10

CambridgeMATHS NSW

STAGE 5 – CORE AND STANDARD PATHS

THIRD EDITION

Stuart Palmer, Karen McDaid
David Greenwood, Sara Woolley
Jenny Goodman, Jennifer Vaughan

CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

First Edition © Stuart Palmer, David Greenwood, Jenny Goodman, Jennifer Vaughan, Sara Woolley 2014
Second Edition © Stuart Palmer, Karen McDaid, David Greenwood, Jenny Goodman, Jennifer Vaughan, Sara Woolley 2019
Third Edition © Stuart Palmer, Karen McDaid, David Greenwood, Jenny Goodman, Jennifer Vaughan, Sara Woolley 2024

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press & Assessment.

First published 2014
Second Edition 2019
Third Edition 2024

20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Cover designed by Denise Lane (Sardine Design)
Typeset by diacriTech
Printed in China by C & C Offset Printing Co., Ltd.

A catalogue record for this book is available from the National Library of Australia at www.nla.gov.au

ISBN 978-1-009-40953-7

Additional resources for this publication at www.cambridge.edu.au/GO

Reproduction and Communication for educational purposes
The Australian *Copyright Act 1968* (the Act) allows a maximum of one chapter or 10% of the pages of this publication, whichever is the greater, to be reproduced and/or communicated by any educational institution for its educational purposes provided that the educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.

For details of the CAL licence for educational institutions contact:

Copyright Agency Limited
Level 12, 66 Goulburn Street
Sydney NSW 2000
Telephone: (02) 9394 7600
Facsimile: (02) 9394 7601
Email: memberservices@copyright.com.au

Reproduction and Communication for other purposes
Except as permitted under the Act (for example a fair dealing for the purposes of study, research, criticism or review) no part of this publication may be reproduced, stored in a retrieval system, communicated or transmitted in any form or by any means without prior written permission. All inquiries should be made to the publisher at the address above.

Cambridge University Press & Assessment has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate. Information regarding prices, travel timetables and other factual information given in this work is correct at the time of first printing but Cambridge University Press & Assessment does not guarantee the accuracy of such information thereafter.

Cambridge University Press & Assessment acknowledges the Aboriginal and Torres Strait Islander peoples of this nation. We acknowledge the traditional custodians of the lands on which our company is located and where we conduct our business. We pay our respects to ancestors and Elders, past and present. Cambridge University Press & Assessment is committed to honouring Aboriginal and Torres Strait Islander peoples' unique cultural and spiritual relationships to the land, waters and seas and their rich contribution to society.

Contents

About the authors viii
Acknowledgements x
Introduction xi
Guide to the working programs in exercises xiii
Guide to this resource xiv

1 Financial mathematics 2

Strand: Number and Algebra

Warm-up quiz 4
1A Review of percentages CONSOLIDATING 5
1B Applying percentages CONSOLIDATING 11
1C Income (Core) 19
1D The PAYG income tax system (Core) 26
1E Using a budget to manage income and expenditure EXTENDING 33
1F Simple interest (Core) 40
Progress quiz 45
1G Compound interest and depreciation (Core) 47
1H Investments and loans (Core) 54
1I Using spreadsheets for investments, loans and depreciation (Core) 62
Maths@Work: Finance manager 66
Working mathematically: Investing in art 69
Puzzles and challenges 71
Chapter summary 72
Chapter checklist with success criteria 73
Chapter review 76

2 Measurement 80

Strand: Measurement and Geometry

Warm-up quiz 82
2A Converting units of measurement CONSOLIDATING 83
2B Accuracy of measuring instruments (Core) 89
2C Perimeter CONSOLIDATING 95
2D Circumference and arc length CONSOLIDATING 101
2E Area of triangles and quadrilaterals CONSOLIDATING 106
2F Area of circles and sectors CONSOLIDATING 113
2G Surface area of prisms (Core) 119
Progress quiz 125
2H Surface area of cylinders (Core) 127
2I Volume of prisms and cylinders (Core) 131

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2J Further problems involving prisms and cylinders (Core) 137
2K Surface area of pyramids and cones (Path) 144
2L Volume of pyramids and cones (Path) 150
2M Volume and surface area of spheres (Path) 155
Maths@Work: Bricklayer 162
Working mathematically: Building playground equipment 165
Puzzles and challenges 167
Chapter summary 168
Chapter checklist with success criteria 169
Chapter review 172

3 Algebraic expressions and indices 176 — Strand: Number and Algebra

Warm-up quiz 178
3A Algebraic expressions CONSOLIDATING 179
3B Simplifying algebraic expressions CONSOLIDATING 186
3C Expanding algebraic expressions (Core) 192
3D Expanding binomial products (Core) 197
3E Factorising algebraic expressions CONSOLIDATING 203
3F Simplifying algebraic fractions: Multiplication and division (Core) 208
3G Simplifying algebraic fractions: Addition and subtraction (Core) 214
Progress quiz 219
3H Index laws for multiplying and dividing (Core) 220
3I Powers of powers, powers of fractions and the zero index (Core) 226
3J Negative indices, algebraic bases EXTENDING 232
3K Scientific notation and significant figures (Core) 238
Maths@Work: Electrical trades 244
Working mathematically: Luxury car investment 246
Puzzles and challenges 248
Chapter summary 249
Chapter checklist with success criteria 250
Chapter review 253

4 Probability 256 — Strand: Statistics and Probability

Warm-up quiz 258
4A Review of probability CONSOLIDATING 259
4B Venn diagrams EXTENDING 266
4C Two-way tables EXTENDING 273
4D Using arrays for two-step experiments (Core) 278
Progress quiz 285
4E Using tree diagrams (Core) 286
4F Dependent events and independent events (Core) 294

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Maths@Work: Business analyst 298
Statistical investigation: Seeing the northern lights 300
Puzzles and challenges 302
Chapter summary 303
Chapter checklist with success criteria 304
Chapter review 306

5 Single variable and bivariate statistics 310

Strand: Statistics and Probability

Warm-up quiz 312
5A Collecting data (Core and Path) 315
5B Column graphs and histograms CONSOLIDATING 320
5C Dot plots and stem-and-leaf plots CONSOLIDATING 330
5D Mean, median, mode and range (Core) 337
5E Quartiles and outliers (Core) 345
5F Box plots (Core) 353
Progress quiz 359
5G Standard deviation (Core) 360
5H Displaying and analysing time-series data CONSOLIDATING 366
5I Bivariate data and scatter plots (Core) 372
5J Line of best fit by eye (Core) 378
Maths@Work: Project manager on a building site 385
Statistical investigation: Modelling weather data 387
Puzzles and challenges 389
Chapter summary 390
Chapter checklist with success criteria 391
Chapter review 394

Semester review 1 398

6 Linear relationships, hyperbolas, parabolas and exponentials 406

Strand: Number and Algebra

Warm-up quiz 408
6A Interpreting straight-line graphs CONSOLIDATING 410
6B Distance–time graphs CONSOLIDATING 419
6C Graphing straight lines CONSOLIDATING 426
6D Exploring gradient (Core) 435
6E Rates from graphs (Core) 444
Progress quiz 451
6F $y = mx + c$ and special lines (Core) 453
6G Parallel lines and perpendicular lines (Core) 460
6H Graphing straight lines using intercepts EXTENDING 467
6I Linear modelling (Core) 472
6J Direct variation (Path) 480

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6K Inverse variation (Path) 490
6L Exploring parabolas (Core) 496
6M Exploring exponential graphs (Core) 505
6N Exponential growth and decay (Core) 509
Maths@Work: Accountant or small business owner 516
Working mathematically: Breaking even in small business 518
Puzzles and challenges 520
Chapter summary 521
Chapter checklist with success criteria 522
Chapter review 527

7 Properties of geometrical figures and networks 536

Strand: Measurement and Geometry

Warm-up quiz 538
7A Parallel lines CONSOLIDATING 540
7B Triangles CONSOLIDATING 546
7C Quadrilaterals EXTENDING 553
7D Polygons EXTENDING 560
Progress quiz 566
7E Similarity and scale drawings (Core) 567
7F Applying similar triangles (Core) 574
7G Introduction to networks (Path) 581
7H Isomorphic and planar graphs (Path) 588
7I Trails, paths and Eulerian circuits (Path) 596
7J Shortest path problems EXTENDING 605
Maths@Work: Pool builder 611
Working mathematically: Calculating inaccessible lengths 613
Puzzles and challenges 616
Chapter summary 617
Chapter checklist with success criteria 618
Chapter review 622

8 Trigonometry 628

Strand: Measurement and Geometry

Warm-up quiz 630
8A Reviewing Pythagoras' theorem CONSOLIDATING 632
8B Finding the length of a shorter side CONSOLIDATING 639
8C Applications of Pythagoras' theorem (Path) 644
8D Trigonometric ratios (Core) 651
8E Finding unknown side lengths (Core) 658
8F Solving for the denominator (Core) 665
Progress quiz 671
8G Finding unknown angles (Core) 673
8H Angles of elevation and depression (Core) 679

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8I Direction and bearings (Core) 688
8J Applications in three dimensions (Path) 696
8K The sine rule (Path) 702
8L The cosine rule (Path) 709
8M Area of a triangle (Path) 714
Maths@Work: Surveyor 719
Working mathematically: Building a roof truss 722
Puzzles and challenges 725
Chapter summary 726
Chapter checklist with success criteria 727
Chapter review 729

9 Equations and formulas 736

Strand: Number and Algebra

Warm-up quiz 738
9A Linear equations with a pronumeral on one side CONSOLIDATING 740
9B Equations with brackets, fractions and pronumerals on both sides (Core) 749
9C Solving equations of the form $ax^2 = c$ CONSOLIDATING 758
9D Using formulas (Core) 764
9E Solving simultaneous equations graphically EXTENDING 769
Progress quiz 775
9F Solving simultaneous equations using substitution EXTENDING 776
9G Solving simultaneous equations using elimination EXTENDING 783
Maths@Work: Nurse 792
Working mathematically: Movie theatre pricing 795
Puzzles and challenges 797
Chapter summary 798
Chapter checklist with success criteria 799
Chapter review 801

Semester review 2 804

Answers *818*
Index *901*

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

About the authors

Stuart Palmer was born and educated in NSW. He is a fully qualified high school mathematics teacher with more than 25 years' experience teaching students from all walks of life in a variety of schools. He has been Head of Mathematics in two schools. He is well known by teachers throughout the state for the professional learning workshops he delivers. Stuart also assists thousands of Year 12 students every year as they prepare for their HSC Examinations. At the University of Sydney, Stuart spent more than a decade running tutorials for pre-service mathematics teachers.

Karen McDaid has more that 20 years' experience teaching mathematics in primary and secondary schools, and as a lecturer teaching mathematics education to primary pre-service teachers at university. As an executive member and Past President of the Mathematics Association of NSW, she has been heavily involved in Mathematics K–10 Curriculum Consultation in NSW and across Australia. Karen co-authored the *CambridgeMATHS NSW GOLD* books Years 7 to 10. Karen is currently working in primary mathematics education at The Australian Catholic University.

David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has 25+ years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 30 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Sara Woolley was born and educated in Tasmania. She completed an Honours degree in Mathematics at the University of Tasmania before completing her education training at the University of Melbourne. She has taught mathematics in Victoria from Years 7 to 12 since 2006 and is currently a Head of Mathematics. She has written more than 15 mathematics titles and specialises in lesson design and differentiation.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Jenny Goodman has taught in schools for over 25 years and is currently teaching at a selective high school in Sydney. Jenny has an interest in the importance of literacy in mathematics education, and in teaching students of differing ability levels. She was awarded the Jones Medal for education at Sydney University and the Bourke Prize for Mathematics. She has written for *CambridgeMATHS NSW* and was involved in the *Spectrum* and *Spectrum Gold* series.

Jennifer Vaughan has taught secondary mathematics for over 30 years in New South Wales, Western Australia, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has had extensive experience in developing resources that make mathematical concepts more accessible, hence facilitating student confidence, achievement and an enjoyment of maths.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Acknowledgements

The author and publisher wish to thank the following sources for permission to reproduce material:

Cover: © Getty Images / Roberto Moiola / Sysaworld

Images: © Getty Images / Taiyou Nomachi, Chapter 1 Opener / bluestocking, p.5 / Xavierarnau, p.10 / AlpamayoPhoto, p.11 / Creative Crop, p.16(1) / mammoth, p.17 / laughingmango, p.18 / Alvarez, p.19 / Maskot, p.24 / PeopleImages, p.26 / Cecilie_Arcurs, p.30 / martin-dm, p.31 / Westend61, p.32 / Wang Yukun, p.36 / SDI Productions, p.37 / Tdo, p.38 / Marje, p.39 / Ilia Shalamaev Wwwfocuswildlifecom, p.44(1) / Luis Alvarez, p.45(1) / FG Trade, p.45(2) / djgunner, p.47 / skynesher, p.53 / Parinda Yatha / EyeEm, p.54 / MarsYu, p.62 / andresr, p.66 / SDI Productions, p.67 / djgunner, p.77 / Juan Silva, p.78 / John Seaton Callahan, Chapter 2 Opener / jhorrocks, p.83 / rclassenlayouts, p.88(1) / Adam Gault, p.88(2) / Photograph by Michael Schwab, p.95 / Eric Meola, p.100(1) / Glasshouse Images, p.100(2) / gilaxia, p.101 / fotoVoyager, p.108 / Martin Barraud, p.111 / Robert Riley / FOAP, p.118 / xxmmxx, p.119 / ferrantraite, p.123 / Peter Cade, p.124 / nicolamargaret, p.126 / ChuckSchugPhotography, p.127 / Maciej Nicgorski / EyeEm, p.129 / Rolfo Brenner / EyeEm, p.130 / Difydave, p.142 / DefenseEngineer, p.144 / Tetra Images, p.162 / Uma Shankar sharma, p.175 / Busakorn Pongparnit, Chapter 3 Opener / Jose Luis Pelaez Inc, p.179 / John Elk III, p.183 / davidf, p.184 / yoh4nn, p.186 / cjp, p.191 / ShutterWorx, p.196 / jada photo, p.207 / I love nature, p.236 / photovideostock, p.238 / KTSDESIGN/SCIENCE PHOTO LIBRARY, p.243 / MPI / Stringer, p.243 / Tetra Images, p.244 / Manuel Godinez, Chapter 4 Opener / vm, p.259 / rubberball, p.264 / Morsa Images, p.265 / Alexander Spatari, p.271 / skynesher, p.272 / BaMa, p.276 / Matelly, p.277(1) / imagedepotpro, p.278 / gilaxia, p.284 / luoman, p.293 / Thomas Trutschel / Contributor, p.298 / Geber86, p.299 / Anna Mardo, p.300, p.301 / wepix, p.307 / pidjoe, p.309 / Lynn Gail, Chapter 5 Opener / Tdo, p.319 / Michael Willson/AFL Photos / Stringer, p.324(1) / Paul Kane / Stringer, p.324(2) / ti-ja, p.327 / davidf, p.329 / Svante Berg / EyeEm, p.336 / SDI Productions, p.337 / wundervisuals, p.341 / Bryn Colton / Contributor, p.343 / Mayur Kakade, p.344 / Jay Yuno, p.345 / 97, p.551(2) / lovro77, p. 353 / fcafotodigital, p.357 / franckreporter, p.359 / nikamata, p.366 / Rosemary Calvert, p.370 / andresr, p.371 / Marnie Burkhart, p.376 / Nigel Killeen, p.377 / Dan Reynolds Photography, p.378 / Tom Merton, p.382 / inoc, p.383(1) / Martin Barraud, p.384 / Astrakan Images, p.385 / Photofusion / Contributor, p.389 / AleksandarNakic, p.394 / cjp, Chapter 6 Opener / Cat Gwynn, p.413 / Haitong Yu, p.415(1) / Brenna Bagley / EyeEm, p.415(2) / Jan Sandvik / EyeEm, p.418 / Yevgen Timashov, p.419 / Alistair Berg, p.420 / Massimo Merlini, p.423 / Prasit photo, p.424 / TJ Blackwell, p.425 / ralucahphotography.ro, p.432 / ozgurcankaya, p.434 / JGI/Jamie Grill, p.444 / Gary John Norman, p.451 / vm, p.471 / Klaus Vedfelt, p.472 / Vicki Smith, p.479 / Aaron Foster, p.484 / Andyworks, p.486 / Craig P. Jewell, p.487 / Sutthiwat Srikhrueadam, p.488(1) / mevans, p.488(2) / Michael Kynes / 500px, p.489 / Westend61, p.490 / Spiderstock, p.493 / Alongkot Sumritjearapol, p.495(1) / sod tatong, p.495(2) / Yongyuan Dai, p.508 / Marianne Purdie, p.514 / thianchai sitthikongsak, p.516(1) / Regina Podolsky / EyeEm, p.516(2) / Matthew Micah Wright, p.527 / filadendron, p.528 / Russell Monk, p.531 / Photo by Aaron Thompson, p.535 / Alexander Dürr / EyeEm, Chapter 7 Opener / Antoine Georges / EyeEm, p.540 / asbe, p.551 / buzbuzzer, p.552 / Reyaz Limalia, p.560 / Image Source, p.564 / Westend61, p.565 / GibsonPictures, p.572 / Yuichiro Chino, p.581 / annick vanderschelden photography, p.587 / Yuichiro Chino, p.588 / ROBERT BROOK/SCIENCE PHOTO LIBRARY, p.595 / Yongyuan Dai, p.596 / Nastasic, p.603 / Gabriel Mello, p.604 / Westend61, p.605 / Anna Gorin, p.610 / ShutterWorx, p.611 / Mike Wëwerka, p.624 / Leanne Vorster / EyeEm, p.627 / Tammy616, p.632 / Anton Petrus, p.637 / sturti, p.638 / Khanti Jantasao / EyeEm, p.639 / EschCollection, p.644 / kali9, p.658 / Jose Martinez, p.664 / Thierry Dosogne, p.665 / koiguo, p.678 / Mordolff, p.681 / YinYang, p.684 / Ascent Xmedia, p.688 / dan_prat, p.693 / Diane Miller, p.694 / Brigitte Blättler, p.695 / Monty Rakusen, p.719 / Prasit photo, p.721 / Mint Images/ Art Wolfe, p.730 / massimo colombo, p.731 / Matthew Micah Wright, p.734(1) / guvendemir, p.734(2) / Daniel Bosma, p.735 / Monty Rakusen, Chapter 9 Opener / Corbis/VCG, p.740 / Matt_Brown, p.747 / Hinterhaus Productions, p.748(1) / Adene Sanchez, p.748(2) / Image Source, p.756 / WANDER WOMEN COLLECTIVE, p.767 / Luis Alvarez, p.768 / Ascent/PKS Media Inc., p.769 / Ian Hitchcock / Stringer, p.773 / Henrik5000, p.774 / Yusuke Nishizawa, p.781 / Lew Robertson, p.782(1) / Nastasic, p.782(2) / Peter Dazeley, p.783 / Eddy Zecchinon / EyeEm, p.790 / Dana Neely, p.792(1) / Luis Alvarez, p.792(2) / pjohnson, p.793 / Tom Werner, p.794.

Every effort has been made to trace and acknowledge copyright. The publisher apologises for any accidental infringement and welcomes information that would redress this situation.

Mathematics K-10 Syllabus © NSW Education Standards Authority for and on behalf of the Crown in right of the State of New South Wales, 2022.

The NSW Education Standards Authority (NESA) does not endorse model answers prepared by or for the publisher and takes no responsibility for errors in the reproduction of the material supplied by the NESA. The publisher wishes to acknowledge that this study guide has been independently published by Cambridge University Press for use by teachers and students, and that although material contained has been reproduced with permission, this publication is in no way endorsed by the NESA.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Introduction

The third edition of CambridgeMATHS NSW Stages 4 and 5 has been carefully prepared for the NSW Syllabus implemented from 2024 onwards. The series is packed with new features. It is intended for students across a wide range of ability levels and has been designed to provide the best possible preparation for students as they make their way from the end of Stage 3 towards Stage 6 mathematics.

Coverage of Core and optional Path topics in Stage 5

As with the previous editions, Stage 5 is covered with two differentiated pairs of books:

- The *Core & Standard Paths* books replace the old Stage 5.1/5.2 books
- The *Core & Advanced/Extension Paths* books replace the Stage 5.1/5.2/5.3 books.

Care has been taken to choose the content in each book carefully and to present it in a way that reflects the spirit of the new syllabus as well as the practicalities of the classroom. 'Paths', in the plural, refers to the fact that the Path topics are optional, and that a student's journey through Stage 5 may incorporate the core content in combination with none, some or all the Path topics in the books, depending upon the student's needs. Detailed teaching programs and Scope and Sequence documents are available via the Online Teaching Suite.

Beyond Stage 5

Students who have used these *Core & Standard Paths* books in Years 9 and 10 will be thoroughly prepared for Year 11 Mathematics Standard. These books are not recommended for students who have aspirations of studying Mathematics Advanced in Stage 6.

Learning intentions, Success criteria checklist, and 'Past, present and future learning'

Every lesson now includes a set of Learning intentions that describe what the student can expect to learn in the lesson. At the end of the chapter, these appear again in the form of a checklist of Success criteria; students can use this to check their progress through the chapter. Every criterion is listed with an example question to remind students of what the mathematics mentioned looks like. These checklists can also be downloaded and printed so that students can physically check them off as they accomplish their goals.

Also included at the beginning of each section, under the Learning intentions, is a short set of dot points that place the topic at hand in the context of **past, present and future learning**.

Now you try

Every worked example now contains additional questions, without solutions, called 'Now you try', which give students immediate practice at the same type of question as the example. We also anticipate these questions will be useful for the teacher to do in front of the class, given that students will not have seen the solution beforehand.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Working Mathematically tasks

A Working mathematically activity now accompanies an Investigation in each chapter, with the goal of familiarising students with using the Working mathematically process to define, solve, verify and then communicate their solutions when solving non-routine problems.

Maths@Work

An activity called 'Maths@Work' is now included near the end of every chapter. These activities show students how the maths learnt in the chapter is relevant to the workplace in a range of industries. The first part of the activity involves students doing the work by hand, while the second part involves the use of technology – often spreadsheets – to allow for greater efficiency, power and accuracy.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Guide to the working programs in exercises

The suggested working programs in the exercises in the Years 9 and 10 Core & Standard Paths books provide two pathways to allow differentiation for Building and Progressing students (schools will have their own names for different ability levels or streams).

Each exercise is structured in subsections that match the Working Mathematically components of Understanding, Fluency, Problem-solving and Reasoning, as well as Enrichment (Challenge). In the exercises, the questions suggested for each pathway are listed in two columns at the top of each subsection:

- The left column (lightest shaded colour) is the Building pathway
- The right column (darkest shaded colour) is the Progressing pathway.

Building	Progressing
UNDERSTANDING	
1–4	4
FLUENCY	
5–7(½)	5–8(½)
PROBLEM-SOLVING AND REASONING	
9–10	10–12
ENRICHMENT: The missing *y*-intercept	
–	13

The working program for Exercise 5G in Year 9. The questions recommended for a Building student are: 1, 2, 3, 4, 5, 6, 7(½), 9 and 10. * See note below.

Gradients within exercises and proficiency strands

The working programs make use of the two difficulty gradients contained within exercises. A gradient runs through the overall structure of each exercise – where there is an increasing level of mathematical sophistication required from Understanding to Fluency through to Problem-solving and Reasoning, and Enrichment – but also within each Working Mathematically component; the first few questions in Fluency, for example, are easier than the last Fluency question.

The right mix of questions

Questions in the working programs are selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Building pathway should use the left tab, which includes all but the hardest Understanding and Fluency questions as well as the easiest Problem-solving and Reasoning questions. The Progressing pathway, while not challenging, spends a little less time on basic Understanding questions and a little more time on Fluency and Problem-solving and Reasoning questions.

Choosing a pathway

There are a variety of ways to determine the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works for them. If required, the Warm-up quiz at the start of each chapter can be used as a tool for helping students select a pathway. The following are recommended guidelines:

- A student who gets 40% or lower should heavily revise core concepts before attempting the Buildings questions.
- A student who gets above 40% and below 75% should complete the Building questions
- A student who gets 75% or higher should complete the Progressing questions.

* The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1–4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e, ... or b, d, f, ...)
- 2–4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- – : complete none of the questions in this section.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Guide to this resource

PRINT TEXTBOOK FEATURES

1 **NSW Syllabus:** content strands, outcome groups and outcomes are listed at the beginning of the chapter to assist with course planning (see teaching programs and Syllabus mapping grids for more detailed guidance)

2 **Chapter introduction:** sets context for students about how the topic connects with the real world and the history of mathematics

3 **Warm-up quiz:** a quiz for students on the prior knowledge and essential skills required before beginning the chapter

4 NEW **Learning intentions:** sets out what a student will be expected to learn in the lesson

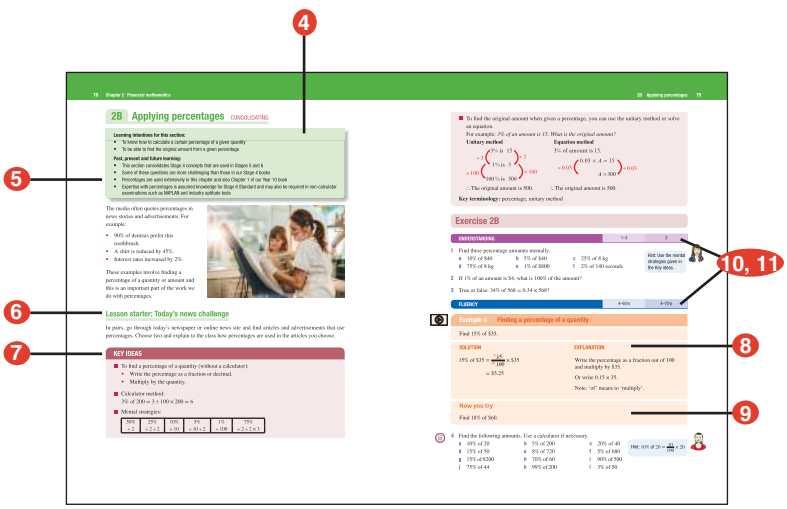
5 NEW **Past, present and future learning**: shows how the lesson builds on previous lessons, or extends to future lessons in the context of the syllabus

6 **Lesson starter:** an activity, which can often be done in groups, to start the lesson

7 **Key ideas:** summarises the knowledge and skills for the lesson

8 **Worked examples:** solutions and explanations of each line of working, along with a clear, description of the mathematics covered by the example

9 NEW **Now you try:** try-it-yourself questions provided after every worked example in exactly the same style as the worked example to give immediate practice



10 **Gentle start to exercises:** the exercise begins at Understanding to ensure comprehension of the lesson's Key ideas before progressing to the other Working Mathematically components

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 **Working programs:** differentiated question sets for two ability levels in exercises

12 NEW **Progress quiz** about two-thirds of the way through each chapter provides a check-in to establish how students are going with the topic so far

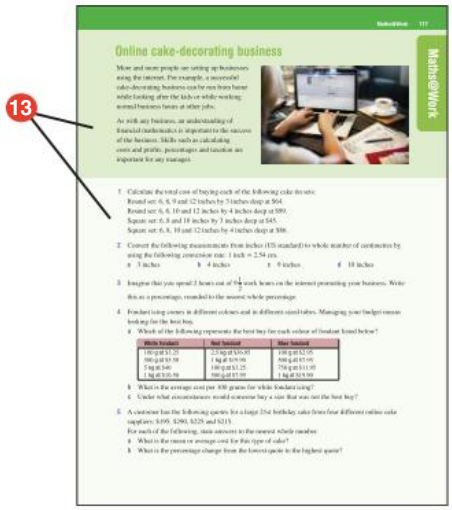
13 NEW **Maths@Work:** a set of questions that gives practice at applying the mathematics of the chapter to real-life contexts

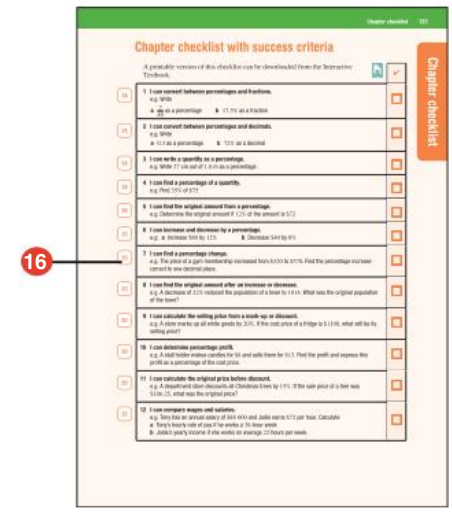
14 NEW **Working Mathematically investigative tasks:** in each chapter that apply the Working Mathematically process to the topic to give students guided practice

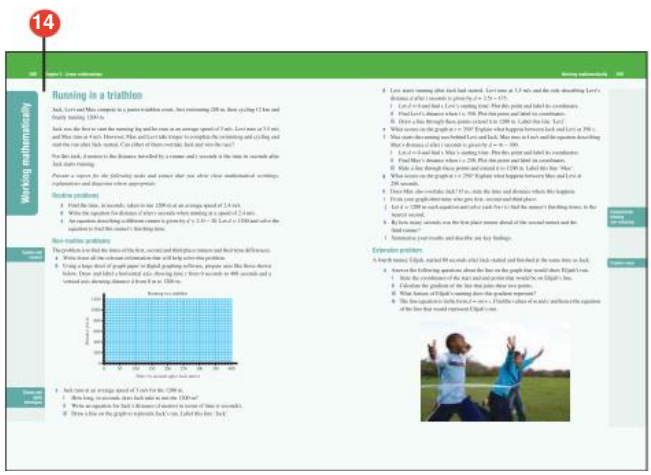
15 **Puzzles and challenges:** in each chapter provide problem-solving practice in the context of puzzles and challenges connected with the topic

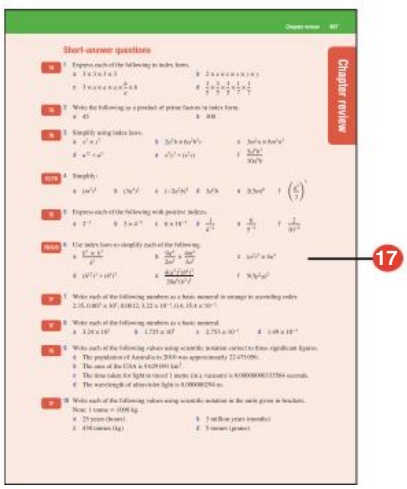
16 NEW **Chapter checklist with success criteria:** a checklist of the learning intentions for the chapter, with example questions

17 **Chapter reviews:** with short-answer, multiple-choice and extended-response questions; questions that are extension are clearly signposted









ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

INTERACTIVE TEXTBOOK FEATURES

18 NEW **Targeted skillsheets**, one for each lesson, focus on a small set of related Fluency-style skills for students who need extra support, with questions linked to worked examples

19 **Workspaces:** almost every textbook question – including all working-out – can be completed inside the Interactive Textbook by using either a stylus, a keyboard and symbol palette, or uploading an image of the work

20 **Self-assessment:** students can then self-assess their own work and send alerts to the teacher. See the Introduction on page xi for more information.

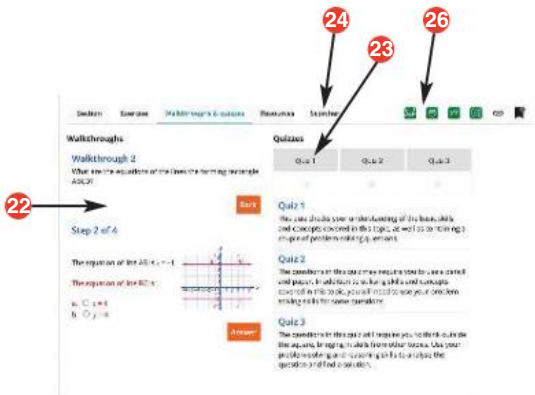
21 **Interactive working programs** can be customised so that only questions included in that working program are shown on the screen

22 **HOTmaths resources:** a huge catered library of widgets, HOTsheets and walkthroughs seamlessly blended with the digital textbook

23 A revised set of **differentiated auto-marked practice quizzes** per lesson with saved scores

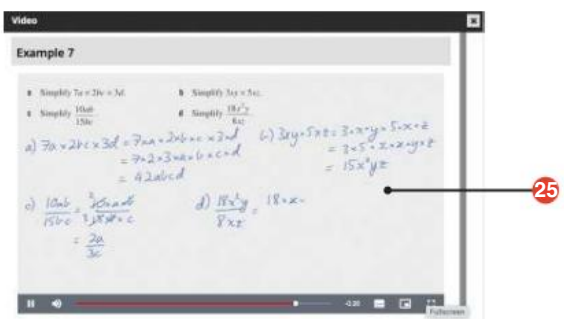
24 **Scorcher:** the popular competitive game

25 **Worked example videos:** every worked example is linked to a high-quality video demonstration, supporting both in-class learning and the flipped classroom



26 **Desmos graphing calculator**, scientific calculator and geometry tool are always available to open within every lesson

27 **Desmos interactives:** a set of Desmos activities written by the authors allow students to explore a key mathematical concept by using the Desmos graphing calculator or geometry tool



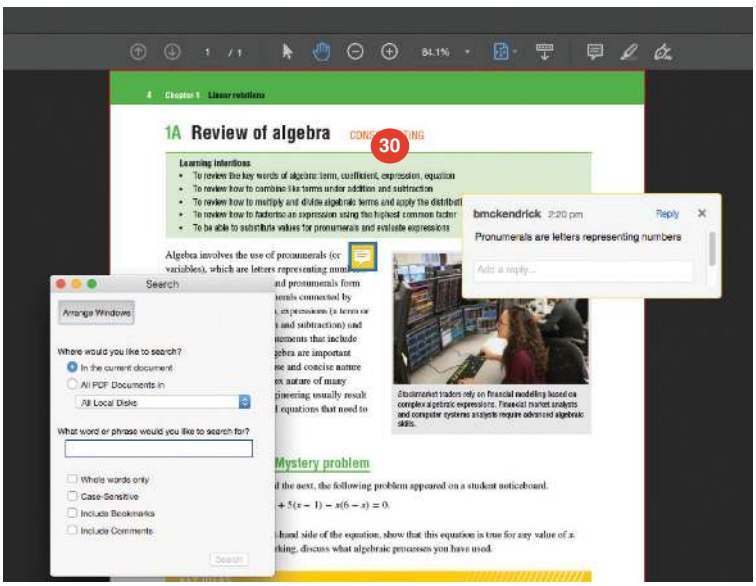
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

28 **Auto-marked warm-up quiz** for testing the knowledge that students will need before starting the chapter

29 **Auto-marked progress quizzes and chapter review multiple-choice questions** in the chapter reviews can now be completed online

DOWNLOADABLE PDF TEXTBOOK

30 In addition to the Interactive Textbook, a **PDF version of the textbook** is provided for times when users cannot go online. PDF search and commenting tools are enabled.



ONLINE TEACHING SUITE

31 NEW **Diagnostic assessment tool** included with the Online Teaching Suite allows for flexible diagnostic testing, reporting and recommendations for follow-up work to assist you to help your students to improve

32 NEW **PowerPoint lesson** summaries contain the main elements of each lesson in a form that can be annotated and projected in front of class

33 **Learning management system** with class and student analytics, including reports and communication tools

34 **Teacher view of student's work and self-assessment** allows the teacher to see their class's work, how students in the class assessed their own work, and any 'red flags' that the class has submitted to the teacher

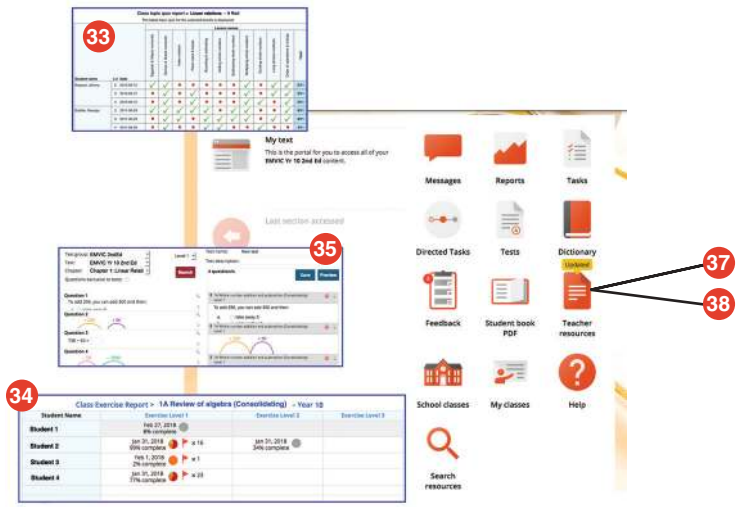
35 **Powerful test generator** with a huge bank of levelled questions as well as ready-made tests

36 **Revamped task manager** allows teachers to incorporate many of the activities and tools listed above into teacher-controlled learning pathways that can be built for individual students, groups of students and whole classes

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

37 **Worksheets and two differentiated chapter tests in every chapter,** provided in editable Word documents

38 **More printable resources:** all Warm-up quizzes, Progress quizzes and Working mathematically tasks are provided in printable worksheet versions



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1

Financial mathematics

Why skills with percentages and consumer arithmetic are important

Money management skills are essential for achieving personal financial independence and security, and for business success. The exchange of money is the basis of our economy, and percentages are used universally to describe changes of value.

- Essential skills for personal money management include making a personal budget, calculating the interest and repayments on loans, comparing discounted prices, and checking pay and tax amounts.
- Service businesses, such as web designers, hairdressers, mobile car mechanics and cake makers, all increase costs by a profit percentage and 10% GST.
- Successful food businesses (e.g. pizza shop, cafe) regularly calculate food costs as a percentage of sales revenue.
- Training for employment in finance, e.g. as a bank teller, includes learning to calculate percentages for loans, investments, insurance, pay scales, commission and tax.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- Investment advisors assist clients to make a budget; analyse investment growth forecasts and loan costs using simple and compound interest rates; and to compare insurance rates and cover.

Chapter contents

1A Review of percentages (CONSOLIDATING)
1B Applying percentages (CONSOLIDATING)
1C Income
1D The PAYG income tax system
1E Using a budget to manage income and expenditure (EXTENDING)
1F Simple interest
1G Compound interest and depreciation
1H Investments and loans
1I Using spreadsheets for investments, loans and depreciation

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- solves financial problems involving simple interest, earning money and spending money (MA5-FIN-C-01)
- solves financial problems involving compound interest and depreciation (MA5-FIN-C-02).

© 2022 NSW Education Standards Authority

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Warm-up quiz

1 Find the following totals.

a $\$15.92 + \$27.50 + \$56.20$ **b** $\$134 + \$457 + \$1021$

c $\$457 \times 6$ **d** $\$56.34 \times 1\frac{1}{2}$

e $\$87\,560 \div 52$ (to the nearest cent)

2 Express the following fractions with denominators of 100.

a $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{1}{5}$ **d** $\frac{17}{25}$ **e** $\frac{9}{20}$

3 Write each of the following fractions as decimals.

a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{5}$ **d** $\frac{20}{50}$ **e** $\frac{1}{3}$

4 Round the following decimals to two decimal places.

a 16.7893 **b** 7.347 **c** 45.3444 **d** 6.8389 **e** 102.8999

5 Give the values of the pronumerals in the following table.

Gross income ($)	Deductions ($)	Net income ($)
4976	456.72	a
72 156	21 646.80	b
92 411	c	62 839
156 794	d	101 916
e	18 472.10	79 431.36

Hint: Net Income = gross income – deductions

6 Calculate the following annual incomes for each of these people.

a Tom: $1256 per week **b** Isla: $15 600 per month

c Henry: $1911 per fortnight

d Crystal: $17.90 per hour, for 40 hours per week, for 50 weeks per year

7 Without a calculator, find:

a 10% of $400 **b** 5% of $5000 **c** 2% of $100

d 25% of $844 **e** 20% of $12.80 **f** 75% of $1000

8 Find the simple interest on the following amounts.

a $400 at 5% p.a. for 1 year

b $5000 at 6% p.a. for 1 year

c $800 at 4% p.a. for 2 years

Hint: $I = Prn$

9 Complete the following table, giving the values of the pronumerals.

Cost price	Deduction	Sale price
$34	$16	a
$460	$137	b
$500	c	$236
d	$45	$67
e	$12.65	$45.27

10 The following amounts include the 10% GST. By dividing each one by 1.1, find the original costs before the GST was added to each.

a $55 **b** $61.60 **c** $605

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1A Review of percentages CONSOLIDATING

Learning intentions for this section:

- To understand that a percentage is a number out of 100
- To be able to convert decimals and fractions to percentages and vice versa
- To be able to find the percentage of a quantity

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stages 5 and 6
- It consolidates and extends the concepts in Sections 2A and 2B of our Year 9 book
- Later in this chapter, we apply this learning to financial mathematics
- Expertise with percentages is assumed knowledge for Stage 6 Standard and may also be required in non-calculator examinations such as industry aptitude tests

It is important that we are able to work with percentages in our everyday lives. Banks, retailers and governments use percentages every day to calculate fees and prices.

Lesson starter: Which option should Jamie choose?

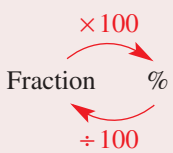
Jamie currently earns $68 460 p.a. (per year) and is given a choice of two different pay rises. Which should she choose and why?

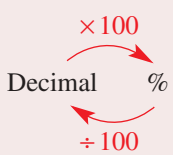
Choice A: Increase of $25 per week

Choice B: Increase of 2% on per annum salary

KEY IDEAS

- A **percentage** means 'out of 100'. It can be written using the symbol %, or as a fraction or a decimal.
  For example: 75 per cent $= 75\% = \frac{75}{100}$ or $\frac{3}{4}$ or 0.75.
- To convert a fraction or a decimal to a percentage, multiply by 100.
- To convert a percentage to a fraction, write it with a **denominator** of 100 and simplify.
  $15\% = \frac{15}{100} = \frac{3}{20}$
- To convert a percentage to a decimal, divide by 100.
  $15\% = 15 \div 100 = 0.15$
- To find a percentage of a quantity, write the percentage as a fraction or a decimal, then multiply by the quantity. For example,
  35% of \$600 $= \frac{35}{100} \times$ \$600.





Key terminology: percentage, denominator

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 1A

UNDERSTANDING | 1–3 | 3

1 Complete the following using the words *multiply* or *divide*.

a To convert a decimal to a percentage _____ by 100.

b To convert a percentage to a decimal _____ by 100.

c To convert a fraction to a percentage _____ by 100.

d To convert a percentage to a fraction _____ by 100.

2 Complete the following to express as a fraction in part **a** and a decimal in part **b**.

a **i** $7\% = \frac{7}{\square}$ **ii** $23\% = \frac{\square}{\square}$

b **i** $18\% = \square$ **ii** $5\% = \square$

3 Use mental strategies to find:

a 10% of \$7.50 **b** 20% of \$400 **c** 50% of \$98

d 75% of \$668 **e** 25% of \$412 **f** 2% of \$60

g 5% of \$750 **h** $33\frac{1}{3}\%$ of \$1200 **i** 30% of \$15

Hint: $10\% = \frac{10}{100}$
'of' means times

FLUENCY | 4–7(½) | 4–8(½)

Example 1 Converting to a percentage

Write each of the following as a percentage.

a $\frac{19}{20}$ **b** $\frac{3}{8}$ **c** 0.07

SOLUTION

a $\frac{19}{20} = \frac{95}{100}$
$= 95\%$

EXPLANATION

Write the fraction using a denominator of 100.

Alternatively, multiply the fraction by 100.

$\frac{19}{\cancel{20}_{1}} \times \frac{\cancel{100}^{5}}{1} = 19 \times 5 = 95$

b $\frac{3}{\cancel{8}_{2}} \times \frac{\cancel{100}^{25}}{1} = \frac{75}{2}$
$= 37.5$

So $\frac{3}{8} = 37.5\%$

Multiply the fraction by 100.

Cancel common factors, then simplify.

c $0.07 \times 100 = 7$
So $0.07 = 7\%$

Multiply the decimal by 100.

Move the decimal point two places to the right.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

Write each of the following as a percentage.

a $\frac{6}{25}$ **b** $\frac{7}{16}$ **c** 0.15

4 Convert each fraction to a percentage.

a $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{1}{4}$ **d** $\frac{1}{10}$

e $\frac{1}{100}$ **f** $\frac{7}{25}$ **g** $\frac{15}{50}$ **h** $\frac{3}{4}$

i $\frac{5}{8}$ **j** $\frac{19}{25}$ **k** $\frac{99}{100}$ **l** $\frac{47}{50}$

Hint: First write using a denominator of 100 or, alternatively, multiply by 100.

5 Write these decimals as percentages.

a 0.17 **b** 0.73 **c** 0.48 **d** 0.09

e 0.06 **f** 0.13 **g** 1.13 **h** 1.01

i 0.8 **j** 0.9 **k** 0.99 **l** 0.175

Hint: To multiply by 100, move the decimal point two places to the right.

Example 2 Writing percentages as simplified fractions

Write each of the following percentages as a simplified fraction.

a 37% **b** 58% **c** $6\frac{1}{2}\%$

SOLUTION

a $37\% = \frac{37}{100}$

b $58\% = \frac{58}{100}$

$= \frac{29}{50}$

c $6\frac{1}{2}\% = \frac{6\frac{1}{2}}{100}$

$= \frac{13}{200}$

EXPLANATION

Write the percentage with a denominator of 100.

Write the percentage with a denominator of 100.

Simplify $\frac{58}{100}$ by cancelling, using the HCF of 58 and 100, which is 2.

$$\frac{\cancel{58}^{29}}{\cancel{100}_{50}} = \frac{29}{50}$$

Write the percentage with a denominator of 100.

Double the numerator $\left(6\frac{1}{2}\right)$ and the denominator (100) so that the numerator is a whole number.

Now you try

Write each of the following percentages as a simplified fraction.

a 23% **b** 64% **c** $10\frac{1}{2}\%$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Write each percentage as a simplified fraction.

a 71%	**b** 80%	**c** 25%
d 55%	**e** 40%	**f** 88%
g 15%	**h** $16\frac{1}{2}\%$	**i** $17\frac{1}{2}\%$
j $2\frac{1}{4}\%$	**k** $5\frac{1}{4}\%$	**l** $52\frac{1}{2}\%$

Hint: Write with a denominator of 100, then simplify if possible.

Example 3 Writing a percentage as a decimal

Convert these percentages to decimals.

a 93%
b 7%
c 30%

SOLUTION	EXPLANATION
a $93\% = 93 \div 100$ $= 0.93$	Divide the percentage by 100. This is the same as moving the decimal point two places to the left.
b $7\% = 7 \div 100$ $= 0.07$	Divide the percentage by 100.
c $30\% = 30 \div 100$ $= 0.3$	Divide the percentage by 100. Write 0.30 as 0.3.

Now you try

Convert these percentages to decimals.

a 28%
b 3%
c 60%

7 Convert these percentages to decimals.

a 61%	**b** 83%	**c** 75%
d 45%	**e** 9%	**f** 90%
g 50%	**h** 16.5%	**i** 7.3%
j 200%	**k** 430%	**l** 0.5%

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 4 Finding a percentage of a quantity

Find 42% of $1800.

SOLUTION

$42\% \text{ of } \$1800 = 0.42 \times 1800$
$= \$756$

EXPLANATION

Remember that 'of' means to multiply.

Write 42% as a decimal or a fraction: $42\% = \frac{42}{100} = 0.42$

Then multiply by the amount.
If using a calculator, enter 0.42×1800.

Without a calculator: $\frac{42}{\cancel{100}_{1}} \times 18\cancel{00} = 42 \times 18 = 756$

Now you try

Find 36% of $2300.

8 Use a calculator to find the following.

a 10% of $250
b 50% of $300
c 75% of $80
d 12% of $750
e 9% of $240
f 43% of 800 grams
g 90% of $56
h 110% of $98
i $17\frac{1}{2}\%$ of 2000 m

PROBLEM-SOLVING AND REASONING 9–11 11–14

9 A 300 g pie contains 15 g of saturated fat.

a What fraction of the pie is saturated fat?
b What percentage of the pie is saturated fat?

Hint: 15 g out of 300 g.

10 About 80% of the mass of a human body is water. If Hugo is 85 kg, how many kilograms of water are in his body?

11 Rema spends 12% of the 6.6 hour school day in maths. How many minutes are spent in the maths classroom?

12 In a cricket match, Brett spent 35 minutes bowling.

His team's total fielding time was $3\frac{1}{2}$ hours.

What percentage of the fielding time, correct to two decimal places, did Brett spend bowling?

Hint: First convert hours to minutes, and then write a fraction comparing times.

13 Lucas lost 8 kg, and now weighs 64 kg. What percentage of his original weight did he lose?

14 47.9% of a local council's budget is spent on garbage collection. If a rate payer pays $107.50 per quarter in total rate charges, how much do they contribute in a year to garbage collection?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: Australia's statistics — 15

15 Below is the data on Australia's population growth, as gathered by the Australian Bureau of Statistics for December 2022.

	Population at 31 December 2022 ('000)	Change over previous year ('000)	Change over previous year (%, one decimal place)
New South Wales	8238.8	138.0	
Victoria	6704.3	137.7	
Queensland	5378.3	116.6	
South Australia	1834.3	28.5	
Western Australia	2825.2	62.7	
Tasmania	571.6	2.9	
Northern Territory	250.1	2.0	
Australian Capital Territory	460.9	8.3	
Australia	26 268.4	496.8	

a Calculate the percentage change for each state and territory shown using the previous year's population, and complete the table.

Hint: $\%\ \text{Change} = \dfrac{\text{change}}{\text{population}} \times \dfrac{100}{1}$

b What percentage of Australia's overall population, correct to one decimal place, is living in:

i NSW? **ii** Vic? **iii** WA?

c Use a spreadsheet to draw a pie chart (i.e. sector graph) showing the populations of the eight states and territories in the table. What percentage of the total is represented by each state/territory? Round your answer to the nearest per cent.

d In your pie chart for part **c**, what is the angle size of the sector representing Victoria, to the nearest degree?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1B Applying percentages CONSOLIDATING

Learning intentions for this section:

- To be able to increase or decrease an amount by a given percentage
- To be able to calculate the percentage by which an amount has increased or decreased
- To be able to solve problems involving percentage profits, losses and discounts

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stages 5 and 6
- It consolidates and extends the concepts in Sections 2C and 2D of our Year 9 book
- Later in this chapter, we apply this learning to financial mathematics
- Expertise with percentages is assumed knowledge for Stage 6 Standard and may also be required in non-calculator examinations such as industry aptitude tests

There are many applications of percentages. Prices are often increased by a percentage to create a profit or decreased by a percentage when on sale.

When goods are purchased by a store, the cost to the owner is called the cost price. The price of the goods sold to the customer is called the selling price. This price will vary according to whether the store is having a sale or decides to make a certain percentage profit.

Lesson starter: Discounts

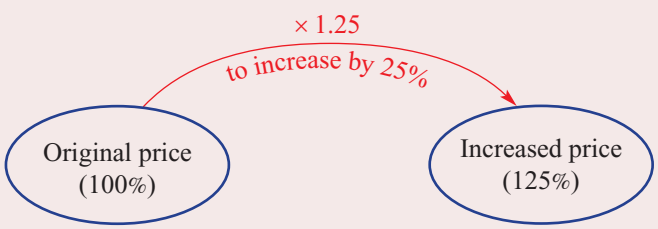
Discuss as a class:

- Which is better: 20% off or a $20 discount?
- If a discount of 20% or $20 resulted in the same price, what was the original price?
- Why are percentages used to show discounts, rather than a dollar amount?

KEY IDEAS

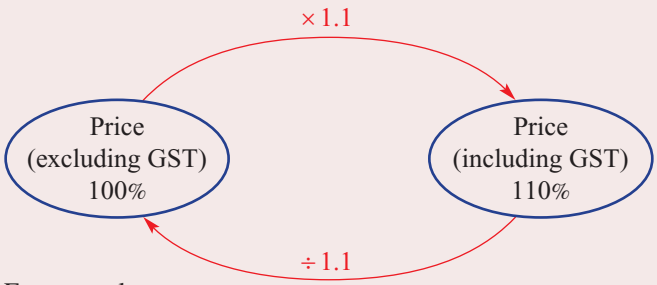
■ To increase an amount by a given percentage:
- Add the percentage increase to 100%.
- Multiply the amount by this new percentage.

For example: to increase by 25%, multiply by 100% + 25% = 125% = 1.25.

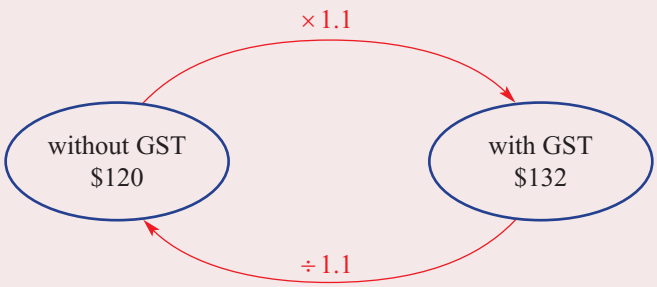


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

■ The **goods and services tax (GST)** adds 10% to prices of goods and services.



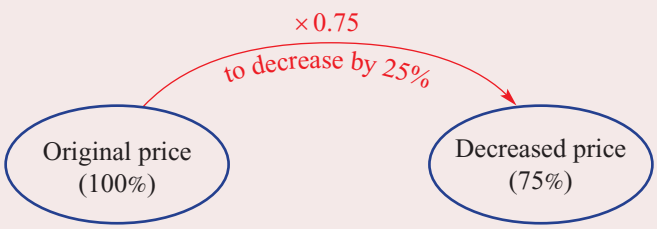
For example:



■ To decrease an amount by a given percentage:

- Subtract the percentage from 100%.
- Multiply the amount by this new percentage.

For example: to decrease by 25%, multiply by 100% − 25% = 75% = 0.75.



■ To find a percentage change, use:

$$\text{Percentage change} = \frac{\text{change in price}}{\text{original price}} \times 100\%$$

■ When goods are purchased by a store, the cost to the owner is called the **cost price**.

■ The price on display in the shop is the **selling price**.

■ **Profit** is the amount of money made on a sale.

Profit = selling price − cost price

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- A **loss** is made when the selling price is less than the cost price.

  Loss = cost price − selling price

- **Mark-up** is the amount added to the cost price to produce the selling price.

  Selling price = cost price + mark-up

- The **percentage profit** or **percentage loss** can be found by dividing the profit or loss by the cost price and multiplying by 100%:

  $$\%\ \text{Profit/Loss} = \frac{\text{profit/loss}}{\text{cost price}} \times 100\%$$

- **Discount** is the amount by which an item is marked down.
  Discount = % discount × original price

  New price = original price − discount

Key terminology: discount, profit, selling price, cost price, loss, mark-up, GST

Exercise 1B

UNDERSTANDING 1–4 | 4

1 By what number do you multiply to increase an amount by:
a 10%? **b** 20%? **c** 2%? **d** 18%?

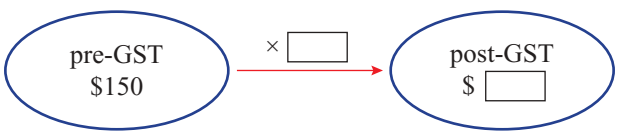
Hint: 100% + 10% = 1.1
100% − 5% = 0.95

2 By what number do you multiply to decrease an amount by:
a 5%? **b** 30%? **c** 15%? **d** 50%?

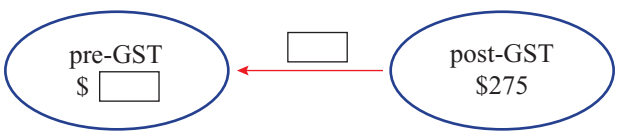
3 Decide how much profit or loss is made in each of the following situations.
a cost price = \$15 selling price = \$20
b cost price = \$17.50 selling price = \$20
c cost price = \$250 selling price = \$234

4 Copy and complete the following, assuming the GST is 10%.

a



b



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 5–7(½), 8–10 | 5–7(½), 8, 9, 11, 12

Example 5 Increasing by a given percentage

Increase $370 by 8%.

SOLUTION

$370 × 1.08 = $399.60

EXPLANATION

100% + 8% = 108%
Write 108% as a decimal (or fraction) and multiply by the amount.
Show two decimal places to represent the cents.

Now you try

Increase $650 by 12%.

5 **a** Increase $90 by 5%.
b Increase $400 by 10%.
c Increase $55 by 20%.
d Increase $490 by 8%.
e Increase $50 by 12%.
f Increase $7000 by 3%.
g Increase $49.50 by 14%.
h Increase $1.50 by 140%.

Hint: To increase by 5%, multiply by 100% + 5% = 1.05.

Example 6 Decreasing by a given percentage

Decrease $8900 by 7%.

SOLUTION

$8900 × 0.93 = $8277.00

EXPLANATION

100% − 7% = 93%
Write 93% as a decimal (or fraction) and multiply by the amount.
Remember to put the units in your answer.

Now you try

Decrease $2700 by 18%.

6 **a** Decrease $1500 by 5%.
b Decrease $400 by 10%.
c Decrease $470 by 20%.
d Decrease $80 by 15%.
e Decrease $550 by 25%.
f Decrease $49.50 by 5%.
g Decrease $119.50 by 15%.
h Decrease $47.10 by 24%.

Hint: To decrease by 5%, multiply by 100% − 5% = 0.95.

Example 7 Calculating profit and percentage profit

The cost price for a new car is $24 780 and it is sold for $27 600.

a Calculate the profit.
b Calculate the percentage profit, to two decimal places.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

a Profit = selling price − cost price
= $27 600 − $24 780
= $2820

b Percentage profit $= \frac{\text{profit}}{\text{cost price}} \times 100$
$= \frac{2820}{24\,780} \times 100$
= 11.38%

EXPLANATION

Write the rule.
Substitute the values and evaluate.
Write the rule.
Substitute the values and evaluate.
Round your answer as instructed.

Now you try

The cost price for a new refrigerator is $888 and it is sold for $997.

a Calculate the profit.
b Calculate the percentage profit, to two decimal places.

7 Copy and complete the table on profits and percentage profit.

	Cost price	Selling price	Profit	Percentage profit
a	$10	$16		
b	$240	$300		
c	$15	$18		
d	$250	$257.50		
e	$3100	$5425		
f	$5.50	$6.49		

Hint: Percentage profit $= \frac{\text{profit}}{\text{cost price}} \times 100$

Example 8 Finding the selling price

A retailer buys some calico material for $43.60 a roll. He wishes to make a 35% profit.

a What will be the selling price per roll?
b If he sells 13 rolls, what profit will he make?

SOLUTION

a Selling price = 135% of $43.60
= 1.35 × $43.60
= $58.86 per roll

b Profit per roll = $58.86 − $43.60
= $15.26

Total profit = $15.26 × 13
= $198.38

EXPLANATION

For a 35% profit the unit price is 135%.
Write 135% as a decimal (1.35) and evaluate.

Selling price − cost price

There are 13 rolls at $15.26 profit per roll.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

A retailer buys swimsuits for $32 per suit. She wishes to make a 30% profit.

a What will be the selling price of each swimsuit?

b If she sells 20 swimsuits, what profit will she make?

8 A retailer buys some Christmas snow globes for $41.80 each. She wishes to make a 25% profit.

a What will be the selling price per snow globe?

b If she sells a box of 25 snow globes, what profit will she make?

9 A second-hand car dealer bought a trade-in car for $1200 and wishes to resell it for a 28% profit.
What is the resale price?

Example 9 Finding the discounted price

A shirt worth $25 is discounted by 15%.

a What is the selling price? **b** How much is the saving?

SOLUTION

a Selling price = 85% of $25
= 0.85 × $25
= $21.25

EXPLANATION

15% discount means there must be 85% left (100% − 15%).
Convert 85% to 0.85 and multiply by the amount.

b Saving = 15% of $25
= 0.15 × $25
= $3.75

You save 15% of the original price.
Convert 15% to 0.15 and multiply by the original price.

Or saving = $25 − $21.25
= $3.75

Saving = original price − discounted price

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

A suitcase worth $220 is discounted by 35%.

a What is the selling price?

b How much is the saving?

10 Samantha buys a wetsuit from the sports store where she works. Its original price was $79.95. Employees receive a 15% discount.

a What is the selling price?

b How much will Samantha save?

11 A travel agent offers a 12.5% discount on airfares if you travel during May or June.
The normal fare to London (return trip) is $2446.

a What is the selling price?

b How much is the saving?

12 A store sells second-hand goods at 40% off the recommended retail price. A lawn mower is valued at $369.

a What is the selling price?

b How much would you save?

PROBLEM-SOLVING AND REASONING

13–16 | 15–18

13 Ski jackets are delivered to a shop in packs of 50 for $3500. The shop owner wishes to make a 35% profit.

a What will be the total profit made on a pack?

b What is the profit on each jacket?

14 A pair of sports shoes is discounted by 47%. The recommended price was $179.

a What is the amount of the discount?

b What will be the discounted price?

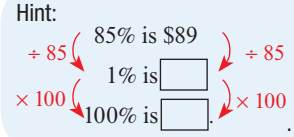
15 Jeans are priced at $89 for a May sale. If this is a saving of 15% off the selling price, what do the jeans normally sell for?

16 Discounted tyres are reduced in price by 35%. They now sell for $69 each. Determine:

a the normal price of one tyre

b the saving if you buy one tyre

Hint:



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

17 The local shop purchases a carton of containers for $54. Each container is sold for $4. If the carton had 30 containers, determine:

- **a** the profit per container
- **b** the percentage profit per container, to two decimal places
- **c** the overall profit per carton
- **d** the overall percentage profit, to two decimal places

18 A retailer buys a book for $50 and wants to sell it for a 26% profit. The 10% GST must then be added onto the cost of the book.

Hint: $\%\ \text{Increase} = \frac{\text{increase}}{\text{cost price}} \times 100$

- **a** Calculate the profit on the book.
- **b** How much GST is added to the cost of the book?
- **c** What is the advertised price of the book, including the GST?
- **d** Find the overall percentage increase of the final selling price compared to the $50 cost price.

ENRICHMENT: Building a gazebo — 19

19 Christopher designs a gazebo for a new house. He buys the timber from a retailer, who sources it at wholesale price and then marks it up before selling to Christopher at retail price. The table below shows the wholesale prices as well as the mark-up for each type of timber.

Quantity	Description	Cost/unit	Mark-up
6	Treated pine posts	$23	20%
11	300 × 50 oregon beams	$75	10%
5	Sheet lattice work	$86	15%
2	300 × 25 oregon fascias	$46	12%
8	Laserlite sheets	$32	10%

- **a** Determine Christopher's overall cost for the material, including the mark-up.
- **b** Determine the profit made by the retailer.
- **c** Determine the retailer's overall percentage profit, to two decimal places.
- **d** If the retailer pays 27% of his profits in tax, how much tax does he pay on this sale?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1C Income

Learning intentions for this section:

- To understand the various ways in which employees can be paid
- To know how net income is calculated from gross income and deductions
- To be able to calculate wages for overtime and shift work
- To be able to calculate income from piecework, commission and royalties

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Financial mathematics A*
- It consolidates and extends the concepts in Section 2E of our Year 9 book
- Financial mathematics is a major topic in Stage 6 Standard

You may have earned money for baby-sitting or delivering newspapers or have a part-time job. As you move into the workforce it is important that you understand how you are paid.

Lesson starter: Casual teaching versus full-time teaching

When one of your normal teachers is absent from school, your school might employ a casual teacher to take their classes for the day.

Teacher A, a casual, was paid $380 for every school day they worked during 2022.

Teacher B, a full-time teacher, was paid about $76 000 during 2022.

- If Teacher A had worked every school day of 2022, how much would they have earned?
- Is this more or less than Teacher B?
- If Teacher A had worked only two days every school week of 2022, how much would they have earned?
- What are some reasons for being a casual teacher compared with a full-time teacher?
- Research the meaning of the following advantages of being a full-time teacher compared with a casual teacher.

Holiday pay	Sick pay	Salary scale	Long service leave
Annual leave loading	Salary packaging	Job security	International teacher exchange

KEY IDEAS

Methods of payment

- Hourly **wages**: You are paid a certain amount per hour worked.
- **Commission**: You are paid a percentage of the total amount of sales. Some people who work for commission are also paid a set weekly amount, called a **retainer**.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- **Royalties:** Some people who write books or music are paid **royalties**; for example, this could be 10% of the sales price of every book sold.
- **Piecework**: You are paid according to number of things you make or do.
- **Salary**: You are paid a set amount per year.
- **Fees**: You are paid according to the charges you set; e.g. doctors, lawyers, contractors.

Terminology

- **Gross income**: The total amount of money you earn before taxes and other deductions.
- **Deductions**: Money taken from your income before you are paid; e.g. taxation, union fees, superannuation.
- **Net income**: The amount of money you actually receive after the deductions are taken from your gross income.

  Net income = gross income − deductions

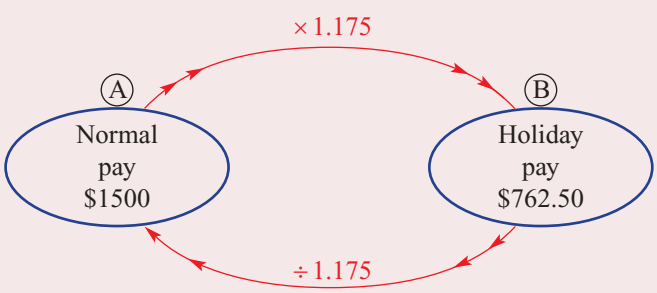
Payments by hourly rate

- If you are paid by the hour you will be paid an amount per hour for your normal working time. Usually, normal working time is 38 hours per week. If you work overtime the rates may be different.
  - Normal: 1.0 × normal rate
  - Time and a half: 1.5 × normal rate
  - Double time: 2.0 × normal rate
- Some people are paid **penalty rates**; e.g. time and a half for working on Saturdays or double time for working on Sundays.

Leave loading

- Some wage and salary earners are paid leave loading. When they are on holidays, they earn their normal pay plus a bonus called leave loading. This is usually 17.5% of their normal pay.

$$100\% + 17.5\% = 117.5\%$$
$$= 1.175$$



Leave loading = 17.5% of (A) = $262.50

Key terminology: wages, commission, salary, fees, gross income, overtime, deductions, net income, time and a half, double time, deductions, penalty rates

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 1C

UNDERSTANDING 1–3 3

1 Match the job description on the left with the method of payment on the right.

a	Ava is paid $85 600 per year	**A**	hourly wage
b	Danielle earns 3% of all the sales she makes	**B**	fee
c	Jett earns $18.90 per hour worked	**C**	commission
d	Stuart charges $450 for a consultation	**D**	salary

2 Callum earns $1090 a week and has annual deductions of $19 838. What is Callum's net income for the year? Assume 52 weeks in a year.

Hint: Net = total – deductions

3 If Tao earns $15.20 per hour, calculate his:

a time-and-a-half rate **b** double-time rate

Hint: Time-and-a-half rate = 1.5× hourly rate

FLUENCY 4–8, 10 5–11

Example 10 Finding gross and net income (including overtime)

Layla is paid $27 per hour at the local stockyard to muck out the stalls. Her normal hours of work are 38 hours per week. She receives time and a half for the next 4 hours worked and double time after that.

a What will be her gross income if she works 50 hours?

b If she pays $417 per week in taxation and $4.75 in union fees, what will be her weekly net income?

SOLUTION	EXPLANATION
a Gross income $= 38 \times \$27$	Normal 38 hours
$+ 4 \times 1.5 \times \$27$	Overtime rate for next 4 hours: time and a half $= 1.5 \times$ normal
$+ 8 \times 2 \times \$27$	Overtime rate for next 8 hours: double time $= 2 \times$ normal
$= \$1740$	
b Net income $= \$1740 - \$417 - \$4.75$	Net income = gross income – deductions
$= \$1318.25$	

Now you try

Caden is paid $24.73 per hour at his supermarket job. His normal hours of work are 38 hours per week. He receives time and a half for the next 6 hours worked and double time after that.

a What will be his gross income if he works 48 hours in a week?

b If he pays $287 per week in taxation and $6.50 in union fees, what will be his weekly net income?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Jack is paid $14.70 per hour. His normal hours of work are 38 hours per week. He receives time and a half for the next 2 hours worked and double time after that.

a What will be his gross income if he works 43 hours?

b If he has $207.20 of deductions, what will be his weekly net income?

5 Copy and complete this table.

	Hourly rate	Normal hours worked	Time and a half hours	Double time hours	Gross income	Deductions	Net income
a	$15	38	0	0		$155	
b	$24	38	2	0		$220	
c	$13.15	38	4	1		$300	
d	$70	40	2	3		$510	
e	$17.55	35	4	6		$184	

Example 11 Calculating shift work

Michael is a shift worker and is paid $31.80 per hour for the morning shift, $37.02 per hour for the afternoon shift and $50.34 per hour for the night shift. Each shift is 8 hours. In a given fortnight he works four morning, two afternoon and three night shifts. Calculate his gross income.

SOLUTION

$$\begin{aligned}\text{Gross income} &= 4 \times 31.80 \times 8 \\ &\quad + 2 \times 37.02 \times 8 \\ &\quad + 3 \times 50.34 \times 8 \\ &= \$2818.08\end{aligned}$$

EXPLANATION

4 morning shifts at $31.80 per hour for 8 hours
2 afternoon shifts at $37.02 per hour for 8 hours
3 night shifts at $50.34 per hour for 8 hours
Gross income because tax has not been paid.

Now you try

Kate is a shift worker and is paid $26.20 per hour for the morning shift, $32.40 per hour for the afternoon shift and $54.25 per hour for the night shift. Each shift is 8 hours. In a given fortnight she works five morning, three afternoon and two night shifts. Calculate her gross income.

6 Greg works shifts at a processing plant. In a given rostered fortnight he works:

- 3 day shifts ($31.80 per hour)
- 4 afternoon shifts ($37.02 per hour)
- 4 night shifts ($50.34 per hour).

a If each shift is 8 hours long, determine Greg's gross income for the fortnight.

b If the answer to part **a** is Greg's average fortnightly income, what will be his gross income for a year (i.e. 52 weeks)?

Hint: A fortnight = 2 weeks

Many hospital workers work shift work.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 12 Calculating income involving commission

Jeff sells memberships to a gym and receives $225 per week plus 5.5% commission on his sales. Calculate his gross income after a 5-day week.

Day	1	2	3	4	5
Sales ($)	680	450	925	1200	1375

SOLUTION

Total sales = $4630

Commission = 5.5% of $4630
= 0.055 × $4630
= $254.65

Gross income = $225 + $254.65
= $479.65

EXPLANATION

Determine the total sales: 680 + 450 + 925 + 1200 + 1375.

Determine the commission on the total sales at 5.5% by multiplying 0.055 by the total sales.

Gross income is $225 plus commission.

Now you try

Jin sells vacuum cleaners and receives $250 per week plus 4.3% commission on her sales. Calculate her gross income after a 5-day week.

Day	1	2	3	4	5
Sales ($)	1230	690	1422	1590	2648

7 A car salesman earns $5000 a month plus 3.5% commission on all sales. In the month of January his sales total was $56 000. Calculate:
- **a** his commission for January
- **b** his gross income for January

8 A real estate agent receives 2.75% commission on the sale of a house valued at $1 250 000. Find the commission earned.

9 Sarah earns an annual salary of $77 000 plus 2% commission on all sales. Find:
- **a** her weekly base salary before sales
- **b** her commission for a week when her sales totalled $7500
- **c** her gross weekly income for the week in part **b**
- **d** her annual gross income if over the year her sales totalled $571 250

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 13 Calculating holiday pay

Rachel is normally paid $1200 per week. When she is on holidays she is paid 17.5% p.a. leave loading.

a How much is her holiday pay for 1 week?

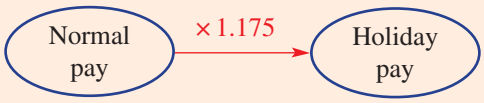
b How much is her leave loading for 1 week?

SOLUTION

a Holiday pay $= \$1200 \times 1.175$
$= \$1410$

b Leave loading $= \$1200 \times 0.175$
$= \$210$

EXPLANATION



17.5% = 0.175
Loading is 17.5% of normal pay.

Now you try

Sophie is a school teacher who earns $100 000 per annum. During December she is paid 4 weeks of leave loading. How much is the leave loading for those 4 weeks?

10 Ashton is normally paid $900 per week. When he is on holidays he is paid leave loading.

a Calculate Ashton's holiday pay for 1 week.

b Calculate Ashton's leave loading for 1 week.

Hint: Leave loading is an extra 17.5% of normal pay.

11 Mary earns $800 per week. Calculate her holiday pay for 4 weeks, including leave loading.

PROBLEM-SOLVING AND REASONING 12, 13 12–14

12 If Simone receives $10 000 on the sale of a property worth $800 000, calculate her rate of commission.

Hint: What percentage of $800 000 is $10 000?

13 Jonah earns a commission on his sales of fashion items. For goods to the value of $2000 he receives 6% and for sales over $2000 he receives 9% on the amount in excess of $2000. In a given week he sold $4730 worth of goods. Find the commission earned.

14 William earns 1.75% commission on all sales at the electrical goods store where he works. If William earns $35 in commission on the sale of one television, how much did the TV sell for?

Hint: 1.75% is $35. Find 1%, then 100%.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: Elmo's payslip — 15

15 Refer to the payslip below to answer the following questions.

Kuger Incorporated			
Employee ID: 75403A			Page: 1
Name: Elmo Rodriguez			Pay Period: 20/05/2023
Pay Method: EFT		Tax Status: Gen Exempt	
Bank account name: E. Rodriguez			
Bank: Mathsville Credit Union			
BSB: 102-196 Account No: 00754031			
Payment Details this pay:			
Amount	Days	Payment Description	Rate/Frequency
2 777.15	14.00	Normal time	$72 454/annum
Before tax deductions:			
This pay		Description	
170		Salary sacrifice: car pre-tax deduction	
Miscellaneous deductions:			
This pay		Description	
52.90		Health fund	
23.10		Union fees	
76.00			
Reconciliation details:			
This pay	YTD	Description	
2 607.15	62 571.60	Taxable gross pay	
616.00	14 784.00	less income tax	
76.00	1 824.00	less miscellaneous deductions	
1 915.15	45 963.60		

a Which company does Elmo work for?

b What is the name of Elmo's bank and what is his account number?

c How much gross pay does Elmo earn in 1 year?

d How often does Elmo get paid?

e How much, per year, does Elmo salary sacrifice?

f How much is Elmo's health fund contribution each week?

g Calculate 1 year's union fees.

h Using the information on this payslip, calculate Elmo's annual tax and also his annual net income.

i If Elmo works Monday to Friday from 9 a.m. to 5 p.m. each day for an entire year, calculate his effective hourly rate of pay. Use Elmo's fortnightly payment as a starting point.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1D The PAYG income tax system

Learning intentions for this section:

- To understand how the key components of the Australian taxation system work
- To be able to calculate a person's taxable income
- To be able to use the ATO table to calculate tax payable

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Financial mathematics A*
- It consolidates and extends the concepts in Section 2F of our Year 9 book
- Financial mathematics is a major topic in Stage 6 Standard

It has been said that there are only two sure things in life: death and taxes! The Australian Taxation Office (ATO) collects taxes on behalf of the government to pay for education, hospitals, roads, railways, airports and services, such as the police and fire brigades.

In Australia, the financial year runs from 1 July to 30 June the following year. People engaged in paid employment are normally paid weekly or fortnightly. Most of them pay some income tax every time they are paid for their work. This is known as the Pay-As-You-Go system (PAYG).

At the end of the financial year (30 June), people who earned an income complete an income tax return to determine if they have paid the correct amount of income tax during the year. If they paid too much, they will receive a refund. If they did not pay enough, they will be required to pay more.

The Australian tax system is very complex and the laws change frequently. This section covers the main aspects only.

Lesson starter: The ATO website

The Australian Taxation Office website has some income tax calculators. Use one to find out how much income tax you would need to pay if your taxable income is:

$5200 per annum (i.e. $100 per week)

$10 400 per annum (i.e. $200 per week)

$15 600 per annum (i.e. $300 per week)

$20 800 per annum (i.e. $400 per week)

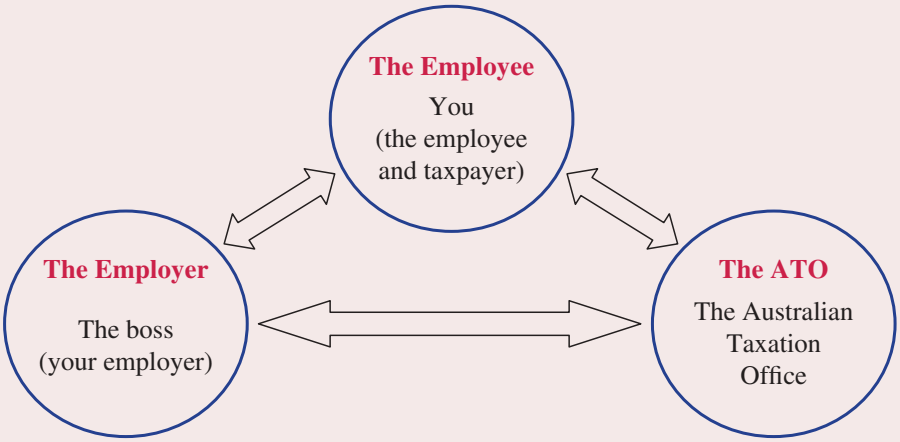
$26 000 per annum (i.e. $500 per week)

Does a person earning $1000 per week pay twice as much tax as a person earning $500 per week?

Does a person earning $2000 per week pay twice as much tax as a person earning $1000 per week?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS



■ The PAYG tax system works in the following way.

- The employee works for and gets paid by the employer every week, fortnight or month.
- The employer calculates the tax that the employee should pay for the amount earned by the employee.
- The employer sends that tax to the ATO every time the employee gets paid.
- The ATO passes the income tax to the federal government.
- On 30 June, the employer gives the employee a payment summary to confirm the amount of tax that has been paid to the ATO on behalf of the employee.
- Between 1 July and 31 October, the employee completes a **tax return** and sends it to the ATO. Some people pay a registered tax agent to do this return for them.
- On this tax return, the employee lists the following.
  - *All* forms of income, including interest from investments.
  - Legitimate deductions shown on receipts and invoices, such as work-related expenses and donations.
- **Taxable income** is calculated using the formula:
  Taxable income = gross income − deductions
- **Income tax payable** can be calculated using the tax table published every year by the Australian Taxation Office (ATO). Every row in the table is called a **tax bracket**. The table below shows the tax rates for the 2022/2023 financial year. The **Medicare levy** is an additional tax that the government uses to support funding for medicines, doctors, hospitals and other aspects of the health care system. Answers to the questions in the following exercise are based on this table, but note that the actual rates may vary from year to year.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $45 000	19 cents for each $1 over $18 200
$45 001 – $120 000	$5 092 plus 32.5 cents for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

The above rates **do not** include the Medicare levy of 2%

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

This table can be used to calculate the amount of tax you *should have* paid (i.e. the tax payable), as opposed to the tax you *did* pay during the year (i.e. the tax withheld). Each row in the table is called a **tax bracket**.

- You may also need to pay the Medicare **levy**. This is a scheme in which all Australian taxpayers share in the cost of running the medical system. For many people this is currently 2% of their taxable income.
- It is possible that you may have paid too much tax during the year and will receive a tax refund.
- It is also possible that you may have paid too little tax and will receive a letter from the ATO asking for the tax liability to be paid.

Key terminology: taxation, employer, employee, tax return, taxable income, tax bracket, levy, deductions, p.a. (per annum)

Exercise 1D

UNDERSTANDING	1–3	2, 3

Note: The questions in this exercise relate to the tax table given in the Key ideas, unless stated otherwise.

1 Complete this statement: Taxable income = _____ income minus _____.

2 Based on the table in the Key ideas, determine if the following statements are true or false.

a A taxable income of $10 400 requires no tax to be paid.

b The highest income earners in Australia pay 45 cents tax for every dollar they earn.

3 In the 2022/2023 financial year, Ann's taxable income was $120 000, which puts her at the very top of the middle tax bracket in the tax table. Ben's taxable income was $120 001, which puts him in a higher tax bracket. Ignoring the Medicare levy, how much extra tax did Ben pay compared to Ann?

FLUENCY	4, 5	4–6

Example 14 Calculating income tax payable

During the 2022/2023 financial year, Richard earned $1050 per week ($54 600 per annum) from his employer and other sources, such as interest on investments. He has receipts for $375 for work-related expenses and donations.

a Calculate Richard's taxable income.

b Use this tax table to calculate Richard's tax payable amount.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $45 000	19 cents for each $1 over $18 200
$45 001 – $120 000	$5 092 plus 32.5 cents for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c Richard must also pay the Medicare levy of 2% of his taxable income. How much is the Medicare levy?
d Add the tax payable and the Medicare levy amounts.
e Express the total tax in part **d** as a percentage of Richard's taxable income, to one decimal place.
f During the financial year, Richard's employer sent a total of $7797 in tax to the ATO. Has Richard paid too much tax or not enough? Calculate his refund or liability.

SOLUTION	EXPLANATION
a Gross income = $54 600 Deductions = $375 Taxable income = $54 225	Taxable income = gross income − deductions
b Tax payable: $5092 + 0.325 × ($54 225 − $45 000) = $8090.13	Richard is in the middle tax bracket in the table, in which it says: $5092 plus 32.5c for each $1 over $45 000 Note: 32.5 cents is $0.325.
c $\frac{2}{100} \times 54\,225 = \$1084.50$	Medicare levy is 2% of the taxable income.
d $8090.13 + $1084.50 = $9174.63	This is the total amount of tax that Richard should have paid.
e $\frac{9174.63}{54\,225} \times 100 = 16.9\%$ (to 1 d.p.)	This implies that Richard paid approximately 16.9% tax on every dollar. This is sometimes read as '16.9 cents in the dollar'.
f Richard paid $7797 in tax during the year. He should have paid $9174.63. Richard has not paid enough tax. He must pay another $1377.63 in tax.	This is known as a shortfall or a liability. He will receive a letter from the ATO requesting payment of the difference. $9174.63 − $7797 = $1377.63

Now you try

During the 2022/2023 financial year, Francesca earned $82 300 per annum from her employer and other sources, such as interest on investments. She has receipts for $530 for work-related expenses and donations.

a Calculate Francesca's taxable income.
b Use the tax table from the Key ideas to calculate Francesca's tax payable amount.
c Francesca must also pay the Medicare levy of 2% of her taxable income. How much is the Medicare levy?
d Add the tax payable and the Medicare levy amounts.
e Express the total tax in part **d** as a percentage of Francesca's taxable income, to one decimal place.
f During the financial year, Francesca's employer sent a total of $17 445 in tax to the ATO. Has Francesca paid too much tax or not enough? Calculate her refund or liability.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 During the 2022/2023 financial year, Liam earned $94 220 from his employer and other sources, such as interest on investments. He has receipts for $615 for work-related expenses and donations.

- **a** Calculate Liam's taxable income.
- **b** Use the tax table from the Key ideas to calculate Liam's tax payable amount.
- **c** Liam must also pay the Medicare levy of 2% of his taxable income. How much is the Medicare levy?
- **d** Add the tax payable and the Medicare levy amounts.
- **e** Express the total tax in part **d** as a percentage of Liam's taxable income, to one decimal place.
- **f** During the financial year, Liam's employer sent a total of $25 249 in tax to the ATO. Has Liam paid too much tax or not enough? Calculate his refund or liability.

5 Use the tax table in the Key ideas to calculate the income tax payable on these taxable incomes.

a $30 000 **b** $60 000 **c** $150 000 **d** $200 000

6 Lee has come to the end of her first financial year employed as a website developer.
On June 30 she made the following notes about the financial year.

Gross income from employer	$58 725
Gross income from casual job	$7500
Interest on investments	$75
Donations	$250
Work-related expenses	$425
Tax paid during the financial year	$13 070

Hint: Taxable income = all incomes − deductions

- **a** Calculate Lee's taxable income.
- **b** Use the tax table shown in Key ideas to calculate Lee's tax payable amount.
- **c** Lee must also pay the Medicare levy of 2% of her taxable income. How much is the Medicare levy?
- **d** Add the tax payable and the Medicare levy.
- **e** Express the total tax in part **d** as a percentage of Lee's taxable income, to one decimal place.
- **f** Has Lee paid too much tax or not enough? Calculate her refund or liability.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING | 7, 8, 10, 11 | 7, 9, 11–13

7 Alec's Medicare levy is $1750. This is 2% of his taxable income. What is Alec's taxable income?

8 Tara is saving for an overseas trip. Her taxable income is usually about $20 000. She estimates that she will need $5000 for the trip, so she is going to do some extra work to raise the money. How much extra will Tara need to earn in order to save the extra $5000 after tax?

9 When Saled used the tax table to calculate his income tax payable, it turned out to be $23 097. What is his taxable income?

Hint: Use the tax table given in Example 14 to determine in which tax bracket Saled falls.

10 Explain the difference between a tax refund and a tax liability.

11 Gordana looked at the last row of the tax table and said, 'It is so unfair that people in that tax bracket must pay 45 cents in every dollar in tax.' Explain why Gordana is incorrect.

12 The two tables provided show a significant change in the tax payable for low income earners from one year to the next. Note that the amounts listed first in each table are often called the tax-free threshold (i.e. the amount that a person can earn before they must pay tax).

2011/2012	
Taxable income	**Tax on this income**
0 – $6000	Nil
$6001 – $37 000	15c for each $1 over $6000
$37 001 – $80 000	$4650 plus 30c for each $1 over $37 000
$80 001 – $180 000	$17 550 plus 37c for each $1 over $80 000
$180 001 and over	$54 550 plus 45c for each $1 over $180 000
2012/2013	
Taxable income	**Tax on this income**
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001 – $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

a There were some significant changes between the financial years 2011/2012 and 2012/2013. Describe three of them.

b The following people had the same taxable income during both financial years. Find the difference in their tax payable amounts and state whether they were advantaged or disadvantaged by the changes, or not affected at all?

i Ali: Taxable income = $5000

ii Xi: Taxable income = $15 000

iii Charlotte: Taxable income = $30 000

iv Diego: Taxable income = $50 000

13 Below is a tax table for people who are not residents of Australia but are working in Australia.

Taxable income	Tax on this income
$0 – $120 000	32.5c for each $1
$120 001 – $180 000	$39 000 plus 37c for each $1 over $120 000
$180 001 and over	$61 200 plus 45c for each $1 over $180 000

Compare this table to the one for Australian residents in the Key ideas.
What difference would it make to the amount of tax paid by these people if they were non-residents rather than residents?

a Amy: Taxable income = $18 000

b Ben: Taxable income = $40 000

c Chi: Taxable income = $150 000

d Dan: Taxable income = $200 000

ENRICHMENT: What are legitimate tax deductions? — 14

14 a Choose an occupation or career in which you are interested. Imagine that you are working in that job. During the year you will need to keep receipts for items you have bought that are legitimate work-related expenses. Do some research on the internet and write down some of the things that you will be able to claim as work-related expenses in your chosen occupation.

b i Imagine your taxable income is $80 000. What is your tax payable amount?

ii You just found a receipt for a $100 donation to a registered charity. This decreases your taxable income by $100. By how much does it decrease your tax payable amount?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1E Using a budget to manage income and expenditure EXTENDING

Learning intentions for this section:

- To know the types of expenses that are included in a budget
- To understand how a budget is affected by fixed and variable expenses
- To be able to calculate savings and other expenses based on the information in a budget
- To be able to calculate the best buy from a range of options

Past, present and future learning:

- These concepts were not addressed in our Year 9 book
- This section goes beyond the syllabus but it is a very useful life skill

Once people have been paid their income for the week, fortnight or month, they must plan how to spend it. Most families work on a budget, allocating money for fixed expenses such as the mortgage or rent and the varying (i.e. changing) expenses of petrol, food and clothing.

Lesson starter: Expenses for the month

Write down everything that you think your family would spend money on for the week and the month, and estimate how much those things might cost for the entire year. Where do you think savings could be made? What would be some additional annual expenses?

KEY IDEAS

- A **budget** is an estimate of income and expenses for a period of time.
- Managing money for an individual is similar to operating a small business. Expenses can be divided into two areas:
  - **Fixed expenses** (these do not change during a time period): payment of loans, mortgages, regular bills etc.
  - **Variable expenses** (these costs change over a time period): clothing, entertainment, food etc. (these are estimates)
- When your budget is completed you should always check that your figures are reasonable estimates.
- By looking at the budget you should be able to see how much money is remaining; this can be used as savings or to buy non-essential items.

Key terminology: budget, fixed expenses, variable expenses

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 1E

UNDERSTANDING	1–3	3

1 Classify each expense listed below as most likely a fixed expense or a variable expense.
   a monthly rent
   b monthly phone bill payment plan
   c take away food
   d stationery supplies for work

2 Binh has an income of $956 a week. His weekly expenses, both fixed and variable, total $831.72 of his income. How much money can Binh save each week?

3 Hailey has the following monthly expenses: mortgage = $1458, mobile phone = $49, internet = $60, council rates = $350, water = $55, electricity = $190. What is the total of Hailey's monthly expenses?

FLUENCY	4–7	4, 5, 7, 8

Example 15 Budgeting using percentages

Fiona has a net annual income of $36 000 after deductions. She allocates her budget on a percentage basis.

	Mortgage	Car loan	Food	Education	Sundries	Savings
Expenses (%)	20	15	25	20	10	10

a Determine the amount of fixed expenses, including the mortgage, car loan and education.
b How much should Fiona save?
c Is the amount allocated for food reasonable?

SOLUTION

a Fixed expenses = 55% of $36 000
= 0.55 × $36 000
= $19 800

EXPLANATION

The mortgage, car loan and education are 55% in total. Change 55% to a decimal and multiply by the net income.

b Savings = 10% of $36 000
= 0.1 × $36 000
= $3600

Savings are 10% of the budget.
Change 10% to a decimal and multiply by the net income.

c Food = 25% of $36 000
= 0.25 × $36 000
= $9000 per year, or
$173 per week

This seems reasonable.

Food is 25% of the budget.
Change 25% to a decimal and calculate.
Divide the yearly expenditure by 52 to make a decision on the reasonableness of your answer.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

Kyle has a net annual income of $64 200 after deductions. He allocates his budget on a percentage basis.

	Rent	Food	Entertainment	Bills	Transport	Sundries	Savings
Expenses (%)	30	10	10	15	5	10	20

a Determine the amount of fixed expenses including the rent and bills.
b How much should Kyle save?
c Is the amount allocated for transport reasonable?

4 Paul has an annual income of $75 000 after deductions. He allocates his budget on a percentage basis.

	Mortgage	Car loan	Personal loan	Clothing	Food	Other
Expenses (%)	20	10	25	5	10	30

a Determine the amount of fixed expenses, including the mortgage and loans.
b How much should Paul have left over after paying for his mortgage, car loan and personal loan?
c Is the amount allocated for food reasonable?

5 Lachlan has an income of $2120 per month. If he budgets 5% for clothes, how much will he actually have to spend on clothes each month?

Example 16 Budgeting using fixed values

Running a certain type of car involves yearly, monthly and weekly expenditure. Consider the following vehicle's costs.

- lease $210 per month
- registration $475 per year
- insurance $145 per quarter
- servicing $1800 per year
- petrol $37 per week

a Determine the overall cost to run this car for a year.
b What percentage of a $70 000 salary would this be, correct to one decimal place?

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

a Overall cost = 210×12
$+ \quad 475$
$+ \quad 145 \times 4$
$+ \quad 1800$
$+ \quad 37 \times 52$
$= \quad \$7299$

The overall cost to run the car is $7299.

b % of salary $= \frac{7299}{70\,000} \times 100$
$= 10.4\%$ (to 1 d.p.)

EXPLANATION

Leasing cost: 12 months in a year
Registration cost
Insurance cost: 4 quarters in a year
Servicing cost
Petrol cost: 52 weeks in a year
The overall cost is found by adding the individual totals.

Percentage $= \frac{\text{car cost}}{\text{total salary}} \times 100$

Round as required.

Now you try

Running a boat involves yearly, monthly and weekly expenditure. Consider the following boat's costs.

- registration $342 per year
- insurance $120 per quarter
- servicing $360 per year
- fuel $300 per month
- storing boat $2400 per year

a Determine the overall cost to run this boat for a year.
b What percentage of a $82 000 salary would this be, correct to one decimal place?

6 Eliana is a student in a share house. She has the following expenses in her budget.

- rent $270 per week
- electricity $550 per quarter
- phone and internet $109 per month
- car $90 per week
- food $170 per week
- insurance $2000 a year

Hint: Use 52 weeks in a year, 12 months in a year and 4 quarters in a year.

a Determine Eliana's costs for a year.
b What percentage of Eliana's net annual salary of $45 000 would this be, correct to one decimal place?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 A school charges the following amounts for every student:

- fees per term (4 terms) $1590
- subject levies per year $615
- building fund per week $45
- uniforms and books per year $459

a Determine the overall cost per year.

b If the school bills twice a year, covering all the items above, what would be the amount of each payment?

c How much should be saved per week to make the biannual payments?

8 A small business owner has the following expenses to budget for.

- rent $1750 a month
- internet provision $73.75 a month
- wages $1500 a week
- electricity $537.50 a quarter
- water $150 a quarter
- insurance $62.50 a month

a What is the annual budget for the small business?

b How much does the business owner need to make each week just to break even?

c If the business earns $6250 a week, what percentage of this needs to be spent on wages?

PROBLEM-SOLVING AND REASONING

9, 10, 12–14 | 10–12, 14, 15

9 Francine's petrol budget is $59 from her weekly income of $437.50.

a What percentage of her budget is this? Give your answer to two decimal places.

b If petrol costs $1.99 per litre, how many litres of petrol, correct to two decimal places, is Francine budgeting for in a week?

10 Grant works a 34-hour week at $23.50 per hour. His net income is 65% of his gross income.

a Determine his net weekly income.

b If Grant spends 12% of his net income on entertainment, determine the amount he actually spends per year on entertainment.

c Grant saves $50 per week. What percentage of his net income is this (to two decimal places)?

11 Dario earns $432 per fortnight at a take-away pizza shop. He budgets 20% for food, 10% for recreation, 13% for transport, 20% for savings, 25% for taxation and 12% for clothing.

a Determine the actual amount budgeted for each category every fortnight.

Dario's wage increases by 30%.

b Determine how much he would now save each week.

c What percentage increase is the answer to part **c** on the original amount saved?

d Determine the extra amount of money Dario saves per year after his wage increase.

e If transport is a fixed expense, its percentage of Dario's budget will change. Determine the new percentage.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 17 Calculating best buys

Soft drink is sold in three convenient packs at the local store.
- carton of 36 (375 mL) cans at $22.50
- a six-pack of (375 mL) cans at $5.00
- 2-litre bottles at $2.80

Determine the cheapest way to buy the soft drink.

SOLUTION

Buying by the carton:
Cost = $22.50 ÷ (36 × 375)
= $0.0017 per mL

Buying by the six-pack:
Cost = $5 ÷ (6 × 375)
= $0.0022 per mL

Buying by the bottle:
Cost = $2.80 ÷ 2000
= $0.0014 per mL

∴ The cheapest way to buy the soft drink is to buy the 2-litre bottle.

EXPLANATION

Total mL = 36 × 375
Divide to work out the cost per mL.

Total mL = 6 × 375

Total mL = 2 × 1000, since 1 L = 1000 mL.

Compare the three costs per mL.

Now you try

A brand of toilet rolls are sold in three pack types at the supermarket.
- a pack of 18 rolls for $8.82
- a pack of 6 rolls for $3.30
- a pack of 4 double length rolls for $3.68

Determine the cheapest way to buy the toilet rolls.

12 Tea bags can be purchased from the supermarket in three forms.
- 25 tea bags at $2.36
- 50 bags at $4.80
- 200 bags at $15.00

What is the cheapest way to buy tea bags?

13 A weekly train concession ticket costs $16. A day ticket costs $3.60. If you are going to school only 4 days next week, is it cheaper to buy one ticket per day or a weekly ticket?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

14 A holiday caravan park offers its cabins at the following rates.

- \$87 per night (Sunday–Thursday)
- \$187 for a weekend (Friday and Saturday)
- \$500 per week

a Determine the nightly rate in each case.

b Which price is the best value?

15 Tomato sauce is priced at:

- 200 mL bottle \$2.35
- 500 mL bottle \$5.24

a Find the cost per mL of the tomato sauce in each case.

b Which is the cheapest way to buy tomato sauce?

c What would be the cost of 200 mL at the 500 mL rate?

d How much would be saved by buying the 200 mL bottle at this rate?

e Suggest why the 200 mL bottle is not sold at this price.

ENRICHMENT: Minimum cost of tennis balls — 16

16 Safeserve has a sale on tennis balls for one month.

When you buy:

- 1 container, it costs \$5
- 6 containers, it costs \$28
- 12 containers, it costs \$40
- 24 containers, it costs \$60

You need 90 containers for your club to have enough for a season.

a Determine the minimum cost if you buy exactly 90 containers.

b Determine the overall minimum cost, and the number of extra containers you will have in this situation.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1F Simple interest

Learning intentions for this section:

- To understand how simple interest is calculated
- To be able to use the simple interest formula to perform calculations

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Financial mathematics A*
- It consolidates and extends the concepts in Sections 2G and 2H of our Year 9 book
- Financial mathematics is a major topic in Stage 6 Standard
- You may be expected to recall the formula from memory in the HSC Examination

If you invest money you will *earn* interest.

If you borrow money you will *pay* interest.

There are two different types of interest:

- simple interest
- compound interest.

Lesson starter: I want to double my money!

Sophie has $100 invested but she would like to have $200.

Her investment increases every year by 10% of the original investment.

- By how much will it increase in the first year?
- Copy and complete the following table.

Year	0	1	2	3	4	5	6	7	8	9	10
Amount	$100	$110	$120								

- How long will it take to reach her goal?
- How long will it take if her investment increases by 5% every year?
- How long will it take if her investment increases by 7.5% every year?
- How long will it take if her investment increases by 5% every year for the first 5 years, then 6% every year thereafter?

KEY IDEAS

- Simple interest is always calculated using the original amount invested or borrowed.
- The terms needed to understand **simple interest** are:
  - **Principal** (P): the amount of money borrowed or invested
  - **Rate of interest** (r): the annual (yearly) percentage rate of interest (e.g. 3% p.a.)
  - Time periods (n): this is usually the number of years.
  - Interest (I): the amount of interest accrued over a given time.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- The formula for calculating simple interest is:

  $I = Prn$

  where

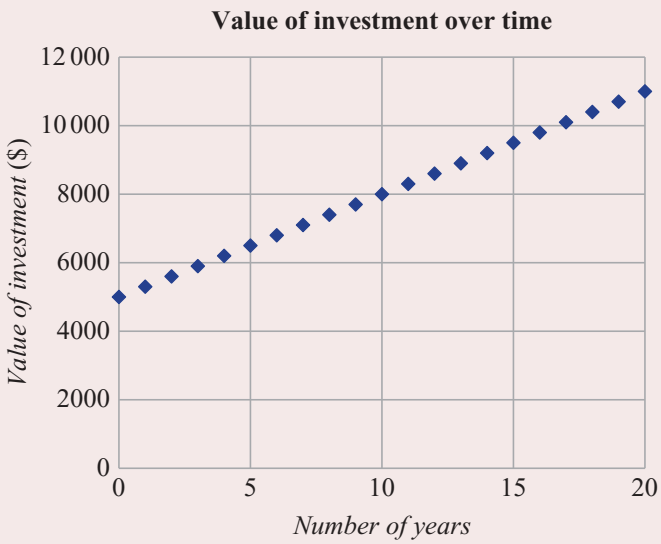
  I = the amount of simple interest (in $)

  P = the principal (the initial amount borrowed or invested)

  r = the interest rate per period, expressed as a decimal

  n = the number of periods

- The graph below shows an investment of $5000 growing with 6% p.a. simple interest, which is a linear relationship.



- In the graph above, the interest earned is $300 per year.
  The points in the graph can be generated on a calculator:
  Enter the number 5000, then press [=].
  Add 300.
  Press [=] [=] [=] [=] etc.

Key terminology: principal, rate of interest, simple interest, annual, invest, borrow

Exercise 1F

UNDERSTANDING	1, 2	2

1 Interest on a loan is fixed at $60 a year. How much interest is due in:

a 2 years?

b 7 years?

c 6 months?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 Simple interest on $7000 is 6% p.a. How much interest is earned in:

a 1 year?

b 2 years?

c 1 month?

FLUENCY 3–6 4–7

Example 18 Using the simple interest formula

Use the simple interest formula, $I = Prn$, to find:

a the interest (I) when $600 is invested at 8% p.a. for 18 months

b the annual interest rate (r) when $5000 earns $150 interest in 2 years

SOLUTION

a $P = 600$

$r = 8 \div 100 = 0.08$

$n = 18 \text{ months} = \frac{18}{12} = 1.5 \text{ years}$

$I = Prn$

$= 600 \times 0.08 \times 1.5$

$= 72$

The interest is $72 in 18 months.

EXPLANATION

Write out the information that you know and the formula.

Express the rate as a decimal (or fraction).

Substitute into the formula, using years for n.

b $P = 5000$

$I = 150$

$n = 2$ years

$I = Prn$

$150 = 5000 \times r \times 2$

$150 = 10\,000\, r$

$r = 150 \div 10\,000 = 0.015$

The simple interest rate is 1.5% per year.

Write the formula and the information known.

Substitute the values into the formula and solve the equation to find r.

Write the rate as a percentage. (i.e. $\times 100$)

Now you try

Use the simple interest formula, $I = Prn$, to find:

a the interest (I) when $450 is invested at 5% p.a. for 30 months

b the annual interest rate (r) when $3500 earns $210 interest in 3 years

3 Use the simple interest formula, $I = Prn$, to find:

a the interest (I) when $500 is invested at 6% p.a. for 24 months

b the annual interest rate (r) when $3000 earns $270 interest in 3 years

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Copy and complete this table of values for I, P, r and n.

	P	r	n	I
a	\$700	5% p.a.	4 years	
b	\$2000	7% p.a.	3 years	
c	\$3500	3% p.a.	22 months	
d	\$750	$2\frac{1}{2}$% p.a.	30 months	
e	\$22 500		3 years	\$2025
f	\$1770		5 years	\$354

Hint: Recall: $I = Prn$
n must be years if rate is p.a.

Example 19 Calculating repayments with simple interest

\$3000 is borrowed at 12% p.a. simple interest for 2 years.

a What is the total amount owed over the 2 years?

b If repayments of the loan are made monthly, how much would each payment need to be?

SOLUTION

a $P = \$3000,\ r = 12 \div 100 = 0.12,\ n = 2$

$I = Prn$

$= 3000 \times 0.12 \times 2$

$= \$720$

Total amount $= \$3000 + \$720$

$= \$3720$

b Amount of each payment $= \$3720 \div 24$

$= \$155$ per month

EXPLANATION

List the information you know.

Write the formula.

Substitute the values and evaluate.

Total amount is the original amount *plus* the interest.

2 years = 24 months, so there are 24 payments to be made.
Divide the total by 24.

Now you try

\$5400 is borrowed at 9% p.a. simple interest for 4 years.

a What is the total amount owed over the 4 years?

b If repayments of the loan are made monthly, how much would each payment need to be?

5 \$5000 is borrowed at 11% p.a. simple interest for 3 years.

a What is the total amount owed over the 3 years?

b If repayments of the loan are made monthly, how much would each payment need to be?

Hint: Calculate the interest first.

6 Under hire purchase, John bought a second-hand car for \$11 500. He paid no deposit and decided to pay the loan off in 7 years. If the simple interest is 6.45%, determine:

a the total interest paid

b the total amount of the repayment

c the payments per month

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 $10 000 is borrowed to buy a second-hand BMW. The interest is calculated at a simple interest rate of 19% p.a. over 4 years.

- **a** What is the total interest on the loan?
- **b** How much is to be repaid?
- **c** What is the monthly repayment on this loan?

PROBLEM-SOLVING AND REASONING

8–10 | 10–13

8 How much interest will Giorgio receive if he invests $7000 in stocks at 3.6% p.a. simple interest for 4 years?

9 Rebecca invests $4000 for 3 years at 5.7% p.a. simple interest paid yearly.

- **a** How much interest will she receive in the first year?
- **b** What is the total amount of interest Rebecca will receive over the 3 years?
- **c** How much money will Rebecca have after the 3-year investment?

10 An investment of $15 000 receives an interest payment over 3 years of $7200. What was the rate of simple interest per annum?

Hint: Substitute into the formula $I = Prn$ and solve the resulting equation.

11 Aiden wishes to invest $3000 at 8% per annum. How long will he need to invest for his total investment to double?

12 Ivan wishes to invest some money for 5 years at 4.5% p.a. paid yearly. If he wishes to receive $3000 in interest payments per year, how much should he invest? Round your answer to the nearest dollar.

13 Gretta's interest payment on her loan totalled $1875. If the interest rate was 5% p.a. and the loan had a life of 5 years, what amount did she borrow?

ENRICHMENT: Which way is best?

— | 14

14 A shed manufacturer offers finance with a rate of 3.5% p.a. paid at the end of 5 years with a deposit of 10%, or a rate of 6.4% repaid over 3 years with a deposit of 20%.
Melania and Donald decide to purchase a fully erected four-square shed for $12 500.

- **a** How much deposit will they need to pay in each case?
- **b** What is the total interest they will incur in each case?
- **c** If they decided to pay per month, what would be their monthly repayment?
- **d** Discuss the benefits of the different types of purchasing methods.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Progress quiz

1A **1** Express:

a 32% as a decimal

b 8% as a simplified fraction

c $\frac{11}{25}$ as a percentage

d $\frac{5}{16}$ as a percentage

e 0.252 as a percentage

f $15\frac{1}{2}\%$ as a fraction

1A **2** Gina puts 36% of her $6000 monthly salary in a savings account. How much does she have left over?

1B **3** Complete the following.

a Increase $230 by 24%

b Increase 180 mL by 8%

c Decrease 156 cm by 15%

1B **4** A $299 coffee machine is discounted by 35%. What is the discounted price?

1B **5** Josh bought a car for $15 000 and later sold it for $21 000. What was his percentage profit?

1C **6** Find the gross income for a particular week in the following work situations.

a Pippa earns $300 per week plus 8% commission on her sales. In a particular week she makes $8200 worth of sales.

b Ari is paid $15.70 per hour in his job as a shop assistant. The first 36 hours he works in a week are paid at the normal hourly rate, the next 4 hours at time and a half and then double time after that. Ari works 42 hours in a particular week.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Progress quiz

1D

7 During the 2022/2023 financial year, Cameron earned $76 300 per annum. He had receipts for $425 for donations and work-related expenses.

a Calculate Cameron's taxable income.

b Use this tax table to calculate Cameron's tax payable amount, to the nearest cent.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $45 000	19 cents for each $1 over $18 200
$45 001 – $120 000	$5 092 plus 32.5 cents for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

c Cameron also must pay the Medicare levy of 2% of his taxable income. How much is the levy, to the nearest cent?

d During the financial year, Cameron's employer sent a total of $15 255 in tax to the ATO on his behalf. By adding together your answers from parts **b** and **c**, calculate the amount Cameron must pay or will be refunded on his tax return.

1E

8 Charli has the following expenses in her household budget.

- rent $320 per week
- phone and internet $119 per month
- electricity $72 per quarter
- car registration $700 per year
- other car costs $120 per month
- food $110 per week
- clothing $260 per month
- medical and other insurance $160 per month

a Determine the overall cost for running the household for the year. (Use 52 weeks in a year.)

b What percentage of an $82 000 annual salary does your answer to part **a** represent? Round your answer to one decimal place.

1F

9 Use the simple interest formula to find:

a the amount owed when $4000 is borrowed at 6% p.a. for 3 years

b the investment period, in years, if an investment of $2500 at 4% p.a. earns $450 in interest

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1G Compound interest and depreciation

Learning intentions for this section:

- To understand how compound interest is calculated
- To be able to use the compound interest formula to solve problems

Past, present and future learning:

- This section introduces the Stage 5 Core topic called *Financial mathematics B*
- These concepts were not addressed in our Year 9 book
- Financial mathematics is a major topic in Stage 6 Standard
- You may be expected to recall the formulas from memory in the HSC Examination

Simple interest is always calculated using the amount invested or borrowed, so the amount of interest earned or charged is the same every year.

In this section, you will see that compound interest on an investment is calculated so that you earn interest on your interest.

Lesson starter: I want to double my money faster!

As noted in Section 1F, Sophie has $100 invested but she would like to have $200.
In this scenario, her investment increases every year by 10% of the **value at the end of the previous year**.

- By how much will her investment increase in the first year?
- How much will her investment be worth at the end of the first year?
- By how much will her investment increase in the second year?
- By how much will her investment increase in the third year?
- Copy and complete the following table.

Year	0	1	2	3	4	5	6	7	8	9	10
Amount	$100	$110									

- How long will it take Sophie to reach her goal?
- How long will it take if her investment increases by 5% every year?
- How long will it take if her investment increases by 7.5% every year?
- How long will it take if her investment increases by 5% every year for the first 5 years, then 6% every year thereafter?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- **Compound interest** is calculated on the current value of an investment (i.e. not the original value).
- The amount of simple interest stays the same for each period. In contrast, the amount of compound interest grows because you earn interest on your interest.
- The table below compares simple interest and compound interest on the same investment.

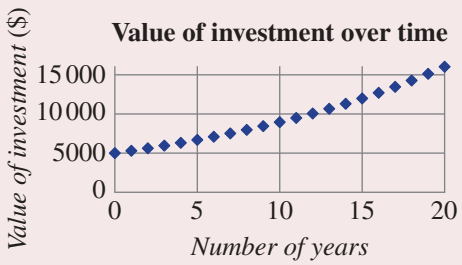
Simple interest 6% p.a.			
Year	Opening balance	Interest	Closing balance
2019	$5000.00	$300.00	$5300.00
2020	$5300.00	$300.00	$5600.00
2021	$5600.00	$300.00	$5900.00
2022	$5900.00	$300.00	$6200.00
2023	$6200.00	$300.00	$6500.00

Compound interest 6% p.a.			
Year	Opening balance	Interest	Closing balance
2019	$5000.00	$300.00	$5300.00
2020	$5300.00	$318.00	$5618.00
2021	$5618.00	$337.08	$5955.08
2022	$5955.08	$357.30	$6312.38
2023	$6312.38	$378.74	$6691.13

- For compound interest: the numbers in the right-hand table above can be generated on a calculator:
  Enter the number 5000 , then press [=].
  100% plus 6% = 106% = 1.06, so multiply by 1.06.
  Press [=] [=] [=] [=] etc.
- The final value also can be calculated as 5000×1.06^5.
- To calculate the amount of compound interest earned, subtract the original value from the final value. In the example above:

  Amount of compound interest = $6691.13 − $5000 = $1691.13

- The graph below shows an investment of $5000 growing as it earns interest of 6% per annum, compounding annually. This is a non-linear relationship. The value of the investment grows exponentially.



- The **future value** of a compound interest investment can be calculated using the formula $FV = PV(1 + r)^n$ where
  FV = future value of the investment
  PV = **present value** of the investment
  r = interest rate as per time period and
  n = number of time periods

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- Amount of compound interest = $FV - PV$
- **Depreciation** is the reverse situation, in which the value of an object decreases by a percentage year after year.
  For example: A \$1000 computer loses 30% in value every year.
  Year 1: \$1000 − (30% of \$1000) = \$700
  Year 2: \$700 − (30% of \$700) = \$490
- In the example above, you can use your calculator to continue the pattern as follows:
  Press 1000 then =
  100% minus 30% = 70% = 0.7, so multiply by 0.7
  Press [=] [=] [=] [=] etc.
- The **salvage value** of an asset can be calculated using the formula
  $S = V_0(1 - r)^n$ where
  S = salvage value
  V_0 = initial value of the asset
  r = depreciation rate per time period and
  n = number of periods

Key terminology: compound interest, principal, rate of interest, present value, future value, depreciation, salvage value

Exercise 1G

UNDERSTANDING 1–3 3

1 Consider \$500 invested at 10% p.a. compounded annually.

Hint: For the second year, you need to use \$500 plus the interest from the first year.

- **a** How much interest is earned in the first year?
- **b** What is the balance of the account once the first year's interest is added?
- **c** How much interest is earned in the second year?
- **d** What is the balance of the account at the end of the second year?
- **e** Use your calculator to work out $500(1.1)^2$.

2 Find the value of the following, correct to two decimal places.

- **a** $\$1000 \times 1.05 \times 1.05$
- **b** $\$1000 \times 1.05^2$
- **c** $\$1000 \times 1.05 \times 1.05 \times 1.05$
- **d** $\$1000 \times 1.05^3$

3 Fill in the missing numbers.

Hint: For compound interest, $FV = PV(1 + r)^n$.

Hint: 15% as a decimal is 0.15.

- **a** \$700 invested at 8% p.a. compounded annually for 2 years.

  $A = \square(1.08)^{\square}$

- **b** \$1000 invested at 15% p.a. compounded annually for 6 years.

  $A = 1000(\square)^6$

- **c** \$850 invested at 6% p.a. compounded annually for 4 years.

  $A = 850(\square)^{\square}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 4, 5–6(½), 7, 8(½) | 4–6(½), 8(½)

Example 20 Using the compound interest formula

Determine the future value after 5 years when $4000 is compounded annually at 8%.

SOLUTION

$PV = 4000,\ n = 5,\ r = 0.08$

$FV = PV(1 + r)^n$

$= 4000(1 + 0.08)^5$

$= 4000(1.08)^5$

$= \$5877.31$

EXPLANATION

List the values for the terms you know.

Write the formula.

Substitute the values.

Simplify and evaluate.
Write your answer to two decimal places, (nearest cent).

Now you try

Determine the future value after 4 years when $3000 is compounded annually at 6%.

4 Determine the amount after 5 years when:

a $4000 is compounded annually at 5%

b $8000 is compounded annually at 8.35%

c $6500 is compounded annually at 16%

d $6500 is compounded annually at 8%

Hint: $FV = PV(1 + r)^n$

5 Determine the amount when $100 000 is compounded annually at 6% for:

a 1 year **b** 2 years **c** 3 years

d 5 years **e** 10 years **f** 15 years

Example 21 Converting rates and time periods

Calculate the number of periods and the rates of interest offered per period for each of the following.

a 6% p.a. over 4 years paid monthly

b 18% p.a. over 3 years paid quarterly

SOLUTION

a $n = 4 \times 12$ $r = (6 \div 12)\%$

$= 48$ $= 0.5\%$

$= 0.05$

b $n = 3 \times 4$ $r = (18 \div 4)\%$

$= 12$ $= 4.5\%$

$= 0.045$

EXPLANATION

4 years is the same as 48 months, as 12 months = 1 year.
6% p.a. = 6% in one year.
Divide by 12 to find the monthly rate.

There are four quarters in 1 year.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

Calculate the number of periods and the rates of interest offered per period for each of the following.

a 3% p.a. over 2 years paid monthly
b 7% p.a. over 4 years paid biannually (twice yearly)

6 Calculate the number of periods (n) and the rates of interest (r) offered per period for the following. (Round the interest rate to three decimal places where necessary.)

a 6% p.a. over 3 years paid biannually
b 12% p.a. over 5 years paid monthly
c 4.5% p.a. over 2 years paid fortnightly
d 10.5% p.a. over 3.5 years paid quarterly
e 15% p.a. over 8 years paid quarterly
f 9.6% p.a. over 10 years paid monthly

Hint: 'Biannually' means 'twice a year'. 26 fortnights = 1 year

Example 22 Finding compounded amounts using months

Tony's investment of $4000 is compounded at 8.4% p.a. over 5 years. Determine the amount he will have after 5 years if the interest is paid monthly.

SOLUTION	EXPLANATION
$PV = 4000$	List the values of the terms you know.
$n = 5 \times 12$ $= 60$	Convert the time in years to the number of periods (in this question, months); i.e. 60 months = 5 years.
$r = 8.4 \div 12 \div 100$ $= 0.007$	Convert the rate per year to the rate per period (i.e. months) by dividing by 12.
$FV = PV(1 + r)^n$	Write the formula.
$= 4000(1 + 0.007)^{60}$	Substitute the values.
$= 4000(1.007)^{60}$	Simplify and evaluate.
$= \$6078.95$	

Now you try

Sally's investment of $6000 is compounded at 4.8% p.a. over 4 years. Determine the amount she will have after 4 years if the interest is paid monthly.

7 An investment of $8000 is compounded at 12% p.a. over 3 years. Determine the amount the investor will have after 3 years if the interest is compounded monthly.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 Calculate the value of the following investments if interest is compounded monthly.

a $2000 at 6% p.a. for 2 years

b $34 000 at 24% p.a. for 4 years

c $350 at 18% p.a. for 8 years

d $670 at 6.6% p.a. for $2\frac{1}{2}$ years

e $250 at 7.2% p.a. for 12 years

Hint: Convert years to months and the annual rate to the monthly rate.

PROBLEM-SOLVING AND REASONING 9, 10 10–12

9 A car worth $50 000 loses 15% in value every year.

a How much will it be worth at the end of 5 years (correct to the nearest dollar)?

b By how much has the value of the car depreciated?

Hint: Use the formula for salvage value: $S = V_0(1 - r)^n$

10 a Calculate the amount of compound interest paid on $8000 at the end of 3 years for each rate below.

i 12% compounded annually

ii 12% compounded biannually (twice a year)

iii 12% compounded monthly

iv 12% compounded weekly

v 12% compounded daily

Hint: Use: 1 year = 12 months
1 year = 52 weeks
1 year = 365 days

b What is the difference in the interest paid between annual and daily compounding in this case?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 The following are expressions relating to compound interest calculations. Determine the present value (P), number of periods (n), rate of interest per period (r), annual rate of interest (R) and the overall time (t).

a $300(1.07)^{12}$, biannually

b $5000(1.025)^{24}$, monthly

c $1000(1.00036)^{65}$, fortnightly

d $3500(1.000053)^{30}$, daily

e $10\,000\ (1.078)^{10}$, annually

Hint: For 12 time periods with interest paid twice a year, this is 6 years

12 Ellen needs to decide whether to invest her $13 500 for 6 years at 4.2% p.a. compounded monthly or 5.3% compounded biannually. Decide which investment would be the best for Ellen.

ENRICHMENT: Double your money — 13

13 You have $100 000 to invest and wish to double that amount. Use trial and error in the following.

a Determine, to the nearest whole number of years, the length of time it will take to do this using the compound interest formula at rates of:

i 12% p.a. **ii** 6% p.a. **iii** 8% p.a.

iv 16% p.a. **v** 10% p.a. **vi** 20% p.a.

b If the amount of investment is $200 000 and you wish to double it, determine the time it will take using the same interest rates as above.

c Are the lengths of time to double your investment the same in part **a** and part **b**?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1H Investments and loans

Learning intentions for this section:

- To understand that a loan can be repaid in instalments that include interest
- To be able to calculate the total payment for a purchase or loan involving repayments

Past, present and future learning:

- This section addresses the Stage 5 Core topics called *Financial mathematics A* and *Financial mathematics B*
- These concepts were not addressed in our Year 9 book
- Financial mathematics is a major topic in Stage 6 Standard
- You may be expected to recall the formulas from memory in the HSC Examination

When you borrow money, interest is charged, and when you invest money, interest is earned. When you invest money, the institution in which you invest (e.g. bank or credit union) pays you interest. However, when you borrow money, the institution from which you borrow charges you interest, so that you must pay back the money you initially borrowed, plus the interest.

Lesson starter: Credit card statements

Refer to Jackson's credit card statement below.

a How many days were there between the closing balance and the due date?

b What is the minimum payment due?

c If Jackson pays only the minimum, on what balance is the interest charged?

d How much interest is charged if Jackson pays $475.23 on 25/5?

Statement Issue Date: 2/5/20		
Date of purchase	**Details**	**Amount**
3/4/20	Opening balance	314.79
5/4/20	Dean's Jeans	59.95
16/4/20	Tyre Warehouse	138.50
22/4/20	Payment made—thank you	−100.00
27/4/20	Cottonworth's Grocery Store	58.64
30/4/20	Interest charges	3.35
2/5/20	Closing balance	475.23
Percentage rate	**Due date**	**Min. payment**
18.95%	25/5/20	23.75

KEY IDEAS

- **Investments** are amounts put into a bank account or similar with the aim of earning interest on the money.
- **Loans** (money borrowed) have interest charged to them on the amount left owing (i.e. the balance).
- **Repayments** (instalments) are amounts paid to the bank, usually each month, to repay a loan and interest within an agreed time period.

Key terminology: investment, loan, repayment, interest, deposit, debit

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 1H

UNDERSTANDING	1–3	3

1 State if the following are examples of investments, loans or repayments.

a Kara pays $160 per month to pay off her holiday loan.

b Sam deposits a $2000 prize in an account with 3% p.a. interest.

c Georgia borrows $6500 from the bank to finance setting up her small business.

2 Donna can afford to repay $220 a month. How much does she repay over:

a 1 year? **b** 18 months? **c** 5 years?

3 Sarafina buys a new bed on a 'buy now, pay later' offer. No interest is charged if she pays for the bed in 2 years. Sarafina's bed costs $2490 and she pays it back over a period of 20 months in 20 equal instalments. How much is each instalment?

FLUENCY	4–8	4, 6–9

Example 23 Repaying a loan

Wendy takes out a personal loan of $7000 to fund her trip to South Africa. Repayments are made monthly for 3 years at $275 a month. Find:

a the total cost of Wendy's trip

b the interest charged on the loan

SOLUTION

a Total cost $= \$275 \times 36$
$= \$9900$

b Interest $= \$9900 - \$7000$
$= \$2900$

EXPLANATION

$3 \text{ years} = 3 \times 12 = 36 \text{ months}$
Cost = 36 lots of $275

Interest = total paid − amount borrowed

Now you try

Jacob takes out a personal loan of $13 000 to buy a car. He makes repayments monthly for 2 years at $680 a month. Find:

a the total cost of the car

b the interest charged on the loan

4 Jason has a personal loan of $10 000. He is repaying the loan over 5 years. The monthly repayment is $310.

a Calculate the total amount Jason repays over the 5 year loan.

b How much interest is he charged?

Hint: How many monthly repayments in 5 years?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Robert borrows $5500 to buy a second-hand motorbike. He repays the loan in 36 equal monthly instalments of $155.

a Calculate the total cost of the loan.

b How much interest does Robert pay?

6 Alma borrows $250 000 to buy a house. The repayments are $1736 a month for 30 years.

a How many repayments does Alma make?

b What is the total amount Alma pays for the house?

c How much interest is paid over the 30 years?

Example 24 Paying off a purchase

Harry buys a new $2100 computer on the following terms.

- 20% deposit
- monthly repayments of $90 for 2 years

Find:

a the deposit paid

b the total paid for the computer

c the interest charged

SOLUTION

a Deposit $= 0.2 \times 2100$
$= \$420$

b Repayments $= \$90 \times 24$
$= \$2160$

Total paid $= \$2160 + \$420$
$= \$2580$

c Interest $= \$2580 - \$2100$
$= \$480$

EXPLANATION

Find 20% of 2100.

2 years = 24 months
Repay 24 lots of $90.

Repay = deposit + repayments

Interest = total paid − original price

Now you try

Sophie pays $3180 for a holiday apartment rental on the following terms.

- 30% deposit
- monthly repayments of $195 for 1 year

Find:

a the deposit paid

b the total paid for the apartment

c the interest charged

7 George buys a car for $12 750 on the following terms: 20% deposit and monthly repayments of $295 for 3 years.

a Calculate the deposit.

b Find the total of all the repayments.

c Find the cost of buying the car on these terms.

d Find the interest George pays on these terms.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 25 Calculating interest

An account has a minimum monthly balance of $200 and interest is credited monthly on this amount at 1.5%.

a Determine the amount of interest to be credited at the end of the month.

b If the bank charges a fixed administration fee of $5 per month and other fees totalling $1.07, what will be the net amount credited or debited to the account at the end of the month?

SOLUTION	EXPLANATION
a Interest = 1.5% of $200	Interest is 1.5% per month.
$= 0.015 \times \$200$	Change 1.5% to a decimal and calculate.
$= \$3$	
b Net amount = $3 - (5 + 1.07)$	Subtract the deductions from the interest.
$= -3.07$	
$3.07 will be debited from the account.	A negative amount is called a debit.

Now you try

An account has a minimum monthly balance of $180 and interest is credited monthly on this amount at 2.2%.

a Determine the amount of interest to be credited at the end of the month.

b If the bank charges a fixed administration fee of $4.50 per month and other fees totalling $1.18, what will be the net amount credited or debited to the account at the end of the month?

8 A bank account has a minimum monthly balance of $300 and interest is credited monthly at 1.5%.

a Determine the amount of interest to be credited each month.

b If the bank charges a fixed administration fee of $3 per month and fees of $0.24, what will be the net amount credited to the account at the end of the month?

9 An account has no administration fee. The monthly balances for May–October are in the table below. If the interest payable on the minimum monthly balance is 1%, how much interest will be added:

a for each separate month? b over the 6-month period?

May	June	July	August	September	October
$240	$300	$12	$500	$208	$73

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING | 10, 12, 13 | 11, 13–15

10 Supersound offers the following two deals on a sound system worth $7500.

- Deal A: no deposit, interest free and nothing to pay for 18 months
- Deal B: 15% off for cash

Hint: 15% off is 85% of the original amount.

a Nick chooses deal A. Find:

- **i** the deposit he must pay
- **ii** the interest charged
- **iii** the total cost if Nick pays the system off within 18 months

b Phil chooses deal B. What does Phil pay for the same sound system?

c How much does Phil save by paying cash?

11 Camden Finance Company charges 35% flat interest on all loans.

a Mei borrows $15 000 from Camden Finance over 6 years.

- **i** Calculate the interest on the loan.
- **ii** What is the total repaid (i.e. loan + interest)?
- **iii** What is the value of each monthly repayment?

b Lancelle borrows $24 000 from the same company over 10 years.

- **i** Calculate the interest on her loan.
- **ii** What is the total repaid?
- **iii** What is the value of each monthly instalment?

12 A list of transactions that Emma made over a 1-month period is shown. The bank calculates interest *daily* at 0.01% and adds the total to the account balance at the end of this period. It has an administrative fee of $7 per month and other fees over this time total $0.35.

a Copy and complete the balance column of the table.

Date	Deposit	Withdrawal	Balance
1 May			$3010
3 May	$490		
5 May		$2300	
17 May	$490		
18 May		$150	
20 May		$50	
25 May		$218	
31 May	$490		

b Determine the amount of interest added over this month.

c Determine the final balance after all calculations have been made.

d Suggest what the regular deposits might be for.

Hint: In part **b**, interest is calculated on the end-of-the-day balance.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

13 The table below shows the interest and monthly repayments on loans when the simple interest rate is 8.5% p.a.

	18-month term		24-month term		36-month term	
Loan amount	Interest ($)	Monthly payments ($)	Interest ($)	Monthly payments ($)	Interest ($)	Monthly payments ($)
1000	127.50	62.64	170.00	48.75	255.00	34.86
1100	140.25	68.90	187.00	53.63	280.50	38.35
1200	153.00	75.17	204.00	58.50	306.00	41.83
1300	165.75	81.43	221.00	63.38	331.50	45.32
1400	178.50	87.69	238.00	68.25	357.00	48.81
1500	191.25	93.96	255.00	73.13	382.50	52.29
1600	204.00	100.22	272.00	78.00	408.00	55.78
1700	216.75	106.49	289.00	82.88	433.50	59.26
1800	229.50	112.75	306.00	87.75	459.00	62.75
1900	242.25	119.01	323.00	92.63	484.50	66.24
2000	255.00	125.28	340.00	97.50	510.00	69.72

a Use the table to find the monthly repayments for a loan of:

i $1500 over 2 years

ii $2000 over 3 years

iii $1200 over 18 months

b Damien and Lisa can afford monthly repayments of $60. What is the most they can borrow and on what terms?

14 Part of a credit card statement is shown here.

Understanding your account

CLOSING BALANCE
$403.80

CLOSING BALANCE
This is the amount you owe at the end of the statement period

MINIMUM PAYMENT DUE
$10.00

MINIMUM PAYMENT DUE
This is the minimum payment that must be made towards this account

PAYABLE TO MINIMISE FURTHER INTEREST CHARGES
$403.80

PAYABLE TO MINIMISE FURTHER INTEREST CHARGES
This amount you must pay to minimise interest charges for the next statement period

a What is the closing balance?

b What is due on the card if only the minimum payment is made on the due date?

c This card charges 21.9% p.a. interest calculated daily on the unpaid balance. To find the daily interest amount, the company multiplies this balance by 0.0006. What does it cost in interest per day if only the minimum payment is made?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

15 Loans usually involve an establishment fee to set up the loan and an interest rate calculated monthly on your balance. You make a monthly or fortnightly payment, which reduces the balance. Bank fees also apply.

Consider the period for the loan statement shown below.

a What is the opening balance for this statement?

b What is the administrative fee charged by the bank for each transaction?

c What is the regular fee charged by the bank for servicing the loan?

d If the term of the loan is 25 years, what will be the total servicing fees charged by the bank?

e What is the regular fortnightly payment made?

f What will be the total fortnightly payments made over the term of the 25-year loan?

Complete Home Loan Transactions – Account number 33164000				
Date	**Transaction description**	**Debits**	**Credits**	**Balance**
	Balance brought forward from previous page			98 822.90 Dr
15 Oct	Repayment/Payment		378.50	
	Administrative fee	0.23		98 444.63 Dr
24 Oct	Interest charged	531.88		98 976.51 Dr
24 Oct	Fee for servicing your loan	8.00		98 984.51 Dr
29 Oct	Repayment/Payment		378.50	
	Administrative fee	0.23		98 606.24 Dr
12 Nov	Repayment/Payment		378.50	
	Administrative fee	0.23		98 227.97 Dr
24 Nov	Interest charged	548.07		98 776.04 Dr
24 Nov	Fee for servicing your loan	8.00		98 784.04 Dr
26 Nov	Repayment/Payment		378.50	
	Administrative fee	0.23		98 405.77 Dr
	⟶ Change in interest rate on 03/12/18 to 06.800% per annum			
10 Dec	Repayment/Payment		378.50	
	Administrative fee	0.23		98 027.50 Dr
24 Dec	Interest charged	543.08		98 570.58 Dr
24 Dec	Fee for servicing your loan	8.00		98 578.58 Dr
24 Dec	Repayment/Payment		378.50	
	Administrative fee	0.23		98 200.31 Dr
31 Dec	Closing balance			98 200.31 Dr

ENRICHMENT: Reducing the balance of loans (a spreadsheet approach) — 16

16 When you take out a loan from a lending institution you will be asked to make regular payments (usually monthly) for a certain period of time to repay the loan completely. The larger the repayment, the shorter the term of the loan.

Loans work mostly on a reducing balance and you can find out how much balance is owing at the end of each month from a statement, which is issued on a regular basis.

Let's look at an example of how the balance is reducing.

If you borrow $15 000 at 17% p.a. and make repayments of $260 per month, at the end of the first month your statement would be calculated as shown.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

$$\text{Interest due} = \frac{15\,000 \times 0.17}{12}$$

$$= \$212.50$$

$$\text{Repayment} = \$260$$

$$\text{Amount owing} = \$15\,000 + \$212.50 - \$260$$

$$= \$14\,952.50$$

This process would be repeated for the next month:

$$\text{Interest due} = \frac{14\,952.50 \times 0.17}{12}$$

$$= \$211.83$$

$$\text{Repayment} = \$260$$

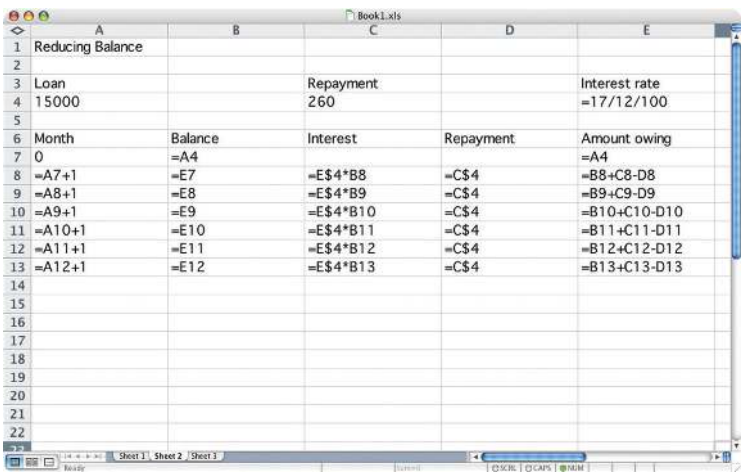
$$\text{Amount owing} = \$14\,952.50 + \$211.83 - \$260$$

$$= \$14\,904.33$$

As you can see, the amount owing is decreasing and so is the interest owed each month. Meanwhile, more of your repayment is actually reducing the balance of the loan.
A statement might look like this:

Balance	Interest	Repayment	Amount owing
15 000	212.50	260	14 952.50
14 952.50	211.83	260	14 904.33
14 904.33	211.14	260	14 855.47
14 855.47	210.45	260	14 805.92
14 805.92	209.75	260	14 755.67

Check to see that all the calculations are correct on the statement above.
As this process is repetitive, the calculations are best done by means of a spreadsheet. To create a spreadsheet for the process, copy the following, extending your sheet to cover 5 years.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1I Using spreadsheets for investments, loans and depreciation

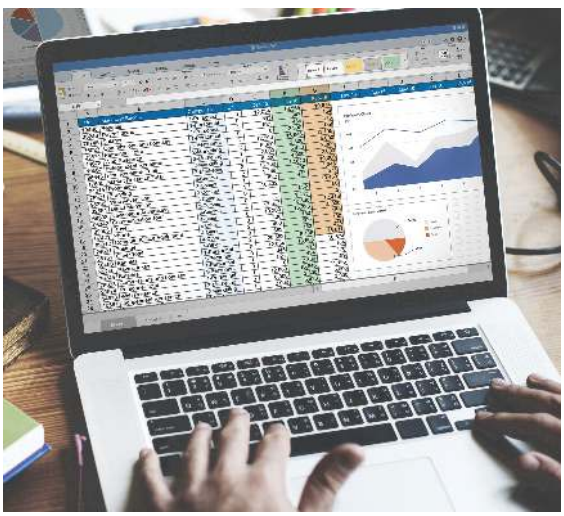
Learning intentions for this section:

- To understand how technology can be used to compare interest calculations
- To be able to use technology to calculate interest and future values

Past, present and future learning:

- This section concludes the Stage 5 Core topics called *Financial mathematics A* and *Financial mathematics B*
- These concepts were not addressed in our Year 9 book
- Financial mathematics is a major topic in Stage 6 Standard
- You may be expected to recall the formulas from memory in the HSC Examination

In this section you will be using spreadsheets to investigate and analyse loans and investments.



Lesson starter: Who earns the most?

- Ceanna invests $500 at 8% p.a., compounded monthly over 3 years.
- Huxley invests $500 at 10% p.a., compounded annually over 3 years.
- Loreli invests $500 at 15% p.a. simple interest over 3 years.
  - How much does each person have at the end of the 3 years?
  - Who earned the most?

KEY IDEAS

- A **spreadsheet** is an excellent tool for investigating financial scenarios. It performs repetitive calculations quickly and accurately. It also makes it simple to explore what happens when interest rates change or loan repayments change.
- A formula in a spreadsheet always begins with =.
  - For example, the formula = B3 + C3 will find the sum of the numbers in cells B3 and C3.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

■ The following table shows important things to know about spreadsheets.

Formula	Explanation
= C3/100	The symbol / is used for division, so this formula will divide the number in cell C3 by 100. This could be useful for converting percentages to decimals.
= C3/100*C2	The symbol * is used for multiplication, so this formula will divide C3 by 100 and then multiply the answer by the number in C2. This could be useful for calculating interest.
= C3^2	The symbol ^ is used for powers of 2, 3, 4 etc. This formula will give you the square of the number in cell C3.
= SUM(C4:C100)	This formula will find the sum of all the numbers in a column from C4 down to C100. This could be useful for finding the sum of the numbers in a column, such as the interest received every year for 20 years.

Key terminology: simple interest, compound interest

Exercise 1I

UNDERSTANDING 1–3 1–3

1 Which of these earns more on an investment of $100 for 2 years?

A simple interest calculated at 5% p.a.

B compound interest calculated at 5% p.a., compounded annually

2 Kaitlin started the spreadsheet below.

	A	B	C	D
1	Year	Opening balance	Interest	Closing balance
2	1	5000		
3	2			
4	3			

a In cell C2 she typed = 4/100*B2, then she pressed the 'Enter' key. What number should appear in cell C2?

b Which formula could be used to give the value in cell D2?

A D2 = B2 + C2

B = B2 + C2

c What value should the formula produce in cell D2?

d The closing balance in Year 1 is also the opening balance in Year 2. What formula can be typed into cell B3 to make this happen?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Consider the spreadsheet shown below.

	A	B	C	D
1	Year	Opening balance	Interest	Closing balance
2	1	5000	300	5300
3	2	5300	300	5600

a How can you tell that this spreadsheet is calculating simple interest, not compounding interest?

b What is the rate of interest?

c Fill in the blanks for the formula that could be in cell C2, using a number and a cell reference.

$= 6/\square * \square$

d Write a formula for cell C3 and cell D3.

FLUENCY 4–7 4–7

Example 26 Using a spreadsheet

Find the total amount of the following investments, using technology.

a $5000 at 5% p.a., compounded annually for 3 years

b $5000 at 5% p.a. simple interest for 3 years

SOLUTION

a $5788.13

b $5750

EXPLANATION

Use the spreadsheets (provided) to compare the closing balances for Year 5.

Now you try

Find the total amount of interest earned on the following investments, using technology.

a $25 000 at 4.5% p.a., compounded annually for 5 years

b $25 000 at 4.5% p.a. simple interest for 5 years

4 a Use a spreadsheet to find the closing balance of the following investments.

- **i** $6000 at 6% p.a., compounded annually for 3 years
- **ii** $6000 at 3% p.a., compounded annually for 5 years
- **iii** $6000 at 3.4% p.a., compounded annually for 4 years
- **iv** $6000 at 10% p.a., compounded annually for 2 years
- **v** $6000 at 5.7% p.a., compounded annually for 5 years

b Which of the investments above yields the most interest?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 **a** Use a spreadsheet to find the closing balance of the following investments.

- **i** $6000 at 6% p.a. simple interest for 3 years
- **ii** $6000 at 3% p.a. simple interest for 6 years
- **iii** $6000 at 3.4% p.a. simple interest for 7 years
- **iv** $6000 at 10% p.a. simple interest for 2 years
- **v** $6000 at 5.7% p.a. simple interest for 5 years

b Which of the above investments yields the most interest?

6 Cars depreciate by 15% per annum. Use the spreadsheet to find the value of a $30 000 car at the end of the:

a first year **b** second year
c third year **d** tenth year

7 If you borrow $30 000 at an interest rate of 12% per annum (i.e. 1% per month) and you agree to repay $1000 per month, how much do you owe at the end of the:

a first month? **b** second month?
c third month? **d** tenth month?

PROBLEM-SOLVING AND REASONING 8–9 9–11

8 **a** When a $30 000 car is depreciating by 15% per annum, how long does it take for the car to lose half of its value?

b When a $20 000 car is depreciating by 15% per annum, how long does it take for the car to lose half of its value?

c Try other values. Do all cars lose half of their value at the same time?

9 Steve is going to borrow $30 000 at an interest rate of 12% per annum (i.e. 1% per month). He agrees to repay $1000 per month.

a How many months will it take him to completely repay the loan and interest?

b How much interest will he pay on the loan?

10 Lauren takes out the same loan as Steve (see Question **9**) but she decides to repay $1200 per month. How much interest will Lauren save compared to Steve?

11 If the interest rate in Question **9** is 15% rather than 12%, how much extra interest will Steve pay on the loan?

ENRICHMENT: $1000 per month versus $500 per fortnight – 12

12 Adam and Bree both borrowed $30 000 at 12% per annum.
Adam's loan requires him to repay $1000 per month. The interest is calculated monthly.
Bree's loan requires her to repay $500 per fortnight. The interest is calculated fortnightly.
The spreadsheet you have been given is designed for Adam's loan. Modify the spreadsheet so it will model Bree's loan.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Maths@Work

Finance manager

A bookkeeper and an accounts manager are both occupations that deal with numbers and budgets. They require employees to have good communication and mathematical skills. Employees also need a commitment to detail and to be honest, as they deal with other people's money.

People employed in these fields need to work with spreadsheets, percentages, tax systems and business plans.

Complete these questions that a finance manager may face in their day-to-day job.

1 Consider the information supplied in a section of a business budget for a 3-month period. Round all answers to two decimal places.

Income	July	August	September
Fixed fee (with 25% reduction)	$52 813	$52 813	$52 813
Variable fee (with 25% reduction)	$53 906	$53 906	$53 906
Associate members fees (with 25% reduction)	$1563	$1563	$1563
Shared costs billed EAL	$0	$0	$0
CBA interest earned	$25	$25	$25
Operating costs	$167	$167	$167
Term deposit	$0	$2500	$0
Miscellaneous income	$0	$0	$0
Total Income	**a**	**b**	**c**

a Calculate the total income for the month of July.
b Calculate the total income for the month of August.
c Calculate the total income for the month of September.
d Which month had the highest income and by how much?
e What contributed to this increase in income?
f What percentage of the total income for the 3 months shown came from a fixed fee?
g What was the monthly fixed fee before the 25% reduction occurred?

Hint: $\%\text{ of total} = \frac{\text{amount}}{\text{total}} \times \frac{100}{1}$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 The office expenses for the same company for the same 3-month period are given below.

Office expenses	July	August	September
Rent	$7200	$7200	$7200
Property outgoings (costs)	$220	$220	$220
Cleaning	$250	$250	$250
Maintenance	$50	$50	$50
Capital expenditure	$100	$100	$100
Electricity	$0	$1200	$0
Total office expenses	**$7820**	**$9020**	**$7820**

a Calculate the percentage of the total office expenses for July spent in rent.

b What is the cost of electricity shown in the table, and in what month is it shown?

c Why does the electricity not appear in the other two months?

d What is the projected cost of electricity for the year?

3 The employment expenses for the three months of October, November and December are shown.

Employment expenses	October	November	December
Superannuation	$4800	$4800	$4800
Employees' salaries	$50 424	$50 424	$50 424
Payroll tax	$1200	$1200	$4500
Consultant	$0	$0	$0
Workers compensation	$0	$0	$0
Performance review	$0	$0	$75 000
Training	$335	$335	$335
Parking	$400	$400	$400
Total employment expenses	**$57 159**	**$57 159**	**a**

a Calculate the total employment expenses for the month of December.

b What is the whole number percentage increase of November's total employment expenses from November to December? What was the cause of this increase?

c The company has 11 full-time employees. What is an employee's average:

i salary per month?

ii annual salary?

d The company has total expenses for the month of November of $92 117. What percentage of the total expenses for November comes from the employment expenses?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Using technology

4 A trucking business has invested in a new prime mover for hauling cattle by road train. It has a bank loan of $230 000 at 9% per annum charged monthly. The business requires an Excel spreadsheet to show the progress of the debt repayment.

a Develop the following table in an Excel spreadsheet by entering formulas into the yellow shaded cells to calculate their values. Use the notes below to help you.

	B	C	D	E	F	G
1	**Debt repayment table**					
2	Payment date	Starting balance	Scheduled payment	Interest due	Principal paid	Ending balance
3	1-Jan	$230,000.00	$5,780.00			
4	1-Feb		$5,780.00			
5	1-Mar		$5,780.00			

Hint: After entering your formulas, check specific results with a calculator.

Notes:

The interest due per month is $\frac{1}{12}$ of 9% of the starting balance for that month.

The principal (i.e. debt) paid will be the scheduled payment minus the interest due.

The ending balances will equal the starting balance minus the principal paid.

The next month's starting balance equals the previous month's ending balance.

b Extend the table for 12 payments and answer the following questions.

i What is the amount of debt remaining on 1 July?

ii What is the interest paid in October?

iii Use an Excel formula to find the difference between the principal paid in December and the principal paid in January.

iv Enter 'sum' formulas to determine the total interest paid in the year and the total principal paid off in the year.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Investing in art

Matilda is a keen art investor and has the opportunity to purchase a new work from an auction house. The auctioneer is saying that the estimated value of the painting is $10 000. Matilda's main investment goal is for each of her investments to at least double in value every 10 years.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

a If Matilda purchases the painting for $10 000 and assumes a growth rate of 5% p.a., calculate the value of the investment after:

i 1 year **ii** 2 years **iii** 3 years.

b The rule connecting the value of the $10 000 investment ($A) growing at 5% p.a. after t years is given by $A = 10\,000\left(1 + \frac{5}{100}\right)^t$.

i Check your answers to part **a** by substituting $t = 1$, $t = 2$ and $t = 3$ into the given rule and evaluating the value of A using your calculator, or a spreadsheet.

ii Construct a similar rule for an investment value of $12 000 and a growth rate of 3%.

iii Construct a similar rule for an investment value of $8000 and a growth rate of 8%.

c Using $A = 10\,000\left(1 + \frac{5}{100}\right)^t$ find the value of a $10 000 investment at 5% p.a. after 10 years.

i.e. Calculate the value of A if $t = 10$.

Non-routine problems

The problem is to determine an investment growth rate that delivers at least a doubling of Matilda's initial investment amount after 10 years.

Explore and connect

a Write down all the relevant information that will help solve this problem. What formula needs to be applied in this task?

b Explain what the numbers 10 000 and 5 mean in the rule $A = 10\,000\left(1 + \frac{5}{100}\right)^t$, in relation to Matilda's investment.

Choose and apply techniques

c Use the rule $A = 10\,000\left(1 + \frac{r}{100}\right)^t$ to determine the value of Matilda's $10 000 investment after 10 years for the following growth rates (r%).

i $r = 4$ **ii** $r = 7$ **iii** $r = 10$

d Choose your own values of r using one decimal place accuracy and determine the growth rate for which the investment doubles in value after 10 years.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Communicate thinking and reasoning

e By considering values of r either side of your chosen value found in part **d** demonstrate that your answer is correct to one decimal place.

f Refine your answer so that it is correct to two decimal places.

g Summarise your findings and describe any key findings. You might like to show your results in a spreadsheet similar to the one below.

Cost of art (P)	Interest rate (r)	Time (t)	(1 + r/100)	(1 + r/100)^t	P(1 + r/100)^t
10000	3	10	1.03	1.343916379	13439.16379
10000	4	10	1.04	1.480244285	14802.44285
10000	5	10	1.05	1.628894627	16288.94627

Extension problems

Problem-solve

a Decide if changing the initial investment value changes the total percentage increase in value after the same number of years. Justify your answer.

b If Matilda only paid \$8000 for the art work but still wanted it to be valued at \$20 000 after 10 years, determine the growth rate that she will need the work to have? Round to two decimal places.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1 Find and define the 10 terms related to consumer arithmetic and percentages hidden in this wordfind.

C	O	M	M	I	S	S	I	O	N	Q	R	W
P	G	S	L	E	R	S	T	B	L	D	U	J
H	L	A	A	P	I	E	C	E	W	O	R	K
U	F	N	U	L	N	Q	B	D	Z	T	J	L
V	K	N	S	T	A	M	O	N	T	H	L	Y
B	H	U	A	I	G	R	O	S	S	U	B	S
N	E	A	C	Y	K	S	Y	E	T	Y	M	D
M	A	L	O	V	E	R	T	I	M	E	Q	T
S	F	O	R	T	N	I	G	H	T	L	Y	S

2 How do you stop a bull charging you? Answer the following problems and match the letters to the answers below to find out.

\$19.47 – \$8.53 E	5% of \$89 T	50% of \$89 I
$12\frac{1}{2}$% of \$100 A	If 5% = \$8.90 then 100% is? S	\$4.48 to the nearest 5 cents R
6% of \$89 W	Increase \$89 by 5% H	10% of \$76 O
\$15 monthly for 2 years D	$12\frac{1}{2}$% as a decimal K	\$50 – \$49.73 U
Decrease \$89 by 5% C	\$15.96 + \$12.42 Y	

☐ ☐ ☐
\$28.38 \$7.60 27c

☐ ☐ ☐ ☐
\$4.45 \$12.50 0.125 \$10.94

☐ ☐ ☐ ☐
\$12.50 \$5.34 \$12.50 \$28.38

☐ ☐ ☐
\$93.45 \$44.50 \$178

☐ ☐ ☐ ☐ ☐ ☐
\$84.55 \$4.50 \$10.94 \$360 \$44.50 \$4.45

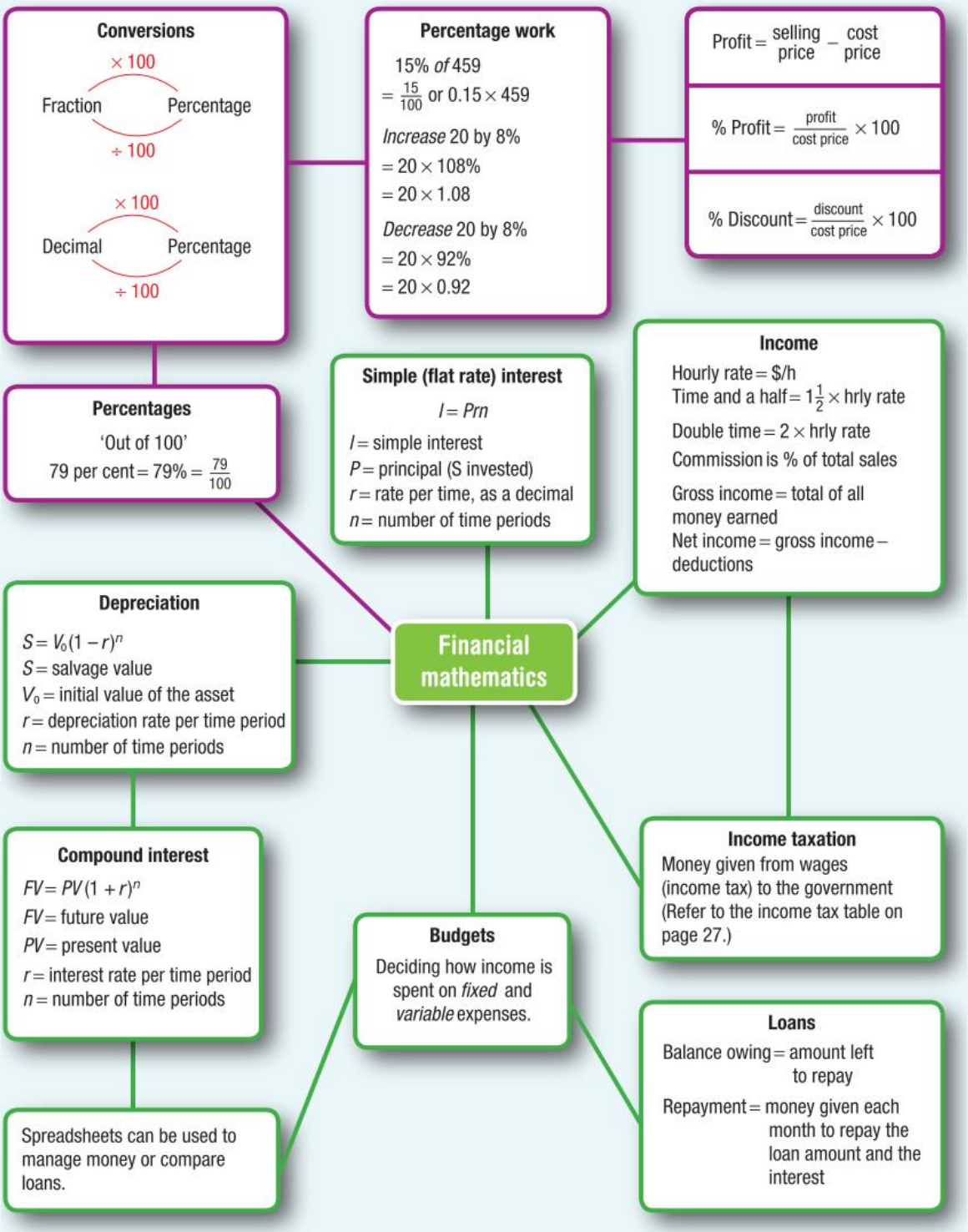
☐ ☐ ☐ ☐
\$84.55 \$12.50 \$4.50 \$360

3 How many years does it take \$1000 to double if it is invested at 10% p.a. compounded annually?

4 The chance of Jayden winning a game of cards is said to be 5%. How many consecutive games should Jayden play to be 95% certain he has won at least one of the games played?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter summary



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
1A	**1 I can convert to a percentage.** e.g. Write each of the following as a percentage. **a** $\frac{7}{40}$ **b** 0.24	☐
1A	**2 I can write percentages as simplified fractions and decimals.** e.g. Write each of the following percentages as both a simplified fraction and a decimal. **a** 53% **b** 4% **c** 10.5%	☐
1A	**3 I can find the percentage of a quantity.** e.g. Find 64% of $1400.	☐
1B	**4 I can increase and decrease by a given percentage.** e.g. For the amount of $800: **a** increase $800 by 6% **b** decrease $800 by 15%	☐
1B	**5 I can calculate percentage profit.** e.g. Jimmy buys a second-hand desk for $145 and restores it to a good condition. If he sells if for $210, calculate his profit and the percentage profit, correct to one decimal place.	☐
1B	**6 I can find the selling price.** e.g. Jo buys t-shirts for $24 each and wishes to make a 28% profit on the purchase. What should be her selling price and what will be the profit on the sale of 20 t-shirts?	☐
1B	**7 I can calculate a discount.** e.g. A $849 television is discounted by 18%. What is the selling price of the television?	☐
1C	**8 I can find gross and net income involving overtime.** e.g. Anika earns $21.40 per hour and has normal working hours of 38 hours per week. She earns time and a half for the next 4 hours worked and double time after that. She pays $190 per week in tax and other deductions. Calculate her gross and net income for a week in which she works 45 hours.	☐
1C	**9 I can calculate income involving commission and royalties.** **a** Tia earns $300 per week plus a commission of 6% on her sales of solar panels. If she sells $8200 worth of solar panels in a week, what is her gross income for the week? **b** Roger's book sells for $30 per copy. If 50 000 were sold and he receives 3% royalties, how much did he earn?	☐
1D	**10 I can calculate income tax payable.** e.g. Noah earns $78 406 per year, including interest on investments. He has receipts for donations and work related expenses of $445. **a** Calculate Noah's taxable income. **b** Use the tax table in the Key ideas in 1D to calculate Noah's tax payable amount, to the nearest cent. **c** If Noah also has to pay $1559 for the Medicare levy, calculate his tax refund if his employer sent $19 200 to the ATO.	☐

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

Chapter checklist with success criteria

1E

11 I can budget using percentages.

e.g. Ash has a net annual income of $54 800 after deductions. She allocates her budget on a percentage basis.

	Rent	Holiday loan	Clothing	Food	Other	Savings
Expenses (%)	25	10	5	10	35	15

a Determine the amount of fixed expenses (rent and the loan).
b Determine how much she budgets to save each month.

1E

12 I can budget from fixed values.

e.g. Running a certain type of motorbike involves the following costs:

registration $520 per year
insurance $120 per quarter
servicing $310 per year
petrol $64 per month

Determine the overall cost to run the bike for a year and what percentage of an $80 000 salary this would be, correct to one decimal place.

1E

13 I can calculate a best buy.

e.g. Packets of chips can be bought in the following ways at the store:

- 20 packs (20 grams each) for $5.50
- 6 packs (20 grams each) for $3.35
- 2 share bags (60 grams each) for $4

Determine the cheapest way to buy the chips.

1F

14 I can use the simple interest formula to find interest.

e.g. Use the simple interest formula to calculate the interest when $800 is invested at 5% p.a. for 3 years.

1F

15 I can calculate repayments using simple interest.

e.g. If a simple interest loan of $4000 is borrowed for 2 years at a simple interest rate of 4% p.a., what is the total amount owed over the 2 years and if repayments are made monthly, how much would each payment need to be?

1F

16 I can use the simple interest formula to find the rate of interest.

e.g. Use the simple interest formula to calculate the rate of interest when $2800 earns $294 interest in 3 years.

1G

17 I can use the compound interest formula.

e.g. Determine the amount after 6 years when $8000 is compounded annually at 3%.

1G

18 I can use compound interest with different time periods.

e.g. An investment of $5500 is compounded at 6% p.a. over 4 years. Determine the amount the investor will have after 4 years if interest is paid monthly.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

1H	**19 I can work with repayments to calculate a purchase cost.** e.g. Vanessa pays for a $8600 travel package with a travel agent with a 30% deposit and monthly repayments of $300 for 2 years. Calculate: **a** the deposit paid, **b** the total amount paid for the travel package and hence the interest paid.	☐
1H	**20 I can calculate interest earned on an account.** e.g. An account has a minimum monthly balance of $140 and interest is credited monthly on this amount at 1.8%. Determine the amount of interest to be credited at the end of the month and the total amount credited or debited if the bank charges $5 per month in account keeping fees.	☐
1I	**21 I can use technology to calculate interest and final amounts.** e.g. Use technology to find the total amount on the following investments. **a** $7000 at 4% p.a. compounded annually for 5 years **b** $7000 at 4% p.a. simple interest for 5 years	☐

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

Short-answer questions

1A 1 Find 16% of $9000.

1B 2 a Increase $968 by 12%.
b Decrease $4900 by 7%.

1B 3 The cost price of an item is $7.60. If this is increased by 50%, determine:
a the retail price
b the profit made.

1B 4 An airfare of $7000 is discounted 40% if you fly off-peak. What would be the discounted price?

1B 5 A sofa is discounted to $375. If this is a 35% discount, find the recommended retail price.

1C 6 Josephine budgets 20% of her income for entertainment. If her yearly income is $37 000, how much could be spent on entertainment in:
a a year?
b a month?
c a week (taking 52 weeks in a year)?

1C 7 Mariah works a 34-hour week at $25.43 per hour. Her net income is 62% of her wage.
a Work out her weekly net income.
b If 15% is spent on clothing, determine the amount she can spend each week.
c If she saves $100, what percentage (to two decimal places) of her gross weekly income is this?

1E 8 Frank has the following expenses to run his car:

- hire purchase payment $350 per month
- registration $885 per year
- insurance $315 per quarter
- servicing $1700 per year
- petrol $90 per week

a Find the total cost of running his vehicle for 1 year.
b What percentage (to the nearest percentage) of the overall cost to run the car is the cost of the petrol?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1E

9 Ronan works 36 hours in a week at $39.20 per hour. He pays $310 in tax and $20.50 in superannuation in the week. Determine:

a his gross wage in a week

b his net pay in a week

1D

10 Lil receives an annual taxable income of $90 000.

a Using the tax table shown, calculate the amount of tax she pays over the year.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $45 000	19 cents for each $1 over $18 200
$45 001 – $120 000	$5 092 plus 32.5 cents for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37 cents for each $1 over $120 000
$180 001 and over	$51 667 plus 45 cents for each $1 over $180 000

b If Lil pays the 2% Medicare levy on her taxable income, find this amount.

1C

11 Zane receives 4.5% commission on sales of $790. Determine the amount of his commission.

1F

12 Find the interest paid on a $5000 investment under the following conditions.

a 8% p.a. simple interest over 4 years

b 7% p.a. simple interest over 3 years and 4 months

1G

13 Find the interest paid on a $5000 investment under the following conditions.

a 4% p.a. compounded annually over 3 years

b 9.75% p.a. compounded annually over 2 years

c 6% p.a. compounded monthly over 2 years

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

Multiple-choice questions

1A 1 28% of $89 is closest to:

A $28.00 B $64.08 C $113.92 D $2492 E $24.92

1A 2 As a percentage, $\frac{21}{60}$ is:

A 21% B 3.5% C 60% D 35% E 12.6%

1E 3 If a budget allows 30% for car expenses, how much is allocated from a weekly wage of $560?

A $201 B $145 C $100 D $168 E $109

1C 4 The gross income for 30 hours at $52.60 per hour is:

A $352.60 B $3892.40 C $1578.00 D $2492.00 E $249.20

1C 5 If Simon receives $2874 on the sale of a property worth $195 800, his rate of commission, to one decimal place, is:

A 21% B 1.5% C 60% D 15% E 12.6%

1C 6 In a given rostered fortnight, Bilal works the following number of 8-hour shifts:

- three day shifts ($42.40 per hour)
- three afternoon shifts ($49.36 per hour)
- five night shifts ($67.12 per hour).

His total income for the fortnight is:

A $4887.04 B $1457.34 C $1000 D $168.84 E $1221.76

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1B 7 A computer tablet is discounted by 26%. What is the price if it was originally $329?

A $85.54 B $243.46 C $156.05 D $77.78 E $206.90

1H 8 A $5000 loan is repaid by monthly instalments of $200 for 5 years. The amount of interest charged is:

A $300 B $7000 C $12 000 D $2400 E $6000

1F 9 The simple interest earned on $600 invested at 5% p.a. for 4 years is:

A $570 B $630 C $120 D $720 E $30

1G 10 The compound interest earned on $4600 invested at 12% p.a. for 2 years is:

A $1104 B $5704 C $4600 D $5770.24 E $1170.24

Extended-response questions

1 $5000 is invested at 4% p.a. compounding annually for 3 years.

a What is the value of the investment after the 3 years?

b How much interest is earned in the 3 years?

c What simple interest rate results in the same amount?

d How much interest is earned on the investment if it is compounded monthly at 4% p.a. for the 3 years?

2 A vehicle worth $7000 is purchased on a finance package. The purchaser pays 15% deposit and $250 per month over 4 years.

a How much deposit is paid?

b What is the total amount repaid?

c How much interest is paid over the term of the loan?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2

Measurement

Why measurement skills are important

Our lives rely on people who can calculate accurate measurements. Qualified experts design and construct our houses, vehicles and water supply systems, and, crucially, they grow our food. Some examples are below.

- Everyday users of area calculations include carpet layers, floor tilers, curtain makers, furniture upholsterers, house painters, landscapers, renovators, carpenters, builders and surveyors.
- Mechanical engineers and sheet metal workers calculate surface areas when designing and constructing commercial kitchens and air-conditioning ducts.
- Architects and builders calculate sector areas when designing and constructing steps for a spiral staircase.
- Plumbers calculate water pressures using a pipe's circular cross-sectional area and its volume.
- Workers in the boat-building trades calculate perimeters, surface areas and volumes when constructing and repairing small craft, cruise ships and naval boats.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter contents

2A Converting units of measurement (CONSOLIDATING)
2B Accuracy of measuring instruments
2C Perimeter (CONSOLIDATING)
2D Circumference and arc length (CONSOLIDATING)
2E Area of triangles and quadrilaterals (CONSOLIDATING)
2F Area of circles and sectors (CONSOLIDATING)
2G Surface area of prisms
2H Surface area of cylinders
2I Volume of prisms and cylinders
2J Further problems involving prisms and cylinders
2K Surface area of pyramids and cones
2L Volume of pyramids and cones
2M Volume and surface area of spheres

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- solves problems involving the surface area of right prisms and practical problems involving the area of composite shapes and solids (MA5-ARE-C-01)
- applies knowledge of the surface area of right pyramids and cones, spheres and composite solids to solve problems (MA5-ARE-P-01)
- solves problems involving the volume of composite solids consisting of right prisms and cylinders (MA5-VOL-C-01)
- applies knowledge of the volume of right pyramids, cones and spheres to solve problems involving related composite solids (MA5-VOL-P-01).

© 2022 NSW Education Standards Authority

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

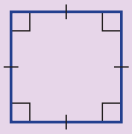
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Warm-up quiz

1 Name these shapes. Choose from the words *trapezium*, *triangle*, *circle*, *rectangle*, *square*, *semicircle*, *parallelogram* and *rhombus*.

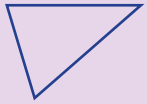
a

b



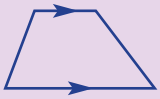
c

d



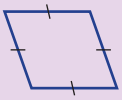
e

f



g

h



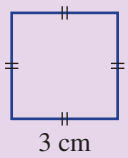
2 Write the missing number.

a 1 km = ☐ m **b** 1 m = ☐ cm **c** 1 cm = ☐ mm

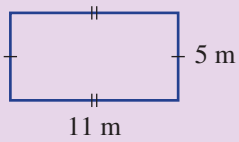
d 1 L = ☐ mL **e** 0.5 km = ☐ m **f** 2.5 cm = ☐ mm

3 Find the perimeter of these shapes.

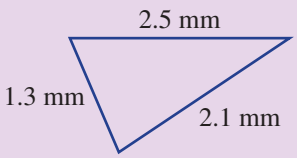
a



b

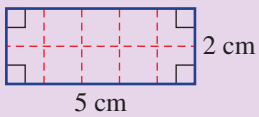


c

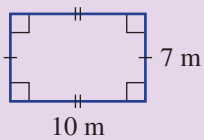


4 Find the area of these shapes.

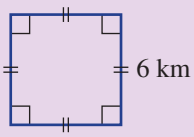
a



b

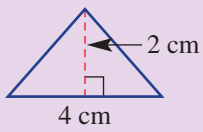


c

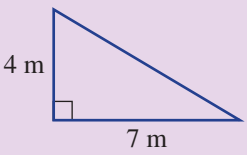


5 Find the area of these triangles using $A = \frac{1}{2}bh$.

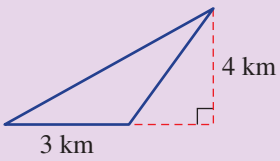
a



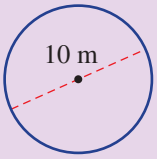
b



c



6 Use $C = \pi d$ and $A = \pi r^2$ to find the circumference and area of this circle. Round your answer to two decimal places.



$d = 10$, $r = 5$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2A Converting units of measurement CONSOLIDATING

Learning intentions for this section:

- To review the metric units of measurement
- To be able to convert between metric units for length, area and volume

Past, present and future learning:

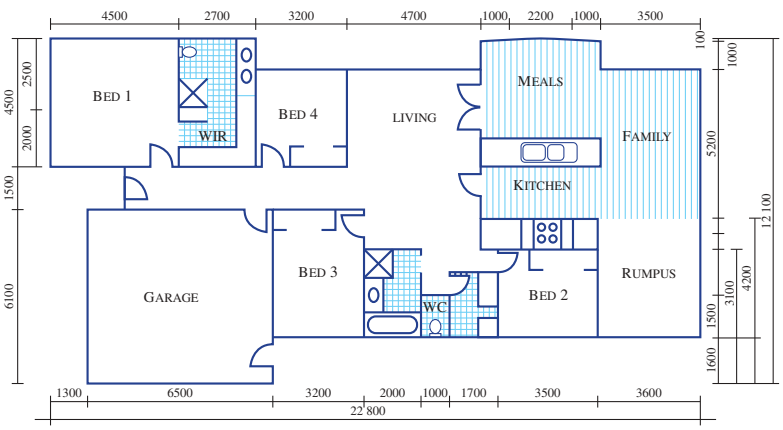
- This section addresses Stage 4 concepts that are used in Stages 5 and 6
- It consolidates and extends the concepts in Chapter 6 of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall these conversions from memory in the HSC Examination

To work with length, area or volume measurements, it is important to be able to convert between different units. For example, timber is widely used in buildings for frames, roof trusses and windows, therefore it is important to order the correct amount so that the cost of the house is minimised. Although plans give measurements in millimetres and centimetres, timber is ordered in metres (often referred to as lineal metres), so we have to convert all our measurements to metres.

Building a house also involves many area and volume calculations and unit conversions.

Lesson starter: House plans

All homes start from a plan, which is usually designed by an architect and shows most of the basic features and measurements that are needed to build the house. Measurements are given in millimetres.

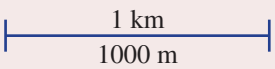


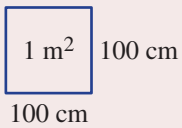
- How many bedrooms are there?
- What are the dimensions of the master bedroom (i.e. BED 1)?
- What are the dimensions of the master bedroom, in metres?
- Will the rumpus room fit a pool table that measures 2.5 m × 1.2 m, and still have room to play?
- Will three cars fit in the garage?

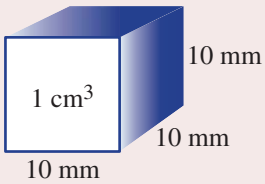
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- To convert units, draw an appropriate diagram and use it to find the conversion factor.







$1\text{ km} = 1000\text{ m}$

$1\text{ m}^2 = 100 \times 100 = 10\,000\text{ cm}^2$

$1\text{ cm}^3 = 10 \times 10 \times 10 = 1000\text{ mm}^3$

- Conversions:

Length: km $\xrightarrow{\times 1000}$ m $\xrightarrow{\times 100}$ cm $\xrightarrow{\times 10}$ mm; $\div 1000$, $\div 100$, $\div 10$

Area: $km^2 \xrightarrow{\times 1000^2} m^2 \xrightarrow{\times 100^2} cm^2 \xrightarrow{\times 10^2} mm^2$; $\div 1000^2$, $\div 100^2$, $\div 10^2$

Volume: $km^3 \xrightarrow{\times 1000^3} m^3 \xrightarrow{\times 100^3} cm^3 \xrightarrow{\times 10^3} mm^3$; $\div 1000^3$, $\div 100^3$, $\div 10^3$

- To multiply by 10, 100, 1000 etc. move the decimal point one place to the right for each zero;
  e.g. $3.425 \times 100 = 342.5$
- To divide by 10, 100, 1000 etc. move the decimal point one place to the left for each zero;
  e.g. $4.10 \div 1000 = 0.0041$

- Metric prefixes in everyday use.

Prefix	Symbol	Factor of 10	Standard form	
tera	T	1 000 000 000 000	10^{12}	1 trillion
giga	G	1 000 000 000	10^{9}	1 billion
mega	M	1 000 000	10^{6}	1 million
kilo	k	1000	10^{3}	1 thousand
hecto	h	100	10^{2}	1 hundred
deca	da	10	10	1 ten
deci	d	0.1	10^{-1}	1 tenth
centi	c	0.01	10^{-2}	1 hundredth
milli	m	0.001	10^{-3}	1 thousandth
micro	μ	0.000001	10^{-6}	1 millionth
nano	n	0.000000001	10^{-9}	1 billionth

Time conversions can be represented on a flow diagram similar to those for length, area and volume.

s – seconds

ms – milliseconds

μs – microseconds

ns – nanoseconds

s $\xrightarrow{\times 1000}$ ms $\xrightarrow{\times 1000}$ μs $\xrightarrow{\times 1000}$ ns; $\div 1000$, $\div 1000$, $\div 1000$

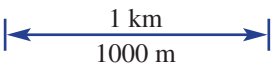
Key terminology: unit, length, area, volume

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 2A

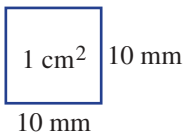
UNDERSTANDING 1–3 | 3

1 Write the missing numbers in these sentences involving length.



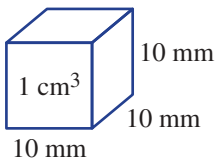
- **a** There are ☐ m in 1 km.
- **b** There are ☐ mm in 1 cm.
- **c** There are ☐ cm in 1 m.

2 Write the missing numbers in these sentences involving area units.



- **a** There are ☐ mm^2 in 1 cm^2.
- **b** There are ☐ cm^2 in 1 m^2.
- **c** There are ☐ m^2 in 1 km^2.

3 Write the missing numbers in these sentences involving volume units.



- **a** There are ☐ mm^3 in 1 cm^3.
- **b** There are ☐ m^3 in 1 km^3.
- **c** There are ☐ cm^3 in 1 m^3.

FLUENCY 4–6 | 4–6(½)

Example 1 Converting length measurements

Convert these length measurements to the units shown in the brackets.

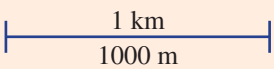
a 8.2 km (m) **b** 45 mm (cm)

SOLUTION

a 8.2 km = 8.2 × 1000
= 8200 m

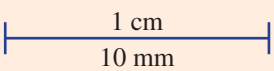
b 45 mm = 45 ÷ 10
= 4.5 cm

EXPLANATION



1 km = 1000 m

Multiply when converting to a smaller unit.



1 cm = 10 mm

Divide when converting to a larger unit.

Now you try

Convert these length measurements to the units shown in the brackets.

a 4.6 m (cm) **b** 3200 m (km)

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Convert the following measurements of length to the units given in the brackets.

a 4.32 cm (mm) **b** 327 m (km) **c** 834 cm (m)
d 0.096 m (mm) **e** 297.5 m (km) **f** 0.0127 m (cm)

Hint: When converting to a smaller unit, multiply. Otherwise, divide.

Example 2 Converting area measurements

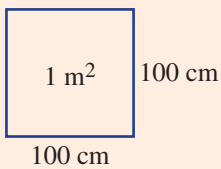
Convert these area measurements to the units shown in the brackets.

a 930 cm^2 (m^2) **b** 0.4 cm^2 (mm^2)

SOLUTION

a $930\ cm^2 = 930 \div 10\,000$
$= 0.093\ m^2$

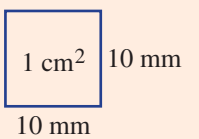
EXPLANATION



$1\ m^2 = 100 \times 100$
$= 10\,000\ cm^2$

When dividing by 10 000, move the decimal point 4 places to the left.

b $0.4\ cm^2 = 0.4 \times 100$
$= 40\ mm^2$



$1\ cm^2 = 10 \times 10$
$= 100\ mm^2$

Now you try

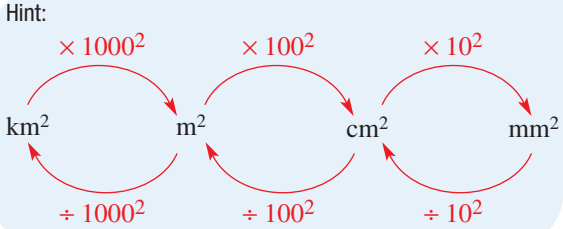
Convert these area measurements to the units shown in the brackets.

a 320 mm^2 (cm^2) **b** 0.00024 km^2 (m^2)

5 Convert the following area measurements to the units given in the brackets.

a 3000 cm^2 (mm^2)
b 0.5 m^2 (cm^2)
c 5 km^2 (m^2)
d 2 980 000 mm^2 (cm^2)
e 537 cm^2 (mm^2)
f 0.023 m^2 (cm^2)

Hint:



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 3 Converting volume measurements

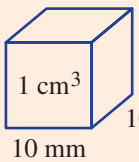
Convert these volume measurements to the units shown in the brackets.

a $3.72\,\text{cm}^3$ (mm^3) **b** $4300\,\text{cm}^3$ (m^3)

SOLUTION

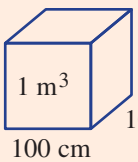
a $3.72\,\text{cm}^3 = 3.72 \times 1000$
$= 3720\,\text{mm}^3$

EXPLANATION



$1\,\text{cm}^3 = 10 \times 10 \times 10$
$= 1000\,\text{mm}^3$

b $4300\,\text{cm}^3 = 4300 \div 1\,000\,000$
$= 0.0043\,\text{m}^3$



$1\,\text{m}^3 = 100 \times 100 \times 100$
$= 1\,000\,000\,\text{cm}^3$

Now you try

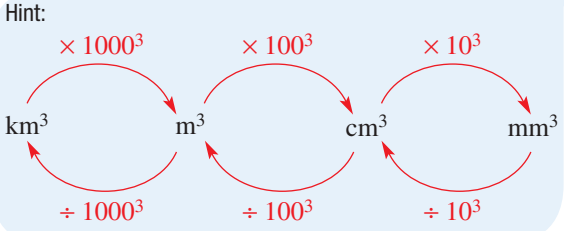
Convert these volume measurements to the units shown in the brackets.

a $0.21\,\text{m}^3$ (cm^3) **b** $94\,000\,\text{mm}^3$ (cm^3)

6 Convert these volume measurements to the units given in the brackets.

a $2\,\text{cm}^3$ (mm^3)
b $0.2\,\text{m}^3$ (cm^3)
c $5700\,\text{mm}^3$ (cm^3)
d $0.015\,\text{km}^3$ (m^3)
e $28\,300\,000\,\text{m}^3$ (km^3)
f $762\,000\,\text{cm}^3$ (m^3)

Hint:

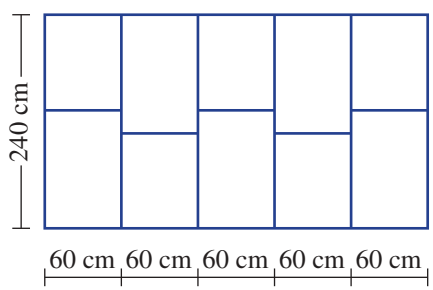


PROBLEM-SOLVING AND REASONING 7–9 8–11

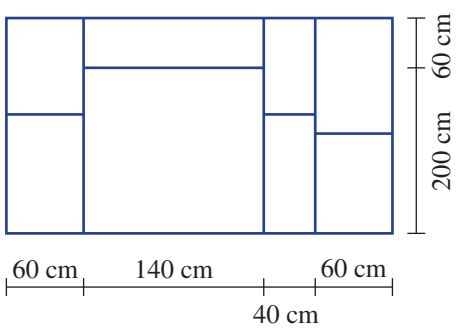
7 An athlete has completed a 5.5 km run. How many metres did the athlete run?

8 Determine the metres of timber needed to construct the following frames.

a



b



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 Find the total sum of the measurements given, expressing your answer in the units given in the brackets.

Hint: Convert to the units in brackets. Add up to find the sum.

- **a** 10 cm, 18 mm (mm)
- **b** 1.2 m, 19 cm, 83 mm (cm)
- **c** 453 km, 258 m (km)
- **d** 400 mm^2, 11.5 cm^2 (cm^2)
- **e** 0.3 m^2, 251 cm^2 (cm^2)
- **f** 0.000 03 km^2, 9 m^2, 37 000 000 cm^2 (m^2)
- **g** 482 000 mm^3, 2.5 cm^3 (mm^3)
- **h** 0.000 51 km^3, 27 300 m^3 (m^3)

10 A snail is moving at a rate of 43 mm every minute. How many centimetres will the snail move in 5 minutes?

11 Why do you think that builders measure many of their lengths using only millimetres, even their long lengths?

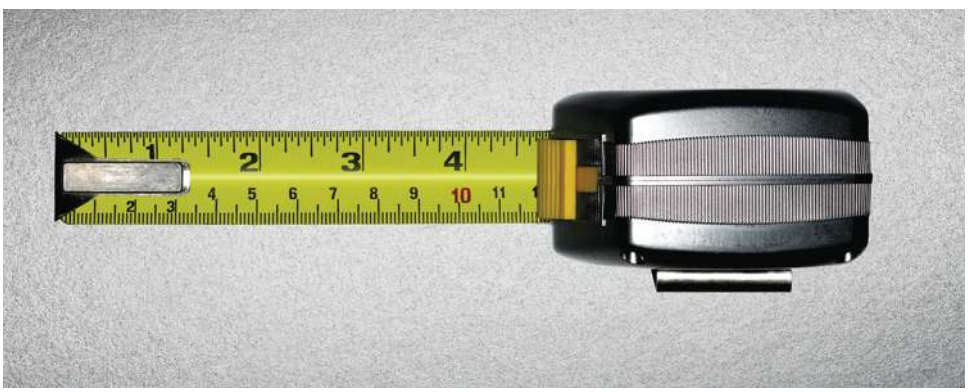
ENRICHMENT: Special units — 12

12 Many units of measurement apart from those relating to mm, cm, m and km are used in our society. Some of these are described here.

Length	Inches	1 inch ≈ 2.54 cm = 25.4 mm
	Feet	1 foot = 12 inches ≈ 30.48 cm
	Miles	1 mile ≈ 1.609 km = 1609 m
Area	Squares	1 square = 100 square feet
	Hectares (ha)	1 hectare = 10 000 m^2
Volume	Millilitres (mL)	1 millilitre = 1 cm^3
	Litres (L)	1 litre = 1000 cm^3

Convert these special measurements to the units given in the brackets. Use the conversion information given above to help.

- **a** 5.5 miles (km)
- **b** 54 inches (feet)
- **c** 10.5 inches (cm)
- **d** 2000 m (miles)
- **e** 5.7 ha (m^2)
- **f** 247 cm^3 (L)
- **g** 8.2 L (mL)
- **h** 5.5 m^3 (mL)
- **i** 10 squares (sq. feet)
- **j** 2 m^3 (L)
- **k** 1 km^2 (ha)
- **l** 152 000 mL (m^3)



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2B Accuracy of measuring instruments

Learning intentions for this section:

- To understand that accuracy depends on how measurements are recorded
- To know the limits of accuracy for a given recorded measurement
- To be able to calculate the limits of accuracy for given measurements

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Numbers of any magnitude*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Humans and machines measure many different things, such as the time taken to swim a race, a length of timber needed for a building and the volume of concrete needed to lay a concrete path around a swimming pool. The degree or level of accuracy required usually depends on what is being measured and what the information is being used for.

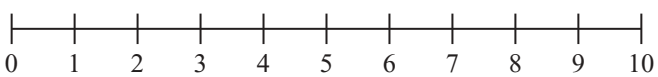
All measurements are approximate. Errors can come from the equipment being used or the person using the measuring device.

Accuracy is how close a measured value is to the 'true' value. **Precision** is how close repeated measurements of the same thing are to each other.

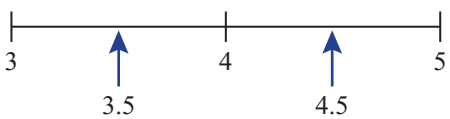
Lesson starter: Limits of accuracy

There is no such thing as an exact measurement. Every measurement is an approximation. Accuracy is limited by the device with which you are measuring.

A measuring device shows the scale below. Objects are measured to the nearest whole number.



If something is measured as 4, the actual measurement could be anything from 3.5 up to (but not including) 4.5, as indicated in the diagram below.



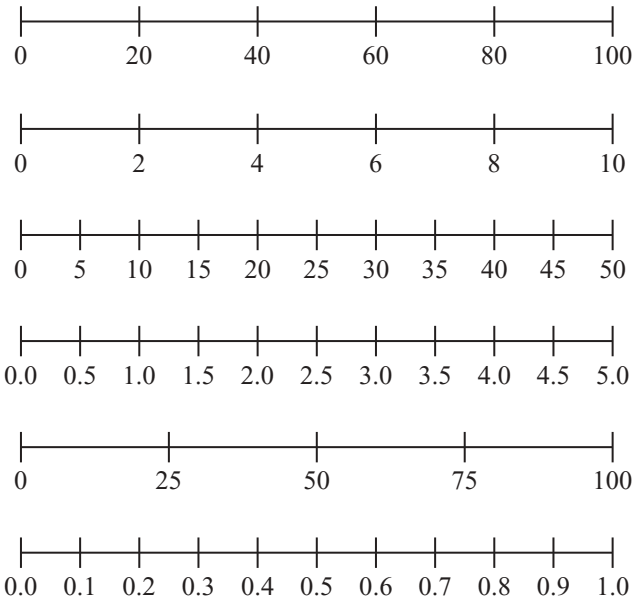
The degree of accuracy is **half a unit (0.5)** each side of the unit of measure.

Therefore the limits of accuracy are ±0.5 because the actual measurement could be half a unit higher or lower than 4.

- Explain why the limits of accuracy of this device are ±5.

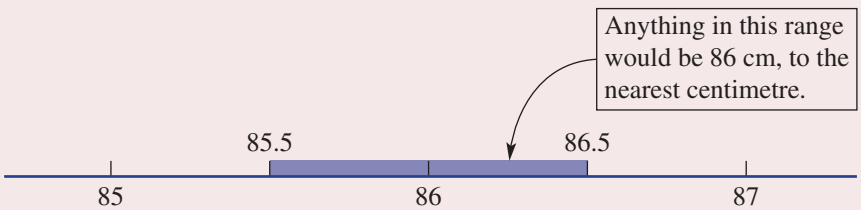
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- What are the limits of accuracy for devices that show these scales?

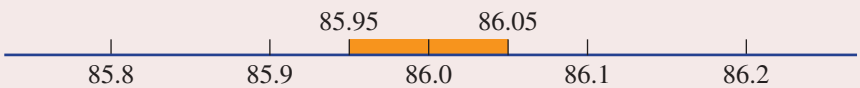


KEY IDEAS

- The limits of **accuracy** tell you what the upper and lower boundaries are for a reported measurement.
  - Usually, it is $0.5 \times$ the smallest unit of measurement.
  - Note that values are stated to one decimal place beyond that of the given measurement. For example, when measuring to the nearest centimetre, 86 cm has limits from 85.5 cm up to 86.5 cm.



- When measuring to the nearest millimetre, the limits of accuracy for 86.0 cm are 85.95 cm to 86.05 cm.



- Absolute error $= \frac{1}{2} \times$ precision
- Upper bound = measurement + absolute error
  Lower bound = measurement − absolute error
- Percentage error of a reported measurement $= \left(\frac{\text{absolute error}}{\text{measurement}} \times 100 \right) \%$

Key terminology: accuracy, precision, rounding

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

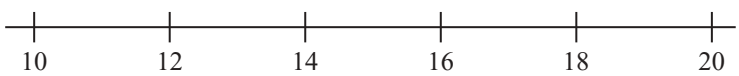
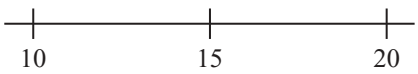
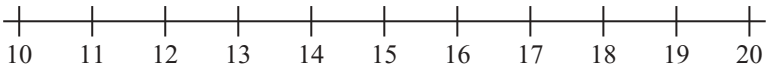
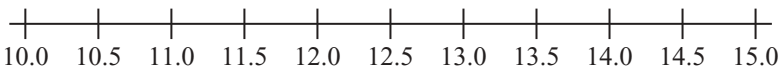
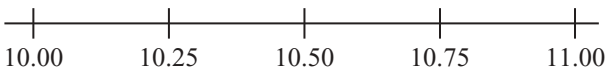
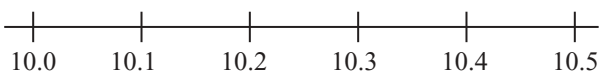
Exercise 2B

UNDERSTANDING 1–3 3

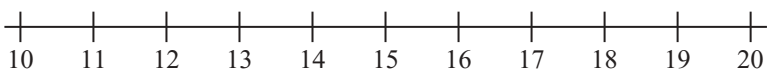
1 What number is halfway between the two numbers given?

a 20 and 30
b 4 and 5
c 0 and 5
d 4 and 6
e 3 and 3.2
f 3 and 3.5
g 3 and 3.1
h 3 and 3.01

2 Match the scale with the limits of accuracy.

	Measurement scale	Limit of accuracy
a		±0.125
b		±0.25
c		±0.05
d		±0.5
e		±15
f		±2.5

3 Copy and complete the following, which refer to the diagram below.



a When using the measurement scale, we are measuring to the nearest _____.
b The limits of accuracy are ± _____.
c If a measurement is quoted as 16, it could be anything from _____ up to, but not including, _____.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 4, 5 4–6

Example 4 Stating the smallest unit of measurement

For each of the following, write down the smallest unit of measurement.

a 89.8 cm **b** 56.85 m

SOLUTION	EXPLANATION
a 0.1 cm or 1 mm	The measurement is given to one decimal place. That means the smallest unit of measurement is tenths.
b 0.01 m or 1 cm	This measurement is given to two decimal places. Therefore, the smallest unit of measurement is hundredths or 0.01.

Now you try

For each of the following, write down the smallest unit of measurement.

a 24.3 m **b** 4.75 km

4 For each of the following, state the smallest unit of measurement.

a 45 cm **b** 6.8 mm **c** 12 m
d 15.6 kg **e** 56.8 g **f** 10 m
g 673 h **h** 9.84 m **i** 12.34 km

Example 5 Finding limits of accuracy

Give the limits of accuracy for these measurements.

a 72 cm **b** 86.6 mm

SOLUTION	EXPLANATION
a $72 \pm 0.5 \times 1$ cm $= 72 - 0.5$ cm to $72 + 0.5$ cm $= 71.5$ cm to 72.5 cm	Smallest unit of measurement is one whole cm. Error $= 0.5 \times 1$ cm This error is subtracted and added to the given measurement to find the limits of accuracy.
b $86.6 \pm 0.5 \times 0.1$ mm $= 86.6 \pm 0.05$ mm $= 86.6 - 0.05$ mm to $86.6 + 0.05$ mm $= 86.55$ mm to 86.65 mm	Smallest unit of measurement is 0.1 mm. Error $= 0.5 \times 0.1$ mm This error is subtracted and added to the given measurement to find the limits of accuracy.

Now you try

Give the limits of accuracy for these measurements.

a 36 cm **b** 15.1 m

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Give the limits of accuracy for each of these measurements.

a 5 m	**b** 8 cm	**c** 78 mm
d 5 ns	**e** 2 km	**f** 34.2 cm
g 3.9 kg	**h** 19.4 kg	**i** 457.9 t
j 18.65 m	**k** 7.88 km	**l** 5.05 s

Hint: Given measurement $\pm$ 0.5 $\times$ smallest unit of measurement.

Example 6 Calculating the percentage error for a reported measurement

A container is said to contain 175 mL, correct to the nearest 5 mL. What are the absolute error and the percentage error, correct to two decimal places?

SOLUTION

The limits of accuracy are 172.5 mL and 177.5 mL.

Absolute error is 2.5 mL

$$\text{Percentage error} = \left(\frac{2.5}{175} \times 100\right)\%$$
$$= 1.43\% \text{ (to 2 d.p.)}$$

EXPLANATION

Smallest unit of measurement is 5 mL.

Half of 5 mL is 2.5 mL

Now you try

A tree was measured to be 80 cm high, to the nearest centimetre. What is the absolute error and the percentage error?

6 Calculate the absolute error and the percentage error for these reported measurements.

a 200 cm, correct to the nearest centimetre
b 200 cm, correct to the nearest 10 centimetres
c 5 kg, correct to the nearest kilogram
d 5 kg, correct to the nearest gram
e 25 cm, correct to the nearest millimetre

PROBLEM-SOLVING AND REASONING 7–9 | 7, 9–12

7 Write the following as a measurement, given that the lower and upper limits of the measurements are as follows.

a 29.5 m to 30.5 m **b** 144.5 g to 145.5 g **c** 4.55 km to 4.65 km

8 Martha writes down the length of her fabric as 150 cm. As Martha does not give her level of accuracy, give the limits of accuracy of her fabric if it was measured correct to the nearest:

a centimetre **b** 10 centimetres **c** millimetre

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 A length of copper pipe is given as 25 cm, correct to the nearest centimetre.

Hint: Use upper and lower limits of accuracy for maximum and minimum lengths.

- **a** What are the limits of accuracy for this measurement?
- **b** Ten pieces of pipe, each with a given length of 25 cm, are joined.
  - **i** What is the minimum length that it could be?
  - **ii** What is the maximum length that it could be?

10 The sides of a square are recorded as 9.2 cm, correct to one decimal place.

- **a** What is the minimum length that each side of this square could be?
- **b** What is the maximum length that each side of this square could be?
- **c** Find the upper and lower boundaries for this square's perimeter.

11 Johan measures the mass of an object to be 6 kg. Amy says the same object is 5.8 kg and Thomas gives his answer as 5.85 kg.

- **a** Explain how all three people could have different answers for the same measurement.
- **b** Write down the level of accuracy being used by each person.
- **c** Are all their answers correct? Discuss.

12 Write down a sentence explaining the need to accurately measure items in our everyday lives and the accuracy that is needed for each of your examples. Give three examples of items that need to be measured correct to the nearest:

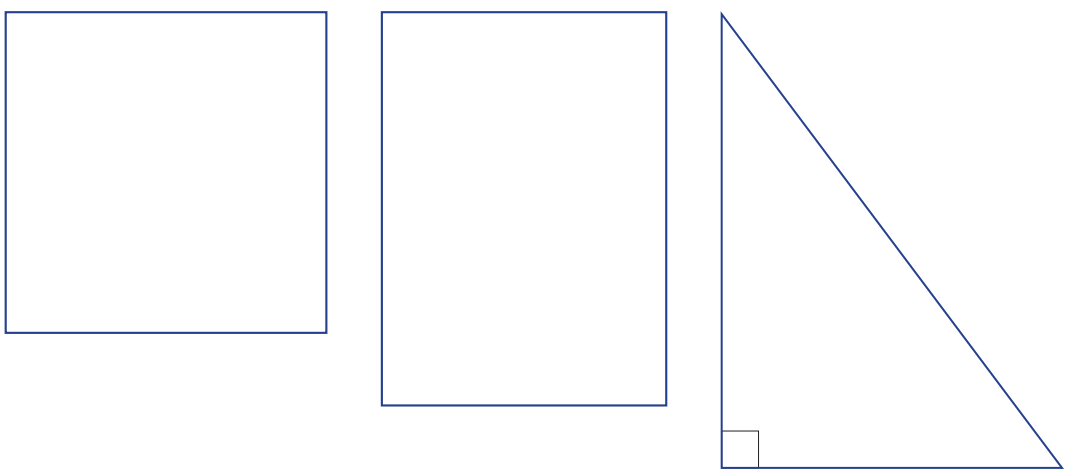
a kilometre **b** millimetre **c** millilitre **d** litre

ENRICHMENT: Practical measurement — 13

13 **a** Measure each of the shapes below, correct to the nearest:

i centimetre **ii** millimetre

- **b** Use your measurements to find the perimeter and area of each shape.
- **c** After collating your classmates' measurements, find the average perimeter for each shape.
- **d** By how much did the lowest and highest perimeters vary? How can this difference be explained?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2C Perimeter CONSOLIDATING

Learning intentions for this section:

- To be able to calculate the perimeter of a shape
- To be able to find an unknown length, given the perimeter

Past, present and future learning:

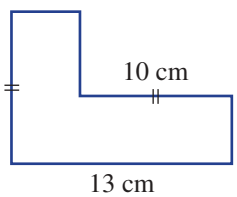
- This section addresses Stage 4 concepts that are used in Stages 5 and 6
- It consolidates and extends the concepts in Section 6A of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formulas from memory in the HSC Examination

Perimeter is a measure of length around the outside of a shape. We calculate perimeter when ordering ceiling cornices for a room or materials for fencing a paddock, or when designing a house.

Lesson starter: L-shaped perimeters

The L-shaped figure on the right includes only right (90°) angles. Only two measurements are given.

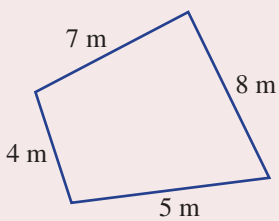
- Can you figure out any other side lengths?
- Is it possible to find its perimeter? Why?



KEY IDEAS

■ **Perimeter** is the distance around the outside of a two-dimensional shape.

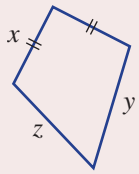
- To find the perimeter, we add all the lengths of the sides in the same units.



$P = 4 + 5 + 7 + 8 = 24\,\text{m}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- When two sides of a shape are the same length they are labelled with the same markings.



$P = 2x + y + z$

Key terminology: perimeter

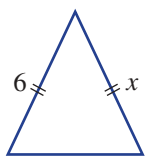
Exercise 2C

UNDERSTANDING	1, 2	2

1 Write the missing word: The distance around the outside of a shape is called the _____.

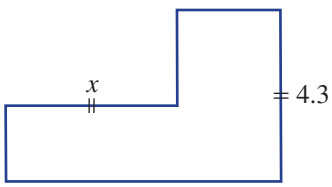
2 Write down the value of x for these shapes.

a



b

c

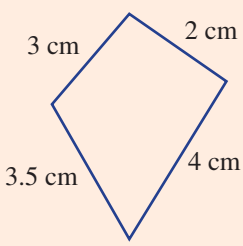


FLUENCY	3, 4	3–4(½)

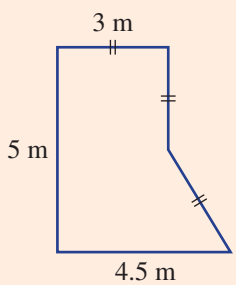
Example 7 Finding perimeters of basic shapes

Find the perimeter of these shapes.

a



b



SOLUTION

a Perimeter $= 3 + 2 + 4 + 3.5$
$= 12.5\,\text{cm}$

b Perimeter $= 5 + 4.5 + 3 \times 3$
$= 18.5\,\text{m}$

EXPLANATION

Add all the lengths of the sides together.

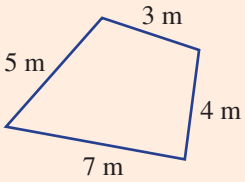
Three lengths have the same markings and therefore are the same length.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

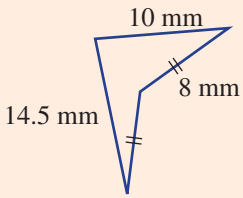
Now you try

Find the perimeter of these shapes.

a



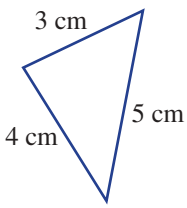
b



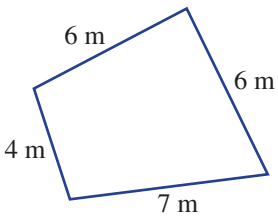
3 Find the perimeter of these shapes.

Hint: Sides with the same markings are the same length.

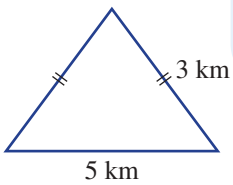
a



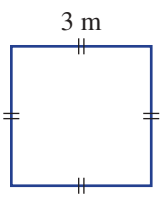
b



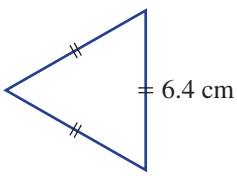
c



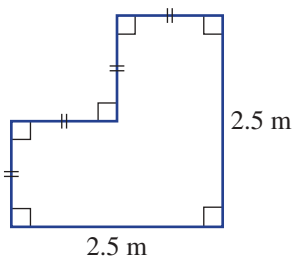
d



e

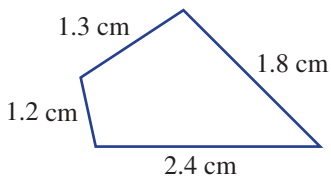


f

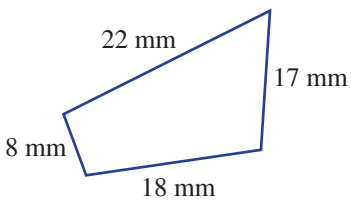


4 Find the perimeter of these shapes.

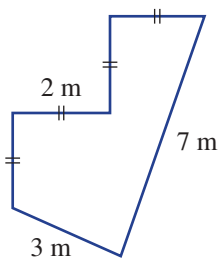
a



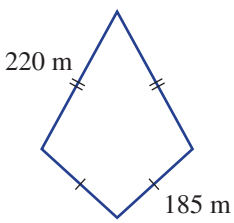
b



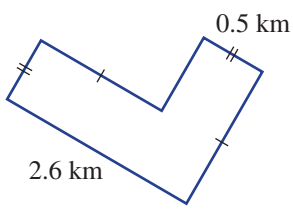
c



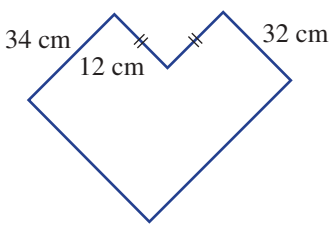
d



e



f

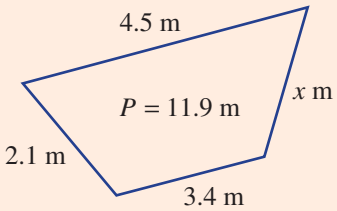


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING 5–7 6–10

Example 8 Finding a missing side length

Find the value of x for this shape with the given perimeter.



SOLUTION

$4.5 + 2.1 + 3.4 + x = 11.9$

$10 + x = 11.9$

$x = 1.9$

EXPLANATION

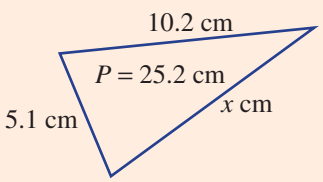
All the sides add to 11.9 in length.

Simplify.

Subtract 10 from both sides to find the value of x.

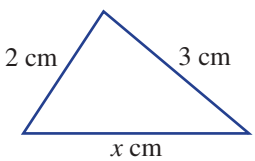
Now you try

Find the value of x for this shape with the given perimeter.



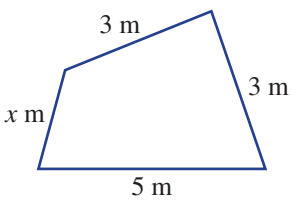
5 Find the value of x for these shapes with the given perimeters.

a



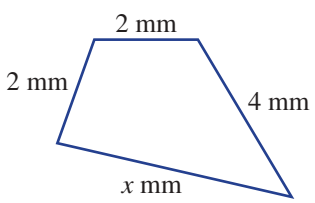
Perimeter = 9 cm

b



Perimeter = 13 m

c

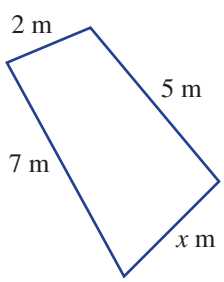


Perimeter = 14 mm

Hint: Add up all the sides and then determine the value of x to suit the given perimeters.

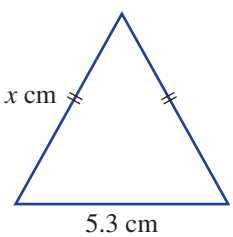
6 Find the value of x for these shapes with the given perimeters.

a



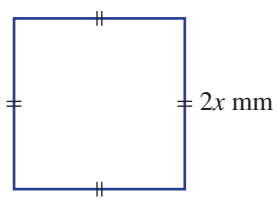
Perimeter = 17 m

b



Perimeter = 22.9 cm

c



Perimeter = 0.8 mm

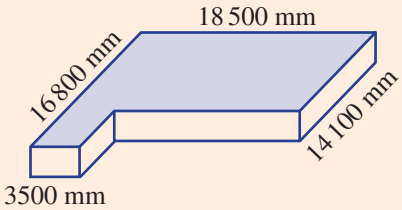
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 9 Working with perimeter with three dimensions

A concrete slab has the measurements shown.

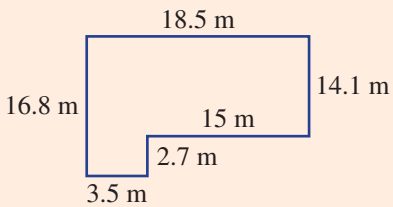
a Draw a new diagram, showing all the measurements in metres.

b Determine the lineal metres of timber needed to surround it.



SOLUTION

a



EXPLANATION

Convert your measurements and place them all on the diagram.

$1\text{ m} = 100 \times 10 = 1000\text{ mm}$

Add or subtract to find the missing measurements.

b Perimeter $= 18.5 + 16.8 + 3.5 + 2.7 + 15 + 14.1$
$= 70.6\text{ m}$

Add all the measurements.

The lineal metres of timber needed is 70.6 m.

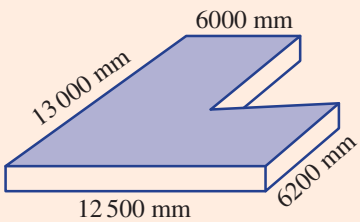
Write your answer in words.

Now you try

A concrete slab has the measurements shown.

a Draw a new diagram showing all the measurements in metres.

b Determine the lineal metres of timber needed to surround it.

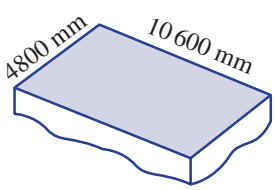


7 Six concrete slabs are shown below.

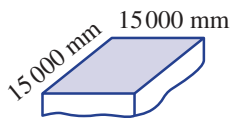
i Draw a new diagram for each with the measurements in metres.

ii Determine the lineal metres of timber needed for each to surround it.

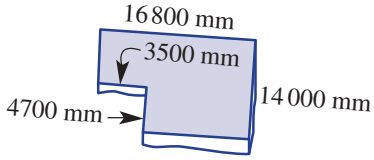
a



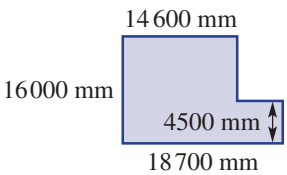
b



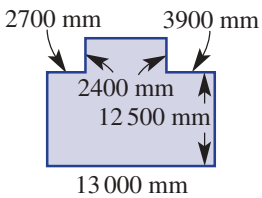
c



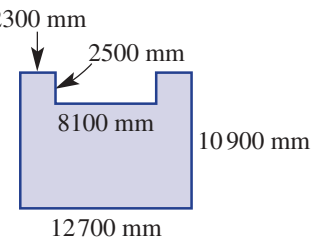
d



e



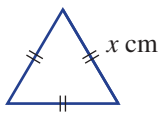
f



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

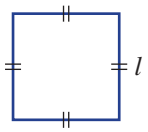
8 A rectangular paddock has perimeter 100 m. Find the breadth of the paddock if its length is 30 m.

9 The equilateral triangle shown has perimeter 45 cm. Find its side length.

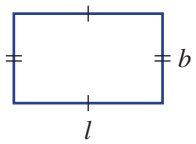


10 Write formulas for the perimeter of these shapes, using the pronumerals given.

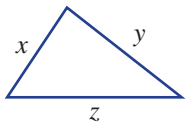
a



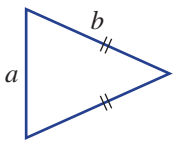
b



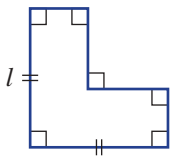
c



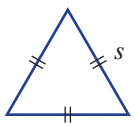
d



e



f



ENRICHMENT: How many different tables? — 11, 12

11 A large dining table is advertised with a perimeter of 12 m. The length and breadth are a whole number of metres (e.g. 1 m, 2 m, …). How many different-sized tables are possible?

12 How many rectangles (using whole number lengths) have perimeters between 16 m and 20 m, inclusive?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2D Circumference and arc length CONSOLIDATING

Learning intentions for this section:

- To know the formulas for the circumference of a circle and arc length
- To be able to calculate the circumference of a circle and length of an arc
- To be able to calculate the perimeter of sectors and composite shapes

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stages 5 and 6
- It consolidates and extends the concepts in Sections 6B and 6E of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formulas from memory in the HSC Examination

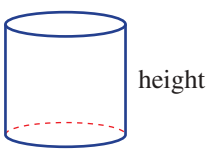
To find the distance around the outside of a circle – the circumference – we use the special number called pi (π). Pi provides a direct link between the diameter of a circle and the circumference of that circle.

The wheel is one of the most useful components in many forms of machinery and its shape, of course, is a circle. One revolution of a vehicle's wheel moves the vehicle a distance equal to the wheel's circumference.

Lesson starter: When circumference = height

Here is an example of a cylinder.

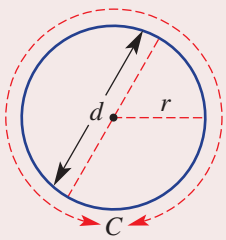
- Try drawing your own cylinder so that its height is equal to the circumference of the circular top.
- How would you check that you have drawn a cylinder with the correct dimensions? Discuss.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

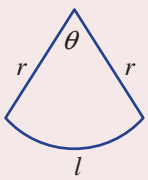
KEY IDEAS

- The **radius** (r) is the distance from the centre of a **circle** to a point on the circle.
- The **diameter** (d) is the distance across a circle through its centre.
  - Radius $= \frac{1}{2}$ diameter or diameter $= 2 \times$ radius
- **Circumference** (C) is the distance around a circle.
  - $C = 2\pi \times$ radius
    $= 2\pi r$
    or $C = \pi \times$ diameter
    $= \pi d$
  - π **(pi)** is a special number and can be found on your calculator.
    It can be approximated by $\pi \approx 3.142$.



- A **sector** is the region inside a circle bounded by two radii and an arc.
  In the diagram:
  r = radius of circle
  θ = number of degrees in angle at centre of circle
  l = length of arc
  Formula for length of arc:

$$l = \frac{\theta}{360} \times 2\pi r$$



Key terminology: circumference, pi, radius, diameter, circle, sector

Exercise 2D

UNDERSTANDING 1–3 3

1 Fill in the missing word in each sentence.

a The distance from the centre of a circle to its outside edge is called the _____.

b The distance across a circle, through its centre is called the _____.

c The distance around a circle is called the _____.

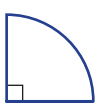
2 Write the formula for the circumference of a circle using:

a d for diameter **b** r for radius

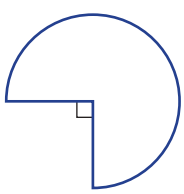
3 What fraction of a circle is shown here?

a

b



c



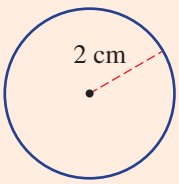
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 4, 5 4–5(½), 6

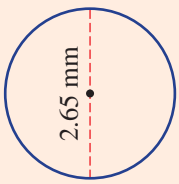
Example 10 Finding the circumference of a circle

Find the circumference of these circles to two decimal places.

a



b

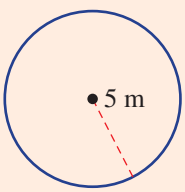


SOLUTION	EXPLANATION
a $C = 2\pi r$	Write the formula involving the radius, r.
$= 2\pi(2)$	Substitute $r = 2$.
$= 12.57$ cm (to 2 d.p.)	Round your answer to two decimal places.
b $C = \pi d$	Write the formula involving diameter.
$= \pi(2.65)$	Substitute $d = 2.65$.
$= 8.33$ mm (to 2 d.p.)	Round your answer to two decimal places.

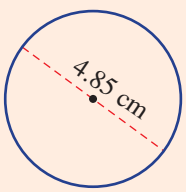
Now you try

Find the circumference of these circles to two decimal places.

a

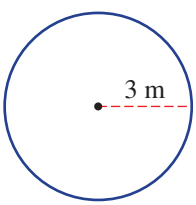


b

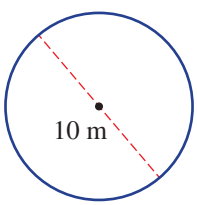


4 Find the circumference of these circles, to two decimal places.

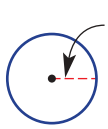
a



b

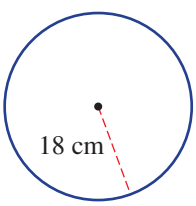


c

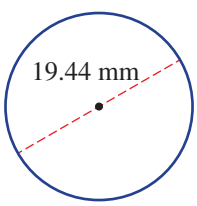


Hint: Use $C = 2\pi r$ or $C = \pi d$.

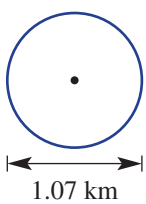
d



e



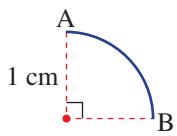
f



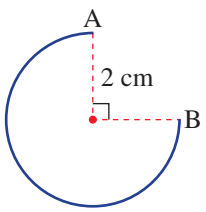
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Find the length of arc AB (to one decimal place).

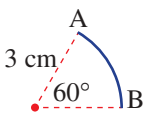
a



b

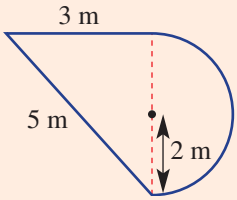


c



Example 11 Finding perimeters of composite shapes

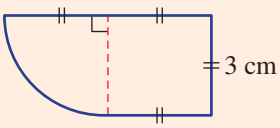
Find the perimeter of this composite shape, to two decimal places.



SOLUTION	EXPLANATION
$P = 3 + 5 + \frac{1}{2} \times 2\pi(2)$	Add all the sides, including half a circle.
$= 8 + 2\pi$	Simplify.
$= 14.28$ m (to 2 d.p.)	Round your answer as instructed.

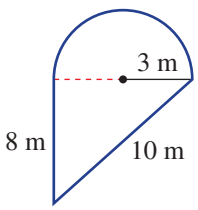
Now you try

Find the perimeter of this composite shape, to two decimal places.

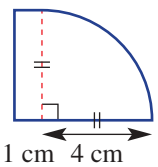


6 Find the perimeter of these composite shapes, correct to two decimal places.

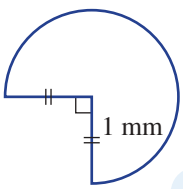
a



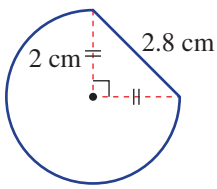
b



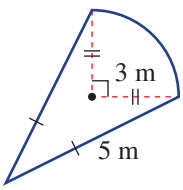
c



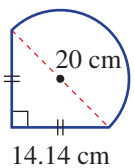
d



e



f



Hint: Don't forget to add the straight sides to the fraction $\left(\frac{1}{4}, \frac{1}{2} \text{ or } \frac{3}{4}\right)$ of the circumference.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

7–9 | 8–11

7 David wishes to build a circular fish pond. The diameter of the pond will be 3 m.

a How many lineal metres of bricks are needed to surround it? Round your answer to two decimal places.

b What is the cost if the bricks are $45 per metre? (Use the rounded answer from part **a**.)

8 The wheels of a bike have a diameter of 1 m.

a How many metres will the bike travel (to two decimal places) after:

i one full turn of the wheels?

ii 15 full turns of the wheels?

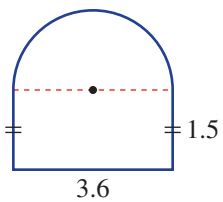
b How many kilometres will the bike travel after 1000 full turns of the wheels? (Give your answer correct to two decimal places.)

Hint: For one revolution, use $C = \pi d$.

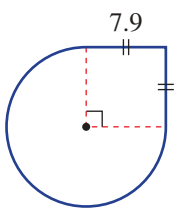
9 What is the minimum number of times a wheel of diameter 1 m needs to spin to cover a distance of 1 km? You will need to find the circumference of the wheel first. Give your answer as a whole number.

10 Find the perimeter of these composite shapes, correct to two decimal places.

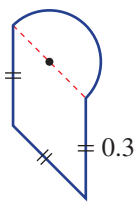
a



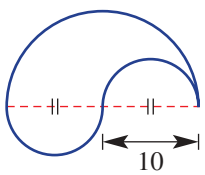
b



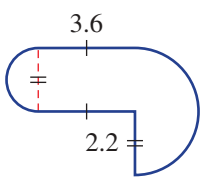
c



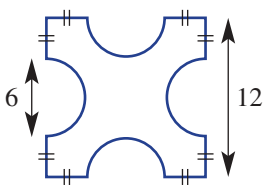
d



e



f



Hint: Make sure you know the radius or diameter of the circle you are dealing with.

11 **a** Rearrange the formula for the circumference of a circle, $C = 2\pi r$, to write r in terms of C.

b Find, to two decimal places, the radius of a circle with the given circumference.

i 35 cm

ii 1.85 m

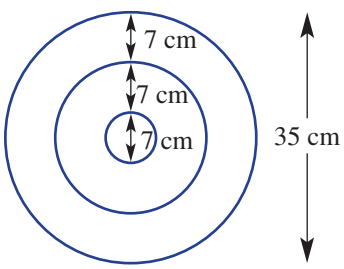
iii 0.27 km

Hint: To make r the subject, divide both sides by 2π.

ENRICHMENT: Target practice

— | 12

12 A target is made up of three rings, as shown.



a Find the radius of the smallest ring.

b Find, to two decimal places, the circumference of the:

i smallest ring

ii middle ring

iii outside ring

c If the circumference of a different ring is 80 cm, what would be its radius, correct to two decimal places?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2E Area of triangles and quadrilaterals CONSOLIDATING

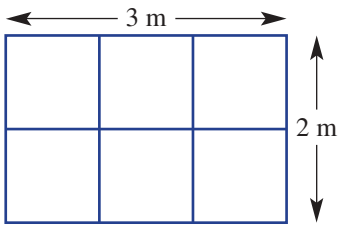
Learning intentions for this section:

- To know the formulas for the area of common shapes
- To be able to find the area of common shapes

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stages 5 and 6
- It consolidates and extends the concepts in Section 6C of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formulas from memory in the HSC Examination

In this simple diagram, a rectangle, with side lengths 2 m and 3 m, has an area of 6 square metres or 6 m^2. This is calculated by counting the number of squares (each measuring a square metre) that make up the rectangle.

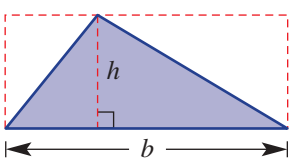


Area = 6 m^2

We use formulas to help us quickly count the number of square units contained within a shape. For this rectangle, for example, the formula $A = lb$ simply tells us to multiply the length by the breadth to find the area.

Lesson starter: How does $A = \frac{1}{2}bh$ work for a triangle?

Look at this triangle, including its rectangular red dashed lines.



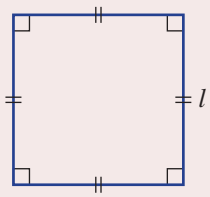
- How does the shape of the triangle relate to the shape of the outside rectangle?
- How can you use the formula for a rectangle to help find the area of the triangle (or parts of the triangle)?
- Why is the rule for the area of a triangle given by $A = \frac{1}{2}bh$?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

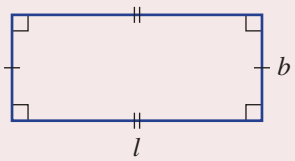
- The **area** of a two-dimensional shape is the number of square units contained within its boundaries.
- Some of the common area formulas are as follows.

Square



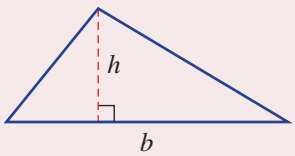
Area = l^2

Rectangle



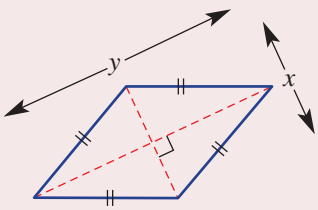
Area = lb

Triangle



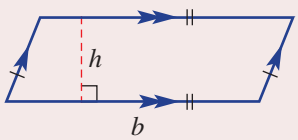
Area = $\frac{1}{2}bh$

Rhombus



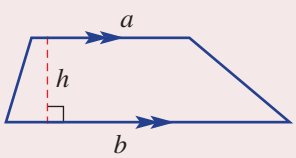
Area = $\frac{1}{2}xy$

Parallelogram



Area = bh

Trapezium



Area = $\frac{h}{2}(a + b)$

- The 'height' in a triangle, parallelogram or trapezium should be **perpendicular** (at 90°) to the base.

Key terminology: area, square, rectangle, triangle, rhombus, parallelogram, trapezium, perpendicular

Exercise 2E

UNDERSTANDING | 1, 2 | 2

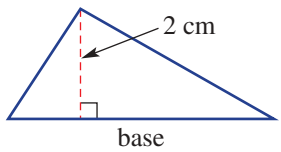
1 Match each shape (**a–f**) with its area formula (**A–F**).

a	square	**A**	$A = \frac{1}{2}bh$
b	rectangle	**B**	$A = lb$
c	rhombus	**C**	$A = bh$
d	parallelogram	**D**	$A = \frac{h}{2}(a + b)$
e	trapezium	**E**	$A = l^2$
f	triangle	**F**	$A = \frac{1}{2}xy$

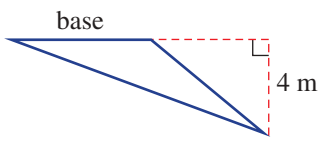
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 These shapes show the base and a height length. Write down the given height of each shape.

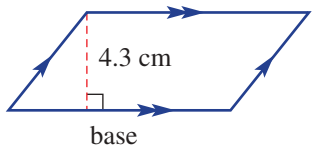
a



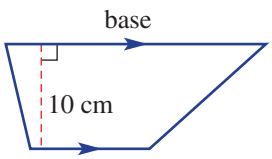
b



c



d

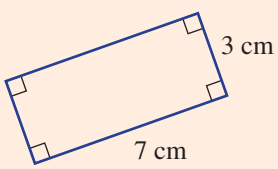


FLUENCY 3, 4 3–4(½)

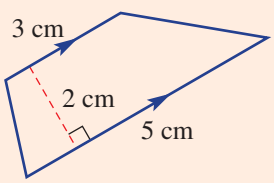
Example 12 Using area formulas

Find the area of these basic shapes.

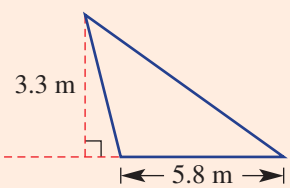
a



b



c



SOLUTION

a Area $= lb$

$= 7 \times 3$

$= 21 \text{ cm}^2$

EXPLANATION

Write the formula for the area of a rectangle.

Substitute the lengths $l = 7$ and $b = 3$.

Simplify and include the units.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b Area $= \frac{h}{2}(a + b)$ — Write the formula for the area of a trapezium.

$= \frac{2}{2}(3 + 5)$ — Substitute the lengths $a = 3$, $b = 5$ and $h = 2$.

$= 8\text{ cm}^2$ — Simplify and include the units.

c Area $= \frac{1}{2}bh$ — Write the formula for the area of a triangle.

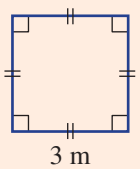
$= \frac{1}{2} \times 5.8 \times 3.3$ — Substitute the lengths $b = 5.8$ and $h = 3.3$.

$= 9.57\text{ m}^2$ — Simplify and include the units.

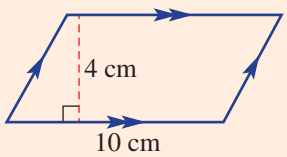
Now you try

Find the area of these basic shapes.

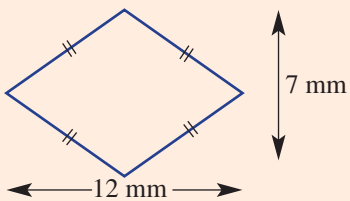
a



b



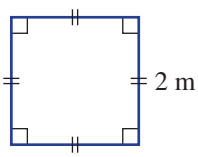
c



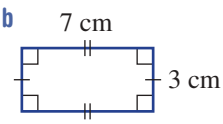
3 Find the area of these shapes.

Hint: First, choose the correct formula and substitute for each pronumeral (letter).

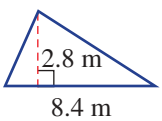
a



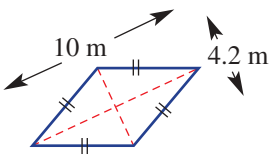
b



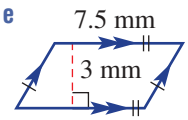
c



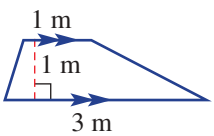
d



e

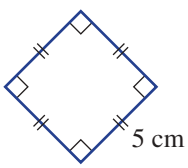


f

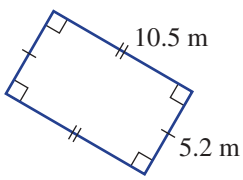


4 Find the area of these shapes.

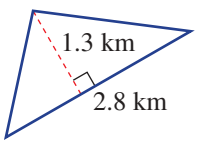
a



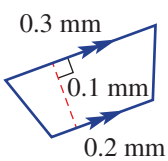
b



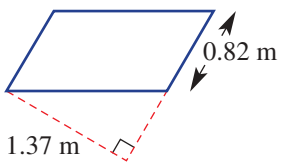
c



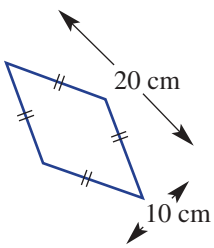
d



e



f



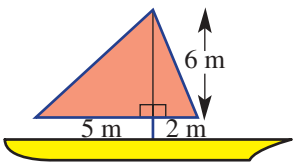
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING 5–8 7–11

5 A rectangular table top is 1.2 m long and 80 cm wide.
Find the area of the table top using:
- **a** square metres (m^2)
- **b** square centimetres (cm^2)

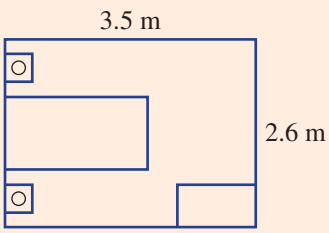
Hint: First convert to the units that you want to work with.

6 Two triangular sails have side lengths as shown. Find the total area of the two sails.



Example 13 Applying area formulas

Christine decides to use carpet to cover the floor of her rectangular bedroom, shown at right. Determine:
- **a** the area of floor to be covered
- **b** the total cost if the carpet costs $32 per square metre



SOLUTION

a Area of floor $= l \times b$
$= 3.5 \times 2.6$
$= 9.1\ m^2$

EXPLANATION

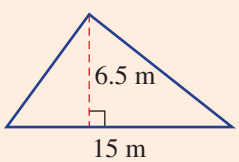
The room is a rectangle, so use $A = l \times b$ to calculate the total floor space.

b Cost of carpet $= 9.1 \times 32$
$= \$291.20$

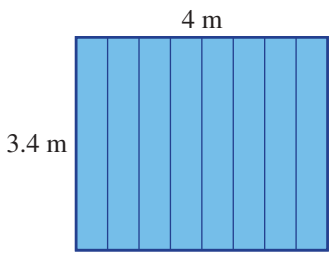
Every square metre of carpet costs $32.

Now you try

Richo decides to lay lawn on his triangular backyard, shown at right. Determine:
- **a** the area of lawn to be laid
- **b** the total cost if lawn costs $11 per square metre



7 Jack's shed is to have a flat rectangular roof, which he decides to cover with metal sheets.
- **a** Determine the total area of the roof.
- **b** If the metal roofing costs $11 a square metre, how much will it cost in total?

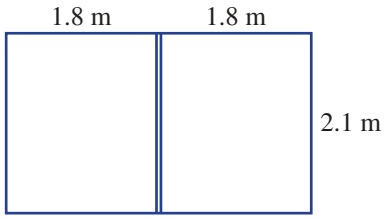


CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 A sliding door has two glass panels. Each of these is 2.1 m high and 1.8 m wide.

a How many square metres of glass are needed?

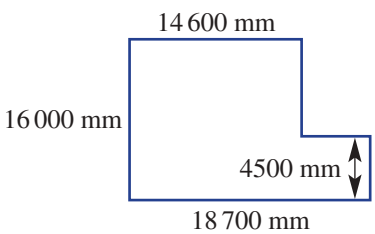
b What is the total cost of the glass if the price is $65 per square metre?



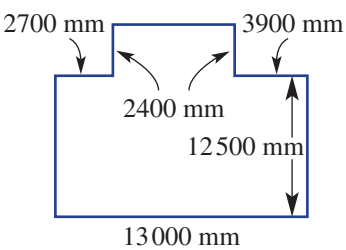
9 A rectangular window has a whole number measurement for its length and breadth and its area is 24 m^2. Write down the possible lengths and breadths for the window.

10 Determine the area of the houses shown (if all angles are right angles), in square metres (correct to two decimal places).

a



b



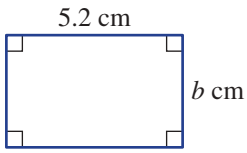
Hint: Note that there are 1000 mm in 1 m.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 Find the value of the pronumeral in these shapes, rounding your answer to two decimal places each time.

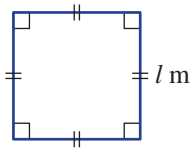
Hint: First, write the appropriate formula and substitute for the area and length pronumerals. Then solve for the unknown.

a



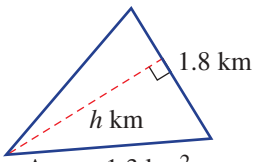
Area = 15 cm^2

b



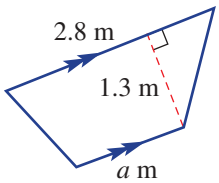
Area = 206 m^2

c



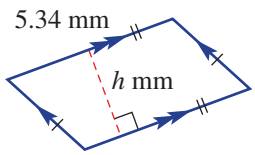
Area = 1.3 km^2

d



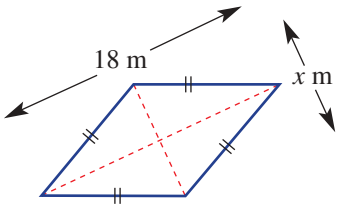
Area = 2.5 m^2

e



Area = 10 mm^2

f

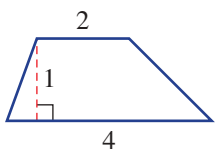


Area = 80 m^2

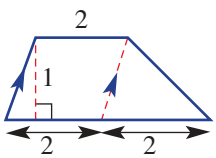
ENRICHMENT: Four ways to find the area of a trapezium — 12

12 Find the area of this trapezium using each of the suggested methods.

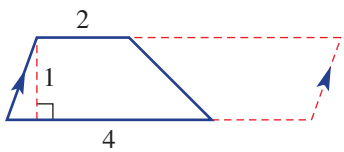
a formula $A = \frac{h}{2}(a + b)$



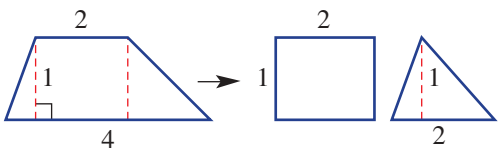
b parallelogram and triangle



c half-parallelogram



d rectangle + triangle



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2F Area of circles and sectors CONSOLIDATING

Learning intentions for this section:

- To know the formulas for the area of a circle and area of a sector
- To be able to find the area of circles and sectors

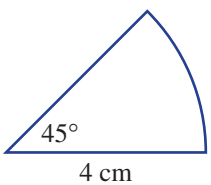
Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stages 5 and 6
- It consolidates and extends the concepts in Sections 6D and 6E of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formulas from memory in the HSC Examination

Like its circumference, a circle's area is linked to the special number pi (π). The area is the product of pi and the square of the radius, so $A = \pi r^2$.

Lesson starter: What fraction is that?

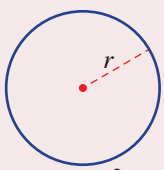
When finding areas of sectors, we first need to decide what fraction of a circle we are dealing with. This sector, for example, has a radius of 4 cm and a 45° angle.



- What fraction of a full circle is shown in this sector?
- How can you use this fraction to help find the area of this sector?
- How would you set out your working to find its area?

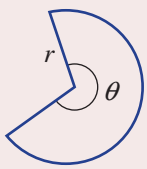
KEY IDEAS

- The formula for finding the area (A) of a circle of radius r is given by the equation $A = \pi r^2$.



- If the diameter (d) of the circle is given, determine the radius before calculating the area of the circle: $r = d \div 2$.
- The area of a sector is given by $A = \frac{\theta}{360} \times \pi r^2$,

  where $\frac{\theta}{360}$ represents the fraction of a full circle.

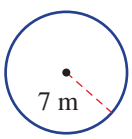


Key terminology: sector, circle, radius, diameter, pi

Exercise 2F

UNDERSTANDING 1–3 3

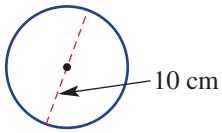
1 Which is the correct working step for the area of this circle?



A $A = \pi(7)$ **B** $A = 2\pi(7)$ **C** $A = \pi(14)^2$
D $A = (\pi 7)^2$ **E** $A = \pi(7)^2$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 Which is the correct working step for the area of this circle?



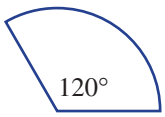
A $A = \pi(10)^2$ **B** $A = (\pi 10)^2$
C $A = \pi(5)^2$ **D** $A = 2\pi(5)$
E $A = 5\pi$

3 What fraction of a circle is shown by these sectors? Simplify your fraction.

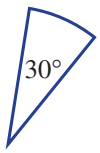
a

b

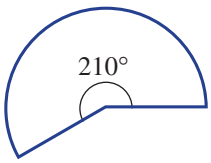
c



d



e



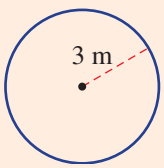
f

FLUENCY 4, 5 4–5(½)

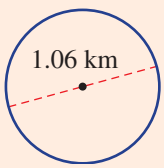
Example 14 Finding areas of circles

Find the area of these circles, correct to two decimal places.

a



b



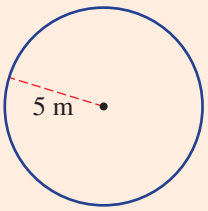
SOLUTION	EXPLANATION
a $A = \pi r^2$	Write the formula.
$= \pi(3)^2$	Substitute $r = 3$.
$= \pi \times 9$	Evaluate $3^2 = 9$ and then multiply by π.
$= 28.27\ \text{m}^2$ (to 2 d.p.)	Round your answer as required.
b Radius $r = 1.06 \div 2 = 0.53$ km	Find the radius, given the diameter of 1.06.
$A = \pi r^2$	Write the formula.
$= \pi(0.53)^2$	Substitute $r = 0.53$.
$= 0.88\ \text{km}^2$ (to 2 d.p.)	Write your answer to two decimal places with units.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

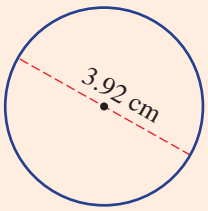
Now you try

Find the area of these circles, correct to two decimal places.

a

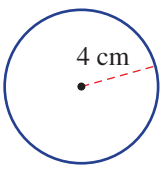


b

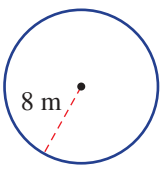


4 Find the area of these circles, correct to two decimal places.

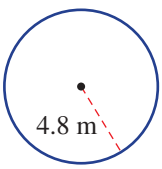
a



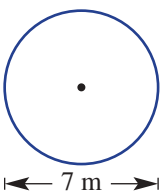
b



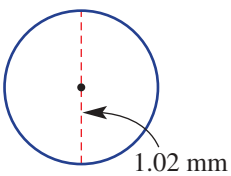
c



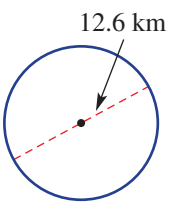
d



e



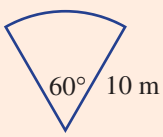
f



Hint: Remember: $r = d \div 2$

Example 15 Finding areas of sectors

Find the area of this sector, correct to two decimal places.



SOLUTION

Fraction of circle $= \frac{60}{360} = \frac{1}{6}$

Area $= \frac{1}{6} \times \pi r^2$

$= \frac{1}{6} \times \pi (10)^2$

$= 52.36\,\text{m}^2$ (to 2 d.p.)

EXPLANATION

The sector uses 60° out of the 360° in a whole circle.

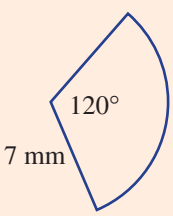
Write the formula, including the fraction.

Substitute $r = 10$.

Write your answer to two decimal places.

Now you try

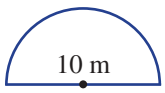
Find the area of this sector, correct to two decimal places.



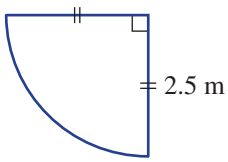
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Find the area of these sectors, correct to two decimal places.

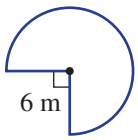
a



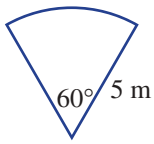
b



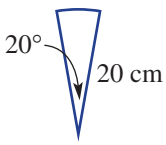
c



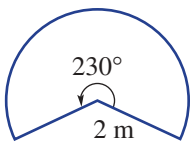
d



e



f



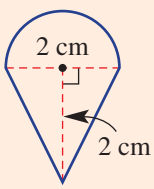
Hint: First determine the fraction of a full circle that you are dealing with.

PROBLEM-SOLVING AND REASONING	6–8	7, 8, 9(½)

6 A pizza with 40 cm diameter is divided into eight equal parts. Find the area of each portion, correct to one decimal place.

Example 16 Finding areas of composite shapes

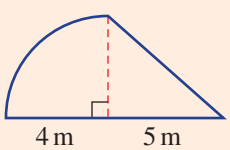
Find the area of this composite shape, correct to two decimal places.



SOLUTION	EXPLANATION
$A = \frac{1}{2}\pi r^2 + \frac{1}{2}bh$	The shape is made up of a semicircle and a triangle. Write the formulas for both shapes.
$= \frac{1}{2}\pi(1)^2 + \frac{1}{2}(2)(2)$	Substitute $r = 1$, $b = 2$ and $h = 2$.
$= 1.5707... + 2$	Write your answer to two decimal places with units.
$= 3.57\text{ cm}^2$ (to 2 d.p.)	

Now you try

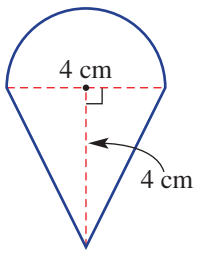
Find the area of this composite shape, correct to two decimal places.



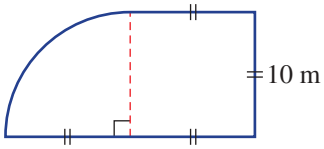
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 Find the area of these composite shapes, correct to two decimal places.

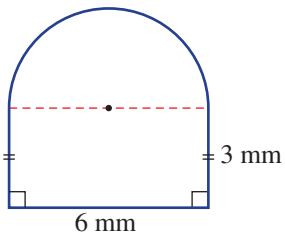
a



b



c

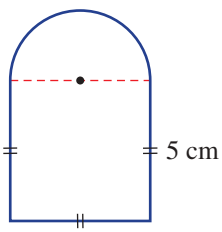


Hint: For Question 7, find the area of each shape that makes up the larger shape, then add them. For example, triangle + semicircle.

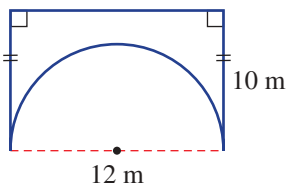
8 The lawn area in a backyard is made up of a semicircular region with diameter 6.5 m and a right-angled triangular region of length 8.2 m, as shown. Find the total area of lawn in the backyard, correct to two decimal places.

9 Find the area of these composite shapes, correct to one decimal place.

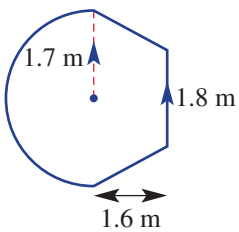
a



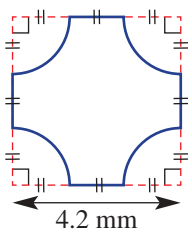
b



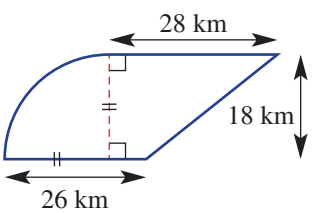
c



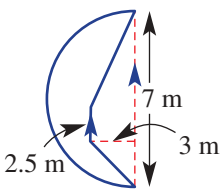
d



e



f

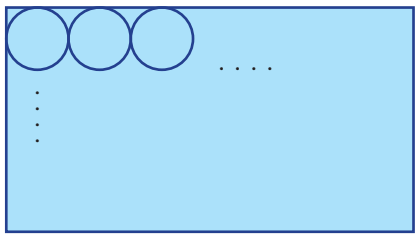


Hint: Use addition or subtraction, depending on the shape given.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: Circular pastries — 10

10 A rectangular piece of pastry is used to create small circular pastry discs for the base of Christmas tarts. The rectangular piece of pastry is 30 cm long and 24 cm wide, and each circular piece has a diameter of 6 cm.



a How many circular pieces of pastry can be removed from the rectangle?

b Find the total area removed from the original rectangle, correct to two decimal places.

c Find the total area of pastry remaining, correct to two decimal places.

d If the remaining pastry was collected and re-rolled to the same thickness, how many circular pieces could be cut? (Assume that the pastry can be re-rolled and cut many times.)

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2G Surface area of prisms

Learning intentions for this section:

- To know that the surface area of a solid can be represented using a net
- To be able to calculate the surface area of a prism

Past, present and future learning:

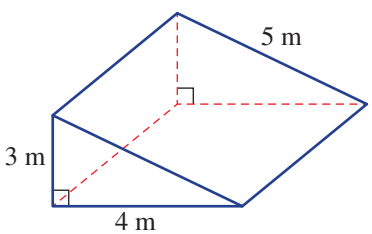
- This section addresses the Stage 5 Core topic called *Area and surface area A*
- It consolidates and extends the concepts in Section 6F of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formulas from memory in the HSC Examination

The surface area of a three-dimensional object can be found by finding the sum of the areas of each of the shapes that make up the surface of the object. A net is a two-dimensional diagram showing all the sides of a three-dimensional object.

Lesson starter: Which net?

The solid below is a triangular prism with a right-angled triangle as its cross-section.

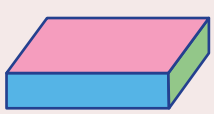
- How many different types of shapes make up its outside surface?
- What is a possible net for the solid? Is there more than one?
- How would you find the surface area?



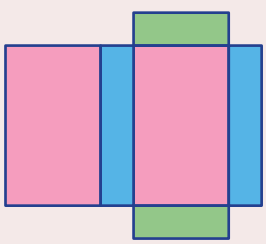
KEY IDEAS

- A **prism** is a solid with a constant **cross-section** shape.
- To calculate the **surface area** (A) of a solid or prism:
  - Draw a **net** (i.e. a two-dimensional drawing that includes all the surfaces).
  - Determine the area of each shape inside the net.
  - Add the areas of each shape together.

Solid



Net



Key terminology: surface area, prism, net, cross-section

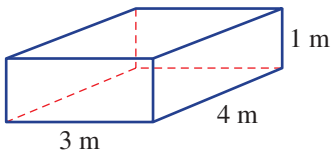
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 2G

UNDERSTANDING 1–3 | 3

1 A two-dimensional drawing of all the faces of a solid is called a _____.

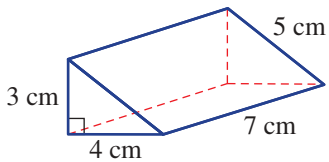
2 For a rectangular prism, answer the following.



a How many faces does the prism have?

b How many *different* rectangles form the surface of the prism?

3 For this triangular prism, answer the following.



a What is the area of the largest surface rectangle?

b What is the area of the smallest surface rectangle?

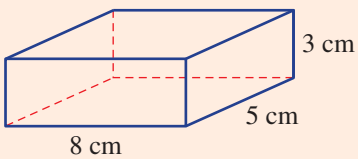
c What is the combined area of the two triangles?

d What is the surface area?

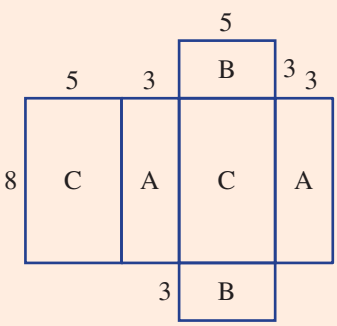
FLUENCY 4, 5 | 4–6

Example 17 Finding the surface area of a rectangular prism

Find the surface area (A) of this rectangular prism by first drawing its net.



SOLUTION



$A = 2 \times$ area of A $+ 2 \times$ area of B
$+ 2 \times$ area of C
$= 2 \times (8 \times 3) + 2 \times (5 \times 3) + 2 \times (8 \times 5)$
$= 158\,\text{cm}^2$

EXPLANATION

Draw the net of the solid, labelling the lengths and shapes of equal areas.

Describe each area.

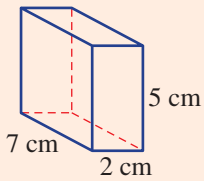
Substitute the correct lengths.

Simplify and include units.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

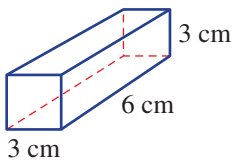
Now you try

Find the surface area (A) of this rectangular prism by first drawing its net.

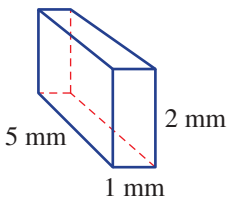


4 Find the surface area (A) of these rectangular prisms by first drawing their nets.

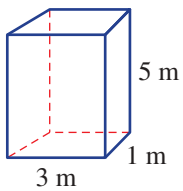
a



b

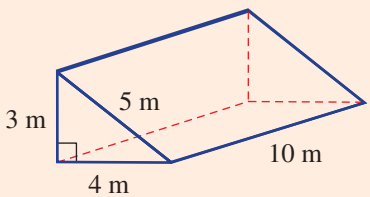


c

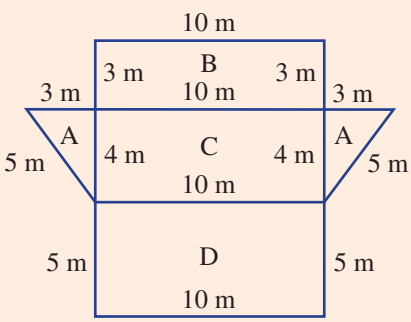


Example 18 Finding the surface area of a triangular prism

Find the surface area of the triangular prism shown.



SOLUTION



Surface area

$= 2 \times \text{area A} + \text{area B} + \text{area C} + \text{area D}$

$= 2 \times \left(\frac{1}{2} \times 3 \times 4\right) + (3 \times 10) + (4 \times 10)$

$+ (5 \times 10)$

$= 12 + 30 + 40 + 50$

$= 132\,\text{m}^2$

EXPLANATION

Draw a net of the object with all the measurements and label the sections to be calculated.

There are two triangles with the same area and three different rectangles.

Substitute the correct lengths.

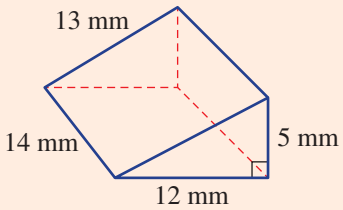
Calculate the area of each shape.

Add the areas together.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

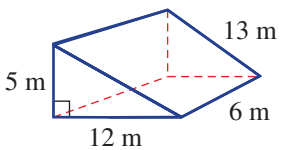
Now you try

Find the surface area of the triangular prism shown.

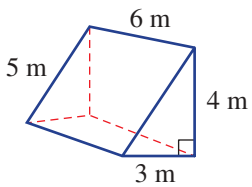


5 Find the surface area of the following prisms.

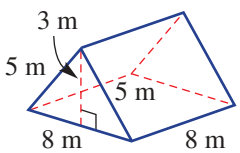
a



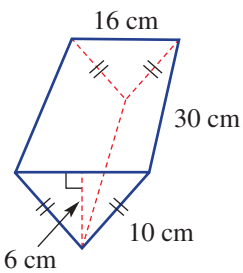
b



c



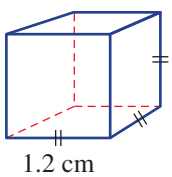
d



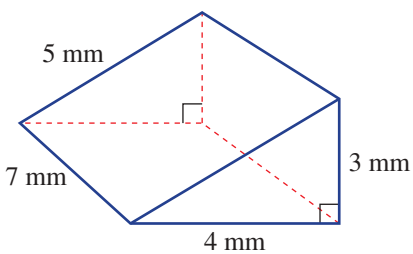
Hint: There are three rectangles and two identical triangles.

6 Find the surface area of these objects by first drawing a net.

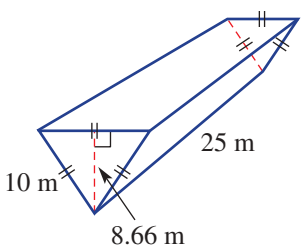
a



b



c

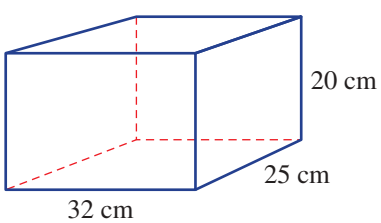


PROBLEM-SOLVING AND REASONING

7–9 | 9–12

7 A cube with side lengths of 8 cm is to be painted all over with bright red paint. What is the surface area that is to be painted?

8 What is the minimum amount of paper required to wrap a box with dimensions 25 cm wide, 32 cm long and 20 cm high?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 An open-topped box is to be covered inside and out with a special material. If the box is 40 cm long, 20 cm wide and 8 cm high, find the minimum amount of material required to cover the box.

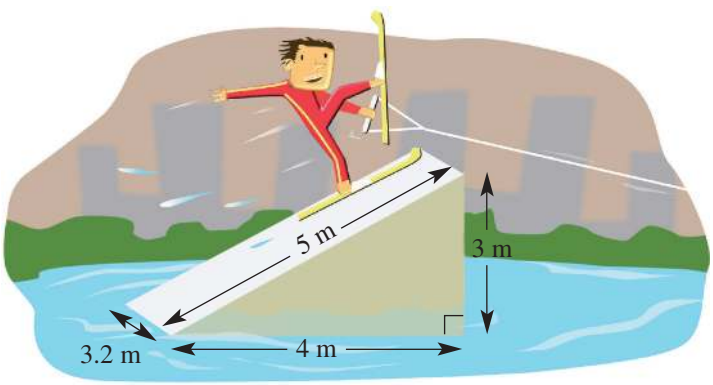
Hint: Include both inside and outside but do not include the top.

10 Mila wants to paint her bedroom. The ceiling and walls are to be the same colour. If the room measures 3.3 m × 4 m and the ceiling is 2.6 m high, find the amount of paint needed if:

a each litre covers 10 square metres

b each litre covers 5 square metres

11 A ski ramp in the shape of a triangular prism needs to be painted before the Moomba Classic waterskiing competition in Melbourne is held. The base and sides of the ramp require a fully waterproof paint, which covers 2.5 square metres per litre. The top needs special smooth paint, which covers only 0.7 square metres per litre.



