the direction and strength of the association.

- (A) strong positive
- (B) strong negative
- (C) weak positive
- (D) weak negative

**2020  
Paper 1  
Section 1  
Question 14**

**Bivariate  
data analysis**

A sample of university staff and students was asked whether they preferred catching public transport or driving their own car to university. The data collected is shown in the table.

	Public transport	Drive own car
Staff	2	18
Students	48	12

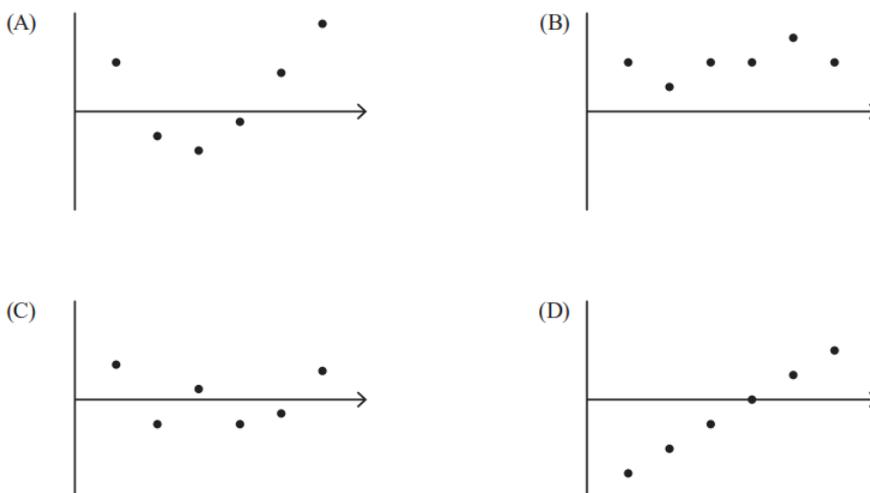
What percentage of university students prefer to drive their own car?

- (A) 12%
- (B) 15%
- (C) 20%
- (D) 40%

**2020  
Paper 1  
Section 1  
Question 1**

**Bivariate  
data analysis**

Four linear models have been developed for a data set. Identify the residual plot that indicates that the developed linear model is justified.



<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 9</b></p> <p><b>Bivariate data analysis</b></p>	<p>It is observed that as the number of ice blocks sold each month increases, the number of fans sold also increases. Which of these statements is therefore true?</p> <p>(A) There is a negative causation between the number of ice blocks sold and the number of fans sold each month.</p> <p>(B) There is a positive causation between the number of ice blocks sold and the number of fans sold each month.</p> <p>(C) There is a negative association between the number of ice blocks sold and the number of fans sold each month.</p> <p>(D) There is a positive association between the number of ice blocks sold and the number of fans sold each month.</p>
--	--

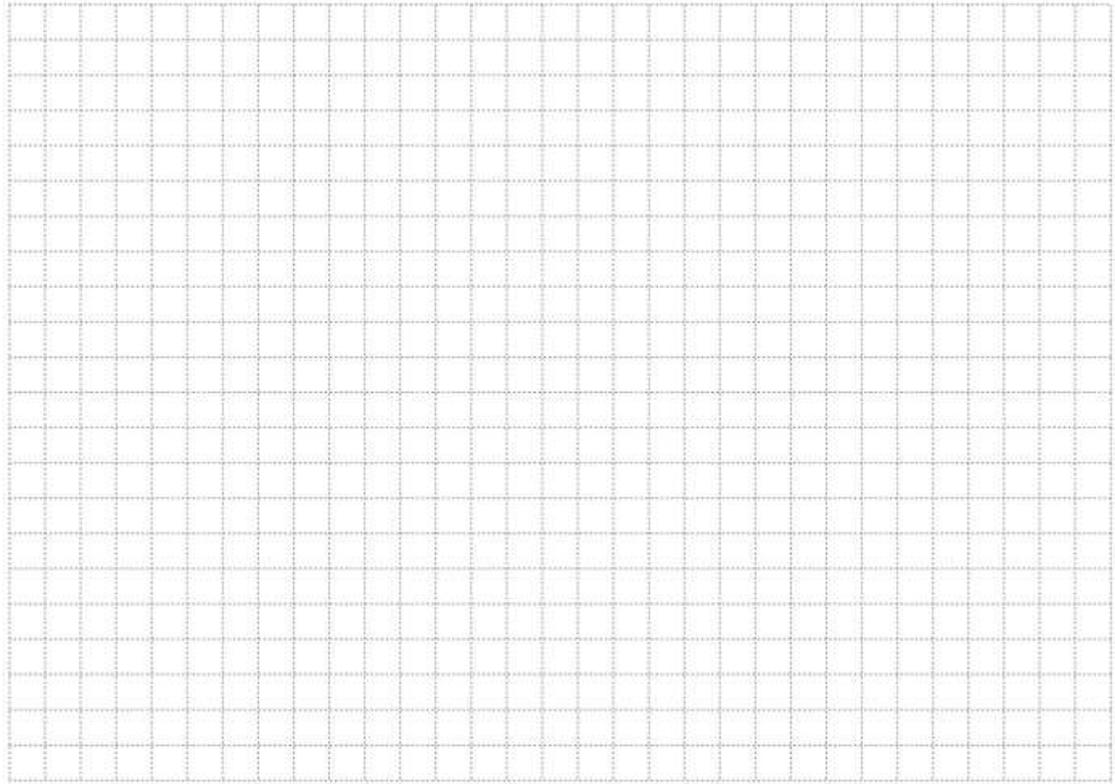
2023  
Paper 1  
Section 2  
Question 22

Bivariate  
data analysis

A person used a fitness tracker to monitor their hours of sleep and the distance they travelled each day.

Sleep (hours)	7	7.5	8	6.5	9.5	6	8.5
Distance (km)	5	7	5	7	4	8	6

a) Using distance as the response variable, display the data in a scatterplot with labelled axes. [3 marks]



b) Identify the direction of the association between hours of sleep and distance travelled. [1 mark]

---

---







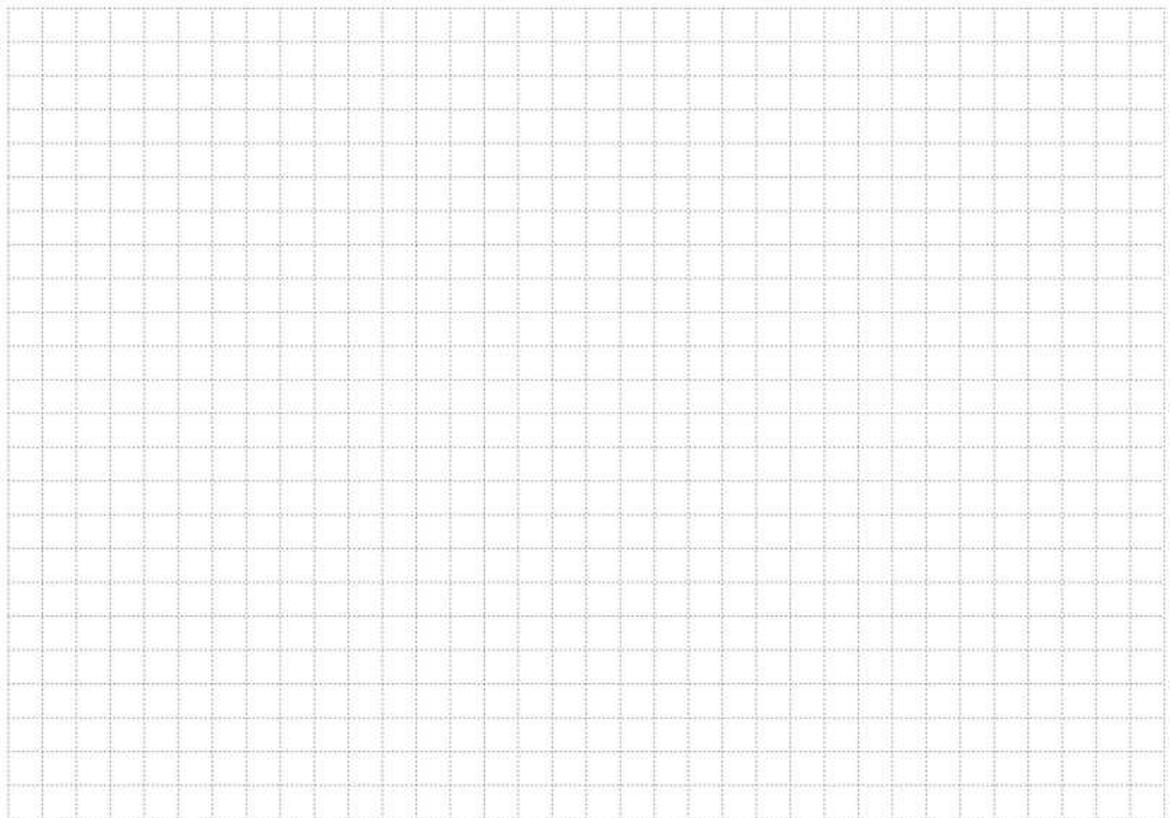
b) Use the equation from Question 23a) to predict the hourly pay of an employee with 15 years experience. [2 marks]

2022  
Paper 1  
Section 2  
Question 24  
  
Bivariate  
data analysis

The maximum temperature and the number of pies sold each day at a bakery are provided in the table.

Maximum temperature (°C)	29	20	31	27	23	25	22	33
Number of pies sold	32	39	25	33	37	35	37	30

a) Construct a scatterplot to display the data on the grid provided. [3 marks]



Note: If you make a mistake in the scatterplot, cancel it by ruling a single diagonal line through your work and use the additional response space at the back of this question and response book.

b) Describe the association between the maximum temperature and the number of pies sold in terms of direction and strength. [2 marks]

---

---

---

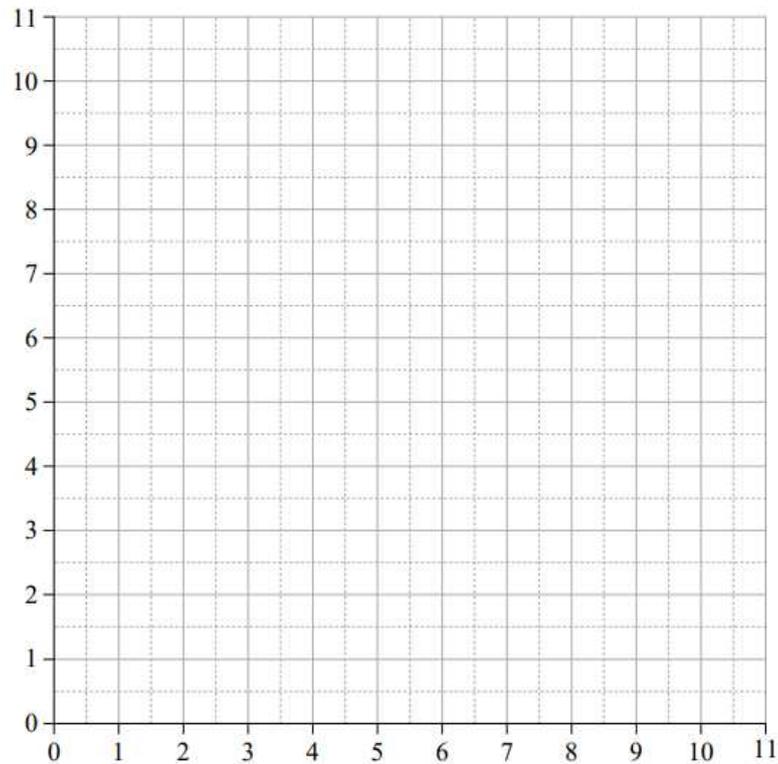
**2021  
Paper 1  
Section 2  
Question 16**

**Bivariate  
data analysis**

The table shows the values of bivariate data with variables  $g$  and  $h$ .

$g$	9	2.5	4.5	8	6	3.5	1
$h$	3.5	6.5	10	6	9.5	9	3

a) Construct a scatterplot of the data with  $g$  as the explanatory variable. [2 marks]



Note: If you make a mistake in the graph, cancel it by ruling a single diagonal line through your work and use the additional graph on page 21 of this question and response book.

b) Describe the association in terms of form and strength. [2 marks]

---

---



**2020  
Paper 1  
Section 2  
Question 22**

**Bivariate  
data analysis**

A store asked its junior and senior staff whether or not they would like to change the store uniform. The results are in the frequency table.

	Change uniform	Do not change uniform
Junior staff	92	28
Senior staff	23	67

a) Convert the two-way table into a percentaged two-way frequency table using column totals. [2 marks]

---

---

---

---

---

---

---

---

b) Explain whether there is an association between staff groups and a desire to change the store uniform. [2 marks]

---

---

---

---

---

---

---

---

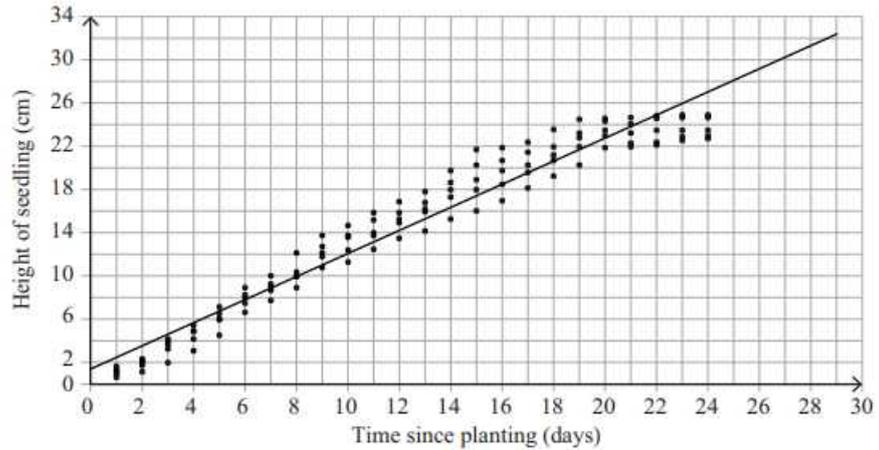
---

---

**2020  
Paper 1  
Section 2  
Question 24**

**Bivariate  
data analysis**

The following data for the height of five seedlings was collected and the least-squares line was developed and graphed.



a) Use the least-squares line to estimate the height of a nine-day-old seedling. [1 mark]

b) Classify the prediction for 24a) as either interpolation or extrapolation. [1 mark]

c) Based on the graph, the following statement was made:  
'A seedling will reach a height of about 32 cm by day 29.'  
Comment on the reasonableness and the possible dangers of this statement. [2 marks]








---



---



---



---



---



---



---



---

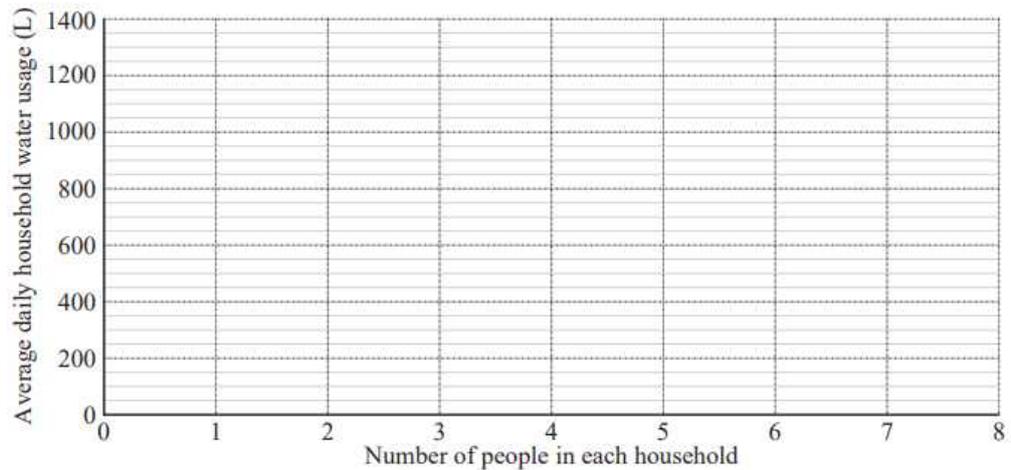
**2020  
Paper 2  
Section 1  
Question 2**

**Bivariate  
data analysis**

The number of people living in each household and the average daily household water usage, measured in litres (L), were recorded for 10 households.

<b>Number of people in each household</b>	6	2	4	5	5	4	3	1	6	7
<b>Average daily household water usage (L)</b>	990	160	320	480	410	280	240	130	940	1340

Calculate Pearson’s correlation coefficient and then evaluate the appropriateness of using this coefficient for the association between daily water usage and the number of people living in a household. (5 marks)



Note: If you make a mistake in the graph, cancel it by ruling a single diagonal line through your work and use the additional graph on page 17 of this question and response book.

---



---



---



---



---



---



## Marking Guide – Paper 1 Section 1

<b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 1</b>  <b>Bivariate data analysis</b>	A linear association with a correlation coefficient of 0.23 is best described as  (A) <b>weak positive. – Answer</b> (B) weak negative. (C) strong positive. (D) strong negative.
--	--

<b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 4</b>  <b>Bivariate data analysis</b>	<p>Athletes were surveyed about their preferred shoe brand: X, Y or Z. The results are shown in the frequency table.</p> <table border="1"><thead><tr><th></th><th>X</th><th>Y</th><th>Z</th><th>Total</th></tr></thead><tbody><tr><th>Field athletes</th><td>26</td><td>12</td><td>2</td><td>40</td></tr><tr><th>Track athletes</th><td>14</td><td>18</td><td>8</td><td>40</td></tr><tr><th>Total</th><td>40</td><td>30</td><td>10</td><td>80</td></tr></tbody></table> <p>The percentage of field athletes who prefer brand Y is</p> <p>(A) 12% (B) 15% (C) <b>30% – Answer</b> (D) 40%</p>		X	Y	Z	Total	Field athletes	26	12	2	40	Track athletes	14	18	8	40	Total	40	30	10	80
	X	Y	Z	Total																	
Field athletes	26	12	2	40																	
Track athletes	14	18	8	40																	
Total	40	30	10	80																	

<b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 5</b>  <b>Bivariate data analysis</b>	<p>A scatterplot is created to identify the nature of the relationship between two variables: vehicle age and distance travelled.</p> <p>Which statement is correct?</p> <p>(A) The vertical axis should show vehicle age as the response variable. (B) <b>The horizontal axis should show vehicle age as the explanatory variable. – Answer</b> (C) The horizontal axis should show distance travelled as the response variable. (D) The vertical axis should show distance travelled as the explanatory variable.</p>
--	---

<b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 7</b>  <b>Bivariate data analysis</b>	<p>Which statement is always true for a causal relationship between an explanatory variable and a response variable?</p> <p>(A) One of the variables is a confounding variable. (B) The relationship is explained by a third variable. (C) There is a positive association between the variables. (D) <b>The response variable is dependent on the explanatory variable. – Answer</b></p>
--	---

<b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 14</b>  <b>Bivariate data analysis</b>	<p>A calculator is used to determine the equation of the least-squares line for the plant growth data in the table.</p> <p>What is the correct equation?</p> <p>(A) <math>d = 0.6h + 5.7</math> (B) <math>h = 0.6d + 5.7</math> (C) <math>d = 5.7h + 0.6</math> (D) <math>h = 5.7d + 0.6</math></p> <p><b>Answer is B.</b></p>
---	--

<p><b>2022</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 8</b></p> <p><b>Bivariate data analysis</b></p>	<p>The scatterplot shows the annual number of visitors to the Great Barrier Reef Marine Park.</p> <p>For 2018, the annual number of visitors could best be</p> <p><b>(A) interpolated as 2.7 million. - Answer</b>          (B) extrapolated as 2.7 million.          (C) interpolated as 3.2 million.          (D) extrapolated as 3.2 million.</p>
--	--

<p><b>2022</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 10</b></p> <p><b>Bivariate data analysis</b></p>	<p>Which example states an explanatory variable followed by a response variable?</p> <p>(A) car manufacturers and car colours          (B) dog breeds and frequency of names          (C) plant growth and amount of fertiliser used  <b>(D) daily temperatures and daily ice cream sales - Answer</b></p>
---	--

<p><b>2022</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 11</b></p> <p><b>Bivariate data analysis</b></p>	<p>The equation of a fitted line for the number of free throws in basketball, <math>t</math>, and the number of hours in a training session, <math>h</math>, is <math>t = 26.781 + 12.974 h</math>.</p> <p>The predicted number of free throws for a 5-hour training session, when rounded to the nearest whole number, is</p> <p>(A) 64          (B) 65          (C) 91  <b>(D) 92 - Answer</b></p>
---	--

**2022**  
**Paper 1**  
**Section 1**  
**Question 12**

**Bivariate data analysis**

Identify the scatterplot that best demonstrates a strong negative association.

(A)

(B)

(C)

(D)

**Answer is C.**

**2022**  
**Paper 1**  
**Section 1**  
**Question 13**

**Bivariate data analysis**

The two-way table summarises the semester 1 results for students enrolled in two courses, Machinery and Electrical. Students achieved either satisfactory (S) or unsatisfactory (U).

		Machinery	
		S	U
Electrical	S	80%	10%
	U	20%	90%

The 10% cell in the table indicates that

(A) 10% of all students achieved satisfactory in Electrical.  
 (B) 10% of all students achieved unsatisfactory in Machinery.  
 (C) 10% of the students who achieved satisfactory in Electrical achieved unsatisfactory in Machinery.  
 (D) **10% of the students who achieved unsatisfactory in Machinery achieved satisfactory in Electrical.**

**2021**  
**Paper 1**  
**Section 1**  
**Question 3**

**Bivariate data analysis**

The table shows the results of a student survey about their preferred movie genre.

Year level	Genre		
	Comedy	Action	Science fiction
7–8	20	25	21
9–10	24	53	21
11–12	36	28	12

Of the students who preferred comedy, what percentage were in Year 9 or higher?

(A) 25%  
 (B) 30%  
 (C) 60%  
 (D) **75% - Answer**

<p><b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 4</b></p> <p><b>Bivariate data analysis</b></p>	<p>A confounding variable is a variable that</p> <p>(A) can only take on a certain number of values.          (B) remains constant throughout a statistical investigation.          (C) is used to predict a difference in the response variable.  <b>(D) other than the explanatory variable, influences the response variable. - Answer</b></p>
--	---

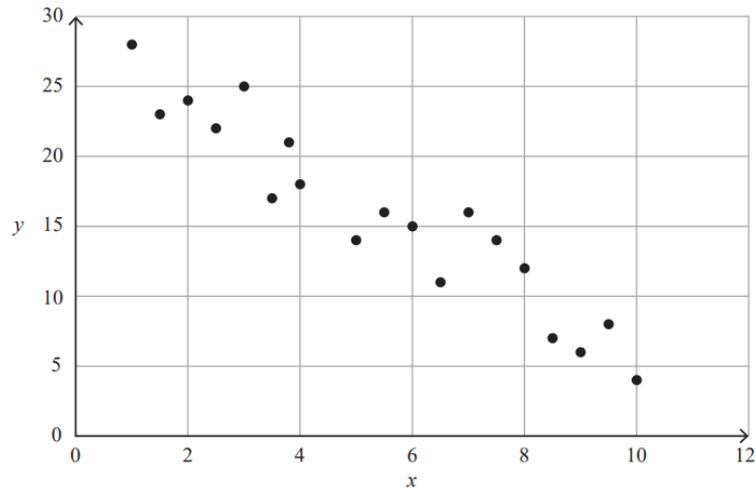
<p><b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 6</b></p> <p><b>Bivariate data analysis</b></p>	<p>These scatterplots show the number of high-rises in a city and their distance from the city centre.</p> <p>Which scatterplot was used to extrapolate that 25 km from the city centre there were 20 high-rises?</p> <div style="display: flex; flex-wrap: wrap;"> <div style="width: 50%;"> <p>(A) </p> </div> <div style="width: 50%;"> <p>(B) </p> </div> <div style="width: 50%;"> <p>(C) </p> </div> <div style="width: 50%;"> <p>(D) </p> </div> </div> <p><b>Answer is A.</b></p>
--	---

<p><b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 11</b></p> <p><b>Bivariate data analysis</b></p>	<p>Which option is an example of bivariate data?</p> <p>(A) The rating given to a brand of meat pies as poor, fair or good.  <b>(B) The number of people in a household and amount of water used. - Answer</b>          (C) The number of cars passing through a particular set of traffic lights.          (D) The time a person spends using a mobile phone on a Friday evening.</p>
---	--

**2020  
Paper 1  
Section 1  
Question 8**

**Bivariate  
data analysis**

The following scatterplot shows a linear association between two numerical variables.



Choose the best description for the direction and strength of the association.

- (A) strong positive
- (B) strong negative - Answer**
- (C) weak positive
- (D) weak negative

**2020  
Paper 1  
Section 1  
Question 14**

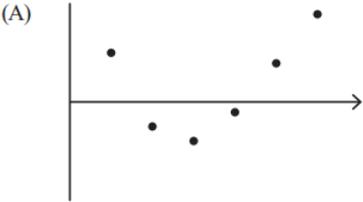
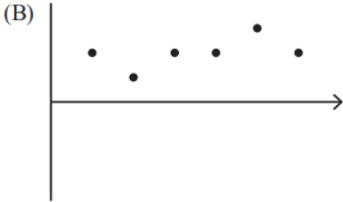
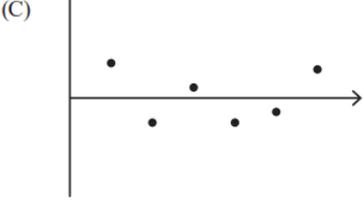
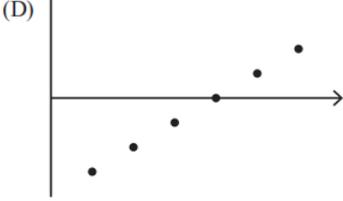
**Bivariate  
data analysis**

A sample of university staff and students was asked whether they preferred catching public transport or driving their own car to university. The data collected is shown in the table.

	Public transport	Drive own car
Staff	2	18
Students	48	12

What percentage of university students prefer to drive their own car?

- (A) 12%
- (B) 15%
- (C) 20% - Answer**
- (D) 40%

<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 1</b></p> <p><b>Bivariate data analysis</b></p>	<p>Four linear models have been developed for a data set. Identify the residual plot that indicates that the developed linear model is justified.</p> <div style="display: flex; flex-wrap: wrap;"> <div style="width: 50%;"> <p>(A) </p> </div> <div style="width: 50%;"> <p>(B) </p> </div> <div style="width: 50%;"> <p>(C) </p> </div> <div style="width: 50%;"> <p>(D) </p> </div> </div> <p><b>Answer is C.</b></p>
--	---

<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 9</b></p> <p><b>Bivariate data analysis</b></p>	<p>It is observed that as the number of ice blocks sold each month increases, the number of fans sold also increases. Which of these statements is therefore true?</p> <p>(A) There is a negative causation between the number of ice blocks sold and the number of fans sold each month.</p> <p>(B) There is a positive causation between the number of ice blocks sold and the number of fans sold each month.</p> <p>(C) There is a negative association between the number of ice blocks sold and the number of fans sold each month.</p> <p><b>(D) There is a positive association between the number of ice blocks sold and the number of fans sold each month. - Answer</b></p>
--	--

2023  
Paper 1  
Section 2  
Question 22

Bivariate  
data analysis

A person used a fitness tracker to monitor their hours of sleep and the distance they travelled each day.

Sleep (hours)	7	7.5	8	6.5	9.5	6	8.5
Distance (km)	5	7	5	7	4	8	6

a) Using distance as the response variable, display the data in a scatterplot with labelled axes. [3 marks]

Sample response	The response
	<ul style="list-style-type: none"> <li>• correctly identifies the axis for the response variable [1 mark]</li> <li>• formats scatterplot with appropriate scaling and labelling for both axes [1 mark]</li> <li>• accurately plots all points [1 mark]</li> </ul>

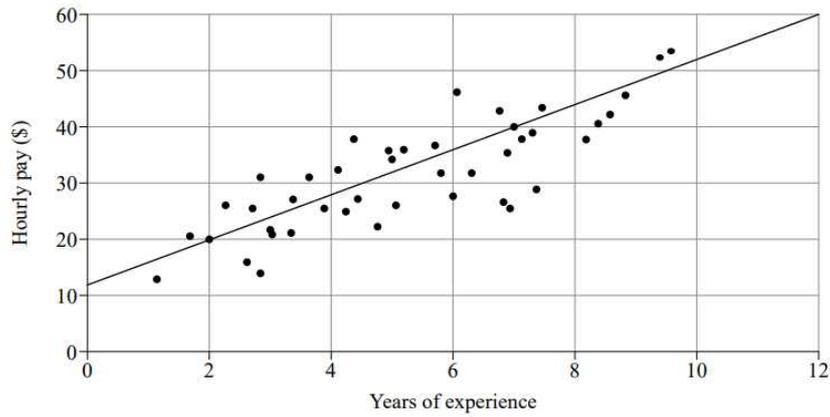
b) Identify the direction of the association between hours of sleep and distance travelled. [1 mark]

Sample response	The response
The association is negative.	• identifies direction [1 mark]

**2022  
Paper 1  
Section 2  
Question 23**

**Bivariate  
data analysis**

The least-squares line has been provided for a scatterplot that shows the association between an employee's years of experience,  $n$ , and their hourly pay,  $p$ .



a) Given that the least-squares line passes directly through the points (2, 20) and (7, 40), determine its equation. [2 marks]

Sample Response	The response
$m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{40 - 20}{7 - 2}$ $= \frac{20}{5}$ $= 4$	<ul style="list-style-type: none"> <li>correctly determines the slope [1 mark]</li> </ul>
$p - p_1 = m(n - n_1)$ $p - 20 = 4(n - 2)$ $p - 20 = 4n - 8$ $p = 4n + 12$	<ul style="list-style-type: none"> <li>determines equation of least-squares line [1 mark]</li> </ul>

b) Use the equation from Question 23a) to predict the hourly pay of an employee with 15 years experience. [2 marks]

Sample Response	The response
$p = 4(15) + 12$ $p = 72$	<ul style="list-style-type: none"> <li>substitutes into equation from Question 23a) [1 mark]</li> </ul>
<p>A person with 15 years experience could expect an hourly pay of \$72.</p>	<ul style="list-style-type: none"> <li>predicts hourly pay, including units [1 mark]</li> </ul>

2022  
Paper 1  
Section 2  
Question 24  
  
Bivariate  
data analysis

The maximum temperature and the number of pies sold each day at a bakery are provided in the table.

Maximum temperature (°C)	29	20	31	27	23	25	22	33
Number of pies sold	32	39	25	33	37	35	37	30

a) Construct a scatterplot to display the data on the grid provided. [3 marks]

Sample Response	The response
	<ul style="list-style-type: none"> <li>• correctly identifies the explanatory and response variables [1 mark]</li> <li>• accurately plots points [1 mark]</li> <li>• formats scatterplot with appropriate scaling and labelling of axes [1 mark]</li> </ul>

Note: If you make a mistake in the scatterplot, cancel it by ruling a single diagonal line through your work and use the additional response space at the back of this question and response book.

b) Describe the association between the maximum temperature and the number of pies sold in terms of direction and strength. [2 marks]

Sample Response	The response
The association is a strong, negative, linear relationship	<ul style="list-style-type: none"> <li>• describes association as               <ul style="list-style-type: none"> <li>- strong [1 mark]</li> <li>- negative [1 mark]</li> </ul> </li> </ul>

**2021  
Paper 1  
Section 2  
Question 16**

**Bivariate  
data analysis**

The table shows the values of bivariate data with variables  $g$  and  $h$ .

