



NELSON QMATHS

for the **Australian Curriculum**



Stephen Swift

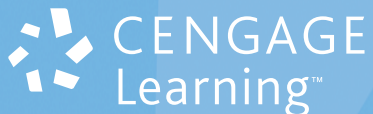
Ross Brodie

Stephen Corcoran

Sue Garner

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NELSON QMATHS

for the **Australian Curriculum**



Stephen Swift | Ross Brodie | Stephen Corcoran | Sue Garner

Nelson QMaths for the Australian Curriculum 9**1st Edition****Stephen Swift****Ross Brodie****Stephen Corcoran****Sue Garner**

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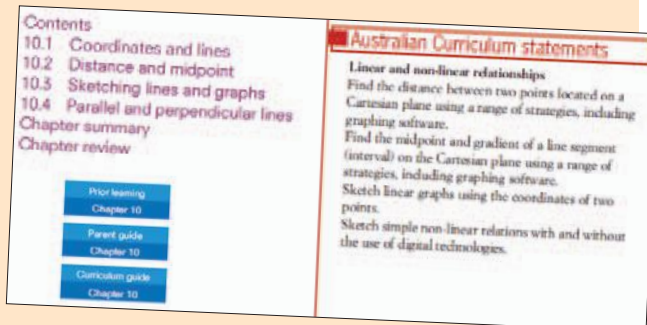
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About this book

At the beginning of each chapter

- Clear outline of chapter contents
- Strong links to the curriculum
- Prior learning provides a quick check of understanding for the individualisation of instructional sequence



Investigate: Average ages across the world

Use a search engine such as Google to find the 'average age' in different countries. You can try searching by using terms such as 'median age in different countries', 'national mean ages', 'world average ages' and so on. Make a list of websites. Without looking at the sources quoted by the websites, which ones do you think are the most reliable? Which ones would you probably regard as inaccurate?

- Parent guide provides vital information for parents
- Curriculum guide provides a rationale in words that students will understand
- Clear sectioning that is consistent with the exercises
- Many sections begin with a concrete investigation, with teacher support materials provided
- Intriguing aids for students provided

In each chapter

- Examples are clearly set out with explanations on the left and expected student answers in bold on the right
- Colour-coded examples make it easy for teachers to direct students
 - Green** – nearly all students will master
 - Gold** – most students will master
 - Blue** – some students should master
- Examples are sequenced in a natural learning progression that promotes chunking and effective use of working memory
- Clear examples enable parents to assist their student with confidence that they are 'doing it the way the teacher explained it'
- Important boxes highlight key terms and concepts

Example 2

Use a table of values to plot the function $y = \frac{1}{2}x - 1$ for $-2 \leq x \leq 6$.

Solution

Draw up a table with x from -2 to 6 .

x	-2	-1	0	1	2	3	4	5	6
y									

Calculate each y -value by substituting the x -value into the function.

$$y = \frac{1}{2} \times -2 - 1$$

$$= -1 - 1$$

$$= -2$$

Example 8

Find the equations of vertical and horizontal lines through $(2, 5)$.

Solution

A vertical line is parallel to the y -axis. **The vertical line is $x = 2$.**
 A horizontal line is parallel to the x -axis. **The horizontal line is $y = 5$.**

Example 25

Find the equation of a line passing through $(2, 5)$

a parallel to $x = 4$ b perpendicular to $x = 4$.

Important

Intercepts

The **x -intercept** of a straight line means the part of the x -axis from the origin to the point where the line crosses the axis. Since the distance is just the x -coordinate of the intersection, this or the point itself is usually taken as the intercept.
 The **y -intercept** is the y -coordinate of the intersection (or the point) of the line and the y -axis.

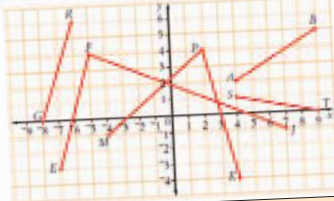
Exercise 10.1 Coordinates and lines

Understanding

1 For each of the following lines, state whether each of the given points is on the line.

- a $3x + 2y + 8 = 0$ $(-4, 2)$ $(3, 4)$ $(-2, 1)$ $(2, 7)$ $(4, 0)$
 b $y = x - 3$ $(-1, 2)$ $(1, -2)$ $(3, 0)$ $(-2, 5)$ $(-3, -6)$
 c $y = 2x + 6$ $(0, -6)$ $(2, 8)$ $(1, 14)$ $(-2, 2)$ $(3, 12)$
 d $y = -3x + 2$ $(1, 1)$ $(-1, 3)$ $(3, 7)$ $(0, -2)$ $(-4, 14)$
 e $3x - y - 6 = 0$ $(-4, 6)$ $(0, -3)$ $(2, 0)$ $(10, -12)$ $(-4, -18)$

2 Find the gradient of each of the line segments AB, ST, GR, PK, PM, EF and FJ shown below.



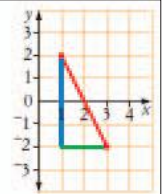
- Colour is used pedagogically to enhance learning

- Initial exercise questions focus on understanding or fluency and refer directly back to examples
- Questions are categorised according to the four proficiency strands — understanding, fluency, problem solving and reasoning
- Questions and parts are colour coded to be consistent with the examples

h Plot the points and join them with a straight line.

The line is *falling* so the gradient is negative.

Draw in the *rise* and the *run*.



Investigate: Midpoint of a line segment

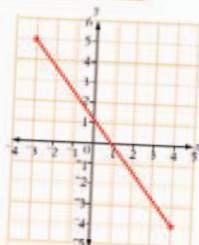
Use graph paper for this investigation.

Plot the points $(-2, 5)$ and $(4, -4)$ on your graph paper. Draw a line segment between the points and find the point halfway along it.

Write down this point. It is the midpoint of the line segment between the points $(-2, 5)$ and $(4, -4)$. Use the same method to find the midpoints of the line segments between:

- $(2, 2)$ and $(5, 8)$
- $(-2, -6)$ and $(4, -3)$
- $(2, 5)$ and $(-1, 0)$

Now look at each pair of points and the midpoint of the line segment. What pattern do you find?



- There is an abundance of carefully designed investigations proven to aid deep understanding

At the end of each chapter

Chapter 10 summary

- The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

- The equation of a circle of radius r centred at the origin is given by $x^2 + y^2 = r^2$.

- The equation of a (simple) parabola symmetrical about the y -axis is given by either $y = x^2 + a$ or $y = -x^2 + a$, where $(0, a)$ is the turning point (vertex) of the parabola. A parabola with a positive x^2 term is cup-shaped and one with a negative x^2 term is hill-shaped.



- A chapter summary that re-states all important concepts

Chapter 10 review

Understanding

1 For the line $4x - 3y + 7 = 0$, state if each of the given points is on the line.

- See Example 3 $(-4, 3)$ $(2, 3)$ $(5, 4)$
 $(5, 9)$ $(-7, -7)$

See Example 4 Find the gradients of the line segments through each of the following pairs of points.
 $(4, 3)$ and $(6, 8)$ $(-4, 3)$ and $(2, -7)$

See Example 8 Find the equations of lines passing through $(7, 3)$
 parallel to the x -axis parallel to the y -axis

About the series

Other books in the series

- There are five books in this Years 7 to 10 series, which has been written specifically for the Australian Mathematics Curriculum, including two books for Year 10 (see back cover).
- There is also a printed Teacher's edition of each textbook with additional guidance and advice on implementing the Australian Mathematics Curriculum in the classroom.
- Accompanying each printed textbook is a digital textbook called the NelsonNetBook (see opposite page).



Icons in the textbook

- In the outside margins of your textbook you will find numerous icons for resources that can be accessed through the NelsonNet website or through the NelsonNetBook.

Alternative method Moving the decimal	Explains different ways of solving a maths problem	Puzzle sheet Find the number	Makes learning maths methods and skills fun
Animated example Stem-and-leaf plots	Shows how to tackle difficult problems	Quiz Expressions	Tests and marks your knowledge of a topic
CAS calculator exercise Fractions and decimals	Develops your CAS calculator skills	Teacher notes Restaurant tables	Gives guidance to teachers
Curriculum guide Chapter 6	Explains what you need to know and be able to do	Technology GeoGebra: Exterior angle of a triangle	Provides a spreadsheet or GeoGebra activity
Extra questions Exercise 6.1	Allows you to do additional practice and drill	TLF learning object Exploring order of operations (L6543)	Provides interactive ways of learning about maths*
Maths clip Statistics	Gives the 'big picture' view of your maths topic	Video tutorial Algebraic expressions	Fully explains maths methods and skills
Maths dictionary	Provides an illustrated explanations of all terms	Weblink Algebra masterclass	Links to a website that makes Maths relevant
Parent guide Chapter 6	Provides guidance for your parent to help you	Worked solutions Exercise 6.1	Gives solutions steps for selected problems
Prior learning Chapter 6	Finds out what you already know and can do	Worksheet Writing a rule	Supplies tasks to be done digitally or by using a pen

* Your teacher will explain how to access The Learning Federation learning objects.

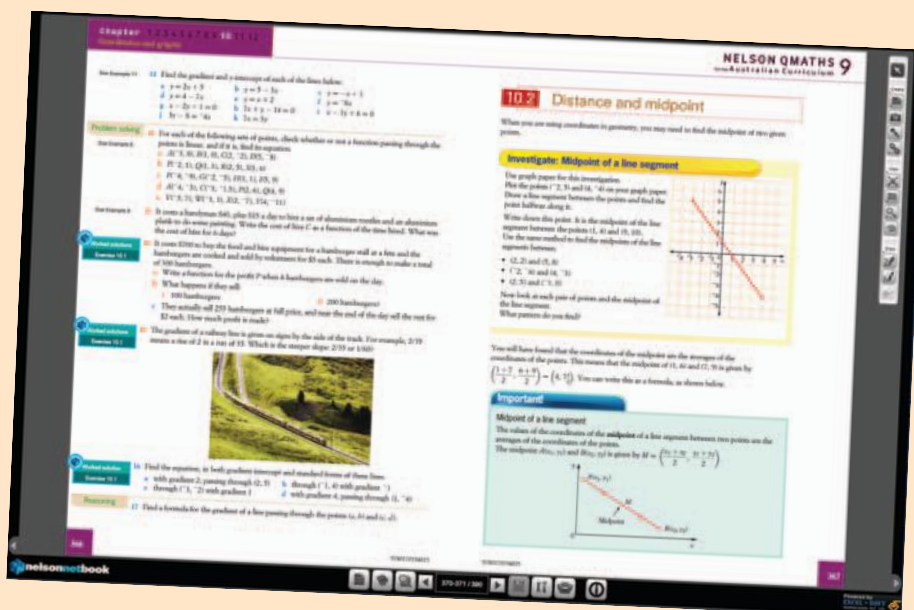
NelsonNet Website

- Go to www.nelsonnet.com.au to log in and find out more.
- The first screen you will see on the student website is called 'Chapter resources'.
- Click on a chapter and a list of different resource types will appear.
- Click on a resource type and a list of specific resources will pop up.
- Clicking on one of those resources will open a pdf file, start a video, or some special maths software.
- Use the blue tabs on the left of the screen to access calculator resources and the maths dictionary.



NelsonNetBook

- This is a web-based ebook that you can customise to meet your own learning needs.
- The icons with the blue NelsonNet logo are 'hotspots'. Click on an icon to open that resource.
- The tools on the vertical toolbar allow you to personalise pages in a variety of ways, including voice recordings, drawings and links to favourite websites. They also enable you to zoom in and out.
- The tools on the horizontal toolbar allow you to navigate around your ebook and change settings.
- Administration tools allow your teacher to manage classes and share annotations with you.



About the authors

Stephen Swift started teaching Mathematics, Science and Computing in 1973 and taught at all levels from Years 7 to 12 in several states, in urban and country schools until retiring in 2010 from the role of Mathematics Head of Department at Wellington Point State High School in Brisbane.

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Stephen Corcoran is an experienced teacher who has been Head of Mathematics in four schools. He is very familiar with current trends in mathematics education. He received a National Excellence in Teaching award for inspiring students and encouraging their abilities through innovative mathematics programs.

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Greek alphabet

A, α alpha	H, η eta	N, ν nu	T, τ tau
B, β beta	Θ, θ theta	Ξ, ξ xi	Υ, υ upsilon
Γ, γ gamma	Ι, ι iota	Ο, ο omicron	Φ, φ phi
Δ, δ delta	Κ, κ kappa	Π, π pi	Χ, χ chi
Ε, ε epsilon	Λ, λ lambda	Ρ, ρ rho	Ψ, ψ psi
Ζ, ζ zeta	Μ, μ mu	Σ, σ sigma	Ω, ω omega

$\therefore$	therefore	$\equiv$	is congruent to
$=$	is equal to	$\parallel$	is similar to
$\neq$	is not equal to	$A \leftrightarrow P$	A corresponds to P
$\approx$	is approximately equal to	$\parallel$	is parallel to
$>$	is greater than	$\perp$	is perpendicular to
$\nless$	is not greater than	$\%$	percentage
$\geq$	is greater than or equal to	$\frac{a}{b}$	fraction, division $a \div b$ or ratio $a : b$
$<$	is less than	$\pm$	plus or minus
$\nless$	is not less than	$\sqrt{\quad}$	square root
$\leq$	is less than or equal to	$\sqrt[3]{\quad}$	cube root
$\propto$	is proportional to	$\sqrt[n]{\quad}$	n th root
$\in$	is a member of	45°	45 degrees
$\notin$	is not a member of	$\bar{x}$	mean, average
m	gradient	π	pi $\approx 3.1415926\dots$
BC	before Christ	$\{\}, \emptyset$	null set, empty set
BCE	before the common era	$\subseteq$	subset of
AD	Anno Domini	$\subset$	proper subset of
CE	common era	$1 : 100$	Scale of 1 to 100
QED	<i>quod erat demonstrandum</i>		right angle
LHS	Left-hand side		complementary angles
RHS	Right-hand side		vertically opposite angles
$\Rightarrow$	it follows that		exterior angle of a triangle
$5.\overline{37214}$ or $5.\dot{3}7214$	shows $5.3721472147214\dots$		opposite angles in a parallelogram
	supplementary angles		corresponding angles
	angles at a point		co-interior (allied) angles
	angles in a triangle	Q_1, Q_1	first quartile
	angles in a quadrilateral	Q_3, Q_3	third quartile
	angles in an isosceles triangle	IQR	interquartile range
	alternate angles	$\pm$	plus or minus
$\triangle ABC$	triangle ABC	RTP	Required to prove
$\angle ABC$	angle ABC		

Metric system

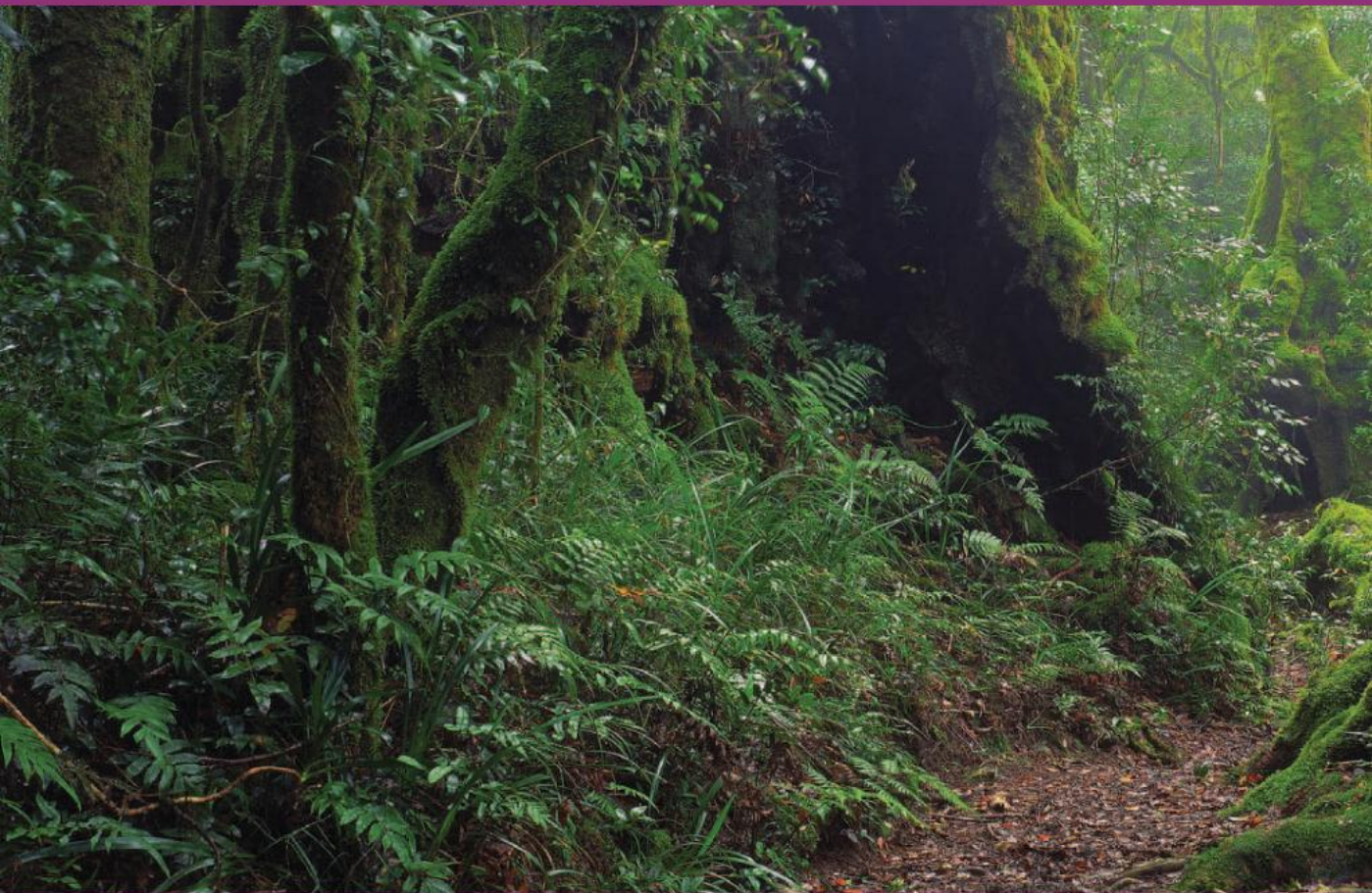
Symbols
millimetre — mm
centimetre — cm
metre — m
kilometre — km
hectare — ha
millilitre — mL
litre — L
kilolitre — kL
megalitre — ML
milligram — mg
gram — g
kilogram — kg
tonne — t

Length
10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

Area
100 mm ² = 1 cm ²
10 000 cm ² = 1 m ²
10 000 m ² = 1 ha
100 ha = 1 km ²

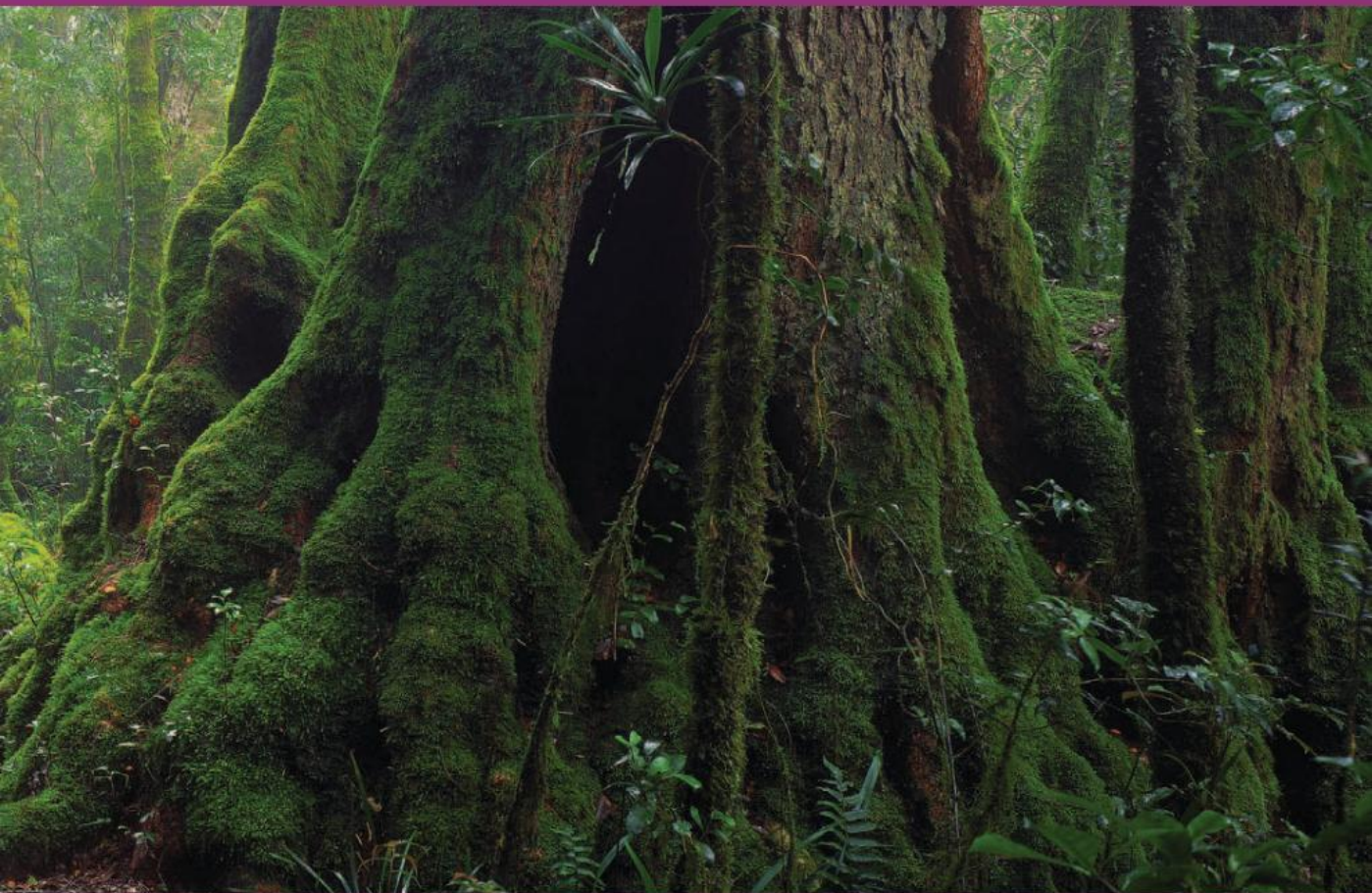
Capacity
1000 mL = 1 L
1000 L = 1 kL
1000 kL = 1 ML

Mass
1000 mg = 1 g
1000 g = 1 kg
1000 kg = 1 t



Statistics and probability

1 Statistics



Contents

1.1 Collecting and describing data

1.2 Data measures

1.3 Data analysis

Chapter summary

Chapter review

Prior learning

Chapter 1

MAT09SPPL00001

Parent guide

Chapter 1

MAT09SPPG00001

Curriculum guide

Chapter 1

MAT09SPCU00001

Australian Curriculum statements

Data representation and interpretation

Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources.

Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bimodal'.

Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread.

Chance

Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians.

Video tutorial
Statistics

MAT09SPVT00001

Weblink
How many blogs are out there?

MAT09SPWB00001

Maths dictionary
MAT09ASDI00001

If medical researchers want to know the health benefits of drinking green tea, they will probably compare the health of people who drink green tea with those who don't. Just collecting data about the people who drink green tea would not be useful. It is often important to compare data about different groups or different areas. Their means or ranges or other measures could be compared. Often, it is not the mean itself that is important, but the comparison of the means or other statistical measures. In this chapter, you will learn more about comparing and describing data.

Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

back-to-back	discrete	ordinal	spread
bias	fair	outlier	survey
bimodal	first quartile	population	symmetrical
bivariate	interquartile range	quartile	systematic sampling
categorical	lower quartile	random sampling	third quartile
census	mean	range	two-way table
central tendency	median	sample	univariate
continuous	mode	scatter plot	upper quartile
cumulative	nominal	side-by-side	
frequency	numerical	skewed	

1.1 Collecting and describing data

You already know that the way that information is collected affects the results. Bias can be introduced by the sample chosen, the choice of what information is collected or by the questions that are asked in surveys. Whenever you see reports on the internet, in newspapers or magazines, you need to ask yourself 'How did they get this?'.

Weblink
How many people can live on planet Earth?
MAT09SPWB00001

Investigate: Average ages across the world

Use a search engine such as Google to find the 'average age' in different countries. You can try searching by using terms such as 'median age in different countries', 'national mean ages', 'world average ages' and so on. Make a list of websites. Without looking at the sources quoted by the websites, which ones do you think are the most reliable? Which ones would you probably regard as inaccurate?



Now look at the websites in detail. Are there any websites that don't say where they obtained the data? Where do they say the information comes from? Does this make a difference to your assessment?

Find accurate information about the mean or median ages of the populations in Australia and some of its neighbours, such as Papua New Guinea, Indonesia, Tonga, Singapore, Malaysia, New Zealand and Cambodia.

Discuss your findings. Your teacher might have some specific questions for you to discuss.

In most cases, it is impractical to collect all the relevant statistical information to answer a question. If you wanted to know whether Melbourne people preferred buses, trams or trains it would be very expensive to ask all of them. You would only ask some of them.

Important!

Bias and samples

When you collect information, the **population** is the complete group that the information would relate to. A **census** obtains data from the whole population.

A **sample** is part of the population. A **survey** obtains data from a sample.

A **fair sample** is like the whole population. A **biased sample** is not typical of the population.

You can also get **bias** by using poor methods or questions.

Example 1

Describe each of the methods below of obtaining information as fair or biased and state whether it is from the population or a sample.

- a** The ABS census in 2006 asked every person in Australia: 'What is your country of birth?'
- b** The Gold Logie award is decided by voting from mobile phones. To vote, you call or send an SMS to particular 1900 numbers at a cost of 55 cents.
- c** A market research company rang a random selection of households in Victoria to ask: 'Do you think the Premier is doing a good job?'
- d** The ABS sends trained investigators to some houses in every capital city in Australia to ask whether the people in the house were in paid employment (worked for 1 hour or more) in the last week to compile employment data.
- e** Every student at a school was asked: 'Which is your favourite subject: Art, English, Maths, Music or Science?'

Weblink

Australian Bureau of
Statistics (ABS)

MAT09SPWB00001

Solution

- a Every person in Australia is asked, and the question is asked in a neutral language.
- b Only people with mobile phones who are willing to pay 55c can vote.
- c Only some people are chosen, but they are chosen at random.
- d Only some people were asked, and the method seems fair.
- e Every student was asked, but they could only choose some subjects.

It is a fair question from the population.

The method uses a sample and it is biased in the way the sample is 'chosen'.

The method uses a sample and the question and sampling method seem fair.

The method uses a sample and the question seems fair.

The population was used, but the method was biased.

There are different sampling methods that you can use. You normally want to make sure your sample is as fair as possible. If you wanted to know how much people at your school would pay to save koalas it would be easiest just to ask the people in your class. This is unlikely to be a fair sample. You could pick every thirtieth person on a list of all the people at school or put everyone's name in a hat and draw out 25 names. These would probably give fair samples.

Important!

Simple sampling methods

Sampling from a convenient group is called **accessibility sampling**.

Random sampling methods ensure that every member of the population has an equal chance of being in the sample.

Sampling by choosing every n th one from a list is called **systematic sampling**.

Sampling by selecting names from a hat (or the equivalent) is called **sampling by lot**.

Investigate: Sampling from your class

For this investigation, you will need a 'hat' and some equal sized pieces of paper with the numbers from 1 to 30 written on them. You can use a pencil case or other convenient container as your 'hat'. If your class has more than 30 people in it, you will have to use more numbers.

Your teacher will go around the class and give each person a number. Show the numbers so that everybody knows who has each number.

- Fold each piece of paper to the same size and put it in your 'hat'.
- Take out five pieces to make a sample of 5.

- Put the information together in a table in your book like the one below by putting tally marks next to each person's name to show the number of times they were sampled.

Number	Name	1st sample	2nd sample	3rd sample	Total
1	John Smith				
2	Mary Jones				

- After you have done the first sample, put the numbers back, shake them up, and pick another 5.
- Do a third sample and add up the number of times that each person was picked.

Look at the results. Do you think that this sampling method is fair? Can you think of how it could not be fair and ways to fix that? Is it practical for a large number of people?

- Now write out the numbers from 1 to 30, or whatever the number of people in your class is.
- Divide the number of people in your class by 5 and round down. If there are 24 people in your class, you would get $24 \div 5 = 4.8$, so you would round down to 4.
- On your list, choose every 4th number until you get a sample of 5.
- Put the results together for the whole class as before.
- Now use the same method to get a sample of 8.
- Finally, use the same method to get a sample of 3.
- Add up the number of times each person was chosen.

Look at the results. Do you think that this sampling method is fair? Can you think of how it could not be fair and ways to fix that? Is it practical for a large number of people?

When you use systematic sampling, a random starting point should be chosen. If you have a population numbered from 1 to 203, and want a sample of 5, then you should select every $\frac{203}{5} = 40.6$ th member of the population. If you round down to 40 and start from a random point in the first 40, you would never pick any of the last 3. If you round up to 41 and start from a random point in the first 41, you might run out before you picked 5. The simplest way out of this problem is to make your starting point anywhere in the population and go back to the beginning when you run out. Then it doesn't matter whether you round up or down.

Example 2

There are 158 Year 9 students at a school. Choose a sample of 12 students, starting from student number 82.

Solution

Find the sampling interval.

$$158 \div 12 \approx 13.17$$

Round off to get the interval.

Sampling interval = 13.

Start from the 82nd student and go up by 13s.

Choose numbers 82, $82 + 13 = 95$

Keep going up by 13s until you run out.

108, 121, 134, 147,

The next would be 160, so take away 158.

The next is $160 - 158 = 2$

Keep going up by 13s until you have the right number of students.

Select students 82, 95, 108, 121, 134, 147, 2, 15, 28, 41, 54 and 67.

Different kinds of data are treated differently in statistical analysis. Sometimes numbers are used to represent values that are not really numbers. You could ask people to pick their favourite colour and use a numerical code for the colour. For example: 1 = White, 2 = Red, 3 = Green, etc. In this case it would not make sense to find the mean of the codes.

Technology Random sampling with your calculator

Scientific calculators have a random function that gives a 'random' number between 0 and 1. You can use this function to generate a series of numbers that are close to being random. They are not actually random because calculators use a rule to find the numbers, and eventually they will repeat the sequence. However, they are good enough for most purposes.

The random number function could be shown as ran#, rand, random or similar. You might have to access it using a statistics or probability button or mode.

If you want random numbers from 20 to 70 inclusive, there are 51 numbers. To get a random selection from these using the function, you multiply the function by 51, add 20, and ignore the decimal places. Once you get the first number, you should be able to get more just by pressing the [=] key again to repeat the calculation. An example is shown below.

Enter as: 51 $\times$ [SHIFT] [.] [+] 20 [=]

51xRan#+20 25.763

Press equals again: [=]

51xRan#+20 52.895

Press equals again: [=]

51xRan#+20 22.142

Press equals again: [=]

51xRan#+20 35.453

This would give the random number sequence 25, 52, 22, 35, and so on.

Work out how you can use your calculator to get a random sequence of numbers from 30 to 99.

Your calculator should give you a different sequence every time you use it to generate random numbers.

Important!

Kinds of data

Categorical data places the information in categories such as colour, sex and nationality.

Nominal data has no numeric meaning or order.

Ordinal data has a natural order, but does not represent a measurement.

Nominal and ordinal data are both categorical.

Numerical data consists of numbers. There are two kinds of numerical data.

Continuous data can take any value between the smallest and largest values, so we may need to show how accurate it is.

Discrete data can only have particular values. In most cases, the values are whole numbers.

Continuous and discrete data are both numerical.

There are some mathematicians who consider that categorical data that can only have particular values, such as nationality, should also be called discrete. In this book, the term discrete is only used for numeric data.

Example 3

Describe the kind of data shown in each of the following.

- a** In the 100 m race, three friends' placings were: Tom 3rd, Thanh 2nd and Gerard 5th.
- b** Ann was 153 cm, Julie 165 cm and Zhara 162 cm tall.
- c** The most popular boys names of babies born in 2010 were Jack, James, Joshua, Oliver, Thomas and William.
- d** Susan said that her dress size was 12.
- e** Lee was 15 years old.
- f** Jane said she was about 500 million seconds old.

Solution

a The data are names, but they have an order.	This is categorical/ordinal data.
b Height can be any number between natural limits.	Height is numerical/continuous data.
c These are just names that have been counted.	This data is categorical/nominal.
d Dress sizes have only particular values, and the sizes have a numeric meaning.	Dress size is numerical/discrete data.
e Age usually means the number of birthdays you have had, but it could also mean the amount of time you have been alive.	The way age is stated here means it is numerical/discrete data.
f This clearly means the amount of time she has been alive.	This age is numerical/continuous data.

Technology International CensusAtSchool

Technology worksheet

Excel worksheet:
CensusAtSchool

MAT09SPCT00016

Go to the CensusAtSchool website and look at the kind of data collected in different countries. Your teacher will probably allocate countries for students to look at.



CensusAtSchool International



Canada



Ireland



Japan



New Zealand



South Africa



United Kingdom



United States

Describe the kinds of data collected in the country you are looking at.

In Australia and the United Kingdom, students were asked for their gender, height, and the time it usually takes them to get to school.

Use the CensusAtSchool site to collect data to answer these questions.

Do you think there would be any difference in heights between boys and girls?

Do you think there would be any difference in heights between students in the UK and in Australia?

Do you think there would be any difference in travel time to school between the UK and Australia?

When you collect data that involves several variables, there are different ways to deal with it. For data involving numerical and categorical variables, it is simplest to draw up separate tables for the numerical results for each value of the categorical variables. You can also use a **two-way table** for frequencies.

Important!

Two-way table

A **two-way table** is made by listing the values of one variable across the top row and the values of the other variable down the left-hand column. The remaining cells are then filled in with data corresponding to both values. Numerical variables are often grouped to reduce the number of boxes.

Example 4

The following data about transport to school and journey times (in minutes) was collected from some Australian Year 9 students. Make a two-way table and comment on the results.

Walk	30	Walk	15	Car	5	Bus	20
Car	50	Car	35	Car	15	Car	15
Car	5	Train/Tram	40	Car	5	Car	11
Car	25	Bus	70	Car	10	Car	10
Train/Tram	10	Bus	25	Walk	15	Train/Tram	55
Walk	10	Bus	30	Car	15	Bus	5
Walk	10	Car	12	Bus	25	Car	30
Car	10	Bicycle	15	Car	13	Walk	30
Bus	25	Bicycle	10	Car	15	Bus	30
Car	30	Car	10	Car	10	Bus	40

Solution

The students travelled in 5 different ways, and the times vary from 5 to 70 minutes. Put the different methods across the top and use 20-minute time intervals.

	Time in minutes			
Transport	1–20	21–40	41–60	61–80
Bicycle				
Bus				
Car				
Train/Tram				
Walk				

Count the number for each cell. You might find it easier to use tally marks first.

	Time in minutes			
Transport	1–20	21–40	41–60	61–80
Bicycle	2			
Bus	2	6		1
Car	15	4	1	
Train/Tram	1	1	1	
Walk	4	2		

Look at the results and make a conclusion.

Most students took 20 minutes or less to get to school. Only students travelling by bus or train/tram generally took more than 20 minutes, but there weren't very many of these. Most students must live close to their school.

TLF Learning object

Healthy life survey
(L3159)

MAT09NAIN00001

Technology World climate and weather

Technology worksheet

Excel worksheet:
World climate

MAT09SPCT00017

The World Climate site <http://www.worldclimate.com> has information about the climate in different parts of the world. For example, it gives the annual rainfall in Kuala Lumpur, Canberra and Port Moresby as 2424, 631 and 1084 mm respectively. Are there parts of Australia that have rainfall similar to Kuala Lumpur? What about other parts of New Guinea? Use the Bureau of Meteorology <http://www.bom.gov.au> and World Climate sites to find the weather in different parts of the world. Compile a table of annual rainfall, average temperature and other data for different parts of the world. What do you find about the climate of places in different latitudes? What about the weather today?



Exercise 1.1 Collecting and describing data

Understanding

Extra questions

Exercise 1.1

MAT09SPEQ00001

See Example 1

- 1 For each of the methods of obtaining information below, state whether the population or a sample has been used.
 - a To find out whether or not to extend the opening hours of a corner shop, all the customers who came into the shop over a period of a month were asked about the possible change.
 - b To find the average heights of 14-year-old Australians, the heights of all 14-year-old students in Perth were measured by qualified nurses.
 - c To find the likely result of a state election, 5000 people in the state were asked who they would vote for. An equal number of people were chosen from the electoral rolls for each electorate.
 - d To find the average depths of backyard pools, the owners of every private pool on the register of pools in Queensland were required to give the maximum depth of their pool.
 - e To find the health needs of teenagers in Victoria, 45 randomly chosen students from each high school in the state were asked to describe themselves as one of: 'very prone to illness', 'average', 'generally unwell', 'generally healthy', or 'never sick'.
- 2 Describe each of the following data items as one of nominal, ordinal, discrete or continuous.
 - a The colour of an egg.
 - b The weight of an egg.
 - c The size (medium, large, extra large, jumbo or king-size) of a carton of eggs.
 - d The minimum mass stated on a carton of eggs.
 - e The price of a dozen eggs.

See Example 3

Fluency

- 3 Describe each of the following data items as categorical or numerical.
- The weight of a car.
 - The colour of a car.
 - The safety rating of a car (1, 2, 3, 4, or 5 – the higher the safer).
 - The number of cylinders in a car.
 - The kind of tyres on a car.
- 4 Choose each of the following samples.
- 7 numbers from 25 to 80 inclusive, starting from 27.
 - 5 numbers from 1 to 50 inclusive, starting from 41.
 - 10 students from 122 in Year 9, starting from the 50th.
 - 5 cars produced on a production line from the 87 produced in the day, starting from the 3rd one produced on the day.
 - 9 houses from the 164 in Victoria Street, starting with number 38.

Worked solution

Exercise 1.1

MAT09SPWS00001

See Example 2

- 5 How could each of the methods in question 1 be changed to make them cheaper?

Problem solving

- 6 How could each of the following be improved?
- A survey asked people arriving in the City at 8:30 whether they would like another bridge across the river.
 - Students at a school were asked what their favourite comic strip was.
 - To gauge public opinion, a TV program asked viewers to SMS Yes or No to capital punishment.
 - A survey of people at the MCG asked whether the government should give more money to deserving sports.



Worked solution

Exercise 1.1

MAT09SPWS00001

- 7 The data below shows the gender and heights in cm of some Year 9 students. Make a two-way table for the information and comment on the results.

See Example 4

F, 148; F, 165; F, 157; M, 171; M, 190; M, 158; M, 171; F, 155; F, 158; M, 173; F, 168; M, 159; F, 157; M, 170; F, 160; F, 160; F, 159; M, 180; M, 166; F, 174; M, 172; F, 142; M, 164; F, 167; F, 162; F, 179; F, 154; F, 160; M, 184; F, 155; M, 153; M, 182; F, 161; M, 166; F, 160; M, 167; M, 171; F, 159; F, 155; M, 175

- 8 For question 1, describe the method as fair or biased and explain the reasons for your answer.
- 9 'Any sample of less than 10 people must be biased!' Give reasons for and against this statement.
- 10 'No sample can be chosen without bias!' Give reasons for and against this statement.

Reasoning

- 11 The results of polls conducted to find the popularity of political decisions rarely have more than 2000 people's opinions. They normally have a statement such as 'within 3 percent' to indicate how reliable they are. Explain if you think that this means the actual results must be within 3% of the results from the poll.
- 12 Explain how your calculator can be used to obtain each of the following random samples.
- a 5 numbers from 200 to 300 inclusive.
 - b 10 numbers from 100 to 200 exclusive.
 - c 4 phone numbers from the White Pages.
 - d 10 students from a school.

Worked solution

Exercise 1.1

MAT09SPWS00001

1.2 Data measures

A set of data can be characterised by its frequency distribution and measures of centre and spread. You have already studied **mean**, **median** and **mode**. Which one of these can be used for nominal data? Which cannot be used for ordinal data? Which is the most commonly used for numerical data? Simple measures of spread such as the **range** are only useful for numerical data.

Important!

The **mode** is the score with the highest frequency. It is the most common score, so it occurs more often than any other score. When there are two scores with equal highest frequencies, we say that the distribution is **bimodal**.

The **median** is the *middle* score. To find the median, all scores are arranged in order from smallest to largest and the middle score is chosen. If there is an even number of scores, then the average of the middle two scores is used. **Cumulative frequency** is the progressive total of frequencies and can be used to work out the median.

The **mean** is the **average** score. It is calculated by adding all the scores and dividing by the number of scores. The symbol $\bar{x}$, pronounced as 'x-bar', is often used for the mean.

The mean, median and mode are known as measures of **central tendency** and are usually very similar. The mean is generally considered as the most typical one as it summarises all the scores. The relationship between the mean and the median can be changed by a few very high or very low scores. If there are a few very high or very low scores that would distort the mean, you should use the median. The mode should be used in cases where the most common score is needed.

The **range** is the difference between the highest and lowest scores and measures the **spread** of the data.

Worksheet

Statistics crossword

MAT09SPWK00006

Example 5

The following data shows the forms of transport used by a group of students and the time in minutes they take to get to school.

Car	t = 5	Car	t = 20	Walk	t = 13	Bus	t = 20
Car	t = 10	Bus	t = 30	Car	t = 30	Car	t = 10
Bus	t = 13	Bus	t = 17	Car	t = 3	Car	t = 5
Car	t = 8	Walk	t = 10	Car	t = 4	Bus	t = 50
Bus	t = 20	Car	t = 3	Bus	t = 33	Bus	t = 26
Bus	t = 14	Car	t = 15	Car	t = 15	Walk	t = 18
Car	t = 16	Car	t = 13	Bus	t = 25	Bus	t = 47
Car	t = 12	Bus	t = 23	Bus	t = 60	Walk	t = 1
Walk	t = 13	Bus	t = 44	Bicycle	t = 10	Bus	t = 30
Bus	t = 35	Bicycle	t = 14	Car	t = 11	Car	t = 5
Bus	t = 5	Bus	t = 28	Bus	t = 32	Boat/Ferry	t = 58
Walk	t = 12	Bus	t = 29	Car	t = 15	Bus	t = 19
Walk	t = 15	Car	t = 5				

- Make frequency tables for the kinds of transport and the time taken in intervals of 5 minutes.
- What is the mode of the transport used?
- Find the mode, mean, median and range of the time taken to get to school.

Solution

- Count each method of transport and complete the table.

Transport	Tally	Frequency
Bicycle		2
Boat/Ferry		1
Bus		21
Car		19
Walk		7
Total		50

Set up the table with groups from 1–5, 6–10, . . . and complete the table.

Time in minutes	Tally	Frequency
1–5		8
6–10		5
11–15		9
16–20		7
21–25		2
26–30		6
31–35		3
36–40		0
41–45		1
46–50		2
51–55		0
56–60		2
Total		50

Worksheet

Statistical match-up

MAT09SPWK00005

Worksheet

Data analysis 1

MAT09SPWK00007

Puzzle sheet

A fish without eyes

MAT09SPPS00001

b The mode has the highest frequency.

c There is a mode and a modal group.

Write all the times in order.

The middle 2 are the 25th and 26th times. Find them.

The middle two scores are both 15.

Add up all the times.

Divide by the number of scores.

Find the range.

Write the answers.

Mode of transport = bus

The mode is 5 minutes, but the modal group is 11–15 minutes.

1, 3, 3, 4, 5, 5, 5, 5, 5, 8, 10, 10, 10, 10, 11, 12, 12, 13, 13, 13, 13, 14, 14, 15, 15, 15, 15, 16, 17, 18, 19, 20, 20, 20, 23, 25, 26, 28, 29, 30, 30, 30, 32, 33, 35, 44, 47, 50, 58, 60

Median = 15

Total = 969 minutes

$$\text{Mean} = \frac{\text{Total of scores}}{\text{Total frequency}} = \frac{969}{50} \approx 19.4 \text{ minutes}$$

Range = highest – lowest = 60 – 1 = 59

The mode is 5 minutes, the median is 15 minutes, the mean is about 19.4 minutes and the range is 59 minutes.

In Example 4, the mean of the time to get to school is noticeably larger than the median. This is because there are a few very high scores that raise the mean. The mode is very low compared to the median, but this might be because there aren't many scores. In this case, the median is a better indicator of the typical time taken than the mean or the mode. As shown in the example below, the median is useful for ordinal data.

Example 6

Teacher notes

Likert scales

MAT09SPTN00001

The results of a survey of Year 9 students about how much they liked lime flavoured ice-cream on a Likert scale are listed below. They could pick one: Hate it (SD), Don't like it (D), Don't care (N), Like it (A) or Love it (SA).

SD, N, D, N, SD, SD, SA, SD, SD, A, SD, D, A, A, SD, N, A, SD, D, D, SA, N, A, A, D, A, SD, N, SA, A, D, A, N, N, SD, N, D, D, A, SD, SD

Make a frequency distribution table and find the mode and median.

Solution

Create the table in order from SD to SA.

Score	Tally	Frequency
SD		12
D		8
N		8
A		10
SA		3
Total		41

The mode is the most common score.
The median is the middle (21st) score.
Write the answers.

**The mode was SD (Hate it).
The first 20 are SD and D, so the 21st is N.
The mode is 'Hate it' and the median score is 'Don't care'.**

Likert scales are named after the person who invented them, Rensis Likert (1903–1981).
For large groups, it is simplest to find the measures of central tendency by using a frequency table.

Example 7

The Science marks for a Year 9 unit test are as shown in the table below.

Result	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Number	8	1	7	6	9	11	22	17	15	17	16	17	13	15	7	9	4	3

- Extend the table to include score $\times$ frequency and cumulative frequency columns.
- Use the table to find the mean, median and mode.
- What is the typical result?

Solution

- Arrange the table in 4 columns with the extra ones for score $\times$ frequency and cumulative frequency.
Multiply each score by its frequency.
Progressively add the frequencies to get the cumulative frequencies.
Add the frequency and score $\times$ frequency columns.

Marks x	Frequency f	xf	Cumulative frequency
3	8	24	8
4	1	4	9
5	7	35	16
6	6	36	22
7	9	63	31
8	11	88	42
9	22	198	64
10	17	170	81
11	15	165	96
12	17	204	113
13	16	208	129
14	17	238	146
15	13	195	159
16	15	240	174
17	7	119	181
18	9	162	190
19	4	76	194
20	3	60	197
Totals	197	2285	

Video tutorial

Statistics from a frequency table

MAT09SPVT10012

CAS TI-Nspire exercise

Statistics

MAT09SPTI00001

CAS ClassPad exercise

Statistics

MAT09SPCP00001

Worksheet

Analysing data

MAT09SPWK00004

Worksheet

Data analysis 2

MAT09SPWK00008

Worksheet

Data analysis 3

MAT09SPWK00009

Worksheet

Data analysis 4

MAT09SPWK00010

Puzzle sheet

Mode, median and mean

MAT09SPPS00003

- b** Calculate the mean.

9 has the highest frequency.

Find the number of scores on each side of the median.

State which score is the median.

There are 96 scores up to and including all the 11s, and the next 17 are 12s.

Give the answers.

- c** The mean and median are very close, so choose the mean.

$$\text{Mean} = \frac{\text{Total of scores}}{\text{Total frequency}} = \frac{2285}{197} \approx 11.6$$

Mode = 9

The median has $\frac{197-1}{2} = 98$ scores on each side.

The median is the 99th score.

The 98th score must be 12.

The mean is 11.6, the mode is 9 and the median is 12.

The typical score is 11.6.

You use the median to find the middle of all the scores. Half the scores are above the median and half are below, so it divides the scores into two portions. It also makes sense to divide the scores into quarters.

Important!

Quartiles and the interquartile range

The **quartiles** of a frequency distribution divide it into quarters.

The **first quartile (lower quartile)** is the score that has a quarter (25%) of the scores below it. Its symbol is Q_1 or Q_1 . It is the median of the first half of the scores (*below* the median).

The **second quartile** is the score that has two quarters (50%) of the scores below it. It is another name for the median. It can be written as Q_2 or Q_2 .

The **third quartile (upper quartile)** is the score that has three quarters (75%) of the scores below it. Its symbol is Q_3 or Q_3 . It is the median of the second half of the scores (*above* the median).

The **interquartile range (IQR)** is the difference between the third and first quartiles.

$$\text{IQR} = Q_3 - Q_1$$

The interquartile range is another measure of spread. It measures the range of the middle half of the data.

If there are an odd number of scores, the median is *not included* in either half of the scores to find the quartiles. Some people call the lowest score the zeroth quartile and the highest score the fourth quartile.

The interquartile range is not affected by a few high or low scores, so it is a more reliable measure of spread than the range.

Animated example

Data measures

MAT09SPAE00001

Worksheet

Collecting and
describing data

MAT09SPWK00001

Example 8

The times taken by some Year 9 students to complete a survey were as follows.

16, 8, 12, 9, 9, 16, 21, 20, 29, 10, 15, 20, 10, 11, 24, 8, 29, 22, 15, 13, 10, 13, 15, 11, 13, 21, 9, 11, 21, 19, 12, 13, 20, 15, 29, 17, 29, 21, 21, 21, 24, 9, 16, 8, 20, 19, 11, 17, 29, 16

- a Find the quartiles.
- b Find the interquartile range.

Solution

- a Count the times.

Arrange all the times in order and divide in half to find the median. There are 25 scores either side of the red line, so the median is 16.

Find the median of the first half of the scores.

Find the first quartile.

Find the median of the second half of the scores.

Find the third quartile.

Write the answer.

- b Calculate the IQR.

Write the answer.

There are 50 times.

8, 8, 8, 9, 9, 9, 9, 10, 10, 10, 11, 11, 11, 11, 12, 12, 13, 13, 13, 13, 15, 15, 15, 15, 16, 16, 16, 16, 17, 17, 19, 19, 20, 20, 20, 20, 21, 21, 21, 21, 21, 21, 22, 24, 24, 29, 29, 29, 29

8, 8, 8, 9, 9, 9, 9, 10, 10, 10, 11, 11, 11, 11, 12, 12, 13, 13, 13, 13, 15, 15, 15, 15, 16

$$Q1 = \frac{11 + 11}{2} = 11$$

16, 16, 16, 17, 17, 19, 19, 20, 20, 20, 20, 21, 21, 21, 21, 21, 21, 22, 24, 24, 29, 29, 29, 29

$$Q3 = \frac{21 + 21}{2} = 21$$

The first quartile is 11 minutes and the third is 21 minutes.

$$IQR = Q3 - Q1 = 21 - 11 = 10$$

The interquartile range is 10 minutes.

CAS TI-Nspire exercise

Statistics

MAT09SPTI00001

CAS ClassPad exercise

Statistics

MAT09SPCP00001

Technology worksheet

Statistical measures calculator

MAT09SPCT00018

Worksheet

Statistical calculations

MAT09SPCT00003

Investigate: Comparing the range and interquartile range

Calculate the range and interquartile range for each of the randomly generated distributions shown below. The distributions all have mean values of about 6.

- 1 Distribution 1: 4, 7, 6, 5, 5, 7, 8, 8, 11, 9, 5, 3, 4, 5, 5, 4, 7, 6, 6, 8
- 2 Distribution 2: 12, 5, 2, 7, 6, 5, 2, 6, 9, 3, 7, 4, 6, 7, 8, 4, 7, 5, 4, 10
- 3 Distribution 3: 5, 2, 6, 8, 8, 6, 4, 5, 5, 7, 7, 3, 7, 3, 6, 6, 8, 8, 8, 4
- 4 Distribution 4: 7, 6, 5, 6, 5, 8, 10, 8, 3, 5, 6, 4, 7, 7, 7, 5, 4, 6, 3, 7
- 5 Distribution 5: 2, 4, 6, 8, 8, 4, 4, 5, 8, 7, 6, 6, 7, 5, 5, 7, 8, 6, 3, 6

For each distribution, calculate the value of $\frac{\text{Range}}{\text{Interquartile range}}$.

What do you find?

The interquartile range (IQR) can be used to quantify the idea of an **outlier**. Instead of just saying that an outlier is a long way from the rest of the data, they are often defined as being more than one-and-a-half times the interquartile range below the first quartile or above the third quartile. As you have seen before, an outlier should be checked to see whether it might be an error.

Example 9

Use the IQR criterion to find any outliers in the following test scores.

8, 16, 9, 17, 8, 19, 9, 14, 3, 12, 13, 13, 7, 11, 8

Solution

Arrange the data in order.

3, 7, 8, 8, 8, 9, 9, 11, 12, 13, 13, 14, 16, 17, 19

Divide into quarters.

3, 7, 8, 8, 8, 9, 9, 11, 12, 13, 13, 14, 16, 17, 19

State the median and IQR.

Median = 11 and IQR = 14 – 8 = 6

Work out the limits for outliers.

Lower limit = $8 - 1.5 \times 6 = -1$

Upper limit = $14 + 1.5 \times 6 = 23$

There are no scores outside the limits.

There are no outliers in the data.

Technology Automatic statistical measures

There are many calculators, computer programs and apps that calculate statistical measures. Your teacher may want you to try out some particular ones. Some of the calculator ones are hard to use because it is easy to make a mistake when entering the data.

There is a spreadsheet calculator (Statistical measures calculator) on NelsonNet that will automatically calculate statistical measures when you type in the data.

Try it out or use one that your teacher has set up for you.

Statistical measures calculator
Whole numbers only

Clear Calculate

Instructions
Press the Clear button
Type your data into the column under the heading Data,
pressing return after each item.
Press the Calculate button

Data	Mean	Median	Mode(s)	Range	Lowest	Highest	First quartile	Third quartile	Interquartile range
5	4.833334	5	No mode, 3 scores with frequency of 2	12	0	12	2	6	4
2									
1									
5									
6									
2									
0									
8									
6									
4									
12									

Exercise 1.2 Data measures

- 1 Find the mean, median, mode and range of each of the following sets of data.
 - a 8, 13, 9, 10, 9, 13, 9, 12, 8, 12, 10, 15, 6, 14, 6, 14, 8, 13, 6, 14
 - b 3, 9, 5, 8, 1, 7, 0, 8, 4, 8, 5, 9, 7, 8, 3, 7, 0, 9
 - c 26, 26, 18, 20, 13, 18, 28, 30, 33, 18, 16, 23, 28, 17, 21, 22, 20, 19, 15, 26, 23, 28, 17, 27
 - d 48, 41, 53, 49, 43, 54, 54, 45, 49, 50, 56, 42, 43, 57, 51, 45, 48, 56, 54, 52, 51, 62
 - e 11, 16, 5, 16, 8, 19, 12, 15, 12, 15, 12, 13, 7, 14, 7, 16, 7, 13, 8, 16, 7, 18, 10, 14
- 2 For each of the following collections of data, state which measures of central tendency can be found.
 - a The star ratings of viewers of a YouTube video.
 - b The popularity (by views) of YouTube videos.
 - c The lengths in seconds of YouTube videos.
 - d The marks given by art critics on a scale of 1–10 for paintings in an art competition.
 - e Marks (0–20) on a spelling test of 20 words.
- 3 Find the quartiles and interquartile range for each of the following sets of data.
 - a 9, 13, 11, 16, 12, 18, 11, 18, 9, 16, 6, 12, 10, 19, 9, 13, 10, 18, 11, 12, 12, 17, 9, 13
 - b 9, 6, 5, 8, 6, 3, 7, 6, 8, 2, 3, 6, 4, 7, 9, 5, 7, 7, 3, 3, 9, 10, 6, 8, 3, 5, 5, 9, 8, 8
 - c 4, 9, 4, 7, 1, 6, 5, 10, 4, 6, 3, 9, 3, 5, 3, 7, 4, 7, 0, 6, 2, 6
 - d 0, 8, 4, 7, 4, 6, 2, 7, 3, 7, 3, 9, 1, 10, 4, 7, 5, 5, 5, 6, 3, 6, 3, 7, 5
 - e 8, 8, 8, 15, 8, 9, 5, 8, 8, 8, 5, 9, 7, 10, 7, 9, 4, 12, 6

Understanding

Extra questions

Exercise 1.2

MAT09SPEQ00002

See Example 5

See Examples 5, 6

See Example 8

- 4 Extend each of the following tables to include score $\times$ frequency and cumulative frequency columns and use the table to find the mean, median and mode of the data.

Fluency

See Example 7

a

Score	1	2	3	4	5	6	7	8	9
Frequency	2	4	6	4	5	7	4	3	2

b

Score	20	21	22	23	24	25	26	27	28	29	30
Frequency	1	2	5	7	8	9	8	8	7	5	4

c

Score	4	5	6	7	8	9	10	11	12	13	14	15
Frequency	2	0	3	6	7	4	8	6	4	2	3	1

d

Score	11	12	13	14	15	16	17	18
Frequency	7	9	12	8	5	6	4	3

e

Score	28	29	30	31	32	33	34	35	36	37
Frequency	2	0	3	5	7	8	5	9	4	2

Worked solution

Exercise 1.2

MAT09SPWS00002

See Example 6

- 5 Find the median for each of the following sets of ordinal data.

a Good, Poor, OK, Terrible, Good, Poor, Poor, Poor, Terrible, OK, Terrible, Good, Terrible, Good, OK, Terrible, OK, Poor, Terrible, Great, Poor, Great, Good, Good, Good

b 

c C, D, A++, C, A+, B, A++, A++, C, A+, A+, B, A, D, A++, C, A, B, A++, C, A, B, C, B, A+, B, A, C, A+, A+

d A, N, D, A, D, SD, N, SA, SA, D, SA, N, D, D, SA, SA, SA, D, D, A, D, SA, A, SA, A, N, SD

e above average, above average, below average, average, below average, short, average, average, above average, tall, average, below average, really short, below average, short, average, short, average, below average, below average, above average, short, very tall, above average, above average, above average, tall, short, below average, really short, average, below average, above average, short, really short, short, below average, below average, above average, above average, below average, below average, below average, really short, average, short, below average, short, short, average, very tall, average, above average, really short, below average, tall, below average, short, short

Worked solution

Exercise 1.2

MAT09SPWS00002

See Example 9

- 6 Find the ratio of the interquartile range to the range of each of the following sets of data in the form 1 : x.

a 7, 5, 6, 7, 12, 17, 13, 16, 8, 11, 10, 6, 7, 12, 9, 3, 1, 11, 10, 17, 12, 8, 6, 11

b 4, 5, 9, 6, 6, 3, 6, 6, 8, 4, 3, 6, 7, 2, 5, 5, 6, 3, 4, 5, 8, 5, 8, 6, 4, 6, 6

c 5, 11, 9, 11, 8, 12, 6, 10, 9, 13, 10, 13, 8, 13, 8, 13, 8, 12, 9, 15

d 12, 20, 11, 17, 13, 20, 12, 16, 11, 17, 13, 17, 11, 14, 10, 19, 10, 14, 13, 17, 11

e 25, 25, 28, 33, 30, 24, 31, 30, 18, 22, 40, 28, 38, 22, 11, 28, 24, 3, 31, 25, 18, 17, 33

- 7 Use the IQR criterion to find any outliers in the following data sets.

a 48, 105, 69, 81, 66, 90, 63, 146, 39, 89, 67, 93, 60, 73, 61, 74, 7, 82, 23, 99, 61, 120, 61, 100, 49

b 10, 14, 8, 12, 11, 16, 3, 18, 11, 15, 9, 20, 4, 17, 9, 22, 10, 15, 9, 14

c 26, 36, 26, 46, 2, 33, 26, 38, 29, 43, 18, 43, 26, 35, 30, 34, 25

- 8 Boxes of oranges imported from California each contain 60 oranges. From 30 boxes, the mean, median and mode of the numbers of rotten oranges in each box for one particular shipment were 8, 4 and 3 respectively. How many rotten oranges were there altogether, and what would be the typical number of rotten oranges?

Problem solving

- 9 The mean time taken by some Year 8 students to complete a test in a 40 minute lesson was 35 minutes and half of the 24 students in the class had finished after 32 minutes. Some students complained that they did not finish the test in the 40 minutes allowed. Two of them said they only needed another 3 minutes and the other one said he could have finished in another 5 minutes. If the class had been kept in until everyone finished, what would the mean and median times have been?
- 10 After a cyclone in northern Queensland, the price of bananas increased by about 250%. A survey of fruit shops and supermarkets found that the prices after the cyclone had a mean of \$13.64 per kilogram and a median price of \$12.20 per kilogram. Before the cyclone, the prices in different shops were all very close to each other. Estimate the mean and median prices before the cyclone and justify your answer.

Worked solution

Exercise 1.2

MAT09SPWS00002



- 11 A 2011 newspaper report of car safety had a headline saying ‘Small cars doing better’. In the story it said that the average rating of small cars in crash test safety had increased from 2.5 to 4 over the last 5 years, and that for larger cars the rating had only increased by 0.3 to 4.5. After each crash test, a rating of 1–5 stars was given, based mainly on the data obtained from the crash test dummies. It stated in the article that ‘Since 1993, ANCAP has published crash test results for over 300 vehicles.’ Explain any problems you see with these statistics.
- 12 Explain if it is possible for the range and interquartile range of a set of data to be the same.

Reasoning

1.3 Data analysis

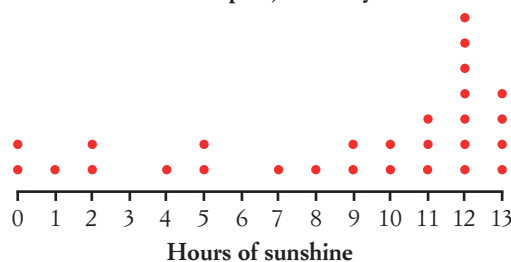
The *shape* of a frequency distribution can tell you something about the data. A simple dot plot or histogram can be used for this purpose. Look at the dot plot below of the hours of sunshine at Essendon Airport in February.

TLF Learning object

Dangerous days
(R12201)

MAT09SPIN00001

Essendon Airport, February 2010



Video tutorial

The shape of a
frequency distribution

MAT09SPVT10013

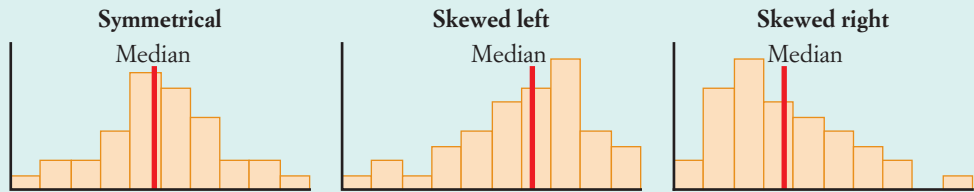
Important!**Symmetrical and skewed distributions**

A **symmetrical** distribution is roughly the same shape on either side of the centre.

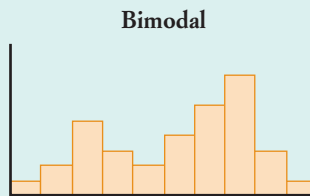
A **skewed** distribution is spread out more on one side of the median than the other.

A distribution that is **skewed to the left** is spread out more below the median than above the median. It is also called a **negative skew** because there is usually more data below (left of) the peak than above it.

If the scores are spread more on the right, it is **skewed to the right** (a **positive skew**). In this case there is more data to the right of the highest part than to the left of it.



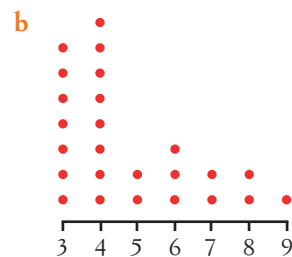
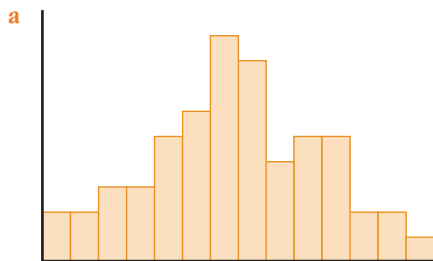
Some distributions have two high parts. These are called **bimodal** distributions, although they don't always have two actual modes.



The dot plot of the hours of sunshine for Essendon Airport is skewed to the left.

Example 10

Describe the distributions below.



c

Stem	Leaf
2	5
3	2 9
4	3 6
5	4 5 9 9
6	0 2 4 7 7 8 9
7	3 6 7
8	1

Solution

- a** This histogram is roughly the same on each side of the centre.
- b** This dot plot has a long 'tail' on the right.
- c** Look at the stem-and-leaf plot sideways. The 'tail' is towards the lower values.

The distribution is symmetrical, so the mean and median would be close to each other.

The distribution is skewed to the right.

The distribution is skewed to the left.

The positions of the mean, median and mode are different in skewed distributions. A distribution with some very high values is usually skewed to the right. You have already seen that the median is a better 'typical' value than the mean for distributions like this.

Example 11

Look at the data set 2, 2, 7, 15, 15. Calculate the mean and median and make a generalisation about the positions of the mean and median in a distribution skewed to the right.

Solution

Find the median.

Find the mean.

State the positions.

Generalise.

Median = 7

Total = 41, mean = $\frac{41}{5} = 8.2$

The mean is to the right of (higher than) the median.

In a distribution skewed to the right, the mean will be higher than the median because the high values 'drag' the mean to the right.

In skewed distributions with a large number of values, the median is usually between the mode and the mean.

The reason you collect data is to analyse it to answer questions. You can use data measures to answer many questions, but sometimes you need to compare different data sets. In this case, the data measures by themselves may not tell you what you want to know.

Of course, you can also use data measures to compare data from different groups.

Example 12

The data below shows the hours worked by sample groups of full-time electricians and waiters. Use statistical data measures to compare the hours worked.

Electricians: 38.5, 44, 38, 43.5, 41.5, 44, 39, 46.5, 38.5, 42.5, 37.5, 45, 39.5, 45, 38.5, 42.5, 39.5, 43.5, 37.5, 42.5

Waiters: 37.5, 40.5, 37.5, 38.5, 37.5, 40, 38, 40, 38, 39, 38.5, 39, 38, 38.5, 38, 41, 38, 39.5, 38, 40.5

Solution

Put the electricians' hours in order and divide into 4 groups.

Find the mean, median, mode and range.

Put the waiters' hours in order and divide into 4 groups.

Find the mean, median, mode and range.

Compare the hours worked.

37.5, 37.5, 38, 38.5, 38.5, 39, 39.5, 39.5, 41.5, 42.5, 42.5, 42.5, 43.5, 43.5, 44, 44, 45, 45, 46.5

$$\text{Mean} = \frac{827}{20} \approx 41.4 \text{ h}$$

$$\text{Median} = \frac{41.5 + 42.5}{2} = 42 \text{ h}$$

$$\text{Modes} = 38.5, 32.5$$

$$\text{Range} = 46.5 - 37.5 = 9 \text{ h}$$

$$\text{IQR} = 43.75 - 38.5 = 6.25 \text{ h}$$

37.5, 37.5, 37.5, 38, 38, 38, 38, 38, 38.5, 38.5, 38.5, 39, 39, 39.5, 40, 40, 40.5, 40.5, 41

$$\text{Mean} = \frac{775.5}{20} \approx 38.8 \text{ h}$$

$$\text{Median} = 38.5 \text{ h}$$

$$\text{Mode} = 38, 32.5$$

$$\text{Range} = 41 - 37.5 = 3.5 \text{ h}$$

$$\text{IQR} = 39.75 - 38 = 1.75 \text{ h}$$

Both professions work at least 37.5 hours, but the electricians, on average, work about 2.6 hours longer than the waiters. Most of the waiters work almost the same hours, as the IQR is very small compared to the electricians, where the middle 50% have over 6 hours of variation in time.

Another way to compare different sets of data is to plot them next to each other.

Important!

A **back-to-back (side-by-side)** display is used to compare two lots of data. One group is shown on the left of a vertical axis, and the other group on the right. Sometimes these displays are shown above and below a horizontal axis. Histograms may be drawn with side-by-side columns to allow for comparisons.

Example 13

The ages of males and females in a self-defence class are shown below. Use a back-to-back stem-and-leaf plot to compare their ages.



Women:	28	45	40	25	45	30	45
	26	50	29	26	23	50	51
	26	29	28	51	53	60	
Men:	37	39	33	37	43	49	36
	45	26	46	37	39	40	41
	54	61	44	48	48	42	

Solution

The lowest age is 23 and the highest is 61, so the stems need to go from 2 to 6.

Ages of self-defence class. Key: 4|3 = 34 = 3|4

Females		Males
	2	
	3	
	4	
	5	
	6	

Complete the plot with the women on the left and the men on the right

Ages of self-defence class. Key: 4|3 = 34 = 3|4

Females		Males
8 5 6 9 6 3 6 9 8	2	6
	3	7 9 3 7 6 7 9
5 0 5 5	4	3 9 5 6 0 1 4 8 8 2
0 0 1 1 3	5	4
	6	1

Worksheet

Stem-and-leaf plots

MAT09SPWK00002

Puzzle sheet

Highway elephants

MAT09SPPS00002

Put the 'leaves' in order.

Ages of self-defence class. Key: 4 3 = 34 = 3 4													
Females							Males						
9	9	8	8	6	6	5	3	2	6				
						0		0	3	3	6	7	7
				5	5	5	0	4	0	1	2	3	4
			3	1	1	0	0	5	4				
						0		0	6	1			

Compare the 'bulges' on each side.

The males are generally in their 30s or 40s, but there are many young women.

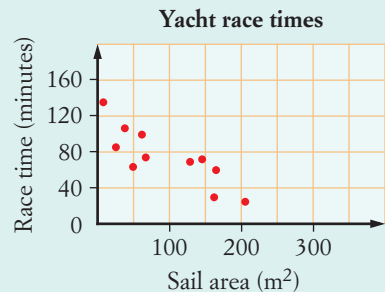
Sometimes you will be interested in information about two aspects of something. For example, health professionals have compiled tables of heights and weights of people. You have previously seen that a line graph may be useful. When the data is 'scattered' you cannot draw a line.

Important!

Bivariate data and scatter plots

Data that has only one measurement involved is called **univariate data**. Data where each item has two measurements is called **bivariate data**. Bivariate data often involves measurements taken over time, and is very common in science.

You can display bivariate data that has two numerical variables as a **scatter plot**. The values are plotted as coordinates. One variable is on the horizontal axis and the other one is on the vertical axis.



Example 14

The following information shows the ages of some people at their last birthday and the numbers of presents they received. Draw a scatter plot to show the information.

Age	13	12	5	9	15	15	5	20	15	16	6	18	7	7
Presents	8	8	5	6	9	3	5	4	6	5	9	6	4	8

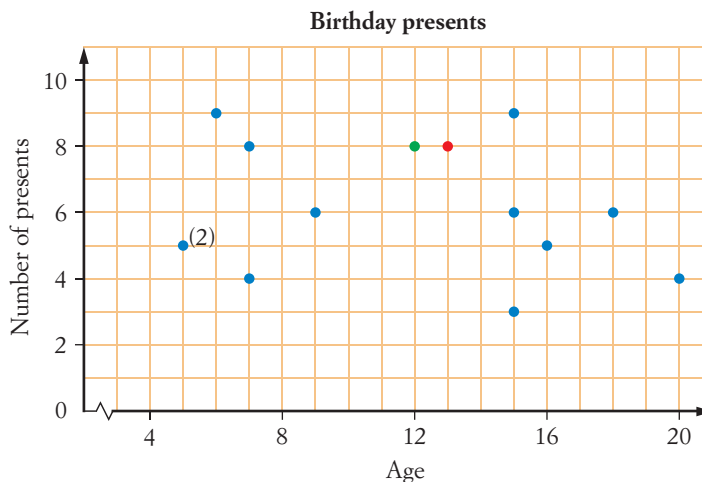
Solution

Mark the axes, putting age along the bottom and presents up the side.

Show age from 4 to 21 and presents from 2 to 10.

Represent each pair of values using a point, with the age first, such as (13, 8). The next point is (12, 8).

Plot the rest of the points. One point, (5, 5) is plotted twice in the same place. You can try to show this by putting a little 2 in brackets next to it.



There is no obvious pattern in the scatter plot shown in Example 14, but sometimes there is a pattern that does not give a line. When you compare the ages and heights of students, younger students are generally shorter than older students. While this is *generally* true, it is not true to say that older students are always taller than younger students.

Example 15

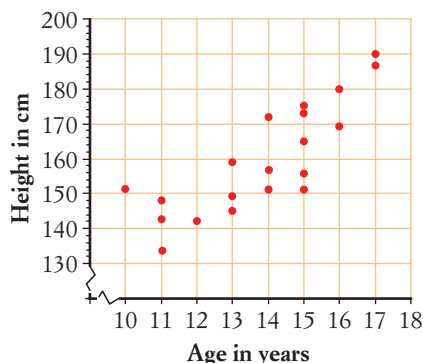
The table below shows the ages and heights of some students. Draw a scatter plot and comment on the results.

Age	10	17	14	15	15	17	12	11	14	16
Height in cm	151	186	157	165	151	190	142	143	151	169

Age	13	11	15	13	15	16	15	13	14	11
Height in cm	159	148	176	149	173	180	156	145	172	134

Solution

Do the ages from 10 to 18 and the heights from 130–200 cm



Make a comment.

The older students are generally taller than the younger students.

CAS TI-Nspire exercise

Statistics

MAT09SPTI00001

CAS ClassPad exercise

Statistics

MAT09SPCP00001

Investigate: Handspan and height

Work in groups to measure the widths of students' palms, pointer finger length, index finger length, thumb thickness, height and weight.

What happens when different people measure the same person's dimensions?
Can you make this less likely?

Make scatter plots of pairs of these measurements and look for relationships between them.

Do you get the same relationships if you do boys and girls separately?

You might like to download some more data from the CensusAtSchool site, which has data of this kind for thousands of students.



Exercise 1.3 Data analysis

Understanding

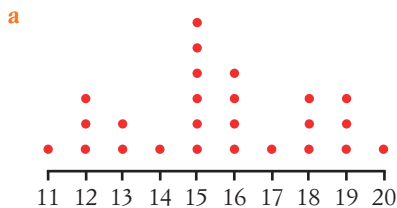
Extra questions

Exercise 1.3

MAT09SPEQ00003

See Example 10

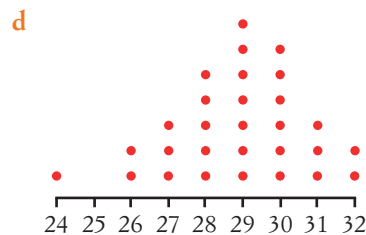
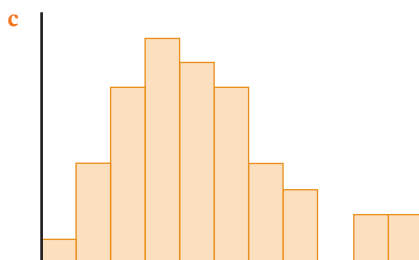
- 1 Describe each of the following distributions.



b

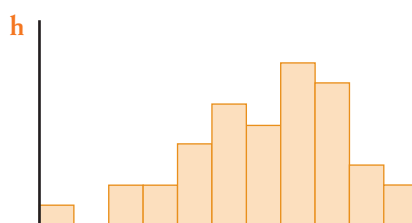
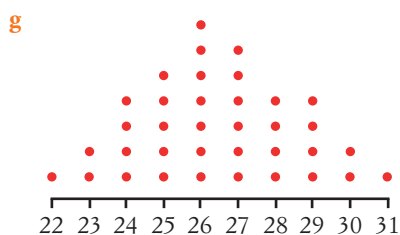
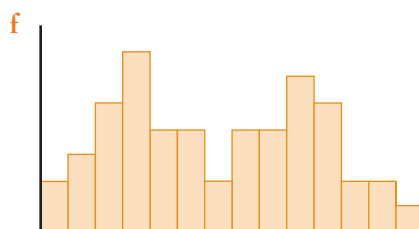
Key: 8|7 = 87

6	7
7	9 5 6
8	0 0 3 7 8 2
9	1 2 5 7
10	4 4 5 7 8
11	1 3
12	1
13	7
14	5



e Key: 7|34 = 734

5	41
6	13 51
7	28 64 81 93 97
8	42 72
9	12 35
10	32 44 79
11	11 23 45 62 71 79
12	07 39 86 94
13	21 58
14	32



2 Draw scatter plots of each of the following distributions.

a

x	16	6	11	10	8	4	14	1	17
y	96	52	83	77	64	42	89	27	100

b

x	11	12	7	10	3	11	1	5	10
y	0.5	1.2	7.3	5.6	12.9	2.8	15.2	9.8	3.6

c

x	25	32	23	8	27	17	6	11	16
y	5.6	5.3	7.9	1	2.9	5.2	6.8	8.8	0.8

Fluency

See Example 14

3 Use statistical measures to compare the times spent on sport in a week reported by the following samples of boys and girls.

Girls: 15, 10, 4, 4, 3, 10, 14, 20, 1, 2, 12, 8, 1, 3, 5, 2, 2, 3, 4, 13, 4, 10, 3, 2, 30, 1, 2, 6, 4, 2

Boys: 11, 26, 8, 5, 9, 2, 13, 10, 4, 16, 5, 4, 1, 7, 9, 4, 1, 5, 12, 7, 10, 5, 15, 5, 5, 3, 0, 4, 2, 16

4 Use statistical measures to compare the numbers of red lights encountered by equal numbers of vehicles travelling north and south on Lutwyche Road at 7:45 a.m. on a week day.

North: 18, 24, 17, 21, 17, 22, 18, 26, 16, 21, 19, 23, 20, 22, 19, 22, 18, 21, 14, 21, 18, 21, 19, 22, 19

South: 14, 26, 14, 25, 15, 16, 15, 24, 15, 21, 14, 20, 15, 21, 13, 21, 13, 15, 13, 17, 14, 18, 14, 19, 11

Problem solving

See Example 12

See Example 13

- 5 The marks of some students for Maths and Science are shown below.

Maths: 27, 68, 40, 65, 54, 88, 30, 69, 61, 72, 23, 80, 56, 77, 57, 72, 58, 67, 56, 74, 41, 69, 44, 70, 61, 73, 61, 77, 15, 73, 47, 74, 44, 69, 31, 82, 35, 81, 16, 67

Science: 32, 59, 45, 83, 33, 56, 42, 73, 32, 66, 29, 72, 40, 57, 38, 63, 53, 84, 47, 90, 12, 66, 20, 78, 45, 71, 18, 66, 28, 60, 18, 90, 50, 59, 45, 57, 12, 57, 43, 74

a Draw a back-to-back stem-and-leaf plot for the data.

b Compare the results.

- 6 The masses of 25 kg bags of potatoes from two different suppliers are shown below.

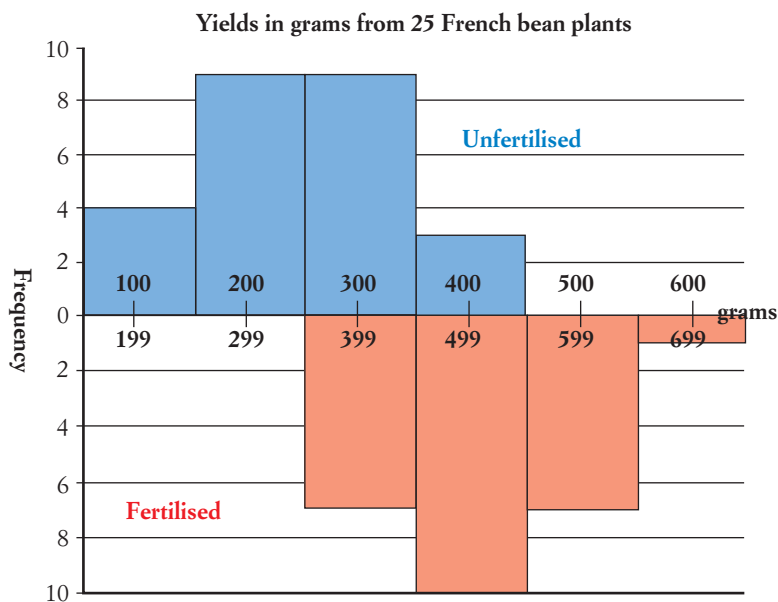
Potato wizards: 25.7, 27.3, 26.5, 25.3, 25.2, 24.6, 28.0, 23.7, 25.5, 25.2, 23.3, 23.2, 25.9, 27.2, 25.0, 26.4, 22.2, 24.6, 23.6, 26.8, 25.8, 24.9, 23.4, 23.2, 25.1

Spud kings: 24.7, 25, 23.8, 26.1, 23.0, 24.1, 24.6, 25.1, 23.9, 25.6, 24.8, 25.1, 25.0, 25.1, 25.0, 25.3, 24.0, 24.9, 23.9, 24.4, 24.2, 24.3, 25.2, 25.7, 25.9

a Draw a back-to-back stem-and-leaf plot for the data.

b Compare the results.

- 7 Use the back-to-back histogram below to compare the yields from fertilised and unfertilised crops of French beans.



- 8 Draw a back-to-back histogram to compare the prices below paid for 'organic' and 'premium' lettuces.

Price	\$1.60– \$1.99	\$2.00– \$2.39	\$2.40– \$2.79	\$2.80– \$3.19	\$3.20– \$3.59	\$3.60– \$3.99	\$4.00– \$4.39
Organic frequency	1	1	3	4	5	2	1
Premium frequency	2	3	4	5	1	1	1

Worked solution

Exercise 1.3

MAT09SPWS00003

- 9 The data below shows the IQ and shoe sizes of a group of Year 9 students.

See Example 15

IQ	123	115	85	96	124	131	135	105	111
Shoe size	8	5	11	5	7	6	7	7	10

- Draw a scatter plot.
- Comment on the results.

- 10 The germination rate for parsnip seeds stored for different times is shown below.

Worked solution

Exercise 1.3

MAT09SPWS00003

Time (weeks)	0	2	4	6	8	10	12	14	16	18	20
Germination (%)	95	91	79	75	83	71	67	67	55	51	47

- Draw a scatter plot to show the results.
- What happens to the germination rate as time goes on?

- 11 The information below shows the cost and weight of some expensive bicycles.

Cost (\$)	3700	1200	1500	2300	2800	3200	2500	5500
Mass (kg)	8.3	9.2	8.8	8.6	8.2	7.9	7.6	7.2

- Draw a scatter plot of the information.
- Comment on the results.



- 12 The data below shows the populations and the average walking speeds of people in some Australian towns.

Worked solution

Exercise 1.3

MAT09SPWS00003

Population	1400	5100	12 300	18 700	3200	8400	15 100
Walking speed (m/s)	0.62	0.78	0.87	0.95	0.70	0.75	0.81

- Draw a scatter plot of the information.
- Comment on the results.

- 13 Make up data sets that are skewed to the left and to the right. Find the median and mean of each and make a generalisation about the positions of the mean and median.

Reasoning

- 14 For each part of question 1, work out the relationship between mean and median, explaining your reasoning.

See Example 11

Chapter 1 summary

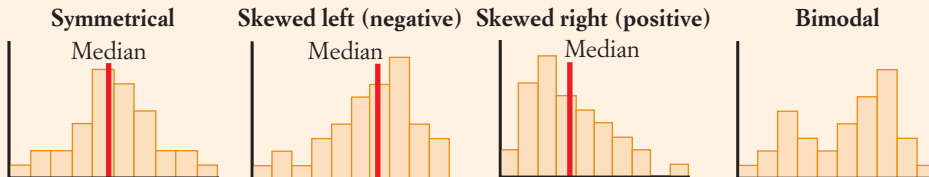
Quiz

Statistics

MAT09SPQZ00001

- The **population** is the complete group that statistical information would relate to. A **census** obtains data from the whole population.
- A **sample** is part of the population. A **survey** obtains data from a sample.
- A **fair sample** is like the whole population. A **biased sample** is not typical of the population.
- You can also get **bias** by using poor methods or questions.
- **Random sampling** ensures that every member of the population has an equal chance of being chosen in the sample.
- **Systematic sampling** means choosing every n th member of a population from a (numbered) list.
- **Sampling by lot** is a form of random sampling and involves drawing a sample from a hat or its equivalent.
- **Categorical** data separates information into different (named) categories: **nominal** data has no numeric meaning or order, **ordinal** data has a natural order, but does not represent a measurement.
- **Numerical** data consists of numbers: **continuous** data can take any value between the smallest and largest values, **discrete** data can only have particular values. In most cases, the values are whole numbers.
- A **two-way table** is used for two variables. The values of one variable go across the top row and the values of the other variable go down the left-hand column.
- The **mean** $\bar{x}$ is the **average** score. It is the total divided by the number of scores.
- The **median** is the middle score. It is found by arranging all the scores in order to choose the middle score. **Cumulative frequency** is the progressive total of frequencies and can be used to work out the median.
- The **mode** is the score with the highest frequency. A **bimodal** distribution has two scores with equal highest frequencies.
- The mean, median and mode are called **measures of central tendency**.
- The mean is most often used as the typical score. However, if there are a few high or low scores that would distort the mean, you should use the median. The mode should be used when the most common score is needed.
- The **range** is the difference between the highest and lowest scores and measures the **spread** of the data.
- The **quartiles** of a frequency distribution divide it into quarters.
- The **first quartile (lower quartile)** is the score that has a quarter (25%) of the scores below it. Its symbol is Q_1 or Q_1 .
- The **second quartile** is the score that has two quarters (50%) of the scores below it. It is another name for the median. It can be written as Q_2 or Q_2 .
- The **third quartile (upper quartile)** is the score that has three quarters (75%) of the scores below it. Its symbol is Q_3 or Q_3 .
- The **interquartile range (IQR)** is the difference between the third and first quartiles.
$$IQR = Q_3 - Q_1$$
- A **symmetrical** distribution is roughly the same shape on either side of the centre.

- A distribution is **skewed to the left** if the scores are spread out more below the median than above the median. If the scores are spread out more on the right, it is **skewed to the right**. A distribution skewed to the left has a **negative skew**, while one skewed to the right has a **positive skew**.
- A **bimodal** distribution has two high parts, although they don't always have two actual modes.



- A **back-to-back** display is used to compare two lots of data. One group is shown to the left of a vertical axis, and the other group is on the right. Sometimes these displays are shown above and below a horizontal axis. Histograms may be shown with **side-by-side** columns to allow for comparisons.
- An **outlier** is a score that has a very different value to the rest of the data. It can be defined as being more than 1.5 times the interquartile range below the first quartile or above the third quartile.
- Data that has only one measurement involved is called **univariate data**.
- Data where each item has two measurements is called **bivariate data**.
- A **scatter plot** of bivariate data has values that are plotted as coordinates. One variable is on the horizontal axis and the other one is on the vertical axis. It is used to find relationships between the two variables.

Chapter 1 review

Understanding

See Example 1

- 1 State whether the population or a sample has been used in each of the following.
 - a All the people who live in a long street are asked: 'Do you want one end of the street blocked to make it safer for children?'.
 - b The people who live in a street are asked: 'Do you want traffic calming methods like speed bumps in the suburb?'.
 - c The people who live in a suburb are asked: 'Do you have strong objections to traffic calming like speed bumps being used in the suburb?'.

See Example 3

- 2 Describe each of the following as nominal, ordinal, discrete or continuous.
 - a The capacity in litres of a new refrigerator.
 - b The energy rating of a new refrigerator.
 - c The price of a new refrigerator.

See Example 5

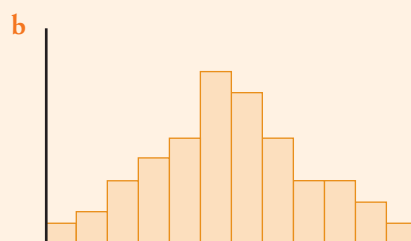
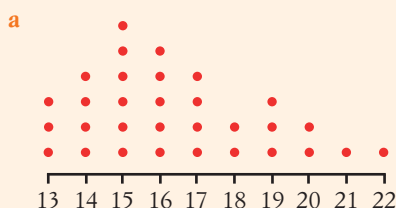
- 3 Find the mean, median, mode and range of each of the following sets of data.
 - a 7, 15, 10, 14, 8, 13, 6, 17, 9, 18, 11, 14, 6, 13, 8, 13, 6, 16, 10, 12, 10, 14, 7, 15, 9
 - b 79, 84, 94, 100, 96, 85, 111, 86, 109, 111, 79, 121, 77, 97, 111, 95, 84
 - c 13, 19, 17, 19, 10, 20, 16, 21, 14, 18, 18, 19, 15, 19, 16, 19, 17, 20, 16, 22, 10, 18, 11, 20, 16, 19, 15, 19, 13

See Examples 5, 6

- 4 For each of the following, state which measures of central tendency can be found.
 - a The numbers of rooms available in motels and flats in Mooloolaba in March.
 - b The RACQ ratings of rooms available in motels and flats in Mooloolaba in March.
 - c The floor space of rooms available in motels and flats in Mooloolaba in March.
 - d The prices of rooms available in motels and flats in Mooloolaba in March.

See Example 10

- 5 Describe each of the following distributions.



c

	Key: 4 6 = 4.6
1	0 3 5
2	2 4 6 8 8
3	3 4 4 6 9 9 9
4	5 6 6 7
5	2 6
6	3 5 7 8 9
7	0 2 2 3 5 5 7 9
8	2 4 5 6
9	0 3

- 6 Find the quartiles and interquartile ranges of each of the following sets of data.
- a 26, 50, 29, 37, 21, 50, 25, 38, 29, 34, 24, 46, 23, 36, 21, 32, 20, 33, 25, 50, 29, 32, 26, 37, 28, 49, 29, 34, 27, 45, 26, 48, 28, 33, 24, 43, 28, 48, 27, 32
 - b 16, 20, 9, 20, 17, 19, 15, 20, 16, 18, 13, 19, 13, 21, 18, 21, 14, 22, 13, 18, 14
 - c 12, 20, 15, 24, 17, 21, 15, 21, 16, 22, 14, 25, 13, 21, 16, 27, 16, 18, 14, 21, 15, 28, 15

See Example 8

- 7 Describe each of the following as either categorical or numerical.
- a The wavelength of light reflected from an object in green light.
 - b The apparent colour of an object in green light.
 - c The apparent brightness of some green light.

Fluency

See Example 3

- 8 Choose a sample of 8 items from numbers between 50 to 250 inclusive, starting from 127.

See Example 2

- 9 Extend each of the following tables and use them to find the mean, median and mode.

See Example 8

a

Score	2	3	4	5	6	7	8	9
Frequency	1	4	7	9	5	4	0	2

b

Score	11	12	13	14	15	16
Frequency	3	9	12	20	17	8

c

Score	22	23	24	25	26	27	28
Frequency	7	8	5	4	7	9	3

- 10 Find the median for each of the following.

See Example 6

- a Ratings of rooms available in motels and flats in central Mooloolaba in March.



- b ABCDE results of students in Year 9 Art.
C, D, C, A, B, B, B, A, D, C, D, C, C, C, C, B, C, D, C, B, D, C, B, C, D, B, B, B, D, E, C, B, D, A, C, D, B
- c SD/D/N/A/SA opinion survey results on whether students should learn about finance.
D, N, N, N, SA, D, N, A, A, N, SD, N, D, N, A, D, A, N, A, SA, SA, D, N, SA, A, SA

Chapter 1 review

See Example 4

- 11** The following data gives the age of some Year 9 students in a school and the number of siblings that they have. Make a two-way table for the information and comment on the results.
15, 1; 14, 2; 15, 3; 15, 2; 14, 3; 15, 1; 15, 4; 15, 3; 15, 5; 16, 0; 15, 3; 14, 3; 15, 1; 14, 1; 16, 0; 16, 1; 16, 3; 16, 4; 15, 2; 16, 1; 15, 1; 15, 0; 15, 3; 15, 2; 14, 3; 16, 5; 15, 0; 16, 2; 16, 2; 16, 2; 14, 2; 16, 3; 14, 2; 16, 2; 16, 3; 14, 3; 15, 4; 16, 1; 15, 2; 14, 3; 15, 0; 14, 2; 15, 1; 16, 1; 15, 0

See Example 9

- 12** Use the IQR criterion to find any outliers in the following sets.
a 16, 23, 16, 24, 16, 20, 11, 17, 15, 27, 16, 19, 15, 24, 16, 18, 15, 19, 17, 20, 17, 19
b 18, 22, 17, 30, 20, 21, 14, 23, 17, 30, 19, 21, 16, 26, 19, 29, 16, 22, 17, 25, 18, 20, 19
c 34, 37, 20, 35, 34, 37, 29, 35, 31, 36, 33, 37, 29, 36, 22, 37, 30, 39, 31

See Example 14

- 13** Draw scatter plots for each of the following.

a

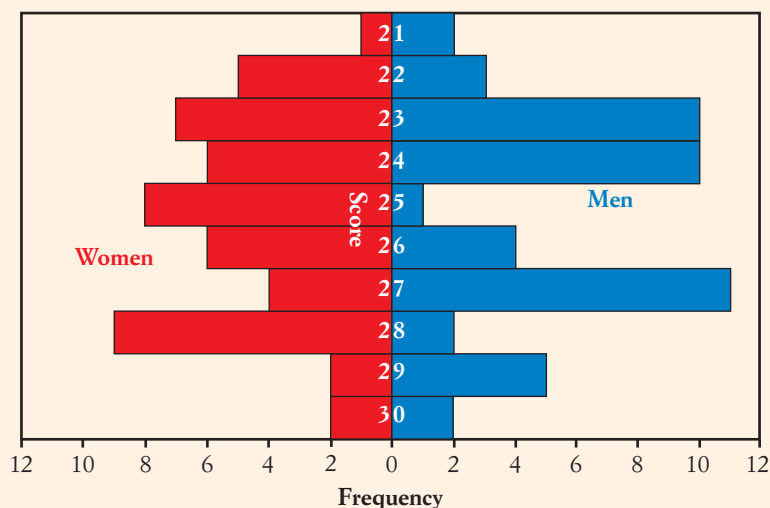
x	17	20	7	8	18	19	7	12	14	5	17	17	15
y	38	39	25	25	41	39	19	27	36	21	45	40	35

b

x	8	13	19	20	13	15	19	10	24	12	12
y	34	10	7	6	36	14	21	30	20	25	20

Problem solving

- 14** How could each of the following be improved?
a A survey of people arriving at a sports ground on Saturday morning were asked: 'Do you agree that the government should give more money for sport?'
b A survey of people picked randomly from the phone book were asked: 'Do you think there should be more buses and trains for people who don't have cars?'
- 15** 23 people had a median height of 175 cm, a mean height of 172 cm and a mode of 169 cm. Three more people of heights 167 cm, 168 cm and 169 cm joined the group. If possible, show how to find the new median, mean and mode.
- 16** The back-to-back bar graph below shows the scores of men and women in Learner Driver's licence tests. Use the graph to compare the performance of men and women.



- 17** Use statistical measures to compare the results below of the Chess Grandmasters and amateurs in separate tournaments of 8 rounds.

See Example 12

Grandmasters: $3\frac{1}{2}$, $5\frac{1}{2}$, $3\frac{1}{2}$, 5, $3\frac{1}{2}$, $4\frac{1}{2}$, $2\frac{1}{2}$, $5\frac{1}{2}$, 4, 5, $2\frac{1}{2}$, $4\frac{1}{2}$, $1\frac{1}{2}$, 6, $3\frac{1}{2}$, $4\frac{1}{2}$, $2\frac{1}{2}$, 4, $2\frac{1}{2}$, $4\frac{1}{2}$, $3\frac{1}{2}$, 4, $3\frac{1}{2}$, $5\frac{1}{2}$, $3\frac{1}{2}$, 5, $2\frac{1}{2}$, 6, $2\frac{1}{2}$, $5\frac{1}{2}$

Amateurs: $4\frac{1}{2}$, 6, 3, $4\frac{1}{2}$, 4, $4\frac{1}{2}$, $2\frac{1}{2}$, $5\frac{1}{2}$, $3\frac{1}{2}$, $6\frac{1}{2}$, 2, 5, 2, 4, 3, 6, 1, 6, $1\frac{1}{2}$, 6, $2\frac{1}{2}$, 5, 1, $5\frac{1}{2}$, 3, $4\frac{1}{2}$, 2, $7\frac{1}{2}$, 2, 6

- 18** The test results below are from a Year 9 Cooking class.

See Example 13

Girls: 31, 60, 30, 81, 52, 66, 51, 60, 20, 69, 48, 87, 52, 84, 24

Boys: 65, 72, 20, 72, 38, 71, 85, 75, 70

a Make a back-to-back stem-and-leaf plot of the data.

b Compare the results.

- 19** Students at a naval academy were asked to decode ten sets of signal flags as quickly as they could. The results below show the times taken by different students and the number of mistakes made.

Time (s)	115	193	196	170	160	51	68	79	143	112	118
Mistakes	2	0	1	2	1	6	5	7	4	5	4
Time (s)	141	123	189	100	29	69	143	93	113	149	
Mistakes	3	2	1	4	7	3	2	5	5	1	

a Make a scatter plot of the results.

b Comment on the results.

- 20** Describe each part of question 1 as fair or biased and explain your reasons.

Reasoning

- 21** Explain why large samples are better than small samples.

- 22** The average approval rating of a school given by students' parents was 3.4 on a scale of 1–5. The following year, the average increased to 3.6, but 20% fewer parents sent back the survey. In both years, the survey was on a tear-off section of the school newsletter. Explain what else you would want to know before accepting that this was a significant change.

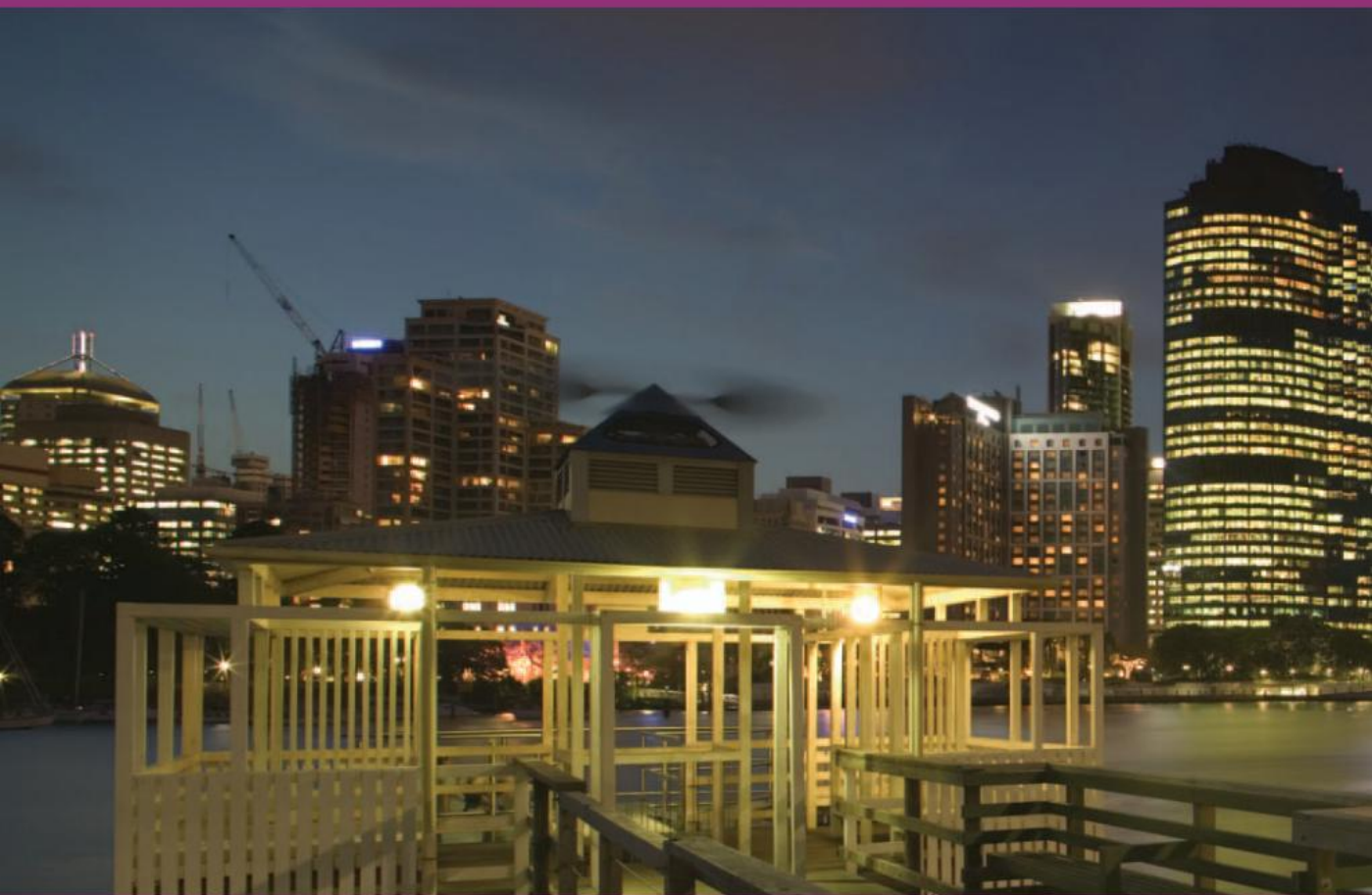
- 23** Explain how you can use a calculator to obtain a random sample of:

a 7 numbers from 8 to 840 inclusive

b 6 patients from those in a hospital.

- 24** The median, range and IQR of a set of data are 24, 24 and 12 respectively. For a second set of data they are 28, 20 and 8 respectively. If it is possible for both sets to include both 10 and 30, show how this could occur.

- 25** For each part of question 5, explain the relationship between the mean and median.



Number and algebra

2

Number and indices



Contents

- 2.1 Using numbers
- 2.2 Index laws
- 2.3 Scientific notation
- 2.4 Using the index laws
- Chapter summary
- Chapter review

Prior learning

Chapter 2

MAT09NAPL00002

Parent guide

Chapter 2

MAT09NAPG00002

Curriculum guide

Chapter 2

MAT09NACU00002

Australian Curriculum statements

Real numbers

Apply index laws to numerical expressions with integer indices.

Express numbers in scientific notation.

Video tutorial

Numbers and indices

MAT09NAVT00002

Weblink

Links to information on different number systems

MAT09NAWB00002

The modern number system has positive and negative numbers, and powers and scientific notation can be used to write large and small numbers. These numbers are needed in modern science and technology. For example, modern computers rely on all kinds of numbers to make them work. It is very important for you to be able to use numbers effectively and to be able to choose the best method of calculation for a particular problem. You must be able to do mental arithmetic, to use a calculator accurately and to use pen and paper to perform mathematics.

Maths dictionary

MAT09ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics.

You may find the glossary or online mathematical dictionary useful for this purpose.

base	index	positive number	square root
cube	integer	power	surds
cube root	magnitude	scientific notation	
directed number	mantissa	significant figure	
exponent	negative number	square	

2.1 Using numbers

Important!

Order of operations

- 1 Work out the terms in the brackets first, from innermost outwards and from left to right. A vinculum (bar) works like brackets for operations on top of it or underneath it.
- 2 Work out powers and roots from left to right.
- 3 Work out multiplications and divisions next, from left to right.
- 4 Work out additions and subtractions last, from left to right.

Example 1

Evaluate the following.

a $3 + 4 \times 5$

b $6 \times [15 + (8 - 3)] \div (2 + 3) + 18 \div 3$

c $\frac{17 + 3 \times 7}{11 - 9}$

d $5 \times \sqrt{55 - 30} \div 5 - 4^3 \div 8$

e $\frac{2 + 6}{2} \times 9 - 20 + 5 \times 3$

Solution

- a** Write the problem.

$$3 + 4 \times 5$$

Perform multiplication first.

$$= 3 + 20$$

Do addition and subtraction last.

$$= 23$$

CAS TI-Nspire exercise

Number and indices

MAT09NATI00002

CAS ClassPad exercise

Number and indices

MAT09NACP00002

- b** Write the problem.

Calculate the brackets first, inside out.

Finish the brackets.

Now perform multiplication and division,
left to right.

Do addition and subtraction last.

- c** Put brackets around the numerator because
it must be worked out first.

Work out the top and bottom separately.

Now perform the division.

- d** Put brackets around the part that must be
worked out first.

Calculate the brackets, starting with the
division.

Do the powers and roots.

Perform multiplication and division.

Do addition and subtraction last.

- e** Write the question with brackets instead
of the bar above.

Calculate the brackets first.

Perform the multiplications.