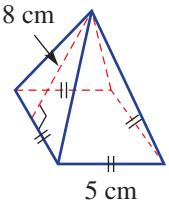
a Determine the amount of each type of paint required. Round your answers to two decimal places where necessary.

b If the waterproof paint is $7 per litre and the special smooth paint is $20 per litre, calculate the total cost of painting the ramp, to the nearest cent. (Use the exact answers from part **a** to help.)

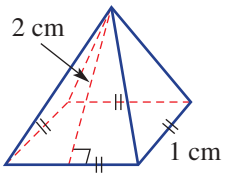
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

12 Find the surface area of these right square-based pyramids.

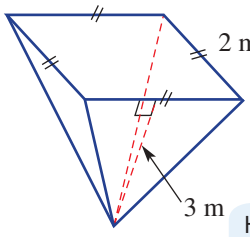
a



b



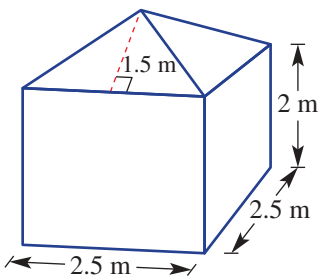
c



Hint: There is one square and four identical triangles.

ENRICHMENT: Will I have enough paint? — 13

13 I have 6 litres of paint and on the tin it says that the coverage is 5.5 m^2 per litre. I wish to paint the four outside walls of a shed and the roof, which has four identical triangular sections. Will I have enough paint to complete the job?



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Progress quiz

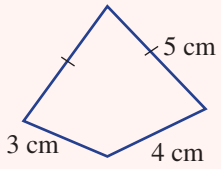
2A **1** Convert the given measurements to the units shown in brackets.

a 9 km (m) **b** 4.5 m^2 (cm^2) **c** 145 cm^3 (mm^3)

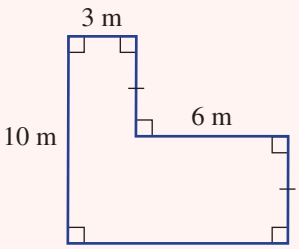
2B **2** An amount of liquid was found to be 178 mL, correct to the nearest millilitre. What is the percentage error, correct to two decimal places?

2C **3** Find the perimeter of these shapes.

a



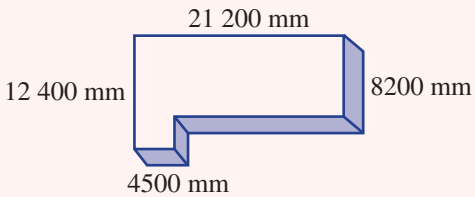
b



2C **4** A concrete slab is shown below.

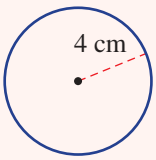
a Draw a new diagram, showing all the measurements in metres.

b Determine the lineal metres of timber needed to surround it.

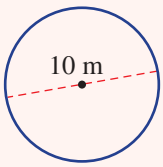


2D/2F **5** Find the circumference (C) and area (A) of these circles, correct to two decimal places.

a

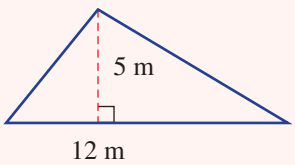


b

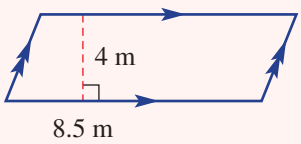


2E/2F **6** Find the area of these shapes. Round your answer to one decimal place in part **d**.

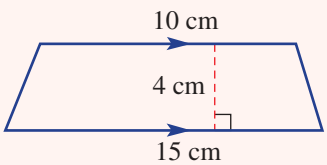
a



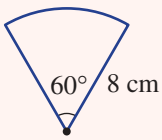
b



c



d



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

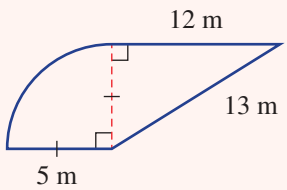
Progress quiz

2E

7 A rectangular kitchen floor is to be replaced with wooden floorboards. If the floorboards cost $46 per square metre, determine the cost to cover the kitchen floor if its dimensions are 4.4 m by 3 m.

2D/2F

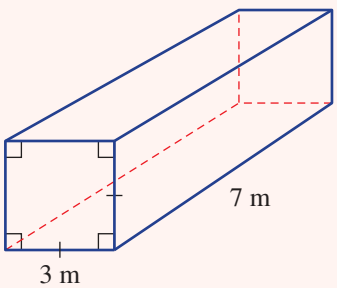
8 Find the area (A) and perimeter (P) of the composite shape shown. Round each answer to one decimal place.



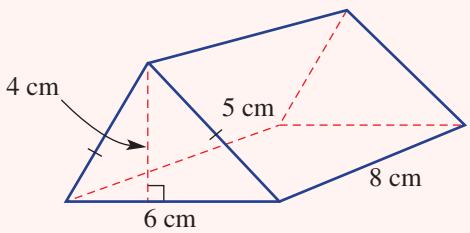
2G

9 Find the surface area of the following prisms. (Drawing a net may help you.)

a



b



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2H Surface area of cylinders

Learning intentions for this section:

- To understand how the net of a cylinder can be drawn to show the surface area
- To know the formula for the surface area of a cylinder
- To be able to calculate the surface area of a cylinder and sections of cylinders

Past, present and future learning:

- This section concludes the Stage 5 Core topic called *Area and surface area A*
- It consolidates and extends the concepts in Section 6G of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formulas from memory in the HSC Examination

Like a prism, a cylinder has a uniform cross-section with identical circles as its two ends. The curved surface of a cylinder can be rolled out to form a rectangle that has a length equal to the circumference of the circle.

A can is a good example of a cylinder. We need to know the area of the ends and the curved surface area in order to cut sections from a sheet of aluminium to manufacture the can.

Lesson starter: Why $2\pi rh$?

We can see from the net of a cylinder (see the diagram below in Key ideas) that the total area of the two circular ends is $2 \times \pi r^2$ or $2\pi r^2$. For the curved part, though, consider the following.

- Why can it be drawn as a rectangle? Can you explain this using a piece of paper?
- Why are the dimensions of this rectangle h and $2\pi r$?
- Where does the formula $A = 2\pi r^2 + 2\pi rh$ come from?

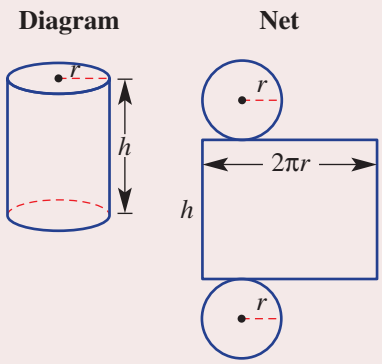
KEY IDEAS

■ A **cylinder** is a solid with a circular **cross-section**.

- The net contains two equal circles and a rectangle. The rectangle has one side length equal to the circumference of the circle.
- $A = 2$ circles $+ 1$ rectangle

  $A = 2\pi r^2 + 2\pi rh$
- This can also be written as

  $A = 2\pi r(r + h)$.



Key terminology: cylinder, area, prism, circumference, net, cross-section

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 2H

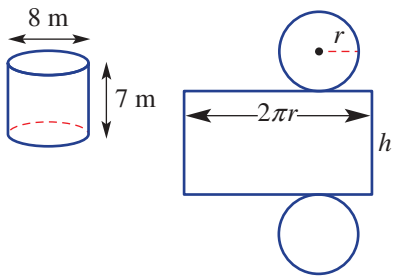
UNDERSTANDING 1, 2 | 2

1 Write the missing word/expression.

a The shape of the cross-section of a cylinder is a _____.

b The surface area of a cylinder is $A = 2\pi r^2 +$ _____.

2 A cylinder and its net are shown here.



a What is the value of:

i r? **ii** h?

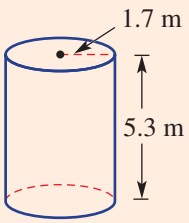
b Find the value of $2\pi r$, correct to two decimal places.

c Use $A = 2\pi r^2 + 2\pi rh$ to find the surface area, correct to two decimal places.

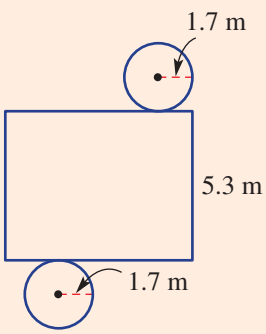
FLUENCY 3, 4 | 3–5

Example 19 Finding the surface area of a cylinder

By first drawing a net, find the surface area of this cylinder, correct to two decimal places.



SOLUTION



EXPLANATION

Draw the net and label the appropriate lengths.

A = 2 circles + 1 rectangle — Write what you need to calculate.

$= 2\pi r^2 + 2\pi rh$ — Write the formula.

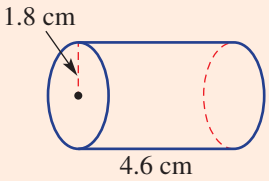
$= 2\pi(1.7)^2 + 2\pi(1.7)(5.3)$ — Substitute the correct values: $r = 1.7$ and $h = 5.3$.

$= 74.77\ \text{m}^2$ (to 2 d.p.) — Round your answer to two decimal places.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

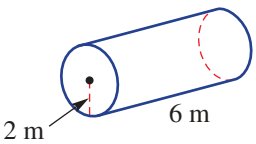
Now you try

By first drawing a net, find the surface area of this cylinder, correct to two decimal places.

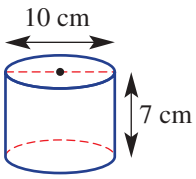


3 By first drawing a net, find the surface area of these cylinders, to two decimal places.

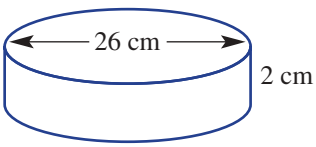
a



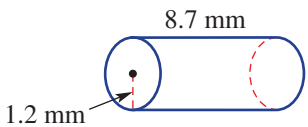
b



c



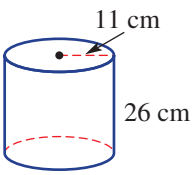
d



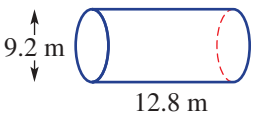
Hint: Remember that radius = diameter ÷ 2.

4 Find the surface area of these cylinders, to one decimal place.

a

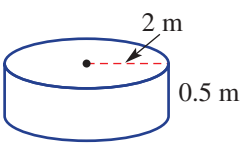


b

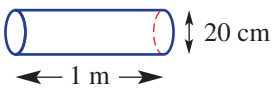


5 Find the area of only the curved surface of these cylinders, to one decimal place.

a



b



Hint: Find only the rectangular part of the net, so use $A = 2\pi rh$. Be careful with the units in part **b**!

PROBLEM-SOLVING AND REASONING

6, 7 | 6–9

6 Find the outside surface area of a pipe of radius 85 cm and length 4.5 m, to one decimal place. Give your answer in square metres.

7 The base and sides of a circular cake tin are to be lined on the inside with baking paper. The tin has a base diameter of 20 cm and is 5 cm high. What is the minimum amount of baking paper required, to one decimal place?

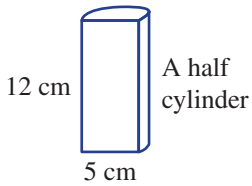
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 The inside and outside of an open-topped cylindrical concrete tank is to be coated with a special waterproofing paint. The tank has diameter 4 m and height 2 m. Find the total area to be coated with the paint. Round your answer to one decimal place.

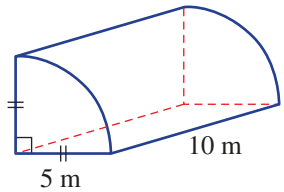
Hint: Include the base but not the top.

9 Find the surface area of these cylindrical portions, to one decimal place.

a

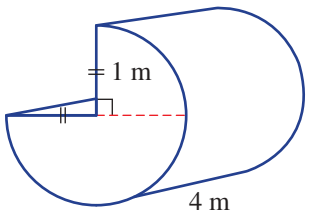


b

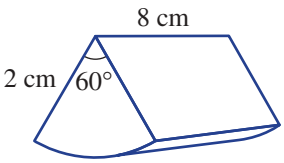


Hint: Carefully consider the fraction of a circle made up by the ends, and the fraction of a full cylinder made up by the curved part.

c



d



ENRICHMENT: The steamroller — 10

10 A steamroller has a large, heavy cylindrical barrel that is 4 m wide and has a diameter of 2 m.

a Find the area of the curved surface of the barrel, to two decimal places.

b After 10 complete turns of the barrel, how much ground would be covered, to two decimal places?

c Find the circumference of one end of the barrel, to two decimal places.

d How many times would the barrel turn after 1 km of distance, to two decimal places?

e What area of ground would be covered if the steamroller travels 1 km?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2I Volume of prisms and cylinders

Learning intentions for this section:

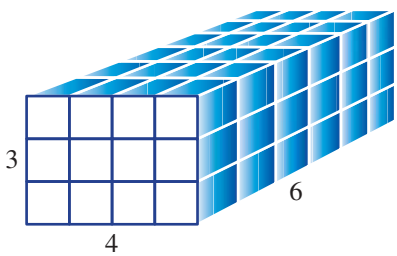
- To understand how the volume of a solid relates to its constant cross-section and height
- To be able to calculate the volume and capacity of a solid with a uniform cross-section

Past, present and future learning:

- This section consolidates Stage 4 and addresses the Stage 5 Core topic called *Volume A*
- It consolidates and extends the concepts in Sections 6H and 6I of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formulas from memory in the HSC Examination

The volume of a solid is the amount of space it occupies within its outside surface. It is measured in cubic units.

For solids with a uniform cross-section, the area of the cross-section multiplied by the perpendicular height gives the volume. Consider the rectangular prism below.



Number of cubic units (base) $= 4 \times 6 = 24$

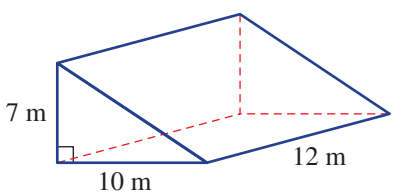
Area (base) $= 4 \times 6 = 24$ units^2

Volume = area (base) $\times 3 = 24 \times 3 = 72$ units^3

Lesson starter: Volume of a triangular prism

This prism has a triangular cross-section.

- What is the area of the cross-section?
- What is the 'height' of the prism?
- How can $V = A \times h$ be applied to this prism, where A is the area of the cross-section?

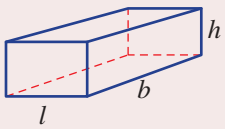


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- **Volume** is the amount of three-dimensional space within an object.
- The volume of a solid with a uniform cross-section is given by $V = A \times h$, where:
  - A is the area of the cross-section.
  - h is the perpendicular (at 90°) height.

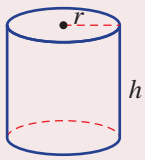
Rectangular prism



$V = A \times h$

$V = lbh$

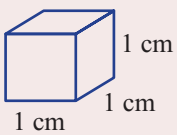
Cylinder



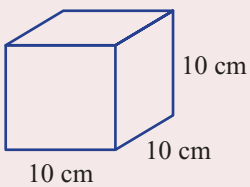
$V = A \times h$

$V = \pi r^2 h$

- Units for capacity include:
  - $1\text{ L} = 1000\text{ mL}$
- One cubic centimetre holds one millilitre.



- This cube holds one litre.



- One cubic metre holds 1000 litres.

Key terminology: solid, volume, cross-section, uniform, prism, cylinder, perpendicular, capacity

Exercise 2I

UNDERSTANDING 1–3 3

1 Match the solid (**a**–**c**) with the volume formula (**A**–**C**).

a	cylinder	**A**	$V = lbh$
b	rectangular prism	**B**	$V = \frac{1}{2}bh \times \text{length}$
c	triangular prism	**C**	$V = \pi r^2 h$

2 Write the missing number.

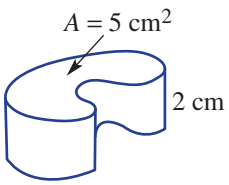
a There are _____ mL in 1 L.

b There are _____ cm^3 in 1 L.

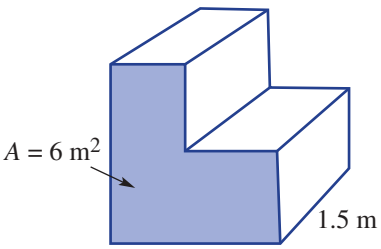
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 The area of the cross-section of this solid is given. Find the solid's volume, using $V = A \times h$.

a



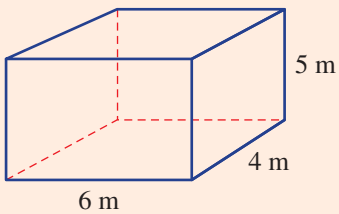
b



FLUENCY 4–6 4–7

Example 20 Finding the volume of a rectangular prism

Find the volume of this rectangular prism.



SOLUTION

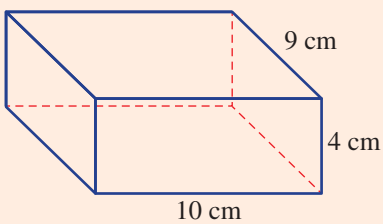
$V = A \times h$
$= 6 \times 4 \times 5$
$= 120\,\text{m}^3$

EXPLANATION

Write the general formula.
$A = 6 \times 4$ and $h = 5$.
Simplify and include units.

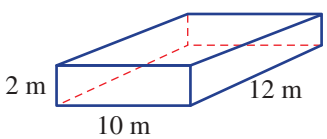
Now you try

Find the volume of this rectangular prism.

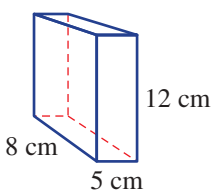


4 Find the volume of these rectangular prisms.

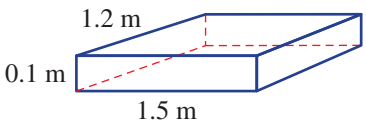
a



b



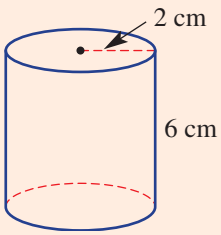
c



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 21 Finding the volume of a cylinder

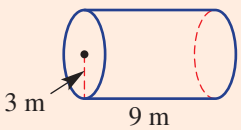
Find the volume of this cylinder, correct to two decimal places.



SOLUTION	EXPLANATION
$V = A \times h$	Write the general formula.
$= \pi r^2 \times h$	The cross-section is a circle.
$= \pi(2)^2 \times 6$	Substitute $r = 2$ and $h = 6$.
$= 75.40\ \text{cm}^3$ (to 2 d.p.)	Simplify and write your answer as required, with units.

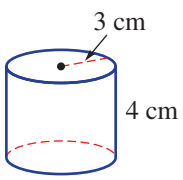
Now you try

Find the volume of this cylinder, correct to two decimal places.

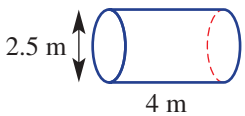


5 Find the volume of these cylinders, correct to two decimal places.

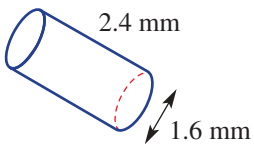
a



b



c

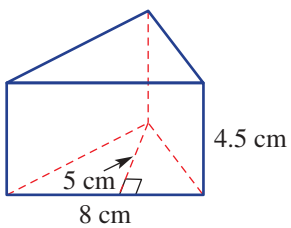


Hint: For a cylinder: $V = \pi r^2 \times h$

6 A triangle with base 8 cm and height 5 cm forms the base of a prism, as shown. If the prism stands 4.5 cm high, find:

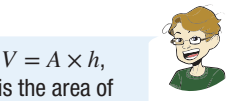
a the area of the triangular base

b the volume of the prism



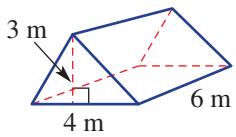
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 Find the volume of these triangular prisms.

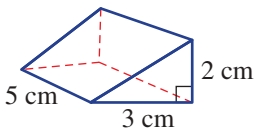


Hint: Use $V = A \times h$, where A is the area of a triangle.

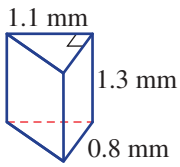
a



b



c

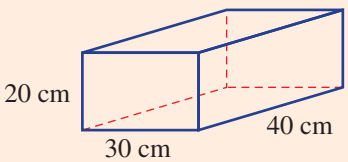


PROBLEM-SOLVING AND REASONING 8–10 9–12

8 A cylindrical drum stands on one end with a diameter of 25 cm and water is filled to a height of 12 cm. Find the volume of water in the drum, in cubic centimetres, correct to two decimal places.

Example 22 Working with capacity

Find the number of litres of water that this container can hold.



SOLUTION

$V = 30 \times 40 \times 20$

$= 24\,000\,\text{cm}^3$

$= 24\,\text{L}$

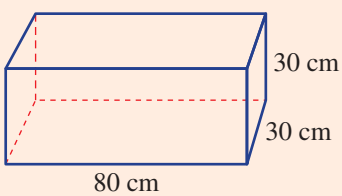
EXPLANATION

First work out the volume in cm^3.

Then divide by 1000 to convert to litres, since $1\,\text{cm}^3 = 1\,\text{mL}$ and there are 1000 mL in 1 litre.

Now you try

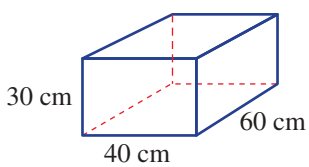
Find the number of litres of water that this container can hold.



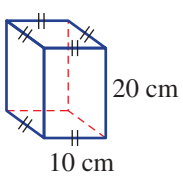
9 Find the number of litres of water that these containers can hold.

Hint: Use $1\,\text{L} = 1000\,\text{cm}^3$.

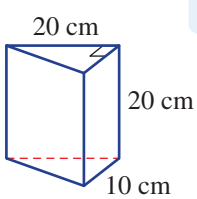
a



b



c

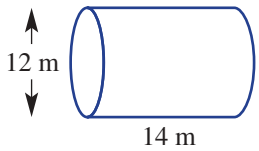


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

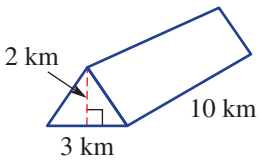
10 Find the volume of these solids, rounding your answers to two decimal places where necessary.

Hint: Find the area of the cross-section first.

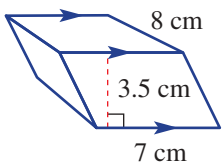
a



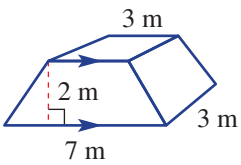
b



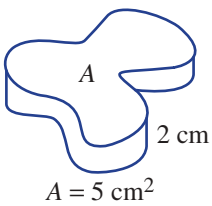
c



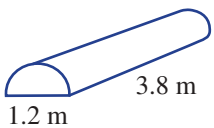
d



e



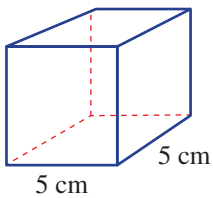
f



11 $100\,\text{cm}^3$ of water is to be poured into this container.

a Find the area of the base of the container.

b Find the depth of water in the container.



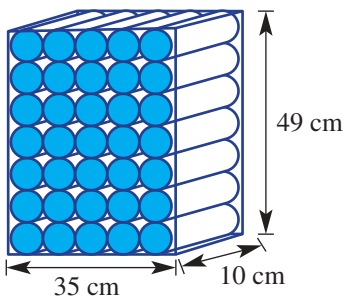
12 In a scientific experiment, solid cylinders of ice are removed from a solid block carved out of a glacier. The ice cylinders have diameter 7 cm and length 10 cm. The dimensions of the solid block are shown in the diagram.

a Find the volume of ice in the original ice block.

b Find the volume of ice in one ice cylinder, to two decimal places.

c Find the number of ice cylinders that can be removed from the ice block, using the configuration shown.

d Find the volume of ice remaining after the ice cylinders are removed from the block, to two decimal places.

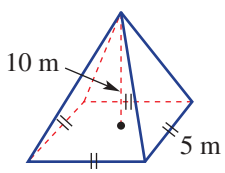


ENRICHMENT: Volume of pyramids and cones — 13

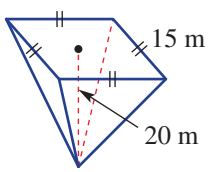
13 The volume of a pyramid or cone is exactly one-third the volume of the prism with the same base area and height; i.e. $V = \frac{1}{3} \times A \times h$.

Find the volume of these pyramids and cones. Round your answer to one decimal place where necessary.

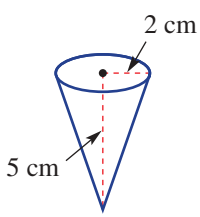
a



b



c



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2J Further problems involving prisms and cylinders

Learning intentions for this section:

- To be able to calculate the surface area and volume of a composite solid
- To be able to solve problems involving composite solids

Past, present and future learning:

- This section concludes the Stage 5 Core topics called *Area and surface area A* and *Volume A*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formulas from memory in the HSC Examination

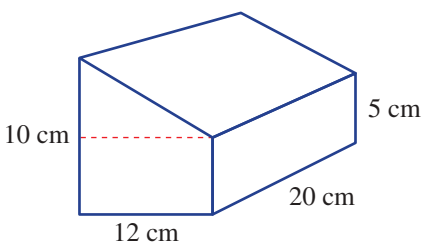
Recall that when working with composite shapes we can find perimeters and areas by considering the combination of the more basic shapes that, together, form the composite shape. Similarly, we can work with composite solids by looking at the combination of more basic solids, like prisms and cylinders. This leads to finding surface areas, volumes and capacity of solids.

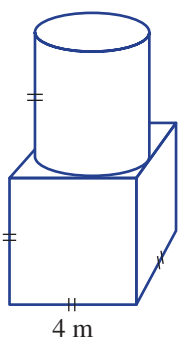
The well-known European artists Christo and Jeanne-Claude had their hands full with composite objects when they wrapped the Reichstag (Parliament building), Berlin in 1995. They used more than 100 000 square metres of fabric and 15 km of rope.

The German parliament, the Reichstag, was wrapped in fabric in 1995 to create a temporary work of art.

Lesson starter: Which solids?

Look at these composite solids.



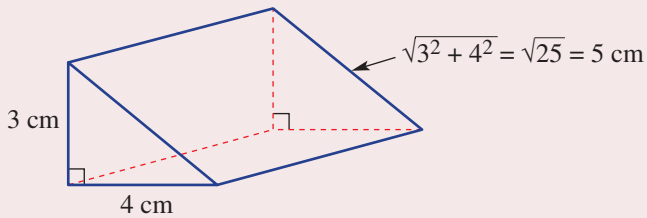


- What are the basic solids that make up each composite solid?
- Explain a method for finding the volume of each solid.
- Explain a method for finding the surface area of each solid.
- Is there enough information provided in each diagram to find the volume and surface area? Discuss.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- Composite solids are made up of more than one basic solid.
- Volumes and surface areas of composite solids can be found by considering the volumes and surface areas (or part there-of) of the basic solids contained within.
- Pythagoras' theorem may be used to help find particular lengths, provided that a right-angled triangle is given.



- Recall these common unit conversions for capacity.
  - $1\text{ L} = 1000\text{ mL} = 1000\text{ cm}^3$
  - $1\text{ m}^3 = 1000\text{ L}$

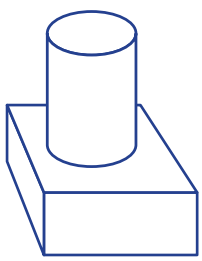
Key terminology: net, composite solid

Exercise 2J

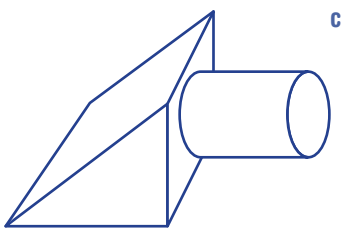
UNDERSTANDING 1 | 1

1 Name the two basic solids that make up each of these composite shapes.

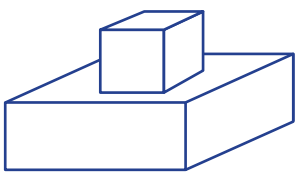
a



b



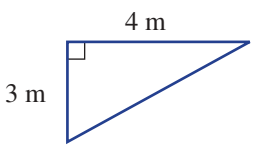
c



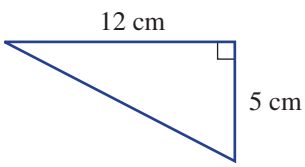
FLUENCY 2–4 | 3–5

2 Use Pythagoras' theorem to find the length of the hypotenuse in these right-angled triangles.

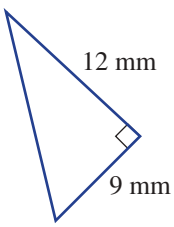
a



b



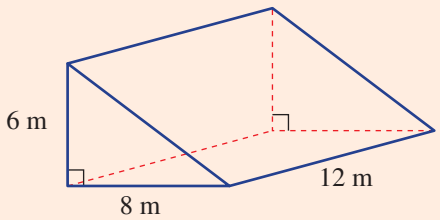
c



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 23 Using Pythagoras' theorem to find the surface area of a triangular prism

Find the surface area of this triangular prism.

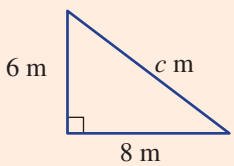


SOLUTION

$c^2 = a^2 + b^2$
$= 6^2 + 8^2$
$= 100$
$c = \sqrt{100}$
$= 10$

EXPLANATION

Use Pythagoras' theorem to find the length of the slanting edge.

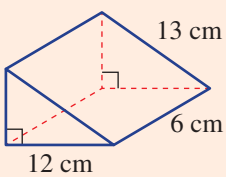


$\text{Surface area} = 2 \times \frac{1}{2} \times 8 \times 6 + 8 \times 12 + 6 \times 12 + 10 \times 12$
$= 48 + 96 + 72 + 120$
$= 336\ \text{m}^2$

The surface area is made up of two congruent triangular ends and three different rectangles.

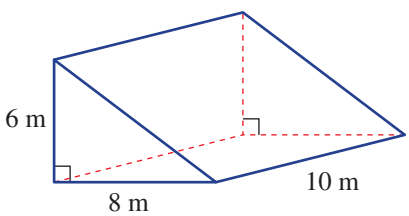
Now you try

Find the surface area of this triangular prism.

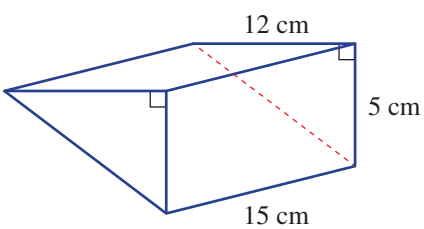


3 Use Pythagoras' theorem to help find the surface area of these triangular prisms.

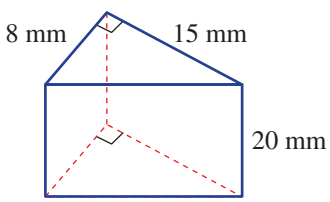
a



b



c

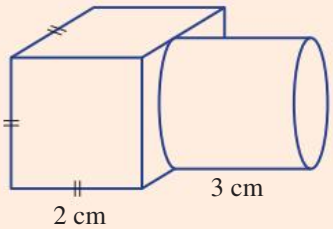


Hint: $c^2 = a^2 + b^2$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 24 Finding the surface area and volume of a composite solid

Find the surface area and volume of this composite solid, correct to two decimal places.



SOLUTION

Surface area $= 6 \times 2^2 + 2\pi(1)(3)$
$= 24 + 6\pi$
$= 42.85\ \text{cm}^2$ (to 2 d.p.)

EXPLANATION

The surface area is made up of 5 square faces plus one more, which is made up of the remaining part of the right side face and the end of the cylinder.

The curved surface of the cylinder ($2\pi rh$) is also included.

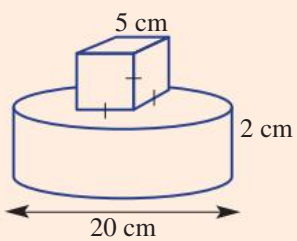
The radius is half the diameter; i.e. $r = 1$.

Volume $= 2^3 + \pi(1)^2(3)$
$= 8 + 3\pi$
$= 17.42\ \text{cm}^3$ (to 2 d.p.)

The volume consists of the sum of a cube (s^3) and a cylinder $(\pi r^2 h)$.

Now you try

Find the surface area and volume of this solid, correct to two decimal places.

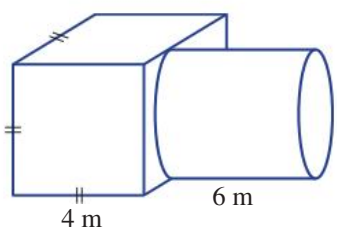


4 For these composite solids, find:

i the surface area **ii** the volume

Give your answer correct to two decimal places.

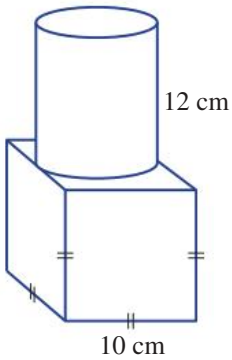
a



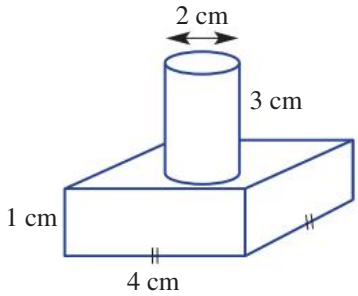
Hint: Include only exposed surfaces in the surface area.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b

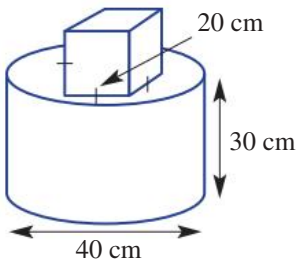


c

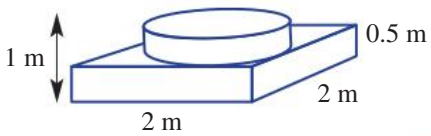


5 Find the capacity of these composite solids, in litres. Round your answer to two decimal places where necessary.

a



b



Hint: Recall:
$1\text{ L} = 1000\text{ cm}^3$
$1\text{ m}^3 = 1000\text{ L}$

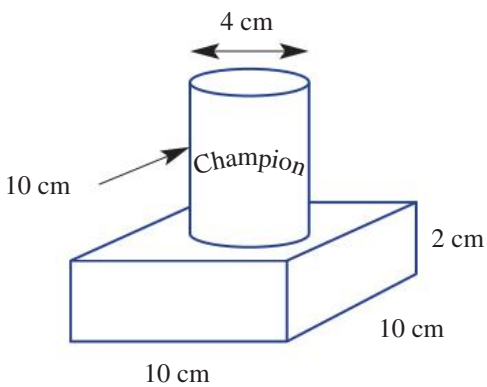
PROBLEM-SOLVING AND REASONING

6–8 | 6, 9–11

6 Here is the design of a glass tennis trophy. The base and the cylindrical part are both made of glass.

a Find the surface area of the trophy, to the nearest square centimetre.

b Find the volume of glass, to the nearest cubic centimetre, required to make the trophy.



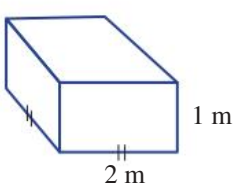
7 When solids are painted, the outer surface area needs to be considered to help find the amount of paint required for the job. Assume that 1 L of paint covers 10 m^2.

Complete for each of these objects.

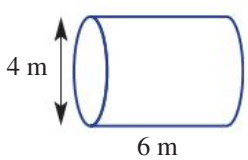
i Find the surface area in square metres, rounding your answer to two decimal places where necessary.

ii Find the amount of paint that must be purchased, assuming that you can buy only a whole number of litres.

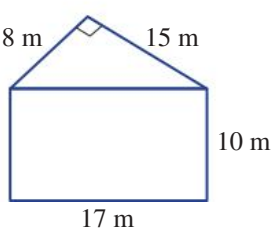
a



b



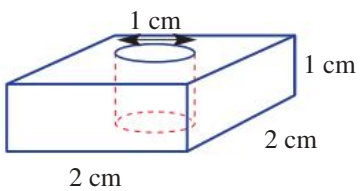
c



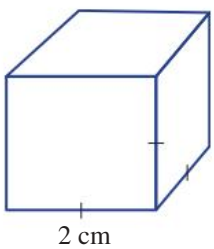
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

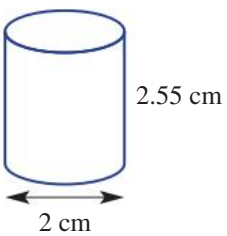
8 When solids are hollow, the inside surface areas are exposed to the air. Find the surface area (i.e. inner and outer combined) of a pipe of diameter 0.3 m and length 3 m. Assume that the inner and outer diameters are the same. (Round your answer to one decimal place.)

9 This nut is a square-based prism with a cylindrical hole removed from the centre. The hole has a diameter of 1 cm. The nut is coated with anti-rust paint. What area is painted, including the inner cylindrical surface? (Round your answer to one decimal place.)



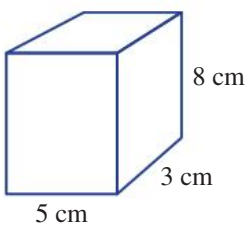
10 These solids have approximately the same volume.

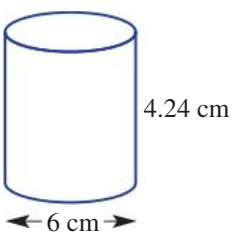




Which has the larger surface area? Do some calculations to find out.

11 A company wishes to design a container for packaging and selling lollies. The two designs are shown here.





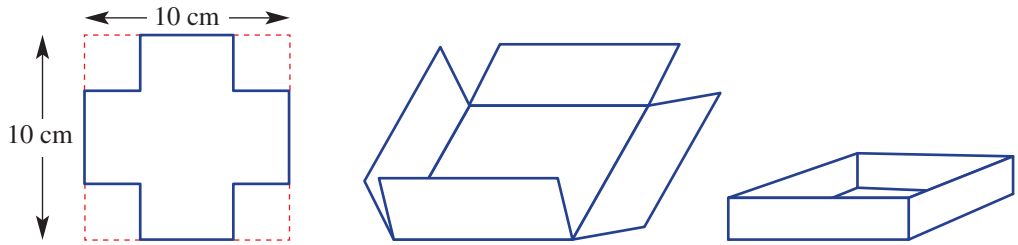
a Make some calculations to show that the two containers have approximately the same volume.

b Which design has the least surface area? Justify your answer.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

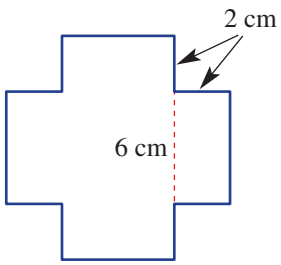
ENRICHMENT: Maximising volume — 12

12 Imagine that a company asks you to make a tray out of a square piece of card, measuring 10 cm by 10 cm, by cutting out four corner squares and folding them to form a tray, as shown.



a What will be the volume of the tray if the side length of the square cut-outs is:

i 1 cm? **ii** 2 cm? **iii** 3 cm?



b Which square cut-out from part **a** gives the largest tray volume?

c Can you find another sized cut-out that gives a larger volume than any of those in part **a**?

d What sized cut-out gives the maximum volume?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2K Surface area of pyramids and cones

Learning intentions for this section:

- To be able to calculate the surface area of pyramids and cones
- To be able to solve problems involving composite solids

Past, present and future learning:

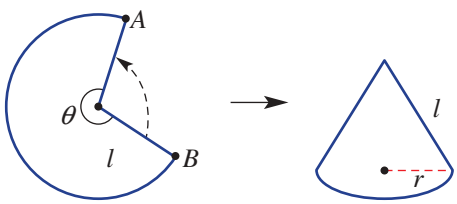
- This section addresses the Stage 5 Path topic called *Area and surface area B*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formulas from memory in the HSC Examination

Pyramids and cones are solids for which we can also calculate the surface area by finding the sum of the areas of all the outside surfaces.

Finding the surface area of a pyramid involves finding the sum of the areas of its base and its triangular faces. The rule for the surface area of a cone can be developed after drawing a net that includes a circle (base) and sector (curved surface), as you will see in the following examples.

Lesson starter: The cone formula

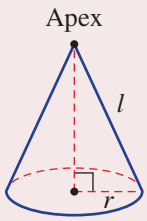
Use a pair of compasses to construct a large sector. Use any sector angle θ that you like. Cut out the sector and join the points A and B to form a cone of radius r.



- Give the rule for the area of the base of the cone.
- Give the rule for the circumference of the base of the cone.
- Give the rule for the circumference of a circle with radius l.
- Use the above to find an expression for the area of the base of the cone as a fraction of the area πl^2.
- Hence, explain why the rule for the surface area of a cone is given by $A = \pi r^2 + \pi rl$.

KEY IDEAS

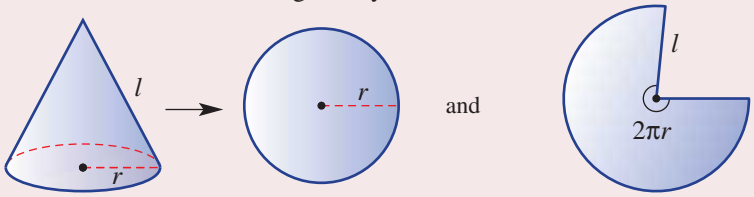
- A **cone** is a solid with a circular base and a curved surface that reaches from the base to a point called the **apex**.
  - A right cone has its apex directly above the centre of the base.
  - The pronumeral l is used for the slant height and r is the radius of the base.



Right cone

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- The surface area of a cone is given by:



Area of base:

$A = \pi r^2$

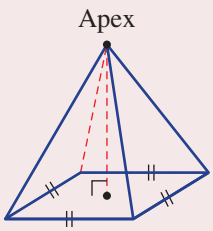
Curved surface area:

$A = \frac{2\pi r}{2\pi l} \times \pi l^2$

$\therefore A = \pi rl$

$\therefore A = \pi r^2 + \pi rl = \pi r(r + l)$

- A **pyramid** has a base that is a polygon and its remaining faces are triangles that meet at the apex.
  - A pyramid is named by the shape of its base.
  - A right pyramid has its apex directly above the centre of the base.



Right square pyramid

Key terminology: apex, curved surface

Exercise 2K

UNDERSTANDING 1–3 1–3

1 Write the formula for the following.

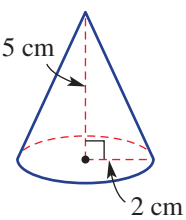
a the area of a triangle

b the surface area of the base of a cone with radius r

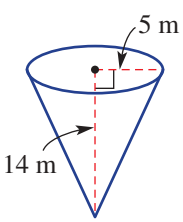
c the surface area of the curved part of a cone with slant height l and radius r

2 Find the exact slant height for these cones, using Pythagoras' theorem. Answer exactly, using a square root sign.

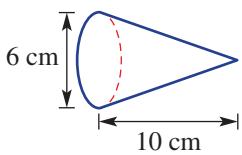
a



b

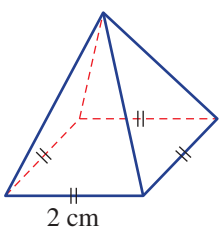


c

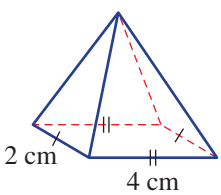


3 Draw a net for each of these closed solids.

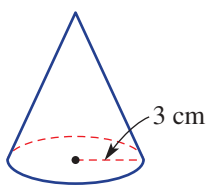
a



b



c



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

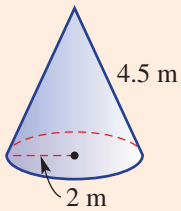
FLUENCY 4–6 4–7

Example 25 Finding the surface area of a cone and a pyramid

Find the surface area of these solids, using two decimal places for part **a**.

a cone with radius 2 m and slant height 4.5 m

b square pyramid with base length 25 mm and triangular face height 22 mm



SOLUTION

a $A = \pi r^2 + \pi rl$

$= \pi (2)^2 + \pi (2) \times (4.5)$

$= 40.84 \text{ m}^2$ (to 2 decimal places)

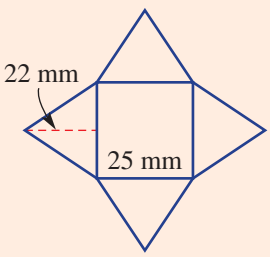
b $A = s^2 + 4 \times \frac{1}{2} bh$

$= 25^2 + 4 \times \frac{1}{2} \times 25 \times 22$

$= 1725 \text{ mm}^2$

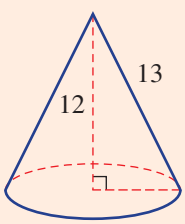
EXPLANATION

The cone includes the circular base plus the curved part. Substitute $r = 2$ and $l = 4.5$.

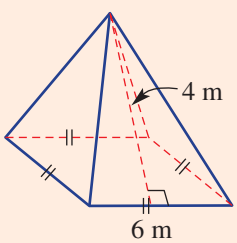


Now you try

a Find the exact external curved surface area of this open cone with slant height 13 cm and perpendicular height 12 cm.



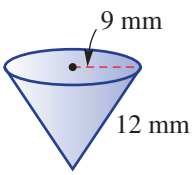
b Find the surface area of the pyramid.



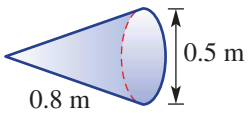
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Find the surface area of these cones, correct to two decimal places.

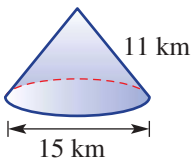
a



b

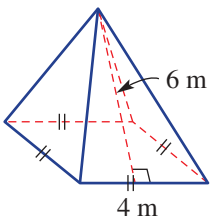


c

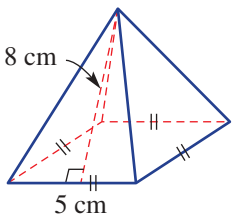


5 Find the surface area of these pyramids.

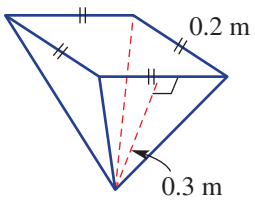
a



b

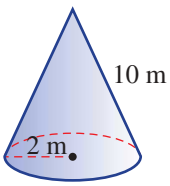


c

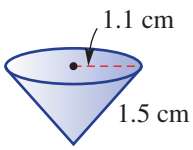


6 For each cone, find the area of the *curved surface* only, correct to two decimal places.

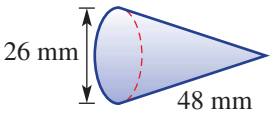
a



b



c



7 A cone has height 10 cm and radius 3 cm.

a Use Pythagoras' theorem to find the slant height of the cone, rounding your answer to two decimal places.

b Find the surface area of the cone, correct to one decimal place.

PROBLEM-SOLVING AND REASONING 8,10,12 9–12

Example 26 Finding the slant height and vertical height of a cone

A cone with radius 3 cm has a curved surface area of 100 cm^2.

a Find the slant height of the cone, correct to one decimal place.

b Find the height of the cone, correct to one decimal place.

SOLUTION

a $A = \pi rl$

$100 = \pi \times 3 \times l$

$l = \dfrac{100}{3\pi}$

$= 10.6$ cm (to 1 decimal place)

EXPLANATION

Substitute the given information into the formula for the curved surface area of a cone and solve for l.

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

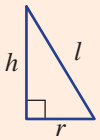
b $h^2 + r^2 = l^2$

$h^2 + 3^2 = \left(\frac{100}{3\pi}\right)^2$

$h^2 = \left(\frac{100}{3\pi}\right)^2 - 9$

$h = \sqrt{\left(\frac{100}{3\pi}\right)^2 - 9}$

$= 10.2$ cm (to 1 decimal place)



Identify the right-angled triangle within the cone and use Pythagoras' theorem to find the height, h. Use the exact value of l from part **a** to avoid accumulating errors.

Now you try

A cone with radius 4 cm has a curved surface area of 200 cm^2.

a Find the slant height of the cone, correct to one decimal place.

b Find the height of the cone, correct to one decimal place.

8 A cone with radius 5 cm has a curved surface area of 400 cm^2.

a Find the slant height of the cone, correct to one decimal place.

b Find the height of the cone, correct to one decimal place.

9 A cone with radius 6.4 cm has a curved surface area of 380 cm^2.

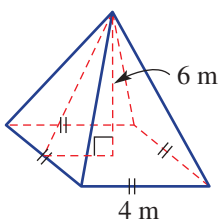
a Find the slant height of the cone, correct to one decimal place.

b Find the height of the cone, correct to one decimal place.

10 This right square pyramid has base side length 4 m and vertical height 6 m.

a Find the height of the triangular faces, correct to one decimal place.

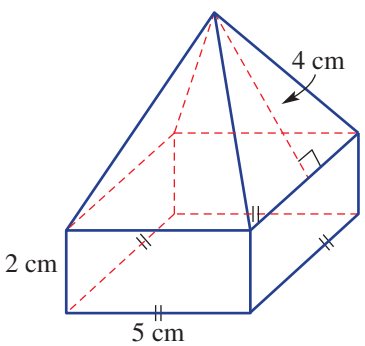
b Find the surface area, correct to one decimal place.



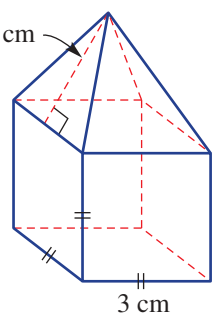
11 Party hats A and B are in the shape of open cones with no base. Hat A has radius 7 cm and slant height 25 cm, and hat B has radius 9 cm and slant height 22 cm. Which hat has the greater surface area?

12 Find the surface area of these composite solids, correct to one decimal place as necessary.

a

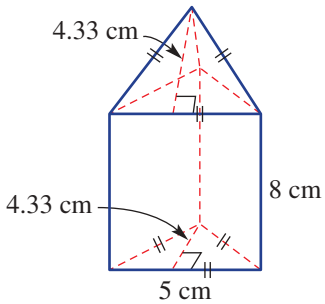


b

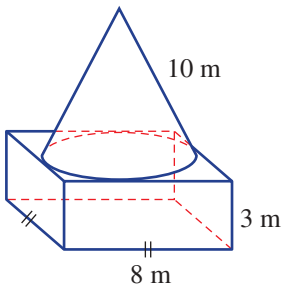


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

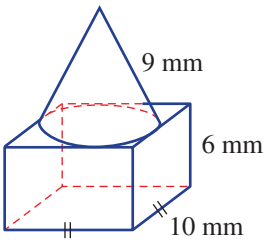
c



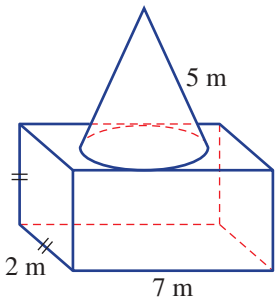
d



e

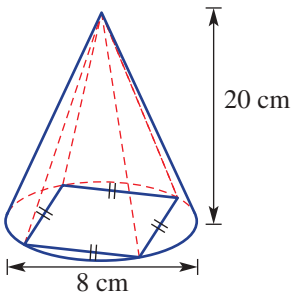


f



ENRICHMENT: Carving pyramids from cones — 13

13 A woodworker uses a rotating lathe to produce a cone with radius 4 cm and height 20 cm. From that cone the woodworker then cut slices off the sides of the cone to produce a square pyramid of the same height.



- **a** Find the exact slant height of the cone.
- **b** Find the surface area of the cone, correct to two decimal places.
- **c** Find the exact side length of the base of the square pyramid.
- **d** Find the height of the triangular faces of the pyramid, correct to 3 decimal places.
- **e** Find the surface area of the pyramid, correct to two decimal places.
- **f** Express the surface area of the pyramid as a percentage of the surface area of the cone. Give the answer correct to the nearest whole percentage.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2L Volume of pyramids and cones

Learning intentions for this section:

- To be able to calculate the volume of pyramids and cones
- To be able to solve problems involving composite solids

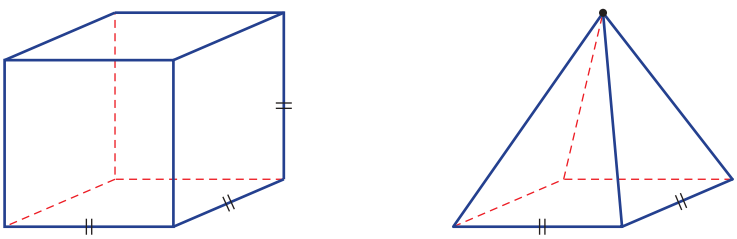
Past, present and future learning:

- This section addresses the Stage 5 Path topic called *Volume B*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formulas from memory in the HSC Examination

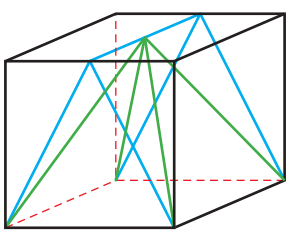
The volume of a cone or pyramid is a certain fraction of the volume of a prism with the same base area. This particular fraction is the same for both cones and pyramids and will be explored in this section.

Lesson starter: Is a pyramid half the volume of a prism?

Here is a cube and a square pyramid with equal base side lengths and equal heights.



- Discuss whether or not you think the pyramid is half the volume of the cube.
- Now consider this diagram of the cube with the pyramid inside.



The cube is black.
The pyramid is green.
The triangular prism is blue.

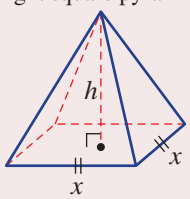
- Compared to the cube, what is the volume of the triangular prism (blue)? Give reasons.
- Is the volume of the pyramid (green) more or less than the volume of the triangular prism (blue)?
- Do you know what the volume of the pyramid is as a fraction of the volume of the cube?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- For pyramids and cones, the volume is given by $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height.

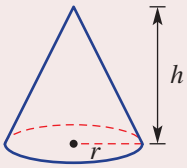
Right square pyramid



$V = \frac{1}{3}Ah$

$= \frac{1}{3}x^2h$

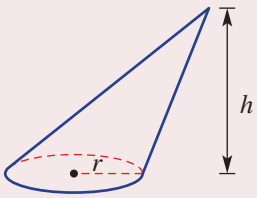
Right cone



$V = \frac{1}{3}Ah$

$= \frac{1}{3}\pi r^2h$

Oblique cone



$V = \frac{1}{3}Ah$

$= \frac{1}{3}\pi r^2h$

Key terminology: right cone, oblique cone

Exercise 2L

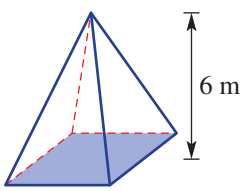
UNDERSTANDING 1–3 1–3

1 A cylinder has volume 12 cm^3. What will be the volume of a cone with the same base area and perpendicular height?

2 A pyramid has volume 5 m^3. What will be the volume of a prism with the same base area and perpendicular height?

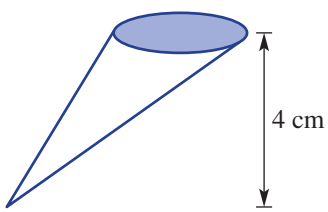
3 Find the volume of these solids with the given base areas.

a



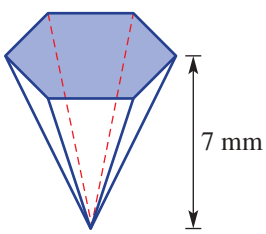
$A = 5$ m^2

b



$A = 2$ cm^2

c



$A = 25$ mm^2

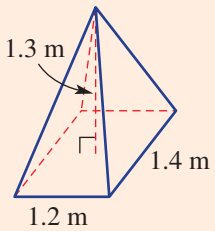
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 4abc 4

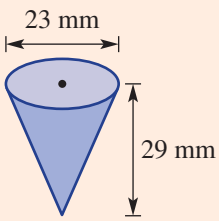
Example 27 Finding the volume of pyramids and cones

Find the volume of this pyramid and cone. Give the answer for part **b** correct to two decimal places.

a



b



SOLUTION

a $V = \frac{1}{3}Ah$

$= \frac{1}{3}(l \times b) \times h$

$= \frac{1}{3}(1.4 \times 1.2) \times 1.3$

$= 0.728 \text{ m}^3$

EXPLANATION

The pyramid has a rectangular base with area $l \times b$.

Substitute $l = 1.4$, $b = 1.2$ and $h = 1.3$.

Evaluate the answer.

b $V = \frac{1}{3}Ah$

$= \frac{1}{3}\pi r^2$

$= \frac{1}{3}\pi(11.5)^2 \times 29$

$= 4016.26 \text{ mm}^3$ (to two decimal places)

The cone has a circular base of area πr^2.

Substitute $r = \frac{23}{2} = 11.5$ and $h = 29$.

Evaluate and round the answer.

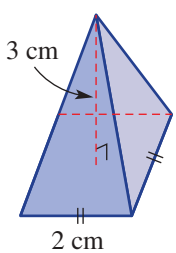
Now you try

a Find the volume of a square pyramid with base edges 3 cm and height 3 cm.

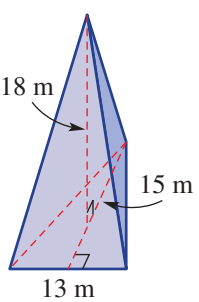
b Find the exact volume of a cone with base radius 3 cm and perpendicular height 3 cm.

4 Find the volume of the following pyramids. For the oblique pyramids (parts **d**, **e**, **f**), use the given perpendicular height.

a

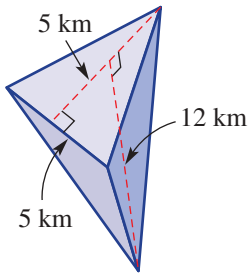


b

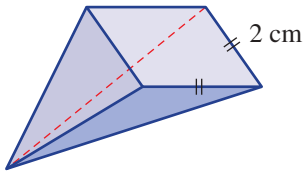


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c

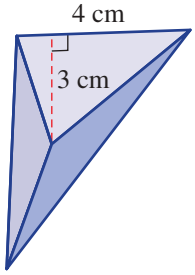


d



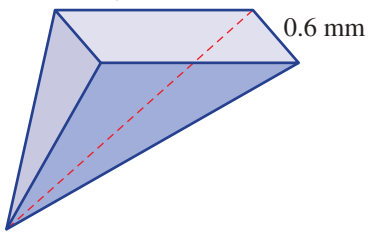
Perpendicular height = 2 cm

e



Perpendicular height = 4 cm

f



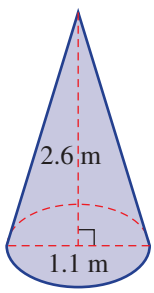
Perpendicular height = 1.2 mm

PROBLEM-SOLVING AND REASONING

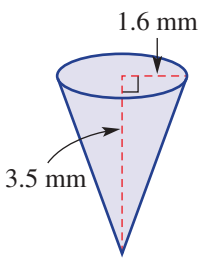
5abc, 6, 7abc, 9 | 7–10

5 Find the volume of the following cones, correct to two decimal places.

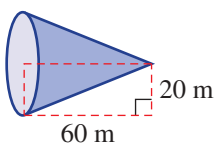
a



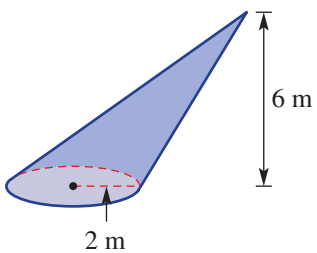
b



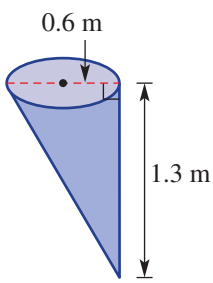
c



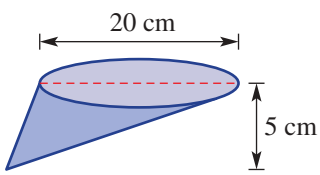
d



e



f

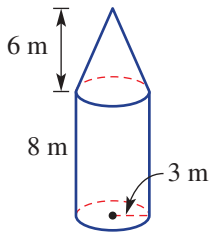


6 A medicine measuring cup is in the shape of a cone with base radius 3 cm and height 5 cm. Find its capacity in millilitres, correct to the nearest millilitre.

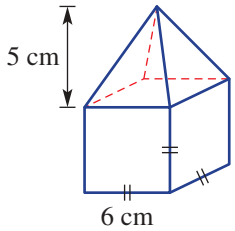
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 Find the volume of these composite objects, rounding to two decimal places where necessary.

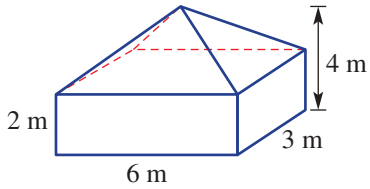
a



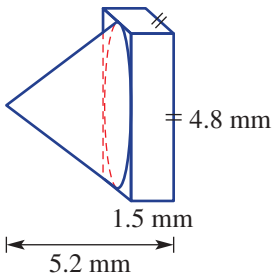
b



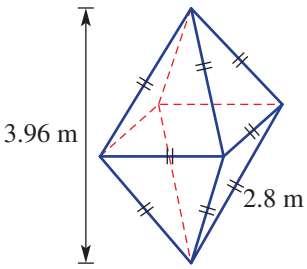
c



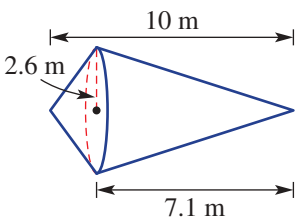
d



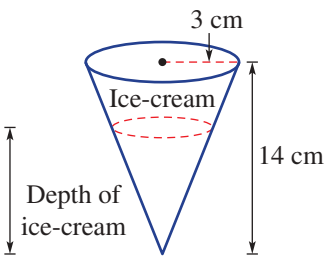
e



f



8 The volume of ice-cream in the cone is half the volume of the cone. The cone has a 3 cm radius and height of 14 cm, as shown at right. What is the depth of the ice-cream, correct to two decimal places?



9 A wooden cylinder is carved to form a cone that has the same base area and same height as the original cylinder. What fraction of the wooden cylinder is wasted? Give a reason for your answer.

10 A square pyramid and a cone are such that the diameter of the cone is equal to the length of the side of the square base of the pyramid. They also have the same height.

a Using x as the side length of the pyramid and h as its height, write a rule for:

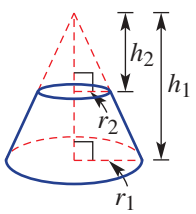
i the volume of the pyramid in terms of x and h

ii the volume of the cone in terms of x and h

b Express the volume of the cone as a fraction of the volume of the pyramid. Give an exact answer.

ENRICHMENT: Truncated cones — 11

11 A truncated cone is a cone that has its apex cut off by an intersecting plane. In this example, the top has radius r_2, the base has radius r_1 and the two circular ends are parallel.



a Give reasons why $\frac{r_1}{r_2} = \frac{h_1}{h_2}$.

b Find a rule for the volume of a truncated cone.

c Find the volume, to one decimal place, of a truncated cone if $r_1 = 2$ cm, $h_1 = 5$ cm and h_2 equals:

i $\frac{1}{2}h_1$ **ii** $\frac{2}{3}h_1$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2M Volume and surface area of spheres

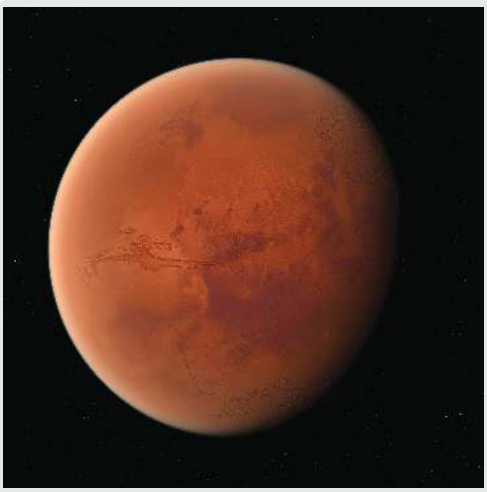
Learning intentions for this section:

- To be able to calculate the volume and surface area of spheres and hemispheres
- To be able to solve problems involving composite solids

Past, present and future learning:

- This section addresses the Stage 5 Path topics called *Area and surface area B* and *Volume B*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formulas from memory in the HSC Examination

Due to the effects of gravity, planets are approximately spherical. This means that we only need one measurement to describe a planet's size: its diameter (or radius). Mars, for example, has a diameter of about half that of Earth, which is about 12 756 km. Earth's volume is about nine times that of Mars and this is because the volume of a sphere varies with the cube of the radius. The surface area of Earth is about 3.5 times that of Mars because the surface area of a sphere varies with the square of the radius.



Mars is approximately spherical.

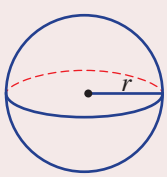
Lesson starter: What percentage of a cube is a sphere?

A sphere of radius 1 unit just fits inside a cube.

- First, guess the percentage of space occupied by the sphere.
- Draw a diagram showing the sphere inside the cube.
- Calculate the volume of the cube and the sphere. For the sphere, use the formula $V = \frac{4}{3}\pi r^3$.
- Now calculate the percentage of space occupied by the sphere. How close was your guess?

KEY IDEAS

- The surface area of a **sphere** depends on its radius, r, and is given by $A = 4\pi r^2$.
- The volume of a sphere depends on its radius, r, and is given by $V = \frac{4}{3}\pi r^3$.



Key terminology: sphere, hemisphere

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 2M

UNDERSTANDING 1ad, 2ad, 3 | 1bcef, 2bcef, 3

1 Evaluate and round your answer to two decimal places.

a $4 \times \pi \times 5^2$ **b** $4 \times \pi \times 2.2^2$ **c** $4 \times \pi \times \left(\frac{1}{2}\right)^2$

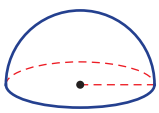
d $\frac{4}{3} \times \pi \times 2^3$ **e** $\frac{4}{3} \times \pi \times 2.8^3$ **f** $\frac{4\pi\,(7)^3}{3}$

2 For each part of Question **1**, write a question to match these calculations.

3 **a** What fraction of a sphere is shown in this diagram?

b What is the formula for the volume of a hemisphere?

c What is the formula for the curved surface area of a hemisphere?



FLUENCY 4abf, 5abdf | 4cef, 5cdf, 6, 7

4 Find the surface area and volume of a sphere with the given dimensions. Give the answer correct to two decimal places.

a radius 3 cm **b** radius 4 m **c** radius 7.4 m

d diameter $\sqrt{5}$ mm **e** diameter $\sqrt{7}$ m **f** diameter 2.2 km

Example 28 Finding the surface area and volume of a sphere

Find the surface area and volume of a sphere of radius 7 cm, correct to two decimal places.

SOLUTION

$A = 4\pi r^2$

$= 4\pi\,(7)^2$

$= 615.75$ cm^2 (to 2 decimal places)

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi\,(7)^3$

$= 1436.76$ cm^3 (to 2 decimal places)

EXPLANATION

Write the formula for the surface area of a sphere and substitute $r = 7$.

Evaluate and round the answer.

Write the formula for the volume of a sphere and substitute $r = 7$.

Evaluate and round the answer.

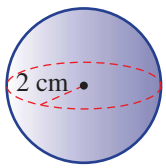
Now you try

Find the exact surface area and volume of a sphere with diameter 6 cm.

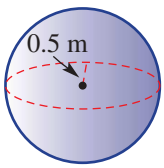
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Find the surface area and volume of the following spheres, correct to two decimal places.

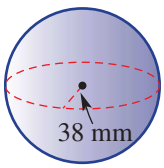
a



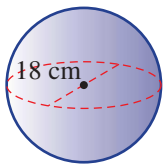
b



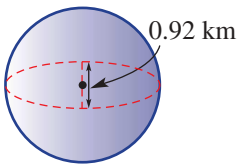
c



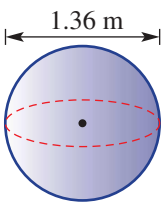
d



e



f

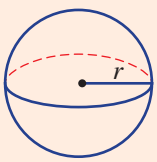


6 **a** Rearrange $A = 4\pi r^2$ to write r in terms of A.

b Rearrange $V = \frac{4}{3}\pi r^3$ to write r in terms of V.

Example 29 Finding the radius of a sphere

Find the radius of a sphere with volume 10 m^3, correct to two decimal places.



$V = 10$ m^3

SOLUTION

$V = \frac{4}{3}\pi r^3$

$10 = \frac{4}{3}\pi r^3$

$30 = 4\pi r^3$

$\frac{15}{2\pi} = r^3$

$\therefore r = \sqrt[3]{\frac{15}{2\pi}}$

$= 1.336\ldots$

$\therefore$ The radius is 1.34 m (to 2 decimal places).

EXPLANATION

Substitute $V = 10$ into the formula for the volume of a sphere.

Solve for r^3 by multiplying both sides by 3 and then dividing both sides by 4π. Simplify $\frac{30}{4\pi} = \frac{15}{2\pi}$.

Take the cube root of both sides to make r the subject, then evaluate.

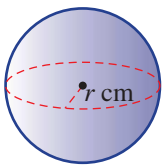
Now you try

The volume of a sphere is exactly 288π cm^3. What is the radius?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

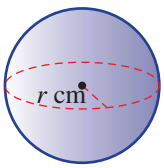
7 **a** Find the radius of these spheres with the given volumes, correct to two decimal places.

i



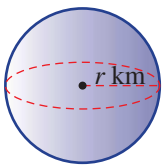
$V = 15$ cm^3

ii



$V = 180$ cm^3

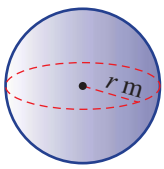
iii



$V = 0.52$ km^3

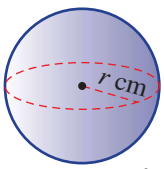
b Find the radius of these spheres with the given surface area, correct to two decimal places.

i



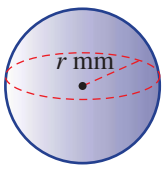
$A = 10$ m^2

ii



$A = 120$ cm^2

iii



$A = 0.43$ mm^2

PROBLEM-SOLVING AND REASONING

8,10,11,13,14ace,15acdf 8–13,14bdf,15–17

8 A box with dimensions 30 cm long, 30 cm wide and 30 cm high holds 50 tennis balls of radius 3 cm. Find:

- **a** the volume of one tennis ball, correct to two decimal places
- **b** the volume of 50 tennis balls, correct to one decimal place
- **c** the volume of the box not taken up by the tennis balls, correct to one decimal place

9 An expanding spherical storage bag has 800 cm^3 of water pumped into it. Find the diameter of the bag, correct to one decimal place, after all the water has been pumped in.

10 A sphere just fits inside a cube. What is the surface area of the sphere as a percentage of the surface area of the cube? Round your answer to the nearest whole percentage.

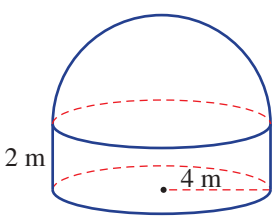
11 Consider a closed hemisphere with radius 2.5 cm. Calculate, correct to one decimal place:

a the volume **b** the curved surface area **c** the total surface area

12 Two sports balls have radii 10 cm and 15 cm. Find the difference in their surface areas, correct to one decimal place.

13 A monolithic structure has a cylindrical base of radius 4 m and height 2 m and a hemispherical top.

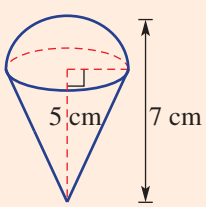
- **a** What is the radius of the hemispherical top?
- **b** Find the total volume of the entire monolithic structure, correct to one decimal place.



Example 30 Finding the surface area and volume of composite solids with sphere portions

This composite object includes a hemisphere and cone, as shown.

- **a** Find the surface area, rounding to two decimal places.
- **b** Find the volume, rounding to two decimal places.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

a Radius $r = 7 - 5 = 2$

$$l^2 = 5^2 + 2^2$$
$$l^2 = 29$$
$$l = \sqrt{29}$$

$$A = \frac{1}{2} \times 4\pi r^2 + \pi rl$$

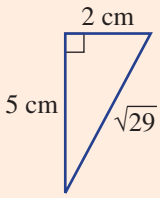
$$= \frac{1}{2} \times 4\pi (2)^2 + \pi (2) \left(\sqrt{29} \right)$$
$$= 8\pi + 2\sqrt{29}\pi$$
$$= 58.97 \text{ cm}^2 \text{(to 2 decimal places)}$$

b $V = \frac{1}{2} \times \frac{4}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$$= \frac{1}{2} \times \frac{4}{3}\pi (2)^3 + \frac{1}{3}\pi (2)^2 (5)$$
$$= \frac{16\pi}{3} + \frac{20\pi}{3}$$
$$= \frac{36\pi}{3}$$
$$= 12\pi$$
$$= 37.70 \text{ cm}^3 \text{ (to 2 decimal places)}$$

EXPLANATION

First, find the radius, r cm, of the hemisphere. Calculate the slant height, l, of the cone using Pythagoras' theorem.



Write the rules for the surface area of each component and note that the top shape is a hemisphere (i.e. half sphere). Only the curved surface of the cone is required.

Substitute $r = 2$ and $h = 5$.

Simplify and then evaluate, rounding as required.

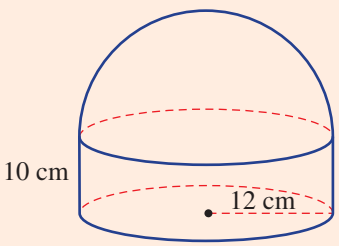
Volume (object) $= \frac{1}{2}$ volume (sphere) + volume (cone)

Substitute $r = 2$ and $h = 5$.

Simplify and then evaluate, rounding as required.

Now you try

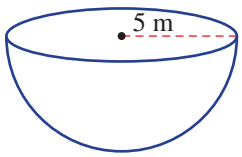
Find the exact surface area and volume of this solid.



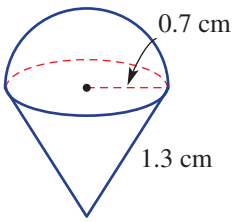
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

14 Find the surface area for these solids, correct to two decimal places.

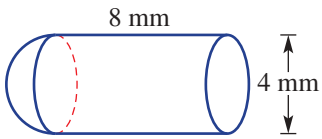
a



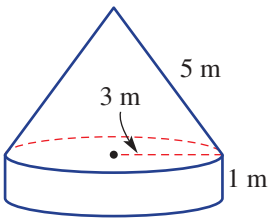
b



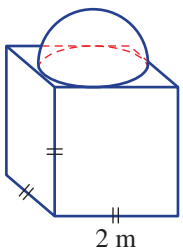
c



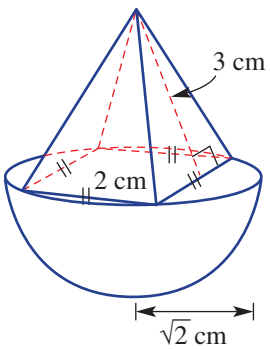
d



e

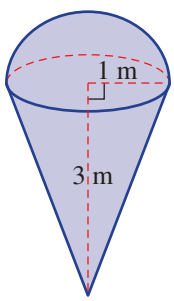


f

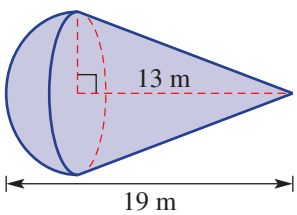


15 Find the volume of the following composite objects, correct to two decimal places.

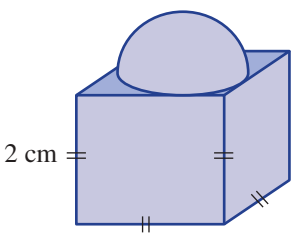
a



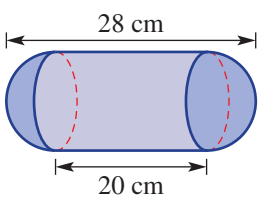
b



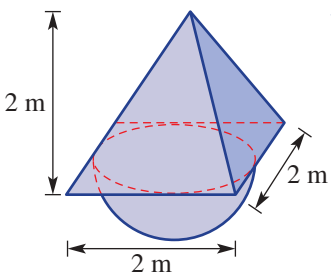
c



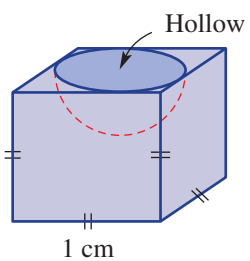
d



e



f



16 A spherical party balloon is blown up to help decorate a room.

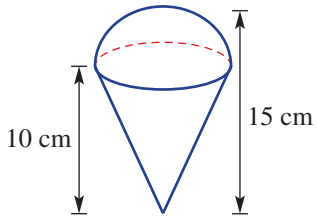
a Find the volume of air, correct to two decimal places, needed for the balloon to be:

i 10 cm wide **ii** 20 cm wide **iii** 30 cm wide

b If the balloon pops when the volume of air reaches 120 000 cm^3, find the diameter of the balloon at that point, correct to one decimal place.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

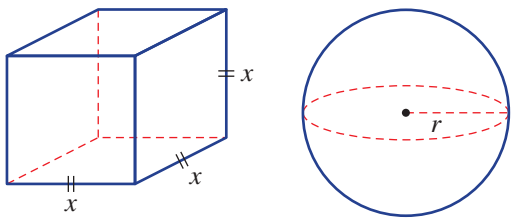
17 A hemisphere sits on a cone and two height measurements are given as shown. Find:



- **a** the radius of the hemisphere
- **b** the exact slant height of the cone, in surd from
- **c** the surface area of the solid, correct to one decimal place

ENRICHMENT: Comparing surface areas — 18

18 Imagine a cube and a sphere that have the same volume.



- **a** If the sphere has volume 1 unit^3, find:
  - **i** the exact radius of the sphere
  - **ii** the exact surface area of the sphere
  - **iii** the value of x (the side length of the cube)
  - **iv** the surface area of the cube
  - **v** the surface area of the sphere as a percentage of the surface area of the cube, correct to one decimal place.
- **b** Now take the radius of the sphere to be r units. Write:
  - **i** the formula for the surface area of the sphere
  - **ii** the formula for x in terms of r, given the volumes are equal
  - **iii** the surface area of the cube in terms of r
- **c** Now write the surface area of the sphere as a fraction of the surface area of the cube, using your results from part **b**.

  Simplify to show that the result is $\sqrt[3]{\frac{\pi}{6}}$.
- **d** Compare your answer from part **a v** with that of part **c**.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Maths@Work

Bricklayer

A bricklayer has a physically challenging job that requires stamina and strength and also good communication skills, as they often work as part of a team.

Bricklayers must have a solid understanding of how the construction process works and the ability to read plans and blueprints.

Mathematical skills are essential in this trade. Bricklayers must understand ratios for mixing mortar and cement. Good measurement skills are also important, as bricklayers must be able to work out the number of bricks required for a job, convert between different units and take accurate measurements at the work site, using the most appropriate tool. An understanding of geometry and trigonometry is also required.

Complete these questions that a bricklayer may face in their day-to-day job.

1 A standard house brick has dimensions 230 mm × 76 mm × 110 mm and the standard thickness of mortar when laying bricks is 10 mm. The bricks are laid so that the outer face is 230 mm × 110 mm.

- **a** What is the length and height of each brick in centimetres and metres?
- **b** Determine the area in square centimetres and square metres of the outer face of one brick.
- **c** Determine the volume in cubic centimetres of each brick.
- **d** Calculate the length, in metres (to two decimal places), when laying the following number of bricks in a line with mortar between each join.
  - **i** 10 bricks
  - **ii** 100 bricks

Hint: For 10 bricks, there would be 9 mortar joins.

- **e** Calculate the height, in millimetres, of a wall of 25 rows of standard house bricks. Remember to consider the thickness of the mortar.
- **f** Estimate how many standard house bricks are needed to build a wall 4 m by 1.5 m, by dividing the area of the wall by the area of a brick's face (ignore the mortar).

2 Ready-mix mortar comes in 20 kg bags that cost $7.95 per bag. One bag of mortar is used to lay 20 standard house bricks.
Using standard house bricks (see dimension details in Question **1**), a brick wall is to be built that has a finished length of 8630 mm, a height of 2750 mm and is one brick deep.

- **a** Calculate the exact number of standard house bricks needed to build this wall. Remember to consider the thickness of the mortar.
- **b** How many ready-mix mortar bags must be purchased for this wall?
- **c** If each house brick costs 60 cents, find the total cost, to the nearest dollar, of the materials needed for this wall.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 A type of large brick is chosen for an outside retaining wall. These bricks are sold only in whole packs and each pack covers 12.5 square metres when laid. How many whole packs of these bricks must be bought to build a wall with dimensions:

a 6 m by 1.5 m?

b 9 m by 2 m?

Hint: You can't buy half a pack, so round up to the next whole number.

4 Fastwall house bricks are larger and lighter than standard bricks and can be used for single-storey constructions. They are sold in pallets of 1000 for $1258.21, including delivery and GST. Each fastwall brick has dimensions 305 mm × 90 mm × 162 mm and the standard thickness of mortar is 10 mm.

a If a pallet has 125 bricks per layer, how many layers does each pallet have?

b If the wooden base of the pallet has a height of 30 cm, what is the total height of the pallet, in centimetres, when loaded with bricks?

c Find the exact number of bricks needed to build a wall that is 20.15 m long and 6.87 m high. Remember to consider the thickness of the mortar.

d Determine the cost of the bricks, to the nearest dollar, required to be bought for building the wall in part **c**.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Using technology

Hint: For E5 the formula would be $= G5 \times A5 + (G5 - 1) \times 10$
For G6 the formula would be $= (E6 + 10)/(230 + 10)$

5 Copy the following table into a spreadsheet. Then enter formulas into the shaded cells and, hence, determine the missing values.

	A	B	C	D	E	F	G	H	I
1	**Brick Wall Calculations**								
2	Note: All dimensions are in mm								
3	Single brick dimensions			Mortar	Brick wall dimensions		Number of bricks used		
4	Length	Width	Height	Width	Length	Height	Per layer (row)	Number of layers	Total
5	230	76	110	10			24	18	
6	230	76	110	10	21350	6230			

6 Copy the following table into your spreadsheet underneath the table from Question **5**. Enter the formulas into the shaded cells and, hence, determine the missing values. Assume there are 1000 bricks per pallet and that one bag of mortar is used per 20 bricks laid.

Hint: Copy the total number of bricks used from the first table.

	A	B	C	D	E	F	G	H	I	J
	Cost calculations									
	Bricks		Cost of pallets				Cost of mortar			
	Number of bricks used	Decimal number of pallets	number of whole pallets	cost per pallet (including GST)	total cost of pallets	Decimal number of mortar bags	number of whole bags	cost per bag (including GST)	total cost of mortar	Total cost of mortar and pallets
12				$1,358.24				$7.95		
13				$1,541.56				$8.45		

In cell B13 enter = ROUNDUP (B12, 0), which will round the number from cell B12 up to the nearest whole number; e.g. 1.3 will be rounded up to 2.

7 Use your spreadsheet tables from Questions **5** and **6** to find the total cost of materials for the following brick walls made from fastwall bricks. (See Question **4** for fastwall brick dimensions.) The spreadsheet formulas will not need to be changed.

- **a** A wall of 44 bricks per layer (row) and 30 layers (rows) if pallets cost $1258.36, including GST, and mortar is $7.55 per bag.
- **b** A wall 20.15 m long by 7.79 m high if pallets cost $1364.32, including GST, and mortar is $9.25 per bag.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Building playground equipment

A group of high school students have raised some money for a volunteer community service project. The students decide to ask the council if they can improve the children's playground equipment in the local park.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

Calculate the following areas, rounding the answers to one decimal place.

a Calculate the area of a circle with diameter 3.5 m.

b Determine the area of the following triangles. In part **ii** you will need to apply Pythagoras' theorem to calculate the triangle's height.

i

ii

c Determine the surface area in square centimetres and square metres of a cylindrical balancing beam with radius 12 cm and length 3 m.

Non-routine problems

You are to prepare a proposal for two improvements to the park's playground equipment:

- constructing a set of low concrete cylinders that children can use as steps, and
- the repainting of an old playground roundabout.

The problem is to determine the volume of concrete and paint needed and to find out if the cost of the project is within a budget of $200.

a With the aid of diagrams, write down all the relevant mathematical formulas that are needed to calculate:

i the volume and surface area of a cylinder

ii the area of an isosceles triangle.

Explore and connect

Choose and apply techniques

Concrete cylinders

b Nine concrete cylinders of radius 20 cm are to be constructed having above-ground heights of: 15 cm, 30 cm, 45 cm, 60 cm, 75 cm, 60 cm, 45 cm, 30 cm, 15 cm. Each cylinder also extends 30 cm below ground for stability.

Copy the table below into an excel spreadsheet and enter formulas into the shaded cells.

Format cells to Number/one decimal place. Use pi() for entering π.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Extend the Excel table to include all nine cylinders and the surface area and volume using units of centimetres and metres.

	A	B	C	D	E	F	G
1	Radius in cm	Height above ground in cm	Circle area in cm^2	Curved area in cm^2	Total surface area above ground in cm^2	Total height in cm	Volume in cm^3
2	20	15				45	
3	20	30				60	
11	*There will be several more lines in your table, making nine cylinders*						
12				Totals using cm			
13				**Totals using m**			

The roundabout

The old roundabout has an octagon shaped top with 8 isosceles triangles; each triangle has a 1 m base and two equal sides of 130 cm. The roundabout's rectangular vertical sides are each 50 cm high and 1 m in length.

- **c** Determine the area of one isosceles triangle and one rectangle in square centimetres. You will need to apply Pythagoras' theorem to calculate the triangle's height.
- **d** Determine the roundabout's surface area, including the top and sides, in square metres to one decimal place.

Communicate thinking and reasoning

The council has offered to pour the concrete cylinders so the student volunteers will only be responsible
for the painting jobs, including the cylinders and the roundabout. Only the painting will be included in the budget.

- **e** If only one colour is used and three coats are applied, find the total area to be painted.
- **f** If non-slip paint covers 8 m^2/litre determine the volume of paint required.
- **g** If non-slip paint comes in 2 L cans at $75 each and 4 L cans at $110 each, calculate the minimum cost of paint, assuming three coats are required, and determine if the project is within budget.
- **h** Summarise your findings stating:
  - **i** the volume of concrete needed for the cylinders
  - **ii** the overall area to be painted
  - **iii** the details of what cans of paint to buy and the cost.

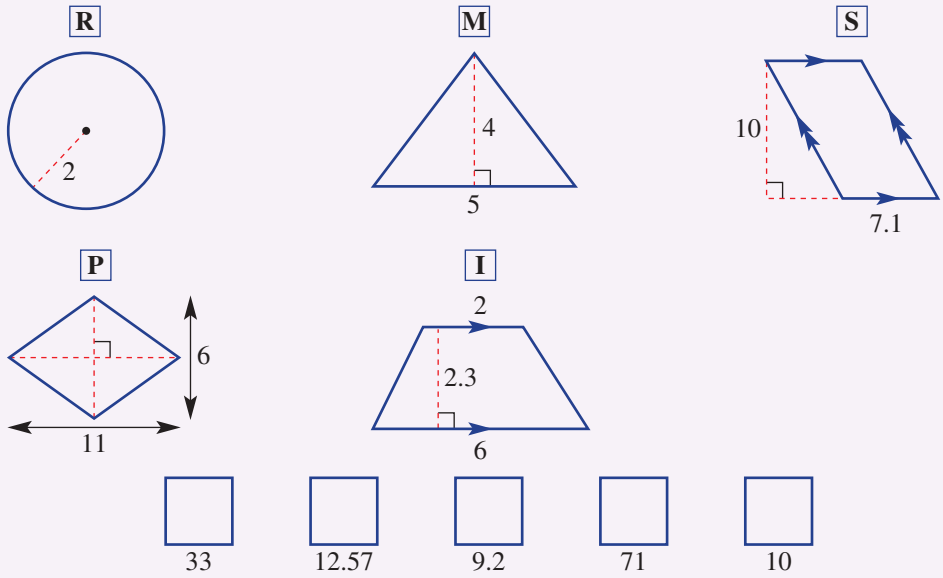
Extension problem

Problem-solve

- **a** In a different playground the steps are half cylinders with radius 30 cm and all are of height 50 cm. They stand up with one vertical flat side, a curved side and a semicircular top. The octagonal roundabout is made up of 8 isosceles triangles, each with base length 1.4 m and two equal sides of 1.83 m. The roundabout's vertical sides are still 50 cm high. Find the surface areas and volumes of the playground objects and calculate the cost of paint for this playground using the same costs as in part **g**.

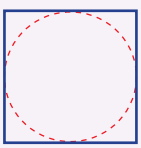
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1 'I am the same shape all the way through. What am I?' Find the area of each shape. Match the letters to the answers below to solve the riddle.

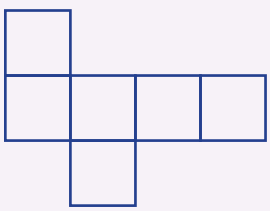


2 One litre of water is poured into a container in the shape of a rectangular prism. The dimensions of the prism are 8 cm by 12 cm by 11 cm. Will the water overflow?

3 A circular piece of pastry is removed from a square sheet with side length 30 cm. What percentage of pastry remains?

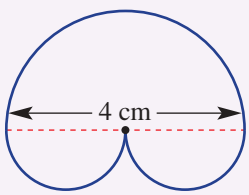


4 How many different nets are there for a cube? Do not count reflections or rotations of the same net. Here is one example.



5 Give the radius of a circle whose value for the circumference is equal to the value for the area.

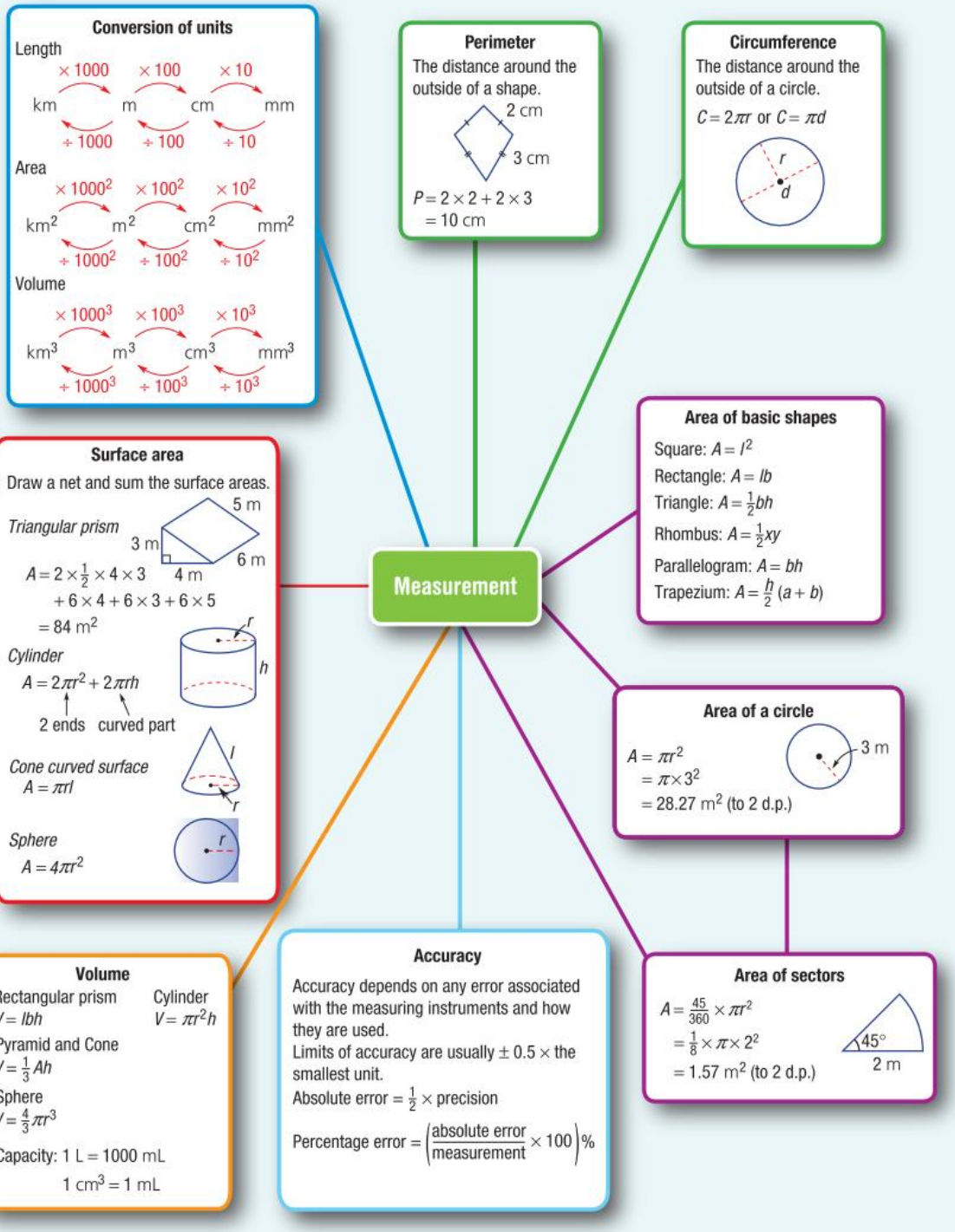
6 Find the area of this special shape.



7 A cube's surface area is 54 cm^2. What is its volume?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

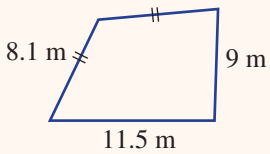
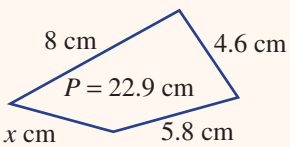
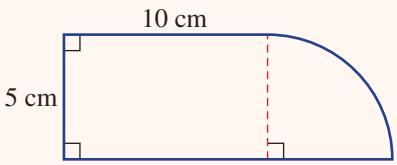
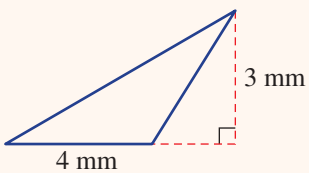
Chapter summary



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
2A	**1 I can convert between metric units of length.** e.g. Convert these measurements to the units shown in the brackets. **a** 2.3 m (cm) **b** 270 000 cm (km)	☐
2A	**2 I can convert between metric units of area.** e.g. Convert these measurements to the units shown in the brackets. **a** 32 000 m^2 (km^2) **b** 7.12 cm^2 (mm^2)	☐
2A	**3 I can convert between metric units of volume.** e.g. Convert these measurements to the units shown in the brackets. **a** 3.7 cm^3 (mm^3) **b** 5 900 000 cm^3 (m^3)	☐
2B	**4 I can state the smallest unit for a given measurement.** e.g. Write down the smallest unit of measurement for 27.3 cm.	☐
2B	**5 I can find the limits of accuracy for a given measurement.** e.g. Give the limits of accuracy for the measurement 65.3 m.	☐
2C	**6 I can find the perimeter of basic shapes.** e.g. Find the perimeter of this shape. 	☐
2C	**7 I can find a missing side length given the perimeter.** e.g. Find the value of x for this shape with the given perimeter. 	☐
2D	**8 I can find the circumference of a circle.** e.g. Find the circumference of a circle with a diameter of 5 m, correct to two decimal places.	☐
2D	**9 I can find the perimeter of composite shapes.** e.g. Find the perimeter of this composite shape, correct to two decimal places. 	☐
2E	**10 I can find the area of squares, rectangles and triangles.** e.g. Find the area of this triangle. 	☐

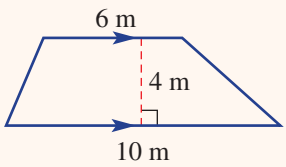
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

Chapter checklist with success criteria

2E

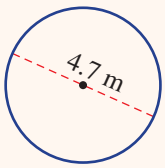
11 I can find the area of rhombuses, parallelograms and trapeziums.
e.g. Find the area of this trapezium.



☐

2F

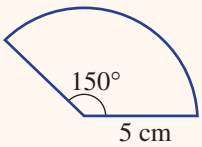
12 I can find the area of a circle.
e.g. Find the area of this circle, correct to two decimal places.



☐

2F

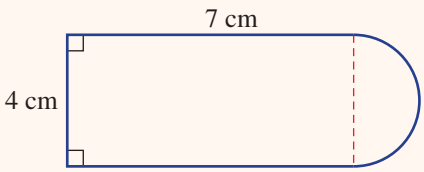
13 I can find the area of a sector.
e.g. Find the area of this sector, correct to two decimal places.



☐

2F

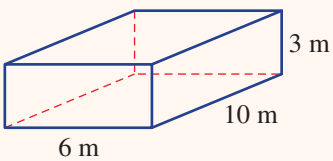
14 I can find the area of simple composite shapes involving sectors.
e.g. Find the area of this composite shape, correct to two decimal places.



☐

2G

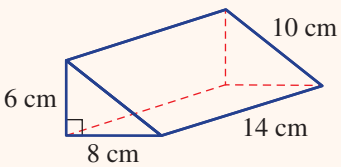
15 I can find the surface area of a rectangular prism using a net.
e.g. Find the surface area of this rectangular prism.



☐

2G

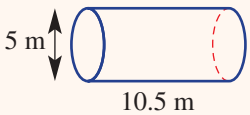
16 I can find the surface area of a triangular prism using a net.
e.g. Find the surface area of this triangular prism.



☐

2H

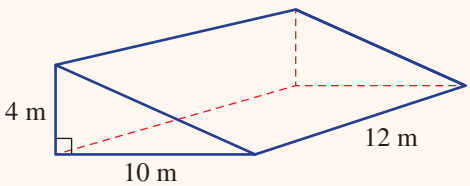
17 I can find the surface area of a cylinder.
e.g. Find the surface area of this cylinder, correct to two decimal places.



☐

2I

18 I can find the volume of a prism.
e.g. Find the volume of this triangular prism.



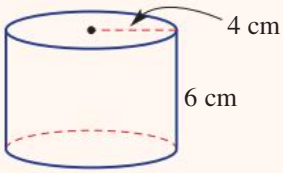
☐

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

2I

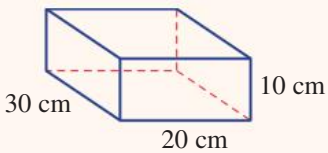
19 I can find the volume of a cylinder.
e.g. Find the volume of this cylinder, correct to two decimal places.



☐

2I

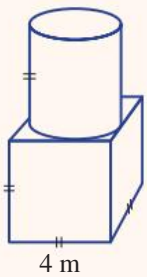
20 I can find the volume of a prism, giving an answer in litres or millilitres.
e.g. Find the volume of this triangular prism in litres.



☐

2J

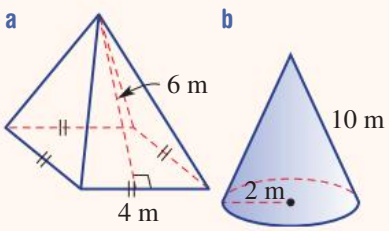
21 I can find the volume and surface area of a composite solid involving cylinders and prisms.
e.g. Find the volume and surface area of this solid.



☐

2K

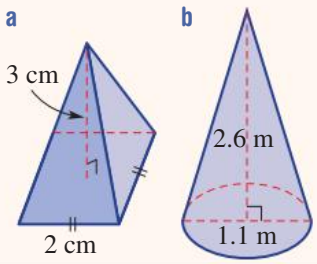
22 I can find the surface area of a pyramid and a cone.
e.g. Find the surface area.



☐

2L

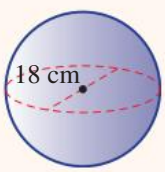
23 I can find the volume of a pyramid and a cone.
e.g. Find the volume.



☐

2M

24 I can find the volume and surface area of a sphere.
e.g. Find the volume and surface area of this sphere.



☐

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

Short-answer questions

2A **1** Convert these measurements to the units shown in the brackets.

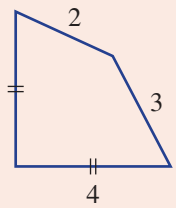
a 5.3 km (m) **b** 27 000 cm^2 (m^2) **c** 0.04 cm^3 (mm^3)

2B **2** Give the limits of accuracy for these measurements.

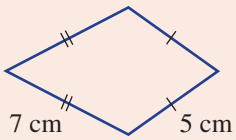
a 6 cm **b** 4.2 kg **c** 16.21 m

2C **3** Find the perimeter of these shapes.

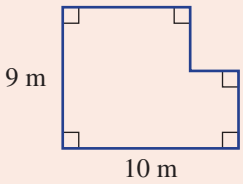
a



b

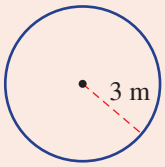


c



2D/2F **4** For the circle shown, find, to two decimal places:

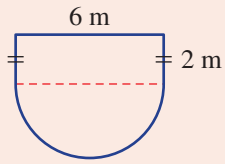
a the circumference **b** the area



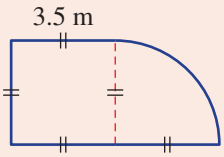
2D/2F **5** For these composite shapes, find, to two decimal places:

i the perimeter **ii** the area

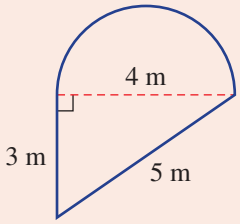
a



b

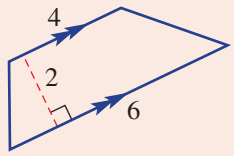


c

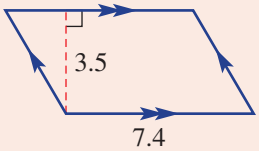


2E **6** Find the area of these shapes.

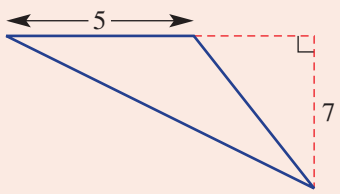
a



b

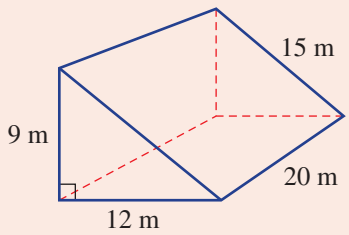


c

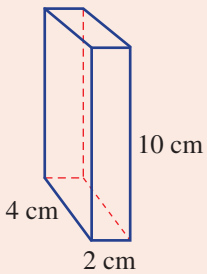


2G **7** Find the surface area of these prisms.

a

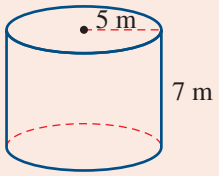


b



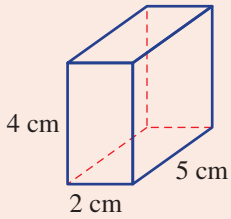
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2H **8** Determine the surface area of this cylinder, to two decimal places.

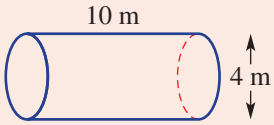


2I **9** Find the volume of these solids, to two decimal places where necessary.

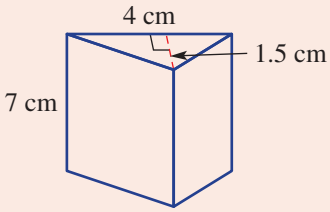
a



b

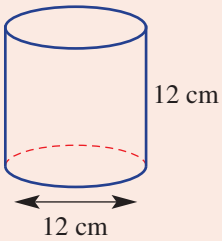


c

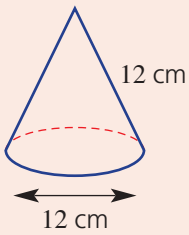


2J/K/L/M **10** Find the exact volume and surface area.

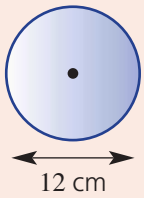
a



b



c



Multiple-choice questions

2A **1** The number of centimetres in a kilometre is:

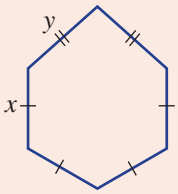
A 10 B 100 C 1000 D 10 000 E 100 000

2B **2** The perimeter of a square with side lengths 2 cm is:

A 4 cm B 8 cm C 4 cm^2 D 8 cm^2 E 16 cm

2B **3** The perimeter of the shape shown is given by the formula:

A $x - y$ B $2x + y$ C $4x + 2y$
D $x - 2y$ E $4x + y$



2C **4** A correct expression for determining the circumference of a circle with diameter 6 cm is:

A $\pi \times 6$ B $\pi \times 3$ C $2 \times \pi \times 6$ D 2×6 E $\pi \times 6^2$

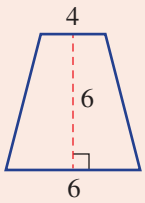
2D **5** The area of a rectangle with side lengths 3 cm and 4 cm is:

A 12 cm^2 B 12 cm C 7 cm^2 D 14 cm E 14 cm^2

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

2D **6** The correct expression for calculating the area of this trapezium is:



A $(6-4)\times 6$ **B** $\frac{1}{2}(6+4)\times 6$ **C** $\frac{1}{2}\times 6\times 4$ **D** $6\times 6-4$

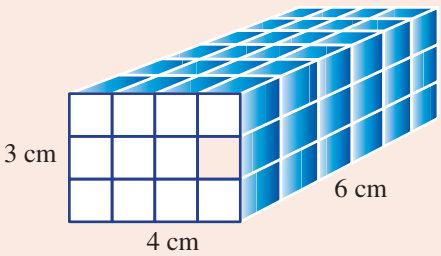
E $6\times 6+6\times 4$

2E **7** A sector's centre angle measures 90°. This is equivalent to:

A $\frac{1}{5}$ of a circle **B** $\frac{1}{2}$ of a circle **C** $\frac{3}{4}$ of a circle

D $\frac{2}{3}$ of a circle **E** $\frac{1}{4}$ of a circle

2H **8** The volume of the shape shown is:



A $13\,cm^3$ **B** $27\,cm^3$

C $72\,cm^2$ **D** $72\,cm^3$

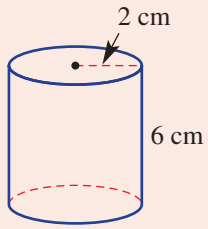
E $27\,cm^2$

2H **9** The volume of a cube of side length 3 cm is:

A $9\,cm^3$ **B** $27\,cm^3$ **C** $54\,cm^2$

D $54\,cm^3$ **E** $27\,cm^2$

2G **10** The curved surface area for this cylinder is closest to:



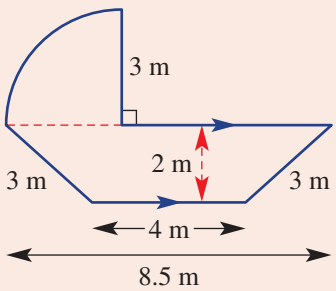
A $87.96\,cm^2$ **B** $12.57\,cm^2$

C $75.40\,cm^2$ **D** $75.39\,cm^2$

E $113.10\,cm^2$

Extended-response questions

1 A new playground is being built with the shape and dimensions as shown in the diagram.



a The playground will be surrounded by wooden planks.

i Determine the perimeter of the playground, correct to two decimal places.

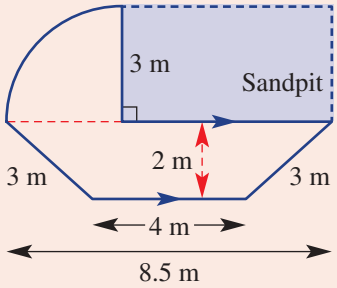
ii If the wood to be used costs \$16.50/m, what will be the cost of surrounding the play area to the nearest dollar?

b The playground area is to be covered with a layer of woodchips. Find the area of the playground, correct to one decimal place.

c If a bag of woodchips from the hardware store covers 7.5 m^2, how many bags would be required to cover the playground area?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

d A rectangular sandpit is to be included as shown. If sand is to be spread flat and filled to a height of 40 cm, determine the volume of sand required in cubic metres.



2 An cylindrical tank has diameter 8 m and height 2 m.

a Find the surface area of the curved part of the tank, to two decimal places.

b Find the surface area, including the top and the base, to two decimal places.

c Find the total volume of the tank, to two decimal places.

d Find the total volume of the tank in litres, to two decimal places.
Note: There are 1000 litres in 1 m^3.

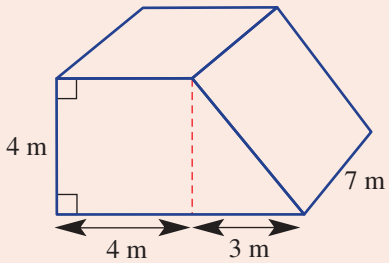
3 An aviary is to be built in the shape shown.

a Use Pythagoras' theorem to find the slant height of the aviary.

b The aviary will be covered in chicken wire. Determine, in square metres, the amount of chicken wire required. Do not include the base.

c If chicken wire costs \$6 per m^2, find the cost of covering the aviary.

d What is the volume of the aviary?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3

Algebraic expressions and indices

Why skills with algebra and index laws are important

Being able to apply algebraic formulas and procedures is an essential skill across the trades and professions, and is especially important for correctly entering and managing formulas in Excel spreadsheets.

- Algebraic formulas are widely used, including by welders (metal shrinkage: $S = \frac{A}{5T} + 0.05d$); nurses (child's dose $C = \frac{AD}{A+12}$); auto mechanics (piston force: $F = \frac{\pi PD^2}{4}$); vets (a horse's weight in kg: $W = \frac{G^2L}{11\,880}$); and financial analysts (investment value: $A = P\left(1 + \frac{r}{100}\right)^n$).
- Technicians in many fields use scientific notation: boiler technicians (hospital sterlisation stream heat energy, kJ/day); lab technicians (red blood cells/L); and air-conditioner technicians (heat transfer through walls in kJ/s).
- Computer programmers first write an algebraic solution to the problem they want a computer to

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

solve; these steps are then coded to form an algorithm. The internet and apps are powered by algebra in the form of algorithms.

- Electricians and electronics engineers require algebra in fields including designing and building microcircuits in robots, autopilots and medical equipment.

Chapter contents

3A Algebraic expressions (CONSOLIDATING)
3B Simplifying algebraic expressions (CONSOLIDATING)
3C Expanding algebraic expressions
3D Expanding binomial products
3E Factorising algebraic expressions (CONSOLIDATING)
3F Simplifying algebraic fractions: Multiplication and division
3G Simplifying algebraic fractions: Addition and subtraction
3H Index laws for multiplying and dividing
3I Powers of powers, powers of fractions and the zero index
3J Negative indices, algebraic bases (EXTENDING)
3K Scientific notation and significant figures

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- simplifies algebraic fractions with numerical denominators and expands algebraic expressions (MA5-ALG-C-01)
- simplifies algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases (MA5-IND-C-01)
- solves measurement problems by using scientific notation to represent numbers and rounding to a given number of significant figures (MA5-MAG-C-01).

© 2022 NSW Education Standards Authority

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Warm-up quiz

1 Write algebraic expressions for the following.

a 3 lots of x **b** one more than a

c 5 less than $2m$ **d** 4 times the sum of x and y

2 Find the value of the following when $x = 4$ and $y = 7$.

a $5x$ **b** $2y + 3$

c $xy - 5$ **d** $2(x + y)$

3 Decide whether the following pairs of terms are like terms.

a $6x$ and 8 **b** $3a$ and $7a$

c $4xy$ and $2yx$ **d** $3x^2$ and $10x$

4 Simplify:

a $3m + 5m$ **b** $8ab - 3ab$

c $4x + 3y + 2x + 5y$ **d** $2 \times 4 \times x$

e $5 \times a \times 3 \times b$ **f** $6y \div 2$

5 Expand:

a $2(x + 5)$ **b** $3(y - 2)$

c $4(2x - 3)$ **d** $x(3x + 1)$

6 Find the HCF (highest common factor) of these pairs of terms.

a 8, 12 **b** 18, 30

c $7a$, $14a$ **d** $2xy$ and $8xz$

e $5x$ and $8x^2$ **f** $7x^2y$ and $21xy^2$

7 Simplify:

a $\frac{3}{8} + \frac{2}{5}$ **b** $\frac{6}{7} - \frac{1}{3}$

c $\frac{5}{9} \times \frac{6}{25}$ **d** $\frac{2}{3} \div \frac{4}{9}$

8 Write each of the following in index form (e.g. $5 \times 5 \times 5 = 5^3$).

a $7 \times 7 \times 7 \times 7$ **b** $m \times m \times m$

c $x \times x \times y \times y \times y$ **d** $3a \times 3a \times 3a \times 3a \times 3a$

9 Evaluate:

a 7^2 **b** 3^3 **c** 2^4 **d** 4^3

10 Write the following as 3 raised to a single power.

a $3^4 \times 3^5$ **b** $3^7 \div 3^5$ **c** $(3^2)^5$ **d** 1 **e** $\frac{1}{3^2}$

11 Complete the following.

a $3.8 \times 10 =$ _____ **b** $2.31 \times 1000 =$ _____

c $17.2 \div 100 =$ _____ **d** $0.18 \div 100 =$ _____

e $3827 \div$ _____ $= 3.827$ **f** $6.49 \times$ _____ $= 64\,900$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3A Algebraic expressions CONSOLIDATING

Learning intentions for this section:

- To review the terminology associated with algebraic expressions
- To be able to convert worded scenarios into algebraic expressions
- To be able to substitute numbers into algebraic expressions

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stages 5 and 6
- It consolidates and extends the concepts in Sections 3A and 9A of our Year 9 book
- Expertise with substitution is assumed knowledge for Stage 6 Standard

Algebra involves the use of pronumerals (also called variables), which are letters that represent numbers. **Terms** form when numbers and pronumerals are connected by multiplication or division. **Expressions** can be made up of one term, or two or more terms that are connected by addition or subtraction.

If a ticket to an art gallery costs \$12, then the cost for y visitors is the expression $12 \times y = 12y$. By substituting values for y we can find the costs for different numbers of visitors. For example, if there are five visitors, then $y = 5$ and $12y = 12 \times 5 = 60$. So total cost = \$60.

Lesson starter: Expressions at the gallery

Ben, Alea and Victoria are visiting the art gallery. The three of them combined have \$$c$ between them. Drinks cost \$$d$ and Ben has bought x postcards in the gift shop.

Write expressions for the following.

- The cost of two drinks
- The amount of money each person has if the money is shared equally
- The number of postcards Alea and Victoria bought if Alea bought three more than Ben, and Victoria bought five less than twice the number Ben bought

KEY IDEAS

- A **pronumeral** (or **variable**) is a letter used to represent an unknown number.
- A **term** is a group of numbers and pronumerals connected by multiplication and division;
  e.g. $2x$, $\frac{y}{4}$, $5x^2$.
- Algebraic **expressions** are made up of one term, or two or more terms connected by addition or subtraction;
  e.g. $2x$, $3a + 7b$, $\frac{x}{2} + 3y$, $3x - 4$.

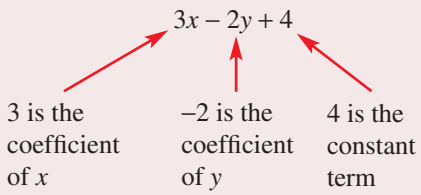
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- A **constant term** is a number with no attached pronumerals; e.g. 7, −3.
- The **coefficient** is the number multiplied by the pronumerals in the term;
  e.g. 3 is the coefficient of y in $2x + 3y$.

  −4 is the coefficient of x in $5 - 4x$.

  1 is the coefficient of x^2 in $2x + x^2$.

  This expression has 3 terms: $3x$, $2y$ and 4.



■ Operations in algebraic expressions:
- The operations for addition and subtraction are written with '+' and '−'.
- Multiplication is written without the sign; e.g. $3 \times y = 3y$.
- Division is written as a fraction; e.g. $y \div 4 = \frac{y}{4}$ or $\frac{1}{4}y$.

■ To find the value of an expression (or to **evaluate**), **substitute** a value for each pronumeral. The order of operations is followed. For example, if $x = 2$ and $y = 3$:

$$\begin{aligned} 4xy - y^2 &= 4 \times 2 \times 3 - 3^2 \\ &= 24 - 9 \\ &= 15 \end{aligned}$$

Key terminology: expression, pronumeral, variable, term, constant term, coefficient, substitute, evaluate

Exercise 3A

UNDERSTANDING 1–3 1–3

1 Fill in the missing word(s) in the sentences, using the words *expression, term, constant term* or *coefficient*.

a An algebraic _____ is made up of one or more terms connected by addition and subtraction.

b A term without a pronumeral part is a _____.

c A number multiplied by the pronumerals in a term is a _____.

d Numbers and pronumerals connected by multiplication and division form a _____.

2 Express the following in simplified mathematical form.

a x plus 3 **b** $5 \times y$ **c** $a \div 5$ **d** $2 \times x \times y$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Substitute $a = -6$ into the following expressions.

a $a + 9$ **b** $a - 8$ **c** $-a$ **d** $-a + 10$

e $12 - a$ **f** $5a$ **g** $-2a$ **h** $\frac{24}{a}$

i a^2 **j** $-a^2$ **k** $(-a)^2$ **l** $(a + 6)^2$

FLUENCY 4, 5, 6(½) 4, 5–6(½)

Example 1 Naming parts of an expression

Consider the expression $\frac{xy}{2} - 4x + 3y^2 - 2$. Determine:

a the number of terms
b the constant term
c the coefficient of:
i y^2 **ii** x

SOLUTION	EXPLANATION
a 4	There are four terms with different combinations of pronumerals and numbers, separated by + or −.
b −2	The term with no pronumerals is −2. The negative is included.
c **i** 3	The number multiplied by y^2 in $3y^2$ is 3.
ii −4	The number multiplied by x in $-4x$ is −4. The negative sign belongs to the term that follows.

Now you try

Consider the expression $4y - \frac{x}{3} - 2x^2 + 1$. Determine:

a the number of terms
b the constant term
c the coefficient of:
i y **ii** x^2

4 For these algebraic expressions, determine:

i the number of terms
ii the constant term
iii the coefficient of y

a $4xy + 5y + 8$

b $2xy + \frac{1}{2}y^2 - 3y + 2$

c $2x^2 - 4 + y$

Hint: The coefficient is the number multiplied by the pronumerals in each term. The constant term has no pronumerals.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 2 Writing algebraic expressions

Write algebraic expressions for the following.

a three more than x
b 4 less than 5 times y
c the sum of c and d is divided by 3
d the product of a and the square of b

SOLUTION	EXPLANATION
a $x + 3$	More than means add (+).
b $5y - 4$	Times means multiply ($5 \times y = 5y$) and less than means subtract (−).
c $\frac{c+d}{3}$	Sum c and d first (+), then divide by 3 (÷). Division is written as a fraction.
d ab^2	'Product' means 'multiply'. The square of b is b^2 (i.e. $b \times b$). $a \times b^2 = ab^2$

Now you try

Write algebraic expressions for the following.

a five more than y
b 7 less than 3 times x
c the sum of a and b is divided by 5
d the product of x and the square of y

5 Write an expression for the following.

a two more than x
b four less than y
c the sum of ab and y
d three less than 2 lots of x
e the product of x and 5
f twice m
g three times the value of r
h half of x
i three-quarters of m
j the quotient of x and y
k the sum of a and b is divided by 4
l the product of the square of x and y

Hint: Quotient means ÷. Product means ×
$\frac{1}{3}y = \frac{y}{3}$.

Example 3 Substituting values

Find the value of these expressions when $x = 2$, $y = 3$ and $z = -5$.

a $xy + 3y$ **b** $y^2 - \frac{8}{x}$ **c** $2x - yz$

SOLUTION	EXPLANATION
a $xy + 3y = 2 \times 3 + 3 \times 3$	Substitute for each pronumeral: $x = 2$ and $y = 3$.
$= 6 + 9$	Recall that $xy = x \times y$ and $3y = 3 \times y$.
$= 15$	Simplify, following order of operations, by multiplying first.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b $y^2 - \frac{8}{x} = 3^2 - \frac{8}{2}$
$= 9 - 4$
$= 5$

Substitute $y = 3$ and $x = 2$.
$3^2 = 3 \times 3$ and $\frac{8}{2} = 8 \div 2$.
Do subtraction last.

c $2x - yz = 2 \times 2 - 3 \times (-5)$
$= 4 - (-15)$
$= 4 + 15$
$= 19$

Substitute for each pronumeral.
$3 \times (-5) = -15$
To subtract a negative number, add its opposite.

Now you try

Find the value of these expressions when $x = 6$, $y = -2$ and $z = 4$.

a $xz + 2x$ **b** $x^2 + \frac{z}{2}$ **c** $3z - xy$

6 Find the value of these expressions when $a = 4$, $b = -2$ and $c = 3$.

a ac **b** $2a - 5$ **c** $3a - c$
d $a^2 - 2c$ **e** $ac + b$ **f** $3b + a$
g $ab + c^2$ **h** $\frac{a}{2} - b$ **i** $\frac{ac}{b}$
j $2a - b$ **k** $a + bc$ **l** $\frac{6bc}{a}$

Hint:
$12 + (-2) = 12 - 2$
$2 - (-2) = 2 + 2$

PROBLEM-SOLVING AND REASONING 7–9 8–11

7 Write an expression for the following.

a The cost of 5 pencils at x cents each
b The cost of y apples at 35 cents each
c One person's share when \$500 is divided among n people
d The cost of a pizza (\$11) equally shared between m people
e Parvinda's age in x years' time if he is 11 years old now

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 A taxi in Sydney has a pick-up charge (i.e. flagfall) of $3.40 and charges $2 per km.

a Write an expression for the taxi fare for a trip of d kilometres.

b Use your expression in part **a** to find the cost of a trip that is:

i 10 km

ii 22 km

Hint: The taxi fare has initial cost + cost per km × number of km.

9 **a** Ye thinks of a number, which we will call x.
Now write an expression for each of the following stages.

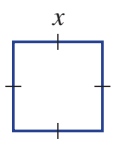
i He doubles the number.

ii He decreases the result by 3.

iii He multiplies the result by 3.

b If $x = 5$, use your answer to part **a iii** to find the final number.

10 A square with side length x is changed to a rectangle by increasing the length by 1 and decreasing the breadth by 1.



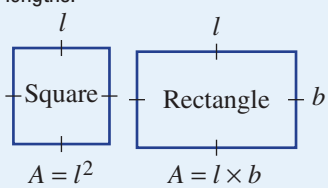
a Write an expression for the new length and breadth of the rectangle.

b Is there any change in the perimeter of the shape?

c **i** Write an expression for the area of the rectangle.

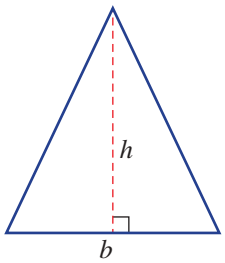
ii Use trial and error to determine whether the area of the rectangle is more or less than the original square. By how much?

Hint: Perimeter is the sum of the side lengths.



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 The area of a triangle is given by $A = \frac{1}{2}bh$.



a If $b = 6$ and $h = 7$, what is the area?

b If the area is 9, what are the possible whole number values for b if h is also a whole number?

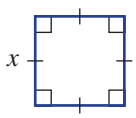
ENRICHMENT: Area and perimeter — 12

12 For the shapes shown, write an expression for:

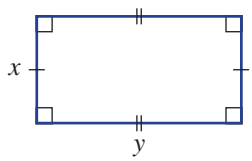
i the perimeter

ii the area

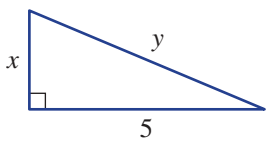
a



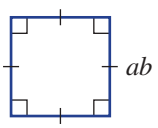
b



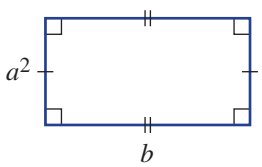
c



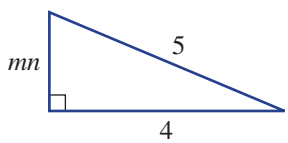
d



e



f



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3B Simplifying algebraic expressions CONSOLIDATING

Learning intentions for this section:

- To be able to identify like terms
- To know that only like terms can be combined under addition and subtraction
- To understand that like terms and order do not matter when multiplying
- To be able to simplify algebraic expressions using the four operations

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stages 5 and 6
- It consolidates and extends the concepts in Sections 3B and 3C of our Year 9 book
- Expertise with these concepts is assumed knowledge for Stage 6 Standard

Many areas of finance and industry involve complex algebraic expressions. Often these expressions can be made simpler by applying the rules of addition, subtraction, multiplication and division.

Just as we would write $3 + 3 + 3 + 3$ as 4×3, we write $x + x + x + x$ as $4 \times x$ or $4x$. Similarly, $3x + 2x = 5x$ and $3x - 2x = 1x$ ($1x$ is written as x).

We also know that $2 \times 3 = 3 \times 2$ and $(2 \times 3) \times 4 = 2 \times 3 \times 4 = 3 \times 4 \times 2$ etc., so $2 \times x \times 4 = 2 \times 4 \times x = 8x$. By writing a division as a fraction we can also cancel common factors. For example, $9x \div 3 = \frac{9x}{3} = 3x$.

Lesson starter: Equivalent expressions

Split these expressions into two groups that are equivalent by simplifying them first.

$3x + 6x$	$17x - 5x$	$x + 7x + x$	$4x + 3 + 5x - 3$
$2 \times 6x$	$\frac{24xy}{2y}$	$3x \times 3$	$3x - 2y + 9x + 2y$
$8x + 6x - 2x$	$18x \div 2$	$\frac{9x^2}{x}$	$6x - (-6x)$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- **Like terms** have the exact same pronumeral factors, including powers; e.g. $3x$ and $7x$, and $4x^2y$ and $-3x^2y$.
  - Since $x \times y = y \times x$, $3xy$ and $2yx$ are like terms.
- Expressions in which like terms are added or subtracted can be simplified.
  For example, $5x + 7x = 12x$

  $7ab - 6ab = 1ab = ab$

  But $3x + 2y$ cannot be simplified.
- Like terms are not required when multiplying and dividing.
  - In multiplication, deal with numerals and pronumerals separately:
    For example, $2 \times 8a = 2 \times 8 \times a = 16a$

    $6x \times 3y = 6 \times 3 \times x \times y = 18xy$
- When dividing, write as a fraction and cancel common factors:

  For example, $\frac{8^{4}x}{2^{1}} = 4x$

  $6x^2 \div (3x) = \frac{6x^2}{3x} = \frac{6^{2} \times x^{1} \times x}{3^{1} \times x^{1}} = 2x$

Key terminology: like terms, pronumeral

Exercise 3B

UNDERSTANDING 1–4 | 4

1 Are the following sets of terms like terms? Answer yes or no.

a $3x, 2x, -5x$ **b** $2ax, 3xa, -ax$ **c** $2ax^2, 2ax, 62a^2x$ **d** $\frac{3}{4}x^2, 2x^2, \frac{x^2}{3}$

2 Simplify the following.

a $8g + 2g$ **b** $3f + 2f$ **c** $12e - 4e$
d $3h - 3h$ **e** $5x + x$ **f** $14st + 3st$

Hint: Add or subtract the numerals in like terms.

3 Simplify the following.

a $3 \times 2x$ **b** $4 \times 3a$ **c** $2 \times 5m$ **d** $-3 \times 6y$

4 Simplify these fractions by cancelling.

a $\frac{4a}{8}$ **b** $\frac{12b}{3}$ **c** $\frac{24c}{8}$
d $\frac{12d}{18}$ **e** $\frac{14e}{21}$ **f** $\frac{35f}{15}$
g $\frac{27g}{36}$ **h** $\frac{18h}{45}$ **i** $\frac{20i}{24}$

Hint: Choose the highest common factor to cancel.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 5–8(½) 5–8(½)

Example 4 Identifying like terms

Write down the like terms in the following lists.

a $3x, 6a, 2ax, 3a, 5xa$ **b** $-2ax, 3x^2a, 3a, -5x^2a, 3x$

SOLUTION

a $6a$ and $3a$

$5xa$ and $2ax$

b $3x^2a$ and $-5x^2a$

EXPLANATION

Both terms contain a.

Both terms contain ax; $x \times a = a \times x$.

Both terms contain x^2a.

Now you try

Write down the like terms in the following lists.

a $4a, 3b, 5ab, 2a, 2ba$ **b** $-x^2y, 3x^2, 2xy, 4x, 4x^2y$

5 Write down the like terms in the following lists.

a $3ac, 2a, 5x, -2ac$ **b** $4pq, 3qp, 2p^2, -4p^2q$

c $7x^2y, -3xy^2, 2xy^2, 4yx^2$ **d** $2r^2, 3rx, -r^2, 4r^2x$

e $-2ab, 5bx, 4ba, 7xa$ **f** $3p^2q, -4pq^2, \frac{1}{2}pq, 4qp^2$

g $\frac{1}{3}lm, 2l^2m, \frac{lm}{4}, 2lm^2$ **h** $x^2y, yx^2, -xy, yx$

Hint: Like terms have the same pronumeral factors.
$x \times y = y \times x$, so $3xy$ and $5yx$ are like terms.

Example 5 Collecting like terms

Simplify the following.

a $4a + 5a + 3$ **b** $3x + 2y + 5x - 3y$ **c** $5xy + 2xy^2 - 2xy + xy^2$

SOLUTION

a $4a + 5a + 3 = 9a + 3$

b $3x + 2y + 5x - 3y = 3x + 5x + 2y - 3y$
$= 8x - y$

c $5xy + 2xy^2 - 2xy + xy^2$
$= 5xy - 2xy + 2xy^2 + xy^2$
$= 3xy + 3xy^2$

EXPLANATION

Collect like terms ($4a$ and $5a$) and add coefficients.

Collect like terms in x ($3 + 5 = 8$) and y ($2 - 3 = -1$). Note: $-1y$ is written as $-y$.

Collect like terms. In xy, the negative belongs to $2xy$. In xy^2, recall that xy^2 is $1xy^2$.

Now you try

Simplify the following.

a $7x + 3x + 2$ **b** $2a + 4b + 3a - 2b$ **c** $4mn + 3m^2n - mn + 2m^2n$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Simplify the following by collecting like terms.

a $4t + 3t + 10$ **b** $5g - g + 1$ **c** $3x - 5 + 4x$

d $4m + 2 - 3m$ **e** $2x + 3y + x$ **f** $3x + 4y - x + 2y$

g $8a + 4b - 3a - 6b$ **h** $2m - 3n - 5m + n$ **i** $3de + 3de^2 + 2de + 4de^2$

j $6kl - 4k^2l - 6k^2l - 3kl$ **k** $3x^2y + 2xy^2 - xy^2 + 4x^2y$ **l** $4fg - 5g^2f + 4fg^2 - fg$

Example 6 Multiplying algebraic terms

Simplify the following.

a $2a \times 7d$ **b** $-3m \times 8mn$

SOLUTION

a $2a \times 7d = 2 \times 7 \times a \times d$
$= 14ad$

EXPLANATION

Multiply coefficients and collect the pronumerals: $2 \times a \times 7 \times d = 2 \times 7 \times a \times d$.

Multiplication can be done in any order.

b $-3m \times 8mn = -3 \times 8 \times m \times m \times n$
$= -24m^2n$

Multiply coefficients ($-3 \times 8 = -24$) and pronumerals. Recall: $m \times m$ can be written as m^2.

Now you try

Simplify the following.

a $4x \times 5w$ **b** $-2a \times 6ac$

7 Simplify the following.

a $3r \times 2s$ **b** $2h \times 3u$ **c** $4w \times 4h$

d $2r^2 \times 3s$ **e** $-2e \times 4s$ **f** $5h \times (-2v)$

g $-3c \times (-4m^2)$ **h** $-7f \times (-5l)$ **i** $2x \times 4xy$

j $3ab \times 8a$ **k** $xy \times 3y$ **l** $-2a \times 8ab$

m $-3m^2n \times 4n$ **n** $-5xy^2 \times (-4x)$ **o** $5ab \times 4ab$

Hint: Multiply the numerals and collect the pronumerals.
$a \times b = ab$

Example 7 Dividing algebraic terms

Simplify the following.

a $\frac{18x}{6}$ **b** $12a^2b \div (8ab)$

SOLUTION

a $\frac{\cancel{18}^{3}x}{\cancel{6}^{1}} = 3x$

EXPLANATION

Cancel highest common factor of numerals, i.e. 6.

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b $12a^2b \div (8ab) = \dfrac{12a^2b}{8ab}$

$= \dfrac{{}^{3}\cancel{12} \times a \times \cancel{a_1} \times \cancel{b_1}}{{}_{2}\cancel{8} \times \cancel{a_1} \times \cancel{b_1}}$

$= \dfrac{3a}{2}$

Write division as a fraction.

Cancel the highest common factor of 12 and 8 and cancel an a and b.

Now you try

Simplify the following.

a $\dfrac{20x}{4}$

b $9s^2t \div (15st)$

8 Simplify by cancelling common factors.

a $\dfrac{6a}{2}$ **b** $\dfrac{7x}{14}$ **c** $3a \div 9$

d $2ab \div 8$ **e** $\dfrac{4ab}{2a}$ **f** $\dfrac{15xy}{5y}$

g $4xy \div (8x)$ **h** $28ab \div (35b)$ **i** $\dfrac{8x^2}{20x}$

j $\dfrac{12xy^2}{18y}$ **k** $30a^2b \div (10a)$ **l** $12mn^2 \div (36mn)$

Hint: Write each division as a fraction first where necessary.

PROBLEM-SOLVING AND REASONING

9, 10 | 9–12

9 A rectangle's length is three times its breadth, x. Write a simplified expression for:

a the rectangle's perimeter

b the rectangle's area

Hint: Draw a rectangle and label the breadth x and the length $3 \times x = 3x$.

10 Fill in the missing term to make the following true.

a $8x + 4 - \square = 3x + 4$

b $3x + 2y - \square + 4y = 3x - 2y$

c $3b \times \square = 12ab$

d $4xy \times (\square) = -24x^2y$

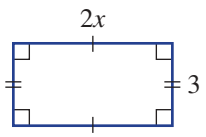
e $12xy \div (\square) = 6y$

f $\square \div (15ab) = \dfrac{2a}{3}$

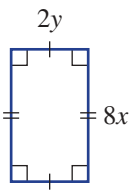
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes.

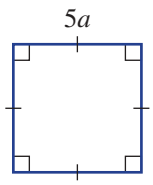
a



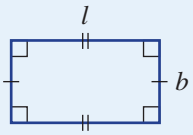
b



c

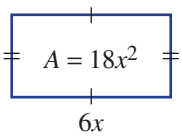


Hint: Perimeter is the sum of all the sides.
Area $= l \times b$



12 A rectangular garden bed has length given by $6x$ and area $18x^2$. What is the breadth of the garden bed?

Hint: The opposite of $\times$ is $\div$.



ENRICHMENT: Order of operations — 13

13 Simplify the following expressions, using order of operations.

a $4 \times 3x \div 2$

b $2 + 4a \times 2 + 5a \div a$

c $5a \times 2b \div a - 6b$

d $8x^2 \div (4x) + 3 \times 3x$

e $2x \times (4x + 5x) \div 6$

f $5xy - 4x^2y \div (2x) + 3x \times 4y$

g $(5x - x) \times (16xy \div (8y))$

h $9x^2y \div (3y) + 4x \times (-8x)$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3C Expanding algebraic expressions

Learning intentions for this section:

- To understand the distributive law for expanding brackets
- To be able to expand expressions involving brackets

Past, present and future learning:

- This section addresses Stage 4 concepts which are used in Stages 5 and 6
- It also includes concepts from the Stage 5 Core topic called *Algebraic Techniques A*
- It consolidates and extends the concepts in Section 3D of our Year 9 book
- Expertise with these concepts is assumed knowledge for Stage 6 Standard

The expression $2(3 + 4)$ means $2 \times (3 + 4)$. It can be simplified in two ways:

$$\begin{aligned} 2 \times (3 + 4) &= 2 \times 7 \\ &= 14 \end{aligned}$$

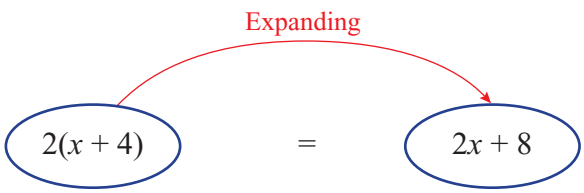
This method *cannot* be used to expand $2(x + 4)$.

OR

$$\begin{aligned} 2 \times (3 + 4) &= 2 \times 3 + 2 \times 4 \\ &= 6 + 8 \\ &= 14 \end{aligned}$$

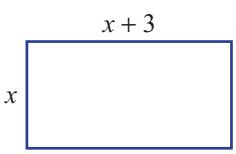
This second method is called the distributive law.
It can also be applied as follows:

$$\begin{aligned} 2(x + 4) &= 2 \times x + 2 \times 4 \\ &= 2x + 8 \end{aligned}$$



Lesson starter: Rectangle brackets

A rectangle's length is 3 more than its breadth.



- Write down as many expressions as you can, both with and without brackets, for the rectangle's:
  - perimeter
  - area
- Can you explain why all the expressions for the perimeter are equivalent?
- Can you explain why all the expressions for the area are equivalent?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- The **distributive law** is used to expand and remove brackets.
- The terms inside the brackets are multiplied by the term outside the brackets.

$$a(b + c) = ab + ac \qquad a(b - c) = ab - ac$$

For example, $2(x + 4) = 2 \times x + 2 \times 4$ and $2(x - 4) = 2 \times x - 2 \times 4$

$= 2x + 8$ $= 2x - 8$

Key terminology: distributive law, expand

Exercise 3C

UNDERSTANDING	1, 2	2

1 Fill in the missing words: The distributive law says that each term inside the _____ is multiplied by the term _____ the brackets.

2 Complete the following.

a $3(x + 4) = 3 \times \square + 3 \times \square$

$= 3x + \square$

b $2(x - 5) = 2 \times \square + \square \times (-5)$

$= \square - 10$

c $2(4x + 3) = 2 \times \square + \square \times 3$

$= \square + 6$

d $x(x - 3) = x \times \square + \square \times \square$

$= \square - \square$

FLUENCY	3–5(½)	3–5(½)

Example 8 Expanding expressions with brackets

Expand the following.

a $2(x + 5)$ **b** $3(2x - 3)$ **c** $3y(2x + 4y)$

SOLUTION

a $2(x + 5) = 2 \times x + 2 \times 5$

$= 2x + 10$

b $3(2x - 3) = 3 \times 2x + 3 \times (-3)$

$= 6x - 9$

c $3y(2x + 4y) = 3y \times 2x + 3y \times 4y$

$= 6xy + 12y^2$

EXPLANATION

Multiply each term inside the brackets by 2.

Multiply $2x$ and -3 by 3.
$3 \times 2x = 3 \times 2 \times x = 6x$.

Multiply $2x$ and $4y$ by $3y$.
$3y \times 2x = 3 \times 2 \times x \times y$ and $3y \times 4y = 3 \times 4 \times y \times y$.

Recall: $y \times y$ is written as y^2.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

Expand the following.

a $3(x + 4)$ **b** $5(3x - 2)$ **c** $4a(2a + 5b)$

3 Expand the following.

a $2(x + 4)$ **b** $3(x + 7)$ **c** $4(y - 3)$

d $5(y - 2)$ **e** $2(3x + 2)$ **f** $4(2x + 5)$

g $3(3a - 4)$ **h** $7(2y - 5)$ **i** $5(2a + b)$

j $3(4a - 3b)$ **k** $2x(x + 5)$ **l** $3x(x - 4)$

m $2a(3a + 2b)$ **n** $2y(3x - 4y)$ **o** $3b(2a - 5b)$

Hint: Use the distributive law:

$a(b + c) = a \times b + a \times c$
$= ab + ac$

$a(b - c) = a \times b + a \times (-c)$
$= ab - ac$

Example 9 Expanding expressions with a negative out the front

Expand the following.

a $-3(x - 4)$ **b** $-2x(3x - 2y)$

SOLUTION

a $-3(x - 4) = -3 \times x + (-3) \times (-4)$
$= -3x + 12$

EXPLANATION

Multiply each term inside the brackets by -3.
$-3 \times (-4) = +12$

If there is a negative sign outside the bracket, the sign of each term inside the brackets is changed when expanded.

b $-2x(3x - 2y) = -2x \times 3x + (-2x) \times (-2y)$
$= -6x^2 + 4xy$

$-2x \times 3x = -2 \times 3 \times x \times x$ and $-2x \times (-2y)$
$= -2 \times (-2) \times x \times y$

Now you try

Expand the following.

a $-4(x - 5)$ **b** $-3y(2x - 4y)$

4 Expand the following.

a $-2(x + 3)$ **b** $-5(m + 2)$ **c** $-3(w + 4)$

d $-4(x - 3)$ **e** $-2(m - 7)$ **f** $-7(w - 5)$

g $-(x + y)$ **h** $-(x - y)$ **i** $-2x(3x + 4)$

j $-3x(2x + 5)$ **k** $-4x(2x - 2)$ **l** $-3y(2y - 9)$

m $-2x(3x - 5y)$ **n** $-3x(3x + 2y)$ **o** $-6y(2x + 3y)$

Hint: A negative out the front will change the sign of each term in the brackets when expanded.
$-2(x - 3) = -2x + 6$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 10 Simplifying expressions by removing brackets

Expand and simplify the following.

a $8 + 3(2x - 3)$ **b** $3(2x + 2) - 4(x + 4)$

SOLUTION

a $8 + 3(2x - 3) = 8 + 6x - 9$
$= 6x - 1$

EXPLANATION

Expand the brackets first: $3 \times 2x + 3 \times (-3) = 6x - 9$.

Collect like terms: $8 - 9 = -1$.

b $3(2x + 2) - 4(x + 4) = 6x + 6 - 4x - 16$
$= 2x - 10$

Expand the brackets first. Note that $-4(x + 4) = -4 \times x + (-4) \times 4 = -4x - 16$.

Collect like terms: $6x - 4x = 2x$ and $6 - 16 = -10$.

Now you try

Expand and simplify the following.

a $5 + 2(4a - 3)$ **b** $5(y + 3) - 2(2y + 5)$

5 Expand and simplify the following.

a $2 + 5(x + 3)$ **b** $3 + 7(x + 2)$

c $5 + 2(x - 3)$ **d** $7 - 2(x + 3)$

e $21 - 5(x + 4)$ **f** $4 + 3(2x - 1)$

g $3 + 2(3x + 4)$ **h** $8 - 2(2x - 3)$ **i** $12 - 3(2x - 5)$

j $3(x + 2) + 4(x + 3)$ **k** $2(p + 2) + 5(p - 3)$ **l** $4(x - 3) + 2(3x + 4)$

m $3(2s + 3) - 2(s + 2)$ **n** $4(3f + 2) - 2(6f + 2)$ **o** $3(2x - 5) - 2(2x - 4)$

Hint: Expand first, then collect like terms.

PROBLEM-SOLVING AND REASONING 6, 7 6–9

6 Fill in the missing term/number to make each statement true.

a $\square(x + 4) = 2x + 8$ **b** $\square(2x - 3) = 8x - 12$

c $\square(2x + 3) = 6x^2 + 9x$ **d** $4(\square + 5) = 12x + 20$

e $4y(\square - \square) = 4y^2 - 4y$ **f** $-2x(\square + \square) = -4x^2 - 6xy$

7 Four rectangular rooms in a house have floor side lengths listed below. Find an expression for the area of each floor in expanded form.

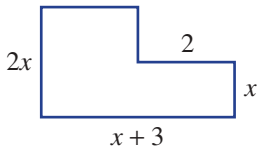
a 2 and $x - 5$ **b** x and $x + 3$

c $2x$ and $x + 4$ **d** $3x$ and $2x - 1$

Hint: Area of a rectangle = length × breadth

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 The deck on a house is constructed in the shape shown. Find the area of the deck in expanded form. (All lines meet at 90°.)



9 Virat earns $\$x$ but does not have to pay tax on the first \$18 200.

a Write an expression for the amount of money Virat is taxed on.

b Virat is taxed 10% of his earnings in part **a**. Write an expanded expression for how much tax he pays.

Hint: To find 10% of an amount, multiply by $\frac{10}{100} = 0.1$.

ENRICHMENT: Expanding binomial products — 10

10 A rectangle has dimensions $(x + 2)$ by $(x + 3)$, as shown. The area can be found by summing the individual areas:

	x	3
x	x^2	$3x$
2	$2x$	6

$$\begin{aligned}(x + 2)(x + 3) &= x^2 + 3x + 2x + 6\\ &= x^2 + 5x + 6\end{aligned}$$

This can also be done using the distributive law:

$$\begin{aligned}(x + 2)(x + 3) &= x(x + 3) + 2(x + 3)\\ &= x^2 + 3x + 2x + 6\\ &= x^2 + 5x + 6\end{aligned}$$

Expand and simplify these binomial products using this method.

a $(x + 4)(x + 3)$ **b** $(x + 3)(x + 1)$ **c** $(x + 2)(x + 5)$

d $(x + 2)(x - 4)$ **e** $(x + 5)(x - 2)$ **f** $(x + 4)(2x + 3)$

g $(2x + 3)(x - 2)$ **h** $(x - 3)(x + 4)$ **i** $(4x - 2)(x + 5)$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3D Expanding binomial products

Learning intentions for this section:

- To know the distributive law in order to expand binomial products
- To be able to expand and simplify binomial products
- To be able to expand perfect squares and be able to form a difference of two squares

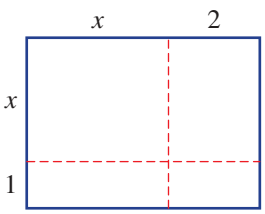
Past, present and future learning:

- This section includes concepts from the Stage 5 Core topic called *Algebraic techniques A*
- It consolidates and extends the concepts in Sections 9B, 9C and 9D of our Year 9 book
- This section has limited relevance to Stage 6 Standard

Expressions that include numerals and pronumerals (or variables) are central to the topic of algebra. Sound skills in algebra are essential for solving most mathematical problems and this includes the ability to expand expressions involving brackets, such as binomial products, perfect squares and the difference of two squares.

Lesson starter: Why does $(x + 1)(x + 2) = x^2 + 3x + 2$?

Look at this rectangle with side lengths $x + 1$ and $x + 2$.



- What are the areas of the four regions?
- Add up the areas to find an expression for the total area.
- Why does this explain that $(x + 1)(x + 2) = x^2 + 3x + 2$?

KEY IDEAS

- **Like terms** have the same pronumeral part.
  - They can be collected (added and subtracted) to form a single term.
    For example: $7x - 11x = -4x$ and $4a^2b - 7ba^2 = -3a^2b$.
- The **distributive law** is used to expand brackets.

$a(b + c) = ab + ac$ $\qquad (a + b)(c + d) = ac + ad + bc + bd$

and $a(b - c) = ab - ac$

	a
b	ab
c	ac

	a	b
c	ac	bc
d	ad	bd

$(a + b)(c + d)$ is called a **binomial** product because each expression in the brackets has two terms.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Perfect squares

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2$$
$$= a^2 + 2ab + b^2$$
$$(a-b)^2 = (a-b)(a-b) = a^2 - ab - ba + b^2$$
$$= a^2 - 2ab + b^2$$

Difference of two squares

$$(a+b)(a-b) = a^2 - ab + ba - b^2$$
$$= a^2 - b^2$$

Key terminology: expand, binomial, product, like terms, distributive law, perfect square, difference of two squares

Exercise 3D

UNDERSTANDING 1–3 3

1 Decide whether each of the following is a perfect square or a difference of two squares.

a $(x+1)^2$ **b** $x^2 - 16$ **c** $4x^2 - 25$ **d** $(2x-3)^2$

2 Copy and complete.

a $a(b+c) =$ _____

b $a(b-c) =$ _____

c $(a+b)(c+d) =$ _____

d $(a+b)(a-b) =$ _____

3 Use each diagram to help expand the expressions.

Hint: Simply add up all the areas inside the rectangular diagram.

a $x(x+2)$

	x
x	x^2
2	$2x$

b $(x+3)(x+1)$

	x	3
x	x^2	$3x$
1	x	3

c $(x+4)^2$

	x	4
x	x^2	$4x$
4	$4x$	16

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 4–7(½) 4–7(½)

Example 11 Expanding simple expressions

Expand and simplify.

a $-3(x-5)$ b $-2x(1-x)$

SOLUTION

a $-3(x-5) = -3 \times x - (-3) \times 5$
$= -3x + 15$

b $-2x(1-x) = -2x \times 1 - (-2x) \times x$
$= -2x + 2x^2$

EXPLANATION

Use the distributive law $a(b-c) = ab - ac$.

A negative times a negative is a positive.

Recall: $x \times x = x^2$.

Now you try

Expand and simplify.

a $-2(x+3)$ b $-5x(3x-1)$

4 Expand and simplify where possible.

a $2(x+5)$ b $3(x-4)$ c $-5(x+3)$
d $-4(x-2)$ e $3(2x-1)$ f $4(3x+1)$
g $-2(5x-3)$ h $-5(4x+3)$ i $x(2x+5)$
j $x(3x-1)$ k $2x(1-x)$ l $3x(2-x)$
m $-2x(3x+2)$ n $-3x(6x-2)$ o $-5x(2-2x)$

Hint: $a(b+c) = ab + ac$

Example 12 Expanding binomial products

Expand the following.

a $(x+5)(x+4)$ b $(2x-1)(3x+5)$

SOLUTION

a $(x+5)(x+4) = x^2 + 4x + 5x + 20$
$= x^2 + 9x + 20$

b $(2x-1)(3x+5) = 6x^2 + 10x - 3x - 5$
$= 6x^2 + 7x - 5$

EXPLANATION

For binomial products use $(a+b)(c+d) = ac + ad + bc + bd$.
Simplify by collecting like terms: $4x + 5x = 9x$

Expand using the distributive law and simplify.

Note that $2x \times 3x = 2 \times 3 \times x \times x = 6x^2$ and $-1 \times 3x = -3x, -1 \times 5 = -5$.

Now you try

Expand the following.

a $(x-1)(x+7)$ b $(3x-2)(4x-1)$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Expand the following.

a $(x+2)(x+8)$ **b** $(x+3)(x+4)$
c $(x+7)(x+5)$ **d** $(x+8)(x-3)$
e $(x+6)(x-5)$ **f** $(x-2)(x+3)$
g $(x-7)(x+3)$ **h** $(x-4)(x-6)$
i $(x-8)(x-5)$ **j** $(2x+1)(3x+5)$
k $(4x+5)(3x+2)$ **l** $(5x+3)(2x+7)$
m $(3x+2)(3x-5)$ **n** $(5x+3)(4x-2)$ **o** $(2x+5)(3x-5)$

Hint: $(a+b)(c+d) = ac + ad + bc + bd$

Example 13 Expanding perfect squares

Expand these perfect squares.

a $(x+2)^2$
b $(x-4)^2$

SOLUTION

a $(x+2)^2 = (x+2)(x+2)$
$= x^2 + 2x + 2x + 4$
$= x^2 + 4x + 4$

or $(x+2)^2 = x^2 + 2(x)(2) + 2^2$
$= x^2 + 4x + 4$

EXPLANATION

First write in expanded form, then use the distributive law.

$(a+b)^2 = a^2 + 2ab + b^2$ with $a = x$ and $b = 2$.

b $(x-4)^2 = (x-4)(x-4)$
$= x^2 - 4x - 4x + 16$
$= x^2 - 8x + 16$

Rewrite and expand, using the distributive law.
$-4 \times (-4) = 16$

or $(x-4)^2 = x^2 - 2(x)(4) + 4^2$
$= x^2 - 8x + 16$

Alternatively for perfect squares
$(a-b)^2 = a^2 - 2ab + b^2$. Here $a = x$ and $b = 4$.

Now you try

Expand these perfect squares.

a $(x+3)^2$
b $(2x-1)^2$

6 Expand these perfect squares.

a $(x+5)^2$ **b** $(x+7)^2$ **c** $(x+6)^2$
d $(x-3)^2$ **e** $(x-8)^2$ **f** $(x-10)^2$
g $(2x+5)^2$ **h** $(5x+6)^2$ **i** $(7x-1)^2$

Hint: Recall:
$(x+5)^2 = (x+5)(x+5)$
$= \ldots$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 14 Expanding to form a difference of two squares

Expand to form a difference of two squares.

a $(x-3)(x+3)$ **b** $(2x+1)(2x-1)$

SOLUTION

a $(x-3)(x+3) = x^2 + 3x - 3x - 9$
$= x^2 - 9$

or $(x-3)(x+3) = x^2 - 3^2$
$= x^2 - 9$

b $(2x+1)(2x-1) = 4x^2 - 2x + 2x - 1$
$= 4x^2 - 1$

or $(2x+1)(2x-1) = (2x)^2 - (1)^2$
$= 4x^2 - 1$

EXPLANATION

$x \times x = x^2, x \times 3 = 3x, -3 \times x = -3x,$
$-3 \times 3 = -9$
Note that the two middle terms cancel.

$(a-b)(a+b) = a^2 - b^2$

Expand, recalling that $2x \times 2x = 4x^2$. Cancel the $-2x$ and $+2x$ terms.

Alternatively, for difference of two squares $(a-b)(a+b) = a^2 - b^2$.

Here $a = 2x$ and $b = 1$ and $(2x)^2 = 2x \times 2x = 4x^2$.

Now you try

Expand to form a difference of two squares.

a $(x+5)(x-5)$ **b** $(3x-2)(3x+2)$

7 Expand to form a difference of two squares.

a $(x+4)(x-4)$ **b** $(x+9)(x-9)$
c $(x+8)(x-8)$ **d** $(3x+4)(3x-4)$
e $(2x-3)(2x+3)$ **f** $(8x-7)(8x+7)$
g $(4x-5)(4x+5)$ **h** $(2x-9)(2x+9)$
i $(5x-7)(5x+7)$ **j** $(7x+11)(7x-11)$

Hint: The two middle terms will cancel to give $(a+b)(a-b) = a^2 - b^2$

PROBLEM-SOLVING AND REASONING 8-9(½) 8-9(½), 10, 11

8 Write the missing number.

a $(x+2)(x-3) = x^2 - x - \square$
b $(x-4)(x-3) = x^2 - \square x + 12$
c $(x-4)(x+4) = x^2 - \square$
d $(2x-1)(2x+1) = \square x^2 - 1$
e $(x+2)^2 = x^2 + \square x + 4$
f $(3x-1)^2 = 9x^2 - \square x + 1$

Hint: Expand if you need to.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 Write the missing number.

a $(x + \square)(x + 2) = x^2 + 5x + 6$

b $(x + \square)(x + 5) = x^2 + 8x + 15$

c $(x + 7)(x - \square) = x^2 + 4x - 21$

d $(x + 4)(x - \square) = x^2 - 4x - 32$

e $(x - 6)(x - \square) = x^2 - 7x + 6$

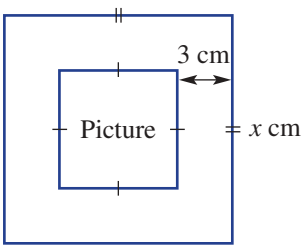
f $(x - \square)(x - 8) = x^2 - 10x + 16$

Hint: Notice how the two numerals in the brackets multiply to give the constant term.

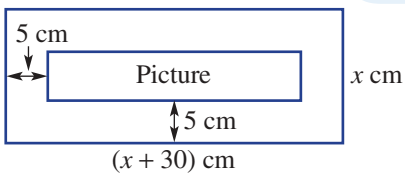
10 Find an expanded expression for the area of the pictures centred in these square and rectangular frames.

Hint: For part **a**, the side length of the picture will be $(x - 6)$ cm.

a



b



11 Each problem below has an incorrect answer. Find the error and give the correct answer.

a $-x(x - 7) = -x^2 - 7x$

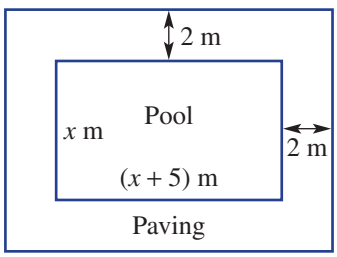
b $3a - 7(4 - a) = -4a - 28$

c $(x - 9)(x + 9) = x^2 - 18x - 81$

d $(2x + 3)^2 = 4x^2 + 9$

ENRICHMENT: Swimming pool algebra — 12

12 A pool company builds rectangular pools that are 5 m longer than they are wide. The company then paves around the outside of the pool using a width of 2 m.



a Find an expanded expression for the:

i pool area

ii total area (including the pool and paving)

iii paved area

b Find the area of the following when $x = 4$.

i the pool

ii the paved area

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3E Factorising algebraic expressions CONSOLIDATING

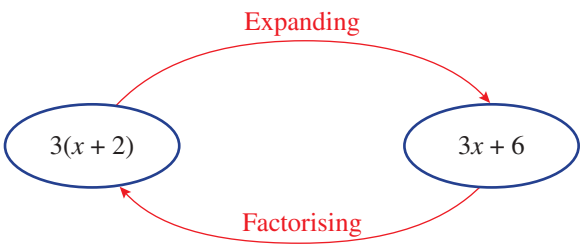
Learning intentions for this section:

- To understand that factorising and expanding are reverse processes
- To be able to identify the highest common factor of terms
- To be able to factorise algebraic expressions involving a common factor

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stages 5 and 6
- It consolidates and extends the concepts in Section 9E of our Year 9 book
- This section has limited relevance to Stage 6 Standard

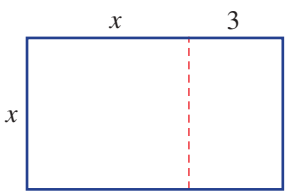
We know that $3(x + 2)$ can be expanded to give $3x + 6$. Similarly, $3x + 6$ can be factorised to give $3(x + 2)$. Factorising is the opposite of expanding.



Lesson starter: Factorised areas

Here is a rectangle of length $(x + 3)$ and breadth x.

- Write an expression for the total area using the given length and breadth.
- Write an expression for the total area by adding up the area of the two smaller regions.
- Are your two expressions equivalent? How could you work from your second expression (expanded) to the first expression (factorised)?



KEY IDEAS

- Factorisation and expansion are opposite processes.
- When **factorising** expressions with common factors, take out the highest common factor (HCF). The HCF could be:
  - a number
    For example: $2x + 10 = 2(x + 5)$
  - a pronumeral
    For example: $x^2 + 5x = x(x + 5)$
  - the product of numbers and pronumerals
    For example: $2x^2 + 10x = 2x(x + 5)$

HCF of $2x$ and 10

$2x + 10 = 2(x + 5)$

Expanded form Factorised form

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

■ A factorised expression can be checked by using expansion.
For example: $2x(x + 5) = 2x^2 + 10x$

Key terminology: highest common factor, factorise, term

Exercise 3E

UNDERSTANDING	1–3	3

1 Write down the highest common factor (HCF) of these pair of numbers.

a 8, 12 **b** 10, 20 **c** 5, 60 **d** 24, 30
e 6, 3 **f** 100, 75 **g** 16, 24 **h** 36, 72

2 For these expressions, state whether it is true (T) or false (F) that the first expression is the factorised form of the second expression. Confirm by expanding.

a $3(x + 2)$, $3x + 6$ **b** $-2(x - 4)$, $-2x - 8$

3 Consider the expression $4x^2 + 8x$.

a Which of the following factorised forms uses the HCF?
A $2(2x^2 + 4x)$ **B** $4(x^2 + 8x)$ **C** $4x(x + 2)$ **D** $2x(2x + 4)$

b What can be said about the terms inside the brackets once the HCF is removed, which is not the case for the other forms?

FLUENCY	4–7(½)	4–7(½)

Example 15 Finding the HCF

Determine the HCF of the following.

a $8a$ and 20 **b** $3x$ and $6x$ **c** $10a^2$ and $15ab$

SOLUTION

a HCF of $8a$ and 20 is 4.

EXPLANATION

Compare numerals and pronumerals separately.

The highest common factor (HCF) of 8 and 20 is 4.

a is not a common factor.

b HCF of $3x$ and $6x$ is $3x$.

HCF of 3 and 6 is 3.

x is also a common factor.

c HCF of $10a^2$ and $15ab$ is $5a$.

HCF of 10 and 15 is 5.

HCF of a^2 and ab is a.

Now you try

Determine the HCF of the following.

a $10x$ and 25 **b** $7x$ and $14x$ **c** $9yz$ and $15y^2$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Determine the HCF of the following.

a $6x$ and 12
b 10 and $15y$
c $8a$ and $12b$
d $9x$ and $18y$
e $5a$ and $20a$
f $10m$ and $22m$
g $14x$ and $21x$
h $8a$ and $40ab$
i $3a^2$ and $9ab$
j $4x^2$ and $10x$
k $16y$ and $24xy$
l $15x^2y$ and $25xy$

Hint: Find the HCF of the numeral and pronumeral factors.

Example 16 Factorising simple expressions

Factorise the following.

a $4x + 20$ **b** $6a - 15b$

SOLUTION

a $4x + 20 = 4(x + 5)$

EXPLANATION

HCF of $4x$ and 20 is 4. Place 4 in front of the brackets and divide each term by 4.

Expand to check: $4(x + 5) = 4x + 20$.

b $6a - 15b = 3(2a - 5b)$

HCF of $6a$ and $15b$ is 3. Place 3 in front of the brackets and divide each term by 3.

Now you try

Factorise the following.

a $3x + 15$ **b** $12m - 18n$

5 Factorise the following.

a $3x + 9$
b $4x - 8$
c $10y - 20$
d $6a + 30$
e $5x + 5y$
f $12a + 4b$
g $18m - 27n$
h $36x - 48y$
i $8x + 44y$
j $24a - 18b$
k $121m + 55n$
l $14k - 63l$

Hint: Check your answer by expanding.
$3(x + 3) = 3x + 9$

Example 17 Factorising expressions with pronumeral common factors

Factorise the following.

a $8y + 12xy$ **b** $4x^2 - 10x$

SOLUTION

a $8y + 12xy = 4y(2 + 3x)$

EXPLANATION

HCF of 8 and 12 is 4, HCF of y and xy is y.

Place $4y$ in front of the brackets and divide each term by $4y$.

Check that $4y(2 + 3x) = 8y + 12xy$.

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b $4x^2 - 10x = 2x(2x - 5)$

HCF of $4x^2$ and $10x$ is $2x$. Place $2x$ in front of the brackets and divide each term by $2x$.

Recall: $x^2 = x \times x$.

Now you try

Factorise the following.

a $9a + 24ab$ **b** $15x^2 - 35x$

6 Factorise the following.

a $14x + 21xy$	**b** $6ab - 15b$	**c** $32y - 40xy$
d $5x^2 - 5x$	**e** $x^2 + 7x$	**f** $2a^2 + 8a$
g $12a^2 + 42ab$	**h** $9y^2 - 63y$	**i** $6x^2 + 14x$
j $9x^2 - 6x$	**k** $16y^2 + 40y$	**l** $10m - 40m^2$

Hint: Place the HCF in front of the brackets and divide each term by the HCF: $14x + 21xy = 7x(___ + ___)$

Example 18 Factorising expressions by removing a common negative

Factorise $-10x^2 - 18x$.

SOLUTION

$-10x^2 - 18x = -2x(5x + 9)$

EXPLANATION

The HCF of $-10x^2$ and $-18x$ is $-2x$, including the common negative. Place $-2x$ in front of the brackets and divide each term by $-2x$.

Dividing by a negative changes the sign of each term.

Now you try

Factorise $-8y^2 - 36y$.

7 Factorise the following, including the common negative.

a $-2x - 6$	**b** $-4a - 8$	**c** $-3x - 6y$
d $-7a - 14ab$	**e** $-x - 10xy$	**f** $-3b - 12ab$
g $-x^2 - 7x$	**h** $-4x^2 - 12x$	**i** $-2y^2 - 10y$
j $-8x^2 - 14x$	**k** $-12x^2 - 8x$	**l** $-15a^2 - 5a$

Hint: Dividing by a negative changes the sign of the term.

PROBLEM-SOLVING AND REASONING 8, 9 8(½), 9–11

8 Factorise these mixed expressions.

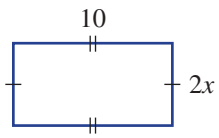
a $7a^2b + ab$	**b** $4a^2b + 20a^2$	**c** $xy - xy^2$
d $x^2y + 4x^2y^2$	**e** $6mn + 18mn^2$	**f** $5x^2y + 10xy^2$
g $-y^2 - 8yz$	**h** $-3a^2b - 6ab$	**i** $-ab^2 - a^2b$

Hint: Be sure to find the highest common factor first.

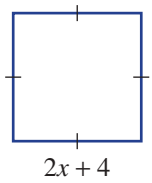
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 Give the perimeter of these shapes in factorised form.

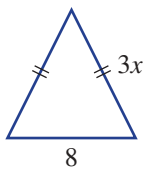
a



b



c



Hint: Find the perimeter first, then factorise.

10 A square sandpit has perimeter $(4x + 12)$ metres. What is the side length of the square?

11 Common factors from expressions involving more than two terms can be removed in a similar way. Factorise these by taking out the HCF.

a $2x + 4y + 6z$

b $3x^2 + 12x + 6$

c $4x^2 + 8xy + 12$

d $6x^2 + 3xy - 9x$

e $10xy - 5xz + 5x$

f $4y^2 - 18y + 14xy$

Hint: $4a + 6b + 10c = 2(2a + 3b + 5c)$

ENRICHMENT: Taking out a binomial factor — 12

12 A common factor may be a binomial term, such as $(x + 1)$.
For example, $3(x + 1) + x(x + 1)$ has HCF $= (x + 1)$, so $3(x + 1) + x(x + 1) = (x + 1)(3 + x)$, where $(3 + x)$ is what remains when $3(x + 1)$ and $x(x + 1)$ are divided by $(x + 1)$.
Use the method above to factorise the following.

a $4(x + 2) + x(x + 2)$

b $x(x + 3) + 2(x + 3)$

c $x(x + 4) - 7(x + 4)$

d $x(2x + 1) - 3(2x + 1)$

e $2x(y - 3) + 4(y - 3)$

f $2x(x - 1) - 3(x - 1)$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3F Simplifying algebraic fractions: Multiplication and division

Learning intentions for this section:

- To know that expressions must be factorised before common factors can be cancelled
- To be able to simplify algebraic fractions by cancelling common factors
- To be able to multiply and divide algebraic fractions

Past, present and future learning:

- This section includes concepts from the Stage 5 Core topic called *Algebraic techniques A*
- It consolidates and extends the concepts in Section 9F of our Year 9 book
- This section has limited relevance to Stage 6 Standard

Since pronumerals represent numbers, the rules for algebraic fractions are the same as those for simple numerical fractions. This includes processes such as cancelling common factors to simplify the calculation and dividing by multiplying by the reciprocal of a fraction.

The process of cancelling requires cancelling of factors, for example:

$$\frac{8}{12} = \frac{2 \times \cancel{4}^{1}}{3 \times \cancel{4}_{1}} = \frac{2}{3}$$

For algebraic fractions, you need to factorise the expressions to identify and cancel common factors.

Lesson starter: Expressions as products of their factors

Factorise these expressions to write them as a product of their factors. Fill in the blanks and simplify.

$\frac{2x+4}{2} = \frac{\square(\square)}{2} = \square$

$\frac{6x+9}{3} = \frac{\square(\square)}{3} = \square$

$\frac{x^2+2x}{x} = \frac{\square(\square)}{x} = \square$

$\frac{4x+4}{4} = \frac{\square(\square)}{4} = \square$

Describe the errors made in these factorisations.

$\frac{\cancel{3}^{1}x+2}{\cancel{3}^{1}} = x+2$

$\frac{x^2+\cancel{3x}^{1}}{\cancel{3x}^{1}} = x^2+1$

$\frac{\cancel{6x}^{1}+6}{\cancel{x}^{1}+1} = \frac{12}{1} = 12$

KEY IDEAS

- Simplify algebraic fractions by cancelling common factors in factorised form.

For example, $\frac{4x+6}{2} = \frac{\cancel{2}_{1}(2x+3)}{\cancel{2}_{1}}$

$= 2x+3$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- To multiply algebraic fractions:
  - Factorise expressions if possible.
  - Cancel common factors.
  - Multiply numerators and denominators together.

$$\frac{(x+1)^{1}}{10_{2}} \times \frac{{}^{1}5x}{4(x+1)_{1}} = \frac{x}{8}$$

- To divide algebraic fractions:
  - Multiply by the **reciprocal** of the fraction following the division sign (e.g. the reciprocal of 6 is $\frac{1}{6}$, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$).
  - Follow the rules for multiplication.

$$\frac{2}{(x-2)} \div \frac{8}{3(x-2)}$$
$$= \frac{2_{1}}{(x-2)_{1}} \times \frac{3(x-2)_{1}}{8_{4}}$$
$$= \frac{3}{4}$$

Key terminology: algebraic fraction, common factor, factorise, numerator, denominator, reciprocal

Exercise 3F

UNDERSTANDING	1–3	2, 3

1 Write these fractions in simplest form by cancelling common factors.

a $\frac{14}{21}$ **b** $\frac{9}{12}$ **c** $\frac{8x}{20}$ **d** $\frac{4x}{10}$

Hint: Be sure to cancel the *highest* common factor.

2 Write the reciprocal of these fractions.

a $\frac{3}{2}$ **b** $\frac{5x}{3}$ **c** 7 **d** $\frac{x+3}{4}$

Hint: The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

3 Follow the rules for multiplication and division to simplify these numeric fractions. Cancel common factors before multiplying.

a $\frac{15}{21} \times \frac{14}{25}$ **b** $\frac{4}{27} \div \frac{16}{9}$

FLUENCY	4–7(½)	4–7(½)

Example 19 Cancelling common factors

Simplify by cancelling common factors.

a $\frac{8xy}{12x}$ **b** $\frac{3(x+2)}{6(x+2)}$

SOLUTION

a $\frac{8xy}{12x} = \frac{{}^{2}\cancel{8} \times \cancel{x}^{1} \times y}{{}^{3}\cancel{12} \times \cancel{x}^{1}}$

$= \frac{2y}{3}$

EXPLANATION

Cancel the highest common factor of 8 and 12 (i.e. 4) and cancel the x.

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b $\dfrac{3(x+2)}{6(x+2)} = \dfrac{{}^{1}\cancel{3} \times \cancel{(x+2)}^{1}}{{}_{2}\cancel{6} \times \cancel{(x+2)}^{1}}$

$= \dfrac{1}{2}$

Cancel the highest common factors: 3 and $(x+2)$.

Now you try

Simplify by cancelling common factors.

a $\dfrac{18ab}{8b}$ **b** $\dfrac{5(x-1)}{15(x-1)}$

4 Simplify by cancelling common factors.

a $\dfrac{6xy}{12x}$ **b** $\dfrac{12ab}{30b}$ **c** $\dfrac{8x^2}{40x}$

d $\dfrac{25x^2}{5x}$ **e** $\dfrac{3(x+1)}{3}$ **f** $\dfrac{7(x-5)}{7}$

g $\dfrac{4(x+1)}{8}$ **h** $\dfrac{5(x-2)}{x-2}$ **i** $\dfrac{4(x-3)}{x-3}$

j $\dfrac{6(x+2)}{12(x+2)}$ **k** $\dfrac{9(x+3)}{3(x+3)}$ **l** $\dfrac{15(x-4)}{10(x-4)}$

Hint: Cancel the HCF of the numerals and pronumerals.

Example 20 Simplifying by factorising

Simplify these fractions by factorising first.

a $\dfrac{9x-12}{3}$ **b** $\dfrac{4x+8}{x+2}$

SOLUTION

a $\dfrac{9x-12}{3} = \dfrac{{}^{1}\cancel{3}(3x-4)}{\cancel{3}_{1}}$

$= 3x-4$

b $\dfrac{4x+8}{x+2} = \dfrac{4\cancel{(x+2)}_{1}}{\cancel{x+2}_{1}}$

$= 4$

EXPLANATION

Factorise the expression in the numerator, which has HCF = 3. Then cancel the common factor of 3.

4 is the HCF in the numerator.
After factorising, $(x+2)$ can be seen as a common factor and can be cancelled.

Now you try

Simplify these fractions by factorising first.

a $\dfrac{16x-8}{8}$ **b** $\dfrac{3x-6}{x-2}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Simplify these fractions by factorising first.

a $\frac{4x+8}{4}$ **b** $\frac{6a-30}{6}$ **c** $\frac{8y-12}{4}$

d $\frac{14b-21}{7}$ **e** $\frac{3x+9}{x+3}$ **f** $\frac{4x-20}{x-5}$

g $\frac{6x+9}{2x+3}$ **h** $\frac{12x-4}{3x-1}$ **i** $\frac{x^2+2x}{x}$

j $\frac{x^2-5x}{x}$ **k** $\frac{2x^2+6x}{2x}$ **l** $\frac{x^2+4x}{x+4}$

m $\frac{x^2-7x}{x-7}$ **n** $\frac{2x^2-4x}{x-2}$ **o** $\frac{3x^2+6x}{x+2}$

Hint: Cancel after you have factorised the numerator.

Example 21 Multiplying algebraic fractions

Simplify these products.

a $\frac{12}{5x} \times \frac{10x}{9}$ **b** $\frac{3(x-1)}{10} \times \frac{15}{x-1}$

SOLUTION

a $\frac{\cancel{12}^{4}}{\cancel{5x}_{1}} \times \frac{\cancel{10x}^{2}}{\cancel{9}_{3}}$

$= \frac{8}{3}\left(= 2\frac{2}{3}\right)$

b $\frac{3\cancel{(x-1)}^{1}}{\cancel{10}^{2}} \times \frac{\cancel{15}_{3}}{\cancel{x-1}_{1}}$

$= \frac{9}{2}\left(= 4\frac{1}{2}\right)$

EXPLANATION

Cancel common factors between numerators and denominators: $5x$ and 3.

Then multiply the numerators and the denominators.

Cancel the common factors, which are $(x-1)$ and 5.

Multiply numerators and denominators.

Now you try

Simplify these products.

a $\frac{20}{3x} \times \frac{6x}{25}$ **b** $\frac{4(x+2)}{9} \times \frac{12}{x+2}$

6 Simplify these products.

a $\frac{3}{x} \times \frac{2x}{9}$ **b** $\frac{4x}{5} \times \frac{15}{8x}$ **c** $\frac{9a}{14} \times \frac{7}{6a}$

d $\frac{2x^2}{5} \times \frac{25}{6x}$ **e** $\frac{4y^2}{7} \times \frac{21}{8y}$ **f** $\frac{x+1}{6} \times \frac{5}{x+1}$

g $\frac{x+3}{9} \times \frac{4}{x+3}$ **h** $\frac{4(y-7)}{2} \times \frac{5}{y-7}$ **i** $\frac{10}{a+6} \times \frac{3(a+6)}{4}$

j $\frac{4(x-2)}{7} \times \frac{14}{5(x-2)}$ **k** $\frac{3(x+2)}{2x} \times \frac{8}{9(x+2)}$ **l** $\frac{4(2x+1)}{3x} \times \frac{9x}{2x+1}$

Hint: Cancel any common factors between numerators and denominators before multiplying.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 22 Dividing algebraic fractions

Simplify the following.

a $\frac{3x^2}{8} \div \frac{9x}{4}$

b $\frac{2(x-2)}{3} \div \frac{x-2}{6}$

SOLUTION

a $\frac{3x^2}{8} \div \frac{9x}{4} = \frac{3x^{\cancel{2}1}}{8_{\cancel{2}}} \times \frac{\cancel{4}_1}{\cancel{3}\cancel{9}x^{\cancel{1}}}$

$= \frac{x}{6}$

EXPLANATION

Multiply by the reciprocal of the second fraction.

The reciprocal of $\frac{9x}{4}$ is $\frac{4}{9x}$.

Cancel common factors: $3x$ and 4.

Note: $\frac{3x^2}{9x} = \frac{\cancel{3}^1 \times x \times \cancel{x}^1}{\cancel{3}\cancel{9} \times \cancel{x}^1}$

Multiply the numerators and the denominators.

b $\frac{2(x-2)}{3} \div \frac{x-2}{6} = \frac{2\cancel{(x-2)}^1}{\cancel{3}^1} \times \frac{\cancel{6}^2}{\cancel{x-2}^1}$

$= 4$

The reciprocal of $\frac{x-2}{6}$ is $\frac{6}{x-2}$.

Cancel the common factors $(x-2)$ and 3, and multiply. Recall: $\frac{4}{1} = 4$.

Now you try

Simplify the following.

a $\frac{7x^2}{10} \div \frac{14x}{5}$

b $\frac{3(x+1)}{4} \div \frac{x+1}{12}$

7 Simplify the following.

a $\frac{x}{5} \div \frac{x}{15}$

b $\frac{3x}{10} \div \frac{x}{20}$

c $\frac{4a^2}{9} \div \frac{a}{18}$

d $\frac{3x^2}{10} \div \frac{6x}{5}$

e $\frac{4a}{9} \div \frac{5a^2}{6}$

f $\frac{2x}{7} \div \frac{x^2}{14}$

g $\frac{x+4}{2} \div \frac{x+4}{6}$

h $\frac{5(x-2)}{8} \div \frac{x-2}{4}$

i $\frac{3(x+4)}{10} \div \frac{6(x+4)}{15}$

j $\frac{2}{5(2x-1)} \div \frac{10}{2x-1}$

k $\frac{2(x-3)}{x-4} \div \frac{x-3}{5(x-4)}$

l $\frac{3(x+1)}{14(x-1)} \div \frac{6(x+1)}{35(x-1)}$

Hint: To divide, multiply by the reciprocal of the fraction following the division sign.
$\frac{x}{5} \div \frac{x}{15} = \frac{x}{5} \times \frac{15}{x}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

8, 9(½) | 8–10(½)

8 Find the error in the simplifying of these fractions and correct it.

a $\dfrac{3x+6}{3} = 3x+2$

b $\dfrac{x^2+2x}{x} = x^2+2$

c $\dfrac{4x}{5} \div \dfrac{10x}{3} = \dfrac{4x}{5} \times \dfrac{10x}{3}$

$= \dfrac{8x^2}{3}$

d $\dfrac{x+4}{15} \times \dfrac{3}{x} = \dfrac{4}{5}$

Hint: Remember that common factors can be easily identified when expressions are in factorised form.

9 Simplify these algebraic fractions by factorising expressions first.

a $\dfrac{7a+14a^2}{21a}$

b $\dfrac{4x+8}{5x+10}$

c $\dfrac{x^2+3x}{4x+12}$

d $\dfrac{2m+4}{15} \times \dfrac{3}{m+2}$

e $\dfrac{5-x}{12} \times \dfrac{14}{15-3x}$

f $\dfrac{x^2+2x}{4} \times \dfrac{8}{3x+6}$

g $\dfrac{2x-1}{10} \div \dfrac{4x-2}{25}$

h $\dfrac{2x+4}{6x} \div \dfrac{3x+6}{x^2}$

i $\dfrac{2x^2-4x}{3x-6} \div \dfrac{6x}{x+5}$

10 By removing a negative factor, further simplifying is sometimes possible.

For example, $\dfrac{-2x-4}{x+2} = \dfrac{-2(x+2)}{x+2} = -2$.

Use this idea to simplify the following.

Hint: Taking out a negative factor changes the sign of each term inside the brackets.

a $\dfrac{-3x-9}{x+3}$

b $\dfrac{-4x-10}{2x+5}$

c $\dfrac{-x^2-4x}{x+4}$

d $\dfrac{-3x^2-6x}{-9x}$

e $\dfrac{-2x+12}{-2}$

f $\dfrac{-10x+15}{-5}$

ENRICHMENT: Cancelling of powers

— | 11

11 Just as $\dfrac{x^2}{x} = x$, $\dfrac{(x+1)^2}{x+1} = x+1$. Use this idea to simplify these algebraic fractions.

Some will need factorising first.

a $\dfrac{(x+1)^2}{8} \times \dfrac{4}{x+1}$

b $\dfrac{(x+1)^2}{7x} \times \dfrac{14x}{3(x+1)}$

c $\dfrac{9}{x-2} \div \dfrac{18}{(x-2)^2}$

d $\dfrac{(x+2)^2}{10} \times \dfrac{5}{4x+8}$

e $\dfrac{(x-3)^2}{9x} \times \dfrac{3x}{4x-12}$

f $\dfrac{15}{8x+4} \div \dfrac{6}{(2x+1)^2}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3G Simplifying algebraic fractions: Addition and subtraction

Learning intentions for this section:

- To know the steps for adding and subtracting algebraic fractions
- To be able to find the lowest common denominator of fractions
- To be able to add and subtract algebraic fractions

Past, present and future learning:

- This section includes concepts from the Stage 5 Core topic called *Algebraic techniques A*
- It consolidates and extends the concepts in Section 9G of our Year 9 book
- This section has limited relevance to Stage 6 Standard

As with multiplying and dividing, the steps for adding and subtracting numerical fractions can be applied to algebraic fractions. A lowest common denominator is required before the fractions can be combined.

Lesson starter: Steps for adding fractions

- Write out the list of steps you would give to someone to show them how to add $\frac{3}{5}$ and $\frac{2}{7}$.
- Follow your steps to add the fractions $\frac{3x}{5}$ and $\frac{2x}{7}$.
- What is different when these steps are applied to $\frac{x+2}{5}$ and $\frac{x}{7}$?

KEY IDEAS

■ To add or subtract algebraic fractions:

- Determine the lowest common denominator (LCD)
  For example, the LCD of 3 and 5 is 15,
  the LCD of 4 and 12 is 12.
- Write each fraction as an equivalent fraction by multiplying the denominator(s) to equal the LCD. When denominators are multiplied, numerators should also be multiplied.
- Add or subtract the numerators.

For example: $\frac{x}{3} + \frac{2x}{5} = \frac{x}{3} \overset{(\times 5)}{\underset{(\times 5)}{}} + \frac{2x}{5} \overset{(\times 3)}{\underset{(\times 3)}{}}$ (LCD of 3 and 5 is 15.)

$$= \frac{5x}{15} + \frac{6x}{15}$$

$$= \frac{11x}{15}$$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

For example: $\frac{2x}{4} - \frac{x}{12} = \frac{2x}{4}\,{}^{(\times 3)} - \frac{x}{2}$ (LCD of 4 and 12 is 12.)

$= \frac{6x}{12} - \frac{x}{12}$

$= \frac{5x}{12}$

- To express $\frac{x+1}{3}$ with a denominator of 12, both the numerator and denominator must be multiplied by 4 with brackets required to multiply the numerator:
  $\frac{(x+1)}{3}\,{}^{(\times 4)}_{(\times 4)} = \frac{4x+4}{12}$

Key terminology: lowest common denominator, equivalent fraction, algebraic fraction, numerator, denominator

Exercise 3G

UNDERSTANDING 1–3 3

1 Write down the lowest common denominator (LCD) for these pairs of fractions.

a $\frac{2x}{5}, \frac{x}{4}$ **b** $\frac{x}{3}, \frac{x}{12}$ **c** $\frac{3x}{10}, \frac{2x}{15}$

Hint: The LCD is not always the two denominators multiplied together; e.g. $3 \times 6 = 18$ but the LCD of 3 and 6 is 6.

2 Complete these equivalent fractions by giving the missing term.

a $\frac{x}{4} = \frac{\square}{12}$ **b** $\frac{2x}{5} = \frac{\square}{15}$ **c** $\frac{x-1}{4} = \frac{\square}{20}(x-1)$

Hint: For equivalent fractions, the numerator must be multiplied by the same amount as the denominator.

3 Complete the following by filling in the boxes.

a $\frac{x}{4} + \frac{x}{5} = \frac{\square}{20} + \frac{\square}{20}$

$= \frac{\square}{20}$

b $\frac{2x}{5} - \frac{x}{10} = \frac{\square}{10} - \frac{\square}{10}$

$= \frac{\square}{10}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 4–6(½) 4–6(½)

Example 23 Adding and subtracting algebraic fractions

Simplify the following.

a $\frac{x}{2}+\frac{x}{3}$ b $\frac{4x}{5}-\frac{x}{2}$ c $\frac{x}{2}-\frac{5}{6}$

SOLUTION

a $\frac{x}{2}\overset{(\times 3)}{}+\frac{x}{3}\overset{(\times 2)}{}=\frac{3x}{6}+\frac{2x}{6}$

$=\frac{5x}{6}$

b $\frac{4x}{5}\overset{(\times 2)}{}-\frac{x}{2}\overset{(\times 5)}{}=\frac{8x}{10}-\frac{5x}{10}$

$=\frac{3x}{10}$

c $\frac{x}{2}\overset{(\times 3)}{}-\frac{5}{6}=\frac{3x}{6}-\frac{5}{6}$

$=\frac{3x-5}{6}$

EXPLANATION

The LCD of 2 and 3 is 6.

Express each fraction with a denominator of 6 and add numerators.

The LCD of 5 and 2 is 10.

Express each fraction with a denominator of 10 and subtract $5x$ from $8x$.

The LCD of 2 and 6 is 6. Multiply the numerator and denominator of $\frac{x}{2}$ by 3 to express with a denominator of 6.

Write as a single fraction; $3x-5$ cannot be simplified.

Now you try

Simplify the following.

a $\frac{x}{4}+\frac{x}{5}$ b $\frac{3x}{2}-\frac{x}{7}$ c $\frac{x}{3}+\frac{7}{9}$

4 Simplify the following.

a $\frac{x}{3}+\frac{x}{4}$ b $\frac{x}{5}+\frac{x}{2}$ c $\frac{x}{3}-\frac{x}{9}$

d $\frac{x}{5}-\frac{x}{7}$ e $\frac{2x}{3}+\frac{x}{5}$ f $\frac{3x}{4}+\frac{5x}{12}$

g $\frac{5x}{6}-\frac{4x}{9}$ h $\frac{7x}{10}-\frac{3x}{8}$ i $\frac{x}{7}-\frac{x}{2}$

j $\frac{x}{10}-\frac{2x}{5}$ k $\frac{5x}{6}-\frac{13x}{15}$ l $\frac{3x}{10}-\frac{3x}{2}$

Hint: Express each fraction with a common denominator using the LCD, then add or subtract numerators.

5 Simplify the following.

a $\frac{x}{2}+\frac{3}{4}$ b $\frac{x}{5}+\frac{2}{3}$ c $\frac{2x}{15}+\frac{7}{20}$

d $\frac{x}{4}-\frac{2}{5}$ e $\frac{2x}{3}-\frac{5}{9}$ f $\frac{5}{6}-\frac{x}{4}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 24 Adding and subtracting with binomial numerators

Simplify the following algebraic expressions.

a $\frac{x+2}{4} - \frac{x}{6}$ **b** $\frac{x+3}{3} + \frac{x-4}{7}$

SOLUTION

a $\frac{(x+2)}{4}\,{}^{(\times 3)} - \frac{x}{6}\,{}^{(\times 2)} = \frac{3(x+2)}{12} - \frac{2x}{12}$

$= \frac{3x+6-2x}{12}$

$= \frac{x+6}{12}$

EXPLANATION

The LCD of 4 and 6 is 12.

Express each fraction with a denominator of 12.

When multiplying $(x+2)$ by 3, brackets are required.

Expand the brackets and collect the terms: $3x+6-2x = 3x-2x+6$

b $\frac{(x+3)}{3}\,{}^{(\times 7)} + \frac{(x-4)}{7}\,{}^{(\times 3)} = \frac{7(x+3)}{21} + \frac{3(x-4)}{21}$

$= \frac{7x+21+3x-12}{21}$

$= \frac{10x+9}{21}$

The LCD of 3 and 7 is 21.

Express each fraction with a denominator of 21.

Expand each pair of brackets first and sum by collecting like terms.

Now you try

Simplify the following algebraic expressions.

a $\frac{x-3}{10} - \frac{x}{15}$ **b** $\frac{x+2}{4} + \frac{x-3}{5}$

6 Simplify these algebraic expressions.

a $\frac{x+2}{3} + \frac{x}{2}$ **b** $\frac{x+4}{5} + \frac{2x}{3}$

c $\frac{x-2}{4} + \frac{3x}{8}$ **d** $\frac{x+4}{3} - \frac{x}{6}$

e $\frac{x+2}{2} - \frac{2x}{5}$ **f** $\frac{6x+7}{12} - \frac{3x}{8}$

g $\frac{x+3}{5} + \frac{x+2}{4}$ **h** $\frac{2x+3}{7} + \frac{x+1}{2}$

i $\frac{x+8}{6} + \frac{x-3}{4}$ **j** $\frac{2x+5}{3} + \frac{x-2}{4}$

k $\frac{x-3}{5} + \frac{x+4}{10}$ **l** $\frac{2x+1}{8} + \frac{x-2}{3}$

Hint: LCD of 2 and 3 is 6:

$\frac{x+2}{3} + \frac{x}{2} = \frac{\square(x+2)}{6} + \frac{\square x}{6}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

7, 8 | 7–9(½)

7 Find the error in each of the following and then correct it.

a $\frac{2x}{3}+\frac{3x}{4}=\frac{5x}{12}$

b $\frac{3x}{5}-\frac{x}{2}=\frac{2x}{3}$

c $\frac{x+2}{5}+\frac{x+4}{3}=\frac{3x+2+5x+4}{15}$

$=\frac{8x+6}{15}$

d $\frac{x+4}{2}+\frac{x-3}{6}=\frac{3x+12+x+3}{6}$

$=\frac{4x+15}{6}$

8 Recall that the expansion of $-5(x-2)$ is $-5x+10$, so $6(x+1)-5(x-2)=6x+6-5x+10=x+16$.
Use this method to simplify these subtractions.

a $\frac{x+1}{5}-\frac{x-2}{6}$

b $\frac{x+2}{3}-\frac{x-4}{5}$

c $\frac{x-3}{4}-\frac{x+2}{5}$

d $\frac{x+8}{2}-\frac{x+7}{4}$

9 The LCD of the fractions $\frac{4}{x}+\frac{2}{3}$ is $3\times x=3x$.
Use this to find the LCD and simplify these fractions.

a $\frac{4}{x}+\frac{2}{3}$

b $\frac{3}{4}+\frac{2}{x}$

c $\frac{2}{5}+\frac{3}{x}$

d $\frac{3}{7}-\frac{2}{x}$

e $\frac{1}{5}-\frac{4}{x}$

f $\frac{3}{x}-\frac{5}{8}$

Hint:
$\frac{4}{x}+\frac{2}{3}=\frac{\square}{3x}+\frac{\square}{3x}$
$=\frac{\square}{3x}$

ENRICHMENT: Pronumerals in the denominator

— | 10

10 As seen in Question **9**, pronumerals may form part of the LCD.
The fractions $\frac{5}{2x}$ and $\frac{3}{4}$ would have an LCD of $4x$, whereas the fractions $\frac{3}{x}$ and $\frac{5}{x^2}$ would have an LCD of x^2.
By first finding the LCD, simplify these algebraic fractions.

a $\frac{3}{4}+\frac{5}{2x}$

b $\frac{1}{6}+\frac{5}{2x}$

c $\frac{3}{10}-\frac{1}{4x}$

d $\frac{3}{x}+\frac{5}{x^2}$

e $\frac{4}{x}+\frac{1}{x^2}$

f $\frac{3}{x^2}-\frac{5}{x}$

g $\frac{3}{2x}+\frac{2}{x^2}$

h $\frac{4}{x}+\frac{7}{3x^2}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Progress quiz

3A **1** For the expression $2x + \frac{y}{2} - 3x^2 + 5$, determine:

a the number of terms

b the constant term

c the coefficient of:

i x^2

ii y

3A **2** Find the value of the following expressions if $a = 2$, $b = -5$ and $c = 8$.

a $ab + 2c$ **b** $b^2 - ac$ **c** $\frac{c}{a} - 2b$

3B **3** Simplify the following by collecting like terms.

a $4x - 3 + 2x$

b $7x - 3y - 2x + 8y$

c $3x^2y + 5xy^2 - x^2y + 2xy^2$

3B **4** Simplify the following.

a $3r \times 4rs$ **b** $\frac{2x}{6}$ **c** $15mn^2 \div (6mn)$

3C **5** Expand the following.

a $3(2x + 3)$

b $4x(5x - 2)$

c $-6(2x - 3)$

3C **6** Expand and simplify the following.

a $4 + 2(4x - 5)$

b $4(2x + 3) - 5(x - 4)$

3D **7** Expand and simplify.

a $(3x + 2)(4x + 5)$ **b** $(3x + 2)(4x - 5)$

c $(3x + 2)^2$ **d** $(3x + 2)(3x - 2)$

3E **8** Factorise the following by first identifying the highest common factor. (Include any common negatives.)

a $6m + 12$ **b** $15a - 20ab$

c $4xy + x$ **d** $6x^2 - 10x$

e $-8x - 20$ **f** $-3y^2 - 6y$

3F **9** Simplify by cancelling common factors. You will need to factorise first in parts **b** and **c**.

a $\frac{8(x - 1)}{4(x - 1)}$ **b** $\frac{15x - 35}{5}$ **c** $\frac{2x^2 + 6x}{x + 3}$

3F **10** Simplify the following algebraic fractions.

a $\frac{8x}{7} \times \frac{21}{16x}$ **b** $\frac{9x}{4(x - 1)} \div \frac{12}{x - 1}$

3G **11** Simplify the following algebraic fractions.

a $\frac{3x}{5} + \frac{x}{4}$ **b** $\frac{2x}{3} - \frac{4}{9}$

3G **12** Simplify $\frac{x + 1}{4} + \frac{x - 2}{6}$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3H Index laws for multiplying and dividing

Learning intentions for this section:

- To understand index notation and know that it is a way of writing repeated multiplication
- To know the index laws for multiplication and division involving a common base
- To be able to apply the index laws for multiplication and division to simplify expressions

Past, present and future learning:

- This section includes concepts from the Stage 5 Core topic called *Indices A*
- It consolidates and extends the concepts in Sections 7A and 7B of our Year 9 book
- This section has limited relevance to Stage 6 Standard

When a product has the same number multiplied by itself over and over, index notation can be used to write a simpler expression. For example:

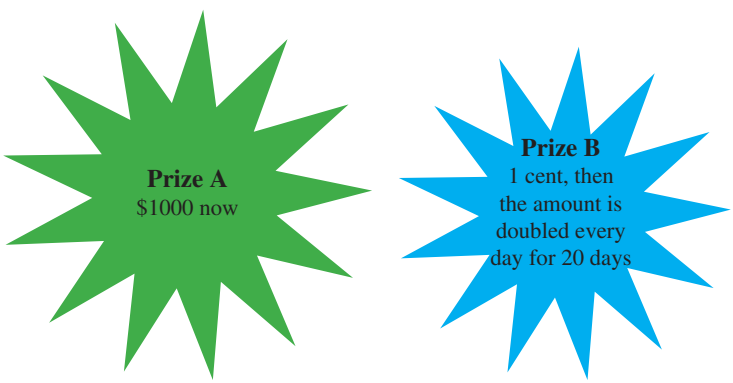
- $5 \times 5 \times 5$ can be written as 5^3
- $x \times x \times x \times x \times x$ can be written as x^5.

Index notation is a way to carry out calculations, such as how much mass is lost over time from ancient stone monuments.

Lesson starter: Who has the most?

A person offers you one of two prizes.

- Which offer would you take?
- Try to calculate the final amount for prize B.
- How might you use index notation to help calculate the value of prize B?
- How can a calculator help to find the amount for prize B using the power button?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- When a number is multiplied by itself many times, that product can be written using **index form**.
  For example:

Expanded form: $\overbrace{2 \times 2 \times 2 \times 2 \times 2}$ = Index form: $\overbrace{2^5}$ = 32

Base: 2; Index: 5; Basic numeral: 32

$x \times x \times x \times x$ = x^4

Base: x; Index: 4

- The **base** is the factor in the product.
- The **index** is the number of times the factor (base number) appears.
  - 2^2 reads '2 to the power of 2' or '2 squared', where $2^2 = 4$.
  - 2^3 reads '2 to the power of 3' or '2 cubed', where $2^3 = 8$.
  - 2^5 reads '2 to the power of 5', where $2^5 = 32$.

  Note that $a^1 = a$. For example: $5^1 = 5$
  - 3^2 does not mean $3 \times 2 = 6$.
- Index law for multiplication: $a^m \times a^n = a^{m+n}$
  - When multiplying terms with the same base, add the indices.
    For example: $7^3 \times 7^2 = 7^{3+2} = 7^5$
- Index law for division: $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$
  - When dividing terms with the same base, subtract the indices.
    For example: $8^5 \div 8^3 = \frac{8^5}{8^3} = 8^{5-3} = 8^2$

Key terminology: index form, exponent, power, base, index

Exercise 3H

UNDERSTANDING | 1–4 | 3, 4

1 Fill in the missing words, using *index, power, multiply, expanded* and *base*.

a In 3^5, 3 is the _____ and 5 is the _____.

b 4^6 is read as 4 to the _____ of 6.

c 7^4 is the _____ form of $7 \times 7 \times 7 \times 7$.

d The power tells you how many times to _____ the base number by itself.

e $6 \times 6 \times 6$ is the _____ form of 6^3.

Hint: $5^{\overset{\text{4 times}}{\textcircled{4}}} = \overbrace{5 \times 5 \times 5 \times 5}$ (Index form / Expanded form)

2 Write the following in expanded form.

a 8^3 **b** 7^5

c x^6 **d** $(ab)^4$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Complete the following to write each as a single term in index form.

a $7^3 \times 7^4 = 7 \times 7 \times 7 \times \square$
$= 7^{\square}$

b $\frac{5^6}{5^2} = \frac{5 \times 5 \times \square}{5 \times 5}$
$= 5^{\square}$

4 Choose from the words *add* or *subtract* to fill in the missing words.

a When two terms with the same base are multiplied, _____ the powers.

b When two terms with the same base are divided, _____ the powers.

FLUENCY 5–7(½) 5–7(½)

Example 25 Writing in index form

Write each of the following in index form.

a $5 \times 5 \times 5$ **b** $4 \times x \times x \times 4 \times x$ **c** $a \times b \times b \times a \times b \times b$

SOLUTION

a $5 \times 5 \times 5 = 5^3$

b $4 \times x \times x \times 4 \times x = 4 \times 4 \times x \times x \times x$
$= 4^2x^3$

c $a \times b \times b \times a \times b \times b$
$= a \times a \times b \times b \times b \times b$
$= a^2b^4$

EXPLANATION

The factor 5 appears 3 times.

Group the factors of 4 and the factors of x together.

The factor x appears 3 times; 4 appears twice.

Group the like pronumerals.

The factor a appears twice and the factor b appears 4 times.

Now you try

Write each of the following in index form.

a $3 \times 3 \times 3 \times 3$ **b** $7 \times y \times 7 \times y \times y$ **c** $m \times m \times n \times m \times n$

5 Write the following in index form.

a $9 \times 9 \times 9 \times 9$

b $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

c $15 \times 15 \times 15$

d $5 \times x \times x \times x \times 5$

e $4 \times a \times 4 \times a \times 4 \times a \times a$

f $b \times 7 \times b \times b \times b$

g $x \times y \times x \times x \times y$

h $a \times b \times a \times b \times b \times b$

i $3 \times x \times 3 \times y \times x \times 3 \times y \times y$

j $4 \times x \times z \times 4 \times z \times x$

Hint: Group different bases together and write each base in index form.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 26 Using the index law for multiplication

Simplify the following.

a $x^7 \times x^4$ **b** $a^2b^2 \times ab^3$ **c** $3x^2y^3 \times 4x^3y^4$

SOLUTION

a $x^7 \times x^4 = x^{7+4}$
$= x^{11}$

b $a^2b^2 \times ab^3 = a^{2+1}b^{2+3}$
$= a^3b^5$

c $3x^2y^3 \times 4x^3y^4 = (3 \times 4)x^{2+3}y^{3+4}$
$= 12x^5y^7$

EXPLANATION

Use $a^m \times a^n = a^{m+n}$ to add the indices.

Add the indices of base a and base b.

Recall that $a = a^1$.

Multiply the coefficients and add indices of the common bases x and y.

Now you try

Simplify the following.

a $x^5 \times x^3$ **b** $x^2y \times x^3y^4$ **c** $5ab^2 \times 2a^3b^4$

6 Simplify the following.

a $x^3 \times x^4$ **b** $p^5 \times p^2$
c $t^7 \times t^2$ **d** $d^4 \times d$
e $g \times g^3$ **f** $f^2 \times f$
g $2p^2 \times p^3$ **h** $3c^4 \times c^4$
i $2s^4 \times 3s^7$ **j** $a^2b^3 \times a^3b^5$
k $d^7f^3 \times d^2f^2$ **l** $v^3z^5 \times v^2z^3$
m $3a^2b \times 5ab^5$ **n** $2x^2y \times 3xy^2$
o $3e^7r^2 \times 6e^2r$ **p** $-4p^3c^2 \times 2pc$
q $-2r^2s^3 \times 5r^5s^5$ **r** $-3d^4f^2 \times (-2f^2d^2)$

Hint: Group common bases and add indices when multiplying.

Example 27 Using the index law for division

Simplify the following.

a $p^5 \div p^3$
b $12m^8 \div (6m^3)$
c $\dfrac{4x^2y^4}{8xy^2}$

SOLUTION

a $p^5 \div p^3 = p^{5-3}$
$= p^2$

EXPLANATION

Use $a^m \div a^n = a^{m-n}$ to subtract the indices.

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b $12m^8 \div (6m^3) = \dfrac{{}^{2}\cancel{12}m^8}{{}^{1}\cancel{6}m^3}$ — Write in fraction form.

$= 2m^{8-3}$ — Cancel the highest common factor of 12 and 6.

$= 2m^5$ — Subtract the indices.

c $\dfrac{4x^2y^4}{8xy^2} = \dfrac{{}^{1}\cancel{4} \times x^2 \times y^4}{{}^{2}\cancel{8} \times x \times y^2}$ — Cancel the common factors of the numerals and subtract the indices of base x and base y.

$= \dfrac{x^{2-1}y^{4-2}}{2}$

$= \dfrac{xy^2}{2}$ $\left(\text{or } \dfrac{1}{2}xy^2\right)$

Now you try

Simplify the following.

a $b^7 \div b^2$ **b** $20a^6 \div (8a^2)$ **c** $\dfrac{6a^3b^4}{9ab^3}$

7 Simplify the following.

a $a^4 \div a^2$ **b** $d^7 \div d^6$

c $r^3 \div r$ **d** $\dfrac{c^{10}}{c^6}$

e $\dfrac{l^4}{l^3}$ **f** $\dfrac{b^5}{b^2}$

g $\dfrac{4d^4}{d^2}$ **h** $\dfrac{f^2}{2f^2}$

i $\dfrac{9n^4}{3n}$ **j** $6p^4 \div (3p^2)$

k $24m^7 \div (16m^3)$ **l** $10d^3 \div (30d)$

m $\dfrac{8t^4r^3}{2tr^2}$ **n** $\dfrac{5h^6d^4}{3d^3h^2}$

o $\dfrac{2p^2q^3}{p^2q}$ **p** $\dfrac{4x^2y^3}{8xy}$

q $\dfrac{3r^5s^2}{9r^3s}$ **r** $6c^4d^6 \div (15c^3d)$

s $2a^4y^2 \div (4ay)$ **t** $13m^4n^6 \div (26m^4n^5)$ **u** $18x^4y^3 \div (-3x^2y)$

Hint:
$a^m \div a^n = a^{m-n}$
or
$\dfrac{a^m}{a^n} = a^{m-n}$
When dividing, subtract indices of common bases.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING | 8 | 8(½), 9

Example 28 Combining index laws

Simplify $\dfrac{2a^3b \times 3a^2b^3}{12a^4b^2}$

SOLUTION

$$\frac{2a^3b \times 3a^2b^3}{12a^4b^2} = \frac{(2 \times 3)a^{3+2}b^{1+3}}{12a^4b^2}$$

$$= \frac{\cancel{6}^{1}a^5b^4}{\cancel{2}\cancel{12}a^4b^2}$$

$$= \frac{a^{5-4}b^{4-2}}{2}$$

$$= \frac{ab^2}{2}$$

EXPLANATION

Simplify the numerator first by multiplying coefficients and adding the indices of a and b.

Cancel common factor of numerals and subtract indices of common bases.

Now you try

Simplify $\dfrac{4ab^2 \times 3a^5b^4}{18a^3b^5}$

8 Simplify the following.

a $\dfrac{x^2y^3 \times x^2y^4}{x^3y^5}$ **b** $\dfrac{m^2w \times m^3w^2}{m^4w^3}$ **c** $\dfrac{r^4s^7 \times r^4s^7}{r^6s^{10}}$

d $\dfrac{16a^8b \times 4ab^7}{32a^7b^6}$ **e** $\dfrac{9x^2y^3 \times 6x^7y^7}{12xy^6}$ **f** $\dfrac{4e^2w^2 \times 12e^2w^3}{12e^4w}$

9 When Stuart uses a calculator to raise -2 to the power 4 he gets -16, when in fact the answer is actually 16. What has he done wrong?

ENRICHMENT: Index laws and calculations | — | 10, 11

10 Consider the following use of negative numbers.

a Evaluate:

i $(-3)^2$ **ii** -3^2

b What is the difference between your two answers in part **a**?

c Evaluate:

i $(-2)^3$ **ii** -2^3

d What do you notice about your answers in part **c**? Explain.

Hint:
$(-3)^2 = -3 \times (-3)$
$(-2)^3 = -2 \times (-2) \times (-2)$
Consider order of operations for -3^2 and -2^3.

11 Evaluate these expressions without the use of a calculator.

a $\dfrac{13^3}{13^2}$ **b** $\dfrac{18^7}{18^6}$ **c** $\dfrac{9^8}{9^6}$ **d** $\dfrac{3^{10}}{3^7}$ **e** $\dfrac{4^8}{4^5}$ **f** $\dfrac{2^{12}}{2^8}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3I Powers of powers, powers of fractions and the zero index

Learning intentions for this section:

- To know the index laws for raising a power to another power and removing brackets
- To know the rule for the zero index
- To be able to simplify expressions using the index laws and the zero index

Past, present and future learning:

- This section includes concepts from the Stage 5 Core topic called *Indices A*
- It consolidates and extends the concepts in Sections 7C and 7D of our Year 9 book
- This section has limited relevance to Stage 6 Standard

The index laws can be used to simplify other expressions.

For example, $(4^2)^3 = 4^2 \times 4^2 \times 4^2$

$= 4^{2+2+2}$

$= 4^6$ (Add indices using the index for multiplication.)

Therefore, $(4^2)^3 = 4^{2\times 3} = 4^6$.

We also have a result for the zero power.

Consider $5^3 \div 5^3$, which clearly equals 1. Using index law for division, we can see that:

$5^3 \div 5^3 = 5^{3-3} = 5^0$

Therefore, $5^0 = 1$, which leads to the zero power rule: $a^0 = 1$ (where $a \neq 0$).

Lesson starter: Indices with brackets

Brackets are used to show that the power outside the brackets applies to each factor inside the brackets.

Consider $(2x)^3 = 2x \times 2x \times 2x$.

- Write this in index form without using brackets.
- Can you suggest the index form of $(3y)^4$ without brackets?

Consider $\left(\frac{3}{5}\right)^4 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$.

- Write the numerator and denominator of this expanded form in index form.
- Can you write the index form of $\left(\frac{x}{4}\right)^5$ without brackets?

Write a rule for removing the brackets of the following.

- $(ab)^m$
- $\left(\frac{a}{b}\right)^m$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- Index law for a power of a power: $(a^m)^n = a^{m \times n} = a^{mn}$
  When raising a term in index form to another power, retain the base and multiply the indices.
  For example: $(x^3)^4 = x^{3 \times 4} = x^{12}$.
  A power outside brackets applies only to the expression inside those brackets
  For example: $5(a^3)^2 = 5a^{3 \times 2} = 5a^6$ but $(5a^3)^2 = 5^2 \times a^6 = 25a^6$

- Power of a product: $(a \times b)^m = a^m \times b^m$
  Apply the index to each factor in the brackets.
  For example, $(3x)^4 = 3^4x^4$.

- Power of a fraction: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
  Apply the index to the numerator and denominator.
  For example: $\left(\frac{y}{3}\right)^5 = \frac{y^5}{3^5}$

- $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$
  If the power is -1, simply write down the reciprocal of the fraction.
  For example: $\left(\frac{5}{3}\right)^{-1} = \frac{3}{5}$

- The zero index: $a^0 = 1$, where $a \neq 0$
  Any number (except zero) to the power of zero is 1.
  For example, $5^0 = 1$, $y^0 = 1$, $4y^0 = 4 \times 1 = 4$.

Key terminology: index/indices, base

Exercise 3I

UNDERSTANDING 1, 2 | 2

1 Complete the following index laws.

a Any number (except 0) to the power of zero is equal to _____.

b $(a^m)^n =$ _____.

c $(a \times b)^m =$ _____ $\times$ _____.

d $\left(\frac{a}{b}\right)^m = \frac{\square}{\square}$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 Copy and complete the following.

a $(2a)^3 = 2a \times \square \times \square$

$= 2 \times \square \times \square \times a \times \square \times \square$

$= 2^{\square}a^{\square}$

b $\left(\frac{4}{7}\right)^4 = \frac{4}{7} \times \square \times \square \times \square$

$= \frac{4 \times \square \times \square \times \square}{7 \times \square \times \square \times \square}$

$= \frac{4^{\square}}{7^{\square}}$

FLUENCY 3–6(½) 3–6(½)

Example 29 Using the zero index: $a^0 = 1$

Evaluate the following.

a $4^0 + 2^0$ **b** $3a^0$ **c** $(-3)^0 + 6x^0$

SOLUTION

a $4^0 + 2^0 = 1 + 1$
$= 2$

b $3a^0 = 3 \times a^0$
$= 3 \times 1$
$= 3$

c $(-3)^0 + 6x^0 = 1 + 6 \times 1$
$= 7$

EXPLANATION

Any number, except zero, to the power zero is 1.

The zero index applies only to a, so $a^0 = 1$.

Any number, except zero, to the power of zero is 1.
$(-3)^0 = 1, 6x^0 = 6 \times x^0 = 6 \times 1$

Now you try

Evaluate the following.

a $5^0 - 3^0$ **b** $4y^0$ **c** $(-2)^0 + 3b^0$

3 Evaluate the following.

a 4^0 **b** 5^0 **c** x^0

d a^0 **e** $3e^0$ **f** $4y^0$

g $3^0 + 6^0$ **h** $10 - 10x^0$ **i** $3d^0 - 2$

j $(-4)^0 + 2x^0$ **k** $\frac{2}{m^0}$ **l** $5a^0 + 4b^0$

Hint: Any number (except zero) to the power of 0 is 1.
$4a^0 = 4 \times a^0 = 4 \times 1 = 4$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 30 Using the index law for the power of a power $(a^m)^n = a^{m \times n}$

Simplify the following.

a $(x^5)^7$ **b** $3(f^4)^3$

SOLUTION

a $(x^5)^7 = x^{5 \times 7}$
$= x^{35}$

b $3(f^4)^3 = 3f^{4 \times 3}$
$= 3f^{12}$

EXPLANATION

Apply the index law $(a^m)^n = a^{m \times n}$ to multiply indices.

Apply the index law to the value inside the bracket only.

Now you try

Simplify the following.

a $(y^3)^6$ **b** $5(m^2)^4$

4 Simplify the following.

a $(b^3)^4$ **b** $(f^5)^4$ **c** $(k^3)^7$

d $3(x^2)^3$ **e** $5(c^9)^2$ **f** $4(s^6)^3$

Example 31 Using the power of a product and power of a fraction rules

Simplify the following by using index laws.

a $(2s)^4$ **b** $(x^2y^3)^5$ **c** $\left(\frac{x}{4}\right)^3$

SOLUTION

a $(2s)^4 = 2^4 \times s^4$
$= 16s^4$

b $(x^2y^3)^5 = (x^2)^5 \times (y^3)^5$
$= x^{10}y^{15}$

c $\left(\frac{x}{4}\right)^3 = \frac{x^3}{4^3}$
$= \frac{x^3}{64}$

EXPLANATION

$(a \times b)^m = a^m \times b^m$.
Evaluate $2^4 = 2 \times 2 \times 2 \times 2$.

Apply the index 5 to each factor in the brackets.
Multiply indices:
$(x^2)^5 = x^{2 \times 5}, (y^3)^5 = y^{3 \times 5}$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Evaluate $4^3 = 4 \times 4 \times 4$.

Now you try

Simplify the following.

a $(3m)^3$ **b** $(a^3b^2)^4$ **c** $\left(\frac{y}{3}\right)^4$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Simplify the following.

a $(3x)^2$ **b** $(4m)^3$ **c** $(5y)^3$

d $(2x^3)^4$ **e** $(x^2y)^5$ **f** $(3a^3)^3$

g $(x^4y^2)^6$ **h** $(a^2b)^3$ **i** $(m^3n^3)^4$

j $\left(\frac{x}{5}\right)^2$ **k** $\left(\frac{y}{3}\right)^4$ **l** $\left(\frac{m}{2}\right)^4$

m $\left(\frac{x^2}{y}\right)^3$ **n** $\left(\frac{x^3}{y^2}\right)^4$ **o** $\left(\frac{x}{y^5}\right)^3$

Hint:
$(3 \times x)^2 = 3^2 \times x^2$
$\left(\frac{x}{5}\right)^2 = \frac{x^2}{5^2}$

Example 32 Combining index laws

Simplify the following.

a $\frac{3x^2y \times 2x^3y^2}{10xy^3}$ **b** $\left(\frac{2x^2}{y}\right)^4$ **c** $(2x^2)^3 + (3x)^0$

SOLUTION

a
$$\frac{3x^2y \times 2x^3y^2}{10xy^3}$$
$$= \frac{6x^5y^3}{10xy^3}$$
$$= \frac{3x^4y^0}{5}$$
$$= \frac{3x^4}{5}$$

EXPLANATION

Simplify the numerator by multiplying coefficients and adding indices.

Cancel the common factor of 6 and 10 and subtract indices of common bases.

The zero index says $y^0 = 1$.

b
$$\left(\frac{2x^2}{y}\right)^4 = \frac{(2x^2)^4}{y^4}$$
$$= \frac{2^4 \times (x^2)^4}{y^4}$$
$$= \frac{16x^8}{y^4}$$

Apply the index to the numerator and denominator.

Multiply indices.

c
$$(2x^2)^3 + (3x)^0 = 2^3 \times x^6 + 3^0 \times x^0$$
$$= 8x^6 + 1 \times 1$$
$$= 8x^6 + 1$$

Apply the power to each factor inside the brackets:
$(x^2)^3 = x^{2\times 3} = x^6$

Any number to the power of zero is 1.

Now you try

Simplify the following.

a $\frac{4a^2b^3 \times 3a^2b}{6a^3b^4}$ **b** $\left(\frac{3x}{y^2}\right)^3$ **c** $(5m^4)^2 + (5m)^0$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Simplify the following

a $\dfrac{m^7w \times m^3w^2}{m^4w^3}$

b $\dfrac{x^3y^2 \times x^2y^7}{10x^5y^4}$

c $\dfrac{b^3c^5 \times 4b^5c^3}{3b^4c^8}$

d $\dfrac{9c^4s^2 \times 3c^3s^5}{2c^3s^7}$

e $\dfrac{(5r^6)^2}{3r^8}$

f $\dfrac{(2p^4)^3}{3p^7}$

g $\left(\dfrac{2s^2}{t^3}\right)^4$

h $\left(\dfrac{r^2}{5s^3}\right)^4$

Hint: First simplify the numerator, then consider the denominator.

PROBLEM-SOLVING AND REASONING

7–8(½) | 7–8(½)

7 Evaluate the following without the use of a calculator.

a $\dfrac{(5^2)^2}{5^4}$

b $\dfrac{36^2}{6^4}$

c $\dfrac{27^2}{3^4}$

d $\dfrac{16^2}{4^3}$

Hint: $36^2 = (6^2)^2$

8 Simplify the following.

a $2p^2q^4 \times pq^3$

b $4(a^2b)^3 \times (3ab)^3$

c $(4r^2y)^2 \times r^2y^4 \times 3(ry^2)^3$

d $2(m^3n)^4 \div m^3$

e $\dfrac{(7s^2y)^2 \times 3sy^2}{7(sy)^2}$

f $\dfrac{3(d^4c^3)^3 \times 4dc}{(2c^2d)^3}$

g $\dfrac{4r^2t \times 3(r^2t)^3}{6r^2t^4}$

h $\dfrac{(2xy)^2 \times 2(x^2y)^3}{8xy \times x^7y^3}$

ENRICHMENT: All laws together

— | 9

9 Simplify the following, expressing your answer with positive indices.

a $(a^3b^2)^3 \times a^2b^4$

b $2x^2y \times (xy^4)^3$

c $2(p^2)^4 \times (3p^2q)^2$

d $\dfrac{2a^3b^2}{a^3} \times \dfrac{2a^2b^5}{b^4}$

e $\dfrac{(3rs^2)^4}{r^3s^4} \times \dfrac{(2r^2s)^2}{s^7}$

f $\dfrac{4(x^2y^4)^2}{x^2y^3} \times \dfrac{xy^4}{2s^2y}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3J Negative indices, algebraic bases EXTENDING

Learning intentions for this section:

- To know how negative indices can be equivalently expressed using positive indices
- To be able to express negative indices in terms of positive indices
- Extending: To be able to use the index laws with negative indices

Past, present and future learning:

- This section includes concepts from the Stage 5 Core topic called *Indices A*
- It consolidates and extends the concepts in Section 7E of our Year 9 book
- It also extends into the Stage 5 Path-to-Advanced topic called *Indices B*
- This section has limited relevance to Stage 6 Standard

In Section 7E of our Year 9 book we studied numerical bases with negative indices. In this section the bases are algebraic.

Consider $\frac{x^3}{x^5}$:

Expanding and simplifying gives

$$\frac{x^3}{x^5} = \frac{x^1 \times x^1 \times x^1}{x \times x \times x \times x \times x \times x}$$

$$= \frac{1}{x \times x}$$

$$= \frac{1}{x^2}$$

Using the index law for division, however, we get $\frac{x^3}{x^5} = x^{3-5}$

$$= x^{-2}$$

$$\therefore x^{-2} = \frac{1}{x^2}.$$

Lesson starter: Continuing the pattern

Complete this table to consider the value of powers of 3, including negative powers.

Index form	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	27	9				$\frac{1}{9} = \frac{1}{3^2}$	

÷ 3 ÷ 3 ÷ 3

Now complete this table.

Index form	x^3	x^2	x^1	x^0	x^{-1}	x^{-2}	x^{-3}
Fraction form	$\frac{x^3}{1}$	$\frac{x^2}{1}$				$\frac{1}{x^2}$	

÷ x ÷ x ÷ x

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- The following is true for numerical bases:

  $3^{-2} = \frac{1}{3^2}$ and $\frac{1}{3^{-2}} = 3^2$

  (3^{-2} is the reciprocal of 3^2)

  Similarly $x^{-2} = \frac{1}{x^2}$ and $\frac{1}{x^{-2}} = x^2$

- The negative index only applies to the pronumeral or term it is a power of.

  For example, $2a^{-4}b^5 = 2 \times \frac{1}{a^4} \times b^5 = \frac{2b^5}{a^4}$

- All the index laws can be applied to negative indices.

Key terminology: index/indices, base

Exercise 3J

UNDERSTANDING	1, 2	2

1 Complete the following.

a $5^{-4} = \frac{1}{5^4}$, so $b^{-4} = \frac{\square}{\square}$

b $10^{-3} = \frac{1}{10^3}$, so $x^{-3} = \frac{\square}{\square}$

2 Complete the following by filling in the boxes to express with positive indices.

a $x^{-3} = \frac{1}{x\square}$

b $5 \times m^{-2} = 5 \times \frac{1}{\square}$

$= \frac{5}{\square}$

FLUENCY	3–5(½)	3–5(½)

Example 33 Expressing negative indices in positive index form

Express the following with positive indices.

a x^{-2} **b** $4y^{-2}$ **c** $2a^{-3}b^2$

SOLUTION

a $x^{-2} = \frac{1}{x^2}$

b $4y^{-2} = 4 \times \frac{1}{y^2}$

$= \frac{4}{y^2}$

EXPLANATION

Use $a^{-m} = \frac{1}{a^m}$.

The negative index applies only to y; i.e. $y^{-2} = \frac{1}{y^2}$.

$4 \times \frac{1}{y^2} = \frac{4}{1} \times \frac{1}{y^2} = \frac{4}{y^2}$

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c $2a^{-3}b^2 = 2 \times \frac{1}{a^3} \times b^2$

$= \frac{2b^2}{a^3}$

$a^{-3} = \frac{1}{a^3}, \frac{2}{1} \times \frac{1}{a^3} \times \frac{b^2}{1} = \frac{2b^2}{a^3}$

Multiply the numerators and the denominators.

Now you try

Express the following with positive indices.

a b^{-4} **b** $3a^{-3}$ **c** $4x^{-2}y^4$

3 Express the following with positive indices.

a y^{-3} **b** x^{-4} **c** x^{-2} **d** a^{-5}

e $3x^{-2}$ **f** $5b^{-3}$ **g** $4x^{-1}$ **h** $2m^{-9}$

i $2x^2y^{-3}$ **j** $3xy^{-4}$ **k** $3a^{-2}b^4$ **l** $5m^{-3}n^2$

Hint: Use $a^{-m} = \frac{1}{a^m}$.

Example 34 Using $\frac{1}{a^{-m}} = a^m$

Rewrite the following with positive indices only.

a $\frac{1}{x^{-3}}$ **b** $\frac{4}{x^{-5}}$ **c** $\frac{5}{a^2b^{-4}}$

SOLUTION

a $\frac{1}{x^{-3}} = x^3$

b $\frac{4}{x^{-5}} = 4 \times \frac{1}{x^{-5}}$

$= 4 \times x^5$

$= 4x^5$

c $\frac{5}{a^2b^{-4}} = \frac{5}{a^2} \times \frac{1}{b^{-4}}$

$= \frac{5}{a^2} \times b^4$

$= \frac{5b^4}{a^2}$

EXPLANATION

Use $\frac{1}{a^{-m}} = a^m$.

The 4 remains unchanged.

Note: $\frac{1}{x^{-5}} = x^5$.

The negative index applies to b only; i.e. $\frac{1}{b^{-4}} = b^4$.

$\frac{5}{a^2} \times b^4 = \frac{5}{a^2} \times \frac{b^4}{1} = \frac{5b^4}{a^2}$

Now you try

Rewrite the following with positive indices only.

a $\frac{1}{y^{-2}}$ **b** $\frac{3}{a^{-4}}$ **c** $\frac{7}{x^{-2}y^3}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Rewrite the following with positive indices only.

a $\frac{1}{b^{-4}}$ **b** $\frac{1}{x^{-7}}$ **c** $\frac{1}{y^{-1}}$

d $\frac{5}{m^{-3}}$ **e** $\frac{2}{y^{-2}}$ **f** $\frac{3}{x^{-4}}$

g $\frac{5a^2}{b^{-3}}$ **h** $\frac{4}{x^2y^{-5}}$ **i** $\frac{10}{a^{-2}b^4}$

Hint: Use $\frac{1}{a^{-m}} = a^m$.

5 Rewrite the following with positive indices only.

a $\frac{4x^{-2}}{y^3}$ **b** $\frac{b^{-3}}{5a^2}$ **c** $\frac{2a^3}{b^{-2}}$

d $\frac{a^4}{3b^{-5}}$ **e** $\frac{y^{-2}}{x^{-3}}$ **f** $\frac{xy^{-3}}{x^{-2}y}$

Hint: For part e, $\frac{y^{-2}}{x^{-3}} = y^{-2} \times \frac{1}{x^{-3}} = \ldots$

PROBLEM-SOLVING AND REASONING 6–7(½), 8 6–7(½), 8, 9(½)

Example 35 Combining index laws with negative indices

Simplify the following, using index laws. Express answers with positive indices.

a $\frac{x^4y^3 \times x^{-2}y^5}{x^5y^4}$ **b** $\frac{4(x^2y^{-1})^{-2}}{y^5}$

SOLUTION

a $\frac{x^4y^3 \times x^{-2}y^5}{x^5y^4} = \frac{x^{4+(-2)}y^{3+5}}{x^5y^4}$

$= \frac{x^2y^8}{x^5y^4}$

$= x^{2-5}y^{8-4}$

$= x^{-3}y^4$

$= \frac{y^4}{x^3}$

EXPLANATION

Add indices of x and y in numerator:

For x: $4 + (-2) = 4 - 2 = 2$

For y: $3 + 5 = 8$

Express with positive indices; i.e. $x^{-3} = \frac{1}{x^3}$.

$x^{-3}y^4 = \frac{1}{x^3} \times \frac{y^4}{1}$

b $\frac{4(x^2y^{-1})^{-2}}{y^5} = \frac{4x^{-4}y^2}{y^5}$

$= 4x^{-4}y^{-3}$

$= \frac{4}{x^4y^3}$

Remove the brackets and distribute the power to each pronumeral: $x^{2\times(-2)}$ and $y^{-1\times(-2)}$.
Subtract the powers with a base of y.

Express with positive indices: $4x^{-4}y^{-3} = 4 \times \frac{1}{x^4} \times \frac{1}{y^3}$

Now you try

Simplify the following, using index laws. Express answers with positive indices.

a $\frac{x^2y^{-1} \times x^2y^4}{x^6y^2}$ **b** $\frac{9(x^{-3}y^2)^{-2}}{x^7}$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Simplify the following, expressing answers using positive indices.

a $\dfrac{a^6b^2 \times a^{-2}b^3}{a^7b}$

b $\dfrac{x^5y^3 \times x^2y^{-1}}{x^3y^5}$

c $\dfrac{x^4y^7 \times x^{-2}y^{-5}}{x^4y^6}$

d $\dfrac{a^5b^{-2} \times a^{-3}b^4}{a^6b}$

Hint: Index laws apply to negative indices also.
$x^5 \times x^{-2} = x^{5+(-2)} = x^3$
$\dfrac{x^4}{x^6} = x^{4-6} = x^{-2} = \dfrac{1}{x^2}$

7 Simplify, using index laws, and express with positive indices.

a $(x^{-4})^2$ **b** $(x^3)^{-2}$

c $(x^{-2})^0$ **d** $(2y^{-2})^3$

e $(ay^{-3})^2$ **f** $(4x^{-3})^{-2}$

g $\dfrac{3(x^{-4}y^3)^{-2}}{4x^7}$ **h** $(a^{-3}b^2)^{-2} \times (a^{-1}b^{-2})^3$

i $\dfrac{(2m^{-3}n)^2}{4m^2n^{-3}}$

Hint: Remove brackets using index laws, then use $a^{-m} = \dfrac{1}{a^m}$ to express with a positive index.

8 The mass of a small insect is 3^{-6} kg. How many grams is this, correct to two decimal places?

9 Evaluate the following without the use of a calculator.

a 2^{-2} **b** 5^{-3}

c $\dfrac{4}{3^{-2}}$ **d** $\dfrac{5}{2^{-3}}$

e -3×2^{-2} **f** $6^4 \times 6^{-6}$

g $\dfrac{2^3}{2^{-3}}$ **h** $8 \times (2^2)^{-2}$

Hint: Express each one with a positive index first.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: The power of −1 — 10

10 Consider the number $\left(\frac{3}{4}\right)^{-1}$. Using a positive index this becomes $\frac{1}{\left(\frac{3}{4}\right)} = 1 \div \frac{3}{4} = 1 \times \frac{4}{3} = \frac{4}{3}$.

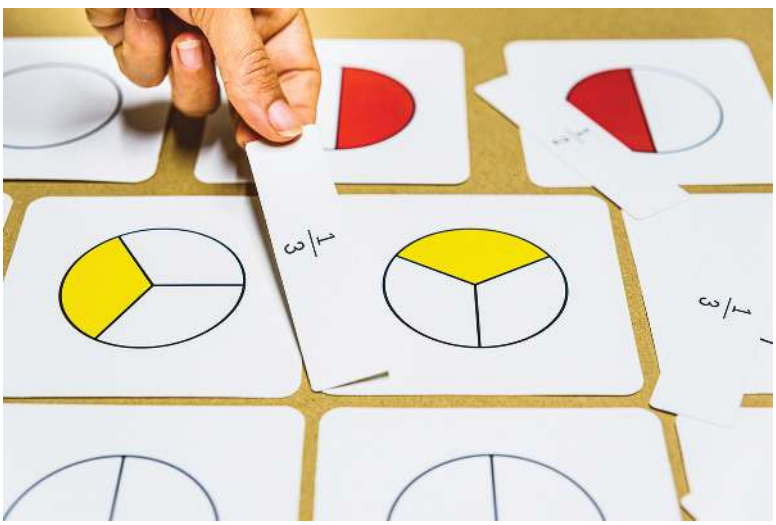
a Complete similar working to simplify the following.

i $\left(\frac{5}{3}\right)^{-1}$ **ii** $\left(\frac{1}{4}\right)^{-1}$ **iii** $\left(\frac{x}{2}\right)^{-1}$ **iv** $\left(\frac{a}{b}\right)^{-1}$

b What conclusion can you come to about simplifying fractions raised to the power of −1?

c Simplify these fractions.

i $\left(\frac{3}{2}\right)^{-2}$ **ii** $\left(\frac{3}{5}\right)^{-2}$ **iii** $\left(\frac{1}{3}\right)^{-3}$ **iv** $\left(\frac{2}{3}\right)^{-4}$



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3K Scientific notation and significant figures

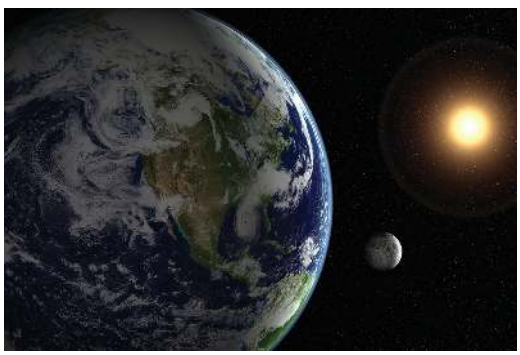
Learning intentions for this section:

- To know that scientific notation is a way of representing very large and very small numbers
- To be able to express numbers using scientific notation and as a basic numeral
- To be able to use and interpret scientific notation on a calculator
- To know how significant figures are counted
- To be able to round to a number of significant figures in scientific notation

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Numbers of any magnitude*
- It consolidates and extends the concepts in Sections 1C, 7F and 7G of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Scientific notation is useful when working with very large or very small numbers. Combined with the use of significant figures, numbers can be written down in a short form with an appropriate degree of accuracy. The approximate distance between Earth and the Sun is 150 million kilometres, or 1.5×10^8 km when written in scientific notation. Negative indices can be used for very small numbers, such as $0.0000382 \text{ g} = 3.82 \times 10^{-5}$ g.



Lesson starter: Amazing facts, large and small

Think of an object, place or living thing that is associated with a very large or small number.

- Give three examples of very large numbers.
- Give three examples of very small numbers.
- Can you remember how to write these numbers using scientific notation? List the rules you remember.

KEY IDEAS

- **Scientific notation** is a way to express very large and very small numbers in a short form.
- In scientific notation:

This is a one-digit number → 3.85×10^7 ← This number is a positive or negative integer

This number is between 1 and 10

This number is always 10

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- To write numbers using scientific notation, place the decimal point after the first non-zero digit and then multiply by the power of 10 that corresponds to how many places the decimal point is moved.
  - Large numbers will use positive powers of 10.
    For example, $24\,800\,000 = 2.48 \times 10^7$ ⟵ Note that there are 7 digits after the first digit
    $8\,020\,000\,000 = 8.02 \times 10^9$ ⟵ The first digit (8) is followed by 9 digits
  - Small numbers will use negative powers of 10.
    For example, $0.00307 = 3.07 \times 10^{-3}$
    $0.000001\,2 = 1.2 \times 10^{-6}$

- **Significant figures** are counted from left to right, starting at the first non-zero digit. Rounding occurs by considering the digit following the last significant digit; 5 or more round up, less than 5 round down. For example:
  - 47 086 120 is written 47 086 000 using five significant figures.
  - 2.03684 is written 2.037 using four significant figures.
  - 0.00143 is written 0.0014 using two significant figures.
  - 0.0014021 is written 0.00140 using three significant figures.

  Zeroes at the end of a number are counted for decimals (see 0.00140 above) but not whole numbers (see 47 086 000 above).

- When using scientific notation, the first significant figure sits to the left of the decimal point. For example:
  - 20 190 000 is written 2.02×10^7 using three significant figures.

- Sometimes your calculator will display an answer in scientific notation. Try this on your calculator:
  - Enter 12 and then press [=].
  - Divide by 10, then press [=] [=] [=] [=] [=] and so on.
  - When the answer is too long to fit on the screen, your calculator will show you the answer in scientific notation. In this case, it will use negative powers of 10.

- Repeat the process above but multiply by 10 rather than dividing by 10.

- Your calculator might have a button like this [$\times 10^x$] or this [EXP]. This can be used to enter numbers into your calculator.
  - For example, for the number 1.2×10^{-6}, you could enter $1.2 \times$ [10^x] $-\,6$. If the answer fits on the screen it will be displayed as a decimal. If not, it will be displayed in scientific notation.

- On some devices, such as a spreadsheet or mobile phone, the output of a calculation might include the character E or e, which stands for 'exponent'.
  - For example, if you calculate 2 to the power of 60, your calculator might display 1.15292 … E +18. This could be written as 1.15×10^{18} (to two decimal places). It represents a very large number that starts with 115 and is followed by 16 digits.
    $1.15 \times 10^{18} = 1\,150\,000\,000\,000\,000\,000$

Key terminology: scientific notation, significant figures

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 3K

UNDERSTANDING | 1–3 | 1, 3

1 Copy and complete this table. The first row has been done for you.

Scientific notation	Power of 10 expanded	Basic numeral
5×10^3	5×1000	5 000
3×10^4		
2×10^5		
7×10^2		
		70 000
		400 000
		6000
		2 000 000

Hint: The power of 10 is equal to the number of zeros.

2 Copy and complete this table. The first row has been done for you.

Scientific notation	Positive power	Fraction	Basic numeral
2×10^{-4}	$\frac{2}{10^4}$	$\frac{2}{10\,000}$	0.0002
3×10^{-2}			
5×10^{-3}			
7×10^{-6}			
			0.009
			0.08
			0.0006
			0.00004

3 Which of the numbers 1000, 10 000 or 100 000 completes each equation?

a $6.2 \times$ _____ $= 62\,000$ **b** $9.41 \times$ _____ $= 9410$

c $1.03 \times$ _____ $= 103\,000$ **d** $3.2 \div$ _____ $= 0.0032$

e $5.16 \div$ _____ $= 0.0000516$

f $1.09 \div$ _____ $= 0.000109$

Hint: The number of zeros tells you how many places to move the decimal point.

FLUENCY | 4–5(½) | 4–5(½)

Example 36 Converting from scientific notation to a basic numeral

Write these numbers as a basic numeral.

a 5.016×10^5 **b** 3.2×10^{-7}

SOLUTION

a $5.016 \times 10^5 = 501\,600$

b $3.2 \times 10^{-7} = 0.00000032$

EXPLANATION

Move the decimal point 5 places to the right, inserting zeroes after the last digit. 5.01600

Move the decimal point 7 places to the left due to the −7, and insert zeroes where necessary.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

Write these numbers as a basic numeral.

a 3.27×10^4 **b** 1.2×10^{-3}

4 Write these numbers as a basic numeral.

a 3.12×10^3 **b** 5.4293×10^4

c 7.105×10^5 **d** 8.213×10^6

e 5.95×10^4 **f** 8.002×10^5

g 1.012×10^4 **h** 9.99×10^6

i 2.105×10^8 **j** 4.5×10^{-3}

k 2.72×10^{-2} **l** 3.085×10^{-4}

m 7.83×10^{-3} **n** 9.2×10^{-5}

o 2.65×10^{-1} **p** 1.002×10^{-4}

q 6.235×10^{-6} **r** 9.8×10^{-1}

Hint: For a positive index, move the decimal point right (number gets bigger). For a negative index, move the decimal point left (number gets smaller).

Example 37 Writing numbers using scientific notation

Write these numbers in scientific notation.

a 5 700 000 **b** 0.0000006

SOLUTION	EXPLANATION
a $5\overset{\curvearrowleft}{7}00000 = 5.7 \times 10^6$	Place the decimal point after the first non-zero digit (5) and then multiply by 10^6, as the decimal point has been moved 6 places to the left.
b $0.\overset{\curvearrowright}{0000006} = 6 \times 10^{-7}$	6 is the first non-zero digit. Multiply by 10^{-7} since the decimal point has been moved 7 places to the right.

Now you try

Write these numbers in scientific notation.

a 320 000 **b** 0.0002

5 Write these numbers in scientific notation.

a 43 000 **b** 712 000

c 901 200 **d** 10 010

e 23 900 **f** 703 000 000

g 0.00078 **h** 0.00101

i 0.00003 **j** 0.03004

k 0.112 **l** 0.00192

Hint: For scientific notation, place the decimal point after the first non-zero digit and multiply by the power of 10.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 38 Converting to scientific notation using significant figures

Write these numbers in scientific notation using three significant figures.

a 5 218 300 **b** 0.0042031

SOLUTION

a $5\,218\,300 = 5.22 \times 10^6$

EXPLANATION

Put the decimal point after 5 and multiply by 10^6:
5.218300

The digit following the third digit (8) is at least 5, so round the 1 up to 2.

b $0.0042031 = 4.20 \times 10^{-3}$

Put the decimal point after 4 and multiply by 10^{-3}:
0.0042031

Round down in this case, since the digit following the third digit (3) is less than 5, but retain the zero to show the value of the third significant figure.

Now you try

Write these numbers in scientific notation using three significant figures.

a 53 721 **b** 0.0003625

PROBLEM-SOLVING AND REASONING 6–8 6–10

6 Write these numbers in scientific notation using three significant figures.

a 6241 **b** 572 644
c 30 248 **d** 423 578
e 10 089 **f** 34 971 863
g 72 477 **h** 356 088
i 110 438 523 **j** 0.002423
k 0.018754 **l** 0.000125
m 0.0078663 **n** 0.0007082
o 0.11396 **p** 0.000006403
q 0.00007892 **r** 0.000129983

Hint: For three significant figures, count from the first non-zero digit. Look at the digit after the third digit to determine whether you should round up or down.

7 Write the following numerical facts using scientific notation.

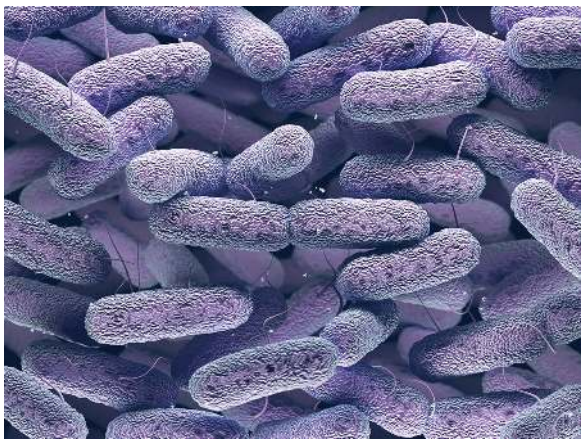
a The area of Australia is about 7 700 000 km^2.
b The number of stones used to build the Pyramid of Khufu is about 2 500 000.
c The greatest distance of Pluto from the Sun is about 7 400 000 000 km.
d A human hair is about 0.01 cm wide.

Hint: Large numbers have a positive index. Small numbers have a negative index.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

e The mass of a neutron is about 0.00000000000000000000000001675 kg.

f The mass of a bacteria cell is about 0.00000000000095 g.



8 Explain why 38×10^7 is not written using scientific notation and then convert it to scientific notation.

9 Use a calculator to evaluate the following, giving the answers in scientific notation using three significant figures.

a $(2.31)^{-7}$ **b** $(5.04)^{-4}$

c $(2.83 \times 10^2)^{-3}$ **d** $5.1 \div (8 \times 10^2)$

e $9.3 \times 10^{-2} \times 8.6 \times 10^8$ **f** $(3.27 \times 10^4) \div (9 \times 10^{-5})$

g $\sqrt{3.23 \times 10^{-6}}$ **h** $\pi(3.3 \times 10^7)^2$

Hint: Locate the $\times 10^x$ or EE or Exp button on your calculator.

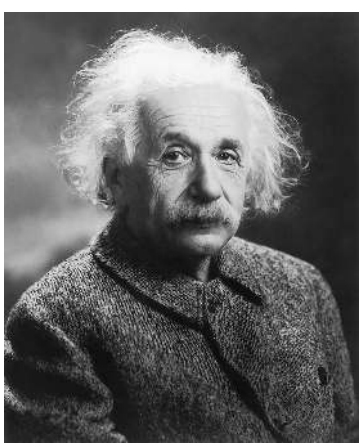
10 The speed of light is approximately 3×10^5 km/s and the average distance between Pluto and the Sun is about 5.9×10^9 km. How long does it take for light from the Sun to reach Pluto? Answer correct to the nearest minute. (Divide by 60 to convert seconds to minutes.)

Hint: Time $= \frac{\text{distance}}{\text{speed}}$

ENRICHMENT: $E = mc^2$ — 11

11 $E = mc^2$ is a formula derived by Albert Einstein (1879–1955). The formula relates the energy (E joules) of an object to its mass (m kg), where c is the speed of light (approximately 3×10^8 m/s).

Use $E = mc^2$ to answer these questions using scientific notation.



a Find the energy, in joules, contained inside an object with these given masses.

i 10 kg **ii** 26 000 kg

iii 0.03 kg **iv** 0.00001 kg

b Find the mass, in kilograms, of an object that contains the given amounts of energy. Give your answer using three significant figures.

i 1×10^{25} J **ii** 3.8×10^{16} J

iii 8.72×10^4 J **iv** 1.7×10^{-2} J

c The mass of Earth is about 6×10^{24} kg. How much energy does this convert to?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Maths@Work

Electrical trades

Electricians must be able to work in teams and also independently. They need to be good at calculating with decimals, as well as using scientific notation. Understanding and working with electrical charges is one example where this is important.

When tertiary institutions are evaluating students' academic readiness for apprenticeship training in the construction trades, which include electricians, plumbers and air conditioning mechanics, scientific notation is seen as important because it appears in different areas of their courses.

Complete these questions, which an apprentice electrician may face during their training.

1 The electrical charge (Q) of an object is determined by the number of electrons it has in excess to the number of protons it has.
The unit for measuring electrical charge is the coulomb (C). One coulomb (1C) is approximately 6.24 quintillion electrons (e).

$$1 \text{ C} = 6\,240\,000\,000\,000\,000\,000\,e$$

a Convert the following electrical charges, in coulombs, to the number of electrons for each. Use scientific notation using three significant figures.

i 1 C **ii** 2 C **iii** 3 C **iv** 250 C **v** $\frac{1}{2}$ C **vi** 12 C

b The charge on one electron in coulombs is $(1 \div 6\,240\,000\,000\,000\,000\,000)$ C. Write down the value of the charge of one electron in scientific notation, using two significant figures.

c Amperes (or amps) are a measure of how much electrical charge in coulombs per second is being transmitted. We call this flow of charge electrical current. This means that an electrical current of 1 A (ampere) has 1 coulomb of charge per second, which is exactly $6.24150975 \times 10^{18}$ electrons per second flowing through a point in the wire at any given time.
Calculate the exact number of electrons per second (e/s) flowing through a wire if the current is:

i 2 A **ii** 10 A **iii** 20 A **iv** $\frac{1}{2}$ A **v** 5 A

2 When working with metal it is important to know how it behaves under increases in temperature. For each degree Celsius increase in temperature of hard steel, it has a linear expansion by a factor of 0.0000132.
Write the following scientific notation answers, using four significant figures.

a Express the value 0.0000132 in scientific notation.

b If a section of hard steel measuring 12 mm thick is subject to a 2°C increase in temperature, what is its increase in length, in millimetres?

c Give one example to illustrate the importance of this information when working with steel.

Hint: Use $12 \times (1 + 0.0000132)^2$, then substract 12 to find the increase.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Using technology

Hint: The formula for cell D3 : = $(1.60217646 \times 10^{-19}) \times$ C3. Excel uses capital E to represent a power of 10.

3 Using an Excel spreadsheet, set up a conversion table between electrical charges measured in coulombs and in electrons, as shown below.

D3 =(1.60217646 *(10^-19))*C3

	A	B	C	D
1	**Conversion table between charge (Q) in coulombs (C) and electrons (e)**			
2	Charge *Q* (in coulombs, C)	Charge *Q* (in number of electrons, e)	Charge *Q* (in number of electrons, e)	Charge *Q* (in coulombs, C)
3	1	6.24150975E+18	1	1.60217646E-19
4	10		10	
5	100		100	
6	1000		1000	
7	10000		10000	
8	100000		100000	
9	1000000		1000000	

Use your spreadsheet to find the answers to the following questions and write them in scientific notation to nine significant figures.

i State the charge of 100 C as a number of electrons.

ii State the charge in coulombs of 1 million electrons.

iii What is the increase in the number of electrons between charges of 1000 C and 1 000 000 C?

4 Using an Excel spreadsheet, set up a conversion table between electrical current in amperes (A), time in seconds and charge in units of coulombs and electrons, as shown below.

Note: Charge in coulombs = amperes × time in seconds

Formulas using amperes will need $ signs since *A*22 is a fixed cell.

Hint: When referring to cell A22, type A22.

	A	B	C	D
20	**Conversion table between amperes, time and electrical charge measured in coulombs and numbers of electrons**			
21	Electrical current (A)	Time (*s*)	Charge *Q* (in coulombs, C)	Charge *Q* (in number of electrons, e)
22	0.5	1		
23		30		
24		60		
25		90		
26		120		
27		150		
28		180		

Use your spreadsheet to find the answers to the following questions and write them in scientific notation, to nine significant figures where possible.

a If a current of 0.50 A flows through a circuit for 90 seconds, how much charge will have passed into the circuit:

i in coulombs? **ii** in number of electrons?

b If a current of 1.5 A flows through a circuit for 150 seconds, how much charge will have passed into the circuit:

i in coulombs? **ii** in number of electrons?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Working mathematically

Luxury car investment

In this task you will apply the algebraic rules that model exponential growth and decay to predict possible future values for some very different cars.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

a Given the interest rate is per year, define the meaning of each pronumeral used in the rules that model exponential growth, $A = A_0\left(1 + \frac{r}{100}\right)^n$, and decay, $A = A_0\left(1 - \frac{r}{100}\right)^n$.

b Suppose that, 30 years ago, your grandfather bought a vintage 1938 Jaguar for $50 000. If its value has increased at 8% p.a., apply the exponential growth rule to determine the value of this car today, to the nearest $1000.

c A farmer buys a new 4-door ute for $50 000. If its value decays (i.e. decreases) at 8% p.a., apply the exponential decay rule to predict the value of this car in 30 years' time, to the nearest $1000.

Non-routine problems

Explore and connect

Harriet and Finlay have different priorities when it comes to their luxury cars. At about the same time, Harriet purchases a vintage 1958 Mercedes Benz for $40 000 and Finlay purchases a brand-new Jaguar Sport automatic for $99 000.

- Harriet's Mercedes Benz grows (i.e. increases) in value at 6% per year.
- Finlay's Jaguar decays (i.e. decreases) in value at 8% per year.

The problem is to find the time that it takes for Finlay's and Harriet's cars to have the same value for the given rates of growth and decay.

a Write down all the relevant information that will help solve this problem.

b Explain why Harriet's vintage car might grow in value and Finlay's brand-new car might decrease in value.

Choose and apply techniques

c **i** State a rule for the value of the Mercedes Benz after n years.

ii Calculate the value of the Mercedes Benz after 10 years.

d **i** State a rule for the value of the Jaguar after n years.

ii Calculate the value of the Jaguar after 10 years.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

e Set up the following table in an Excel spreadsheet, enter formulas in the shaded cells and extend the table to 12 years. Format currency values to whole numbers.

Use $ signs to anchor a cell, e.g. C3 keeps a fixed reference to C3.

Use ^ to raise to a power.

	A	B	C
1	**Years of ownership *n***	**Value of Mercedes Benz**	**Value of Jaguar**
2	0	$40,000	$99,000
3	1		
4	2		
5	*You will need to extend this table.*		

f Insert a graph in the Excel spreadsheet. Select the whole table, including the headings, then select Insert/Scatter/Smooth Lines and Markers. Give the chart a title and label the axes.

g Using the graph, estimate the number of years, n, after which Harriet's and Finlay's cars have the same value.

h Using the algebraic rules for exponential growth and decay calculate the value of Harriet's car and Finlay's car for the value of n found in part **g** above.

Communicate thinking and reasoning

i Using the algebraic rules, explore whether a different value of n gives a better estimate of the number of years it takes for Harriet's and Finlay's cars to have equal value.

j Summarise your findings on a document. Copy the table and graph and write a conclusion about when the cars are expected to have equal value.

Extension problems

Refine the prediction using technology

Problem-solve

a Refine your Excel table near the year the car values become equal and increase time by months instead of years, i.e. increase each cell value by 1/12 of a year (e.g. = C18 + 1/12).

b Create a new graph. Select the whole table, including the headings, then select Insert/Scatter/Smooth Lines and Markers. To format a graph axis in months, enter the major unit as 0.08333 (i.e. 1/12 as a decimal).

c Using the graph, estimate the number of years and months after purchase that Harriet's and Finlay's car have the same value.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Puzzles and challenges

1 In this magic square, each row and column adds to a sum that is an algebraic expression. Complete the square to find the sum.