$g$	9	2.5	4.5	8	6	3.5	1
$h$	3.5	6.5	10	6	9.5	9	3

a) Construct a scatterplot of the data with  $g$  as the explanatory variable. [2 marks]

Sample Response	The response
	<ul style="list-style-type: none"> <li>• correctly formats the Cartesian plane with <math>g</math> along the <math>x</math>-axis and <math>h</math> along the <math>y</math>-axis [1 mark]</li> <li>• correctly plots all the data points [1 mark]</li> </ul>

Note: If you make a mistake in the graph, cancel it by ruling a single diagonal line through your work and use the additional graph on page 21 of this question and response book.

b) Describe the association in terms of form and strength. [2 marks]

Sample Response	The response
non-linear and strong	<ul style="list-style-type: none"> <li>• identifies form [1 mark]</li> <li>• identifies strength [1 mark]</li> </ul>

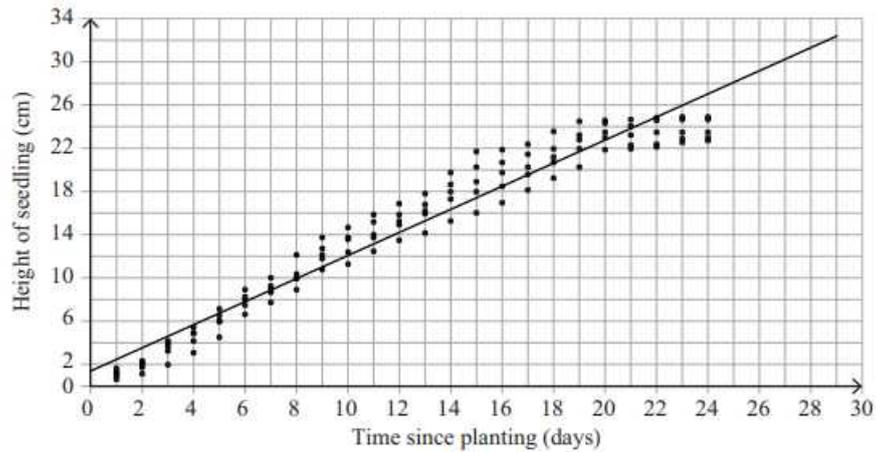
<p><b>2020</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 19</b></p> <p><b>Bivariate data analysis</b></p>	<p>Data was collected relating the number of hours spent fishing and the total number of fish caught. The linear model for this data was found to be <math>y = 2.3x + 31.4</math>, where <math>x</math> is the number of hours spent fishing, and <math>y</math> is the total number of fish caught.</p> <p>a) Use the model to predict the number of fish caught if 12 hours were spent fishing. [1 mark]</p>							
	<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td> <math>x = 12</math>  <math>\therefore y = 2.3(12) + 31.4</math>  <math>= 59</math> </td> <td> <ul style="list-style-type: none"> <li>correctly calculates 59 [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	$x = 12$ $\therefore y = 2.3(12) + 31.4$ $= 59$	<ul style="list-style-type: none"> <li>correctly calculates 59 [1 mark]</li> </ul>			
	Sample Response	The response						
	$x = 12$ $\therefore y = 2.3(12) + 31.4$ $= 59$	<ul style="list-style-type: none"> <li>correctly calculates 59 [1 mark]</li> </ul>						
<p>b) The correlation coefficient for this data is 0.688 and the coefficient of determination is 0.473. Use each of these to describe the strength of the linear association between the two variables and decide if your prediction is valid. [3 marks]</p>								
<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td>A correlation coefficient of 0.688 suggests a moderate association, which means that as the hours spent fishing increase so do the number of fish caught.</td> <td> <ul style="list-style-type: none"> <li>correctly describes the strength as either moderate or strong [1 mark]</li> </ul> </td> </tr> <tr> <td>A coefficient of determination of 0.473 means that 47% of the variation in results can be explained by the variation of hours spent fishing.</td> <td> <ul style="list-style-type: none"> <li>correctly describes the meaning of the coefficient of determination [1 mark]</li> </ul> </td> </tr> <tr> <td>Therefore the prediction of catching 59 fish after fishing for 12 hours may be valid, however other factors will also come into play.</td> <td> <ul style="list-style-type: none"> <li>evaluates the reasonableness of the solution [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	A correlation coefficient of 0.688 suggests a moderate association, which means that as the hours spent fishing increase so do the number of fish caught.	<ul style="list-style-type: none"> <li>correctly describes the strength as either moderate or strong [1 mark]</li> </ul>	A coefficient of determination of 0.473 means that 47% of the variation in results can be explained by the variation of hours spent fishing.	<ul style="list-style-type: none"> <li>correctly describes the meaning of the coefficient of determination [1 mark]</li> </ul>	Therefore the prediction of catching 59 fish after fishing for 12 hours may be valid, however other factors will also come into play.	<ul style="list-style-type: none"> <li>evaluates the reasonableness of the solution [1 mark]</li> </ul>
Sample Response	The response							
A correlation coefficient of 0.688 suggests a moderate association, which means that as the hours spent fishing increase so do the number of fish caught.	<ul style="list-style-type: none"> <li>correctly describes the strength as either moderate or strong [1 mark]</li> </ul>							
A coefficient of determination of 0.473 means that 47% of the variation in results can be explained by the variation of hours spent fishing.	<ul style="list-style-type: none"> <li>correctly describes the meaning of the coefficient of determination [1 mark]</li> </ul>							
Therefore the prediction of catching 59 fish after fishing for 12 hours may be valid, however other factors will also come into play.	<ul style="list-style-type: none"> <li>evaluates the reasonableness of the solution [1 mark]</li> </ul>							

<p><b>2020</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 22</b></p> <p><b>Bivariate data analysis</b></p>	<p>A store asked its junior and senior staff whether or not they would like to change the store uniform. The results are in the frequency table.</p>																	
	<table border="1"> <thead> <tr> <th></th> <th>Change uniform</th> <th>Do not change uniform</th> </tr> </thead> <tbody> <tr> <th>Junior staff</th> <td>92</td> <td>28</td> </tr> <tr> <th>Senior staff</th> <td>23</td> <td>67</td> </tr> </tbody> </table>		Change uniform	Do not change uniform	Junior staff	92	28	Senior staff	23	67								
		Change uniform	Do not change uniform															
	Junior staff	92	28															
Senior staff	23	67																
<p>a) Convert the two-way table into a percentaged two-way frequency table using column totals. [2 marks]</p>																		
<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td>           Total # change uniform = 115            Total # do not change = 95         </td> <td> <ul style="list-style-type: none"> <li>correctly determines column totals [1 mark]</li> </ul> </td> </tr> <tr> <td> <table border="1"> <thead> <tr> <th></th> <th>Change uniform</th> <th>Do not change uniform</th> </tr> </thead> <tbody> <tr> <th>Junior staff</th> <td>80%</td> <td>29.5%</td> </tr> <tr> <th>Senior staff</th> <td>20%</td> <td>70.5%</td> </tr> <tr> <td></td> <td>100%</td> <td>100%</td> </tr> </tbody> </table> </td> <td> <ul style="list-style-type: none"> <li>correctly represents the data in a percentaged two-way table [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	Total # change uniform = 115 Total # do not change = 95	<ul style="list-style-type: none"> <li>correctly determines column totals [1 mark]</li> </ul>	<table border="1"> <thead> <tr> <th></th> <th>Change uniform</th> <th>Do not change uniform</th> </tr> </thead> <tbody> <tr> <th>Junior staff</th> <td>80%</td> <td>29.5%</td> </tr> <tr> <th>Senior staff</th> <td>20%</td> <td>70.5%</td> </tr> <tr> <td></td> <td>100%</td> <td>100%</td> </tr> </tbody> </table>		Change uniform	Do not change uniform	Junior staff	80%	29.5%	Senior staff	20%	70.5%		100%	100%	<ul style="list-style-type: none"> <li>correctly represents the data in a percentaged two-way table [1 mark]</li> </ul>
Sample Response	The response																	
Total # change uniform = 115 Total # do not change = 95	<ul style="list-style-type: none"> <li>correctly determines column totals [1 mark]</li> </ul>																	
<table border="1"> <thead> <tr> <th></th> <th>Change uniform</th> <th>Do not change uniform</th> </tr> </thead> <tbody> <tr> <th>Junior staff</th> <td>80%</td> <td>29.5%</td> </tr> <tr> <th>Senior staff</th> <td>20%</td> <td>70.5%</td> </tr> <tr> <td></td> <td>100%</td> <td>100%</td> </tr> </tbody> </table>		Change uniform	Do not change uniform	Junior staff	80%	29.5%	Senior staff	20%	70.5%		100%	100%	<ul style="list-style-type: none"> <li>correctly represents the data in a percentaged two-way table [1 mark]</li> </ul>					
	Change uniform	Do not change uniform																
Junior staff	80%	29.5%																
Senior staff	20%	70.5%																
	100%	100%																
<p>b) Explain whether there is an association between staff groups and a desire to change the store uniform. [2 marks]</p>																		
<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td>There does appear to be an association between the staff groups and wanting to change the uniform.</td> <td> <ul style="list-style-type: none"> <li>suggests the presence of an association [1 mark]</li> </ul> </td> </tr> <tr> <td>The data suggests that junior staff want to change the uniform (80% as opposed to 20% of senior staff) and senior staff do not want to change (70.5% compared with 29.5% of junior staff).</td> <td> <ul style="list-style-type: none"> <li>provides reasons to support conclusion [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	There does appear to be an association between the staff groups and wanting to change the uniform.	<ul style="list-style-type: none"> <li>suggests the presence of an association [1 mark]</li> </ul>	The data suggests that junior staff want to change the uniform (80% as opposed to 20% of senior staff) and senior staff do not want to change (70.5% compared with 29.5% of junior staff).	<ul style="list-style-type: none"> <li>provides reasons to support conclusion [1 mark]</li> </ul>												
Sample Response	The response																	
There does appear to be an association between the staff groups and wanting to change the uniform.	<ul style="list-style-type: none"> <li>suggests the presence of an association [1 mark]</li> </ul>																	
The data suggests that junior staff want to change the uniform (80% as opposed to 20% of senior staff) and senior staff do not want to change (70.5% compared with 29.5% of junior staff).	<ul style="list-style-type: none"> <li>provides reasons to support conclusion [1 mark]</li> </ul>																	

**2020  
Paper 1  
Section 2  
Question 24**

**Bivariate  
data analysis**

The following data for the height of five seedlings was collected and the least-squares line was developed and graphed.



a) Use the least-squares line to estimate the height of a nine-day-old seedling. [1 mark]

Sample Response	The response
11 cm	• provides the correct value including units [1 mark]

b) Classify the prediction for 24a) as either interpolation or extrapolation. [1 mark]

Sample Response	The response
Interpolation	• correctly classifies the prediction as interpolation [1 mark]

c) Based on the graph, the following statement was made:

‘A seedling will reach a height of about 32 cm by day 29.’

Comment on the reasonableness and the possible dangers of this statement. [2 marks]

Sample Response	The response
The least-squares line provided does suggest that at 29 days, the seedling will be 32 cm high.  However, the data values are levelling off at about 25 cm, so extrapolation is unwise.	• identifies that the least-squares line supports the statement [1 mark] • identifies potential dangers of extrapolation [1 mark]

Marking Guide – Paper 2 Section 1

2023  
Paper 1  
Section 2  
Question 22  
  
Bivariate  
data analysis

A person used a fitness tracker to monitor their hours of sleep and the distance they travelled each day.

Sleep (hours)	7	7.5	8	6.5	9.5	6	8.5
Distance (km)	5	7	5	7	4	8	6

a) Using distance as the response variable, display the data in a scatterplot with labelled axes. [3 marks]

Sample response	The response
	<ul style="list-style-type: none"> <li>• correctly identifies the axis for the response variable [1 mark]</li> <li>• formats scatterplot with appropriate scaling and labelling for both axes [1 mark]</li> <li>• accurately plots all points [1 mark]</li> </ul>

b) Identify the direction of the association between hours of sleep and distance travelled. [1 mark]

Sample response	The response
The association is negative.	<ul style="list-style-type: none"> <li>• identifies direction [1 mark]</li> </ul>

2023  
Paper 2  
Section 1  
Question 4

Bivariate  
data analysis

Hiroki believes that more fish are caught on warmer days. Jiro believes that the number of fish caught in a day is more dependent on the number of people fishing.

Bivariate datasets for six days are shown.

Temperature, $t$ ( $^{\circ}\text{C}$ )	32	26	20	27	23	29
Number of fish caught, $f$	530	400	320	220	180	120

Number of people fishing, $p$	46	58	38	34	30	28
Number of fish caught, $f$	530	400	320	220	180	120

Calculate the correlation coefficient for each dataset and use the results to identify the explanatory variable for the stronger linear association. Use the least-squares line equation for the stronger linear association to predict the number of fish caught on a 25  $^{\circ}\text{C}$  day when 50 people are fishing. (5 marks)

Sample response	The response						
<p>Calculate correlation coefficient for each dataset.</p> <table border="1"> <thead> <tr> <th>Dataset</th> <th>Correlation coefficient, <math>r</math></th> </tr> </thead> <tbody> <tr> <td><math>t \text{ vs } f</math></td> <td>0.3</td> </tr> <tr> <td><math>p \text{ vs } f</math></td> <td>0.8</td> </tr> </tbody> </table> <p><math>0.8 &gt; 0.3</math> The <b>explanatory variable</b> for the stronger linear association is <math>p</math>, number of people fishing.</p> <p><math>y = a + bx</math> Using calculator, <math>a = -130</math>, <math>b = 11</math> Equation in terms of given variables is <math>f = -130 + 11p</math></p> <p><math>= -130 + 11 \times 50</math> <math>= 420</math> It is predicted that 420 fish will be caught.</p>	Dataset	Correlation coefficient, $r$	$t \text{ vs } f$	0.3	$p \text{ vs } f$	0.8	<ul style="list-style-type: none"> <li>correctly calculates correlation coefficient for each dataset [1 mark]</li> <li>identifies explanatory variable for stronger linear association [1 mark]</li> </ul> <hr/> <ul style="list-style-type: none"> <li>determines least-squares line equation for dataset with stronger linear association [1 mark]</li> <li>substitutes value for relevant explanatory variable [1 mark]</li> <li>predicts number of fish caught [1 mark]</li> </ul>
Dataset	Correlation coefficient, $r$						
$t \text{ vs } f$	0.3						
$p \text{ vs } f$	0.8						

**2023  
Paper 2  
Section 1  
Question 6**

**Bivariate  
data  
analysis**

The table shows the average superannuation account balance for workers of various ages in two different industries. The coefficient of determination,  $R^2$ , for age versus account balance is 0.95 for industry A and 0.96 for industry B. 40-year-old Leigh works in the industry for which age explains a higher percentage of the account balance variation. Tony is 10 years older than Leigh and works in the other industry.

Age (years)	Account balance (\$)	
	Industry A	Industry B
22	7500	8100
32	42 000	60 000
42	98 000	120 000
52	160 000	210 000
62	290 000	360 000
72	400 000	480 000

Use linear models to predict the difference in current superannuation account balances for Leigh and Tony. (7 marks)

Sample response	The response
Compare $R^2$ values: $0.95 < 0.96$ . So, age explains a higher percentage of the account balance variation for the industry B dataset.	<ul style="list-style-type: none"> <li>correctly identifies dataset for which age explains a higher percentage of the account balance variation [1 mark]</li> </ul>
Linear model for industry A: Let $x = \text{age}$ , $y = \text{account balance}$ $y = bx + a$ Using calculator, $b = 7910$ and $a = -205\,520$ $y = 7910x + -205\,520$	<ul style="list-style-type: none"> <li>correctly determines linear model for age vs account balance for industry A data [1 mark]</li> </ul>
Linear model for industry B: Let $x = \text{age}$ , $y = \text{account balance}$ $y = bx + a$ Using calculator, $b = 9570$ and $a = -243\,440$ $y = 9570x + -243\,440$	<ul style="list-style-type: none"> <li>correctly determines linear model for age vs account balance for industry B data [1 mark]</li> </ul>
40-year-old Leigh works in industry B; substitute $x = 40$ $y = 9570 \times 40 + -243\,440$ $= 139\,360$  Tony's age = $40 + 10 = 50$ Tony works in industry A; substitute $x = 50$ $y = 7910 \times 50 + -205\,520$ $= 189\,980$	<ul style="list-style-type: none"> <li>correctly determines linear model for age vs account balance for industry B data [1 mark]</li> <li>substitutes <math>x = 40</math> into appropriate equation and calculates Leigh's current account balance [1 mark]</li> <li>substitutes <math>x = 50</math> into appropriate equation and calculates Tony's current account balance [1 mark]</li> </ul>
Difference = $189\,980 - 139\,360$ $= 50\,620$ The difference in account balances for Leigh and Tony is predicted to be \$50 620.	<ul style="list-style-type: none"> <li>calculates difference in current account balances for Leigh and Tony [1 mark]</li> <li>shows logical organisation communicating key steps [1 mark]</li> </ul>

**2022  
Paper 2  
Section 1  
Question 3**

**Bivariate  
data analysis**

In a company's first 10 years of operation, the average annual profit ( $\bar{y}$ ) was \$9660 with a standard deviation ( $s_y$ ) of \$3010. Fitting a least-squares line to the data comparing annual profit ( $y$ ) to the year of operation ( $x$ ) produced a correlation coefficient of 0.9987.

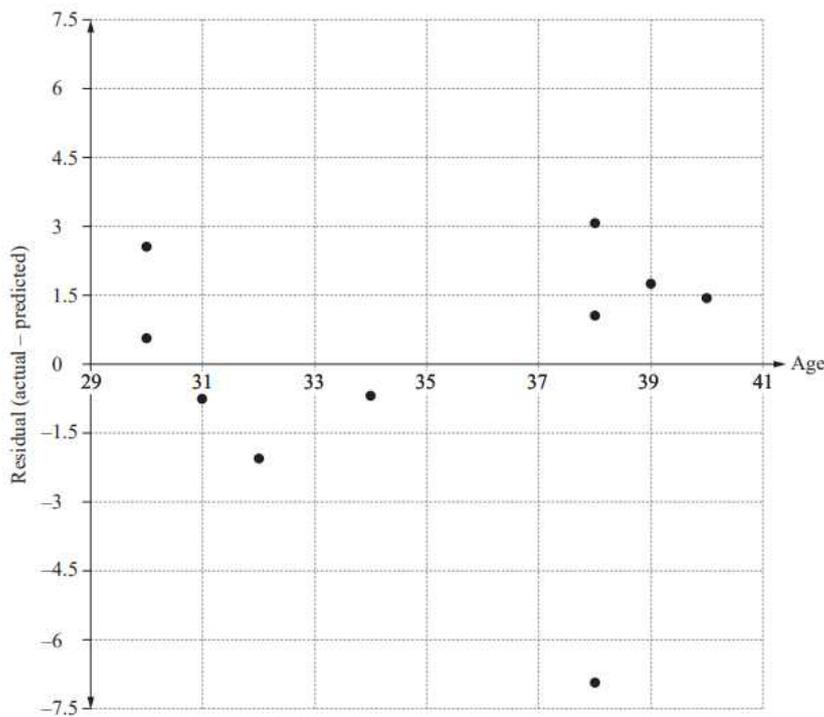
Show that the predicted profit, to the nearest dollar, for this company in the 11th year of operation will be \$15 121. (5 marks)

Sample Response	The response
$x$ parameters $x = 1, 2, \dots, 10$ $\bar{x} = 5.5$ $s_x = 3.02765$ Given $\bar{y} = 9660$ $s_y = 3010$ $r = 0.9987$	<ul style="list-style-type: none"> <li>correctly determines <math>\bar{x}</math> and <math>s_x</math> [1 mark]</li> </ul>
$b = r \frac{s_y}{s_x}$ $= 0.9987 \times \frac{3010}{3.02765}$ $= 992.878$	<ul style="list-style-type: none"> <li>determines <math>b</math> [1 mark]</li> </ul>
$a = \bar{y} - b\bar{x}$ $= 9660 - 992.878 \times 5.5$ $= 4199.17$	<ul style="list-style-type: none"> <li>determines <math>a</math> [1 mark]</li> </ul>
Profit in the 11th year $y = a + bx$ $= 4199.17 + 992.878(11)$ $= 15\,120.83$ $= \$15\,121$  Predicted profit in the 11th year is \$15 121.	<ul style="list-style-type: none"> <li>determines 11th year profit to the nearest dollar [1 mark]</li> <li>shows logical organisation communicating key steps [1 mark]</li> </ul>

**2021  
Paper 2  
Section 1  
Question 5**

**Bivariate  
data analysis**

Researchers gathered a set of data to determine if a model could reliably predict systolic blood pressure given a person's age. One candidate was 31 years old and had a systolic blood pressure of 119. For this data, the correlation coefficient ( $r$ ) is 0.875, the standard deviation for the person's age ( $s_x$ ) is 4, and the standard deviation for the systolic blood pressure ( $s_y$ ) is 6. A residual plot was produced for the model.



Determine the actual systolic blood pressure, to the nearest whole number, for the oldest person in the sample (40 years old). (6 marks)

Sample Response	The response
Predicted data @ $x = 31$ $y_A - y_P = -0.75$ $119 - y_P = -0.75$ $\therefore y_P = 119.75$	<ul style="list-style-type: none"> <li>correctly determines the <math>y_P</math> value [1 mark]</li> </ul>
Find $b$ $b = r \frac{s_y}{s_x}$ $= 0.875 \times 6.4$ $= 1.3125$	<ul style="list-style-type: none"> <li>correctly determines the <math>b</math> value [1 mark]</li> </ul>
Find $a$ $y = bx + a$ $119.75 = 1.3125 \times 31 + a$ $\therefore 79.0625 = a$	<ul style="list-style-type: none"> <li>determines <math>a</math> value [1 mark]</li> </ul>
Model: $y = 1.3125x + 79.0625$  Oldest patient @ $x = 40$ $y = 1.3125 \times 40 + 79.0625$ $= 131.5625$  Residual = 1.4 $y = 131.5625 + 1.4$ $y = 132.9625$	<ul style="list-style-type: none"> <li>determines predicted <math>y</math> value for oldest person [1 mark]</li> </ul>
The oldest person in the sample has a systolic blood pressure of 133.	<ul style="list-style-type: none"> <li>determines actual systolic blood pressure as a whole number [1 mark]</li> <li>shows logical organisation communicating key steps [1 mark]</li> </ul>

**2020  
Paper 2  
Section 1  
Question 2**

**Bivariate  
data analysis**

The number of people living in each household and the average daily household water usage, measured in litres (L), were recorded for 10 households.

<b>Number of people in each household</b>	6	2	4	5	5	4	3	1	6	7
<b>Average daily household water usage (L)</b>	990	160	320	480	410	280	240	130	940	1340

Calculate Pearson's correlation coefficient and then evaluate the appropriateness of using this coefficient for the association between daily water usage and the number of people living in a household. (5 marks)

Sample Response	The response
	<ul style="list-style-type: none"> <li>• correctly constructs a scatterplot [1 mark]</li> </ul>
From the calculator $r = 0.886$	<ul style="list-style-type: none"> <li>• correctly determines the correlation coefficient [1 mark]</li> </ul>
A correlation coefficient of 0.886 indicates that the relationship is a very strong positive relationship.	<ul style="list-style-type: none"> <li>• interprets the value of the correlation coefficient [1 mark]</li> </ul>
However, this relationship as shown in the scatterplot does not appear to be linear, therefore the correlation coefficient should not be used.	<ul style="list-style-type: none"> <li>• correctly identifies that the scatterplot is not linear [1 mark]</li> <li>correctly identifies that the correlation coefficient should not be used [1 mark]</li> </ul>

**2020  
Paper 2  
Section 1  
Question 3**

**Bivariate  
data analysis**

The least-squares line for a sample of five data points was found to be  $y = 2.1875x + 0.0625$ , with a correlation coefficient of  $r = 0.875$ .

Determine a set of values for  $p$  and  $q$ , given that these values differ by 3. (6 marks)

$x$	4	3	8	4	6
$y$	$p$	4	16	8	$q$

Sample Response	The response
$y = 2.1875x + 0.0625$ $\therefore b = 2.1875$ $a = 0.0625$	<ul style="list-style-type: none"> <li>correctly identifies the <math>a</math> and <math>b</math> values [1 mark]</li> </ul>
From the table of values $\bar{x} = 5$	<ul style="list-style-type: none"> <li>correctly determines <math>\bar{x}</math> [1 mark]</li> </ul>
Using $a$ $a = \bar{y} - b\bar{x}$ $0.0625 = \bar{y} - 2.1875 \times 5$ $\therefore \bar{y} = 11$	<ul style="list-style-type: none"> <li>determines <math>\bar{y}</math> [1 mark]</li> </ul>
$\bar{y} = \frac{\sum y}{n}$ $\therefore 11 = \frac{4 + 8 + p + q + 16}{5}$ $\therefore 55 = 28 + p + q$ $\therefore p + q = 27$	<ul style="list-style-type: none"> <li>determines sum of missing values [1 mark]</li> </ul>
If $q = p + 3$ then $p + p + 3 = 27$ $\therefore 2p = 24$ $\therefore p = 12$ $\therefore q = 15$	<ul style="list-style-type: none"> <li>determines values for <math>p</math> and <math>q</math> [1 mark]</li> <li>shows logical organisation, communicating key steps [1 mark]</li> </ul>

## Unit 3 – Topic 2: Time series analysis

### Paper 1 Section 1

**2023**  
**Paper 1**  
**Section 1**  
**Question 2**

**Time series analysis**

A time series plot is shown.

Time (years)

It could best be described as

(A) cyclical.  
(B) seasonal.  
(C) irregular.  
(D) increasing.

**2023**  
**Paper 1**  
**Section 1**  
**Question 10**

**Time series analysis**

Annual sales data and related quarterly indices are shown. The quarterly indices were calculated by applying the average percentage method using the mean.

Quarter	Q1	Q2	Q3	Q4
Sales	160	$x$	128	200
Index	1.0	0.95	$y$	1.25

Determine the values for  $x$  and  $y$ .

	$x$	$y$
(A)	122	0.8
(B)	122	3.2
(C)	152	0.8
(D)	152	3.2

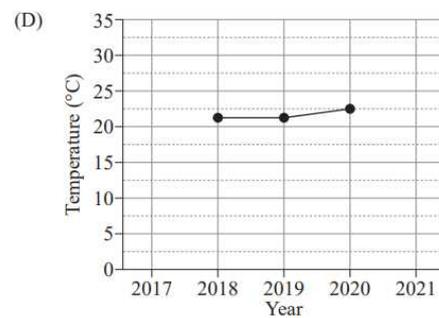
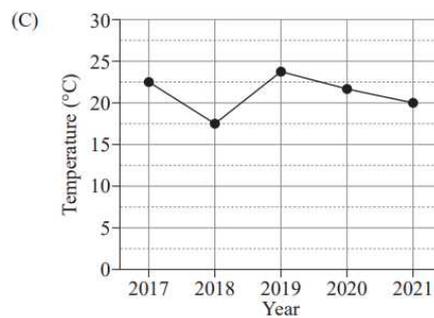
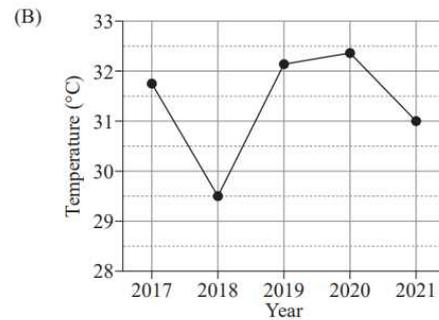
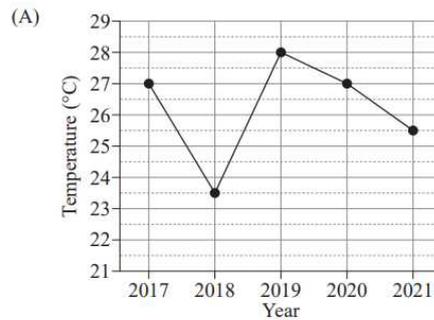
**2022  
Paper 1  
Section 1  
Question 3**

**Time series  
analysis**

The table shows the minimum and maximum temperatures on January 1 each year in Bundaberg.

	Min (°C)	Max (°C)
2017	22.1	31.8
2018	17.8	29.6
2019	24.1	32.1
2020	22.1	32.3
2021	19.9	30.9

Which time series plot best represents the mean temperatures?



**2022  
Paper 1  
Section 1  
Question 5**

**Time series  
analysis**

The table lists the number of books sold per month by an online bookstore. If the simple 3-point moving average in October is 54, what is the simple 3-point moving average in May?

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
45	52	68	65	89	65	53	33	40	45	77	92

- (A) 69
- (B) 73
- (C) 74
- (D) 89

**2021  
Paper 1  
Section 1  
Question 1**

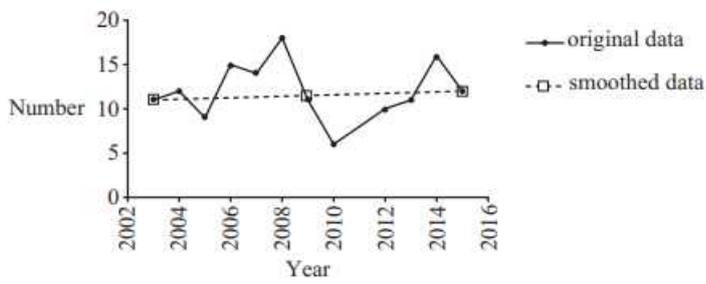
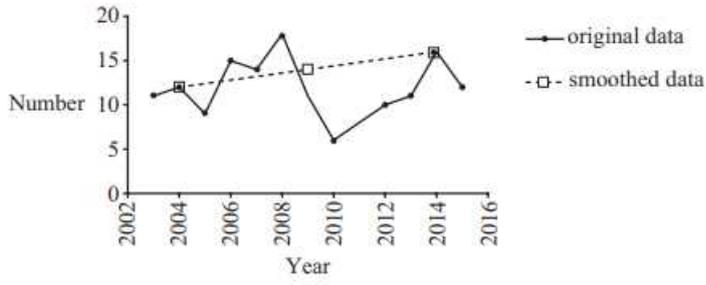
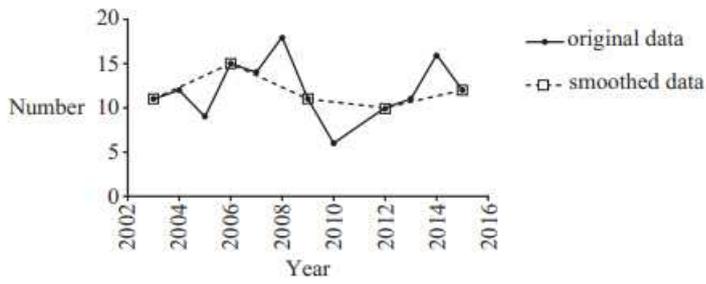
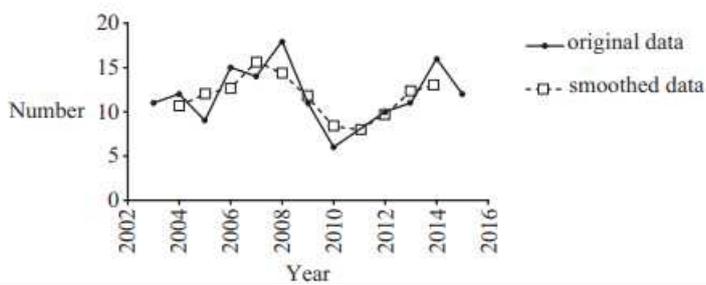
**Time series  
analysis**

The second smoothed value for the 3-point moving average is

Day	1	2	3	4	5	6	7
Value	5	10	18	32	52	70	90

- (A) 32
- (B) 25
- (C) 20
- (D) 18

<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 5</b></p> <p><b>Time series analysis</b></p>	<p>Determine the equation of the least-squares line where <math>r = 0.926</math>, <math>\bar{x} = 5.2</math>, <math>s_x = 2.3</math>, <math>\bar{y} = 68.6</math> and <math>s_y = 41.7</math>.</p> <p>(A) <math>y = 16.79x - 1146.51</math>          (B) <math>y = 16.79x - 18.70</math>          (C) <math>y = 0.05x - 68.33</math>          (D) <math>y = 0.05x - 1.70</math></p>
---	---

<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 10</b></p> <p><b>Time series analysis</b></p>	<p>Which of the following graphs represents a time series plot with a three-point moving average?</p> <p>(A) </p> <p>(B) </p> <p>(C) </p> <p>(D) </p>
--	--

**2022  
Paper 1  
Section 2  
Question 16**  
  
**Time series  
analysis**

The table shows the number of sales for a small business in their first six months of trading.

Time in months, $t$	Number of sales, $n$
1	86
2	180
3	160
4	226
5	240
6	335

a) Use your calculator to determine the equation of the least-squares line. [1 mark]

---



---

b) Use the equation from Question 16a) to predict the number of sales in the 21st month. [2 marks]

---



---



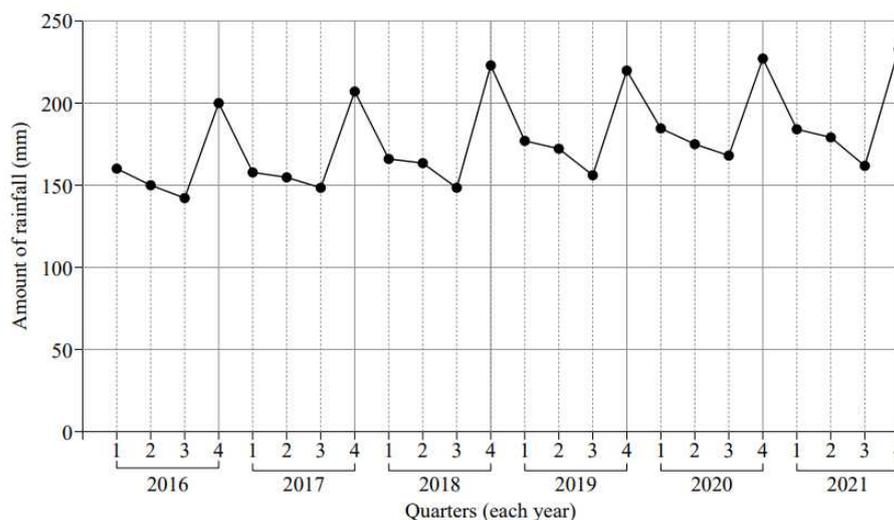
---



---

**2022  
Paper 1  
Section 2  
Question 19**  
  
**Time series  
analysis**

The graph shows the amount of rainfall (in mm) for each quarter from 2016 to 2021.



a) Describe the long-term trend and seasonality of the time series data. [2 marks]

---



---



---



---

b) A least-squares line was fitted to the data, with  $y$  representing the amount of rainfall and  $x$  representing the number of quarters since the beginning of 2016 (e.g.  $x = 5$  for the first quarter of 2017).

$$y = 1.763x + 156.5$$

Interpret the  $y$ -intercept and slope of the fitted line. [2 marks]

---

---

---

---

---

---

---

**2021  
Paper 1  
Section 2  
Question 18**  
  
**Time series  
analysis**

The table shows the profit made each year (in thousands of dollars) by a small business.

Year	Profit (\$'000s)
2015	42.1
2016	36.9
2017	48.4
2018	52.3
2019	56.1
2020	59.8

a) Use a mathematical model to determine the equation of the least-squares line to fit this data. [2 marks]

---

---

---

---

---

---

---

b) Use the least-squares line to forecast the profit in 2021, to the nearest hundred dollars. [2 marks]

---

---

---

---

---

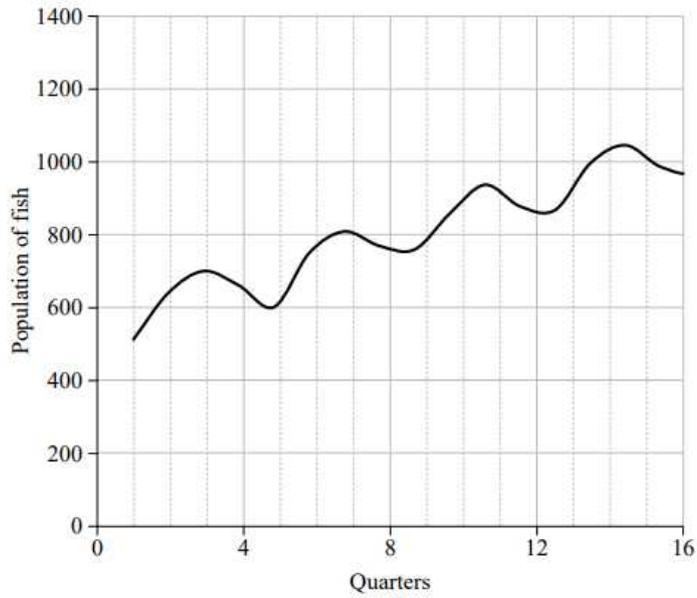
---

---

**2021  
Paper 1  
Section 2  
Question 24**

**Time series  
analysis**

The population of fish in a fish farm pond was recorded every three months. The graph shows the data..



Describe the time series plot by identifying three key features. (3 marks)

---

---

---

---

---

---

---

---

---

---



**2022**  
**Paper 2**  
**Section 1**  
**Question 1**

**Time series analysis**

The table shows a swimwear company's seasonally adjusted swimsuit sales (in thousands).

	Season			
	Spring	Summer	Autumn	Winter
Seasonally adjusted swimsuit sales (in thousands)	33.3	34.8	36.4	35.8

The long-term seasonal indices for spring, summer and winter are 1.11, 1.42 and 0.62 respectively. Determine the actual swimsuit sales for autumn. (4 marks)

---

---

---

---

---

---

---

---

---

---

---

---

**2020**  
**Paper 2**  
**Section 1**  
**Question 4**

**Time series analysis**

The following data shows the profits per quarter for a company for the last two years.