Do addition and subtraction last.

$$6 \times [15 + (8 - 3)] \div (2 + 3) + 18 \div 3$$

$$= 6 \times [15 + 5] \div (2 + 3) + 18 \div 3$$

$$= 6 \times 20 \div 5 + 18 \div 3$$

$$= 24 + 6$$

$$= 30$$

$$\frac{(17 + 3 \times 7)}{11 - 9}$$

$$= \frac{(17 + 21)}{2}$$

$$= \frac{38}{2}$$

$$= 19$$

$$5 \times \sqrt{(55 - 30 \div 5)} - 4^3 \div 8$$

$$= 5 \times \sqrt{(55 - 6)} - 4^3 \div 8$$

$$= 5 \times \sqrt{49} - 4^3 \div 8$$

$$= 5 \times 7 - 64 \div 8$$

$$= 35 - 8$$

$$= 27$$

$$(2 + 6) \times 9 - 20 + 5 \times 3$$

$$= 8 \times 9 - 20 + 5 \times 3$$

$$= 72 - 20 + 15$$

$$= 67$$

When you use a calculator to perform multiple operations, it is very important that you use brackets correctly. In some cases, such as those involving square roots and fractions, it is necessary to insert brackets to ensure that the calculator does the operations in the correct order. You may find it easier to write out the problem on one line before entering it in your calculator.

Alternatively, you could break the problem into several steps and use the memory of your calculator to store answers for the next steps.

In real life, you need to be able to decide what calculations are needed to solve a practical problem. This may involve more than one calculation.

Example 2

There are 50 matches in a box, and 10 boxes in a wrapped packet. A carton contains 120 packets. How many matches are in a carton?



Solution

Multiply to get the number of matches in a wrapped packet.

$$\begin{aligned}\text{Number in a packet} &= 10 \times 50 \\ &= 500\end{aligned}$$

Multiply to get the number of matches in a carton.

$$\begin{aligned}\text{Number in a carton} &= 120 \times 500 \\ &= 60\,000\end{aligned}$$

Write the answer.

There are 60 000 matches in a carton.

On your calculator, you could do the calculation in Example 2 in any of the following ways:

$$10 \times 50 \times 120 =$$

$10 \times 50 \times 120$	60000

$$10 \times 50 =$$

10×50	500

$$\times 120 =$$

$\text{Ans} \times 120$	60000

$$10 \times 50 \text{ STO } A$$

$10 \times 50 \rightarrow A$	500

$$\text{Then do } 120 \times \text{RCL } A =$$

$120 \times A$	60000

The exact sequence of steps for storing and recalling answers will depend on the calculator model you are using.

Example 3

Peter is travelling at an average speed of 85 km/h from Brisbane to Bundaberg, a distance of 375 km. How much further has he left to go after 3 hours?

Solution

Multiply to find the distance.

$$\begin{aligned}\text{Distance travelled} &= 85 \times 3 \\ &= 255 \text{ km}\end{aligned}$$

Subtract to find the distance left.

$$\begin{aligned}\text{Distance left} &= 375 - 255 \\ &= 120 \text{ km}\end{aligned}$$

Write the answer.

He has 120 km left to go.

Worksheet

Modelling and problem solving

MAT09NAWK00013

Worksheet

Solving number problems

MAT09NAWK00017

Important!

Integers

Integers are whole numbers with a sign. **Negative numbers** have a raised $-$ sign. **Positive numbers** have a raised $+$ sign. The $+$ sign is usually left off positive numbers. Integers are also known as **directed numbers**.

All positive numbers are greater than zero. All negative numbers are less than zero.

Integers can be **added** by:

- starting from the first number and counting towards the second number, in the direction of its sign; or
- for numbers with the same sign, adding normally and keeping the sign; and
- for numbers with opposite signs, taking away and using the sign of the part left over.

Integers can be **subtracted** by:

- starting from the first number and counting towards the second number, in the *opposite* direction to its sign; or
- adding the *opposite* of the number after the subtraction sign.

When two integers of the same sign are **multiplied** or **divided**, the answer is positive. When two integers with different signs are **multiplied** or **divided**, the answer is negative.

Example 4

Work out the following.

a $-8 + -3 \times -5$

Solution

a Write the problem.

Perform multiplication before addition.

b Write the problem.

Calculate the terms inside the brackets first.

Perform multiplication and division first, then do addition.

Now do the rest of the calculations.

Do multiplication before subtraction.

b $6 \times -5 - (3 \times -5 + -18 \div 2)$

$$-8 + -3 \times -5$$

$$= -8 + 15$$

$$= 7$$

$$6 \times -5 - (3 \times -5 + -18 \div 2)$$

$$= 6 \times -5 - (-15 + -9)$$

$$= 6 \times -5 - -24$$

$$= -30 - -24$$

$$= -6$$

Worksheet

Using numbers 1

MAT09NAWK00011

Worksheet

Integers 1

MAT09NAWK00014

Worksheet

Integers 2

MAT09NAWK00016

You can use your calculator for operations with integers. The negative key is the $(-)$ button on most calculators. On some calculators, to enter a negative number, you press the $+/-$ key *after* the number to make it negative. Scientific calculators correctly follow the order of operations, so you could do Example 4b above as shown below, but be careful to enter it exactly.

6 $\times$ $(-)$ 5 $-$ (3 $\times$ $(-)$ 5 $+$ $(-)$ 18 $\div$ 2) $\boxed{6 \times 5 - (3 \times 5 + 18 \div 2)}$ $\boxed{-6}$

You should choose between mental, written or calculator methods when doing fraction and decimal calculations, depending on the actual calculation.

Example 5

Calculate the following in your head.

a $0.471 + 0.256$

b $32.5 - 17.7$

c $64.7 + 2.684$

d $7.6 - 4.742$

Solution

a Treat as $471 + 256 (= 470 + 257)$ thousandths.

$$0.471 + 0.256 = 0.727$$

b Treat as $325 - 177 (= 328 - 180)$ tenths.

$$32.5 - 17.7 = 14.8$$

c Add the overlap ($4.7 + 2.6$) and leave the rest.

$$64.7 + 2.684 = 67.384$$

d Treat as $7.6 - 4.8 (= 7.8 - 5)$ plus $0.100 - 0.042$.

$$7.6 - 4.742 = 2.858$$

Example 6

Use written methods for the following.

a $125.4078 + 24.267 + 4.96752 + 45.77057$

b $5152.97 - 348.4963$

c $28.7 + 32.76 - 15.35 + 4.098 - 21.076 - 4.278$

d $44.63 - 3.46 + 19.21 - 4.7$

e $5.8 - 22.37 + 23.28$

Solution

a Set out vertically with the decimal points under each other.

Add extra zeros so all the decimals are the same length.

Add in the usual way, and put the decimal point in place.

$$\begin{array}{r} \overset{1}{1} \overset{2}{2} \overset{2}{5} \overset{2}{4} \overset{2}{0} \overset{2}{7} \overset{1}{8} \overset{0}{0} \\ 24.26700 \\ 4.96752 \\ + 45.77057 \\ \hline 200.41289 \end{array}$$

b Set out vertically with the decimal points under each other.

Add extra zeros so all the decimals are the same length.

Add in the usual way, and put the decimal point in place.

$$\begin{array}{r} 448169 \\ 5152.9710 \\ - 348.4963 \\ \hline 4804.4737 \end{array}$$

- c Write the problem.

Rearrange for efficiency.

$$\begin{aligned} & 28.7 + 32.76 - 15.35 + 4.098 - 21.076 - 4.278 \\ &= 28.7 + 32.76 + 4.098 - 15.35 - 21.076 - 4.278 \\ &= 28.7 + 32.76 + 4.098 - (15.35 + 21.076 + 4.278) \end{aligned}$$

Add extra zeros and add each part.

$$\begin{array}{r} ^1 ^1 ^1 \\ 28.7 \\ 32.76 \\ + 4.098 \\ \hline 65.558 \end{array} \quad \begin{array}{r} ^1 ^2 ^1 \\ 15.35 \\ 21.076 \\ + 4.278 \\ \hline 40.704 \end{array}$$

Now subtract the totals.

$$\begin{array}{r} ^4 \\ 65.558 \\ - 40.704 \\ \hline 24.854 \end{array}$$

- d Write the problem.

Change the negative numbers and operations.

Rearrange.

$$\begin{aligned} & 44.63 - 3.46 + ^{-}19.21 - ^{-}4.7 \\ &= 44.63 - 3.46 - 19.21 + 4.7 \\ &= 44.63 + 4.7 - 3.46 - 19.21 \\ &= 44.63 + 4.7 - (3.46 + 19.21) \end{aligned}$$

Do the additions.

$$\begin{array}{r} ^1 \\ 44.63 \\ + 4.7 \\ \hline 49.33 \end{array} \quad \begin{array}{r} ^1 \\ 3.46 \\ + 19.21 \\ \hline 22.67 \end{array}$$

Now subtract the totals.

$$\begin{array}{r} ^8 ^2 \\ 49.33 \\ - 22.67 \\ \hline 26.66 \end{array}$$

- e Write the problem.

Change the negative number and operation.

Rearrange.

Add a zero and do the addition.

$$\begin{aligned} & ^{-}5.8 - 22.37 + ^{-}23.28 \\ &= ^{-}5.8 - 22.37 - 23.28 \\ &= ^{-}(5.8 + 22.37 + 23.28) \end{aligned}$$

$$\begin{array}{r} ^1 ^1 ^1 \\ 5.8 \\ 22.37 \\ + 23.28 \\ \hline 51.45 \end{array}$$

Write the answer with the correct sign.

$$^{-}5.8 - 22.37 + ^{-}23.28 = ^{-}51.45$$

The calculation $44.63 - 3.46 + ^{-}19.21 - ^{-}4.7$ would be entered on your calculator as

44.63 **-** 3.46 **+** **(-)** 19.21 **-** **(-)** 4.7 **=**

44.63-3.46+-19.21--4.7 26.66

Example 7

Calculate the following.

a $5\frac{2}{5} - 2\frac{7}{10}$

c $6\frac{5}{8} - 1\frac{11}{12}$

b $2\frac{2}{3} + 4\frac{1}{2} + 5\frac{3}{4}$

d $4\frac{5}{14} - 2\frac{5}{6}$

Solution

- a** Write the problem.

Subtract the whole numbers and change the fractions to the lowest common denominator.

Change $3\frac{4}{10}$ to $2\frac{14}{10}$ so that you can subtract.

Subtract the numerators.

- b** Write the problem.

Add the whole numbers and change the fractions to the lowest common denominator, 12.

Add the numerators.

Simplify.

- c** Write the problem.

Subtract the whole numbers and change the fractions to the lowest common denominator, 24.

Change $5\frac{15}{24}$ to $4\frac{39}{24}$ so that you can subtract.

Subtract the numerators.

- d** Write the problem.

Subtract the whole numbers and change the fractions to the lowest common denominator, 42.

Change $2\frac{15}{42}$ to $1\frac{57}{42}$ so that you can subtract.

Subtract the numerators.

Simplify.

$$\begin{aligned} 5\frac{2}{5} - 2\frac{7}{10} \\ &= (5 - 2) + \left(\frac{2}{5} - \frac{7}{10}\right) \\ &= 3 + \frac{4}{10} - \frac{7}{10} \\ &= 2\frac{14}{10} - \frac{7}{10} \\ &= 2\frac{7}{10} \end{aligned}$$

$$\begin{aligned} 2\frac{2}{3} + 4\frac{1}{2} + 5\frac{3}{4} \\ &= 11 + \frac{8}{12} + \frac{6}{12} + \frac{9}{12} \\ &= 11\frac{23}{12} \\ &= 12\frac{11}{12} \end{aligned}$$

$$\begin{aligned} 6\frac{5}{8} - 1\frac{11}{12} \\ &= 5 + \frac{15}{24} - \frac{22}{24} \\ &= 4\frac{39}{24} - \frac{22}{24} \\ &= 4\frac{17}{24} \end{aligned}$$

$$\begin{aligned} 4\frac{5}{14} - 2\frac{5}{6} \\ &= 2 + \frac{15}{42} - \frac{35}{42} \\ &= 1\frac{57}{42} - \frac{35}{42} \\ &= 1\frac{22}{42} \\ &= 1\frac{11}{21} \end{aligned}$$

Example 8

Work out the following.

a $32.4 \div 600$

c 42.27×0.000412

b 0.025×73.5

d $-37 \div -0.07$

Solution

a Treat 600 as 100×6 , so decrease the values by two places and divide by 6.

b Treat 0.025 as $\frac{1}{40}$, so decrease the values by one place and divide by 4.

c Ignore the decimal points and set out as 4227×412 , so put the 7 over the 2.

Multiply out.

Count the total number of decimal places and put in the decimal point.

Write the answer.

d Ignore the signs and do as $37 \div 0.07$.

Change the decimal places.

Do the division.

Take account of signs and write the answer, using a bar over the recurring digits.

$$32.4 \div 600 = 0.324 \div 6 \\ = 0.054$$

$$0.025 \times 73.5 = 7.35 \div 4 \\ = 1.8375$$

$$\begin{array}{r} 4227 \\ \times 0.000412 \\ \hline 8454 \\ 42270 \\ 1690800 \\ \hline 0.01741524 \end{array} \quad \begin{array}{l} 2 \text{ places} \\ 6 \text{ places} \\ 8 \text{ places} \end{array}$$

$$42.27 \times 0.000412 = 0.01741524$$

$$37 \div 0.07 \\ = 3700 \div 7 \\ \begin{array}{r} 528.5714285... \\ 7 \overline{) 37000.00000000} \end{array}$$

$$-37 \div -0.07 = 528.\overline{571428}$$

Example 9

Use your calculator to work out the following.

a -0.3574×62.9653

b $42.674 \div -0.0573$

Solution

a Enter as $(-)$ 0.3574 $\times$ 62.9653 $=$

$$\begin{array}{r} -0.3574 \times 62.9653 \\ -22.50379822 \end{array}$$

b Enter as 42.674 $\div$ $(-)$ 0.0573 $=$

$$\begin{array}{r} 42.674 \div -0.0573 \\ -744.7469459 \end{array}$$

Example 10

Work out the following in your head.

a $\frac{1}{4} \times \frac{5}{6}$

Solution

- a** Think of the 6ths as 24ths so that the numerator can be divided by 4.

Now do the multiplication by taking a quarter of the shaded 24ths.

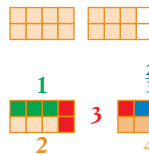
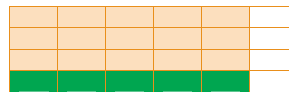
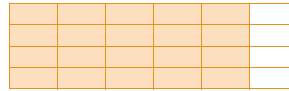
Write the answer.

b $1\frac{3}{4} \div \frac{3}{8}$

- b** Think of $1\frac{3}{4}$ in 8ths.

Think of the division as 'how many lots of three 8ths in fourteen 8ths?'

Write the answer.



$$\frac{5}{6} = \frac{20}{24}$$

$$\frac{1}{4} \times \frac{20}{24} = \frac{5}{24}$$

$$\frac{1}{4} \times \frac{5}{6} = \frac{5}{24}$$

$$1\frac{3}{4} = \frac{7}{4} = \frac{14}{8}$$

$$\frac{14}{8} \div \frac{3}{8} = 4\frac{2}{3}$$

$$1\frac{3}{4} \div \frac{3}{8} = 4\frac{2}{3}$$

Example 11

Work out the following using written methods.

a $\frac{1}{3} \times \frac{6}{11}$

b $\frac{5}{6} \times \frac{7}{8}$

c $\frac{6}{7} \times \frac{-14}{15}$

d $1\frac{3}{4} \times 2\frac{1}{6}$

e $\frac{5}{18} \times 3\frac{3}{4} \times -2\frac{1}{5}$

Solution

- a** Write the problem.

It is quickest to cancel first if possible.

Multiply the tops and multiply the bottoms.

$$\begin{aligned} \frac{1}{3} \times \frac{6}{11} &= \frac{1}{\cancel{3}} \times \frac{\cancel{6}^2}{11} \\ &= \frac{1 \times 2}{1 \times 11} \\ &= \frac{2}{11} \end{aligned}$$

- b** Write the problem.

There is no cancellation, so multiply.

$$\begin{aligned} \frac{5}{6} \times \frac{7}{8} &= \frac{5 \times 7}{6 \times 8} \\ &= \frac{35}{48} \end{aligned}$$

- c Write the problem without the signs.

Cancel where possible.

Multiply the tops and multiply the bottoms.

Write the answer, taking the signs into account.

- d Write the problem.

Change to improper fractions.

There is no cancellation, so multiply.

Simplify.

- e Write the problem without the signs.

Change to improper fractions.

Cancel where possible.

Multiply.

Simplify.

Write the answer, taking the signs into account.

$$\begin{aligned} \frac{6}{7} \times \frac{14}{15} \\ = \frac{\cancel{2}^2 \cancel{6}^1}{\cancel{1}^1 \cancel{7}^1} \times \frac{\cancel{14}^2 \cancel{15}^1}{\cancel{15}^1 \cancel{5}^1} \\ = \frac{2 \times 2}{1 \times 5} \\ = \frac{4}{5} \end{aligned}$$

$$\frac{6}{7} \times \frac{-14}{15} = \frac{-4}{5}$$

$$\begin{aligned} 1\frac{3}{4} \times 2\frac{1}{6} \\ = \frac{7}{4} \times \frac{13}{6} \end{aligned}$$

$$= \frac{91}{24}$$

$$= 3\frac{19}{24}$$

$$\frac{5}{18} \times 3\frac{3}{4} \times 2\frac{1}{5}$$

$$= \frac{5}{18} \times \frac{15}{4} \times \frac{11}{5}$$

$$= \frac{\cancel{5}^1}{\cancel{6}^1 \cancel{18}^1} \times \frac{\cancel{15}^3 \cancel{5}^1}{4} \times \frac{11}{\cancel{5}^1}$$

$$= \frac{5 \times 1 \times 11}{6 \times 4 \times 1}$$

$$= \frac{55}{24}$$

$$= 2\frac{7}{24}$$

$$\frac{5}{18} \times 3\frac{3}{4} \times -2\frac{1}{5} = -2\frac{7}{24}$$

Investigate: The number system

Use the library or a search engine such as Google to find out where numbers came from. You should try to find the following.

- What the first zero looked like.
- The early Arabic symbols for numbers.
- The first uses of fractions by Babylonians and Egyptians.
- The invention of decimal fractions and their popularisation by John Napier.
- Other ways of writing fractions and decimals.

Your teacher may ask you to write a brief report.

Example 12

A local hardware store stocks clear plastic tubes. They sell more tubes in the 8 mm diameter than anything else, and it comes in 300 m rolls. On Monday they started a new roll and sold 3 m, 0.4 m, 2.5 m, 1.2 m and 1.6 m. They do twice as much business on each weekend day as during the week.

Assuming that a similar amount is bought each week day as on the Monday, when will they need to order more 8 mm tube?



Solution

Work out the amount sold on Monday in your head.

Double for the weekend days.

Work out the sales for a week.

Write down the intermediate calculations.

Work out the number of weeks a roll will last.

Enter as $300 \div 78.3 =$.

Allow time for delivery.

$$\begin{aligned}\text{Amount} &= 3 + 0.4 + 2.5 + 1.2 + 1.6 \\ &= 8.7 \text{ m}\end{aligned}$$

$$\text{Weekend day sales} \approx 17.4 \text{ m}$$

$$\begin{aligned}\text{Week sales} &\approx 5 \times 8.7 + 2 \times 17.4 \\ &= 43.5 + 34.8 \\ &= 78.3 \text{ m.}\end{aligned}$$

$$\text{Time} = 300 \div 78.3$$

$$300 \div 78.3 = 3.831417625$$

They should order more in 3 weeks.

In Example 12, calculating $30 \div 8$ in your head would probably give a good enough estimate of the time.

Example 13

A car's fuel gauge has Empty, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and Full marked, with divisions halfway between these marks. The tank has a capacity of 60 litres. When the gauge is halfway between $\frac{1}{4}$ and $\frac{1}{2}$, the driver puts \$30 of fuel in the tank at \$1.65/litre. What will the gauge show when she gets back into the car?

Solution

Work out the amount of fuel added.

$$\begin{aligned}\text{Fuel added} &= 30 \div 1.65 \\ &= 18.\overline{18} \text{ L}\end{aligned}$$

Estimate the fraction of fuel added.

$$\begin{aligned}\frac{18.\overline{18}}{60} &\approx \frac{18}{60} \\ &= \frac{3}{10}\end{aligned}$$

Simplify.

Work out initial gauge position.

Halfway between $\frac{1}{4}$ and $\frac{1}{2}$

$$= \frac{\frac{1}{4} + \frac{1}{2}}{2}$$

$$= \frac{\frac{1}{4} + \frac{2}{4}}{2}$$

$$= \frac{\frac{3}{4}}{2}$$

Simplify.

$$= \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

Add fractions together.

$$\begin{aligned}\frac{3}{10} + \frac{3}{8} &= \frac{12 + 15}{40} \\ &= \frac{27}{40}\end{aligned}$$

Convert fraction to decimal.

$$\frac{27}{40} = 0.675$$

0.675 is between $0.5 = \frac{1}{2}$ and $0.75 = \frac{3}{4}$.

Find the midpoint between $\frac{1}{2}$ and $\frac{3}{4}$.

Halfway between $\frac{1}{2}$ and $\frac{3}{4}$

$$= \frac{\frac{1}{2} + \frac{3}{4}}{2}$$

$$= \frac{\frac{2}{4} + \frac{3}{4}}{2}$$

$$= \frac{\frac{5}{4}}{2}$$

Simplify.

$$= \frac{1}{2} \times \frac{5}{4} = \frac{5}{8} = 0.625$$

Compare the fraction of fuel with the $\frac{5}{8}$ mark.

$$0.675 > 0.625$$

Give answer as just above/below gauge mark.

The gauge will be just past the mark between $\frac{1}{2}$ and $\frac{3}{4}$.

Exercise 2.1 Using Numbers

Understanding

Extra questions

Exercise 2.1

MAT09NAEQ00004

See Example 1

1 Work out each of the following.

a $3 + 2 \times 5$

b $(3 + 2) \times 5$

c $21 \div 3 + 4$

d $21 \div (3 + 4)$

e $16 - 2 \times 5$

f $4^2 + 3 \times 2$

g $(4^2 + 3) \times 2$

h $5^3 - 3 \times (12 - 2 \times 4)$

i $6 + 4 \times [8 + (2 + 3)]$

j $\frac{48}{8 - 2}$

k $2^3 + 3^3 + 4^3$

l $\sqrt{5^2 + 4^3 + 5 \times 4 - 3^2}$

2 Work out each of the following (without a calculator).

a -18×-34

b $1542 \div -6$

c $-36 + -96$

d $52 - 109$

e -27×15

f $-798 \div -14$

g $56 + -132$

h $-32 - 73$

i $-75 + 161$

j $-96 - -45$

k $-59 + -97$

l $-1862 \div 19$

m -1876×-28

n $-87 - -131$

o $-140 \div -35$

p $9120 \div -38$

See Example 5

3 Calculate the following in your head.

a $28.4 + 17.8$

b $0.742 - 0.337$

c $0.0296 + 0.0572$

d $8.06 - 4.74$

e $0.0784 - 0.0386$

f $45.5 - 21.8$

g $0.463 + 0.909$

h $2.69 + 9.88$

i $0.008\ 89 - 0.003\ 97$

j $36.8 + 84.6$

k $0.509 + 0.798$

l $86.2 - 27.5$

See Example 6

4 Use written methods to find the following.

a $5.109 + 6.0408 + 523.616 + 44.095 + 2116.29 + 285.783$

b $927.129 - 268.0761$

c $74.2 + 192\ 645.8 + 461.6101 + 18.094\ 07 + 7.6107$

d $3417.05 - 2210.0249$

e $4.6382 + 0.066\ 097 + 1.962\ 049 + 407.82 + 2431.617 + 9.033\ 42$

f $725.21 - 209.062\ 00324$

See Example 8

5 Work out the following in your head.

a $0.76 \div 0.5$

b 49×0.063

c 0.04×3.24

d $\frac{6.27}{0.05}$

e 79×0.053

f $0.0058 \div 0.025$

g 0.0099×25

h 0.069×0.035

See Example 8

6 Use written methods to work out the following.

a -6.7×54.7

b $2.35 \div -0.8$

c 5.3×-0.0054

d $\frac{-0.043}{-0.9}$

e $-7.908 \div 0.13$

f -0.0524×-0.014

g $\frac{0.324}{-0.006}$

h -0.068×0.36

i -9.5×-2.096

j 0.237×-0.52

k $-6.8 \div -0.005$

l $\frac{-0.00577}{1.8}$

See Example 9

7 Use a calculator to solve the following.

a -0.0678×-3.974

b $22.68 \div -4.85$

c $\frac{-0.029}{0.84}$

d $0.0531 \times -0.023\ 67$

e $-528.04 \div -17.37$

f -205.79×18.304

g $\frac{-81.29}{248.6}$

h -3.85×-5.2107

8 Do these calculations in your head.

See Example 10

a $4\frac{2}{3} \div \frac{7}{8}$

b $3\frac{1}{3} \times \frac{4}{5}$

c $4\frac{2}{5} \div \frac{3}{5}$

d $4\frac{1}{4} \div \frac{1}{2}$

e $5\frac{1}{3} \times \frac{3}{8}$

f $6\frac{1}{2} \div \frac{3}{4}$

g $5 \div 1\frac{1}{4}$

h $4\frac{2}{3} \div 1\frac{1}{3}$

i $2\frac{3}{5} \div 3\frac{1}{4}$

j $5\frac{1}{2} \div \frac{3}{4}$

k $6\frac{1}{4} \div \frac{3}{4}$

l $2\frac{1}{5} \times \frac{1}{22}$

9 Calculate the following.

Fluency

See Example 7

a $5\frac{3}{4} + 4\frac{5}{6}$

b $3\frac{1}{4} - 2\frac{3}{5}$

c $7\frac{1}{5} - 5\frac{1}{2}$

d $\frac{1}{12} + \frac{3}{4} + \frac{1}{3}$

e $\frac{1}{3} - \frac{1}{4} + \frac{3}{8}$

f $\frac{3}{7} + \frac{5}{14} - \frac{1}{2}$

g $\frac{5}{6} + \frac{3}{4} + \frac{5}{12}$

h $7\frac{4}{9} - 2\frac{1}{6} - 3\frac{5}{18}$

i $3\frac{2}{5} + 5\frac{7}{15} - 3\frac{7}{10}$

j $5\frac{7}{10} + 1\frac{1}{3} + 8\frac{7}{15}$

k $5\frac{1}{2} - 2\frac{8}{9} + 5\frac{7}{36}$

l $7\frac{2}{3} + 6\frac{7}{12} - 8\frac{5}{8}$

10 Use written methods to work out the following.

See Example 11

a $\frac{1}{3} \times \frac{4}{7}$

b $5\frac{2}{3} \times \frac{4}{5}$

c $6\frac{1}{3} \div -4\frac{1}{2}$

d $\frac{-2}{5} \times \frac{3}{4} \times \frac{-5}{8}$

e $-6\frac{3}{4} \div -4\frac{1}{5}$

f $2\frac{3}{5} \div 5\frac{1}{2}$

g $6\frac{1}{4} \times 3\frac{1}{3}$

h $5\frac{2}{3} \times 2\frac{3}{5}$

i $-1\frac{1}{4} \div 6\frac{1}{3}$

j $-4\frac{1}{2} \div -1\frac{3}{4}$

k $3\frac{1}{3} \times -6\frac{1}{4}$

l $1\frac{5}{8} \div -2\frac{2}{3}$

11 Evaluate the following.

See Example 1

a $5 + 6 \times 4 - 18 \div 2 + 4$

b $(5 + 6) \times 4 - 18 \div (2 + 4)$

c $27 - 16 \div 8 + 7 \times 3 - 5 \times 2$

d $24 \times 25 + 16 \times 14 - 210 \div 15$

e $5^2 \times 2^5 - (2 \times 5) \div 2 \times 5$

f $27 + 3 \times 4 - 6 \times 5 + 8 \times 6$

g $4^3 - 5^2 \times \frac{12 + 8}{2^5 - 3 \times 2}$

h $45 + 5 \times \frac{(2^3 + 2^2) \times 3^2}{5^3 - (3 + 2)}$

i $\sqrt[3]{11^2 - 6 + (9^2 + 4 \times 5)} \div \frac{3 + 5}{4} + 3^3 \div 2$

j $(45 \times 3^{-2} + \sqrt{25})^{-3} \times 10^5 \div 2^2$

12 Work out each of the following.

See Example 4

a $6 - ^{-}3 + ^{-}8$

b $^{-}3 + 8 - 6 - ^{-}4$

c $^{-}8 - ^{-}2 - 11 + ^{-}6$

d $6 + 7 + ^{-}4 - 7 - ^{-}9$

e $^{-}6 \times ^{-}8 \times 5$

f $^{-}12 \times 15 \div ^{-}18$

g $^{-}5 \times ^{-}4 \div 10 \times ^{-}6$

h $2 \times ^{-}3 - 5$

i $2 - ^{-}3 \times 5$

j $^{-}4 \times ^{-}5 + 20 \div ^{-}2$

k $5 - (^{-}2 + 8) \times ^{-}12$

l $(^{-}3 + 5) \times (7 - ^{-}4)$

m $^{-}3 + 5 \times 7 - ^{-}4$

n $^{-}24 \div (^{-}3 - 5) - ^{-}18$

o $1 - 21 \div ^{-}3 \times 7$

p $3 \times (8 - ^{-}3) \times (9 - 11)$

- 13 A school walkathon follows a course 500 m across the oval, 2460 m down one street, 432 m along another, 4624 m along a bush track, back along another street for 3042 m and 1324 m on the last street back to school.

Problem solving

Worked solution

Exercise 2.1

MAT09NAWS00004

a How long is the course?

b If Kylie went 1680 m out of her way to visit her home, and then rejoined the walkathon, how far did she walk?

c Andrew took a short cut and walked only 2340 m on the bush track section. How far did he walk?

- 14 A mango tree is estimated to have 4500 mangoes. Flying foxes eat 2746 of them. How many are left?

- 15 A house has four rainwater tanks. They have capacities of 4100 L, 7640 L, 8942 L and 2960 L. When they are all full, how much water is available?
- 16 A country show is held over 2 days. There are 427 exhibitors and on the first day 7380 people paid to see the show. On the second day 4290 paid, but 2385 of these also came on the first day. The exhibitors were given free entry for themselves and a partner. They all used their free entry, but 109 of them did not bring a partner.
- a How many people paid to see the show?
- b How many of these paid twice?
- c How many saw it without paying to get in?
- d How many paid for only one day?
- e How many people saw the show altogether?
- 17 At a weighbridge a truck is found to weigh 15 906 kg. When empty, the truck weighs 4628 kg. What is the mass of the load?
- 18 A windsurfer is travelling directly downwind at 16 km/h. The true wind speed is 27 km/h. What speed does the wind appear to be to the windsurfer?
- 19 Samantha buys 15 sheets of chipboard, each 19 mm thick. How high is the stack?
- 20 Al's mini-gym is 3400 cm wide. If tiles are 15 cm long, how many are needed for each row of tiles across the floor?
- 21 If Germaine earns \$134 each day, how much would she earn in a year with 241 working days?
- 22 A bulk ore truck carries 48 tonnes of ore in a load. How many loads are needed to shift 7000 tonnes?
- 23 There are 50 matches in a box, and 12 boxes in each packet. How many matches are there in 15 packets?
- 24 A train from Melbourne to Sydney travels at an average speed of 92 km/h. How long does the train take to travel the 963 km?
- 25 At 4 a.m. it was 4°C in Hobart. At 2 p.m. it was 38°C. By how much had the temperature risen?
- 26 It costs \$28 a metre to have a fence built. How much will it cost to fully fence a rectangular block 40 m long and 23 m wide?
- 27 A light bulb lasts for about 1000 hours. It is usually on for about 3 hours a day. For how many months will it last?
- 28 Sandra is overdrawn by \$700 at the bank and Stephen is \$800 in the red. Rianna has \$400 in the bank and Joshua has \$300.
- a Who has the most money?
- b Who has the least money?



See Example 2

- 29** Altitude is measured up or down from sea-level. A shipwreck is located 40 m below sea-level. Near it are a reef 2 m below sea-level and a submerged rock 10 m below sea-level. A nearby coral island is generally at an altitude of 5 m, although it does have some sand dunes up to 20 m above sea-level.



- a** What is the lowest thing mentioned?
b What is the highest?
c Arrange everything mentioned in order of altitude (lowest first).
- 30** The temperatures at 2 p.m. in some parts of the world on 12 November are shown below.
Townsville 30°C Jakarta 38°C Chicago -6°C
Moscow -10°C Paris 7°C
- a** What was the highest temperature? **b** What was the lowest?
- 31 a** Find the first number that has remainders of 1, 2, 3 and 4 when divided by 2, 3, 4 and 5 respectively. What is the next number?
b Find a number that has divisors of 2, 3, 4, 5 and 6.
- 32** The plants for a new hedge are placed so that there are 5 plants in every metre. The hedge has to go down the sides and across the back of a block that is 21 m wide and 29 m deep. The landscape gardener brought 30 dozen plants to the site. How many more plants are needed?
- 33** It takes a new robotic machine 12 seconds to assemble a hard drive. The machine has to be checked every 10 minutes during its first day of operation to ensure that it is adjusted properly. The check takes 5 minutes. How many hard drives will the machine assemble in its first 6 hours of operation?
- 34** Penny travels for 2 hours at an average speed of 80 km/h, for half an hour at 50 km/h and then for another hour and a half at 70 km/h. How far does she travel altogether, and what is her overall average speed?
- 35** Eleanor was permitted an overdraft on her business cheque account. She started March with a balance of \$2700. Work out her balance after each of the following transactions.
- | | |
|-------------------------------------|-------------------------------------|
| a March 3—cheque for \$900 | b March 8—deposit of \$1500 |
| c March 10—cheque for \$3200 | d March 11—cheque for \$4700 |
| e March 16—cheque for \$1640 | f March 29—deposit of \$6280 |
- 36** A minisub is at a depth of 30 m below sea-level. Work out its altitude relative to sea-level after each of the following movements in sequence.
- | | | |
|---------------------|---------------------|---|
| a Rises 29 m | b Dives 40 m | c Travels upriver, rising by 50 m with the river |
| d Rises 3 m | e Dives 8 m | f Travels back downstream to its starting point |
- 37** How long is it from 320 BCE to 570 CE?
38 How much colder is -45°C than 8°C ?
39 A lift starts from the 5th floor and travels to the 7th underground level. How many floors does it travel?

Worked solution

Exercise 2.1

MAT09NAWS00004

- 40 If Pieter is \$56 overdrawn on his cheque account, what will his balance be after he writes a cheque for \$76 and deposits \$108?
- 41 Julie's car built up an electric charge of $-98 \mu\text{C}$ (microcoulombs) from friction with the air while she was driving on a hot day. When Julie stepped out she got a shock and acquired a charge of $-27 \mu\text{C}$. She shook hands with Peter and they both got a shock as $15 \mu\text{C}$ was transferred from Peter to her. Peter had no charge to start with.
- a What is the charge on the car? b What is the charge on Julie?
- c What is the charge on Peter?
- 42 A model car can travel at 8 km/h. It is raced up the corridor of a train that is travelling at 25 km/h. What is the true speed of the car when it is travelling:
- a towards the front of the train? b towards the back of the train?
- 43 A small bank has about 1400 customers with average deposits of \$900. Another 50 customers have average overdrafts of \$1200. Write an expression for, and work out, the total assets of the bank.
- 44 An electrical spark at a poor connection carries $-16 \mu\text{C}$ of charge. How many sparks are needed to carry $-288 \mu\text{C}$?
- 45 Jakob and Elena start saving to buy a block of land. After 6 months they have saved $\frac{1}{4}$ of the deposit. They economise and save another $\frac{2}{3}$ of the deposit in the next 6 months.
- a How much of the deposit have they saved?
- b How much is there left to go?
- 46 The Ross family give all their children the same amount of pocket money. When the children go out with their parents, they have to pay more towards the cost of treats from their pocket money as they grow older. Mary, who is 5, has $\frac{3}{5}$ of the cost of her treats paid by her parents. Peter, who is 8, has $\frac{3}{8}$ of his cost paid, and Theresa, who is 10, has $\frac{3}{10}$ of her cost paid.
- a If the children each have one ice-cream, what fraction of the cost do they each pay?
- b If the ice-creams are \$2.40 each, how much do Mum and Dad pay?
- c If the prices of ice-creams do not change, how much will the parents pay next year?
- 47 At 10 a.m. the tide covers two-thirds of the sand on a beach and at 11 a.m. it covers a quarter. How much has been exposed in this hour?
- 48 The couple in question 45 have a disaster. Their car breaks down and they have to spend $\frac{1}{5}$ of the deposit on repairs. How much of the deposit is left now?
- 49 A $\frac{1}{18}$ fraction of Derek's wages is deducted for superannuation and $\frac{1}{5}$ goes on income tax; $\frac{1}{4}$ of his money is used to pay off his house. How much does he have left?
- 50 Two-thirds of the cost of building a house is labour costs and $\frac{1}{15}$ is spent on site preparation. If the rest of the cost goes on materials, what fraction is spent on materials?



- 51** In a school library, $\frac{1}{12}$ of the books are for teacher reference, $\frac{1}{3}$ are on humanities subjects and $\frac{2}{5}$ are on other subjects. The rest are fiction.
- a** What fraction are non-fiction? **b** What fraction are fiction?
- 52** A medicine bottle contains 45 mL. A baby has only 0.75 mL at a time in an eye-dropper. How many baby doses are in the bottle?
- 53** Sand costs \$32.40 per m^3 . How much will 5.8 m^3 cost?
- 54** A wave travels at 2.9 m/s.
- a** How far will it travel in 5 minutes?
b How long will it take to travel 5 km?



- 55** Australian students spend about 5 hours a day in school lessons. About 200 days of the year are school days.
- a** What fraction of the day is spent in lessons?
b What fraction of the year is spent in lessons?
c In 12 years, how many hours are spent in lessons?
d How many hours are spent on other things in 12 years?
- 56** Two-thirds of a house is carpeted and the rest is tiled. Three-quarters of the tiles are cork and the rest are ceramic.
- a** What fraction of the house has ceramic tiles?
b What fraction has cork tiles?
- 57** Cheese costs \$7.98 per kg.
- a** How much will 5.27 kg cost?
b How much cheese can you get for \$20?
- 58** A roll of blind cord has 257 m left on it. A length of 1.7 m is needed for each blind. How many blinds can you make with what is left on the roll?



Reasoning

- 59 Most books are printed on both sides of very large pieces of paper, which are then folded and cut to make 32 pages (16 leaves) of the book. This means it is often cheaper to print books in multiples of 32 pages and leave some blank pages than to print the correct number. A new book has 247 pages, including contents and title pages. How many blank pages will it have?
- 60 A brick wall is to be 12 m long and 2 m high. Including allowance for mortar, the bricks to be used are 20 cm long and 8 cm high. The bricks are on special, but must be bought in pallets of 280 bricks. How many bricks will be left over?
- 61 Six mandarins weigh the same as four apples. Five bananas weigh the same as four pears. Ten bananas weigh the same as eight mandarins. What are the smallest numbers of apples and pears you would need so that you had equal masses of each?
- 62 A room is 12 m long, 3 m high and 4 m wide. A specially bred fly crawls only along joins between the walls, floor and ceiling.
- What is the total length of joins?
 - The fly has to go over some joins twice to crawl along every join at least once. How many does it have to go over twice, and what is the shortest distance it can crawl?
- 63 Trudy and some friends are playing cards at a round table. In a particular game, 25 cards are dealt out in turn. Trudy gets the first and last cards dealt. How many people may be playing cards?
- 64 Maria made some four-legged tables and three-legged stools. She used 37 legs altogether. How many tables and stools could she have made?
- 65 An astronaut is repairing her spaceship when she drifts away from it. To get back to the ship she throws away her toolbox at 10 km/h, but it seems to travel at 12 km/h. What is her speed?



- 66 An aircraft is flying at a height of 14 000 m. It runs into turbulence and falls three times, dropping by 800 m each time. It is also taken up by currents four times, going up 500 m each time. After coming through the turbulence the pilots decide to increase the altitude by 500 m/minute for 6 minutes. What is the altitude at the end of this time?

Worked solution

Exercise 2.1

MAT09NAWS00004

- 67 A water tower can be filled at a rate of $\frac{1}{4}$ of its capacity every hour. During the day (6 a.m.–6 p.m.), it supplies water at a rate of $\frac{2}{7}$ of its capacity every hour. At night the rate of supply is only $\frac{1}{8}$ of its capacity every hour.
- What is the rate at which it empties during the day?
 - What is the rate at which it fills at night?
 - If it is full at 6 a.m., when is it full again?
 - On a particularly hot day, the water consumption in the town doubles. At what time does the water run out?

2.2 Index laws

We use **powers** to write repeated multiplication.

Important!

Indices

A **power** is written with the **base** number in normal size and a small number at the top right called the **index** or **exponent**. It shows repeated multiplication. The index shows the number of times the base is in the multiplication.

$$\begin{array}{ccc} \text{Base} & & \text{Index} \\ \swarrow & & \searrow \\ 4 & 5 & \\ \text{Power form} & = & \text{Extended form} \\ 4^5 & = & 4 \times 4 \times 4 \times 4 \times 4 \end{array}$$

The power shown is said as ‘four to the fifth’, ‘the fifth power of four’ or ‘four to the power of five’. We use the special names **square** and **cube** for 2nd and 3rd powers respectively.

We can also assign meanings to zero and negative indices by considering repeated division. Every time we multiply by the base, the power increases by 1, so division must decrease the power.

$$\begin{array}{llll} 4^5 \times 4 = 4^6 & \text{so} & 4^6 \div 4 = 4^5 & \text{or} & 4096 \div 4 = 1024 \\ 4^4 \times 4 = 4^5 & \text{so} & 4^5 \div 4 = 4^4 & \text{or} & 1024 \div 4 = 256 \\ 4^3 \times 4 = 4^4 & \text{so} & 4^4 \div 4 = 4^3 & \text{or} & 256 \div 4 = 64 \end{array}$$

Continuing the pattern gives us the following.

$$\begin{array}{ll} 4^3 \div 4 = 4^2 & \text{or} & 64 \div 4 = 16 \\ 4^2 \div 4 = 4^1 & \text{or} & 16 \div 4 = 4 \\ 4^1 \div 4 = 4^0 & \text{or} & 4 \div 4 = 1 \\ 4^0 \div 4 = 4^{-1} & \text{or} & 1 \div 4 = \frac{1}{4} = \frac{1}{4^1} \\ 4^{-1} \div 4 = 4^{-2} & \text{or} & \frac{1}{4} \div 4 = \frac{1}{16} = \frac{1}{4^2} \\ 4^{-2} \div 4 = 4^{-3} & \text{or} & \frac{1}{16} \div 4 = \frac{1}{64} = \frac{1}{4^3} \end{array}$$

Example 14

Find the value of each of the following.

a 5^4

b 8 cubed

c 6^{-2}

d 7^0

Solution

- a Write in extended form.
Calculate the answer.

$$5^4 = 5 \times 5 \times 5 \times 5 \\ = 625$$

- b Write in extended form.
Calculate the answer.

$$8 \text{ cubed} = 8 \times 8 \times 8 \\ = 512$$

- c Write in reciprocal form.

$$6^{-2} = \frac{1}{6^2}$$

Write in extended form.

$$= \frac{1}{6 \times 6}$$

Calculate the answer.

$$= \frac{1}{36}$$

- d Use the rule for a^0 .

$$7^0 = 1$$

When you have powers with the same base in a problem, you can work out the answer by writing each part in extended form. This can also be done for problems where the index applies to two numbers or where there is a power of a power.

Example 15

Simplify each of the following and leave your answer in index form.

a $6^3 \times 6^4$

b $8^6 \div 8^2$

c $(5 \times 3)^4$

d $(7^3)^4$

Solution

- a Write the problem.

$$6^3 \times 6^4$$

Write each power in extended form.

$$= 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$$

Write the answer in index form.

$$= 6^7$$

- b Write the problem in fraction format.

$$= \frac{8^6}{8^2}$$

Write each power in extended form.

$$= \frac{8 \times 8 \times 8 \times 8 \times 8 \times 8}{8 \times 8}$$

Cancel two of the 8s.

$$= 8 \times 8 \times 8 \times 8$$

Write the answer in index form.

$$= 8^4$$

- c Write the problem.

$$(5 \times 3)^4$$

Write the power in extended form.

$$= (5 \times 3) \times (5 \times 3) \times (5 \times 3) \times (5 \times 3)$$

Rearrange the product.

$$= 5 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3$$

Write the answer in index form.

$$= 5^4 \times 3^4$$

- d Write the problem.

$$(7^3)^4$$

Write the outer power in extended form.

$$= 7^3 \times 7^3 \times 7^3 \times 7^3$$

Write each power in extended form.

$$= 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \\ \times 7 \times 7 \times 7$$

Write the answer in index form.

$$= 7^{12}$$

Important!

Zero and negative indices

These definitions fit with the patterns of indices that we have observed.

We define $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$ for any integer n and any base a except zero ($a \neq 0$).

Index laws

For any bases a and b and any indices n and m :

$$a^n \times a^m = a^{n+m} \qquad a^n \div a^m = a^{n-m}$$

$$(a^n)^m = a^{nm} \qquad (ab)^n = a^n b^n$$

Note: n and m are integers and $a \neq 0$, $b \neq 0$.

The opposite of finding a power is finding a **root**.

Important!

Roots

The **surd** symbol $\sqrt{}$ is used to show roots. The n th root of a number x is shown as $\sqrt[n]{x}$ and is the number y so that $y^n = x$. For example, $\sqrt[4]{16} = 2$ because $2^4 = 16$.

The **square root** ($\sqrt{}$) is normally written without the index, so $\sqrt{25} = 5$.

The **cube root** ($\sqrt[3]{}$) is the opposite of finding a cube, so $\sqrt[3]{216} = 6$.

Example 16

Find each of the following.

a $\sqrt{100}$

b $\sqrt[4]{625}$

Solution

a Think of the square that gives 100.

Write the answer.

$$10^2 = 100$$

$$\sqrt{100} = 10$$

b Try some possibilities.

$$4^4 = 4 \times 4 \times 4 \times 4 = 256$$

$$5^4 = 5 \times 5 \times 5 \times 5 = 625$$

Write the answer.

$$\sqrt[4]{625} = 5$$

The power key on your calculator could look like $x^\square$, x^y or y^x . The n th root could be shown as $\sqrt[n]{}$, $\sqrt[n]{}$ or $y^{1/x}$. Many calculators also have square and square root keys shown as

x^2 and $\sqrt{}$. In most cases, roots are obtained using the 2nd function key.

The order in which you need to enter powers may vary between calculators.

Example 17

Use a calculator to find:

a 15^3

b $\sqrt{135}$

c 6^{-3}

d $\sqrt[4]{300}$

e $4^{1.3}$

Solution

a Enter as $15 \text{ } x^y \text{ } 3 \text{ } =$.

Write the answer.

$$15^3 = 3375$$

b Enter as $\sqrt{} \text{ } 135 \text{ } =$.

Round and write the answer.

$$\sqrt{135} \approx 11.619$$

c Enter as $6 \text{ } x^y \text{ } (-) \text{ } 3 \text{ } =$.

Round and write the answer.

$$6^{-3} \approx 0.00462962963$$

d Enter as $4 \text{ } \sqrt[4]{} \text{ } 300 \text{ } =$.

Round and write the answer.

$$\sqrt[4]{300} \approx 4.162$$

e Enter as $4 \text{ } x^y \text{ } 1.3 \text{ } =$.

Round and write the answer.

$$4^{1.3} \approx 6.063$$

Example 18

Use the index laws to simplify the following, leaving your answers with positive indices.

a $9^3 \times 9^8$

b $17^{15} \div 17^3$

c $4^3 \div 4^8$

d $(2^5)^3$

e $(7^2)^{-3}$

f $5^{11} \div 5^{11}$

Solution

a Write the problem.

Use the index law.

Work out the answer in index form.

$$9^3 \times 9^8$$

$$= 9^{3+8}$$

$$= 9^{11}$$

b Write the problem.

Use the index law.

Work out the answer in index form.

$$17^{15} \div 17^3$$

$$= 17^{15-3}$$

$$= 17^{12}$$

c Write the problem.

Use the index law.

Work out the answer in index form.

Change to a positive index.

$$4^3 \div 4^8$$

$$= 4^{3-8}$$

$$= 4^{-5}$$

$$= \frac{1}{4^5}$$

d Write the problem.

Use the index law.

Work out the answer in index form.

$$(2^5)^3$$

$$= 2^{5 \times 3}$$

$$= 2^{15}$$

- e** Write the problem. $(7^2)^{-3}$
Use the index law. $= 7^{2 \times -3}$
Work out the answer in index form. $= 7^{-6}$
Change to a positive index. $= \frac{1}{7^6}$
- f** Write the problem. $5^{11} \div 5^{11}$
Use the index law. $= 5^{11-11}$
Work out the answer in index form. $= 5^0$
Simplify using the definition $a^0 = 1$. $= 1$

Exercise 2.2 Index laws

1 Work out each of the following.

a 5^3	b 7^5	c 3^5
d 4^3	e 8^4	f 2^{10}
g 4^5	h 13^2	i 5^4

2 Use a calculator to find each of the following.

a 15^4	b 13^3	c 26^2
d 19^3	e 24^4	f 18^5
g 34^2	h 43^3	i 16^4

3 Work out each of the following.

a $\sqrt{64}$	b $\sqrt{25}$	c $\sqrt{100}$
d $\sqrt{400}$	e $\sqrt{144}$	f $\sqrt{81}$
g $\sqrt{225}$	h $\sqrt{10\,000}$	i $\sqrt{16\,000}$

4 Use a calculator to find each of the following.

a $\sqrt{50}$	b $\sqrt{250}$	c $\sqrt{148}$
d $\sqrt{85}$	e $\sqrt{7}$	f $\sqrt{24}$
g $\sqrt{42}$	h $\sqrt{1000}$	i $\sqrt{420\,000}$

5 Find the value of each of the following.

a 9^3	b 3^{-5}	c 6^0
d 9^{-2}	e 11^{-1}	f 6^{-3}
g 14^{-2}	h 13^0	i 5^{-4}

6 Simplify each of the following and leave your answer in index form.

a $4^3 \times 4^7$	b $7^6 \times 7$	c $12^3 \times 12^3$
d $6^4 \times 6^5$	e $10^4 \times 10^{12}$	f $9^5 \times 9^3 \times 9$
g $14^3 \times 14^2 \times 14^7$	h $3^6 \times 3 \times 3^5$	i $25^3 \times 25^3 \times 25^4$

7 Simplify each of the following and leave your answer in index form.

a $8^5 \div 8^3$	b $3^6 \div 3^2$	c $7^5 \div 7$
d $11^8 \div 11^7$	e $6^{12} \div 6^6$	f $15^7 \div 15^2$
g $14^9 \div 14^3$	h $17^{15} \div 17^5$	i $12^3 \div 12^2$

Understanding

Extra questions

Exercise 2.2

MAT09NAEQ00005

See Example 14

See Example 17

See Example 16

See Example 17

Worked solution

Exercise 2.2

MAT09NAWS00005

See Example 14

See Example 15

See Example 15

See Example 15

8 Simplify each of the following and leave your answer in index form.

a $(7 \times 5)^4$

b $(6 \times 3)^6$

c $(9 \times 11)^4$

d $(6 \times 8)^7$

e $(12 \times 13)^8$

f $(2 \times 7)^{12}$

See Example 15

9 Simplify each of the following and leave your answer in index form.

a $(4^5)^3$

b $(8^2)^5$

c $(5^7)^4$

d $(12^2)^4$

e $(9^4)^6$

f $(3^3)^3$

Worked solution

Exercise 2.2

MAT09NAWS00005

See Example 16

10 Find each of the following.

a $\sqrt[3]{125}$

b $\sqrt{169}$

c $\sqrt[4]{16}$

d $\sqrt[3]{64}$

e $\sqrt[6]{64}$

f $\sqrt[4]{81}$

g $\sqrt[6]{1\,000\,000}$

h $\sqrt{225}$

i $\sqrt[3]{8000}$

See Example 17

11 Use a calculator to find each of the following.

a 5^{-3}

b $\sqrt{48}$

c $\sqrt[4]{48}$

d 2.8^4

e $\sqrt{29.7}$

f $\sqrt[3]{247.8}$

g 6.3^{-1}

h 7.85^{-3}

i $\sqrt[3]{1.75}$

Fluency

See Example 17

12 Use a calculator to find each of the following.

a $4^{1.8}$

b $3^{1.8}$

c $16^{0.5}$

d $5^{1.6}$

e $1.4^{2.6}$

f $3.75^{4.1}$

See Example 18

13 Use the index laws to simplify the following, leaving your answers with positive indices.

a $15^7 \times 15^3$

b $3^9 \times 3^2$

c $19^8 \times 19^2$

d $17^8 \times 17^3$

e $11^6 \times 11^2$

f $3^9 \times 3^5$

g $7^2 \times 7^4 \times 7^4$

h $14^3 \times 14^5 \times 14^9$

i $4^6 \times 4^4 \times 4^4$

See Example 18

14 Use the index laws to simplify the following, leaving your answers with positive indices.

a $15^9 \div 15^3$

b $6^9 \div 6^5$

c $19^4 \div 19^2$

d $4^8 \div 4^4$

e $7^2 \div 7^8$

f $13^8 \div 13^4$

g $11^5 \div 11^9$

h $6^{15} \div 6^4$

i $11^2 \div 11^9$

See Example 18

15 Use the index laws to simplify the following, leaving your answers with positive indices.

a $(4 \times 13)^6$

b $(14 \times 19)^9$

c $(7 \times 3)^9$

d $(11 \times 19)^2$

e $(13 \times 7)^2$

f $(17 \times 14)^6$

See Example 18

16 Use the index laws to simplify the following, leaving your answers with positive indices.

a $(4^5)^3$

b $(8^2)^5$

c $(5^7)^4$

d $(12^2)^4$

e $(9^4)^6$

f $(3^3)^3$

See Example 18

17 Use the index laws to simplify the following, leaving your answers with positive indices.

a $8^{10} \times 8^{25}$

b $6^{-7} \times 6^3$

c $9^8 \div 9^8$

d $(4^{-3})^5$

e $(7^{-6})^{-1}$

f $(5^{-3} \times 4^6)^3$

g $(6^{-1} \times 8^4)^{-5}$

h $(11^{-3} \times 7^4)^{-2} \times 7^5$

i $8^6 \times 8^{-4} \div 8^{-3}$

Problem solving

18 **a** Find the values of $\sqrt{36}$, $\sqrt{9}$ and $\sqrt{4}$ and a relationship between them.

b Find the values of $\sqrt{100}$, $\sqrt{25}$ and $\sqrt{4}$ and a relationship between them.

c What is the relationship between $\sqrt{ab}$, $\sqrt{a}$ and $\sqrt{b}$ for numbers a and b ?

Worked solution

Exercise 2.2

MAT09NAWS00005

- 19 a Find the values of $\sqrt{16}$, $\sqrt[3]{16}$, $\sqrt[4]{16}$, $\sqrt[5]{16}$ and $\sqrt[6]{16}$.
 b Find the values of $\sqrt{0.0625}$, $\sqrt[3]{0.0625}$, $\sqrt[4]{0.0625}$, $\sqrt[5]{0.0625}$ and $\sqrt[6]{0.0625}$.
 c What happens to the values of the roots of a positive number as the root power is increased?

2.3 Scientific notation

Very large and very small numbers are usually written in **scientific (standard) notation**.

Important!

Scientific notation

Numbers written in scientific notation are shown with a **mantissa** from 1 to less than 10, multiplied by a power of 10. The **order of magnitude** of the number is the power. For example:

$$\begin{array}{lcl}
 25\,000\,000 = 2.5 \times 10^7 & \text{has mantissa } 2.5 \text{ and order } 7. \\
 \quad \quad \quad \swarrow \quad \searrow & & \\
 \text{mantissa} & & \text{order} \\
 0.006\,37 = 6.37 \times 10^{-3} & \text{has mantissa } 6.37 \text{ and order } -3.
 \end{array}$$

Example 19

Write the following in scientific notation.

- a 764 000 000 000
 c 0.0008

- b 6000
 d 0.000 000 472

Solution

- a Write with one digit before the decimal point, with the place value as a 'tens' number.
 Write with a power of 10.
 b Write the value digit, 6, with the place value as a 'tens' number.
 Write with a power of 10.
 c Write the value digit, 8, with the place value as a 'tens' number.
 Write with a power of 10.
 d Write with one digit before the decimal point, with the place value as a 'tens' number.
 Write with a power of 10.

$$\begin{array}{l}
 764\,000\,000\,000 \\
 = 7.64 \times 100\,000\,000\,000 \\
 = 7.64 \times 10^{11} \\
 6000 \\
 = 6 \times 1000 \\
 = 6 \times 10^3 \\
 0.0008 \\
 = 8 \times 0.0001 \\
 = 8 \times 10^{-4} \\
 0.000\,000\,472 \\
 = 4.72 \times 0.000\,0001 \\
 = 4.72 \times 10^{-7}
 \end{array}$$

TLF Learning object

Exploring powers of 10
(L6548)

MAT09NAIN00002

Video tutorial

Scientific notation

MAT09NAVT10001

Technology worksheet

Excel worksheet:
Comparing numbers –
Scientific notation

MAT09NACT00019

TLF Learning object

Exploring exponents
and scientific notation
(L6550)

MAT09MGIN00002

Some people change to scientific notation by moving the decimal point to make a number from 1 to less than 10 and counting how many places it has to move.

Example 20

Worksheet

Scientific notation 2

MAT09NAWK00019

Puzzle sheet

Scientific notation for superheroes

MAT09NAPS00004

Puzzle sheet

Scientific notation: Accomplishing great things

MAT09NAPS00005

Write the following as ordinary numbers.

a 6.25×10^4

b 9.1×10^{-2}

Solution

a Write the problem.

$$6.25 \times 10^4$$

Write the power as a 'tens' number.

$$= 6.25 \times 10\,000$$

Write as an ordinary number.

$$= 62\,500$$

b Write the problem.

$$9.1 \times 10^{-2}$$

Write the power as a 'tens' number.

$$= 9.1 \times 0.01$$

Write as an ordinary number.

$$= 0.091$$

Most calculators have an **EXP** key to enter numbers in scientific notation. They automatically show large or small answers in scientific notation and can be set to show all answers in scientific notation. The display varies between calculators, but generally shows the order of magnitude as a superscript or as a number following an E.

Investigate: Large and small numbers

Weblink

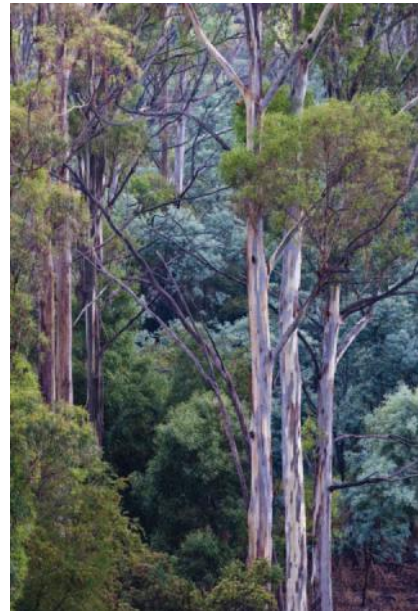
Absolute astronomer

MAT09NAWB00002

Work in groups of three or four for this investigation and estimate each of the following.

- The mass of the school building, including foundations (in kg).
- The mass of an ant (in kg).
- The distance in metres to the nearest beach.
- The thickness in metres of a single hair.
- The number of leaves on a large gum tree.

- 1 Now write all your estimates in scientific notation.
- 2 Discuss the advantages of scientific notation.



Example 21

Use a calculator to work out the following.

a $9.8 \times 10^{-6} \times 6.1 \times 10^{18}$

Solution

a Enter

9.8 **EXP** **(-)** 6 **×** 6.1 **EXP** 18 **=**.

$$\frac{9.8 \times 10^{-6} \times 6.1 \times 10^{18}}{5.978 \times 10^{13}}$$

Write the answer.

$$9.8 \times 10^{-6} \times 6.1 \times 10^{18} = 5.978 \times 10^{13}$$

b Enter 5.16 **EXP** 28 **×** 2640 **÷** 3.72 **EXP** **(-)** 19 **=**.

$$\frac{5.16 \times 10^{28} \times 2640 \div 3.72 \times 10^{-19}}{3.661935484 \times 10^{50}}$$

Round and write the answer.

$$\frac{5.16 \times 10^{28} \times 2640}{3.72 \times 10^{-19}} \approx 3.662 \times 10^{50}$$

Worksheet

Scientific notation 1

MAT09NAWK00012

Worksheet

Scientific notation 3

MAT09NAWK00020

CAS TI-Nspire exercise

Number and indices

MAT09NATI00002

CAS ClassPad exercise

Number and indices

MAT09NACP00002

A measurement accurate to the nearest metre could be stated as 57 m, 0.057 km or 57 000 mm. Each of these has the same accuracy, and this is very clear if they are written in scientific notation as 5.7×10^1 m, 5.7×10^{-2} km or 5.7×10^4 mm.

Important!

Accuracy

The accuracy of a number can be stated in terms of the **number of significant figures**, which is the number of digits in the mantissa of a number written in scientific notation.

Example 22

Which is the most accurately written number from 0.0065, 57 800 and 3 000 000?

Solution

Think of each number in scientific notation.

$$0.0065 = 6.5 \times 10^{-3}$$

$$57\,800 = 5.78 \times 10^4$$

$$3\,000\,000 = 3 \times 10^6$$

Think of the number of significant figures.

6.5 has 2 digits, 5.78 has 3 digits, 3 has 1 digit.

Write the answer.

57 800 is the most accurately written.

Exercise 2.3 Scientific notation

Understanding

Extra questions

Exercise 2.3

MAT09NAEQ00006

See Example 19

Worked solution

Exercise 2.3

MAT09NAWS00006

See Example 20

- 1 Write in scientific notation.

a 6300

b 142 000 000

c 0.0075

d 0.000 000 938

e 500 000

f 0.0006

g 210

h 0.79

i 31 090 000

j 0.040 62

k 9 623 000

l 0.000 000 002

- 2 Write as ordinary numbers.

a 4.3×10^5

b 2.1×10^{-3}

c 9.84×10^6

d 3.19×10^{-1}

e 6×10^{-9}

f 2×10^4

g 4.2×10^1

h 5.8×10^{-2}

i 1.62×10^0

j 9.5×10^{-5}

k 4.64×10^2

l 2.1×10^3

Fluency

See Example 21

- 3 Use a calculator to find each of the following and write your answer in scientific notation, correct to 4 significant figures.

a $4.6 \times 10^{-8} \times 3.2 \times 10^9$

b $6.1 \times 10^9 \div (8.34 \times 10^{-2})$

c $5.64 \times 10^7 \times 3.9 \times 10^{11}$

d $8.6 \times 10^{-3} \times 2.9 \times 10^{-12}$

e $8.1 \times 10^4 \div (3.61 \times 10^{-9})$

f $4.74 \times 10^{-3} \div (1.8 \times 10^{15})$

- 4 State the number of significant figures in each of the following.

a 24 700

b 0.000 35

c 700

d 0.000 005

e 10

f 9.65×10^{11}

g 3.047×10^{-9}

h 105 030 000

i 510 000

- 5 Choose the most accurately written figure in each set of numbers.

a 42, 3000, 1.6, 475

b 103, 7.8, 0.000 54, 32 000

c 5.047, 0.0006, 129 000, 4500

d 0.003, 96 000, 1 057 000, 48.3

e 19, 247.6, 3.95, 0.064

f 8 000 000, 21 000, 0.0005, 0.000 000 644

g 1, 100, 100.5, 0.0057

h 0.060 47, 2.94, 3500, 24 600

- 6 Use a calculator to find each of the following and write the answer in scientific notation, correct to 4 significant figures.

a $\frac{4861 \times 2 \times 10^9}{3.58 \times 10^{-5}}$

b $\frac{7.3 \times 10^{-8} \times 1.6 \times 10^5}{3925}$

c $\frac{3.1 \times 10^6 \times 8.2 \times 10^{12}}{5.8 \times 10^{11} \times 0.067}$

d $\frac{9.85 \times 10^9 \times 9.84 \times 10^9}{9.85 \times 10^{-9} - 9.84 \times 10^{-9}}$

e $\frac{5.1 \times 10^{-10} \times 1.2 \times 10^{-3}}{3 \times 10^{-5} + 8 \times 10^{-7}}$

f $\frac{4.71 \times 10^6 \times 2.95 \times 10^7}{3.5 \times 10^9 \times \pi}$

2.4 Using the index laws

Some expressions need to be simplified using more than one index law. This often arises in Physics and Chemistry when numbers in scientific notation are substituted into expressions with powers. The normal order of operations still applies, although you may have to insert brackets where they are only implied in the original expression.

Example 23

Simplify each of the following expressions and leave in power form.

a $(3^2 \times 5^{-3})^4$

b $\frac{2^3 \times 3^2}{(3^{-4} \times 5^2)^3}$

c $\left(\frac{3}{4}\right)^2 \div \left(\frac{3^{-2}}{5}\right)^3$

Solution

a Write the problem.

Use an index law.

Write the answer with positive powers.

$$\begin{aligned}(3^2 \times 5^{-3})^4 \\&= 3^8 \times 5^{-12} \\&= \frac{3^8}{5^{12}}\end{aligned}$$

b Write the problem.

Move the denominator to the top and change the sign of the power.

Apply an index law to remove the brackets.

Apply another index law to the repeated base of 3.

Change to a positive index.

$$\begin{aligned}\frac{2^3 \times 3^2}{(3^{-4} \times 5^2)^3} \\&= 2^3 \times 3^2 \times (3^{-4} \times 5^2)^{-3} \\&= 2^3 \times 3^2 \times 3^{12} \times 5^{-6} \\&= 2^3 \times 3^{14} \times 5^{-6} \\&= \frac{2^3 \times 3^{14}}{5^6}\end{aligned}$$

c Write the problem.

Use an index law.

Invert the 2nd fraction as a result of changing the division sign to a multiplication sign.

Apply an index law to the common base of 3 and change base 4 to its lowest prime base.

Tidy up and use an index law.

$$\begin{aligned}\left(\frac{3}{4}\right)^2 \div \left(\frac{3^{-2}}{5}\right)^3 \\&= \frac{3^2}{4^2} \div \frac{3^{-6}}{5^3} \\&= \frac{3^2}{4^2} \times \frac{5^3}{3^{-6}} \\&= \frac{3^{2+6}}{(2^2)^2} \times \frac{5^3}{1} \\&= \frac{3^8 \times 5^3}{2^4}\end{aligned}$$

CAS TI-Nspire exercise

Number and indices

MAT09NATI00002

CAS ClassPad exercise

Number and indices

MAT09NACP00002

Example 24

The kinetic energy of an object is given by the formula $E = \frac{1}{2}mv^2$. If m is in kg and v is in m/s, then E is in joules. What is the kinetic energy of an electron (mass = 9.1×10^{-31} kg) travelling at a quarter of the speed of light (speed of light, $c = 3 \times 10^8$ m/s)?

Solution

Find the velocity (v) of the electron.

$$\begin{aligned}\frac{c}{4} &= \frac{3 \times 10^8}{4} \\&= 7.5 \times 10^7 \text{ m/s}\end{aligned}$$

Write the formula.

$$E = \frac{1}{2}mv^2$$

Substitute in the appropriate values. (Check your units)

$$= \frac{9.1 \times 10^{-31} \times (7.5 \times 10^7)^2}{2}$$

Apply an index law to remove the brackets.

$$= \frac{9.1 \times 10^{-31} \times 7.5^2 \times 10^{14}}{2}$$

Use another index law and tidy up.

$$= \frac{9.1 \times 7.5^2 \times 10^{-17}}{2}$$

Enter as: (9.1 × 7.5 x² × 10 y^x (-) 17) ÷ 2

$$\frac{(9.1 \times 7.5^2 \times 10^{-17}) \div 2}{2.559375 \times 10^{-15}}$$

Write your answer.

The kinetic energy of the electron is 2.559375×10^{-15} joules.

Some expressions with indices can be simplified more easily by changing the numbers to prime factorisations first.

Example 25

Animated example

Using index laws

MAT09NAAE00002

Simplify each of the following and leave in power form.

a $15^4 \times 8 \times 30^{-2}$

b $\frac{8^3 \times 4^5}{(2^{-4} \times 16^3)^2}$

Solution

a Write the problem.

Rewrite the bases as a product of their prime factors.

Use an index law and tidy up where possible.

Apply another index law to the repeated bases.

Work out the answer in power form.

$$15^4 \times 8 \times 30^{-2}$$

$$= (3 \times 5)^4 \times (2 \times 2 \times 2) \times (2 \times 3 \times 5)^{-2}$$

$$= 3^4 \times 5^4 \times 2^3 \times 2^{-2} \times 3^{-2} \times 5^{-2}$$

$$= 2^{3-2} \times 3^{4-2} \times 5^{4-2}$$

$$= 2 \times 3^2 \times 5^2$$

$$\frac{8^3 \times 4^5}{(2^{-4} \times 16^3)^2}$$

$$= 8^3 \times 4^5 \times (2^{-4} \times 16^3)^{-2}$$

$$= 8^3 \times 4^5 \times 2^8 \times 16^{-6}$$

$$= (2 \times 2 \times 2)^3 \times (2 \times 2)^5 \times 2^8 \times (2 \times 2 \times 2 \times 2)^{-6}$$

$$= (2^3)^3 \times (2^2)^5 \times 2^8 \times (2^4)^{-6}$$

$$= 2^9 \times 2^{10} \times 2^8 \times 2^{-24}$$

$$= 2^{9+10+8-24}$$

$$= 2^3$$

b Write the problem.

Move the denominator to the top and change the sign of the power.

Apply an index law to remove the brackets.

Rewrite the bases as a product of their prime factors.

Tidy up where possible inside the brackets.

Apply an index law.

Apply another index law.

Work out the answer in index form.

The index laws can also be used with algebraic expressions.

Example 26

Simplify each of the following expressions. Write your answers with positive powers.

a $(xy^3)^4$

b $a^4 \times a^{-6} \div a^3$

c $(3m^{-2}n^4) \div (9m^{-3}n^6)$

Solution

- a** Write the problem.

Use an index law.

$$(xy^3)^4 = x^4 y^{12}$$

- b** Write the problem.

Use an index law.

$$a^4 \times a^{-6} \div a^3 = a^{4-6-3}$$

$$= a^{-5} = \frac{1}{a^5}$$

Write as a positive power.

- c** Write the problem.

Change the 9 to its smallest prime base.

Use an index law.

$$(3m^{-2}n^4) \div (9m^{-3}n^6) = (3m^{-2}n^4) \div (3^2m^{-3}n^6)$$

$$= 3^{1-2}m^{-2+3}n^{4-6}$$

$$= 3^{-1}mn^{-2} = \frac{m}{3n^2}$$

Simplify.

Write as positive powers.

Video tutorial

Negative indices

MAT09NAVT10010

Worksheet

Power calculations

MAT09NAWK00018

CAS TI-Nspire exercise

Number and indices

MAT09NATI00002

CAS ClassPad exercise

Number and indices

MAT09NACP00002

Exercise 2.4 Using the index laws

- 1** Simplify each of the following expressions and leave in power form.

a $(4^2 \times 5^{-2})^3$

b $(3^{-2} \times 2^4)^2$

c $(5^2 \times 9^{-3})^{-3}$

d $\frac{4^3 \times 3^2}{(3^4 \times 2^2)^3}$

e $\frac{2^{-3} \times 5^2}{(5^{-3} \times 4^2)^3}$

f $\frac{7^{-2} \times 3^{-2}}{(7^{-4} \times 3^2)^{-3}}$

g $\left(\frac{3}{5^4}\right)^3 \times \left(\frac{5^2}{3}\right)^4$

h $\left(\frac{7}{4}\right)^2 \div \left(\frac{7^{-2}}{8}\right)^3$

i $\left(\frac{3}{5}\right)^6 \div \left(\frac{3^{-2}}{5^{-2}}\right)^{-3}$

- 2** Simplify each of the following expressions and leave in power form.

a $18^2 \times 8 \times 27^{-2}$

b $21^3 \times 7^{-3} \times 14^{-2}$

c $15^4 \times 125 \times 50^{-2} \times 5^{-3}$

d $\frac{7^3 \times 2^{20}}{(7^{-4} \times 16^3)^2}$

e $\frac{5^{-3} \times 3^5}{(3^{-4} \times 25^3)^{-3}}$

f $\frac{4^{-9} \times 11^8}{(22^{-4} \times 16^3)^{-2}}$

- 3** Simplify each of the following expressions. Write your answers with positive powers.

a $(x^2y^3)^3$

b $(xy^{-2})^4$

c $(x^{-2}y^{-1})^{-3}$

d $b^4 \times b^3 \div b^6$

e $a^{-4} \times a^{-5} \div a^{-9}$

f $c^4 \div c^{-6} \times c^{-12}$

g $(8m^{-3}n^4) \div (16m^{-2}n)$

h $(5p^4q^{-3}) \div (125p^4q^{-7})$

i $(216r^{-2}s^4t^3) \div (36r^{-3}s^6t)$

Understanding

Extra questions

Exercise 2.4

MAT09NAEQ00007

See Example 23

Worked solution

Exercise 2.4

MAT09NAWS00007

See Example 25

Fluency

Worked solution

Exercise 2.4

MAT09NAWS00007

See Example 26

Problem solving

See Example 24

- 4 The Large Hadron Collider accelerates protons around a 27 km long circular tunnel under the border of Switzerland and France. They reach speeds of 0.999 999 991 times the speed of light.

At this speed, the mass of a proton is

$$\frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \text{ times its rest mass.}$$

The speed of light is 3×10^8 m/s and the rest mass of a proton is 1.672×10^{-27} kg.

What is its mass at:

- half the speed of light $\left(\frac{v}{c} = 0.5\right)$?
 - nine-tenths the speed of light $\left(\frac{v}{c} = 0.9\right)$?
 - 0.999 999 991 times the speed of light?
- 5 The force of gravity is given by $F = \frac{Gm_1m_2}{r^2}$, where F is in newtons, $G = 6.6730 \times 10^{-11}$, m_1 and m_2 are the masses of the objects in kg and r is the distance between them in m. The masses of the Earth and Moon are 5.9722×10^{24} kg and 7.3477×10^{22} kg respectively. The average distance from Earth to the Moon is 384 403 000 m. Find the force of attraction between the Earth and the Moon.



Reasoning

- The mass of the Earth is 5.9722×10^{24} kg and its diameter is 12 756 200 m. The mass of the Sun is 1.9889×10^{30} kg and its diameter is 1 392 000 km. Considering them as spheres, find:
 - the volume of the Earth
 - the volume of the Sun
 - the density of the Earth
 - the density of the Sun.
 - Explain whether you would consider the Earth or the Sun 'heavier'.
- The mass of a hydrogen atom is 1.67264×10^{-27} kg. In the Sun, 4 hydrogen atoms are combined to make one helium nucleus, which has a mass of 6.68241×10^{-27} kg. The missing mass is converted to energy which makes the Sun shine. The mass is changed to energy according to the formula $E = mc^2$, where $c = 3 \times 10^8$ m/s is the speed of light, m is in kg and E is in joules. 1 kg of hydrogen has about 6×10^{27} atoms. A typical power station produces between 500 to 900 MW of electricity, which is 5×10^8 to 9×10^8 joules per second. If a fusion power station converted 1 kg of hydrogen to helium every day, how many power stations could it replace? Explain your steps carefully.

- The **order of operations** is:
 - 1 Work out brackets first, from innermost outwards and from left to right. A vinculum works like brackets for operations on top or underneath.
 - 2 Work out powers and roots from left to right.
 - 3 Work out multiplications and divisions next, from left to right.
 - 4 Work out additions and subtractions last, from left to right.
- **Integers (directed numbers)** are whole numbers with a sign. **Negative numbers** are shown by a raised $-$ symbol and **positive numbers** can be shown by a raised $+$, but the positive sign is often omitted. All positive numbers are greater than 0 and all negative numbers are less than 0.
- When two integers of the same sign are multiplied or divided, the answer is positive.
- When two integers of different signs are multiplied or divided, the answer is negative.
- A repeated multiplication may be written as a **power**. The number multiplied is written once as the **base**. The number of times it is in the multiplication is written as the small **index (exponent)** at the top right. The **extended form** of a power has the multiplication written out in full.
- A **square** is a power with index 2, and a **cube** has index 3.
- Powers with **zero** and **negative exponents** are defined for non-zero bases as $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$ for any integer n and any base a except zero ($a \neq 0$).
- The index laws are that for any numbers a and b and any indices n and m :

$$a^n \times a^m = a^{n+m} \qquad a^n \div a^m = a^{n-m} \qquad (a^n)^m = a^{nm} \qquad (ab)^n = a^n b^n$$
- The n th **root** of a number x is written using the **surd** symbol as $\sqrt[n]{x}$. It is a number y that must be raised to the n th power to give x , so $y^n = x$. The **square root**, $\sqrt{x}$, is written without the index number. The third root, $\sqrt[3]{x}$, is called the **cube root**.
- Numbers written in **scientific notation (standard notation)** are shown with a **mantissa** from 1 to less than 10, multiplied by a power of 10. The **order of magnitude** of the number is the power.
- The accuracy of a number can be given in terms of the **number of significant figures**, which is the number of digits in the mantissa of a number written in scientific notation.

Quiz

Number and indices

MAT09NAQZ00002

Chapter 2 review

Understanding

See Example 1

- 1 Work out each of the following.

a $28 - 4 \times 5$

b $4^2 + [46 - 3 \times (7 - 5)]$

c $\sqrt{1 + 6 \times 8}$

- 2 Calculate the following in your head.

a $16.3 + 28.9$

b $0.749 - 0.387$

c $\frac{5}{8} + \frac{7}{8}$

d $\frac{3}{5} - \frac{1}{10}$

- 3 Use written methods to solve the following.

a $516.8 + 0.6854 + 28.907 + 7.0634 + 96$

b $452.1 - 97.327$

c $16.94 - 4.804 + 428.57 - 304.67 + 26.719$

d $2\frac{2}{3} + 5\frac{3}{4}$

e $8\frac{3}{5} - 5\frac{1}{10}$

f $6\frac{1}{3} - 2\frac{3}{5}$

- 4 Work out the following on your calculator.

a $2080.75 + 314.2806 - 404.7693 - 29.7906 + 984.76$

b $9\frac{3}{5} - 2\frac{7}{8} + 5\frac{13}{20}$

- 5 Calculate the following in your head.

a 0.0059×25

b $65.2 \div 4000$

c $\frac{1}{4} \times \frac{8}{15}$

d $2\frac{2}{3} \div \frac{1}{6}$

- 6 Use written methods to solve the following.

a 5.79×45.8

b $1.695 \div 0.06$

c $\frac{5}{16} \times \frac{3}{10}$

d $\frac{8}{9} \div \frac{4}{7}$

- 7 Work out the following on your calculator.

a $3\frac{3}{5} \times 2\frac{2}{3}$

b $4\frac{1}{2} \div 5\frac{1}{4}$

See Example 14

- 8 Work out each of the following without using a calculator.

a 2^6

b 3^4

c 7^3

d $\sqrt{49}$

e $\sqrt{121}$

f $\sqrt{900}$

See Example 17

- 9 Use a calculator to find each of the following.

a 15^3

b 21^2

c 34^4

d $\sqrt{48}$

e $\sqrt{213}$

f $\sqrt{234\,000}$

See Example 23

- 10 Simplify each of the following expressions and leave in power form.

a $(4^2 \times 5^{-2})^3$

b $\frac{2^3 \times 3^{-2}}{(3^{-3} \times 5^2)^2}$

c $\left(\frac{3}{5}\right)^6 \div \left(\frac{3^{-3}}{5^2}\right)^3$

See Example 24

- 11 Simplify each of the following and leave in power form.

a $10^4 \times 8 \times 20^{-3}$

b $\frac{9^3 \times 3^5}{(3^{-4} \times 27^3)^2}$

Fluency

- 12 Calculate the following in your head.

a $28.96 + 2.8472$

b $418.6 - 2.927$

c $-\frac{3}{5} + \frac{7}{10}$

d $-\frac{2}{3} - \frac{4}{5}$

- 13 Use written methods to solve the following.

a $-428.6 + 87.93$

b $-3.975 - -8.984$

c $3\frac{4}{5} + 5\frac{3}{8}$

d $9\frac{5}{6} - 4\frac{7}{9}$

e $5\frac{3}{5} - 2\frac{5}{6} + 7\frac{8}{15}$

14 Work out the following on your calculator.

a $-4\frac{5}{8} + 3\frac{7}{9}$

b $6\frac{1}{12} - 5\frac{5}{18}$

c $4\frac{5}{9} + -3\frac{1}{6} - 5\frac{3}{10}$

15 Work out the following in your head.

a $6.8 \div 0.8$

b 39×1.42

c $5\frac{1}{2} \times 1\frac{1}{11}$

d $4\frac{2}{3} \div 2\frac{4}{5}$

16 Use written methods to solve the following.

a -6.97×-0.492

b $1.584 \div -0.12$

c $2\frac{4}{5} \times -5\frac{1}{2}$

d $-6\frac{2}{3} \div -1\frac{1}{2}$

17 Work out the following on your calculator.

a -4.987×7.096

b $4.9065 \div -45.9$

c $3\frac{4}{7} \times \frac{-3}{20} \div -5\frac{2}{5}$

18 Evaluate each of the following.

a $\frac{4^3 - 5 \times 4}{5 + 2 \times 3}$

b $9 \times 8 \div 3 \times 5 - 7 \times (6 + 3)$

c $2^{-3} \times (7 + 3^2) \times 5^3 \div 10 \times 2$

See Example 1

19 Work out each of the following without a calculator.

a $47 - -83$

b -14×-24

c $15\,600 \div -39$

d $7 - -8 + -5$

e $4 - 3 \times -6$

f $15 \div -3 - -4 \times (6 + -2)$

See Example 4

20 Work out each of the following without using a calculator.

a 14^3

b 9^0

c 4^{-2}

d $\sqrt[3]{216}$

e $\sqrt[4]{81}$

f $\sqrt{250\,000}$

See Example 17

21 Simplify each of the following and leave your answer in index form.

a $14^8 \times 14^7$

b $20^{12} \div 20^4$

c $(3 \times 7)^6$

d $5^3 \times 5^6$

e $14^6 \div 14^2$

f $(13 \times 11)^8$

See Example 18

22 Use a calculator to find each of the following.

a 15^{-2}

b $\sqrt[3]{9}$

c 4.8^3

d $\sqrt{13.96}$

e $\sqrt[5]{424}$

f 6.27^{-3}

See Example 17

23 Find each of the following.

a $6^{1.5}$

b $8^{2.3}$

c $6^{-3.2}$

See Example 17

24 Use the index laws to simplify the following, leaving your answers with positive indices.

a $14^8 \times 14^7$

b $20^{12} \div 20^4$

c $(3 \times 7)^6$

d $5^3 \times 5^6$

e $14^6 \div 14^2$

f $(13 \times 11)^8$

g $(5^6)^3$

h $(13^2)^{-4}$

i $(9^{-3})^{-5}$

See Example 18

25 Write in scientific notation.

a $0.000\,06$

b $237\,000\,000$

c $0.005\,342$

See Example 19

26 Write as ordinary numbers.

a 6.8×10^8

b 4.35×10^{-6}

c 7.125×10^2

See Example 20

Chapter 2 review

See Example 21

- 27** Use a calculator to find each of the following and write your answer in scientific notation, correct to four significant figures.

a $3.6 \times 10^8 \div (1.5 \times 10^{-6})$

b $6.7 \times 10^{-5} \times 5.2 \times 10^{15}$

c $4.71 \times 10^{-3} \div (5.6 \times 10^{12})$

- 28** State the number of significant figures in each number.

a 4250

b 0.000 67

c 21 002 000

See Example 22

- 29** Choose the most accurately written number from the following.

28.6, 4.7, 0.0062, 435 200, 154, 68 000 000

- 30** Use a calculator to find each of the following and write your answer in scientific notation, correct to four significant figures.

a $\frac{4.5 \times 10^4 \times 5.3 \times 10^{16}}{2.5 \times 10^{-7} \times 0.095}$

b $\frac{8.6 \times 10^{-5} + 9.2 \times 10^{-6}}{6.8 \times 10^8 - 9.2 \times 10^7}$

c $\frac{\pi \times 4.1 \times 10^9}{6.2 \times 10^{-8} \times 2.3 \times 10^5}$

See Example 26

- 31** Simplify each of the following expressions. Write your answers with positive powers.

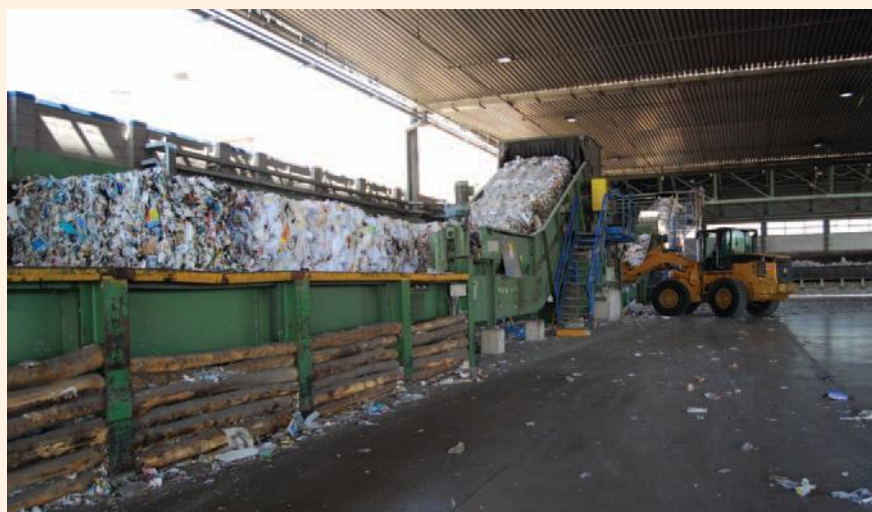
a $(xy^2)^3$

b $a^3 \times a^{-5} \div a$

c $(33m^{-3}n^4) \div (11m^{-2}n^7)$

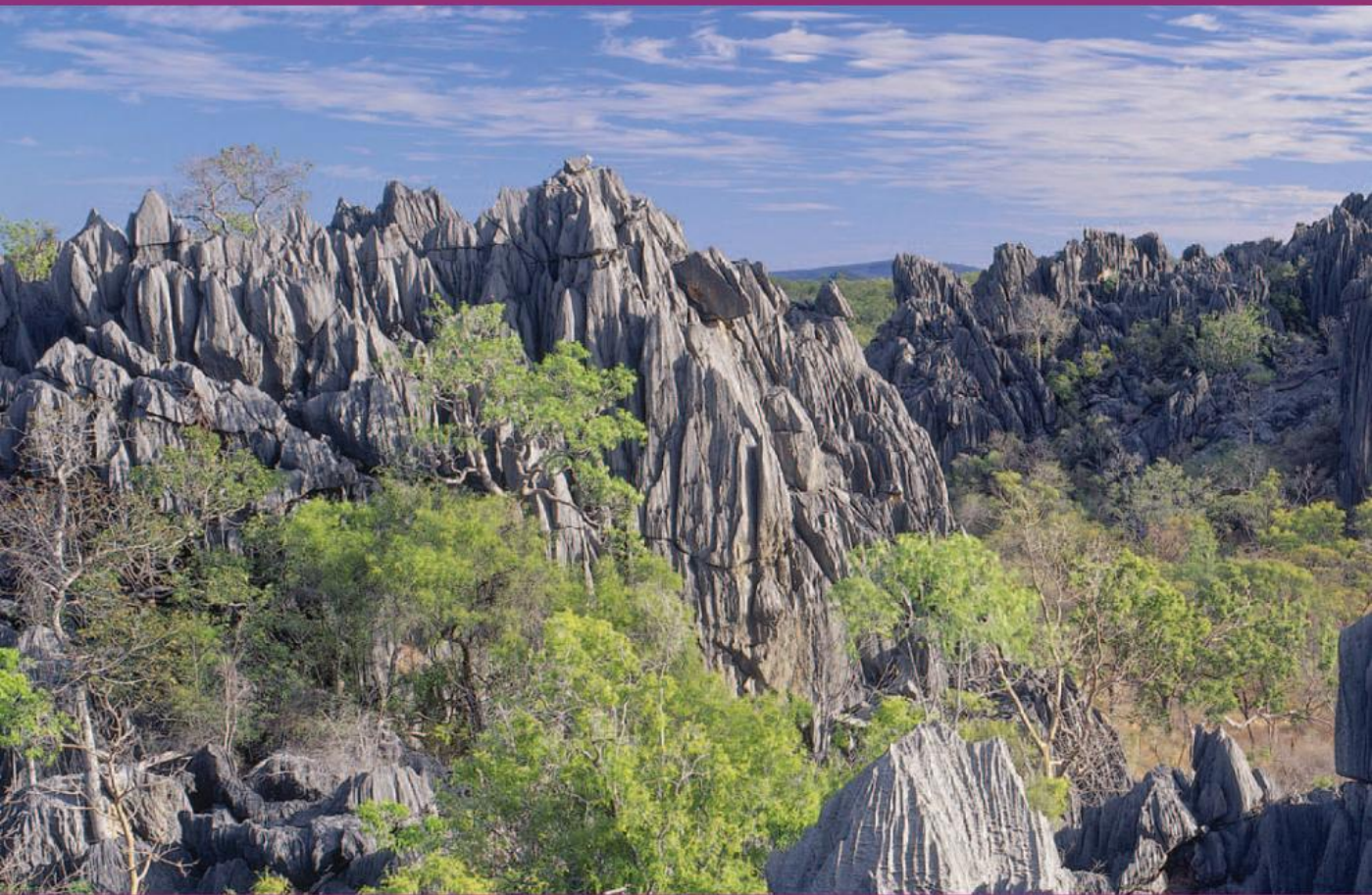
Problem solving

- 32** A load of 1600 bricks was delivered to a building site on Monday. On Tuesday another 2470 bricks were delivered and on Wednesday 1450 more were delivered. Early on Friday a small order of 453 bricks was delivered to enable the job to be finished. How many bricks were used altogether?
- 33** At the recycling centre, a car and loaded trailer were weighed in at 1624 kg. When leaving, their mass was shown as 1247 kg. What was the mass of the load of rubbish?



- 34** A rocket has a speed of 2300 km/minute. How far can it go in 3 hours?
- 35** The average growth of a very slow-growing tree is 35 cm a year. How long would it take this tree to grow 14 m?
- 36** Which is the highest temperature: -15°C , 9°C , -10°C , or 12°C ?
- 37** A miner is at a depth of 5700 m. He then travels 1600 m up the shaft. Write an expression with integers to find his final depth.
- 38** A lightning strike carries -6 C (coulombs) of charge from the ground to a cloud. How many such strikes are needed to carry a charge of -42 C ?
- 39** In Margaret's kitchen cupboard there are 3 opened packets of flour. There is half left in one packet, two-fifths in another and one-third in the last. All the packets were originally 2 kg.
- a** What fraction of a packet is left altogether?
- b** Margaret uses 600 g of flour in a cake. How much flour does she now have?
- 40** A laboratory technician spills 12.85 L of an acid. It takes 2.14 kg of sodium carbonate to neutralise 1 L of this acid. How much sodium carbonate does the technician need to neutralise the spill?
- 41** A roll of extension cord has $22\frac{3}{4}\text{ m}$ left on it; $2\frac{1}{3}\text{ m}$ is needed to make short leads, allowing for connections at the ends.
- a** How many such leads can be made?
- b** How much cord is left over?
- 42** The kinetic energy of an object is given by the formula $E = \frac{1}{2}mv^2$. If m is in kg and v is in m/s, then E is in joules. What is the kinetic energy of an electron (mass = $9.1 \times 10^{-31}\text{ kg}$) travelling at a fifth of the speed of light (speed of light, $c = 3 \times 10^8\text{ m/s}$).
- 43** A pool can be filled from either of two hoses. The first hose takes 20 hours to fill the pool and the second hose takes 12 hours.
- a** What fraction of the pool does each hose fill in 1 hour?
- b** How long would it take both hoses to fill the pool together?

Reasoning



Measurement and geometry

3 Measurement



Contents

3.1 Area

3.2 Surface area

3.3 Volume and capacity

3.4 Solving time problems

Chapter summary

Chapter review

Prior learning

Chapter 3

MAT09MGPL00003

Parent guide

Chapter 3

MAT09MGPG00003

Curriculum guide

Chapter 3

MAT09MGCU00003

Australian Curriculum statements

Using units of measurement

Calculate the areas of composite shapes.

Calculate the surface area and volume of cylinders and solve related problems.

Solve problems involving the surface area and volume of right prisms.

Investigate very small and very large time scales and intervals.

Video tutorial
Measurement

MAT09MGVT00003

There are many activities in your life that rely on your ability to measure quantities such as length, area, volume and time. If you want to paint the walls of a room, you need to calculate the area to be painted and then use this to find the volume of paint needed. Your doctor needs to accurately calculate the volume of medicine you need to take if you are sick. When cooking, you need to calculate the time that the food needs to be in the oven.
In this chapter, you will learn how to calculate areas, volumes, capacities and time intervals.

Mathematical literacy

Maths dictionary

MAT09ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

area	Australian Western	International Date	pyramid
Australian Central	Standard Time	Line	rhombus
Daylight Time	capacity	kilolitre	sector
Australian Central	cubic centimetre	leap year	square centimetre
Standard Time	cubic metre	megalitre	square metre
Australian Eastern	cubic millimetre	microsecond	surface area
Daylight Time	curved surface area	millisecond	time zone
Australian Eastern	daylight saving	nanosecond	trapezium
Standard Time	decade	parallelogram	
	hectare	prism	

3.1 Area

The **area** of a flat shape is the space enclosed by the boundary of the shape. It is measured by working out how many squares are needed to cover the shape.

Important!

Area units

A **square millimetre** (mm^2) is the area of a square 1 mm long and 1 mm wide.

A **square centimetre** (cm^2) is the area of a square 1 cm long and 1 cm wide.

A **square metre** (m^2) is the area of a square 1 m long and 1 m wide.

A **square kilometre** (km^2) is the area of a square 1 km long and 1 km wide.

A **hectare** (**ha**) is the area of a square 100 m long and 100 m wide.

1 cm^2

Square millimetres and square centimetres are used for small areas, and square metres are used for the areas of rooms, buildings and blocks of land. Hectares are used for measuring areas such as farms, schools or golf courses, and square kilometres are used for very large areas such as cattle stations or countries.

The metric units of measurement used in this chapter and their conversions are shown in the following table.

Quantity	Unit	Abbreviation	Conversion
Length	kilometre	km	$1 \text{ km} = 1000 \text{ m}$
	metre	m	$1 \text{ m} = 100 \text{ cm}$
	centimetre	cm	$1 \text{ cm} = 10 \text{ mm}$
	millimetre	mm	$1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm}$
Area	hectare	ha	$1 \text{ ha} = 100 \text{ m} \times 100 \text{ m} = 10\,000 \text{ m}^2$
	square metre	m^2	$1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2$
	square centimetre	cm^2	$1 \text{ cm}^2 = 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$
	square millimetre	mm^2	$1 \text{ mm}^2 = 1 \text{ mm} \times 1 \text{ mm}$
Volume	cubic metre	m^3	$1 \text{ m}^3 = 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$ $= 1\,000\,000 \text{ cm}^3$
	cubic centimetre	cm^3	$1 \text{ cm}^3 = 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm}$ $= 1000 \text{ mm}^3$
	cubic millimetre	mm^3	$1 \text{ mm}^3 = 1 \text{ mm} \times 1 \text{ mm} \times 1 \text{ mm}$
Capacity	megalitre	ML	$1 \text{ ML} = 1000 \text{ kL}$
	kilolitre	kL	$1 \text{ kL} = 1000 \text{ L} = 1 \text{ m}^3$
	litre	L	$1 \text{ L} = 1000 \text{ mL}$
	millilitre	mL	$1 \text{ mL} = 1 \text{ cm}^3$
Time	day	d	$1 \text{ day} = 24 \text{ h}$
	hour	h	$1 \text{ h} = 60 \text{ min}$
	minute	min	$1 \text{ min} = 60 \text{ s}$
	second	s	

Example 1

What is the most appropriate unit to measure the area of a garage door?



Solution

A typical single garage door is about 3 m by 2 m.

The area should be measured in m^2 .

Investigate: Area conversion

Work in groups for this investigation.

- 1 Use large sheets of paper to make a square metre. (You may have to join some together.)
- 2 Use a metre ruler to draw lines on your square metre to divide it into square centimetres.
- 3 Count or work out the number of square centimetres needed to make a square metre.
- 4 Divide one of the square centimetres into square millimetres.
- 5 Count the number of square millimetres in a square centimetre.

Work out a method for converting from:

- mm^2 to cm^2 and cm^2 to mm^2
- cm^2 to m^2 and m^2 to cm^2
- mm^2 to m^2 and m^2 to mm^2

The previous investigation shows that the area units are changed from one to another by squaring the linear units.

$$1 \text{ m}^2 = 1 \text{ m} \times 1 \text{ m} = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2$$

The conversions are as follows.

Important!**Metric area unit conversions**

$$1 \text{ cm}^2 = 100 \text{ mm}^2$$

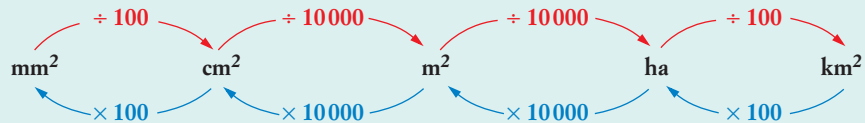
$$1 \text{ m}^2 = 10\,000 \text{ cm}^2 = 1\,000\,000 \text{ mm}^2$$

$$1 \text{ ha} = 10\,000 \text{ m}^2$$

$$1 \text{ km}^2 = 100 \text{ ha} = 1\,000\,000 \text{ m}^2$$

If you are changing to a *smaller* unit, *multiply*.

If changing to a *larger* unit, *divide*.

**Example 2**

Convert the measurements to the units indicated.

a $20\,000 \text{ cm}^2$ in m^2

b 15 cm^2 in mm^2

c 4.5 ha in m^2

d 5000 ha in km^2

Solution

- a** Write the measurement.

Changing to a **larger unit**. The number will be smaller, so **divide**.

$$20\,000\text{ cm}^2$$

$$= 20\,000 \div 10\,000\text{ m}^2$$

$$= 2\text{ m}^2$$

- b** Write the measurement.

Changing to a **smaller unit**. The number will be larger, so **multiply**.

$$15\text{ cm}^2$$

$$= 15 \times 100\text{ mm}^2$$

$$= 1500\text{ mm}^2$$

- b** Write the measurement.

Smaller unit $\rightarrow$ larger number, so **multiply**.

$$4.5\text{ ha}$$

$$= 4.5 \times 1000\text{ m}^2$$

$$= 45\,000\text{ m}^2$$

- d** Write the measurement.

Larger unit $\rightarrow$ smaller number, so **divide**.

$$5000\text{ ha}$$

$$= 5000 \div 100\text{ km}^2$$

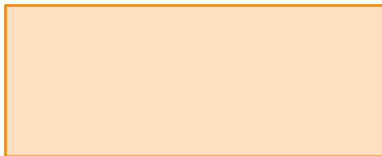
$$= 50\text{ km}^2$$

The area of a shape can be worked out by finding how many squares will fit into the shape. For irregular shapes, this may have to be estimated.

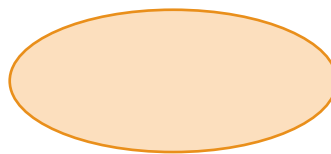
Example 3

Find the areas of these shapes.

a



b

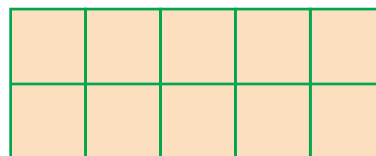


Solution

- a** Divide the shape into square centimetres.

Count the squares.

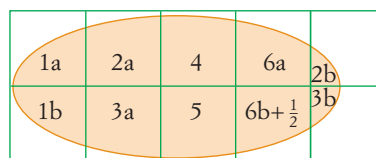
Write the area.



The area is 10 cm^2 .

- b** Divide the shape into square centimetres as much as possible.

Number parts that combine to make whole squares.



Count up the whole squares.

The area is about $6\frac{1}{2}\text{ cm}^2$.

The calculation of areas of regular shapes can be done using special formulas.

Important!

Teacher notes

Area rules

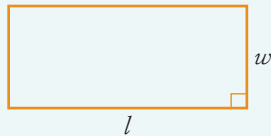
MAT09MGTN00003

Area rules

Rectangle

$$A = l \times w$$

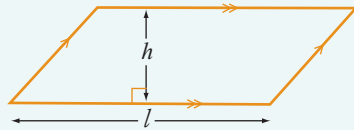
$$= \text{length} \times \text{width}$$



Parallelogram

$$A = l \times h$$

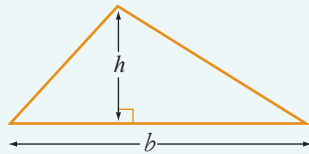
$$= \text{length} \times \text{height}$$



Triangle

$$A = \frac{1}{2} \times b \times h = \frac{b \times h}{2}$$

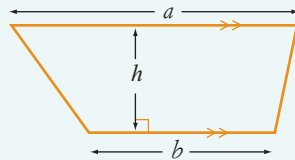
$$= \text{half}(\text{base} \times \text{height})$$



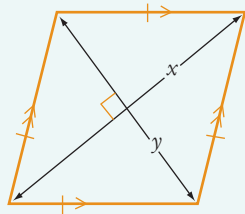
Trapezium (trapezoid)

$$A = \frac{1}{2} \times (a + b) \times h = \frac{(a + b)h}{2}$$

$$= \text{average of parallel sides} \times \text{height}$$



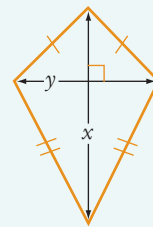
Rhombus



$$A = \frac{1}{2}xy$$

$$= \frac{1}{2} \times (\text{product of diagonals})$$

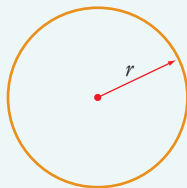
Kite



Circle

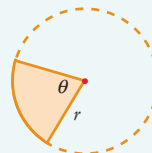
$$A = \pi r^2, \text{ where } \pi \approx 3.14$$

$$= \pi \times \text{radius} \times \text{radius}$$



Sector of a circle

If θ is the sector angle, then the fraction of the circle $= \frac{\theta}{360}$ and the area of the sector, $A = \frac{\theta}{360} \times \pi r^2$



Video tutorial

Perimeter and area of a sector

MAT09MGVT10014

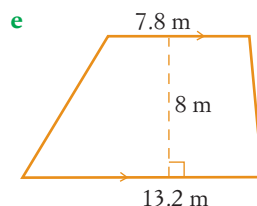
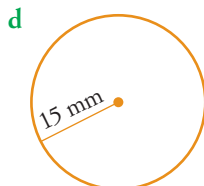
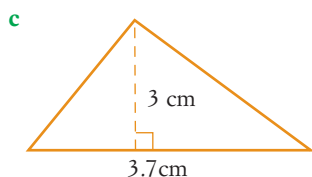
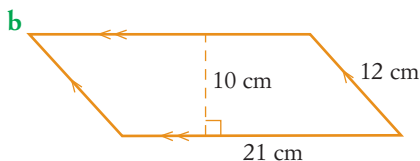
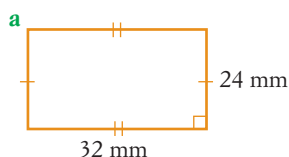
Technology

Geogebra: Area and perimeter of a sector

MAT09MGTC00002

Example 4

Find the area of each of the following, using appropriate formulas.



Solution

- a** Write the area formula for a rectangle.

Put in the values.

Work out the answer.

- b** Write the area formula for a parallelogram.

Put in the values (not 12 cm).

Work out the answer.

- c** Write the area formula for a triangle.

Put in the values.

Work out the answer.

- d** Write the area formula for a circle.