$\frac{4x^2}{2x}$	$-y$	$x+3y$
$x-2y$		$2y$

2 Write $3^{n-1} \times 3^{n-1} \times 3^{n-1}$ as a single power of 3.

3 You are offered a choice of two prizes:

- One million dollars right now, or
- You can receive 1 cent on the first day of a 30-day month, double your money every day for 30 days and receive the total amount on the 30th day.

Which prize offers the most money?

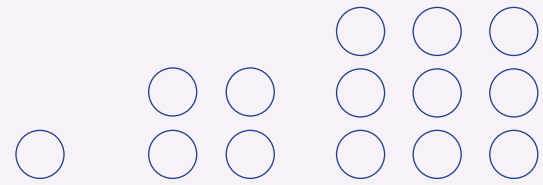
4 Simplify $\frac{25^6 \times 5^4}{125^5}$ without the use of a calculator.

5 Write $(((2^1)^2)^3)^4$ as a single power of 2.

6 How many zeroes are there in 100^{100} in expanded form?

7 Simplify $\frac{x}{2}+\frac{3x}{5}-\frac{4x}{3}+\frac{x+1}{6}$.

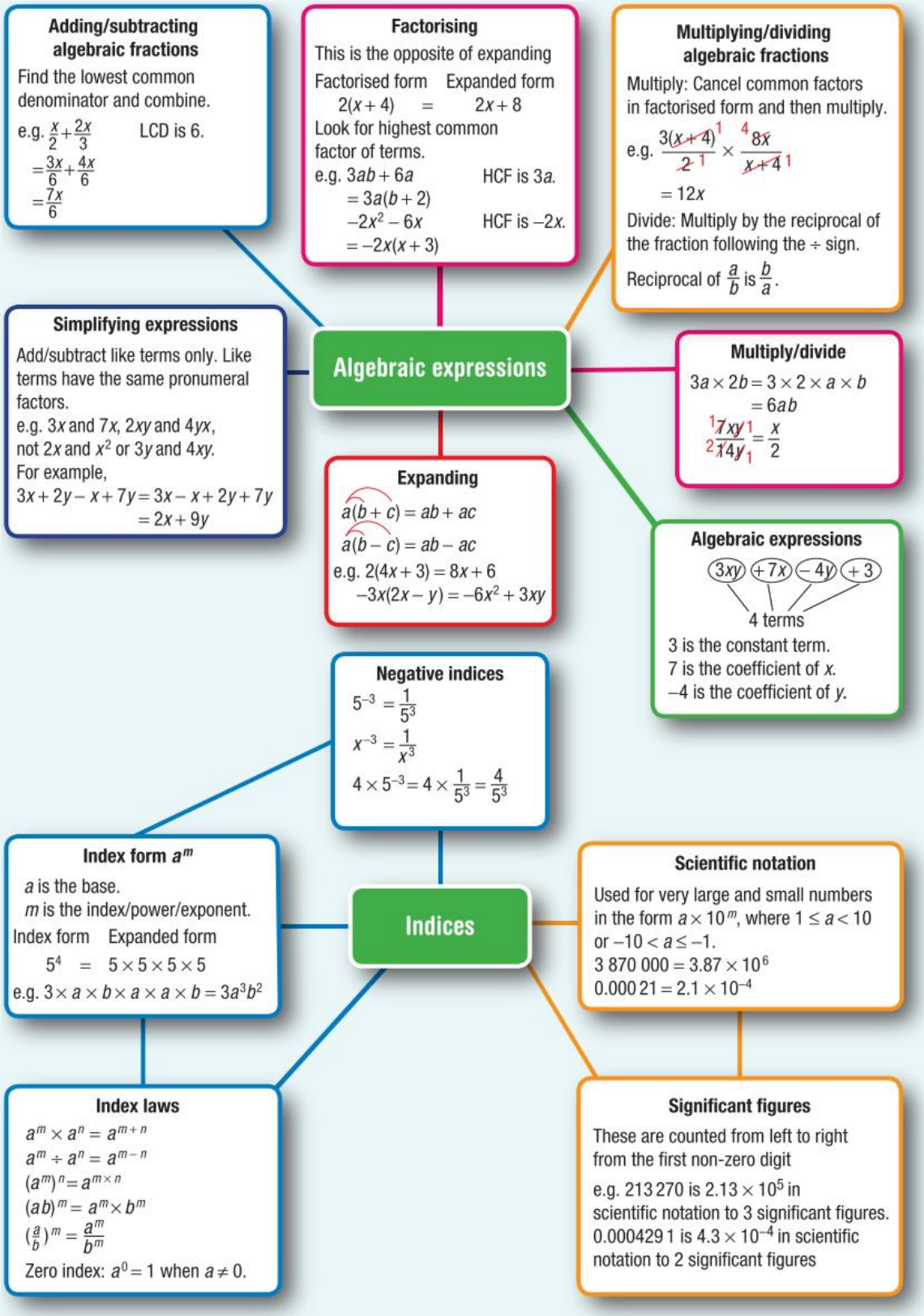
8 Write a rule for the number of counters in the nth term of the pattern below. Use this to find the number of counters in the 15th term.



Term 1 Term 2 Term 3

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter summary



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
3A	**1 I can identify the parts of an algebraic expression.** e.g. For the expression $4a + 3b - 7$, state the: **a** number of terms **b** constant term **c** coefficient of b	☐
3A	**2 I can form an algebraic expression.** e.g. Write an algebraic expression for: **a** 3 more than 2 lots of a **b** the product of x and y, divided by 2	☐
3A	**3 I can evaluate an algebraic expression using substitution.** e.g. If $x = 2$, $y = 5$ and $z = -3$, evaluate: **a** $xy + 2z$ **b** $y^2 - xz$	☐
3B	**4 I can identify like terms.** e.g. Write down the like terms in the following list: $ax, 7b, 6x, -3b, -2xa$	☐
3B	**5 I can collect like terms.** e.g. Simplify: **a** $5b + 4b - 2$ **b** $4xy + 3x - 5xy + 3x$	☐
3B	**6 I can multiply and divide algebraic terms.** e.g. Simplify: **a** $3a \times 5ab$ **b** $9xy \div (18x)$	☐
3C	**7 I can expand expressions with brackets.** e.g. Expand the following. **a** $4(3x - 2)$ **b** $-2y(5x - 7y)$	☐
3C	**8 I can simplify expressions by removing brackets.** e.g. Expand and simplify $3(2x + 5) - 2(x + 2)$.	☐
3D	**9 I can expand binomial products including perfect squares.** e.g. Expand the following. **a** $(2x - 1)(x + 5)$ **b** $(2x + 3)^2$	☐
3D	**10 I can expand binomial products to form a difference of two squares.** e.g. Expand $(3x - 1)(3x + 1)$.	☐
3E	**11 I can determine the HCF.** e.g. Determine the HCF of the following. **a** $6x$ and $24x$ **b** $8ab$ and $20b^2$	☐
3E	**12 I can factorise expressions with positive or negative common factors.** e.g. Factorise the following. **a** $8a + 12$ **b** $6x^2 - 10xy$ **c** $-4ab - 18a$	☐

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

3F	**13 I can simplify algebraic fractions.** e.g. Simplify this fraction by factorising first: $\frac{4x-12}{x-3}$.	☐
3F	**14 I can multiply algebraic fractions.** e.g. Simplify this product: $\frac{3(x-2)}{4x} \times \frac{10x}{x-2}$.	☐
3F	**15 I can divide algebraic fractions.** e.g. Simplify $\frac{5x^2}{9} \div \frac{10x}{3}$.	☐
3G	**16 I can add and subtract simple algebraic fractions.** e.g. Simplify $\frac{3x}{4} - \frac{x}{16}$.	☐
3G	**17 I can add and subtract with binomial numerators.** e.g. Simplify $\frac{x+2}{4} - \frac{x}{10}$.	☐
3H	**18 I can write in index form.** e.g. Write the following in index form. **a** $7 \times 7 \times 7 \times 7$ **b** $x \times y \times x \times x \times y$	☐
3H	**19 I can use the index law for multiplication.** e.g. Simplify the following. **a** $y^3 \times y^5$ **b** $3a^3b \times 5a^2b^3$	☐
3H	**20 I can use the index law for division.** e.g. Simplify the following. **a** $\frac{4m^5}{2m^3}$ **b** $5x^5y^4 \div (15xy^2)$	☐
3H	**21 I can combine index laws.** e.g. Simplify $\frac{2x^5y^3 \times 3x^2y^2}{9x^3y^4}$.	☐
3I	**22 I can use the zero index.** e.g. Evaluate $5^0 + a^0$.	☐
3I	**23 I can use the index law for power of a power.** e.g. Simplify $4(x^2)^5$.	☐
3I	**24 I can use index laws to remove brackets.** e.g. Simplify, using index laws. **a** $(2m)^3$ **b** $\left(\frac{x^2}{3}\right)^4$	☐
3I	**25 I can combine index laws.** e.g. Simplify: **a** $\frac{(m^2n)^5}{4m^3n^5}$ **b** $(3a)^0 - (4a^3)^2$	☐

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

Chapter checklist with success criteria

3J	**26 I can express negative indices in positive index form.** e.g. Rewrite the following with positive indices only. **a** $3y^{-2}$ **b** $\frac{4}{x^2y^{-3}}$	☐
3J	**27 I can use index laws with negative indices.** e.g. Simplify, expressing with positive indices, $\frac{a^4b^3 \times a^{-2}(b^2)^{-3}}{a^6}$.	☐
3K	**28 I can convert from scientific notation to a basic numeral.** e.g. Write these numbers as a basic numeral. **a** 3.02×10^4 **b** 7.21×10^{-5}	☐
3K	**29 I can write numbers using scientific notation.** e.g. Write these numbers in scientific notation. **a** 64 000 **b** 0.000035	☐
3K	**30 I can write in scientific notation rounding to significant figures.** e.g. Write these numbers in scientific notation using three significant figures. **a** 472 815 **b** 0.0053821	☐

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Short-answer questions

3A **1** Consider the expression $3xy - 3b + 4x^2 + 5$.

a How many terms are in the expression?

b What is the constant term?

c State the coefficient of:

i x^2 ii b

3A **2** Write an algebraic expression for the following.

a 3 more than y

b 5 less than the product of x and y

c the sum of a and b is divided by 4

3A **3** Evaluate the following if $x = 3, y = 5$ and $z = -2$.

a $3x + y$ b xyz c $y^2 - 5z$

3B **4** Simplify the following expressions.

a $4x - 5 + 3x$ b $4a - 5b + 9a + 3b$ c $3xy + xy^2 - 2xy - 4y^2x$

d $3m \times 4n$ e $-2xy \times 7x$ f $\dfrac{8ab}{12a}$

3C **5** Expand the following and collect like terms where necessary.

a $5(2x + 4)$ b $-2(3x - 4y)$ c $3x(2x + 5y)$

d $3 + 4(a + 3)$ e $3(y + 3) + 2(y + 2)$ f $5(2t + 3) - 2(t + 2)$

3D **6** Expand and simplify.

a $(2x + 1)^2$ b $(2x + 1)(3x + 2)$

c $(2x + 1)(3x - 2)$ d $(2x + 1)(2x - 1)$

3E **7** Factorise the following expressions.

a $16x - 40$ b $10x^2y + 35xy^2$

c $4x^2 - 10x$ d $-2xy - 18x$ (include the common negative)

3G **8** Simplify the following algebraic fractions involving addition and subtraction.

a $\dfrac{2x}{3} + \dfrac{4x}{15}$ b $\dfrac{3}{7} - \dfrac{a}{2}$ c $\dfrac{x+4}{4} + \dfrac{x-3}{5}$

3F **9** Simplify these algebraic fractions by first cancelling common factors in factorised form.

a $\dfrac{5x}{12} \times \dfrac{9}{10x}$ b $\dfrac{x+2}{4} \times \dfrac{16x}{x+2}$ c $\dfrac{12x-4}{4}$ d $\dfrac{x-3}{4} \div \dfrac{3(x-3)}{8}$

3H **10** Simplify the following.

a $3x^5 \times 4x^2$ b $4xy^6 \times 2x^3y^2$ c $\dfrac{b^7}{b^3}$ d $\dfrac{4a^3b^5}{6ab^2}$

3I **11** Simplify the following.

a $(b^2)^4$ b $(2m^2)^3$ c $\left(\dfrac{x}{7}\right)^2$ d $\left(\dfrac{4y^2}{z^4}\right)^3$

3J **12** Simplify the following.

a 7^0 b $4x^0$ c $5a^0 + (2y)^0$ d $(x^2 + 4y)^0$

3J **13** Express the following, using positive indices.

a $4x^{-3}$ b $3r^4s^{-2}$ c $\dfrac{2x^{-3}y^4}{3}$ d $\dfrac{4}{m^{-5}}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

3J **14** Simplify the following, using index laws. Express all answers with positive indices.

a $\dfrac{3x^2y^4 \times 5xy^7}{12x^3y^5}$ **b** $\dfrac{(5x^2y)^2 \times 4xy^2}{8(xy)^2}$

c $\dfrac{2x^3y^2 \times 5xy^2}{x^7y^4}$ **d** $\dfrac{4x^2y^5}{8(x^2)^{-3}y^8}$

3K **15** Write these numbers as a basic numeral.

a 4.25×10^3 **b** 3.7×10^7 **c** 2.1×10^{-2} **d** 7.25×10^{-5}

3K **16** Convert these numbers to scientific notation, using three significant figures.

a 123 574 **b** 39 452 178 **c** 0.0000090241 **d** 0.00045986

Multiple-choice questions

3A **1** The coefficient of x in $3xy - 4x + 7$ is:

A 4 **B** 7 **C** -4 **D** 3 **E** -1

3B **2** The simplified form of $7ab + 2b - 5ab + b$ is:

A $2ab + 2b^2$ **B** $2ab + 3b$ **C** $5ab$ **D** $2ab + b$ **E** $12ab + 3b$

3C **3** The expanded form of $2x(3x - 5)$ is:

A $6x^2 - 5$ **B** $6x - 10$ **C** $6x^2 - 10x$ **D** $5x^2 - 10x$ **E** $-4x$

3E **4** The fully factorised form of $8xy - 24y$ is:

A $4y(2x - 6y)$ **B** $8(xy - 3y)$ **C** $8y(x - 24)$ **D** $8y(x - 3)$ **E** $8x(y - 24)$

3F **5** The simplified form of $\dfrac{2(x+1)}{5x} \times \dfrac{15}{x+1}$ is:

A $\dfrac{6}{x+1}$ **B** $\dfrac{6}{x}$ **C** $\dfrac{3(x+1)}{x}$ **D** $6x$ **E** $\dfrac{3x}{x+1}$

3G **6** The sum of the algebraic fractions $\dfrac{3x}{8} + \dfrac{x}{12}$ is:

A $\dfrac{x}{5}$ **B** $\dfrac{x}{6}$ **C** $\dfrac{x}{24}$ **D** $\dfrac{11x}{24}$ **E** $\dfrac{9x}{24}$

3H **7** $3x^3y \times 2x^5y^3$ is equal to:

A $5x^{15}y^3$ **B** $6x^{15}y^3$ **C** $6x^8y^4$ **D** $5x^8y^4$ **E** $6x^8y^3$

3I **8** $(2x^4)^3$ can be written as:

A $2x^{12}$ **B** $2x^7$ **C** $6x^{12}$ **D** $8x^{12}$ **E** $8x^7$

3I **9** $5x^0 - (2x)^0$ is equal to:

A 4 **B** 0 **C** 3 **D** 2 **E** -1

3J **10** $12a^4 \div (4a^7)$ simplifies to:

A $3a^3$ **B** $8a^3$ **C** $3a^{11}$ **D** $\dfrac{8}{a^3}$ **E** $\dfrac{3}{a^3}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

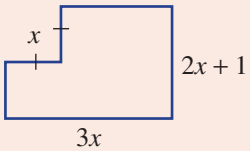
3K

11 417 000 converted to scientific notation is:

A 4.17×10^{-5} **B** 417×10^{3} **C** 4.17×10^{5}
D 0.417×10^{6} **E** 41.7×10^{-2}

Extended-response questions

1 A room in a house has the shape and dimensions, in metres, shown. All lines meet at 90°.



a Find the perimeter of the room, in factorised form.

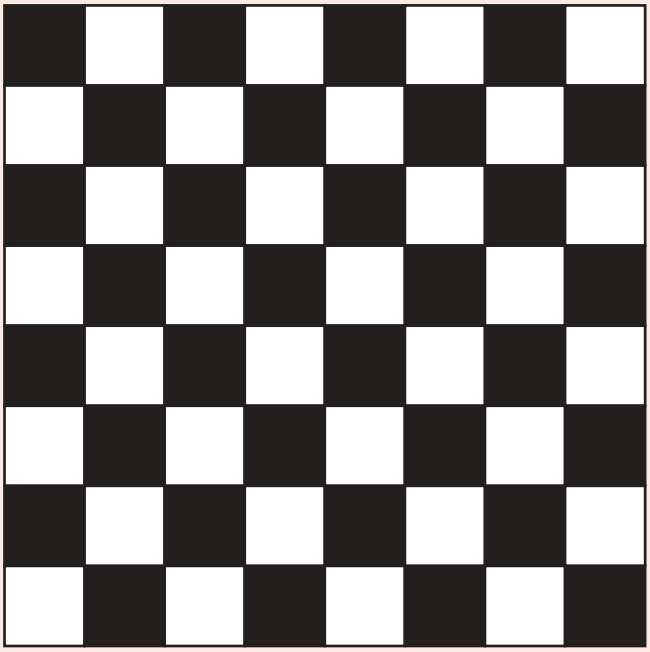
b If $x = 3$, what is the room's perimeter?

The floor of the room is to be recarpeted.

c Give the area of the floor in terms of x and in expanded form.

d If the carpet costs $20 per square metre and $x = 3$, what is the cost of laying the carpet?

2 It is thought that the game of chess was invented by an ancient Indian mathematician. The king was so pleased with the game that he offered the inventor any reward of his choice: rare jewels, bags of gold or even a large property.
To the king's surprise the Indian mathematician asked for some wheat! He asked for 1 grain for the first square of the chess board, 2 grains for the second square, 4 grains for the third square, 8 grains for the fourth square etc., continuing in this way right up to the 64th square.



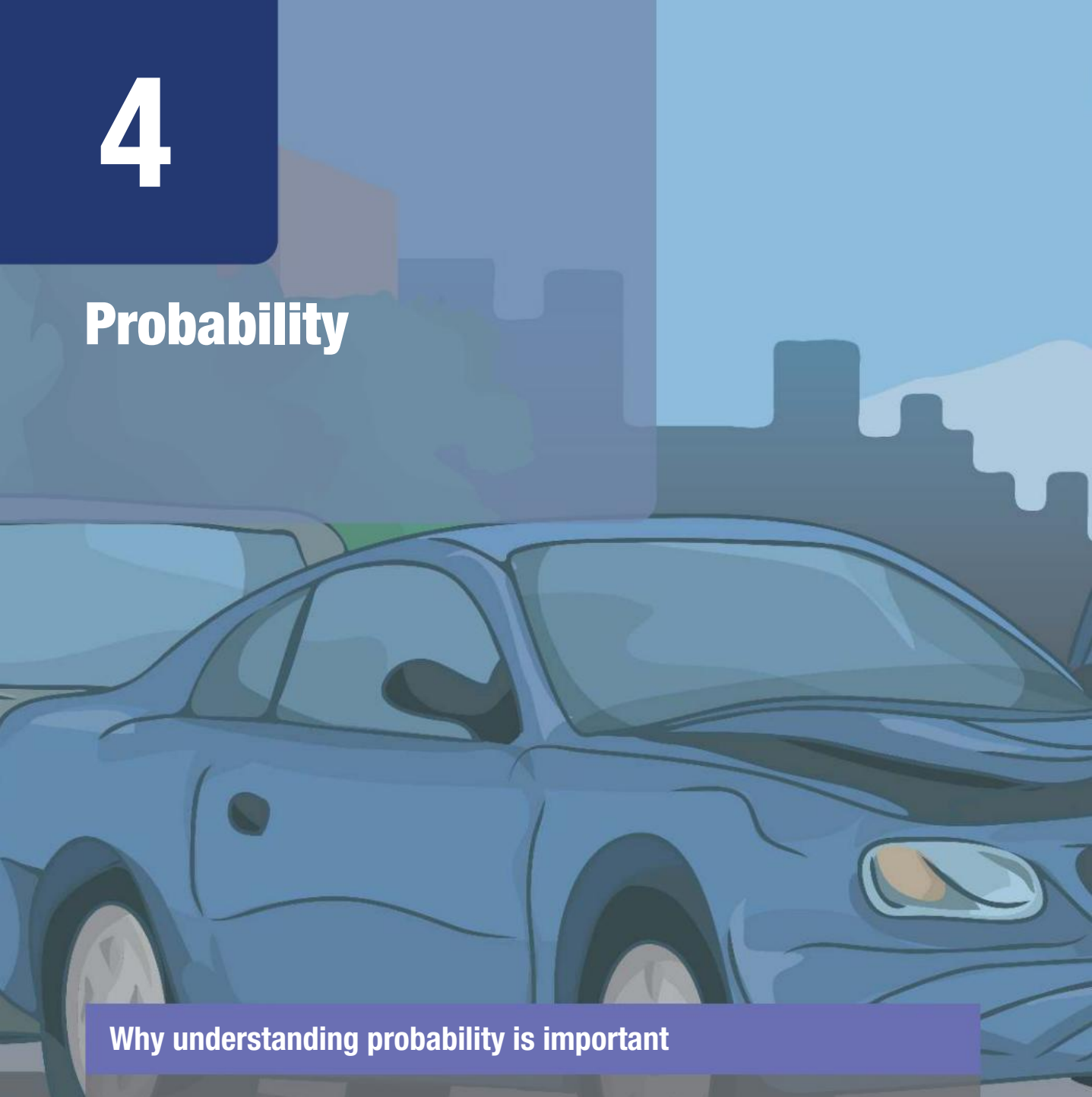
a If one grain of wheat weighs 2×10^{-8} tonnes, what weight of wheat would the inventor have received for the 64th square? Answer in scientific notation, correct to 3 significant figures.

b How much money would this wheat be worth at $275 per tonne?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4

Probability



Why understanding probability is important

Probability indicates the likely occurrence or risk of a particular event. Awareness of risk helps people to make informed decisions. Being able to analyse data and interpret probabilities are important skills and essential for success in many businesses.

Vast amounts of data can be hard to make sense of, so techniques are used to discover the various proportions (i.e. probabilities) within results. For example, Venn diagrams and two-way tables can show the numbers of people:

- aged under 30 and who watch sport or comedy or both
- aged under 25 who have car accidents from drink driving or speeding or both
- who buy utes or are trade workers or both
- who smoke or die from cancer or both.

Using probabilities, informed management decisions can be made about a range of things: TV scheduling; money spent on advertising; shops selecting what stock to by; and insurance premiums based on age, accident probabilities and even whether the person is a smoker.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter contents

4A Review of probability (CONSOLIDATING)
4B Venn diagrams (EXTENDING)
4C Two-way tables (EXTENDING)
4D Using arrays for two-step experiments
4E Using tree diagrams
4F Dependent events and independent events

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- solves problems involving probabilities in multistage chance experiments and simulations (MA5-PRO-C-01)
- solves problems involving Venn diagrams, 2-way tables and conditional probability (MA5-PRO-P-01).

© 2022 NSW Education Standards Authority

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Warm-up quiz

1 A letter is selected from the word PROBABILITY.

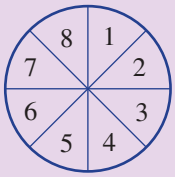
a How many letters are there in total?

b Find the chance (i.e. probability) of selecting:

i the letter R **ii** the letter B **iii** a vowel

iv not a vowel **v** a T or an I **vi** neither a B nor a P

2 A spinning wheel has eight equal sectors numbered 1 to 8. On one spin of the wheel, find the following probabilities.



a $P(5)$ **b** $P(\text{even})$

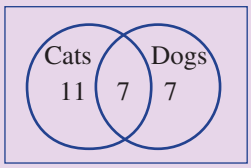
c $P(\text{not even})$ **d** $P(\text{multiple of 3})$

e $P(\text{factor of 12})$ **f** $P(\text{odd or a factor of 12})$

g $P(\text{both odd and a factor of 12})$

3 Arrange from lowest to highest: $\frac{1}{2}$, 0.4, 1 in 5, 39%, $\frac{3}{4}$, 1, 0, $\frac{9}{10}$, 0.62, 71%.

4 This Venn diagram shows the number of people in a group of 25 who own cats and/or dogs.



a State the number of people who own:

i a dog **ii** a cat or a dog (including both)

iii only a cat

b If a person is selected at random from this group, find the probability that they will own:

i a cat **ii** a cat and a dog **iii** only a dog

5 Drew shoots from the free-throw line on a basketball court. After 80 shots he counts 35 successful throws.

a Estimate the probability, in simplified form, that his next throw will be successful.

b Estimate the probability, in simplified form, that his next throw will not be successful.

6 Two 4-sided dice are rolled and the sum of the two numbers obtained is noted. Copy and complete this grid to help answer the following.

Roll 2 \ Roll 1	1	2	3	4
1				
2				
3				
4				

a What is the total number of outcomes?

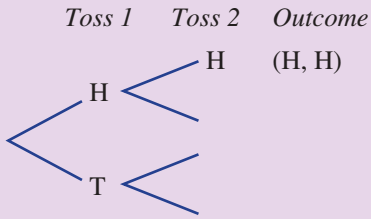
b Find the probability that the total sum is:

i 2 **ii** 4 **iii** less than 5

iv less than or equal to 5 **v** at most 6

vi no more than 3

7 Two coins are tossed. Copy and complete this tree diagram to help answer the following.



a State the total number of outcomes.

b Find the probability of obtaining:

i 2 heads **ii** no heads **iii** 1 tail

iv at least 1 tail **v** 1 of each, a head and a tail

vi at most 2 heads

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4A Review of probability CONSOLIDATING

Learning intentions for this section:

- To understand the concept of probability and how it is described numerically
- To know the terminology of probability
- To be able to find the probability of an event in an experiment with equally likely outcomes

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- It consolidates and extends the concepts in Sections 10A and 10E of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Probability is an area of mathematics concerned with the likelihood of particular random events. In some situations, such as rolling a die, we can determine theoretical probabilities because we know the total number of outcomes and the number of favourable outcomes. In other cases we can use statistics and experimental results to describe the chance that an event will occur. The chance that a particular soccer team will win its next match, for example, could be estimated using various results from previous games.

Lesson starter: More or less than 50%

When a coin is tossed once, the chance of tossing heads is 50%. Describe an experiment and outcome that has the following probability.

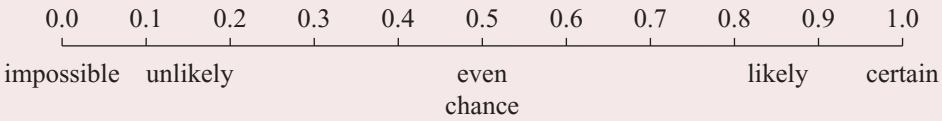
a 0% **b** less than 50% **c** more than 50% **d** 100%

KEY IDEAS

- Key terms used in probability are given below.
  - A **chance experiment** (often shortened to just *experiment*) is an activity that may produce a variety of different results that occur randomly. Rolling a die is a single-step experiment.
  - A **trial** is a single occurrence of an experiment, such as a single roll of a die.
  - The **sample space** is the list of all possible outcomes from an experiment.
  - An **outcome** is a possible result of an experiment.
  - An **event** is either one outcome or a collection of outcomes.
  - **Equally likely outcomes** are events that have the same chance of occurring.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- In the study of probability, a numerical value based on a scale from 0 to 1 is used to describe levels of **chance**.



- The **theoretical probability** (or **expected probability**) of an event in which outcomes are equally likely is calculated as follows.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- A probability can be written as a fraction, decimal or percentage.
- **Experimental probability** (or **observed probability**) is calculated in the same way as theoretical probability but uses the results of an experiment:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of trials}}$$

  - If the number of trials is large, the experimental probability should be very close to the theoretical probability.

Key terminology: theoretical probability, experimental probability, trial, sample space, outcome, event, chance, long run proportion, equally likely outcomes

Exercise 4A

UNDERSTANDING 1–3 3

1 Complete the following by filling in the blanks.

a A list of all the possible outcomes from an experiment is called the _____.

b The numerical values used to describe levels of chance are between _____ and _____.

c An event with a probability of 0.8 would be described as _____ to occur.

d Obtaining a tail from the toss of a coin is called an _____ of the experiment.

2 Order these events (**A–D**) from least likely to most likely.

A The chance that it will rain every day for the next 10 days.

B The chance that a member of class is ill on the next school day.

C The chance that school is cancelled next year.

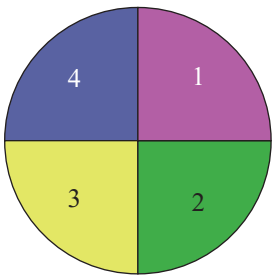
D The chance that the Sun comes up tomorrow.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

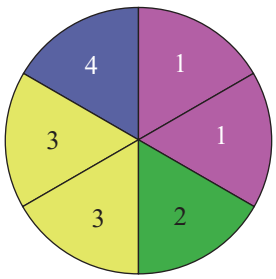
3 For the following spinners, find the probability that the outcome will be a 4.

Hint: Since each section is equal, $P(4) = \frac{\text{number of 4s}}{\text{total number of sections}}$

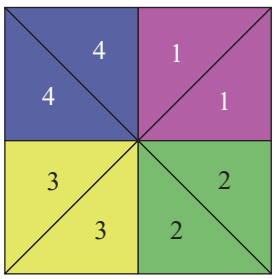
a



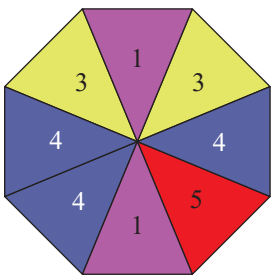
b



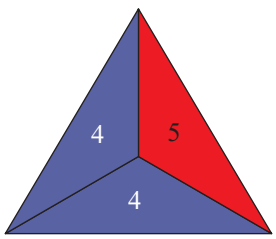
c



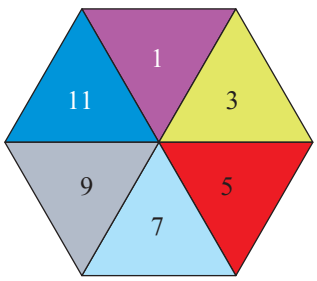
d



e



f



FLUENCY 4–6 4, 6, 7

Example 1 Calculating simple theoretical probabilities

A letter is chosen randomly from the word TELEVISION.

a How many letters are there in the word TELEVISION?

b Find the probability that the letter is:

i a V **ii** an E

iii not an E **iv** an E or a V

SOLUTION

a 10

EXPLANATION

The sample space includes 10 letters.

b i $P(\text{V}) = \frac{1}{10} (= 0.1)$

$P(\text{V}) = \frac{\text{number of Vs}}{\text{total number of letters}}$

ii $P(\text{E}) = \frac{2}{10}$

$= \frac{1}{5} (= 0.2)$

There are 2 Es in the word TELEVISION.

Simplify the fraction or convert to decimal.

iii $P(\text{not an E}) = \frac{8}{10}$

$= \frac{4}{5} (= 0.8)$

If there are 2 Es in the word TELEVISION with 10 letters, then there must be 8 letters that are not E.

iv $P(\text{an E or a V}) = \frac{3}{10} (= 0.3)$

The number of letters that are either E or V is 3.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

A letter is chosen randomly from the word DINNER.

a How many letters are there in the word DINNER?

b Find the probability that the letter is:

i a D
ii an N
iii not an N
iv a D or an N

4 A letter is chosen randomly from the word TEACHER.

a How many letters are there in the word TEACHER?

b Find the probability that the letter is:

i an R
ii an E
iii not an E
iv an R or an E

5 A letter is chosen randomly from the word EXPERIMENT. Find the probability that the letter is:

a an E
b a vowel
c not a vowel
d an X or a vowel

Hint: The vowels are A, E, I, O and U.

Example 2 Calculating simple experimental probabilities

An experiment involves tossing three coins and counting the number of heads. Here are the results after running the experiment 100 times.

Number of heads	0	1	2	3
Frequency	11	40	36	13

a How many times did 2 heads occur?

b How many times did fewer than 2 heads occur?

c Find the experimental probability of obtaining:

i 0 heads
ii 2 heads
iii fewer than 2 heads
iv at least 1 head

SOLUTION

a 36

b $11 + 40 = 51$

EXPLANATION

From the table, you can see that 2 heads has a frequency of 36.

Fewer than 2 means obtaining 0 heads or 1 head.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c i $P(0 \text{ heads}) = \frac{11}{100}$
$= 0.11$

$P(0 \text{ heads}) = \frac{\text{number of times 0 heads is observed}}{\text{total number of trials}}$

ii $P(2 \text{ heads}) = \frac{36}{100}$
$= 0.36$

$P(2 \text{ heads}) = \frac{\text{number of times 2 heads is observed}}{\text{total number of trials}}$

iii $P(\text{fewer than 2 heads})$
$= \frac{11 + 40}{100}$
$= \frac{51}{100} = 0.51$

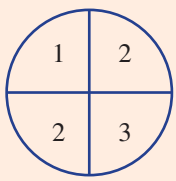
Fewer than 2 heads means to observe 0 or 1 head.

iv $P(\text{at least 1 head})$
$= \frac{40 + 36 + 13}{100}$
$= \frac{89}{100} = 0.89$

At least 1 head means that 1, 2 or 3 heads can be observed.

Now you try

An experiment involves spinning the spinner shown three times and counting the number of 2s. Here are the results after running the experiment 100 times.



Number of 2s	0	1	2	3
Frequency	15	34	42	9

a How many times did one 2 occur?
b How many times did more than one 2 occur?
c Find the experimental probability of obtaining:
i no 2s
ii three 2s
iii fewer than two 2s
iv at least one 2

6 An experiment involves tossing three coins and counting the number of heads. Here are the results after running the experiment 100 times.

Hint: The total number of outcomes is 100.

Number of heads	0	1	2	3
Frequency	9	38	43	10

a How many times did 2 heads occur?
b How many times did fewer than 2 heads occur?
c Find the experimental probability of obtaining:
i 0 heads
ii 2 heads
iii fewer than 2 heads
iv at least 1 head

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 An experiment involves rolling two dice and counting the number of 6s. Here are the results after running the experiment 100 times.

Number of 6s	0	1	2
Frequency	62	35	3

Find the experimental probability of obtaining:

a no 6s
b two 6s
c fewer than two 6s
d at least one 6

PROBLEM-SOLVING AND REASONING

8, 9 | 9–11

8 A 10-sided die, numbered 1 to 10, is rolled once. Find these probabilities.

1 2 3
4 5
6 7 8
9 10

a $P(8)$
b $P(\text{odd})$
c $P(\text{even})$
d $P(\text{less than 6})$
e $P(\text{prime})$
f $P(3 \text{ or } 8)$
g $P(8, 9 \text{ or } 10)$

Hint: Prime numbers less than 10 are 2, 3, 5 and 7.

9 Amelia is a prizewinner in a competition and will be randomly awarded a single prize chosen from a collection of 50 prizes. The type and number of prizes to be handed out are listed below.

Prize	Car	Holiday	iPad	Blu-ray player
Number	1	4	15	30

Find the probability that Amelia will be awarded the following.

a a car
b an iPad
c a prize that is not a car

Hint: Remember that the total number of prizes is 50.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 Many of the 50 cars inspected at an assembly plant contain faults. The results of the inspection are as follows.

Number of faults	0	1	2	3	4
Number of cars	30	12	4	3	1

Find the experimental probability that a car selected from the assembly plant will have:

- **a** 1 fault
- **b** 4 faults
- **c** fewer than 2 faults
- **d** 1 or more faults
- **e** 3 or 4 faults
- **f** at least 2 faults

11 A bag contains red and yellow counters. A counter is drawn from the bag and then replaced. This happens 100 times and 41 of the counters drawn were red.

Hint: $\frac{41}{100}$ were red.

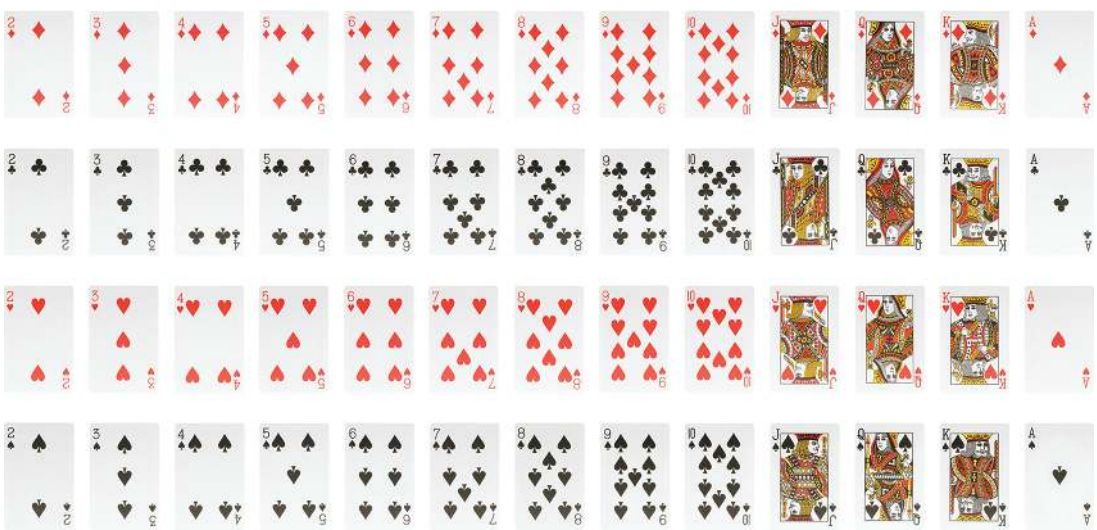
- **a** How many counters drawn were yellow?
- **b** If there were 10 counters in the bag, how many do you expect were red? Give a reason.
- **c** If there were 20 counters in the bag, how many do you expect were red? Give a reason.

ENRICHMENT: Cards probability — 12

12 A card is chosen from a standard deck of 52 playing cards that includes 4 aces, 4 kings, 4 queens and 4 jacks. Find the following probabilities.

- **a** P(heart)
- **b** P(king)
- **c** P(king of hearts)
- **d** P(heart or club)
- **e** P(king or jack)
- **f** P(heart or king)
- **g** P(not a king)
- **h** P(neither a heart nor a king)

Hint: There are 4 suits in a deck of cards: hearts, diamonds, spades and clubs.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4B Venn diagrams EXTENDING

Learning intentions for this section:

- To understand how Venn diagrams show all combinations of two attributes
- To know the notation and regions that represent the union, intersection and complement
- To be able to use a Venn diagram to display the distribution of two sets
- To be able to use a Venn diagram to calculate probabilities of events

Past, present and future learning:

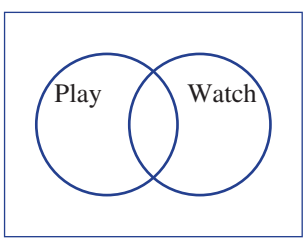
- This section is included as an Extending topic
- It consolidates and extends the concepts in Section 10B our Year 9 book
- Venn diagrams are not mentioned in Stage 4, Stage 5 Core or Stage 6 Standard
- Venn diagrams are in a Stage 5 Path topic for students intending to do Stage 6 Advanced

Sometimes we need to work with situations where there are overlapping events. A sports channel, for example, might be collecting statistics regarding whether or not a person watches cricket and/or tennis or neither over a certain period of time. The estimated probability that a person will watch cricket *or* tennis will therefore depend on how many people responded *yes* to watching both cricket *and* tennis. Venn diagrams are a useful tool when dealing with such events.

Lesson starter: How many like both?

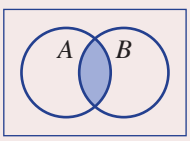
Of 20 students in a class, 12 people like to play tennis and 15 people like to watch tennis. Two people like neither playing nor watching tennis. Some like both playing and watching tennis.

- Is it possible to represent this information in a Venn diagram?
- How many students like to play and watch tennis?
- How many students only like to watch tennis?
- From the group of 20 students, what would be the probability of selecting a person that likes watching tennis only?



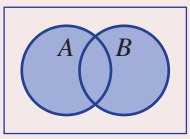
KEY IDEAS

- A **Venn diagram** illustrates how all elements in the sample space are distributed among the events.
  - The **intersection**: all elements that belong to both *A and B*

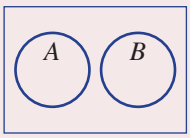


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- The **union**: all elements that belong to either events A *or* B. (Note: this includes events that are in both A and B.)



- The two sets A and B are **mutually exclusive** if they have no elements in common.



- For an event A, the **complement** of A is 'not A' (sometimes shown as A'). $P(\text{not } A) = 1 - P(A)$

Key terminology: Venn diagram, union, intersection, complement, mutually exclusive

Exercise 4B

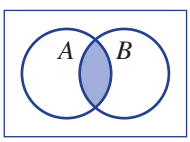
UNDERSTANDING 1–3 3

1 Match the descriptions (**a**–**e**) to the pictures (**A**–**E**).

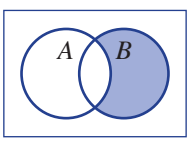
a A or B **b** A **c** not A

d A and B **e** B only

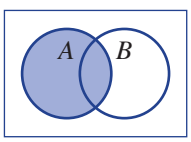
A



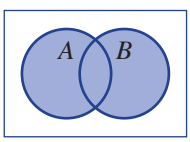
B



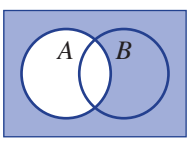
C



D



E



2 Decide whether the events A and B are mutually exclusive.

Hint: Mutually exclusive events have nothing in common.

a $A = \{1, 3, 5, 7\}$
$B = \{5, 8, 11, 14\}$

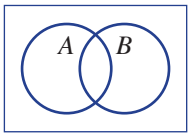
b $A = \{-3, -2, \ldots, 4\}$
$B = \{-11, -10, \ldots, -4\}$

c $A = \{\text{prime numbers}\}$
$B = \{\text{even numbers}\}$

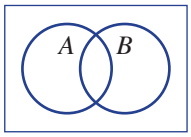
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Copy these Venn diagrams and shade the region described by each of the following.

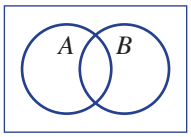
a A



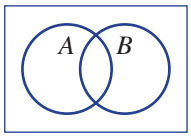
b B



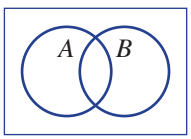
c A and B



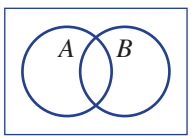
d A or B



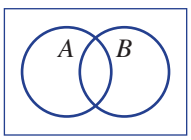
e A only



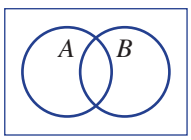
f B only



g not A



h neither A nor B



FLUENCY 4–6 5–7

Example 3 Listing sets

Consider the given events A and B, which involve numbers taken from the first 10 positive integers.
$A = \{1, 2, 3, 4, 5, 6\}$ $B = \{1, 3, 7, 8\}$

a Represent the two events A and B in a Venn diagram.

b List the sets:

i A and B **ii** A or B

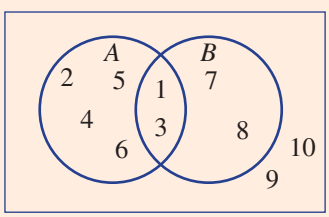
c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.

i A **ii** A and B **iii** A or B

d Are the events A and B mutually exclusive? Why or why not?

SOLUTION

a



EXPLANATION

The elements 1 and 3 are common to both sets A and B.

The elements 9 and 10 belong to neither set A nor set B.

b **i** A and $B = \{1, 3\}$

1 and 3 are in A and B.

ii A or $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

These numbers are in either A or B or both.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c **i** $P(A) = \frac{6}{10} = \frac{3}{5}$ There are 6 numbers in A.

ii $P(A \text{ and } B) = \frac{2}{10} = \frac{1}{5}$ A and B contains 2 numbers.

iii $P(A \text{ or } B) = \frac{8}{10} = \frac{4}{5}$ A or B contains 8 numbers.

d The sets A and B are not mutually exclusive since there are numbers inside A and B. The set A and B contains at least 1 number.

Now you try

Consider the given events A and B, which involve numbers taken from the first 10 positive integers.

$A = \{2, 4, 6, 8, 10\}$ $B = \{2, 3, 5, 7\}$

a Represent the two events A and B in a Venn diagram.

b List the following sets.

i A and B **ii** A or B

c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.

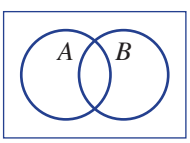
i A **ii** A and B **iii** A or B

d Are the events A and B mutually exclusive? Why or why not?

4 Consider the given events A and B, which involve numbers taken from the first 10 positive integers.

$A = \{1, 2, 4, 5, 7, 8, 10\}$ $B = \{2, 3, 5, 6, 8, 9\}$

a Represent events A and B in a Venn diagram, as shown below.



b List the following sets.

i A and B **ii** A or B

c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.

i A **ii** A and B **iii** A or B

d Are the events A and B mutually exclusive? Why or why not?

5 The elements of the events A and B described below are numbers taken from the first 10 prime numbers.

$A = \{2, 5, 7, 11, 13\}$ $B = \{2, 3, 13, 17, 19, 23, 29\}$

a Represent events A and B in a Venn diagram.

b List the elements belonging to the following.

i A and B **ii** A or B

c If a number from the first 10 prime numbers is selected, find the probability that these events occur.

i A **ii** B **iii** A and B **iv** A or B

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 4 Using Venn diagrams

From a class of 30 students, 12 enjoy cricket (C), 14 enjoy netball (N) and 6 enjoy both cricket and netball.

a Illustrate this information in a Venn diagram.

b State the number of students who enjoy:

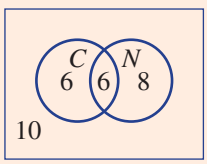
- **i** netball only
- **ii** neither cricket nor netball

c Find the probability that a person chosen randomly from the class will enjoy:

- **i** netball
- **ii** netball only
- **iii** both cricket and netball

SOLUTION

a



b **i** 8

ii 10

c **i** $P(N) = \frac{14}{30} = \frac{7}{15}$

ii $P(N \text{ only}) = \frac{8}{30} = \frac{4}{15}$

iii $P(C \text{ and } N) = \frac{6}{30} = \frac{1}{5}$

EXPLANATION

First place the 6 in the intersection (i.e. 6 enjoy cricket and netball), then determine the other values according to the given information.

The total must be 30, with 12 in the cricket circle and 14 in netball.

Includes students in N but not in C.

These are the students outside both C and N.

14 of the 30 students enjoy netball.

8 of the 30 students enjoy netball but not cricket.

6 students like both cricket and netball.

Now you try

From a survey of 20 families, 7 enjoy camping (C), 10 enjoy beach holidays (B) and 2 enjoy both camping and beach holidays.

a Illustrate this information in a Venn diagram.

b State the number of families who enjoy:

- **i** camping only
- **ii** neither camping nor beach holidays

c Find the probability that a randomly chosen family from the survey enjoys:

- **i** camping
- **ii** camping only
- **iii** both camping and beach holidays

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 From a group of 50 adults, 35 enjoy reading fiction (F), 20 enjoy reading non-fiction (N) and 10 enjoy reading both fiction and non-fiction.

Hint: First enter the '10' in the intersection, then balance all the other regions.

- **a** Illustrate the information in a Venn diagram.
- **b** State the number of people who enjoy:
  - **i** fiction only
  - **ii** neither fiction nor non-fiction
- **c** Find the probability that a person chosen randomly from the group will enjoy reading:
  - **i** non-fiction
  - **ii** non-fiction only
  - **iii** both fiction and non-fiction

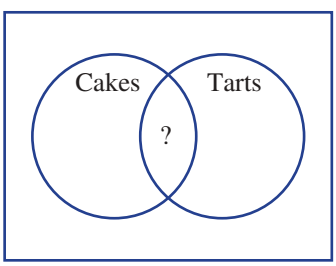
7 At a show, 45 children have the choice of riding on the Ferris wheel (F) and/or the Big Dipper (B). 35 of the children wish to ride on the Ferris wheel, 15 children want to ride on the Big Dipper and 10 children want to ride on both.

- **a** Illustrate the information in a Venn diagram.
- **b** State the number of children who want to:
  - **i** ride on the Ferris wheel only
  - **ii** ride on neither the Ferris wheel nor the Big Dipper
- **c** For a child chosen at random from the group, find the probability that they will want to ride on:
  - **i** the Ferris wheel
  - **ii** both the Ferris wheel and the Big Dipper
  - **iii** the Ferris wheel or the Big Dipper
  - **iv** not the Ferris wheel
  - **v** neither the Ferris wheel nor the Big Dipper

PROBLEM-SOLVING AND REASONING

8, 9 | 8–10

8 In a group of 12 chefs, all enjoy baking cakes and/or tarts. In fact, 7 enjoy baking cakes and 8 enjoy baking tarts. Find out how many chefs from the group enjoy baking both cakes and tarts.



9 In a group of 32 car enthusiasts, all collect either vintage cars or modern sports cars. 18 collect vintage cars and 19 collect modern sports cars. How many from the group collect both vintage cars and modern sports cars?

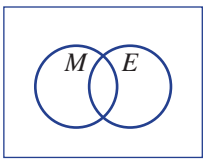
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 Mario and Elisa are choosing a colour to paint the interior walls of their house. They have six colours to choose from: white (w), cream (c), navy (n), sky blue (s), maroon (m) and violet (v).

Mario would be happy with white or cream and Elisa would be happy with cream, navy or sky blue. As they can't decide, a colour is chosen at random for them.

Let M be the event that Mario will be happy with the colour and let E be the event that Elisa will be happy with the colour.

a Represent the events M and E in a Venn diagram.



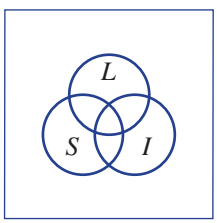
b Find the probability that the following events occur.

- **i** Mario will be happy with the colour choice; i.e. find $P(M)$.
- **ii** Mario will not be happy with the colour choice.
- **iii** Both Mario and Elisa will be happy with the colour choice.
- **iv** Mario or Elisa will be happy with the colour choice.
- **v** Neither Mario nor Elisa will be happy with the colour choice.

ENRICHMENT: Courier companies — 11

11 Of 15 chosen courier companies, 9 offer a local service (L), 7 offer an interstate service (S) and 6 offer an international service (I). Two companies offer all three services, 3 offer both local and interstate services, 5 offer only local services and 1 offers only an international service.

a Draw a Venn diagram displaying the given information.



b Find the number of chosen courier companies that offer neither local, interstate nor international services.

c If a courier is chosen at random from the 15 initially examined, find the following probabilities.

- **i** $P(L)$
- **ii** $P(L$ only)
- **iii** $P(L$ or $S)$
- **iv** $P(L$ and S only)

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4C Two-way tables EXTENDING

Learning intentions for this section:

- To know that a two-way table is another way to show all combinations of two attributes
- To understand how the rows and columns of a two-way table work
- To be able to fill out a two-way table either from a problem or from a Venn diagram
- To be able to use a two-way table to find associated probabilities

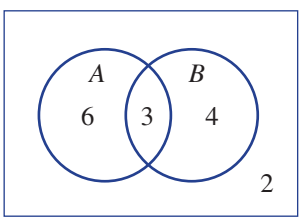
Past, present and future learning:

- This section is included as an Extending topic
- It consolidates and extends the concepts in Section 10B of our Year 9 book
- Two-way tables are mentioned briefly in Stage 6 Standard
- Two-way tables are in a Stage 5 Path topic for students intending to do Stage 6 Advanced

Like Venn diagrams, two-way tables are useful tools for the organisation of overlapping events. The totals at the end of each column and row help to find the unknown numbers required to solve various problems.

Lesson starter: Comparing Venn diagrams with two-way tables

Here is a Venn diagram and an incomplete two-way table.



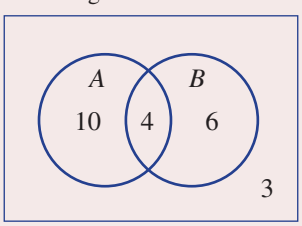
	A	not *A*	Total
B		4	
not *B*			8
Total	9		15

- First, can you complete the two-way table?
- Describe what each box in the two-way table means.
- Was it possible to find all the missing numbers in the two-way table without referring to the Venn diagram?

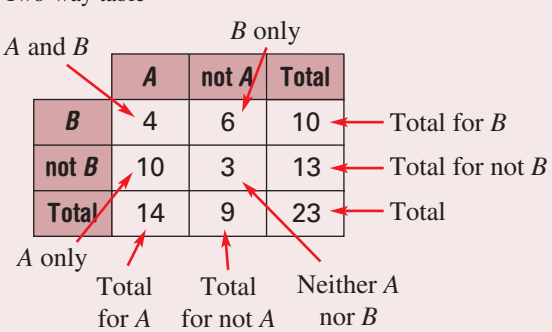
KEY IDEAS

■ **Two-way tables** use rows and columns to describe the number of elements in different regions of overlapping events.

Venn diagram



Two-way table



Key terminology: two-way table, Venn diagram

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 4C

UNDERSTANDING | 1, 2 | 2

1 Match the shaded two-way tables (**A–D**) with each description (**a–d**).

a A and B **b** B only **c** A **d** A or B

A

	A	not A	Total
B			
not B			
Total			

B

	A	A	Total
B			
not B			
Total			

C

	A	not A	Total
B			
not B			
Total			

D

	A	not A	Total
B			
not B			
Total			

2 Look at this two-way table.

	A	not A	Total
B	4	3	7
not B	6	1	7
Total	10	4	14

a State the number of elements in these events.

i A and B **ii** A only
iii B only **iv** neither A nor B
v A **vi** B
vii not A **viii** not B

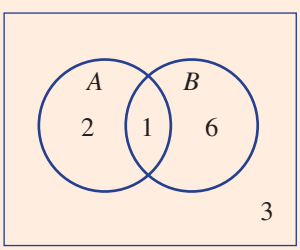
b A or B includes A and B, A only and B only.
Find the total number of elements in A or B.

Hint: A only means A and not B.

FLUENCY | 3, 4 | 3–5

Example 5 Using two-way tables

The Venn diagram shows the distribution of elements in two sets, A and B.



a Transfer the information in the Venn diagram to a two-way table.

b Find the number of elements for these regions.

i A and B **ii** B only **iii** A only
iv neither A nor B **v** A **vi** not B
vii A or B

c Find:

i $P(A \text{ and } B)$ **ii** $P(\text{not } A)$ **iii** $P(A \text{ only})$

SOLUTION

a

	A	not A	Total
B	1	6	7
not B	2	3	5
Total	3	9	12

EXPLANATION

	A	not A	Total
B	A and B	B only	Total the row
not B	A only	Neither A nor B	Total the row
Total	Total the column	Total the column	Overall total

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b **i** 1 — In both A and B
ii 6 — In B but not A
iii 2 — In A but not B
iv 3 — In neither A nor B
v 3 — Total of A: $2 + 1 = 3$
vi 5 — Total not in B: $2 + 3 = 5$
vii $2 + 1 + 6 = 9$ — In A only or B only or both (three regions)

c **i** $P(A \text{ and } B) = \frac{1}{12}$

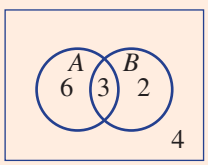
ii $P(\text{not } A) = \frac{9}{12} = \frac{3}{4}$

iii $P(A \text{ only}) = \frac{2}{12} = \frac{1}{6}$

When calculating probabilities, you will need to divide the number of elements in each set by the number of elements in the sample space, which is 12.

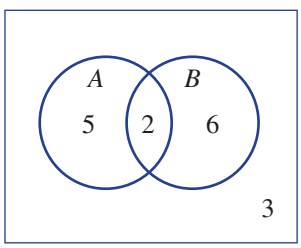
Now you try

The Venn diagram shows the distribution of elements in two sets, A and B.



a Transfer the information in the Venn diagram to a two-way table.
b Find the number of elements for these regions.
i A and B **ii** B only **iii** A only
iv neither A nor B **v** A **vi** not B
vii A or B
c Find:
i $P(A \text{ and } B)$ **ii** $P(\text{not } A)$ **iii** $P(A \text{ only})$

3 The Venn diagram shows the distribution of elements in two sets, A and B.



Hint: A two-way table has these headings:

	A	not *A*	Total
B			
not *B*			
Total			

a Transfer the information in the Venn diagram to a two-way table.
b Find the number of elements in these regions.
i A and B **ii** B only **iii** A only **iv** neither A nor B
v A **vi** not B **vii** A or B
c Find:
i $P(A \text{ and } B)$ **ii** $P(\text{not } A)$ **iii** $P(A \text{ only})$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 From a total of 10 people, 5 like oranges (O), 6 like grapes (G) and 4 like both oranges and grapes.

Hint: Once you have your Venn diagram, you can transfer the information to the two-way table.

a Draw a Venn diagram for the 10 people.

b Draw a two-way table for the 10 people.

c Find the number of people who like:

i only grapes **ii** oranges

iii oranges and grapes **iv** oranges or grapes

d Find:

i $P(G)$ **ii** $P(O \text{ and } G)$ **iii** $P(O \text{ only})$ **iv** $P(\text{not } G)$ **v** $P(O \text{ or } G)$

5 Of 12 people interviewed at a train station, 7 like staying in hotels, 8 like staying in apartments and 4 like staying in hotels and apartments.

a Draw a two-way table for the 12 people.

b Find the number of people who like:

i only hotels

ii neither hotels nor apartments

c Find the probability that one of the people interviewed likes:

i hotels or apartments

ii only apartments

PROBLEM-SOLVING AND REASONING 6–8 7–10

6 Complete the following two-way tables.

a

	A	not *A*	Total
B		3	6
not *B*			
Total		4	11

Hint: All the rows and columns should add up correctly.

b

	A	not *A*	Total
B	2	7	
not *B*			3
Total	4		

7 In a class of 24 students, 13 like Mathematics, 9 like English and 3 like both.

a Find the probability that a randomly selected student from the class likes both Mathematics and English.

b Find the probability that a randomly selected student from the class likes neither Mathematics nor English.

8 Two sets, A and B, are mutually exclusive.

a Find $P(A \text{ and } B)$.

b Now complete this two-way table.

	A	not *A*	Total
B		6	
not *B*			12
Total	10		18

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 Of 32 cars at a show, 18 cars have four-wheel drive, 21 are sports cars and 27 have four-wheel drive or are sports cars.

a Find the probability that a randomly selected car at the show is both four-wheel drive and a sports car.

b Find the probability that a randomly selected car at the show is neither four-wheel drive nor a sports car.

10 A card is selected from a deck of 52 playing cards.
Find the probability that the card is:

a a heart or a king **b** a club or a queen

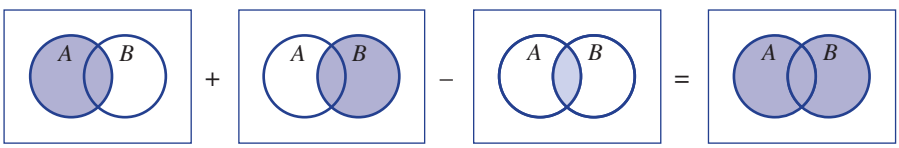
c a black card or an ace **d** a red card or a jack

Hint: Make sure you don't count some cards twice; e.g. the king of hearts in part **a**.

ENRICHMENT: The addition rule — 11

For some of the problems above you will have noticed the following:

$A + B - A \text{ and } B = A \text{ or } B$



This is called the **addition rule**.

11 Use the addition rule to find A or B in these problems.

a Of 20 people at a sports day, 12 people like hurdles (H), 14 like discus (D) and 8 like both hurdles and discus (H and D). How many like hurdles or discuss?

b Of 100 households, 84 have wide screen TVs, 32 have high-definition TVs and 41 have both. How many have wide screen or high-definition TVs?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4D Using arrays for two-step experiments

Learning intentions for this section:

- To know how to list the sample space of a two-step experiment in an array
- To understand the difference between 'with replacement' and 'without replacement'
- To be able to construct arrays and then find associated probabilities

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Probability A*
- It consolidates and extends the concepts in Section 10C of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Some experiments involve two steps. Examples include:

- tossing two coins
- rolling two dice
- tossing a coin once and rolling a die once
- choosing two chocolates from a box

This section will look at probabilities associated with such events.

Lesson starter: Two prizes, three people

Two special prizes are to be awarded in some way to Bill, May and Li for their efforts in helping at the school fête. This table shows how the prizes might be awarded.

		2nd prize		
		Bill	**May**	**Li**
1st prize	Bill	(B, B)	(B, M)	(B, L)
	May	(M, B)		
	Li			

- Complete the table to show how the two prizes can be awarded.
- Does the table show that the same person can be awarded both prizes?
- What is the probability that Bill and Li are both awarded a prize?
- How would the table change if the same person could not be awarded both prizes?
- How do the words 'with replacement' and 'without replacement' relate to the situation above? Discuss.

KEY IDEAS

- An array is a table that can be used to list the sample space for two-step experiments.
- In some two-step experiments, the first item chosen *can* be chosen again. This is called 'with replacement'.
- If replacement is allowed, then outcomes from each selection can be repeated.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

■ If selections are made without replacement, then outcomes from each selection cannot be repeated.
For example: Two selections are made from the digits $\{1, 2, 3\}$.

With replacement

		1st		
		1	2	3
2nd	1	(1,1)	(2,1)	(3,1)
	2	(1,2)	(2,2)	(3,2)
	3	(1,3)	(2,3)	(3,3)

9 outcomes

Without replacement

		1st		
		1	2	3
2nd	1	×	(2,1)	(3,1)
	2	(1,2)	×	(3,2)
	3	(1,3)	(2,3)	×

6 outcomes

Key terminology: with replacement, without replacement, two-step experiments, sample space

Exercise 4D

UNDERSTANDING 1–3 1, 3

1 Choose either *with replacement* or *without replacement* to complete the following.

a Two chocolates are selected from a box and eaten. This is an example of _____.

b Two cards are selected from a pack one after the other and their suit recorded. Each card is returned to the pack after its suit is recorded. This is an example of _____.

2 Two letters are chosen from the word DOG.

a Complete a table listing the sample space if selections are made:

i with replacement

		1st		
		D	O	G
2nd	D	(D, D)	(0, D)	
	O			
	G			

ii without replacement

		1st		
		D	O	G
2nd	D	×	(0, D)	
	O		×	
	G			×

b State the total number of outcomes if selection is made:

i with replacement **ii** without replacement

c If selection is made with replacement, find the probability that:

i the two letters are the same

ii there is at least one D

iii there is not an O

iv there is a D or a G

v there is a D and a G

Hint: Count up your favourable outcomes and divide by the total.

d If selection is made without replacement, find the probability that:

i the two letters are the same

ii there is at least one D

iii there is not an O

iv there is a D or a G

v there is a D and a G

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Two digits are selected from the set $\{2, 3, 4\}$ to form a two-digit number. Find the number of two-digit numbers that can be formed if the digits are selected:

a with replacement

	2	3	4
2	22	32	
3			
4			

b without replacement

	2	3	4
2	×	32	
3		×	
4			×

FLUENCY 4–6 4–6

Example 6 Constructing a table with replacement

A fair 6-sided die is rolled twice.

a List all the outcomes, using a table.

b State the total number of outcomes.

c Find the probability of obtaining the outcome $(1, 5)$.

d Find:

i $P(\text{double})$ **ii** $P(\text{sum of at least 10})$ **iii** $P(\text{sum not equal to 7})$

SOLUTION

a

		Roll 2					
		1	**2**	**3**	**4**	**5**	**6**
Roll 1	**1**	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

EXPLANATION

Be sure to place the number from roll 1 in the first position for each outcome.

b 36 outcomes

There is a total of $6 \times 6 = 36$ outcomes.

c $P(1, 5) = \frac{1}{36}$

Only one outcome is $(1, 5)$.

d **i** $P(\text{double}) = \frac{6}{36}$

$= \frac{1}{6}$

Six outcomes have the same number repeated.

ii $P(\text{sum of at least 10}) = \frac{6}{36} = \frac{1}{6}$

Six outcomes have a sum of either 10, 11 or 12.

iii $P(\text{sum not equal to 7}) = 1 - \frac{6}{36}$

$= \frac{30}{36}$

$= \frac{5}{6}$

This is the complement of having a sum of 7.

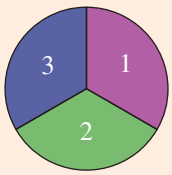
Six outcomes have a sum of 7.

$P(\text{not } A) = 1 - P(A)$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

The spinner shown is spun twice.



a List all the outcomes, using a table.
b State the total number of outcomes.
c Find the probability of obtaining the outcome (3, 2).
d Find:
- **i** P(same 2 numbers)
- **ii** P(product is more than 3)
- **iii** P(product is not equal to 9)

4 A fair 4-sided die is rolled twice.

Hint: Questions **4** and **5** are making selections 'with replacement' because outcomes can be repeated.

a List all the outcomes, using a table.

		1st roll			
		1	**2**	**3**	**4**
2nd roll	**1**	(1, 1)	(2, 1)		
	2				
	3				
	4				

b State the total number of possible outcomes.
c Find the probability of obtaining the outcome (2, 4).
d Find the probability of:
- **i** a double
- **ii** a sum of at least 5
- **iii** a sum not equal to 4

5 Two coins are tossed, each landing with a head (H) or tail (T).

		1st toss	
		H	**T**
2nd toss	**H**	(H, H)	(T, H)
	T		

a List all the outcomes, using a table.
b State the total number of possible outcomes.
c Find the probability of obtaining the outcome (H, T).
d Find the probability of obtaining:
- **i** exactly one tail
- **ii** at least one tail

e If the two coins are tossed 1000 times, how many times would you expect to get two tails?

Example 7 Constructing a table without replacement

Two letters are chosen from the word KICK, without replacement.
a Construct a table to list the sample space.
b Find the probability of:
- **i** obtaining the outcome (K, C)
- **ii** selecting two letters that are K
- **iii** selecting a K and a C

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

a

		1st			
		K	I	C	K
2nd	K	×	(I, K)	(C, K)	(K, K)
	I	(K, I)	×	(C, I)	(K, I)
	C	(K, C)	(I, C)	×	(K, C)
	K	(K, K)	(I, K)	(C, K)	×

b i $P(\text{K, C}) = \frac{2}{12}$

$= \frac{1}{6}$

ii $P(\text{K, K}) = \frac{2}{12}$

$= \frac{1}{6}$

iii $P(\text{K and C}) = \frac{4}{12}$

$= \frac{1}{3}$

EXPLANATION

Selection is without replacement, so the same letter (from the same position) cannot be chosen twice.

Two of the 12 outcomes are (K, C).

Two of the outcomes are K and K, which use different Ks from the word KICK.

Four outcomes contain a K and a C.

Now you try

Two letters are chosen from the word TREE, without replacement.

a Construct a table to list the sample space.

b Find the probability of:

i obtaining the outcome (T, E)

ii selecting two letters that are E

iii selecting an E and a T

6 Two letters are chosen from the word SETS, without replacement.

a Complete this table to list the sample space.

		1st			
		S	E	T	S
2nd	S	×	(E, S)	(T, S)	(S, S)
	E		×		
	T			×	
	S				×

b Find the probability of:

i obtaining the outcome (E, S)

ii selecting one T

iii selecting two letters that are S

iv selecting an S and a T

v selecting an S or a T

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

7, 8 | 8–10

7 A letter is chosen from the word LEVEL without replacement and then a second letter is chosen from the same word.

Hint: Remember that this is 'without replacement'.

- **a** Draw a table displaying the sample space for the pair of letters chosen.
- **b** State the total number of outcomes possible.
- **c** State the number of outcomes that contain exactly one of the following letters.
  - **i** V **ii** L **iii** E
- **d** Find the probability that the outcome will contain exactly one of the following letters.
  - **i** V **ii** L **iii** E
- **e** Find the probability that the two letters chosen will be the same.

8 In a quiz, Min guessed that the probability of rolling a sum of 10 or more from two 6-sided dice is 10%. Complete the following to decide whether or not this guess is correct.

		Die 2					
		1	**2**	**3**	**4**	**5**	**6**
Die 1	**1**	2	3	…			
	2	3	…				
	3	4					
	4	⋮					
	5	⋮					
	6						

- **a** Copy and complete the table representing all the outcomes for possible totals that can be obtained.
- **b** State the total number of outcomes.
- **c** Find the number of the outcomes that represent a sum of:
  - **i** 3
  - **ii** 7
  - **iii** less than 7
- **d** Find the probability that the following sums are obtained.
  - **i** 7 **ii** less than 5 **iii** greater than 2 **iv** at least 11
- **e** Find the probability that the sum is at least 10, and decide whether or not Min's guess is correct.

9 A coin and a 6-sided die are tossed. Heads on the coin is worth 2 points and a tail is worth 4 points. These points are added to the score on the die. Use a table to find the probability that the total score is:

a 4 **b** 6 **c** greater than 7 **d** at most 6

10 Decide whether the following situations would naturally involve selections with replacement or without replacement.

- **a** selecting two people to play in a team
- **b** tossing a coin twice
- **c** rolling two dice
- **d** choosing two chocolates to eat

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: Random weights — 11

11 In a gym, Justine considers choosing two weights to fit onto a leg weights machine to make the load heavier. She can choose from 2.5 kg, 5 kg, 10 kg or 20 kg, and there are plenty of each weight available. Justine's friend randomly chooses both weights, with equal probability that she will choose each weight, and places them on the machine. Justine then attempts to operate the machine without knowing which weights were chosen.

a Complete a table that displays all possible total weights that could be placed on the machine.

b State the total number of outcomes.

c How many of the outcomes deliver a total weight described by the following?

- **i** equal to 10 kg
- **ii** less than 20 kg
- **iii** at least 20 kg

d Find the probability that Justine will be attempting to lift the following weights.

- **i** 20 kg
- **ii** 30 kg
- **iii** no more than 10 kg
- **iv** less than 10 kg

e If Justine is unable to lift more than 22 kg, what is the probability that she will not be able to operate the leg weights machine?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Progress quiz

4A **1** A letter is chosen from the word ELEPHANT. Find the probability that the letter is:
a an E **b** a T or an E **c** not an E **d** a vowel

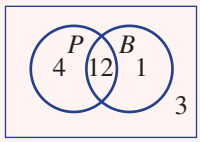
4A **2** An experiment involves rolling two dice and counting the number of even numbers. Here are the results after running the experiment 100 times.

Number of even numbers	0	1	2
Frequency	21	46	33

a How many times did more than 1 even number occur?
b Find the experimental probability of obtaining:
i 0 even numbers
ii fewer than 2 even numbers
iii at least 1 even number

4B **3** From a group of 25 students on a school camp, 18 enjoy sailing (S), 15 enjoy bushwalking (B) and 8 enjoy both sailing and bushwalking.
a Illustrate this information in a Venn diagram.
b State the number of students who enjoy:
i sailing only
ii neither sailing nor bushwalking
c Find the following probabilities for a student chosen at random from the group.
i $P(B)$ **ii** $P(S \text{ only})$ **iii** $P(B \text{ and } S)$ **iv** $P(\text{not } B)$

4C **4** This Venn diagram shows the distribution of 20 guests staying at a resort in Noosa. Some guests liked to swim at the hotel pool (P), others liked to swim at the beach (B) and others liked both.



a Transfer the information to a two-way table.
b Find the number of guests who like:
i only swimming at the hotel pool
ii swimming at either the beach or the hotel pool
c Find:
i $P(P)$ **ii** $P(B \text{ only})$ **iii** $P(P \text{ and } B)$

4D **5** A fair 6-sided die is rolled and a coin is tossed.

	1	2	3	4	5	6
H	(1, H)	(2, H)…				
T						

a List all the outcomes, using a table.
b State the total number of outcomes.
c Find the probability of obtaining the outcome (3, H).
d Find the probability of obtaining:
i an even number and a tail
ii at least a 2 and a head

4D **6** Two counters are chosen randomly from a bag containing 2 red, 1 blue and 1 green counter, without replacement.
a Construct a table to list the sample space and state the number of outcomes.
b Find the probability of:
i obtaining the outcome (R, G)
ii selecting 2 red counters
iii selecting 1 blue and 1 red counter

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4E Using tree diagrams

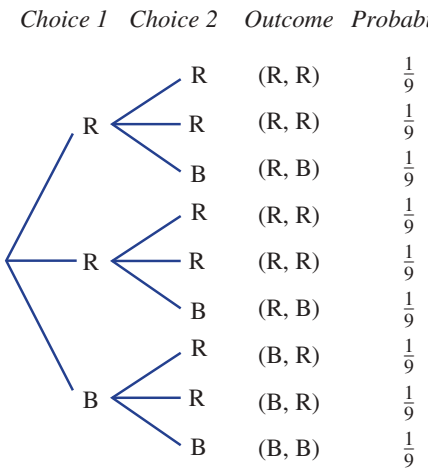
Learning intentions for this section:

- To know how to use a tree diagram to list the sample space from multistage experiments
- To understand the difference between 'with replacement' and 'without replacement'
- To be able to use a tree diagram to find the probability of certain events

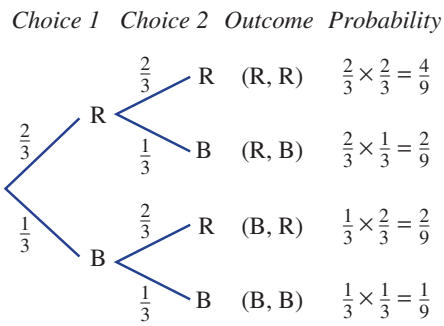
Past, present and future learning:

- This section addresses the Stage 5 Core Topic called *Probability A*
- It consolidates and extends the concepts in Section 10D of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Tree diagrams can be used to help list outcomes for multi-stage experiments. Suppose that a bag contains two red counters and one blue counter and that two counters are selected at random with replacement. One way to display the outcomes is with a tree diagram in which all equally likely outcomes are listed in columns, as shown in the first diagram below. A more efficient way, however, is to group similar outcomes and write their corresponding probabilities on the branches, as shown in the second diagram.



$$P(\text{R, B}) = \frac{2}{9}$$



$$P(\text{R, B}) = \frac{2}{9}$$

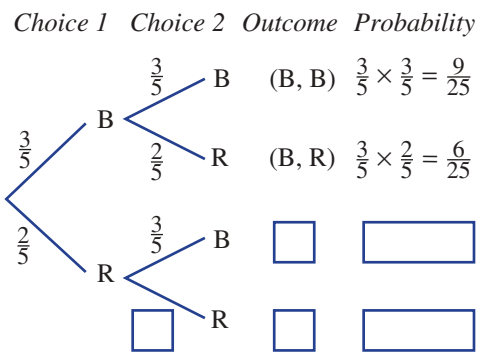
In the second tree diagram, the probability of each outcome is obtained by multiplying the branch probabilities. This also applies when selection is made without replacement.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

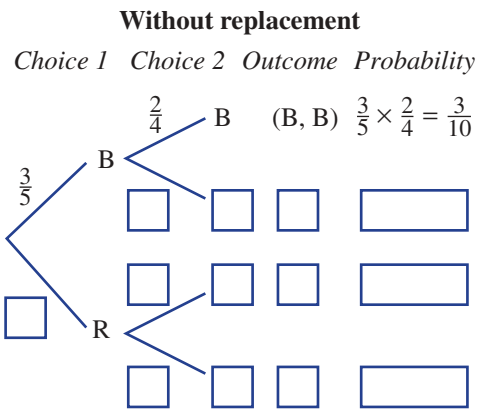
Lesson starter: Trees with and without replacement

Suppose that two selections are made from a group of 2 red and 3 blue beads.

With replacement



Without replacement



- Complete these two tree diagrams to show how these selections can be made, both with and without replacement.
- Explain where the branch probabilities come from on each branch of the tree diagrams.
- What is the total of all the probabilities on each tree diagram?

KEY IDEAS

■ **Tree diagrams** can be used to list the sample space for experiments involving two or more components.

- Branch probabilities are used to describe the chance of each outcome at each step.
- The probability of each outcome for the experiment is obtained by multiplying the branch probabilities.
- Branch probabilities will depend on whether selection is made with or without replacement.
  For experiments *with replacement*, probabilities do not change.
  For experiments *without replacement*, probabilities do change.

Key terminology: tree diagram, with replacement, without replacement, sample space

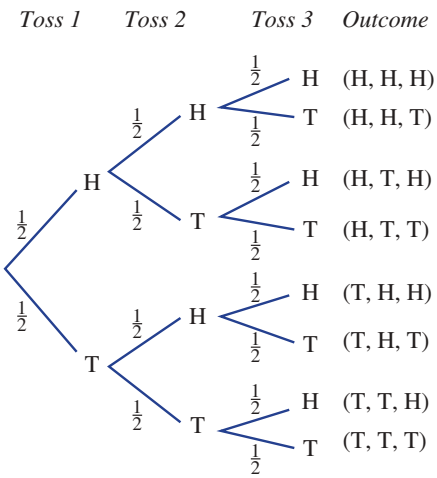
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 4E

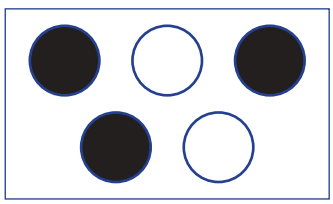
UNDERSTANDING 1, 2 | 2

1 A coin is tossed three times and a head or tail is obtained each time as shown in the tree diagram.

- **a** How many outcomes are there?
- **b** What is the probability of the outcome HHH?
- **c** How many outcomes obtain:
  - **i** 2 tails?
  - **ii** 2 or 3 heads?



2 A box contains 2 white (W) and 3 black (B) counters.



- **a** A single counter is drawn at random. Find the probability that it is:
  - **i** white **ii** black
- **b** Two counters are now drawn at random. The first one is replaced before the second one is drawn. Find the probability that the second counter is:
  - **i** white **ii** black
- **c** Two counters are drawn and the first counter is not replaced before the second one is drawn. If the first counter is white, find the probability that the second counter is:
  - **i** white **ii** black

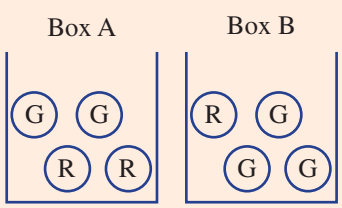
Hint: After one white counter is taken out, how many of each remain?

FLUENCY 3, 4 | 3–5

Example 8 Constructing a tree diagram for multi-stage experiments

Boxes A and B contain 4 counters each. Box A contains 2 red and 2 green counters and box B contains 1 red and 3 green counters. A box is chosen at random and then a single counter is selected.

- **a** What is the probability of selecting a red counter from box A?
- **b** What is the probability of selecting a red counter from box B?
- **c** Represent the options available as a tree diagram that shows all possible outcomes and related probabilities.
- **d** What is the probability of selecting box B and a red counter?
- **e** What is the probability of selecting a red counter?



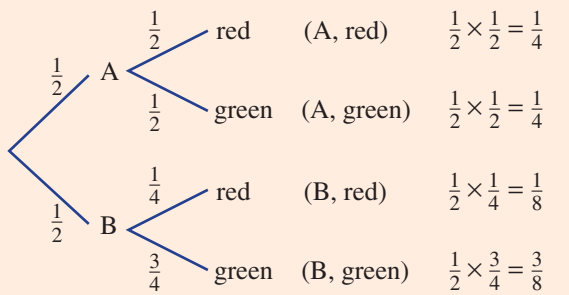
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

a $P(\text{red from box A}) = \frac{2}{4} = \frac{1}{2}$

b $P(\text{red from box B}) = \frac{1}{4}$

c



d $P(\text{B, red}) = \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{8}$

e $P(\text{1 red}) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{4} + \frac{1}{8}$

$= \frac{3}{8}$

EXPLANATION

2 of the 4 counters in box A are red.

1 of the 4 counters in box B is red.

First selection is a box followed by a counter.

Multiply each of the probabilities along the branch pathways to find the probability of each outcome.

The probability of choosing box B is $\frac{1}{2}$ and a red counter from box B is $\frac{1}{4}$, so multiply the probabilities for these two outcomes together.

The outcomes (A, red) and (B, red) both contain 1 red counter, so add together the probabilities for these two outcomes.

Now you try

Boxes A and B contain 5 counters each. Box A contains 2 red and 3 blue counters and box B contains 1 red and 4 blue counters.

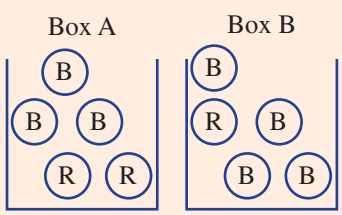
a What is the probability of selecting a red counter from box A?

b What is the probability of selecting a red counter from box B?

c Represent the options available as a tree diagram that shows all possible outcomes and related probabilities.

d What is the probability of selecting box B and a red counter?

e What is the probability of selecting a red counter?



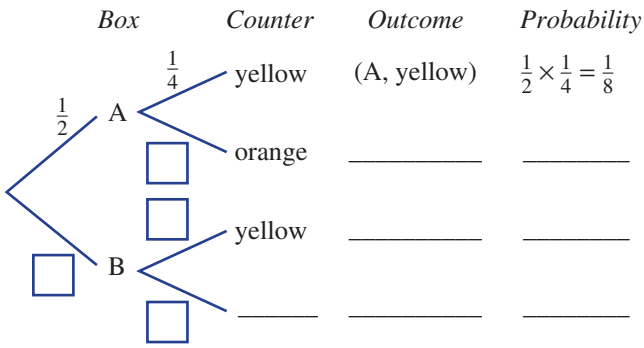
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Boxes A and B contain 4 counters each. Box A contains 1 yellow and 3 orange counters and box B contains 3 yellow and 1 orange counter. A box is chosen at random and then a single counter is selected.

a If box A is chosen, what is the probability of selecting a yellow counter?

b If box B is chosen, what is the probability of selecting a yellow counter?

c Represent the options available by completing this tree diagram.



d What is the probability of selecting box B and a yellow counter?

e What is the probability of selecting 1 yellow counter?

Hint: For part **e**, add the probabilities for both outcomes that have a yellow counter.

Example 9 Using a tree diagram without replacement

A bag contains 5 blue (B) and 3 white (W) marbles, and 2 marbles are selected without replacement.

a Draw a tree diagram showing all outcomes and probabilities.

b Find the probability of selecting:

i a blue marble followed by a white marble; i.e. the outcome (B, W)

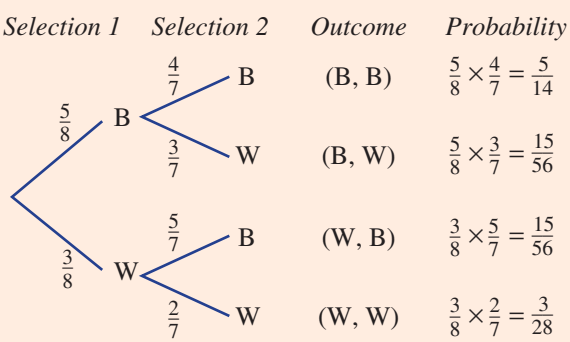
ii 2 blue marbles

iii exactly 1 blue marble

c If the experiment is repeated with replacement, find the answers to each question in part **b**.

SOLUTION

a



EXPLANATION

After 1 blue marble is selected there are 7 marbles remaining: 4 blue and 3 white.

After 1 white marble is selected there are 7 marbles remaining: 5 blue and 2 white.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b **i** $P(\text{B, W}) = \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$

Multiply the probabilities on the (B, W) pathway.

ii $P(\text{B, B}) = \frac{5}{8^{2}} \times \frac{4^{1}}{7} = \frac{5}{14}$

Only 4 blue marbles remain after the first selection. Multiply the probabilities on the (B, B) pathway.

iii $P(\text{1 blue}) = \frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{5}{7}$

$= \frac{15}{28}$

The outcomes (B, W) and (W, B) both have 1 blue marble. Multiply probabilities to find individual probabilities, then sum for the final result.

c **i** $P(\text{B, W}) = \frac{5}{8} \times \frac{3}{8}$

$= \frac{15}{64}$

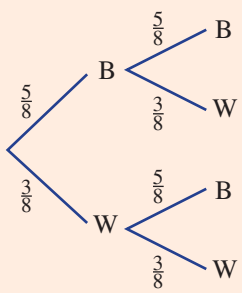
When selecting objects with replacement, remember that the number of marbles in the bag remains the same for each selection.

ii $P(\text{B, B}) = \frac{5}{8} \times \frac{5}{8}$

$= \frac{25}{64}$

iii $P(\text{1 blue}) = \frac{5}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{5}{8}$

$= \frac{15}{32}$



Now you try

A jar contains 4 toffees (T) and 3 mints (M), and 2 lollies are selected without replacement.

a Draw a tree diagram showing all outcomes and probabilities.

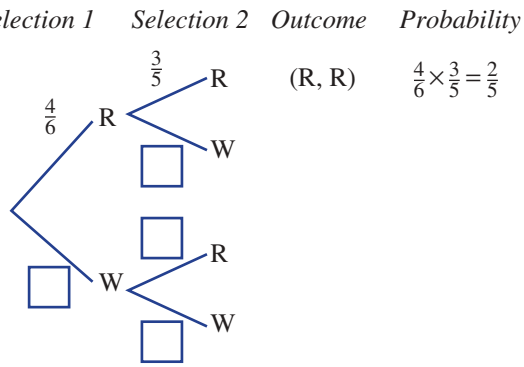
b Find the probability of selecting:

- **i** a toffee followed by a mint: i.e. the outcome (T, M)
- **ii** 2 mints
- **iii** exactly 1 toffee

c If the experiment is repeated with replacement, find the answers to each question in part **b**.

4 A bag contains 4 red (R) and 2 white (W) marbles, and 2 marbles are selected without replacement.

a Complete this tree diagram, showing all outcomes and probabilities.



b Find the probability of selecting:

- **i** a red marble and then a white marble (R, W)
- **ii** 2 red marbles
- **iii** exactly 1 red marble

c If the experiment is repeated with replacement, find the answers to each question in part **b**. You may need to redraw the tree diagram.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Two students are selected from a group of 3 class A (M) and 4 class B (F) students, without replacement.

a Complete this tree diagram to help find the probability of selecting:

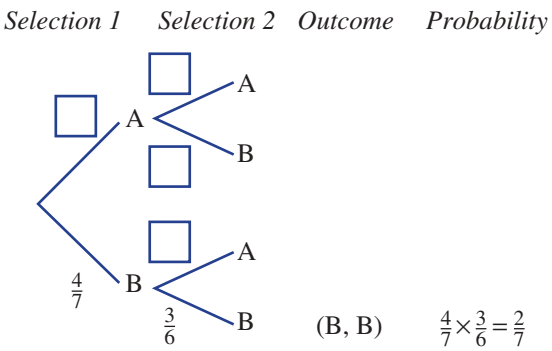
i 2 class A students

ii 2 class B students

iii 1 class A student and 1 class B student

iv 2 people either both class A or both from class B

b If the experiment is repeated with replacement, find the answers to each question in part **a**.



PROBLEM-SOLVING AND REASONING

6, 7 | 6–8

6 A fair 4-sided die is rolled twice and the pair of numbers is recorded.

a Complete this tree diagram to list the outcomes.

b State the total number of outcomes.

c Find the probability of obtaining:

i a 4 then a 1; i.e. the outcome (4, 1)

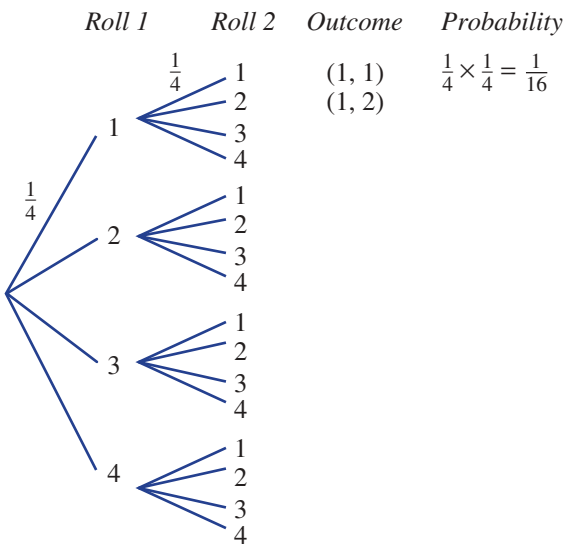
ii a double

d Find the probability of obtaining a sum described by the following:

i equal to 2

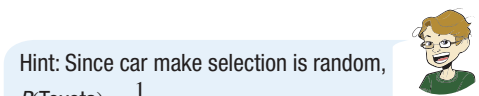
ii equal to 5

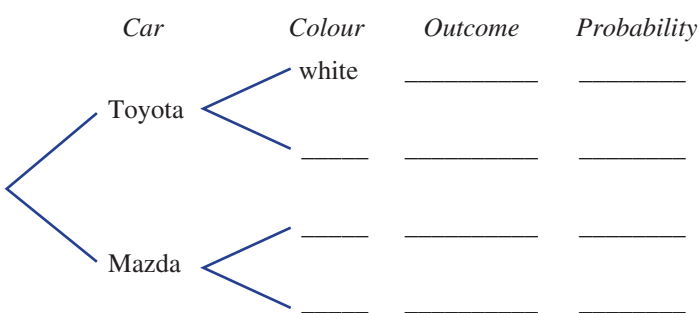
iii less than or equal to 5



7 As part of a salary package, a person can select either a Toyota or a Mazda. There are 3 white Toyotas, 1 silver Toyota, 2 white Mazdas and 1 red Mazda to choose from.

a Complete a tree diagram showing a random selection of a car make, then a colour.





ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b Find the probability that the person chooses:

i a white Toyota
ii a red Mazda
iii a white car
iv a car that is not white
v a silver car or a white car
vi a car that is neither a Toyota nor red

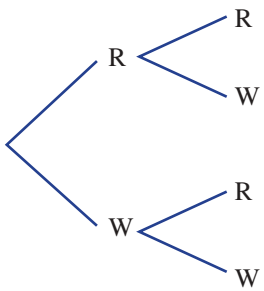
8 Two bottles of wine are randomly selected for tasting from a box containing 2 red and 2 white wines. Use a tree diagram to help answer the following.

a If the first bottle is replaced before the second is selected, find:

i $P(2 \text{ red})$
ii $P(1 \text{ red})$
iii $P(\text{not } 2 \text{ white})$
iv $P(\text{at least } 1 \text{ white})$

b If the first bottle is not replaced before the second is selected, find:

i $P(2 \text{ red})$
ii $P(1 \text{ red})$
iii $P(\text{not } 2 \text{ white})$
iv $P(\text{at least } 1 \text{ white})$



ENRICHMENT: Rainy days — 9

9 Imagine that the probability of rain next Monday is 0.2. The probability of rain on a day after a rainy day is 0.85, whereas the probability of rain on a day after a non-rainy day is 0.1.

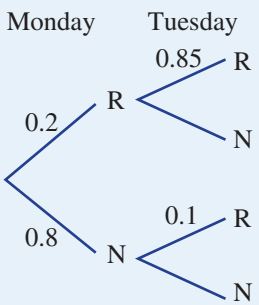
a Next Monday and Tuesday, find the probability of having:

i 2 rainy days
ii exactly 1 rainy day
iii at least 1 dry day

b Next Monday, Tuesday and Wednesday, find the probability of having:

i 3 rainy days
ii exactly 1 dry day
iii at most 2 rainy days

Hint: Draw a tree diagram, like the one below.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4F Dependent events and independent events

Learning intentions for this section:

- To understand the concept of independent events
- To be able to determine if two events are independent
- To know how with and without replacement affects independent events

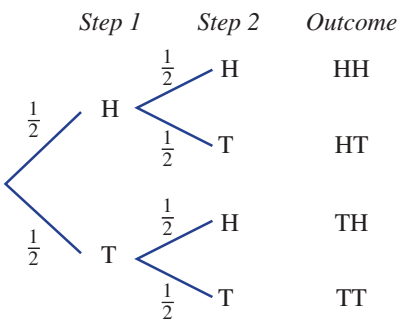
Past, present and future learning:

- This section concludes the Stage 5 Core topic called *Probability A*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

In some two-step experiments, the result in the second step is independent of the result in the first step. This means that the result of the first step has no effect on the probability or outcome of the second step.

Lesson starter: Are they independent?

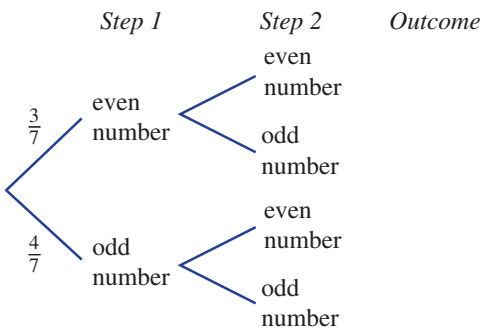
Rachel is tossing a fair coin repeatedly.



Every time she tosses the coin, the probabilities are the same.

- What is the probability of heads on Rachel's first toss?
- What is the probability of heads on Rachel's second toss?
- Is the probability of a head on the second toss affected by the result of the first toss?
- Do you think this means that 'heads on first toss' and 'heads on second toss' are independent events?

Sophie places seven balls, which are numbered from 1 to 7, into a box and then selects one randomly. She does not replace it before she chooses another.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

For each selection the probabilities change.

- What is the probability that the second ball is even if the first ball was:
  - **a** even?
  - **b** odd?
- Are your answers to **a** and **b** different?
- Do you think that the events 'even number on first ball' and 'even number on second ball' are dependent or independent? Why?

KEY IDEAS

- Two events are **independent** if the outcome of the first step does not change the probability of obtaining the other event in the second step.
- For multistage events where selection is made *with* replacement, successive events are independent.
- For multistage events where selection is made *without* replacement, successive events are not independent.
- If a fair coin is tossed many times, the result is always independent of the previous results.

Key terminology: independent events, with replacement, without replacement

Exercise 4F

UNDERSTANDING 1–2 2

1 Would these experiments contain independent events? Answer yes or no.

Hint: Is the probability the same each time?

- **a** Rolling a die 3 times
- **b** Choosing 10 jellybeans from a jar, one at a time, and eating them.
- **c** Choosing 5 cards from a standard deck and removing them
- **d** Tossing a coin 3 times
- **e** Tossing a coin and rolling a die

2 Complete each sentence.

Hint: Choose from 'with' or 'without'.

- **a** For multistage events, successive events are independent if selections are made _____ replacement.
- **b** For multistage events, successive events are not independent if selections are made _____ replacement.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 3, 5 3–5

Example 10 Deciding whether events are independent

Decide whether the following events A and B are independent.

a A die is rolled twice. Let A be the event 'rolling a 6 on the first roll' and let B be the event 'rolling a 3 on the second roll'.

b Two playing cards are randomly selected from a standard deck, without replacement. Let A be the event 'the first card is a heart' and let B be the event 'the second card is a heart'.

SOLUTION	EXPLANATION
a Yes, events A and B are independent.	The outcome of the first roll of the die does not affect the outcome of the second roll.
b No, events A and B are not independent.	The type of card selected first affects the probability of selecting a heart when choosing the second card.

Now you try

In one of these examples, A and B are independent events. Which one is it?
Example 1: Choose a ball from a bag, replace it, then choose a ball from the same bag. Let A be choosing the first ball and B be choosing the second ball.
Example 2: Choose a ball from a bag, set it aside, then choose a ball from the same bag. Let A be choosing the first ball and B be choosing the second ball.

3 A coin is tossed twice. Let A be the event 'the first toss gives a tail'. Let B be the event 'the second toss gives a tail'.
Are A and B independent events?

4 **a** Kathryn tossed a fair coin and it showed heads. What is the probability that the coin shows heads on the next toss?

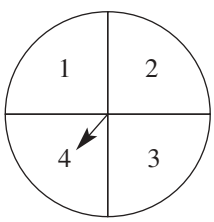
b Felix tossed heads six times in a row. What is the probability that the next toss shows heads?

5 **a** What is the probability of tossing heads three times in a row?

b On the spinner shown, what is the probability of spinning a 4 three times in a row?

c What is the probability of rolling the number 2 on a die three times in a row?

Hint: Looking at the relevant part of a tree diagram may help.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

6 | 6, 7

6 The company that manufactures a popular ice-cream is running a promotion. The company claims that 1 in 6 of its ice-creams contains a stick that gives the owner a free ice-cream.

- **a** Tilly says: "I am going to buy 6 ice-creams. One of them will definitely be a winner." Discuss the validity of Tilly's statement.
- **b** Cody says: "I bought 5 ice-creams last week and none of them were winners, so there is a really high chance that the next one will be a winner." Is he correct?
- **c** Lucy says: "I bought 6 ice-creams last week and all of them were winners." Is this possible?

7 A coin is tossed 5 times. Find the probability of obtaining:

- **a** 5 heads
- **b** at least one tail
- **c** at least one head

Hint: Coin tosses are independent. From two coins, the probability of two heads is $\frac{1}{2} \times \frac{1}{2}$.

Hint: 'At least one tail' includes all cases except 5 heads.

ENRICHMENT: Roll a 6, then stop rolling

— | 8

8 In some games, you must roll a 6 before you can start moving on the board. Try rolling a die and count how many rolls you need to roll a 6. What is the probability that:

- **a** it happens on the first roll?
- **b** you need 2 rolls?
- **c** you need 1, 2, 3 or 4 rolls?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Maths@Work

Business analyst

Businesses employ analysts to look at how their businesses run, and how they can run more efficiently. Analysts use probabilities, relative frequencies and graphs to look at data, undertake calculations and make recommendations.

Being able to summarise and interpret data and understand likelihoods are important skills for analysts and business owners to have.

Complete these questions that a business analyst may face in their day-to-day job.

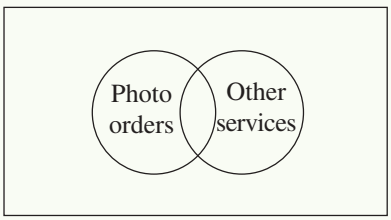
1 An online photo printing company wishes to analyse their customer orders and needs based on the products that are currently purchased from them. They looked at the results for a typical week over two areas of the business: photos and other services, such as canvases, mugs and calendars.
The results of one week's orders are shown in the table below.

	Other services	No other services	Total orders
Photos ordered	561	1179	1740
No photos ordered	260	-	260
Totals	821	1179	2000

a Percentages are often used in probability. Rewrite the table above using percentages, where 100% equals the total orders for the week.

Hint: $\frac{561}{2000} \times \frac{100}{1} = 28.05\%$

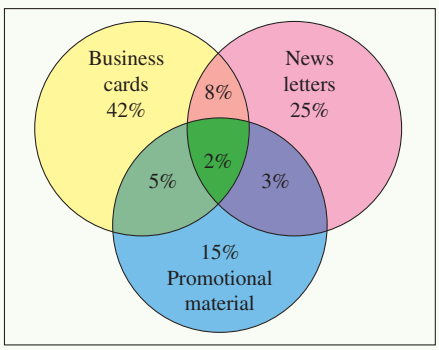
b Show this information in a Venn diagram, using percentages.



c Each customer ordering photos averages 30 prints (size 4×6). The company charges 12 cents a print. How much is the weekly income generated by photos, using the week's orders shown above?

d The company wishes to invest money in some advertising. Should this advertising be for the promotion of their photo printing service or should it be aimed at the other services they supply? Explain your answer.

2 Consider the information of the services offered by a different company wishing to look into how its business operates. They offer three types of products: business cards, newsletter printing and promotional lines, which include calendars, mugs etc. This company does not offer separate photo printing.
The breakdown of their business can be seen in the Venn diagram on the right.



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

a If 1000 customers were surveyed, how many of them purchased:
- **i** only one of the three product lines on offer?
- **ii** exactly two of the product lines?
- **iii** at least two of the product lines?
- **iv** all three product lines available?

b What is the single most important product that the company offers? State the statistics for your selection and explain the significance to the company of this information.

c If you were the owners or share holders in this company, ideally how would you like to see the Venn diagram change over time to see growth within the company?

Using technology

3 'Get Set' is an online business selling sportswear. It has been very successful over the past year and its quarterly sales are shown in this table.

	A	B	C	D	E	F	G
1	Sportswear sales from Get Set online business						
2	**Sport**	**QUARTER 1**	**QUARTER 2**	**QUARTER 3**	**QUARTER 4**	**Totals**	**Percentages**
3	Golf	159	98	104	137		
4	Running	278	312	320	287		
5	Cycling	209	276	248	268		
6	Dance	94	77	107	68		
7	Gym	169	176	195	284		
8	Yoga	29	47	26	32		
9	**Totals**						
10	**Percentages**						

a Copy the data into an Excel spreadsheet and enter appropriate formulas into the shaded cells to find their values.

Hint: Use dollar signs (e.g. H12) in a formula that links to a fixed cell.

b Which type of sportswear is:
- **i** most likely to be sold?
- **ii** least likely to be sold?

Hint: Select the sports and sales area of the table and click on Insert and then Column.

c Insert a column graph showing the quarterly sales for each sport.

d In the fourth quarter, Get Set offered special prices for one line of sportswear as part of an advertising drive. Which sportswear do you think was on special? Was the advertising successful? Give reasons for your answer.

Hint: Hold the Ctrl button while selecting the titles Quarter 1 to Quarter 4 and also their total percentages row.

e Insert a column graph showing the total percentages for each quarter of sales.

f Businesses use graphs to show comparisons between datasets. What is the increase in total percentage from Quarter 1 to Quarter 4? Comment on whether your graph in part **e** exaggerates this increase and, if so, explain why. Make an adjustment on the graph so that it is not misleading.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Statistical investigation

Seeing the northern lights

Wallace travels to Alaska in the hope of seeing an aurora called the northern lights. An aurora is a light show caused by collisions between electrically charged particles.

He takes part in a tour which ventures out for 10 nights.

Wallace is hoping to see the lights at least 7 times. The company that operates the 10-night adventure can show statistically that the probability of witnessing the lights on any given night is 0.5.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

a On average how many times can you expect Wallace to see the lights across the 10 nights?

b Estimate with a guess the probability that Wallace will see the lights at least 7 times.

Non-routine problems

The problem is to use simulation to help estimate the probability that Wallace will see the northern lights at least 7 times.

Explore and connect

Choose and apply techniques

a Use 10 coins (with heads set as 'seeing the lights' and tails as 'not seeing the lights') to simulate a 10-night adventure and count the number of times the lights are observed.

b Describe how the coin is used in the simulation to produce the results.

c Use your coins to simulate the 10-night adventure, counting the number of times the lights are observed.

d Repeat the simulation to generate results for a total of twenty 10-night adventures.

e Use your results to determine the experimental probability that Wallace will see the northern lights at least 7 times.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

f Try to improve your results from part **e** by increasing the number of times the 10-night adventure is simulated.
Use a table like the one shown below to record your results.

Communicate thinking and reasoning

Number of times lights are seen on each 10-night adventure	0	1	2	3	4	5	6	7	8	9	10
Tally											
Frequency											

g Compare your estimated probability with the theoretical value of 0.172 (correct to three decimal places).

h Describe the simulation tool and how it is used to predict success.
What conclusions can be made about the likelihood that Wallace will be happy with his adventure?

Extension problems

Problem-solve

a What other random simulation tools could be used to generate the data needed.

b If the probability of witnessing the lights on any given night is altered to say 0.4, describe how an experiment can be conducted and run your experiment to estimate the probability that Wallace will see the lights at least 7 times on a 10-night adventure.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

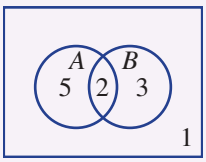
Puzzles and challenges

1 'I have nothing in common.' Match the answers to the letters in parts **a** and **b** to uncover the code.

$\frac{5}{14}$	5	2	5	7	10	10	$\frac{7}{11}$

$\frac{5}{11}$	$\frac{3}{14}$	$\frac{1}{2}$	10	5	$\frac{10}{11}$	$\frac{1}{7}$	3	$\frac{5}{11}$

a These questions relate to the Venn diagram at right.



T How many elements are in A and B?
L How many elements are in A or B?
V How many elements are in B only?
Y Find $P(A)$.
S Find $P(A \text{ or } B)$.
E Find $P(A \text{ only})$.

b These questions relate to the two-way table at right.

	P	not P	
Q	U	4	9
not Q	2		
		A	14

U What number should be in place of the letter U?
A What number should be in place of the letter A?
M Find $P(P \text{ and } Q)$.
C Find $P(\text{not } P)$.
X Find $P(\text{neither } P \text{ nor } Q)$.
I Find $P(P \text{ only})$.

2 What is the chance of getting a sum of at least 10 from rolling two 6-sided dice?

3 *Game for two people*: You will need a bag or pocket and coloured counters.

- One person places 8 counters of 3 different colours in a bag or pocket. The second person must not look!
- The second person then selects a counter from the bag. The colour is noted, then the counter is returned to the bag. This is repeated 100 times.
- Complete this table.

Colour	Tally	Frequency	Guess
Total:	100	100	

- Using the experimental results, the second person now tries to guess how many counters of each colour are in the bag.

4 Two digits are chosen without replacement from the set $\{1, 2, 3, 4\}$ to form a two-digit number. Find the probability that the two-digit number is:

a 32 **b** even **c** less than 40 **d** at least 22

5 A coin is tossed 4 times. What is the probability that at least 1 tail is obtained?

6 Two leadership positions are to be filled from a group of 2 class A and 3 class B students. What is the probability that the positions will be filled by 1 class A and 1 class B student?

7 The letters of the word DOOR are jumbled randomly. What is the probability that the final arrangement will spell DOOR?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

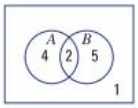
Chapter summary

Probability

Review

- Sample space is the list of all possible outcomes
- $P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

Venn diagram

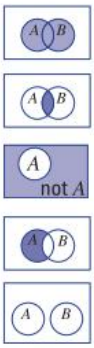


Two-way table

	A	not A	Total
B	2	5	7
not B	4	1	5
Total	6	6	12

Notation

- A or B
- A and B
- Complement of A is not A.
- A only
- Mutually exclusive events



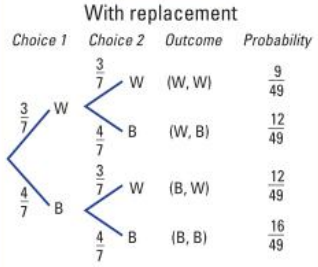
Independent events

- Multistage events are independent if the result of the first step does not change the probabilities of the steps that follow.
- For experiments *with replacement*, successive events are independent.
- For experiments *without replacement*, successive events are not independent.

Tree diagrams

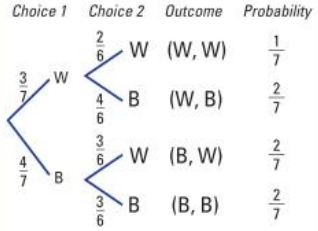
3 white
4 black

With replacement



$$P(\text{W, B}) = \frac{3}{7} \times \frac{4}{7} = \frac{12}{49}$$

Without replacement



$$P(\text{1 white}) = \frac{2}{7} + \frac{2}{7} = \frac{4}{7}$$

Arrays

With replacement

	A	B	C
A	(A, A)	(B, A)	(C, A)
B	(A, B)	(B, B)	(C, B)
C	(A, C)	(B, C)	(C, C)

Without replacement

	A	B	C
A	×	(B, A)	(C, A)
B	(A, B)	×	(C, B)
C	(A, C)	(B, C)	×

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook. ✔

4A

1 I can calculate a theoretical probability.

e.g. A letter is chosen from the word ALLIGATOR. Find the probability that the letter is:

a an A

b not a G

c an A or a T

4A

2 I can find an experimental probability.

e.g. An experiment involves tossing 3 coins and counting the number of tails. The results from running the experiment 100 times are shown.

Number of tails	0	1	2	3
Frequency	15	39	37	9

Find the experimental probability of obtaining:

a 1 tail

b at least 1 tail

4B

3 I can list sets from a Venn diagram.

e.g. Events A and B involve numbers taken from the first 10 positive integers:
$A = \{1, 4, 5, 7\}$ and $B = \{4, 5, 6, 7, 8\}$.
Represent the two events in a Venn diagram and hence:

a list the set A or B

b find the probability that a randomly selected number from the first 10 positive integers is in A and B

c decide if the events are mutually exclusive

4B

4 I can use a Venn diagram.

e.g. From a group of 20 people in a swimming squad, 12 train for freestyle (F), 6 train for butterfly (B) and 3 train for both freestyle and butterfly.
Illustrate this information in a Venn diagram and hence:

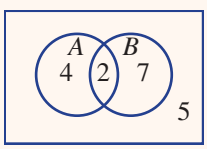
a state the number in the squad who train for butterfly only

b find the probability a randomly selected swimmer from the squad trains for neither freestyle nor butterfly

4C

5 I can use a two-way table.

e.g. The Venn diagram shows the distribution of elements from two events A and B.



Transfer the information into a two-way table and hence, find:

a the number of elements in A and B

b the number of elements in neither A nor B

c P(not B) and $P(A$ only)

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

4D	**6 I can construct an array with replacement.** e.g. A fair 5-sided die is rolled twice. List all the outcomes using a table and find: **a** $P(2 \text{ even numbers})$ **b** $P(\text{sum of at least } 7)$ **c** $P(\text{sum not equal to } 9)$	☐
4D	**7 I can construct an array without replacement.** e.g. Two letters are chosen from the word TENT, without replacement. Construct a table to list the sample space and find the probability of: **a** obtaining the outcome (T, E) **b** selecting a T and an N	☐
4E	**8 I can construct a tree diagram.** e.g. Two jars contain 4 jelly beans each. Jar A has 3 black and 1 pink jelly bean and Jar B contains 2 black and 2 pink jelly beans. A jar is chosen at random followed by a single jelly bean. Represent the options available in a tree diagram that shows all outcomes and the associated probabilities. Find the probability of selecting: **a** jar A and a black jelly bean **b** a black jelly bean	☐
4E	**9 I can use a tree diagram without replacement.** e.g. A mixed bag of chocolates contains 2 Mars bars (M) and 4 Snickers (S). Draw a tree diagram to show the outcomes and probabilities of the selection of two chocolates without replacement. Find the probability of selecting: **a** 2 Snickers **b** exactly 1 Mars bar	☐
4F	**10 I can determine if events are independent.** e.g. A selection of 10 soccer club memberships found that 5 included a free scarf and 3 included a free soccer ball. One had both a free scarf and a free soccer ball. Let A be the event 'soccer membership had a free scarf', and B be the event 'soccer membership had a free soccer ball'. Summarise the information in a Venn diagram and find $P(A\|B)$ and $P(B\|A)$ to determine whether or not events A and B are independent.	☐

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

Short-answer questions

4A **1** A fair 6-sided die is rolled once. Find:

a $P(4)$ **b** $P(\text{even})$ **c** $P(\text{at least 3})$

4A **2** A letter is chosen from the word INTEREST. Find the probability that the letter will be:

a I **b** E **c** a vowel

d not a vowel **e** E or T

4A **3** An engineer inspects 20 houses in a street for cracks. The results are summarised in this table.

Number of cracks	0	1	2	3	4
Frequency	8	5	4	2	1

a From these results, estimate the probability that the next house inspected in the street will have the following number of cracks.

i 0 **ii** 1 **iii** 2 **iv** 3 **v** 4

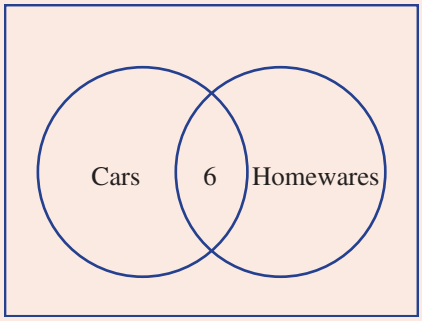
b Estimate the probability that the next house will have:

i at least 1 crack

ii no more than 2 cracks

4B/C **4** Of 36 people, 18 have an interest in cars, 11 have an interest in homewares and 6 have an interest in both cars and homewares.

a Complete this Venn diagram.



b Complete this two-way table.

	C	not *C*	Total
H	6		
not *H*			
Total			

c State the number of people from the group who do not have an interest in either cars or homewares.

d If a person is chosen at random from the group, find the probability that the person will:

i have an interest in cars and homewares

ii have an interest in homewares only

iii not have any interest in cars

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4B/C **5** All 26 birds in an aviary have clipped wings and/or a tag. In total, 18 birds have tags, 14 have clipped wings and 6 have both clipped wings and a tag.

a Find the number of birds that have only clipped wings.

b Find the probability that a bird chosen at random will have a tag only.

4D **6** A letter is chosen at random from the word HAPPY and a second letter is chosen from the word HEY.

a List the sample space by completing this table.

		1st				
		H	A	P	P	Y
2nd	H	(H, H)	(A, H)	(P, H)		
	E					
	Y					

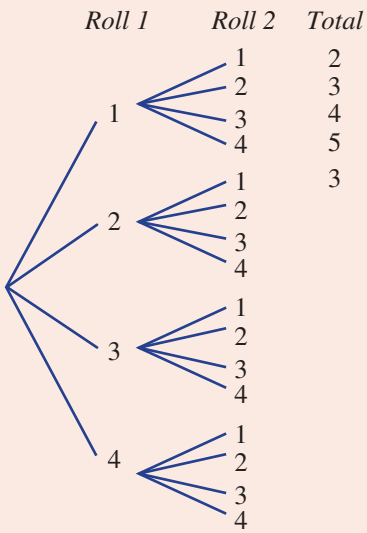
b State the total number of outcomes.

c Find the probability that the two letters chosen will be:

i H then E **ii** the same **iii** not the same

4E **7** A fair 4-sided die is rolled twice and the total is noted.

a Complete this tree diagram to list the sample space.



b Find these probabilities.

i $P(2)$

ii $P(5)$

iii $P(1)$

iv $P(\text{not } 1)$

4E **8** Two pets are selected from a group of 2 ferrets and 3 mice, without replacement. Use a tree diagram to find the probability of selecting:

a a ferret on the first selection

b 2 mice

c 1 mouse

d at least 1 ferret

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

Multiple-choice questions

4A **1** A letter is chosen from the word SUCCESS. The probability that the letter is not a C is:

A $\frac{2}{7}$ **B** $\frac{3}{5}$ **C** $\frac{5}{7}$ **D** $\frac{4}{7}$ **E** $\frac{3}{7}$

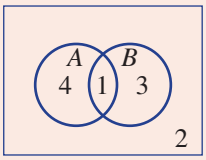
4A **2** The number of manufacturing errors spotted in a car plant on 20 randomly selected days is given by this table.

Number of errors	0	1	2	3	Total
Frequency	11	6	2	1	20

An estimate of the probability that on the next day no errors will be observed is:

A $\frac{3}{10}$ **B** $\frac{9}{20}$ **C** $\frac{11}{20}$ **D** $\frac{17}{20}$ **E** $\frac{3}{20}$

4B **3** For this Venn diagram, $P(A \text{ or } B)$ is equal to:



A $\frac{4}{5}$ **B** $\frac{1}{2}$ **C** $\frac{5}{8}$ **D** $\frac{1}{4}$ **E** $\frac{1}{10}$

4B **4** 15 people like apples or bananas. Of those 15 people, 10 like apples and 3 like both apples and bananas. How many from the group like only apples?

A 5 **B** 3 **C** 13 **D** 7 **E** 10

4D **5** A letter is chosen from each of the words CAN and TOO. The probability that the pair of letters will not have an O is:

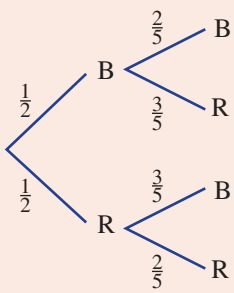
	C	A	N
T	(C, T)	(A, T)	(N, T)
O	(C, O)	(A, O)	(N, O)
O	(C, O)	(A, O)	(N, O)

A $\frac{2}{3}$ **B** $\frac{1}{2}$ **C** $\frac{1}{3}$ **D** $\frac{1}{9}$ **E** $\frac{5}{9}$

4B **6** The sets A and B are known to be mutually exclusive. Which of the following is therefore true?

A $P(A) = P(B)$ **B** $P(A \text{ and } B) = 0$ **C** $P(A) = 0$
D $P(A \text{ and } B) = 1$ **E** $P(A \text{ or } B) = 0$

4E **7** For this tree diagram, what is the probability of the outcome (B, R)?



A $\frac{1}{5}$ **B** $\frac{3}{10}$ **C** $\frac{3}{7}$ **D** $\frac{1}{10}$ **E** $\frac{6}{11}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4C **8** For this two-way table, $P(A \text{ and } B)$ is:

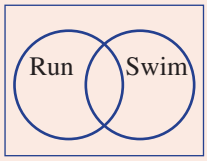
	A	not *A*	
B		1	3
not *B*			4
		4	

A $\frac{2}{3}$ **B** $\frac{1}{4}$ **C** $\frac{1}{7}$

D $\frac{1}{3}$ **E** $\frac{2}{7}$

Extended-response questions

1 Of 15 people surveyed to find out if they run or swim for exercise, 6 said they run, 4 said they swim and 3 said they both run and swim.



a How many people surveyed neither run nor swim?

b One of the 15 people is selected at random. Find the probability that they:

i run or swim **ii** only swim

c Represent the information in a two-way table.

2 A bakery sells three types of bread: raisin (R) at \$2 each, sourdough (S) at \$3 each, and white (W) at \$1.50 each. Judy is in a hurry. She randomly selects 2 loaves and quickly takes them to the counter. (Assume an unlimited loaf supply.)

		1st		
		R	***S***	***W***
2nd	***R***	*(R, R)*	*(S, R)*	*(W, R)*
	S			
	W			

a Complete this table, showing the possible combination of loaves that Judy could have selected.

b Find the probability that Judy selects:

i 2 raisin loaves **ii** 2 loaves that are the same

iii at least 1 white loaf **iv** not a sourdough loaf

Judy has only \$4 in her purse.

c How many different combinations of bread will Judy be able to afford?

d Find the probability that Judy will not be able to afford her two chosen loaves.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5

Single variable and bivariate statistics

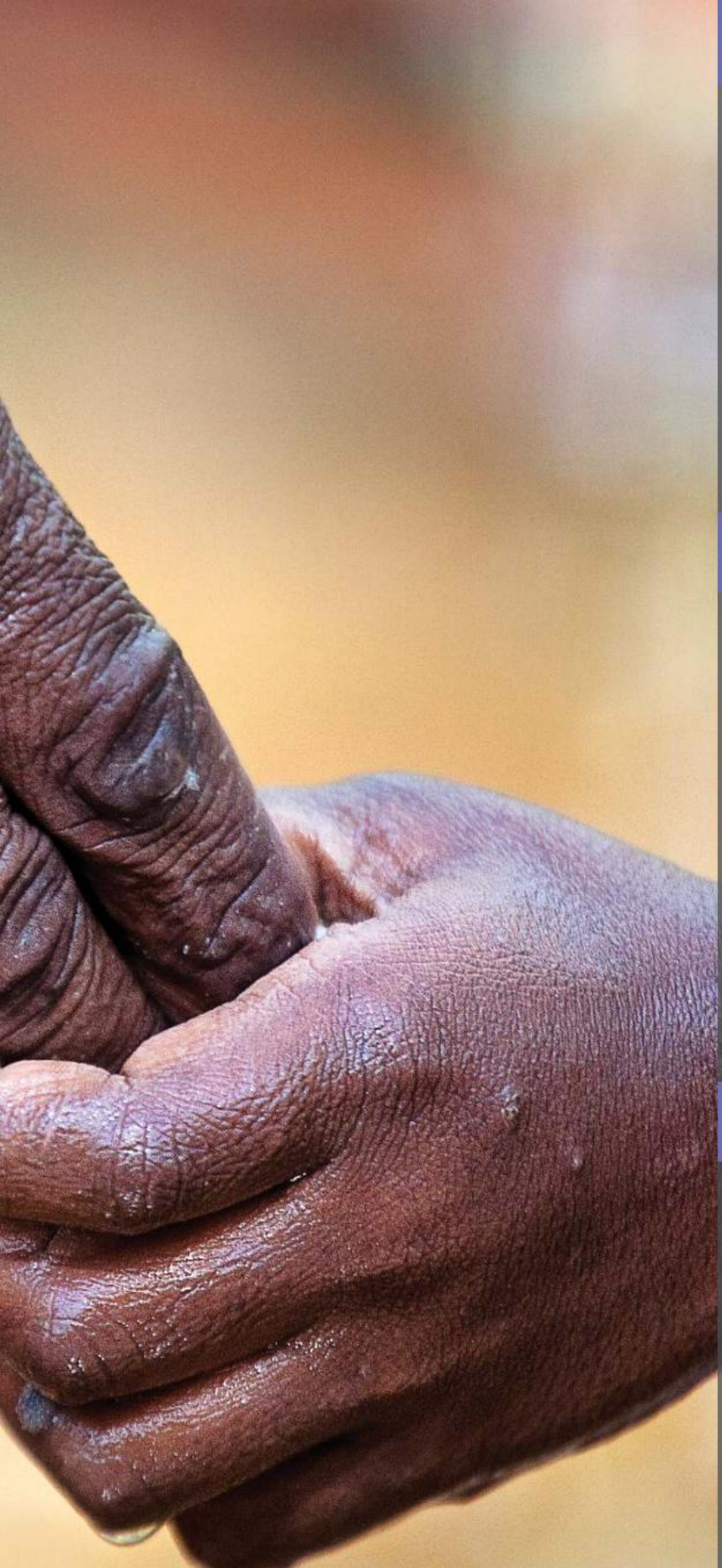
Why skills with statistics are important

Vast quantities of data are collected from online records and surveys. To find meaning, statistics such as typical measures and graphs are used to show trends and allow comparisions.

Statistical calculations and interpratations provide essential information for decision making by governments, farmers, research scientists, technicians, and business and finance personnel.

- The Australian Government uses statistics to compare the health of Indigenous and non-Indigenous Australians to improve policy; for example, time-series graphs of average lifespan versus year.
- Farmers use digital sensors to record data which is then statistically analysed. For example, crop and soil moisture levels are used to automate irrigation and improve crop production.
- Stem-and-leaf plots can be used to record fuel economies and tailpipe emissions of carbon dioxide for various vehicle models.
- Medical researchers use parallel box plots that show significantly lower birth weights for babies with mothers who smoke compared to non-smoking mothers. Low birth weights can cause health problems.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.



Chapter contents

5A Collecting data
5B Column graphs and histograms (CONSOLIDATING)
5C Dot plots and stem-and-leaf plots (CONSOLIDATING)
5D Mean, median, mode and range
5E Quartiles and outliers
5F Box plots
5G Standard deviation
5H Displaying and analysing time-series data (CONSOLIDATING)
5I Bivariate data and scatter plots
5J Line of best fit by eye

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MA0-WM-01)
- compares and analyses datasets using summary statistics and graphical representations (MA5-DAT-C-01)
- displays and interprets datasets involving bivariate data (MA5-DAT-C-02)
- plans, conducts and reviews a statistical inquiry into a question of interest (MA5-DAT-P-01).

© 2022 NSW Education Standards Authority

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Warm-up quiz

1 Below is a list of statistical tools and a list of diagrams. Match each tool (**a**–**r**) with the most appropriate diagram (**A**–**R**).

a frequency distribution table
b dot plot
c stem-and-leaf plot
d sector graph (pie chart)
e divided bar graph
f column graph
g histogram
h box plot
i time-series graph
j mean
k median
l mode
m lower quartile and upper quartile
n minimum and maximum
o range
p interquartile range
q symmetrical data
r bimodal data

A

B $\dfrac{\text{sum of all data values}}{\text{number of data values}}$

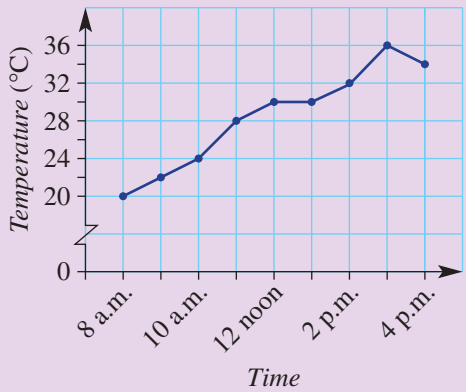
C 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 7, 18

D 5 8 10 15 20 12 10 20

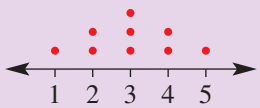
E 5 8 10 15 20 12 10 50

$50 - 5 = 45$

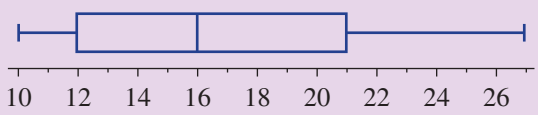
F



G



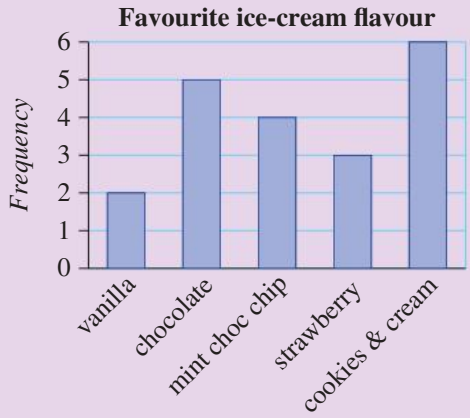
H



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Warm-up quiz

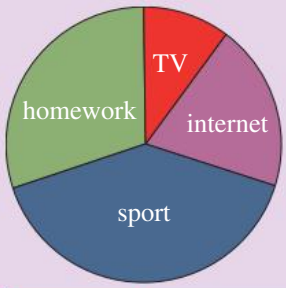
I



J

Interval	Frequency	Percentage frequency
0–4	3	15
5–9	7	35
10–14	6	30
15–19	4	20
Total	20	100

K



L

M 4 5 3 6 4 3 3

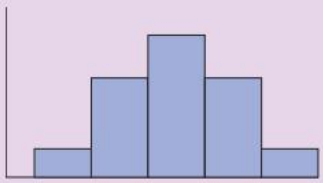
N 3, 15, 12, 9, 12, 15, 6, 8

O 10 11 12 14 15 | 17 18 21 26 27

P 10 11 12 14 15 | 17 18 21 26 27

$21 - 12 = 9$

Q

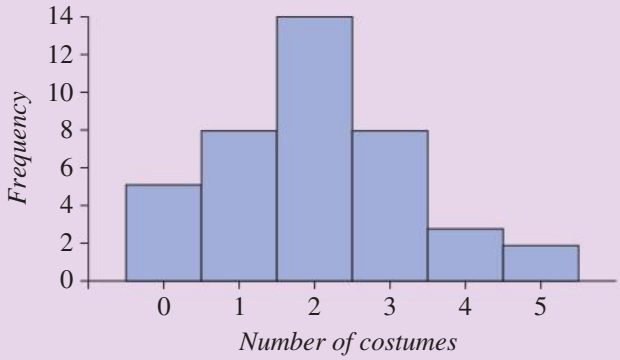


R

Stem	Leaf
9	8
10	2 6
11	1 1 4 9
12	3 6
13	8 9 9
14	0 2 5

2 The number of costumes rented per person at a store is shown in this graph.

- **a** How many customers rented three costumes?
- **b** How many customers were surveyed?
- **c** How many costumes were rented during the survey?
- **d** How many customers rented fewer than two costumes?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Warm-up quiz

3 This table shows the frequency of scores in a test.

Score	Frequency
0–19	2
20–39	3
40–59	6
60–79	12
80–100	7

a How many scores were in the 40 to 59 range?

b How many scores were:

i at least 60?

ii less than 80?

c How many scores were there in total?

d What percentage of scores were in the 20 to 39 range?

4 Calculate:

a $\frac{6+10}{2}$

b $\frac{8+9}{2}$

c $\frac{2+4+5+9}{4}$

d $\frac{3+5+8+10+14}{5}$

5 For each of these datasets, find:

i the mean (i.e. by dividing the sum by the number of scores)

ii the mode (most frequent)

iii the median (middle value of ordered data)

iv the range (difference between highest and lowest)

a 38, 41, 41, 47, 58

b 2, 2, 2, 4, 6, 6, 7, 9, 10, 12

6 This stem-and-leaf plot shows the weight, in grams, of some small calculators.

Stem	Leaf
9	8
10	2 6
11	1 1 4 9
12	3 6
13	8 9 9
14	0 2 5

13 | 6 means 136 grams

a How many calculators are represented in the plot?

b What is the mode (most frequent)?

c What is the minimum calculator weight and maximum weight?

d Find the range (i.e. maximum value – minimum value).

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5A Collecting data

Learning intentions for this section:

- To understand what is required to construct a good survey
- To know the difference between a population and a sample
- To be able to categorise types of statistical data

Past, present and future learning:

- This section addresses Stage 4 and the Stage 5 Path topic called *Data analysis C*
- Some of these concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

One part of a statistician's job is to design surveys. Statisticians also collect, analyse and interpret data. They assist the government, companies and other organisations to make decisions and plan for the future.

Statisticians:

- **Formulate** and **refine** questions for a survey.
- Choose some **subjects** (i.e. people) to complete the survey.
- **Collect** the data.
- **Organise and display** the data using the most appropriate graphs and tables.
- **Analyse** the data.
- **Interpret the data** and draw conclusions.

There are many reports in the media that begin with the words 'A recent study has found that…'. These are usually the result of a survey or investigation that a researcher has conducted to collect information about an important issue, such as unemployment, crime or obesity.

Lesson starter: Critiquing survey questions

Here is a short survey. It is not very well constructed.

Question 1: How old are you?

Question 2: How much time did you spend sitting in front of the television or a computer yesterday?

Question 3: Some people say that teenagers like you are lazy and spend too much time sitting around when you should be outside exercising. What do you think of that comment?
Have a class discussion about the following.

- What will the answers to Question 1 look like? How could they be displayed?
- What will the answers to Question 2 look like? How could they be displayed?
- Is Question 2 going to give a realistic picture of your normal daily activity?
- How could Question 2 be improved?
- What will the answers to Question 3 look like? How could they be displayed?
- How could Question 3 be improved?

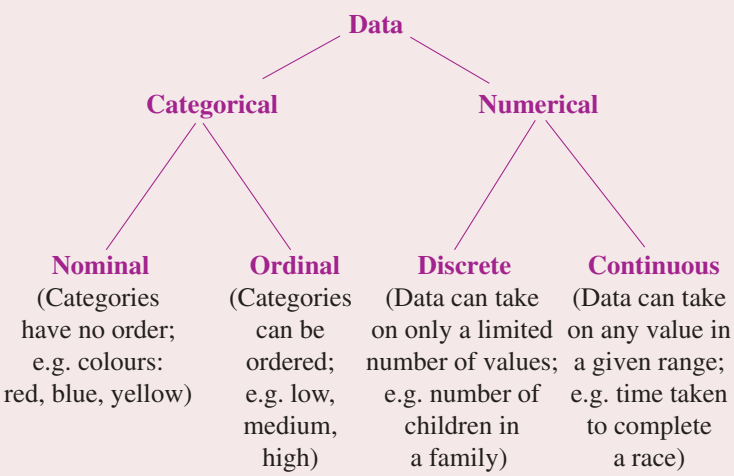
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- **Surveys** are used to collect statistical data by asking randomly selected people questions.
  - Survey questions need to be constructed carefully so that the person knows exactly what sort of answer to give. They should use simple language and should not be ambiguous.
  - Survey questions should not be worded so that they deliberately try to provoke a certain kind of response.
  - If the question contains an option to be chosen from a list, the number of options should be an odd number, so that there is a 'neutral' choice. For example, the options could be:

strongly disagree	disagree	unsure	agree	strongly agree

- A **population** is a group of people, animals or objects with something in common. Some examples of populations are:
  - all the people in Australia on Census night (a **census** is a set of statistics collected from the entire population)
  - all the students in your school
  - all the boys in your Maths class
  - all the tigers in the wild in Sumatra
  - all the cars in Sydney
  - all the wheat farms in NSW
- A **sample** is a group that has been chosen from a population. Sometimes information from a sample is used to describe the whole population, so it is important to choose the sample carefully.
- If information is collected from a sample of a population so that some members are less likely to be included, then the sample is possibly biased.
- **Statistical data** is information gathered by observation, survey or measurement. It can be categorised as follows.



Key terminology: survey, statistical data, categorical data, numerical data, nominal data, ordinal data, discrete data, continuous data, population, sample, census

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 5A

UNDERSTANDING	1–3	2, 3

1 A popular Australian 'current affairs' television show recently investigated the issue of spelling. They suspected that people in their twenties are not as good at spelling as people in their fifties, so they decided to conduct a statistical investigation. They chose a sample of 12 people aged 50–59 years and 12 people aged 20–29 years.
Answer the following questions on paper, then discuss in a small group or as a whole class.

- **a** Do you think that the number of people surveyed is enough?
- **b** How many people do you think there are in Australia aged 20–29 years?
- **c** How many people do you think there are in Australia aged 50–59 years?
- **d** Use the website of the Australian Bureau of Statistics to look up the answers to parts **b** and **c** .
- **e** Do you think it is fair and reasonable to compare the spelling ability of these two groups of people?
- **f** How would you go about comparing the spelling ability of these two groups of people?
- **g** Would you give the two groups the same set of words to spell?
- **h** How could you give the younger people an unfair advantage?
- **i** What sorts of words would you include in a spelling test for the survey?
- **j** How and where would you choose the people to do the spelling test?

2 Match each word (**a**–**h**) with its definition (**A**–**H**).

a	population	**A**	a group chosen from a population
b	census	**B**	a tool used to collect statistical data
c	sample	**C**	the state of being secret
d	survey	**D**	an element or feature that can vary
e	data	**E**	all the people or objects in question
f	variable	**F**	statistics collected from an entire population
g	statistics	**G**	the practice of collecting and analysing data
h	confidentiality	**H**	the factual information collected from a survey or other source

3 Match each word (**a**–**f**) with its definition (**A**–**F**).

a	numerical	**A**	categorical data that has no order
b	continuous	**B**	data that are numbers
c	discrete	**C**	numerical data that take on a limited number of values
d	categorical	**D**	data that can be divided into categories
e	ordinal	**E**	numerical data that take any value in a given range
f	nominal	**F**	categorical data that can be ordered

FLUENCY	4–6	4, 5, 7

Example 1 Describing types of data

What type of data would the following survey questions generate?

- **a** How many televisions do you have in your home?
- **b** To what type of music do you most like to listen?

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION	EXPLANATION
a Numerical and discrete	The answer to the question is a number with a limited number of values; in this case, a whole number.
b Categorical and nominal	The answer is a type of music and these categories have no order.

Now you try

What type of data would the following survey questions generate?

a How would you rate your hotel stay (poor, satisfactory, good, excellent)?

b How many times in a week do you exercise?

4 Which one of the following survey questions would generate numerical data?

A What is your favourite colour?

B What type of car does your family own?

C How long does it take for you to travel to school?

D What type of dog do you own?

5 Which one of the following survey questions would generate categorical data?

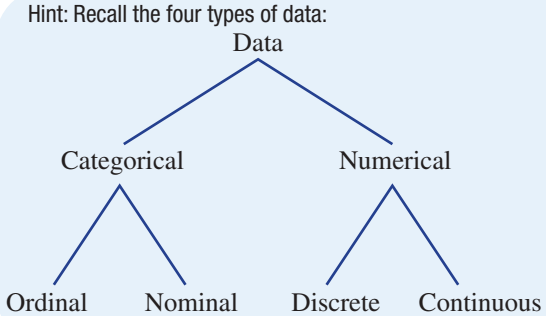
A How many times do you eat at your favourite fast-food place in a typical week?

B How much money do you usually spend buying your favourite fast food?

C How many items did you buy last time you went to your favourite fast-food place?

D Which is your favourite fast food?

Hint: Recall the four types of data:



6 Year 10 students were asked the following questions in a survey. Describe what type of data each question generates.

a How many people under the age of 18 years are there in your immediate family?

b How many letters are there in your first name?

c Which company is the carrier of your mobile telephone calls?
Optus/Telstra/Vodafone/Boost/Other (Please specify.)

d What is your height?

e How would you describe your level of application in Maths? (Choose from *very high*, *high*, *medium* or *low*.)

7 Every student in Years 7 to 12 votes in the prefect elections. The election process is an example of:

A a population

B continuous data

C a representative sample

D a census

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING | 8, 9 | 9–11

8 TV 'ratings' are used to determine the shows that are the most popular. Every week some households are chosen at random and a device is attached to their television. The device keeps track of the shows the households are watching during the week.
The company that chooses the households should always attempt to find:

A a census.
B continuous data.
C a representative sample.
D ungrouped data.

9 The principal decides to survey Year 10 students to determine their opinion of mathematics. In order to increase the chance of choosing a representative sample, the principal should:

A Give a survey form to the first 30 Year 10 students who arrive at school.
B Give a survey form to all the students studying the most advanced maths subject.
C Give a survey form to five students in every maths class.
D Give a survey form to 20% of the students in every class.

10 Discuss some of the problems with the selection of a survey sample for each given topic.

a A survey at the train station of how Australians get to work.
b An email survey on people's use of computers.
c Phoning people on the electoral roll to determine Australia's favourite sport.

11 Choose a topic in which you are especially interested, such as football, cricket, movies, music, cooking, food, computer games or social media.
Make up a survey about your topic that you could give to the other students in your class. It must have *four* questions.
Question 1 must produce data that is categorical and ordinal.
Question 2 must produce data that is categorical and nominal.
Question 3 must produce data that is numerical and discrete.
Question 4 must produce data that is numerical and continuous.

ENRICHMENT: The Australian Census | — | 12

12 The Australian Census is conducted by the Australian Bureau of Statistics every five years.
Research the 2021 Australian Census on the website of the Australian Bureau of Statistics. Find out something interesting from the results of the Census.
Write a short news report or record a 3-minute news report on your findings.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5B Column graphs and histograms CONSOLIDATING

Learning intentions for this section:

- To be able to construct a frequency table from a set of data
- To know the difference between a column graph and a histogram
- To be able to construct and analyse column graphs and histograms
- To be able to describe data as symmetrical or skewed

Past, present and future learning:

- This section addresses Stage 4 and the Stage 5 Core topic called *Data analysis A*
- It consolidates and extends the concepts in Sections 10F, 10G and 10H of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Data can be collected in a number of ways, including surveys, experiments, recording the performance of a sportsperson or just counting. As a simple list, data can be difficult to interpret. Sorting the data into a frequency table allows us to make sense of it and draw conclusions from it.

Statistical graphs are an essential part of the analysis and representation of data. By looking at statistical graphs, we can draw conclusions about the numbers or categories in the dataset.

Lesson starter: A packet of smarties

Smarties are sold in small packets.

- How many Smarties would you expect to find in one packet?
- Would you expect every packet to contain the same number of Smarties?
- How many different colours would you expect to find in your packet?

Consider these photos of the Smarties found in one packet.

- Which of the following tools could be used to display and analyse the data produced by the question: What are the colours of the Smarties in your packet?

A frequency distribution table
B dot plot
C stem-and-leaf plot
D sector graph (pie chart)
E divided bar graph
F column graph
G histogram
H mean
I median
J mode
K range

- Open a small packet of Smarties and use some of the tools listed above to analyse its contents. Compare your results with your initial expectations.

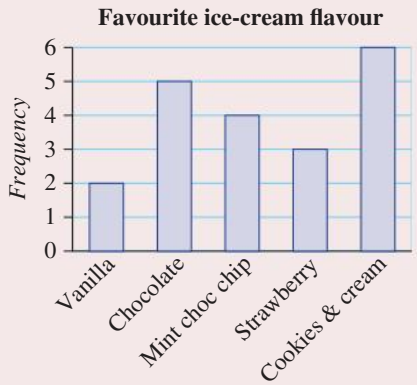
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- A **frequency table** displays data by showing the number of values within a set of categories or class intervals. It may include a tally column to help count the data.

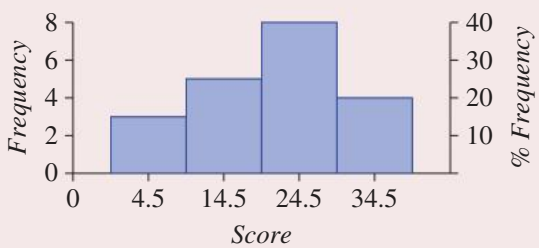
Favourite ice-cream flavour	Tally	Frequency
Vanilla	||	2
Chocolate	𝍸	5
Mint choc chip	||||	4
Strawberry	|||	3
Cookies and cream	𝍸 |	6

- A **column graph** can be used for a single set of categorical or discrete data to show the frequency.

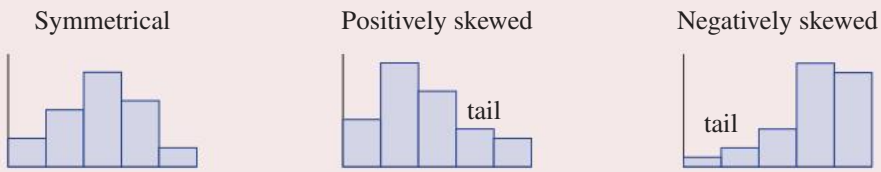


- **Histograms** can be used for grouped discrete or continuous numerical data. The frequency of particular class intervals is recorded.
  - In the following tables, the interval 10–19 includes all numbers from 10 (including 10) to less than 20.
  - The percentage frequency is calculated as $\%\ \text{Frequency} = \dfrac{\text{frequency}}{\text{total}} \times 100$.

Class interval	Frequency	Percentage frequency
1–9	3	$\frac{3}{20} \times 100 = 15\%$
10–19	5	$\frac{5}{20} \times 100 = 25\%$
20–29	8	40%
30–39	4	20%
Total	20	100%



- Data can be **symmetrical** (same shape either side of the middle) or **skewed** (data weighted to the left or the right).



Key terminology: frequency table, column graph, histogram, symmetrical, skewed, data

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 5B

UNDERSTANDING 1–3 3

1 Classify each set of data as categorical or numerical.

- **a** 4.7, 3.8, 1.6, 9.2, 4.8
- **b** red, blue, yellow, green, blue, red
- **c** low, medium, high, low, low, medium
- **d** 3 g, 7 g, 8 g, 7 g, 4 g, 1 g, 10 g

2 Complete these frequency tables.

a

Car colour	Tally	Frequency
Red	|||	
White	𝍸	
Green	||	
Silver	||	
Total		

Hint: In the tally, 𝍸 is 5.

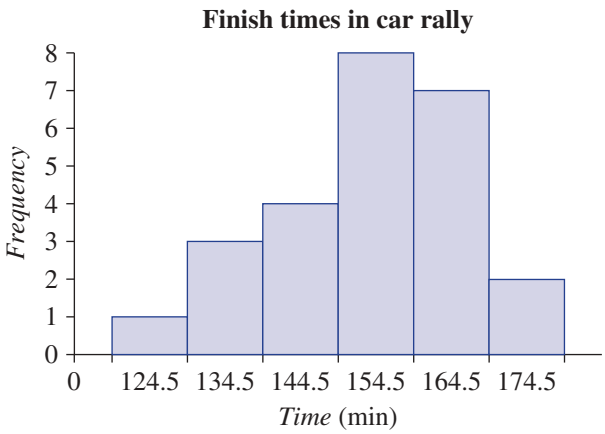
b

Class interval	Frequency	Percentage frequency
80–84	8	$\frac{8}{50} \times 100 = 16\%$
85–89	23	
90–94	13	
95–100		
Total	50	

3 This frequency histogram shows how many competitors in a car rally race finished within a given time interval.

- **a** How many competitors finished in a time between 150 and 160 minutes?
- **b** How many cars were there in the race?

Hint: There was one competitor in the 120–129 interval, three competitors in the 130–139 interval etc.



- **c** Determine the following.
  - **i** How many competitors finished in fewer than 150 minutes?
  - **ii** What percentage of competitors finished in fewer than 150 minutes?

Hint:
Percentage $= \frac{\text{number} < 150}{\text{total}} \times 100$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 4–6 4, 5, 7

Example 2 Constructing a frequency table and column graph

Twenty people checking out at a hotel were surveyed on the level of service provided by the hotel staff. The results were:

Poor	First class	Poor	Average	Good
Good	Average	Good	First class	First class
Good	Good	First class	Good	Average
Average	Good	Poor	First class	Good

a Construct a frequency table to record the data, with headings Category, Tally and Frequency.

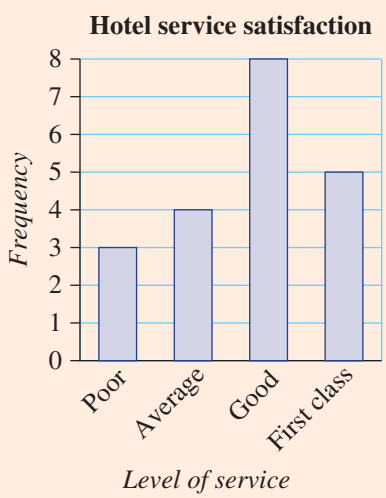
b Construct a column graph for the data.

SOLUTION

a

Category	Tally	Frequency
Poor	|||	3
Average	||||	4
Good	𝍸 |||	8
First class	𝍸	5
Total	20	20

b



EXPLANATION

Construct a table with the headings Category, Tally, Frequency.

Fill in each category shown in the data. Work through the data in order, recording a tally mark (|) next to the category. It is a good idea to tick the data as you go, to keep track.

On the 5th occurrence of a category, place a diagonal line through the tally marks (𝍸). Then start again on the 6th. Do this every five values, as it makes the tally marks easy to count up.

Once all data is recorded, count the tally marks for the frequency.

Check that the frequency total adds up to the number of people surveyed (20).

Draw a set of axes with frequency going up to 8.

For each category, draw a column with height up to its frequency value.

Leave gaps between each column.

Give your graph an appropriate heading.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

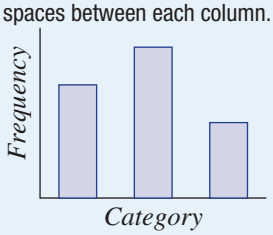
A class of 24 students was surveyed on their favourite genre of movie. The results were:

Action	Comedy	Comedy	Romance	Action	Sci-Fi
Horror	Sci-Fi	Comedy	Action	Comedy	Romance
Comedy	Action	Romance	Horror	Action	Comedy
Action	Romance	Horror	Action	Comedy	Action

a Construct a frequency table to record the data, with headings Category, Tally and Frequency.
b Construct a column graph for the data.

4 Use the data below (obtained from surveys) to complete this question.

Hint: In the column graph, leave spaces between each column.



i Copy and complete this frequency table.

Category	Tally	Frequency
⋮	⋮	⋮

ii Construct a column graph and include a heading.

a The results from 10 subjects on a student's school report showing their level of application are:

Good	Low	Good	Good	Excellent
Very Low	Low	Good	Good	Low

b The favourite sports of a class of students are:

Football	Tennis	Basketball	Tennis	Football
Netball	Football	Tennis	Football	Basketball
Basketball	Tennis	Netball	Football	Football
Football	Basketball	Football	Netball	Tennis

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 3 Constructing and analysing a histogram

Twenty people were surveyed to find out how many times they use the internet in a week. The raw data is listed.

12, 19, 5, 10, 15, 18, 31, 40, 32, 25

11, 28, 31, 29, 16, 2, 13, 33, 14, 24

a Organise the data into a frequency table using class intervals of 10. Include a percentage frequency column.

b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.

c Which interval is the most frequent?

d What percentage of people used the internet 20 times or more?

SOLUTION

a

Class interval	Tally	Frequency	Percentage frequency
0–9	||	2	10%
10–19	𝍸 |||	8	40%
20–29	𝍸	5	25%
30–39	||||	4	20%
40–49	|	1	5%
Total	20	20	100%

EXPLANATION

Work through the data and place a tally mark in the correct interval each time.

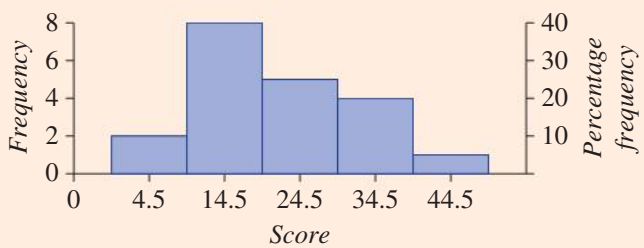
The interval 10–19 includes all numbers from 10 (including 10) to less than 20, so 10 is in this interval but 20 is not.

Count the tally marks to record the frequency.

Add the frequency column to ensure all 20 values have been recorded.

Calculate each percentage frequency by dividing the frequency by the total (i.e. 20) and multiplying by 100; i.e. $\frac{2}{20} \times 100 = 10$.

b **Number of times the internet is accessed**



In a histogram there are no gaps between the columns. When data is grouped into classes, the class centre is sometimes displayed in the middle of the base of the column. For instance, in the 40–49 class interval, the class centre is $\frac{(49 + 40)}{2} = 44.5$.

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c The 10–19 interval is the most frequent.

The frequency (8) is highest for this interval. It is the highest bar on the histogram.

d 50% of those surveyed used the internet 20 or more times.

Sum the percentages for the class intervals from 20–49 and above.
$25 + 20 + 5 = 50$

Now you try

The numbers of points scored by a basketball team in its 25-game season are shown below:

74	82	77	101	91
66	86	87	90	88
79	108	94	89	70
75	81	72	89	97
86	78	78	82	88

a Organise the data into a frequency table, using class intervals of 10. Include a percentage frequency column. Start with 60–69, 70–79 etc.
b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.
c Which interval is the most frequent?
d What percentage of their games did they score 80 or more?

5 The Maths test results of a class of 25 students were recorded as:

74	65	54	77	85	68	93	59	75
71	82	57	98	73	66	88	76	
92	70	77	65	68	81	79	80	

a Organise the data into a frequency table, using class intervals of 10. Include a percentage frequency column.
b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.
c Which interval is the most frequent?
d If an A is awarded for a mark of 80 or more, what percentage of the class received an A?

Hint: Construct a frequency table like this:

Class interval	Tally	Frequency	Percentage frequency
50–59	|||	3	$\frac{\text{freq.}}{\text{total}} \times 100$
60–69			
70–79			
80–89			
90–99			
Total			

6 The number of wins scored this season is given for 20 hockey teams. Here is the raw data.
4, 8, 5, 12, 15, 9, 9, 7, 3, 7
10, 11, 1, 9, 13, 0, 6, 4, 12, 5

a Organise the data into a frequency table, using class intervals of 5. Start with 0–4, then 5–9 etc. and include a percentage frequency column.
b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.
c Which interval is the most frequent?
d What percentage of teams scored 5 or more wins?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 This frequency table displays the way in which 40 people travel to and from work.

Type of transport	Frequency	Percentage frequency
Car	16	
Train	6	
Tram	8	
Walking	5	
Bicycle	2	
Bus	3	
Total	40	

a Copy and complete the table.

b Use the table to find the:

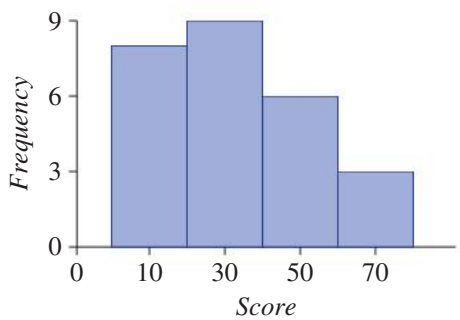
- **i** frequency of people who travel by train
- **ii** most popular form of transport
- **iii** percentage of people who travel by car
- **iv** percentage of people who walk or cycle to work
- **v** percentage of people who travel by public transport, including trains, buses and trams

Hint: Percentage frequency $= \frac{\text{frequency}}{\text{total}} \times 100$

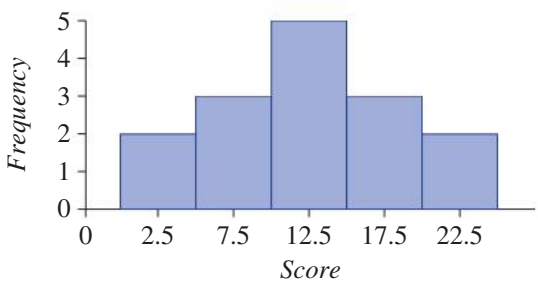
PROBLEM-SOLVING AND REASONING 8, 9 8, 10, 11

8 State whether each of these histograms shows a symmetrical dataset or a skewed dataset.

a



b



9 This tally records the number of mice that were weighed (correct to the nearest gram) and categorised into particular mass intervals for a scientific experiment.

Mass (grams)	Tally
10–14	|||
15–19	𝍸 |
20–24	𝍸 𝍸 𝍸 |
25–29	𝍸 𝍸 𝍸 𝍸 |
30–34	||||

a Construct a table using these column headings: Mass, Frequency and Percentage frequency.

b Find the total number of mice weighed in the experiment.

c State the percentage of mice that were in the 20–24 gram interval.

d Which was the most common mass interval?

e What percentage of mice were in the most common mass interval?

f What percentage of mice had a mass of 15 grams or more?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 A school orchestra contains four musical sections: string, woodwind, brass and percussion. The number of students playing in each section is summarised in this tally.

Section	Tally
String	𝍸 𝍸 𝍸 𝍸 |
Woodwind	𝍸 |||
Brass	𝍸 ||
Percussion	||||

a Construct and complete a percentage frequency table for the data.

b What is the total number of students in the school orchestra?

c What percentage of students play in the string section?

d What percentage of students do not play in the string section?

e If the number of students in the string section increased by three, what would be the percentage of students who play in the percussion section? Assume that the three students did not move from another musical section. (Round your answer to one decimal place.)

11 Describe the information that is lost when displaying data using a histogram.

ENRICHMENT: Interpreting other graphical displays — 12, 13

12 The graph shown compares the life expectancy of males and females in 10 different countries. Use the graph to answer the following questions.

a Which country has the biggest difference in life expectancy for males and females? Approximately how many years is this difference?

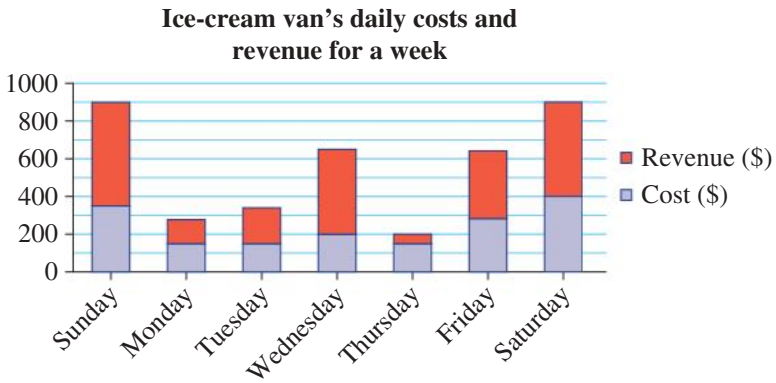
b List three countries in which men and women have roughly the same life expectancy (within four years).

c From the information in the graph, write a statement comparing the life expectancy of males and females.

d Life expectancy in Chad is clearly lower than in other countries. Provide some reasons why you think this may be the case.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

13 This graph shows the amount spent (cost) on the purchase and storage of ice-cream each day by an ice-cream vendor, and the amount of money made from the daily sales of ice-cream (revenue) over the course of a week.



a On which days was the cost highest for the purchase and storage of ice-cream? Why do you think the vendor chose these days to spend the most?

b Wednesday had the greatest revenue for any weekday. What factors may have led to this?

c Daily profit is determined by the difference between revenue and cost. Identify:

i on which day the largest profit was made and state this profit (in dollars)

ii on which day the vendor suffered the biggest financial loss

d Describe some problems associated with this type of graph.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5C Dot plots and stem-and-leaf plots CONSOLIDATING

Learning intentions for this section:

- To be able to construct and interpret a dot plot
- To be able to construct and interpret a stem-and-leaf plot
- To know when it is appropriate to use a dot plot or stem-and-leaf plot
- To be able to describe the distribution as symmetrical or skewed

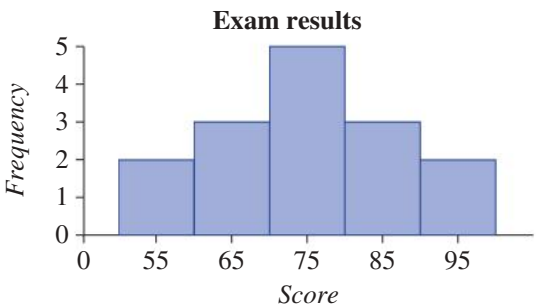
Past, present and future learning:

- This section addresses Stage 4 and the Stage 5 Core topic called *Data analysis A*
- It consolidates and extends the concepts in Sections 10G and 10H of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

In addition to column graphs, dot plots and stem-and-leaf plots can be used to display categorical or discrete data. They can also display two related sets for comparison. Like histograms, they help to show how the data is distributed. A stem-and-leaf plot has the advantage of still displaying all the individual data items.

Lesson starter: Alternative representations

The histogram and stem-and-leaf plot below represent the same set of data. They show the scores achieved by a class in an exam.



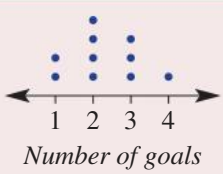
Stem	Leaf
5	1 3
6	5 7 8
7	1 2 4 6
8	3 4 7
9	6

6 | 8 means 68

- Describe the similarities in what the two graphs display.
- What does the stem-and-leaf provide that the histogram does not? What is the advantage of this?
- Which graph do you prefer?
- Discuss any other types of graphs that could be used to present the data.

KEY IDEAS

- A **dot plot** records the frequency of each category or discrete value in a dataset.
  - Each occurrence of the value is marked with a dot.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

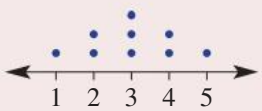
- A **stem-and-leaf plot** displays each value in the dataset using a stem number and a leaf number.
  - The data is displayed in two parts: a stem and a leaf.
  - The 'key' tells you how to interpret the stem and leaf parts.
  - The graph is similar to a histogram with class intervals, but the original data values are not lost.
  - The stem-and-leaf plot is ordered to allow for further statistical calculations.
  - Back-to-back stem-and-leaf plots, with leaves either side of the stem, can be used to compare two related sets of data.

Stem	Leaf
1	0 1 1 5
2	3 7
3	4 4 6
4	2 9

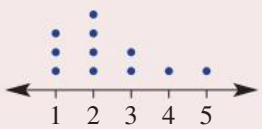
2 | 3 means 23

Key

- The shape of each of these graphs gives information about the distribution of the data.
  - A graph that is even either side of the centre is symmetrical.



  - A graph that is bunched to one side of the centre is skewed.



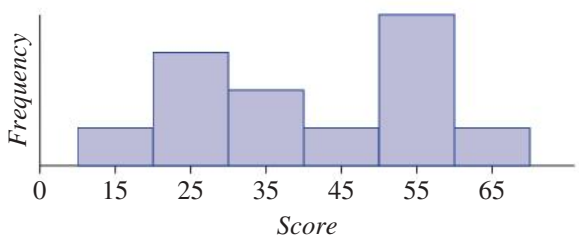
Key terminology: dot plot, stem-and-leaf plot, symmetrical data, skewed data

Exercise 5C

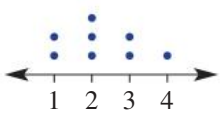
UNDERSTANDING 1–3 3

1 Name each of these types of graphs.

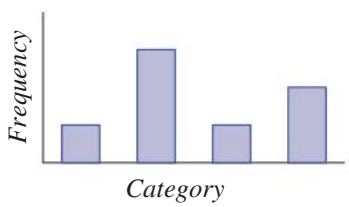
a



b



c



d

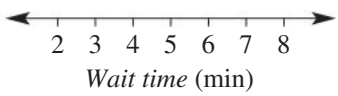
Stem	Leaf
0	1 1 3
1	2 4 7
2	0 2 2 5 8
3	1 3

2 | 5 means 25

2 A student records the following wait times, in minutes, for his school bus over 4 school weeks.

5 4 2 8 4 2 7 5 3 3
5 4 2 5 4 5 8 7 2 6

Copy and complete this dot plot of the data by placing a dot for each occurrence of a value.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 List all the data shown in these stem-and-leaf plots (e.g. 32, 35, …).

a

Stem	Leaf
3	2 5
4	1 3 7
5	4 4 6
6	0 2
7	1 1

4 | 1 means 41

b

Stem	Leaf
0	2 3 7
1	4 4 8 9
2	3 6 6
3	0 5

2 | 3 means 2.3

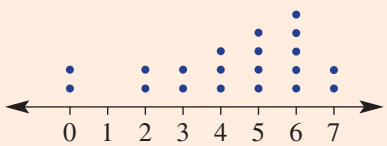
Hint: Look at the key '4 | 1 means 41' to see how the stems and leaves go together.

FLUENCY 4, 5, 6–7(½) 4, 5, 6–7(½)

Example 4 Interpreting a dot plot

This dot plot shows the number of goals per game scored by a team during the soccer season.

a How many games were played?
b What was the most common number of goals per game?
c How many goals were scored for the season?
d Describe the data in the dot plot.



Number of goals per game

SOLUTION

a There were 20 matches played.

EXPLANATION

Each dot represents a match. Count the number of dots.

b 6 goals in a game occurred most often.

The most common number of goals has the most dots.

c $2 \times 0 + 2 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + 2 \times 7$
$= 0 + 4 + 6 + 12 + 20 + 30 + 14$
$= 86$ goals

Count the number of games (dots) for each number of goals and multiply by the number of goals. Add these together.

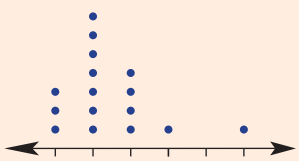
d Two games resulted in no goals but the data were generally skewed towards a higher number of goals.

Consider the shape of the graph; it is bunched towards the 6 end of the goal scale.

Now you try

This dot plot shows the number of pets owned by a number of households in a street.

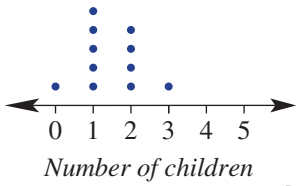
a How many households were surveyed?
b What was the most common number of pets?
c How many pets are there in the street?
d Describe the data in the dot plot.



Number of pets

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 A number of families were surveyed to find the number of children in each. The results are shown in this dot plot.



a How many families were surveyed?

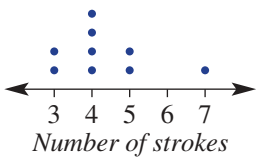
b What was the most common number of children in a family?

c How many children were there in total?

d Describe the data in the dot plot.

Hint: 4 families had 2 children (4 dots), so that represents 8 children from these families.

5 This dot plot shows the number of strokes a golfer played, each hole, in her round of golf.



a How many holes did she play?

b How many strokes did she play in the round?

c Describe her round of golf.

Example 5 Constructing a stem-and-leaf plot

Consider this dataset.

22 62 53 44 35 47 51 64 72
32 43 57 64 70 33 51 68 59

a Organise the data into an ordered stem-and-leaf plot.

b Describe the distribution of the data as symmetrical or skewed.

SOLUTION

a

Stem	Leaf
2	2
3	2 3 5
4	3 4 7
5	1 1 3 7 9
6	2 4 4 8
7	0 2

5 | 1 means 51

EXPLANATION

For two-digit numbers, select the tens value as the stem and the units as the leaves.

The data ranges from 22 to 72, so the graph will need stems 2 to 7.

Work through the data and record the leaves in the order of the data.

Stem	Leaf
2	2
3	5 2 3
4	4 7 3
5	3 1 7 1 9
6	2 4 4 8
7	2 0

5 | 1 means 51

51 occurs twice, so the leaf 1 is recorded twice in the 5 stem row.

Once data is recorded, redraw and order the leaves from smallest to largest.

Include a key to explain how the stem and leaf go together; e.g. 5 | 1 means 51.

b The distribution of the data is symmetrical.

The shape of the graph is symmetrical (i.e. evenly spread) either side of the centre.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

Consider this dataset.

15 11 18 22 31 24 24 26
14 63 54 40 44 32 28 10

a Organise the data into an ordered stem-and-leaf plot.
b Describe the distribution of the data as symmetrical or skewed.

6 Consider the following sets of data.
- **i** Organise the data into an ordered stem-and-leaf plot.
- **ii** Describe the distribution of the data as symmetrical or skewed.

Hint: Remember to include a key such as '4 | 6 means 46'.

a 46 22 37 15 26 38 52 24
31 20 15 37 21 25 26

b 35 16 23 55 38 44 12 48 21 42
53 36 35 25 40 51 27 31 40 36 32

c 153 121 124 117 125 118 135 137 162
145 147 119 127 149 116 133 160 158

d 4.9 3.7 4.5 5.8 3.8 4.3 5.2 7.0 4.7
4.4 5.5 6.5 6.1 3.3 5.4 2.0 6.3 4.8

Hint:

Symmetrical	
Stem	Leaf
1	1 2
2	1 2 3
3	1 2 3 4
4	1 2 7
5	3

Skewed	
Stem	Leaf
1	2 5 7 8
2	3 4 7 6
3	1 2
4	5

Example 6 Constructing back-to-back stem-and-leaf plots (EXTENDING)

Two television sales representatives sell the following number of televisions each week over a 15-week period.

Employee 1
23 38 35 21 45 27 43 36
19 35 49 20 39 58 18

Employee 2
28 32 37 20 30 45 48 17
32 37 29 17 49 40 46

a Construct an ordered back-to-back stem-and-leaf plot.
b Describe the distribution of each employee's sales.

SOLUTION

a

Employee 1 leaf	Stem	Employee 2 leaf
9 8	1	7 7
7 3 1 0	2	0 8 9
9 8 6 5 5	3	0 2 2 7 7
9 5 3	4	0 5 6 8 9
8	5	

3 | 7 means 37

b Sales by employee 1 are symmetrical, whereas sales by employee 2 are skewed.

EXPLANATION

Construct an ordered stem-and-leaf plot with the sales by employee 1 on the left-hand side and the sales by employee 2 on the right-hand side. Include a key.

Observe the shape of each employee's graph. If appropriate, use the words symmetrical (i.e. spread evenly around the centre) or skewed (i.e. bunched to one side of the centre).

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

Two friends spent the following amounts in dollars on take-away food over a 12-week period.

Friend 1						*Friend 2*					
54	44	30	32	46	62	16	24	30	44	29	19
22	66	41	36	57	48	22	28	18	52	32	41

a Construct an ordered back-to-back stem-and-leaf plot.

b Describe the distribution of each friend's spending.

7 Consider the following sets of data.

i Draw a back-to-back stem-and-leaf plot.

ii Comment on the distribution of the two datasets.

Hint: For part **a** use a key like '3 | 7 means 37' and for part **b** use a key like '15 | 6 means 156'.

a Set 1: 61 38 40 53 48 57 64
39 42 59 46 42 53 43

Set 2: 41 55 64 47 35 63 61
52 60 52 56 47 67 32

b Set 1: 176 164 180 168 185 187 195 166 201
199 171 188 175 192 181 172 187 208

Set 2: 190 174 160 170 186 163 182 171
167 187 171 165 194 182 163 178

PROBLEM-SOLVING AND REASONING

8, 9 | 8, 10, 11

8 Two football players, Logan and Max, compare their personal tallies of the number of goals scored for their team over a 12-match season. Their tallies are as follows.

Game	1	2	3	4	5	6	7	8	9	10	11	12
Logan	0	2	2	0	3	1	2	1	2	3	0	1
Max	0	0	4	1	0	5	0	3	1	0	4	0

a Draw a dot plot to display Logan's goal-scoring achievement.

b Draw a dot plot to display Max's goal-scoring achievement.

c How would you describe Logan's scoring habits?

d How would you describe Max's scoring habits?

9 This stem-and-leaf plot shows the times, in minutes, that Chris has achieved in the past 14 fun runs she competed in.

Stem	Leaf
20	5 7
21	1 2 6
22	2 4 6 8
23	4 5 6
24	3 6

22 | 4 means 22.4

a What is the difference between her slowest and fastest times?

b Just by looking at the stem-and-leaf plot, what would you estimate to be Chris' average time?

c If Chris records another time of 24.9 minutes, how would this affect your answer to part **b**?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 The data below show the distances travelled (in km) by students at an inner-city and an outer-suburb school.

Inner city:						
3	10	9	14	21	6	
1	12	24	1	19	4	

Outer suburb:						
12	21	18	9	34	19	
24	3	23	41	18	4	

a Draw a back-to-back stem-and-leaf plot for the data.

b Comment on the distribution of distances for each school.

c Give a practical reason for the distribution of the data.

11 Determine the possible values of the pronumerals in the following ordered stem-and-leaf plots.

a

Stem	Leaf
1	2 4
2	3 6 9 b
a	1 4
4	7 c 8

2 | 3 means 2.3

b

Stem	Leaf
20	a 1 4
21	2 2 9
22	0 b 5 7
23	1 4

22 | 7 means 227

Hint: The stems and leaves are ordered from smallest to largest. A leaf can appear more than once.

ENRICHMENT: Splitting stems — 12

12 The back-to-back stem-and-leaf plot on the right shows the maximum daily temperature for two cities over a 2-week period.

Maximum temperature

City A leaf	Stem	City B leaf
	0	
9 8 8	0*	
4 3 3 1 1 1	1	
8 8 6 6 5	1*	7 9
	2	0 2 2 3 4 4
	2*	5 6 7 7 8
	3	1

1 | 4 means 14

1* | 5 means 15

a Describe the difference between the stems 1 and 1*.

b To which stem would these numbers be allocated?

i 12°C **ii** 5°C

c Why might you use this process of splitting stems, like that used for 1 and 1*?

d Compare and comment on the differences in temperatures between the two cities.

e What might be a reason for these different temperatures?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5D Mean, median, mode and range

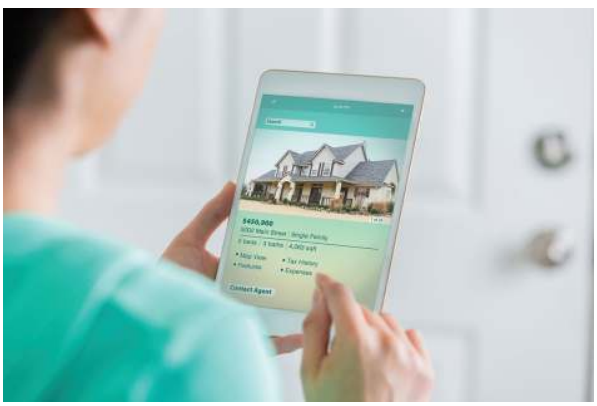
Learning intentions for this section:

- To know that mean, median, mode and range are used to summarise a dataset
- To know how the median is found for datasets with an odd or even number of values
- To be able to find the mean, median, mode and range of a dataset
- To understand the effect of outliers on the values of the summary statistics

Past, present and future learning:

- This section addresses Stage 4 and the Stage 5 Core topic called *Data analysis A*
- It consolidates and extends the concepts in Section 10F of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

In the previous sections you have seen how to summarise data in the form of a frequency table and to display data using graphs. Key summary statistics also allow us to describe the data using a single numerical value. The mean (i.e. average), for example, may be used to describe a student's performance over a series of tests. The median (i.e. middle value when data is ordered) is often used when describing the house prices in a suburb. These are termed *measures of centre*. The range is a measure that provides information about the spread of the data. It measures the difference between the maximum and minimum values.



Lesson starter: Mean, median or mode?

The following data represent the number of goals scored by Ellie in each game of a 9-game netball season.

24 18 25 16 3 23 27 19 25

It is known that the figures below represent, in some order, the mean, median and mode.

25 20 23

- Without doing any calculations, can you suggest which statistic is which? Explain.
- From the data, what gives an indication that the mean (i.e. average) will be less than the median (i.e. middle value)?
- Describe how you would calculate the mean, median and mode from the data values.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- Statisticians use **summary statistics** to highlight important aspects of a dataset. These are summarised below.

- Some summary statistics are called **measures of location.**
  - The two most commonly used measures of location are the **mean** and the **median**. These are also called '**measures of centre**' or '**measures of central tendency**'. Mean and median can only be applied to numerical data.
  - The **mean** is sometimes called the '**arithmetic mean**' or the '**average**'. The formula used for calculating the mean, $\bar{x}$, is:

    $$\bar{x} = \frac{\text{sum of data values}}{\text{number of data values}}$$

    For example: In the following dataset

5	7	2	5	1

    the mean is $\frac{5+7+2+5+1}{5} = 20 \div 5 = 4$
  - The **median** divides an ordered dataset into two sets, each of which contain the same number of data values. It is often called the 'middle value'. The median is found by firstly ensuring the data values are in ascending order, then selecting the 'middle' value.
    If the number of values is odd, simply choose the one in the middle.
    2 2 3 4 6 9 9
    Median = 4
    If the number of values is even, find the average of the two in the middle.
    2 2 3 4 7 9 9 9
    Median = $(4 + 7) \div 2 = 5.5$
  - The **mode** of a dataset is the most frequently occurring data value. There can be more than one mode. When there are two modes, the data is said to be **bimodal**. The mode can be very useful for categorical data. It can also be used for numerical data, but it may not be an accurate measure of centre. For example, in the dataset below the mode is 10, but it is the largest data value.

1	2	3	10	10

  - An **outlier** is a data value which is significantly greater than or less than the other data values in the dataset. The inclusion of an outlier in a dataset can inflate or deflate the mean and give a false impression of the 'average'. For example:
    In the dataset 1, 2, 3, 4, 5, the mean is 3 and median is 3.
    If the data value 5 is changed to 20, it is now an outlier.
    In the dataset 1, 2, 3, 4, 20, the median is still 3, but the mean is now 6, which is greater than four of the five data values. In this case, the median is a better measure of centre than the mean.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- Some summary statistics are called **measures of spread**.
  - The **range** is a measure of spread.
  - The range of a dataset is given by the formula:
    range = maximum data value – minimum data value
    For example: In the dataset below, the range is $7 - 1 = 6$
    5 7 2 5 1
  - In later sections of this chapter you will encounter some other measures of spread, such as interquartile range and standard deviation.

Key terminology: mode, bimodal, mean, range, median, stem-and-leaf plot, dot plot

Exercise 5D

UNDERSTANDING	1–3	1, 3

1 Use the words from this list to fill in the missing word in these sentences: *mean, median, mode, bimodal, range*.

- **a** The _____ is the most frequently occurring value in a dataset.
- **b** Dividing the sum of all the data values by the total number of values gives the _____.
- **c** The middle value of a dataset ordered from smallest to largest is the _____ .
- **d** A dataset with two most common values is _____ .
- **e** A dataset has a maximum value of 7 and a minimum value of 2. The _____ is 5.

2 Circle the middle value(s) of these ordered datasets.

- **a** 2 4 6 7 8 10 11
- **b** 6 9 10 14 17 20

Hint: Recall that an even number of data values will have two middle values.

3 Aaron drinks the following number of cups of coffee each day in a week.

4 5 3 6 4 3 3

- **a** How many cups of coffee does he drink in the week (sum of the data values)?
- **b** How many days are in the week (total number of data values)?
- **c** What is the mean number of cups of coffee Aaron drinks each day (part **a** ÷ part **b**)?

FLUENCY	4–5(½)	4–5(½), 6

Example 7 Finding the mean, mode and range

For the following datasets, find:

- **i** the mean
- **ii** the mode
- **iii** the range

a 2, 4, 5, 8, 8

b 3, 15, 12, 9, 12, 15, 6, 8

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

a i $\text{Mean} = \dfrac{2+4+5+8+8}{5}$

$= \dfrac{27}{5}$

$= 5.4$

ii The mode is 8.

iii Range $= 8 - 2$

$= 6$

b i $\text{Mean} = \dfrac{3+15+12+9+12+15+6+8}{8}$

$= \dfrac{80}{8}$

$= 10$

ii There are two modes: 12 and 15.

iii Range $= 15 - 3$

$= 12$

EXPLANATION

$\text{Mean} = \dfrac{\text{sum of all data values}}{\text{number of data values}}$

Add all the data and divide by the number of values (5).

The mode is the most common value in the data.

Range = maximum value − minimum value

$\text{Mean} = \dfrac{\text{sum of all data values}}{\text{number of data values}}$

Add all the data and divide by the number of values (8).

The dataset is bimodal: 12 and 15 are the most common data values.

Range = maximum value − minimum value

Now you try

For the following datasets, find:

i the mean
ii the mode
iii the range

a 8, 10, 2, 6, 12, 3, 10, 1 **b** 22, 25, 18, 22, 3, 18

4 For each of the following datasets, find:

i the mean
ii the mode
iii the range

a 2 4 5 8 8
b 5 8 10 15 20 12 10 50
c 55 70 75 50 90 85 50 65 90
d 27 30 28 29 24 12
e 2.0 1.9 2.7 2.9 2.6 1.9 2.7 1.9
f 1.7 1.2 1.4 1.6 2.4 1.3

Hint:
Mean $= \dfrac{\text{sum of data values}}{\text{number of data values}}$
Mode is the most common value.
Range = maximum − minimum

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 8 Finding the median

Find the median of each dataset.

a 4, 7, 12, 2, 9, 15, 1 **b** 16, 20, 8, 5, 21, 14

SOLUTION

a 1 2 4 (7) 9 12 15

Median = 7

EXPLANATION

The data must first be ordered from smallest to largest.

The median is the middle value.

For an odd number of data values, there will be one middle value.

b 5 8 14 16 20 21

$$\text{Median} = \frac{14 + 16}{2}$$

$$= 15$$

Order the data from smallest to largest.

For an even number of data values, there will be two middle values.

The median is the average of these two values (i.e. the value halfway between the two middle numbers).

Now you try

Find the median of each dataset.

a 10, 16, 2, 7, 1, 18, 5, 10, 14 **b** 10, 35, 18, 24, 12, 28, 16, 31

5 Find the median of each dataset.

a 1 4 7 8 12

b 1 2 2 4 4 7 9

c 11 13 6 10 14 13 11

d 62 77 56 78 64 73 79 75 77

e 2 4 4 5 6 8 8 10 12 22

f 1 2 2 3 7 12 12 18

g 30 36 31 38 27 40

h 2.4 2.0 3.2 2.8 3.5 3.1 3.7 3.9

Hint: First make sure that the data values are in order. For two middle values, find their average.

6 Nine people watch the following number of hours of television on a weekend.

4 4 6 6 6 8 9 9 11

a Find the mean number of hours of television watched.

b Find the median number of hours of television watched.

c Find the range of the television hours watched.

d What is the mode number of hours of television watched?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING 7–9 9–11

7 Eight students compare the amount of pocket money they receive. The data is as follows.

$12 $15 $12 $24 $20 $8 $50 $25

a Find the range of pocket money received.

b Find the median amount of pocket money.

c Find the mean amount of pocket money.

d Why is the mean larger than the median?

Example 9 Calculating summary statistics from a stem-and-leaf plot

For the data in this stem-and-leaf plot, find the:

a range

b mode

c mean

d median

Stem	Leaf
2	5 8
3	1 2 2 2 6
4	0 3 3
5	2 6

5 | 2 means 52

SOLUTION

a Minimum value = 25
Maximum value = 56
Range = 56 − 25
= 31

EXPLANATION

In an ordered stem-and-leaf plot, the first data item is the minimum and the last is the maximum. Use the key '5 | 2 means 52' to see how to put the stem and leaf together.
Range = maximum value − minimum value

b Mode = 32

The mode is the most common value. The leaf 2 appears three times with the stem 3.

c $$\text{Mean} = \frac{25 + 28 + 31 + 32 + 32 + 32 + 36 + 40 + 43 + 43 + 52 + 56}{12}$$

$$= \frac{450}{12}$$

$$= 37.5$$

Form each data value from the graph and add them all together. Then divide by the number of data values in the stem-and-leaf plot.

d $$\text{Median} = \frac{32 + 36}{2}$$

$$= 34$$

There is an even number of data values: 12. The median will be the average of the middle two values (i.e. the 6th and 7th data values).

Now you try

For the data in this stem-and-leaf plot, find the:

a range

b mode

c mean

d median

Stem	Leaf
2	1 3 7
3	2 8 9 9
4	4 6 8

3 | 2 means 32

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 For the data in these stem-and-leaf plots, find the:

- **i** range
- **ii** mode
- **iii** mean (rounded to one decimal place)
- **iv** median

Hint: Use the key to see how the stem and leaf go together.

a

Stem	Leaf
0	4 4
1	0 2 5 9
2	1 7 8
3	2

2 | 7 means 27

b

Stem	Leaf
10	1 2 4
11	2 6
12	5

11 | 6 means 116

c

Stem	Leaf
3	0 0 5
4	2 7
5	1 3 3
6	0 2

3 | 2 means 3.2

9 This back-to-back stem-and-leaf plot shows the results of two students, Hugh and Mark, on their end-of-year examination in each subject.

a For each student, find:

- **i** the mean
- **ii** the median
- **iii** the range

b Compare the performance of the two students using your answers to part **a**.

Hugh leaf	Stem	Mark leaf
8 8 5	6	4
7 3	7	4 7
5 4 2 1 1	8	2 4 6 8
	9	2 4 5

7 | 4 means 74%

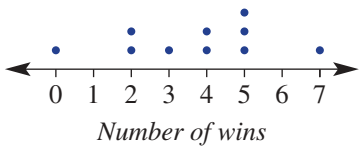
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 A real estate agent recorded the following amounts for the sale of five properties:

$120 000 $210 000 $280 000 $370 000 $1 700 000

The mean is $536 000 and the median is $280 000.
Which is a better measure of the centre of the five property prices: the mean or the median?
Give a reason.

11 This dot plot shows the number of wins recorded by a school sports team in the past 10 eight-game seasons.



a What was the median number of wins?

b What was the mean number of wins?

c The following season, the team records 3 wins. What effect will this have (i.e. increase/decrease/no change) on the:

i median? **ii** mean?

ENRICHMENT: Moving run average — 12

12 A moving average is determined by calculating the average of all data values up to a particular time or place in the dataset.
Consider a batter in cricket with the following runs scored from 10 completed innings.

Innings	1	2	3	4	5	6	7	8	9	10
Score	26	38	5	10	52	103	75	21	33	0
Moving average	26	32								

In the table, 26 is the average after 1 innings and 32 is the average after 2 innings.

a Complete the table by calculating the moving average for innings 3–10. Round your answer to the nearest whole number where required.

b Plot the score and moving averages for the batter on the same set of axes, with the innings number on the horizontal axis. Join the points to form two line graphs.

c Describe the behaviour of the:

i score graph **ii** moving average graph

d Describe the main difference in the behaviour of the two graphs. Give reasons.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5E Quartiles and outliers

Learning intentions for this section:

- To know which statistical measures are used in the 5-number summary
- To be able to calculate the quartiles of a dataset
- To be able to find the interquartile range and know what it represents
- To know what is meant by an outlier and be able to identify any

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Data analysis A*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

In addition to the median of a single set of data, there are two related statistics, called the upper and lower quartiles. If data is placed in order, then the lower quartile is central to the lower half of the data. The upper quartile is central to the upper half of the data. These quartiles are used to calculate the interquartile range, which helps to describe the spread of the data, and show whether or not any data points do not fit the rest of the data (outliers).

Lesson starter: House prices

A real estate agent tells you that the median house price for a suburb in 2020 was $753 000 and the mean was $948 000.

- Is it possible for the median and the mean to differ by so much?
- Under what circumstances could this occur? Discuss.

KEY IDEAS

■ The **5-number summary** uses the following statistical measures to summarise a set of data:

- Minimum value (min) — the lowest value
- **Lower quartile** (Q_1) — the number above 25% of the ordered data
- Median (Q_2) — the middle value above 50% of the ordered data
- **Upper quartile** (Q_3) — the number above 75% of the ordered data
- Maximum value (max) — the highest value

Odd number of data values

1 2 2 3) ⑤ (6 6 7 9

$Q_1 = \frac{2+2}{2} = 2$ Q_2 $Q_3 = \frac{6+7}{2} = 6.5$

Even number of data values

2 3 ③ 4 7 | 8 8 ⑨ 9 9

$Q_1 = 3$ $Q_2 = 7.5$ $Q_3 = 9$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- Another measure of the spread of the data is the **interquartile range (IQR)**.
  IQR = upper quartile − lower quartile
  $= Q_3 - Q_1$
- **Outliers** are data elements outside the vicinity of the rest of the data.
  A data point is an outlier if it is either:
  - less than the **lower fence**, where lower fence $= Q_1 - 1.5 \times \text{IQR}$ or
  - greater than the **upper fence**, where upper fence $= Q_3 + 1.5 \times \text{IQR}$
- Outliers significantly affect the range of a dataset but have limited to no effect on the IQR.

Key terminology: 5-number summary, upper quartile, lower quartile, median, interquartile range, outlier, upper fence, lower fence

Exercise 5E

UNDERSTANDING	1–3	3

1 **a** State the five values that need to be calculated for a 5-number summary.
b Explain the difference between the range and the interquartile range.
c What is an outlier?

2 The data shows, in order, the numbers of cars owned by 10 families surveyed.

0, 1, 1, 1, 1, 2, 2, 2, 3, 3

Hint: For an even number of data values, average the two middle values for the median.

a Find the median (the middle value).
b By splitting the data in half, determine the:
i lower quartile Q_1 (middle of lower half)
ii upper quartile Q_3 (middle of upper half)

3 For the dataset with $Q_1 = 3$ and $Q_3 = 8$:
a Find IQR $= Q_3 - Q_1$.
b Calculate $Q_1 - 1.5 \times \text{IQR}$ (lower fence) and $Q_3 + 1.5 \times \text{IQR}$ (upper fence).
c Identify the name that would be given to the value 18 in the dataset.

FLUENCY	4–6	4–6, 7(½)

Example 10 Finding quartiles and IQR for an even number of data values

Consider this dataset:
2, 2, 4, 5, 6, 8, 10, 13, 16, 20

a Find the upper quartile (Q_3) and the lower quartile (Q_1).
b Determine the IQR.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

a 2 2 (4) 5 6 | 8 10 (13) 16 20

$Q_2 = \frac{6+8}{2}$

$= 7$

Q_1 Q_3

$Q_1 = 4$ and $Q_3 = 13$

b IQR $= 13 - 4$

$= 9$

EXPLANATION

The data is already ordered. Since there is an even number of values, split the data in half to locate the median.

Q_1 is the middle value of the lower half:
2 2 (4) 5 6

Q_3 is the middle value of the upper half:
8 10 (13) 16 20

IQR $= Q_3 - Q_1$

Now you try

Consider this dataset: 3, 6, 6, 10, 12, 13, 15, 19, 24, 27, 28, 31, 33, 33

a Find the upper quartile (Q_3) and the lower quartile (Q_1).

b Determine the IQR.

4 For these datasets, find the:

i upper quartile (Q_3) and the lower quartile (Q_1)

ii IQR

a 3, 4, 6, 8, 8, 10

b 10, 10, 11, 14, 14, 15, 16, 18, 20, 21

c 41, 49, 53, 58, 59, 62, 62, 65

d 1.2, 1.7, 1.9, 2.2, 2.4, 2.5, 2.9, 3.2

Hint: For an even number of data values, split the ordered data in half:
2 4 7 | 8 10 12

Q_1 Q_3

IQR $= Q_3 - Q_1$

Example 11 Finding quartiles and IQR for an odd number of data values

Consider this dataset: 2.2, 1.6, 3.0, 2.7, 1.8, 3.6, 3.9, 2.8, 3.8

a Find the upper quartile (Q_3) and the lower quartile (Q_1).

b Determine the IQR.

SOLUTION

a (1.6 1.8 | 2.2 2.7) 2.8 (3.0 3.6 | 3.8 3.9)

$Q_1 = \frac{1.8+2.2}{2}$

$= \frac{4.0}{2}$

$= 2.0$

Q_2

$Q_3 = \frac{3.6+3.8}{2}$

$= \frac{7.4}{2}$

$= 3.7$

b IQR $= 3.7 - 2.0$

$= 1.7$

EXPLANATION

First order the data and locate the median (Q_2).

Split the data in half; i.e. either side of the median.

Q_1 is the middle value of the lower half; for two middle values, average the two numbers.

Q_3 is the middle value of the upper half.

IQR $= Q_3 - Q_1$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

Consider this dataset:
14.4, 15.2, 16.0, 13.7, 18.2, 21.4, 19.7, 19.9, 12.8, 20.6, 21.4

a Find the upper quartile (Q_3) and the lower quartile (Q_1).
b Determine the IQR.

5 For these datasets, find:
- **i** the upper quartile (Q_3) and the lower quartile (Q_1)
- **ii** the IQR

a 1, 2, 4, 8, 10, 11, 14
b 10, 7, 14, 2, 5, 8, 3, 9, 2, 12, 1
c 0.9, 1.3, 1.1, 1.2, 1.7, 1.5, 1.9, 1.1, 0.8
d 21, 7, 15, 9, 18, 16, 24, 33, 4, 12, 13, 18, 24

Hint: For an odd number of data values, split ordered data in half, leaving out the middle value.
0 ② 4) 7 (9 ⑭ 16
↑ Q_1 ↑ Q_3

Example 12 Finding the 5-number summary and outliers

The following dataset represents the number of flying geese spotted on each day of a 13-day tour of England.
5, 1, 2, 6, 3, 3, 18, 4, 4, 1, 7, 2, 4

a For the data, find the:
- **i** minimum and maximum number of geese spotted
- **ii** median
- **iii** upper and lower quartiles
- **iv** IQR

b Find any outliers.
c Can you give a possible reason for why the outlier occurred?
d If the outlier's value is corrected to 8, determine the range and IQR of the dataset. Explain the similarities or differences compared to the original dataset.

SOLUTION

a i Min = 1, max = 18

ii 1, 1, 2, 2, 3, 3, ④, 4, 4, 5, 6, 7, 18
∴ Median = 4

iii Lower quartile (Q_1) $= \frac{2+2}{2}$
$= 2$

Upper quartile(Q_3) $= \frac{5+6}{2}$
$= 5.5$

iv IQR $= 5.5 - 2$
$= 3.5$

EXPLANATION

Look for the largest and smallest numbers and order the data:

1 1 2 | 2 3 3) 4 (4 4 5 | 6 7 18
↑ Q_1 ↑ Q_2 ↑ Q_3

Since Q_2 falls on a data value, it is not included in the lower or higher halves when Q_1 and Q_3 are calculated.

IQR $= Q_3 - Q_1$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b Lower fence $= Q_1 - 1.5 \times \text{IQR}$
$= 2 - 1.5 \times 3.5$
$= 2 - 5.25$
$= -3.25$

A data point is an outlier if it is less than the lower fence $= Q_1 - 1.5 \times \text{IQR}$ or greater than the upper fence $= Q_3 + 1.5 \times \text{IQR}$.

Upper fence $= Q_3 + 1.5 \times \text{IQR}$
$= 5.5 + 1.5 \times 3.5$
$= 5.5 + 5.25$
$= 10.75$

There are no numbers less than -3.25 but 18 is greater than 10.75.

$\therefore$ The outlier is 18.

c Perhaps a flock of geese was spotted that day.

d 1, 1, 2, 2, 3, 4, 4, 4, 5, 6, 7, 8
Range $= 8 - 1$
$= 7$
IQR $= 5.5 - 2$
$= 3.5$

18 is replaced in the dataset with 8 and the range and IQR calculated.

The range is significantly reduced by correcting the outlier. The IQR here is unchanged. Extreme values do not significantly affect the IQR.

The outlier affected the range as the maximum value is changed but the IQR is not affected by the value of the maximum.

Now you try

The following dataset represents the number of hours a ballkid worked at the Australian Open over the 14-day event.
4, 6, 6, 5, 8, 7, 8, 14, 5, 4, 2, 3, 5, 6

a For the data, find the:
- **i** minimum and maximum number of hours worked
- **ii** median
- **iii** upper and lower quartiles
- **iv** IQR

b Find any outliers.

c Can you give a possible reason for why the outlier occurred?

d If the outlier's value is corrected to 9, determine the range and IQR of the dataset. Explain the similarities or differences compared to the original dataset.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 The following numbers of cars were counted on each day for 15 days, travelling on a quiet suburban street.

10, 9, 15, 14, 10, 17, 15, 0, 12, 14, 8, 15, 15, 11, 13

Hint:
Outliers are:
more than $Q_3 + 1.5 \times IQR$
or
less than $Q_1 - 1.5 \times IQR$

a For the given data, find the:
- **i** minimum and maximum number of cars counted
- **ii** median
- **iii** lower and upper quartiles (Q_1 and Q_3)
- **iv** IQR

b Find any outliers.

c Give a possible reason for the outlier.

d The outlier value is changed to 8. Find the new range and IQR and comment on any similarities or differences compared to the original dataset.

7 Summarise the datasets below by finding:
- **i** the minimum and maximum values
- **ii** the median (Q_2)
- **iii** the lower and upper quartiles (Q_1 and Q_3)
- **iv** the IQR
- **v** any outliers

a 4, 5, 10, 7, 5, 14, 8, 5, 9, 9

b 24, 21, 23, 18, 25, 29, 31, 16, 26, 25, 27

c 10, 13, 2, 11, 10, 8, 24, 12, 13, 15, 12

d 3, 6, 10, 11, 17, 4, 4, 1, 8, 4, 10, 8

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

8, 9 | 8, 10–12

8 Twelve different calculators had the following numbers of buttons:

36, 48, 52, 43, 46, 53, 25, 60, 128, 32, 52, 40

a For the given data, find:

- **i** the minimum and maximum number of buttons on the calculators
- **ii** the median
- **iii** the lower and upper quartiles (Q_1 and Q_3)
- **iv** the IQR
- **v** any outliers
- **vi** the mean

b Can you give a possible reason why the outlier has occurred?

c Which is a better measure of the centre of the data: the mean or the median? Explain.

d Which is a better measure of the spread of the data: the range or the IQR? Explain.

9 At an airport, Paul checks the weight of 20 luggage items. If the weight of a piece of luggage is an outlier, then the contents undergo a further check. The weights in kilograms are:

1 4 5 5 6 7 7 7 8 8
10 10 10 13 15 16 17 19 32 33

How many luggage items will undergo a further check?

10 The prices of nine fridges are displayed in a sale catalogue. They are:

$350 $1000 $850 $900 $1100 $1200
$1100 $1000 $1700

How many of the fridge prices could be considered outliers?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 For the data in this stem-and-leaf plot, find:

a the IQR

b any outliers

c any outliers if the number 32 was added to the list

Stem	Leaf
0	1
1	6 8
2	0 4 6 8
3	0

2 | 4 means 24

Hint: Split the data in half to find Q_2, then find Q_1 and Q_3.

12 For the data in this stem-and-leaf plot the value 42 was incorrectly recorded. What are the possible values it should have been if the:

a median is not changed?

b IQR is not changed?

Stem	Leaf
1	7 8
2	1 4 6
3	5 5
4	2 9
5	0 3

4 | 2 means 42

ENRICHMENT: Some research — 13

13 Use the internet to search for data about a topic that interests you. Try to choose a single set of data that includes between 15 and 50 values.

a Organise the data using a:

i stem-and-leaf plot

ii frequency table and histogram

b Find the mean and the median.

c Find the range and the interquartile range.

d Write a brief report describing the centre and spread of the data, referring to parts **a** to **c** above.

e Present your findings to your class or a classmate.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5F Box plots

Learning intentions for this section:

- To know that a box plot can be used to represent the 5-number summary of a dataset
- To understand that each section of a box plot represents approximately 25% of the data
- To be able to construct box plots both with and without outliers
- To know that parallel box plots can be used to compare two or more datasets

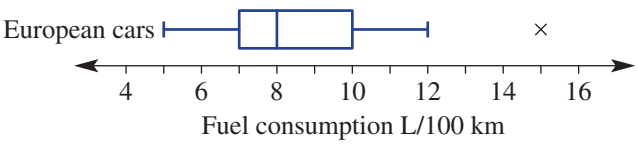
Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Data analysis A*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

The 5-number summary (min, Q_1, Q_2, Q_3, max) can be represented in graphical form as a box plot. Box plots are graphs that summarise single datasets. They clearly display the minimum and maximum values, the median, the quartiles and any outliers. Q_1, Q_2 and Q_3 divide the data into quarters. Box plots also give a clear indication of how data is spread, as the IQR (interquartile range) is shown by the width of the central box.

Lesson starter: Fuel consumption

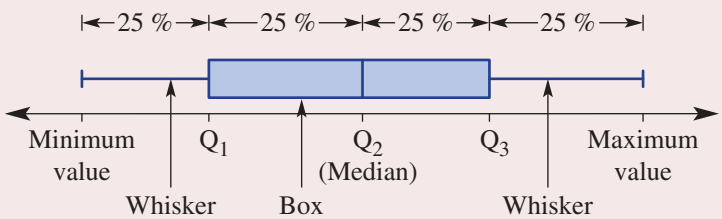
This box plot summarises the average fuel consumption (litres per 100 km) for a group of European-made cars.



- What does each part of the box plot represent in terms of the 5-number summary?
- What do you think the cross (×) represents?
- Describe how you can use the box plot to find the IQR.
- For the top 25% of cars, what would you expect the fuel consumption to be above?

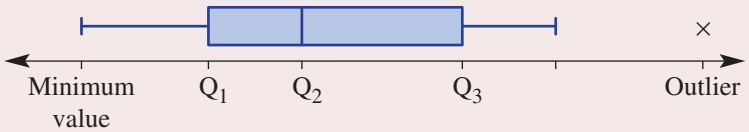
KEY IDEAS

■ A **box plot** (also called a box-and-whisker plot) can be used to summarise a dataset and show the spread of the data. It displays the 5-number summary (min, Q_1, Q_2, Q_3, max), as shown.

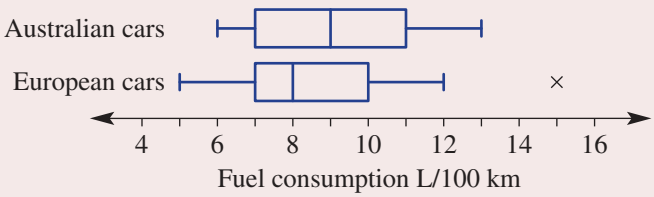


CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- An outlier may be marked with a cross (×).
  - An outlier is greater than $Q_3 + 1.5 \times IQR$ (upper fence) or less than $Q_1 - 1.5 \times IQR$ (lower fence).
  - $IQR = Q_3 - Q_1$
  - The whiskers stretch to the lowest and highest data values that are not outliers.



- **Parallel box plots** are box plots drawn on the same scale. They are used to compare datasets within the same context.

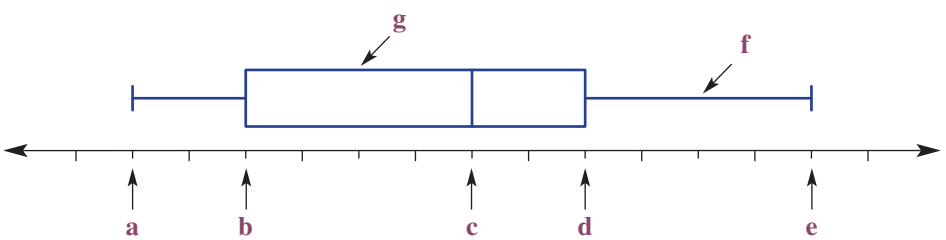


Key terminology: box plot, parallel box plot, outlier, 5-number summary, upper quartile, lower quartile, median, interquartile range, upper fence, lower fence

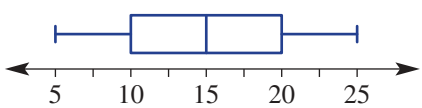
Exercise 5F

UNDERSTANDING 1–4 4

1 Label the parts **a**–**g** of the box plot below.



2 For this simple box plot, find the:

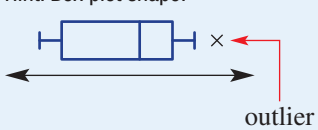


- **a** median (Q_2)
- **b** minimum
- **c** maximum
- **d** range
- **e** lower quartile (Q_1)
- **f** upper quartile (Q_3)
- **g** interquartile range (IQR)

3 Construct a box plot showing these features.

- **a** Min = 1, Q_1 = 3, Q_2 = 4, Q_3 = 7, max = 8
- **b** Outlier = 5, minimum above outlier = 10, Q_1 = 12, Q_2 = 14, Q_3 = 15, max = 17

Hint: Box plot shape:



Include an even scale.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Select from the list below to fill in the blanks.

minimum, Q_1, Q_2, Q_3, *maximum*.

a The top 25% of data is above _____.

b The middle 50% of data is between _____ and _____.

c The lowest or first 25% of data is between the _____ and _____.

d The highest or last 25% of data is between _____ and the _____.

FLUENCY 5–6(½) 5–6(½)

Example 13 Constructing box plots with no outliers

Consider the given dataset. 12 26 14 11 15 10 18 17 21 27

a Find the 5-number summary (i.e. the minimum, lower quartile (Q_1), median (Q_2), upper quartile (Q_3) and the maximum).

b Draw a box plot to summarise the data.

SOLUTION

a 10 11 (12) 14 15 | 17 18 (21) 26 27

Q_1 $Q_2 = \frac{15 + 17}{2}$ Q_3

Min $= 16$ Max

Minimum = 10

$Q_1 = 12$, $Q_2 = 16$, $Q_3 = 21$

Maximum = 27

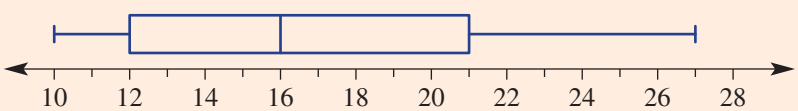
EXPLANATION

Order the data from smallest to largest.

Locate the median (Q_2) first. For 10 data values there are two middle values, 15 and 17. Average these to find Q_2.

Split the data in half at the median. Q_1 is the middle value of the lower half. Q_3 is the middle value of the upper half. The minimum is the smallest value and the maximum is the largest value.

b



Draw an even scale, covering the minimum and maximum values.

Mark the minimum (10), Q_1 (12), Q_2 (16), Q_3 (21) and the maximum (27) to draw the box plot.

Now you try

Consider the given dataset. 56 32 44 45 60 55 54 38 40 44 54 52

a Find the 5-number summary (i.e. the minimum, lower quartile (Q_1), median (Q_2), upper quartile (Q_3) and the maximum).

b Draw a box plot to summarise the data.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Consider the datasets below.

i Find the 5-number summary (minimum, Q_1, Q_2, Q_3, maximum).

ii Draw a box plot to summarise the data.

a 11, 15, 18, 17, 1, 2, 8, 12, 19, 15

b 0, 1, 5, 4, 4, 4, 2, 3, 3, 1, 4, 3

c 124, 118, 119, 117, 120, 120, 121, 118, 122

d 62, 85, 20, 34, 40, 66, 47, 82, 25, 32, 28, 49, 41, 30, 22

Example 14 Constructing box plots with outliers

Consider the given dataset. 5, 9, 4, 3, 5, 6, 6, 5, 7, 12, 2, 3, 5

a Determine the quartiles Q_1, Q_2 and Q_3.

b Determine whether any outliers exist.

c Draw a box plot to summarise the data, marking outliers if they exist.

SOLUTION

a 2 3 3 4 5 5 (5) (5 6 6 7 9 12

Q_1 Q_2 Q_3

$Q_1 = \frac{3+4}{2}$

$= 3.5$

$Q_3 = \frac{6+7}{2}$

$= 6.5$

EXPLANATION

Order the data to help find the quartiles.

Locate the median Q_2 (i.e. the middle value), then split the data in half above and below this value.

Q_1 is the middle value of the lower half and Q_3 is the middle value of the upper half. Average the two middle values to find the median.

b $\text{IQR} = 6.5 - 3.5 = 3$

$Q_1 - 1.5 \times \text{IQR} = 3.5 - 1.5 \times 3$

$= -1$

$Q_3 + 1.5 \times \text{IQR} = 6.5 + 1.5 \times 3$

$= 11$

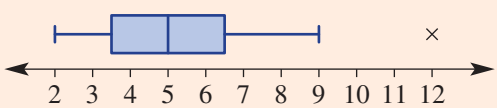
$\therefore$ 12 is an outlier.

Determine $\text{IQR} = Q_3 - Q_1$.

Check for any outliers; i.e. numbers below $Q_1 - 1.5 \times \text{IQR}$ or above $Q_3 + 1.5 \times \text{IQR}$.

There is no data below -1, but $12 > 11$.

c



Draw a line and mark in a uniform scale, reaching from 2 to 12. Sketch the box plot by marking the minimum 2 and the outlier 12, and Q_1, Q_2 and Q_3. The end of the 5-number summary is the nearest value below 11; i.e. 9.

Now you try

Consider the given dataset. 10, 8, 11, 9, 8, 22, 15, 10, 12

a Determine the quartiles Q_1, Q_2 and Q_3.

b Determine whether any outliers exist.

c Draw a box plot to summarise the data, marking outliers if they exist.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Consider the datasets below.

- **i** Determine the quartiles Q_1, Q_2 and Q_3.
- **ii** Determine whether any outliers exist.
- **iii** Draw a box plot to summarise the data, marking outliers if they exist.

a 4, 6, 5, 2, 3, 4, 4, 13, 8, 7, 6

b 1.8, 1.7, 1.8, 1.9, 1.6, 1.8, 2.0, 1.1, 1.4, 1.9, 2.2

c 21, 23, 18, 11, 16, 19, 24, 21, 23, 22, 20, 31, 26, 22

d 37, 48, 52, 51, 51, 42, 48, 47, 39, 41, 65

Hint:
Outliers are:
more than $Q_3 + 1.5 \times IQR$
or
less than $Q_1 - 1.5 \times IQR$.
Mark with a X.
The next value above or below an outlier is used as the new end of the whisker.

PROBLEM-SOLVING AND REASONING

7–9 | 7–10

7 A butcher records the weight (in kilograms) of a dozen parcels of sausages sold on one morning.

1.6 1.9 2.0 2.0 2.1 2.2
2.2 2.4 2.5 2.7 3.8 3.9

a Write down the value of:

- **i** the minimum
- **ii** Q_1
- **iii** Q_2
- **iv** Q_3
- **v** the maximum
- **vi** IQR

b Find any outliers.

c Draw a box plot for the weight of the parcels of sausages.

8 Joel the gardener records the number of days that it takes for 11 special bulbs to germinate. The results are:

8 14 15 15 16 16 16 17 19 19 24

a Write down the value of:

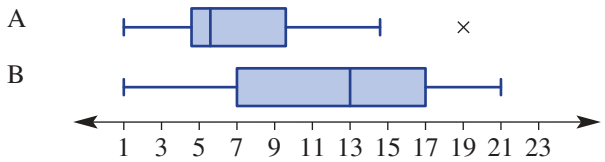
- **i** the minimum
- **ii** Q_1
- **iii** Q_2
- **iv** Q_3
- **v** the maximum
- **vi** IQR

b Are there any outliers? If so, what are they?

c Draw a box plot for the number of days it takes for the bulbs to germinate.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 Consider these parallel box plots, A and B.



Hint: Parallel box plots are two box plots that can be compared using the same scale.
Compare the box plots at each point of the 5-number summary.

a What statistical measure do these box plots have in common?

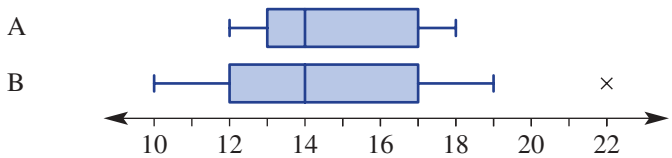
b Which dataset (A or B) has a wider range of values?

c Find the IQR for:

i dataset A **ii** dataset B

d How would you describe the main difference between the two sets of data from which the parallel box plots have been drawn?

10 Two datasets can be compared using parallel box plots on the same scale, as shown below.



a What statistical measures do these box plots have in common?

b Which dataset (A or B) has a wider range of values?

c Find the IQR for:

i dataset A

ii dataset B

d How would you describe the main difference between the two sets of data from which the parallel box plots have been drawn?

ENRICHMENT: Creating parallel box plots — 11

11 Fifteen essays are marked for spelling errors by a particular examiner and the following numbers of spelling errors are counted.

3, 2, 4, 6, 8, 4, 6, 7, 6, 1, 7, 12, 7, 3, 8

The same 15 essays are marked for spelling errors by a second examiner and the following numbers of spelling errors are counted.

12, 7, 9, 11, 15, 5, 14, 16, 9, 11, 8, 13, 14, 15, 13

a Draw parallel box plots for the data.

b Do you believe there is a major difference in the way the essays were marked by the two examiners? If yes, describe this difference.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Progress quiz

5A **1** Name the type of data that would be generated by the following survey questions.

a What is your favourite sport?

b How many times did you exercise in the past week?

5B **2** Twenty-five people were surveyed as to the number of hours of sleep they had the previous night. The data is listed below.

6	5	8	9	7	10	5	8	8
11	7	9	4	2	9	8	7	
10	8	7	5	7	10	6	9	

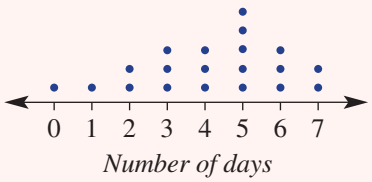
a Organise the data into a frequency table using class intervals of 3, starting with 0–2, then 3–5 etc. Include a percentage frequency column.

b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.

c What percentage of people surveyed had fewer than 6 hours sleep?

d Which interval is the most frequent?

5C/5D **3** This dot plot shows the number of days a group of students did homework in a week.



a How many students were surveyed?

b What was the mode number of days of homework?

c What was the median number of days of homework?

d What was the mean number of days of homework?

e What was the range of the number of days of homework?

5C/5D **4** Consider the given dataset.

7	11	46	42	34	40	24	45	15	22
21	16	49	25	33	30	47	30	3	48

a Organise the data into an ordered stem-and-leaf plot.

b Describe the distribution of the data as symmetrical or skewed.

c Use the stem-and-leaf plot to find:

i the mode ii the median

5D **5** For the following dataset, find the:

8, 15, 23, 12, 3, 19, 42, 33

a mean b median c range

5E **6** Consider the given dataset.

10 13 8 15 24 18 11 20

a Find the upper quartile (Q_3) and lower quartile (Q_1).

b Determine the IQR.

5F **7** The data below show the number of cars that travel down a particular road between 5:30 p.m. and 6 p.m. each day for a week.

0 82 75 49 102 110 97

a Determine the quartiles Q_1, Q_2 and Q_3.

b Determine whether any outliers exist.

c Draw a box plot to summarise the data.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5G Standard deviation

Learning intentions for this section:

- To understand how the standard deviation is calculated
- To be able calculate standard deviation using technology
- To compare datasets using standard deviation

Past, present and future learning:

- This section concludes the Stage 5 Core topic called *Data analysis A*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- Students are not required to recall the formula from memory in the HSC Examination

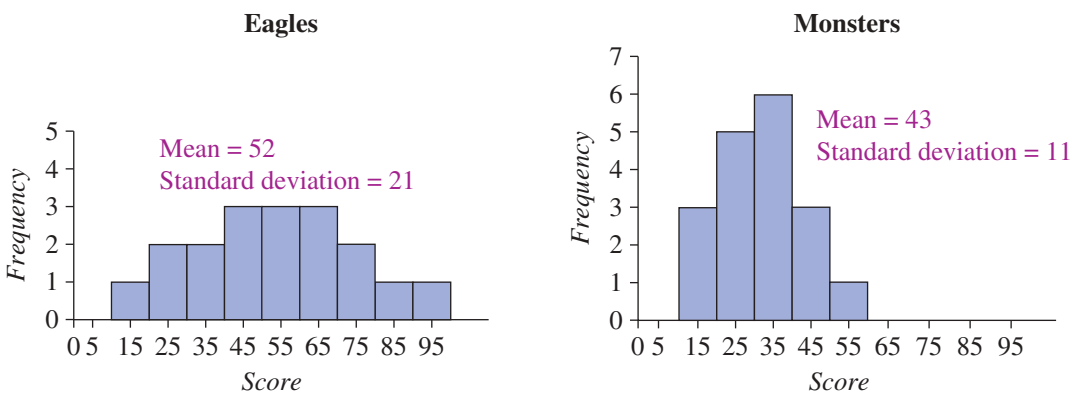
We have already used the range and interquartile range to describe the spread of the data. Another statistic commonly used to describe spread is standard deviation. The standard deviation is a number that describes how far data values are from the mean. A dataset with a relatively small standard deviation will have data values concentrated about the mean, and if a dataset has a relatively large standard deviation then the data values will be more spread out from the mean.

Standard deviation can be calculated by hand but, given the tedious nature of the calculation, technology can be used for more complex datasets. You will be able to find a function on your calculator (often denoted σ) that can be used to calculate the standard deviation.

Lesson starter: Which is the better team?

These histograms show the number of points scored by the Eagles and Monsters basketball teams in an 18-round competition. The mean and standard deviation are given for each team.

- Which team has the higher mean? What does this say about the team's performance?
- Which team has the smaller standard deviation? What does this say about the team's performance? Discuss.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- The **standard deviation** is a number that describes how far data values deviate from the mean.
  - The symbol used for standard deviation is σ (sigma).
  - In a dataset in which all the data values are equal, the standard deviation is zero.
  - If data values are concentrated about the mean, then the standard deviation is relatively small.
  - If data values are spread out from the mean, then the standard deviation is relatively large.
- Most calculators have a button that calculates population standard deviation. Find it and learn how to use it.
  - Your calculator will also have a button for sample standard deviation. This is used when a dataset is collected from a sample.
- These are the steps that your calculator uses to calculate the standard deviation of a dataset collected from a population.
  - Find the mean $(\bar{x})$.
  - Find the difference between each value and the mean (called the deviation).
  - Square each deviation.
  - Sum the squares of each deviation.
  - Divide by the number of data values, n.
  - Take the square root.

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}}$$

Key terminology: standard deviation

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 5G

UNDERSTANDING | 1–4 | 1–4

Example 15 Interpreting the standard deviation

This back-to-back stem-and-leaf plot shows the distribution of distances that 17 people in Darwin and Sydney travel to work. The means and standard deviations are given.

Darwin leaf	Stem	Sydney leaf
8 7 4 2	0	1 5
9 9 5 5 3	1	2 3 7
8 7 4 3 0	2	0 5 5 6
5 2 2	3	2 5 9 9
	4	4 4 6
	5	2

3 | 5 means 35 km

Sydney
$\bar{x} = 27.9$
$\sigma = 14.7$

Darwin
$\bar{x} = 19.0$
$\sigma = 9.8$

Consider the position and spread of the data and then answer the following.

a By looking at the stem-and-leaf plot, suggest why Darwin's mean is less than that of Sydney.

b Why is Sydney's standard deviation larger than that of Darwin?

c Give a practical reason for the difference in centre and spread for the data for Darwin and Sydney.

SOLUTION

a The maximum score for Darwin is 35. Sydney's mean is affected by several values larger than 35.

b The data for Sydney are more spread out from the mean. Darwin's scores are closely clustered near its mean.

c Sydney is a larger city and more spread out, so people have to travel farther to get to work.

EXPLANATION

The mean depends on every value in the dataset.

Sydney has more scores with a large distance from its mean. Darwin's scores are closer to the Darwin mean.

Higher populations often lead to larger cities and longer travel distances.

Now you try

William and Olivia both conducted a survey. They asked: 'How old will you be on your next birthday?'
William chose a sample of students from his high school.
Olivia chose a sample of people at her local shopping centre.

a Whose dataset will have the highest mean? Explain.

b Whose dataset will have the highest standard deviation? Explain.

1 Insert the word *smaller* or *larger* into each sentence.

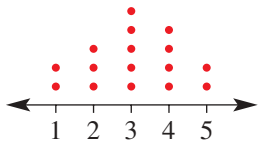
a If data values are more spread out from the mean, then the standard deviation is _____.

b If data values are more concentrated about the mean, then the standard deviation is _____.

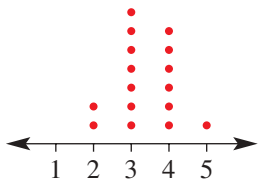
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 Here are two dot plots, A and B.

A



B

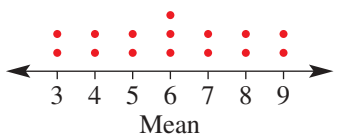


a Which dataset (A or B) would have the higher mean?

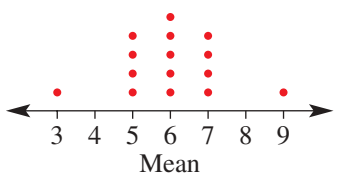
b Which dataset (A or B) would have the higher standard deviation?

3 These dot plots show the results for a class of 15 students who sat tests A and B. Both sets of results have the same mean and range.

A



B



a Which dataset (A or B) would have the higher standard deviation?

b Give a reason for your answer to part **a**.

4 This back-to-back stem-and-leaf plot compares the number of trees or shrubs in the backyards of homes in the suburbs of Gum Heights and Oak Valley.

Gum Heights leaf	Stem	Oak Valley leaf
7 3 1	0	6
9 8 6 4 0	1	0 5
9 8 7 2	2	0 2 3 6 8 8 9
6 4	3	4 6 8
	4	3

2 | 8 means 28

a Which suburb has the smaller mean number of trees or shrubs? Do not calculate the actual means.

b Which suburb has the smaller standard deviation?

FLUENCY

5–6 | 5–6

5 Calculate the mean and standard deviation for these small datasets. Round the standard deviation to one decimal place where necessary.

a 3, 5, 6, 7, 9

b 1, 1, 4, 5, 7

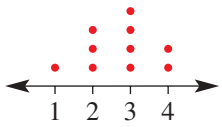
c 2, 5, 6, 9, 10, 11, 13

d 28, 29, 32, 33, 36, 37

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Calculate the mean and standard deviation for the data in these graphs, correct to one decimal place.

a



b

Stem	Leaf
0	4
1	1 3 7
2	0 2

1 | 7 means 17

PROBLEM-SOLVING AND REASONING 7–9 7–10

7 This back-to-back stem-and-leaf plot shows the distribution of distances travelled by students at an inner-city and an outer-suburb school. The means and standard deviations are given.

Inner city leaf	Stem	Outer suburb leaf
9 6 4 3 1 1	0	3 4 9
9 4 2 0	1	2 8 8 9
7 1	2	1 3 4
	3	4
	4	1

2 | 4 means 24

Inner city
$\bar{x} = 10.6$
$\sigma = 8.0$

Outer suburb
$\bar{x} = 18.8$
$\sigma = 10.7$

Consider the position and spread of the data and then answer the following.

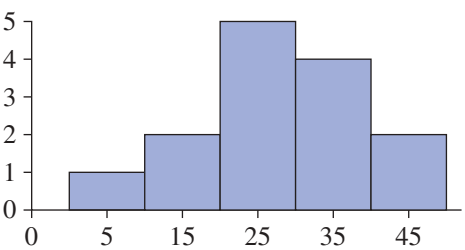
a Why is the mean for the outer-suburb school larger than that for the inner-city school?

b Why is the standard deviation for the inner-city school smaller than that for the outer-suburb school?

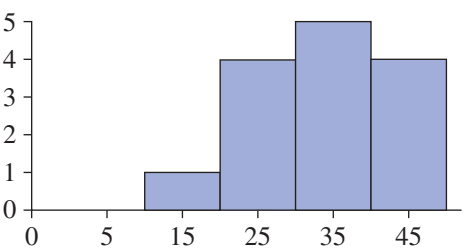
c Give a practical reason for the differences in centre and spread for the two schools.

8 Consider these two histograms, and then state if the following statements are true (T) or false (F).

A



B



a The mean for set A is greater than the mean for set B.

b The range for set A is greater than the range for set B.

c The standard deviation for set A is greater than the standard deviation for set B.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 Find the mean and standard deviation for the scores in these frequency tables. Round the standard deviations to one decimal place.

a

Score	Frequency
1	3
2	1
3	3

b

Score	Frequency
4	1
5	4
6	3

10 Two simple datasets, A and B, are identical except for the maximum value, which is an outlier for set B.

A: 4, 5, 7, 9, 10

B: 4, 5, 7, 9, 20

a Is the range for set A equal to the range for set B?

b Is the mean for both datasets the same?

c Is the median for both datasets the same?

d Would the standard deviation be affected by the outlier? Explain.

ENRICHMENT: Study scores — 11

11 The Maths study scores (out of 100) for 50 students in a school are as listed.

71, 85, 62, 54, 37, 49, 92, 85, 67, 89
96, 44, 67, 62, 75, 84, 71, 63, 69, 81
57, 43, 64, 61, 52, 59, 83, 46, 90, 32
94, 84, 66, 70, 78, 45, 50, 64, 68, 73
79, 89, 80, 62, 57, 83, 86, 94, 81, 65

The mean is 69.16 and the standard deviation is 16.0.

a Calculate:

i $\bar{x} + \sigma$

ii $\bar{x} - \sigma$

iii $\bar{x} + 2\sigma$

iv $\bar{x} - 2\sigma$

v $\bar{x} + 3\sigma$

vi $\bar{x} - 3\sigma$

b Use your answers from part **a** to find the percentage of students with a score within:

i one standard deviation from the mean

ii two standard deviations from the mean

iii three standard deviations from the mean

c **i** Research what it means when we say that the data is 'normally distributed'. Give a brief explanation.

ii For data that is normally distributed, find out what percentage of data is within one, two and three standard deviations from the mean. Compare this with your results for part **b** above.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5H Displaying and analysing time-series data CONSOLIDATING

Learning intentions for this section:

- To understand that time-series data is data recorded at regular time intervals
- To be able to plot a time-series graph
- To be able to describe any trends in the data of a time-series graph

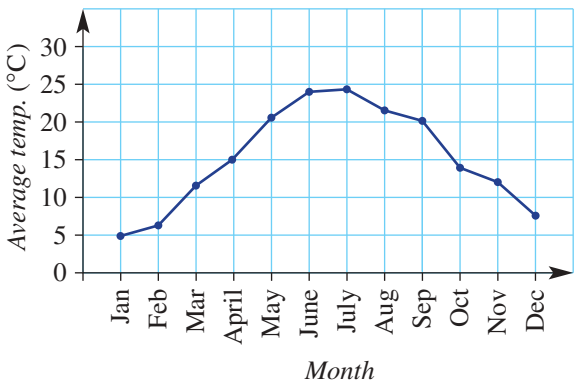
Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stages 5 and 6
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

A time series is a sequence of data values that are recorded at regular time intervals. Examples include temperature recorded on the hour, speed recorded every second, population recorded every year and profit recorded every month. A line graph can be used to represent time-series data. This can help to analyse the data, describe trends and make predictions about the future.

Lesson starter: Changing temperatures

The average monthly maximum temperature for a city is illustrated by this graph.

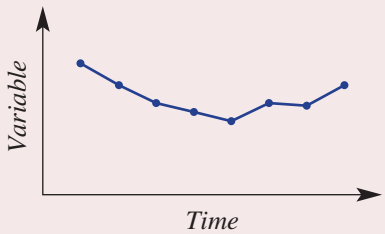


- Describe the trend in the data at different times of the year.
- Explain why the average maximum temperature for December is close to the average maximum temperature for January.
- Do you think this graph is for an Australian city? Explain.
- If another year of temperatures was included on this graph, what would you expect the shape of the graph to look like?
- Do you think this city is in the Northern Hemisphere or the Southern Hemisphere? Give a reason.

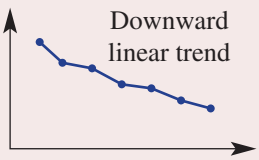
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- **Time-series data** is data that is recorded at regular time intervals.
- The graph or plot of a time series uses:
  - time on the horizontal axis
  - line segments connecting points on the graph



- If the time-series plot results in points being on or near a straight line, then we say that the **trend** is **linear**.



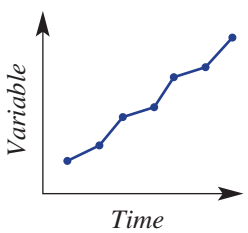
Key terminology: time-series data, linear, trend

Exercise 5H

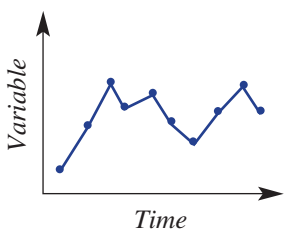
UNDERSTANDING 1, 2 2

1 Describe the following time-series plots as having a linear (i.e. straight line) trend, non-linear trend (i.e. a smooth curve) or no trend.

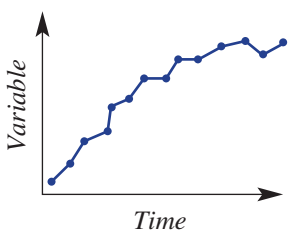
a



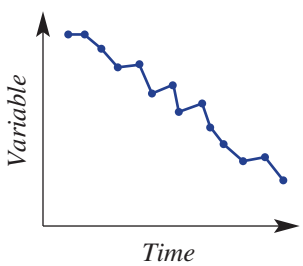
b



c



d



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 This time-series graph shows the temperature over the course of 8 hours of a day.

a State the temperature at:

i 8 a.m.

ii noon

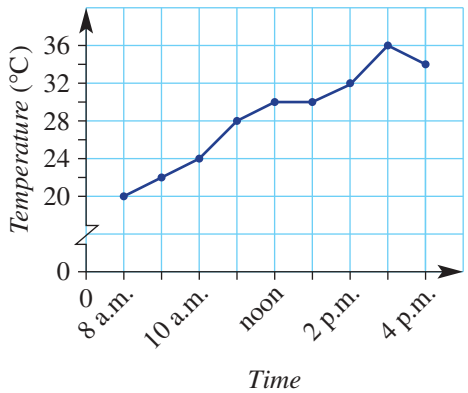
b What was the maximum temperature?

c During what times did the temperature:

i stay the same?

ii decrease?

d Describe the general trend in the temperature for the 8 hours.



FLUENCY 3, 4 3, 4

Example 16 Plotting and interpreting a time-series plot

The approximate population of a small town was recorded from 2008 to 2023.

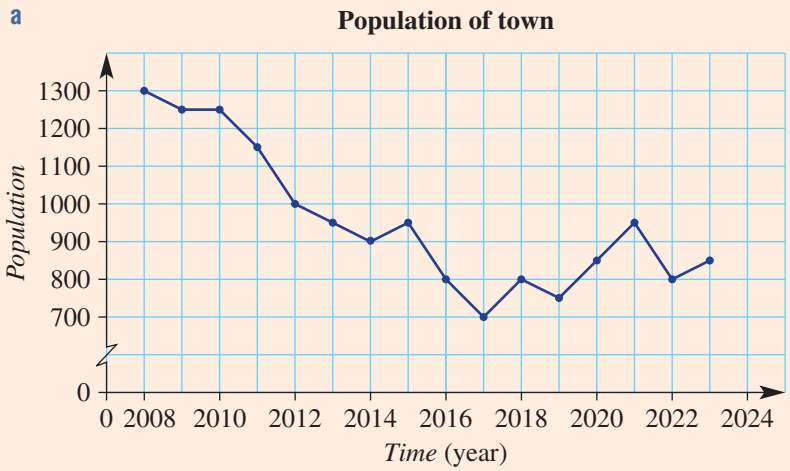
Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Population	1300	1250	1250	1150	1000	950	900	950	800	700	800	750	850	950	800	850

a Plot the time-series graph.

b Describe the trend in the data over the 16 years.

SOLUTION

a



b The population declines steadily for the first 10 years. The population rises and falls in the last 6 years, resulting in a slight upwards trend.

EXPLANATION

Use time on the horizontal axis. Break the y-axis so as not to include 0–700. Label an even scale on each axis. Join points with line segments.

Interpret the overall rise and fall of the lines on the graph.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

The approximate population of a small town was recorded from 2013 to 2023.

Year	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Population	550	500	550	600	700	650	750	750	850	950	900

a Plot the time-series graph. Break the y-axis so it does not include 0–500.
b Describe the general trend in the data over the 11 years.

3 A company's share price over 12 months is recorded in this table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Price ($)	1.30	1.32	1.35	1.34	1.40	1.43	1.40	1.38	1.30	1.25	1.22	1.23

a Plot the time-series graph. Break the y-axis to exclude values from \$0 to \$1.20.
b Describe the way in which the share price has changed over the 12 months.
c What is the difference between the maximum and minimum share price in the 12 months?

Hint: The scale on the vertical axis will need to include from \$1.20 to \$1.43. Choose an appropriate scale. Month will be on the horizontal axis.

4 The pass rate (%) for a particular examination is given in this table.

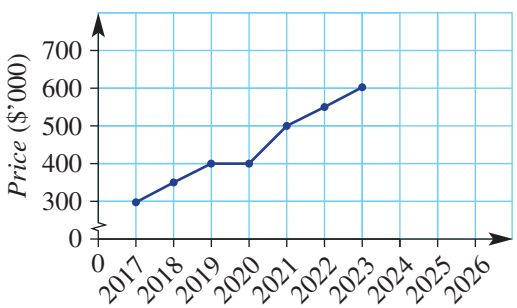
Year	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Pass rate (%)	74	71	73	79	85	84	87	81	84	83

a Plot the time-series graph for the 10 years.
b Describe the way in which the pass rate for the examination has changed in the given time period.
c In what year is the pass rate a maximum?
d By how much has the pass rate improved from 2014 to 2023?

PROBLEM-SOLVING AND REASONING

5, 6 | 5, 7, 8

5 This time-series plot shows the upwards trend of house prices in an Adelaide suburb over 7 years from 2017 to 2023.



Hint: Recall that a linear trend has the points on or near a straight line.

a Would you say that the general trend in house prices is linear or non-linear?
b Assuming that the trend in house prices continues for this suburb, what would you expect the house price to be in:
i 2024? **ii** 2026?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 The following data show the monthly sales of strawberries ($'000s) for a particular year.

Hint: $'000s means 22 represents $22 000.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales ($'000s)	22	14	9	11	12	9	7	9	8	10	18	25

a Plot the time-series graph for the year.

b Describe any trends in the data over the year.

c Give a reason why you think the trends you observed may have occurred.

7 The two top-selling book stores for a company list their sales figures for the first 6 months of the year. Sales amounts are in thousands of dollars.

	July	August	September	October	November	December
City Central ($'000)	12	13	12	10	11	13
Southbank ($'000)	17	19	16	12	13	9

a What is the difference in the sales volume for:

i August?

ii December?

b How many months did the City Central store sell more books than the Southbank store?

c Construct a time-series plot for both stores on the same set of axes.

d Describe the trend of sales for the 6 months for:

i City Central

ii Southbank

Hint: Use different colours for the two line graphs.

e Based on the trend for the sales for the Southbank store, what would you expect the approximate sales volume to be in January?

8 Two pigeons (Green Tail and Blue Crest) each have a beacon that communicates with a recording machine. The distance of each pigeon from the machine is recorded every hour for 8 hours.

a State the distance from the machine at 3 p.m. of:

i Blue Crest

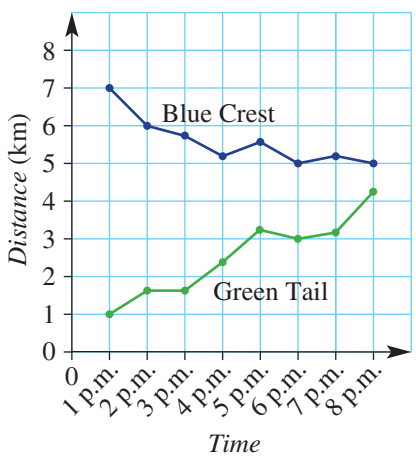
ii Green Tail

b Describe the trend in the distance from the recording machine for:

i Blue Crest

ii Green Tail

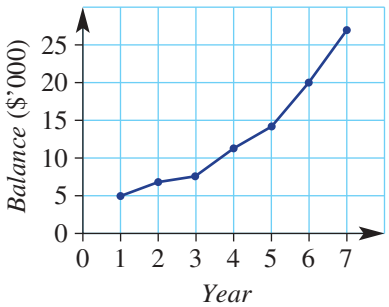
c Assuming that the given trends continue, predict the time when the pigeons will be the same distance from the recording machine.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: Non-linear trends — 9, 10

9 The balance of an investment account is shown in this time-series plot.



a Describe the trend in the account balance over the 7 years.

b Give a practical reason for the shape of the curve that models the trend in the graph.

10 A drink at room temperature is placed in a fridge that is at 4°C.

a Sketch a time-series plot that might show the temperature of the drink after it has been placed in the fridge.

b Would the temperature of the drink ever get to 3°C? Why?

c Record the temperature at regular intervals of a drink at room temperature that is placed in a fridge. Plot your results and compare them to your answer in part **a**.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5I Bivariate data and scatter plots

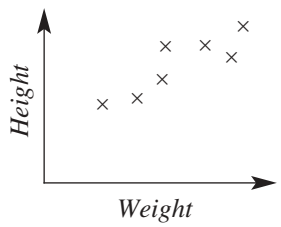
Learning intentions for this section:

- To know that bivariate data is data that involves two variables
- To be able to construct a scatterplot for bivariate data
- To be able to describe the correlation of the data in a scatter plot

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Data analysis B*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

When we collect information about two variables in a given context we are collecting bivariate data. As there are two variables involved in bivariate data, we use a number plane to graph the data. These graphs are called scatter plots and are used to show a relationship that may exist between the variables. Scatter plots make it very easy to see the strength of the relationship between the two variables.



Lesson starter: A relationship or not?

Consider the two variables in each part below.

- Would you expect there to be some relationship between the two variables in each of these cases?
- If you feel that a relationship exists, would you expect the second-listed variable to increase or to decrease as the first variable increases?

a *Height of person* and *Weight of person*
b *Temperature* and *Life of milk*
c *Length of hair* and *IQ*
d *Depth of topsoil* and *Brand of motorcycle*
e *Years of education* and *Income*
f *Spring rainfall* and *Crop yield*
g *Size of ship* and *Cargo capacity*
h *Fuel economy* and *CD track number*
i *Amount of traffic* and *Travel time*
j *Cost of* 2 *litres of milk* and *Ability to swim*
k *Background noise* and *Amount of work completed*

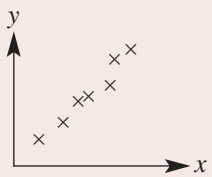
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- **Bivariate data** is data that involves two variables.
  - The two variables are usually related; for example, height and weight.
- A **scatter plot** is a graph on a number plane in which the axes variables correspond to the two variables from the bivariate data. Points are marked with a cross, which could, for example, represent the age and height of one person.
- The words **relationship**, **correlation** and **association** are used to describe the way in which the variables are related and potentially causal, although this would need further investigation.
  Relationship – In summer, as the temperature increases during the day, there may be a relationship between 'temperature' and 'number of ice-creams sold per hour'
  Correlation – Is a measure of the strength of the relationship
  Association – Two variables may have an association. This may described as causal, if one is believed to cause the other.
- Types of correlation:
  - The correlation is positive if the y variable generally increases as the x variable increases.
  - The correlation is negative if the y variable generally decreases as the x variable increases.

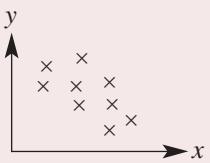
  Examples:

Strong positive correlation



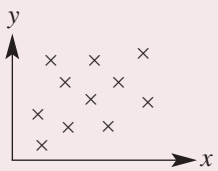
As x increases, y clearly increases.

Weak negative correlation



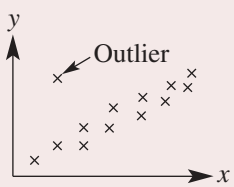
As x increases, y generally decreases.

No correlation



As x increases, there is no particular effect on y.

- An outlier can be clearly identified as a data point that is isolated from the rest of the data.



Key terminology: bivariate data, scatter plot, correlation, association, outlier

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 5I

UNDERSTANDING	1–3	3

1 Decide whether it is likely or unlikely that there will be a strong relationship between these pairs of variables.

- **a** *Height of door* and *Width of door*
- **b** *Weight of car* and *Fuel consumption*
- **c** *Temperature* and *Length of phone calls*
- **d** *Colour of flower* and *Strength of perfume*
- **e** *Amount of rain* and *Size of vegetables in the vegetable garden*

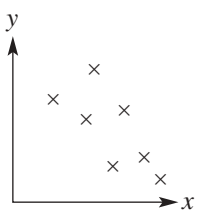
2 Complete the following using the word *increases* or *decreases*.

- **a** For a positive correlation the y variable generally _____ as the x variable increases.
- **b** For a negative correlation the y variable generally _____ as the x variable increases.

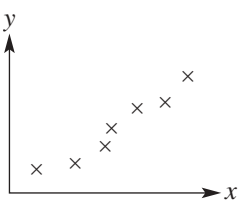
3 For these scatter plots, choose two words from those listed below to best describe the correlation between the two variables.

strong *weak* *positive* *negative*

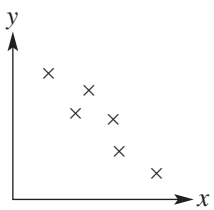
a



b



c



FLUENCY	4, 5	4, 6

Example 17 Constructing and interpreting scatter plots

Consider this simple bivariate dataset.

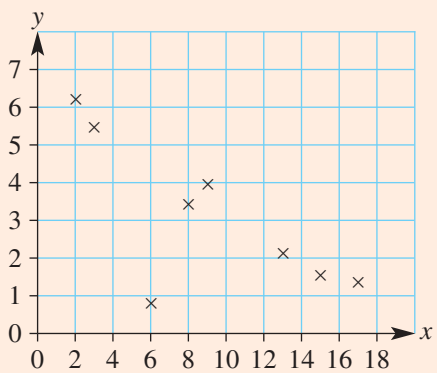
x	13	9	2	17	3	6	8	15
y	2.1	4.0	6.2	1.3	5.5	0.9	3.5	1.6

- **a** Draw a scatter plot for the data.
- **b** Describe the correlation between x and y as positive or negative.
- **c** Describe the correlation between x and y as strong or weak.
- **d** Identify any outliers.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

a



EXPLANATION

Draw an appropriate scale on each axis by looking at the data:

- x is up to 17
- y is up to 6.2

The scale must be spread evenly on each axis. Plot each point using a cross symbol on graph paper.

b Negative correlation

Looking at the scatter plot, as x increases y decreases.

c Strong correlation

The downwards trend in the data is clearly defined.

d The outlier is $(6, 0.9)$.

This point defies the trend.

Now you try

Consider this simple bivariate dataset.

x	9	6	4	3	8	2	1	10
y	11	8	4	4	3	2	3	12

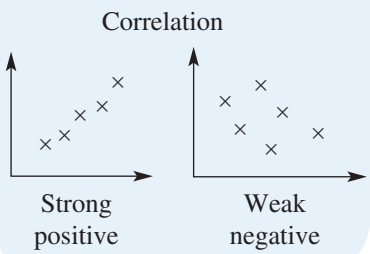
a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.

4 Consider this simple bivariate dataset.

x	1	2	3	4	5	6	7	8
y	1.0	1.1	1.3	1.3	1.4	1.6	1.8	1.0

a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.

Hint:



5 Consider this simple bivariate dataset.

x	14	8	7	10	11	15	6	9	10
y	4	2.5	2.5	1.5	1.5	0.5	3	2	2

a Draw a scatter plot for the data.
b Describe the correlation between x and y as positive or negative.
c Describe the correlation between x and y as strong or weak.
d Identify any outliers.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 By completing scatter plots for each of the following datasets, describe the correlation between x and y as *positive*, *negative* or *none*.

a

x	1.1	1.8	1.2	1.3	1.7	1.9	1.6	1.6	1.4	1.0	1.5
y	22	12	19	15	10	9	14	13	16	23	16

b

x	4	3	1	7	8	10	6	9	5	5
y	115	105	105	135	145	145	125	140	120	130

c

x	28	32	16	19	21	24	27	25	30	18
y	13	25	22	21	16	9	19	25	15	12

PROBLEM-SOLVING AND REASONING

7, 8 | 7, 9, 10

7 A tomato grower experiments with a new organic fertiliser and sets up five separate garden beds: A, B, C, D and E. The grower applies different amounts of fertiliser to each bed and records the diameter of each tomato picked.

The average diameter of a tomato from each garden bed and the corresponding amount of fertiliser are recorded below.

Bed	A	B	C	D	E
Fertiliser (grams per week)	20	25	30	35	40
Average diameter (cm)	6.8	7.4	7.6	6.2	8.5

a Draw a scatter plot for the data with 'Diameter' on the vertical axis and 'Fertiliser' on the horizontal axis. Label the points A, B, C, D and E.

b Which garden bed appears to go against the trend?

c According to the given results, would you be confident saying that the amount of fertiliser fed to tomato plants does affect the size of the tomato produced?

8 For common motor vehicles, consider the two variables *Engine size* (cylinder volume) and *Fuel economy* (number of kilometres travelled for every litre of petrol).

a Do you expect there to be some relationship between these two variables?

b As the engine size increases, would you expect the fuel economy to increase or decrease?

c The following data were collected for 10 vehicles.

Car	A	B	C	D	E	F	G	H	I	J
Engine size	1.1	1.2	1.2	1.5	1.5	1.8	2.4	3.3	4.2	5.0
Fuel economy	21	18	19	18	17	16	15	20	14	11

i Do the data generally support your answers to parts **a** and **b** above?

ii Which car gives a fuel economy reading that does not support the general trend?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 On 14 consecutive days a local council measures the volume of sound heard from a freeway at various points in a local suburb. The volume (V) of sound, in decibels, is recorded against the distance (d), in metres, between the freeway and the point in the suburb.

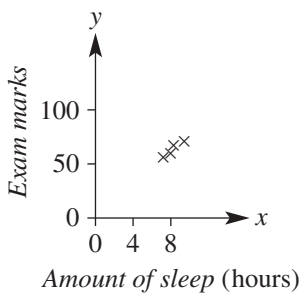
d	200	350	500	150	1000	850	200	450	750	250	300	1500	700	1250
V	4.3	3.7	2.9	4.5	2.1	2.3	4.4	3.3	2.8	4.1	3.6	1.7	3.0	2.2

a Draw a scatter plot of V against d, plotting V on the vertical axis and d on the horizontal axis.

b Describe the correlation between d and V as positive, negative or none.

c Generally, as d increases, does V increase or decrease?

10 A person presents you with this scatter plot and suggests to you that there is a strong correlation between the amount of sleep and exam marks. What do you suggest is the problem with the person's graph and conclusions?



ENRICHMENT: Crime rates and police — 11

11 A government department is interested in convincing the electorate that a large number of police on patrol leads to lower crime rates. Two separate surveys are completed over a 1-week period and the results are listed in the table below.

	Area	A	B	C	D	E	F	G
Survey 1	Number of police	15	21	8	14	19	31	17
	Incidence of crime	28	16	36	24	24	19	21
Survey 2	Number of police	12	18	9	12	14	26	21
	Incidence of crime	26	25	20	24	22	23	19

a Using scatter plots, determine whether or not there is a relationship between the number of police on patrol and the incidence of crime, using the data in:

i survey 1 **ii** survey 2

Hint: *Number of police* will be on the horizontal axis.

b Which survey results do you think the government will use to make its point? Why?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5J Line of best fit by eye

Learning intentions for this section:

- To know that a straight line can be fitted by eye to bivariate data
- To know how to fit a line of best fit by eye
- To be able to use a line of best fit to interpolate and extrapolate

Past, present and future learning:

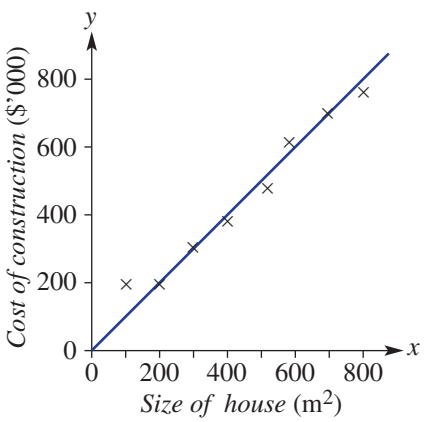
- This section concludes the Stage 5 Core topic called *Data analysis B*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

When bivariate data has a strong linear correlation, we can model the data with a straight line. This line is called a trend line or line of best fit. When we fit the line 'by eye', we try to balance the number of data points above the line with the number of points below the line. This trend line can then be used to construct other data points inside and outside the existing data points.

Lesson starter: Size versus cost

This scatter plot shows the estimated cost of building a house of a given size by a building company. A trend line has been added to the scatter plot.

- Why is it appropriate to fit a trend line to this data?
- Do you think the trend line is a good fit to the points on the scatter plot? Why?
- How can you predict the cost of a house of 1000 m^2 with this building company?

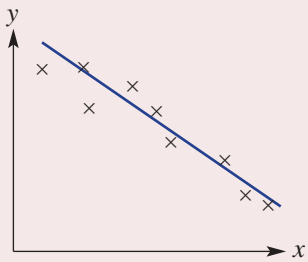


KEY IDEAS

- For bivariate data showing a clearly defined positive or negative correlation, a straight line can be fitted by eye.
- A **line of best fit** (or trend line) is positioned by eye by balancing the number of points above the line with the number of points below the line.
  - The distance of each point from the trend line also needs to be taken into account.
  - Outliers should be ignored.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- The line of best fit can be used for:
  - **interpolation:** finding unknown points within the given data range
  - **extrapolation:** finding unknown points outside the given data range, although there may be limits to this



Key terminology: bivariate data, line of best fit (trend line), interpolation, extrapolation, linear, correlation

Exercise 5J

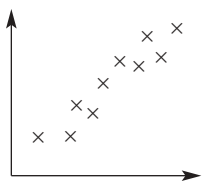
UNDERSTANDING	1–3	2(½), 3

1 **a** When is it suitable to add a line of best fit to a scatterplot?

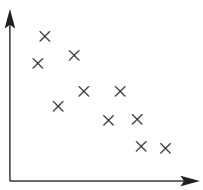
b Describe the general guideline for placing a line of best fit.

2 Copy these graphs and practise fitting a line of best fit on these scatter plots by trying to balance the number of points above the line with the number of points below the line. (Using a pencil might help.)

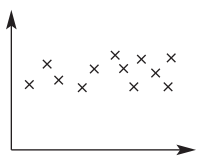
a



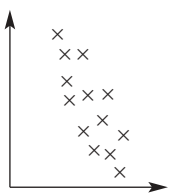
b



c



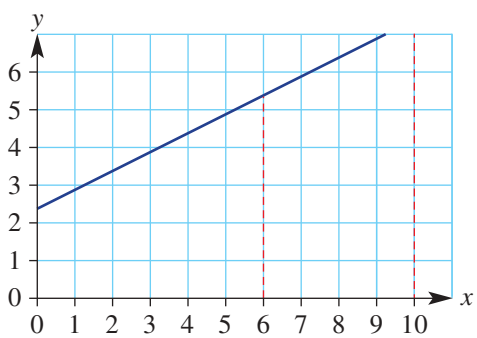
d



3 For the graph on the right, with the given line of best fit shown, choose from *interpolation* or *extrapolation* to complete the following.

a Estimating the y value when $x = 6$ is an example of _____.

b Estimating the y value when $x = 10$ is an example of _____.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 4–6 4, 6, 7

Example 18 Fitting a line of best fit

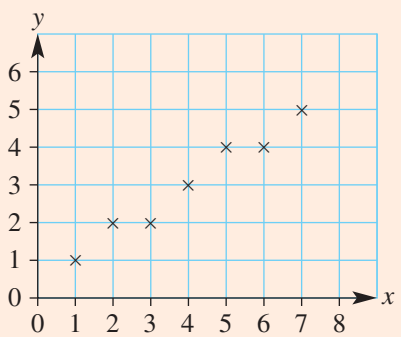
Consider the variables x and y and the corresponding bivariate data.

x	1	2	3	4	5	6	7
y	1	2	2	3	4	4	5

a Draw a scatter plot for the data.
b Is there positive, negative or no correlation between x and y?
c Fit a line of best fit by eye to the data on the scatter plot.
d Use your line of best fit to estimate:
i y when $x = 3.5$ ii y when $x = 0$
iii x when $y = 1.5$ iv x when $y = 5.5$

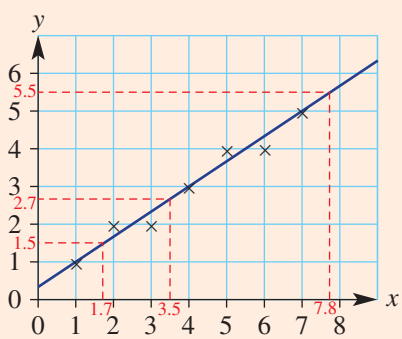
SOLUTION

a



b Positive correlation

c



d i $y \approx 2.7$
ii $y \approx 0.4$
iii $x \approx 1.7$
iv $x \approx 7.8$

EXPLANATION

Plot the points on graph paper.

As x increases, y increases.

Since a relationship exists, draw a line on the plot, keeping as many points above as below the line. (There are no outliers in this case.)

Start at $x = 3.5$. Draw a vertical line to the line of best fit, then draw a horizontal line to the y-axis and read off your solution.
Extend vertical and horizontal lines from the values given and read off your solution.
As they are approximations, we use the $\approx$ sign and not the $=$ sign.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

Consider the variables x and y and the corresponding bivariate data.

x	1	2	3	4	5	6
y	10	7	6	6	5	3

a Draw a scatter plot for the data.

b Is there positive, negative or no correlation between x and y?

c Fit a line of best fit by eye to the data on the scatter plot.

d Use your line of best fit to estimate:

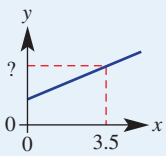
i y when $x = 4.5$ **ii** y when $x = 0$

iii x when $y = 7.5$ **iv** x when $y = 1.5$

4 Consider the variables x and y and the corresponding bivariate data.

x	1	2	3	4	5	6	7
y	2	2	3	4	4	5	5

Hint: Locate $x = 3.5$ and read off the y-value for part **d i**.



a Draw a scatter plot for the data.

b Is there positive, negative or no correlation between x and y?

c Fit a line of best fit by eye to the data on the scatter plot.

d Use your line of best fit to estimate:

i y when $x = 3.5$ **ii** y when $x = 0$ **iii** x when $y = 2$ **iv** x when $y = 5.5$

5 Consider the variables x and y and the corresponding data below.

x	1	2	4	5	7	8	10	12
y	20	16	17	16	14	13	9	10

a Draw a scatter plot for the data.

b Is there a positive, negative or no correlation between x and y?