	Quarter	Profit (\$'000s)
<b>2018</b>	1	64
	2	98
	3	116
	4	122
<b>2019</b>	1	87
	2	156
	3	180
	4	177

Deseasonalise the data and plot this on the same set of axes as the original data in the graph on the next page. (5 marks)

---

---

---

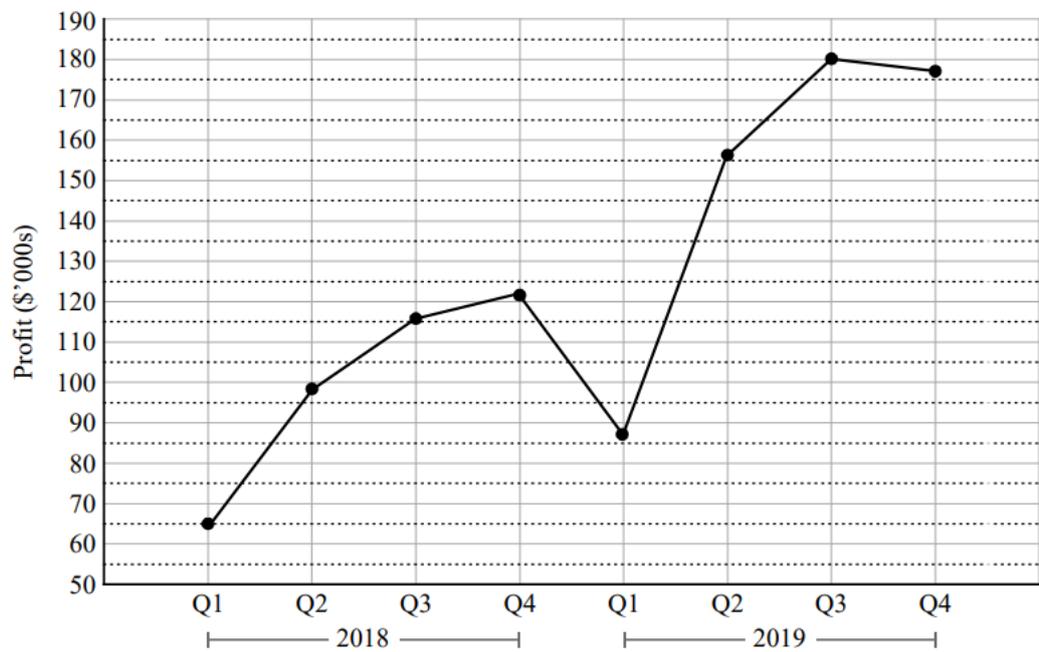
---

---

---

---

---



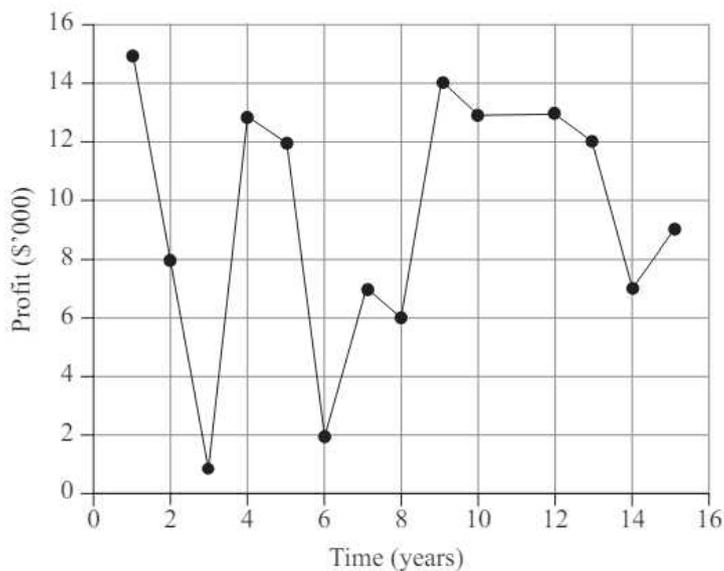
Note: If you make a mistake in the graph, cancel it by ruling a single diagonal line through your work and use the additional graph on page 18 of this question and response book.

Marking Guide – Paper 1 Section 1

2023  
Paper 1  
Section 1  
Question 2

Time series  
analysis

A time series plot is shown.



It could best be described as

- (A) cyclical.
- (B) seasonal.
- (C) irregular. – Answer**
- (D) increasing.

2023  
Paper 1  
Section 1  
Question 10

Time series  
analysis

Annual sales data and related quarterly indices are shown. The quarterly indices were calculated by applying the average percentage method using the mean.

Quarter	Q1	Q2	Q3	Q4
Sales	160	$x$	128	200
Index	1.0	0.95	$y$	1.25

Determine the values for  $x$  and  $y$ .

	$x$	$y$
(A)	122	0.8
(B)	122	3.2
(C)	152	0.8
(D)	152	3.2

Answer is C.

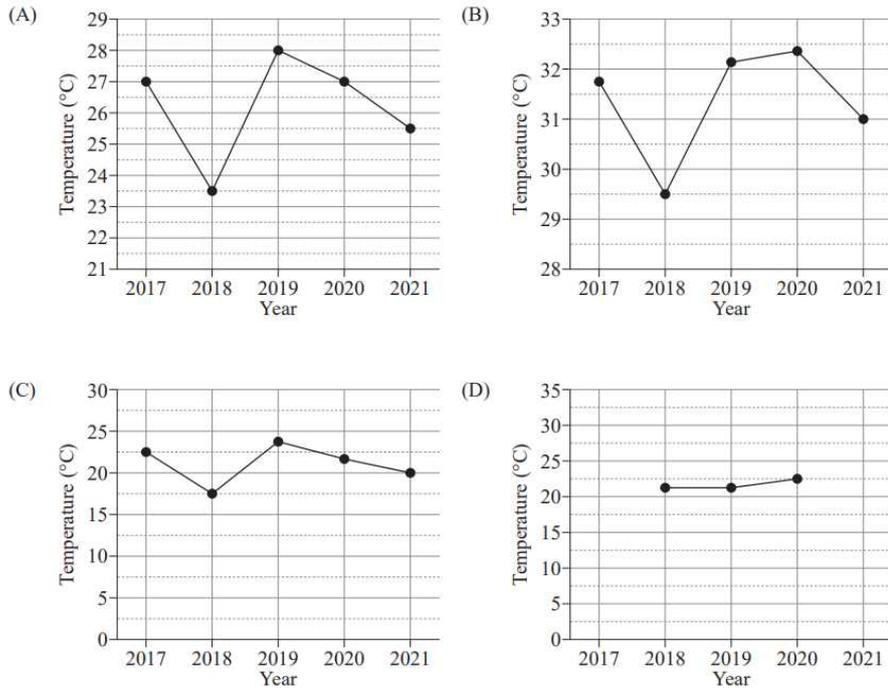
**2022**  
**Paper 1**  
**Section 1**  
**Question 3**

**Time series analysis**

The table shows the minimum and maximum temperatures on January 1 each year in Bundaberg.

	Min (°C)	Max (°C)
2017	22.1	31.8
2018	17.8	29.6
2019	24.1	32.1
2020	22.1	32.3
2021	19.9	30.9

Which time series plot best represents the mean temperatures?



**Answer is A.**

**2022**  
**Paper 1**  
**Section 1**  
**Question 5**

**Time series analysis**

The table lists the number of books sold per month by an online bookstore. If the simple 3-point moving average in October is 54, what is the simple 3-point moving average in May?

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
45	52	68	65	89	65	53	33	40	45	77	92

- (A) 69
- (B) 73 - Answer**
- (C) 74
- (D) 89

**2021**  
**Paper 1**  
**Section 1**  
**Question 1**

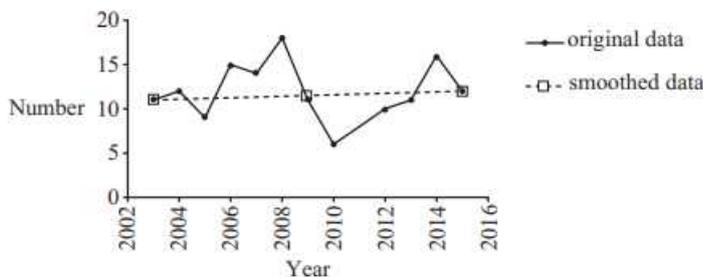
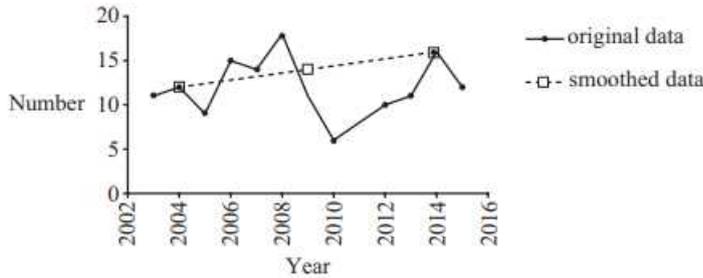
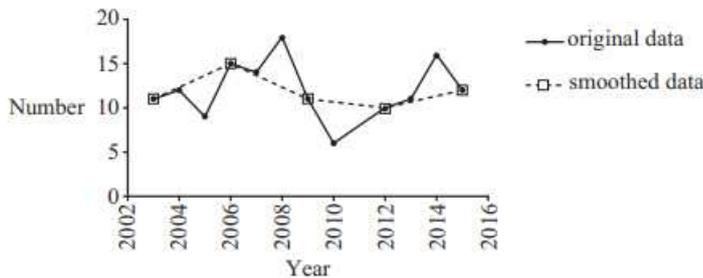
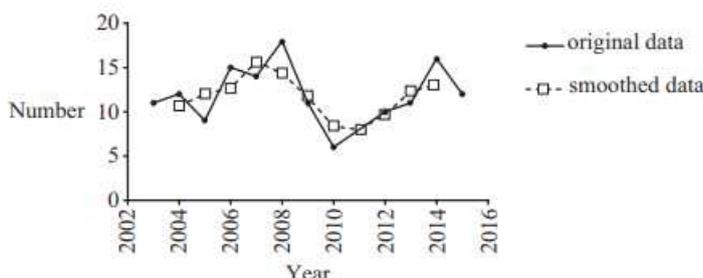
**Time series analysis**

The second smoothed value for the 3-point moving average is

Day	1	2	3	4	5	6	7
Value	5	10	18	32	52	70	90

- (A) 32
- (B) 25
- (C) 20 - Answer**
- (D) 18

<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 5</b></p> <p><b>Time series analysis</b></p>	<p>Determine the equation of the least-squares line where <math>r = 0.926</math>, <math>\bar{x} = 5.2</math>, <math>s_x = 2.3</math>, <math>\bar{y} = 68.6</math> and <math>s_y = 41.7</math>.</p> <p>(A) <math>y = 16.79x - 1146.51</math>  <b>(B) <math>y = 16.79x - 18.70</math> - Answer</b>  (C) <math>y = 0.05x - 68.33</math>  (D) <math>y = 0.05x - 1.70</math></p>
---	---

<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 10</b></p> <p><b>Time series analysis</b></p>	<p>Which of the following graphs represents a time series plot with a three-point moving average?</p> <p>(A) </p> <p>(B) </p> <p>(C) </p> <p>(D) </p> <p><b>Answer is D.</b></p>
--	---

2022  
Paper 1  
Section 2  
Question 16  
  
Time series  
analysis

The table shows the number of sales for a small business in their first six months of trading.

Time in months, $t$	Number of sales, $n$
1	86
2	180
3	160
4	226
5	240
6	335

a) Use your calculator to determine the equation of the least-squares line. [1 mark]

Sample Response	The response
$n = 42.6t + 55.4$	<ul style="list-style-type: none"> <li>correctly determines the equation of the least squares line [1 mark]</li> </ul>

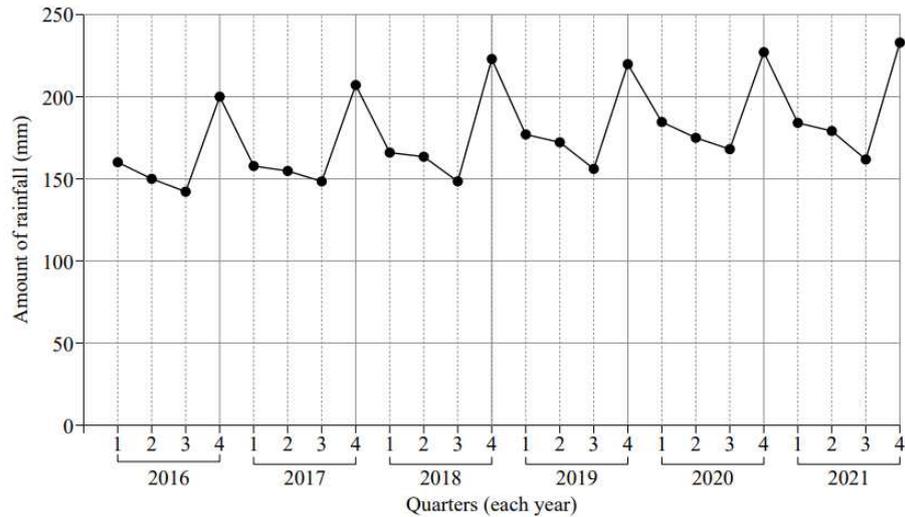
b) Use the equation from Question 16a) to predict the number of sales in the 21st month. [2 marks]

Sample Response	The response
Let $t = 21$ $n = 42.6(21) + 55.4$ $= 950$	<ul style="list-style-type: none"> <li>substitutes into equation from Question 16a) [1 mark]</li> </ul>
The predicted number of sales is 950.	<ul style="list-style-type: none"> <li>predicts number of sales [1 mark]</li> </ul>

**2022  
Paper 1  
Section 2  
Question 19**

**Time series  
analysis**

The graph shows the amount of rainfall (in mm) for each quarter from 2016 to 2021.



a) Describe the long-term trend and seasonality of the time series data. [2 marks]

Sample Response	The response
Trend — long term is positive because the amount of rainfall generally increases as time increases.	• appropriately describes the long-term trend [1 mark]
Seasonality — The data is seasonal with a high 4th quarter every year.	• appropriately describes the seasonality [1 mark]

b) A least-squares line was fitted to the data, with  $y$  representing the amount of rainfall and  $x$  representing the number of quarters since the beginning of 2016 (e.g.  $x = 5$  for the first quarter of 2017).

$$y = 1.763x + 156.5$$

Interpret the  $y$ -intercept and slope of the fitted line. [2 marks]

Sample Response	The response
$y$ -intercept — The model predicts that 156.5 mm of rainfall was falling in the 4th quarter of 2015.	• appropriately interprets the $y$ -intercept [1 mark]
Slope — On average an additional 1.763 mm of rainfall was precipitated each quarter	• appropriately interprets the slope [1 mark]

**2021  
Paper 1  
Section 2  
Question 18**

**Time series  
analysis**

The table shows the profit made each year (in thousands of dollars) by a small business.

Year	Profit (\$'000s)
2015	42.1
2016	36.9
2017	48.4
2018	52.3
2019	56.1
2020	59.8

a) Use a mathematical model to determine the equation of the least-squares line to fit this data. [2 marks]

Sample Response	The response
Let $x$ = the number of years since 2014 Let $y$ = the business's annual profit (in \$'000s)	• correctly defines the variables [1 mark]
$y = 4.286x + 34.267$	• correctly determines the equation of the least-squares line [1 mark]

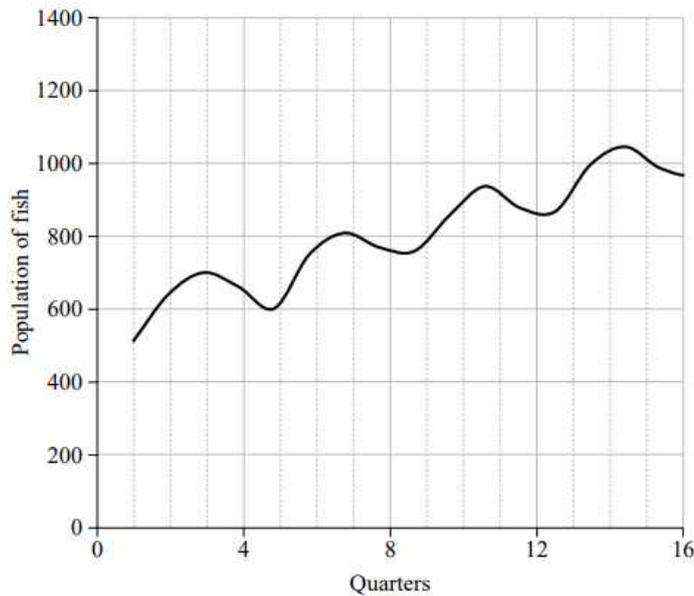
b) Use the least-squares line to forecast the profit in 2021, to the nearest hundred dollars. [2 marks]

Sample Response	The response
For 2021, $x = 7$ $\therefore y = 4.286 \times 7 + 34.267$ $= 64.269$	• correctly determines the $x$ value [1 mark]
The business will make \$64 300.	• determines profit [1 mark]

**2021  
Paper 1  
Section 2  
Question 24**

**Time series  
analysis**

The population of fish in a fish farm pond was recorded every three months. The graph shows the data..



Describe the time series plot by identifying three key features.

Sample Response	The response
1. Non-linear form	• correctly identifies the non-linear form [1 mark]
2. Seasonal cycle every 12 months	• correctly identifies a seasonal pattern [1 mark]
3. Positive long-term trend	• correctly identifies a positive long-term trend [1 mark]

Marking Guide – Paper 2 Section 1

**2023**  
**Paper 2**  
**Section 1**  
**Question 2**

**Time series analysis**

Buffalo fly bites cause skin wounds on cattle. The table shows the average number of skin wounds per animal in a herd for two years.

	Autumn	Winter	Spring	Summer
2021	285	28	195	460
2022	276	22	170	392

Deseasonalise the data. (4 marks)

Sample response							The response
Year	Season	Number of skin wounds	Yearly average	Number Yearly average	Seasonal indices	Deseasonalised number	<ul style="list-style-type: none"> <li>correctly determines the yearly averages [1 mark]</li> <li>determines number/yearly average values [1 mark]</li> <li>determines seasonal indices [1 mark]</li> <li>determines deseasonalised numbers [1 mark]</li> </ul>
2021	Autumn	285	242	1.1776 ...	1.2307...	232	
	Winter	28		0.1157 ...	0.1090...	257	
	Spring	195		0.8057 ...	0.7982...	244	
	Summer	460		1.9008 ...	1.8620...	247	
2022	Autumn	276	215	1.2837 ...	1.2307...	224	
	Winter	22		0.1023 ...	0.1090...	202	
	Spring	170		0.7906 ...	0.7982...	213	
	Summer	392		1.8232 ...	1.8620...	211	

**2022**  
**Paper 2**  
**Section 1**  
**Question 1**

**Time series analysis**

The table shows a swimwear company's seasonally adjusted swimsuit sales (in thousands).

	Season			
	Spring	Summer	Autumn	Winter
Seasonally adjusted swimsuit sales (in thousands)	33.3	34.8	36.4	35.8

The long-term seasonal indices for spring, summer and winter are 1.11, 1.42 and 0.62 respectively. Determine the actual swimsuit sales for autumn. (4 marks)

Sample Response	The response
Let $x$ = autumn's seasonal index	<ul style="list-style-type: none"> <li>correctly identifies the sum of all the seasonal indices [1 mark]</li> </ul>
Total of seasonal indices: $1.11 + 1.42 + 0.62 + x = 4$	
$\therefore x = 0.85$	<ul style="list-style-type: none"> <li>correctly determines autumn's seasonal index [1 mark]</li> </ul>
Actual value for autumn  actual value = deseasonalised value $\times$ seasonal index $= 36.4 \times 0.85$ $= 30.94$	
In autumn they had actual sales of 30 940 swimsuits.	<ul style="list-style-type: none"> <li>determines actual sales for autumn [1 mark]</li> </ul>

**2020**  
**Paper 2**  
**Section 1**  
**Question 4**

**Time series analysis**

The following data shows the profits per quarter for a company for the last two years.

	Quarter	Profit (\$'000s)
2018	1	64
	2	98
	3	116
	4	122
2019	1	87
	2	156
	3	180
	4	177

Deseasonalise the data and plot this on the same set of axes as the original data in the graph on the next page. (5 marks)

Sample Response							The response
Year	Quarter	Profit (in \$1000s)	Yearly average	Profit/ yearly average	Seasonal indices	Deseasonalised	<ul style="list-style-type: none"> <li>correctly determines the yearly averages [1 mark]</li> <li>determines profit/yearly average values [1 mark]</li> <li>determines seasonal indices [1 mark]</li> <li>determines deseasonalised values [1 mark]</li> </ul>
2018	1	64	100	0.64	0.61	104.92	
	2	98		0.98	1.01	97.03	
	3	116		1.16	1.18	98.31	
	4	122		1.22	1.2	101.67	
2019	1	87	150	0.58	0.61	142.62	
	2	156		1.04	1.01	154.46	
	3	180		1.2	1.18	152.54	
	4	177		1.18	1.2	147.5	

- accurately plots deseasonalised data on provided graph [1 mark]

## Unit 3 – Topic 3: Growth and decay in sequences

### Paper 1 Section 1

<b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 6</b>  <b>Growth and decay in sequences</b>	<p>In January 2022, 40 fish were released into a new dam that has the capacity to support 10 000 fish. It is predicted that the dam will reach its capacity in January 2030 if the fish population doubles every year.</p> <p>Which sequence rule models the prediction?</p> <p>(A) <math>t_n = t_1 r^{(n-1)}</math>, where <math>t_1 = 40</math>, <math>r = 2</math>, <math>n = 8</math></p> <p>(B) <math>t_n = t_1 r^{(n-1)}</math>, where <math>t_1 = 40</math>, <math>r = 2</math>, <math>n = 9</math></p> <p>(C) <math>t_n = t_1 + (n-1)d</math>, where <math>t_1 = 40</math>, <math>d = 2</math>, <math>n = 8</math></p> <p>(D) <math>t_n = t_1 + (n-1)d</math>, where <math>t_1 = 40</math>, <math>d = 2</math>, <math>n = 9</math></p>								
<b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 9</b>  <b>Growth and decay in sequences</b>	<p>Determine the 6th term of the arithmetic sequence that begins 3, 9, ...</p> <p>(A) 21 (B) 33 (C) 45 (D) 729</p>								
<b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 15</b>  <b>Growth and decay in sequences</b>	<p>The resale value of a boat shows geometric decay.</p> <table border="1" data-bbox="496 1055 1283 1167"><tbody><tr><td>Years after purchase</td><td>0</td><td>1</td><td>2</td></tr><tr><td>Resale value (\$)</td><td>50 000</td><td>40 000</td><td>32 000</td></tr></tbody></table> <p>Determine the resale value four years after purchase.</p> <p>(A) \$16 000 (B) \$20 480 (C) \$22 000 (D) \$25 600</p>	Years after purchase	0	1	2	Resale value (\$)	50 000	40 000	32 000
Years after purchase	0	1	2						
Resale value (\$)	50 000	40 000	32 000						
<b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 9</b>  <b>Growth and decay in sequences</b>	<p>A staircase is to be extended by installing <math>n</math> additional stairs. Each stair is 25 cm high. If the existing staircase reaches 1.2 m off the ground, which rule models the total height the stairs will reach in centimetres?</p> <p>(A) <math>t_n = 25 + (n-1) \times 1.45</math></p> <p>(B) <math>t_n = 1.2 + (n-1) \times 25</math></p> <p>(C) <math>t_n = 145 + (n-1) \times 25</math></p> <p>(D) <math>t_n = 25 + (n-1) \times 120</math></p>								
<b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 14</b>  <b>Growth and decay in sequences</b>	<p>A town's current population of 15 480 is predicted to grow steadily at an annual rate of 12%. The predicted population after 10 years is approximately</p> <p>(A) 48 079 (B) 34 056 (C) 18 576 (D) 17 338</p>								

<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 3</b></p> <p><b>Growth and decay in sequences</b></p>	<p>For the sequence 4, 2, 0, -2, -4 ... the common difference is</p> <p>(A) 4 (B) 2 (C) -2 (D) -4</p>
--	---

<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 4</b></p> <p><b>Growth and decay in sequences</b></p>	<p>Which of the following graphs could be modelled using a geometric sequence?</p> <div style="display: flex; flex-wrap: wrap;"> <div style="width: 50%;"> <p>(A)</p> </div> <div style="width: 50%;"> <p>(B)</p> </div> <div style="width: 50%;"> <p>(C)</p> </div> <div style="width: 50%;"> <p>(D)</p> </div> </div>
--	---













**Marking Guide – Paper 1 Section 1**

**2023  
Paper 1  
Section 1  
Question 6**

**Growth and decay in sequences**

In January 2022, 40 fish were released into a new dam that has the capacity to support 10 000 fish. It is predicted that the dam will reach its capacity in January 2030 if the fish population doubles every year.

Which sequence rule models the prediction?

(A)  $t_n = t_1 r^{(n-1)}$ , where  $t_1 = 40$ ,  $r = 2$ ,  $n = 8$

(B)  $t_n = t_1 r^{(n-1)}$ , where  $t_1 = 40$ ,  $r = 2$ ,  $n = 9$

(C)  $t_n = t_1 + (n-1)d$ , where  $t_1 = 40$ ,  $d = 2$ ,  $n = 8$

(D)  $t_n = t_1 + (n-1)d$ , where  $t_1 = 40$ ,  $d = 2$ ,  $n = 9$

**Answer is B.**

**2023  
Paper 1  
Section 1  
Question 9**

**Growth and decay in sequences**

Determine the 6th term of the arithmetic sequence that begins 3, 9, ...

(A) 21  
(B) **33 – Answer**  
(C) 45  
(D) 729

**2023  
Paper 1  
Section 1  
Question 15**

**Growth and decay in sequences**

The resale value of a boat shows geometric decay.

<b>Years after purchase</b>	0	1	2
<b>Resale value (\$)</b>	50 000	40 000	32 000

Determine the resale value four years after purchase.

(A) \$16 000  
(B) **\$20 480 – Answer**  
(C) \$22 000  
(D) \$25 600

**2021  
Paper 1  
Section 1  
Question 9**

**Growth and decay in sequences**

A staircase is to be extended by installing  $n$  additional stairs. Each stair is 25 cm high. If the existing staircase reaches 1.2 m off the ground, which rule models the total height the stairs will reach in centimetres?

(A)  $t_n = 25 + (n-1) \times 1.45$

(B)  $t_n = 1.2 + (n-1) \times 25$

(C)  $t_n = 145 + (n-1) \times 25$

(D)  $t_n = 25 + (n-1) \times 120$

**Answer is C.**

<p><b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 14</b></p> <p><b>Growth and decay in sequences</b></p>	<p>A town's current population of 15 480 is predicted to grow steadily at an annual rate of 12%. The predicted population after 10 years is approximately</p> <p>(A) 48 079 - Answer (B) 34 056 (C) 18 576 (D) 17 338</p>
---	---

<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 3</b></p> <p><b>Growth and decay in sequences</b></p>	<p>For the sequence 4, 2, 0, -2, -4 ... the common difference is</p> <p>(A) 4 (B) 2 (C) -2 - Answer (D) -4</p>
--	--

<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 4</b></p> <p><b>Growth and decay in sequences</b></p>	<p>Which of the following graphs could be modelled using a geometric sequence?</p> <div style="display: flex; flex-wrap: wrap;"> <div style="width: 50%;"> <p>(A) </p> </div> <div style="width: 50%;"> <p>(B) </p> </div> <div style="width: 50%;"> <p>(C) </p> </div> <div style="width: 50%;"> <p>(D) </p> </div> </div> <p><b>Answer is D.</b></p>
--	--

**2022**  
**Paper 1**  
**Section 2**  
**Question 18**

**Growth and decay in sequences**

The number of songs on a person’s playlist,  $n$ , in each week since joining a music streaming service,  $t$ , forms an arithmetic sequence, as shown by the graph.

Sample Response	The response
Arithmetic sequence $t_1 = 87$	• correctly determines $t_1$ [1 mark]
$d = t_2 - t_1$ $= 209 - 87$ $= 122$	• correctly determines $d$ [1 mark]
$t_n = t_1 + (n - 1)d$ $\therefore t_n = 87 + 122(n - 1)$	• uses an arithmetic sequence [1 mark]
At 25 weeks, $n = 25$ $t_{25} = 87 + 122 \times 24$ $t_{25} = 3015$	• predicts number of songs at 25 weeks [1 mark]

Use the arithmetic sequence to predict the number of songs on this person’s playlist 25 weeks after joining the streaming service. (4 marks)

**2021**  
**Paper 1**  
**Section 2**  
**Question 20**

**Growth and decay in sequences**

A farmer bought a tractor for \$45100 at the start of 2012. It depreciates by \$2700 each year.

Identify and use a mathematical model to determine the value of the tractor at the start of 2021. (4 marks)

Sample Response	The response
<b>Option 1: Arithmetic sequence</b> $t_1 = 45\ 100$ $d = -2700$ $n = 10$ $t_n = ?$	• correctly identifies the model [1 mark] • correctly identifies the parameters $t_1$ , $d$ and $n$ [1 mark]
$t_n = t_1 + (n - 1)d$ $\therefore t_n = 45\ 100 - 2700(10 - 1)$ $\therefore = 20\ 800$	• substitutes values into appropriate model [1 mark]
The tractor will be worth \$20 800.	• determines value of tractor, including units [1 mark]
<b>Option 2: Linear function</b> $c = 45\ 100$ $m = -2700$ $x = 9$	• correctly identifies the model [1 mark] • correctly identifies the parameters $c$ , $m$ and $x$ [1 mark]
$y = mx + c$ $\therefore y = -2700 \times 9 + 45\ 100$ $= 20\ 800$	• substitutes values into appropriate model [1 mark]
The tractor will be worth \$20 800.	• determines value of tractor, including units [1 mark]

<p><b>2020</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 18</b></p> <p><b>Growth and decay in sequences</b></p>	<p>Exhibition organisers believe that the number of attendees increases each day as an arithmetic sequence. The organisers know that 353 people attended the first day and 439 people attended the third day.</p>							
	<p>a) Determine the common difference. [2 marks]</p>							
	<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td>           Arithmetic sequence  <math>t_1 = 35</math>  <math>t_3 = 439</math>             Find <math>d</math>  <math>t_3 = t_1 + 2d</math>  <math>439 = 353 + 2d</math>  <math>86 = 2d</math> </td> <td> <ul style="list-style-type: none"> <li>correctly provides mathematical reasoning to support the answer [1 mark]</li> </ul> </td> </tr> <tr> <td> <math>43 = d</math> </td> <td> <ul style="list-style-type: none"> <li>correctly determines the common difference [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	Arithmetic sequence $t_1 = 35$ $t_3 = 439$  Find $d$ $t_3 = t_1 + 2d$ $439 = 353 + 2d$ $86 = 2d$	<ul style="list-style-type: none"> <li>correctly provides mathematical reasoning to support the answer [1 mark]</li> </ul>	$43 = d$	<ul style="list-style-type: none"> <li>correctly determines the common difference [1 mark]</li> </ul>	
	Sample Response	The response						
Arithmetic sequence $t_1 = 35$ $t_3 = 439$  Find $d$ $t_3 = t_1 + 2d$ $439 = 353 + 2d$ $86 = 2d$	<ul style="list-style-type: none"> <li>correctly provides mathematical reasoning to support the answer [1 mark]</li> </ul>							
$43 = d$	<ul style="list-style-type: none"> <li>correctly determines the common difference [1 mark]</li> </ul>							
<p>b) Use the result from 18a) to predict the number of people who will attend the sixth day. [2 marks]</p>								
	<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td>           Find <math>t_6</math>  <math>t_6 = t_1 + 5d</math>  <math>= 353 + 5 \times 43</math>  <math>= 568</math> </td> <td> <ul style="list-style-type: none"> <li>substitutes into an appropriate rule [1 mark]</li> </ul> </td> </tr> <tr> <td>           They would expect 568 people to attend the sixth day.         </td> <td> <ul style="list-style-type: none"> <li>determines value [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	Find $t_6$ $t_6 = t_1 + 5d$ $= 353 + 5 \times 43$ $= 568$	<ul style="list-style-type: none"> <li>substitutes into an appropriate rule [1 mark]</li> </ul>	They would expect 568 people to attend the sixth day.	<ul style="list-style-type: none"> <li>determines value [1 mark]</li> </ul>	
Sample Response	The response							
Find $t_6$ $t_6 = t_1 + 5d$ $= 353 + 5 \times 43$ $= 568$	<ul style="list-style-type: none"> <li>substitutes into an appropriate rule [1 mark]</li> </ul>							
They would expect 568 people to attend the sixth day.	<ul style="list-style-type: none"> <li>determines value [1 mark]</li> </ul>							

<p><b>2020</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 26</b></p> <p><b>Growth and decay in sequences</b></p>	<p>A scientist observed that the population of a specific bird species is decreasing by 17% each year and that at the beginning of 2016, there were 483 birds.</p>									
	<p>a) Use a geometric sequence to model the bird population. [2 marks]</p>									
	<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td>           Determine the common ratio  <math>r = 1 - 0.17</math>  <math>= 0.83</math> </td> <td> <ul style="list-style-type: none"> <li>correctly determines the common ratio [1 mark]</li> </ul> </td> </tr> <tr> <td>           Determine the model            let <math>n</math> = the number of years since 2015 and  <math>t_n</math> = the number of birds  <math>t_n = t_1 r^{(n-1)}</math>  <math>= 483 \times 0.83^{n-1}</math> </td> <td> <ul style="list-style-type: none"> <li>determines geometric model [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	Determine the common ratio $r = 1 - 0.17$ $= 0.83$	<ul style="list-style-type: none"> <li>correctly determines the common ratio [1 mark]</li> </ul>	Determine the model let $n$ = the number of years since 2015 and $t_n$ = the number of birds $t_n = t_1 r^{(n-1)}$ $= 483 \times 0.83^{n-1}$	<ul style="list-style-type: none"> <li>determines geometric model [1 mark]</li> </ul>			
	Sample Response	The response								
Determine the common ratio $r = 1 - 0.17$ $= 0.83$	<ul style="list-style-type: none"> <li>correctly determines the common ratio [1 mark]</li> </ul>									
Determine the model let $n$ = the number of years since 2015 and $t_n$ = the number of birds $t_n = t_1 r^{(n-1)}$ $= 483 \times 0.83^{n-1}$	<ul style="list-style-type: none"> <li>determines geometric model [1 mark]</li> </ul>									
<p>b) Using the model from 26a), predict the number of birds remaining at the beginning of 2021. [3 marks]</p>										
	<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td> <math>n = 6</math> </td> <td> <ul style="list-style-type: none"> <li>correctly determines the <math>n</math> value [1 mark]</li> </ul> </td> </tr> <tr> <td> <math>t_6 = 483 \times 0.83^5 = 190.255 \dots</math> </td> <td> <ul style="list-style-type: none"> <li>determines <math>t_6</math> [1 mark]</li> </ul> </td> </tr> <tr> <td>           Expect 190 birds remaining.         </td> <td> <ul style="list-style-type: none"> <li>states a reasonable answer [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	$n = 6$	<ul style="list-style-type: none"> <li>correctly determines the <math>n</math> value [1 mark]</li> </ul>	$t_6 = 483 \times 0.83^5 = 190.255 \dots$	<ul style="list-style-type: none"> <li>determines <math>t_6</math> [1 mark]</li> </ul>	Expect 190 birds remaining.	<ul style="list-style-type: none"> <li>states a reasonable answer [1 mark]</li> </ul>	
Sample Response	The response									
$n = 6$	<ul style="list-style-type: none"> <li>correctly determines the <math>n</math> value [1 mark]</li> </ul>									
$t_6 = 483 \times 0.83^5 = 190.255 \dots$	<ul style="list-style-type: none"> <li>determines <math>t_6</math> [1 mark]</li> </ul>									
Expect 190 birds remaining.	<ul style="list-style-type: none"> <li>states a reasonable answer [1 mark]</li> </ul>									

Marking Guide – Paper 2 Section 1

<p><b>2022 Paper 2 Section 1 Question 6</b></p> <p><b>Growth and decay in sequences</b></p>	<p>The first three lines in a pattern have the equations given. Their slopes form the terms of one sequence and their y-intercepts form the terms of another sequence. Each sequence is either arithmetic or geometric.</p> <p>Line 1: <math>y = -0.8x + 1.2</math>                  Line 2: <math>y = 0.4x + 2.7</math>                  Line 3: <math>y = -0.2x + 4.2</math></p> <p>Determine the coordinates of the point where Line 5 in the pattern intersects Line 1. (7 marks)</p>												
	<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td>                     Slope sequence  <math>-0.8, 0.4, -0.2, \dots</math>                      This forms a geometric sequence with <math>t_1 = -0.8</math> and <math>r = -0.5</math>.   <math>\therefore t_n = -0.8 \times (-0.5)^{(n-1)}</math> </td> <td> <ul style="list-style-type: none"> <li>correctly determines the geometric sequence parameters for the slopes [1 mark]</li> </ul> </td> </tr> <tr> <td>                     y-intercept sequence  <math>1.2, 2.7, 4.2, \dots</math>                      This forms an arithmetic sequence with <math>t_1 = 1.2</math> and <math>d = 1.5</math>.   <math>\therefore t_n = 1.2 + (n - 1) \times 1.5</math> </td> <td> <ul style="list-style-type: none"> <li>correctly determines the arithmetic sequence parameters for the y-intercepts [1 mark]</li> </ul> </td> </tr> <tr> <td>                     The equation for Line 5  <math>m = -0.8 \times (-0.5)^4</math>  <math>= -0.05</math> </td> <td> <ul style="list-style-type: none"> <li>determines slope for Line 5 [1 mark]</li> </ul> </td> </tr> <tr> <td> <math>c = 1.2 + 4 \times 1.5</math>  <math>= 7.2</math>   <math>\therefore y_5 = -0.05x + 7.2</math> </td> <td> <ul style="list-style-type: none"> <li>determines y-intercept for Line 5 [1 mark]</li> </ul> </td> </tr> <tr> <td>                     Solve simultaneously  <math>y_1 = y_5</math>  <math>\therefore -0.8x + 1.2 = -0.05x + 7.2</math>  <math>\therefore -0.75x = 6</math>  <math>\therefore x = -8</math>                      sub into <math>y_1</math>  <math>\therefore y = -0.8(-8) + 1.2</math>  <math>\therefore y = 7.6</math>                      The intersection point is <math>(-8, 7.6)</math>.                 </td> <td> <ul style="list-style-type: none"> <li>determines x-coordinate of intersection point [1 mark]</li> <li>determines y-coordinate of intersection point [1 mark]</li> <li>shows logical organisation communicating key steps [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	Slope sequence $-0.8, 0.4, -0.2, \dots$ This forms a geometric sequence with $t_1 = -0.8$ and $r = -0.5$ .  $\therefore t_n = -0.8 \times (-0.5)^{(n-1)}$	<ul style="list-style-type: none"> <li>correctly determines the geometric sequence parameters for the slopes [1 mark]</li> </ul>	y-intercept sequence $1.2, 2.7, 4.2, \dots$ This forms an arithmetic sequence with $t_1 = 1.2$ and $d = 1.5$ .  $\therefore t_n = 1.2 + (n - 1) \times 1.5$	<ul style="list-style-type: none"> <li>correctly determines the arithmetic sequence parameters for the y-intercepts [1 mark]</li> </ul>	The equation for Line 5 $m = -0.8 \times (-0.5)^4$ $= -0.05$	<ul style="list-style-type: none"> <li>determines slope for Line 5 [1 mark]</li> </ul>	$c = 1.2 + 4 \times 1.5$ $= 7.2$  $\therefore y_5 = -0.05x + 7.2$	<ul style="list-style-type: none"> <li>determines y-intercept for Line 5 [1 mark]</li> </ul>	Solve simultaneously $y_1 = y_5$ $\therefore -0.8x + 1.2 = -0.05x + 7.2$ $\therefore -0.75x = 6$ $\therefore x = -8$ sub into $y_1$ $\therefore y = -0.8(-8) + 1.2$ $\therefore y = 7.6$ The intersection point is $(-8, 7.6)$ .	<ul style="list-style-type: none"> <li>determines x-coordinate of intersection point [1 mark]</li> <li>determines y-coordinate of intersection point [1 mark]</li> <li>shows logical organisation communicating key steps [1 mark]</li> </ul>
	Sample Response	The response											
	Slope sequence $-0.8, 0.4, -0.2, \dots$ This forms a geometric sequence with $t_1 = -0.8$ and $r = -0.5$ .  $\therefore t_n = -0.8 \times (-0.5)^{(n-1)}$	<ul style="list-style-type: none"> <li>correctly determines the geometric sequence parameters for the slopes [1 mark]</li> </ul>											
	y-intercept sequence $1.2, 2.7, 4.2, \dots$ This forms an arithmetic sequence with $t_1 = 1.2$ and $d = 1.5$ .  $\therefore t_n = 1.2 + (n - 1) \times 1.5$	<ul style="list-style-type: none"> <li>correctly determines the arithmetic sequence parameters for the y-intercepts [1 mark]</li> </ul>											
	The equation for Line 5 $m = -0.8 \times (-0.5)^4$ $= -0.05$	<ul style="list-style-type: none"> <li>determines slope for Line 5 [1 mark]</li> </ul>											
$c = 1.2 + 4 \times 1.5$ $= 7.2$  $\therefore y_5 = -0.05x + 7.2$	<ul style="list-style-type: none"> <li>determines y-intercept for Line 5 [1 mark]</li> </ul>												
Solve simultaneously $y_1 = y_5$ $\therefore -0.8x + 1.2 = -0.05x + 7.2$ $\therefore -0.75x = 6$ $\therefore x = -8$ sub into $y_1$ $\therefore y = -0.8(-8) + 1.2$ $\therefore y = 7.6$ The intersection point is $(-8, 7.6)$ .	<ul style="list-style-type: none"> <li>determines x-coordinate of intersection point [1 mark]</li> <li>determines y-coordinate of intersection point [1 mark]</li> <li>shows logical organisation communicating key steps [1 mark]</li> </ul>												

**2021  
Paper 2  
Section 1  
Question 7**

**Growth and  
decay in  
sequences**

The table shows the total number of times a new song is played on a music service in the days following its first release.