Put in the values, using π from your calculator.

Work out the answer, using a calculator.

Round off the answer to one decimal place.

- e** Write the area formula for a trapezium.

Put in the values.

Work out the answer.

$$\begin{aligned} A &= l \times w \\ &= 32 \times 24 \text{ m}^2 \\ &= 768 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A &= l \times h \\ &= 21 \times 10 \text{ cm}^2 \\ &= 210 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A &= \frac{b \times h}{2} \\ &= \frac{3.7 \times 3}{2} \text{ cm}^2 \\ &= 5.55 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 15 \times 15 \text{ mm}^2 \\ &= 706.858... \text{ mm}^2 \\ &\approx 706.9 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} A &= \frac{(a + b)h}{2} \\ &= \frac{(7.8 + 13.2) \times 8}{2} \text{ m}^2 \\ &= 84 \text{ m}^2 \end{aligned}$$

CAS TI-Nspire exercise

Measurement

MAT09MGTI00003

CAS ClassPad exercise

Measurement

MAT09MGCP00003

Worksheet

Back-to-front problems

MAT09MGWK00022

Worksheet

Area and perimeter investigations

MAT09MGWK00025

TLF Learning object

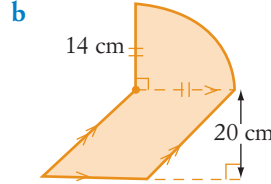
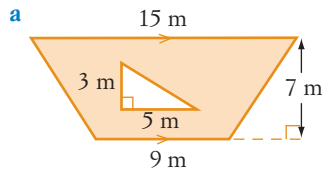
Measures: scaling up (L2310)

MAT09MGIN00003

The areas of other shapes can sometimes be calculated by dividing an irregular shape into regular parts and using combinations of the area rules.

Example 5

Find the shaded areas of the following shapes.



Solution

- a** The shape is a trapezium with a triangle cut out.

First work out the area of the trapezium.

Write the area formula for a trapezium.

Put in the values.

Work out the answer.

Next, work out the area of the triangle.

Write the area formula for a triangle.

Put in the values.

Work out the answer.

The area of the shape is the difference in the areas of the trapezium and the triangle.

$$\begin{aligned} A &= \frac{(a + b)h}{2} \\ &= \frac{(9 + 15) \times 7}{2} \text{ m}^2 \end{aligned}$$

$$\text{Trapezium area} = 84 \text{ m}^2$$

$$\begin{aligned} A &= \frac{(b \times h)}{2} \\ &= \frac{5 \times 3}{2} \text{ m}^2 \end{aligned}$$

$$\text{Triangle area} = 7.5 \text{ m}^2$$

$$\begin{aligned} \text{Area of shape} &= \text{trapezium area} - \text{triangle area} \\ &= 84 - 7.5 \text{ m}^2 \\ &= 76.5 \text{ m}^2 \end{aligned}$$

- b** This shape is a parallelogram with a circle sector added on.

First, work out the area of the parallelogram.

Top length is 14 cm.

Work out the answer.

Next, work out the area of the circle sector.

Write the area formula for a sector.

Put in the values.

$$\begin{aligned} A &= l \times h \\ &= 14 \times 20 \text{ cm}^2 \end{aligned}$$

$$\text{Parallelogram area} = 280 \text{ cm}^2$$

$$\begin{aligned} A &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{90}{360} \times \pi \times 14 \times 14 \text{ cm}^2 \end{aligned}$$

Worksheet

Circular shapes

MAT09MGWK00021

Worksheet

Applications of area 1

MAT09MGWK00023

Worksheet

Applications of area 2

MAT09MGWK00024

Puzzle sheet

Carpet talk

MAT09MGPS00007

Work out the answer.

Round off the answer to one decimal place.

The area of the shape is the sum of the areas of the parallelogram and sector.

$$= 153.93... \text{ cm}^2$$

$$\approx 153.9 \text{ cm}^2$$

Area of shape

$$= \text{parallelogram area} + \text{sector area}$$

$$\approx 280 + 153.9 \text{ cm}^2$$

$$= 433.9 \text{ cm}^2$$

Exercise 3.1 Area

1 Convert each of the measurements below to the unit shown in brackets.

a $50\,000 \text{ cm}^2 (\text{m}^2)$

b $6 \text{ cm}^2 (\text{mm}^2)$

c $2.5 \text{ m}^2 (\text{cm}^2)$

d $180 \text{ mm}^2 (\text{cm}^2)$

e $600\,000 \text{ mm}^2 (\text{m}^2)$

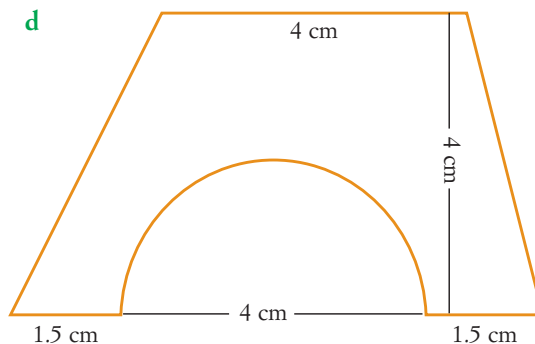
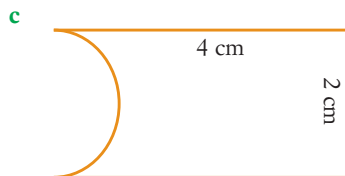
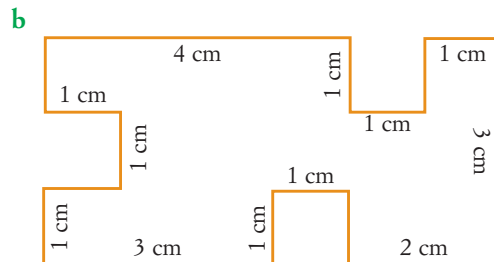
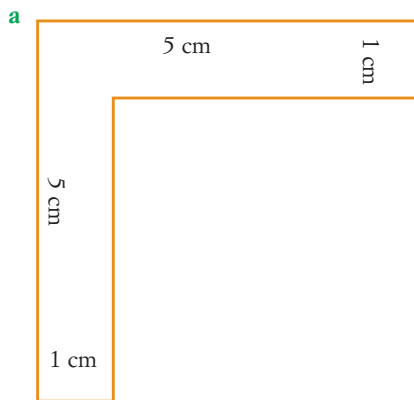
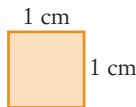
f $4000 \text{ cm}^2 (\text{m}^2)$

g $25 \text{ cm}^2 (\text{mm}^2)$

h $7000 \text{ m}^2 (\text{cm}^2)$

i $32 \text{ cm}^2 (\text{m}^2)$

2 Estimate the area of each of the following in cm^2 .



Understanding

Extra questions

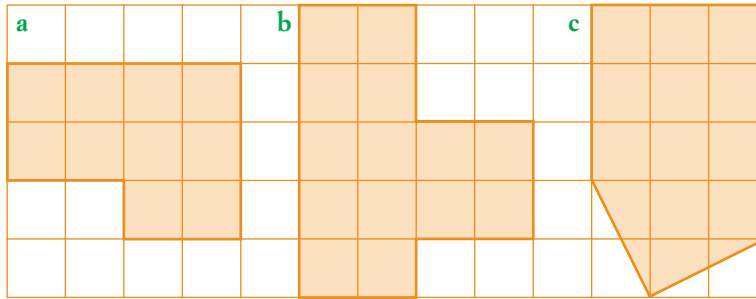
Exercise 3.1

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See Example 2

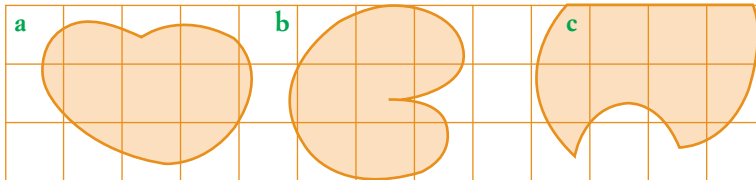
See Example 3

- 3 Find the area of each of the following shapes if each square represents 1 cm^2 .



See Example 3

- 4 Find the approximate area of each of the shapes if each square represents 1 cm^2 .



See Example 2

- 5 Convert the measurements below to the units shown in brackets.

a 6 ha (m^2)	b 15 km^2 (m^2)	c 28 ha (km^2)
d 15 000 m^2 (ha)	e 500 000 m^2 (km^2)	f 4200 ha (m^2)
g 0.03 km^2 (ha)	h 0.25 ha (m^2)	i 0.7 ha (km^2)

Fluency

- 6 Find the area of each of the following.

a Square of side 12 cm	b Rectangle 7 m by 9 m
c Rectangle 2.1 m by 6.3 m	d Square of side 3.2 km
e Square with side 0.4 m	f Rectangle 2.3 km by 4 km

See Example 4

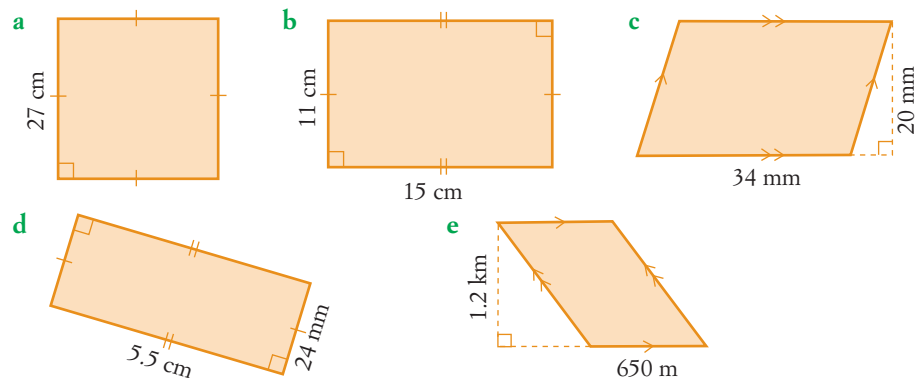
See Example 4

- 7 Calculate the areas of the following parallelograms using the dimensions given.

a Length 12 cm and height 8 cm	b Length 25 m and height 9 m
c Length 7 mm and height 13 mm	d Length 14 km and height 11 km
e Length 3.8 cm and height 20 mm	f Length 560 m and height 2.5 km

See Example 4

- 8 Find the area of each of the shapes shown below.



9 Find the areas of the triangles with the following measurements.

See Example 4

- a** Base = 5 cm and height = 8 cm **b** Base = 4.3 m and height = 12 m
c Base = 12 km and height = 3.5 km **d** Base = 18 mm and height = 15 mm
e Base = 76 cm and height = 4 m **f** Base = 4.6 mm and height = 3 cm

10 Find the area of each of the following circles using the given measurements.

See Example 4

- a** Radius 20 mm **b** Diameter 12 cm
c Radius 10.5 cm **d** Diameter 9.2 m
e Diameter 12.3 cm **f** Radius 0.63 km

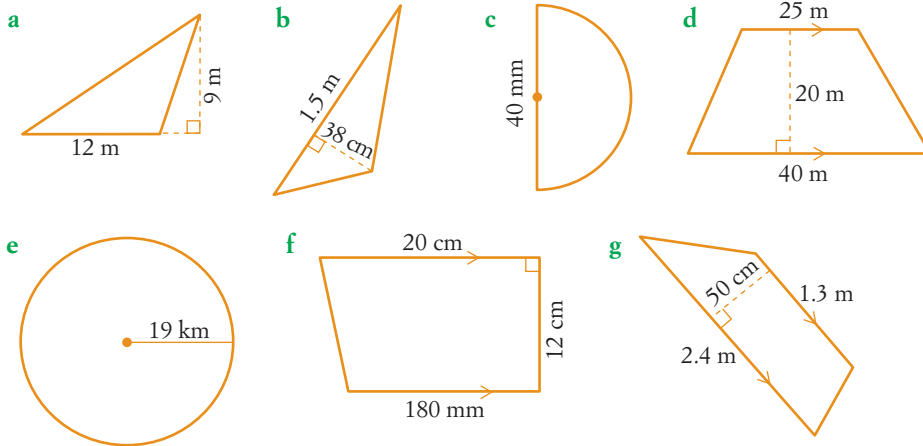
11 Find the areas of trapeziums with the following measurements.

See Example 4

- a** Parallel sides of 4 cm and 9 cm and height of 12 cm
b Parallel sides of 15 mm and 8 mm and height of 6 mm
c Parallel sides of 20 m and 14 m and height of 8 m
d Parallel sides of 3.2 km and 4.8 km and height of 2 km
e Parallel sides of 65 mm and 4.2 cm and height of 30 mm
f Parallel sides of 1.4 m and 150 cm and height of 60 cm

12 Find the area of each of the shapes shown below.

See Example 4

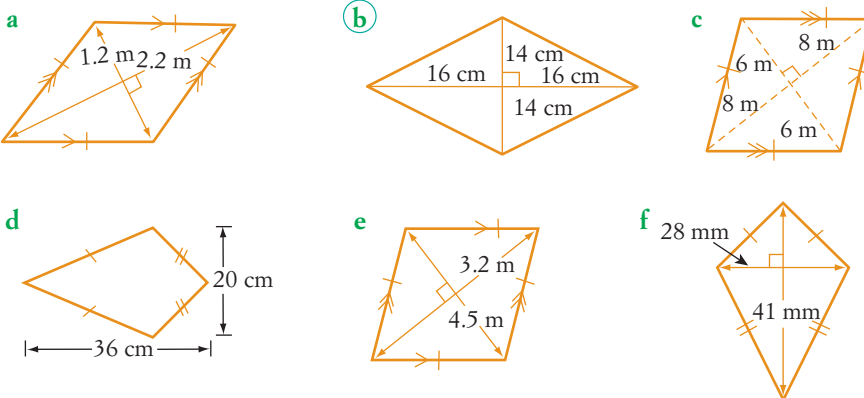


13 Find the area of each of the shapes shown below.

Worked solutions

Exercise 3.1

MAT09MGWS00008



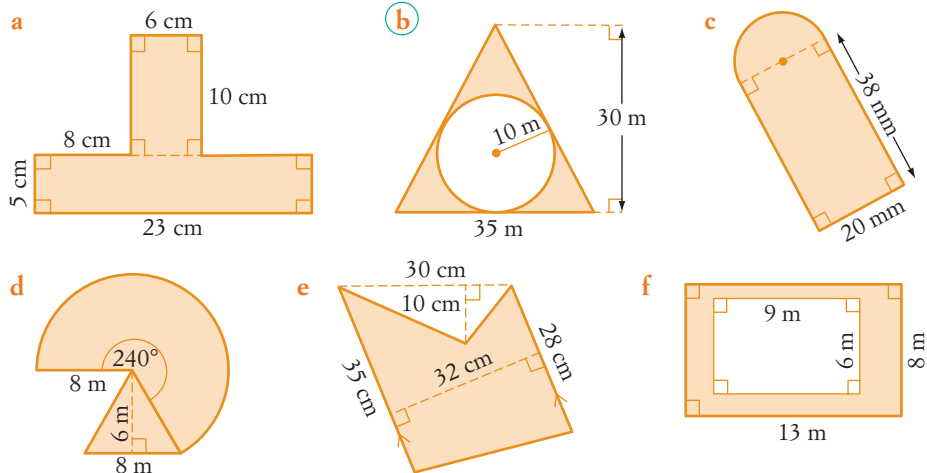
Problem solving

- 14 What is the most appropriate unit to measure the area of:
- a the classroom?
 - b a 5-cent piece?
 - c Queensland?
 - d your desk top?
 - e a suburban block of land?
 - f a postage stamp?
 - g Brisbane and its suburbs?
 - h a playing card?
 - i a three-bedroom house?
 - j Australia's fishing zone?
- 15 A rectangular farm measures 3 km by 780 m. Calculate its area in hectares.
- 16 A rectangular rugby league field is 106.8 m by 68.8 m. Find the area of the field, to the nearest square metre.
- 17 A market garden has an area of 2 ha. The amount of seed needed to give particular yields of different crops is shown in the table below.

Crop	Seeds/g	Kg of seed/ha	Yield/ha
Asparagus	55	1	4700 bunches (1 bunch = 750 g)
Corn	4	20	500 cartons (1 carton = 40 cobs)
Onions	270	7	1400 bags (1 bag = 20 kg)
Peas	7	160	140 sacks (1 sack = 22.5 kg)

Calculate:

- a the mass of asparagus seed required to plant this area
 - b the mass of onion seed required to plant this area
 - c the yield (in kg) if asparagus were planted
 - d the yield (in kg) if onions were planted
 - e the number of peas needed to plant this area.
- 18 Find the shaded area of each of the following shapes.



Worked solutions

Exercise 3.1

MAT09MGWS00008

- 19 Use the fact that the area of Tasmania is approximately $68\,400\text{ km}^2$ and the map below to estimate the area of:



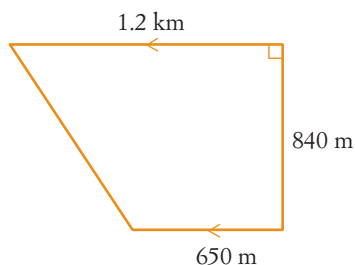
a Queensland

b Victoria

c Western Australia

- 20 A very large block of land has an area of 1 ha. The length is 150 m longer than the width. What is the width?

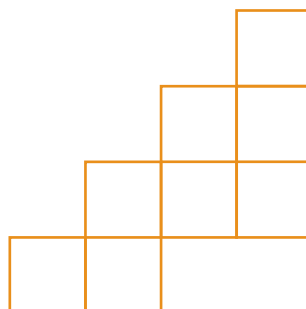
- 21 Use the information from question 17 to find the mass of asparagus seed that would be needed to plant the paddock shown on the right.



- 22 A courtyard is made up of 8 squares as shown below. The external perimeter of the courtyard is 36 m.

Calculate:

- a the dimensions of each square
b the area of the courtyard.



Reasoning

Worked solutions

Exercise 3.1

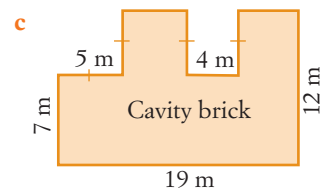
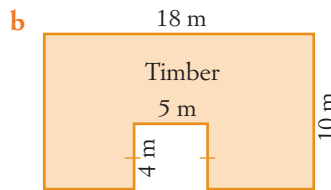
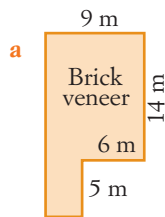
MAT09MGWS00008

- 23 The Sydney Cricket Ground (SCG), shown below, is approximately circular.
- Find the area of the SCG to the nearest m^2 .
 - Find its area to the nearest ha.
 - The ground can be mown at the rate of $5500 \text{ m}^2/\text{h}$. How long does it take to mow?



- 24 The approximate costs of building different types of homes are shown on the right. Find the costs of building the homes with the floor plans shown below.

Type	Cost/ m^2
Cavity brick	\$1700
Brick veneer	\$1380
Timber	\$1500



- 25 A rectangular garden measures 20 m by 10 m. A 1 m wide concrete path is laid around the garden. If the path costs \$43.25 per square metre, calculate the cost of the path.

3.2 Surface area

Weblink

Human body surface area calculations

MAT09MGWB00003

In order to make, cover or paint three-dimensional (3D) shapes, you may need to know the total area of the surfaces of the shape.

Important!

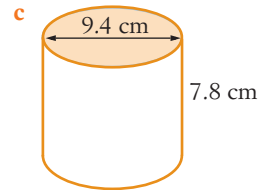
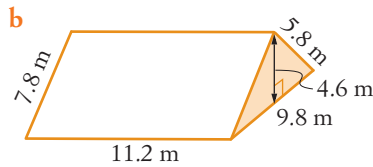
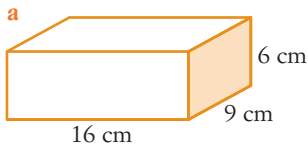
Surface area

The **surface area (SA)** of a shape is the total area of all its faces. It is calculated by adding the areas of the faces.

The surface area is often calculated using the **net** of a shape. The net is the flat shape you get by 'unfolding' a 3D shape.

Example 6

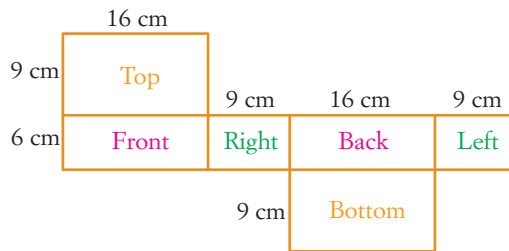
Find the surface areas of the following shapes.



Solution

- a** Draw the net of the shape.

Write the dimensions on the net.



Work out the area of each face.

$$\text{Front} = l \times w = 16 \times 6 \\ = 96 \text{ cm}^2$$

$$\text{Right} = l \times w = 9 \times 6 \\ = 54 \text{ cm}^2$$

$$\text{Top} = l \times w = 16 \times 9 \\ = 144 \text{ cm}^2$$

$$\text{Back} = l \times w = 16 \times 9 \\ = 96 \text{ cm}^2$$

$$\text{Left} = l \times w = 9 \times 6 \\ = 54 \text{ cm}^2$$

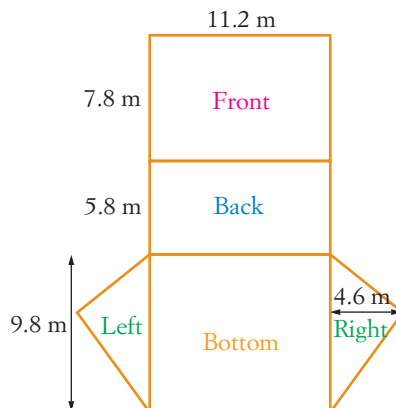
$$\text{Bottom} = l \times w = 16 \times 9 \\ = 144 \text{ cm}^2$$

$$\text{SA} = 96 + 54 + 144 + 96 + 54 + 144 = 588 \text{ cm}^2$$

Find the total.

- b** Draw the net of the shape.

Write the dimensions on the net.



Video tutorial

Surface area of a prism

MAT09MGVT10015

Video tutorial

Surface area of a cylinder

MAT09MGVT10016

Worksheet

Surface area

MAT09MGWK00026

Worksheet

Formula matching game

MAT09MGWK00027

Puzzle sheet

Car song

MAT09MGPS00006

Technology worksheet

Excel worksheet: Surface area calculator

MAT09MGCT00020

Work out the area of each face.

$$\begin{aligned}\text{Left} &= \frac{b \times h}{2} = \frac{9.8 \times 4.6}{2} \\ &= 22.54 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Front} &= l \times w = 11.2 \times 7.8 \\ &= 87.36 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Back} &= l \times w = 11.2 \times 5.8 \\ &= 64.96 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Bottom} &= l \times w = 11.2 \times 9.8 \\ &= 109.76 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Left} &= \frac{b \times h}{2} = \frac{9.8 \times 4.6}{2} \\ &= 22.54 \text{ m}^2\end{aligned}$$

Find the total.

$$\begin{aligned}\text{SA} &= 22.54 + 87.36 + 64.96 + 109.76 + 22.54 \text{ m}^2 \\ &= 307.16 \text{ m}^2\end{aligned}$$

Round off to write the answer.

The surface area is about 307 m².

- c Mentally open out the top and bottom and unroll the cylinder to make a net.

The net is a rectangle (the curved surface of the cylinder) and two circles (the top and bottom of the cylinder).

Write the dimensions on the net.

The length of the rectangle is the circumference of the circle.

Work out the length of the rectangle, keeping the full answer on your calculator.

$$\begin{aligned}C &= \pi D \\ &= \pi \times 9.4 \\ &= 29.53... \text{ cm}\end{aligned}$$

Work out the area of the rectangle and store the full answer.

$$\begin{aligned}\text{Area of rectangle} &= l \times w \\ &= 29.53... \times 7.8 \\ &= 230.3... \text{ cm}^2\end{aligned}$$

Find the circle's area.

The radius = $9.4 \div 2 = 4.7$ cm.

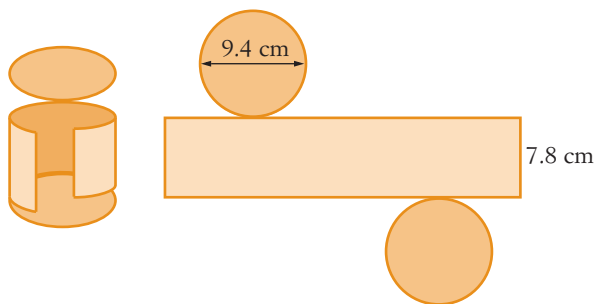
$$\begin{aligned}\text{Area of circle} &= \pi r^2 \\ &= \pi \times 4.7 \times 4.7 \\ &= 69.39... \text{ cm}^2\end{aligned}$$

Find the total, using the stored answers.

$$\begin{aligned}\text{SA} &= \text{area of rectangle} + 2 \times \text{area of circle} \\ &= 230.3... + 2 \times 69.39... \text{ cm}^2 \\ &= 369.13... \text{ cm}^2\end{aligned}$$

Round and write the answer.

The surface area is about 369 cm².



The cylinder in the previous example is called a **closed cylinder** because it has two circular ends. An **open cylinder** has only one circular end.

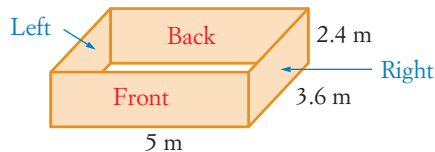
Example 7

A room has ceilings 2.4 m high and is 5 m by 3.6 m. Ignoring doors and windows, find the surface area of the walls and the cost of painting the walls at \$13.50/m².

Solution

Draw a diagram showing the walls.

Place the dimensions on the diagram.



Work out the total area of the walls.

Area of walls

$$\begin{aligned} &= \text{front} + \text{right} + \text{back} + \text{left} \\ &= 5 \times 2.4 + 3.6 \times 2.4 + 5 \times 2.4 + 3.6 \times 2.4 \\ &= 41.28 \text{ m}^2 \end{aligned}$$

Multiply to find the cost.

$$\begin{aligned} \text{Cost of painting} &= 41.28 \times \$13.50 \\ &= \$557.28 \end{aligned}$$

Write the answer.

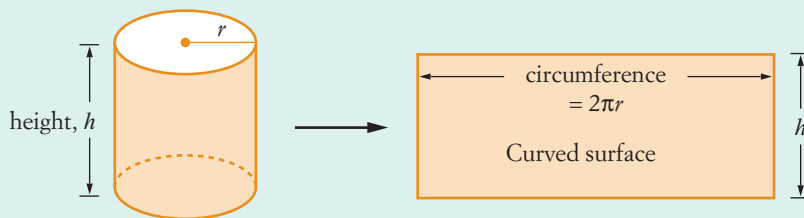
The cost of painting the walls is \$557.28.

It is generally more convenient to use a rule for working out the surface area of a cylinder.

Important!

Surface area of a cylinder

The curved area of a cylinder, without the top and bottom, is called the **curved surface area**.



For a cylinder of radius r and height h , the curved surface area is given by
Curved surface area (CSA) $= 2\pi rh$

Example 8

A cylinder for storing industrial chemicals has a bottom but no lid. It is 3.2 m high and 4.6 m in diameter. Find its surface area.

Solution

Write the curved surface area formula.

$$\begin{aligned}\text{CSA} &= 2\pi rh \\ &= 2 \times \pi \times 2.3 \times 3.2 \\ &= 46.24... \text{ m}^2\end{aligned}$$

Substitute the values $r = 2.3$ m and $h = 3.2$ m.

Use your calculator and store the answer.

The base is a circle.

$$\begin{aligned}\text{Area of base} &= \pi r^2 \\ &= \pi \times 2.3 \times 2.3 \\ &= 16.61... \text{ m}^2\end{aligned}$$

Use $r = 2.3$ m.

Find the total, using your stored answers.

$$\begin{aligned}\text{Surface area} &= 46.24... + 16.61... \\ &= 62.86... \text{ m}^2\end{aligned}$$

Round and write the answer.

The surface area is about 63 m^2 .

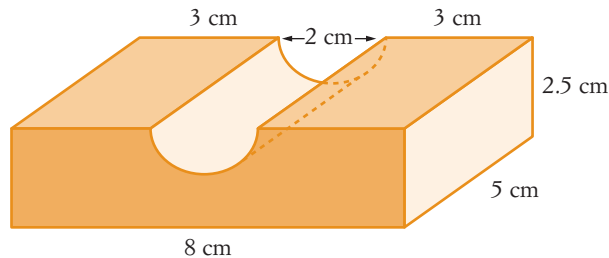
Example 9

Animated example

Surface area

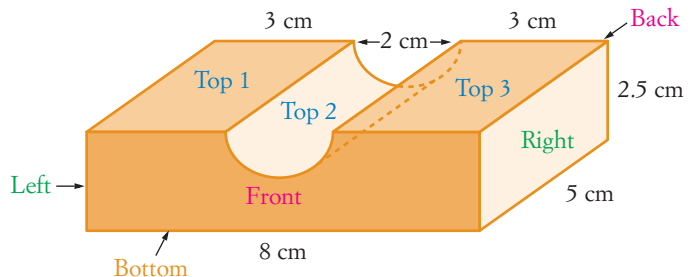
MAT09MGAE00003

A small rectangular metal block has a section in the shape of half a cylinder removed. The entire block is to be electroplated with silver. Calculate the surface area to be coated.



Solution

Label the different sides of the solid.



The left and right sides are the same. Calculate the area of the left and right sides.

$$\text{Area of left} = l \times w = 5 \times 2.5 = 12.5 \text{ cm}^2$$

Calculate the area of the bottom.

$$\text{Area of bottom} = l \times w = 8 \times 5 = 40 \text{ cm}^2$$

The front and back are the same.
The front is a rectangle with a semicircle removed. The radius of the half cylinder is 1 cm.

Calculate the area of the front and back and store the answer.

The top consists of 3 parts:
2 equal rectangles and half of the curved part of a cylinder.

Calculate the area of the top.

The height of the half-cylinder is 5 cm.

Find the total, using stored values.

Round and write the answer.

$$\begin{aligned}\text{Area of front} &= l \times w - \frac{1}{2} \times \pi r^2 \\ &= 8 \times 2.5 - \frac{1}{2} \times \pi \times 1^2\end{aligned}$$

$$= 18.42... \text{ cm}^2$$

$$\begin{aligned}\text{Area of top} &= \text{top 1} + \text{top 2} + \text{top 3} \\ &= 2 \times \text{top 1} + \text{top 2}\end{aligned}$$

$$= 2 \times l \times w + \frac{1}{2} \times 2\pi h$$

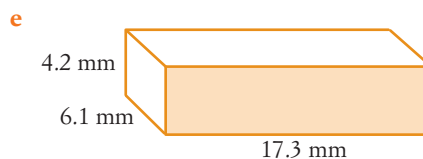
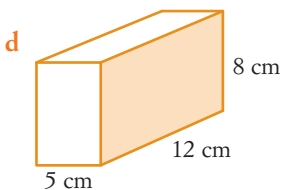
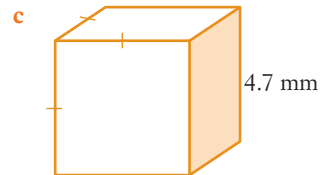
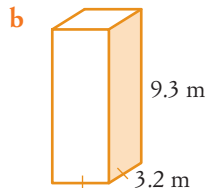
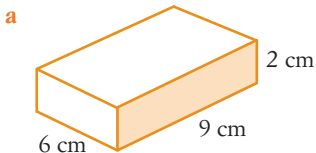
$$\begin{aligned}&= 2 \times 5 \times 3 + \frac{1}{2} \times 2 \times \pi \times 5 \\ &= 45.70... \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{SA} &= 2 \times \text{left} + \text{bottom} + 2 \times \text{front} + \text{top} \\ &= 2 \times 12.5 + 40 + 2 \times 18.42... + 45.70... \text{ cm}^2 \\ &= 147.56... \text{ cm}^2\end{aligned}$$

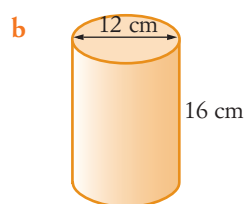
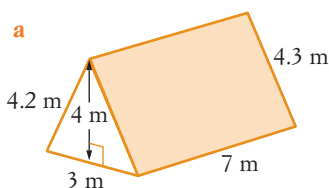
The surface area of the block is about 148 cm².

Exercise 3.2 Surface area

- 1 Find the surface area of each of the following.



- 2 Find the surface area of each of the following.



Fluency

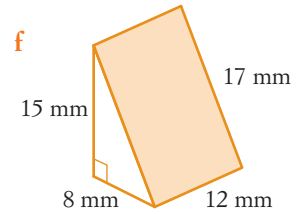
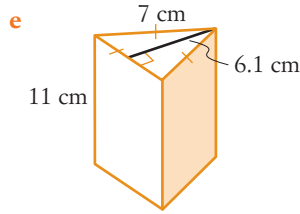
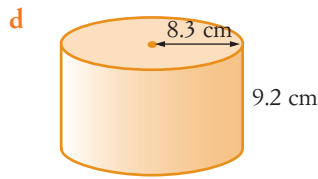
Extra questions

Exercise 3.2

MAT09MGEQ00009

See Example 6

See Example 6



See Example 8

- 3 Calculate the curved surface area of the cylinders below using the dimensions given. Give your answers correct to two decimal places.

a radius 8 cm, height 10 cm

b diameter 45 mm, height 28 mm

c diameter 1.8 m, height 6 m

d radius 12.9 cm, height 34.6 cm

- 4 Calculate the total surface areas of the following closed cylinders using the given dimensions. Give your answers correct to one decimal place.

a diameter 20 cm, height 15 cm

b radius 1.5 m, height 8 m

c radius 36 mm, height 62 mm

d diameter 31.7 cm, height 66.4 cm

- 5 Calculate the total surface areas of the following open cylinders using the given dimensions. Give your answers correct to two decimal places.

a radius 1 m, height 6 m

b diameter 26 cm, height 49 cm

c diameter 28 mm, height 62 mm

d radius 17.6 cm, height 45.2 cm

Worked solution

Exercise 3.2

MAT09MGWS00009

See Example 8

See Example 8

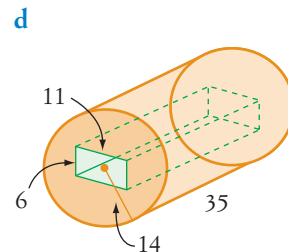
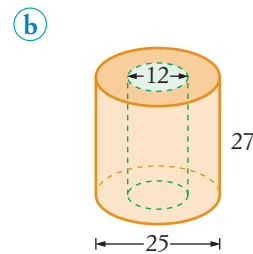
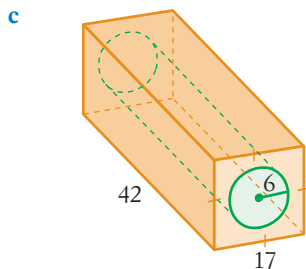
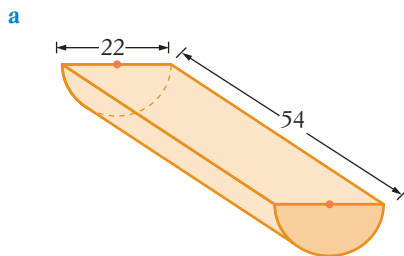
Problem solving

Worked solution

Exercise 3.2

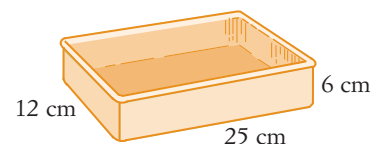
MAT09MGWS00009

- 6 Calculate the total surface area of each of the following solids. All dimensions shown are in centimetres.



See Example 7

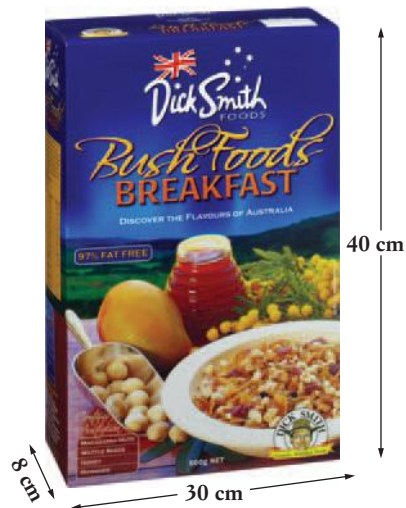
- 7 This cake tin was made by a Year 9 student. Find the total area of metal needed to make the tin if you neglect the extra required for joins.



- 8 A cylindrical 'wheel' of cheese has a diameter of 25 cm and a height of 6.5 cm. It also has a wedge cut out as shown below. Calculate the total surface area of the cheese remaining.



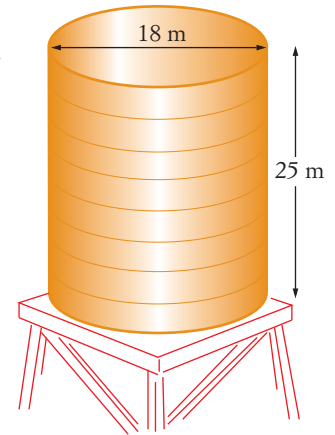
- 9 A cereal manufacturer uses boxes as shown.
- Find the surface area of the box.
 - If 10% extra is allowed for joins, how much cardboard is needed for one box?
 - How many square metres of cardboard are needed for 20 000 boxes?



- 10 An oil company is making 2500 petrol cans that measure 140 mm by 160 mm by 250 mm. For each can, 0.04 m^2 is allowed for overlap, wastage, etc. How many square metres of metal are needed to make the cans?
- 11 A ballroom is 12 m long and 10 m wide with ceilings 3.2 m high. Find the cost of painting the walls if painting costs $\$8.40/\text{m}^2$.
- 12 A machinery shed is 30 m long, 20 m wide and 5 m high. The shed does not have any windows and the front closes completely. How much will it cost to paint the outside walls if it costs $\$4.20/\text{m}^2$?
- 13 A rectangular sheet of metal has been curved and then welded to form a cylindrical pipe to act as a culvert. The pipe is 5.3 m long, with a diameter of 3 m.
- Find the dimensions of the smallest sheet of metal that could have been used for the job.
 - What is the outside area of the pipe?

Reasoning

- 14 The water supply for a small town is held in the open-topped cylindrical tank shown here. The internal dimensions of the tank are shown. Each year the inside of the tank needs to be recoated with a special paint to stop bacteria from growing.
- What area needs to be coated?
 - If 1 L of the paint covers 5 m^2 , how many litres will be required for the job?



- 15 A paint manufacturer has just brought out a new line of timber stain to be sold in tins like the one on the right.
- If the label completely covers the curved surface and has a 1 cm overlap, find the total area of the label.
 - Calculate the maximum number of labels that could be made from a rectangular sheet of paper that measures 1 m by 5 m.

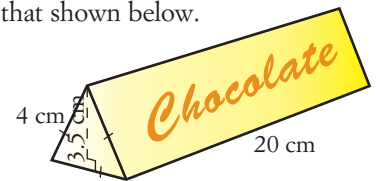


Worked solution

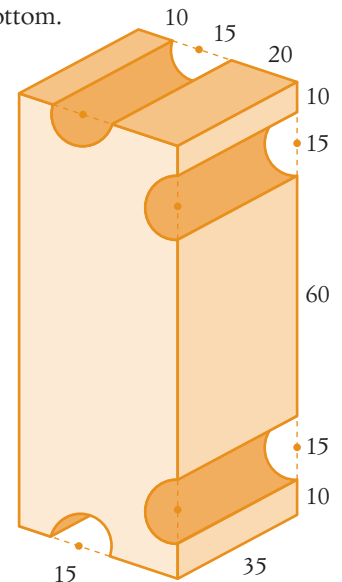
Exercise 3.2

MAT09MGWS00009

- 16 A popular brand of chocolate is sold in triangular boxes like that shown below.
- What is the surface area of the box?
 - If 15% is allowed for waste, how much cardboard is needed for one box?
 - The cardboard costs $\$3.50/\text{m}^2$, including printing. How much does it cost to make 8000 boxes?



- 17 An oil refinery stores petrol in large cylindrical tanks that are 22 m high and 35 m in diameter.
- Find the surface area of one petrol tank, not counting the bottom.
 - The top and sides of the tanks are to be painted with paint that covers $16 \text{ m}^2/\text{L}$. How many litres of paint are needed for each tank?
- 18 The rectangular metal block shown here has a number of sections removed that are in the shape of half a cylinder. All dimensions are shown in centimetres. The entire block is to be coated with a special reflective substance. Calculate the surface area to be coated.



3.3 Volume and capacity

The **cubic volume** of a 3D shape is a measure of the space it occupies. It is normal to measure volume by working out how many cubes of standard size would be needed to fill the same space. **Capacity** is really another word for volume, but we think of capacity as the inside space of a container, particularly one that is used to hold liquids. In the metric system, the units of capacity were chosen to make it easy to change from cubic volume to capacity.

Important!

Volume and capacity

The **volume** of a 3D object measures the amount of space it occupies. It is calculated by working out how many cubes of unit size would be needed to fill the same space. Volume is measured in **cubic** units, such as **cubic centimetres** (cm^3 or **cc**) or **cubic metres** (m^3).

The **capacity** of a container measures the volume inside.

Capacity is measured in **millilitres (mL)**, **litres (L)**, **kilolitres (kL)** or **megalitres (ML)**.

1 litre = 1000 millilitres

1 kilolitre = 1000 L

1 megalitre = 1000 kL = 1 000 000 L

In the metric system:

$1 \text{ cm}^3 = 1 \text{ mL}$ and $1 \text{ m}^3 = 1000 \text{ L} = 1 \text{ kL}$

TLF Learning object

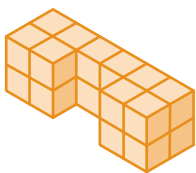
Inside a cubic metre
(L164)

MAT09MGIN00003

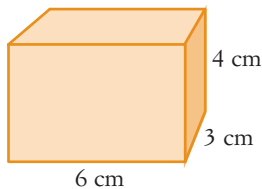
Example 10

Find the volume of each of the following shapes, where each small block is 1 cm^3 .

a



b



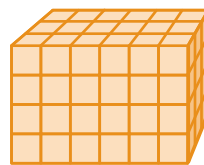
Solution

- a** Count the blocks in the top layer.
Count the blocks in the second layer.
Write the total volume.
- b** Divide the shape into 1 cm^3 blocks.

There are 10 blocks in the top layer.

There are 10 blocks in the second layer.

Volume = $10 + 10 = 20 \text{ cm}^3$



There are 18 blocks in the top layer.

There are 4 identical layers.

Volume = $4 \times 18 = 72 \text{ cm}^3$

Count the blocks in the top layer.

Count the layers.

Work out the volume.

The volumes of common shapes can be worked out using formulas.

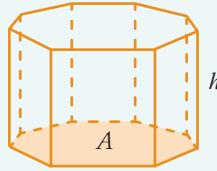
Important!

Volume formulas

Prism

$$V = Ah$$

Volume = area of the base $\times$ height

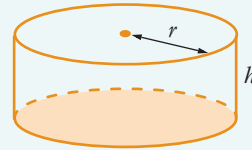


Cylinder

$$V = Ah$$

$$= \pi r^2 h$$

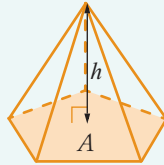
$$= \pi \times \text{radius}^2 \times \text{height}$$



Pyramid

$$V = \frac{1}{3}Ah = \frac{Ah}{3}$$

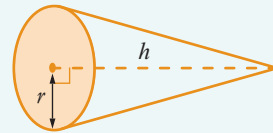
Volume = $\frac{1}{3} \times$ area of the base $\times$ height



Cone

$$V = \frac{1}{3}Ah = \frac{1}{3}\pi r^2 h = \frac{\pi r^2 h}{3}$$

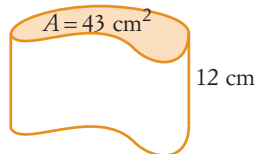
$$= \frac{1}{3} \times \pi \times \text{radius}^2 \times \text{height}$$



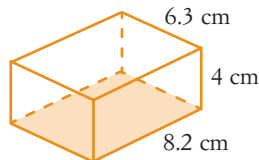
Example 11

Work out the volume of each of the following shapes.

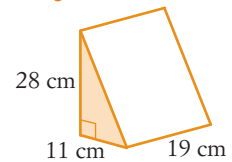
a



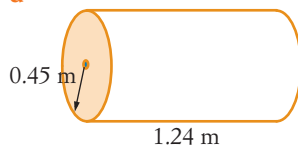
b



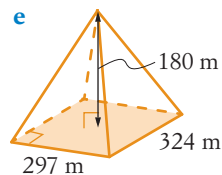
c



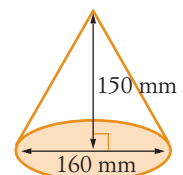
d



e



f



Puzzle sheet

Volume

MAT09MGPS00008

Solution

- a** The shape has a constant cross-section, so it is a prism.

Write the volume formula for a prism.

Substitute the given values.

Write the answer.

- b** Write the volume formula for a prism.

The base is a rectangle.

Substitute the given values.

Round and write the answer.

- c** The base of the prism is a triangle.

Substitute $b = 11$ cm and $h = 28$ cm.

Write the volume formula for a prism.

Substitute the values.

Write the answer.

- d** Write the volume formula for a cylinder.

Substitute $r = 0.45$ m and $h = 1.24$ m.

Use your calculator for π .

Round and write the answer.

- e** Write the volume formula for a pyramid.

The pyramid has a rectangular base.

Substitute the given values.

Write the answer.

- f** Write the volume formula for a cone.

Substitute $r = 8$ cm and $h = 15$ cm.

Use your calculator.

Round and write the answer.

$$V = Ah$$

$$= 43 \times 12 = 516 \text{ cm}^3$$

The volume is 516 cm^3 .

$$V = Ah$$

$$V = l \times w \times h$$

$$= 8.2 \times 6.3 \times 4$$

$$= 206.64 \text{ m}^3$$

The volume is about 207 m^3 .

$$A = \frac{b \times h}{2}$$

$$= \frac{11 \times 28}{2}$$

$$= 154 \text{ cm}^2$$

$$V = Ah$$

$$= 154 \times 19$$

$$= 2926 \text{ cm}^2$$

The volume is 2926 cm^3 .

$$V = \pi r^2 h$$

$$= \pi \times 0.45 \times 0.45 \times 1.24$$

$$= 0.7888... \text{ m}^3$$

The volume is about 0.79 m^3 .

$$V = \frac{Ah}{3}$$

$$V = \frac{l \times w \times h}{3}$$

$$= \frac{297 \times 324 \times 180}{3}$$

$$= 5\,773\,680 \text{ m}^3$$

The volume is $5\,773\,680 \text{ m}^3$.

$$V = \frac{1}{3} \pi r^2 h$$

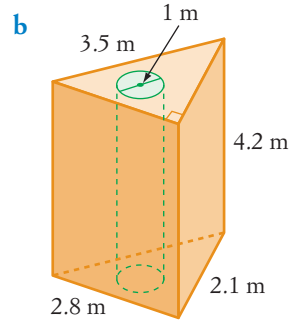
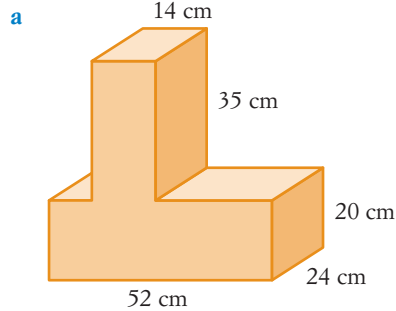
$$= \pi \times 8 \times 8 \times 15 \div 3$$

$$= 1005.30... \text{ cm}^3$$

The volume is about 1005 cm^3 .

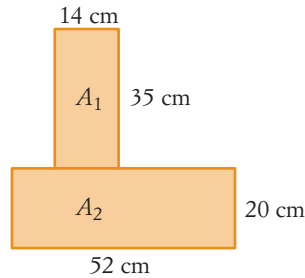
Example 12

Calculate the volume of each of these solids.



Solution

- a** The cross-section is made up of two rectangles.



Calculate the area of the cross-section.

Substitute given values.

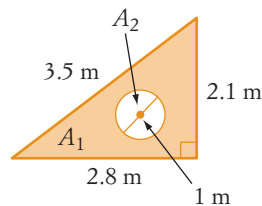
$$\begin{aligned}\text{Cross-section area} &= A_1 + A_2 \\ &= 14 \times 35 + 52 \times 20 \\ &= 1530 \text{ cm}^2\end{aligned}$$

Write the volume formula for a prism.

Substitute given values.

$$\begin{aligned}V &= Ah \\ &= 1530 \times 24 \\ &= 36\,720 \text{ cm}^3\end{aligned}$$

- b** The cross-section is made up of a triangle with a circle cut out.



Calculate the area of the cross-section.

Use the formulas for the area of a triangle and a circle.

Substitute given values.

$$\begin{aligned}\text{Cross-section area} &= A_1 - A_2 \\ &= \frac{b \times h}{2} - \pi r^2 \\ &= \frac{2.8 \times 2.1}{2} - \pi \times 0.5 \times 0.5 \\ &= 2.154... \text{ m}^2\end{aligned}$$

Use your calculator and store the answer.

Write the volume formula for a prism.

Substitute values.

$$\begin{aligned}V &= Ah \\ &= 2.154... \times 4.2 \\ &= 9.049... \text{ m}^3\end{aligned}$$

Round and write the answer.

The volume is about 9.0 m^3 .

Worksheet

Volume and capacity

MAT09MGWK00028

Worksheet

Measurement in the home

MAT09MGWK00029

Investigate: Capacities of common containers

You will need some containers from home, such as empty fruit tins, juice tins, laundry buckets, mixing bowls, cups, breakfast bowls and toy crates. Your teacher may also give you some other common containers. You will need measuring tapes and measuring jugs as well.



Work in groups of about three students.

- 1 For regular shapes, measure the internal dimensions and calculate the capacities.
- 2 For irregular shapes, use measuring jugs to measure how much water is needed to fill the container. This will give the capacity.
- 3 Make a table of the capacities of common containers.

To decide on suitable units, you need to have an idea of their practical sizes. A medicine glass holds about 20 mL, a large bottle of soft-drink is usually 2 L, a large drum holds about 150 L, the inside of a normal car holds about 600 L of air (about 0.6 m^3) and a normal classroom has a size of about 100 m^3 . The amount of water held in a backyard swimming pool is usually measured in cubic metres or kilolitres, while the sizes of dams are measured in megalitres.

Example 13

What is the most suitable unit to measure the capacity of:

a a drinking glass?

b a shipping container?

Solution

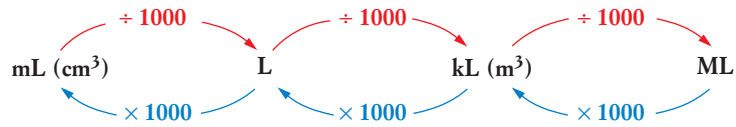
a A drinking glass holds a small amount. Most drinking glasses hold less than a litre.

Use millilitres.

b A shipping container holds a large amount.

Use cubic metres.

Use the following when converting units.



If you are changing to a *smaller* unit, *multiply*. If changing to a *larger* unit, *divide*.

Example 14

Convert each of the following measurements to the unit shown in brackets.

- a** 20 000 L (m^3) **b** 15 cm^3 (mL) **c** 50 ML (m^3) **d** 50 000 cm^3 (kL)

Solution

- a** Write the measurement.

Larger unit $\rightarrow$ smaller number,
so **divide**.

$$\begin{aligned} &20\,000\text{ L} \\ &= 20\,000 \div 1000\text{ m}^3 \\ &= 20\text{ m}^3 \end{aligned}$$

- b** Write the measurement.

These are equal units. $1\text{ cm}^3 = 1\text{ mL}$.

$$\begin{aligned} &15\text{ cm}^3 \\ &= 15\text{ mL} \end{aligned}$$

- c** Since $1\text{ m}^3 = 1\text{ kL}$, change to kL first.

Write the measurement.

Smaller unit $\rightarrow$ larger number,
so **multiply**.

Now use $1\text{ kL} = 1\text{ m}^3$.

$$\begin{aligned} &50\text{ ML} \\ &= 50 \times 1000\text{ kL} \\ &= 50\,000\text{ kL} \\ &= 50\,000\text{ m}^3 \end{aligned}$$

- d** Write the measurement.

Use $1\text{ cm}^3 = 1\text{ mL}$.

Change to L. **Larger** unit $\rightarrow$ smaller number,
so **divide**.

Now change to kL. **Larger** unit $\rightarrow$ smaller number,
so **divide**.

$$\begin{aligned} &50\,000\text{ cm}^3 \\ &= 50\,000\text{ mL} \\ &= 50\,000 \div 1000\text{ L} \\ &= 50\text{ L} \\ &= 50 \div 1000\text{ kL} \\ &= 0.05\text{ kL} \end{aligned}$$

Example 15

This large can was used for buying bulk turps. What is its capacity if it has the diameter and height shown?

Solution

Write the volume formula for a cylinder.

$$V = \pi r^2 h$$

Substitute $r = 9\text{ cm}$ and

$h = 19.7\text{ cm}$.

Use your calculator.

$$\begin{aligned} &= \pi \times 9 \times 9 \times 19.7 \\ &= 5013.03... \text{ cm}^3 \end{aligned}$$



We want to know the capacity,
so change to millilitres.

$$= 5013.03... \text{ mL}$$

Change to litres (i.e. $\div 1000$).

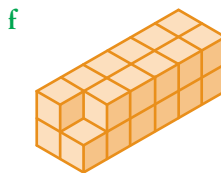
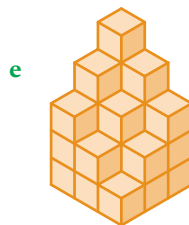
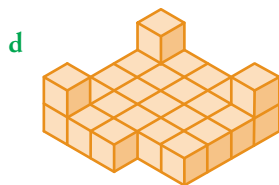
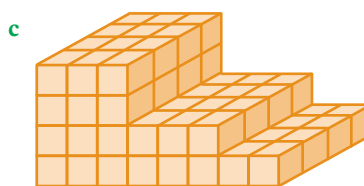
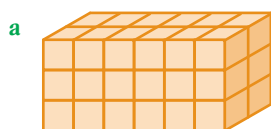
$$= 5.01303... \text{ L}$$

Round and write the answer.

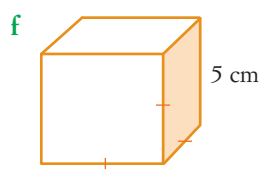
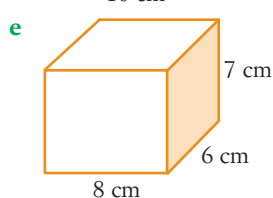
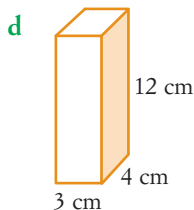
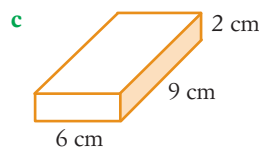
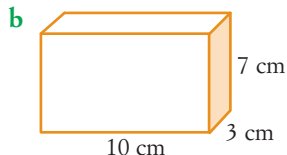
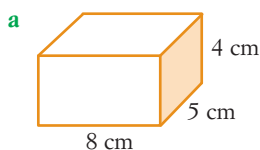
The capacity of the can is about 5 L.

Exercise 3.3 Volume and capacity

- 1 Find the volume of each of the following, given that each block is 1 cm^3 .



- 2 Work out the volume of each of the following.



- 3 Convert each of the following measurements to the unit shown in brackets.

a 500 mL (L)

b 1.27 L (cm^3)

c 78 cm^3 (mL)

d 4300 L (m^3)

e 24 000 cm^3 (L)

f 0.008 m^3 (mL)

- 4 Convert each of the following measurements to the unit shown in brackets.

a 0.004 kL (cm^3)

b 1200 L (kL)

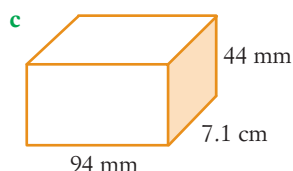
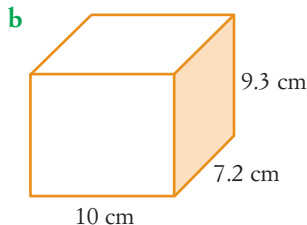
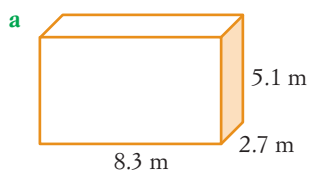
c 1.05 kL (L)

d 45.3 m^3 (kL)

e 446 ML (m^3)

f 6400 L (ML)

- 5 Work out the volume of each of the following.



Understanding

Extra questions

Exercise 3.3

MAT09MGEQ00010

See Example 10

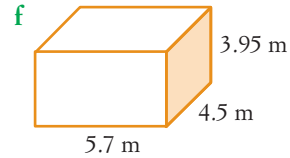
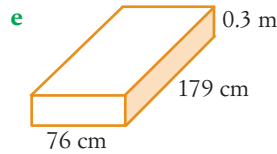
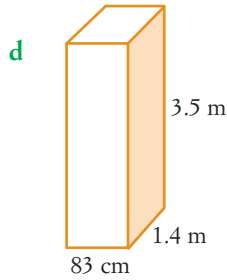
See Example 10

See Example 14

See Example 14

Fluency

See Example 14



- 6 Find the volumes of the following prisms using the measurements given.
- a** Area of base = 16 cm^2 and height = 4 cm
 - b** Area of base = 28 mm^2 and height = 9 mm
 - c** Area of base = 3.2 m^2 and height = 0.7 m
 - d** Area of base = 54 cm^2 and height = 13 cm
- 7 Find the volumes of cylinders with the following dimensions.
- a** Radius 8 cm and height 7 cm
 - b** Diameter 3.4 m and height 2.5 m
 - c** Diameter 2.5 m and height 3.4 m
 - d** Radius 2.5 m and height 3.4 m
- 8 Find the volumes of pyramids with the following measurements.
- a** Area of base = 21 cm^2 and height = 6 cm
 - b** Area of base = 26 mm^2 and height = 9 mm
 - c** Area of base = 5.4 m^2 and height = 1.2 m
 - d** Area of base = 504 m^2 and height = 23 m
- 9 Find the volumes of cones with the following dimensions.
- a** Radius 12 m and height 15 m
 - b** Diameter 5.1 cm and height 6.2 cm
 - c** Diameter 4.5 m and height 3.5 m
 - d** Radius 15 cm and height 21 cm

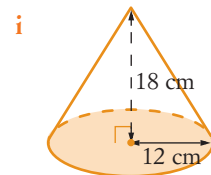
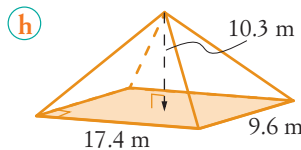
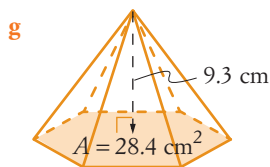
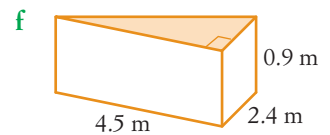
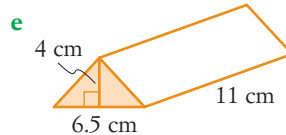
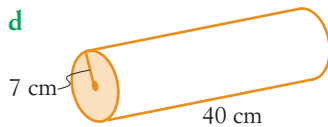
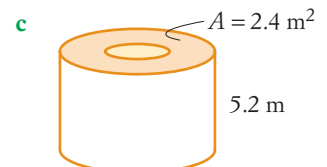
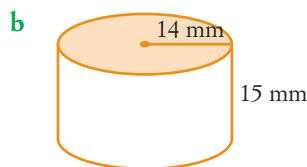
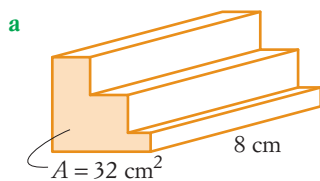
Worked solutions

Exercise 3.3

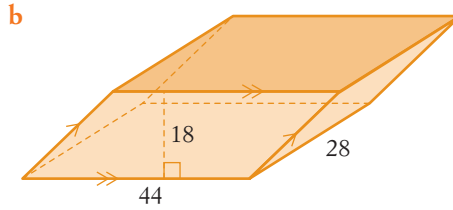
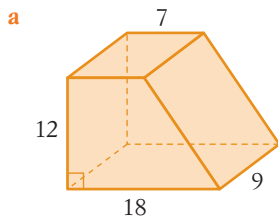
MAT09MGWS00010

See Example 11

- 10 Work out the volume of each of the following.

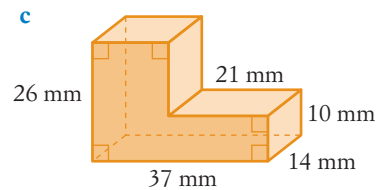
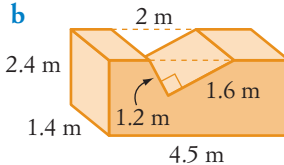
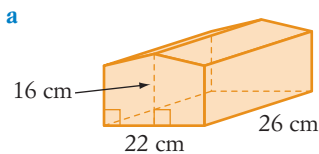


- 11 Calculate the volume of each of the prisms shown below. All measurements are in centimetres.

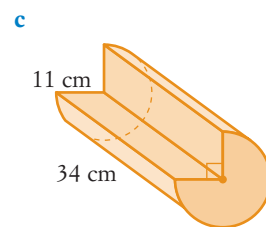
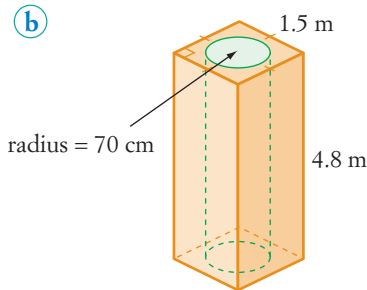
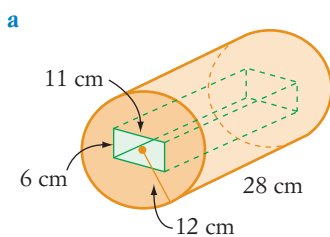


- 12 Calculate the volume of each of these solids.

See Example 12



- 13 Calculate the volume of each of the following solids.

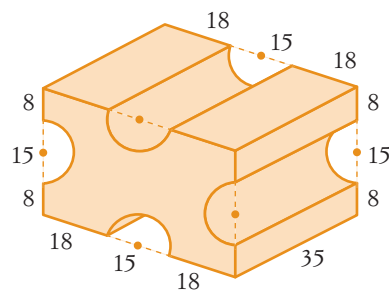


Worked solutions

Exercise 3.3

MAT09MGWS00010

- 14 All of the dimensions shown on this diagram are in centimetres. Calculate the volume of the solid.



- 15 Choose the most suitable unit to measure the capacity of:

a a shoe box	b a swimming pool	c a thimble
d a plastic milk container	e a garbage bin	f a hot-water system
g Moogerah Dam	h a basketball	i a refrigerator.

Problem solving

See Example 13

- 16 Jamal and Salama know that their Esky has a capacity of 48 L. Calculate its capacity in cubic centimetres.

- 17 Salim wanted to know the capacity of his wheelbarrow. He filled it up with water, measuring it as he filled. He found that it holds 258 L of water. What is its capacity in:

- a** cm^3 ? **b** m^3 ?

- 18 When she ordered concrete for the floor of her garden shed, Ming asked for 3.2 m^3 . What would this be in:

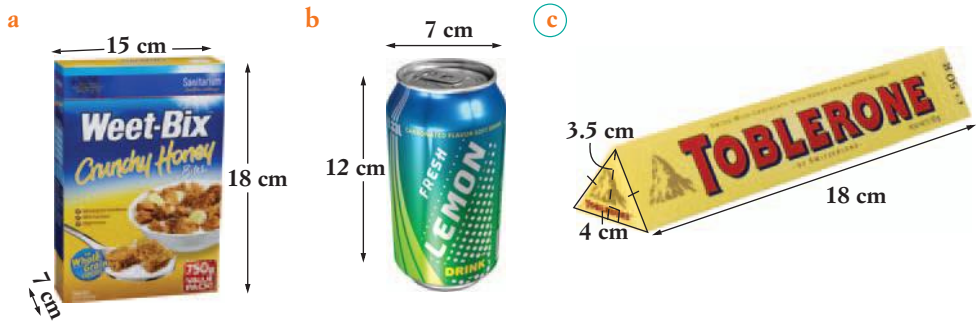
- a** kL? **b** L?

Worked solution

Exercise 3.3

MAT09MGWS00010

- 19 Find the volume of each of the following containers.



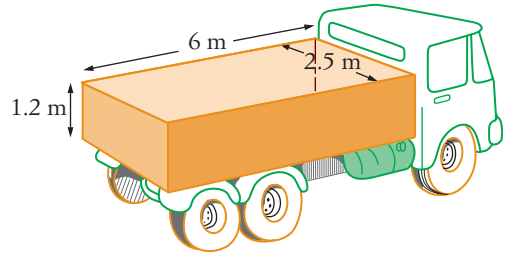
Reasoning

- 20 Cans of soft drink have a capacity of 375 mL. Soft drink is delivered to supermarkets on pallets holding 81 cartons. A carton contains 24 cans.

- a** Calculate the volume (in litres) of soft drink in a carton.
- b** Calculate the volume (in litres) on a pallet.

- 21 The driver of this gravel truck is paid \$13.50 for every cubic metre delivered.

- a** Find the volume of the truck's tray.
- b** How much would the driver be paid for delivering half a load?
- c** What would the driver be paid for a day's work if he delivered five full loads?

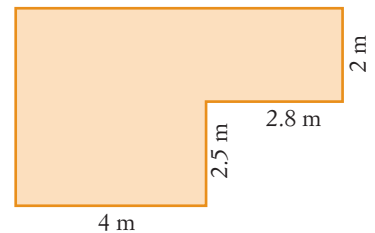


- 22 A bale of straw is in the shape of a cube with a side of 50 cm. A number of these bales are stored in a flat-topped shed 12 m long, 8.5 m wide and 3 m high.

- a** Find the volume of one bale, in cubic metres.
- b** What is the volume of the shed?
- c** How many bales can be stored in the shed?

- 23 Gemma and Chang want to build a garden shed with this floor plan. The first step is to lay a concrete slab 10 cm thick.

- a** What volume of concrete will they require?
- b** If they have the concrete delivered, it will cost \$309/m³. How much will it cost for the concrete in the slab?



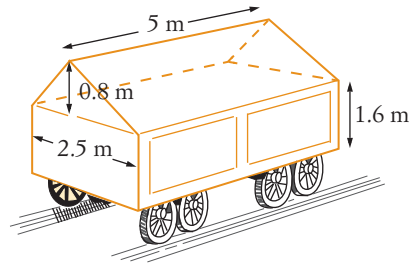
- 24 A farm tank holds the diesel for the vehicles on a farm. The tank is cylindrical, with a length of 4.2 m and a diameter of 2 m.

- a** Find the volume of the tank.
- b** How many litres of fuel does the tank hold when full?

- 25 A waste heap at a sawmill is in the shape of a cone with a diameter of 12 m and a height of 5 m. Wet sawdust is dangerous, so the pile must be moved. The wet sawdust weighs 14 kg/L. What mass of sawdust must be moved?

26 This railway wagon is used for transporting bulk wheat.

- What volume of wheat is this wagon able to carry when completely full?
- If 750 m^3 of wheat has to be moved, how many of these wagons will be needed?



27 A wheat silo is in the shape of a large concrete cylinder with internal diameter 3.6 m and height 13 m.

- What volume of wheat can the silo hold (to the nearest cubic metre)?
- During a wet spell, the empty silo became three-fifths full of water. What was the volume of water?
- How many litres of water were in the silo?
- If 1 L of water has a mass of 1 kg, what mass of water (in tonnes) was in the silo?

3.4 Solving time problems

Many different units are used to measure time intervals. The main one is the **day** – the time taken for the Earth to rotate once on its axis.

Important!

Units of time

1 century = 100 years (y)	1 day = 24 hours (h)
1 decade = 10 years	1 hour = 60 minutes (min)
1 year = 12 months $\approx$ 52 weeks (wk)	1 minute = 60 seconds (s)
1 year = 365 days (d)	1 millisecond = $0.001 \text{ s} = 1 \times 10^{-3} \text{ s}$
1 leap year = 366 days	1 microsecond = $0.000\,001 \text{ s} = 1 \times 10^{-6} \text{ s}$
1 fortnight = 2 weeks	1 nanosecond = $0.000\,000\,001 \text{ s} = 1 \times 10^{-9} \text{ s}$
1 week = 7 days	

The number of days in each month can be remembered using the following rhyme:

*Thirty days has September, April, June and November.
All the rest have 31, except February alone
Which has 28 days clear
And 29 days each leap year.*

You should know the following abbreviations.

a.m. (*ante meridiem*) – before noon

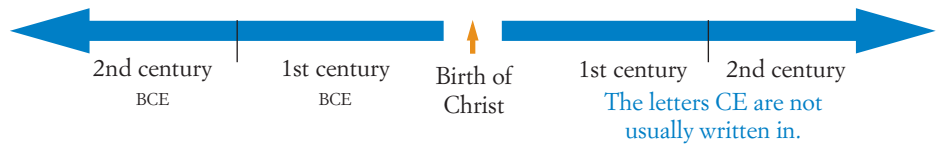
p.m. (*post meridiem*) – after noon

BCE (before the Common Era) – used for dates before the birth of Christ

CE (during the Common Era) – used for dates after the birth of Christ

Some people prefer the abbreviations BC (before Christ) and AD (*Anno Domini*) instead of BCE and CE.

Years are grouped into centuries in the following way.



Note that there is no 'year 0'.

Investigate: Wait a minute

You've probably heard someone use the phrase 'Wait a minute!' A minute doesn't sound long, but you may be surprised just how long it is when you are concentrating on it. Work in pairs or a group of three for this investigation. *You will need a stopwatch, a watch with a sweep hand or some other suitable timing device.*

- 1 Nominate one person as the timer and another as guesser. A third person may be the recorder.
- 2 On the guesser's command, the timer starts the stopwatch. When the guesser thinks 1 minute has passed, he or she tells the timer to stop.
- 3 Record the actual time that has passed, next to 'Guess 1' in a table like the one below.
- 4 Now repeat the procedure three more times, recording the guess after each attempt.

Guess	Number of seconds over or under 1 min.	% error
Guess 1		
Guess 2		
Guess 3		
Guess 4		

- 5 Complete the '% error' column.
- 6 Is there any trend you can see? Comment.

Variation

This variation that can be done with a larger group. The timer says 'Go', starts the stopwatch and then says 'Stop' as he or she stops timing. The others have to guess the elapsed time. The closest guess wins.



When calculating time intervals between given dates, be careful of whether you need to include the given dates. 'From . . . to . . .' generally means include the second date but not the first. 'Inclusive' means include both dates.

Example 16

Calculate the time interval from:

- a** 5:40 p.m. Monday to 4:20 a.m. Tuesday
- b** 25 January to 15 April in the same non-leap year
- c** 11/5/11 to 17/3/12
- d** 45 BCE to 1066 CE inclusive.

Solution

- a** Break the time interval into the revolutions of the clock. The first part is from 5:40 p.m. Monday to midnight. To subtract the minutes, change 12:00 a.m. to 11:60.

$$\begin{array}{r} 11 : 60 \\ - 5 : 40 \\ \hline 6 : 20 \end{array}$$

Calculate the second interval.

Calculate the total.

Write the answer.

Midnight to 4:20 a.m. Tuesday = **4 h 20 min**

$$\begin{aligned} \text{Total time} &= 6 \text{ h } 20 \text{ min} + 4 \text{ h } 20 \text{ min} \\ &= 10 \text{ h } 40 \text{ min} \end{aligned}$$

It is 10 hours 40 minutes from 5:40 p.m. Monday to 4:20 a.m. Tuesday.

- b** Work out the days in January, *not* including 25 January.
Add the days of the months between.
Work out the days in April, including 15 April.
Find the total.
Write the answer.

$$\text{January days} = 31 - 25 = \mathbf{6d}$$

$$\text{February} + \text{March} = 28 + 31 = \mathbf{59 d}$$

$$\text{April days} = \mathbf{15 d}$$

$$\text{Total time} = 6 + 59 + 15 = 80 \text{ d}$$

There are 80 days from 25 January to 15 April in the same non-leap year.

- c** Work out the days in May 2011, *not* including 11 May.
Add the days for the rest of 2011 and the first two months of 2012.

$$\text{May 2011 days} = 31 - 11 = \mathbf{20 d}$$

June 2011 to February 2012 inclusive

$$= 30 + 31 + 31 + 30 + 31 + 30 + 31 + 31 + 29$$

$$= \mathbf{274 d}$$

Work out the days in March 2012, including 17 March.

Find the total.

Write the answer.

$$\text{March 2012 days} = \mathbf{17 d}$$

$$\text{Total time} = 20 + 274 + 17 = 311 \text{ d}$$

There are 311 days from 11/5/11 to 17/3/12.

- d** Add the BCE and CE years, including both 45 BCE and 1066 CE.

Write the answer.

$$\text{Total time} = 45 + 1066 = 1111 \text{ y}$$

It is 1111 years from 45 BCE to 1066 CE inclusive.

Some times are stated in decimal form.

Example 17

Convert the following time intervals to non-decimal form.

a 7.75 h

b 3.2 days

Solution

a Convert the decimal part (1 h = 60 min).

$$0.75 \text{ h} = 0.75 \times 60 \text{ min} = 45 \text{ min}$$

State the result.

$$7.75 \text{ h} = 7 \text{ h } 45 \text{ min}$$

b Convert the decimal part (1 d = 24 h).

$$0.2 \text{ d} = 0.2 \times 24 \text{ h} \\ = 4.8 \text{ h}$$

Convert the decimal part.

$$0.8 \text{ h} = 0.8 \times 60 \text{ min} \\ = 48 \text{ min}$$

State the result.

$$3.2 \text{ d} = 3 \text{ d } 4 \text{ h } 48 \text{ min}$$

Times stated as a.m. or p.m. are **12-hour times**. It is sometimes possible to confuse a.m. and p.m. times. To overcome this problem, **24-hour time** is used. 24-hour time is widely used in the military and in aviation but is also used in most programmable devices such as microwaves and DVDs.

Important!

24-hour time

24-hour times use 4 digits and have an 'h' written at the end. The first two digits show the number of hours after midnight, and the last two digits show the number of minutes after the hour. We say 'hours' at the end of 24-hour times.

So 1:20 p.m. is 1320 h and is said 'thirteen-twenty hours' and 7:00 a.m. is 0700 h and is said 'oh-seven-hundred hours'.

Example 18

Convert:

a 7:38 p.m. to 24-hour time

b 0313 h to 12-hour time.

Solution

a It is a p.m. time, so add 1200 h.

$$7:38 \text{ p.m.} = 1938 \text{ h}$$

b It is before 1200 h, so just separate to a.m. time.

$$0313 \text{ h} = 3:13 \text{ a.m.}$$

Example 19

Use 24-hour time to construct a time line for the following events at a school camp.

- 4:00 p.m. Free time
- 12:40 p.m. Lunch
- 10:20 a.m. Swimming
- 5:45 p.m. Pre-dinner shower
- 10:00 a.m. Morning tea
- 7:30 a.m. Breakfast
- 1:50 p.m. Mini Olympics
- 8:10 a.m. Bush walk
- 7:15 a.m. Get up



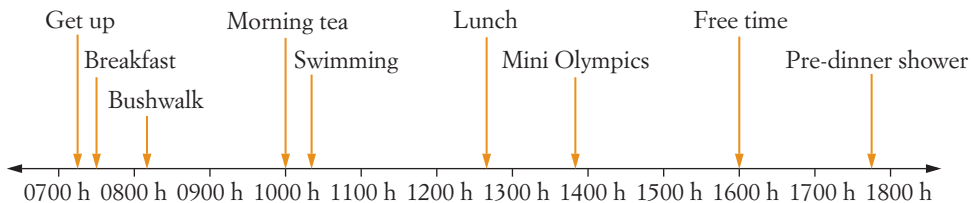
Solution

Convert to 24-hour times.

- 1600 h Free time
- 0730 h Breakfast
- 1240 h Lunch
- 1350 h Mini Olympics
- 1020 h Swimming
- 0810 h Bush walk
- 1745 h Pre-dinner shower
- 0715 h Get up
- 1000 h Morning tea

Choose a scale for the time line.

Put the events in order on the time line.



Investigate: Very large and very small time intervals

Large time intervals

Did you know that the Earth is about 4.6 billion years old? That's 4 600 000 000 years!

This investigation will help you understand just how long 4.6 billion years really is.

You will need a roll of toilet paper and access to a very long hallway or verandah and some sticky notes to complete the investigation. Try to use a double length roll of toilet paper; the more sheets the better.

- 1 Unroll the full roll of toilet paper down the hallway. Double rolls have about 400 sheets, and each sheet is about 11 cm long.
- 2 For the purposes of this investigation we will assume that the Earth is about 5 billion years old.
- 3 If the unrolled toilet paper represents the age of the Earth, work out the period of time represented by 1 sheet.
- 4 Locate the following important events in the Earth's history and place sticky notes on the unrolled sheets.
 - 4.5 billion years ago: Moon is formed from the collision of Earth with the planet Theia
 - 3.7 billion years ago: First cells appear
 - 3.5 billion years ago: Life first appears in the oceans
 - 3 billion years ago: First photosynthesising cells appear, oxygen starts to be produced
 - 2.4 billion years ago: Oceans contain significant amounts of oxygen
 - 1.2 billion years ago: Sexual reproduction starts, increasing rate of evolution
 - 716 million years ago: Snowball Earth – glaciers cover the whole Earth
 - 570 million years ago: First arthropods - insect-like creatures
 - 300 million years ago: Reptiles first appear
 - 200 million years ago: Mammals first appear
 - 150 million years ago: Continents begin drifting apart
 - 65 million years ago: Age of dinosaurs ends
 - 3.5 million years ago: The first ancestors of humans appear
 - 25 000 years ago: Last Neanderthals die out
 - 10 000 years ago: Recorded human history begins
- 5 Estimate the fraction of the Earth's history represented by:
 - the period since the first simple animals appeared
 - the period of time it took for photosynthesis to begin in the oceans
 - the period of time that humans have been on Earth



Small time intervals

Scientists are also interested in very small time periods.

The smallest commonly used unit of time in the metric system is the nanosecond (ns).

$$1 \text{ nanosecond} = 0.000\,000\,001 \text{ s} = 1 \times 10^{-9} \text{ s}$$

Times of this magnitude are commonly encountered in telecommunications, pulsed lasers and some areas of electronics.

Just as very large time intervals are difficult to visualise, so too are very small time intervals like the nanosecond. The following will help you to get some understanding of small time intervals.

A light year is a unit of length; it is equal to the distance that light travels in a year. The speed of light is about 300 000 km/s.

- 1 Calculate the distance that light travels in a year.
- 2 Calculate how long it would take light to travel 1 m.
- 3 How far would light travel in a nanosecond?
- 4 Find out about a unit of time called an attosecond.
- 5 Find the smallest time interval that has been directly measured.

Time zones

If you have seen a live news report or a sporting event from overseas, you would know something about **time zones**. The world is divided into 24 different time zones, with the reference point being Greenwich, near London.



The map on page 121 shows that as you move *east* of Greenwich, an hour is *added* for each new time zone. As you move *west* of Greenwich, an hour is *subtracted* for each new time zone. The **International Date Line** shows where the old day finishes and the new day begins.

The map on page 121 has been greatly simplified and does not show the many minor variations due to the borders of states and countries. If you look at the maps on page 120, you can see some examples of these minor variations in Australia.

Important!

Australian time zones

Australia is divided into three time zones along state and territory borders:

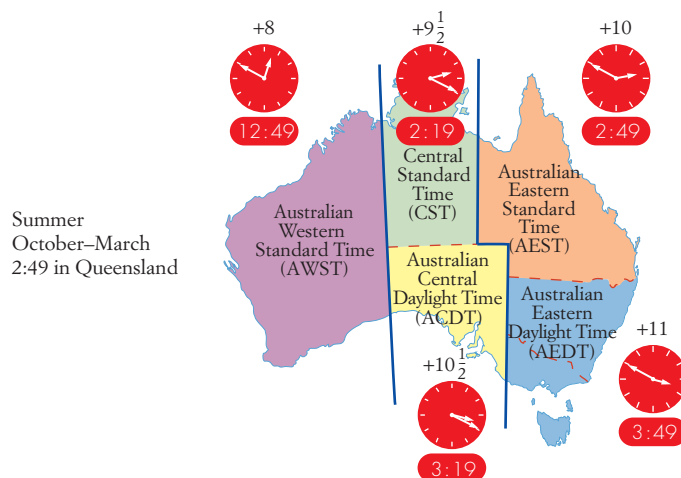
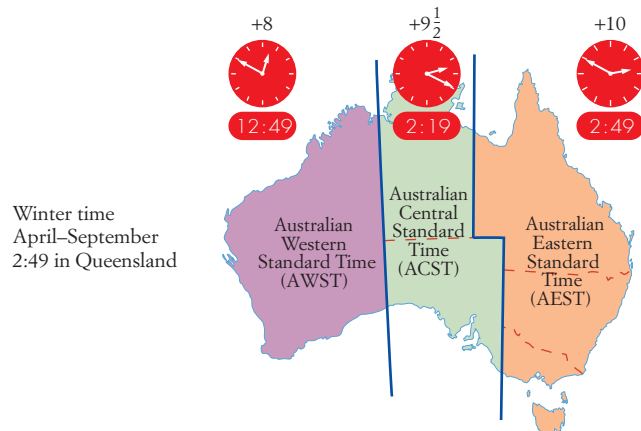
- **Australian Eastern Standard Time (AEST)** applies to Queensland, New South Wales, the Australian Capital Territory, Victoria and Tasmania.
- **Australian Central Standard Time (ACST)** applies to the Northern Territory and South Australia.
- **Australian Western Standard Time (AWST)** applies to Western Australia.

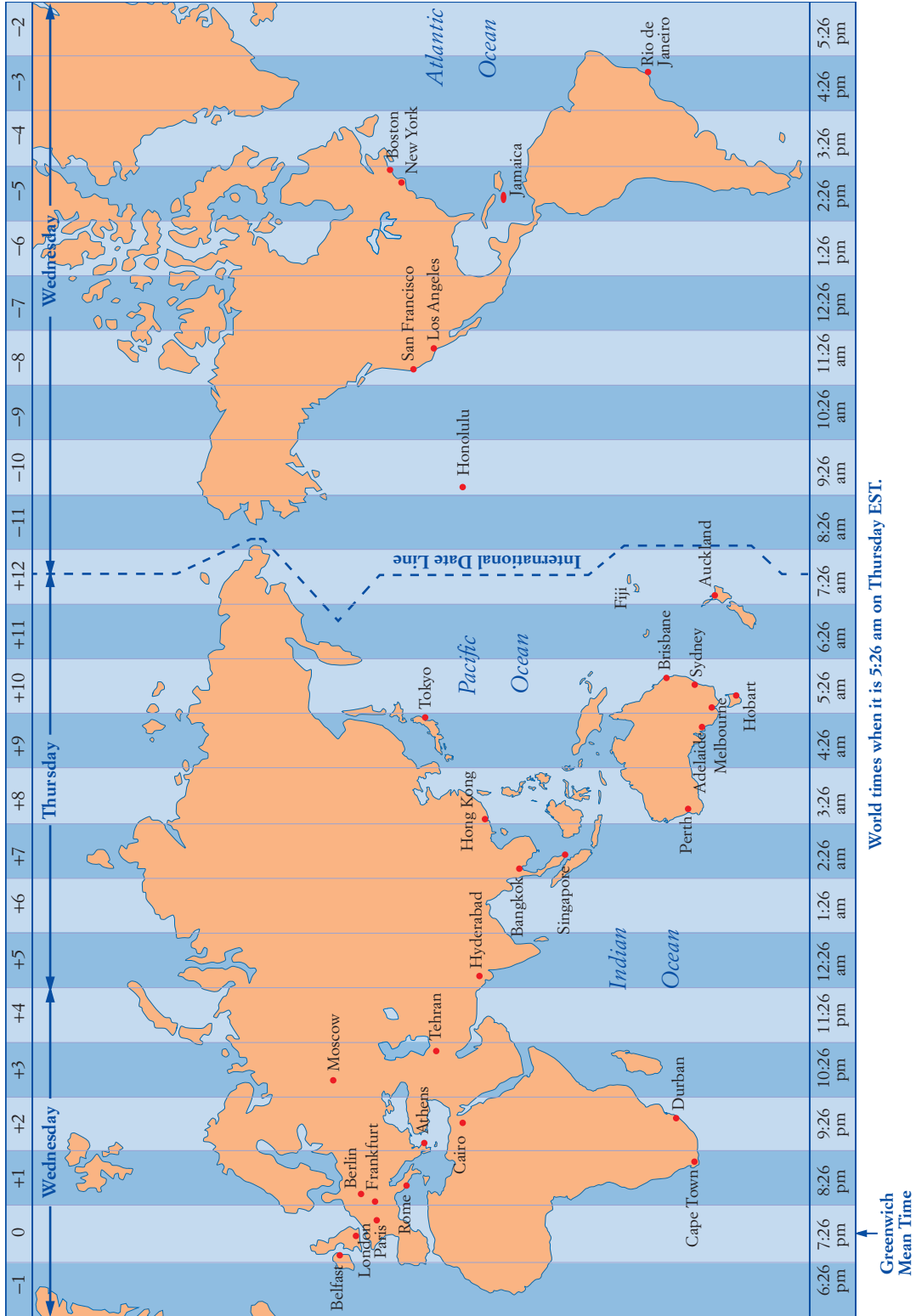
AEST is 30 minutes ahead of ACST and 2 hours ahead of AWST.

During summer, some states move their clocks 1 hour forward. This is called daylight saving. **Daylight saving** begins on the first Sunday in October and ends on the first Saturday in April.

During daylight saving, New South Wales, the Australian Capital Territory, Victoria and Tasmania move from AEST to **Australian Eastern Daylight Time (AEDT)**, while South Australia moves from ACST to **Australian Central Daylight Time (ACDT)**.

Daylight Saving Time is not observed in Queensland, the Northern Territory or Western Australia.





Example 20

Jasmine flew from Cairns to Adelaide via Brisbane to visit relatives for Christmas. She left Cairns at 5:35 a.m. The trip took 5 h 30 min. What time was it in Adelaide when she arrived?

Solution

Find the arrival time in Adelaide.

$$\text{Arrival time} = 5:35 \text{ a.m.} + 5 \text{ h } 30 \text{ min}$$

Daylight saving is in place in December.

Adelaide is 30 minutes ahead of Brisbane during daylight saving.

$$= 11:05 \text{ a.m. Brisbane time}$$

$$= 11:35 \text{ a.m. Adelaide time}$$

Write the answer.

Jasmine arrived in Adelaide at 11:35 a.m.

Investigate: Time zones

Some of Australia's Asian neighbours include:

Cambodia	Vietnam	East Timor
China	Papua New Guinea	Singapore
Indonesia	Brunei	Sri Lanka
South Korea	Japan	Burma
Hong Kong	Malaysia	Philippines
Thailand	Laos	Taiwan

- Locate each of the places listed above on a map.
- Compare the positions of these places with the eastern, central and western states of Australia.
- Use this information to group the places into those with time zones the same as AEST, ACST and AWST.
- Use the internet or some other source to compare the actual time zones of the places.
- Explain the possible differences between the actual time zones and those you calculated.

Exercise 3.4 Solving time problems

You will need to refer to the time zone maps on pages 120 and 121 to answer some of the questions in this exercise. The following map will also be useful.



Understanding

Extra questions

Exercise 3.4

MAT09MGEQ00011

- How many seconds are there in each of these time intervals?

a 2 min	b 10 min	c 3 h	d $4\frac{1}{2}$ min
e $1\frac{1}{4}$ min	f $3\frac{3}{4}$ min	g $4\frac{1}{4}$ h	h 1 d
- How many minutes are there in each of these time intervals?

a 2 h	b 12 h	c 120 s	d 450 s
e $2\frac{1}{2}$ h	f 30 s	g $1\frac{1}{2}$ d	h $10\frac{1}{4}$ d
- Calculate the time interval from:

a 7:45 p.m. today to 4:50 a.m. tomorrow	
b 3:46 a.m. today to 7:48 p.m. tomorrow	
c 11:42 a.m. Monday to 7:37 a.m. Wednesday	
d 9:21 p.m. Friday to 5:29 p.m. Monday	
e 10:13 a.m. Saturday to 11:43 a.m. Tuesday.	
- Refer to this year's calendar and find the date:

a 2 weeks before Christmas Day	b 18 days after Boxing Day
c 2 weeks before 17 April	d 21 days after 12 November
e 3 weeks and 4 days after Anzac Day	f 7 weeks and 5 days before Australia Day.
- Change these 12-hour times to 24-hour time.

a 7:30 a.m.	b 5:12 p.m.
c 11:50 a.m.	d 11:50 p.m.
e half past ten at night	f a quarter to six in the morning
g six-thirty in the evening	h ten to eight in the morning
i 10:12 p.m.	j 1:34 a.m.

See Example 16

See Example 16

See Example 18

6 Convert these 24-hour times to 12-hour times.

a 0630 h	b 1440 h	c 1035 h
d 1920 h	e 1110 h	f 0910 h
g 2324 h	h oh-five-twenty hours	i fourteen hundred hours
j twenty-one-twelve hours	k oh-nine-fifty-six hours	l eleven-twenty-three hours

Fluency

7 Change each of these time intervals to the unit indicated in brackets.

a 35 days (h)	b 3 years (d)	c 5 decades (y)
d 86 days (wk)	e 1074 years (centuries)	f 5 days (s)
g 325 years (decades)	h 588 hours (wk)	i 15 300 seconds (h)

See Example 16

8 Find the time interval from:

a 12 May to 18 September	b 3 April to 15 October inclusive
c 2 November to 28 December	d 11/9/68 to 15/6/03
e February 2007 to October 2014	f 243 BCE to 1456 CE inclusive
g 156 BCE to 682 BCE	h 1 April 1995 to 1 November 2001
i 7/10/99 to 15/12/05	

9 In which century is each of these years?

a 452 BC	b 1830	c 673 BCE
d 1567 BC	e 2013	f 1489 CE

See Example 17

10 Convert the following time intervals to non-decimal form.

a 3.5 h	b 6.25 h	c 33.2 min
d 6.4 d	e 5.45 min	f 22.4 decades
g 9.6 h	h 5.34 h	i 5.15 d

11 If it is 10:30 a.m. in Brisbane in July, what time is it in:

a Sydney?	b Adelaide?	c Marble Bar?
d Darwin?	e Hobart?	f Cairns?

12 If it is 8:45 p.m. in Sydney in January, what time is it in:

a Brisbane?	b Oodnadatta?	c Perth?
d Alice Springs?	e Broken Hill?	f Launceston?

Worked solution

Exercise 3.4

MAT09MGWS00011

See Example 20

13 What time is it in winter in:

- a** Perth if it is 4:00 a.m. in Brisbane?
b Mt Isa if it is noon in Sydney?
c Adelaide if it is 1530 h in Melbourne?
d Broome if it is midnight in Tennant Creek?
e Cairns if the 6:00 p.m. news has just started in Whyalla?
f Alice Springs if it is 0540 h in Weipa?
g Brisbane if it is five-thirty in the evening in Adelaide?

Problem solving

14 The time in London is 10 hours behind the time in Brisbane. What time is it in London when it is:

a 2:00 p.m. in Brisbane?	b 8:00 a.m. in Brisbane?
c 11:00 a.m. in Brisbane?	d 6:15 p.m. in Brisbane?

Worked solution

Exercise 3.4

MAT09MGWS00011

15 What time is it in Brisbane when it is:

- a 5:00 a.m. in London? b 2:00 p.m. in London?
c 10:00 p.m. in London? d 12:30 a.m. in London?

16 Copy the following table and complete the 'Suitable unit' column first using h, min, s, etc. Then estimate the time taken for each event. Now discuss your estimates with others or do some research to work out reasonable times, and complete the table. There is no one correct answer for any of these events.

Time taken for:	Suitable unit	Estimate of time taken	Reasonable time taken
A direct flight from Brisbane to Melbourne			
An Olympic 100 m dash			
You to walk 4 km			
A drive from Brisbane to Rockhampton			
A typist to type 500 words			
A glacier to move 10 m			
A hailstone to fall 1000 m			
A cricket ball to reach a batsman			
The erosion of the Grand Canyon			
A student to clean a blackboard			
You to ride to school on a bike			

17 Construct a time line to show the following events in Australia's history.

See Example 19

- 1893 First peoples' convention held
- 1854 Eureka Stockade
- 1900 Commonwealth of Australia proclaimed
- 1896 Sir Henry Parkes dies
- 1883 Federal Council of Australasia formed
- 1859 Queensland separates from NSW
- 1846 Federal Government first suggested
- 1851 Victoria separates from NSW
- 1891 First Constitution drafted

18 Construct a time line to show the following events in the history of mathematics:

See Example 19

1629	Pierre de Fermat establishes differential calculus.
530 BCE	Pythagoras discovers that $\sqrt{2}$ is irrational.
750	Al-Khawarizmi lays the foundations for modern algebra.
260 BCE	Archimedes computes π using geometry.
550	Hindu mathematicians give zero a numerical representation in the number system.
1736	Leonhard Euler solves the Königsberg bridge problem.
1614	John Napier establishes logarithms.
140 BCE	Hipparchus develops the basis for trigonometry.
250	Diophantus uses symbols for unknown numbers.
200 BCE	Eratosthenes uses the sieve algorithm to isolate prime numbers.

Worked solution

Exercise 3.4

MAT09MGWS00011

- 19 A fast printer can print 800 pages in a minute. Estimate the times taken to print one word and a single letter.
- 20 The date 17 May 1985 was special because, if it is written in the form 17/5/85, we find that $17 \times 5 = 85$ or $\text{day} \times \text{month} = \text{year}$
- Did any other dates that year have this property?
 - Do any dates this year have this property?
- 21 Find the sunrise and sunset times for your area. Calculate the period of daylight for the current time of the season. Use the Internet to contact the weather bureau to find the same information for other seasons. Is the period of daylight different? Comment.
- 22 A Qantas flight schedule for a flight from Cairns to Adelaide and return is shown below. All times shown are local.
- What is the flying time between Cairns and Brisbane?
 - What is the flying time between Brisbane and Adelaide?
 - What is the flying time between Adelaide and Melbourne?
 - Compare the durations of the forward and return journeys.
 - Compare the total flying times for the forward and return journeys.

See Example 20

QANTAS
Spirit of Australia

1. Search 2. Select Flights 3. Review 4. Passengers 5. Payment 6. Confirmation

Step 3 - Review your flights

Your Search

From: Cairns
To: Adelaide
Depart: Sep 11 Mon 05
Return: Sep 11 Fri 16
Adults: 1
Children: 0
Infants: 0
Search Options: Must Travel
Go

Your Flights

Flight Out	From	To	Flight	Departure Terminal	Fare Type	Baggage
Mon 5 Sep 11 10:05	Cairns	12:05 Brisbane	QF709	Domestic	Super Saver	Checked: 1 piece
15:45 Brisbane	18:05 Adelaide	QF643	Domestic			
Flight Back						
Fri 16 Sep 11 11:40	Adelaide	13:25 Melbourne	QF602	1	Red e-Deal	Checked: 1 piece
16:20 Melbourne	19:45 Cairns	JQ348	1	JetSaver	Checked: 20kg	

Key: QF = Qantas flight JQ = Flight Operated by Jetstar

Notes: Qantas flights QF1-QF399 depart from international terminals. All other Qantas flights depart from domestic terminals. Jetstar flights JQ1-JQ399 depart from international terminals, except JQ61, JQ73 and JQ81 which depart from domestic terminals. All other Jetstar flights depart from domestic terminals. Food, beverages and entertainment are not included on Jetstar services unless you book international StarClass. Items may be purchased inflight if available, and pre-purchased on international flights (excluding New Zealand). On Jetstar international services, amenity kits (blanket, neck support, eyeshades and socks) may also be purchased.

Change Flights

- 23 A teleconference is being arranged for 2:15 p.m. on 20 October, Brisbane time. The participants are located in Perth, Alice Springs, Melbourne and Hobart. At what local time should each be advised to expect the call?
- 24 A flight from Townsville to Adelaide takes 5 h 5 min, including a stopover in Brisbane. At what times do flights departing Townsville at 10:25 a.m. arrive in Adelaide in June and December?

- The **area** of a flat shape is the space enclosed by the boundary of the shape and is measured by working out how many squares are needed to cover the shape.
- Small areas are measured in **square millimetres (mm²)** or **square centimetres (cm²)**, and large areas in **square metres (m²)**, **hectares (ha)** or **square kilometres (km²)**.

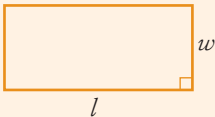
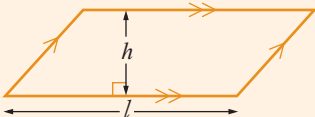
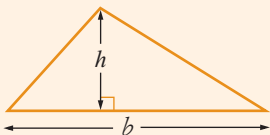
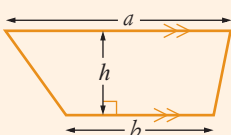
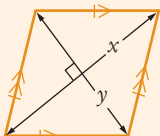
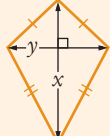
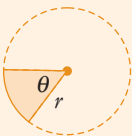
$$1 \text{ cm}^2 = 100 \text{ mm}^2$$

$$1 \text{ m}^2 = 10\,000 \text{ cm}^2 = 1\,000\,000 \text{ mm}^2$$

$$1 \text{ ha} = 10\,000 \text{ m}^2$$

$$1 \text{ km}^2 = 100 \text{ ha} = 1\,000\,000 \text{ m}^2$$

- Areas of regular shapes may be calculated using the following formulas.

Rectangle $A = l \times w$ = length $\times$ width 	Parallelogram $A = l \times h$ = length $\times$ height 
Triangle $A = \frac{b \times h}{2}$ = half(base $\times$ height) 	Trapezium (trapezoid) $A = \frac{(a + b)h}{2}$ = average of parallel sides $\times$ height 
Rhombus  $A = \frac{1}{2}xy$ = $\frac{1}{2} \times$ (product of diagonals)	Kite 
Circle $A = \pi r^2$, where $\pi \approx 3.14$ = $\pi \times$ radius $\times$ radius 	Sector of a circle If θ is the sector angle, then the fraction of the circle = $\frac{\theta}{360}$ and the area of the sector, $A = \frac{\theta}{360} \times \pi r^2$ 

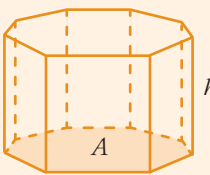
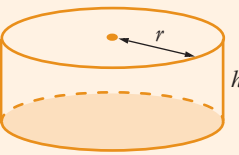
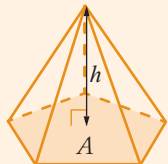
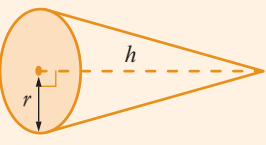
Quiz

Measurement

MAT09MGQZ00003

Chapter 3 summary

- The areas of **combined shapes** are found by dividing them into other shapes.
- A **net** of a 3D shape is a 2D shape that can be folded up to make the shape.
- The **surface area** of a 3D shape is the total area of all the faces. It is calculated by adding the areas of the faces.
- The **curved surface area** of a cylinder, without top and bottom, is given by $CSA = 2\pi rh$.
- The **volume** of a 3D object measures the amount of space it occupies. It is calculated by working out how many cubes of unit size would be needed to fill the same space.
- Volume is measured in **cubic** units such as **cubic centimetres** (cm^3 or **cc**) or **cubic metres** (m^3).
- The basic volume formulas are shown below.

 Prism $V = Ah$	 Cylinder $V = \pi r^2 h$
 Pyramid $V = \frac{1}{3}Ah = \frac{Ah}{3}$	 Cone $V = \frac{1}{3}\pi r^2 h = \frac{\pi r^2 h}{3}$

- The **capacity** of a container means the space inside.
- Capacity is normally measured in **millilitres (mL)**, **litres (L)**, **kilolitres (kL)** or **megalitres (ML)**.

1 L = 1000 mL
1 kL = 1000 L
1 ML = 1000 kL = 1 000 000 L
- The metric system was designed so that:

1 cm^3 = 1 mL
and 1 m^3 = 1000 L = 1 kL
- The standard unit of **time** in the metric system is the **second (s)**. Years before the birth of Christ have **BC** or **BCE** (before the Common Era) at the end. Years after the birth of Christ may be shown by **AD** or **CE** (during the Common Era).

1 century = 100 years (y)

1 decade = 10 years

1 year $\approx$ 52 weeks (wk)

1 year = 365 days

1 leap year = 366 days

1 fortnight = 2 weeks

1 week = 7 days (d)

1 day = 24 hours (h)

1 hour = 60 minutes (min)

1 minute = 60 seconds (s)
- **12-hour** times use a.m. and p.m. **24-hour** times use four digits and have an 'h' at the end.

- There are 24 time zones in the world. **Greenwich Mean Time (GMT)** is the reference for international times. Australia has three time zones: **Australian Eastern Standard Time (AEST)**, **Australian Central Standard Time (ACST)** and **Australian Western Standard Time (AWST)**.
- During summer, some states move their clocks 1 hour forward. This is called **daylight saving**. Daylight saving begins on the first Sunday in October and ends on the first Saturday in April. During daylight saving NSW, the ACT, Vic. and Tas. move from AEST to **Australian Eastern Daylight Time (AEDT)**, while SA moves from ACST to **Australian Central Daylight Time (ACDT)**.

Chapter 3 review

Understanding

See Example 2

- 1 Convert each of the following measurements to the unit shown in brackets.

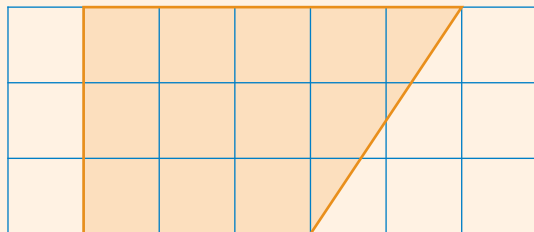
a 6000 cm^2 (m^2)

b 16 cm^2 (mm^2)

c 5 m^2 (cm^2)

See Example 3

- 2 Find the area of the shaded shape, drawn on a centimetre square grid.