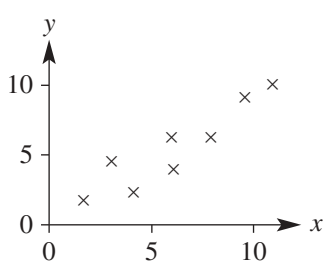
c Fit a line of best fit by eye to the data on the scatter plot.

d Use your line of best fit to estimate:

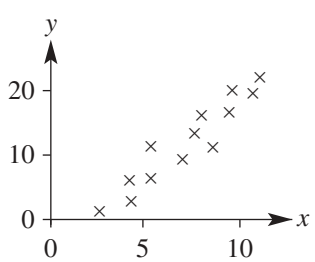
i y when $x = 7.5$ **ii** y when $x = 0$ **iii** x when $y = 12$ **iv** x when $y = 15$

6 For the following scatter plots, pencil in a line of best fit by eye, and then use your line to estimate the value of y when $x = 5$.

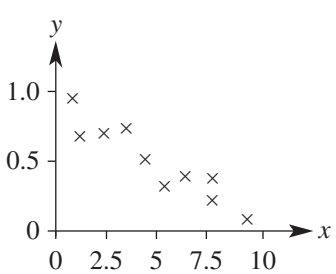
a



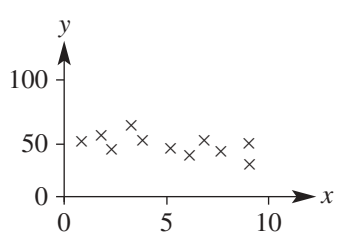
b



c

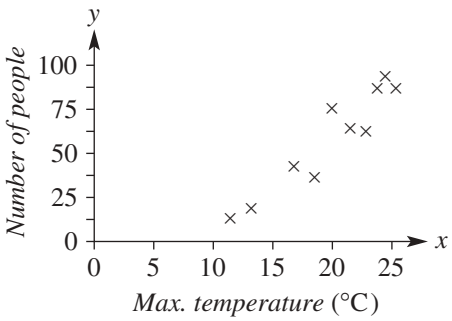


d



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 This chart shows data for the number of people entering a suburban park and the corresponding maximum temperature for 10 spring days.



- **a** Generally, as the maximum daily temperature increases, does the number of people who enter the park increase or decrease?
- **b** Draw a line of best fit by eye on the given chart.
- **c** Use your line of best fit to estimate:
  - **i** the number of people expected to enter the park if the maximum daily temperature is 20°C
  - **ii** the maximum daily temperature when the total number of people who visit the park on a particular day is 25

PROBLEM-SOLVING AND REASONING

8, 9 | 8, 10

8 A small bookshop records its profit and number of customers for the past 8 days.

Number of customers	6	12	15	9	8	5	8
Profit ($)	200	450	550	300	350	250	300

- **a** Draw a scatter plot for the data, using profit on the vertical axis.
- **b** Fit a line of best fit by eye.
- **c** Use your line of best fit to predict the profit for 17 customers.
- **d** Use your line of best fit to predict the number of customers for a $100 profit.

Hint: For 17 customers, you will need to extend your line beyond the data. This is called extrapolation.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 Over eight consecutive years, a city nursery has measured the growth of an outdoor bamboo species for that year. The annual rainfall in the area where the bamboo is growing was also recorded. The data is listed in the table.

Rainfall (mm)	450	620	560	830	680	650	720	540
Growth (cm)	25	45	25	85	50	55	50	20

a Draw a scatter plot for the data, showing growth on the vertical axis.

b Fit a line of best fit by eye.

c Use your line of best fit to estimate the growth expected for the following rainfall readings.

i 500 mm **ii** 900 mm

d Use your line of best fit to estimate the rainfall for a given year if the growth of the bamboo is:

i 30 cm **ii** 60 cm

10 At a suburban sports club, the distance record for the men's hammer throw has increased over time. The first recorded value was 72.3 m in 1967. The most recent record was 118.2 m in 1996. Further details are in this table.

Year	1967	1968	1969	1976	1978	1983	1987	1996
New record (m)	72.3	73.4	82.7	94.2	99.1	101.2	111.6	118.2

a Draw a scatter plot for the data.

b Fit a line of best fit by eye.

c Use your line of best fit to estimate the distance record for the men's hammer throw for:

i 2000 **ii** 2015

d Would you say that it is realistic to use your line of best fit to estimate distance records beyond 2015? Why?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: Heart rate and age — 11

11 Two independent scientific experiments confirmed a correlation between *Maximum heart rate* and *Age*. The data for the two experiments are in this table.

Experiment 1													
Age (years)	15	18	22	25	30	34	35	40	40	52	60	65	71
Max. heart rate (bpm)	190	200	195	195	180	185	170	165	165	150	125	128	105
Experiment 2													
Age (years)	20	20	21	26	27	32	35	41	43	49	50	58	82
Max. heart rate (bpm)	205	195	180	185	175	160	160	145	150	150	135	140	90

a Sketch separate scatter plots for experiment 1 and experiment 2, with age on the horizontal axis.

b By fitting a line of best fit by eye to your scatter plots, estimate the maximum heart rate for a person aged 55 years, using the results from:

i experiment 1 **ii** experiment 2

c Estimate the age of a person who has a maximum heart rate of 190 bpm, using the results from:

i experiment 1 **ii** experiment 2

d For a person aged 25 years, which experiment estimates a lower maximum heart rate?

e Research the average maximum heart rate of people by age and compare with the results given above.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Maths@Work

Project manager on a building site

Project managers can be responsible for coordinating a building site. They need to be organised, must be able to read and interpret plans, manage staff, coordinate the supply and costs of labour and equipment, as well as solve problems when unexpected situations arise on the worksite.

Project managers also need to evaluate their staff. Performance reviews are often used to evaluate staff performance. In some companies, bonuses and other incentives are linked to these reviews.

The performance review below is from a worksite, and used by the project manager to review their staff.

Use the performance review data in the table on the next page to answer the following questions that a project manager may face in their day-to-day job.

1 Consider trainee A.
   - a Calculate their mean score on this review.
   - b What is their median score?
   - c Write down the 5-number summary from the data given in this review.

Hint: Don't forget to order the data before finding the median.

2 Consider trainee B.
   - a Calculate their mean score on this review.
   - b What is their median score?
   - c Write down the 5-number summary from the data given in this review.

Hint: The 5-number summary includes min, Q_1, Q_2, Q_3 and max.

3 Display the data for all three trainees in parallel box plots on the same scale.

4 Compare the competency of trainees A, B and C by listing the following for *each* trainee. Then decide who is the most competent trainee, giving reasons.
   - a mean
   - b median
   - c lowest score
   - d highest score
   - e interquartile range
   - f range

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Performance review

Competency - please score 0 to 10	Trainee A	Trainee B	Trainee C
1. Please score the individual's punctuality when arriving to work.	9	8	8
2. Please score the individual's attitude when performing required tasks.	8.5	9	6.5
3. Once the individual has completed a task, do they ask for additional work?	8	8	5
4. How would you score the individual's willingness to learn?	9.5	8	6
5. How would you score the individual's application to competencies or requirements, as taught by the site team?	8	7	6
6. Please score the individual's accuracy when performing required tasks.	8.5	7.5	6.5
7. Please score the individual's skills in email, faxes and letter writing.	7	8	8.5
8. Please score the individual's skills in drafting site instructions.	7.5	7	5
9. Please score the individual's skills in drafting or issuing RFIs.	8	6.5	5
10. Please score the individual's respect earned within the site/department team for their required role.	8	7.5	5.5
11. Please score the individual's respect earned with subcontractors for their required role.	7.5	7	5.5
12. Please score the individual's respect earned with consultants for their required role.	8	7	5
13. How would you score the individual's application/focus to the required tasks you have assigned?	8.5	8	5
14. Do you consider that the individual has integrated well with the site/department team?	9	8	4.5
15. Do you find that the individual has improved their skills while under your supervision?	8	7.5	5.5
16. Would you take the individual to your next project to continue their competency training in other aspects of construction? Yes - score 10 Maybe - score 5 No - score 0	10	5	0
17. How would you score the individual's progression through their required tasks?	9	8.5	3

Using technology

5 The following data gives the competency scores (C) for trainee D.

C 1	C 2	C 3	C 4	C 5	C 6	C 7	C 8	C 9
9	7	7	7	6	7	7	6	6
C 10	C 11	C 12	C 13	C 14	C 15	C 16	C 17	
7	8	6	7	9	8	5	8	

a Use technology to find the 5-number summary for trainees B and D and draw parallel box plots.

b Compare the IQRs for trainee B and D. Which IQR shows more consistent competency? Give reasons for your answer.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

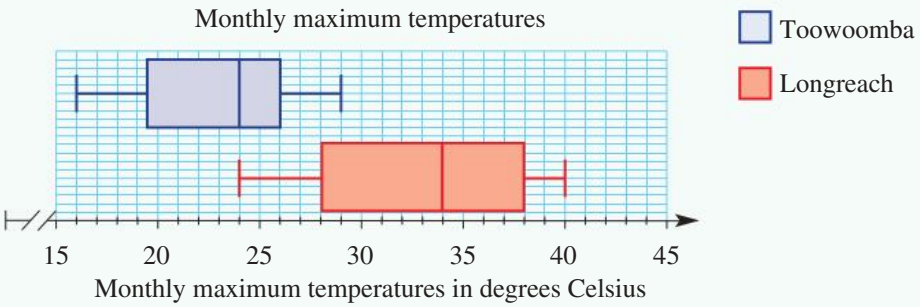
Modelling weather data

Measurements of the weather, such as temperatures and rainfall, are collected daily by the BOM (Bureau of Meteorology). Weather data and corresponding tables and graphs are used to find trends and make forecasts that are especially important for farmers.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

The box plots below represent recent monthly maximum temperatures for Toowoomba and Longreach, Queensland.



a List the 5-number summaries (min, Q_1, Q_2, Q_3, max) for Toowoomba and Longreach.

b Which temperature is common to each 5-number summary? What statistical measure is this value for Toowoomba and what statistical measure is it for Longreach?

c State the range and IQR for Toowoomba and Longreach.

d Which town do you consider has the hotter climate? Compare two statistical measures from the box plots to justify your conclusion.

Non-routine problems

For several years, including 2018, areas of Queensland and NSW suffered a devastating drought.

The problem is to prepare a report that includes statistical models and comparisons of the monthly rainfall for 2018 to the overall mean monthly rainfall, for Narromine, NSW.

Narromine monthly rainfall (mm)												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean	51	47	44	38	38	41	40	38	34	43	45	47
2018	60	12	2	5	7	20	2	23	14	42	86	24

(Wyanga station data, source: http://www.bom.gov.au/climate/data/)

Explore and connect

a Write down all the relevant styles of tables and graphs that are useful when comparing two sets of data.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Choose and apply techniques

Back-to-back stem-and-leaf plot

- **b** Construct an ordered back-to-back stem-and-leaf plot for the rainfall data given in the table on the previous page. Include a title and a key.
- **c** Describe how each distribution is bunched. Is the data symmetrical or skewed?
- **d** For each set of data, determine the 5-number summary (min, Q_1, Q_2, Q_3, max).
- **e** Calculate each IQR and determine if there is an outlier in either set of data.

Parallel box plots

- **f** Draw parallel box plots to summarise the data, marking outliers if they exist.
- **g** Consider the parallel box plots and compare the:
  - **i** median values
  - **ii** interquartile ranges (IQRs)
  - **iii** minimum values

Time-series graphs

- **h** Use Excel to develop a time-series graph for the two sets of data above.
  - **i** Copy the Narromine monthly rainfall data, including the months, into an Excel spreadsheet.
  - **ii** Select the table, Insert/2D Line with Markers. Label the axes.

Communicate thinking and reasoning

- **i** Describe the similarities and differences that you observe in the two time-series graphs.
- **j** Consider the parallel box plots and compare the 2018 Q_3 to the mean minimum value. Determine the proportion of months in 2018 that each received less rainfall than the minimum mean value.
- **k** Write a conclusion about the severity of the drought in this area.

Extension problems

Problem-solve

Prepare a report comparing the time-series graphs for the rainfall or temperature of both the mean and for a particular year in your hometown, or a town of your choice.

To select data from BOM:

- On the BOM website http://www.bom.gov.au/climate/data/ select the type of data you want to find, enter the town's name and choose a nearby weather station.
- Click 'Get Data' and find last year's monthly data and the overall monthly means.
  - **a** Complete your report in a Word document, including the table and graph.
  - **b** Describe the similarities, differences and any trends that you observe in the two time-series graphs.
  - **c** Write a conclusion.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1 The mean mass of six boys is 71 kg. The mean mass of five girls is 60 kg. Find the mean mass of all 11 children together.

2 Sean has a current four-topic average of 78% for Maths. What score does he need in the fifth topic to have an overall average of 80%?

3 I am a dataset made up of five whole number values. My mode is 2 and both my mean and median are 5. What is my biggest possible range?

4 A single dataset has 3 added to every value. Describe the change in the:

- **a** mean
- **b** median
- **c** range

5 Find the interquartile range for a set of data if 75% of the data is above 2.6 and 25% of the data is above 3.7.

6 I am a dataset with four whole number values.

- I have a range of 8.
- I have a mode of 3.
- I have a median of 6.

What are my four values?

7 A single-ordered dataset includes the following data.
$2, 4, 5, 6, 8, 10, x$
What is the largest possible value of x if it is not an outlier?

8 Describe what happens to the mean, median and mode of a dataset if each value in the set is:

- **a** increased by 10
- **b** multiplied by 10

9 At the end of 2019, the average rainfall in a town for the ten-year period just ended was 546 mm. A year later, the ten-year average was 562 mm and 654 mm had fallen in 2020.
What was the rainfall in millimetres in 2010?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter summary

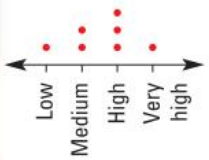
Single variable and bivariate statistics

Collecting data
A survey can be used to collect data from a population. The sample of the population chosen to be surveyed should be selected without bias and should be representative of the population.

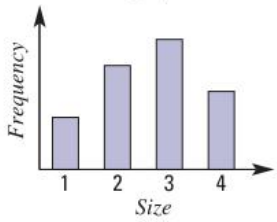
Data
Categorical
- Nominal (red, blue, ...)
- Ordinal (low, medium, ...)

Numerical
- Discrete (1, 2, 3, ...)
- Continuous (0.31, 0.481, ...)

Graphs for single set of categorical or discrete data
Dot plot



Column graphs



Stem-and-leaf plot

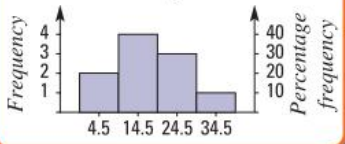
Stem	Leaf
0	1 6
1	2 7 9
2	3 8
3	4

2|3 means 23

Grouped data

Class interval	Frequency	Percentage frequency
0–9	2	20
10–19	4	40
20–29	3	30
30–39	1	10
Total	10	100

Histogram



Measures of centre
- Mean $(\bar{x}) = \frac{\text{sum of all values}}{\text{number of scores}}$
- Median (Q_2) = middle value
- Mode = most common value

Measures of spread
- Range = max – min
- Interquartile range (IQR) = $Q_3 - Q_1$
- Standard deviation (σ) for n data values

$$= \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}}$$

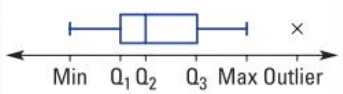
– If σ is relatively small, the data values are concentrated about the mean.
– If σ is relatively large, the data values are spread out from the mean.

Quartiles
Q_1: above 25% of the data
Q_3: above 75% of the data

2 3 5 7 8 | 9 11 12 14 15

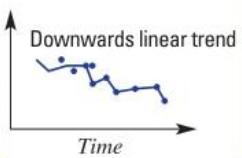
Q_1 $Q_2 = 8.5$ Q_3

Box plots

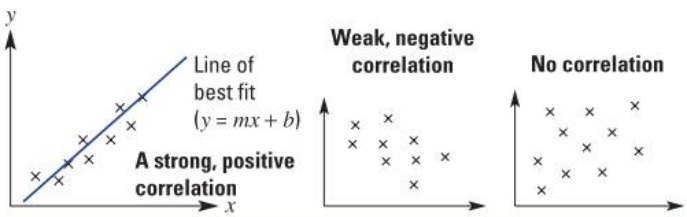


Outliers
- Single dataset
  – less than $Q_1 - 1.5 \times IQR$ or more than $Q_3 + 1.5 \times IQR$
- Bivariate
  – not in the vicinity of the rest of the data

Time-series data



Bivariate data
– Two related variables
– Scatter plot



Linear regression with technology
– Least squares line

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

✔

5A

1 I can describe the type of data generated by a survey question.
e.g. What type of data is generated by the survey questions, 'What is your favourite fruit?' and 'How many mobile phones are there in your household?'

5B

2 I can construct a frequency table and column graph.
e.g. Twenty students were surveyed on their favourite school subject. The results were:

Maths	PE	Science	Maths	Language
Maths	History	English	PE	History
Science	English	History	Language	Science
Maths	PE	Maths	History	Maths

Construct a frequency table including the tally and frequency and draw a column graph for the data.

5B

3 I can work with a histogram.
e.g. A sample of 25 people leaving a movie are asked their age. The data is listed:

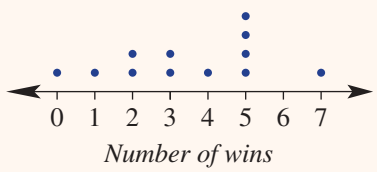
16	19	22	23	42	44	36	41	28
26	28	34	52	55	46	20	22	
34	35	48	46	50	29	26	35	

Organise the data into a frequency table, using class intervals of 10 and including a percentage frequency column. Construct a histogram showing both the frequency and percentage frequency and determine the percentage of people surveyed who were 30 years or older.

5C

4 I can interpret a dot plot.
e.g. This dot plot shows the number of wins by a tennis player in each of the grand slams they entered over 3 years.

a How many grand slams did they play in?
b What was the most common number of wins they had and the total number of wins in the 3 years?
c Describe the data in the dot plot.



5C

5 I can construct a stem-and-leaf plot or back-to-back stem-and-leaf plot.
e.g. Two sausage sizzles at two hardware stores sell the following number of sausages each week for a 10-week period.

Store 1

32	44	28	36	52
56	31	45	42	47

Store 2

27	35	32	43	52
34	29	24	38	22

Construct an ordered back-to-back stem-and-leaf plot and describe the distribution of each store's sausage sales.

5D

6 I can find the mean, mode and range.
e.g. For the dataset 12, 7, 11, 14, 3, 7, find the:
a mode **b** mean **c** range

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

Chapter checklist with success criteria

5D

7 I can find the median.
e.g. Find the median for the following datasets.
a 18, 14, 25, 28, 7, 11, 15
b 14, 19, 25, 16, 8, 1, 10, 30

☐

5D

8 I can calculate summary statistics from a graphical display.
e.g. For the data in this stem-and-leaf plot, find the range, mode, median and mean.

Stem	Leaf
1	2 5
2	1 3 4 7
3	0 0 5
4	2

2 | 3 means 23

☐

5E

9 I can find the upper and lower quartiles and the interquartile range of a dataset.
e.g. For the datasets listed below, find the upper quartile (Q_3) and the lower quartile (Q_1) and hence the IQR.
a 4, 8, 12, 18, 16, 20, 24, 19
b 10.2, 11.4, 12.6, 9.8, 10.0, 15.6, 18

☐

5E

10 I can find the 5-number summary and outliers.
e.g. The following dataset represents the price of 10 different refrigerators in a department store.

$620	$540	$642	$457	$585
$877	$1599	$918	$724	$840

For the dataset, find:
a the 5-number summary
b the IQR
c any outliers, and give a possible reason why they are outliers

☐

5F

11 I can construct a box plot with and without outliers.
e.g. For the datasets below, find the 5-number summary and identify whether any outliers exist. Draw a box plot to summarise the data, including outliers if they exist.
a 15 20 21 16 15 22 18 23 25 20
b 2.9 3.5 2.6 4.1 3.2 2.4 1.8 3.1 4.4 9.2 5.6 3.0 2.8

☐

5G

12 I can calculate standared deviation and use it to describe a dataset.
Calculate the standard deviation of this dataset, correct to one decimal place:
1 3 2 2 2 3 4 3 2 2 2 3 3 2 3

☐

5H

13 I can plot and interpret a time-series graph.
e.g. The approximate rainfall (in mm) for the 12 months of a year was recorded for Brisbane.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Rainfall (mm)	110	125	115	130	95	70	60	40	30	80	95	105

Plot the time-series graph and describe the trend in the data over the 12 months.

☐

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

5I

14 I can construct and interpret a scatter plot.

e.g. For the bivariate dataset below, draw a scatter plot of the data, identify any outliers and describe the correlation between x and y, choosing from the words: *positive*, *negative*, *weak*, *strong*.

x	0.5	1.1	0.9	1.6	1.2	1.8	0.1	2.5	2.0
y	8	7	6	6	7	9	10	1	3

☐

5J

15 I can fit a line of best fit by eye.

e.g. Consider the bivariate data in the table.

x	1	2	4	5	8	10	12	15
y	2	4	4	6	10	9	12	16

Draw a scatter plot for the data and fit a line of best fit by eye. Use the line of best fit to estimate:

a y when $x = 6$ **b** x when $y = 18$

☐

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

Short-answer questions

5B **1** A group of 16 people was surveyed to find the number of hours of media they stream in a week. The raw data is listed:
6, 5, 11, 13, 24, 8, 1, 12, 7, 6, 14, 10, 9, 16, 8, 3

a Organise the data into a table with classes 0–4, 5–9 etc. Include a tally, frequency and percentage frequency column.

b Construct a histogram for the data, showing both the frequency and percentage frequency on the graph.

c Would you describe the data as symmetrical or skewed?

5C **2** A basketball team scores the following points per match for a season.
20, 19, 24, 37, 42, 34, 38, 49, 28, 15, 38, 32, 50, 29

a Construct an ordered stem-and-leaf plot for the data.

b Describe the distribution of scores.

5D **3** For the following sets of data, determine:
i the mean **ii** the range **iii** the median

a 2, 7, 4, 8, 3, 6, 5

b 10, 55, 67, 24, 11, 16

c 1.7, 1.2, 1.4, 1.6, 2.4, 1.3

5D **4** Thirteen adults compare their ages at a party. They are:
40, 41, 37, 32, 48, 43, 32, 76, 29, 33, 26, 38, 87

a Find the mean age of the adults, to one decimal place.

b Find the median age of the adults.

c Why do you think the mean age is larger than the median age?

5E **5** Determine Q_1, Q_2 and Q_3 for these sets of data.

a 4, 5, 8, 10, 10, 11, 12, 14, 15, 17, 21

b 14, 6, 2, 23, 11, 6, 15, 14, 12, 18, 16, 10

5E **6** For the dataset: 3, 7, 2, 10, 6, 21, 5, 9, 6, 2, 8, 10.

a Find the range and IQR of the dataset

b Find any outlier in the dataset

c Remove the outlier from the dataset and find the new range and IQR. What do you notice?

5F **7** For each set of data below, complete the following tasks.
i Find the lower quartile (Q_1) and the upper quartile (Q_3).
ii Find the interquartile range ($IQR = Q_3 - Q_1$).
iii Locate any outliers.
iv Draw a box plot.

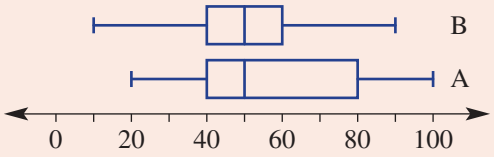
a 2, 2, 3, 3, 3, 4, 5, 6, 12

b 11, 12, 15, 15, 17, 18, 20, 21, 24, 27, 28

c 2.4, 0.7, 2.1, 2.8, 2.3, 2.6, 2.6, 1.9, 3.1, 2.2

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5F **8** Compare these parallel box plots, A and B, and answer the following as true (T) or false (F).



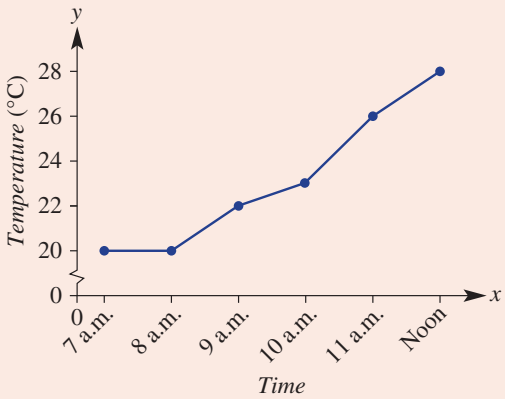
- **a** The range for A is greater than the range for B.
- **b** The median for A is equal to the median for B.
- **c** The interquartile range is smaller for B.
- **d** 75% of the data for A sits below 80.

5G **9** Calculate the mean and standard deviation for these small datasets. Round the standard deviation to one decimal place.

- **a** 4, 5, 7, 9, 10
- **b** 1, 1, 3, 5, 5, 9

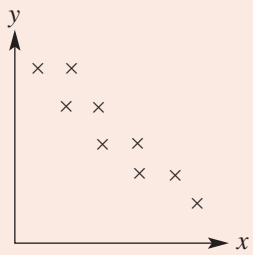
5H **10** This time series plot shows six temperature readings taken over 5 hours.

- **a** Would you describe the trend as linear or non-linear?
- **b** During which hour does the largest temperature increase occur?

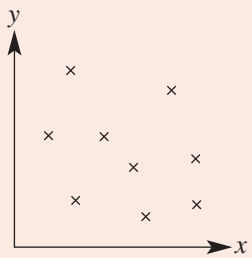


5I **11** For the scatter plots below, describe the correlation between x and y as positive, negative or none.

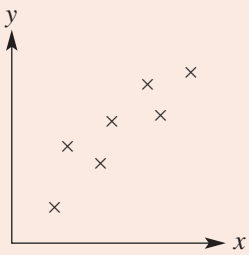
a



b



c



5J **12** Consider the simple bivariate dataset.

x	1	4	3	2	1	4	3	2	5	5
y	24	15	16	20	22	11	5	17	6	8

- **a** Draw a scatter plot for the data.
- **b** Describe the correlation between x and y as positive or negative.
- **c** Describe the correlation between x and y as strong or weak.
- **d** Identify any outliers.

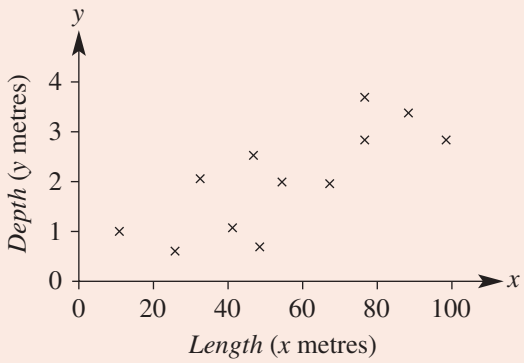
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

5J **13** The given scatter plot shows the maximum length (x metres) and depth (y metres) of 11 public pools around town.

a Draw a line of best fit by eye.

b Use your line to estimate the maximum depth of a pool that is 50 m in length.



Multiple-choice questions

5A **1** The numbers of pets in 10 households are recorded after a survey is conducted. The type of data recorded could be described as:

A categorical and discrete **B** categorical and ordinal

C categorical and nominal **D** numerical and discrete

E numerical and continuous

Questions **2** and **3** refer to the stem-and-leaf plot below, at right.

Stem	Leaf
2	4 9
3	1 1 7 8
4	2 4 6
5	0 4

4 | 2 means 42

5C **2** The minimum score in the data is:

A 4 **B** 0 **C** 24 **D** 38 **E** 54

5C **3** The mode is:

A 3 **B** 31 **C** 4 **D** 38 **E** 1

5D **4** The range and mean of 2, 4, 3, 5, 10 and 6 are:

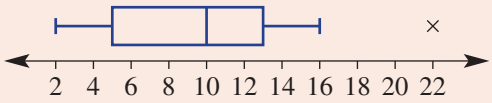
A range = 8, mean = 5 **B** range = 4, mean = 5 **C** range = 8, mean = 4

D range = 2 − 10, mean = 6 **E** range = 8, mean = 6

5D **5** The median of 29, 12, 18, 26, 15 and 22 is:

A 18 **B** 22 **C** 20 **D** 17 **E** 26

Questions **6**–**8** refer to the box plot below.



5E/5F **6** The interquartile range (IQR) is:

A 8 **B** 5 **C** 3 **D** 20 **E** 14

5F **7** The outlier is:

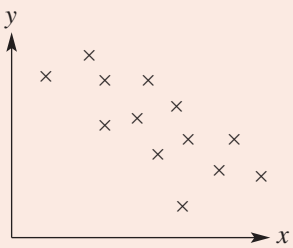
A 2 **B** 0 **C** 20 **D** 16 **E** 22

5F **8** The median is:

A 2 **B** 3 **C** 10 **D** 13 **E** 16

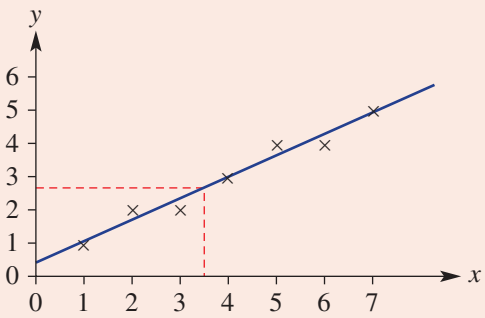
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5H **9** The variables x and y in this scatter plot could be described as having:



A no correlation
B strong positive correlation
C strong negative correlation
D weak negative correlation
E weak positive correlation

5I **10** According to this scatter plot, when x is 3.5, y is approximately:



A 4.4 **B** 2.7
C 2.5 **D** 3.5
E 5

Extended-response questions

1 The number of flying foxes taking refuge in a fig tree is recorded over a period of 14 days. The data collected is given here.

Number of flying foxes	73	50	36	82	15	24	73	57	65	86	51	32	21	39

a Find the IQR.
b Identify any outliers.
c Draw a box plot for the data.

2 A newsagency records the number of customers and profit for 14 working days.

Number of customers	18	13	15	24	29	12	18	16	15	11	4	32	26	21
Profit ($)	150	70	100	210	240	90	130	110	120	80	30	240	200	190

a Draw a scatter plot for the data and draw a line of best fit by eye. Place number of customers on the horizontal axis.
b Use your line of best fit to predict the profit for:
i 10 customers **ii** 20 customers **iii** 30 customers
c Use your line of best fit to predict the number of customers for a:
i $50 profit **ii** $105 profit **iii** $220 profit

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter 1: Financial mathematics

Short-answer questions

1 Sven earns $28.40 per hour working in a bar. What would he earn for working a 6-hour shift?

2 Tobias earns $55.20 an hour as a mechanic. Calculate his:

- a time-and-a-half rate
- b double-time rate
- c weekly wage for 38 hours at normal rate
- d weekly wage for 38 hours at normal rate plus 3 hours at time and a half

3 Fatima earns $60.80 an hour on weekdays and double time on the weekends. Calculate her weekly pay if she works 9 a.m.–3 p.m. Monday to Friday and 9 a.m. till 11:30 a.m. on Saturday.

4 Cara invests 10% of her net annual salary for 1 year into an investment account earning 4% p.a. simple interest for 5 years. Calculate the simple interest earned if her annual net salary is $77 500.

5 A $120 set of headphones is discounted by 15%. What is the sale price?

6 Marina has a taxable income of $92 600. Calculate her income tax if she falls into the following tax bracket.

$5092 plus 32.5c for each $1 over $45 000

7 Each fortnight, Raj earns $1430 gross income and pays $34.94 in superannuation, $23.40 in union fees and $493.60 in tax.

- a What is Raj's annual gross income?
- b How much tax does Raj pay each year?
- c What is Raj's net annual income?
- d What is Raj's net weekly income?

8 Find the final value of an investment of $7000 at 6% p.a., compounded annually for 4 years.

9 Jason earns $952.43 in a week, where he works 35 hours at his normal hourly rate and 4 hours at time and a half. Calculate Jason's normal hourly rate.

Multiple-choice questions

1 Nigel earns $1256 a week. His annual income is:

A $24.15 B $32 656 C $65 312 D $15 072 E $12 560

2 Who earns the most in a year?

- A Priya: $56 982 p.a.
- B Suresh: $1986 per fortnight
- C Henry: $1095 per week
- D Yasmin: $32.57 per hour, 38-hour weeks for 44 weeks
- E Bill: $20 000 p.a.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Adrian works 35 hours a week, earning $575.75. His wage for a 38-hour week is:

A $16.45 B $21 878.50 C $625.10 D $530.30 E $575.75

4 Jake earns a retainer of $420 per week plus a 2% commission on all sales. His fortnightly pay when his sales total $56 000 for the fortnight is:

A $2240 B $840 C $1540 D $1960 E $56 420

5 Antonia earns $4700 gross a month. She has annual deductions of $14 100 in tax and $1664 in health insurance. Her net monthly income is:

A $3386.33 B $11 064 C $40 636 D $72 164 E $10 000

Extended-response question

A computer tablet with a recommended retail price of $749 is offered for sale in three different ways:

Method A	Method B	Method C
5% discount for cash	3% fee for a credit card payment	20% deposit and then $18.95 per month for 3 years

a Jai buys a tablet for cash. How much does he pay?

b Thalia buys a tablet using her mother's credit card. How much more does Thalia pay for her tablet compared to Jai?

c Georgia chooses to pay for her tablet using method C.

i Calculate the deposit Georgia must pay.

ii What is the final cost of purchasing the tablet on these terms?

iii How much interest does Georgia pay on her purchase?

iv What percentage of the recommended retail price is Georgia's interest? Round your answer to two decimal places.

Chapter 2: Measurement

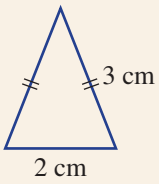
Short-answer questions

1 Convert these measurements to the units shown in the brackets.

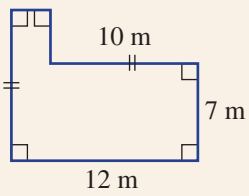
a 0.43 m (cm) b 32 000 mm^2 (cm^2) c 0.03 m^3 (cm^3)

2 Find the perimeter of each of these shapes.

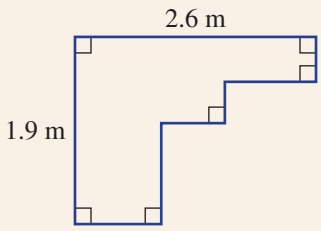
a



b



c

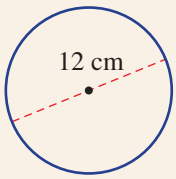


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

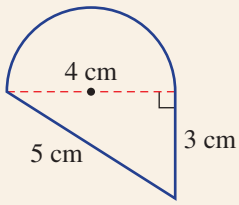
3 For these shapes, find, correct to two decimal places:

i the perimeter ii the area

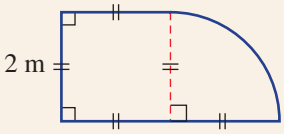
a



b

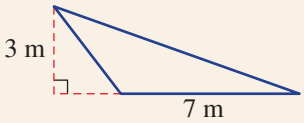


c

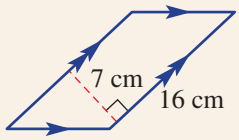


4 Find the area of each of these shapes.

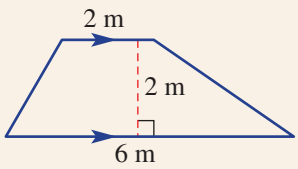
a



b



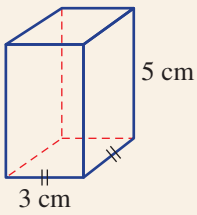
c



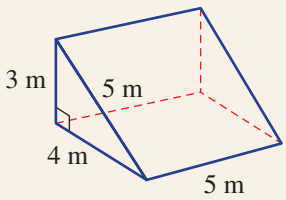
5 For these solids, find (correct to two decimal places where necessary):

i the volume ii the surface area

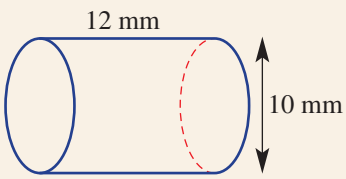
a



b



c



Multiple-choice questions

1 The number of centimetres in 2.8 metres is:

A 0.28 **B** 28 **C** 280 **D** 2.8 **E** 2800

2 A rectangle has length 7 cm and perimeter 22 cm. Its breadth is:

A 7.5 cm **B** 15 cm **C** 14 cm **D** 8 cm **E** 4 cm

3 The area of a circle with diameter 10 cm is given by:

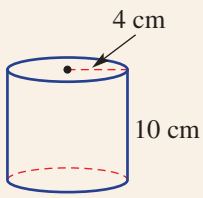
A $\pi(10)^2$ cm^2 **B** $\pi(5)^2$ cm^2 **C** 10π cm^2

D $5 \times \pi$ cm^2 **E** 25 cm^2

4 The surface area of this cylinder is closest to:

A 351.9 cm^2 **B** 301.6 cm^2 **C** 175.9 cm^2

D 276.5 cm^2 **E** 183.4 cm^2



5 The area of the triangular cross-section of a prism is 8 mm^2 and the prism's height is 3 mm. The prism's volume is:

A 48 mm^3 **B** 12 mm^3 **C** 24 mm^2

D 24 mm^3 **E** 12 mm^2

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

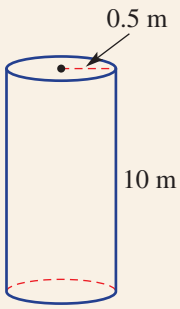
Extended-response question

A concrete cylindrical pole has radius 0.5 m and height 10 m. The outside curved surface only is to be painted. Answer the following, to two decimal places.

a What volume of concrete was used to make the pole?

b What area is to be painted?

c A litre of paint covers 6 m^2. Paint costs \$12 per litre and there are 18 poles to be painted. What is the cost of paint required? Round your answer to the nearest \$10.



Chapter 3: Algebraic expressions and indices

Short-answer questions

1 Simplify the following.

a $2xy + 7x + 5xy - 3x$ **b** $-3a \times 7b$ **c** $\dfrac{4a^2b}{8ab}$

2 **a** Expand and simplify the following.

i $-4(x - 3)$ **ii** $3x(5x + 2)$ **iii** $4(2x + 1) + 5(x - 2)$

b Factorise the following.

i $18 - 6b$ **ii** $3x^2 + 6x$ **iii** $-8xy - 12y$

3 Simplify these algebraic fractions.

a $\dfrac{6x + 18}{6}$ **b** $\dfrac{3(x - 1)}{8x} \div \dfrac{x - 1}{2x}$ **c** $\dfrac{x}{2} + \dfrac{2x}{5}$ **d** $\dfrac{x}{4} - \dfrac{3}{8}$

4 Use index laws to simplify the following. Express with positive indices.

a $2x^2 \times 5x^4$ **b** $\dfrac{12x^3y^2}{3xy^5}$ **c** $(2m^4)^3$ **d** $3x^0 + (4x)^0$

e $\left(\dfrac{3a}{b^4}\right)^2$ **f** $3a^{-5}b^2$ **g** $\dfrac{4}{t^{-5}}$ **h** $\dfrac{4x^5y^3 \times 5x^{-2}y}{10x^7y^2}$

5 **a** Write the following as basic numerals.

i 4.73×10^5 **ii** 5.21×10^{-3}

b Convert these to scientific notation, using three significant figures.

i 0.000027561 **ii** 8 707 332

Multiple-choice questions

1 The expanded and simplified form of $4(2x - 3) - 4$ is:

A $8x - 7$ **B** $6x - 11$ **C** $8x - 16$ **D** $8x - 8$ **E** $6x - 7$

2 The fully factorised form of $4x^2 + 12x$ is:

A $4(x^2 + 3x)$ **B** $4x(x + 12)$ **C** $4(x^2 + 12x)$ **D** $4x(x + 3)$ **E** $2x(x + 6)$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 $\frac{5(x-2)}{3} \times \frac{12}{x-2}$ simplifies to:

A 20 **B** -6 **C** $\frac{20}{x}$ **D** $16(x-2)$ **E** $\frac{(x-2)}{6}$

4 Using index laws, $\frac{3x^2y \times 2x^3y^2}{xy^3}$ simplifies to:

A $\frac{5x^5}{y}$ **B** $6x^5$ **C** $\frac{6x^2}{y}$ **D** $6x^4$ **E** $\frac{6x^4}{y}$

5 $(5^3)^4 \times 4^{-2}$ expressed with positive indices is:

A $\frac{5^7}{4^2}$ **B** $\frac{5^{12}}{4^2}$ **C** $\frac{1}{5^7 \times 4^2}$ **D** $\frac{5^{12}}{4^{-2}}$ **E** $5^7 \times 4^2$

Extended-response question

Consider the pattern $1, 2, 4, 8, 16, 32, \ldots$
The first term is 1, the second term is 2, the third term is 4 and so on.
Every term is a power of 2. This pattern can be written as $2^0, 2^1, 2^2, 2^3, \ldots$

a What is the sum of:
- **i** the first two terms?
- **ii** the first three terms?
- **iii** the first four terms?
- **iv** the first five terms?

b Look for a pattern in part **a** and use it to express, in index form, the sum of the first 100 numbers. Hint: Compare the answers to part **a i**, **ii**, **iii** and **iv** to the original pattern.

c Using the same pattern $1, 2, 4, 8, 16, \ldots$, what is the sum of:
- **i** the first term and the second term?
- **ii** the second term and the third term?
- **iii** the third term and the fourth term?
- **iv** the fourth term and the fifth term?

d Look for a pattern in part **c** and use it to express, in index form, the sum of the 100th term and the 101st term. Hint: Every sum has a factor of 3.

Chapter 4: Probability

Short-answer questions

1 A keen birdwatcher records the number of different species of birds in her backyard over a 20-day period.

Number of species	0	1	2	3	4	5	6
Frequency	0	2	3	8	4	2	1

From these results, estimate the probability that on the next day the birdwatcher will observe the following number of species.

a 3 **b** 2 or 3 **c** fewer than 5 **d** at least 2

2 Of 25 students, 18 are wearing jackets, 14 are wearing hats and 10 are wearing both jackets and hats.

a Represent this information in a Venn diagram.

b Represent this information in a two-way table.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c How many students are wearing neither a hat nor a jacket?

d If a student is chosen randomly from the group, find the probability that the student will be wearing:

i a hat and not a jacket **ii** a hat or a jacket **iii** a hat and a jacket

3 Two 6-sided dice are rolled and the total is recorded.

a Complete the table to find the total number of outcomes.

b Find:

i $P(5)$ **ii** $P(7)$

iii $P(\text{at least } 7)$ **iv** $P(\text{at most } 4)$

Die 2 \ Die 1	1	2	3	4	5	6
1	2	3	4			
2	3	4				
3						
4						
5						
6						

4 Two animals are chosen without replacement from a group of 2 sheep and 4 goats. Use a tree diagram to help find the probability of selecting:

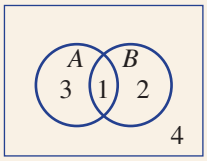
a 2 sheep **b** 1 sheep and 1 goat **c** at least 1 goat

Multiple-choice questions

1 A letter is chosen from the word PROBABILITY. What is the probability that it will not be a vowel?

A $\frac{3}{11}$ **B** $\frac{4}{11}$ **C** $\frac{7}{11}$ **D** $\frac{1}{2}$ **E** $\frac{8}{11}$

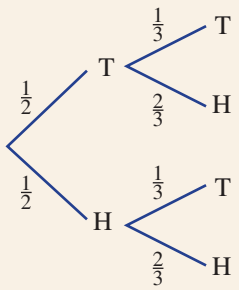
2 For this Venn diagram, $P(A \text{ or } B)$ is equal to:



A 1 **B** $\frac{1}{6}$ **C** $\frac{1}{10}$

D $\frac{3}{10}$ **E** $\frac{3}{5}$

3 For this tree diagram, what is the probability of the outcome (T, H)?



A $\frac{1}{3}$ **B** $\frac{1}{6}$ **C** $\frac{1}{2}$

D $\frac{1}{4}$ **E** $\frac{2}{3}$

4 The number of faults in a computer network over a period of 10 days is recorded in this table.

Number of faults	0	1	2	3
Frequency	1	5	3	1

An estimate for the probability that on the next day there would be at least two errors is:

A $\frac{3}{10}$ **B** $\frac{1}{5}$ **C** $\frac{4}{5}$ **D** $\frac{2}{5}$ **E** $\frac{1}{10}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Extended-response question

A hot dog stall produces two types of hot dogs: traditional (T) at $4 each and Aussie (A) at $5 each. Gareth randomly selects two hot dogs.

a Complete this table to show the possible selections.

	T	A
T	(T, T)	
A		

b Find the probability of selecting:

i 2 Aussie hot dogs

ii at least 1 Aussie hot dog

c Gareth has only $8. What is the probability that he will be able to afford two hot dogs?

Chapter 5: Single variable and bivariate statistics

Short-answer questions

1 Twenty people were surveyed to find out how many days in the past completed month they used public transport. The results are as follows:

7, 16, 22, 23, 28, 12, 18, 4, 0, 5, 8, 19, 20, 22, 14, 9, 21, 24, 11, 10

a Organise the data into a frequency table with class intervals of 5 and include a percentage frequency column.

b Construct a histogram for the data, showing both the frequency and the percentage frequency on the one graph.

c **i** State the frequency of people who used public transport on 10 or more days.

ii State the percentage of people who used public transport on fewer than 15 days.

iii State the most common interval of days for which public transport was used. Can you think of a reason for this?

2 The dataset shows the number of apps downloaded by students in a school class.

12 24 36 17 8 24 9 4 15 32 41 26 15 18 7

a Display this data using a stem-and-leaf plot.

b Describe the distribution of the data as symmetrical or skewed.

3 For the dataset 8, 10, 2, 17, 6, 30, 12, 7, 12, 15, 4:

a Order the data.

b Determine:

i the minimum and maximum values

ii the median

iii the lower quartile (Q_1) and the upper quartile (Q_3)

iv IQR ($= Q_3 - Q_1$)

v any outliers

c Draw a box plot of the data.

4 Farsan's bank balance over 12 months is recorded below.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Balance ($)	1500	2100	2300	2500	2200	1500	1200	1600	2000	2200	1700	2000

a Plot the time series for the 12 months.

b Describe the way in which the bank balance has changed over the 12 months.

c Between which consecutive months did the biggest change in the bank balance occur?

d What is the overall change in the bank balance over the year?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Multiple-choice questions

1 The values of a and b in this frequency table are:

Colour	Frequency	Percentage frequency
Blue	4	16
Red	7	b
Green	a	12
White	6	24
Black	5	20
Total	25	

A $a = 3, b = 28$
B $a = 4, b = 28$
C $a = 4, b = 19$
D $a = 6, b = 20$
E $a = 3, b = 30$

2 The mean, median and mode of the dataset 3, 11, 11, 7, 1, 9 are:
A mean = 7, median = 9, mode = 11
B mean = 6, median = 9, mode = 11
C mean = 7, median = 8, mode = 11
D mean = 7, median = 11, mode = 8
E mean = 8, median = 7, mode = 11

3 For the given stem-and-leaf plot, the range and median, respectively, of the data are:

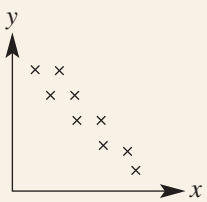
Stem	Leaf
0	2 2 6 7
1	0 1 2 3 5 8
2	3 3 5 7 9

1 | 5 means 15

A 20, 12.5 **B** 7, 12 **C** 27, 12.5
D 29.3 **E** 27, 13

4 The interquartile range (IQR) for the dataset 2, 3, 3, 7, 8, 8, 10, 13, 15 is:
A 5 **B** 8.5 **C** 7 **D** 13 **E** 8

5 The best description of the correlation between the variables for the scatter plot shown is:



A weak negative **B** strong positive
C strong negative **D** weak positive
E no correlation

Extended-response question

The heights of plants in a group of the same species after a month of watering with a set number of millimetres of water per day are recorded below.

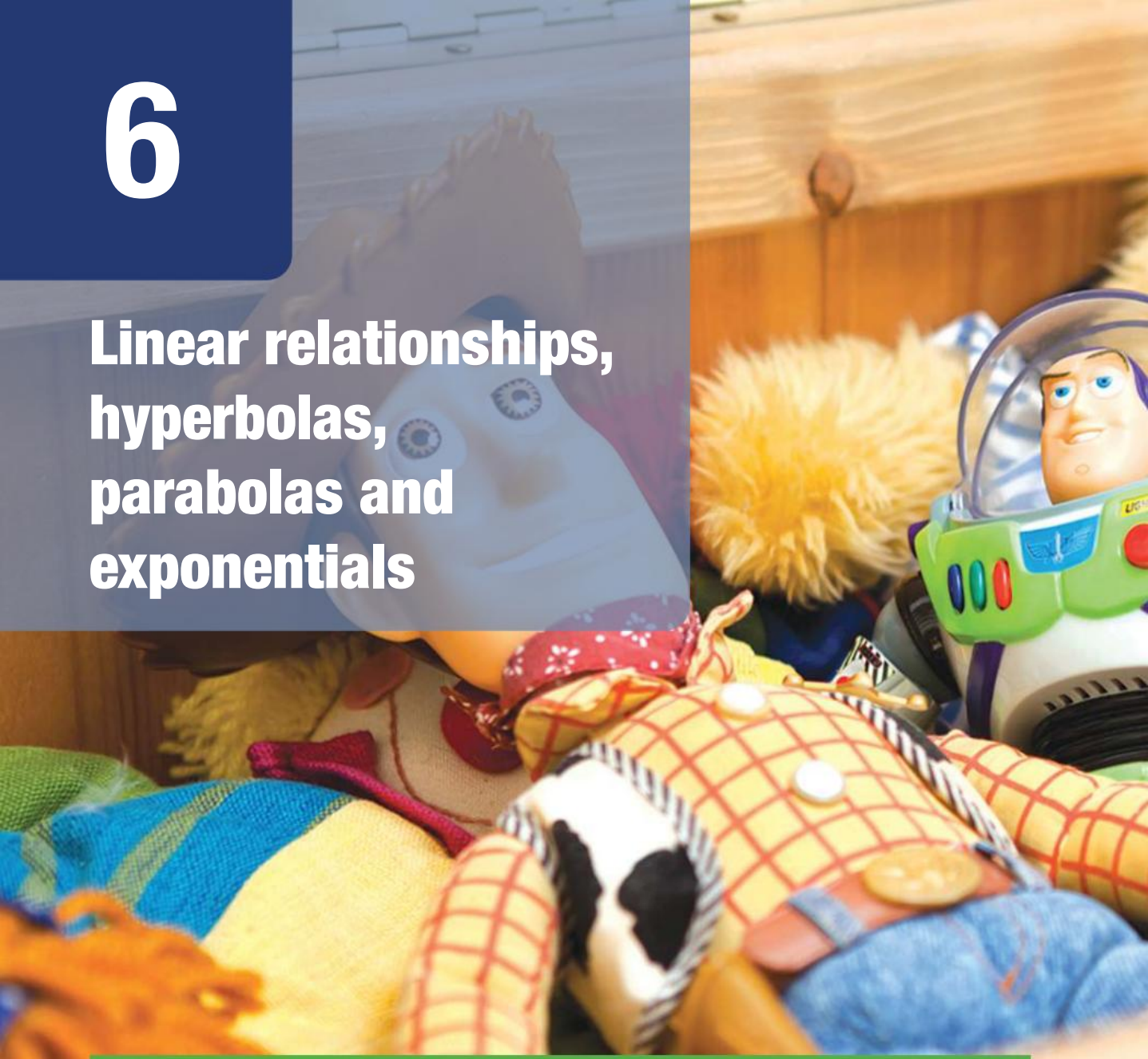
Water (mL)	8	5	10	14	12	15	18
Height (cm)	25	27	34	40	35	38	45

a Draw a scatter plot for the data, using 'Water' for the x-axis.
b Describe the correlation between water and height as positive, negative or none.
c Fit a line of best fit by eye to the data on the scatter plot.
d Use your line of best fit to estimate the:
i height of a plant watered with 16 mL of water per day
ii daily amount of water given to a plant of height 50 cm

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6

Linear relationships, hyperbolas, parabolas and exponentials



Why skills with straight-line graphs are important

Many graph applications are essential to our quality of life and are found in the professions, trades and finance industry.

- Surveyors and civil engineers apply skills using straight-line gradient and advanced algebra when designing and constructing roads, railways and tunnels.
- Engineers and plumbers design and build clean water supply and sanitation systems that are essential for avoiding infectious disease epidemics. Plumbers apply straight-line gradient skills to calculate the 'fall' of underground pipes, so wastewater flows downhill under gravity.
- Animators and game developers apply straight-line algebra in algorithms to move a virtual object by shifting, rotating, enlarging or shrinking. Segment midpoints and lengths are used to create 3D curved shapes.
- Small businesses (e.g. couriers, carpet cleaners, landscapers) claim a tax deduction for annual equipment depreciation (its decrease in value).

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

A straight-line graph joins the y-intercept (the original cost) to the x-intercept (total years of usage). The gradient gives the depreciation per year.

- Parabolic radio telescope dishes focus incoming waves to a position from which they can be collected.
- Scientists use exponential curves to make interpretations of the past and predictions about the future.

Chapter contents

6A Interpreting straight-line graphs (CONSOLIDATING)
6B Distance–time graphs (CONSOLIDATING)
6C Graphing straight lines (CONSOLIDATING)
6D Exploring gradient
6E Rates from graphs
6F $y = mx + c$ and special lines
6G Parallel lines and perpendicular lines
6H Graphing straight lines using intercepts (EXTENDING)
6I Linear modelling
6J Direct variation
6K Inverse variation
6L Exploring parabolas
6M Exploring exponential graphs
6N Exponential growth and decay

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MA0-WM-01)
- graphs and interprets linear relationships using the gradient/slope–intercept form (MA5-LIN-C-02)
- identifies and solves problems involving direct and inverse variation and their graphical representations (MA5-RAT-P-01)
- identifies connections between algebraic and graphical representations of quadratic and exponential relationships in various contexts (MA5-NLI-C-01)
- identifies and compares features of parabolas and exponential curves in various contexts (MA5-NLI-C-02).

© 2022 NSW Education Standards Authority

Online resources

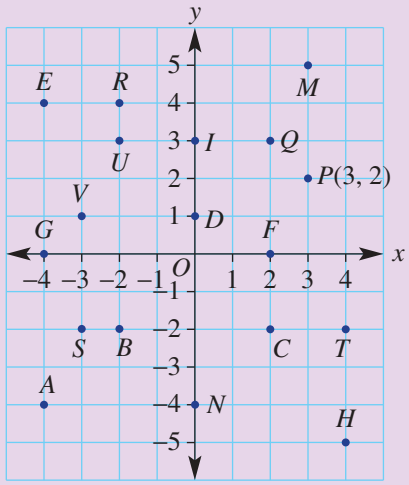
A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Warm-up quiz

1 The coordinates of P on this graph are $(3, 2)$.
Write down the coordinates of:

a M **b** T **c** A
d V **e** C **f** F



2 Name the point with coordinates:

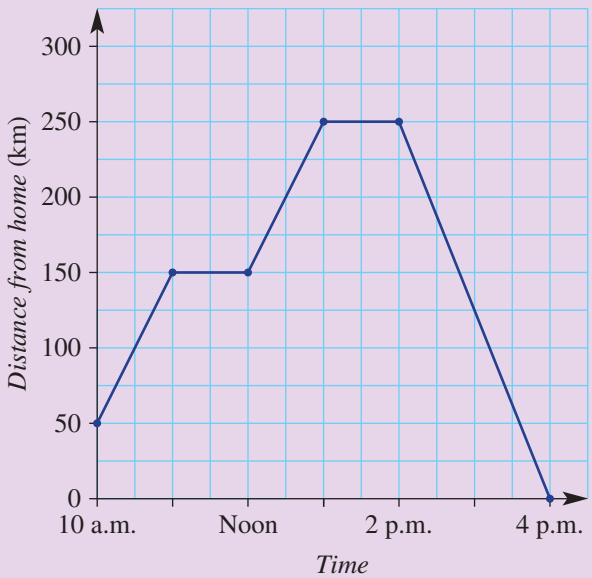
a $(-4, 0)$ **b** $(0, 1)$ **c** $(-2, -2)$
d $(-3, -2)$ **e** $(0, -4)$ **f** $(2, 3)$

3 Draw up a four-quadrant number plane and plot the following points.
What shape do they form? (Choose from: *hexagon*, *rectangle*, *square*, *pentagon*, *equilateral triangle*, *isosceles triangle*.)

a $(0, 0), (0, 5), (5, 5), (5, 0)$
b $(-3, -1), (-3, 1), (4, 0)$
c $(-2, 3), (-4, 1), (-2, -3), (2, -3), (4, 1), (2, 3)$

4 The following graph shows the Heart family's journey.

a For how many minutes did they stop?
b How far had the Heart family travelled by 1 p.m., after starting at 10 a.m.?
c What was their speed in the first hour of travel?



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Copy and complete the table of values for each equation given.

a $y = x + 3$

x	0	1	2	3
y				

b $y = x - 2$

x	0	1	2	3
y				

c $y = 2x$

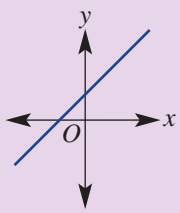
x	0	1	2
y			

d $y = 4 - x$

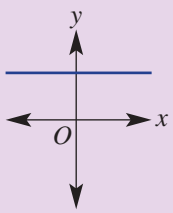
x	-2	-1	0
y			

6 Select a word (i.e. *positive*, *negative*, *zero or undefined*) to describe the gradient (slope) of the following lines.

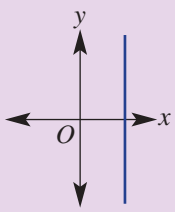
a



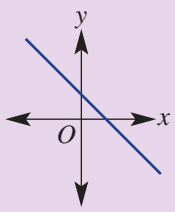
b



c



d



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6A Interpreting straight-line graphs CONSOLIDATING

Learning intentions for this section:

- To understand that two variables with a linear relationship produce a straight-line graph
- To be able to interpret information from a graph using the given variables
- To be able to read off values from a graph, extending the line if necessary

Past, present and future learning:

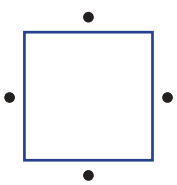
- This section addresses Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- Some of these concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Hospital patients are often given medicine or other fluids through a drip. This is so that the patient receives the liquid in small amounts at a constant rate. This is an example of a linear relationship, which can be represented using a straight-line graph.

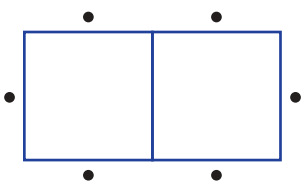
A drip like this is able to dispense fluid in very small amounts – less than 1 millilitre.

Lesson starter: Café tables

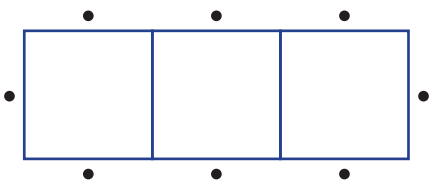
A café has many square tables. Four people can sit around one table, like this.



For a group of six people, two tables can be used like this.



Three tables can accommodate eight people.

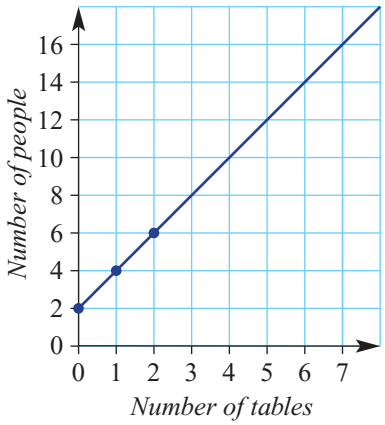


- How many people can sit around 4 tables?
- Copy and complete the table below.

Number of tables	1	2	3	4	5	6	7
Number of people	4	6					

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- Plot the values in the table onto this graph. Then use a ruler to confirm that the points form a straight line.

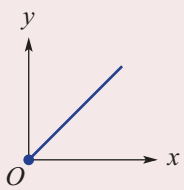


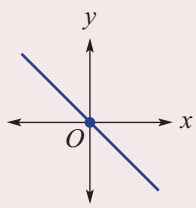
KEY IDEAS

- There are many real-life situations that involve two variables, such as the time spent cycling and the distance travelled.
- A table is often used to show some values that satisfy a relationship. In this example, the values in the top row are increasing by 1 and the numbers in the bottom row are increasing by 5. This is an example of a **linear relationship**.

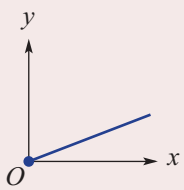
Time	0	1	2	3	4	5
Distance	0	5	10	15	20	25

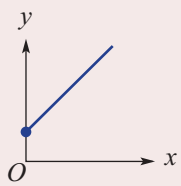
- A graph on the Cartesian plane is used to display all the values that satisfy a relationship between two variables. Linear relationships are always straight-line graphs.
  - Some straight-line graphs pass through the origin (O).



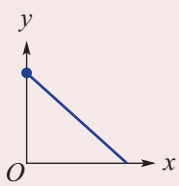


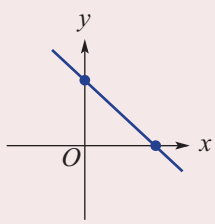
  - Some straight-line graphs indicate that both variables are increasing.





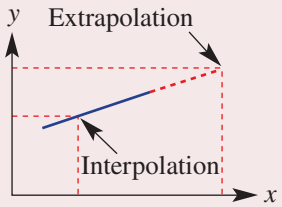
  - Some straight-line graphs indicate that one variable decreases as the other increases.





CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- Information can be found from:
  - reading within a graph (**interpolation**) or
  - reading off an extended graph (**extrapolation**)



Key terminology: variable, linear relationship, interpolation, extrapolation

Exercise 6A

UNDERSTANDING	1, 2	2

1 Will the information in this table form a straight line if it is graphed?

x	0	1	2	3	4
y	4	7	10	13	16

2 What number is missing from this linear relationship?

x	0	1	2	3	4	5
y		5	8	11	14	17

FLUENCY	3–5	3–5

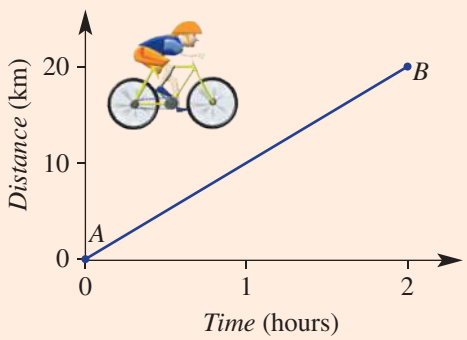
Example 1 Reading information from a graph

This graph shows the journey of a cyclist from one place (A) to another (B).

a How far did the cyclist travel?

b How long did it take the cyclist to complete the journey?

c If the cyclist were to ride from A to B and then halfway back to A, how far would the journey be?



SOLUTION

a 20 km

EXPLANATION

Draw an imaginary line from the endpoint (B) to the vertical axis; i.e. 20 km.

b 2 hours

Draw an imaginary line from the endpoint (B) to the horizontal axis; i.e. 2 hours.

c $20 + 10 = 30$ km

Cyclist would ride 20 km out and 10 km back.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

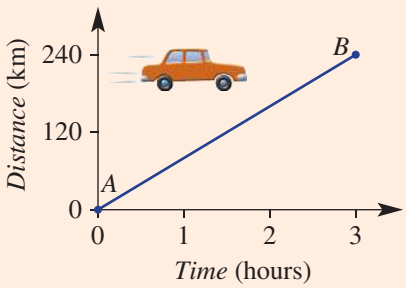
Now you try

This graph shows a car journey from one place (A) to another (B).

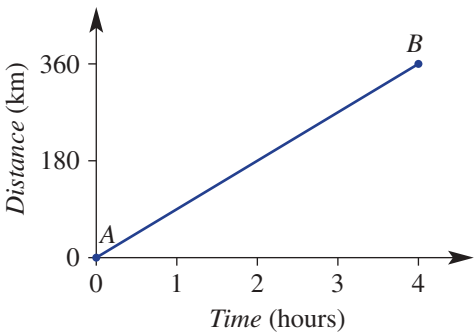
a How far did the car travel?

b How long did it take to complete the journey?

c If the car were to be driven from A to B, then halfway back to A, how far would the journey be?



3 This graph shows a motorcycle journey from one place (A) to another (B).



Hint: To find the total time taken to go from A to B, look on the time scale that is aligned with the right endpoint.

a How far did the motorcycle travel?

b How long did it take to complete the journey?

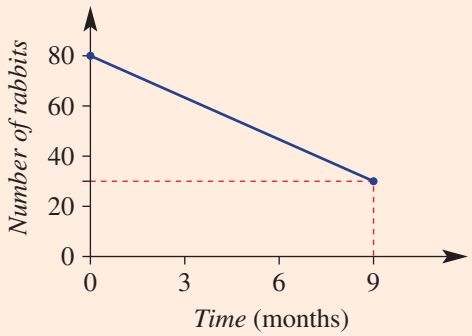
c If the motorcycle were to be driven from A to B, then halfway back to A, how far would the journey be?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 2 Interpreting information from a graph

The number of rabbits in a colony has decreased according to this graph.

a How many rabbits were there in the colony to begin with?
b How many rabbits were there after 9 months?
c How many rabbits disappeared from the colony during the 9-month period?

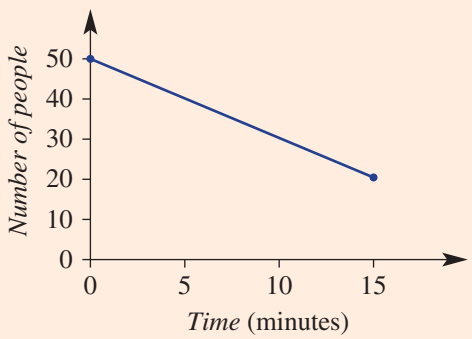


SOLUTION	EXPLANATION
a 80 rabbits	At $t = 0$ there were 80 rabbits.
b 30 rabbits	Read the number of rabbits from the graph at $t = 9$.
c $80 - 30 = 50$ rabbits	There were 80 rabbits at the start and 30 after 9 months.

Now you try

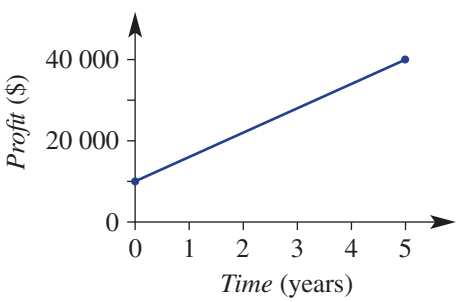
The number of people in a queue has decreased according to this graph.

a How many people were in the queue to begin with?
b How many people were in the queue after 15 minutes?
c How many people left the queue during the 15 minutes shown?



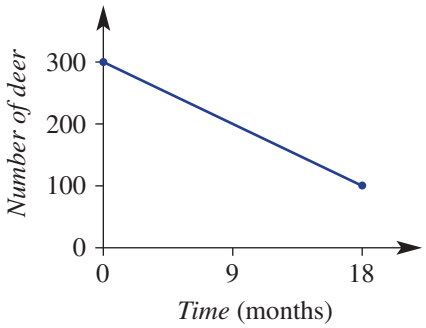
4 This graph shows a company's profit result over a 5-year period.

a What is the company's profit at:
  i the beginning of the 5-year period?
  ii the end of the 5-year period?
b Has the profit increased or decreased over the 5-year period?
c How much has the profit increased over the 5 years?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 The number of deer in a particular forest has decreased over recent months according to the graph shown.



a How many deer were there to begin with?

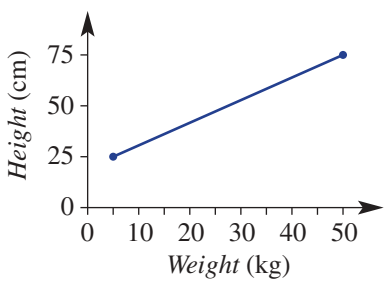
b How many deer were there after 18 months?

c How many deer disappeared from the colony during the 18-month period?

Hint: 'To begin with' means time = 0.

PROBLEM-SOLVING AND REASONING 6–9 7, 9, 10

6 A height versus weight graph for a golden retriever dog breed is shown.



a From the smallest to the largest dog, use the graph to find the total increase in:

i height **ii** weight

b Fill in the missing numbers.

i The largest weight is ___ times the smallest weight.

ii The largest height is ___ times the smallest height.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

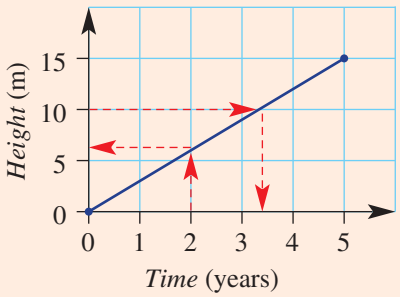
Example 3 Reading within a graph (interpolation)

This graph shows the growth of a tree over 5 years.

a How many metres has the tree grown over the 5 years?

b Use the graph to find how tall the tree is after 2 years.

c Use the graph to find how long it took for the tree to grow to 10 metres.



SOLUTION	EXPLANATION
a 15 metres	The endpoint of the graph is at 15 metres.
b 6 metres	Draw a dotted line at 2 years and read the height.
c 3.3 years	Draw a dotted line at 10 metres and read the time.

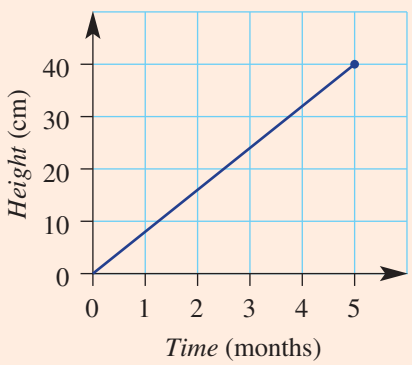
Now you try

This graph shows the growth of a seedling over 5 months.

a How many centimetres has the seed grown over the 5 months?

b Use the graph to find the height of the seedling after 2 months.

c Use the graph to find how long it took for the seedling to grow to 30 cm.

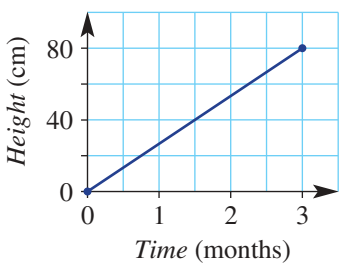


7 This graph shows the height of a tomato plant over 3 months.

a How many centimetres has the plant grown over 3 months?

b Use the graph to find how tall the tomato plant is after $1\frac{1}{2}$ months.

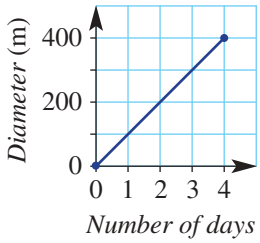
c Use the graph to find how long it took for the plant to grow to 60 centimetres.



Hint: Start at 60 cm on the height axis, then go across to the straight line and down to the time axis. Read off the time.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 The diameter of an oil slick increased every day after an oil tanker hit some rocks. Use the graph to answer the following.

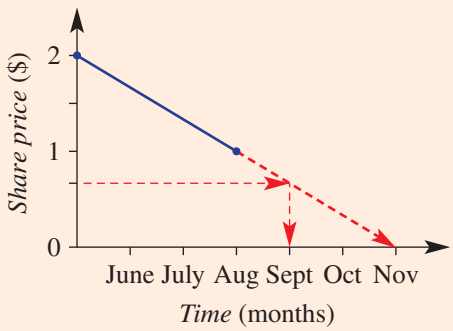


- **a** How wide is the oil slick after 4 days?
- **b** How wide is the oil slick after 2.5 days?
- **c** How many days does it take for the oil slick to reach a diameter of 350 m?

Example 4 Reading off an extended graph (extrapolation)

Due to poor performance, the value of a company's share price is falling.

- **a** By the end of August, how much has the share price fallen?
- **b** At the end of November what would you estimate the share price to be?
- **c** Near the end of which month would you estimate the share price to be 70 cents?

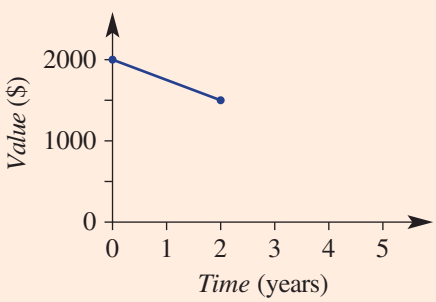


SOLUTION	EXPLANATION
a Price has dropped by $1.	By August the price has changed from $2 to $1.
b $0	Use a ruler to extend your graph (shown by the dotted line) and read the share price for November.
c September	Move across from 70 cents to the extended line and read the month.

Now you try

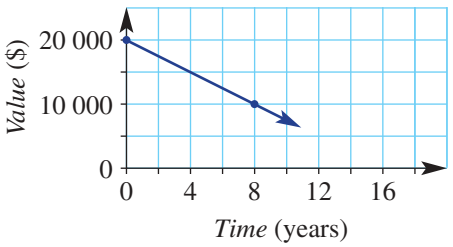
The value of a computer is decreasing over time.

- **a** By the end of 2 years, how much has the value fallen?
- **b** At the end of 4 years, what would the estimated value be?
- **c** Near the end of which year would you estimate the value to be $1200?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 The value of a car decreases with time.



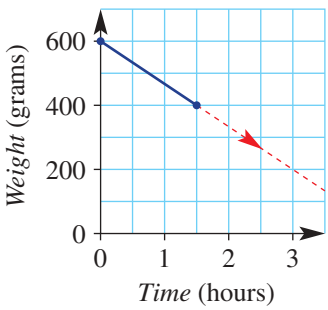
Hint: Use your ruler on the line to 'extend' it.

a By the end of 8 years, how much has the car's value fallen?

b At the end of 16 years, what would you estimate the car's value to be?

c Near the end of which year would you estimate the car's value to be $5000?

10 The weight of a wet sponge is reduced after it is left in the sun to dry.



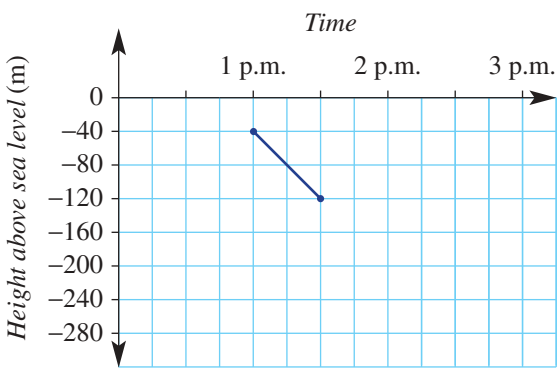
a The weight of the sponge has been reduced by how many grams over the first 1.5 hours?

b What would you estimate the weight of the sponge to be after 3 hours?

c How many hours would it take for the sponge to weigh 300 g?

ENRICHMENT: Submarine depth — 11

11 A submarine goes to depths below sea level, as shown in this graph.



a How long does it take for the submarine to drop from 40 m to 120 m below sea level?

b At what time of day is the submarine at:

i −40 m? **ii** −80 m?

iii −60 m? **iv** −120 m?

c What is the submarine's depth at:

i 1:30 p.m.?

ii 1:15 p.m.?

d Extend the graph to find the submarine's depth at:

i 12:45 p.m. **ii** 1:45 p.m. **iii** 2:30 p.m.

e Use your extended graph to estimate the time when the submarine is at:

i 0 m **ii** −200 m **iii** −320 m

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6B Distance–time graphs CONSOLIDATING

Learning intentions for this section:

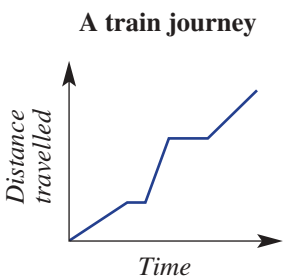
- To understand that a distance–time graph can be made up of line segments joined together
- To be able to interpret the features of a distance–time graph
- To be able to sketch a distance–time graph with movement at a constant rate

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

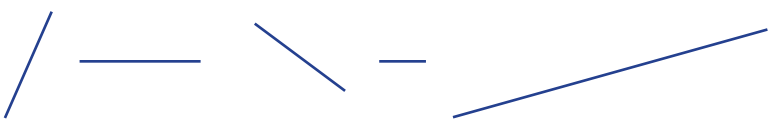
Some of the graphs considered in the previous section were distance–time graphs, which show the distance on the vertical axis and the time on the horizontal axis. Many important features of a journey can be displayed on such graphs. Each section of a journey that is at a constant rate of movement can be graphed with a straight-line segment. Several different line segments can make up a total journey.

For example, a train journey could be graphed with a series of sloping line segments that show travel between stations and flat line segments that show when the train is stopped at a station.



Lesson starter: An imaginary journey

Here are five line segments.

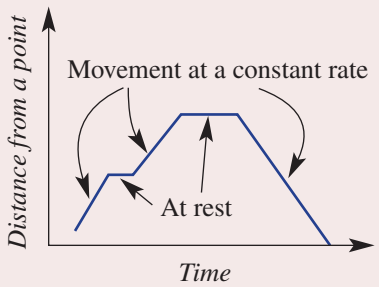


- Use five similar line segments, arranged in any order you choose, and draw a distance–time graph. Each segment must be joined to the one next to it.
- Write a summary of the journey shown by your distance–time graph.
- Swap graphs with a classmate and explain the journey that you think your classmate's graph is showing.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- Graphs of distance versus time usually consist of **line segments**.
- Each segment shows whether the object is moving or at rest (**stationary**).
- To draw a graph of a journey, use time on the horizontal axis and distance on the vertical axis.

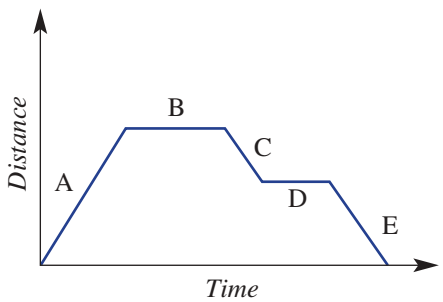


Key terminology: line segment, stationary, horizontal axis, vertical axis

Exercise 6B

UNDERSTANDING 1, 2 2

1 For the distance versus time graph shown, for each line segment A – E, describe the movement of the object as either moving at a constant rate or at rest.

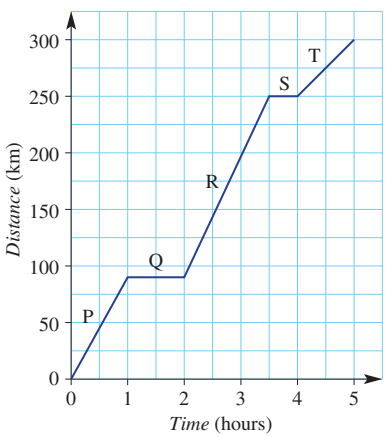


2 The Martino family makes a 300-km car journey, which takes 5 hours. The distance–time graph of this journey is shown. For each description below, choose the line segment of the graph that matches it.

- **a** A half-hour rest break is taken after travelling 250 km.
- **b** In the first hour, the car travels 90 km.
- **c** The car is at rest for 1 hour, 90 km from the start.
- **d** The car takes 1.5 hours to travel from 90 km to 250 km.
- **e** The distance from 250 km to 350 km takes 1 hour.
- **f** The distance travelled stays constant at 250 km for half an hour.

Hint: A flat line segment shows that the car has stopped.

Distance–time graph of car journey



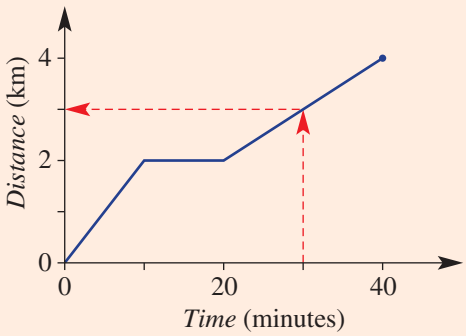
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 3–5 | 3, 5, 6

Example 5 Interpreting a distance–time graph

This distance–time graph shows a car's journey from home, to school and then to the local shopping centre.

a What is the total distance travelled?
b How long is the car resting while at the school?
c What is the total distance travelled after 30 minutes?

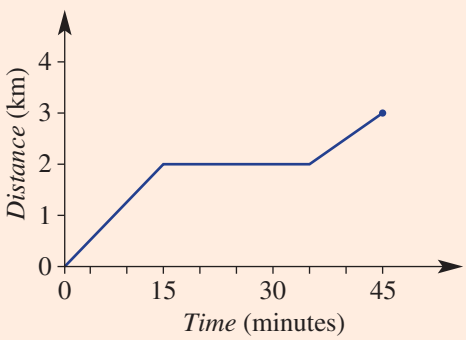


SOLUTION	EXPLANATION
a 4 km	Read the distance from the endpoint of the graph.
b 10 minutes	The rest starts at 10 minutes and finishes at 20 minutes.
c 3 km	Draw a line from 30 minutes and read off the distance.

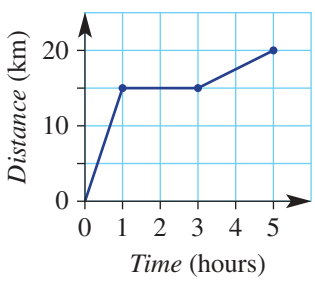
Now you try

The distance–time graph shows a person's walk from their house to the cafe for a rest and then to the post office.

a What is the total distance travelled?
b How long is the person resting at the cafe?
c What is the total distance travelled after 40 minutes?



3 A bicycle journey is shown on this distance–time graph.



a What is the total distance travelled?
b How long is the cyclist at rest?
c How far has the cyclist travelled after 4 hours?

Hint: From the end of the line, go across to the distance scale. This will show the total distance travelled.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 The weight of a water container increases while water is poured into it from a tap.

a What is the total weight of the container after:

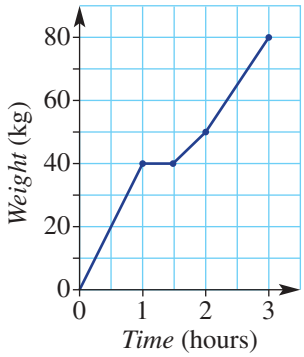
i 1 hour?

ii 2 hours?

iii 3 hours?

b During the 3 hours, how long is the container not actually being filled with water?

c During which hour is the container filling the fastest?



Hint: A flat line segment shows that the weight is not changing, so no water is being poured in at that time.

5 The graph below shows a shopper's short walk in a shopping mall.

a What is the total distance the shopper travelled?

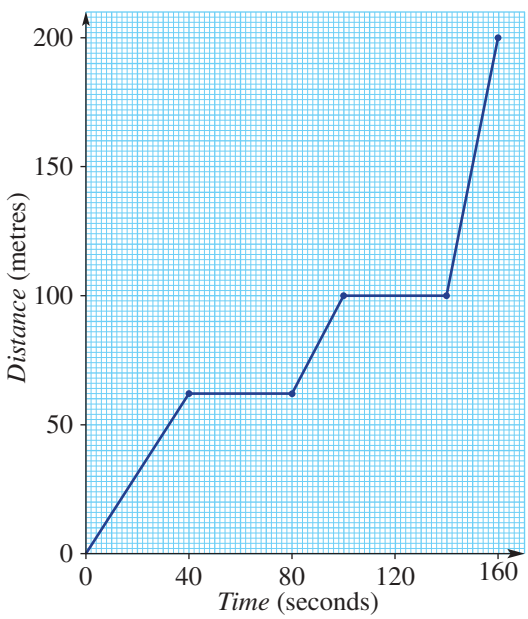
b How long was the shopper not walking?

c What was the total distance the shopper travelled by the following times?

i 20 seconds

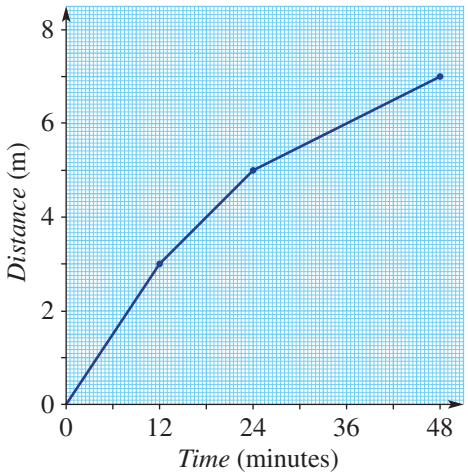
ii 80 seconds

iii 2.5 minutes



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 A snail makes its way across a footpath, then a garden bed and then a lawn according to this graph.



a How far does the snail travel on:
- **i** the footpath?
- **ii** the garden bed?
- **iii** the lawn?

b On which surface does the snail spend the most time?

c Use your graph to find how far the snail travelled after:
- **i** 6 minutes
- **ii** 18 minutes
- **iii** 42 minutes

Hint: The line segment that has the largest horizontal change is the surface that the snail spent most time on.

PROBLEM-SOLVING AND REASONING 7–9 9–11

7 This graph shows the distance of a train from the city station over a period of time.

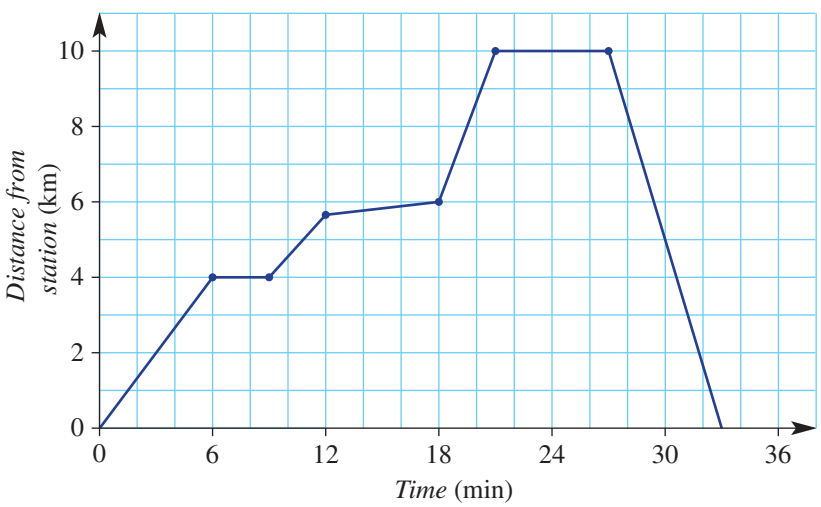
a What is the farthest distance the train travelled from the station?

b What is the total distance travelled?

c After how many minutes does the train begin to return to the station?

d What is the total number of minutes the train was at rest?

Hint: Remember to include the return trip in the total distance travelled.

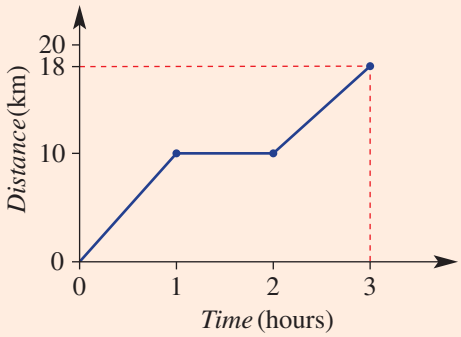


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 6 Sketching a distance–time graph

Sketch a distance–time graph displaying all of the following information.
- total distance covered is 18 km in 3 hours
- 10 km covered in the first hour
- a 1-hour long rest after the first hour

SOLUTION



EXPLANATION

Draw axes with time on the horizontal (up to 3 hours) and distance on the vertical (up to 18 km).

Start at time zero.

Draw the first hour with 10 km covered.

Draw the rest stop, which lasts for 1 hour.

Draw the remainder of the journey, so that 18 km is completed after 3 hours.

Now you try

Sketch a distance–time graph displaying all of the following information.
- total distance covered is 20 km in 2 hours
- 14 km covered in the first hour
- a half-hour rest stop after the first hour

8 Sketch a distance–time graph displaying all of the following information.
- total distance covered is 100 km in 2 hours
- 50 km covered in the first hour
- a half-hour rest stop after the first hour

Hint: Draw axes with time on the horizontal (up to 2 hours) and distance on the vertical (up to 100 km).

9 Sketch a graph to illustrate a journey described by the following.
- total distance covered is 15 metres in 40 seconds
- 10 metres covered in the first 10 seconds
- a 25-second rest after the first 10 seconds

Hint: Always use a ruler to draw line segments.

10 A bus travels 5 km in 6 minutes, stops for 2 minutes, then travels 10 km in 8 minutes, stops for another 2 minutes and then completes the journey by travelling 5 km in 4 minutes.

a What is the total distance travelled?

b What is the total time taken?

c Sketch a distance–time graph for the journey.

Hint: Find the total time taken to determine the scale for the horizontal axis. Find the totaldistance travelled to determine the scale for the vertical axis.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 A 1-day, 20 km bush hike included the following features.

- a 3-hour hike to waterfalls (10 km distance)
- a half-hour rest at the falls
- a 2-hour hike to the mountain peak (5 km distance)
- a $1\frac{1}{2}$-hour hike to the camp site

Sketch a distance–time graph for the journey.

ENRICHMENT: Pigeon flight — 12

12 The distance travelled by a pigeon is described by these points.

- a half-hour flight, covering a distance of 18 km
- a 15-minute rest
- a 15-minute flight, covering 12 km
- a half-hour rest
- turning and flying 10 km back towards 'home' over the next half hour
- a rest for a quarter of an hour
- reaching 'home' after another 45-minute flight

a Sketch a graph illustrating the points above using distance on the vertical axis.

b What is the fastest speed (in km/h) that the pigeon flew? $\left(\text{Speed} = \frac{\text{distance}}{\text{time}}\right)$

c Determine the pigeon's average speed, in km/h. $\left(\text{Average speed} = \frac{\text{total distance}}{\text{total flying time}}\right)$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6C Graphing straight lines CONSOLIDATING

Learning intentions for this section:

- To review the Cartesian plane and how to position and describe points using coordinates
- To understand that a straight-line graph is formed from a linear relationship
- To be able to plot a graph of a linear relationship using a table of values
- To be able to use a straight-line graph to determine the values of variables

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- It consolidates and extends the concepts in Section 5A of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Some equations have only one solution. The equation $10 = 2x + 4$ has only one value of x that makes it true. That value is $x = 3$.

On the other hand, the equation $y = 2x + 4$ has many solutions, such as $x = 5$, $y = 14$ and $x = 0$, $y = 4$. When these solutions are graphed on the Cartesian plane they form a straight line. The line shows every possible solution.

These equations have one solution	These equations have many solutions
$10 = 5x$	$y = 5x$
$9 = 2x - 4$	$y = 2x - 4$
$1 = x - 5$	$y = x - 5$
$y = 1$	$y = x$
$x - 5 = 2$	$x - y = 2$

Lesson starter: One equation, two variables, multiple solutions

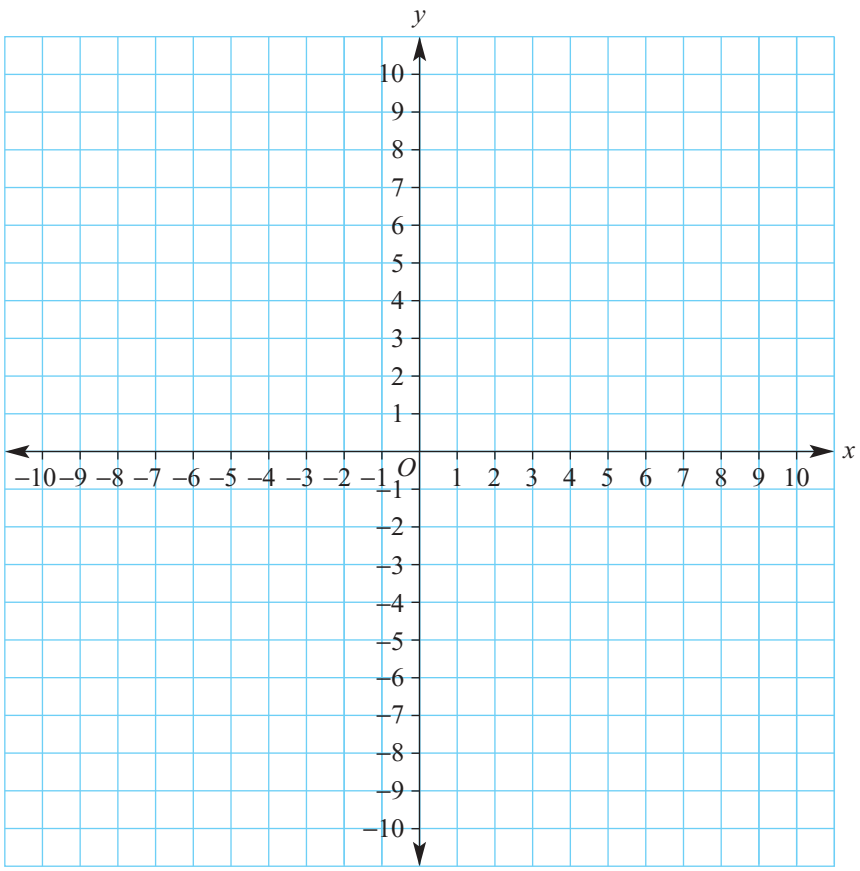
Consider the statement, 'The sum of two numbers is 5.'

If the two numbers are called x and y, this statement could be written as $x + y = 5$.

- Write down ten pairs of numbers that have a sum of 5, like this:
  $4 + 1 = 5$, so $x = 4$ and $y = 1$, which is the point (4, 1).

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

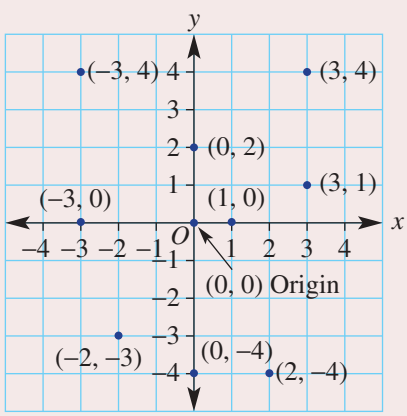
- Plot the ten points on this Cartesian plane, then use a ruler to join them with a single straight line.



- Look at the ten points another student has chosen. Did they choose the same points as you? Did they draw the same line as you?

KEY IDEAS

- A **number plane** or **Cartesian plane** includes a vertical ***y*-axis** and a horizontal ***x*-axis** intersecting at right angles.
- A point on a number plane has coordinates (x, y).
  - The ***x*-coordinate** is listed first, followed by the ***y*-coordinate**.
- The point $(0, 0)$ is called the **origin** (O).
- $(x, y) = \begin{pmatrix} \text{horizontal units from origin} \\ , \\ \text{vertical units from origin} \end{pmatrix}$

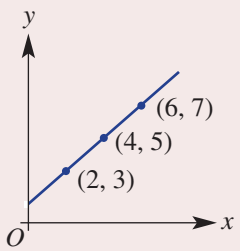


- A rule is an equation connecting two or more variables.
- A straight-line graph will result from an equation that is **linear**.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- The equation for a linear relationship is usually written with y as the subject. For example, $y = 2x - 3$ or $y = -x + 7$.
- To graph a linear relationship using a rule:
  - Construct a table of values finding a y-coordinate for each given x-coordinate. Substitute each x-coordinate into the rule.
  - Plot the points given in the table on a set of axes.
  - Draw a line through the points to complete the graph.

x	2	4	6
y	3	5	7



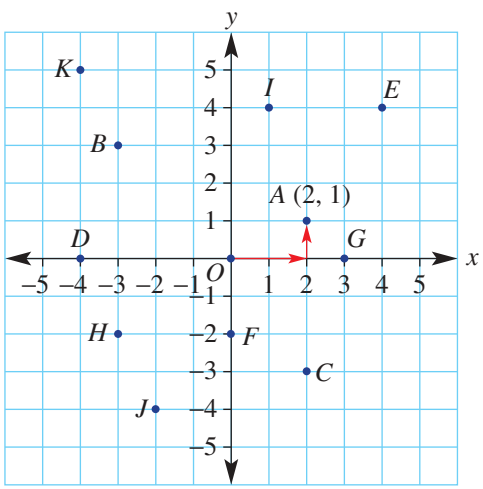
- The **point of intersection** of two lines is the point where the lines cross over each other.

Key terminology: Cartesian or number plane, x-coordinate, y-coordinate, point of intersection, origin, linear

Exercise 6C

UNDERSTANDING 1–3 3

1 **a** List the coordinates of each point (A–K) plotted on the number plane.
b Which points are on the x-axis?
c Which points are on the y-axis?
d What are the coordinates of the point called the 'origin'?



Hint:
$(x, y) = \left(\begin{matrix} \text{right} \\ \text{or} \\ \text{left} \end{matrix} , \begin{matrix} \text{up} \\ \text{or} \\ \text{down} \end{matrix} \right)$
The 'origin' is the point where the x-axis and y-axis meet.

2 Write the coordinates for each point listed in this table.

x	−2	−1	0	1	2
y	1	−1	−3	−5	−7

Hint: Coordinates are written as (x, y).

x	−2
y	1

$\}$ $(-2, 1)$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Ethan is finding the coordinates of some points that are on the line $y = -2x + 4$. Copy and complete these calculations, stating the coordinates for each point.

a $x = -3, y = -2 \times (-3) + 4 = 6 + 4 =$ $(-3, \quad)$

b $x = -1, y = -2 \times (-1) + 4 =$ $= (\quad, \quad)$

c $x = 0, y = -2 \times 0 + 4 =$ $= (\quad, \quad)$

d $x = 2, y = -2 \times 2 + 4 =$ $= (\quad, \quad)$

Hint: In the order of operations, first do any multiplication, then do addition or subtraction from left to right. When multiplying, same signs make a positive and different signs make a negative.

FLUENCY 4, 5(½) 4–5(½), 6

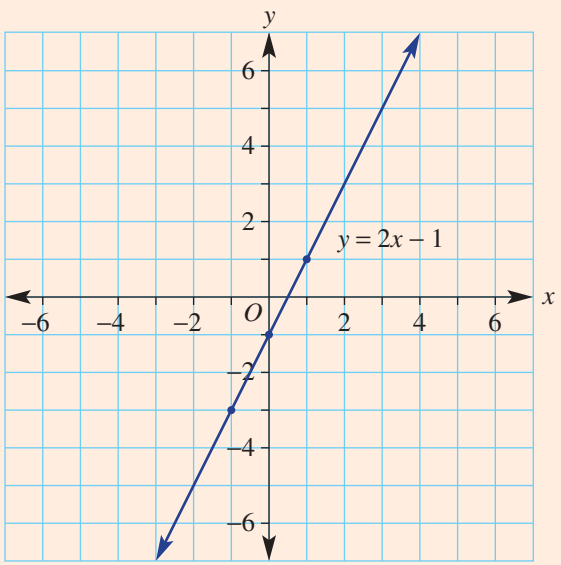
Example 7 Plotting a graph from a rule

Plot the graph of $y = 2x - 1$ by first completing the table of values.

x	−1	0	1
y			

SOLUTION

x	−1	0	1
y	−3	−1	1



EXPLANATION

Substitute each value into the equation:

$x = -1, y = 2 \times (-1) - 1 = -3 \quad (-1, -3)$

$x = 0, y = 2 \times 0 - 1 = -1 \quad (0, -1)$

$x = 1, y = 2 \times 1 - 1 = 1 \quad (1, 1)$

Plot the points and draw the line with a ruler.

When labelling axes, put the numbers on the grid lines, not in the spaces.

Now you try

Plot the graph of $y = 3x + 2$ by first completing the table of values.

x	−1	0	1
y			

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Complete the following tables, then plot the graph of each one on a separate number plane.

a $y = 2x$

x	-1	0	1
y			

b $y = x + 4$

x	0	1	2
y			

Hint: When multiplying, same signs make a positive; e.g. $-2 \times (-1) = 2$

c $y = 2x - 3$

x	0	1	2
y			

d $y = -2x$

x	-1	0	1
y			

e $y = x - 4$

x	1	2	3
y			

f $y = 6 - x$

x	0	1	2
y			

5 Complete the following tables, then plot the graph of each pair on the same axes.

a **i** $y = x + 2$

x	0	2	4
y			

ii $y = -x + 2$

x	0	2	4
y			

Hint: For each part, draw line **i** and line **ii** on the same axes.

b **i** $y = x - 4$

x	0	1	2
y			

ii $y = 4 - x$

x	0	1	2
y			

c **i** $y = 2 + 3x$

x	-3	0	3
y			

ii $y = 3x - 4$

x	-3	0	3
y			

6 By plotting the graphs of each of the following pairs of lines on the same axes, find the coordinates of the point of intersection. Use a table of values, with x from -2 to 2.

a $y = 2x$ and $y = x$

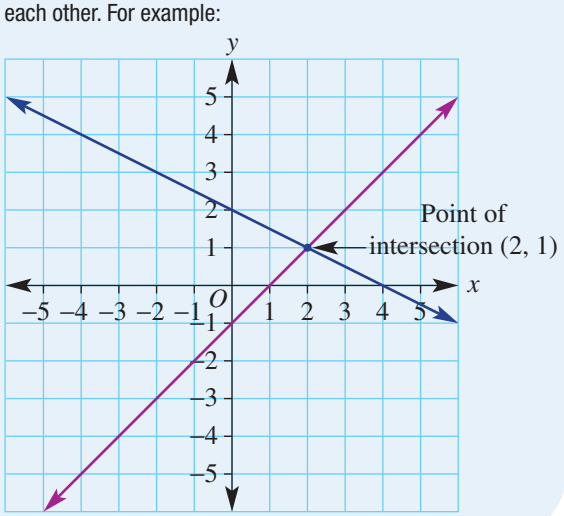
b $y = x + 3$ and $y = 2x + 2$

c $y = 2 - x$ and $y = 2x + 5$

d $y = 2 - x$ and $y = x + 2$

e $y = 2x - 3$ and $y = x - 4$

Hint: The point of intersection of two lines is where they cross each other. For example:



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING	7, 9	8, 10

Example 8 Interpreting a graph when given a table of values

Jasmine is organising a school dance. The venue is chosen and the costs are shown in the table.

Number of students (S)	0	100	200	300
Total cost in dollars (C)	500	1000	1500	2000

a Plot a graph of the total cost against the number of students.

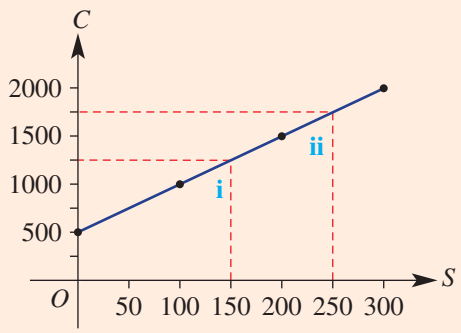
b Use the graph to determine:

i the total cost for 150 students

ii how many students could attend the dance if Jasmine has $1750 to spend

SOLUTION

a



EXPLANATION

Construct a set of axes using S between 0 and 300 and C between 0 and 2000.

Number of students is placed on the horizontal axis.

Plot each point using the information in the table.

b i The total cost for 150 students is $1250.

Draw a vertical dotted line at $S = 150$ to meet the graph, then draw another dotted line horizontally to the C-axis.

ii 250 students could attend the dance if the budget is $1750.

Draw a horizontal dotted line at $C = 1750$ to meet the graph, then draw a dotted line vertically to the S-axis.

Now you try

Hayley is organising a party for her child. The venue is chosen and the costs are shown in the table.

Number of children (n)	0	10	20	30
Total cost in dollars (C)	200	400	600	800

a Plot a graph of the total cost against the number of children.

b Use the graph to determine:

i the total cost for 25 children

ii how many children could be invited if Hayley has $500 to spend

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 A furniture removalist charges per hour. His rates are shown in the table below.

No. of hours (n)	0	1	2	3	4	5
Cost (C)	200	240	280	320	360	400

a Plot a graph of cost against hours.

b Use the graph to determine the:

i total cost for 2.5 hours of work

ii number of hours the removalist will work for $380

Hint: Place 'Number of hours' on the horizontal axis.

8 Olive oil is sold in bulk for $8 per litre.

No. of litres (L)	1	2	3	4	5
Cost (C)	8	16	24	32	40

a Plot a graph of cost against number of litres.

b Use the graph to determine the:

i total cost for 3.5 litres of oil

ii number of litres of oil you can buy for $20

Example 9 Constructing a table and graph for interpretation

A gardener charges $50 per visit and $60 an hour for labour.

a Complete the table of values.

No. of hours (n)	0	1	2	3	4	5
Cost (C)						

b Plot a graph of cost against number of hours.

c Use the graph to determine:

i the cost for 4.5 hours of work

ii how long the gardener will work for $260

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

a

No. of hours (n)	0	1	2	3	4	5
Cost (C)	50	110	170	230	290	350

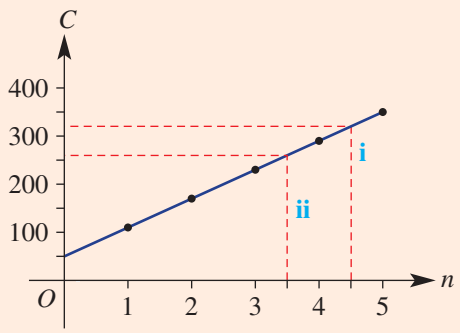
EXPLANATION

Cost for 1 hour = \$50 + \$60 = \$110

Cost for 2 hours = \$50 + 2 × \$60 = \$170

Cost for 3 hours = \$50 + 3 × \$60 = \$230 etc.

b



Plot the points from the table using C on the vertical axis and n on the horizontal axis. Join all the points to form the straight line.

c **i** The cost is \$320.

Draw a vertical dotted line at $n = 4.5$ to meet the graph, then draw a line horizontally to the C-axis.

ii The gardener will work for 3.5 hours.

Draw a horizontal dotted line at $C = 260$ to meet the graph, then draw vertically to the n-axis.

Now you try

A balloon artist charges \$120 to attend a child's party and \$60 per hour in attendance.

a Complete the table of values.

No. of hours (n)	0	1	2	3	4
Cost (C)					

b Plot a graph of cost against number of hours.

c Use the graph to determine:

i the cost for 2.5 hours of hire

ii how long the balloon artist will work for \$330

9 A car rental firm charges \$200 plus \$1 for each kilometre travelled.

a Complete the table of values below.

No. of km (k)	0	100	200	300	400	500
Cost (C)						

b Plot a graph of cost against kilometres.

c Use the graph to determine:

i the cost if you travel 250 km

ii how many kilometres you can travel on a budget of \$650

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 Matthew delivers pizza for a fast-food outlet. He is paid $20 a shift plus $3 per delivery.

a Complete the table of values below.

No. of deliveries (d)	0	5	10	15	20
Pay (P)					

b Plot a graph of Matthew's pay against number of deliveries.

c Use the graph to determine the:

i amount of pay for 12 deliveries

ii number of deliveries made if Matthew is paid $74

ENRICHMENT: Which mechanic? — 11

11 Two people charge different rates for their labour. Yuri charges $75 for a service call plus $50 per hour. Sherry charges $90 for a service call plus $40 per hour.

a Create a table for each worker for up to 5 hours of work.

b Plot a graph for the total charge against the number of hours worked for Yuri and Sherry, on the same axes.

c Use the graph to determine the:

i cost of hiring Yuri for 3.5 hours

ii cost of hiring Sherry for 1.5 hours

iii number of hours of work if Yuri charges $100

iv number of hours of work if Sherry charges $260

v number of hours of work if the cost from Yuri and Sherry is the same

d Write a sentence describing who is cheaper for different hours of work.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6D Exploring gradient

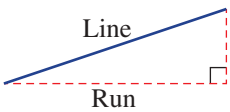
Learning intentions for this section:

- To know that gradient is a constant numerical measure of the slope of a line
- To be able to identify on a graph if the gradient is positive, negative, zero or undefined
- To be able to find the gradient of a line from a graph or using two given points

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Linear relationships A*
- It consolidates and extends the concepts in Section 5E of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formula from memory in the HSC Examination

The gradient of a line is a measure of its slope. It is a number that shows the steepness of a line. It is calculated by knowing how far a line rises or falls (called the *rise*) within a certain horizontal distance (called the *run*). The gradient is equal to the rise divided by the run. The letter m is used to represent gradient.

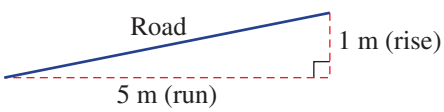


$$\text{Gradient of line} = m = \frac{\text{rise}}{\text{run}}$$

Engineers apply their knowledge of gradients when designing roads, bridges, railway lines and buildings. Some mountain railways have a gradient greater than 1, which is a slope far too steep for a normal train or even a powerful car.

For example, a train takes tourists to the Matterhorn, a mountain in Switzerland. To cope with the very steep slopes it has an extra wheel with teeth, which grips a central notched line.

Lesson starter: What's the gradient?



A road that rises by 1 m for each 5 m of horizontal distance has a gradient of 0.2 or 20%.

Trucks would find this gradient very steep.

The gradient is calculated by finding the rise divided by the run.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{1}{5} = 0.2 = 20\%$$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1 Find the gradient for each of these roads. Give the answer as a decimal and a percentage.

a Baldwin Street, Dunedin, New Zealand is known as the steepest street in the world. For each 2.86 m of horizontal (run) distance, the road rises by 1 m.

b Gower Street, Toowong, is Brisbane's steepest street. For each 3.2 m of horizontal (run) distance, the road rises by 1 m.

2 The Scenic Railway, Katoomba, NSW has a maximum gradient of 122% as it passes through a gorge in the cliff. What is its vertical distance (rise) for each 1 m of horizontal distance (run)?

Use computer software (dynamic geometry) to produce a set of axes and grid.

- Construct a line segment with endpoints on the grid. Show the coordinates of the endpoints.
- Calculate the rise (i.e. vertical distance between the endpoints) and the run (i.e. horizontal distance between the endpoints).
- Calculate the gradient as the rise divided by the run.
- Now drag the endpoints and explore the effect on the gradient.
- Can you drag the endpoints but retain the same gradient value? Explain why this is possible.
- Can you drag the endpoints so that the gradient is zero or undefined? Describe how this can be achieved.

KEY IDEAS

■ Gradient is given by the formula $m = \frac{\text{rise}}{\text{run}}$.

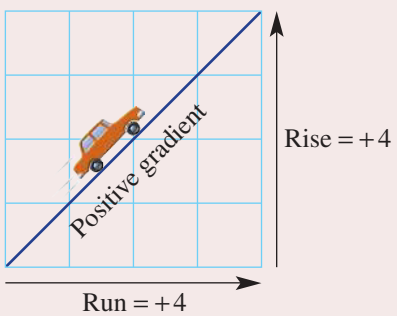
The value of the gradient can be expressed as:

- a fraction
- a decimal
- a percentage
- a ratio

Always move from left to right when considering the rise and the run.

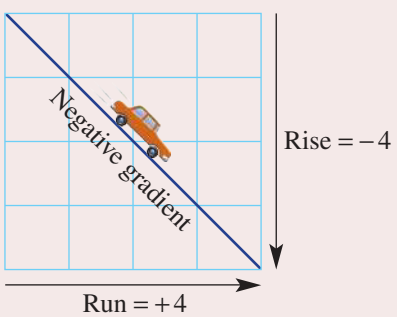
- The horizontal **run** always goes to the right and is always positive. The vertical **rise** can go up (positive) or down (negative).
- If the line slopes up from left to right, the rise is positive and the gradient is positive.

e.g. $m = \frac{\text{rise}}{\text{run}} = \frac{+4}{+4} = 1$



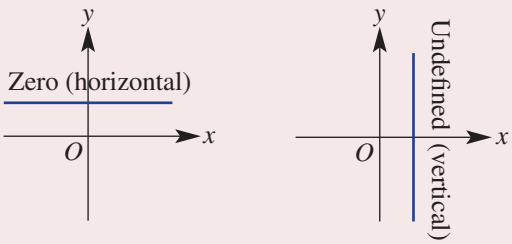
- If the line slopes down from left to right, the rise is considered to be negative and the gradient is negative.

e.g. $m = \frac{\text{rise}}{\text{run}} = \frac{-4}{+4} = -1$



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

■ The gradient can also be zero (when a line is horizontal) or undefined (when a line is vertical).



Key terminology: gradient, slope, rise, run

Exercise 6D

UNDERSTANDING	1, 2	1

1 Use the words *positive*, *negative*, *zero* or *undefined* to complete each sentence.

a The gradient of a horizontal line is _____.

b The gradient of the line joining (0, 3) and (5, 0) is _____.

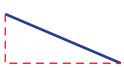
c The gradient of the line joining (−6, 0) and (1, 1) is _____.

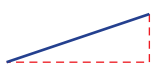
d The gradient of a vertical line is _____.

Hint: Lines going downhill from left to right have a negative gradient.

2 Decide whether each of the following lines would have a positive or negative gradient.

a

b 

c 

d

e

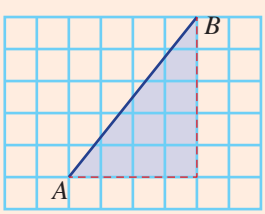
f

FLUENCY	3–5, 7	3, 4, 5(½), 6, 7

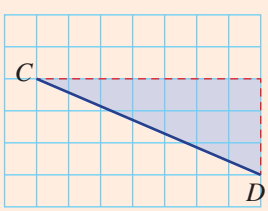
Example 10 Finding the gradient from a grid

Find the gradient of the following line segments, where each grid box equals 1 unit.

a



b



Continued on next page

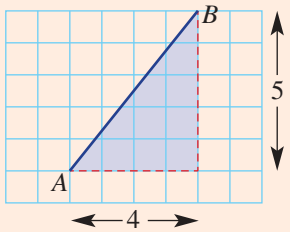
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

a Gradient of $AB = \frac{\text{rise}}{\text{run}}$

$= \frac{5}{4}$

EXPLANATION



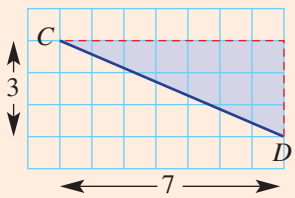
The slope is upwards, therefore the gradient is positive.

The rise is 5 and the run is 4.

b Gradient of $CD = \frac{\text{rise}}{\text{run}}$

$= \frac{-3}{7}$

$= -\frac{3}{7}$



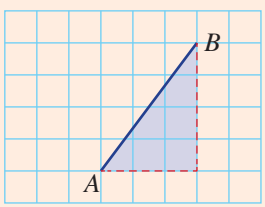
The slope is downwards, therefore the gradient is negative.

The fall is 3, so we write rise $= -3$, and the run is 7.

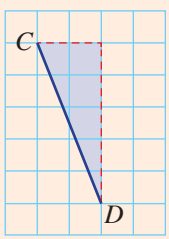
Now you try

Find the gradient of the following line segments, where each grid box equals 1 unit.

a

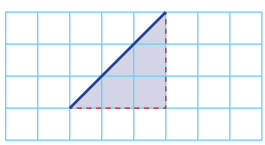


b

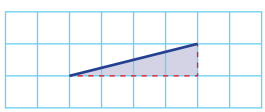


3 Find the gradient of the following line segments.

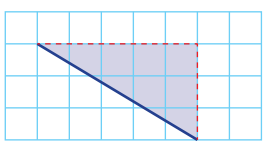
a



b

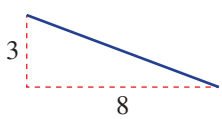


c

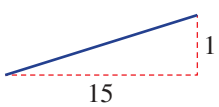


4 Find the gradient of the following.

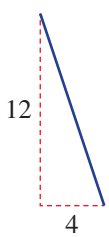
a



b



c



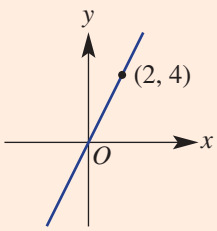
Hint: The gradient is written as a fraction or a whole number.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

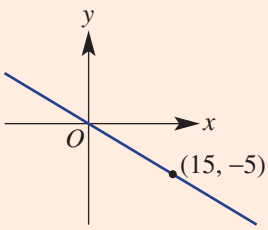
Example 11 Finding the gradient from graphs

Find the gradient of the following lines.

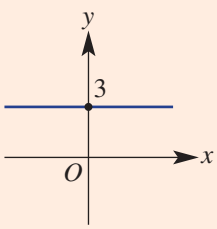
a



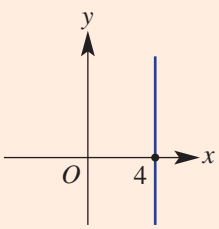
b



c



d



SOLUTION

EXPLANATION

a Gradient $= \frac{\text{rise}}{\text{run}}$

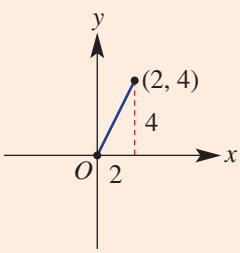
Write the rule each time.

$= \frac{4}{2}$

The rise is 4 and the run is 2 between the two points $(0, 0)$ and $(2, 4)$.

$= 2$

Simplify by cancelling.



b Gradient $= \frac{\text{rise}}{\text{run}}$

Note this time that, when working from left to right, there will be a slope downwards.

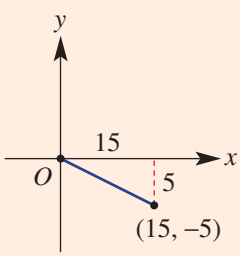
$= \frac{-5}{15}$

The fall is 5 (rise $= -5$) and the run is 15.

$= \frac{-1}{3}$

Simplify.

$= -\frac{1}{3}$



c Gradient $= 0$

Horizontal lines have a zero gradient.

d Gradient is undefined.

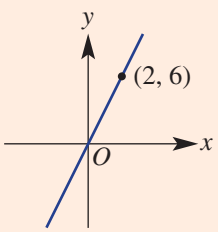
Vertical lines have an undefined gradient.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

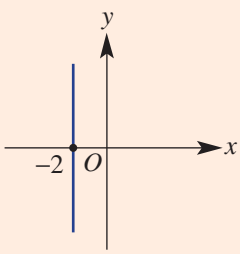
Now you try

Find the gradient of the following lines.

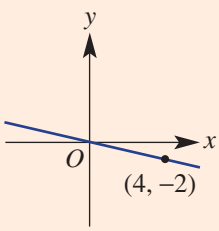
a



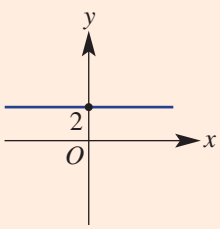
b



c

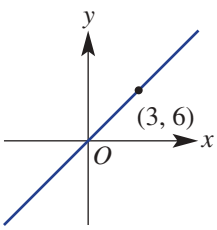


d

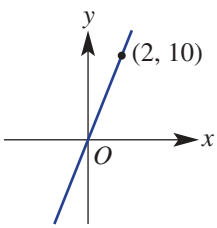


5 Find the gradient of the following lines.

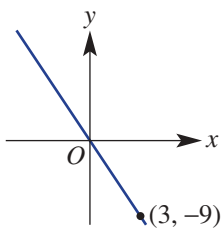
a



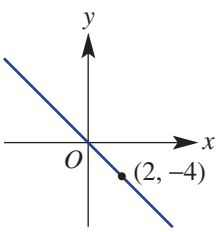
b



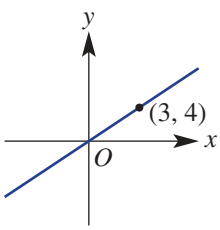
c



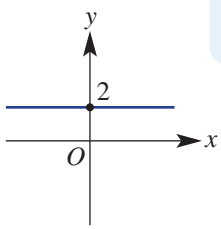
d



e

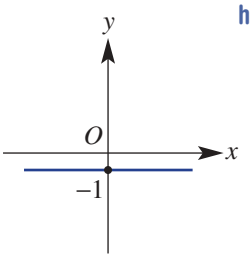


f

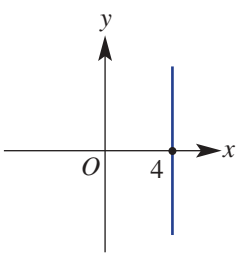


Hint: A horizontal line has zero rise, so its gradient is zero.

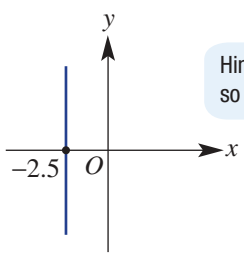
g



h



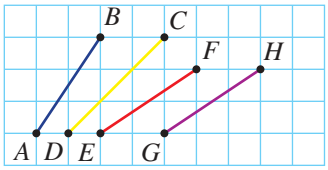
i



Hint: A vertical line has no 'run', so it has undefined gradient.

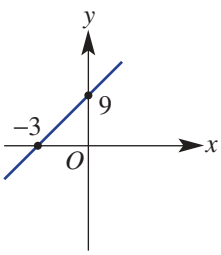
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Use the grid to find the gradient of these line segments. Then order the segments from least to steepest gradient.

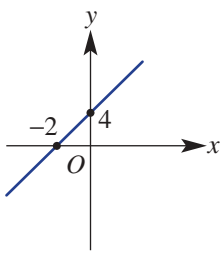


7 Determine the gradient of the following lines.

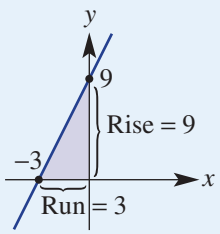
a



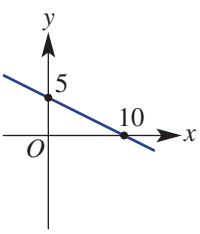
b



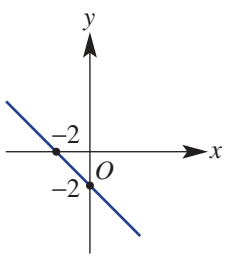
Hint:



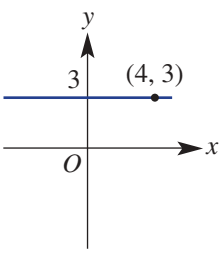
c



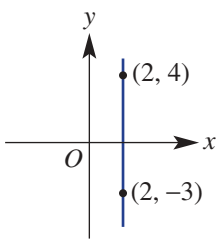
d



e



f

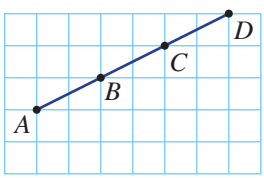


PROBLEM-SOLVING AND REASONING

8, 9 | 9–10(½), 11

8 a Copy and complete the table below.

Line segment	Rise	Run	Gradient
AB			
AC			
AD			
BC			
BD			
CD			



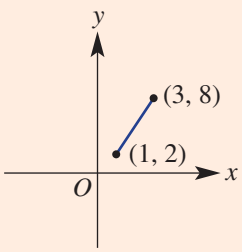
b What do you notice about the gradient between points on the same line?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

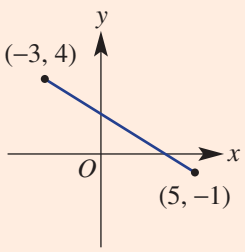
Example 12 Using a formula to calculate gradient (EXTENDING)

Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line segments between the following pairs of points.

a



b



SOLUTION

a $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{8 - 2}{3 - 1}$

$= \frac{6}{2}$

$= 3$

EXPLANATION

Write the rule.

$(1, 2) \quad (3, 8)$

$x_1 \; y_1 \quad x_2 \; y_2$

It does not matter which point is labelled (x_1, y_1) and which is (x_2, y_2).

b $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1 - 4}{5 - (-3)}$

$= \frac{-5}{8}$

$= -\frac{5}{8}$

Write the rule.

$(-3, 4) \quad (5, -1)$

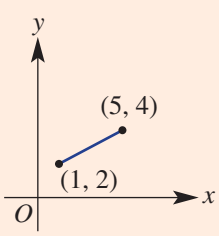
$x_1 \; y_1 \quad x_2 \; y_2$

Remember that $5 - (-3) = 5 + 3 = 8$.

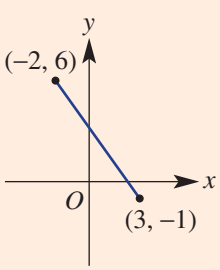
Now you try

Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient of the line segments between the following pairs of points.

a



b

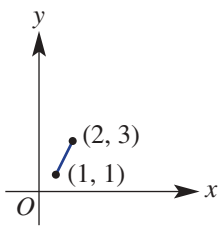


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

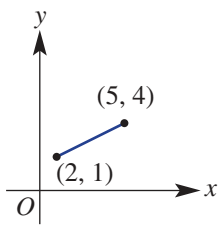
9 Use the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the gradient between these pairs of points.

Hint: First copy the coordinates and label them.
e.g. (1, 1) (2, 3)
x_1 y_1 x_2 y_2

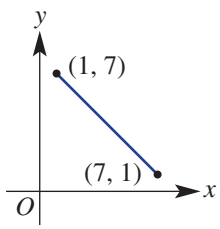
a



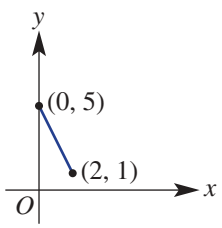
b



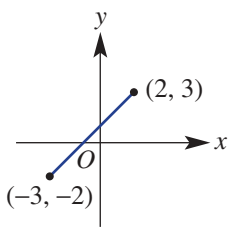
c



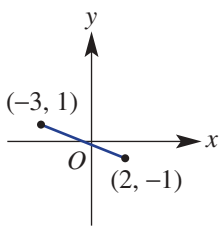
d



e



f



Hint: You can choose either point to be (x_1, y_1).

10 Find the gradient between the following pairs of points.

a (1, 3) and (5, 7)
b (−1, −1) and (3, 3)
c (−3, 4) and (2, 1)
d (−6, −1) and (3, −1)
e (1, −4) and (2, 7)
f (−4, −2) and (−1, −1)

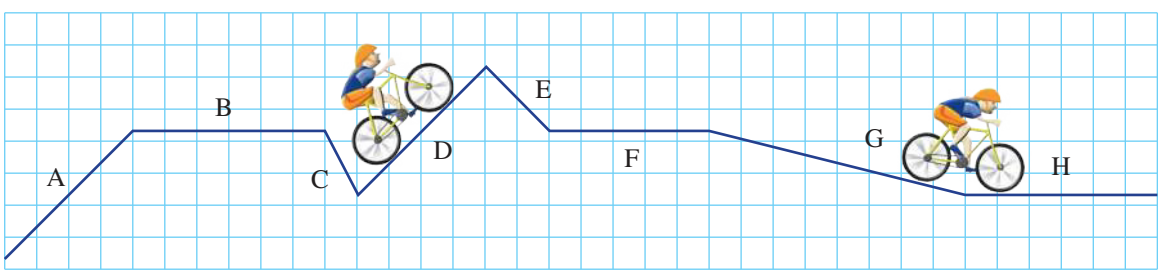
Hint: Use $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

11 The first section of the Cairns Skyrail travels from Caravonica terminal, at 5 m above sea level, to Red Peak terminal, which is 545 m above sea level. This is across a horizontal distance of approximately 1.57 km. What is the overall gradient of this section of the Skyrail? Round your answer to three decimal places.

Hint: Both distances must be in the same units.

ENRICHMENT: From Bakersville to Rolland — 12

12 A transversal map for a bike ride from Bakersville to Rolland is shown.



a Which sections, A, B, C, D, E, F, G or H, indicate travelling a positive gradient?
b Which sections indicate travelling a negative gradient?
c Which will be the hardest section to ride?
d Which sections show a zero gradient?
e Which section is the flattest of the downhill rides?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6E Rates from graphs

Learning intentions for this section:

- To know that a rate compares two quantities
- To understand the connection between rate and the gradient of a straight line
- To be able to calculate speed and other rates from a graph using the gradient

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Linear relationships B*
- It consolidates and extends the concepts in Section 5F of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

The speed or rate at which something changes can be analysed by looking at the gradient (steepness) of a graph. Two common rates are kilometres per hour (km/h) and litres per second (L/s).

Graphs of a patient's records provide valuable information for a doctor. For example, from a graph of temperature versus time, the rate of temperature change in degrees Celsius per minute (°C/min) can be calculated. This rate provides important information to help a doctor diagnose an illness.

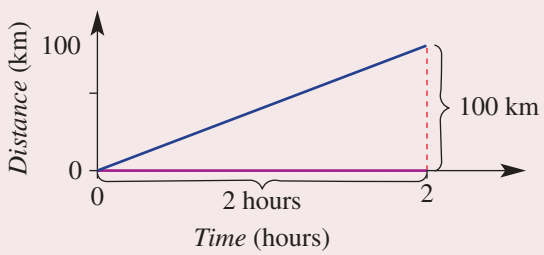
Lesson starter: What's the rate?

Calculate each of these rates.

a $60 000 for 200 tonnes of wheat.
b Lee travels 840 km in 12 hours.
c A foal grows 18 cm in height in 3 months.
d Petrol costs $96 for 50 litres.
e Before take-off, a hot-air balloon of volume 6000 m^3 is filled in 60 seconds.

KEY IDEAS

■ A **rate** compares two quantities. Many rates show how a quantity changes over time.



$$\text{Rate} = \frac{\text{rise}}{\text{run}} = \frac{100}{2} = 50 \text{ km/h}$$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- Rate = change in quantity ÷ change in time
  (L, kg, ...) (seconds, hours, ...)
- The gradient of a line gives the rate.
- A common rate is **speed**.
  - Speed = change in distance ÷ change in time
    (cm, km, ...) (seconds, hours, ...)
- To determine a rate from a linear graph, calculate the gradient and include the units (y unit/x unit); e.g. km/h.

Key terminology: rate, gradient, speed

Exercise 6E

UNDERSTANDING 1, 2 2

1 Complete the sentences.

a A rate is found from a linear graph by calculating the _____ of the line.

b A rate compares _____ quantities.

c A rate has two _____.

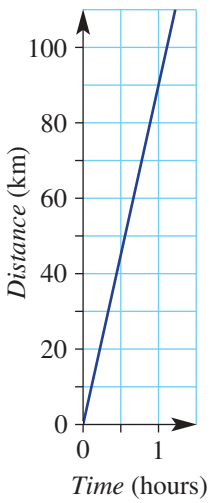
d A speed of 60 kilometres per hour is written as 60 _____.

e If the rate of filling a bath is 50 litres per minute, this is written as 50 _____.

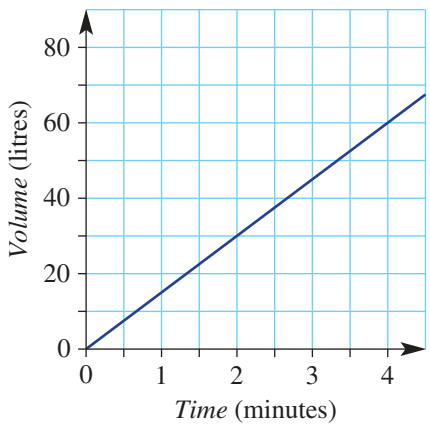
Hint: Choose from: *km/h, gradient, units, L/min, two.*

2 Write down the rate by calculating the gradient of each line graph. Include units.

a



b



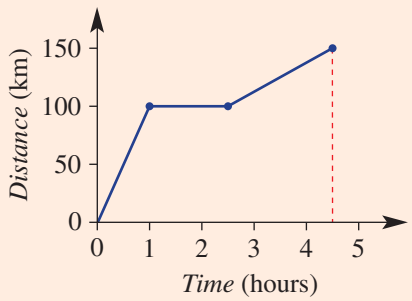
Hint: A rate = a gradient with units
Gradient = $\frac{\text{rise}}{\text{run}}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 3–5 3, 5, 6

Example 13 Calculating speed from a graph

A 4WD vehicle completes a journey, which is described by this graph.



a For the first hour, find:
  i the total distance travelled
  ii the speed

b How fast was the 4WD travelling during:
  i the first hour?
  ii the second section?
  iii the third section?

SOLUTION

a i 100 km
  ii 100 km/1 h = 100 km/h

b i 100 km/1 h = 100 km/h
  ii 0 km/1.5 h = 0 km/h
  iii (150 − 100) km/(4.5 − 2.5) h
  = 50 km/2 h
  = 25 km/h

EXPLANATION

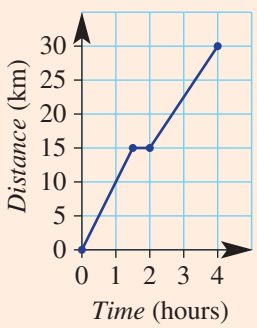
Read the distance at 1 hour.
Speed = distance ÷ time

Speed = distance ÷ time
The vehicle is at rest.
Determine the distance travelled and the amount of time, then apply the rate formula.

Speed = distance ÷ time
50 km in 2 hours is $\frac{50}{2}$ = 25 km in 1 hour.

Now you try

A long-distance runner completes a training run, which is described by this graph.

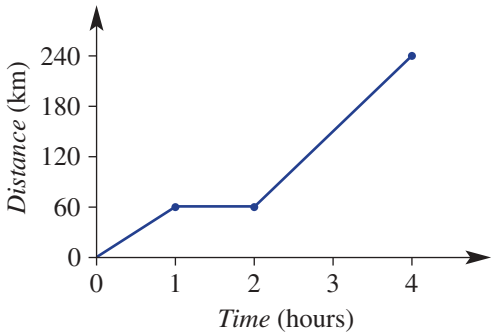


a For the first hour, find:
  i the total distance travelled
  ii the speed

b How fast was the runner travelling during:
  i the first section?
  ii the second section?
  iii the third section?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

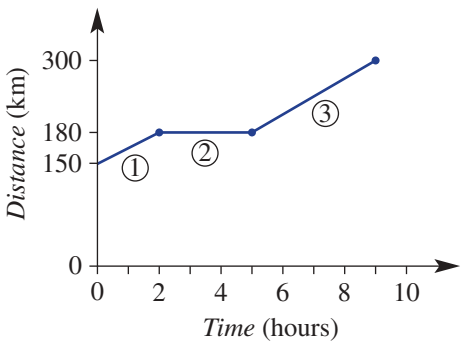
3 A car completes a journey, which is described by this graph.



a For the first hour, find:
- **i** the total distance travelled
- **ii** the speed

b How fast was the car travelling during:
- **i** the first hour?
- **ii** the second section?
- **iii** the third section?

4 A cyclist training for a professional race includes a rest stop between two travelling sections.

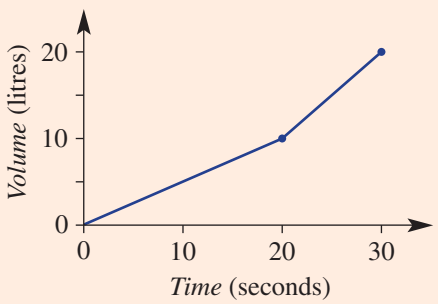


a For the first hour, find:
- **i** the total distance travelled
- **ii** the speed

b How fast was the cyclist travelling during:
- **i** the second section?
- **ii** the third section?

Example 14 Calculating the rate of change of volume in L/s

A container is being filled with water from a hose.



a How many litres are filled during:
- **i** the first 10 seconds?
- **ii** the final 10 seconds?

b How fast (i.e. what rate in L/s) is the container being filled:
- **i** during the first 10 seconds?
- **ii** during the final 10 seconds?
- **iii** between the 10 and 20 second marks?

SOLUTION	EXPLANATION
a i 5 litres	Read the number of litres after 10 seconds.
ii 10 litres	Read the change in litres from 20 to 30 seconds.
b i 5 L/10 s = 0.5 L/s	5 litres is added in the first 10 seconds.
ii 10 L/10 s = 1 L/s	10 litres is added in the final 10 seconds.
iii 5 L/10 s = 0.5 L/s	5 litres is added between 10 and 20 seconds.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

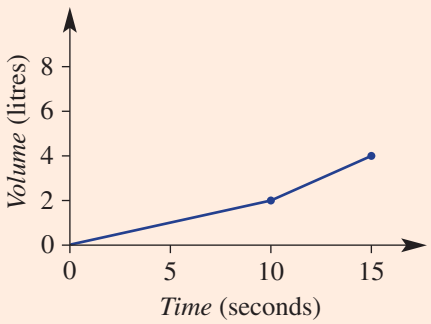
A bucket is being filled with water from a tap.

a How many litres are filled during:

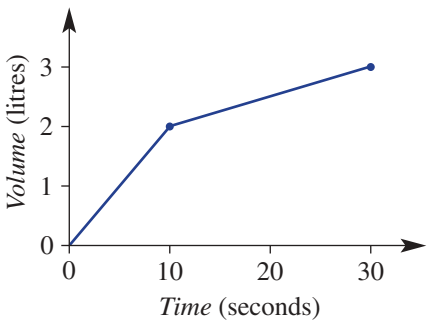
- **i** the first 10 seconds?
- **ii** the final 5 seconds?

b How fast (i.e. what rate in L/s) is the bucket being filled:

- **i** during the final 5 seconds?
- **ii** between the 5 and 10 second marks?



5 A large carton is being filled with milk.



Hint:
Rate = volume ÷ time

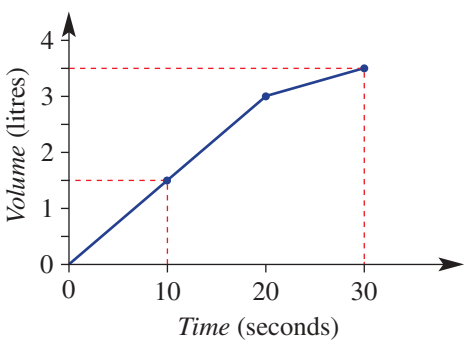
a How many litres are filled during:

- **i** the first 10 seconds?
- **ii** the final 10 seconds?

b How fast (i.e. what rate in L/s) is the container being filled:

- **i** during the first 10 seconds?
- **ii** during the final 10 seconds?
- **iii** between the 10 and 20 second marks?

6 A large bottle with a long, narrow neck is being filled with water.



a How many litres are filled during:

- **i** the first 10 seconds?
- **ii** the final 10 seconds?

b How fast (i.e. what rate in L/s) is the bottle being filled:

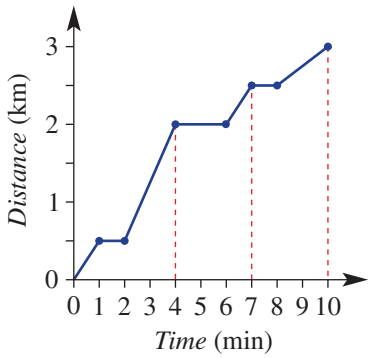
- **i** during the first 10 seconds?
- **ii** during the final 10 seconds?
- **iii** between the 10 and 20 second marks?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

7, 8 | 8–10

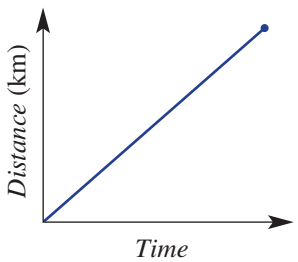
7 A postal worker stops to deliver mail to each of three houses along a country lane.



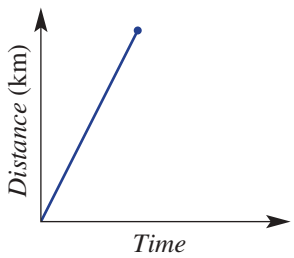
- **a** What is the total length of the country lane?
- **b** What is the total time the postal worker spends standing still?
- **c** Find the speed (use km/min) of the postal worker at the following times.
  - **i** before the first house
  - **ii** between the first and the second house
  - **iii** between the second and the third house
  - **iv** after his delivery to the third house

8 Three friends, Anna, Billy and Cianne, travel 5 km from school to the library. Their journeys are displayed in the three graphs below. All three graphs are drawn to the same scale.

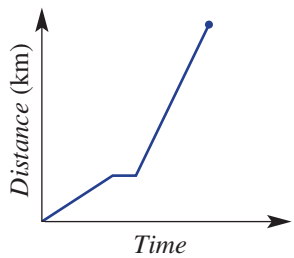
A



B



C



- **a** If Anna walked a short distance before getting picked up by her mum, which graph represents her trip?
- **b** If Cianne arrived at the library last, which graph best represents her journey?
- **c** Which graph represents the fastest journey? Explain your answer.

9 **a** Draw your own graph to show the following journey: 10 km/h for 2 hours, then rest for 1 hour, and then 20 km/h for 2 hours.

- **b** Then use your graph to find the total distance travelled.

Hint:
Mark each segment one at a time. 10 km/h for 2 hours covers a distance of $10 \times 2 = 20$ km.

10 A lift starts on the ground floor (height 0 m) and moves to floor 3 at a rate of 3 m/s for 5 seconds. After waiting at floor 3 for 9 seconds, the lift rises 45 m to floor 9 in 9 seconds. The lift rests for 11 seconds before returning to ground level at a rate of 6 m/s. Draw a graph to help find the total time taken to complete the movements described. Use time, in seconds, on the horizontal axis.

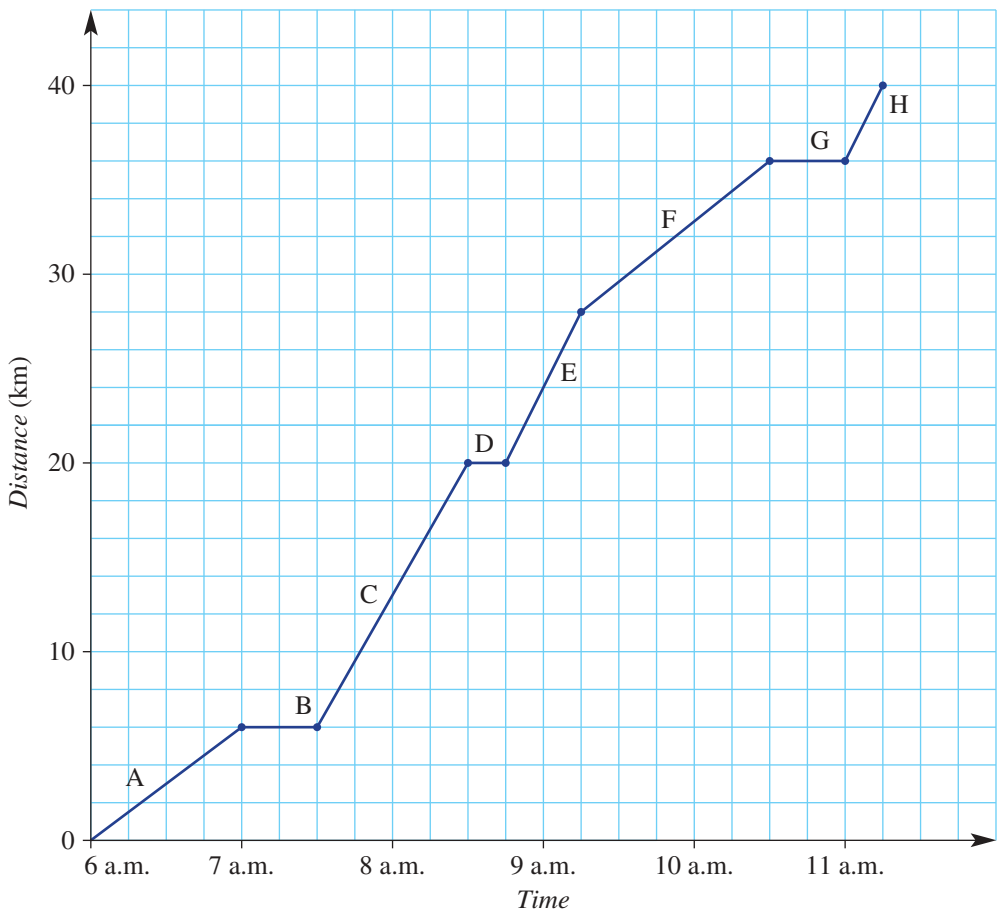
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: Sienna's training — 11

11 Sienna is training for the Sydney Marathon. Her distance–time graph is shown below.

- **a** How many stops does Sienna make?
- **b** How far does she jog between:
  - **i** 6 a.m. and 7 a.m.?
  - **ii** 7:30 a.m. and 8:30 a.m.?
- **c** Which sections of the graph have a zero gradient?
- **d** Which sections of the graph have the steepest gradient?
- **e** At what speed does Sienna run in section:
  - **i** A? **ii** C? **iii** E? **iv** F? **v** H?
- **f** In which sections is Sienna travelling at the same speed? How does the graph show this?
- **g** How long does the training session last?
- **h** What is the total distance travelled by Sienna during the training session?
- **i** What is her average speed for the entire trip, excluding rest periods?

Sienna's training



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Progress quiz

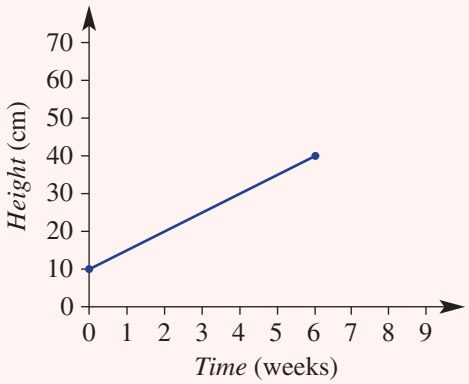
6A **1** The graph at right shows the growth of a plant measured at weekly intervals.

a What is the initial height of the plant?

b How much does the plant grow in 6 weeks?

c Use the graph to find the height of the plant after 4 weeks.

d Extend the graph to estimate the height of the plant after 8 weeks.

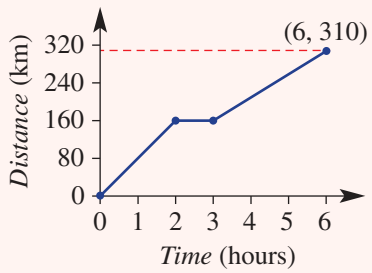


6B/6E **2** The graph at right shows the journey of a car.

a What is the distance travelled in the first 2 hours?

b What is the speed in the first 2 hours?

c What is the speed in the final section after the rest break?



6B **3** Sketch a distance–time graph for a runner's training session. Display the following information in your graph:

- covers 6 km in 30 minutes
- 4 km covered in first 15 minutes
- a 5-minute rest after 15 minutes.

6C **4** Plot the graphs of the following by first completing the table of values.

a $y = 3x + 1$

x	−2	−1	0	1	2
y					

b $y = -2x + 3$

x	−2	−1	0	1	2
y					

6C **5** A sailing boat can be hired for \$60 per hour with an initial fee of \$40.

a Complete the table of values.

No. of hours (n)	0	1	2	3	4	5
Cost (\$$C$)						

b Plot a graph of cost against number of hours.

c Use the graph to determine:

i the cost for 2.5 hours of hire

ii how long the sailing boat was hired if the cost was \$250

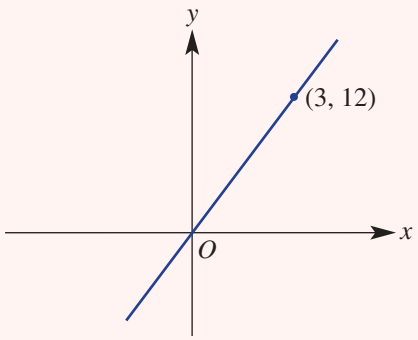
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Progress quiz

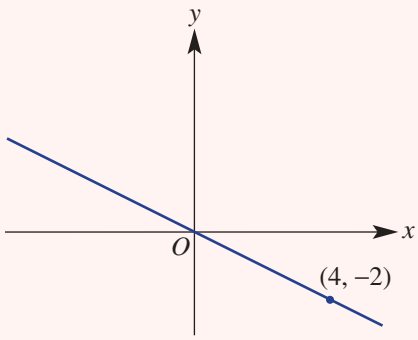
6D

6 Find the gradient of the following lines.

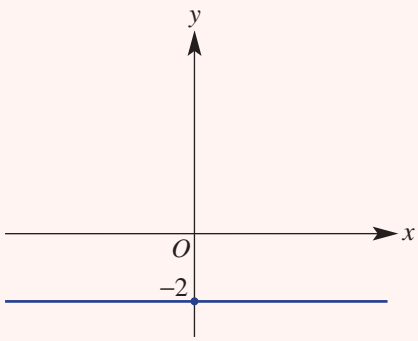
a



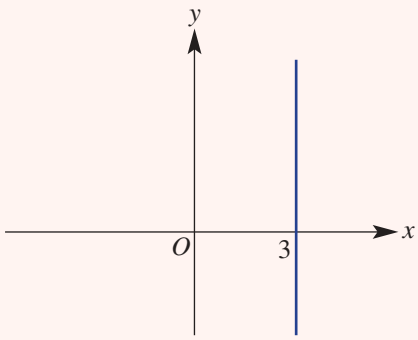
b



c



d



6D

7 Calculate the gradient of the line segment with endpoints $(-3, 2)$ and $(2, 6)$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6F $y = mx + c$ and special lines

Learning intentions for this section:

- To know how the gradient and y-intercept can be determined from the equation of a line
- To be able to sketch a straight-line graph using the gradient and the y-intercept
- To be able to sketch vertical and horizontal lines and lines passing through the origin
- To be able to determine if a point is on a line

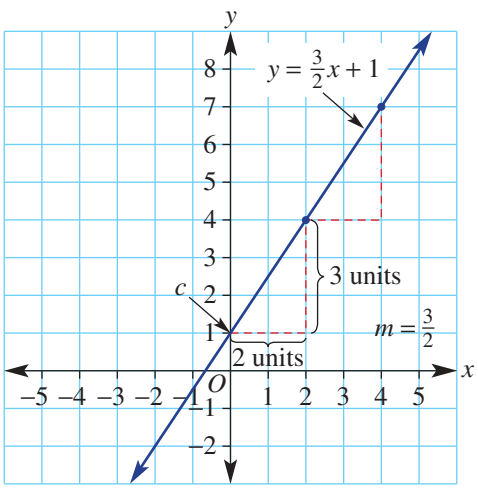
Past, present and future learning:

- This section addresses the Stage 5 Core topics called *Linear relationships A* and *Linear relationships B*
- It consolidates and extends the concepts in Sections 5D, 5G and 5H of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formula from memory in the HSC Examination

It is possible to use the equation of a line to predict the position of the line. The y-intercept is the y value of the point where the line cuts the y-axis.

Here is a graph of $y = \frac{3}{2}x + 1$

gradient $m = \frac{3}{2}$ y-intercept = 1

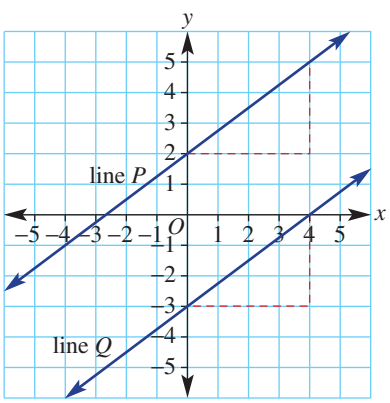


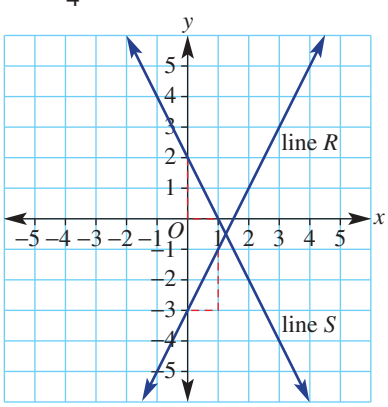
Lesson starter: Matching lines with equations

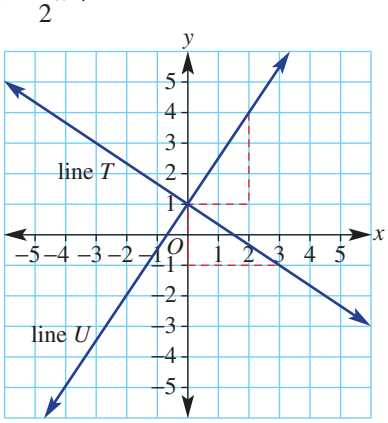
Below are some equations of lines and some graphs. Work with a classmate and help each other to match each equation with its correct line graph.

a $y = 2x - 3$ **b** $y = \frac{3}{4}x + 2$ **c** $y = -\frac{2}{3}x + 1$

d $y = -2x + 2$ **e** $y = \frac{3}{4}x - 3$ **f** $y = \frac{3}{2}x + 1$





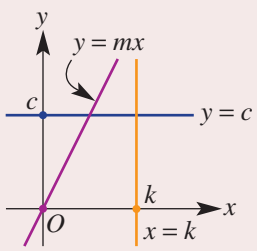


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- $y = mx + c$ is called the **gradient–intercept form**, where m and c are constants.
  Examples are $y = 3x + 4$, $y = \frac{1}{2}x - 3$ and $y = 2$.
  - The **gradient** or slope equals m (the **coefficient** of x).
  - The **y-intercept** is the y value at the point where the line cuts the y-axis.
    In $y = mx + c$, the y-intercept is c.

- Some special lines include:
  - **horizontal** lines: $y = c$ ($m = 0$)
  - **vertical** lines: $x = k$ (m is undefined)
  - lines passing through the origin $(0, 0)$: $y = mx$ ($c = 0$)



Key terminology: gradient–intercept form, gradient, coefficient, y-intercept, horizontal, vertical

Exercise 6F

UNDERSTANDING 1–3 2, 3

1 Complete the sentences.

a $y = mx + c$ is called the _____ – _____ form of a straight line.

b The symbol m stands for the _____.

c In the equation, the gradient, m, is the _____ of x.

d The symbol c stands for the _____.

e A horizontal line has a _____ gradient.

Hint:
Choose from: *gradient, intercept, zero, y-intercept, coefficient.*

2 Form rules of the form $y = mx + c$ for the following straight-line graphs:

a gradient $= 3$, y-intercept $= -1$

b gradient $= -\frac{3}{2}$, y-intercept $= 2$

c gradient $= -1$, y-intercept $= 0$

Hint: In $y = mx + c$
m is the gradient
c is the y-intercept.

3 Identify the following special lines as: *horizontal*, *vertical* or *passes though the origin*.

a $y = 2$

b $x = -3$

c $y = 2x$

d $x = 0$

e $y = -x$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 4, 5, 6(½) 4–7(½)

Example 15 Reading the gradient and y-intercept from an equation

For the following equations, state:

i the gradient ii the y-intercept

a $y = 3x + 4$

b $y = -\frac{3}{4}x - 7$

SOLUTION

a i Gradient is 3.

ii y-intercept is 4.

b i Gradient is $-\frac{3}{4}$.

ii y-intercept is -7.

EXPLANATION

The coefficient of x is 3; i.e. $m = 3$.
The value of c is $+4$.

The coefficient of x is $-\frac{3}{4}$; i.e. $m = -\frac{3}{4}$.

The value of c is -7; don't forget to include the – sign.

Now you try

For the following equations, state the:

i gradient ii y-intercept

a $y = 2x - 5$

b $y = -\frac{1}{3}x + 2$

4 For each of the following equations, state the:

i gradient ii y-intercept

a $y = 2x + 4$

b $y = 6x - 7$

c $y = -\frac{2}{3}x + 7$

d $y = -7x - 3$

e $y = \frac{3}{5}x - 8$

f $y = 9x - 5$

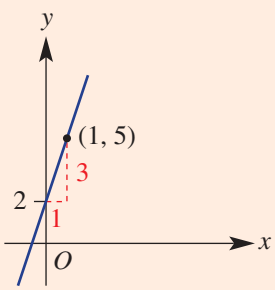
Hint: The gradient is the coefficient of x, which is the number multiplied by x. It does not include the x. Write the y-intercept, including the sign of c.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 16 Sketching a line using the *y*-intercept and gradient

Sketch the graph of $y = 3x + 2$ by considering the y-intercept and the gradient.

SOLUTION



EXPLANATION

Consider $y = mx + c$; the value of c is 2 and therefore the y-intercept is 2.

The value of m is 3 and therefore the gradient is 3 or $\frac{3}{1}$.

Start at the y-intercept 2 and, with the gradient of $\frac{3}{1}$, move 1 right (run) and 3 up (rise) to the point $(1, 5)$.

Draw a line passing through the two points.

Now you try

Sketch the graph of $y = 4x - 1$ by considering the y-intercept and the gradient.

5 Sketch the graph of the following by considering the y-intercept and the gradient.

a $y = 2x + 3$ **b** $y = 3x - 12$

c $y = x + 4$ **d** $y = -2x + 5$

e $y = -5x - 7$ **f** $y = -x - 4$

Hint: Plot the y-intercept first. For a line with $m = -2$:
$m = -2 = \frac{-2}{1} = \frac{\text{down } 2}{\text{right } 1}$.
From the y-intercept, go right 1 then down 2 to plot the next point.

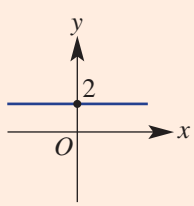
Example 17 Sketching special lines

Sketch the graphs of these equations.

a $y = 2$ **b** $x = -3$ **c** $y = -2x$

SOLUTION

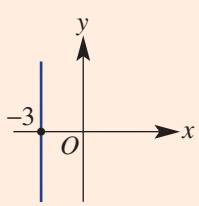
a



EXPLANATION

Sketch a horizontal line passing through $(0, 2)$.

b



Sketch a vertical line passing through $(-3, 0)$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

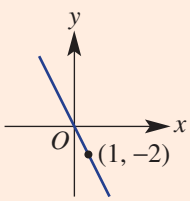
c When $x = 0$, $y = -2(0) = 0$.

When $x = 1$, $y = -2(1) = -2$.

The line passes through the origin $(0, 0)$.

Use $x = 1$ to find another point.

Sketch the graph passing through $(0, 0)$ and $(1, -2)$.



Now you try

Sketch the graphs of these equations.

a $y = -3$ **b** $x = 1$ **c** $y = 4x$

6 Sketch the following lines.

a $y = 4$ **b** $y = -2$ **c** $y = 5$

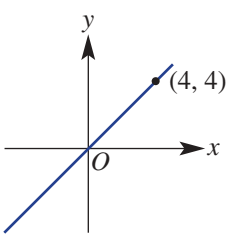
d $x = 5$ **e** $x = -2$ **f** $x = 9$

g $y = 3x$ **h** $y = 6x$ **i** $y = -2x$

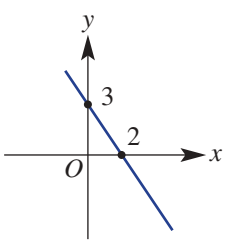
Hint: For the line equation $y = 2$, every point on the line has a y value of 2; e.g. $(-3, 2)$ $(0, 2)$ $(1, 2)$ $(3, 2)$. For the line equation $x = -3$, every point on the line has an x value of -3; e.g. $(-3, 1)$ $(-3, 0)$ $(-3, -4)$.

7 Determine the gradient and y-intercept for the following lines.

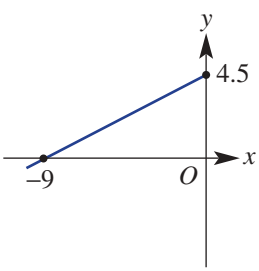
a



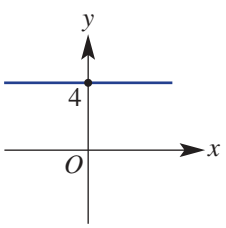
b



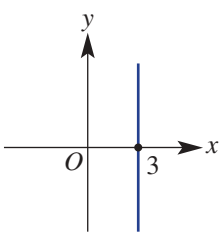
c



d



e



Hint: Use $m = \frac{\text{rise}}{\text{run}}$ for the gradient between two known points.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

8, 9, 10(½), 12 | 8, 11–13

8 Match each of the following linear equations to one of the sketches shown.

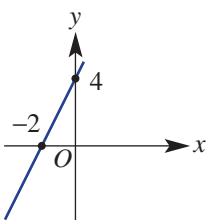
a $y = -\frac{2}{3}x + 2$ **b** $y = -x + 4$ **c** $y = x + 3$

d $y = 2x + 4$ **e** $y = 4$ **f** $y = 7x$

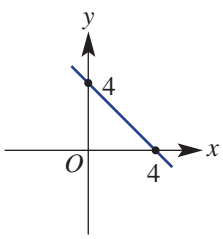
g $y = -3x + 6$ **h** $x = 2$ **i** $y = -3x$

Hint: A linear equation is an equation that gives a straight-line graph.

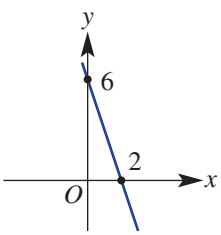
i



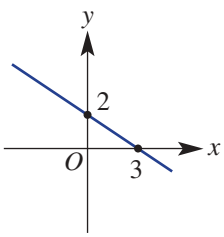
ii



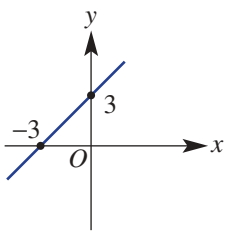
iii



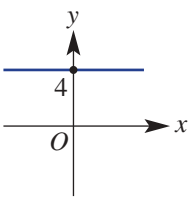
iv



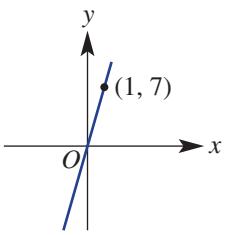
v



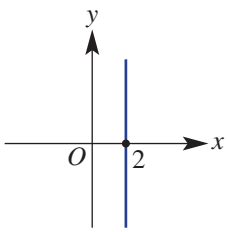
vi



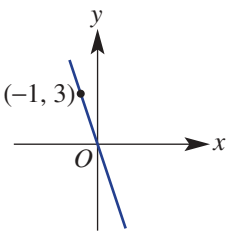
vii



viii



ix



Hint: For a negative gradient, move the negative sign to the numerator.
$m = -\frac{2}{3} = \frac{-2}{3} = \frac{\text{down } 2}{\text{right } 3}$

9 Write down three different equations that have a graph with a y-intercept of:

a 5 **b** -2

10 Write down three different equations that have a graph with:

a a gradient of 3 **b** a gradient of -1

c a gradient of 0 **d** an undefined gradient

11 **a** Which of the following points lie on the line $y = 2$?

i $(2, 3)$ **ii** $(1, 2)$

iii $(5, 2)$ **iv** $(-2, -2)$

b Which of the following points lie on the line $x = 5$?

i $(5, 3)$ **ii** $(3, 5)$

iii $(1, 7)$ **iv** $(5, -2)$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 18 Identifying points on a line

Does the point $(3, -4)$ lie on the line $y = 2x - 7$?

SOLUTION

$y = 2x - 7$
$y = 2 \times 3 - 7$
$y = -1$
$\neq 4$

No, $(3, -4)$ is not on the line.

EXPLANATION

Copy the equation and substitute $x = 3$.
The y value for $x = 3$ is $y = -1$.
Compare the y values.
The point $(3, -1)$ is *on* the line.
So $(3, -4)$ is *not* on the line.

Now you try

Does the point $(-2, 5)$ lie on the line $y = 3x + 2$?

12 **a** Does the point $(3, 2)$ lie on the line $y = x + 2$?
b Does the point $(-2, 0)$ lie on the line $y = x + 2$?
c Does the point $(1, -5)$ lie on the line $y = 3x + 2$?
d Does the point $(2, 2)$ lie on the line $y = x$?
e Does the line $y - 2x = 0$ pass through the origin?

Hint: Substitute the x value into the equation and compare the two y values. When the y values are the same, the point is on the line.

13 Draw each of the following on a number plane and write down the equation of the line.

a

x	0	1	2	3
y	4	5	6	7

b

x	0	1	2	3
y	−1	0	1	2

c

x	−2	0	4	6
y	−1	0	2	3

d

x	−2	0	2	4
y	−3	1	5	9

Hint: Use your graph to find the gradient between two points (m) and locate the y-intercept (c). Then use $y = mx + c$.

ENRICHMENT: Graphs using technology — 14, 15

14 Use technology to sketch a graph of these equations.

a $y = x + 2$ **b** $y = -4x - 3$ **c** $y = \frac{1}{2}x - 1$ **d** $y = 1.5x + 3$
e $y = 2x - 5$ **f** $y = 0.5x + 5$ **g** $y = -0.2x - 3$ **h** $y = 0.1x - 1.4$

15 **a** On the same set of axes, plot graphs of $y = 2x$, $y = 2x + 1$, $y = 2x + 4$, $y = 2x - 2$ and $y = 2x - 3$, using a calculator.
Discuss what you see and describe the connection with the given equations.

b On the same set of axes, plot graphs of $y = x - 1$, $y = 2x - 1$, $y = 3x - 1$, $y = \frac{1}{2}x - 1$ and $y = \frac{3}{4}x - 1$, using a calculator.
Discuss what you see and describe the connection with the given equations.

c The equations of families of graphs can be entered into a calculator using one line only. For example, $y = 2x + 1$, $y = 2x + 2$ and $y = 2x + 3$ can be entered as $y = 2x + \{1, 2, 3\}$ using set brackets. Use this notation to draw the graphs of the equations in parts **a** and **b**.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6G Parallel lines and perpendicular lines

Learning intentions for this section:

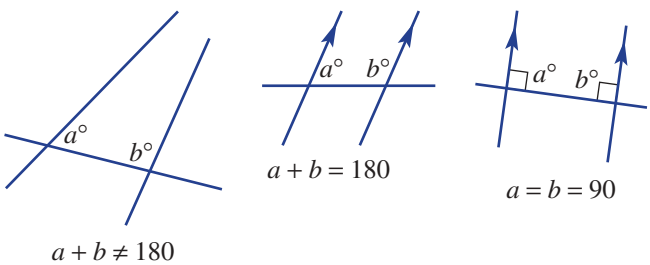
- To know that parallel lines have the same gradient
- To know the relationship between the gradients of perpendicular lines
- To be able to determine if lines are parallel or perpendicular using their gradients
- To be able to find the equation of parallel and perpendicular lines

Past, present and future learning:

- This section addresses the Stage 5 Core topics called *Linear relationships A* and *Linear relationships B*
- These concepts were not addressed in our Year 9 book
- The skills presented are occasionally useful in Stage 6 Standard

Euclid of Alexandria (300 BCE) was a Greek mathematician and is known as the 'Father of geometry'. In his texts, known as *Euclid's Elements*, his work is based on five simple axioms.

His fifth axiom, the Parallel Postulate, says that if cointerior angles do not sum to 180° then the two lines are not parallel. Furthermore, if the two interior angles are equal and also sum to 180°, then the third line must be perpendicular to the other two.

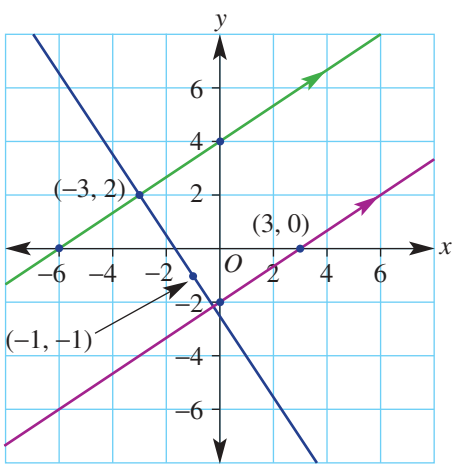


Parallel and perpendicular lines, including their gradient and rule, are the focus of this section.

Lesson starter: Gradient connection

Shown here is a pair of parallel lines and a third line that is perpendicular to the other two lines.

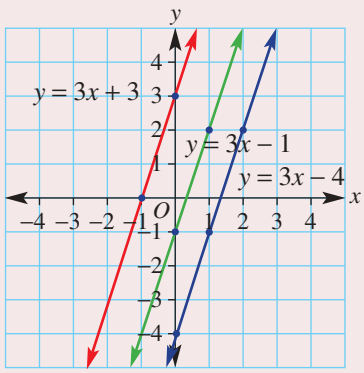
- Find the gradient of each line using the coordinates shown on the graph.
- What is common about the gradients for the two parallel lines?
- Is there any connection between the gradients of the parallel lines and the perpendicular line? Can you write down this connection as a formula?



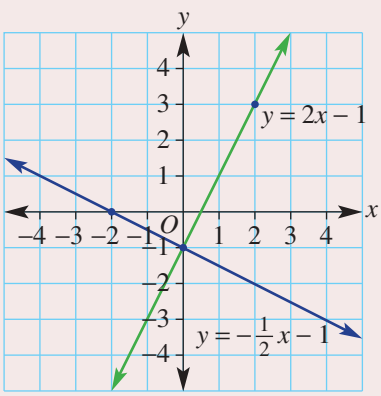
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- Two **parallel lines** have the same gradient as they point in the same direction.
  For example, $y = 3x - 1$ and $y = 3x + 3$ have the same gradient: 3.



- Two **perpendicular lines** (lines that are at right angles to each other) with gradients m_1 and m_2 satisfy the following rule:
  $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$ (i.e. m_2 is the negative reciprocal of m_1).
  In the graph shown, $m_1 \times m_2 = 2 \times \left(-\frac{1}{2}\right) = -1$.



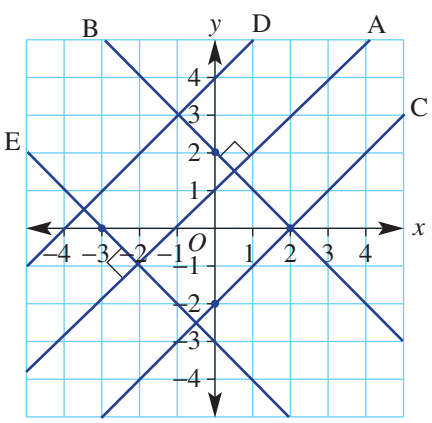
- Equations of parallel or perpendicular lines can be found by:
  - first finding the gradient (m)
  - then substituting a point to find c in $y = mx + c$

Key terminology: parallel, perpendicular, gradient, reciprocal, y-intercept

Exercise 6G

UNDERSTANDING 1–3 | 3

1 For this diagram:



- **a** Which lines, B, C, D or E, are parallel to line A?
- **b** Which lines, B, C, D or E, are perpendicular to line A?
- **c** Are lines C and D parallel?
- **d** Are lines B and E parallel?
- **e** Are lines E and C perpendicular?

2 Write down the gradient of a line that is parallel to the graph of these equations.

- **a** $y = 4x - 6$
- **b** $y = -7x - 1$
- **c** $y = -\frac{3}{4}x + 2$
- **d** $y = \frac{8}{7}x - \frac{1}{2}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Use $m_2 = -\frac{1}{m_1}$ to find the gradient of the line that is perpendicular to the graphs of these equations.

a $y = 3x - 1$ **b** $y = -2x + 6$

c $y = \frac{7}{8}x - \frac{2}{3}$ **d** $y = -\frac{4}{9}x - \frac{4}{7}$

Hint: In part **a**, $m_1 = 3$ so find m_2, which is the perpendicular gradient. Note: $-\frac{1}{\left(\frac{7}{8}\right)} = -1 \div \frac{7}{8} = -1 \times \frac{8}{7}$

FLUENCY 4–6(½) 4–6(½)

Example 19 Deciding if lines are parallel or perpendicular

Decide if the graphs of each pair of rules will be parallel, perpendicular or neither.

a $y = 4x + 2$ and $y = 4x - 6$ **b** $y = -3x - 8$ and $y = \frac{1}{3}x + 1$

SOLUTION	EXPLANATION
a $y = 4x + 2, m = 4$ (1) $y = 4x - 6, m = 4$ (2)	Note that both equations are in the form $y = mx + c$.
So the lines are parallel.	Both lines have a gradient of 4, so the lines are parallel.
b $y = -3x - 8, m = -3$ (1) $y = \frac{1}{3}x + 1, m = \frac{1}{3}$ (2)	Both equations are in the form $y = mx + c$.
$-3 \times \frac{1}{3} = -1$	Test $m_1 \times m_2 = -1$.
So the lines are perpendicular.	

Now you try

Decide if the graphs of each pair of rules will be parallel, perpendicular or neither.

a $y = -3x + 2$ and $y = -3x - 7$ **b** $y = 4x - 2$ and $y = -\frac{1}{4}x + 1$

4 Decide if the line graphs of each pair of rules will be parallel, perpendicular or neither.

a $y = 2x - 1$ and $y = 2x + 1$

b $y = 3x + 3$ and $y = -\frac{1}{3}x + 1$

c $y = 5x + 2$ and $y = 6x + 2$

d $y = -4x - 1$ and $y = \frac{1}{5}x - 1$

e $y = 3x - 1$ and $y = 3x + 7$

f $y = \frac{1}{2}x - 6$ and $y = \frac{1}{2}x - 4$

Hint: If the gradients are equal, then the lines are parallel. If $m_1 \times m_2 = -1$, then the lines are perpendicular.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

g $y = -\frac{2}{3}x + 1$ and $y = \frac{2}{3}x - 3$

h $y = -4x - 2$ and $y = x - 7$

i $y = -\frac{3}{7}x - \frac{1}{2}$ and $y = \frac{7}{3}x + 2$

j $y = -8x + 4$ and $y = \frac{1}{8}x - 2$

Example 20 Finding the equation of a parallel or perpendicular line when given the *y*-intercept

Find the equation of the line, given the following description.

a A line passes through $(0, 2)$ and is parallel to another line with gradient 3.

b A line passes through $(0, -1)$ and is perpendicular to another line with gradient -2.

SOLUTION

a $m = 3$ and $c = 2$

So $y = 3x + 2$.

b $m = -\left(\frac{1}{-2}\right) = \frac{1}{2}$ and $c = -1$

So $y = \frac{1}{2}x - 1$.

EXPLANATION

A parallel line has the same gradient.

Use $y = mx + c$ with $m = 3$ and y-intercept 2.

Being perpendicular, use $m_2 = -\frac{1}{m_1}$.

Note also that the y-intercept is -1.

Now you try

Find the equation of the line, given the following description.

a A line passes through $(0, 4)$ and is parallel to another line with gradient 2.

b A line passes through $(0, 3)$ and is perpendicular to another line with gradient 4.

5 Find the equation of the lines with the following description.

Hint: In $y = mx + c$, m is the gradient and c is the y-intercept.

a A line passes through $(0, 2)$ and is parallel to another line with gradient 4.

b A line passes through $(0, 4)$ and is parallel to another line with gradient 3.

c A line passes through $(0, -3)$ and is parallel to another line with gradient -1.

d A line passes through $(0, 3)$ and is perpendicular to another line with gradient 2.

e A line passes through $(0, -5)$ and is perpendicular to another line with gradient 3.

f A line passes through $(0, -10)$ and is perpendicular to another line with gradient $\frac{1}{2}$.

g A line passes through $(0, 6)$ and is perpendicular to another line with gradient $\frac{1}{6}$.

h A line passes through $(0, -7)$ and is perpendicular to another line with gradient $-\frac{1}{4}$.

Hint: $-\frac{1}{\left(\frac{1}{2}\right)} = -2$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 21 Finding the equation of a parallel or perpendicular line

Find the equation of the line that is:

a parallel to $y = -2x - 7$ and passes through $(1, 9)$

b perpendicular to $y = \frac{1}{4}x - 1$ and passes through $(3, -2)$

SOLUTION

a $y = mx + c$

$m = -2$

$y = -2x + c$

Substitute $(1, 9)$: $9 = -2(1) + c$

$11 = c$

$\therefore y = -2x + 11$

EXPLANATION

Since the line is parallel to $y = -2x - 7$, $m = -2$. Write the equation $y = mx + c$ with $m = -2$.

Substitute the given point $(1, 9)$ where $x = 1$ and $y = 9$ and solve for c.

b $y = mx + c$

$m = -\dfrac{1}{\frac{1}{4}}$

$= -1 \times \frac{4}{1}$

$= -4$

$y = -4x + c$

The gradient is the negative reciprocal of $\frac{1}{4}$.

$-1 \div \frac{1}{4} = -1 \times \frac{4}{1}$.

Substitute $(3, -2)$: $-2 = -4(3) + c$

$-2 = -12 + c$

$c = 10$

$\therefore y = -4x + 10$

Substitute $(3, -2)$ and solve for c.

Now you try

Find the equation of the line that is:

a parallel to $y = 3x - 4$ and passes through $(2, 3)$

b perpendicular to $y = -\frac{1}{2}x + 2$ and passes through $(-1, 4)$

6 Find the equation of the line that is:

a parallel to $y = x + 3$ and passes through $(1, 5)$

b parallel to $y = -x - 5$ and passes through $(1, 7)$

c parallel to $y = -4x - 1$ and passes through $(-1, 3)$

d parallel to $y = \frac{2}{3}x + 1$ and passes through $(3, -4)$

Hint: First find m. Then write $y = mx + c$. Substitute a point to find c; e.g. for $(1, 5)$ substitute $x = 1$ and $y = 5$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

e perpendicular to $y = 2x + 3$ and passes through $(2, 5)$

f perpendicular to $y = -4x + 1$ and passes through $(-4, -3)$

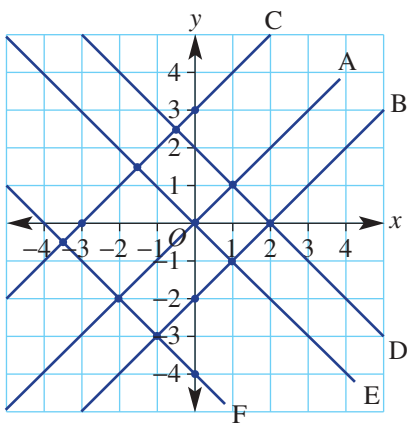
g perpendicular to $y = \frac{2}{3}x - 4$ and passes through $(4, -1)$

h perpendicular to $y = -\frac{2}{7}x - \frac{3}{4}$ and passes through $(-8, 3)$

PROBLEM-SOLVING AND REASONING

7, 8 | 7, 9, 10

7 The lines on this grid are either parallel or perpendicular to each other.



a What is the gradient of each of these lines?

i A **ii** D

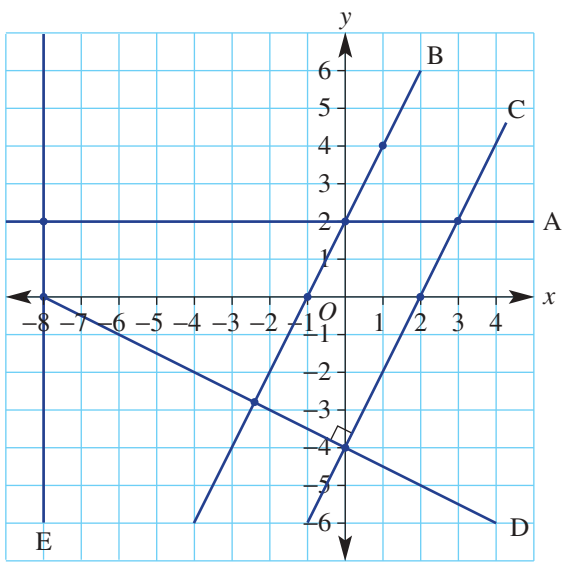
b What is the equation for the following lines?

i A **ii** B

iii C **iv** D

v E **vi** F

8 In its original position, the line A has equation $y = 2$, as shown.



a Line A is rotated to form line B. What is its new equation?

b Line B is shifted to form line C. What is its new equation?

c Line C is rotated 90° to form line D. What is its new equation?

d Line A is rotated 90° to form line E. What is its new equation?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 Recall that the negative reciprocal of, say, $\frac{2}{3}$ is $-\frac{3}{2}$.

Use this to help find the equation of a line that:

a passes through $(0, 7)$ and is perpendicular to $y = \frac{2}{3}x + 3$

b passes through $(0, -2)$ and is perpendicular to $y = \frac{3}{2}x + 1$

c passes through $(0, 2)$ and is perpendicular to $y = -\frac{4}{5}x - 3$

d passes through $(1, -2)$ and is perpendicular to $y = -\frac{2}{3}x - 1$

10 Decide if the graphs of each pair of rules will be parallel, perpendicular or neither.

a $2y + x = 2$ and $y = -\frac{1}{2}x - 3$

b $x - y = 4$ and $y = x + \frac{1}{2}$

c $8y + 2x = 3$ and $y = 4x + 1$

d $3x - y = 2$ and $x + 3y = 5$

Hint: First write each rule in the form $y = mx + c$.

ENRICHMENT: Perpendicular and parallel geometry — 11, 12

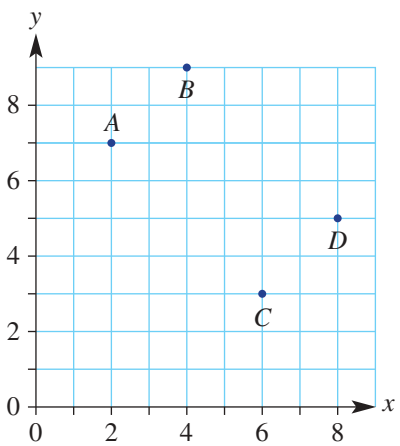
11 A quadrilateral, $ABCD$, has vertex coordinates $A(2, 7)$, $B(4, 9)$, $C(6, 3)$ and $D(8, 5)$.

a Find the gradient of these line segments.

i AB **ii** CD **iii** BD **iv** AC

b What do you notice about the gradient of the opposite sides?

c What type of quadrilateral is $ABCD$?



12 The vertices of triangle ABC are $A(0, 0)$, $B(3, 4)$ and $C\left(\frac{25}{3}, 0\right)$.

a Find the gradient of these line segments.

i AB **ii** BC **iii** CA

b What type of triangle is $\triangle ABC$?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6H Graphing straight lines using intercepts EXTENDING

Learning intentions for this section:

- To know how to find the x- and y-intercepts of a straight-line graph from its equation
- To know that only two points are needed to sketch a straight line
- To be able to sketch a linear graph using its x- and y-intercepts

Past, present and future learning:

- This section addresses a Stage 5 Path-to-Advanced topic called *Linear relationships C*
- It consolidates and extends the concepts in Sections 5B and 5C of our Year 9 book
- The skills presented are occasionally useful in Stage 6 Standard

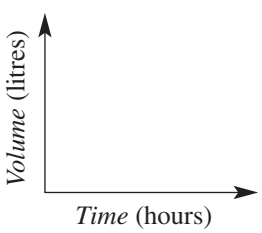
Only two points are required to define a straight line. Two convenient points are the x- and y-intercepts. These are the points where the graph crosses the x- and y-axis, respectively.

The axis intercepts are quite significant in practical situations. For example, imagine that 200 m^3 of dirt needs to be removed from a construction site before foundations for a new building can be laid. The graph of volume remaining versus time taken to remove the dirt has a y-intercept of 200, showing the total volume to be removed, and an x-intercept showing the time taken for the job.

Lesson starter: Leaking water

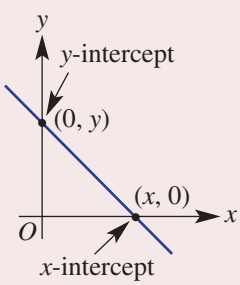
A family that is camping has 20 litres of water in a plastic container. The water begins to slowly leak at a rate of 2 litres per hour.

- Sketch two axes and label them 'Volume' and 'Time'.
- At $t = 0$, what is the volume of water in the container? Mark and label this point on the volume axis.
- How long will it take the water container to be empty? Mark this point on the time axis.
- Draw a straight line passing through the two points.
- Write the coordinates of the volume axis intercept and the time axis intercept. Follow the order (i.e. time, volume).
- Can you suggest a rule for finding the volume of the water in the container after t hours?



KEY IDEAS

- A straight line can be sketched by finding axis intercepts.
- The ***x*-intercept** (where the line cuts the x-axis) is where $y = 0$.
  Find the x-intercept by substituting $y = 0$ into the equation.
- The ***y*-intercept** is where $x = 0$.
  Find the y-intercept by substituting $x = 0$ into the equation.
- Once the axis intercepts are found, plot the points and draw a line passing through them to form the straight-line graph.



Key terminology: x-intercept, y-intercept, x-axis, y-axis

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 6H

UNDERSTANDING	1, 2	1

1 Copy and complete these sentences.

a The x-intercept is where _____ $= 0$.

b The y-intercept is where _____ $= 0$.

2 Plot the following x- and y-intercept coordinates and draw a straight line passing through them to form the graph.

a $(0, 3)$ and $(-2, 0)$ **b** $(0, -1)$ and $(2, 0)$

c $(0, -4)$ and $(-1, 0)$ **d** $(0, 3)$ and $(5, 0)$

FLUENCY	3(½), 4, 5	3(½), 4, 5(½), 6

Example 22 Sketching lines in the form $ax + by = d$, using x- and y-intercepts

Sketch a graph of $3x - 4y = 12$ by finding the x- and y-intercepts.

SOLUTION

$3x - 4y = 12$

x-intercept ($y = 0$):

$3x - 4(0) = 12$

$3x = 12$

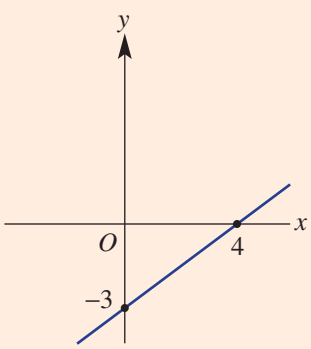
$x = 4$

y-intercept ($x = 0$):

$3(0) - 4y = 12$

$-4y = 12$

$y = -3$



EXPLANATION

Find the x-intercept by substituting $y = 0$.

Simplify. Any number multiplied by 0 is 0.

Divide both sides by 3 to solve for x.

Find the y-intercept by substituting $x = 0$.

Simplify and retain the negative sign.

Divide both sides by -4 to solve for y.

Sketch the graph by first marking the x-intercept $(4, 0)$ and the y-intercept $(0, -3)$ and then drawing a line passing through them.

Now you try

Sketch a graph of $5x + 3y = 15$ by finding the x- and y-intercepts.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Sketch graphs of the following equations by finding the x- and y-intercepts.

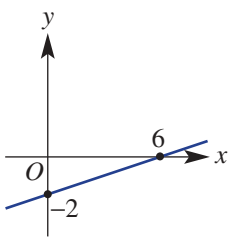
a $3x - 2y = 6$ **b** $2x + 6y = 12$
c $3x - 4y = 12$ **d** $5x - 2y = 20$
e $-2x + 7y = 14$ **f** $-x + 3y = 3$
g $-x - 2y = 8$ **h** $-5x - 9y = 90$

Hint: Two calculations are required: Substitute $y = 0$ and solve for x-intercept. Substitute $x = 0$ and solve for y-intercept.

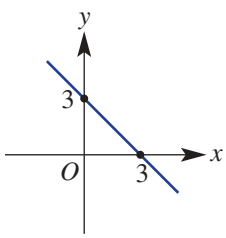
4 Match each of the following linear equations of the form $ax + by = d$ to one of the graphs shown.

a $x + y = 3$ **b** $2x - y = 4$ **c** $x - 3y = 6$

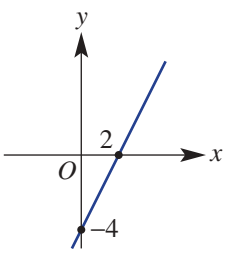
i



ii



iii



Hint: First find the x- and y-intercepts for the line equation.

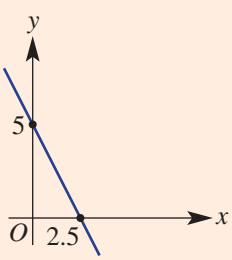
Example 23 Sketching lines in the form $y = mx + c$, using x- and y-intercepts

Sketch the graph of $y = -2x + 5$ by finding the x- and y-intercepts.

SOLUTION

$y = -2x + 5$

		EXPLANATION
x-intercept ($y = 0$):	$0 = -2x + 5$	Substitute $y = 0$.
	$-5 = -2x$	Subtract 5 from both sides.
	$x = 2.5$	Divide both sides by -2.
y-intercept is ($x = 0$):	$y = -2(0) + 5$	Substitute $x = 0$.
	$= 5$	Simplify.



Sketch the graph by first marking the x-intercept $(2.5, 0)$ and the y-intercept $(0, 5)$.

Now you try

Sketch the graph of $y = 3x - 9$ by finding the x- and y-intercepts.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Sketch graphs of the following equations by finding the x- and y-intercepts.

a $y = 2x + 1$ **b** $y = 3x - 2$ **c** $y = -4x - 3$

d $y = -x + 2$ **e** $y = -\frac{1}{2}x + 1$ **f** $y = \frac{3}{2}x - 3$

Hint: **5e:** $0 = -\frac{1}{2}x + 1$

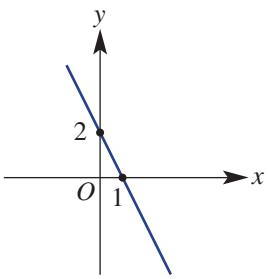
$\frac{1}{2}x = 1$

Now multiply both sides by 2.

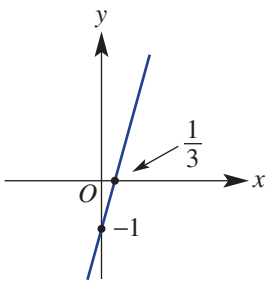
6 Match each of the following linear equations of the form $y = mx + c$ to one of the sketches shown.

a $y = x + 1$ **b** $y = 3x - 1$ **c** $y = -2x + 2$

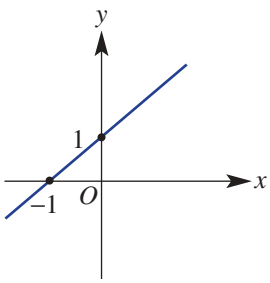
i



ii



iii



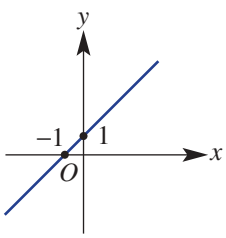
PROBLEM-SOLVING AND REASONING 7–9 8–11

7 Match each of the following linear equations to one of the graphs shown.

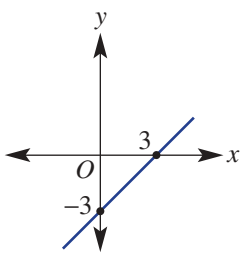
a $2x + y = 4$ **b** $x - y = 3$ **c** $y = x + 1$

d $y = -2x - 3$ **e** $3x - 5y = 15$ **f** $y = \frac{2}{5}x - 1$

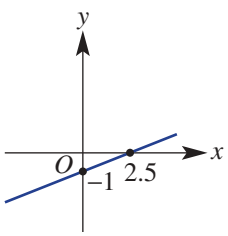
i



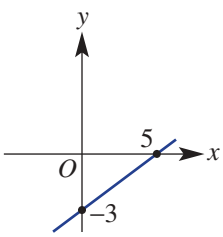
ii



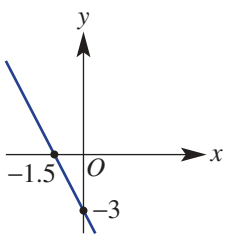
iii



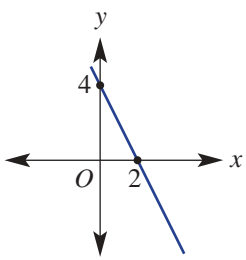
iv



v



vi



8 By first finding the x- and y-intercepts of the graphs of these equations, find the gradient in each case.

a $2x + y = 4$ **b** $x - 5y = 10$

c $4x - 2y = 5$ **d** $-1.5x + 3y = 4$

Hint: A quick sketch of each line and the axis intercepts will help to show the rise and run.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 For the graphs of each of the following equations, find:

- **i** the x- and y-intercepts
- **ii** the area of the triangle enclosed by the x- and y-axes and the graph of each equation

Remember that the area of a triangle is $A = \frac{1}{2}bh$.

a $2x - y = 4$ **b** $-3x + 3y = 6$ **c** $y = -2x - 3$ **d** $y = \frac{1}{2}x + 2$

10 The height, h, in metres, of a lift above ground after t seconds is given by $h = 90 - 12t$.

a How high is the lift initially (i.e. at $t = 0$)?

b How long does it take for the lift to reach the ground (i.e. at $h = 0$)?

11 If $ax + by = d$, find a set of numbers for a, b and d that give an x-intercept of $(2, 0)$ and y-intercept of $(0, 4)$.

Hint: Use trial and error to start.

ENRICHMENT: Axes intercepts using technology — 12

12 For the following equations, use technology to sketch a graph and find the x- and y-intercepts.

a $y = 2x - 4$ **b** $y = -2x - 10$ **c** $y = -x + 1$

d $y + 2x = 4$ **e** $2y - 3x = 12$ **f** $3y - 2x = 2$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6I Linear modelling

Learning intentions for this section:

- To be able to find the equation of a straight-line graph
- To be able to form a linear model from a scenario relating two variables
- To be able to use the equation, table or graph to solve a problem

Past, present and future learning:

- This section concludes the Stage 5 Core topic called *Linear relationships B*
- It consolidates and extends the concepts in Section 5J of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

When given at least two points, you can find the equation of a straight line. If the relationship between two variables is linear, then:

- the graph of the relationship is a straight line
- a rule can be written in the form $y = mx + c$.

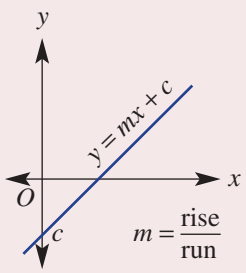
Lesson starter: Meeting room hire and catering

Josh hires a function room for a meeting. The price is \$50 for the room plus \$8 per person for refreshments. The cost can be modelled using the equation $C = 50 + 8n$, where C is the total cost and n is the number of people who attend.

- Explain why the n is attached to the 8 in the equation.
- Find the cost of the meeting if 100 people attend.
- Josh has allowed \$1000 in his budget for this meeting. What is the maximum number of people who can attend, within his budget?

KEY IDEAS

- The equation of a straight line can be determined using:
  - $y = mx + c$
  - gradient $= m = \dfrac{\text{rise}}{\text{run}}$
  - y-intercept $= c$



- If the y-intercept is not obvious, then it can be found by substituting a point.
- Vertical and horizontal lines:
  - vertical lines have the equation $x = k$, where k is the x-intercept
  - horizontal lines have the equation $y = c$, where c is the y-intercept

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

■ **Modelling** may involve:
- writing a rule linking two variables
- sketching a graph
- using the rule or the graph to help solve related problems

Key terminology: linear, gradient, *y*-intercept, modelling

Exercise 6I

UNDERSTANDING	1, 2	2

1 Each week Ava gets paid $300 plus $45 per hour. Decide which rule shows the relationship between Ava's total weekly pay, P, and the number of hours she works, n.

A $P = 300 + n$ **B** $P = 45n$ **C** $P = 300 + 45$ **D** $P = 300 + 45n$

2 Riley is 100 km from home and is cycling home at 20 km/h. Decide which rule shows the relationship between Riley's distance from home, d km, and the number of hours he has been cycling, t.

A $P = 100t$ **B** $P = 100 - 20$ **C** $P = 100 - 20t$ **D** $P = 100 + 20t$

FLUENCY	3–6(½)	3–7(½)

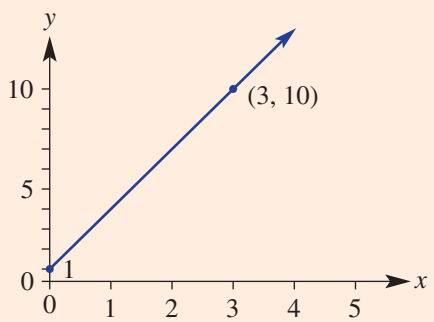
Example 24 Finding the equation of a line from a graph with a known *y*-intercept

A straight line passes through the points shown.

a Determine its gradient.

b Find the *y*-intercept.

c Write the equation of the line.



SOLUTION

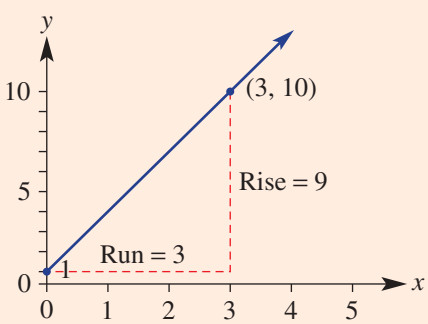
a $m = \frac{\text{rise}}{\text{run}}$

$= \frac{9}{3}$

$= 3$

EXPLANATION

Draw a triangle on the graph and decide whether the gradient is positive or negative.



Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b The y-intercept is 1, so $c = 1$. Look at where the graph meets the y-axis.

c $y = 3x + 1$ Substitute m and c into $y = mx + c$.

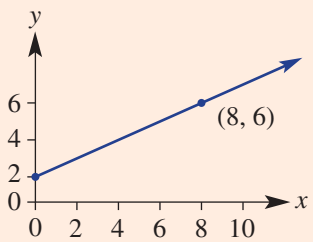
Now you try

A straight line passes through the points shown.

a Determine its gradient.

b Find the y-intercept.

c Write the equation of the line.



3 The graphs below show straight lines.

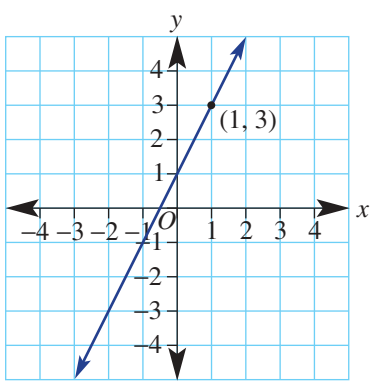
i Determine the gradient of each.

ii Find the y-intercept.

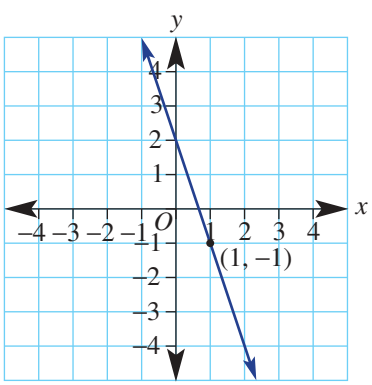
iii Write the equation of the line.

Hint: To find the rise and run, form a right-angled triangle using the y-intercept and the second point.

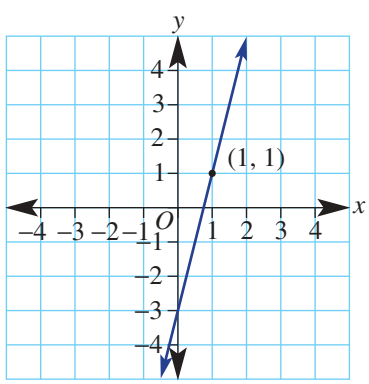
a



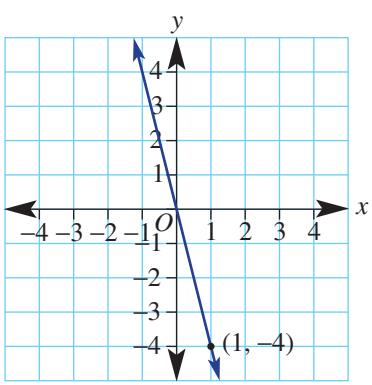
b



c

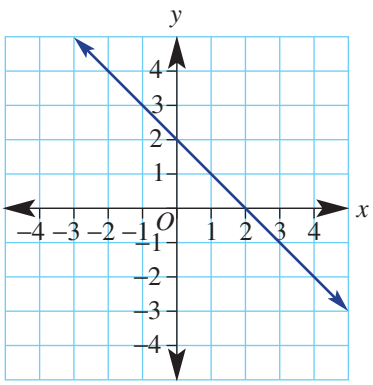


d

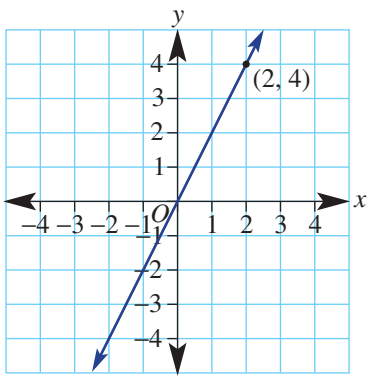


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

e



f



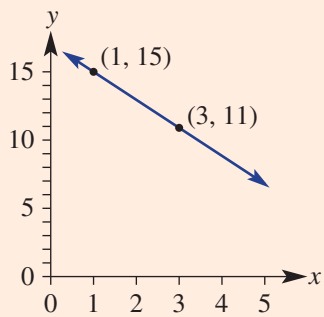
Example 25 Finding the equation of a line when given a graph with two known points

A straight line passes through the points shown.

a Determine the gradient.

b Find the y-intercept.

c Write the equation of the line.



SOLUTION

a $m = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{-4}{2}$

$= -2$

EXPLANATION

The gradient is negative.
Run $= 3 - 1 = 2$
Rise $= 11 - 15 = -4$
A 'fall' of 4 means rise $= -4$.
Simplify.

b $y = -2x + c$

$11 = -2(3) + c$

$11 = -6 + c$

$17 = c$, so the y-intercept is 17.

Write $y = mx + c$ using $m = -2$.
Substitute a chosen point into $y = -2x + c$, (use (3, 11) or (1, 15)). Here, $x = 3$ and $y = 11$.
Simplify and solve for c.

c $y = -2x + 17$

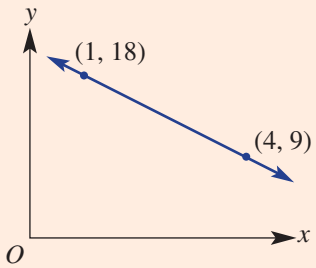
Substitute $m = -2$ and $c = 17$ into $y = mx + c$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

A straight line passes through the points shown.

a Determine the gradient.
b Find the y-intercept.
c Write the equation of the line.



4 Straight lines passing through two points are shown below.

i Determine the gradient.
ii Find the y-intercept.
iii Write the equation of the line.

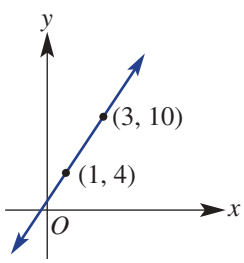
Hint:
For m, find the rise and run between the two given points. Choose either point to substitute when finding c. If $m = 4$ and $(3, 5) = (x, y)$:

$y = mx + c.$

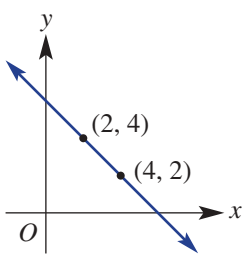
$5 = 4 \times 3 + c$

Solve for c.

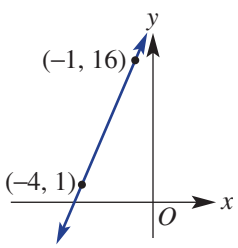
a



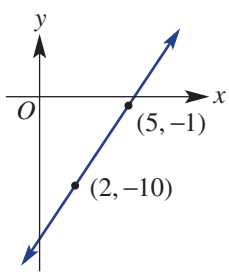
b



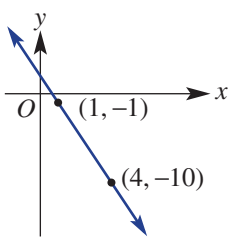
c



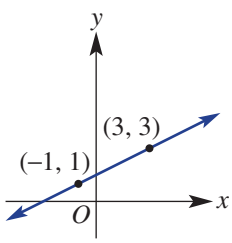
d



e



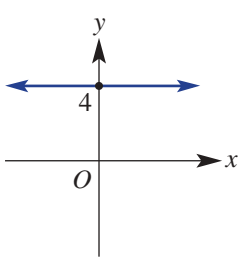
f



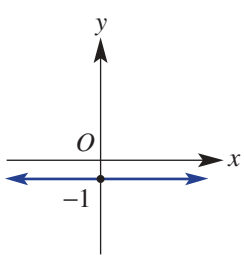
5 Determine the equation of each of the following lines. Remember from Section 6F that vertical and horizontal lines have special equations.

Hint: Vertical lines cut the x-axis and have an equation such as $x = 3$. Horizontal lines cut the y-axis and have an equation such as $y = -4$.

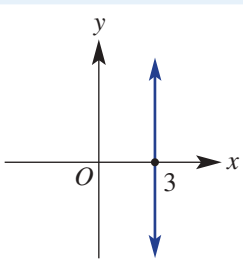
a



b

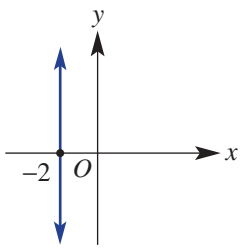


c

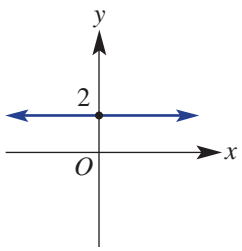


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

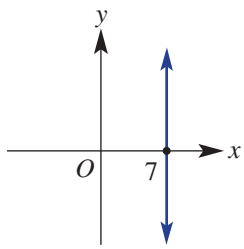
d



e

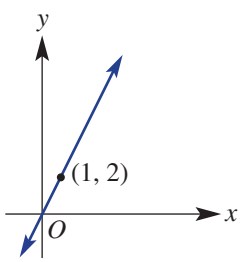


f

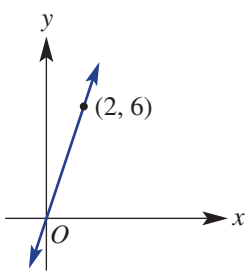


6 Remember that equations of graphs that pass through the origin are of the form $y = mx$ (since $c = 0$). Find the equation of each of these graphs.

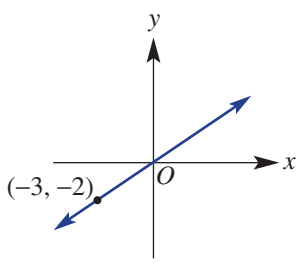
a



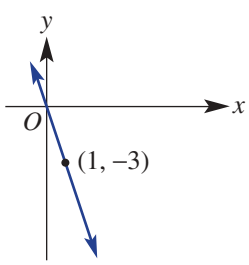
b



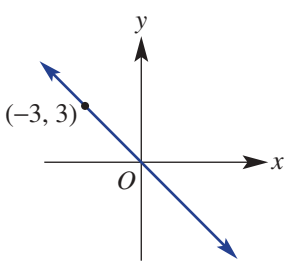
c



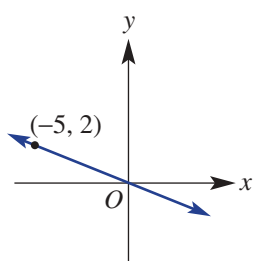
d



e



f



7 For the line that passes through the following pairs of points, find:

i the gradient **ii** the equation of the line

a (0, 0) and (1, 7) **b** (0, 0) and (2, –3)

c (–1, 1) and (1, 3) **d** (–2, 3) and (2, –3)

e (–4, 2) and (7, 2) **f** (3, –3) and (3, 1)

Hint: Substitute the gradient and a point in $y = mx + c$ to find c in part **ii**.

PROBLEM-SOLVING AND REASONING 8, 9 8, 10, 11

Example 26 Modelling with linear graphs

A model gets paid \$200 plus \$100 for each hour of work. If they earn \$$C$ for t hours of work, complete the following.

a Write a rule for C in terms of t.

b Sketch a graph, using t between 0 and 10.

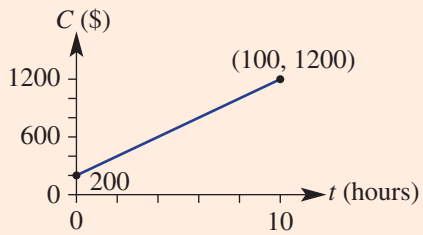
c Use your rule to find:

i the amount earned after working 8 hours

ii the number of hours worked if \$1800 is earned

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION	EXPLANATION
a $C = 100t + 200$	\$100 is earned for each hour and \$200 is a fixed amount.
b 	200 is the y-intercept and the gradient is $100 = \frac{100}{1}$. For $t = 10$, $C = 100(10) + 200 = 1200$.
c i $C = 100(8) + 200$	Substitute $t = 8$ into $C = 100t + 200$.
$= 1000$	Simplify.
\$1000 is earned.	Write your answer in words.
ii $1800 = 100t + 200$	Substitute $C = 1800$ into $C = 100t + 200$.
$1600 = 100t$	Subtract 200 from both sides.
$t = 16$	Divide both sides by 100.
16 hours of work is completed.	Write your answer in words.

Now you try

A tennis court costs \$100 to hire plus \$20 per hour of use. If \$$C$ is the cost for t hours of hire, complete the following.

a Write a rule for C in terms of t.

b Sketch a graph, using t between 0 and 5.

c Use your rule to find:

- **i** the cost of hiring for 3 hours
- **ii** the number of hours of hire if the cost is \$260

8 A shearer is paid \$100 travel costs plus \$4 per sheep. The shearer's pay for shearing n sheep is \$$P$.

a Write the equation for P in terms of n.

b Use the equation to find the shearer's pay for shearing 500 sheep in a week.

c How many sheep need to be shorn to earn \$1000 per week?

Hint:
Rate of pay
$y = mx + c$
P n Fixed amount

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 A landscaper charges \$100 for the initial consultation plus \$60 per hour thereafter. If the architect earns \$$A$ for t hours of work, complete the following.

Hint:
Draw a line between the points at $t = 0$ and $t = 15$.

- **a** Write a rule for A in terms of t.
- **b** Sketch a graph of A against t for t between 0 and 15.
- **c** Use your rule to find:
  - **i** the amount earned after working for 12 hours
  - **ii** the number of hours worked if the architect earns \$700

10 A man's weight when holding two empty buckets of water is 80 kg. 1 kg is added for each litre of water poured into the buckets. If the man's total weight is W kg with l litres of water, complete the following.

- **a** Write a rule for W in terms of l.
- **b** Sketch a graph of W against l for l between 0 and 20.
- **c** Use your rule to find:
  - **i** the man's weight after 7 litres of water are added
  - **ii** the number of litres of water added if the man's weight is 109 kg

11 The amount of water (W litres) in a leaking tank after t hours is given by the rule $W = -2t + 1000$.

- **a** State the gradient and y-intercept for the graph of the rule.
- **b** Sketch a graph of W against t for t between 0 and 500.
- **c** State the initial water volume at $t = 0$.
- **d** Find the volume of water after:
  - **i** 320 hours **ii** 1 day **iii** 1 week
- **e** Find the time taken, in hours, for the water volume to fall to:
  - **i** 300 litres **ii** 185 litres

ENRICHMENT: Production lines — 12

12 An assembly plant needs to order some new parts. Three companies can supply them but at different rates.

- Mandy's Millers charge: set-up fee \$0 + \$1.40 per part
- Terry's Turners charge: set-up fee \$3000 + \$0.70 per part
- Lenny's Lathes charge: set-up fee \$4000 + \$0.50 per part

- **a** Complete a table of values similar to the following for each of the companies.

No. of parts (p)	0	1000	2000	3000	4000	5000	6000	7000	8000	9000
Cost (C)										

- **b** Plot a graph of the total cost against the number of parts for each company on the same set of axes. Make your axes quite large as there are three graphs to complete.
- **c** Use the graphs to find the lowest price for:
  - **i** 1500 parts **ii** 1000 parts **iii** 6500 parts **iv** 9500 parts
- **d** Advise the assembly plant when it is best to use Mandy's, Terry's or Lenny's company.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6J Direct variation

Learning intentions for this section:

- To understand what it means if two variables are directly proportional
- To know the form of the equation and graph of two variables in direct proportion
- To be able to form and use an equation for two directly proportional variables
- To be able to find the constant of proportionality for two variables in direct proportion

Past, present and future learning:

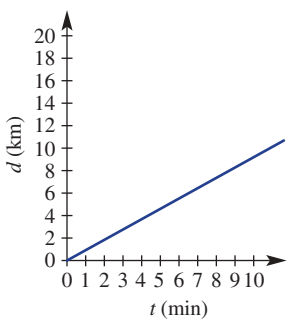
- This section addresses the Stage 5 Path topic called *Variation and rates of change A*
- It consolidates and extends the concepts in Section 5F of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formula from memory in the HSC Examination

Direct variation occurs when the rate of change of one variable with respect to the other is constant. So if one variable increases, then the other also increases and at the same rate. For example, distance is directly proportional to speed because, in a given time, if the speed is doubled then the distance travelled is doubled also.

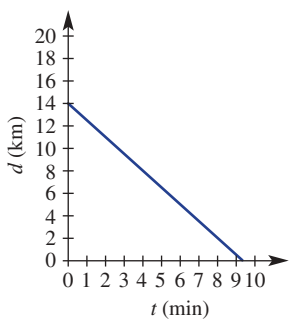
Lesson starter: Discovering the features of direct variation

Here are five different travel graphs showing how the distance from home varies with time.

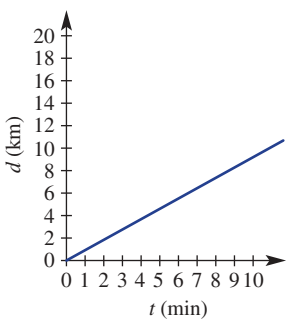
A



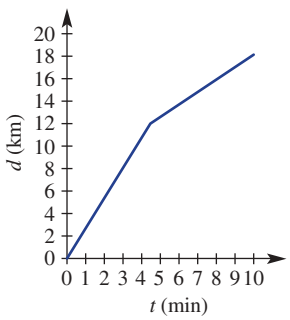
B



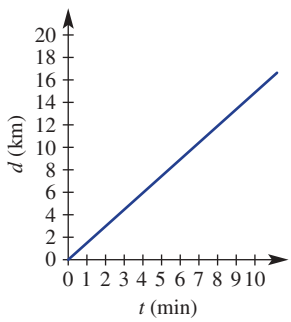
C



D



E



List all the graphs that show the following features.

- The distance from home increases as the time increases.
- The distance from home decreases as the time increases.
- The person travels at a constant speed throughout the trip.
- The person starts from home.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- The person starts from home and also travels at a constant speed throughout the trip.
- The graph starts at $(0, 0)$ and the gradient is constant.
- The distance is directly proportional to the time (i.e. the equation is of the form $y = mx$).

List three features of graphs that show when two variables are directly proportional to each other.

KEY IDEAS

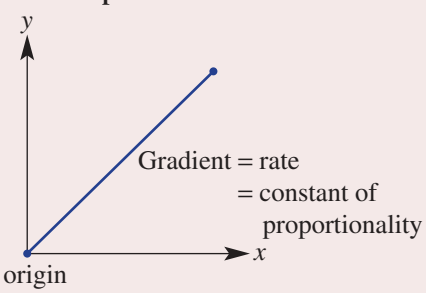
- **Direct variation** is also known as direct proportion.
- In this table, the y-value is twice the x-value.

x	0	1	2	3	4	5
y	0	2	4	6	8	10

  We can say: x and y vary directly,
  or y is directly proportional to x,
  or y varies directly as x.
  We can write $y \propto x$ or $y = kx$,
  where k is the constant of variation.

- For two variables that are **directly proportional**:
  - Both variables will increase together or decrease together at the same rate. For example, the cost of buying some sausages is directly proportional to the weight of the sausages. If the weight increases, then the cost increases; when the weight decreases, the cost decreases.

Graph of direct variation



  - The rate of change of one variable with respect to the other is constant.
  - The graph is a straight line passing through the origin $(0, 0)$; i.e. the equation is of the form $y = mx$.
  - The equation is usually written as $y = kx$, where k is the constant of proportionality.
  - The **constant of proportionality**, k, is the gradient with units, it is the same as the rate.

- Everyday examples of direct variation include:
  - Renting a house – 1 week for $400, 2 weeks for $800, etc.
  - Buying tickets – 1 for $2, 2 for $4, etc.
  - Driving at a constant speed – 1 hour, 60 km; 2 hours 120 km, etc.
  - Converting money from Australian dollars to US dollars
  - Converting inches to centimetres

Key terminology: direct variation, directly proportional, constant of proportionality

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 6J

UNDERSTANDING | 1–3 | 2

1 Write in the missing words for these statements: *increases* or *decreases*.

a As the volume of fuel decreases, the distance a car can travel _____.

b As the volume of fuel decreases, the cost of filling the tank _____.

2 **a** Which of these equations show that y is directly proportional to x?

i $y = 2x + 4$ **ii** $y = 3x$ **iii** $y = x - 2$

iv $y = 8x + 5$ **v** $y = 70x$

Hint: The direct proportion equation is $y = kx$, where k is the constant of proportionality.

b State the value of k, the constant of proportionality, for each of these direct proportion equations.

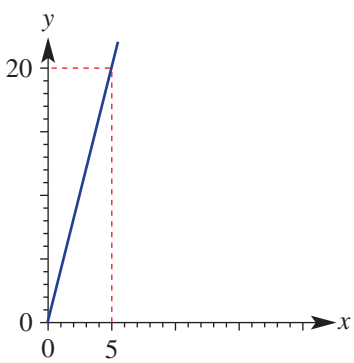
i $y = 5x$ **ii** $y = 12x$ **iii** $d = 40t$

iv $V = 22t$ **v** $C = 7.5w$

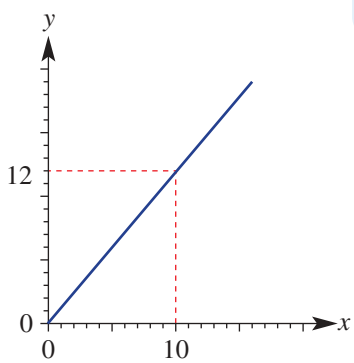
3 For each of the following graphs of direct variation, determine the value of k and write the equation in the form $y = kx$.

Hint: k = gradient = $\frac{\text{rise}}{\text{run}}$

a



b



FLUENCY | 4–6, 8 | 5–7, 8(½)

Example 27 Forming direct proportion equations when given the constant of proportionality, *k*

For a fixed price per litre, the cost (C) in dollars of buying fuel is directly proportional to the number (n) of litres.

a Write the direct proportion equation, given that $k = \$1.45$/L.

b Use this equation to calculate the cost of 63 L.

SOLUTION	EXPLANATION
a $C = 1.45n$	$y = kx$ becomes $C = kn$, where $k = 1.45$.
b $C = 1.45 \times 63$	Substitute $n = 63$ into the equation.
$= \$91.35$	Write the answer in dollars.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

For a fixed price per litre, the cost (C) in dollars of buying fuel is directly proportional to the number (n) of litres of fuel pumped.

a Write the direct proportion equation, given that $k = \$1.38/\text{L}$.

b Use this equation to calculate the cost of 48 L.

4 For a fixed rate of pay, wages (W) in dollars are directly proportional to the number (n) of hours worked.

a Write the direct proportion equation, given that $k = \$11.50/\text{h}$.

b Use this equation to calculate the wages earned for 37.5 hours worked.

5 At a fixed flow rate, the volume (V) in litres of water flowing from a tap is directly proportional to the amount of time (t) the tap has been turned on.

a Write the direct proportion equation, given that $k = 6$ L/min.

b Use this equation to calculate the volume of water, in litres, flowing from a tap for 4 hours.

c Change the constant of proportionality to units of L/day and rewrite the equation with this new value of k.

d Use this equation to calculate the volume of water, in litres, flowing from a tap for 1 week.

Hint: There are $60 \times 24 = 1440$ minutes in a day.

Example 28 Forming direct proportion equations from given information

The amount of wages Sonali earns is in direct proportion to the number of hours she works.

a Find the constant of proportionality, k, given that Sonali earned \$166.50 in 6 hours.

b Write the direct proportion equation relating Sonali's wages (W) in dollars and the number of hours (n) that she worked.

c Calculate the wages earned for 8 hours and 45 minutes of work.

d Calculate the number of hours and minutes Sonali must work to earn \$259.

SOLUTION	EXPLANATION
a $k = \frac{166.50}{6}$	The constant of proportionality, k, is the rate of pay.
$= \$27.75/\text{h}$	Include units in the answer.
b $W = 27.75n$	$y = kx$ becomes $W = kn$, where $k = 27.75$.
c $W = 27.75n$	Write the equation.
$= 27.75 \times 8.75$	45 min $= 45 \div 60 = 0.75$ h. Substitute $n = 8.75$.
$= \$242.81$	Write \$ in the answer and round to two decimal places.

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

d

$W = 27.75n$	Write the equation.
$259 = 27.75n$	Substitute $W = 259$.
$\frac{259}{27.75} = n$	Divide both sides by 27.75.
$n = 9.\dot{3}$	
∴ Sonali must work for 9 hours and 20 minutes.	Write the answer in words.
Check: $W = 27.75 \times 9.\dot{3} = \$259$	Check that your answer is correct.

Now you try

Daniel's wages earned are in direct proportion to the hours he works at the local service station.

a Find the constant of proportionality, k, given that Daniel earned \$200 in 8 hours.

b Write the direct proportion equation relating Daniel's wages (W) in dollars and the number of hours (n) worked.

c Calculate the wage earned for 6 hours of work.

d Calculate the number of hours Daniel must work to earn a wage of \$237.50.

6 The amount that a farmer earns from selling wheat is in direct proportion to the number of tonnes harvested.

a Find the constant of proportionality, k, given that a farmer receives \$8296 for 34 tonnes of wheat.

b Write the direct proportion equation relating selling price (P) in dolllars and number of tonnes (n).

c Calculate the selling price of 136 tonnes of wheat.

d Calculate the number of tonnes of harvested wheat that is sold for \$286 700.

7 When flying at a constant speed, the distance that an aeroplane has travelled is in direct proportion to the time it has been flying.

a Find the constant of proportionality, k, given that the plane flies 1161 km in 1.5 h.

b Write the direct proportion equation relating distance (d) in km and time (t) in hours.

c Calculate the time taken, in hours, for the aeroplane to fly from Sydney to Perth, a distance of around 3300 km. Round your answer to two decimal places.

d Calculate the distance that the plane would fly in 48 minutes.

Hint: Convert 48 mins to hours in part **d**.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

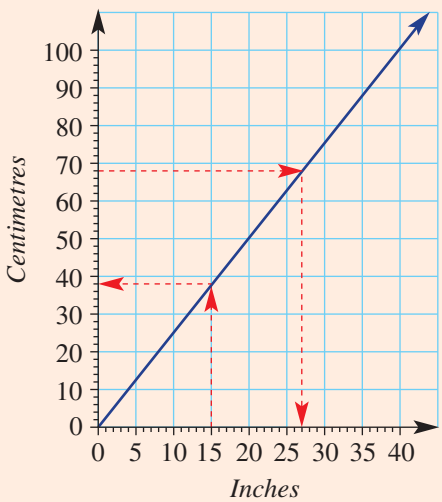
Example 29 Using a unit conversion graph

A length measured in centimetres is directly proportional to that length in inches. Use the graph below to make the following unit conversions.

a 15 inches to centimetres

b 68 cm to inches

Length unit conversion graph



SOLUTION

a 38 cm

EXPLANATION

Start at 15 inches. Now move up to the line and then across to the centimetre scale.

b 27 inches

Start at 68 cm. Now move across to the line and then down to the inches scale. Round your answer to the nearest whole number.

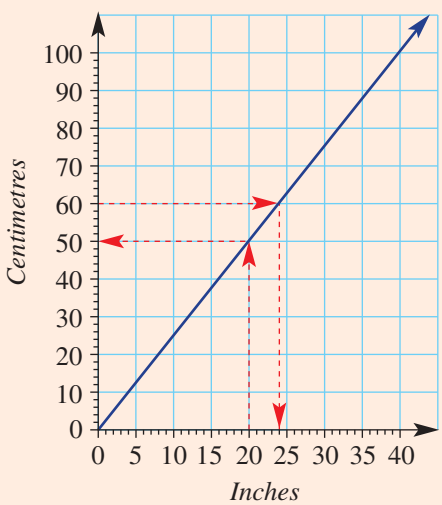
Now you try

A length, measured in centimetres, is directly proportional to that length in inches. Use the graph below to make the following unit conversions.

a 20 inches to cm

b 60 cm to inches

Length unit conversion graph



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 Use the graph in Example 29 to make the following unit conversions. Round your answers to the nearest whole number.

- **a** 19 inches to cm
- **b** 25 cm to inches
- **c** 1 foot (12 inches) to cm
- **d** 1 hand (4 inches) to cm
- **e** the height, in inches, of the world's verified shortest man, who was 54.6 cm tall
- **f** the height, in cm, of a miniature pony that is 7.5 hands high
- **g** the height, in cm, of the world's shortest living woman, who is about 2 feet and 1 inch tall
- **h** the length, in cm, of a giant Australian earthworm that is 39.5 inches long

PROBLEM-SOLVING AND REASONING

9–11 | 10–12, 13(½), 14

9 Use the length unit conversion graph in Example 29 to answer these questions.

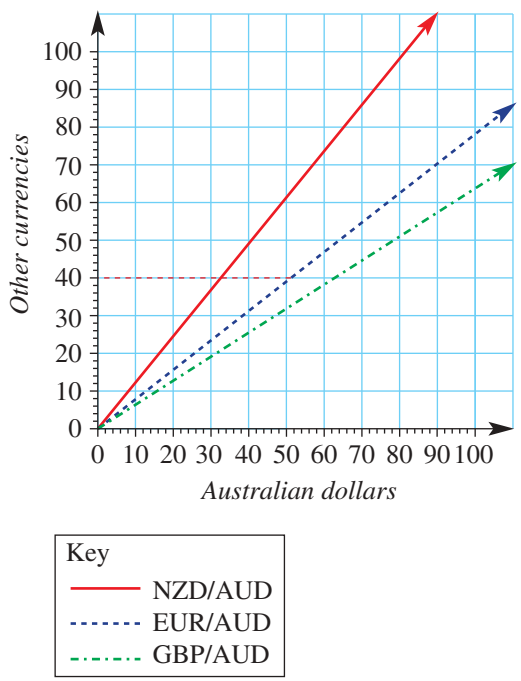
Hint: Gradient $= \frac{\text{rise}}{\text{run}} = \frac{\text{cm}}{\text{inches}}$

k = rate
= gradient with units

- **a** Convert 94 cm to inches and use these values to find the gradient of the line, to two decimal places.
- **b** State the conversion rate in cm/inch, to two decimal places.
- **c** State the value of k, the constant of proportionality, to two decimal places.
- **d** Write the direct proportion equation between centimetres (y) and inches (x).
- **e** Use the equation to calculate the number of centimetres in 50 inches.

10 At any given time, an amount in Australian dollars is directly proportional to that amount in a foreign currency. This graph shows the direct variation (at a particular time) between Australian dollars (AUD) and New Zealand dollars (NZD), European euros (EUR) and Great British pounds (GBP).

Currency conversion graph



- **a** Use the graph to make these currency conversions.
  - **i** 80 AUD to NZD
  - **ii** 80 AUD to EUR
  - **iii** 80 AUD to GBP
  - **iv** 50 NZD to AUD
  - **v** 32 EUR to AUD
  - **vi** 26 GBP to AUD

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b Answer these questions, using the line that shows the direct variation between the euro (EUR) and the Australian dollar (AUD).

- **i** Find 40 EUR in AUD and, hence, find the gradient of the line, to one decimal place.
- **ii** State the conversion rate in EUR/AUD, to one decimal place.
- **iii** State the value of k, the constant of proportionality, to one decimal place.
- **iv** Write the direct proportion equation between EUR (y) and AUD (x).
- **v** Use the equation to calculate the value in euros of 625 Australian dollars.

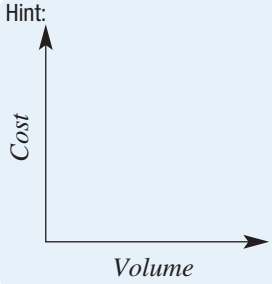
Hint: The rate in EUR/AUD = ? euros, per 1 Australian dollar. The rate = the constant of proportionality, k.

11 The cost (C) of buying fuel is directly proportional to the volume (V) of fuel pumped.

a Copy and complete this table for the cost of diesel, using the rule $C = 1.5V$.

Volume (V) of diesel, in litres	0	10	20	30	40	50
Cost (C), in dollars						

Hint:



b Plot these points and then use a ruler to draw a line through them to form a neat graph of volume (V) vs cost (C).

c Find the gradient, m, of the line.

d At what rate is the cost of fuel increasing, in \$/L?

e What is the constant of proportionality, k, between cost and volume?

12 For each of the following pair of variables, describe, using sentences, why the variables are in direct proportion to each other or why they are not in direct proportion.

a The number of hours worked and wages earned at a fixed rate per hour.

b The cost of buying tomatoes and the number of kilograms at a fixed price per kilogram.

c The speed and time taken to travel a certain distance.

d The size of a movie file and the time taken to download it to a computer at a constant rate of kB/s.

e The cost of a taxi ride and the distance travelled. The cost includes flag fall (i.e. a starting charge) and a fixed rate of \$/km.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

13 Convert the following rates to the units given in brackets.

a \$9/h (cents/min)

b \$24/h (cents/min)

c 10.8 L/h (mL/s)

d 18 L/h (mL/s)

e 72 km/h (m/s)

f 18 km/h (m/s)

g \$15/kg (cents/g)

h \$32/kg (cents/g)

i 400 g/month (kg/year)

j 220 g/day (kg/week)

Hint:
\$9/h = 900 cents in 1 hour
= 900 cents in 60 mins
= 15 cents in 1 min
= 15 cents/min

14 For a fixed speed, the distance (d) that a car travels is directly proportional to time (t).

a Write the direct proportion equation, given that $k = 90$ km/h.

b Change the constant of proportionality to units of m/s and rewrite the equation with this new value of k.

c Use this equation to calculate the distance, in metres, that a car would travel in 4 seconds.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: Currency conversions — 15

15 At any given time, an amount of money in a foreign currency is in direct proportion to the corresponding amount in Australian dollars.
For example, if 8 Hong Kong dollars is equivalent to 1 Australian dollar, the conversion rate is HK $8/AUD and the direct proportion equation is $HK = 8 \times AUD$.

- To change A$24 to Hong Kong dollars, we must substitute 24 for AUD:
  $HK = 8 \times 24$
  $= \$192$
- To change HK $24 to Australian dollars, we substitute 24 for HK:
  $24 = 8 \times AUD$
  $\frac{24}{8} = AUD$
  $AUD = \$3$

Follow the example above to complete the following questions.

a Singapore dollar (SGD)
- **i** Write the direct proportion equation, given the conversion rate is 1.2 SGD/AUD.
- **ii** Convert AUD 240 to SGD.
- **iii** Convert SGD 240 to AUD.

b Chinese yuan (CNY)
- **i** Write the direct proportion equation, given the conversion rate is 6.47 CNY/AUD.
- **ii** Convert AUD 75 to CNY.
- **iii** Convert CNY 75 to AUD.

c South African rand (ZAR)
- **i** Write the direct proportion equation, given the conversion rate is 9.5 ZAR/AUD.
- **ii** Convert AUD 50 to ZAR.
- **iii** Convert ZAR 50 to AUD.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6K Inverse variation

Learning intentions for this section:

- To understand what it means if two variables are inversely proportional
- To be able to find the constant of proportionality for two inversely proportional variables
- To be able to form and use an equation for inversely proportional variables

Past, present and future learning:

- This section concludes the Stage 5 Path topic called *Variation and rates of change A*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formula from memory in the HSC Examination

In some situations, when the value of x is doubled, the value of y is halved.

For example:
x = number of slices one pizza is cut into
y = area of each slice

Lesson starter: Pizza slices

If the area of the whole pizza is 500 square centimetres, complete this table.

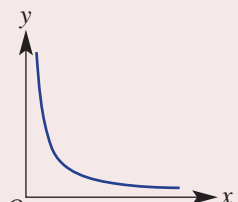
Number of slices	1	2	3	4	5	6	7	8
Area of each slice	500	250						

Note: each pair has a product of 500

KEY IDEAS

- For two **variables** with **inverse variation**:
  - When one variable is multiplied by a number, then the other variable is divided by that same number.
  - We say: 'y is inversely proportional to x', or 'y varies inversely as x'
  - We write $y \propto \frac{1}{x}$ or $y = \frac{k}{x}$

Graph of indirect or inverse variation



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- The graph is a curve showing that as x increases, then y decreases. For example, speed is indirectly or inversely proportional to travelling time. As the speed decreases, the time taken to travel a particular distance increases; as the speed increases, the time taken to travel a particular distance decreases.
- The equation is usually written as $y = \frac{k}{x}$, where k is the constant of proportionality.
- The graph is a hyperbola.

Key terminology: inverse variation, inversely proportional, constant of proportionality, variables

Exercise 6K

UNDERSTANDING	1–3	2, 3

1 For each part, state whether the variables are directly or indirectly (i.e. inversely) proportional.

a As the number of questions correct increases, the total mark for the test increases.

b As the speed decreases, the time taken to travel a particular distance increases.

c As the number of hours worked increases, the pay for that work increases.

d As the size of a computer file decreases, the time required to transfer it decreases.

e As the rate of typing words per minute increases, the time needed to type an assignment decreases.

2 Which of the following equations shows inverse proportion?

a $y = \frac{2}{x}$ **b** $y = 3x$ **c** $T = \frac{2.7}{s}$

Hint: Inverse proportion equation is of the form $y = \frac{k}{x}$.

3 Determine the value of k in $y = \frac{k}{x}$ when:

a $x = 2$ and $y = 3$ **b** $y = 2$ and $x = 0.5$

FLUENCY	4, 5	4–6

Example 30 Working with inverse proportion

The length of a rectangle, l metres, with a fixed area of 2 m^2, varies inversely with the breadth, b metres, such that $l = \frac{2}{b}$.

a Complete this table of values.

b	1	2	3	4
l				

b Sketch a graph of l vs b using your table of values to help. Use b on the x-axis.

c Find the length of the rectangle if the breadth is 5 m.

d Find the breadth of the rectangle if the length is 0.5 m.

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

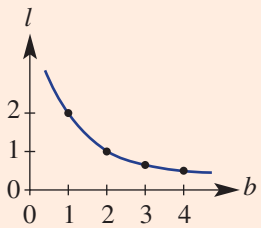
EXPLANATION

a

b	1	2	3	4
l	2	1	$\frac{2}{3}$	$\frac{1}{2}$

Substitute each value of b into the rule $l = \frac{2}{b}$.

b



Plot the points from the table and join the points in a smooth curve.

c

$$l = \frac{2}{b}$$

$$l = \frac{2}{5}$$

length = 0.4 metres

Substitute $b = 5$ into the given rule.

d

$$l = \frac{2}{b}$$

$$0.5 = \frac{2}{b}$$

$$0.5b = 2$$

$$b = 4$$

breadth = 4 metres

Substitute $l = 0.5$ into the given rule then solve for b. Note that $2 \div 0.5 = 4$.

Now you try

The length of a rectangle, l metres, with a fixed area of 12 m^2, varies inversely with the breadth, b metres, such that $l = \frac{12}{b}$.

a Complete this table of values.

b	1	3	6	12
l				

b Sketch a graph of l vs b using your table of values to help. Use b on the x-axis.

c Find the length of the rectangle if the breadth is 4 m.

d Find the breadth of the rectangle if the length is 2.5 m.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 The length of a rectangle, l metres, with a fixed area of 6 m^2, varies inversely with the breadth, b metres, such that $l = \frac{6}{b}$.

a Complete this table of values.

b	1	2	3	6
l				

b Sketch a graph of l vs b using your table of values to help. Use b on the x-axis.

c Find the length of the rectangle if the breadth is 8 m.

d Find the breadth of the rectangle if the length is 1.5 m.

5 The time taken (t days) to paint a house is inversely proportional to the number of painters (n) such that $t = \frac{16}{n}$.

a Find the time taken to paint a house for the following number of painters, n.

i 1

ii 4

iii 8

b Sketch a graph of t vs n with n on the x-axis.

c Find how long it will take for 10 painters to paint a house.

d Find the number of painters required to paint a house in 8 days.

6 y is inversely proportional to x such that $y = \frac{20}{x}$.

a Find the value of y if:

i $x = 5$

ii $x = 8$

b Find the value of x if:

i $y = 10$

ii $y = 40$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING 7–9 7–10

Example 31 Finding and using the constant of proportionality

y is inversely proportional to x; i.e. $y = \frac{k}{x}$.

a Find the constant of proportionality, k, if when $x = 2$, $y = 3$.
b Find the value of y if $x = 6$.

SOLUTION

a $y = \frac{k}{x}$

$3 = \frac{k}{2}$

$k = 2 \times 3$

$= 6$

EXPLANATION

Substitute $x = 2$ and $y = 3$ into the rule and solve for k.

b $y = \frac{6}{x}$

Write the rule using $k = 6$.

$y = \frac{6}{6}$

Substitute $x = 6$ and simplify to find the value of y.

$y = 1$

Now you try

y is inversely proportional to x; i.e. $y = \frac{k}{x}$.

a Find the constant of proportionality, k, if when $x = 4$, $y = 2$.
b Find the value of y if $x = 1$.

7 y is inversely proportional to x; i.e. $y = \frac{k}{x}$.

a Find the constant of proportionality, k, if when $x = 5$, $y = 10$.
b Find the value of y if $x = 2$.

8 For each of the following determine the constant of proportionality, k, if $y = \frac{k}{x}$. Then complete the table of values.

a

x	1	2	3	4
y	12	6		

b

x	0.5	1	2	4
y	8		2	

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 The volume, V cm^3, of gas in a container is inversely proportional to the pressure, P kg/cm^3. When the pressure is 2 kg/cm^3 the volume is 20 cm^3.

a Find the constant of proportionality k if $V = \frac{k}{P}$.

b Find the volume if the pressure is 4 kg/cm^3.

c Find the pressure if the volume is 16 cm^3.

10 The cost to rent a house for a week is $500.

a Find the cost per person if:

i 2 people rent the house

ii 5 people rent the house

b Find a rule linking the cost per person, $C, with the number of people, n.

c Find the number of people sharing the house if the cost per person is $62.50.

ENRICHMENT: Electrical current — 11

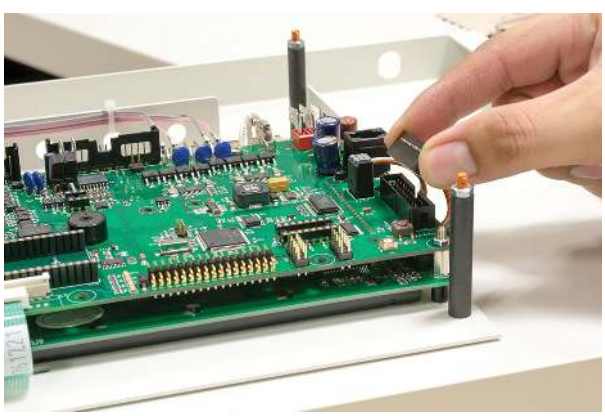
11 The current (I amps) that flows from a device is inversely proportional to the resistance (R ohms). When the current is 4 amps, the resistance is 60 ohms.

a Find a rule linking I and R.

b Find the current if the resistance is 40 ohms.

c Find the resistance if the current is 120 amps.

d What percentage increase in resistance causes a 50% reduction in current?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6L Exploring parabolas

Learning intentions for this section:

- To understand that a parabola is a graph of a quadratic relationship
- To be able to identify key features of a parabola
- To be able to sketch transformations using simple dilations, reflections and translations

Past, present and future learning:

- This section addresses the Stage 5 Core topics called *Non-linear relationships A* and *Non-linear relationships B*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Sometimes the relationship between two variables does not produce a straight-line graph. Such relationships are called non-linear relationships. The first non-linear graphs we will investigate are called parabolas.

When water comes out of a pipe or hose it forms a shape that looks very much like a parabola.

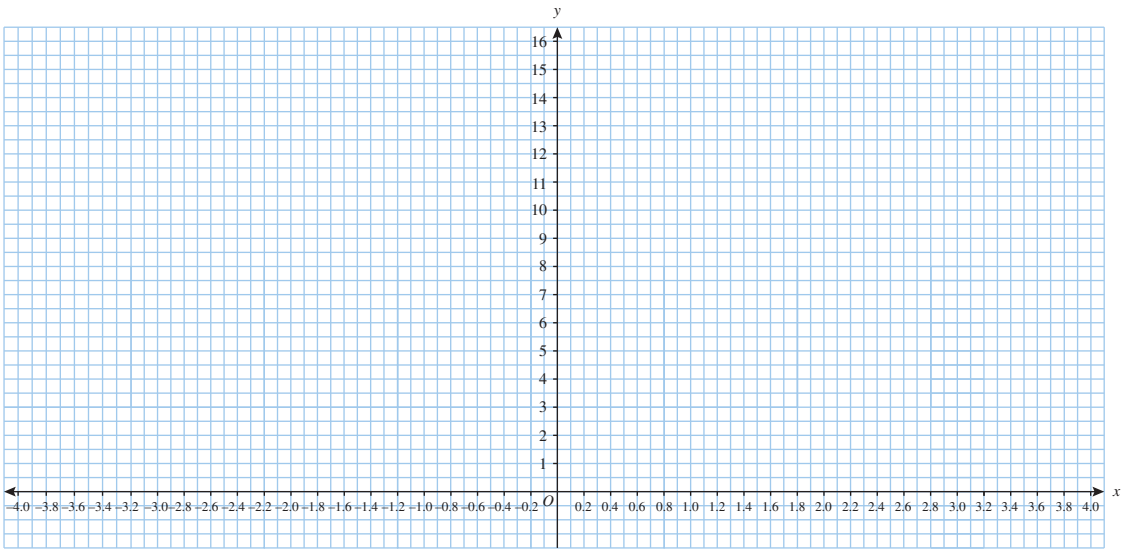
Lesson starter: Square your number!

Everybody in your class will be given one or more of these numbers.

–4	–3	–2	–1	0	0.1	0.2	0.3	0.4	0.5
0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
1.5	1.6	1.7	1.8	1.9	2	2.5	3	3.5	4

- Square the number you have been given and write down the result.
- Was your result the same as the number you were given? For which numbers is this true?
- Did your result have a different sign to the number you were given? For which numbers is this true?
- Is it possible for the result to be negative?
- Was your result greater than the number you were given? For which numbers is this true?
- Was your result less than the number you were given? For which numbers is this true?
- Use your number and the result to make a point, such as (3, 9). All students in the class will place a point on a copy of the following chart:

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

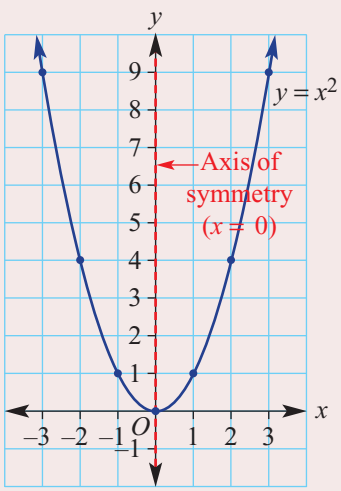


KEY IDEAS

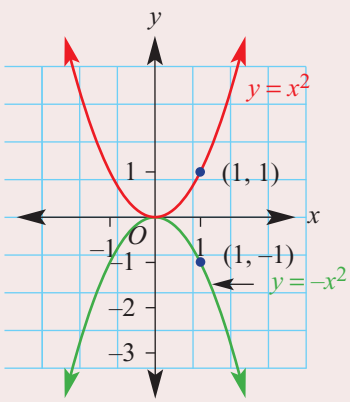
- A **parabola** is the graph of a non-linear relationship. The basic parabola has the equation $y = x^2$.
  Key points on $y = x^2$:

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

  - The **vertex** (or turning point) is (0, 0).
  - It is a minimum turning point.
  - **Axis of symmetry** is $x = 0$ (the y-axis).



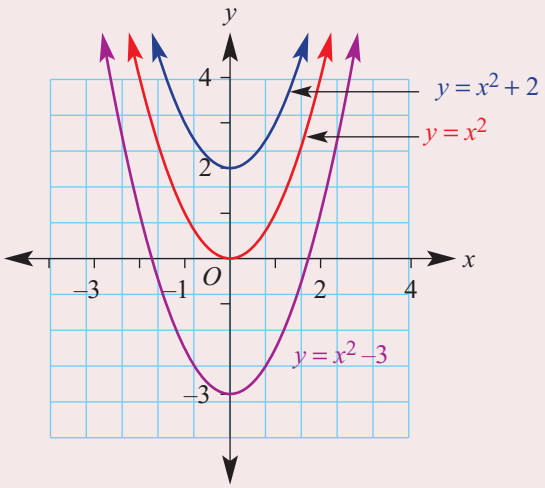
If $y = x^2$ is reflected across the x-axis, the equation becomes $y = -x^2$.



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

The parabola $y = x^2$ can be translated up or down the Cartesian plane by adding a positive or negative number to the right-hand side.

- $y = x^2 + 2$ translates $y = x^2$ up by 2 units.
- $y - x^2 - 3$ translates $y = x^2$ down by 3 units.



- Graphing software is a convenient tool for drawing graphs.
- Computer software, such as Desmos, can be very useful for investigating parabolas

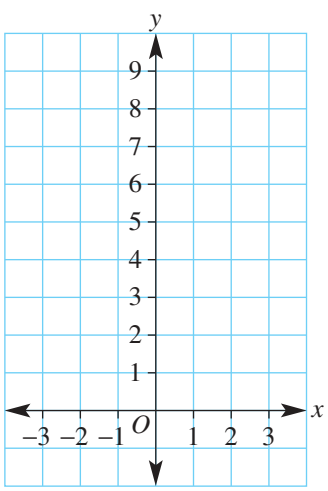
Key terminology: parabola, non-linear, plot, turning point, intercept, vertex, minimum, maximum, transformations, dilation, reflection, translation, axis of symmetry

Exercise 6L

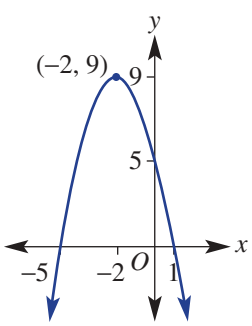
UNDERSTANDING 1, 2 2

1 Complete this table and grid to plot the graph of $y = x^2$.

x	−3	−2	−1	0	1	2	3
y	9						



2 Write the missing features for this graph in the sentences below.



a The parabola has a _____ (maximum or minimum) turning point.

b The coordinates of the turning point are _____ .

c The y-intercept is _____.

d The x-intercepts are _____ and _____.

e The axis of symmetry is _____.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 3–5 3–5

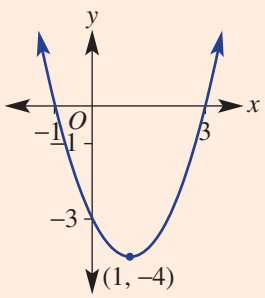
Example 32 Identifying key features of parabolas

Determine the following key features of each of the given graphs:

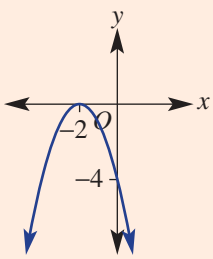
i turning point and whether it is a maximum or minimum

ii axis of symmetry iii x-intercepts iv y-intercept

a



b



SOLUTION

a i Turning point is a minimum at $(1, -4)$.

ii Axis of symmetry is $x = 1$.

iii x-intercepts at -1 and 3.

iv y-intercept at -3.

b i Turning point is a maximum at $(-2, 0)$.

ii Axis of symmetry is $x = -2$.

iii x-intercept at -2.

iv y-intercept at -4.

EXPLANATION

Lowest point of graph is at $(1, -4)$.

Line of symmetry is through the x-coordinate of the turning point.

x-intercepts lie on the x-axis ($y = 0$) and the y-intercept on the y-axis ($x = 0$).

Graph has a highest point at $(-2, 0)$.

Line of symmetry is through the x-coordinate of the turning point.

Turning point is also the one x-intercept.

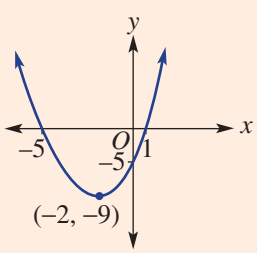
Now you try

Determine the following key features of each of the given graphs:

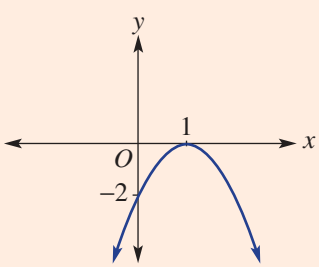
i turning point and whether it is a maximum or minimum

ii axis of symmetry iii x-intercepts iv y-intercept

a



b



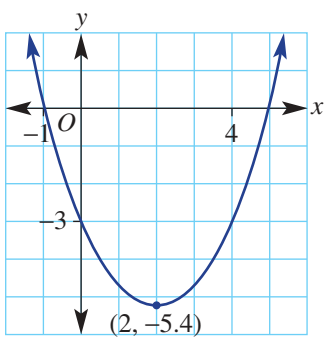
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Determine these key features of the following graphs.

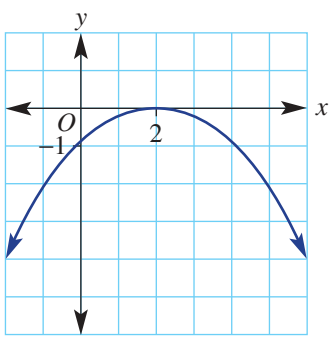
- **i** turning point and whether it is a maximum or minimum
- **ii** axis of symmetry
- **iii** x-intercepts
- **iv** y-intercept

Hint: An axis of symmetry is described by an equation such as $x = 2$ or $x = -3$.