Number of days since first release	5	10	15	20
Total number of times played ('000s)	8	12	18	27

The songwriter is paid 0.175 cents every time their song is played and will be paid after 60 days. They predict that by that time, they will be owed at least \$1000.

Given that the number of times the song is played is increasing exponentially, evaluate the reasonableness of this prediction. (6 marks)

Sample Response	The response
Let $n = \frac{\# \text{ of days}}{5}$ Let $t_n = \text{the total number of plays}$ $\therefore t_1 = 8$	• correctly defines the variables [1 mark]
$r = \frac{12}{8}$ $= 1.5$	• correctly determines the parameter $r$ [1 mark]
$\therefore t_n = 8 \times 1.5^{(n-1)}$ At 60 days $n = \frac{60}{5}$ $= 12$	• correctly determines a geometric (exponential) model [1 mark]
Total number of plays (in 1000s) $\therefore t_{12} = 8 \times 1.5^{11} = 691.98$	• determines total number of plays [1 mark]
Total predicted income Income = $0.175 \times 691\ 980$ $= 121\ 096.5$ cents $= \$1210.97$	• determines income [1 mark]
At least \$1000 is a reasonable prediction if plays continue as a geometric progression.	• evaluates reasonableness of solution [1 mark]

**2020  
Paper 2  
Section 1  
Question 1**

**Growth and  
decay in  
sequences**

A water tank contains 12 500 L of water. The tap is accidentally left on and the tank loses 135 L per minute. The tap is turned off when the tank has 5000 L of water left.

Use a mathematical model to determine how long the tap was left on to the nearest minute. (5 marks)

Sample Response	The response
<p><b>Option 1: Arithmetic sequence</b>  <math>n</math> = the number of minutes starting at 1  <math>t_n</math> = the amount of water in the tank</p>	<ul style="list-style-type: none"> <li>correctly defines the variables [1 mark]</li> </ul>
<p><math>t_1 = 12\,500</math>  <math>d = -135</math>  <math>t_n = 5000</math>  <math>n = ?</math></p>	<ul style="list-style-type: none"> <li>correctly identifies the parameters <math>t_1</math>, <math>d</math> and <math>tn</math> [1 mark]</li> </ul>
<p>Find <math>n</math>  <math>t_n = t_1 + (n - 1) d</math>  <math>\therefore 5000 = 12\,500 - 135(n - 1)</math>  <math>\therefore 135(n - 1) = 7500</math>  <math>\therefore n - 1 = 55.5556</math>  <math>\therefore n = 56.5556</math></p>	<ul style="list-style-type: none"> <li>substitutes values into appropriate model [1 mark]</li> <li>determines <math>n</math> value [1 mark]</li> </ul>
<p>The tap was left on until the 57th term.  The tap was left on for about 56 minutes.</p>	<ul style="list-style-type: none"> <li>states a reasonable answer rounded to the nearest minute [1 mark]</li> </ul>
<p><b>Option 2: Linear function</b>  <math>x</math> = the time that the tap has been on  <math>y</math> = the amount of water in the tank</p>	<ul style="list-style-type: none"> <li>correctly defines the variables [1 mark]</li> </ul>
<p><math>c = 12\,500</math>  <math>m = -135</math></p>	
<p><math>y = mx + c</math>  <math>\therefore y = -135x + 12\,500</math>  Find <math>x</math> when <math>y = 5000</math></p>	<ul style="list-style-type: none"> <li>correctly identifies the parameters <math>y</math>, <math>m</math> and <math>c</math> [1 mark]</li> </ul>
<p><math>\therefore 5000 = -135x + 12500</math>  <math>\therefore 135x = 7500</math></p>	<ul style="list-style-type: none"> <li>substitutes values into appropriate model [1 mark]</li> </ul>
<p><math>\therefore x = 55.5556</math></p>	<ul style="list-style-type: none"> <li>determines <math>x</math> value [1 mark]</li> </ul>
<p>The tap was left on for about 56 minutes.</p>	<ul style="list-style-type: none"> <li>states a reasonable answer rounded to the nearest minute [1 mark]</li> </ul>

## Unit 3 – Topic 4: Earth geometry and time zones

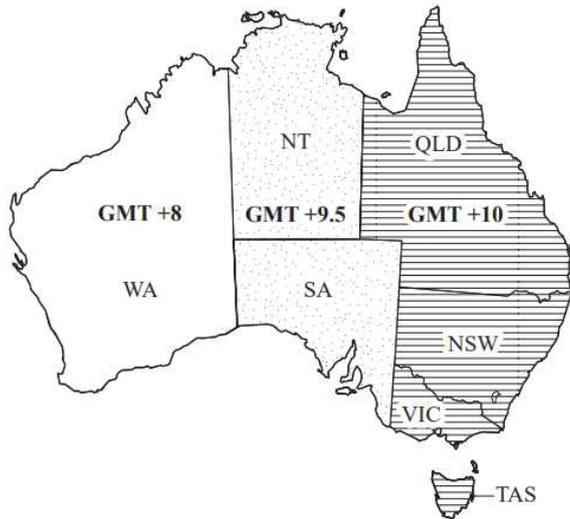
### Paper 1 Section 1

<b>2022</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 14</b>  <b>Earth</b> <b>geometry</b> <b>and time</b> <b>zones</b>	<p>A rugby fan in Perth (Australia) plans to watch a live match played in New Zealand next winter. The time zone for Perth is UTC +8. The time zone for New Zealand is UTC +13 in winter and UTC +12 in summer. If the match is played at 6:30 pm New Zealand time, what time will the match be viewed in Perth?</p> <p>(A) 1:30 pm (B) 2:30 pm (C) 10:30 pm (D) 11:30 pm</p>
<b>2022</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 15</b>  <b>Earth</b> <b>geometry</b> <b>and time</b> <b>zones</b>	<p>The actual distance between two cities has been correctly calculated as 556 km. The latitude and longitude respectively of these two cities could be</p> <p>(A) 2° N 104° W and 3° S 104° W. (B) 2° N 104° W and 3° N 104° W. (C) 25° N 150° E and 30° S 150° E. (D) 25° N 145° E and 25° N 150° E.</p>
<b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 10</b>  <b>Earth</b> <b>geometry</b> <b>and time</b> <b>zones</b>	<p>City A is located at 55° N, 120° E and City B is located at 40° N, 165° E. The sun will rise in City A approximately</p> <p>(A) 1 hour before it rises in City B. (B) 1 hour after it rises in City B. (C) 3 hours before it rises in City B. (D) 3 hours after it rises in City B.</p>

**2020  
Paper 1  
Section 1  
Question 2**

**Earth  
geometry  
and time  
zones**

The standard Australian time zones are shown on the map.



All states and territories, except Western Australia (WA), Queensland (QLD) and the Northern Territory (NT), have daylight saving in summer. Daylight saving time is 1 hour ahead of standard time.

When it is 10:00 am daylight saving time in New South Wales (NSW), it is

- (A) 9:30 am in South Australia (SA) and 9:00 am in QLD.
- (B) 9:30 am in SA and 11:00 am in QLD.
- (C) 10:30 am in SA and 9:00 am in QLD.
- (D) 10:30 am in SA and 11:00 am in QLD

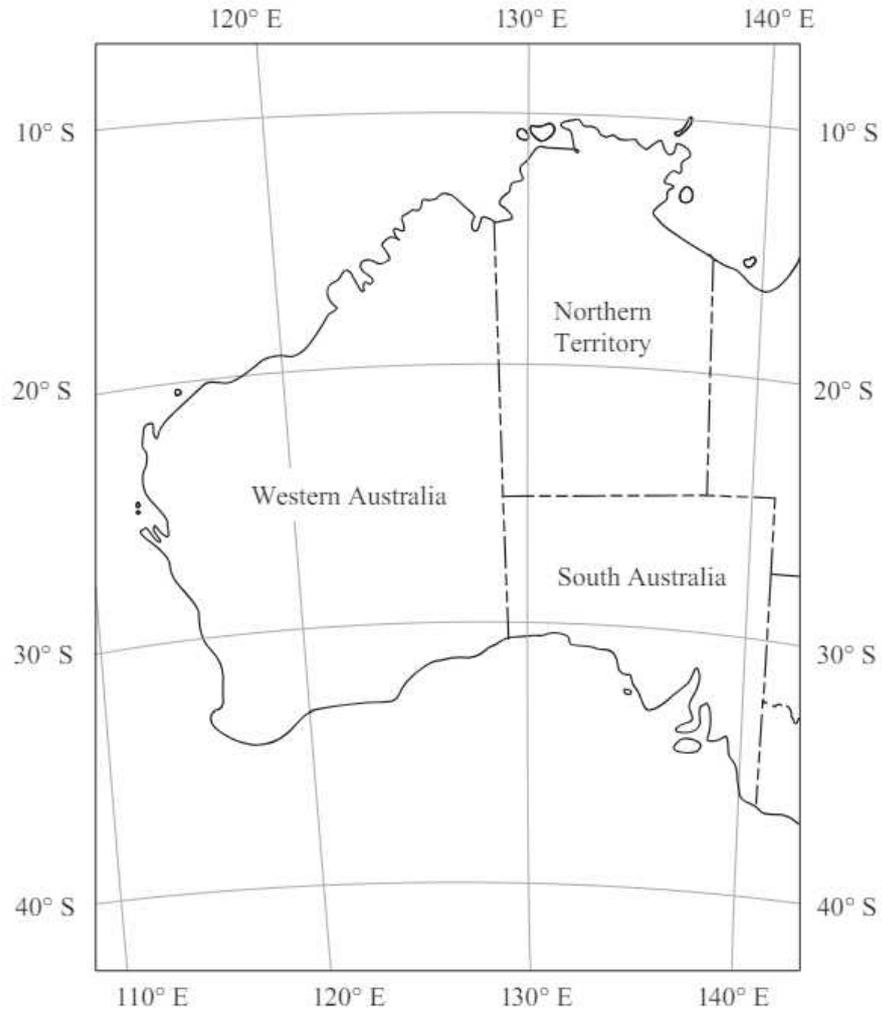


2023  
Paper 1  
Section 2  
Question 18

Earth  
geometry  
and time  
zones

The locations of three space research sites in Australia are listed in the table.

Site	Coordinates
A	32° S 116° E
B	32° S 136° E
C	12° S 136° E



a) Which site is closest to the equator? [1 mark]

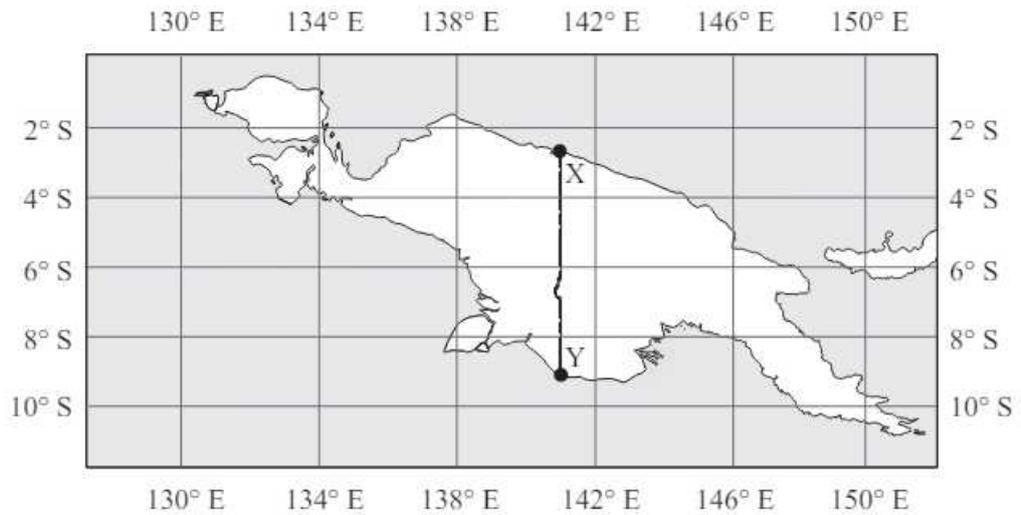
b) Name the state or territory in which site A is located. [1 mark]

c) Determine and explain which two sites are in the same standard time zone. [2 marks]

2023  
Paper 1  
Section 2  
Question 25

Earth  
geometry  
and time  
zones

The map shows a land border from X to Y.



a) State the latitude and longitude of X to the nearest degree. [1 mark]

---

b) State the latitude and longitude of Y to the nearest degree. [1 mark]

---

c) Calculate the distance between X and Y in kilometres. [3 marks]

---

---

---

---

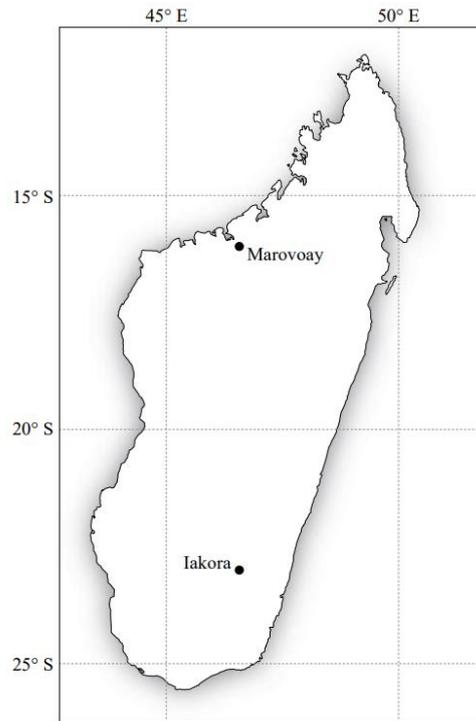
---

---

**2022  
Paper 1  
Section 2  
Question 22**

**Earth  
geometry  
and time  
zones**

Marovoay and Iakora are located on the same meridian at  $46.6^\circ$  E, as shown on the map of Madagascar.



a) Determine the latitudes of Marovoay and Iakora. [1 mark]

---

---

b) Use the result from Question 22a) to determine the shortest distance between Marovoay and Iakora. [3 marks]

---

---

---

---

---

---









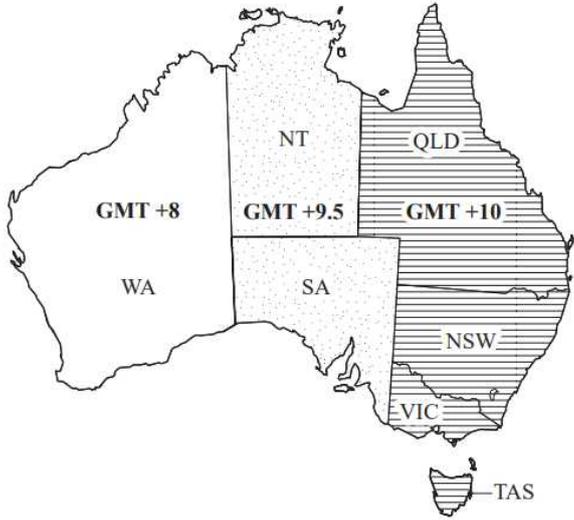


**Marking Guide – Paper 1 Section 1**

<p><b>2022</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 14</b></p> <p><b>Earth geometry and time zones</b></p>	<p>A rugby fan in Perth (Australia) plans to watch a live match played in New Zealand next winter. The time zone for Perth is UTC +8. The time zone for New Zealand is UTC +13 in winter and UTC +12 in summer. If the match is played at 6:30 pm New Zealand time, what time will the match be viewed in Perth?</p> <p><b>(A) 1:30 pm - Answer</b> (B) 2:30 pm (C) 10:30 pm (D) 11:30 pm</p>
---	---

<p><b>2022</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 15</b></p> <p><b>Earth geometry and time zones</b></p>	<p>The actual distance between two cities has been correctly calculated as 556 km. The latitude and longitude respectively of these two cities could be</p> <p><b>(A) 2° N 104° W and 3° S 104° W. - Answer</b> (B) 2° N 104° W and 3° N 104° W. (C) 25° N 150° E and 30° S 150° E. (D) 25° N 145° E and 25° N 150° E.</p>
---	--

<p><b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 10</b></p> <p><b>Earth geometry and time zones</b></p>	<p>City A is located at 55° N, 120° E and City B is located at 40° N, 165° E. The sun will rise in City A approximately</p> <p>(A) 1 hour before it rises in City B. (B) 1 hour after it rises in City B. (C) 3 hours before it rises in City B. <b>(D) 3 hours after it rises in City B. - Answer</b></p>
---	--

<p><b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 2</b></p> <p><b>Earth geometry and time zones</b></p>	<p>The standard Australian time zones are shown on the map.</p>  <p>All states and territories, except Western Australia (WA), Queensland (QLD) and the Northern Territory (NT), have daylight saving in summer. Daylight saving time is 1 hour ahead of standard time.</p> <p>When it is 10:00 am daylight saving time in New South Wales (NSW), it is</p> <p><b>(A) 9:30 am in South Australia (SA) and 9:00 am in QLD. - Answer</b> (B) 9:30 am in SA and 11:00 am in QLD. (C) 10:30 am in SA and 9:00 am in QLD. (D) 10:30 am in SA and 11:00 am in QLD</p>
--	--

Marking Guide – Paper 1 Section 2

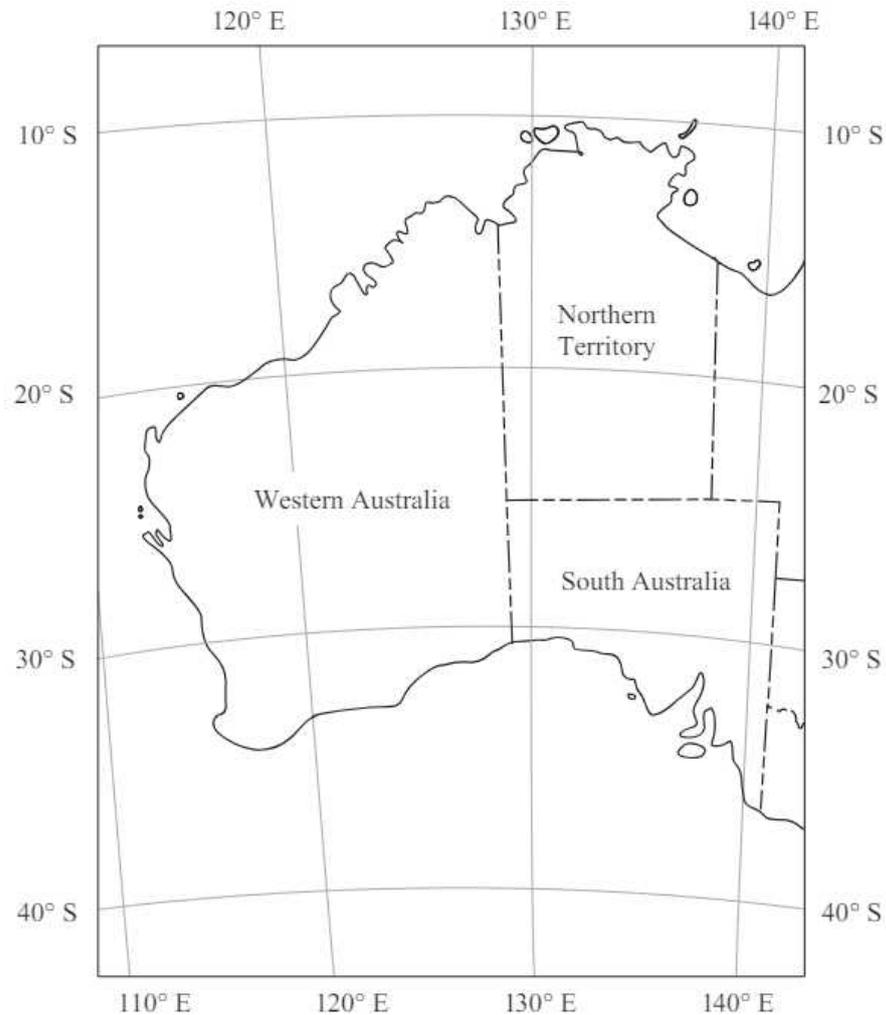
<p><b>2023</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 16</b></p> <p><b>Earth geometry and time zones</b></p>	<p>If it is 2:00 am local time in town A (30° N 90° W), calculate the local time in town B (26° S 120° E). (3 marks)</p>	
	<p><b>Sample response</b></p>	<p><b>The response</b></p>
<p><b>Method 1</b> Angular difference = <math>90^\circ + 120^\circ</math> = <math>210^\circ</math> Time difference = <math>\frac{210^\circ}{15^\circ / \text{h}}</math> = <b>14 hours</b> Town B is east of town A, so town B is 14 hours ahead of town A. Local time in town B = 2:00 am + 14 hours = <b>4:00 pm</b></p>		<ul style="list-style-type: none"> <li>• correctly determines the angular difference [1 mark]</li> <li>• determines absolute time difference between town A and town B [1 mark]</li> <li>• determines local time in town B [1 mark]</li> </ul>
<p><b>Method 2</b> Town B's longitude is east, so its time is ahead of UTC. Time difference = <math>\frac{120^\circ}{15^\circ / \text{h}}</math> = <b>8 hours (UTC<sup>+8</sup>)</b> Town A's longitude is west, so its time is behind UTC. Time difference = <math>\frac{90^\circ}{15^\circ / \text{h}}</math> = <b>6 hours (UTC<sup>-6</sup>)</b> Time difference = <math>+8 - -6 =</math> <b>14 hours</b> Town B is east of town A, so town B is 14 hours ahead of town A. Local time in town B = 2:00 am + 14 hours = <b>4:00 pm</b></p>		<ul style="list-style-type: none"> <li>• correctly determines the time difference for each of town A and town B compared to 0° [1 mark]</li> <li>• determines absolute time difference between town A and town B [1 mark]</li> <li>• determines local time in town B [1 mark]</li> </ul>

**2023  
Paper 1  
Section 2  
Question 18**

**Earth  
geometry  
and time  
zones**

The locations of three space research sites in Australia are listed in the table.

Site	Coordinates
A	32° S 116° E
B	32° S 136° E
C	12° S 136° E



a) Which site is closest to the equator? [1 mark]

Sample response	The response
Site C	<ul style="list-style-type: none"> <li>correctly names the site [1 mark]</li> </ul>

b) Name the state or territory in which site A is located. [1 mark]

Sample response	The response
Western Australia	<ul style="list-style-type: none"> <li>correctly names the state [1 mark]</li> </ul>

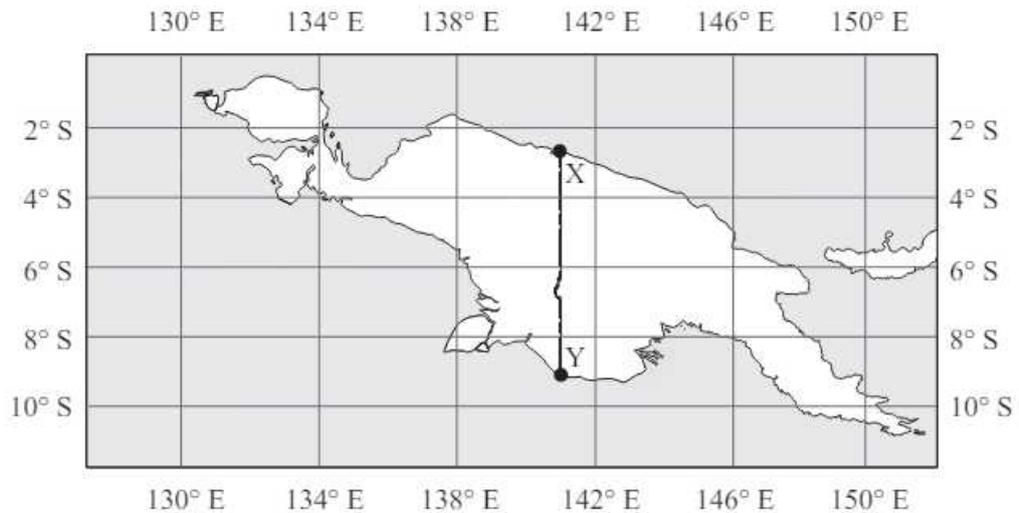
c) Determine and explain which two sites are in the same standard time zone. [2 marks]

Sample response	The response
Sites B and C are in the same standard time zone because they have the same longitude.	<ul style="list-style-type: none"> <li>correctly determines sites B and C are in the same standard time zone [1 mark]</li> <li>correctly explains using longitude [1 mark]</li> </ul>

2023  
Paper 1  
Section 2  
Question 25

Earth  
geometry  
and time  
zones

The map shows a land border from X to Y.



a) State the latitude and longitude of X to the nearest degree. [1 mark]

Sample response	The response
Latitude and longitude of X = 3° S 141° E	<ul style="list-style-type: none"> <li>correctly determines the latitude and longitude of X to the nearest degree [1 mark]</li> </ul>

b) State the latitude and longitude of Y to the nearest degree. [1 mark]

Sample response	The response
Latitude and longitude of Y = 9° S 141° E	<ul style="list-style-type: none"> <li>correctly determines the latitude and longitude of Y to the nearest degree [1 mark]</li> </ul>

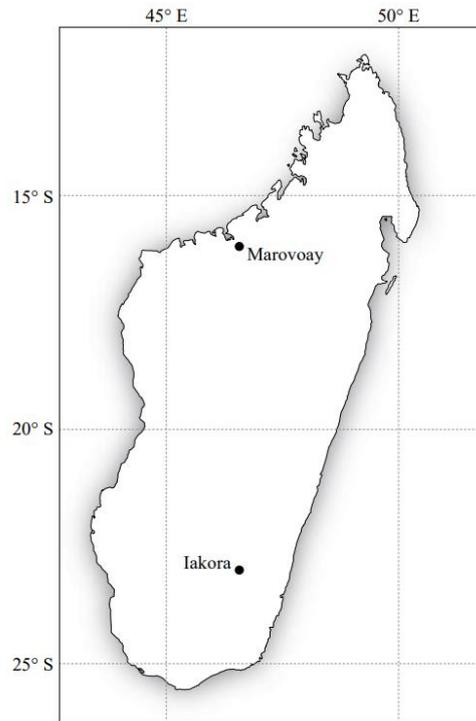
c) Calculate the distance between X and Y in kilometres. [3 marks]

Sample response	The response
$\text{Angular distance} = 9^\circ - 3^\circ$ $= 6^\circ$ $D = 111.2 \times \text{angular distance}$ $= 111.2 \times 6^\circ$ $\approx 667.2 \text{ km}$ The distance between X and Y is 667.2 km.	<ul style="list-style-type: none"> <li>determines angular distance [1 mark]</li> <li>substitutes into appropriate rule [1 mark]</li> <li>determines distance [1 mark]</li> </ul>

**2022  
Paper 1  
Section 2  
Question 22**

**Earth  
geometry  
and time  
zones**

Marovoay and Iakora are located on the same meridian at  $46.6^\circ$  E, as shown on the map of Madagascar.



a) Determine the latitudes of Marovoay and Iakora. [1 mark]

Sample Response	The response
Marovoay $16.1^\circ$ S $46.6^\circ$ E Iakora $23.1^\circ$ S $46.6^\circ$ E	• correctly determines the latitudes for both locations within $\pm 0.2^\circ$ [1 mark]

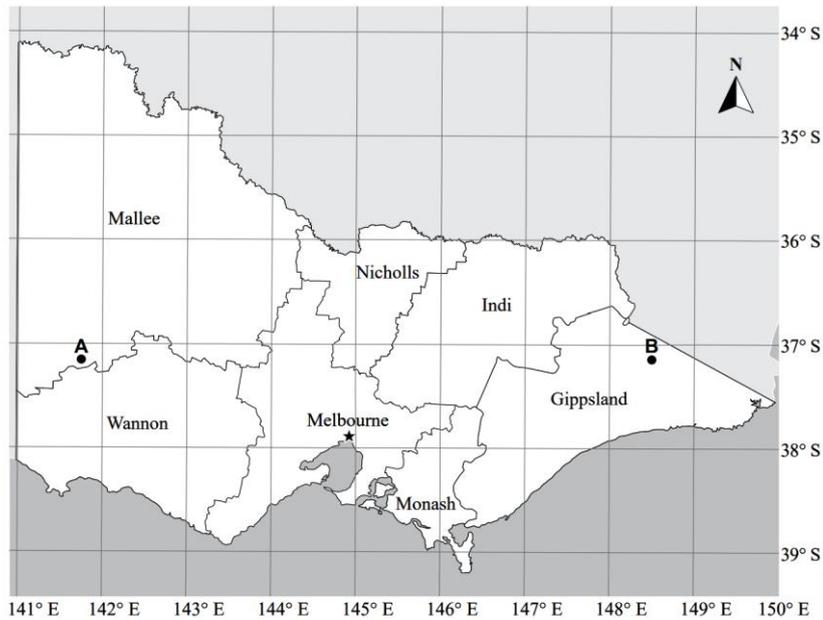
b) Use the result from Question 22a) to determine the shortest distance between Marovoay and Iakora. [3 marks]

Sample Response	The response
Angular distance = $23.1 - 16.1$ = 7	• determines angular distance [1 mark]
Distance = $111.2 \times$ angular distance = $111.2 \times 7$ = 778.4	• substitutes into appropriate distance formula [1 mark]
Marovoay is approximately 778 km north of Iakora.	• determines distance, including units [1 mark]

**2021  
Paper 1  
Section 2  
Question 21**

**Earth  
geometry  
and time  
zones**

The map shows regional federal electorates in Victoria.



a) In which federal electorate is the position  $37^{\circ} 10' S, 146^{\circ} 30' E$ ? [1 mark]

Sample Response	The response
Indi	• correctly identifies the federal electorate [1 mark]

b) Determine the distance between point A in the electorate of Mallee and point B in the electorate of Gippsland along the same parallel of latitude, to the nearest 100 km. [5 marks]

Sample Response	The response
Point A: $37.25^{\circ} S 141.75^{\circ} E$ Point B: $37.25^{\circ} S 148.5^{\circ} E$	• correctly identifies the latitude for A and B ( $37.1^{\circ} S$ to $37.3^{\circ} S$ ) [1 mark] • correctly identifies the longitudes for A ( $141.6^{\circ} E$ to $141.9^{\circ} E$ ) and B ( $148.3^{\circ} E$ to $148.7^{\circ} E$ ) [1 mark]
angular distance = $6.75^{\circ}$	• determines angular distance [1 mark]
Distance is E–W $D = 111.2 \times \cos\theta \times \text{angular distance}$ $= 111.2 \times \cos(37.25^{\circ}) \times 6.75^{\circ}$ $= 597.48$	• substitutes values into appropriate rule [1 mark]
The points are approximately 600 km apart.	• states answer rounded to the nearest 100 km [1 mark]