See Example 2

- 3 Convert each of the following measurements to the unit shown in brackets.

a $600\,000 \text{ m}^2$ (ha)

b 4.2 km^2 (ha)

c 5 ha (m^2)

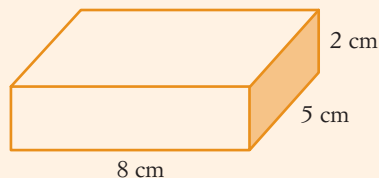
See Example 10

- 4 Find the volumes of the following shapes, where each small block is 1 cm^3 .

a



b



See Example 14

- 5 Convert each of the following measurements to the unit shown in brackets.

a $600\,000 \text{ cm}^3$ (m^3)

b 4.2 L (mL)

c 5 m^3 (L)

See Example 14

- 6 Convert each of the following measurements to the unit shown in brackets.

a 66 kL (cm^3)

b 250 m^3 (ML)

c 0.073 m^3 (kL)

- 7 Change each of the following time intervals to the unit shown in brackets.

a 150 seconds (min)

b $2 \text{ weeks } 5 \text{ days}$ (d)

c 75 years (decades)

- 8 Change to 24-hour time.

a $8:15 \text{ a.m.}$

b $9:32 \text{ p.m.}$

- 9 Change to 12-hour time.

a 2020 h

b 0743 h

c $\text{ten-forty-three hours}$

Fluency

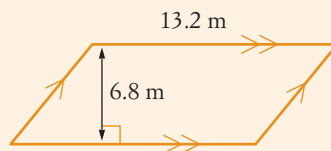
See Example 4

- 10 Find the area of each of the following.

a a rectangle 5.2 m by 6.4 m

c a square of side 6.3 cm

b

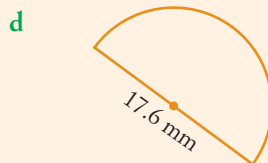
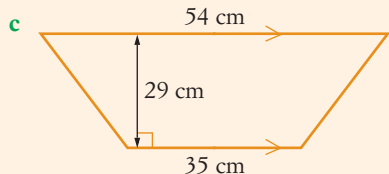


11 Find the area of each of the following.

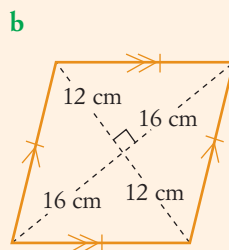
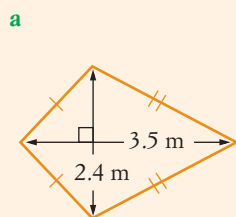
See Example 4

a Triangle with base 5.6 m and height 6.5 m

b Circle of radius 7.4 cm



12 Find the area of each of the following.

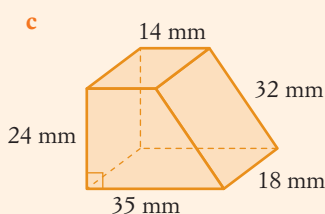
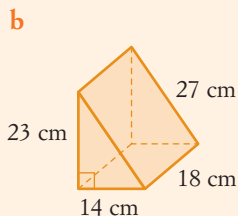
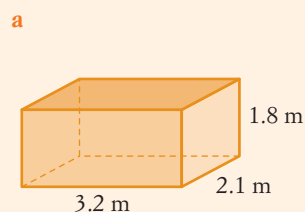


13 Find the surface area of a rectangular container that is 5.6 m long, 3.1 m high and 2.4 m wide.

14 A cylinder 2.6 m in radius and 4.7 m high is closed at one end. What is the surface area of the cylinder?

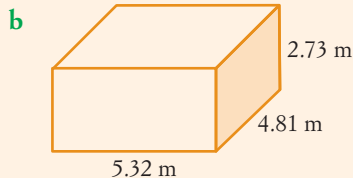
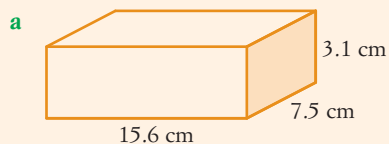
See Example 8

15 Find the surface area of each of the following solids.



16 Work out the volume of each of the following.

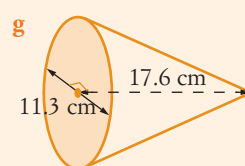
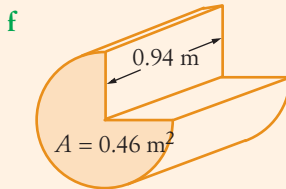
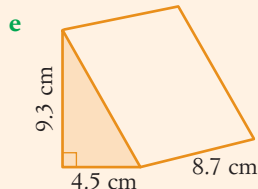
See Example 11



Chapter 3 review

See Example 11

- 17 Find the volume of each of the following.
- a Prism with base area 25 cm^2 and height 6 cm
 - b Pyramid with base area 24 m^2 and height 3.4 m
 - c Cylinder with radius 2.7 m and height 6.2 m
 - d Cone with diameter 12.7 cm and height 7.6 cm

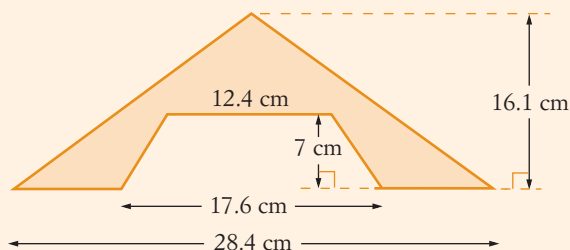


See Example 16

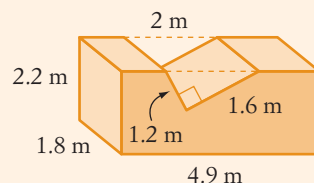
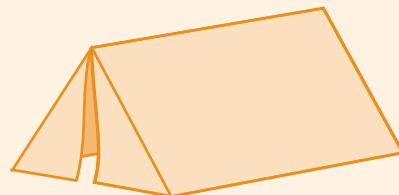
- 18 Find the time interval from:
- a 7:30 a.m. Monday to 3:15 p.m. Tuesday
 - b 23 March to 12 May
 - c 6/12/04 to 5/3/05 inclusive
 - d 318 BCE to 26 BCE
- 19 It is 2:35 p.m. in winter in Albany, Western Australia. What time is it in Townsville, Queensland? (Use the maps on pages 120 and 121 for help.)

Problem solving

- 20 What is the most appropriate unit to measure the area of:
- a a football field?
 - b a page in a book?
- 21 Find the area of the shape shown below.

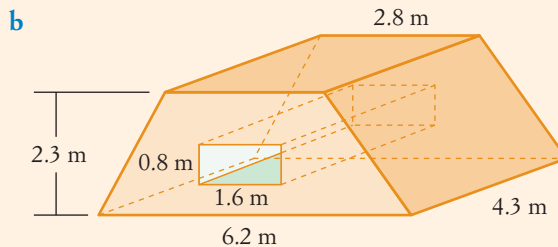
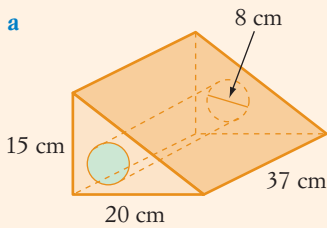


- 22 A cylindrical container is 20 cm in diameter and 25 cm high. The label wraps right around the container, but does not cover the top or bottom. What area must be printed on the label if the overlap is not printed?
- 23 A triangular tent (with no floor) is 2.6 m long and 1.6 m wide at the base, and tapers to a top. Its top is 1.5 m from the ground and the sloping distance from the top of the tent to the ground is 1.7 m. Allowing 10% for waste and overlap, how much material is needed to make the tent?
- 24 A rectangular prism has a V-shaped section removed. Calculate the surface area of the resulting solid and compare it with the surface area of the prism before the section was removed.



- 25 Calculate the volume of each of the following solids.

See Example 12

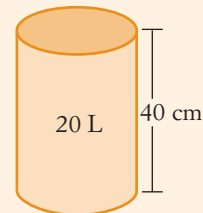


- 26 What is the most appropriate unit to measure the capacity of:

- a** a tablespoon?
b a petrol tanker?



- 27 The gas from a gas lamp can be dangerous in a closed tent. If gas is produced at a rate of 45 L/h, how long would the tent in question 21 take to fill with gas?
- 28 A cylindrical can has a capacity of 20 L. If the can is 40 cm high, calculate its diameter correct to the nearest centimetre.



- 29 A rectangular paddock is 300 m wide and 400 m long. Fertiliser is spread at the rate of 0.03 kg/m^2 . How much fertiliser is needed for the paddock?
- 30 A school oval is approximately circular with a diameter of 140 m. The gardener can mow 20 m^2 every minute with a tractor-drawn mower. How long does it take to mow the oval?
- 31 Children's wooden toy blocks are 12 cm long, 5 cm wide and 3 cm high. They must be painted with a non-toxic coating that costs \$25 to cover a square metre.
- a** What is the surface area of one block?
b How much will it cost to paint 10 000 blocks?
- 32 A flat-roofed garden shed is 5 m long, 2.1 m high and 3.4 m wide. To fumigate the shed, it must be completely filled with gas. A cylinder of compressed gas holds 4000 L of gas. Find the capacity of the shed, and the number of cylinders of gas required.
- 33 A swimming pool is 25 m long and 10 m wide and increases evenly in depth from 1.2 m at the shallow end to 2.7 m at the deep end. Chlorine is added at a rate of 3 g/kL. How many kilograms of chlorine are used?

Reasoning



Number and algebra

4 Algebra



Contents

- 4.1 Simplification of expressions
- 4.2 Expansion of brackets
- 4.3 Factorisation
- 4.4 Algebraic fractions
- Chapter summary
- Chapter review

Prior learning

Chapter 4

MAT09NAPL00004

Parent guide

Chapter 4

MAT09NAPG00004

Curriculum guide

Chapter 4

MAT09NACU00004

Australian Curriculum statements

Patterns and algebra

Extend and apply the index laws to variables, using positive integer indices and the zero index.

Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.

Video tutorial

Algebra

MAT09NAVT00004

Weblink

Numb3rs scene:
Everything is numbers,
math is everywhere

MAT09NAWB00004

Algebra is the most commonly used modelling method in mathematics. There is also a large amount of maths about general algebraic methods. This is very important because it allows you to change models around. This means that you can recognise algebra in models and use your knowledge to simplify and solve problems. For example, the short cut for finding the square of a number ending in 5 in your head is based on a simple algebraic expansion. You will see how this works later in the chapter.

Maths dictionary

MAT09ASDI00001

Weblink

Cut the knot

MAT09NAWB00004

Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

algebraic	collecting like terms	factorisation	simplification
expression	common factor	formula	simplify
algebraic fraction	distributive law	highest common	subject
binomial brackets	expand	factor	term
coefficient	expression	like terms	variable

4.1 Simplification of expressions

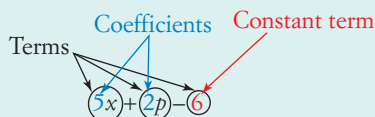
You have already done some work with algebraic expressions. It is important to know the correct names for expressions so that you know what your teacher means when he/she uses them.

Important!

Variables, expressions and formulas

A **variable** is a letter or symbol that stands for a number. A **constant** is a number.

An **expression** has variables and/or numbers connected by arithmetic operations like $+$, $-$, $\times$, $\div$ and powers. An expression that just uses numbers is an **arithmetic expression**, while one with variables is an **algebraic expression**. The numbers are called constants. The numbers that are multiplied by variables are called **coefficients**, the parts separated from the rest by $+$ or $-$ are called **terms** and a number on its own is called a **constant term**.



In algebra, you usually leave out the multiply symbol $\times$ between a constant and a variable or between variables. So in the expression shown above, $5x$ means $5 \times x$ and $2p$ means $2 \times p$. To **evaluate** an algebraic expression you use **substitution**. You put in values for the variables (**substitute** values) and work out the answer.

A **formula** is an equation with a variable on the left and an algebraic expression on the right. The variable on the left is the **subject** of the formula. The value of the subject variable is calculated by evaluating the expression on the right using given values for the variables in the expression.

Example 1

The cost, in dollars, of running an appliance is given by the formula $C = \frac{wtp}{100\,000}$, where p is the price of electricity in cents/kwH, w is the power in watts needed to run the appliance, and t is the running time in hours.

Find the cost of running a fan heater rated at 2400 W for 6 hours at 16.4 c/kwH.

Solution

Write the formula.

$$C = \frac{wtp}{100\,000}$$

Substitute the values.

$$= \frac{2400 \times 6 \times 16.4}{100\,000}$$

Enter as 2400 $\times$ 6 $\times$ 16.4 $\div$ 100 000 $=$

2400 $\times$ 6 $\times$ 16.4 $\div$ 100000 2.3616

Round and write the answer.

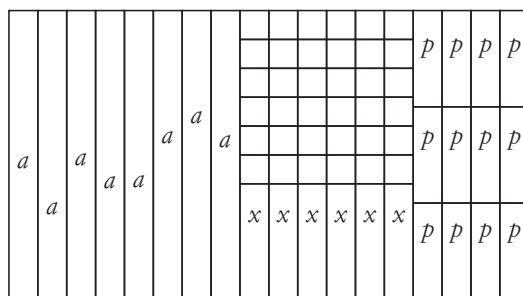
It costs \$2.36 to run the fan heater for 6 hours.

Complicated algebraic expressions can often be made simpler. This is called **simplification**.

Investigate: Simplifying expressions

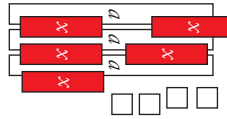
You can use a simple model for expressions. The model is made from strips and squares of paper like those shown below.

- Draw lines 1 cm apart on a blank piece of paper in the pattern shown.
- It doesn't matter how long the strips are, as long as they are not an exact centimetre length. Make the squares exactly 1 cm each. Label the strips of equal length a , p and x as shown below.

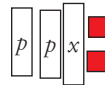


- Use a highlighter or coloured pencil to shade the other side of the paper.
- Cut out the squares and strips and label the reverse sides of the strips.

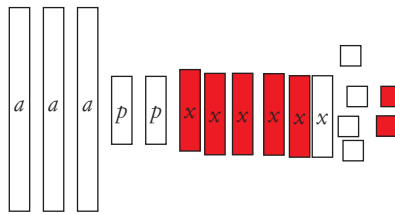
The white side represents positive and the coloured side represents negative.
Make a model for $3a - 5x + 4$ like this:



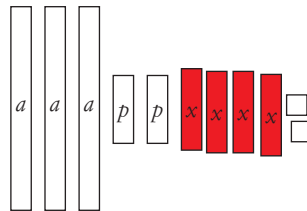
Now do a model for $2p + x - 2$ like this:



Put the models together and rearrange to show $3a - 5x + 4 + 2p - x - 2$.



One white and one red x (meaning $x - x$) cancel out.
The same applies to two white and two red squares ($2 - 2$).
This leaves the model like this:



What does the result show that $3a - 5x + 4 + 2p - x - 2$ simplifies to?
Your teacher probably has some other ones for you to try.

The investigation above shows that some algebraic expressions can be simplified. Parts with the same variables or constants can be put together to make a simpler expression.

Important!

Like terms

We call terms in an algebraic expression that have exactly the same variables **like terms**. Each variable has to be to the same power. Like terms can be added or subtracted to **simplify** the expression. This is called **collecting like terms**.

Example 2

Identify the like terms in the following expressions.

- a** $3a, 5b, -6a, b, 4ab, 5$
b $-6x, 4x^2, 5x^3, -x^2, 7x$
c $3m^2, -5m^2n, 6, 3mn^2, m^2n, n^2, -2, 4m^2n$

Solution

- a** There are two terms with only a and a separate pair with only b .
b There are two terms with x and two with x^2 .
c Three terms have m^2 and n , and there are two constants.
- 3a and -6a are like terms. So are 5b and b.**
-6x and 7x are like terms and 4x² and -x² are as well.
The like terms are -5m²n, m²n and 4m²n; 6 and -2.

It is easier to identify like terms if the variables in each term are in the same order. It is easiest to put them in alphabetical order. You should consider the sign in front as part of the term.

Example 3

Simplify the following if possible.

- a** $2mj + 3jm$
b $20r - 3r \times 5r$
c $8zy - 6ay - 2yz + ay + 8$
d $3a^4 - 2a^6 + a^4$

Solution

- a** Write the expression.
 Use the commutative law to change $2mj$ to $2jm$.
 Collect like terms.

$$2mj + 3jm = 2jm + 3jm = 5jm$$
b Write the expression.
 Simplify the product.
 One term has r and the other has r^2 .

$$20r - 3r \times 5r = 20r - 15r^2$$
There are no like terms.
c Write the expression with each term in the correct order.
 Identify the like terms.
 Collect the like terms ($8yz - 2yz$) and ($-6ay + ay$).

$$8yz + 6ay - 2yz + ay + 8 = 8yz - 6ay - 2yz + ay + 8 = 6yz - 5ay + 8$$
d $3a^4$ and a^4 are like terms.
 Simplify.

$$3a^4 - 2a^6 + a^4 = 4a^4 - 2a^6$$

CAS TI-Nspire exercise

Algebra

MAT09NATI00004

CAS ClassPad exercise

Algebra

MAT09NACP00004

Expressions involving multiplication and division can often be simplified.

Example 4

Simplify the following.

a $4b \times 5fd$

b $6nj \times ^{-7}nj^3$

c $24s^3d \div 18s^2$

Solution

- a** Write the problem with $\times$ signs.

$$4 \times b \times 5 \times f \times d$$

Separate coefficients with the commutative law.

$$= 4 \times 5 \times b \times f \times d$$

Finish with the variables in alphabetical order.

$$= 20bdf$$

- b** Write the problem with $\times$ signs.

$$6 \times n \times j \times ^{-7}n \times j \times j \times j$$

Separate parts with the commutative law.

$$= 6 \times ^{-7}n \times j \times j \times j \times j \times n \times n$$

Finish with the correct powers.

$$= ^{-42}j^4n^2$$

- c** Write as a fraction with $\times$ signs.

$$\frac{24 \times s \times s \times s \times d}{18 \times s \times s}$$

Cancel numbers and letters where possible.

$$= \frac{^4\cancel{24} \times s \times \cancel{s} \times \cancel{s} \times d}{_3\cancel{18} \times \cancel{s} \times \cancel{s}}$$

Finish and leave as a fraction.

$$= \frac{4ds}{3}$$

You can use the index laws in simplifications involving multiplications and divisions.

Example 5

Simplify the following.

a $18m^4n^3 \times 4m^3n$

b $18p^6q^5 \div 30p^2q^7$

Solution

- a** Write the expression with $\times$ signs.

$$18 \times m^4 \times n^3 \times 4 \times m^3 \times n$$

Use the commutative law.

$$= 18 \times 4 \times m^4 \times m^3 \times n^3 \times n$$

Multiply the coefficients and use the index laws.

$$= 72 \times m^{4+3} \times n^{3+1}$$

Finish the problem.

$$= 72m^7n^4$$

- b** Write as a fraction with $\times$ signs.

$$\frac{18 \times p^6 \times q^5}{30 \times p^2 \times q^7}$$

Cancel the coefficients and use the index laws.

$$= \frac{^3\cancel{18} \times p^{6-2} \times q^{5-7}}{_5\cancel{30}}$$

Complete the indices calculations.

$$= \frac{3p^4q^{-2}}{5}$$

Write with positive indices.

$$= \frac{3p^4}{5q^2}$$

Some expressions with multiplications and divisions may have brackets, numerators or denominators simplified first.

Example 6

Simplify $\frac{8a + a}{7b^2 - b^2} \times (3b + 2b)$.

Solution

Write with brackets in the numerator and denominator and write $(3b + 2b)$ as a fraction.

Simplify each set of brackets.

Cancel 9 and 6; also do b and b^2 .

Simplify the numerator and the denominator.

$$\begin{aligned} & \frac{(8a + a)}{(7b^2 - b^2)} \times \frac{(3b + 2b)}{1} \\ &= \frac{9a}{6b^2} \times \frac{5b}{1} \\ &= \frac{\overset{3}{\cancel{9}}a}{\underset{2}{\cancel{6}}\overset{1}{\cancel{b^2}}} \times \frac{\overset{1}{\cancel{5}}b}{1} \\ &= \frac{15a}{2b} \end{aligned}$$

Exercise 4.1 Simplification of expressions

1 Choose the terms on the right that are like the blue term in each of the following.

a $5c$: $5, 4c, 2cd, -2c^2, fc$

b $3mn$: $3m, -7mn, 4nm, 5m^2n, -7nmp$

c $6r^4$: $r^2, -2r^4, 7r^3k, 25r^4, 17rq^3$

d $2pq^3$: $2p, 5q^3p, 6pq, 3q^3p^3, 12q^3$

e $2ghg$: $4g^2, 6ghg, 11gh^2, 3hg^2, -2g^3h$

f $4ad^3e$: $6ad, -9ade, 5ae^2d, ead^3, 10a^2ed$

2 Select any like terms from each group.

a $3a^2, -3a, a^2, 4a^2$

b $9y^6, 6y^2, -5y^6, 3y^2y^4$

c $2w, -8w^4, 7w^2w, wwww$

d $-5g^3, 2g^2g, ag^3, 3g^2$

e $2a^2j, -5j^2a, a^2j^2, 7jaj$

f $4b^2c, (bc)^2, 3bc^2, -cb^2c$

3 Simplify each of the following.

a $3a + 7a$

b $4q + 12q$

c $11mn - 4mn$

d $5rt + 7rt$

e $6s^2u - 4us^2$

f $10db^2 - 8b^2d$

g $17acg + 4gac$

h $14q^2tp - 11pq^2t$

i $15jk^2m - 9k^2mj$

4 Simplify these products.

a $2n \times 4e$

b $3d \times 6y$

c $4df \times 3a$

d $5bq \times 4t$

e $7yu \times 3ux$

f $cv \times 4vt$

g $8jk \times 3jb$

h $11ht \times 3eh$

i $5fn \times 3hfn$

5 Simplify these quotients.

a $24g \div 4$

b $18ab \div 3$

c $12gh \div 6$

d $21er \div 7$

e $14jk \div 2k$

f $22tr \div 2t$

g $30mp \div 6p$

h $36hy \div 15y$

i $28pr \div 21rt$

Understanding

Extra questions

Exercise 4.1

MAT09NAEQ000012

See Example 2

See Example 3

See Example 4

6 Simplify the following by adding or subtracting like terms.

a $3m^2 + 4m^2$

d $10h^2 - h^2$

g $8x^3y^2 - 8x^3y^2$

j $9r^2 - 2r - 5r$

b $9f^3r + 3f^3r$

e $5cd^3 + 3cd^3$

h $4x^3 + 7x - x^3$

k $8k^2 + 4ck - 3k^2$

c $12mg^2 - 4mg^2$

f $8m^5 + m^5$

i $4f^2 + 3f^3 + 2f^2$

l $-4n^3 - n^3$

Fluency

7 Simplify the following where possible.

a $3x^2y + 7xy - 2xy + x^2y$

c $12 + 4c^2q^2 - 2c^2q^4 - 2$

e $7f^3r + 3rf - 5rf^3 + 6fr$

g $9 - 14n + 5nj^2 - 2n + 8$

b $8m^3 - 4mb^2 + m^3 - 3mb^2$

d $14d + 7d^3 - 10d + d^3$

f $6gy + 4y^3 - 8y + 2y^3$

h $e^2w^3 - ew^3 + 6w^3e - 5w^3e^2$

8 Simplify these products.

a $3a \times -2t$

d $5b \times 2m \times 3t$

g $mnp \times npq \times -9$

j $4wh^2 \times 6w^4h$

b $5h \times 4u \times q$

e $4hk \times 3kj \times -7$

h $7gy^2 \times 4gx^3$

k $3d^2e \times 9de^4f$

c $3p \times 4r \times -2$

f $5er \times 3fg \times -3eg$

i $6h^2 \times 8hg^4$

l $-7r^2j^5 \times 4rj$

9 Simplify these quotients.

a $28v \div 7$

d $45def \div 9ef$

g $-54fy \div -18y$

b $-25hk \div 5$

e $12mn \div 6m$

h $21cq \div -12qc$

c $27gh \div 9h$

f $-48pqr \div 12q$

i $48cd^2 \div 18cd$

10 Simplify the following algebraic expressions.

a $4 \times 3d \div 6$

d $6 \times -3cd \div 9d$

g $-7u \times 3u^2 \times u^3$

j $12n^2m \times 4nm^3 \div 20nm$

b $6m \times 2n \div -8mn$

e $3q \times 6q^2 \div 2q$

h $8h \times 7hj \div 14j$

k $16xy^3 \times -2x^4y \div 8y^2z$

c $pq \times 8qr \div 4q$

f $4xy \times 12yz \div -16xyz$

i $20n \times 3np \div 15pr$

l $8a^2bc \times 15a^3b \div -20bc^2$

11 Simplify the following algebraic expressions.

a $8a + 2 \times 7a$

d $17cd - 4 \times 3cd$

g $9xyz + 7xy \times 2z$

j $8mn - 4m \times 5n$

b $5g + 6 \times 2g$

e $22hj + 4h \times 3j$

h $19hab - 6ab \times 3h$

k $9rt - 7t \times 2r$

c $21p - 3 \times 5p$

f $28xy - 5y \times 4x$

i $5q - 7q \times 3$

l $11cd - 3cd \times 5$

See Example 5

12 Simplify each of the following.

a $3m^5h^4 \times 5m^3h^6$

d $9t^5q^5 \times 3t^6q^3$

g $2d^5m^4 \times 4d^4m^3$

j $w^4r^7 \times 7w^2r^2$

m $6p^3x^4 \times p^3x^3$

p $3r^5x^7 \times 3r^3x^5$

s $2b^7m^2 \times 8b^6m^3$

b $k^7p^3 \times 2k^5p^2$

e $h^7d^7 \times 5h^3d^7$

h $3u^4p^4 \times 6u^6p^6$

k $9a^4x^2 \times 4a^6x^5$

n $5p^4w^3 \times 7p^2w^2$

q $6u^4y^3 \times u^7y^6$

t $18k^9m^2 \times 3k^2m^4$

c $g^5a^7 \times 7g^5a^2$

f $q^7h^5 \times 3q^7h^5$

i $7d^5g^6 \times 2d^5g^3$

l $7h^2d^2 \times 7h^3d^2$

o $4k^5h^2 \times 8k^7h^6$

r $7z^5h^3 \times 5z^6h^4$

13 Simplify each of the following.

a $40c^4n^4 \div 8c^7n^3$

d $18h^3y^5 \div 69h^6y^7$

g $12z^7c^6 \div 33z^7c^2$

j $4t^5a^7 \div t^7a^4$

m $18x^5a^2 \div 8x^3a^2$

p $3t^7z^2 \div 33t^2z^5$

s $5q^7w^4 \div 15q^4w^2$

b $4t^4m^6 \div 16t^3m^7$

e $32t^7z^7 \div 48t^2z^6$

h $44w^5b^3 \div 8w^3b^5$

k $4v^5y^3 \div 12v^7y^6$

n $25g^3b^6 \div 15g^4b^7$

q $40k^7h^2 \div 50k^6h^7$

t $28h^2k^5 \div 42h^7k^3$

c $14x^3t^5 \div 12x^4t^3$

f $36q^3z^2 \div 8q^5z^4$

i $9b^4z^5 \div 18b^2z^2$

l $63d^5a^5 \div 27d^7a^7$

o $4a^6v^7 \div 26a^4v^7$

r $35n^4c^2 \div 25n^2c^4$

14 Simplify each of the following.

a $\frac{5ab + 9ab}{10a^2 - 3a^2}$

c $(2n + 5n) \frac{2m^2n^3 + 13m^2n^3}{14m^3 - 9m^3}$

e $\frac{11pqr + 9pqr}{24mrw - 9mrw} \times \frac{18wm - 10wm}{4v + 3v}$

b $\frac{12x - 4x}{5xz - xz} \times (3a + 2a)$

d $\frac{9p^2 - 3p^2}{4q^4 + 5q^4} \times \frac{2q^2 + q^2}{9p - 7p}$

Worked solution

Exercise 4.1

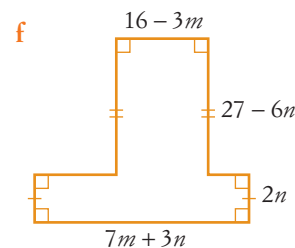
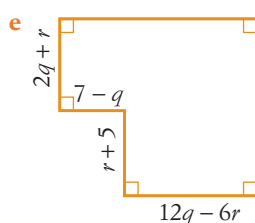
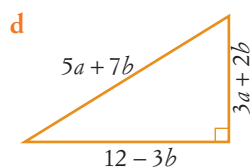
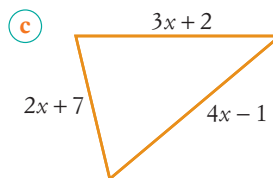
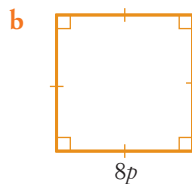
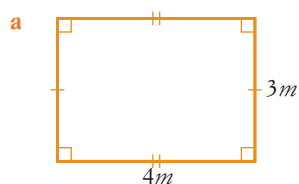
MAT09NAWS00012

See Example 6

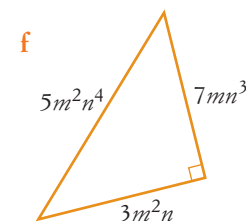
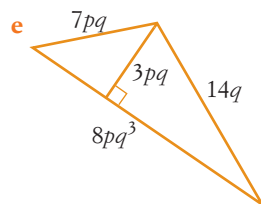
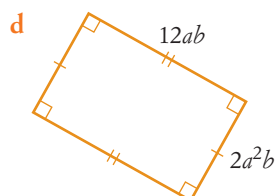
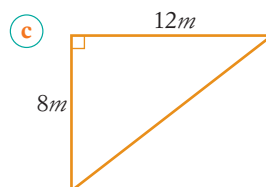
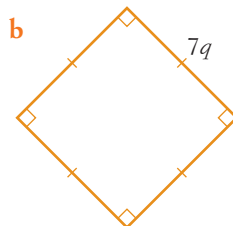
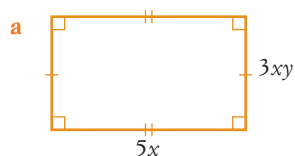
15 The formula $F = \frac{m(v_2 - v_1)}{t}$ gives the force F in newtons exerted on an object to accelerate it from v_1 to v_2 m/s in t seconds. A man of mass 80 kg takes 3 seconds to fall 7.36 m from the top of a landing craft to the surface of the Moon. He lands at 4.9 m/s. Find the force on the man on the Moon. How does this compare to the force he would experience on Earth, where it would take him only 0.5 seconds to reach the same speed after falling just 1.2 m?

16 The (flat-rate) interest rate on a loan of \$ L that is repaid over n years at \$ P per month is given by $r = \frac{100(12nP - L)}{nL}$ %. Find the interest rate that Henry was charged for a personal loan of \$6000 that he paid off over 5 years at \$190 per month.

17 Find an expression for the perimeter of each of these shapes. Simplify your answers.



18 Find an expression for the area of each of these shapes. Simplify your answers.



Worked solutions

Exercise 4.1

MAT09NAWS00012

Worked solutions

Exercise 4.1

MAT09NAWS00012

4.2 Expansion of brackets

You cannot normally collect like terms inside brackets because there are not usually any like terms in them. Instead, you use the **distributive law** to simplify an algebraic expression with brackets.

Important!

The distributive law

The product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each of the two numbers. This can be written in symbols as:

$$a \times (b + c) = a \times b + a \times c$$

and

$$a \times (b - c) = a \times b - a \times c.$$

The distributive law can be summarised as: $a(b \pm c) = ab \pm ac$.

When you are **expanding** brackets, it is a good idea to write subtractions as addition of a negative. This helps you avoid mistakes with signs.

Example 7

Simplify each of the following.

a $3(2x + 5)$

b $6b(4a - 3b)$

c $-2(3q + 2p)$

d $-7g(4g - 5h)$

e $6(3k - 2m + 8)$

Solution

- a** Write the expression with a $\times$ sign.

Use the distributive law.

Simplify the terms.

$$3 \times (2x + 5)$$

$$= 3 \times 2x + 3 \times 5$$

$$= 6x + 15$$

- b** Write the subtraction as adding a negative.

Use the distributive law.

Simplify the terms.

Write the second term in the usual way.

$$6b \times (4a + -3b)$$

$$= 6b \times 4a + 6b \times -3b$$

$$= 24ab + -18b^2$$

$$= 24a - 18b^2$$

- c** Write the expression.

Use the distributive law.

Simplify the terms.

Write the second term in the usual way.

$$-2 \times (3q + 2p)$$

$$= -2 \times 3q + -2 \times 2p$$

$$= -6q + -4p$$

$$= -6q - 4p$$

- d** Write the subtraction as adding a negative.

Use the distributive law.

Simplify the terms.

Write the positive term first.

- e** Write the subtraction as adding a negative.

Use the distributive law.

Simplify the terms.

$$\begin{aligned} & -7g \times (4g + ^{-}5h) \\ &= ^{-}7g \times 4g + ^{-}7g \times ^{-}5h \\ &= ^{-}28g^2 + 35gh \\ &= 35gh - 28g^2 \\ & 6 \times (3k + ^{-}2m + 8) \\ &= 6 \times 3k + 6 \times ^{-}2m + 6 \times 8 \\ &= 18k - 12m + 48 \end{aligned}$$

Example 8

Simplify each of the following.

a $4x^2(7x^3 - 6y)$

b $^{-}7a^2b^3(5ac^2 - 6b^4)$

Solution

- a** Write with a $\times$ sign.

Use the distributive law.

Simplify and write the negative as subtraction.

- b** Write the expression with a $\times$ sign.

Use the distributive law.

Simplify and write the positive term first.

$$\begin{aligned} & 4x^2 \times (7x^3 - 6y) \\ &= 4x^2 \times 7x^3 + 4x^2 \times ^{-}6y \\ &= 28x^5 - 24x^2y \\ & ^{-}7a^2b^3 \times (5ac^2 - 6b^4) \\ &= ^{-}7a^2b^3 \times 5ac^2 + ^{-}7a^2b^3 \times ^{-}6b^4 \\ &= 42a^2b^7 - 35a^3b^3c^2 \end{aligned}$$

Example 9

Expand and simplify each of the following.

a $6(7m - 4n + 4) - 3(8m - 7n - 5)$

b $4ac^2(5a + 2c - 8ac) + 4c^2(2a^2 - 5a^2c + 4)$

Solution

- a** Write the problem.

Use the distributive law.

Collect like terms.

- b** Write the problem.

Use the distributive law.

Simplify each term.

Collect like terms.

$$\begin{aligned} & 6 \times (7m + ^{-}4n + 4) + ^{-}3 \times (8m + ^{-}7n + ^{-}5) \\ &= 42m + ^{-}24n + 24 + ^{-}24m + 21n + 15 \\ &= 18m - 3n + 39 \\ & 4ac^2 \times (5a + 2c + ^{-}8ac) + 4c^2 \times (2a^2 + ^{-}5a^2c + 4) \\ &= 4ac^2 \times 5a + 4ac^2 \times 2c + 4ac^2 \times ^{-}8ac + 4c^2 \times 2a^2 \\ &\quad + 4c^2 \times ^{-}5a^2c + 4c^2 \times 4 \\ &= 20a^2c^2 + 8ac^3 + ^{-}32a^2c^3 + 8a^2c^2 + ^{-}20a^2c^3 + 16c^2 \\ &= 28a^2c^2 + 8ac^3 - 52a^2c^3 + 16c^2 \end{aligned}$$

You can expand brackets multiplied together in various ways. The first method we will look at is called the 'area model'.

Teacher notes

Area model expansion

MAT09NATN00002

Investigate: Area model expansion

How can we expand the brackets $(2x + 3)(3x + 5)$?

You can think of this as the area of a rectangle with height $(2x + 3)$ and width $(3x + 5)$. (Remember that the area of a rectangle is the product of the lengths of its sides.)

We can draw a diagram as shown on the right and divide the rectangle into parts as shown by the dashed lines.

$$\begin{aligned}\text{Area } A &= (2x + 3) \times (3x + 5) \\ &= (2x + 3)(3x + 5)\end{aligned}$$

The area of the rectangle is the sum of the areas of the separate parts.

$$\text{Area } A_1 = 3x \times 2x = 6x^2$$

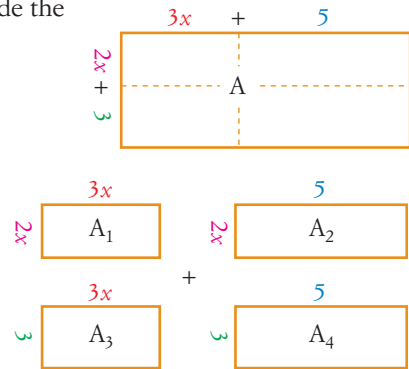
$$\text{Area } A_2 = 2x \times 5 = 10x$$

$$\text{Area } A_3 = 3 \times 3x = 9x$$

$$\text{Area } A_4 = 3 \times 5 = 15$$

Obviously, $A = A_1 + A_2 + A_3 + A_4$,
so we get

$$(2x + 3)(3x + 5) = 6x^2 + 10x + 9x + 15 = 6x^2 + 19x + 15$$



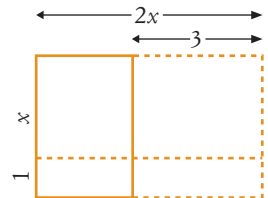
Try $(4x + 5)(3x + 1)$ and $(x + 8)(4x + 7)$ for yourself!

You can use the area model even when there are negatives.

For example, you could picture $(x + 1)(2x - 3)$ as shown on the right, but this is pretty messy.

It is much easier to consider the $(2x - 3)$ as $(2x + -3)$ and use a negative label on the box.

When you multiply, just take the signs into account.



Example 10

Worksheet

Expanding expressions
using area diagrams

MAT09NAWK00032

Technology

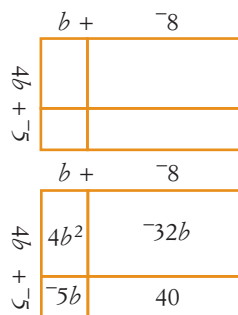
Geogebra: Binomial
expansion

MAT09NATC00003

Use the area model to simplify $(4b - 5)(b - 8)$.

Solution

Draw a diagram



Do the multiplications and write the answers in the boxes.

Write the result.

Collect the like terms.

$$\begin{aligned}(4b - 5)(b - 8) &= 4b^2 + -32b + -5b + 40 \\ &= 4b^2 - 37b + 40\end{aligned}$$

Two brackets multiplied together are called **binomial brackets**.
You can also use the distributive law several times to expand binomial brackets.

Example 11

Use the distributive law to simplify $(3x - 4)(4x + 3)$.

Solution

Write with negatives instead of subtraction. $(3x + ^{-}4) \times (4x + 3)$

Use the distributive law with the first bracket.

$$= 3x \times (4x + 3) + ^{-}4 \times (4x + 3)$$

Use the distributive law on both brackets.

$$= 3x \times 4x + 3x \times 3 + ^{-}4 \times 4x + ^{-}4 \times 3$$

Simplify each term.

$$= 12x^2 + 9x + ^{-}16x + ^{-}12$$

Collect like terms.

$$= 12x^2 - 7x - 12$$

Example 12

Expand and simplify $(3x + 4y)(4x - 3y) - (5x - 2y)(x + 4y)$.

Solution

Write extra brackets.

$$[(3x + 4y)(4x + ^{-}3y)] - [(5x + ^{-}2y)(x + 4y)]$$

Do the first brackets in each.

$$= [3x(4x + ^{-}3y) + 4y(4x + ^{-}3y)] - [5x(x + 4y) + ^{-}2y(x + 4y)]$$

Do the next brackets in each.

$$= [12x^2 + ^{-}9xy + 16xy + ^{-}12y^2] - [5x^2 + 20xy + ^{-}2xy + ^{-}8y^2]$$

Simplify and write subtraction as adding a negative.

$$= [12x^2 + 7xy + ^{-}12y^2] + ^{-}1 \times [5x^2 + 18xy + ^{-}8y^2]$$

Expand brackets.

$$= 12x^2 + 7xy + ^{-}12y^2 - 5x^2 - 18xy + 8y^2$$

Simplify.

$$= 7x^2 - 11xy - 4y^2$$

Video tutorial

Expanding binomial products

MAT09NAVT10006

Worksheet

Binomial products

MAT09NAWK00033

Worksheet

Special products

MAT09NAWK00034

Worksheet

Mixed expansions

MAT09NAWK00035

Puzzle sheet

Expanding brackets

MAT09NAPS00009

Puzzle sheet

Enough time

MAT09NAPS00011

Puzzle sheet

Products and factors squaresaw

MAT09NAPS00019

Puzzle sheet

Factoronimoes

MAT09NAPS00020

Technology worksheet

Excel worksheet: Expanding binomials

MAT09NACT00021

Understanding

Extra questions

Exercise 4.2

MAT09NAEQ00013

Exercise 4.2 Expansion of brackets

1 Complete each of the following.

a $3(m + 1)$
 $= 3 \times \dots + 3 \times \dots$
 $= \dots + \dots$

c $5(6 + q)$
 $= \dots \times 6 + 5 \times \dots$
 $= \dots + \dots$

b $2(d - 5)$
 $= 2 \times \dots - 2 \times \dots$
 $= \dots - \dots$

d $4(6 + y)$
 $= 4 \times \dots + \dots \times y$
 $= \dots + \dots$

$$\begin{aligned} \mathbf{e} \quad & 2(5r - 7) \\ & = 2 \times \dots - 2 \times \dots \\ & = \dots - \dots \end{aligned}$$

$$\begin{aligned} \mathbf{g} \quad & d(3p + 5) \\ & = \dots \times 3p + \dots \times \dots \\ & = \dots + \dots \end{aligned}$$

$$\begin{aligned} \mathbf{f} \quad & 7(3 - 4q) \\ & = \dots \times \dots - 7 \times \dots \\ & = \dots - \dots \end{aligned}$$

$$\begin{aligned} \mathbf{h} \quad & (8b + 6)a \\ & = \dots \times \dots + \dots \times \dots \\ & = \dots + \dots \end{aligned}$$

See Example 7

2 Simplify each of the following.

a $2(3 + a)$

d $4(a + 5)$

g $3(12 - 3e)$

j $5(4g + 7)$

b $1(b - 2)$

e $2(7 - 4a)$

h $4(11 + 9h)$

k $8(2y - 10)$

c $3(1 + h)$

f $6(2d + 5)$

i $2(3 - 6u)$

l $7(6 - 3k)$

3 Simplify each of the following.

a $f(f + 2)$

d $p(9 + p)$

g $e(3 + u)$

j $y(5 - y)$

b $y(1 - y)$

e $g(r + 1)$

h $(q + 1)r$

k $a(a - 4)$

c $(z + 4)z$

f $m(p - z)$

i $s(w + 7)$

l $p(7 - p)$

See Example 9

4 Expand and simplify each of the following.

a $3(g + 1) + 2(g - 1)$

c $2(q + 3) + 5(q - 2)$

e $3(b + 5) + 2(b - 1)$

b $4(c + 2) + 3(c + 1)$

d $6(k + 1) + 4(k - 3)$

f $5(y - 2) + 4(y - 3)$

See Example 10

5 Use the area model to simplify each of the following.

a $(4v - 5)(2v + 1)$

c $(2q + 9)(5q - 2)$

e $(2n - 5)(5n - 2)$

g $(c + 5)(3c - 1)$

i $(4m + 1)(4m - 3)$

k $(2r + 3)(4r + 1)$

m $(m - 6)(4m - 3)$

o $(4k + 7)(5k + 6)$

q $(4x + 3)(3x + 7)$

s $(3c - 4)(2c + 9)$

b $(m - 1)(3m + 7)$

d $(e - 9)(3e + 1)$

f $(3m + 4)(5m - 1)$

h $(4d + 9)(2d - 5)$

j $(5r - 1)(5r - 4)$

l $(3r + 7)(3r + 8)$

n $(3c - 5)(2c - 7)$

p $(4t + 3)(2t - 7)$

r $(2p + 1)(5p - 8)$

t $(4u - 1)(5u - 7)$

Fluency

6 Simplify each of the following.

a $2a(a + 3)$

d $3k(3 - 2w)$

g $-2e(1 - 3e)$

j $5z(4 + 3z)$

b $5h(1 - h)$

e $6g(4 + 2g)$

h $4j(b + 7a)$

k $-4d(-d - 7)$

c $(c + 3)4c$

f $3m(2 - 6m)$

i $-3t(6 - 4r)$

l $-3f(-2f + 9)$

7 Simplify each of the following.

a $2d(3d + 1)$

d $2a(5a + 3)$

g $-7g(3g + 1)$

j $4n(2m - 3n)$

b $4a(2a + 5)$

e $3g(2g - 6)$

h $-9k(2 - 5k)$

k $3x(2x + 5y)$

c $2r(4r + 3)$

f $5s(3s - 2)$

i $6q(4q + 7p)$

l $-6x(3y - 5x)$

8 Simplify each of the following.

- a** $8(6r - c - 8m)$
c $3(6t + z - 4x)$
e $6(3c - 11g - 2h)$
g $5(5a + 7e + 5m)$
i $3(8x - 5c + 6h)$
k $^{-}8(4q - 8y - 6n)$
m $5(3h - 7w - 8g)$
o $9(8b - n - 6u)$
q $^{-}4(2r + 8k + d)$
s $5(y - 13v - 7y)$

- b** $^{-}8(9q - 6r - 10k)$
d $^{-}7(3q - a - 7m)$
f $6(3v + q + 13q)$
h $^{-}8(9k - 9r + 7t)$
j $^{-}7(2a + 5u - 3b)$
l $2(3u - 5y - 8r)$
n $^{-}9(g + 6u + 9r)$
p $^{-}8(g - 9w - 3h)$
r $3(3z + 6p + 4p)$
t $4(4g - 7p - 5t)$

9 Simplify each of the following.

- a** $9a^3r(8a^2 + 8r^2)$
c $4y(3y^2n^2 + 10)$
e $7pg^3(3 + p^3)$
g $9w(7w^3 + m)$
i $8pw^2(p^3w^2 - 2)$
k $2e^2m^3(7 - 3e^2m)$
m $7uv^2(1 - 15u^2)$
o $3g^3n^2(5 - n^2)$
q $7(4p^4u^3 - 5)$
s $3d^3p^4(2d^2 - 5)$

- b** $2n^3v^2(6n^2 - 5v^2)$
d $4e(8h - 5e)$
f $6p^4v(4 - v^3)$
h $4ac(7ac + 3)$
j $2v^3(7c^5 + 3v)$
l $4z^4m^2(8m - 8)$
n $8h(3h^3w^4 - 4)$
p $7m(3m - 10y)$
r $4e^4(3 + ec^2)$
t $9q(q^4 + 3g)$

See Example 8

10 Simplify each of the following.

- a** $^{-}8n^3(4b^8 + n^4)$
c $4y^6g^5(4y - 3g^2)$
e $^{-}10y^3n(8 - 8yn)$
g $6c^3a^5(3 + 4c^5a^3)$
i $^{-}7h^4e(4h^2e^6 - 3)$
k $6k^4(5k^2 + 12u^4)$
m $10k^5u^2(8u^6 + 1)$
o $^{-}11r^2h^5(5r^5 - 12h^2)$
q $^{-}6b^6z(z^4 - 2b^2)$
s $6u^2(7b^8 + 13u)$

- b** $^{-}5n^4y^4(5n^4 - 2)$
d $10k^3g^5(5g^3 - 12k^3)$
f $^{-}4u^2z^2(9z^3 + u^3)$
h $3n^4u^3(9n^4 + 1)$
j $^{-}5z^5(6z - e^4)$
l $^{-}4k^5(5y^4 - 3k^3)$
n $2k^3(3 + 2kg^3)$
p $2h(9 - 10x^8)$
r $7v^6(5e^6 + v^2)$
t $3c^4(7c + 13v^2)$

11 Simplify each of the following.

- a** $2w(4w + 3w^2 + 7x)$
c $5a(2 - 6am + a^2m)$
e $4d(2d + d^2g + 3dg)$
g $^{-}3x(2xy + 3x^2 - 5y)$
i $^{-}2y(4yz + 5z^2 + 3z)$

- b** $3m(1 + 3mp - 6m^2)$
d $3h(2h - 4h^2 - 5hj)$
f $^{-}2u(4ut - 3u^3 - 6t)$
h $4d(5 - 2de^2 - 7ed)$
j $5r(2r + 3r^2 - rj^2)$

12 Expand and simplify each of the following.

- a** $4(p + 3) - 2(5 - p)$
c $3(s - 2) - 5(2 - s)$
e $10(z - 2) - 3(z + 5)$

- b** $7(n + 2) + 3(n - 5)$
d $3(e - 4) + 5(e - 2)$
f $3(2x - 4) - 6(5 - 3x)$

13 Expand and simplify each of the following.

a $2x + 3(x + 5)$

c $3(5d + 1) - 8d$

e $6 - 3(4a - 1)$

g $3(r + 5) + 2(r - 7)$

i $6z - 3(2 - 5z) + 8$

k $4(6 - u) - (4u + 7)$

m $3a(2a + 5) - 2(a - 8)$

o $-5(3 - 4b) + 3(6 - 5b)$

b $3(2q - 4) + 9$

d $11 - 2(5j + 3)$

f $12 - (7 - 6e)$

h $10n - 4n(7 - 3n)$

j $3g - 4(1 - 5g) + 8$

l $2(3f + 4) - 3(2f - 5)$

n $6a(3 - 2a) + 4(a - 8)$

p $2h(8 - 3h) + 12h + 10$

14 Expand and simplify each of the following.

a $5z(3c^3 - 9m - 10mc) + 3(5z - 2mz + mzc)$

b $8h(5w + 12u + 9uh^2w) - 5uvw(2uh^3w^2 + 5u^2h^3 + 5h^3)$

c $9gq^2(9z^2g^2q^2 - g - 10zq^2) + 11gq(6zggq^2 + 4g^2q + 5zq^3)$

d $7z^2(3m^3z + 10u^2m^2z - 4) - 3(3m^3z^3 + 4u^2 - 4u^2z^2)$

e $9p(8 + 8c^3e^2 - 5pe^4) - 7ce(c^2pe + 5ce^2 + 2p^3)$

f $9za(5a - 2z^2a + y^2za) + 4y^2z(1 - 2za^2 - 6zya)$

g $10ay^2n(n + 4 + an^2) - 9(7a^2y^2n^2 + 3a^3 + 13yn)$

h $10a^2(5b^2c^4 - 12b^2a + 8ac^2) + 6(9b^2a^2c^4 + a + 8b^2ac^3)$

i $4(3y^2v^3 + 4d - 4d^2v) + 2y^2v^3(4 + 9d^3 + 8yv)$

j $4(5h - 12a + 6avh) - 5vh(9vh - 4h^3 + 6a)$

See Example 11

15 Use the distributive law to simplify each of the following.

a $(9q - 2)(9q - 7)$

c $(7q + 6)(8q + 7)$

e $(8m + 9)(8m - 1)$

g $(9x + 2)(x + 1)$

i $(6u + 5)(9u - 4)$

k $(8d - 7)(8d + 9)$

m $(5e - 1)(e + 6)$

o $(4u + 1)(u - 1)$

q $(5u + 9)(9u + 8)$

s $(4x - 7)(5x - 7)$

b $(9y - 1)(6y - 5)$

d $(8v - 1)(5v + 7)$

f $(8g - 9)(4g - 1)$

h $(2y - 5)(y + 5)$

j $(8d + 9)(2d + 5)$

l $(8t + 5)(3t - 7)$

n $(2g + 7)(g + 7)$

p $(9p - 8)(7p - 1)$

r $(2q + 9)(q + 1)$

t $(7m + 1)(7m - 1)$

16 Use the distributive law to simplify each of the following.

a $(3k + r)(5k + 3r)$

c $(8g + 3c)(5g - 2c)$

e $(3x + 5b)(8x - b)$

g $(7r + 3v)(7r + 3v)$

i $(8g - 7e)(9g + 7e)$

k $(6n + e)(8n + 3e)$

m $(8t - 3w)(4t + 7w)$

o $(5v + 3d)(4v + 3d)$

q $(2m - e)(3m - e)$

s $(9x - 5n)(x + 9n)$

b $(8v + 9y)(5v + y)$

d $(4u + 7h)(8u + 9h)$

f $(4e - 9r)(8e + 7r)$

h $(5t - 9e)(5t + 7e)$

j $(w + 5e)(7w + e)$

l $(9t + 2a)(5t - 8a)$

n $(3r - 4d)(2r + d)$

p $(9w + 2n)(4w - 9n)$

r $(4n - 7c)(7n + 9c)$

t $(4g - 7n)(9g - 2n)$

17 Expand and simplify each of the following.

See Example 12

- a** $(9q - 7t)(5q - t) - (3q - 4t)(2q + 5t)$ **b** $(5v - 2e)(6v - 7e) - (9v - 8e)(9v + 7e)$
c $(7y + 5k)(4y - 5k) + (4y + 5k)(3y + 7k)$ **d** $(9g - t)(7g + t) + (9g + 7t)(9g - 5t)$
e $(7k + 8a)(7k - 9a) - (4k + 3a)(4k - a)$ **f** $(8c + 7n)(9c - 7n) + (4c + n)(c + 6n)$
g $(8r - 9m)(r - 4m) - (9r - m)(6r + m)$ **h** $(7y - 6h)(y - 4h) + (8y - 9h)(8y + 7h)$
i $(5x + 4z)(8x - 3z) - (6x - 7z)(8x - z)$ **j** $(8g - 9u)(7g - 9u) + (2g + u)(7g + 9u)$
k $(5m + 9g)(6m - 7g) - (9m - 4g)(2m + 7g)$ **l** $(7y + 8g)(8y + g) - (4y - 9g)(8y - g)$
m $(3m - 8x)(3m - x) + (2m + 3x)(6m - 5x)$ **n** $(2h + 3u)(4h - u) - (2h - 9u)(5h - 8u)$
o $(9d + 4x)(7d + 2x) - (d + 2x)(5d + 6x)$ **p** $(x + 5u)(2x - 9u) + (6x - 7u)(3x + 7u)$
q $(k + 7e)(2k + e) + (6k - 5e)(k + 6e)$ **r** $(3c + e)(3c - 4e) + (6c - e)(c + 6e)$
s $(7q - 5e)(6q - 5e) - (2q - e)(4q + 9e)$ **t** $(9x - 7m)(x - 3m) - (5x + 9m)(2x + 9m)$

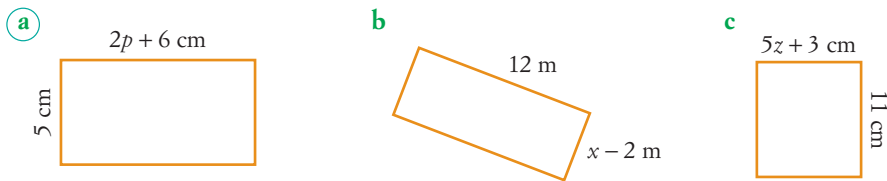
18 Find expressions for the areas of each of the following shapes and simplify your expressions.

Problem solving

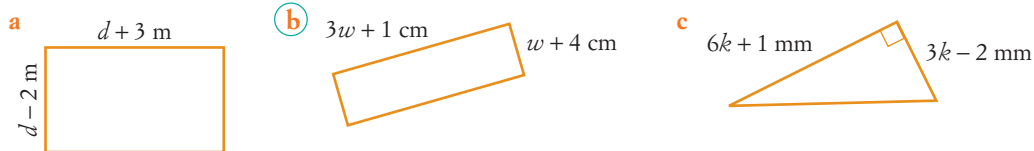
Worked solutions

Exercise 4.2

MAT09NAWS00013



19 Find expressions for the areas of each of the following shapes and simplify your expressions.

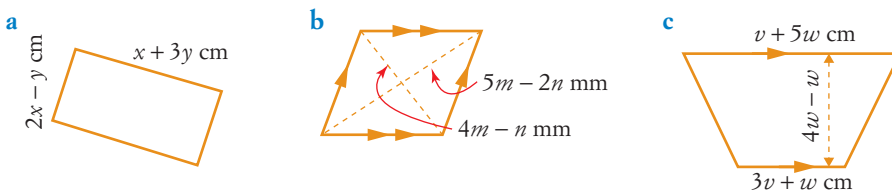


20 Find expressions for the areas of each of the following shapes and simplify your expressions.

Worked solutions

Exercise 4.2

MAT09NAWS00013



- 21 a** Expand each of the following: $(a + 1)(a - 1)$, $(m - 8)(m + 8)$, $(4p + 5)(4p - 5)$
b Write a rule for the expansion of expressions like $(x + y)(x - y)$ and prove your rule.
22 a Expand each of the following: $(z + 1)^2$, $(5w + 1)^2$, $(3v + 7)^2$
b Write a rule for the expansion of expressions like $(x + y)^2$ and prove your rule.
23 a Expand each of the following: $(m - 8)^2$, $(3q - 4)^2$, $(4k - 5)^2$.
b Write a rule for the expansion of expressions like $(x - y)^2$ and prove your rule.

Reasoning

Worked solutions

Exercise 4.2

MAT09NAWS00013

4.3 Factorisation

In all mathematics, we like to be able to work backwards and forwards on the same kind of problem. The opposite to expansion of brackets is **factorisation**.

Investigate: Area model factorisation

In the previous section, you used the area model to expand binomial brackets. If you used the model to expand $5(2x - 7)$, it would look like this:

$$\begin{array}{|c|c|} \hline 2x & + & -7 \\ \hline 5 & 5 \times 2x & 5 \times -7 \\ \hline \end{array}$$

Working out the areas gives the second rectangle.

$$\begin{aligned} \text{Overall, you get } 5(2x - 7) &= 10x + -35 \\ &= 10x - 35 \end{aligned}$$

$$\begin{array}{|c|c|} \hline 2x & + & -7 \\ \hline 5 & 10x & -35 \\ \hline \end{array}$$

How can you factorise the expression $12x + 20$ using this method? You would have $12x$ and 20 in the rectangle, like this:

$$\begin{array}{|c|c|} \hline & + & \\ \hline 12x & & 20 \\ \hline \end{array}$$

What would you have on the top and at the side?

$$\begin{array}{|c|c|} \hline ? & + & ? \\ \hline ? & 12x & 20 \\ \hline \end{array}$$

The number at the side has to divide into *both* $12x$ and 20 , so it is 4 .

$$\begin{array}{|c|c|} \hline ? & + & ? \\ \hline 4 & 12x & 20 \\ \hline \end{array}$$

To get the correct areas, the others must be as shown here.

$$\begin{array}{|c|c|} \hline 3x & + & 5 \\ \hline 4 & 12x & 20 \\ \hline \end{array}$$

Overall, you get $12x + 20 = 4(3x + 5)$.

Try $18x + 24$ and $16x - 10$ for yourself.

Your teacher probably has some others for you to try.

Important!

Factorisation

Factorisation is the opposite process to multiplying out brackets. In the **common factor** method, the **highest common factor** of the terms is put outside the brackets, leaving the other factors of each term inside the brackets.

You can do factorisations without the rectangle by thinking of the terms separately and dividing each one by the highest common factor (HCF).

Example 13

Factorise each of the following.

a $6y - 18$

b $45 - 30h$

c $24a + 18$

Solution

- a** Think of '−18' as '+ −18' and find the HCF.

Write the problem with the negative 18.

Dividing each term by 6 gives $6y \div 6 = 1y$ and $-18 \div 6 = -3$.

Write the answer in the usual way.

- b** Write the problem with the '−30h' written as '+ −30h'.

Divide each term by the HCF, 15.

Write the answer in the usual way.

- c** Write the problem.

Divide each term by the HCF, 6.

The HCF of $6y$ and -18 is 6.

$$6y + -18$$

$$= 6(1y + -3)$$

$$= 6(y - 3)$$

$$45 + -30h$$

$$= 15(3 + -2h)$$

$$= 15(3 - 2h)$$

$$24a + 18$$

$$= 6(4a + 3)$$

Video tutorial

Factorising expressions

MAT09NAVT10005

Example 14

Factorise each of the following.

a $-9t - 15$

b $-30m + 12$

c $28g - 12p + 20$

Solution

- a** Write the problem with the '−15' written as '+ −15'.

Divide each term by the highest common factor, 3.

- b** Write the problem.

There are two possible HCFs, −6 or 6.

Write in the usual way.

- c** Write the problem.

Divide by the HCF, 4.

$$-9t + -15$$

$$= -3(3t + 5)$$

$$-30m + 12$$

$$= -6(5m + -2) \text{ or } = 6(-5m + 2)$$

$$= -6(5m - 2) \text{ or } = 6(2 - 5m)$$

$$28g + -12p + 20$$

$$= 4(7g + -3p + 5)$$

In many cases, you need to take out an algebraic factor. You still need to take out the highest common factor, which may include a power.

Example 15

Factorise each of the following.

a $5p^2 - 20pq$

b $36b^4e^4 - 12b^2c^3e^2 + 18c^5e^2$

Solution

a Write the problem.

The highest common factor is $5p$.

$$5p^2 - 20pq$$

$$= 5p(p - 4q)$$

b Write the problem.

The highest common factor is $6e^2$.

$$36b^4e^4 - 12b^2c^3e^2 + 18c^5e^2$$

$$= 6e^2(6b^4e^2 - 2b^2c^3 + 3c^5)$$

Exercise 4.3 Factorisation

Understanding

Extra questions

Exercise 4.3

MAT09NAEQ00014

See Example 13

See Example 14

See Example 15

1 Factorise each of the following.

a $8a - 16$

d $42q - 18$

g $28m + 4$

j $48k - 40$

m $12g + 15$

p $7m + 21$

s $27e + 6$

b $12g + 18$

e $40m - 8$

h $25y + 15$

k $15h - 6$

n $35y + 84$

q $20e + 12$

t $45k - 5$

c $56m - 35$

f $16e + 24$

i $63p - 9$

l $18k + 15$

o $18c - 45$

r $27u + 3$

2 Factorise each of the following.

a $36t - 6n$

d $-7d + 14y$

g $16p + 2w$

j $18c + 9u$

m $64y - 8g$

p $-18n - 9h$

s $8y + 24n$

b $3p + 24m$

e $36a - 9u$

h $35z - 28q$

k $-18k + 42n$

n $14r - 21n$

q $-14d - 49d$

t $-72n - 8e$

c $8u + 4d$

f $12x + 3h$

i $81g - 18g$

l $-54b - 45t$

o $-64c + 24w$

r $6d + 2p$

3 Factorise each of the following.

a $16g^2 - 8hg$

d $4q^2 + 28aq$

g $3y^2 + 18dy$

j $5w^2 + 5ew$

m $42q^2 - 78vq$

p $-28w^3 - 52xw^2$

s $-63v^2 + 36rv$

b $35x^2 + 14ex$

e $25h^2 - 40h$

h $45n^2 + 72qn$

k $18r^3 + 72hr^2$

n $-14q^4 + 4zq^2$

q $-12y^2 + 8by$

t $-12m^4 - 18nm^2$

c $14b^2 + 7xb$

f $36d^2 - 63qd$

i $24x^2 + 24ax$

l $-14m^3 - 10qm^2$

o $40z^3 - 25tz$

r $28u^3 + 21nu$

4 Factorise each of the following.

a $90v + 20$	b $63y + 117$	c $45u - 150$
d $27y - 90$	e $75v - 30$	f $16x + 2$
g $8p + 28$	h $6h + 24$	i $45t + 20$
j $70k - 168$	k $48p + 40$	l $6k + 36$
m $36u - 12$	n $60v - 105$	o $18d - 27$
p $66x + 11$	q $20u + 48$	r $28p + 16$
s $21v + 49$	t $78r - 13$	

5 Factorise each of the following.

a $49u - 7r$	b $-70x + 30z$	c $-9q + 21n$
d $75k - 15z$	e $56g - 42d$	f $30c - 25n$
g $72q - 9y$	h $36m + 40r$	i $-20h + 80c$
j $77e - 66p$	k $25e + 10a$	l $18n + 9y$
m $-6z - 2c$	n $78k - 13y$	o $20u + 12b$
p $-30d + 25t$	q $75w + 120g$	r $-135w + 60q$
s $-3q - 15w$	t $-80g + 30m$	

6 Factorise each of the following.

a $72h + 45q$	b $-112y - 98r$	c $105p - 120v$
d $-9d - 39k$	e $-81z - 18a$	f $8g + 44p$
g $-112b + 112w$	h $30g - 35u$	i $9d + 108q$
j $15c + 36w$	k $4y - 44h$	l $-18x - 33u$
m $45d - 65m$	n $-14u - 56g$	o $54e - 45g$
p $-78a + 143k$	q $18t - 27m$	r $72r - 64b$
s $-18x + 15z$	t $-24c + 36m$	

7 Factorise each of the following.

a $140g - 98y$	b $156b + 12u$	c $-105e + 7x$
d $9k + 81n$	e $-26t - 24z$	f $56w + 20e$
g $-16g + 104k$	h $-55r - 65c$	i $-30q + 70x$
j $-4g + 10u$	k $12w - 2f$	l $130h - 100p$
m $22q - 88t$	n $-63b + 91r$	o $-26q - 91u$
p $104w - 13r$	q $25p - 40c$	r $-180p + 96a$
s $72g + 108r$	t $-45e + 99f$	

8 Factorise each of the following.

a $24a - 4x + 32$	b $63q - 21t + 28$	c $72n - 32w + 8$
d $40c - 25k - 35$	e $45w - 63b - 27$	f $18q - 18y - 45$
g $2c - 14m + 10$	h $36g + 4v - 28$	i $18h - 60z - 18$
j $12w + 12k - 20$	k $8h - 20p + 24$	l $6p - 15c - 21$
m $7d - 7y - 28$	n $36v - 12q - 30$	o $32e - 64t + 32$
p $6c + 9w - 21$	q $72v + 8g + 48$	r $56w + 40w - 16$
s $21c + 39w - 18$	t $40x + 45a + 35$	

9 Factorise each of the following.

a $3r + 36y - 6u$	b $12z - 42t - 66p$
c $64k - 88b - 48e$	d $12r + 12e - 30r$
e $-14p + 8p + 16g$	f $24b - 4z - 20m$

Worked solutions

Exercise 4.3

MAT09NAWS00014

g $-15k - 50y + 50v$

i $32q - 64e + 80p$

k $-12u + 20d - 48v$

m $-48a + 18c + 42u$

o $32v + 104m - 8z$

q $20z - 52e + 24w$

s $40t + 15x - 55g$

h $-35h - 7z - 84r$

j $18g - 16x - 20u$

l $-18y - 54g - 27b$

n $45m + 99n + 36r$

p $-45n - 54b - 108z$

r $12w - 21d - 39t$

t $72e + 9t + 72y$

Worked solutions

Exercise 4.3

MAT09NAWS00014

10 Factorise each of the following.

a $42p^3 - 7gp^2$

d $36c^3 - 72vc$

g $16z^4 - 16kz^2$

j $28y^5 - 8ay^3$

m $36a^3 + 4ta$

p $-81b^6 + 63zb^2$

s $-4u^5 - 14eu^3$

b $14g^6 - 26vg^2$

e $15b^6 + 50db^2$

h $27x^5 - 90vx^4$

k $24g^7 + 20ag^3$

n $4b^2 + 14rb$

q $-72g^3 - 40zg^2$

t $54z^6 + 45xz^2$

c $42m^6 - 30hm^4$

f $-32x^5 + 40xy$

i $-4y^4 - 12gy^2$

l $14m^5 - 26wm$

o $12d^3 + 6ud$

r $-36n^7 - 8hn^4$

Worked solutions

Exercise 4.3

MAT09NAWS00014

11 Factorise each of the following.

a $48rx^3 + 12k^4rx^3 - 48x^3$

c $40c^5d^3 + 40n^5c^2d - 25n^2c^2d^2$

e $72m^5u^2d - 24m^4u^5d^2 + 80ud^2$

g $-8dv - 24vy^2 + 8d^3v^5y^5$

i $-18n^3t + 36n^3r^4t^2 - 27n^3r^4$

k $6vnm^4 - 16v^3n^3 - 2v^4m$

m $-72q^2 - 72b^3q^2w^4 + 72bq^3w$

o $63e^5q^5 - 27u^2q^3 + 18q^4$

q $72q^5r^5b^3 + 40q^5r + 32q^4r^2b^5$

s $12ta^5x^4 + 24a^3x^2 - 9a^3x^5$

b $-20r^3t^5y^2 + 25r^5t^4y^5 + 45r^3t^5y^5$

d $-18yn^4 + 15 + 27b^2y^3$

f $56q^2h^3 - 8eq^5h - 16e^3q^5h^4$

h $8u^3x^5m^3 + 24u^5x^5m^4 + 64x^3m^3$

j $12b^4d^2a^2 - 48b^3d^2a^3 - 48b^5d^5a^3$

l $-36ry^3 + 4r^2y^5n^4 - 12ry^3n$

n $7wu^5y^5 - 14w^5y^2 - 35w^3u^2y$

p $36qd + 16dm^2 + 24q^2d^2m^3$

r $-27n^2dt^5 - 12n^4t^5 + 9nd^5$

t $18n^5w^5 - 15t^2w^5 - 9t^2w^3$

Reasoning

12 Use the formulas for the circumference and area of a circle and the area of a rectangle to prove that the surface area of a closed cylinder is given by $SA = 2\pi r(r + h)$, where r is the radius and h is the height of the cylinder.

4.4 Algebraic fractions

You can have fractions involving algebraic expressions. They are called **algebraic fractions**.

Operations involving algebraic fractions are simplified in the same way as normal fractions. You do additions and subtractions using the lowest common denominator. You should put brackets around any numerators or denominators with more than one term to avoid errors with multiplications and signs.

Example 16

Simplify each of the following.

a $\frac{a+5}{3} + \frac{2a-4}{5}$

Solution

a Put brackets on the numerators.

Change to the LCD, 15.

Put over the common denominator.

Simplify the brackets.

Simplify the numerator.

b Put brackets on the numerators.

Change to the LCD, 12.

Put over the common denominator.

Simplify the brackets, taking care with signs.

Simplify the numerator.

Write in one of these forms.

b $\frac{m-5}{6} - \frac{3m-4}{4}$

$$\begin{aligned} & \frac{(a+5)}{3} + \frac{(2a-4)}{5} \\ &= \frac{5(a+5)}{5 \times 3} + \frac{3(2a-4)}{3 \times 5} \\ &= \frac{5(a+5) + 3(2a-4)}{15} \\ &= \frac{5a + 25 + 6a - 12}{15} \\ &= \frac{11a + 13}{15} \\ & \frac{(m-5)}{6} - \frac{(3m-4)}{4} \\ &= \frac{2(m-5)}{2 \times 6} - \frac{3(3m-4)}{3 \times 4} \\ &= \frac{2(m-5) - 3(3m-4)}{12} \\ &= \frac{2m - 10 - 9m + 12}{12} \\ &= \frac{-7m + 2}{12} \\ &= \frac{2 - 7m}{12} \text{ or } \frac{-(7m - 2)}{12} \end{aligned}$$

You multiply algebraic fractions by multiplying the numerators and multiplying the denominators. This might involve expansion of brackets.

Example 17

Simplify $\frac{2p-1}{3} \times \frac{3p+4}{4}$.

Solution

Put brackets on the numerators.

Multiply the fractions in the normal way.

Multiply out the brackets.

Complete the multiplications, being careful with signs.

Simplify the numerator.

$$\begin{aligned} & \frac{(2p-1)}{3} \times \frac{(3p+4)}{4} \\ &= \frac{(2p-1) \times (3p+4)}{3 \times 4} \\ &= \frac{2p(3p+4) - 1(3p+4)}{12} \\ &= \frac{6p^2 + 8p - 3p - 4}{12} \\ &= \frac{6p^2 + 5p - 4}{12} \end{aligned}$$

Animated example

Adding and subtracting fractions

MAT09NAAE00004

Video tutorial

Adding and subtracting algebraic fractions

MAT09NAVT10004

CAS TI-Nspire exercise

Algebra

MAT09NATI00004

CAS ClassPad exercise

Algebra

MAT09NACP00004

Worksheet

Simplifying expressions

MAT09NAWK00031

Puzzle sheet

Algebraic fractions

MAT09NAPS00010

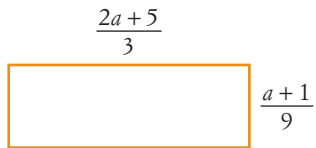
Puzzle sheet

Upside down fractions

MAT09NAPS00018

Example 18

Find an expression for the perimeter of this rectangle.



Solution

Add the sides or use the rule for the perimeter of a rectangle.

Change to the LCD, 9 and use brackets.

Put over the common denominator.

Simplify the brackets.

Simplify the numerator.

$$\begin{aligned}
 & 2 \times \frac{2a+5}{3} + 2 \times \frac{a+1}{9} \\
 &= \frac{2 \times 3(2a+5)}{3 \times 3} + \frac{2 \times (a+1)}{9} \\
 &= \frac{6(2a+5) + 2(a+1)}{9} \\
 &= \frac{12a + 30 + 2a + 2}{9} \\
 &= \frac{14a + 32}{9}
 \end{aligned}$$

You will sometimes find that the top line of an algebraic fraction can be factorised to cancel with the denominator. In Example 18, the numerator would factorise to $2(7a + 16)$, but the 2 does not cancel with the 9.

Exercise 4.4 Algebraic fractions

Fluency

- 1 Simplify each of the following.

a $\frac{2x-1}{8} + \frac{3x-5}{6}$

c $\frac{2b-2}{7} + \frac{b-5}{2}$

e $\frac{4k+5}{7} + \frac{3k-1}{5}$

g $\frac{c+2}{3} + \frac{c+3}{4}$

b $\frac{4m-4}{9} + \frac{4m+5}{3}$

d $\frac{2w-5}{5} + \frac{2w-5}{8}$

f $\frac{3p-5}{5} + \frac{4p+3}{8}$

h $\frac{4d+2}{7} + \frac{d+2}{2}$

- 2 Simplify each of the following.

a $\frac{5a+1}{5} - \frac{4a+5}{2}$

c $\frac{f+1}{8} - \frac{2f+2}{3}$

e $\frac{4t-5}{3} - \frac{t-2}{4}$

g $\frac{e-3}{10} - \frac{3e+3}{4}$

b $\frac{5n+1}{4} - \frac{5n-1}{12}$

d $\frac{q+2}{5} - \frac{q-1}{4}$

f $\frac{4b-5}{5} - \frac{2b-5}{2}$

h $\frac{4s+1}{9} - \frac{4s+1}{4}$

Extra questions

Exercise 4.4

MAT09NAEQ00015

See Example 16

3 Simplify each of the following.

a $\frac{3v+4}{5} + \frac{4v+5}{4}$

c $\frac{4y-5}{7} - \frac{y-1}{6}$

e $\frac{3u+1}{5} - \frac{3u-4}{20}$

g $\frac{5k-6}{9} - \frac{7k-3}{12}$

i $\frac{w-3}{10} - \frac{3w-4}{6}$

k $\frac{3m-5}{10} - \frac{5m+3}{15}$

b $\frac{4c+2}{7} - \frac{7c-1}{6}$

d $\frac{4g-3}{3} + \frac{g-1}{4}$

f $\frac{6z-4}{9} + \frac{7z+3}{6}$

h $\frac{j+3}{6} + \frac{4j-6}{12}$

j $\frac{3a+1}{9} - \frac{6a+5}{12}$

l $\frac{b-1}{3} + \frac{5b-7}{12}$

Worked solutions

Exercise 4.4

MAT09NAWS00015

4 Simplify each of the following.

a $\frac{6n+3}{4} \times \frac{2n-5}{12}$

c $\frac{7p+7}{5} \times \frac{p+2}{6}$

e $\frac{5q+3}{3} \times \frac{3q+5}{5}$

g $\frac{7y+7}{2} \times \frac{4y-7}{6}$

b $\frac{x-2}{2} \times \frac{x+2}{6}$

d $\frac{7t+2}{2} \times \frac{t+1}{2}$

f $\frac{2s+3}{4} \times \frac{s-2}{2}$

h $\frac{4b+4}{9} \times \frac{7b+7}{4}$

Worked solutions

Exercise 4.4

MAT09NAWS00015

See Example 17

5 Find the perimeters of the following.

a $\frac{7x-4}{5}$  $\frac{2x+1}{6}$

b $\frac{13x+2}{21}$  $\frac{3x-4}{7}$ $\frac{2x+1}{6}$ $\frac{3x+1}{3}$

c $\frac{2x-1}{4}$  $\frac{x+5}{6}$

Problem solving

Worked solutions

Exercise 4.4

MAT09NAWS00015

See Example 18

6 Find the area of the shape in question 5a.

7 Show the steps to find the areas of the shapes in questions 5b and 5c.

Reasoning

Worked solutions

Exercise 4.4

MAT09NAWS00015

Chapter 4 summary

Quiz

Algebra

MAT09NAQZ00004

- A **variable** is a letter or symbol that stands for a number. A **constant** is a number.
- An **expression** has variables and/or numbers connected by arithmetic operations like $+$, $\div$ and powers. An expression with numbers only is an **arithmetic expression**, while one with variables is an **algebraic expression**.
- The numbers in an expression that are multiplied by variables are called **coefficients**, the parts separated from the rest by $+$ or $-$ are called **terms** and a number on its own is called a **constant term**.
- To **evaluate** an algebraic expression, you **substitute** values for the variables and work out (**evaluate**) the answer.
- A **formula** is an equation with a variable on the left and an algebraic expression on the right. The variable on the left is the **subject** of the formula.
- Algebraic terms with exactly the same variables are called **like terms**. Like terms can be added or subtracted to **simplify** the expression. This is called **collecting like terms**.
- When you multiply or divide algebraic terms, the coefficients and each different variable are multiplied or divided separately.
- The **distributive law** states that the product of the sum (or difference) of two numbers with another number is the same as the sum (or difference) of the products of that number with each one separately. This can be written in symbols as:
$$a \times (b + c) = a \times b + a \times c \quad \text{and} \quad a \times (b - c) = a \times b - a \times c.$$
- **Expansion of brackets** means the removal of brackets using the distributive law. It is also called **simplification**.
- **Binomial products** are double brackets like $(x + 2)(x - 5)$. They can be expanded using the distributive law several times.
- The **highest common factor** of two algebraic terms is the largest expression that will divide exactly into the given terms.
- **Factorisation** is the opposite process to multiplying out brackets. In the **common factor** method, the highest common factor of the terms is put outside the brackets, leaving the other factors of each term inside the brackets.
- An **algebraic fraction** is a fraction that has at least one algebraic expression. They are simplified in the same way as normal fractions. The top line of an algebraic fraction sometimes factorises to make a factor that will cancel with the denominator.

Understanding

1 Choose any like terms from each group.

a $3n, 5n^2, -8nk, 4nn, 7nm^2$ **b** $7nm, 4m^2, -3mn, 6n^2m, -4nm^2, 3m^2n$

See Example 2

2 Simplify each of the following.

a $5z + 3z$ **b** $7t - 10t$ **c** $1b^2 - 6b^2$ **d** $8gk - 6gk$

See Example 3

3 Simplify each of the following.

a $4x \times 5p$ **b** $9mn \times 7np$ **c** $\frac{24t}{4}$
d $\frac{30pq}{24qt}$ **e** $3m^2 + 4m^2$ **f** $2ay^2 + 4ay + 6y^2a$

See Example 4

4 Simplify each of the following.

a $5(g + 4)$ **b** $6(7 - 4r)$ **c** $h(h + 4)$ **d** $m(8 - m)$

See Example 7

5 Factorise each of the following.

a $5v + 15$ **b** $6c - 12$ **c** $3w + 45$ **d** $9d - 36$

See Example 13

6 Expand and simplify each of the following.

a $4(e + 8) + 3(e - 9)$ **b** $5(p + 3) + 7(p + 4)$

See Example 9

7 Use the area model to simplify each of the following.

a $(3y + 8)(2y + 7)$ **b** $(3w + 2)(1w - 8)$
c $(4w - 3)(2w + 9)$ **d** $(2n - 7)(3n - 1)$

See Example 10

8 Factorise each of the following.

a $24t + 20r$ **b** $4v + 14z$
c $-25c + 10p$ **d** $-18b - 4m$

See Example 14

9 Factorise each of the following.

a $-10e^2t - 15t^2$ **b** $-12ga^3 - 18g^3a^3$
c $54k^3b + 12k^3$ **d** $-2p^2w^2 + 4p^3$

See Example 15

Fluency

10 Simplify the following where possible.

a $3h^2y - 5hy + 3y^2h + 10yh$ **b** $2m^2n^2 - m^3 - 8m^2n^2 + 9m^3$
c $24 - uv - 3uv + 9v$

11 Simplify the following.

a $-5p \times 7q$ **b** $-2k \times 8e \times -3k$
c $-12y \div -2$ **d** $28dg \div -35g$

12 Simplify the following.

a $2g(g + 7)$ **b** $-3t(7 - t)$
c $4m(2e - 5m)$ **d** $-5(3p + 4q - r)$

13 Factorise each of the following.

a $12w + 10$ **b** $15k + 10$
c $18v - 42$ **d** $20y + 16$

Chapter 4 review

See Example 8

14 Simplify the following.

a $3tz^4(4t^2 - 5)$

c $4by^2(c^3 + b^2 + 8b^2y)$

b $-11x^2b^2(1 - 3x^2b)$

d $-11rq(9r - 9hq + 10h^2rq)$

15 Expand and simplify each of the following.

a $2(r + 8) - 5(r - 7)$

c $5mc^3(4 - 2nc + n^2) - 3nc^2(2m^2n^2 - 3mnc + c^2m)$

b $-3(q - 7) - 2(6 - 4q)$

See Example 11

16 Use the distributive law to simplify each of the following.

a $(5v - 1)(v - 7)$

b $(3p + 4)(9p + 7)$

17 Factorise each of the following.

a $40t + 48d$

c $88a + 104b$

b $-56k + 84q$

d $-10y - 65m$

See Example 5

18 Simplify each of the following.

a $7a^4n^3 \times 9a^7n^4$

c $36m^5h^4 \div 4m^3h^6$

e $\frac{5m + 3m}{2t + 3t} \times \frac{18t^3 - 3t^3}{12m^2 - 10m^2}$

b $8x^5u^4 \div 4x^2u^3$

d $54m^2p^3 \div 72m^2p^{12}$

19 Use the distributive law to simplify each of the following.

a $(7r - q)(3r - 4q)$

b $(m - 8c)(7m + 5c)$

See Example 12

20 Expand and simplify each of the following.

a $(5g + 2d)(8g + 9d) + (g - 9d)(8g + 5d)$

b $(2r + 7m)(9r - 8m) - (8r + 5m)(3r - 2m)$

21 Factorise each of the following.

a $-15n^3p^3 + 3n^6p^2$

c $14n^2y^5 + 56ny^4$

b $77n^6m - 33n^4m^5$

d $-63a^4n^3 - 27a^6n$

22 Factorise each of the following.

a $40m^2ww^4 + 96m^4w^2w^6 - 56w^5$

c $-50g^5t^2h^4 - 10g^4t^4h + 10g^2t^4h^2$

b $99r^5 + 33r^4v^3h^2 - 11r^5v^5$

d $28c^6a^6 - 20g^5c^4a^5 + 4g^5c^5a^2$

See Examples 16, 17

23 Simplify each of the following.

a $\frac{2m - 4}{5} + \frac{6m - 2}{9}$

b $\frac{5q + 6}{9} - \frac{5q - 3}{6}$

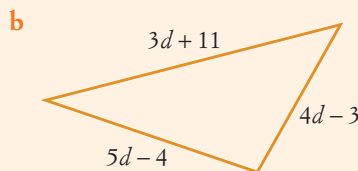
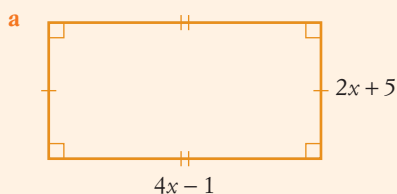
c $\frac{3y - 2}{4} \times \frac{5y + 1}{6}$

Problem solving

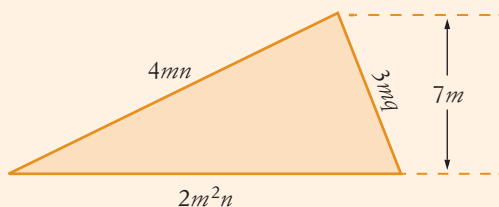
See Example 1

24 The formula for the price paid by a retailer who buys furniture direct from the manufacturer is $P = 0.011C(m + 100)$. $\$P$ is the price including GST paid by the shop for furniture that costs $\$C$ to make, assuming the manufacturer has a profit margin of $m\%$. What price does the retailer pay for chairs that cost $\$200$ each to make when the producer uses a mark-up of 40% ?

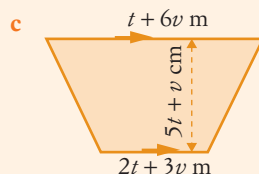
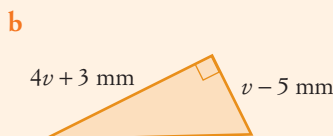
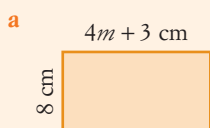
- 25 Find an expression for the perimeter of each of the following shapes.



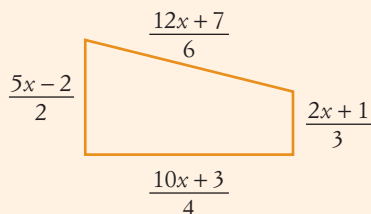
- 26 Find an expression for the area of this triangle.



- 27 Find expressions for the areas of each of the following shapes and simplify your expressions.



- 28 Find the perimeter of this shape.



- 29 Show that if two discounts of $d_1\%$ and $d_2\%$ are given in succession, then the combined discount is given by $(d_1 + d_2 - 0.01d_1d_2)$. To do this, use the distributive law to expand an expression for the price paid.
- 30 Show the logical steps to expand $(a - 7)(a^2 + 7a + 49)$ and make a generalisation for a factorisation.
- 31 Show the steps to find the area of the shape shown in question 28.

Reasoning



Number and algebra

5 Ratio and proportion



Contents

5.1 Rates

5.2 Ratios

5.3 Direct proportion

5.4 Inverse and joint proportion

Chapter summary

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Chapter 5

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Parent guide

Chapter 5

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Curriculum guide

Chapter 5

MAT09NACU00005

Australian Curriculum statements

Real numbers

Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems.

Video tutorial

Ratio and proportion

MAT09NAVT00005

We use rates, ratios and proportion in many areas of mathematics, science and technology, as well as in everyday life. Many prices, speeds and other quantities are actually rates because they compare different, related quantities. The ratio of the circumference of a circle to its diameter has a special symbol, π . Whenever we use a scale drawing or a map, we are applying a ratio to the size of the real object.

The golden ratio, ϕ , is a good guide for design. It is evident in the proportions of classical buildings and paintings. Some believe the human concept of beauty is based on proportions close to the golden ratio. The head forms a golden rectangle with the eyes at its midpoint. The mouth and nose are each placed at golden sections of the distance between the eyes and the bottom of the chin.

Mathematical literacy

Maths dictionary

MAT09ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

constant	fraction	joint proportion	speed
cross-multiplication	hyperbola	rate	unitary method
direct proportion	inverse proportion	ratio	variation
equivalent ratio			

5.1 Rates

A rate compares related measurements. The measurements that are compared within a rate are usually referred to as 'quantities'.

Important!

Rates

A **rate** is obtained by dividing one quantity by a different, related quantity. A rate can be written as a fraction. The measurement units for the quantities in a rate are separated by the word 'per' or the symbol /, with the second unit being the denominator.

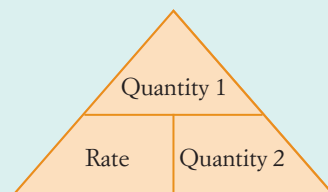
Prices, pay rates and **speeds** are the most common rates.

$$\text{Price} = \frac{\text{charge}}{\text{amount}} \quad \text{Pay rate} = \frac{\text{payment}}{\text{time}} \quad \text{Average speed} = \frac{\text{distance}}{\text{time}}$$

Rates can be used to calculate one quantity from the other.

A triangle of the rate can be used to remember the operation.

$$\begin{aligned} \text{Rate} &= \frac{\text{quantity 1}}{\text{quantity 2}} \\ \text{or Quantity 1} &= \text{rate} \times \text{quantity 2} \\ \text{or Quantity 2} &= \frac{\text{quantity 1}}{\text{rate}} \end{aligned}$$

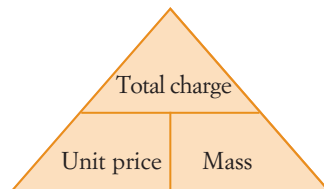


Example 1

Tomatoes cost \$5.45/kg. Calculate the mass of tomatoes you could buy for \$35.

Solution

Show the rate triangle for the price.



Use the rate triangle to write the equation.

Substitute values.

Calculate and round off.

Write the answer.

$$\begin{aligned}\text{Mass} &= \frac{\text{total}}{\text{unit price}} \\ &= \frac{\$35}{\$5.45/\text{kg}}\end{aligned}$$

$$\approx 6.42 \text{ kg}$$

You could buy about 6.42 kg of tomatoes.

CAS TI-Nspire exercise

Ratio and proportion

MAT09NATI00005

CAS ClassPad exercise

Ratio and proportion

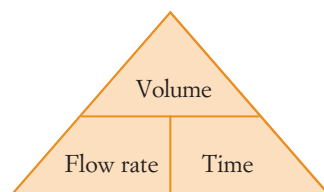
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Example 2

A recycling pump for a water feature delivers 21 kL in 3 hours. Express the flow rate in litres per second.

Solution

Show the rate triangle for the flow rate.



Use the rate triangle to write the equation.

Substitute values.

Change to the required units.

Calculate and round off.

Write the answer.

$$\begin{aligned}\text{Flow rate} &= \frac{\text{volume}}{\text{time}} \\ &= \frac{21 \text{ kL}}{3 \text{ h}} \\ &= \frac{21\,000 \text{ L}}{10\,800 \text{ s}} \\ &\approx 1.94 \text{ L/s}\end{aligned}$$

The flow rate is about 1.9 L/s.

Worksheet

Rates exercise

MAT09NAWK00041

Example 3

- a** A good athlete ran 200 m in 28 seconds.
Calculate the athlete's speed in m/s.
- b** Change the speed to km/h and comment on the result.



Solution

- a** Write the formula.

Substitute values.

Calculate and round off.

- b** Change to the required units.

Invert division to get multiplication.

Calculate and round off.

Write a realistic answer.

$$\begin{aligned}\text{Speed} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{200 \text{ m}}{28 \text{ s}} \\ &\approx 7.14 \text{ m/s}\end{aligned}$$

$$\begin{aligned}\text{Speed} &= \frac{200 \div 1000 \text{ km}}{28 \div 60 \div 60 \text{ h}} \\ &= \frac{200 \times 60 \times 60}{28 \times 1000} \text{ km/h} \\ &\approx 25.71 \text{ km/h}\end{aligned}$$

The athlete's speed was about 26 km/h over 200 m. This speed could not be maintained.

You can use rates to work out the best buy for different quantities of goods. In this case, you need to make sure that you are comparing goods of similar and acceptable quality. A cheap shoe is not necessarily a good buy if it falls apart quickly.

Example 4

Which bag of brushed potatoes is the better buy: 2 kg for \$3.80 or 5 kg for \$9.30?

Solution

Calculate the price/kg for each bag.

$$\begin{aligned}\text{Price (5 kg bag)} &= \frac{\text{charge}}{\text{amount}} \\ &= \frac{\$9.30}{5 \text{ kg}} \\ &= \$1.86/\text{kg} \\ \text{Price (2 kg bag)} &= \frac{\$3.80}{2 \text{ kg}} \\ &= \$1.90/\text{kg}\end{aligned}$$

Compare the two rates.

Write the answer.

\$1.86/kg is cheaper than \$1.90/kg.

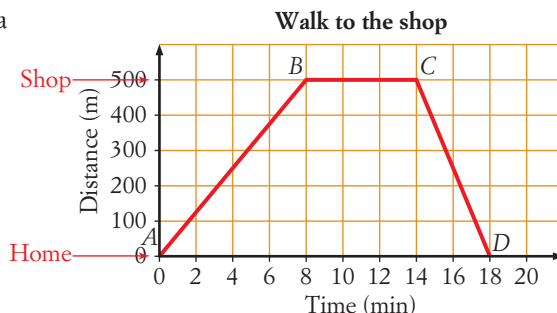
The 5 kg bag is the better buy.

Graphs can be used to find or compare rates. The graph in Example 5 shows the distance covered by Peta and Jose when they walked to the shop, 500 m from their home, and returned. On this graph, the distance from home is shown vertically and time is shown horizontally.

Example 5

Use the graph on the right showing Peta and Jose's walk to work out:

- a how fast they walked to the shop
- b how long they stayed at the shop
- c how fast they walked back home.



Solution

- a Write down the distance to the shop.

Write down the time to the shop.

Calculate the speed.

Write the answer.

- b They were at the shop from B to C.

Write the answer.

- c Write down the distance from the shop.

Write down the time from the shop.

Calculate the speed.

Write the answer.

$$\text{Distance } AB = 500 \text{ m}$$

$$\text{Time } AB = 8 \text{ minutes}$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{500 \text{ m}}{8 \text{ min}}$$

$$= 62.5 \text{ m/min}$$

They walked to the shop at 62.5 m/min.

$$\text{Time from } B \text{ to } C = 14 - 8$$

$$= 6 \text{ minutes}$$

They were at the shop for 6 minutes.

$$\text{Distance } CD = 500 \text{ m}$$

$$\text{Time } CD = 4 \text{ minutes}$$

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{500 \text{ m}}{4 \text{ min}}$$

$$= 125 \text{ m/min.}$$

They walked back home at 125 m/min.

Worksheet

Distance time graphs

MAT09NAWK00042

TLF Learning object

Mobile phone plans:
several companies
(L1110)

MAT09NAIN00005

TLF Learning object

Triathlon: assessment
(L8274)

MAT09NAIN00005

Exercise 5.1 Rates

Understanding

Extra questions

Exercise 5.1

MAT09NAEQ00016

See Example 2

- 1 Express each of the following rates in the units indicated.
 - a Unit cost if \$15.60 buys 12 kg of potatoes (\$/kg)
 - b Unit cost if 2.4 m of timber costs \$9.00 (\$/m)
 - c Petrol consumption if 480 km is travelled on 60 L (L/100 km)
 - d Average speed if 260 km is travelled in 4 h (m/s)
 - e Heating rate if temperature rises by 60°C in 24 minutes (°C/min)
 - f \$120 for 375g (\$/kg)
 - g \$740 for 20 people (\$/head)
 - h 256 runs for 8 wickets (runs/wicket)
 - i 220 g for 5.5 m² (g/m²)
 - j 64 runs from 80 balls (runs/over), if an over is 6 balls.

See Example 2

- 2 Change the following rates to the units indicated.
 - a 20 m/s to km/h
 - b 16 km/h to m/s
 - c 20 L/s to kL/h
 - d 7.2 m/s to km/h
 - e 52 kL/h to L/s
 - f 14 ML/week to L/min
 - g \$28/kg to cents/g
 - h 15 cents/g to \$/kg
 - i 96 km/h to m/s

Fluency

- 3 Water flows in a creek at 28 kL/h.
 - a How much water will flow in 5 hours?
 - b How long would it take for 600 kL to pass?
 - c How long would it take for 800 L to pass?
- 4 Isabella bought 5.6 kg of bananas for \$13.95.
 - a What was the unit cost (\$/kg)?
 - b How much would 2.1 kg cost?
- 5 An athlete jogged 5 km in 20 minutes. What was the athlete's speed in:
 - a m/s?
 - b km/h?

Worked solutions

Exercise 5.1

MAT09NAWS00016

See Example 3

Problem solving

- 6 Here are the charges for a car rental company.

Car type	Insurance	Daily charge	Mileage charge
Large car	\$60	\$89	35c per km
Medium car	\$40	\$69	
Small car	\$14	\$59	

Find the cost of renting:

- a a large car for 4 days if 1500 km is travelled
- b a small car for 7 days if 800 km is travelled
- c a medium car for 3 days if 570 km is travelled.

- 7 A car's petrol consumption is 15 L/100 km in the city, and 9.6 L/100 km on long trips.
- How much petrol would be used in a country drive from Brisbane to Bundaberg (450 km)?
 - How much would the petrol cost to go from Ipswich to the Brisbane suburb of Bulimba (32 km) and back, if petrol costs \$1.65/L?

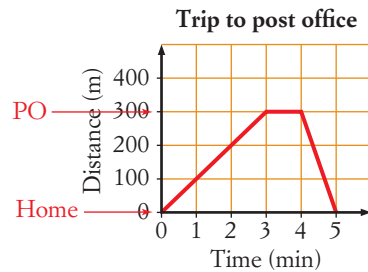
- 8 Choose the better buy from each of the following, assuming that quality is similar.

See Example 4

- Decaf coffee: 150 g for \$9.12 or 200 g for \$11.60
- Free-range chickens: 1.6 kg for \$11.60 or 1.8 kg for \$13.80
- Pineapple pieces: 450 g for \$2.30 or 240 g for \$1.12
- Sausages: 750 g for \$5.98 or 2.5 kg for \$17.50

- 9 The distance-time graph on the right shows Debbie's trip to the post office near her home.

- Find how fast she went to the post office.
- How long did she stay at the post office?
- What was the total distance travelled?
- What was her speed on the way home?
- Did she walk or run home? Explain.



Worked solutions

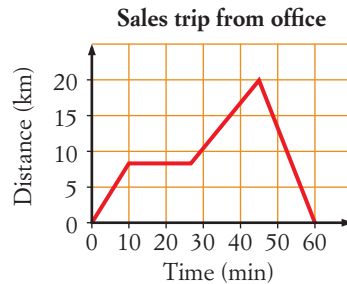
Exercise 5.1

MAT09NAWS00016

See Example 5

- 10 This distance-time graph shows the travels of a sales representative from his office.

- What was the farthest he travelled from his office?
- How long did the entire journey take?
- Find the average speed for the first 10 minutes.
- Find the average speed for the last 15 minutes.
- What happened between the 10th and 25th minutes?
- What was the average speed for the entire trip?



See Example 5

- 11 The stages of a trip made by Sandra to see her friend Alphonse were as follows.

- Sandra left home and travelled at 100 m per minute for 3 minutes.
- She stopped to buy a drink. This took 5 minutes.
- She travelled for another 2 minutes at 250 m/min to reach Alphonse's house.
- She talked to him for 15 minutes.
- She returned home at 400 m/min.

Use the information to draw a distance-time graph for Sandra's trip.

Reasoning

Worked solution

Exercise 5.1

MAT09NAWS00016

5.2 Ratios

Important!

Ratios

A **ratio** compares quantities of the *same kind* in a definite order, such as 5 : 8 lollies.

A ratio is stated without units, using

- the word 'to', as in 8 to 5
- the symbol :, as in 8 : 5, or
- **fraction** notation with the second quantity as the denominator, as in $\frac{8}{5}$.

Ratios are normally expressed using whole numbers, but to compare the size of ratios it is easier to express them as decimals; for example, $8 : 5 = 1.6$ or $1.6 : 1$ or $1 : 0.625$.

Ratios that are the same are called **equivalent ratios**.

Example 6

Weblink

All about ratios

MAT09NAWB00005

Write the ratio of apples to bananas.



Solution

Write the ratio.

$$\text{Apples : bananas} = 3 : 6$$

Simplify by dividing both parts of the ratio by 3.

$$= 1 : 2$$

Example 7

Lucy's favourite cheesecake contains 250 mL of cream cheese, 125 mL of cream and 50 mL of orange juice. Write the ingredients as a ratio in simplest form.

Solution

Write the ratio.

$$\text{Cheese : cream : juice} = 250 : 125 : 50$$

Simplify by dividing each part of the ratio by 25.

$$= 10 : 5 : 2$$

Example 8

Write these ratios in simplest form.

a $18 : 24$

d $2\frac{1}{4} : 2\frac{2}{5}$

b $80 \text{ cm} : 2 \text{ m}$

e $4\frac{3}{8} : 6\frac{2}{3}$

c $1.2 : 0.4$

Solution

a Write the ratio.

Divide each part of the ratio by the HCF, 6.

$$18 : 24$$

$$= 18 \div 6 : 24 \div 6$$

$$= 3 : 4$$

b Write the ratio.

Change to the same units.

Write as a fraction and cancel by the HCF, 40.

$$80 \text{ cm} : 2 \text{ m}$$

$$= 80 : 200$$

$$= \frac{80^2}{200^5}$$

$$= 2 : 5$$

Write in ratio form.

c Write the ratio.

Multiply each part of the ratio by 10 to get rid of the decimals.

Divide by 4.

$$1.2 : 0.4$$

$$= 12 : 4$$

$$= 3 : 1$$

d Write the ratio.

Write as improper fractions.

Multiply each part of the ratio by the LCD, 20 (i.e. $\frac{20}{1}$), and cancel within each part of the ratio.

Simplify.

Divide by 3.

$$2\frac{1}{4} : 2\frac{2}{5}$$

$$\frac{9}{4} : \frac{12}{5}$$

$$= \frac{9}{\cancel{4}^1} \times \frac{\cancel{20}^5}{1} : \frac{12}{\cancel{5}^1} \times \frac{\cancel{20}^4}{1}$$

$$= 45 : 48$$

$$= 15 : 16$$

e Write the ratio.

Write as improper fractions.

Multiply each part of the ratio by the LCD, 24, and cancel within each part of the ratio.

Simplify.

Divide by 5.

$$4\frac{3}{8} : 6\frac{2}{3}$$

$$\frac{35}{8} : \frac{20}{3}$$

$$= \frac{35}{\cancel{8}^3} \times \frac{\cancel{24}^3}{1} : \frac{20}{\cancel{3}^8} \times \frac{\cancel{24}^8}{1}$$

$$= 105 : 160$$

$$= 21 : 32$$

Puzzle sheet

Simplifying ratios

MAT09NAPS00012

Technology worksheet

Excel worksheet:
Simplifying ratios

MAT09NACT00022

Puzzle sheet

Simplifying ratios with
fractions

MAT09NAPS00013

Weblink

The Golden Ratio

MAT09NAWB00005

Example 9

In 2010, the numbers of prisoners in Victorian, Western Australian and Queensland gaols were as follows.

	Vic	WA	Qld
Male	4223	4367	5168
Female	314	404	449

Which state had the highest ratio of female : male prisoners?

Solution

Write the ratio for each state.

Vic WA Qld
 $314 : 4223$ $404 : 4367$ $449 : 5168$

Write as decimals.

$\approx 0.0744 : 1$ $\approx 0.0925 : 1$ $\approx 0.0869 : 1$

Write the answer.

Western Australia had the highest ratio of female : male prisoners.

In cases that require a quantity to be divided into shares in a particular ratio, it is simplest to use the **unitary method**, finding the value of one part of the total parts of a ratio.

Important!**Unitary method**

A unitary method involves two main steps:

- 1 Calculate the value of a single part of the ratio.
- 2 Multiply this value by (each) the number of parts to calculate the result(s).

Example 10

Antonio and Gabriella run a craft shop. Antonio works for 20 hours a week and Gabriella works for 35 hours. Their profit one week was \$1402.50. How much should each person receive as their fair share?

**Solution**

Write the ratio of hours worked.

Antonio : Gabriella = 20 : 35

Simplify.

= 4 : 7

Puzzle sheet

Dividing an amount in
a given ratio

MAT09NAPS00014

TLF Learning object

Vitality drinks:
assessment (L8867)

MAT09NAIN00005

Work out the total number of parts.
Calculate the amount for one part.

Calculate the first share.

Calculate the second share.

Check total.

Write the answer.

$$\text{Total parts} = 4 + 7 = 11$$

$$\begin{aligned}\text{Value of one part} &= \$1402.50 \div 11 \\ &= \$127.50\end{aligned}$$

$$\begin{aligned}\text{First share} &= 4 \text{ parts} \\ &= 4 \times \$127.50 = \$510\end{aligned}$$

$$\begin{aligned}\text{Second share} &= 7 \text{ parts} \\ &= 7 \times \$127.50 \\ &= \$892.50\end{aligned}$$

$$\begin{aligned}\text{Total amount} &= \$510 + \$892.50 \\ &= \$1402.50 \checkmark \text{ OK}\end{aligned}$$

Antonio should get \$510 and
Gabriella should get \$892.50.

Exercise 5.2 Ratios

1 Use the photograph to write the ratio of:

- a exercise books : pens
- b black pens : rulers
- c rulers : exercise books
- d red pens : black pens
- e blue pens : rulers.



2 Write the following ratios in simplest form.

a 20 : 8	b 14 : 28	c 24 cm : 18 mm
d 250 kg : 3 t	e 10 cm : 1 m	f $5\frac{1}{2} : 3\frac{3}{4}$
g 6.3 : 7.5	h 8.4 : 2.4	i $3\frac{1}{2} : 2\frac{1}{10}$
j 18 : 24	k 12 : 15	l 0.45 : 0.15
m 1.08 : 4.5	n $7\frac{1}{2} : 2\frac{2}{5}$	o $1\frac{1}{2} : 5\frac{1}{4}$
p \$3.50 : 70c	q 140 s : 1.5 min	r 12 days : 4 weeks
s 42 : 14	t 6.3 : 1.8	

3 Divide each of the following quantities in the given ratio.

a \$80 in the ratio 5 : 3	b 15 L in the ratio 2 : 1
c 500 cm^3 in the ratio 1 : 4	d \$96 in the ratio 11 : 4
e 55 kg in the ratio 2 : 9	f 128 m in the ratio 3 : 5
g \$90 in the ratio 5 : 4	h \$120 in the ratio 11 : 13
i \$60 in the ratio 7 : 8	

Understanding

Extra questions

Exercise 5.2

MAT09NAEQ00017

See Example 6

See Example 8

Worked solutions

Exercise 5.2

MAT09NAWS00017

Fluency

- 4 A nut mixture claims to have 200 g of peanuts, 100 g of macadamias and 80 g of cashews. Write the ratio of nuts in simplest form.