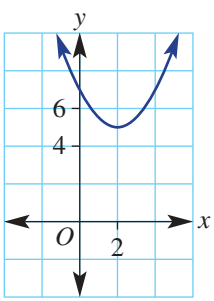
a



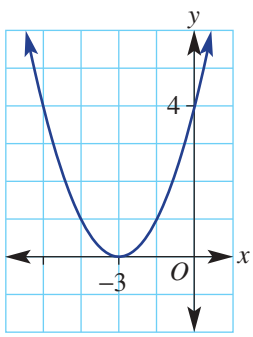
b



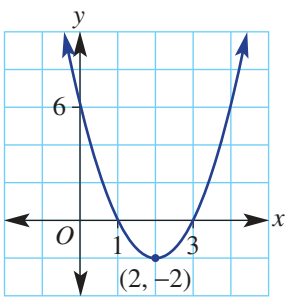
c



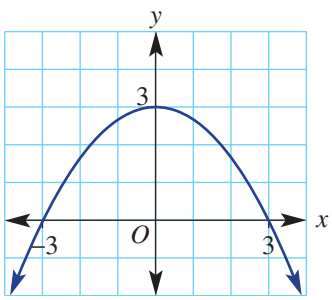
d



e



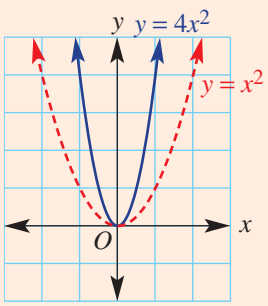
f



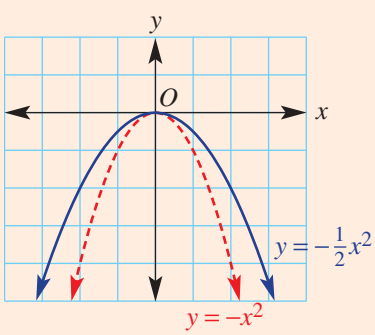
Example 33 Dilating and reflecting parabolas

Copy and complete the table for the following graphs.

a



b



	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 4x^2$					
b	$y = -\frac{1}{2}x^2$					

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 4x^2$	Minimum	No	$(0, 0)$	4	Narrower
b	$y = -\frac{1}{2}x^2$	Maximum	Yes	$(0, 0)$	$-\frac{1}{2}$	Wider

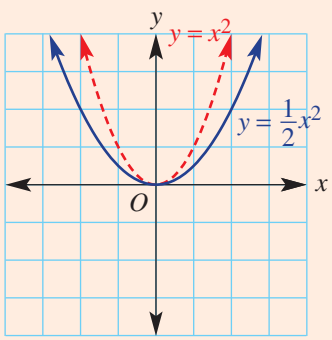
EXPLANATION

Read features from graphs and consider the effect of each change in equation on the graph.

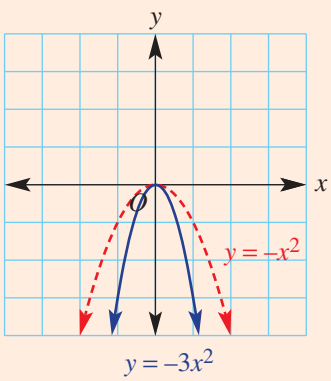
Now you try

Copy and complete the table for the following graphs.

a



b



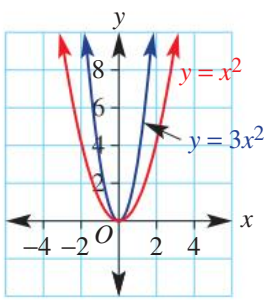
	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = \frac{1}{2}x^2$					
b	$y = -3x^2$					

4 Copy and complete the table below for the graphs on the following page.

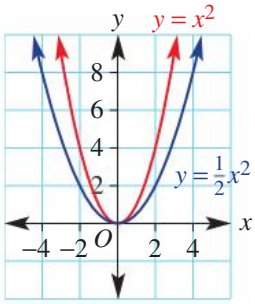
	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 3x^2$					
b	$y = \frac{1}{2}x^2$					
c	$y = 2x^2$					
d	$y = -4x^2$					
e	$y = -\frac{1}{3}x^2$					
f	$y = -2x^2$					

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

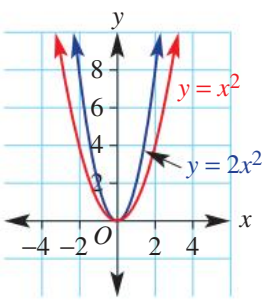
a



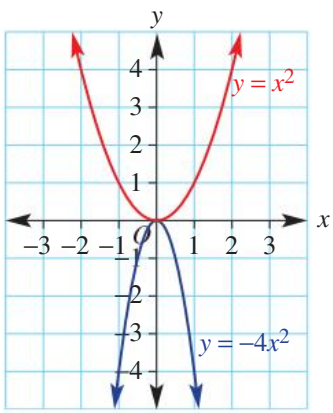
b



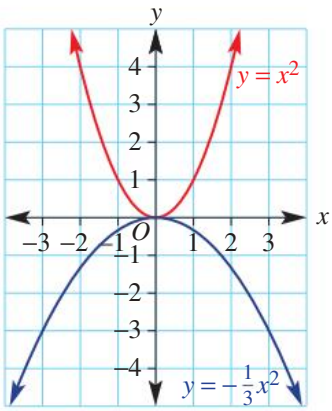
c



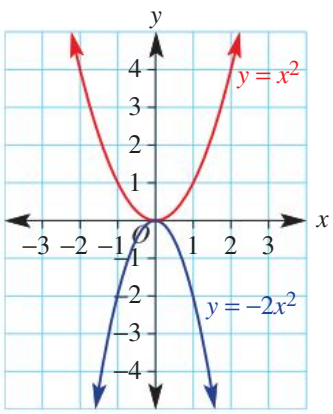
d



e



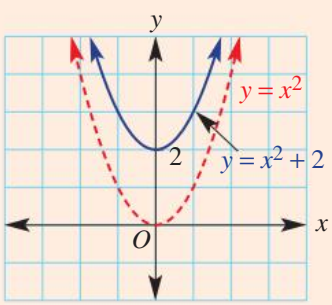
f



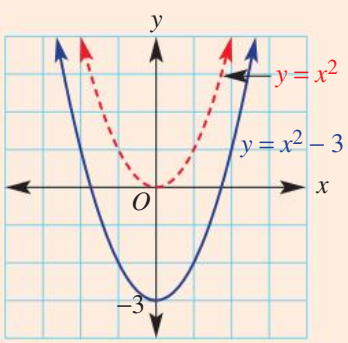
Example 34 Translating vertically

Copy and complete the table for the following graphs.

a



b



	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = x^2 + 2$					
b	$y = x^2 - 3$					

SOLUTION

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = x^2 + 2$	Minimum	No	$(0, 2)$	3	Same
b	$y = x^2 - 3$	Minimum	No	$(0, -3)$	-2	Same

EXPLANATION

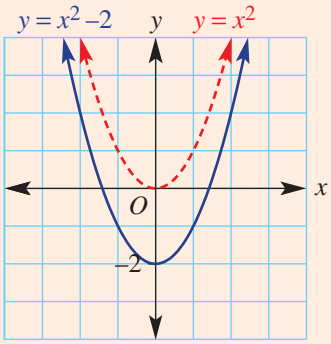
The effect is to shift up or down; up for $y = x^2 + 2$ and down for $y = x^2 - 3$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

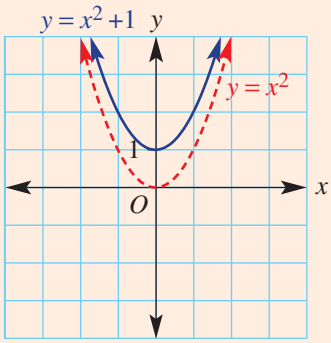
Now you try

Copy and complete the table for the following graphs.

a



b

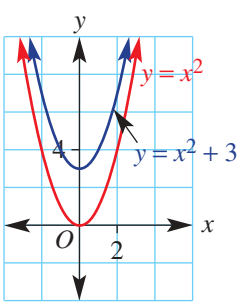


	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = x^2 - 2$					
b	$y = x^2 + 1$					

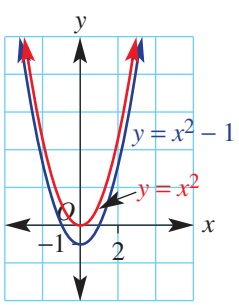
5 Copy and complete the table for the graphs that follow.

	Formula	Turning point	y-intercept ($x = 0$)	y-value when $x = 1$
a	$y = x^2 + 3$			
b	$y = x^2 - 1$			
c	$y = x^2 + 2$			
d	$y = x^2 - 4$			

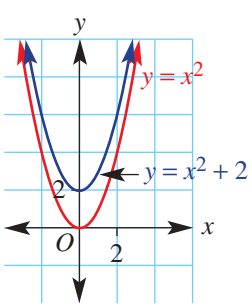
a



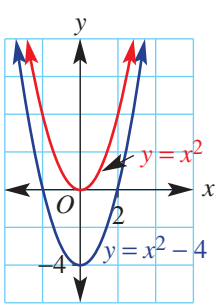
b



c



d



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

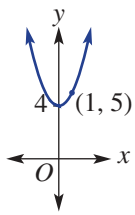
PROBLEM-SOLVING AND REASONING

6 | 6, 7

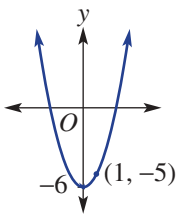
6 Match each of the following equations to one of the graphs below.

a $y = 2x^2$ **b** $y = x^2 - 6$ **c** $y = -5x^2$ **d** $y = x^2 + 4$

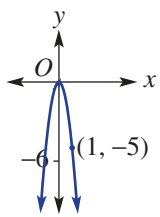
i



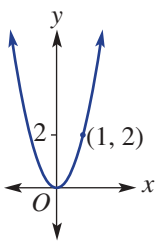
ii



iii



iv



7 Write an equation for a parabola with each feature.

a Same shape as $y = x^2$, minimum turning point $(0, 2)$

b Same shape as $y = x^2$, maximum turning point $(0, 0)$

c Same shape as $y = x^2$, minimum turning point $(5, 0)$

Hint: What turns $y = x^2$ into a graph with a maximum turning point?

ENRICHMENT: Parabolas with technology

— | 8–10

8 **a** Using technology, plot the following pairs of graphs on the same set of axes for values from $x = -5$ to $x = 5$ and compare their tables of values.

i $y = x^2$ and $y = 4x^2$ **ii** $y = x^2$ and $y = \frac{1}{3}x^2$ **iii** $y = x^2$ and $y = 6x^2$

iv $y = x^2$ and $y = \frac{1}{4}x^2$ **v** $y = x^2$ and $y = 7x^2$ **vi** $y = x^2$ and $y = \frac{2}{5}x^2$

b Suggest how the constant a in $y = ax^2$ transforms the graph of $y = x^2$.

9 **a** Using technology, plot the following sets of graphs on the same set of axes for values from $x = -5$ to $x = 5$ and compare the turning point of each.

i $y = x^2, y = (x + 1)^2, y = (x + 2)^2, y = (x + 3)^2$

ii $y = x^2, y = (x - 1)^2, y = (x - 2)^2, y = (x - 3)^2$

b Explain how the constant h in $y = (x + h)^2$ transforms the graph of $y = x^2$.

10 **a** Using technology, plot the following sets of graphs on the same set of axes for values from $x = -5$ to $x = 5$ and compare the turning point of each.

i $y = x^2, y = x^2 + 1, y = x^2 + 2, y = x^2 + 3$

ii $y = x^2, y = x^2 - 1, y = x^2 - 3, y = x^2 - 5$

b Explain how the constant k in $y = x^2 + k$ transforms the graph of $y = x^2$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6M Exploring exponential graphs

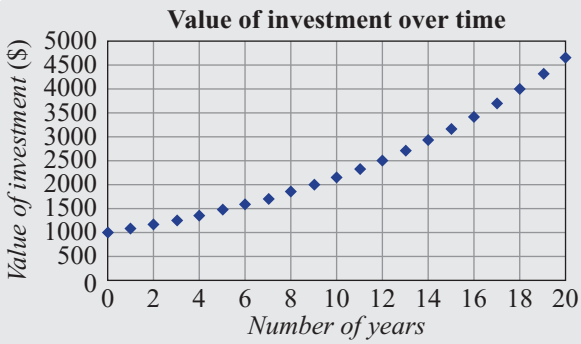
Learning intentions for this section:

- To know the general rule for an exponential relationship
- To be able to identify key features of an exponential graph
- To be able to draw exponential graphs, given the equation

Past, present and future learning:

- This section addresses the Stage 5 Core topics called *Non-linear relationships A* and *Non-linear relationships B*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Equations such as $y = 2^x$ are called exponential functions. They have graphs like the one on the right.

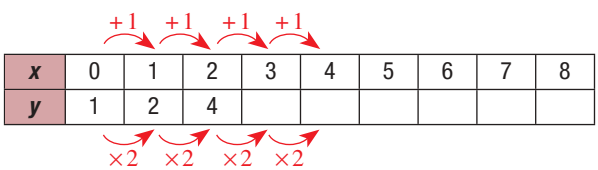


When an investment grows due to compound interest, the relationship between time and money is exponential.

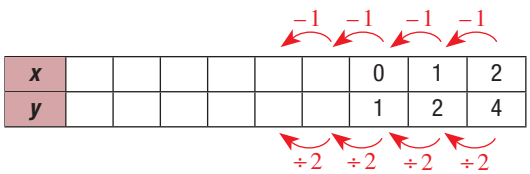
Lesson starter: Increase *x* by 1 and double the value of *y*

In the table on the right, the first value of x is 0. The value of y is 1.

As we move from left to right, x will increase by 1 and y will double.



- Copy and complete the table.
- The pattern of y values can be generated on your calculator by pressing 1 then [=], then ×2, then [=] [=] [=] [=] [=], etc. Try it. What is the biggest number that fits on your screen without being converted to scientific notation?
- If the number 25 appeared in the top row, what number would appear in the bottom row?
- If the number 1 073 741 824 appeared in the bottom row, what number would appear in the top row?
- Which one of the following equations is true for every pair of numbers in the table?
  i $y = x^2$ ii $y = x^3$ iii $y = 2x$ iv $y = 3x$
- Continue the table in the opposite direction (i.e. right to left) to see what happens for negative values of x.



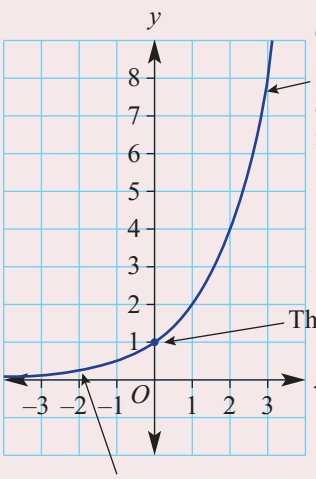
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

■ The simplest **exponential function** is $y = 2^x$.
To find a value for y, start with the number 2 and raise it to the power of x.
For example, if $x = 3$, y will be 2 to the power of 3, which is 8.

x	−3	−2	−1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

The graph of $y = 2^x$



This end of the curve gets steeper and steeper. This is often a good model for population increase.

The y-intercept is 1.

This end of the curve continues on forever and gets closer and closer to the x-axis but never makes contact with it.

Key terminology: exponential

Exercise 6M

UNDERSTANDING 1, 2 | 1, 2

1 Complete this table for $y = 3^x$.

x	0	1	2	3	4	5	6
y	$3^0 = 1$	$3^1 = 3$	$3^2 =$				

×3 ×3

2 Which of the following is an exponential relationship?

A $y = 2x$ **B** $y = 2^x$ **C** $y = x^2$ **D** $y^2 + x^2 = 2$

FLUENCY 3, 4 | 3, 4

Example 35 Plotting an exponential graph

Consider the equation $y = 2^x$.

a Complete this table.

b Plot points to form its graph.

x	−3	−2	−1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$				

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

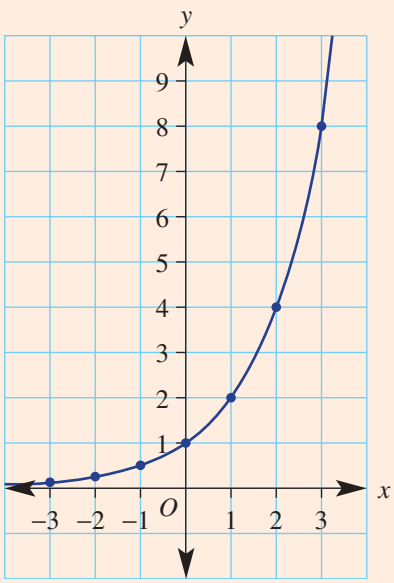
a

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

EXPLANATION

$2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8.$

b



Plot each point and join to form a smooth curve.

Now you try

Consider the equation $y = 5^x$.

a Complete this table.

b Plot points to form its graph.

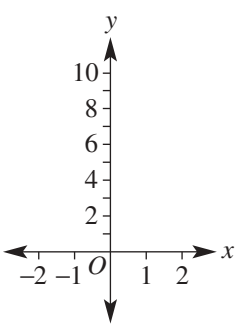
x	-2	-1	0	1	2
y	$\frac{1}{25}$	$\frac{1}{5}$			

3 Consider the exponential equation $y = 3^x$.

a Complete this table.

x	-2	-1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$			

b Plot the points in the table on axes like the ones shown to form the graph of $y = 3^x$.



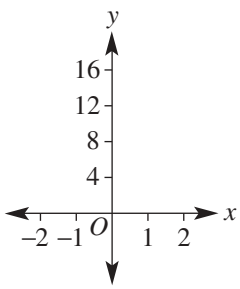
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Consider the exponential equation $y = 4^x$.

a Complete this table.

x	−2	−1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$			

b Plot the points in the table on axes like the ones shown to form the graph of $y = 4^x$.



PROBLEM-SOLVING AND REASONING 5 5

5 **a** Use graphing software to graph the following on the same set of axes:

i $y = 2^x$

ii $y = 4^x$

iii $y = 5^x$

b What do you notice about the y-intercept on each graph?

c What does increasing the base number do to each graph?

ENRICHMENT: Graphs of hyperbolas — 6

6 A study shows that the population of a town is modelled by the rule $P = 2^t$, where t is in years and P is the number of people, in thousands of people.

a State the number of people in the town at the start of the study (i.e. $t = 0$).

b State the number of people in the town after:

i 1 year **ii** 3 years

c When is the town's population expected to reach:

i 4000 people? **ii** 16 000 people?

Hint: If $P = 3$, there are 3000 people.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6N Exponential growth and decay

Learning intentions for this section:

- To understand the concept and equation of exponential growth and decay
- To be able to form an exponential growth or decay equation for a real-life scenario
- To be able to apply an exponential equation to a real-life scenario

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Linear relationships A* and *Linear relationships B*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Exponential change occurs when a quantity is continually affected by a constant multiplying factor. The change in quantity is not the same amount each time.

If you have a continual percentage increase, it is called exponential growth. If you have a continual percentage decrease, it is called exponential decay.

Some examples include:

- compound interest at a rate of 5% per year, where the interest is calculated as 5% of the investment value each year, including the previous year's interest
- a radioactive element has a 'half-life' of 5 years, which means the element decays at a rate of 50% every 5 years.

Lesson starter: A compound rule

Imagine that you have an investment valued at $100 000 and you hope that it will return 10% p.a. (per annum).
The 10% return is to be added to the investment balance each year.

- Discuss how to calculate the investment balance in the first year.
- Discuss how to calculate the investment balance in the second year.
- Complete this table.

Year	0	1	2	3
Balance ($)	100 000	$100\,000 \times 1.1 =$ _____	$100\,000 \times 1.1 \times$ _____ $=$ _____	_____ $=$ _____

- Recall how indices can be used to calculate the balance after the second year.
- Discuss how indices can be used to calculate the balance after the 10th year.
- What might be the rule connecting the investment balance ($\$A$) and the time, n years?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- **Exponential growth** and **decay** can be modelled by the equation $A = A_0(1 \pm r)^n$.
  - A is the amount.
  - A_0 is the initial amount (the subscript zero represents time zero).
  - r is the percentage rate of increase or decrease, expressed as a decimal.
  - n is time; how many times the percentage increase/decrease is applied.

  For example

  For a population increasing at 2% per year, $P = P_0(1.02)^n$.

  For a population decreasing at 3% per year, $P = P_0(0.97)^n$.
- For a growth rate of r p.a., use $1 + r$.
- For a decay rate of r p.a., use $1 - r$.
- Compound interest involves *adding* any interest earned to the balance at the end of each year or other period. The equation for the investment amount ($\$A$) is given by: $A = P(1 + r)^n$.
  - P is the initial amount or principal.
  - r is the interest rate, expressed as a decimal.
  - n is the time.

Key terminology: exponential growth, exponential decay, compound interest, principal

Exercise 6N

UNDERSTANDING 1–3 | 3

1 An investment of $1000 is to grow by 5% per year. Round your answers to the nearest cent.

Hint: 5% of $x = 0.05 \times x$.

- **a** Find the interest earned in the first year; i.e. 5% of $1000.
- **b** Find the investment balance at the end of the first year; i.e. investment + interest.
- **c** Find the interest earned in the second year; i.e. 5% of answer to part **b**.
- **d** Find the interest earned in the third year.

2 The mass of a limestone 5 kg rock exposed to the weather is decreasing at a rate of 2% per annum.

- **a** Find the mass of the rock at the end of the first year.
- **b** Copy and complete the rule for the mass of the rock (M kg) after n years.

  $M = 5(1 - _____)^n = 5 \times _____^n$
- **c** Use your rule to calculate the mass of the rock after 5 years, correct to two decimal places.

Hint: For decrease, use $1 - r$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Decide whether the following represent exponential *growth* or exponential *decay*.

a $A = 1000 \times 1.3^n$ **b** $A = 200 \times 1.78^n$ **c** $A = 350 \times 0.9^n$

d $P = 50\,000 \times 0.85^n$ **e** $P = P_0\left(1 + \frac{3}{100}\right)^n$ **f** $T = T_0\left(1 - \frac{7}{100}\right)^n$

FLUENCY 4–6 4(½),5–7

Example 36 Writing exponential rules

Form exponential rules for the following situations.

a John invests his \$100 000 in savings at a rate of 14% per annum.

b A city's initial population of 50 000 is decreasing by 12% per year.

SOLUTION

a Let A = the amount of money at any time
n = the number of years the money is invested
P = initial amount invested or principal
$r = 0.14$
$P = 100\,000$
$A = 100\,000(1 + 0.14)^n$
$\therefore A = 100\,000(1.14)^n$

EXPLANATION

Define your variables.

The basic formula is $A = P(1 + r)^n$.

Substitute $r = 0.14$ and $P = 100\,000$ and use '+' since we have growth.

b Let P = the population at any time
n = the number of years the population decreases
P_0 = starting population
$r = 0.12$
$P = 50\,000(1 - 0.12)^n$
$\therefore P = 50\,000(0.88)^n$

Define your variables.

The basic formula is $P = P_0(1 - r)^n$.

Substitute $r = 0.12$ and $P_0 = 50\,000$ and use '−' since we have decay.

$1 - 0.12 = 0.88$.

Now you try

Form exponential rules for the following situations.

a A town population of 3000 is increasing by 2% per year.

b A car purchased for \$36 000 is losing value at 6% per year.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Define variables and form exponential equations for the following situations.

a $200 000 is invested at 17% per annum.

b A house initially valued at $530 000 is losing value at 5% per annum.

c The value of a car, bought for $14 200, is decreasing at 3% per annum.

d A population, initially 172 500, is increasing at 15% per year.

e A tank with 1200 litres of water is leaking at a rate of 10% of the water in the tank every hour.

f A cell of area 0.01 cm^2 doubles its size every minute.

g An oil spill, initially covering an area of 2 square metres, is increasing at 5% per minute.

h A substance of mass 30 g is decaying at a rate of 8% per hour.

Hint: The exponential rule is of the form $A = A_0(1 + r)^n$
- A is the amount
- A_0 is the initial amount
- r is the percentage increase/decrease, as a decimal
- n is the time

Use '+' for growth and '−' for decay.

Example 37 Applying exponential rules

House prices are rising at 9% per year and Zoe's flat is currently valued at $600 000.

a Determine a rule for the value of Zoe's flat ($\$V$) in n years' time.

b What will be the value of her flat:

i next year?

ii in 3 years' time?

c Use trial and error to find when Zoe's flat will be valued at $900 000, to one decimal place.

SOLUTION

a Let V = value of Zoe's flat at any time

V_0 = starting value $600 000

n = number of years from now

$r = 0.09$

$V = V_0(1 + 0.09)^n$

$\therefore V = 600\,000(1.09)^n$

EXPLANATION

Define your variables.

The basic formula is $V = V_0(1 \pm r)^n$

$9\% = 0.09$

Use '+' since we have growth.

b i $n = 1$, so $V = 600\,000(1.09)^1$

$= 654\,000$

Zoe's flat would be valued at $654 000 next year.

Substitute $n = 1$ for next year.

Answer in words.

ii $n = 3$, so $V = 600\,000(1.09)^3$

$= 777\,017.40$

In 3 years' time Zoe's flat will be valued at about $777 017.

For 3 years, substitute $n = 3$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c

n	4	5	4.6	4.8	4.7
V	846 949	923 174	891 894	907 399	899 613

Zoe's flat will be valued at \$900 000 in about 4.7 years' time.

Try a value of n in the rule. If V is too low, increase your n value; if V is too high, decrease your n value. Continue this process until you get close to 900 000.

Now you try

An investment is increasing at 4% per year and is currently valued at \$20 000.

a Determine a rule for the value of the investment (\$$V$) in n years' time.

b What will be the value of the investment

i next year?

ii in 5 years' time? (Round to the nearest cent.)

c Use trial and error to find when the investment will be valued at \$25 000, to one decimal place.

5 The value of a house purchased for \$500 000 is expected to grow by 10% per year. Let \$$A$ be the value of the house after n years..

a Copy and complete the rule connecting A and n. $A = 500\,000 \times$ _____^{n}

b Use your rule to find the expected value of the house after the following number of years. Round your answers to the nearest cent

i 3 years

ii 10 years

iii 20 years

Hint: An increase of 10% is $1 + \frac{10}{100}$.

c Use trial and error to estimate when the house will be worth \$1 million. Round your answer to one decimal place.

6 A share portfolio initially worth \$300 000 is reduced by 15% p.a. over a number of years. Let \$$A$ be the share portfolio value after n years.

Hint: A decrease of 15% is $1 - \frac{15}{100}$.

a Copy and complete the rule connecting A and n.

$A =$ _____ $\times 0.85^n$

b Use your rule to find the value of the shares after the following number of years. Round your answers to the nearest cent.

i 2 years

ii 7 years

iii 12 years

c Use trial and error to estimate when the share portfolio will be valued at \$180 000. Round your answer to one decimal place.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 A water tank containing 15 000 L has a small hole that reduces the amount of water by 6% per hour.

a Determine an equation for the volume of water (V litres) left after n hours.

b Calculate (to the nearest litre) the amount of water left in the tank after:

i 3 hours **ii** 7 hours

c How much water is left after 2 days? Round your answer to two decimal places.

d Using trial and error, determine when the tank holds less than 500 L of water, to one decimal place.

PROBLEM-SOLVING AND REASONING 8, 9 8, 10

8 A certain type of bacteria grows according to the equation $N = 3000(2.6)^n$, where N is the number of cells present after n hours.

Hint: 'At the start' is $n = 0$ and $a^0 = 1$.

a How many bacteria are there at the start?

b Determine the number of cells (round to the whole number) present after:

i 0 hours **ii** 2 hours **iii** 4.6 hours

c If 5 000 000 bacteria are needed to make a drop of serum, determine by trial and error how long you will have to wait to make a drop (to the nearest minute).

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 A car tyre has 10 mm of tread when new. It is considered unroadworthy when there is only 3 mm left. The rubber wears at 12.5% every 10 000 km.

a Write an equation relating the depth of tread (D) for every 10 000 km travelled.

b If a tyre lasts 80 000 km ($n = 8$) before becoming unroadworthy, it is considered to be a 'good' tyre. Is this a good tyre?

c Using trial and error, determine when the tyre becomes unroadworthy ($D = 3$), to the nearest 10 000 km.

Hint: Use $D = D_0(1 - 0.125)^n$ where n is $\frac{\text{number of km}}{10\,000}$ and D_0 is initial tread. In part **b**, is D greater than 3 when $n = 8$?

10 A cup of coffee has an initial temperature of 90°C.

a If the temperature reduces by 8% every minute, determine a rule for the temperature of the coffee (T) after n minutes.

b What is the temperature of the coffee (to one decimal place) after:

i 2 minutes? **ii** 90 seconds?

Hint: The rule is of the form: $T = T_0(1 - r)^n$

c Using trial and error, when is the coffee suitable to drink if it is best consumed at a temperature of 68.8°C? Give your answer to the nearest second.

ENRICHMENT: Time periods — 11, 12

11 Interest earned on investments can be calculated using different time periods. Consider \$1000 invested at 10% p.a. over 5 years.

- If interest is compounded annually, then $r = 0.1$ and $t = 5$, so $A = 1000(1.1)^5$.
- If interest is compounded monthly, then $r = \frac{0.1}{12}$ and $t = 5 \times 12 = 60$, so

$$A = 1000\left(1 + \frac{0.1}{12}\right)^{60}.$$

a If interest is calculated annually, find the value of the investment, to the nearest cent, after:

i 5 years **ii** 8 years **iii** 15 years

b If interest is calculated monthly, find the value of the investment, to the nearest cent, after:

i 5 years **ii** 8 years **iii** 15 years

12 You are given \$2000 and you invest it in an account that offers 7% p.a. compound interest. What will the investment be worth, to the nearest cent, after 5 years if interest is compounded:

a annually? **b** monthly? **c** weekly? (Assume 52 weeks in the year.)

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Maths@Work

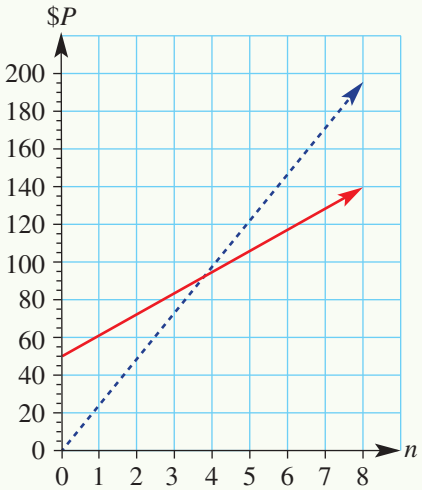
Accountant or small business owner

Accountants and small business owners need to have an understanding of operating costs and income. It is important to understand break-even points, which are when the costs or expenses equal the income. This is essential knowledge for small business owners, given that many start-up businesses fail in their first 3 years.

Reading and drawing graphs helps accountants and business owners understand break-even points. Many business situations can be displayed using linear functions.

Complete these questions that an accountant or small business owner may face in their day-to-day job.

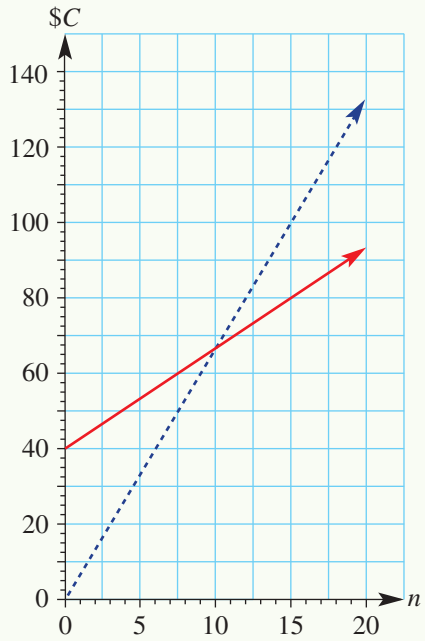
1 Chloe buys and sells necklaces at a local community weekend market. She sells each necklace for \$25 and her costs, C, are calculated using the formula $C = 50 + 12n$, where n is the number of necklaces Chloe buys and \$50 is the fixed cost of hiring the stand each weekend.



- **a** For the graph shown at right, which line represents Chloe's costs and which line represents Chloe's income?
- **b** State the possible numbers of necklaces to be sold each weekend that will result in Chloe making a loss.
- **c** What is the loss at the break-even point? What is the profit at the break-even point?
- **d** State the minimum number of necklaces to be sold each weekend for Chloe to start making a profit.
- **e** Sketch a copy of the graph and shade the region that represents the area of profit.
- **f** Write down a rule, in terms of n, for calculating Chloe's profit, \$$P$, each weekend.
- **g** How much profit is made by selling:
  - **i** 10 necklaces?
  - **ii** 15 necklaces?
  - **iii** 30 necklaces?
  - **iv** 50 necklaces?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 Consider this graph for a company creating retro drinking glasses.



- **a** What is the value of the y-intercept for the costs relationship, and what could it represent in this situation?
- **b** What is the wholesale price of each glass?
- **c** What is the gradient of the income line and what does this represent?
- **d** How many glasses must be sold for the company to break even?
- **e** Write an equation for the:
  - **i** cost
  - **ii** income
  - **iii** profit

Using technology

3 An industrial plant produces car parts, which they sell to car manufacturers for \$80 per car part. The costs of production for the plant are \$200 plus \$50 per car part produced each day.

- **a** Write an equation for the costs (\$$C$) of production of n car parts.
- **b** Write an equation for the income (\$$I$) generated.
- **c** Use technology to graph these two equations.
- **d** From the graphs, what is the break-even point for the plant each day?
- **e** Determine the profit when 32 car parts are sold on any given day.

4 Alex has a start-up company. He estimates his costs, including staff, rent, electricity and insurance, to be \$3000 per month. He buys computer parts at \$4 per part and sells them on at a retail price of \$$a$ per part.

Use a graphics calculator or a digital graphing tool to answer these questions.

- **a** By drawing pairs of graphs, find the number of parts, x, and Alex's costs, at each break-even point when the retail price, \$$a$, per part is:
  - **i** \$10 **ii** \$12 **iii** \$14 **iv** \$16

  Hint: Enter all the graph equations into the graphics calculator or computer graphing tool and then select two equations at a time to graph.
- **b** Alex buys a batch of 500 parts. Write an equation for Alex's profit when these parts are sold at the retail price \$$a$ per part.
- **c** Calculate the profit made when the parts are sold at each value of \$$a$ given in part **a** above.
- **d** Give one reason why Alex must limit the retail price charged per part.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Working mathematically

Breaking even in small business

Callum is trying to start up a new business as his passion is woodwork.

Callum is planning on making wooden children's tables to be sold through online market places. He predicts that it cost $80 in materials per table and fixed costs are $500. Callum is planning on selling each table for $140, not including delivery.

Callum thinks that he is capable of making 5 sets a week and estimates 20 a month.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

a How much does it cost Callum to make the following number of tables?
i 1 table **ii** 2 tables **iii** 5 tables **iv** 10 tables

b How much does he earn when selling the following number of tables?
i 1 table **ii** 2 tables **iii** 5 tables **iv** 10 tables

c How much profit or loss is made on the sale of the following number of tables?
i 1 table **ii** 2 tables **iii** 5 tables **iv** 10 tables

Non-routine problems

Explore and connect

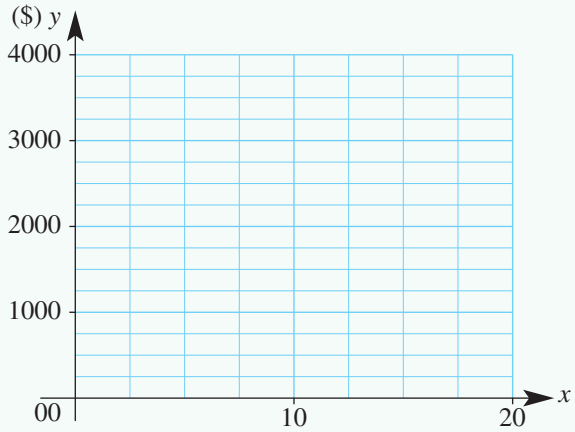
You are asked to form two equations and use the graphs of the two equations to find the break-even point for the potential business.

a What is the cost of making each table?
b What are the fixed costs for the potential business?
c Write down the linear equation to represent Callum's costs ($\$y$) for making x tables. What does the gradient represent and what does the y-intercept represent in the equation?
d Write down the linear equation to represent the income for selling x tables.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

e Graph both linear equations on the same number plane. Use a number plane similar to the one shown below, and ensure that you label the graph of each equation. (An online graphing tool such as Desmos could be used.)

Choose and apply techniques



f Determine how many tables Callum needs to sell before he can say that his business is profitable. This is the break-even point.

g Explain how you obtained your answer to part **f**, making reference to the graphs and equations.

h Determine how long Callum must expect to work before he will make a profit. Explain your answer.

Communicate thinking and reasoning

i Communicate all of your solutions above in written form.

j Explain if this is a business model that you would recommend to Callum.

Extension problems

Problem-solve

Callum considers changing his business model to making bird houses. The fixed costs of this venture is $1000 and cost of materials is $15 per house. Callum would set the sale price of a single bird house at $45.

Callum expects to make three times as many bird houses as tables each month.

a Create two new linear equations and decide, by referring to their graphs and the break-even point, if Callum is better off making, and selling, bird houses rather than tables.

b Reflect on the suitability of each plan and identify the strengths and weaknesses either might have.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Puzzles and challenges

1 What is not so devious? Solve the puzzle to find the answer.
Match the letter beside each question to the answers below.

Find where each line cuts the x-axis:

O $y = 3x - 24$

! $y = -\frac{3}{2}x - 9$

N $4x - 2y = -20$

I This line passes through $(0, 4)$ and $(5, -1)$.

S $x = -7$

Find the gradient of each line:

E $y = 3x - 4$

L $y = -\frac{5}{2}x + 7$

P This line passes through the origin and $(3, -6)$.

G This line passes through $(2, 5)$ and $(-4, 11)$.

T $y = 5$

-2	-2.5	8	0	0	4	-5	-1	-2.5	4	-5	3	-7	-6

2 Solve the wordfind below.

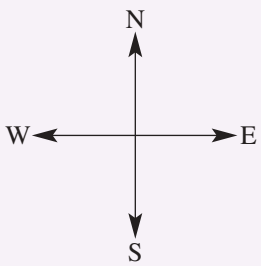
T	F	Z	T	V	M	V	Z	J	H	E	R
O	W	I	M	J	G	R	J	O	A	L	A
T	M	G	T	E	R	K	R	U	T	B	T
E	C	N	A	T	S	I	D	N	E	A	E
S	T	M	P	R	Z	A	E	S	I	I	F
R	P	H	G	O	X	M	E	O	Z	R	E
D	Y	E	N	Z	G	J	U	R	X	A	F
H	G	T	E	E	J	Q	W	G	C	V	H
I	A	L	S	D	R	U	S	G	U	N	A
L	P	G	R	A	P	H	A	P	R	P	I

- DISTANCE
- GRAPH
- HORIZONTAL
- INCREASE
- RATE
- SEGMENT
- SPEED
- TIME
- VARIABLE

3 Cooper and Sophie are in a cycling orienteering competition.

- From the starting point, Cooper cycles 7 km east, then 3 km south to checkpoint 1. From there, Cooper cycles 5 km east and 8 km north to checkpoint 2.
- Sophie cycles 10 km north from the starting point to checkpoint 3.

Use calculations to show that the distance between where Sophie and Cooper are now is the same as the direct distance that Cooper is now from the starting point.



4 Lucas and Charlotte want to raise money for their school environment club, so they have volunteered to run a strawberry ice-cream stall at their town's annual show.
It costs \$200 to hire the stall and they make \$1.25 profit on each ice-cream sold.

a How many ice-creams must be sold to make zero profit (i.e. not a loss)?

b If they make \$416.25 profit, how many ice-creams were sold?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

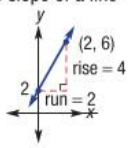
Chapter summary

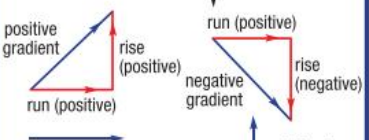
Linear relationships, hyperbolas, parabolas and exponentials

Gradient of a line

Gradient measures the slope of a line

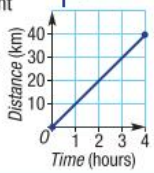
Gradient, $m = \frac{\text{rise}}{\text{run}}$ e.g. $m = \frac{4}{2} = 2$





A rate equals the gradient with units.

e.g. Speed $= \frac{40}{4} = 10$ km/h



Equation of a line

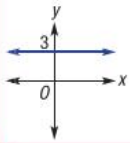
$y = mx + c$

gradient y-intercept

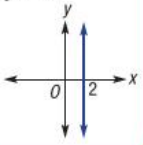
- The rule is a linear equation.
- The graph is made up of points in a straight line.

Special lines

Horizontal lines e.g. $y = 3$



Vertical lines e.g. $x = 2$



Parallel and perpendicular lines

- Parallel lines have the same gradient; e.g. $y = 3x - 4$ and $y = 3x + 1$
- For perpendicular lines, the product of their gradients is -1, so $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$.

Sketching a line

Plotting straight-line graphs:

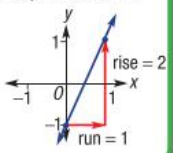
- Complete a table of values.
- Plot points and draw a line through them to form a straight line.

Using the y-intercept and gradient:

- Plot the y-intercept (c).
- Use the gradient to plot the next point.
- Draw a line through the points to form a straight line.

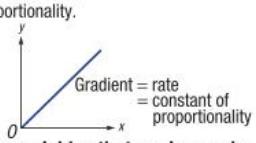
e.g. $y = 2x - 1$

$c = -1$ $m = \frac{2}{1}$



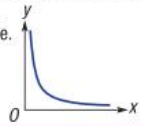
For two variables that are directly proportional

- Both variables will increase or decrease together at the same rate.
- The rule is $y = kx$, where k is the constant of proportionality.

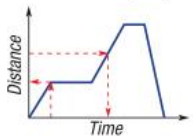


For two variables that are inversely (or indirectly) proportional

- When one variable increases, then the other variable decreases.
- The graph is a curve.



Distance–time graph



- Flat segment means the object is at rest.

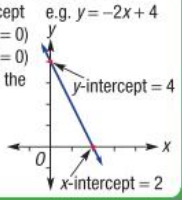
Reading from a graph:

- Start on given distance; move across to line and then down to time scale (or in reverse).

Using the axes intercepts

- Plot each axis intercept
  - x-intercept (when $y = 0$)
  - y-intercept (when $x = 0$)
- Draw a line through the points to form a straight line.

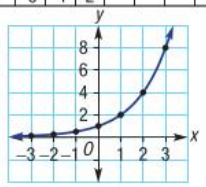
e.g. $y = -2x + 4$



Exponential graphs

For example, $y = 2^x$

x	–3	–2	–1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

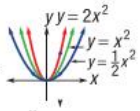


Transformations of a parabola

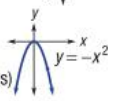
$y = ax^2$ dilates graph compared to $y = x^2$

$0 < a < 1$ graph is wider

$a > 1$ graph is narrower



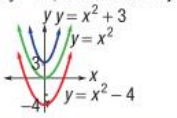
$a < 0$ graph is inverted (i.e reflected in x-axis)



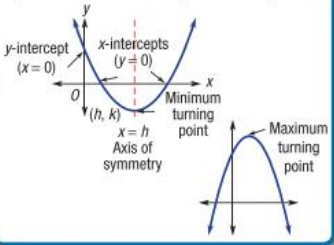
$y = x^2 + k$ translates $y = x^2$, k units vertically

$k > 0$ move up

$k < 0$ move down



A parabola's rule is a quadratic equation



Linear modelling

- Find an equation in the form $y = mx + c$, using the appropriate pronumerals.
- Sketch a graph.
- Use the equation to solve problems.
- Answer the problem in words.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook. ✔

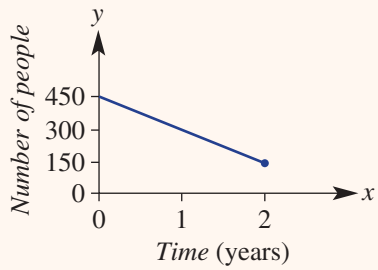
6A

1 I can interpret information from a graph.

e.g. The number of people living on a small island has decreased over recent years according to the graph shown.

a How many people were there to begin with?

b How many people left the island during the 2-year period?



6A

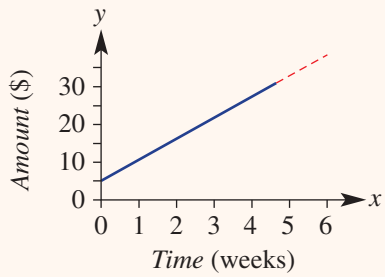
2 I can read off a graph using interpolation and extrapolation.

e.g. This graph shows the increase in Chloe's pocket money savings over 4 weeks.

a How much has she saved over the 4 weeks?

b Use the graph to find out how much she saved after 2 weeks.

c After how long does the graph suggest she will have saved $35?



6B

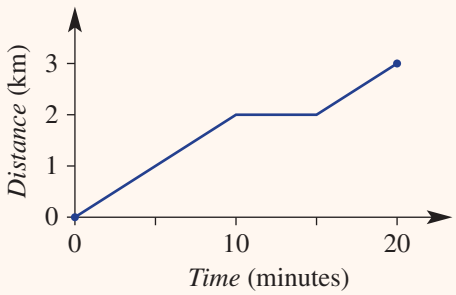
3 I can interpret a distance–time graph.

e.g. The distance–time graph shows a student's bike ride from school, to the corner store for an ice-cream and then to home. Determine:

a the total distance covered

b how long the student stopped at the store

c the total distance travelled after 17.5 minutes



6B

4 I can sketch a distance–time graph.

e.g. Sketch a distance–time graph displaying all the following information.

- total distance covered is 12 km in 3 hours
- 6 km covered in the first hour
- a half-hour rest stop after the first hour

6C

5 I can plot a graph from a rule.

e.g. Plot the graph of $y = 3x - 2$ by first completing the table of values.

x	-1	0	1
y			

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

6C

6 I can construct a table and graph and interpret it.

e.g. A TV technician charges $110 for a service call and $70 per hour for labour.
Complete the table of values and plot a graph of cost against number of hours.

No. of hours (n)	0	1	2	3	4
Cost (C)					

Use the graph to determine:

a the cost for 3.5 hours of work

b how long the technician worked on a job that cost $285

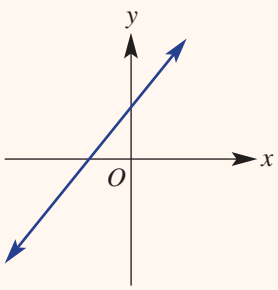
☐

6D

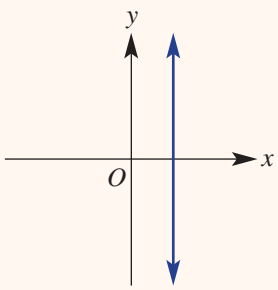
7 I can describe the gradient of a graph.

e.g. Describe the gradients of these lines as positive, negative, zero or undefined.

a



b



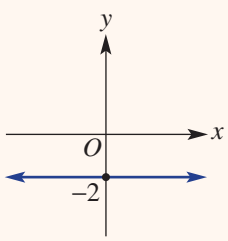
☐

6D

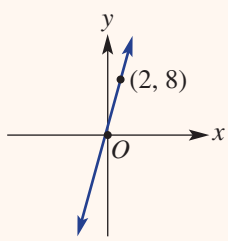
8 I can calculate the gradient from a graph.

e.g. Find the gradients of the following lines and line segments.

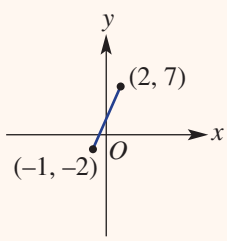
a



b



c



☐

6E

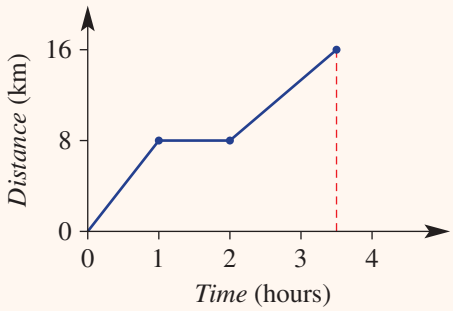
9 I can calculate speed from a distance–time graph.

e.g. A cyclist completes a journey which is described by this graph.
Find how fast the cyclist was travelling during the:

a first hour

b second section

c third section

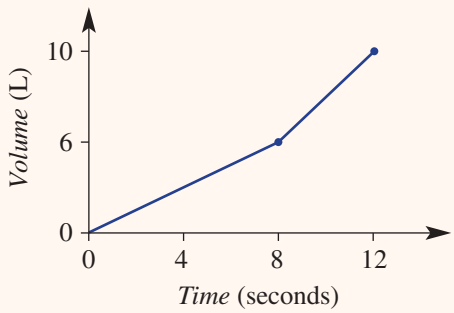


☐

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

Chapter checklist with success criteria

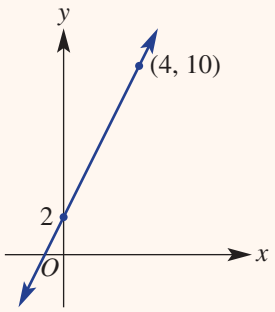
6E	**10 I can calculate a rate from a graph.** e.g. A kettle is being filled with water from a tap as shown. How fast (in L/s) is the kettle being filled in the first 4 seconds, final 4 seconds and between the 4- and 8-second mark?  	☐
6F	**11 I can determine the gradient and *y*-intercept from an equation.** e.g. For $y = \frac{1}{2}x - 4$, state the gradient and the y-intercept.	☐
6F	**12 I can sketch a line using the *y*-intercept and gradient.** e.g. Sketch the graph of $y = 2x + 3$ by considering the y-intercept and gradient.	☐
6F	**13 I can sketch horizontal and vertical lines.** e.g. Sketch the graph of the equations: **a** $x = 4$ **b** $y = -3$.	☐
6F	**14 I can sketch lines passing through the origin.** e.g. Sketch the graph of $y = -3x$.	☐
6G	**15 I can determine if lines are parallel or perpendicular.** e.g. State whether the following pair of lines are parallel, perpendicular or neither: $y = -2x - 3$ and $y = \frac{1}{2}x + 1$	☐
6G	**16 I can find the equation of a line that is parallel or perpendicular to another line, given the *y*-intercept.** e.g. A line passes through $(0, -3)$. Give the equation of the line if it is: **a** parallel to a line with gradient 2 **b** perpendicular to another line with gradient 3	☐
6G	**17 I can find the equation of a line that is parallel or perpendicular to a line.** e.g. A line passes through $(2, -3)$. Find the equation of the line if it is: **a** parallel to the line with equation $y = -3x + 1$ **b** perpendicular to the line with equation $y = \frac{1}{2}x + 1$	☐
6H	**18 I can sketch lines in the form *ax* + *by* = *d* using *x*- and *y*-intercepts.** e.g. Sketch a graph of $2x - 5y = 15$ by finding the x- and y-intercepts.	☐
6H	**19 I can sketch lines in the form *y* = *mx* + *c* using *x*- and *y*-intercepts.** e.g. Sketch a graph of $y = -3x + 12$ by finding the x- and y-intercepts.	☐

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

6H

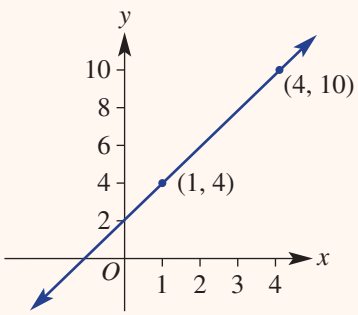
20 I can find the equation of a line from a graph with a known *y*-intercept.
e.g. For the straight line shown, determine the gradient and y-intercept and write the equation of the line.



☐

6I

21 I can find the equation of a line from a graph given two points.
e.g. For the straight line shown, determine the gradient and y-intercept and write the equation of the line.



☐

6I

22 I can form a linear model and graph for a problem.
e.g. An employee gets paid $\$50$ plus $\$15$ for each hour of work. If she earns $\$C$ for t hours of work, write a rule for C in terms of t and sketch the graph for t between 0 and 8. Use the rule to find:
a the amount earned after working 6 hours
b the number of hours worked if $\$200$ is earned

☐

6J

23 I can determine the constant of proportionality and find an equation connecting two variables that are directly proportional.
e.g. The volume of water in a bucket is in direct proportion to the number of seconds it has been filled for. The bucket was filled with 6 L of water in 12 seconds.
a Find the constant of proportionality, k.
b Use this to write the direct proportion equation relating the volume (V litres) of water in the bucket and the number of seconds (s) it is filled for.
c Use the rule to find the volume after 8 seconds and the number of seconds to fill the bucket with 20 L of water.

☐

6K

24 I can use an equation connecting inversely proportional variables to sketch a graph and find the value of an unknown.
e.g. In a rectangle of area 8 cm^2, the length, l cm, is inversely proportional to the breadth, b cm, such that $l = \frac{8}{b}$. Sketch a graph of l vs b and find the length if $b = 2.5$.

☐

6K

25 I can determine the constant of proportionality and find an equation connecting two variables that are inversely proportional.
e.g. If y is inversely proportional to x and when $x = 3$, $y = 4$, find an equation linking y and x. Then find the value of x if $y = 6$.

☐

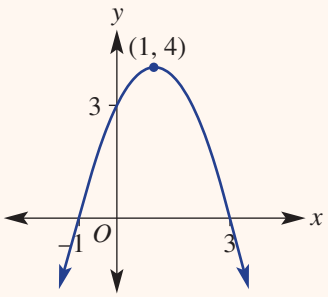
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

Chapter checklist with success criteria

6L

26 I can identify key features of a parabola.
e.g. List the key features of this parabola, including the type and coordinates of the turning point, the axis of symmetry and the x- and y-intercepts.

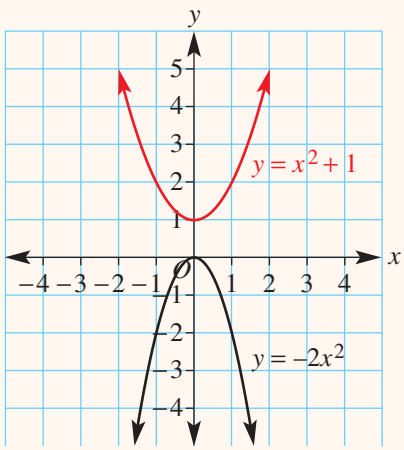


☐

6L

27 I can identify key features of a parabola after $y = x^2$ is reflected, dilated or translated.
e.g. List the key features of the following parabolas, including the type and coordinates of the turning point, axis of symmetry and x- and y-intercepts.

a $y = -2x^2$ **b** $y = x^2 + 1$



☐

6M

28 I can identify key features and am able to sketch a simple exponential relationship.
e.g. Sketch the graph of $y = 3^x$.

☐

6N

29 I can form exponential equations.
e.g. Write a rule for this statement: A substance of mass 450 g is decaying at a rate of 14% per day.

☐

6N

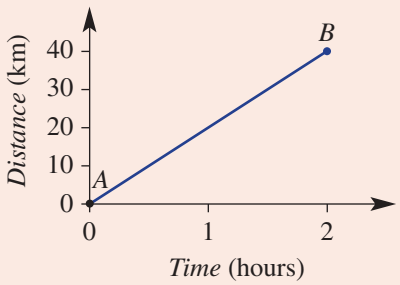
30 I can apply exponential equations.
e.g. A share portfolio valued at \$80 000 is expected to grow by 6% per year. Determine a rule for the value of the portfolio (\$$V$) in n years' time then use this to find the value in 4 years' time. Use trial and error to find when the portfolio will be valued at \$115 000, correct to one decimal place.

☐

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

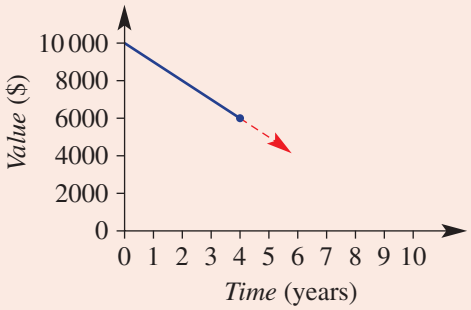
Short-answer questions

6A/6B **1** This graph shows the journey of a cyclist from place *A* to place *B*.



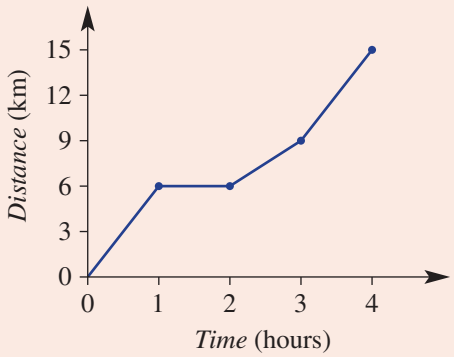
- **a** How far does the cyclist travel?
- **b** How long does it take the cyclist to complete the journey?
- **c** If the cyclist were to ride from *A* to *B* and then halfway back to *A*, how far would the journey be?

6A **2** The value of a poor investment has decreased according to this graph.



- **a** Find the value of the investment after:
  - **i** 4 years **ii** 2 years
  - **iii** 1 year
- **b** Extend the graph and use it to estimate the value of the investment after:
  - **i** 8 years **ii** 6 years
  - **iii** 5 years
- **c** After how many years will the investment be valued at $0?

6B **3** The distance travelled by a walker is described by this graph.



- **a** What is the total distance walked?
- **b** For how long does the person actually walk?
- **c** How far has the person walked after:
  - **i** 1 hour? **ii** 2 hours?
  - **iii** 3 hours? **iv** 4 hours?
- **d** How long does it take the walker to walk a distance of 12 km?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

6B **4** Sketch a graph to show a journey described by:

- a total distance of 60 m in 15 seconds
- 30 m covered in the first 6 seconds
- a 5 second rest after the first 6 seconds

6C **5** Francene delivers burgers for a fast-food outlet. She is paid $10 a shift plus $5 per delivery.

a Complete the table of values.

No. of deliveries (d)	0	5	10	15	20
Pay (P)					

b Plot a graph of amount paid against number of deliveries.

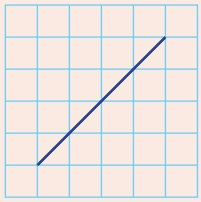
c Use the graph to determine:

i the amount of pay for 12 deliveries

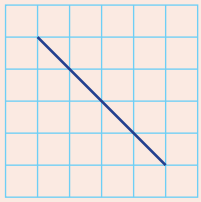
ii the number of deliveries made if Francene is paid $95

6D **6** Find the gradient of the following lines.

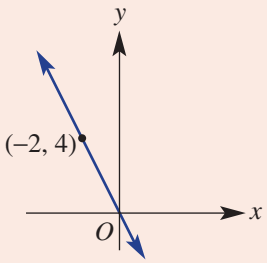
a



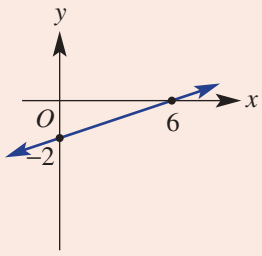
b



c



d



6F **7** State the gradient and y-intercept of the following lines.

a $y = 3x + 4$ b $y = -2x$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6F **8** Sketch the following lines by considering the y-intercept and the gradient.

a $y = 2x + 3$ **b** $y = -4x$ **c** $y = 2$ **d** $x = -1$

6H **9** Sketch the following lines by considering the x- and y-intercepts.

a $3x + 4y = 12$ **b** $2x - y = 6$ **c** $y = 3x - 9$

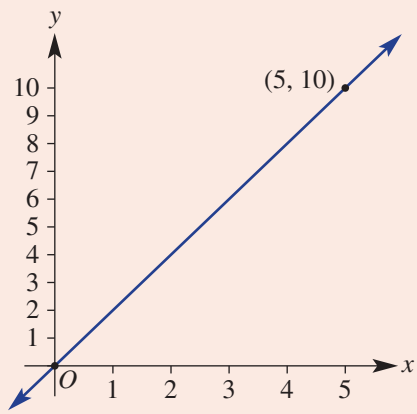
6F **10** For each of the straight lines shown:

i Determine the gradient.

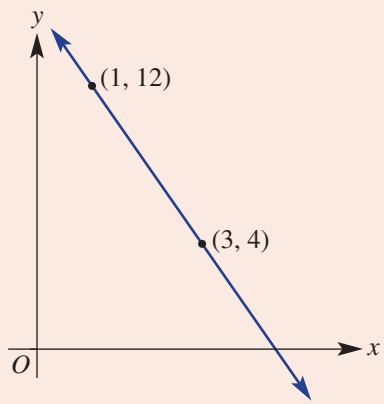
ii Find the y-intercept.

iii Write the equation of the line.

a



b

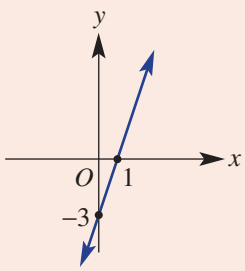


6F/6H **11** Match each of the linear equations to the lines shown.

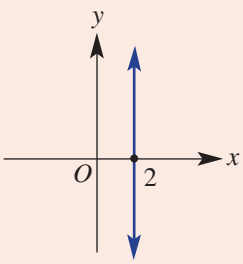
a $y = 3x - 3$ **b** $y = 5x$ **c** $5x + 4y = 20$

d $x = 2$ **e** $-2x + 5y = 10$ **f** $y = -4$

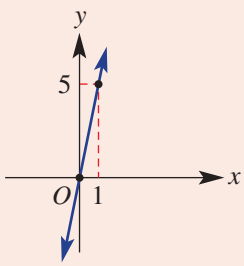
i



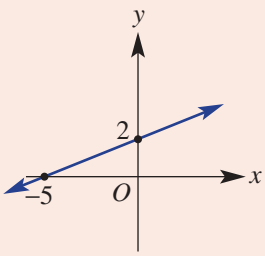
ii



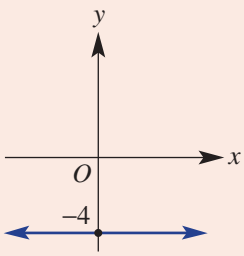
iii



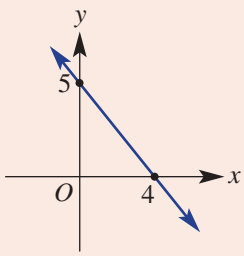
iv



v



vi



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

6I **12** A fruit picker earns \$50 plus \$20 per bin of fruit picked. If the picker earns \$$E$ for n bins picked, complete the following.

- **a** Write a rule for E in terms of n.
- **b** Sketch a graph for n between 0 and 6.
- **c** Use your rule to find:
  - **i** the amount earned after picking four bins of fruit
  - **ii** the number of bins of fruit picked if \$160 is earned

6G **13** Find the equation of the lines with the given description.

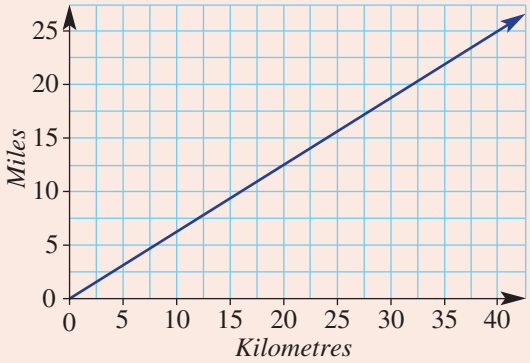
- **a** A line passes through $(0, 3)$ and is parallel to another line with gradient 2.
- **b** A line passes through $(0, -1)$ and is parallel to another line with gradient $\frac{1}{2}$.
- **c** A line passes through $(0, 2)$ and is perpendicular to another line with gradient 1.
- **d** A line passes through $(0, -7)$ and is perpendicular to another line with gradient $\frac{3}{4}$.
- **e** A line passes through $(1, 2)$ and is parallel to another line with gradient -4.
- **f** A line passes through $(-2, 5)$ and is perpendicular to another line with gradient 2.

6J **14** An employee earns wages in direct proportion to the hours he works.

- **a** Find the constant of proportionality, k, given that he earned \$198 in 12 hours.
- **b** Write the direct proportion equation relating the wages (W) in dollars and the number of hours (n) worked.
- **c** Use the rule in part **b** to calculate:
  - **i** the wages earned for 8 hours of work
  - **ii** the number of hours the employee must work to earn a wage of \$264

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6J **15** This graph shows the direct proportional relationship between miles and kilometres.



a Use the graph to convert 5 miles to kilometres.
b Use the graph to convert 35 kilometres to miles.
c Given that 15 miles is 24.14 km, find the gradient, to three decimal places.
d State the conversion rate in miles/km, to three decimal places.
e Determine the constant of proportionality, k, to three decimal places.
f Write the direct proportion equation between miles (y) and kilometres (x).
g Use this equation to find the number of miles in 100 km.
h Use this equation to find the number of kilometres in 100 miles.

6K **16** State whether these variables are in direct or inverse proportion and give a reason why.

a Cost of buying cricket balls and the number of balls.
b Cost per person of renting a beach house for a week and the number of people sharing it.

6K **17** The length of a rectangle, l metres, with a fixed area, varies inversely with the breadth, b metres, such that $l = \frac{k}{b}$.

a If when $b = 3$, $l = 5$ find the value of k, the constant of proportionality.
b Complete this table of values.

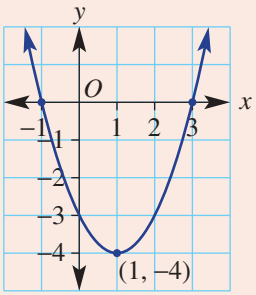
b	1	3	5	15
l				

c Sketch a graph of l vs b using your table of values to help. Use b on the x-axis.
d Find the length of the rectangle if the breadth is 10 m.
e Find the breadth of the rectangle if the length is 4 m.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

6L **18** State the following features of the quadratic graph shown:

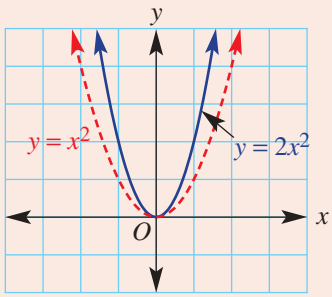


- **a** turning point and whether it is a maximum or a minimum
- **b** axis of symmetry
- **c** x-intercepts
- **d** y-intercept

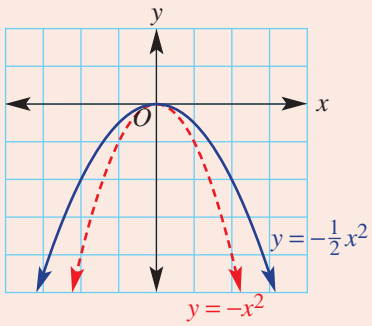
6L **19** Copy and complete the table for the following graphs.

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 2x^2$					
b	$y = -\frac{1}{2}x^2$					
c	$y = x^2 - 2$					

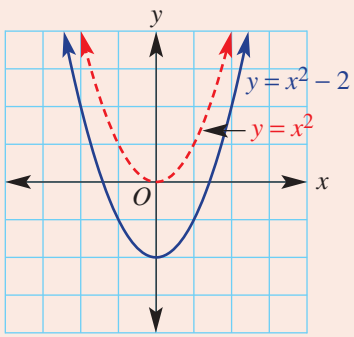
a



b



c



6M **20** Sketch the following graphs, labelling the y-intercept and the point where $x = 1$.

- **a** $y = 2^x$
- **b** $y = 4^x$

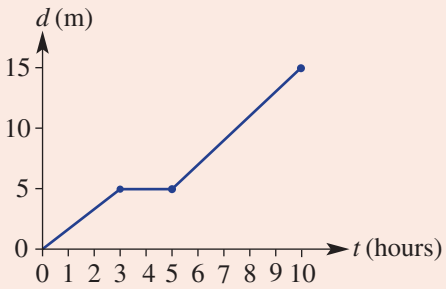
6N **21** Form an exponential equation for the following.

- **a** The population of a colony of kangaroos, which starts at 20 and is increasing at a rate of 10%.
- **b** The amount of petrol in a petrol tank fuelling a generator if it starts with 100 000 litres and uses 15% of its fuel every hour.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Multiple-choice questions

Questions 1–4 refer to the following graph of the movement of a snail.



6A **1** The total number of hours the snail is at rest is:

A 2 **B** 4 **C** 5
D 6 **E** 10

6B **2** The distance travelled in the first 3 hours is:

A 3 m **B** 3 hours **C** 7 m
D 4 m **E** 5 m

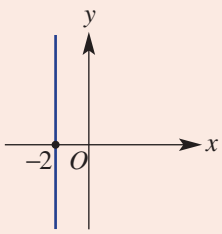
6F **3** The speed of the snail in the last 5 hours is:

A 5 hours **B** 10 m **C** 10 m/h **D** 2 m/h **E** 5 m/h

6B **4** The total distance travelled by the snail is:

A 15 m **B** 10 m **C** 5 m **D** 12 m **E** 8 m

6F **5** The equation of the line shown at the right is:



A $x = 2$ **B** $x = -2$ **C** $y = 1$
D $y = -2$ **E** $y = -2x$

6F **6** The graph of $C = 10t + 5$ would pass through which of the following points?

A $(1, 10)$ **B** $(1, 20)$ **C** $(2, 20)$
D $(4, 50)$ **E** $(5, 55)$

6D **7** The gradient of the line that passes through $(0, 0)$ and $(2, -6)$ is:

A 2 **B** 3 **C** -3 **D** 6 **E** -6

6D **8** A vertical line has gradient:

A undefined **B** zero **C** positive **D** negative **E** 1

6D **9** A line passes through $(-2, 7)$ and $(1, 2)$. The gradient of the line is:

A -3 **B** $-\frac{5}{3}$ **C** 3 **D** $\frac{5}{3}$ **E** $-\frac{3}{5}$

6H **10** The x- and y-intercepts of the graph of the equation $3x - y = 4.5$ are, respectively:

A $(0, 3.5)$ and $(4.5, 0)$ **B** $(-1.5, 0)$ and $(4.5, 0)$ **C** $(1.5, 0)$ and $(0, 4.5)$
D $(1.5, 0)$ and $(0, -4.5)$ **E** $(0, 3.5)$ and $(-4.5, 0)$

6F **11** Which of the following equations has a gradient of 2 and a y-intercept of -1?

A $2y + x = 2$ **B** $y - 2x = 1$ **C** $y = -2x + 1$ **D** $y = 2x - 1$ **E** $2x + y = 1$

6H **12** A line has x- and y-intercepts of, respectively, 1 and 2. Its equation is:

A $2x - y = 2$ **B** $y = -x + 2$ **C** $y = 2x + 2$ **D** $x + 2y = 1$ **E** $y = -2x + 2$

6G **13** The gradient of the line that is perpendicular to the line with equation $y = -2x - 5$ is:

A 2 **B** -2 **C** $\frac{1}{2}$ **D** $-\frac{1}{2}$ **E** $\frac{1}{5}$

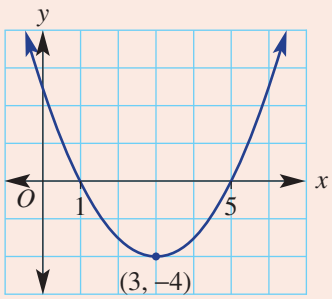
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

6K **14** Which equation shows that y is directly proportional to x?

A $y = 5x - 6$ **B** $y = \frac{6}{x}$ **C** $y = 2x + 4$ **D** $y = 12x$ **E** $y = 20 - 3x$

6L **15** The equation of the axis of symmetry of the graph shown is:



A $y = -4$ **B** $x = 3$ **C** $x = -4$ **D** $y = 3$ **E** $y = 3x$

6M **16** The graph of $y = 3^x$ crosses the y-axis at:

A $(0, 3)$ **B** $(3, 0)$ **C** $(0, 1)$

D $(1, 3)$ **E** $\left(0, \frac{1}{3}\right)$

6N **17** An equation for the amount of money, A, in an account after n years, if \$1200 is invested at 4% per year, is:

A $A = 1200(4)^n$ **B** $A = 1200(1.4)^n$ **C** $A = 1200(0.96)^n$

D $A = 1200(1.04)^n$ **E** $A = 1200(0.04)^n$

Extended-response questions

1 David was 720 km from home. He drove home at an average speed of 90 km per hour.

a Complete the table of values below from 0 to 8 hours.

Time in hours (t)	0	2	4	6	8
km from home	720				

b Plot a graph of the number of kilometres from home against time.

c David started his trip at 6 a.m. If he decides to stop for breakfast 270 km from his starting point, what time would he stop for breakfast?

d If the car needs refilling every 630 km, how long could David drive for before refilling the car?

e What would be the total driving time if he didn't stop at all?

f If the total number of breaks, including food and petrol stops, is 2 hours, when would David arrive home?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 A young maths whiz in the back seat of a car is counting down the distance to the nearest town, which initially is 520 km away. The car is travelling at an average speed of 80 km per hour.

a Find the distance to the town after:

i 1 hour **ii** 3 hours

b D km is the distance to the town after t hours.

i Write a rule for D in terms of t.

ii Sketch a graph for t between 0 and 6.5.

c Use your rule to find:

i the distance to the town after 4.5 hours

ii the time it takes for the distance to the town to be 340 km

3 During the growing season, a certain type of water lily spreads by 9% per week. The water lily covers an area of 2 m^2 at the start of the growing season.

a Write a rule for the area, A m^2, covered by the water lily after n weeks.

b Calculate the area covered, correct to four decimal places, after:

i 2 weeks

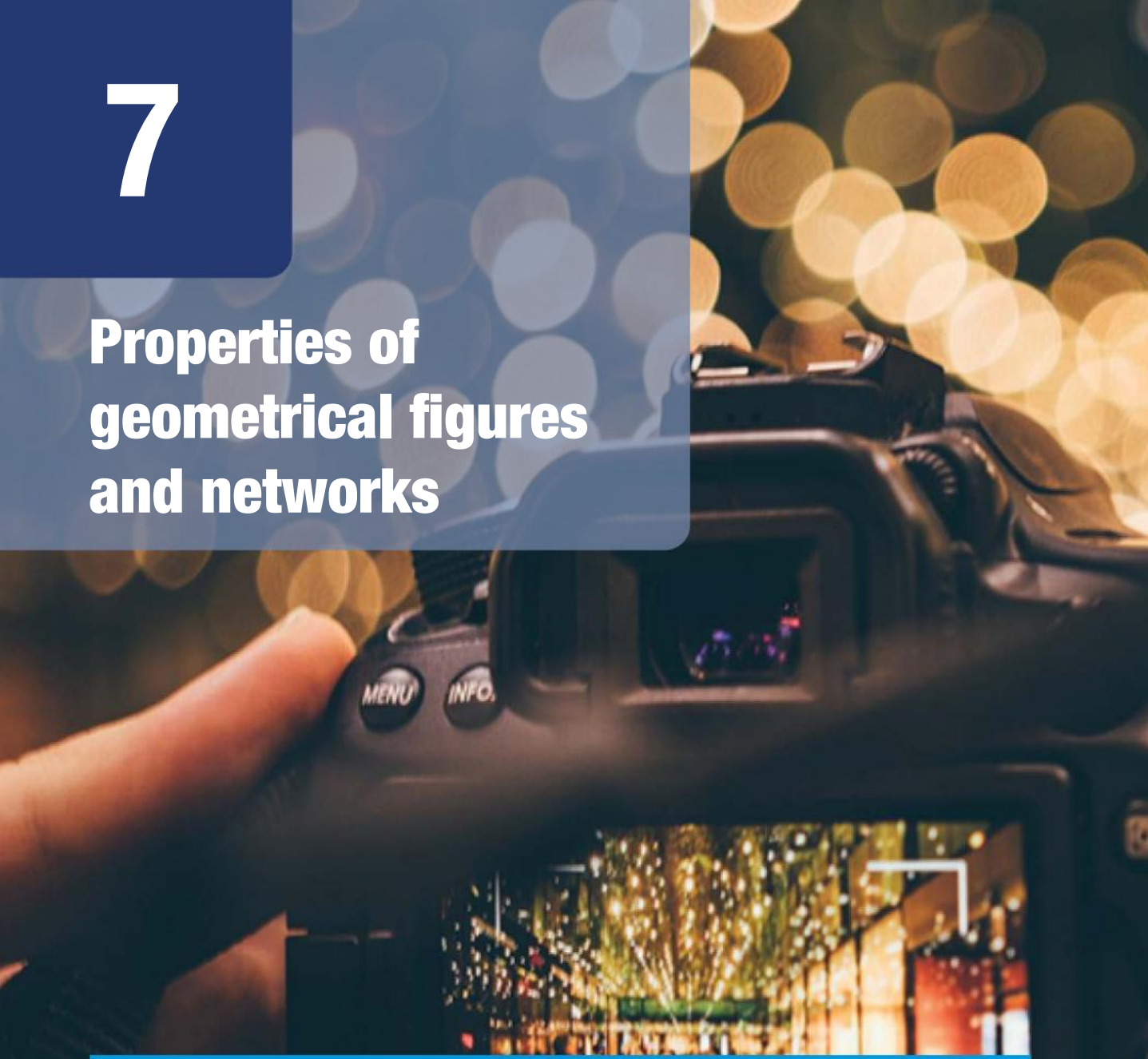
ii 5 weeks

c Use trial and error to determine, to one decimal place, when there will be a coverage of 50 m^2.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7

Properties of geometrical figures and networks



Why geometry skills are important

Geometry skills are fundamental to almost every aspect of the built environment. They are used in the construction of houses, hospitals, high-rise buildings, bridges, roads and railways, towers for power transmission and communication antennas, and clean water supply and sanitation systems.

- Making use of the geometry of similar and congruent triangles, combined with trigonometry, is essential for builders, architects, engineers, surveyors, navigators and astronomers.
- When builders and carpenters construct a house, it is essential that roof rafters are parallel, ceiling joists are horizontal and parallel, and wall studs are vertical and parallel.
- Triangles are the strongest form of support. Skills using congruent and similar triangles are essential for engineers and construction workers when building communication towers, electricity pylons, steel joist girders, cranes and bridges.
- Designers of eyeglasses, cameras, microscopes, telescopes and projectors all apply similar triangle geometry to calculate the size of a virtual image formed when a lens bends light rays.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter contents

7A Parallel lines (CONSOLIDATING)
7B Triangles (CONSOLIDATING)
7C Quadrilaterals (EXTENDING)
7D Polygons (EXTENDING)
7E Similarity and scale drawings
7F Applying similar triangles
7G Introduction to networks
7H Isomorphic and planar graphs
7I Trails, paths and Eulerian circuits
7J Shortest path problems (EXTENDING)

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- identifies and applies the properties of similar figures and scale drawings to solve problems (MA5-GEO-C-01)
- solves problems involving the characteristics of graphs/networks, planar graphs and Eulerian trails and circuits (MA5-NET-P-01).

© 2022 NSW Education Standards Authority

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Warm-up quiz

1 Write the missing word. Choose from: *right*, *reflex*, *straight*, *complementary*, *acute*, *revolution*, *obtuse*, *supplementary*.

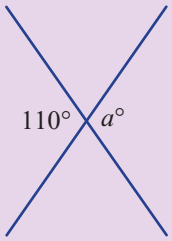
a _____ angles are between 0° and 90°.
b A _____ angle is 90°.
c An _____ angle is between 90° and 180°.
d A 180° angle is called a _____ angle.
e A _____ angle is between 180° and 360°.
f A _____ is 360°.
g _____ angles sum to 90°.
h _____ angles sum to 180°.

2 Match the type of triangle (**A–F**) with the given properties (**a–f**).

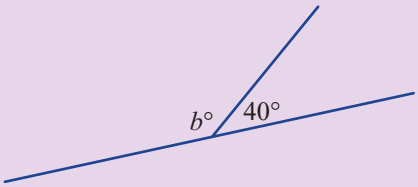
a	all sides of different length	**A**	obtuse angled
b	two sides of the same length	**B**	isosceles
c	one right angle	**C**	equilateral
d	one obtuse angle	**D**	acute angled
e	three sides of equal length	**E**	scalene
f	all angles acute	**F**	right angled

3 Find the values of the pronumerals.

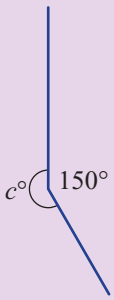
a



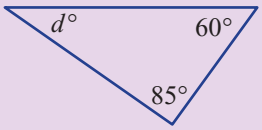
b



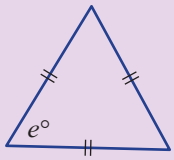
c



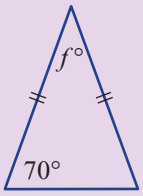
d



e



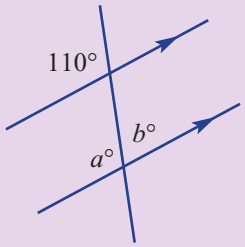
f



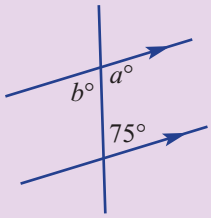
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Find the value of the pronumerals in these sets of parallel lines.

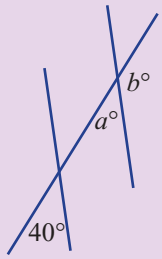
a



b



c

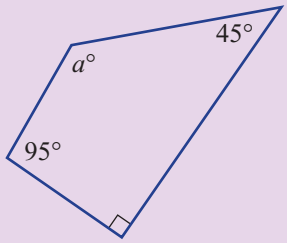


5 How many special quadrilaterals have these properties?

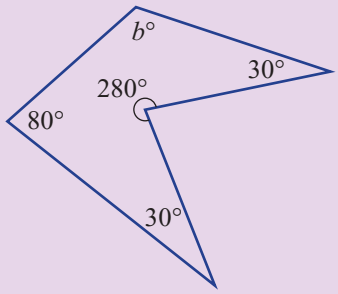
- **a** All sides equal and all angles 90°
- **b** Two pairs of parallel sides
- **c** Two pairs of parallel sides and all angles 90°
- **d** Two pairs of parallel sides and all sides equal
- **e** One pair of parallel sides
- **f** Two pairs of equal-length sides and no sides parallel

6 Use the angle sum formula, $S = (n - 2) \times 180°$, to find the angle sum of these polygons and the value of the pronumeral.

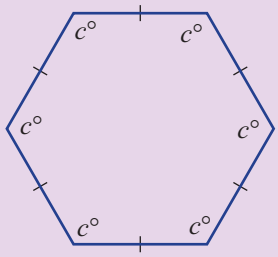
a



b



c



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7A Parallel lines CONSOLIDATING

Learning intentions for this section:

- To know the special pairs of angles formed when a transversal cuts two other lines
- To know the relationships between angles formed by parallel lines and a transversal
- To be able to calculate unknown angles associated with parallel lines

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- It consolidates and extends the concepts in Section 8B of our Year 9 book
- These concepts are assumed knowledge for Stage 6 Standard and are occasionally useful

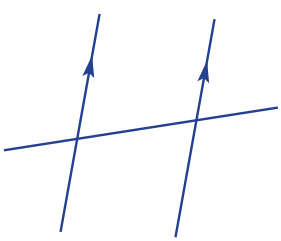
Parallel lines are everywhere – in buildings, in nature and on clothing patterns. Steel or concrete uprights at road intersections are an example.

Parallel lines are always the same distance apart and never meet. In diagrams, arrows are used to show that lines are parallel.

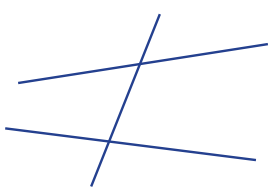
Lesson starter: 2, 4 or 8 different angles

Here are two pairs of lines crossed by a transversal. One pair is parallel and the other is not.

A



B

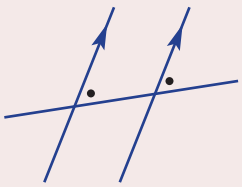


- How many angles of different size are in set A?
- How many angles of different size are in set B?
- If only one angle is known in set A, can you determine all other angles? Give reasons.

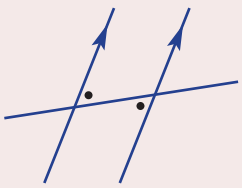
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

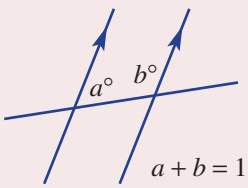
- A **transversal** is a line cutting two or more other lines.
- For parallel lines:
  - **Corresponding** angles are equal.



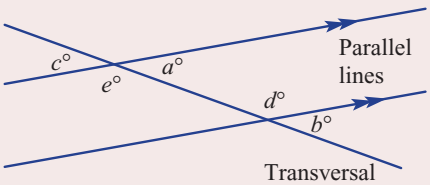
  - **Alternate** angles are equal.



  - **Cointerior** angles are **supplementary** (sum to 180°).



$a + b = 180$



$a = b$	Corresponding angles
$a = c$	**Vertically opposite** angles
$d = e$	Alternate angles
$a + e = 180$	Supplementary angles
$a + d = 180$	Cointerior angles

Key terminology: parallel, transversal, corresponding, alternate, cointerior, vertically opposite, supplementary

Exercise 7A

UNDERSTANDING | 1, 2 | 2

1 Write the missing word or number.

a Supplementary angles add to _____.

b Vertically opposite angles are _____.

c When two lines are parallel and are crossed by a transversal, then:

i Corresponding angles are _____.

ii Alternate angles are _____.

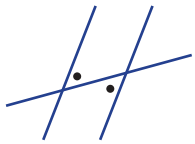
iii Cointerior angles are _____.

Hint: Choose from: 180°, *equal, supplementary*

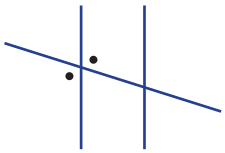
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 For the given diagrams, decide whether the given pair of marked angles are corresponding, alternate, cointerior or vertically opposite.

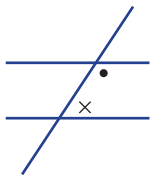
a



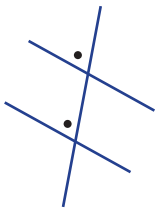
b



c



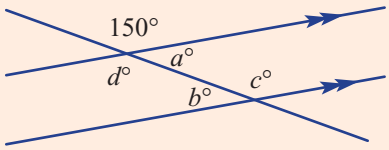
d



FLUENCY 3–5 3, 4, 5(½)

Example 1 Finding angles in parallel lines

Find the values of the pronumerals in this diagram. Write down the reason in each case.



SOLUTION

$a + 150 = 180$ (angles on a straight line)

$a = 30$

$b = a = 30$ (alternate angles in parallel lines)

$a + c = 180$ (cointerior angles in parallel lines)

$30 + c = 180$

$c = 150$

$d = 150$ (vertically opposite angles)

EXPLANATION

Two angles on a straight line sum to 180°.

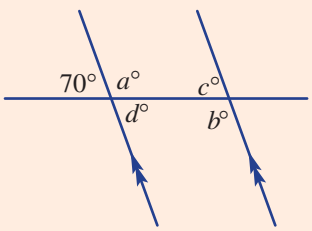
$a°$ and $b°$ are alternate angles in parallel lines.

$a°$ and $c°$ are cointerior angles in parallel lines.

Vertically opposite angles are equal.

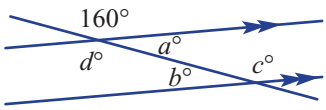
Now you try

Find the values of the pronumerals in this diagram. Write down the reason in each case.

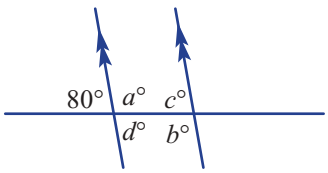


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Find the values of the pronumerals in this diagram. Write down the reason in each case.

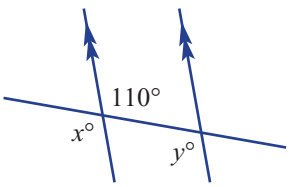


4 Find the values of the pronumerals in this diagram. Write down the reason in each case.

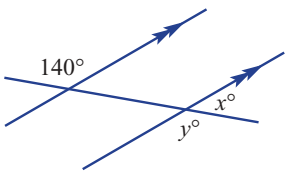


5 Find the values of x and y in these diagrams.

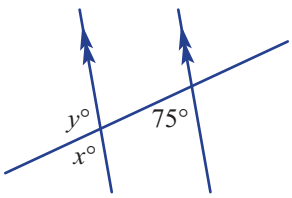
a



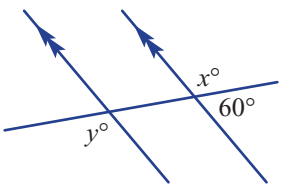
b



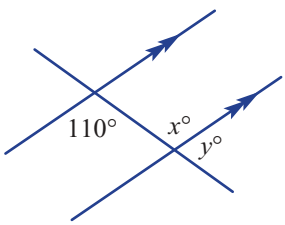
c



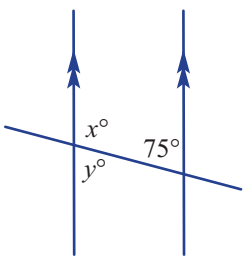
d



e



f



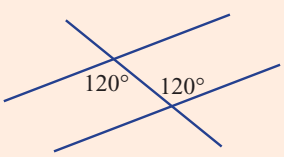
Hint: When lines are parallel:
- Corresponding angles are equal.
- Alternate angles are equal.
- Cointerior angles add to 180°.

PROBLEM-SOLVING AND REASONING 6, 7–8(½), 9 | 6–8(½), 9, 10

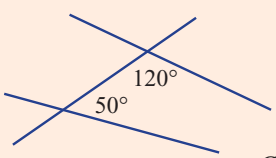
Example 2 Proving that two lines are parallel

Decide, with reasons, whether the given pairs of lines are parallel.

a



b



Continued on next page

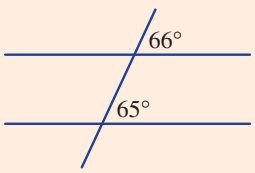
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION	EXPLANATION
a Yes, alternate angles are equal.	If alternate angles are equal, then lines are parallel.
b No, cointerior angles are not supplementary.	If lines are parallel, then cointerior angles should add to 180°, but 120° + 50° = 170°.

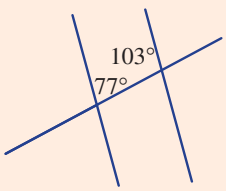
Now you try

Decide, with reasons, whether the given pairs of lines are parallel.

a

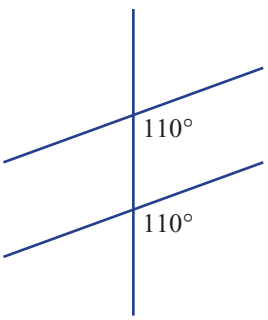


b

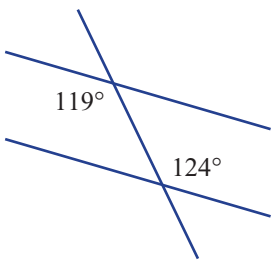


6 Decide, with reasons, whether the given pairs of lines are parallel.

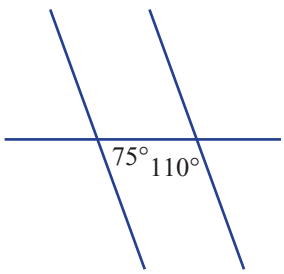
a



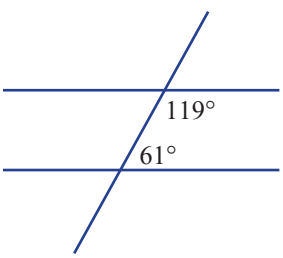
b



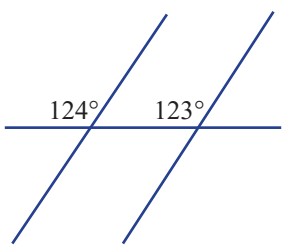
c



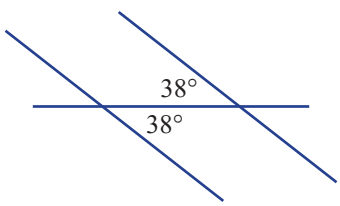
d



e

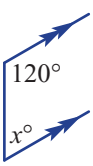


f

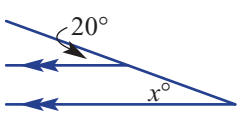


7 These diagrams have a pair of parallel lines. Find the value of x.

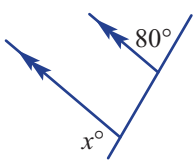
a



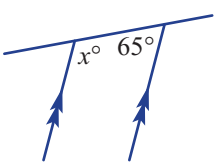
b



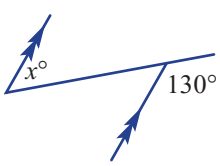
c



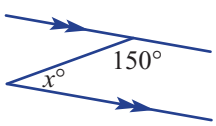
d



e



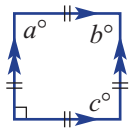
f



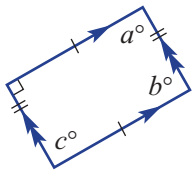
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 These common shapes consist of parallel lines. One or more internal angles are given. Find the values of the pronumerals.

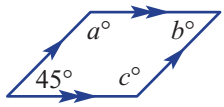
a



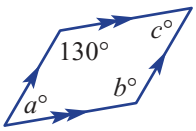
b



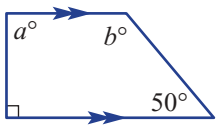
c



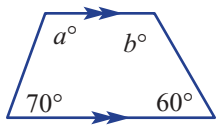
d



e



f



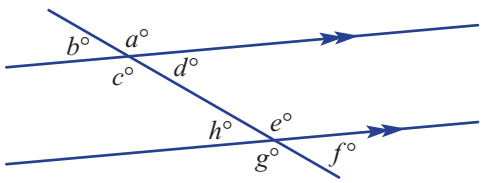
9 For this diagram, list all pairs of angles that are:

a corresponding

b alternate

c cointerior

d vertically opposite

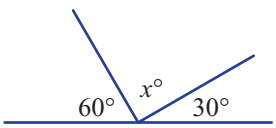


Hint: One example for part **a** is (a, e).

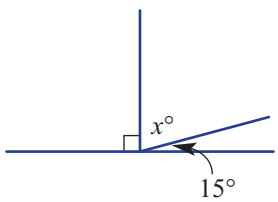
10 Find the unknown value, x, in each of these cases.

Hint: Angles on a straight line add to 180°.

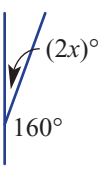
a



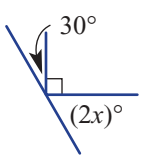
b



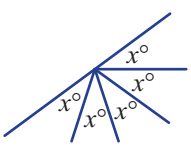
c



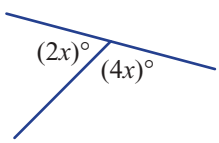
d



e



f



ENRICHMENT: The roof truss — 11

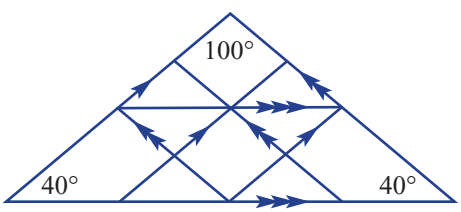
11 This diagram is of a roof truss with three groups of parallel supports.

How many of the angles are:

a 100°?

b 40°?

c 140°?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7B Triangles CONSOLIDATING

Learning intentions for this section:

- To know the angle sum of a triangle and the properties of special types of triangles
- To know the exterior angle theorem
- To be able to calculate unknown interior and exterior angles of a triangle

Past, present and future learning:

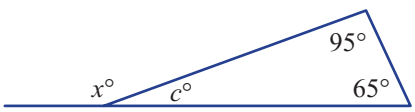
- This section addresses Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- It consolidates and extends the concepts in Section 8A of our Year 9 book
- These concepts are assumed knowledge for Stage 6 Standard and are occasionally useful

One of the best known and most useful properties of triangles is the internal angle sum (180°). You can check this by measuring and adding up the three internal angles of any triangle.

Lesson starter: Exterior angle proof

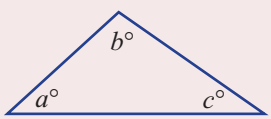
Consider this triangle with exterior angle $x°$.

- Use the angle sum of a triangle to find the value of c.
- Now find the value of x.
- What do you notice about $x°$ and the two given angles? Is this true for other triangles? Give examples and reasons.

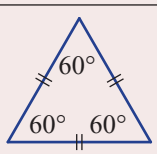
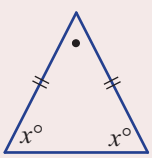
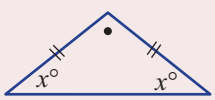
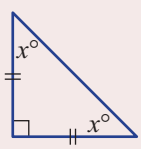
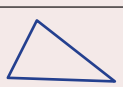
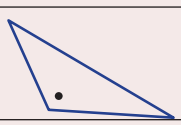
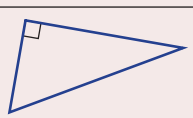


KEY IDEAS

- The sum of all three internal angles of a triangle is 180°.
- Triangles can be classified by their side lengths or their internal angles.

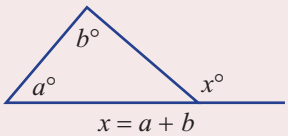


$a + b + c = 180$

		Classified by internal angles		
		Acute-angled triangles (all angles acute, $< 90°$)	Obtuse-angled triangles (one angle obtuse, $> 90°$)	Right-angled triangles (one right angle, $90°$)
Classified by side lengths	Equilateral triangles (three equal side lengths)		Not possible	Not possible
	Isosceles triangles (two equal side lengths)			
	Scalene triangles (no equal side lengths)			

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

■ The **exterior angle theorem**:
The exterior angle is equal to the sum of the two opposite interior angles.



Key terminology: triangle, acute, right, obtuse, scalene, isosceles, equilateral, exterior angle theorem

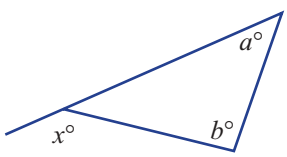
Exercise 7B

UNDERSTANDING 1–3 3

1 State the missing word or angle size.

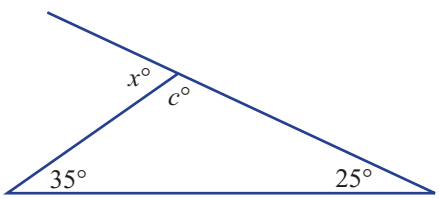
- **a** The angle sum of a triangle is _____.
- **b** An _____ triangle has two equal side lengths.
- **c** An equilateral triangle's interior angles are all _____.
- **d** An _____ -angled triangle has one obtuse angle.
- **e** A _____ triangle has all sides of different length.
- **f** An acute-angled triangle has all angles less than _____.

2 Choose the correct expression for this exterior angle.



A $a = x + b$ **B** $b = x + a$
C $x = a + b$ **D** $a + b = 180$
E $2a + b = 2x$

3 The two given interior angles for this triangle are 25° and 35°.

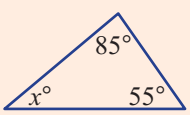


- **a** Use the angle sum (180°) to find the value of c.
- **b** Hence, find the value of x.
- **c** What do you notice about the value of x and the two given interior angles?

FLUENCY 4–6(½) 4–6(½)

Example 3 Using the angle sum

Find the value of the unknown angle (x) in this triangle.



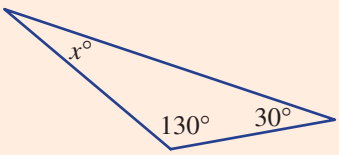
Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION	EXPLANATION
$x + 85 + 55 = 180$	The sum of the three internal angles in a triangle is 180°.
$x + 140 = 180$	Simplify before solving for x.
$x = 40$	Solve for x by subtracting 140 from both sides of the equals sign.
∴ The unknown angle is 40°.	

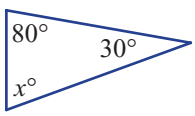
Now you try

Find the value of the unknown angle, x, in this triangle.

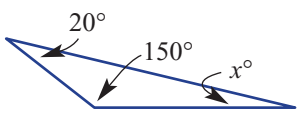


4 Find the value of the unknown angle (x) in these triangles.

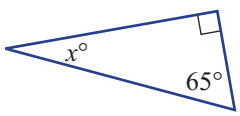
a



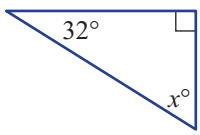
b



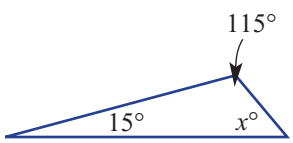
c



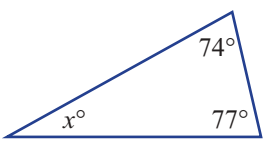
d



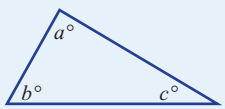
e



f



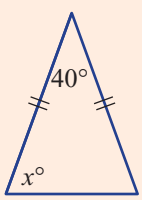
Hint:



$a + b + c = 180$

Example 4 Working with an isosceles triangle

Find the value of x in this isosceles triangle.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

$x + x + 40 = 180$
$2x + 40 = 180$
$2x = 140$
$x = 70$

$\therefore$ The unknown angle is 70°.

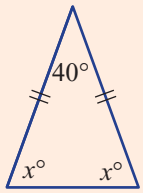
EXPLANATION

The triangle is isosceles and therefore the two base angles are equal.

Collect like terms.

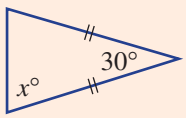
Subtract 40 from both sides.

Divide both sides by 2.



Now you try

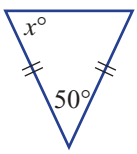
Find the value of x in this isosceles triangle.



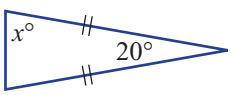
5 Find the value of the unknown angle (x) in these triangles.

Hint: Label the third angle first.

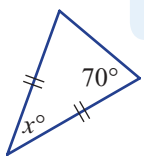
a



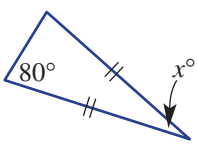
b



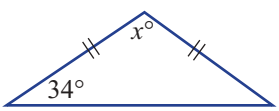
c



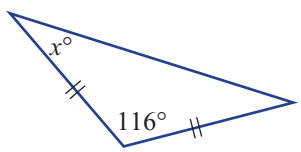
d



e



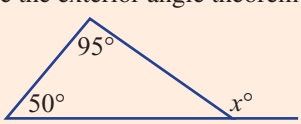
f



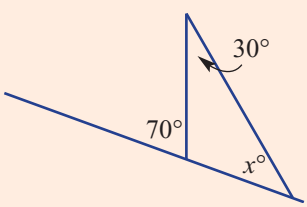
Example 5 Using the exterior angle theorem

Use the exterior angle theorem to find the value of x.

a



b



SOLUTION

a $x = 95 + 50$
$= 145$

b $x + 30 = 70$
$x = 40$

EXPLANATION

The exterior angle $x°$ is the sum of the two opposite interior angles.

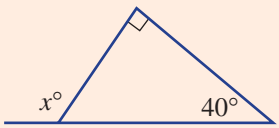
The two opposite interior angles are $x°$ and 30°, and 70° is the exterior angle.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

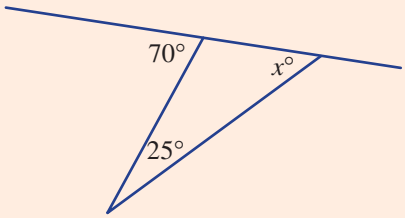
Now you try

Use the exterior angle theorem to find the value of x.

a

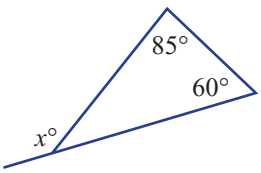


b

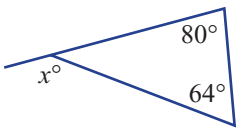


6 Use the exterior angle theorem to find the value of x.

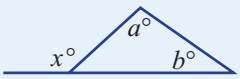
a



b

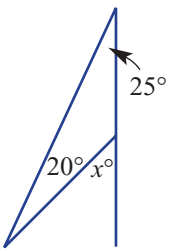


Hint:

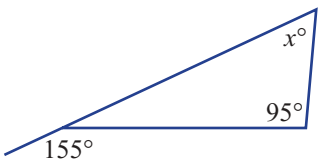


$x = a + b$

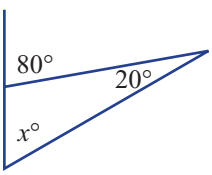
c



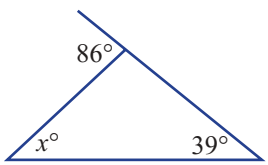
d



e



f



PROBLEM-SOLVING AND REASONING

7, 8 | 7, 9, 10

7 Decide whether the following are possible. If so, make a drawing.

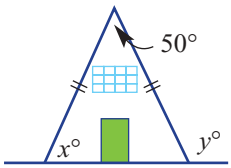
- **a** acute scalene triangle
- **b** acute isosceles triangle
- **c** obtuse equilateral triangle
- **d** acute equilateral triangle
- **e** obtuse isosceles triangle
- **f** obtuse scalene triangle
- **g** right equilateral triangle
- **h** right isosceles triangle
- **i** right scalene triangle

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 An architect draws the cross-section of a new ski lodge, which includes a very steep roof, as shown. The angle at the top is 50°. Find:

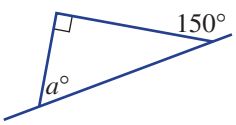
a the acute angle the roof makes with the floor ($x°$)

b the obtuse angle the roof makes with the ground outside ($y°$)

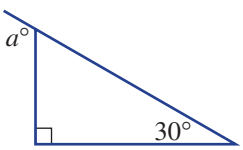


9 Use your knowledge of parallel lines and triangles to find out the value of the pronumerals in these diagrams.

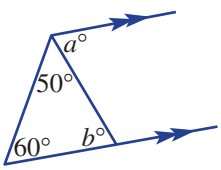
a



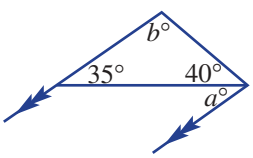
b



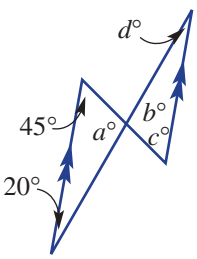
c



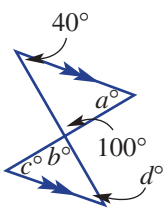
d



e



f

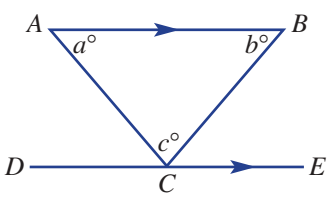


10 For this diagram, AB is parallel to DE.

a Which pronumeral has the same size as $\angle ACD$? Give a reason.

b Which pronumeral has the same size as $\angle BCE$? Give a reason.

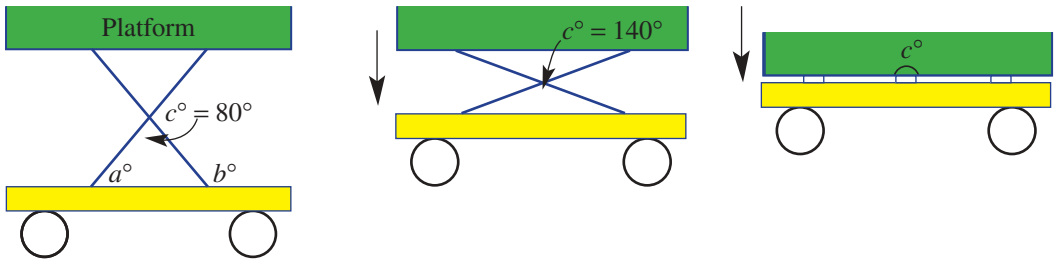
c Since $\angle DCE = 180°$, what does this tell us about a, b and c?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: The hydraulic platform — 11

11 A hydraulic platform includes a movable 'X' shape support system, as shown. When the platform is at its highest point, the angle at the centre ($c°$) of the 'X' is 80°, as shown.



a Find the following when the platform is at its highest position.

i the acute angle the 'X' makes with the platform ($a°$)

ii the obtuse angle the 'X' makes with the platform ($b°$)

b The platform now moves down so that the angle at the centre ($c°$) of the 'X' changes from 80° to 140°. With this platform position, find the values of:

i the acute angle the 'X' makes with the platform ($a°$)

ii the obtuse angle the 'X' makes with the platform ($b°$)

c The platform now moves down to the base so that the angle at the centre ($c°$) of the 'X' is now 180°. Find:

i the angle the 'X' makes with the platform ($a°$)

ii the angle the 'X' makes with the platform ($b°$)

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7C Quadrilaterals EXTENDING

Learning intentions for this section:

- To know the properties of special quadrilaterals
- To know the angle sum of a quadrilateral
- To be able to calculate unknown angles inside a quadrilateral

Past, present and future learning:

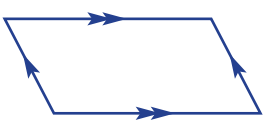
- This section addresses Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- It consolidates and extends the concepts in Section 8C of our Year 9 book
- Properties of quadrilaterals are extension work from a Stage 5 Path-to-advanced topic
- These concepts are occasionally useful in Stage 6 Standard

Quadrilaterals are shapes that have four straight sides with a special angle sum of 360°. There are six special quadrilaterals, each with their own special set of properties. If you look around any old or modern building, you will see examples of these shapes.

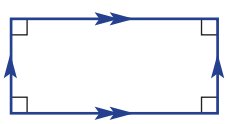
Mosaics are made up of quadrilaterals along with many other polygons.

Lesson starter: Why is a rectangle a parallelogram?

By definition, a parallelogram is a quadrilateral with two pairs of parallel sides.



Parallelogram



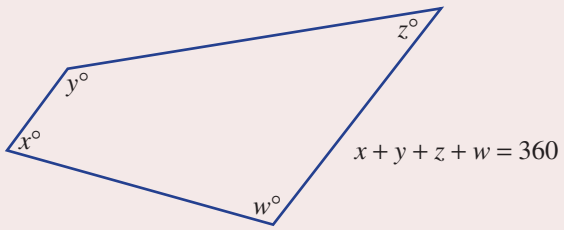
Rectangle

- Using this definition, do you think that a rectangle is also a parallelogram? Why?
- What properties does a rectangle have that a general parallelogram does not?
- What other special shapes are parallelograms? What are their properties?

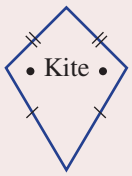
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

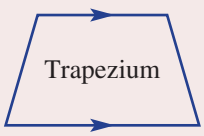
- The sum of the interior angles of any quadrilateral is 360°.



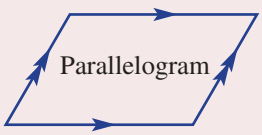
- Formal definitions:
  - A kite is a quadrilateral with two pairs of adjacent sides that are equal and one pair of opposite equal angles.



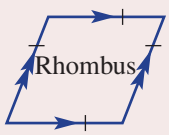
  - A trapezium is a quadrilateral with at least one pair of opposite sides that are parallel.



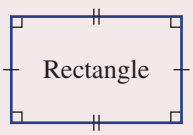
  - A parallelogram is a quadrilateral with both pairs of opposite sides that are parallel.



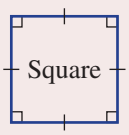
  - A rhombus is a parallelogram with two adjacent sides that are equal in length.



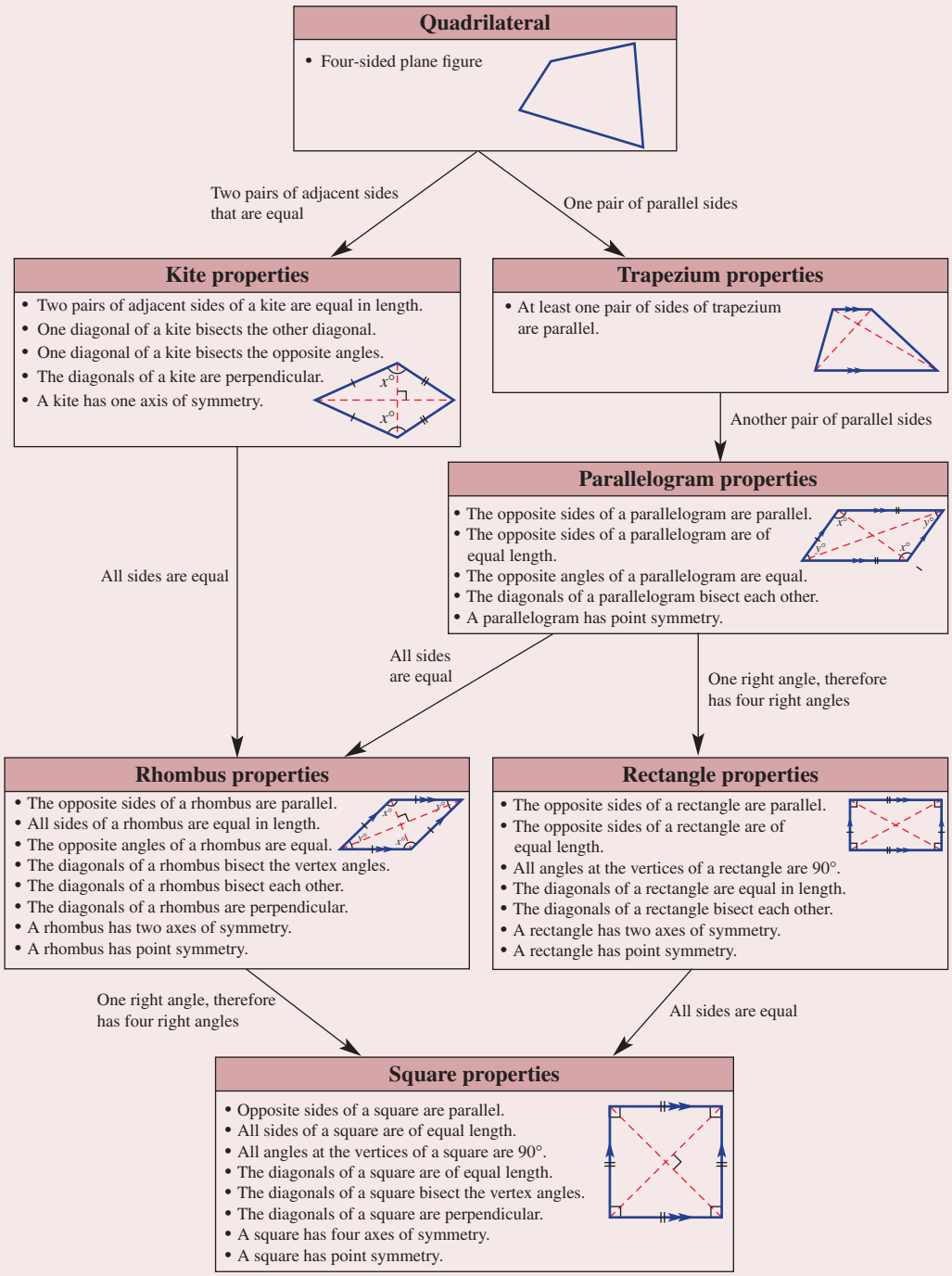
  - A rectangle is a paralleogram with one angle that is a right angle.



  - A square is a rectangle with two adjacent sides that are equal.



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.



Key terminology: quadrilateral, parallelogram, square, rectangle. rhombus, trapezium, kite, diagonal, parallel

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 7C

UNDERSTANDING	1–3	2, 3

1 Which special quadrilaterals are parallelograms?

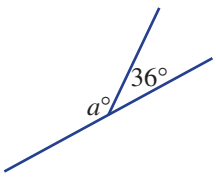
Hint: Refer to the Key ideas for help.

2 List all the quadrilaterals that have the following properties.

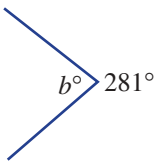
a two pairs of parallel sides
b two pairs of equal length sides
c equal opposite angles
d one pair of parallel sides
e one pair of equal angles
f all angles 90°
g equal length diagonals
h diagonals intersecting at right angles

3 Find the value of the pronumerals.

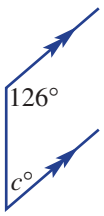
a



b



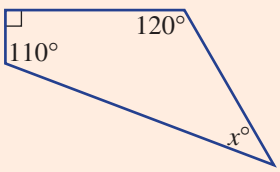
c



FLUENCY	4–5(½)	4–5(½)

Example 6 Using the angle sum of a quadrilateral

Find the unknown angle in this quadrilateral.



SOLUTION

$x + 110 + 120 + 90 = 360$
$x + 320 = 360$
$x = 40$

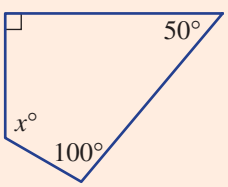
∴ The unknown angle is 40°.

EXPLANATION

The sum of the interior angles is 360° in a quadrilateral.
Simplify.
Subtract 320 from both sides.

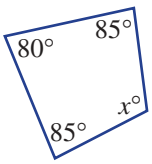
Now you try

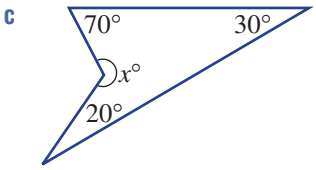
Find the unknown angle in this quadrilateral.

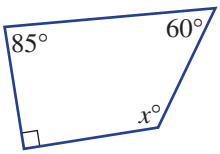


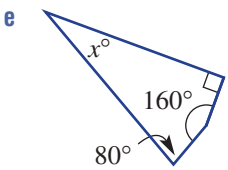
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

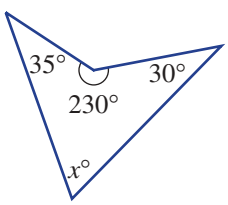
4 Find the unknown angles in these quadrilaterals.







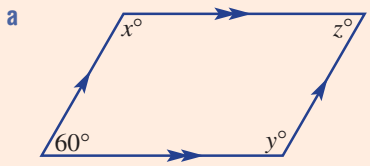


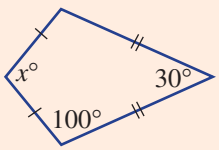


Hint: The angle sum of a quadrilateral is 360°

Example 7 Finding angles in special quadrilaterals

Find the value of the pronumerals in these special quadrilaterals, giving reasons.





SOLUTION

a $x + 60 = 180$ (cointerior angles in parallel lines)

$x = 120$

$\therefore y = 120$ (opposite angles in a parallelogram)

$\therefore z = 60$ (opposite angles in a parallelogram)

EXPLANATION

$x°$ and $60°$ are cointerior angles and sum to $180°$.

Subtract 60 from both sides.

$y°$ is opposite and equal to $x°$.

$z°$ is opposite and equal to $60°$.

b $x + 100 + 100 + 30 = 360$ (angle sum of a quadrilateral)

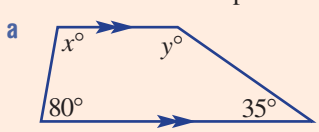
$x + 230 = 360$

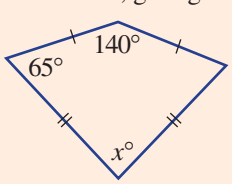
$x = 130$

A kite has a pair of equal, opposite angles, so there are two $100°$ angles.
The total sum is still $360°$.

Now you try

Find the value of the pronumerals in these special quadrilaterals, giving reasons.



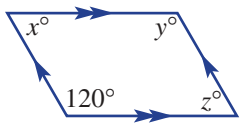


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

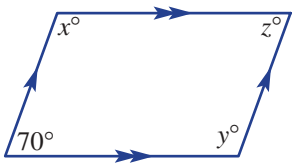
5 Find the value of the pronumerals in these special quadrilaterals, giving reasons.

Hint: Refer to the properties of special quadrilaterals for help.

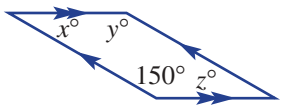
a



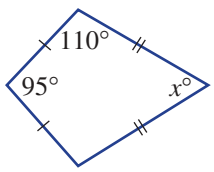
b



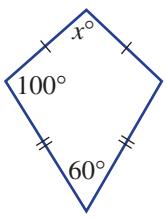
c



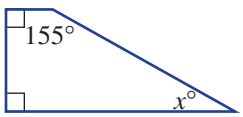
d



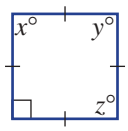
e



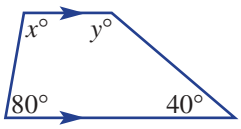
f



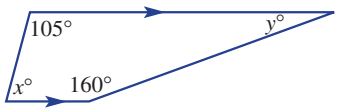
g



h



i

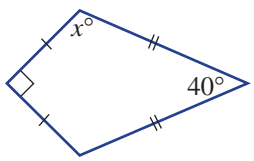


PROBLEM-SOLVING AND REASONING

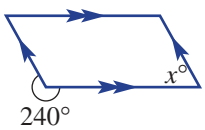
6, 7 | 6–9

6 Find the value of the pronumerals in these shapes.

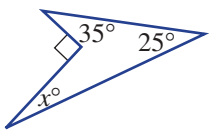
a



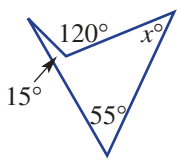
b



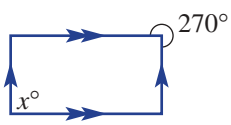
c



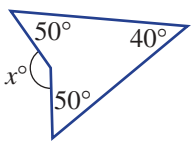
d



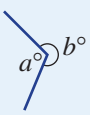
e



f



Hint:
Angles in a revolution add to 360°.



$a + b = 360$

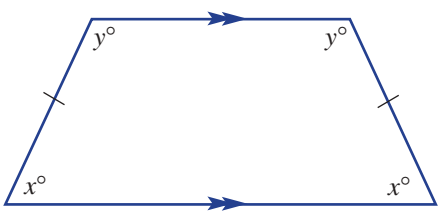
7 This shape is called an isosceles trapezium.

a Why do you think it is called an isosceles trapezium?

b **i** If $x = 60$, find the value of y.

ii If $y = 140$, find the value of x.

c List the properties of an isosceles trapezium.

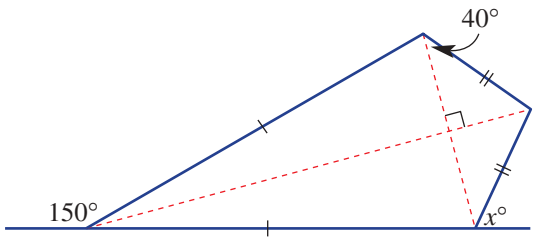


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 A modern hotel is in the shape of a kite. Some angles are given in the diagram.

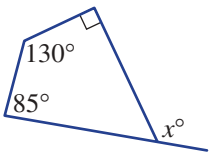
a Draw a copy of just the kite shape, including the diagonals.

b Find the angle that the right-hand wall makes with the ground ($x°$).

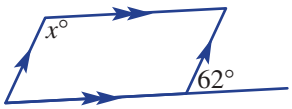


9 These quadrilaterals also include exterior angles. Find the value of x.

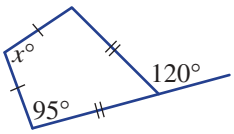
a



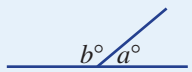
b



c



Hint:

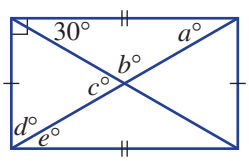


$a + b = 180$

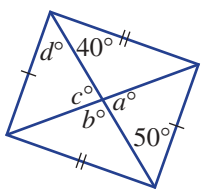
ENRICHMENT: Quadrilaterals and triangles — 10

10 The following shapes combine quadrilaterals with triangles. Find the values of the pronumerals.

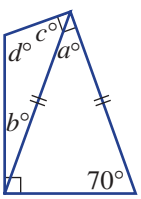
a



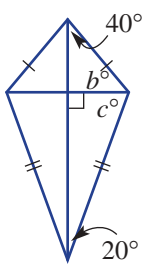
b



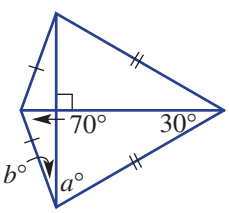
c



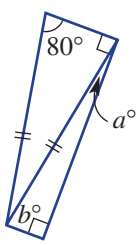
d



e



f



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7D Polygons EXTENDING

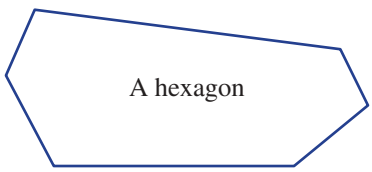
Learning intentions for this section:

- To know the rule for the angle sum of a polygon
- To be able to calculate unknown angles inside a polygon
- To be able to calculate the interior angle of regular polygons

Past, present and future learning:

- This section goes beyond the Path topics for Standard but has been included for good reason
- It consolidates and extends the concepts in Section 8D of our Year 9 book
- The skills presented are occasionally useful in Stage 6 Standard

A closed shape with all straight sides is called a polygon. Like triangles and quadrilaterals (which are both polygons), each other type of polygon has a special angle sum.



Lesson starter: Remember the names

From previous years you should remember some of the names for polygons.
See if you can remember them by completing this table.

Number of sides	Name
3	
4	
5	
6	
7	Heptagon

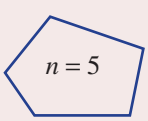
Number of sides	Name
8	
9	
10	
11	
12	

KEY IDEAS

- A **polygon** is a shape with straight sides.
  - Polygons are named by their number of sides.
- The sum of internal angles (S) of a polygon is given by this rule:

  $S = (n - 2) \times 180°$

  where n is the number of sides

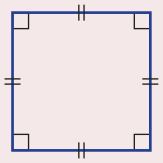


$S = (n - 2) \times 180°$
$S = (5 - 2) \times 180°$
$= 3 \times 180°$
$= 540°$

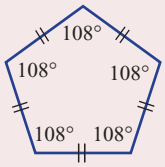
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

■ A **regular polygon** has sides of equal length and equal angles.

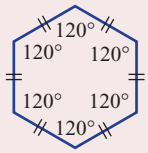
Regular quadrilateral (square) (four sides)



Regular pentagon (five sides)



Regular hexagon (six sides)



Key terminology: polygon, regular polygon

Exercise 7D

UNDERSTANDING	1–3	2(½), 3

1 How many sides do these shapes have?

a quadrilateral **b** octagon **c** decagon **d** heptagon

e nonagon **f** hexagon **g** pentagon **h** dodecagon

2 Use the angle sum rule, $S = (n - 2) \times 180°$, to find the angle sum of these polygons.

a pentagon ($n = 5$) **b** hexagon ($n = 6$) **c** heptagon ($n = 7$)

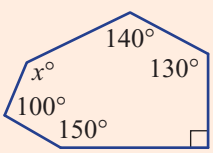
d octagon ($n = 8$) **e** nonagon ($n = 9$) **f** decagon ($n = 10$)

3 What is always true about a polygon that is regular?

FLUENCY	4(½), 5	4(½), 6

Example 8 Finding and using the angle sum of a polygon

For this polygon, find the angle sum and then the value of x.



SOLUTION

$S = (n - 2) \times 180°$
$= (6 - 2) \times 180°$
$= 720°$

$x + 100 + 150 + 90 + 130 + 140 = 720$
$x + 610 = 720$
$x = 110$

EXPLANATION

Use the angle sum rule first, with $n = 6$ since there are 6 sides.

Find the angle sum.

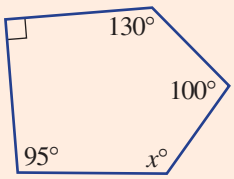
Use the total angle sum to find the value of x.

Solve for the value of x.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

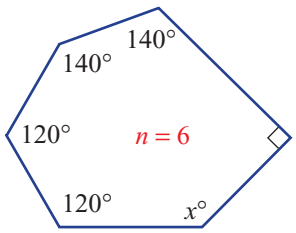
Now you try

For this polygon, find the angle sum and then the value of x.

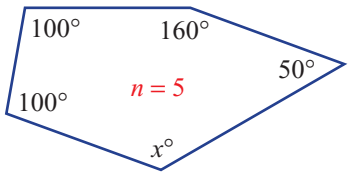


4 For these polygons, find the angle sum and then find the value of x.

a

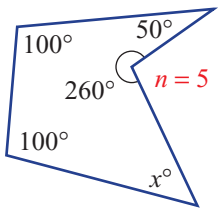


b

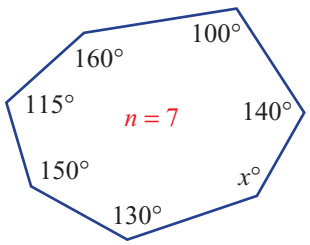


Hint: First use $S = (n - 2) \times 180°$

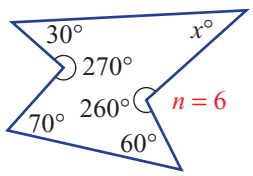
c



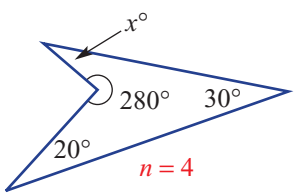
d



e

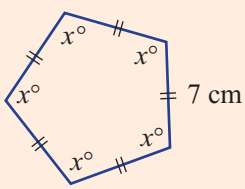


f



Example 9 Working with regular polygons

Shown here is a regular pentagon with a straight edge side length of 7 cm.



a Find the perimeter of the pentagon.
b Find the total internal angle sum (S).
c Find the size of each internal angle, $x°$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION	EXPLANATION
a 35 cm	There are five sides at 7 cm each.
b $S = (n - 2) \times 180°$ $= (5 - 2) \times 180°$ $= 3 \times 180°$ $= 540°$	Write the general rule for the sum of internal angles for a polygon. $n = 5$ since there are five sides. Simplify and evaluate.
c $540° \div 5 = 108°$ $x° = 108°$	There are five equally sized angles since it is a regular pentagon.

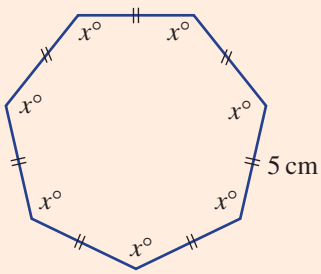
Now you try

Shown here is a regular heptagon with a straight edge side length of 5 cm.

a Find the perimeter of the heptagon.

b Find the total internal angle sum (S).

c Find the size of each internal angle, $x°$, correct two decimal places.

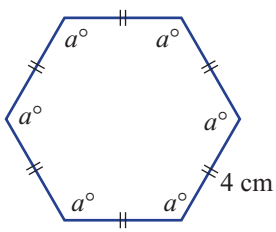


5 Shown here is a regular hexagon with straight-edge side length 4 cm.

a Find the perimeter of the hexagon.

b Find the total internal angle sum (S).

c Find the size of each internal angle, $a°$.

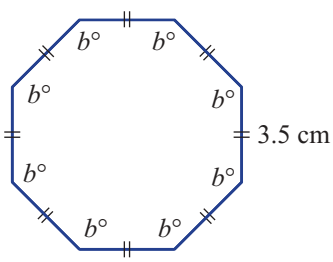


6 Shown here is a regular octagon with straight-edge side length 3.5 cm.

a Find the perimeter of the octagon.

b Find the total internal angle sum (S).

c Find the size of each internal angle, $b°$.



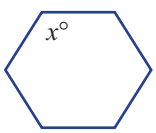
PROBLEM-SOLVING AND REASONING

7, 8, 10 | 9–11

7 The cross-section of a pencil is a regular hexagon.

a Find the sum of the interior angles.

b Find the interior angle ($x°$).



8 Find the total internal angle sum for a polygon with:

a 11 sides

b 20 sides

Hint: Remember: $S = (n - 2) \times 180°$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 Find the size of a single internal angle for a regular polygon with:

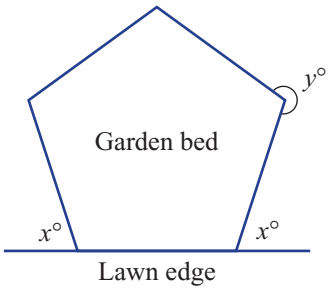
a 10 sides

b 25 sides

10 A garden bed is to be designed in the shape of a regular pentagon and sits adjacent to a lawn edge, as shown.

a Find the angle the lawn edge makes with the garden bed ($x°$).

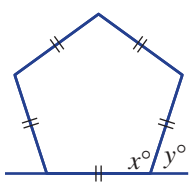
b Find the outside angle for each corner ($y°$).



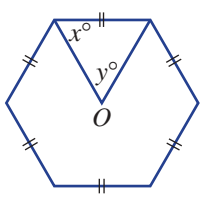
11 For these diagrams, find the values of the unknowns. The shapes are regular.

Hint: In parts **b** and **c**, the point at O is the centre.

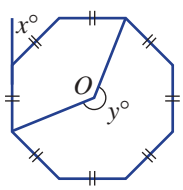
a



b



c

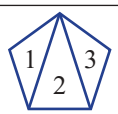


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: Develop the angle sum rule — 12

12 a Copy and complete this table. For the diagram, use diagonals to divide the shape into triangles, as shown for the pentagon.

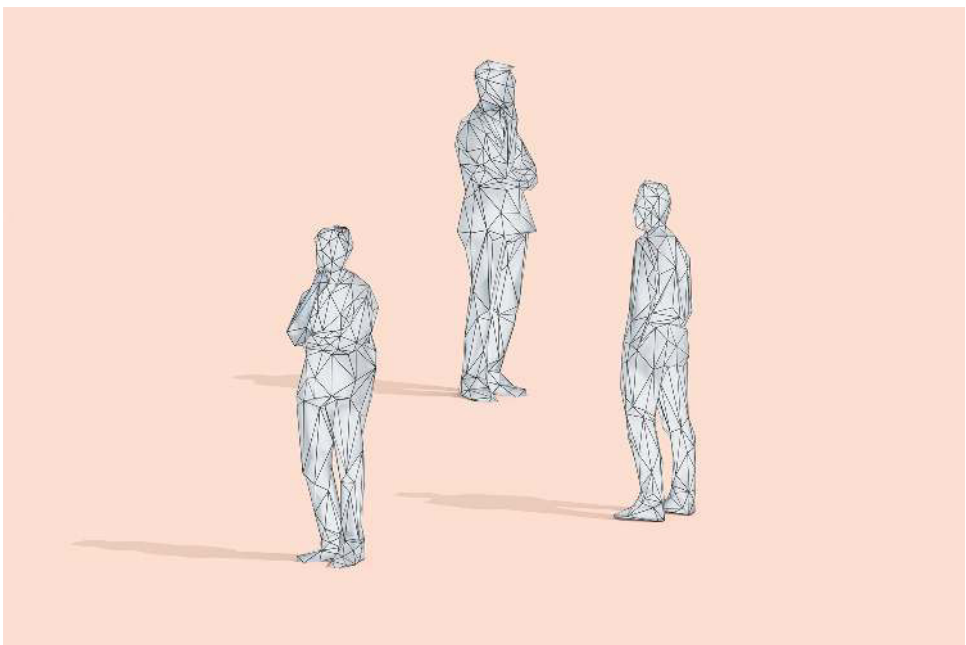
Hint: For the diagram, choose one vertex and draw lines to all other vertices.

Regular Polygon	Number of sides	Diagram	Number of triangles	Interior angle sum *(S)*	Single interior angle *(A)*
Triangle					
Quadrilateral					
Pentagon	5		3	$3 \times 180° = 540°$	$540° \div 5 = 108°$
Hexagon					
…					
n-gon	n				

b Complete these sentences by writing the rule.

i For a polygon with n sides, the interior angle sum, S, is given by $S =$ _____.

ii For a regular polygon with n sides, a single interior angle, A, is given by $A =$ _____.

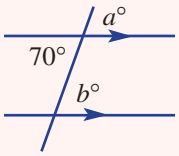


CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

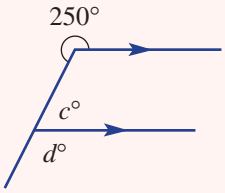
Progress quiz

7A **1** Find the values of the pronumerals in these diagrams with parallel lines, giving reasons.

a

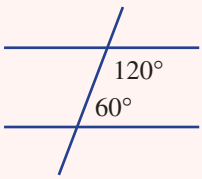


b

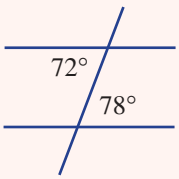


7A **2** State, with reasons, if the following pairs of lines are parallel.

a

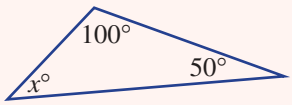


b

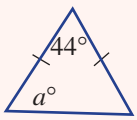


7B/7C **3** Find the values of the pronumerals in these triangles and quadrilaterals.

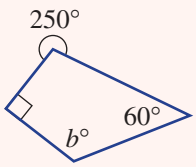
a



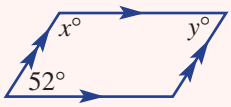
b



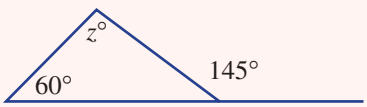
c



d

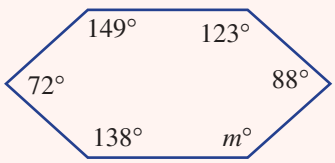


7B **4** Use the exterior angle theorem to find the value of z.

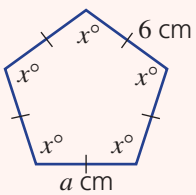


7D **5** Find the angle sum of these polygons and then find the value of each pronumeral.

a



b



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7E Similarity and scale drawings

Learning intentions for this section:

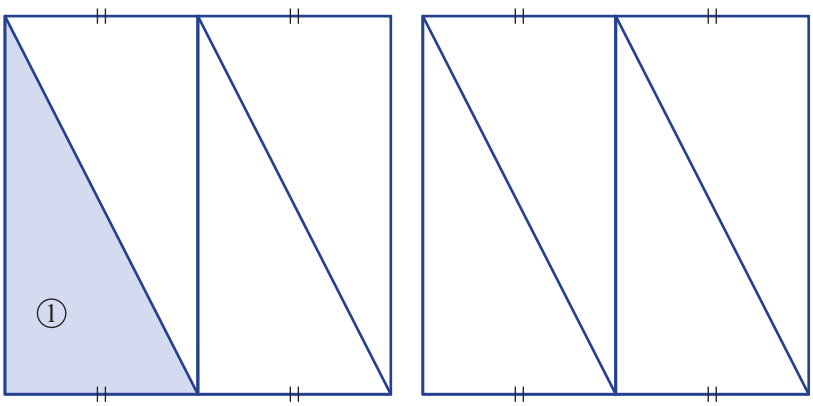
- To know that two similar figures are connected by a scale factor
- To understand that the enlargement transformation produces similar figures
- To be able to calculate a scale factor from similar figures
- To be able to find unknown side lengths and distances in similar figures and scale drawings

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Properties of geometrical figures A*
- It consolidates and extends the concepts in Sections 8E and 8F of our Year 9 book
- Students are not required to know or use the four tests for proving similarity
- In Stage 6 Standard these concepts will be revised, briefly, then extended

When two objects are similar, they are the same shape but not necessarily the same size. For example, a computer image reproduced on a large screen will show all aspects of the image in the same way except in size. The computer image and screen image are said to be similar figures.

Lesson starter: Same shape, different size

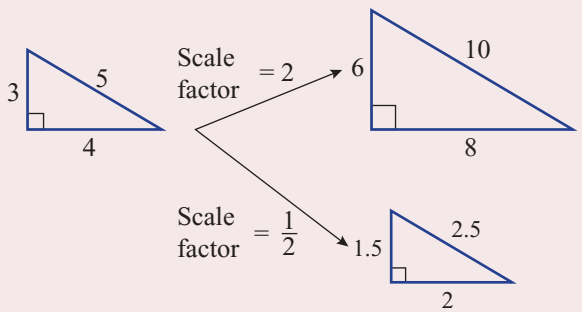


- Start with two congruent squares. Cut them into eight congruent triangles, as shown in the diagrams above.
- Use four triangles to make a larger triangle that is similar to triangle 1.
- Try adding a fifth triangle to make another triangle similar to triangle 1.
- It is possible to make a triangle similar to triangle 1 using nine triangles? Work with a partner to create the triangle.
- Can you make a similar triangle using 16 triangles?

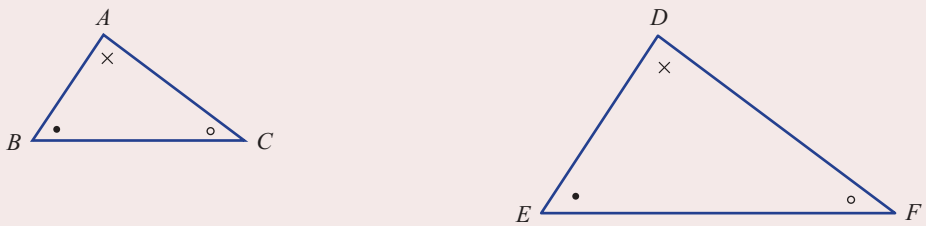
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- Two figures are **similar** if one can be enlarged to be congruent to the other (or they are the same shape and size).
  - Matching angles are equal.
  - Pairs of matching sides are in the same proportion or ratio.
- The **scale factor** $= \dfrac{\text{image length}}{\text{original length}}$
- **Enlargement** is a transformation that involves the increase or decrease in size of an object.
  - The shape of the object is unchanged.
  - Enlargement is done by multiplying all dimensions of a shape by a **scale factor**.



- If ΔABC is similar to ΔDEF, then we write $\Delta ABC \,|||\, \Delta DEF$.



- A scale drawing, such as the plan of a house, is smaller than the actual house. The scale factor is usually written as a ratio, such as 1 : 100.
  1 : 100 means that 1 mm on the plan represents 100 mm of the house.

Key terminology: similar, scale factor, enlargement

Exercise 7E

UNDERSTANDING 1–3 3

1 Two figures are said to be _____ when they are the same shape, but not necessarily the same size.

2 Use the words *factor, equal, ratio* and *scale* to fill in the blanks. In similar figures:

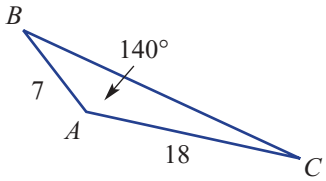
a Matching angles are _____.

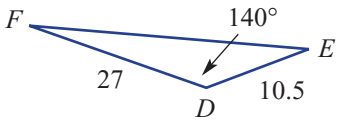
b Matching sides are in _____.

c The _____ _____ can be used to make the sides of the image longer or shorter than the original.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Consider this pair of triangles.





a Work out $\frac{DE}{AB}$.

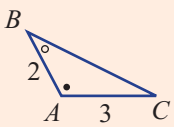
b Work out $\frac{DF}{AC}$. What do you notice?

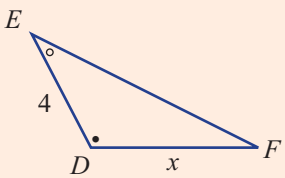
c What is the scale factor?

FLUENCY 4–6 4(½), 6

Example 10 Using similarity to find unknown values

If the given triangles are known to be similar, find the value of x.





SOLUTION

Scale factor $= \frac{DE}{AB} = \frac{4}{2} = 2$

$x = 3 \times 2$

$\therefore x = 6$

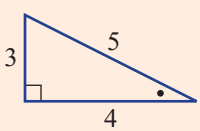
EXPLANATION

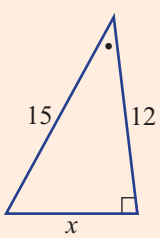
First, find the scale factor using a pair of corresponding sides. Divide the larger number by the smaller number.

Multiply the corresponding length of the smaller triangle using the scale factor.

Now you try

If the given triangles are known to be similar, find the value of x.



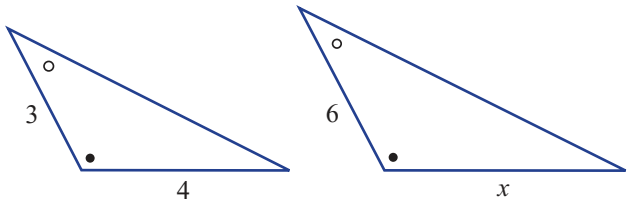


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

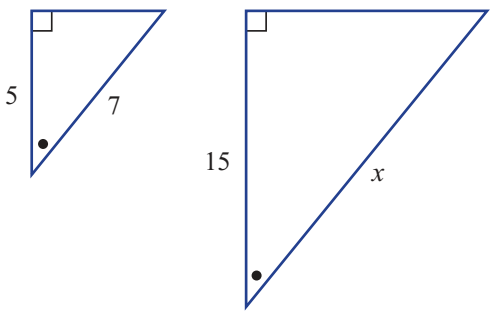
4 If the given pairs of triangles are known to be similar, find the value of x.

Hint: For parts **c** and **d**, use division to find x.

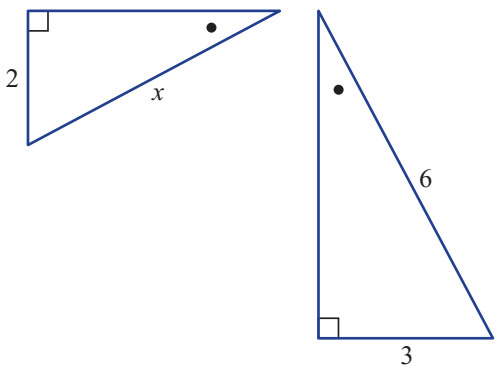
a



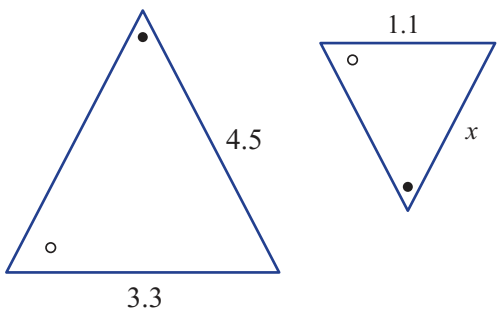
b



c



d



Example 11 Measuring to find actual lengths

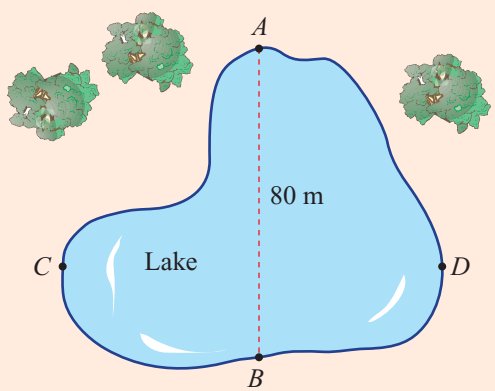
The given diagram is a simple map of a park lake.

a Use a ruler to measure the distance across the lake (AB). (Answer in cm.)

b Find the scale factor between the map and ground distance.

c Use a ruler to measure the map distance across the lake (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the lake (CD). (Answer in m.)



SOLUTION

a 4 cm

b $\frac{8000}{4} = 2000$

EXPLANATION

Check with your ruler.

Using the same units, divide the real distance (80 m = 8000 cm) by the measured distance (4 cm).

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c 5 cm — Check with your ruler.

d $5 \times 2000 = 10\,000$ cm
$= 100$ m — Multiply the measured distance by the scale factor and convert to metres by dividing by 100.

Now you try

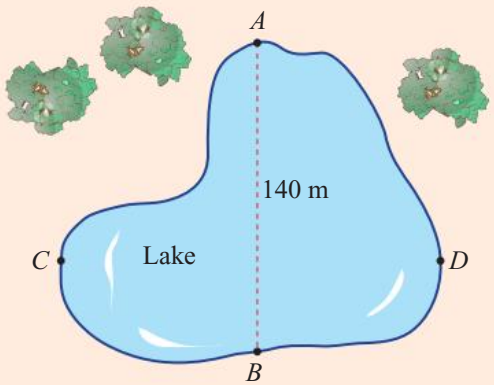
The given diagram is a simple map of a park lake.

a Use a ruler to measure the distance across the lake (AB). (Answer in cm.)

b Find the scale factor between the map and ground distance.

c Use a ruler to measure the map distance across the lake (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the lake (CD). (Answer in m.)



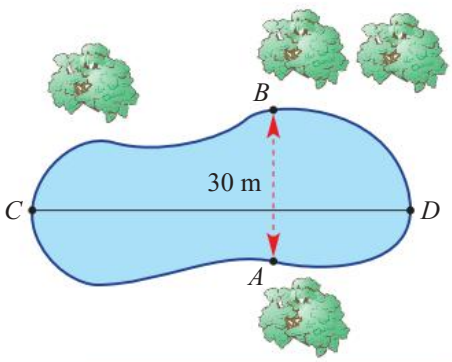
5 The given diagram is a map of a park lake.

a Use a ruler to measure the distance across the lake (AB). (Answer in cm.)

b Find the scale factor between the map and ground distance.

c Use a ruler to find the map distance across the lake (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the lake (CD). (Answer in m.)



Hint: Use the measured distance AB and the actual distance AB to find the scale factor.

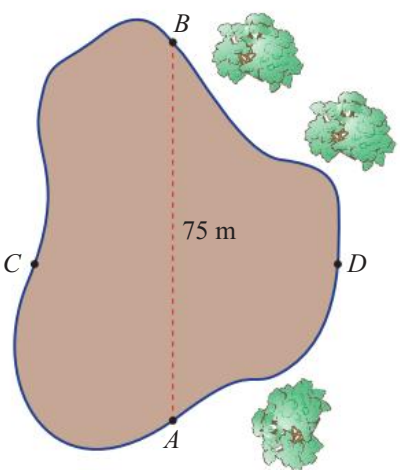
6 The given diagram is a map of a children's play area.

a Use a ruler to measure the distance across the children's play area (AB). (Answer in cm.)

b Find the scale factor between the map and ground distance.

c Use a ruler to find the map distance across the children's play area (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the children's play area (CD). (Answer in m.)



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

7, 8 | 8, 9

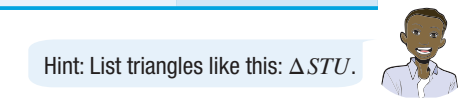
7 A ski ramp has a vertical support, as shown.

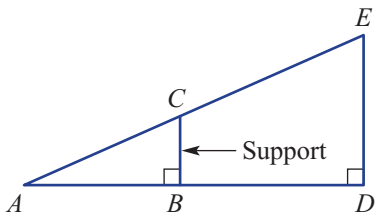
a List the two triangles that are similar.

b Why are the two triangles similar?

c If $AB = 4$ m and $AD = 10$ m, find the scale factor.

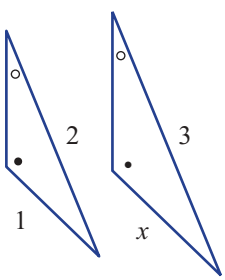
d If $BC = 1.5$ m, find the height of DE.



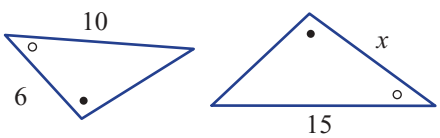


8 The pairs of triangles are similar. Determine the value of x in each case.

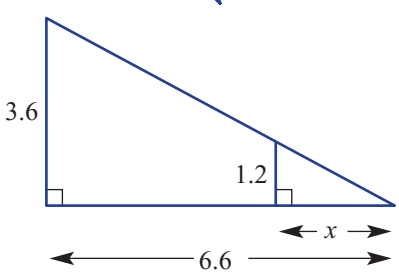
a



b



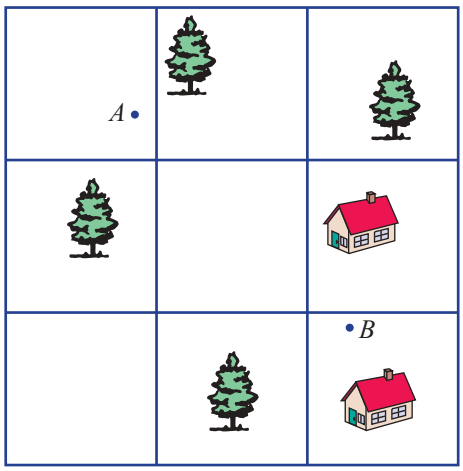
c



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 The given map has a scale factor of 50 000 (ratio 1 : 50 000).

Hint: 1 m = 100 cm
1 km = 1000 m



a How far on the ground, in km, is represented by these map distances?

i 2 cm

ii 6 cm

b How far on the map, in cm, is represented by these ground distances?

i 5 km

ii 0.5 km

c What is the actual ground distance, in km, between the two points A and B?

ENRICHMENT: Triangles in parallel lines – 10

10 In the given diagram, AB is parallel to DE.

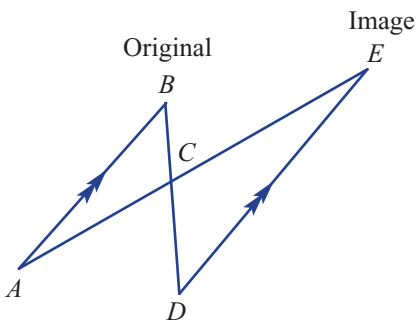
a List the three pairs of angles that are equal and give reasons.

b If $AB = 8$ cm and $DE = 12$ cm, find:

i the scale factor

ii DC if $BC = 4$ cm

iii AC if $EC = 9$ cm



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7F Applying similar triangles

Learning intentions for this section:

- To know that two similar figures are connected by a scale factor
- To understand that the enlargement transformation produces similar figures
- To be able to calculate a scale factor from similar figures
- To be able to apply similarity to real-life contexts

Past, present and future learning:

- This section concludes the Stage 5 Core topic called *Properties of geometrical figures A*
- It consolidates and extends the concepts in Sections 8E and 8F of our Year 9 book
- Students are not required to know or use the four tests for proving similarity
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Once it is established that two triangles are similar, the scale factor between side lengths can be used to find unknown side lengths.

Similar triangles have many applications in the real world. One application is finding the distance across a deep canyon from one side of the canyon.

The distance across this canyon can be found without having to actually measure it.

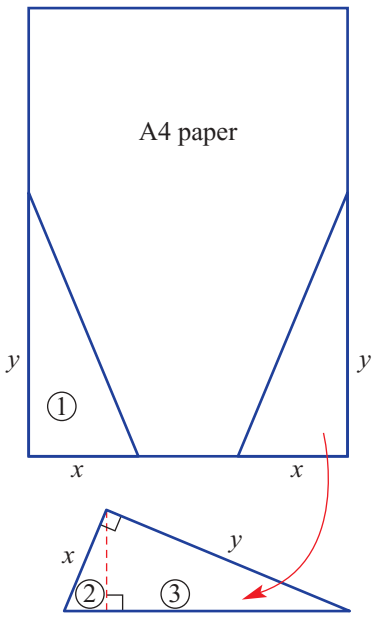
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Lesson starter: One-two-three similar triangles

This mathematical experiment requires careful drawing and accurate measurements.

Start with an A4 sheet of paper.

- Use a ruler to carefully draw two congruent triangles, as shown in the diagram at right.



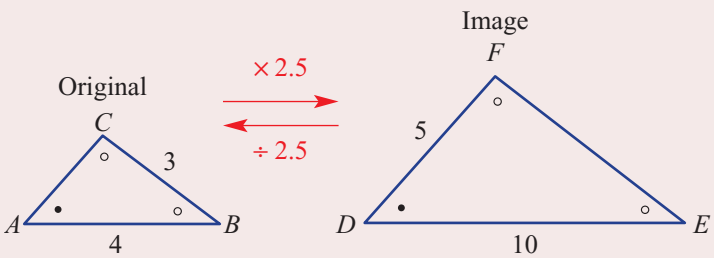
Choose your own values for x and y. Make sure they are different to those chosen by the student next to you.

- Cut your triangles off the sheet.
- Using a protractor, carefully measure the angles in triangle 1 and write them on the shape.
- Carefully measure the sides of triangle 1, in millimetres. Write them on the shape.
- Take the other triangle and rotate it so that the hypotenuse is horizontal, as in the diagram at right. Carefully fold it along a line that is perpendicular to the hypotenuse.
- Cut along that line to divide your triangle into triangle 2 and triangle 3, where triangle 2 is the smaller triangle.
- Measure all the sides and angles of triangle 2 and triangle 3 and write them on the shapes.
- If you have done this carefully, triangles 1, 2 and 3 will contain the same three angles (and therefore they will be similar).
  If this did not happen, check your measurements.
- Use your side lengths to find an approximation for the scale factor between triangle 2 and triangle 3.
- Use your side lengths to find an approximation for the scale factor between triangle 2 and triangle 1.

KEY IDEAS

■ For similar triangles, the ratio of the corresponding side lengths written as a single number is called the **scale factor**.

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$



$$\text{Scale factor} = \frac{DE}{AB} = \frac{10}{4} = \frac{5}{2} = 2.5$$

$\therefore$ All sides must be multiplied by 2.5.

$\therefore EF = 3 \times 2.5 = 7.5$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

■ Once the scale factor is known, it can be used to find unknown side lengths.
AC can be calculated by working backwards (i.e. by dividing by 2.5)
$\therefore AC = 5 \div 2.5$
$= 2$

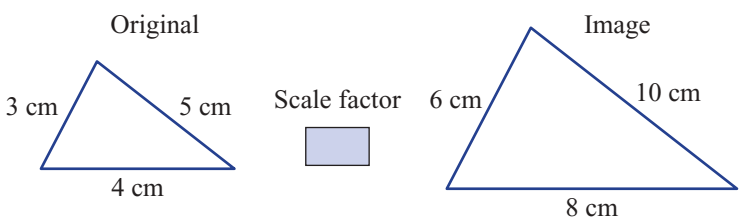
Key terminology: scale factor, original, image

Exercise 7F

UNDERSTANDING | 1, 2 | 1, 2

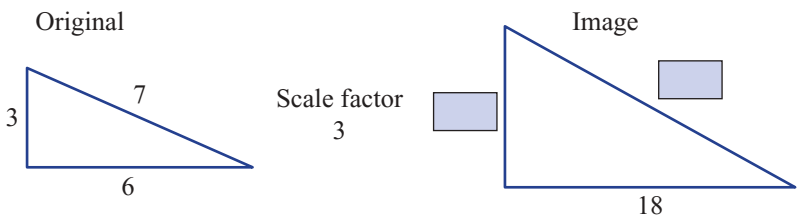
1 Write down the missing numbers that should go in the blue rectangles for these similar triangles.

a

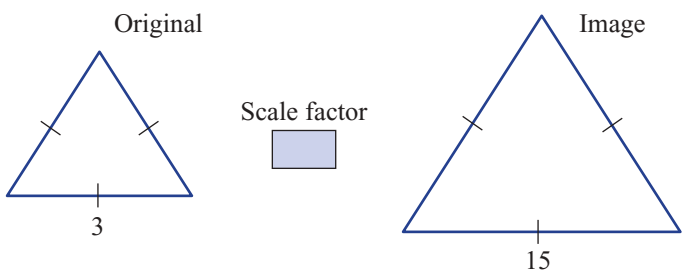


Hint: Compare AB and AD for the scale factor.

b



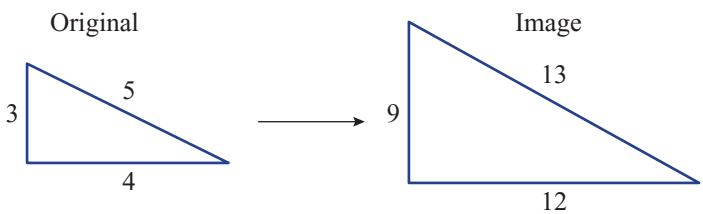
c



2 For the triangles shown below:

a Has the same scale factor been applied to all sides of the original triangle?

b Are the triangles similar?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

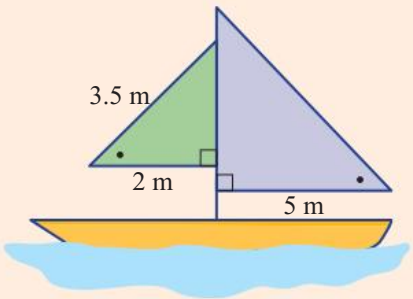
FLUENCY 3–5 4, 5

Example 12 Applying similar triangles

A home-made raft consists of two similar sails with measurements and angles as shown in this diagram.

a Find the scale factor for the side lengths of the sails.

b Find the length of the longest side of the large sail.



SOLUTION

a Scale factor $= \frac{5}{2} = 2.5$

b Longest side $= 3.5 \times 2.5$
$= 8.75$ m

EXPLANATION

Choose two corresponding sides with known lengths and divide the larger by the smaller.

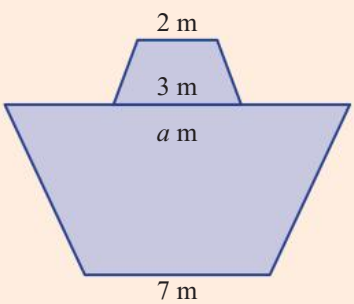
Multiply the corresponding side on the smaller triangle by the scale factor.

Now you try

The diagram shows a logo for a hat company. The two trapeziums are similar.

a Find the scale factor which enlarges the smaller trapezium to the larger one.

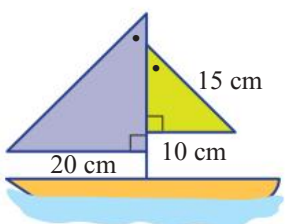
b Find the value of a.



3 A toy yacht consists of two similar sails with measurements and angles as shown in this diagram.

a Find the scale factor for the side lengths of the sails.

b Find the length of the longest side of the large sail.

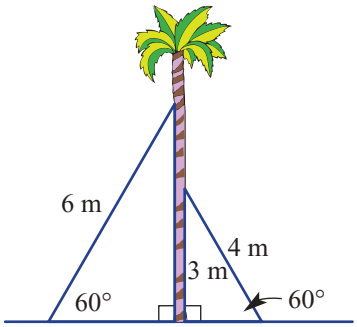


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 A tall palm tree is held in place with two cables of length 6 m and 4 m, which create similar triangles, as shown.

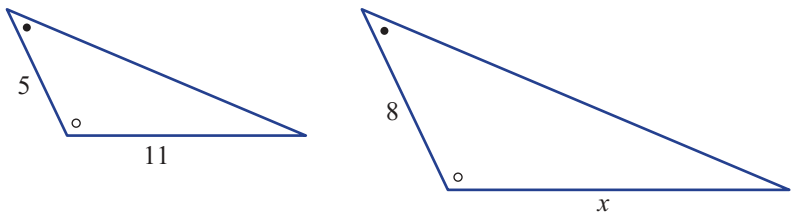
a Find the scale factor for the side lengths of the cables.

b Find the height of the point above the ground where the longer cable is attached to the palm tree.

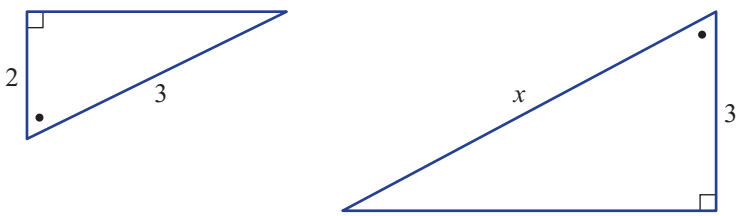


5 These pairs of triangles are known to be similar. By finding the scale factor, find the value of x.

a



b



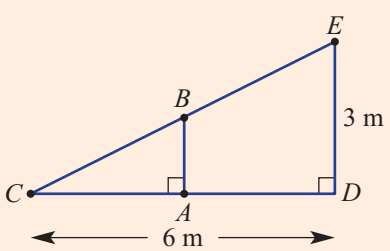
PROBLEM-SOLVING AND REASONING 6–8 8–10

Example 13 Working with combined triangles

A ramp is supported by a vertical stud, AB, where A is at the centre of CD. It is known that $CD = 6$ m and that the ramp is 3 m high.

a Draw the triangle separately and include all known lengths.

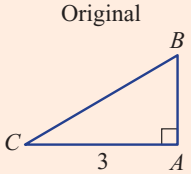
b Use the scale factor to find the length of the stud AB.



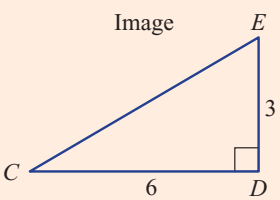
SOLUTION

a

Original



Image



EXPLANATION

CA is half of CD.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b Scale factor $= \frac{CD}{CA} = \frac{6}{3} = 2$

$\therefore AB = 3 \div 2$

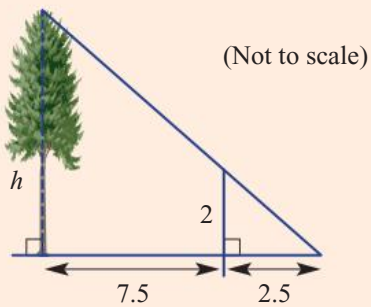
$= 1.5$ m

$CD = 6$ m and $CA = 3$ m.

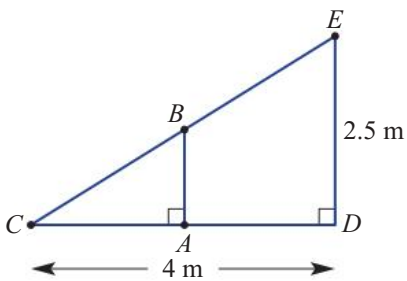
Divide the larger side length, DE, by the scale factor.

Now you try

A tree is h metres tall. Given the information in the diagram, find the value of h. All measurements are in metres.



6 A ramp is supported by a vertical stud, AB, where A is at the centre of CD. It is known that $CD = 4$ m and that the ramp is 2.5 m high. Find the length of the stud AB.

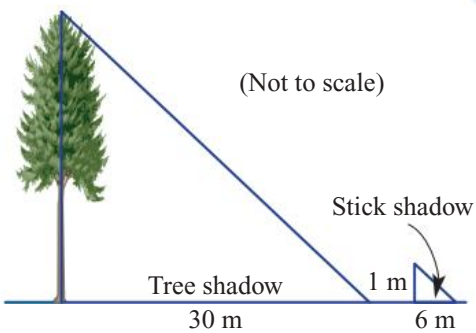


7 A 1 m vertical stick and a tree cast their shadows at a particular time in the day. The shadow lengths are shown in this diagram.

a Find the scale factor for the side lengths of the triangles.

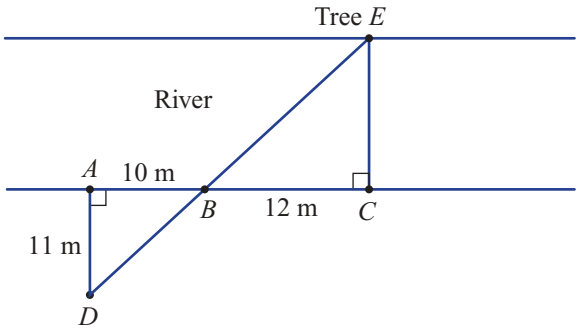
b Find the height of the tree.

Hint: At the same time of day, the angle that the light makes with the ground will be the same.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 From a place on the river (C), a tree (E) is spotted on the opposite bank. The distances between selected trees A, B, C and D are measured as shown.



a Find the scale factor.

b Find the width of the river.

Hint: AB corresponds to CB and AD corresponds to CE.

9 At a particular time of day, Aaron casts a shadow 1.3 m long, whereas Theo, who is 1.75 m tall, casts a shadow 1.2 m long. Find the height of Aaron, to two decimal places.

Hint: Draw a diagram to find the scale factor.

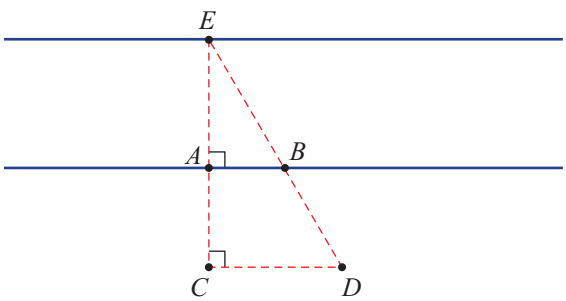
10 Try this activity with a partner but ensure that at least one person knows their height.

a Go out into the sunlight and measure the length of each person's shadow.

b Use these measurements plus the known height of one person to find the height of the other person.

ENRICHMENT: Gorge challenge – 11

11 Mandy sets up a series of rocks alongside a straight section of a deep gorge. She places rocks A, B, C and D as shown. Rock E sits naturally on the other side of the gorge. She then measures the following distances.



- $AB = 10$ m
- $AC = 10$ m
- $CD = 15$ m

a What is the scale factor?

b Use trial and error to find the distance across the gorge from rocks A to E.

c Can you find instead the length AE by setting up an equation?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7G Introduction to networks

Learning intentions for this section:

- To know what is meant by a network graph
- To know the key features of a network graph
- To be able to find the degree of a vertex and the sum of degrees for a graph
- To be able to describe simple walks through a network using the vertex labels

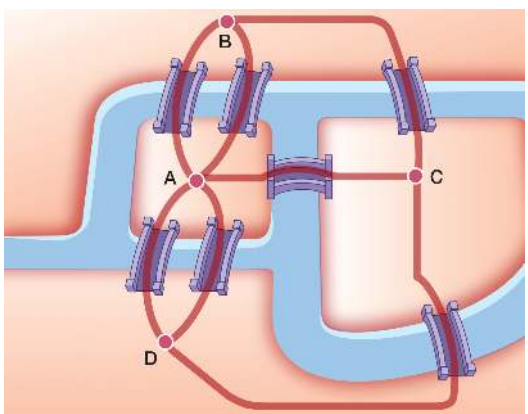
Past, present and future learning:

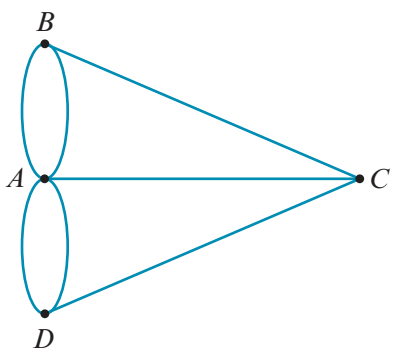
- This section addresses the Stage 5 Path topic called *Introduction to networks*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

A network is a collection of points (vertices or nodes) that can be connected by lines (edges). Networks are used to help solve a range of real-world problems, including travel and distance problems, intelligence and crime problems, computer network problems and even metabolic network problems associated with the human body. In Mathematics, a network diagram can be referred to as a graph, not to be confused with the graph of a rule like $y = x^2 + 3$.

Lesson starter: The Konigsberg bridge problem

The seven bridges of Konigsberg is a well-known historical problem solved by Leonhard Euler, who laid the foundations of graph theory. It involves two islands at the centre of an old German city connected by seven bridges over a river, as shown in these diagrams.

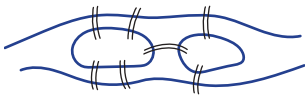




ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

The problem states: Is it possible to start at one point and pass over every bridge exactly once and return to your starting point?

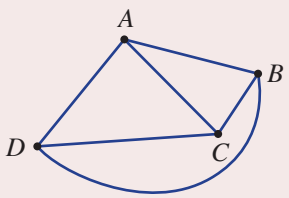
- Make a copy of this simplified map of the seven bridges of Konigsberg and try tracing out a walk that crosses all bridges exactly once. Try starting at different places.



- Investigate if there might be a solution to this problem if one of the bridges is removed.
- Investigate if there might be a solution to this problem if one bridge is added.

KEY IDEAS

- A **network** or **graph** is a diagram connecting points using lines.
  - The points are called **vertices** (or nodes). Vertex is singular, vertices is plural.
  - The lines are called **edges**.
- The **degree** of a vertex is the number of edges connected to it.
  - A vertex is odd if the number of edges connected to it is odd.
  - A vertex is even if the number of edges connected to it is even.
- The sum of degrees is calculated by adding up of the degrees of all vertices in a graph.
  - It is also equal to twice the number of edges.
- A **walk** is any type of route through a network.
  - A walk can be defined using the vertex labels.
  - Example: *A*-*B*-*C*-*A*-*D*.

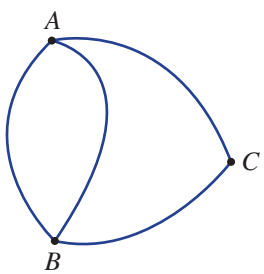


Key terminology: network, graph, vertex (node), degree, walk, edge

Exercise 7G

UNDERSTANDING 1, 2 2

1 Here is a graph representing roads connecting three towns *A*, *B* and *C*.



a How many different roads (edges) does this graph have?

b How many different vertices (nodes) does this graph have?

c If no road (edge) is used more than once and no town (vertex/node) is visited more than once, how many different walks are there if travelling from:

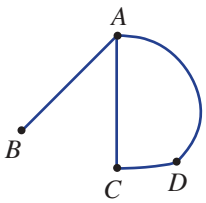
i *A* to *C*? **ii** *A* to *B*? **iii** *B* to *C*?

d How many roads connect to:

i town *A*? **ii** town *B*? **iii** town *C*?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 This graph uses four edges to connect four vertices.

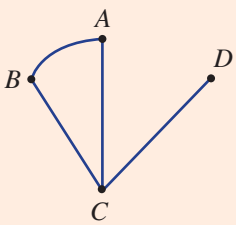


- **a** How many edges connect to vertex A?
- **b** What is the degree of vertex A?
- **c** State the total number of edges on the graph.
- **d** By finding the number of edges connected to each vertex find the sum of degrees of the graph.
- **e** What do you notice about the total number of edges and the sum of degrees for this graph?

FLUENCY 3–5 5–7

Example 14 Determining features of a graph

For the graph shown complete the following.



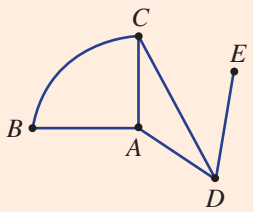
- **a** State the total number of:
  - **i** vertices (nodes) **ii** edges.
- **b** State the degree of:
  - **i** vertex A **ii** vertex B
  - **iii** vertex C **iv** vertex D.
- **c** Find the sum of degrees for the graph.
- **d** State which vertices are:
  - **i** odd **ii** even.

SOLUTION	EXPLANATION
a i 4 **ii** 4	The four vertices are A, B, C and D. The four edges are AB, AC, BC and CD.
b i 2 **ii** 2 **iii** 3 **iv** 1	Simply count how many edges connect to each vertex.
c $2 + 2 + 3 + 1 = 8$	Add up all the degrees from all vertices.
d i C and D **ii** A and B	The degrees of vertices C and D are 3 and 1. The degrees of vertices A and B be are both 2.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

For the graph shown complete the following.



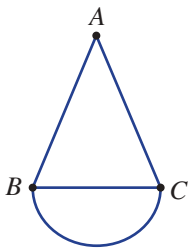
a State the total number of:
- **i** vertices (nodes)
- **ii** edges.

b State the degree of:
- **i** vertex A
- **ii** vertex B
- **iii** vertex C
- **iv** vertex D
- **v** vertex E.

c Find the sum of degrees for the graph.

d State which vertices are:
- **i** odd
- **ii** even.

3 For the graph shown complete the following.



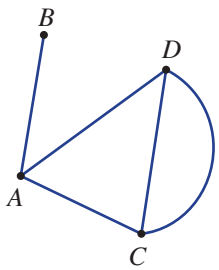
a State the total number of:
- **i** vertices (nodes)
- **ii** edges.

b State the degree of:
- **i** vertex A
- **ii** vertex B
- **iii** vertex C.

c Find the sum of degrees for the graph.

d State which vertices are:
- **i** odd
- **ii** even.

4 For the graph shown complete the following.



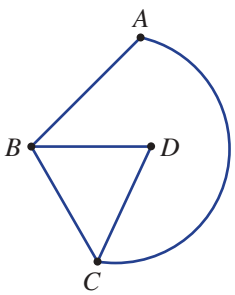
a State the total number of:
- **i** vertices (nodes)
- **ii** edges.

b State the degree of:
- **i** vertex A
- **ii** vertex B
- **iii** vertex C
- **iv** vertex D.

c Find the sum of degrees for the graph.

d State which vertices are:
- **i** odd
- **ii** even.

5 For the graph shown complete the following.



a State the total number of:
- **i** vertices
- **ii** edges.

b State the degree of:
- **i** vertex A
- **ii** vertex B
- **iii** vertex C
- **iv** vertex D.

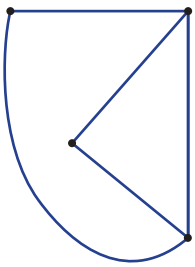
c Find the sum of degrees for the graph.

d State which vertices are:
- **i** odd
- **ii** even.

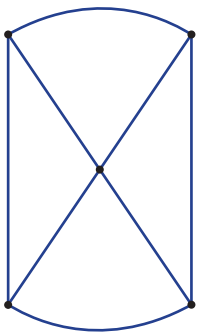
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Which of the following graphs has vertices that are all odd?

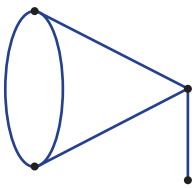
A



B

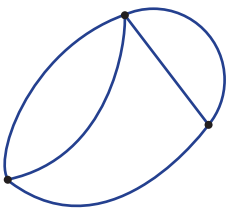


C

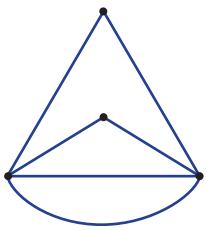


7 Which of the following graphs has vertices that are all even?

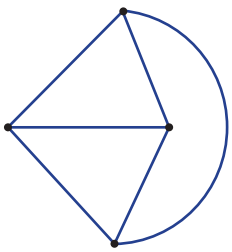
A



B



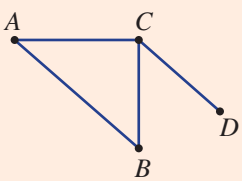
C



PROBLEM-SOLVING AND REASONING 8–10 10–13

Example 15 Finding a walk

Consider this graph connecting the four vertices A, B, C and D. Without visiting a vertex (node) more than once or using an edge more than once, find how many walks there are connecting to the following pairs of vertices.



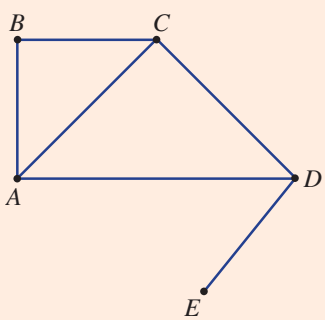
a A and D

b B and D

SOLUTION	EXPLANATION
a 2	The two walks are A-B-C-D and A-C-D.
b 2	The two walks are B-A-C-D and B-C-D.

Now you try

Consider this graph connecting the five vertices A, B, C, D and E. Without visiting a vertex more than once or using an edge more than once, find how many walks there are connecting to the following pairs of vertices.



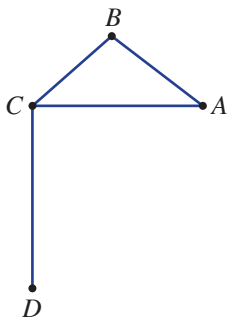
a A and E

b B and D

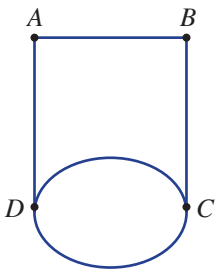
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 Consider this graph connecting the four vertices A, B, C and D. Without visiting a vertex (node) more than once or using an edge more than once, find how many walks there are connecting vertex A with vertex D.

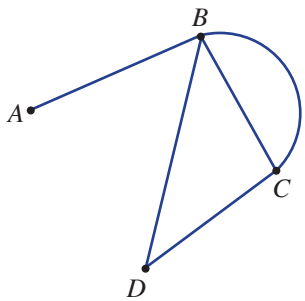
a



b

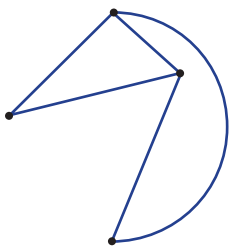


c

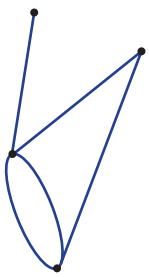


9 Which of the following graphs has the greatest sum of degrees?

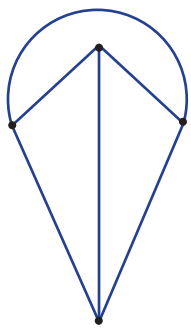
A



B

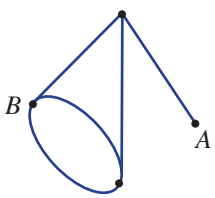


C

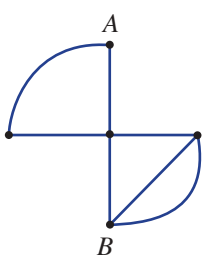


10 Which of the following graphs has the greatest number of walks connecting vertices A and B if no edge or vertex can be used more than once?

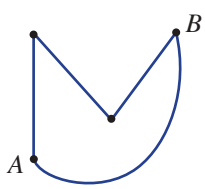
A



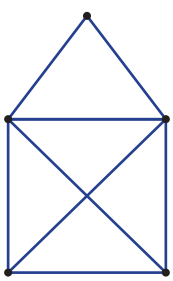
B



C



11 Is it possible to find a walk around this graph so that each edge is used exactly once? There is no restriction on the number of times vertices are visited or where you should start and finish. If the answer is 'yes', draw your walk. The place in the middle of the square is not a vertex, just an intersection of two edges. You cannot change direction at this position.

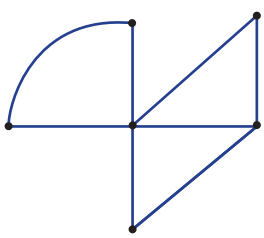


12 Consider the given graph.

a How many edges does the graph have?

b Find the sum of degrees for the graph.

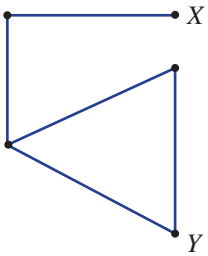
c What do you notice about your answers to part **a** and part **b** above? Decide if your conclusion is true for other graphs by drawing your own.



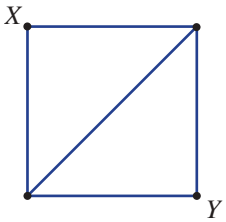
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

13 Decide if it is possible to find a walk from X to Y in these graphs by visiting each vertex exactly once and using each edge exactly once.

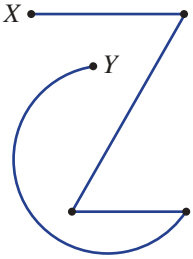
a



b

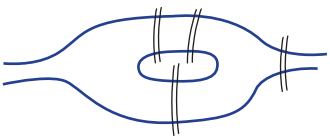


c



ENRICHMENT: Modifying the Konigsberg bridge problem — 14

14 Similar to the Konigsberg bridge problem, this diagram shows an island in the middle of a river and four bridges.



a Is it possible to start on the island and finish on the island by crossing each of the four bridges exactly once?

b Is it possible to start on one side of the river (not on the island) and finish on the same side by crossing each of the four bridges exactly once?

c Is it possible to start on one side of the river (not on the island) and finish on the opposite side by crossing each of the four bridges exactly once?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7H Isomorphic and planar graphs

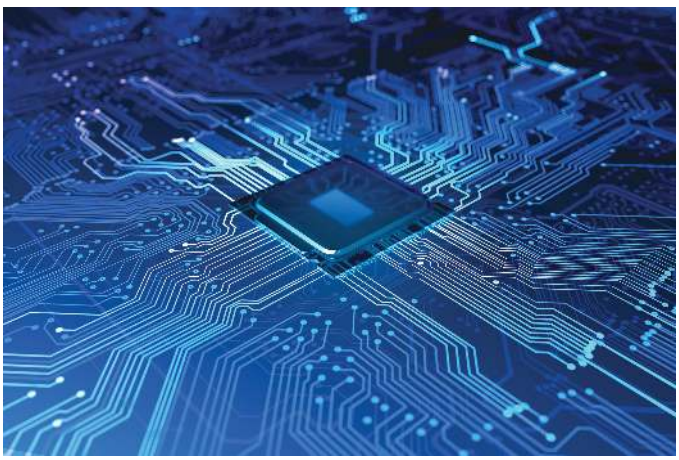
Learning intentions for this section:

- To know what isomorphic and planar graphs are
- To be able to identify isomorphic and planar graphs
- To be able to verify Euler's formula and use it to determine features of a graph

Past, present and future learning:

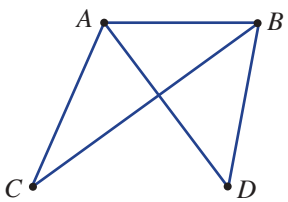
- This section addresses the Stage 5 Path topic called *Introduction to networks*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Many of the graphs considered so far may look different but in fact contain exactly the same information. Such graphs are said to be isomorphic and could be redrawn to look the same. Further, most of the graphs previously considered have been drawn with no intersecting edges. Such graphs are called planar graphs and often occur naturally in problems involving electronic circuits, railway networks and utility lines.



Lesson starter: Exploring Euler's formula with planar graphs

Consider this graph representing the connection between four vertices. There is no vertex at the intersection of the edges AD and BC.

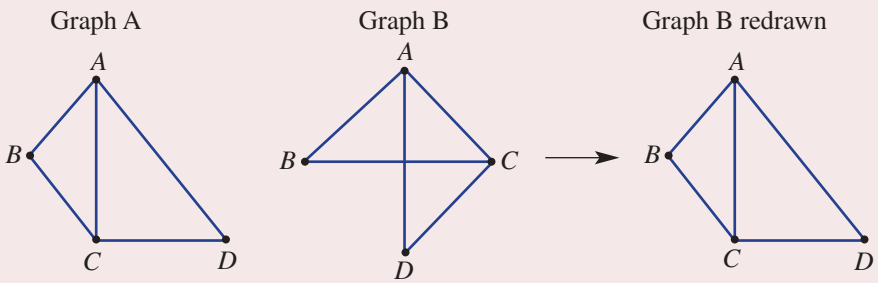


- Is it possible to redraw the graph so that all the vertices and edges are retained but edges do not intersect? Try this with a new drawing.
- Your new drawing should be a planar graph with no intersecting edges. How many different regions make up the graph including the outside region? These are called faces.
- Calculate the following.
  - $v + f$ (The sum of the number of vertices and the number of faces)
  - $e + 2$ (Two more than the number of edges)
- What do you notice about the two calculations above?
- Try drawing a different planar graph to verify your conclusion.

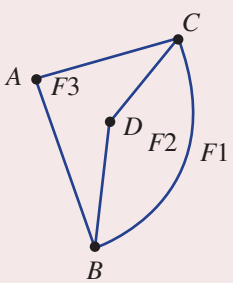
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- **Isomorphic graphs** contain the same information including:
  - the same number of vertices
  - the same number of edges
  - the same edge connections
- Isomorphic graphs can be drawn to look the same, such as Graph A and Graph B below



- A **planar graph** can be drawn so that it has no intersecting edges.
  - Many graphs may not look planar initially but could be redrawn without any intersecting edges.
  - A non-planar graph cannot be drawn without some edges crossing.
- A **face** of a graph is a region bound by a set of edges and vertices. By convention, the unbounded area outside the graph is counted as a face.



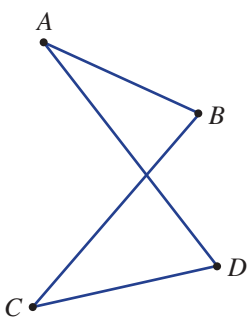
- **Euler's formula** applies to planar graphs and is such that $v + f = e + 2$ where:
  - v is the number of vertices
  - f is the number of faces including the outside face
  - e is the number of edges.
  - Euler's formula is often written as $v - e + f = 2$.

Key terminology: isomorphic graph, planar, face, Euler's formula

Exercise 7H

UNDERSTANDING 1, 2 | 1, 2

1 Consider this graph connecting the four vertices A, B, C and D.

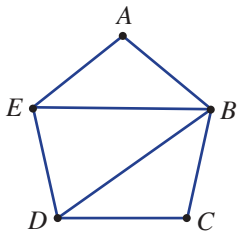


a Try redrawing the graph with no edges crossing.

b Would you therefore say that the graph is planar?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 Consider this planar graph.



a State the number of:

i vertices v

ii edges e

iii faces f (Don't forget to count the outside face)

b Calculate the following.

i $v + f$

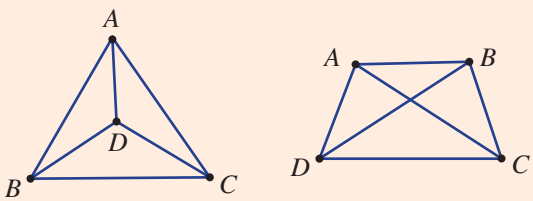
ii $e + 2$

c What do you notice about your answers to part **b**?

FLUENCY 3–5 3(½), 4–6

Example 16 Deciding if graphs are isomorphic

Decide if the following two graphs are isomorphic.



SOLUTION

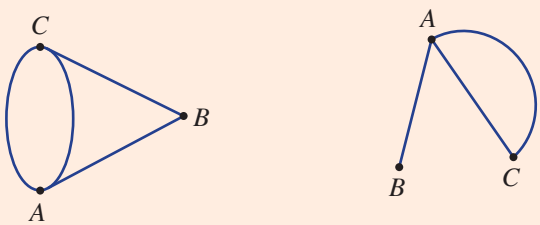
Yes, the graphs are isomorphic.

EXPLANATION

Both graphs have the same number of vertices and edges and the same connections. They could be redrawn to look identical.

Now you try

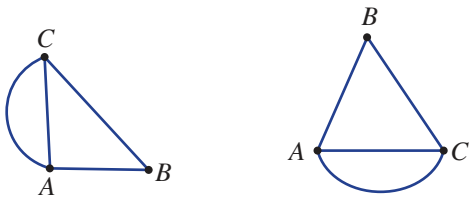
Decide if the following two graphs are isomorphic.



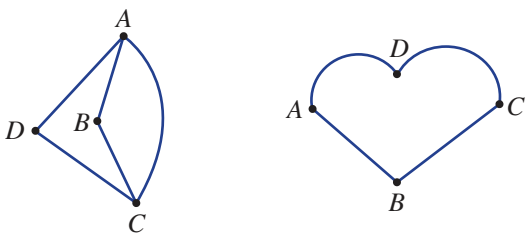
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Decide if the following pairs of graphs are isomorphic.

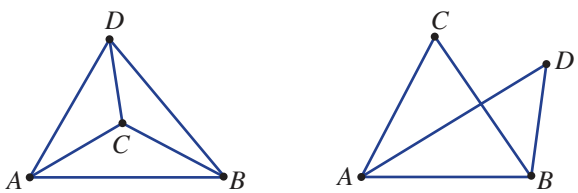
a



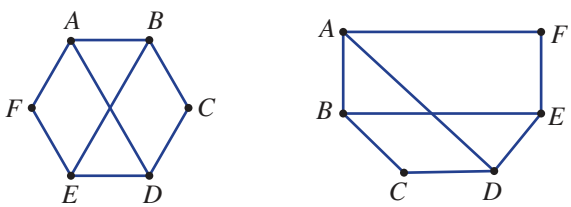
b



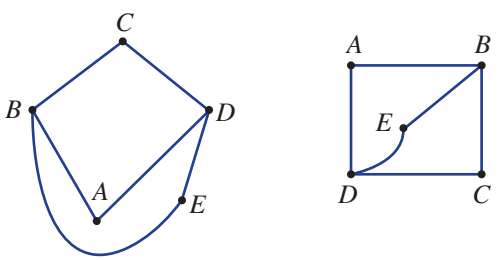
c



d



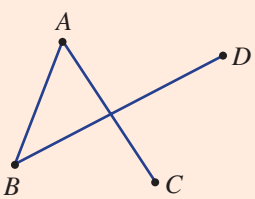
e



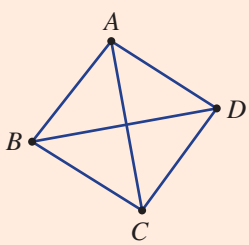
Example 17 Deciding if graphs are planar

Decide if the following graphs are planar or non-planar.

a



b



Continued on next page

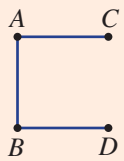
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

a Yes, the graph is planar.

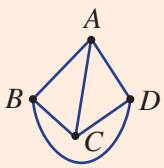
EXPLANATION

The graph can be drawn without any intersecting edges.



b Yes, the graph is planar.

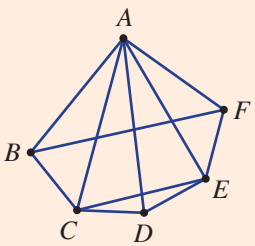
The graph can be drawn without any intersecting edges.



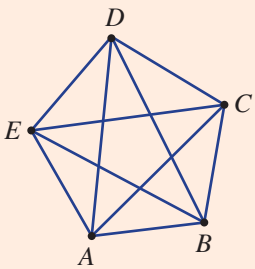
Now you try

Decide if the following graphs are planar or non-planar.

a

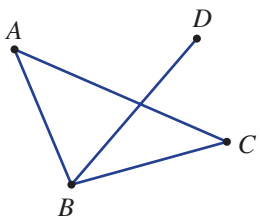


b

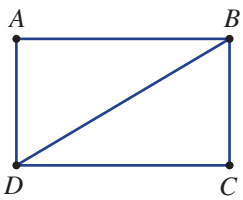


4 Decide if the following graphs are planar or non-planar.

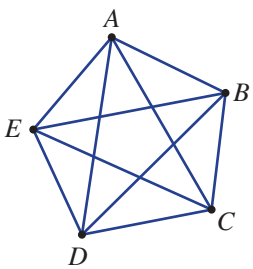
a



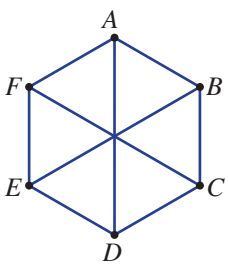
b



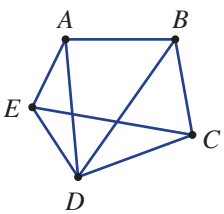
c



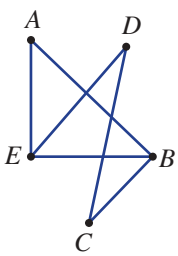
d



e



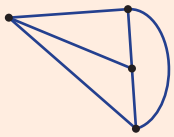
f



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 18 Verifying Euler's formula

Consider this planar graph.



a Find the number of:

- **i** vertices, v
- **ii** edges, e
- **iii** faces, f

b Verify Euler's formula using the information in the given planar graph.

SOLUTION

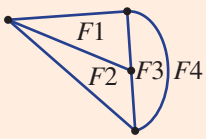
a **i** 4

ii 6

iii 4

EXPLANATION

Simply count the number of vertices, edges and faces. Don't forget to count the infinite, outside face.



b $v + f = e + 2$ or $v - e + f = 2$

$4 + 4 = 6 + 2$ $4 - 6 + 4 = 2$

$8 = 8$ $2 = 2$

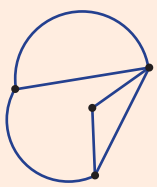
Therefore, Euler's formula is verified.

Substitute the values of v, e, and f into Euler's formula and check that the left-hand side equals the right-hand side.

Using $v + f = e + 2$ is equivalent to using $v - e + f = 2$.

Now you try

Consider this planar graph.

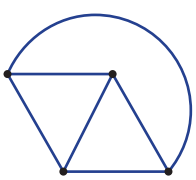


a Find the number of:

- **i** vertices, v
- **ii** edges, e
- **iii** faces, f

b Verify Euler's formula using the information in the given planar graph.

5 Consider this planar graph.



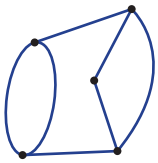
a Find the number of:

- **i** vertices, v
- **ii** edges, e
- **iii** faces, f

b Verify Euler's formula using the information in the given planar graph.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Consider this planar graph.



a Find the number of:

i vertices, v

ii edges, e

iii faces, f

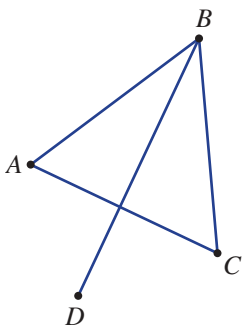
b Verify Euler's formula using the information in the given planar graph.

PROBLEM-SOLVING AND REASONING

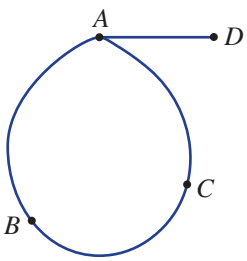
7–9 | 8–11

7 Of the three graphs, A, B and C, which pair is isomorphic?

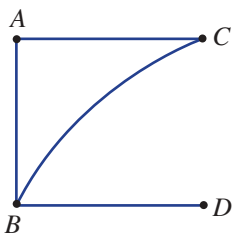
A



B

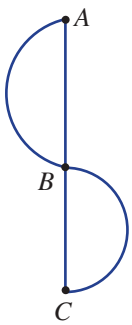


C

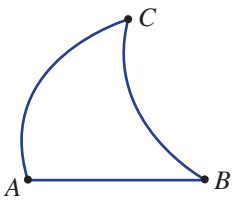


8 Of the three graphs, A, B and C, which pair is isomorphic?

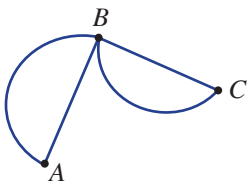
A



B



C



9 Use Euler's formula to answer the following, which involve planar graphs.

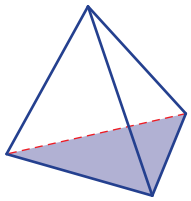
a If the number of vertices is 6 and the number of faces is 8 then find the number of edges.

b If the number of vertices is 8 and the number of edges is 12 then find the number of faces.

c If the number of edges is 10 and the number of faces is 8 then find the number of vertices.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

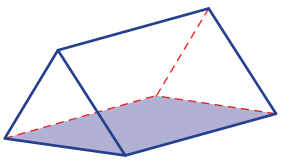
10 Consider this four-sided, three-dimensional polyhedron called a tetrahedron.



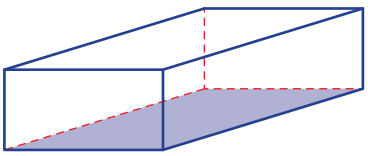
- **a** State the number of vertices, edges and faces.
- **b** Verify Euler's formula for the tetrahedron.
- **c** Now draw the tetrahedron as a two-dimensional planar graph. It does not need to look like a tetrahedron, just a planar graph with the same vertex, edge and face information.
- **d** Verify Euler's formula using your planar graph from part **c**.

11 Complete parts **a**–**d** from Question **10** above for the following three-dimensional polyhedra.

a Pentahedron



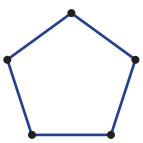
b Hexahedron



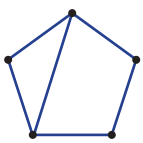
ENRICHMENT: Planar pentagons — 12

12 The following pentagons include some diagonal edges. Decide which ones are planar.

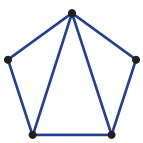
A



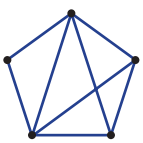
B



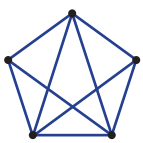
C



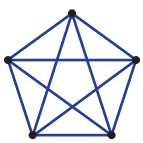
D



E



F



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7I Trails, paths and Eulerian circuits

Learning intentions for this section:

- To know the difference between a walk, trail, circuit, path and cycle in a network
- To know what Eulerian trails and cycles are
- To be able to identify different types of walks within a network
- To be able to find Eulerian trails and circuits within a network

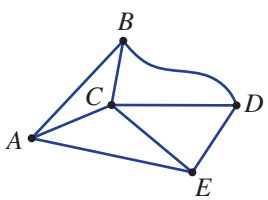
Past, present and future learning:

- This section concludes the Stage 5 Path topic called *Introduction to networks*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

Depending on the application of graph theory the desired walk through a network may involve a wide range of factors. The number of times that a vertex can be visited, or an edge is used, may or may not be important. In one application, it may be a requirement that a walk starts and ends at the same vertex. Alternatively, each edge might have to be used exactly once. In this section we will define and explore the different types of walks through a network and focus on a special group of them called Eulerian trails.

Lesson starter: Exploring Buxton

The small town of Buxton has five key tourist destinations labelled here with the letters A, B, C, D and E. They are connected by footpaths as shown in this network diagram.



A particular trail is defined by B-C-D-E-C.

- Are there any footpaths (edges) used more than once in this trail?
- Are there any destinations (vertices) visited more than once?
- What extensions to the trail could be added (at the end) so that it becomes a circuit that ends where it began? Is it possible to do this without using a particular footpath more than once?

A path is defined by A-B-C-E.

- Are there any footpaths (edges) used more than once in this trail?
- Are there any destinations (vertices) visited more than once?
- What extensions to the path could be added (at the end) so that it becomes a cycle that ends where it began? Is it possible to do this without using a particular footpath (edge) or destination (vertex) more than once?

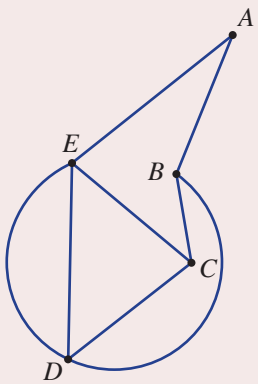
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now imagine trying to visit every destination in Buxton.

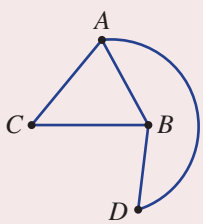
- Is it possible to visit all the destinations on a given path so that all destinations are visited exactly once and no footpath is used more than once? If so, how?

KEY IDEAS

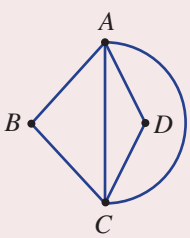
- A **walk** is any type of route through a network. Examples:
  - *A-B-D-C-B*
  - *E-D-B-C-D-E*



- A **trail** is a walk where edges are not repeated. Examples:
  - *A-E-D-C-E*
  - *C-E-D-C-B*
- A **circuit** is a trail that begins and ends at the same vertex. Examples:
  - *E-D-B-C-D-E*
  - *B-C-D-E-D-B*
- A **path** is a walk where vertices and edges are not repeated. Examples:
  - *E-D-C-B*
  - *B-C-D-E-A*
- A **cycle** is a path that begins and ends at the same place. Examples:
  - *E-D-C-E*
  - *A-E-D-B-A*
- An **Eulerian trail** is a walk where every edge is included exactly once. Vertices are allowed to be revisited.
  - Eulerian trails exist if there are zero or exactly two vertices of odd degree.
  - If exactly two vertices are of odd degree, then Eulerian trails start at one of these vertices and end at the other.
  - If there are zero vertices of odd degree, then all Eulerian trails are circuits.
  - This graph has two vertices of odd degree and an example of an Eulerian trail is *B-D-A-C-B-A*.



- An **Eulerian circuit** is an Eulerian trail which starts and ends at the same vertex.
  - An Eulerian circuit exists if and only if zero vertices are of odd degree.
  - This graph has zero vertices of odd degree and an example of an Eulerian circuit is *A-B-C-A-D-C-A*.
  - An Eulerian circuit can start at any vertex.



Key terminology: walk, trail, circuit, path, cycle, Eulerian trail, Eulerian circuit

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

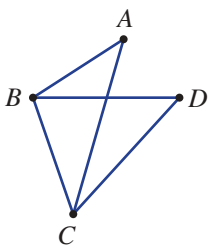
Exercise 7I

UNDERSTANDING	1, 2	1, 2

1 Give the formal name (walk, trail, circuit, path or cycle) of each of the following.

- **a** Any type of route through a network.
- **b** A trail that begins and ends at the same vertex.
- **c** A path that begins and ends at the same vertex.
- **d** A walk where edges are only used once.
- **e** A walk where vertices and edges are only used once.

2 Consider this simple graph.



- **a** Which vertices are of odd degree?
- **b** Find a trail so that all edges are used exactly once. Write down your trail using the letters A, B, C and/or D.
- **c** What is the name of the type of walk you discovered in part **b**?

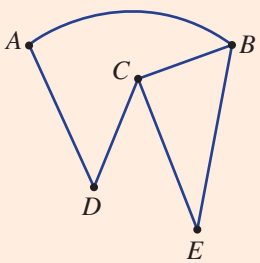
Assume now that the edge AD is added to the graph.

- **d** Is it now possible to find an Eulerian trail through the network?
- **e** How many vertices are of odd degree?
- **f** How many vertices need to be of odd degree if a trail is to be Eulerian?

FLUENCY	3, 4, 6	3, 5, 6

Example 19 Defining walks

Consider this graph.



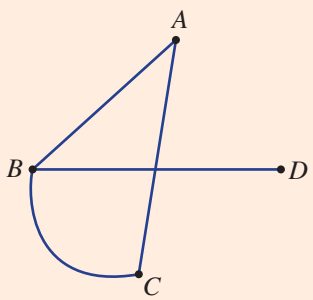
- **a** Decide if the following walks are trails. Answer Yes or No.
  - **i** *A-D-C-B*
  - **ii** *A-B-C-E-B*
- **b** Decide if the following walks are circuits. Answer Yes or No.
  - **i** *A-B-C-D-A*
  - **ii** *B-C-D-C-B*
- **c** Decide if the following walks are paths. Answer Yes or No.
  - **i** *C-E-B-C-D*
  - **ii** *E-B-C-D-A*
- **d** Decide if the following walks are cycles. Answer Yes or No.
  - **i** *C-D-A-B-C*
  - **ii** *E-B-A-D-C-B-E*

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION		EXPLANATION
a i	Yes	Edges are only used once.
ii	Yes	Edges are only used once.
b i	Yes	Edges are only used once and it starts and ends at the same vertex.
ii	No	The edges *BC* and *CD* are used twice.
c i	No	The vertex *C* is visited twice.
ii	Yes	Edges are used only once and vertices are visited once.
d i	Yes	The cycle is a path that starts and ends at the same vertex.
ii	No	The walk visits vertex *B* twice and uses the edge *BE* twice.

Now you try

Consider this graph.



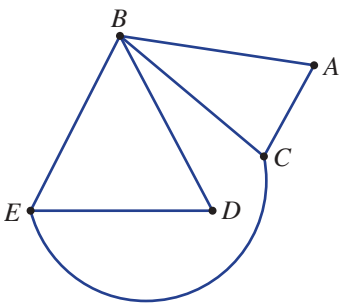
a Decide if the following walks are trails. Answer Yes or No.
- **i** *D-B-A-C-A*
- **ii** *C-B-D*

b Decide if the following walks are circuits. Answer Yes or No.
- **i** *D-B-C-B-D*
- **ii** *B-C-A-B*

c Decide if the following walks are paths. Answer Yes or No.
- **i** *D-B-A-C*
- **ii** *C-B-A-C-B*

d Decide if the following walks are cycles. Answer Yes or No.
- **i** *D-B-C-B*
- **ii** *B-C-A-B*

3 Consider this graph.



a Decide if the following walks are trails. Answer Yes or No.
- **i** *A-B-D-E-B* **ii** *E-C-A-B-E-D*

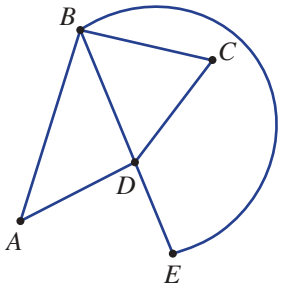
b Decide if the following walks are circuits. Answer Yes or No.
- **i** *B-A-C-B-D-E-B* **ii** *D-E-C-B-E-D*

c Decide if the following walks are paths. Answer Yes or No.
- **i** *A-B-E-D-B-C* **ii** *A-C-B-E-D*

d Decide if the following walks are cycles. Answer Yes or No.
- **i** *D-E-B-D* **ii** *C-A-B-E-D-B-C*

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Consider this graph.



a Decide if the following walks are trails. Answer Yes or No.

i *A-B-C-D-B-E*

ii *A-D-C-B-A-D*

b Decide if the following walks are circuits. Answer Yes or No.

i *A-D-E-B-D-C-B-A*

ii *D-C-B-A-D-E*

c Decide if the following walks are paths. Answer Yes or No.

i *D-E-B-D-C*

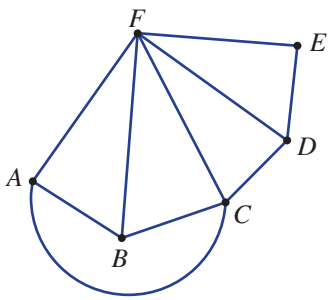
ii *C-D-E-B-A*

d Decide if the following walks are cycles. Answer Yes or No.

i *D-E-B-C-D*

ii *A-D-E-B-D-A*

5 Consider this graph.



a Decide if the following walks are trails. Answer Yes or No.

i *A-B-C-A-F-D*

ii *D-C-A-F-C-D*

b Decide if the following walks are circuits. Answer Yes or No.

i *F-D-C-A-B-C-F*

ii *B-F-C-A-F-B*

c Decide if the following walks are paths. Answer Yes or No.

i *A-B-C-D-E*

ii *F-C-B-F-A*

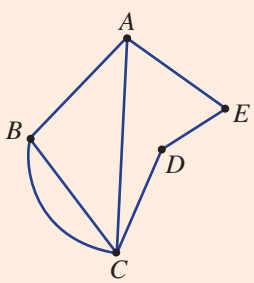
d Decide if the following walks are cycles. Answer Yes or No.

i *D-F-A-C-D*

ii *C-F-B-A-F-C*

Example 20 Exploring Eulerian trails

Decide if this graph has an Eulerian trail. If so decide if all such trails will be circuits.



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION

Yes e.g. *A-E-D-C-A-B-C-B*

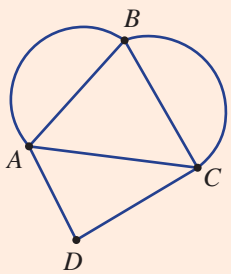
Eulerian trails will not be circuits.

EXPLANATION

There are two vertices of odd degree and therefore an Eulerian trail exists.
For an Eulerian circuit to exist there must be zero vertices of odd degree.

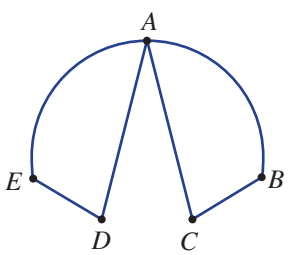
Now you try

Decide if this graph has an Eulerian trail. If so decide if all such trails will be circuits.

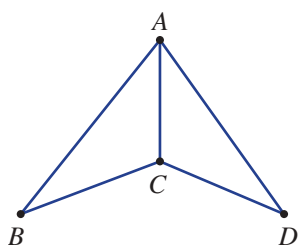


6 Decide if the following graphs have an Eulerian trail. If so decide if all such trails will be circuits.

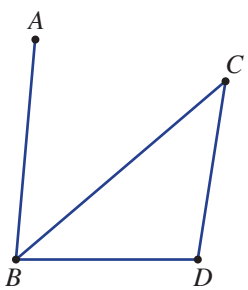
a



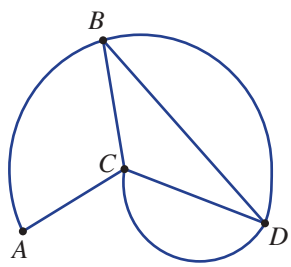
b



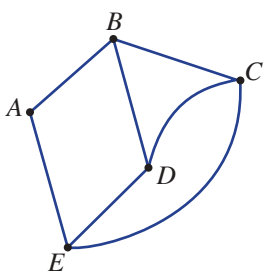
c



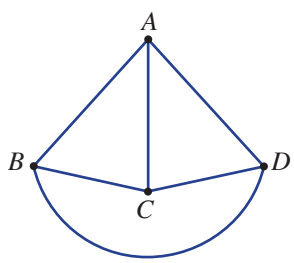
d



e



f



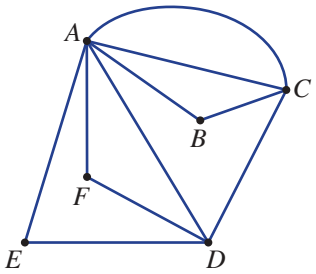
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

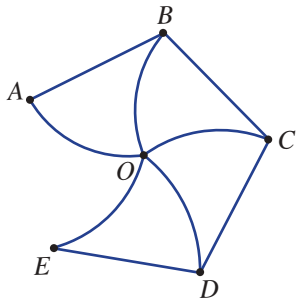
7–9 | 8–11

7 By determining the degree of each vertex decide if the following graphs have Eulerian trails.

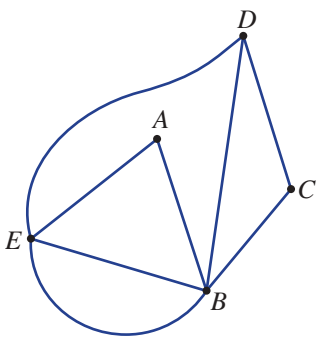
a



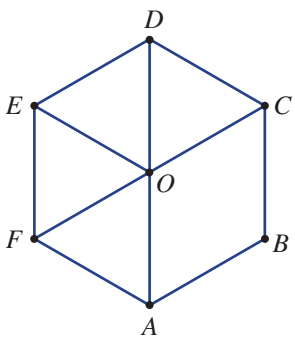
b



c



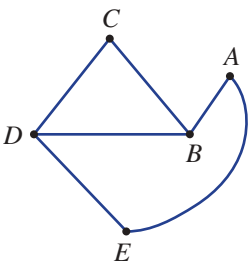
d



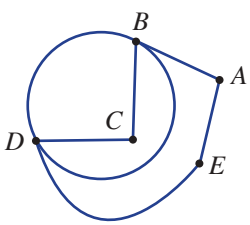
8 Consider the possible Eulerian trails in this graph.

a List the different Eulerian trails starting at vertex *B* and ending at vertex *D*.

b List the different Eulerian trails starting at vertex *D* and ending at vertex *B*.

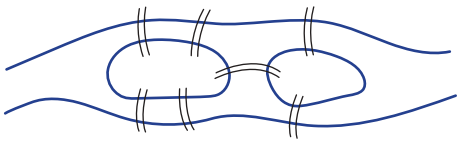


9 How many Eulerian circuits can you find through this network, starting at vertex *A*? Do not count the same circuit in reverse.

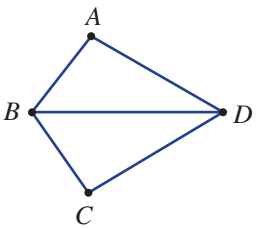


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

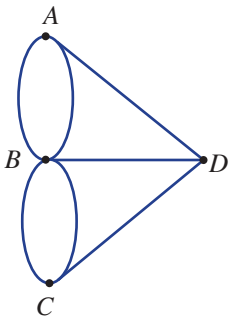
10 Let's revisit the Konigsberg bridge problem using this diagram.



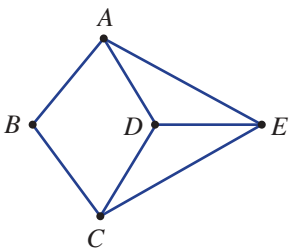
Graph 1



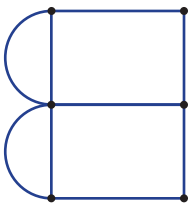
Graph 2



Graph 3



Graph 4

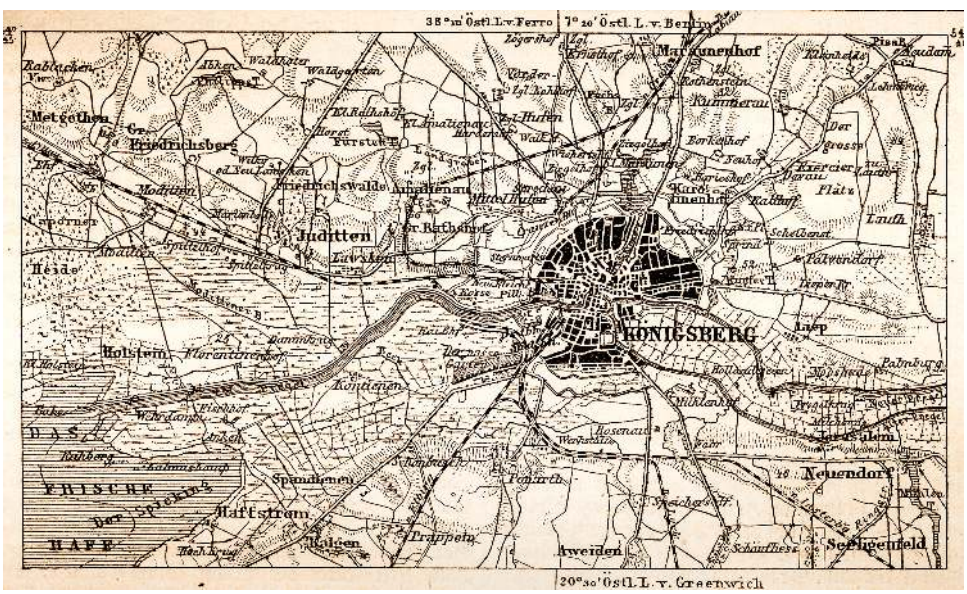


a Which of Graphs 1 to 4 is valid for the Konigsberg bridge problem?

b What do the edges on the network represent in the Konigsberg bridge problem?

c How many vertices on the graph representing the problem are of odd degree?

d What does your answer to part **c** tell you about the Konigsberg bridge problem?

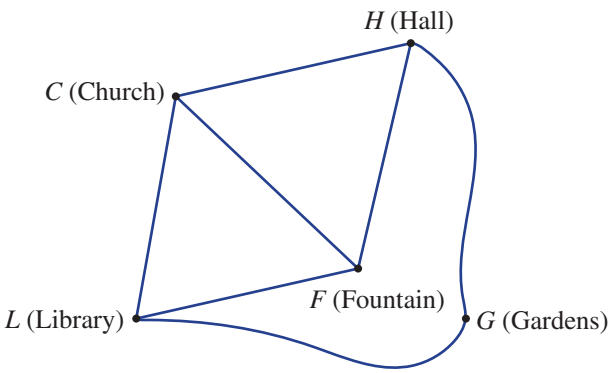

KÖNIGSBERG
Juditten
Friedrichsberg
Metgethen
Holstein
Haffstrom
Neuendorf
Sackheim
Palvendorf
Lauth
Amalienau
Trappeyen
Schalauer
Sackheimer
Kneiphof
Löbenicht
Altstadt
Vorstadt
Tragheim
Roßgarten
Steindamm
Fischhof
Kalthof
Lauk
Palmburg
Lep
Schalauer Thor
Sackheimer Thor
Roßgärter Thor
Steindamm Thor
Fischhof Thor
Kalthof Thor
Lauk Thor
Palmburg Thor
Lep Thor
Neuendorf Thor
Sackheim Thor
Tragheim Thor
Löbenicht Thor
Altstadt Thor
Vorstadt Thor
Kneiphof Thor

11 Answer true (T) or false (F).

- **a** All paths are walks.
- **b** All cycles are paths.
- **c** All walks are trails.
- **d** All trails are circuits.
- **e** All paths are trails.
- **f** All circuits are cycles.

ENRICHMENT: Margo's village walk — 12

12 Margo wishes to visit every attraction in a village. The attractions are connected with walking paths as shown.



- **a** Is it possible for Margo to find a walk which visits all the attractions, starting and ending at the church, without passing any given attraction twice? If so, how many ways can this be achieved? List the walks.
- **b** Is it possible for Margo to use every footpath exactly once and visit every attraction at least once? If so, list the trail.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7J Shortest path problems EXTENDING

Learning intentions for this section:

- To understanded what a weighted graph is
- To interpret weighted graphs
- To be able to find a shortest path through a network

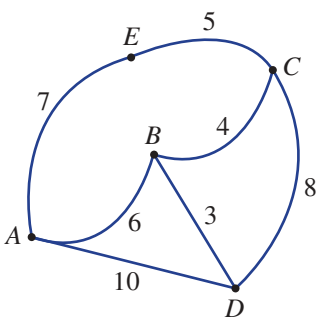
Past, present and future learning:

- This section goes beyond the Stage 5 Path topic called *Introduction to networks*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

A weighted graph is a network where each edge is labelled with a number. These numbers could represent the cost to transport goods between points, the voltages in an electrical circuit or distances between towns on a map. In this section we will focus on networks including distances, namely, shortest path problems. It is common in network theory to try to minimise the distances between two points. An example is a bus network where we might be interested in stopping at a range of points across a town using the minimum possible distance.

Lesson starter: Village distances

This simplified map shows the distance, in kilometres, between five villages Almore (A), Bellan (B), Coldstom (C), Denont (D) and Elimono (E).



- Find the total distance between Denont and Elimono if travelling via:
  - Bellan and Coldstrom
  - Bellan and Almore.
- Find the shortest path from Denont to Elimono and state this minimum distance.
- If the distance from Denont to Bellan was instead 4 km, would this change your mind when finding the minimum distance between villages Denont and Elimono? Discuss.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

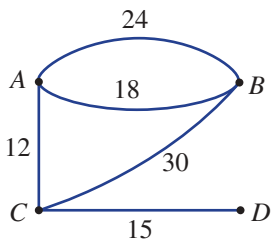
- A **weighted graph** is a graph with numbers attached to each of the edges.
  - These numbers could represent for example, costs, volumes, times or distances.
- A **shortest path** problem involves finding a walk through a network which provides a minimum distance.

Key terminology: weighted graph, shortest path

Exercise 7J

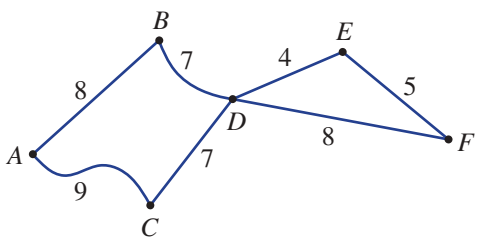
UNDERSTANDING	1, 2	1, 2

1 This graph represents a rail map joining four towns A, B, C and D. Distances are in kilometres.



- **a** How many edges are labelled representing distances greater than 20 km?
- **b** How many different ways could you travel from town A to town C without using an edge more than once?
- **c** How many different ways could you travel from town A to town C without visiting a town more than once?
- **d** What is the length of the shortest path between towns A and C?
- **e** What is the length of the shortest path between towns A and D?

2 This graph represents the distance in kilometres using trails between points on a bushwalking map.

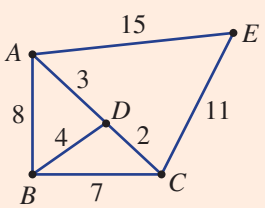


- **a** Find the distance between points A and D via:
  - **i** B
  - **ii** C
- **b** Find the distance between points D and F via point E.
- **c** What points would be visited if the shortest path is taken from point A to point F?

FLUENCY	3–5	3, 5

Example 21 Interpreting a weighted graph

This graph shows the distance, in metres, between vertices in a pipe network.



- **a** How far is it from point B to C via D on the network?
- **b** Calculate the distance for the path A-D-B-C-E.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

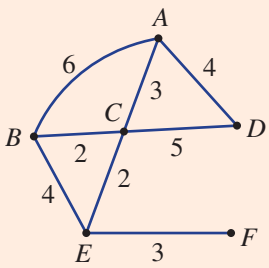
SOLUTION	EXPLANATION
a $4 + 2 = 6$ m	The distance from B to D is 4 m and from D to C is 2 m.
b $3 + 4 + 7 + 11 = 25$ m	Add all the distances connecting the points in the correct order.

Now you try

This graph shows the distance, in metres, between vertices on an electrical circuit.

a How far is it from point A to F via only B and E on the circuit?

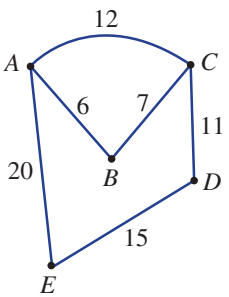
b Calculate the distance for the path A-D-C-E-F.



3 This graph shows the distance, in kilometres, for a simple road network between four points A, B, C, D and E.

a How far is it from point E to C via D on the network?

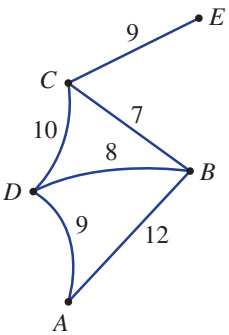
b Calculate the distance for the path A-B-C-D.



4 This graph shows the distance, in metres, between junctions on a cable network.

a How far is it from point A to B via D and C on the network?

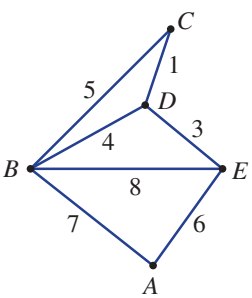
b Calculate the distance for the path A-B-D-C-E.



5 This graph shows the distance, in centimetres, between points on an electrical circuit.

a How far is it from point A to B via E, D and C on the network?

b Calculate the distance for the path A-E-B-C.

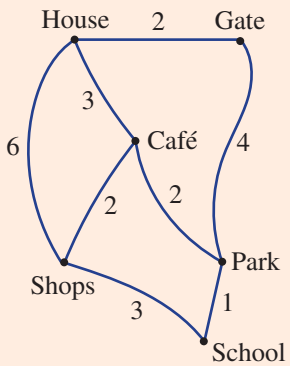


CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING 6–8 6–10

Example 22 Finding the shortest path

This graph represents a cycling network between a student's house and their school. Distances are in kilometres.



a How far is the walk from the house to the school via:
   i the shops without passing by the café?
   ii the gate and the park without passing by the café?

b Find the minimum distance from the house to the school via:
   i the shops
   ii any possible walk.

SOLUTION

a i $6 + 3 = 9$ km
   ii $2 + 4 + 1 = 7$ km

b i $3 + 2 + 3 = 8$ km
   ii $3 + 2 + 1 = 6$ km

EXPLANATION

The path house-shops-school includes the distances 6 km and 3 km.

The path house-gate-park-school includes the distances 2 km, 4 km and 1 km.

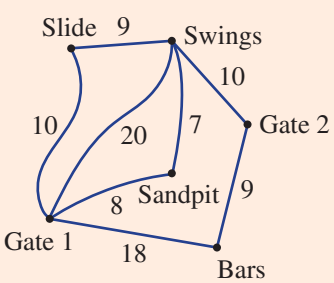
It is shorter to get to the shops via the café.

Some possible walks are:

- house-gate-park-school (7 km)
- house-shops-school (9 km)
- house-café-shops-school (8 km)
- house-café-park-school (6 km)

Now you try

This graph represents a path network connecting equipment in a playground. Distances are in metres.

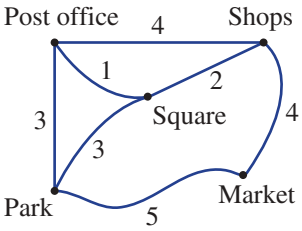


a How far is the path from gate 1 to the swings via:
   i the slide?
   ii the sand pit?

b Find the minimum distance from gate 1 to gate 2 via:
   i the slide?
   ii any possible walk?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 This graph represents a street map connecting a number of key places in a town. Distances are in kilometres.



a How far is the walk from the post office to the park via:

i the square without passing by the shops?

ii the square, shops and market?

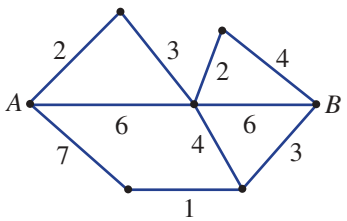
b Find the minimum distance from the shops to the park via:

i the post office

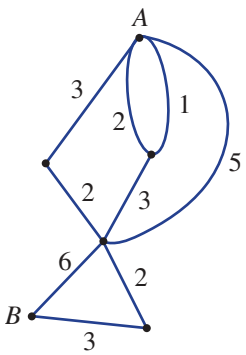
ii any possible walk.

7 Find the shortest possible distance to travel from point A to point B in these graphs. Distances are in kilometres.

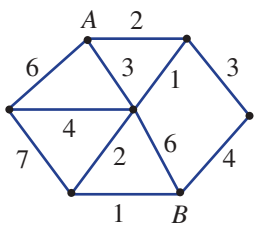
a



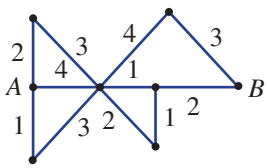
b



c

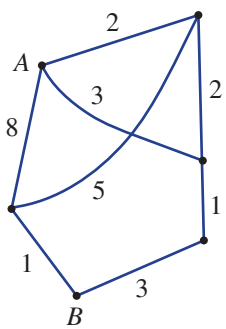


d

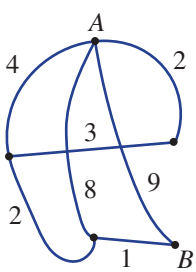


8 These graphs have intersecting edges but are in fact planar. Find the shortest distance from point A to B. Distances are in centimetres.

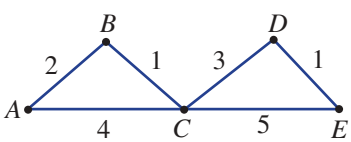
a



b



9 Explain why the shortest distance between the points A and E in this graph involves going via points B and D rather than just via point C.

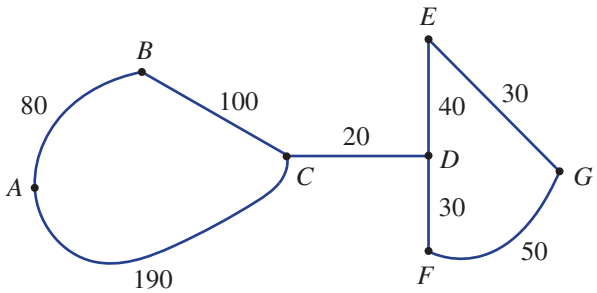


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 This weighted graph represents the cost of transporting goods via a network. The units are dollars.

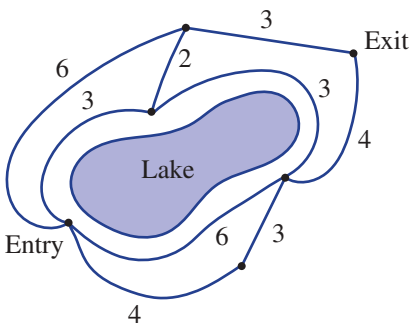
a Which points should be visited to transport goods with minimum cost from points A to G?

b What is the minimum cost if transporting the goods from A to G?



ENRICHMENT: Walking the lake — 11

11 A network of walking paths around a lake is represented in this graph with distances in kilometres. Find the shortest distance between the entry and exit points.



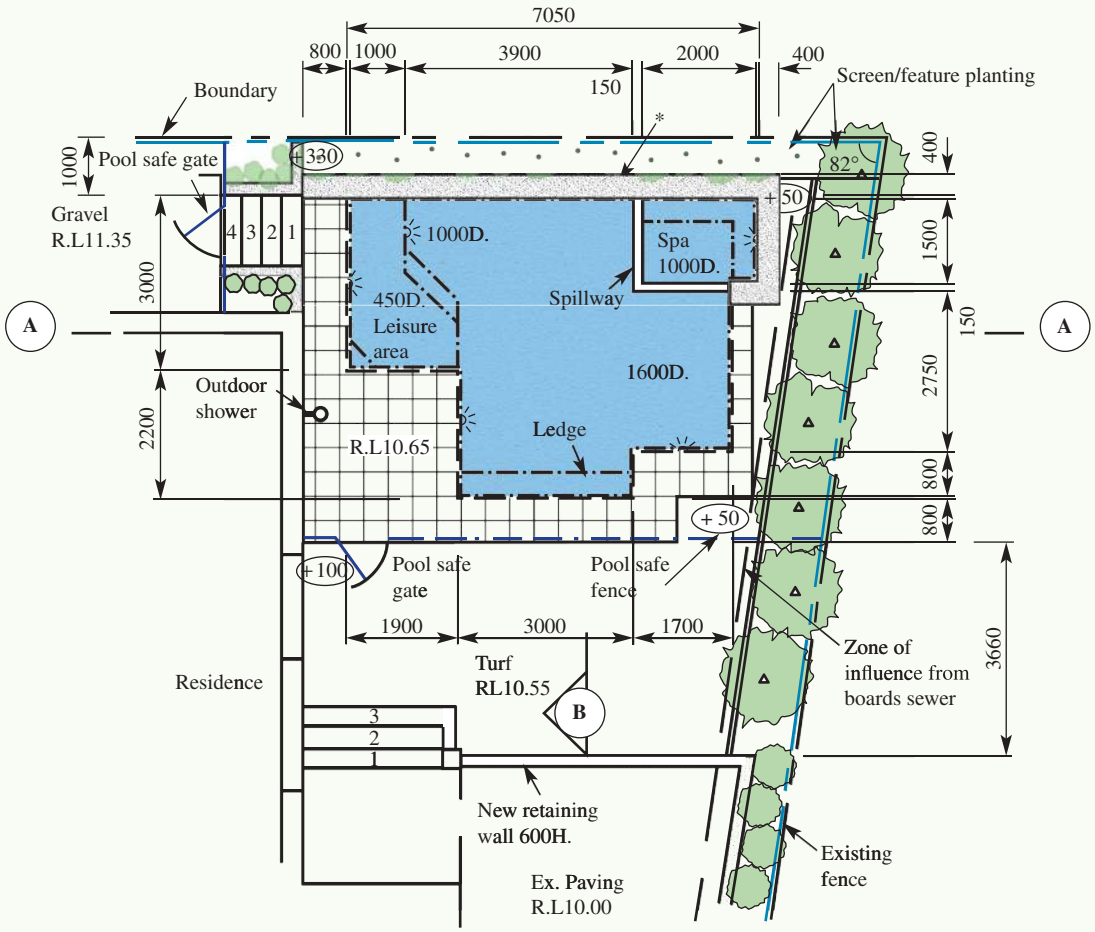
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Pool builder

Pool builders and designers need to understand many aspects of measurement and geometry to be able to accurately undertake the task of designing and building pools that meet the specifications of the client. They must be able to read plans, measure the angles, create similar figures and work with parallel lines, especially when they create and work from the pool plans and drawings.

Maths@Work

Below are the plans for a pool drawn by a designer. All lengths are given in millimetres. Answer the questions on the next page relating to the design of this pool.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1 Refer to the plans to answer the following questions.

- **a** What is the total length of the pool when looking across the top of the plans, in metres?
- **b** How deep is the proposed spa, in metres?
- **c** What is the height of the new retaining wall, in centimetres?
- **d** What is the area taken up by the internal dimensions of the spa? Include the seated area and give your answer in square metres.
- **e** What is the angle between the back fence and the right side fence?
- **f** If you walked around the outside of the pool/spa area, how many right angles would you turn? Assume you start and finish at the same point, facing in the same direction.

2 Consider the section of the pool called Leisure area, including the two steps which are a part of this area.

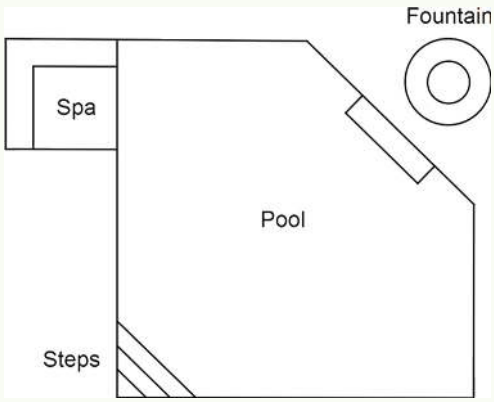
- **a** By counting the number of outside edges, decide what shape the proposed leisure area is.
- **b** What is the sum of the interior angles of the leisure area, using your knowledge of the angle sum of any polygon?
- **c** Use a protractor to measure the angles of the leisure area to check that the total is what you expect from part **b**.
- **d** Using your geometrical equipment, ruler and protractor, create an enlargement of the leisure area. Use the enlargement factor of 4. Your drawing will be mathematically similar to the original. Add all the labelled measurements from the original drawing.

3 Consider this pool tile, which will be used in the spa of this pool.

- **a** What are two shapes being used in its design?
- **b** Are both shapes regular?
- **c** What are the internal angles of the two shapes?
- **d** This pattern is a tessellation of two shapes. Why does there need to be two different shapes used here? Explain.
- **e** Design your own tessellation using two regular shapes, ensuring they are different from the ones used here.

Using technology

4 Use a geometry package, such as GeoGebra, Desmos, Cabri or Geometer's sketchpad, to come up with your own pool design. You may wish to add length and angle measurements to add detail. Here is a simple example.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Calculating inaccessible lengths

Without the use of sophisticated equipment, it is possible to use similar triangles as geometric models to estimate inaccessible lengths such as tree heights and distances across an impassable object like a gorge or a river.

You and a work-mate Hugo are surveyors in Australia's north, working for a tourism company that requires measurements and calculations for some new tourist facilities.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

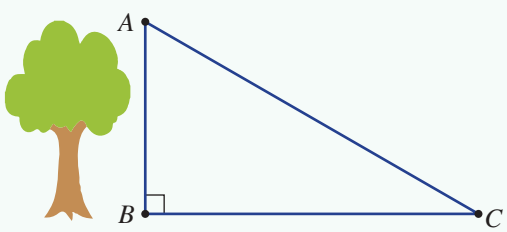
Routine problems

The height of a tree

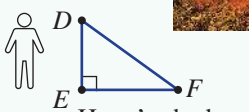
In the Kimberley, W.A., a tourist sign needs to include the height of an old boab tree and of a nearby termite mound.

Late one afternoon Hugo measured the shadow of the boab tree to be 16 m, and at the same time, his own shadow was 2.5 m long. Hugo's height is 1.8 m.

a Copy these two right-angled triangles and label the known lengths: $BC = 16$ m, $ED = 1.8$ m and $EF = 2.5$ m.



Tree's shadow



Hugo's shadow

b Explain why $\angle ACB = \angle DFE$

c Prove the ΔABC is similar to ΔDEF.

d Find the scale factor linking the two triangles.

e Calculate the height of the boab tree.

f If, at the same time, a nearby termite mound had a shadow of length 5 m, find its height using similar triangles.

Non-routine problems

The tourism company requires a gorge and a river to be surveyed at a location where a bridge could be built for tourists.

The problem is to apply appropriate mathematical procedures and calculate the width of the gorge and of the river.

Explore and connect

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Describe the mathematical procedures appropriate for this task by answering the following questions.

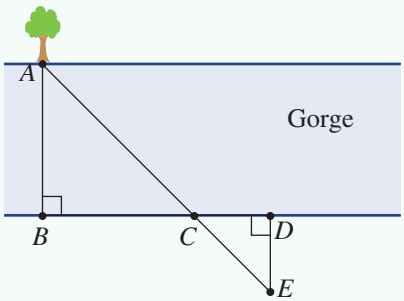
- **a** When a pair of triangles are similar, explain how the corresponding angles are related and how the corresponding sides are related.
- **b** Explain how the scale factor is calculated for similar triangles.

Choose and apply techniques

The width of a gorge

At Knox Gorge in Karijini National Park, W.A., you notice a tree (A) directly across the other side of the gorge and you proceed to place rocks (B, C, D and E) on your side of the gorge in special positions as shown in the diagram below.

You measure $BC = 12$ m, $CD = 4$ m and $DE = 6$ m.



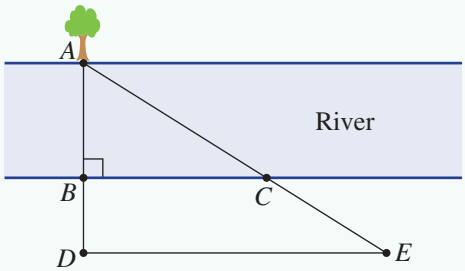
- **d** Copy the triangles, label known lengths and pairs of equal angles.
- **e** Prove that $\Delta ABC \parallel\!\parallel \Delta EDC$.
- **f** Find the scale factor linking the two triangles.
- **g** Find the distance across this gorge.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

The width of a river

In Nitmiluk National Park, N.T., Hugo wants to measure the width of Katherine River. He sees a tree (A) directly across the other side of the river and places rocks (B, C, D and E) on his side of the river in special positions as shown in the diagram below.

He measures $BC = 15$ m, $BD = 5$ m and $DE = 18$ m.



h Copy the triangles, label known lengths and equal angles.

i Explain why $\Delta ABC \Vert\Vert \Delta ADE$.

j Find the scale factor linking the two triangles.

k Use trial and error to find the distance across the river from the tree at A to the rock placed at B.

l Check that you have answered each question appropriately with the use of labelled diagrams, clear mathematical workings and sentence answers.

m Write a brief recommendation to the tourism company including the gorge and river widths.

Communicate thinking and reasoning

Extension problem

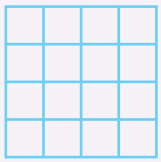
Use an equation, instead of trial and error, to find the width of Katherine River (i.e. the length AB in the river diagram above).

Problem-solve

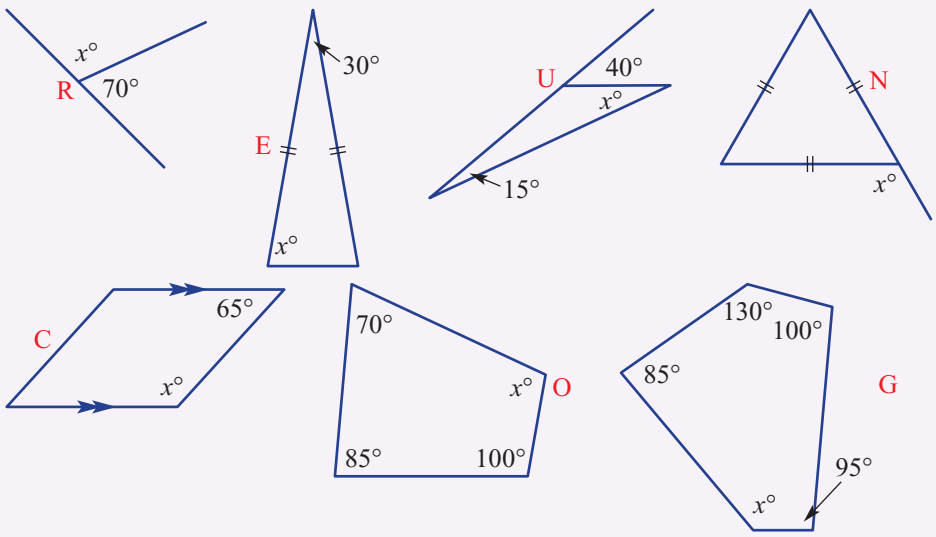
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Puzzles and challenges

1 How many squares can you see in this diagram?

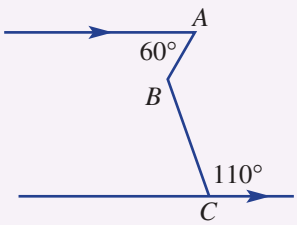


2 'I think of this when I look in the mirror.' Find the value of x in each diagram, then match the letters beside the diagrams to the answers below.



$\overline{115}$ $\overline{105}$ $\overline{120}$ $\overline{130}$ $\overline{110}$ $\overline{25}$ $\overline{75}$ $\overline{120}$ $\overline{115}$ $\overline{75}$

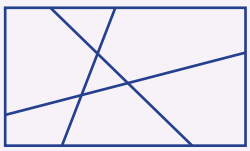
3 What is the size of the obtuse angle $\angle ABC$ in this diagram?



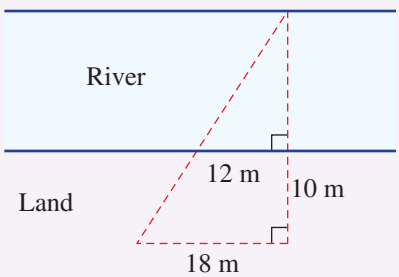
4 This rectangle is subdivided by three straight lines.

a How many regions are formed?

b What is the maximum number of regions formed if four lines are used instead of three?



5 Find the distance across the river.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter summary

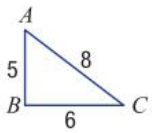
Properties of geometrical figures

Similar triangles

Identical in shape and sides are in proportion

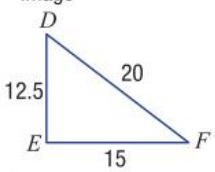
A similarity statement:
$\Delta ABC \Vert\!\Vert\!\Vert \Delta DEF$

Original



Scale factor is 2.5 →

Image



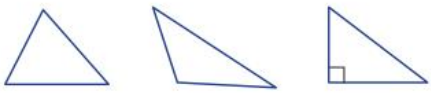
$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

Triangles

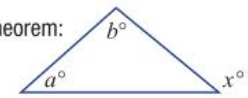
Angle sum = 180°

Scalene | Isosceles | Equilateral

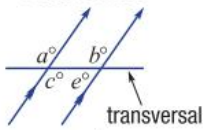
Acute | Obtuse | Right angled



Exterior angle theorem:
$x = a + b$



Parallel lines



$a = b$ corresponding
$b = c$ alternate
$c + e = 180$ cointerior
$a = c$ vertically opposite

Quadrilaterals

Angle sum = 360°

Special types include:
- parallelogram
- square
- rectangle
- rhombus
- kite
- trapezium

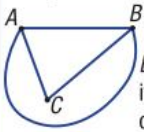
Polygons

Interior angle sum $S = (n - 2) \times 180°$

Regular polygons have:
- equal side lengths
- equal angles

Networks

A network is a diagram connecting vertices (nodes) using lines (edges). The degree of a vertex is the number of edges connected to it.



B is an odd vertex as it has an odd number of edges connected to it.

Walks

A walk is a sequence of edges through a network:
- trail – a walk with no repeated edges
- path – walk with no repeated edges or vertices
- cycle – a path that starts and ends at the same vertex
- circuit – a trail that starts and ends at the same vertex
- Eulerian trail – every edge is used exactly once, needs 0 or 2 odd vertices to exist
- Eulerian circuit – an Eulerian trail that starts and ends at the same vertex, requires no odd vertices.

Planar graphs and Euler's fomula

Isomorphic graphs (same edges, vertices and connections) can be redrawn to look the same.
A planar graph can be drawn so it has no intersecting edges.
Euler's formula for planar graphs says:
$v + f = e + 2$ or alternatively $v - e + f = 2$
v = no. of vertices, f = no. of faces, e = no. of edges

Weighted graphs

A weighted graph has a number attached to each edge which might represent distances, time, costs etc.

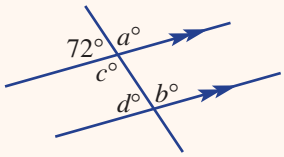
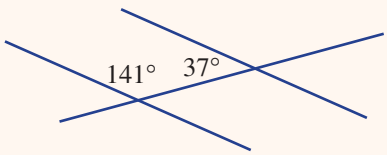
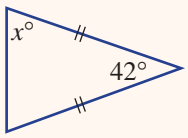
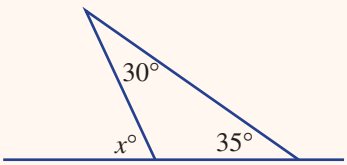
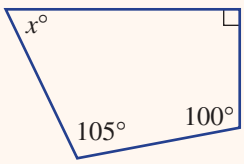
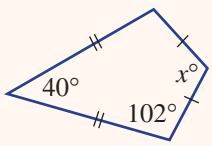
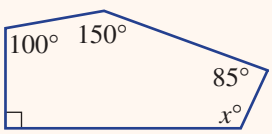
A shortest path problem involves finding a walk through the network of minimum distance.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

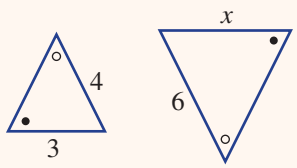
		✔
7A	**1 I can find unknown angles in parallel lines.** e.g. Find the values of the pronumerals in this diagram and give reasons for your answers. 	☐
7A	**2 I can prove that two lines are parallel.** e.g. Decide, with reasons, whether the given pair of lines are parallel. 	☐
7B	**3 I can find unknown angles in any type of triangle.** e.g. Find the value of x in this triangle. 	☐
7B	**4 I can use the exterior angle theorem to find unknown angles.** e.g. Use the exterior angle theorem to find the value of x in this diagram. 	☐
7C	**5 I can find an unknown angle in a quadrilateral.** e.g. Find the value of x in this quadrilateral. 	☐
7C	**6 I can find an unknown angle in a special quadrilateral.** e.g. Find the value of x in this kite. 	☐
7D	**7 I can find an angle sum of a polygon and an unknown angle in a polygon.** e.g. Find the value of x in this pentagon after finding the angle sum. 	☐
7D	**8 I can find the internal angle in a regular polygon.** e.g. Find the size of an internal angle inside a regular hexagon.	☐

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

7E

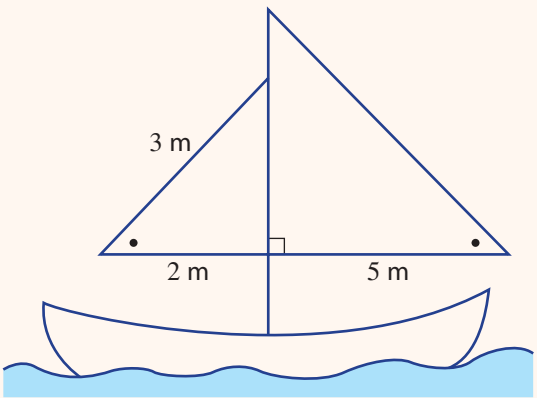
9 I can find a scale factor and use this to find an unknown length.
e.g. If the given pair of triangles are known to be similar, find the value of x.