<b>2021</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 25</b>  <b>Earth geometry and time zones</b>	<p>A conference is being held in Singapore (UTC +8).</p> <p>a) A conference attendee got a flight from Brisbane (UTC +10) at 10:30 am Brisbane time on Monday 7 December.</p> <p>If the flight from Brisbane to Singapore took 7 hours and 40 minutes, determine the time, day and date in Singapore when the flight lands. [2 marks]</p>						
	<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td>           Depart Brisbane 10:30 Mon 7/12            Flight: + 7:40            Arrive Singapore 18:10            UTC correction –2:00            = 16:10         </td> <td> <ul style="list-style-type: none"> <li>correctly adds the flight time [1 mark]</li> </ul> </td> </tr> <tr> <td>4:10 pm in Singapore on Mon 7/12</td> <td> <ul style="list-style-type: none"> <li>correctly determines the local time, day and date in Singapore [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	Depart Brisbane 10:30 Mon 7/12 Flight: + 7:40 Arrive Singapore 18:10 UTC correction –2:00 = 16:10	<ul style="list-style-type: none"> <li>correctly adds the flight time [1 mark]</li> </ul>	4:10 pm in Singapore on Mon 7/12	<ul style="list-style-type: none"> <li>correctly determines the local time, day and date in Singapore [1 mark]</li> </ul>
	Sample Response	The response					
	Depart Brisbane 10:30 Mon 7/12 Flight: + 7:40 Arrive Singapore 18:10 UTC correction –2:00 = 16:10	<ul style="list-style-type: none"> <li>correctly adds the flight time [1 mark]</li> </ul>					
4:10 pm in Singapore on Mon 7/12	<ul style="list-style-type: none"> <li>correctly determines the local time, day and date in Singapore [1 mark]</li> </ul>						
<p>b) A conference attendee from Dubai (UTC +4) arrived in Singapore at 5 pm Singapore time on Monday 7 December.</p> <p>If the flight from Dubai to Singapore took 8 hours and 25 minutes, determine the time, day and date in Dubai when the flight departed. [2 marks]</p>							
<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td>           Arrive Singapore 17:00 Mon 7/12            Flight: – 8:25            Depart Dubai 8:35            UTC correction –4:00            = 4:35         </td> <td> <ul style="list-style-type: none"> <li>correctly subtracts the flight time [1 mark]</li> </ul> </td> </tr> <tr> <td>4:35 am in Dubai on Mon 7/12</td> <td> <ul style="list-style-type: none"> <li>correctly determines the local time, day and date in Dubai [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	Arrive Singapore 17:00 Mon 7/12 Flight: – 8:25 Depart Dubai 8:35 UTC correction –4:00 = 4:35	<ul style="list-style-type: none"> <li>correctly subtracts the flight time [1 mark]</li> </ul>	4:35 am in Dubai on Mon 7/12	<ul style="list-style-type: none"> <li>correctly determines the local time, day and date in Dubai [1 mark]</li> </ul>	
Sample Response	The response						
Arrive Singapore 17:00 Mon 7/12 Flight: – 8:25 Depart Dubai 8:35 UTC correction –4:00 = 4:35	<ul style="list-style-type: none"> <li>correctly subtracts the flight time [1 mark]</li> </ul>						
4:35 am in Dubai on Mon 7/12	<ul style="list-style-type: none"> <li>correctly determines the local time, day and date in Dubai [1 mark]</li> </ul>						

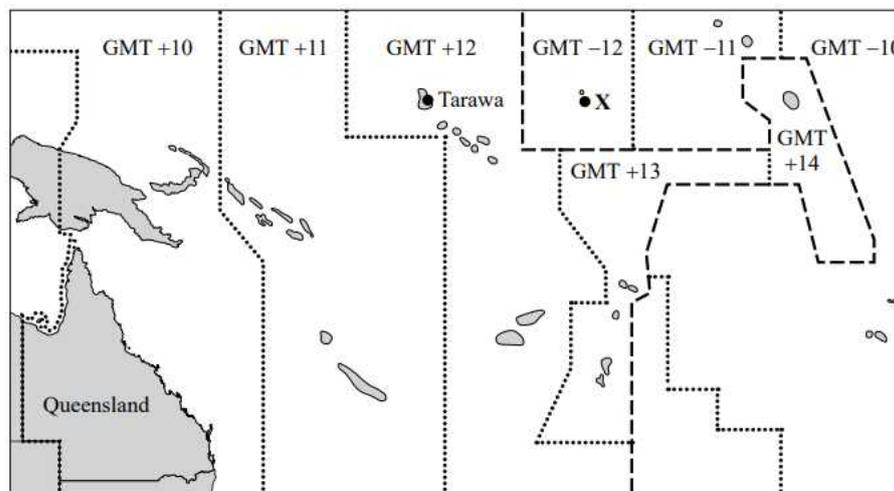
<b>2020</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 17</b>  <b>Earth geometry and time zones</b>	<p>Calculate the distance along the parallel of latitude between Mount Gambier, South Australia (37° 50' S, 140° 47' E) and Bairnsdale, Victoria (37° 50' S, 147° 37' E) to the nearest kilometre. (3 marks)</p>								
	<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td>           Distance is east / west  <math>\therefore \text{distance} = 111.2 \times \cos\theta \times \text{angular dist.}</math>             angular dist. = <math>147^\circ 37' - 140^\circ 47'</math>            = <math>6^\circ 50'</math> </td> <td> <ul style="list-style-type: none"> <li>correctly calculates the angular distance [1 mark]</li> </ul> </td> </tr> <tr> <td> <math>\text{distance} = 111.2 \times \cos\theta \times \text{angular dist.} =</math>  <math>= 111.2 \times \cos(37^\circ 50') \times (6^\circ 50')</math>            = 600.14         </td> <td> <ul style="list-style-type: none"> <li>provides evidence of substituting into the appropriate distance rule [1 mark]</li> </ul> </td> </tr> <tr> <td>It is approximately 600 km between Mount Gambier and Bairnsdale.</td> <td> <ul style="list-style-type: none"> <li>calculates distance to the nearest km [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	Distance is east / west $\therefore \text{distance} = 111.2 \times \cos\theta \times \text{angular dist.}$  angular dist. = $147^\circ 37' - 140^\circ 47'$ = $6^\circ 50'$	<ul style="list-style-type: none"> <li>correctly calculates the angular distance [1 mark]</li> </ul>	$\text{distance} = 111.2 \times \cos\theta \times \text{angular dist.} =$ $= 111.2 \times \cos(37^\circ 50') \times (6^\circ 50')$ = 600.14	<ul style="list-style-type: none"> <li>provides evidence of substituting into the appropriate distance rule [1 mark]</li> </ul>	It is approximately 600 km between Mount Gambier and Bairnsdale.	<ul style="list-style-type: none"> <li>calculates distance to the nearest km [1 mark]</li> </ul>
	Sample Response	The response							
	Distance is east / west $\therefore \text{distance} = 111.2 \times \cos\theta \times \text{angular dist.}$  angular dist. = $147^\circ 37' - 140^\circ 47'$ = $6^\circ 50'$	<ul style="list-style-type: none"> <li>correctly calculates the angular distance [1 mark]</li> </ul>							
$\text{distance} = 111.2 \times \cos\theta \times \text{angular dist.} =$ $= 111.2 \times \cos(37^\circ 50') \times (6^\circ 50')$ = 600.14	<ul style="list-style-type: none"> <li>provides evidence of substituting into the appropriate distance rule [1 mark]</li> </ul>								
It is approximately 600 km between Mount Gambier and Bairnsdale.	<ul style="list-style-type: none"> <li>calculates distance to the nearest km [1 mark]</li> </ul>								

<b>2020</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 23</b>  <b>Earth</b> <b>geometry</b> <b>and time</b> <b>zones</b>	A plane leaves Brisbane (UTC +10) at 10:45 pm on Monday and takes 14 hours and 35 minutes to fly to Dubai (UTC +4).	
	Determine the local time and day in Dubai when the plane arrives. (4 marks)	
	<b>Sample Response</b>	<b>The response</b>
	Depart Brisbane 22:45 Monday Brisbane time Travel +14:35	<ul style="list-style-type: none"> <li>• correctly adds travel time [1 mark]</li> </ul>
Arrive Dubai 37:20 Monday next day – 24:00 13:20 Tuesday	<ul style="list-style-type: none"> <li>• calculates arrival time from Brisbane’s perspective [1 mark]</li> <li>• correctly subtracts time difference [1 mark]</li> </ul>	
UTC correction – 6:00 7:20 am on Tuesday in Dubai	<ul style="list-style-type: none"> <li>• calculates arrival time and day from Dubai’s perspective [1 mark]</li> </ul>	

2022  
Paper 2  
Section 1  
Question 7

Earth  
geometry  
and time  
zones

You live in Queensland and your friend is on a cruise ship holiday. As their ship departs from X to travel 1350 km due west to Tarawa, your friend sends you a message saying ‘Local time 6:12 am Wednesday and enjoying the sunrise as our ship begins its trip to Tarawa’.



Not to scale

Key --- International date line ..... Time zone

You plan to phone your friend as soon as they arrive in Tarawa. Assuming their ship is travelling at 50 km/h, determine the time in Queensland when you will phone your friend. (7 marks)

Sample Response	The response
Ship's travel time from X to Tarawa $\text{speed} = \frac{\text{distance}}{\text{time}}$ $50 = \frac{1350}{\text{time}}$	<ul style="list-style-type: none"> <li>correctly substitutes into an appropriate rule [1 mark]</li> </ul>
$\therefore \text{time} = 27 \text{ hours}$ Ship's travel time is 27 hours.	<ul style="list-style-type: none"> <li>correctly calculates ship's travel time of 27 hours [1 mark]</li> </ul>
Time difference between Tarawa (GMT +12) and X (GMT -12) $+12 - (-12) = 24 \text{ hours}$ $\therefore \text{Tarawa is 24 hours ahead of X.}$	<ul style="list-style-type: none"> <li>correctly determines the time difference between Tarawa and X [1 mark]</li> </ul>
Time difference between Queensland (GMT +10) and Tarawa (GMT +12) $+10 - (+12) = -2 \text{ hours}$ $\therefore \text{Queensland is 2 hours behind Tarawa.}$	<ul style="list-style-type: none"> <li>correctly determines the time difference between Queensland and Tarawa [1 mark]</li> </ul>
Tarawa time at time of message = 6:12 am Wednesday + 24 hours = 6:12 am Thursday Tarawa time when ship arrives in Tarawa = 6:12 am Thursday + 27 hours = 9:12 am Friday Queensland time when ship arrives in Tarawa = 9:12 am Friday - 2 hours	<ul style="list-style-type: none"> <li>appropriately applies ship's travel time and both time differences to 6:12 am Wednesday [1 mark]</li> </ul>
= 7:12 am Friday	<ul style="list-style-type: none"> <li>determines time and day in Queensland at time of ship's arrival in Tarawa [1 mark]</li> <li>shows logical organisation, communicating key steps [1 mark]</li> </ul>

**2021  
Paper 2  
Section 1  
Question 1**

**Earth  
geometry  
and time  
zones**

A sailor anchors her yacht near Rocky Island at  $14^{\circ}, 52' \text{ S}$ ,  $145^{\circ} 29' \text{ E}$ . Her yacht is at the same latitude as her home, but the sun rises exactly 1 hour and 13 minutes later at home.

What are the coordinates of her home? (4 marks)

Sample Response	The response
Home latitude = $14^{\circ}52' \text{ S}$	• correctly identifies the latitude [1 mark]
Change time difference to angular difference Angle = $1\frac{13}{60} \times 15^{\circ}$ = $18.25^{\circ}$	• correctly determines the angle [1 mark]
Home longitude = $145^{\circ}29' - 18^{\circ}15'$ = $127^{\circ}14'$	• subtracts angle from longitude in same format [1 mark]
Home coordinates are $14^{\circ}52' \text{ S}$ , $127^{\circ}41' \text{ E}$	• determines longitude [1 mark]

## Unit 4: Investing and Networking

### Unit 4 – Topic 1: Loans, investments and annuities

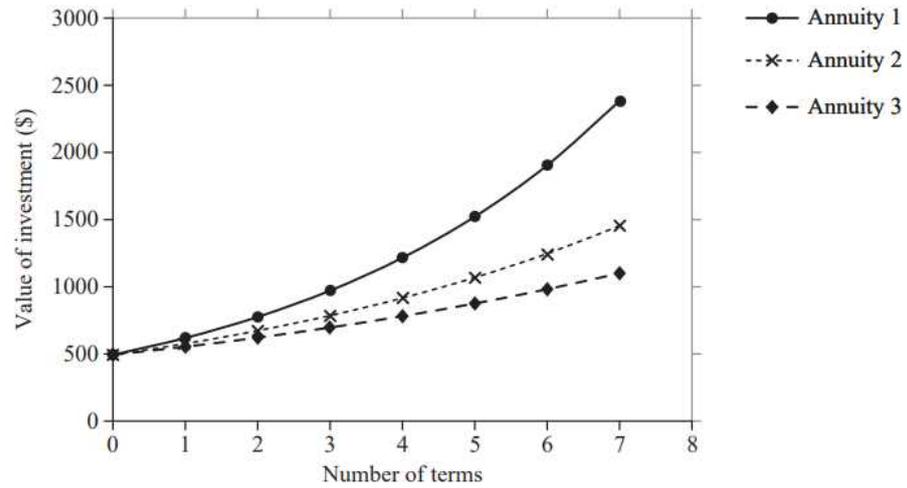
#### Paper 1 Section 1

<b>2023 Paper 1 Section 1 Question 11</b>  <b>Loans, investments and annuities</b>	<p>An annuity with an initial zero balance has \$500 deposited at the end of every month. The annuity earns 4.8% p.a. interest, compounding monthly. At the end of the fourth month, the balance is closest to</p> <p>(A) \$2002 (B) \$2008 (C) \$2012 (D) \$2014</p>
<b>2023 Paper 1 Section 1 Question 12</b>  <b>Loans, investments and annuities</b>	<p>A reducing balance loan with an initial balance of \$6000 is modelled by the recurrence relation</p> $A_{n+1} = \left(1 + \frac{0.03}{12}\right)A_n - 400, \text{ where } n \text{ is the number of months.}$ <p>The loan balance at the end of two months is closest to</p> <p>(A) \$5100 (B) \$5200 (C) \$5215 (D) \$5230</p>
<b>2021 Paper 1 Section 1 Question 15</b>  <b>Loans, investments and annuities</b>	<p>Which of the following investment options gives the best return?</p> <p>(A) 5.93% p.a. compounding daily (B) 5.95% p.a. compounding monthly (C) 5.97% p.a. compounding quarterly (D) 5.99% p.a. compounding six-monthly</p>
<b>2020 Paper 1 Section 1 Question 6</b>  <b>Loans, investments and annuities</b>	<p>A loan of \$10 000 has interest charged on a reducing balance at 6% p.a. compounding quarterly with quarterly repayments of \$700. The balance after 6 months is</p> <p>(A) \$8696.75 (B) \$8891.75 (C) \$8900.00 (D) \$9794.00</p>

**2020  
Paper 1  
Section 1  
Question 15**

**Loans,  
investments  
and  
annuities**

The graph shows the value of three different annuities over time.



Which of the following statements gives a plausible explanation for the different values after seven terms?

- (A) Annuity 1 and Annuity 2 have higher regular deposits than Annuity 3.
- (B) Annuity 1 and Annuity 2 have shorter interest terms than Annuity 3.
- (C) Annuity 2 and Annuity 3 have a lower initial value than Annuity 1.
- (D) Annuity 2 and Annuity 3 have higher interest rates than Annuity 1.




<b>2022 Paper 1 Section 2 Question 17</b>  <b>Loans, investments and annuities</b>	An investment of \$50 000 that compounds interest monthly is modelled by the recurrence relation $A_{n+1} = 1.00375A_n$ where $A_0 = 50\,000$ .
	a) What would be the advertised interest rate per annum, compounding monthly? [2 marks]
	b) How many months would it take for the value of the investment to exceed \$51 000? [2 marks]



















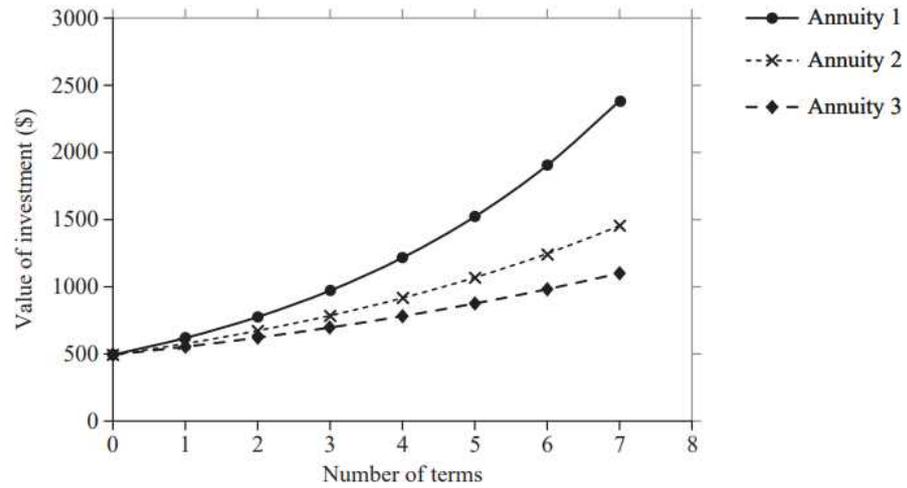
## Marking Guide – Paper 1 Section 1

<b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 11</b>  <b>Loans,</b> <b>investments</b> <b>and</b> <b>annuities</b>	An annuity with an initial zero balance has \$500 deposited at the end of every month. The annuity earns 4.8% p.a. interest, compounding monthly. At the end of the fourth month, the balance is closest to  (A) \$2002 (B) \$2008 <b>(C) \$2012 – Answer</b> (D) \$2014
<b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 12</b>  <b>Loans,</b> <b>investments</b> <b>and</b> <b>annuities</b>	A reducing balance loan with an initial balance of \$6000 is modelled by the recurrence relation $A_{n+1} = \left(1 + \frac{0.03}{12}\right)A_n - 400$ , where $n$ is the number of months.  The loan balance at the end of two months is closest to  (A) \$5100 (B) \$5200 (C) \$5215 <b>(D) \$5230 – Answer</b>
<b>2021</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 15</b>  <b>Loans,</b> <b>investments</b> <b>and</b> <b>annuities</b>	Which of the following investment options gives the best return?  (A) 5.93% p.a. compounding daily <b>(B) 5.95% p.a. compounding monthly - Answer</b> (C) 5.97% p.a. compounding quarterly (D) 5.99% p.a. compounding six-monthly
<b>2020</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 6</b>  <b>Loans,</b> <b>investments</b> <b>and</b> <b>annuities</b>	A loan of \$10 000 has interest charged on a reducing balance at 6% p.a. compounding quarterly with quarterly repayments of \$700. The balance after 6 months is  (A) \$8696.75 <b>(B) \$8891.75 - Answer</b> (C) \$8900.00 (D) \$9794.00

**2020  
Paper 1  
Section 1  
Question 15**

**Loans,  
investments  
and  
annuities**

The graph shows the value of three different annuities over time.



Which of the following statements gives a plausible explanation for the different values after seven terms?

- (A) **Annuity 1 and Annuity 2 have higher regular deposits than Annuity 3. - Answer**
- (B) Annuity 1 and Annuity 2 have shorter interest terms than Annuity 3.
- (C) Annuity 2 and Annuity 3 have a lower initial value than Annuity 1.
- (D) Annuity 2 and Annuity 3 have higher interest rates than Annuity 1.

Marking Guide – Paper 1 Section 2

<p><b>2023</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 17</b></p> <p><b>Loans, investments and annuities</b></p>	<p>Terome paid a \$50 000 deposit on a house valued at \$570 000 and borrowed the remainder as a reducing balance loan at 6.6% p.a. compounding monthly. Determine the monthly repayment required to pay off the loan over 25 years. (4 marks)</p>	
	<b>Sample response</b>	<b>The response</b>
	$i = \frac{6.6}{12 \times 100}$ $= 0.0055$ $n = 25 \times 12$ $= 300$	<ul style="list-style-type: none"> <li>correctly determines the <math>i</math> and <math>n</math> values [1 mark]</li> </ul>
	<p>Amount borrowed, <math>A = 570\,000 - 50\,000</math></p> $= 520\,000$	<ul style="list-style-type: none"> <li>correctly determines the amount borrowed [1 mark]</li> </ul>
	$A = M \left( \frac{1 - (1+i)^{-n}}{i} \right)$ $520\,000 = M \left( \frac{1 - (1 + 0.0055)^{-300}}{0.0055} \right)$	<ul style="list-style-type: none"> <li>substitutes into appropriate annuity rule [1 mark]</li> </ul>
$M = \frac{520\,000}{\left( \frac{1 - (1 + 0.0055)^{-300}}{0.0055} \right)}$ $= 3543.64$ <p>Monthly repayment is \$3543.64</p>	<ul style="list-style-type: none"> <li>determines monthly repayment [1 mark]</li> </ul>	

<p><b>2023</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 19</b></p> <p><b>Loans, investments and annuities</b></p>	<p>Ngarra compares two investment options and decides option A will provide the better return.</p> <ul style="list-style-type: none"> <li>Option A: 5.60% p.a. compounding monthly</li> <li>Option B: 5.62% p.a. compounding quarterly</li> </ul> <p>Use the effective annual rate of interest formula to evaluate the reasonableness of Ngarra’s decision. (4 marks)</p>	
	<b>Sample response</b>	<b>The response</b>
	<p>Option A: <math>i = 0.056, n = 12</math></p> $i_{\text{effective}} = \left( 1 + \frac{i}{n} \right)^n - 1$ $= \left( 1 + \frac{0.056}{12} \right)^{12} - 1$ $\approx 0.05745\dots$ <p>Option B: <math>i = 0.0562, n = 4</math></p> $i_{\text{effective}} = \left( 1 + \frac{i}{n} \right)^n - 1$ $= \left( 1 + \frac{0.0562}{4} \right)^4 - 1$ $\approx 0.05739\dots$ <p><math>0.05745 &gt; 0.05739</math></p> <p>Ngarra's decision is reasonable because option A has a higher effective interest rate.</p>	<ul style="list-style-type: none"> <li>correctly substitutes into appropriate rule for either option [1 mark]</li> <li>calculates effective interest rate for option A [1 mark]</li> <li>calculates effective interest rate for option B [1 mark]</li>   <li>provides a statement of reasonableness linked to effective interest rate [1 mark]</li> </ul>

<b>2022 Paper 1 Section 2 Question 17</b>  <b>Loans, investments and annuities</b>	An investment of \$50 000 that compounds interest monthly is modelled by the recurrence relation $A_{n+1} = 1.00375A_n$ where $A_0 = 50\,000$ .							
	a) What would be the advertised interest rate per annum, compounding monthly? [2 marks]							
	<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td> <math display="block">r = 1 + \frac{i}{n}</math> <math display="block">1.00375 = 1 + \frac{i}{12}</math> <math display="block">0.00375 = \frac{i}{12}</math> <math display="block">i = 0.045</math> </td> <td> <ul style="list-style-type: none"> <li>correctly substitutes into an appropriate rule [1 mark]</li> </ul> </td> </tr> <tr> <td>           Therefore, the annual interest rate is 4.5% p.a. compounding monthly.         </td> <td> <ul style="list-style-type: none"> <li>calculates annual interest rate [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	$r = 1 + \frac{i}{n}$ $1.00375 = 1 + \frac{i}{12}$ $0.00375 = \frac{i}{12}$ $i = 0.045$	<ul style="list-style-type: none"> <li>correctly substitutes into an appropriate rule [1 mark]</li> </ul>	Therefore, the annual interest rate is 4.5% p.a. compounding monthly.	<ul style="list-style-type: none"> <li>calculates annual interest rate [1 mark]</li> </ul>	
	Sample Response	The response						
$r = 1 + \frac{i}{n}$ $1.00375 = 1 + \frac{i}{12}$ $0.00375 = \frac{i}{12}$ $i = 0.045$	<ul style="list-style-type: none"> <li>correctly substitutes into an appropriate rule [1 mark]</li> </ul>							
Therefore, the annual interest rate is 4.5% p.a. compounding monthly.	<ul style="list-style-type: none"> <li>calculates annual interest rate [1 mark]</li> </ul>							
b) How many months would it take for the value of the investment to exceed \$51 000? [2 marks]								
<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td>           Method 1: Recursion  <math>A_0 = 50\,000</math>  <math>A_1 = 50\,187.50</math>  <math>A_2 = 50\,375.70</math>  <math>A_3 = 50\,564.61</math>  <math>A_4 = 50\,754.23</math>  <math>A_5 = 50\,944.56</math>  <math>A_6 = 51\,135.60</math>             Method 2: Compound interest rule  <math>A = P(1 + i)^n</math>  <math>51\,000 = 50\,000 \times 1.00375^n</math>            Using trial and error: when <math>n = 5</math>, <math>A = 50\,944.56</math>  <math>n = 6</math>, <math>A = 51\,135.60</math> </td> <td> <ul style="list-style-type: none"> <li>correctly uses an appropriate method [1 mark]</li> </ul> </td> </tr> <tr> <td>           Therefore, the investment would exceed \$51 000 at 6 months.         </td> <td> <ul style="list-style-type: none"> <li>determines when the investment would exceed \$51 000 [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	Method 1: Recursion $A_0 = 50\,000$ $A_1 = 50\,187.50$ $A_2 = 50\,375.70$ $A_3 = 50\,564.61$ $A_4 = 50\,754.23$ $A_5 = 50\,944.56$ $A_6 = 51\,135.60$  Method 2: Compound interest rule $A = P(1 + i)^n$ $51\,000 = 50\,000 \times 1.00375^n$ Using trial and error: when $n = 5$ , $A = 50\,944.56$ $n = 6$ , $A = 51\,135.60$	<ul style="list-style-type: none"> <li>correctly uses an appropriate method [1 mark]</li> </ul>	Therefore, the investment would exceed \$51 000 at 6 months.	<ul style="list-style-type: none"> <li>determines when the investment would exceed \$51 000 [1 mark]</li> </ul>		
Sample Response	The response							
Method 1: Recursion $A_0 = 50\,000$ $A_1 = 50\,187.50$ $A_2 = 50\,375.70$ $A_3 = 50\,564.61$ $A_4 = 50\,754.23$ $A_5 = 50\,944.56$ $A_6 = 51\,135.60$  Method 2: Compound interest rule $A = P(1 + i)^n$ $51\,000 = 50\,000 \times 1.00375^n$ Using trial and error: when $n = 5$ , $A = 50\,944.56$ $n = 6$ , $A = 51\,135.60$	<ul style="list-style-type: none"> <li>correctly uses an appropriate method [1 mark]</li> </ul>							
Therefore, the investment would exceed \$51 000 at 6 months.	<ul style="list-style-type: none"> <li>determines when the investment would exceed \$51 000 [1 mark]</li> </ul>							

<b>2022 Paper 1 Section 2 Question 25</b>  <b>Loans, investments and annuities</b>	A couple borrow money to complete home renovations. Their bank has loaned the amount at 2.4% p.a. compounding monthly with repayments of \$993.14 each month for 15 years.									
	a) Determine the amount of money borrowed. [3 marks]									
	<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td> <math display="block">i = \frac{2.4}{1200}</math> <math display="block">= 0.002</math> <math display="block">n = 15 \times 12</math> <math display="block">= 180</math> <math display="block">M = 993.14</math> </td> <td> <ul style="list-style-type: none"> <li>correctly determines the <math>i</math>, <math>n</math> and <math>M</math> values [1 mark]</li> </ul> </td> </tr> <tr> <td> <math display="block">A = M \left( \frac{1 - (1 + i)^{-n}}{i} \right)</math> <math display="block">= 993.14 \left( \frac{1 - (1 + i)^{-180}}{i} \right)</math> <math display="block">= 150\,000.29</math> </td> <td> <ul style="list-style-type: none"> <li>substitutes into the appropriate annuity formula [1 mark]</li> </ul> </td> </tr> <tr> <td>           They borrowed \$150 000.         </td> <td> <ul style="list-style-type: none"> <li>determines amount of money borrowed, including units [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	$i = \frac{2.4}{1200}$ $= 0.002$ $n = 15 \times 12$ $= 180$ $M = 993.14$	<ul style="list-style-type: none"> <li>correctly determines the <math>i</math>, <math>n</math> and <math>M</math> values [1 mark]</li> </ul>	$A = M \left( \frac{1 - (1 + i)^{-n}}{i} \right)$ $= 993.14 \left( \frac{1 - (1 + i)^{-180}}{i} \right)$ $= 150\,000.29$	<ul style="list-style-type: none"> <li>substitutes into the appropriate annuity formula [1 mark]</li> </ul>	They borrowed \$150 000.	<ul style="list-style-type: none"> <li>determines amount of money borrowed, including units [1 mark]</li> </ul>	
	Sample Response	The response								
$i = \frac{2.4}{1200}$ $= 0.002$ $n = 15 \times 12$ $= 180$ $M = 993.14$	<ul style="list-style-type: none"> <li>correctly determines the <math>i</math>, <math>n</math> and <math>M</math> values [1 mark]</li> </ul>									
$A = M \left( \frac{1 - (1 + i)^{-n}}{i} \right)$ $= 993.14 \left( \frac{1 - (1 + i)^{-180}}{i} \right)$ $= 150\,000.29$	<ul style="list-style-type: none"> <li>substitutes into the appropriate annuity formula [1 mark]</li> </ul>									
They borrowed \$150 000.	<ul style="list-style-type: none"> <li>determines amount of money borrowed, including units [1 mark]</li> </ul>									
b) Write a recurrence relation for the amount owing after $n$ months. [2 marks]										
<table border="1"> <thead> <tr> <th>Sample Response</th> <th>The response</th> </tr> </thead> <tbody> <tr> <td> <math display="block">A_{n+1} = rA_n - R</math> <math display="block">A_{n+1} = \left( 1 + \frac{2.4}{1200} \right) A_n - 993.14</math> </td> <td> <ul style="list-style-type: none"> <li>correctly selects the appropriate formula [1 mark]</li> </ul> </td> </tr> <tr> <td> <math display="block">A_{n+1} = 1.002A_n - 993.14</math> </td> <td> <ul style="list-style-type: none"> <li>determines recurrence relation [1 mark]</li> </ul> </td> </tr> </tbody> </table>	Sample Response	The response	$A_{n+1} = rA_n - R$ $A_{n+1} = \left( 1 + \frac{2.4}{1200} \right) A_n - 993.14$	<ul style="list-style-type: none"> <li>correctly selects the appropriate formula [1 mark]</li> </ul>	$A_{n+1} = 1.002A_n - 993.14$	<ul style="list-style-type: none"> <li>determines recurrence relation [1 mark]</li> </ul>				
Sample Response	The response									
$A_{n+1} = rA_n - R$ $A_{n+1} = \left( 1 + \frac{2.4}{1200} \right) A_n - 993.14$	<ul style="list-style-type: none"> <li>correctly selects the appropriate formula [1 mark]</li> </ul>									
$A_{n+1} = 1.002A_n - 993.14$	<ul style="list-style-type: none"> <li>determines recurrence relation [1 mark]</li> </ul>									

<b>2021 Paper 1 Section 2 Question 17</b>  <b>Loans, investments and annuities</b>	Determine the monthly repayment on a reducing balance loan of \$720 000 at 4.8% p.a. over 25 years. Give your answer to the nearest dollar. (4 marks)	
	Sample Response	The response
	$A = 720\,000$ $M = ?$  $i = \frac{0.048}{12} = 0.004$ $n = 25 \times 12 = 300$	<ul style="list-style-type: none"> <li>correctly determines the <math>i</math> and <math>n</math> values [1 mark]</li> </ul>
	$A = M \left( \frac{1 - (1 + i)^{-n}}{i} \right)$ $A = M \left( \frac{1 - (1 + 0.004)^{-300}}{0.004} \right)$	<ul style="list-style-type: none"> <li>substitutes into appropriate annuity rule [1 mark]</li> </ul>
	$720\,000 = M \times 174.520 \dots$ $M = \frac{720\,000}{174.520 \dots}$ $M = 4.125.758 \dots$	<ul style="list-style-type: none"> <li>determines monthly repayment [1 mark]</li> </ul>
The monthly repayment will be \$4126 each month for 25 years.		<ul style="list-style-type: none"> <li>states solution to the nearest dollar [1 mark]</li> </ul>

<b>2021 Paper 1 Section 2 Question 22</b>  <b>Loans, investments and annuities</b>	Rosa borrowed \$32 000 as a reducing balance loan at 4.8% p.a. compounding monthly, with monthly repayments of \$278.	
	How much will Rosa owe after 2 months, to the nearest cent? (4 marks)	
	Sample Response	The response
	<b>Option 1: Recursion</b> $i = \frac{4.8}{1200} = 0.004$	<ul style="list-style-type: none"> <li>correctly determines the <math>i</math> value [1 mark]</li> </ul>
	$\therefore r = 1.004$ $R = 278$ $A_0 = 32\,000$ $An + 1 = rA_n - R$ $\therefore A_1 = 1.004 \times 32\,000 - 278 = 31\,850$	<ul style="list-style-type: none"> <li>correctly substitutes into an appropriate rule [1 mark]</li> </ul>
	$\therefore A_2 = 1.004 \times 31\,850 - 278 = 31\,699.4$	<ul style="list-style-type: none"> <li>substitutes for <math>A_2</math> using result from <math>A_1</math> [1 mark]</li> </ul>
	After 2 months, Rosa owes \$31 699.40	<ul style="list-style-type: none"> <li>provides answer rounded to the nearest cent [1 mark]</li> </ul>
	<b>Option 2: Annuity</b> $i = \frac{4.8}{1200} = 0.004$	<ul style="list-style-type: none"> <li>correctly determines the <math>i</math> value [1 mark]</li> </ul>
$\therefore r = 1.004$ $R = 278$ $P = 32\,000$ $A_n = P(1 + i)^n - M \left( \frac{(1 + i)^n - 1}{i} \right)$ $\therefore A_2 = 32\,000(1.004)^2 - 278 \left( \frac{(1.004)^2 - 1}{0.004} \right) = 31\,699.4$	<ul style="list-style-type: none"> <li>correctly substitutes into an appropriate compound interest rule [1 mark]</li> <li>correctly substitutes into an appropriate annuity rule [1 mark]</li> </ul>	
After 2 months, Rosa owes \$31 699.40	<ul style="list-style-type: none"> <li>provides answer rounded to the nearest cent [1 mark]</li> </ul>	

<b>2020</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 20</b>  <b>Loans,</b> <b>investments</b> <b>and</b> <b>annuities</b>	Determine the monthly repayment on a \$350 000 home loan over 25 years with 6.5% p.a. fixed interest compounded monthly. (4 marks)	
	Sample Response	The response
	$A = 350\,000$ $M = ?$  $i = \frac{0.065}{12}$ $= 0.005416 \dots$  $n = 25 \times 12$ $= 300$	<ul style="list-style-type: none"> <li>correctly determines the <math>i</math> and <math>n</math> values [1 mark]</li> </ul>
	$A = M \left( \frac{(1+i)^n - 1}{i} \right)$  $A = M \left( \frac{(1 + 0.005416 \dots)^{300}}{0.005416 \dots} \right)$  $350\,000 = M \times 148.102 \dots$  $M = \frac{350\,000}{148.102}$	<ul style="list-style-type: none"> <li>substitutes into appropriate annuity rule [1 mark]</li> </ul>
$M = 2363.225 \dots$  The monthly repayment will be \$2363.23 each month for 25 years.	<ul style="list-style-type: none"> <li>determines monthly repayment [1 mark]</li> <li>states solution with correct units and appropriate rounding [1 mark]</li> </ul>	

<b>2020</b> <b>Paper 1</b> <b>Section 2</b> <b>Question 25</b>  <b>Loans,</b> <b>investments</b> <b>and</b> <b>annuities</b>	A financial institution offers two investment options: Option 1: 7% p.a. compounding quarterly Option 2: 6.8% p.a. compounding monthly  Use the effective interest rate formula to determine the option that will provide the better return. (5 marks)	
	Sample Response	The response
	Option 1 $i_{e1} = \left( 1 + \frac{i}{n} \right)^n - 1$ $= \left( 1 + \frac{0.07}{4} \right)^4 - 1$	<ul style="list-style-type: none"> <li>correctly substitutes into appropriate rule [1 mark]</li> </ul>
	$\approx 0.07186$	<ul style="list-style-type: none"> <li>calculates effective interest rate for Option 1 [1 mark]</li> </ul>
	Option 2 $i_{e2} = \left( 1 + \frac{i}{n} \right)^n - 1$ $= \left( 1 + \frac{0.068}{12} \right)^{12} - 1$	<ul style="list-style-type: none"> <li>correctly substitutes into appropriate rule [1 mark]</li> </ul>
	$\approx 0.07016$	<ul style="list-style-type: none"> <li>calculates the effective interest rate for Option 2 [1 mark]</li> </ul>
Option 1 is better because it has a slightly higher effective interest rate.	<ul style="list-style-type: none"> <li>states better option [1 mark]</li> </ul>	

Marking Guide – Paper 2 Section 1

<p><b>2023 Paper 2 Section 1 Question 7</b></p> <p><b>Loans, investments and annuities</b></p>	<p>Five years ago, a retiree invested \$100 000 in a compound interest account earning 3.8% p.a. compounding monthly. They now intend to use the balance of the account to begin a perpetuity that will return 4% p.a. compounding annually and pay them \$6000 each year.</p>	
	<p>Provide advice to the retiree about whether their compound interest investment is large enough to finance the perpetuity. (5 marks)</p>	
	<b>Sample response</b>	<b>The response</b>
	<p><b>Method 1</b> Compound interest investment</p> $A = P(1+i)^n$ $= 100\,000 \left(1 + \frac{3.8}{12 \times 100}\right)^{5 \times 12}$ $= 120\,888.66$ <p>The balance of the investment account is \$120 888.66.</p>	<ul style="list-style-type: none"> <li>• correctly substitutes into an appropriate rule for compound interest investment [1 mark]</li> <li>• determines balance of investment account [1 mark]</li> </ul>
	<p>Perpetuity</p> $M = A \times i$ $6000 = A \times 0.04$ $A = \frac{6000}{0.04}$ $= 150\,000$ <p>The present value of the perpetuity needs to be \$150 000. 120 888.66 &lt; 150 000</p>	<ul style="list-style-type: none"> <li>• correctly substitutes into an appropriate rule for perpetuity [1 mark]</li> <li>• determines present value of perpetuity [1 mark]</li> </ul>
<p>The compound interest investment will not provide enough money to finance the perpetuity.</p>	<ul style="list-style-type: none"> <li>• determines if the compound interest investment is large enough to finance the perpetuity [1 mark]</li> </ul>	
<p><b>Method 2</b> Perpetuity</p> $M = A \times i$ $6000 = A \times 0.04$ $A = \frac{6000}{0.04}$ $= 150\,000$ <p>The present value of the perpetuity needs to be \$150 000.</p>	<ul style="list-style-type: none"> <li>• correctly substitutes into an appropriate rule for perpetuity [1 mark]</li> <li>• determines present value of perpetuity [1 mark]</li> </ul>	
<p>Compound interest investment Find principal, <math>P</math>, for balance needing to be at least \$150 000.</p> $A = P(1+i)^n$ $150\,000 = P \left(1 + \frac{3.8}{12 \times 100}\right)^{5 \times 12}$ $P = \frac{150\,000}{\left(1 + \frac{3.8}{12 \times 100}\right)^{5 \times 12}}$ $= 124\,081.11$ <p>The principal needs to be \$124 081.11.</p> <p>100 000 &lt; 124 081.11 The compound interest investment will not provide enough money to finance the perpetuity.</p>	<ul style="list-style-type: none"> <li>• correctly substitutes into an appropriate rule for compound interest investment [1 mark]</li> <li>• determines required principal for investment account [1 mark]</li> <li>• determines if the compound interest investment is large enough to finance the perpetuity [1 mark]</li> </ul>	

**2022  
Paper 2  
Section 1  
Question 2**

**Loans,  
investments  
and  
annuities**

Tam deposits a fixed amount at the end of each month into an account paying 8.6% p.a. compounding monthly. From an initial zero balance, she accumulates \$51 343.85 in four years.