See Example 7

5 Write each of the following ratios in simplest form.

a $15 : 25 : 30$

b $16 : 24 : 10$

c $25 \text{ cm} : 1.5 \text{ m} : 70 \text{ cm}$

d $6.5 : 3 : 4.5$

e $5\frac{2}{3} : 7\frac{5}{6}$

f $4\frac{3}{4} : 5\frac{1}{2} : 2\frac{7}{8}$

g $14.8 : 6.2 : 2.4$

h $2.8 \text{ kg} : 500 \text{ g} : 1 \text{ kg}$

i $3 \text{ h} : 4 \text{ h } 20 \text{ min} : 1 \text{ h } 40 \text{ min}$

6 Choose the larger ratio from each of the following pairs.

a $6 : 14$ and $7 : 16$

b $21 : 48$ and $43 : 89$

c $1534 : 346$ and $18412 : 379$

d $42 : 65$ and $56 : 80$

Problem solving

7 Consider two squares as shown.

a Write the ratio of their side lengths.

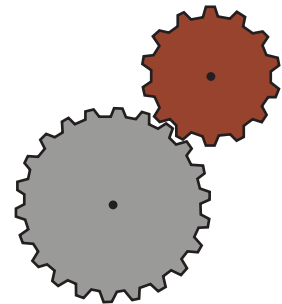
b Write the ratio of their perimeters.

c Write the ratio of their areas.



8 Two cogs, *A* and *B*, are positioned as shown. Copy and complete the table below.

	Number of teeth		Ratio of teeth	Number of full turns	
	Cog <i>A</i>	Cog <i>B</i>		Cog <i>A</i>	Cog <i>B</i>
a	20	40	$1 : 2$	2	1
b	10	?	$1 : 3$	6	2
c	9	36	?	20	?
d	12	60	?	?	7
e	32	8	?	?	40
f	21	?		11	33
g	?	56	?	8	2

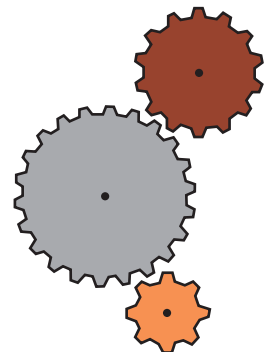


9 Katrina and Andrew bought a 1.5 kg box of chocolates. Andrew put in \$5.60 while Katrina contributed \$6.40. What mass of chocolates should Katrina receive?

10 Two numbers are in the ratio $7 : 2$. If their sum is 81, what are the numbers?

11 Three cogs, *A*, *B* and *C*, are positioned as shown. Copy and complete the table below.

	Number of teeth			Ratio of teeth	Number of full turns		
	Cog <i>A</i>	Cog <i>B</i>	Cog <i>C</i>		Cog <i>A</i>	Cog <i>B</i>	Cog <i>C</i>
a	30	20	10	$3 : 2 : 1$	2	3	6
b	5	10	20	?	?	?	3
c	9	24	12	?	?	?	18
d	?	?	24	$2 : 5 : 3$	15	?	?
e	?	24	?	?	6	2	3
f	?	?	30	?	3	10	9
g	35	?	?	$7 : 2 : 5$	10	?	?



12 A mix of concrete has cement, sand and gravel in the ratio $2 : 5 : 8$ by mass. How many kilograms of each will be needed to make 3 t of concrete?

Worked solutions

Exercise 5.2

MAT09NAWS00017

Worksheet

Ratio

MAT09NAWK00043

- 13 A fruit cake contains sultanas, mixed peel and dates in the ratio 5 : 1 : 4 by mass. If 250 g of sultanas are used, what mass of each of the other two ingredients will be needed?



See Example 10

- 14 Iceland, Ireland and Cuba have populations of 269 000, 20.2 million and 11 million respectively. Their areas are 103 000 km², 70 300 km² and 111 000 km² respectively.
- Determine the order of size for the population ratios
 - Iceland : Ireland
 - Iceland : Cuba
 - Cuba : Ireland.
 - Do the same for their area ratios.
 - Which island has the greatest *population density* (population/km²)?

Worked solutions

Exercise 5.2

MAT09NAWS00017

See Example 9

5.3 Direct proportion

Quantities that always increase or decrease in the same ratio (by the same factor) are said to be in direct proportion.

Important!

Direct proportion

Quantities x and y are **directly proportional** to each other if, whenever one quantity changes, the other changes by the same factor. If x is the independent variable, then we write

$$y \propto x$$

When two quantities are in direct proportion, the ratios of the corresponding quantities in direct proportion are equal. This can be written as

$$\frac{x_1}{x_2} = \frac{y_1}{y_2}$$

We say that ratios are *in proportion* if they are equivalent. We can use **cross-multiplication** to check whether ratios are in proportion. If ratios are in proportion, then

$$x_1 y_2 = x_2 y_1$$

For quantities x and y in direct proportion,

$$y = kx$$

where the rate k is called the **constant of proportion** or **constant of variation**.

The graph of quantities in direct proportion is a straight line passing through the origin. Problems involving direct proportion can be solved by using ratios, the constant of proportion or the unitary method.

Puzzle sheet

Rolling a coin

MAT09NAPS00015

Example 11

An old grandfather clock has two gears that turn in direct proportion to one another. The ratio of turns of the large gear to the smaller gear is 4 : 5. In 1 minute the smaller gear turns 15 times. How many times does the large gear turn?

**Solution****Ratio method**

Write the ratio equation.

Use y for large gear turns and x for small.

Substitute values.

Cross-multiply to find large_2 .

Write the answer.

Proportion constant method

Write the proportion equation.

Use y for large gear turns and x for small.

Substitute the ratio values.

Find the value of k .

Write the equation again.

Substitute values.

Write the answer.

Unitary method

Compare quantities.

Find the large turns for 1 small turn.

Multiply to find the number of large turns for 15 small turns.

Write the answer.

$$\frac{y_2}{y_1} = \frac{x_2}{x_1}$$

$$\frac{\text{large}_2}{\text{large}_1} = \frac{\text{small}_2}{\text{small}_1}$$

$$\frac{\text{large}_2}{4} = \frac{15}{5}$$

$$5 \times \text{large}_2 = 4 \times 15$$

$$5 \times \text{large}_2 = 60$$

$$\text{large}_2 = 60 \div 5$$

$$= 12 \text{ turns}$$

The large gear turns 12 times.

$$y = kx$$

$$\text{large} = k \times \text{small}$$

$$4 = k \times 5$$

$$k = 0.8$$

$$\text{large} = k \times \text{small}$$

$$= 0.8 \times 15$$

$$= 12 \text{ turns}$$

The large gear turns 12 times.

$$\begin{aligned} \text{Large turns for 1 small turn} &= 4 \div 5 \\ &= 0.8 \end{aligned}$$

$$0.8 \times 15 = 12$$

The large gear turns 12 times.

The ratio method, or a graph, is most commonly used to show that quantities are in direct proportion, while the other methods are mostly used to solve problems.

Example 12

The current (A) through a resistor is proportional to the voltage (V) in the circuit. At 18 volts, a current of 1.2 amps passes.

- a** What current would pass at a voltage of 25 volts?
- b** What voltage is needed to produce a current of 5 amps through the resistor?

Solution

- a** Write the proportion equation.

$$A = kV$$

Substitute values to find k .

$$1.2 = k \times 18$$

$$k = 1.2 \div 18$$

$$\approx 0.0667$$

Keep the exact value on your calculator.

Write the equation again.

$$A = kV$$

Substitute for V , using the exact value of k .

$$\approx 0.0667 \times 25$$

$$\approx 1.67 \text{ amps}$$

Write the answer.

The current would be about 1.67 amps.

- b** Write the equation.

$$A = kV$$

Substitute for A , using the exact value of k .

$$5 \approx 0.0667 \times V$$

Rearrange to find V .

$$V \approx \frac{5}{0.0667}$$

$$= 75 \text{ volts}$$

Write the answer.

75 volts is needed.

Video tutorial

Direct proportion

MAT09NAVT10021

Puzzle sheet

Direct proportion

MAT09NAPS00016

CAS TI-Nspire exercise

Ratio and proportion

MAT09NATI00005

CAS ClassPad exercise

Ratio and proportion

MAT09NACP00005

Investigate: Shoe size and height

For this investigation you will need to measure the heights of everybody in your class. You will also need to check the shoe size for every person.

Draw a graph showing shoe sizes and heights. Each person in the class will be represented by a point on the graph, which should list shoe size across a row and height down a column. It is useful to put all the boys in one colour and all the girls in another. What do you find about the relationship between shoe size and height?



Example 13

Determine whether the following pairs of ratios are in proportion (equivalent).

a $4 : 6$ and $12 : 20$

b $14 : 10$ and $\frac{21}{15}$

c $6a : 27$ and $4ab : 18b$

Solution

a Write the ratios as fractions.

Cross-multiply.

Write the answer.

b Write the ratios as fractions.

Cross-multiply.

Write the answer.

c Write the ratios as fractions.

Cross-multiply.

Write the answer.

$$\frac{4}{6} \text{ and } \frac{12}{20}$$

$$\frac{4}{6} \text{ and } \frac{12}{20}$$

$$4 \times 20 = 80 \text{ and } 6 \times 12 = 72$$

The cross-products are different, so the ratios are not in proportion.

$$\frac{14}{10} \text{ and } \frac{21}{15}$$

$$\frac{14}{10} \text{ and } \frac{21}{15}$$

$$14 \times 15 = 210 \text{ and } 10 \times 21 = 210$$

The cross-products are the same, so the ratios are in proportion.

$$\frac{6a}{27} \text{ and } \frac{4ab}{18b}$$

$$\frac{6a}{27} \text{ and } \frac{4ab}{18b}$$

$$6a \times 18b = 108ab \text{ and } 27 \times 4ab = 108ab$$

The cross-products are the same, so the ratios are in proportion.

Example 14

Find the values of the variables in the following proportions.

a $\frac{8}{a} = \frac{12}{18}$

b $15 : a : 3 = m : 6 : 1$

Solution

a Write the proportion.

Cross-multiply, putting the unknown on the left.

Simplify.

Solve.

b Write as two proportions.

Cross-multiply, putting the unknowns on the left.

Simplify and solve.

$$\frac{8}{a} = \frac{12}{18}$$

$$a \times 12 = 8 \times 18$$

$$12a = 144$$

$$a = 12$$

$$\frac{a}{3} = \frac{6}{1} \text{ and } \frac{15}{3} = \frac{m}{1}$$

$$a \times 1 = 3 \times 6 \text{ and } 3 \times m = 15 \times 1$$

$$a = 18 \text{ and } m = 5$$

Problems involving direct proportion are most easily solved using the constant of variation, although they can also be solved using ratios or the unitary method.

Example 15

For a gas at constant pressure, the volume ($V \text{ cm}^3$) varies directly with the temperature ($T^\circ\text{K}$). (The temperature must be in kelvins.) Suppose that there are 2000 cm^3 of gas at 400°K .

- a** What volume would be present if the temperature was reduced to 300°K ?
b At what temperature would 500 cm^3 of gas be present?

Solution

- a** Write the proportion equation.

$$V = kT$$

Substitute values to find k .

$$2000 = k \times 400$$

Solve for k .

$$k = 5 \text{ cm}^3/\text{K}$$

Write the equation again.

$$V = kT$$

Substitute for k and T .

$$= 5 \times 300$$

Evaluate V .

$$= 1500 \text{ cm}^3$$

Write the answer.

The volume would be 1500 cm^3 .

- b** Write the equation.

$$V = kT$$

Substitute for k and V .

$$500 = 5 \times T$$

Solve for T .

$$T = \frac{500}{5}$$

$$= 100$$

Write the answer.

The temperature would be 100°K .

Exercise 5.3 Direct proportion

- 1 Determine whether the following pairs of ratios are in proportion.

a $2 : 3$ and $6 : 9$

b $10 : 8$ and $5 : 4$

c $\frac{40}{28}$ and $16 : 14$

d $4 : 9$ and $\frac{20}{45}$

e $3 : 15$ and $\frac{9}{42}$

f $2.5 : 1.5$ and $10 : 6$

- 2 Determine whether the following pairs of ratios are in proportion.

a $8a : 16$ and $12a : 24$

b $\frac{4}{6m}$ and $\frac{14}{21m}$

c $\frac{20c}{12c}$ and $18c : 8c$

d $18x : 4x$ and $\frac{27}{6}$

e $3pq : 8p$ and $12qt : 36qt$

f $\frac{2y}{5x}$ and $\frac{8y^2}{20xy}$

- 3 Copper pipe is often used in plumbing.

- a** Copy and complete the following table of mass (M) and length (L) of piping.

$M \text{ (kg)}$	1	2	3	4	5	6	7	8
$L \text{ (m)}$	4	8						

Understanding

Extra questions

Exercise 5.3

MAT09NAEQ00018

See Example 13

See Example 13

Fluency

- b** Construct a graph using the information from your completed table.
c From your graph, are the quantities directly proportional?
d What mass would 22 m of pipe have?
e How many kilograms of copper would there be in 15 m of pipe?
f What length of pipe would have a mass of 4.5 kg?
g How long would a 6.25 kg piece of pipe be?
- 4** The distance (d) travelled by a cyclist and the number of pedal turns (n) are shown in the table below.

Number of pedal turns, n	1	2	3
Distance, d (m)	2.5	5	7.5

- a** Copy and complete this table for values up to $n = 10$.
b Draw a graph of the relationship between the two quantities.
c From your graph, are the quantities directly proportional?
d What distance would be covered after 7 pedal turns?
e How far would the cyclist travel in $9\frac{1}{2}$ pedal turns?
f How many pedal turns would there be if the bicycle travelled 20 m?
g How many times would the pedals turn if the bicycle travelled 14.5 m?



See Example 14

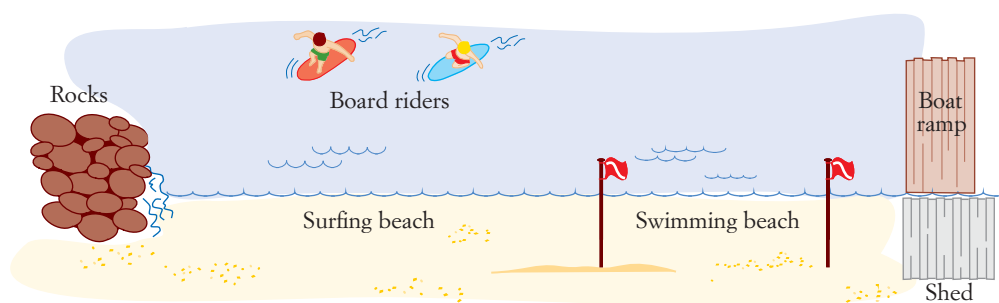
- 5** Find the values of the variables in the following proportions.
- a** $\frac{b}{1.5} = \frac{3}{5}$ **b** $\frac{5.4}{9} = \frac{2.1}{2m}$ **c** $3 : 12 : x = 6 : y : 4$
d $\frac{7}{3m} = \frac{14}{18}$ **e** $x : 3 : 6 = 15 : 5 : 10$ **f** $x : 4 : 2 = 18 : y : 6$
- 6** Find the constant of variation in each of the following cases.
- a** $m \propto p$ and $m = 42$ when $p = 14$ **b** $y \propto x$ and $y = 20$ when $x = 8$
c $b \propto a$ and $b = 18$ when $a = 3$ **d** $g \propto m$ and $g = 24$ when $m = 5$
e $y \propto x$ and $y = -6$ when $x = -2$ **f** $f \propto h$ and $f = -8$ when $h = 5$

Problem solving

See Example 11

- 7** Use direct proportion to solve the following.
- a** The ratio of children to adults at a theatre is $7 : 4$. If there are 96 adults, how many children are there?
b A street directory is drawn to a scale of $1 : 25\,000$. How far would 8 cm on the map actually be?
c In a recipe, sugar and flour are used in the ratio $2 : 3$. How much flour should be added if 250 g of sugar is used?
d The ratio of red to yellow Smarties in a packet is $12 : 7$. If there are 91 yellow Smarties, how many red ones are there?
e A hairdressing salon serves 8 female customers for every 3 male customers. In one week there were 54 males. How many females were there in that week?

- 8 The amount of medicine given to a patient varies according to the mass of the patient. 5 mL is given for every 10 kg of body mass. Find the correct doses for people with the following masses.
a 60 kg b 25 kg c 47 kg
- 9 A truck uses 17 L of diesel fuel for every 100 km travelled. How much fuel would be used on a 1200 km journey?
- 10 Libby and Mario run a catering business. They know that for a luncheon for 40 people they need 55 chicken wings. How many chicken wings should they prepare for a luncheon with:
a 120 people? b 25 people? c 283 people?
- 11 A landscape service charges \$500 to supply and lay 120 m^2 of turf. Any quote given is proportional to the area to be covered.
a How much would a 350 m^2 lawn cost?
b What would it cost for a 0.24 ha lawn? ($1 \text{ ha} = 10\,000 \text{ m}^2$)
- 12 A photograph measuring 12 cm by 18 cm is to be enlarged so that its longer side will be 25 cm. What will its shorter side measure, to the nearest centimetre?
- 13 An old grandfather clock lost 5 seconds in 32 hours. How much time will it lose in a week if it continues to lose at the same rate?
- 14 The size of a real object is directly proportional to its size in a scale drawing. The length of the beach in this scale drawing is actually 100 m from the rocks to the boat ramp.
a How far apart are the flags?
b What length of beach is available to surfboard riders?
c How far from the beach are the board riders?



- 15 If constant braking pressure is applied, the time taken to stop a car is proportional to its speed. A car travelling at 80 km/h takes 4.5 seconds to stop completely. How long would it take the car travelling at 120 km/h to stop?
- 16 When two of the floodgates of a dam are open, the water level drops by 86 cm in a day. How much would the level drop in a day if six floodgates were left open?



- ## Worked solutions

Exercise 5.3

MAT09NAWS00018

- See Example 12

- See Example 15

-

Reasoning

- ## Worked solutions

Exercise 5.3

MAT09NAWS00018

- 28** A photographic slide measures 24 mm by 36 mm. It is to be projected onto a screen measuring 1.22 m by 1.38 m. What will be the dimensions of the largest possible image of the slide on the screen? (The *whole* image must be on the screen.)

5.4 Inverse and joint proportion

The faster you run a 100 m race, the less time it takes to reach the finish line. As your speed increases, the time decreases. This is an example of **inverse proportion**. Inverse proportion and joint variation are outside the scope of this course. However, they may help your understanding of direct proportion. Science contains many examples of inverse proportion and joint variation.

Important!

Inverse proportion

Quantities x and y are **inversely proportional** to each other if, whenever one quantity changes, the other changes by the inverse factor. If x is the independent variable, then

$$y \propto \frac{1}{x}$$

When two quantities are in inverse proportion, the ratios of the corresponding quantities are opposite to each other. This can be written as

$$\frac{x_1}{x_2} = \frac{y_2}{y_1}$$

We can multiply ratios to check whether they are inversely proportional. If ratios are in inverse proportion,

$$\frac{x_1}{x_2} \times \frac{y_1}{y_2} = 1$$

A **constant of inverse proportion** (or **constant of inverse variation**) can be used to write an equation from the proportion, so

$$y = k \times \frac{1}{x} = \frac{k}{x}$$

This can also be written as $xy = k$, so the product of quantities in inverse proportion is the constant.

Example 16

Determine whether the following pairs of ratios are inversely proportional.

- a** $2 : 8$ and $24 : 6$ **b** $15 : 10$ and $\frac{18}{24}$ **c** $9p : 21$ and $14q : 6pq$

Solution

- a** Write the ratios as fractions.

$$\frac{2}{8} \text{ and } \frac{24}{6}$$

Multiply the ratios.

$$\frac{2}{8} \times \frac{24}{6} = \frac{48}{48} = 1$$

Write the answer.

The product is 1, so the ratios are in inverse proportion.

- b Write the ratios as fractions.

$$\frac{15}{10} \text{ and } \frac{18}{24}$$

Multiply the ratios.

$$\frac{15}{10} \times \frac{18}{24} = \frac{270}{240} \neq 1$$

Write the answer.

Product is not 1, so the ratios are not in inverse proportion.

- c Write the ratios as fractions.

$$\frac{9p}{21} \text{ and } \frac{14q}{6pq}$$

Multiply the ratios.

$$\frac{9p}{21} \times \frac{14q}{6pq} = \frac{126pq}{126pq} = 1$$

Write the answer.

The product is 1, so the ratios are in inverse proportion.

Example 17

Technology

Geogebra: Inverse proportion

MAT09NATC00004

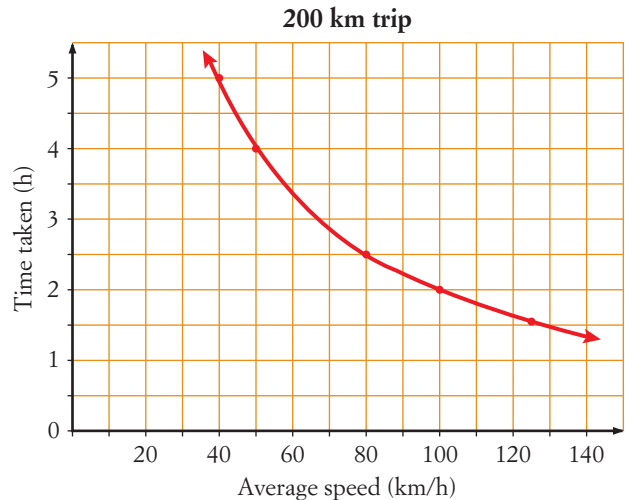
The times taken for a new car to complete a 200 km trip at different average speeds are shown in the table below. Draw a graph of time against speed and describe the graph.

Average speed (km/h)	40	50	80	100	125
Time taken (h)	5	4	2.5	2	1.6

Solution

Draw the graph.

Remember that time against speed means that speed is on the horizontal axis and time is on the vertical axis.



Describe the graph.

The graph is a curve that bulges towards the origin.

The graph of inversely proportional quantities is *not a straight line*. It is a **hyperbola**. The products of variables can be used to show that they are inversely proportional.

Example 18

Determine which of the following show inverse variation. If applicable, find the constant of inverse variation.

a

x	2	5	10	20
y	60	24	12	6

b

p	3	6	10	20
q	24	12	8	4

Solution

- a** Find the products of the variables.

$$\begin{aligned}x_1y_1 &= 2 \times 60 = 120 \\x_2y_2 &= 5 \times 24 = 120 \\x_3y_3 &= 10 \times 12 = 120 \\x_4y_4 &= 20 \times 6 = 120\end{aligned}$$

Write the conclusion.

The product of the variables is constant, so they are inversely proportional and $k = xy = 120$.

- b** Find the products of the variables.

$$\begin{aligned}p_1q_1 &= 3 \times 24 = 72 \\p_2q_2 &= 6 \times 12 = 72 \\p_3q_3 &= 10 \times 8 = 80 \\p_4q_4 &= 20 \times 4 = 80\end{aligned}$$

Write the conclusion.

The product of the variables is not constant, so they are not inversely proportional.

Example 19

The time that a container of molasses will last depends inversely on how many cattle are being fed molasses. A container lasts 20 cattle 6 days. How long will a container last 12 cattle?

Solution

Let t represent time (days) and c the number of cattle.

Write the inverse proportion.

$$t \propto \frac{1}{c}$$

Write the inverse proportion equation.

$$t = \frac{k}{c} \text{ or } k = ct$$

Substitute the first values to find k .

$$k = 20 \times 6$$

Evaluate.

$$= 120$$

Write the inverse proportion equation again.

$$t = \frac{k}{c}$$

Substitute for k and c to find t , and evaluate.

$$= \frac{120}{12} = 10 \text{ days}$$

Write the answer.

The container will last 12 cattle for 10 days.

Puzzle sheet

Inverse proportion

MAT09NAPS00017

CAS TI-Nspire exercise

Ratio and proportion

MAT09NATI00005

CAS ClassPad exercise

Ratio and proportion

MAT09NACP00005

Example 20

Write each of the following as a statement of proportion and as a proportion equation.

- a** The mass (m) of a piece of paper varies directly with its area (a).
b The time taken (t) to complete a job varies inversely with the number of workers (n).

Solution

- a** Write the direct proportion.

$$m \propto a$$

Change to an equation with a constant.

$$m = ka$$

- b** Write the inverse proportion.

$$t \propto \frac{1}{n}$$

Change to an equation with a constant.

$$t = \frac{k}{n}$$

In many application questions, you will need to use your common sense to work out whether to use direct or inverse variation. Remember that, for direct proportion, the quantities increase and decrease together. However, for inverse variation, the quantities change inversely (that is, one goes up while the other goes down).

Example 21

A seatbelt stretches by 15 mm when a force of 5000 N (newtons) is applied. By how much would it stretch if a force of 12 000 N was applied?

**Solution**

Choose variable names.

Work out the variation.

Write the equation.

Substitute values to find k .

Solve for k .

Write the equation again.

Substitute for k and F .

Evaluate L .

Write the answer.

Let force = F and stretch = L .

As force increases, so does stretch,
so it is direct proportion.

$$L = kF$$

$$15 = k \times 5000$$

$$k = 15 \div 5000 = 0.003 \text{ mm/N}$$

$$L = kF$$

$$= 0.003 \times 12\,000$$

$$= 36 \text{ mm}$$

The seatbelt would stretch by 36 mm.

Example 22

Lifeboats carry emergency supplies. The water in a lifeboat will last 5 people for 12 days. For how long would it last 7 people?



Solution

Choose variable names.

Work out the variation.

Write the equation.

Substitute the initial values to find k .

Evaluate.

Write the equation again.

Substitute for k and n to find t .

Evaluate.

Round and write the answer.

Let time = t and number of people = n .

As the number increases, the time decreases, so it is inverse proportion.

$$t = \frac{k}{n} \text{ or } k = nt$$

$$k = 5 \times 12$$

$$= 60$$

$$t = \frac{k}{n}$$

$$= 60 \div 7$$

$$= 8.5714 \dots \text{ days}$$

The water would last 7 people for about $8\frac{1}{2}$ days.

You should now be able to write proportion equations for both direct and inverse variation. This is called **joint proportion** or **joint variation** because one variable varies *jointly* with two or more other variables. In many real cases, the variation of one quantity depends on a number of others.

Important!

Joint variation

A quantity that varies directly or inversely with more than one other quantity is said to vary **jointly** with the other quantities. The proportion equations are combined with one constant.

$$\text{If } z \propto x \text{ and } z \propto y, \text{ then } z = kxy.$$

$$\text{If } z \propto x \text{ and } z \propto \frac{1}{y}, \text{ then } z = \frac{kx}{y}.$$

Example 23

Animated example

Ratio and proportion

MAT09NAAE00005

The cost of a club newsletter varies directly to the number of pages and inversely to the number of advertisements. It costs \$1.50/copy to publish a 25-page newsletter with 50 ads. What is the maximum number of pages that the newsletter can have to reduce the cost to \$1/copy, with the number of ads increased to 60?

Solution

Choose variable names.

Let cost = C , number of pages = p
and number of ads = a

Write the joint proportion equation.

$$C = \frac{kp}{a}$$

Substitute the given values to find k .

$$1.5 = \frac{k \times 25}{50}$$

Solve for k .

$$k = 3$$

Write the joint proportion equation again.

$$C = \frac{kp}{a}$$

Substitute for C , k and a .

$$1 = \frac{3p}{60}$$

Solve for p .

$$p = 20 \text{ pages}$$

Write the answer.

The club can have a maximum of 20 pages in its newsletter.

Exercise 5.4 Inverse and joint proportion

Understanding

Extra questions

Exercise 5.4

MAT09NAEQ00019

Worked solutions

Exercise 5.4

MAT09NAWS00019

See Example 16

See Example 18

- 1 Determine whether the following pairs of ratios are inversely proportional.

a $5 : 8$ and $16 : 10$

b $24 : 16$ and $40 : 64$

c $\frac{18}{39}$ and $52 : 24$

d $15 : 55$ and $\frac{132}{36}$

e $2.8 : 1$ and $\frac{70}{25}$

f $56 : 21$ and $32 : 10$

- 2 Determine which of the following tables show inverse variation and, if applicable, find the constant of inverse variation.

a

x	1	2	4	5
y	40	20	10	8

b

x	1	3	6	12
y	12	4	2	1

c

n	1	2	3	5
c	10	9	8	6

d

a	2	3	4	5
b	30	20	15	12

e

g	$\frac{1}{4}$	$\frac{1}{2}$	3	$\frac{1}{3}$
h	24	12	2	18

f

t	2	8	5	10
s	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{10}$	$\frac{1}{20}$

Fluency

See Example 20

- 3 Write each of the following as a statement of proportion and as a proportion equation.
- a Distance travelled (d) varies directly with time (t).
 - b Circumference (C) of a circle varies directly as its radius (r).
 - c Area (A) of a triangle varies directly as its height (h).
 - d Perimeter (P) of a square varies directly as its side length (s).
 - e Travel time (t) varies inversely as speed (s).
 - f Volume (V) of a gas varies inversely as the pressure (P) on it.
 - g Light intensity (I) varies inversely as the square of the distance (d) from its source.
 - h Force (F) of a moving object varies inversely as the square of the time (t) it moves.

Problem solving

See Example 17

- 4 A group of students are asked to organise a letter-drop to 4000 households in the local area. So that they can share the work equally, they draw up this table.

Number of students	1	2	4	5	6	7	10
Number of letters	4000						

- a Copy and complete the table.
- b Draw a graph of the information in the table.
- c Is this an example of direct or inverse proportion?
- d About how many letters would be delivered by each student if 3 students were used?
- e How many students would be needed so that each delivers 500 letters?
- f How many students would be needed so that each delivers no more than 1250 letters?

- 5 An important highway bridge has been damaged. A Main Roads engineer calculates that it will take 400 working hours (i.e. one worker 400 hours) to repair the damage. A maximum of 12 workers are available for the repair work.



See Example 17

- a Copy and complete the following table.

Number of workers	1	2		5	6			12
Time to finish job (h)	400		100			50	40	

- b Draw a graph of the information in the table, using an appropriate set of axes.
- c Is this an inversely or directly proportional relationship?
- d How long would it take 8 workers to finish the job?
- e Approximately how long would it take 7 workers to finish the job?
- f If the repairs have to be completed within 48 working hours, how many workers will be needed?
- g Could the work be completed within 24 working hours? Why/why not?
- h How much time would be saved if 8 rather than 5 workers were used?

See Example 17

- 6 The Sharmas are planning their annual holiday to the country. It is a 540 km trip.
a Copy and complete this table of speeds and times.

Average speed (km/h)	120	90	80		50		20
Time for trip (h)	4.5			9		13.5	

- b Draw a graph of the information in the table, using an appropriate set of axes.
c What type of proportion is this?
d How long would it take to complete the trip at 30 km/h?
e How long would the trip take at an average speed of 100 km/h?
f If the family have to be at their destination in less than 10 hours, what is the slowest speed they should average?
g What would be the average speed for the trip to take 6 hours?

Worked solutions

Exercise 5.4

MAT09NAWS00019

- 7 The volume and pressure of a mass of gas vary at constant temperature as shown in the table below.

Volume, v (cm ³)	25	50	150	200	250
Pressure, p (mm of Hg)	450	225	75	56.25	45

- a Draw a graph of pressure against volume and describe the graph.
b Use your graph to find the pressure when the volume is:
i 37.5 cm³ ii 125 cm³ iii 175 cm³ iv 112.5 cm³
c Use your graph to find the volume when the pressure is:
i 250 mm of Hg ii 200 mm of Hg
iii 60 mm of Hg iv 125 mm of Hg.

- 8 The volume and density of polyurethane foam insulation are shown below.

Density, d (g/cm ³)	0.5	0.3	0.2	0.1	0.05
Volume, v (cm ³)	240	400	600	1200	2400

- a Draw a graph of volume against density and describe the graph.
b Use your graph to find the volume when the density is:
i 0.4 g/cm³ ii 0.25 g/cm³ iii 0.08 g/cm³ iv 0.35 g/cm³
c Use your graph to find the density when the volume is:
i 300 cm³ ii 500 cm³ iii 1000 cm³ iv 2000 cm³

See Example 19

- 9 Time varies inversely as speed for a given journey. If it takes 4 hours to complete a journey travelling at 80 km/h, how long would the journey take travelling at 100 km/h?

Worked solutions

Exercise 5.4

MAT09NAWS00019

- 10 The frequency of sound varies inversely as the wavelength. The frequency of a musical note is 276 cycles per second when the wavelength is 1.2 m.
a Find the frequency when the wavelength is:
i 2.5 m ii 0.6 m
b Find the wavelength when the frequency is 600 cycles per second.



- 11** An aircraft's landing speed varies inversely as the wing area. The landing speed of an aircraft with a wing area of 45 m^2 is 84 km/h . What would be the landing speed of an aircraft with a wing area of 60 m^2 ?
- 12** When 3 hoses are used to empty a backyard pool, it empties in 10 hours. How long would it take to empty if 5 hoses were used?
- 13** A bank of solar cells with a surface area of 5 m^2 gathers 6000 mAh (milliampere-hours) of energy. How much energy would be gathered in the same time under similar weather conditions by a bank with a surface area of 12 m^2 ?

See Example 22

See Example 21



- 14** It took 12 hours to trickle-charge the battery of a remote-control car with a current of 4 mA (milliamperes). How long would the battery have taken to charge using a current of 7 mA ?
- 15** A space probe travels $30\,000 \text{ km}$ in 6 hours. At that speed, how far would it travel in 3 days?



- 16** The time (t) taken to complete a journey varies directly as the distance (d) and inversely as the average speed (s). If it takes 3 h to complete a 280 km trip at 80 km/h , how long will it take to travel 546 km at 65 km/h ?
- 17** The kinetic energy (E) of a body varies jointly as its mass (m) and the square of its velocity (v). A 2.5 kg mass travelling at 40 m/s has a kinetic energy of 2000 J (joules). Find the kinetic energy of a 22 kg mass travelling at 16 m/s .
- 18** A clear piece of cylindrical plastic pipe holds 15 times as much liquid as a black piece of the pipe. The radius of the black pipe is one-quarter of the radius of the clear pipe. The black pipe is 5 cm long. Which piece of pipe is longer, and by how much?

Reasoning

Chapter 5 summary

Quiz

Ratio and proportion

MAT09NAQZ00005

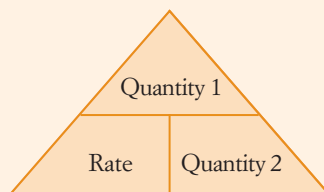
- A **rate** is obtained by dividing one quantity by a different, related quantity. A rate can be written as a fraction. The measurement units of a rate are separated by the symbol / with the second quantity being the denominator.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Price} = \frac{\text{charge}}{\text{amount}}$$

$$\text{Pay rate} = \frac{\text{payment}}{\text{time}}$$

- A rate triangle can be used to remember how to obtain one quantity from the other.



- Graphs can be used to find or compare rates.
- A ratio compares two or more quantities of the same kind in a definite order. It is stated without units using the word 'to' or the symbol : between the parts.
- A **ratio** may be shown in fraction notation, but it is easier to compare ratios when they are written as decimals.
- Equivalent ratios** are the same.
- A **unitary method** can be used to help divide a quantity into a particular ratio. The value of one part of the ratio is found first, and then the value of each share is calculated.
- Quantities x and y are in **direct proportion** if, whenever one increases or decreases, the other increases or decreases by the same factor. We write

$$y \propto x \quad \text{or} \quad y = kx$$

where k is called the **constant of proportion** (or **constant of variation**).

- The ratios of corresponding quantities in direct proportion are equal, so

$$\frac{x_1}{x_2} = \frac{y_1}{y_2}$$

- The equivalence of ratios or proportionality may be checked by **cross-multiplication**, as

$$x_1 y_2 = x_2 y_1$$

- The graph of quantities in direct proportion is a straight line passing through the origin.
- Quantities x and y are **inversely proportional** if, whenever one quantity changes, the other changes by the inverse factor. We write

$$y \propto \frac{1}{x} \quad \text{or} \quad y = \frac{k}{x} \quad \text{or} \quad xy = k$$

where k is called the **constant of inverse proportion** (or **constant of inverse variation**).

- The ratios of corresponding quantities in inverse proportion are opposite to one another, so

$$\frac{x_1}{x_2} = \frac{y_2}{y_1}$$

- Inverse proportionality of ratios may be checked by multiplying the ratios, as

$$\frac{x_1}{x_2} \times \frac{y_1}{y_2} = 1$$

- The graph of inversely proportional quantities is *not a straight line*. It is a **hyperbola**.
- Problems involving inverse or direct proportions may be solved using the proportion equation or by using a unitary method.
- Proportion equations can involve direct and inverse variation. This is called **joint proportion** or **joint variation** because one variable varies *jointly* with two or more other variables. A quantity that varies directly or inversely with more than one other quantity is said to vary **jointly** with the other quantities. The proportion equations are combined with one constant.

If $z \propto x$ and $z \propto y$, then $z = kxy$.

If $z \propto x$ and $z \propto \frac{1}{y}$, then $z = \frac{kx}{y}$.

Chapter 5 review

Understanding

Worksheet

Ratio and proportion review

MAT09NAWK00044

See Example 8

See Example 13

- 1 A bowler conceded 96 runs while getting out 3 batsmen. What was the strike rate in runs/wicket?
- 2 Write in simplest form:
 - a $14 : 20$
 - b $4\frac{1}{2} : 3\frac{3}{4}$
 - c $2\text{h} : 35\text{min}$
 - d $1.8 : 3.2$
- 3 Divide \$56 in the ratio 5 : 3.
- 4 Are the ratios 6 : 15 and 14 : 35 in proportion?

Fluency

See Example 3

See Example 13

See Example 14

See Example 16

See Example 18

- 5 Change 15 km/h to m/s.
- 6 Which is the larger of the ratios 15 : 24 and 25 : 45?
- 7 Determine whether the following pairs of ratios are in proportion.
 - a $6mn : 16n$ and $15mpq : 40pq$
 - b $\frac{7y^2}{21xy}$ and $\frac{8x^2}{24xy}$
- 8 Find the value of the variable in each of the following proportions.
 - a $\frac{8}{12m} = \frac{5}{15}$
 - b $x : 8 : 10 = 18 : y : 15$
- 9 $a \propto b$ and $a = 24$ when $b = 5$. Find the constant of variation.
- 10 Determine whether the following pairs of ratios are inversely proportional.
 - a 25 : 35 and 49 : 35
 - b 24 : 9 and 18 : 40
 - c $\frac{84}{28}$ and 15 : 5
- 11 Find which of the following tables show inverse variation and, if applicable, find the constant of inverse variation.
 - a

x	5	8	10	20
y	8	5	6	3
 - b

x	2	3	4	6
y	18	12	9	6
- 12 Write each of the following as a statement of proportion and as a proportion equation.
 - a The energy (E) of a moving object varies directly with its mass (m).
 - b The reaction time (t) of a chemical varies inversely as its temperature (T).
 - c The energy (E) of a spring varies directly with the square of its compression (d).

See Example 20

Problem solving

See Example 10

- 13 If \$320 is to be divided between Jackie and Sam in the ratio 5 : 3, find the value of Sam's share.
- 14 The interest that Sharmian earns on her bank account is directly proportional to the amount of money she has in the account. Last year she had \$8500 in the account, which earned \$382.50 for the year. She spent some of the money and left \$6000 in the account. If the interest rate does not change, how much will she earn this year?

- 15 Which is the better buy: 350 g of ham for \$4.30, or 750 g of ham for \$9.20?

See Example 4

- 16 The height reached by a fireworks rocket is directly proportional to the amount of powder used in the rocket. A rocket with 200 g of powder reaches a height of 30 m.

See Example 12

- a Find the height reached by a rocket with 240 g of powder.
b How much powder would be needed to ensure that a rocket reached a height of 50 m?

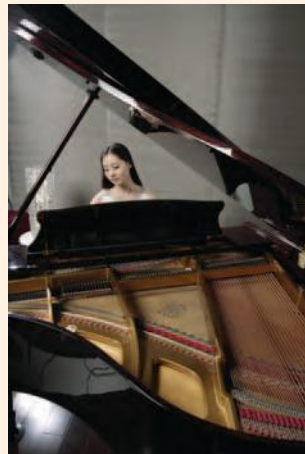
- 17 The pressure and volume of some neon gas are shown below.

See Example 17

Pressure, P (kPa)	5	10	15	20	25
Volume, V (cm ³)	1440	720	480	360	288

- a Draw a graph of volume against pressure and describe the graph.
b Use your graph to find the volume when the pressure is 12 kPa.
c Use your graph to find the pressure when the volume is 900 cm³.
- 18 The number of vibrations in a piano string varies inversely as the length of the string. If there are 400 vibrations per second with a length of 1.2 m, how many vibrations per second will there be when the string is lengthened to 1.8 m?

See Example 19



- 19 An office with complete walls and corridors can accommodate 40 clerks with an average floor space use of 5 m²/clerk. The same floor space can be divided with low partitions and plants in an open-plan layout to use 4 m²/clerk. How many clerks can be accommodated in an open-plan layout?

See Example 22

- 20 A tree casts a shadow 27 m long at the same time that a 4.5 m stick casts a 6 m shadow. How tall is the tree?

See Example 21

- 21 James leaves home and travels for 3 minutes at 50 m/min. Next he stops for 5 minutes to look at a butterfly. He then continues for 2 minutes at 100 m/min, before finally returning home in 4 minutes.

Reasoning

- a Draw a distance-time graph to show James's trip.
b What is the greatest average speed reached by James on any part of his trip?



Measurement and geometry

6

Congruence and similarity



Contents

- 6.1 Transformations, congruence and similarity
- 6.2 Congruent triangles
- 6.3 Similar triangles
- 6.4 Applications of similarity
- Chapter summary
- Chapter review

Prior learning

Chapter 6

MAT09MGPL00006

Parent guide

Chapter 6

MAT09MGPG00006

Curriculum guide

Chapter 6

MAT09MGCU00003

9780170194815

Australian Curriculum statements

Geometric reasoning

Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar.

Solve problems using ratio and scale factors in similar figures.

Video tutorial

Congruence and
similarity

MAT09MGVT00006

Weblink

National library of
virtual manipulatives

MAT09MGWB00006

Transformations, similarity and congruence are closely related concepts. We can use transformations to explore similar and congruent figures. In fact, you can regard congruent figures as transformations of each other by rotation, reflection and translation, while similar figures also involve enlargement or reduction. Similar and congruent figures are extensively used in the proofs of geometric properties, as well as in establishing important principles in science and technology. In this chapter you will look at the tests that establish similarity and congruence of triangles. Since all polygons can be broken into triangles, these tests are used to establish the properties of other figures as well.

Mathematical literacy

Maths dictionary

MAT09ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

centre of magnification	enlargement enlargement factor	reduction reduction factor	scale factor
centre of rotation	image	reflection	similar
congruent	line of reflection	rotation	transformation
corresponding dilation	magnification object	scale scale drawing	translation

6.1 Transformations, congruence and similarity

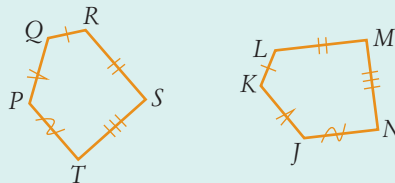
Two figures are **congruent** if they are exactly the same shape and size.

Important!

Congruent figures

Two figures are congruent if one is an exact copy of the other.

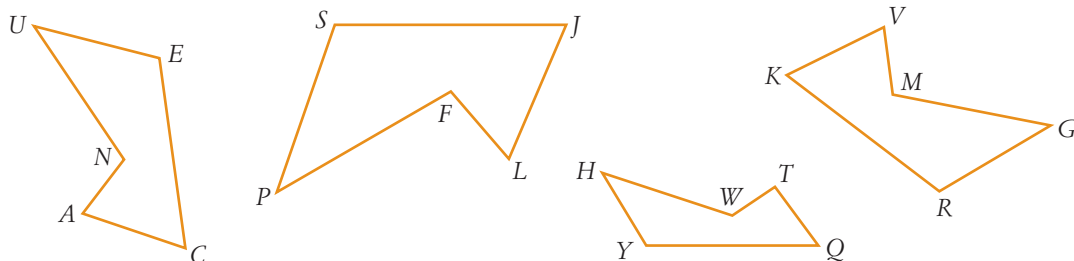
The symbol ' $\equiv$ ' means 'is congruent to'. When naming congruent figures, corresponding vertices must be named in the same order. The two pentagons here are congruent, so we write $PQRST \equiv JKLMN$



One way to check if shapes are congruent is to compare their angle and side measurements.

Example 1

Name any congruent shapes in the following diagrams. Measure sides and angles if necessary.



Solution

Compare the shape and size of each figure.

Measure the corresponding sides.

Measure the corresponding angles.

Write the result.

Only ANUEC and VMGRK appear to be the same shape and size.

$$\begin{aligned} AN &= VM & NU &= MG \\ UE &= GR & EC &= RK \\ CA &= KV \end{aligned}$$

$$\begin{aligned} \angle A &= \angle V & \angle N &= \angle M & \angle U &= \angle G \\ \angle E &= \angle R & \angle C &= \angle K \end{aligned}$$

$$ANUEC \cong VMGRK$$

Another way to check if two shapes are congruent is to see if it is possible to exactly superimpose one on the other. We can do this using **transformations**.

A transformation changes a shape (**object**) or its position to form an **image**.



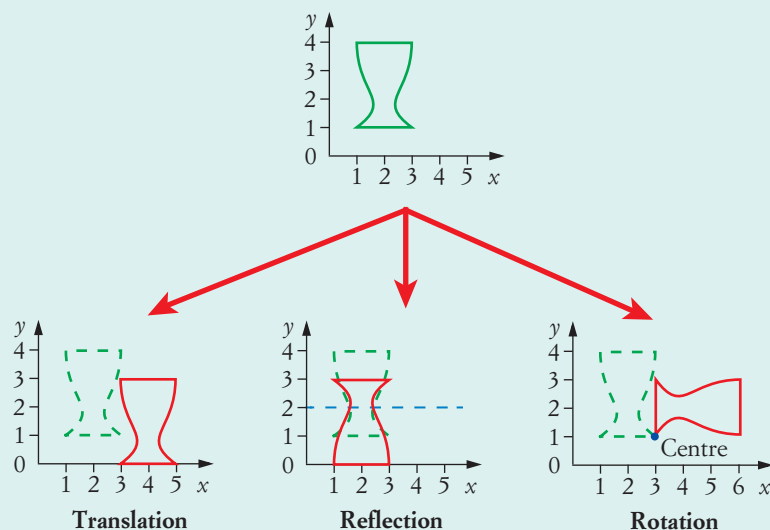
Important!

Transformations

A **translation** is a sliding movement. The vertices of the object are all changed by the same amount to form the image.

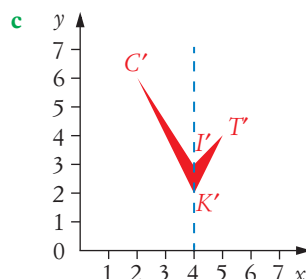
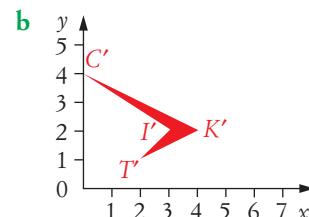
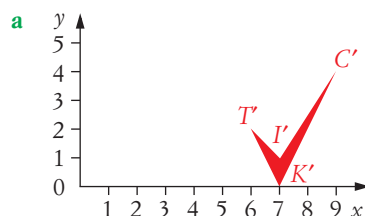
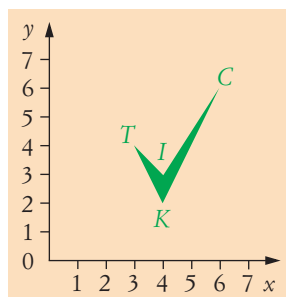
A **rotation** is a turn (**clockwise** or **anticlockwise**) through an angle. Each point stays the same distance from the **centre of rotation**.

When a shape is **reflected**, all points swap to the other side of the **line of reflection**.



Example 2

Use coordinates to describe the transformations of the tick shape shown below. Identify the transformation in each case.



Solution

- a** Describe the object and image coordinates.

Object points move 3 units to the right and 2 units down.

- b** Describe the object and image coordinates.

K does not move. All other points are rotated anticlockwise.

- c** Describe the object and image coordinates.

Neither I nor K moves. T and C swap positions on either side of the line through IK .

$$\begin{aligned} T(3, 4) &\rightarrow T'(6, 2) \\ I(4, 3) &\rightarrow I'(7, 1) \\ C(6, 6) &\rightarrow C'(9, 4) \\ K(4, 2) &\rightarrow K'(7, 0) \end{aligned}$$

It is a translation of 3 right and 2 down.

$$\begin{aligned} T(3, 4) &\rightarrow T'(2, 1) \\ I(4, 3) &\rightarrow I'(3, 2) \\ C(6, 6) &\rightarrow C'(0, 4) \\ K(4, 2) &\rightarrow K'(4, 2) \end{aligned}$$

It is a rotation 90° anticlockwise with centre K .

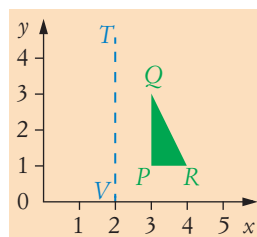
$$\begin{aligned} T(3, 4) &\rightarrow T'(5, 4) \\ I(4, 3) &\rightarrow I'(4, 3) \\ C(6, 6) &\rightarrow C'(2, 6) \\ K(4, 2) &\rightarrow K'(4, 2) \end{aligned}$$

It is a reflection across IK (vertical line through $x = 4$).

Example 3

Use number lattices to show the following transformations of this figure.

- a** Translation of 2 left and 1 up
b Rotation 90° clockwise, centre Q
c Reflection across TV



Solution

- a** For each vertex of the object, mark a corresponding image vertex 2 left and 1 up:

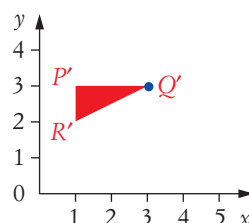
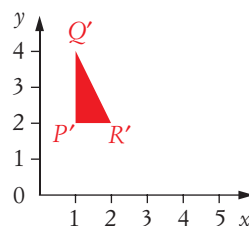
$$\begin{aligned} P(3, 1) &\rightarrow P'(1, 2), Q(3, 3) \rightarrow Q'(1, 4) \text{ and} \\ R(4, 1) &\rightarrow R'(2, 2). \end{aligned}$$

Join up the vertices to form the image.

- b** For each vertex of the object, mark a corresponding image vertex rotated 90° clockwise with centre Q :

$$\begin{aligned} P(3, 1) &\rightarrow P'(1, 3), Q(3, 3) \rightarrow Q'(3, 3) \text{ and} \\ R(4, 1) &\rightarrow R'(1, 2). \end{aligned}$$

Join up the vertices to form the image.

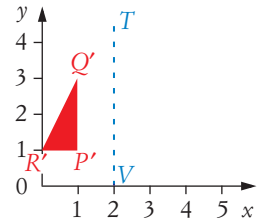


- c Mark image vertices the same distance on the left of TV as the corresponding object vertices are on the right:

$$P(3, 1) \rightarrow P'(1, 1), Q(3, 3) \rightarrow Q'(1, 3) \text{ and}$$

$$R(4, 1) \rightarrow R'(0, 1).$$

Join up the vertices to form the image.



Sometimes more than one transformation may be required to see if it is possible to superimpose one figure on another.

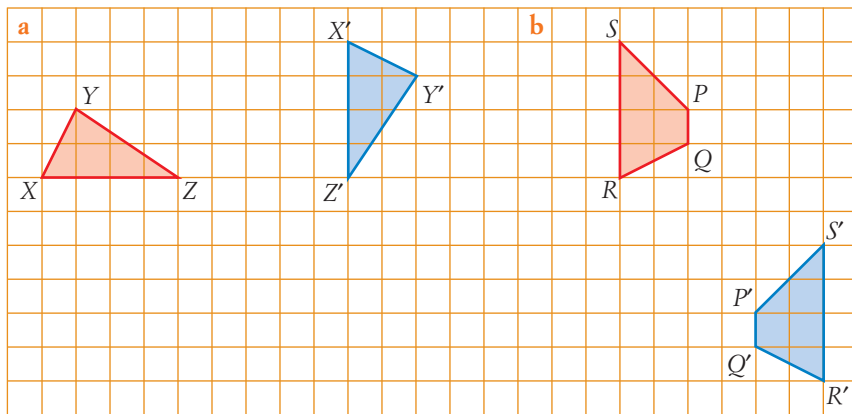
Example 4

Worksheet

Transformations

MAT09MGWK00038

Describe the transformations that could be used to superimpose the red figure onto the congruent blue figure in each case.



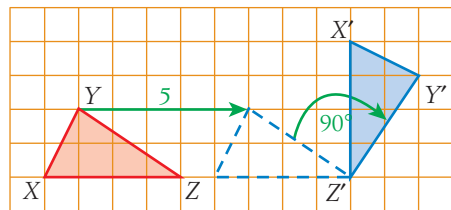
Solution

- a It looks like $\triangle XYZ$ has been slid to the right (translated) and then rotated.

Either try to visualise these transformations or cut out a triangle that is roughly the same shape as $\triangle XYZ$ and use it to work out the transformations.

$\triangle XYZ$ has been translated right and then rotated clockwise about Z .

State the result.

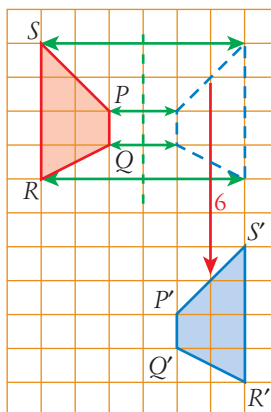


$\triangle XYZ$ has been translated 5 units to the right and then rotated 90° clockwise about Z .

- b** It looks like $PQRS$ has been reflected and then slid down.

The line of reflection is parallel to PQ .

The reflected image has then been translated 6 units down.



State the result.

$PQRS$ has been reflected in a vertical line 1 unit to the right of PQ and then translated 6 units down.

It may be possible to apply more than one combination of transformations to an object to result in the same image. For example in Example 4b, it is also possible to rotate $PQRS$ 180° clockwise and then translate it down to $P'Q'R'S'$.

For a translation, reflection or rotation the object and resulting image are congruent. There is another type of transformation called a **dilation** where the object and image are not congruent.

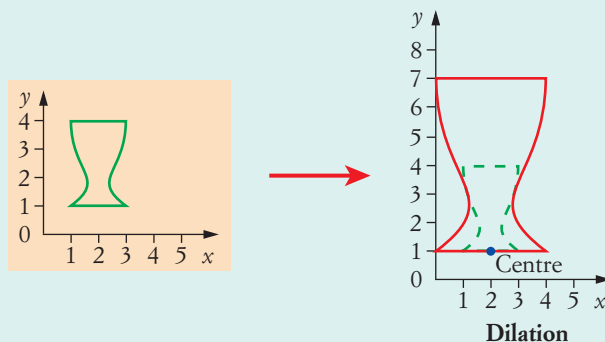
Important!

Dilation

A **dilation** is a change of size without a change in shape.

The **enlargement** or **reduction** is measured by the scale factor or **magnification**.

The **centre of magnification** keeps the same coordinates.



Puzzle sheet

Cartoon enlargement

MAT09MGPS00021

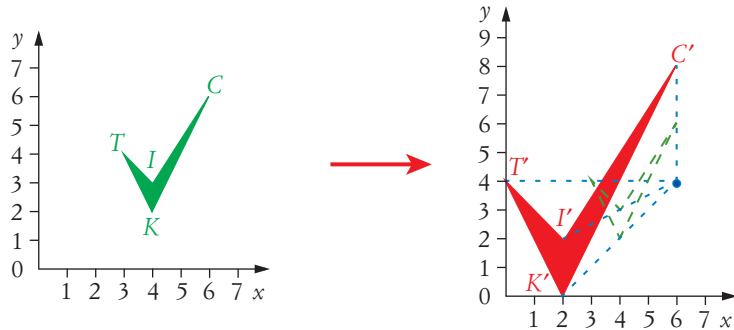
Worksheet

Dilations

MAT09MGWK00045

Example 5

Use coordinates to describe and identify the transformation of the tick shape shown here.



Solution

Describe the object and image coordinates.

$$T(3, 4) \rightarrow T'(0, 4)$$

$$I(4, 3) \rightarrow I'(2, 2)$$

$$C(6, 6) \rightarrow C'(6, 8)$$

$$K(4, 2) \rightarrow K'(2, 0)$$

The image is twice the size of the object.

Draw in lines to find the centre of magnification.

It is an enlargement (dilation) with magnification 2 and centre of magnification (6, 4).

Example 6

Puzzle sheet

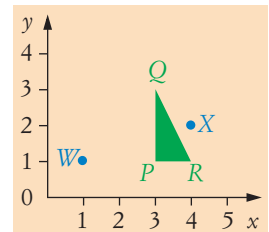
Transformations

MAT09MGPS00022

Use number lattices to show the following transformations of this figure.

a Reduction by half, centre W

b Enlargement by 3, centre X



Solution

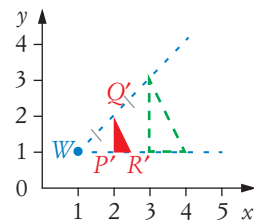
a Using W as a centre, draw lines through the object vertices.

Each image vertex is halfway between the object vertex and W :

$$P(3, 1) \rightarrow P'(2, 1), Q(3, 3) \rightarrow Q'(2, 2) \text{ and}$$

$$R(4, 1) \rightarrow R'(2.5, 1).$$

Join up the vertices to form the image.



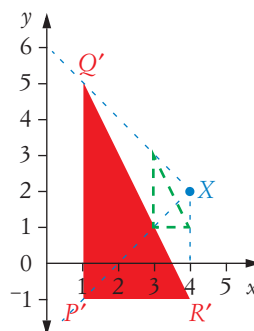
- b** Using X as a centre, draw lines through the object vertices.

Each image vertex is three times the distance between the object vertex and X :

$$P(3, 1) \rightarrow P'(1, -1), Q(3, 3) \rightarrow Q'(1, 5) \text{ and}$$

$$R(4, 1) \rightarrow R'(4, -1).$$

Join up the vertices to form the image.



An enlargement or reduction of an object results in an image that is the same shape but a different size. In these cases, the object and its image are called **similar** figures.

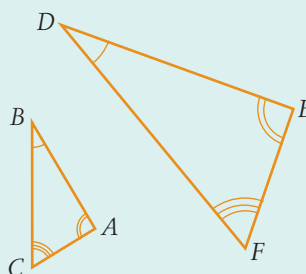
Important!

Similar figures

Similar figures are exactly the same shape but not necessarily the same size.

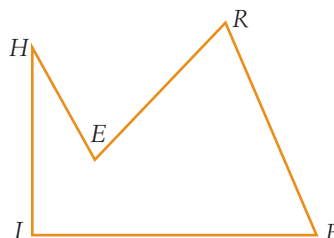
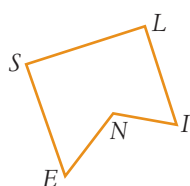
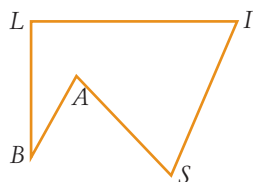
The angles in corresponding positions (**corresponding angles**) in each figure are the same. The sides opposite corresponding angles are called **corresponding sides**.

The symbol ' $\parallel$ ' means 'is similar to'. When naming similar figures, corresponding vertices must be named in the same order. The two triangles here are similar and so we write $\triangle ABC \parallel \triangle EDF$



Example 7

Name the similar shapes shown here.



Solution

Compare the shape and size of each figure – they appear to be the same shape but different sizes.

Measure the corresponding angles.

Write the result.

Only **BASIL** and **HERBI** appear to be the same shape.

$$\begin{aligned} \angle B &= \angle H & \angle B &= \angle E & \angle B &= \angle R \\ \angle I &= \angle B & \angle L &= \angle I \end{aligned}$$

$$\mathbf{BASIL \parallel HERBI}$$

TLF Learning object

Measures: scaling up solids (L2313)

MAT09MGIN00006

For similar figures, the lengths of corresponding sides are proportional. Corresponding distances are related by a **scale factor**.

Important!

Scale factor

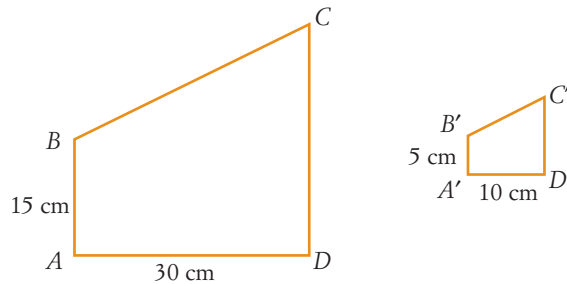
The scale factor is the ratio of corresponding lengths in an object and its image.

$$\text{Scale factor} = \frac{\text{image length}}{\text{object length}}$$

The scale factor may also be referred to as the **enlargement factor** or the **reduction factor**.

Example 8

If $ABCD \parallel A'B'C'D'$, calculate the scale factor for the reduction shown.



Solution

Write the rule for scale factor.

$$\text{Scale factor} = \frac{\text{image length}}{\text{object length}}$$

Any corresponding lengths will do.

$$= \frac{A'D'}{AD} = \frac{10}{30} = \frac{1}{3}$$

State the result.

$$\text{The scale factor is } \frac{1}{3}.$$



Exercise 6.1 Transformations, congruence and similarity

1 For each of the following pairs of congruent shapes:

- i list the matching angles
- ii list the matching sides
- iii write a statement of congruency

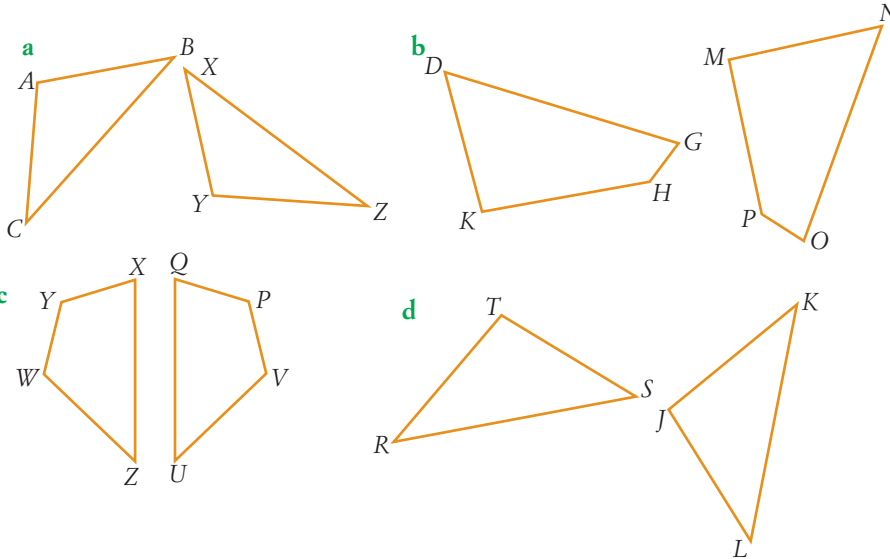
Understanding

Extra questions

Exercise 6.1

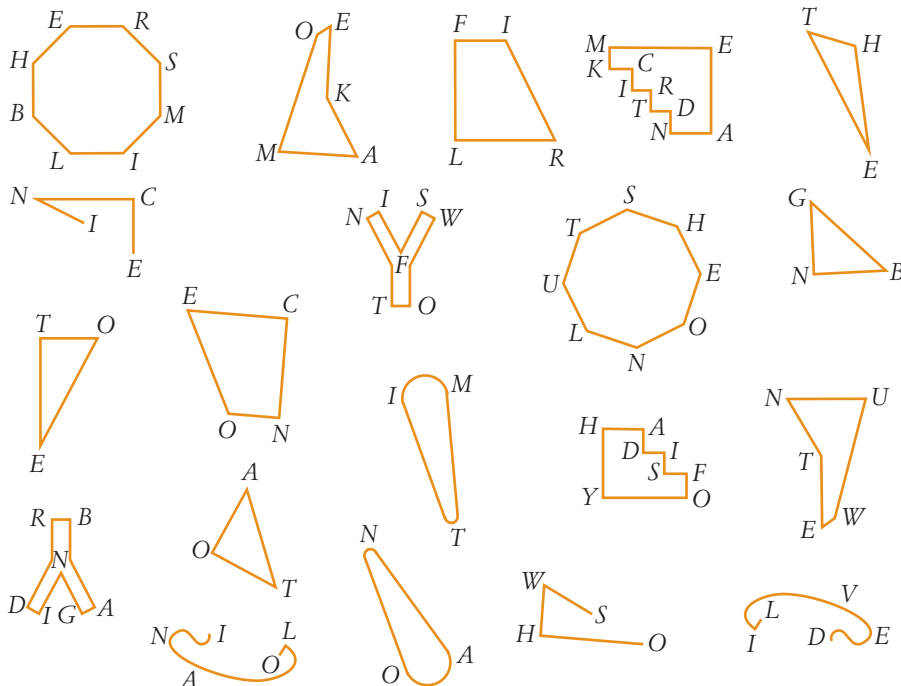
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See Example 1



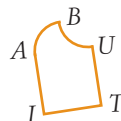
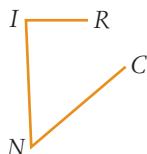
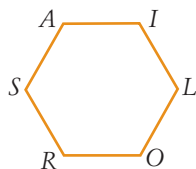
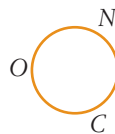
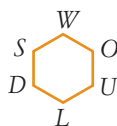
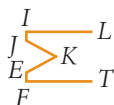
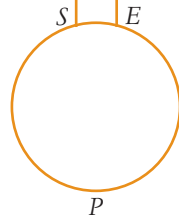
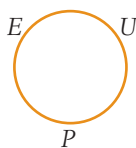
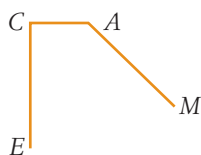
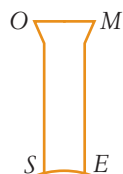
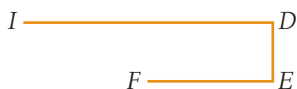
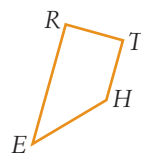
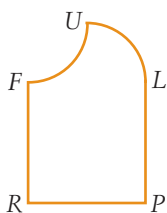
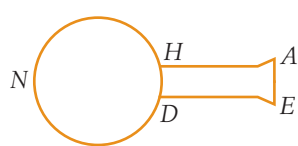
2 Name any congruent shapes in the diagrams below. Measure sides and angles if necessary.

See Example 1



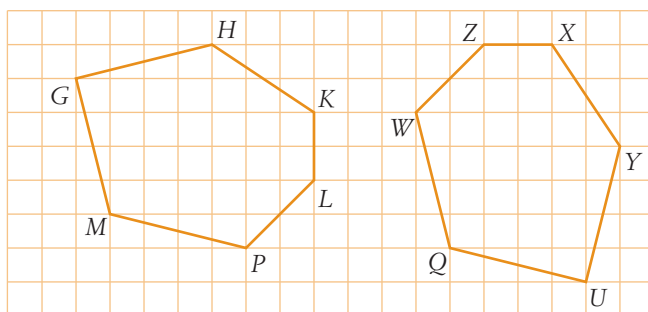
See Example 7

3 Name any similar figures below.



Fluency

4 a Why is the figure $GHKLPM$ congruent to the figure $UYXZWO$?



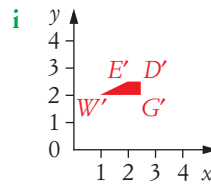
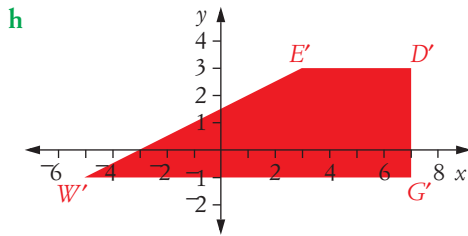
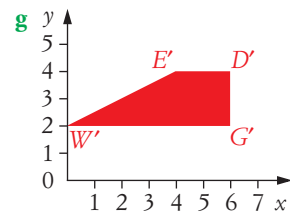
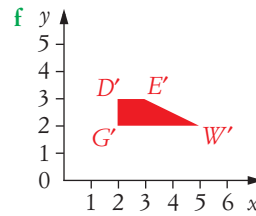
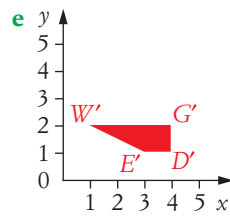
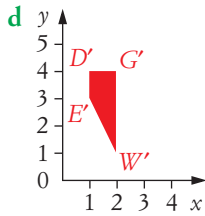
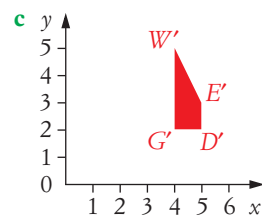
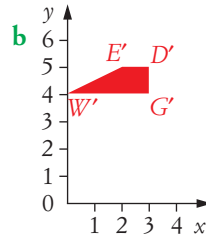
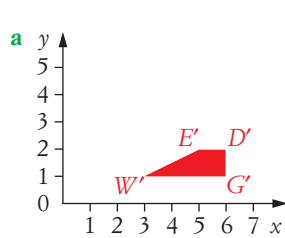
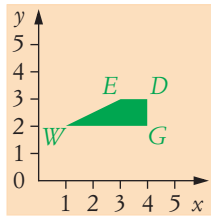
b Find:

- the perimeter of each figure (in millimetres)
- the area of each figure (in square units).

c What do you notice about the perimeters and areas?

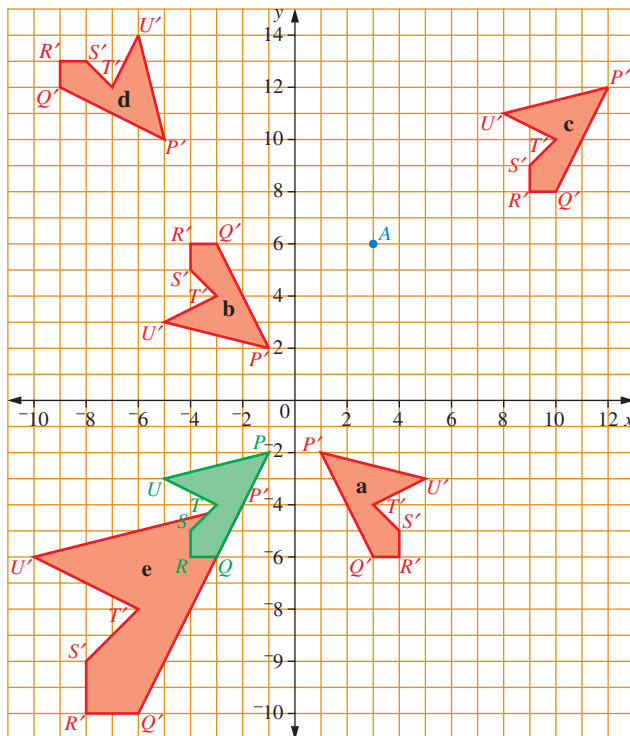
5 Use coordinates to describe each transformation shown below.

See Example 2



6 The diagram below shows the effect of a number of transformations on figure $PQRSTU$. Describe the transformation that produced each image $P'Q'R'S'T'U'$.

See Examples 2, 3



Worked solutions

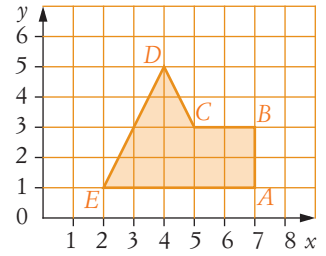
Exercise 6.1

MAT09NAWS00020

See Examples 3, 5

7 Copy the figure $ABCDE$ shown here onto a number lattice, then:

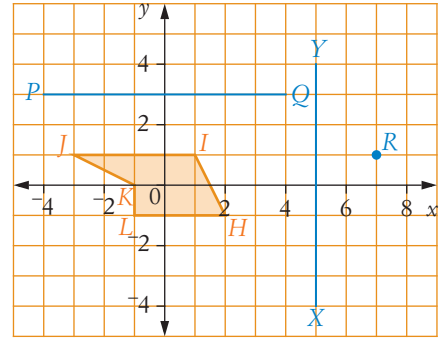
- a translate it 7 left and 4 up
- b translate it 5 right and 8 up
- c rotate it 90° anticlockwise about X
- d reflect it in the x -axis
- e enlarge it with magnification 3 and centre A
- f rotate it 180° clockwise about E .



See Examples 3, 5

8 Copy the figure $Hijkl$ shown here onto a number lattice, then:

- a translate it 5 right and 10 up
- b translate it 8 left and 2 down
- c reflect it in the line XY
- d reflect it in the line PQ
- e rotate it 90° anticlockwise about R
- f enlarge it with magnification 4 and centre $(0, 0)$.



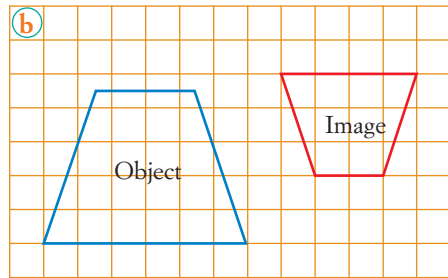
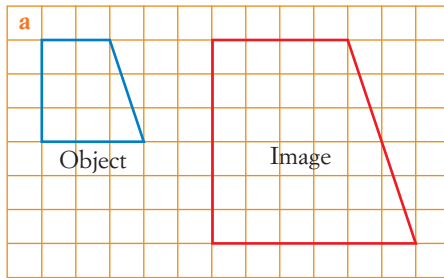
Worked solutions

Exercise 6.1

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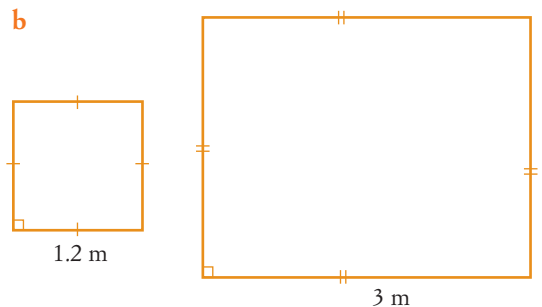
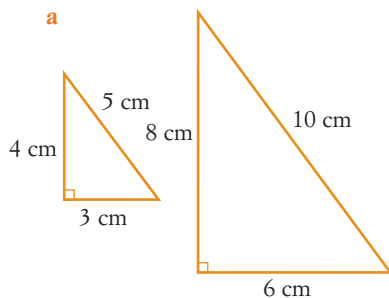
See Example 8

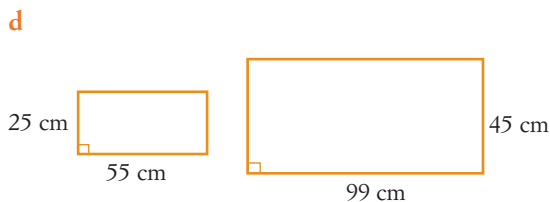
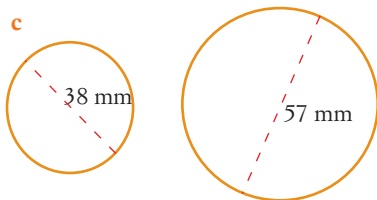
9 Calculate the scale factor for each dilation shown below.



See Example 8

10 In each case the object is on the left and the image is on the right.



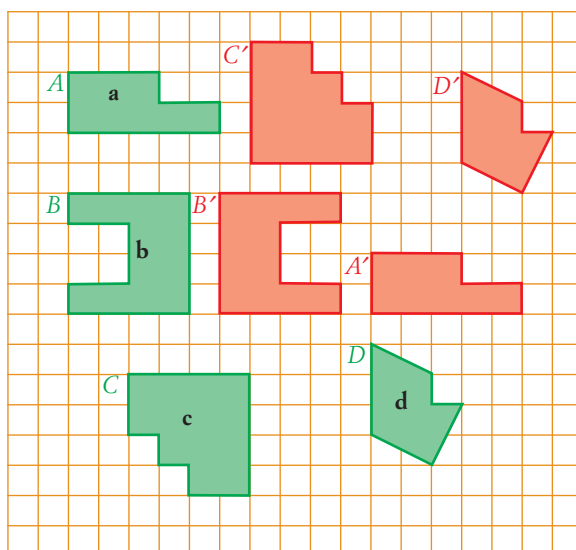


- i** Calculate the scale factor for each enlargement shown.
- ii** Calculate the perimeter of each figure.
- iii** Calculate the area of each figure.

- 11** What type of transformation has been used to change each figure below (A , B , C and D) into its image (A' , B' , C' and D')?

Problem solving

See Examples 2, 3



- 12** Describe each of the transformations of $WEDG$ shown in question 5.
- 13** Refer back to your answer for question 4. Would you expect the same result for any pair of congruent figures?
- 14** Refer back to your answer for question 10 to answer and complete the following.
 - a** Perimeter of image = $\dots \times$ perimeter of object
 - b** Area of image = $\dots \times$ area of object

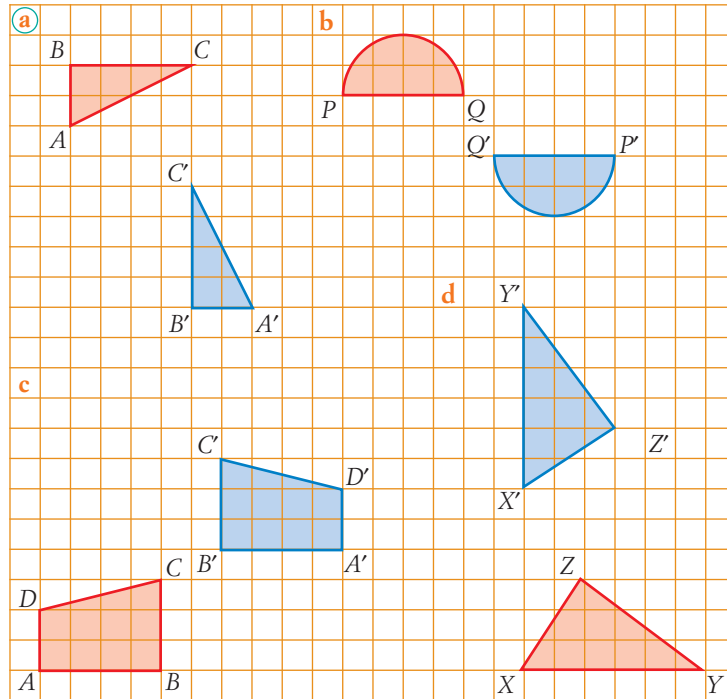
Worked solutions

Exercise 6.1

MAT09MGWS00020

See Example 4

- 15 Describe the combination of two transformations that could be used to superimpose the red figure onto the congruent blue figure in each case.



6.2 Congruent triangles

We can check if two figures are congruent by measuring *all* their sides and angles. In the case of triangles, we don't always need to do this.

Investigate: Congruent triangles

Activity 1

You will need coloured straws, sticky tape, scissors and a ruler.

- Step 1:** Cut three red straws to lengths of 18 cm, 15 cm and 10 cm. Use sticky tape to join them together to make a triangle.
- Step 2:** Now make a green triangle the same way.
- Step 3:** Make a third triangle using blue straws and the same measurements.

Compare the three triangles. What do you find?

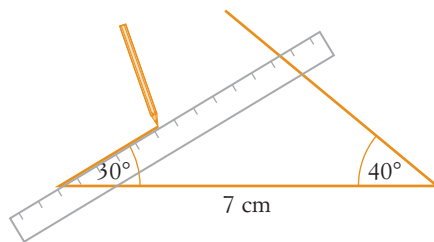
Choose three different side lengths and repeat the process. What do you find?

You will need a protractor, pencil, paper, a compass and a ruler for the remaining activities.

Activity 2

Let's see how many different triangles can be drawn with a side of 7 cm between angles of 30° and 40° . The 7 cm side is called the contained side because it is contained between the two angles.

- Step 1:* Draw a side of length 7 cm.
Step 2: From one end draw a side that makes an angle of 30° with the first side.
Step 3: From the other end draw a side that makes an angle of 40° with the first side and complete the triangle.

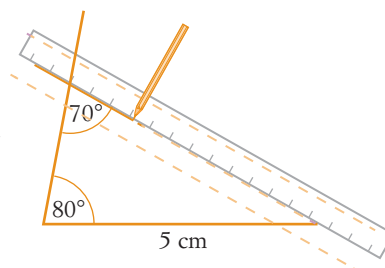


Is it possible to draw a different triangle with angles of 30° and 40° that contains a side of length 7 cm?

Activity 3

Let's see how many different triangles can be drawn with angles of 70° and 80° and a side of 5 cm not contained between the angles.

- Step 1:* Draw a side of length 5 cm.
Step 2: From one end draw a second side that makes an angle of 80° with the first side.
Step 3: Now draw a light pencil line that makes an angle of 70° with the second side.
Step 4: Slide your ruler parallel with the light pencil line until it meets the 5 cm side and complete the triangle.

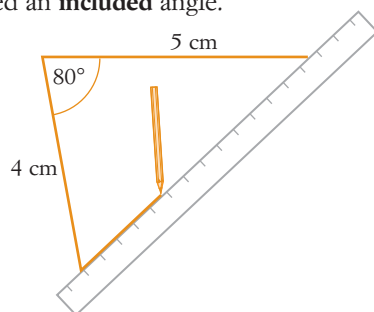


Is it possible to draw a different triangle with angles of 80° and 70° and a non-contained side of length 5 cm?

Activity 4

Let's see how many different triangles can be drawn with sides of 4 cm and 5 cm and an angle of 80° between the two sides. The 80° angle is called an **included angle**.

- Step 1:* Draw a side of length 4 cm.
Step 2: From one end draw a second side that makes an angle of 80° with the first side.
Step 3: Measure 5 cm down this side and complete the triangle.

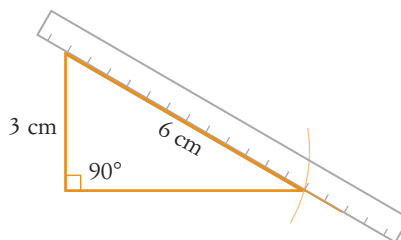


Is it possible to draw a different triangle with sides of 4 cm and 5 cm and an included angle of 80° ?

Activity 5

Let's see how many different triangles can be drawn with a right angle, a hypotenuse of 6 cm and another side of 3 cm.

- Step 1:* Draw a side of length 3 cm.
Step 2: From one end draw a second side that makes an angle of 90° with the first side.
Step 3: Set the compass to a distance of 6 cm.
Step 4: Place the point of the compass at the other end of the 3 cm side and mark off a point on the second side of the triangle.
 Now draw in the hypotenuse of 6 cm.



Is it possible to draw a different right-angled triangle with a hypotenuse of 6 cm and another side of 3 cm?

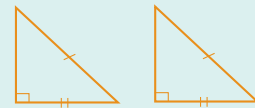
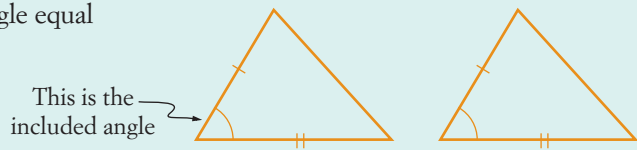
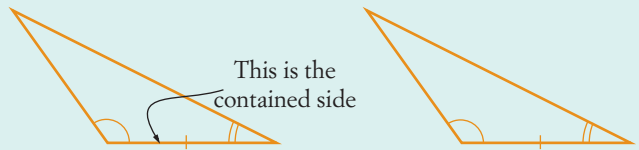
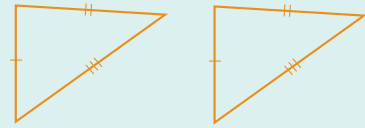
By completing the previous investigation, you can see that there are a number of ways in which congruent triangles can be identified using angle and side properties.

Important!

Congruent triangles

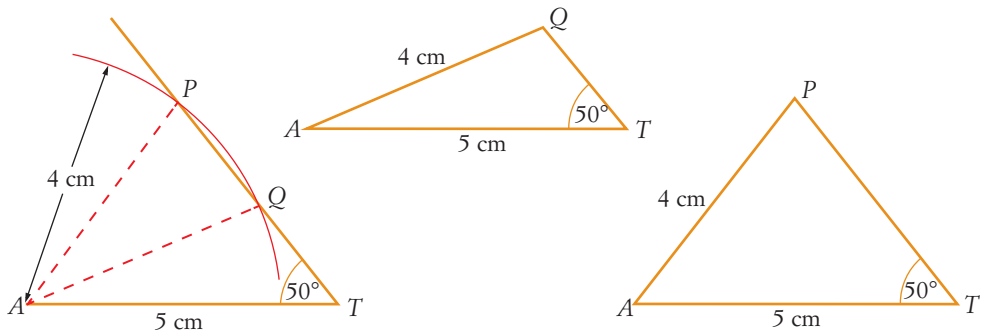
Triangles must be congruent if they have:

- three corresponding sides equal (**side, side, side** or **SSS**); or
- two angles and the side they contain equal (**angle, side, angle** or **ASA**); or
- two sides and the included angle equal (**side, angle, side** or **SAS**); or
- a right angle and the hypotenuse and a corresponding side equal (**right angle, hypotenuse, side** or **RHS**).



These are the four **tests for congruent triangles**.

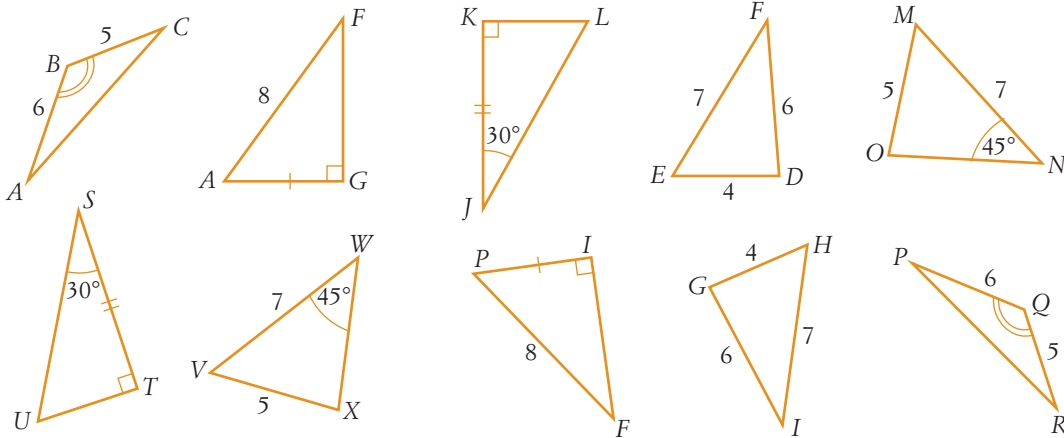
You have seen that if two triangles have two sides and the included angle equal, then they must be congruent. What about the situation where the two sides do *not* include the known angle? The diagrams below show the triangles that can be drawn with sides of 4 cm and 5 cm if an angle of 50° is drawn at the end of the 5 cm side.



There are two possible triangles: $\triangle ATP$ and $\triangle ATQ$. This means that if two triangles have two sides and an angle *not included by* (*between*) *the sides* the same, they are not necessarily congruent.

Example 9

Identify any pairs of congruent triangles below. Explain why they are congruent.



Solution

Begin with any triangle and see if there is a triangle that appears congruent. Then see if one of the tests for congruent triangles can be applied.

$\triangle ABC$ could be congruent with $\triangle PQR$.

You can see that:

$AB = PQ$, $BC = QR$ and $\angle ABC = \angle PQR$.

$\triangle ABC \equiv \triangle PQR$ because two sides and the included angle are equal (SAS).

$\triangle AFG$ could be congruent with $\triangle PFI$.

You can see that:

$AF = PF$ (hypotenuse), $AG = PI$ and $\angle AGF = \angle PIF = 90^\circ$.

$\triangle AFG \equiv \triangle PFI$ because they are right-angled triangles with the hypotenuse and a corresponding side equal (RHS).

$\triangle KJL$ could be congruent with $\triangle STU$.

You can see that:

$KJ = TS$, $\angle KJL = \angle TSU$ and $\angle JKL = \angle STU$.

$\triangle KJL \equiv \triangle STU$ because two angles and the side they contain are equal (ASA).

$\triangle EFD$ could be congruent with $\triangle HIG$.

You can see that:

$FE = IH$, $ED = HG$ and $FD = IG$.

$\triangle EFD \equiv \triangle HIG$ because all three sides are equal (SSS).

$\triangle MNO$ could be congruent with $\triangle VWX$.

You can see that:

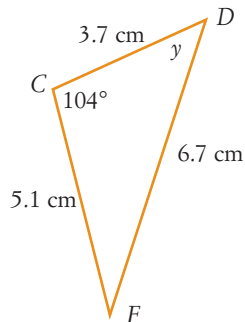
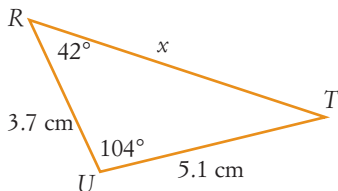
$OM = XV$, $MN = VW$ and $\angle MNO = \angle VWX$, but $\angle MNO$ and $\angle VWX$ are not included by the pairs of equal sides.

$\triangle MNO$ and $\triangle VWX$ are not necessarily congruent because the 45° angle is not included by the pairs of equal sides.

The tests for congruent triangles can be used to solve mathematical problems and establish properties of geometric shapes.

Example 10

Find the values of the unknown angle and side indicated below.



Solution

The two triangles appear to be congruent.
Identify equal sides and angles.

Refer to the tests for congruent triangles.

Identify corresponding vertices.

Write the congruency statement.

We need to calculate the value of x . Find the side in $\triangle CFD$ that corresponds to x .

We need to calculate the value of y . Find the angle in $\triangle RTU$ that corresponds to y .

$$RU = DC$$

$$UT = CF$$

$$\angle U = \angle C$$

The triangles satisfy the SAS test, so they are congruent.

$$R \leftrightarrow D, T \leftrightarrow F, U \leftrightarrow C$$

$$\triangle RTU \equiv \triangle DFC$$

$$RT = DF$$

$$x = 6.7 \text{ cm}$$

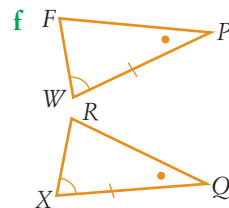
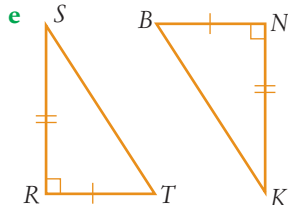
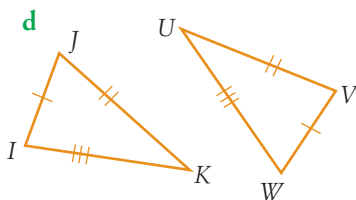
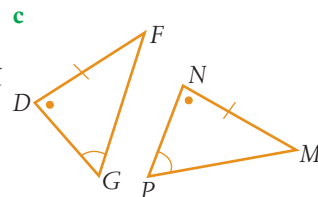
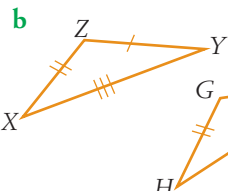
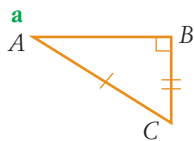
$$\angle R = \angle D$$

$$y = 42^\circ$$

Exercise 6.2 Congruent triangles

Understanding

- 1 Each of the following pairs of triangles is congruent. Write a statement of congruency (e.g. $\triangle ABC \equiv \triangle XYZ$) for each pair of triangles.



Extra questions

Exercise 6.2

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2 Which of the tests for congruency (SSS, SAS, ASA or RHS) can be used to prove that each of the pairs of triangles in Question 1 is congruent?

3 Determine which pairs of triangles are congruent. Give reasons for your answers.

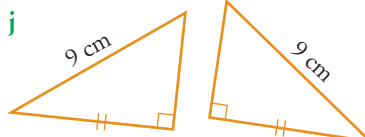
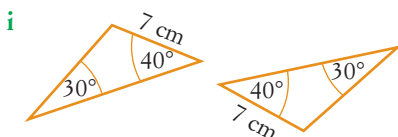
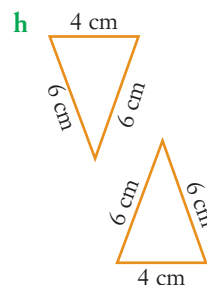
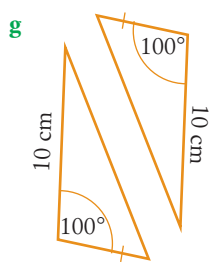
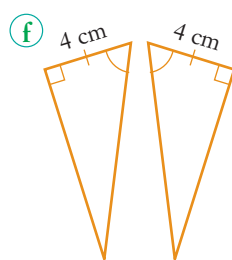
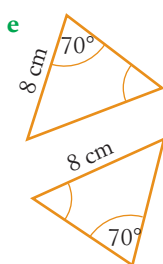
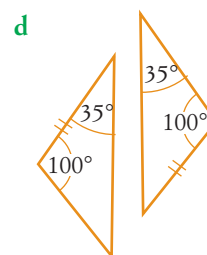
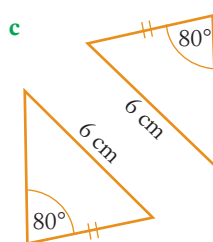
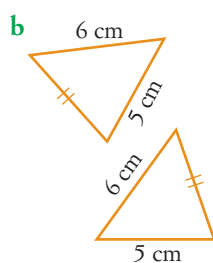
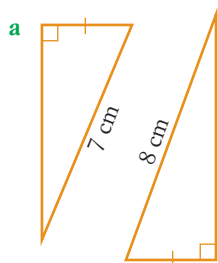
Fluency

Worked solutions

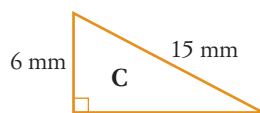
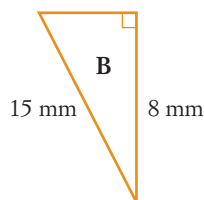
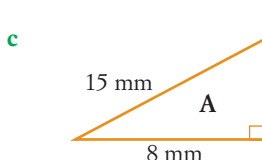
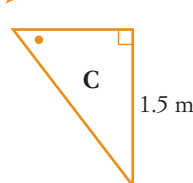
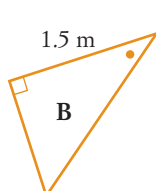
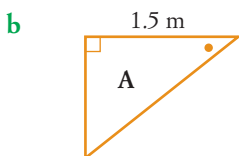
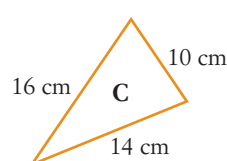
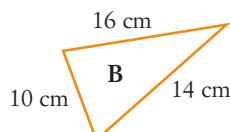
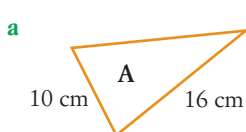
Exercise 6.2

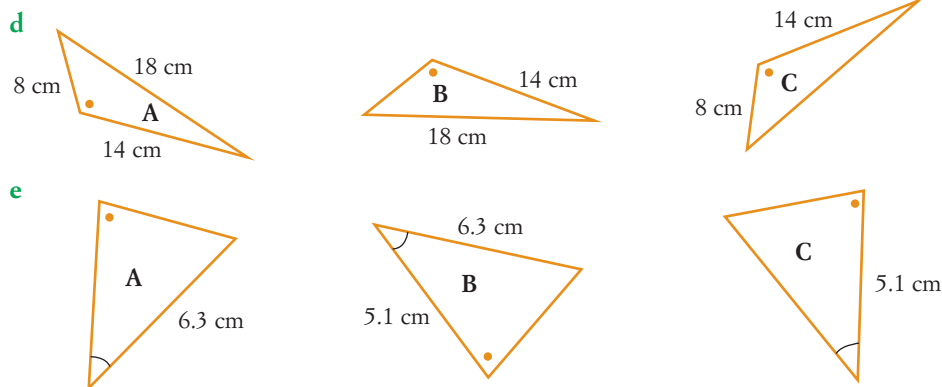
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See Example 9



4 For each set of three triangles, decide which two triangles are congruent and give the reason for your answer (SSS, SAS, ASA, RHS).



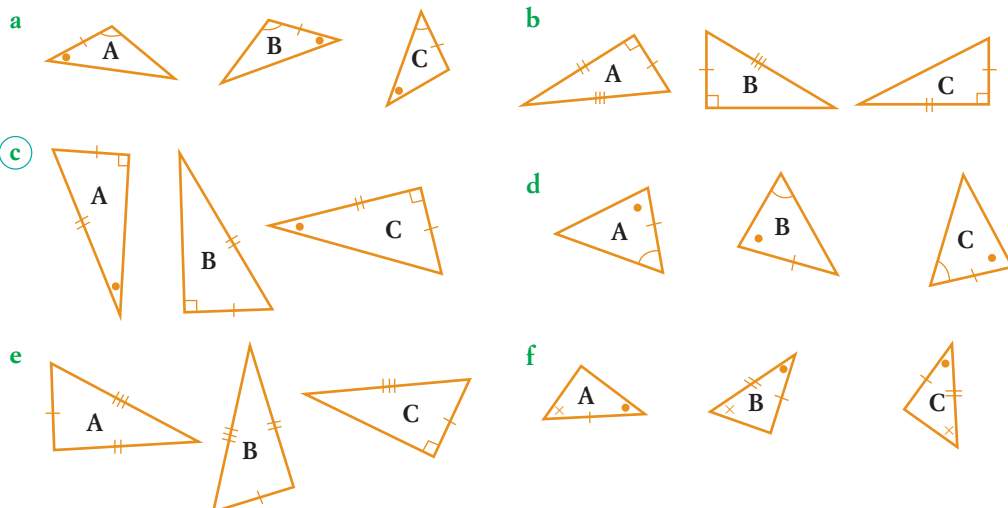


Worked solutions

Exercise 6.2

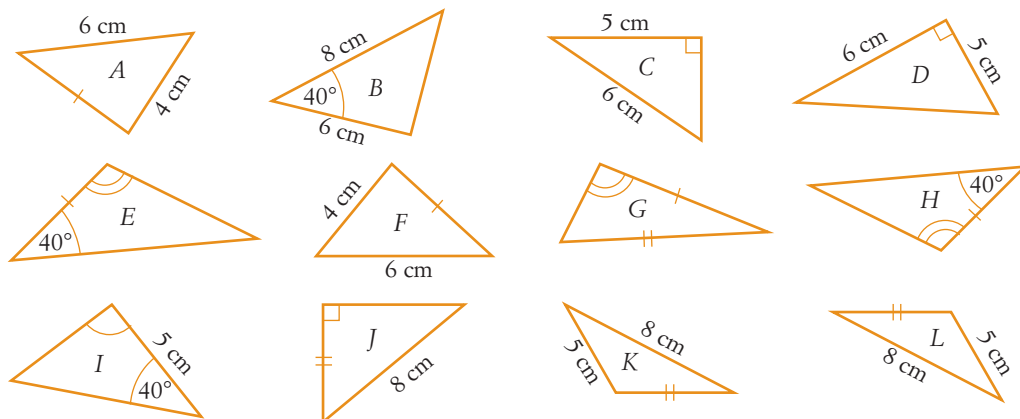
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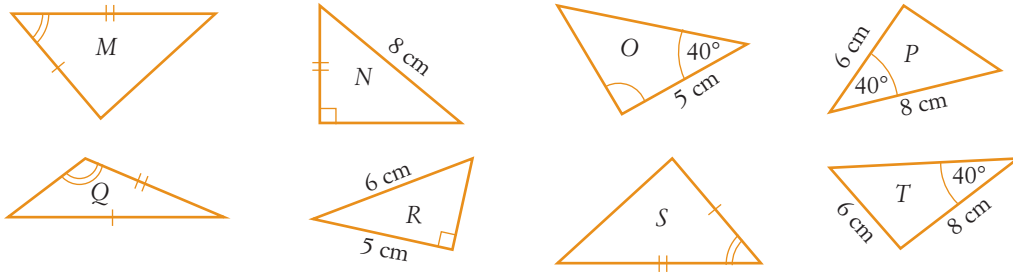
- 5 For each set of three triangles below, decide which two triangles are congruent and give the reason for your answer (SSS, SAS, ASA, RHS).



See Example 9

- 6 Identify congruent triangles in the following diagrams. State why they are congruent in each case.





- 7** Which of the following statements would be sufficient information on its own to identify congruent triangles?
- One triangle can be exactly fitted (superimposed) over the other triangle.
 - All of the angles in one triangle are the same as the angles of the other triangle.
 - The lengths of two sides and one angle of one triangle are exactly the same as the lengths of two sides and one angle of the other triangle.
 - The lengths of the sides of one triangle are the same as the lengths of the sides of the other triangle.
 - A right-angled triangle has two sides that are the same lengths as two sides of another right-angled triangle.
- 8** Find the values of the unknown angles and sides indicated in the diagrams below. Give reasons for your answers.

Problem solving

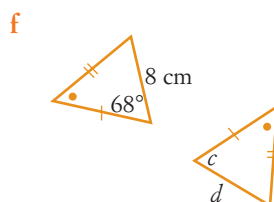
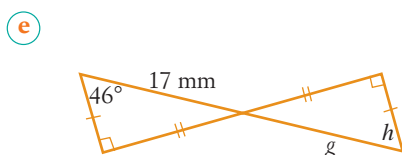
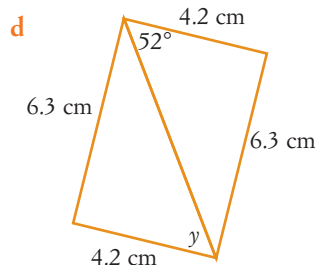
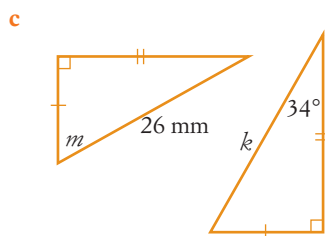
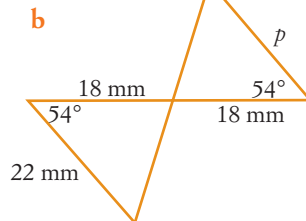
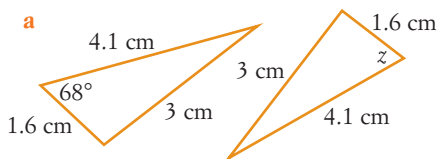
See Example 9

Worked solutions

Exercise 6.2

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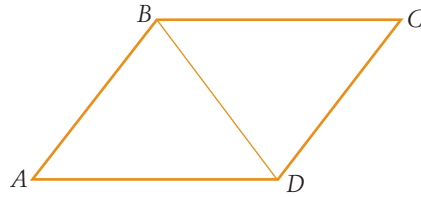
See Example 9



Reasoning

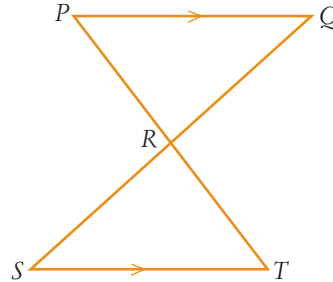
9 If $BC = AD$ and $AB = CD$, prove:

- a $\triangle DBA \equiv \triangle BCD$
- b $\angle ABD = \angle CDB$
- c $AB = CD$



10 If $PQ \parallel ST$ and SQ bisects PT at R , prove:

- a $\triangle PQR \equiv \triangle STR$
- b $\angle PQS = \angle QST$
- c $PR = TR$



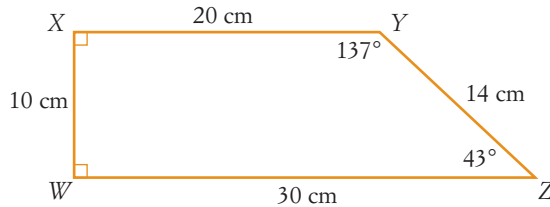
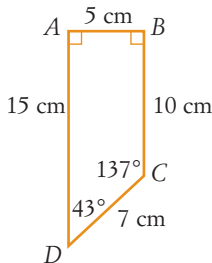
6.3 Similar triangles

Video tutorial

Areas of similar figures

MAT09MGVT10024

You have previously seen that **similar** figures have the same shape but different sizes.