☐

7F

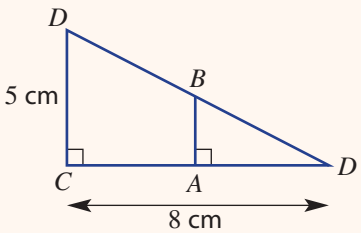
10 I can use the scale factor for similar triangles to find an unknown length in a real context.
e.g. For the triangles formed from these yacht sails, give a reason why they are similar then use the scale factor to find the length of the hypotenuse on the larger sail.



☐

7F

10 I can use the scale factor for similar triangles to find an unknown length inside combined triangles.
e.g. If the point A is at the centre of CD in this diagram, find the length AB.

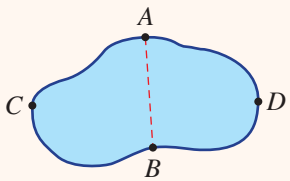


☐

7F

11 I can use measurement and the map scale factor to find a real distance on a map.
e.g. Use measurement and the map scale factor to find the distance across the pool (CD).

Scale $1:300$

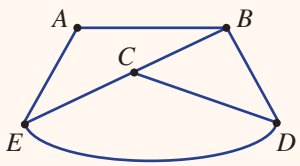


☐

7G

12 I can determine the features of a network.
e.g. For the graph shown, state:

- **i** the number of vertices
- **ii** the degree of vertex C.



☐

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

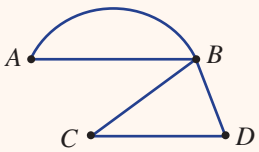
Chapter checklist

Chapter checklist with success criteria

7G

13 I can find walks between vertices.

e.g. For the graph shown, find how many walks there are connecting A and C without visiting a vertex more than once or using an edge more than once.

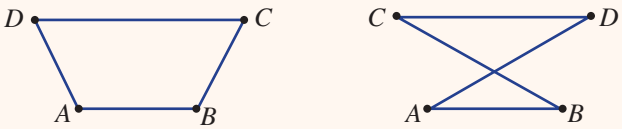


☐

7H

14 I can decide if graphs are isomorphic.

e.g. Decide if the following two graphs are isomorphic.

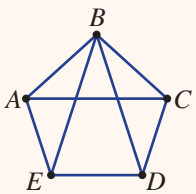


☐

7H

15 I can decide if a graph is planar or non-planar.

e.g. Decide if the following graph is planar or non-planar.

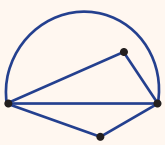


☐

7H

16 I can verify Euler's formula.

e.g. Find the number of vertices, edges and faces in this planar graph and verify Euler's formula.



☐

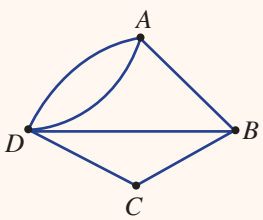
7I

17 I can define a type of walk.

e.g. For the given graph, decide if:

i A–B–C–D–B is a trial

ii A–B–D–C–B–A is a cycle.



☐

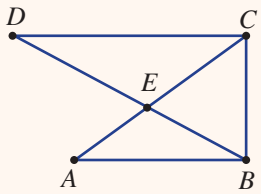
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

7I

18 I can decide if a graph has an Eulerian trail or circuit.

e.g. Decide if this graph has an Eulerian trail and if so whether all such trails will be circuits.

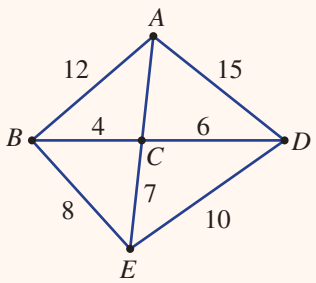


☐

7J

19 I can interpret a weighted graph.

e.g. This graph shows the distances in kilometres along the roads connecting five towns, A to E. Calculate the distance for the path *A–B–C–E–D*.

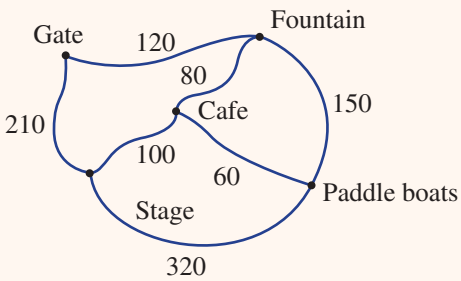


☐

7J

20 I can find the shortest path on a network graph.

e.g. This graph represents a path network connecting landmarks in the botanical gardens. Distances are in metres. Find the minimum distance from the gate to the paddle boats.



☐

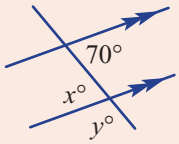
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

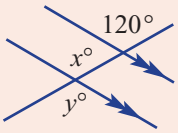
Short-answer questions

7A **1** Find the value of x and y in these diagrams.

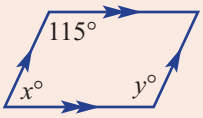
a



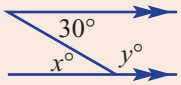
b



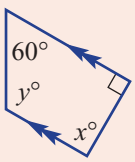
c



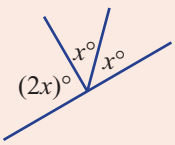
d



e

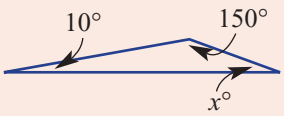


f

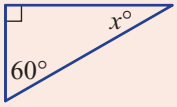


7B **2** Find the value of x in these triangles.

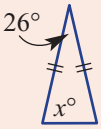
a



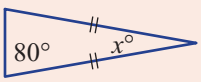
b



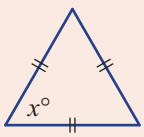
c



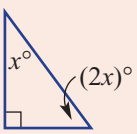
d



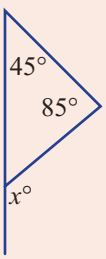
e



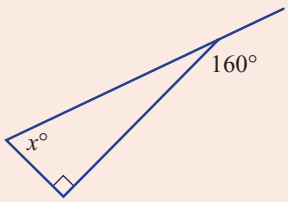
f



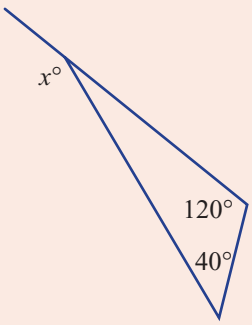
g



h



i



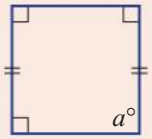
7C **3** List all the quadrilaterals that have:

- **a** two pairs of parallel lines
- **b** opposite angles that are equal
- **c** one pair of equal angles
- **d** diagonals intersecting at right angles

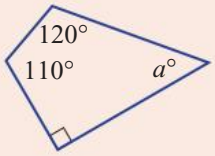
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7C **4** Find the values of the pronumerals in these quadrilaterals.

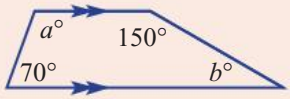
a



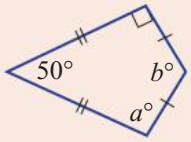
b



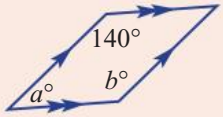
c



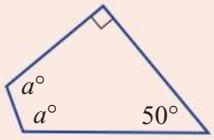
d



e

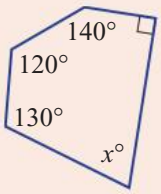


f

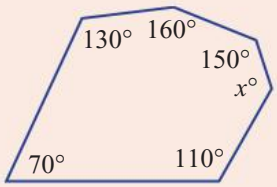


7D **5** Find the value of x by first finding the angle sum. Use $S = (n - 2) \times 180°$.

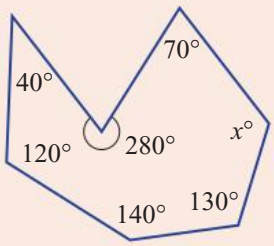
a



b



c

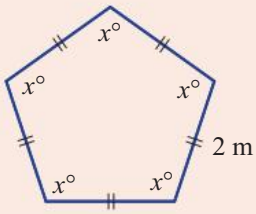


7D **6** Shown here is an example of a regular pentagon ($n = 5$) with side lengths 2 m.

a Find the perimeter of the pentagon.

b Find the total internal angle sum (S).

c Find the size of each internal angle (x).

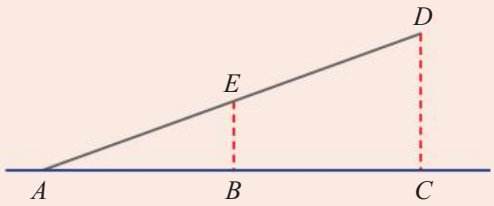


7G **7** A skateboard ramp is supported by two vertical struts, BE (2 m) and CD (5 m).

a Name two triangles that are similar, using the letters A, B, C, D and E.

b Find the scale factor from the smallest to the largest triangle.

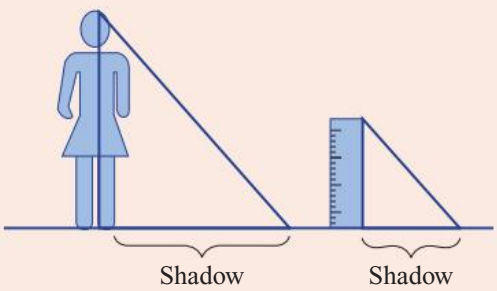
c If the length AB is 3 m, find the horizontal length of the ramp AC.



7G **8** The shadow of Ming standing in sunlight is 1.5 m long, and the shadow of a 30 cm ruler is 24 cm.

a Find the scale factor between the two similar triangles.

b How tall is Ming?

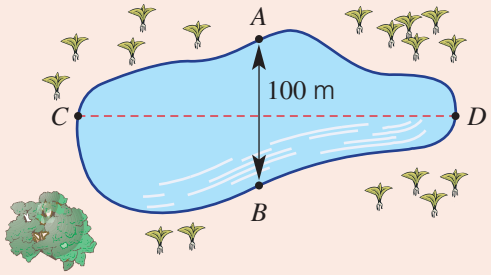


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

7H

9 The given diagram is a simple map of a swamp in bushland.



a Use a ruler to measure the distance across the swamp (AB). (Answer in cm.)

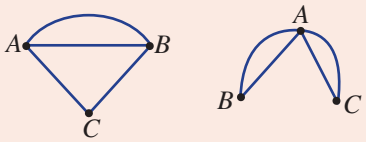
b Find the scale factor between the map and actual ground distance.

c Use a ruler to find the map distance across the swamp (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the swamp (CD). (Answer in m.)

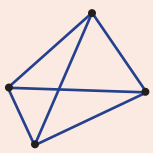
7H

10 **a** Decide if the following two graphs are isomorphic.

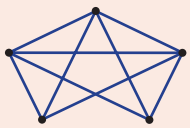


b Decide if the following graphs are planar or non-planar.

i

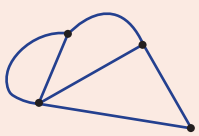


ii



7G/H

11 Consider the planar graph shown.



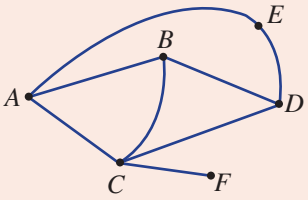
a State the number of edges and the number of vertices.

b State the number of faces.

c Hence, verify Euler's formula for the graph.

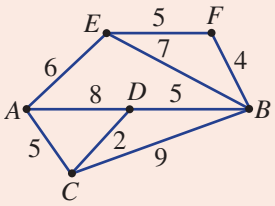
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7I **12** Consider the graph shown.



- **a** Decide if the walk $A–C–B–D–C–A$ is a trail.
- **b** Explain why this graph does not have an Eulerian trail.
- **c** Add one edge to the graph from D so that it has an Eulerian trail.
- **d** State the Eulerian trail from part **c** and decide if it is a circuit.

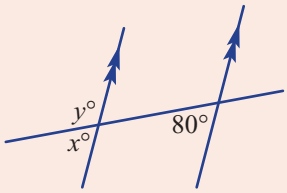
7J **13** This graph shows the distances between points on an electrical circuit in centimetres.



- **a** Calculate the distance for the path $A–E–F–B$.
- **b** What is the shortest distance from A to B?

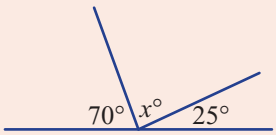
Multiple-choice questions

7A **1** The values of x and y in this diagram are, respectively:



A 100, 100 **B** 80, 100
C 80, 80 **D** 60, 120
E 80, 60

7A **2** The unknown value x in this diagram is:

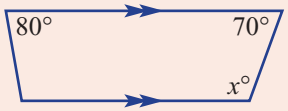


A 85 **B** 105 **C** 75
D 80 **E** 90

7B **3** A triangle has one angle of 60° and another angle of 70°. The third angle is:

A 60° **B** 30° **C** 40°
D 50° **E** 70°

7C **4** The value of x in this quadrilateral is:



A 130 **B** 90 **C** 100
D 120 **E** 110

7D **5** The sum of the interior angles of a hexagon is:

A 180° **B** 900° **C** 360°
D 540° **E** 720°

7E **6** Two similar triangles have a scale factor of 1.5. If one side on the smaller triangle is 5 cm, the length of the corresponding side on the larger triangle is:

A 3 cm **B** 7.5 cm **C** 9 cm
D 8 cm **E** 6 cm

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

7F **7** A stick of length 2 m and a tree of unknown height stand vertically in the sunlight. The shadow lengths cast by each are 1.5 m and 30 m, respectively. The height of the tree is:

A 40 m **B** 30 m **C** 15 m **D** 20 m **E** 60 m

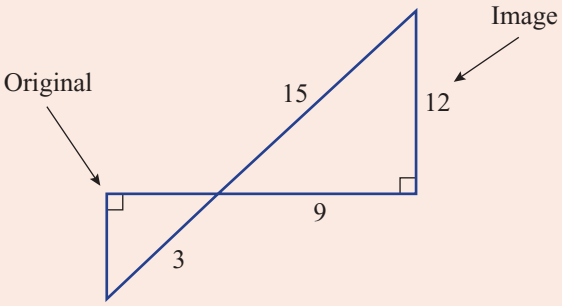
7G **8** How many sides in a dodecagon?

A 5 **B** 10 **C** 11 **D** 12 **E** 20

7H **9** What is the scale factor of the similar triangles shown?

A $\frac{1}{5}$ **B** $\frac{1}{4}$ **C** 3

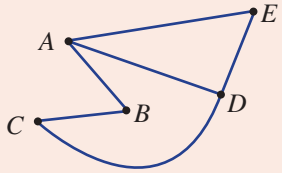
D 4 **E** 5



7G **10** How many odd vertices does the graph shown have?

A 0 **B** 1 **C** 2

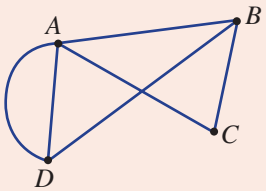
D 3 **E** 4



7I **11** So that the graph shown has an Eulerian circuit, which edge should be removed?

A *A–D* **B** *A–C* **C** *B–D*

D *A–B* **E** *B–C*



7I **12** Which of the following is not a path for the graph shown?

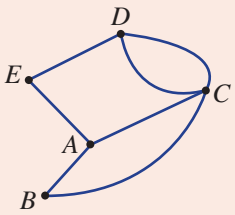
A *A–B–C–D*

B *A–C–D–E*

C *A–B–C–D*

D *A–C–D–C–B*

E *E–D–C–A–B*



7J **13** The shortest path through this network from *A* to *D* via *B*, where distance is in kilometres, is:

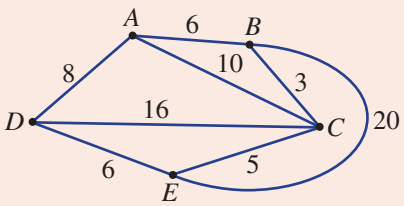
A 20 km

B 25 km

C 32 km

D 8 km

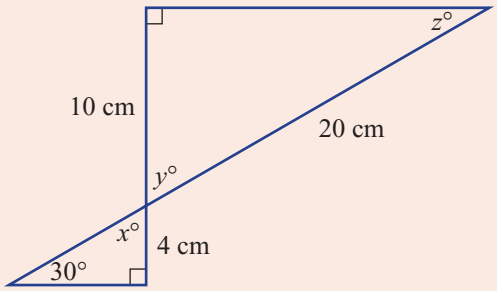
E 9 km



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Extended-response questions

1 A company logo contains two similar triangles, as shown.

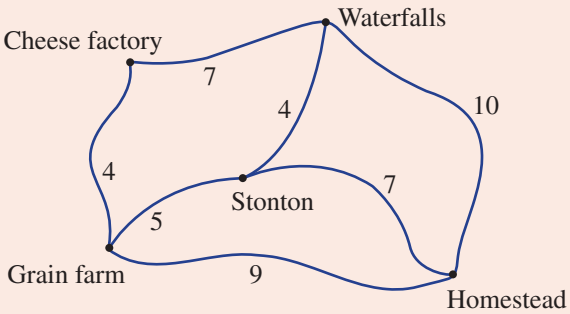


- **a** Write down the value of x, y and z.
- **b** Write down the scale factor for length.
- **c** Find the length of the longest side of the smaller triangle.

2 A toy model of a car is 8 cm long and the actual car is 5 m long.

- **a** Write as a fraction the scale factor for the length ratio of the toy car to the actual car.
- **b** If the toy car is 4.5 cm wide, what is the width in centimetres of the actual car?

3 This graph represents a map of the countryside near the village of Stonton. Distances are in kilometres.

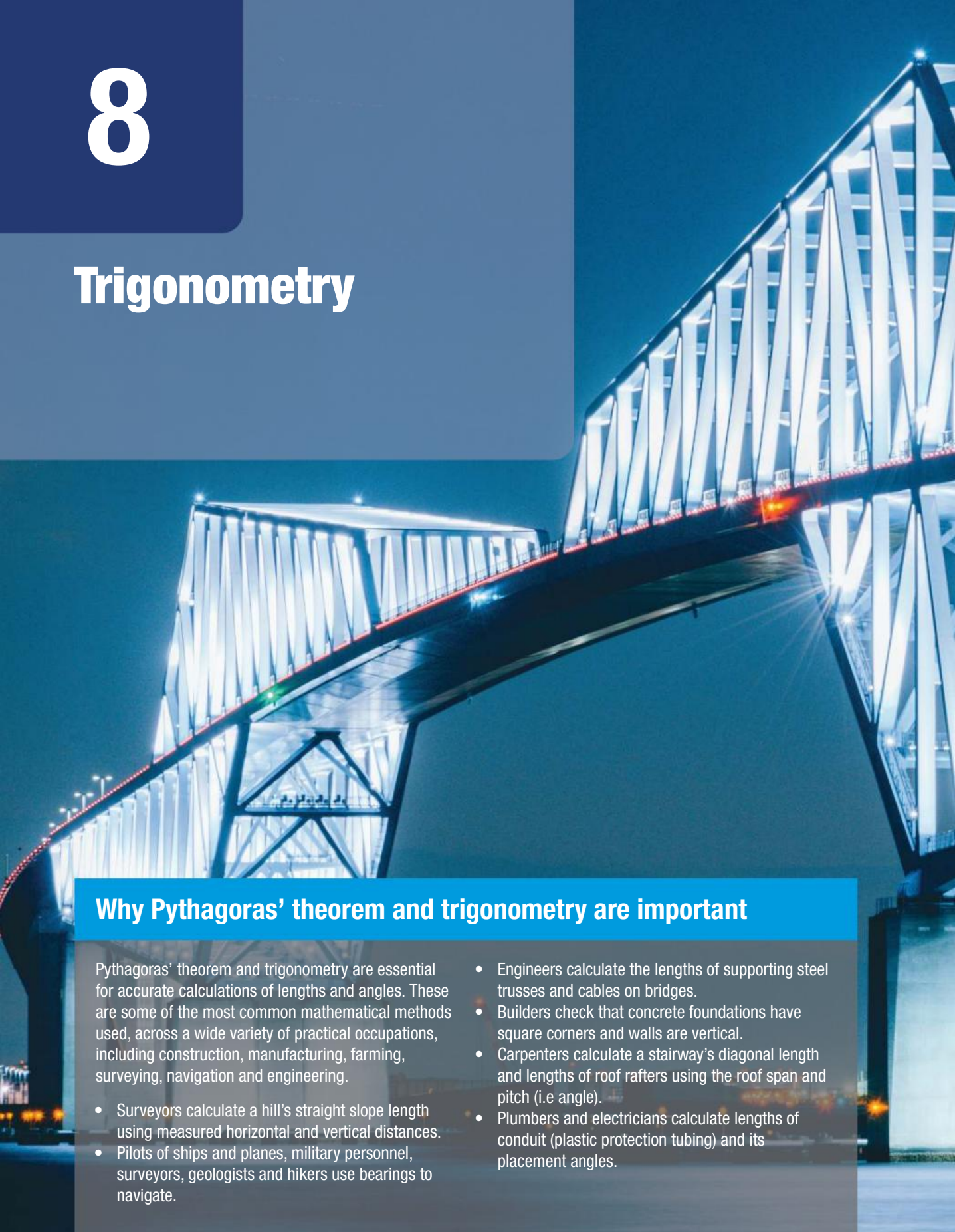


- **a** Find the sum of degrees of all the vertices.
- **b** How far is the walk Stonton-grain farm-cheese factory?
- **c** Confirm that the graph satisfies Euler's formula $v + f = e + 2$.
- **d** Decide if the trail Stonton-waterfalls-homestead-grain farm-cheese factory is Eulerian. Give a reason.
- **e** Find the length of the shortest path between the following places.
  - **i** grain farm and waterfalls
  - **ii** homestead and cheese factory if going via Stonton

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8

Trigonometry



Why Pythagoras' theorem and trigonometry are important

Pythagoras' theorem and trigonometry are essential for accurate calculations of lengths and angles. These are some of the most common mathematical methods used, across a wide variety of practical occupations, including construction, manufacturing, farming, surveying, navigation and engineering.

- Surveyors calculate a hill's straight slope length using measured horizontal and vertical distances.
- Pilots of ships and planes, military personnel, surveyors, geologists and hikers use bearings to navigate.
- Engineers calculate the lengths of supporting steel trusses and cables on bridges.
- Builders check that concrete foundations have square corners and walls are vertical.
- Carpenters calculate a stairway's diagonal length and lengths of roof rafters using the roof span and pitch (i.e angle).
- Plumbers and electricians calculate lengths of conduit (plastic protection tubing) and its placement angles.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter contents

8A Reviewing Pythagoras' theorem (CONSOLIDATING)
8B Finding the length of a shorter side (CONSOLIDATING)
8C Applications of Pythagoras' theorem
8D Trigonometric ratios
8E Finding unknown side lengths
8F Solving for the denominator
8G Finding unknown angles
8H Angles of elevation and depression
8I Direction and bearings
8J Applications in three dimensions
8K The sine rule
8L The cosine rule
8M Area of a triangle

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- applies trigonometric ratios to solve right-angled triangle problems (MA5-TRG-C-01)
- applies trigonometry to solve problems, including bearings and angles of elevation and depression (MA5-TRG-C-02)
- applies Pythagoras' theorem and trigonometry to solve 3-dimensional problems and applies the sine, cosine and area rules to solve 2-dimensional problems, including bearings (MA5-TRG-P-01).

© 2022 NSW Education Standards Authority

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Warm-up quiz

1 Round the following decimals, correct to two decimal places.

a 15.84312 **b** 164.8731
c 0.86602 **d** 0.57735
e 0.173648 **f** 0.7071
g 12.99038 **h** 14.301

2 Find the value of each of the following.

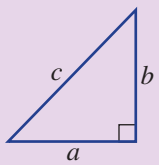
a 5^2 **b** 6.8^2
c 19^2 **d** $9^2 + 12^2$
e $3.1^2 + 5.8^2$ **f** $41^2 - 40^2$

3 Write the following as a decimal, correct to one decimal place.

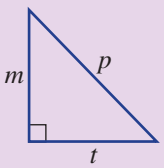
a $\sqrt{8}$ **b** $\sqrt{7}$
c $\sqrt{15}$ **d** $\sqrt{10}$
e $\sqrt{12.9}$ **f** $\sqrt{8.915}$
g $\sqrt{3.8}$ **h** $\sqrt{200}$

4 Write down the letter or letters matching the hypotenuse (i.e. the side opposite the right angle) on the following triangles.

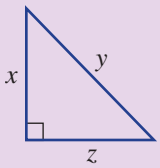
a



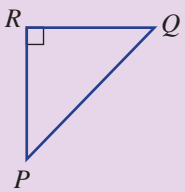
b



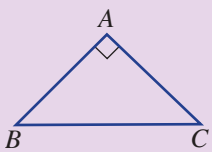
c



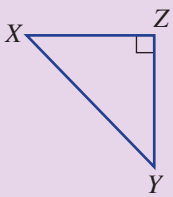
d



e



f



5 Solve for x.

a $3x = 9$ **b** $4x = 16$
c $\frac{x}{5} = 7$ **d** $\frac{2x}{3} = 6$

6 Solve for m.

a $7m = 25.55$ **b** $9m = 10.8$
c $\frac{m}{1.3} = 4$ **d** $\frac{m}{5.4} = 1.06$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 Solve each of the following equations, correct to one decimal place.

a $\frac{3}{x} = 5$ **b** $\frac{4}{x} = 17$

c $\frac{32}{x} = 15$ **d** $\frac{3.8}{x} = 9.2$

e $\frac{15}{x} = 6.2$ **f** $\frac{29.3}{x} = 3.2$

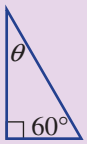
8 If x is a positive integer, solve:

a $x^2 = 16$ **b** $x^2 = 400$

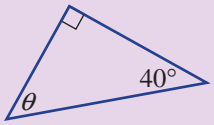
c $x^2 = 5^2 + 12^2$ **d** $x^2 + 3^2 = 5^2$

9 Find the size of the angle θ in the following diagrams.

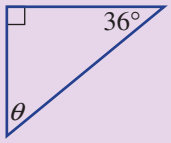
a



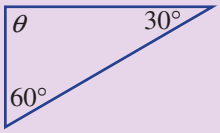
b



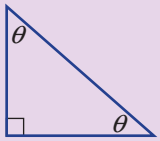
c



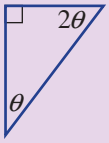
d



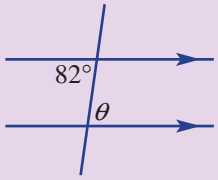
e



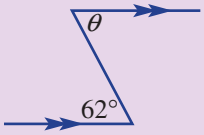
f



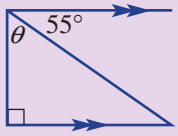
g



h



i



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8A Reviewing Pythagoras' theorem CONSOLIDATING

Learning intentions for this section:

- To know that Pythagoras' theorem connects the three side lengths of a right-angled triangle
- To be able to find the length of the hypotenuse given the other two sides
- To be able to find hypotenuse lengths as rounded decimals or exact values

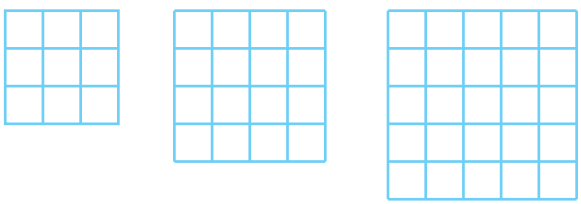
Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- It consolidates and extends the concepts in Sections 4A and 4B of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formula from memory in the HSC Examination

The ancient Egyptians knew of the relationship between the numbers 3, 4 and 5 and how they could be used to form a right-angled triangle.

Greek philosopher and mathematician Pythagoras expanded on this idea and the theorem we use today is named after him.

Lesson starter: Three, four and five



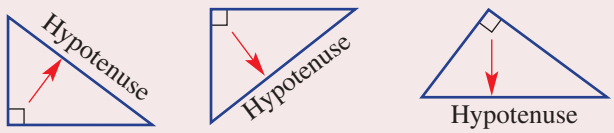
- On square grid paper, construct three squares as shown above.
- Cut them out and place the middle-sized square on top of the largest square. Then cut the smallest square into 9 smaller squares and also place them onto the largest square to finish covering it.
- What does this show about the numbers 3, 4 and 5?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- A right-angled triangle has its longest side opposite the right angle. This side is called the **hypotenuse**.

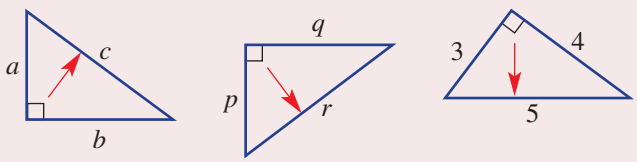
  For example:



- **Pythagoras' theorem** states:

  The square of the hypotenuse is equal to the sum of the squares on the other two sides.

  For example:



$c^2 = a^2 + b^2$ $\quad r^2 = p^2 + q^2$ $\quad 5^2 = 3^2 + 4^2$

Key terminology: Pythagoras' theorem, right angle, hypotenuse, square

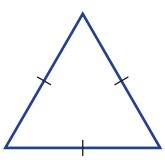
Exercise 8A

UNDERSTANDING | 1–4 | 3, 4

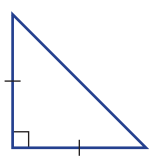
1 Which of the following triangles have a side known as the hypotenuse?

Hint: Only right-angled triangles have a hypotenuse.

A

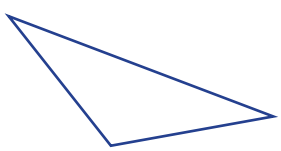


B



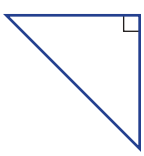
C

D

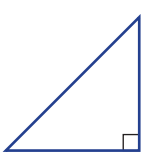


2 Copy these triangles into your workbook and label the hypotenuse.

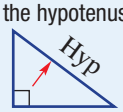
a



b



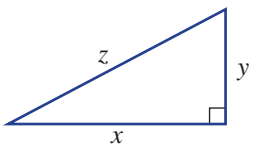
Hint: Draw an arrow across from the right angle to find the hypotenuse (hyp).



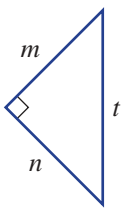
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Write the relationship between the sides of these triangles.

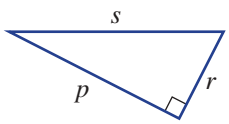
a



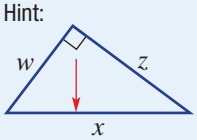
b



c



Hint:



$x^2 = w^2 + z^2$

4 Find the value of $a^2 + b^2$ when:

a $a = 3$ and $b = 4$

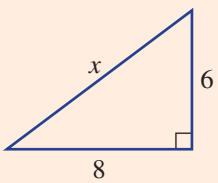
b $a = 3$ and $b = 5$

c $a = 3$ and $b = 6$

FLUENCY 5–6(½) 5–7(½)

Example 1 Finding the length of the hypotenuse

Find the length of the hypotenuse (x) of the triangle shown.



SOLUTION

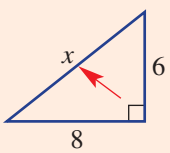
$x^2 = 6^2 + 8^2$

$= 36 + 64$

$= 100$

EXPLANATION

Write the relationship for the given triangle using Pythagoras' theorem.



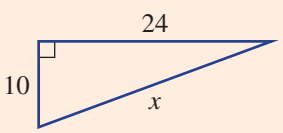
$x = \sqrt{100}$

$= 10$

Take the square root to find x.

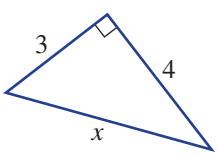
Now you try

Find the length of the hypotenuse (x) of the triangle shown.

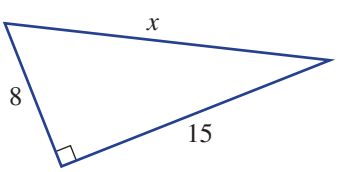


5 Find the length of the hypotenuse in these right-angled triangles.

a

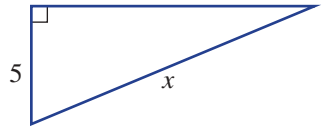


b

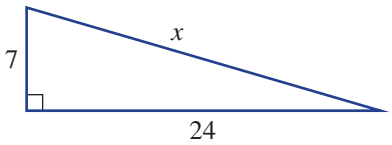


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

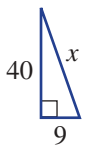
c



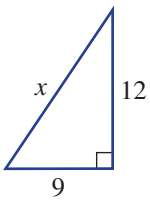
d



e

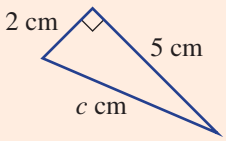


f



Example 2 Finding the length of the hypotenuse as a decimal

Find the length of the hypotenuse in this triangle, correct to one decimal place.



SOLUTION

$c^2 = 5^2 + 2^2$

$= 25 + 4$

$= 29$

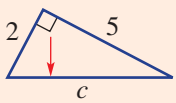
$c = \sqrt{29}$

$c = 5.38516\ldots$

$c = 5.4$ (to 1 d.p.)

EXPLANATION

Write the relationship for this triangle, where c is the length of the hypotenuse.



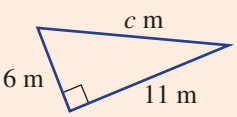
Simplify.

Take the square root to find c.

Round 5.3 ⑧ 516 … to one decimal place by rounding up.

Now you try

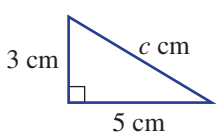
Find the length of the hypotenuse in this triangle, correct to one decimal place.



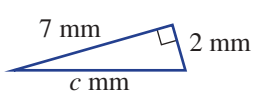
6 Find the length of the hypotenuse in these triangles, correct to one decimal place.

Hint: If $c^2 = 34$, then $c = \sqrt{34}$. Use a calculator to find the decimal.

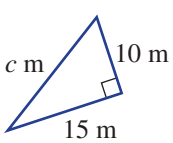
a



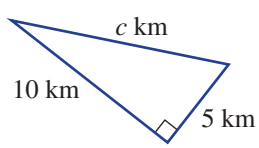
b



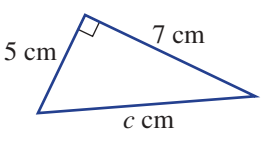
c



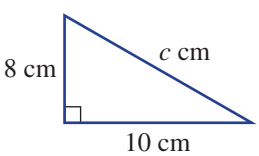
d



e



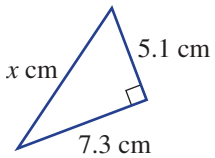
f



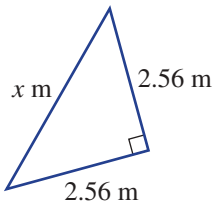
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 Find the value of the hypotenuse in these triangles, correct to two decimal places.

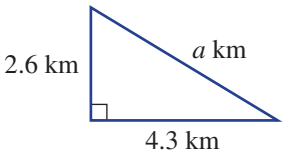
a



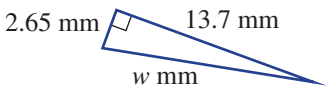
b



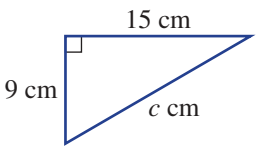
c



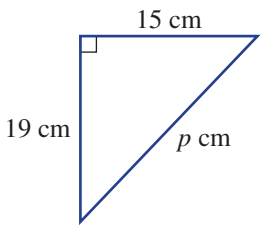
d



e



f

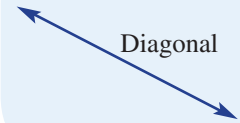


PROBLEM-SOLVING AND REASONING

8–11 | 11–14

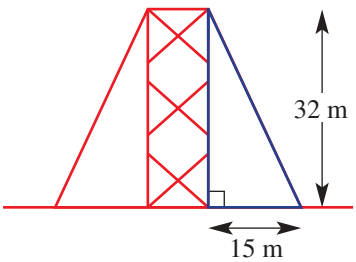
8 A TV is 154 cm long and 96 cm high. Calculate the length of its diagonal, correct to one decimal place.

Hint:

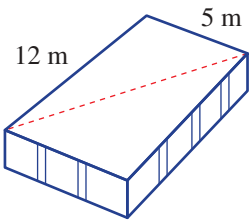


9 A 32 m tower is supported by cables from the top to a position on the ground 15 m from the base of the tower. Determine the length of each cable needed to support the tower, correct to one decimal place.

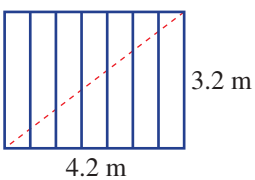
Hint: Set up and solve using Pythagoras' theorem.



10 Boris the builder uses Pythagoras' theorem to check the corners of his concrete slab. What will be the length of the diagonal when the angle is 90°?

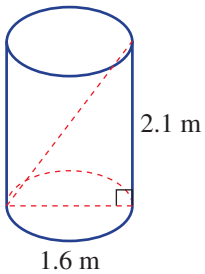


11 Find the length of the diagonal steel brace needed to support a gate of length 4.2 m and width 3.2 m, correct to two decimal places.

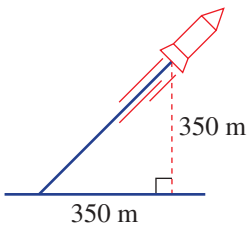


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

12 Find the length of the longest rod that will fit in a cylindrical container of height 2.1 m and diameter 1.6 m, correct to two decimal places.

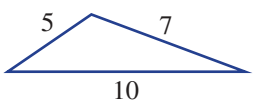


13 A rocket blasts off and after a few seconds it is 350 m above the ground. At this time it has covered a horizontal distance of 350 m. How far has the rocket travelled, correct to two decimal places?

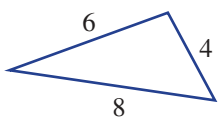


14 Determine whether these triangles contain a right angle.

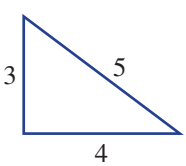
a



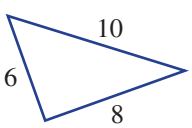
b



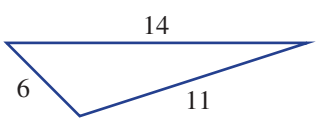
c



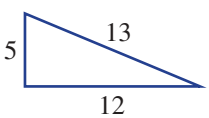
d



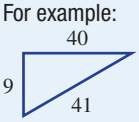
e



f



Hint: If Pythagoras' theorem works, then the triangle has a right angle.
For example:



$41^2 = 1681$
$40^2 + 9^2 = 1681$
$\therefore 41^2 = 40^2 + 9^2$ and the triangle has a right angle, opposite the 41.

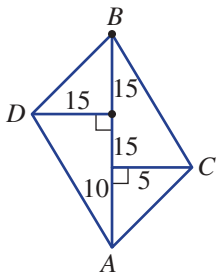
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: An offset survey — 15, 16

15 An offset survey measures distances perpendicular to the baseline offset. A notebook entry is made showing these distances, and then perimeters and areas are calculated. All measurements are in metres.

		B		
		40		
D	15	25		
		10	5	C
		0		
		A		

Notebook entry



Field diagram (Not to scale)

a Using the diagrams above, find these lengths, correct to one decimal place.

i AC **ii** BC **iii** DB **iv** AD

b Find the perimeter of the field $ACBD$, correct to the nearest metre.

c Find the area of the field.

16 At right is a notebook entry. Draw the field diagram and find the perimeter of the field, to one decimal place. All measurements are in metres.

		B		
		60		
		40	10	E
D	25	30		
C	15	10		
		0		
		A		

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8B Finding the length of a shorter side CONSOLIDATING

Learning intentions for this section:

- To be able to set up Pythagoras' theorem where the unknown is a shorter side
- To be able to solve the resulting equation to find the length of a shorter side

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- It consolidates and extends the concepts in Section 4C of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formula from memory in the HSC Examination

Using Pythagoras' theorem, we can determine the length of the shorter sides of a right-angled triangle. The angled support beams on a rollercoaster ride, for example, create right-angled triangles with the ground. The vertical and horizontal distances are the shorter sides of the triangle.

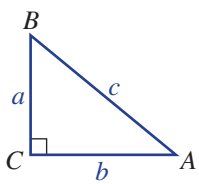
Lesson starter: Choosing the correct numbers

For the triangle ABC, Pythagoras' theorem is written $c^2 = a^2 + b^2$.

Choose the 3 numbers from each group that work for $c^2 = a^2 + b^2$.

Group 1 : 6, 7, 8, 9, 10 Group 2 : 15, 16, 20, 25

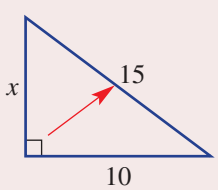
Group 3 : 9, 10, 12, 15 Group 4 : 9, 20, 21, 40, 41



KEY IDEAS

- We can use Pythagoras' theorem to determine the length of one of the shorter sides if we know the length of the hypotenuse and the other side.

For example:



$$15^2 = x^2 + 10^2$$
$$x^2 + 10^2 = 15^2$$
$$x^2 + 100 = 225$$
$$x^2 = 125 \quad (-100 \text{ from both sides})$$
$$x = \sqrt{125}$$

Key terminology: hypotenuse

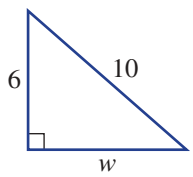
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 8B

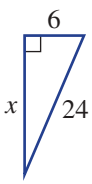
UNDERSTANDING 1–3 3

1 What is the length of the hypotenuse in each of these triangles?

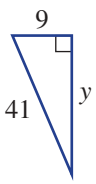
a



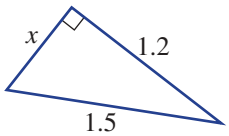
b



c



d

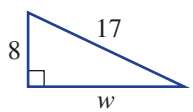


2 Copy and complete the following.

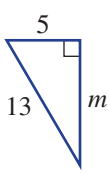
$$10^2 = w^2 + 6^2$$
$$w^2 + 6^2 = 10^2$$
$$w^2 + \square = \square \quad (-\square \text{ on both sides})$$
$$w^2 = \square \quad (-\square \text{ on both sides})$$
$$w = \square$$

3 Substitute the numbers and pronumerals into Pythagoras' theorem $c^2 = a^2 + b^2$ for each of these triangles. Do not solve for the unknown.

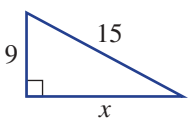
a



b



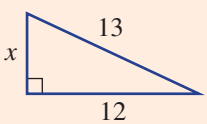
c



FLUENCY 4–5(½) 4–6(½)

Example 3 Calculating a shorter side

Determine the value of x in the triangle shown, using Pythagoras' theorem.



SOLUTION

$13^2 = x^2 + 12^2$

$x^2 + 12^2 = 13^2$

$x^2 = 13^2 - 12^2$

$= 169 - 144$

$= 25$

$x = \sqrt{25}$

$\therefore x = 5$

EXPLANATION

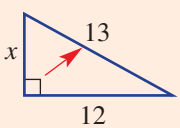
Write the relationship.

Swap LHS and RHS.

Rewrite the rule with the x^2 on the left-hand side.

Simplify.

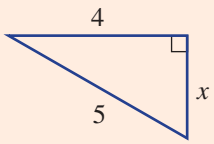
Find the square root to find x.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

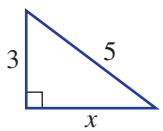
Now you try

Determine the value of x in the triangle shown, using Pythagoras' theorem.

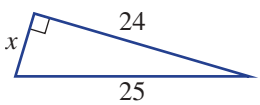


4 Determine the value of x in these triangles, using Pythagoras' theorem.

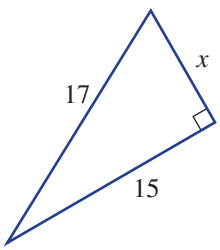
a



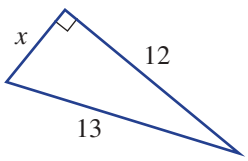
b



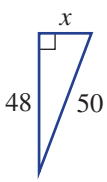
c



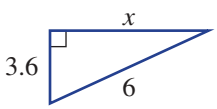
d



e



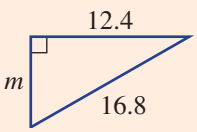
f



Hint: In $c^2 = a^2 + b^2$, c is always the hypotenuse.

Example 4 Finding a shorter side length as a decimal value

Determine the value of m in the triangle, correct to one decimal place.



SOLUTION

$16.8^2 = m^2 + 12.4^2$

$m^2 + 12.4^2 = 16.8^2$

$m^2 = 16.8^2 - 12.4^2$

$= 128.48$

$m = \sqrt{128.48}$

$= 11.3349\ldots$

$m = 11.3$ (to 1 d.p.)

EXPLANATION

Write the relationship for this triangle.

Swap LHS and RHS.

Make m^2 the subject.

Simplify, using your calculator.

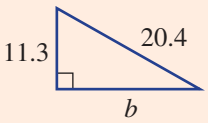
Take the square root of both sides to find m.

Round your answer to one decimal place.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

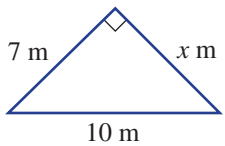
Determine the value of b in the triangle, correct to one decimal place.



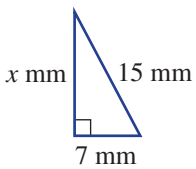
5 Determine the value of x in these triangles, using Pythagoras' theorem. Answer correct to one decimal place.

Hint: To round to one decimal place, look at the second decimal place. If it is 5 or more, round up. If it is 4 or less, round down. For example, 7.1 ④ 14 … rounds to 7.1.

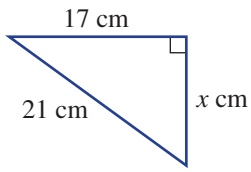
a



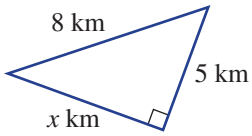
b



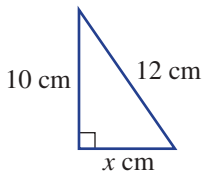
c



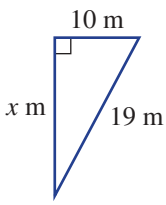
d



e

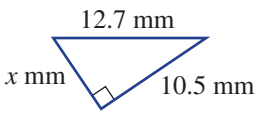


f

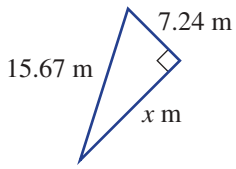


6 Determine the value of x in these triangles, using Pythagoras' theorem. Answer correct to two decimal places.

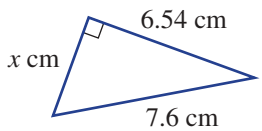
a



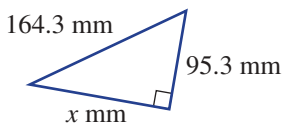
b



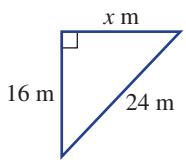
c



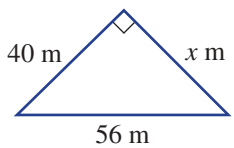
d



e

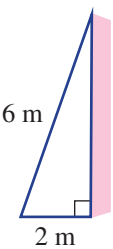


f



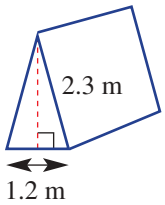
PROBLEM-SOLVING AND REASONING 7–9 9–11

7 A 6 m ladder leans against a wall. If the base of the ladder is 2 m from the wall, determine how high the ladder is up the wall, correct to two decimal places.

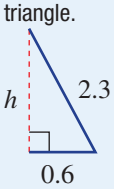


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

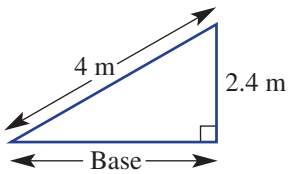
8 A tent has sloping sides of length 2.3 m and a base of 1.2 m. Determine the height of the tent pole, correct to one decimal place.



Hint: Identify the right-angled triangle.



9 A city council wants to build a skateboard ramp measuring 4 m long and 2.4 m high. How long should the base of the ramp be, correct to one decimal place?



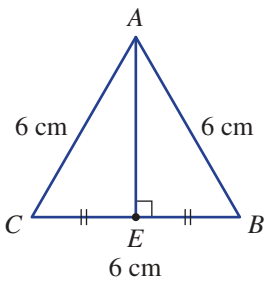
10 Triangle ABC is equilateral. AE is an axis of symmetry.

a Find the length of:

i EB

ii AE, to one decimal place

b Find the area of triangle ABC, to one decimal place.

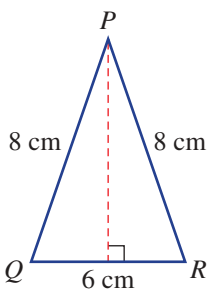


Hint: An equilateral triangle has 3 equal sides.

Hint: Remember: $A = \frac{1}{2}bh$ is the area of a triangle.

Hint: Pythagoras' theorem applies only to right-angled triangles.

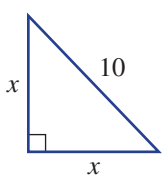
11 What is the height of this isosceles triangle, to one decimal place?



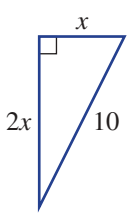
ENRICHMENT: Two unknown sides — 12

12 Find the value of x in each of the following. Answer to one decimal place.

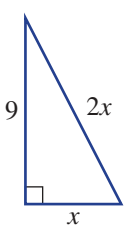
a



b



c



Hint: Remember to square the entire side. The square of $2x$ is $(2x)^2$ or $4x^2$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8C Applications of Pythagoras' theorem

Learning intentions for this section:

- To be able to identify right-angled triangles in a two-dimensional situation
- To be able to use Pythagoras' theorem to solve a problem

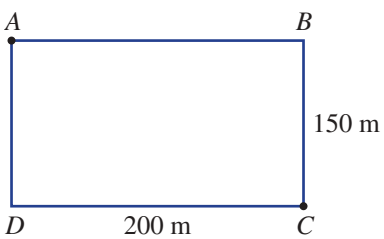
Past, present and future learning:

- This section addresses Stage 4 concepts and the Stage 5 Path topic called *Trigonometry C*
- It consolidates and extends the concepts in Section 4D of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the formula from memory in the HSC Examination

Pythagoras' theorem has many applications, some of which you may have noticed already in this chapter. Some areas where Pythagoras' theorem is useful include drafting, building and navigation.

Lesson starter: Finding the shortest path

A rectangular field is 200 m by 150 m. Marco wants to walk from the corner of the field marked A to the corner of the field marked C. How many metres are saved by walking along the diagonal AC rather than walking along AB and then BC?



KEY IDEAS

- When applying Pythagoras' theorem follow these steps.
  - Identify and draw the right-angled triangle or triangles you need in order to solve the problem.
  - Label the triangle and place a pronumeral (letter) on the side length that is unknown.
  - Use Pythagoras' theorem to find the value of the pronumeral.
  - Answer the question. (Written questions should have written answers.)

Key terminology: identify, pronumeral

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 8C

UNDERSTANDING | 1–3 | 3

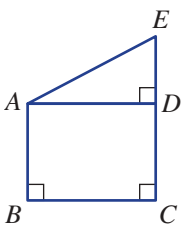
1 Draw a diagram for each of the following questions. You don't need to answer the question.

Hint: Each one involves a right-angled triangle.

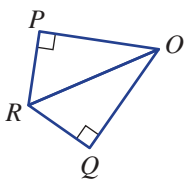
- **a** A 2.4 m ladder is placed 1 m from the foot of a building. How far up the building will the ladder reach?
- **b** The diagonal of a rectangle with length 18 cm is 24 cm. How wide is the rectangle?
- **c** Sebastian walks 5 km north, then 3 km west. How far is he from his starting point?

2 Name the right-angled triangles in each of the following diagrams; e.g. ΔABC.

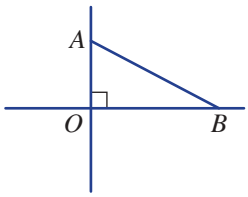
a



b

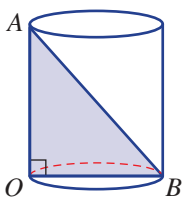


c

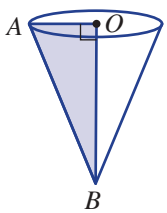


3 Name the hypotenuse in each of the shaded right-angled triangles found within these three-dimensional shapes; e.g. FG.

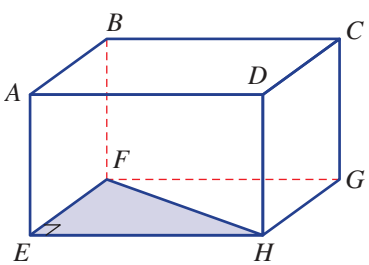
a



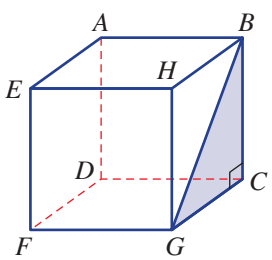
b



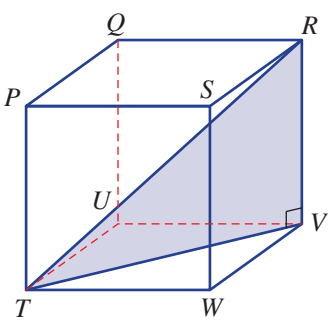
c



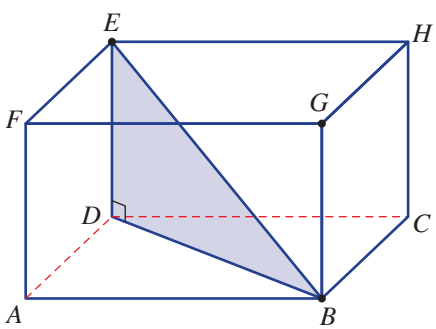
d



e



f

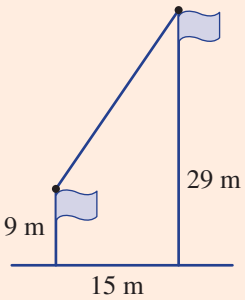


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 4–7 4, 5, 7, 8

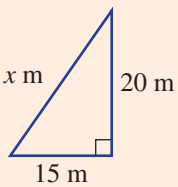
Example 5 Applying Pythagoras' theorem

Two flagpoles are 15 m apart and a rope links the tops of the poles. Find the length of the rope if one flagpole is 9 m tall and the other is 29 m tall.



SOLUTION

Let x metres be the length of rope.

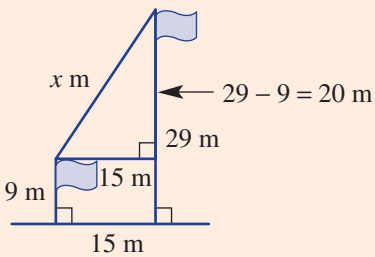


$x^2 = 15^2 + 20^2$
$= 225 + 400$
$= 625$
$x = \sqrt{625}$
$= 25$

The rope is 25 m long.

EXPLANATION

Locate and draw the right-angled triangle, showing all measurements. Introduce a pronumeral for the missing side.



Write the relationship, using Pythagoras' theorem.

Simplify.

Take the square root to find x.

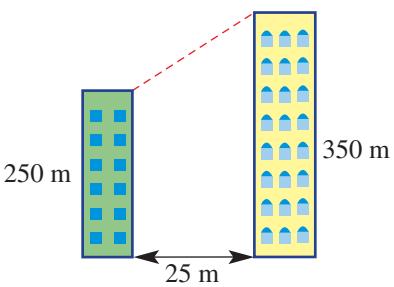
Answer the question.

Now you try

Two vertical poles are 2 m and 5 m high and are 4 m apart. Find the distance between the tops of the two poles.

4 Two skyscrapers are 25 m apart and a cable runs from the top of one building to the top of the other. One building is 350 m tall and the other is 250 m.

- **a** Determine the difference in the heights of the buildings.
- **b** Draw an appropriate right-angled triangle you could use to find the length of the cable.
- **c** Find the length of the cable, correct to two decimal places.

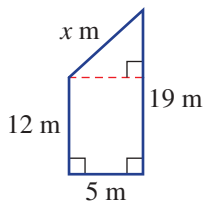


(Not to scale)

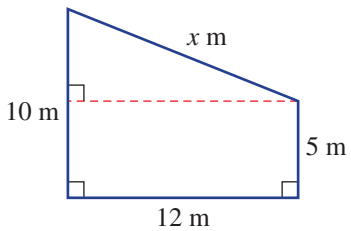
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Find the value of x in each of the following, correct to one decimal place where necessary.

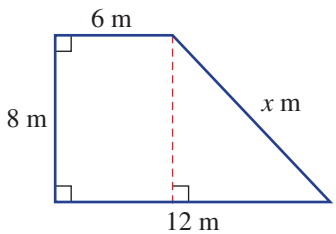
a



b



c

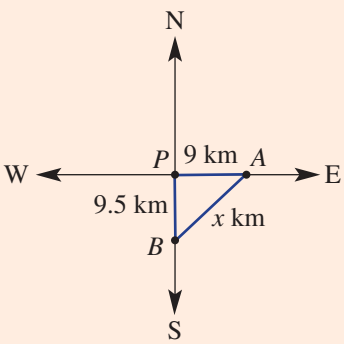


Hint: Label the two known lengths of each triangle first.

Example 6 Using direction with Pythagoras' theorem

Two hikers leave their camp (P) at the same time. One walks due east for 9 km; the other walks due south for 9.5 km. How far apart are the two hikers at this point? (Answer to one decimal place.)

SOLUTION



$\therefore x^2 = 9^2 + 9.5^2$

$x^2 = 171.25$

$x = \sqrt{171.25}$

$= 13.086$

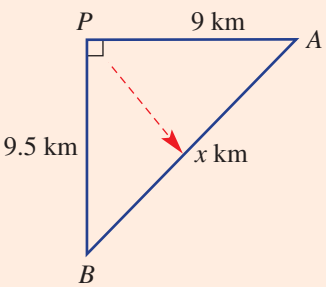
$= 13.1$ (to 1 d.p.)

$\therefore$ The hikers are 13.1 km apart.

EXPLANATION

Draw a diagram.

Consider ΔPAB.



Write Pythagoras' theorem and evaluate.

Square root to find x.

Round to one decimal place.

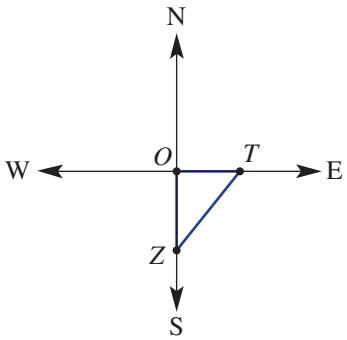
Answer the question in words.

Now you try

From a floating oil rig, one ship travels due north for 20 km and another travels due west for 30 km. How far apart are the two ships at this point? (Answer to one decimal place.)

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Tranh (T) walks 4.5 km east while Zara (Z) walks 5.2 km south. How far from Tranh is Zara? Answer to one decimal place.

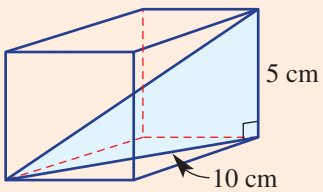


7 Find the distance between Sui and Kevin if:

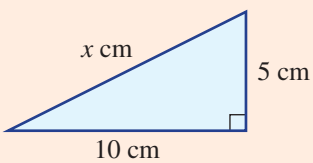
- **a** Sui walks 6 km north from camp O and Kevin walks 8 km west from camp O.
- **b** Sui walks 40 km east from point A and Kevin walks 9 km south from point A.
- **c** Kevin walks 15 km north-west from O and Sui walks 8 km south-west also from O.

Example 7 Using Pythagoras' theorem in 3D

Find the distance from one corner of this rectangular prism to the opposite corner, correct to two decimal places.



SOLUTION



$x^2 = 5^2 + 10^2$

$= 25 + 100$

$x = \sqrt{125}$

$= 11.18$ cm (to 2 d.p.)

$\therefore$ The distance between the opposite corners is 11.18 cm.

EXPLANATION

Draw the triangle you need and mark the lengths.

Write the relationship for this triangle.

Simplify.

Take the square root to find x.

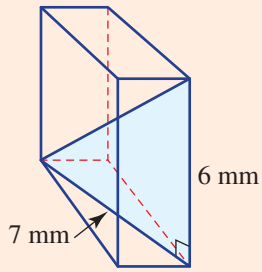
Round your answer to two decimal places.

Write the answer.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

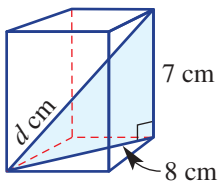
Now you try

Find the distance from one corner of this rectangular prism to the opposite corner, correct to two decimal places.

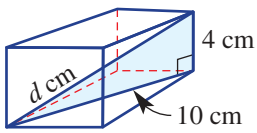


8 Find the distance of d from one corner to the opposite corner in the following rectangular prisms, correct to one decimal place.

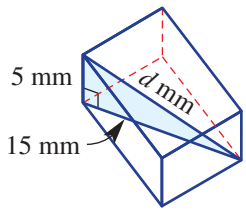
a



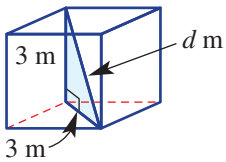
b



c



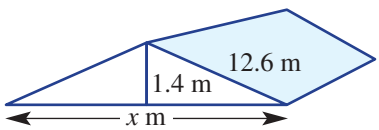
d



PROBLEM-SOLVING AND REASONING

9–11 | 11–14

9 The height of a roof is 1.4 m. If the length of a gable (the diagonal) is to be 12.6 m, determine the length of the horizontal beam needed to support the roof, correct to two decimal places.

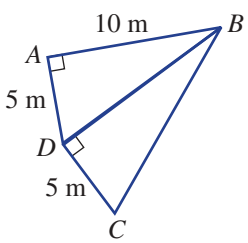


Hint: Find the base length of the right-angled triangle first.

10 For the diagram shown, find the lengths of:

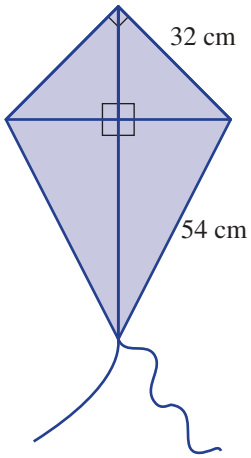
a BD, correct to two decimal places

b BC, correct to one decimal place



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

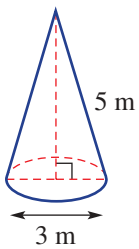
11 A kite is constructed with six pieces of wooden dowel and covered in fabric. The four pieces around the edge have two 32 cm rods and two 54 cm rods. If the top of the kite is right angled, find the length of the horizontal and vertical rods, correct to two decimal places.



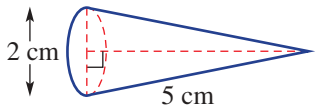
Hint: Find the length of the horizontal rod first. What type of triangle is the top of the kite? Find the length of the vertical rod using two calculations.

12 Find the height of the following cones, correct to two decimal places.

a

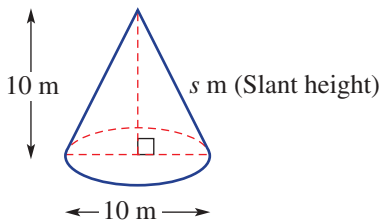


b

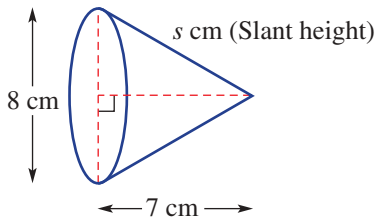


13 Find the slant height of the following, correct to one decimal place.

a

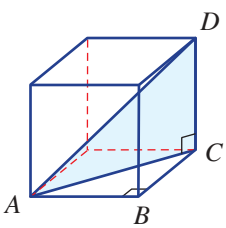


b



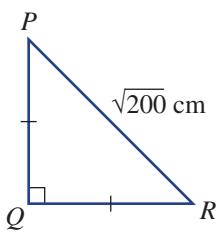
14 This cube has 1 cm sides. Find, correct to two decimal places, the lengths of:

a AC **b** AD



ENRICHMENT: How much do you know? — 15

15 Write down everything you know about $\triangle PQR$, including the things that you can calculate.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8D Trigonometric ratios

Learning intentions for this section:

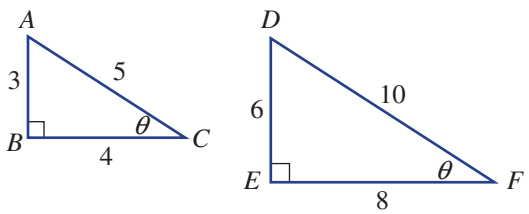
- To know the three trigonometric ratios for sine, cosine and tangent
- To be able to use two sides, an angle and a trigonometric ratio to write an equation

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Trigonometry A*
- It consolidates and extends the concepts in Section 4E of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the three ratios from memory in the HSC Examination

The study of trigonometry explores the relationship between the angles and side lengths of triangles. Trigonometry can be applied to simple problems, such as finding the angle of elevation of a kite, to solving complex problems in surveying and design.

Trigonometry is built upon the three ratios sine, cosine and tangent. These ratios do not change for triangles that are similar, such as ΔABC and ΔDEF.



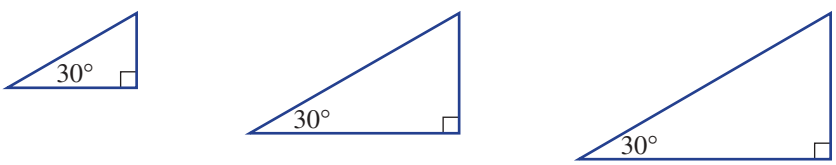
$\sin\theta = \frac{3}{5}$ $\quad$ $\sin\theta = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$ $\quad$ $\cos\theta = \frac{8}{10} = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$ $\quad$ $\tan\theta = \frac{6}{8} = \frac{3}{4}$

Lesson starter: Thirty degrees

- Draw three different right-angled triangles that each have a 30° angle.

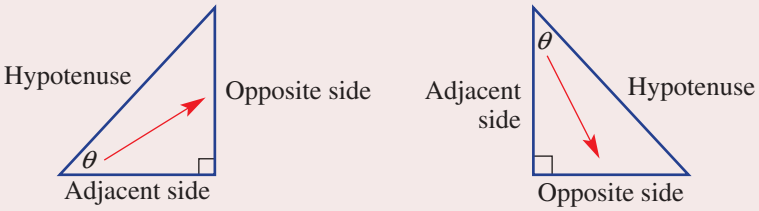


- Measure each side of each triangle and add these measurements to your diagrams.
- The hypotenuse, as we know, is opposite the right angle. The side opposite the 30° is called the opposite side. For each of your three triangles, write down the ratio of the opposite side divided by the hypotenuse. What do you notice?
- Type 'sin 30°' into your calculator. What do you notice?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

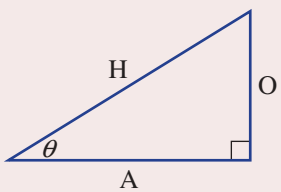
KEY IDEAS

- Any right-angled triangle has three sides: the hypotenuse, adjacent and opposite.
  - The **angle of reference** is the angle in a right-angled triangle that is used to determine the opposite side and the adjacent side.
  - The **hypotenuse** is always opposite the right angle.
  - The **adjacent** side is next to the angle of reference.
  - The **opposite** side is opposite the angle of reference.



- For a right-angled triangle with a given angle θ (theta), the three trigonometric ratios of **sine (sin)**, **cosine (cos)** and **tangent (tan)** are given by:
  - sine of angle θ: $\sin\theta = \dfrac{\text{length of opposite side}}{\text{length of the hypotenuse}}$
  - cosine of angle θ: $\cos\theta = \dfrac{\text{length of adjacent side}}{\text{length of the hypotenuse}}$
  - tangent of angle θ: $\tan\theta = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$

- When working with right-angled triangles, label each side of the triangle O (opposite), A (adjacent) or H (hypotenuse).



- The three trigonometric ratios are:
  $\sin\theta = \dfrac{O}{H}$ $\cos\theta = \dfrac{A}{H}$ $\tan\theta = \dfrac{O}{A}$
  We can remember this as **SOH CAH TOA**.

- Angles are measured in degrees (°), minutes (′) and seconds (″).
  - 1 degree = 60 minutes
  - 1 minute = 60 seconds
  - For example: 24°21′54″ is 24 degrees, 21 minutes and 54 seconds.
  - On most calculators you can enter angle values in this way.

Key terminology: trigonometry, sine, cosine, tangent, hypotenuse, opposite, adjacent, angle of reference

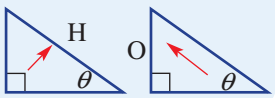
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 8D

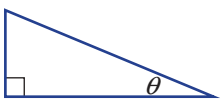
UNDERSTANDING 1–4 4

1 By referring to the angles marked, copy each triangle and label the sides opposite, adjacent and hypotenuse, using O, A and H.

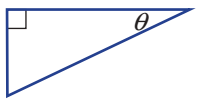
Hint: Arrows help you find the hypotenuse and the opposite side:



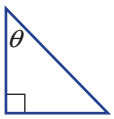
a



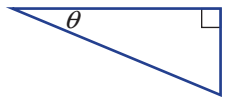
b



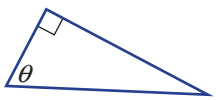
c



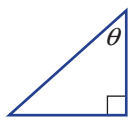
d



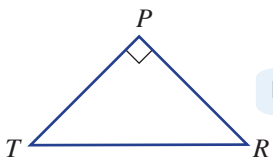
e



f



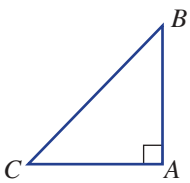
2 Referring to triangle *PTR*, name the:



Hint: 'Adjacent' means 'next to'.

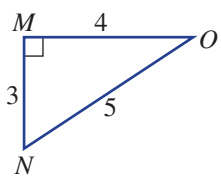
- **a** side opposite the angle at *T*
- **b** side adjacent to the angle at *T*
- **c** side opposite the angle at *R*
- **d** side adjacent to the angle at *R*
- **e** hypotenuse
- **f** angle opposite the side *PR*

3 Referring to triangle *ABC*, name the:



- **a** hypotenuse
- **b** side opposite the angle at *B*
- **c** side opposite the angle at *C*
- **d** side adjacent to the angle at *B*

4 In triangle *MNO*, write the ratio (i.e. fraction) of:



- **a** $\dfrac{\text{the side oposite angle } O}{\text{hypotenuse}}$
- **b** $\dfrac{\text{the side opposite angle } N}{\text{hypotenuse}}$
- **c** $\dfrac{\text{the side adjacent angle } O}{\text{hypotenuse}}$

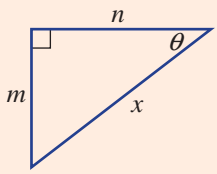
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 5–6(½) 5–6(½)

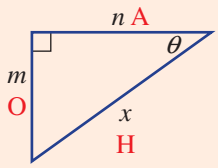
Example 8 Writing trigonometric ratios

Label the sides of the triangle O, A and H and write the ratios for:

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$



SOLUTION



a $\sin\theta = \frac{m}{x}$

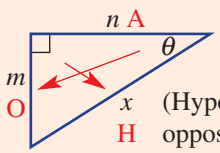
b $\cos\theta = \frac{n}{x}$

c $\tan\theta = \frac{m}{n}$

EXPLANATION

Use arrows to label the sides correctly.

(opposite angle θ)



(Hypotenuse is always opposite the right angle.)

SOH CAH TOA

$\sin\theta = \frac{\text{O}}{\text{H}} = \frac{m}{x}$

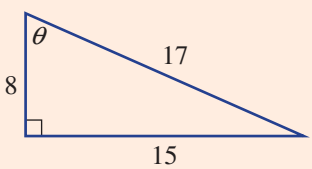
$\cos\theta = \frac{\text{A}}{\text{H}} = \frac{n}{x}$

$\tan\theta = \frac{\text{O}}{\text{A}} = \frac{m}{n}$

Now you try

Label the sides of the triangle O, A and H and write the ratios for:

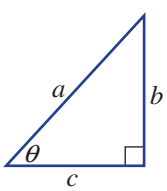
a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$



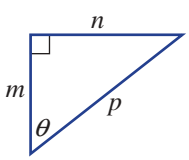
5 For each of the following triangles, write a ratio for:

i $\sin\theta$ **ii** $\cos\theta$ **iii** $\tan\theta$

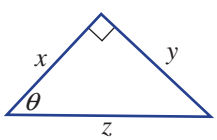
a



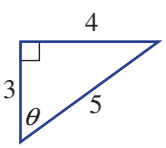
b



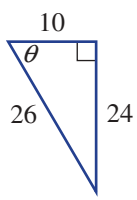
c



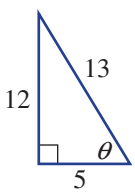
d



e



f

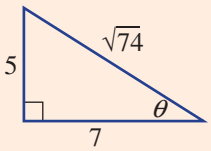


Hint: Use SOH CAH TOA after labelling the sides as O, A and H.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 9 Writing a trigonometric ratio

Write down the ratio of $\cos\theta$ for this triangle.



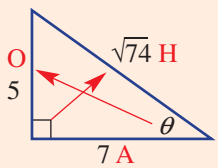
SOLUTION

$\cos\theta = \frac{\text{A}}{\text{H}}$

$\cos\theta = \frac{7}{\sqrt{74}}$

EXPLANATION

Label the sides of the triangle.

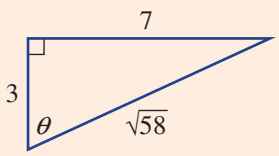


SOH CAH TOA tells us $\cos\theta$ is $\frac{\text{adjacent}}{\text{hypotenuse}}$.

Substitute the values for the adjacent (A) and hypotenuse (H).

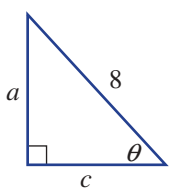
Now you try

Write down the ratio of $\sin\theta$ for this triangle.



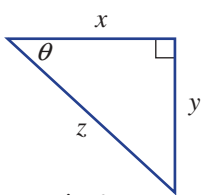
6 Write the trigonometric ratio asked for in each of the following.

a



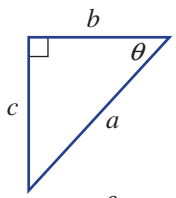
$\tan\theta =$

b



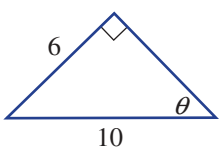
$\sin\theta =$

c



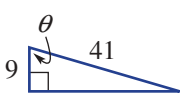
$\cos\theta =$

d



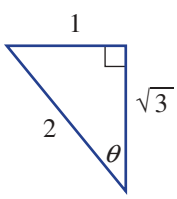
$\sin\theta =$

e



$\sin\theta =$

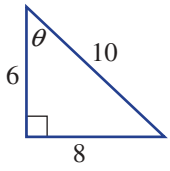
f



$\tan\theta =$

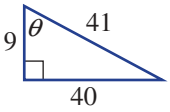
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

g



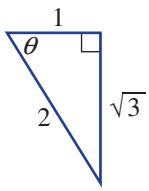
$\tan\theta =$

h



$\cos\theta =$

i



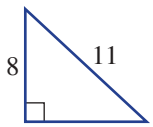
$\tan\theta =$

PROBLEM-SOLVING AND REASONING

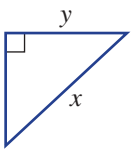
7–9 | 8–11

7 Copy each of these triangles and mark the angle θ that will enable you to write a ratio for $\sin\theta$.

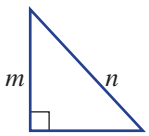
a



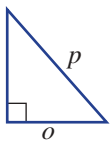
b



c



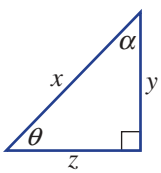
d



8 For the triangle shown, write a ratio for:

a $\sin\theta$ **b** $\sin\alpha$ **c** $\cos\theta$

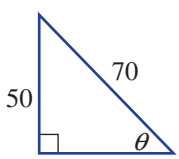
d $\cos\alpha$ **e** $\tan\theta$ **f** $\tan\alpha$



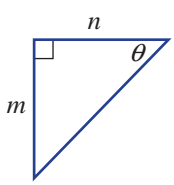
Hint: θ and α are letters of the Greek alphabet that are used to mark angles.

9 For each of the triangles below, decide which trigonometric ratio (sin, cos or tan) you would use for the given information.

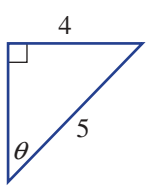
a



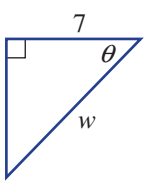
b



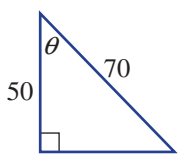
c



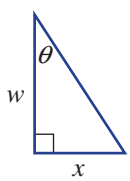
d



e



f

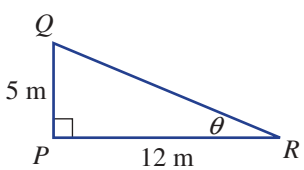


Hint: First decide which two sides are involved.

10 Consider the triangle PQR.

a Use Pythagoras' theorem to find the length of QR.

b Write down the ratio of $\sin\theta$.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 For a given right-angled triangle, $\sin\theta = \frac{1}{2}$.

- **a** Draw a right-angled triangle and show this information.
- **b** What is the length of the third side? Use Pythagoras' theorem.
- **c** Find the value of:
  - **i** $\cos\theta$
  - **ii** $\tan\theta$

ENRICHMENT: Relationship between sine and cosine — 12

12 Use your calculator to complete the table, answering to three decimal places where necessary.

Angle (θ)	sin θ	cos θ
0°		
5°		
10°		
15°		
20°		
25°		
30°		
35°		
40°		
45°		
50°		
55°		
60°		
65°		
70°		
75°		
80°		
85°		
90°		

- **a** For what angle is $\sin\theta = \cos\theta$?
- **b** Copy and complete:
  - **i** $\sin 5° = \cos$ ____°
  - **ii** $\sin 10° = \cos$ ____°
  - **iii** $\sin 60° = \cos$ ____°
  - **iv** $\sin 90° = \cos$ ____°

Hint: For most calculators, you enter the values in the same order as they are written. That is, $\sin 30° \rightarrow$ [sin] $30 = 0.5$.

- **c** Write down a relationship, in words, between sin and cos.
- **d** Why do you think it's called cosine?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8E Finding unknown side lengths

Learning intentions for this section:

- To be able to identify which trigonometric ratio can be used to set up an equation
- To be able to find a missing length on a right-angled triangle given an angle and another side

Past, present and future learning:

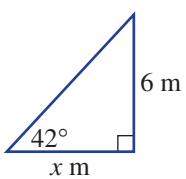
- This section addresses the Stage 5 Core topic called *Trigonometry A*
- It consolidates and extends the concepts in Section 4F of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

In any right-angled triangle, given one of the acute angles and a side length, you can find the length of the other two sides. This can help builders find special lengths in right-angled triangles if they know an angle and the length of another side.

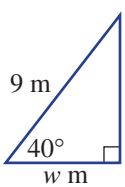
Lesson starter: Is it sin, cos or tan?

Of the six triangles below, only two provide enough information to directly use the sine ratio. Which two triangles are they?

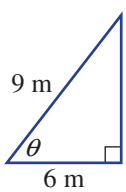
A



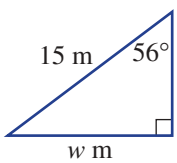
B



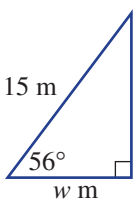
C



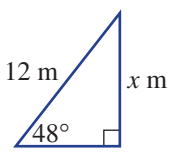
D



E



F



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- To find a missing side when given a right-angled triangle with one acute angle and one of the sides:
  - Label the triangle using O (opposite), A (adjacent) and H (hypotenuse).
  - Use SOH CAH TOA to decide on the correct trigonometric ratio.
  - Write down the relationship.
  - Solve the equation, using your calculator, to find the unknown.

  For example:

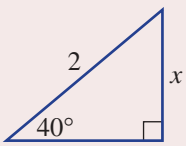
$$\sin 40° = \frac{x}{2}$$

$$\frac{x}{2} = \sin 40°$$

$$x = 2 \times \sin 40°$$

$$x = 1.2855\ldots.$$

$$x = 1.3 \text{ (to 1 d.p.)}$$



- Always check that your answer is reasonable. The hypotenuse (the largest side) is 2, so x must be less than 2.

- Degrees and minutes
  - Sometimes angles cannot be expressed with whole numbers only.

    For example: half of 45° is 22.5° or $22\frac{1}{2}°$.

    This is sometimes written as 22 degrees 30 minutes or as 22°30′.
  - There are 60 minutes in 1 degree.

    These conversions can be performed on your calculator:

$2.5° = 2°30′$

$3.25° = 3°15′$

$22.75° = 22°45′$

$15.4° = 15°24′$

Key terminology: sine, cosine, tangent, hypotenuse, opposite, adjacent

Exercise 8E

UNDERSTANDING | 1, 2(½), 3 | 2(½), 3

1 Use a calculator to find the value of each of the following, correct to four decimal places.

a sin 10°	**b** cos 10°	**c** tan 10°
d tan 30°	**e** cos 40°	**f** sin 70°
g cos 80.1°	**h** tan 40.5°	**i** sin 80.75°
j sin 60°15′	**k** cos 50°12′	**l** tan 60°5′

Hint: Locate the sin, cos and tan buttons on your calculator.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

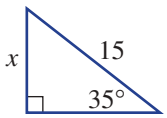
2 Evaluate each of the following, correct to two decimal places.

a $12 \tan 10°$ **b** $12 \sin 25°$
c $18 \tan 60°$ **d** $56 \sin 56°$
e $8 \tan 45°$ **f** $20 \sin 70°$

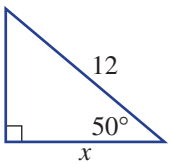
Hint: On your calculator, enter $12 \tan 10°$ as $12 \times \tan 10$.

3 Decide which of the three trigonometric ratios (sin, cos or tan) is suitable for these triangles. Do not solve.

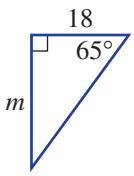
a



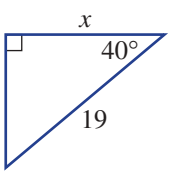
b



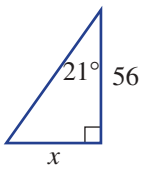
c



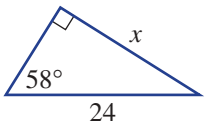
d



e



f



Hint: Remember to label the triangle and think SOH CAH TOA. Consider which two sides are involved.

FLUENCY

4–8(½) 4–9(½)

Example 10 Solving a trigonometric equation

Find the value of x, correct to two decimal places, for $\cos 30° = \frac{x}{12}$.

SOLUTION

$\cos 30° = \frac{x}{12}$

$x = 12 \times \cos 30°$

$= 10.39230 \ldots$

$= 10.39$ (to 2 d.p.)

EXPLANATION

Multiply both sides by 12 to get x on its own.

$12 \times \cos 30° = \frac{x}{\cancel{12}_1} \times \cancel{12}_1$

Use your calculator.

Round as indicated.

Now you try

Find the value of x, correct to two decimal places, for $\sin 40° = \frac{x}{6}$.

4 Find the value of x in these equations, correct to two decimal places.

a $\sin 20° = \frac{x}{4}$ **b** $\cos 43° = \frac{x}{7}$ **c** $\tan 85° = \frac{x}{8}$

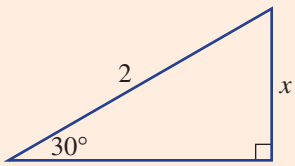
d $\tan 30° = \frac{x}{24}$ **e** $\sin 50° = \frac{x}{12}$ **f** $\cos 40° = \frac{x}{12}$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

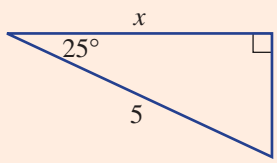
Example 11 Finding a missing side, using SOH CAH TOA

Find the value of the unknown length (x) in these triangles. Round to two decimal places where necessary.

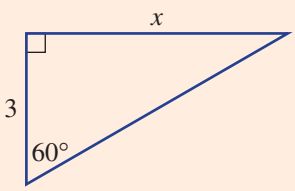
a



b



c



SOLUTION

a

$$\sin\theta = \frac{\text{O}}{\text{H}}$$

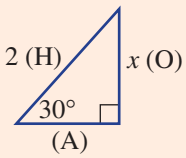
$$\sin 30° = \frac{x}{2}$$

$$2 \times \sin 30° = x$$

$$\therefore x = 1$$

EXPLANATION

Label the triangle and decide on your trigonometric ratio SOH CAH TOA.



Write the ratio.

Substitute values and solve the equation, using your calculator.

b

$$\cos\theta = \frac{\text{A}}{\text{H}}$$

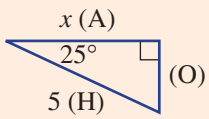
$$\cos 25° = \frac{x}{5}$$

$$5 \times \cos 25° = x$$

$$x = 4.5315\ldots$$

$$\therefore x = 4.53 \text{ (to 2 d.p.)}$$

Label the triangle.
SOH CAH TOA



Write the ratio.

Substitute values and solve the equation, using your calculator.

Round to two decimal places.

c

$$\tan\theta = \frac{\text{O}}{\text{A}}$$

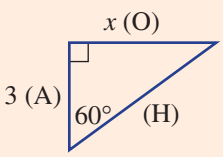
$$\tan 60° = \frac{x}{3}$$

$$3 \times \tan 60° = x$$

$$x = 5.1961\ldots$$

$$\therefore x = 5.20 \text{ (to 2 d.p.)}$$

Label the triangle.
SOH CAH TOA



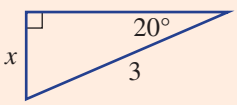
Write the ratio.

Substitute values and solve the equation, using your calculator.
Round to two decimal places.

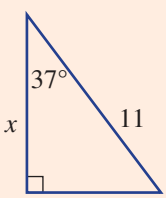
Now you try

Find the value of the unknown length (x) in these triangles. Round to two decimal places where necessary.

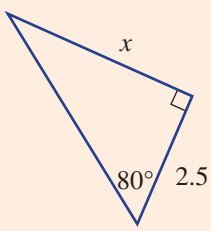
a



b



c



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Triangles with one unknown side are given below.

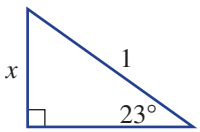
i Copy each one and label the three sides opposite (O), adjacent (A) and hypotenuse (H).

ii Decide on a trigonometric ratio.

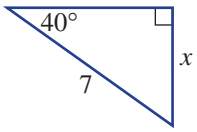
iii Find the value of each pronumeral, correct to two decimal places.

Hint: Use SOH CAH TOA to help you decide which ratio to use. If O and H are involved, use sin etc.

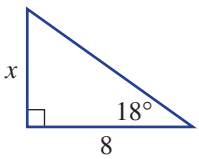
a



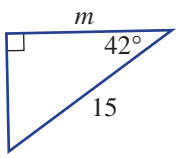
b



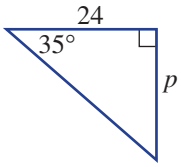
c



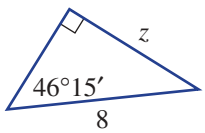
d



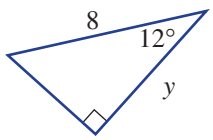
e



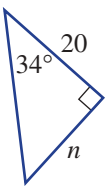
f



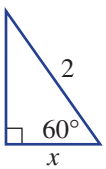
g



h



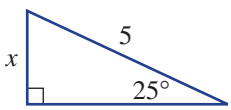
i



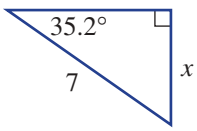
6 Find the value of the unknown length (x) in these triangles. Round to two decimal places.

Hint: What ratio did you use for each of these?

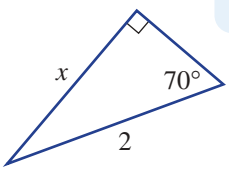
a



b

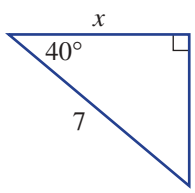


c

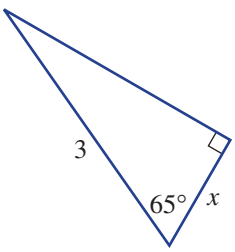


7 Find the value of the unknown length (x) in these triangles. Round to two decimal places.

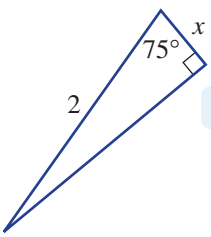
a



b



c

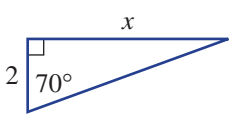


Hint: These three all use cos.

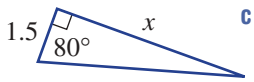
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 Find the value of the unknown length (x) in these triangles. Round to two decimal places.

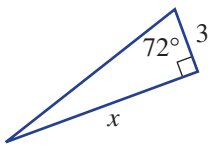
a



b



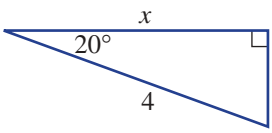
c



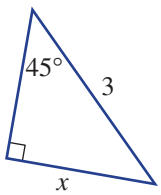
Hint: These all use tan.

9 Decide whether to use sin, cos or tan, then find the value of x in these triangles. Round to two decimal places.

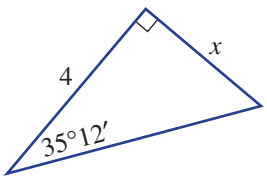
a



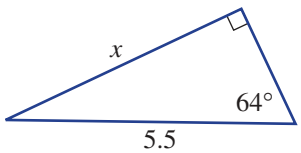
b



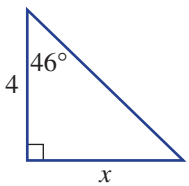
c



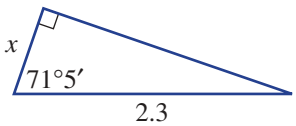
d



e



f

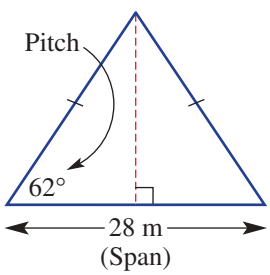


PROBLEM-SOLVING AND REASONING

10, 11 | 10, 12

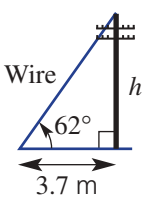
10 **a** Find the height of this isosceles triangle, which is similar to a roof truss, to two decimal places.

b If the span doubles to 56 m, what is the height of the roof, to two decimal places?



Hint: In an isosceles triangle, the perpendicular cuts the base in half.

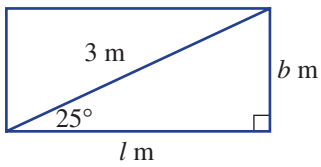
11 The stay wire of a power pole joins the top to the ground. It makes an angle of 62° with the ground. It is fixed to the ground 3.7 m from the bottom of the pole. How high is the pole, correct to two decimal places?



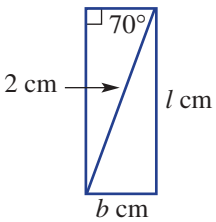
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

12 Find the length and breadth of these rectangles, correct to two decimal places.

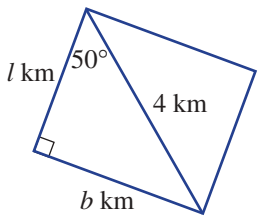
a



b



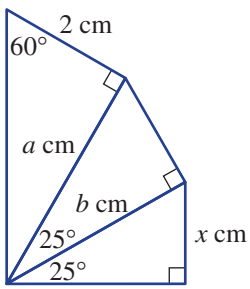
c



Hint: Use the hypotenuse in each calculation.

ENRICHMENT: Accuracy and errors — 13

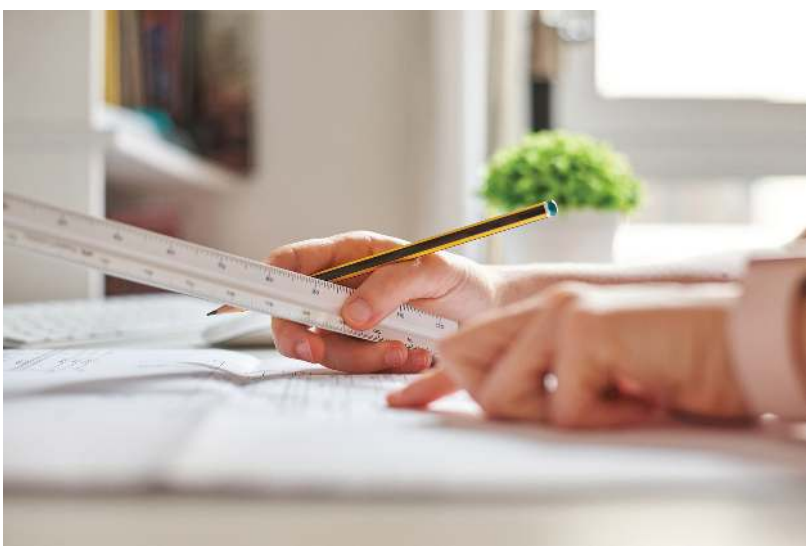
13 Our aim is to find the value of x, correct to two decimal places, by first finding the value of a and b.



a Find the value of a, then b and then x, using one decimal place for a and b.

b Repeat this process, finding a and b, correct to three decimal places each, before finding x.

c Does it make any difference to your final answer for x if you round off the values of a and b during calculations? Explain.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8F Solving for the denominator

Learning intentions for this section:

- To be able to use a trigonometric ratio, then find an unknown in the denominator

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Trigonometry A*
- It consolidates and extends the concepts in Section 4G of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the three ratios from memory in the HSC Examination

So far, we have been dealing with equations that have the pronumeral in the numerator. However, sometimes the unknown is in the denominator. Such problems can be solved with an extra step in your mathematical working.

Lesson starter: Solving equations with x in the denominator

Consider the equations $\frac{x}{3} = 4$ and $\frac{3}{x} = 4$.

- Do the equations have the same solution?
- What steps are used to solve the equations?
- Now solve $\frac{4}{x} = \sin 30°$ and $\frac{2}{x} = \cos 40°$.

KEY IDEAS

- When the unknown value is in the **denominator**, you need to do two algebraic steps to find the unknown. For example,

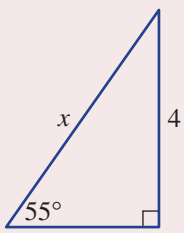
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 55° = \frac{4}{x}$$

$$x \times \sin 55° = 4$$

$$x = \frac{4}{\sin 55°}$$

$$= 4.88 \text{ (to two decimal places)}$$



Key terminology: denominator, hypotenuse, opposite, adjacent

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 8F

UNDERSTANDING | 1–3 | 2, 3

1 Find the value, correct to two decimal places, of:

a $\dfrac{10}{\tan 30°}$ **b** $\dfrac{12}{\sin 60°}$

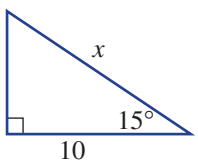
c $\dfrac{15}{\tan 8°}$ **d** $\dfrac{12.4}{\tan 32°}$

e $\dfrac{15.2}{\sin 38°}$ **f** $\dfrac{9}{\cos 47°58'}$

Hint: For part **a**, enter $10 \div \tan 30$ into your calculator.

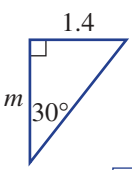
2 For each of these triangles, complete the required trigonometric ratio.

a



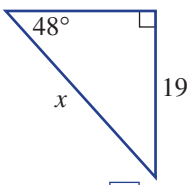
$\cos 15° = \dfrac{\square}{\square}$

b



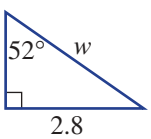
$\tan 30° = \dfrac{\square}{\square}$

c



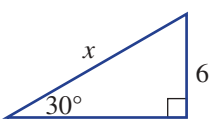
$\sin 48° = \dfrac{\square}{\square}$

d



$\sin 52° = \dfrac{\square}{\square}$

3 Which one of the following is the correct working to solve for x in this triangle?



A
$$\cos 30° = \frac{6}{x}$$
$$x \cos 30° = 6$$
$$x = \frac{6}{\cos 30°}$$
$$= 6.93$$

B
$$\sin 30° = \frac{6}{x}$$
$$x = 6 \times \sin 30°$$
$$= 3$$

C
$$\sin 30° = \frac{6}{x}$$
$$x \sin 30° = 6$$
$$x = \frac{6}{\sin 30°}$$
$$= 12$$

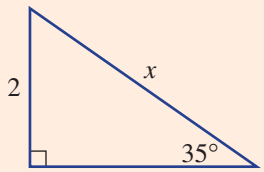
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 4–6 4–6, 7(½)

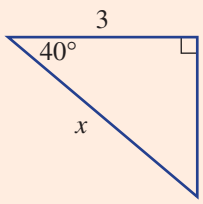
Example 12 Finding the value of the denominator

Find the value of the unknown length (x) in these right-angled triangles. Round your answer to two decimal places.

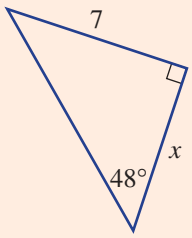
a



b



c



SOLUTION **EXPLANATION**

a $\sin 35° = \frac{2}{x}$

Use $\sin\theta = \frac{O}{H}$, as we can use the opposite (2) and hypotenuse (x).

$x \times \sin 35° = 2$

Multiply both sides by x.

$x = \frac{2}{\sin 35°}$

Divide both sides by $\sin 35°$ to get x on its own.

Recall that $\sin 35°$ is just a number.

$\therefore x = 3.49$ (to 2 d.p.)

Evaluate and round your answer.

b $\cos 40° = \frac{3}{x}$

Use $\cos\theta = \frac{A}{H}$, as we can use the adjacent (3) and hypotenuse (x).

$x \times \cos 40° = 3$

Multiply both sides by x.

$x = \frac{3}{\cos 40°}$

Divide both sides by $\cos\ 40°$ to get x on its own.

$\therefore x = 3.92$ (to 2 d.p.)

Evaluate and round your answer.

c $\tan 48° = \frac{7}{x}$

Use $\tan\theta = \frac{O}{A}$, as we can use the adjacent (x) and opposite (7).

$x \times \tan 48° = 7$

Multiply both sides by x.

$x = \frac{7}{\tan 48°}$

Divide both sides by $\tan 48°$ to get x on its own.

$\therefore x = 6.30$ (to 2 d.p.)

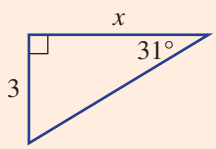
Evaluate and round your answer.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

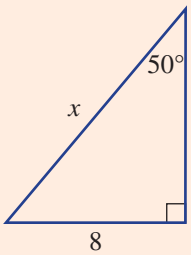
Now you try

Find the value of the unknown length (x) in these right-angled triangles. Round your answer to two decimal places.

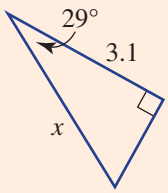
a



b



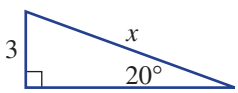
c



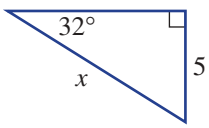
4 Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.

Hint: In $\sin 20° = \frac{3}{x}$, multiply both sides by x first.

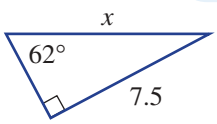
a



b



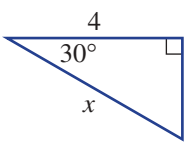
c



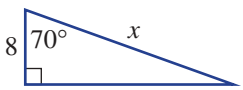
5 Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.

Hint: $\cos\theta = \frac{A}{H}$

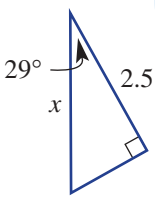
a



b



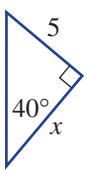
c



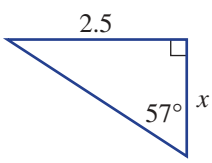
6 Find the value of the unknown length (x) in these right-angled triangles. Round your answer to two decimal places.

Hint: $\tan\theta = \frac{O}{A}$

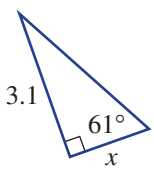
a



b



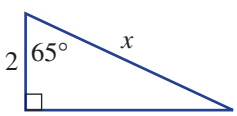
c



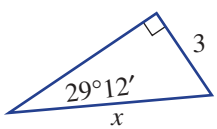
7 By first deciding whether to use $\sin\theta$, $\cos\theta$ or $\tan\theta$, find the value of x in these triangles. Round to two decimal places.

Hint: SOH CAH TOA

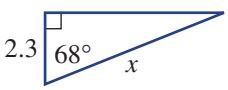
a



b

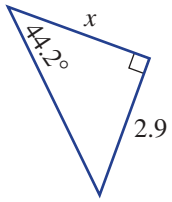


c

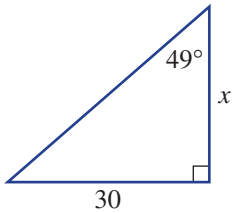


CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

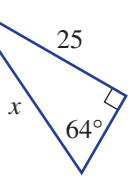
d



e



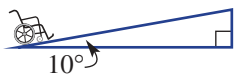
f



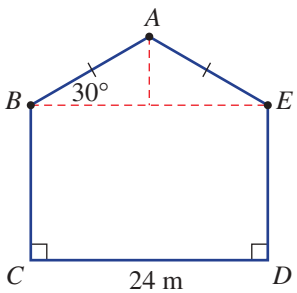
PROBLEM-SOLVING AND REASONING

8–10 | 9–12

8 The recommended angle of a wheelchair ramp to the horizontal is approximately 10°. How long is the ramp if the horizontal distance is 2.5 metres? Round your answer to two decimal places.

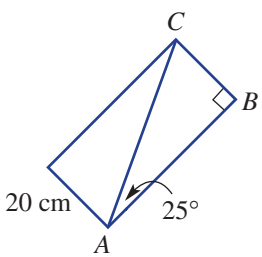


9 The roof of this barn has a pitch of 30°, as shown. Find the length of roof section AB, to one decimal place.

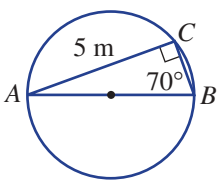


10 Find the length AB and BC in these shapes. Round your answers to two decimal places.

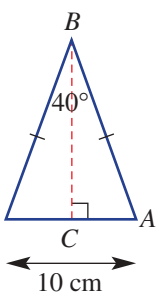
a



b



c

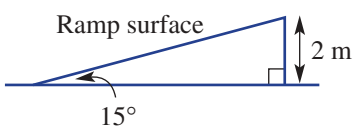


11 The ramp shown has an incline angle of 15° and a height of 2 m. Find, correct to three decimal places:

a the base length of the ramp

b the length of the ramp surface

Hint: The 'incline' is the angle to the horizontal.

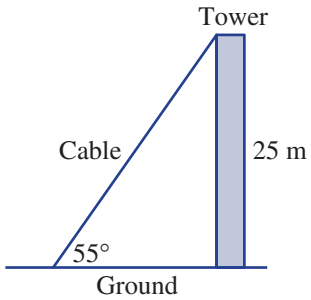


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

12 For this communications tower, find, correct to one decimal place:

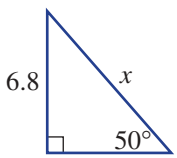
a the length of the cable

b the distance from the base of the tower to the point where the cable is attached to the ground



ENRICHMENT: Inverting the fraction — 13

13 Shown below is another way of solving trigonometric equations with x in the denominator.



Find x, to two decimal places.

$\sin 50° = \frac{6.8}{x}$ First, write the correct ratio.

$\frac{1}{\sin 50°} = \frac{x}{6.8}$ Invert both fractions so x is in the numerator.

$x = \frac{1}{\sin 50°} \times 6.8$ $\frac{1}{\sin 50°} \times 6.8 = \frac{x}{6.8} \times 6.8$

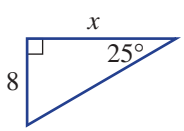
$x = \frac{6.8}{\sin 50°}$ Multiply both sides by 6.8 to get x on its own.

$x = 8.876769\ldots$ Use your calculator.

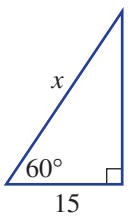
$x = 8.88$ (to 2 d.p.) Round your answer as required.

Use the method shown above to find the value of x, to two decimal places where necessary, in each of the following.

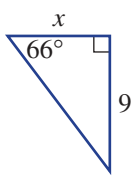
a



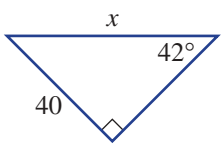
b



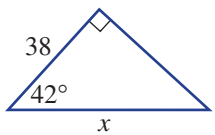
c



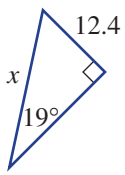
d



e



f

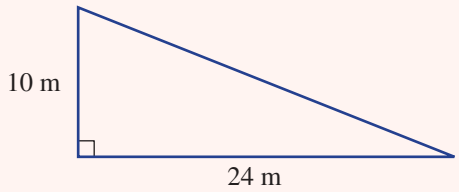


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

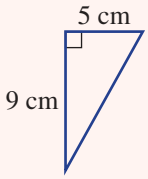
Progress quiz

8A **1** Find the length of the hypotenuse in the following right-angled triangles. Round to one decimal place in part **b**.

a

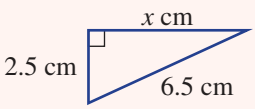


b

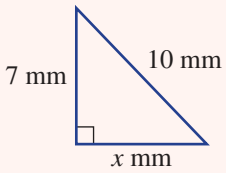


8B **2** Find the value of the pronumeral in the following triangles. Round to one decimal place in part **b**.

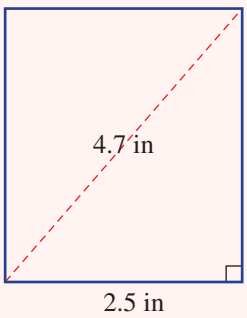
a



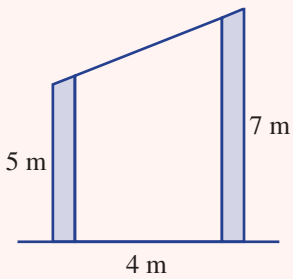
b



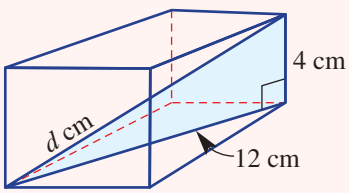
8B **3** A phone has a screen size of 4.7 inches (i.e. this is the distance from corner to corner). If the screen has a width of 2.5 inches, what is the screen height, correct to one decimal place?



8C **4** A straight wire connects the tops of two telegraph poles. The poles are 7 m and 5 m high and are located 4 m apart. What is the length of the wire, correct to two decimal places?



8C **5** Find the distance (d cm) from one corner to the opposite corner of this rectangular prism, correct to one decimal place.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Progress quiz

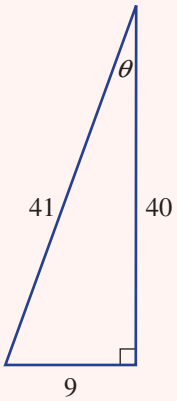
8D

6 For the triangle shown, write down the ratio for:

a $\cos\theta$

b $\sin\theta$

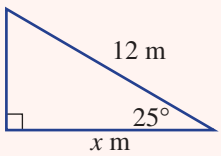
c $\tan\theta$



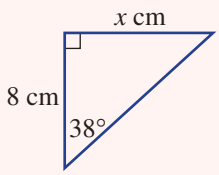
8E

7 Find the value of x in these triangles. Round your answers to one decimal place.

a



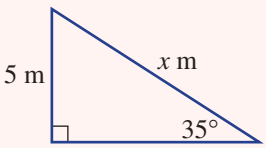
b



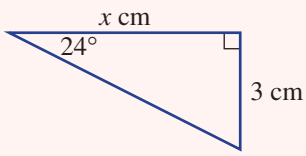
8F

8 Find the value of x in these triangles. Round your answers to one decimal place.

a



b



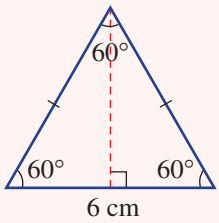
8B/8F

9 Use each of the following methods to find the area of the equilateral triangle shown. Give your answer to one decimal place.

(Hint: First you will need to find the height of the triangle.)

a Pythagoras' theorem

b Trigonometry



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8G Finding unknown angles

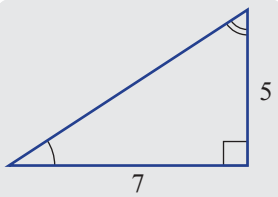
Learning intentions for this section:

- To know that two sides must be known to find an unknown angle in a right-angled triangle
- To be able to evaluate the unknown angle using a calculator

Past, present and future learning:

- This section concludes the Stage 5 Core topic called *Trigonometry A*
- It consolidates and extends the concepts in Section 4H of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the three ratios from memory in the HSC Examination

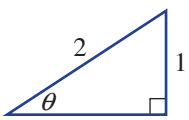
When given two or three side lengths of a right-angled triangle, you can find either of the acute angles. Given a statement like $\sin\theta = \frac{5}{7}$, your calculator can given you a value for θ.



This is enough information to calculate the size of the unknown angles.

Lesson starter: Knowing the angle

Imagine a triangle that produces $\sin\theta = 0.5$.



- Use your calculator and trial and error to find a value of θ for which $\sin\theta = 0.5$.
- Repeat for $\tan\theta = 1$ and $\cos\theta = \frac{\sqrt{3}}{2}$.
- Do you know of a quicker method, rather than using trial and error?

KEY IDEAS

- There are buttons on your calculator that find unknown angles:

  $\boxed{\sin^{-1}}$ $\boxed{\cos^{-1}}$ $\boxed{\tan^{-1}}$

- If solving $\sin\theta = \frac{3}{4}$ on your calculator, enter $\sin^{-1}\left(\frac{3}{4}\right) =$ to find θ.

Key terminology: inverse trigonometric function or ratio

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 8G

UNDERSTANDING | 1–3 | 3

1 Calculate $\sin 30°$. Now use the $\boxed{\sin^{-1}}$ button to solve $\sin\theta = 0.5$.

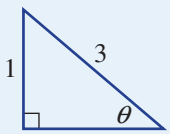
2 Use your calculator to find θ.

a $\sin\theta = \frac{1}{2}$ **b** $\cos\theta = \frac{1}{2}$ **c** $\tan\theta = 1$

Hint: Most calculators use $\boxed{\text{shift}}$ to access $\sin^{-1}$ or $\cos^{-1}$ or $\tan^{-1}$.

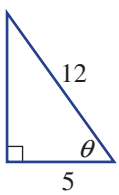
3 Write down the trigonometric ratio for these triangles,
e.g. $\sin\theta = \frac{2}{3}$.

Hint: Look for: SOH CAH TOA

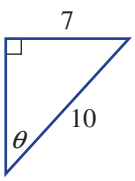


$\sin\theta = \frac{1}{3}$

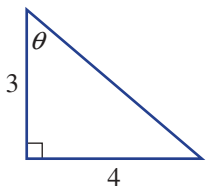
a



b



c



FLUENCY | 4–5(½) | 4–6(½)

Example 13 Finding an unknown angle, correct to the nearest degree

Find the angle θ, correct to the nearest degree, in each of the following.

a $\sin\theta = \frac{2}{3}$ **b** $\cos\theta = \frac{1}{2}$ **c** $\tan\theta = 1.7$

SOLUTION

a $\sin\theta = \frac{2}{3}$

$\theta = 41.8103\ldots°$

$\theta = 42°$ (to the nearest degree)

b $\cos\theta = \frac{1}{2}$

$\theta = 60°$

c $\tan\theta = 1.7$

$\theta = 59.53\ldots°$

$\theta = 60°$ (to nearest degree)

EXPLANATION

Look for the $\sin^{-1}$ button on your calculator.

Evaluate $sin^{-1}\left(\frac{2}{3}\right)$ on your calculator.

Round your answer as required.

Look for the $\cos^{-1}$ button on your calculator.

Evaluate $\cos^{-1}\left(\frac{1}{2}\right)$

Look for the $\tan^{-1}$ button on your calculator.

Round your answer to the nearest degree.

Now you try

Find the angle θ, correct to the nearest degree where necessary, in each of the following.

a $\cos\theta = 0.7$ **b** $\tan\theta = 2.5$ **c** $\sin\theta = \frac{4}{5}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Find the angle θ, to the nearest degree, for the following.

a $\sin\theta = \frac{1}{2}$ **b** $\cos\theta = \frac{3}{5}$ **c** $\sin\theta = \frac{7}{8}$

d $\tan\theta = 1$ **e** $\tan\theta = \frac{7}{8}$ **f** $\sin\theta = \frac{8}{10}$

g $\cos\theta = \frac{2}{3}$ **h** $\sin\theta = \frac{1}{10}$ **i** $\cos\theta = \frac{4}{5}$

j $\tan\theta = 6$ **k** $\cos\theta = \frac{3}{10}$ **l** $\tan\theta = \sqrt{3}$

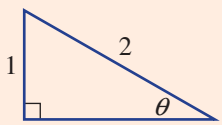
m $\sin\theta = \frac{4}{6}$ **n** $\cos\theta = \frac{4}{6}$ **o** $\tan\theta = \frac{4}{6}$

Hint:
Remember: use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ on the calculator.

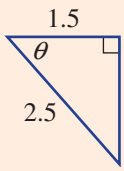
Example 14 Using SOH CAH TOA to find angles

Find θ in the following right-angled triangles, correct to two decimal places where necessary.

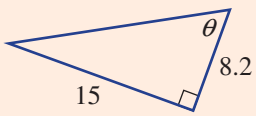
a



b



c



SOLUTION

a $\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

EXPLANATION

Use $\sin\theta$ since we know the opposite and the hypotenuse.

Substitute O = 1 and H = 2.

Use your calculator to find $\sin^{-1}\left(\frac{1}{2}\right)$.

b $\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{1.5}{2.5}$

$\theta = 53.1301\ldots°$

$= 53.13°$ (to 2 d.p.)

Use $\cos\theta$ since we know the adjacent and the hypotenuse.

Substitute A = 1.5 and H = 2.5.

Use your calculator to find $\cos^{-1}\left(\frac{1.5}{2.5}\right)$.

Round your answers to two decimal places.

c $\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{15}{8.2}$

$\theta = 61.336\ldots°$

$\theta = 61.34°$ (to 2 d.p.)

Use $\tan\theta$ since we know the opposite and the adjacent.

Substitute O = 15 and A = 8.2.

Use your calculator to find $\tan^{-1}\left(\frac{15}{8.2}\right)$.

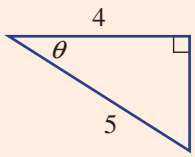
Round your answers to two decimal places.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

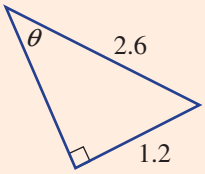
Now you try

Find θ in the following right-angled triangles, correct to two decimal places where necessary.

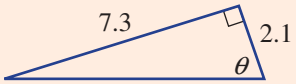
a



b

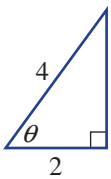


c

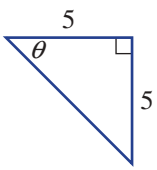


5 Use one of sin, cos or tan to find θ in these triangles, rounding to two decimal places where necessary.

a

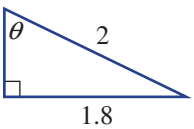


b

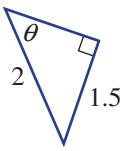


Hint: SOH CAH TOA

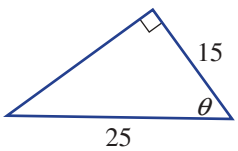
c



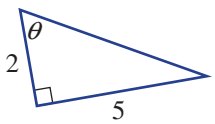
d



e

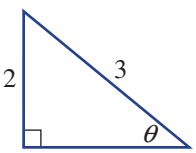


f

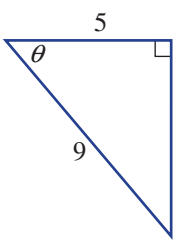


6 Find the angle θ, correct to the nearest minute, in these triangles. You will need to decide whether to use $\sin\theta$, $\cos\theta$ or $\tan\theta$.

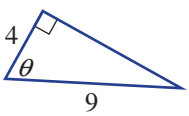
a



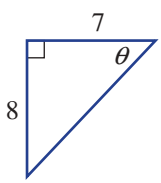
b



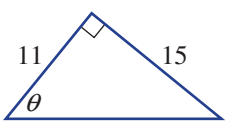
c



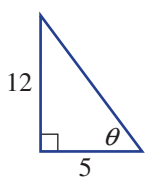
d



e



f

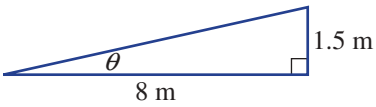


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

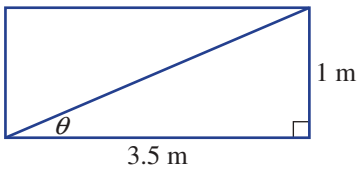
PROBLEM-SOLVING AND REASONING

7, 8 | 8–10

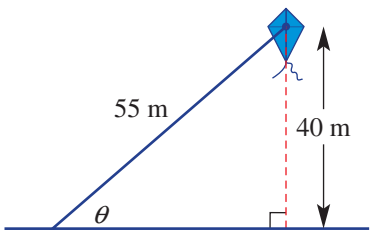
7 A ramp is 8 m long horizontally and 1.5 m high. Find the angle the ramp makes with the ground, correct to the nearest minute.



8 A rectangular piece of wood 1 m wide and 3.5 m long is to be cut across the diagonal. Find the angle the cut makes with the long side (correct to the nearest minute).



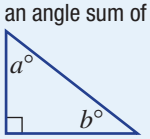
9 At what angle to the ground is a kite (shown) with height 40 m and string length 55 m? Round to the nearest minute.



10 Find the two acute angles in a right-angled triangle with the given side lengths, correct to one decimal place.

- **a** hypotenuse 5 cm, other side 3 cm
- **b** hypotenuse 7 m, other side 4 m
- **c** hypotenuse 0.5 mm, other side 0.3 mm
- **d** the two shorter side lengths are 3 cm and 6 cm
- **e** the two shorter side lengths are 10 m and 4 m

Hint:
- Draw a picture.
- Use SOH CAH TOA.
- Find one acute angle using trigonometry.
- Remember: all triangles have an angle sum of 180°.

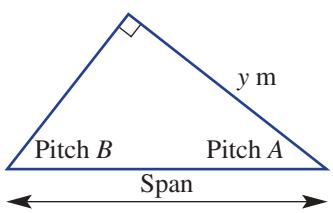


$a + b + 90 = 180$

ENRICHMENT: Building and construction

— | 11–13

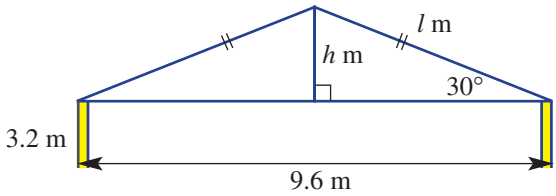
11 A roof is pitched so that the angle at its peak is 90°. If each roof truss spans 10.5 m and distance y is 7.2 m, find the pitch angles A and B, to the nearest degree.



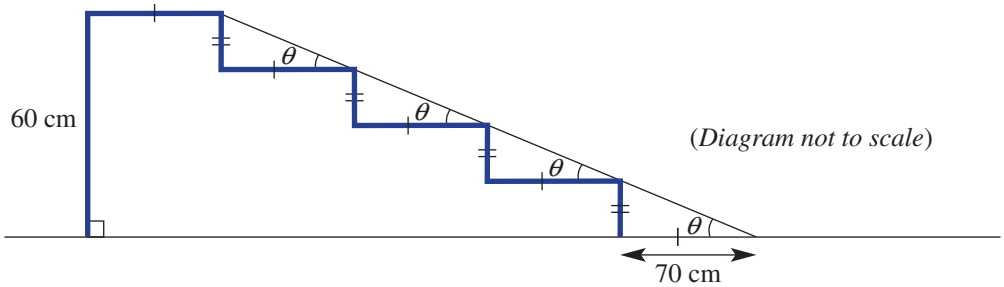
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

12 **a** Find the length of the slats (l metres) needed along each hypotenuse for this roof cross-section, correct to two decimal places.

b Find the height of the highest point of the roof above ground level, correct to two decimal places.



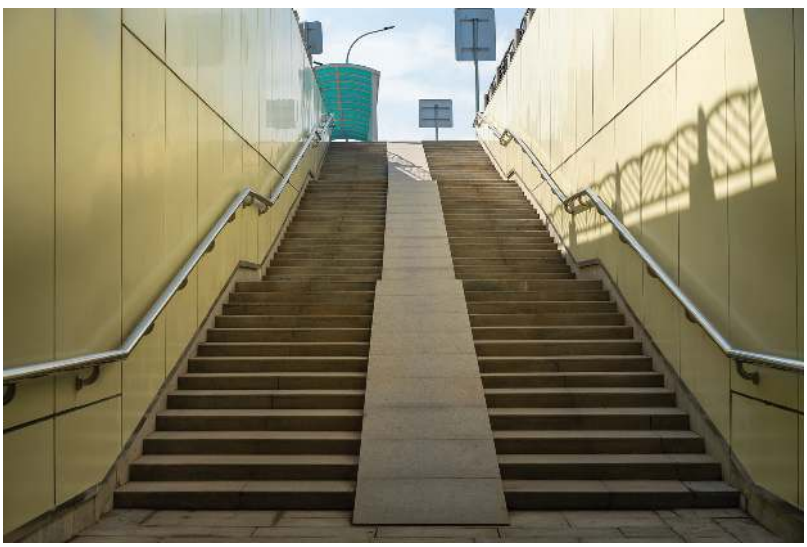
13 A ramp is to be constructed to allow disabled access over a set of existing stairs, as shown.



a What angle does the ramp make with the ground, to the nearest degree?

b Government regulations state that the ramp cannot be more than 13° to the horizontal. Does this ramp meet these requirements?

c Use Pythagoras' theorem to find the length of the ramp.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8H Angles of elevation and depression

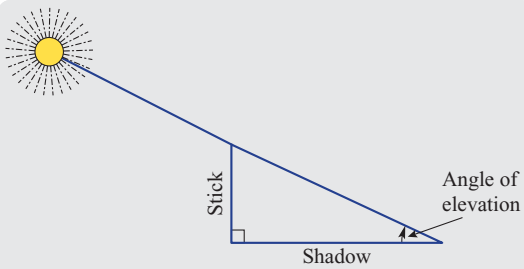
Learning intentions for this section:

- To know how angles of elevation and depression can be identified in a real context
- To apply trigonometry to solve problems involving angles of elevation and depression

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Trigonometry B*
- It consolidates and extends the concepts in Section 4I of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the three ratios from memory in the HSC Examination

Many applications of trigonometry involve angles of elevation and angles of depression. These angles are measured up or down from a horizontal level.

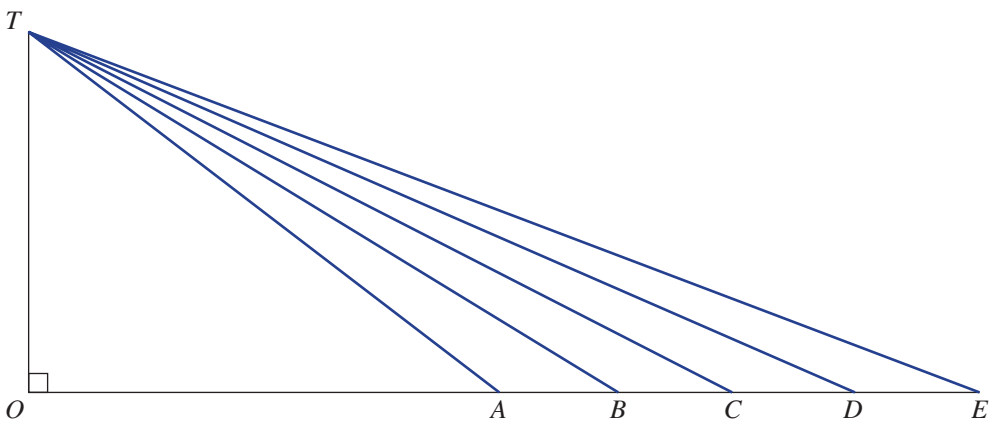


On flat ground, a vertical stick and the shadow it makes can be used to calculate the angle of elevation of the Sun.

Lesson starter: How close should you sit?

The diagram below shows an outdoor movie screen (OT). The point T is the top of the screen. The points $A-E$ are the five rows of seats in the theatre, from which a person's line of sight is taken. The line OE is the horizontal line of sight.

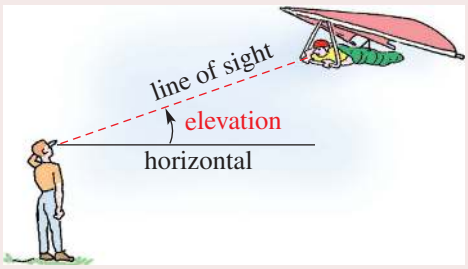
- Use your protractor to measure the angle of elevation from each point along the horizontal to the top of the movie screen.
- Where should you sit if you wish to have an angle of elevation between 25° and 20° and not be in the first or last row of the theatre?



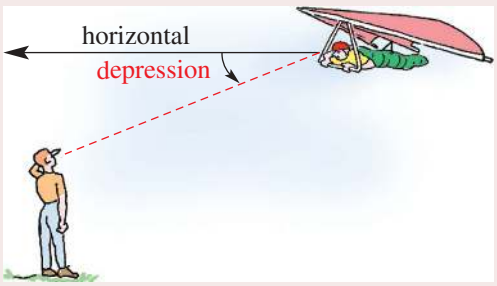
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

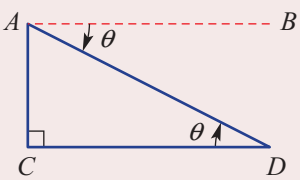
KEY IDEAS

- Looking up at an object forms an **angle of elevation** above the horizontal.



- Looking down at an object forms an **angle of depression** below the horizontal.



- 

AB is parallel to CD.
$\therefore \angle BAD = \angle ADC$ because they are alternate angles in parallel lines.
$\therefore$ Angle of elevation = Angle of depression

Key terminology: elevation, depression, horizontal

Exercise 8H

UNDERSTANDING | 1–3 | 2, 3

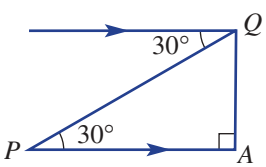
1 Give the missing word.

a Looking down at an object forms an angle of _____.

b Looking up at an object forms an angle of _____.

c Angles of elevation and depression are measured up or down from the _____.

2 This diagram shows angles of elevation and depression.



a What is the angle of elevation of Q from P?

b What is the angle of depression of P from Q?

c What is the size of $\angle PQA$?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 For each description, draw a triangle diagram that matches the information given.

a The angle of elevation to the top of a tower from a point 50 m from its base is 55°.

b The angle of depression from the top of a 200 m cliff to a boat out at sea is 22°.

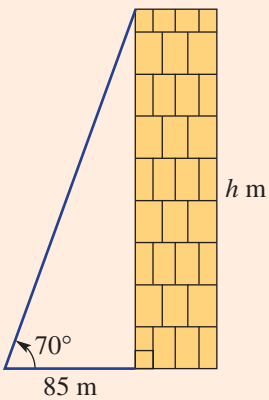
c The angle of elevation of the top of a castle wall from a point on the ground 30 m from the castle wall is 33°.

Hint: Always measure angles of elevation and depression from the horizontal.

FLUENCY 4–6(½) 4–6(½)

Example 15 Using an angle of elevation

To find the height of a tall building, Johal stands 85 m away from its base and measures the angle of elevation to the top of the building as 70°. Find the height of the building, correct to the nearest metre.



SOLUTION

$\tan\theta = \frac{\text{O}}{\text{A}}$

$\tan 70° = \frac{h}{85}$

$h = 85 \times \tan 70°$

$= 233.53558\ldots$

$= 234$ m (to the nearest metre)

$\therefore$ The building is 234 m tall.

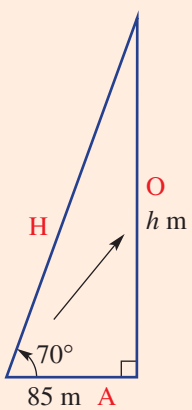
EXPLANATION

Label the triangle with O, A and H.
Use tan since the opposite and adjacent are given.

Find h by solving the equation.

$85 \times \tan 70° = \frac{h}{85} \times 85$

Round to the nearest metre.

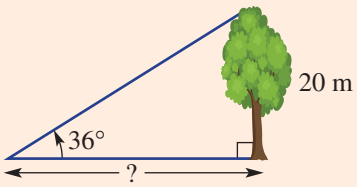


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

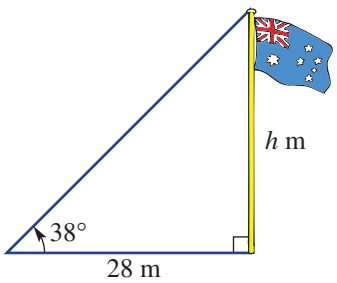
A 20 m high tree casts a shadow of unknown length.

Find the length of the shadow if the angle of elevation of the top of the tree from the end of the shadow is 36°. Round to the nearest metre.

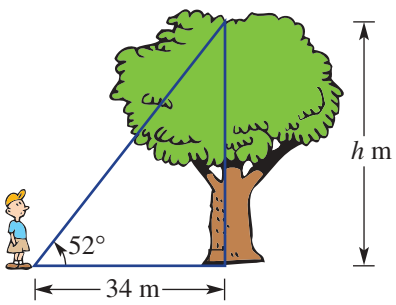


4 Answer the following questions about angles of elevation.

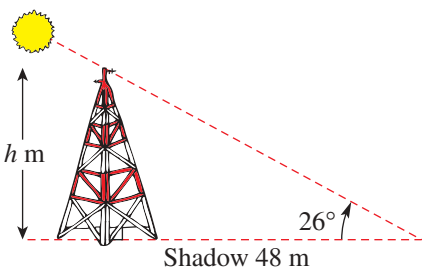
a The angle of elevation to the top of a flagpole from a point 28 m from its base is 38°. How tall is the flagpole, correct to two decimal places?



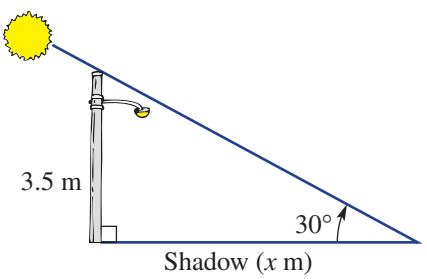
b Alvin is 34 m away from a tree and the angle of elevation to the top of the tree from the ground is 52°. What is the height of the tree, correct to one decimal place?



c The Sun's rays shining over a tower make an angle of elevation of 26° and cast a 48 m shadow on the ground. How tall, to two decimal places, is the tower?

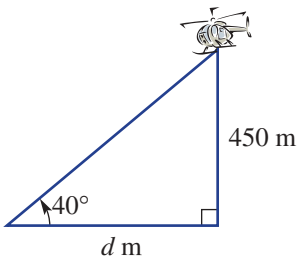


d The Sun makes an angle of elevation of 30° with a lamp post that is 3.5 m tall. How long is the shadow on the ground, correct to two decimal places?

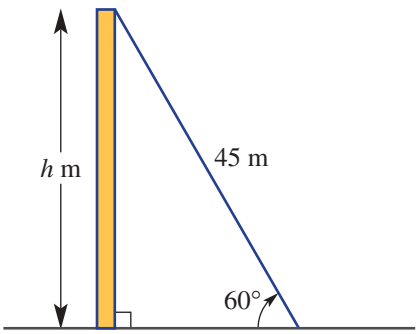


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

e The altitude of a hovering helicopter is 450 m, and the angle of elevation from the helipad to the helicopter is 40°. Find the horizontal distance from the helicopter to the helipad, correct to two decimal places.

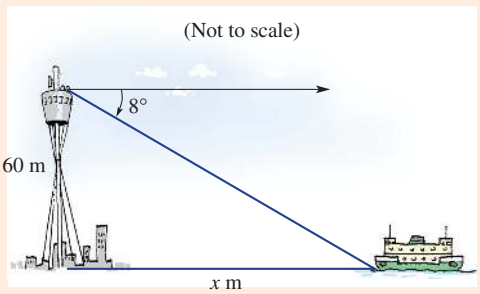


f A cable of length 45 m is anchored from the ground to the top of a communications mast. The angle of elevation of the cable to the top of the mast is 60°. Find the height of the communications mast, correct to two decimal places.

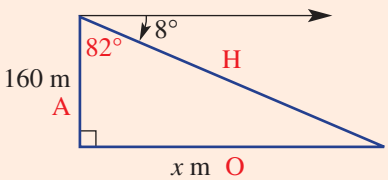


Example 16 Using an angle of depression

From the observation room of Centrepoint Tower in Sydney, which has a height of 160 m, the angle of depression of a boat moored at Circular Quay is observed to be 8°. How far from the base of the tower is the boat, correct to the nearest metre?



SOLUTION



$$\tan\theta = \frac{\text{O}}{\text{A}}$$

$$\tan 82° = \frac{x}{160}$$

$$x = 160 \times \tan 82°$$
$$= 1138.459\ldots$$
$$= 1138 \text{ (to the nearest metre)}$$

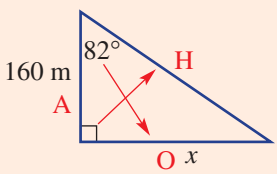
$\therefore$ The boat is about 1138 m from the base of the tower.

EXPLANATION

Draw the triangle and find the angle inside the triangle:
$90° - 8° = 82°$

Use this angle to label the triangle.

Use tan since we have the opposite and adjacent.



Find x by solving the equation.

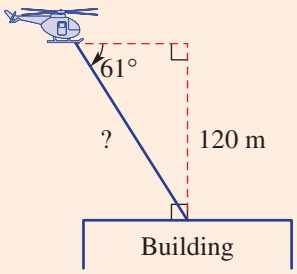
Round to the nearest metre.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

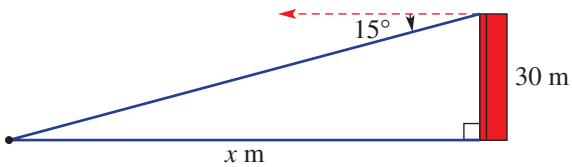
The angle of depression from a hovering helicopter to a point on the building's roof is 61°.

If the vertical height of the helicopter above the building is 120 m, find the direct distance from the helicopter to the point on the roof. Round to one decimal place.



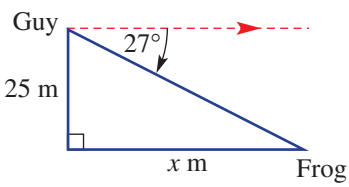
5 Answer these problems relating to angles of depression.

a The angle of depression from the top of a tower 30 m tall to a point x m from its base is 15°. Find the value of x, correct to one decimal place.



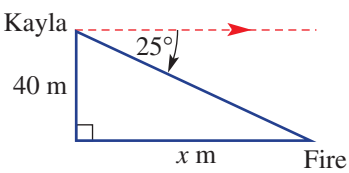
Hint: Use 15° to label an angle inside the triangle.

b From a bridge 25 m above a stream, Guy spots two frogs on a lilypad. He estimates the angle of depression to the frogs to be 27°. How far from the bridge are the frogs, to the nearest metre?

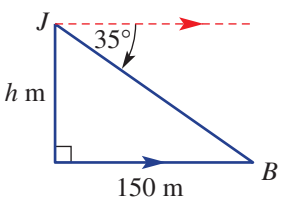


Hint: The angle of depression is the angle below the horizontal, looking down at an object.

c From a lookout tower, Kayla spots a bushfire at an angle of depression of 25°. If the lookout tower is 40 m tall, how far away (to the nearest metre) is the bushfire from the base of the tower?

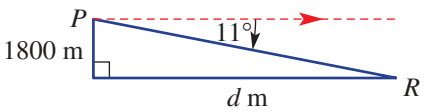


d From the top of a vertical cliff, Jung spots a boat 150 m out to sea. The angle of depression from Jung to the boat is 35°. How many metres (to the nearest whole number) above sea level is Jung?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

e A plane is flying 1800 m above the ground. At the time the pilots spot the runway, the angle of depression to the edge of the runway is 11°. How far does the plane have to fly to be above the edge of the runway at its current altitude? Answer to the nearest whole number.

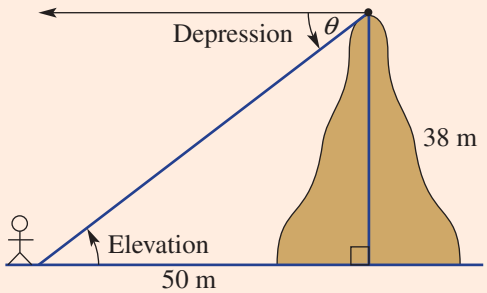


Hint: 'Altitude' means height.

Example 17 Finding angles of elevation and depression

a Find the angle of depression from the top of the hill to a point on the ground 50 m from the middle of the hill. Answer to the nearest degree.

b What is the angle of elevation from the point on the ground to the top of the 38 m hill? Give your answer to the nearest degree.



SOLUTION

a $\tan\theta = \frac{O}{A}$

$\tan\theta = \frac{38}{50}$

$\theta = 37.2348\ldots°$

$\theta = 37°$ (to the nearest degree)

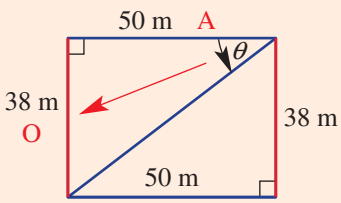
Angle of depression is 37°.

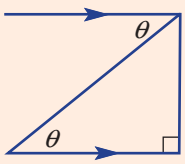
b Angle of elevation is 37°.

EXPLANATION

Aim to find θ. Redraw the diagram as a rectangle so that θ is inside the triangle.

Label the triangle: opposite and adjacent. Use tan.

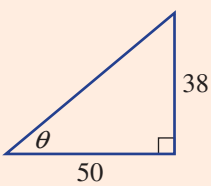




Alternate angles are equal when lines are parallel.

Angle of elevation = Angle of depression

Alternatively, use $\tan\theta = \frac{38}{50}$



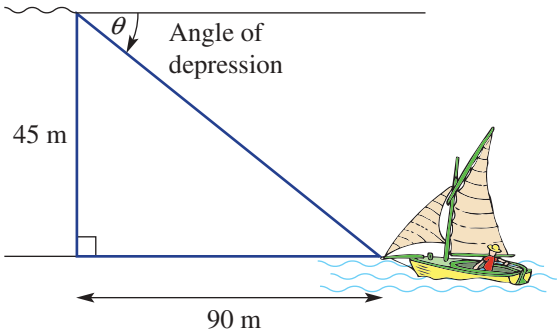
Now you try

From a 20 m high lookout platform, you can see a fire that is 150 m away horizontally. Find the angle of depression from the lookout platform to the fire, correct to the nearest degree.

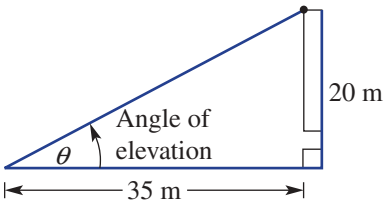
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Answer these questions about finding angles of elevation and depression. Round all answers to one decimal place.

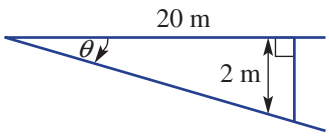
a From the top of a vertical cliff, Jacquie spots a boat 90 m out to sea. If the top of the cliff is 45 m above sea level, find the angle of depression from the top of the cliff to the boat.



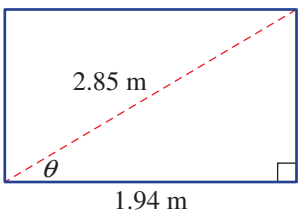
b Find the angle of elevation from a person sitting 35 m from a movie screen to the top of the screen at 20 m above the ground.



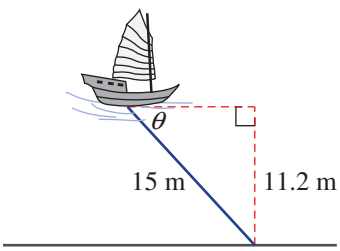
c A person sits 20 m away from a stage that is 2 m below the horizontal viewing level. Find the angle of depression of the person's viewing level to the stage.



d A diagonal cut 2.85 m long is to be made on a piece of plaster board attached to a wall, as shown. The base of the plaster board measures 1.94 m. Find the angle of elevation of the diagonal cut from the base.



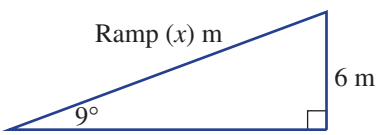
e A 15 m chain with an anchor attached, as shown, is holding a boat in a position against a current. If the water depth is 11.2 m, find the angle of depression from the boat to where the anchor is fixed to the seabed.



PROBLEM-SOLVING AND REASONING

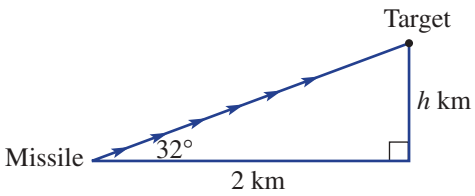
7, 9 | 8–10

7 A ramp for wheelchairs is to be constructed onto a footbridge 6 m high. The angle of elevation is to be 9°. What will be the length of the ramp, correct to two decimal places?

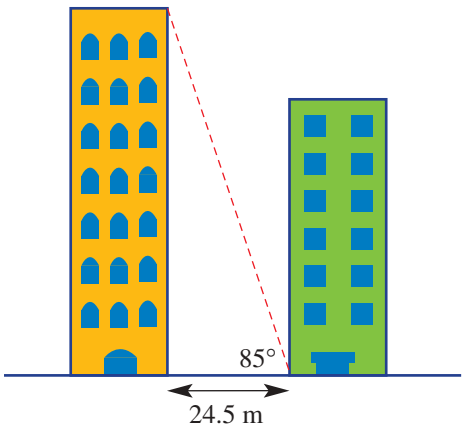


CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 A missile is launched at an angle of elevation of 32°. If the target is 2 km away on the horizontal, how far above ground level is the target, correct to two decimal places?

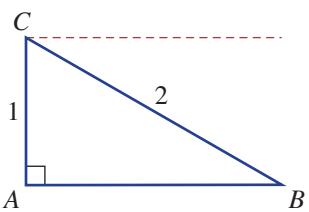


9 The distance between two buildings is 24.5 m, as shown. Find the height of the taller building, correct to two decimal places, if the angle of elevation from the base of the shorter building to the top of the taller building is 85°.



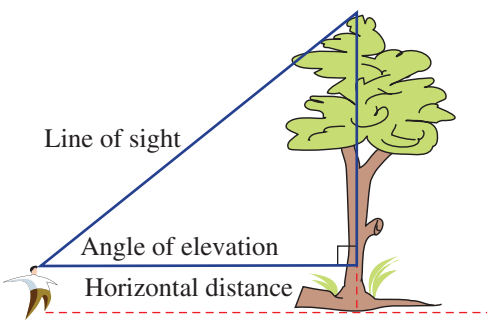
10 For this triangle:

- **a** Find the angle of elevation from B to C.
- **b** State the angle of depression from C to B.
- **c** Describe the relationship that exists between these two angles.
- **d** Find the length AB, correct to one decimal place.



ENRICHMENT: Practical trigonometry: measuring heights — 11

11 It is not always possible or practical to measure the height of an object directly. Here you will find the height of an object that is difficult to measure. Select a building or other structure (e.g. a statue or flagpole) for height calculation. You must be able to measure right up to the base of the structure.



- **a** Choose a position from which you can see the top of your structure and measure the angle of elevation, θ, from your eye level. (Use an inclinometer, if your teacher has one, or simply estimate the angle using a protractor.)
- **b** Measure the distance along the ground (d) from your location to the base of the structure.
- **c** Calculate the height of the structure. *Remember to make an adjustment for the height of your eye level from the ground.*
- **d** Move to another position and repeat the measurements. Calculate the height using your new measurements.
- **e** Was there much difference between the calculated heights? Suggest reasons for any differences.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8I Direction and bearings

Learning intentions for this section:

- To identify and interpret true bearings and compass bearings
- To be able to solve practical problems involving bearings

Past, present and future learning:

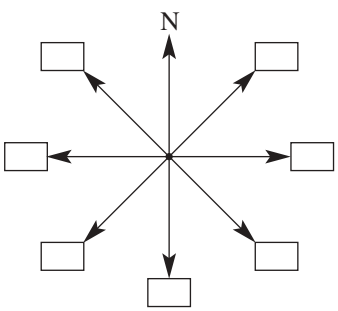
- This section concludes the Stage 5 Core topic called *Trigonometry B*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the three ratios from memory in the HSC Examination

True bearings are used to communicate a direction and are important in navigation. Ships, planes, bushwalkers and the military all use bearings when communicating direction.

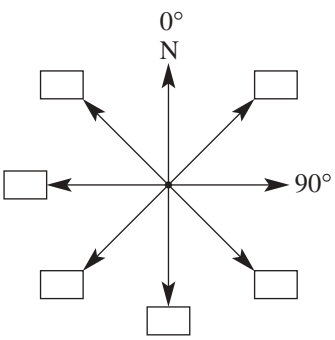
Lesson starter: Compass bearings

Work together as a class to label the 8-point compass rose using letters/words in **1** and angles in **2**.

1



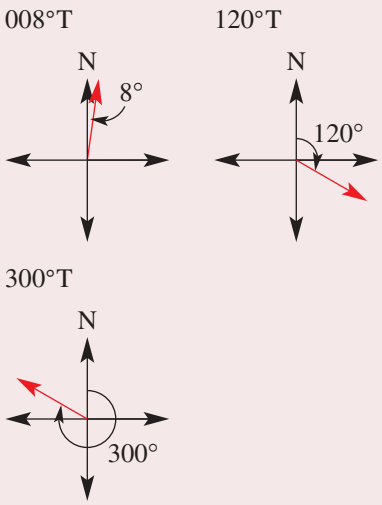
2

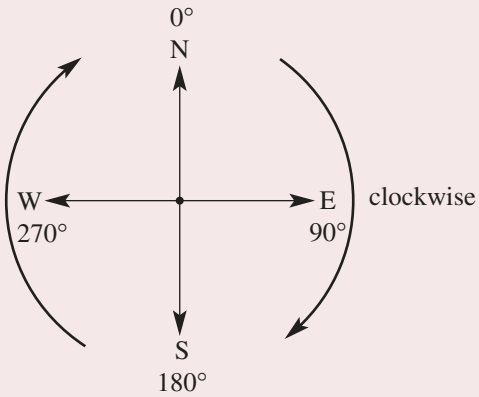


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

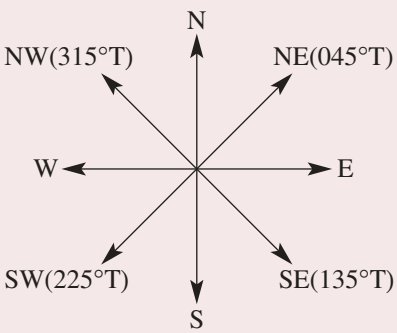
KEY IDEAS

- A **true bearing** (°T) is an angle measured clockwise from north.
  - It is written using three digits.
    For example,

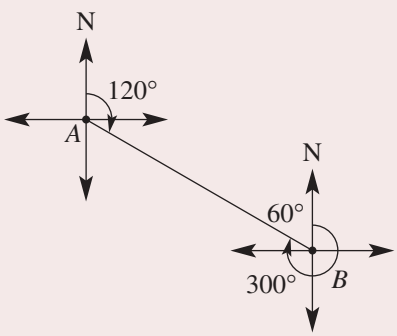




- This diagram shows eight commonly used directions.



  - The word *from* indicates the starting point from which a bearing is being taken.
    For example,



The bearing of *B* **from** ***A*** is 120°T.

The bearing of *A* **from** ***B*** is 300°T.

  - When solving problems relating to bearings, always draw a diagram using N, S, E and W each time a bearing is used.

Key terminology: true bearing, clockwise

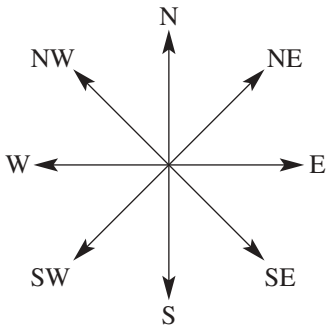
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 8I

UNDERSTANDING | 1–4 | 3, 4

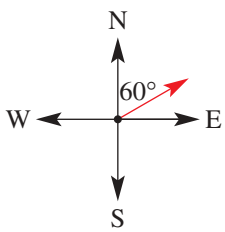
1 Give the bearing for each of these directions.

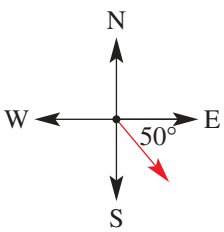
a N
b NE
c E
d SE
e S
f SW
g W
h NW

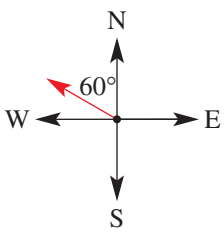


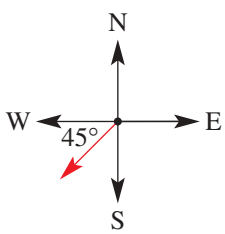
2 Match each diagram below with the correct true bearing from the list below.

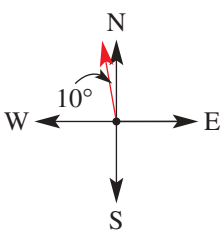
i 300°
ii 260°
iii 225°
iv 140°
v 060°
vi 350°

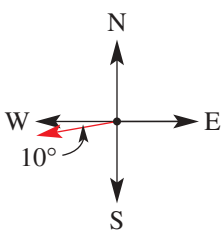
a


b


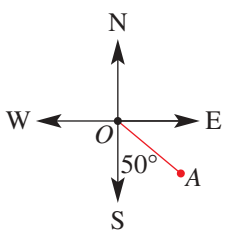
c


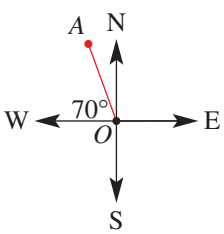
d


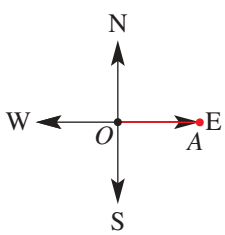
e


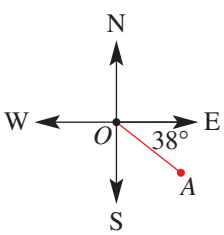
f


3 Write down the true bearings of A from O, as shown in these diagrams.

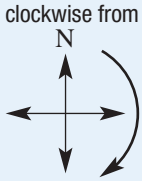
a


b


c


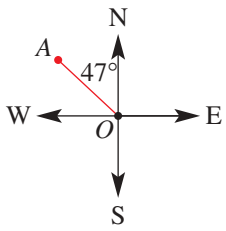
d


Hint: Remember to use 3 digits and to go clockwise from north.

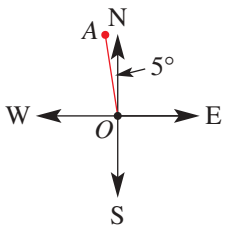


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

e



f



4 Fill in the missing terms and values for the diagram shown.

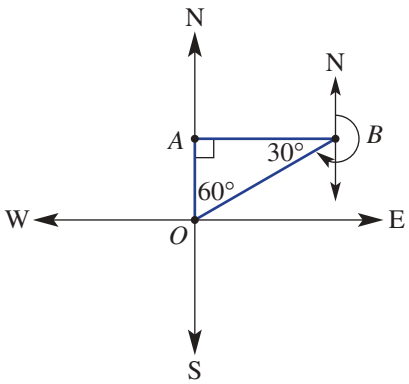
a A is due _____ of O.

b B is due _____ of A.

c A is due _____ of B.

d The true bearing of B from O is _____.

e The true bearing of O from B is _____.



Hint: For part **e**, start from north and move clockwise to the line BO.

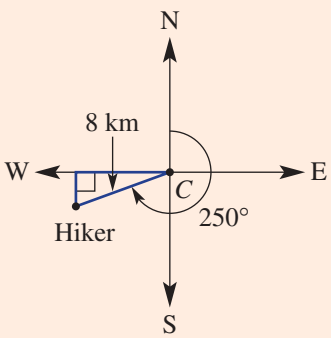
FLUENCY

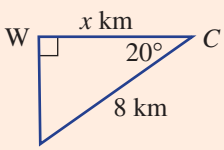
5–7 | 5, 6, 8

Example 18 Drawing a diagram

A hiker leaves camp (C) and walks on a bearing of 250°T for 8 km. How far west of camp (x km) is the hiker? Show all this information on a right-angled triangle. You do not need to solve for x.

SOLUTION





EXPLANATION

Draw the compass points first.

Start your diagram with the camp at the centre.

Mark in 250° clockwise from north, 8 km.

Draw a line from the hiker to the west line at right angles.

Redraw the triangle, showing any angles and lengths known ($270° - 250° = 20°$). Place a pronumeral on the required side.

Now you try

A plane leaves the airport and heads on a bearing of 165°T for 100 km. How far south of the airport (x km) is the plane?

Show all this information on a right-angled triangle. You do not need to solve for x.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

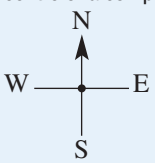
5 Draw a right-angled triangle for each of the situations outlined below.

a Zahra runs on a true bearing of 300° from her home for 6 km.
How far north of home is she when she stops?

b Bailey walks 12.5 km from camp, C, on a bearing of 135°T.
How far south is he now from camp?

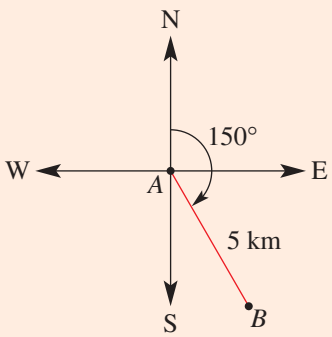
c Samir walks due south 10 km, then turns and walks due east 12 km.
What is his bearing from O, his starting point?

Hint: Start by making the starting point at the centre of a compass.

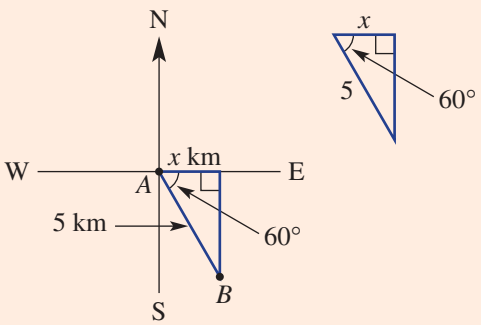


Example 19 Finding distances with bearings

A bushwalker walks 5 km on a bearing of 150°T from point A to point B. Find how far east point B is from point A.



SOLUTION



EXPLANATION

Copy the diagram and draw a line from B up to the east line.

Use the pronumeral x along the east line.

Find the angle within the triangle:
$150° - 90° = 60°$

Redraw the triangle.

$\cos\theta = \frac{\text{A}}{\text{H}}$

Since the adjacent (A) and hypotenuse (H) are given, use cos.

$\cos 60° = \frac{x}{5}$

$x = 5 \times \cos 60°$

$x = 2.5$

Solve the equation to find x.

$\therefore$ Point B is 2.5 km east of point A.

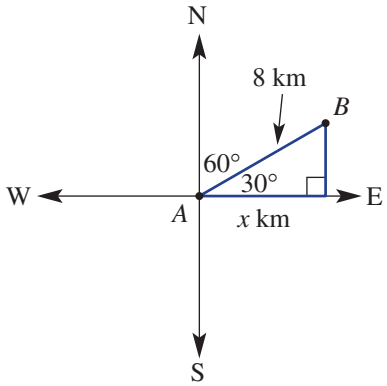
Answer the question.

Now you try

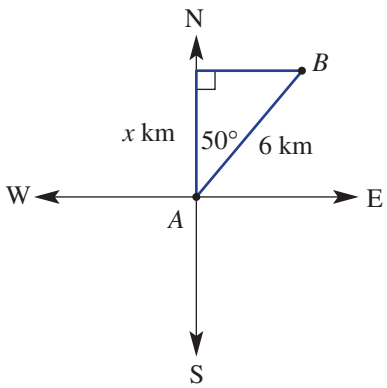
A ship travels 200 km on a true bearing of 290° from point A to point B. Find how far west the point B is from point A. Round to one decimal place.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

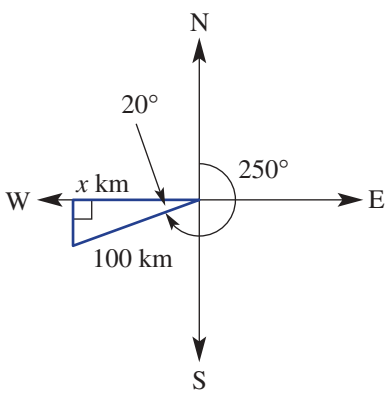
6 Rihanna walks 8 km from point A to point B on a bearing of 060°T. How far east, correct to one decimal place, is point B from point A?



7 A bushwalker walks 6 km on a bearing of 050°T from point A to point B. Find how far north point B is from point A, correct to two decimal places.



8 A speed boat travels 100 km on a true bearing of 250°. Find how far west of its starting point the speed boat is, correct to two decimal places.



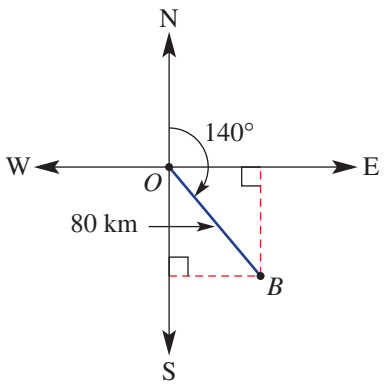
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

9, 10 | 9, 11

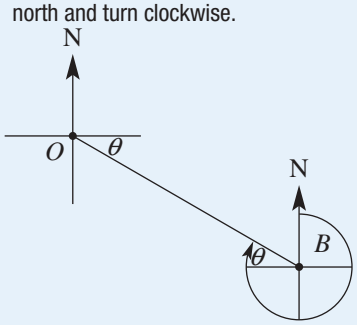
9 A fishing boat starts from point O and sails 80 km on a true bearing of 140° to point B, as shown below.

- **a** How far east of point O is point B? (Answer to two decimal places.)
- **b** How far south of point O is point B? (Answer to two decimal places.)
- **c** What is the bearing of point O from point B?



Hint:
Remember what 'from' means!

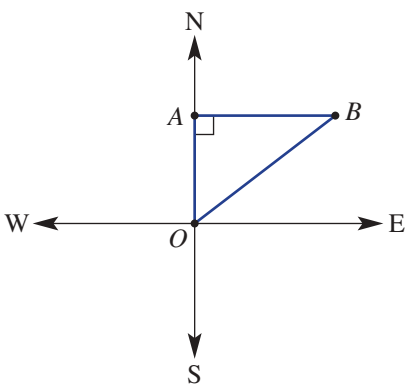
Hint:
Remember: to find a bearing, face north and turn clockwise.



Bearings are given as a 3-digit angle.

10 A plane flies from point O 50 km due north to point A, and then turns and flies 60 km east to point B.

- **a** Copy the diagram below and mark in the lengths 50 km and 60 km.



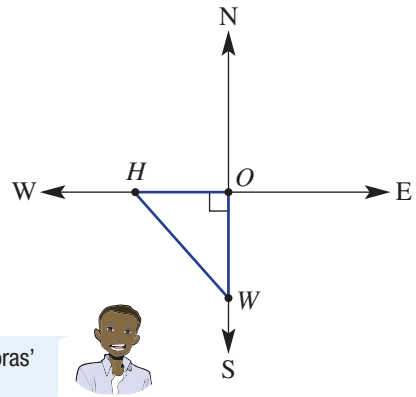
Hint: Pythagoras' theorem:
$c^2 = a^2 + b^2$

- **a** Use Pythagoras' theorem to find the distance of B from O, correct to two decimal places.
- **b** Use trigonometry to find the size of angle AOB. Round to the nearest degree.
- **c** What is the bearing of B from O?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 William and Harry both leave camp, O, at the same time. William walks south from O for 10 km. Harry walks west from O for 8 km.

- **a** Copy and complete the diagram for this question.
- **b** How far is Harry from William (to one decimal place)?
- **c** Find the size of angle OWH, correct to the nearest degree.
- **d** What is the bearing of Harry from William correct to the nearest degree?



Hint: You can use Pythagoras' theorem here.

ENRICHMENT: Drawing your own diagrams — 12–14

12 Huang walks on a true bearing of 210° for 6.5 km. How far west of his starting point is he?

13 A plane flies on a true bearing of 320° from an airport, A, for 150 km. At this time how far north of the airport is the plane? Answer to the nearest kilometre.

14 Point A is 10 km due east of point O, and point B is 15 km due south of point A.

- **a** How far is it, correct to two decimal places, from point B to point O?
- **b** What is the bearing, to the nearest degree, of point B from point O?

Hint: Remember: The word 'from' indicates where the bearing is being taken.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8J Applications in three dimensions

Learning intentions for this section:

- To identify right-angled triangles in three-dimensional situations
- To solve problems involving right-angled triangles in three-dimensional situations

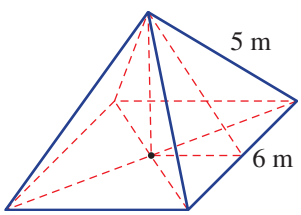
Past, present and future learning:

- This section addresses the Stage 5 Path topic called *Trigonometry C*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- You may be expected to recall the three ratios from memory in the HSC Examination

Although a right-angled triangle is a two-dimensional shape, it can also be used to solve problems in three dimensions. Being able to visualise right-angled triangles included in three-dimensional diagrams is an important part of the process of finding angles and lengths associated with three-dimensional objects.

Lesson starter: How many right-angled triangles?

A right square pyramid has the apex above the centre of the base. In this example, the base length is 6 m and slant height is 5 m. Other important lines are dashed.

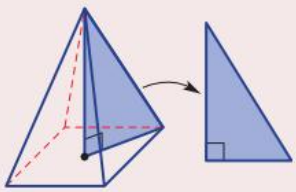


- Using the given dashed lines and the edges of the pyramid, how many different right-angled triangles can you draw?
- Is it possible to determine the exact side lengths of all your right-angled triangles?
- Is it possible to determine all the angles inside all your right-angled triangles?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

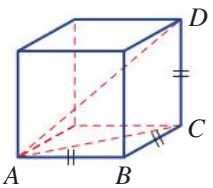
- Using trigonometry to solve problems in three dimensions involves:
  - visualising and drawing any relevant two-dimensional triangles
  - using trigonometric ratios to find unknowns
  - relating answers from two-dimensional diagrams to the original three-dimensional object.



Exercise 8J

UNDERSTANDING 1

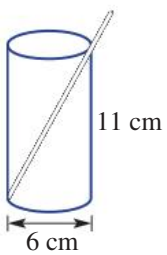
1 The cube shown here has a side length of 2 m.



- **a** Draw the right-angled triangle ABC and find and label all the side lengths. Pythagoras' theorem can be used. Answer using exact values (e.g. $\sqrt{5}$).
- **b** Draw the right-angled triangle ACD and find and label all the side lengths. Pythagoras' theorem can be used. Answer using exact values.
- **c** Use trigonometry to find $\angle DAC$, correct to one decimal place.
- **d** Find $\angle CAB$.

FLUENCY 2–4 3–6

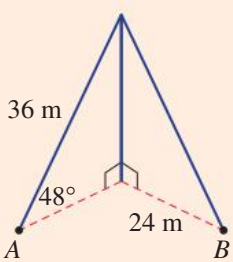
2 Find the angle of elevation that this drinking straw makes with the base of the can, which has diameter 6 cm and height 11 cm. Round your answer to one decimal place.



Example 20 Applying trigonometry in three dimensions

A vertical mast is supported at the top by two cables reaching from two points, A and B. The cable reaching from point A is 36 m long and is at an angle of 48° to the horizontal. Point B is 24 m from the base of the mast.

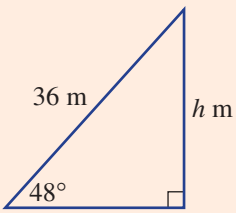
- **a** Find the height of the mast, correct to three decimal places.
- **b** Find the angle to the horizontal of the cable from point B, to two decimal places.



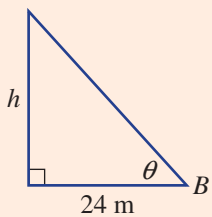
Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION	EXPLANATION
a Let h be the height of the mast, in metres.	First, draw the right-angled triangle, showing the information given.



$\sin 48° = \frac{h}{36}$	The opposite (O) and hypotenuse (H) are given, so use sine.
$h = 36 \times \sin 48°$ $= 26.7532\ldots$	Multiply both sides by 36.
The height of the mast is 26.753 m (to 3 decimal places).	Answer the question in words.
b	Draw the second triangle, including the answer from part **a**.



$\tan\theta = \frac{26.7532\ldots}{24}$	Use the inverse tan function to find the angle.
$\theta = 48.1050\ldots$	Use the more precise answer from part **a** (i.e. 26.7532 …) that is stored in your calculator.
The cable reaching from point B is at an angle of 48.11° to the horizontal (to 2 decimal places).	Answer the question in words, rounding your answer appropriately.

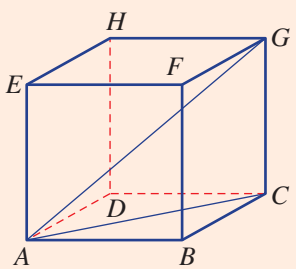
Now you try

The diagram shows a cube with edges 10 metres long.

a Use Pythagoras' theorem to find the length of AC, in exact form.

b Use Pythagoras' theorem to find the length of AG, in exact form.

c Use trigonometry to find the size of angle CAG, correct to the nearest degree.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 A vertical mast is supported at the top by two cables reaching from two points, A and B. The cable reaching from point A is 43 metres long and is at an angle of 61° to the horizontal. Point B is 37 metres from the base of the mast.

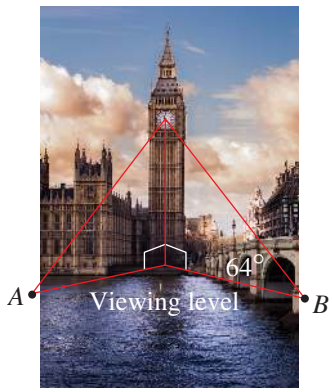
a Find the height of the mast, correct to three decimal places.

b Find the angle to the horizontal of the cable from point B, to two decimal places.

4 Viewing points A and B are at a horizontal distance of 36 metres and 28 metres, respectively, from a clock tower. The viewing angle to the clockface at point B is 64°.

a Find the height of the clockface above the viewing level, to three decimal places.

b Find the viewing angle to the clockface at point A, to two decimal places.

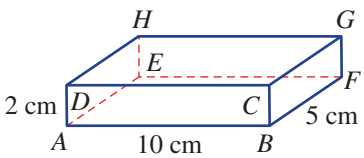


5 A rectangular prism, $ABCDEFGH$, is 5 cm wide, 10 cm long and 2 cm high.

a By drawing the triangle ABF find, to two decimal places:

i $\angle BAF$ **ii** AF

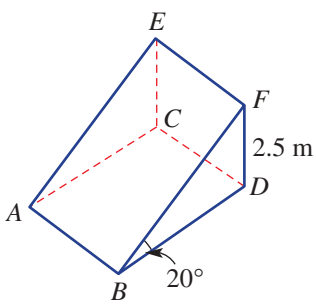
b By drawing the triangle AGF, find $\angle GAF$, to two decimal places.



6 A ramp, $ABCDEF$, rests at an angle of 20° to the horizontal and the highest point on the ramp is 2.5 m above the ground, as shown. Give your answers to two decimal places in the following questions.

a Find the length of the ramp BF.

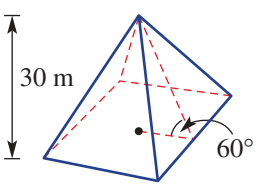
b Find the length of the horizontal BD.



PROBLEM-SOLVING AND REASONING

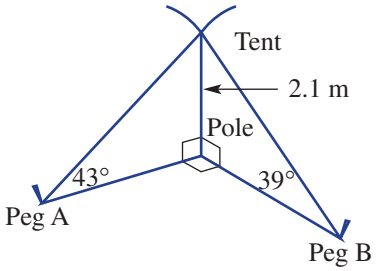
7–9 | 8–12

7 The triangular faces of a right square pyramid are at an angle of 60° to the base. The height of the pyramid is 30 m. Find the perimeter of the base of the pyramid, correct to two decimal places.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 A tent pole measuring 2.1 m tall is secured by ropes in two directions. The ropes are held by pegs A and B at angles of 43° and 39°, respectively, from the horizontal. The line from the base of the pole to peg A is at right angles to the line from the base of the pole to peg B. Round your answers to two decimal places in these questions.



a Find the distance from the base of the tent pole to:
i peg A **ii** peg B

b Find the angle at peg B formed by peg A, peg B and the base of the pole.

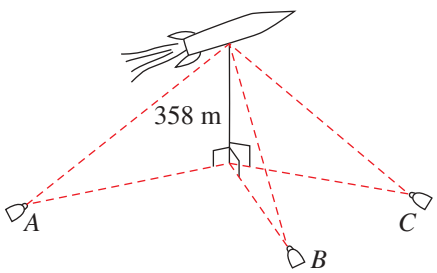
c Find the distance between peg A and peg B.

9 The communities of Wood Town and Green Village live in a valley. Communication between the two communities is enhanced by a repeater station on the summit of a nearby mountain. It is known that the angles of depression from the repeater station to Wood Town and Green Village are 44.6° and 58.2°, respectively. Also, the horizontal distances from the repeater to Wood Town and Green Village are 1.35 km and 1.04 km, respectively.

a Find the vertical height, to the nearest metre, between the repeater station and:
i Wood Town **ii** Green Village

b Find the difference in height between the two communities, to the nearest metre.

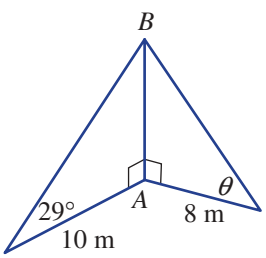
10 Three cameras operated at ground level view a rocket being launched into space.
At 5 seconds immediately after launch, the rocket is 385 m above ground level and the three cameras, A, B and, C are positioned at an angle of 28°, 32° and 36°, respectively, to the horizontal.
At the 5 second mark, find:



a which camera is closest to the rocket

b the distance between the rocket and the closest camera, to the nearest centimetre

11 It is important to use a high degree of accuracy for calculations that involve multiple parts. For this three-dimensional (3D) diagram, complete these steps.



a Find AB, correct to one decimal place.

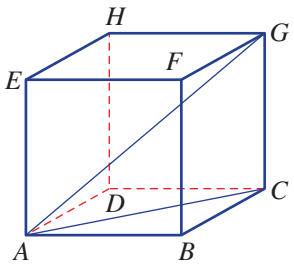
b Use your answer from part **a** to find θ, correct to one decimal place.

c Now recalculate θ using a more accurate value for AB. Round θ to one decimal place.

d What is the difference between the answers for parts **b** and **c**?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

12 For a cube $ABCDEFGH$ with side length 1 unit, as shown, use trigonometry to find the following, correct to two decimal places where necessary. Be careful that errors do not accumulate.

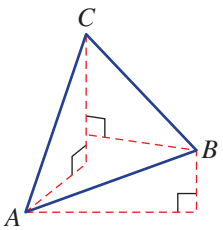


a $\angle BAC$ **b** AC

c $\angle CAG$ **d** AG

ENRICHMENT: Three points in 3D — 13–15

13 Three points, A, B and C, in three-dimensional (3D) space are such that $AB = 6$, $BC = 3$ and $AC = 5$. The angles of elevation from A to B and from B to C are 15° and 25°, respectively. Round your answer to two decimal places in the following.



a Find the vertical difference in height between:

i A and B **ii** B and C **iii** A and C

b Find the angle of elevation from A to C.

14 The points A, B and C in 3D space are such that:

- $AB = 10$ mm, $AC = 17$ mm, and $BC = 28$ mm.
- The angle of elevation from A to B is 20°.
- The angle of elevation from A to C is 55°.

Find the angle of elevation from B to C, to the nearest degree.

15 From a point, A, due east of the foot of a tower, O, the angle of elevation is 40°. From another point, B, due south of the tower and 100 m from point A, the angle of elevation is 42°. Let the top of the tower be T, and the height of the tower be h metres.

a Use triangle TOA to show that $\tan 50° = \frac{OA}{h}$.

b Find an expression using triangle TOB for the length of OB, in terms of tan.

c Use Pythagoras' theorem and triangle AOB to find the height of the tower, correct to one decimal place.

Triangulation points or 'trig stations' such as this are used in geodetic surveying to mark points at which measurements are made to calculate local altitude. The calculations involved are similar to those in the Enrichment questions above.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8K The sine rule

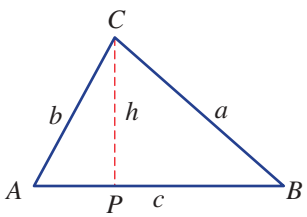
Learning intentions for this section:

- To use the sine rule to calculate unknown sides and angles in non-right-angled triangled

Past, present and future learning:

- This section addresses the Stage 5 Path topic called *Trigonometry C*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- The formula for the sine rule may be provided in the HSC Examination

The use of sine, cosine and tangent functions can be extended to non right-angled triangles. First, consider this triangle with sides a, b and c and with opposite angles $\angle A$, $\angle B$ and $\angle C$. Height h is also shown.



From ΔCPB, $\frac{h}{a} = \sin B$

so $h = a \sin B$

From ΔCPA, $\frac{h}{b} = \sin A$

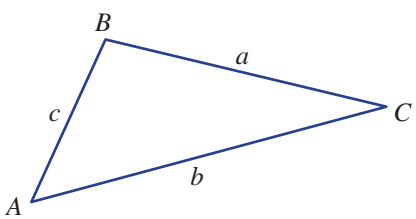
so $h = b \sin A$

$\therefore a \sin B = b \sin A$ or $\frac{a}{\sin A} = \frac{b}{\sin B}$

Similarly, it can be shown that $\frac{a}{\sin A} = \frac{c}{\sin C}$ and $\frac{b}{\sin B} = \frac{c}{\sin C}$.

Lesson starter: Explore the sine rule

Use a ruler and a protractor or dynamic geometry software to measure the side lengths (a, b and c) in centimetres, correct to one decimal place, and the angles (A, B and C), correct to the nearest degree, for this triangle.

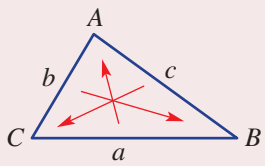


- Calculate the following.
  - **a** $\frac{a}{\sin A}$
  - **b** $\frac{b}{\sin B}$
  - **c** $\frac{c}{\sin C}$
- What do you notice about the three answers above?
- Draw your own triangle and check to see if your observations are consistent for any triangle.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- When using the sine rule, label triangles with capital letters for vertices and the corresponding lower-case letter for the side opposite the angle.



- The **sine rule** states that the ratios of each side of a triangle to the sine of the opposite angle are equal.

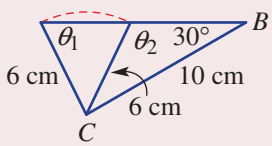
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

  - The sine rule holds true for both acute- and obtuse-angled triangles.

- The sine rule is useful when you know:
  - the size of one angle
  - the length of the side opposite that angle, and
  - another side or angle.

(EXTENDING)

- The **ambiguous case** may arise when we are given two sides and an angle that is not the included angle.
  - This example shows a diagram with two given side lengths and one angle. Two different triangles contain this information.
  - Using $\frac{6}{\sin 30°} = \frac{10}{\sin \theta}$ could give two results for θ (i.e. θ_1 or θ_2). Sometimes there are two valid angles (i.e. one acute and one obtuse). The obtuse angle is the supplement of the acute angle. Sometimes the obtuse angle is not a valid solution.



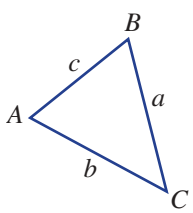
Key terminology: sine rule, opposite, ambiguous

Exercise 8K

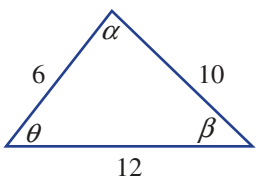
UNDERSTANDING 1 1

1 Write the sine rule for each triangle

a



b



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 2–5 2–5

2 Solve each equation for a, b or c, correct to one decimal place.

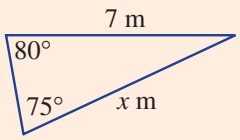
a $\frac{a}{\sin 47°} = \frac{2}{\sin 51°}$ **b** $\frac{b}{\sin 31°} = \frac{7}{\sin 84°}$ **c** $\frac{5}{\sin 63°} = \frac{c}{\sin 27°}$

3 Find θ, correct to one decimal place, if θ is acute.

a $\frac{4}{\sin 38°} = \frac{5}{\sin \theta}$ **b** $\frac{11}{\sin 51°} = \frac{9}{\sin \theta}$ **c** $\frac{1.2}{\sin \theta} = \frac{1.8}{\sin 47°}$

Example 21 Finding a side length using the sine rule

Find the value of x in this triangle, correct to one decimal place.



SOLUTION

$$\frac{x}{\sin 80°} = \frac{7}{\sin 75°}$$

$$x = \frac{7}{\sin 75°} \times \sin 80°$$

$$= 7.1368\ldots$$

$$= 7.1 \text{ (to 1 decimal place)}$$

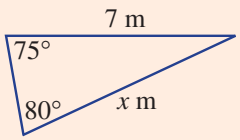
EXPLANATION

Use the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B}$.

Multiply both sides by 80°.

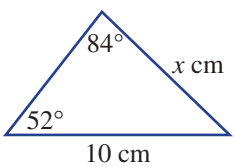
Now you try

Find the value of x in this triangle, correct to one decimal place.

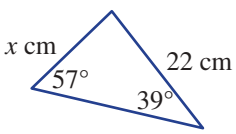


4 Find the value of x in these triangles, correct to one decimal place.

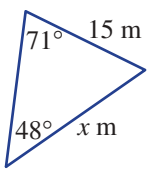
a



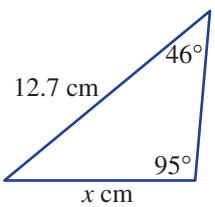
b



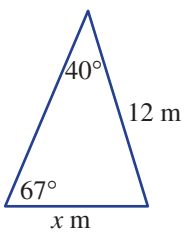
c



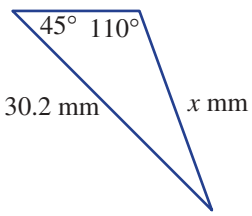
d



e



f

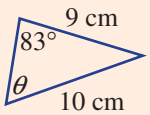


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 22 Finding acute angles using the sine rule

Find the value(s) of θ in these triangle, correct to one decimal place.

θ is acute.



SOLUTION

$$\frac{10}{\sin 83°} = \frac{9}{\sin \theta}$$

$$10 \times \sin \theta = 9 \times \sin 83°$$

$$\sin \theta = \frac{9 \times \sin 83°}{10}$$

$$\theta = 63.289\ldots$$

$$= 63.3° \text{ (to 1 decimal place)}$$

EXPLANATION

Substitute known values into the sine rule and rearrange to make $\sin \theta$ the subject.

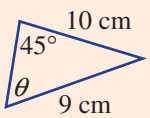
Use $\sin^{-1}$ on your calculator to find the value of θ.

Note: The obtuse angle would be $180° - 63.3° = 116.7°$ but this is not possible.

Now you try

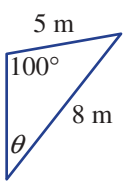
Find the value(s) of θ in these triangle, correct to one decimal place.

θ is acute.

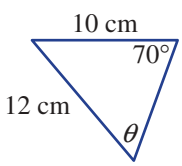


5 Find the value of θ, correct to one decimal place, if θ is acute.

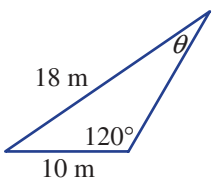
a



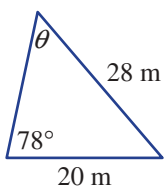
b



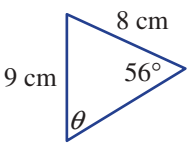
c



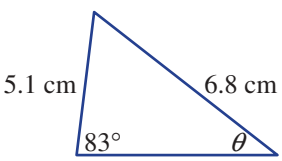
d



e

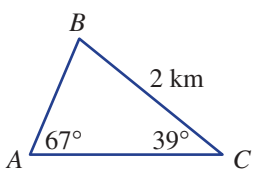


f



PROBLEM-SOLVING AND REASONING 6, 7, 9, 10 6–14

6 Three markers, A, B and C, map out the course for a cross-country race. The angles at A and C are 67° and 39°, respectively, and BC is 2 km.



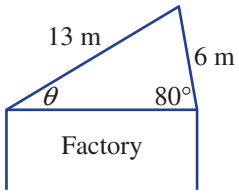
a Find the length AB, correct to three decimal places.

b Find the angle at B.

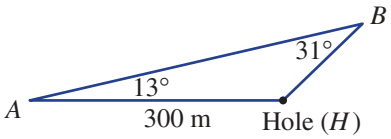
c Find the length AC, correct to three decimal places.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 A factory roof has a steep 6 m section at 80° to the horizontal and another 13 m section. What is the angle of elevation of the 13 m section of roof? Give your answer to one decimal place.



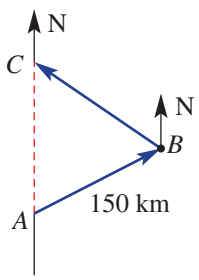
8 A golf ball is hit off-course by 13° to point B. The shortest distance to the hole is 300 m and the angle formed by the new ball position is 31°, as shown. Find the new distance to the hole (BH), correct to one decimal place.



9 An aeroplane is flying due north but, to avoid a storm, it flies 150 km on a bearing of 060° and then on a bearing of 320° until it reaches its original course.

a Find the angles $\angle ABC$ and $\angle ACB$.

b As a result of the diversion, how much farther did the aeroplane have to fly? Round your answer to the nearest kilometre.

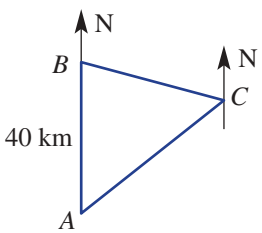


10 A ship heads due north from point A for 40 km to point B, and then heads on a bearing of 100° to point C. The bearing from C to A is 240°.

a Find $\angle ABC$.

b Find the distance from A to C, correct to one decimal place.

c Find the distance from B to C, correct to one decimal place.

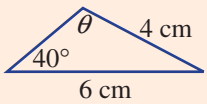


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 23 Finding obtuse angles using the sine rule

Find the value(s) of θ in this triangle, correct to one decimal place.

θ is obtuse.



SOLUTION

$$\frac{4}{\sin 40°} = \frac{6}{\sin \theta}$$

$$4 \times \sin \theta = 6 \times \sin 40°$$

$$\sin \theta = \frac{6 \times \sin 40°}{4}$$

$\theta = 74.6°$ or $180° - 74.6° = 105.4°$

θ is obtuse, so $\theta = 105.4°$.

EXPLANATION

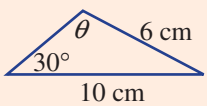
Solve for θ using the sine rule.

The obtuse solution is the supplement of the acute solution.

Now you try

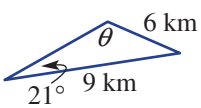
Find the value(s) of θ in these triangle, correct to one decimal place.

θ is obtuse

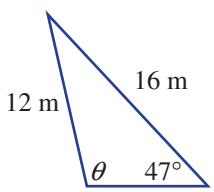


11 Find the value of θ, correct to one decimal place, if θ is obtuse.

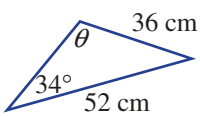
a



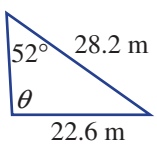
b



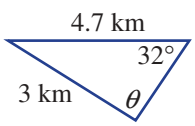
c



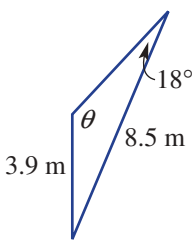
d



e

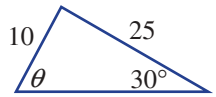


f



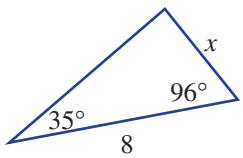
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

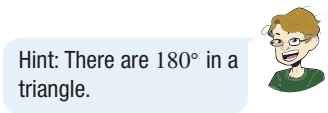
12 Try to find the angle θ in this triangle. What do you notice? Can you explain this result?



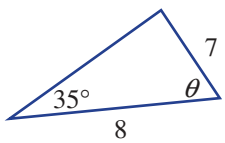
13 A triangle ABC has $\angle C = 25°$, $AC = 13$ cm and $AB = 9$ cm. Find all possible values of $\angle B$, correct to one decimal place.

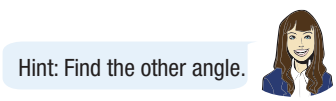
14 **a** Find the value of x.





b Find the value of θ.





ENRICHMENT: More on the ambiguous case — 15

15 When finding a missing angle θ in a triangle, the number of possible solutions for θ can be one or two, depending on the given information.

Two solutions: A triangle ABC has $AB = 3$ cm, $AC = 2$ cm and $\angle B = 35°$.

a Find the possible values of $\angle C$, correct to one decimal place.

b Draw a triangle for each angle for $\angle C$ in part **a**.

One solution: A triangle ABC has $AB = 6$ m, $AC = 10$ m and $\angle B = 120°$.

c Find the possible values of $\angle C$, correct to one decimal place.

d Explain why there is only one solution for $\angle C$ and not the extra supplementary angle, as in parts **a** and **b** above.

e Draw a triangle for your solution to part **c**.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8L The cosine rule

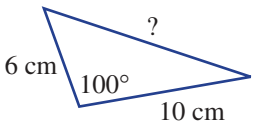
Learning intentions for this section:

- To use the cosine rule to calculate unknown sides and angles in non-right-angled triangles

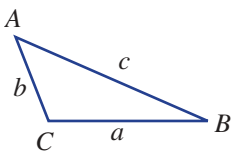
Past, present and future learning:

- This section addresses the Stage 5 Path topic called *Trigonometry C*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- The formula for the cosine rule may be provided in the HSC Examination

When a triangle is defined by two sides and the included angle, the sine rule is unhelpful in finding the length of the third side because at least one of the other two angles is needed.



In such situations, the cosine rule can be used. It relates all three side lengths and the cosine of one angle. This means that the cosine rule can also be used to find an angle inside a triangle when given all three sides.

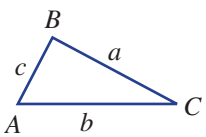


The proof of the cosine rule will be considered in the Enrichment question of this section.

Lesson starter: Cosine rule in three ways

One way to write the cosine rule is like this:

$c^2 = a^2 + b^2 - 2ab\cos C$, where c^2 is the subject of the rule.



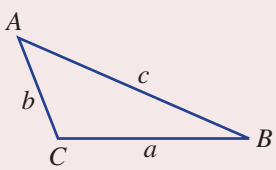
- Rewrite the cosine rule by replacing c with a, a with c and C with A.
- Rewrite the cosine rule by replacing c with b, b with c and C with B.

KEY IDEAS

- The **cosine rule** relates one angle and three sides of any triangle.
- The cosine rule is used to find:
  - the third side of a triangle when given two sides and the included angle
  - an angle when given three sides

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



- When checking that your answer is reasonable, keep in mind that:
  - the longest side is opposite the largest angle
  - the shortest side is opposite the smallest angle

Key terminology: cosine rule, included

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

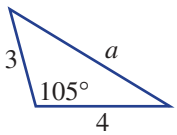
Exercise 8L

UNDERSTANDING	1	1

1 Copy and complete the cosine rule for each triangle below.

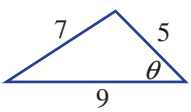
a $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 3^2 + _____ - 2 \times _____ \times _____ \times \cos _____$



b $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos\theta = \dfrac{\square + \square - \square}{2 \times \square \times \square}$



FLUENCY	2–4	2–4

2 Simplify and solve for the unknown (a or θ) in these equations, correct to one decimal place.

a $a^2 = 4^2 + 7^2 - 2 \times 4 \times 7 \times \cos 120°$

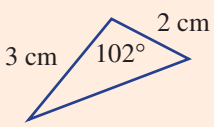
b $a^2 = 1.5^2 + 1.1^2 - 2 \times 1.5 \times 1.1 \times \cos 70°$

c $\cos\theta = \dfrac{7^2 + 6^2 - 9^2}{2 \times 7 \times 6}$

d $\cos\theta = \dfrac{21^2 + 30^2 - 18^2}{2 \times 21 \times 30}$

Example 24 Finding a side length using the cosine rule

Find the length of the third side in this triangle, correct to two decimal places.



SOLUTION

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 3^2 + 2^2 - 2(3)(2)\cos 102°$

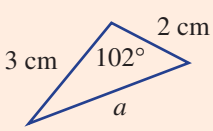
$= 15.49494\ldots$

$\therefore a = 3.9363\ldots$

The length of the third side is 3.94 cm (to 2 decimal places).

EXPLANATION

We have been given two sides and the included angle, so the cosine rule applies.

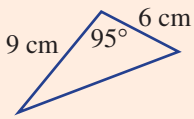


a is the length of the unknown side.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

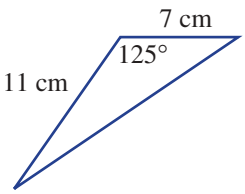
Now you try

Find the length of the third side in this triangle, correct to two decimal places.

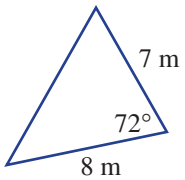


3 Find the length of the third side, correct to two decimal places.

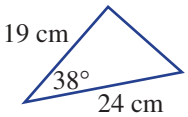
a



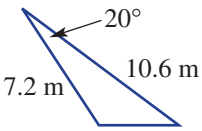
b



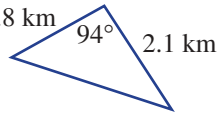
c



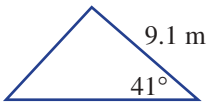
d



e

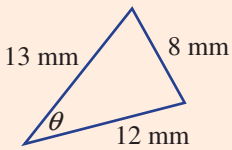


f



Example 25 Finding an angle using the cosine rule

Find the angle θ in this triangle, correct to the nearest minute.



SOLUTION

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

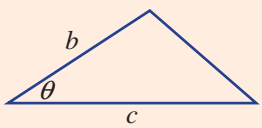
$$\cos\theta = \frac{13^2 + 12^2 - 8^2}{2 \times 13 \times 12}$$

$$\cos\theta = \frac{249}{312}$$

$$\theta = 37.053\ldots$$

$\theta = 37°3'$ (to the nearest minute)

EXPLANATION

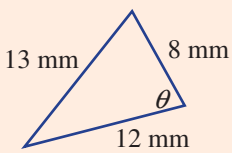


b and c are the two sides nearest to the angle θ.

Use $\cos^{-1}$ to solve for θ.

Now you try

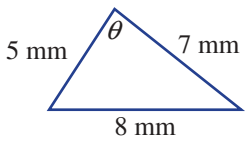
Find the angle θ in this triangle, correct to the nearest minute.



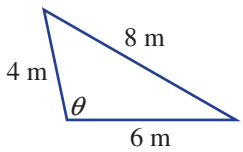
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Find the angle θ, correct to two decimal places.

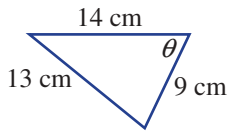
a



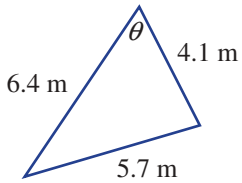
b



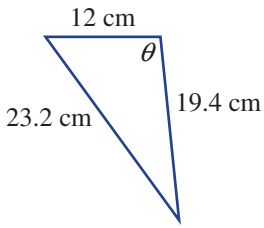
c



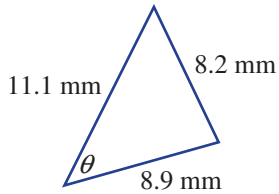
d



e



f



PROBLEM-SOLVING AND REASONING

5, 7, 8, 10 | 7–10

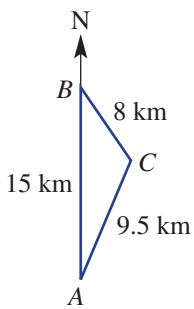
5 A triangular goat paddock has two sides of lengths 320 m and 170 m, and a 71° angle between them. Find the length of the third side, correct to the nearest metre.

6 Find the size of all three angles in a triangle that has side lengths 10 m, 7 m and 13 m. Round each angle to one decimal place.

7 Three camp sites, *A*, *B* and *C*, are planned for a hike and the distances between the camp sites are 8 km, 15 km and 9.5 km, as shown. If campsite *B* is due north of camp site *A*, find the following, correct to one decimal place.

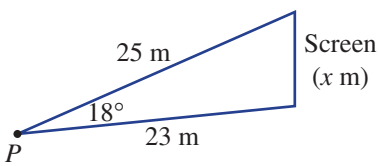
a the bearing from campsite *B* to campsite *C*

b the bearing from campsite *C* to campsite *A*



8 A helicopter on a joy flight over Kakadu National Park travels due east for 125 km, then on a bearing of 215° for 137 km before returning to its starting point. Find the total length of the journey, correct to the nearest kilometre.

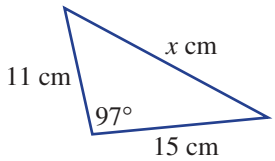
9 The viewing angle to a vertical screen is 18° and the distances between the viewing point *P* and the top and bottom of the screen are 25 m and 23 m, respectively. Find the height of the screen (x m), correct to the nearest centimetre.



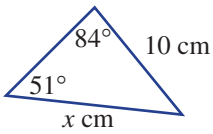
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 Decide whether the cosine rule or sine rule would be used to calculate the value of x in these triangles. Then find the value of x, correct to one decimal place.

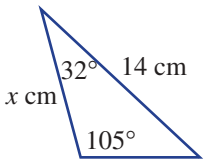
a



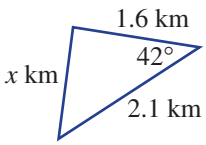
b



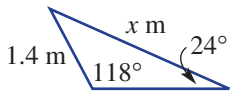
c



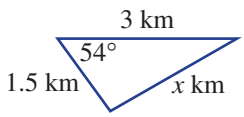
d



e

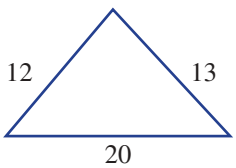


f



ENRICHMENT: Proof of the cosine rule — 11, 12

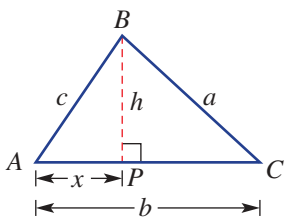
11 Consider the triangle shown.



a Use the cosine rule to find the size of the largest angle.

b Use the sine rule to find the size of the smallest angle.

12 Triangle ABC shown here includes point P such that $PB \perp CA$, $BP = h$ and $AP = x$.



a Write an expression for length CP.

b Use Pythagoras' theorem and ΔABP to write an equation in c, x and h.

c Use Pythagoras' theorem and ΔCPB to write an equation in b, a, x and h.

d Combine your equations from parts **b** and **c** to eliminate h. Simplify your result.

e Use $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ to write an expression for $\cos A$.

f Combine your equations from parts **d** and **e** to prove $a^2 = b^2 + c^2 - 2bc\cos A$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8M Area of a triangle

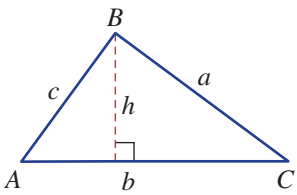
Learning intentions for this section:

- To use two sides and their included angle to calculate the area of a triangle

Past, present and future learning:

- This section concludes the Stage 5 Path topic called *Trigonometry C*
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended
- The formula for the area may be provided in the HSC Examination

We can use trigonometry to establish a rule for the area of a triangle using two sides and the included angle.

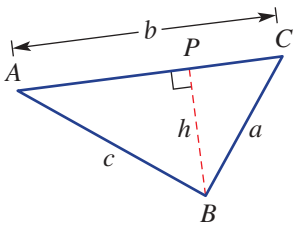


We can see in this triangle that $\frac{h}{a} = \sin C$, so $h = a \sin C$.

$\therefore A = \frac{1}{2}bh$ becomes $A = \frac{1}{2}ba \sin C$.

Lesson starter: Calculating area in two ways

Draw any triangle ABC and construct the height PB. Measure the following as accurately as possible.



i AC
ii BC
iii BP
iv $\angle C$

Now calculate the area using:

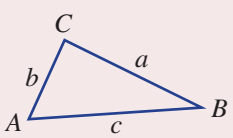
- Area $= \frac{1}{2}bh$
- Area $= \frac{1}{2}ba \sin C$

How close are your answers? They should be equal!

KEY IDEAS

- The area of a triangle is equal to half the product of two sides and the sine of the included angle.

  Area $= \frac{1}{2}ab \sin C$



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 8M

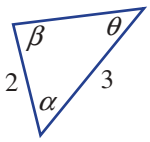
UNDERSTANDING | 1, 2 | 1–3

1 Evaluate $\frac{1}{2}ab\sin C$, correct to one decimal place, for the given values of a, b and C.

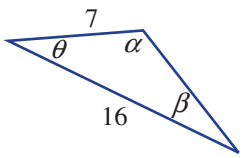
a $a = 3,\ b = 4,\ C = 38°$ **b** $a = 6,\ b = 10,\ C = 74°$ **c** $a = 15,\ b = 7,\ C = 114°$

2 Name the included angle between the two given sides in these triangles.

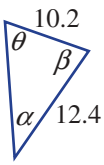
a



b



c



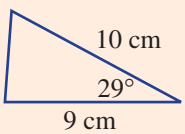
3 Solve these equations for C. Round your answer to two decimal places.

a $10 = \frac{1}{2} \times 4 \times 6 \times \sin C$ **b** $25 = \frac{1}{2} \times 7 \times 10 \times \sin C$ **c** $42 = \frac{1}{2} \times 11 \times 9 \times \sin C$

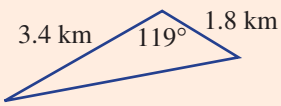
Example 26 Finding the area of a triangle

Find the area of these triangles, correct to one decimal place.

a



b



SOLUTION

a Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 9 \times 10 \times \sin 29°$

$= 21.8\ \text{cm}^2$ (to 1 decimal place)

EXPLANATION

Substitute the two known sides (a and b) and the included angle into the rule.

b Area $= \frac{1}{2}ab\sin C$

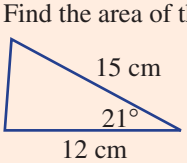
$= \frac{1}{2} \times 1.8 \times 3.4 \times \sin 119°$

$= 2.7\ \text{km}^2$ (to 1 decimal place)

If C is obtuse, then $\sin C$ is positive and the rule can still be used for obtuse-angled triangles.

Now you try

Find the area of this triangle, correct to one decimal place.

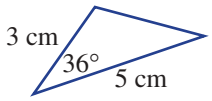


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

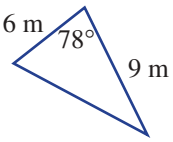
FLUENCY 4–6 4–6

4 Find the area of these triangles, correct to one decimal place.

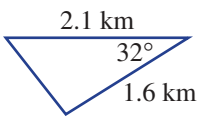
a



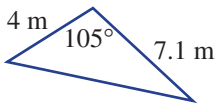
b



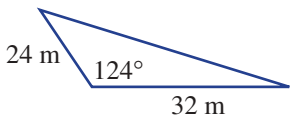
c



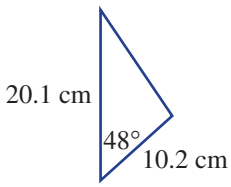
d



e



f



5 Find the area of these triangles, correct to one decimal place.

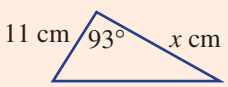
a ΔXYZ if $XY = 5$ cm, $XZ = 7$ cm and $\angle X = 43°$

b ΔSTU if $ST = 12$ m, $SU = 18$ m and $\angle S = 78°$

c ΔEFG if $EF = 1.6$ km, $FG = 2.1$ km and $\angle F = 112°$

Example 27 Finding a side length when given the area

Find the value of x, correct to two decimal places, given that the area of this triangle is 70 cm^2.



SOLUTION

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$70 = \frac{1}{2} \times 11 \times x \times \sin 93°$$

$$140 = 11x \sin 93°$$

$$x = \frac{140}{11 \times \sin 93°}$$

$$= 12.74 \text{ (to 2 decimal places)}$$

EXPLANATION

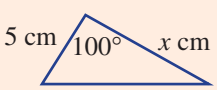
Substitute all the given information into the rule, letting $a = 11$ and $b = x$. Use $\angle C = 93°$ as the included angle.

Multiply both sides by 2 and then divide by $11 \times \sin 93°$.

Solve for x.

Now you try

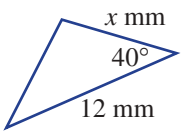
Find the value of x, correct to two decimal places, given that the area of this triangle is 20 cm^2.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

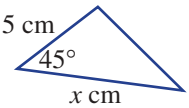
6 Find the value of x, correct to one decimal place, for these triangles with given areas.

a



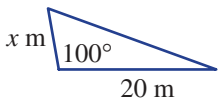
Area = 22 mm^2

b



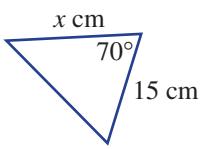
Area = 14 cm^2

c



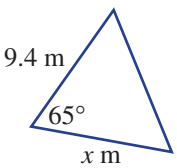
Area = 90 m^2

d



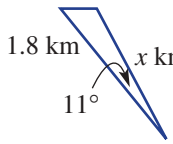
Area = 128 cm^2

e



Area = 45 m^2

f



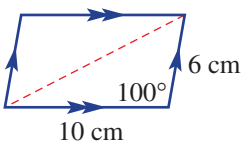
Area = 0.23 km^2

PROBLEM-SOLVING AND REASONING

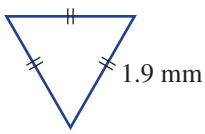
7–10 | 7–13

7 Find the area of these shapes, correct to two decimal places.

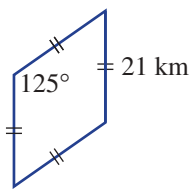
a



b

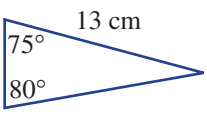


c

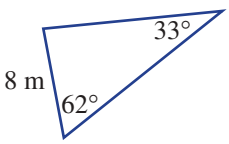


8 First use the sine rule to find another side length, and then find the area of these triangles, correct to two decimal places.

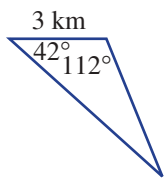
a



b

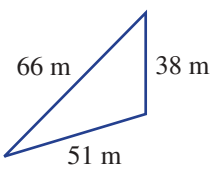


c

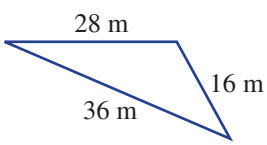


9 First use the cosine rule to find an angle, and then calculate the area of these triangles, correct to two decimal places.

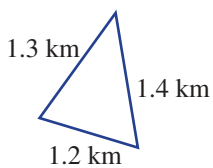
a



b

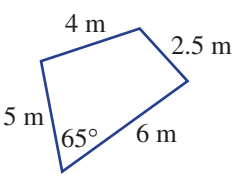


c

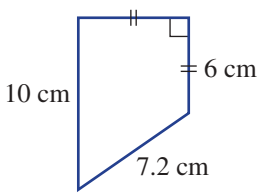


10 Find the area of these quadrilaterals, correct to one decimal place.

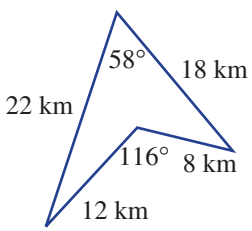
a



b



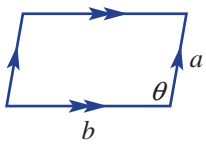
c



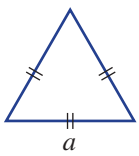
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 Write a rule for the area of these shapes, using the given pronumerals.

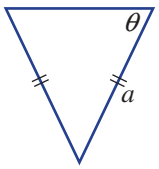
a



b

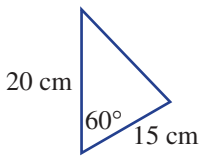


c

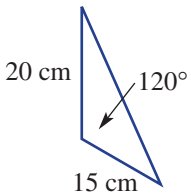


12 a Find the area of these two triangles, correct to one decimal place.

i

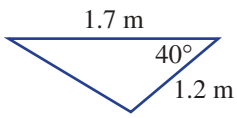


ii



b What do you notice about your answers in part **a**? How can you explain this?

c Draw another triangle that has the same two given lengths and area as the triangle on the right.

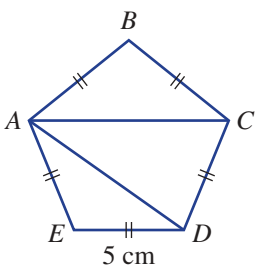


13 a Use the rule Area $= \frac{1}{2}ab\sin C$ to find the two possible values of θ in the triangle described below. Round your answer to one decimal place.
ΔABC with $AB = 11$ m, $AC = 8$ m, included angle θ and area $= 40$ m^2.

b Draw the two triangles for the two sets of results found in part **a**.

ENRICHMENT: Polygon areas — 14

14 You will recall that the sum (S) of the interior angles of a polygon with n sides is given by $S = (n - 2) \times 180°$.



a This regular pentagon has each side measuring 5 cm.

i Calculate the angle sum of a pentagon.

ii Calculate the size of one interior angle of a regular pentagon.

iii Find the area of ΔAED, correct to two decimal places.

iv Find the length AD, correct to two decimal places.

v Find $\angle ADC$ and $\angle DAC$.

vi Find the area of ΔADC, correct to one decimal place.

vii Find the total area of the pentagon, correct to one decimal place.

b Use a similar approach to find the area of a regular hexagon of side length 5 cm, correct to one decimal place.

c Can this method be used for other regular polygons? Explore and give examples.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Surveyor

Surveyors collect and measure data about land and the environment, including measurements for subdivisions of new housing estates, new roads and open-pit mines. They work with plans, files, charts and reports.

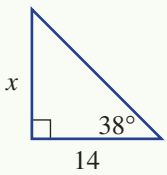
To become a surveyor you can either do a university course, requiring Year 12 mathematics, or apply to TAFE and complete the training there. All courses require a student to have studied mathematics.

Maths@Work

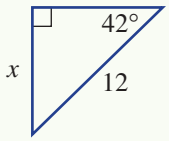
Mathematical skills needed in becoming a land surveyor include geometry and trigonometry. Complete these questions that a surveyor may face in their day-to-day job.

1 Refresh your trigonometry skills by finding the value of each pronumeral in the following triangles. All measurements are in metres, and give answers correct to two decimal places.

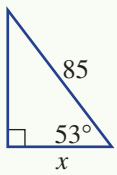
a



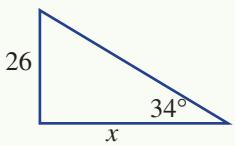
b



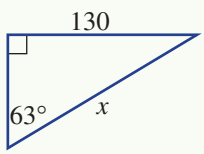
c



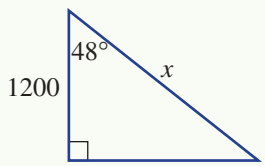
d



e

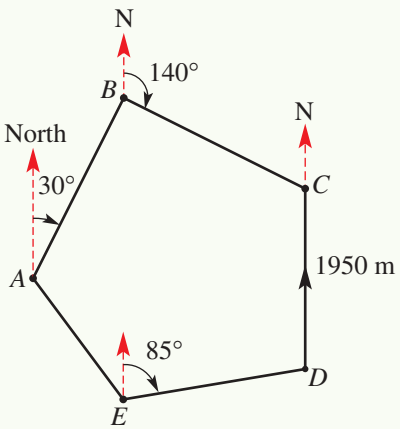


f



In surveying, there are two types of traverse surveys: the closed traverse survey and the open traverse survey.

2 The example below is of a closed traverse survey that could be around a large contained development, such as a school or airport. The diagram is not to scale.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

State answers to the following questions in metres, to two decimal places unless told otherwise.

Hint: For part **a**, start by forming a right-angled triangle with AB as the hypotenuse.

a Given that marker B is 650 m east of A, calculate the distance AB.

b Given that marker C is 900 m south of B, calculate the distance BC.

c Given that marker E is 160 m south of D, calculate the distance DE.

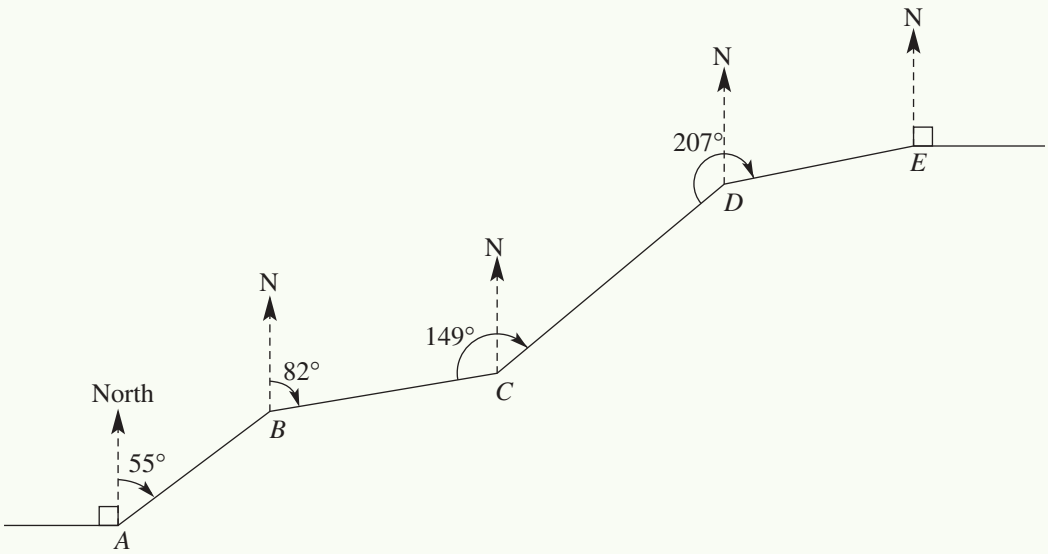
d Marker E is 600 m east of A and 900 m south of A. Calculate the distance AE.

e Calculate the perimeter of the figure $ABCDE$, correct to the nearest metre.

f Calculate the true bearing of B from C.

g Calculate the true bearing of E from D.

3 Below is an example of an open traverse survey that could be the simplified model of a section of road. The diagram is not to scale.



a Complete this table of measurements for the open traverse survey above.
Hints: Draw a large, ruled and labelled diagram with a north–south line at each marker. Recall the rules for angles between parallel lines.

Marker	Distance to the next marker (metres)	True bearing of the next marker from the current marker	Angle between line segments at marker (clockwise)	Distance east to the next marker (m)	Distance north to the next marker (m)
A	98.00	055°			
B	125.36	082°			
C	157.80		149°		
D	104.42		207°		
E	–	–		–	–

b How far east is marker E from A? Give your answer in metres to two decimal places.

c How far north of marker A is E? Give your answer in metres to two decimal places.

d By considering a right-angle triangle with hypotenuse AE, what is the true bearing of E from A? Give your answer to the nearest degree.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Using technology

4 Use a digital drawing program to create a labelled diagram of a closed traverse survey for a town park. Your diagram of the park must include:

- a true north line on each marker
- true north bearings of each next marker from each marker
- the length, in metres, of each boundary segment
- internal angles between line segments (these need to be calculated)
- the perimeter of the park

Your diagram doesn't need to be to scale but it should be a close representation of the survey data.

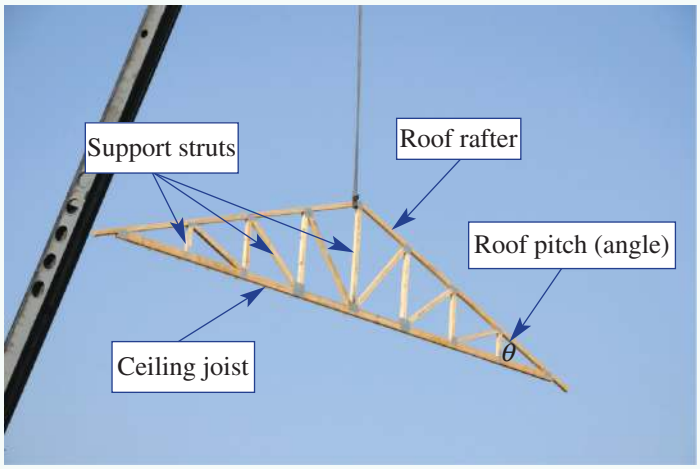
Closed traverse survey data for Penny Park			
Marker	**True north bearing of next marker from current marker**	**Distance from current marker to next marker**	**Internal angle between line segments at marker**
A	30°	394 m	
B	99°	658 m	
C	180°	500 m	
D	266°	590 m	
E	324°	360 m	

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Working mathematically

Building a roof truss

A house roof is supported by wooden frames called trusses. A truss has a sloping rafter on each side and a ceiling joist along the bottom. Several shorter support struts are attached in between the rafters and the ceiling joist.



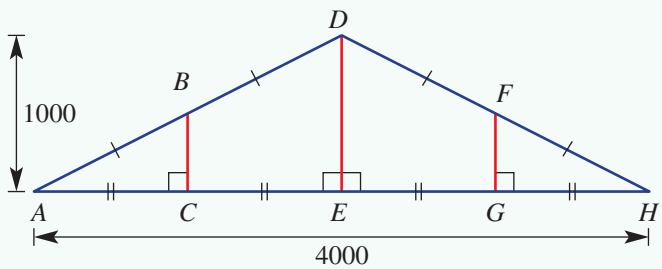
This roof truss is being lowered into position by a crane. How many rafters are there? How many support struts can you count? Is the roof pitch an angle or a length?

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Note: Each truss shape is symmetrical. Diagrams are not to scale. Ignore the width of all boards. Measurements are in millimetres. Round lengths to the nearest millimetre and angles to two decimal places.

Routine problems

The truss shown below has ceiling joist $AH = 4000$ mm and support strut $DE = 1000$ mm. There are many equal lengths, as marked on the diagram.



- **a** Calculate the length of the roof rafters, DA and DH, that are on each side of this truss.
- **b** Calculate the pitch of the roof, which is the angle between each rafter and the horizontal ceiling joist, such as $\angle DAE$.
- **c** Draw and label $\triangle ABC$ and calculate the length of the support strut BC.
- **d** State the length of each support strut, e.g. $DE = 1000$ mm.
- **e** Determine the area of $\triangle ADH$ in square metres.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Non-routine problems

Explore and connect

A builder's house plans are an example of a mathematical model.

You are to prepare a report with the requested measurements of the given truss styles.

The problem is to determine which of the given truss styles gives the greatest volume of ceiling space.

a With the aid of diagrams, write down all the relevant mathematical rules that can be used to calculate lengths and angles in right-angled triangles.

b With the aid of diagrams, write down the relevant mathematical rules for calculating the volume of a triangular prism.

Choose and apply techniques

For each truss style below, calculate:

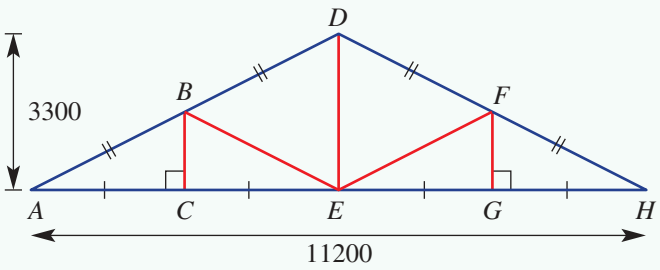
i length of the rafter AD in millimetres.

ii the roof pitch $\angle BAC$ to two decimal places.

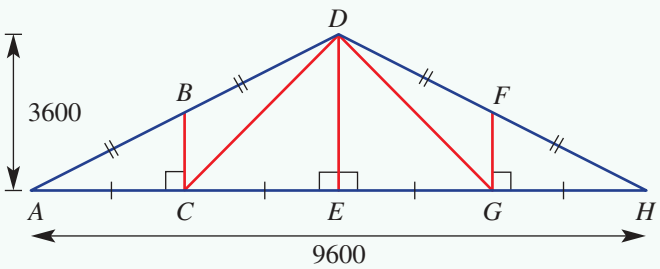
iii the length of each support strut in millimetres. (e.g. $DE = 3300$ mm to nearest millimetre where necessary).

iv the area of the truss triangle in square metres to two decimal places (e.g. area of ΔADH).

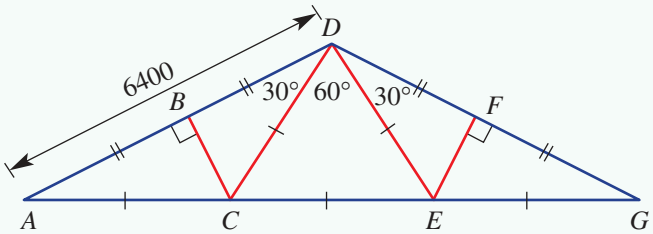
c Howe Truss



d Pratt Truss



e Fink Truss



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Communicate thinking and reasoning

Assume a house of length 15 m uses the same truss style along the full length of the house.

f What type of prism is formed by the ceiling space under the roof?

g Determine which truss style above (in **c**, **d** and **e**) will allow the greatest volume of ceiling space. Justify your conclusion with appropriate calculations of volume in cubic metres to two decimal places.

h Summarise your results in a table with the following headings.

	Truss area (m^2)	Ceiling space volume (m^3)
Howe Truss		
Pratt Truss		
Fink Truss		

Extension problems

Problem-solve

Assume a house of length 15 m uses the same truss style along the full length of the house.

a What shape is each side of its roof surface?

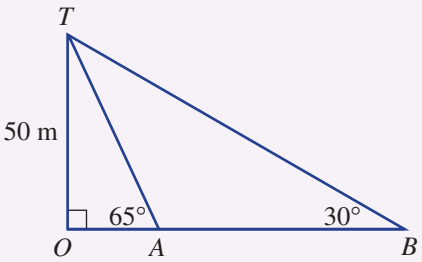
b Consider each of the truss styles above (in **c**, **d** and **e**) and determine which style will allow the greatest surface area of the roof. Justify your conclusion with appropriate calculations of area in square metres to two decimal places.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1 What is the opposite direction to:

a East? **b** NE? **c** SE? **d** 018°? **e** 300°?

2 Use two different right-angled triangles to find the distance from A to B in this diagram, correct to two decimal places.



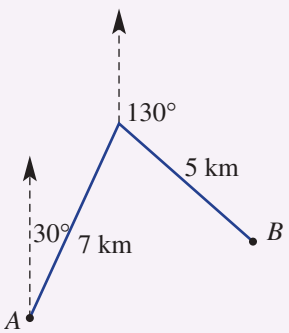
3 Make up your own saying using the letters in SOH CAH TOA as the first letter of each word.

S ___ O ___ H ___ C ___ A ___ H ___ T ___ O ___ A ___

4 In the wordfind below there are 17 terms that are used in this chapter. See if you can locate all 17 terms and write a definition or draw a diagram for each of them.

S	I	D	R	T	Y	I	P	Y	T	S	O	H	T	H
I	D	E	P	R	E	S	S	I	O	N	D	Y	O	Y
N	T	E	I	N	T	E	W	P	Y	A	D	J	H	P
E	I	O	E	E	D	S	V	B	Y	T	P	U	W	O
Q	U	O	T	R	S	I	A	D	J	A	C	E	N	T
A	D	A	N	G	E	D	E	P	R	A	N	G	L	E
S	E	N	A	R	T	E	E	G	R	I	L	K	O	N
D	P	T	R	I	A	N	G	L	E	H	E	I	P	U
C	T	R	I	G	O	N	O	M	E	T	R	Y	P	S
B	A	C	G	B	L	I	N	O	Y	A	A	W	O	E
H	D	O	H	T	E	A	N	G	L	N	T	H	S	H
U	J	S	T	R	G	A	O	K	Y	G	I	I	I	Y
J	E	I	W	F	L	N	R	M	K	E	O	D	T	P
K	N	N	T	J	T	R	N	I	O	N	P	Z	E	O
E	L	E	V	A	T	I	O	N	N	T	P	A	M	B
E	T	B	A	S	P	Y	T	H	A	G	O	R	A	S

5 Find the bearing from A to B in this diagram.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

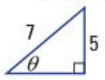
Chapter summary

Trigonometry

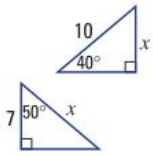
Finding an angle

$\sin\theta = \frac{5}{7}$

$\theta = 45°35'$ (to the nearest minute)



Finding lengths



$\sin 40° = \frac{x}{10}$

$\therefore x = 10 \times \sin 40°$

$\cos 50° = \frac{7}{x}$

$x \times \cos 50° = 7$

$x = \frac{7}{\cos 50°}$

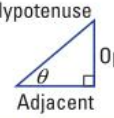
Trigonometric ratios

SOHCAHTOA

$\sin\theta = \frac{O}{H}$

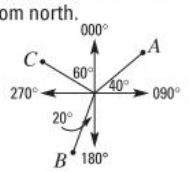
$\cos\theta = \frac{A}{H}$

$\tan\theta = \frac{O}{A} = \frac{\sin\theta}{\cos\theta}$



Bearings

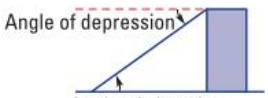
Bearings are measured clockwise from north.



A: 050°
B: 200°
C: 300°

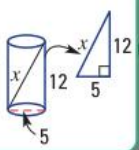
2D applications

- Draw the right-angled triangle with key information.
- Use trigonometry to find unknown.



3D applications

- Identify and redraw right-angled triangle.
- Use trigonometric ratios.
- Answer in words.



Cosine rule

Rule is used to find:

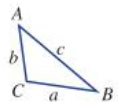
- third side, when given two sides and included angle

$a^2 = b^2 + c^2 - 2bc\cos A$

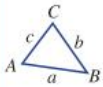
or

- an angle, when given all three sides

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$



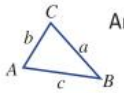
Sine rule



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

When finding angles, determine whether you require the acute or the obtuse angle.

Area of a triangle

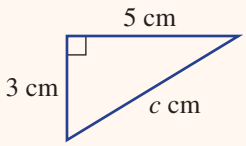
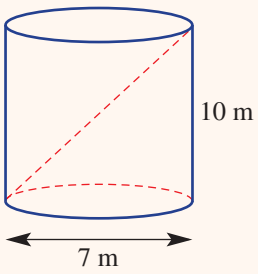
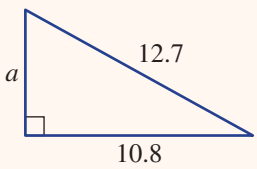
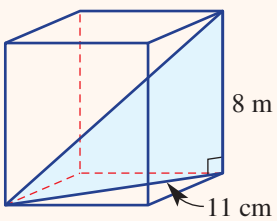
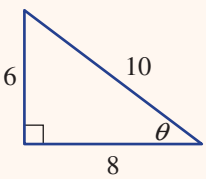
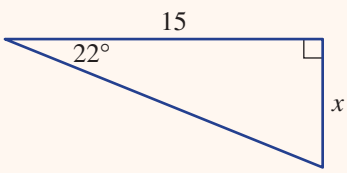


Area $= \frac{1}{2}ab\sin C$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

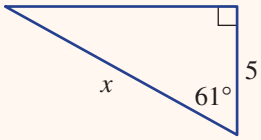
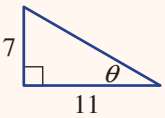
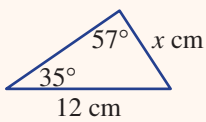
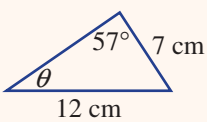
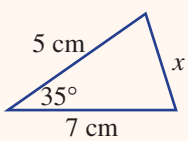
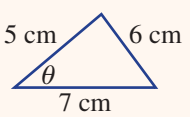
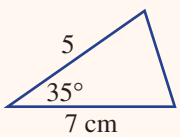
A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
8A	**1 I can find the length of the hypotenuse for a right-angled triangle given the length of the other two sides.** e.g. Find the length of the hypotenuse of this triangle, correct to one decimal place. 	☐
8A	**2 I can apply Pythagoras' theorem and find the length of the hypotenuse in a real situation.** e.g. Find the length of the longest rod that will fit into this cylinder, correct to two decimal places. 	☐
8B	**3 I can use Pythagoras' theorem to find the length of a shorter side in a right-angled triangle.** e.g. Find the value of a in this triangle, correct to one decimal place. 	☐
8C	**4 I can apply Pythagoras' theorem to solve a range of real-life problems.** e.g. Two bushwalkers leave their camp at the same time. One walks due south for 5 km and the other walks due west for 4 km. How far apart are the bushwalkers at this point? Round to two decimal places.	☐
8C	**5 I can apply Pythagoras' theorem to find a length in a three-dimensional solid.** e.g. Find the distance from one corner of this rectangular prism to the opposite corner, correct to two decimal places. 	☐
8D	**6 I can write a ratio for sine, cosine and tangent of an angle in a triangle with given side lengths.** e.g. Write a ratio for $\sin\theta$, $\cos\theta$ and $\tan\theta$ for this triangle. 	☐
8E	**7 I can find a missing length on a right-angled triangle given an angle and another side.** e.g. Find the value of x in this triangle, correct to two decimal places. 	☐

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

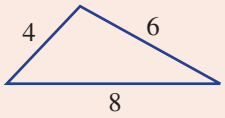
Chapter checklist with success criteria

8F	**8 I can find a missing length in a right-angled triangle if the pronumeral sits in the denominator of the fraction.** e.g. Find the value of x in this triangle, correct to two decimal places. 	☐
8G	**9 I can find an angle in a right-angled triangle given any two side lengths.** e.g. Find the angle θ in this triangle, correct to one decimal place. 	☐
8H	**10 I can solve problems in trigonometry using angles of elevation and depression.** e.g. The angle of elevation to the top of a vertical pole from a point on the ground 50 m away horizontally is $27°$. Find the height of the pole, correct to the nearest metre.	☐
8H	**11 I can find angles of elevation or depression in a real situation if given two other lengths from a right-angled triangle.** e.g. Find the angle of elevation of a 40 m high tower from a point which is 30 m horizontally from the base of the tower. Round to one decimal place.	☐
8I	**12 I can interpret a bearings problem and draw a suitable right-angled triangle.** e.g. A yacht travels 10 km on a true bearing of $210°$. How far east of the starting point is the yacht? Show all this information on a diagram using a right-angled triangle. You do not need to solve the problem.	☐
8I	**13 I can find an unknown distance in a problem involving bearings.** e.g. A hiker leaves base camp and walks 8 km on a true bearing of $235°$. How far west of base camp is the hiker at this point? Round to the nearest degree.	☐
8J	**14 I can use Pythagoras' theorem and trigonometry to solve three-dimensional problems.** e.g. Find the longest pencil that fits into a box measuring 5 cm $\times$ 6 cm $\times$ 8 cm.	☐
8K	**15 I can use the sine rule to find an unknown side of a non-right-angled triangle.** e.g. Find the value of x 	☐
8K	**16 I can use the sine rule to find an unknown angle in a non-right-angled triangle.** e.g. Find the value of θ 	☐
8L	**17 I can use the cosine rule to find an unknown side of a non-right-angled triangle.** e.g. 	☐
8L	**18 I can use the cosine rule to find an unknown angle in a non-right-angled triangle.** e.g. 	☐
8M	**19 I can use the formula for the area of a triangle.** e.g. Find the area of the triangle. 	☐

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

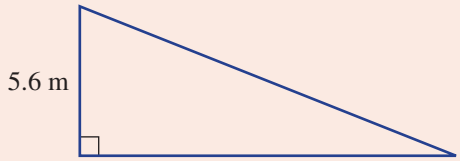
Short-answer questions

8A **1** Determine whether the triangle shown contains a right angle.

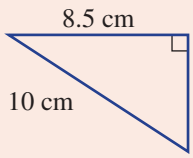


8A/8B **2** Find the missing length, correct to two decimal places, in these triangles.

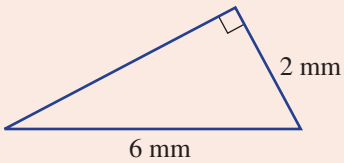
a



b



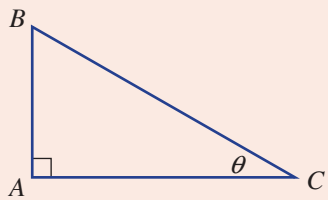
c



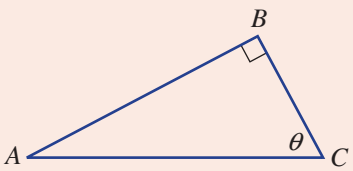
8D **3** Which side (AB, AC or BC) of these triangles is:

i the hypotenuse? **ii** opposite to θ? **iii** adjacent to θ?

a



b

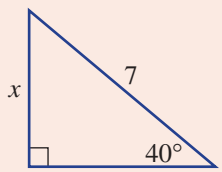


8D **4** Use a calculator to find the value of the following, rounding to two decimal places.

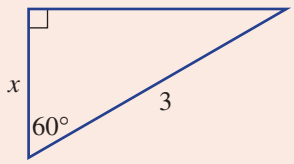
a $\sin 35°$ **b** $\cos 17°$ **c** $\tan 83°15'$

8E **5** Find the value of the unknown length (x) in these triangles. Round to two decimal places where necessary.

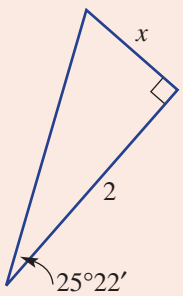
a



b

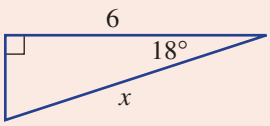


c

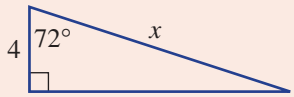


8F **6** Find the value of the unknown length (x) in these right-angled triangles. Round to two decimal places.

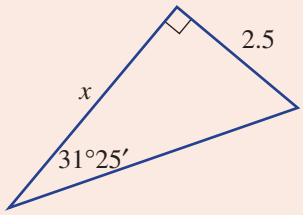
a



b



c



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

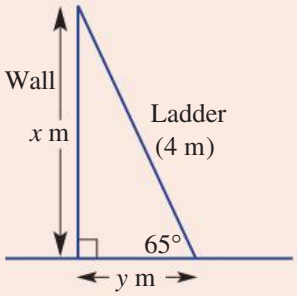
Chapter review

8E

7 A 4 m ladder leans, as shown, against a wall at an angle of 65° to the horizontal.

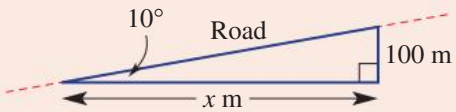
a Find how high up the wall the ladder reaches (x m), correct to two decimal places.

b Find how far the bottom of the ladder is from the wall (y m), correct to two decimal places.



8F

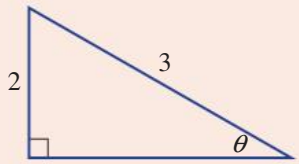
8 A section of road has a slope of 10° and gains 100 m in height. Find the horizontal length of the road (x m), correct to two decimal places.



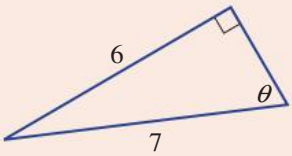
8G

9 Find θ in the following right-angled triangles, correct to the nearest minute.

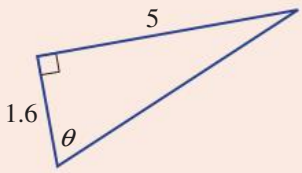
a



b



c

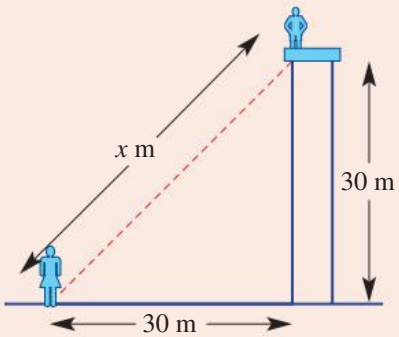


8H

10 Raymond and Sofia view each other from two different places, as shown. Raymond is on a viewing platform, whereas Sofia is 30 m from the base of the platform, on the ground. The platform is 30 m above the ground.

a Find the angle of elevation from Sofia's feet to Raymond's feet.

b Using your answer to part a, find the distance (x) between Sofia and Raymond, correct to one decimal place.



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8I

11 A hiker walks on a true bearing of 220° from point A for 3 km to point B.

a Find how far south the hiker has walked, correct to one decimal place.

b Find how far west the hiker has walked, correct to one decimal place.

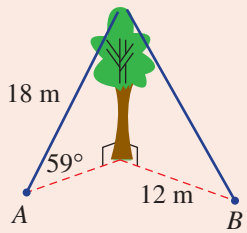
c What is the bearing of point A from B?

8I

12 A tree is being supported by two ropes, as shown. The rope to point A is 18 m long and makes an angle of 59° with the ground. Point B is 12 m from the base of the tree.

a Find the height of the tree, to two decimal places.

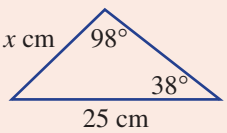
b Find the angle the rope to point B makes with the ground, to the nearest degree.



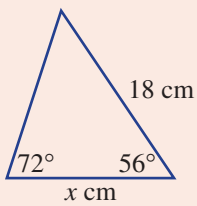
8K/L/M

13 For these triangles, find the following, correct to one decimal place.

a **i** the value of x

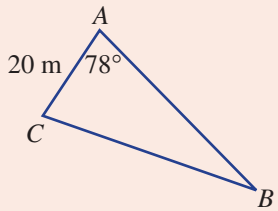


b **ii** the area of the triangle



8M

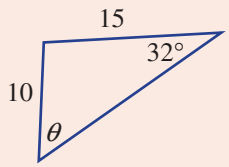
14 Three fences are used to form a triangular pig pen with known dimensions as shown in the diagram. If the area of the pig pen is 275 m^2, what is the length AB? Round your answer to one decimal place.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

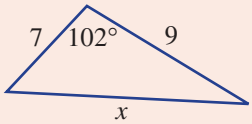
Chapter review

8K **15** Use the sine rule to find the value of θ, correct to one decimal place.

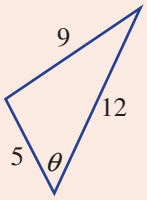


8L **16** Use the cosine rule to find the value of the pronumeral, to one decimal place.

a

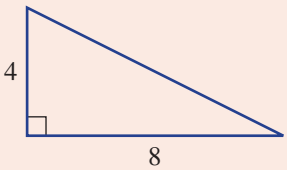


b



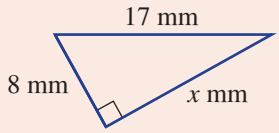
Multiple-choice questions

8A **1** The length of the hypotenuse in the triangle shown is closest to:



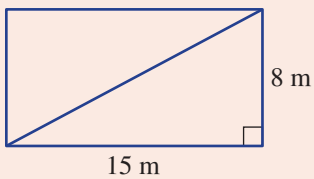
A 10 **B** 9 **C** 4
D 100 **E** 64

8B **2** The length of the side marked x in the triangle shown is:



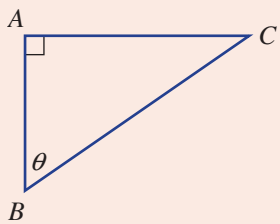
A 23 **B** 17 **C** 12
D 19 **E** 15

8C **3** For the shape shown to be a rectangle, the length of the diagonal must be:



A 15 m **B** 8 m **C** 17 m
D 23 m **E** 32 m

8D **4** Which side (AB, AC or BC) is adjacent to θ in this triangle?

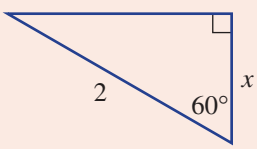


A AC **B** AB **C** BC
D hypotenuse **E** opposite

8D **5** The value of cos 21° is closest to:

A −0.55 **B** 0.9 **C** 0.9336
D 0.93 **E** 0.934

8E **6** The value of x in this triangle is:



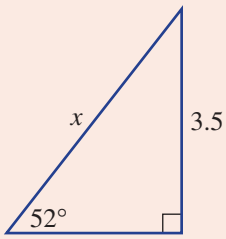
A $2 \div \cos 60°$ **B** $2 \div \sin 60°$
C $2 \times \cos 60°$ **D** $2 \times \sin 60°$
E $2 \times \tan 60°$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8F

7 The value of x in this triangle is closest to:

A 2.76 **B** 4.48 **C** 5.68
D 4.44 **E** 2.73



8F

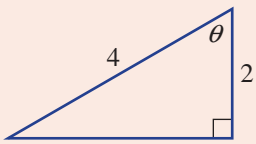
8 A metal brace sits at 55° to the horizontal and reaches 4.2 m up a wall. The distance between the base of the wall and the base of the brace is closest to:

A 6.00 m **B** 2.41 m **C** 7.32 m
D 5.13 m **E** 2.94 m

8G

9 The angle θ in this triangle is:

A 60° **B** 30° **C** 26.57°
D 20° **E** none of the above



8H

10 The angle of depression from the roof of a building to a trampoline is 75°. If the roof is 12 m above the level of the trampoline, then the distance of the trampoline from the building is closest to:

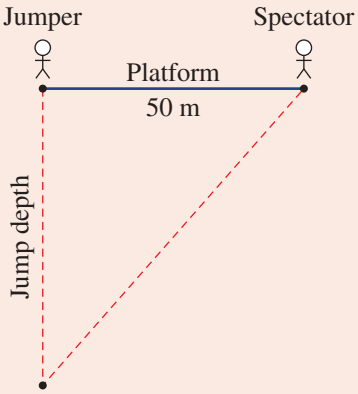
A 12.42 m **B** 11.59 m **C** 3.22 m
D 44.78 m **E** 3.11 m

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter review

Extended-response questions

1 A spectator is viewing bungee jumping from a point 50 m to the side but level with the jumping platform.



a The first bungee jumper has a maximum fall of 70 m. Find the angle of depression from the spectator to the bungee jumper at the maximum depth, correct to two decimal places.

b The second bungee jumper's maximum angle of depression from the spectator is 69°. Find the jumper's maximum depth, correct to two decimal places.

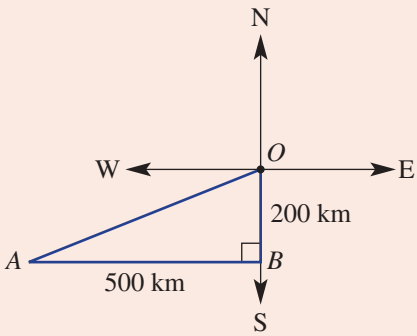
c The third jumper wants to do the 'Head Dunk' into the river below. This occurs when the spectator's angle of depression to the river is 75°. Find, correct to the nearest metre, the height of the platform above the river.

2 A military plane flies 200 km from point O to point B, then west 500 km to point A.

a How far is A from O, to the nearest kilometre?

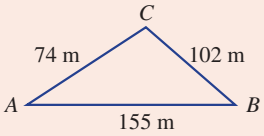
b What is angle BOA, correct to the nearest degree?

c What is the bearing of A from O?



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 A paddock, ABC, is fenced off, as shown in the figure.



a Find $\angle A$, to three decimal places.

b Hence, find the area enclosed by the fences. Round your answer to two decimal places.

It is planned to divide the paddock into two triangular paddocks by constructing a fence from point C to meet AB at right angles at a point D.

c Determine how many metres of fencing will be required along CD, to the nearest centimetre.

d How far is point D from point A, to the nearest centimetre?

e The person who constructs the fence CD misinterprets the information and builds a fence that does not meet AB at right angles. The fence is 45 metres long.

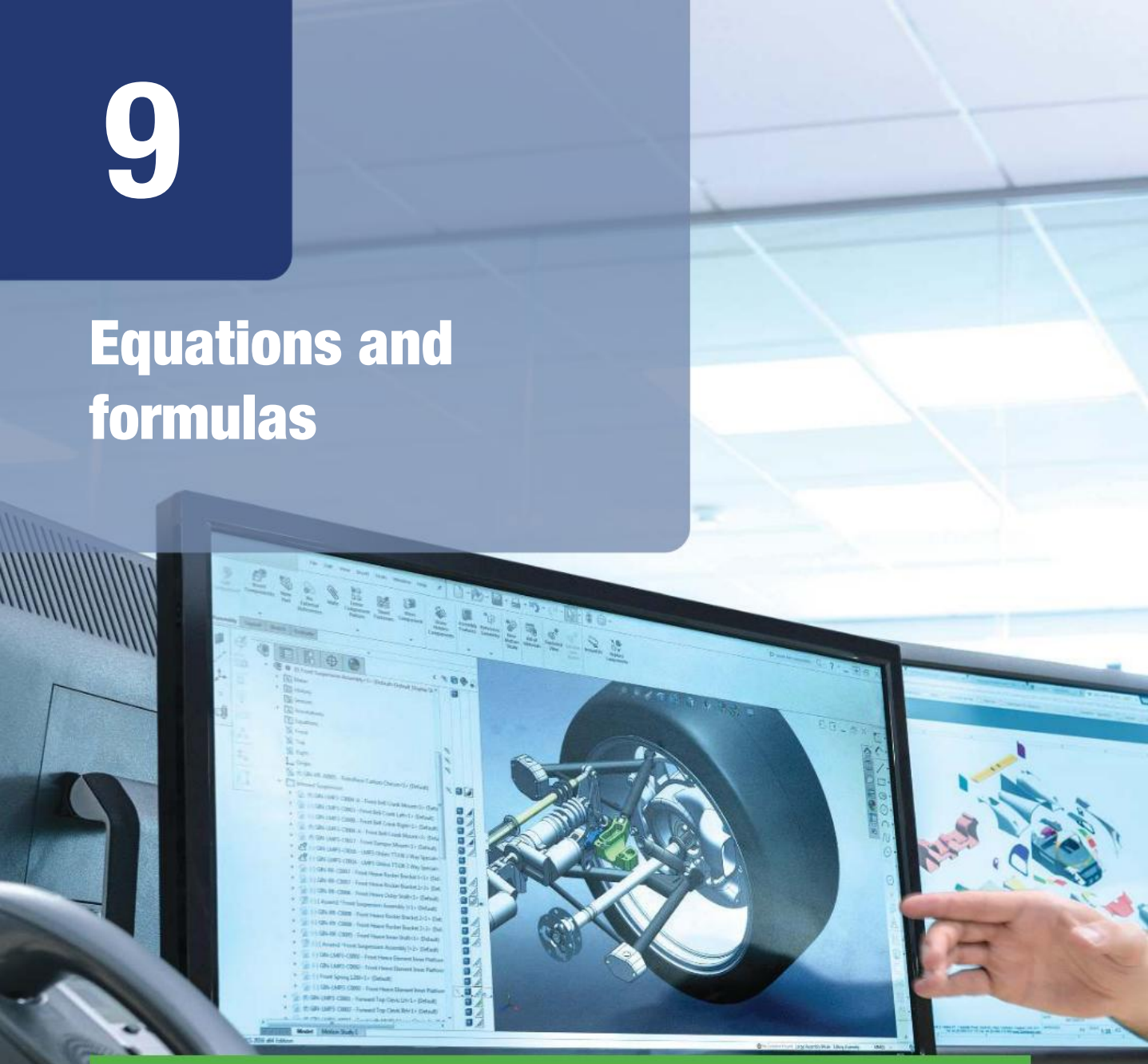
i Determine, to two decimal places, the two possible angles (i.e. acute and obtuse) this fence line makes with AB.

ii Hence, find the two possible distances of fence post D from A. Round your answer to one decimal place.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9

Equations and formulas



Why solving equations is important

Solving equations happens all the time in professional sport, almost every type of trade, and in every business.

- Car designers and engineers use equations to optmise the strength of the materials, flow of fluids through the engine, fuel efficiency, friction on the tyres, safety of the car in a collision and many other uses.
- Personal finance decisions can be assisted by solving simultaneous equations. For example, finding the best deal between various rental properties, running costs for cars or quotes from trade workers.
- Construction workers such as engineers, electricians, builders, carpenters and concreters solve equations to find the cost of materials, time a job will need and profit.
- Financial analysts create straight line graphs of profit and costs vs number of sales. A profit occurs after the point of intersection, where the profit line rises above the costs line.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- To be successful, businesses analyse money flow. Solving equations can determine affordable stock and staff levels.

Chapter contents

9A Linear equations with a pronumeral on one side (CONSOLIDATING)
9B Equations with brackets, fractions and pronumerals on both sides
9C Solving equations of the form $ax^2 = c$ (CONSOLIDATING)
9D Using formulas
9E Solving simultaneous equations graphically (EXTENDING)
9F Solving simultaneous equations using substitution (EXTENDING)
9G Solving simultaneous equations using elimination (EXTENDING)

NSW Syllabus

In this chapter, a student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly (MAO-WM-01)
- solves linear equations of up to 3 steps, limited to one algebraic fraction (MA5-EQU-C-01)
- solves linear equations of more than 3 steps, monic and non-monic quadratic equations, and linear simultaneous equations (MA5-EQU-P-02).

© 2022 NSW Education Standards Authority

Online resources

A host of additional online resources are included as part of your Interactive Textbook, including HOTmaths content, video demonstrations of all worked examples, auto-marked quizzes and much more.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Warm-up quiz

1 When $a = 6$ and $b = -3$, evaluate the following.

a $a - b$ **b** ab

c b^2 **d** $3(a + 2b)$

2 When $m = 4$, $n = 7$ and $p = -2$, evaluate the following.

a $4m + p$ **b** $p(4 - n)$

c $\frac{8m}{p}$ **d** $2m^2$

3 Simplify the following.

a $a + 2a$ **b** $4m - m$

c $6p + 2p$ **d** $7m - 7m$

e $2m - 7m$ **f** $8x + y - x$

g $8p + 4p - 3p$ **h** $7m - 4m + 3m$

4 Simplify the following.

a $5x \times 3$ **b** $4p \times 4$ **c** $8x \times 4y$

d $6a \times (-5)$ **e** $a \times b$ **f** $6x \div 6$

g $m \div m$ **h** $6a \div 3$ **i** $\frac{15a}{5a}$

5 Complete the following.

a $x + 5 - \square = x$ **b** $w - 3 + \square = w$

c $p - 5 + \square = p$ **d** $z + 1 - \square = z$

e $w \times 4 \div \square = w$ **f** $a \div 2 \times \square = a$

g $m - 3 + \square = m$ **h** $2m \div \square = m$

i $\frac{x}{4} \times \square = x$ **j** $\frac{m}{3} \times \square = m$

k $6a \div \square = a$ **l** $10x \div \square = x$

6 Write an expression for each of the following.

a the sum of x and 3 **b** six more than n

c double w **d** half of x

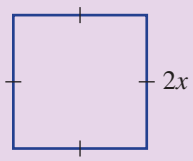
e six more than double x **f** seven less than x

g three more than x and then doubled **h** one more than triple x

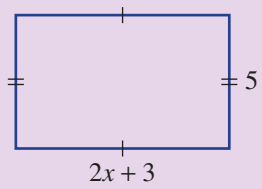
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 Write an expression for the perimeter of the following.

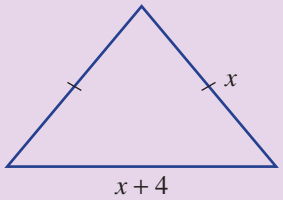
a



b



c



8 Answer true (T) or false (F) for whether the following are equations.

a $x + 3$ **b** $3x - 6 = 9$ **c** $x^2 - 8$

d $2x$ **e** $3a = 12$ **f** $x^2 = 100$

g $1 = x - 3$ **h** $m - m$ **i** $2p = 0$

9 Solve the following simple equations.

a $m + 3 = 10$ **b** $y - 7 = 12$

c $3x = 15$ **d** $\frac{b}{4} = 3$

10 Answer true (T) or false (F) to the following.

a $x = 3$ is a solution of the equation $x^2 = 9$

b $x = -3$ is a solution of the equation $x^2 = 9$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9A Linear equations with a pronumeral on one side CONSOLIDATING

Learning intentions for this section:

- To be able to solve simple linear equations with a pronumeral on one side only
- To know what a solution to an equation is and to be able to verify a solution to an equation

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- It consolidates and extends the concepts in Section 3E of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

A cricket batter will put on socks, then cricket shoes and, finally, pads, in that order. When the game is over, these items are removed in reverse order: first the pads, then the shoes and, finally, the socks. A similar reversal occurs when solving equations.