A financial planner has advised Tam that she would have been at least \$3000 better off if she had instead deposited half of the fixed amount at the end of each fortnight into an account paying 7.9% p.a. compounding fortnightly.

Evaluate the reasonableness of this advice. (5 marks)

Sample Response	The response
$A = M \left( \frac{(1+i)^n - 1}{i} \right)$ $51\,343.85 = M \left( \frac{\left(1 + \frac{0.086}{12}\right)^{48} - 1}{\frac{0.086}{12}} \right)$	<ul style="list-style-type: none"> <li>correctly substitutes parameters into the appropriate annuity rule [1 mark]</li> </ul>
$51\,343.85 = M \times 57.0487$ $\therefore M = 900$	<ul style="list-style-type: none"> <li>correctly determines the monthly amount [1 mark]</li> </ul>
<p>Fortnightly annuity balance</p> $A = M \left( \frac{(1+i)^n - 1}{i} \right)$ $A = 450 \left( \frac{\left(1 + \frac{0.079}{26}\right)^{104} - 1}{\frac{0.079}{26}} \right)$	<ul style="list-style-type: none"> <li>determines value of fortnightly annuity [1 mark]</li> </ul>
$\text{Diff} = 54\,941.61 - 51\,343.85$ $= 3597.76$	<ul style="list-style-type: none"> <li>determines difference in annuity balances [1 mark]</li> </ul>
<p>The advice that she would have been at least \$3000 better off is reasonable as \$3597.76 &gt; \$3000.</p>	<ul style="list-style-type: none"> <li>compares values to evaluate the reasonableness of the advice [1 mark]</li> </ul>

**2021  
Paper 2  
Section 1  
Question 3**

**Loans,  
investments  
and  
annuities**

Jo contributes \$2500 per quarter to an annuity earning 3.6% p.a. compounding quarterly.

At the end of 4 years, Jo makes a one-off extra contribution of \$10 000 and continues with the regular quarterly contributions.

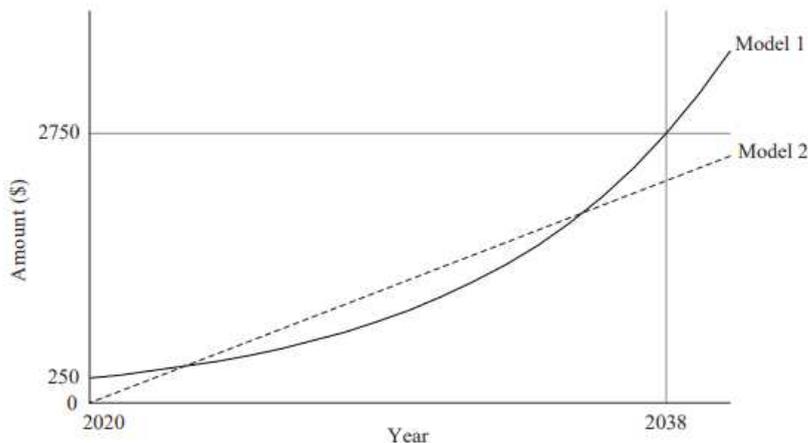
Determine the value of the annuity at the end of 6 years, to the nearest dollar. (5 marks)

Sample Response	The response
Value of regular contributions $M = 2500$ $i = \frac{3.6}{400}$ $= 0.009$ $n = 6 \times 4$ $= 24$	<ul style="list-style-type: none"> <li>correctly determines the <math>i</math> and <math>n</math> values [1 mark]</li> </ul>
$A = M \left( \frac{(1+i)^n - 1}{i} \right)$ $= 2500 \left( \frac{(1.009)^{24} - 1}{0.009} \right)$ $= 66\,639.94$	<ul style="list-style-type: none"> <li>substitutes into appropriate annuity rule [1 mark]</li> </ul>
Value of extra payment $P = 10\,000$ $i = \frac{3.6}{400}$ $= 0.009$ $n = 2 \times 4$ $= 8$  $A = P(1+i)^n$ $= 10\,000(1.009)^8$ $= 10\,743.09$	<ul style="list-style-type: none"> <li>substitutes into appropriate rule [1 mark]</li> </ul>
Total value = $66\,639.94 + 10\,743.09$ $= 77\,383.03$ $= \$77\,383$	<ul style="list-style-type: none"> <li>determines sum of two values [1 mark]</li> <li>determines total value, rounded to the nearest dollar [1 mark]</li> </ul>

**2021  
Paper 2  
Section 1  
Question 4**

**Loans,  
investments  
and  
annuities**

The graph shows two investment models. Model 1 is compounding annually, while Model 2 increases linearly by \$126 per year.



Determine the difference between the two investment strategies in 2030, to the nearest dollar. (6 marks)

Sample Response	The response
<p>Let <math>n</math> = the number of years since 2019 Let <math>t_n</math> = the amount of money</p> <p>In 2020, <math>n = 1</math> and <math>t_1 = 250</math> In 2038, <math>n = 19</math> and <math>t_{19} = 2750</math></p> <p>Find <math>r</math> <math>t_n = t_1 r^{(n-1)}</math> <math>\therefore 2750 = 250 \times r^{18}</math> <math>\therefore 11 = r^{18}</math> <math>\therefore r = 1.1425</math></p>	<ul style="list-style-type: none"> <li>correctly substitutes the values into a geometric rule [1 mark]</li> </ul>
<p>The geometric model for Model 1 <math>\therefore t_n = 250 \times 1.1425^{(n-1)}</math></p>	<ul style="list-style-type: none"> <li>determines geometric model for Model 1 [1 mark]</li> </ul>
<p>The arithmetic model for Model 2 <math>t_n = t_1 + (n-1)d</math> <math>\therefore t_n = 126(n-1)</math></p>	<ul style="list-style-type: none"> <li>correctly determines an arithmetic model for Model 2 [1 mark]</li> </ul>
<p>Comparison of investments in 2030, <math>n = 11</math> Model 1's amount in 2030, <math>t_{11} = 250 \times 1.1425^{10} = 947.33</math></p> <p>Model 2's amount in 2030, <math>t_{11} = 126 \times 10 = 1260</math></p>	<ul style="list-style-type: none"> <li>determines the amounts for both models in 2030 [1 mark]</li> </ul>
<p>Difference = <math>1260 - 947.33</math> <math>= 312.67</math></p> <p>In 2030 Model 2 is \$313 more than Model 1.</p>	<ul style="list-style-type: none"> <li>determines difference to nearest dollar [1 mark]</li> <li>shows logical organisation communicating key steps [1 mark]</li> </ul>

**2020  
Paper 2  
Section 1  
Question 7**

**Loans,  
investments  
and  
annuities**

A couple saved for their retirement by making the same monthly payments for 20 years into an account that earned 4.2% p.a. compounded monthly. At the age of 65, the couple retired and used all their savings to purchase a perpetuity with an interest rate of 5.76% p.a. compounded monthly, paying \$3600 each month. How much did they save each month to prepare for their retirement? (7 marks)

Sample Response	The response
Perpetuity — find the size of the savings $M = 3600$ $i = \frac{0.0576}{12}$ $= 0.0048$	<ul style="list-style-type: none"> <li>correctly determines the <math>i</math> value [1 mark]</li> </ul>
$A = ?$ $A = \frac{M}{i}$ $= \frac{3600}{0.0048}$ $= 750\,000$	<ul style="list-style-type: none"> <li>correctly recalls the perpetuity rule [1 mark]</li> <li>determines purchase price of perpetuity [1 mark]</li> </ul>
Use the total savings to find the size of the monthly payment $A = 750\,000$ $M = ?$ $i = \frac{0.042}{12}$ $= 0.0035$ $n = 20 \times 12$ $= 240$ $A = M \left( \frac{(1+i)^{-n} - 1}{i} \right)$	<ul style="list-style-type: none"> <li>correctly determines the <math>i</math> and <math>n</math> values [1 mark]</li> <li>correctly selects the appropriate annuity rule [1 mark]</li> </ul>
$750\,000 = M \times 375.13 \dots$ $M = 1999.281$	<ul style="list-style-type: none"> <li>determines payment [1 mark]</li> </ul>
The monthly savings were \$1999.29.	<ul style="list-style-type: none"> <li>shows logical organisation, communicating key steps [1 mark]</li> </ul>

## Unit 4 – Topic 2: Graphs and networks

### Paper 1 Section 1

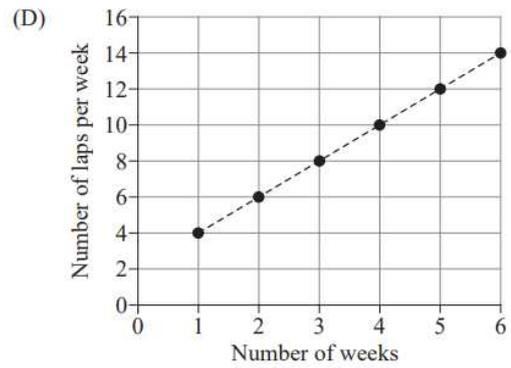
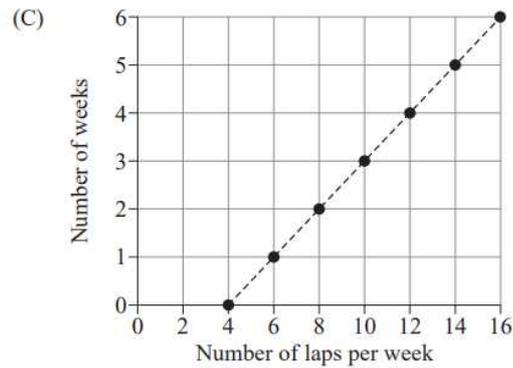
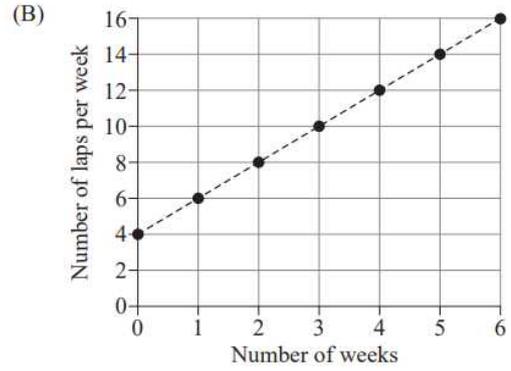
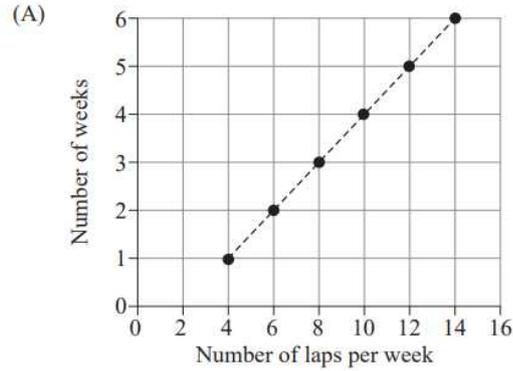
<p><b>2023</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 13</b></p> <p><b>Graphs and networks</b></p>	<p>Four athletes, Eoin (E), Fedir (F), Gede (G) and Hana (H), compete in three events: javelin (J), long jump (L) and sprints (S).</p> <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 5px;">Athlete</th> <th style="padding: 5px;">Events</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">Eoin</td> <td style="padding: 5px;">L, S</td> </tr> <tr> <td style="padding: 5px;">Fedir</td> <td style="padding: 5px;">J</td> </tr> <tr> <td style="padding: 5px;">Gede</td> <td style="padding: 5px;">S</td> </tr> <tr> <td style="padding: 5px;">Hana</td> <td style="padding: 5px;">J, L</td> </tr> </tbody> </table> <p>Which bipartite graph represents this information?</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>(A)</p> </div> <div style="text-align: center;"> <p>(B)</p> </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div style="text-align: center;"> <p>(C)</p> </div> <div style="text-align: center;"> <p>(D)</p> </div> </div>	Athlete	Events	Eoin	L, S	Fedir	J	Gede	S	Hana	J, L
Athlete	Events										
Eoin	L, S										
Fedir	J										
Gede	S										
Hana	J, L										

<p><b>2022</b> <b>Paper 1</b> <b>Section 1</b> <b>Question 2</b></p> <p><b>Graphs and networks</b></p>	<p>The total number of vertices in this graph is</p> <div style="text-align: center; margin: 10px 0;"> </div> <p>(A) 3 (B) 5 (C) 6 (D) 7</p>
--	--

**2022  
Paper 1  
Section 1  
Question 4**

**Graphs and networks**

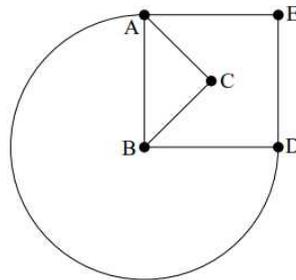
A swimmer has a weekly training routine to improve their fitness as modelled by the recursive function  $T_{n+1} = T_n + 2$ , where  $T_n$  is the number of laps they swim in week  $n$  and  $T_1 = 4$ . Which graph best represents the swimmer's routine?



**2021  
Paper 1  
Section 1  
Question 2**

**Graphs and networks**

How many faces does this planar graph have?

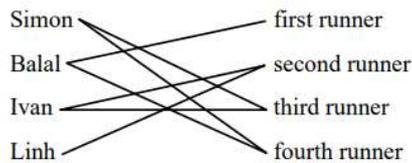


- (A) 3
- (B) 4
- (C) 5
- (D) 7

**2021  
Paper 1  
Section 1  
Question 5**

**Graphs and networks**

The coach of a four-person relay team is deciding on the order of the runners. Use the bipartite graph to determine which statement is correct.



- (A) Linh should be the first runner.
- (B) Simon should be the second runner.
- (C) Ivan should be the third runner.
- (D) Balal should be the fourth runner.

**2021  
Paper 1  
Section 1  
Question 8**

**Graphs and  
networks**

A basketball competition has six teams that have completed three rounds of competition as shown.

	Bears	Eagles	Lions	Meerkats	Tigers	Wombats
Bears	—	—	✓	—	✓	✓
Eagles	—	—	✓	✓	—	✓
Lions	✓	✓	—	✓	—	—
Meerkats	—	✓	✓	—	✓	—
Tigers	✓	—	—	✓	—	✓
Wombats	✓	✓	—	—	✓	—

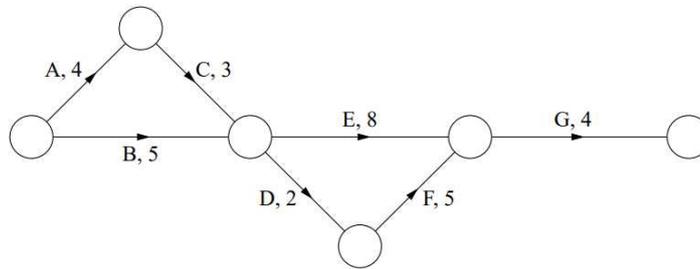
The graph to represent this information has

- (A) 6 edges.
- (B) 9 edges.
- (C) 15 edges.
- (D) 18 edges.

**2021  
Paper 1  
Section 1  
Question 13**

**Graphs and  
networks**

Based on this project network, what is the minimum number of days required to complete the project?

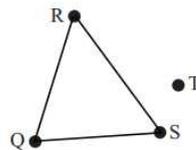


- (A) 16
- (B) 17
- (C) 19
- (D) 31

**2020  
Paper 1  
Section 1  
Question 11**

**Graphs and  
networks**

Determine the adjacency matrix that represents this graph.



(A) 
$$\begin{matrix} & Q & R & S \\ Q & \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \\ R & \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \\ S & \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \end{matrix}$$

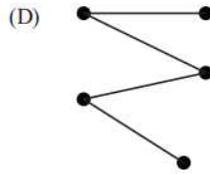
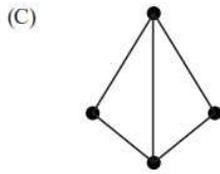
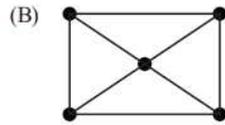
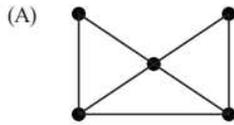
(B) 
$$\begin{matrix} & Q & R & S & T \\ Q & \begin{pmatrix} 0 & 1 & 1 & 0 \end{pmatrix} \\ R & \begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix} \\ S & \begin{pmatrix} 1 & 1 & 0 & 0 \end{pmatrix} \\ T & \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

(C) 
$$\begin{matrix} & Q & R & S & T \\ Q & \begin{pmatrix} 0 & 1 & 1 & 0 \end{pmatrix} \\ R & \begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix} \\ S & \begin{pmatrix} 1 & 1 & 0 & 1 \end{pmatrix} \\ T & \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} \end{matrix}$$

(D) 
$$\begin{matrix} & Q & R & S & T \\ Q & \begin{pmatrix} 0 & 2 & 2 & 0 \end{pmatrix} \\ R & \begin{pmatrix} 2 & 0 & 2 & 0 \end{pmatrix} \\ S & \begin{pmatrix} 2 & 2 & 0 & 0 \end{pmatrix} \\ T & \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

**2020**  
**Paper 1**  
**Section 1**  
**Question 13**  
**Graphs and**  
**networks**

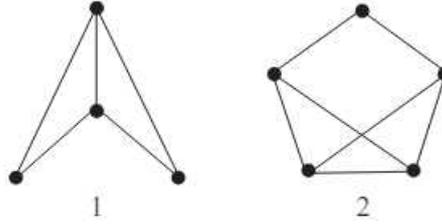
Which of the following is a planar graph with 5 vertices and 4 faces?



2023  
Paper 1  
Section 2  
Question 20

Graphs and  
networks

Graphs 1 and 2 are shown.



a) Show that Euler's formula applies to graph 1. [2 marks]

---

---

---

---

---

b) What feature in the drawing should be changed to represent graph 2 as planar? [1 mark]

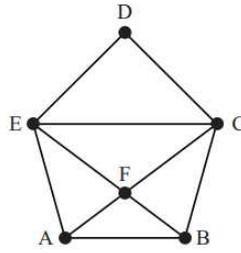
---

c) Redraw graph 2 to represent it as a planar graph. [1 mark]



**2020**  
**Paper 1**  
**Section 2**  
**Question 16**  
  
**Graphs and**  
**networks**

Use the graph to identify whether each of the following is a cycle, an open walk, an open trail or a closed trail.



a) AFCFB [1 mark]

---

---

b) AFCEFBA [1 mark]

---

---

c) ABCDEFA [1 mark]

---

---



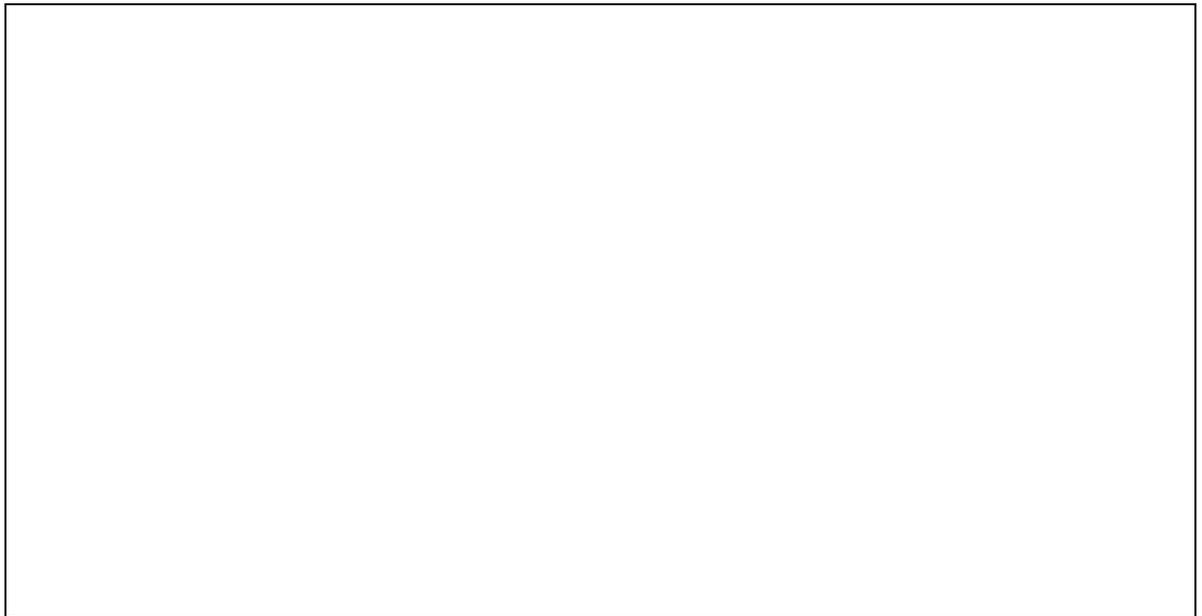
2022  
Paper 2  
Section 1  
Question 4

Graphs and  
networks

The table shows the current road length (in kilometres) between six towns.

	Manon	Veria	Bolint	Farra	Recen	Alin
Manon		16	34	—	—	33
Veria			12	—	—	15
Bolint				—	10	—
Farra					15	23
Recen						15
Alin						

The government plans to build a direct road between Manon and Farra. Use a network diagram to determine the length of the direct road if it is to be 4 km shorter than the length of the current shortest road route between Manon and Farra. (5 marks)



Note: If you make a mistake in the diagram, cancel it by ruling a single diagonal line through your work and use the additional response space at the back of this question and response book.

---

---

---

---

---

---

---

---

---

---

Marking Guide – Paper 1 Section 1

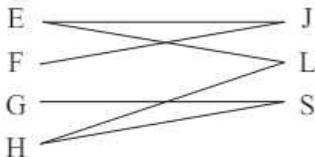
**2023  
Paper 1  
Section 1  
Question 13**

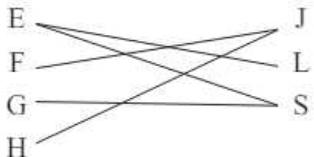
**Graphs and networks**

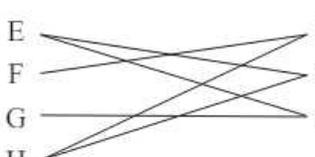
Four athletes, Eoin (E), Fedir (F), Gede (G) and Hana (H), compete in three events: javelin (J), long jump (L) and sprints (S).

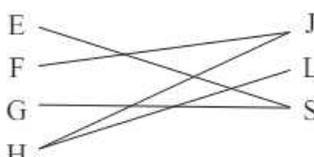
Athlete	Events
Eoin	L, S
Fedir	J
Gede	S
Hana	J, L

Which bipartite graph represents this information?

(A) 

(B) 

(C) 

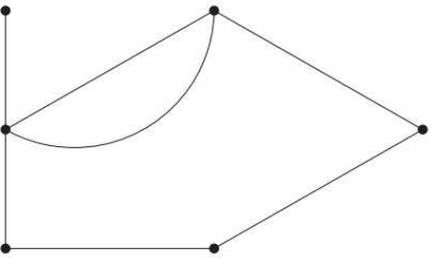
(D) 

**Answer is C.**

**2022  
Paper 1  
Section 1  
Question 2**

**Graphs and networks**

The total number of vertices in this graph is

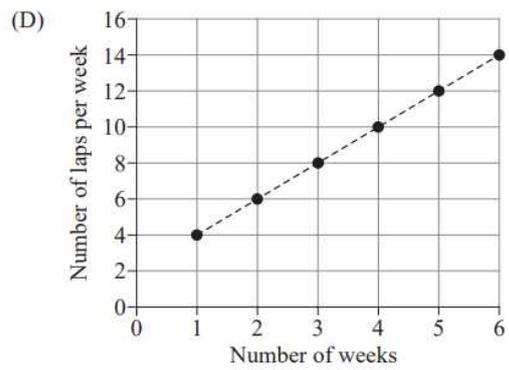
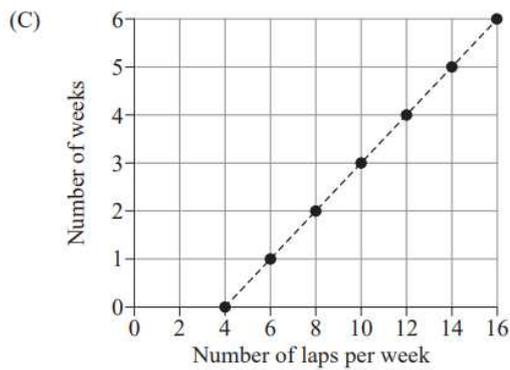
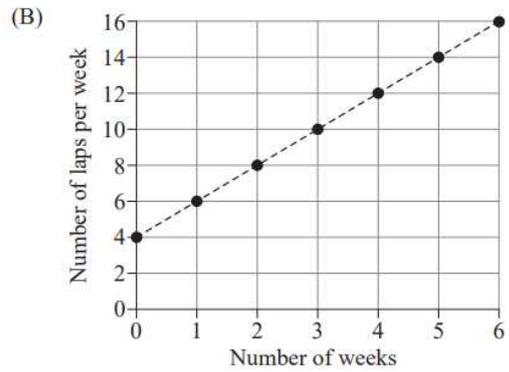
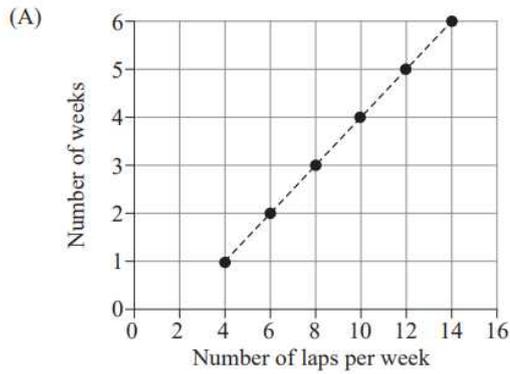


(A) 3  
(B) 5  
(C) 6 - Answer  
(D) 7

2022  
Paper 1  
Section 1  
Question 4

Graphs and networks

A swimmer has a weekly training routine to improve their fitness as modelled by the recursive function  $T_{n+1} = T_n + 2$ , where  $T_n$  is the number of laps they swim in week  $n$  and  $T_1 = 4$ . Which graph best represents the swimmer's routine?

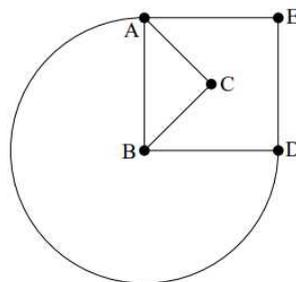


Answer is D.

2021  
Paper 1  
Section 1  
Question 2

Graphs and networks

How many faces does this planar graph have?



- (A) 3
- (B) 4 - Answer
- (C) 5
- (D) 7

**2021 Paper 1 Section 1 Question 5**  
**Graphs and networks**

The coach of a four-person relay team is deciding on the order of the runners. Use the bipartite graph to determine which statement is correct.

(A) Linh should be the first runner.  
 (B) Simon should be the second runner.  
**(C) Ivan should be the third runner. - Answer**  
 (D) Balal should be the fourth runner.

**2021 Paper 1 Section 1 Question 8**  
**Graphs and networks**

A basketball competition has six teams that have completed three rounds of competition as shown.

	Bears	Eagles	Lions	Meerkats	Tigers	Wombats
Bears	—	—	✓	—	✓	✓
Eagles	—	—	✓	✓	—	✓
Lions	✓	✓	—	✓	—	—
Meerkats	—	✓	✓	—	✓	—
Tigers	✓	—	—	✓	—	✓
Wombats	✓	✓	—	—	✓	—

The graph to represent this information has

(A) 6 edges.  
**(B) 9 edges. - Answer**  
 (C) 15 edges.  
 (D) 18 edges.

**2021 Paper 1 Section 1 Question 13**  
**Graphs and networks**

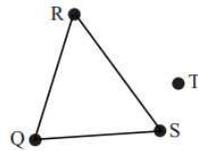
Based on this project network, what is the minimum number of days required to complete the project?

(A) 16  
 (B) 17  
**(C) 19 - Answer**  
 (D) 31

**2020  
Paper 1  
Section 1  
Question 11**

**Graphs and  
networks**

Determine the adjacency matrix that represents this graph.



(A) 
$$\begin{matrix} & Q & R & S \\ Q & 0 & 1 & 1 \\ R & 1 & 0 & 1 \\ S & 1 & 1 & 0 \end{matrix}$$

(B) 
$$\begin{matrix} & Q & R & S & T \\ Q & 0 & 1 & 1 & 0 \\ R & 1 & 0 & 1 & 0 \\ S & 1 & 1 & 0 & 0 \\ T & 0 & 0 & 0 & 0 \end{matrix}$$

(C) 
$$\begin{matrix} & Q & R & S & T \\ Q & 0 & 1 & 1 & 0 \\ R & 1 & 0 & 1 & 0 \\ S & 1 & 1 & 0 & 1 \\ T & 0 & 0 & 1 & 0 \end{matrix}$$

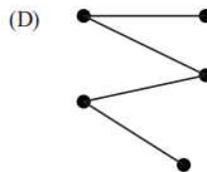
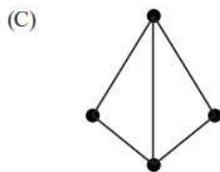
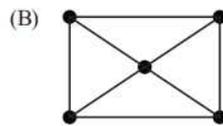
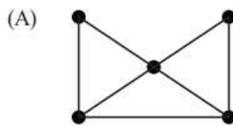
(D) 
$$\begin{matrix} & Q & R & S & T \\ Q & 0 & 2 & 2 & 0 \\ R & 2 & 0 & 2 & 0 \\ S & 2 & 2 & 0 & 0 \\ T & 0 & 0 & 0 & 0 \end{matrix}$$

**Answer is B.**

**2020  
Paper 1  
Section 1  
Question 13**

**Graphs and  
networks**

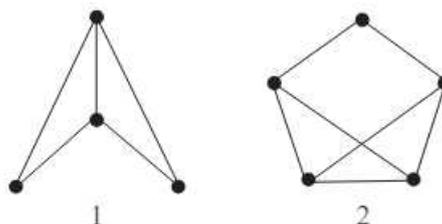
Which of the following is a planar graph with 5 vertices and 4 faces?