Corresponding points in similar figures are ones that are in matching positions. In the figures above, you can see that:

$$A \leftrightarrow W, B \leftrightarrow X, C \leftrightarrow Y \text{ and } D \leftrightarrow Z$$

Comparing the quadrilaterals above, you can see that corresponding angles are equal:

$$\angle A = \angle W, \angle B = \angle X, \angle C = \angle Y, \angle D = \angle Z$$

You can also see that corresponding sides are in the same ratio:

$$\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW} = \frac{1}{2}$$

So the figures must be similar. When we write a statement of similarity, corresponding points must be written in the same order for each figure:

$$ABCD \parallel WXYZ$$

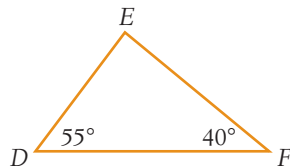
There are certain conditions that need to be met for two triangles to be congruent. Let us look at the conditions that need to be met for us to say that two triangles are similar.

Investigate: Conditions for similarity

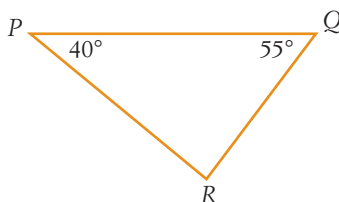
Work in groups of 3 or 4 for this activity

Case 1

- Draw $\triangle DEF$ with $\angle D = 55^\circ$ and $\angle F = 40^\circ$.



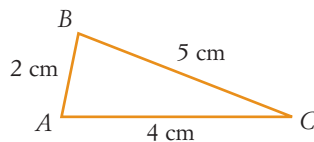
- Draw $\triangle QRP$ with $\angle Q = 55^\circ$ and $\angle P = 40^\circ$.



- Calculate the value of the third angle in each triangle.
- Are the triangles the same shape?
- Are the triangles similar?
- Would any two triangles that have two angles equal be similar?

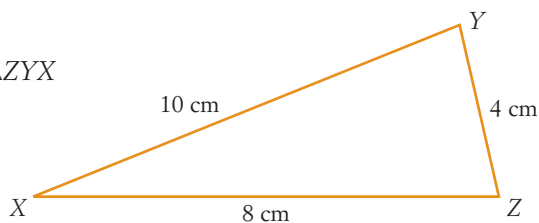
Case 2

- Draw $\triangle ABC$ with $AC = 4$ cm, $AB = 2$ cm and $BC = 5$ cm.
- Is it possible to draw another triangle with sides of 4 cm, 2 cm and 5 cm that is different from $\triangle ABC$? Can you say that $\triangle ABC$ is unique?
- Now draw $\triangle ZYX$ with $ZX = 8$ cm, $ZY = 4$ cm and $YX = 10$ cm.



- Complete the following statements.
The corresponding sides of $\triangle ABC$ and $\triangle ZYX$ are in the . . . ratio.
 $\triangle ABC$ and $\triangle ZYX$ are . . . triangles.

- Would any triangles with corresponding sides in the same ratio be similar?



Case 3

- Draw $\triangle LMN$ with $LN = 5$ cm, $LM = 3$ cm and $\angle L = 70^\circ$.
- Is it possible to draw another triangle with sides of 5 cm, 3 cm and the included angle equal to 70° that is different from $\triangle LMN$? Can you say that $\triangle LMN$ is unique?
- Now draw $\triangle RTS$ with $RS = 8$ cm, $RT = 4.8$ cm and $\angle R = 70^\circ$.

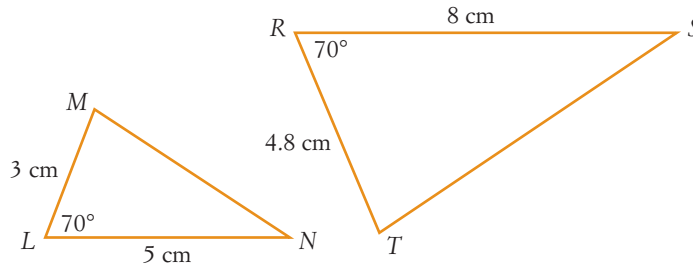
- Complete the following statements.

Two corresponding sides of $\triangle LMN$ and $\triangle RTS$ are in the . . . ratio.

The angles included by these sides are . . .

$\triangle LMN$ and $\triangle RTS$ are . . . triangles.

- Would any two triangles that have two corresponding sides in the same ratio and the included angles equal be similar?



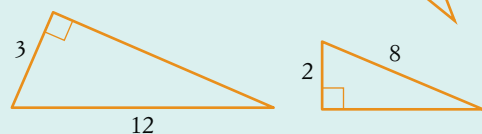
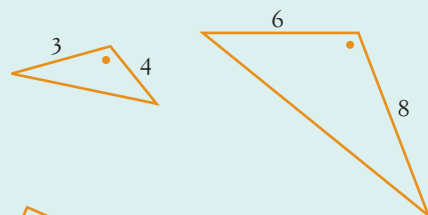
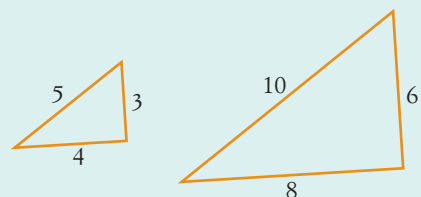
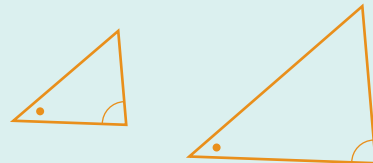
Just as we have special tests to show that triangles are congruent, we have tests to check if triangles are similar.

Important!

Tests for similar triangles

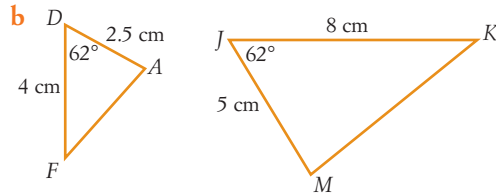
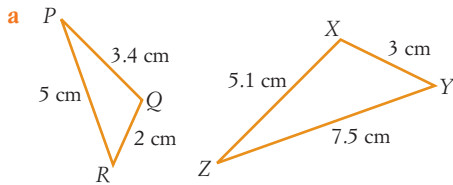
The tests for similar triangles are as follows.

- Angle, angle, angle or AAA**
Triangles are similar if all corresponding angles are equal.
- Side, side, side or SSS**
Triangles are similar if all pairs of corresponding sides are in the same ratio.
- Side, side, angle or SSA**
Triangles are similar if two pairs of corresponding sides are in the same ratio and the included angles are equal.
- Right angle, hypotenuse, side or RHS**
Triangles are similar if they are both right-angled and the hypotenuse and another side are in the same ratio.



Example 11

In each case below, determine if the pair of triangles is similar.



Solution

- a** Calculate the ratio of the smallest sides.

$$\frac{QR}{XY} = \frac{2}{3}$$

Calculate the ratio of the largest sides.

$$\frac{PR}{ZY} = \frac{5}{7.5} = \frac{50}{75} = \frac{2}{3}$$

Calculate the ratio of the remaining sides.

$$\frac{PQ}{ZX} = \frac{3.4}{5.1} = \frac{34}{51} = \frac{2}{3}$$

Identify corresponding vertices.

$$P \leftrightarrow Z, Q \leftrightarrow X, R \leftrightarrow Y$$

All pairs of corresponding sides are in the same ratio.

$$\triangle PQR \parallel \triangle ZXY \text{ (SSS)}$$

- b** Two angles are the same.

$$\angle FDA = \angle KJM$$

Calculate the ratio of the smaller sides.

$$\frac{DA}{JM} = \frac{2.5}{5} = \frac{25}{50} = \frac{1}{2}$$

Calculate the ratio of the larger sides.

$$\frac{DF}{JK} = \frac{4}{8} = \frac{1}{2}$$

Identify corresponding vertices.

$$D \leftrightarrow J, A \leftrightarrow M, F \leftrightarrow K$$

Two pairs of corresponding sides are in the same ratio and the included angles are equal.

$$\triangle DAF \parallel \triangle JMK \text{ (SSA)}$$

Worksheet

Similar triangles

MAT09MGWK00039

A statement of similarity identifies corresponding points in figures.

Example 12

If $\triangle MAC \parallel \triangle JOG$, complete the following.

- a** $\angle ACM = \angle \dots$

- b** $\angle OJG = \angle \dots$

Solution

- a** $\triangle MAC \parallel \triangle JOG$.

$$M \leftrightarrow J, A \leftrightarrow O \text{ and } C \leftrightarrow G.$$

Identify the angle that corresponds to $\angle ACM$.

$$\angle ACM \leftrightarrow \angle OGJ$$

Corresponding angles in similar triangles are equal.

$$\angle ACM = \angle OGJ$$

- b** Use the corresponding vertices. $\angle OJG \leftrightarrow \angle AMC$.

$$\angle OJG = \angle AMC$$

Example 13

If $\triangle RTS \parallel \triangle KLM$, $RT = 12$ cm, $TS = 5$ cm and $KL = 30$ cm, calculate the value of LM .

Solution

$$\triangle RTS \parallel \triangle KLM.$$

Identify the corresponding sides in the triangles.

Calculate the ratio of RT and KL .

The ratio of corresponding sides is equal.

Use the result $\frac{RT}{KL} = \frac{2}{5}$

Substitute for TS .

Cross-multiply.

Divide both sides by 2.

State the result.

$$R \leftrightarrow K, T \leftrightarrow L \text{ and } S \leftrightarrow M.$$

$$RT \leftrightarrow KL$$

$$TS \leftrightarrow LM$$

$$SR \leftrightarrow MK$$

$$\frac{RT}{KL} = \frac{12}{30} = \frac{2}{5}$$

$$\frac{RT}{KL} = \frac{TS}{LM}$$

$$\frac{2}{5} = \frac{TS}{LM}$$

$$\frac{2}{5} = \frac{5}{LM}$$

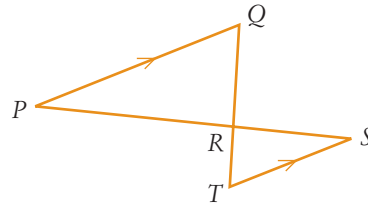
$$2 LM = 25$$

$$LM = 12.5$$

$$LM \text{ is } 12.5 \text{ cm long.}$$

Example 14

Prove that $\triangle PQR$ is similar to $\triangle STR$.



Solution

$\angle P$ and $\angle S$ are alternate angles on parallel lines.

$\angle Q$ and $\angle T$ are alternate angles on parallel lines.

$\angle QRP$ and $\angle TRS$ are vertically opposite angles.

Refer to the tests for similar triangles.

$$\angle P = \angle S$$

$$\angle Q = \angle T$$

$$\angle PQR = \angle STR$$

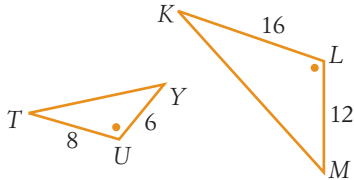
$$\triangle PQR \parallel \triangle STR$$

The AAA test for similar triangles was used in the previous example. While the test states that triangles are similar if all corresponding angles are equal, it is sufficient to show that only two pairs of corresponding angles are equal. Why?

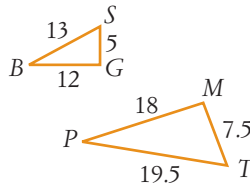
Exercise 6.3 Similar triangles

- 1 Each of the following pairs of triangles is similar. Write a statement of similarity (e.g. $\triangle ABC \sim \triangle XYZ$) in each case.

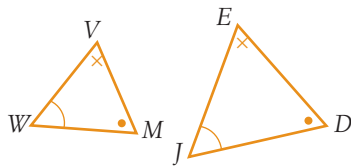
a



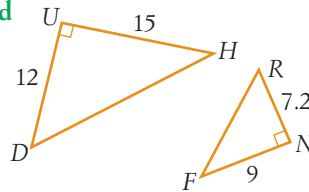
b



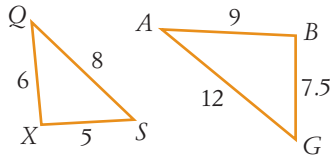
c



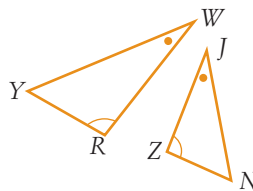
d



e



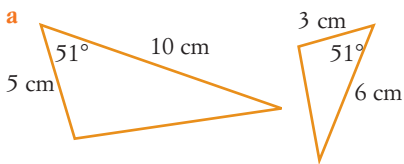
f



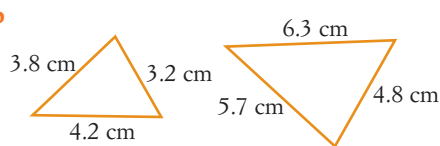
- 2 Which of the tests for similarity (SSS, AAA or SSA) can be used to prove that each of the pairs of triangles in question 1 is congruent?
- 3 Determine if each of the following pairs of triangles is similar. Give a reason for each answer (e.g. AAA, SSS or SSA).

See Example 11

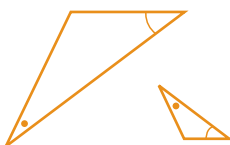
a



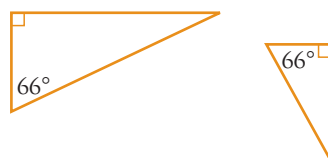
b



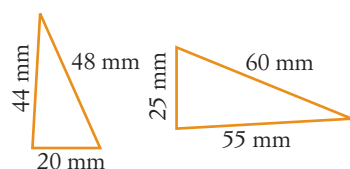
c



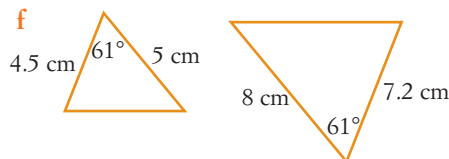
d



e



f



Understanding

Extra questions

Exercise 6.3

MAT09MGEQ00022

Worksheet

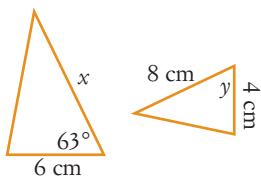
Congruence and
similarity review

MAT09MGWK00046

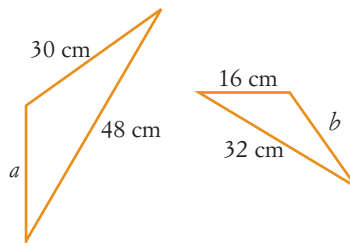
Fluency

- 4 Each of the following pairs of triangles is similar. Find the value of the unknown in each case.

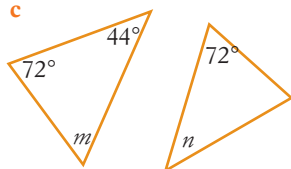
a



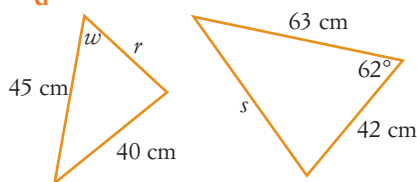
b



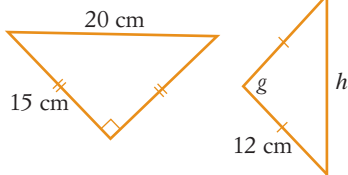
c



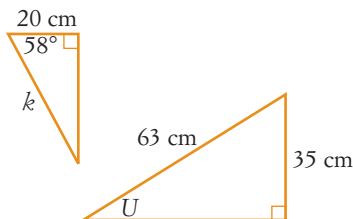
d



e



f



See Example 12

- 5 If $\triangle CRT \sim \triangle NMP$ select from **A**, **B**, **C** and **D** below to complete the statement $\angle T = \dots$
A $\angle N$ **B** $\angle M$ **C** $\angle P$ **D** $\angle R$

See Example 12

- 6 If $\triangle DAF \sim \triangle WRX$ select from **A**, **B**, **C** and **D** below to complete the statement $\frac{DA}{WR} = \dots$
A $\frac{AD}{RX}$ **B** $\frac{DF}{WX}$ **C** $\frac{FA}{WR}$ **D** $\frac{AF}{RW}$

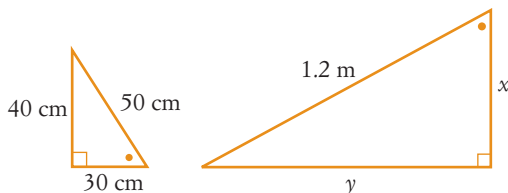
Worked solutions

Exercise 6.3

MAT09MGWS00022

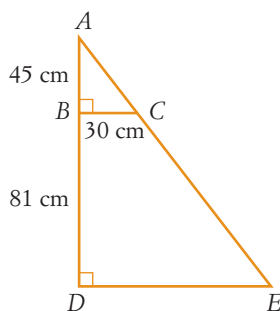
See Example 11

- 7 Show that these two triangles are similar and then calculate the values of x and y .



See Example 11

- 8 a Show that $\triangle ABC \sim \triangle ADE$.
 b Calculate the value of DE .



- 9 If $\triangle ABC \parallel \triangle PQR$, $AB = 28$ mm, $AC = 15$ mm and $PR = 27$ mm, calculate the value of PQ .
10 If $\triangle KLM \parallel \triangle FDE$, $KL = 62$ cm, $MK = 46.5$ cm and $DF = 40$ cm, calculate the value of FE .

See Example 13

See Example 13

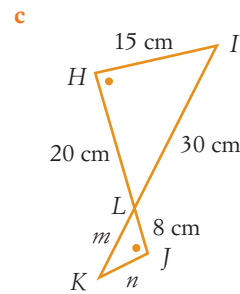
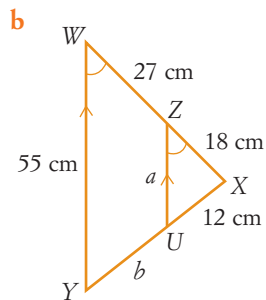
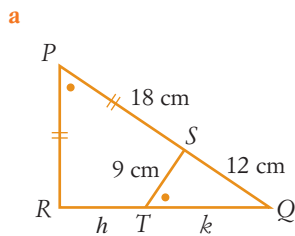
- 11 i Identify similar triangles in each of the following and state the test for similarity.
ii Calculate the value of the unknown sides in each case.

Problem solving

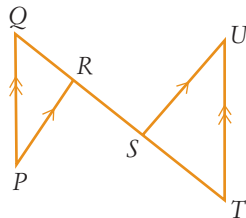
Puzzle sheet

Nested similar triangles

MAT09MGPS00025



- 12 Prove that the triangles shown here are similar. Give reasons for each step in the proof.



Reasoning

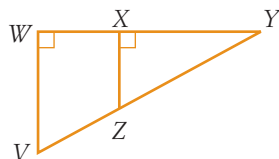
Worked solutions

Exercise 6.3

MAT09MGWS00022

See Example 14

- 13 Prove that the triangles shown here are similar. Give reasons for each step in the proof.



See Example 14



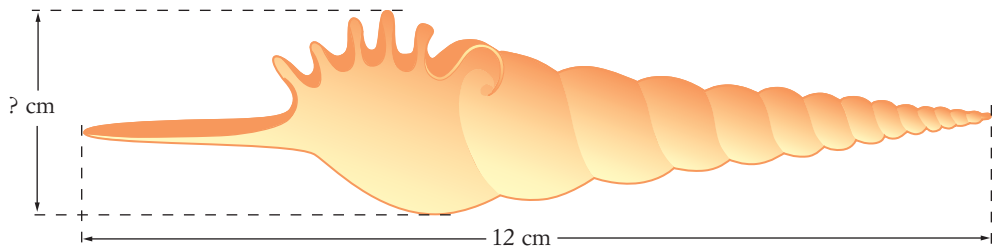
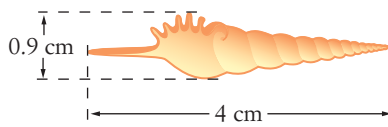
6.4 Applications of similarity

You have seen that the lengths of corresponding sides are proportional in similar figures. Corresponding distances are related by the scale factor. The scale factor is the number by which distances in one figure are multiplied in order to produce the corresponding distances in the similar figure.

Example 15

An original drawing of a shell and an enlargement are shown here.

- Find the scale factor.
- Find the width of the shell in the enlargement without measuring.



Solution

- Write down the shell length in each drawing.

Length in original = 4 cm
Length in enlargement = 12 cm

The scale factor is the ratio of the lengths in the enlargement and the original.

$$\text{Scale factor} = \frac{12}{4}$$

Simplify.

$$= 3$$

- Calculate the width.

$$\begin{aligned} \text{Width in enlargement} &= \text{width in original} \\ &\quad \times \text{scale factor} \\ &= 0.9 \text{ cm} \times 3 \\ &= 2.7 \text{ cm} \end{aligned}$$

When we draw something *to scale*, we are making an enlargement or reduction. The **scale drawing** is similar to the real shape. In scale drawings the scale factor is just called the **scale**. The scale is written as a ratio, with the size of the drawing first.

Example 16

What scale should be used for drawings of:

- a** a house? **b** a skin cell? **c** a street map of Brisbane?

Choose from:

10 000 : 1 100 : 1 1 : 100 1 : 10 000.

Solution

- a** The drawing will need to be smaller than a real house, so we can choose from 1 : 100 or 1 : 10 000. For a scale of 1 : 100, 1 cm on the drawing represents 100 cm (1 m) on the house. This is reasonable.

Write the answer.

House scale = 1 : 100

- b** The drawing will need to be larger than a real skin cell, so we can choose from 100 : 1 or 10 000 : 1. For a scale of 100 : 1, 100 mm (10 cm) on the drawing represents 1 mm on the skin cell. This is reasonable.

Write the answer.

Skin cell scale = 100 : 1

- c** The drawing will need to be much smaller than Brisbane, so we can choose from 1 : 100 or 1 : 10 000. For a scale of 1 : 10 000, 1 cm on the drawing represents 10 000 cm (100 m) on Brisbane. This is reasonable.

Write the answer.

Street map scale = 1 : 10 000

Investigate: How high is it?

Work in groups of 3 or 4 for this investigation.

- Go into the playground or school oval where you can see a tall tree or school building.
- Identify a tall object that is a measurable distance from you.
- Work out a method using similar triangles to calculate the height of the object you have identified.
- Use your method to calculate the height of the object.
- Compare your results with those of other groups.
- Write a report explaining your method and showing the calculations you used to calculate the height of the object.



Example 17

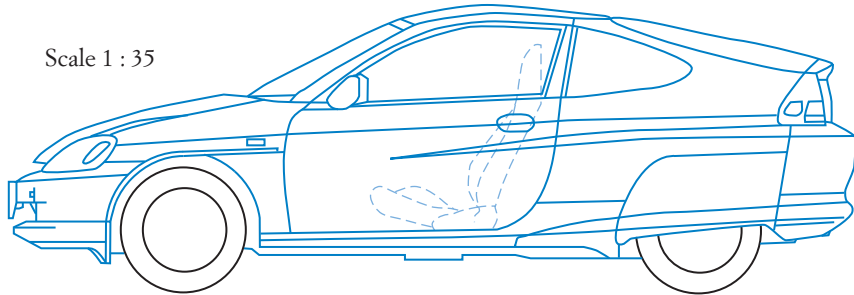
Worksheet

Scale diagrams

MAT09MGWK00040

Calculate the length of the car represented by this scale drawing.

Scale 1 : 35



Solution

Measure the length of the drawing.

The scale is 1 : 35.

The real car is bigger, so we multiply.

State the result.

Drawing length = 11.2 cm

1 cm on the drawing = 35 cm on the car

Car length = $11.2 \text{ cm} \times 35 = 392 \text{ cm}$

The real car is 3.92 m long.

Example 18

This is a micrograph of the end of a polar bear hair. The scale of the picture is 300 : 1. What is the largest diameter of the hole in the hair?



Solution

Measure the hole on the picture.

The real hole is smaller, so we divide.

State the result.

Largest diameter of hole = 15 mm

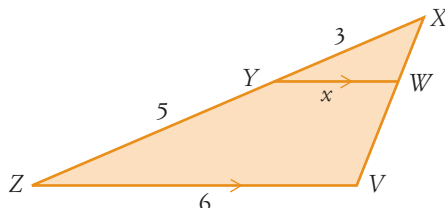
**Real hole diameter $\approx 15 \text{ mm} \div 300$
 $= 0.05 \text{ mm}$**

The largest diameter of the real hole is about 0.05 mm.

When solving problems involving similar triangles, we may first have to use the similarity tests to show that the triangles are similar.

Example 19

Prove that these triangles are similar and then calculate the value of x .



Solution

Use the fact that $ZV \parallel YW$ to find angles that are equal.

$\angle XZV$ and $\angle XYW$ are alternate angles.

$\angle XVZ$ and $\angle XWY$ are alternate angles.

Refer to the tests for similar triangles.

The ratio of corresponding sides is equal.

Use the information from the diagram.

Simplify.

Multiply both sides by 6.

Cancel.

Evaluate.

$$\angle XZV = \angle XYW$$

$$\angle XVZ = \angle XWY$$

$$\triangle XYW \parallel \triangle XZV \text{ (AAA)}$$

$$\frac{YW}{ZV} = \frac{XY}{XZ}$$

$$\frac{x}{6} = \frac{3}{5+3}$$

$$\frac{x}{6} = \frac{3}{8}$$

$$\frac{x}{6} \times 6 = \frac{3}{8} \times 6$$

$$\frac{x}{\cancel{6}^1} \times \cancel{6}^1 = \frac{3}{8_4} \times \cancel{6}^3$$

$$x = \frac{9}{4} \\ = 2\frac{1}{4}$$

Animated example

Similarity

MAT09MGAE00006

CAS TI-Nspire exercise

Congruence and
similarity

MAT09MGTI00006

CAS ClassPad exercise

Congruence and
similarity

MAT09MGCP00006

Exercise 6.4 Applications of similarity

Understanding

Extra questions

Exercise 6.4

MAT09MGEQ00023

See Example 16

1 Here are some scales:

1 000 000 : 1

10 000 : 1

100 : 1

10 : 1

1 : 10

1 : 100

1 : 10 000

1 : 1 000 000

Choose the best scale for a drawing of:

a a bedroom

c Tasmania

e a western Queensland cattle station

g a car engine

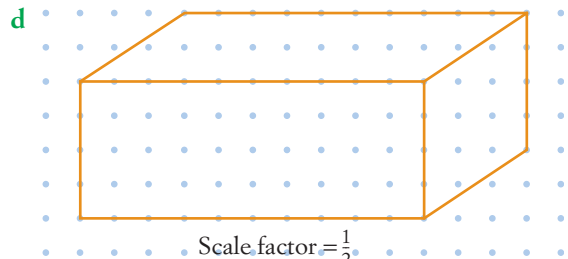
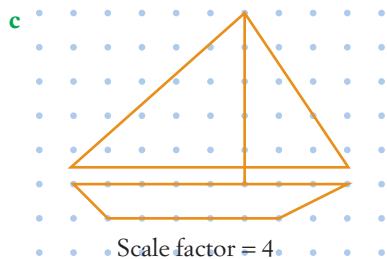
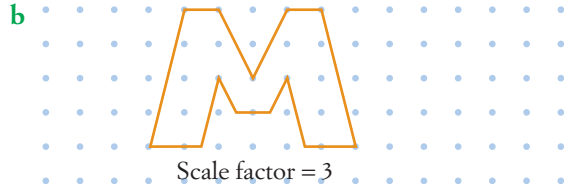
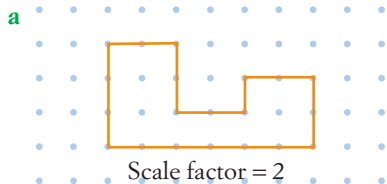
b an adult's leg

d the ear bones of a human

f a yeast cell

h a bacterium

2 For each, use dot or grid paper to draw a similar figure with the given scale factor.



3 Find the value of the unknown in each of the following, giving your answer correct to 1 decimal place if necessary.

a $\frac{x}{3} = \frac{22}{6}$

b $\frac{k}{8} = \frac{24}{32}$

c $\frac{p}{24} = \frac{1.5}{8}$

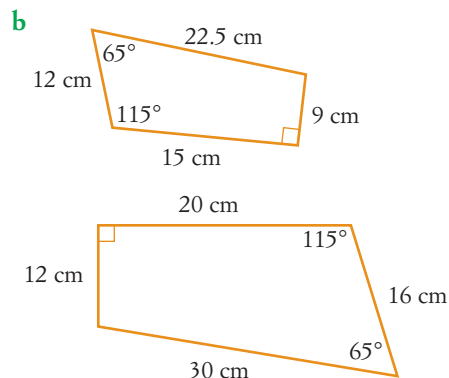
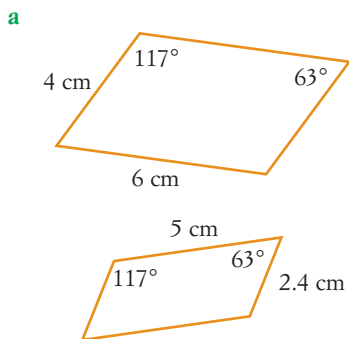
d $\frac{m}{4} = \frac{27}{36}$

e $\frac{9.6}{12.4} = \frac{h}{1.55}$

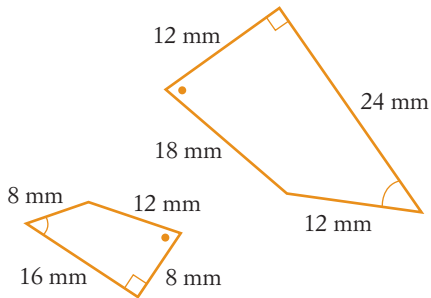
f $\frac{4.8}{16} = \frac{d}{24}$

Fluency

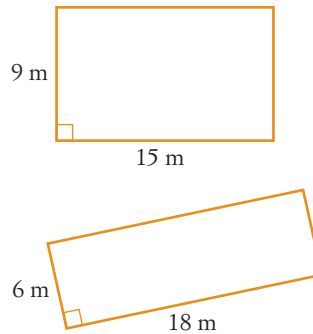
4 Which of the following pairs of figures are similar?



c



d



- 5 Use the scale drawing of the car in Example 17 to work out (to the nearest centimetre):
- a the diameter of the front wheel
 - b the height of the car
 - c the greatest diagonal of the door panel
 - d the greatest height of the front door window
- 6 This is a photograph of a mosquito.
The scale of the photograph is 15 : 1.
Calculate (to the nearest 0.1 mm) the real:
- a length of the wing
 - b length of the body
 - c distance across the body
 - d distance from the body to the first joint in the back leg

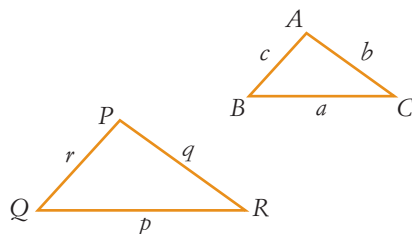
See Example 17

See Example 18



- 7 Use the micrograph of the polar bear hair in Example 18 to work out (to the nearest 0.01 mm):
- a the narrowest diameter of the hole
 - b the thickness of the wall of the hair
 - c the greatest visible diameter of the hair
- 8 Here, $\triangle ABC \parallel \triangle PQR$. For each of the following sets of measures, find the measures of the remaining sides.

See Example 17



Worked solutions

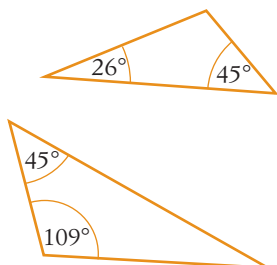
Exercise 6.4

MAT09MGWS00023

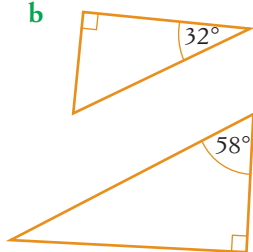
- a $a = 8, b = 7, c = 6, p = 16$
- b $a = 12, b = 10, c = 4, r = 6$
- c $p = 40, q = 36, r = 32, b = 9$
- d $p = 45, q = 36, r = 30, c = 10$

- 9 Determine if each of the following pairs of triangles are similar. Diagrams are not drawn accurately.

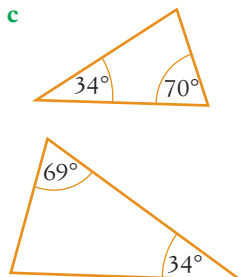
a



b



c



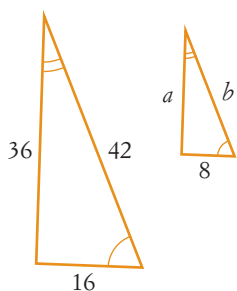
Worked solutions

Exercise 6.4

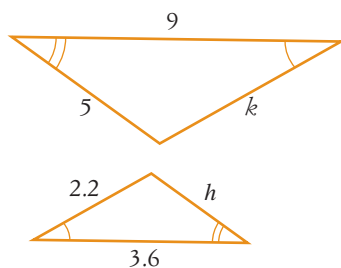
MAT09MGWS00023

- 10 Find the unknown side lengths in each of the following pairs of similar triangles. Diagrams are not drawn accurately.

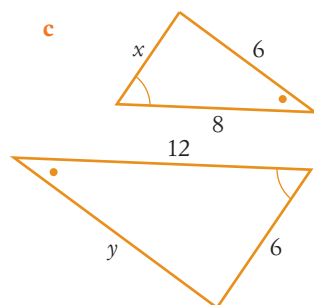
a



b



c



See Example 15

- 11 This *Tyrannosaurus rex* skull is actually 1.2 m high (when the jaw is closed). Calculate the scale factor for this photograph.



- 12 This section of a map of Moreton Bay is drawn to a scale of 1 : 1 000 000.
- How long is North Stradbroke Island?
 - How far is it from Point Lookout to Brisbane?
 - How far is it by road from Beenleigh to Redcliffe?
 - Estimate the area of Moreton Island.
 - Estimate the area of North Stradbroke Island.



Problem solving

See Example 17

CAS TI-Nspire exercise

Congruence and similarity

MAT09MGTI00006

CAS ClassPad exercise

Congruence and similarity

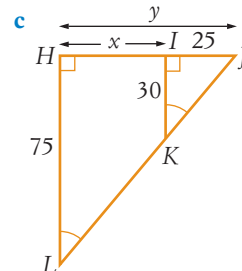
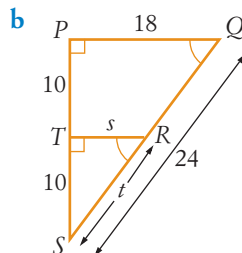
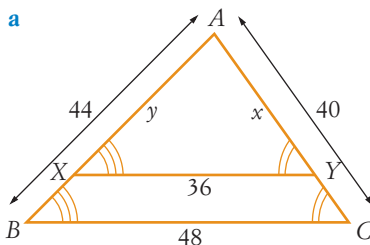
MAT09MGCP00006

Technology worksheet

Excel worksheet: Map scales

MAT09MGCT00023

- 13 Draw separate similar triangles for each of the following diagrams, then calculate the missing lengths.



See Example 19

Reasoning

Worked solutions

Exercise 6.4

MAT09MGWS00023

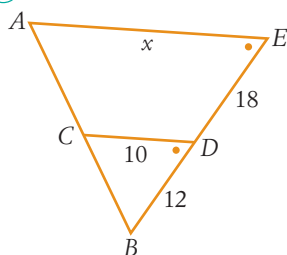
See Example 19

14 In each case below:

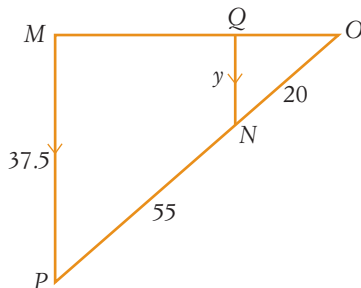
i show that the two triangles are similar

ii calculate the value of the unknown side.

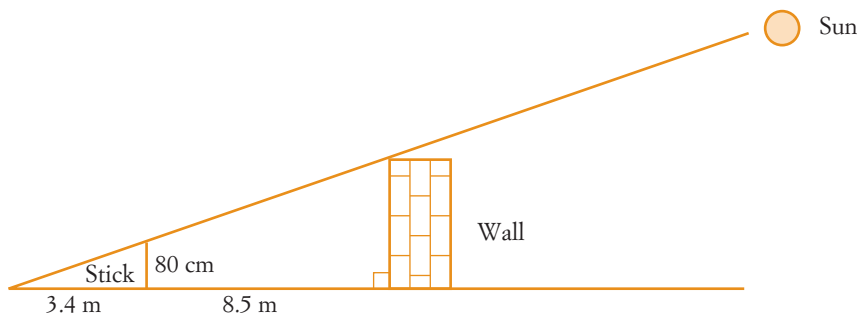
a



b

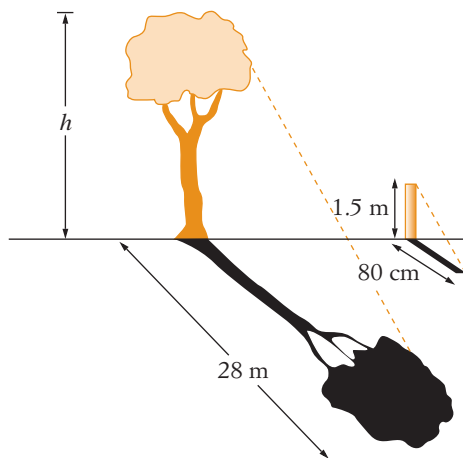


15 A vertical stick is placed 8.5 m from a wall. The shadow cast by the stick is 3.4 m long. The end of the stick's shadow is at the same point as the end of the shadow of the wall. Calculate the height of the wall.



16 A 1.5 m long stick casts a shadow 80 cm long when placed upright on the ground. At the same time, a tree casts a shadow 28 m long as shown in the diagram on the right. Find the height, h , of the tree.

Sun



- **Congruent** figures are the same shape and size.
- There are four tests for **congruent triangles** – three corresponding sides equal (**side, side, side** or **SSS**); two angles and the side they contain equal (**angle, side, angle** or **ASA**); two sides and the included angle equal (**side, angle, side** or **SAS**); and a right angle and the hypotenuse and a corresponding side equal (**right angle, hypotenuse, side** or **RHS**).
- A **transformation** changes the position or appearance of a shape. Coordinates can be used to show a transformation.
- A **translation** is a sliding movement. A **rotation** is a turn, **clockwise** or **anticlockwise**, around the **centre of rotation**. A **reflection** swaps all the points across the **line of reflection**, like a mirror. A **dilation** is an enlargement or reduction from a **centre of magnification**. The **magnification** is the scale factor.
- **Similar** figures are the same shape but not necessarily the same size. Similar triangles have corresponding angles equal and the ratios of corresponding sides equal. The **scale factor**, **magnification** or **enlargement** measures the size increase or decrease of similar shapes.
- There are three tests for **similar triangles** – all corresponding angles are equal (**angle, angle, angle** or **AAA**); all pairs of corresponding sides are in the same ratio (**side, side, side** or **SSS**); two pairs of corresponding sides are in the same ratio and the included angles are equal (**side, side, angle** or **SSA**).
- The magnification in **scale drawings** is called the **scale**. It is given as a ratio, with the size of the drawing first. We multiply or divide by the scale to find the sizes of real things shown in the drawing.

Quiz

Congruence and
similarity

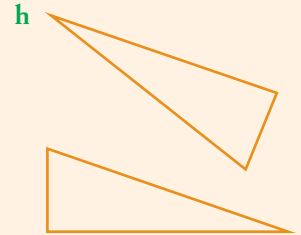
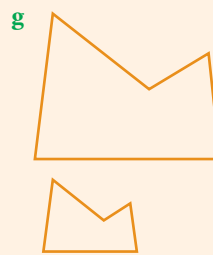
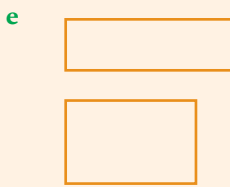
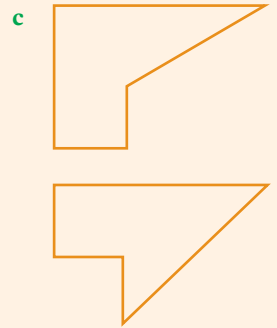
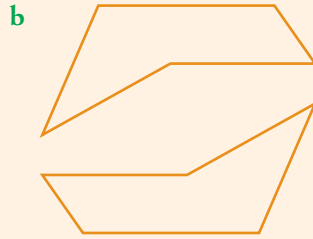
MAT09MGQZ00006

Chapter 6 review

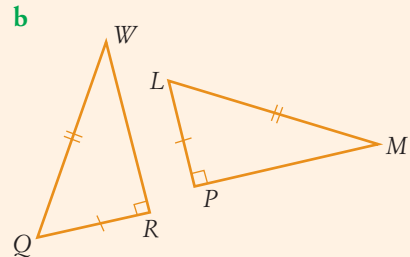
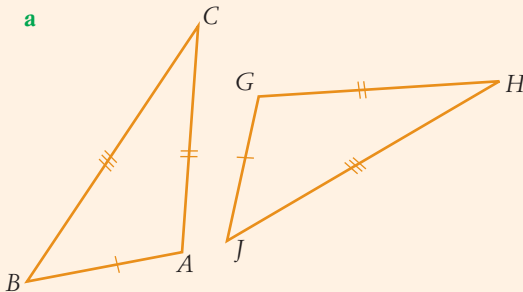
Understanding

See Examples 1, 7

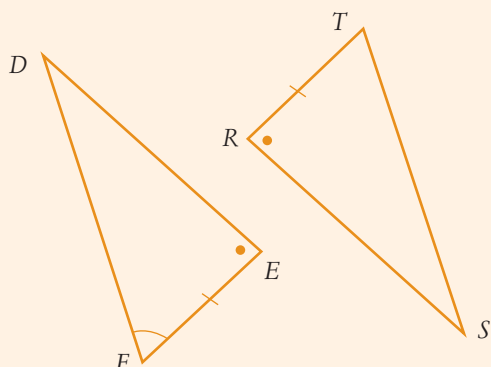
- 1 State if each of the following pairs of shapes are similar, congruent or neither.



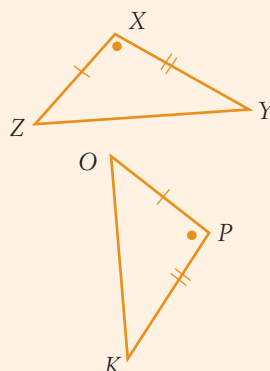
- 2 State whether each of the following is true or false.
- a** Congruent shapes are also similar.
 - b** Similar shapes are also congruent.
 - c** Similar figures are the same shape but different sizes.
 - d** Congruent shapes can be exactly superimposed.
- 3 Each of the following pairs of triangles is congruent. Write a statement of congruency (e.g. $\triangle ABC \equiv \triangle XYZ$) for each pair of triangles.



c

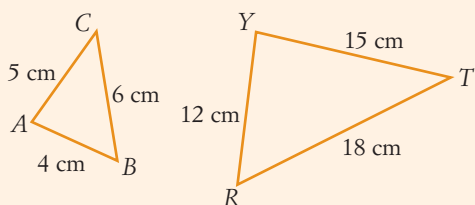


d

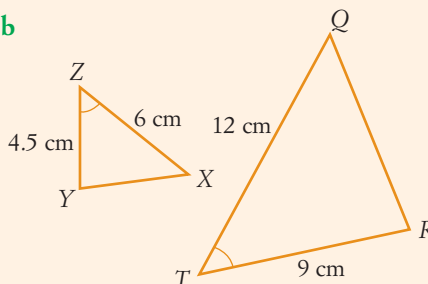


- 4 Which of the tests for congruency (SSS, SAS, ASA or RHS) can be used to prove that each of the pairs of triangles in question 3 is congruent?
- 5 Each of the following pairs of triangles is similar. Write a statement of similarity (e.g. $\triangle ABC \sim \triangle XYZ$) for each pair of triangles.

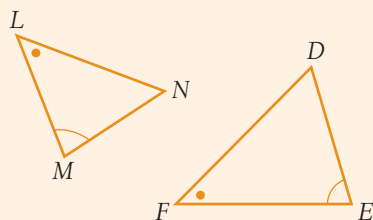
a



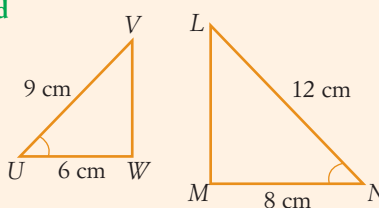
b



c



d



- 6 Which of the tests for similarity (SSS, AAA or SSA) can be used to prove that each of the pairs of triangles in question 5 is similar?
- 7 Here are some scales:
- | | | | |
|---------------|------------|------------|---------------|
| 1 000 000 : 1 | 10 000 : 1 | 100 : 1 | 10 : 1 |
| 1 : 10 | 1 : 100 | 1 : 10 000 | 1 : 1 000 000 |

See Example 16

Choose the best scale for a drawing of:

- a your suburb b your classroom
c a flea d Australia

Chapter 6 review

- 8 Calculate the value of the unknown in each of the following.

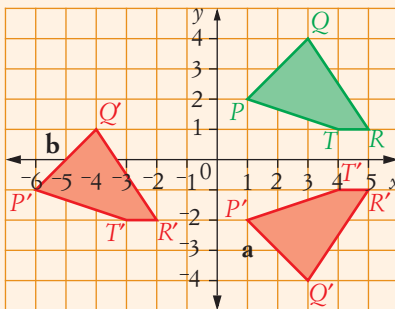
a $\frac{s}{4} = \frac{15}{20}$

b $\frac{8}{x} = \frac{32}{56}$

c $\frac{5}{16} = \frac{m}{24}$

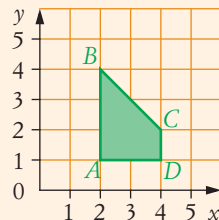
See Example 2

- 9 $PQRT$ has undergone two transformations, as shown here. Identify the transformations.



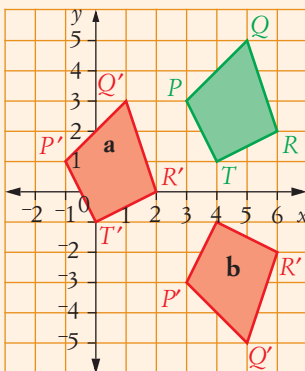
See Example 3

- 10 Draw figure $ABCD$ on some grid paper, then:
- translate it 3 units right and 4 units down
 - reflect it in the y -axis
 - dilate it with centre $(0, 0)$ and magnification 3
 - rotate it 270° anticlockwise around B .



See Example 2

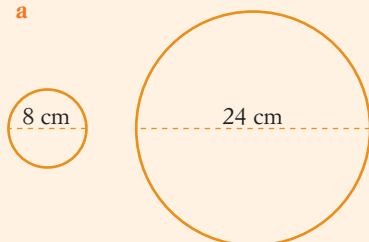
- 11 $PQRT$ has undergone two transformations, as shown here. For each transformation:
- identify the transformation
 - describe the transformation using coordinates.



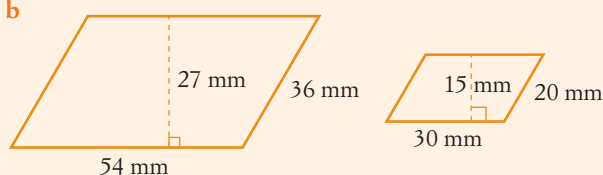
- 12** In each case the object is on the left and the image is on the right.

See Example 8

a



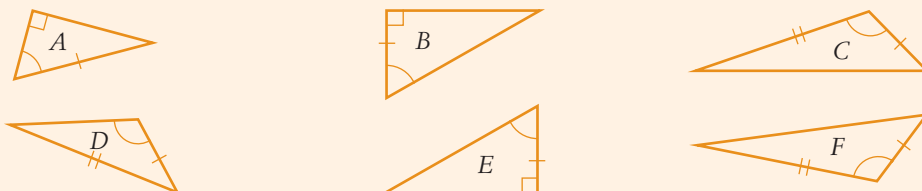
b



- Calculate the scale factor for each enlargement shown above.
 - Calculate the perimeter of each figure.
 - Calculate the area of each figure.
- 13** Draw figure $PQRT$ from question 11 on some grid paper, then:
- translate it 2 units right and 5 units down
 - reflect it in the y -axis
 - dilate it with centre $(0, 0)$ and magnification 2
 - rotate it 90° anticlockwise around T .

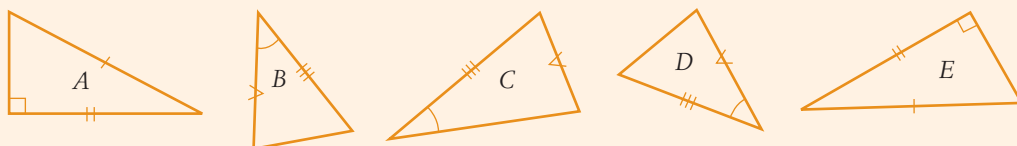
- 14** Name any congruent triangles below, giving reasons for your answers.

See Example 9



- 15** Name any congruent triangles below, giving reasons for your answers.

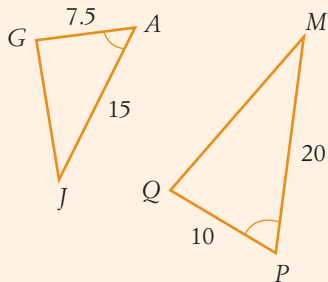
See Example 9



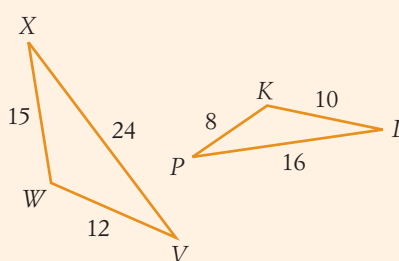
- 16** Determine if each of the following pairs of triangles is similar. Write a statement of similarity and give a reason for each answer (e.g. AAA, SSS or SSA).

See Example 11

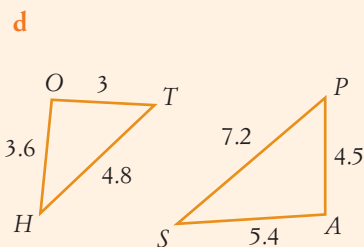
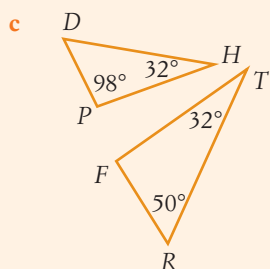
a



b



Chapter 6 review



See Example 12

- 17** If $\triangle KLP \parallel \triangle FDW$, select from **A**, **B**, **C** and **D** below to complete the statement $\angle P = \dots$

A $\angle W$

B $\angle F$

C $\angle D$

D $\angle L$

See Example 12

- 18** If $\triangle YUM \parallel \triangle AEC$, select from **A**, **B**, **C** and **D** below to complete the statement $\frac{YU}{AE} = \dots$

A $\frac{MU}{EC}$

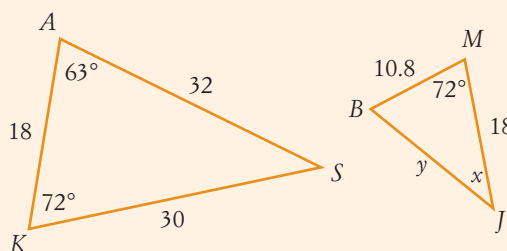
B $\frac{YM}{AE}$

C $\frac{UY}{CE}$

D $\frac{MY}{CA}$

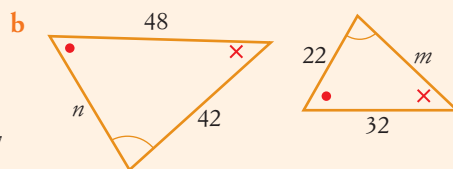
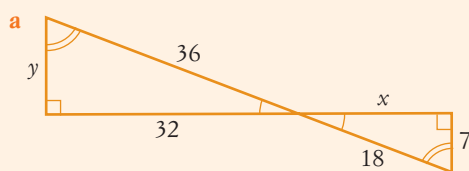
See Example 11

- 19** Show that these two triangles are similar and then calculate the values of x and y .

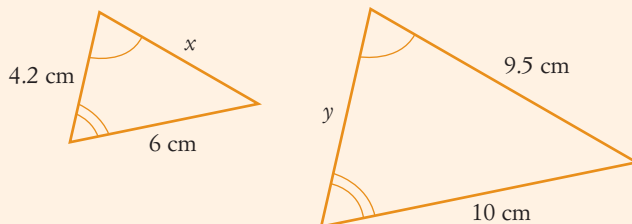


See Example 11

- 20** Find the missing lengths in each pair of similar triangles.

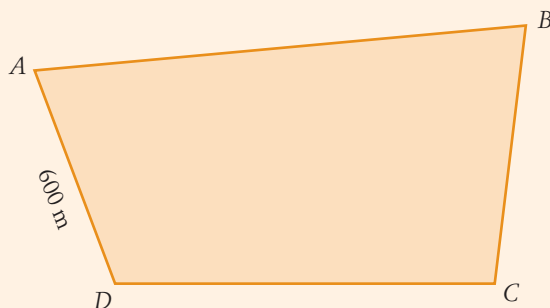


- 21** Find the missing lengths in this pair of similar triangles.



- 22 This is a scale drawing of a plot of land.

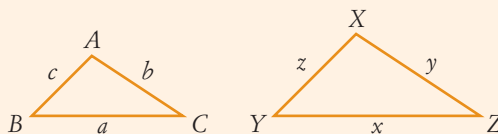
- Calculate the scale factor.
- What are the real lengths of the boundaries of the land shown in the drawing by AB and CD ?



See Example 15

- 23 Here, $\triangle ABC \parallel \triangle XYZ$. In each of the following cases, find the measures of the remaining sides.

- $a = 16$, $b = 7$, $c = 12$, $x = 48$
- $b = 44$, $c = 20$, $x = 32.4$, $y = 26.4$



- 24 Transformations can be represented by changing the coordinates of the vertices of shapes. In Example 5 on page 206, the coordinates of $TICK$ change to those of $T'I'C'K'$. Some transformations can be expressed as a rule for the coordinates.

Problem solving

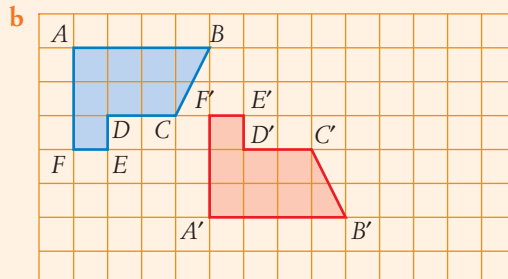
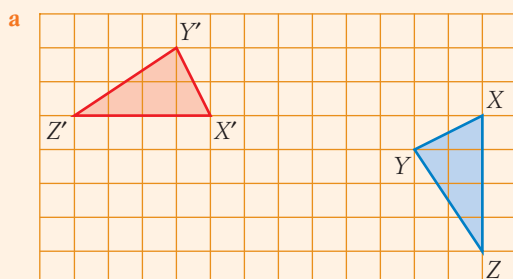
- Add $(+3, -2)$ to each of the coordinates of $T(3, 4)$, $I(4, 3)$, $C(6, 6)$ and $K(4, 2)$ to get $T'I'C'K'$. Draw a sketch of $T'I'C'K'$. What type of transformation is this?
- Multiply each of the coordinates of $TICK$ by 3 to get $T'I'C'K'$. Draw a sketch of $T'I'C'K'$. What type of transformation is this?
- $TICK$ is translated 4 right and 5 down. Without drawing a diagram, find the coordinates of $T'I'C'K'$.
- $TICK$ is enlarged with magnification 5 and centre $(0,0)$. Without drawing a diagram, find the coordinates of $T'I'C'K'$.

- 25 The points $A(3, 5)$, $B(4, 6)$ and $C(6, 1)$ form a triangle. Without drawing, find the coordinates after each of the following transformations.

- Translation 2 up and 4 left
- Reflection in the x -axis
- Dilation by a factor of 3 with centre at $(0, 0)$
- Rotation by 90° clockwise around $(0, 0)$

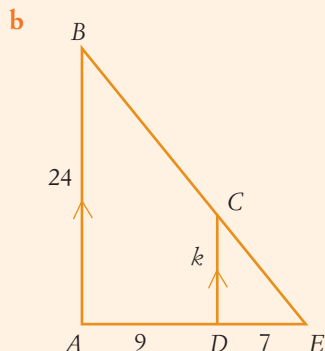
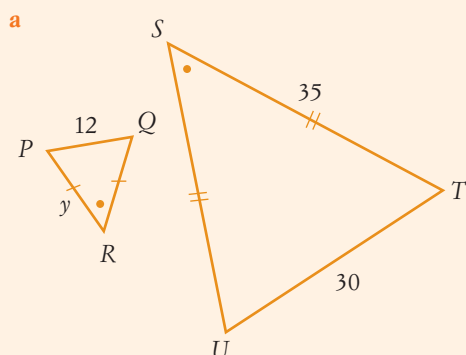
- 26 Describe the two transformations that could be used to superimpose the green figure onto the congruent red figure in each case.

See Example 4



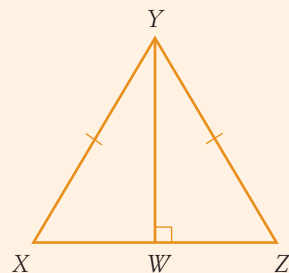
Chapter 6 review

27 Calculate the value of the unknown sides in each case.

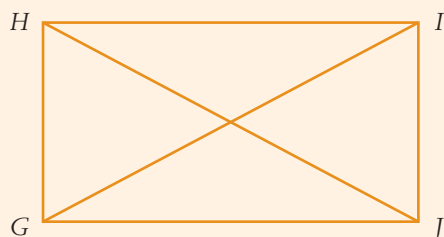


Reasoning

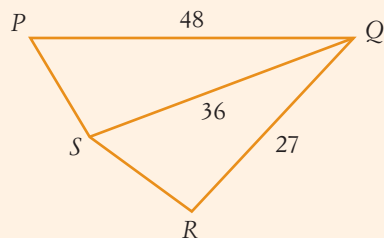
28 In $\triangle XYZ$, $XY = YZ$ and $YW \perp XZ$. Prove that $XW = ZW$.



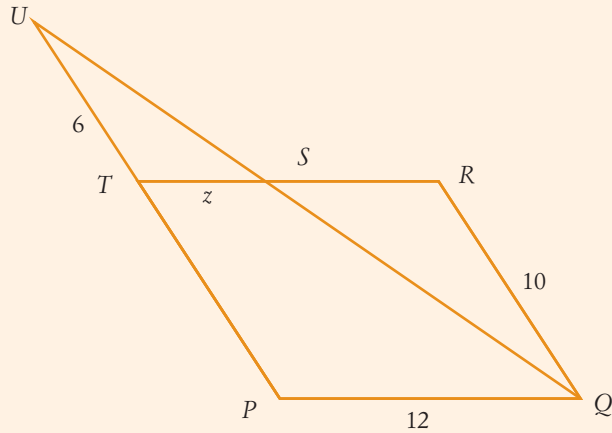
29 $GHIJ$ is a rectangle. Prove that $HJ = IG$.



30 SQ bisects $\angle PQR$. Prove that $\triangle PQS \cong \triangle SQR$.



- 31 $TRQP$ is a parallelogram. Calculate the value of z .



- 32 A stick is perpendicular to the ground near a wall. The top of the stick is 1.5 m above the ground and it casts a shadow 1.2 m long. How long is the shadow cast by the wall?



Sun





Statistics and probability

7

Probability



Contents

- 7.1 Calculating probability
- 7.2 Combined events
- 7.3 Multi-step probabilities
- Chapter summary
- Chapter review

Prior learning

Chapter 7

MAT09SPPL00007

Parent guide

Chapter 7

MAT09SPPG00007

Curriculum guide

Chapter 7

MAT09SPCU00007

Australian Curriculum statements

Chance

List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events. Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'.

Video tutorial
Probability

MAT09SPVT00007

Knowing the likelihood of something happening in the future is very important for planning. If you live in the north of Australia, you have a high risk of being hit by a tropical cyclone, so it is worth taking precautionary measures. While the risk in any one year is fairly low, the potential damage is very high. Houses near the coast in the north of Australia are built to withstand cyclones because it is likely that a house will be hit by a cyclone during its lifetime. In Melbourne, houses are insulated to protect families from very cold or very hot temperatures. It is unlikely that a house in Tasmania will need to be cool enough for a heatwave of over 40° temperatures for a week. In this chapter, you will further develop your ideas about probability.

Mathematical literacy

Maths dictionary

MAT09ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

complement	favourable	sample space	trial
element	outcomes	simplified tree	two-way table
event	frequency	diagram	universal set
expected frequency	mutually exclusive	theoretical	Venn diagram
experimental	relative frequencies	probability	
probability	sample point	tree diagram	

7.1 Calculating probability

Weblink

The world clock

MAT09SPWB00007

We can determine the likelihood of something happening in the future by checking what happens in experiments, collecting data or working out how many times things could happen.

Weblink

The Spanish flu

MAT09SPWB00007

Investigate: Card hands

You should work in groups of five or six. *Each group will need a normal pack of playing cards.* Before you start, each group should draw up a table like the one below to record results.

Deal number	No pairs	One pair	Two pairs	Three of a kind or better

- 1 Shuffle and deal 5 cards to each member of the group, and check the hands that result.
- 2 Make a note of the number of hands that have no pairs, a pair, two pairs, or three of a kind or better.
- 3 Pass the deal on and repeat. Do 4 or 5 deals each so that over 100 hands are counted.
- 4 Now total each column and work out the total number of hands that were dealt.
- 5 Work out the percentage of hands that had no pairs. Work out the percentage that had a pair, and so on.

6 Answer these questions.

- a If 500 hands were dealt, how many would you expect to have a pair?
- b If you were dealt a hand of 5 cards, what do you think would be your chance of getting a pair?

The words that you use to describe probability, such as likely, unlikely and so on have particular meanings.

Important!

Probability descriptions

An **event** is something that might or might not happen that you can describe exactly.

An event that can never happen is said to be **impossible**.

An event that could sometimes happen is **possible**. The **possible outcomes (events)** of a situation are all the things that could happen.

A **certain** event will always happen.

An event that occurs less than half the time is **unlikely** or **improbable**.

An event that is just as likely to occur as not occur is said to have an **even chance** or a **fifty-fifty chance**.

An event that occurs more than half the time is **likely** or **probable**.

When we compare two possible events, we say that the event with a greater chance of occurring is the **more likely** event and the other is the **less likely** event.

Possible events that have the same chance of occurring are called **equally likely** events.

Example 1

Describe the probability of each of the following events.

- a You will live to be 110 years old.
- b A student chosen at random in your class was born on a weekend.
- c A person chosen at random is a female.
- d A student chosen at random in your class has had their 14th birthday.
- e A student chosen at random in your class was born on a day with a 'd' in the name.

Solution

a Very few people live to be 110.	This is very unlikely.
b There are 2 weekend days out of 7.	It is unlikely that a student was born on a weekend.
c About half of the population is female.	There is a fifty-fifty chance of selecting a female.
d Most students turn 15 in Year 9.	It is likely that a Year 9 student has had their 14th birthday.
e Every day has a 'd' in it.	It is certain that the day has a 'd' in the name.

The kind of information used to work out chances makes a difference to what we call the probability.

Important!

Experimental and theoretical probability

Experimental probability uses real data to work out chances or likelihoods. Each item of data is called a **trial**.

Theoretical probability is based on a list that contains all the possible outcomes. The list is called the **sample space**. Every outcome in the sample space is listed exactly as many times as it can occur. A single outcome is called an **element** or **sample point**.

An **event** is a collection of outcomes, whether they come from trials or a sample space. The set could have any number of elements, from none up to the total number.

The outcomes in an event for which you are calculating the probability are called **favourable outcomes**. The number of elements in an event is written as $n(\text{event})$ or is the **frequency** of the event.

The probability of an event is a number from 0 to 1, written as $P(\text{event})$ and worked out using the formulas below.

$$P(\text{event}) = \frac{\text{Frequency of event}}{\text{Total Frequency}} = \frac{\text{Number of favourable outcomes}}{\text{Number of trials}}$$

$$\text{or } P(\text{event}) = \frac{n(\text{event})}{n(\text{sample space})}$$

Experimental probabilities are also called **relative frequencies**. Probabilities can be written as fractions, decimals or percentages.

A probability measures the expected proportion of the time an event will occur. **Expected frequency** is the product of the probability and the number of trials.

Example 2

Students took turns at throwing a matchbox into the air five times and noting whether it landed flat (F), on its end (E), or on its side (S). The following results were obtained.

FFFFF	FFFSS	FFSFF	FFSEF
FFFSS	FFFFS	FFFFF	FFFFS
FFFSF	FFFFF	FFSEF	FSFFF
SFSFF			



Side



Flat



End

- Draw up a frequency table.
- Which is more likely: flat or side?
- Find the probability that a matchbox will not land flat.
- Estimate the number of times a matchbox will land flat in 230 tosses.

CAS TI-Nspire exercise

Probability

MAT09SPTI00007

CAS ClassPad exercise

Probability

MAT09SPCP00007

Worksheet

Relative frequencies

MAT09SPWK00047

TLF Learning object

Exploring probability
(L6567)

MAT09SPIN00007

Solution

- a** There are three outcomes (scores), so set up the table in the normal way, with outcome and frequency columns. You can also use a tally column to help.

Outcome	Frequency
Flat	50
End	2
Side	13
Total (trials)	65

- b** Landing flat has the higher frequency.

Landing flat is more likely than on the side.

- c** Write the formula.

$$P(\text{not flat}) = \frac{f(\text{not flat})}{\text{number of trials}}$$

$$= \frac{15}{65}$$

Substitute values.

Enter as 15 $\div$ 65 $=$.

15 ÷ 65 0.2307692308

Write the answer.

The probability of not landing flat is about 23%.

- d** Calculate the probability of landing flat.

$$P(\text{flat}) = \frac{f(\text{flat})}{\text{number of trials}}$$

$$= \frac{50}{65} = \frac{10}{13} \approx 0.77$$

Multiply by the number of trials.

Expected frequency = $P(\text{flat}) \times \text{no. of trials}$

Substitute values.

$$= \frac{10}{13} \times 230$$

$$= 176 \frac{12}{13}$$

Enter as 10 $\frac{a}{b/c}$ 13 $\times$ 230 $=$ SHIFT $\frac{a}{b/c} \leftrightarrow \frac{d}{e}$.

Round and write the answer.

$\frac{10}{13} \times 230$ 176 $\frac{12}{13}$

The matchbox will land flat about 177 times.

In Example 2d you could calculate the probability by doing 50 $\div$ 65 on your calculator, but then you should *keep the exact value* to use in the last calculation. You can also change $176 \frac{12}{13}$ using the changing function on your calculator, which is usually shown as $F \leftrightarrow D$ or similar.

Example 3

A marble is chosen from a bag containing 3 red, 2 blue and 5 green marbles.

- List the sample space.
- Find the probability of a red marble.
- If this is repeated 27 times (replacing each time), how many reds would you expect?

Solution

- Use the first letters to list the sample space.

Sample space = {R, R, R, B, B, G, G, G, G, G}

- Write the formula.

$$P(\text{event}) = \frac{n(\text{event})}{n(\text{sample space})}$$

Substitute values.

$$= \frac{3}{10} = 0.3 = 30\%$$

Write the answer.

The probability of red is $\frac{3}{10}$.

- Calculate the expected number.

Expected frequency = $P(\text{red}) \times \text{no. of trials}$

Substitute values.

$$= 0.3 \times 27 = 8.1$$

Round and write the answer.

You would expect about 8 reds from 27 trials.

In Example 2, the events 'landing flat' and 'not landing flat' were opposite to each other. Events like this divide the outcomes or sample space into two separate parts.

Important!

Complementary events

For any event E, the **complement** of the event consists of all the outcomes *not* in E and is shown as E' or $\bar{E}$.

The sum of the probability of an event and its complement is 1.

$$P(E) + P(E') = 1, \text{ so } P(E') = 1 - P(E) \text{ and } P(E) = 1 - P(E')$$

You can use complementary events to find some probabilities.

Example 4

A T-shirt designer has a website for online orders. 97.7% of those who visit the site do not place an order. What is the probability that the next hit on the site results in a sale?

Solution

Write the probability that you know.

$$P(\text{no sale}) = 0.977$$

Write the relationship.

'No sale' and 'Sale' are complementary events.

Calculate the desired probability.

$$P(\text{Sale}) = 1 - P(\text{No sale})$$

$$= 1 - 0.977 = 0.023$$

Write the answer.

The probability of a sale is 0.023, or 2.3%.

Investigate: Comparing probabilities

You will need a soft drink bottle, and beads (or marbles) of several different colours.

Work in groups of two or three.

- 1 Place coloured beads in the bottle, and make a note of how many of each colour are in the bottle. Make sure that there is a different number of each colour.
- 2 Draw up a table like the one below.

Colour	Tally	Number shaken out of bottle

- 3 Shake one bead from the bottle and note its colour.
- 4 Put a tally mark in the row of the table for that colour.
- 5 Replace the bead in the bottle and repeat the experiment. Do this about 50 or 100 times.
- 6 Now work out the number of times that beads of each colour were shaken out.
- 7 Calculate the relative frequency of getting each colour.
- 8 Compare the relative frequencies with the theoretical probabilities.
- 9 Combine the class's results and recalculate the relative frequencies. What do you find?



Exercise 7.1 Calculating probability

- 1 Students in a Year 9 class measured the lengths, in millimetres, of their index-finger nails. The results were:

9 14 16 13 8 10 9 7 10 9 11 12 8 11 11
16 15 8 14 8 11 10 13 13 8 16 12 13 9

- a Draw up a frequency table.
- b Which is more likely: 10 mm or 11 mm?
- c What is the most likely length?
- d Is 13 mm or 14 mm more likely?

Work out:

- e the relative frequency of each length
- f the number from 500 Year 9 students you would expect to have an index-finger nail of length 11 mm
- g the number from 500 you would expect to have a length less than 10 mm.



Fluency

Extra questions

Exercise 7.1

MAT09SPEQ00024

See Examples 1, 2

- 2 A market research company did a survey of shoppers in an arcade. The first day they interviewed 224 people; 27 of them said they were going to buy clothes and 32 said they were on their way somewhere else. The next day the interviewers started earlier and interviewed 345 people; 50 of these said they were buying food and 27 said they were going to buy CDs or DVDs.

- a If another shopper is asked, are they more likely to be buying clothes or on their way somewhere else?
b Are they more likely to be buying food or CDs or DVDs?

Work out:

- c the relative frequency for each type of shopper
d from 700 shoppers, the expected number of each type of shopper
e the most popular type of shopping
f the least popular type of shopping.
- 3 A chocolate factory was testing its production line. In a run of 540, it found that 23 chocolates were too misshapen to sell, and used these for chocolate coatings.
- a What was the relative frequency of misshapen chocolates?
b How many misshapen chocolates would you expect in a run of 12 000?
- 4 A biscuit factory checked the quality of its jam-coated biscuits. There were 13 broken biscuits from 400 that were checked, and 15 biscuits out of 420 had no jam coating.
- a What was the relative frequency of broken biscuits?
b What was the relative frequency of biscuits with no jam on them?

If 5000 biscuits were produced:

- c how many broken ones would you expect?
d how many with no jam would you expect?

See Example 3

- 5 A bag containing coloured playing discs has 4 red discs, 5 yellow, 2 blue, 3 black and 7 white. One disc is taken out.
- a Is it more likely to be red or yellow?
b Which is more likely: a primary colour (red, yellow or blue) or a neutral colour (black or white)?
c Calculate the probability of each colour.
d The experiment is repeated 200 times. How many times do you expect a yellow disc?
- 6 A bag contains 3 green, 5 red and 4 yellow marbles. One marble is taken out at random.
- a Work out the probability of getting:
- | | | |
|------------------|-----------------|---------------------|
| i a green marble | ii a red marble | iii a yellow marble |
|------------------|-----------------|---------------------|
- b The procedure is repeated 80 times. Work out the expected number of:
- | | | |
|----------|---------|-------------|
| i greens | ii reds | iii yellows |
|----------|---------|-------------|
- 7 A bag contains 3 opals, 5 pearls and 4 topazes. One jewel is taken out at random.
- a Work out the probability of getting:
- | | | |
|-----------|------------|-------------|
| i an opal | ii a pearl | iii a topaz |
|-----------|------------|-------------|
- b The procedure is repeated 50 times. Work out the expected number of:
- | | | |
|---------|-----------|-------------|
| i opals | ii pearls | iii topazes |
|---------|-----------|-------------|

- 8 A Year 9 student has an 85% chance of getting a C or above in Maths. What is the probability of getting a D or E?
- 9 The probability of getting a drawn match in Aussie Rules is about 2%. What is the complement of the event 'a drawn match' and what is its probability?

See Example 4

- 10 The watches of a class were checked against the real time. The following data shows how many minutes fast (+) or slow (–) they were:

+2	+1	–1	0	0	0	–3	–2	–1	+5	–2	0	0	–2
–2	+1	–3	+2	+1	0	0	–1	–1	–3	+4	+2	0	0

- a Draw up a frequency table and calculate the relative frequency of each time.
- b If the watches of the whole school (970 pupils have watches) were checked, how many would you expect to show the correct time?
- c How many in the school would you expect to be fast?
- d How many would you expect to be slow?
- e How many would you expect to be less than 3 minutes out?
- 11 There are, on average, 7 wet days in June in Mackay.
- a Which is more likely in June in Mackay: a fine day or a wet day?
- b What is the probability of a wet day in Mackay in June?

- 12 Ursula has the Australian coins shown on the right. Work out the probability that a coin chosen at random is:

- a 1-dollar
- b 2-dollar
- c 5-cent
- d 10-cent
- e 20-cent
- f 50-cent
- g an Australian coin
- h more than 10 cents
- i less than 10 cents
- j more than \$1



- 13 A litter of puppies has 2 brown, 1 black and 3 black-and-brown. If the colours are equally popular, work out the probability that the first puppy sold will be:

- a brown b black c black-and-brown



Problem solving

Worked solutions

Exercise 7.1

MAT09SPWS00024

Worked solutions

Exercise 7.1

MAT09SPWS00024

Reasoning

Worked solutions

Exercise 7.1

MAT09SPWS00024

- 14 Use the information from question 1 to find how many Year 9 students you would expect to check before obtaining 40 with an index-finger nail length longer than 10 mm.
- 15 For a survey, a marketing company wanted to interview 100 people who were shopping for clothes. Explain your steps in finding the number of shoppers in the arcade in question 2 that you would need to interview to obtain this number of clothes shoppers.
- 16 After a survey of Year 9 students' viewing habits, a teacher concluded that the relative frequency of students regularly watching Channel 2 was 0.15. Find how many students should be interviewed to find 30 who regularly watch Channel 2 and explain your logic.

7.2 Combined events

You have probably seen statistical probabilities such as 'Obese people have a 50% higher probability of heart disease than the general population'. This is an example of a result worked out from collecting statistics from the population. The probability of the combined event 'Obesity and Heart disease' is compared to the probability of 'Heart disease'. This is often done with a **two-way table**. Remember that a two-way table has the values of one variable across the top row and the values of the other variable down the side, with frequencies in the cells.

Example 5

The information in the two-way table below shows the results of a survey of high school students (Years 7–12) about the language they spoke at home and the time they said they spent doing homework.

	Hours spent on homework in a week				
Language	0–4	5–9	10–14	15–19	≥ 20
English	150	68	22	11	12
Other	12	6	4	3	1

Compare the probabilities of students spending less than 5 hours a week on homework for students who speak English at home to those who speak other languages at home.

Solution

Find the total number of English-speaking students.

$$\text{Total English} = 263$$

Calculate the probability.

$$P(\text{English and } < 5) = \frac{150}{263}$$

Enter as 150 ÷ 263 = .

$$150 \div 263 = 0.570342053$$

$$\approx 0.57$$

Find the total number of Other languages.

$$\text{Total Other} = 26$$

Calculate the probability.

$$P(\text{Other and } < 5) = \frac{12}{26}$$

Enter as 12 $\div$ 26 $=$.

12 $\div$ 26

0.4615384615

≈ 0.462

Write the answer.

Students who speak English at home have an 11% greater probability of spending less than 5 hours a week on homework than other students.

The reliability of the results in the survey reported in Example 5 could be questionable. Why? Would you know if only the 'answer' was reported in the media? It is important that you understand the meanings of 'and' and 'or' in mathematics.

Important!

The meanings of 'and', 'or'

In mathematics, ' $\dots$ **and** $\dots$ ' means *both*, but ' $\dots$ **or** $\dots$ ' means *at least one*.

Peter has blue eyes and fair hair means that he has *both* blue eyes and fair hair.

Daria has brown eyes or fair hair means she could have brown eyes and fair hair, brown eyes and dark hair, blue eyes and dark hair, green eyes and dark hair, or other combinations as long as she has *at least one* of brown eyes and dark hair.

Investigate: Vitamins and health

You probably have learnt that Captain Cook's sailors suffered less from scurvy than other sailors of the time because he insisted that they ate as much fresh fruit and vegetables as possible. Every time they were in a port he brought supplies on board. This is attributed to their intake of Vitamin C, as we now know that lack of Vitamin C causes scurvy. Dietary deficiencies are known to cause other diseases as well, such as Beriberi and Ricketts. Nowadays, many people take vitamin tablets because they think that extra vitamins will help them stay healthy. Is this true?

In this investigation, you will collect information to find out.

- Ask students at your school the following questions.
'Do you take vitamin tablets or capsules regularly?'
'If so, how many did you take last week?'
'How many days did you have off school last term because you were sick?'
- Make a two-way table for your information.
- Calculate probabilities to make a decision about vitamins and health (You will have to decide what number of 'sick days' you will classify as healthy, based on the information you collect).
- Discuss your results.

The investigation above is an example of an epidemiological study. These are widely used in medical research, but results have to be interpreted cautiously because they do not ‘prove’ results. For example, it is well-known that people who eat carrots eventually die. This does not prove that eating carrots kills you, because you will die eventually whether or not you eat carrots. A two-way table can also be useful when you are doing theoretical probabilities.



Example 6

Animated example

Combined events

MAT09SPAE00007

Worksheet

Two-way probability tables

MAT09SPWS00048

TLF Learning object

Performance of diagnostic tests (R12187)

MAT09SPIN00007

- Two normal dice are rolled together.
- a Construct a two-way table for the totals shown on the dice.
 - b Find the probability that one of the dice shows a number less than 3 and the other shows a number greater than 4.
 - c Find the probability of getting a double or a total bigger than 8.
 - d Find the probability of getting a 4 on one of the dice when the other is not bigger than 4.



Solution

- a Make the two-way table, and work out the totals.
- b Find the part of the table for ‘one of the dice is less than 3’.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Find the part of the table for
'a number greater than 4'.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

'and' means both must be true.
Find the part(s) for which this is
the case.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Count the number and work out
the probability.

$$P(< 3 \text{ and } > 4) = \frac{8}{36} = \frac{2}{9} \approx 0.22$$

- c** 'or' means that at least one must be true. Find the part of the table for doubles and combine with the part for totals bigger than 8.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Count the number and work out
the probability.

$$P(\text{double or total} > 8) = \frac{14}{36} = \frac{7}{18} \approx 0.39$$

- d** This means getting a 4 AND the other is less than 5. Find the part of the table for which they are both true.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Count the number and work out
the probability.

$$P(4 \text{ and } < 5) = \frac{7}{36} \approx 0.19$$

You can also represent combined events using a **Venn diagram**.

Important!

Venn diagrams

A Venn diagram shows two or more sets as circles, with the elements of each set inside its circle. The intersections show the elements that are common to the overlapping sets. If there is no intersection, then the sets are **mutually exclusive**, which means that they have no common elements. The circles are usually enclosed in a rectangle that represents all the elements being considered and is called the **universal set**.

Example 7

Puzzle sheet

Venn diagrams

MAT09SPPS00028

Worksheet

Three set Venn diagrams

MAT09SPWS00049

From 24 students in a class, 6 have fair hair and freckles. There are 10 people altogether with fair hair (red hair counts as fair) and 14 people with freckles.

a Draw a Venn diagram for this situation.

Find the probability that a student chosen at random has:

- b** fair hair or freckles
- c** fair hair or freckles but not both
- d** neither fair hair nor freckles
- e** no freckles but fair hair.

Solution

a Draw the Venn diagram. The rectangle represents all the people in the class, and the circles show the people with fair hair and the people with freckles. Label the circles and the rectangle.

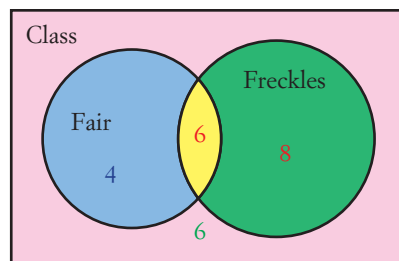
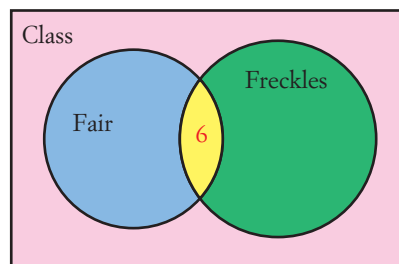
There are 6 people with both fair hair and freckles, so the intersection has 6 students.

There are 10 with fair hair, so there must be $10 - 6 = 4$ in the rest of the fair hair circle.

There are $14 - 6 = 8$ in the rest of the freckles circle.

Since there are 18 people altogether in the circles, there must be $24 - 18 = 6$ outside the circles.

b Fair hair or freckles means at least one of these, so they are the 18 people inside the circles. Calculate the probability.



$$P(\text{Fair or freckles}) = \frac{18}{24} = \frac{3}{4} = 0.75$$

- c** Fair hair or freckles but not both excludes the 6 people in the intersection.

$P(\text{Fair or freckles but not both})$

$$= \frac{12}{24} = \frac{1}{2} = 0.5$$

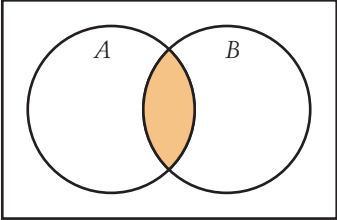
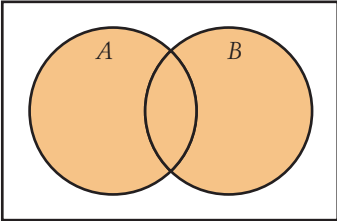
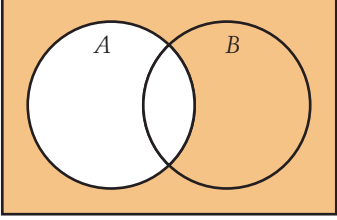
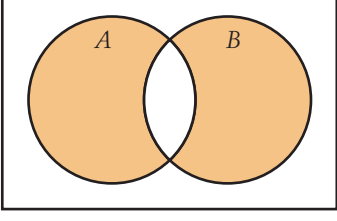
- d** Neither fair hair nor freckles means the six people outside the circles.

$$P(\text{Not fair and not freckles}) = \frac{6}{24} = \frac{1}{4} = 0.25$$

- e** No freckles but fair hair means the 4 people in the fair circle outside the intersection.

$$P(\text{Fair and not freckles}) = \frac{4}{24} = \frac{1}{6} \approx 0.167$$

Example 7, part **c** is an example of ‘**exclusive or**’, which means one or the other, but not both. This is sometimes written as **XOR**. There are special words for other logical combinations as well. The Venn diagrams below show the appearance of various combinations of A and B . They can be regarded as logical combinations or combinations of unions and intersections of sets.

Logical combination	Set combination	Venn diagram
A and B	$A \cap B$ (Intersection)	
A or B	$A \cup B$ (Union)	
Not A	$\bar{A}$ (not A)	
A or B but not A and B	$(A \cup B) \cap \overline{(A \cap B)}$	

You can break some combined events into a couple of stages. In this case, you can work out the probability of the combined event with a **tree diagram**.

Important!

Tree diagrams

A tree diagram begins from a single point. It has lines branching to the separate outcomes that can occur. At the end of each line, there may be another stage with further outcomes indicated by more branching.



When you use tree diagrams, there are two possible scenarios. The situation could be the same for the second stage as for the first stage. When a second card is taken from a pack after the first card is replaced, the second stage is the same as the first, so the outcomes are the same. The second stage could be different to the first stage, as is the case if a second card is taken from a pack without the first being replaced.

Example 8

Two marbles are drawn in succession (without replacement) from a bag containing 3 red and 2 green marbles.

- Draw a tree diagram for this situation
- What is the probability of getting 2 red marbles?
- What is the probability of getting 2 green marbles?
- What is the probability that the marbles are different?

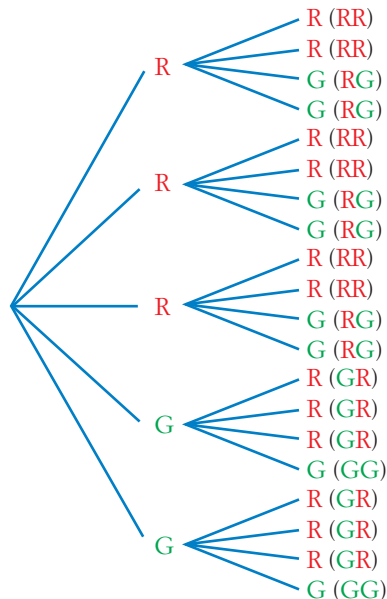
Solution

- There are 5 possibilities for the first marble.



Whatever the first marble is, there are 4 possibilities for the second marble.

There are a total of 20 combined possibilities.



- b** Write the number of RR outcomes. $n(\text{RR}) = 6$
Calculate the probability. $P(\text{RR}) = \frac{6}{20} = 0.3$
- c** Write the number of GG outcomes. $n(\text{GG}) = 2$
Calculate the probability. $P(\text{GG}) = \frac{2}{20} = 0.1$
- d** Use complements. $P(\text{different colours}) = 1 - P(\text{same colour})$
Same colour means RR or GG. $= 1 - (P(\text{RR}) + P(\text{GG}))$
RR and GG are mutually exclusive. $= 1 - 0.4$
 $= 0.6$
- Write the answer. **The probability of different colours is 60%.**

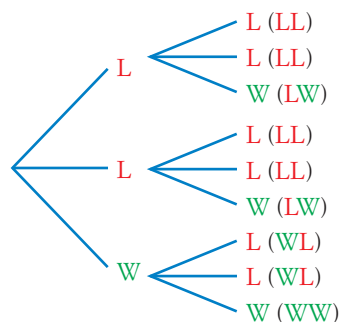
Example 9

The 'Thimble game' is played as follows. The operator places a pea under one of three thimbles on a table and then quickly shuffles the thimbles around. The player watches and pays to bet on which thimble the pea is under. If the player is correct, they double their money, otherwise they lose. Assuming that the game is played fairly,

- a** draw a tree diagram to show the possible outcomes of two plays
b find the probability of a player winning both times
c find the probability of the player losing both times.

Solution

- a** There are 3 branches, one wins and the other two lose. The second stage is the same.



- b** There is one outcome with 2 wins. $P(\text{WW}) = \frac{1}{9} \approx 0.111$
- c** There are 4 outcomes with 2 losses. $P(\text{LL}) = \frac{4}{9} \approx 0.444$

The game in Example 9 is almost never played fairly. The operator is often skilled at sleight-of-hand and places the pea under one of the thimbles the player does not pick as the operator picks them up.

In many cases, you are interested in finding the probability of an event if another event has already occurred.

Important!

Given

In probability, the word '**given**' means that you assume that the event specified has already occurred. The sample space considered is smaller than the general case.

Example 10

Two normal dice are rolled together. Given that the total is more than 7, what is the probability that one of the dice is less than 4?

Solution

Shade the restricted sample space as part of a two-way table.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Write the formula for the probability.

From the 15 outcomes with totals more than 7, 6 have one die less than 4.

Write the answer.

$P(\text{one less than 4 given total} > 7)$

$$= \frac{n(\text{one less than 4})}{n(\text{total} > 7)}$$

$$= \frac{6}{15} = \frac{2}{5} = 0.4$$

Given that the total of two dice is more than 7, the probability that one of the dice is less than 4 is 0.4 or 40%.

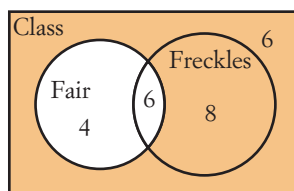
Example 11

From 24 students in a class, 6 have fair hair and freckles. There are 10 people altogether with fair hair (red hair counts as fair) and 14 people with freckles. Given that a person does not have fair hair, what is the probability that they have freckles?

Solution

Show the Venn diagram.

Shade the restricted sample space of people who are not fair-haired.



Puzzle sheet

Conditional probabilities

MAT09SPPS00029

Worksheet

Probability review

MAT09SPWK00050

Write the formula for the probability.

From 14 people in the restricted sample space, there are 8 with freckles.

Write the answer.

$$P(\text{freckles given not fair hair}) = \frac{n(\text{freckles})}{n(\text{not fair hair})}$$

$$= \frac{8}{14} = \frac{4}{7}$$

$$\approx 0.572$$

The probability of a student having freckles, given that they don't have fair hair is $\frac{4}{7}$ or about 57.2%.

Exercise 7.2 Combined events

- Normal red and green dice are labelled 1 to 6. Use a two-way table to show the sum of the dice when they are thrown. Find the probability that:
 - you get a total less than 7
 - one of the dice gives a number less than 3
 - you get a total less than 7 and one of the dice has a number less than 3
 - you get a total less than 7 or one of the dice has a number less than 3
 - exactly one of the dice has a number less than 3.
- Draw a Venn diagram to show the numbers from 1 to 15 inclusive. Show set $A = \{\text{odd numbers}\}$ and $B = \{\text{prime numbers}\}$.
- The weather may be hot, warm, cool or cold in combination with fine, cloudy or raining.
 - Show the sample space for weather conditions with a two-way table.
 - Show the sample space for weather conditions with a tree diagram.
- Two multiple choice questions have answers A, B, C and D.
 - Use a two-way table to show the sample space.
 - Use a tree diagram to show the sample space.
 - How many elements are in the sample space?
- A box contains 4 light bulbs, of which one is faulty. Draw a tree diagram to show the sample space for taking out two light bulbs from the box.
- Use a tree diagram to show the sample space for tossing:
 - two normal coins
 - three normal coins
- Use a two-way table for the totals from throwing two normal dice to find each of the following probabilities.
 - At least one of the dice is a 1 or the total is less than 4.
 - One of the dice is a five and the other is less than 3.
 - The total is more than 9 or one of the dice is a six.
 - Both dice are 5 or less and the total is not 5.
 - The total is 4 or at least one of the dice is a 6.

Understanding

Extra questions

Exercise 7.2

MAT09SPEQ00025

See Example 5

See Example 7

See Example 8

See Example 9

Fluency

See Example 6

- 8 A coin is tossed and a normal die is rolled at the same time.
- Show the sample space as a two-way table.
 - What is the probability of getting a head and a four?
 - What is the probability of getting a head or a four?
 - What is the probability of getting a head or a four, but not a head and a four?
 - What is the probability of getting a head, but not a four?
- 9 For the two multiple choice questions from question 4, find the probability that if they are answered at random:
- both answers are correct
 - both answers are wrong
- 10 Repeat question 9 for the case where the two questions have 3 options: A, B and C.
- 11 Repeat question 9 for the case where the two questions have 5 options: A, B, C, D and E.
- 12 A box contains 2 yellow, 3 green and 2 red beads. Two beads are shaken out. What is the probability of getting:
- 2 yellows?
 - 2 greens?
 - a green and a yellow?
 - a red and a yellow?
- 13 A bag contains 2 red and 2 green marbles. Two marbles are taken out, one after the other, with the first being replaced before the second is taken out.
- What is the probability that they are both red?
 - What is the probability that they are different colours?
 - What is the probability that they are both green?
 - What happens to the probabilities if the first is not replaced before the second is drawn?
- 14 Two four-sided dice numbered 1–4 are thrown and the numbers they land on are noted. Find the probability that:
- one lands on 3 and the other on 4
 - one lands on 3 or the other on 4
 - the total is 6
 - one lands on a number less than 3 and the other lands on 3 or more
 - one lands on a number less than 3 or the other lands on 3 or more
 - one lands on a number less than 3 or the other lands on 3 or more, but not both
 - the total is 6 and one of the numbers is not 4
 - the total is 6 or one of the numbers is not 4.
- 15 Use the Venn diagram from question 2 to find the following probabilities for a number chosen at random from 1 to 15.
- It is odd and prime.
 - It is even and prime.
 - It is even or prime.
- 16 a Draw a Venn diagram to show the names of the months of the year classified into the following sets.
- $A = \{\text{Months containing the letter 'a' in the name}\}$
- $Y = \{\text{Months ending in the letter 'y'}\}$

See Example 11

Now use your diagram to find the probability that a person chosen at random has their birthday in a month:

- in sets A and Y
- in set A or set Y
- in set A or Y but not both
- in neither sets A nor Y
- in set A but not Y
- in set Y but not in A

- 17 a** Draw a Venn diagram to show the numbers from 1 to 60 inclusive in each of the sets $A = \{\text{numbers divisible by 2}\}$, $B = \{\text{numbers divisible by 3}\}$ and $C = \{\text{numbers divisible by 5}\}$.

A number is chosen at random from 1 to 60. Use your Venn diagram to find each of the following probabilities.

- b** The number is divisible by 2 or 3.
- c** It is divisible by 2 and 3.
- d** It is divisible by 2 or 3, but not both.
- e** The number is divisible by 2 or 5.
- f** It is divisible by 2 and 5.
- g** It is divisible by 2 or 5, but not both.
- h** It is divisible by 3 or 5.
- i** It is divisible by 3 and 5.
- j** It is divisible by 3 or 5, but not both.
- k** It is divisible by 2 or 3 or 5.
- l** It is divisible by 2 and 3 and 5.

- 18 a** Draw a two-way table to show the total obtained from rolling two dice, one a normal die and the other an octahedral with eight sides numbered from 1 to 8.

- b** Given that one die shows a 3, what is the probability that the total is 8?
- c** Given that the total is 8, what is the probability that one die shows a 3?
- d** Given that one die shows a 2, what is the probability that the total is 6?
- e** Given that the total is 6, what is the probability that one die shows a 2?
- f** Given that one die shows a 2, what is the probability that the total is 10?
- g** Given that the total is 10, what is the probability that one die shows a 2?
- h** Given that the total is 10, what is the probability that one die shows a 1?
- i** Given that the total is 14, what is the probability that one die shows a 6?

- 19 a** Draw a Venn diagram to show positive integers less than 31 such that: $A = \{\text{even numbers}\}$, $B = \{\text{numbers divisible by 3}\}$, $C = \{\text{numbers divisible by 7}\}$. For a number chosen at random, find each of the following probabilities.

- b** Given it is even, it is divisible by 3.
- c** Given it is divisible by 3, it is even.
- d** Given it is even, it is divisible by 7.
- e** Given it is divisible by 7, it is even.
- f** Given it is divisible by 3, it is divisible by 7.
- g** Given it is divisible by 7, it is divisible by 3.

- 20** A table has jumpers 'reduced to clear', but there are limited styles and colours. From 40 jumpers, 10 are V-necks and of these 8 are red. There are 24 red jumpers altogether. If a jumper is chosen at random, find the following probabilities.

- a** It is a red V-neck.
- b** Given that it is red, that it is a V-neck.
- c** It is neither red nor a V-neck.
- d** Given that it is not red, that it is not a V-neck.
- e** It is a V-neck or red.



Worked solutions

Exercise 7.2

MAT09SPWS00025

See Example 10

See Example 11

Problem solving

- 21 The information in the table below shows the reaction time for the dominant hands and the stated hours spent watching TV on school nights of a large group of students.

		Reaction time (seconds)			
		0.2–0.29	0.3–0.39	0.4–0.49	0.5+
TV hours	0–2	20	44	21	16
	3–5	17	58	19	9
	6–10	15	55	25	14
	11–15	14	25	12	6
	16–20	5	13	8	3
	> 20	6	16	8	5

- What is the probability that a student has a reaction time under 0.3 seconds?
 - What is the probability that a student watches more than 15 hours of TV on school nights and has a reaction time under 0.3 seconds?
 - Given that a student watches more than 15 hours of TV on school nights, what is the probability of a reaction time under 0.3 seconds?
 - Given that a student watches less than 16 hours of TV on school nights, what is the probability of a reaction time under 0.3 seconds?
 - Given that a student watches more than 15 hours of TV on school nights, what is the probability of a reaction time of at least 0.4 seconds?
 - Given that a student watches less than 16 hours of TV on school nights, what is the probability of a reaction time of at least 0.4 seconds?
 - Compare the probabilities of reaction times under 0.3 seconds of students who watch less than 6 hours of TV on school nights with those who watch more than 10 hours.
 - Can you make any conclusions about excessive TV-watching and reaction times?
- 22 Year 9 students at a particular school have a choice of some subjects. From a Maths class of 24 students, 12 do Woodwork and 12 take Art. Of those who take Art, 3 do Woodwork and one of these also studies Music. There are 8 students who study Music and 2 of these do neither Art nor Woodwork. 6 of the Woodwork students take neither Art nor Music. Find each of the following probabilities.
- A student takes Art and Woodwork.
 - A student takes Art or Woodwork.
 - A student does all three subjects Art, Woodwork and Music.
 - A student does none of Art, Music or Woodwork.
 - A student taking Art does not do Woodwork.
 - A student doing Woodwork does not take Art.
 - Given that a student does Music, the student takes Art.
 - Given that a student takes Art, the student studies Music.
 - Given that a student does Woodwork, that the student doesn't do Art or Music.
 - That a student not taking Art does Woodwork.

Reasoning

- 23 From a group of 40 students, 16 students had fair hair and 10 had fair skin. 7 had both fair hair and fair skin. One of the students incorrectly worked out that the probability of having fair hair or fair skin was 0.65. What did the student do incorrectly?

Worked solutions

Exercise 7.2

MAT09SPWS00025

- 24 a Considering the students in question 23, calculate the probabilities of having fair hair, fair skin, both fair hair and fair skin and either fair hair or fair skin.
- b What is the relationship between the probabilities in a?
- 25 a Two sets A and B have n and m elements respectively and there are p elements in $A \cap B$. The universal set has u elements. Find the probabilities $P(A)$, $P(B)$, $P(A \text{ and } B)$ and $P(A \text{ or } B)$ for an element chosen at random from the universal set.
- b What is the relationship between $P(A)$, $P(B)$, $P(A \text{ and } B)$ and $P(A \text{ or } B)$?
- c Can you make a generalisation?

Worked solutions

Exercise 7.2

MAT09SPWS00025

7.3 Multi-step probabilities

Venn diagrams and two-way tables are not very useful for events that have more than two stages involved. In these cases, we need to use tree diagrams to find probabilities of particular outcomes.

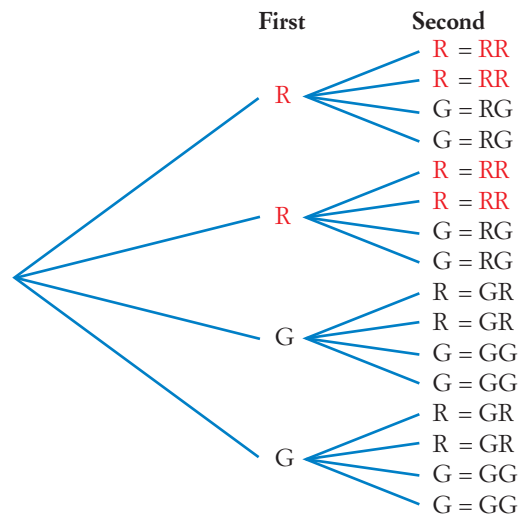
Example 12

A bag contains 2 red and 2 green marbles. Two marbles are taken out, one after the other, with the first being replaced before the second is taken out.

- a Draw a tree diagram to show the sample space.
- b Find the probability that both marbles taken are red.

Solution

- a Draw the tree diagram.



- b Count the sample space.

Write the formula.

Substitute values.

Write the answer.

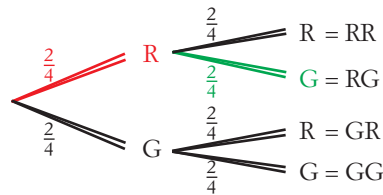
There are 16 outcomes altogether.

$$P(2R) = \frac{n(2R)}{n(\text{sample space})}$$

$$= \frac{4}{16} = \frac{1}{4} = 0.25$$

The probability that both marbles are red is 0.25.

The tree shown in Example 12 has 16 branches, but many of the branches are duplicated. We can simplify the tree by putting some of these together, with a clear indication of how many have been combined, and how many branches there are at each join. It is usual to show this in fraction form, as shown below. *The order is very important.*



What is the probability of red then green?

The red branch has 2 out of 4 branches, and the green branch also has 2 out of 4 branches, so there are 2 lots of 2 branches that give the result RG.

This gives $n(RG) = 2 \times 2 = 4$.

Similarly, $n(\text{sample space}) = 4 \times 4 = 16$

Thus $P(RG) = \frac{4}{16} = \frac{1}{4} = 0.25$, so the probability of getting a red and then a green is 0.25.

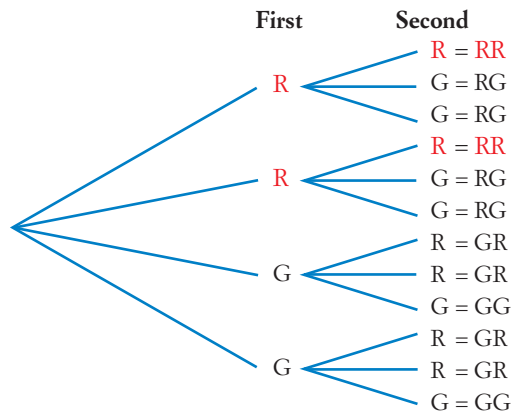
Example 13

A bag contains 2 red and 2 green marbles. Two marbles are taken out, one after the other, with the first *not* replaced before the second is taken out.

- Draw a tree diagram to show the sample space.
- Find the probability that both marbles taken are red.

Solution

- Draw the tree diagram.



- Count the sample space.

Write the formula.

Substitute values.

Write the answer.

There are 12 outcomes altogether.

$$P(2R) = \frac{n(2R)}{n(\text{sample space})} = \frac{2}{12} = \frac{1}{6} \approx 0.167$$

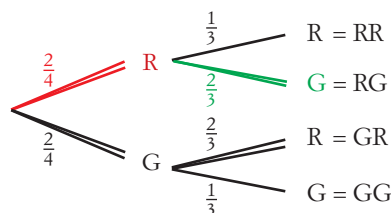
The probability that both marbles are red is about 0.167.

We can simplify the tree in Example 13 in the same way as for Example 12 to obtain the tree on the right. What is the probability in this case of getting red then green?

In this case, instead of 2 out of 4 branches going to 2 out of 4, there are 2 out of 4 red branches going to 2 out of 3 (green) branches.

Thus we have that $n(RG) = 4$, but $n(\text{sample space}) = 4 \times 3 = 12$.

Thus $P(RG) = \frac{4}{12} = \frac{1}{3} \approx 0.333$ and the probability of red then green is about 0.333.



Important!

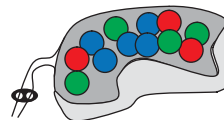
Simplified tree diagrams

A **simplified tree diagram** shows repeated branches on a normal tree diagram as single branches, with a fraction showing the number of branches that have been combined, out of the total number at that join.

Example 14

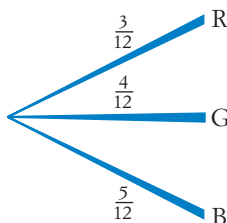
A bag contains 3 red, 4 green and 5 blue marbles. Two marbles are taken out of the bag, with replacement.

- Draw a simplified tree diagram to show the sample space.
- Find the probability that the marbles taken are green, then blue.
- Find the probability that the marbles taken are green and blue.



Solution

- Draw the first part of the tree diagram. It has 12 branches, with 3 red, 4 green and 5 blue. Show the fractions on each branch.



Puzzle sheet

Final 6 investigation

MAT09SPPS00026

Puzzle sheet

A different final 6 investigation

MAT09SPPS00027

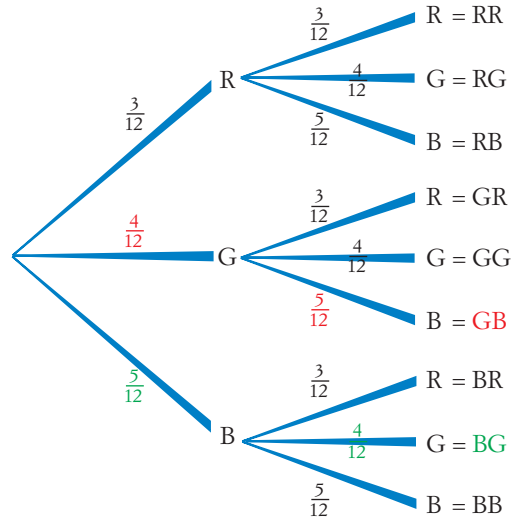
Puzzle sheet

Tree diagrams

MAT09SPPS00030

With replacement, the branches at the second join are the same as the branches at the first join, and the fractions are the same.