We can undo the operations around the pronumeral (e.g. x) by applying the opposite operations in the reverse order to how they have been applied to the pronumeral. To keep each equation balanced, we always apply the same operation to both sides of an equation.

For example:

Applying operations to $x = 7$

$x = 7$

$\times 2$ $\times 2$

$2x = 14$

$+12$ $+12$

$2x + 12 = 26$

Solving an equation

$2x + 12 = 26$

-12 -12

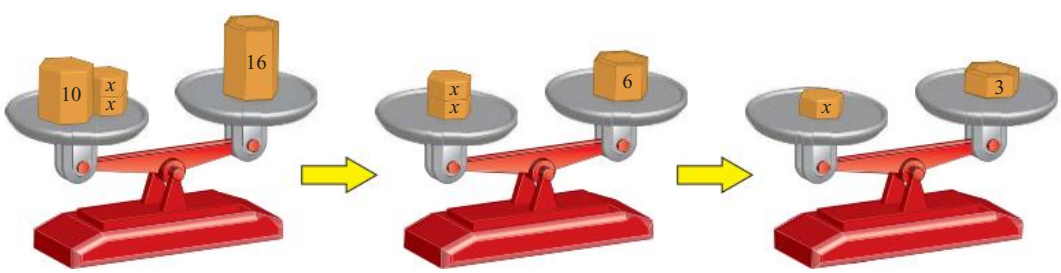
$2x = 14$

$\div 2$ $\div 2$

$x = 7$

Lesson starter: Keeping it balanced

Three scales are each balanced with various weights on the left and right pans.



Balanced scales 1 Balanced scales 2 Balanced scales 3

- What weight has been removed from each side of scales 1 to get to scales 2?
- What has been done to both the left and right sides of scales 2 to get to scales 3?
- What equations are represented in each of the balanced scales shown above?
- What methods can you recall for solving equations?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

KEY IDEAS

- An **equation** is a mathematical statement that includes an equals sign. The equation will be true only when the left-hand side is equal to the right-hand side.
  For example: $\frac{5x}{6} = -2$, $3p + 2t = 6$ are equations.
  $6x - 13$ is not an equation.

- A **linear equation** contains a pronumeral (e.g. x) to the power of 1 and no other powers.
  For example: $3x - 5 = 7$, $4(m - 3) = m + 6$ are linear equations.
  $x^2 = 49$ is not linear.

- Every equation has a left-hand side (LHS) and a right-hand side (RHS).
  For example: $\underbrace{2x + 12}_{\text{LHS}} = \underbrace{26}_{\text{RHS}}$

- To **solve** an equation, undo the operations built around the pronumeral by doing the opposite operation in the reverse order.
  - Always perform the same operation to both sides of an equation so it remains balanced.

  Equations are solved like this:

  $5x + 2 = 17$
  -2 -2
  $5x = 15$
  $\div 5$ $\div 5$
  $x = 3$

  $x = 3$ is called 'the solution'.

- To **verify** means to check that the solution is correct by substituting the answer to see if it makes the equation true.
  For example:
  Verify that $x = 3$ is a solution to $5x + 2 = 17$, using substitution.

  $\text{LHS} = 5x + 2 \qquad \text{RHS} = 17$
  $= 5 \times 3 + 2$
  $= 17$
  $\text{LHS} = \text{RHS}$
  $\therefore x = 3$ is a solution.

- Sometimes it is a good idea to swap the LHS and RHS.
  For example: $15 = 2x + 3$ is the same as $2x + 3 = 15$.

Key terminology: equation, linear equation, solve, variable, pronumeral, verify, substitute, solution

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 9A

UNDERSTANDING | 1–3 | 3

1 State the missing word or number.

a An equation is a statement that contains an _____ sign.

b A linear equation contains a variable to the power of _____.

2 Consider the equation $2x + 3 = 7$.

a Complete this table by evaluating $2x + 3$ for the given values of x.

x	0	1	2	3
$2x + 3$				

b By looking at your table of values, which value of x is the solution to $2x + 3 = 7$?

3 Decide whether $x = 2$ is a solution to these equations.

a $x + 3 = 5$ **b** $2x = 7$ **c** $x - 1 = 4$

d $2x - 1 = 10$ **e** $3x + 2 = 8$ **f** $2 - x = 0$

Hint: Substitute $x = 2$ to see whether LHS = RHS.

FLUENCY | 4–9(½) | 6–9(½)

Example 1 Solving one-step equations

Solve:

a $x + 7 = 12$ **b** $x - 9 = 3$

c $3x = 12$ **d** $\frac{x}{4} = 20$

SOLUTION

a $x + 7 = 12$
$x = 12 - 7$
$x = 5$

Verify: LHS $= 5 + 7$ RHS $= 12$
$= 12$

EXPLANATION

Write the equation. The opposite of $+7$ is -7.
Subtract 7 from both sides.
Simplify.

Check that your answer is correct.

b $x - 9 = 3$
$x = 3 + 9$
$x = 12$

Verify: LHS $= 12 - 9$ RHS $= 3$
$= 3$

Write the equation. The opposite of -9 is $+9$.
Add 9 to both sides.
Simplify.

Check that your answer is correct.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c $3x = 12$ — Write the equation. The opposite of $\times 3$ is $\div 3$.

$x = \frac{12}{3}$ — Divide both sides by 3.

$x = 4$ — Simplify.

Verify: LHS $= 3 \times 4$ RHS $= 12$

$= 12$ — Check that your answer is correct.

d $\frac{x}{4} = 20$ — Write the equation. The opposite of $\div 4$ is $\times 4$.

$x = 20 \times 4$ — Multiply both sides by 4.

$x = 80$ — Check that your answer is correct.

Verify: LHS $= \frac{80}{4}$ RHS $= 20$ — Simplify.

$= 20$

Now you try

Solve:

a $x + 5 = 21$ **b** $x - 6 = 12$ **c** $4x = 36$ **d** $\frac{x}{3} = -4$

4 Solve the following.

Hint: $8 + x = 14$ is the same as $x + 8 = 14$. $16 = m + 1$ is the same as $m + 1 = 16$.

a $t + 5 = 8$ **b** $m + 4 = 10$ **c** $8 + x = 14$

d $m + 8 = 40$ **e** $a + 1 = -5$ **f** $16 = m + 1$

g $x - 3 = 3$ **h** $x - 7 = 2$ **i** $x - 8 = 9$

j $x - 3 = 0$ **k** $x - 2 = -8$ **l** $x - 5 = 7$

5 Solve the following.

Hint: $27 = 3d$ is the same as $3d = 27$.

a $8p = 24$ **b** $5c = 30$ **c** $27 = 3d$

d $15p = 15$ **e** $6m = -42$ **f** $-10 = 20p$

g $\frac{x}{5} = 10$ **h** $\frac{m}{3} = 7$ **i** $\frac{a}{6} = -2$

j $\frac{z}{7} = 0$ **k** $\frac{w}{3} = \frac{1}{2}$ **l** $\frac{m}{2} = \frac{1}{4}$

Hint: $3 \times \frac{1}{2} = \frac{3}{1} \times \frac{1}{2} = \frac{3}{2}$

6 Solve the following equations.

Hint: Carry out the 'opposite' operation to solve for x.

a $x + 9 = 12$ **b** $x + 3 = 12$ **c** $x + 15 = 4$

d $x - 7 = 3$ **e** $x - 2 = 12$ **f** $x - 5 = 5$

g $3x = 9$ **h** $4x = 16$ **i** $2x = 100$

j $\frac{x}{5} = 4$ **k** $\frac{x}{3} = 7$ **l** $\frac{x}{7} = 1$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 2 Solving two-step equations

Solve $4x + 5 = 17$.

SOLUTION

$4x + 5 = 17$

$4x = 12$

$x = \frac{12}{4}$

$x = 3$

Verify: LHS $= 4(3) + 5$ RHS $= 17$

$= 17$

EXPLANATION

Write the equation.

Subtract 5 from both sides first.

Divide both sides by 4.

Simplify.

Check your answer.

Now you try

Solve $5x - 1 = 19$.

7 Solve the following equations.

a $2x + 5 = 7$ **b** $3x + 2 = 11$

c $4x - 3 = 9$ **d** $6x + 13 = 1$

e $8x + 16 = 8$ **f** $10x + 92 = 2$

g $3x - 4 = 8$ **h** $2x - 7 = 9$

i $5x - 4 = 36$ **j** $2x - 6 = -10$

k $7x - 3 = -24$ **l** $6x - 3 = 27$

Hint: First choose to add or subtract a number from both sides and then divide by the coefficient of x.

Example 3 Solving two-step equations involving simple fractions

Solve $\frac{x}{5} - 3 = 4$.

SOLUTION

$\frac{x}{5} - 3 = 4$

$\frac{x}{5} = 7$

$x = 35$

Verify: LHS $= \frac{35}{5} - 3$ RHS $= 4$

$= 4$

EXPLANATION

Write the equation.

Add 3 to both sides.

Multiply both sides by 5.

Check that your answer is correct.

Now you try

Solve $\frac{x}{7} + 2 = 6$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 Solve the following equations.

a $\frac{x}{3} + 2 = 5$ **b** $\frac{x}{6} + 3 = 3$ **c** $\frac{x}{7} + 4 = 12$

d $\frac{x}{4} - 3 = 2$ **e** $\frac{x}{5} - 4 = 3$ **f** $\frac{x}{10} - 2 = 7$

g $\frac{x}{8} - 2 = -6$ **h** $\frac{x}{4} - 3 = -8$ **i** $\frac{x}{2} - 1 = -10$

Hint: When solving equations, the order of steps is important. For $\frac{x}{3} - 5$, undo the -5 first, then undo the $\div 3$.

Example 4 Solving more two-step equations

Solve $\frac{x+4}{2} = 6$.

SOLUTION	EXPLANATION
$\frac{x+4}{2} = 6$	Write the equation.
$x + 4 = 12$	In $\frac{x+4}{2}$ we first add 4 and then divide by 2. So to undo we first multiply both sides by 2.
$x = 8$	Subtract 4 from both sides.
Verify: LHS $= \frac{8+4}{2}$ RHS $= 6$ $= 6$	Check that your answer is correct.

Now you try

Solve $\frac{x-3}{4} = 1$.

9 Solve the following equations.

a $\frac{m+1}{2} = 3$ **b** $\frac{a-1}{3} = 2$

c $\frac{x+5}{2} = 3$ **d** $\frac{x+5}{3} = 2$

e $\frac{n-4}{5} = 1$ **f** $\frac{m-6}{2} = 8$

g $\frac{w+4}{3} = -1$ **h** $\frac{m+3}{5} = 2$

i $\frac{w-6}{3} = 7$ **j** $\frac{a+7}{4} = 2$

k $\frac{a-3}{8} = -5$ **l** $\frac{m+5}{8} = 0$

Hint: When solving equations, the order of steps is important. For $\frac{x+7}{3}$, undo the $\div 3$ first, then undo the $+7$. Never cancel a number joined by $+$ or $-$ to an x. In $\frac{x+8}{4}$, you cannot cancel the 4 into the 8.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING | 10, 11 | 11–13

Example 5 Writing equations from word problems

For each of the following statements, write an equation and solve for the pronumeral.

a When 7 is subtracted from x, the result is 12.

b When x is divided by 5 and then 6 is added, the result is 10.

c When 4 is subtracted from x and that answer is divided by 2, the result is 9.

SOLUTION	EXPLANATION
a $x - 7 = 12$	Subtract 7 from x means to start with x and then subtract 7.
$x = 19$	'The result' means '='.
b $\frac{x}{5} + 6 = 10$	Divide x by 5, then add 6 and make it equal to 10.
$\frac{x}{5} = 4$	Solve the equation by subtracting 6 from both sides first.
$x = 20$	
c $\frac{x-4}{2} = 9$	Subtracting 4 from x gives $x - 4$, and divide that answer by 2.
$x - 4 = 18$	Undo $\div 2$ by multiplying both sides by 2, then add 4 to both sides.
$x = 22$	

Now you try

For each of the following statements, write an equation and solve for the pronumeral.

a When 3 is added to x, the result is 9.

b When x is divided by 3 then 7 is subtracted, the result is 0.

c When 6 is subtracted from x and that answer is divided by 3, the result is 10.

10 For each of the following statements, write an equation and solve for the pronumeral.

a When 4 is added to x, the result is 6.

b When x is added to 12, the result is 8.

c When 5 is subtracted from x, the result is 5.

d When x is divided by 3 and then 2 is added, the result is 8.

e Twice the value of x is added to 3 and the result is 9.

f $(x - 3)$ is divided by 5 and the result is 6.

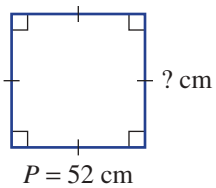
g 3 times x plus 4 is equal to 16.

Hint: 5 subtracted from x is $x - 5$.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

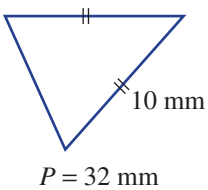
11 Write an equation and solve it for each of these questions.

a The perimeter of a square is 52 cm. Determine the length of the side.



Hint: Draw a diagram and choose a pronumeral to represent the unknown side. Then write an equation and solve it.

b The perimeter of an isosceles triangle is 32 mm. If the equal sides are both 10 mm, determine the length of the other side.



12 Convert the following into equations, then solve them for the unknown number.

a n is multiplied by 2, then 5 is added. The result is 11.

b Four times a certain number is added to 9 and the result is 29. What is the number?

c Half of a number less 2 equals 12. What is the number?

d A number plus 6 has been divided by 4. The result is 12. What is the number?

e 12 is subtracted from a certain number and the result is divided by 5. If the answer is 14, what is the number?

Hint: Choose a pronumeral to represent the unknown number, then write an equation using the pronumeral. $\frac{1}{2}$ of x can be written as $\frac{x}{2}$.

13 Write an equation and solve it for each of these questions.

a The sum of two consecutive whole numbers is 23. What are the numbers?

b If I add 5 to twice a number, the result is 17. What is the number?

c Three less than five times a number is 12. What is the number?

d One person is 19 years older than another person. Their age sum is 69. What are their ages?

e Andrew threw the shot-put 3 m more than twice the distance Barry threw it. If Andrew threw the shot-put 19 m, how far did Barry throw it?

Hint: Consecutive whole numbers are one number apart; e.g. 3, 4, 5, 6 etc. The next consecutive number after x is $x + 1$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: Modelling with equations	—	14, 15

14 A service technician charges $40 up front and $60 for each hour she works.

- **a** Find a linear equation for the total charge, $\$C$, of any job for h hours worked.
- **b** What will a 4-hour job cost?
- **c** If the technician works on a job for 3 days and averages 6 hours per day, what will be the overall cost?
- **d** If a customer is charged $400, how long did the job take?

15 A petrol tank holds 71 litres of fuel. It originally contained 5 litres. If a petrol pump fills it at 6 litres per minute, find:

- **a** a linear equation for the amount of fuel (V litres) in the tank at time t minutes
- **b** how long it will take to fill the tank to 23 litres
- **c** how long it will take to fill the tank

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9B Equations with brackets, fractions and pronumerals on both sides

Learning intentions for this section:

- To know the order in which steps need to be applied to solve equations
- To be able to collect pronumerals on one side in order to solve a linear equation
- To be able to solve a simple word problem by setting up and solving a linear equation

Past, present and future learning:

- This section addresses the Stage 5 Core topic called *Equations A*
- It consolidates and extends the concepts in Sections 3F, 3G and 3H of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

More complex linear equations may have variables on both sides of the equation and/or brackets. Examples are $6x = 2x - 8$ or $5(x + 3) = 12x + 4$.

Brackets can be removed by expanding. Equations with variables on both sides can be solved by collecting variables to one side, using addition or subtraction of a term.

More complex linear equations of this type are used when constructing buildings and in science and engineering.

Lesson starter: Steps in the wrong order

The steps to solve $8(x + 2) = 2(3x + 12)$ are listed here in the incorrect order.

$$8(x + 2) = 2(3x + 12)$$
$$x = 4$$
$$2x + 16 = 24$$
$$8x + 16 = 6x + 24$$
$$2x = 8$$

- Arrange them in the correct order, working from the problem to the solution.
- By considering all the steps in the correct order, write what has happened in each step.

KEY IDEAS

■ When solving linear equations:

1 First, **expand** any brackets.
In this example, multiply the 3 into the first bracket and the 2 into the second bracket.

$$3(2x - 1) + 2(x + 2) = 22$$
$$6x - 3 + 2x + 4 = 22$$

2 Collect any **like terms** on the LHS and any like terms on the RHS.
Collecting like terms on the left side of the example below.
$5x - 3x = 2x$ and $-4 - 9 = -13$.

$$5x - 4 - 3x - 9 = x - 5 + 2x + 10$$
$$2x - 13 = 3x + 5$$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Sometimes it is good to swap the LHS and RHS.
For example: $2x - 13 = 3x + 5$
2 is less than 3
$\therefore$ swap LHS and RHS.
$2x - 13 = 3x + 5$
becomes
$3x + 5 = 2x - 13$

4 If an equation has variables on both sides, collect to one side by adding or subtracting one of the terms.
For example, when solving the equation $12x + 7 = 5x + 19$, first subtract $5x$ from both sides:

$$\begin{array}{l} 12x + 7 = 5x + 19 \quad (-5x) \\ 7x + 7 = 19 \end{array}$$

5 Start to perform the opposite operation to both sides of the equation.
6 Repeat Step 5 until the equation is solved.
7 Verify that the answer is correct, using substitution.

■ To solve a word problem using algebra:
- Read the problem and find out what the question is asking for.
- Define a variable and write a statement such as: 'Let x be the number of …'. The variable is often what you have been asked to find in the question.
- Write an equation using your defined variable.
- Solve the equation and check that the solution is reasonable.
- Answer the question in words.

Key terminology: expand, like terms, product, equivalent

Exercise 9B

UNDERSTANDING 1–3 2, 3

1 Choose from the words *collect*, *expand* and *one* to complete the following when solving linear equations.
a First _____ any brackets.
b _____ any like terms.
c If variables are on both sides, collect to _____ side.

2 When $-2(x - 1)$ is expanded, the result is:
A $-2x - 2$
B $-2x + 1$
C $-2x + 2$
D $2x + 2$
E $2x + 1$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 When $2x$ is subtracted from both sides, $5x + 1 = 2x - 3$ becomes:

A $3x - 1 = 3$ **B** $7x + 1 = -3$ **C** $7x + 1 = 3$

D $3x + 1 = 3$ **E** $3x + 1 = -3$

FLUENCY 4–9(½) 4–9(½)

Example 6 Solving equations with brackets

Solve $4(x - 1) = 16$.

SOLUTION

$4(x - 1) = 16$

$4x - 4 = 16$

$4x = 20$

$x = 5$

EXPLANATION

Expand the brackets: $4 \times x$ and $4 \times (-1)$.

Add 4 to both sides.

Divide both sides by 4.

Now you try

Solve $3(x + 1) = 15$.

4 Solve each of the following equations by first expanding the brackets.

a $3(x + 2) = 9$ **b** $4(x - 1) = 16$

c $3(x + 5) = 12$ **d** $4(a - 2) = 12$

e $5(a + 1) = 10$ **f** $2(x - 10) = 10$

g $6(m - 3) = 6$ **h** $3(d + 4) = 15$

i $7(a - 8) = 14$ **j** $10(a + 2) = 20$

k $5(3 + x) = 15$ **l** $2(a - 3) = 0$

Example 7 Solving equations with two sets of brackets

Solve $3(2x + 4) + 2(3x - 2) = 20$.

SOLUTION

$3(2x + 4) + 2(3x - 2) = 20$

$6x + 12 + 6x - 4 = 20$

$12x + 8 = 20$

$12x = 12$

$x = 1$

EXPLANATION

Use the distributive law to expand each set of brackets.

Collect like terms on the LHS.

Subtract 8 from both sides.

Divide both sides by 12.

Now you try

Solve $2(3x - 1) - 3(x - 4) = 16$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Solve the following equations.

a $3(2x + 3) + 2(x + 4) = 25$
b $2(2x + 3) + 4(3x + 1) = 42$
c $2(2x + 3) + 3(4x - 1) = 51$
d $3(2x - 2) + 5(x + 4) = 36$
e $4(2x - 3) + 2(x - 4) = 10$
f $2(3x - 1) + 3(2x - 3) = 13$
g $2(x - 4) + 3(x - 1) = -21$
h $4(2x - 1) + 2(2x - 3) = -22$

Hint: Expand each pair of brackets and collect like terms before solving.

6 Solve the following equations.

a $3(2x + 4) - 4(x + 2) = 6$
b $2(5x + 4) - 3(2x + 1) = 9$
c $2(3x - 2) - 3(x + 1) = -7$
d $2(x + 1) - 3(x - 2) = 8$
e $8(x - 1) - 2(3x - 2) = 2$
f $5(2x - 3) - 2(3x - 1) = -9$
g $5(2x + 1) - 3(x - 3) = 35$
h $4(2x - 3) - 2(3x - 1) = -14$

Hint:
$-4(x + 2) = -4x - 8$
$-4(x - 2) = -4x + 8$

Example 8 Solving equations with variables on both sides

Solve $7x + 9 = 2x - 11$ for x.

SOLUTION	EXPLANATION
$7x + 9 = 2x - 11$	Subtract $2x$ from both sides.
$5x + 9 = -11$	Subtract 9 from both sides.
$5x = -20$	Divide both sides by 5.
$x = -4$	

Now you try

Solve $10x + 3 = 8x - 1$ for x.

7 Find the value of x in the following.

a $7x = 2x + 10$
b $10x = 9x + 12$
c $8x = 4x - 12$
d $6x = 2x + 80$
e $2x = 12 - x$
f $2x = 8 + x$
g $3x + 4 = x + 12$
h $4x + 9 = x - 3$
i $2x - 9 = x - 10$
j $6x - 10 = 12 + 4x$
k $9x = 10 - x$
l $1 - x = x + 3$

Hint: Remove the term containing x on the RHS. For parts **e**, **k** and **l**, you will need to add x to both sides.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 9 Solving equations with fractions

Solve $\frac{2x+3}{4} = 2$ for x.

SOLUTION

$\frac{2x+3}{4} = 2$

$2x + 3 = 8$

$2x = 5$

$x = \frac{5}{2}$

EXPLANATION

Multiply both sides by 4.

Subtract 3 from both sides.

Divide both sides by 2.

Now you try

Solve $\frac{4x-3}{2} = 4$.

8 Solve the following equations.

a $\frac{x+2}{3} = 5$ **b** $\frac{x+4}{2} = 5$ **c** $\frac{x-1}{3} = 4$

d $\frac{x-5}{3} = 2$ **e** $\frac{2x+1}{7} = 3$ **f** $\frac{2x+2}{3} = 4$

g $\frac{5x-3}{3} = 9$ **h** $\frac{3x-6}{2} = 9$ **i** $\frac{5x-2}{4} = -3$

Hint: First multiply by the denominator.

Example 10 Solving equations with more difficult fractions

Solve $\frac{3x}{2} - 4 = 2$ for x.

SOLUTION

$\frac{3x}{2} - 4 = 2$

$\frac{3x}{2} = 6$

$3x = 12$

$x = 4$

EXPLANATION

Add 4 to both sides.

Multiply both sides by 2.

Divide both sides by 3.

Now you try

Solve $\frac{5x}{3} + 1 = 6$ for x.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 Solve the following equations.

a $\frac{x}{3} + 1 = 5$ **b** $\frac{x}{3} + 1 = 7$ **c** $\frac{x}{4} - 5 = 10$

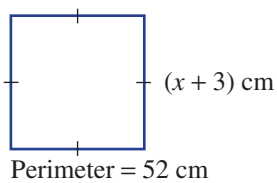
d $\frac{3x}{4} - 2 = 5$ **e** $\frac{2x}{5} - 3 = -1$ **f** $\frac{3x}{2} - 5 = -14$

Hint: First add or subtract a number from both sides.

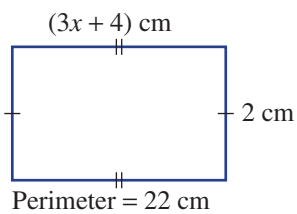
PROBLEM-SOLVING AND REASONING 10–13 11–14

10 For each of these questions, write an equation and solve it for x.

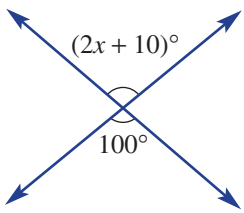
a



b



c



11 Solve the following equations using trial and error (guess, check and refine). Substitute your chosen values of x until you have found a value that makes the equation true.

Hint: Vertically opposite angles are equal.

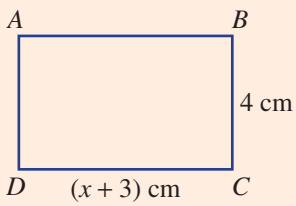
a $\frac{x + 22}{3} = 4x$

b $5(3 - x) = 2(x + 7.5)$

c $\frac{2x - 1}{4} = 2 - x$

Example 11 Solving a word problem

Find the value of x if the area of rectangle $ABCD$ shown is 24 cm^2.

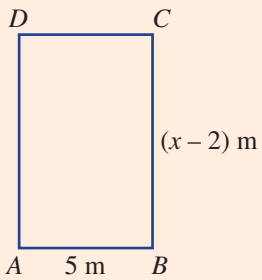


SOLUTION	EXPLANATION
$A = l \times b$	Write an equation for area.
$24 = (x + 3) \times 4$	Substitute: $l = (x + 3)$, $b = 4$, $A = 24$.
$24 = 4x + 12$	Expand the brackets: $(x + 3) \times 4 = 4(x + 3)$.
$12 = 4x$	Subtract 12 from both sides.
$3 = x$	Divide both sides by 4.
$x = 3$	Write the answer.

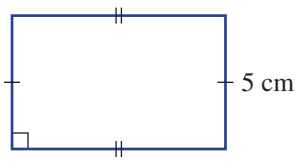
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Now you try

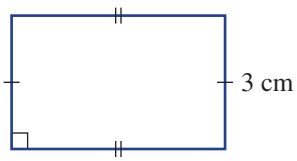
Find the value of x if the area of rectangle $ABCD$ shown is 40 m^2.



12 a Find the value of x if the area is 35 cm^2.

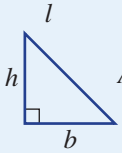


b Find the value of x if the area is 27 cm^2.



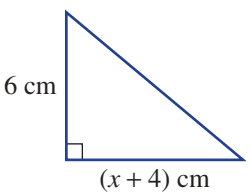
Hint: Form the area equation first.

$A = lb$

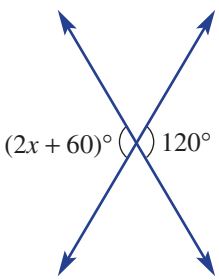


$A = \frac{1}{2}bh$

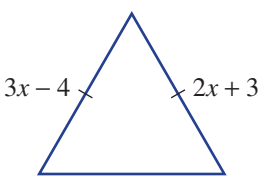
c Find the value of x if the area is 42 cm^2.



d Vertically opposite angles are equal. Find the value of x.



e Find the value of x.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Hint:
- 'Product' means multiply
- The product of 5 and 1 more than a number means $5(x + 1)$.
- '6 less than 3 lots of a number is doubled' will require brackets.
- 'Tripled' means three times a number.
- 'Equivalent to' means 'equal to'.

13 Using x for the unknown number, write down an equation and then solve it to find the number.

a The product of 5 and 1 more than a number is 40.

b The product of 5 and 6 less than a number is -15.

c When 6 less than 3 lots of a number is doubled, the result is 18.

d When 8 more than 2 lots of a number is tripled, the result is 36.

e 10 more than 4 lots of a number is equivalent to 6 lots of the number.

f 5 more than 4 times a number is equivalent to 1 less than 5 times the number.

g 6 more than a doubled number is equivalent to 5 less than 3 lots of the number.

14 Valentina and Harrison are planning to hire a car for their wedding day. 'Vehicles For You' have the following deal: \$850 hiring fee plus a charge of \$156 per hour.

a Write an equation for the cost (\$$C$) of hiring a car for h hours.

b If Valentina and Harrison have budgeted for the car to cost a maximum of \$2000, find the maximum number of full hours they can hire the car.

c If the car picks up the bride at 1:15 p.m., at what time must the event finish if the cost is to remain within budget?

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

ENRICHMENT: More than one fraction — 15

15 Consider:

$$\frac{4x-2}{3}=\frac{3x-1}{2}$$

$\frac{{}^{2}\cancel{6}(4x-2)}{\cancel{3}_{1}}=\frac{{}^{3}\cancel{6}(3x-1)}{\cancel{2}_{1}}$ (Multiply both sides by 6 (LCM of 2 and 3) to get rid of the fractions.)

$2(4x-2)=3(3x-1)$ (Simplify.)

$8x-4=9x-3$ (Expand both sides.)

$-4=x-3$ (Subtract $8x$ from both sides.)

$-1=x$ (Add 3 to both sides.)

$\therefore x=-1$

Solve the following equations using the method shown above.

a $\frac{x+2}{3}=\frac{x+1}{2}$ **b** $\frac{x+1}{2}=\frac{x}{3}$ **c** $\frac{3x+4}{4}=\frac{x+6}{3}$

d $\frac{5x+2}{3}=\frac{3x+4}{2}$ **e** $\frac{2x+1}{7}=\frac{3x-5}{4}$ **f** $\frac{5x-1}{3}=\frac{x-4}{4}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9C Solving equations of the form $ax^2 = c$ CONSOLIDATING

Learning intentions for this section:

- To know that a quadratic equation can have 0, 1 or 2 solutions
- To be able to solve a quadratic equation of the form $ax^2 = c$

Past, present and future learning:

- This section addresses Stage 4 concepts that are used in Stage 5 and Stage 6 Standard
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard this concept will be applied to formulas for volume and surface area

In this section we will solve equations such as these:

$x^2 = 16$ $\quad 4x^2 = 16$ $\quad x^2 - 16 = 0$ $\quad 4x^2 - 16 = 0$

These are called quadratic equations.

Lesson starter: How many possible solutions?

- List all the pairs of numbers that multiply to 16.
- List all the pairs of numbers that multiply to -16.
- What do you notice about the signs of your pairs of numbers that multiply to 16? What about the signs of the numbers that multiply to -16?
- How many of your listed pairs satisfy $x \times x = 16$?
- Are there any pairs that satisfy $x \times x = -16$?
- From what you have found above, how many values of x would make the following true?

a $x^2 = 9$ **b** $x^2 = -9$

KEY IDEAS

■ A **quadratic equation** is an equation in which the highest power of the pronumeral is 2.

For example: $x^2 = 9$, $2x^2 - 6 = 0$ and $x^2 - 2x = 8$.

- A quadratic equation may have 0, 1 or 2 possible solutions.

■ The square of a number is always positive or 0; i.e. $x^2 \geq 0$ for all values of x.

For example: $(2)^2 = 2 \times 2 = 4$ and $(-3)^2 = -3 \times (-3) = 9$.

■ The inverse operation of squaring is the square root, $\sqrt{\ }$.

- $\sqrt{4} = 2$ since $2 \times 2 = 4$.
- $\sqrt{7} \times \sqrt{7} = 7$

■ When $x^2 = c$, then $x = \pm\sqrt{c}$ provided that $c \geq 0$.

- $\pm$ represents two possible solutions: $+\sqrt{c}$ and $-\sqrt{c}$. For example: if $x^2 = 9$, then $x = \pm 3$
- $c = 0$ gives just one solution since $\sqrt{0} = 0$. For example: if $x^2 = 0$, then $x = 0$

■ When $x^2 = c$ and $c < 0$, there are no real solutions. For example: $x^2 = -9$ has no solution.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

- Square roots that do not reduce to whole numbers are called **surds** and can be left in this exact form; i.e. $\sqrt{2}$, $\sqrt{10}$, … etc.
  For example: $\sqrt{25}$ is not a surd as 25 is a square number and $\sqrt{25} = 5$.
- To solve equations of the form $ax^2 = c$, make x^2 the subject and then take the square root of both sides to solve for x.
- In practical problems such as those involving measurement, it may make sense to reject the negative solution obtained.

Key terminology: quadratic equation, solve, exact, surd, square root

Exercise 9C

UNDERSTANDING 1–4 | 4

1 List the first 12 square numbers.

Hint: Start at $1^2 = 1$, $2^2 = \ldots$

2 Evaluate the following.

a 4^2 **b** 9^2 **c** 7^2
d $(-3)^2$ **e** $(-8)^2$ **f** $(-1)^2$

Hint: $4^2 = 4 \times 4 = \ldots$
$(-3)^2 = -3 \times (-3) = \ldots$

3 Evaluate the following.

a $\sqrt{36}$ **b** $-\sqrt{49}$ **c** $-\sqrt{100}$
d $\sqrt{0}$ **e** $\sqrt{400}$ **f** $\sqrt{1600}$

Hint: $\sqrt{4} = 2$ since $2^2 = 4$.

4 Complete the following. The equation $x^2 = c$ has:

a two solutions when _____ **b** one solution when _____
c no solutions when _____

FLUENCY 5–7(½) | 5–7(½)

Example 12 Solving equations of the form $ax^2 = c$

Find all possible solutions to the following equations.

a $x^2 = 16$ **b** $3x^2 = 12$ **c** $x^2 = -25$

SOLUTION

a $x^2 = 16$
$\therefore x = \pm\sqrt{16}$
$x = \pm 4$

EXPLANATION

Take the square root of both sides to solve for x. If $x^2 = c$, where $c > 0$, then $x = \pm\sqrt{c}$.

Since 16 is a square number, $\sqrt{16}$ is a whole number.
$\pm$ represents two solutions: $+4$ since $(+4)^2 = 16$ and -4 since $(-4)^2 = 16$.

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b $3x^2 = 12$ — First make x^2 the subject by dividing both sides by 3.

$x^2 = 4$ — Take the square root of both sides to solve for x.

$\therefore x = \pm\sqrt{4}$ — This gives two possible values for x: $+2$ and -2.

$x = \pm 2$

c $x^2 = -25$ — Since $x^2 \geq 0$ for all values of x, there are no values of x that will make $x^2 = -25$.

There are no solutions.

Now you try

Find all possible solutions to the following equations.

a $x^2 = 64$ **b** $2x^2 = 18$ **c** $x^2 = -36$

5 Find the possible solutions for x in the following equations.

a $x^2 = 25$ **b** $x^2 = 81$ **c** $x^2 = 36$

d $x^2 = 49$ **e** $x^2 = -16$ **f** $x^2 = -100$

g $x^2 = 400$ **h** $x^2 = 144$ **i** $2x^2 = 50$

j $3x^2 = 48$ **k** $-5x^2 = -5$ **l** $-5x^2 = 20$

m $\frac{x^2}{2} = 32$ **n** $\frac{x^2}{3} = 27$ **o** $\frac{1}{2}x^2 = 18$

Hint: When taking the square root, don't forget $\pm$.

Hint: $\frac{1}{2}x^2$ is the same as $\frac{x^2}{2}$.

Example 13 Solving equations with answers that are surds

Find all possible solutions to the following equations.

a $x^2 = 10$ (Give your answer in exact surd form.)

b $3x^2 = 9$ (Give your answer to one decimal place.)

SOLUTION	EXPLANATION
a $x^2 = 10$	Take the square root of both sides.
$\therefore x = \pm\sqrt{10}$	Since 10 is not a square number, leave your answer in exact surd form $\pm\sqrt{10}$, as required.
b $3x^2 = 9$	Divide both sides by 3.
$x^2 = 3$	Take the square root of both sides.
$\therefore x = \pm\sqrt{3}$	Use a calculator to evaluate $\pm\sqrt{3} = 1.73205\ldots$ and round your answer to one decimal place, as required.
$x = \pm 1.7$ (to 1 d.p.)	

Now you try

Find all possible solutions to the following equations.

a $x^2 = 29$ (Give your answer in exact surd form.)

b $5x^2 = 55$ (Give your answer to one decimal place.)

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 a Solve the following equations, where possible. Leave your answer in exact surd form.

i $x^2 = 14$ **ii** $x^2 = 22$

iii $x^2 = 17$ **iv** $x^2 = -7$

v $3x^2 = 15$ **vi** $4x^2 = 24$

vii $\frac{x^2}{3} = 7$ **viii** $\frac{x^2}{2} = 3$

ix $\frac{1}{5}x^2 = 3$

Hint: $\sqrt{14}$ is exact surd form.

b Solve these equations, giving your answers to one decimal place.

i $x^2 = 12$

ii $x^2 = 35$

iii $3x^2 = 90$

iv $\frac{x^2}{9} = 7$

Example 14 Solving equations of the form $ax^2 + b = 0$

Find the exact solution(s), where possible, to the following equations.

a $x^2 - 9 = 0$ **b** $x^2 + 12 = 0$

c $\frac{x^2}{2} - 3 = 4,\ x > 0$ **d** $7 - x^2 = 6,\ x < 0$

SOLUTION	EXPLANATION
a $x^2 - 9 = 0$	
$x^2 = 9$	Make x^2 the subject by adding 9 to both sides.
$\therefore x = \pm\sqrt{9}$	Solve for x by taking the square root of both sides.
$x = \pm 3$	
b $x^2 + 12 = 0$	
$x^2 = -12$	Make x^2 the subject by subtracting 12 from both sides.
There is no real solution.	Since x^2 is positive for all values of x, $x^2 = c$, where c is negative has no real solution.
c $\frac{x^2}{2} - 3 = 4$	
$\frac{x^2}{2} = 7$	Solve for x^2 by adding 3 to both sides and then multiplying both sides by 2.
$x^2 = 14$	Take the square root of both sides to solve for x. $\sqrt{14}$ is the exact form of the answer.
$\therefore x = \pm\sqrt{14}$	Note the restriction that x must be a positive value (i.e. > 0), so reject the solution $x = -\sqrt{14}$.
$x = \sqrt{14}$ since $x > 0$.	

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

d $7 - x^2 = 6$

$-x^2 = -1$

$x^2 = 1$

$\therefore x = \pm\sqrt{1}$

$x = \pm 1$

$x = -1$ since $x < 0$.

Subtract 7 from both sides and then divide both sides by -1.

Alternatively, add x^2 to both sides to give $7 = x^2 + 6$ and then solve.

$\sqrt{1} = 1$ since $1^2 = 1$.

Reject the solution $x = 1$ because $x < 0$.

Now you try

Find the exact solution(s), where possible, to the following equations.

a $x^2 - 25 = 0$

b $x^2 + 7 = 0$

c $\frac{x^2}{3} - 1 = 4,\ x > 0$

d $3 - x^2 = -6,\ x < 0$

7 Find the exact solution(s), where possible, of these equations.

Hint: First get your equation in the form $x^2 = c$.

a $x^2 - 4 = 0$

b $x^2 - 81 = 0$

c $x^2 + 16 = 0$

d $x^2 + 10 = 0$

e $x^2 + 6 = 15$

f $x^2 - 5 = 10$

g $2x^2 - 6 = 0,\ x > 0$

h $3x^2 + 10 = 13$

i $\frac{x^2}{2} + 4 = 5,\ x > 0$

j $10 - x^2 = 3,\ x < 0$

k $5 - x^2 = 7$

l $4 - x^2 = -1,\ x > 0$

PROBLEM-SOLVING AND REASONING

8–12 | 8(½), 11–16

8 Solve these equations by first collecting like terms.

Hint: Rewrite the equation in the form $x^2 = c$ by collecting x^2 terms on one side and the number on the other.

$3x^2 = 2x^2 + 16$

$-2x^2$ → $x^2 = 16$ ← $-2x^2$

a $3x^2 = 2x^2 + 16$

b $4x^2 = 2x^2 + 18$

c $4x^2 - 11 = x^2 + 16$

d $x^2 = 50 - x^2$

e $2x^2 + 5 = 17 - x^2$

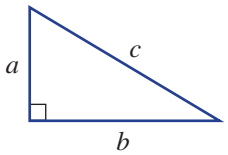
f $4x^2 + 8 = 2x^2 + 5$

9 The area of a square is 36 m^2. What is its perimeter?

10 The distance, d metres, that a bird dives in t seconds is given by $d = 5t^2$. How long does the bird take to dive 80 m?

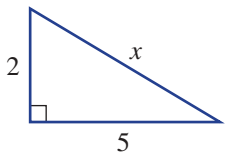
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 Recall Pythagoras' theorem: $c^2 = a^2 + b^2$.

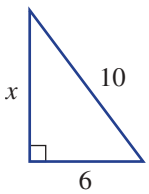


Use this to find the unknown side length in the following triangles. Give your answers in exact form.

a

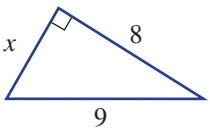


b

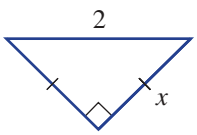


Hint: Set up the Pythagorean equation first.

c



d



12 The volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder. If the volume of a cylindrical water tank is 285 m^3 and its height is 15 m, determine its radius, to one decimal place.

Hint: $\pi r^2(15)$ can be written as $15\pi r^2$.

13 **a** Solve the equation $\pi r^2 = 24$, to find two possible values for r, rounded to one decimal place.

b Given that $A = \pi r^2$ is the formula for the area of a circle of radius r, what restriction does this place on your answer to part **a** if the area of the circle is 24 units^2?

14 Determine for which values of b the equation $x^2 - b = 0$ has:

a two solutions **b** one solution **c** no solutions

Hint: For Questions **14** and **15**, first solve for x^2.

15 Determine for which values of b the equation $x^2 + b = 0$ has:

a two solutions **b** one solution **c** no solutions

16 **a** Solve the equation $x^2 - 25 = 0$.

b Hence, determine the range of values of x for which:

i $x^2 - 25 < 0$ **ii** $x^2 - 25 > 0$

Hint: In part **b**, make use of your answer to part **a**.

ENRICHMENT: Square powers and brackets — 17(½)

17 Consider the following example.

Solve $(x + 1)^2 = 16$

$x + 1 = \pm\sqrt{16}$ (Take the square root of both sides.)

$x + 1 = \pm 4$

$x + 1 = 4$ or $x + 1 = -4$ (Write each equation separately.)

$x = 3$ or $x = -5$ (Subtract 1 from both sides.)

Use this method to solve these equations.

a $(x + 1)^2 = 9$ **b** $(x + 3)^2 = 49$ **c** $(x - 2)^2 = 4$ **d** $(x - 5)^2 = 16$

e $(2x + 1)^2 = 25$ **f** $(2x - 1)^2 = 1$ **g** $(3x - 2)^2 = 4$ **h** $(4x - 3)^2 = 81$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9D Using formulas

Learning intentions for this section:

- To understand that a relationship between variables can be described using formulas
- To be able to substitute into a formula and evaluate
- To be able to solve an equation after substitution into a formula

Past, present and future learning:

- This section concludes the Stage 5 Core topic called *Equations A*
- It consolidates and extends the concepts in Sections 3I and 3J of our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

A formula is an equation that relates two or more variables. You can find the value of one of the variables if you are given the value of all other unknowns.

A metal worker building pipes applies circle, area and volume formulas.

You will already be familiar with many formulas. For example:

- $C = 2\pi r$ is the formula for finding the circumference, C, of a circle given its radius, r.
- $F = \frac{9}{5}C + 32$ is the formula for converting degrees Celsius, C, to degrees Fahrenheit, F.
- $s = \frac{d}{t}$ is the formula for finding the speed, s, given the distance, d, and time, t.

C, F and s are said to be the subjects of the formulas given above.

Lesson starter: Jumbled solution

Problem: The formula for the area of a trapezium is $A = \frac{h}{2}(a + b)$.

Xavier was asked to find a given that $A = 126$, $b = 10$ and $h = 14$, and to write the explanation beside each step of the solution.

Xavier's solution and explanation are on the next page. His solution is correct but he has jumbled up the steps in the explanation. Copy Xavier's solution and write the correct instruction(s) beside each step.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION	EXPLANATION
$A = \frac{h}{2}(a+b)$	Write the answer.
$126 = \frac{14}{2}(a+10)$	Subtract 70 from both sides.
$\frac{14}{2}(a+10) = 126$	Divide both sides by 7.
$7(a+10) = 126$	Substitute the given values.
$7a + 70 = 126$	Copy the formula.
$7a = 56$	Simplify $\frac{14}{2}$.
$a = 8$	Expand the brackets.
	Swap LHS and RHS.

KEY IDEAS

- A **formula** is an equation that relates two or more variables.
- The **subject** of a formula is a variable that usually sits on its own on the left-hand side. For example, the C in $C = 2\pi r$ is the subject of the formula.
- A variable in a formula can be evaluated by substituting numbers for all other variables.
- A formula can be rearranged to make another variable the subject. $C = 2\pi r$ can be rearranged to give $r = \frac{C}{2\pi}$.

Key terminology: subject, formula, variable, substitute, evaluate

Exercise 9D

UNDERSTANDING 1, 2 | 2

1 State the letter that is the subject in these formulas.

a $I = Prn$ **b** $F = ma$ **c** $V = \frac{4}{3}\pi r^3$

d $A = \pi r^2$ **e** $c = a^2 + b^2$ **f** $P = 2x + 2y$

Hint: The subject of a formula is the letter on its own, on the left-hand side.

2 Substitute the given values into each of the following formulas to find the value of each subject. Round the answer to one decimal place where appropriate.

Hint: Copy each formula, substitute the given values and then calculate the answer.

a $m = \frac{F}{a}$, when $F = 180$ and $a = 3$

b $A = lb$, when $l = 6$ and $b = 8$

c $A = \frac{h}{2}(a+b)$, when $a = 6$, $b = 12$ and $h = 4$

d $v^2 = u^2 + 2as$, when $u = 6$, $a = 12$ and $s = 7$ **e** $m = \frac{x}{y}$, when $x = 56$ and $y = 4$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

FLUENCY 3–8(½) 4–5(½), 7–9(½)

Example 15 Substituting values and solving equations

If $v = u + at$, find t when $v = 16$, $u = 4$ and $a = 3$.

SOLUTION	EXPLANATION
$v = u + at$	Substitute each value into the formula. $v = 16, u = 4, a = 3$
$16 = 4 + 3t$	An equation now exists. Solve this equation for t.
$12 = 3t$	Subtract 4 from both sides.
$4 = t$	Divide both sides by 3.
$t = 4$	Answer with the pronumeral on the left-hand side.

Now you try

If $A = \frac{1}{2}xy$, find y when $A = 12$ and $x = 4$.

3 If $v = u + at$, find t when:

a $v = 16$, $u = 8$ and $a = 2$ **b** $v = 20$, $u = 8$ and $a = 3$

c $v = 100$, $u = 10$ and $a = 9$ **d** $v = 84$, $u = 4$ and $a = 10$

Hint: First copy the formula. Then substitute the given values. Then solve the equation.

4 If $p = 2(l + b)$, find b when:

a $P = 60$ and $l = 10$ **b** $P = 48$ and $l = 6$

c $P = 96$ and $l = 14$ **d** $P = 12.4$ and $l = 3.6$

5 If $V = lbh$, find h when:

a $V = 100$, $l = 5$ and $b = 4$ **b** $V = 144$, $l = 3$ and $b = 4$

c $V = 108$, $l = 3$ and $b = 12$ **d** $V = 280$, $l = 8$ and $b = 5$

Hint: For $90 = \frac{1}{2} \times b \times 12$,

$\frac{1}{2} \times b \times 12 = \frac{1}{2} \times 12 \times b$

$= 6b$

So, $90 = 6b$

Solve for b.

6 If $A = \frac{1}{2}bh$, find b when:

a $A = 90$ and $h = 12$

b $A = 72$ and $h = 9$

c $A = 108$ and $h = 18$

d $A = 96$ and $h = 6$

7 If $A = \frac{h}{2}(a + b)$, find h when:

a $A = 20$, $a = 4$ and $b = 1$

b $A = 48$, $a = 5$ and $b = 7$

c $A = 108$, $a = 9$ and $b = 9$

d $A = 196$, $a = 9$ and $b = 5$

Hint: When solving the equation first undo the division by 2 by multiplying both sides by 2.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 $E = mc^2$. Find m when:

- **a** $E = 100$ and $c = 5$
- **b** $E = 4000$ and $c = 10$
- **c** $E = 72$ and $c = 1$
- **d** $E = 144$ and $c = 6$

Hint: Square the c value before solving the equation.

9 If $V = \pi r^2 h$, find h (to one decimal place) when:

- **a** $V = 160$ and $r = 3$
- **b** $V = 400$ and $r = 5$
- **c** $V = 1460$ and $r = 9$
- **d** $V = 314$ and $r = 2.5$

Hint: For $160 = 9\pi h$, divide both sides by 9π to find h:
$h = \frac{160}{9\pi}$
Then evaluate on a calculator.

PROBLEM-SOLVING AND REASONING

10–12 | 10, 12–14

10 The formula $F = \frac{9C}{5} + 32$ is used to convert temperature from degrees Celsius (°C) (which is used in Australia) to degrees Fahrenheit (°F) (which is used in the USA).

Hint: When finding C, you will have an equation to solve.

- **a** When it is 30°C in Sydney, what is the temperature in Fahrenheit?
- **b** How many degrees Celsius is 30° Fahrenheit? Answer to one decimal place.
- **c** Water boils at 100°C. What is this temperature in degrees Fahrenheit?
- **d** What is 0°F in degrees Celsius? Answer to one decimal place.

11 The cost, in dollars, of a taxi is $C = 3 + 1.45d$, where d is the distance travelled, in kilometres.

- **a** What is the cost of a 20 km trip?
- **b** How many kilometres can be travelled for $90?

12 $I = \frac{Prt}{100}$ calculates interest on an investment. Find:

- **a** P when $I = 60$, $r = 8$ and $t = 1$
- **b** t when $I = 125$, $r = 5$ and $P = 800$
- **c** r when $I = 337.50$, $P = 1500$ and $t = 3$

13 The number of tablets a nurse must give a patient is found using the formula:

$$\text{Tablets} = \frac{\text{strength required}}{\text{tablet strength}}$$

- **a** 750 milligrams of a drug must be administered to a patient. How many 500 milligram tablets should the nurse give the patient?
- **b** If the nurse administers 2.5 of these tablets to another patient, how much of the drug did the patient take?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

14 A drip is a way of pumping a liquid drug into a patient's blood. The flow rate of the pump, in millilitres per hour, is calculated using the formula: $\text{rate} = \frac{\text{volume (mL)}}{\text{time (h)}}$.

a A patient needs 300 mL of the drug administered over 4 hours. Calculate the rate, in mL/h, which needs to be delivered by the pump.

b A patient was administered 100 mL of the drug at a rate of 300 mL/h. How long was the pump running?

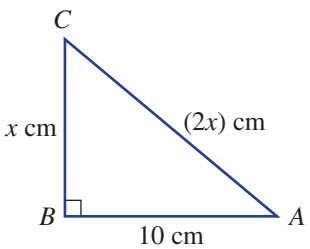
ENRICHMENT: Calculation challenges — 15–17

15 A tax agent charges \$680 for an 8-hour day. The agent uses the formula $F = \frac{680x}{8}$ to calculate a fee to a client, in dollars.

a What does the x represent?

b If the fee charged to a client is \$637.50, how many hours, to one decimal place, did the agent spend working on the client's behalf?

16 Find the area and perimeter of triangle ABC, shown. Do not round values during calculations, but round your final answer to two decimal places.



Hint: Use Pythagoras' theorem to find x.

17 Iqra is 10 years older than Urek. In 3 years' time, she will be twice as old as Urek. How old are they now?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9E Solving simultaneous equations graphically EXTENDING

Learning intentions for this section:

- To understand that the solution to simultaneous linear equations is the intersection point
- To be able to locate an intersection point graphically
- To be able to solve a problem by finding the intersection point

Past, present and future learning:

- This section goes beyond the Path topics for Standard but has been included to help students prepare for Stage 6 Standard
- These concepts were not addressed in our Year 9 book
- In Stage 6 Standard these concepts will be revised, briefly, then extended

When we approach an intersection while driving, we near the shared position of two or more roads.

Like two roads, two straight lines in the same plane will always intersect unless they are parallel.

If we try to find the point of intersection, we are said to be solving the equations simultaneously.

Lesson starter: Which job has better pay?

You start working as a delivery person for the Hasty Tasty Pizza Company. You're paid $25 per shift and $4 per pizza delivery.

A second pizza company, More-2-Munch Pizzas, offers you a job at $15 per shift and $5 per pizza delivery.

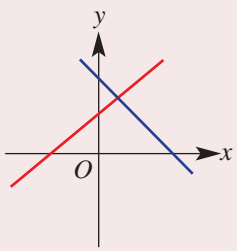
- How much does each company pay for delivery of 7 pizzas in one shift? How much does each company pay for delivery of 12 pizzas in one shift?
- For each pizza company, draw up a table to show the money you could earn for delivery of up to 15 pizzas delivered in one shift.
- On the same sheet, draw a graph of the information in your tables for each pizza company. Draw the graph for each pizza company on the same set of axes.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

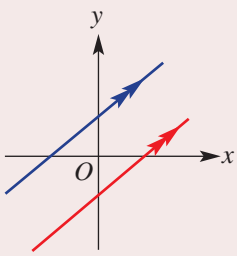
- What does the point of intersection show us?
- Write a sentence describing which job pays better for different numbers of pizzas delivered.
- Write down one advantage of using a graph to compare these two wages.

KEY IDEAS

- At a point of **intersection**, two lines will have the same coordinates.
- The point of intersection represents the solution of two linear simultaneous equations.
- To find the point of intersection, sketch each straight line and read off the coordinates of where the lines meet.
- When two lines are **parallel**, they have the same gradient and there is no point of intersection.



1 point of intersection



0 points of intersection

Key terminology: intersection point, coordinates, parallel, gradient, simultaneous

Exercise 9E

UNDERSTANDING 1–3 2, 3

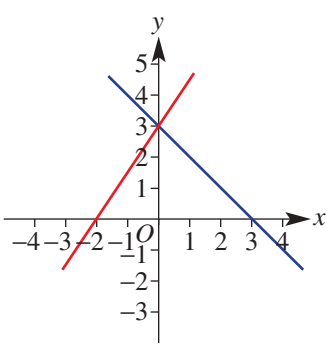
1 State the missing number.

a If two lines are parallel, then there are _____ points of intersection.

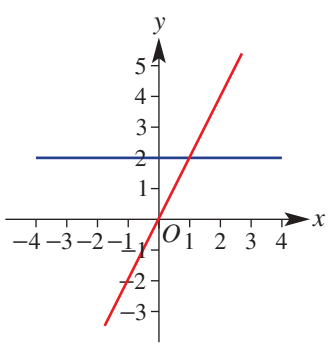
b If two lines are not parallel, then there is _____ point of intersection.

2 State the point of intersection (x, y) for the following lines, if there is one.

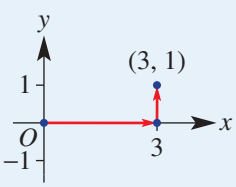
a



b



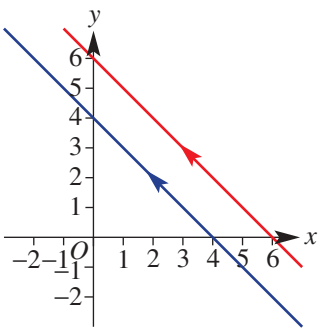
Hint:



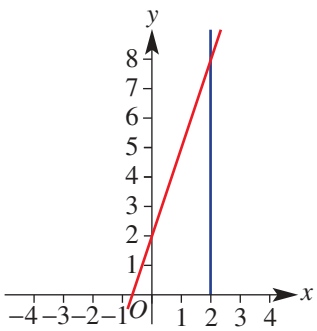
(3, 1) = (across 3 up 1)

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

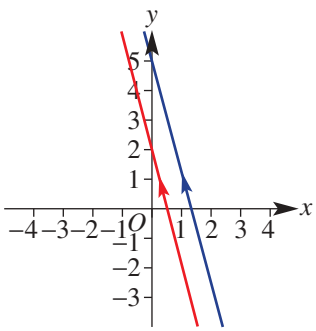
c



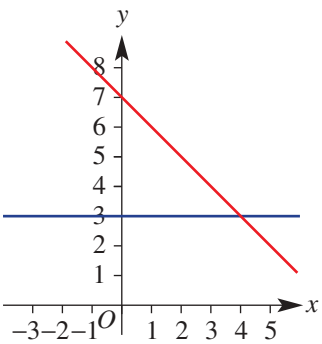
d



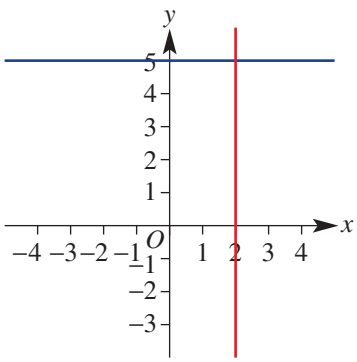
e



f



g



3 Use the method of trial and error (guess, check and refine) to find the point of intersection for these pairs of linear equations. Remember, your chosen point must satisfy both equations, so substitute your x and y values into the equations to see if both are true.

a $y = 2x - 1$ and $y = 5 - x$

b $y = x - 3$ and $2x + y = -6$

FLUENCY 4(½), 5, 6 | 4(½), 5–7

Example 16 Finding the point of intersection by graphing

Find the point of intersection (x, y) of $y = 2x + 4$ and $3x + y = 9$ by sketching accurate graphs on the same axes.

SOLUTION

$y = 2x + 4$

y-intercept at $x = 0$: $y = 2(0) + 4 = 4$

x-intercept at $y = 0$: $0 = 2x + 4$

$2x = -4$

$x = -2$

$3x + y = 9$

y-intercept at $x = 0$: $3(0) + y = 9$

$y = 9$

EXPLANATION

First, find the x- and y-intercepts of each graph.

Substitute $x = 0$.

Substitute $y = 0$.

Subtract 4 from both sides.

Divide both sides by 2.

Substitute $x = 0$.

Simplify.

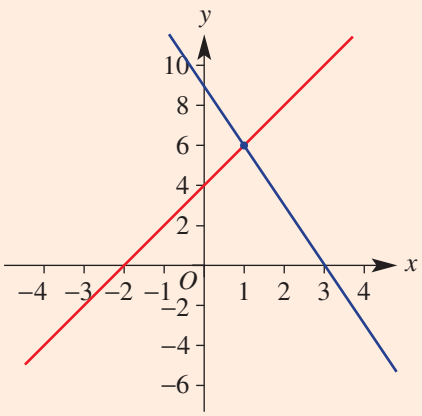
Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

x-intercept at $y = 0$: $3x + (0) = 9$ — Substitute $y = 0$.

$3x = 9$ — Simplify.

$x = 3$ — Divide both sides by 3.



Sketch the graphs using the x- and y-intercepts.

For $y = 2x + 4$, the x-intercept $= -2$ and the y-intercept $= 4$.

For $3x + y = 9$, the x-intercept $= 3$ and the y-intercept $= 9$.

The point of intersection is $(1, 6)$.

Read off the intersection point, listing x followed by y.

Now you try

Find the point of intersection (x, y) of $y = x - 2$ and $2x - y = 3$ by sketching accurate graphs on the same axes.

4 Find the point of intersection (x, y) of each pair of equations by plotting an accurate graph.

- **a** $y = x + 1$ and $3x + 2y = 12$
- **b** $y = 3x + 2$ and $2x + y = 12$
- **c** $y = 2x + 9$ and $3x + 2y = 18$
- **d** $y = x + 11$ and $4x + 3y = 12$

Hint:
y-intercept: $x = 0$
x-intercept: $y = 0$

5 Find the point of intersection of each pair of equations by plotting an accurate graph.

- **a** $y = 3$ and $x = 2$
- **b** $y = -2$ and $x = 3$

Hint: $y = 3$ cuts the y-axis at 3 and is horizontal.
$x = 2$ cuts the x-axis at 2 and is vertical.

6 Find the point of intersection of each pair of equations by plotting an accurate graph.

- **a** $y = 3x$ and $y = 2x + 3$
- **b** $y = -3x$ and $y = 2x - 5$

7 Find the point of intersection of each pair of equations by plotting an accurate graph.

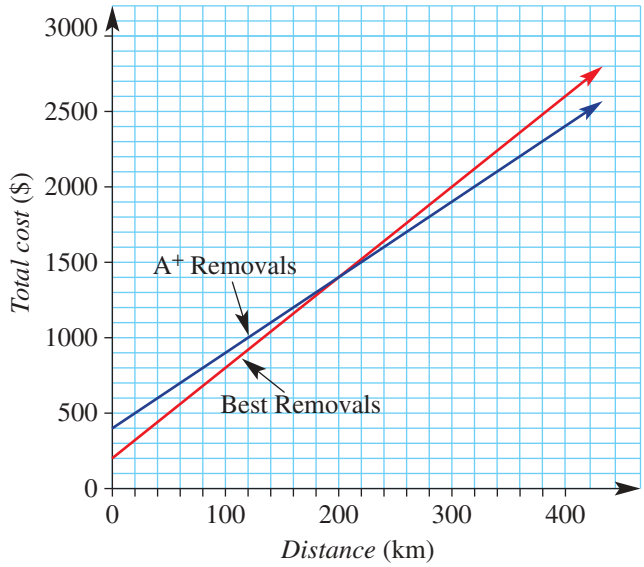
- **a** $y = 2x - 6$ and $y = 3x - 7$
- **b** $y = -2x + 3$ and $y = 3x - 2$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING

8, 9 | 9–11

8 This graph represents the cost of hiring two different removalist companies to move a person's belongings for various distances.



a Determine the number of kilometres for which the total cost of the removalists is the same.

Hint: The cost is the same at the point of intersection.

b What is the price when the total cost is equal?

c If a person wanted to move 100 km, which company would be cheaper and by how much?

d If a person wanted to move 400 km, which company would be cheaper and by how much?

9 The hiring fees charged by companies A and B are given by the following.
Company A: $20 per hour
Company B: $45 plus $15 per hour

a Complete two tables, showing the pay by each company for up to 12 hours.

b Draw a graph of the fee charged by each company (on the vertical axis) versus time, in hours (on the horizontal axis). Draw the graphs for both companies on the same set of axes.

c State the number of hours of hire for which the fee is the same for the two companies.

d State the fee when it is the same for the two companies.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 **a** Graph these three lines on the same coordinate axes by plotting the axis intercepts for each: $y = 3, y = x + 1, y = 1 - x$.

b Write the coordinates of the points of intersection.

c Find the length of each line segment formed between the intersection points. Give your answers as exact values.

d What type of triangle is formed by these line segments?

Hint: Use Pythagoras' theorem to find the length of a line segment.

11 The value of two cars is depreciating (i.e. decreasing) at a constant rate according to the information in this table.

Hint: Annual depreciation means how much the car's value goes down by each year.

Car	Initial value	Annual depreciation
Luxury sports coupe	$70 000	$5000
Family sedan	$50 000	$3000

a Complete two tables showing the value of each car every second year from zero to 12 years.

b Draw a graph of the value of each car (on the vertical axis) versus time, in years (on the horizontal axis). Draw both graphs on the same set of axes.

c From the graph, determine the time taken for the cars to have the same value.

d State the value of the cars when they have the same value.

ENRICHMENT: Multiple intersections — 12, 13

Use a calculator to complete these questions.

12 On the same axes, plot the graphs of $y = 2x, y = 2x + 1, y = 2x + 2$ and $y = 2x + 3$.

a Are there any points of intersection?

b Suggest a reason for your answer to part **a**.

c Plot the graph of $y = 3x + 6$.

d Determine the points of intersection of the graphs already drawn and $y = 3x + 6$.

13 On the same axes, plot $y = x - 1, y = 2x - 1, y = 3x - 1$ and $y = 4x - 1$.

a Are there any points of intersection?

b Suggest a reason for your answer to part **a**.

c Plot the graph of $y = 2x + 1$.

d Determine the points of intersection of the graphs already drawn and $y = 2x + 1$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Progress quiz

9A **1** Solve the following one-step equations.

a $x + 12 = 18$ **b** $y - 9 = 8$ **c** $3m = 21$ **d** $\frac{x}{5} = -2$

9A/9B **2** Solve the following equations.

a $2x - 3 = 15$ **b** $1 - 2y = 9$ **c** $\frac{x}{3} + 4 = 7$

d $\frac{x-2}{4} = 1$ **e** $\frac{2y+2}{5} = 4$ **f** $2 - \frac{x}{5} = 3$

9B **3** Solve these equations with brackets.

a $3(x - 1) = 6$ **b** $5(2a + 3) = -45$

c $3(y + 4) + 2(2y - 3) = 27$ **d** $4(2x - 1) - 3(x - 2) = 22$

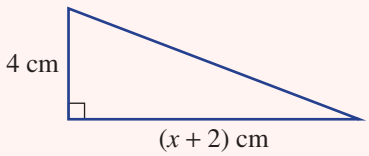
9A/9B **4** Write equations for the following and solve for the pronumeral.

a When x is doubled and then 3 is added, the result is 25.

b When 3 is subtracted from x and the answer is divided by 2, the result is 6.

c When 2 is subtracted from x and the answer is multiplied by 3, the result is 12.

d The area of the triangle shown is 16 cm^2.



9B **5** Solve these equations with variables on both sides.

a $8x = 2x - 12$ **b** $9x + 4 = 4x + 14$ **c** $3x + 4 = 20 - x$

9C **6** Find all possible solutions to the following equations. Give exact answers.

a $x^2 = 25$ **b** $2x^2 = 72$ **c** $x^2 = 14$

d $x^2 + 9 = 0$ **e** $x^2 - 2 = 9$ **f** $6 - x^2 = 2, x < 0$

9D **7** Find the value of the unknown in these formulas by substituting the given values.

a For $A = \frac{1}{2}bh$, find b when $A = 28$ and $h = 14$.

b For $s = ut + \frac{1}{2}at^2$, find u when $a = 2$, $t = 3$ and $s = 24$.

c For $S = 2\pi rh$, find h (to one decimal place) when $S = 30$ and $r = 2$.

9E **8** Find the point of intersection of $x + y = 4$ and $y = 2x + 1$ by sketching accurate graphs on the same axes.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9F Solving simultaneous equations using substitution EXTENDING

Learning intentions for this section:

- To understand that a solution to a pair of simultaneous equations can be found algebraically
- To use the method of substitution to solve problems involving simultaneous equations

Past, present and future learning:

- This section shows another way to solve simultaneous equations
- It addresses the Stage 5 Path-to-Advanced topic called *Equations C*
- These concepts were not addressed in our Year 9 book
- Students of Stage 6 Standard are expected to solve simultaneous equations graphically

Two simultaneous equations can be made when there are two unknown quantities (variables) and two lots of information relating these quantities. The solution gives the values that make both equations true.

In the previous section, the solution was found from the point of intersection of two line graphs. In this section, you will learn how to find the solution using the algebraic method of *substitution*.

An example of two variables is the cost of a wedding reception and the number of invited guests. Two simultaneous equations could be made about the costs of two different catering companies. The solution will be the number of guests that make the costs equal for the two companies. Using equations gives an accurate comparison of two deals.

Simultaneous equations can be used to compare the costs of two different catering companies for a wedding reception.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Lesson starter: Equations and solutions

Match each set of simultaneous equations with the correct solution.

SIMULTANEOUS EQUATIONS	
J	*M*
$a + b = 100$	$y = 500 + 70x$
$12a + 5b = 920$	$y = 700 + 68x$
K	*N*
$x = 4y$	$a + b = 50$
$2x + 2y = 1000$	$a - b = 6$
L	*O*
$3x + 2y = 29$	$a = b - 5$
$2x - y = 3$	$2a + b = 35$

SOLUTIONS

a $x = 5$ and $y = 7$
b Elias' age (a) is 10 years and Maria's age (b) is 15 years.
c Nicole's age (a) is 28 years and Julian's age (b) is 22 years.
d The length (x) of a playing field is 400 m and the width (y) is 100 m.
e 60 adults (a) and 40 children (b) attended a rugby match. The tickets cost \$12 per adult and \$5 per child.
f A wedding reception hosting 100 guests (x) costs \$7500 (y).

KEY IDEAS

- The algebraic method of **substitution** is generally used when at least one of the linear equations has x or y as the subject;

  e.g. $y = 3x + 4$ and $3x + y = 2$ or $y = -2x + 6$ and $y = -x - 1$ or $x = 2$ and $2x - y = 5$

- The method involves:
  - substituting one equation into the other
  - solving for the remaining variable
  - substituting to find the value of the second variable

- When problem-solving with simultaneous linear equations, follow these steps.
  - Define/describe two unknowns using pronumerals
  - Write down two equations using your pronumerals
  - Solve the equations using the method of substitution
  - Answer the original question in words

Key terminology: substitute, subject, define

Exercise 9F

UNDERSTANDING 1, 2 2

1 Write the missing words to complete each statement. Choose from *intersection*, *substituted*, *simultaneous* and *substitution*.

a _____ equations involve two or more equations and two variables.
b When two equations have been graphed, the x and y values that make both equations true are the coordinates of the point of _____.
c If x (or y) is replaced with a number, then we have _____ that number for x (or y).
d If x (or y) is replaced with an algebraic expression, then we have _____ that expression for x (or y).
e When we algebraically substitute one equation into another, this is called solving simultaneous equations by the method of _____.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 Choose the correct option.

a When $y = x - 1$ is substituted into $2x + y = 6$, the result is:

A $2x + (x - 1) = 6$
B $2x - 1 = 6$
C $x - 1 = 6$
D $2x - x + 1 = 6$
E $3x = 6$

Hint: In part **a**, replace y in $2x + y = 6$ with $x + 1$.

b When $y = 2x + 3$ is substituted into $x - 3y = 1$, the result is:

A $x + 3(2x + 3) = 1$
B $3(2x + 3) = 1$
C $x - 3(2x + 3) = 1$
D $x - (2x + 3) = 1$
E $2x - 3 = 1$

FLUENCY 3–4(½), 6 | 3–4(½), 5–7

Example 17 Using the substitution method to solve simultaneous equations

Determine the point of intersection of $y = 5x$ and $y = 2x + 6$.

SOLUTION	EXPLANATION
$y = 5x$ [1] $y = 2x + 6$ [2]	Label the two equations.
Substitute [1] into [2]:	Explain how you are substituting the equations.
$5x = 2x + 6$	Replace y in the second equation with $5x$.
$3x = 6$	Subtract $2x$ from both sides.
$x = 2$	Divide both sides by 3.
Substitute $x = 2$ into [1]:	Alternatively, substitute into equation [2].
$y = 5(2)$	Replace x with the number 2.
$y = 10$	Simplify.
The point of intersection is $(2, 10)$.	Write the solution.
Check: $10 = 2(2) + 6$	Substitute your solution into the other equation to check.

Now you try

Determine the point of intersection of $y = 7x$ and $y = 2x + 5$.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Determine the point of intersection for the following pairs of lines.

a $y = 5x$
$y = 3x + 4$

b $y = 3x$
$y = 2x - 5$

c $y = 2x$
$y = 4x + 8$

d $y = 4x$
$y = -3x + 7$

e $y = x$
$y = -5x + 12$

f $y = 6x$
$y = -2x + 16$

Hint:
$y = 5x$
$y = 3x + 4$
$5x = 3x + 4$

Example 18 Solving simultaneous equations with the substitution method

Solve the simultaneous equations $y = x + 3$ and $2x + 3y = 19$ using the substitution method; i.e. find the point of intersection.

SOLUTION	EXPLANATION
$y = x + 3$ [1] $2x + 3y = 19$ [2]	Label the two equations.
Substitute [1] into [2]:	Explain how you are substituting the equations.
$2x + 3(x + 3) = 19$	Replace y in the second equation with $(x + 3)$.
$2x + 3x + 9 = 19$	Expand the brackets.
$5x + 9 = 19$	Simplify.
$5x = 10$	Subtract 9 from both sides.
$x = 2$	Divide both sides by 5.
Substitute $x = 2$ into [1]:	Alternatively, substitute into equation (2).
$y = 2 + 3$	Replace x with the number 2.
$y = 5$	Simplify.
The point of intersection is $(2, 5)$.	Write the solution.
Check: $2(2) + 3(5) = 19$	Substitute your solution into the other equation to check.

Now you try

Solve the simultaneous equations $y = x - 1$ and $3x + 2y = -12$ using the substitution method; i.e. find the point of intersection.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Solve the following pairs of simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = x + 3$ and $2x + 3y = 19$

b $y = x + 2$ and $3x + y = 6$

c $y = x - 1$ and $3x + 2y = 8$

d $y = x - 1$ and $3x + 5y = 27$

e $y = x + 2$ and $2x + 3y = -19$

f $y = x + 5$ and $5x - y = -1$

g $y = x - 3$ and $5x - 2y = 18$

h $y = x - 4$ and $3x - y = 2$

Hint: In part **a**, replace y in the second equation with $(x + 3)$. It is important to use brackets.

$y = (x + 3)$

$2x + 3(y) = 19$

Remember that

$3(x + 3) = 3x + 9$

5 Solve the following pairs of simultaneous equations; i.e. find the point of intersection.

a $y = 2$

$y = 2x + 4$

b $y = -1$

$y = 2x - 7$

c $y = 4$

$2x + 3y = 20$

Hint: Replace y in the second equation with 2.

$y = (2)$

$(y) = 2x + 4$

6 Determine the point of intersection for the following.

a $x = 2$

$3x + 2y = 14$

b $x = -3$

$y = -2x - 4$

c $x = 7$

$4x - 3y = 31$

Hint: Replace x in the second equation with 2.

$x = (2)$

$3(x) + 2y = 14$

Remember that $3x$ means $3 \times x$.

7 Solve the following pairs of simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = 2x + 3$

$11x - 5y = -14$

b $y = 3x - 2$

$7x - 2y = 8$

c $y = 3x - 5$

$3x + 5y = 11$

d $y = 4x + 1$

$2x - 3y = -23$

Hint: Be careful with signs when expanding brackets.

$-5 \times (+3) = -15$

$11x - 5(2x + 3)$

$= 11x - 10x - 15$

When multiplying numbers with different signs, the answer is negative.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

PROBLEM-SOLVING AND REASONING	8, 9	8, 10, 11

Example 19 Solving word problems with simultaneous equations (substitution)

Jade is 5 years older than Marian. If their combined age is 33, find their ages.

SOLUTION	EXPLANATION
Let j be Jade's age and m be Marian's age.	Define two pronumerals using words.
$j = m + 5$ [1]	The first piece of information is that Jade is 5 years older than Marian.
$j + m = 33$ [2]	The second piece of information is that their combined age is 33.
$(m + 5) + m = 33$	Substitute $m + 5$ for j in the second equation.
$2m + 5 = 33$	Collect any like terms, so $m + m = 2m$.
$2m = 28$	Subtract 5 from both sides.
$m = 14$	Divide both sides by 2.
$j = m + 5$ [1] $j = 14 + 5$ $j = 19$	Use the first equation, $j = m + 5$, to find j.
Jade is 19 years old and Marian is 14 years old.	Answer the original question in words.

Now you try

A rectangle's length is 3 cm more than its width. If its perimeter is 32 cm, determine its dimensions.

8 Kye is 5 years older than Viviana. If their combined age is 81, determine their ages.

Hint: First define a pronumeral for Kye's age and another pronumeral for Viviana's age. Then write two equations before solving.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 The length of a rectangle is three times the width. If the perimeter of the rectangle is 48 cm, determine its dimensions.

Hint: Draw a diagram to help form the perimeter equation.

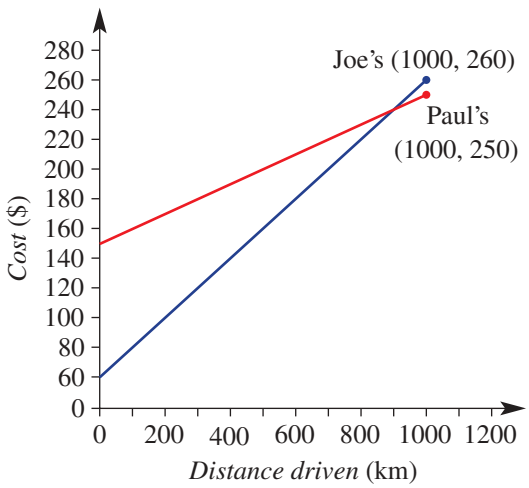
10 A vanilla thick shake is \$2 more than a fruity twirl. If three vanilla thick shakes and five fruity twirls cost \$30, determine their individual prices.

Hint: If a fruity twirl costs $\$x$, then 5 will cost $\$5x$.

11 Carlo's age is 3 more than twice Ella's age. If the sum of their ages is 54 years, determine their ages.

ENRICHMENT: Rentals — 12

12 The given graph represents the rental cost of a new car from two car rental firms called Paul's Motor Mart and Joe's Car Rental.



a Determine:

- **i** the initial rental cost from each company
- **ii** the cost in cents per kilometre when renting from each company
- **iii** the linear equations for the total cost from each company
- **iv** the number of kilometres at which the total cost is the same for both rental firms, using the method of substitution.

b Describe when you would use Joe's or Paul's rental firm.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9G Solving simultaneous equations using elimination EXTENDING

Learning intentions for this section:

- To use the method of elimination to solve problems involving simultaneous equations

Past, present and future learning:

- This section shows another way to solve simultaneous equations
- It addresses the Stage 5 Path-to-Advanced topic called *Equations C*
- These concepts were not addressed in our Year 9 book
- Students of Stage 6 Standard are expected to solve simultaneous equations graphically

A second method for solving simultaneous equations, called elimination, can sometimes be more efficient, depending on how the equations are structured in the first place.

When setting up equations for real situations, we should define the unknown quantities using pronumerals. When solving simultaneous linear equations there should be only two unknown quantities for two equations formed from the given information.

For example, two related variables are the cost of owning a car and the number of kilometres driven. For two different cars, two equations could be made relating these variables. The simultaneous solution gives the number of kilometres that makes the total running costs of each car equal. Solving simultaneous equations provides information for an accurate comparison of costs between two vehicles.

Lesson starter: Eliminating a variable

One step in the elimination method involves adding or subtracting two equations in order to eliminate one of the variables. When adding, we write [1] + [2]; when subtracting, we write [1] − [2].

- A student has either added or subtracted pairs of equations, but has many incorrect answers.
- Determine which answers are incorrect and write the correct answer for these. (Note: Do not solve the equations for x or y.)

A $5x + 3y = 34$ [1]
$7x - 3y = 26$ [2]
[1] + [2] gives:
$12x + 0 = 60$

B $3x + 2y = 18$ [1]
$2x - 2y = 2$ [2]
[1] + [2] gives:
$5x - 4y = 20$

C $3x - 3y = 9$ [1]
$2x - 3y = 4$ [2]
[1] − [2] gives:
$5x + 0 = 5$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

D $2x - 2y = 8$ [1]
$4x - 2y = 24$ [2]
[1] − [2] gives:
$2x - 4y = 16$

E $4x + 3y = 16$ [1]
$-4x + 2y = 3$ [2]
[1] + [2] gives:
$0 + y = 19$

F $3x + 2y = 25$ [1]
$2x + 2y = 18$ [2]
[1] − [2] gives:
$x + 0 = 43$

G $5x + 3y = 31$ [1]
$5x - 3y = 19$ [2]
[1] + [2] gives:
$0 + 0 = 12$

H $x + 3y = 15$ [1]
$x + 2y = 12$ [2]
[1] − [2] gives:
$2x + y = 3$

I $7x - y = 5$ [1]
$3x - y = -2$ [2]
[1] − [2] gives:
$4x = 3$

KEY IDEAS

- **Elimination** is generally used to solve simultaneous equations when both equations are in the form $ax + by = d$.
  e.g. $2x - y = 6$ and $3x + y = 10$ or $-5x + y = -2$ and $6x + 3y = 5$
- Adding or subtracting multiples of these two equations allows one of the variables to be eliminated.
  - Add $x - y = 10$ and $3x + y = 34$ to eliminate y.
  - Subtract $5x + 2y = 7$ and $5x + y = 6$ to eliminate the x.
  - Form a matching pair by multiplying by a chosen factor.
    For example, $2x - y = 3 \Rightarrow \times 2 \Rightarrow 4x - 2y = 6$
    $5x + 2y = 12 \Rightarrow 5x + 2y = 12$
- When problem-solving with simultaneous linear equations, follow these steps.
  - Define/describe two variables using letters.
  - Write down two equations using your variables.
  - Solve the equations using the method of elimination.
  - Answer the original question in words.

Key terminology: elimination, simultaneous, multiple

Exercise 9G

UNDERSTANDING 1–3 2, 3

1 What operation (i.e. + or −) will make these equations true?
a $2x$ _____ $2x = 0$ **b** $-3y$ _____ $3y = 0$ **c** $4x$ _____ $(-4x) = 0$

2 Multiply both sides of the equation $3x - 2y = -1$ by the following numbers. Write the new equations.
a 2 **b** 3 **c** 4

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Choose the correct option.

a When $2x + y = 3$ is added to $5x - y = 11$ the result is:

A $7x = 11$ **B** $7x = 14$ **C** $3x = 14$

D $3x = 1$ **E** $7x = 8$

b When $x + y = 5$ is subtracted from $3x + y = 7$ the result is:

A $2x = 7$ **B** $4x = 2$ **C** $4x = 12$

D $x = 2$ **E** $2x = 2$

FLUENCY 4, 5, 6–9(½) 6–11(½)

Example 20 Eliminating a variable by addition of equations then solving

Add equation [1] to equation [2] and then solve for x and y.

$x + 2y = 10$ [1]

$x - 2y = 2$ [2]

SOLUTION	EXPLANATION
$x + 2y = 10$ [1] $x - 2y = 2$ [2]	Copy equations with the labels [1] and [2].
[1] + [2] gives:	Write the instruction to add: [1] + [2].
$2x + 0 = 12$	Add the x column: $x + x = 2x$.
$2x = 12$	Add the y column: $2y + (-2y) = 0$.
$x = 6$	Add the RHS: $10 + 2 = 12$ and then divide both sides by 2.
Substitute $x = 6$ into [1]: $6 + 2y = 10$	In equation [1], replace x with 6. Equation [2] could also have been used.
$2y = 4$	Subtract 6 from both sides.
$y = 2$	Divide both sides by 2.
Solution is $(6, 2)$.	Write the solution as an ordered pair.
Check: [2] $6 - 2 \times 2 = 2$, true	Check that the solution satisfies equation [2].

Now you try

Add equation [1] to equation [2] then solve for x and y.

$3x + y = 11$ [1]

$x - y = 5$ [2]

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 Copy each pair of equations, add equation [1] to [2], then solve for x and y.

a $x + y = 7$ [1]
$x - y = 5$ [2]
[1] + [2]

b $x + 2y = 11$ [1]
$x - 2y = -5$ [2]
[1] + [2]

c $3x + 2y = 20$ [1]
$-3x + y = 1$ [2]
[1] + [2]

Hint:
Adding equations:
[1] + [2]

$x + y = 7$ [1]
$x - y = 5$ [2]

$2x + 0 = 12$

Remember that
$+y + (-y) = +y - y = 0$

5 Copy each pair of equations, subtract equation [2] from equation [1] and then solve for x and y, showing all steps.

a $2x + y = 16$ [1]
$x + y = 9$ [2]
[1] − [2]

b $3x + 5y = 49$ [1]
$3x + 2y = 25$ [2]
[1] − [2]

c $5x - 4y = 16$ [1]
$2x - 4y = 4$ [2]
[1] − [2]

Hint:
Subtracting equations:
[1] − [2]

$5x - 2y = 16$ [1]
$2x - 2y = 4$ [2]

$3x + 0 = 12$

Remember that
$-2y - (-2y)$
$= -2y + 2y = 0$

6 Determine the point of intersection of the following lines, using the elimination method.

a $x + y = 7$ and $5x - y = 5$

b $x + y = 5$ and $3x - y = 3$

c $x - y = 2$ and $2x + y = 10$

d $x - y = 0$ and $4x + y = 10$

Hint: Label the two equations, one under the other, and decide whether to eliminate x or y; i.e. eliminate whichever variable has the same number in each equation. Remember that $+y + (-y) = 0$. The point of intersection is the same as the simultaneous solution of the equations.

7 Solve the following pairs of simultaneous equations, using the elimination method. You will need to subtract the equations to eliminate one of the variables.

a $3x + 4y = 7$
$2x + 4y = 6$

b $4x + 3y = 11$
$x + 3y = 5$

c $2x + 3y = 1$
$2x + 5y = -1$

Hint: Always label the equations and write the instruction; e.g. [1] − [2] or [2] − [1].

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 21 Forming a matching pair

Determine the point of intersection of the lines $x + y = 6$ and $3x + 2y = 14$, using the elimination method.

SOLUTION	EXPLANATION
$x + y = 6$ [1]	Label the two equations and decide where to form a matching pair.
$3x + 2y = 14$ [2]	
$[1] \times 2: \quad 2x + 2y = 12$ [3]	
$[2]: \quad 3x + 2y = 14$ [2]	
$[2] - [3]:$	Subtract the two equations because $2y - 2y = 0$,
$x = 2$	$3x - 2x = x$ and $14 - 12 = 2$
Substitute $x = 2$ into [1]:	Alternatively, substitute into equation [2].
$2 + y = 6$	Replace x with the number 2.
$y = 4$	Subtract 2 from both sides.
Point of intersection is $(2, 4)$.	Write the solution as an ordered pair.
Check: $3(2) + 2(4) = 14$	Check that the solution satisfies the other equation.

Now you try

Determine the point of intersection of the lines $x + y = 4$ and $5x + 2y = 11$, using the elimination method.

Hint: Multiply one equation by a number to form a matching pair.

8 Solve these simultaneous equations by first forming a matching pair.

a $x - 3y = 1$
$2x + y = 9$

b $4x + 2y = 10$
$x + 3y = 10$

c $3x + 4y = 19$
$x - 3y = 2$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 22 Forming a matching pair by multiplying both equations

Solve the simultaneous equations $3x + 2y = 6$ and $5x + 3y = 11$, using the elimination method.

SOLUTION			EXPLANATION
	$3x + 2y = 6$	[1]	Label the two equations and decide whether to eliminate x or y.
	$5x + 3y = 11$	[2]	
[1] $\times 3$:	$9x + 6y = 18$	[3]	Multiplying the first equation by 3 and the second by 2 results in a matching pair $6y$ in each equation.
[2] $\times 2$:	$10x + 6y = 22$	[4]	
[4] $-$ [3] :	$x = 4$		Subtract the equations since $6y - 6y = 0$.
Substitute $x = 4$ into [1]:			Alternatively, substitute into equation [2].
$3(4) + 2y = 6$			Replace x with the number 4.
$2y = -6$			Subtract 12 from both sides, since $3 \times 4 = 12$.
$y = -3$			Divide both sides by 2.
Solution is $(4, -3)$.			Write the solution as an ordered pair.
Check: $5(4) + 3(-3) = 11$			Check the solution with the other equation.

Now you try

Solve the simultaneous equations $5x + 3y = 1$ and $2x + 5y = 8$, using the elimination method.

9 Solve the following pairs of simultaneous equations, using the elimination method.

a $3x + 2y = 6$ and $5x + 3y = 11$

b $3x + 2y = 5$ and $2x + 3y = 5$

c $2x + y = 4$ and $5x + 2y = 10$

d $2x + 5y = 7$ and $x + 3y = 4$

Hint: When multiplying an equation by a number, multiply every term on the LHS and RHS by that number. Always write the instruction for multiplying; e.g. [2] $\times 4$.

10 Solve the following pairs of simultaneous equations, using the elimination method.

a $3x + 5y = 8$

$x - 2y = -1$

b $2x + y = 10$

$3x - 2y = 8$

c $4x - 3y = 0$

$3x + 4y = 25$

Hint: Choose to eliminate x or y. The coefficients need to be the same size (with $+$ or $-$); e.g. $-4x$ and $4x$ or $-5y$ and $5y$. Choose to add or subtract the equations to eliminate one variable.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

11 Solve the following pairs of simultaneous equations.

a $5x + 3y = 18$ and $3y - x = 0$

b $3x - y = 13$ and $x + y = -9$

c $2x + 7y = -25$ and $5x + 7y = -31$

d $2x + 6y = 6$ and $3x - 2y = -2$

e $4x - 5y = -14$ and $7x + y = -5$

f $7x - 3y = 41$ and $3x - y = 17$

PROBLEM-SOLVING AND REASONING	12–15	12, 13, 16–18

Example 23 Solving word problems with simultaneous equations (elimination)

Kathy is older than Blake. The sum of their ages is 17 years and the difference is 5 years. Find Kathy and Blake's ages.

SOLUTION	EXPLANATION
Let k be Kathy's age and b be Blake's age.	Define two variables.
$k + b = 17$ [1]	The first piece of information is 'the sum of their ages is 17'.
$k - b = 5$ [2]	The second is 'the difference is 5 and Kathy is older than Blake'.
[1] + [2]: $2k = 22$ $k = 11$	Add the two equations to eliminate b or, alternatively, subtract to eliminate k.
Substitute $k = 11$ into [1]: $11 + b = 17$ $b = 6$	Alternatively, substitute into [2]. Subtract 11 from both sides.
Kathy is 11 years old and Blake is 6.	Answer the original question in words.

Now you try

The sum of two numbers is 97 and their difference is 13. Find the two numbers.

12 Ayden is older than Tamara. The sum of their ages is 56 years and the difference is 16 years. Use simultaneous equations to find Ayden and Tamara's ages.

Example 24 Problem solving with simultaneous equations

Reese purchases three daffodils and five petunias from the local nursery and the cost is $25. Giuliana buys four daffodils and three petunias and the cost is $26. Determine the cost of each type of flower.

Continued on next page

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

SOLUTION	EXPLANATION
Let $\$d$ be the cost of a daffodil and $\$p$ be the cost of a petunia.	Define your variables.
$3d + 5p = 25$ [1]	Three daffodils and five petunias from the local nursery cost \$25.
$4d + 3p = 26$ [2]	Four daffodils and three petunias cost \$26.
[1] $\times 4$: $12d + 20p = 100$ [3]	Multiply [1] by 4 and [2] by 3 to obtain a matching pair ($12d$ and $12d$).
[2] $\times 3$: $12d + 9p = 78$ [4]	
[3] $-$ [4] : $11p = 22$	Subtract the equations to eliminate d.
$p = 2$	Divide both sides by 2.
Substitute $p = 2$ into [1]:	Alternatively, substitute into [2].
$3d + 5(2) = 25$	Replace p with the number 2.
$3d + 10 = 25$	Simplify.
$3d = 15$	Subtract 10 from both sides.
$d = 5$	Divide both sides by 3.
Check: $4(5) + 3(2) = 26$	Check your solutions by substituting into the second equation.
Daffodils cost \$5 and petunias cost \$2 each.	Answer the question in words.

Now you try

Jess purchases 4 buckets of chips and 3 drinks for \$23.50, while Nigel purchases 3 buckets of chips and 4 drinks for \$22. Find the price of a bucket of chips and a drink.

13 A market stall sells two fruit packs:
Pack 1 : 10 apples and 5 mangoes (\$12.50)
Pack 2 : 15 apples and 4 mangoes (\$13.50)

- **a** Define two pronumerals and set up a pair of linear equations to eventually find the cost of each fruit.
- **b** Solve the two simultaneous equations to determine the individual prices of each piece of fruit.
- **c** Determine the cost of one apple and five mangoes.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

14 Tickets to a basketball game cost \$3 for children and \$7 for adults. If 5000 people attended the game and the total takings at the door was \$25 000, determine the number of children and adults who attended the game.

Hint: What you are being asked to find is often how you define your variables.

15 A Maths test contains multiple-choice questions worth 2 marks each and short-answer questions worth 3 marks each. The test is out of 50 marks and there are 22 questions.

Hint: Total marks is 50. Number of questions is 22.

a Define two pronumerals to represent the number of each question type.

b Set up two linear equations.

c Solve the two equations simultaneously to determine the number of multiple-choice questions.

16 Let x and y be two numbers that satisfy the following statements. Set up two linear equations according to the information and solve them simultaneously to determine the numbers in each case.

a Their sum is 16 and their difference is 2.

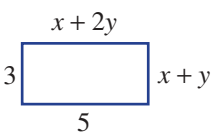
b Their sum is 30 and their difference is 10.

c Twice the larger number plus the smaller is 12 and their sum is 7.

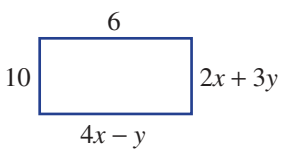
17 Find the value of x and y in the following rectangles. You will need to write two equations and solve using the elimination method.

Hint: Opposite sides of rectangles are equal.

a



b



18 Gordon is currently 31 years older than his daughter. In 30 years' time he will be twice his daughter's age. Using g for Gordon's current age and d for Gordon's daughter's current age, complete the following.

a Write down expressions for:

i Gordon's age in 30 years' time

ii Gordon's daughter's age in 30 years' time

b Write down two linear equations, using the information at the start.

c Solve the equations to find the current ages of Gordon and his daughter.

ENRICHMENT: Using technology — 19

19 Use technology to solve these simultaneous equations.

a $3x + 2y = 6$ and $5x + 3y = 11$

b $3x + 2y = 5$ and $2x + 3y = 5$

c $4x - 3y = 0$ and $3x + 4y = 25$

d $2x + 3y = 10$ and $3x - 4y = -2$

e $-2y - 4x = 0$ and $3y + 2x = -2$

f $-7x + 3y = 22$ and $3x - 6y = -11$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Maths@Work

Nurse

Nursing is a career that is both challenging and rewarding. It requires a person to be caring and empathetic. Good communication skills, as well as an understanding of mathematics and science, are also important.

Nurses need to be competent in many mathematical areas, including fractions, ratios, converting units of measurement and equations. They must be able to calculate medical dosages, substitute into equations and also know how to program the correct flow rate for intravenous (IV) drips.

Complete these questions, which are typical calculations that a nurse might complete when they are administering medication.

1 Use these formulas to answer each of the following questions.

$$\text{Volume required} = \frac{\text{strength needed}}{\text{strength in stock}} \times \text{volume of stock solution}$$

$$\text{Number of tablets} = \frac{\text{strength required}}{\text{strength per tablet in stock}}$$

Hint: For 100 mg in 2 mL, need 75 mg in ? mL.

a What volume, in millilitres, of pethidine should be given if the patient is prescribed 75 mg and the existing stock contains 100 mg in 2 mL?

b Calculate the volume, in millilitres, of insulin that is required for a patient who has been prescribed 60 units of the drug, if the stock is 100 units/1 mL.

c Pethidine 50 mg has been ordered to alleviate a patient's pain. The stock strength is 75 mg/1.5 mL. How much pethidine should be given?

d How many tablets does a nurse need to give for a prescription of 500 mg of amoxicillin per day, if the stock available in the ward is 250 mg per capsule?

e How many tablets are needed for a dosage of 125 mg, if the stock available is labelled 25 mg per tablet?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 Paediatrics is a branch of medicine dealing with young children. Different formulas are used to calculate the doses suitable for children. Apply the rules given below to complete the following calculations and state each answer to the nearest milligram.

Clarke's body weight rule:

$$\text{Child's dose} = \frac{\text{weight of child (kg)}}{\text{average adult weight (70 kg)}} \times \text{adult dose}$$

Clarke's body surface area rule:

$$\text{Child's dose} = \frac{\text{surface area of child (m}^2\text{)}}{\text{average adult surface area (1.7 m}^2\text{)}} \times \text{adult dose}$$

Fried's rule (used for infants under 1 year old):

$$\text{Child's dose} = \frac{\text{age in months}}{150} \times \text{adult dose}$$

Young's rule (used for children aged 2 to 12 years):

$$\text{Child's dose} = \frac{\text{age in years}}{\text{age} + 12} \times \text{adult dose}$$

a Use Young's rule to calculate the amoxicillin dose needed for a 10-year-old boy, if the adult dose of the drug amoxicillin is 250 mg.

b Use Clarke's body weight rule to find a child's dose for the drug ampicillin, given the child's weight is 15 kg and an adult's dose is 500 mg.

c Use Fried's rule to calculate the dose required for an 8-month-old baby girl for the drug ciprofloxacin, given that an adult's dose is 500 mg.

d Use Clarke's body surface area formula to find the dose of penicillin, in milligrams, required for a child whose surface area is 8000 cm^2, given that the adult dose is 1 gram.

Hint: Recall 1 m^2 = 10 000 cm^2

3 Drugs that are given with an intravenous (IV) drip use a different set of equations to calculate the time needed or the drop rate per minute.

$$\text{Time (in minutes)} = \frac{\text{volume (mL)}}{\text{flow rate (drops/min)}} \times \text{drip factor}$$

$$\text{Flow rate (drops/minute)} = \frac{\text{volume (mL)}}{\text{time (mins)}} \times \text{drip factor}$$

Use the equations above to answer these questions about IV drug dosage. The drip factor is in drops/mL. State all answers rounded to one decimal place.

Hint: Recall 1 L = 1000 mL

a Find the flow rate, in drops per minute, when a $\frac{1}{2}$ litre bag of saline solution is run over 4 hours with the IV machine set at a drip factor of 20 drops per mL.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b An IV drip of saline solution started at 4:15 p.m. Tuesday. The machine has 700 mL to run and is set at 40 drops/min with a drip factor of 18 drops per mL. At what time will the IV be finished?

c How long will it take 180 mL of zero negative blood to flow through an IV at 36 drops/minute when the blood supply machine is set at a drip factor of 15 drops/mL?

Using technology

4 Set up this Excel worksheet to calculate the ending times of intravenous drips for various patients. You will need to copy the given data and enter formulas into the shaded cells.

	A	B	C	D	E	F	G
1	**Intravenous drip administration**						
2	**IV bag**	**IV machine settings**		**Times**			
3	Total volume in mL	Flow rate in drops per minute	Drip factor drops per mL	Starting time	Time in minutes for IV	Time in hours and minutes for IV	Ending time
4	700	40	18	4:15 PM			
5	500	36	16	10:00 AM			
6	800	30	24	1:25 PM			
7	1500	40	20	6:00 PM			
8	2000	45	22	2:00 AM			

Hint:
- Format starting and ending times as *Number Category: Custom, Type: h:mm AM/PM*.
- To calculate time in hours and minutes, divide the time in minutes by the number of minutes in 24 hours, and format cells as *Number Category: Custom and Type: h:mm*.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Movie theatre pricing

A small boutique movie theatre is opening soon and will be publishing adult and child ticket prices on its website. After some analysis, the owner decides to make the following assumptions:

- The sum of the cost of a child ticket and an adult ticket will be $30
- 40 children and 25 adults are expected to attend each evening.

Present a report for the following tasks and ensure that you show clear mathematical workings, explanations and diagrams where appropriate.

Routine problems

a If the cost of a child movie ticket is $6, find the cost of an adult ticket.

b Find the revenue (total amount earned) for the movie theatre on one particular evening using the assumption that 40 children and 25 adults attend and a child ticket is $6.

c Solve the following simultaneous equations.
Use the elimination method, or rewrite the first equation and use the substitution method.

$$a + c = 30$$
$$25a + 40c = 855$$

Non-routine problems

Explore and connect

The problem is to determine a suitable price for the adult and child tickets to achieve a revenue of at least $1000.

a Write down all the relevant information that will help solve this problem.

b Using $\$a$ for the cost of an adult ticket and $\$c$ for the cost of a child ticket.
Determine the equations for,

- **i** The sum of the cost of a child ticket and an adult ticket will be $30.
- **ii** The revenue ($\$R$) if 40 children and 25 adults attend on one particular evening.
- **iii** If the total revenue, R, for the evening is $900, substitute this into the equation above to form a new equation, relating a and c.

Choose and apply techniques

c Solve the two simultaneous equations in part **a** to find the cost of one adult ticket and one child ticket.

d By changing the value of R in the equation for revenue in part **b ii**, solve two simultaneous equations, and, find the cost of an adult ticket and a child ticket if the revenue is;

i $825 **ii** $960 **iii** $984 **iv** $1086

e Explore the following special cases for the total revenue earned and decide if they represent reasonable pricing structures. Write a sentence detailing if you think that each revenue results in a good pricing structure.

i $975 **ii** $765

Communicate thinking and reasoning

f Explore the possibility of the revenue being $1200 based on the assumptions made by the company. Explain why you reached your conclusion

g Is it reasonable for the company to assume they can earn over $1000 each evening with their other assumptions?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Summarise your results by completing a table such as the one below.

Equation 1 _____ **Equation 2** _____

Revenue $R	Adult ticket price	Child Ticket price
900		
825		
960 . .		

Extension problems

Problem-solve

a Consider using an online graphing tool, such as Demos or Geogebra, to show how graphs can be used to solve the problem replacing the a with x and the c with y.

b If the sum of the cost of a child ticket and an adult ticket is changed to $40, do you think it would be reasonable to expect that more than $1000 revenue can be earned each evening? Explain your choices and calculations to justify your answer.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1 The answers to these equations will form a magic square, where each row, column and diagonal will add to the same number. Draw a 4 by 4 square for your answers and check that they do make a magic square.

$x - 3 = 6$	$x + 15 = 10$	$\frac{x}{2} = -2$	$5x = 30$
$3x + 7 = 1$	$\frac{x}{4} - 8 = -7$	$\frac{x+7}{2} = 5$	$3(x + 4) = x + 14$
$\frac{x}{2} - 5 = -4$	$4x - 9 = -9$	$x + 7 = 4x + 10$	$2(3x - 12) - 5 = 1$
$\frac{9-3x}{3} = 6$	$-2(3 - x) = x + 1$	$x - 16 = -x$	$5x + 30 - 3x = -3x$

2 Write an equation and solve it to help you find each unknown number in these puzzles.

a Three-quarters of a number plus 16 is equal to 64.

b A number is increased by 6, then that answer is doubled and the result is four more than triple the number.

c The average of a number and its triple is equal to 58.6.

d In 4 years' time, Ahmed's age will be double the age he was 7 years ago. How old is Ahmed now?

3 By applying at least two operations to x, write three different equations so that each equation has the solution $x = -2$. Verify that $x = -2$ makes each equation true.
For example, $3 \times (-2) + 10 = 4$, so one possible equation would be $3x + 10 = 4$.

4 Write two sets of simultaneous equations so that each pair has the solution $(3, -2)$.

5 Jules and Enzo are participating in a long-distance bike race. Jules rides at 18 km/h and has a 2 hour head start. Enzo travels at 26 km/h.

a How long does it take for Enzo to catch up to Jules? (Use: Distance = speed $\times$ time.)

b How far did they both ride before Enzo caught up to Jules?

6 Emily travelled a distance of 138 km by jogging for 2 hours and cycling for 5 hours. She could have travelled the same distance by jogging for 4 hours and cycling for 4 hours. Find the speed at which she was jogging and the speed at which she was cycling.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter summary

Equations and formulas

Solving linear equations

Solving involves finding the value that makes an equation true.

e.g. $2x + 5 = 9$
$2x = 4$ (subtract 5)
$x = 2$ (divide by 2)

Solving linear equations that have brackets

- Expand all brackets.
- Collect like terms on each side of the equation.
- Collect terms with a pronumeral to one side (usually the LHS).
- Solve for unknown.

e.g.
$12(x + 1) - 2(3x - 3) = 4(x + 10)$
$12x + 12 - 6x + 6 = 4x + 40$
$6x + 18 = 4x + 40$
$2x + 18 = 40$
$2x = 22$
$x = 11$

Equations with fractions

e.g.
$\frac{3x}{4} - 2 = 7$
$\frac{3x}{4} = 9$ (first $+ 2$ to both sides)
$3x = 36$ ($\times 4$ both sides)
$x = 12$ ($\div 3$ both sides)

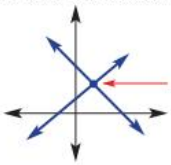
e.g.
$\frac{2x-5}{3} = 7$
$2x - 5 = 21$ (first $\times 3$ to both sides)
$2x = 26$ ($+5$ to both sides)
$x = 13$ ($\div 2$ to both sides)

Solving word problems

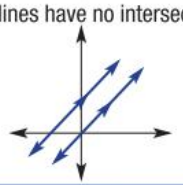
1 Define variable(s).
2 Set up equation(s).
3 Solve equation(s).
4 Check each answer and write in words.

Graphical solutions of simultaneous equations

Graph each line and read off point of intersection.



Parallel lines have no intersection point.



Formulas

Some common formulas
e.g. $A = \pi r^2$, $C = 2\pi r$
An unknown value can be found by substituting values for the other variables.
A formula can be rearranged to make a different variable the subject; i.e. the variable is out the front on its own.
e.g. $E = mc^2$, find m when $E = 320$ and $c = 4$.
$320 = m \times 4^2$ (substitute values)
$320 = 16m$
$20 = m$ (divide both sides by 16)
$m = 20$ (Write the answer with m on the left.)

Simultaneous equations (EXTENDING)

Use substitution or elimination to find the solution that satisfies two equations.

Substitution

e.g. $2x + y = 12$ [1]
$y = x + 3$ [2]

In [1] replace y with [2]:
$2x + (x + 3) = 12$
$3x + 3 = 12$
$3x = 9$
$x = 3$
Sub. $x = 3$ to find y.
In [2] $y = 3 + 3 = 6$
Solution is (3, 6).

Elimination

Ensure both equations have a matching pair. Add two equations if matching pair has different sign; subtract if same sign.

e.g. $x + 2y = 2$ [1]
$2x + 3y = 5$ [2]
[1] $\times 2$: $2x + 4y = 4$ [3]
[3] $-$ [2]: $y = -1$
In [1]: $x + 2(-1) = 2$
$x - 2 = 2$
$x = 4$
Solution is (4, -1).

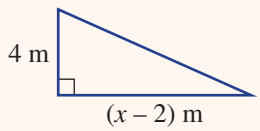
Solving $ax^2 = c$

e.g. $2x^2 = 8$
$x^2 = 4$
$x = \pm 2$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

A printable version of this checklist can be downloaded from the Interactive Textbook.

		✔
9A	**1 I can solve simple linear equations.** e.g. Solve these linear equations. **a** $2x - 1 = 7$ **b** $\frac{x+4}{2} = 12$	☐
9A	**2 I can set up and solve worded problems using linear equations.** e.g. 5 less than twice a number is equal to 31. Find the number.	☐
9B	**3 I can solve equations with brackets.** e.g. Solve $3(x - 2) = 21$.	☐
9B	**4 I can solve equations with variables on both sides.** e.g. Solve $5x - 2 = 3x + 6$.	☐
9B	**5 I can solve equations with fractions.** e.g. Solve $\frac{2x-1}{4} = -1$.	☐
9B	**6 I can set up and solve an equation with brackets from a real situation.** e.g. Find the value of x if the area of this triangle is 32 m^2.  	☐
9C	**7 I can solve quadratic equations of the form $ax^2 = c$.** e.g. Solve $2x^2 = 72$.	☐
9C	**8 I can solve quadratic equations of the form $ax^2 + b = c$.** e.g. Solve $2x^2 - 2 = 10$.	☐
9C	**9 I can substitute into a formula and solve for a variable.** e.g. If $A = \frac{h}{2}(a + b)$ and $A = 20$, $h = 4$ and $b = 3$ find the value of a.	☐
9E	**10 I can find an intersection point of two linear relationships, which represents the solution to a pair of simultaneous equations.** e.g. Find the point of intersection (x, y) of these equations by plotting a graph. $y = 4x - 9$ and $2x + y = 3$	☐
9F	**11 I can use the method of substitution to find the solution to a pair of simultaneous equations.** e.g. Solve the simultaneous equations $y = x - 2$ and $2x + 3y = -1$ using the substitution method; i.e. find the point of intersection.	☐
9F	**12 I can use the method of substitution to find a solution to a real problem involving simultaneous equations.** e.g. Jonah is 9 years older then Penny and their combined ages is 47. Find their ages.	☐

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist

Chapter checklist with success criteria

9G	**13 I can use the method of elimination to find the solution to a pair of simultaneous equations.** e.g. Solve the simultaneous equations $x + y = 1$ and $4x + 3y = 5$ using the elimination method.	☐
9G	**14 I can use the method of elimination to find a solution to a real problem involving simultaneous equations.** e.g. Jill buys 5 pens and 2 pencils from her favourite store for $\$13$, while Michael buys 4 pens and 3 pencils from the same store for $\$12.50$. Find the cost of a pen and a pencil from this store.	☐

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Short-answer questions

9A **1** Solve the following.

a $4a = 32$ b $\frac{m}{5} = -6$ c $x + 9 = 1$

d $x + x = 16$ e $9m = 0$ f $w - 6 = 9$

g $8m = -1.6$ h $\frac{w}{4} = 1$ i $r - 3 = 3$

9B **2** Find the solution to the following.

a $2m + 7 = 11$ b $3w - 6 = 18$ c $\frac{m}{2} + 1 = 6$

d $\frac{5w}{4} - 3 = 7$ e $\frac{m - 6}{2} = 4$ f $\frac{3m + 2}{6} = 1$

g $6a - 9 = 0$ h $4 - x = 3$ i $9 = x + 6$

9C **3** Solve the following by first expanding the brackets.

a $3(m + 1) = 12$ b $4(a - 3) = 16$

c $5(2 + x) = 30$ d $4(2x + 1) = 16$

e $2(3m - 3) = 9$ f $2(1 + 4x) = 9$

g $2(2x + 3) + 3(5x - 1) = 41$ h $3(2x + 4) - 4(x - 7) = 56$

9B **4** Find the value of p in the following.

a $7p = 5p + 8$ b $2p = 12 - p$ c $6p + 9 = 5p$

d $2p + 10 = p + 8$ e $3p + 1 = p - 9$ f $4p - 8 = p - 2$

9A **5** Write an equation for the following and then solve it.

a Six times a number equals 420. What is the number?

b Eight more than a number equals 5. What is the number?

c A number divided by 9 gives 12. What is the number?

d Seven more than a number gives 3. What is the number?

e The sum of a number and 2.3 equals 7. What is the number?

9C **6** Solve the following, giving exact values.

a $x^2 = 11$ b $2x^2 = 6$ c $3x^2 - 1 = 14$

9D **7** a For $A = \frac{1}{2}hb$, find b when $A = 24$ and $h = 6$.

b For $V = lbh$, find b when $V = 84$, $l = 6$ and $h = 4$.

c For $A = \frac{x + y}{2}$, find x when $A = 3.2$ and $y = 4$.

d For $E = mc^2$, find m when $E = 40$ and $c = 2$.

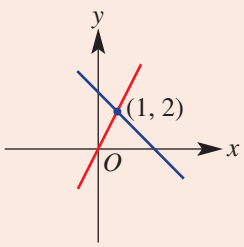
e For $F = \frac{9}{5}C + 32$, find C when $F = 95$.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

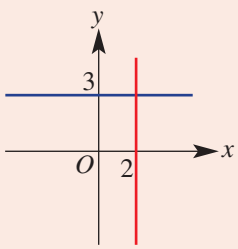
Chapter review

9E **8** Determine the point of intersection of the following lines.

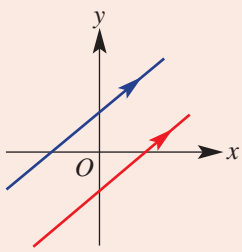
a



b



c



9E **9** Find the point of intersection (x, y) of the following by plotting an accurate graph.

a $y = 2x + 4$
$3x + y = 9$

b $y = 2$
$x = 3$

c $y = 3x$
$y = -3x$

9F **10** Solve the simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = 5x - 13$
$2x + 3y = 12$

b $y = -1$
$y = 2x - 11$

9G **11** Determine the point of intersection of the following lines, using the elimination method.

a $2x + 7y = -25$
$5x + 7y = -31$

b $3x + 2y = 8$
$x - 2y = 0$

9G **12** The sum of two numbers is 15 and their difference is 7. Use simultaneous equations to find the two numbers.

9G **13** A money box contains 20 cent and 50 cent coins. The amount in the money box is \$50 and there are 160 coins.

a Define two variables and set up a pair of linear equations.

b Solve the two simultaneous equations to determine the number of 20 cent and 50 cent coins.

9F **14** There are twice as many adults as children at a local grand final football match. It costs \$10 for adults and \$2 for children to attend the match. If the football club collected \$1100 at the entrance gates, how many children went to see the match?

Multiple-choice questions

9A **1** The solution to $x + 7 = 9$ is:

A $x = 16$ **B** $x = -2$ **C** $x = 2$ **D** $x = 1$ **E** $x = -16$

9B **2** To solve the equation $3(2x + 4) - 4(x + 2) = 6$, you would first:

A divide both sides by 12
B expand the brackets
C subtract 6 from both sides
D multiply both sides by 6
E add $4(x + 2)$ to both sides

9A **3** A number is increased by 6 and then doubled. The result is 36. This translates to:

A $6x + 2 = 36$ **B** $2x + 6 = 36$ **C** $2(x + 6) = 36$
D $2(x - 6) = 36$ **E** $x + 12 = 36$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9B **4** If $4a - 6 = 2a$, then a equals:

A -1 **B** 1 **C** 6 **D** 3 **E** -3

9B **5** The solution to $\frac{5x}{9} - 4 = 1$ is:

A $x = 6$ **B** $x = -9$ **C** $x = -5$ **D** $x = 9$ **E** $x = 5$

9E **6** The intersection point for the graphs of $y = 2$ and $x = 3$ is:

A $(-1, 2)$ **B** $(2, 2)$ **C** $(3, 2)$ **D** $(3, 3)$ **E** $(2, 3)$

9B **7** The solution to $3(x - 1) = 12$ is:

A $x = -1$ **B** $x = 2$ **C** $x = 0$ **D** $x = 5$ **E** $x = 4$

9F **8** $y = 3x$ and $x + y = 4$ has the solution:

A $(1, 3)$ **B** $(3, 1)$ **C** $(2, 6)$ **D** $(2, 2)$ **E** $(-1, 5)$

9F **9** Substituting $y = x - 1$ into $x + 2y = 3$ gives:

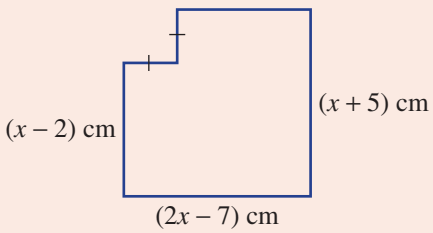
A $x - 2x - 2 = 3$ **B** $x + 2y - 2 = 3$ **C** $x - x - 1 = 3$
D $x + 2x - 1 = 3$ **E** $x + 2(x - 1) = 3$

9G **10** Adding $x + y = 3$ to $x - y = 4$ gives:

A $2x - 2y = 7$ **B** $2x = 7$ **C** $x = 7$ **D** $y = 7$ **E** $2y = 7$

Extended-response questions

1 All the lines meet at 90° in this shape.



a Determine the equation of its perimeter, P.

b **i** If the perimeter is 128 cm, determine the value of x.

ii Find the actual side lengths.

c Repeat part **b** for perimeters of:

i 152 cm **ii** 224 cm

2 Two computer repairers have an up-front fee plus an hourly rate. Rhys charges \$50 plus \$70 per hour, whereas Agnes charges \$100 plus \$60 per hour.

a Using \$$C$ for the cost and t hours for the time, write a rule for the cost of hiring:

i Rhys **ii** Agnes

b If Agnes charges \$280, solve an equation to find how long she was hired for.

c By drawing a graph of C versus t for both Rhys and Agnes on the same set of axes, find the coordinates of the intersection point.

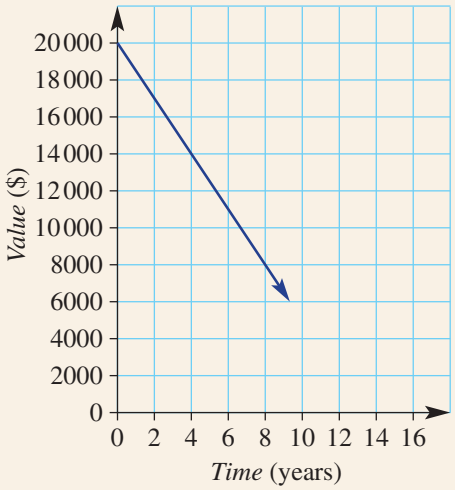
d Use the algebraic method of substitution to solve the simultaneous equations and confirm your answer to part **c**.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter 6: Linear relationships, hyperbolas, parabolas and exponentials

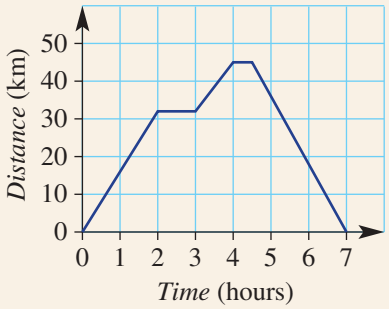
Short-answer questions

1 This graph shows how the value of a car decreases with time.



- **a** By the end of 4 years, how much has its value fallen?
- **b** After how many years is the car worth $8000?
- **c** Estimate the car's worth after 13 years.

2 This distance–time graph shows the journey of a cyclist from home to a location and back again.



- **a** How many km has the cyclist travelled after:
  - **i** 1 hour?
  - **ii** 1.5 hours?
  - **iii** 3 hours?
- **b** Calculate the cyclist's speed over the first 2 hours.
- **c** What is the total time in rest breaks?
- **d** What is the cyclist's greatest distance from home?
- **e** How long does the return trip take?
- **f** Calculate the cyclist's speed for the return journey.
- **g** What is the total distance cycled?

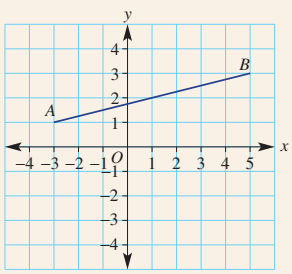
3 **a** Copy and complete this table for the rule $y = 2x - 1$.

x	−2	−1	0	1	2	3
y						

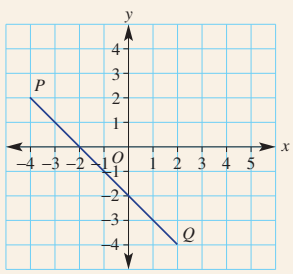
- **b** List the coordinates for each point in the table.
- **c** Draw x- and y-axes, each labelled between −5 and 5. Plot the points and join them with a ruler.

4 For each of the graphs that follow, find the gradient of the line segment.

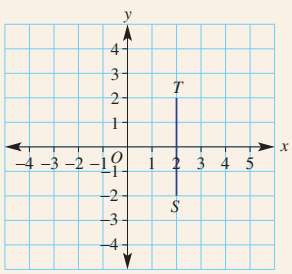
a



b



c



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 Plot and join each pair of points and find the gradient, m, of the line segment joining these points.

a $A(3, 2)$ and $B(5, 6)$ **b** $K(1, -3)$ and $L(-2, 6)$

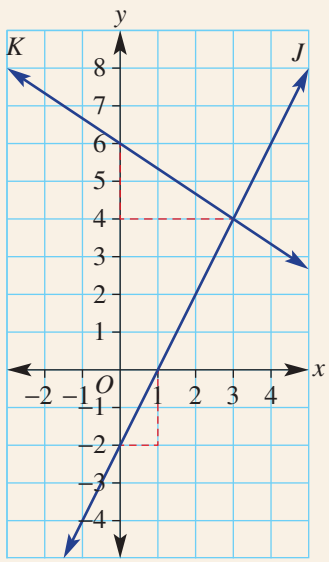
6 **a** Consider the lines, J and K, graphed here, and for each line:

i Find the y-intercept.

ii Determine the gradient.

iii Write the equation of the line.

b State the coordinates of the point of intersection of lines J and K.



7 Sketch each of these lines by considering the y-intercept and gradient.

a $y = -3x + 4$ **b** $y = \frac{2}{3}x + 1$

c $y = -2$ **d** $x = -3$

8 Find the x- and y-intercepts for each of these lines and sketch them. Label all axes intercepts.

a $y = 2x - 6$ **b** $2x - 3y = 12$

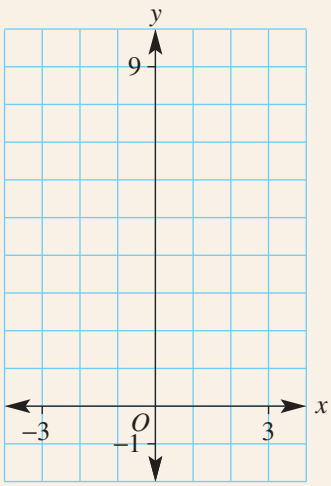
9 **a** Complete this table for $y = x^2$.

x	−3	−2	−1	0	1	2	3
y	9						

b Plot a graph of $y = x^2$ and state if its shape is:

A linear (i.e. a straight line)

B non-linear (i.e. not a straight line)

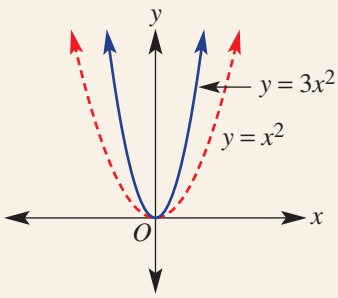


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

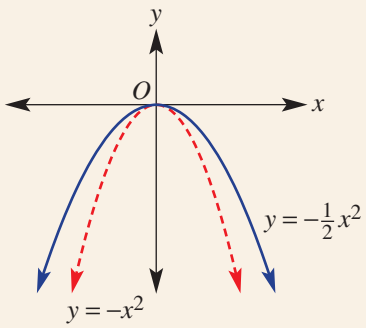
10 Copy and complete the table for these parabolas.

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 3x^2$					
b	$y = -\frac{1}{2}x^2$					
c	$y = x^2 - 3$					

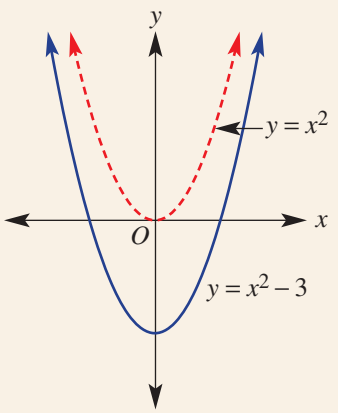
a



b



c



Multiple-choice questions

1 If a straight line has a gradient of -3 and a y-intercept of 5, its equation is:

A $y = 5$ **B** $y = 3x + 5$ **C** $y = 5x - 3$
D $y = -3x + 5$ **E** $m = -3$

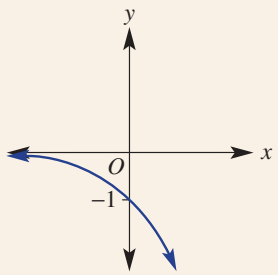
2 The gradient of the line joining $(0, 6)$ and $(2, -4)$ is:

A -2 **B** 6 **C** 5 **D** $\frac{1}{5}$ **E** -5

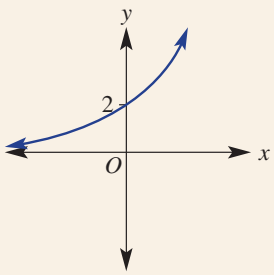
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 The graph of $y = 2^x$ could be:

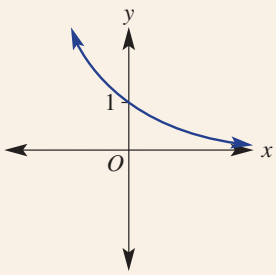
A



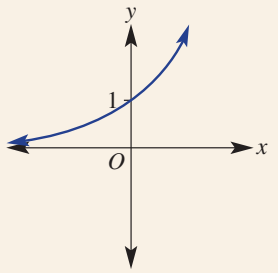
B



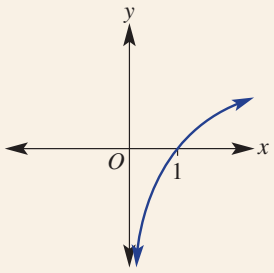
C



D



E



4 The equation and gradient of the vertical line through the point (1, 3) are:

A $x = 1$; gradient undefined
B $x = 1$; gradient zero
C $y = 3$; gradient positive
D $y = 3$; gradient negative
E $y = 3$; gradient undefined

5 A landscape company charges \$80 delivery plus \$73 per cubic metre of soil. If \$$C$ is the cost of n cubic metres of soil, then:

A $C = 80n + 73$
B $C = 73n - 80$
C $n = 73C + 80$
D $C = 73n + 80$
E $C = 80n - 73$

Extended-response question

You have a \$100 gift voucher for downloading movies from the internet. Each movie costs \$2.40. After downloading n movies you have a balance of \$$B$ on your voucher.

a Write a rule for the balance, B, on your voucher in terms of n.

b Use your rule to find:

i the balance on the voucher after 10 movies are downloaded
ii the number of movies bought that will leave a balance of \$28

c Copy and complete this table.

Number of movies (n)	0	5	10	15	20
Balance (B)					

d Sketch a graph of B versus n, using the values in the table above.

e What is the maximum number of movies you could buy and how much would be left on your voucher?

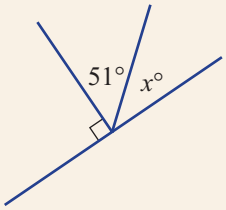
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter 7: Properties of geometrical figures and networks

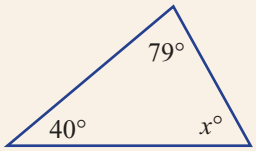
Short-answer questions

1 Find the value of x in these diagrams.

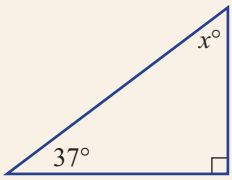
a



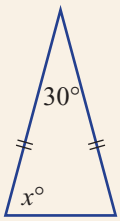
b



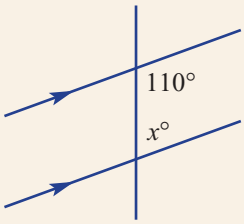
c



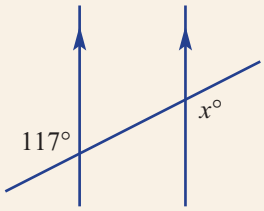
d



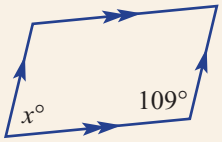
e



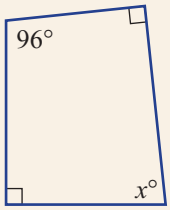
f



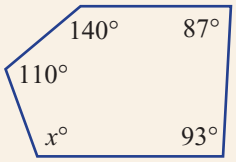
g



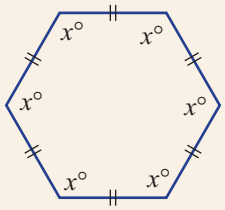
h



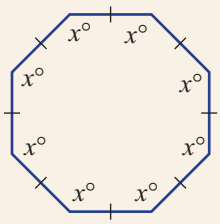
i



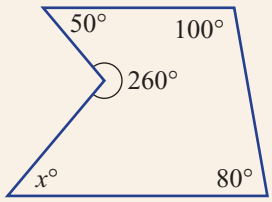
j



k



l

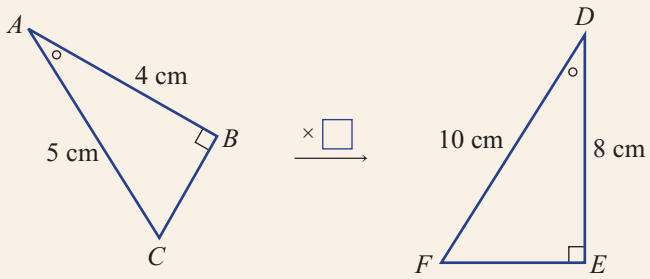


CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

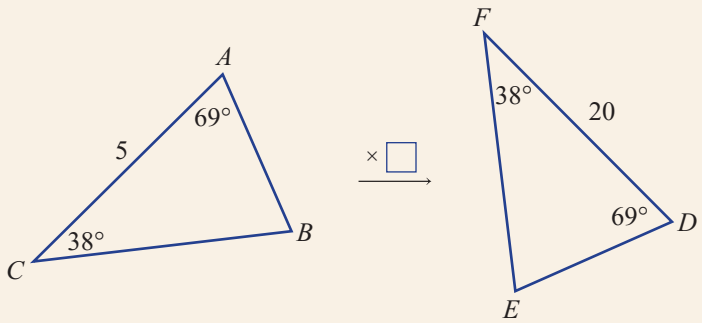
2 Use the formula $S = (n - 2) \times 180°$ to find the interior angle sum of a decagon.

3 Find the scale factor for each pair of similar triangles.

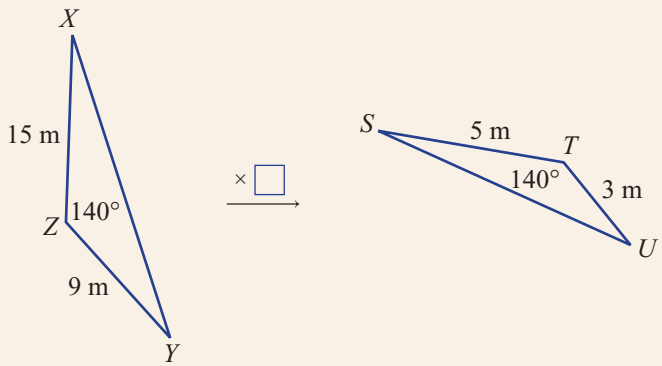
a



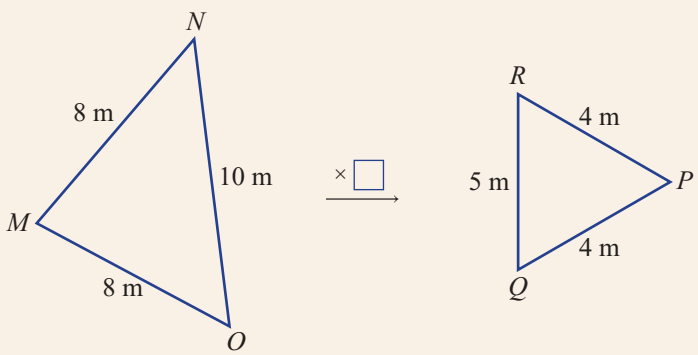
b



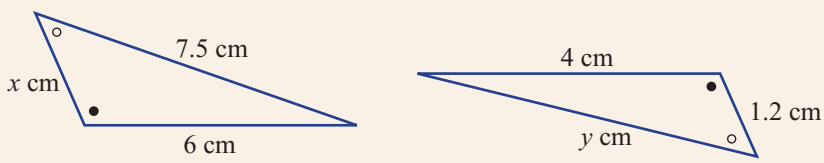
c



d

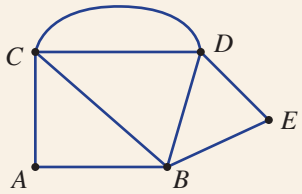


4 The given pair of triangles are known to be similar. Find the scale factor and value of x and y.



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

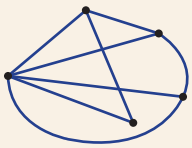
5 Consider the network graph shown.



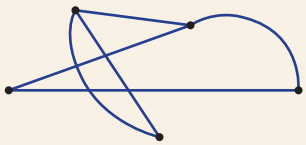
- **a** State the number of odd vertices (nodes).
- **b** Verify Euler's formula using the information given in the graph.
- **c** Decide if the walk *B*–*A*–*C*–*B* is a path.
- **d** Decide if the graph has an Eulerian trail, with reason and if all Eulerian trails will be circuits.

6 Decide if the following graphs are planar or non-planar.

a



b



Multiple-choice questions

1 If two lines are parallel, then cointerior angles will:

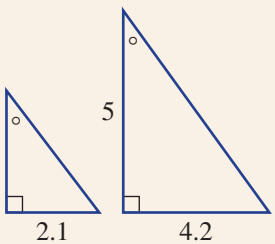
A be equal **B** sum to 90° **C** sum to 180° **D** sum to 360° **E** sum to 270°

2 The sum of the internal angles of a hexagon is:

A 360° **B** 540° **C** 1080° **D** 900° **E** 720°

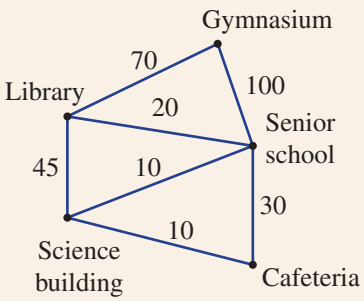
3 The scale factor for these similar triangles is:

A 2 **B** 4 **C** 5
D 0.1 **E** 0.4



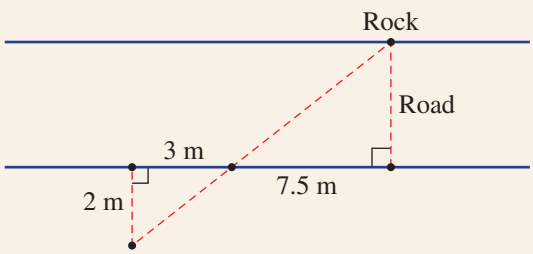
4 This weighted graph shows the distances in metres along paths between the main buildings in a school. The shortest distance between the library and the cafeteria is:

A 50 m **B** 55 m **C** 40 m
D 200 m **E** 85 m



Extended-response question

A chicken wants to know the distance across the road without having to cross it. The chicken places 4 pebbles in various positions on its own side of the road, as shown. There is a rock on the other side of the road aligned with one of the pebbles.



- **a** Find the scale factor.
- **b** What is the distance across the road?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter 8: Trigonometry

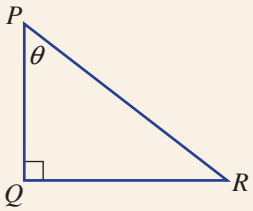
Short-answer questions

1 Refer to the angle marked θ and name the:

a hypotenuse

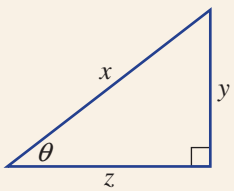
b side opposite θ

c side adjacent to θ



2 Use the triangle shown to help you write a fraction for:

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$



3 Use a calculator to find the value of each of the following, correct to two decimal places.

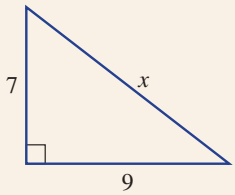
a $\sin 40°$ **b** $\cos 51°$ **c** $\tan 18°$

d $12\tan 32°$ **e** $40\tan 38°$ **f** $5.6\sin 55°$

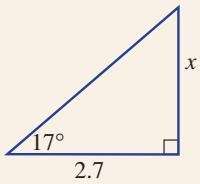
g $\dfrac{15}{\sin 24°}$ **h** $\dfrac{28}{\cos 30°}$ **i** $\dfrac{12.5}{\tan\ 52°}$

4 Find the value of each pronumeral, correct to two decimal places.

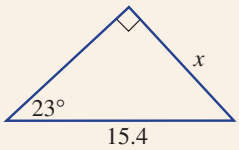
a



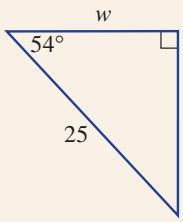
b



c

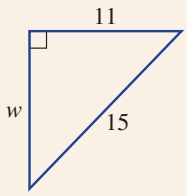


d

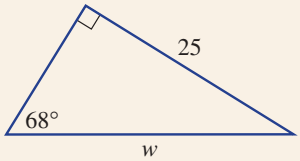


5 Find the value of each pronumeral, correct to one decimal place.

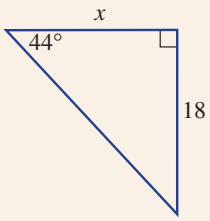
a



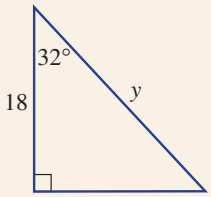
b



c

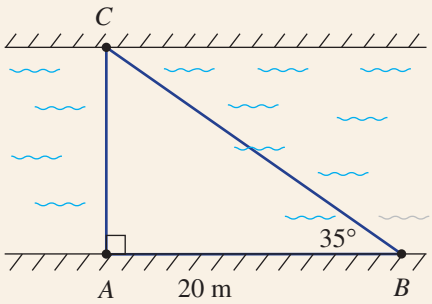


d

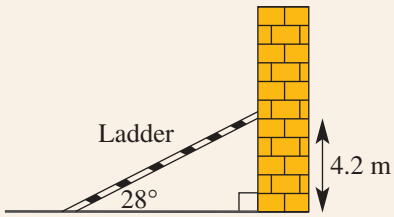


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 Kara wants to measure the width of a river. She places two markers, A and B, 20 m apart along one side. C is a point directly opposite marker A. Kara measures angle ABC as 35°. How wide is the river, to the nearest metre?

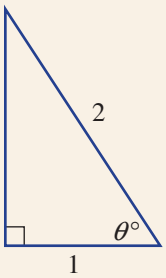


7 A ladder is inclined at an angle of 28° to the ground. If the ladder reaches 4.2 m up the wall, what is the length of the ladder, correct to two decimal places?

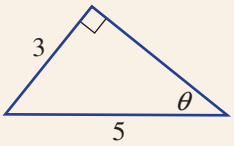


8 Find the angle θ in the following triangles, correct to the nearest degree.

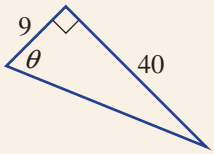
a



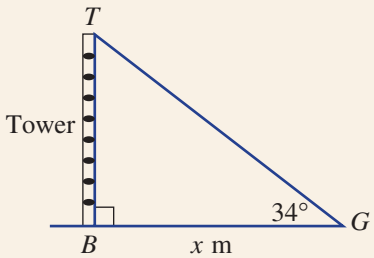
b



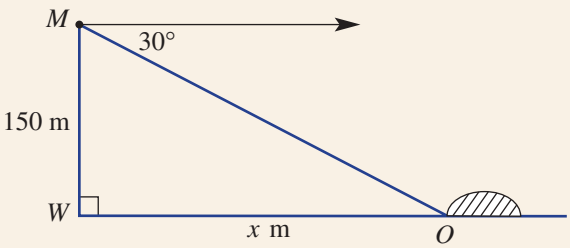
c



9 Elliott measures the angle of elevation to the top of a 120 m tower to be 34°. How many metres is Elliott from the base of the tower? Round your answer to one decimal place.

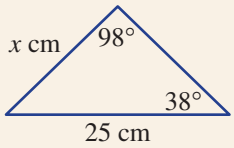


10 Xi-Rui is sitting on top of a bridge 150 m above the water level of the river. He notices an object floating on the river some distance away. If the angle of depression to the object is thought to be 30°, how many metres (x) from the bridge is the object? Round your answer to one decimal place.

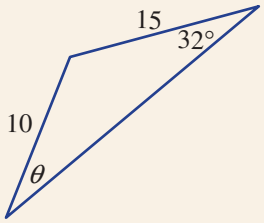


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

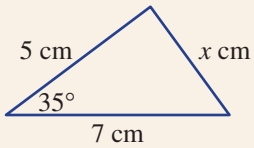
11 Use the sine rule to find the value of x, correct to one decimal place.



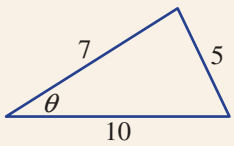
12 Use the sine rule to find the value of θ, correct to the nearest degree.



13 Use the cosine rule to find the value of x, correct to one decimal place.

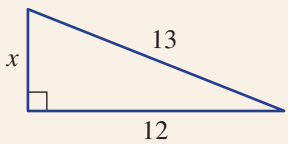


14 Use the cosine rule to find the value of θ, correct to the nearest degree.



Multiple-choice questions

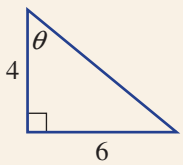
1 The value of x in the triangle shown is:



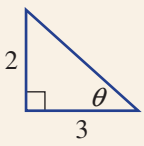
A 1 **B** 11 **C** 4
D 10 **E** 5

2 In which of the following triangles does $\cos\theta = \frac{2}{3}$?

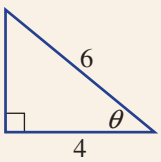
A



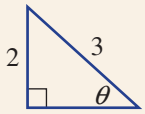
B



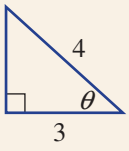
C



D

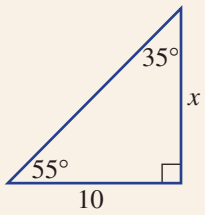


E



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 Choose the correct trigonometric statement for the diagram shown.

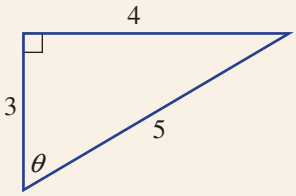


A $\tan 55° = \frac{x}{10}$ **B** $\tan 35° = \frac{x}{10}$

C $\sin 55° = \frac{x}{10}$ **D** $\sin 35° = \frac{x}{10}$

E $\cos 35° = \frac{x}{10}$

4 For the triangle shown, $\sin\theta$ is equal to:

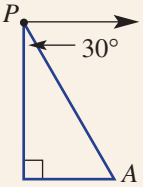


A $\frac{3}{5}$ **B** $\frac{4}{5}$ **C** $\frac{5}{3}$

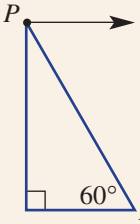
D $\frac{3}{4}$ **E** $\frac{4}{3}$

5 In which diagram is the angle of depression of A from P equal to 30°?

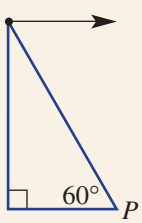
A



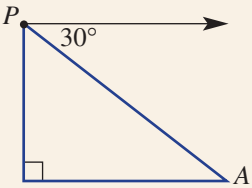
B



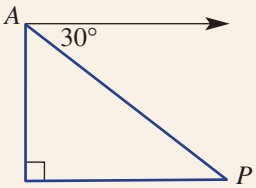
C



D



E



Extended-response question

A plane flies from the airport on a bearing of 136° for 450 km.

a Draw a diagram showing the plane's flight.

b How far east of the airport is the plane? Round to one decimal place.

c How far south of the airport is the plane? Round to one decimal place.

d What is the true bearing of the airport from the plane, correct to the nearest degree?

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter 9: Equations and formulas

Short-answer questions

1 Solve the following one-step equations.

a $x + 12 = 3$ **b** $x - 5 = 21$ **c** $\frac{m}{4} = 8$

d $5a = 20$ **e** $-2k = 12$ **f** $-x = 8$

2 Solve the following two-step equations.

a $2p + 3 = 7$ **b** $3a - 10 = 2$ **c** $\frac{x}{2} + 3 = 9$ **d** $\frac{x - 8}{4} = 3$

3 Solve the following equations.

a $2(x - 4) = 8$ **b** $3(k - 2) + 4k = 15$

c $3m - 13 = m + 5$ **d** $\frac{3x + 1}{2} = 8$

e $\frac{3a - 2}{7} = -2$ **f** $4x + 7 + 3x - 12 = 5x + 3$

4 For each of the following statements, write an equation and then solve it for the pronumeral.

a When 5 is subtracted from x, the result is 8

b When 8 is added to the product of 4 and x, the result is 20.

c When 6 less than 3 lots of x is doubled, the result is 18.

5 Solve

a $x^2 - 4 = 0$ **b** $4x^2 - 9 = 0$ **c** $3x^2 = 0$

6 Find the value of the unknown in each of the following formulas.

a For $A = \frac{1}{2}bh$, find b when $A = 120$ and $h = 24$.

b For $I = \frac{Prt}{100}$, find P when $I = 80$, $r = 5$ and $t = 4$.

7 Find the point of intersection (x, y) of each pair of equations below by plotting an accurate graph. First draw x- and y-axes, each labelled from -5 to 5.

a $x = 3$, $y = 2$

b $y = 2x - 4$ and $3x + 2y = 6$

8 Solve the following pairs of simultaneous equations using the substitution method; i.e. find the point of intersection.

a $y = 2x$

$x + y = 3$

b $x + y = 12$

$y = x + 6$

c $y = 3 - x$

$3x + 2y = 5$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 Solve the following pairs of simultaneous equations using the elimination method; i.e. find the point of intersection.

a $x + 2y = 3$
$-x + 3y = 2$

b $3x + y = 10$
$x + y = 6$

c $2x - 3y = 3$
$3x - 2y = 7$

10 Oliver is older than Ruby. The sum of their ages is 45 years and the difference of their ages is 7 years.

a Define pronumerals to represent each person's age.

b Set up a pair of simultaneous equations based on the information given.

c Solve the simultaneous equations to find Oliver and Ruby's ages.

Multiple-choice questions

1 Which of the following is *not* an equation?

A $x - 3 = 5$ **B** $2x + 4 = 5x$ **C** $y + 7x - 4$ **D** $y = 3x - 5$ **E** $y = 8$

2 A number is decreased by 8 and then doubled. The result is equal to 24. This can be written as:

A $2x - 8 = 24$ **B** $x - 8 \times 2 = 24$ **C** $x - 8 = 2 \times 24$

D $2(x - 8) = 24$ **E** $\frac{x - 8}{2} = 24$

3 The solution to $\frac{x - 9}{3} = 6$ is:

A $x = 27$ **B** $x = 45$ **C** $x = 9$ **D** $x = 11$ **E** $x = 3$

4 The solution to $3(x - 1) = 5x + 7$ is:

A $x = -4$ **B** $x = -5$ **C** $x = 5$ **D** $x = 3$ **E** $x = 1$

5 The solution of $x^2 = 9$ is:

A $x = 3$ **B** $x = -3$ **C** $x = \pm 3$ **D** $x = \sqrt{9}$ **E** $x = 81$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Extended-response question

Ishan and Mia normally have an electricity bill of $200 per month. Now that they have installed solar panels (which cost $6000, including installation), the solar energy is providing for all their power usage with some left over. Their excess power is sold to the town's electricity supplier. On average, they receive a payment of $50 per month for the sale of this solar-generated power.

a Copy and complete this table, which compares the total cost of normal town electricity to solar power.

Number of months, *n*	0	6	12	18	24	30	36	42	48
Total cost of electricity (at $200 per month), *E*	0								
Total cost of solar power (reducing by $50 per month), *S*	$6000								

b Draw a graph of the information given in the table.

c On the graph, show the point at which the total cost of solar power is the same as the total cost of electricity for that period of time. State the value of n and the cost.

d Write an equation for E (i.e. total cost of electricity) in terms of n and another equation for S (i.e. total cost of solar power) in terms of n.

e Solve the equations in part **d** for n, when $E = S$ (i.e. the total costs of each power supply are equal).

f After 4 years, how much money has been saved by using solar power?

g Suppose that, in a wet climate, the payment for excess power sold to the town supply is reduced to $25 per month. Using equations, find how many months it takes for the total costs to be equal. (Round to the nearest month.)

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter 1

Warm-up quiz

1 **a** \$99.62 **b** \$1612 **c** \$2742
d \$84.51 **e** \$1683.85

2 **a** $\frac{50}{100}$ **b** $\frac{75}{100}$ **c** $\frac{20}{100}$ **d** $\frac{68}{100}$ **e** $\frac{45}{100}$

3 **a** 0.5 **b** 0.25 **c** 0.2 **d** 0.4 **e** $0.\dot{3}$

4 **a** 16.79 **b** 7.35 **c** 45.34 **d** 6.84
e 102.90

5 $a = 4519.28$, $b = 50\,509.20$, $c = 29\,572$,
$d = 54\,878$, $e = 97\,903.46$

6 **a** \$65 312 **b** \$187 200 **c** \$49 686 **d** \$35 800

7 **a** \$40 **b** \$250 **c** \$2 **d** \$211
e \$2.56 **f** \$750

8 **a** \$20 **b** \$300 **c** \$64

9 $a = \$18$, $b = \$323$, $c = \$264$, $d = \$112$, $e = \$57.92$

10 **a** \$50 **b** \$56 **c** \$550

1A

Now you try

Example 1
a 24% **b** 43.75% **c** 15%

Example 2
a $\frac{23}{100}$ **b** $\frac{16}{25}$ **c** $\frac{21}{200}$

Example 3
a 0.28 **b** 0.03 **c** 0.6

Example 4
\$828

Exercise 1A

1 **a** multiply **b** divide **c** multiply **d** divide

2 **a** **i** 100 **ii** $\frac{23}{100}$
b **i** 0.18 **ii** 0.05

3 **a** \$0.75 **b** \$80 **c** \$49 **d** \$501
e \$103 **f** \$1.20 **g** \$37.50 **h** \$400
i \$4.50

4 **a** 50% **b** 20% **c** 25% **d** 10%
e 1% **f** 28% **g** 30% **h** 75%
i $62\frac{1}{2}\%$ **j** 76% **k** 99% **l** 94%

5 **a** 17% **b** 73% **c** 48% **d** 9%
e 6% **f** 13% **g** 113% **h** 101%
i 80% **j** 90% **k** 99% **l** 17.5%

6 **a** $\frac{71}{100}$ **b** $\frac{4}{5}$ **c** $\frac{1}{4}$ **d** $\frac{11}{20}$
e $\frac{2}{5}$ **f** $\frac{22}{25}$ **g** $\frac{3}{20}$ **h** $\frac{33}{200}$
i $\frac{7}{40}$ **j** $\frac{9}{400}$ **k** $\frac{21}{400}$ **l** $\frac{21}{40}$

7 **a** 0.61 **b** 0.83 **c** 0.75 **d** 0.45
e 0.09 **f** 0.9 **g** 0.5 **h** 0.165
i 0.073 **j** 2 **k** 4.3 **l** 0.005

8 **a** \$25 **b** \$150 **c** \$60 **d** \$90
e \$21.60 **f** 344 grams **g** \$50.40 **h** \$107.80
i 350 m

9 **a** $\frac{15}{300} = \frac{1}{20}$ **b** 5%

10 68 kg

11 47.52 minutes

12 16.67%

13 $11\frac{1}{9}\%$

14 \$205.97

15 **a** 1.7% **b** **i** 31.4% **c** NSW – 31% **d** 94°
2.1% **ii** 25.5% Vic – 26%
2.2% **iii** 10.8% Qld – 20%
1.6% SA – 7%
2.2% WA – 11%
0.5% Tas – 2%
0.8% NT – 1%
1.8% ACT – 2%
1.9%

1B

Now you try

Example 5
\$728

Example 6
\$2214

Example 7
a \$109 **b** 12.27%

Example 8
a \$41.60 **b** \$192

Example 9
a \$143 **b** \$77

Exercise 1B

1 **a** 1.1 **b** 1.2 **c** 1.02 **d** 1.18

2 **a** 0.95 **b** 0.7 **c** 0.85 **d** 0.5

3 **a** Profit : \$5 **b** Profit : \$2.50 **c** Loss : \$16

4 **a** 1.1, \$165 **b** \$250, ÷ 1.1

5 **a** \$94.50 **b** \$440 **c** \$66 **d** \$529.20
e \$56 **f** \$7210 **g** \$56.43 **h** \$3.60

6 **a** \$1425 **b** \$360 **c** \$376 **d** \$68
e \$412.50 **f** \$47.03 **g** \$101.58 **h** \$35.80

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7

a	$6	60%
b	$60	25%
c	$3	20%
d	$7.50	3%
e	$2325	75%
f	$0.99	18%

8 a $52.25 b $261.25
9 $1536
10 a $67.96 b $11.99
11 a $2140.25 b $305.75
12 a $221.40 b $147.60
13 a $1225 b $24.50
14 a $84.13 b $94.87
15 $104.71
16 a $106.15 b $37.15
17 a $2.20 b 122.22% c $66 d 122.22%
18 a $13 b $6.30 c $69.30 d 38.6%
19 a $1952.24 b $211.24 c 12.13% d $57.03

1C

Now you try

Example 10
a $1360.15 b $1066.50

Example 11
$2693.60

Example 12
$575.94

Example 13
$1346.15

Exercise 1C

1 a D b C c A d B
2 $36 842
3 a $22.80 b $30.40
4 a $690.90 b $483.70
5

	Gross income	Net income
a	$570	$415
b	$984	$764
c	$604.90	$304.90
d	$3430	$2920
e	$930.15	$746.15

6 a $3558.72 b $92 526.72
7 a $1960 b $6960
8 $34 375.00
9 a $1480.77 b $150 c $1630.77 d $88 425
10 a $1057.50 b $157.50
11 $3760
12 1.25%
13 $365.70
14 $2000
15 a Kuger Incorporated
b Mathsville Credit Union, 00 754 031 c $72 454
d Fortnightly e $4420 f $26.45 g $600.60
h $16 016 tax, net = $49 793.90 i $34.71/h

1D

Now you try

Example 14
a $81 770 b $17 042.25 c $1635.40 d $18 677.65
e 22.8% f Not enough; must pay another $1232.65

Exercise 1D

1 Taxable income = gross income minus deductions
2 a True b False
3 37c
4 a $93 605 b $20 888.63 c $1872.10
d $22 760.73 e 24.3% (to 1 d.p.)
f Refund of $2488.27
5 a $2242 b $9967 c $40 567 d $60 667
6 a $65 625 b $11 795.13 c $1312.50
d $13 107.63 e 20.0% (to 1 d.p.)
f Not enough paid; owes $37.63
7 $87 500
8 $6172.84
9 $100 400
10 If a person pays too much tax during the year they will receive a tax refund. If they do not pay enough tax during the year they will have a tax liability to pay.
11 They only pay 45 cents for every dollar over $180 000.
12 a The tax-free threshold increased from $6000 to $18 200. In the second tax bracket, the **marginal** rate changed from 15c to 19c. In the third tax bracket, the **marginal** rate changed from 30c to 32.5c.

b

	2011/2012	2012/2013	
Ali	$0	$0	No change
Xi	$1350	$0	$1350 less tax to pay
Charlotte	$3600	$2242	$1358 less tax to pay
Diego	$8550	$7797	$753 less tax to pay

13

	Resident	Non-resident	
Amy	$0	$5850	Non-residents pay a lot more tax than residents.
Ben	$4142	$13 000	
Chi	$40 567	$50 100	
Dan	$60 667	$70 200	

14 a Answers will vary.
b i $16 467, if there are no deductions in part a.
ii $32.50, so this means that the $100 donation really costs you only $67.50.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

1E

Now you try

Example 15
a $28 890 **b** $12 840
c Yes, approx $62 per week

Example 16
a $7182 **b** 8.8%

Example 17
The 4 double length rolls

Exercise 1E

1 **a** Fixed **b** Fixed **c** Variable **d** Variable
2 $124.28
3 $2162
4 **a** $41 250 **b** $33 750 **c** Yes, $144 per week
5 $106
6 **a** $33 068 **b** 73.5%
7 **a** $9774 **b** $4887 **c** $187.96
8 **a** $103 385 **b** $1988.17 **c** 24%
9 **a** 13.49% **b** 29.65 L
10 **a** $519.35 **b** $3240.74 **c** 9.63%
11 **a**

Food	$86.40
Recreation	$43.20
Transport	$56.16
Savings	$86.40
Taxation	$108.00
Clothing	$51.84

b $56.16 **c** 30% **d** $673.92 **e** 10%
12 200 tea bags
13 Daily
14 **a** Mon–Thurs – $87
Fri–Sat – $93.50
Weekly – $71.43
b Weekly
15 **a** 200 mL bottle $0.011 75, 500 mL bottle $0.01048
b 500 mL bottle **c** $2.10 **d** $0.25 **e** Cost of packaging
16 **a** $248 **b** $240, 6 containers

1F

Now you try

Example 18
a $56.25 **b** 2%

Example 19
a $7344 **b** $153

Exercise 1F

1 **a** $120 **b** $420 **c** $30
2 **a** $420 **b** $840 **c** $35
3 **a** $60 **b** 3%
4 **a** $140 **b** $420 **c** $192.50
d $46.88 **e** 3% p.a. **f** 4% p.a.
5 **a** $6650 **b** $184.72
6 **a** $5192.25 **b** $16 692.25 **c** $198.72
7 **a** $7600 **b** $17 600 **c** $366.67
8 $1008
9 **a** $228 **b** $684 **c** $4684
10 16%
11 12.5 years
12 $66 667
13 $7500
14 **a** $1250, $2500 **b** $1968.75, $1920.00
c $220.31, $331.11 **d** Answers will vary

Progress quiz

1 **a** 0.32 **b** $\frac{2}{25}$ **c** 44%
d 31.25% **e** 25.2% **f** $\frac{31}{200}$
2 $3840
3 **a** $285.20 **b** 194.4 mL **c** 132.6 cm
4 $194.35
5 40%
6 **a** $956 **b** $722.20
7 **a** $75 875 **b** $15 126.38 **c** $1517.50
d Cameron must pay $1388.88
8 **a** $31 256 **b** 38.1%
9 **a** $4720 **b** 4.5 years

1G

Now you try

Example 20
$3787.43

Example 21
a $n = 24, r = 0.25$ **b** $n = 8, r = 3.5$

Example 22
$7267.24

Exercise 1G

1 **a** $50 **b** $550 **c** $55
d $605 **e** $605
2 **a** $1102.50 **b** $1102.50 **c** $1157.63 **d** $1157.63
3 **a** $700(1.08)^2$ **b** $1000(1.15)^6$ **c** $850(1.06)^4$
4 **a** $5105.13 **b** $11 946.33 **c** $13 652.22 **d** $9550.63
5 **a** $106 000 **b** $112 360 **c** $119 101.60
d $133 822.56 **e** $179 084.77 **f** $239 655.82
6 **a** 6, 3% **b** 60, 1% **c** 52, 0.173%
d 14, 2.625% **e** 32, 3.75% **f** 120, 0.8%
7 $11 446.15
8 **a** $2254.32 **b** $87 960.39 **c** $1461.53 **d** $789.84
e $591.63
9 **a** $22 185 **b** $27 815

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 a i $3239.42 ii $3348.15 iii $3446.15
iv $3461.88 v $3465.96
b $226.54
11 a $P = 300, n = 12, r = 7\%$, $r = 14\%, t = 6$ years
b $P = 5000, n = 24, r = 2.5\%$, $r = 30\%, t = 2$ years
c $P = 1000, n = 65, r = 0.036\%$, $r = 0.936\%, t = 2.5$ years
d $P = 3500, n = 30, r = 0.0053\%$, $r = 1.9345\%, t = 30$ days
e $P = 10\,000, n = 10, r = 7.8\%$, $r = 7.8\%, t = 10$ years
12 5.3% compounded bi-annually
13 a i Approx. 6 years ii Approx. 12 years
iii Approx. 9 years iv Approx. 5 years
v Approx. 7 years vi Approx. 4 years
b Same answer as part **a**.
c Yes

1H

Now you try

Example 23
a $16 320 **b** $3320

Example 24
a $954 **b** $3294 **c** $114

Example 25
a $3.96 **b** $1.72 debited

Exercise 1H

1 a Repayment **b** Investment **c** Loan
2 a $2640 **b** $3960 **c** $13 200
3 $124.50
4 a $18 600 **b** $8600
5 a $5580 **b** $80
6 a 360 **b** $624 960 **c** $374 960
7 a $2550 **b** $10 620 **c** $13 170 **d** $420
8 a $4.50 **b** $1.26
9 a

May	June	July	August
$2.40	$3.00	$0.12	$5.00

September	October
$2.08	$0.73

b $13.33
10 a i $0 ii $0 iii $7500
b $6375
c $1125
11 a i $5250 ii $20 250 iii $281.25
b i $8400 ii $32 400 iii $270

12 a

Date	Deposit	Withdrawal	Balance
1 May			$3010
3 May	$490		$3500
5 May		$2300	$1200
17 May	$490		$1690
18 May		$150	$1540
20 May		$50	$1490
25 May		$218	$1272
31 May	$490		$1762

b $4.90 **c** $1759.55 **d** Wages
13 a i $73.13 ii $69.72 iii $75.17
b $1700 over 3 years
14 a $403.80 **b** $393.80 **c** 24 cents a day
15 a $98 822.90 **b** $0.23 **c** $8.00
d $2400 **e** $378.50 **f** $246 025

1I

Now you try

Example 26
a $6154.55 **b** $5625

Exercise 1I

1 B
2 a 200 **b** B **c** 5200 **d** = D2
3 a The interest in Year 1 is equal to the interest in Year 2.
b 6% p.a. **c** = 6/100 * B2
d Formula for cell C3 is either = 6/100 * B3 or = C2
Formula for cell D3 is = B3 + C3
4 a i $7146.10 ii $6955.64 iii $6858.57
iv $7260 v $7916.37
b 6000 at 5.7% p.a., for 5 years
5 a i $7080 ii $7080 iii $7428
iv $7200 v $7710
b 6000 at 5.7% p.a., for 5 years
6 a $25 500 **b** $21 675 **c** $18 423.75 **d** $5906.23
7 a $29 300 **b** $28 593
c $27 878.93 **d** $22 676.45
8 a 4 to 5 years **b** 4 to 5 years **c** Yes
9 a approx. 35–36 months **b** approx. $5850
10 Lauren will pay $4700 in interest, which is approximately $1150 less than Steve.
11 Steve will pay approximately $7830 in interest, which is approximately $1980 more than Question 9.
12 Check with your teacher

Maths@Work: Finance manager

1 a $108 474 **b** $110 974 **c** $108 474
d August by $2500 **e** The term deposit
f 48.32% **g** $70 417.33
2 a 92.07% **b** $1200 in August
c It may be billed only every 2 months. **d** $7200

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 **a** $135 459

b 137% increase of November's total.
The Payroll tax increased by $3300 and a performance review cost $75 000.

c **i** $4584 **ii** $55 008

d 62.05%

4 **a**

Starting balance	Scheduled payment	Interest due	Principal paid	Ending balance
$230 000.00	$5780.00	$1725.00	$4055.00	$225 945.00
$225 945.00	$5780.00	$1694.59	$4085.41	$221 859.59
$221 859.59	$5780.00	$1663.95	$4116.05	$217 743.53

b **i** $205 209.22 **ii** $1442.93
iii $347.37 **iv** $18 641.74; $50 718.26

Puzzles and challenges

1 Commission, fortnightly, overtime, piecework, annual, gross, net, monthly, casual, salary

2 You take away his credit card.

3 7 years 4 months

4 $0.95^n = 0.05$
$n = 58.4$
$\therefore$ 59 games

Chapter checklist with success criteria

1 **a** 17.5% **b** 24%

2 **a** $\frac{53}{100}$, 0.53 **b** $\frac{1}{25}$, 0.04 **c** $\frac{21}{200}$, 0.105

3 $896

4 **a** $848 **b** $680

5 $65, 44.8%

6 $30.72, $134.40

7 $619.18

8 $1070, $880

9 **a** $792 **b** $45 000

10 **a** $77 961 **b** $15 804.33 **c** $1836.67

11 **a** $19 180 **b** $685

12 $2078, 2.6%

13 20 packs for $5.50

14 $920

15 $4320, $180

16 3.5%

17 $9552.42

18 $6987.69

19 **a** $2580 **b** $9780, $1180

20 $2.52, $ − 2.48

21 **a** $8516.57 **b** $8400

Chapter 1 review

Short-answer questions

1 $1440

2 **a** $1084.16 **b** $4557

3 **a** $11.40 **b** $3.80

4 $4200

5 $576.92

6 **a** $7400 **b** $616.67 **c** $142.31

7 **a** $536.06 **b** $80.41 **c** 11.57%

8 **a** $12 725 **b** 37%

9 **a** $1411.20 **b** $1080.70

10 **a** $19 717 **b** $1800

11 $35.55

12 **a** $1600 **b** $1166.67

13 **a** $624.32 **b** $1022.53 **c** $635.80

Multiple-choice questions

1 E **2** D **3** D **4** C **5** B **6** A
7 B **8** B **9** C **10** E

Extended-response questions

1 **a** $5624.32 **b** $624.32 **c** 4.16% **d** $636.36

2 **a** $1050 **b** $12 000 **c** $6050

Chapter 2

Warm-up quiz

1 **a** Circle **b** Square **c** Parallelogram
d Triangle **e** Rectangle **f** Trapezium
g Semicircle **h** Rhombus

2 **a** 1000 **b** 100 **c** 10 **d** 1000
e 500 **f** 25

3 **a** 12 cm **b** 32 m **c** 5.9 mm

4 **a** 10 cm^2 **b** 70 m^2 **c** 36 km^2

5 **a** 4 cm^2 **b** 14 m^2 **c** 6 km^2

6 $C = 31.42$ m
$A = 78.54$ m^2

2A

Now you try

Example 1
a 460 cm **b** 3.2 km

Example 2
a 3.2 cm^2 **b** 240 m^2

Example 3
a 210 000 cm^3 **b** 94 cm^3

Exercise 2A

1 **a** 1000 **b** 10 **c** 100

2 **a** 100 **b** 10 000 **c** 1 000 000

3 **a** 1000 **b** 1 000 000 000
c 1 000 000

4 **a** 43.2 mm **b** 0.327 km **c** 8.34 m
d 96 mm **e** 0.2975 km **f** 1.27 cm

5 **a** 300 000 mm^2 **b** 5000 cm^2
c 5 000 000 m^2 **d** 29 800 cm^2
e 53 700 mm^2 **f** 230 cm^2

6 **a** 2000 mm^3 **b** 200 000 cm^3 **c** 5.7 cm^3
d 15 000 000 m^3 **e** 0.0283 km^3 **f** 0.762 m^3

7 5500 m

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 **a** 23.4 m **b** 22 m
9 **a** 118 mm **b** 147.3 cm **c** 453.258 km
d 15.5 cm^2 **e** 3251 cm^2 **f** 3739 m^2
g 484 500 mm^3 **h** 537 300 m^3
10 21.5 cm
11 For a high level of accuracy
12 **a** 8.85 km **b** 4.5 feet **c** 26.67 cm
d 1.243 miles **e** 57 000 m^2 **f** 0.247 L
g 8200 mL **h** 5 500 000 mL **i** 1000 sq feet
j 2000 L **k** 100 ha **l** 0.152 m^3

2B

Now you try

Example 4
a 0.1 m or 10 cm **b** 0.01 km or 10 m

Example 5
a 35.5 cm to 36.5 cm **b** 15.05 m to 15.15 m

Example 6
Absolute error 0.5 cm, percentage error 0.625%

Exercise 2B

1 **a** 25 **b** 4.5 **c** 2.5 **d** 5
e 3.1 **f** 3.25 **g** 3.05 **h** 3.005
2 **a** ± 1 **b** ± 2.5 **c** ± 0.5 **d** ± 0.25
e ± 0.125 **f** ± 0.05
3 **a** Whole number **b** 0.5 **c** 15.5, 16.5
4 **a** 1 cm **b** 0.1 mm **c** 1 m
d 0.1 kg **e** 0.1 g **f** 1 m
g 1 h **h** 0.01 m or 1 cm
i 0.01 km or 10 m
5 **a** 4.5 m to 5.5 m **b** 7.5 cm to 8.5 cm
c 77.5 mm to 78.5 mm **d** 4.5 ns to 5.5 ns
e 1.5 km to 2.5 km **f** 34.15 cm to 34.25 cm
g 3.85 kg to 3.95 kg **h** 19.35 kg to 19.45 kg
i 457.85 t to 457.95 t **j** 18.645 m to 18.655 m
k 7.875 km to 7.885 km **l** 5.045 s to 5.055 s
6 **a** 0.5 cm, 0.25% **b** 5 cm, 2.5%
c 0.5 kg, 10% **d** 0.5 g, 0.01% **e** 0.5 mm, 0.2%
7 **a** 30 m **b** 145 g **c** 4.6 km
8 **a** 149.5 cm to 150.5 cm
b 145 cm to 155 cm
c 149.95 cm to 150.05 cm
9 **a** 24.5 cm to 25.5 cm
b **i** 245 cm **ii** 255 cm
10 **a** 9.15 cm
b 9.25 cm
c 36.6 cm to 37 cm
11 **a** If they all choose a different level of accuracy, then they will have different answers. Also, human error plays a part.
b Johan: nearest kg; Amy: nearest 100 g; Thomas: nearest 10 g.
c Yes; however, the more decimal places being considered then the more accurate that the measurement will be when used in further calculations, if they are required.
12 **a** Distance between towns, cities, or airplane rides; length of major rivers
b House plans, plumbing plans and building, in general
c Mixing chemicals, administering cough mixture to children, matching paint colours, pay for petrol
d Filling a swimming pool, describing the fuel tank of a car or plane
13 Compare your results with those of other students. Answers may vary.

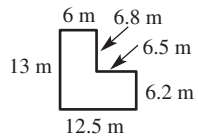
2C

Now you try

Example 7
a 19 m **b** 40.5 mm

Example 8
9.9 cm

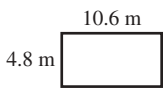
Example 9
a



b The lineal metres of timber needed is 51 m

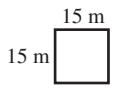
Exercise 2C

1 perimeter
2 **a** 6 **b** 7.1 **c** 4.3
3 **a** 12 cm **b** 23 m **c** 11 km
d 12 m **e** 19.2 cm **f** 10 m
4 **a** 6.7 cm **b** 65 mm **c** 18 m
d 810 m **e** 9.4 km **f** 180 cm
5 **a** $x = 4$ **b** $x = 2$ **c** $x = 6$
6 **a** $x = 3$ **b** $x = 8.8$ **c** $x = 0.1$
7 **a** **i**



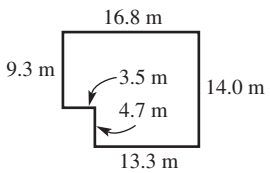
ii 30.8 m

b **i**



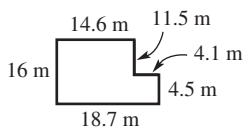
ii 60 m

c **i**



ii 61.6 m

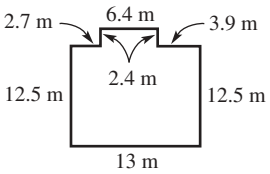
d **i**



ii 69.4 m

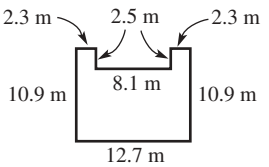
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

e **i**



ii 55.8 m

f **i**



ii 52.2 m

8 20 m

9 15 cm

10 **a** $P = 4l$ **b** $P = 2l + 2b$ **c** $P = x + y + z$
d $P = a + 2b$ **e** $P = 4l$ **f** $P = 3s$

11 3

12 13

2D

Now you try

Example 10

a 31.42 m **b** 15.24 cm

Example 11

16.71 cm

Exercise 2D

1 **a** radius **b** diameter **c** circumference

2 **a** $C = \pi d$ **b** $C = 2\pi r$

3 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$

4 **a** 18.85 m **b** 31.42 m **c** 31.42 km
d 113.10 cm **e** 61.07 mm **f** 3.36 km

5 **a** 1.6 cm **b** 9.4 cm **c** 3.1 cm

6 **a** 27.42 m **b** 16.28 cm **c** 6.71 mm **d** 12.22 cm
e 14.71 m **f** 59.70 cm

7 **a** 9.42 m **b** \$423.90

8 **a** **i** 3.14 m **ii** 47.12 m
b 3.14 km

9 319 times

10 **a** 12.25 **b** 53.03 **c** 1.37 **d** 62.83
e 19.77 **f** 61.70

11 **a** $r = \frac{C}{2\pi}$
b **i** 5.57 cm **ii** 0.29 m **iii** 0.04 km

12 **a** 3.5 cm
b **i** 21.99 cm **ii** 65.97 cm **iii** 109.96 cm
c 12.73 cm

2E

Now you try

Example 12

a 9 m^2 **b** 40 cm^2 **c** 42 mm^2

Example 13

a 48.75 m^2 **b** \$536.25

Exercise 2E

1 **a** E **b** B **c** F **d** C
e D **f** A

2 **a** 2 cm **b** 4 m **c** 4.3 cm **d** 10 cm

3 **a** 4 m^2 **b** 21 cm^2 **c** 11.76 m^2 **d** 21 m^2
e 22.5 mm^2 **f** 2 m^2

4 **a** 25 cm^2 **b** 54.6 m^2 **c** 1.82 km^2
d 0.025 mm^2 **e** 1.1234 m^2 **f** 100 cm^2

5 **a** 0.96 m^2 **b** 9600 cm^2

6 21 m^2

7 **a** 13.6 m^2 **b** \$149.60

8 **a** 7.56 m^2 **b** \$491.40

9 1 m and 24 m, 2 m and 12 m, 3 m and 8 m, 4 m and 6 m

10 **a** 252.05 m^2 **b** 177.86 m^2

11 **a** $b = 2.88$ **b** $l = 14.35$ **c** $h = 1.44$ **d** $a = 1.05$
e $h = 1.87$ **f** $x = 8.89$

12 All answers = 3

2F

Now you try

Example 14

a 78.54 m^2 **b** 12.07 cm^2

Example 15

51.31 mm^2

Example 16

22.57 m^2

Exercise 2F

1 E

2 C

3 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$ **d** $\frac{1}{12}$
e $\frac{7}{12}$ **f** $\frac{5}{6}$

4 **a** 50.27 cm^2 **b** 201.06 m^2 **c** 72.38 m^2
d 38.48 m^2 **e** 0.82 mm^2 **f** 124.69 km^2

5 **a** 39.27 m^2 **b** 4.91 m^2 **c** 84.82 m^2
d 13.09 m^2 **e** 69.81 cm^2 **f** 8.03 m^2

6 157.1 cm^2

7 **a** 14.28 cm^2 **b** 178.54 m^2 **c** 32.14 mm^2

8 43.24 m^2

9 **a** 34.8 cm^2 **b** 63.5 m^2 **c** 8.7 m^2
d 11.5 mm^2 **e** 578.5 km^2 **f** 5.0 m^2

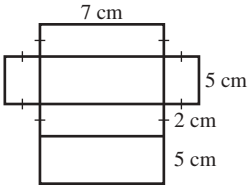
10 **a** 20 **b** 565.49 cm^2 **c** 154.51 cm^2 **d** 5

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2G

Now you try

Example 17



118 cm^2

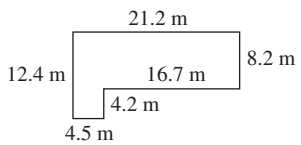
Example 18
480 mm^2

Exercise 2G

1 net
2 **a** 6 **b** 3
3 **a** 35 cm^2 **b** 21 cm^2 **c** 12 cm^2 **d** 96 cm^2
4 **a** 90 cm^2 **b** 34 mm^2 **c** 46 m^2
5 **a** 240 m^2 **b** 84 m^2 **c** 168 m^2 **d** 1176 cm^2
6 **a** 8.64 cm^2 **b** 96 mm^2 **c** 836.6 m^2
7 384 cm^2
8 3880 cm^2
9 3520 cm^2
10 **a** 5.116 L **b** 10.232 L
11 **a** Waterproof 13.76 L
Smooth paint 22.86 L
b $553.46
12 **a** 105 cm^2 **b** 5 cm^2 **c** 16 m^2
13 Yes, only 5 L required

Progress quiz

1 **a** 9000 m **b** 45 000 cm^2 **c** 145 000 mm^3
2 0.28%
3 **a** 17 cm **b** 38 m
4 **a**



b 67.2 m
5 **a** $C = 25.13$ cm, $A = 50.27$ cm^2
b $C = 31.42$ m, $A = 78.54$ m^2
6 **a** 30 m^2 **b** 34 m^2 **c** 50 cm^2 **d** 33.5 cm^2
7 $607.20
8 $A = 49.6$ m^2, $P = 37.9$ m
9 **a** 102 m^2 **b** 152 cm^2

2H

Now you try

Example 19
72.38 cm^2

Exercise 2H

1 **a** circle **b** $2\pi rh$
2 **a** **i** 4 m **ii** 7 m
b 25.13 m
c 276.46 m^2
3 **a** 100.53 m^2 **b** 376.99 cm^2
c 1225.22 cm^2 **d** 74.64 mm^2
4 **a** 2557.3 cm^2 **b** 502.9 m^2
5 **a** 6.3 m^2 **b** 6283.2 cm^2
6 24.0 m^2
7 628.3 cm^2
8 75.4 m^2
9 **a** 173.9 cm^2 **b** 217.8 m^2
c 31.6 m^2 **d** 52.9 cm^2
10 **a** 25.13 m^2 **b** 251.33 m^2 **c** 6.28 m
d 159.15 times **e** 4000 m^2

2I

Now you try

Example 20
360 cm^3

Example 21
254.47 m^3

Example 22
72 L

Exercise 2I

1 **a** C **b** A **c** B
2 **a** 1000 **b** 1000
3 **a** 10 cm^3 **b** 9 m^3
4 **a** 240 m^3 **b** 480 cm^3 **c** 0.18 m^3
5 **a** 113.10 cm^3 **b** 19.63 m^3 **c** 4.83 mm^3
6 **a** 20 cm^2 **b** 90 cm^3
7 **a** 36 m^3 **b** 15 cm^3 **c** 0.572 mm^3
8 5890.49 cm^3
9 **a** 72 L **b** 2 L **c** 2 L
10 **a** 1583.36 m^3 **b** 30 km^3 **c** 196 cm^3
d 30 m^3 **e** 10 cm^3 **f** 2.15 m^3
11 **a** 25 cm^2 **b** 4 cm
12 **a** 17 150 cm^3 **b** 384.85 cm^3 **c** 35
d 3680.42 cm^3
13 **a** 83.3 m^3 **b** 1500 m^3 **c** 20.9 cm^3

2J

Now you try

Example 23
240 cm^2

Example 24
853.98 cm^2, 753.32 cm^3

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 2J

1 **a** Rectangular prism, cylinder
b Triangular prism, cylinder
c Rectangular prism, cube

2 **a** 5 m **b** 13 cm **c** 15 mm

3 **a** 288 m^2 **b** 510 cm^2 **c** 920 mm^2

4 **a** **i** $A = 171.40\ m^2$ **ii** $V = 139.40\ m^3$
b **i** $A = 976.99\ cm^2$ **ii** $V = 1942.48\ cm^3$
c **i** $A = 66.85\ cm^2$ **ii** $V = 25.42\ cm^3$

5 **a** 45.70 L **b** 3570.80 L

6 **a** 406 cm^2 **b** 326 cm^3

7 **a** **i** 16 m^2 **ii** 2 L
b **i** 100.53 m^2 **ii** 11 L
c **i** 520 m^2 **ii** 52 L

8 5.7 m^2

9 17.6 cm^2

10 Cube

11 **a** Approx. 120 cm^3
b Cylinder; surface area of cylinder $= 136.47\ cm^2$, surface area of prism $i = 158\ cm^2$

12 **a** **i** 64 cm^3 **ii** 72 cm^3 **iii** 48 cm^3
b 2 cm cut-out
c Yes (close to 1.7 cm)
d Cut-out length of $\frac{5}{3}$ cm

2K

Now you try

Example 25
a $65\pi\ cm^2$ **b** $84\ cm^2$

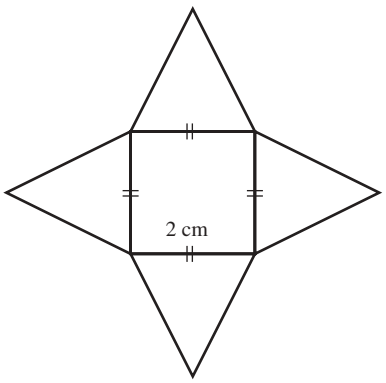
Example 26
a 15.9 cm (to 1 d.p.) **b** 15.4 cm^2 (to 1 d.p.)

Exercise 2K

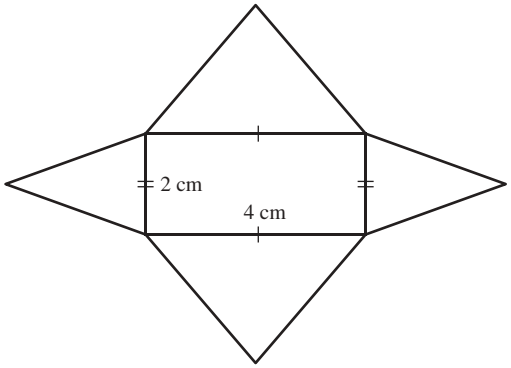
1 **a** $\frac{1}{2}bh$ **b** πr^2 **c** πrl

2 **a** $\sqrt{29}$ cm **b** $\sqrt{221}$ m **c** $\sqrt{109}$ cm

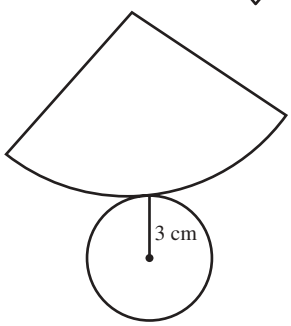
3 **a**



b



c



4 **a** 593.76 mm^2 **b** 0.82 m^2 **c** 435.90 km^2

5 **a** 64 m^2 **b** 105 cm^2 **c** 0.16 m^2

6 **a** 62.83 m^2 **b** 5.18 cm^2 **c** 1960.35 mm^2

7 **a** 10.44 cm **b** 126.7 cm^2

8 **a** 25.5 cm **b** 25.0 cm

9 **a** 18.9 cm **b** 17.8 cm

10 **a** 6.3 m **b** 66.6 m^2

11 Hat B

12 **a** 105 cm^2 **b** 63 cm^2 **c** 163.3 cm^2
d 299.4 m^2 **e** 502.8 mm^2 **f** 76.6 m^2

13 **a** $4\sqrt{26}$ cm **b** 306.57 cm^2 **c** $4\sqrt{2}$ cm
d 20.199 cm **e** 260.53 cm^2 **f** 85%

2L

Now you try

Example 27
a 9 cm^3 **b** $9\pi\ cm^3$

Exercise 2L

1 4 cm^3

2 15 m^3

3 **a** 10 m^3 **b** $\frac{8}{3}$ cm^3 **c** $58\frac{1}{3}$ mm^3

4 **a** 4 cm^3 **b** 585 m^3 **c** 50 km^3
d $\frac{8}{3}$ cm^3 **e** 8 cm^3 **f** 0.336 mm^3

5 **a** 0.82 m^3 **b** 9.38 mm^3 **c** 25 132.74 m^3
d 25.13 m^3 **e** 0.12 m^3 **f** 523.60 cm^3

6 47 mL

7 **a** 282.74 m^3 **b** 276 cm^3 **c** 48 m^3
d 56.88 mm^3 **e** 10.35 m^3 **f** 70.79 m^3

8 11.11 cm

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 $\frac{2}{3}$

Wood wasted = volume of cylinder − volume of cone

Wood wasted $= \pi r^2 h - \frac{1}{3}\pi r^2 h$

Wood wasted $= \frac{2}{3}\pi r^2 h$

Wood wasted $= \frac{2}{3}$ of the volume of cylinder

10 a i $V = \frac{1}{3}x^2 h$ **ii** $V = \frac{1}{12}\pi x^2 h$

b $\frac{\pi}{4}$

11 a Similar triangles are formed, so corresponding sides are in the same ratio.

b $\frac{1}{3}\pi\left(r_1^2 h_1 - r_2^2 h_2\right)$

c i 18.3 cm^3 **ii** 14.7 cm^3

2M

Now you try

Example 28
36π cm^2, 36π cm^3

Example 29
6 cm

Example 30
672π cm^2, 2592π cm^3

Exercise 2M

1 a 314.16 **b** 60.82 **c** 3.14 **d** 33.51
e 91.95 **f** 1436.76

2 a Calculate the surface area of a sphere with radius 5 cm.
b Calculate the surface area of a sphere with radius 2.2 cm.
c Calculate the surface area of a sphere with radius 0.5 cm.
d Calculate the volume of a sphere with radius 2 cm.
e Calculate the volume of a sphere with radius 2.8 cm.
f Calculate the volume of a sphere with radius 7 cm.

3 a $\frac{1}{2}$ **b** $V = \frac{2}{3}\pi r^3$ **c** $A = 2\pi r^2$

4 a 113.10 cm^2, 113.10 cm^3 **b** 201.06 m^2, 268.08 m^3
c 688.13 m^2, 1697.40 m^3 **d** 15.71 mm^2, 5.85 mm^3
e 21.99 m^2, 9.70 m^3 **f** 15.21 km^2, 5.58 km^3

5 a 50.27 cm^2, 33.51 cm^3 **b** 3.14 m^2, 0.52 m^3
c 18 145.84 mm^2, 229 847.30 mm^3
d 1017.88 cm^2, 3053.63 cm^3 **e** 2.66 km^2, 0.41 km^3
f 5.81 m^2, 1.32 m^3

6 a $r = \sqrt{\frac{A}{4\pi}}$ **b** $\sqrt[3]{\frac{3V}{4\pi}}$

7 a i 1.53 cm **ii** 3.50 cm **iii** 0.50 km
b i 0.89 m **ii** 3.09 cm **iii** 0.18 mm

8 a 113.10 cm^3 **b** 5654.9 cm^3 **c** 21 345.1 cm^3

9 11.5 cm

10 52%

11 a 32.7 cm^3 **b** 39.3 cm^2 **c** 58.9 cm^2

12 1570.8 cm^2

13 a 4 m **b** 234.6 m^3

14 a 235.62 m^2 **b** 5.94 cm^2 **c** 138.23 mm^2 **d** 94.25 m^2
e 27.14 m^2 **f** 26.85 cm^2

15 a 5.24 m^3 **b** 942.48 m^3 **c** 10.09 cm^3
d 1273.39 cm^3 **e** 4.76 m^3 **f** 0.74 cm^3

16 a i 523.60 cm^3 **ii** 4188.79 cm^3 **iii** 14 137.17 cm^3
b 61.2 cm

17 a 5 cm **b** $5\sqrt{5}$ cm **c** 332.7 cm^2

18 a i $\sqrt[3]{\frac{3}{4\pi}}$ **ii** $\sqrt[3]{36\pi}$ **iii** 1
iv 6 units^2 **v** 80.6%

b i $4\pi r^2$ **ii** $\sqrt[3]{\frac{4\pi}{3}}r$ **iii** $6\left(\frac{4\pi}{3}\right)^{\frac{2}{3}} r^2$

c Proof required. Example:

$$\frac{4\pi r^2}{6\left(\frac{4\pi}{3}\right)^{\frac{2}{3}} r^2} = \frac{2\pi}{3^{\frac{1}{3}}(4\pi)^{\frac{2}{3}}} = \frac{2\pi^{\frac{1}{3}}}{8^{\frac{1}{3}} \times 6^{\frac{1}{3}}} = \sqrt[3]{\frac{\pi}{6}} \text{ as required.}$$

d They are the same.

Maths@Work: Bricklayer

1 a $l = 23$ cm $= 0.23$ m; $h = 11$ cm $= 0.11$ m
b 253 cm^2; 0.0253 m^2
c 1922.8 cm^3
d i 2.39 m **ii** 23.99 m
e 2990 mm
f Approx. 237 bricks

2 a 36×23 bricks $= 828$ bricks **b** 42 bags
c \$831

3 a 1 pack **b** 2 packs

4 a 8 layers
b 159.6 cm
c 64×40 bricks $= 2560$ bricks
d \$3775

5

Brick wall dimensions		Numbers of bricks used		
Length	Height	Per layer (row)	Number of layers	Total
5750	2150	24	18	432
21350	6230	89	52	4628

6

Bricks	Cost of pallets			
Number of bricks used	Decimal number of pallets	Number of whole pallets	Cost per pallet (including GST)	Total cost of pallets
432	0.432	1	\$1358.24	\$1358.24
4628	4.628	5	\$1541.56	\$7707.80

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Cost of mortar				Total cost of mortar and pallets
Decimal number of mortar bags	Number of whole bags	Cost per bag (including GST)	Total cost of mortar	
21.6	22	$7.95	$174.90	$1533.14
231.4	232	$8.45	$1960.40	$9668.20

7 **a** $3015.02 **b** $10 711.17

Puzzles and challenges

1 PRISM **2** No **3** 21.46% **4** 11
5 $r = 2$ **6** 9.42 cm^2 **7** 27 cm^3

Chapter checklist with success criteria

1 **a** 230 **b** 2.7 km
2 **a** 0.032 km^2 **b** 712 mm^2
3 **a** 3700 mm^3 **b** 5.9 m^3
4 0.1 cm
5 65.35 m, 65.25m
6 36.7 m
7 4.5 cm
8 15.71 m^2
9 37.85 cm
10 6 mm^2
11 32 m^2
12 17.35 m^2
13 32.72 m^2
14 34.28 cm^2
15 216 m^2
16 384 cm^2
17 204.20 m^2
18 240 m^3
19 301.59 cm^3
20 6 L
21 114.27 m^3, 146.27 m^2
22 **a** 64 m^2 **b** 75.4 m^2
23 **a** 4 cm^3 **b** 0.82 m^3
24 3053.63 cm^3, 1017.88 cm^2

Chapter 2 review

Short-answer questions

1 **a** 5300 m **b** 2.7 m^2 **c** 40 mm^3
2 **a** 5.5 cm to 6.5 cm **b** 4.15 kg to 4.25 kg
c 16.205 m to 16.215 m
3 **a** 13 units **b** 24 cm **c** 38 m
4 **a** 18.85 m **b** 28.27 m^2
5 **a** **i** 19.42 m **ii** 26.14 m^2
b **i** 19.50 m **ii** 21.87 m^2
c **i** 14.28 m **ii** 12.28 m^2
6 **a** 10 units **b** 25.9 units **c** 17.5 units
7 **a** 828 m^2 **b** 136 cm^2
8 376.99 m^2
9 **a** 40 cm^3 **b** 125.66 m^3 **c** 21 cm^3
10 **a** 432π cm^3, 216π cm^2
b $12\sqrt{108}\pi$ or $72\sqrt{3}\pi$ cm^3, 108π cm^2
c 288π cm^3, 144π cm^2

Multiple-choice questions

1 E **2** B **3** C **4** A
5 A **6** B **7** E **8** D
9 B **10** C

Extended-response questions

1 **a** **i** 23.21 m **ii** $383
b 19.6 m^2
c 3
d 6.6 m^3
2 **a** 50.27 m^2 **b** 150.80 m^2
c 100.53 m^3 **d** 100 530.96 L
3 **a** 5 m **b** 135 m^2
c $810 **d** 154 m^3

Chapter 3

Warm-up quiz

1 **a** $3x$ **b** $a + 1$ **c** $2m - 5$ **d** $4(x + y)$
2 **a** 20 **b** 17 **c** 23 **d** 22
3 **a** No **b** Yes **c** Yes **d** No
4 **a** $8m$ **b** $5ab$ **c** $6x + 8y$ **d** $8x$
e $15ab$ **f** $3y$
5 **a** $2x + 10$ **b** $3y - 6$ **c** $8x - 12$ **d** $3x^2 + x$
6 **a** 4 **b** 6 **c** $7a$ **d** $2x$
e x **f** $7xy$
7 **a** $\frac{31}{40}$ **b** $\frac{11}{21}$ **c** $\frac{2}{15}$ **d** $\frac{3}{2}$
8 **a** 7^4 **b** m^3 **c** x^2y^3 **d** 3^5a^5
9 **a** 49 **b** 27 **c** 16 **d** 64
10 **a** 3^9 **b** 3^2 **c** 3^{10} **d** 3^0
e 3^{-2}
11 **a** 38 **b** 2310 **c** 0.172 **d** 0.0018
e 1000 **f** 10 000

3A

Now you try

Example 1

a 4
b 1
c **i** 4 **ii** -2

Example 2

a $y + 5$ **b** $3x - 7$ **c** $\frac{a+b}{5}$ **d** xy^2

Example 3

a 36 **b** 38 **c** 24

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 3A

1 **a** Expression **b** Constant term
c Coefficient **d** Term

2 **a** $x + 3$ **b** $5y$ **c** $\frac{a}{5}$ **d** $2xy$

3 **a** 3 **b** -14 **c** 6 **d** 16
e 18 **f** -30 **g** 12 **h** -4
i 36 **j** -36 **k** 36 **l** 0

4 **a** **i** 3 **ii** 8 **iii** 5
b **i** 4 **ii** 2 **iii** -3
c **i** 3 **ii** -4 **iii** 1

5 **a** $x + 2$ **b** $y - 4$ **c** $ab + y$ **d** $2x - 3$
e $5x$ **f** $2m$ **g** $3r$ **h** $\frac{1}{2}x$
i $\frac{3}{4}m$ **j** $\frac{x}{y}$ **k** $\frac{a+b}{4}$ **l** x^2y

6 **a** 12 **b** 3 **c** 9 **d** 10 **e** 10 **f** -2
g 1 **h** 4 **i** -6 **j** 10 **k** -2 **l** -9

7 **a** $5x$ c = \$0.5x **b** $35y$ c = \$0.35y
c $\frac{\$500}{n}$ **d** $\frac{\$11}{m}$ **e** $11 + x$

8 **a** $\$(3.40 + 2d)$
b **i** \$23.40 **ii** \$47.40

9 **a** **i** $2x$ **ii** $2x - 3$ **iii** $3(2x - 3)$
b 21

10 **a** $x + 1$ and $x - 1$
b No
c **i** $(x + 1)(x - 1)$
ii Less by one square metre.

11 **a** 21 sq. units **b** 1, 2, 3, 6, 9, 18

12 **a** **i** $4x$ **ii** x^2
b **i** $2x + 2y$ **ii** xy
c **i** $x + y + 5$ **ii** $\frac{5x}{2}$
d **i** $4ab$ **ii** a^2b^2
e **i** $2a^2 + 2b$ **ii** a^2b
f **i** $mn + 9$ **ii** $2mn$

3B

Now you try

Example 4

a $4a$ and $2a$
$5ab$ and $2ba$
b $-x^2y$ and $4x^2y$

Example 5

a $10x + 2$ **b** $5a + 2b$ **c** $3mn + 5m^2n$

Example 6

a $20xw$ **b** $-12a^2c$

Example 7

a $5x$ **b** $\frac{3s}{5}$

Exercise 3B

1 **a** Yes **b** Yes **c** No **d** Yes

2 **a** $10g$ **b** $5f$ **c** $8e$ **d** 0
e $6x$ **f** $17st$

3 **a** $6x$ **b** $12a$ **c** $10m$ **d** $-18y$

4 **a** $\frac{a}{2}$ **b** $4b$ **c** $3c$ **d** $\frac{2d}{3}$ **e** $\frac{2e}{3}$
f $\frac{7f}{3}$ **g** $\frac{3g}{4}$ **h** $\frac{2h}{5}$ **i** $\frac{5i}{6}$

5 **a** $3ac$ and $-2ac$ **b** $4pq$ and $3qp$
c $7x^2y$ and $4yx^2$, $-3xy^2$ and $2xy^2$
d $2r^2$ and $-r^2$ **e** $-2ab$ and $4ba$
f $3p^2q$ and $4qp^2$ **g** $\frac{1}{3}lm$ and $\frac{lm}{4}$
h x^2y and yx^2, $-xy$ and yx

6 **a** $7t + 10$ **b** $4g + 1$ **c** $7x - 5$ **d** $m + 2$
e $3x + 3y$ **f** $2x + 6y$ **g** $5a - 2b$
h $-3m - 2n$ **i** $5de + 7de^2$ **j** $3kl - 10k^2l$
k $7x^2y + xy^2$ **l** $3fg - fg^2$

7 **a** $6rs$ **b** $6hu$ **c** $16wh$ **d** $6r^2s$
e $-8es$ **f** $-10hv$ **g** $12cm^2$ **h** $35fl$
i $8x^2y$ **j** $24a^2b$ **k** $3xy^2$ **l** $-16a^2b$
m $-12m^2n^2$ **n** $20x^2y^2$ **o** $20a^2b^2$

8 **a** $3a$ **b** $\frac{x}{2}$ **c** $\frac{a}{3}$ **d** $\frac{ab}{4}$ **e** $2b$ **f** $3x$
g $\frac{y}{2}$ **h** $\frac{4a}{5}$ **i** $\frac{2x}{5}$ **j** $\frac{2xy}{3}$ **k** $3ab$ **l** $\frac{n}{3}$

9 **a** $8x$ **b** $3x^2$

10 **a** $5x$ **b** $8y$ **c** $4a$
d $-6x$ **e** $2x$ **f** $10a^2b$

11 **a** $P = 4x + 6, A = 6x$ **b** $P = 4y + 16x, A = 16xy$
c $P = 20a, A = 25a^2$

12 $3x$

13 **a** $6x$ **b** $8a + 7$ **c** $4b$ **d** $11x$
e $3x^2$ **f** $15xy$ **g** $8x^2$ **h** $-29x^2$

3C

Now you try

Example 8

a $3x + 12$ **b** $15x - 10$ **c** $8a^2 + 20ab$

Example 9

a $-4x + 20$ **b** $-6xy + 12y^2$

Example 10

a $8a - 1$ **b** $y + 5$

Exercise 3C

1 brackets, outside

2 **a** $3(x + 4) = 3 \times x + 3 \times 4$
$= 3x + 12$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b $2(x-5) = 2 \times x + 2 \times (-5)$
$= 2x - 10$

c $2(4x+3) = 2 \times 4x + 2 \times 3$
$= 8x + 6$

d $x(x-3) = x \times x + x \times (-3)$
$= x^2 - 3x$

3 **a** $2x + 8$ **b** $3x + 21$ **c** $4y - 12$
d $5y - 10$ **e** $6x + 4$ **f** $8x + 20$
g $9a - 12$ **h** $14y - 35$ **i** $10a + 5b$
j $12a - 9b$ **k** $2x^2 + 10x$ **l** $3x^2 - 12x$
m $6a^2 + 4ab$ **n** $6xy - 8y^2$ **o** $6ab - 15b^2$

4 **a** $-2x - 6$ **b** $-5m - 10$ **c** $-3w - 12$
d $-4x + 12$ **e** $-2m + 14$ **f** $-7w + 35$
g $-x - y$ **h** $-x + y$ **i** $-6x^2 - 8x$
j $-6x^2 - 15x$ **k** $-8x^2 + 8x$ **l** $-6y^2 + 27y$
m $-6x^2 + 10xy$ **n** $-9x^2 - 6xy$ **o** $-12xy - 18y^2$

5 **a** $5x + 17$ **b** $7x + 17$ **c** $2x - 1$ **d** $1 - 2x$
e $1 - 5x$ **f** $6x + 1$ **g** $6x + 11$ **h** $14 - 4x$
i $27 - 6x$ **j** $7x + 18$ **k** $7p - 11$ **l** $10x - 4$
m $4s + 5$ **n** 4 **o** $2x - 7$

6 **a** 2 **b** 4 **c** $3x$
d $3x$ **e** $y, 1$ **f** $2x, 3y$

7 **a** $2x - 10$ **b** $x^2 + 3x$ **c** $2x^2 + 8x$ **d** $6x^2 - 3x$

8 $2x^2 + 4x$

9 **a** $x - 18\,200$ **b** $0.1x - 1820$

10 **a** $x^2 + 7x + 12$ **b** $x^2 + 4x + 3$ **c** $x^2 + 7x + 10$
d $x^2 - 2x - 8$ **e** $x^2 + 3x - 10$ **f** $2x^2 + 11x + 12$
g $2x^2 - x - 6$ **h** $x^2 + x - 12$ **i** $4x^2 + 18x - 10$

3D

Now you try

Example 11
a $-2x - 6$ **b** $-15x + 5$

Example 12
a $x^2 + 6x - 7$ **b** $12x^2 - 11x + 2$

Example 13
a $x^2 + 6x + 9$ **b** $4x^2 - 4x + 1$

Example 14
a $x^2 - 25$ **b** $9x^2 - 4$

Exercise 3D

1 **a** Perfect square **b** Difference of two squares
c Difference of two squares **d** Perfect square
2 **a** $ab + ac$ **b** $ab - ac$ **c** $ac + ad + bc + bd$
d $a^2 - b^2$
3 **a** $x^2 + 2x$ **b** $x^2 + 4x + 3$ **c** $x^2 + 8x + 16$
4 **a** $2x + 10$ **b** $3x - 12$ **c** $-5x - 15$
d $-4x + 8$ **e** $6x - 3$ **f** $12x + 4$
g $-10x + 6$ **h** $-20x - 15$ **i** $2x^2 + 5x$
j $3x^2 - x$ **k** $2x - 2x^2$ **l** $6x - 3x^2$
m $-6x^2 - 4x$ **n** $-18x^2 + 6x$ **o** $-10x + 10x^2$
5 **a** $x^2 + 10x + 16$ **b** $x^2 + 7x + 12$
c $x^2 + 12x + 35$ **d** $x^2 + 5x - 24$
e $x^2 + x - 30$ **f** $x^2 + x - 6$
g $x^2 - 4x - 21$ **h** $x^2 - 10x + 24$
i $x^2 - 13x + 40$ **j** $6x^2 + 13x + 5$
k $12x^2 + 23x + 10$ **l** $10x^2 + 41x + 21$
m $9x^2 - 9x - 10$ **n** $20x^2 + 2x - 6$
o $6x^2 + 5x - 25$
6 **a** $x^2 + 10x + 25$ **b** $x^2 + 14x + 49$
c $x^2 + 12x + 36$ **d** $x^2 - 6x + 9$
e $x^2 - 16x + 64$ **f** $x^2 - 20x + 100$
g $4x^2 + 20x + 25$ **h** $25x^2 + 60x + 36$
i $49x^2 - 14x + 1$
7 **a** $x^2 - 16$ **b** $x^2 - 81$ **c** $x^2 - 64$
d $9x^2 - 16$ **e** $4x^2 - 9$ **f** $64x^2 - 49$
g $16x^2 - 25$ **h** $4x^2 - 81$ **i** $25x^2 - 49$
j $49x^2 - 121$
8 **a** 6 **b** 7 **c** 16 **d** 4 **e** 4 **f** 6
9 **a** 3 **b** 3 **c** 3 **d** 8 **e** 1 **f** 2
10 **a** $(x^2 - 12x + 36)$ cm^2 **b** $(x^2 + 10x - 200)$ cm^2
11 **a** $-x^2 + 7x$ **b** $10a - 28$
c $x^2 - 81$ **d** $4x^2 + 12x + 9$
12 **a** **i** $x^2 + 5x$ **ii** $x^2 + 13x + 36$ **iii** $8x + 36$
b **i** 36 m^2 **ii** 68 m^2

3E

Now you try

Example 15
a 5 **b** $7x$ **c** $3y$

Example 16
a $3(x + 5)$ **b** $6(2m - 3n)$

Example 17
a $3a(3 + 8b)$ **b** $5x(3x - 7)$

Example 18
$-4y(2y + 9)$

Exercise 3E

1 **a** 4 **b** 10 **c** 5 **d** 6
e 3 **f** 25 **g** 8 **h** 36
2 **a** T **b** F
3 **a** C **b** They have no common factor.
4 **a** 6 **b** 5 **c** 4 **d** 9 **e** $5a$ **f** $2m$
g $7x$ **h** $8a$ **i** $3a$ **j** $2x$ **k** $8y$ **l** $5xy$
5 **a** $3(x + 3)$ **b** $4(x - 2)$ **c** $10(y - 2)$
d $6(a + 5)$ **e** $5(x + y)$ **f** $4(3a + b)$
g $9(2m - 3n)$ **h** $12(3x - 4y)$ **i** $4(2x + 11y)$
j $6(4a - 3b)$ **k** $11(11m + 5n)$ **l** $7(2k - 9l)$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 **a** $7x(2+3y)$ **b** $3b(2a-5)$ **c** $8y(4-5x)$
d $5x(x-1)$ **e** $x(x+7)$ **f** $2a(a+4)$
g $6a(2a+7b)$ **h** $9y(y-7)$ **i** $2x(3x+7)$
j $3x(3x-2)$ **k** $8y(2y+5)$ **l** $10m(1-4m)$

7 **a** $-2(x+3)$ **b** $-4(a+2)$ **c** $-3(x+2y)$
d $-7a(1+2b)$ **e** $-x(1+10y)$ **f** $-3b(1+4a)$
g $-x(x+7)$ **h** $-4x(x+3)$ **i** $-2y(y+5)$
j $-2x(4x+7)$ **k** $-4x(3x+2)$ **l** $-5a(3a+1)$

8 **a** $ab(7a+1)$ **b** $4a^2(b+5)$ **c** $xy(1-y)$
d $x^2y(1+4y)$ **e** $6mn(1+3n)$ **f** $5xy(x+2y)$
g $-y(y+8z)$ **h** $-3ab(a+2)$ **i** $-ab(b+a)$

9 **a** $4(x+5)$ **b** $8(x+2)$ **c** $2(3x+4)$

10 $(x+3)$ metres

11 **a** $2(x+2y+3z)$ **b** $3(x^2+4x+2)$
c $4(x^2+2xy+3)$ **d** $3x(2x+y-3)$
e $5x(2y-z+1)$ **f** $2y(2y-9+7x)$

12 **a** $(x+2)(4+x)$ **b** $(x+3)(x+2)$
c $(x+4)(x-7)$ **d** $(2x+1)(x-3)$
e $(y-3)(2x+4)$ **f** $(x-1)(2x-3)$

3F

Now you try

Example 19

a $\frac{9a}{4}$ **b** $\frac{1}{3}$

Example 20

a $2x-1$ **b** 3

Example 21

a $\frac{8}{5}$ **b** $\frac{16}{3}$

Example 22

a $\frac{x}{4}$ **b** 9

Exercise 3F

1 **a** $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** $\frac{2x}{5}$ **d** $\frac{2x}{5}$

2 **a** $\frac{2}{3}$ **b** $\frac{3}{5x}$ **c** $\frac{1}{7}$ **d** $\frac{4}{x+3}$

3 **a** $\frac{2}{5}$ **b** $\frac{1}{12}$

4 **a** $\frac{y}{2}$ **b** $\frac{2a}{5}$ **c** $\frac{x}{5}$ **d** $5x$
e $x+1$ **f** $x-5$ **g** $\frac{x+1}{2}$ **h** 5
i 4 **j** $\frac{1}{2}$ **k** 3 **l** $\frac{3}{2}$

5 **a** $x+2$ **b** $a-5$ **c** $2y-3$ **d** $2b-3$
e 3 **f** 4 **g** 3 **h** 4
i $x+2$ **j** $x-5$ **k** $x+3$ **l** x
m x **n** $2x$ **o** $3x$

6 **a** $\frac{2}{3}$ **b** $\frac{3}{2}$ **c** $\frac{3}{4}$ **d** $\frac{5x}{3}$
e $\frac{3y}{2}$ **f** $\frac{5}{6}$ **g** $\frac{4}{9}$ **h** 10
i $\frac{15}{2}$ **j** $\frac{8}{5}$ **k** $\frac{4}{3x}$ **l** 12

7 **a** 3 **b** 6 **c** $8a$ **d** $\frac{x}{4}$
e $\frac{8}{15a}$ **f** $\frac{4}{x}$ **g** 3 **h** $\frac{5}{2}$
i $\frac{3}{4}$ **j** $\frac{1}{25}$ **k** 10 **l** $\frac{5}{4}$

8 **a** Must factorise first, $x+2$.
b Factorise first, $x+2$.
c Need to multiply by the reciprocal of the fraction after division sign, $\frac{6}{25}$.
d x is not a common factor, cannot cancel, $\frac{x+4}{5x}$.

9 **a** $\frac{1+2a}{3}$ **b** $\frac{4}{5}$ **c** $\frac{x}{4}$ **d** $\frac{2}{5}$ **e** $\frac{7}{18}$
f $\frac{2x}{3}$ **g** $\frac{5}{4}$ **h** $\frac{x}{9}$ **i** $\frac{x+5}{9}$

10 **a** -3 **b** -2 **c** $-x$
d $\frac{x+2}{3}$ **e** $x-6$ **f** $2x-3$

11 **a** $\frac{x+1}{2}$ **b** $\frac{2(x+1)}{3}$ **c** $\frac{x-2}{2}$
d $\frac{x+2}{8}$ **e** $\frac{x-3}{12}$ **f** $\frac{5(2x+1)}{8}$

3G

Now you try

Example 23

a $\frac{9x}{20}$ **b** $\frac{19x}{14}$ **c** $\frac{3x+7}{9}$

Example 24

a $\frac{x-9}{30}$ **b** $\frac{9x-2}{20}$

Exercise 3G

1 **a** 20 **b** 12 **c** 30

2 **a** $3x$ **b** $6x$ **c** 5

3 **a** $\frac{x}{4}+\frac{x}{5}=\frac{5x}{20}+\frac{4x}{20}$

$=\frac{9x}{20}$

b $\frac{2x}{5}-\frac{x}{10}=\frac{4x}{10}-\frac{x}{10}$

$=\frac{3x}{10}$

4 **a** $\frac{7x}{12}$ **b** $\frac{7x}{10}$ **c** $\frac{2x}{9}$ **d** $\frac{2x}{35}$
e $\frac{13x}{15}$ **f** $\frac{7x}{6}$ **g** $\frac{7x}{18}$ **h** $\frac{13x}{40}$
i $\frac{-5x}{14}$ **j** $\frac{-3x}{10}$ **k** $\frac{-x}{30}$ **l** $\frac{-6x}{5}$

5 **a** $\frac{2x+3}{4}$ **b** $\frac{3x+10}{15}$ **c** $\frac{8x+21}{60}$ **d** $\frac{5x-8}{20}$
e $\frac{6x-5}{9}$ **f** $\frac{10-3x}{12}$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 **a** $\frac{5x+4}{6}$ **b** $\frac{13x+12}{15}$ **c** $\frac{5x-4}{8}$ **d** $\frac{x+8}{6}$
e $\frac{x+10}{10}$ **f** $\frac{3x+14}{24}$ **g** $\frac{9x+22}{20}$ **h** $\frac{11x+13}{14}$
i $\frac{5x+7}{12}$ **j** $\frac{11x+14}{12}$ **k** $\frac{3x-2}{10}$ **l** $\frac{14x-13}{24}$

7 **a** Need to multiply numerators also when getting common denominator, $\frac{17x}{12}$.
b Need common denominator before subtracting, $\frac{x}{10}$.
c $3(x+2)=3x+6$ and $5(x+4)=5x+20$, $\frac{8x+26}{15}$
d $-$ sign changed to $+$, $\frac{4x+9}{6}$

8 **a** $\frac{x+16}{30}$ **b** $\frac{2x+22}{15}$ **c** $\frac{x-23}{20}$ **d** $\frac{x+9}{4}$

9 **a** $\frac{12+2x}{3x}$ **b** $\frac{3x+8}{4x}$ **c** $\frac{2x+15}{5x}$ **d** $\frac{3x-14}{7x}$
e $\frac{x-20}{5x}$ **f** $\frac{24-5x}{8x}$

10 **a** $\frac{3x+10}{4x}$ **b** $\frac{x+15}{6x}$ **c** $\frac{6x-5}{20x}$ **d** $\frac{3x+5}{x^2}$
e $\frac{4x+1}{x^2}$ **f** $\frac{3-5x}{x^2}$ **g** $\frac{3x+4}{2x^2}$ **h** $\frac{12x+7}{3x^2}$

Progress quiz

1 **a** 4
b 5
c **i** -3 **ii** $\frac{1}{2}$

2 **a** 6 **b** 9 **c** 14

3 **a** $6x-3$ **b** $5x+5y$ **c** $2x^2y+7xy^2$

4 **a** $12r^2s$ **b** $\frac{x}{3}$ **c** $\frac{5n}{2}$

5 **a** $6x+9$ **b** $20x^2-8x$ **c** $-12x+18$

6 **a** $8x-6$ **b** $3x+32$

7 **a** $12x^2+23x+10$ **b** $12x^2-7x-10$
c $9x^2+12x+4$ **d** $9x^2-4$

8 **a** $6(m+2)$ **b** $5a(3-4b)$ **c** $x(4y+1)$
d $2x(3x-5)$ **e** $-4(2x+5)$ **f** $-3y(y+2)$

9 **a** 2 **b** $3x-7$ **c** $2x$

10 **a** $\frac{3}{2}$ or $1\frac{1}{2}$ **b** $\frac{3x}{16}$

11 **a** $\frac{17x}{20}$ **b** $\frac{6x-4}{9}$ or $\frac{2(3x-2)}{9}$

12 $\frac{5x-1}{12}$

3H

Now you try

Example 25
a 3^4 **b** 7^2y^3 **c** m^3n^2

Example 26
a x^8 **b** x^5y^5 **c** $10a^4b^6$

Example 27
a b^5 **b** $\frac{5a^4}{2}$ **c** $\frac{2a^2b}{3}$

Example 28
$\frac{2a^3b}{3}$

Exercise 3H

1 **a** base, index or power **b** power **c** index
d multiply **e** expanded

2 **a** $8\times8\times8$ **b** $7\times7\times7\times7\times7$
c $x\times x\times x\times x\times x\times x\times x$ **d** $ab\times ab\times ab\times ab$

3 **a** $7\times7\times7\times7$, 7^7 **b** $5\times5\times5\times5$, 5^4

4 **a** add **b** subtract

5 **a** 9^4 **b** 3^6 **c** 15^3 **d** 5^2x^3 **e** 4^3a^4
f $7b^4$ **g** x^3y^2 **h** a^2b^4 **i** $3^3x^2y^3$ **j** $4^2x^2z^2$

6 **a** x^7 **b** p^7 **c** t^9 **d** d^5
e g^4 **f** f^3 **g** $2p^5$ **h** $3c^8$
i $6s^{11}$ **j** a^5b^8 **k** d^9f^5 **l** v^5z^8
m $15a^3b^6$ **n** $6x^3y^3$ **o** $18e^9r^3$ **p** $-8p^4c^3$
q $-10r^7s^8$ **r** $6d^6f^4$

7 **a** a^2 **b** d **c** r^2 **d** c^4 **e** l
f b^3 **g** $4d^2$ **h** $\frac{1}{2}$ **i** $3n^3$ **j** $2p^2$
k $\frac{3m^4}{2}$ **l** $\frac{d^2}{3}$ **m** $4t^3r$ **n** $\frac{5h^4d}{3}$ **o** $2q^2$
p $\frac{xy^2}{2}$ **q** $\frac{r^2s}{3}$ **r** $\frac{2cd^5}{5}$ **s** $\frac{a^3y}{2}$ **t** $\frac{n}{2}$
u $-6x^2y^2$

8 **a** xy^2 **b** m **c** r^2s^4 **d** $2a^2b^2$
e $\frac{9x^8y^4}{2}$ **f** $4w^4$

9 Stuart hasn't put brackets around -2; i.e. $(-2)^4$.

10 **a** **i** 9 **ii** -9
b **i** is $-3\times(-3)$ and **ii** is $-(3\times3)$
c **i** -8 **ii** -8
d They are both -8 since $-2\times(-2)\times(-2)=-8$.

11 **a** 13 **b** 18 **c** 81 **d** 27 **e** 64 **f** 16

3I

Now you try

Example 29
a 0 **b** 4 **c** 4

Example 30
a y^{18} **b** $5m^8$

Example 31
a $27m^3$ **b** $a^{12}b^8$ **c** $\frac{y^4}{81}$

Example 32
a $2a$ **b** $\frac{27x^3}{y^6}$ **c** $25m^8+1$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 3I

1 **a** 1 **b** a^{mn} **c** $a^m \times b^m$ **d** $\frac{a^m}{b^m}$

2 **a** $(2a)^3 = 2a \times 2a \times 2a$
$= 2 \times 2 \times 2 \times a \times a \times a$
$= 2^3 a^3$

b $\left(\frac{4}{7}\right)^4 = \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7}$
$= \frac{4 \times 4 \times 4 \times 4}{7 \times 7 \times 7 \times 7}$
$= \frac{4^4}{7^4}$

3 **a** 1 **b** 1 **c** 1 **d** 1
e 3 **f** 4 **g** 2 **h** 0
i 1 **j** 3 **k** 2 **l** 9

4 **a** b^{12} **b** f^{20} **c** k^{21} **d** $3x^6$
e $5c^{18}$ **f** $4s^{18}$

5 **a** $9x^2$ **b** $64m^3$ **c** $125y^3$ **d** $16x^{12}$
e $x^{10}y^5$ **f** $27a^9$ **g** $x^{24}y^{12}$ **h** a^6b^3
i $m^{12}n^{12}$ **j** $\frac{x^2}{25}$ **k** $\frac{y^4}{81}$ **l** $\frac{m^4}{16}$
m $\frac{x^6}{y^3}$ **n** $\frac{x^{12}}{y^8}$ **o** $\frac{x^3}{y^{15}}$

6 **a** m^6 **b** $\frac{y^5}{10}$ **c** $\frac{4b^4}{3}$ **d** $\frac{27c^4}{2}$
e $\frac{25r^4}{3}$ **f** $\frac{8p^5}{3}$ **g** $\frac{16s^8}{t^{12}}$ **h** $\frac{r^8}{625s^{12}}$

7 **a** 1 **b** 1 **c** 9 **d** 4

8 **a** $2p^3q^7$ **b** $108a^9b^6$ **c** $48r^9y^{12}$ **d** $2m^9n^4$
e $21s^3y^2$ **f** $\frac{3c^4d^{10}}{2}$ **g** $2r^6$ **h** y

9 **a** $a^{11}b^{10}$ **b** $2x^5y^{13}$ **c** $18p^{12}q^2$ **d** $4a^2b^3$
e $\frac{324r^5}{s}$ **f** $\frac{2x^3y^8}{s^2}$

3J

Now you try

Example 33
a $\frac{1}{b^4}$ **b** $\frac{3}{a^3}$ **c** $\frac{4y^4}{x^2}$

Example 34
a y^2 **b** $3a^4$ **c** $\frac{7x^2}{y^3}$

Example 35
a $\frac{y}{x^2}$ **b** $\frac{9}{xy^4}$

Exercise 3J

1 **a** $\frac{1}{b^4}$ **b** $\frac{1}{x^3}$

2 **a** 3 **b** m^2, m^2

3 **a** $\frac{1}{y^3}$ **b** $\frac{1}{x^4}$ **c** $\frac{1}{x^2}$ **d** $\frac{1}{a^5}$
e $\frac{3}{x^2}$ **f** $\frac{5}{b^3}$ **g** $\frac{4}{x}$ **h** $\frac{2}{m^9}$
i $\frac{2x^2}{y^3}$ **j** $\frac{3x}{y^4}$ **k** $\frac{3b^4}{a^2}$ **l** $\frac{5n^2}{m^3}$

4 **a** b^4 **b** x^7 **c** y
d $5m^3$ **e** $2y^2$ **f** $3x^4$
g $5a^2b^3$ **h** $\frac{4y^5}{x^2}$ **i** $\frac{10a^2}{b^4}$

5 **a** $\frac{4}{x^2y^3}$ **b** $\frac{1}{5a^2b^3}$ **c** $2a^3b^2$
d $\frac{a^4b^5}{3}$ **e** $\frac{x^3}{y^2}$ **f** $\frac{x^3}{y^4}$

6 **a** $\frac{b^4}{a^3}$ **b** $\frac{x^4}{y^3}$ **c** $\frac{1}{x^2y^4}$ **d** $\frac{b}{a^4}$

7 **a** $\frac{1}{x^8}$ **b** $\frac{1}{x^6}$ **c** 1
d $\frac{8}{y^6}$ **e** $\frac{a^2}{y^6}$ **f** $\frac{x^6}{16}$
g $\frac{3x}{4y^6}$ **h** $\frac{a^3}{b^{10}}$ **i** $\frac{n^5}{m^8}$

8 1.37 g

9 **a** $\frac{1}{4}$ **b** $\frac{1}{125}$ **c** 36 **d** 40
e $-\frac{3}{4}$ **f** $\frac{1}{36}$ **g** 64 **h** $\frac{1}{2}$

10 **a** **i** $\frac{3}{5}$ **ii** 4 **iii** $\frac{2}{x}$ **iv** $\frac{b}{a}$
b It results in the reciprocal of the fraction.
c **i** $\frac{4}{9}$ **ii** $\frac{25}{9}$ **iii** 27 **iv** $\frac{81}{16}$

3K

Now you try

Example 36
a 32 700 **b** 0.0012

Example 37
a 3.2×10^5 **b** 2×10^{-4}

Example 38
a 5.37×10^4 **b** 3.63×10^{-4}

Exercise 3K

1

Scientific notation	Power of 10 expanded	Basic numeral
5×10^3	5×1000	5000
3×10^4	$3 \times 10\,000$	30 000
2×10^5	$2 \times 100\,000$	200 000
7×10^2	7×100	700
7×10^4	$7 \times 10\,000$	70 000
4×10^5	$4 \times 100\,000$	400 000
6×10^3	6×1000	6000
2×10^6	$2 \times 1\,000\,000$	2 000 000

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2

Scientific notation	Positive power	Fraction	Basic numeral
2×10^{-4}	$\frac{2}{10^4}$	$\frac{2}{10\,000}$	0.0002
3×10^{-2}	$\frac{3}{10^2}$	$\frac{3}{100}$	0.03
5×10^{-3}	$\frac{5}{10^3}$	$\frac{5}{1000}$	0.005
7×10^{-6}	$\frac{7}{10^6}$	$\frac{7}{1\,000\,000}$	0.000007
9×10^{-3}	$\frac{9}{10^3}$	$\frac{9}{1000}$	0.009
8×10^{-2}	$\frac{8}{10^2}$	$\frac{8}{100}$	0.08
6×10^{-4}	$\frac{6}{10^4}$	$\frac{6}{10\,000}$	0.0006
4×10^{-5}	$\frac{4}{10^5}$	$\frac{4}{100\,000}$	0.00004

3 **a** 10 000 **b** 1000 **c** 100 000
d 1000 **e** 100 000 **f** 10 000

4 **a** 3 120 **b** 54 293 **c** 710 500
d 8 213 000 **e** 59 500 **f** 800 200
g 10 120 **h** 9 990 000 **i** 210 500 000
j 0.0045 **k** 0.0272 **l** 0.000 308 5
m 0.007 83 **n** 0.000 092 **o** 0.265
p 0.000 100 2 **q** 0.000 006 235 **r** 0.98

5 **a** 4.3×10^4 **b** 7.12×10^5 **c** 9.012×10^5
d 1.001×10^4 **e** 2.39×10^4 **f** 7.03×10^8
g 7.8×10^{-4} **h** 1.01×10^{-3} **i** 3×10^{-5}
j 3.004×10^{-2} **k** 1.12×10^{-1} **l** 1.92×10^{-3}

6 **a** 6.24×10^3 **b** 5.73×10^5 **c** 3.02×10^4
d 4.24×10^5 **e** 1.01×10^4 **f** 3.50×10^7
g 7.25×10^4 **h** 3.56×10^5 **i** 1.10×10^8
j 2.42×10^{-3} **k** 1.88×10^{-2} **l** 1.25×10^{-4}
m 7.87×10^{-3} **n** 7.08×10^{-4} **o** 1.14×10^{-1}
p 6.40×10^{-6} **q** 7.89×10^{-5} **r** 1.30×10^{-4}

7 **a** 7.7×10^6 km^2 **b** 2.5×10^6
c 7.4×10^9 km **d** 1×10^{-2} cm
e 1.675×10^{-27} kg **f** 9.5×10^{-13} g

8 The numeral part has to be between 1 and 10, i.e. 3.8×10^8.

9 **a** 2.85×10^{-3} **b** 1.55×10^{-3} **c** 4.41×10^{-8}
d 6.38×10^{-3} **e** 8.00×10^7 **f** 3.63×10^8
g 1.80×10^{-3} **h** 3.42×10^{15}

10 328 minutes

11 **a** **i** 9×10^{17} J **ii** 2.34×10^{21} J
iii 2.7×10^{15} J **iv** 9×10^{11} J
b **i** 1.11×10^8 kg **ii** 4.22×10^{-1} kg
iii 9.69×10^{-13} kg **iv** 1.89×10^{-19} kg
c 5.4×10^{41} J

Maths@Work: Electrical trades

1 **a** **i** 1 C $= 6.24 \times 10^{18}$ e **ii** 2 C $= 1.25 \times 10^{19}$ e
iii 3 C $= 1.87 \times 10^{19}$ e **iv** 250 C $= 1.56 \times 10^{21}$ e
v $\frac{1}{2}$C $= 3.12 \times 10^{18}$ e **vi** 12 C $= 7.49 \times 10^{19}$ e

b 1 e $= 1.6 \times 10^{-19}$ C

c **i** $1.24830195 \times 10^{19}$ e/s **ii** $6.24150975 \times 10^{19}$ e/s
iii $1.24830195 \times 10^{20}$ e/s **iv** $3.12075488 \times 10^{18}$ e/s
v $3.12075488 \times 10^{19}$ e/s

2 **a** 1.32×10^{-5}
b 3.168×10^{-4} mm
c Any construction using steel needs to allow for the expansion of steel with a temperature increase. For example, gaps need to be left in a bridge to allow for steel expansion and, hence stop, the road buckling.

3

Conversion table between coulomb and electron charges			
Charge (coulombs, C)	Charge (number of electrons, e)	Charge (number of electrons, e)	Charge (coulombs, C)
1	6.24150975E+18	1	1.60217646E-19
10	6.24150975E+19	10	1.60217646E-18
100	6.24150975E+20	100	1.60217646E-17
1000	6.24150975E+21	1000	1.60217646E-16
10000	6.24150975E+22	10000	1.60217646E-15
100000	6.24150975E+23	100000	1.60217646E-14
1000000	6.24150975E+24	1000000	1.60217646E-13

i $6.24150975 \times 10^{20}$ e **ii** $1.60217646 \times 10^{-13}$ C
iii $6.23526824 \times 10^{24}$ e

4

Conversion table between amperes, time and electrical charge measured in coulombs and numbers of electrons			
Electrical current (in amps)	Time (in seconds)	Charge Q (in coulombs C)	Charge Q number of electrons, e)
0.5	1	5.000E-01	3.12075488E+18
	30	1.500E+01	9.36226463E+19
	60	3.000E+01	1.87245293E+20
	90	4.500E+01	2.80867939E+20
	120	6.000E+01	3.74490585E+20
	150	7.500E+01	4.68113231E+20
	180	9.000E+01	5.61735878E+20

a **i** 4.5×10^1 C **ii** $2.80867939 \times 10^{20}$ e
b **i** 2.25×10^2 C **ii** $1.40433969 \times 10^{21}$ e

Puzzles and challenges

1 Magic square sum $= 3x + 2y$

$\frac{4x^2}{2x}$	$-y$	$x + 3y$
$4y$	$x + y$	$2x - 3y$
$x - 2y$	$2x + 2y$	$2y$

2 3^{3n-3}

3 1c and then double each day

4 5

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 2^{24}
6 200
7 $\frac{5-2x}{30}$
8 n^2, 225

Chapter checklist with success criteria

1 a 3 **b** −7 **c** 3
2 a $2a+3$ **b** $\frac{xy}{2}$
3 a 4 **b** 19
4 ax, $-2ax$ and $7b$, $-3b$
5 a $9b-2$ **b** $-xy+6x$
6 a $15a^2b$ **b** $\frac{1}{2}y$
7 a $12x-8$ **b** $-10yx+14y^2$
8 $4x-11$
9 a $2x^2+9x-5$ **b** $4x^2+12x+9$
10 $9x^2-1$
11 a $6x$ **b** $4b$
12 a $4(2a+3)$ **b** $2x(3x-5y)$ **c** $-2a(2b+9)$
13 4
14 $\frac{15}{2}$
15 $\frac{x}{3}$
16 $\frac{11x}{16}$
17 $\frac{3x+10}{20}$
18 a 7^4 **b** x^3y^2
19 a y^8 **b** $15a^5b^4$
20 a $2m^2$ **b** $\frac{x^4y^2}{3}$
21 $\frac{2}{3}x^4y$
22 2
23 $4x^{10}$
24 a $8m^3$ **b** $\frac{x^8}{81}$
25 a $\frac{1}{4}m^7$ **b** $1-16a^6$
26 a $\frac{3}{y^2}$ **b** $\frac{4y^3}{x^2}$
27 $\frac{1}{a^4b^3}$
28 a 30 200 **b** 0.0000721
29 a 6.4×10^4 **b** 3.5×10^{-5}
30 a 4.73×10^5 **b** 5.38×10^{-3}

Chapter 3 review

Short-answer questions

1 a 4 **b** 5 **c** i 4 ii −3
2 a $y+3$ **b** $xy-5$ **c** $\frac{a+b}{4}$
3 a 14 **b** −30 **c** 35
4 a $7x-5$ **b** $13a-2b$ **c** $xy-3xy^2$
d $12mn$ **e** $-14x^2y$ **f** $\frac{2b}{3}$
5 a $10x+20$ **b** $-6x+8y$ **c** $6x^2+15xy$
d $4a+15$ **e** $5y+13$ **f** $8t+11$
6 a $4x^2+4x+1$ **b** $6x^2+7x+2$
c $6x^2-x-2$ **d** $4x^2-1$
7 a $8(2x-5)$ **b** $5xy(2x+7y)$
c $2x(2x-5)$ **d** $-2x(y+9)$
8 a $\frac{14x}{15}$ **b** $\frac{6-7a}{14}$ **c** $\frac{9x+8}{20}$
9 a $\frac{3}{8}$ **b** $4x$ **c** $3x-1$ **d** $\frac{2}{3}$
10 a $12x^7$ **b** $8x^4y^8$ **c** b^4 **d** $\frac{2a^2b^3}{3}$
11 a b^8 **b** $8m^6$ **c** $\frac{x^2}{49}$ **d** $\frac{64y^6}{z^{12}}$
12 a 1 **b** 4 **c** 6 **d** 1
13 a $\frac{4}{x^3}$ **b** $\frac{3r^4}{s^2}$ **c** $\frac{2y^4}{3x^3}$ **d** $4m^5$
14 a $\frac{5y^6}{4}$ **b** $\frac{25x^3y^2}{2}$ **c** $\frac{10}{x^3}$ **d** $\frac{x^8}{2y^3}$
15 a 4250 **b** 37 000 000
c 0.021 **d** 0.0000725
16 a 1.24×10^5 **b** 3.95×10^7
c 9.02×10^{-6} **d** 4.60×10^{-4}

Multiple-choice questions

1 C **2** B **3** C **4** D
5 B **6** D **7** C **8** D
9 A **10** E **11** C

Extended-response questions

1 a $2(5x+1)$ m **b** 32 m
c $(5x^2+3x)$ m^2 **d** \$1080
2 a 1.84×10^{11} tonnes
b More than 50 trillion dollars (approx 5.0729×10^{13})

Chapter 4

Warm-up quiz

1 a 11
b i $\frac{1}{11}$ ii $\frac{2}{11}$ iii $\frac{4}{11}$
iv $\frac{7}{11}$ v $\frac{3}{11}$ vi $\frac{8}{11}$
2 a $\frac{1}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{4}$
e $\frac{5}{8}$ **f** $\frac{7}{8}$ **g** $\frac{1}{4}$
3 0, 1 in 5, 39%, 0.4, $\frac{1}{2}$, 0.62, 71%, $\frac{3}{4}$, $\frac{9}{10}$, 1
4 a i 14 ii 25 iii 11
b i $\frac{18}{25}$ ii $\frac{7}{25}$ iii $\frac{7}{25}$
5 a $\frac{7}{16}$ **b** $\frac{9}{16}$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 a 16

b i $\frac{1}{16}$ ii $\frac{3}{16}$ iii $\frac{3}{8}$

iv $\frac{5}{8}$ v $\frac{13}{16}$ vi $\frac{3}{16}$

7 a 4

b i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{1}{2}$

iv $\frac{3}{4}$ v $\frac{1}{2}$ vi 1

4A

Now you try

Example 1

a 6

b i $\frac{1}{6}$ ii $\frac{1}{3}$ iii $\frac{2}{3}$ iv $\frac{1}{2}$

Example 2

a 34

b 51

c i 0.15 ii 0.09 iii 0.49 iv 0.85

Exercise 4A

1 a Sample space b 0, 1 c Likely d Outcome

2 C, A, B, D

3 a $\frac{1}{4}$ b $\frac{1}{6}$ c $\frac{1}{4}$ d $\frac{3}{8}$ e $\frac{2}{3}$ f 0

4 a 7

b i $\frac{1}{7}$ ii $\frac{2}{7}$ iii $\frac{5}{7}$ iv $\frac{3}{7}$

5 a $\frac{3}{10}$ b $\frac{2}{5}$ c $\frac{3}{5}$ d $\frac{1}{2}$

6 a 43

b 47

c i 0.09 ii 0.43 iii 0.47 iv 0.91

7 a 0.62 b 0.03 c 0.97 d 0.38

8 a $\frac{1}{10}$ b $\frac{1}{2}$ c $\frac{1}{2}$ d $\frac{1}{2}$

e $\frac{2}{5}$ f $\frac{1}{5}$ g $\frac{3}{10}$

9 a $\frac{1}{50}$ b $\frac{3}{10}$ c $\frac{49}{50}$

10 a $\frac{6}{25}$ b $\frac{1}{50}$ c $\frac{21}{25}$

d $\frac{2}{5}$ e $\frac{2}{25}$ f $\frac{4}{25}$

11 a 59 b 4, as $\frac{41}{100}$ of 10 is closest to 4.

c 8, as $\frac{41}{100}$ of 20 is closest to 8.

12 a $\frac{1}{4}$ b $\frac{1}{13}$ c $\frac{1}{52}$ d $\frac{1}{2}$

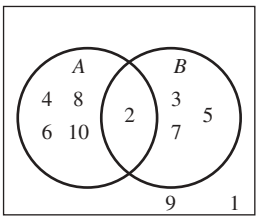
e $\frac{2}{13}$ f $\frac{4}{13}$ g $\frac{12}{13}$ h $\frac{9}{13}$

4B

Now you try

Example 3

a



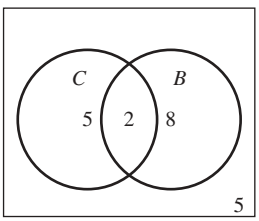
b i {2} ii {2, 3, 4, 5, 6, 7, 8, 10}

c i $\frac{1}{2}$ ii $\frac{1}{10}$ iii $\frac{4}{5}$

d They are not mutually exclusive because there is a number in A and B.

Example 4

a



b i 5 ii 5

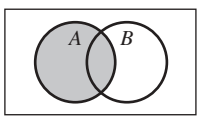
c i $\frac{7}{20}$ ii $\frac{1}{4}$ iii $\frac{1}{10}$

Exercise 4B

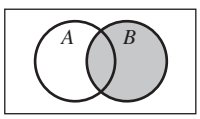
1 a D b C c E d A e B

2 a No b Yes c No

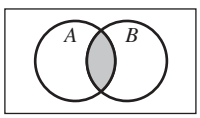
3 a



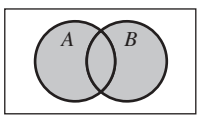
b



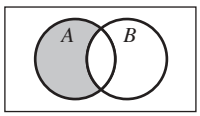
c



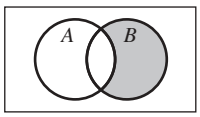
d



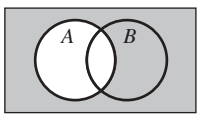
e



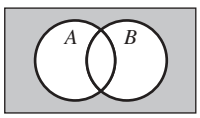
f



g

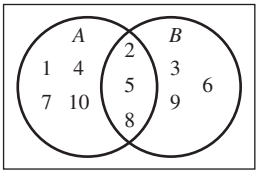


h



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 a



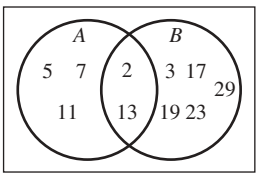
b i A and $B = \{2, 5, 8\}$

ii A or $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

c i $\frac{7}{10}$ ii $\frac{3}{10}$ iii 1

d No, they are not mutually exclusive because there is at least one number in A and B.

5 a

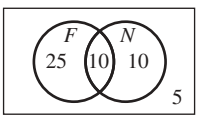


b i A and $B = \{2, 13\}$

ii A or $B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

c i $\frac{1}{2}$ ii $\frac{7}{10}$ iii $\frac{1}{5}$ iv 1

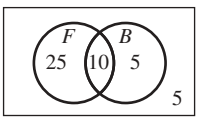
6 a



b i 25 ii 5

c i $\frac{2}{5}$ ii $\frac{1}{5}$ iii $\frac{1}{5}$

7 a



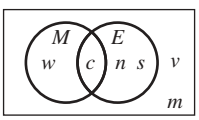
b i 25 ii 5

c i $\frac{7}{9}$ ii $\frac{2}{9}$ iii $\frac{8}{9}$ iv $\frac{2}{9}$ v $\frac{1}{9}$

8 3

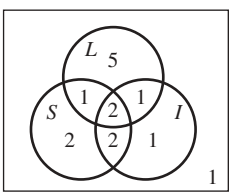
9 5

10 a



b i $\frac{1}{3}$ ii $\frac{2}{3}$ iii $\frac{1}{6}$ iv $\frac{2}{3}$ v $\frac{1}{3}$

11 a



b 1

c i $\frac{3}{5}$ ii $\frac{1}{3}$ iii $\frac{13}{15}$ iv $\frac{1}{15}$

4C

Now you try

Example 5

a

	A	not A	
B	3	2	5
not B	6	4	10
	9	6	15

b i 3 ii 2 iii 6 iv 4

v 9 vi 10 vii 11

c i $\frac{1}{5}$ ii $\frac{2}{5}$ iii $\frac{2}{5}$

Exercise 4C

1 a B **b** A **c** D **d** C

2 a i 4 ii 6 iii 3 iv 1

v 10 vi 7 vii 4 viii 7

b 13

3 a

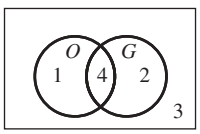
	A	not A	
B	2	6	8
not B	5	3	8
	7	9	16

b i 2 ii 6 iii 5 iv 3

v 7 vi 8 vii 13

c i $\frac{1}{8}$ ii $\frac{9}{16}$ iii $\frac{5}{16}$

4 a



b

	O	not O	
G	4	2	6
not G	1	3	4
	5	5	10

c i 2 ii 5 iii 4 iv 7

d i $\frac{3}{5}$ ii $\frac{2}{5}$ iii $\frac{1}{10}$ iv $\frac{2}{5}$

v $\frac{7}{10}$

5 a

	A	not A	
H	4	3	7
not H	4	1	5
	8	4	12

b i 3 ii 1

c i $\frac{11}{12}$ ii $\frac{1}{3}$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 a

	A	not *A*	
B	3	3	6
not *B*	4	1	5
	7	4	11

b

	A	not *A*	
B	2	7	9
B	2	1	3
	4	8	12

7 a $\frac{1}{8}$ b $\frac{5}{24}$

8 a 0

b

	A	not A	
B	0	6	6
not *B*	10	2	12
	10	8	18

9 a $\frac{3}{8}$ b $\frac{5}{32}$

10 a $\frac{4}{13}$

b $\frac{4}{13}$

c $\frac{7}{13}$

d $\frac{7}{13}$

11 a 18

b 75

4D

Now you try

Example 6

a

		Spin 2		
		1	2	3
Spin 1	1	(1, 1)	(1, 2)	(1, 3)
	2	(2, 1)	(2, 2)	(2, 3)
	3	(3, 1)	(3, 2)	(3, 3)

b 9

c $\frac{1}{9}$

d i $\frac{1}{3}$ ii $\frac{4}{9}$ iii $\frac{8}{9}$

Example 7

a

		1st			
		T	R	E	E
2nd	T	X	(R, T)	(E, T)	(E, T)
	R	(T, R)	X	(E, R)	(E, R)
	E	(T, E)	(R, E)	X	(E, E)
	E	(T, E)	(R, E)	(E, E)	X

b i $\frac{1}{6}$ ii $\frac{1}{6}$ iii $\frac{1}{3}$

Exercise 4D

1 a without replacement b with replacement

2 a i

		1st		
		D	O	G
2nd	D	(D, D)	(O, D)	(G, D)
	O	(D, O)	(O, O)	(G, O)
	G	(D, G)	(O, G)	(G, G)

ii

		1st		
		D	O	G
2nd	D	×	(O, D)	(G, D)
	O	(D, O)	×	(G, O)
	G	(D, G)	(O, G)	×

b i 9 ii 6

c i $\frac{1}{3}$ ii $\frac{5}{9}$ iii $\frac{4}{9}$ iv $\frac{8}{9}$

v $\frac{2}{9}$

d i 0 ii $\frac{2}{3}$ iii $\frac{1}{3}$ iv 1 v $\frac{1}{3}$

3 a 9 b 6

4 a

		1st roll			
		1	2	3	4
2nd roll	1	(1, 1)	(2, 1)	(3, 1)	(4, 1)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)

b 16 c $\frac{1}{16}$

d i $\frac{1}{4}$ ii $\frac{5}{8}$ iii $\frac{13}{16}$

5 a

		1st toss	
		H	T
2nd toss	H	(H, H)	(T, H)
	T	(H, T)	(T, T)

b 4

c $\frac{1}{4}$

d i $\frac{1}{2}$ ii $\frac{3}{4}$

e 250

6 a

		1st			
		S	E	T	S
2nd	S	×	(E, S)	(T, S)	(S, S)
	E	(S, E)	×	(T, E)	(S, E)
	T	(S, T)	(E, T)	×	(S, T)
	S	(S, S)	(E, S)	(T, S)	×

b i $\frac{1}{6}$ ii $\frac{1}{2}$ iii $\frac{1}{6}$ iv $\frac{1}{3}$ v 1

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 a

		1st				
		L	**E**	**V**	**E**	**L**
2nd	**L**	×	(E, L)	(V, L)	(E, L)	(L, L)
	E	(L, E)	×	(V, E)	(E, E)	(L, E)
	V	(L, V)	(E, V)	×	(E, V)	(L, V)
	E	(L, E)	(E, E)	(V, E)	×	(L, E)
	L	(L, L)	(E, L)	(V, L)	(E, L)	×

b 20

c **i** 8 **ii** 12 **iii** 12

d **i** $\frac{2}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{3}{5}$

e $\frac{1}{5}$

8 a

		Die 2					
		1	**2**	**3**	**4**	**5**	**6**
Die 1	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b 36

c **i** 2 **ii** 6 **iii** 15

d **i** $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{35}{36}$ **iv** $\frac{1}{12}$

e $\frac{1}{6}$. Min's guess is wrong.

9 a $\frac{1}{12}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3}$ **d** $\frac{1}{2}$

10 a Without

b With

c With

d Without

11 a

		1st			
		2.5	**5**	**10**	**20**
2nd	**2.5**	5	7.5	12.5	22.5
	5	7.5	10	15	25
	10	12.5	15	20	30
	20	22.5	25	30	40

b 16

c **i** 1 **ii** 8 **iii** 8

d **i** $\frac{1}{16}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{4}$ **iv** $\frac{3}{16}$

e $\frac{7}{16}$

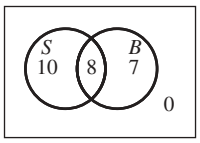
Progress quiz

1 a $\frac{1}{4}$ **b** $\frac{3}{8}$ **c** $\frac{3}{4}$ **d** $\frac{3}{8}$

2 a 33

b **i** $\frac{21}{100}$ **ii** $\frac{67}{100}$ **iii** $\frac{79}{100}$

3 a



b **i** 10 **ii** 0

c **i** $\frac{3}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{8}{25}$ **iv** $\frac{2}{5}$

4 a

	P	**not *P***	
B	12	1	13
not *B*	4	3	7
	16	4	20

b **i** 4 **ii** 17

c **i** $\frac{4}{5}$ **ii** $\frac{1}{20}$ **iii** $\frac{3}{5}$

5 a

	1	**2**	**3**	**4**	**5**	**6**
H	(1, H)	(2, H)	(3, H)	(4, H)	(5, H)	(6, H)
T	(1, T)	(2, T)	(3, T)	(4, T)	(5, T)	(6, T)

b 12

c $\frac{1}{12}$

d **i** $\frac{1}{4}$ **ii** $\frac{5}{12}$

6 a

	R	**R**	**B**	**G**
R	×	(R, R)	(B, R)	(G, R)
R	(R, R)	×	(B, R)	(G, R)
B	(R, B)	(R, B)	×	(G, B)
G	(R, G)	(R, G)	(B, G)	×

12 outcomes

b **i** $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{3}$

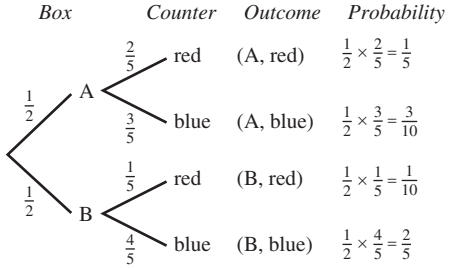
4E

Now you try

Example 8

a $\frac{2}{5}$ **b** $\frac{1}{5}$

c



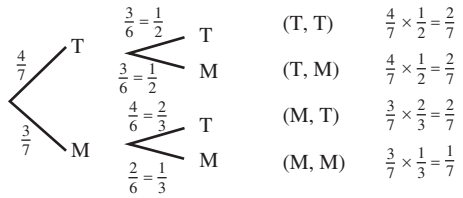
d $\frac{1}{10}$ **e** $\frac{3}{10}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 9

a

Selection 1	*Selection 2*	*Outcome*	*Probability*
T	T	(T, T)	$\frac{4}{7} \times \frac{1}{2} = \frac{2}{7}$
	M	(T, M)	$\frac{4}{7} \times \frac{1}{2} = \frac{2}{7}$
M	T	(M, T)	$\frac{3}{7} \times \frac{2}{3} = \frac{2}{7}$
	M	(M, M)	$\frac{3}{7} \times \frac{1}{3} = \frac{1}{7}$



b i $\frac{2}{7}$ ii $\frac{1}{7}$ iii $\frac{4}{7}$

c i $\frac{12}{49}$ ii $\frac{9}{49}$ iii $\frac{24}{49}$

Exercise 4E

1 a 8 **b** $\frac{1}{8}$

c i 3 ii 4

2 a i $\frac{2}{5}$ ii $\frac{3}{5}$

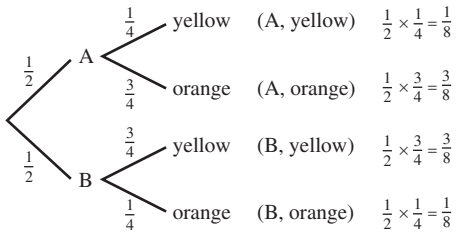
b i $\frac{2}{5}$ ii $\frac{3}{5}$

c i $\frac{1}{4}$ ii $\frac{3}{4}$

3 a $\frac{1}{4}$ **b** $\frac{3}{4}$

c

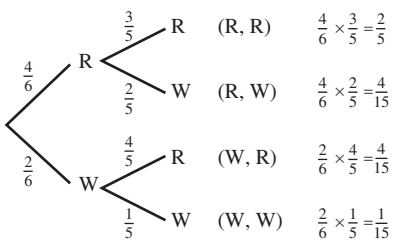
Box	*Counter*	*Outcome*	*Probability*
A	yellow	(A, yellow)	$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$
	orange	(A, orange)	$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$
B	yellow	(B, yellow)	$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$
	orange	(B, orange)	$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$



d $\frac{3}{8}$ **e** $\frac{1}{2}$

4 a

Selection 1	*Selection 2*	*Outcome*	*Probability*
R	R	(R, R)	$\frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$
	W	(R, W)	$\frac{4}{6} \times \frac{2}{5} = \frac{4}{15}$
W	R	(W, R)	$\frac{2}{6} \times \frac{4}{5} = \frac{4}{15}$
	W	(W, W)	$\frac{2}{6} \times \frac{1}{5} = \frac{1}{15}$

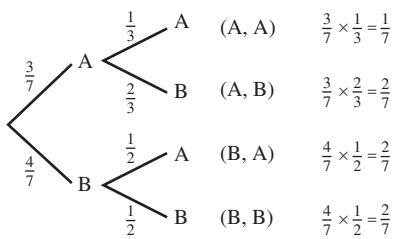


b i $\frac{4}{15}$ ii $\frac{2}{5}$ iii $\frac{8}{15}$

c i $\frac{2}{9}$ ii $\frac{4}{9}$ iii $\frac{4}{9}$

5 a

Selection 1	*Selection 2*	*Outcome*	*Probability*
A	A	(A, A)	$\frac{3}{7} \times \frac{1}{3} = \frac{1}{7}$
	B	(A, B)	$\frac{3}{7} \times \frac{2}{3} = \frac{2}{7}$
B	A	(B, A)	$\frac{4}{7} \times \frac{1}{2} = \frac{2}{7}$
	B	(B, B)	$\frac{4}{7} \times \frac{1}{2} = \frac{2}{7}$

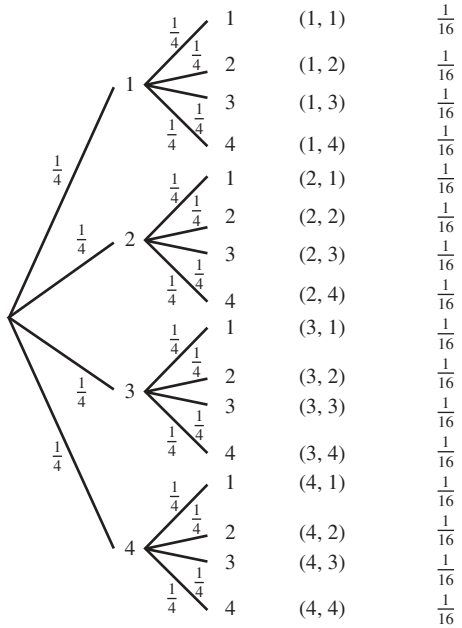


i $\frac{1}{7}$ **ii** $\frac{2}{7}$ **iii** $\frac{4}{7}$ **iv** $\frac{3}{7}$

b i $\frac{9}{49}$ ii $\frac{16}{49}$ iii $\frac{24}{49}$ iv $\frac{25}{49}$

6 a

1st roll	*2nd roll*	*Outcome*	*Probability*
1	1	(1, 1)	$\frac{1}{16}$
	2	(1, 2)	$\frac{1}{16}$
	3	(1, 3)	$\frac{1}{16}$
	4	(1, 4)	$\frac{1}{16}$
2	1	(2, 1)	$\frac{1}{16}$
	2	(2, 2)	$\frac{1}{16}$
	3	(2, 3)	$\frac{1}{16}$
	4	(2, 4)	$\frac{1}{16}$
3	1	(3, 1)	$\frac{1}{16}$
	2	(3, 2)	$\frac{1}{16}$
	3	(3, 3)	$\frac{1}{16}$
	4	(3, 4)	$\frac{1}{16}$
4	1	(4, 1)	$\frac{1}{16}$
	2	(4, 2)	$\frac{1}{16}$
	3	(4, 3)	$\frac{1}{16}$
	4	(4, 4)	$\frac{1}{16}$



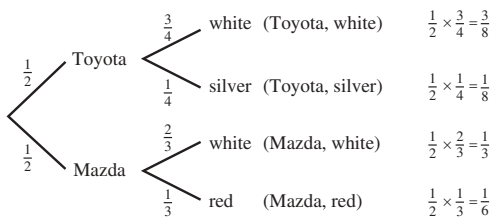
b 16

c i $\frac{1}{16}$ ii $\frac{1}{4}$

d i $\frac{1}{16}$ ii $\frac{1}{4}$ iii $\frac{5}{8}$

7 a

Car		*Outcome*	*Probability*
Toyota	white	(Toyota, white)	$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$
	silver	(Toyota, silver)	$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$
Mazda	white	(Mazda, white)	$\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$
	red	(Mazda, red)	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

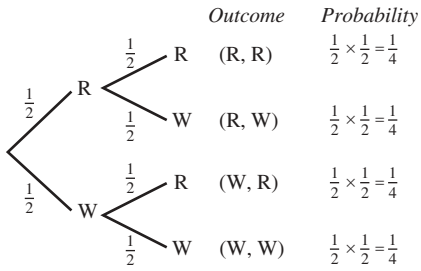


b i $\frac{3}{8}$ ii $\frac{1}{6}$ iii $\frac{17}{24}$ iv $\frac{7}{24}$

v $\frac{5}{6}$ **vi** $\frac{1}{3}$

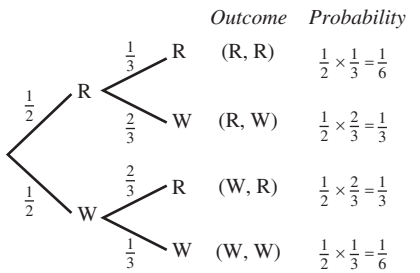
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 a



i $\frac{1}{4}$ ii $\frac{1}{2}$ iii $\frac{3}{4}$ iv $\frac{3}{4}$

b



i $\frac{1}{6}$ ii $\frac{2}{3}$ iii $\frac{5}{6}$ iv $\frac{5}{6}$

9 a i 0.17 ii 0.11 iii 0.83

b i 0.1445 ii 0.0965 iii 0.8555

4F

Now you try

Example 10
Example 1

Exercise 4F

1 a Yes **b** No **c** No **d** Yes **e** Yes

2 a With **b** Without

3 Yes

4 a $\frac{1}{2}$ **b** $\frac{1}{2}$

5 a $\frac{1}{8}$ **b** $\frac{1}{64}$ **c** $\frac{1}{216}$

6 If we assume that the '1 in 6' claim is true and they are chosen from a very large pool, then:

a Tilly is incorrect because she might be unlucky.

b Cody is incorrect since the probability is still $\frac{1}{6}$.

c It is possible to win six times in a row, although it is unlikely in this case.