**Answer is A.**

2023  
Paper 1  
Section 2  
Question 20  
  
Graphs and  
networks

Graphs 1 and 2 are shown.



a) Show that Euler's formula applies to graph 1. [2 marks]

Sample response	The response
$v = 4$ $f = 3$ $e = 5$  $v + f - e = 4 + 3 - 5 = 2$	<ul style="list-style-type: none"> <li>correctly identifies the number of vertices, faces and edges for graph 1 [1 mark]</li> <li>applies Euler's formula to graph 1 [1 mark]</li> </ul>

b) What feature in the drawing should be changed to represent graph 2 as planar? [1 mark]

Sample response	The response
Show no crossing edges.	<ul style="list-style-type: none"> <li>correctly identifies the feature to be changed [1 mark]</li> </ul>

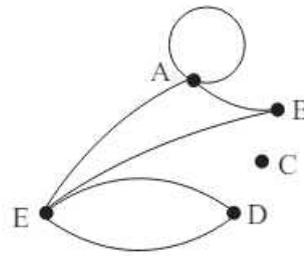
c) Redraw graph 2 to represent it as a planar graph. [1 mark]

Sample response	The response
	<ul style="list-style-type: none"> <li>correctly draws graph 2 as a simple connected graph with seven edges that do not cross and five vertices (one degree 2 vertex, four degree 3 vertices) [1 mark]</li> </ul>

2023  
Paper 1  
Section 2  
Question 23

Graphs and networks

A network graph is shown.



a) State the degree of vertex E. [1 mark]

Sample response	The response
Degree = 4	<ul style="list-style-type: none"> <li>correctly states the degree [1 mark]</li> </ul>

b) State the number of edges joining D and E. [1 mark]

Sample response	The response
2 edges	<ul style="list-style-type: none"> <li>correctly states the number of edges [1 mark]</li> </ul>

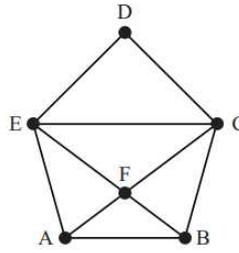
c) Construct an adjacency matrix from the graph with the vertices in alphabetical order. [3 marks]

Sample response	The response
$  \begin{array}{c}  \begin{array}{ccccc}  & A & B & C & D & E \\  A & \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \end{bmatrix} \\  B & \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \end{bmatrix} \\  C & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\  D & \begin{bmatrix} 0 & 0 & 0 & 0 & 2 \end{bmatrix} \\  E & \begin{bmatrix} 1 & 1 & 0 & 2 & 0 \end{bmatrix}  \end{array}  \end{array}  $	<ul style="list-style-type: none"> <li>correctly completes entries for one row or one column in a 5 x 5 matrix [1 mark]</li> <li>correctly enters 1 for number of edges joining A to A [1 mark]</li> <li>completes adjacency matrix [1 mark]</li> </ul>

**2020**  
**Paper 1**  
**Section 2**  
**Question 16**

**Graphs and networks**

Use the graph to identify whether each of the following is a cycle, an open walk, an open trail or a closed trail.



a) AFCFB [1 mark]

Sample Response	The response
AFCFB starts and ends at a different vertex edge repeats: FC = CF ∴ Open walk	<ul style="list-style-type: none"> <li>correctly identifies an open walk [1 mark]</li> </ul>

b) AFCEFBA [1 mark]

Sample Response	The response
AFCEFBA starts and ends at the same vertex no edges are repeated vertex F is repeated ∴ Closed trail	<ul style="list-style-type: none"> <li>correctly identifies a closed trail [1 mark]</li> </ul>

c) ABCDEFA [1 mark]

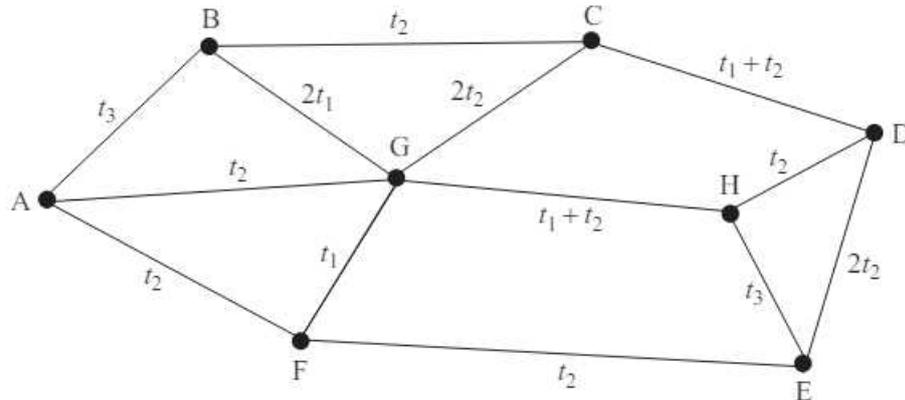
Sample Response	The response
ABCDEFA starts and ends at the same vertex no edges are repeated no vertices are repeated ∴ Cycle	<ul style="list-style-type: none"> <li>correctly identifies a cycle [1 mark]</li> </ul>

Marking Guide – Paper 2 Section 1

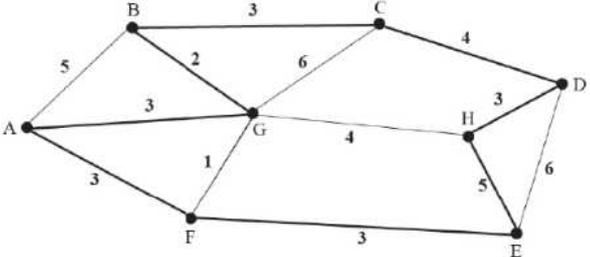
2023  
Paper 2  
Section 1  
Question 5  
Graphs and networks

At 9:00 am, a security guard begins their patrol of the eight work sites represented in the network diagram, starting and ending at site A. They drive at 40 km/h on the roads between sites and check every site once for 15 minutes.

The length (km) of each road corresponds to the terms of the arithmetic sequence  $t_n = t_1 + 2(n - 1)$ , where  $t_1 = 1$ .



Determine the earliest possible time the security guard can finish their patrol, and identify the route they must follow. (7 marks)

Sample response	The response
<p>Hamiltonian cycle starting and ending at A (by trial and error).</p>  <p>Shortest distance = <math>3 + 2 + 3 + 4 + 3 + 5 + 3 + 3</math> = 26 km</p>	<ul style="list-style-type: none"> <li>correctly applies arithmetic sequence to determine all road lengths [1 mark]</li> <li>identifies Hamiltonian cycle beginning at A [1 mark]</li> <li>calculates shortest total distance [1 mark]</li> </ul>
<p>Time to drive the shortest distance</p> $t = \frac{d}{s}$ $= \frac{26}{40}$ $= 0.65 \text{ h}$ $= 39 \text{ min}$ <p>Time to check eight stations = <math>8 \times 15 = 120 \text{ min}</math></p> <p>Total time for patrol = <math>39 + 120</math> = 159 min = 2h 39 min</p> <p>Time when guard finishes patrol = 9:00 am + 2 h 39 min = 11:39 am</p>	<ul style="list-style-type: none"> <li>calculates total time to drive shortest distance [1 mark]</li> <li>calculates total time spent at eight sites [1 mark]</li> <li>calculates total time for patrol [1 mark]</li> <li>determines time of day [1 mark]</li> </ul>

2022  
Paper 2  
Section 1  
Question 4

Graphs and  
networks

The table shows the current road length (in kilometres) between six towns.

	Manon	Veria	Bolint	Farra	Recen	Alin
Manon		16	34	—	—	33
Veria			12	—	—	15
Bolint				—	10	—
Farra					15	23
Recen						15
Alin						

The government plans to build a direct road between Manon and Farra. Use a network diagram to determine the length of the direct road if it is to be 4 km shorter than the length of the current shortest road route between Manon and Farra. (5 marks)

Sample Response	The response
<p>Network diagram</p>	<ul style="list-style-type: none"> <li>• correctly represents the connected towns as a network [1 mark]</li> <li>• correctly includes lengths on the network [1 mark]</li> </ul>
<p>Shortest path = M-V-B-R-F = 16 + 12 + 10 + 15 = 53</p>	<ul style="list-style-type: none"> <li>• determines shortest current path [1 mark]</li> <li>• identifies length of shortest path [1 mark]</li> </ul>
<p>New road length = 53 – 4 = 49 The new road will be 49 km long.</p>	<ul style="list-style-type: none"> <li>• determines new road length [1 mark]</li> </ul>

## Unit 4 – Topic 3: Networks and decision mathematics

### Paper 1 Section 1

**2023 Paper 1 Section 1 Question 3**  
**Networks and decision mathematics**

The duration, in minutes, of all activities in a project are shown.

Activity	P	Q	R	S	T	U	V
Duration	38	42	32	34	16	14	26

The critical path for the project is PRSV.

What is the earliest completion time for the project if it starts at 11:00 am?

(A) 12:30 pm  
 (B) 1:10 pm  
 (C) 1:30 pm  
 (D) 2:10 pm

**2023 Paper 1 Section 1 Question 8**  
**Networks and decision mathematics**

Activities P and Q are the critical activities for a project.

Activity	Duration	Prerequisite activity
P	3	–
Q	6	P

What are the earliest starting time (EST) and latest starting time (LST) for Activity Q?

	EST	LST
(A)	3	3
(B)	3	6
(C)	6	6
(D)	6	9

**2022 Paper 1 Section 1 Question 1**  
**Networks and decision mathematics**

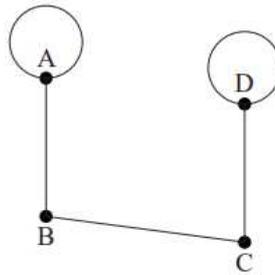
The float time, in minutes, for the non-critical activity of this project network is

(A) 10  
 (B) 20  
 (C) 23  
 (D) 30

2022  
Paper 1  
Section 1  
Question 6

Graphs and networks

This semi-Eulerian graph can be changed to a Eulerian graph by



- (A) adding a loop to vertex B.
- (B) removing the loop at vertex A.
- (C) adding an edge between vertices A and D.
- (D) removing the edge between vertices B and C.

2022  
Paper 1  
Section 1  
Question 7

Networks and decision mathematics

This matrix was obtained after applying the Hungarian algorithm to determine the optimal allocation of three people, Elandra (E), Farid (F) and Grace (G), to three tasks: legal (L), monitoring (M) and verification (V).

	L	M	V
E	0	0	7
F	0	3	8
G	1	0	0

The optimal allocation is

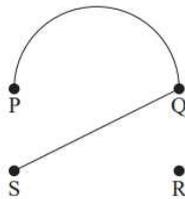
- (A) E to V, F to M and G to L.
- (B) E to V, F to L and G to M.
- (C) E to M, F to L and G to V.
- (D) E to M, F to V and G to L.

2022  
Paper 1  
Section 1  
Question 9

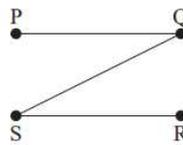
Networks and decision mathematics

Identify the graph that is a spanning tree.

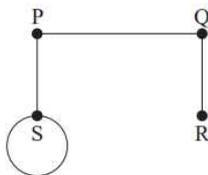
(A)



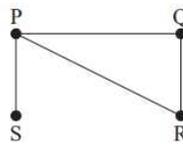
(B)



(C)



(D)



**2021 Paper 1 Section 1 Question 7**  
**Networks and decision mathematics**

Which cut ( $C_1$ ,  $C_2$ ,  $C_3$  or  $C_4$ ) could be used to determine the maximum flow from the source to the sink in this network?

Not to scale

(A)  $C_1$   
 (B)  $C_2$   
 (C)  $C_3$   
 (D)  $C_4$

**2021 Paper 1 Section 1 Question 12**  
**Networks and decision mathematics**

Which statement is correct?

(A) A minimum spanning tree must contain a loop.  
 (B) A minimum spanning tree must contain a cycle.  
 (C) Every network has only one minimum spanning tree.  
 (D) A minimum spanning tree has one more vertex than the number of edges.

**2020 Paper 1 Section 1 Question 7**  
**Networks and decision mathematics**

What is the length of the minimum spanning tree for this network? All distances are in kilometres (km).

(A) 22 km  
 (B) 14 km  
 (C) 12 km  
 (D) 3 km

**2020 Paper 1 Section 1 Question 12**  
**Networks and decision mathematics**

The activity times in the project network shown are in days.

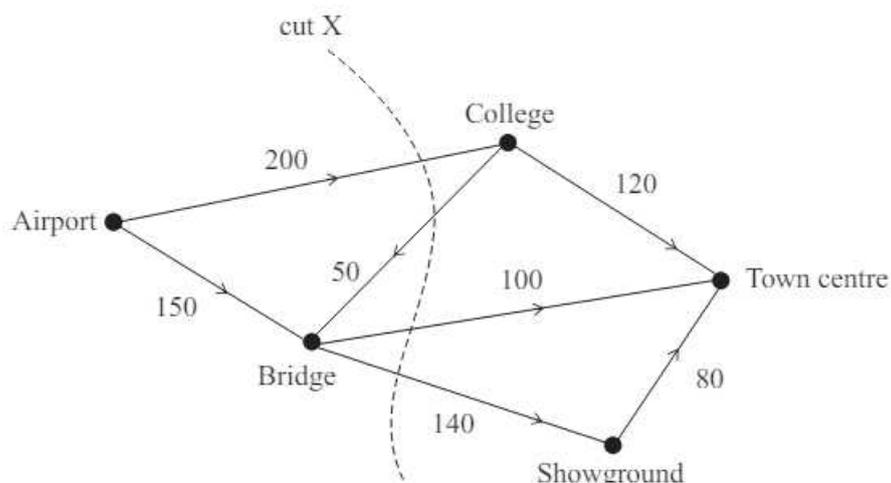
The greatest float time for a non-critical activity in this network is

(A) 2 days  
 (B) 4 days.  
 (C) 5 days.  
 (D) 12 days.

2023  
Paper 1  
Section 2  
Question 21

Networks  
and decision  
mathematics

The road network shows the number of vehicles per hour travelling from the airport to the town centre.



a) Determine the capacity of cut X. [1 mark]

---



---

b) Determine the maximum flow from the airport to the town centre. [2 marks]

---



---



---



---



---



---



---



---

c) During a weather emergency, all roads to and from the bridge are closed to vehicles. Determine the maximum flow from the airport to the town centre during this time. [1 mark]

---



---

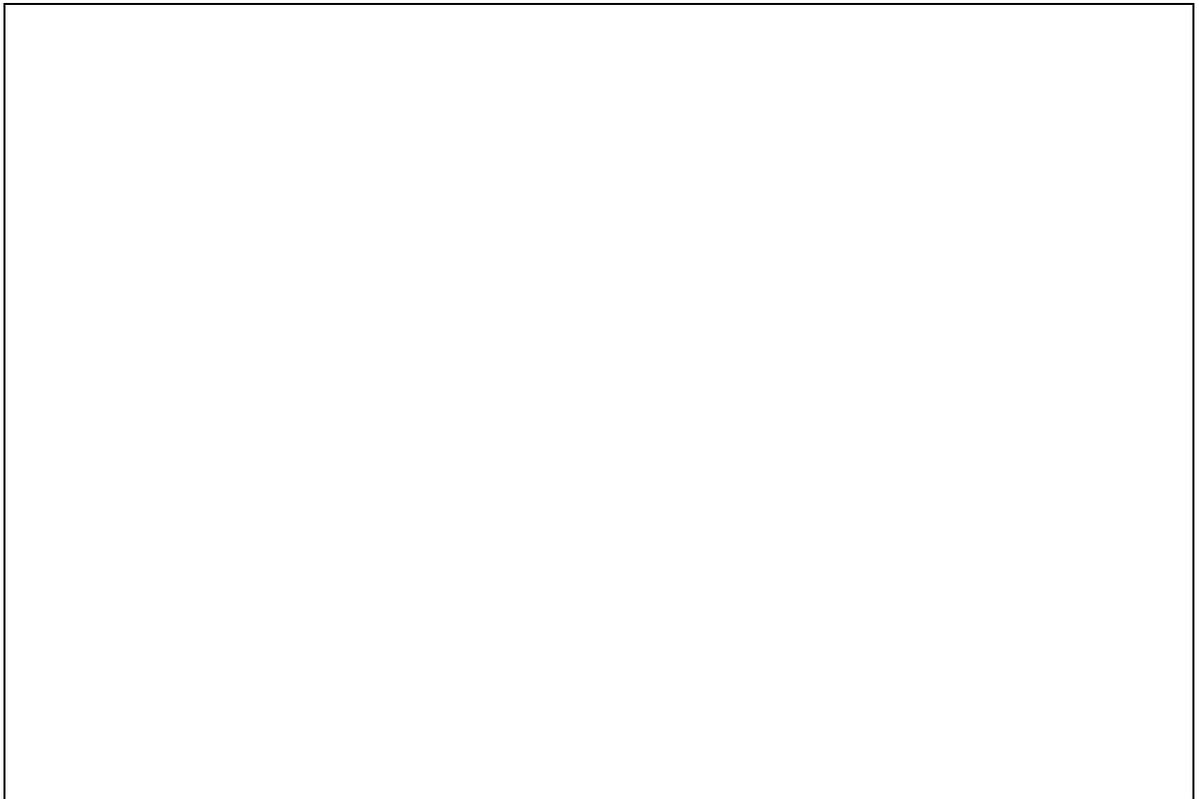
**2023  
Paper 1  
Section 2  
Question 24**

**Networks  
and decision  
mathematics**

Jed is preparing and serving a meal of curry and rice with naan bread. The table shows the duration and prerequisites for all the tasks they must complete.

Task	Task description	Duration (minutes)	Prerequisite
A	Assemble equipment	2	—
B	Boil rice	20	A
C	Prepare curry ingredients	6	A
D	Make naan bread	8	A
E	Simmer curry	40	C
F	Fry naan bread	4	D
G	Serve meal	2	B, E, F

a) Construct a network diagram to show the sequence of tasks from start to finish, labelling all tasks and durations. Use forward and backward scanning to show the earliest and latest starting times for all tasks. [3 marks]



b) Determine the critical activities and minimum completion time for preparing and serving the meal. [2 marks]

---

---

---

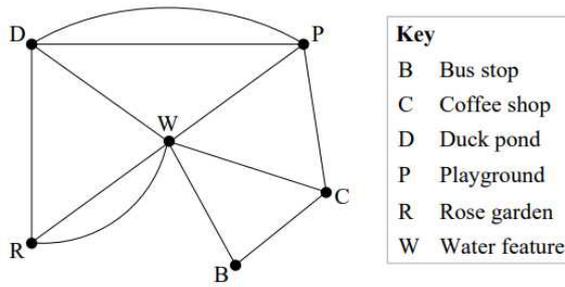
---



**2022  
Paper 1  
Section 2  
Question 21**

**Graphs and  
networks**

The paths connecting various landmarks in a park are shown.



a) Identify one cycle that passes through the rose garden and the playground. [1 mark]

---

b) Identify whether the graph is Eulerian or semi-Eulerian. Justify your response. [2 marks]

---



---



---

c) Construct an adjacency matrix from the graph, using the vertex order listed in the key. [2 marks]

---



---



---



---



---



---



---



---

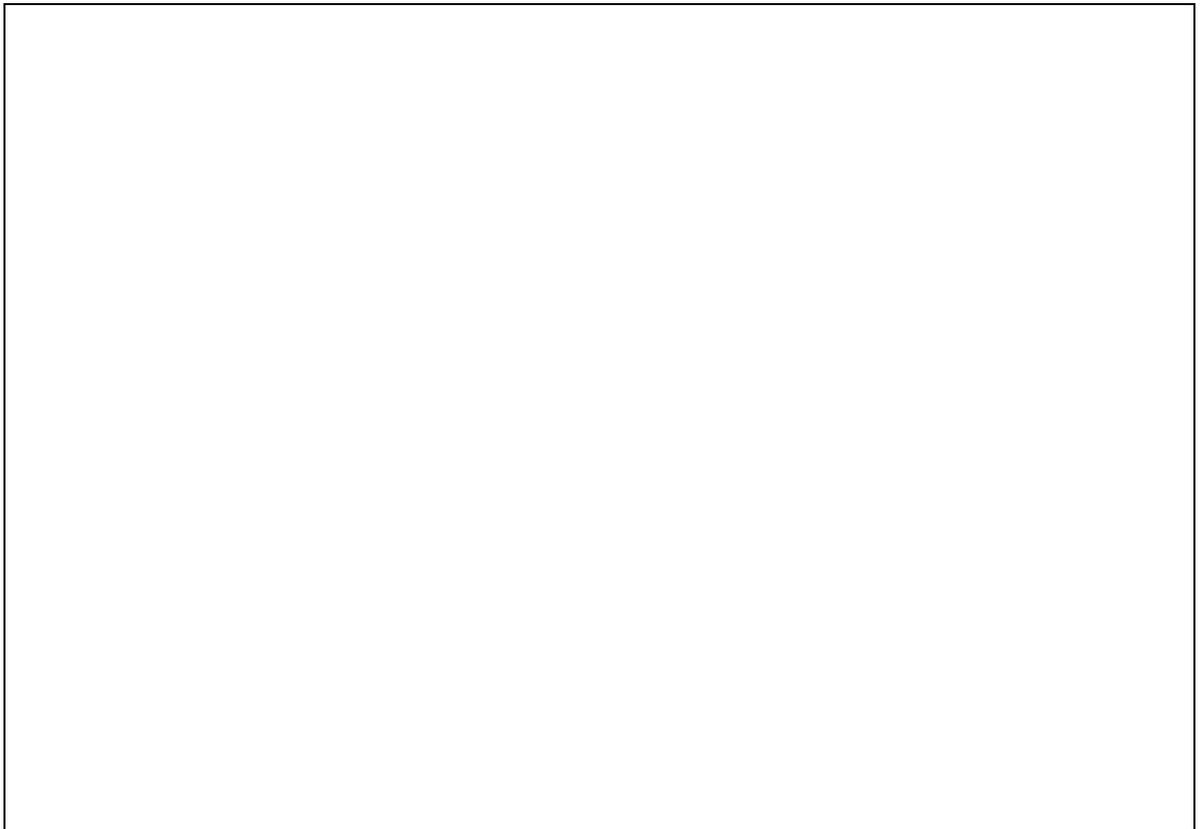
**2021  
Paper 1  
Section 2  
Question 19**

**Networks  
and decision  
mathematics**

The activity table for a project is shown.

Activity	Prerequisite activity	Time (days)
A	—	2
B	—	4
C	A	3
D	B	6
E	D	3
F	C, E	4
G	D	8
H	F, G	4

a) Use the activity table to construct a network diagram, including earliest and latest starting times. [3 marks]



Note: If you make a mistake in the diagram, cancel it by ruling a single diagonal line through your work and use the additional response space on page 20 of this question and response book.

b) Determine the critical path. [1 mark]

---

---

c) Determine the shortest completion time for the project. [1 mark]

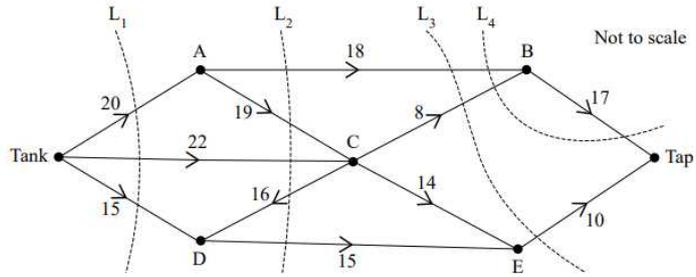
---

---

**2021  
Paper 1  
Section 2  
Question 23**

**Networks  
and decision  
mathematics**

The network diagram shows the flow of water from the tank (source) to the kitchen tap (sink).



a) Explain which dashed line ( $L_1$ ,  $L_2$ ,  $L_3$  or  $L_4$ ) is not a valid cut. [1 mark]

---



---

b) Calculate the capacity of each dashed line that is a valid cut. [3 marks]

---



---



---



---



---



---



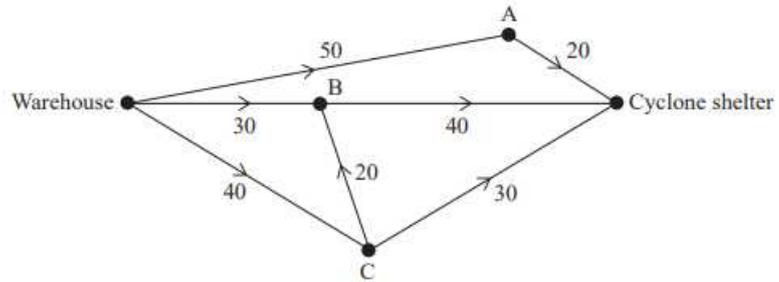
---



---

**2020**  
**Paper 1**  
**Section 2**  
**Question 21**  
  
**Networks**  
**and decision**  
**mathematics**

This network shows the maximum number of supplies (in tonnes) that can be transported from a warehouse to a cyclone shelter along each road each day during an emergency.



Note: If you make a mistake in the network, cancel it by ruling a single diagonal line through your work and use the additional network on page 18 of this question and response book.

a) Use the ‘maximum flow, minimum cut’ theorem to determine the maximum amount of supplies that can reach the cyclone shelter each day. [3 marks]

---

---

---

---

---

---

---

---

b) During a cyclone, the intersection at vertex A is damaged and no longer allows for any supplies to pass through it. What is the new maximum amount of supplies each day that can reach the cyclone shelter? [2 marks]

---

---

---

---

















**Marking Guide – Paper 1 Section 1**

**2023 Paper 1 Section 1 Question 3**  
**Networks and decision mathematics**

The duration, in minutes, of all activities in a project are shown.

Activity	P	Q	R	S	T	U	V
Duration	38	42	32	34	16	14	26

The critical path for the project is PRSV.

What is the earliest completion time for the project if it starts at 11:00 am?

(A) 12:30 pm  
**(B) 1:10 pm – Answer**  
 (C) 1:30 pm  
 (D) 2:10 pm

**2023 Paper 1 Section 1 Question 8**  
**Networks and decision mathematics**

Activities P and Q are the critical activities for a project.

Activity	Duration	Prerequisite activity
P	3	–
Q	6	P

What are the earliest starting time (EST) and latest starting time (LST) for Activity Q?

	EST	LST
(A)	3	3
(B)	3	6
(C)	6	6
(D)	6	9

**Answer is A.**

**2022 Paper 1 Section 1 Question 1**  
**Networks and decision mathematics**

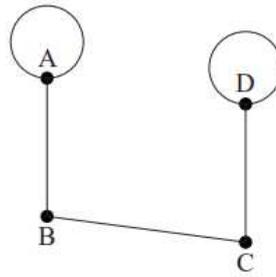
The float time, in minutes, for the non-critical activity of this project network is

(A) 10  
**(B) 20 - Answer**  
 (C) 23  
 (D) 30

2022  
Paper 1  
Section 1  
Question 6

Graphs and networks

This semi-Eulerian graph can be changed to a Eulerian graph by



- (A) adding a loop to vertex B.
- (B) removing the loop at vertex A.
- (C) adding an edge between vertices A and D. - Answer**
- (D) removing the edge between vertices B and C.

2022  
Paper 1  
Section 1  
Question 7

Networks and decision mathematics

This matrix was obtained after applying the Hungarian algorithm to determine the optimal allocation of three people, Elandra (E), Farid (F) and Grace (G), to three tasks: legal (L), monitoring (M) and verification (V).

	L	M	V
E	0	0	7
F	0	3	8
G	1	0	0

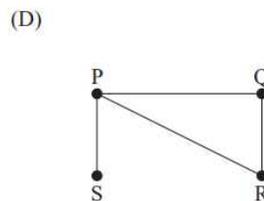
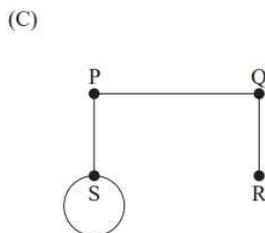
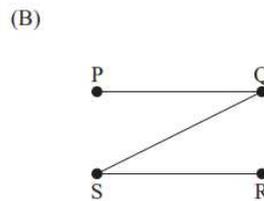
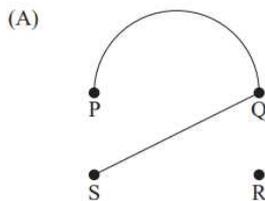
The optimal allocation is

- (A) E to V, F to M and G to L.
- (B) E to V, F to L and G to M.
- (C) E to M, F to L and G to V. - Answer**
- (D) E to M, F to V and G to L.

2022  
Paper 1  
Section 1  
Question 9

Networks and decision mathematics

Identify the graph that is a spanning tree.



**Answer is B.**

**2021 Paper 1 Section 1 Question 7**

**Networks and decision mathematics**

Which cut ( $C_1$ ,  $C_2$ ,  $C_3$  or  $C_4$ ) could be used to determine the maximum flow from the source to the sink in this network?

(A)  $C_1$  - Answer  
 (B)  $C_2$   
 (C)  $C_3$   
 (D)  $C_4$

**2021 Paper 1 Section 1 Question 12**

**Networks and decision mathematics**

Which statement is correct?

(A) A minimum spanning tree must contain a loop.  
 (B) A minimum spanning tree must contain a cycle.  
 (C) Every network has only one minimum spanning tree.  
 (D) **A minimum spanning tree has one more vertex than the number of edges. - Answer**

**2020 Paper 1 Section 1 Question 7**

**Networks and decision mathematics**

What is the length of the minimum spanning tree for this network? All distances are in kilometres (km).

(A) 22 km  
 (B) 14 km  
 (C) **12 km - Answer**  
 (D) 3 km

**2020 Paper 1 Section 1 Question 12**

**Networks and decision mathematics**

The activity times in the project network shown are in days.

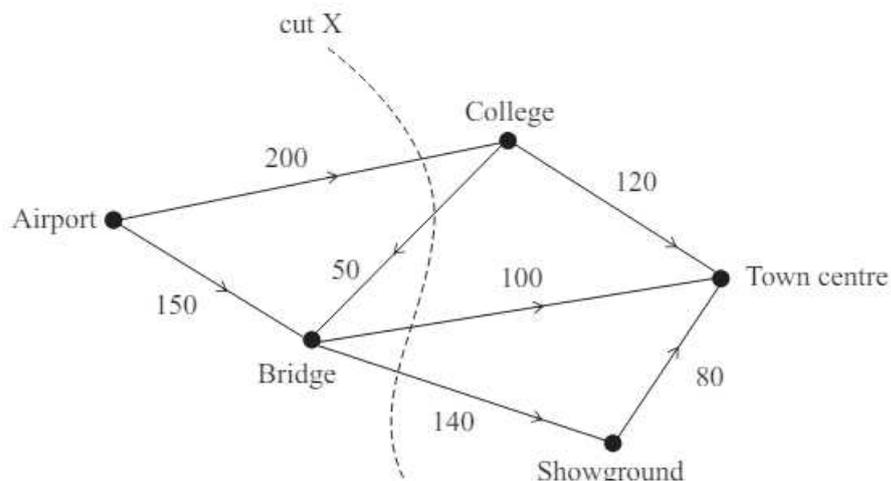
The greatest float time for a non-critical activity in this network is

(A) **2 days. - Answer**  
 (B) 4 days.  
 (C) 5 days.  
 (D) 12 days.

2023  
Paper 1  
Section 2  
Question 21

Networks  
and decision  
mathematics

The road network shows the number of vehicles per hour travelling from the airport to the town centre.



a) Determine the capacity of cut X. [1 mark]

Sample response	The response
Capacity of cut X = $200 + 100 + 140$ = 440 vehicles per hour	<ul style="list-style-type: none"> <li>correctly determines the capacity of cut X [1 mark]</li> </ul>

b) Determine the maximum flow from the airport to the town centre. [2 marks]

Sample response	The response
<p>Maximum flow = <math>120 + 100 + 80</math> = 300 vehicles per hour</p>	<ul style="list-style-type: none"> <li>correctly identifies an appropriate method [1 mark]</li> <li>identifies maximum flow [1 mark]</li> </ul>

c) During a weather emergency, all roads to and from the bridge are closed to vehicles. Determine the maximum flow from the airport to the town centre during this time. [1 mark]

Sample response	The response
Maximum flow during weather emergency = 120 vehicles per hour	<ul style="list-style-type: none"> <li>correctly determines the maximum flow [1 mark]</li> </ul>

**2023  
Paper 1  
Section 2  
Question 24**

**Networks  
and decision  
mathematics**

Jed is preparing and serving a meal of curry and rice with naan bread. The table shows the duration and prerequisites for all the tasks they must complete.

Task	Task description	Duration (minutes)	Prerequisite
A	Assemble equipment	2	—
B	Boil rice	20	A
C	Prepare curry ingredients	6	A
D	Make naan bread	8	A
E	Simmer curry	40	C
F	Fry naan bread	4	D
G	Serve meal	2	B, E, F

a) Construct a network diagram to show the sequence of tasks from start to finish, labelling all tasks and durations. Use forward and backward scanning to show the earliest and latest starting times for all tasks. [3 marks]

Sample response	The response
	<ul style="list-style-type: none"> <li>correctly constructs a network diagram showing the appropriate sequence for all tasks [1 mark]</li> <li>labels all tasks and durations on network diagram [1 mark]</li> <li>shows earliest and latest starting times for all tasks [1 mark]</li> </ul>

b) Determine the critical activities and minimum completion time for preparing and serving the meal. [2 marks]

Sample response	The response
<p>Critical activities: A, C, E, G.</p> <p>Minimum completion time = 2 + 6 + 40 + 2 = 50 minutes</p>	<ul style="list-style-type: none"> <li>determines critical activities [1 mark]</li> <li>determines minimum completion time, including units [1 mark]</li> </ul>

**2022  
Paper 1  
Section 2  
Question 20**

**Networks  
and decision  
mathematics**

The table summarises the distances in kilometres (km) between three flower stores and three delivery locations: A, B and C.