Draw the rest of the simplified tree diagram, with three branches at each join. Show the fractions on each branch.



- b** Find the number of GB outcomes.

Find the total number.

Write the formula.

Write the answer.

- c** Green and Blue means G then B or B then G.

Write the formula.

Substitute values and perform the calculation.

Write the answer.

$$n(\text{GB}) = 4 \times 5 = 20$$

$$n(\text{sample space}) = 12 \times 12 = 144$$

$$P(\text{GB}) = \frac{n(\text{GB})}{n(\text{sample space})} \\ = \frac{20}{144} = \frac{5}{36} \approx 0.139$$

The probability of green then blue is about 0.139.

$$n(\text{G and B}) = 4 \times 5 + 5 \times 4 = 40$$

$$P(\text{G and B}) = \frac{n(\text{G and B})}{n(\text{sample space})} \\ = \frac{40}{144} = \frac{5}{18} \approx 0.278$$

The probability of green and blue is about 0.278.

Example 15

A box of sweet biscuits has 6 orange creams and 4 ginger snaps left. Three biscuits are taken out at random, without replacement.

- a** Draw a simplified tree diagram to show the sample space.
b Find the probability that the biscuits taken out are all the same.

Weblink

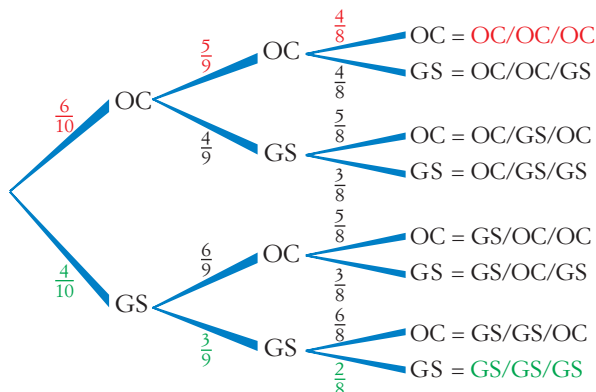
Birth month paradox

MAT09SPWB00007

Solution

- a** There are 10 possibilities for the first biscuit: 6 orange creams and 4 ginger snaps.

Without replacement, the branches at the second join have only 9 possibilities, and those at the third join have 8 possibilities. Draw the simplified tree diagram and put the fractions on the branches.



- b** There are two groups of outcomes that have all the biscuits the same: OC/OC/OC and GS/GS/GS. Work out the numbers.

Work out the total number.

Write the formula.

Substitute values and do the calculation.

Write the answer.

$$n(\text{OC/OC/OC}) = 6 \times 5 \times 4 = 120$$

$$n(\text{GS/GS/GS}) = 4 \times 3 \times 2 = 24$$

$$n(\text{sample space}) = 10 \times 9 \times 8 = 720$$

$$P(3 \text{ the same}) = \frac{n(3 \text{ the same})}{n(\text{sample space})}$$

$$= \frac{120 + 24}{720} = \frac{144}{720} = \frac{1}{5} = 0.2$$

The probability that the biscuits are all the same is 20%.

Example 16

Two cards are taken from a normal pack at the same time. Find the probability that they are both aces.

Solution

You are interested in aces, so draw a simplified tree diagram with branches showing 'ace' and 'not ace'. For the first card, there are 4 aces out of 52 cards.

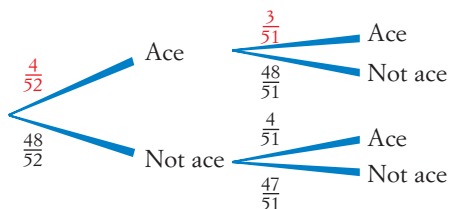
Calculate the number of 'both aces'.

Calculate the total number.

Write the formula.

Substitute values and perform the calculation.

Write the answer.



$$n(\text{both aces}) = 4 \times 3 = 12$$

$$n(\text{sample spaces}) = 52 \times 51 = 2652$$

$$P(2 \text{ aces}) = \frac{n(2 \text{ aces})}{n(\text{sample space})}$$

$$= \frac{12}{2652} = \frac{1}{221} = 0.004524 \dots$$

The probability of 2 aces is about 0.45%.

Puzzle sheet

Tree diagrams (without replacement)

MAT09SPPS00031

Exercise 7.3 Multi-step probabilities

Understanding

Extra questions

Exercise 7.3

MAT09SPEQ00026

See Examples 12, 13

See Examples 14, 15

- 1 Use a tree diagram to show the sample space for:
 - a tossing 2 coins
 - b tossing 3 coins
 - c tossing 4 coins
 - d tossing 5 coins
- 2 The numbers 1 to 5 are written on cards. A student takes two cards in succession, replacing the first one before taking the second. Use a tree diagram to show the sample space.
- 3 Repeat question 2 for the situation where the first card is *not* replaced.
- 4 A normal pack of cards contains 13 of each of the suits spades ♠, hearts ♥, diamonds ♦ and clubs ♣.
Draw simplified tree diagrams to show the suits of cards taken from a shuffled pack under the following conditions.
 - a 2 cards are taken in succession with replacement.
 - b 2 cards are taken in succession without replacement.
- 5 Draw simplified tree diagrams to show the results when marbles are taken from a bag containing 4 green, 2 black, and 5 yellow marbles under the following conditions.
 - a 2 marbles are taken in succession with replacement.
 - b 2 marbles are taken in succession without replacement.
 - c 3 marbles are taken in succession with replacement.
 - d 3 marbles are taken in succession without replacement.
- 6 A bag contains 1 red, 2 yellow and 3 green buttons. Use a simplified tree diagram to show the sample space for choosing two buttons if each is replaced before the next is taken.
- 7 Repeat question 6 for the situation where the first button is *not* replaced.

Fluency

- 8 Two normal coins are tossed. Work out the probability of getting:
 - a 2 heads
 - b 2 tails
 - c a head and then a tail
 - d a head and tail.
 - e The coins are tossed 50 times. How many times would you expect 2 heads?
- 9 A coin is tossed three times and the results are noted.
 - a What is the probability of a head, then 2 tails?
 - b What is the probability of exactly 2 heads in a row?
 - c What is the probability of 3 heads in a row?
- 10 Three coins are tossed.
 - a Is a result of 3 heads, or 2 heads and a tail, more likely?
Work out the probability of getting:
 - b 3 heads
 - c 2 heads and a tail
 - d at least 2 heads
 - e at least 1 head
- 11 Four coins are tossed. Work out the probability of getting:
 - a 4 heads
 - b 2 heads and 2 tails
 - c 2 heads, then 2 tails
 - d exactly 1 head

- 12** A box contains 2 yellow, 3 green and 2 red beads. Two beads are shaken out. What is the probability of getting:
- a** 2 yellows?
 - b** 2 greens?
 - c** a green and a yellow?
 - d** a red and a yellow?
- 13** Five building blocks are left in a toy box. Two are red and 3 are green. A child takes two of the blocks at random, with replacement.
- a** What is the probability that both blocks taken are red?
 - b** What is the probability of a red, then a green?
 - c** What is the probability of a red and a green?
 - d** What is the probability that they are the same colour?
- 14** Repeat question **13** without replacement.
- 15** A bag of marbles contains 3 red, 2 green and 5 blue marbles. Two marbles are drawn, with replacement. Find the probability of drawing:
- a** 2 green marbles
 - b** 2 red marbles
 - c** 2 blue marbles
 - d** different-coloured marbles
 - e** marbles of the same colour
- 16** Repeat question **15** without replacement.
- 17** A card is drawn at random from a normal 52-card pack and it is noted whether it is a picture card or a number card. The card is replaced and the draw is repeated. (Jacks, queens and kings are known as 'picture cards'.) Find the probability of drawing:
- a** 2 picture cards
 - b** 2 number cards
 - c** a number card, then a picture card
- 18** Repeat question **17** without replacement.
- 19** The procedure in question **17** is done for 3 cards, with replacement. Find the probability of drawing:
- a** 3 picture cards
 - b** 3 number cards
 - c** number, number, picture
 - d** exactly 2 picture cards
 - e** 2 picture cards in a row
 - f** exactly 1 picture card
- 20** Repeat question **19** without replacement.
- 21** Five questions may be answered true or false. The pass mark is set at 3 marks. 200 people just guess the answers.
- a** What is the probability of getting all questions correct?
 - b** What is the probability of passing?
 - c** How many people would you expect to pass?
- 22** To play the card game '500' with four players, the twos, threes and black fours are removed from a normal pack of cards. A joker is added. The cards are dealt so that every player has 10 cards and the kitty has 3 cards.
- a** Which is more likely in the kitty: 2 red cards and a black, or 2 black cards and a red?
 - b** Are players likely to have more red cards or more black cards?
 - c** What is the probability that the first card dealt to a player is a red four?
 - d** What is the probability that a player's first card is a king?

Worked solutions

Exercise 7.3

MAT09SPWS00026

Problem solving

- 23** The dealer changes in the game ‘pontoon’ when a player is dealt a two-card hand with ‘pontoon’—that is, a combination of an ace and a 10 or picture card.
- a** Find the probability of getting pontoon.
 - b** On average, how often would you expect pontoon to occur?
- 24** Rose is playing blindman’s buff with Donna, Julie, Peter, Tim, Alina, Thomas, Mary, Candice, Evelyn and Samuel. She has to catch someone and guess if the person is a boy or girl. They can then join back in.
- a** What is the probability that Rose will catch 2 girls in a row?
 - b** What is the probability that she will catch 2 boys in a row?
 - c** What is the probability of catching a boy, then a girl?
- 25** Repeat question **24**. But this time, once someone has been caught, they are ‘out’.
- 26** What is the probability of being dealt 4 aces in row from a normal pack of cards?

Reasoning

- 27** A coin has been tossed twice and there have been 2 heads.
- a** List the sample space for the third toss and find the probability of getting a third head.
 - b** List the sample space for tossing a coin three times in a row and find the probability of getting 3 heads.
 - c** Explain the difference between your answers in parts **a** and **b**.

Worked solutions

Exercise 7.3

MAT09SPWS00026

- 28** The scrabble letters S, P, A, C and E are placed at random on a letter holder. Explain the steps in calculating the probability that they spell the word ‘SPACE’.
- 29** In the card game poker, a straight is a run of 5 cards in succession, such as 7-8-9-10-J. The normal order is 2-3-4-5-6-7-8-9-10-J-Q-K-A. In the draw poker version, you can throw away some cards and get new ones. New players are advised not to try to draw to an ‘inside straight’, which means not trying to get a card to complete a straight when the card needed is between others. Show that you have a lower chance of completing an inside straight than a straight needing a card at either end.

Worked solutions

Exercise 7.3

MAT09SPWS00026

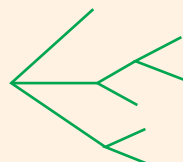
- 30** All pathology tests, such as testing someone’s blood to see if they have diabetes, can give wrong results. A ‘false positive’ means that a test has given an incorrect positive result and is generally considered to be less dangerous than a ‘false negative’. For rare diseases, false positives are common. For a particular disease, only 1 person out of every 200 tested actually has the disease, and the test itself is right 95% of the time. Work out the probabilities of a false positive and a true positive. Give reasons why you think this is a problem or not.

- **Experimental probability** uses real data to work out probability. Each item of data is called a **trial**.
- **Theoretical probability** is based on a list containing all the possible outcomes called the **sample space**. Every outcome in the sample space is listed exactly as many times as it occurs. A single outcome is called an **element** or **sample point**.
- An **event** is a collection of outcomes, whether they come from trials or a sample space. The outcomes in an event for which you are calculating the probability are called **favourable outcomes**. The number of elements in an event is written as $n(\text{event})$ and is the **frequency** of the outcome.
- The probability of an event is a number from 0 to 1, written as $P(\text{event})$ and worked out using the formulas below.

$$P(\text{event}) = \frac{\text{Frequency of event}}{\text{Total Frequency}} = \frac{\text{Number of favourable outcomes}}{\text{Number of trials}}$$

$$\text{or } P(\text{event}) = \frac{n(\text{event})}{n(\text{sample space})}$$

- Experimental probabilities are also called **relative frequencies**.
- **Expected frequency** is the product of the probability and the number of trials.
- For any event E, the **complement** of the event is shown as E' or $\bar{E}$. The sum of the probabilities of an event and its complement is 1. $P(E) + P(E') = 1$, so $P(E') = 1 - P(E)$ and $P(E) = 1 - P(E')$
- A **two-way table** has the values of one variable across the top row and the values of the other variable down the side, with frequencies in the cells.
- In mathematics, '**... and ...**' means *both*, but '**... or ...**' means *at least one*.
- A Venn diagram shows two or more sets as circles, with the elements of each set inside its circle. The intersections show the elements that are common to the overlapping sets. If there is no intersection, then the sets are **mutually exclusive**, which means that they have no common elements. The circles are usually enclosed in a rectangle representing all the elements being considered, called the **universal set**.
- '**Exclusive or**' means one or the other, but not both. This is sometimes written as **XOR**.
- A **tree diagram** begins from a single point. It has lines branching to the separate outcomes that can occur. At the end of each line, there may be another stage with further outcomes indicated by more branching.
- In probability, the word '**given**' means that you assume that the event specified has already occurred. The sample space considered is smaller than the general case.
- A **simplified tree diagram** shows repeated branches on a normal tree diagram as single branches, with a fraction showing the number of branches that have been combined, out of the total number at that join.



Chapter 7 review

Understanding

See Example 6

- 1 Two normal six-sided dice are thrown. Use a two-way table to show the products of the numbers on the dice.

See Example 7

- 2 Draw a Venn diagram to show the numbers from 40 to 60 inclusive. Show set $A = \{\text{odd numbers}\}$ and set $B = \{\text{numbers divisible by 3}\}$.

See Example 8

- 3 Draw a tree diagram to show the sample space for choosing two discs from a box containing 1 red and 2 blue discs under the following conditions.
- a The first disc is replaced before the second is drawn.
 - b The discs are drawn without replacement.

See Example 9

- 4 A card is taken from a normal pack and it is noted whether it is red or black. The card is then replaced. Use a tree diagram to show the sample space for the following cases.
- a 2 cards are taken
 - b 3 cards are taken

See Example 12

- 5 Students on a debating team could be boys or girls. In an English class, the teacher chooses debating teams randomly. Use a tree diagram to show the genders of students chosen for debating teams with the following numbers of students.
- a 2 students
 - b 3 students
 - c 4 students
 - d 5 students

See Example 13

- 6 A pencil case contains 2 blue and 2 black pens. Draw a tree diagram to show the sample space when two pens are taken out at random (without replacement).

See Example 14

- 7 A box contains 4 diodes and 5 small resistors. Draw a simplified tree diagram to show the sample space if two components are taken out at random, with the first being replaced before the second is taken out.

See Example 15

- 8 CDs are chosen at random from a selection containing 14 Jazz, 16 Classical and 10 Musical albums. Draw simplified tree diagrams to show the sample space under the following conditions.
- a 2 CDs are chosen without replacement.
 - b 3 CDs are chosen with replacement.
 - c 3 CDs are chosen without replacement

Fluency

See Example 1

- 9 Describe the probability of each of the following events.
- a A person chosen at random from your class is at least three years old.
 - b A person chosen at random from your class lives with both parents.
 - c A person chosen at random from your class has two or more living grandparents.
 - d A person chosen at random from your class has four or more living great-grandparents.
 - e A person chosen at random from your class was born on a day beginning with 'B'.

See Example 2

- 10 The students in a Year 9 class measured the width of their classroom and got these results in metres.
- 6.1, 6.2, 6.3, 5.8, 5.9, 5.8, 6, 5.8, 5.9, 6.1, 6.3, 6.1, 6.2, 6, 5.9, 5.6, 6.2, 6.3, 6, 6.2, 6.2, 5.8, 6.2, 6, 6.2
- a Draw up a frequency table.
 - b Which is more likely: 5.8 m or 5.9 m?
 - c What is the most likely width?

Work out:

- d** the relative frequency of each length
 - e** the number from 300 Year 9 students that you would expect to measure the width as 6.2 m
 - f** the number from 500 that you would expect to measure the width as less than 6 m.
- 11** A bag contains 2 blue, 7 pink and 6 white marbles. One marble is taken out at random. See Example 3
- a** Which is the least likely colour?
 - b** Work out the probability of getting:
 - i** a blue marble **ii** a pink marble **iii** a white marble
 - c** The procedure is repeated 120 times. Work out the expected number of:
 - i** blues **ii** pinks **iii** whites
- 12** The probability that a student selected at random has 2 or more siblings is 55%. What is the probability that a student selected at random has less than 2 siblings? See Example 4
- 13** Use a two-way table for the totals from throwing two normal dice to find the probability that:
- a** you get a total less than 5
 - b** one of the dice gives a number more than 3
 - c** you get a total less than 6 and one of the dice has a number less than 4
 - d** you get a total less than 6 or one of the dice has a number less than 4
 - e** exactly one of the dice has a number more than 2
- 14** A coin is tossed and an 8-sided die numbered 1–8 is rolled at the same time.
- a** Show the sample space as a two-way table.
 - b** What is the probability of getting a head and a four?
 - c** What is the probability of getting a head or a four?
 - d** What is the probability of getting a head or a four, but not a head and a four?
 - e** What is the probability of getting a head, but not a four?
- 15** Two wooden balls each have three flat sides cut on them. The flats are numbered 1 to 3 on each ball. The balls are rolled and the numbers that they land on are totalled.
- a** Which is less likely: a total of 4 or a total of 5?
 - b** What is the probability of getting a total of 1?
 - c** What is the probability of getting a total of 3?
 - d** How many times would you expect a total of 4 from 50 rolls?
- 16** A bag contains 3 pink and 2 yellow marbles. Two marbles are taken out, one after the other, with the first being replaced before the second is taken out.
- a** What is the probability that they are both pink?
 - b** What is the probability that they are different colours?
 - c** What is the probability that they are both yellow?
 - d** What happens to the probabilities if the first is not replaced before the second is drawn?
- 17** Use the Venn diagram from question 2 to find the following probabilities for a number chosen at random between 40 and 60 inclusive.
- a** It's in sets A and B.
 - b** It's in set A or set B.
 - c** It's in set A or B but not both.
 - d** It's in neither sets A nor B.
 - e** It's in set A but not B.
 - f** It's in set B but not in A.

Chapter 7 review

See Example 10

- 18 a** Draw a two-way table to show the total obtained from rolling two dice: one normal and one with ten sides numbered from 1 to 10.
- b** Given that one die shows a 3, what is the probability that the total is 9?
- c** Given that the total is 9, what is the probability that one die shows a 3?
- d** Given that one die shows a 4, what is the probability that the total is 6?
- e** Given that the total is 6, what is the probability that one die shows a 4?
- f** Given that one die shows a 4, what is the probability that the total is 10?
- g** Given that the total is 10, what is the probability that one die shows a 4?
- h** Given that the total is 10, what is the probability that one die shows a 3?
- i** Given that the total is 16, what is the probability that one die shows a 10?

See Example 11

- 19** a Draw a Venn diagram to show numbers from 20 to 50 inclusive with: $A = \{\text{even numbers}\}$, $B = \{\text{numbers divisible by 3}\}$, $C = \{\text{numbers divisible by 5}\}$. For a number chosen at random, find each of the following probabilities.
- b Given it is even, it is divisible by 3.
- c Given it is divisible by 3, it is even.
- d Given it is even, it is divisible by 5.
- e Given it is divisible by 5, it is even.
- f Given it is divisible by 3, it is divisible by 5.
- g Given it is divisible by 5, it is divisible by 3.
- 20** A child's card game has 5 red and 5 green cards; 3 of the red cards and 2 of the green cards have pictures of lions on them. The other cards have pictures of tigers. One card is selected at random. Work out the probability that the card is:
- a a tiger card
- b a red card
- c a red tiger card
- d a lion card or a green card
- 21** A bag of marbles contains 5 yellow, 4 orange and 7 purple marbles. Two marbles are drawn, with replacement. Find the probability of getting each of the following.
- a 2 yellow marbles
- b 2 orange marbles
- c different-coloured marbles.
- 22** Repeat question **21** without replacement.

Problem solving

- 23** In a survey of 200 people, 40 said that they did not like chocolate and the rest said that they liked it; 50 actually said that they loved it.
- a** Which was more likely: liking or disliking chocolate?
 - b** What is the relative frequency of disliking chocolate?
 - c** If 500 people were surveyed, how many would you expect to dislike chocolate?
 - d** If 2140 people were asked, how many would you expect to be chocolate-lovers?
- 24** The ages of people riding a roller coaster on the Gold Coast were as follows:
32, 16, 16, 20, 17, 15, 16, 15, 28, 21, 21, 22, 16, 16, 16, 27, 17, 16, 24, 25, 17, 14, 22
- a** Make a frequency table to show the data.
 - b** Why were there no 10-year-olds?
 - c** Which is more likely: under 20 or over 20?

- 25** A mixed crate of soft drinks has 3 orange, 2 raspberry, 5 cola and 4 lemonade bottles. One bottle is taken out at random.
- a** Give the sample space.
 - b** Which bottle is the most likely to be taken out?
 - c** Work out the probability that it is lemonade.
- 26** A drinks dispenser has been filled with 50 unsorted bottles of juice. There are 20 bottles of orange juice, and of these 8 are 600 mL bottles. The rest are apple juice and of these 10 are 750 mL bottles. All the bottles are 750 mL or 600 mL. If a bottle is dispensed, work out the following probabilities.
- a** It is a 600 mL orange juice.
 - b** Given that it is 600 mL, that it is orange juice.
 - c** Given that it is orange juice, that it is 600 mL.
 - d** It is 600 mL or orange juice.
 - e** It is neither orange juice, nor 600 mL.
 - f** It is 750 mL and apple juice.
 - g** It is 750 mL or apple juice.
 - h** Given that it is apple juice, that it is 750 mL.
- 27** The information in the table below shows the test marks for some Year 9 students in English and History. Both tests were out of 20.

See Example 5

		English						
		0–2	3–5	6–8	9–11	12–14	15–17	18–20
History	0–2	3	2	2	2	3	0	0
	3–5	1	2	1	1	1	1	0
	6–8	2	2	3	3	3	0	1
	9–11	4	1	2	1	0	4	1
	12–14	0	1	5	4	7	4	7
	15–17	0	0	1	4	4	4	4
	18–20	0	0	1	1	5	3	9

- a** What is the probability that a student scores under 12 in English?
- b** What is the probability that a student scores under 12 in English and under 12 in History?
- c** Given that a student scores under 12 in English what is the probability of the student scoring under 12 in History?
- d** Given that a student scores under 6 in English what is the probability of the student scoring under 6 in History?
- e** Given that a student scores over 11 in English what is the probability of the student scoring over 11 in History?
- f** Given that a student scores strictly between 5 and 15 in English what is the probability of the student scoring between 5 and 15 in History?
- g** Given that a student scores strictly between 5 and 15 in History what is the probability of the student scoring between 5 and 15 in English?
- h** Can you make any conclusions about scores in English and History?

Chapter 7 review

- 28** Year 11 students from a Specialist Maths class of 24 students were asked about Chemistry, Music and Physics. Of the 10 taking Physics, 2 did Music and 5 took Chemistry. Only one student did all three subjects and 4 took none of them. There were 3 students who did only Chemistry and 5 who took only Music. Find the probabilities of each of the following for a student chosen at random from the class.
- a** The student takes Chemistry and Physics.
 - b** The student takes Chemistry or Physics.
 - c** The student does all three of Chemistry, Physics and Music.
 - d** The student does not take any of Chemistry, Music or Physics.
 - e** Given that a student takes Chemistry they do not do Physics.
 - f** Given that a student takes Physics they do not do Chemistry.
 - g** Given that the student takes Music, the student does Chemistry.
 - h** Given that the student takes Chemistry, the student does Music.
 - i** Given that the student takes Physics, that the student does not do Chemistry or Music.
 - j** Given that the student does not take Chemistry, that the student studies Physics.
- 29** A box has 12 coloured glass balls in it: 3 red, 1 gold, 5 blue and 3 green. The streamers for the Christmas tree are in a bag. There are 2 green streamers, 1 yellow and 3 red. A glass ball and then a streamer are taken out at random.
- a** What is the probability of getting a blue ball and a red streamer?
 - b** What is the probability of getting a red ball and a yellow streamer?
- 30** In a hand of five playing cards, what is the probability that all the cards are the same suit?
- 31** Two people are playing 'Snap'. In this game, each player starts with a (shuffled) pile of cards face down. Each player places his cards, face down, in a pile in front of him. On his turn, each player turns over the top card from his face-down pile. When someone turns over a card that matches a card already face up on another player's pile, the first player to call "Snap!" wins both piles and adds them to the bottom of his face-down pile. Work out the following probabilities.
- a** 'Snap' occurs after 2 cards have been played.
 - b** 'Snap' occurs after 3 cards have been played.
 - c** 'Snap' occurs after 4 cards have been played.

Reasoning

- 32** Express buses and ordinary buses both go past Cyrano's bus stop. They all stop there and go into town. There are 14 express buses and 14 ordinary buses each day. Cyrano works out the sample space and concludes that the probability of getting an express bus is 0.5. Sixty times in a fortnight, he goes to the bus stop and catches the first bus to come along to test his theory. He finds that he has caught the express bus 50 times and the ordinary one only 10 times. What is wrong with his analysis?
- 33** In question 11, how many times would you expect to repeat the experiment to get 40 blue marbles?
- 34** Using the information from question 23, how many people would you expect to ask before finding 90 who dislike chocolate?

- 35** In a group of 45 Year 9 students, 16 liked heavy metal and 38 liked chocolate. 4 of the heavy metal fans did not like chocolate. One of the students said that the probability that a student who liked chocolate would also like heavy metal was 0.75. What is wrong with this and how did the student make a mistake?
- 36** Considering the students in question **35**, calculate the following probabilities.
- a** A student liking heavy metal.
 - b** A student liking heavy metal and chocolate.
 - c** Given that a student likes heavy metal, that the student will also like chocolate.
- 37** Can you find a relationship between the probabilities in question **36**?
- 38** From a class of 24 Year 9 students, 15 preferred using pens rather than pacers and 18 preferred using templates to draw circles instead of compasses. Of the pacer-likers, 4 preferred compasses to templates. Find the probabilities that a student chosen at random prefers
- a** pens or templates
 - b** pens and templates
 - c** pens or templates, but not both.
- 39** Can you find a relationship between the probabilities in question **38**?
- 40** A popular TV game has rounds where someone is given four choices (A, B, C, D) for the answer to a question. You have the choice of choosing an answer or passing to the next player. There are quite a few players before you get another turn if you pass. From watching the show, you know you will be able to get about half the answers right, but will not be sure about the others. In half the rest of the questions, you can eliminate one answer, but will have to guess after that. Explain your chances of winning by getting 4 questions right in a row. What if you have to get 5 questions right in a row?



Number and algebra

8 Equations



Contents

- 8.1 Solving linear equations
- 8.2 Applying linear equations
- 8.3 Linear inequalities
- Chapter summary
- Chapter review

Prior learning

Chapter 8

MAT09NAPL00008

Parent guide

Chapter 8

MAT09NAPG00008

Curriculum guide

Chapter 8

MAT09NACU00008

Australian Curriculum statements

Linear and non-linear relationships

Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution. (Year 8)
Solve problems involving linear equations, including those derived from formulas. (Year 10)

Video tutorial

Equations

MAT09NAVT00008

Weblink

Jonny Heeley's Master class on Algebra

MAT09NAWB00008

The ability to solve equations is a key mathematical skill that is used in a wide range of applications. Many complex real world problems can be reduced to fairly simple equations. In many cases, the equation may be an approximation to a situation, but it will give an answer that is 'good enough'. On the rare occasions when you need an exact answer, an equation may be the only way to get it. Most physical and chemical laws are expressed in terms of equations and formulas, including the most famous equation of all, $E = mc^2$, the relationship between mass and energy. In this chapter, you will learn more about solving equations.

Mathematical literacy

Maths dictionary

MAT09ASDI00001

Weblink

The RITEMATHS project: Linear functions

MAT09NAWB00008

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

backtracking	left-hand side	QED	satisfy
equation	linear	right-hand side	solution
flow chart	lowest common	RTP	solve
inequality	denominator		
inverse operation	proof		

8.1 Solving linear equations

You have previously studied solving equations. You need to make sure that you are familiar with the meanings of the following important terms.

Important!

Linear equations

An **equation** is a mathematical sentence. It has an equals sign with expressions on both sides. The simplest algebraic equations are equations with *one variable*.

A **linear equation** is one like $\frac{2x - 5}{4} - 8 = 0$ that would produce a straight line if it was plotted on graph paper for different values of the variable. Linear equations do not have any powers, roots or reciprocals of the variable.

A **solution** of an algebraic equation is a value of the variable that makes the equation true. The process of finding the solution to an equation is called **solving** an equation. We say that a solution **satisfies** the equation.

Example 1

State whether or not each of the following is a linear equation in one variable.

a $5x + 3 = 2x - 5$

b $7m(m + 2) = 9$

c $\frac{5}{b-6} = \frac{b+2}{5}$

d $\frac{5b-7}{3} + 8 = 2(b+1) + \frac{b}{5}$

e $3p - 4q = 2(p - 5q)$

f $3x^2 - \sqrt{x} - 8 = 0$

Solution

- a** There are no powers, roots or reciprocals of the variable.

$5x + 3 = 2x - 5$ is linear.

- b** When you expand the brackets, you get $7m^2$.

$7m(m + 2) = 9$ is not linear.

- c** $\frac{5}{b-6}$ has the variable in the denominator.

$\frac{5}{b-6} = \frac{b+2}{5}$ is not linear.

- d** There are no powers, roots or reciprocals of the variable.

$\frac{5b-7}{3} + 8 = 2(b+1) + \frac{b}{5}$ is linear.

- e** There are two variables.

This is not an equation in only one variable.

- f** There is both a square and a root of the variable.

$3x^2 - \sqrt{x} - 8 = 0$ is not linear.

For very simple equations, a simple method is used.

Important!

Solution by inspection

To solve an equation **by inspection**, you look at what the equation means. Then you try different numbers until you find one that satisfies the equation.

This is also called **guess and check** or **trial and error**.



Example 2

CAS TI-Nspire exercise

Equations

MAT09NAT100008

CAS ClassPad exercise

Equations

MAT09NACP00008

Solve the following by inspection.

- a** $p + 5 = 17$ **b** $v + 8 = 2$
c $e - 8 = 3$ **d** $3x = 24$
e $\frac{m}{7} = -2$



Solution

- a** What does the equation mean?

Try $p = 22$.

Whoops, try $p = 12$.

State the answer.

- b** What does the equation mean?

Try $v = -6$.

State the answer.

- c** What does the equation mean?

Try $e = 5$

Try $e = 11$.

State the answer.

- d** What does the equation mean?

Try $x = 8$.

State the answer.

- e** What does the equation mean?

Try $m = 14$.

Try $m = -14$.

State the answer.

When you add 5 to p , you get 17.

$$\text{LHS} = 22 + 5 = 27 \neq \text{RHS}$$

$$\text{LHS} = 12 + 5 = 17 = \text{RHS}$$

The solution is $p = 12$.

When you add 8 to v , you get 2.

$$\text{LHS} = -6 + 8 = 2 = \text{RHS}$$

The solution is $v = -6$.

When you take 8 from e , you get 3.

$$\text{LHS} = 5 - 8 = -3 \neq \text{RHS}$$

$$\text{LHS} = 11 - 8 = 3 = \text{RHS}$$

The solution is $e = 11$.

3 times the number is 24

$$\text{LHS} = 3 \times 8 = 24 = \text{RHS}$$

The solution is $x = 8$.

When divide the number by 7, you get -2.

$$\text{LHS} = \frac{14}{7} = 2 \neq \text{RHS}$$

$$\text{LHS} = \frac{-14}{7} = -2 = \text{RHS}$$

The solution is $m = -14$.

You can solve more complicated equations using graphs. You can almost always use this method for equations in one variable, but other methods are often more efficient.

Important!

Solution by graphing

To solve an equation by **graphing**, you change the equation so that there is only a number on the **right-hand side**. The easiest number to have on the RHS is 0. Then you draw a graph of the expression on the **left-hand side** to find the value of the variable that satisfies the equation.

Example 3

Solve $3p - 5 = 7$ by graphing.

Solution

Write the equation to graph.

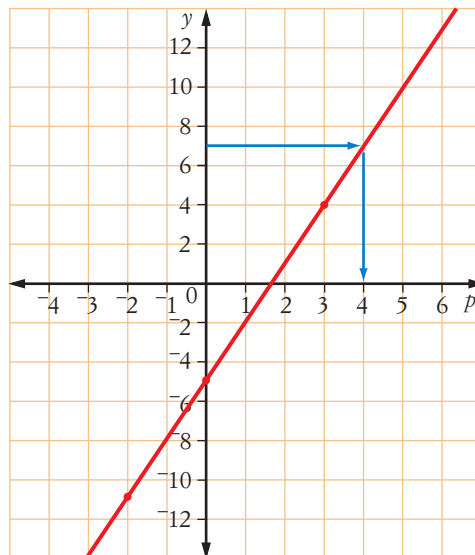
Make a table of values using at least 3 values.

Draw a graph of $y = 3p - 5$

p	-2	0	3
y	-11	-5	4

Plot the points, and extend the line past the ends.

You want $y = 7$. Find this on the graph, then look at the x value.



Write the result.

Check in $3p - 5 = 7$.

Write your answer.

$$p = 4$$

$$\text{LHS} = 3 \times 4 - 5 = 12 - 5 = 7 = \text{RHS} \quad \checkmark \text{ OK}$$

The solution of $3p - 5 = 7$ is $p = 4$.

It is generally quicker to solve equations where the variable only appears once on the left-hand side by backtracking rather than by drawing graphs.

Important!

Solution by backtracking

To solve an equation using **backtracking** you set up a **flow chart** showing what has been done to the variable on the left to give the value on the right of the equation. You then work backwards from the right-hand side, reversing each operation as you go.

Example 4

Solve the equation $\frac{3d+1}{-7} + 4 = 6$ by backtracking.

Solution

Put the variable in the first box.

d

First, the variable is multiplied by 3.

$d \cdot 3 \rightarrow \square$

Put $\times 3$ on the first arrow.

Now put $+ 1$ on the next arrow.

$d \cdot 3 \rightarrow \square + 1 \rightarrow \square$

The expression is next divided by -7 .

$d \cdot 3 \rightarrow \square + 1 \rightarrow \square \div -7 \rightarrow \square$

Finally, 4 is added to give 6.

$d \cdot 3 \rightarrow \square + 1 \rightarrow \square \div -7 \rightarrow \square + 4 \rightarrow 6$

Now backtrack the last operation, reversing the $+ 4$ to $- 4$ to get 2.

$d \cdot 3 \rightarrow \square + 1 \rightarrow \square \div -7 \rightarrow 2 \xleftarrow{-4} 6$

Reverse $\div -7$ to $\times -7$ to get -14 .

$d \cdot 3 \rightarrow \square + 1 \rightarrow 14 \xleftarrow{\times -7} 2 \xleftarrow{-4} 6$

Reverse $+ 1$ to $- 1$ to get -15 .

$d \cdot 3 \rightarrow -15 \xleftarrow{-1} 14 \xleftarrow{\times -7} 2 \xleftarrow{-4} 6$

Reverse $\times 3$ to $\div 3$ to get -5 .

$-5 \xleftarrow{\div 3} -15 \xleftarrow{-1} 14 \xleftarrow{\times -7} 2 \xleftarrow{-4} 6$

Write the answer.

The solution to $\frac{3d+1}{-7} + 4 = 6$ is $d = -5$.

Worksheet

Backtracking

MAT09NAWK00051

Puzzle sheet

Backtracking

MAT09NAPS00032

The formal solution of equations is done algebraically by reversing steps, similar to backtracking and the balance model you looked at last year.

Important!

Formal solution of equations

Operation	Symbol	Inverse operation	Symbol
Add	+	Subtract	−
Subtract	−	Add	+
Multiply	×	Divide	÷
Divide	÷	Multiply	×
Square	$\square^2$	Square root	$\sqrt{\quad}$
Square root	$\sqrt{\quad}$	Square	$\square^2$
Cube	$\square^3$	Cube root	$\sqrt[3]{\quad}$
Cube root	$\sqrt[3]{\quad}$	Cube	$\square^3$

To formally solve equations, use **inverse operations** in the reverse order to the normal order of operations to get the variable on its own on the LHS with only an arithmetic expression on the RHS.

Whatever you do to one side must be done to the other.

You must show your steps.



Puzzle sheet

Solving equations

MAT09NAPS00033

CAS TI-Nspire exercise

Equations

MAT09NATI00008

CAS ClassPad exercise

Equations

MAT09NACP00008

Example 5

Formally solve the following equations.

a $3b - 2 = 16$

c $5k + 8 = 27$

Solution

a Write the equation.

Add 2 to both sides.

Simplify.

Divide both sides by 3.

Simplify to get the solution.

b Write the equation.

Subtract 4 from both sides.

Simplify.

Multiply both sides by 5.

Simplify to get the solution.

c Write the equation.

Subtract 8 from both sides.

Simplify.

Divide both sides by 5.

Simplify to get the solution.

d Write the equation.

Subtract 7 from both sides.

Simplify.

Multiply both sides by 3.

Simplify to get the solution.

b $\frac{m}{5} + 4 = 11$

d $\frac{x}{3} + 7 = 2$

$$3b - 2 = 16$$

$$3b - 2 + 2 = 16 + 2$$

$$3b = 18$$

$$\frac{3b}{3} = \frac{18}{3}$$

$$b = 6$$

$$\frac{m}{5} + 4 = 11$$

$$\frac{m}{5} + 4 - 4 = 11 - 4$$

$$\frac{m}{5} = 7$$

$$5 \times \frac{m}{5} = 5 \times 7$$

$$m = 35$$

$$5k + 8 = 27$$

$$5k + 8 - 8 = 27 - 8$$

$$5k = 19$$

$$\frac{5k}{5} = \frac{19}{5}$$

$$k = 3\frac{4}{5}$$

$$\frac{x}{3} + 7 = 2$$

$$\frac{x}{3} + 7 - 7 = 2 - 7$$

$$\frac{x}{3} = -5$$

$$3 \times \frac{x}{3} = 3 \times -5$$

$$x = -15$$

Some equations have variables on both sides. In this case, you move all the variable terms to the left-hand side. This changes the equation into ones like those in the previous example.

Example 6

Formally solve the following equations.

a $5y - 6 = 3y + 16$

Solution

a Write the equation.

Subtract $3y$ from both sides.

Simplify.

Add 6 to both sides.

Simplify.

Divide by 2 to get the solution.

b Write the equation.

Subtract $7g$ and 9 from both sides.

Simplify.

Divide both sides by 4.

Simplify to get the solution.

b $11g + 9 = 7g - 17$

$$5y - 6 = 3y + 16$$

$$5y - 6 - 3y = 3y + 16 - 3y$$

$$2y - 6 = 16$$

$$2y - 6 + 6 = 16 + 6$$

$$2y = 22$$

$$y = 11$$

$$11g + 9 = 7g - 17$$

$$11g + 9 - 7g - 9 = 7g - 17 - 7g - 9$$

$$4g = -26$$

$$\frac{4g}{4} = \frac{-26}{4}$$

$$g = -6\frac{1}{2}$$

Puzzle sheet

Equations with
unknowns on both
sides

MAT09NAPS00035

To solve an equation with brackets, is usually easier to multiply out the brackets before doing anything else. This makes the equation just like the ones you have already seen.

Example 7

Formally solve the following equations.

a $3(2x - 5) + 4 = 8 + 4(x + 9)$

b $2(z + 8) - 6(2z - 5) = 8(z + 9)$

Solution

a Write the equation.

Expand the brackets, taking care with signs.

Simplify.

Add 11 and take $4x$ from both sides.

Simplify.

Divide by 2 to get the solution.

$$3(2x - 5) + 4 = 8 + 4(x + 9)$$

$$6x - 15 + 4 = 8 + 4x + 36$$

$$6x - 11 = 4x + 44$$

$$6x - 11 + 11 - 4x = 4x + 44 + 11 - 4x$$

$$2x = 55$$

$$x = 27\frac{1}{2}$$

Video tutorial

Equations with brackets

MAT09NAVT10022

Puzzle sheet

Equations

MAT09NAPS00036

- b** Write the equation.

Expand the brackets, taking care with signs.

Simplify.

Take $8x$ and 46 from both sides.

Simplify.

Divide by -18 to get the solution.

$$2(z + 8) - 6(2z - 5) = 8(z + 9)$$

$$2z + 16 - 12z + 30 = 8z + 72$$

$$-10z + 46 = 8z + 72$$

$$-10z + 46 - 8z - 46 = 8z + 72 - 8z - 46$$

$$-18z = 26$$

$$z = -1\frac{4}{9}$$

When you have equations with fractions, you should multiply both sides of the equation by the denominator. If there is more than one fraction, multiply by the **lowest common denominator (LCD)**.

Example 8

Formally solve the following equations.

a $\frac{a}{3} + 4 = 7$

b $\frac{4x - 2}{5} = 6$

c $\frac{y}{3} - \frac{2y}{5} + 1 = 5$

Solution

- a** Write the equation.

Multiply *each term* by the denominator, 3.

Simplify.

Subtract 12 to get the solution.

- b** Write the equation.

Multiply *both terms* by the denominator, 5.

Simplify.

Add 2 to both sides.

Divide by 4 to get the solution.

- c** Write the equation.

Multiply *each term* by the lowest common denominator, 15.

Simplify the products.

Simplify the variables and subtract 15.

Divide by -1 to get the solution.

$$\frac{a}{3} + 4 = 7$$

$$3 \times \frac{a}{3} + 3 \times 4 = 3 \times 7$$

$$a + 12 = 21$$

$$a = 9$$

$$\frac{4x - 2}{5} = 6$$

$$5 \times \frac{4x - 2}{5} = 5 \times 6$$

$$4x - 2 = 30$$

$$4x = 32$$

$$x = 8$$

$$\frac{y}{3} - \frac{2y}{5} + 1 = 5$$

$$15 \times \frac{y}{3} - 15 \times \frac{2y}{5} + 15 \times 1 = 15 \times 5$$

$$5y - 6y + 15 = 75$$

$$-1y = 60$$

$$y = -60$$

In Example 8, you could solve the equations a little more easily by subtracting the constant on the left-hand side before multiplying by the LCD.

Worksheet

Mixed equations

MAT09NAWK00052

CAS TI-Nspire exercise

Equations

MAT09NATI00008

CAS ClassPad exercise

Equations

MAT09NACP00008

You should also be able to translate sentences involving mathematical operations into equations and then solve them.

Example 9

The sum of three times a number and 7 is twice the sum of the number and 8. What is the number?

Solution

Write as an equation using x as the variable.

$$3x + 7 = 2(x + 8)$$

Expand the brackets.

$$3x + 7 = 2x + 16$$

Subtract $2x$ and 7 from both sides.

$$3x + 7 - 2x - 7 = 2x + 16 - 2x - 7$$

Simplify.

$$x = 9$$

Write the answer.

The number is 9.

Exercise 8.1 Solving linear equations

1 State whether or not each of the following is a linear equation in one variable.

a $3x^2 - 5 = 11$

b $4m + 62 = 48$

c $\frac{p}{3} + 6(p - 5) = \frac{4}{5}$

d $4 - \frac{3}{z} = z + 7$

e $\sqrt{5}(h - 5) + 7 = 9$

f $3(b - 3)(b + 2) + 6(b + 8) = 7$

g $5\sqrt{x} + 8 - x = 14$

h $2x^2 = 18$

i $3(c + 2) - 5(c - 1) = 7(c + 3) + 8$

j $3(3y - 4) + 8 + 4x = 15$

2 Solve each of the following equations by inspection.

a $5x = 30$

b $m - 5 = 9$

c $\frac{h}{3} = 6$

d $g + 5 = 16$

e $g + 8 = 3$

f $2k = -12$

g $t - 6 = -2$

h $v + 8 = -4$

i $\frac{p}{-4} = -6$

j $-3m = 24$

3 Solve each of the following equations by graphing.

a $2x - 5 = 3$

b $e + 2 = -3$

c $7 - 2m = 1$

d $3x - 5 = -8$

e $\frac{z}{2} + 2 = 5$

4 Solve each of the following equations by drawing a flow chart and backtracking.

a $3(q - 2) + 4 = 13$

b $\frac{k + 5}{3} - 2 = 4$

c $\frac{2y - 9}{7} + 3 = 8$

d $\frac{4(h + 3)}{5} - 2 = 10$

e $6(3m - 7) + 3 = -21$

Understanding

Extra questions

Exercise 8.1

MAT09NAEQ00027

See Example 1

Fluency

See Example 2

See Example 3

See Example 4

See Example 5

5 Formally solve each of the following equations.

a $2m - 9 = 7$

b $\frac{a}{4} - 1 = 5$

c $5t + 3 = 38$

d $\frac{p}{2} + 1 = 6$

e $9y - 15 = 3$

f $7t - 3 = 39$

g $24 - 3e = 9$

h $\frac{c}{3} - 5 = -3$

i $3d + 11 = 29$

j $\frac{k}{3} - 6 = 6$

6 Formally solve each of the following equations.

a $2q - 3 = -21$

b $x - 2 = -14$

c $4v + 1 = -39$

d $4r - 5 = -45$

e $-r + 4 = -2$

f $4r - 8 = -28$

g $4n + 3 = -17$

h $3y - 3 = 24$

i $4x + 5 = -39$

j $-2z - 7 = -1$

7 Formally solve each of the following equations.

a $16x + 8 = 12$

b $-8r + 5 = -39$

c $-10w + 2 = -10$

d $8m + 6 = 34$

e $4b - 8 = -5$

f $-15h + 9 = 3$

g $3a - 7 = -6$

h $12d - 1 = 29$

i $16a + 4 = 16$

j $4w + 7 = 8$

See Example 6

8 Formally solve each of the following equations.

a $7c - 3 = 6c + 4$

b $8x + 3 = 7x + 12$

c $2d + 5 = 4d - 11$

d $6p - 4 = 5p + 4$

e $4q + 9 = 5q + 6$

f $5y - 7 = 2y + 17$

g $2y - 3 = 5y - 39$

h $4a - 1 = 3a + 6$

i $5e + 7 = 6e - 3$

j $4p - 6 = 2p + 18$

9 Formally solve each of the following equations.

a $13g + 9 = 9g - 39$

b $9p - 2 = 5p + 30$

c $-12p - 3 = -8p + 29$

d $4h - 6 = 3h - 10$

e $11e - 5 = 7e - 33$

f $e - 8 = 3e - 20$

g $8k - 6 = 7k - 10$

h $9w + 5 = 6w + 20$

i $-8z - 1 = -6z - 17$

j $r - 9 = 2r - 2$

See Example 7

10 Formally solve each of the following equations.

a $4(g + 2) = 29 + g$

b $3(2w - 1) = 9(w + 2)$

c $2(3a + 5) = 4(a - 2)$

d $6(w - 2) + 10 = 4(2w + 3)$

e $2(4p + 3) - 3 = 5(p + 6)$

f $2(z - 4) + 7 = -1(z - 5)$

g $3(2d + 5) = 5(d + 8) + 6d$

h $3(u - 3) = 2(2u - 5) - 7$

i $43 - 4(c + 3) = 4 - 1(c + 9)$

j $11 - 5(p + 1) = 6 - 3(3p - 4)$

11 Formally solve each of the following equations.

a $-6w - 1 = 21$

b $12a - 2 = -23$

c $16p - 5 = 35$

d $-12w - 2 = -29$

e $2a + 3 = -15$

f $12d + 8 = -25$

g $10z + 9 = -3$

h $-10y - 9 = -13$

i $-9n + 8 = -22$

j $-3z + 5 = -4$

12 Formally solve each of the following equations.

a $-21t + 2 = -6t - 22$

b $7y + 7 = 5y - 2$

c $-3x + 1 = -x - 23$

d $-5q + 2 = 4q + 32$

e $4w - 8 = 8w - 19$

f $-24y - 9 = -8y - 49$

g $-11n - 4 = -5n + 18$

h $-11g - 1 = -9g + 10$

i $17z + 1 = 9z - 23$

j $-3t - 3 = 5t + 33$

13 Formally solve each of the following equations.

a $\frac{n}{4} - 5 = 27$

b $\frac{u+3}{3} + 5 = 4$

c $\frac{4q}{5} + 1 = -3\frac{4}{5}$

d $\frac{-2m}{3} + 2 = \frac{m}{3} - 2$

e $\frac{z}{2} + 2 = \frac{3z-4}{8} + 3$

f $\frac{3z-3}{6} = \frac{z}{3} - 1$

g $\frac{11b-6}{12} = \frac{2b}{3} + 2$

h $\frac{q+4}{2} = \frac{q+2}{10} + 3$

i $\frac{2w-4}{3} + 1 = \frac{2w+4}{2}$

j $\frac{9k-1}{8} = k - 1\frac{1}{8}$

14 Formally solve each of the following equations.

a $\frac{a}{2} + \frac{a}{3} = 15$

b $\frac{x}{3} - \frac{x}{4} = 1$

c $\frac{b}{2} - \frac{b}{5} = 2$

d $\frac{m}{3} + \frac{m}{4} = 4$

e $\frac{3y}{2} - \frac{2y}{3} = 5$

f $\frac{2h}{5} - \frac{3h}{10} = 6$

g $\frac{2d}{3} - \frac{3d}{4} = \frac{1}{2}$

h $\frac{2m}{5} - \frac{m}{3} = 1$

i $\frac{2a}{5} + \frac{3a}{10} = 7$

j $\frac{4r}{3} - \frac{r}{2} = 4$

k $\frac{d}{5} - \frac{7d}{10} = 8$

l $\frac{4b}{8} + \frac{2b}{3} = \frac{7}{3}$

15 Formally solve each of the following equations.

a $4 - \frac{x}{2} = \frac{x}{3} + 6$

b $7 - y = \frac{y}{6} - \frac{2y}{3}$

c $4 - \frac{5a}{12} = 2\left(1 - \frac{4a}{3}\right)$

d $\frac{b+3}{2} + \frac{b+5}{3} = 4$

e $\frac{m+2}{3} + \frac{m+2}{4} = 28$

f $\frac{w-3}{2} + \frac{w-4}{3} = 12$

g $\frac{2x-3}{4} + \frac{1+3x}{3} = 15$

h $\frac{2a-3}{2} + \frac{a+4}{5} = 53$

i $3 - 2d = \frac{4d+4}{8}$

j $\frac{a+3}{2} + \frac{2a+12}{3} = a - 2(a+7)$

Worked solutions

Exercise 8.1

MAT09NAWS00027

16 Write each of the following as an equation and formally solve it.

a Four times the sum of triple a number and 5 is the same as 16 times the number.

b The sum of a number and twice its successor is 48.

c Thrice the difference between a number and 20 is the same as 5 times the number.

d The product of 5 and four less than three times a number is the same as 8 more than the number.

e Four less than the product of a number and 5 is the same as triple 8 more than the number.

Problem solving

Worked solutions

Exercise 8.1

MAT09NAWS00027

See Example 8

Reasoning

Worked solutions

Exercise 8.1

MAT09NAWS00027

- 17 Check each of the following solutions. If there is an error, identify the first line on which an error occurred and state the correct answer.

a $\frac{a}{3} - 4 = 10$

$\frac{a}{3} = 6$ (A)

$3 \times \frac{a}{3} = 3 \times 6$ (B)

$a = 18$ (C)

c $\frac{3a-5}{4} = 8$

$\frac{4(3a-5)}{4} = 4 \times 8$ (A)

$3a - 5 = 32$ (B)

$3a = 27$ (C)

$a = 9$ (D)

b $\frac{y}{4} - 5 = 8$

$4\left(\frac{y}{4} - 5\right) = 4 \times 8$ (A)

$y - 20 = 32$ (B)

$y = 12$ (C)

d $\frac{x-14}{2} + 4 = 12$

$2 \times \frac{x-14}{2} + 2 \times 4 = 2 \times 12$ (A)

$2x - 28 + 8 = 24$ (B)

$2x - 20 = 24$ (C)

$2x = 44$ (D)

$x = 22$ (E)

e $\frac{4m+5}{5} - 14 = 7$

$5 \times \frac{4m+5}{5} - 14 = 5 \times 7$ (A)

$4m + 5 - 14 = 35$ (B)

$4m - 9 = 35$ (C)

$4m = 44$ (D)

$m = 11$ (E)

f $\frac{13a+18}{6} - 4 = \frac{2a}{3} + 5$

$6 \times \frac{13a+18}{6} - 6 \times 4 = 6 \times \frac{2a}{3} + 6 \times 5$ (A)

$13a + 18 - 24 = 2a + 30$ (B)

$13a - 6 = 2a + 30$ (C)

$15a = 36$ (D)

$a = 2\frac{3}{5}$ (E)

8.2 Applying linear equations

You can solve many different kinds of problems by changing the problem into a mathematical statement in the form of an equation. The basic steps are as follows.

Important!

Using equations to solve problems

Choose one of the quantities you need to find as the variable. If you can, choose the smaller value for your variable.

Write all other unknowns (if there are any) as expressions involving the variable.

Write the rest of the information as an equation involving the variable.

Solve the equation.

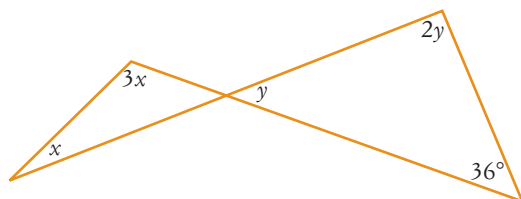
Check your answer(s) in the original problem.

If the solution doesn't work, go back and check your reasoning, especially the equation you wrote.

Some of the problems that you will encounter will involve other mathematics, such as geometry.

Example 10

Find the unknowns in the following diagram.



Solution

y is in the triangle with the known angle.

There is only one variable in the triangle.

Solve the equation.

Divide by 3.

The angle vertically opposite is also y .

Substitute the value for y .

Solve for x .

Divide by 4.

Check in the original problem.

Find y first.

$$y + 2y + 36^\circ = 180^\circ \text{ (sum of angles in a triangle)}$$

$$3y = 144^\circ$$

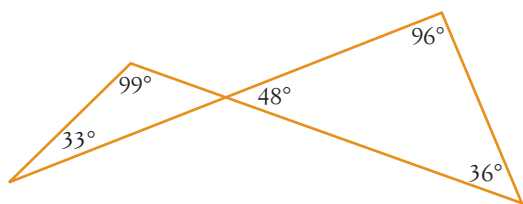
$$y = 48^\circ$$

$$x + 3x + y = 180^\circ \text{ (sum of angles in triangle)}$$

$$4x + 48^\circ = 180^\circ$$

$$4x = 132^\circ$$

$$x = 33^\circ$$



Add the angles in the triangle on the right.

$$48^\circ + 96^\circ + 36^\circ = 180^\circ \checkmark \text{ OK}$$

In the triangle on the left, the unmarked angle is 48° (vertically opposite). Check.

$$33^\circ + 99^\circ + 48^\circ = 180^\circ \checkmark \text{ OK}$$

State the solutions.

$$x = 33^\circ \text{ and } y = 48^\circ.$$

You can also get equations from measurement problems. In these problems, make sure you draw a diagram.

Example 11

Technology

Geogebra: Equation problem

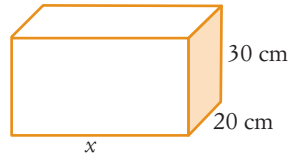
MAT09MGTC00005

An oblong carton needs to have a volume of 0.024 m^3 . The maximum permissible height is 30 cm and the maximum width is 20 cm. What length must it be?

Solution

Draw a diagram and mark the known measurements.

Choose the length variable as x .



Write the volume formula.

$$V = l \times w \times h$$

Change to the same units.

$$20 \text{ cm} = 0.2 \text{ m}, 30 \text{ cm} = 0.3 \text{ m}$$

Substitute values.

$$0.024 = x \times 0.2 \times 0.3$$

Simplify.

$$0.06x = 0.024$$

Divide by 0.06.

$$x = \frac{0.024}{0.06} = 0.4$$

Check.

$$\text{Volume} = 0.4 \times 0.2 \times 0.3 = 0.024 \checkmark \text{ OK}$$

Write the solution in the original units.

The carton must be 40 cm long.

Like Example 11, you will also find that other problems involve solving formulas for an unknown. You will also find that many problems relating to money can be solved using equations.

Example 12

Worksheet

Word problems

MAT09NAWK00053

Adam and Britney have \$480 between them. Adam has seven times as much as Britney. How much does each have?

Solution

Choose the smaller amount as the variable.

Let Britney's amount = b

Write the other unknown in terms of b .

Adam's amount = $7b$

Write the information as an equation.

$$b + 7b = 480$$

Simplify.

$$8b = 480$$

Divide by 8.

$$b = 60$$

Work out Adam's amount.

$$\text{Adam has } 7 \times \$60 = \$420$$

Check in the original problem.

$$\text{Total amount} = \$60 + \$420 = \$480 \checkmark \text{ OK}$$

Write the solution.

Adam has \$420 and Britney has \$60.

Example 13

Elena bought a coat for \$136, with a discount of 15% on the marked price for cash. How much did she save by paying cash?

Solution

You want the amount saved.

Express the other unknown in terms of d .

The discount is 15% of the marked price.
15% is 15 hundredths.

Simplify the fraction.

Multiply by the denominator.

Simplify.

Subtract $3d$ from both sides.

Divide by 17.

Check in the original problem.

Check the percentage.

Do the calculation.

State the solution.

Let the discount = d

Marked price = $136 + d$

$$d = \frac{15}{100} \times (136 + d)$$

$$d = \frac{3(136 + d)}{20}$$

$$20 \times d = 20 \times \frac{3(136 + d)}{20}$$

$$20d = 408 + 3d$$

$$17d = 408$$

$$d = 24$$

Marked price = \$136 + \$24 = \$160.

$$\text{Discount} = \frac{24}{160} \times \frac{100}{1} \%$$

$$= 15\% \checkmark \text{ OK}$$

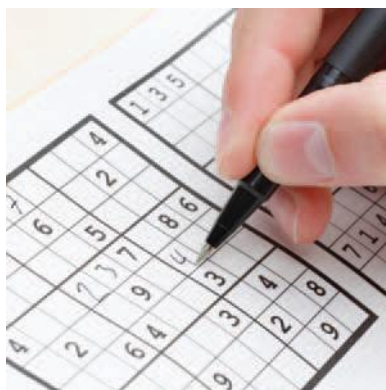
Elena saved \$24 by paying cash.

Example 13 also illustrates problems involving percentages. Many newspapers include problems that can also be solved using simple equations.

Investigate: Newspaper problems

Look in newspapers and magazines in the school library for some problems in the puzzle sections.

Make a display poster showing the problems that can be solved using equations.



Example 14

Animated example

Linear equations

MAT09NAAE00008

Puzzle sheet

Writing and solving
equations

MAT09NAPS00034

Mamoud is five times older than his sister Natasha. In twelve years time, he will be 6 years younger than double Natasha's age. Find their ages.



Solution

Choose the smaller number as the variable.

Write Mamoud's present age in terms of n .

Write their ages in 12 years time.

Write the other information in terms of n .

Write an equation.

Multiply out the brackets.

Isolate the variables.

Divide by 3.

Work out the other ages.

Check with the problem's information.

Write the answer.

Let Natasha's present age = n

Mamoud's present age = $5n$

Natasha's age in 12 years = $n + 12$

Mamoud's age in 12 years = $5n + 12$

Mamoud's age in 12 years =
 $2(n + 12) - 6$

$2(n + 12) - 6 = 5n + 12$

$2n + 24 - 6 = 5n + 12$

$3n = 6$

$n = 2$

Mamoud's present age = $5 \times 2 = 10$

Natasha's age in 12 years = 14

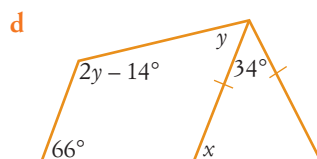
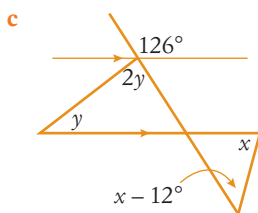
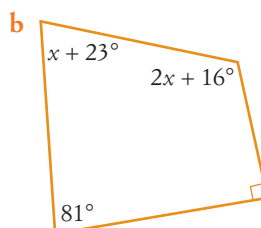
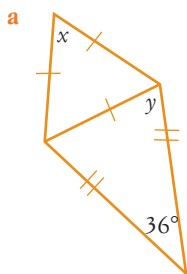
Mamoud's age in 12 years = 22

$2 \times 14 - 6 = 28 - 6 = 22$ ✓ OK

Natasha is 2 and Mamoud is 10.

Exercise 8.2 Applying linear equations

- 1 Find the unknowns in the following diagrams.



- 2 A rectangle has a perimeter of 68 cm and its length is 5 cm more than its width. What are its length and width?
- 3 A cylinder of volume 600 mL has a radius of 5 cm. What is its height?
- 4 The label for a can of fruit was found to have an area of 240 cm^2 . The diameter of the can is 8 cm. What is the height and volume of the can?
- 5 Some incomplete packs of cards were used for building card houses. The probability of getting a king from the cards was 0.025. When all the cards were used, the tower was counted and there were 160 cards altogether. How many were kings?

Problem solving

Extra questions

Exercise 8.2

MAT09NAEQ00028

See Example 10

See Example 11

Worked solutions

Exercise 8.2

MAT09NAWS00028



- 6 6 people had an average mass of 78 kg. When another person joined the group, the average mass went up to 83 kg. What was the mass of the new person?

See Example 12

- 7 Suzanna and Karli earned \$150 for picking strawberries. Suzanna actually worked for 6 hours and Karli worked for 4 hours. How much should they each get?

See Example 13

- 8 A sofa sold for \$480 at a mark-up of 60%. How much was the markup?

Worked solutions

Exercise 8.2

MAT09NAWS00028

- 9 David worked on a commission of 8% with a retainer of \$200 a week. He made \$720 in one week. What was the value of his sales?

- 10 After getting a discount of 20%, Alfred said that he saved \$34 on buying a new suit. What did he pay?

See Example 14

- 11 Tom is 4 times as old as his son. In another 4 years he will only be 3 times older than his son. How old are they now?

Worked solutions

Exercise 8.2

MAT09NAWS00028

- 12 Andrew is 3 years younger than his wife. 40 years ago, he would have been half her age. How old are they?

Reasoning

- 13 Work out a formula for finding the retainer that someone gets when they work on a commission plus retainer. Assume that you know the earnings for a week, the commission percentage and the sales.

- 14 Instead of being given a discount and marked price, work out a formula for finding the discount from the final price and the discount percentage.

8.3 Linear inequalities

There are many situations where you will want to know about a situation where, instead of having an equation you will have an **inequality**. You will need to solve the inequality. A typical situation where this occurs is in business modelling.

Important!

Linear inequalities

A **linear** inequality is like a linear equation, but instead of having an equals sign, it has one of the signs $>$, $<$, $\geq$ or $\leq$.

You can use methods similar to those for equations to solve linear inequalities. However, you need to be particularly careful when multiplying or dividing inequalities by constants.

Important!

Properties of inequalities

For any inequality $x > y$

- 1 $x + a > y + a$ for any number a
- 2 $x - a > y - a$ for any number a
- 3 $az > ay$ for any *positive* number a
- 4 $az < ay$ for any *negative* number a .

The same laws hold for $\geq$, $<$ and $\leq$.

Example 15

Solve $x + 8 \geq 3$.

Solution

Write the inequality.

$$x + 8 \geq 3$$

Use inequality rule 2.

$$x + 8 - 8 \geq 3 - 8$$

Write the solution.

$$x \geq -5$$

CAS TI-Nspire exercise

Equations

MAT09NATI00008

CAS ClassPad exercise

Equations

MAT09NACP00008

Example 16

Solve $m - 6 < 13$.

Solution

Write the inequality.

$$m - 6 < 13$$

Use inequality rule 1.

$$m - 6 + 6 < 13 + 6$$

Write the solution.

$$m < 19$$



Example 17

Solve $3g \geq -15$

Solution

Write the inequality. $3g \geq -15$

Use inequality rule 3. $\frac{3g}{3} \geq \frac{-15}{3}$

Write the solution. $g \geq -5$

Example 18

Solve $-4k \leq 12$

Solution

Write the inequality. $-4k \leq 12$

Use inequality rule 4. $\frac{-4k}{-4} \geq \frac{12}{-4}$

Write the solution. $k \geq -3$

Puzzle sheet

Inequations

MAT09NAPS00037

Important!

Symbols for mathematical proofs

When you prove things, you use some special abbreviations and symbols.

RTP means 'required to prove': It is what you have to prove. It is placed at the beginning of the **proof** with a statement of what you have to prove.

QED stands for *quod erat demonstrandum*, Latin for 'that which was required'. It is placed at the end of the proof to show that it is complete.

$\therefore$ is the mathematical symbol that means 'therefore'.

$\Rightarrow$ is the mathematical symbol that means 'it follows that', or 'then'.

The use of some of these symbols is shown in the following example.

Example 19

Prove that if $x \geq y$, then for $a < 0$, $ax \leq ay$.

Solution

Write the requirement.

Write the inequality.

Write in terms of a number property.

Use the product of positive and negative numbers.

Multiply out the brackets.

Change to an inequality.

RTP: If $x \geq y$, $a < 0 \Rightarrow ax \leq ay$.

$x \geq y$

$\Rightarrow x - y$ is positive or zero

$\Rightarrow a(x - y)$ is negative or zero

$\Rightarrow ax - ay$ is negative or zero

$\Rightarrow ax \leq ay$ **QED**

Exercise 8.3 Linear inequalities

1 Solve each of the following inequalities.

a $m + 3 < 9$

b $r + 7 \geq 16$

c $q + 8 < 11$

d $r + 1 \leq 5$

e $w + 8 \geq 16$

f $c + 8 < 9$

g $z + 7 \leq 16$

h $v + 2 \geq 11$

i $m + 7 \leq 12$

j $r + 3 < 6$

2 Solve each of the following inequalities.

a $y - 3 < 2$

b $p - 5 \leq 6$

c $m - 1 \geq 6$

d $r - 9 \leq 6$

e $q - 4 < 9$

f $q - 7 < 6$

g $v - 7 \geq 8$

h $r - 9 > 9$

i $n - 2 > 7$

j $t - 3 \geq 1$

3 Solve each of the following inequalities.

a $5d < 30$

b $2g \geq 18$

c $6q < 24$

d $4p \leq 48$

e $4z \geq -36$

f $7q < 70$

g $9z \leq -27$

h $3m \geq 36$

i $4m \leq -16$

j $7z < 84$

4 Solve each of the following inequalities.

a $-8a > -80$

b $-6x \geq -72$

c $-5x \leq -20$

d $-6x \geq 18$

e $-6e > -36$

f $-8g > -8$

g $-6y \leq 18$

h $-4c < 16$

i $-9b < -18$

j $-3w \geq 24$

Understanding

Extra questions

Exercise 8.3

MAT09NAEQ00029

See Example 15

See Example 16

See Example 17

See Example 18

Fluency

5 Solve each of the following inequalities.

a $-4k - 6 > -10$

b $-8w - 3 > -91$

c $8z - 1 \geq 39$

d $-3t + 1 \geq -5$

e $-4c - 5 > 7$

f $6z + 9 \geq 45$

g $2c - 6 \geq 18$

h $4y + 4 \leq 12$

i $-7w + 1 > 71$

j $-6m - 5 \geq 1$

Reasoning

6 Prove that if $x \geq y$, then for $a > 0$, $ax \geq ay$.7 Prove that if $x \geq y$, then, $x + a \geq y + a$.8 Prove that if $x < y$, then for $a > 0$, $ax < ay$.9 Prove that if $x < y$, then for $a < 0$, $ax > ay$.10 Prove that if $x \geq y$, then, $x - a \geq y - a$.

See Example 19

Worked solutions

Exercise 8.3

MAT09NAWS00029

- An **equation** is a mathematical sentence with an equals sign and expressions on either side. The simplest algebraic equations are equations with *one variable*.
- A **linear equation** produces a straight line when plotted on graph paper for different values of the variable. Linear equations do not have any powers, roots or reciprocals of the variable.
- A **solution** of an algebraic equation is a value of the variable that makes the equation true. The process of finding the solution to an equation is called **solving** an equation.
- To solve an equation **by inspection** you try different numbers until you find one that satisfies the equation. This is also called **guess and check** or **trial and error**.
- To solve an equation by **graphing**, you draw a graph of the expression on the left-hand side to find the value of the variable that satisfies the equation.
- To solve an equation using **backtracking** you set up a **flow chart** showing what has been done to the variable on the left to give the value on the right of the equation. You then work backwards from the right-hand side, reversing each operation as you go.
- To **formally solve equations**, use **inverse operations** in the reverse order to the normal order of operations to get the variable on its own on the LHS with only an arithmetic expression on the RHS. Whatever you do to one side must be done to the other.
- A **linear inequality** is like a linear equation, but instead of having an equals sign, it has one of the signs $>$, $<$, $\geq$ or $\leq$.
- When a constant is added or subtracted from both sides of an inequality, the inequality is unchanged.
- When an inequality is multiplied by a positive constant, the inequality is unchanged.
- When an inequality is multiplied by a negative constant, the inequality is reversed.

Quiz

Equations

MAT09NAQZ00008

Worksheet

Equations review

MAT09NAWK00054

Chapter 8 review

Understanding

See Example 1

- 1 State whether or not each of the following is a linear equation in one variable.

a $\frac{1}{x} + 3x - 6 = 15$

b $3(m - 5) - 2 = 4(3m - 5)(m + 2)$

c $\frac{g}{4} - 3(g - 7) = 8$

See Examples 15–18

- 2 Solve each of the following inequalities.

a $h + 25 \geq 12$

b $p - 4 < 15$

c $3k \leq 24$

d $-5h > 20$

Fluency

See Example 2

- 3 Solve each of the following equations by inspection.

a $7x = 42$

b $v - 15 = 8$

c $\frac{m}{7} = 4$

d $y + 8 = 2$

See Example 3

- 4 Solve $2h - 5 = 3$ by graphing.

See Example 4

- 5 Solve $\frac{3p - 4}{2} - 3 = 4$ by drawing a flow chart and backtracking.

See Example 5

- 6 Formally solve each of the following equations.

a $3p + 8 = 26$

b $\frac{k}{3} - 2 = 5$

c $\frac{m - 5}{3} + 4 = 11$

d $3(x - 8) + 2 = 8$

- 7 Formally solve each of the following equations.

a $3m + 14 = 11$

b $2q - 6 = 5$

See Examples 6, 7

- 8 Formally solve each of the following equations.

a $4t + 5 = 2t + 17$

b $5p + 8 = 2p - 13$

c $4(3d - 5) = 2(d + 8)$

d $4(e - 7) - 4 = 2(3e - 5)$

- 9 Formally solve each of the following equations.

a $4g - 8 = 9g + 21$

b $7w + 9 = 3w + 8$

See Example 8

- 10 Formally solve each of the following equations.

a $\frac{m - 7}{3} + 7 = \frac{2m + 6}{4} + 3$

b $\frac{c - 4}{3} + \frac{c}{5} = \frac{4c + 2}{3} + 9$

- 11 Solve each of the following inequalities.

a $6x - 8 < -32$

b $7 - 3m \geq 64$

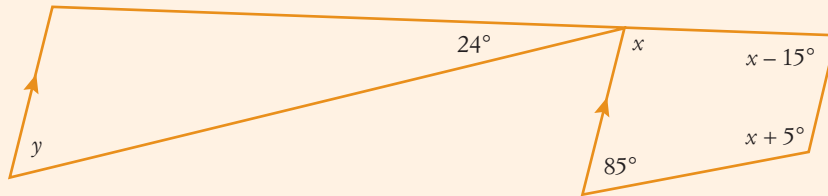
Problem solving

See Example 9

- 12 Half the sum of a number and 11 is the same as 8 less than double the number. Write an equation and solve it to find the number.

- 13 Find the unknowns in the diagram below.

See Example 10



- 14 The volume of a triangular prism is 28 cm^3 and the height of the prism is 8 cm. One of the shorter sides in the right-angled triangle that forms the base is 4 cm long. How long is the other side?
- 15 Andy, Barry and Carla have \$50 between them. Barry has twice as much as Carla and Andrew has \$10 less than Carla. How much does Carla have?
- 16 A used-car dealership made a profit of 25% when it sold a car for \$7250. How much profit did it make?
- 17 Graham is 12 years older than Patrick. In 4 years time he will be three times as old as Patrick. How old are they now?

See Example 11

See Example 12

See Example 13

See Example 14

- 18 Check the following solutions. If there is an error, identify the line on which it first occurs and state the correct answer.

Reasoning

a $2f + 8 = 5f - 4$

$3f - 4 = 8$ (A)

$3f = -12$ (B)

$f = -4$ (C)

b

$$\frac{3x - 2}{4} - 2x + 5 = \frac{x}{3} + 1$$

$12 \times \left(\frac{3x - 2}{4} \right) - 12 \times 2x + 12 \times 5 = 12 \times \frac{x}{3} + 12 \times 1$ (A)

$9x - 8 - 24x + 60 = 4x + 12$ (B)

$-11x - 6 = 4x + 12$ (C)

$-15x = 18$ (D)

$x = -1\frac{1}{5}$ (E)

- 19 Work out a formula for the height of a closed cylinder of radius r if you are given its total surface area A .
- 20 Prove that if $x \geq y$, then, $x - a \geq y - a$.

See Example 19



Measurement and geometry

9

Trigonometry



Contents

- 9.1 Pythagoras' theorem
- 9.2 Trigonometric ratios
- 9.3 Applications of triangles
- Chapter summary
- Chapter review

Prior learning

Chapter 9

MAT09MGPL00009

Parent guide

Chapter 9

MAT09MGPG00009

Curriculum guide

Chapter 9

MAT09MGCU00009

Australian Curriculum statements

Pythagoras and trigonometry

Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles.

Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles.

Apply trigonometry to solve right-angled triangle problems.

Video tutorial

Trigonometry

MAT09MGVT00009

Weblink

Math is fun!

MAT09MGWB00009

Trigonometry is the study of angles. Interest in trigonometry goes back to ancient times when people noticed that shadow lengths and directions changed as the day grew older and as seasons changed. It may seem surprising, but as early as 200 BCE a Greek, named Eratosthenes, used trigonometry to accurately estimate the radius of the Earth. Nowadays, trigonometry is used in a vast number of fields of study from architecture to seismology.

Pythagoras was a Greek philosopher and mathematician who lived about 2500 years ago. Builders and surveyors who lived before Pythagoras' time knew that a triangle with sides measuring 3 units, 4 units and 5 units respectively formed a right-angled triangle. Pythagoras was the first person to prove why certain combinations of three side lengths resulted in a right-angled triangle. The rule (or theorem) regarding the lengths of sides of a right-angled triangle was subsequently named after him. Since then, hundreds of proofs for the theorem have been developed.

Mathematical literacy

Maths dictionary

MAT09ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

adjacent

converse

cosine (cos)

degrees

hypotenuse

minutes

opposite

Pythagoras'

theorem

Pythagorean triple

Pythagorean triad

right-angled

seconds

sine (sin)

square root

surd

tangent (tan)

theorem

trigonometric ratio

trigonometry

9.1 Pythagoras' theorem

TLF Learning object

Exploring the
Pythagorean theorem
(L6559)

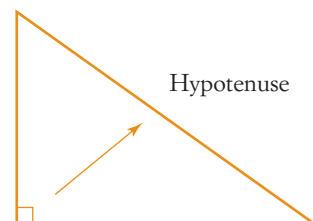
MAT09MGIN00009

Puzzle sheet

Trigonometry

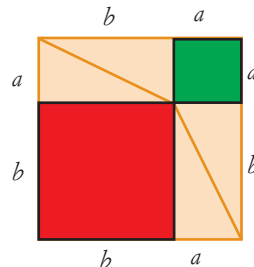
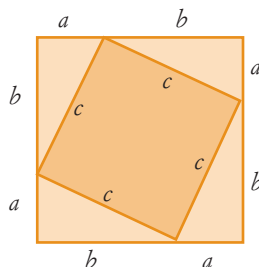
MAT09MGPS00038

A right-angled triangle has one angle measuring 90° (a right angle) and two smaller angles. The side opposite the right angle is the longest side and is called the **hypotenuse**.



Investigate: Pythagoras' theorem

Look at the two squares drawn below. The areas of the large squares are both $(a + b)^2$.



Each square has been divided into different-sized squares and right-angled triangles. Let's calculate the area of the square on the left by adding the parts together.

Area of left-hand square = area of dark orange square + area of 4 light orange triangles

$$\begin{aligned} &= c \times c + 4 \times \left(\frac{1}{2} b \times a \right) \\ &= c^2 + 2 \times ba \\ &= c^2 + 2ba \end{aligned}$$

Now let's calculate the area of the square on the right from its parts.

Area of right-hand square = area of red square + area of green square
+ area of 4 light orange triangles

$$\begin{aligned} &= b \times b + a \times a + 4 \times \left(\frac{1}{2} b \times a \right) \\ &= b^2 + a^2 + 2ba \end{aligned}$$

The areas of the two squares are equal.

Area of left-hand square = Area of right-hand square

$$\begin{aligned} c^2 + 2ba &= b^2 + a^2 + 2ba \\ c^2 &= b^2 + a^2 \end{aligned}$$

This result is known as **Pythagoras' theorem**.

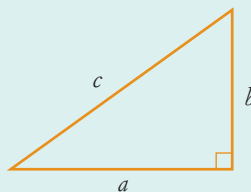
In mathematics, a **theorem** is a statement that can be shown to be true by mathematical operations.

Important!

Pythagoras' theorem

For any **right-angled** triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$c^2 = a^2 + b^2$$



Technology

Geogebra: Pythagoras' theorem

MAT09MGTC00007

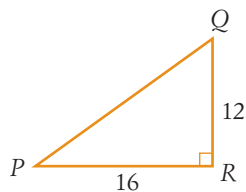
When you perform calculations involving Pythagoras' theorem you will need to find the squares and square roots of numbers. The **square root** ($\sqrt{\quad}$) of a number is the positive value which, if squared will give that number.

For example, the square root of 49 ($\sqrt{49}$) is 7 because $7^2 = 49$.

Example 1

For the triangle shown here:

- a write Pythagoras' theorem
- b calculate the length of PQ .



Solution

- a Identify the sides of the right-angled triangle.

PQ is the hypotenuse and the other sides are QR and RP .

Use the sides to complete Pythagoras' theorem.

$$PQ^2 = QR^2 + RP^2$$

- b Substitute the values for QR and RP .

$$PQ^2 = 12^2 + 16^2$$

Evaluate.

$$= 144 + 256$$

$$PQ^2 = 400$$

Take the square root of both sides of the equation.

$$PQ = 20$$

You can also calculate a side other than the hypotenuse using Pythagoras' theorem.

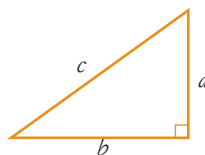
Technology Calculating the hypotenuse

Technology

We are going to use a spreadsheet to calculate the length of the hypotenuse (c) of a right-angled triangle using the lengths of the other two sides (a and b).

Step 1: For this triangle we know that:

$$c^2 = a^2 + b^2$$



For a spreadsheet, this can be shown as:

$$c^2 = a^2 + b^2$$

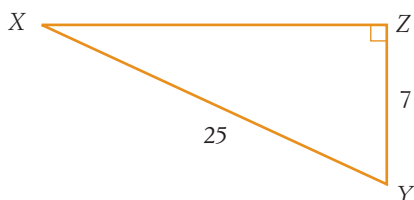
Step 2: Set up a spreadsheet like the one shown below by entering the headings shown into cells A1, B1, C1 and D1 and entering the numbers shown into columns A and B.

	A	B	C	D	E
1	Calculating the hypotenuse				
2	a	b	$a^2 + b^2$	c	
3	3	4			
4	4	7.5			
5	5	12			
6	6	8			
7	7	24			
8	8	15			
9	9	12			
10	10	24			
11	11	60			
12	12	16			
13	13	84			
14					

- Step 3: Type the formula $=A2^2+B2^2$ into cell C2. Copy this formula into cells C3 to C13.
- Step 4: To calculate each hypotenuse, we need to find the square root of the values in column C. In cell D2, enter $=SQRT(C2)$. Then copy this formula into cells D3 to D13.
- Step 5: Extend the spreadsheet by entering two new side lengths into A14 and B14, and then copying cells C13 and D13 into the cells below (C14 and D14 respectively). You can continue this process to calculate the values of hypotenuses in other right-angled triangles.

Example 2

Use Pythagoras' theorem to calculate the length of XZ.



Solution

Identify the sides of the right-angled triangle.

XY is the hypotenuse and the other sides are XZ and ZY .

Use the sides to complete Pythagoras' theorem.

$$XY^2 = XZ^2 + ZY^2$$

Substitute the values for QR and RP .

$$25^2 = XZ^2 + 7^2$$

Evaluate.

$$625 = XZ^2 + 49$$

Subtract 49 from both sides.

$$576 = XZ^2$$

Reverse the equation and take the square root of both sides.

$$XZ^2 = 576$$

$$XZ = 24$$

When you are finding the *hypotenuse*, you *add* the squares of the other two sides. When you are finding a *short side*, you *subtract* the square of the other short side from the square of the hypotenuse. In both cases, you then find the square root.

Not all square roots have exact values. For example:

$$\sqrt{15} = 3.872\,983\,346 \dots$$

$$\approx 3.9 \text{ (correct to 1 decimal place)}$$

Numbers such as $\sqrt{15}$ are called **surd**s. They are irrational numbers that are the roots of rational numbers, particularly positive integers.

Important!

Surds

A surd is an irrational number that can only be expressed exactly using the square root sign ($\sqrt{}$). A surd has an infinite number of non-recurring decimals.

Puzzle sheet

Pythagoras 1

MAT09MGPS00039

Puzzle sheet

Pythagoras 2

MAT09MGPS00040

Example 3

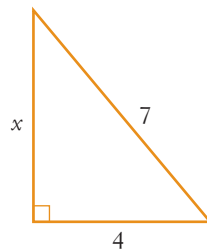
Video tutorial

Pythagoras' theorem

MAT09MGVT10001

Calculate the length of the unknown side in this triangle.

Give your answer both as an exact value and as an approximate value correct to 1 decimal place.



Solution

Use Pythagoras' theorem.

Evaluate.

Subtract 16 from both sides.

Reverse the equation and take the square root of both sides.

Use your calculator to find an approximate value.

Round the result to 1 decimal place.

State the result.

$$7^2 = x^2 + 4^2$$

$$49 = x^2 + 16$$

$$33 = x^2$$

$$x = \sqrt{33}$$

$$\approx 5.744\ 562\ 246$$

$$\approx 5.7$$

The unknown side measures $\sqrt{33}$ or 5.7 (correct to 1 decimal place).

We know that in any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. The reverse of Pythagoras' theorem is known as the **converse**.

Important!

Converse of Pythagoras' theorem

If the square of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle must be right-angled.

We can use the converse of Pythagoras' theorem to check to see if a triangle is right-angled.

Example 4

Video tutorial

Testing for right-angled triangles

MAT09MGVT10002

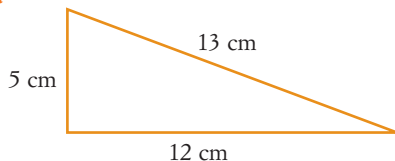
Worksheet

Pythagorean triads

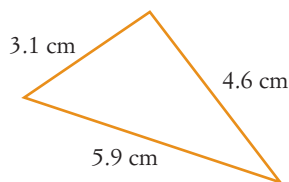
MAT09MGWK00056

Test whether each of the triangles below is right-angled.

a



b



Solution

- a** Calculate the square of the longest side.

$$13^2 = 169$$

Calculate the squares of the other two sides and add them.

$$5^2 + 12^2 = 25 + 144 = 169$$

Apply the converse of Pythagoras' theorem.

The triangle is right-angled because the square of the longest side is equal to the sum of the squares of the other two sides.

- b** Calculate the square of the longest side.

$$5.9^2 = 34.81$$

Calculate the squares of the other two sides and add them.

$$3.1^2 + 4.6^2 = 9.61 + 21.16 = 30.77$$

Apply the converse of Pythagoras' theorem.

The triangle is not right-angled because the square of the longest side does not equal the sum of the squares of the other two sides.

There are special groups of three numbers that obey Pythagoras' theorem. Such a group of three whole numbers is called a **Pythagorean triple** or **triad**. The first Pythagorean triple is (3, 4, 5). You can see that (3, 4, 5) is a Pythagorean triple because $3^2 + 4^2 = 5^2$, i.e. $9 + 16 = 25$.

There is an infinite number of Pythagorean triples. Here are some more whole number ones:

(5, 12, 13)

(7, 24, 25)

(9, 40, 41)

Check them to confirm that they also obey Pythagoras' theorem.

There is a formula that can be used to develop Pythagorean triples. If the first number in a triple is x , then the other two numbers will be $\frac{1}{2}(x^2 - 1)$ and $\frac{1}{2}(x^2 + 1)$.

An interesting fact is that any whole number multiple of a Pythagorean triple will also be a triple.

Check the following multiples of the (3, 4, 5) triple to confirm that they are also triples.

(6, 8, 10)

(9, 12, 15)

(15, 20, 25)

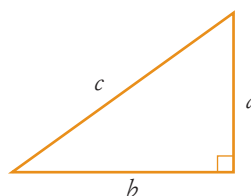
(18, 24, 30)

Technology Pythagorean triples

We are going to use a spreadsheet to generate some Pythagorean triples.

Step 1: Set up a spreadsheet like the one shown below.

	A	B	C
1	Pythagorean triples		
2	a	b	c
3	3		
4	4		
5	5		
6	6		
7	7		
8	8		
9	9		
10	10		



Technology

Excel worksheet:
Pythagorean triples

MAT09MGCT00025

Step 2: In cell B3, enter a formula to calculate length b by typing '=0.5*(A3^2 - 1)'. Copy this formula into cells B4 through to B10.

Step 3: In cell C3, enter a formula to calculate length c by typing '=0.5*(A3^2 + 1)'. Copy this formula into cells C4 through C10.

Step 4: Extend the spreadsheet to identify the first 10 Pythagorean triples.

Exercise 9.1 Pythagoras' theorem

Understanding

Extra questions

Exercise 9.1

MAT09MGEQ00030

- 1 Use your calculator to find the value of each of the following.

a 6^2

b 14^2

c 22^2

d 5.2^2

e 3.8^2

f 7.09^2

g $8^2 + 11^2$

h $7.6^2 - 4.3^2$

i $9.16^2 - 4.7^2$

- 2 Use your calculator to find the values of:

a $\sqrt{289}$

b $\sqrt{196}$

c $\sqrt{441}$

d $\sqrt{324}$

e $\sqrt{576}$

f $\sqrt{1369}$

- 3 Calculate the square root of each of these numbers, correct to two decimal places.

a 14

b 156

c 327

d 401

e 222

f 1097

g 1346

h 2029

- 4 Find the value of each of the following, correct to one decimal place.

a $\sqrt{8^2 - 5^2}$

b $\sqrt{6^2 + 11^2}$

c $\sqrt{13^2 - 8^2}$

d $\sqrt{9.6^2 + 4.3^2}$

e $\sqrt{11.6^2 - 7.5^2}$

f $\sqrt{17.2^2 + 8.3^2}$

- 5 Select the surds from the following square roots.

a $\sqrt{366}$

b $\sqrt{144}$

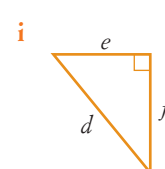
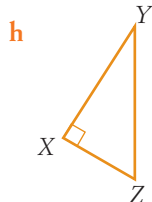
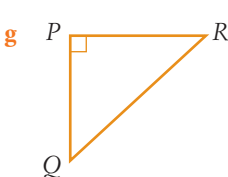
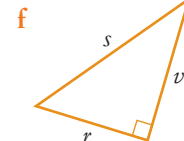
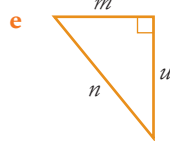
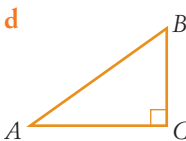
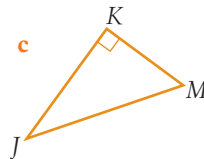
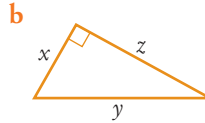
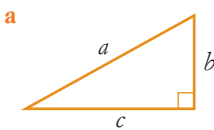
c $\sqrt{677}$

d $\sqrt{731}$

e $\sqrt{841}$

f $\sqrt{488}$

- 6 Name the hypotenuse in each of these triangles.



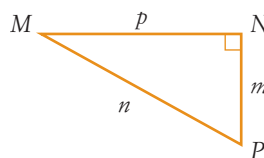
- 7 Which of the following is Pythagoras' theorem for this triangle? Select **A**, **B**, **C** or **D**.

A $MN^2 = MP^2 + NP^2$

B $m^2 = p^2 + n^2$

C $NP^2 = MN^2 + MP^2$

D $n^2 = p^2 + m^2$



See Example 1

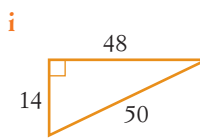
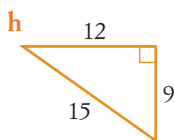
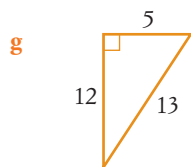
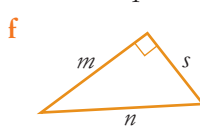
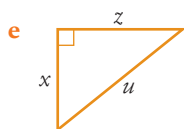
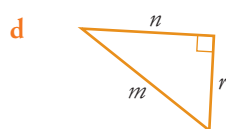
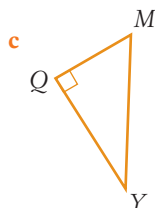
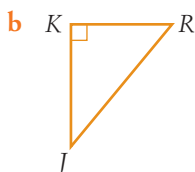
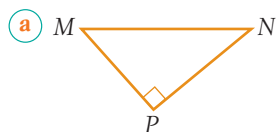
- 8 Write Pythagoras' theorem for each of the following triangles.

Worked solutions

Exercise 9.1

MAT09MGWS00030

See Example 1



- 9 Which of the following is a Pythagorean triple?

a (9, 12, 15)

b (12, 22, 28)

c (25, 60, 65)

d (15, 27, 35)

e (30, 72, 78)

f (15, 36, 39)

g (21, 72, 75)

h (14, 42, 48)

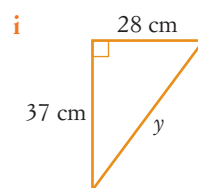
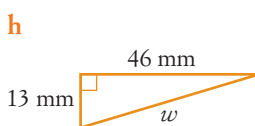
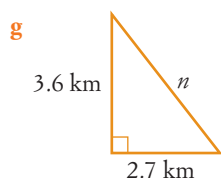
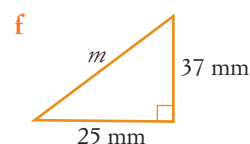
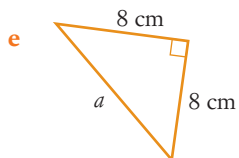
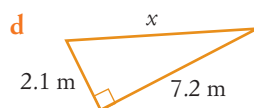
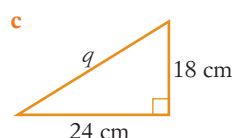
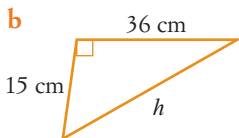
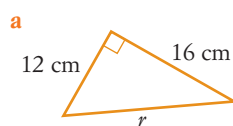
i (42, 144, 150)

For the remaining questions in this exercise, give answers correct to one decimal place where necessary.

- 10 Find the length of the hypotenuse for each of these triangles.

Fluency

See Examples 1, 3



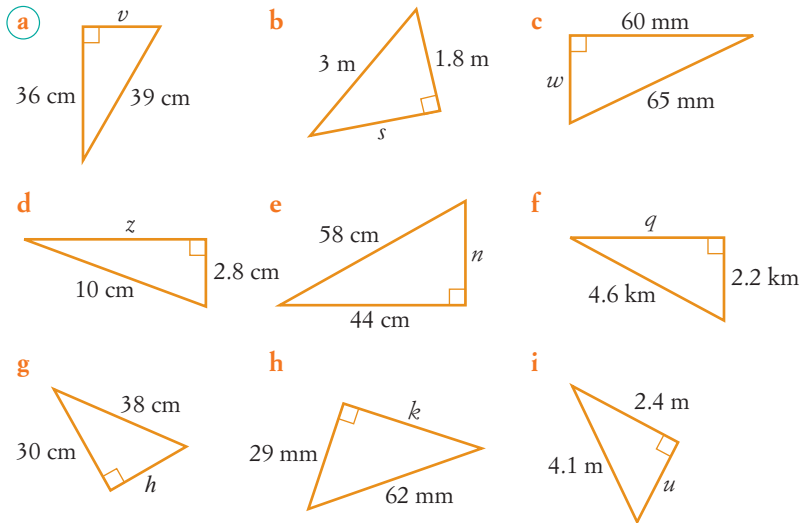
Worked solutions

Exercise 9.1

MAT09MGWS00030

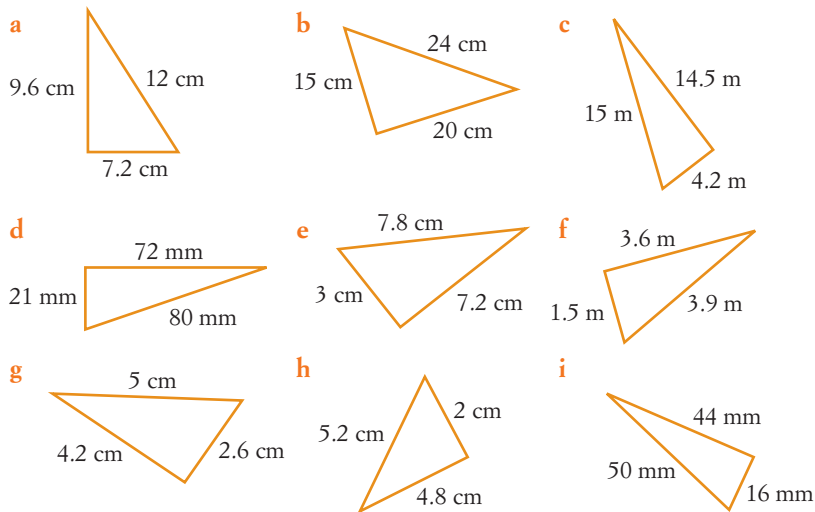
See Examples 2, 3

- 11 Find the length of the unknown side for each of these triangles.



See Example 4

- 12 Use Pythagoras' theorem to test whether each of the following triangles is right-angled.



Problem solving

Worked solutions

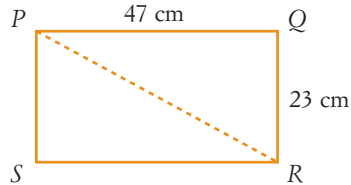
Exercise 9.1

MAT09MGWS00030

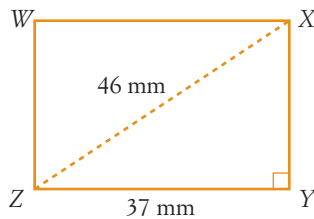
- 13 In $\triangle XYZ$, $\angle XYZ = 90^\circ$, $XY = 68$ cm, and $YZ = 106$ cm. Find XZ in centimetres, correct to one decimal place.
- 14 In $\triangle KFC$, $KF = KC = 13.4$ cm, and $\angle FKC = 90^\circ$. Find the length of CF in millimetres, correct to one decimal place.
- 15 In $\triangle RTS$, $\angle R = 90^\circ$, $RS = 141$ cm, and $RT = 8.2$ m. Find TS in centimetres, correct to the nearest centimetre.
- 16 In $\triangle CDE$, $\angle ECD = 90^\circ$, $CD = 44$ mm, and $DE = 6.8$ cm. Calculate EC correct to the nearest millimetre.
- 17 In $\triangle UVW$, $\angle U = 90^\circ$, $VW = 1.8$ m, and $UW = 112$ cm. Calculate UV correct to the nearest centimetre.
- 18 In $\triangle MNO$, $\angle N = 90^\circ$, $MO = 2.48$ m, and $NO = 188$ cm. Find NM in centimetres, correct to one decimal place.

19 Chang was asked to construct a right-angled triangle. The dimensions of his triangle were 15 cm, 21 cm and 26 cm. Without seeing the triangle that Chang drew, determine if it could be right-angled.

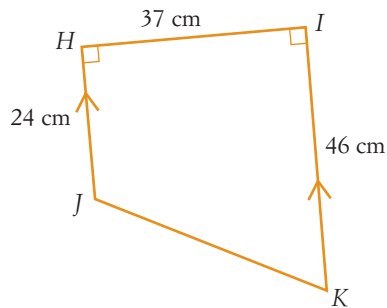
20 How long is the diagonal PR in this rectangle? Give your answer correct to 1 decimal place.



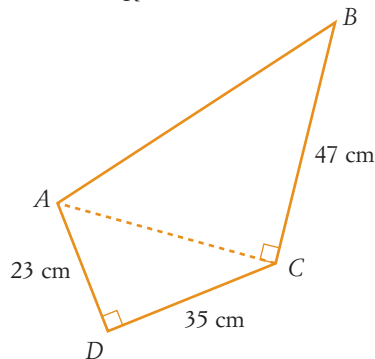
21 Find the area of this rectangle (correct to one decimal place).



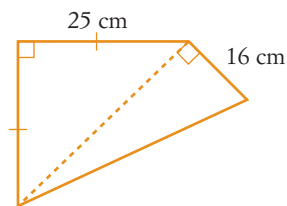
22 Find the length of JK in this trapezium, correct to one decimal place.



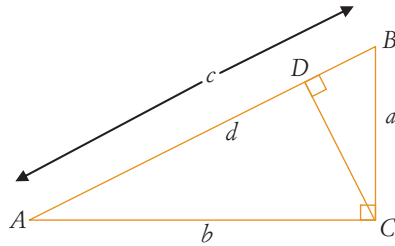
23 Find the length of AB in this quadrilateral.



24 Find the perimeter of the shape below correct to one decimal place.

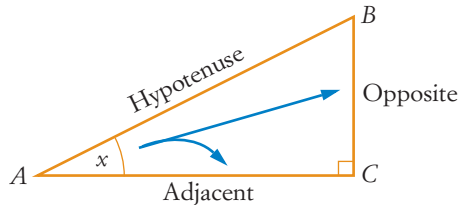


- 25 Show that the three triangles $\triangle ABC$, $\triangle CBD$ and $\triangle ACD$ in the diagram below are similar. Write a statement about the ratio of the sides AC and AD in $\triangle ACD$ and sides in $\triangle ABC$. Write a statement about the ratio of the sides BC and BD in $\triangle CBD$. Use these statements to prove Pythagoras' theorem.



9.2 Trigonometric ratios

The study of **trigonometry** concerns right-angled triangles. You have already seen that in a right-angled triangle, the **hypotenuse** is the longest side. It is *opposite* the right angle.



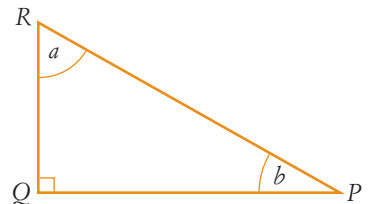
So in $\triangle ABC$ above, AB is the hypotenuse. When using **trigonometric (trig) ratios**, we need to mark in an angle so that we can refer to it. Angle x is indicated in this triangle.

For angle x in this right-angled triangle:

- BC is the side **opposite** x
- AC is the side *next to* x so we say that AC is **adjacent** to x

Example 5

For the angles a and b indicated, name the hypotenuse, the opposite side and the adjacent side.



Solution

Locate the longest side.

Look at a .

Look at b .

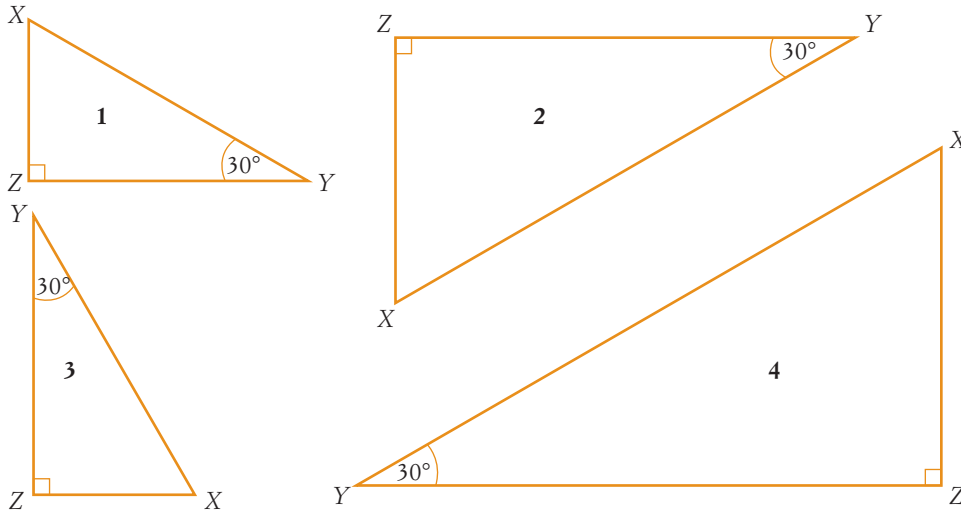
For both angles, PR is the hypotenuse.

For a , the adjacent side is QR and the opposite side is PQ .

For b , the adjacent side is PQ and the opposite side is QR .

Investigate: Ratios of sides of similar right-angled triangles

Below are four different sized right-angled triangles numbered 1 through 4.



- 1 How do you know that the triangles are similar?
- 2 Copy the table below into your workbook.

Triangle	Side length (mm)			Ratios of sides with respect to 30°		
	XZ	ZY	XY	$\frac{\text{opposite}}{\text{hypotenuse}} = \frac{XZ}{XY}$	$\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{ZY}{XY}$	$\frac{\text{opposite}}{\text{adjacent}} = \frac{XZ}{ZY}$
1						
2						
3						
4						

- 3 For each triangle, measure the sides XZ, ZY and XY to the nearest millimetre and record the measurements in the table in your workbook.
- 4 Now use your measurements and a calculator to find the ratios shown in the table.
- 5 What do you notice about the values of each ratio $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$ and $\frac{\text{opposite}}{\text{adjacent}}$ for the different triangles?
- 6 Compare your results with others in the class.
- 7 Now draw a large triangle with angles 30°, 60° and 90°. You could do this on the blackboard, white board or even on a piece of butcher's paper or a piece of A3 paper.
- 8 Measure the sides of the triangle and calculate the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$, $\frac{\text{adjacent}}{\text{hypotenuse}}$ and $\frac{\text{opposite}}{\text{adjacent}}$ for the 30° angle.
- 9 Compare the values of the ratios with the ones you calculated earlier and also with the values calculated by others in your class.

Technology

Geogebra:
Trigonometric ratios

MAT09MGTC00006

Trigonometric ratios

In the previous investigation you calculated the ratios of the sides for a number of right-angled triangles. The ratios of any two sides of a right-angled triangle are known as **trigonometric** (or trig) **ratios**. The three most commonly used trigonometric ratios are:

- the **sine** ratio – abbreviated to **sin** but pronounced as *sign*
- the **cosine** ratio – abbreviated to **cos** but pronounced as *cosign*
- the **tangent** ratio – abbreviated to **tan**.

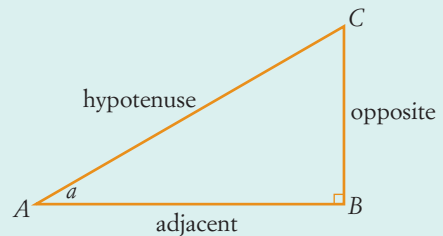
Important!**Trigonometric ratios**

For the right-angled $\triangle ABC$:

$$\sin a = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\cos a = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\tan a = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AB}$$



Some students find that the mnemonic **SOH CAH TOA** (pronounced ‘so-car-towa’) is useful for remembering the three ratios.