7 a $\frac{1}{32}$ **b** $\frac{31}{32}$ **c** $\frac{31}{32}$

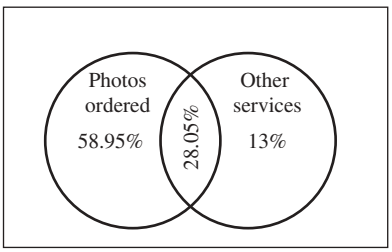
8 a $\frac{1}{6}$ **b** $\frac{5}{36}$ **c** $\frac{671}{1296} = 0.52$

Maths@Work: Business analyst

1 a

	Other services	No other services	Total orders
Photos ordered	28.05%	58.95%	87%
No photos	13%	-	13%
Total	41.05%	58.95%	100%

b



c $6264

d Advertising should focus on photo print services as 87% of the weekly orders are for photo prints.

2 a i 820 ii 160 iii 180 iv 20

b Business cards are ordered by 57% of customers with 42% ordering only business cards. This is greater than either newsletter printing (38%, 25% only newsletters) and promotional material (25%, 15% only promotional material). There is a strong market for business cards so the company must focus on offering a quality product with a competitive price.

c Greater percentages in newsletters and promotional material to increase the company's business over these product lines.

3 a *See table at top of next page.*

b i Running ii Yoga

c *See figure on next page.*

d Gym wear increased by 89 sales from Q3 to Q4. The advertising was successful as this is a much larger increase than the other increases of: Q1 to Q2, 7 sales; Q2 to Q3, 19 sales.

e *See figure on next page.*

f A 3.4% increase. The graph in part **e** shows the Quarter 4 column is twice as high as the Quarter 1 column. The reason why this is misleading is because the percentage scale does not start at 0. With the percentage scale starting at zero, comparisons are more realistic.

See figure on page 842.

Puzzles and challenges

1 MUTUALLY EXCLUSIVE

2 $\frac{1}{6}$

3 Results may vary.

4 a $\frac{1}{12}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$ **d** $\frac{2}{3}$

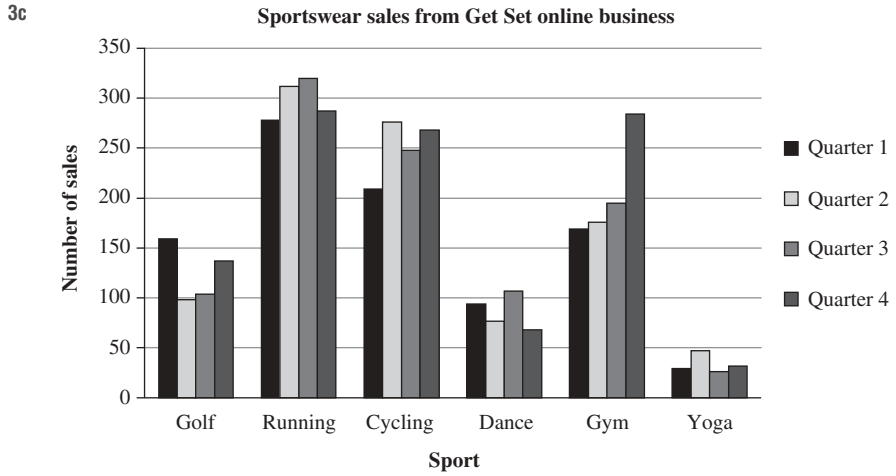
5 $\frac{15}{16}$ **6** $\frac{3}{5}$ **7** $\frac{1}{12}$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

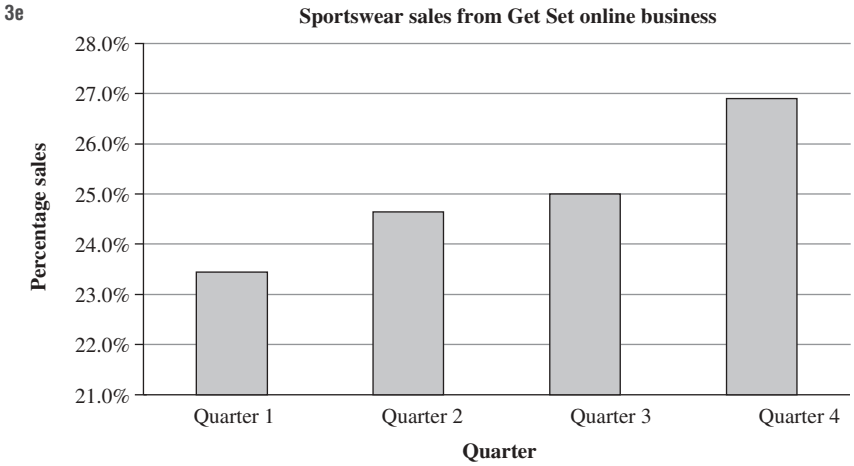
3a

Sportswear sales from Get Set online business						
Sport	QUARTER 1	QUARTER 2	QUARTER 3	QUARTER 4	Totals	Percentages
Golf	159	98	104	137	498	12.5%
Running	278	312	320	287	1197	29.9%
Cycling	209	276	248	268	1001	25.0%
Dance	94	77	107	68	346	8.7%
Gym	169	176	195	284	824	20.6%
Yoga	29	47	26	32	134	3.4%
Totals	938	986	1000	1076	4000	100.0%
Percentages	23.5%	24.7%	25.0%	26.9%	100.0%	

3c

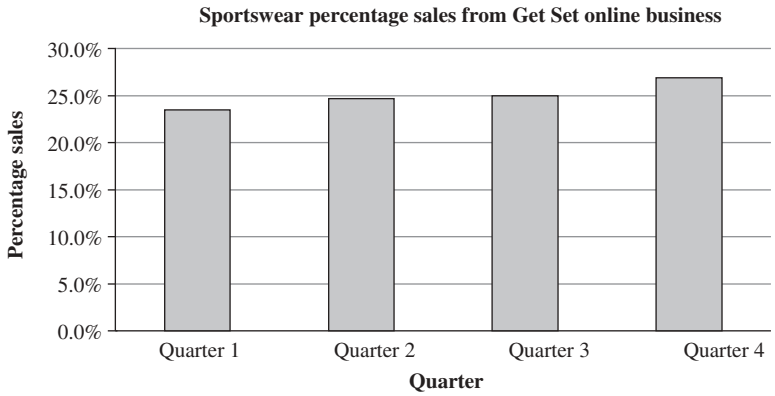


3e



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3f



Chapter checklist with success criteria

1 a $\frac{2}{9}$ b $\frac{8}{9}$ c $\frac{1}{3}$

2 a 0.39 b 0.85

3 a $\{1, 4, 5, 6, 7, 8\}$ b $\frac{3}{10}$

c They are not because there is a number in A and B

4 a 3 b $\frac{1}{4}$

5 a 13 b 5 c $\frac{5}{18}$, $\frac{2}{9}$

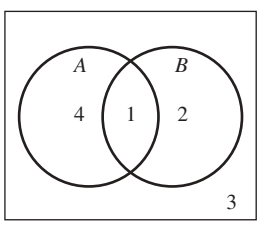
6 a $\frac{4}{25}$ b $\frac{9}{25}$ c $\frac{23}{25}$

7 a $\frac{1}{6}$ b $\frac{1}{3}$

8 a $\frac{3}{8}$ b $\frac{5}{8}$

9 a $\frac{3}{5}$ b $\frac{8}{15}$

10



$P(A|B) = \frac{1}{3}, P(B|A) = \frac{1}{5}$. The events are not independent.

Chapter 4 review

Short-answer questions

1 a $\frac{1}{6}$ b $\frac{1}{2}$ c $\frac{2}{3}$

2 a $\frac{1}{8}$ b $\frac{1}{4}$ c $\frac{3}{8}$ d $\frac{5}{8}$

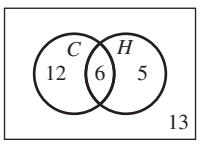
e $\frac{1}{2}$

3 a i $\frac{2}{5}$ ii $\frac{1}{4}$ iii $\frac{1}{5}$

iv $\frac{1}{10}$ v $\frac{1}{20}$

b i $\frac{3}{5}$ ii $\frac{17}{20}$

4 a



b

	C	not *C*	
H	6	5	11
not *H*	12	13	25
	18	18	36

c 13

d i $\frac{1}{6}$ ii $\frac{5}{36}$ iii $\frac{1}{2}$

5 a 8 b $\frac{6}{13}$

6 a

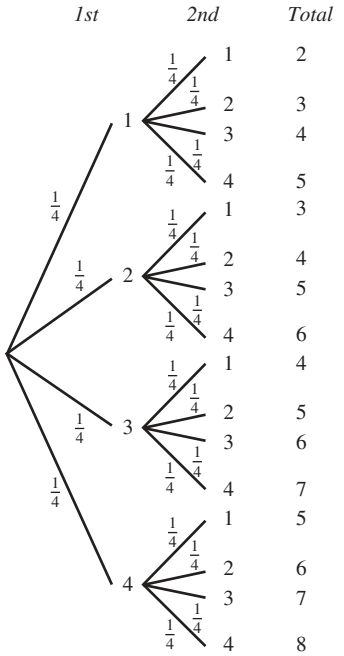
		1st				
		H	A	P	P	Y
2nd	H	(H, H)	(A, H)	(P, H)	(P, H)	(Y, H)
	E	(H, E)	(A, E)	(P, E)	(P, E)	(Y, E)
	Y	(H, Y)	(A, Y)	(P, Y)	(P, Y)	(Y, Y)

b 15

c i $\frac{1}{15}$ ii $\frac{2}{15}$ iii $\frac{13}{15}$

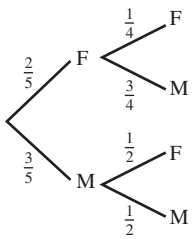
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 a



b i $\frac{1}{16}$ ii $\frac{1}{4}$ iii 0 iv 1

8



a $\frac{2}{5}$ **b** $\frac{3}{10}$ **c** $\frac{3}{5}$ **d** $\frac{7}{10}$

Multiple-choice questions

1 C **2** C **3** A **4** D
5 C **6** B **7** B **8** E

Extended-response questions

1 a 8

b i $\frac{7}{15}$ ii $\frac{1}{15}$

c

	R	not R	
S	3	1	4
not S	3	8	11
	6	9	15

2 a

		1st		
		R	S	W
2nd	R	(R, R)	(S, R)	(W, R)
	S	(R, S)	(S, S)	(W, S)
	W	(R, W)	(S, W)	(W, W)

b i $\frac{1}{9}$ ii $\frac{1}{3}$ iii $\frac{5}{9}$ iv $\frac{4}{9}$

c 4

d $\frac{5}{9}$

Chapter 5

Warm-up quiz

1 a J **b** G **c** R **d** K
e L **f** I **g** A **h** H
i F **j** B **k** C **l** M
m O **n** D **o** E **p** P
q Q **r** N

2 a 8 **b** 40 **c** 82 **d** 13

3 a 6

b i 19 ii 23

c 30

d 10%

4 a 8 **b** 8.5 **c** 5 **d** 8

5 a i mean = 45 ii mode = 41 iii median = 41
iv range = 20

b i mean = 6 ii mode = 2 iii median = 6
iv range = 10

6 a 15

b 111 g, 139 g are most frequent.

c min = 98 g, max = 145 g

d 47 g

5A

Now you try

Example 1

a Categorical and ordinal

b Numerical and discrete

Exercise 5A

1 Check with your teacher.

2 a E **b** F **c** A **d** B
e H **f** D **g** G **h** C

3 a B **b** E **c** C **d** D
e F **f** A

4 C

5 D

6 a Numerical and discrete

b Numerical and discrete

c Categorical and nominal

d Numerical and continuous

e Categorical and ordinal

7 D

8 C

9 D

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 a Carrying out survey at a train station will create a very high proportion of train users in survey's results.
b Survey will reach only those people who use computers.
c Survey will access only people over 18 years of age.

11 Check with your teacher.

12 Check with your teacher.

5B

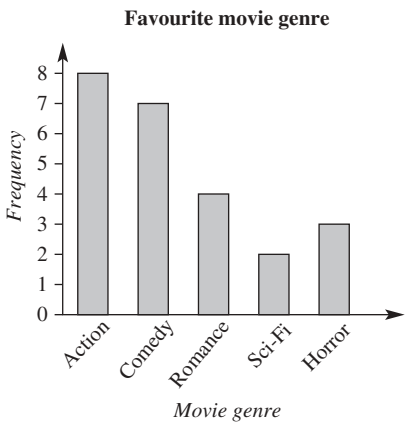
Now you try

Example 2

a

Movie Category	Tally	Frequency
Action	𝍸 |||	8
Comedy	𝍸 ||	7
Romance	||||	4
Sci-Fi	||	2
Horror	|||	3
Total	**24**	**24**

b

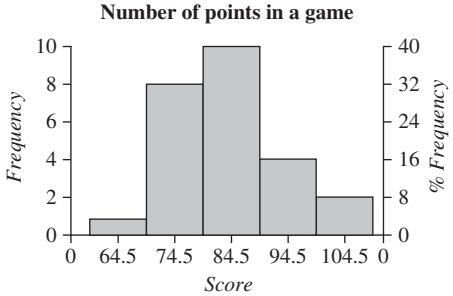


Example 3

a

Class interval	Tally	Frequency	Percentage frequency
60 – 69	|	1	4%
70 – 79	𝍸 |||	8	32%
80 – 89	𝍸 𝍸	10	40%
90 – 99	||||	4	16%
100–109	||	2	8%
Total	25	25	100%

b



c 80–89

d 64%

Exercise 5B

1 a numerical b categorical (nominal)
c categorical (ordinal) d numerical

2 a

Car colour	Tally	Frequency
Red	|||	3
White	𝍸	5
Green	||	2
Silver	||	2
Total	**12**	**12**

b

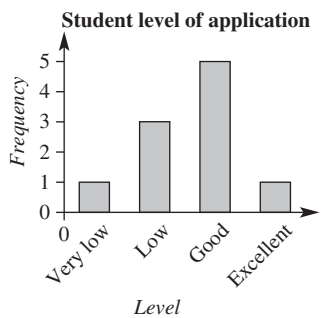
Class interval	Frequency	Percentage frequency
80–84	8	16%
85–89	23	46%
90–94	13	26%
95–100	6	12%
Total	**50**	**100%**

3 a 8
b 25
c i 8 ii 32%

4 a i

Application	Tally	Frequency
Very low	|	1
Low	|||	3
Good	𝍸	5
Excellent	|	1
Total	**10**	**10**

ii

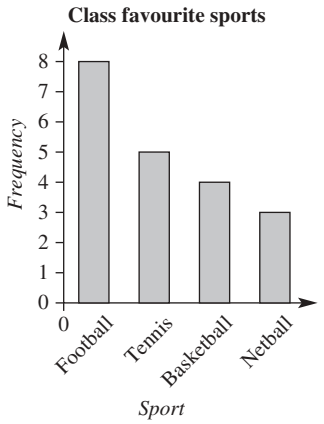


b i

Favourite sport	Tally	Frequency
Football	𝍸 |||	8
Tennis	𝍸	5
Basketball	||||	4
Netball	|||	3
Total	**20**	**20**

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

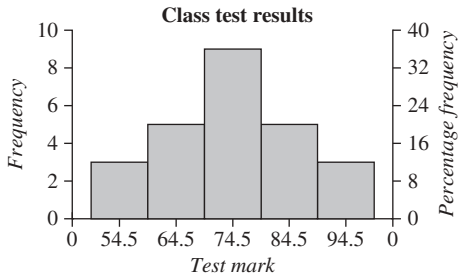
ii



5 a

Class interval	Tally	Frequency	Percentage frequency
50–59	|||	3	12%
60–69	𝍸	5	20%
70–79	𝍸 ||||	9	36%
80–89	𝍸	5	20%
90–99	|||	3	12%
Total	**25**	**25**	**100%**

b



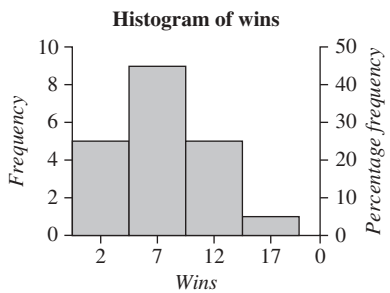
c The 70–79 interval

d 32%

6 a

Class interval	Tally	Frequency	Percentage frequency
0–4	𝍸	5	25%
5–9	𝍸 ||||	9	45%
10–14	𝍸	5	25%
15–19	|	1	5%
Total		**20**	**100%**

b



c The 5–9 interval d 75%

7 a

Type of transport	Frequency	Percentage frequency
Car	16	40%
Train	6	15%
Tram	8	20%
Walking	5	12.5%
Bicycle	2	5%
Bus	3	7.5%
Total	**40**	**100%**

b i 6 ii Car iii 40%
iv 17.5% v 42.5%

8 a Skewed b Symmetrical

9 a

Mass	Frequency	Percentage frequency
10–14	3	6%
15–19	6	12%
20–24	16	32%
25–29	21	42%
30–34	4	8%
Total	**50**	**100%**

b 50 c 32% d 25–29 g.
e 42% f 94%

10 a

Section	Frequency	Percentage frequency
String	21	52.5%
Woodwind	8	20%
Brass	7	17.5%
Percussion	4	10%
Total	**40**	**100%**

b 40 c 52.5% d 47.5% e 9.3%

11 Frequencies of individual scores are lost if the histogram displays only categories of scores.

12 a Russia, ~ 10 years
b Australia, New Zealand, UK, India, Chad
c In all countries, female life expectancy is greater than male life expectancy.
d Extreme poverty, lack of resources and prevalance of disease.

13 a Saturday and Sunday; vendor would expect greater sales at the weekend.
b May have been a particularly warm day or a public holiday.
c i Wednesday; $250 ii Thursday
d The graph does not help us to visualise the profit and loss.

5C

Now you try

Example 4

a 16
b 1

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c 23

d One household had 5 pets but the data were generally skewed towards a lower number of pets.

Example 5

a

Stem	Leaf
1	0 1 4 5 8
2	2 4 4 6 8
3	1 2
4	0 4
5	4
6	3

3 | 1 means 31

b The distribution of the data is skewed.

Example 6

a

Friend 1 Leaf	Stem	Friend 2 Leaf
	1	6 8 9
2	2	2 4 8 9
6 2 0	3	0 2
8 6 4 1	4	1 4
7 4	5	2
6 2	6	

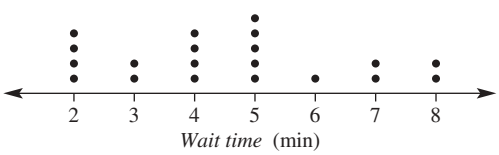
5 | 4 means 54

b Friend 1's spending is symmetrical whereas friend 2's spending is skewed.

Exercise 5C

1 a Histogram **b** Dot plot **c** Column graph
d Stem-and-leaf plot

2



Wait time (min)

3 a 32, 35, 41, 43, 47, 54, 54, 56, 60, 62, 71, 71

b 0.2, 0.3, 0.7, 1.4, 1.4, 1.8, 1.9, 2.3, 2.6, 2.6, 3.0, 3.5

4 a 11 **b** 1 **c** 16

d One family had 3 children but the data were generally symmetrical.

5 a 9

b 39

c She had one bad hole with 7 strokes, but generally the data were consistent, between 3 and 5 strokes.

6 a i

Stem	Leaf
1	5 5
2	0 1 2 4 5 6 6
3	1 7 7 8
4	6
5	2

1 | 5 means 15

ii Skewed

b i

Stem	Leaf
1	2 6
2	1 3 5 7
3	1 2 5 5 6 6 8
4	0 0 2 4 8
5	1 3 5

3 | 2 means 32

ii Symmetrical

c i

Stem	Leaf
11	6 7 8 9
12	1 4 5 7
13	3 5 7
14	5 7 9
15	3 8
16	0 2

13 | 5 means 135

ii Skewed

d i

Stem	Leaf
2	0
3	3 7 8
4	3 4 5 7 8 9
5	2 4 5 8
6	1 3 5
7	0

3 | 7 means 3.7

ii Symmetrical

7 a i

Set 1 leaf	Stem	Set 2 leaf
9 8	3	2 5
8 6 3 2 2 0	4	1 7 7
9 7 3 3	5	2 2 5 6
4 1	6	0 1 3 4 7

5 | 2 means 52

ii Set 1 is symmetrical and set 2 is skewed with more data at the higher end.

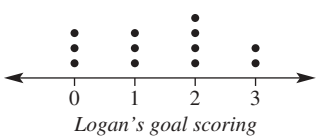
b i

Set 1 leaf	Stem	Set 2 leaf
8 6 4	16	0 3 3 5 7
6 5 2 1	17	0 1 1 4 8
8 7 7 5 1 0	18	2 2 6 7
9 5 2	19	0 4
8 1	20	

19 | 5 means 195

ii Set 1 is symmetrical and set 2 is skewed with most of the data at the lower numbers.

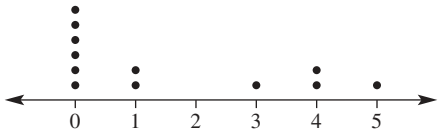
8 a



Logan's goal scoring

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b



Max's goal scoring

c Well spread performance

d Irregular performance, skewed

9 a 4.1 minutes **b** ~ 22.5 minutes

c This would increase the average time.

10 a

Inner city leaf	Stem	Outer suburb leaf
9 6 4 3 1 1	0	3 4 9
9 4 2 0	1	2 8 8 9
4 1	2	1 3 4
	3	4
	4	1

2 | 1 means 21 km

b For the inner city, the data values are closer together and bunched around the lower distances. The outer suburb data values are more spread out.

c In the outer suburbs, students will be travelling farther distances to their school, whereas at inner city schools they are more likely to live close to the school.

11 a $a = 3, b = 9, c = 7$ or 8

b $a = 0$ or $1, b = 0, 1, 2, 3, 4$ or 5

12 a The stem 1 is allocated the leaves 0–4 (included) and 1* is allocated 5–9 (included).

b **i** 1 **ii** 0*

c For city B, for example, most temperatures are in the 20s; splitting into 20–24 and 25–29 allows better analysis of the data and still means that a stem-and-leaf is an appropriate choice of graph.

d City A experienced cooler weather, with temperatures between 8°C and 18°C. City B had warmer weather and a wider range of temperatures, between 17°C and 31°C.

e The cities may have been experiencing different seasons; maybe winter and summer.

5D

Now you try

Example 7

a **i** 6.5 **ii** 10 **iii** 11

b **i** 18 **ii** 22 **iii** 22

Example 8

a 10

b 21

Example 9

a 27

b 39

c 35.7

d 38.5

Exercise 5D

1 a mode **b** mean **c** median **d** bimodal **e** range

2 a 7 **b** 10 and 14

3 a 28 **b** 7 **c** 4

4 a **i** 5.4 **ii** 8 **iii** 6

b **i** 16.25 **ii** 10 **iii** 45

c **i** 70 **ii** 50, 90 **iii** 40

d **i** 25 **ii** no mode **iii** 18

e **i** 2.325 **ii** 1.9 **iii** 1

f **i** 1.6 **ii** no mode **iii** 1.2

5 a 7 **b** 4 **c** 11 **d** 75

e 7 **f** 5 **g** 33.5 **h** 3.15

6 a 7 **b** 6 **c** 7 **d** 6

7 a \$42 **b** \$17.50 **c** \$20.75

d Due to the \$50 value, which is much larger than the other amounts.

8 a **i** 28 **ii** 4 **iii** 17.2 **iv** 17

b **i** 24 **ii** no mode **iii** 110 **iv** 108

c **i** 3.2 **ii** 3.0, 5.3 **iii** 4.6 **iv** 4.9

9 a Mark:

i 83.6 **ii** 85 **iii** 31

Hugh:

i 76.4 **ii** 79 **iii** 20

b Mark's scores varied more greatly, with a higher range, whereas Hugh's results were more consistent. Mark had the higher mean and median though, as he had several high scores.

10 The median, since the mean is affected by the one large value (\$1 700 000).

11 a 4

b 3.7

c **i** The median is unchanged in this case.

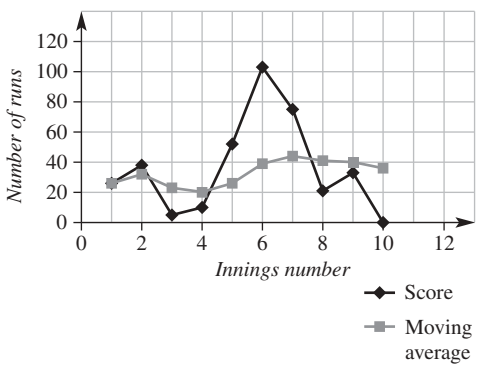
ii The mean is decreased.

12 a

Innings	1	2	3	4	5
Score	26	38	5	10	52
Moving average	26	32	23	20	26

Innings	6	7	8	9	10
Score	103	75	21	33	0
Moving average	39	44	41	40	36

b



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

c **i** The score fluctuates wildly.
ii The graph is fairly constant with small increases and decreases.
d The moving average graph follows the trend of the score graph but the fluctuations are much less significant.

5E

Now you try

Example 10

a $Q_1 = 10, Q_3 = 28$
b 18

Example 11

a $Q_1 = 14.4, Q_3 = 20.6$
b 6.2

Example 12

a **i** Min $= 2$, max $= 14$
ii 5.5
iii $Q_1 = 4$ $Q_3 = 7$
iv 3
b 14 is an outlier.
c Matches went late on a particular day.
d Range $= 7$
IQR $= 3$
Outlier affects the range significantly but does not overly affect the IQR.

Exercise 5E

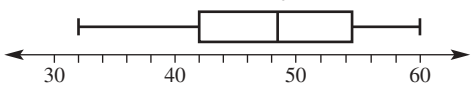
1 **a** Min, lower quartile (Q_1), median (Q_2), upper quartile (Q_3), max
b Range is max − min; IQR is $Q_3 - Q_1$. Range is the spread of all the data, IQR is the spread of the middle 50% of data.
c An outlier is a data point (element) outside the vicinity of the rest of the data.
2 **a** 1.5
b **i** 1 **ii** 2
3 **a** 5 **b** −4.5 and 15.5 **c** An outlier
4 **a** **i** $Q_1 = 4, Q_3 = 8$ **ii** 4
b **i** $Q_1 = 11, Q_3 = 18$ **ii** 7
c **i** $Q_1 = 51, Q_3 = 62$ **ii** 11
d **i** $Q_1 = 1.8, Q_3 = 2.7$ **ii** 0.9
5 **a** **i** $Q_1 = 2, Q_3 = 11$ **ii** 9
b **i** $Q_1 = 2, Q_3 = 10$ **ii** 8
c **i** $Q_1 = 1.0, Q_3 = 1.6$ **ii** 0.6
d **i** $Q_1 = 10.5, Q_3 = 22.5$ **ii** 12
6 **a** **i** Min $= 0$, max $= 17$ **ii** Median $= 13$
iii $Q_1 = 10, Q_3 = 15$ **iv** IQR $= 5$
b 0 is an outlier.
c Road may have been closed that day.
d Range $= 9$
IQR $= 5$
Range is reduced but IQR is unaffected.
7 **a** **i** Min $= 4$, max $= 14$ **ii** 7.5
iii $Q_1 = 5, Q_3 = 9$ **iv** IQR $= 4$
v No outliers
b **i** Min $= 16$, max $= 31$ **ii** 25
iii $Q_1 = 21, Q_3 = 27$ **iv** IQR $= 6$
v No outliers
c **i** Min $= 2$, max $= 24$ **ii** 12
iii $Q_1 = 10, Q_3 = 13$ **iv** IQR $= 3$
v 24 and 2
d **i** Min $= 1$, max $= 17$ **ii** 7
iii $Q_1 = 4, Q_3 = 10$ **iv** IQR $= 6$
v No outliers
8 **a** **i** Min $= 25$, max $= 128$ **ii** 47
iii $Q_1 = 38, Q_3 = 52.5$ **iv** IQR $= 14.5$
v Yes, 128 **vi** 51.25
b A more advanced calculator was used.
c Median, as it is not affected dramatically by the outlier.
d The IQR, as it is not affected dramatically by the outlier.
9 2 bags; bags that weigh 32 kg and 33 kg will be weighed again
10 2 fridges; \$350 and \$1700 are outliers
11 **a** IQR $= 10$
b Yes, 1 is an outlier.
c No
12 **a** 35 or more
b 35–49 inclusive
13 Check with your teacher

5F

Now you try

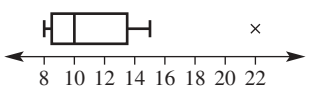
Example 13

a Min $= 32$, $Q_1 = 42$, $Q_2 = 48.5$, $Q_3 = 54.5$, max $= 60$
b



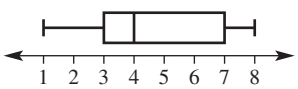
Example 14

a $Q_1 = 8.5$, $Q_2 = 10$, $Q_3 = 13.5$
b 22 is an outlier.
c

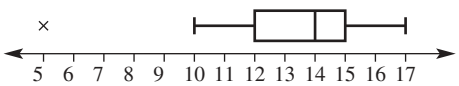


Exercise 5F

1 **a** Minimum **b** Q_1 **c** Q_2(median) **d** Q_3
e Maximum **f** Whisker **g** Box
2 **a** 15 **b** 5 **c** 25 **d** 20
e 10 **f** 20 **g** 10
3 **a**



b

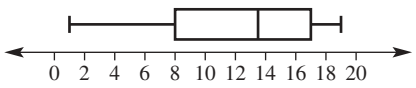


CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 **a** Q_3 **b** Q_1, Q_3 **c** minimum, Q_1
d Q_3, maximum

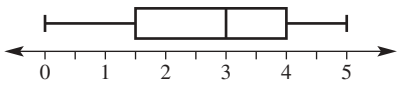
5 **a** **i** Min = 1, Q_1 = 8, Q_2 = 13.5, Q_3 = 17, max = 19

ii



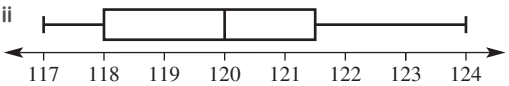
b **i** Min = 0, Q_1 = 1.5, Q_2 = 3, Q_3 = 4, max = 5

ii



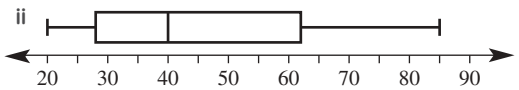
c **i** Min = 117, Q_1 = 118, Q_2 = 120, Q_3 = 121.5, max = 124

ii



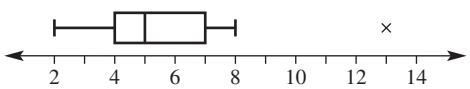
d **i** Min = 20, Q_1 = 28, Q_2 = 40, Q_3 = 62, max = 85

ii



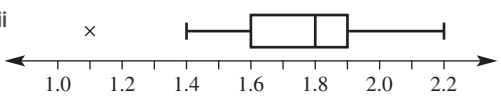
6 **a** **i** Q_1 = 4, Q_2 = 5, Q_3 = 7 **ii** Outlier is 13.

iii



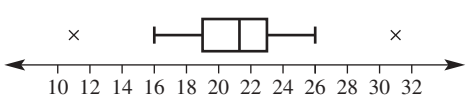
b **i** Q_1 = 1.6, Q_2 = 1.8, Q_3 = 1.9 **ii** Outlier is 1.1.

iii



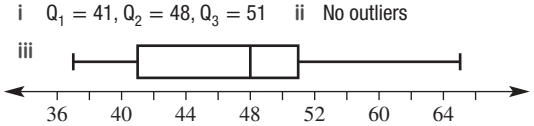
c **i** Q_1 = 19, Q_2 = 21.5, Q_3 = 23 **ii** Outliers are 11 and 31.

iii



d **i** Q_1 = 41, Q_2 = 48, Q_3 = 51 **ii** No outliers

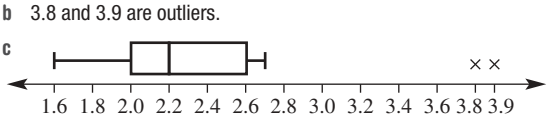
iii



7 **a** **i** 1.6 **ii** 2.0 **iii** 2.2 **iv** 2.6
v 3.9 **vi** 0.6

b 3.8 and 3.9 are outliers.

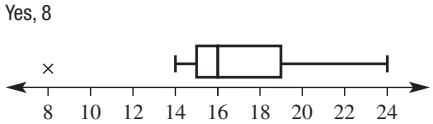
c



8 **a** **i** 8 **ii** 15 **iii** 16 **iv** 19
v 24 **vi** 4

b Yes, 8

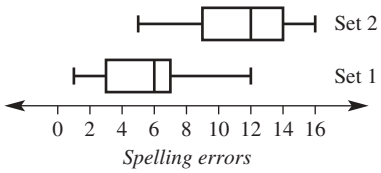
c



9 **a** Same minimum of 1.
b B
c **i** 5 **ii** 10
d Data points for B are more evenly spread than those for A.

10 **a** They have the same median and upper quartile.
b B
c **i** 4 **ii** 5
d Dataset B is more spread out.

11 **a**



b Yes, median number of errors found by examiner 2 was 12, median number of errors found by examiner 1 was 6.

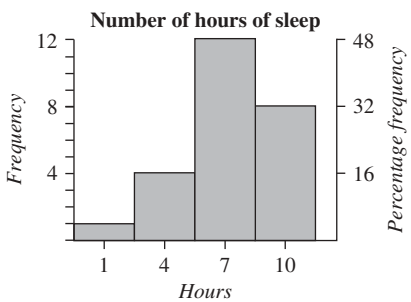
Progress quiz

1 **a** Categorical and nominal **b** Numerical and discrete

2 **a**

Class Interval	Tally	Frequency	Percentage frequency
0–2	|	1	4%
3–5	||||	4	16%
6–8	𝍸 𝍸 ||	12	48%
9–11	𝍸 |||	8	32%
Total	25	25	100%

b



c 20% of those surveyed had fewer than 6 hours of sleep.
d The 6–8 interval is the most frequent.

3 **a** 20 **b** 5 days **c** 4.5 days **d** 4.15 days
e 7

4 **a**

Stem	Leaf
0	3 7
1	1 5 6
2	1 2 4 5
3	0 0 3 4
4	0 2 5 6 7 8 9

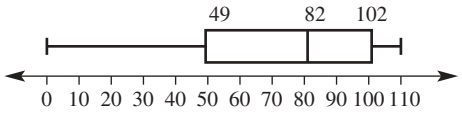
4 | 2 means 42

b Data is skewed.
c **i** 30 **ii** 30

5 **a** 19.375 **b** 17 **c** 39

6 **a** Q_1 = 10.5, Q_3 = 19 **b** 8.5

7 **a** Q_1 = 49, Q_2 = 82 and Q_3 = 102
b There are no outliers.
c



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5G

Now you try

Example 15

a Olivia's, because many shoppers are much older than school students.

b Olivia's, because high school students are all aged 12 to 18 but shoppers can be aged from 0 to over 100 years old, which is a much greater spread of ages.

Exercise 5G

1 **a** larger **b** smaller

2 **a** B **b** A

3 **a** A

b The data values in A are spread farther from the mean than the data values in B.

4 **a** Gum Heights **b** Gum Heights

5 **a** mean = 6, $\sigma = 2$ **b** mean = 3.6, $\sigma = 2.3$

c mean = 8, $\sigma = 3.5$ **d** mean = 32.5, $\sigma = 3.3$

6 **a** mean = 2.7, $\sigma = 0.9$ **b** mean = 14.5, $\sigma = 6.0$

7 **a** The outer suburb has more data values in the higher range.

b There is less spread. Data values are closer to the mean.

c Students at outer-suburb schools may live further from school than inner-city students.

8 **a** false **b** true **c** true

9 **a** mean = 2, $\sigma = 0.9$ **b** mean = 5.25, $\sigma = 0.7$

10 **a** no **b** no **c** yes

d Yes, the standard deviation would be greater since the data values are spread out further from the mean.

11 **a** **i** 85.16 **ii** 53.16 **iii** 101.16 **iv** 37.16

v 117.16 **vi** 21.16

b **i** 66% **ii** 96% **iii** 100%

c **i** Student's own research required.

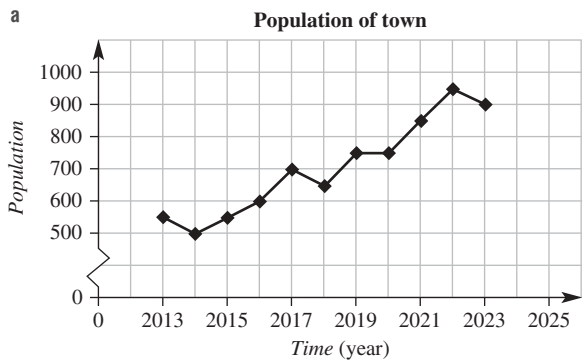
ii One SD from the mean = 68%
Two SDs from the mean = 95%
Three SDs from the mean = 99.7%
Close to answers found.

5H

Now you try

Example 16

a



b Generally linear in a positive direction.

Exercise 5H

1 **a** Linear

b No trend

c Non-linear

d Linear

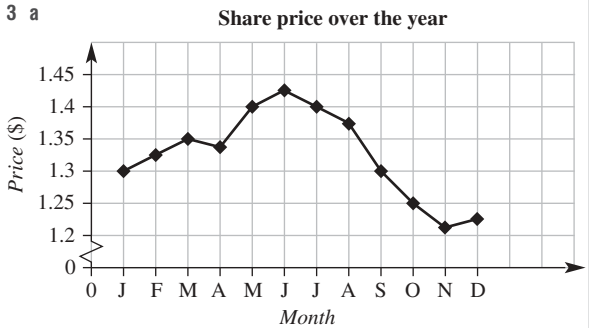
2 **a** **i** 20°C **ii** 30°C

b 36°C

c **i** Noon to 1 p.m. **ii** 3 to 4 p.m.

d Temperature is increasing from 8 a.m. to 3 p.m. in a generally linear way. At 3 p.m. the temperature starts to drop.

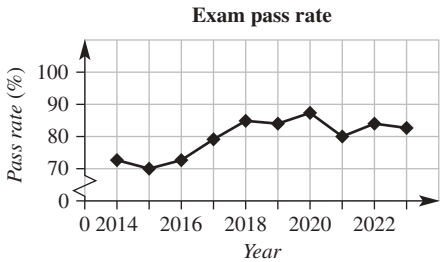
3 **a**



b The share price generally increased until it peaked in June and then continually decreased to a yearly low in November before trending upwards again in the final month.

c $0.21

4 **a**



b The pass rate for the examination has increased marginally over the 10 years, with a peak in 2020.

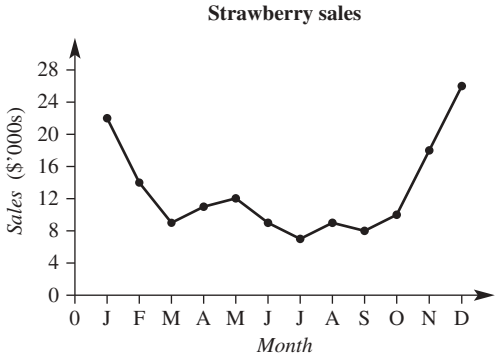
c 2020

d 9%

5 **a** Linear

b **i** $650 000 **ii** $750 000

6 **a**



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

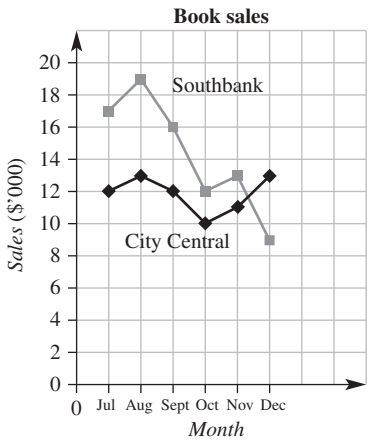
b The sales start high and decrease during the middle of the year, before increasing again towards the end of the year.

c Strawberries are in season in the warmer months but not in the cooler winter months.

7 a i \$6000 **ii** \$4000

b 1

c



d i The sales trend for City Central for the 6 months is fairly constant.

ii Sales for Southbank peaked in August before taking a downturn.

e About \$5000

8 a i 5.8 km **ii** 1.7 km

b i Blue Crest slowly gets closer to the machine.

ii Green Tail starts near the machine and gets farther from it.

c 8:30 p.m.

9 a Increases continually, rising more rapidly as the years progress.

b Compound interest – exponential growth

10 a Graphs may vary but it should decrease from room temperature to the temperature of the fridge.

b No. Drink cannot cool to a temperature *lower* than that of the internal environment of the fridge.

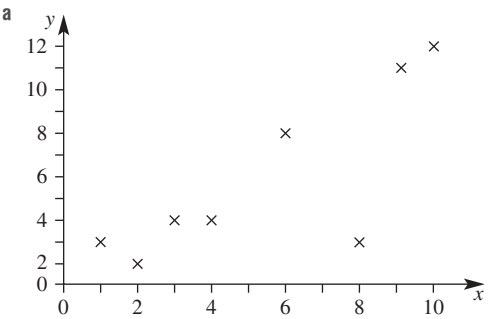
c Check with your teacher

5I

Now you try

Example 17

a



b Positive correlation

c Strong correlation

d Outlier is (8, 3)

Exercise 5I

1 a Likely **b** Likely **c** Unlikely

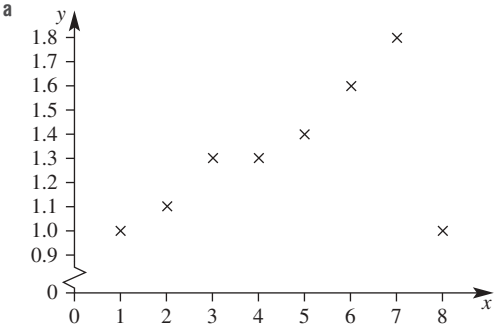
d Unlikely **e** Likely

2 a increases **b** decreases

3 a Weak negative **b** Strong positive

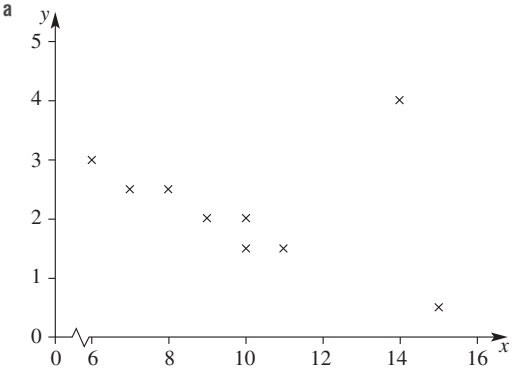
c Strong negative

4 a



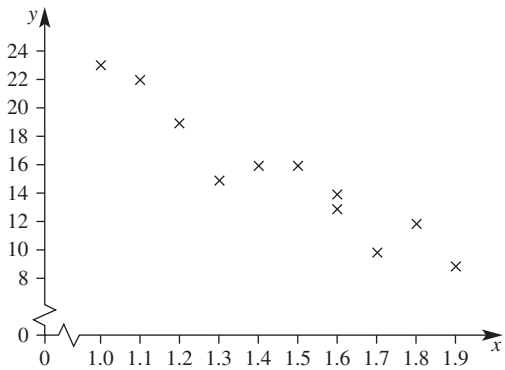
b Positive **c** Strong **d** (8, 1.0)

5 a



b Negative **c** Strong **d** (14, 4)

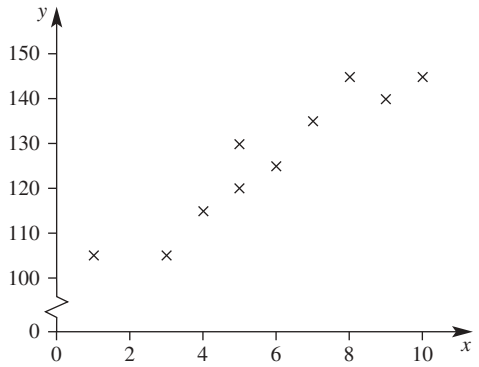
6 a Negative



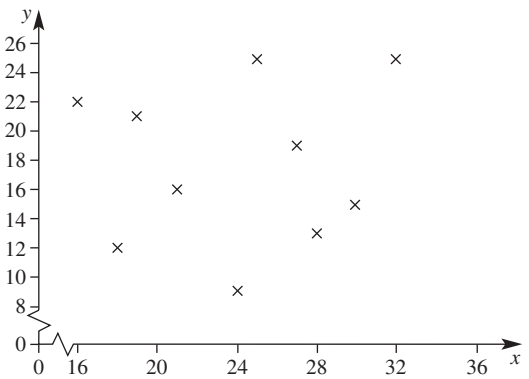
5I

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

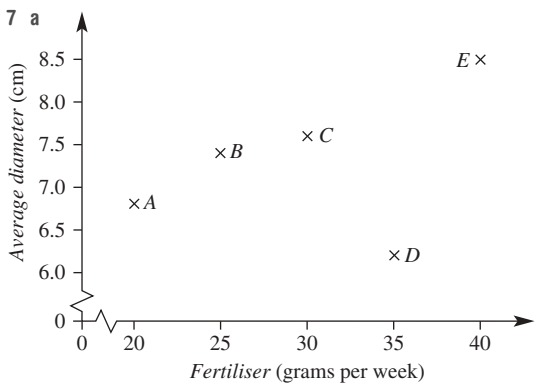
b Positive



c None



7 a



b D

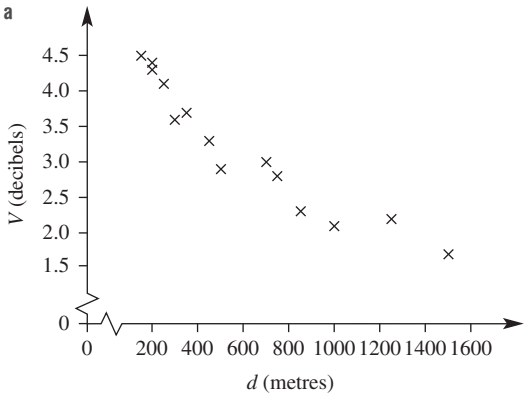
c Yes, although weak correlation and small sample size does lead to doubt.

8 a Yes

b Decrease

c **i** Yes **ii** Car H

9 a

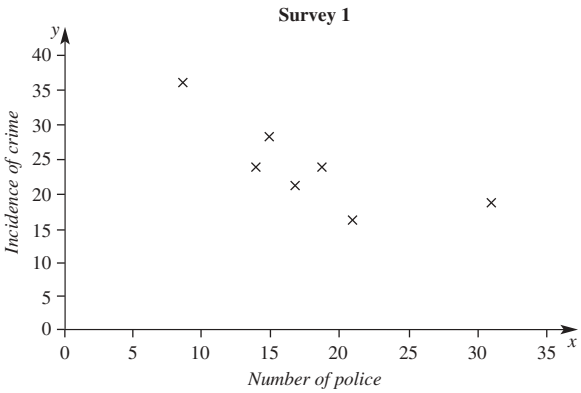


b Negative

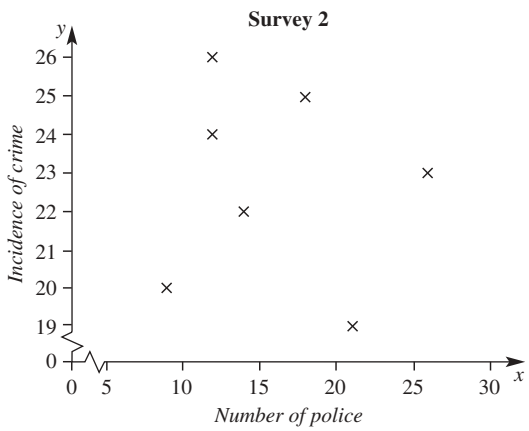
c As distance increases, volume decreases.

10 Each axis needs a better scale. All data are between 6 and 8 hours sleep and shows only a minimum change in exam marks. Also, there are only 4 observations presented on the plot, which is not enough to form strong conclusions.

11 a **i** Weak negative correlation



ii No correlation



b Survey 1, as this shows an increase in the number of police has seen a decrease in the incidence of crime.

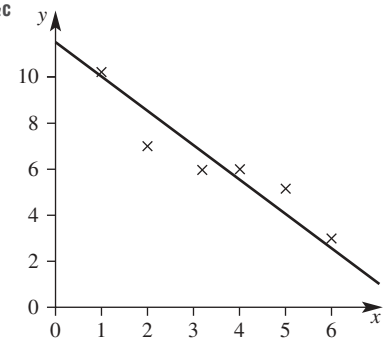
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5J

Now you try

Example 18

a&c



b Negative correlation

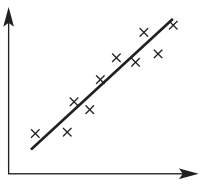
d i $y \approx 4.8$ ii $y \approx 11.5$ iii $x \approx 2.8$ iv $x \approx 7.5$

Exercise 5J

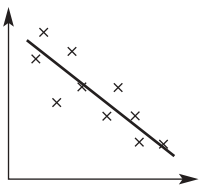
1 a When the data appear to fit on or near a straight line, it shows a definite linear trend.

b Balance points evenly either side of the line; ignore outliers when taking distance from the line into account.

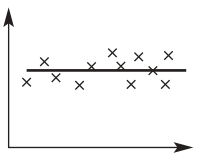
2 a



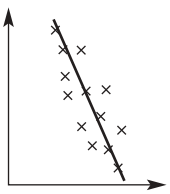
b



c

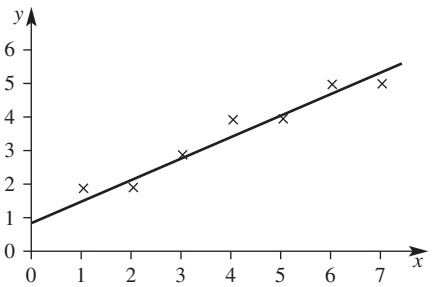


d



3 a interpolation **b** extrapolation

4 a



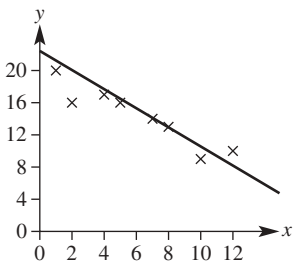
b Positive correlation

c As above

d All answers are approximate.

i 3.2 **ii** 0.9 **iii** 1.8 **iv** 7.4

5 a



b Negative

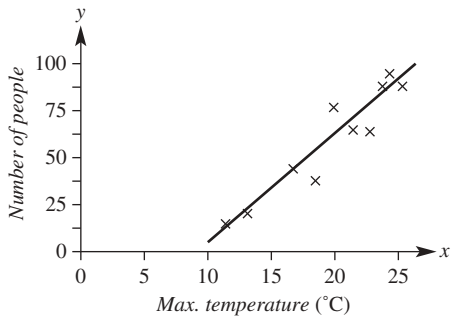
c As above

d i ≈ 13.5 ii ≈ 23 iii ≈ 9 iv ≈ 7

6 a ≈ 4.5 **b** ≈ 6 **c** ≈ 0.5 **d** ≈ 50

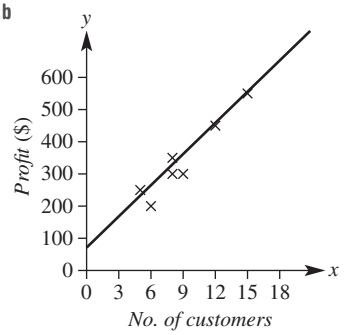
7 a Increases

b



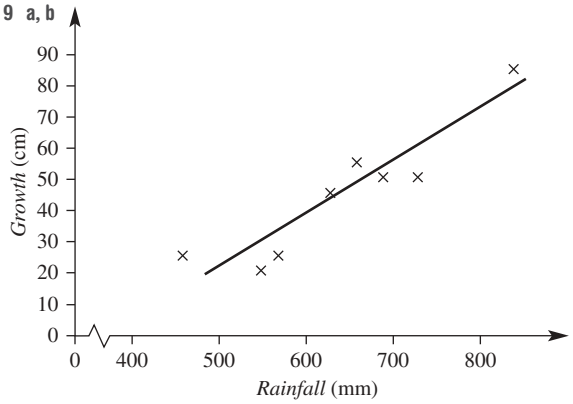
c i ≈ 65 ii $\approx 14°C$

8 a, b



c $600 **d** 2

9 a, b

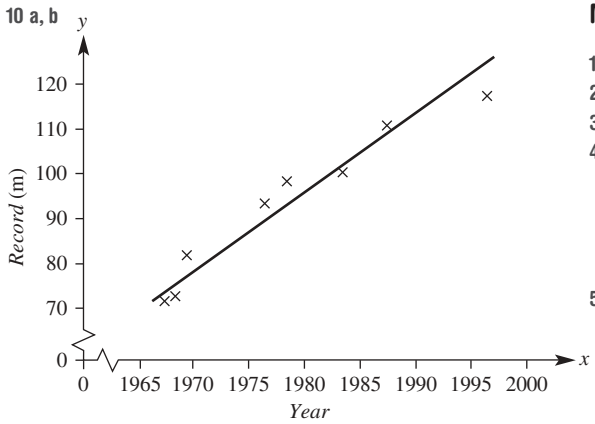


c i ≈ 25 cm ii ≈ 85 cm

d i ≈ 520 mm ii ≈ 720 mm

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

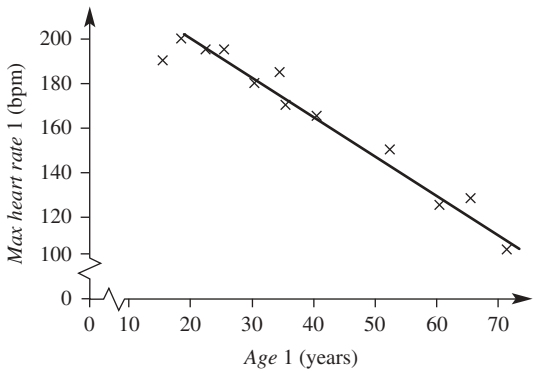
10 a, b



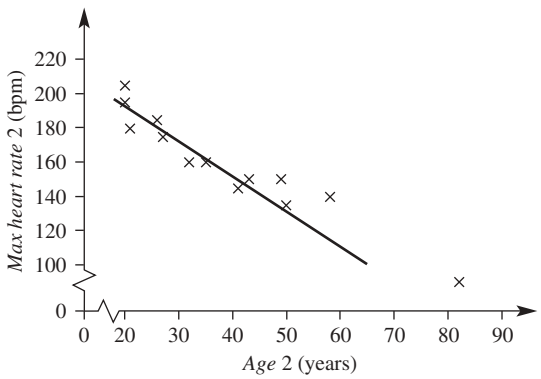
c **i** ≈ 130 m **ii** ≈ 150 m

d No, records are not likely to continue to increase at this rate.

11 a Experiment 1



Experiment 2



b **i** ≈ 140 **ii** ≈ 125

c **i** ≈ 25 **ii** ≈ 22

d Experiment 2

e Check with your teacher

Maths@Work: Project manager on a building site

1 **a** 8.4 **b** 8 **c** 7, 8, 8, 9, 10

2 **a** 7.5 **b** 7.5 **c** 5, 7, 7.5, 8, 9

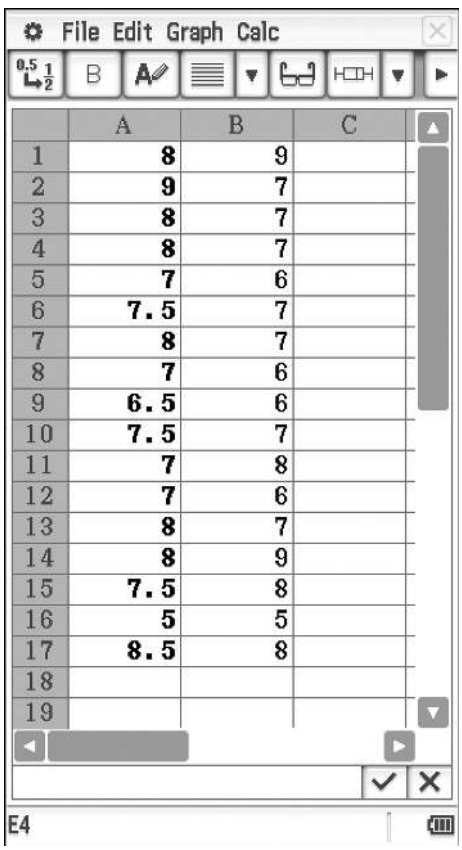
3 *See figure at base of page.*

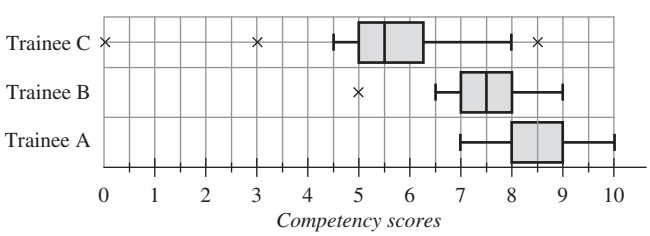
4 Trainee A is the most competent: they have the highest mean and median scores and are the most consistent, with a small range and IQR. Trainee C is the least competent.

a 8.4, 7.5, 5.3 **b** 8, 7.5, 5.5 **c** 7, 5, 0 **d** 10, 9, 8.5

e 1, 1, 1.25 **f** 3, 4, 8.5

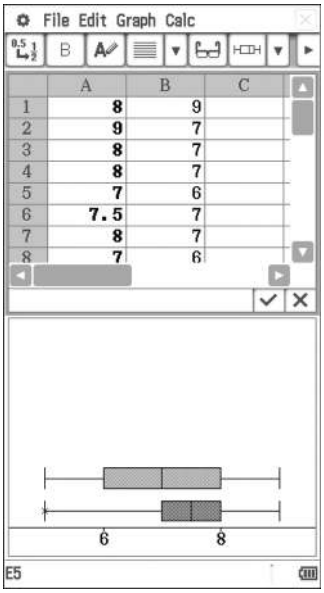
5 *Note that a Class Pad II is used for the CAS screenshots below.* Listing the data:





ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Drawing the parallel box plots:



a Trainee B: 5, 7, 7.5, 8, 9
Trainee D: 5, 6, 7, 8, 9

b Trainee B IQR = 1; trainee D IQR = 2
Trainee B has more consistent results as his/her IQR shows that the middle 50% of scores only vary by 1 unit but trainee D's middle 50% of scores vary by 2 units.

Puzzles and challenges

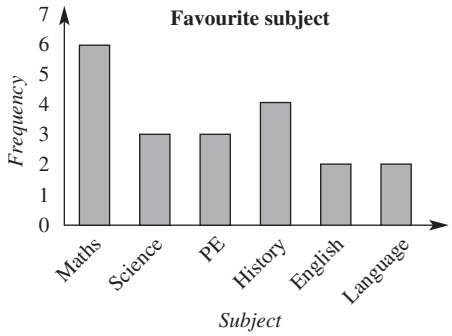
1 66 kg
2 88%
3 8
4 **a** Larger by 3 **b** Larger by 3 **c** No change
5 1.1
6 3, 3, 9, 11
7 19
8 **a** Mean ↑ by 10; median ↑ by 10; mode ↑ by 10
b Mean is × by 10; median is × by 10; mode is × by 10
9 494 mm

Chapter checklist with success criteria

1 Categorical and Nominal, Numerical and Discrete

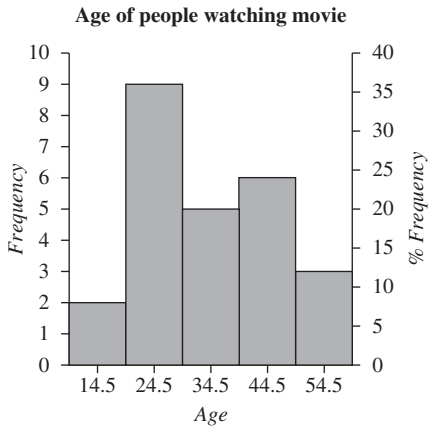
2

Subject	Tally	Frequency
Maths	𝍸 |	6
Science	|||	3
PE	|||	3
History	||||	4
English	||	2
Language	||	2
Total	**20**	**20**



3

Class interval	Frequency	Percentage frequency
10–19	2	8%
20–29	9	36%
30–39	5	20%
40–49	6	24%
50–59	3	12%
Total	**25**	**100%**



56%

4 **a** 12
b 5, 42
c the distribution of wins is skewed towards a higher number of goals

5

Store 1 Leaf	Stem	Store 2 Leaf
8	2	2 4 7 9
6 2 1	3	2 4 5 8
7 5 4 2	4	3
6 2	5	2

5 | 2 means 52
Store 1 is skewed towards a higher number of sales while store 2 is skewed towards a lower number of sales.

6 **a** 7 **b** 9 **c** 9
7 **a** 15 **b** 15
8 30, 30, 25.5, 25.9
9 **a** $Q_3 = 19.25, Q_1 = 11, IQR = 8.25$
b $Q_3 = 14.1, Q_1 = 10.1, IQR = 4$

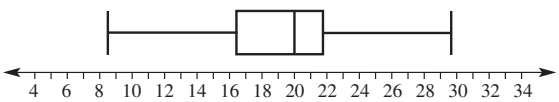
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

10 a Min = \$457, Q_1 = \$593.75, Q_2 = \$683, Q_3 = \$867.75, Max = \$1599

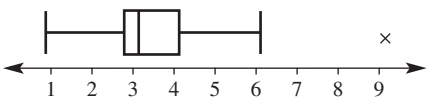
b IQR = \$274

c \$1599 is an outlier, this may be a luxury brand.

11 a Min = 15, Q_1 = 16.5, Q_2 = 20, Q_3 = 21.75, Max = 25, there are no outliers

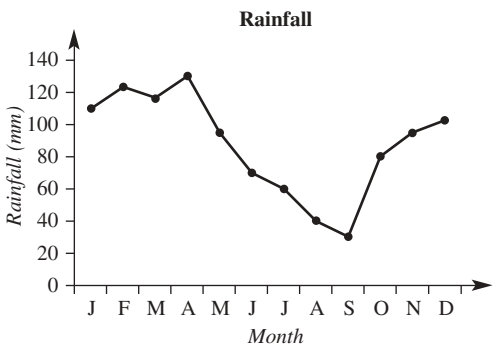


b Min = 1.8, Q_1 = 2.8, Q_2 = 3.1, Q_3 = 4.1, Max = 9.2, 9.2 is the only outlier

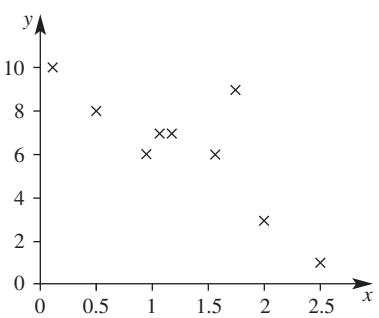


12 0.7

13

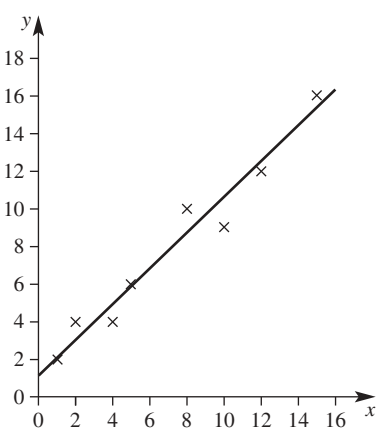


14



(1.8, 9) is an outlier and there is a strong negative correlation.

15



a 7 **b** 18

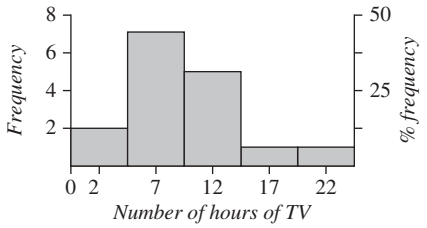
Chapter 5 review

Short-answer questions

1 a

Class interval	Tally	Frequency	Percentage frequency
0–4	||	2	12.5%
5–9	𝍸 ||	7	43.75%
10–14	𝍸	5	31.25%
15–19	|	1	6.25%
20–24	|	1	6.25%
Total		**16**	**100%**

b



c Skewed

2 a

Stem	Leaf
1	5 9
2	0 4 8 9
3	2 4 7 8 8
4	2 9
5	0

3 | 2 means 32

b The dataset is symmetrical about scores in the 30s.

3 a i 5 ii 6 iii 5

b i 30.5 ii 57 iii 20

c i 1.6 ii 1.2 iii 1.5

4 a 43.2 years

b 38 years

c The mean is affected by the high ages 76 and 87.

5 a Q_1 = 8, Q_2 = 11, Q_3 = 15

b Q_1 = 8, Q_2 = 13, Q_3 = 15.5

6 a Range = 19, IQR = 5.5

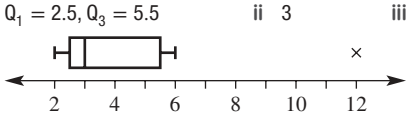
b Outlier = 21

c Range = 8, IQR = 6

Range is significantly reduced, the effect on the IQR is smaller.

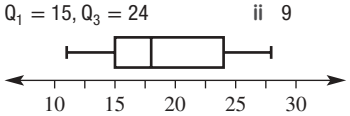
7 a i Q_1 = 2.5, Q_3 = 5.5 ii 3 iii 12

iv



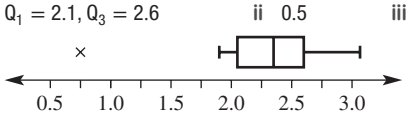
b i Q_1 = 15, Q_3 = 24 ii 9 iii none

iv



c i Q_1 = 2.1, Q_3 = 2.6 ii 0.5 iii 0.7

iv



8 a False **b** True **c** True **d** True

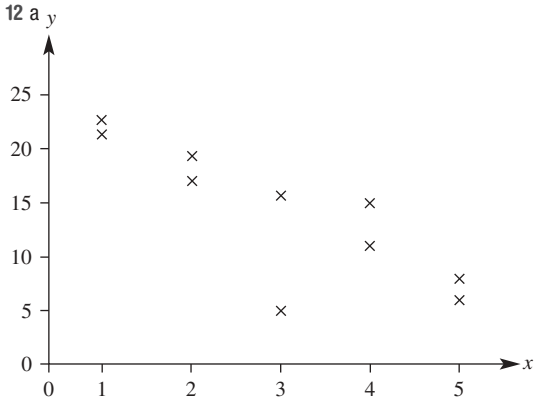
9 a Mean = 7, σ = 2.3 **b** Mean = 4, σ = 2.8

10 a Linear **b** 10 a.m.–11 a.m.

11 a Negative **b** None **c** Positive

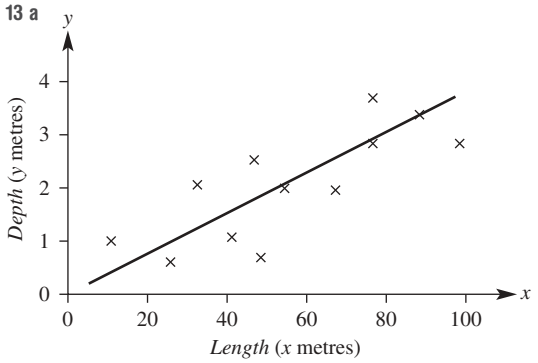
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

12 a



b Negative **c** Strong **d** (3, 5)

13 a



b ≈ 1.8 m

Multiple-choice questions

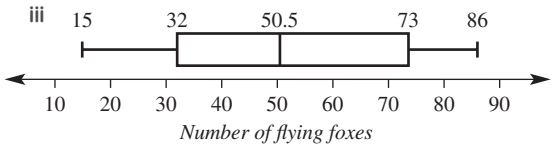
1 D	**2** C	**3** B	**4** A
5 C	**6** A	**7** E	**8** C
9 D	**10** B		

Extended-response questions

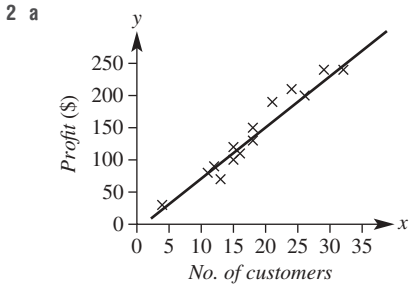
1 i 41

ii No outliers

iii



2 a



b i ≈ $80 **ii** ≈ $150 **iii** ≈ $240

c i ≈ 7 **ii** ≈ 16 **iii** ≈ 27

Semester review 1

Chapter 1: Financial mathematics

Short-answer questions

1 $170.40

2 a $82.80 **b** $110.40 **c** $2097.60 **d** $2346

3 $2128

4 $1550

5 $102

6 $20 562

7 a $37 180 **b** $12 833.60 **c** $22 829.56 **d** $439.03

8 $8837.34

9 $23.23

Multiple-choice questions

1 C **2** A **3** C **4** D **5** A

Extended-response question

a $711.55

b $59.92

c i $149.80 **ii** $832 **iii** $83 **iv** 11.08%

Chapter 2: Measurement

Short-answer questions

1 a 43 cm **b** 320 cm^2 **c** 30 000 cm^3

2 a 8 cm **b** 44 m **c** 9 m

3 a i 37.70 cm **ii** 113.10 cm^2

b i 14.28 cm **ii** 12.28 cm^2

c i 11.14 m **ii** 7.14 m^2

4 a 10.5 m^2 **b** 112 cm^2 **c** 8 m^2

5 a i 45 cm^3 **ii** 78 m^2

b i 30 m^3 **ii** 72 m^2

c i 942.48 mm^3 **ii** 534.07 mm^2

Multiple-choice questions

1 C **2** E **3** B **4** A **5** D

Extended-response question

a 7.85 m^3 **b** 31.42 m^2 **c** $1130

Chapter 3: Algebraic expressions

Short-answer questions

1 a $7xy + 4x$ **b** $-21ab$ **c** $\frac{a}{2}$

2 a i $-4x + 12$ **ii** $15x^2 + 6x$ **iii** $13x - 6$

b i $6(3 - b)$ **ii** $3x(x + 2)$ **iii** $-4y(2x + 3)$

3 a $x + 3$ **b** $\frac{3}{4}$ **c** $\frac{9x}{10}$ **d** $\frac{2x - 3}{8}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

4 **a** $10x^6$ **b** $\frac{4x^2}{y^3}$ **c** $8m^{12}$ **d** 4

e $\frac{9a^2}{b^8}$ **f** $\frac{3b^2}{a^5}$ **g** $4t^5$ **h** $\frac{2y^2}{x^4}$

5 **a** **i** 473 000 **ii** 0.00521

b **i** 2.76×10^{-5} **ii** 8.71×10^6

Multiple-choice questions

1 C **2** D **3** A **4** D **5** B

Extended-response question

a **i** 3 **ii** 7 **iii** 15 **iv** 31

b Every sum is one less than a power of 2; e.g. the sum of the first five terms is $31 = 2^5 - 1$, so the sum of the first 100 terms is $2^{100} - 1$

c **i** 3 **ii** 6 **iii** 12 **iv** 24

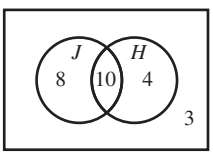
d Every sum is the product of 3 and a power of 2; e.g. the sum of the 4th and 5th terms is $24 = 3 \times 2^3$, so the sum of the 100th and 101st term is 3×2^{99}

Chapter 4: Probability

Short-answer questions

1 **a** $\frac{2}{5}$ **b** $\frac{11}{20}$ **c** $\frac{17}{20}$ **d** $\frac{9}{10}$

2 **a**



b

	H	not *H*	
J	10	8	18
not *J*	4	3	7
	14	11	25

c 3

d **i** $\frac{4}{25}$ **ii** $\frac{22}{25}$ **iii** $\frac{2}{5}$

3 **a** 36

b **i** $\frac{1}{9}$ **ii** $\frac{1}{6}$ **iii** $\frac{7}{12}$ **iv** $\frac{1}{6}$

4 **a** $\frac{1}{15}$ **b** $\frac{8}{15}$ **c** $\frac{14}{15}$

Multiple-choice questions

1 C **2** E **3** A **4** D

Extended-response question

a

	T	A
T	(T, T)	(A, T)
A	(T, A)	(A, A)

b **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$

c $\frac{1}{4}$

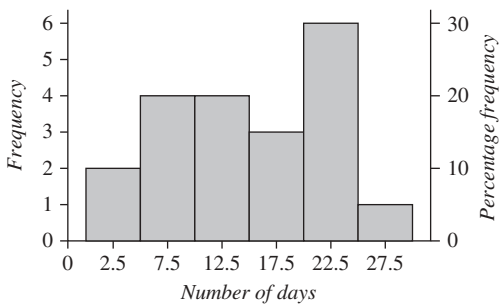
Chapter 5: Single variable and bivariate statistics

Short-answer questions

1 **a**

Class interval	Frequency	Percentage frequency
0–4	2	10%
5–9	4	20%
10–14	4	20%
15–19	3	15%
20–24	6	30%
25–29	1	5%
Total	**20**	**100%**

b



c **i** 14

ii 50%

iii 20–24 days, those that maybe catch public transport to work or school each week day.

2 **a**

Stem	Leaf
0	4 7 8 9
1	2 5 5 7 8
2	4 4 6
3	2 6
4	1

3 | 6 means 36

b Skewed

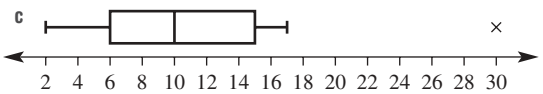
3 **a** 2 4 6 7 8 10 12 12 15 17 30

b **i** Min = 2, max = 30 **ii** 10

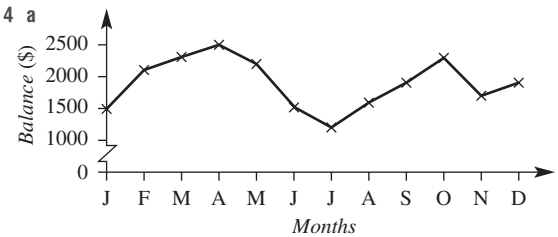
iii $Q_1 = 6, Q_3 = 15$ **iv** 9

v Yes, 30

c



4 **a**



b Balance fluctuated throughout the year but ended up with more money after 12 months.

c From May to June

d Increase of $500.

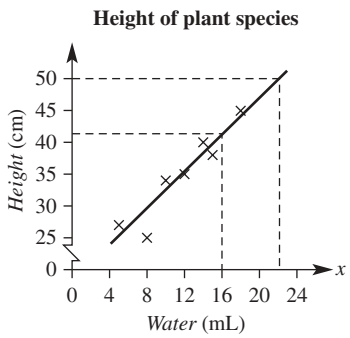
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Multiple-choice questions

1 A 2 C 3 E 4 B 5 C

Extended-response question

a, c



b Positive

d i 41 cm **ii** 22 mL

Chapter 6

Warm-up quiz

1 a (3, 5) **b** (4, −2) **c** (−4, −4) **d** (−3, 1)
e (2, −2) **f** (2, 0)

2 a G **b** D **c** B **d** S
e N **f** Q

3 a Square **b** Isosceles triangle **c** Hexagon

4 a 120 min **b** 200 km **c** 100 km/h

5 a 3, 4, 5, 6 **b** −2, −1, 0, 1
c 0, 2, 4 **d** 6, 5, 4

6 a Positive **b** Zero **c** Undefined **d** Negative

6A

Now you try

Example 1
a 240 km **b** 3 hours **c** 360 km

Example 2
a 50 **b** 20 **c** 30

Example 3
a 40 cm **b** ≈ 16 cm **c** ≈ 3.75 months

Example 4
a $500 **b** $1000 **c** 3 years

Exercise 6A

1 Yes

2 2

3 a 360 km **b** 4 h **c** 540 km

4 a i $10 000 **ii** $40 000
b Increased
c $30 000

5 a 300 deer **b** 100 deer **c** 200 deer

6 a i 50 cm **ii** 45 kg
b i 10 **ii** 3

7 a 80 cm **b** 40 cm **c** Approx. $2\frac{1}{4}$ months

8 a 400 m **b** Approx. 250 m
c Approx. $3\frac{1}{2}$ days

9 a $10 000 **b** $0 **c** 12 years

10 a 200 g **b** 200 g **c** $2\frac{1}{4}$ h (2 h 15 min)

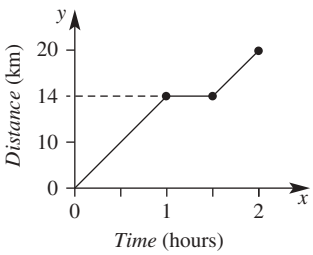
11 a $\frac{1}{2}$ h (30 min)
b i 1 p.m. **ii** 1:15 p.m. **iii** Approx. 1:08 p.m.
iv 1:30 p.m.
c i −120 m **ii** −80 m
d i 0 m **ii** −160 m **iii** −280 m
e i 12:45 p.m. **ii** 2 p.m. **iii** 2:45 p.m.

6B

Now you try

Example 5
a 3 km **b** 20 minutes **c** 2.5 km

Example 6



Exercise 6B

1 A, C, E – moving at a constant rate; B, D – at rest

2 a S **b** P **c** Q **d** R
e T **f** S

3 a 20 km **b** 2 h **c** approx. 17.5 km

4 a i 40 kg **ii** 50 kg **iii** 80 kg
b $\frac{1}{2}$ h
c 1st hour

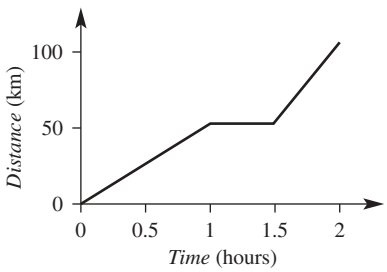
5 a 200 m
b 80 s
c i Approx. 31 m **ii** 62 m **iii** 150 m

6 a i 3 m **ii** 2 m **iii** 2 m
b The lawn
c i 1.5 m **ii** 4 m **iii** 6.5 m

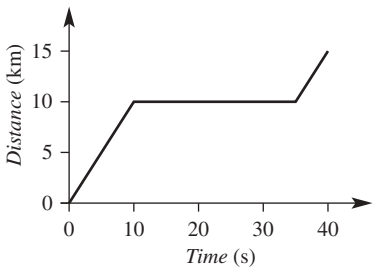
7 a 10 km **b** 20 km **c** 27 min **d** 9 min

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8

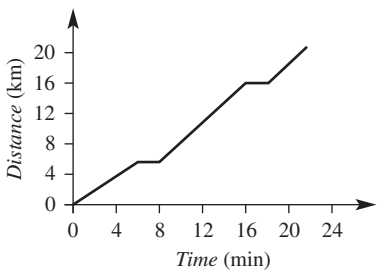


9

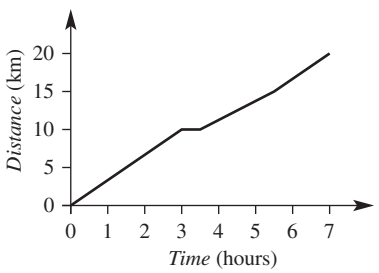


10 a 20 km **b** 22 min

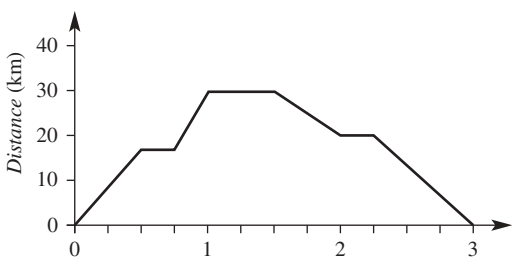
c



11



12 a



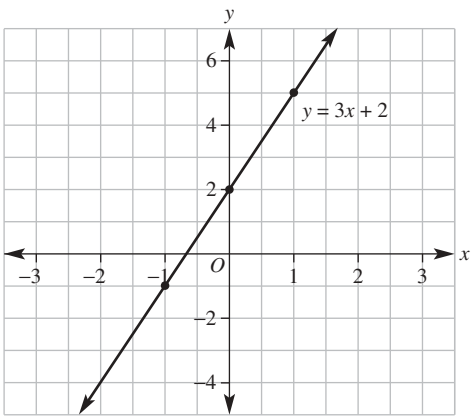
b 48 km/h **c** 20 km/h

6C

Now you try

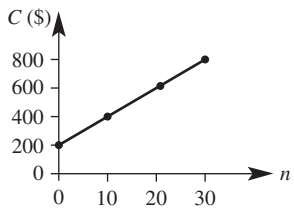
Example 7

x	−1	0	1
y	−1	2	5



Example 8

a



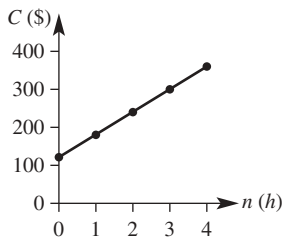
b i $700 **ii** 15

Example 9

a

No. of hours (n)	0	1	2	3	4
Cost (C)	120	180	240	300	360

b



c i $270 **ii** 3.5 hours

Exercise 6C

1 a $A(2, 1)$ $B(-3, 3)$ $C(2, -3)$ $D(-4, 0)$ $E(4, 4)$ $F(0, -2)$ $G(3, 0)$ $H(-3, -2)$ $I(1, 4)$ $J(-2, -4)$ $K(-4, 5)$

b D, 0, G **c** 0, F **d** (0, 0)

2 $(-2, 1)$ $(-1, -1)$ $(0, -3)$ $(1, -5)$ $(2, -7)$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

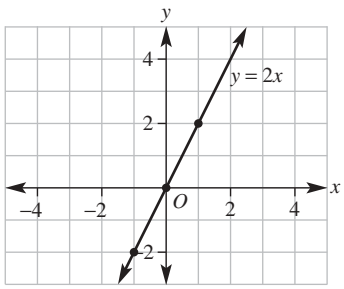
3 a $6 + 4 = 10, (-3, 10)$

b $2 + 4 = 6, (-1, 6)$

c $0 + 4 = 4, (0, 4)$

d $-4 + 4 = 0, (2, 0)$

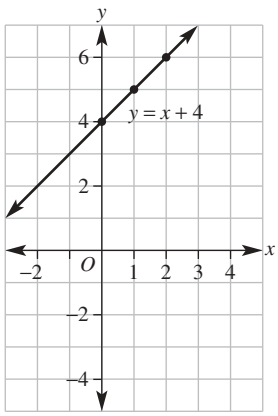
4 a



$y = 2x$

x	−1	0	1
y	−2	0	2

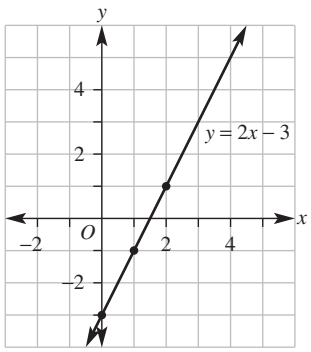
b



$y = x + 4$

x	0	1	2
y	4	5	6

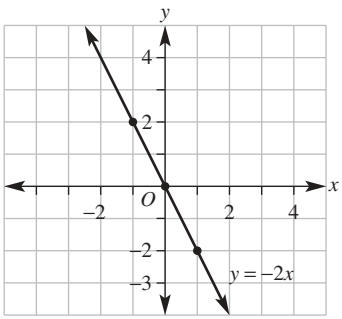
c



$y = 2x - 3$

x	0	1	2
y	−3	−1	1

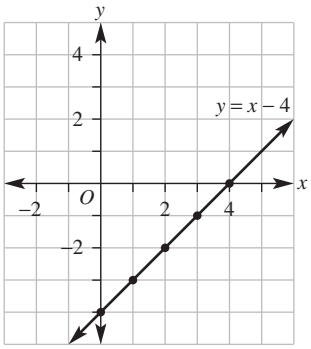
d



$y = -2x$

x	−1	0	1
y	2	0	−2

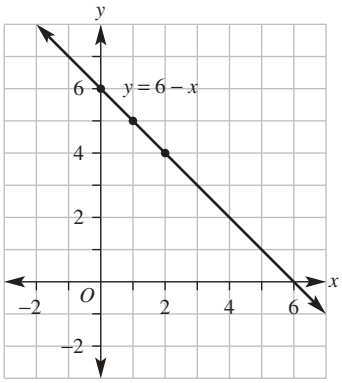
e



$y = x - 4$

x	1	2	3
y	−3	−2	−1

f

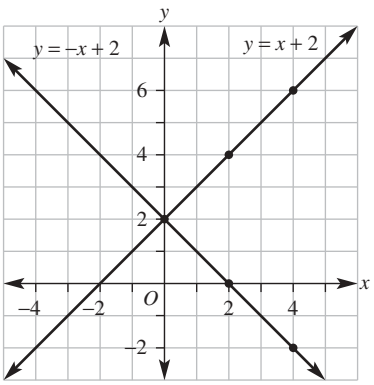


$y = 6 - x$

x	0	1	2
y	6	5	4

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 a



i

$y = x + 2$

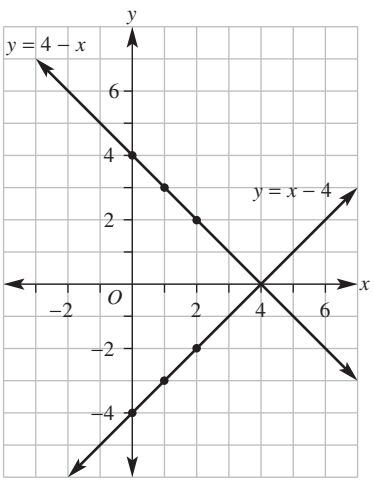
x	0	2	4
y	2	4	6

ii

$y = -x + 2$

x	0	2	4
y	2	0	−2

b



i

$y = x - 4$

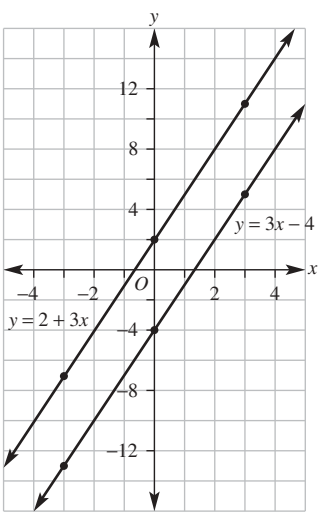
x	0	1	2
y	−4	−3	−2

ii

$y = 4 - x$

x	0	1	2
y	4	3	2

c



i

$y = 2 + 3x$

x	−3	0	3
y	−7	2	11

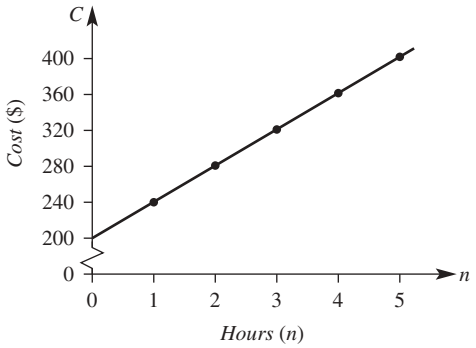
ii

$y = 3x - 4$

x	−3	0	3
y	−13	−4	5

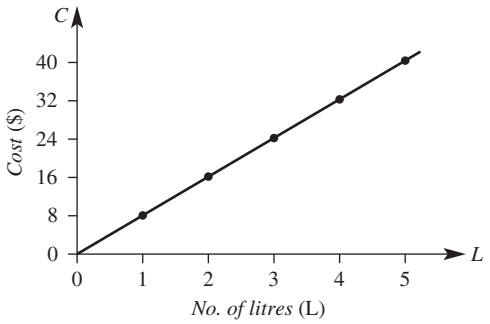
6 a (0, 0) **b** (1, 4) **c** (−1, 3) **d** (0, 2)
e (−1, −5)

7 a



b i $300 ii 4.5 h

8 a

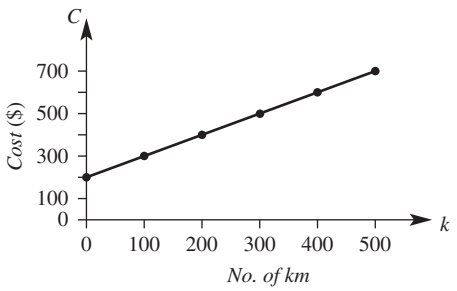


b i $28 ii 2.5 litres

9 a

No. of km (k)	0	100	200	300	400	500
Cost (C)	200	300	400	500	600	700

b



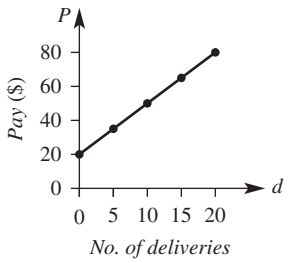
c i $450 ii 450 km

10 a

No. of deliveries (d)	0	5	10	15	20
Pay (P)	20	35	50	65	80

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b



c **i** $56 **ii** 18

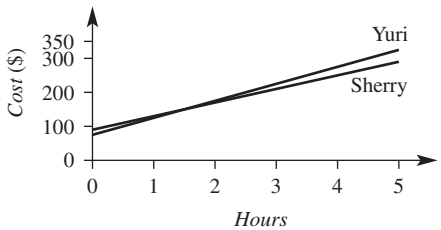
11 a Yuri

No. of hours work	0	1	2	3	4	5
Cost (C)	75	125	175	225	275	325

Sherry

No. of hours work	0	1	2	3	4	5
Cost (C)	90	130	170	210	250	290

b



c **i** $250 **ii** $150 **iii** 0.5 h
iv 4.25 h **v** 1.5 h

d Yuri is cheaper only for 1.5 hours or less.

6D

Now you try

Example 10

a $\frac{4}{3}$ **b** $-\frac{5}{2}$

Example 11

a 3 **b** Undefined **c** $-\frac{1}{2}$ **d** 0

Example 12

a $\frac{1}{2}$ **b** $-\frac{7}{5}$

Exercise 6D

1 a zero **b** negative **c** positive **d** undefined
2 a + **b** − **c** + **d** −
e − **f** +
3 a 1 **b** $\frac{1}{4}$ **c** $-\frac{3}{5}$
4 a $-\frac{3}{8}$ **b** $\frac{1}{15}$ **c** −3
5 a 2 **b** 5 **c** −3 **d** −2
e $\frac{4}{3}$ **f** 0 **g** 0 **h** Undefined
i Undefined

6 $EF\frac{2}{3}, GH\frac{2}{3}, DC\,1, AB\,\frac{3}{2}$

7 a 3 **b** 2 **c** $-\frac{1}{2}$ **d** −1
e 0 **f** Undefined

8 a

Line segment	Rise	Run	Gradient
AB	1	2	$\frac{1}{2}$
AC	2	4	$\frac{1}{2}$
AD	3	6	$\frac{1}{2}$
BC	1	2	$\frac{1}{2}$
BD	2	4	$\frac{1}{2}$
CD	1	2	$\frac{1}{2}$

b They have the same gradient.

9 a 2 **b** 1 **c** −1 **d** −2 **e** 1 **f** $-\frac{2}{5}$

10 a 1 **b** 1 **c** $-\frac{3}{5}$ **d** 0
e 11 **f** $\frac{1}{3}$

11 Gradient = 0.344

12 a A, D **b** C, E, G **c** D
d B, F, H **e** G

6E

Now you try

Example 13

a **i** 10 km **ii** 10 km/h
b **i** 10 km/h **ii** 0 km/h **iii** 7.5 km/h

Example 14

a **i** 2 L **ii** 2 L
b **i** 0.4 L/s **ii** 0.2 L/s

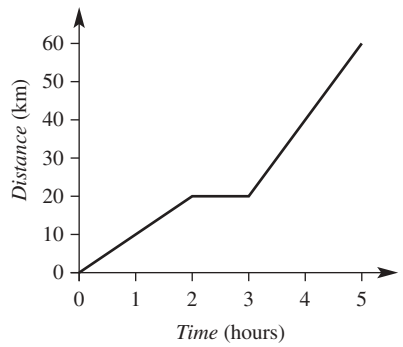
Exercise 6E

1 a gradient **b** two **c** units
d km/h **e** L/min
2 a 90 km/h **b** 15 L/min
3 a **i** 60 km **ii** 60 km/h
b **i** 60 km/h **ii** 0 km/h **iii** 90 km/h
4 a **i** 15 km **ii** 15 km/h
b **i** 0 km/h **ii** 30 km/h
5 a **i** 2 L **ii** 0.5 L
b **i** 0.2 L/s **ii** 0.05 L/s **iii** 0.05 L/s
6 a **i** 1.5 L **ii** 0.5 L
b **i** 0.15 L/s **ii** 0.05 L/s **iii** 0.15 L/s
7 a 3 km
b 4 min
c **i** 0.5 km/min **ii** 0.75 km/min
iii 0.5 km/min **iv** 0.25 km/min

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

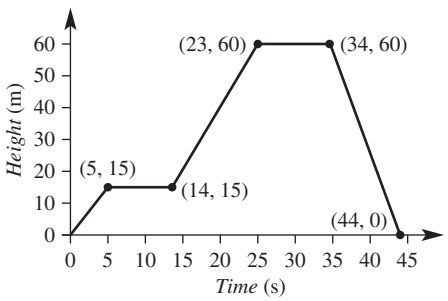
8 **a** C **b** A **c** B steepest

9 **a**



b 60 km

10



Time is 44 seconds.

11 **a** 3

b **i** 6 km **ii** 14 km

c B, D, G

d E, H

e **i** 6 km/h **ii** 14 km/h **iii** 16 km/h
iv 6.4 km/h **v** 16 km/h

f E and H, same gradient

g $5\frac{1}{4}$ h

h 40 km

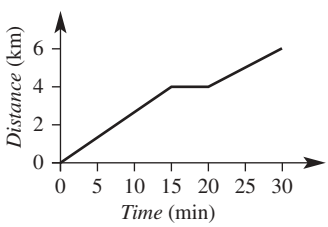
i 10 km/h

Progress quiz

1 **a** 10 cm **b** 30 cm **c** 30 cm **d** 50 cm

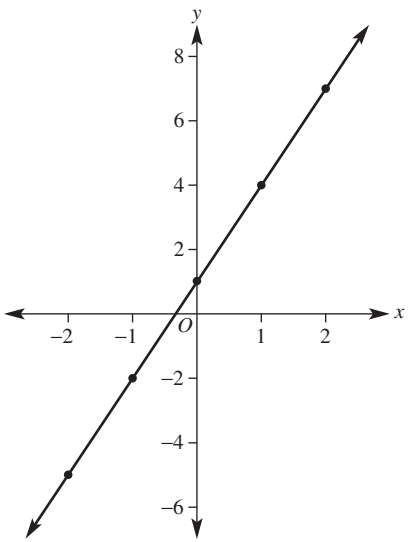
2 **a** 160 km **b** 80 km/h **c** 50 km/h

3



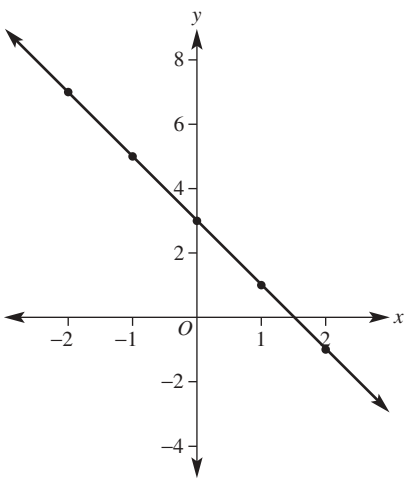
4 **a** $y = 3x + 1$

x	−2	−1	0	1	2
y	−5	−2	1	4	7



b $y = -2x + 3$

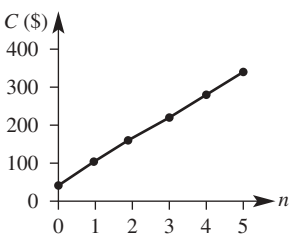
x	−2	−1	0	1	2
y	7	5	3	1	−1



5 **a**

No. of hours (n)	0	1	2	3	4	5
Cost ($C)	40	100	160	220	280	340

b



c **i** $190 **ii** 3.5 hours

6 **a** 4 **b** $-\frac{1}{2}$ **c** 0

d Undefined

7 $\frac{4}{5}$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6F

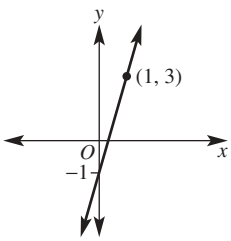
Now you try

Example 15

a **i** 2 **ii** −5

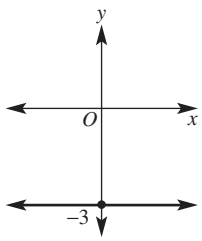
b **i** $-\frac{1}{3}$ **ii** 2

Example 16

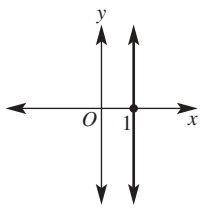


Example 17

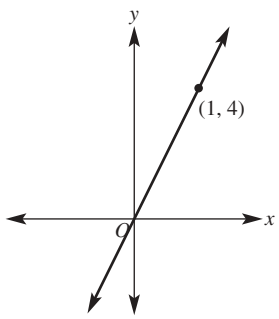
a



b



c



Example 18

No

Exercise 6F

1 **a** gradient–intercept **b** gradient **c** coefficient **d** y-intercept **e** zero

2 **a** $y = 3x - 1$ **b** $y = -\frac{3}{2}x + 2$ **c** $y = -x$

3 **a** Horizontal

b Vertical

c Passes through the origin

d Vertical and passes through the origin

e Passes through the origin

4 **a** **i** 2 **ii** 4

b **i** 6 **ii** −7

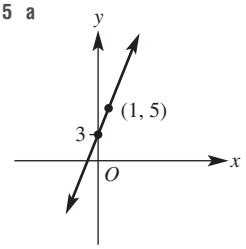
c **i** $-\frac{2}{3}$ **ii** 7

d **i** −7 **ii** −3

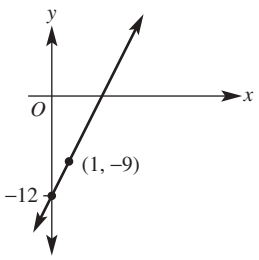
e **i** $\frac{3}{5}$ **ii** −8

f **i** 9 **ii** −5

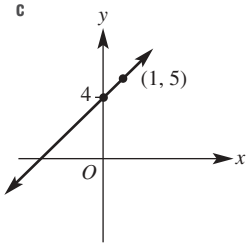
5 **a**



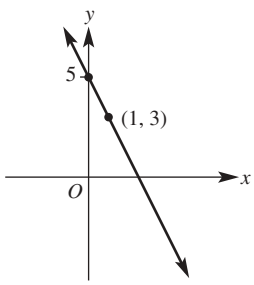
b



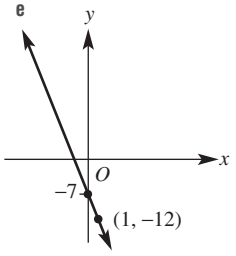
c



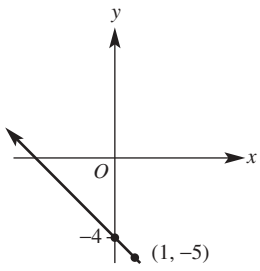
d



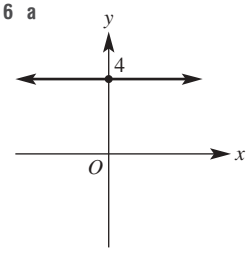
e



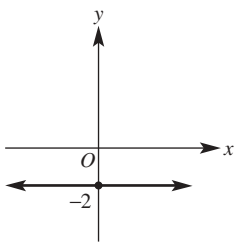
f



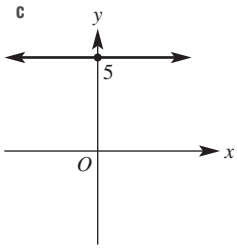
6 **a**



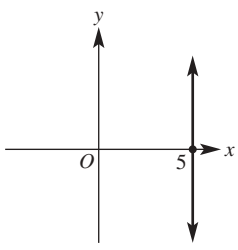
b



c

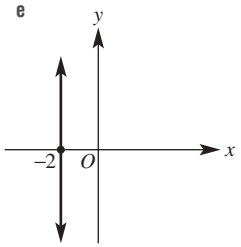


d

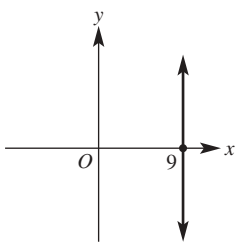


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

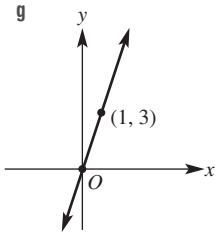
e



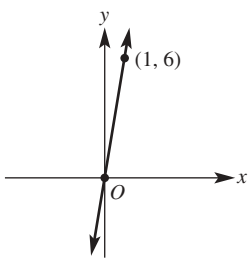
f



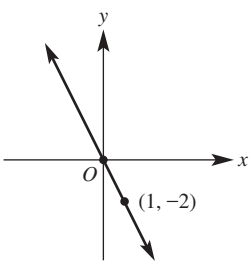
g



h



i



7 a 1, 0

b $-\frac{3}{2}$, 3

c $\frac{1}{2}$, 4.5

d 0, 4

e Undefined, none

8 a iv **b** ii **c** v **d** i
e vi **f** vii **g** iii **h** viii
i ix

9 a $c = 5$ in each equation; e.g. $y = 2x + 5, y = -3x + 5$ etc.

b $c = -2$ in each equation; e.g. $y = 7x - 2, y = x - 2$ etc.

10 a $m = 3$ in each equation;
e.g. $y = 3x - 1, y = 3x, y = 3x + 4$ etc.

b $m = -1$ in each equation;
e.g. $y = -x, y = -x + 7, y = -x - 3$ etc.

c $m = 0$ in each equation; e.g. $y = 4, y = -2$ etc.

d m is undefined in each equation; e.g. $x = -7$ etc.

11 a ii and iii **b** i and iv

12 a No **b** Yes **c** No **d** Yes
e Yes

13 a $y = x + 4$ **b** $y = x - 1$
c $y = \frac{x}{2}$ **d** $y = 2x + 1$

14 Check with your teacher.

15 a They all have the same gradient.

b They all have a y-intercept at -1.

c Check with your teacher.

6G

Now you try

Example 19

a Parallel **b** Perpendicular

Example 20

a $y = 2x + 4$ **b** $y = -\frac{1}{4}x + 3$

Example 21

a $y = 3x - 3$ **b** $y = 2x + 6$

Exercise 6G

1 a C, D **b** B, E **c** Yes **d** Yes
e Yes

2 a 4 **b** -7 **c** $-\frac{3}{4}$ **d** $\frac{8}{7}$

3 a $-\frac{1}{3}$ **b** $\frac{1}{2}$ **c** $-\frac{8}{7}$ **d** $\frac{9}{4}$

4 a Parallel **b** Perpendicular **c** Neither
d Neither **e** Parallel **f** Parallel
g Neither **h** Neither **i** Perpendicular
j Perpendicular

5 a $y = 4x + 2$ **b** $y = 3x + 4$
c $y = -x - 3$ **d** $y = -\frac{1}{2}x + 3$
e $y = -\frac{1}{3}x - 5$ **f** $y = -2x - 10$
g $y = -6x + 6$ **h** $y = 4x - 7$

6 a $y = x + 4$ **b** $y = -x + 8$
c $y = -4x - 1$ **d** $y = \frac{2}{3}x - 6$
e $y = -\frac{1}{2}x + 6$ **f** $y = \frac{1}{4}x - 2$
g $y = -\frac{3}{2}x + 5$ **h** $y = \frac{7}{2}x + 31$

7 a **i** 1 **ii** -1
b **i** $y = x$ **ii** $y = x - 2$ **iii** $y = x + 3$
iv $y = -x + 2$ **v** $y = -x$ **vi** $y = -x - 4$

8 a $y = 2x + 2$ **b** $y = 2x - 4$
c $y = -\frac{1}{2}x - 4$ **d** $x = -8$

9 a $y = -\frac{3}{2}x + 7$ **b** $y = -\frac{2}{3}x - 2$
c $y = \frac{5}{4}x + 2$ **d** $y = \frac{3}{2}x - \frac{7}{2}$

10 a Parallel **b** Parallel **c** Perpendicular
d Perpendicular

11 a **i** 1 **ii** 1 **iii** -1 **iv** -1
b Opposite sides are parallel; adjacent sides are perpendicular.
c Rectangle

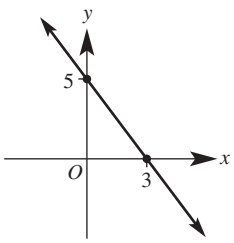
12 a **i** $\frac{4}{3}$ **ii** $-\frac{3}{4}$ **iii** 0
b Right-angled triangle because AB is perpendicular to BC.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

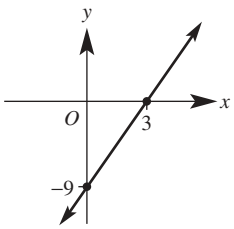
6H

Now you try

Example 22



Example 23

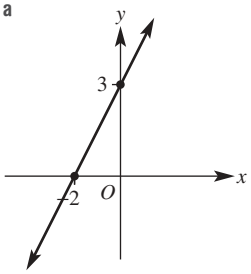


Exercise 6H

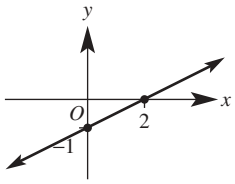
1 **a** The x-intercept is where $y = 0$.

b The y-intercept is where $x = 0$.

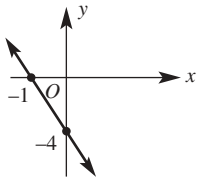
2 **a**



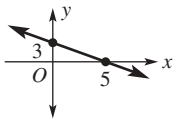
b



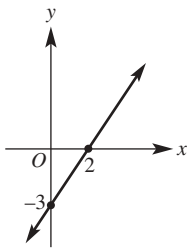
c



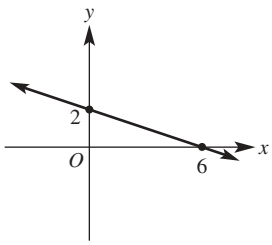
d



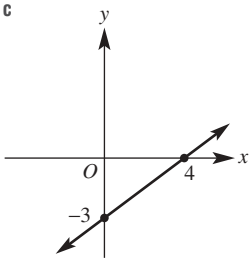
3 **a**



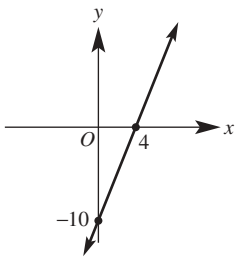
b



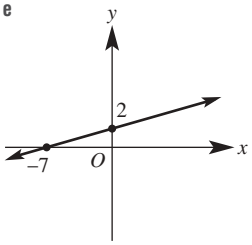
c



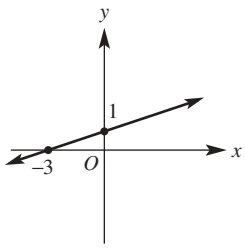
d



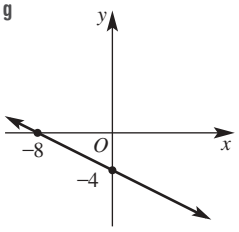
e



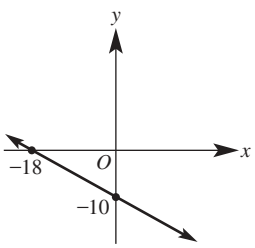
f



g

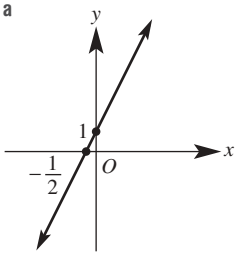


h

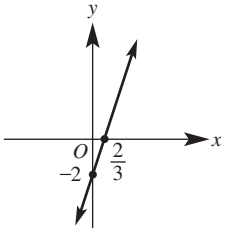


4 **a** ii **b** iii **c** i

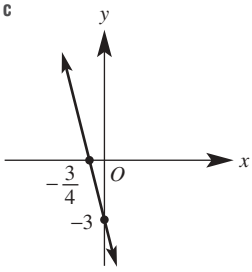
5 **a**



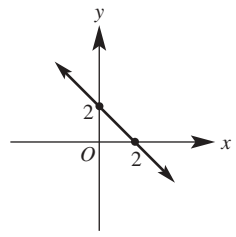
b



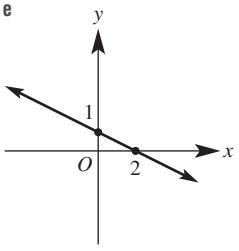
c



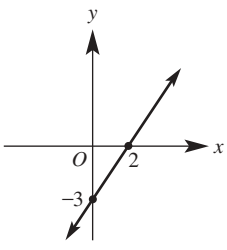
d



e



f



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 **a** iii **b** ii **c** i

7 **a** vi **b** ii **c** i **d** v
e iv **f** iii

8 **a** -2 **b** $\frac{1}{5}$ **c** 2 **d** $\frac{1}{2}$

9 **a** **i** $2, -4$ **ii** $A = 4$ units^2
b **i** $-2, 2$ **ii** $A = 2$ units^2
c **i** $-1.5, -3$ **ii** $A = 2.25$ units^2
d **i** $-4, 2$ **ii** $A = 4$ units^2

10 **a** 90 m **b** $7\frac{1}{2}$ s

11 Many answers; e.g. $2x + y = 4, a = 2, b = 1, d = 4$

12 **a** $2, -4$ **b** $-5, -10$ **c** $1, 1$
d $2, 4$ **e** $-4, 6$ **f** $-1, \frac{2}{3}$

6I

Now you try

Example 24

a $\frac{1}{2}$ **b** 2 **c** $y = \frac{1}{2}x + 2$

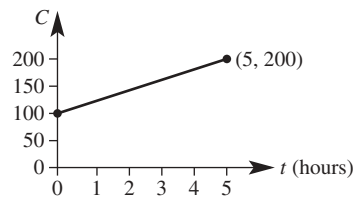
Example 25

a -3 **b** 21 **c** $y = -3x + 21$

Example 26

a $C = 20t + 100$

b



c **i** \$160 **ii** 8 hours

Exercise 6I

1 D

2 C

3 **a** **i** $m = 2$ **ii** 1 **iii** $y = 2x + 1$
b **i** $m = -3$ **ii** 2 **iii** $y = -3x + 2$
c **i** $m = 4$ **ii** -3 **iii** $y = 4x - 3$
d **i** $m = -4$ **ii** 0 **iii** $y = -4x$
e **i** $m = -1$ **ii** 2 **iii** $y = -x + 2$
f **i** $m = 2$ **ii** 0 **iii** $y = 2x$

4 **a** **i** 3 **ii** 1 **iii** $y = 3x + 1$
b **i** -1 **ii** 6 **iii** $y = -x + 6$
c **i** 5 **ii** 21 **iii** $y = 5x + 21$
d **i** 3 **ii** -16 **iii** $y = 3x - 16$
e **i** -3 **ii** 2 **iii** $y = -3x + 2$
f **i** $\frac{1}{2}$ **ii** $\frac{3}{2}$ **iii** $y = \frac{1}{2}x + \frac{3}{2}$

5 **a** $y = 4$ **b** $y = -1$ **c** $x = 3$ **d** $x = -2$
e $y = 2$ **f** $x = 7$

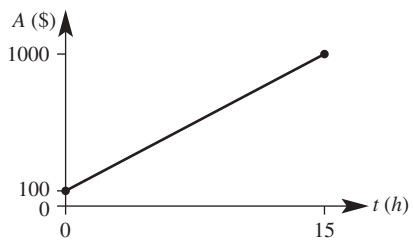
6 **a** $y = 2x$ **b** $y = 3x$ **c** $y = \frac{2}{3}x$ **d** $y = -3x$
e $y = -x$ **f** $y = -\frac{2}{5}x$

7 **a** **i** 7 **ii** $y = 7x$
b **i** $-\frac{3}{2}$ **ii** $y = -\frac{3}{2}x$
c **i** 1 **ii** $y = x + 2$
d **i** $-\frac{3}{2}$ **ii** $y = -\frac{3}{2}x$
e **i** 0 **ii** $y = 2$
f **i** Undefined **ii** $x = 3$

8 **a** $P = 4n + 100$
b \$2100
c 225

9 **a** $A = 60t + 100$

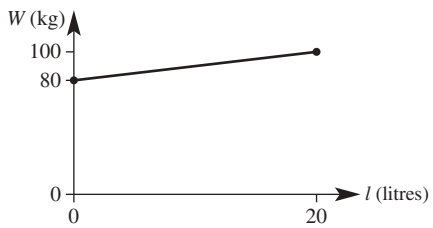
b



c **i** \$820 **ii** 10 h

10 **a** $W = l + 80$

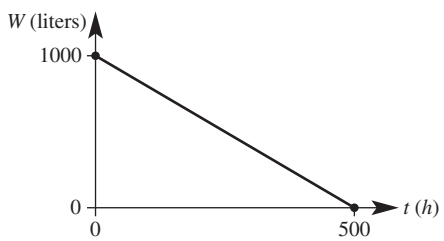
b



c **i** 87 kg **ii** 29 L

11 **a** -2, $(0, 1000)$

b



c 1000 L

d **i** 360 L **ii** 952 L **iii** 664 L

e **i** 350 h **ii** 407.5 h

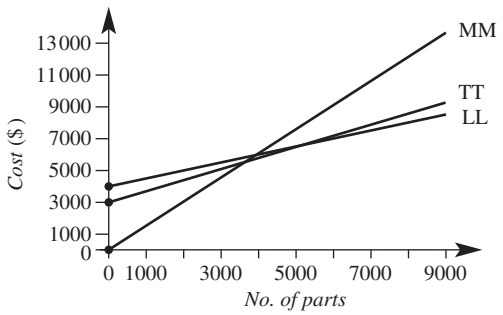
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

12 a

p	0	1000	2000	3000	4000
Cost, MM	0	1400	2800	4200	5600
Cost, TT	3000	3700	4400	5100	5800
Cost, LL	4000	4500	5000	5500	6000

p	5000	6000	7000	8000	9000
Cost, MM	7000	8400	9800	11 200	12 600
Cost, TT	6500	7200	7900	8 600	9 300
Cost, LL	6500	7000	7500	8 000	8 500

b



c i \$2100 ii \$1400 iii \$7250 iv \$8750

d Mandy's is best for parts less than or equal to 4285. Terry's is best for between 4286 and 5000 parts and equal to Lenny's at 5000. Lenny's is best for parts > 5000.

6J

Now you try

Example 27

a $C = 1.38n$ **b** \$66.24

Example 28

a $k = 25$ **b** $W = 25n$

c \$150 **d** 9.5 hours

Example 29

a 50 cm **b** 24 inches

Exercise 6J

1 a decreases **b** increases

2 a ii and v

b i $k = 5$ ii $k = 12$ iii $k = 40$

iv $k = 22$ v $k = 7.5$

3 a $k = 4, y = 4x$ **b** $k = \frac{6}{5}, y = \frac{6}{5}x$

4 a $W = 11.5n$ **b** \$431.25

5 a $V = 6t$ **b** 1440 L **c** $k = 8640,\ V = 8640t$

d 60 480 L

6 a $k = 244$ **b** $P = 244n$ **c** \$33 184

d 1175 tonnes

7 a $k = 774$

b $d = 774t$

c 4.26 h

d 619.2 km

8 a 48 cm **b** 10 inches **c** 30 cm **d** 10 cm

e 22 inches **f** 76 cm **g** 63 cm (or 64 cm)

h 100 cm

9 a 94 cm = 37 inches, $m = \frac{94}{37} = 2.54$

b 2.54 cm/inch **c** $k = 2.54$

d $y = 2.54x$ **e** 127 cm

10 a i 98 NZD ii 62 EUR iii 51 GBP

iv 41 AUD v 41 AUD vi 41 AUD

b i 40 EUR = 50 AUD, $m = \frac{40}{50} = 0.8$

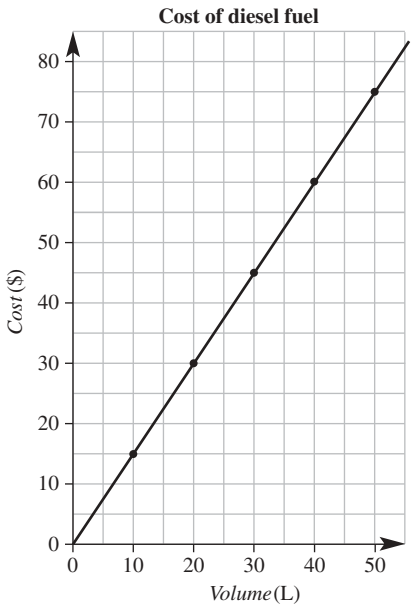
ii 0.8 EUR/AUD iii $k = 0.8$ EUR/AUD

iv $y = 0.8x$ v 500 EUR

11 a

Volume (V) of diesel, in litres	0	10	20	30	40	50
Cost (C), in dollars	0	15	30	45	60	75

b



c $m = 1.5$ **d** Rate = \$1.5/L **e** $k = 1.5$

12 a Number of hours worked and wages earned are in direct proportion. If the number of hours worked doubles, then the wages earned also doubles.

b The cost of buying tomatoes and the number of kg are in direct proportion. If the number of kg doubles, then the cost also doubles.

c The speed and time taken are not in direct proportion. If the speed increases, then the time taken decreases. Alternatively, if the speed decreases, then the time taken increases.

d The size of the movie file and the time taken to download it are in direct proportion. If the size of the movie file is doubled, then the time to download will also double.

e The cost of a taxi ride and the distance travelled are not in direct proportion because the graph does not start at (0, 0). The y-intercept is the flag fall cost.

13 a 15 cents/min **b** 40 cents/min **c** 3 mL/s

d 5 mL/s **e** 20 m/s **f** 5 m/s

g 1.5 cents/g **h** 3.2 cents/g **i** 4.8 kg/year

j 1.54 kg/week

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

14 a $d = 90t$

b $k = 25$, $d = 25t$

c 100 m

15 a Singapore dollar (SGD)

i SGD $= 1.2 \times$ AUD **ii** SGD \$288

iii AUD \$200

b Chinese yuan (CNY)

i CNY $= 6.47 \times$ AUD **ii** CNY 485.25

iii AUD \$11.59

c South African rand (ZAR)

i ZAR $= 9.5 \times$ AUD **ii** ZAR 475

iii AUD \$5.26

6K

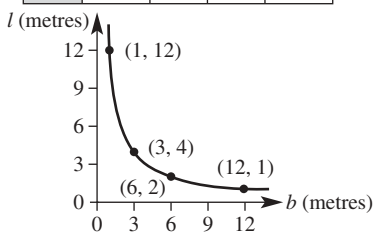
Now you try

Example 30

a

b	1	3	6	12
l	12	4	2	1

b



c 3 metres

d 4.8 metres

Example 31

a $k = 8$ **b** 8

Exercise 6K

1 a Direct **b** Indirect **c** Direct **d** Direct

e Indirect

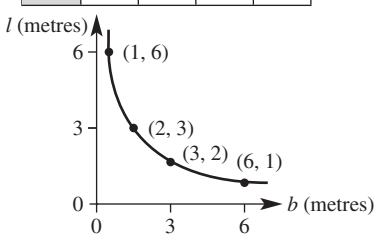
2 a Yes **b** No **c** Yes

3 a 6 **b** 1

4 a

b	1	2	3	6
l	6	3	2	1

b

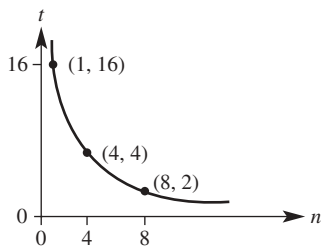


c 0.75 metres

d 4 metres

5 a **i** 16 days **ii** 4 days **iii** 2 days

b



c 1.6 days

d 2 painters

6 a **i** 4 **ii** 2.5

b **i** 2 **ii** 0.5

7 a $k = 50$

b 25

8 a $k = 12$ and $\{4, 3\}$ **b** $k = 4$ and $\{4, 1\}$

9 a $k = 40$ **b** 10 cm^3 **c** 2.5 kg/cm^3

10 a **i** \$250 **ii** \$100

b $C = \frac{500}{n}$

c 8 people

11 a $I = \frac{240}{R}$ **b** 6 amps **c** 2 ohms **d** 100%

6L

Now you try

Example 32

a **i** Minimum $(-2, -9)$ **ii** $x = -2$

iii $x = -5, 1$ **iv** $y = -5$

b **i** Maximum $(1, 0)$ **ii** $x = 1$ **iii** $x = 1$

iv $y = -2$

Example 33

See table at top of next page.

Example 34

See table at top of next page.

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Example 33

	Formula	Max or min	Ref in x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = \frac{1}{2}x^2$	Min	No	(0, 0)	$y = \frac{1}{2}$	Wider
b	$y = -3x^2$	Max	Yes	(0, 0)	$y = -3$	Narrower

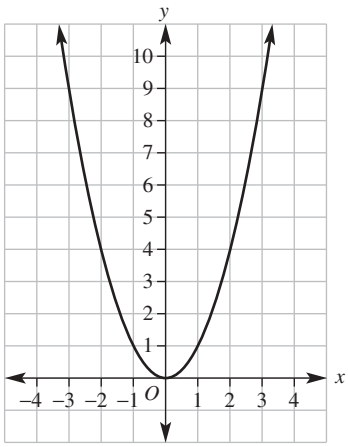
Example 34

	Formula	Max or min	Ref in x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = x^2 - 2$	Min	No	(0, −2)	$y = -1$	Same
b	$y = x^2 + 1$	Min	No	(0, 1)	$y = 2$	Same

Exercise 6L

1

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9



2 **a** maximum **b** (−2, 9) **c** 5 **d** −5, 1
e $x = -2$

3 **a** **i** (2, −5.4), minimum **ii** $x = 2$ **iii** −1, 5 **iv** −3
b **i** (2, 0), maximum **ii** $x = 2$ **iii** 2 **iv** −1
c **i** (2, 5), minimum **ii** $x = 2$ **iii** No x-intercepts **iv** 7
d **i** (−3, 0), minimum **ii** $x = -3$ **iii** −3 **iv** 4
e **i** (2, −2), minimum **ii** $x = 2$ **iii** 1, 3 **iv** 6
f **i** (0, 3), maximum **ii** $x = 0$ **iii** −3, 3 **iv** 3

4 *See table at base of this page.*

5 *See table at base of this page.*

6 **a** iv **b** ii **c** iii **d** i

7 **a** $y = x^2 + 2$ **b** $y = -x^2$ **c** $y = x^2 + 5$

4

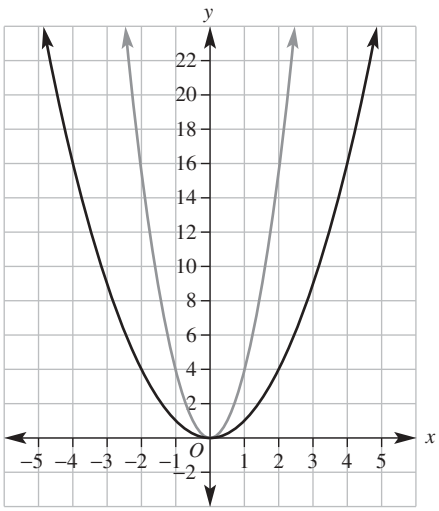
	Formula	Max or min	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 3x^2$	Min	No	(0, 0)	$y = 3$	Narrower
b	$y = \frac{1}{2}x^2$	Min	No	(0, 0)	$y = \frac{1}{2}$	Wider
c	$y = 2x^2$	Min	No	(0, 0)	$y = 2$	Narrower
d	$y = -4x^2$	Max	Yes	(0, 0)	$y = -4$	Narrower
e	$y = -\frac{1}{3}x^2$	Max	Yes	(0, 0)	$y = -\frac{1}{3}$	Wider
f	$y = -2x^2$	Max	Yes	(0, 0)	$y = -2$	Narrower

5

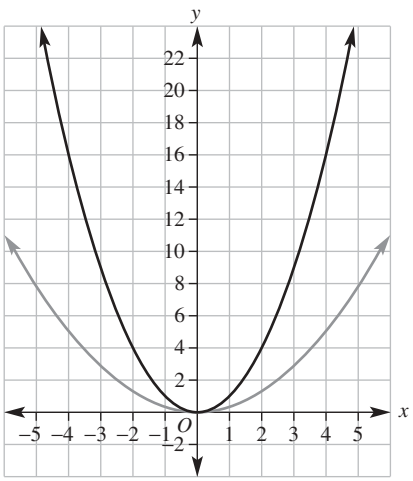
	Formula	Turning point	y-intercept ($x = 0$)	y value when $x = 1$
a	$y = x^2 + 3$	(0, 3)	3	$y = 4$
b	$y = x^2 - 1$	(0, −1)	−1	$y = 0$
c	$y = x^2 + 2$	(0, 2)	2	$y = 3$
d	$y = x^2 - 4$	(0, −4)	−4	$y = -3$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

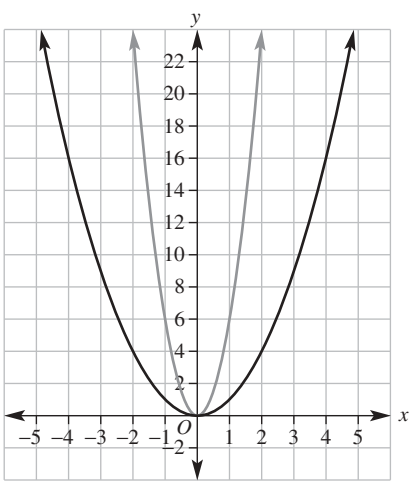
8 a i



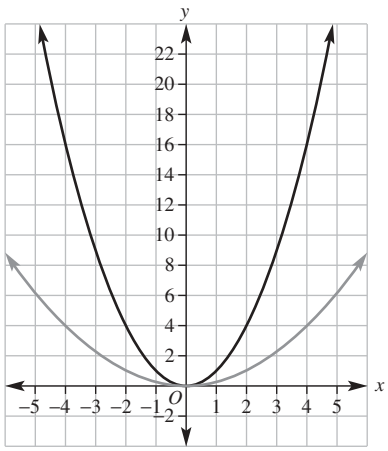
ii



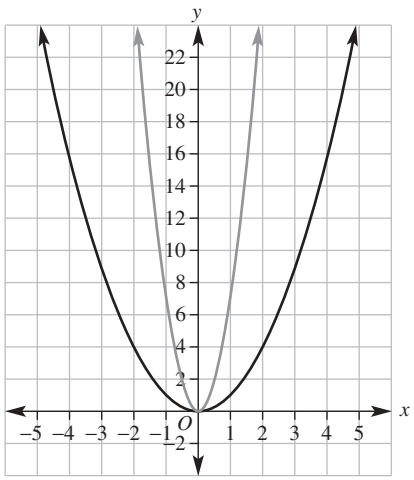
iii



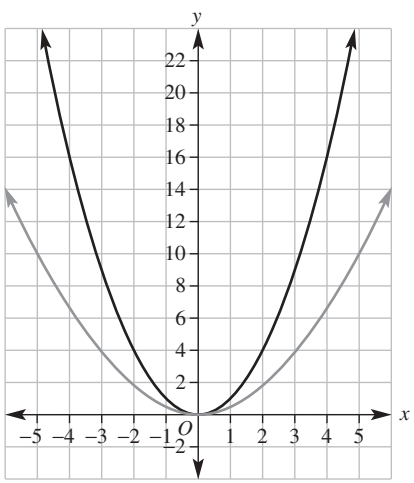
iv



v



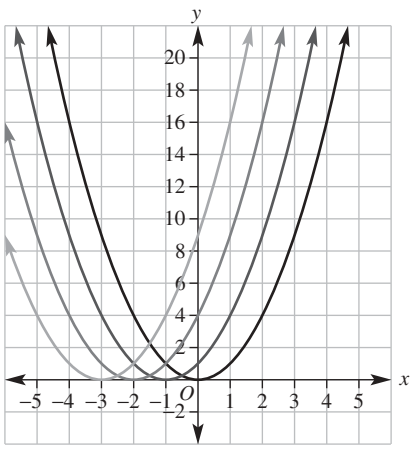
vi



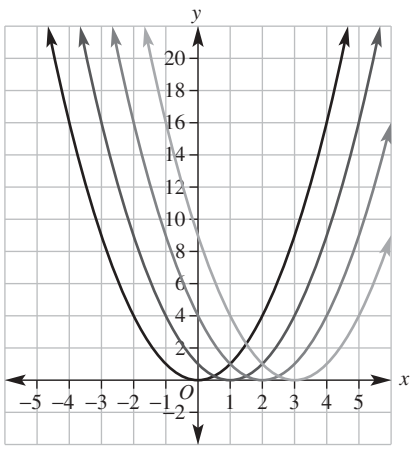
b The constant a determines the narrowness of the graph.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9 a i

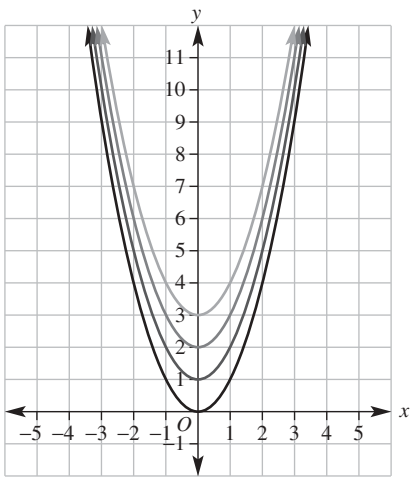


ii

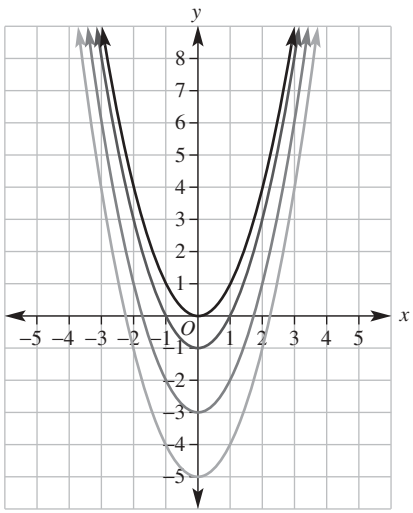


b The constant h determines whether the graph moves left or right from $y = x^2$.

10 a i



ii



b The constant k determines whether the graph moves up or down from $y = x^2$.

6M

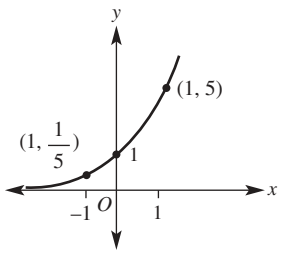
Now you try

Example 35

a

x	−2	−1	0	1	2
y	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25

b



Exercise 6M

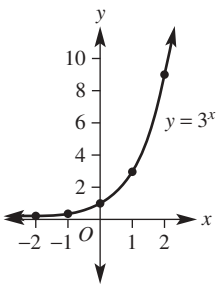
1 9, 27, 81, 243, 729

2 B

3 a

x	−2	−1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

b

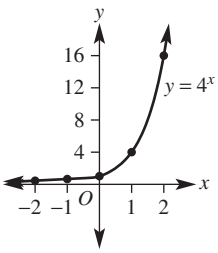


ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

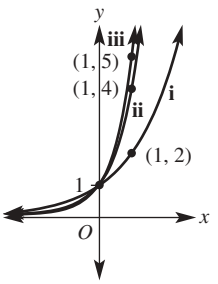
4 a

x	−2	−1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

b



5 a



b The same point $(0, 1)$.

c Makes it rise more quickly.

6 a 1000

b i 2000 ii 8000

c i 2 years ii 4 years

6N

Now you try

Example 36

a $P = 3000\,(1.02)^n$ b $V = 36\,000\,(0.94)^n$

Example 37

a $V = 20\,000\,(1.04)^n$

b i \$20 800 ii \$24 333.06

c 5.7 years

Exercise 6N

1 a \$50 b \$1050 c \$ 52.50 d \$55.13

2 a 4.9 kg b $\frac{2}{100}$, 0.98 c 4.52 kg

3 a Growth

b Growth

c Decay

d Decay

e Growth

f Decay

4 a A = amount of money at any time,
n = number of years of investment
$A = \$200\,000 \times 1.17^n$

b A = house value at any time,
n = number of years since initial valuation
$A = \$530\,000 \times 0.95^n$

c A = car value at any time,
n = number of years since purchase
$A = \$14\,200 \times 0.97^n$

d A = population at any time,
n = number of years since initial census
$A = 172\,500 \times 1.15^n$

e A = litres in tank at any time,
n = number of hours elapsed
$A = 1200 \times 0.9^n$

f A = cell size at any time,
n = number of minutes elapsed
$A = 0.01\ \text{cm}^2 \times 2^n$

g A = size of oil spill at any time,
n = number of minutes elapsed
$A = 2\ \text{m}^2 \times 1.05^n$

h A = mass of substance at any time,
n = number of hours elapsed
$A = 30\ \text{g} \times 0.92^n$

5 a $A = 500\,000 \times 1.1^n$

b i \$665 500 ii \$1 296 871.23
iii \$3 363 749.98

c After 7.3 years

6 a $A = 300\,000 \times 0.85^n$

b i \$216 750 ii \$96 173.13 iii \$42 672.53

c 3.1 years

7 a $V = 15\,000 \times 0.94^n$

b i 12 459 L ii 9727 L

c 769.53 L

d 55.0 hours

8 a 3000

b i 3000 ii 20 280 iii 243 220

c 7 hours 46 minutes

9 a $D = 10 \times 0.875^t$, where t = number of 10 000 kms travelled

b Yes, since it has 3.44 mm of tread after 80 000 km.

c 90 000

10 a $T = 90^\circ C \times 0.92^t$

b i 76.2°C ii 79.4°C

c 3 minutes 13 seconds

11 a i \$1610.51 ii \$2143.59 iii \$4177.25

b i \$1645.31 ii \$2218.18 iii \$4453.92

12 a \$2805.10 b \$2835.25 c \$2837.47

Maths@Work: Accountant or small business owner

1 a Income: blue line; costs: red line

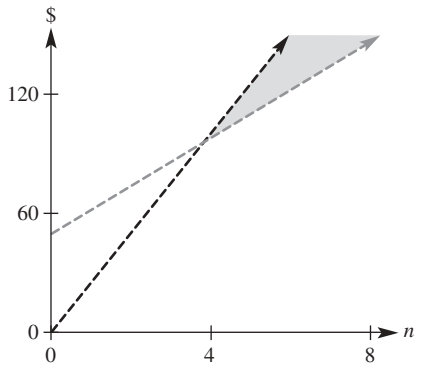
b 0, 1, 2 or 3

c \$0, \$0

d 4

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

e



f $P = 13n - 50$

g **i** \$80 **ii** \$145 **iii** \$340 **iv** \$600

2 **a** 40; cost of advertisement = \$40

b \$2.67

c 6.67; \$6.67 retail price per glass

d 10

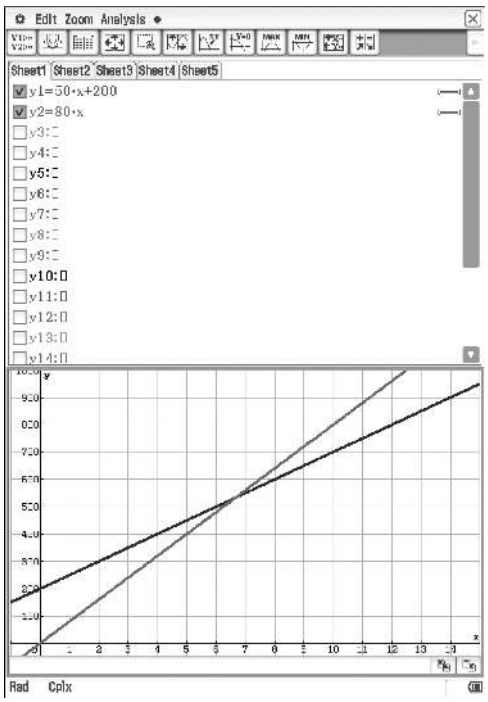
e **i** $C = 2.67n + 40$ **ii** $I = 6.67n$

iii $P = 4n - 40$

3 **a** $C = 200 + 50n$

b $I = 80n$

c



d 7 car parts

e \$760

4 **a** **i** 500, \$5000 **ii** 375, \$4500 **iii** 300, \$4200

iv 250, \$4000

b $P = 500a - 5000$

c **i** \$0 **ii** \$1000 **iii** \$2000 **iv** \$3000

d People will stop buying parts if they are more expensive than the price from another business or if they are just too expensive for the customer to afford.

Puzzles and challenges

1 PLOTTING LINES!

2

			T						H	E	R
		I						O		L	A
	M			E			R		T	B	T
E	C	N	A	T	S	I	D	N		A	E
S					Z	A	E			I	
	P			O		M	E			R	
		E	N		G			R		A	
		T	E	E					C	V	
	A		S	D						N	
L		G	R	A	P	H					I

3 Both 13 km apart

4 **a** 160 ice-creams for zero profit **b** 493 ice-creams sold

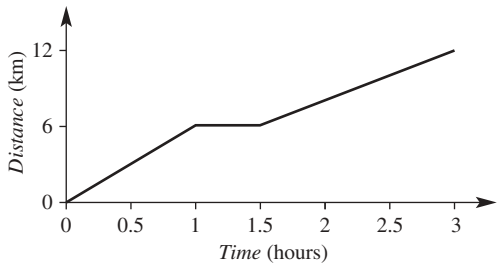
Chapter checklist with success criteria

1 **a** 450 **b** 300

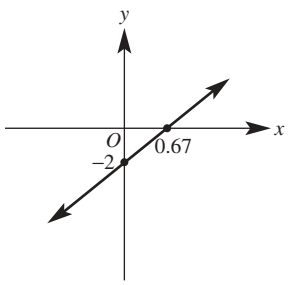
2 **a** \$25 **b** \$15 **c** 6 weeks

3 **a** 3 km **b** 5 minutes **c** 2.5 km

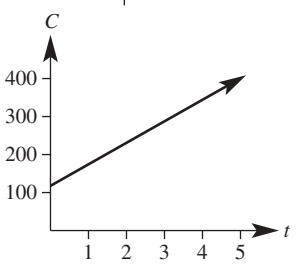
4



5



6



7 **a** positive **b** undefined

8 **a** zero **b** 4 **c** 3

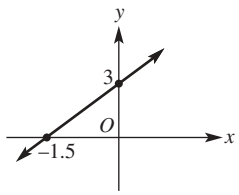
9 **a** 8 km/h **b** 0 km/h **c** 5.33 km/h

10 $\frac{3}{4}$ L/s, 1 L/s, $\frac{3}{4}$ L/s

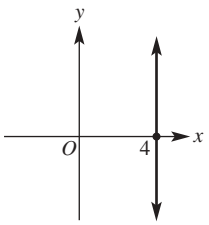
11 $\frac{1}{2}, -4$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

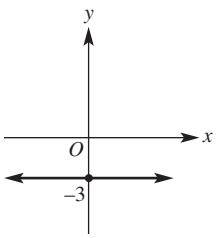
12



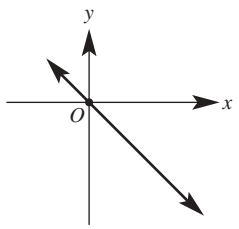
13 a



b



14

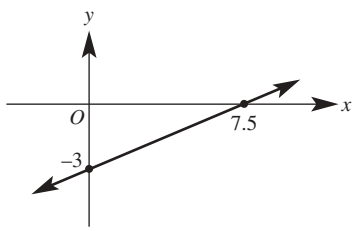


15 Perpendicular

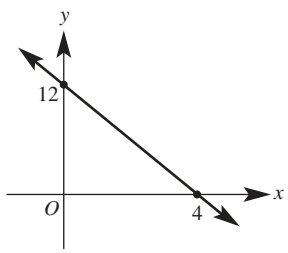
16 a $y = 2x - 3$ **b** $y = -\frac{1}{3}x - 3$

17 a $y = -3x + 3$ **b** $y = -2x + 1$

18



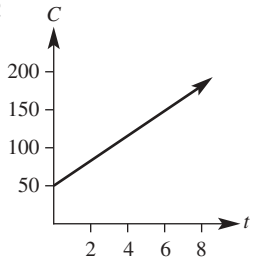
19



20 $y = 2x + 2$

21 $y = 2x + 2$

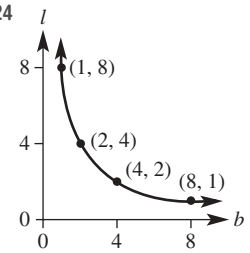
22



a \$150 **b** 10 hours

23 a 0.5 L/s **b** $V = 0.5$ s **c** 4L, 40 seconds

24



$l = 3.2$ cm

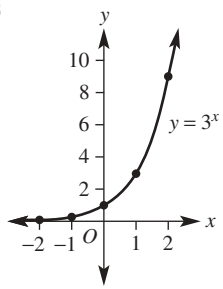
25 $y = \frac{12}{x}$

26 Turning point at $(1, 4)$ is a maximum, axis of symmetry is $x = 1$, x-intercepts are $(-1, 0)$ and $(3, 0)$, y-intercept is $(0, 3)$

27 a Turning point at $(0, 0)$ is a maximum, axis of symmetry is $x = 0$, x-intercept is $(0, 0)$, y-intercept is $(0, 0)$

b Turning point at $(0, 1)$ is a minimum, axis of symmetry is $x = 0$, there are no x-intercepts, y-intercept is $(0, 1)$

28



29 $M = 450\,(0.86)^n$

30 $A = 80\,000\,(1.06)^n$, \$100 998.16, 6.23 years

Chapter 6 review

Short-answer questions

1 a 40 km **b** 2 hours **c** 60 km

2 a i \$6000 **ii** \$8000 **iii** \$9000

b i $\approx$ \$2000 **ii** $\approx$ \$4000 **iii** $\approx$ \$5000

c $\approx$ 10 years

3 a 15 km

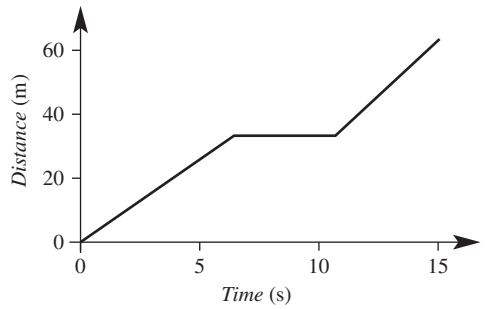
b 3 h

c i 6 km **ii** 6 km **iii** 9 km **iv** 15 km

d $3\frac{1}{2}$ h

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

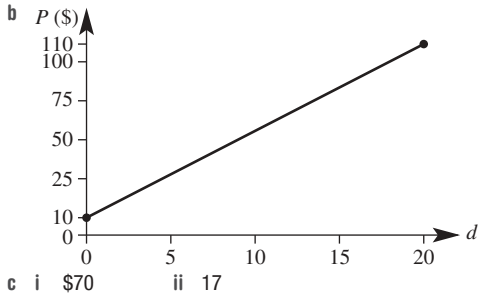
4



5 a

d	0	5	10	15	20
P	10	35	60	85	110

b



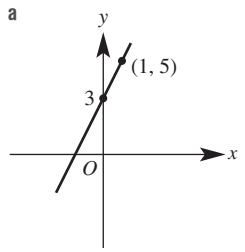
c i \$70 ii 17

6 a 1 **b** −1 **c** −2 **d** $\frac{1}{3}$

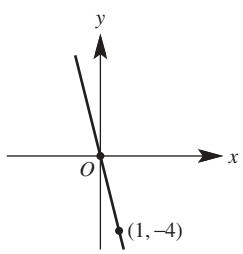
7 a Gradient $= 3$, y-intercept $= 4$

b Gradient $= -2$, y-intercept $= 0$

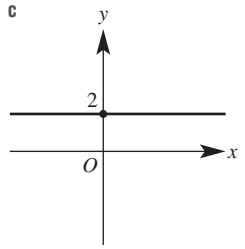
8 a



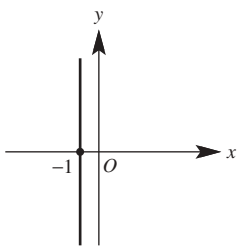
b



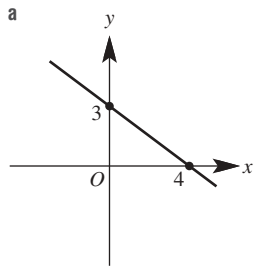
c



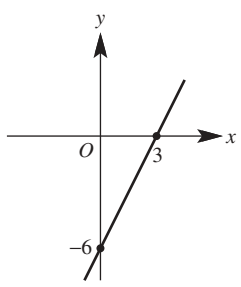
d



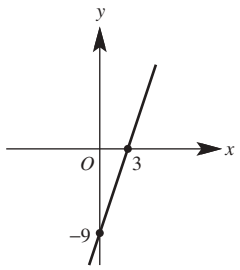
9 a



b



c



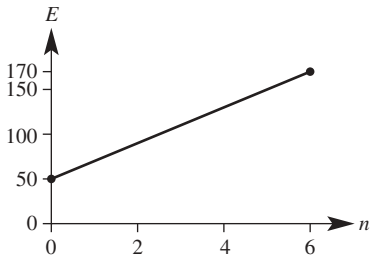
10 a i 2 ii 0 iii $y = 2x$

b i −4 ii 16 iii $y = -4x + 16$

11 a i **b** iii **c** vi **d** ii **e** iv **f** v

12 a $E = 20n + 50$

b



c i \$130 ii 5.5 bins

13 a $y = 2x + 3$ **b** $y = \frac{1}{2}x - 1$ **c** $y = -x + 2$

d $y = -\frac{4}{3}x - 7$ **e** $y = -4x + 6$ **f** $y = -\frac{1}{2}x + 4$

14 a $k = \$16.50/\text{h}$

b $W = 16.5n$

c i \$132 ii 16 hours

15 a 8 km **b** 21 miles **c** $m = \frac{15}{24.14} = 0.621$

d 0.621 miles/km **e** $k = 0.621$ miles/km

f $y = 0.621x$ **g** 62.1 miles **h** 161 km

16 a Direct proportion. If the number of cricket balls increases, then the cost will also increase.

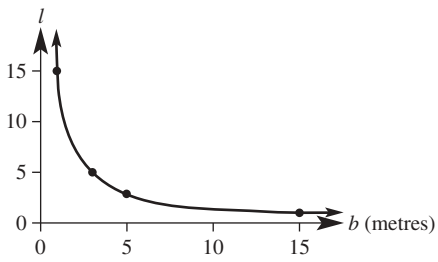
b Indirect proportion. If the number of people increases, then the cost per person decreases.

17 a $k = 15$

b

b	1	3	5	15
l	15	5	3	1

c



d 1.5 m

e 3.75 m

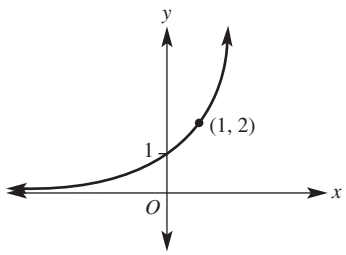
18 a Minimum at $(1, -4)$ **b** $x = 1$

c −1 and 3 **d** −3

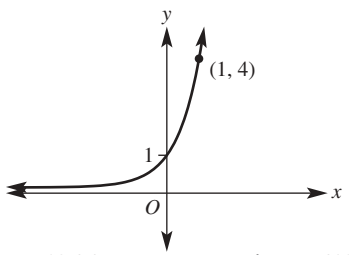
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

19 *See table at bottom of this page.*

20 a



b



21 a $P = 20(1.1)^n$ **b** $A = 100\,000\,(0.85)^n$

Multiple-choice questions

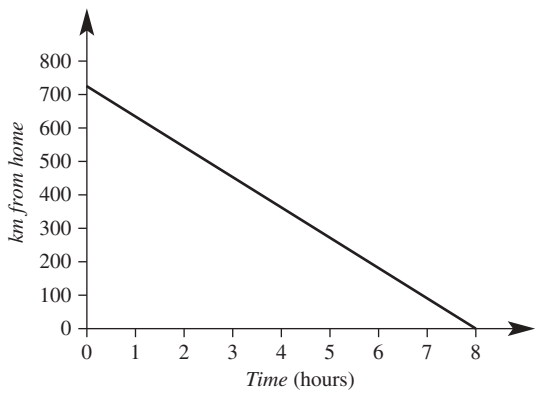
1 A	**2** E	**3** D	**4** A
5 B	**6** E	**7** C	**8** A
9 B	**10** D	**11** D	**12** E
13 C	**14** D	**15** B	**16** C
17 D			

Extended-response questions

1 a

Time in hours (t)	0	2	4	6	8
km from home	720	540	360	180	0

b

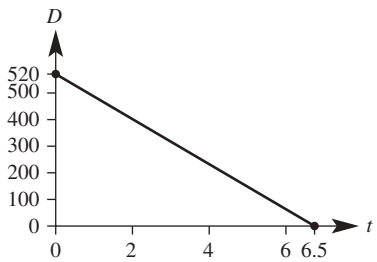


c 9 a.m. **d** 7 h **e** 8 h **f** 4 p.m.

2 a i 440 km **ii** 280 km

b i $D = -80t + 520$ or $D = 520 - 80t$

ii



c i 160 km **ii** 2.25 hours

3 a $A = 2\,(1.09)^t$

b i 2.3762 m^2 **ii** 3.0772 m^2

c 37.4 weeks

Chapter 7

Warm-up quiz

1 a Acute **b** right **c** obtuse **d** straight
e reflex **f** revolution **g** Complementary
h Supplementary

2 a E **b** B **c** F **d** A
e C **f** D

3 a $a = 110$ **b** $b = 140$ **c** $c = 210$ **d** $d = 35$
e $e = 60$ **f** $f = 40$

4 a $a = 110, b = 70$ **b** $a = 105, b = 75$
c $a = 40, b = 140$

5 a 1 **b** 4 **c** 2 **d** 2
e 1 **f** 1

6 a $S = 360°, a = 130$ **b** $S = 540°, b = 120$
c $S = 720°, c = 120$

7A

Now you try

Example 1

$a = 110$ (supplementary),
$b = 110$ (alternate)
$c = 70$ (corresponding or cointerior)
$d = 70$ (vertically opposite)

Example 2

a No – corresponding angles are not equal.
b Yes – cointerior angles are supplementary.

19

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 2x^2$	Min	No	(0, 0)	2	Narrower
b	$y = -\frac{1}{2}x^2$	Max	Yes	(0, 0)	$-\frac{1}{2}$	Wider
c	$y = x^2 - 2$	Min	No	(0, −2)	−1	Same

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 7A

1 **a** 180°
b equal
c **i** equal **ii** equal **iii** supplementary
2 **a** Alternate **b** Vertically opposite
c Cointerior **d** Corresponding
3 $a = 20$ supplementary, $b = 20$ alternate,
$c = 160$ corresponding, $d = 160$ vertically opposite
4 $a = 100$ supplementary, $b = 100$ alternate,
$c = 80$ corresponding, $d = 80$ vertically opposite
5 **a** $x = 110$, $y = 110$ **b** $x = 40$, $y = 140$
c $x = 75$, $y = 105$ **d** $x = 120$, $y = 120$
e $x = 110$, $y = 70$ **f** $x = 105$, $y = 75$
6 **a** Yes, corresponding angles are equal.
b No, alternate angles are not equal.
c No, cointerior angles are not supplementary.
d Yes, cointerior angles are supplementary.
e No, corresponding angles are not equal.
f Yes, alternate angles are equal.
7 **a** 60 **b** 20 **c** 100 **d** 115
e 50 **f** 30
8 **a** $a = 90$, $b = 90$, $c = 90$
b $a = 90$, $b = 90$, $c = 90$
c $a = 135$, $b = 45$, $c = 135$
d $a = 50$, $b = 130$, $c = 50$
e $a = 90$, $b = 130$
f $a = 110$, $b = 120$
9 **a** (a, e), (d, f), (b, h), (c, g)
b (d, h), (c, e)
c (c, h), (d, e)
d (a, c), (b, d), (e, g), (f, h)
10 **a** 90 **b** 75 **c** 10 **d** 30 **e** 36 **f** 30
11 **a** 12 **b** 14 **c** 10

7B

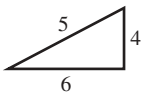
Now you try

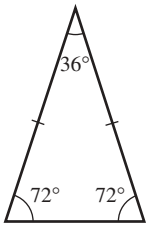
Example 3
20

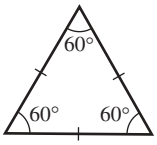
Example 4
75

Example 5
a 130 **b** 45

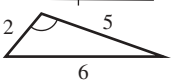
Exercise 7B

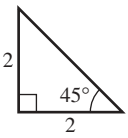
1 **a** 180° **b** isosceles **c** 60°
d obtuse **e** scalene **f** 90°
2 C
3 **a** $c = 120$ **b** $x = 60$ **c** $x = 25 + 35$
4 **a** 70 **b** 10 **c** 25 **d** 58 **e** 50 **f** 29
5 **a** 65 **b** 80 **c** 40 **d** 20 **e** 112 **f** 32
6 **a** 145 **b** 144 **c** 45 **d** 60 **e** 60 **f** 47
7 **a** Yes 

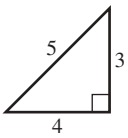
b Yes 

c No
d Yes 

e Yes

f Yes 

g No
h Yes 

i Yes 

8 **a** 65° **b** 115°
9 **a** $a = 60$ **b** $a = 120$ **c** $a = 70$, $b = 70$
d $a = 35$, $b = 105$
e $a = 115$, $b = 115$, $c = 45$, $d = 20$
f $a = 40$, $b = 100$, $c = 40$, $d = 40$
10 **a** $a°$, alternate **b** $b°$, alternate **c** Sum to 180°
11 **a** **i** 50° **ii** 130°
b **i** 20° **ii** 160°
c **i** 0° **ii** 180°

7C

Now you try

Example 6
120°

Example 7
a $x = 100$, $y = 145$; cointerior angles
b $x = 90$; kite has a pair of equal, opposite angles, and angle sum of a quadrilateral

Exercise 7C

1 Parallelograms, incl. squares, rectangles and rhombuses.
2 **a** square, rectangle, rhombus, parallelogram
b rectangle, square, parallelogram, rhombus, kite
c rhombus, square, parallelogram, rectangle
d trapezium
e kite
f square, rectangle
g square, rectangle
h square, rhombus, kite
3 **a** $a = 144$ **b** $b = 79$ **c** $c = 54$
4 **a** $x = 20$ **b** $x = 110$ **c** $x = 240$
d $x = 125$ **e** $x = 30$ **f** $x = 65$
5 **a** $x = 60$ (co-interior angles in parallel lines)
$\therefore y = 120$ (opposite angles in a parallelogram)
$z = 60$ (co-interior angles in parallel lines)

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b $x = 110$ (co-interior angles in parallel lines)
$y = 110$ (co-interior angles in parallel lines)
$z = 70$ (co-interior angles in parallel lines)
c $x = 30$ (co-interior angles in parallel lines)
$y = 150$ (co-interior angles in parallel lines)
$z = 30$ (co-interior angles in parallel lines)
d $x = 45$ (angle sum of a quadrilateral)
e $x = 100$ (angle sum of a quadrilateral)
f $x = 25$ (angle sum of a quadrilateral)
g $x = y = z = 90$ (angles in a square)
h $x = 100$ (co-interior angles in parallel lines)
$y = 140$ (co-interior angles in parallel lines)
i $x = 75$ (co-interior angles in parallel lines)
$y = 20$ (co-interior angles in parallel lines)

6 **a** 115 **b** 60 **c** 30
d 50 **e** 90 **f** 140

7 **a** It has two equal side lengths.
b **i** 120 **ii** 40
c It has two equal side lengths and two pairs of equal angles and one pair of parallel sides.

8 **b** 65°

9 **a** 125 **b** 118 **c** 110

10 **a** $a = 30,\ b = 120,\ c = 60,\ d = 60,\ e = 30$
b $a = 80,\ b = 100,\ c = 80,\ d = 50$
c $a = 40,\ b = 20,\ c = 50,\ d = 110$
d $b = 50,\ c = 70$
e $a = 60,\ b = 20$
f $a = 10,\ b = 80$

7D

Now you try

Example 8
$S = 540°, x = 125$

Example 9
a 35 cm **b** 900° **c** 128.57°

Exercise 7D

1 **a** 4 **b** 8 **c** 10 **d** 7
e 9 **f** 6 **g** 5 **h** 12

2 **a** 540° **b** 720° **c** 900° **d** 1080°
e 1260° **f** 1440°

3 All sides and all angles equal.

4 **a** 720°, 110 **b** 540°, 130 **c** 540°, 30
d 900°, 105 **e** 720°, 30 **f** 360°, 30

5 **a** 24 cm **b** 720° **c** 120°

6 **a** 28 cm **b** 1080° **c** 135°

7 **a** 720° **b** 120°

8 **a** 1620° **b** 3240°

9 **a** 144° **b** 165.6°

10 **a** 72° **b** 252°

11 **a** $x = 108,\ y = 72$
b $x = 60,\ y = 60$
c $x = 45,\ y = 225$

12 **a** *See table at base of this page.*

12 **a**

Polygon	No. of sides	Diagram	No. of triangles	Interior angle sum (S)	Single interior angle (A)
triangle	3		1	180°	60°
quadrilateral	4		2	360°	90°
pentagon	5		3	540°	108°
hexagon	6		4	720°	120°
…					
n-gon	n		$n - 2$	$(n - 2) \times 180°$	$\frac{(n - 2) \times 180°}{n}$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b i $S = 180° \times (n - 2)$

ii $A = \dfrac{180° \times (n - 2)}{n}$

Progress quiz

1 a $a = 70$ (vertically opposite), $b = 70$ (alternate angles in parallel lines)

b $c = 70$ (cointerior in parallel lines), $d = 110$ (corresponding angles in parallel lines or supplementary)

2 a Yes, cointerior angles sum to 180°.

b No, alternate angles are not equal.

3 a $x = 30$ **b** $a = 68$ **c** $b = 100$

d $x = 128, y = 52$

4 85

5 a Sum $= 720°$, $m = 150$

b Sum $= 540°$, $a = 6$, $x = 108$

7E

Now you try

Example 10

9

Example 11

a 4 cm **b** 3500 **c** 5 cm **d** 175 m

Exercise 7E

1 similar

2 a equal **b** ratio **c** scale factor

3 a 1.5 **b** 1.5, the same **c** 1.5

4 a 8 **b** 21 **c** 4 **d** 1.5

5 a 2 cm **b** 1500 **c** 5 cm **d** 75 m

6 a 5 cm **b** 1500 **c** 4 cm **d** 60 m

7 a $\Delta ABC, \Delta ADE$

b $\angle A$ is common and $\angle ABC = \angle ADE$

c 2.5

d 3.75 m

8 a $x = 1.5$

b $x = 9$

c $x = 2.2$

9 a i 1 km ii 3 km

b i 10 cm ii 1 cm

c 2 km

10 a $\angle BAC = \angle DEC$ (alternate), $\angle ABC = \angle EDC$ (alternate), $\angle ACB = \angle ECD$ (vertically opposite)

b i 1.5 ii $DC = 6$ cm iii $AC = 6$ cm

7F

Now you try

Example 12

a 3.5

b 10.5

Example 13

$h = 8$

Exercise 7F

1 a 2 **b** 9, 21 **c** 5

2 a No **b** No

3 a 2 **b** 30 cm

4 a 1.5 **b** 4.5 m

5 a $\dfrac{88}{5} = 17.6$ **b** $x = 4.5$

6 1.25 m

7 a 5 or $\dfrac{1}{5}$ **b** 5 m

8 a $\dfrac{6}{5} = 1.2$ **b** 13.2 m

9 1.90 m

10 Answers will vary.

11 a 1.5

b 20 m

c Let $AE = x$

$1.5x = x + 10$

$\therefore x = 20$

7G

Now you try

Example 14

a i 5 ii 6

b i 3 ii 2 iii 3 iv 3 v 1

c 12

d i A, C, D and E ii B

Example 15

a 3 **b** 4

Exercise 7G

1 a 4

b 3

c i 3 ii 3 iii 3

d i 3 ii 3 iii 2

2 a 3 **b** 3 **c** 4 **d** 8

e The sum of degrees is twice the total number of edges.

3 a i 3 ii 4

b i 2 ii 3 iii 3

c 8

d i B and C ii A

4 a i 4 ii 5

b i 3 ii 1 iii 3 iv 3

c 10

d i A, B, C and D ii none

5 a i 4 ii 5

b i 2 ii 3 iii 3 iv 2

c 10

d i B and C ii A and D

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

6 C
7 B
8 **a** 2 **b** 3 **c** 3
9 C
10 B
11 Yes
12 **a** 8 **b** 16
c The sum of degrees is twice the number of edges.
13 **a** No **b** No **c** Yes
14 **a** No **b** No **c** No

7H

Now you try

Example 16
No

Example 17

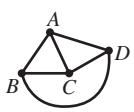
a Yes
b No

Example 18

a **i** 4 **ii** 6 **iii** 4
b $v + f = 8$ and $e + 2 = 8$

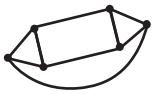
Exercise 7H

1 **a** Answers will vary, a rectangle of ABCD would be an example
b Yes
2 **a** **i** 5
ii 7
iii 4
b **i** 9
ii 9
c They are equal.
3 **a** Yes **b** No **c** No
d Yes **e** Yes
4 **a** Yes **b** Yes **c** No
d No **e** Yes **f** Yes
5 **a** **i** 4 **ii** 6 **iii** 4
b $v + f = 8$ and $e + 2 = 8$
6 **a** **i** 5 **ii** 7 **iii** 4
b $v + f = 9$ and $e + 2 = 9$
7 A and C
8 A and C
9 **a** 12 **b** 6 **c** 4
10 **a** $v = 4,\ e = 6$ and $f = 4$
b $v + f = 8$ and $e + 2 = 8$
c For example



d $v + f = 8$ and $e + 2 = 8$

11 **a** $v = 6,\ e = 9$ and $f = 5$
$v + f = 11$ and $e + 2 = 11$
one possible graph



b $v = 8,\ e = 12$ and $f = 6$
$v + f = 14$ and $e + 2 = 14$
one possible graph

12 A, B, C, D and E

7I

Now you try

Example 19

a **i** No **ii** Yes
b **i** No **ii** Yes
c **i** Yes **ii** No
d **i** No **ii** Yes

Example 20
Yes, Eulerian trails will be circuits, since there are zero vertices of odd degree.

Exercise 7I

1 **a** Walk **b** Circuit **c** Cycle **d** Trail **e** Path
2 **a** B and C
b e.g. $B - A - C - B - D - C$
c Eulerian trail
d No
e 4
f 0 or 2
3 **a** **i** Yes **ii** Yes
b **i** Yes **ii** No
c **i** No **ii** Yes
d **i** Yes **ii** No
4 **a** **i** Yes **ii** No
b **i** Yes **ii** No
c **i** No **ii** Yes
d **i** Yes **ii** No
5 **a** **i** Yes **ii** No
b **i** Yes **ii** No
c **i** Yes **ii** No
d **i** Yes **ii** No
6 **a** Yes, Yes **b** Yes, No **c** Yes, No
d Yes, Yes **e** No **f** No
7 **a** Yes **b** No
c Yes **d** No

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8 a $B-A-E-D-B-C-D$
$B-A-E-D-C-B-D$
$B-C-D-E-A-B-D$
$B-C-D-B-A-E-D$
$B-D-C-B-A-E-D$
$B-D-E-A-B-C-D$

b $D-C-B-D-E-A-B$
$D-C-B-A-E-D-B$
$D-B-C-D-E-A-B$
$D-B-A-E-D-C-B$
$D-E-A-B-C-D-B$
$D-E-A-B-D-C-B$

9 6

10 a Graph 2 **b** Bridges **c** 4
d Does not have an Eulerian trail

11 a T **b** T **c** F
d F **e** T **f** F

12 a Yes, 4
C, F, L, G, H, C
C, H, G, L, F, C
C, L, G, H, F, C
C, F, H, G, L, C

b No

7J

Now you try

Example 21

a 13 m
b 14 m

Example 22

a i 19 m **ii** 15 m
b i 29 m **ii** 25 m

Exercise 7J

1 a 2 **b** 4 **c** 3 **d** 12 km
e 27 km

2 a i 15 km **ii** 16 km
b 9 km
c A, B, D and F

3 a 26 km **b** 24 km

4 a 26 m **b** 39 m

5 a 15 cm **b** 19 cm

6 a i 4 km **ii** 12 km
b i 6 km **ii** 5 km

7 a 11 km **b** 9 km
c 6 km **d** 7 km

8 a 7 cm **b** 7 cm

9 A-B-C is shorter than A-C and C-D-E is shorter than C-E.

10 a A-B-C-D-E-G
b $ 270

11 8 km

Maths@Work: Pool builder

1 a 7.05 m **b** 1 m
c 60 cm **d** 3 m^2
e 82° **f** 10

2 a Hexagon (6 sides)
b 720°
c 720°
d Drawing should be the same shape but 4 times larger.

3 a Octagon and square (a quadrilateral)
b The octagon is not but the square is
c Octagon 135° and square 90°
d If the square was not used, there would be gaps and hence no tessellation.
e They should all join up with no gaps.

Puzzles and challenges

1 30
2 CONGRUENCE
3 130°
4 a 7 **b** 11
5 20 m

Chapter checklist with success criteria

1 $a° = 108°, b° = 108°, c° = 108°, d° = 72°$
2 No, co-interior angles are supplementary.
3 69°
4 65°
5 65°
6 116°
7 115°
8 720°
9 4.5
10 7.5 m
11 2.5 cm
12 4.2 m
13 i 5 **ii** 3
14 4
15 Yes
16 Planar
17 $v = 4, e = 6, f = 4$
18 i yes **ii** yes
19 Yes, no
20 33 km
21 260 m

Chapter 7 review

Short-answer questions

1 a $x = 70, y = 110$ **b** $x = 120, y = 120$
c $x = 65, y = 115$ **d** $x = 30, y = 150$
e $x = 90, y = 120$ **f** $x = 45$

2 a 20 **b** 30 **c** 77 **d** 20
e 60 **f** 30 **g** 130 **h** 70
i 160

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 **a** Square, rectangle, rhombus, parallelogram
b Parallelogram, square, rhombus, rectangle
c Kite
d Square, rhombus, kite

4 **a** $a = 90$ **b** $a = 40$ **c** $a = 110, b = 30$
d $a = 90, b = 130$ **e** $a = 40, b = 140$
f $a = 110$

5 **a** 540°, 60 **b** 720°, 100 **c** 900°, 120

6 **a** 10 m **b** 540° **c** 108°

7 **a** $\Delta ABE, \Delta ACD$ **b** 2.5 **c** 7.5

8 **a** 6.25 **b** 187.5 cm

9 **a** 2 cm **b** 5000 **c** 5 cm **d** 250 m

10 **a** No, *BC* not joined by an edge
b **i** Planar **ii** Planar

11 **a** $e = 6, v = 4$
b $f = 4$
c Yes, $v + f = e + 2$

12 **a** No, *AC* repeated
b It has more than 2 odd vertices.
c *DF* or *DB* or *DA*
d Answer will vary. It will not be an Eulerian circuit because it has two odd vertices. (A Eulerian circuit has no odd vertices.)

13 **a** 15 cm **b** 12 cm

Multiple-choice questions

1 B **2** A **3** D **4** E **5** E **6** B
7 A **8** D **9** E **10** C **11** C **12** D
13 A

Extended-response questions

1 **a** 60, 60, 30 **b** 2.5 **c** 8 cm

2 **a** $\frac{500}{8}$ **b** 281.25 cm

3 **a** 14
b 9 km
c $v + f = 5 + 4 = 9$ and $e + 2 = 7 + 2 = 9$
d No, every edge is not included exactly once.
e **i** 9 km **ii** 16 km

Chapter 8

Warm-up quiz

1 **a** 15.84 **b** 164.87 **c** 0.87 **d** 0.58
e 0.17 **f** 0.71 **g** 12.99 **h** 14.30

2 **a** 25 **b** 46.24 **c** 361
d 225 **e** 43.25 **f** 81

3 **a** 2.8 **b** 2.6 **c** 3.9 **d** 3.2
e 3.6 **f** 3.0 **g** 1.9 **h** 14.1

4 **a** c **b** p **c** y **d** PQ
e BC **f** XY

5 **a** $x = 3$ **b** $x = 4$ **c** $x = 35$ **d** $x = 9$

6 **a** $m = 3.65$ **b** $m = 1.2$
c $m = 5.2$ **d** $m = 5.724$

7 **a** $x = 0.6$ **b** $x = 0.2$ **c** $x = 2.1$ **d** $x = 0.4$
e $x = 2.4$ **f** $x = 9.2$

8 **a** $x = 4$ **b** $x = 20$ **c** $x = 13$ **d** $x = 4$

9 **a** 30° **b** 50° **c** 54° **d** 90° **e** 45°
f 30° **g** 82° **h** 62° **i** 35°

8A

Now you try

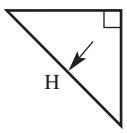
Example 1
26

Example 2
12.5 m

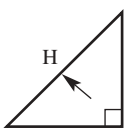
Exercise 8A

1 **B** and **C**

2 **a**



b



3 **a** $z^2 = x^2 + y^2$ **b** $t^2 = m^2 + n^2$ **c** $s^2 = p^2 + r^2$

4 **a** 25 **b** 34 **c** 45

5 **a** 5 **b** 17 **c** 13
d 25 **e** 41 **f** 15

6 **a** 5.8 cm **b** 7.3 mm **c** 18.0 m
d 11.2 km **e** 8.6 cm **f** 12.8 cm

7 **a** 8.91 cm **b** 3.62 m **c** 5.02 km
d 13.95 mm **e** 17.49 cm **f** 24.21 cm

8 181.5 cm

9 35.3 m

10 13 m

11 5.28 m

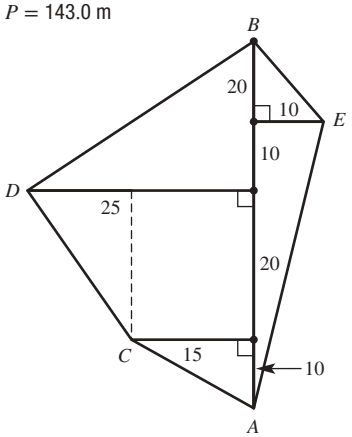
12 2.64 m

13 494.97 m

14 **a** No **b** No **c** Yes
d Yes **e** No **f** Yes

15 **a** **i** $AC = 11.2$ **ii** $BC = 30.4$
iii $DB = 21.2$ **iv** $AD = 29.2$
b 92 m
c 400 m^2

16 $P = 143.0$ m



CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8B

Now you try

Example 3
3

Example 4
17.0

Exercise 8B

1 **a** 10 **b** 24 **c** 41 **d** 1.5

2
$$10^2 = w^2 + 6^2$$
$$w^2 + 6^2 = 10^2$$
$$w^2 + 36 = 100 \quad (-36)$$
$$w^2 = 64 \quad (-36)$$
$$w = 8$$

3 **a** $17^2 = 8^2 + w^2$ **b** $13^2 = m^2 + 5^2$
c $15^2 = x^2 + 9^2$

4 **a** 4 **b** 7 **c** 8
d 5 **e** 14 **f** 4.8

5 **a** 7.1 **b** 13.3 **c** 12.3
d 6.2 **e** 6.6 **f** 16.2

6 **a** 7.14 **b** 13.90 **c** 3.87
d 133.84 **e** 17.89 **f** 39.19

7 5.66 m

8 2.2 m

9 3.2 m

10 **a** **i** 3 cm **ii** 5.2 cm
b 15.6 cm^2

11 7.4 cm

12 **a** 7.1 **b** 4.5 **c** 5.2

8C

Now you try

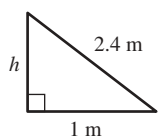
Example 5
5 m

Example 6
36.1 km

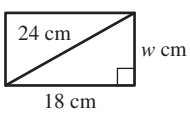
Example 7
9.22 mm

Exercise 8C

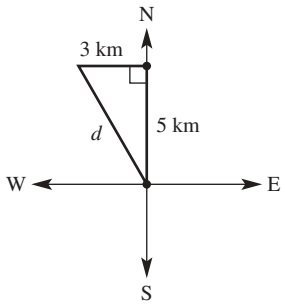
1 **a**



b



c

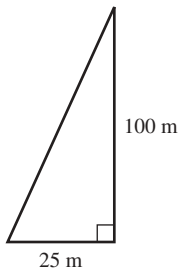


2 **a** ΔADE **b** ΔPRO and ΔRQO **c** ΔAOB

3 **a** AB **b** AB **c** FH
d BG **e** RT **f** EB

4 **a** 100 m

b



c 103.08 m

5 **a** 8.6 m **b** 13 m **c** 10 m

6 6.9 km

7 **a** 10 km **b** 41 km **c** 17 km

8 **a** 10.6 **b** 10.8 **c** 15.8 **d** 4.2

9 25.04 m

10 **a** 11.18 m **b** 12.2 m

11 Horizontal = 45.25 cm
Vertical = 71.66 cm

12 **a** 4.77 m **b** 4.90 cm

13 **a** 11.2 m **b** 8.1 cm

14 **a** 1.41 cm **b** 1.73 cm

15
- ΔPQR is a right-angled isosceles triangle
- With $\angle P = \angle R = 45°$
- The hypotenuse = $\sqrt{200}$ cm
- $PQ = QR = 10$ cm
- Perimeter = 34.1 cm
- Area = 50 cm^2

8D

Now you try

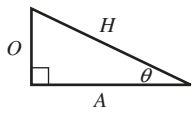
Example 8
a $\frac{15}{17}$ **b** $\frac{8}{17}$ **c** $\frac{15}{8}$

Example 9
$\frac{7}{\sqrt{58}}$

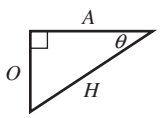
CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Exercise 8D

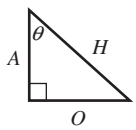
1 a



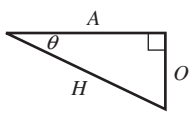
b



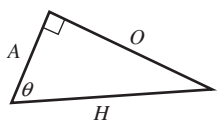
c



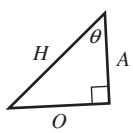
d



e



f



2 a PR **b** TP **c** TP
d PR **e** TR **f** $\angle T$

3 a BC **b** CA **c** BA **d** BA

4 a $\frac{3}{5}$ **b** $\frac{4}{5}$ **c** $\frac{4}{5}$

5 a i $\frac{b}{a}$ **ii** $\frac{c}{a}$ **iii** $\frac{b}{c}$

b i $\frac{n}{p}$ **ii** $\frac{m}{p}$ **iii** $\frac{n}{m}$

c i $\frac{y}{z}$ **ii** $\frac{x}{z}$ **iii** $\frac{y}{x}$

d i $\frac{4}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{4}{3}$

e i $\frac{24}{26} = \frac{12}{13}$ **ii** $\frac{10}{26} = \frac{5}{13}$ **iii** $\frac{24}{10} = \frac{12}{5}$

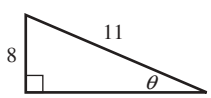
f i $\frac{12}{13}$ **ii** $\frac{5}{13}$ **iii** $\frac{12}{5}$

6 a $\frac{a}{c}$ **b** $\frac{y}{z}$ **c** $\frac{b}{a}$

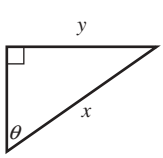
d $\frac{6}{10} = \frac{3}{5}$ **e** $\frac{40}{41}$ **f** $\frac{1}{\sqrt{3}}$

g $\frac{8}{6} = \frac{4}{3}$ **h** $\frac{9}{41}$ **i** $\sqrt{3}$

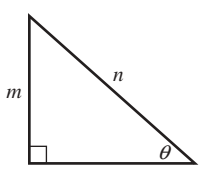
7 a



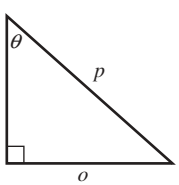
b



c



d



8 a $\frac{y}{x}$ **b** $\frac{z}{x}$ **c** $\frac{z}{x}$

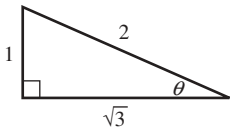
d $\frac{y}{x}$ **e** $\frac{y}{z}$ **f** $\frac{z}{y}$

9 a $\sin\theta$ **b** $\tan\theta$ **c** $\sin\theta$

d $\cos\theta$ **e** $\cos\theta$ **f** $\tan\theta$

10 a $QR = 13$ m **b** $\sin\theta = \frac{5}{13}$

11 a



b $\sqrt{3}$

c i $\cos\theta = \frac{\sqrt{3}}{2}$ **ii** $\tan\theta = \frac{1}{\sqrt{3}}$

12

Angle (θ)	sin θ	cos θ
0°	0	1
5°	0.087	0.996
10°	0.174	0.985
15°	0.259	0.966
20°	0.342	0.940
25°	0.423	0.906
30°	0.5	0.866
35°	0.574	0.819
40°	0.643	0.766
45°	0.707	0.707
50°	0.766	0.643
55°	0.819	0.574
60°	0.866	0.5
65°	0.906	0.423
70°	0.940	0.342
75°	0.966	0.259
80°	0.985	0.174
85°	0.996	0.087
90°	1	0

a 45°

b i 85 **ii** 80 **iii** 30 **iv** 0

c If angles θ and α sum to 90°, $\sin\theta = \cos\alpha$.

d It's the same as the complement of sine.

8E

Now you try

Example 10
3.86

Example 11
a 1.03 **b** 8.78 **c** 14.18

Exercise 8E

1 a 0.1736 **b** 0.9848 **c** 0.1763 **d** 0.5774
e 0.7660 **f** 0.9397 **g** 0.1719 **h** 0.8541
i 0.9870 **j** 0.8682 **k** 0.6401 **l** 1.7379

2 a 2.12 **b** 5.07 **c** 31.18
d 46.43 **e** 8 **f** 18.79

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

3 **a** sin **b** cos **c** tan
d cos **e** tan **f** sin
4 **a** $x = 1.37$ **b** $x = 5.12$ **c** $x = 91.44$
d $x = 13.86$ **e** $x = 9.19$ **f** $x = 9.19$
5 **a** 0.39 **b** 4.50 **c** 2.60 **d** 11.15 **e** 16.80
f 5.78 **g** 7.83 **h** 13.49 **i** 1
6 **a** 2.11 **b** 4.04 **c** 1.88
7 **a** 5.36 **b** 1.27 **c** 0.52
8 **a** 5.49 **b** 8.51 **c** 9.23
9 **a** 3.76 **b** 2.12 **c** 2.82
d 4.94 **e** 4.14 **f** 0.75
10 **a** 26.33 m **b** 52.66 m **11** 6.96 m
12 **a** $b = 1.27, l = 2.72$ **b** $b = 0.68, l = 1.88$
c $b = 3.06, l = 2.57$
13 **a** $a = 3.5, b = 3.2, x = 1.4$
b $a = 3.464, b = 3.139, x = 1.327$
c Yes, it makes a difference. It is better not to round off during the process as sometimes it can change the final answer.

8F

Now you try

Example 12
a 4.99 **b** 10.44 **c** 3.54

Exercise 8F

1 **a** 17.32 **b** 13.86 **c** 106.73 **d** 19.84
e 24.69 **f** 13.44
2 **a** $\frac{10}{x}$ **b** $\frac{1.4}{m}$ **c** $\frac{19}{x}$ **d** $\frac{2.8}{w}$
3 C
4 **a** 8.77 **b** 9.44 **c** 8.49
5 **a** 4.62 **b** 23.39 **c** 2.86
6 **a** 5.96 **b** 1.62 **c** 1.72
7 **a** 4.73 **b** 6.15 **c** 6.14
d 2.98 **e** 26.08 **f** 27.82
8 2.54 m **9** 13.9 m
10 **a** $AB = 42.89$ cm, $BC = 20$ cm
b $AB = 5.32$ m, $BC = 1.82$ m
c $AB = 14.62$ cm, $BC = 13.74$ cm
11 **a** 7.464 m **b** 7.727 m
12 **a** 30.5 m **b** 17.5 m
13 **a** 17.16 **b** 30 **c** 4.01 **d** 59.78
e 51.13 **f** 38.09

Progress quiz

1 **a** 26 m **b** 10.3 m
2 **a** $x = 6$ **b** $x = 7.1$
3 4.0 inches
4 4.47 m
5 12.6 cm
6 **a** $\cos\theta = \frac{40}{41}$ **b** $\sin\theta = \frac{9}{41}$ **c** $\tan\theta = \frac{9}{40}$
7 **a** $x = 10.9$ **b** $x = 6.3$
8 **a** $x = 8.7$ **b** $x = 6.7$
9 **a, b** 15.6 cm^2

8G

Now you try

Example 13
a 46° **b** 68° **c** 53°

Example 14
a 36.87° **b** 27.49° **c** 73.95°

Exercise 8G

1 $\theta = 30°$
2 **a** $\theta = 30°$ **b** $\theta = 60°$ **c** $\theta = 45°$
3 **a** $\cos\theta = \frac{5}{12}$ **b** $\sin\theta = \frac{7}{10}$ **c** $\tan\theta = \frac{4}{3}$
4 **a** 30° **b** 53° **c** 61° **d** 45°
e 41° **f** 53° **g** 48° **h** 6°
i 37° **j** 81° **k** 73° **l** 60°
m 42° **n** 48° **o** 34°
5 **a** 60° **b** 45° **c** 64.16° **d** 48.59°
e 53.13° **f** 68.20°
6 **a** 41°49′ **b** 56°15′ **c** 63°37′ **d** 48°49′
e 53°45′ **f** 67°23′
7 10°37′
8 15°57′
9 46°39′
10 **a** 36.9°, 53.1° **b** 34.8°, 55.2°
c 36.9°, 53.1° **d** 26.6°, 63.4°
e 68.2°, 21.8°
11 Pitch $A = 47°, B = 43°$
12 **a** 5.54 m **b** 5.97 m
13 **a** 12°
b Yes, as $\theta \approx 9.7°$
c 286.4 cm

8H

Now you try

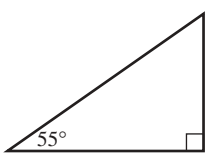
Example 15
28 m

Example 16
137.2 m

Example 17
8°

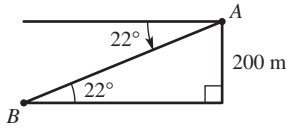
Exercise 8H

1 **a** depression **b** elevation **c** horizontal
2 **a** 30° **b** 30° **c** 60°
3 **a**

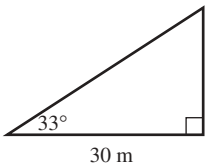


CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b



c



4 **a** 21.88 m **b** 43.5 m **c** 23.41 m
d 6.06 m **e** 536.29 m **f** 38.97 m

5 **a** 112.0 m **b** 49 m **c** 86 m **d** 105 m **e** 9260 m

6 **a** 26.6° **b** 29.7° **c** 5.7° **d** 47.1° **e** 48.3°

7 38.35 m **8** 1.25 km

9 280.04 m

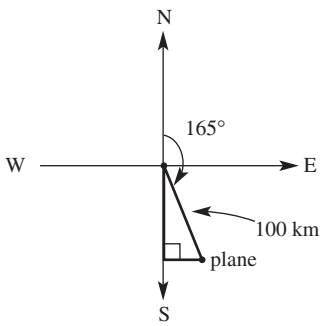
10 **a** 30° **b** 30° **c** Equal due to parallel lines **d** 1.7

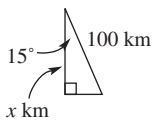
11 Answers will vary.

8I

Now you try

Example 18





Example 19

187.9 km

Exercise 8I

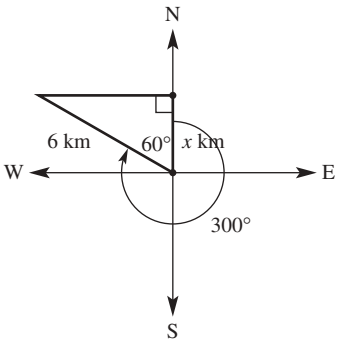
1 **a** 000°T **b** 045°T **c** 090°T **d** 135°T
e 180°T **f** 225°T **g** 270°T **h** 315°T

2 **a** v **b** iv **c** i
d iii **e** vi **f** ii

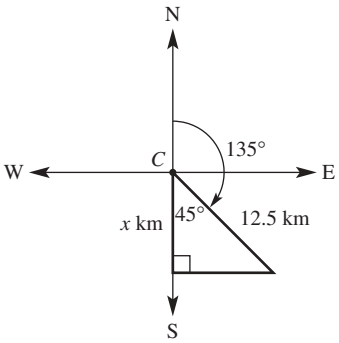
3 **a** 130° **b** 340° **c** 090°
d 128° **e** 313° **f** 355°

4 **a** north **b** east **c** west
d 060° **e** 240°

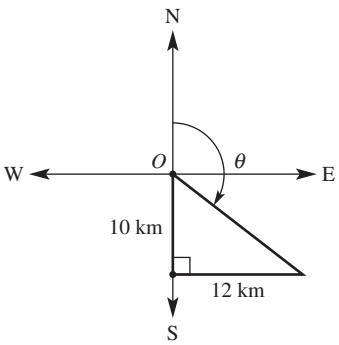
5 **a**



b



c



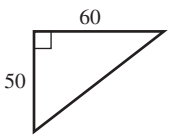
6 6.9 km

7 3.86 km

8 93.97 km

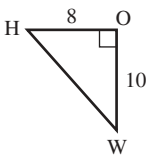
9 **a** 51.42 km **b** 61.28 km **c** 320°

10 **a**



b 78.10 km **c** 50° **d** 050°

11 **a**



b 12.8 km **c** 39° **d** 321°

12 3.25 km

13 115 km

14 **a** 18.03 km **b** 146°

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

8J

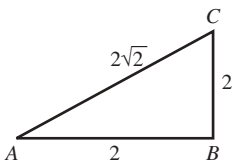
Now you try

Example 20

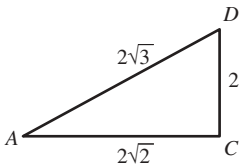
a $\sqrt{200}$ m **b** $\sqrt{300}$ m **c** 35°

Exercise 8J

1 a



b



c 35.3°

d 45°

2 61.4°

3 a 37.609 m **b** 45.47°

4 a 57.409 m **b** 57.91°

5 a i 26.57° **ii** 11.18 cm

b 10.14°

6 a 7.31 m **b** 6.87 m

7 138.56 m

8 a i 2.25 m **ii** 2.59 m

b 40.98°

c 3.43 m

9 a i 1.331 km **ii** 1.677 km

b 0.346 km

10 a camera C **b** 609.07 m

11 a 5.5 m **b** 34.5° **c** 34.7° **d** 0.2°

12 a 45° **b** 1.41 units **c** 35.26°

d 1.73 units

13 a i 1.55 **ii** 1.27 **iii** 2.82

b 34.34°

14 22°

15 a $\tan T = \frac{OA}{OT}$

b $OB = h \tan 48°$ or $OB = \frac{h}{\tan 42°}$

c 61.4 m

8K

Now you try

Example 21

6.9

Example 22

51.8°

Example 23

123.6°

Exercise 8K

1 a $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

b $\frac{12}{\sin \alpha} = \frac{6}{\sin \beta} = \frac{10}{\sin \theta}$

2 a 1.9 **b** 3.6 **c** 2.5

3 a 50.3° **b** 39.5° **c** 29.2°

4 a 7.9 **b** 16.5 **c** 19.1

d 9.2 **e** 8.4 **f** 22.7

5 a 38.0° **b** 51.5° **c** 28.8°

d 44.3° **e** 47.5° **f** 48.1°

6 a 1.367 km **b** 74° **c** 2.089 km

7 27.0°

8 131.0 m

9 a $\angle ABC = 80°$, $\angle ACB = 40°$ **b** 122 km

10 a $\angle ABC = 80°$

b 61.3 km

c 53.9 km

11 a 147.5° **b** 102.8° **c** 126.1°

d 100.5° **e** 123.9° **f** 137.7°

12 Impossible to find θ as such a triangle does not exist.

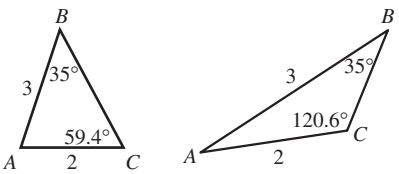
13 37.6° or 142.4°

14 a 6.1

b 104.0°

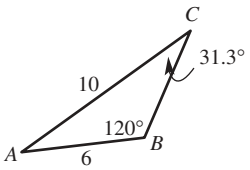
15 a 59.4° or 120.6°

b



c 31.3° **d** A triangle can have only one obtuse angle.

e



8L

Now you try

Example 24

11.24 cm

Example 25

78°17′

Exercise 8L

1 a $a^2 = 3^2 + 4^2 - 2 \times 3 \times 4 \times \cos 105°$

b $\cos \theta = \frac{5^2 + 9^2 - 7^2}{2 \times 5 \times 9}$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

2 **a** 9.6 **b** 1.5 **c** 87.3° **d** 36.2°
3 **a** 16.07 cm **b** 8.85 m **c** 14.78 cm
d 4.56 m **e** 2.86 km **f** 8.14 m
4 **a** 81.79° **b** 104.48° **c** 64.62°
d 61.20° **e** 92.20° **f** 46.83°
5 310 m
6 32.2°, 49.6°, 98.2°
7 **a** 145.9° **b** 208.2°
8 383 km
9 7.76 m
10 **a** cosine rule, 19.7
b sine rule, 12.8
c sine rule, 9.9
d cosine rule, 1.4
e sine rule, 3.0
f cosine rule, 2.4
11 **a, b** 106°, 35°
12 **a** $CP = b - x$
b $c^2 = x^2 + h^2$
c $a^2 = h^2 + (b - x)^2$
d $a^2 = c^2 - x^2 + (b - x)^2 = c^2 + b^2 - 2bx$
e $\cos A = \frac{x}{c}$
f $x = c\cos A$ substitute into part **d**.

8M

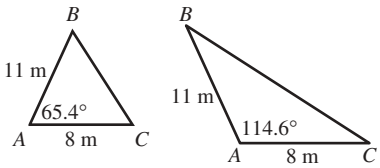
Now you try

Example 26
32.3 cm^2

Example 27
8.12 cm

Exercise 8M

1 **a** 3.7 **b** 28.8 **c** 48.0
2 **a** α **b** θ **c** β
3 **a** 56.44° **b** 45.58° **c** 58.05°
4 **a** 4.4 cm^2 **b** 26.4 m^2 **c** 0.9 km^2
d 13.7 m^2 **e** 318.4 m^2 **f** 76.2 cm^2
5 **a** 11.9 cm^2 **b** 105.6 m^2 **c** 1.6 km^2
6 **a** 5.7 **b** 7.9 **c** 9.1
d 18.2 **e** 10.6 **f** 1.3
7 **a** 59.09 cm^2 **b** 1.56 mm^2 **c** 361.25 km^2
8 **a** 35.03 cm^2 **b** 51.68 m^2 **c** 6.37 km^2
9 **a** 965.88 m^2 **b** 214.66 m^2 **c** 0.72 km^2
10 **a** 17.3 m^2 **b** 48 m^2 **c** 124.8 km^2
11 **a** Area $= ab\sin\theta$
b Area $= \frac{1}{2}a^2\sin 60° = \frac{\sqrt{3}}{4}a^2$
c Area $= \frac{1}{2}a^2\sin(180° - 2\theta) = \frac{1}{2}a^2\sin 2\theta$
12 **a** **i** 129.9 cm^2 **ii** 129.9 cm^2
a They are equal because sin 60° and sin 120° are equal.
b For example, same side lengths with included angle 140°.
13 **a** 65.4°, 114.6°
b



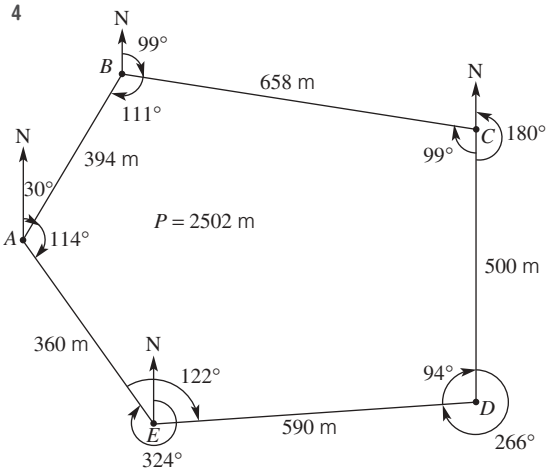
14 **a** **i** 540° **ii** 108° **iii** 11.89 cm^2 **iv** 8.09 cm
v 72°, 36° **vi** 19.2 cm^2 **vii** 43.0 cm^2
b 65.0 cm^2
c Answers may vary.

Maths@Work: Surveyor

1 **a** 10.94 m **b** 8.03 m **c** 51.15 m
d 38.55 m **e** 145.90 m **f** 1793.37 m
2 **a** 1300 m **b** 1174.87 m **c** 1835.79 m
d 1081.67 m **e** 7342.33 m **f** 320° **g** 265°
3 **a**

Marker	True bearing	Angle	Distance east	Distance north
A	055°	145°	80.28	56.21
B	082°	207°	124.14	17.45
C	051°	149°	122.63	99.31
D	078°	207°	102.14	21.71
E	-	192°	-	-

b 429.19 m
c 194.68 m
d 066°
4



ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Puzzles and challenges

1 **a** West **b** SW **c** NW **d** 198° **e** 120°

2 63.29 m

3 Answers will vary.

4

S														H
I	D	E	P	R	E	S	S	I	O	N				Y
N		E												P
E			G			S								O
				R		I	A	D	J	A	C	E	N	T
					E	D				A	N	G	L	E
						E							O	N
		T	R	I	A	N	G	L	E				P	U
	T	R	I	G	O	N	O	M	E	T	R	Y	P	S
		C	G	B						A	A		O	E
		O	H		E					N	T		S	
		S	T			A				G	I		I	
		I					R			E	O		T	
		N						I		N			E	
E	L	E	V	A	T	I	O	N	N	T				
					P	Y	T	H	A	G	O	R	A	S

5 068.8°T

Chapter checklist with success criteria

1 5.8 cm

2 12.21 cm

3 6.7 cm

4 6.40 km

5 13.60 cm

6 $\frac{3}{5}, \frac{4}{5}, \frac{3}{4}$

7 6.06

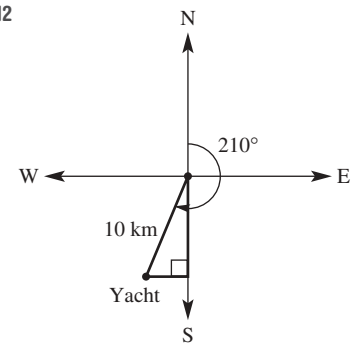
8 10.31

9 32.5°

10 22.7 m

11 53.1°

12



13 6.6 km

14 11.18 cm

15 8.21 cm

16 29.28°

17 4.08 cm

18 57.12°

19 10.04 cm^2

Chapter 8 review

Short-answer questions

1 No, $8^2 \neq 4^2 + 6^2$

2 **a** 15.17 m **b** 5.27 cm **c** 5.66 mm

3 **a** **i** BC **ii** AB **iii** AC

b **i** AC **ii** AB **iii** BC

4 **a** 0.57 **b** 0.96 **c** 8.45

5 **a** 4.50 **b** 1.5 **c** 0.95

6 **a** 6.31 **b** 12.94 **c** 4.09

7 **a** 3.63 m **b** 1.69 m

8 567.13 m

9 **a** 41°49′ **b** 59°00′ **c** 72°15′

10 **a** 45° **b** 42.4 m

11 **a** 2.3 km **b** 1.9 km **c** 040° T

12 **a** 15.43 m **b** 52°

13 **a** **i** 15.5 cm **ii** 135.0 cm^2

b **i** 14.9 cm **ii** 111.3 cm^2

14 28.1 m

15 52.6°

16 **a** 12.5 **b** 42.8°

Multiple-choice questions

1 B **2** E **3** C **4** B **5** C **6** C

7 D **8** E **9** A **10** C

Extended-response questions

1 **a** 54.46° **b** 130.25 m **c** 187 m

2 **a** 539 km **b** 68° **c** 248° T

3 **a** 33.346

b 3177.53 m^2 or 3177.54 m^2 if unrounded figures are used.

c $CD = 41.00$ m

d $AD = 61.60$ m

e **i** 65.66°, 114.34° **ii** 43.05 m, 80.15 m

Chapter 9

Warm-up quiz

1 **a** 9 **b** −18 **c** 9 **d** 0

2 **a** 14 **b** 6 **c** −16 **d** 32

3 **a** $3a$ **b** $3m$ **c** $8p$ **d** 0

e $-5m$ **f** $7x + y$ **g** $9p$ **h** $6m$

4 **a** $15x$ **b** $16p$ **c** $32xy$ **d** $-30a$ **e** ab

f x **g** 1 **h** $2a$ **i** 3

5 **a** 5 **b** 3 **c** 5 **d** 1

e 4 **f** 2 **g** 3 **h** 2

i 4 **j** 3 **k** 6 **l** 10

6 **a** $x + 3$ **b** $n + 6$ **c** $2w$ **d** $\frac{x}{2}$

e $2x + 6$ **f** $x - 7$ **g** $2(x + 3)$ **h** $3x + 1$

7 **a** $8x$ **b** $4x + 16$ or $4(x + 4)$ **c** $3x + 4$

8 **a** F **b** T **c** F **d** F **e** T

f T **g** T **h** F **i** T

9 **a** $m = 7$ **b** $y = 19$ **c** $x = 5$ **d** $b = 12$

10 **a** T **b** T

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9A

Now you try

Example 1
a $x = 16$ **b** $x = 18$ **c** $x = 9$ **d** $x = -12$

Example 2
$x = 4$

Example 3
$x = 28$

Example 4
$x = 7$

Example 5
a $x + 3 = 9, x = 6$ **b** $\frac{x}{3} - 7 = 0, x = 21$

c $\frac{x - 6}{3} = 10, x = 36$

Exercise 9A

1 **a** equals **b** one

2 **a**

x	0	1	2	3
$2x + 3$	3	5	7	9

b $x = 2$

3 **a** Yes **b** No **c** No **d** No
e Yes **f** Yes

4 **a** $t = 3$ **b** $m = 6$ **c** $x = 6$ **d** $m = 32$
e $a = -6$ **f** $m = 15$ **g** $x = 6$ **h** $x = 9$
i $x = 17$ **j** $x = 3$ **k** $x = -6$ **l** $x = 12$

5 **a** $p = 3$ **b** $c = 6$ **c** $d = 9$ **d** $p = 1$
e $m = -7$ **f** $p = -\frac{1}{2}$ **g** $x = 50$ **h** $m = 21$
i $a = -12$ **j** $z = 0$ **k** $w = \frac{3}{2}$ **l** $m = \frac{1}{2}$

6 **a** $x = 3$ **b** $x = 9$ **c** $x = -11$ **d** $x = 10$
e $x = 14$ **f** $x = 10$ **g** $x = 3$ **h** $x = 4$
i $x = 50$ **j** $x = 20$ **k** $x = 21$ **l** $x = 7$

7 **a** $x = 1$ **b** $x = 3$ **c** $x = 3$ **d** $x = -2$
e $x = -1$ **f** $x = -9$ **g** $x = 4$ **h** $x = 8$
i $x = 8$ **j** $x = -2$ **k** $x = -3$ **l** $x = 5$

8 **a** $x = 9$ **b** $x = 0$ **c** $x = 56$
d $x = 20$ **e** $x = 35$ **f** $x = 90$
g $x = -32$ **h** $x = -20$ **i** $x = -18$

9 **a** $m = 5$ **b** $a = 7$ **c** $x = 1$ **d** $x = 1$
e $n = 9$ **f** $m = 22$ **g** $w = -7$ **h** $m = 7$
i $w = 27$ **j** $a = 1$ **k** $a = -37$ **l** $m = -5$

10 **a** $x + 4 = 6, x = 2$ **b** $x + 12 = 8, x = -4$
c $x - 5 = 5, x = 10$ **d** $\frac{x}{3} + 2 = 8, x = 18$
e $2x + 3 = 9, x = 3$ **f** $\frac{x - 3}{5} = 6, x = 33$
g $3x + 4 = 16, x = 4$

11 **a** 13 cm **b** 12 mm

12 **a** 3 **b** 5 **c** 28 **d** 42 **e** 82

13 **a** 11, 12 **b** 6 **c** 3 **d** 25, 44 **e** 8 m

14 **a** $C = 60h + 40$ **b** \$280 **c** \$1120 **d** 6 h

15 **a** $v = 6t + 5$ **b** 3 min **c** 11 min

9B

Now you try

Example 6
$x = 4$

Example 7
$x = 2$

Example 8
$x = -2$

Example 9
$x = \frac{11}{4}$

Example 10
$x = 3$

Example 11
$x = 10$

Exercise 9B

1 **a** expand **b** Collect **c** one

2 C

3 E

4 **a** $x = 1$ **b** $x = 5$ **c** $x = -1$ **d** $a = 5$
e $a = 1$ **f** $x = 15$ **g** $m = 4$ **h** $d = 1$
i $a = 10$ **j** $a = 0$ **k** $x = 0$ **l** $a = 3$

5 **a** $x = 1$ **b** $x = 2$ **c** $x = 3$ **d** $x = 2$
e $x = 3$ **f** $x = 2$ **g** $x = -2$ **h** $x = -1$

6 **a** $x = 1$ **b** $x = 1$ **c** $x = 0$ **d** $x = 0$
e $x = 3$ **f** $x = 1$ **g** $x = 3$ **h** $x = -2$

7 **a** $x = 2$ **b** $x = 12$ **c** $x = -3$ **d** $x = 20$
e $x = 4$ **f** $x = 8$ **g** $x = 4$ **h** $x = -4$
i $x = -1$ **j** $x = 11$ **k** $x = 1$ **l** $x = -1$

8 **a** $x = 13$ **b** $x = 6$ **c** $x = 13$ **d** $x = 11$
e $x = 10$ **f** $x = 5$ **g** $x = 6$ **h** $x = 8$
i $x = -2$

9 **a** $x = 12$ **b** $x = 18$ **c** $x = 60$ **d** $x = 9\frac{1}{3}$
e $x = 5$ **f** $x = -6$

10 **a** $x = 10$ **b** $x = \frac{5}{3}$ **c** $x = 45$

11 **a** $x = 2$ **b** $x = 0$ **c** $x = \frac{3}{2}$

12 **a** $x = 3$ **b** $x = 5$ **c** $x = 10$ **d** $x = 30$
e $x = 7$

13 **a** $x = 7$ **b** $x = 3$ **c** $x = 5$ **d** $x = 2$
e $x = 5$ **f** $x = 6$ **g** $x = 11$

14 **a** $C = 850 + 156h$ **b** 7 h **c** 8:15 p.m.

15 **a** $x = 1$ **b** $x = -3$ **c** $x = 2\frac{2}{5}$ **d** $x = 8$
e $x = 3$ **f** $x = -\frac{8}{17}$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

9C

Now you try

Example 12
a $x = \pm 8$ **b** $x = \pm 3$ **c** No solution

Example 13
a $x = \pm\sqrt{29}$ **b** $x = \pm\sqrt{11} \approx \pm 3.3$

Example 14
a $x = \pm 5$ **b** No solution **c** $x = \sqrt{15}$ **d** $x = -3$

Exercise 9C

1 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144
2 **a** 16 **b** 81 **c** 49 **d** 9
e 64 **f** 1
3 **a** 6 **b** -7 **c** -10 **d** 0
e 20 **f** 40
4 **a** $c > 0$ **b** $c = 0$ **c** $c < 0$
5 **a** $x = \pm 5$ **b** $x = \pm 9$ **c** $x = \pm 6$
d $x = \pm 7$ **e** No solutions **f** No solutions
g $x = \pm 20$ **h** $x = \pm 12$ **i** $x = \pm 5$
j $x = \pm 4$ **k** $x = \pm 1$ **l** No solutions
m $x = \pm 8$ **n** $x = \pm 9$ **o** $x = \pm 6$
6 **a** **i** $x = \pm\sqrt{14}$ **ii** $x = \pm\sqrt{22}$ **iii** $x = \pm\sqrt{17}$
iv No solutions **v** $x = \pm\sqrt{5}$ **vi** $x = \pm\sqrt{6}$
vii $x = \pm\sqrt{21}$ **viii** $x = \pm\sqrt{6}$ **ix** $\pm\sqrt{15}$
b **i** $x = \pm 3.5$ **ii** $x = \pm 5.9$
iii $x = \pm 5.5$ **iv** $x = \pm 7.9$
7 **a** $x = \pm 2$ **b** $x = \pm 9$ **c** No solutions
d No solutions **e** $x = \pm 3$ **f** $x = \pm\sqrt{15}$
g $x = \sqrt{3}$ **h** $x = \pm 1$ **i** $x = \sqrt{2}$
j $x = -\sqrt{7}$ **k** No solutions **l** $x = \sqrt{5}$
8 **a** $x = \pm 4$ **b** $x = \pm 3$ **c** $x = \pm 3$
d $x = \pm 5$ **e** $x = \pm 2$ **f** No solutions
9 24 m
10 4 seconds
11 **a** $x = \sqrt{29}$ **b** $x = 8$ **c** $x = \sqrt{17}$ **d** $x = \sqrt{2}$
12 2.5 m
13 **a** $r = \pm 2.8$
b Since, in this practical scenario, r is the radius, then r must be greater than 0; i.e. $r = 2.8$.
14 **a** $b > 0$ **b** $b = 0$ **c** $b < 0$
15 **a** $b < 0$ **b** $b = 0$ **c** $b > 0$
16 **a** $x = \pm 5$
b **i** $-5 < x < 5$ **ii** $x > 5$ or $x < -5$
17 **a** $x = -4, 2$ **b** $x = -10, 4$ **c** $x = 0, 4$
d $x = 1, 9$ **e** $x = -3, 2$ **f** $x = 0, 1$
g $x = 0, \frac{4}{3}$ **h** $x = -\frac{3}{2}, 3$

9D

Now you try

Example 15
$y = 6$

Exercise 9D

1 **a** I **b** F **c** V
d A **e** c **f** P
2 **a** $m = 60$ **b** $A = 48$ **c** $A = 36$ **d** $v = 14.3$
e $m = 14$
3 **a** $t = 4$ **b** $t = 4$ **c** $t = 10$ **d** $t = 8$
4 **a** $b = 20$ **b** $b = 18$ **c** $b = 34$ **d** $b = 2.6$
5 **a** $h = 5$ **b** $h = 12$ **c** $h = 3$ **d** $h = 7$
6 **a** $b = 15$ **b** $b = 16$ **c** $b = 12$ **d** $b = 32$
7 **a** $h = 8$ **b** $h = 8$ **c** $h = 12$ **d** $h = 28$
8 **a** $m = 4$ **b** $m = 40$ **c** $m = 72$ **d** $m = 4$
9 **a** $h = 5.7$ **b** $h = 5.1$ **c** $h = 5.7$ **d** $h = 16.0$
10 **a** 86°F **b** -1.1°C **c** 212°F **d** -17.8°C
11 **a** \$32 **b** 60 km
12 **a** $P = 750$ **b** $t = 3.125$ **c** $r = 7.5$
13 **a** 1.5 tablets **b** 1250 mg
14 **a** 75 mL/h **b** $\frac{1}{3}$ h = 20 min
15 **a** Number of hours **b** 7.5 h
16 $P = 27.32$ cm, $A = 28.87$ cm^2
17 Iqra is now 17 and Urek is 7.

9E

Now you try

Example 16
$(1, -1)$

Exercise 9E

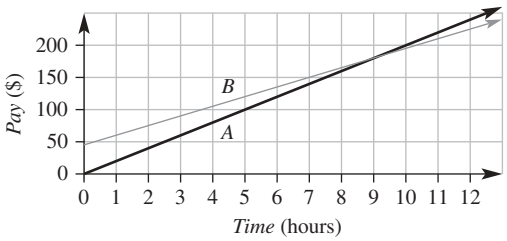
1 **a** 0 **b** 1
2 **a** $(0, 3)$ **b** $(1, 2)$ **c** No point of intersection
d $(2, 8)$ **e** No point of intersection **f** $(4, 3)$
g $(2, 5)$
3 **a** $(2, 3)$ **b** $(-1, -4)$
4 **a** $(2, 3)$ **b** $(2, 8)$ **c** $(0, 9)$ **d** $(-3, 8)$
5 **a** $(2, 3)$ **b** $(3, -2)$
6 **a** $(3, 9)$ **b** $(1, -3)$
7 **a** $(1, -4)$ **b** $(1, 1)$
8 **a** 200 km **b** \$1400
c Best removals, \$100 cheaper
d A+ removals, \$200 cheaper
9 **a**

Time in hours	0	1	2	3	4	5	6
Pay of company A	\$0	\$20	\$40	\$60	\$80	\$100	\$120
Time in hours	7	8	9	10	11	12	
Pay of company A	\$140	\$160	\$180	\$200	\$220	\$240	

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Time in hours	0	1	2	3	4	5	6
Pay of company B	$45	$60	$75	$90	$105	$120	$135
Time in hours	7	8	9	10	11	12	
Pay of company B	$150	$165	$180	$195	$210	$225	

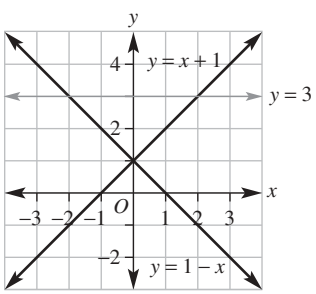
b



c 9 h

d $180

10 a



b (−2, 3)(0, 1)(2, 3) **c** 4 units, $\sqrt{8}$ units, $\sqrt{8}$ units

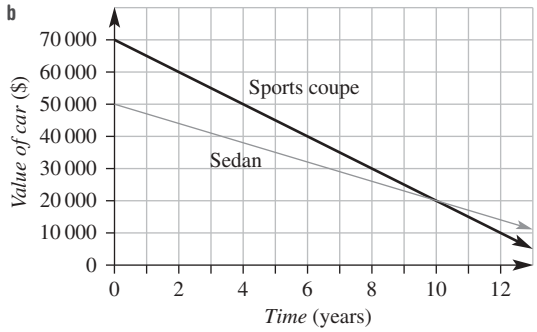
d Isosceles right-angled triangle

11 a

Time in years	0	2	4	6
Value of luxury sports coupe	$70 000	$60 000	$50 000	$40 000
Time in years	8	10	12	
Value of luxury sports coupe	$30 000	$20 000	$10 000	

Time in years	0	2	4	6
Value of sedan	$50 000	$44 000	$38 000	$32 000
Time in years	8	10	12	
Value of sedan	$26 000	$20 000	$14 000	

b



c 10 years

d $20 000

12 a No **b** Parallel lines

d (−6, −12), (−5, −9), (−4, −6), (−3, −3)

13 a Yes, (0, −1) **b** Same y-intercept

d (−2, −3), no points of intersection because parallel, (2, 5), (1, 3)

Progress quiz

1 a $x = 6$ **b** $y = 17$ **c** $m = 7$ **d** $x = -10$

2 a $x = 9$ **b** $y = -4$ **c** $x = 9$ **d** $x = 6$

e $y = 9$ **f** $x = -5$

3 a $x = 3$ **b** $a = -6$ **c** $y = 3$ **d** $x = 4$

4 a $2x + 3 = 25; x = 11$ **b** $\frac{x-3}{2} = 6; x = 15$

c $3(x - 2) = 12; x = 6$ **d** $\frac{4(x+2)}{2} = 16; x = 6$

5 a $x = -2$ **b** $x = 2$ **c** $x = 4$

6 a $x = \pm 5$ **b** $x = \pm 6$ **c** $x = \pm\sqrt{14}$

d No real solution **e** $x = \pm\sqrt{11}$ **f** $x = -2$

7 a $b = 4$ **b** $u = 5$ **c** $h = 2.4$

8 (1, 3)

9F

Now you try

Example 17

(1, 7)

Example 18

(−2, −3)

Example 19

The width is 6.5 cm and the length is 9.5 cm.

Exercise 9F

1 a Simultaneous **b** intersection

c substituted **d** substituted

e substitution

2 a A **b** C

3 a (2, 10) **b** (−5, −15) **c** (−4, −8)

d (1, 4) **e** (2, 2) **f** (2, 12)

4 a (2, 5) **b** (1, 3) **c** (2, 1) **d** (4, 3)

e (−5, −3) **f** (1, 6) **g** (4, 1) **h** (−1, −5)

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10
ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

5 **a** $(-1, 2)$ **b** $(3, -1)$ **c** $(4, 4)$
6 **a** $(2, 4)$ **b** $(-3, 2)$ **c** $(7, -1)$
7 **a** $(1, 5)$ **b** $(4, 10)$ **c** $(2, 1)$ **d** $(2, 9)$
8 Kye: 43 years old, Viviana: 38 years old
9 Length $= 18$ cm, width $= 6$ cm
10 Vanilla thick shake: \$5, fruity twirl: \$3
11 Carlos: 37 years old, Ella: 17 years old
12 **a** **i** Joe's: \$60, Paul's \$150
ii Joe's: 20 c/km, Paul's 10 c/km
iii Joe's $C = 0.2k + 60$, Paul's $C = 0.1k + 150$
iv 900 km
b Joe's if you are travelling less than 900 km and Paul's for more than 900 km.

9G

Now you try

Example 20
$(4, -1)$

Example 21
$(1, 3)$

Example 22
$(-1, 2)$

Example 23
The two numbers are 42 and 55.

Example 24
Buckets of chips cost \$4 and drinks cost \$2.50.

Exercise 9G

1 **a** $-$ **b** $+$ **c** $+$
2 **a** $6x - 4y = -2$ **b** $9x - 6y = -3$
c $12x - 8y = -4$
3 **a** B **b** E
4 **a** $(6, 1)$ **b** $(3, 4)$ **c** $(2, 7)$
5 **a** $(7, 2)$ **b** $(3, 8)$ **c** $(4, 1)$
6 **a** $(2, 5)$ **b** $(2, 3)$ **c** $(4, 2)$ **d** $(2, 2)$
7 **a** $(1, 1)$ **b** $(2, 1)$ **c** $(2, -1)$
8 **a** $(4, 1)$ **b** $(1, 3)$ **c** $(5, 1)$
9 **a** $(4, -3)$ **b** $(1, 1)$ **c** $(2, 0)$ **d** $(1, 1)$
10 **a** $(1, 1)$ **b** $(4, 2)$ **c** $(3, 4)$
11 **a** $(3, 1)$ **b** $(1, -10)$ **c** $(-2, -3)$ **d** $(0, 1)$
e $(-1, 2)$ **f** $(5, -2)$
12 Ayden: 36 years old, Tamara: 20 years old
13 **a** Let a be the number of apples and m be the number of mangoes.
$10a + 5m = 1250$,
$15a + 4m = 1350$
b Apples: 50c, mangoes: \$1.50
c \$8.00
14 Children: 2500, adults: 2500
15 **a** Let m be the number of multiple-choice and s be the number of Short-answer questions.
b $2m + 3s = 50, m + s = 22$
c 16 Multiple-choice questions
16 **a** $x + y = 16, x - y = 2, x = 9, y = 7$
b $x + y = 30, x - y = 10, x = 20, y = 10$
c $2x + y = 12, x + y = 7, x = 5, y = 2$
17 **a** $x = 1, y = 2$ **b** $x = 2, y = 2$
18 **a** **i** $g + 30$ **ii** $d + 30$
b $g = d + 31, g + 30 = 2(d + 30)$
c Gordon is aged 32 and his daughter is aged 1.
19 **a** $(4, -3)$ **b** $(1, 1)$ **c** $(3, 4)$ **d** $(2, 2)$
e $(\frac{1}{2}, -1)$ **f** $(-3, \frac{1}{3})$

Maths@Work: Nurse

1 **a** 1.5 mL **b** 0.6 mL **c** 1 mL
d 2 capsules **e** 5 tablets
2 **a** 114 mg **b** 107 mg **c** 27 mg **d** 471 mg
3 **a** 41.7 drops per minute
b 9:30 p.m. Tuesday
c 75 min or 1 h 15 min
4

Starting time	Time in minutes for IV	Time in hours and minutes for IV	Ending time
4:15 p.m.	315	5:15	9:30 p.m.
10 a.m.	222.2222	3:42	1:42 p.m.
1:25 p.m.	640	10:40	12:05 a.m.
6 p.m.	750	12:30	6:30 a.m.
2 a.m.	977.7778	16:18	6:18 p.m.

Puzzles and challenges

1 Each row, column and diagonal adds to 6.

9	−5	−4	6
−2	4	3	1
2	0	−1	5
−3	7	8	−6

2 **a** 64 **b** 8
c 29.3 **d** 18 years old
3 Many possible equations; e.g. $3x + 2 = -4$; $\frac{5x}{2} = -5$;
$2(x + 5) = 6$.
4 Many possible simultaneous equations; e.g. $x + y = 1, 2x - y = 8$.
5 **a** 4.5 **b** 117 km
6 Emily was jogging at 11.5 km/h and cycling at 23 km/h.

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Chapter checklist with success criteria

1 **a** $x = 4$ **b** $x = 20$
2 $x = 18$
3 $x = 9$
4 $x = 4$
5 $x = \frac{-3}{2}$
6 $x = 18$ m
7 $x = 6$
8 $x = \pm\sqrt{6}$
9 $a = 7$
10 $(2, -1)$
11 $(1, -1)$
12 28 and 19
13 $(2, -1)$
14 \$2 and \$1.50

Chapter 9 review

Short-answer questions

1 **a** $a = 8$ **b** $m = -30$ **c** $x = -8$
d $x = 8$ **e** $m = 0$ **f** $w = 15$
g $m = -0.2$ **h** $w = 4$ **i** $r = 6$
2 **a** $m = 2$ **b** $w = 8$ **c** $m = 10$
d $w = 8$ **e** $m = 14$ **f** $m = \frac{4}{3} = 1\frac{1}{3}$
g $a = \frac{3}{2} = 1\frac{1}{2}$ **h** $x = 1$ **i** $x = 3$
3 **a** $m = 3$ **b** $a = 7$ **c** $x = 4$
d $x = \frac{3}{2} = 1\frac{1}{2}$ **e** $m = 2\frac{1}{2}$ **f** $x = \frac{7}{8}$
g $x = 2$ **h** $x = 8$
4 **a** $p = 4$ **b** $p = 4$ **c** $p = -9$
d $p = -2$ **e** $p = -5$ **f** $p = 2$
5 **a** $6x = 420$, the number is 70.
b $x + 8 = 5$, the number is -3.
c $\frac{a}{9} = 12$, the number is 108.
d $x + 7 = 3$, the number is -4.
e $x + 2.3 = 7$, the number is 4.7.
6 **a** $x = \pm\sqrt{11}$ **b** $x = \pm\sqrt{3}$
c $x = \pm\sqrt{5}$
7 **a** $b = 8$ **b** $w = 3.5$ **c** $x = 2.4$
d $m = 10$ **e** $c = 35$
8 **a** $(1, 2)$ **b** $(2, 3)$ **c** No point of intersection
9 **a** $(1, 6)$ **b** $(3, 2)$ **c** $(0, 0)$
10 **a** $(3, 2)$ **b** $(5, -1)$
11 **a** $(-2, -3)$ **b** $(2, 1)$
12 4 and 11
13 **a** x: number of 20-cent coins, y: number of 50-cent coins;
$x + y = 160$, $20x + 50y = 5000$
b 100 20-cent coins and 60 50-cent coins
14 50 children

Multiple-choice questions

1 C **2** B **3** C **4** D
5 D **6** C **7** D **8** A
9 E **10** B

Extended-response questions

1 **a** $P = 6x - 4$
b **i** $x = 22$ **ii** 20 cm, 27 cm, 37 cm, 7 cm, 30 cm
c **i** $x = 26$; 24 cm, 31 cm, 45cm, 7 cm, 38 cm
ii $x = 38$; 36 cm, 43 cm, 69 cm, 7 cm, 62 cm
2 **a** **i** $C = 70t + 50$ **ii** $C = 100 + 60t$
b 3 h **c** $(5, 400)$

Semester review 2

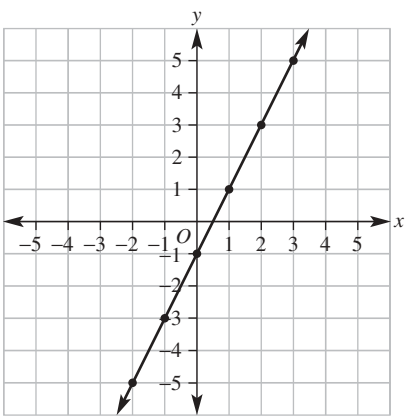
Chapter 6: Linear relationships, hyperbolas, parabolas and exponentials

Short-answer questions

1 **a** \$6000 **b** 8 years **c** $\approx$ \$500
2 **a** **i** $\approx$ 16 km **ii** $\approx$ 24 km **iii** $\approx$ 32 km
b 16 km/h **c** 1.5 hours **d** 45 km
e 2.5 hours **f** 18 km/h **g** 90 km
3 **a**

x	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5

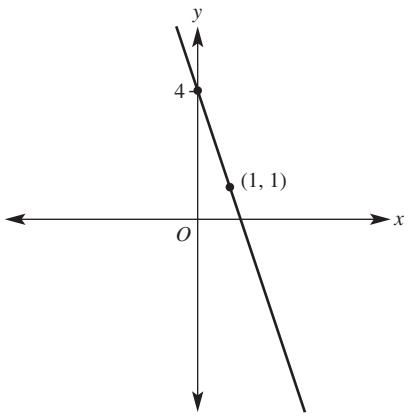
b $(-2, -5), (-1, -3), (0, -1), (1, 1), (2, 3), (3, 5)$
c



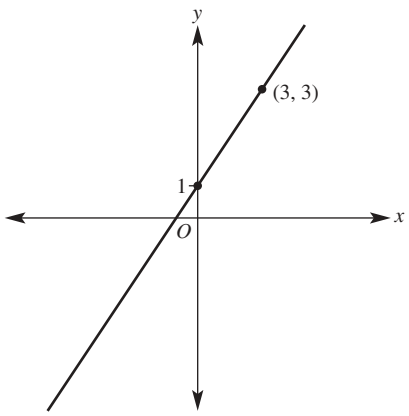
4 **a** $m = \frac{1}{4}$
b $m = -1$
c Gradient undefined
5 **a** $m = 2$ **b** $m = -3$
6 **a** Line J: y-intercept $= -2$, $m = 2$, $y = 2x - 2$
Line K: y-intercept $= 6$, $m = -\frac{2}{3}$, $y = -\frac{2}{3}x + 6$
b $(3, 4)$

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

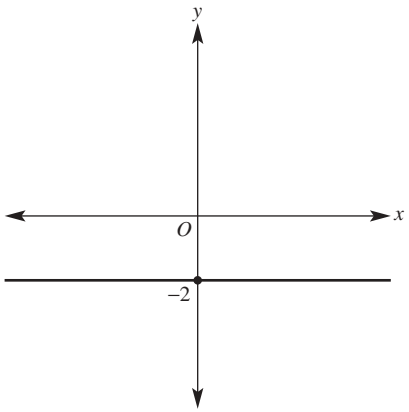
7 a



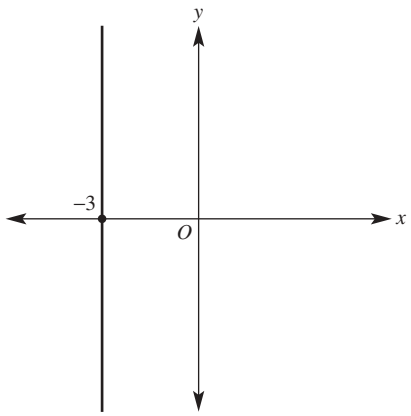
b



c

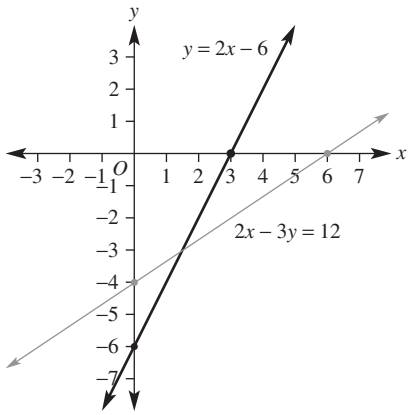


d



8 a x-intercept $(3, 0)$, y-intercept $(0, -6)$

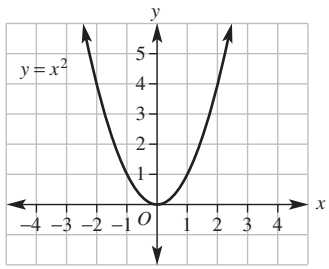
b x-intercept $(6, 0)$, y-intercept $(0, -4)$



9 a

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

b B



10 *See table at base of page.*

	Formula	Maximum or minimum	Reflected in x-axis (yes/no)	Turning point	y value when $x = 1$	Wider or narrower than $y = x^2$
a	$y = 3x^2$	Minimum	No	$(0, 0)$	3	Narrower
b	$y = -\frac{1}{2}x^2$	Maximum	Yes	$(0, 0)$	$-\frac{1}{2}$	Wider
c	$y = x^2 - 3$	Minimum	No	$(0, -3)$	-2	Same

CambridgeMATHS NSW Stage 5 – Core and Standard Paths Year 10 ISBN 978-1-009-40953-7 © Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Multiple-choice questions

1 D 2 E 3 D
4 A 5 D

Extended-response question

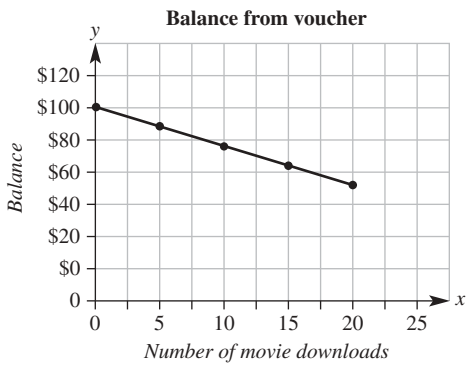
a $B = 100 - 2.4n$

b i \$76 ii 30 movies

c

Number of movies, n	0	5	10	15	20
Balance, B	\$100	\$88	\$76	\$64	\$52

d



e 41 movies, \$1.60 remaining on voucher

Chapter 7: Properties of geometrical figures

Short-answer questions

1 a 39 b 61 c 53 d 75
e 70 f 117 g 71 h 84
i 110 j 120 k 135 l 50

2 1440°

3 a 2 b 4 c $\frac{1}{3}$ d $\frac{1}{2}$

4 $\frac{2}{3}, x = 1.8,\ y = 5$

5 a Zero

b $e = 8, v = 5, f = 5$, so $e + 2 = 10 = v + f$

c Yes, it is a path that begins and ends at the same place (a cycle).

d Yes, this graph has an Eulerian circuit, which is a special type of Eulerian trail that starts and ends at the same place. It has no vertices with odd degree, therefore all Eulerian trails are circuits.

6 a Yes b Yes

Multiple-choice questions

1 C 2 E 3 A 4 C

Extended-response question

a 2.5 b 5 m

Chapter 8: Trigonometry

Short-answer questions

1 a PR b QR c PQ

2 a $\frac{y}{x}$ b $\frac{z}{x}$ c $\frac{y}{z}$

3 a 0.64 b 0.63 c 0.32 d 7.50
e 31.25 f 4.59 g 36.88 h 32.33
i 9.77

4 a 11.40 b 0.83 c 6.02 d 14.69

5 a 10.2 b 27.0 c 18.6 d 21.2

6 14 m

7 8.95 m

8 a 60° b 37° c 77°

9 177.9 m

10 259.8 m

11 15.5

12 53°

13 4.1

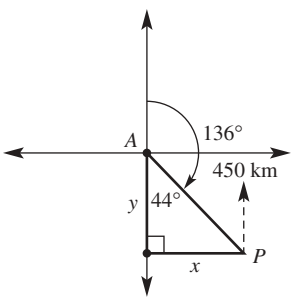
14 28

Multiple-choice questions

1 E 2 C 3 A 4 B 5 D

Extended-response question

a



b 312.6 km c 323.7 km d 316° T

Chapter 9: Equations and formulas

Short-answer questions

1 a $x = -9$ b $x = 26$ c $m = 32$ d $a = 4$
e $k = -6$ f $x = -8$

2 a $p = 2$ b $a = 4$ c $x = 12$ d $x = 20$

3 a $x = 8$ b $k = 3$ c $m = 9$ d $x = 5$
e $a = -4$ f $x = 4$

4 a $x - 5 = 8; x = 13$

b $4x + 8 = 20; x = 3$

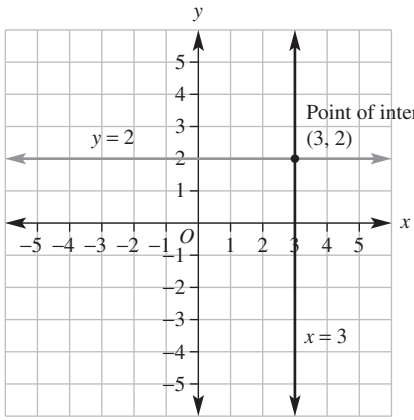
c $2(3x - 6) = 18; x = 5$

5 a $x = \pm 2$ b $x = \pm\frac{3}{2}$ c $x = 0$

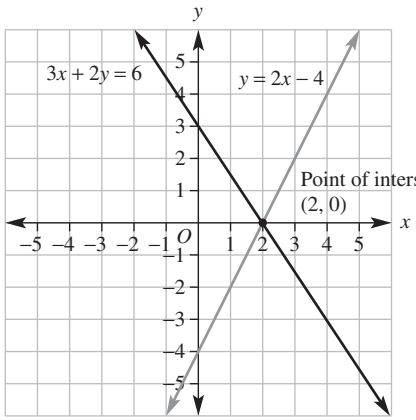
6 a $b = 10$ b $P = 400$

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

7 a (3, 2)



b (2, 0)



8 a (1, 2) **b** (3, 9) **c** (−1, 4)

9 a (1, 1) **b** (2, 4) **c** (3, 1)

10 a Let a = Oliver's age, b = Ruby's age (any pronumeral selection is correct)

b $a - b = 7, a + b = 45$

c Oliver is 26 years old, Ruby is 19 years old

Multiple-choice questions

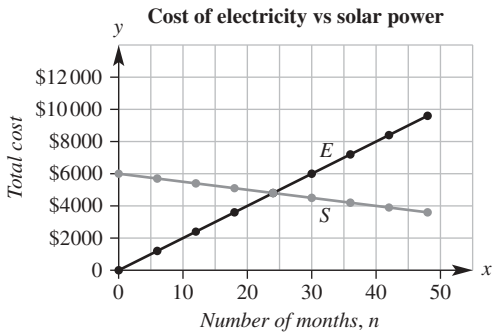
1 C **2** D **3** A **4** B **5** C

Extended-response question

a

n	*E*	*S*
0	$0	$6000
6	$1200	$5700
12	$2400	$5400
18	$3600	$5100
24	$4800	$4800
30	$6000	$4500
36	$7200	$4200
42	$8400	$3900
48	$9600	$3600

b



c $n = 24$ months, $E = S = \$4800$

d $E = 200n$

$S = 6000 - 50n$

e 24 months **f** $6000 **g** 27 months

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

Index

absolute error 90, 93
accuracy 89–90, 92, 238
acute angles 673
addition 179–80, 186, 214, 216–17, 749, 785
adjacent side 652, 665
algebra 197, 750
algebraic bases 232
algebraic expressions 179, 182, 186, 189–90, 192
algebraic fractions 208, 210, 212, 214, 216
alternate angles 541
ambiguity 316, 703
angle of reference 652
angle sums 547–8, 553–4, 556, 560–1
angles 542, 557, 652, 673, 675, 711
angles of elevation/depression 679–81, 683, 685
apex 144
arcs/arc length 101–2
area 83–4, 86, 106–10, 113, 150, 203, 714–16, 754
arithmetic means 338
arrays 278
arrows 540
association 373
averages/averaging 338
axes 427, 436, 467
axis of symmetry 497, 499

back-to-back stem-and-leaf plots 334, 362
balanced equations 740–1
bases 145, 150, 221, 227, 232–3
bearings 688, 691–2
bimodal data 338
binomial numerators 217
binomial products 197, 199
bivariate data 372–3, 380
boundaries 90
box plots 353–6
brackets 193, 195, 197, 215, 226–7, 749, 751
branch probabilities 287
breadth 106, 491–2
budgets 33–6, 38

calculators 239, 360–1, 659, 673
cancelling 186–7, 208–10
capacity 132, 135, 138
Cartesian planes 411, 426–7, 498
categorical data 316, 330
causal association 373
census 316
chance 259–60
change 447, 481, 509
circles 101–3, 113, 144
circuits 597, 600–1
circumference 101, 103, 127
coefficients 180–1, 454
cointerior angles 460, 541
column graphs 320–1, 323
commission 19, 23
common factors 186–7, 203, 205–6, 208–10
common negatives 206
comparison 776, 783
complements 267
composite shapes/solids 116, 137–8, 140, 158–9
compound interest/formula 47–51, 64, 510
cones 144, 146–8, 150–2
constant of proportionality 481–4
constant terms 180–1, 480
continuous data 316
conversion 5–6, 8, 50, 63, 83–7, 138, 240, 485, 659
coordinates 427, 436, 770
correlation 373
corresponding angles 541
cosine/rule 651–2, 655, 658, 673–4, 709–11
cost price 12
cross-section 119, 127, 132
cubed numbers 221
cubes 150
cubic units 131
cycles 597
cylinders 127–8, 131, 134

data points 345
data spread 345, 353, 360, 362
data/data collection 315, 317, 366
decimal points/places 84, 90, 103–4, 239, 641
decimals 5, 8, 239, 260, 436, 510, 635, 641
deductions 20
degrees 582–3, 652, 659, 674
denominators 5, 209, 214–15, 226–7, 665, 667
dependent events 294
depreciation 47–9
deviation 361
diagrams 84, 106, 266, 286–7, 290, 540, 581, 691
diameter 102, 113
difference of two squares 197–8, 201
dilation 500–1
direct variation/proportion 480–4
direction 461, 647, 688–9
discounts 11, 13, 16
discrete data 316, 330
distance 101, 421, 424, 540, 570–1, 574, 605–6, 608, 639, 648, 692
distance–time graphs 419, 421, 424
distribution (of data) 331, 333–4
distributive law 192–3, 197
division 63, 84, 179, 186, 189–90, 208–9, 212, 220–1, 223–4, 232
dot plots 330, 332

edges 581–3, 585, 588–9, 593, 596–7
elimination 783–5, 788–9
endpoints 436
enlargement 568
equally likely outcomes 259
equations 453, 455, 461, 463–4, 472–5, 482–3, 510, 660, 746, 751–3, 759–62, 785, 788
equivalent fractions 214–15
error 89–90, 93
Eulerian trails 596–7, 600–1
Euler's formula 588–9, 593
evaluation 180, 228
events 259, 273, 294–6
expanding 192–3, 195, 197, 199–201, 203–4, 232, 749
experimental probability 260, 262–3
experiments 320
exponential functions 505–7
exponential growth/decay 509–13
expressions 179–83, 186, 189–90, 192–4, 199, 205–6
exterior angle theorem 547, 549
extrapolation 379, 412, 417

faces 588–9, 593
factorising 203–6, 208, 210
factors 208–10, 221
fair die/coins 280, 294–5
fees 20
5-number summary 345, 348–9, 353
fixed expenses 33–5
flow diagrams 84
formulas 108–10, 144, 338–9, 442, 500–2, 764–5
fractions 5, 7, 113, 150, 180, 186–7, 208, 210, 212, 214, 226–7, 229, 260, 436, 744, 749, 753
frequency tables 320–1, 323
future value 48, 50

goods and services tax (GST) 12
gradient–intercept form 454
gradients 435–7, 439, 442, 444–5, 453–6, 461, 472, 770
graphs 366–7, 372, 410–14, 416–17, 426, 431–3, 439, 444, 456–7, 473–5, 505–7, 581, 771–2
gravity 155
grids 436–7
gross income 20–2

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

groups/grouping 286
growth/decay 509–13

height 101, 107, 132, 144, 147–8, 714
highest common factor (HCF) 203–4
histograms 320–1, 325–6, 360
holiday pay 24
horizontal axis 420, 427
horizontal lines 454, 472
hourly rates 20
hyperbola 491
hypotenuse 633–5, 639, 651–2, 659, 665

income 19, 23, 28–9
independent events 294–6
index form 221–2, 226
index laws 220–1, 223–5, 227, 229–30, 235
index notation 220
intercepts 454–6
interest 40–1, 47, 54–7, 63, 510
interior angles 460, 546–7
internal angle sum 546
interpolation 379, 412, 416
interquartile range (IQR) 346–9, 353–4
intersection 266, 428, 769–72, 778, 787
inverse variation 490–2
investment 40–1, 47–8, 54, 62
isomorphic graphs 588–90
isosceles triangles 548–9

kites 554–5

labels/labelling 96, 582, 605, 644, 652, 654, 659, 703
leave loading 20
left-hand side (LHS) 741, 749–50
length 83–5, 95, 106, 485, 570–1, 577–8, 634–5, 646, 651, 658, 661, 702, 704, 709–10
letters 179, 703
like terms 187–8, 749
limits of accuracy 89–90, 92
line graphs 366
line of best fit (by eye) 378, 380
line segments 367–8, 419–20, 436–8, 442
linear correlation 378
linear equations 740–1, 749, 777, 783
linear graphs 477–8
linear modelling 472–3
linear relationship 410–11, 427–8, 472
linear trends 367
lines 467–8, 581–2
loans 54–5, 62
loss 13
lower fence 346, 354
lower quartiles 345–9, 355
lowest common denominator (LCD) 214

mark-up 13
matching pairs 787–8
maximum values 337, 339, 345, 348–9, 355, 366
mean 337–40, 342, 360–1
measurement 83–7, 89–90, 92, 95
measures of centre 337–8
measures of spread 339
median 337–8, 341–2, 348–9, 355
Medicare levy 28–9
metric prefixes 84
minimum values 337, 339, 345, 348–9, 355
minutes 652, 659, 711
mode 337–40, 342
modelling 472–3, 477–8, 506, 510
multiplication 63, 84, 179, 186, 189, 193, 208–10, 220–1, 223, 227, 788
multi-stage events 295
multi-stage experiments 286, 288–9
mutually exclusive 267

negative correlation 373
negative indices 232–5
negative signs 194
net income 20–1
nets 119–20, 127, 144
networks 581–2, 596, 605–6
nominal data 316
non-linear relationships 48, 496–7
notation 220, 238
number planes 372, 427
numbers 179, 203, 221, 238, 241, 337
numerals 197, 240
numerators 209, 214–15, 226–7
numerical data 316

oblique cones 151
obtuse angles 707
one-step equations 742–3
operations 180
opposite side 652, 665
order (of operations) 180, 740–1
ordered data 316, 333, 337, 345
origin 427, 454, 481
outcomes 259, 278–9, 286–7, 290–1, 296
outliers 345–6, 348–9, 354–6, 373, 378
overtime 21

parabolas 496–9
parallel box plots 354
parallel lines 460–4, 540–4, 770
parallelograms 107, 553–5
paths 597
PAYG income tax system 26–7
penalty rates 20
percentage error 93
percentage profit/loss 13
percentages 5–9, 11, 13–15, 19, 34–5, 93, 155, 260, 436
perfect squares 197–8, 200
perimeter 95–6, 99, 104
perpendicular lines 460–4
perpendicularity 107

pi 101–2, 113
piecework 20
planar graphs 588–9, 591–2
plots/plotting 367–8, 427–9, 506–7
points 459, 467, 472, 475, 581
points of intersection 428, 769–72, 778, 787
polygons 560–1
population 316, 361, 506, 510
positive correlation 373
positive indices 233–5
powers 63, 221, 226–7, 229, 232, 239, 506, 758
precision 89–90
prediction 366, 453
present value 48
price 11–12
principal 40–1, 510
prisms 119, 150–1
probability 259, 270, 278–80, 286–91, 294
problem solving 273, 473, 581, 660, 665, 696–8, 741–2, 751–4, 758–62, 766, 769, 776–9, 781, 784–5, 789–90
product 113, 182, 203, 208, 210, 227, 229, 714
profit 11–12, 14–15
pronumerals 179, 187, 197, 203, 205–6, 542, 557, 644, 665, 740, 749, 758, 783
proportionality 481–4, 490–2
pyramids 144–6, 150, 152
Pythagoras' theorem 138–9, 632–3, 639–40, 644, 646–8

quadratic equations 758
quadrilaterals 106, 553–7, 560
quantity 5, 9, 444–5, 509

radii 102, 113, 144, 155, 157
random events 259
range 339–40, 342, 379
rate of change 481
rate of interest 40–1
rates 20, 50, 444–5, 447
ratios 436, 651–2, 654–5, 659
reciprocal 209
rectangles 106–10, 127, 554–5
rectangular prisms 120, 132–3
reflection 497, 500–1
regular polygons 561–3
relations/relationships 372–3, 410, 472, 496–7, 651, 764–5
repayment 43, 54–6
replacement 278–82
retainers 19
rhombuses 107, 554–5
right angles 461
right cones 151
right pyramids 145, 151
right-angled triangles 138, 632–3, 639, 644, 651–2, 658, 667, 675, 696

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

right-hand side (RHS) 741, 749–50
rise over run 435–6, 472
rounding 239, 667
royalties 20
rules 106, 186, 208–9, 226, 229, 427–9, 461, 473, 511–13, 560, 714

salary 20
salvage value 49
sample spaces 259
samples 316
scale 89–90, 354
scale drawings 567
scale factor 568, 570–1, 574–9
scatter plots 372–5, 380
scientific notation 238–42
seconds 652
sectors 102, 113, 115, 144
selling price 12, 15–16
sets (data) 268–9, 330, 333, 338, 341, 353, 355–6
shapes 95–6, 104, 108, 116, 137
shortest path problems 605–6, 608, 644
sides/lengths (of shapes) 96, 98, 119, 127, 546, 560, 575, 639–41, 644, 651, 658, 661, 702, 704, 709–10, 716
significant figures 238–9, 242
signs 180, 194, 209
similar triangles 574–5, 577
similarity 567–9
simple interest/formula 40–2, 47–8, 64
simplification 5, 7, 186–7, 189–90, 192, 195, 199, 210, 212, 214, 216–17, 220, 223–6, 229–30, 232, 235
simultaneous equations 769, 776, 778–9, 781, 783–4, 789–90
sine/rule 651–2, 658, 665, 673–4, 702–5, 707
skewed data 321, 331
slant height 144, 147–8
SOH CAH TOA 652, 659, 661, 675
solids 119, 137, 144
space 131–2
special lines 453, 456–7
speed 445–6
spheres 155–9
spreadsheets 62–4
square roots 361, 758
squared numbers 63, 113, 182, 221, 361, 496, 758
squares 107, 554–5, 561
standard deviation 360–2
stationary objects 420
statistical data 316, 320, 331
stem-and-leaf plots 330–2, 342
straight lines 367, 419, 426, 467, 472, 481
straight-line graphs 410–12, 467
subjects 315, 428, 759, 765, 777
substitution 108–9, 180, 182–3, 428, 461, 467, 472, 741, 750, 765–6, 776–9, 781
subtraction 179–80, 186, 214, 216–17, 749
sum 144, 182, 338, 361
summary statistics 338–9, 342
supplementary angles 541
surds 759–60
surface area 119–21, 127–8, 139–40, 144–6, 155–6, 158–9
surveys 315–16, 320
symbols 5, 63, 84, 96, 361
symmetrical data 321, 331
symmetry 497, 499

tables 315, 320–1, 323, 429, 431–3
tangent 651–2, 658, 673–4
tax brackets 28
taxation 12, 27–9
terms 181, 187–8, 193, 227
theoretical probability 259–61
three-dimensional (3D) objects 99, 119, 696–8
three-dimensional space 132, 648
time-series data 366–8
time/time period 33, 40–1, 48, 50, 366, 420, 480, 510
trails 596–7
transformation 568
translation 498, 502
transversals 540–1
trapeziums 107–9, 554–5
tree diagrams 286–7, 290
trends 366–8
trial and error 673
trials 259–60, 262–3
triangles 106–9, 145, 546, 560, 578–9, 702, 704, 714–15
triangular prisms 121, 131, 139
trigonometric ratios 651–2, 654–5, 659–60, 697
true bearings 688
true equations 776
turning points 497, 499–502
two-dimensional (2D) objects 107, 697
two-step equations 744–5
two-step experiments 278
two-way tables 273–5

union 267
unit conversions 485
units 83–7, 92, 107, 131, 138, 445
unknown values 179, 273, 379, 547–8, 556, 569, 576, 658–9, 661, 665, 667, 673–4, 764, 776, 783
upper fence 346, 354
upper quartiles 345–9, 355

value(s) 35–6, 47–9, 182–3, 337, 339, 345, 347, 361, 429, 431, 754, 766
variable expenses 33
variables 179, 372, 380, 410–11, 427, 473, 481, 490–2, 749–50, 752, 764–5, 783, 785
Venn diagrams 266, 270
verifying 741, 750
vertical axis 420, 427
vertical lines 454, 472
vertically opposite angles 541
vertices 497, 582–3, 585, 588–9, 593, 596–7
volume 83–4, 87, 131–4, 140, 150–1, 156, 158–9, 447

wages 19
walks 582, 585, 596–9, 606
weighted graphs 605–7
whole numbers 239, 659
with/without replacement 278–82, 287, 290–1, 295
word problems 746, 750, 754, 781, 789

x-axis 427, 467
x-intercepts 467–9, 499

y-axis 427, 467
y-intercepts 453–6, 463, 467–9, 473–5, 499

zero 239
zero index 226–8

ISBN 978-1-009-40953-7
© Palmer et al. 2024 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.