Use the Hungarian algorithm to determine the minimum total distance needed to deliver flowers to all locations if each store delivers flowers to only one location. (4 marks)

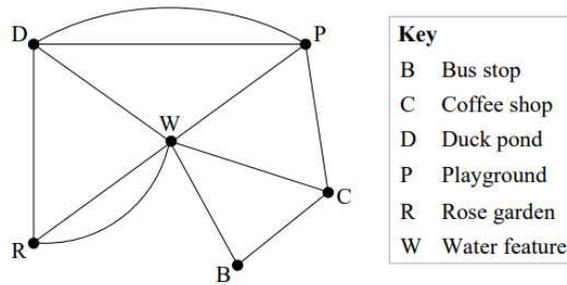
	A	B	C
Store 1	19	17	24
Store 2	15	14	22
Store 3	23	16	40

Sample Response	The response																				
<p>Method 1</p> <table> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>Row reduction</th> </tr> </thead> <tbody> <tr> <td>Store 1</td> <td>19</td> <td>17</td> <td>24</td> <td>-17</td> </tr> <tr> <td>Store 2</td> <td>15</td> <td>14</td> <td>22</td> <td>-14</td> </tr> <tr> <td>Store 3</td> <td>23</td> <td>16</td> <td>40</td> <td>-16</td> </tr> </tbody> </table>		A	B	C	Row reduction	Store 1	19	17	24	-17	Store 2	15	14	22	-14	Store 3	23	16	40	-16	<ul style="list-style-type: none"> <li>correctly reduces each row [1 mark]</li> </ul>
	A	B	C	Row reduction																	
Store 1	19	17	24	-17																	
Store 2	15	14	22	-14																	
Store 3	23	16	40	-16																	
<p>Column reduction</p> <table> <tbody> <tr> <td></td> <td><math>\begin{bmatrix} 2 &amp; 0 &amp; 7 \\ 1 &amp; 0 &amp; 8 \\ 7 &amp; 0 &amp; 24 \end{bmatrix}</math></td> </tr> <tr> <td></td> <td><math>\begin{bmatrix} 1 &amp; 0 &amp; 0 \\ 0 &amp; 0 &amp; 1 \\ 6 &amp; 0 &amp; 17 \end{bmatrix}</math></td> </tr> </tbody> </table>		$\begin{bmatrix} 2 & 0 & 7 \\ 1 & 0 & 8 \\ 7 & 0 & 24 \end{bmatrix}$		$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 6 & 0 & 17 \end{bmatrix}$	<ul style="list-style-type: none"> <li>correctly reduces each column [1 mark]</li> </ul>																
	$\begin{bmatrix} 2 & 0 & 7 \\ 1 & 0 & 8 \\ 7 & 0 & 24 \end{bmatrix}$																				
	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 6 & 0 & 17 \end{bmatrix}$																				
<p>Therefore for minimum distance, Store 1 must deliver to C, Store 2 must deliver to A and Store 3 must deliver to B.</p>	<ul style="list-style-type: none"> <li>identifies which store delivers to which location [1 mark]</li> </ul>																				
<p>Minimum total distance = <math>24 + 15 + 16 = 55</math> km</p>	<ul style="list-style-type: none"> <li>determines minimum total distance [1 mark]</li> </ul>																				

Sample Response	The response																				
<p>Method 2</p> <table> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> </tr> </thead> <tbody> <tr> <td>Store 1</td> <td>19</td> <td>17</td> <td>24</td> </tr> <tr> <td>Store 2</td> <td>15</td> <td>14</td> <td>22</td> </tr> <tr> <td>Store 3</td> <td>23</td> <td>16</td> <td>40</td> </tr> </tbody> </table> <p>Column reduction</p> <table> <tbody> <tr> <td></td> <td><math>\begin{bmatrix} 4 &amp; 3 &amp; 2 \\ 0 &amp; 0 &amp; 0 \\ 8 &amp; 2 &amp; 18 \end{bmatrix}</math></td> </tr> <tr> <td></td> <td><math>\begin{bmatrix} 2 &amp; 1 &amp; 0 \\ 0 &amp; 0 &amp; 0 \\ 6 &amp; 0 &amp; 16 \end{bmatrix}</math></td> </tr> </tbody> </table>		A	B	C	Store 1	19	17	24	Store 2	15	14	22	Store 3	23	16	40		$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 0 & 0 \\ 8 & 2 & 18 \end{bmatrix}$		$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 6 & 0 & 16 \end{bmatrix}$	<ul style="list-style-type: none"> <li>correctly reduces each column [1 mark]</li> </ul>
	A	B	C																		
Store 1	19	17	24																		
Store 2	15	14	22																		
Store 3	23	16	40																		
	$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 0 & 0 \\ 8 & 2 & 18 \end{bmatrix}$																				
	$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 6 & 0 & 16 \end{bmatrix}$																				
<p>Row reduction</p> <table> <tbody> <tr> <td></td> <td><math>\begin{bmatrix} 4 &amp; 3 &amp; 2 \\ 0 &amp; 0 &amp; 0 \\ 8 &amp; 2 &amp; 18 \end{bmatrix}</math></td> </tr> <tr> <td></td> <td><math>\begin{bmatrix} 2 &amp; 1 &amp; 0 \\ 0 &amp; 0 &amp; 0 \\ 6 &amp; 0 &amp; 16 \end{bmatrix}</math></td> </tr> </tbody> </table>		$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 0 & 0 \\ 8 & 2 & 18 \end{bmatrix}$		$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 6 & 0 & 16 \end{bmatrix}$	<ul style="list-style-type: none"> <li>correctly reduces each row [1 mark]</li> </ul>																
	$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 0 & 0 \\ 8 & 2 & 18 \end{bmatrix}$																				
	$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 6 & 0 & 16 \end{bmatrix}$																				
<p>Therefore for minimum distance, Store 1 must deliver to C, Store 2 must deliver to A and Store 3 must deliver to B.</p>	<ul style="list-style-type: none"> <li>identifies which store delivers to which location [1 mark]</li> </ul>																				
<p>Minimum total distance = <math>24 + 15 + 16 = 55</math> km</p>	<ul style="list-style-type: none"> <li>determines minimum total distance [1 mark]</li> </ul>																				

2022  
Paper 1  
Section 2  
Question 21  
  
Graphs and  
networks

The paths connecting various landmarks in a park are shown.



a) Identify one cycle that passes through the rose garden and the playground. [1 mark]

Sample Response	The response
RWPDR	• correctly identifies one cycle [1 mark]

b) Identify whether the graph is Eulerian or semi-Eulerian. Justify your response. [2 marks]

Sample Response	The response
The graph is semi-Eulerian because it has two odd degree vertices and the remaining vertices are even degree	• correctly identifies the graph as semi-Eulerian [1 mark] • justifies the decision [1 mark]

c) Construct an adjacency matrix from the graph, using the vertex order listed in the key. [2 marks]

Sample Response	The response
$  \begin{matrix} & \begin{matrix} B & C & D & P & R & W \end{matrix} \\ \begin{matrix} B \\ C \\ D \\ P \\ R \\ W \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 & 1 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 & 2 & 0 \end{bmatrix} \end{matrix}  $	<ul style="list-style-type: none"> <li>• correctly constructs a valid adjacency matrix with same horizontal and vertical labels [1 mark]</li> <li>• correctly determines values in the adjacency matrix [1 mark]</li> </ul>

**2021  
Paper 1  
Section 2  
Question 19**

**Networks  
and decision  
mathematics**

The activity table for a project is shown.

Activity	Prerequisite activity	Time (days)
A	—	2
B	—	4
C	A	3
D	B	6
E	D	3
F	C, E	4
G	D	8
H	F, G	4

a) Use the activity table to construct a network diagram, including earliest and latest starting times. [3 marks]

Note: If you make a mistake in the diagram, cancel it by ruling a single diagonal line through your work and use the additional response space on page 20 of this question and response book.

Sample Response	The response
	<ul style="list-style-type: none"> <li>• correctly translates the information into a network diagram [1 mark]</li> <li>• correctly labels each activity letter and duration [1 mark]</li> <li>• provides evidence of forward and backward scanning [1 mark]</li> </ul>

b) Determine the critical path. [1 mark]

Sample Response	The response
BDGH	• determines critical path [1 mark]

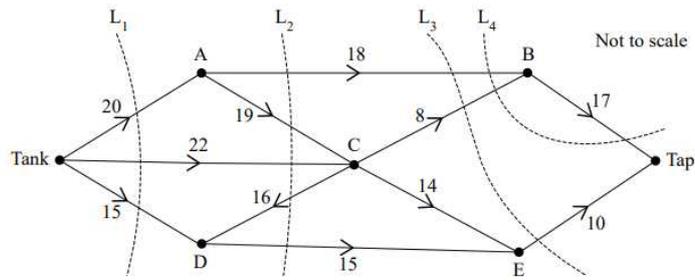
c) Determine the shortest completion time for the project. [1 mark]

Sample Response	The response
22 days	• determines shortest time [1 mark]

**2021  
Paper 1  
Section 2  
Question 23**

**Networks  
and decision  
mathematics**

The network diagram shows the flow of water from the tank (source) to the kitchen tap (sink).



a) Explain which dashed line ( $L_1$ ,  $L_2$ ,  $L_3$  or  $L_4$ ) is not a valid cut. [1 mark]

Sample Response	The response
$L_4$ is not valid because the tank and the tap are on the same side of the line.	<ul style="list-style-type: none"> <li>correctly explains why <math>L_4</math> is not a valid cut [1 mark]</li> </ul>

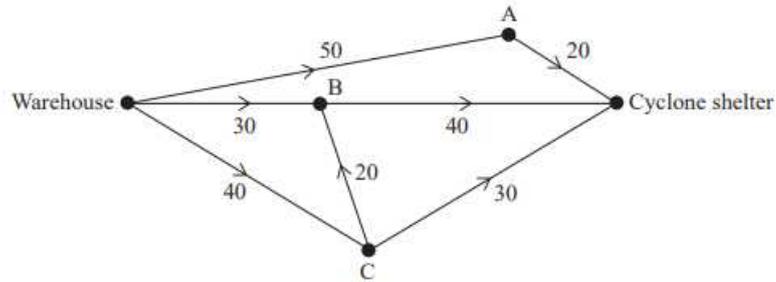
b) Calculate the capacity of each dashed line that is a valid cut. [3 marks]

Sample Response	The response
$L_1$ capacity = $20 + 22 + 15 = 57$	<ul style="list-style-type: none"> <li>correctly determines the <math>L_1</math> capacity [1 mark]</li> </ul>
$L_2$ capacity = $18 + 19 + 22 + 15 = 74$	<ul style="list-style-type: none"> <li>correctly determines the <math>L_2</math> capacity [1 mark]</li> </ul>
$L_3$ capacity = $18 + 8 + 10 = 36$	<ul style="list-style-type: none"> <li>correctly determines the <math>L_3</math> capacity [1 mark]</li> </ul>

**2020  
Paper 1  
Section 2  
Question 21**

**Networks  
and decision  
mathematics**

This network shows the maximum number of supplies (in tonnes) that can be transported from a warehouse to a cyclone shelter along each road each day during an emergency.



Note: If you make a mistake in the network, cancel it by ruling a single diagonal line through your work and use the additional network on page 18 of this question and response book.

a) Use the ‘maximum flow, minimum cut’ theorem to determine the maximum amount of supplies that can reach the cyclone shelter each day. [3 marks]

Sample Response	The response
<p>                     Cut 1 <math>\rightarrow 50+30+40 = 120</math>                      Cut 2 <math>\rightarrow 50+30+20+30 = 130</math>                      Cut 3 <math>\rightarrow 50+40+40 = 130</math>                      Cut 4 <math>\rightarrow 50+40+30 = 120</math>                      Cut 5 <math>\rightarrow 20+30+40 = 90</math>                      Cut 6 <math>\rightarrow 20+40+40 = 100</math>                      Cut 7 <math>\rightarrow 20+40+30 = 90</math> </p>	<ul style="list-style-type: none"> <li>• correctly identifies all the possible cuts [1 mark]</li> <li>• correctly calculates the flow across all the cuts [1 mark]</li> </ul>
<p>The capacity of the minimum cut is 90, so the maximum flow of this network is 90.</p>	<ul style="list-style-type: none"> <li>• states maximum flow across minimum cut [1 mark]</li> </ul>

b) During a cyclone, the intersection at vertex A is damaged and no longer allows for any supplies to pass through it. What is the new maximum amount of supplies each day that can reach the cyclone shelter? [2 marks]

Sample Response	The response
<p>Cut 1, 2, 3, 4, 5, 6 and 7 change to 70, 80, 80, 70, 70, 80 and 70.</p>	<ul style="list-style-type: none"> <li>• recalculates flow across minimum cut [1 mark]</li> </ul>
<p>The maximum flow is now 70.</p>	<ul style="list-style-type: none"> <li>• states new maximum flow [1 mark]</li> </ul>

Marking Guide – Paper 2 Section 1

2023  
Paper 2  
Section 1  
Question 1

Networks  
and decision  
mathematics

A triathlon relay has three sections: swim (S), cycle (C) and run (R). The matrix shows the average number of minutes for three athletes, Jane (J), Knox (K) and Levi (L), to complete each section.

$$\begin{matrix} & \text{S} & \text{C} & \text{R} \\ \text{J} & \begin{bmatrix} 40 & 56 & 66 \end{bmatrix} \\ \text{K} & \begin{bmatrix} 36 & 60 & 72 \end{bmatrix} \\ \text{L} & \begin{bmatrix} 25 & 48 & 78 \end{bmatrix} \end{matrix}$$

Use the Hungarian algorithm to predict the minimum total relay time if assigning each athlete to completing one section. (5 marks)

Sample response	The response																																																									
<p><b>Method 1</b></p> <table style="margin-left: 20px;"> <tr><td></td><td>S</td><td>C</td><td>R</td><td></td></tr> <tr><td>J</td><td>40</td><td>56</td><td>66</td><td></td></tr> <tr><td>K</td><td>36</td><td>60</td><td>72</td><td></td></tr> <tr><td>L</td><td>25</td><td>48</td><td>78</td><td></td></tr> <tr><td>column reduction</td><td>-25</td><td>-48</td><td>-66</td><td></td></tr> </table> <table style="margin-left: 20px;"> <tr><td></td><td></td><td></td><td></td><td>row reduction</td></tr> <tr><td></td><td>15</td><td>8</td><td>0</td><td>-0</td></tr> <tr><td></td><td>11</td><td>12</td><td>6</td><td>-6</td></tr> <tr><td></td><td>0</td><td>0</td><td>12</td><td>-0</td></tr> </table> <table style="margin-left: 20px;"> <tr><td></td><td>15</td><td>8</td><td>0</td></tr> <tr><td></td><td>5</td><td>6</td><td>0</td></tr> <tr><td></td><td>0</td><td>0</td><td>12</td></tr> </table>		S	C	R		J	40	56	66		K	36	60	72		L	25	48	78		column reduction	-25	-48	-66						row reduction		15	8	0	-0		11	12	6	-6		0	0	12	-0		15	8	0		5	6	0		0	0	12	<ul style="list-style-type: none"> <li>correctly reduces each column [1 mark]</li> <li>reduces each row [1 mark]</li> </ul>
	S	C	R																																																							
J	40	56	66																																																							
K	36	60	72																																																							
L	25	48	78																																																							
column reduction	-25	-48	-66																																																							
				row reduction																																																						
	15	8	0	-0																																																						
	11	12	6	-6																																																						
	0	0	12	-0																																																						
	15	8	0																																																							
	5	6	0																																																							
	0	0	12																																																							
<p>Number of lines needed to cover all zeroes &lt; number of tasks 2 &lt; 3, so continue algorithm steps. Smallest uncovered number is 5. Subtract 5 from all uncovered numbers and add 5 to number covered twice.</p> <table style="margin-left: 20px;"> <tr><td></td><td>10</td><td>3</td><td>0</td></tr> <tr><td></td><td>0</td><td>1</td><td>0</td></tr> <tr><td></td><td>0</td><td>0</td><td>17</td></tr> </table> <p>Number of lines needed to cover all zeroes = number of tasks 3 = 3, so assign tasks. To minimise the total relay time, assign <b>Jane to run, Knox to swim and Levi to cycle.</b></p>		10	3	0		0	1	0		0	0	17	<ul style="list-style-type: none"> <li>continues algorithm steps until number of lines needed to cover all zeroes equals number of tasks [1 mark]</li> <li>assigns each athlete to complete one section [1 mark]</li> </ul>																																													
	10	3	0																																																							
	0	1	0																																																							
	0	0	17																																																							
<p>Predicted minimum total relay time = 66 + 36 + 48 = 150 min = 2 h 30 min</p>	<ul style="list-style-type: none"> <li>predicts minimum total relay time including units [1 mark]</li> </ul>																																																									
<p><b>Method 2</b></p> <table style="margin-left: 20px;"> <tr><td></td><td>S</td><td>C</td><td>R</td><td>row reduction</td></tr> <tr><td>J</td><td>40</td><td>56</td><td>66</td><td>-40</td></tr> <tr><td>K</td><td>36</td><td>60</td><td>72</td><td>-36</td></tr> <tr><td>L</td><td>25</td><td>48</td><td>78</td><td>-25</td></tr> </table> <table style="margin-left: 20px;"> <tr><td></td><td>0</td><td>16</td><td>26</td></tr> <tr><td></td><td>0</td><td>24</td><td>36</td></tr> <tr><td></td><td>0</td><td>23</td><td>53</td></tr> </table> <table style="margin-left: 20px;"> <tr><td>column reduction</td><td>-0</td><td>-16</td><td>-26</td></tr> </table> <table style="margin-left: 20px;"> <tr><td></td><td>0</td><td>0</td><td>0</td></tr> <tr><td></td><td>0</td><td>8</td><td>10</td></tr> <tr><td></td><td>0</td><td>7</td><td>27</td></tr> </table>		S	C	R	row reduction	J	40	56	66	-40	K	36	60	72	-36	L	25	48	78	-25		0	16	26		0	24	36		0	23	53	column reduction	-0	-16	-26		0	0	0		0	8	10		0	7	27	<ul style="list-style-type: none"> <li>correctly reduces each row [1 mark]</li> <li>reduces each column [1 mark]</li> </ul>									
	S	C	R	row reduction																																																						
J	40	56	66	-40																																																						
K	36	60	72	-36																																																						
L	25	48	78	-25																																																						
	0	16	26																																																							
	0	24	36																																																							
	0	23	53																																																							
column reduction	-0	-16	-26																																																							
	0	0	0																																																							
	0	8	10																																																							
	0	7	27																																																							

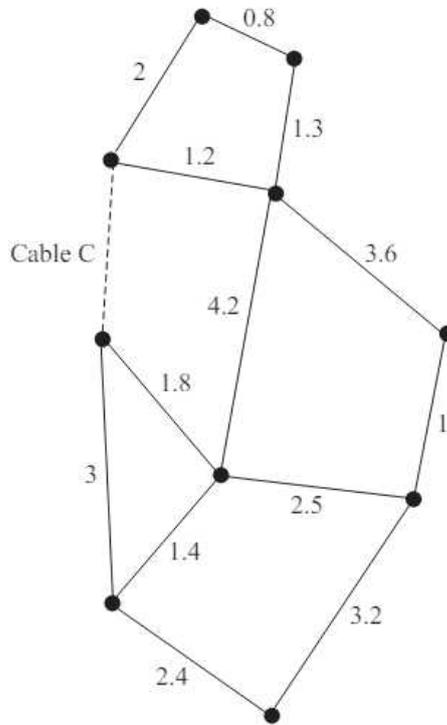
	<p>Number of lines needed to cover all zeroes &lt; number of tasks  <math>2 &lt; 3</math>, so continue algorithm steps.          Smallest uncovered number is 7. Subtract 7 from all uncovered numbers and add 7 to number covered twice.</p> $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 20 \end{bmatrix}$ <p>Number of lines needed to cover all zeroes = number of tasks  <math>3 = 3</math>, so assign tasks.          To minimise the total relay time, assign <b>Jane to run</b>, <b>Knox to swim</b> and <b>Levi to cycle</b>.</p>	<ul style="list-style-type: none"> <li>continues algorithm steps until number of lines needed to cover all zeroes equals number of tasks <b>[1 mark]</b></li> <li>assigns athlete to complete one section <b>[1 mark]</b></li> </ul>
	<p>Predicted minimum total relay time = <math>66 + 36 + 48</math>          = <b>150 min</b>          = <b>2 h 30 min</b></p>	<ul style="list-style-type: none"> <li>predicts minimum total relay time including units <b>[1 mark]</b></li> </ul>

**2023  
Paper 2  
Section 1  
Question 3**

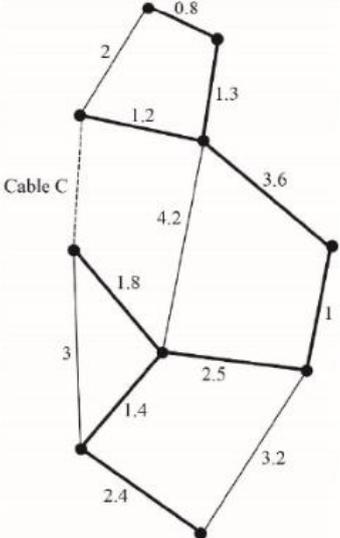
**Networks  
and decision  
mathematics**

The diagram represents a network of 10 ski stations connected by chairlift cables. The length (km) of each cable is shown, except for cable C, which is closed for maintenance. When cable C reopens, the minimum total cable length required to connect all stations will decrease by 1 km.

Determine the length of cable C and the minimum total cable length required to connect all stations when cable C reopens. (5 marks)



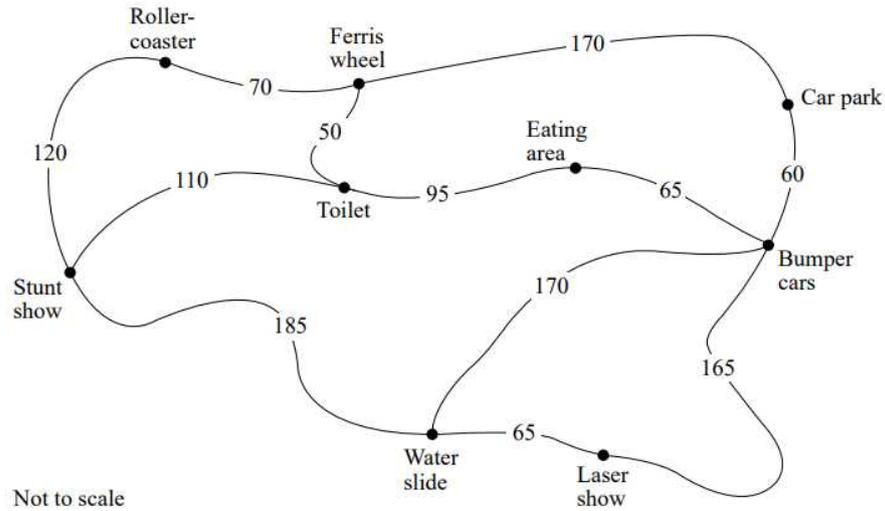
Sample response	The response
<p><b>Method 1</b> Minimum spanning tree without cable C</p> <p>Minimum total cable length without cable C  <math>= 0.8 + 1.3 + 1.2 + 3.6 + 1 + 2.5 + 1.8 + 1.4 + 2.4</math>  <math>= 16 \text{ km}</math></p>	<ul style="list-style-type: none"> <li>• correctly identifies minimum spanning tree without cable C [1 mark]</li> <li>• correctly determines minimum total cable length without cable C [1 mark]</li> </ul>

	<p>Minimum total cable length with cable C  <math>= 16 - 1</math>  <math>= 15 \text{ km}</math>  Length of cable C  <math>= 15 - (0.8 + 1.3 + 1.2 + 1 + 2.5 + 1.8 + 1.4 + 2.4)</math>  <math>= 2.6 \text{ km}</math></p>	<ul style="list-style-type: none"> <li>• correctly determines minimum total cable length without cable C [1 mark]</li> <li>• determines minimum total cable length with cable C [1 mark]</li> <li>• determines length of cable C [1 mark]</li> <li>• shows logical organisation communicating key steps [1 mark]</li> </ul>
	<p><b>Method 2</b>  Minimum spanning tree without cable C</p>  <p>For all stations to be connected by cables, the <b>3.6 km cable is not required</b> when cable C reopens.</p>	<ul style="list-style-type: none"> <li>• correctly identifies minimum spanning tree without cable C [1 mark]</li> <li>• correctly identifies the 3.6 km cable is not required when cable C reopens [1 mark]</li> </ul>
	<p>Length of cable C = <math>3.6 - 1 = 2.6 \text{ km}</math>  Minimum total cable length with cable C  <math>= 0.8 + 1.3 + 1.2 + 2.6 + 1 + 2.5 + 1.8 + 1.4 + 2.4</math>  <math>= 15 \text{ km}</math></p>	<ul style="list-style-type: none"> <li>• determines length of cable C [1 mark]</li> <li>• determines minimum total cable length with cable C [1 mark]</li> <li>• shows logical organisation communicating key steps [1 mark]</li> </ul>

**2022  
Paper 2  
Section 1  
Question 5**

**Networks  
and decision  
mathematics**

The map details the length (in metres) of paths between nine key locations in a theme park. The annual cost to maintain the paths is \$214 per metre. The theme park manager believes at least \$138 000 can be saved each year if some paths are removed, while still allowing visitors to access every key location using paths.



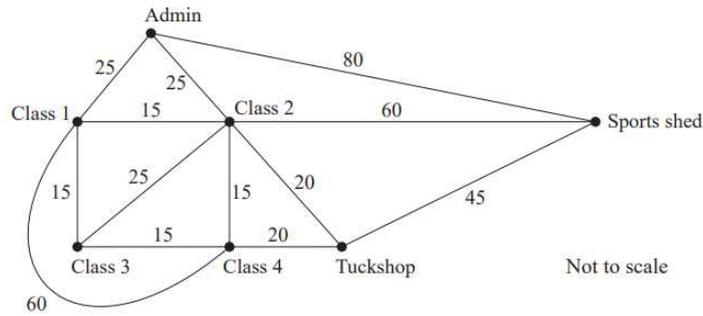
Evaluate the reasonableness of the manager's belief. (5 marks)

Sample Response	The response
<p>Minimum spanning tree</p> <p>Not to scale</p>	<ul style="list-style-type: none"> <li>• correctly identifies minimum spanning tree [1 mark]</li> </ul>
<p>Total length of removed paths = <math>120 + 185 + 170 + 170</math> = 645</p>	<ul style="list-style-type: none"> <li>• correctly calculates total length of the removed paths [1 mark]</li> </ul>
<p>Annual savings = <math>645 \times 214</math> = 138 030</p>	<ul style="list-style-type: none"> <li>• calculates annual savings [1 mark]</li> </ul>
<p><math>\therefore</math> The manager is correct and they will save more than \$138 000 if they remove the unnecessary paths to all nine key locations.</p>	<ul style="list-style-type: none"> <li>• compares values to evaluate the reasonableness of the belief [1 mark]</li> <li>• shows logical organisation communicating key steps [1 mark]</li> </ul>

**2021  
Paper 2  
Section 1  
Question 2**

**Networks  
and decision  
mathematics**

All buildings in a school are connected by underground electricity cables, indicated by the network. All measurements are in metres.



The electricity cables need replacing and will cost \$1200 per metre. The school wants to minimise costs by replacing the shortest length of cable necessary to connect all buildings.

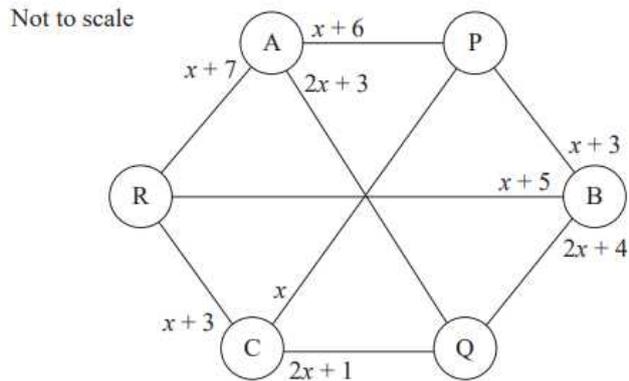
If the school has a budget of \$155 000, evaluate whether they can afford this project. (4 marks)

Sample Response	The response
<p>Minimum spanning tree  <math>= A - C1 - C3 - C4 - C2 - T - SS</math></p>	<ul style="list-style-type: none"> <li>correctly identifies a minimum spanning tree [1 mark]</li> </ul>
<p>Total length = <math>(15 \times 3) + 20 + 25 + 45 = 135</math> m</p>	<ul style="list-style-type: none"> <li>determines - total length of minimum spanning tree [1 mark]</li> <li>OR</li> <li>- cost of each arc of minimum spanning tree [1 mark]</li> </ul>
<p>Total cost = <math>135 \times 1200 = \\$162\ 000</math></p>	<ul style="list-style-type: none"> <li>determines total cost [1 mark]</li> </ul>
<p>Since \$155 000 is less than \$162 000, the school cannot afford the project.</p>	<ul style="list-style-type: none"> <li>determines if the school can afford the project [1 mark]</li> </ul>

**2021  
Paper 2  
Section 1  
Question 6**

**Networks  
and decision  
mathematics**

A tow truck company has three tow trucks (A, B and C) and receives calls from three motorists (P, Q and R), who have broken down. The network shows the distances in kilometres (km) from each of the tow trucks to each of the motorists, where  $x$  represents the distance between Tow Truck C and Motorist P.



The minimum total distance travelled by the three tow trucks in order for each tow truck to visit exactly one motorist is 32 km.

Use the Hungarian algorithm to determine the distance between Tow Truck C and Motorist P. (7 marks)

Sample Response	The response																
<p><b>Hungarian algorithm</b></p> <p>Matrix form</p> <table border="1"> <tr> <td></td> <td>P</td> <td>Q</td> <td>R</td> </tr> <tr> <td>A</td> <td><math>x+6</math></td> <td><math>2x+3</math></td> <td><math>x+7</math></td> </tr> <tr> <td>B</td> <td><math>x+3</math></td> <td><math>2x+4</math></td> <td><math>x+5</math></td> </tr> <tr> <td>C</td> <td><math>x</math></td> <td><math>2x+1</math></td> <td><math>x+3</math></td> </tr> </table>		P	Q	R	A	$x+6$	$2x+3$	$x+7$	B	$x+3$	$2x+4$	$x+5$	C	$x$	$2x+1$	$x+3$	<ul style="list-style-type: none"> <li>correctly converts the network information into a matrix form [1 mark]</li> </ul>
	P	Q	R														
A	$x+6$	$2x+3$	$x+7$														
B	$x+3$	$2x+4$	$x+5$														
C	$x$	$2x+1$	$x+3$														
<p>Row reduction: <math>R_1 - (x+6), R_2 - (x+3), R_3 - x</math></p> <table border="1"> <tr> <td>0</td> <td><math>x-3</math></td> <td>1</td> <td></td> </tr> <tr> <td>0</td> <td><math>x+1</math></td> <td>2</td> <td></td> </tr> <tr> <td>0</td> <td><math>x+1</math></td> <td>3</td> <td></td> </tr> </table>	0	$x-3$	1		0	$x+1$	2		0	$x+1$	3		<ul style="list-style-type: none"> <li>determines each matrix element by reducing each row [1 mark]</li> </ul>				
0	$x-3$	1															
0	$x+1$	2															
0	$x+1$	3															
<p>Column reduction: <math>C_2 - (x-3), C_3 - 1</math></p> <table border="1"> <tr> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>4</td> <td>1</td> </tr> <tr> <td>0</td> <td>4</td> <td>2</td> </tr> </table>	0	0	0	0	4	1	0	4	2	<ul style="list-style-type: none"> <li>determines each matrix element by reducing each column [1 mark]</li> </ul>							
0	0	0															
0	4	1															
0	4	2															
<p>Only 2 lines are needed to cover all the 0s; therefore, need to use Hungarian algorithm with minimum of 1. Add 1 to overlap, subtract 1 from uncovered.</p> <table border="1"> <tr> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>3</td> <td>0</td> </tr> <tr> <td>0</td> <td>3</td> <td>1</td> </tr> </table>	1	0	0	0	3	0	0	3	1	<ul style="list-style-type: none"> <li>correctly applies Hungarian algorithm [1 mark]</li> </ul>							
1	0	0															
0	3	0															
0	3	1															
<p>Bipartite graph</p> <table border="1"> <tr> <td>A</td> <td>P</td> </tr> <tr> <td>B</td> <td>Q</td> </tr> <tr> <td>C</td> <td>R</td> </tr> <tr> <td>AQ</td> <td>BR</td> <td>CP</td> </tr> </table>	A	P	B	Q	C	R	AQ	BR	CP	<ul style="list-style-type: none"> <li>determines minimum allocation [1 mark]</li> </ul>							
A	P																
B	Q																
C	R																
AQ	BR	CP															
<p>Total distance = <math>2x+3+x+5+x</math></p> <p><math>32 = 4x+8</math></p> <p><math>24 = 4x</math></p> <p><math>x = 6</math></p> <p>It is 6 km from C to P.</p>	<ul style="list-style-type: none"> <li>determines <math>x</math> [1 mark]</li> <li>shows logical organisation communicating key steps [1 mark]</li> </ul>																

**2020  
Paper 2  
Section 1  
Question 5**

**Networks  
and decision  
mathematics**

A company has three tasks to allocate to three contractors. Each of the contractors has a quote recorded for each task, shown in the table. The quotes are in thousands of dollars (\$'000s).

Contractor	Task 1	Task 2	Task 3
A	3	3	1
B	4	7	2
C	4	4	1

Use a matrix method to determine the minimum cost if each contractor is allocated one task. (5 marks)

Sample Response	The response															
<p>Matrix form</p> $\begin{matrix} 3 & 3 & 1 \\ 4 & 7 & 2 \\ 4 & 4 & 1 \end{matrix}$ <p>row reduction: <math>R_1 - 1, R_2 - 2, R_3 - 1</math></p> $\begin{matrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 3 & 3 & 0 \end{matrix}$	<ul style="list-style-type: none"> <li>correctly reduces each row [1 mark]</li> </ul>															
<p>only need 1 line to cover all the 0s, ∴ column reduction: <math>C_1 - 2, C_2 - 2</math></p> $\begin{matrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 1 & 1 & 0 \end{matrix}$ <p>need 3 lines to cover all the 0s, ∴ bipartite graph:</p>	<ul style="list-style-type: none"> <li>correctly reduces each column [1 mark]</li> </ul>															
<table border="1"> <thead> <tr> <th>contractor</th> <th>task</th> <th>cost</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>2</td> <td>3</td> </tr> <tr> <td>B</td> <td>1</td> <td>4</td> </tr> <tr> <td>C</td> <td>3</td> <td>1</td> </tr> <tr> <td>Total</td> <td></td> <td>8</td> </tr> </tbody> </table>	contractor	task	cost	A	2	3	B	1	4	C	3	1	Total		8	<ul style="list-style-type: none"> <li>allocates each task to one contractor [1 mark]</li> </ul>
contractor	task	cost														
A	2	3														
B	1	4														
C	3	1														
Total		8														
<p>∴ Minimum cost is \$8000.</p>	<ul style="list-style-type: none"> <li>determines minimum cost [1 mark]</li> <li>shows logical organisation, communicating key steps [1 mark]</li> </ul>															

**2020  
Paper 2  
Section 1  
Question 6**

**Networks  
and decision  
mathematics**

A company needs to complete the following project as quickly as possible. Each task can only be completed by a single employee and must be completed before that employee can start the next task.

Task	Time (days)	Prerequisite
A	3	—
B	4	—
C	2	A
D	8	C
E	5	C
F	4	B
G	3	B
H	1	E, F
I	2	G
J	3	H, I

The owner believes that this project can be completed in minimal time with only three employees. Evaluate the reasonableness of this belief. (7 marks)

Sample Response	The response																																																																																										
	<ul style="list-style-type: none"> <li>correctly translates the information into a network [1 mark]</li> <li>determines LST for each activity [1 mark]</li> <li>determines EST for each activity [1 mark]</li> </ul>																																																																																										
<p>Determine minimum completion time. Shortest path is 14, so with a large enough workforce the job could be completed on the 14th day.</p>	<ul style="list-style-type: none"> <li>determines minimum completion time [1 mark]</li> </ul>																																																																																										
<p>Find how many employees required. At the start of the project only tasks A and B can be done, so employing more than 2 people at the start would be wasteful. If the company employed 3 people as suggested, the following job allocation could be used. Worker 1 follows the critical path to complete the job on day 14. Worker 2 works on non-critical jobs that are available. Day 5 is the first day where having 3 employees would be useful.</p> <table border="1"> <thead> <tr> <th></th> <th colspan="14">Day</th> </tr> <tr> <th></th> <th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th><th>11</th><th>12</th><th>13</th><th>14</th> </tr> </thead> <tbody> <tr> <td>worker 1</td> <td>A</td><td>A</td><td>A</td><td>C</td><td>C</td><td>E</td><td>E</td><td>E</td><td>E</td><td>E</td><td>H</td><td>J</td><td>J</td><td>J</td> </tr> <tr> <td>worker 2</td> <td>B</td><td>B</td><td>B</td><td>B</td><td>F</td><td>F</td><td>F</td><td>F</td><td></td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>worker 3</td> <td></td><td></td><td></td><td></td><td>G</td><td>G</td><td>G</td><td>I</td><td>I</td><td></td><td></td><td></td><td></td><td></td> </tr> <tr> <td>worker 4</td> <td></td><td></td><td></td><td></td><td></td><td></td><td>D</td><td>D</td><td>D</td><td>D</td><td>D</td><td>D</td><td>D</td><td>D</td> </tr> </tbody> </table>		Day															1	2	3	4	5	6	7	8	9	10	11	12	13	14	worker 1	A	A	A	C	C	E	E	E	E	E	H	J	J	J	worker 2	B	B	B	B	F	F	F	F							worker 3					G	G	G	I	I						worker 4							D	D	D	D	D	D	D	D	<ul style="list-style-type: none"> <li>determines whether three workers are sufficient [1 mark]</li> </ul>
	Day																																																																																										
	1	2	3	4	5	6	7	8	9	10	11	12	13	14																																																																													
worker 1	A	A	A	C	C	E	E	E	E	E	H	J	J	J																																																																													
worker 2	B	B	B	B	F	F	F	F																																																																																			
worker 3					G	G	G	I	I																																																																																		
worker 4							D	D	D	D	D	D	D	D																																																																													
<p>But with 3 workers, activity D could not be completed by day 14. The owner's belief is incorrect: at least 4 workers must be employed.</p>	<ul style="list-style-type: none"> <li>evaluates reasonableness of the claim [1 mark]</li> <li>shows logical organisation, communicating key steps [1 mark]</li> </ul>																																																																																										