- SOH $\rightarrow$ Sin = Opposite/Hypotenuse
- CAH $\rightarrow$ Cos = Adjacent/Hypotenuse
- TOA $\rightarrow$ Tan = Opposite/Adjacent

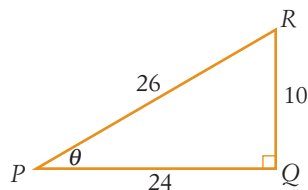
The reason that trigonometric ratios are useful is that no matter how big or small the triangle is, the value of each ratio remains the same.



In both of the situations above, the values of $\sin \theta$, $\cos \theta$ and $\tan \theta$ are the same.

Example 6

Calculate the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$ in $\triangle PQR$.



Solution

Identify the sides that relate to θ .

Hypotenuse = $PR = 26$

Opposite side = $RQ = 10$

Adjacent side = $PQ = 24$

Use the rule for $\sin \theta$.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

Substitute for known values.

$$= \frac{10}{26}$$

Simplify.

$$= \frac{5}{13}$$

Use the rule for $\cos \theta$.

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Substitute for known values.

$$= \frac{24}{26}$$

Simplify.

$$= \frac{12}{13}$$

Use the rule for $\tan \theta$.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Substitute for known values.

$$= \frac{10}{24}$$

$$= \frac{5}{12}$$

Video tutorial

The trigonometric ratios

MAT09MGVT10007

In geometry, angles are measured in **degrees**, **minutes** and **seconds**.

Important!

Degrees, minutes and seconds

1 degree = 60 minutes ($1^\circ = 60'$)

1 minute = 60 seconds ($1' = 60''$)

You can use the $\circ$ ' ' or DMS key on a scientific calculator when working with angles.

Example 7

Convert:

a $27^{\circ}36'$ to a decimal

Solution

a Enter as $27 \text{ } \circ' \text{ } 36 \text{ } \circ' \text{ } =$.

Press $\text{ } \circ' \text{ } \text{ }.$

State the result.

b Enter as $43.72 \text{ } = \text{ } \circ' \text{ } \text{ }.$

State the result.

b 43.72° to degrees, minutes and seconds

$27^{\circ}36' \quad 27^{\circ}36'0''$

$27^{\circ}36' \quad 27.6$

$27^{\circ}36' = 27.6^{\circ}$

$43.72 \quad 43^{\circ}43'12''$

$43.72^{\circ} = 43^{\circ}43'12''$

You can use a scientific calculator to find the values of trigonometric ratios.

Example 8

Calculate, correct to four decimal places:

a $\cos 62^{\circ}17'$

Solution

a Enter as $\cos 62 \text{ } \circ' \text{ } 17 \text{ } \circ' \text{ } =$.

State the result.

b Enter as $\sin 19.8 \text{ } =$.

State the result.

b $\sin 19.8^{\circ}$

$\cos(62^{\circ}17')$
0.4650995766

$\cos 62^{\circ}17' \approx 0.4651$

$\sin(19.8)$
0.3387379202

$\sin 19.8^{\circ} \approx 0.3387$

It is also possible to use a scientific calculator to find an angle if you know its trigonometric ratio. To use a calculator to find the angle from its trigonometric ratio, you must use a special key. The key may be SHIFT or 2ndF or INV or ARC . In this book we use the symbol SHIFT .

Example 9

a Find x in degrees if $\tan x = 0.893$.

b Find θ in degrees and minutes if $\sin \theta = 0.8339$.

Solution

a Enter as $\text{SHIFT} \text{ tan } 0.893 \text{ } =$.

State the result.

$\tan^{-1}(0.893)$
41.76485376

$x \approx 41.8^{\circ}$

Technology

Excel worksheet:
Trigonometry values

MAT09MGCT00026

CAS TI-Nspire exercise

Trigonometry

MAT09MGTI00009

CAS ClassPad exercise

Trigonometry

MAT09MGCP00009

b Enter as **SHIFT** **sin** 0.8339 **=**.

Press **◁ ▷**.

State the result.

$\sin^{-1}(0.8339)$
56.50147182

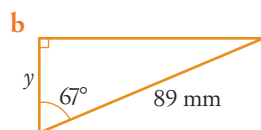
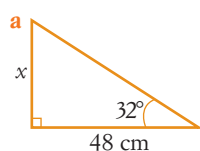
$\sin^{-1}(0.8339)$
56°30'5.3"

$\theta \approx 56^\circ 30'$

Trigonometric ratios can be used to find the sides of a right-angled triangle.

Example 10

Find the lengths of sides x and y in these two triangles.



Solution

a Look at the sides relative to the 32° angle.

Use SOH CAH **TOA**.

Write the rule for $\tan 32^\circ$.

Substitute for known sides.

Multiply both sides by 48.

Simplify.

Reverse the equation.

Enter as 48 **×** **tan** 32 **=**.

Round and write the result.

b Look at the sides relative to the 67° angle.

Use SOH **CAH** TOA.

Write the rule for $\cos 67^\circ$.

Substitute for known sides.

Multiply both sides by 89.

Simplify.

Reverse the equation.

x is **opposite** and the 48 cm side is **adjacent**.

TOA → the **tan** ratio can be used.

$$\tan 32^\circ = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 32^\circ = \frac{x}{48}$$

$$48 \times \tan 32^\circ = \frac{x}{48} \times 48$$

$$48 \times \tan 32^\circ = x$$

$$x = 48 \times \tan 32^\circ$$

48x\tan(32)
29.99372889

$$x \approx 30 \text{ cm}$$

y is **adjacent** and the 89 mm side is the **hypotenuse**.

CAH → the **cos** ratio can be used.

$$\cos 67^\circ = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 67^\circ = \frac{y}{89}$$

$$89 \times \cos 67^\circ = \frac{y}{89} \times 89$$

$$89 \times \cos 67^\circ = y$$

$$y = 89 \times \cos 67^\circ$$

CAS TI-Nspire exercise

Trigonometry

MAT09MGTI00009

CAS ClassPad exercise

Trigonometry

MAT09MGCP00009

Enter as $89 \times \cos 67 =$.

$89 \times \cos(67)$
34.77507044

Round and write the result.

$$y \approx 34.8 \text{ mm}$$

In some cases the unknown side will appear in the denominator of the trigonometric equation being solved.

Example 11

Video tutorial

Finding an unknown side

MAT09MGVT10008

Puzzle sheet

Trigonometry equations 1

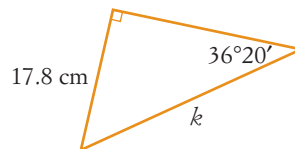
MAT09MGPS00041

Puzzle sheet

Trigonometry: Finding sides

MAT09MGPS00043

Calculate the value of k in the triangle below correct to one decimal place.



Solution

Look at the sides relative to the $36^\circ 20'$ angle.

Use SOH CAH TOA.

Write the rule for $\sin 36^\circ 20'$.

Substitute for known sides.

Multiply both sides by k .

Simplify.

Divide both sides by $\sin 36^\circ 20'$.

Simplify.

Enter as:

$17.8 \div \sin 36^\circ 20' =$

Round and write the result.

The 17.8 cm side is **opposite** and k is the **hypotenuse**.

SOH $\rightarrow$ the sin ratio can be used.

$$\sin 36^\circ 20' = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 36^\circ 20' = \frac{17.8}{k}$$

$$k \times \sin 36^\circ 20' = \frac{17.8}{k} \times k$$

$$k \times \sin 36^\circ 20' = 17.8$$

$$k \times \frac{\sin 36^\circ 20'}{\sin 36^\circ 20'} = \frac{17.8}{\sin 36^\circ 20'}$$

$$k = \frac{17.8}{\sin 36^\circ 20'}$$

$17.8 \div \sin(36^\circ 20')$
30.0431094

$$k \approx 33.0 \text{ cm}$$

In some cases, drawing a diagram will help to solve the problem.

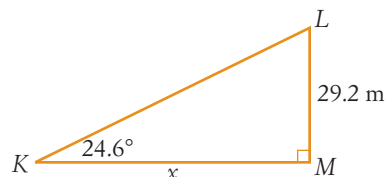
Example 12

$\triangle MLK$ is a right-angled triangle with the right angle at M . $LM = 29.2$ m long and $\angle LKM = 24.6^\circ$. Find KM correct to two decimal places.

Solution

Use the information to sketch the triangle.

Mark KM as x .



Look at the sides relative to the 24.6° angle.

Use SOH CAH TOA.

Write the rule for $\tan 24.6^\circ$.

Substitute for known sides.

Multiply both sides by x .

Simplify.

Divide both sides by $\tan 24.6^\circ$.

Simplify.

Enter as $29.2 \div \tan 24.6 =$.

Round and write the result.

Alternative method

The length of KM can also be found using $\angle KLM$.

Use the sum of the angles in a triangle rule.

Substitute for $\angle LKM$.

Subtract 114.6° from both sides.

Look at the sides relative to the 65.4° angle.

Use SOH CAH TOA.

Write the rule for $\tan 65.4^\circ$.

Substitute for known sides.

Multiply both sides by 29.2.

Simplify.

Reverse the equation.

Enter as $29.2 \times \tan 65.4 =$.

Round and write the result.

The 29.2 m side is **opposite** and x is the **adjacent**.

TOA $\rightarrow$ the $\tan$ ratio can be used.

$$\begin{aligned}\tan 24.6^\circ &= \frac{\text{opposite}}{\text{adjacent}} \\ \tan 24.6^\circ &= \frac{29.2}{x} \\ x \times \tan 24.6^\circ &= \frac{29.2}{x} \times x \\ x \times \tan 24.6^\circ &= 29.2 \\ x \times \frac{\tan 24.6^\circ}{\tan 24.6^\circ} &= \frac{29.2}{\tan 24.6^\circ} \\ x &= \frac{29.2}{\tan 24.6^\circ}\end{aligned}$$

$$\begin{array}{r} 29.2 \div \tan(24.6) \\ 63.77833154 \end{array}$$

$$x \approx 63.78 \text{ m}$$

$$\angle KLM + \angle LKM + 90^\circ = 180^\circ$$

$$\angle KLM + 24.6^\circ + 90^\circ = 180^\circ$$

$$\angle KLM + 114.6^\circ = 180^\circ$$

$$\angle KLM = 65.4^\circ$$

x is **opposite** and the 29.2 m side is the **adjacent**.

TOA $\rightarrow$ the $\tan$ ratio can be used.

$$\begin{aligned}\tan 65.4^\circ &= \frac{\text{opposite}}{\text{adjacent}} \\ \tan 65.4^\circ &= \frac{x}{29.2} \\ 29.2 \times \tan 65.4^\circ &= \frac{x}{29.2} \times 29.2 \\ 29.2 \times \tan 65.4^\circ &= x \\ x &= 29.2 \times \tan 65.4^\circ\end{aligned}$$

$$\begin{array}{r} 29.2 \times \tan(65.4) \\ 63.77833154 \end{array}$$

$$x \approx 63.78 \text{ m}$$

You can calculate the angles of a right-angled triangle if you know the lengths of at least two sides.

Example 13

Video tutorial

Finding an unknown angle

MAT09MGVT10009

TLF Learning object

Trigonometry: finding angles (L2335)

MAT09MGIN00009

Puzzle sheet

Trigonometry equations 2

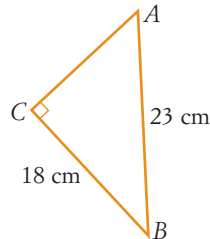
MAT09MGPS00042

Puzzle sheet

Trigonometry: Finding angles

MAT09MGPS00044

Calculate the size of angle A in the triangle below correct to one decimal place.



Solution

Look at the sides relative to angle A .

Use **SOH** CAH TOA.

Write the rule for $\sin A$.

Substitute for known sides.

Enter as **SHIFT** **sin** 18 **a^{b/c}** 23 **=**.

Round and write the result.

The 18 cm side is **opposite** and the 23 cm side is the **hypotenuse**.

SOH → the sin ratio can be used.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{18}{23}$$

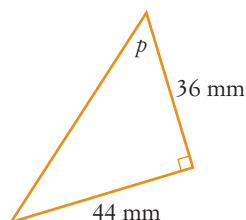
$$\sin^{-1}\left(\frac{18}{23}\right) = 51.50004959$$

$$A \approx 51.5^\circ$$

If you look above the **sin** **cos** and **tan** buttons on your calculator you will see $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ respectively. Pressing **SHIFT** **sin** on your calculator is the same as $\sin^{-1}$. The expression $\sin^{-1}0.5$ is a shorthand way of saying 'the angle whose sine is 0.5'.

Example 14

Calculate the size of angle p in the triangle below correct to the nearest minute.



Solution

Look at the sides relative to p .

Use SOH CAH TOA.

Write the rule for $\tan p$.

Substitute for known sides.

Enter as **SHIFT** **tan** 44 **a^{b/c}** 36 **=** **o[°] "**.

The 44 mm side is **opposite** and the 36 mm side is **adjacent**.

TOA → the **tan** ratio can be used.

$$\tan p = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan p = \frac{44}{36}$$

$$\tan^{-1}\left(\frac{44}{36}\right) = 50^{\circ}42'38.14''$$

Round and write the result.

$$p \approx 50^{\circ}43'$$

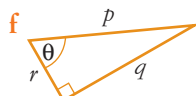
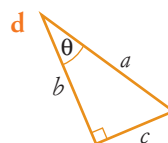
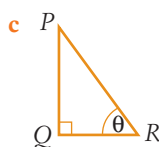
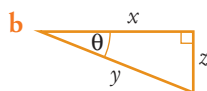
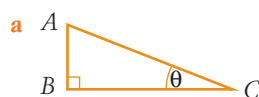
Exercise 9.2 Trigonometric ratios

1 For each of the triangles below, name:

i the hypotenuse

ii the side opposite θ

iii the side adjacent to θ



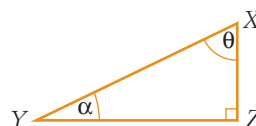
2 Name the angle in $\triangle XYZ$ on the right that is:

a opposite YZ

b adjacent to XZ

c opposite XY

d adjacent to YZ



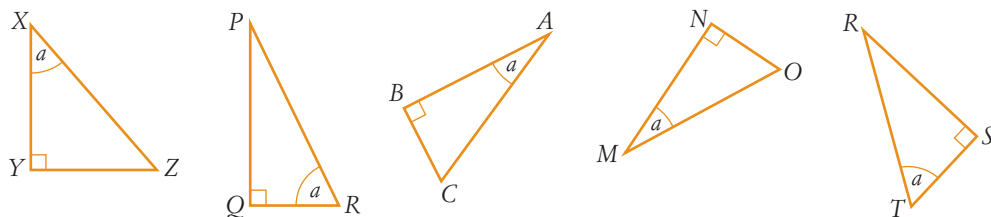
Understanding

Extra questions

Exercise 9.2

MAT09MGEQ00031

- 3 Use the following triangles below to complete the table.



	Triangle	Opposite side	Adjacent side	Hypotenuse	$\sin a$	$\cos a$	$\tan a$
a	XYZ	YZ	...	...	$\frac{YZ}{XZ}$	$\frac{YZ}{XY}$	$\frac{YZ}{XY}$
b	PQR	...	QR	...	$\frac{PQ}{PR}$	$\frac{QR}{PR}$	$\frac{PQ}{QR}$
c	ABC	...	...	AC	$\frac{BC}{AC}$	$\frac{AB}{AC}$	$\frac{BC}{AB}$
d	MNO	...	MN	...	$\frac{NO}{MO}$	$\frac{MN}{MO}$	$\frac{NO}{MN}$
e	RST	RS	...	...	$\frac{RT}{RS}$	$\frac{ST}{RS}$	$\frac{RT}{ST}$

See Examples 7, 8

- 4 Use your calculator to express the following as decimals correct to 1 decimal place.

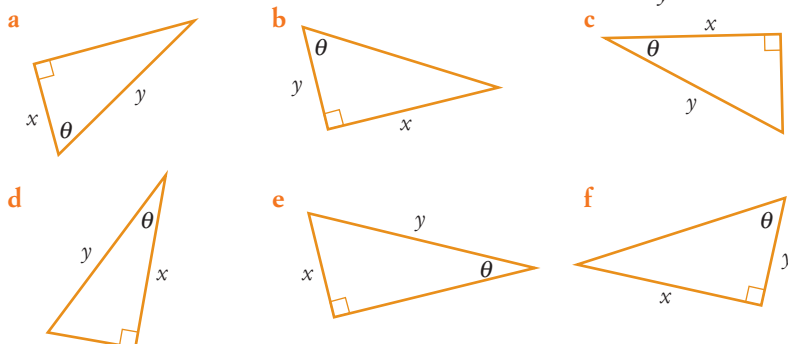
- a** $22^\circ 53'$ **b** $82^\circ 16'$ **c** $5^\circ 58'$
d $51^\circ 10'$ **e** $44^\circ 48'$ **f** $77^\circ 18'$
g $13^\circ 47' 26''$ **h** $78^\circ 9' 25''$ **i** $67^\circ 23' 41''$

See Examples 7, 8

- 5 Use your calculator to express the following in degrees, minutes and seconds.

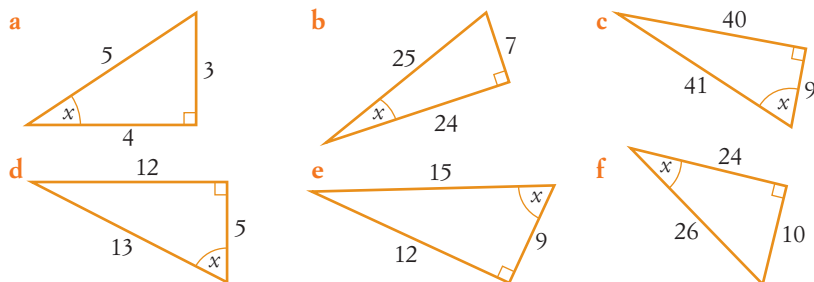
- a** 17.3° **b** 67.5° **c** 20.25°
d 55.8° **e** 88.15° **f** 39.33°
g 9.58° **h** 22.11° **i** 77.04°

- 6 Which trigonometric ratio ($\sin \theta$, $\cos \theta$ or $\tan \theta$) is equal to $\frac{x}{y}$ in each case?



- 7 Find the value of $\sin x$, $\cos x$ and $\tan x$ in the following (correct to 3 decimal places if necessary).

See Example 6



- 8 Find the value of the following (correct to 3 decimal places).

a $\tan 10^\circ$	b $\cos 20^\circ$	c $\sin 40^\circ$	d $\tan 60^\circ$
e $\sin 70^\circ$	f $\tan 80^\circ$	g $\cos 12^\circ$	h $\sin 24^\circ$
i $\tan 84^\circ$	j $\sin 45^\circ$	k $\cos 6^\circ$	l $\tan 79^\circ$

- 9 Find (correct to 1 decimal place) the value x if:

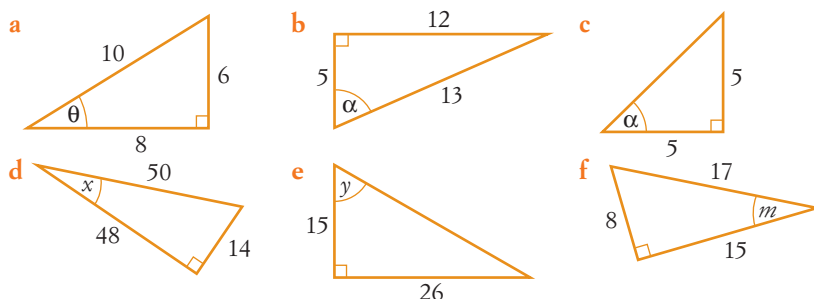
See Example 9

a $\tan x = 0.9$	b $\sin x = 0.07$	c $\cos x = 0.51$
d $\sin x = 0.26$	e $\tan x = 1$	f $\cos x = 0.81$
g $\cos x = 0.02$	h $\tan x = 1.34$	i $\sin x = 0.95$
j $\tan x = 7.23$	k $\sin x = 0.697$	l $\cos x = 0.572$

- 10 Find the value of the angle indicated in each triangle, correct to 1 decimal place.

Fluency

See Example 6



- 11 Work out the value of each of the following, correct to 3 decimal places.

a $12 \times \tan 30^\circ$	b $30 \times \sin 45.3^\circ$	c $\cos 78.06^\circ \times 65$
d $20 \times \tan 7^\circ 15'$	e $\tan 80^\circ 49' \times 18$	f $12.4 \times \tan 68^\circ 11' 35''$
g $\frac{37}{\cos 53^\circ}$	h $\frac{102}{\tan 24.9^\circ}$	i $\frac{82.5}{\sin 72^\circ 17'}$
j $\frac{31.07}{\tan 27.08^\circ}$	k $\frac{0.753}{\sin 12^\circ 41' 45''}$	l $\frac{28.9}{\cos 27.06^\circ}$

- 12 Find the value of a correct to 1 decimal place.

a $\tan 36^\circ = \frac{a}{60}$	b $\sin 45^\circ = \frac{a}{76}$	c $\cos 22^\circ = \frac{a}{15}$
d $\cos 67^\circ = \frac{a}{14}$	e $\tan 82^\circ = \frac{23}{a}$	f $\sin 54.7^\circ = \frac{57}{a}$
g $\cos 32^\circ 12' = \frac{a}{15.3}$	h $\sin 78^\circ 23' = \frac{a}{52.6}$	i $\tan 12^\circ = \frac{a}{104.4}$

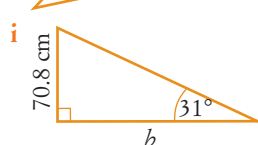
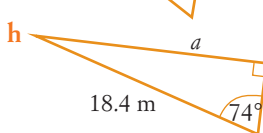
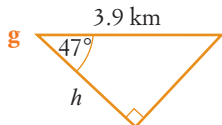
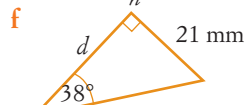
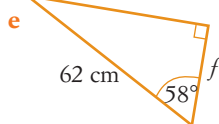
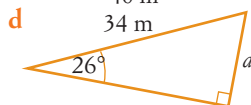
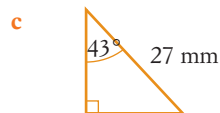
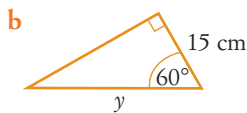
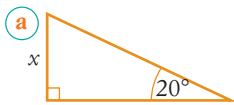
Worked solutions

Exercise 9.2

MAT09MGWS00031

See Examples 10, 11

- 13 Find the value of the unknown side in each triangle, correct to 1 decimal place.



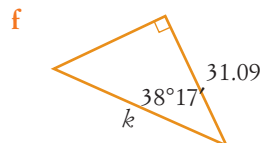
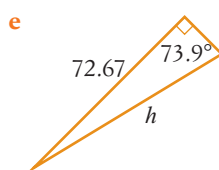
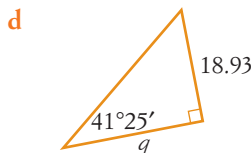
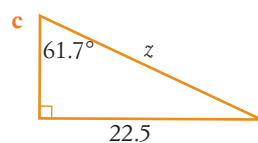
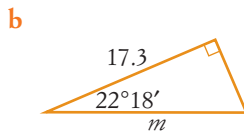
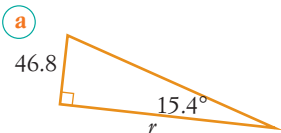
Worked solutions

Exercise 9.2

MAT09MGWS00031

See Examples 10, 11

- 14 Use the given angle to find the value of the unknown side in each triangle, correct to 1 decimal place.



- 15 Calculate the value of the unknown side in each triangle in question 14 by using the complement of the given angle.

Problem solving

- 16 A trigonometric ratio for a right-angled triangle ABC is given in each case below. In each case:
i calculate the length of the third side (leave as a surd if necessary)
ii draw a sketch of $\triangle ABC$
iii calculate the other two ratios for the given angle, leaving your answers as fractions.

a $\cos A = \frac{12}{13}$

b $\sin B = \frac{4}{5}$

c $\cos C = \frac{15}{17}$

c $\sin B = \frac{3}{5}$

d $\cos C = \frac{1}{3}$

e $\tan A = 1$

MAT09MGWS00031

See Example 12

See Example 12

See Example 12

See Example 12

See Example 12

See Example 12

- 17 In $\triangle PQR$, $\angle Q = 90^\circ$, $\angle P = 24.7^\circ$ and $RQ = 37.44$ cm. Find the length of PR , correct to two decimal places.

- 18 $\triangle XYZ$ is right-angled at X , $\angle Z = 67.8^\circ$ and $ZX = 75.3$ m. Find the length of ZY , correct to one decimal place.

- 19 $\triangle ABC$ is right-angled at C , $AC = 176$ mm and $\angle A = 53^\circ 22'$. Find the length of AB , correct to the nearest millimetre.

- 20 In $\triangle MNO$, $\angle N = 90^\circ$, $MN = 117$ cm and $\angle O = 78.3^\circ$. Find MO , correct to the nearest centimetre.

- 21 $\triangle KLM$ is right-angled at L , $\angle M = 42.05^\circ$ and $LM = 9.87$ m. Find the length of side KL , correct to two decimal places.

- 22 In $\triangle UVW$, $\angle W = 90^\circ$, $\angle V = 22^\circ 34'$ and side $UW = 63.8$ cm. Find the length of side UV , correct to one decimal place.

Worked solutions

Exercise 9.2

MAT09MGWS00031

See Example 12

See Example 12

See Example 12

See Example 12

See Example 12

See Example 12

9.3 Applications of triangles

Pythagoras' theorem and trigonometry may be applied to many everyday situations to solve problems. Trigonometry is especially useful in situations where it is not possible or not practical to make actual measurements, for example, to measure a tall building.

When you solve these types of problems using Pythagoras' theorem, it is helpful to draw a sketch to represent the situation and label the hypotenuse and other sides.

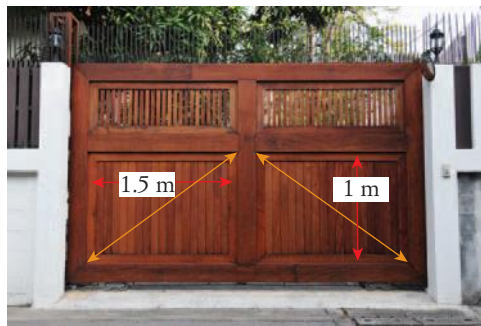
Example 15

Mandy built a gate for her driveway.

The two bottom panels of the gate are the same size and measure 2.4 m by 1.6 m as shown in the photo on the right.

Mandy decided to brace the bottom panels for extra strength so she decided to nail in two supports as indicated by the orange lines in the photo.

What length of timber will she need to buy to complete the job?



Solution

First draw a sketch to represent the situation and label it.

This is a case where Pythagoras' theorem can be applied.

Write Pythagoras' theorem using the labelled sides in the diagram.

Substitute known values.

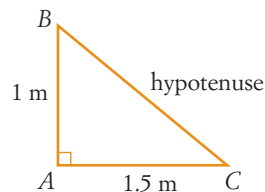
Evaluate.

Take the square root of each side.

Round off.

Timber is usually purchased in lengths that are multiples of 30 cm.

Two supports are required. State the result.



$$BC^2 = BA^2 + AC^2$$

$$= 1^2 + 1.5^2$$

$$BC^2 = 3.25$$

$$BC = 1.8027 \dots$$

$$\approx 1.9 \text{ m}$$

**The timber needed for each
brace = 2.1 m**

**Mandy will need to purchase 3.9 m
of timber to complete the job.**

When solving problems involving trigonometry, it is also helpful to make a sketch. In these cases, it is useful to mark in the angles involved as well as labelling the sides as opposite, adjacent or hypotenuse. This will help to determine which trigonometric ratio to use.

Example 16

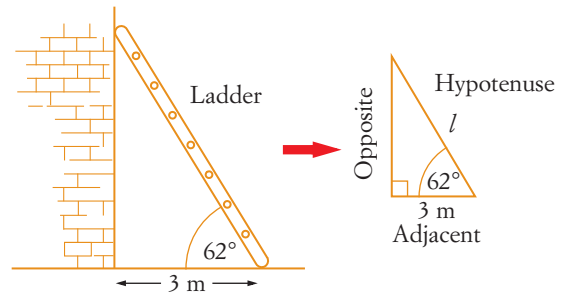
The end of a ladder is 3 m from the base of a wall. It is leaning so that it makes an angle of 62° with the ground. How long (to the nearest cm) is the ladder?

Solution

First make a sketch and put the important information on it.

Next use the sketch to draw a triangle diagram with the same information.

Let the length of the ladder be l .



The adjacent side and hypotenuse are involved, so use $\cos 62^\circ$.

Substitute for known values.

Multiply both sides by l .

Cancel.

Divide both sides by $\cos 62^\circ$.

Cancel.

Enter as $3 \div \cos 62 =$.

$$\cos 62^\circ = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{3}{l}$$

$$l \times \cos 62^\circ = \frac{3}{l} \times l$$

$$= 3$$

$$\frac{l \times \cos 62^\circ}{\cos 62^\circ} = \frac{3}{\cos 62^\circ}$$

$$l = \frac{3}{\cos 62^\circ}$$

$$3 \div \cos(62)$$

$$6.390163405$$

Round off to the nearest cm.

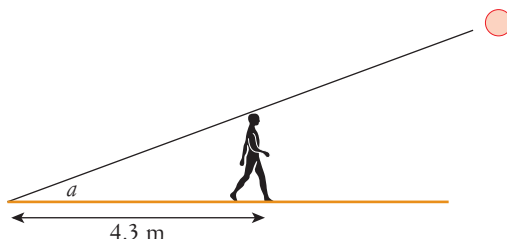
State the result.

$$\approx 6.39 \text{ m}$$

The ladder is about 6.39 m (639 cm) long.

Example 17

A person walking towards the sun casts a shadow 4.3 m long as shown in the diagram below. If the person is 178 cm tall, calculate the value of angle a .

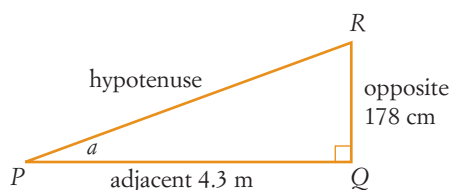


Solution

Do a sketch.

Label the vertices of the triangle.

Insert the information that is known.



The adjacent and opposite sides are involved, so use $\tan a$.

Substitute known values.

Convert 178 cm to metres.

Enter as **SHIFT** **tan** 1.78 **a^{b/c}** 4.3 **=**.

$$\begin{aligned}\tan a &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{1.78}{4.3}\end{aligned}$$

$$\tan^{-1}\left(\frac{1.78}{4.3}\right) = 22.48727991$$

Round off to one decimal place.

State the result.

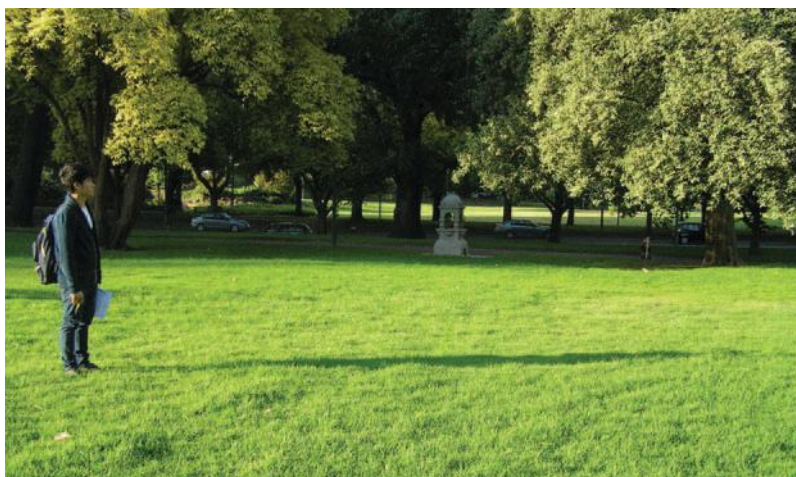
$$a \approx 22.5^\circ$$

The angle is about 22.5° .

Animated example

Trigonometry

MAT09MGAE00009



Exercise 9.3 Applications of triangles

Problem solving

Extra questions

Exercise 9.3

MAT09MGEQ00032

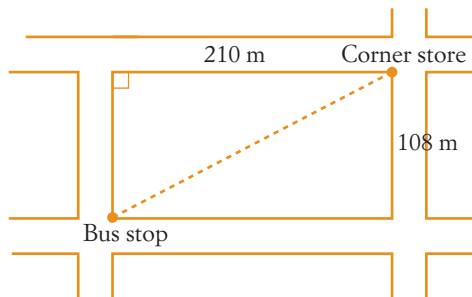
See Example 15

See Example 15

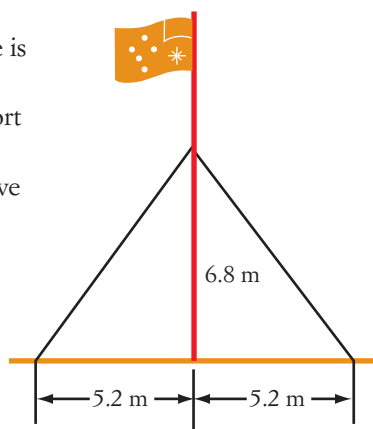
See Example 15

- 1 A bus stop and a corner store are at diagonally opposite corners of a suburban block as shown. Tammy gets off the bus and takes a short cut to the store on her way home from school. Find:

- the distance she travels by taking the short cut
- the distance she saves by not going the long way to the corner store.



- 2 A flagpole is supported by two guy-wires that are attached to the pole 6.8 m above the ground. Each wire is attached to the ground 5.2 m from the base of the flagpole. Find the total length of the wire used to support the flagpole if an additional 30 cm is required for each guy-wire to attach them to the pole and the ground. Give your answer correct to the nearest centimetre.

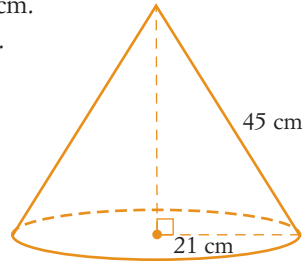


- 3 Jose is a keen fisherman who lives on a lake. As shown in the photograph below, 670 m directly west of his house is the boat ramp he uses when he launches his boat into the water. 820 m directly north of the boat ramp is a shed that he uses to store fishing gear and carry out minor repairs to his boat. How far is the shed from Jose's house?



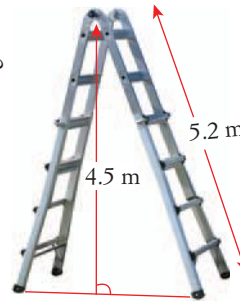
- 4 The slant height of a cone is 45 cm and the radius is 21 cm.
Find the height of the cone correct to one decimal place.

See Example 15



- 5 The arms of this fold out ladder are 5.2 m long.
When fully spread, the ladder has a height of 4.5 m.
How far apart are the arms of the ladder when it is fully extended?

See Example 15



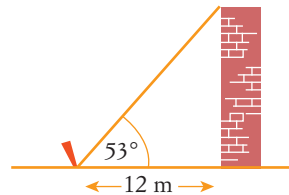
- 6 A survey peg is placed 12 m from the base of a wall. The base of the peg makes an angle of 53° with the top of the wall.
Find the height of the wall, to the nearest centimetre.

Worked solutions

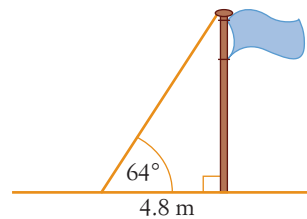
Exercise 9.3

MAT09MGWS00032

See Example 16

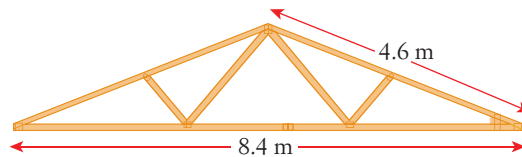


- 7 This flagpole is held in place with a guy-wire. The wire is secured 4.8 m from the base of the pole.
a What is the height of the pole, to the nearest centimetre?
b How long is the guy-wire, to the nearest centimetre?



- 8 Calculate the height of this roof truss.

See Example 15



Worked solutions

Exercise 9.3

MAT09MGWS00032

See Example 16

- 9 This set of stairs makes an angle of 23° with the horizontal. The horizontal distance between the top tread and the bottom tread is 3.6 m. What is the vertical distance between these two treads, correct to the nearest centimetre?



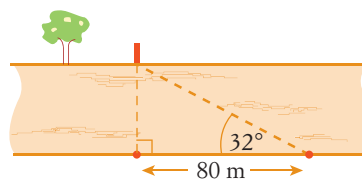
See Example 16

- 10 Find the length of this rectangular gate if the diagonal support makes an angle of 58° with the height.



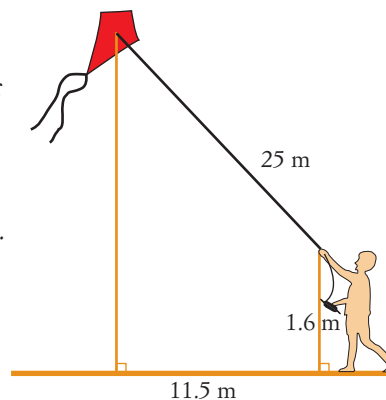
See Example 16

- 11 Two survey pegs are 80 m apart on one bank of a river. On the opposite bank is a navigation beacon. It makes an angle of 32° with one of the pegs and is directly opposite the other peg. How wide is the river (correct to 1 decimal place)?

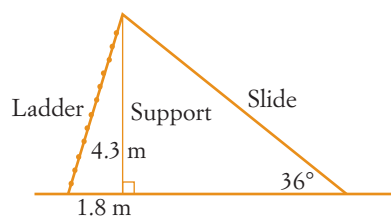


See Examples 16, 17

- 12 A kite is attached to a 25 m piece of rope, as shown. The rope is held 1.6 m above the ground and the horizontal distance between the kite and the kite flyer is 11.5 m. Find:
a the height of the kite above the ground, correct to 1 decimal place
b the angle that the string makes with the horizontal.



- 13 A playground slide makes an angle of 36° with the horizontal. The top of the slide is accessed by a ladder. The base of the ladder is 1.8 m away from a 4.3 m high vertical support as shown. Find:
a the length of the slide to the nearest centimetre
b the angle that the ladder makes with the horizontal correct to 1 decimal place.



Worked solutions

Exercise 9.3

MAT09NMGS00032

See Examples 16, 17

- 14** This awning is held in position by a supporting strut 1.2 m long. Calculate (correct to the nearest centimetre):

See Example 16

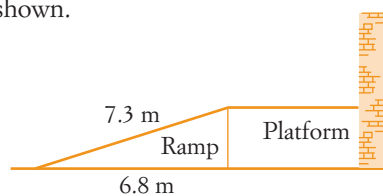
- the distance between the top of the awning and the point where the strut meets the wall
- the length of the edge of the awning between the wall and the strut.



- 15** A ramp is used to deliver material to a platform as shown. Calculate:

See Examples 16, 17

- the height of the ramp
- the angle that the ramp makes with the horizontal.



- 16** Lifesavers climb the ladder shown here to gain access to the enclosure from which they can observe the beach. The ladder is 5 m long and the base of the ladder is horizontally 1.8 m away from the top of the ladder.

See Examples 16, 17

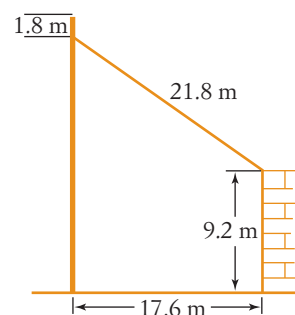
- What angle does the ladder make with the vertical?
- How high above the ground is the ladder?



- 17** A vertical telecommunications pole is erected 17.6 m from the base of a wall that is 9.2 m high. To stabilise the pole, a taut piece of wire is connected from the top of the wall to a point 1.8 m from the top of the pole as shown. Calculate:

See Examples 16, 17

- the overall length of the pole that is above the ground
- the angle (correct to 1 decimal place) that the wire makes with the pole.



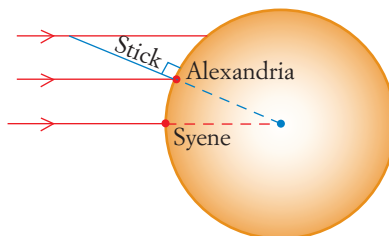
See Examples 16, 17

- 18** A crane is positioned on top of a high-rise building as shown.
- What angle does the boom of the crane make with the horizontal?
 - How long (correct to 1 decimal place) is the boom of the crane?



Reasoning

- 19** A ladder leans against a wall so that it forms an angle of 58° with the ground. If the ladder reaches 8.6 m up the wall, how far from the base of the wall is the foot of the ladder? (Answer correct to the nearest 10 cm.)
- 20** A rectangular frame has a diagonal support that forms an angle of 34° with its length. The length is 45 cm. What is the width of the frame, correct to 1 decimal place?
- 21** Here is some information similar to that used by Eratosthenes (275–194 BCE) to calculate an approximate value for the radius of the Earth. At noon on a certain day in Syene, the Sun was directly overhead. At the same time, 804.6 km north at Alexandria, a one metre vertical stick cast a 13 cm shadow. Rays from the Sun can be considered to be parallel.

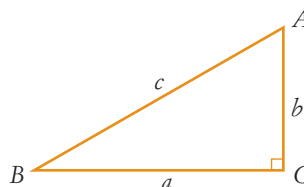


Use the same information to calculate a value for the radius of the Earth. How close is your value to the actual measurement (about 6400 km)?

- 22** Use the method of geometric proof to prove:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

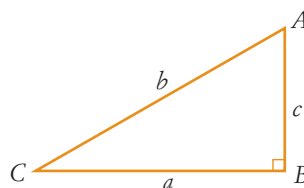
for the triangle shown here.



- 23** Use the method of geometric proof to prove:

$$\text{Area of } \triangle ABC = \frac{1}{2}ab \sin C$$

for the triangle shown here.

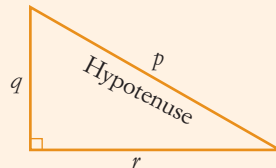


- The **hypotenuse** of a right-angled triangle is the longest side. It is also the side opposite the right angle.
- A **surd** is an irrational number that can only be expressed exactly using the square root sign ($\sqrt{\quad}$).

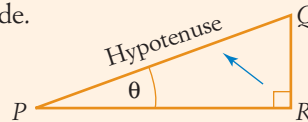
- Pythagoras's theorem** states that the square on the hypotenuse is equal to the sum of the squares on the other two sides.

So for this triangle,

$$p^2 = q^2 + r^2$$



- The **converse of Pythagoras' theorem** is that if the square of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle must be right-angled.
- A **Pythagorean triple** or **triad** such as (3, 4, 5) is a set of three numbers that obey Pythagoras' theorem.
- Trigonometry** involves the study of angles and the ratio of sides in right-angled triangles.
- In a right-angled triangle, the **hypotenuse** is the longest side. It is opposite the right angle. In $\triangle PQR$, PQ is the hypotenuse, QR is the side **opposite** θ and PR is the side **adjacent** to θ .



- Three common trigonometric ratios are **sine** (**sin**), **cosine** (**cos**) and **tangent** (**tan**). In $\triangle PQR$,

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{QR}{PQ}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{PR}{PQ}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{QR}{PR}$$

- The sides involved with each of the trigonometric ratios can be remembered using the mnemonic SOH CAH TOA.
- The values of trigonometric ratios are constant for a particular angle in any right-angled triangle. Values of trigonometric ratios may be found using a calculator.
- Angles are measured in **degrees** ($^{\circ}$). Degrees may be further divided into **minutes** ($'$) and **seconds** ($''$). $1^{\circ} = 60'$ and $1' = 60''$.

Chapter 9 review

Understanding

Worksheet

Trigonometry review

MAT09MGWK00057

- 1 Use your calculator to find the value (correct to 2 decimal places if necessary) of each of the following.

a 16^2

b $8^2 + 11^2$

c $7.6^2 - 4.3^2$

d $\sqrt{84}$

e $\sqrt{11^2 + 17^2}$

f $\sqrt{20^2 - 14^2}$

- 2 Select the surds from the following square roots.

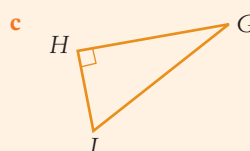
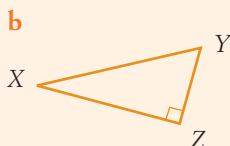
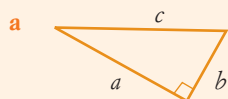
a $\sqrt{36}$

b $\sqrt{291}$

c $\sqrt{529}$

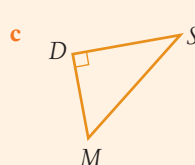
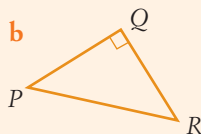
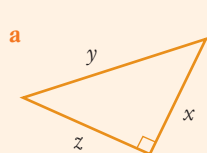
d $\sqrt{361}$

- 3 Name the hypotenuse of each of these triangles.



See Example 1

- 4 Write Pythagoras' theorem for each of the following triangles.



- 5 Which of the following is a Pythagorean triple?

a (7, 24, 27)

b (9, 42, 43)

c (9, 12, 15)

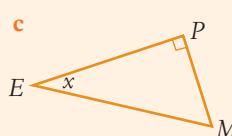
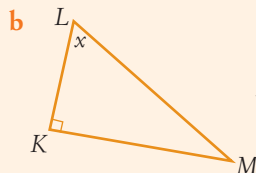
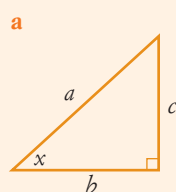
See Example 5

- 6 For each of the triangles below, name:

i the hypotenuse

ii the side opposite x

iii the side adjacent to x



See Example 8

- 7 Use your calculator to express the following as decimals correct to 1 decimal place.

a $48^\circ 27'$

b $22^\circ 9'$

c $63^\circ 11' 48''$

See Example 7

- 8 Use your calculator to express the following in degrees, minutes and seconds.

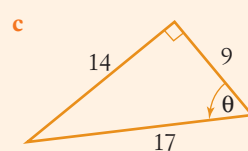
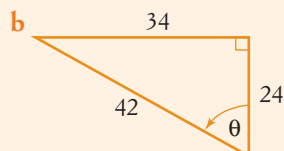
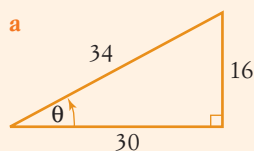
a 81.4°

b 12.7°

c 52.33°

See Example 6

- 9 Find the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$ in the following (correct to 3 decimal places if necessary).



- 10 Find the value of the following (correct to 3 decimal places).

a $\tan 22.5^\circ$

b $\cos 81.9^\circ$

c $\sin 17^\circ 41'$

- 11 Find (correct to 1 decimal place) the value x if:

a $\tan x = 0.623$

b $\sin x = 0.512$

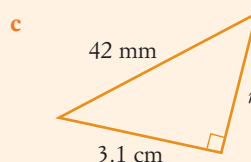
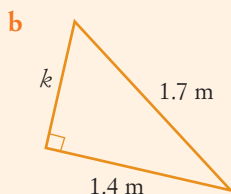
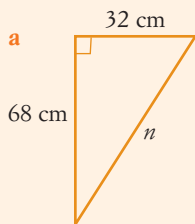
c $\cos x = 0.711$

See Example 9

- 12 Find the length (correct to 1 decimal place) of the unknown side for each of these triangles.

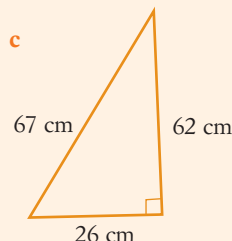
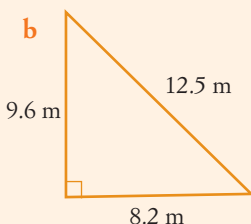
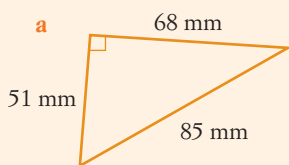
Fluency

See Examples 1–3



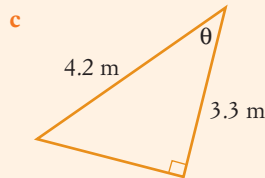
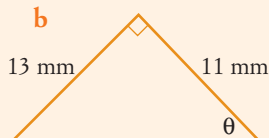
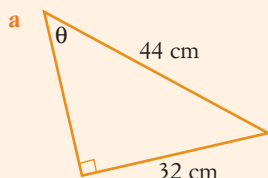
- 13 Use Pythagoras' theorem to test whether each of the following triangles is right-angled.

See Example 4



- 14 Find the value of θ in each triangle, correct to 1 decimal place.

See Example 13



- 15 Work out the value of each of the following, correct to 3 decimal places.

a $21 \times \tan 43^\circ$

b $17 \times \sin 62.8^\circ$

c $\cos 26^\circ 36' \times 42$

d $\frac{51}{\cos 38^\circ}$

e $\frac{37}{\sin 41.8^\circ}$

f $\frac{71}{\cos 35^\circ 28'}$

- 16 Find the value of x correct to 1 decimal place.

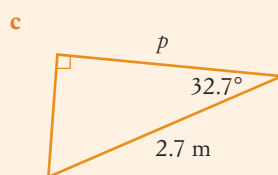
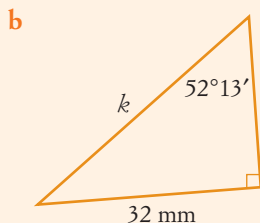
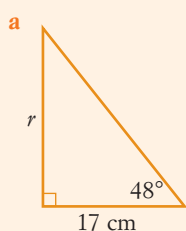
a $\tan 29^\circ = \frac{x}{43}$

b $\sin 62.8^\circ = \frac{37}{x}$

c $\cos 28^\circ 43' = \frac{x}{44.9}$

- 17 Find the value of the unknown side in each triangle, correct to 1 decimal place.

See Examples 10, 11



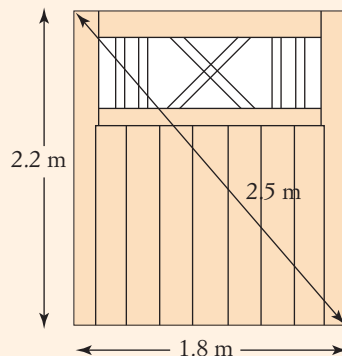
Chapter 9 review

- 18 Calculate the value of the unknown side in each triangle in question 17 by using the complement of the given angle.

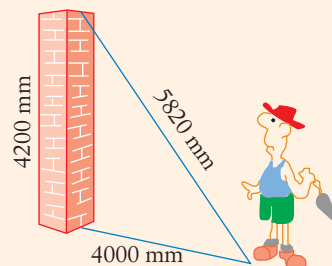
Problem solving

- 19 In $\triangle HUT$, $\angle U = 90^\circ$, $HT = 1.72$ m, and $UT = 118$ cm. Find UH , correct to the nearest centimetre.

- 20 Sasha built a gate for her side fence. When she finished, she measured its length, width and diagonal as shown in the diagram on the right. A gate is said to be 'square' if it has corners that are right angles. Without using a protractor, determine if Sasha's gate is square.

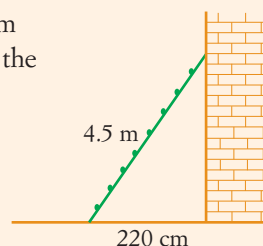


- 21 A bricklayer made the measurements shown to see if this brick support was perpendicular. Is it perpendicular? If not, does it lean towards the bricklayer or away from him?



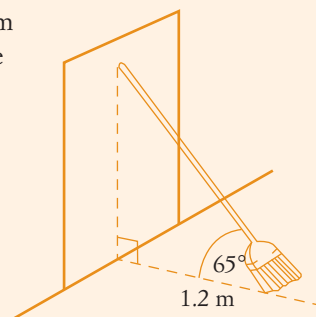
See Example 15

- 22 The foot of a 4.5 m long ladder is placed 220 cm from the base of a wall. How far does the ladder reach up the wall? (Give your answer correct to the nearest centimetre.)

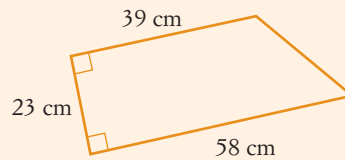


See Example 16

- 23 A broom is leaning against a wall at an angle of 65° from the floor. One end of the broom is 1.2 m away from the base of the wall. How far up the wall does the broom reach?

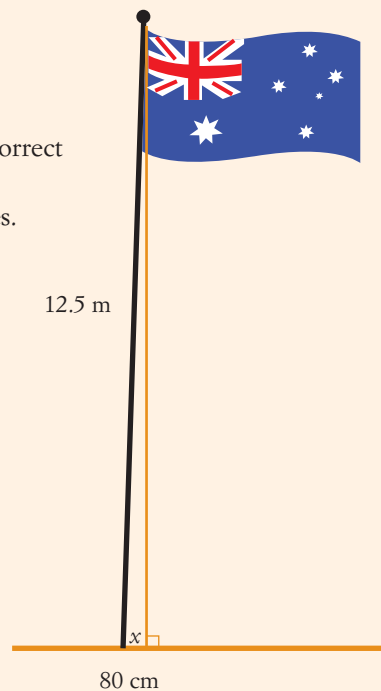


- 24 Calculate the perimeter of the trapezium on the right correct to 1 decimal place.



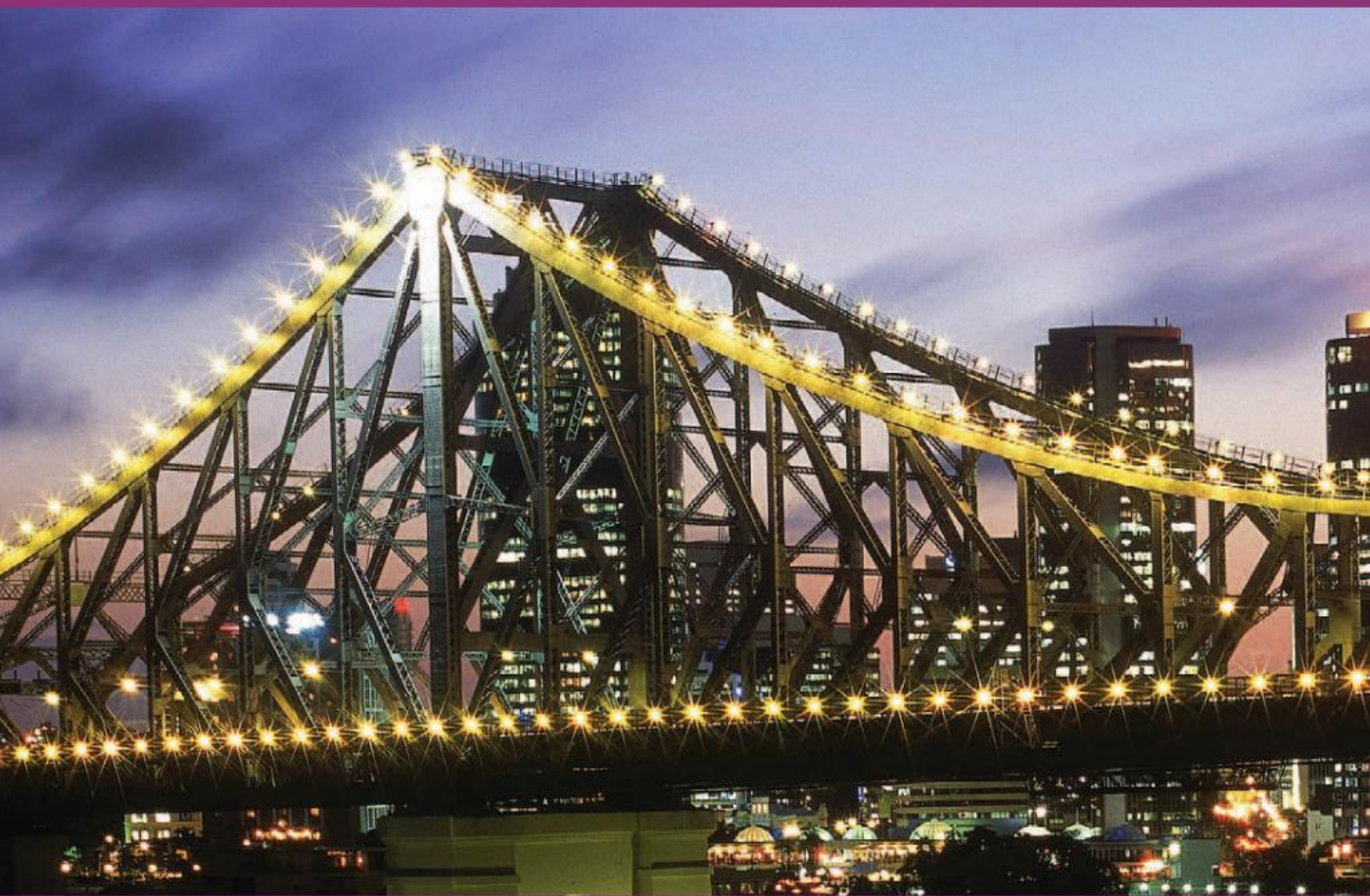
- 25 In $\triangle NJL$, $\angle J = 90^\circ$, $JL = 12$ m and $NJ = 27$ m. Find the size of angle L , correct to one decimal place.
- 26 A straight stretch of highway rises 48 m for every 500 m travelled along the road. Find the angle (correct to one decimal place) at which the road is inclined to the horizontal.
- 27 A 12.5 m long flagpole is not perpendicular to the ground as shown here. The top of the pole overhangs the bottom of the pole by 80 cm. Calculate:

See Examples 16, 17



- 28 A conical pile of refined sugar has a diameter of 28 m and measures 22 m up the slope to the apex of the pile. Calculate, correct to 1 decimal place:
- the angle that the sloping side of the pile makes with the horizontal
 - the height of the pile.
- 29 In $\triangle ABC$, $\angle B = 90^\circ$ and $\angle A = 42^\circ$. The hypotenuse, AC , is 25 cm long. Calculate the area of the triangle.

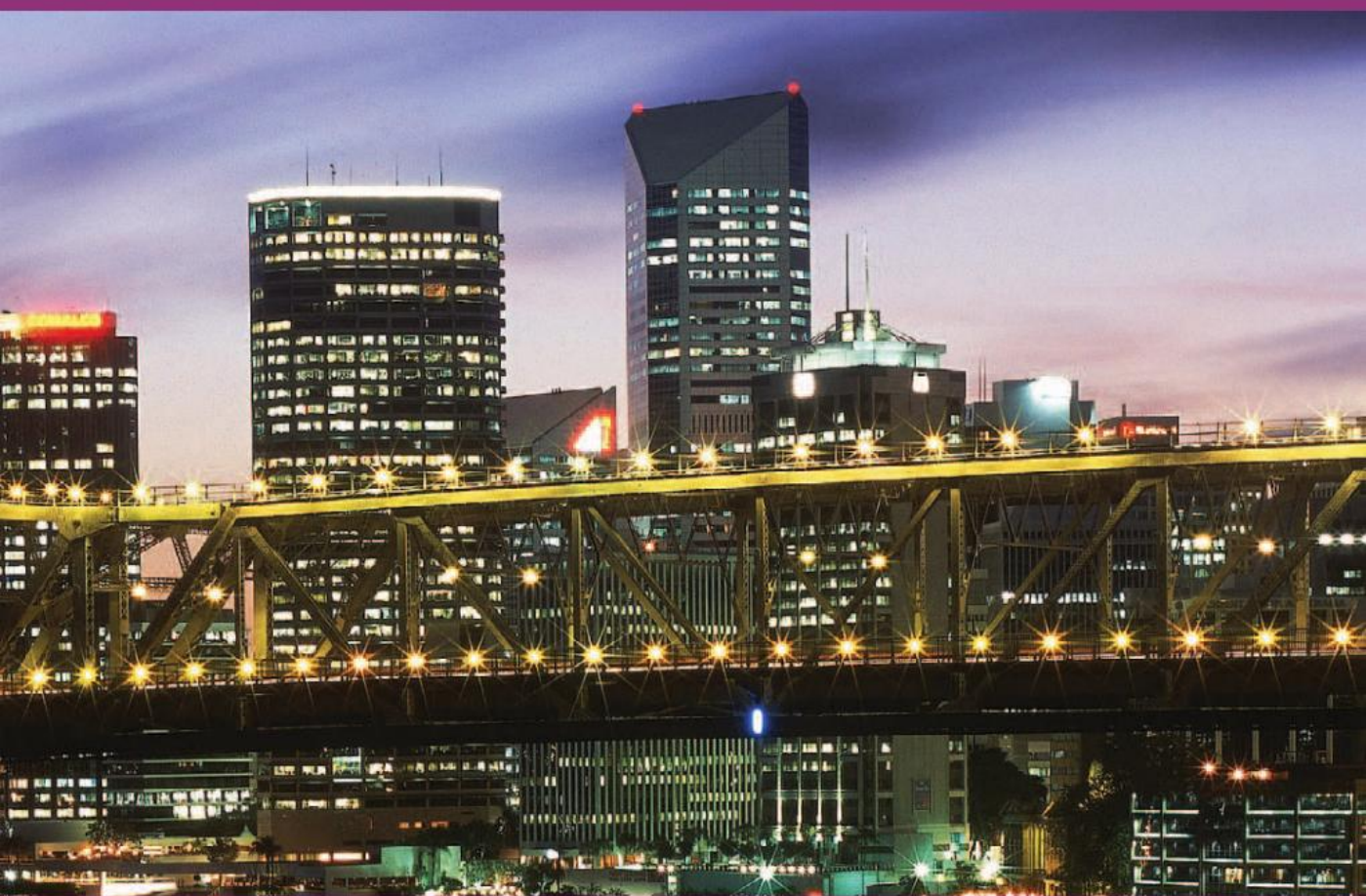
Reasoning



Number and algebra

10

**Coordinates
and graphs**



Contents

- 10.1 Coordinates and lines
- 10.2 Distance and midpoint
- 10.3 Drawing lines and graphs
- 10.4 Parallel and perpendicular lines
- Chapter summary
- Chapter review

Prior learning

Chapter 10

MAT09NAPL00010

Parent guide

Chapter 10

MAT09NAPG00010

Curriculum guide

Chapter 10

MAT09NACU00010

Australian Curriculum statements

Linear and non-linear relationships

Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software.

Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including graphing software.

Sketch linear graphs using the coordinates of two points and solve linear equations.

Graph simple non-linear relations with and without the use of digital technologies and solve simple related equations.

Video tutorial

Coordinates and graphs

MAT09NAVT00010

Weblink

Maths trails

MAT09NAWB00010

Maths dictionary

MAT09ASDI00001

Weblink

Why math?

MAT09NAWB00010

You have already seen how you can use graphs and algebra to model real-life situations. This is particularly useful in Business, Economics, Science and Technology. The connection between Algebra and Geometry in graphing is a very powerful part of mathematics. In this chapter you will learn more about graphs, particularly graphs of straight lines. The work you did in previous years on coordinates is crucial for this kind of modelling.

Mathematical literacy

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

Cartesian plane	gradient–intercept	rise	vertex
circle	form	rising line	x -axis
coordinates	linear function	run	x -coordinate
distance	linear graph	satisfy	x -intercept
equation of a line	midpoint	slope	y -axis
falling line	origin	standard form of	y -coordinate
general form	parabola	equation	y -intercept
gradient	parallel lines	table of values	
	perpendicular lines	turning point	

10.1 Coordinates and lines

You have previously seen how points are plotted on the Cartesian plane.

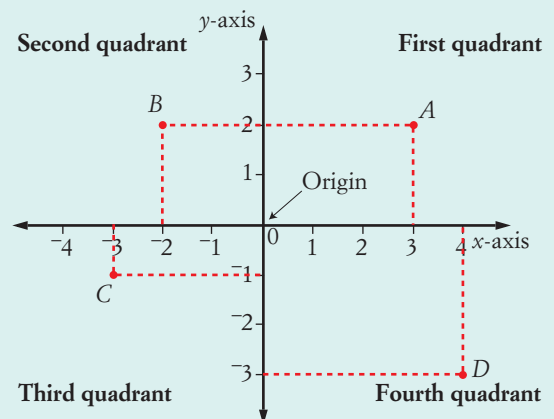
Important!

The Cartesian plane

The **Cartesian plane** has two axes at right angles. It is also called a **number lattice**. The **first axis** is horizontal and is called the **x -axis**. The **second axis** is vertical and is known as the **y -axis**. The axes cross at the **origin**.

The position of a point on the Cartesian plane is determined by its **coordinates**. The x -coordinate gives the horizontal position of the point from the origin, and the y -coordinate gives the vertical position of the point from the origin.

Coordinates are shown in parentheses (round brackets) with a comma between them. The **x -coordinate** is always shown *first* and the **y -coordinate** *last*. The origin is the point (0, 0). Each quarter of the Cartesian plane is called a **quadrant**. In the diagram, Point A is called (3, 2), B is (−2, 2), C is (−3, −1) and D is (4, −3).



It is very common to plot the points in a table of values to look at the relationship between the variables.

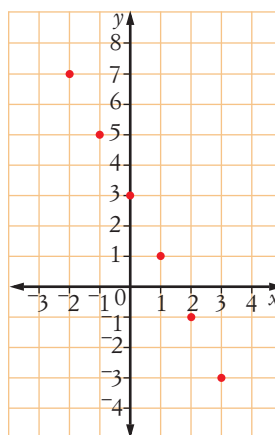
Example 1

Plot the following points and describe the relationship.

x	-2	-1	0	1	2	3
y	7	5	3	1	-1	-3

Solution

The point $(-2, 7)$ is 2 units left and 7 units up from the origin. The other points are plotted in a similar way.



Describe the relationship using the shape of the graph.

The values of y go down by 2 for each 1 that the x -values increase by. It is a straight line. The function is a decreasing linear relationship.

Graphs like the one in Example 1 are called **linear graphs**, which just means a straight line.

Important!

Linear graphs

You can plot points using a **table of values**. A rule that produces a straight line is called a **linear function** and the graph is called a **linear graph**. Linear functions can always be expressed in the form $y = ax + b$, where a and b are constants. a and b can be any kinds of numbers, including fractions, decimals, positive and negative numbers. The equation of a straight line is often referred to as the **equation of the line**.

Example 2

CAS TI-Nspire exercise

Coordinates and
graphs

MAT09NATI00010

CAS ClassPad exercise

Coordinates and
graphs

MAT09NACP00010

Use a table of values to plot the function $y = \frac{1}{2}x - 1$ for $-2 \leq x \leq 6$.

Solution

Draw up a table with x from -2 to 6 .

x	-2	-1	0	1	2	3	4	5	6
y									

Calculate each y -value by substituting the x -value into the function.

$$\begin{aligned} y &= \frac{1}{2} \times -2 - 1 \\ &= -1 - 1 \\ &= -2 \end{aligned}$$

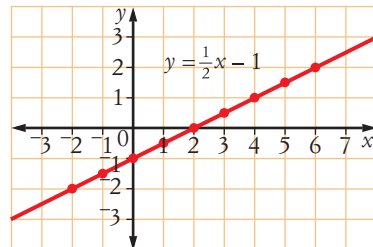
Complete the table of values.

x	-2	-1	0	1	2	3	4	5	6
y	-2	$-1\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2

Plot the function and label it with the equation.

Join the points with a straight line.

Extend the line past the points to show that the function continues.



It is common to refer to a function or a straight line by its equation. In Example 2 you could call the line $y = \frac{1}{2}x - 1$.

To work out whether or not a point is on a line, you only need to check if it works in the equation. If it does, we say it **satisfies** the equation and must lie on the line.

Example 3

Worksheet

Straight-line equations

MAT09NAWK00059

- a** Is $A(5, 3)$ on the line $y = 2x - 4$?
b Is $B(-1, 2)$ on the line $3x - 2y + 7 = 0$?

Solution

- a** Write the equation.

$$y = 2x - 4$$

Substitute the x -coordinate of $A(5, 3)$ for x .

$$= 2 \times 5 - 4$$

Evaluate.

$$= 10 - 4 = 6$$

Compare with the y -coordinate of $A(5, 3)$.

$$6 \neq 3$$

State the result.

$$A(5, 3) \text{ is not on the line } y = 2x - 4.$$

b Write the equation.

Substitute the coordinates of $B(-1, 2)$ into the left-hand side (LHS) of the equation.

Evaluate.

Compare with the RHS.

Since $\text{LHS} = \text{RHS}$, $B(-1, 2)$ satisfies the equation.

$$3x - 2y + 7 = 0$$

$$\text{LHS} = 3 \times -1 - 2 \times 2 + 7$$

$$= -3 - 4 + 7 = 0$$

$$= \text{RHS}$$

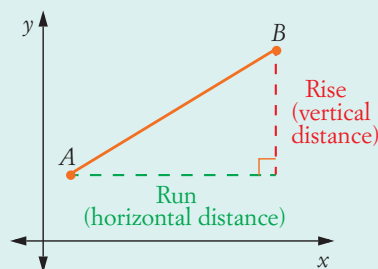
$B(-1, 2)$ is on the line $3x - 2y + 7 = 0$.

You may have previously looked at the **gradient** of a line.

Important!

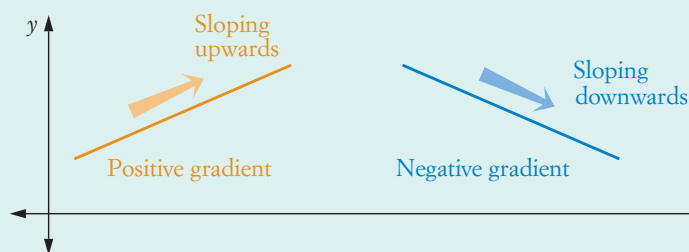
Gradient of a line

For two points on a straight line in the Cartesian plane, the **rise** is the vertical distance between the points and the **run** is the horizontal distance between the points. The **gradient (slope)** of a straight line is calculated as $\text{gradient} = \frac{\text{rise}}{\text{run}}$ for any two points on the line. The usual symbol for gradient is m .



The line of an increasing function slopes upwards from left to right. You call it a **rising line** and it has a *positive gradient*.

The line of a decreasing function slopes downwards from left to right. You call it a **falling line** and it has a *negative gradient*.



Example 4

Video tutorial

The gradient of a line

MAT09NAVT10019

Technology

Geogebra: Slope, distance, midpoint

MAT09NATC00008

Puzzle sheet

Find the gradient 1

MAT09NAPS00049

Worksheet

Gradient between two points

MAT09NAWK00058

Find the gradients of the lines through the following pairs of points.

a $(-2, 2)$ and $(4, 6)$

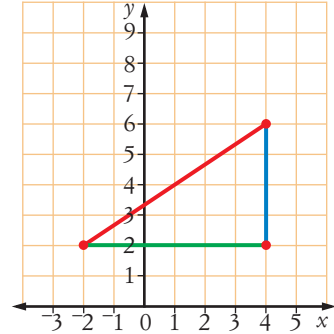
b $(1, 2)$ and $(3, -2)$

Solution

a Plot the points and join them with a straight line.

The line is *rising* so the gradient is positive.

Draw in the **rise** and the **run**.



$$\text{Rise} = 6 - 2 = 4$$

$$\text{Run} = 4 - (-2) = 6$$

$$\begin{aligned} \text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{4}{6} = \frac{2}{3} \end{aligned}$$

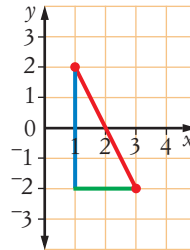
The gradient of the line through

$(-2, 2)$ and $(4, 6)$ is $m = \frac{2}{3}$.

b Plot the points and join them with a straight line.

The line is *falling* so the gradient is negative.

Draw in the **rise** and the **run**.



$$\text{Rise} = -2 - 2 = -4$$

$$\text{Run} = 3 - 1 = 2$$

$$\begin{aligned} \text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-4}{2} = -2 \end{aligned}$$

The gradient of the line through

$(1, 2)$ and $(3, -2)$ is $m = -2$.

Calculate the rise.

Calculate the run.

Write the formula.

Substitute values and simplify.

Write the answer.

The rise is actually going down.

Calculate the run.

Write the formula.

Substitute values and simplify.

Write the answer.

Important!

Intercepts

The ***x*-intercept** of a straight line means the part of the *x*-axis from the origin to the point where the line crosses the axis. Since the distance is just the *x*-coordinate of the intersection, this or the point itself is usually taken as the intercept.

The ***y*-intercept** is the *y*-coordinate of the intersection (or the point) of the line and the *y*-axis.

Example 5

- a** Draw the graph of $2y + 3x = 6$ for $-4 \leq x \leq 4$.
b Choose two points on the line to calculate the gradient and find the intercepts.

Solution

- a** Write the equation.

Change to the form $y = \dots$ for convenience.

Simplify the first step.

Divide by 2 to get $y = \dots$

Choose easy values between -4 and 4 and work out the table of values.

$$2y + 3x = 6$$

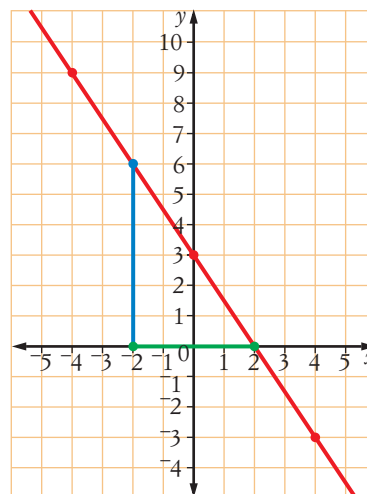
$$2y + 3x - 3x = 6 - 3x$$

$$2y = 6 - 3x$$

$$y = 3 - \frac{3}{2}x \text{ or } y = 3 - 1\frac{1}{2}x$$

<i>x</i>	-4	0	4
<i>y</i>	9	3	-3

Plot and draw the line.



- b** Choose, say, $(-2, 6)$ and $(2, 0)$ to find the **rise** and **run**.

Find the gradient.

The line goes through the *x*-axis at $(2, 0)$.

The line goes through the *y*-axis at $(0, 3)$.

$$\text{Rise} = -6 \text{ (going down)}$$

$$\text{Run} = 4$$

$$m = \frac{\text{rise}}{\text{run}} = \frac{-6}{4} = \frac{-3}{2}$$

$$\text{x-intercept} = 2$$

$$\text{y-intercept} = 3$$

CAS TI-Nspire exercise

Coordinates and
graphs

MAT09NATI00010

CAS ClassPad exercise

Coordinates and
graphs

MAT09NACP00010

Investigate: Equations of lines

1 Draw graphs of the lines represented by the following equations.

a $y = 2x + 5$

b $y = -x - 3$

c $y = -3x + 4$

d $y = 3x - 1$

e $y = \frac{1}{2}x - 3$

f $y = -\frac{2}{3}x + 1$

2 Work out the gradient and y -intercept for each.

3 Compare the values of the gradient and y -intercept with the equations for the lines.

4 What do you notice? Does this always work? Can you explain why?

In the above investigation, you should have seen that the equation of a straight line can be written using its gradient and y -intercept. The gradient (slope) is the coefficient of the x term and the y -intercept is the constant term.

In Example 5 and the investigation above, you can see that the equation of a line can be written using its gradient and y -intercept, but this form often has fractions.

Important!**Forms of the equation of a line**

The **gradient–intercept** (or **slope–intercept**) **form** of the equation of a straight line is $y = mx + c$, where m is the gradient and c is the y -intercept.

The **standard** (or **general**) **form of the equation** of a straight line is written with no fractions and all the terms on one side, as $ax + by + c = 0$, where a , b and c are constants (numbers).

In many cases, you will have some points or information about a situation and want to model it as a linear function. The simplest way to do this is to find the gradient and y -intercept. To find out whether or not a function is linear, you can check that the gradient between any two points is the same.

Example 6

Check that the line through the points $A(3, 7)$, $B(4, 5)$, $C(6, 1)$ and $D(8, -3)$ is linear and find its equation.

Solution

Find the rise from A to B .

$$\text{Rise from } A \text{ to } B = 5 - 7 = -2$$

Find the run from A to B .

$$\text{Run from } A \text{ to } B = 4 - 3 = 1$$

Find the gradient from A to B .

$$m_{AB} = \frac{-2}{1} = -2$$

Find the rise from B to C .

$$\text{Rise from } B \text{ to } C = 1 - 5 = -4$$

Find the run from B to C .

$$\text{Run from } B \text{ to } C = 6 - 4 = 2$$

Find the gradient from B to C .

$$m_{BC} = \frac{-4}{2} = -2$$

Find the rise from C to D .

Find the run from C to D .

Find the gradient from C to D .

Write your conclusion.

Write what you know about the function.

Write the equation with the value of m and substitute A , say.

Simplify and solve for c .

Turn the equation around.

Simplify.

Write the equation.

$$\text{Rise from } C \text{ to } D = -3 - 1 = -4$$

$$\text{Run from } C \text{ to } D = 8 - 6 = 2$$

$$m_{CD} = \frac{-4}{2} = -2$$

Since the gradients between each pair of points is the same, the function is linear.

The function is of the form $y = mx + c$ and $m = -2$. Each of the points A , B , C and D also satisfies the equation.

$$y = -2x + c$$

$$7 = -2 \times 3 + c$$

$$7 = -6 + c$$

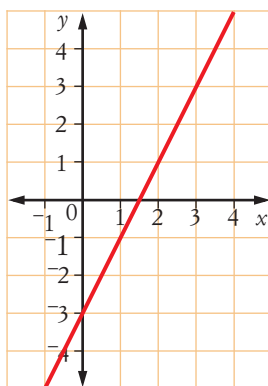
$$c + -6 + 6 = 7 + 6$$

$$c = 13$$

The equation of the function is $y = -2x + 13$ or $2x + y - 13 = 0$.

Example 7

Find the equation of the line shown on the right.



Solution

Find the y -intercept.

Choose two easy points.

Find the rise.

Find the run.

Find the gradient.

Substitute in $y = mx + c$.

The line passes through the y -axis at $y = -3$.

Use $(0, -3)$ and $(2, 1)$ to find the gradient.

$$\text{Rise} = 4$$

$$\text{Run} = 2$$

$$m = \frac{4}{2} = 2$$

The equation is $y = 2x - 3$.

Video tutorial

Finding the equation of a line

MAT09NAVT10020

Puzzle sheet

Straight-line equations

MAT09NAPS00045

TLF Learning object

EagleCat: linear graph (L10090)

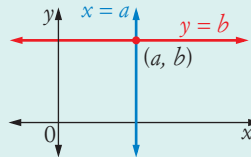
MAT09NAIN00010

The gradient of a vertical line is not defined because it has no run. In fact, the equations of lines parallel to the axes are both special cases with very simple equations.

Important!

Lines parallel to the axes

The line parallel to the x -axis through (a, b) has the equation $y = b$.



The line parallel to the y -axis through (a, b) has the equation $x = a$.

Example 8

Find the equations of vertical and horizontal lines through $(2, 5)$.

Solution

A vertical line is parallel to the y -axis.

The vertical line is $x = 2$.

A horizontal line is parallel to the x -axis.

The horizontal line is $y = 5$.

In many practical situations you will want to solve linear equations that arise from linear functions.

Example 9

It costs \$45 plus \$10 an hour to rent a windsurfer on the Gold Coast.

a Write the hire cost C as a function of the hire time in hours h .

b Find the hire cost for 3 hours.

c How long could you hire it for if you had \$90?

Solution

a Find the y -intercept.

For 0 hours the cost is \$45, so $c = 45$.

Find the gradient.

For each hour (run) the cost goes up by \$10, so $m = 10$.

Substitute in $y = mx + c$.

The function is $C = 10h + 45$.

b Substitute $h = 3$.

$$C = 10 \times 3 + 45 = 75$$

Write the answer.

It costs \$75 to hire a windsurfer for 3 hours.

c Substitute $C = 90$ in the function.

$$90 = 10h + 45$$

Subtract 45 from both sides.

$$10h + 45 - 45 = 90 - 45$$

Simplify.

$$10h = 45$$

Divide by 10.

$$h = 4.5$$

Write the answer.

You could hire it for $4\frac{1}{2}$ hours.

TLF Learning object

Exploring linear equations (L6553)

MAT09NAIN00010

Example 10

Write the equation of the lines with the properties below in standard form.

a gradient = 7 and y -intercept = -2

b gradient = $-\frac{3}{4}$ and y -intercept = $(0, 5)$

Solution

a Write the gradient–intercept form of an equation. $y = mx + c$

Substitute $m = 7$ and $c = -2$.

$$y = 7x + -2$$

Simplify.

$$y = 7x - 2$$

Move all the terms to the LHS.

$$y - 7x + 2 = 7x - 2 - 7x + 2$$

Simplify.

$$y - 7x + 2 = 0$$

Write with the x term first.

$$-7x + y + 2 = 0 \text{ or } 7x - y - 2 = 0$$

b Write the gradient–intercept form.

$$y = mx + c$$

Substitute $m = -\frac{3}{4}$ and $c = 5$.

$$y = -\frac{3}{4}x + 5$$

Multiply by 4 to get rid of the fraction.

$$4 \times y = 4 \times -\frac{3}{4}x + 4 \times 5$$

Simplify.

$$4y = -3x + 20$$

Move all the terms to the LHS.

$$4y + 3x - 20 = -3x + 20 + 3x - 20$$

Put the x term first.

$$3x + 4y - 20 = 0$$

Puzzle sheet

Equation of a line

MAT09NAPS00046

Example 11

Find the gradient and the y -intercept of each of these lines.

a $y = -4x + 9$

b $3x + 2y - 6 = 0$

Solution

a Write the equation.

$$y = -4x + 9$$

Compare to the gradient–intercept form.

$$y = mx + c$$

Write the gradient and y -intercept.

The gradient, $m = -4$ and the y -intercept, $c = 9$.

b Write the equation.

$$3x + 2y - 6 = 0$$

Isolate the y -term.

$$3x + 2y - 6 - 3x + 6 = -3x + 6$$

Simplify.

$$2y = -3x + 6$$

Divide by 2.

$$\frac{2y}{2} = \frac{-3x}{2} + \frac{6}{2}$$

Simplify.

$$y = -\frac{3}{2}x + 3$$

Compare to the gradient–intercept form.

$$y = mx + c$$

Write the gradient and y -intercept.

The gradient, $m = -\frac{3}{2}$ and the y -intercept, $c = 3$.

Exercise 10.1 Coordinates and lines

Understanding

Extra questions

Exercise 10.1

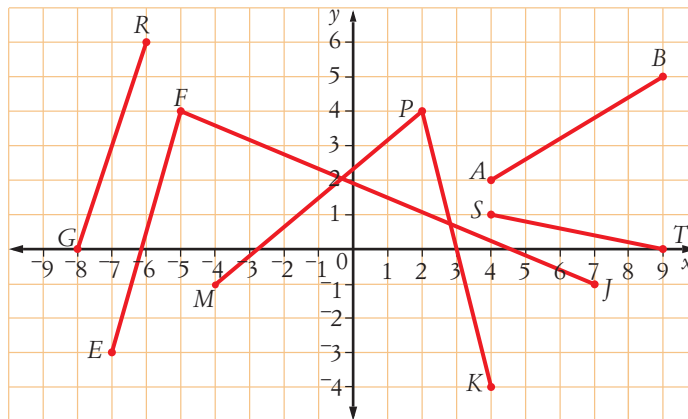
MAT09NAEQ00033

See Example 3

- 1 For each of the following lines, state whether each of the given points is on the line.

a $3x + 2y + 8 = 0$	$(-4, 2)$	$(3, 4)$	$(-2, 1)$	$(2, -7)$	$(4, 0)$
b $y = x - 3$	$(-1, 2)$	$(1, -2)$	$(3, 0)$	$(-2, 5)$	$(-3, -6)$
c $y = 2x + 6$	$(0, -6)$	$(2, 8)$	$(1, 14)$	$(-2, 2)$	$(3, 12)$
d $y = -3x + 2$	$(1, 1)$	$(-1, 3)$	$(3, -7)$	$(0, -2)$	$(-4, 14)$
e $3x - y - 6 = 0$	$(-4, 6)$	$(0, -3)$	$(2, 0)$	$(10, -12)$	$(-4, -18)$

- 2 Find the gradient of each of the line segments AB , ST , GR , PK , PM , EF and FJ shown below.



See Example 4

- 3 Find the gradients of the line segments through each of the following pairs of points.

a $(2, 3)$ and $(8, 5)$	b $(4, 0)$ and $(7, 4)$
c $(-5, 5)$ and $(-2, 1)$	d $(-9, -4)$ and $(-4, 1)$
e $(1, 1)$ and $(5, -4)$	f $(-4, -3)$ and $(2, 1)$
g $(-8, -1)$ and $(2, -4)$	h $(-7, 2)$ and $(4, 5)$
i $(-6, -3)$ and $(-1, 3)$	

See Example 8

- 4 Find the equations of the lines parallel to the x -axis and y -axis that pass through each of the following.

a $(3, -2)$	b $(-1, 5)$	c $(3, 6)$

Fluency

See Example 1

- 5 Plot each of the following sets of points and describe the relationships.

a

x	-2	0	2	4	6	8
y	-4	-1	2	5	8	11

b

x	-2	-1	0	1	2
y	4	3	2	1	0

c

x	-4	-2	0	2	4	6
y	4	1	-2	-5	-8	-11

d

x	-4	-2	0	2	4
y	-5	-1	3	7	11

6 Use tables of values to plot each of the following functions for the stated values of x .

See Example 2

- a** $y = 2x - 3$ for $-2 \leq x \leq 3$ **b** $y = x + 1$ for $-3 \leq x \leq 2$
c $y = -x + 2$ for $-1 \leq x \leq 4$ **d** $y = -2x - 3$ for $-3 \leq x \leq 2$
e $y = 3x - 1$ for $-1 \leq x \leq 3$ **f** $y = -x - 3$ for $-4 \leq x \leq 1$

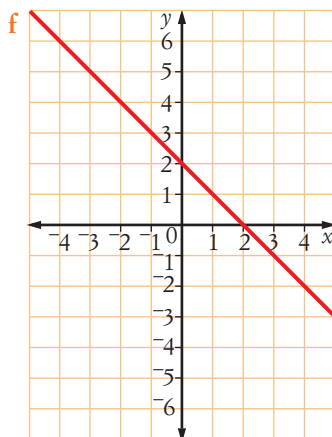
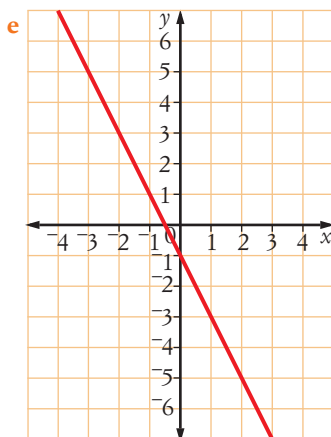
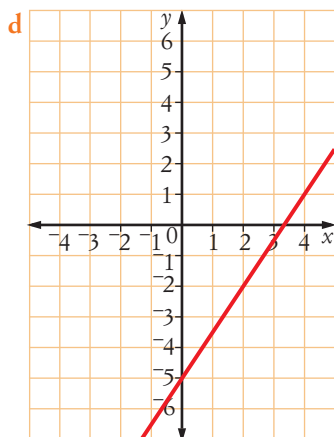
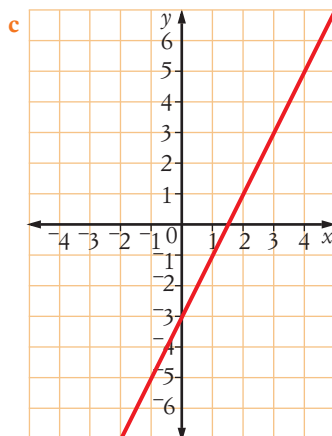
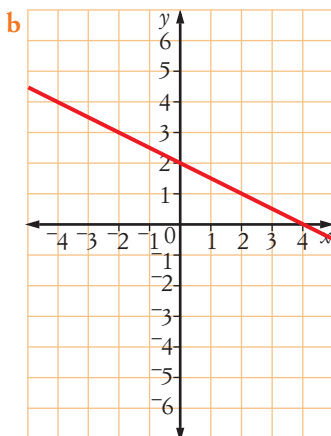
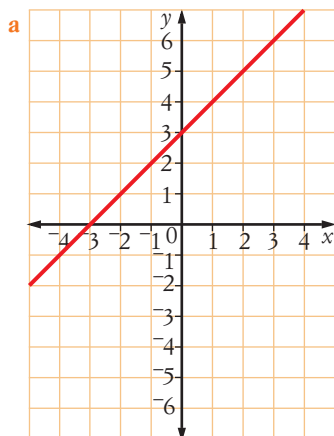
7 Draw graphs of each of the following lines for suitable values of x . Choose two points to calculate the gradient and find the intercepts.

See Example 5

- a** $y = 3x - 2$ **b** $y = x + 4$ **c** $y = -2x + 3$ **d** $x + y + 1 = 0$
e $y = -\frac{1}{4}x + 1$ **f** $2y = 3x - 4$ **g** $2y = 4x + 1$ **h** $3y = -2x - 3$
i $5y = -7x + 8$ **j** $4x - y + 8 = 0$ **k** $x + 3y - 4 = 0$ **l** $4x - 3y = 0$

8 Find the equations of each of the lines shown below.

See Example 7



9 Write the equation of each of the lines with the properties below in gradient-intercept form.

- a** gradient = 2 and y -intercept = $(0, 4)$ **b** gradient = 3 and y -intercept = $(0, 1)$
c y -intercept = -6 and gradient = 5 **d** gradient = 3 and y -intercept = -2
e y -intercept = $(0, -3)$ and gradient = 0 **f** gradient = -4 and y -intercept = 0

10 Write the equation of each of the lines with the properties below in standard form.

See Example 10

- a** gradient = 4 and y -intercept = -1 **b** gradient = $\frac{2}{3}$ and y -intercept = $(0, -2)$
c y -intercept = -1 and gradient = $\frac{4}{3}$ **d** y -intercept = $(0, 2)$ and gradient = $2\frac{1}{2}$
e gradient = $3\frac{1}{3}$ and y -intercept = -4 **f** y -intercept = $1\frac{1}{4}$ and gradient = $-1\frac{3}{4}$

See Example 11

- 11 Find the gradient and
- y
- intercept of each of the lines below.

a $y = 2x + 5$

b $y = 5 - 3x$

c $y = -x + 1$

d $y = 4 - 7x$

e $y = x + 2$

f $y = -8x$

g $x - 2y + 1 = 0$

h $7x + y - 14 = 0$

i $x - 3y + 6 = 0$

j $3y - 8 = -4x$

k $7x = 3y$

Problem solving

See Example 6

- 12 For each of the following sets of points, check whether or not a function passing through the points is linear, and if it is, find its equation.

a $A(-3, 8), B(1, 0), C(2, -2), D(5, -8)$

b $P(-2, 1), Q(1, 3), R(2, 5), S(3, 6)$

c $F(-4, -9), G(-2, -5), H(1, 1), I(5, 9)$

d $A(-4, -3), C(-3, -1.5), P(2, 6), Q(4, 9)$

e $V(-5, 7), W(-3, 3), X(2, -7), Y(4, -11)$

See Example 9

- 13 It costs a handyman \$40, plus \$15 a day to hire a set of aluminium trestles and an aluminium plank to do some painting.

a Write the cost of hire C as a function of the time hired.**b** What was the cost of hire for 6 days?**c** How long could he hire them for \$100?

Worked solutions

Exercise 10.1

MAT09NAWS00033

- 14 It costs \$700 to buy the food and hire equipment for a hamburger stall at a fete and the hamburgers are cooked and sold by volunteers for \$5 each. There is enough to make a total of 300 hamburgers.

a Write a function for the profit P when h hamburgers are sold on the day.**b** What happens if they sell:**i** 100 hamburgers**ii** 200 hamburgers?**c** How many hamburgers would they need to sell to make a profit of \$400?**d** They actually sell 255 hamburgers at full price, and near the end of the day sell the rest for \$2 each. How much profit is made?

Worked solutions

Exercise 10.1

MAT09NAWS00033

- 15 The gradient of a railway line is given on signs by the side of the track. For example,
- $2/35$
- means a rise of 2 in a run of 35. Which is the steeper slope:
- $2/35$
- or
- $1/60$
- ?



Worked solution

Exercise 10.1

MAT09NAWS00033

- 16 Find the equation, in both gradient–intercept and standard forms, of these lines.

a with gradient 2, passing through $(2, 5)$ **b** through $(-1, 4)$ with gradient -1 **c** through $(-3, -2)$ with gradient 1**d** with gradient 4, passing through $(1, -4)$

Reasoning

- 17 Find a formula for the gradient of a line passing through the points
- (a, b)
- and
- (c, d)
- .

10.2 Distance and midpoint

When you are using coordinates in geometry, you may need to find the midpoint of two given points.

Investigate: Midpoint of a line segment

Use graph paper for this investigation.

Plot the points $(-2, 5)$ and $(4, -4)$ on your graph paper. Draw a line segment between the points and find the point halfway along it.

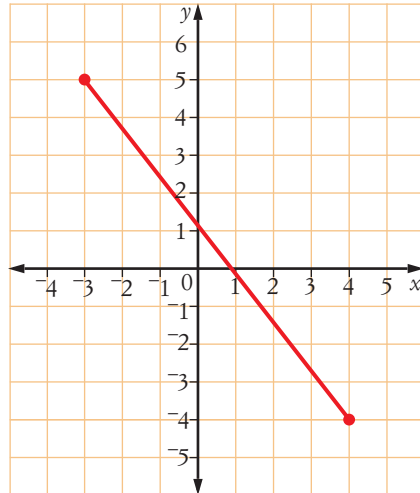
Write down this point. It is the midpoint of the line segment between the points $(1, 4)$ and $(9, 10)$.

Use the same method to find the midpoints of the line segments between:

- $(2, 2)$ and $(5, 8)$
- $(-2, -6)$ and $(4, -3)$
- $(2, 5)$ and $(-1, 0)$

Now look at each pair of points and the midpoint of the line segment.

What pattern do you find?



You will have found that the coordinates of the midpoint are the averages of the coordinates of the points. This means that the midpoint of $(1, 6)$ and $(7, 9)$ is given by

$$\left(\frac{1+7}{2}, \frac{6+9}{2}\right) = \left(4, 7\frac{1}{2}\right).$$

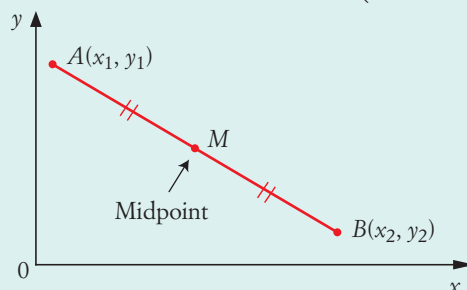
You can write this as a formula, as shown below.

Important!

Midpoint of a line segment

The values of the coordinates of the **midpoint** of a line segment between two points are the averages of the coordinates of the points.

The midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.



Example 12

State the midpoint formula for (a, b) and (c, d) .

Solution

Identify the two points.

Choose $(a, b) = (x_1, y_1)$ and $(c, d) = (x_2, y_2)$.

Write the formula.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{a + c}{2}, \frac{b + d}{2} \right)$$

Substitute values ($x_1 = a$, $x_2 = c$, etc.).

Example 13

Find the midpoint of $(4, 10)$ and $(7, -2)$.

Solution

Identify the two points.

Choose $(4, 10) = (x_1, y_1)$ and $(7, -2) = (x_2, y_2)$.

Write the formula.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{4 + 7}{2}, \frac{10 + -2}{2} \right)$$

Substitute values ($x_1 = 4$, $x_2 = 7$, etc.).

$$= \left(5\frac{1}{2}, 4 \right)$$

Calculate the result.

Write the answer.

The midpoint of $(4, 10)$ and $(7, -2)$ is $(5\frac{1}{2}, 4)$.

Example 14

The midpoint of A and B is $(5, 4)$. If $A = (-2, 6)$ find B .

Solution

Identify the two points.

Choose $A = (-2, 6) = (x_1, y_1)$ and $B = (x_2, y_2)$.

Identify the midpoint.

$$M = (5, 4)$$

Write the formula.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$(5, 4) = \left(\frac{-2 + x_2}{2}, \frac{6 + y_2}{2} \right)$$

Substitute into the formula.

Write equations for x_2 and y_2 .

$$\frac{-2 + x_2}{2} = 5 \text{ and } \frac{6 + y_2}{2} = 4$$

Solve for x_2 and y_2 .

$$-2 + x_2 = 10 \text{ and } 6 + y_2 = 8$$

Keep going.

$$x_2 = 12 \text{ and } y_2 = 2$$

Write the answer.

$$B = (12, 2)$$

Technology

Excel worksheet:
Finding midpoints

MAT09NACT00027

The distance between two points is often used in science and technology applications. For example, if you want to know the overall result of forces acting in different directions, you will need to find the distance between points.

Since the coordinate system is based on perpendicular axes, you can use Pythagoras' theorem. Suppose you have two points $A(1, 2)$ and $B(6, 14)$.

Plot them on a grid as shown below on the right, and draw extra lines to make the right-angled triangle $\triangle ABC$.

In $\triangle ABC$:

$$AC = 6 - 1 = 5$$

$$BC = 14 - 2 = 12$$

Using Pythagoras' theorem:

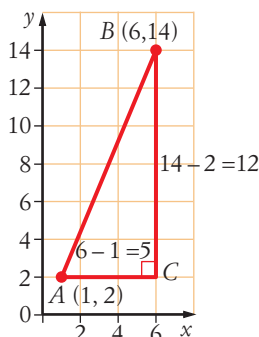
$$AB^2 = AC^2 + BC^2$$

$$= 5^2 + 12^2$$

$$\text{So } AB = \sqrt{5^2 + 12^2}$$

$$= \sqrt{169}$$

$$= 13$$



Now suppose $A = (x_1, y_1)$ and $B = (x_2, y_2)$, as shown in the diagram on the right.

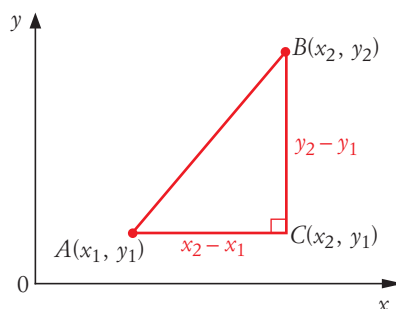
$$AC = x_2 - x_1$$

$$BC = y_2 - y_1$$

$$AB^2 = AC^2 + BC^2$$

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

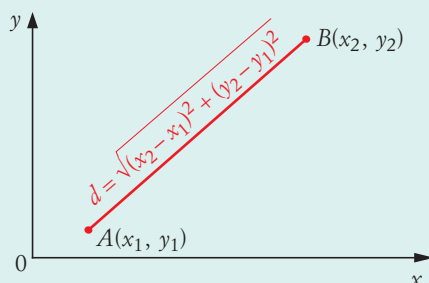


Important!

Distance formula

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 15

State the distance formula for $E(e, f) = (x_1, y_1)$ and $G(g, h) = (x_2, y_2)$.

Solution

Identify the two points.

Choose $E(e, f) = (x_1, y_1)$ and $G(g, h) = (x_2, y_2)$.

Write the formula.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute values ($x_1 = a$, $x_2 = c$, etc).

$$EG = \sqrt{(e - g)^2 + (f - h)^2}$$

Example 16

Find the distance between the points $(3, 8)$ and $(5, -2)$ correct to 2 decimal places.

Solution

Identify the points.

Choose $A = (3, 8) = (x_1, y_1)$ and
 $B = (5, -2) = (x_2, y_2)$.

Write the formula.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute the values.

$$= \sqrt{(5 - 3)^2 + (-2 - 8)^2}$$

Work out the brackets.

$$= \sqrt{2^2 + (-10)^2}$$

Find the squares.

$$= \sqrt{4 + 100}$$

Add.

$$= \sqrt{104}$$

Round to 2 decimal places.

$$\approx 10.20$$

On a calculator:

Enter as:

$$\sqrt{\quad} \left(\left(5 - 3 \right)^2 + \left(-2 - 8 \right)^2 \right) =$$

$$\sqrt{[(5-3)^2 + (-2-8)^2]} \\ 10.19803903$$

Write the answer.

The distance from $(3, 8)$ to $(5, -2)$ is
about 10.20.

Video tutorial

The length of an
interval

MAT09NAVT10018

Puzzle sheet

Length of a line
segment

MAT09NAPS00047

Exercise 10.2 Distance and midpoint

1 State the midpoint formula for each of the following pairs of points.

- a** (v, w) and (x, y) **b** (p, r) and (q, s) **c** (k, m) and (n, e)
d (r, s) and (t, u) **e** (d, a) and (g, f)

2 State the distance formula for each of the following pairs of points.

- a** (v, w) and (x, y) **b** (p, r) and (q, s) **c** (k, m) and (n, e)
d (r, s) and (t, u) **e** (d, a) and (g, f)

3 Find the midpoint of each of the following pairs of points.

- a** $(3, -2)$ and $(7, 4)$ **b** $(1, 5)$ and $(-3, -4)$
c $(4, 5)$ and $(-4, 7)$ **d** $(3, -3)$ and $(-2, 4)$
e $(-3, 6)$ and $(-7, 4)$ **f** $(-3, 9)$ and $(4, -6)$
g $(-7, 0)$ and $(0, 5)$ **h** $(0, 0)$ and $(9, -4)$
i $(-3, -9)$ and $(-6, -7)$ **j** $(4, 2)$ and $(-3, -8)$

4 Find the distance between each of the following pairs of points.

- a** $(3, 6)$ and $(5, 8)$ **b** $(-2, 4)$ and $(4, 0)$
c $(7, -4)$ and $(-5, 1)$ **d** $(8, 6)$ and $(-7, 13)$
e $(-3, -5)$ and $(6, 4)$ **f** $(6, 2)$ and $(-3, -7)$
g $(4, 1)$ and $(-2, -4)$ **h** $(-3, 5)$ and $(-1, -7)$
i $(6, 1)$ and $(3, -7)$ **j** $(5, -5)$ and $(-3, 8)$

5 The midpoint of AB is $(-5, 3)$. If A is $(2, 9)$, find B .

6 The midpoint of PQ is $(8, 7)$. If P is $(5, 10)$, find Q .

7 The midpoint of RT is $(-4, -5)$. If T is $(-10, 3)$, find R .

8 Show that the points $A(4, 2)$, $B(8, 5)$, $C(5, 1)$ and $D(1, -2)$ form a rhombus.

9 Show that the points $A(-1, 9)$, $B(8, 21)$, $C(13, 9)$ and $D(8, -3)$ form a rhombus.

10 Show that it doesn't matter which way around (x_1, y_1) and (x_2, y_2) are chosen for two given points, the answer for the midpoint is the same.

Understanding

Extra questions

Exercise 10.2

MAT09NAEQ00034

See Example 12

See Example 15

Fluency

See Example 13

See Example 16

Problem solving

See Example 14

Worked solutions

Exercise 10.2

MAT09NAWS00034

Reasoning

Worked solutions

Exercise 10.2

MAT09NAWS00034

10.3 Drawing lines and graphs

It is often the case that you do not need to make an exact drawing of a line or graph, but only need a general idea of the shape, with a few points plotted and their coordinates labelled.

For a straight line, two points are sufficient to determine the line. If the graph does not pass through the origin, it is usually convenient to find the intercepts and sketch the line using these as a guide. The x -intercept is found by substituting $y = 0$ into the equation and the y -intercept by substituting $x = 0$ into the equation.

When we sketch a graph, we do not usually draw in a grid.

Example 17

Worksheet

Sketch graphs of
straight lines

MAT09NAWK00060

Find the intercepts of $4x - 3y = 6$.**Solution**Substitute $y = 0$ in the equation.

$$4x - 3 \times 0 = 6$$

Simplify.

$$4x = 6$$

Divide by 4.

$$x = 1\frac{1}{2}, \text{ so the } x\text{-intercept is } (1\frac{1}{2}, 0)$$

Substitute $x = 0$ in the equation.

$$4 \times 0 - 3y = 6$$

Simplify.

$$-3y = 6$$

Divide by -3 .

$$y = -2, \text{ so the } y\text{-intercept is } (0, -2)$$

State the answer.

$$\text{The } x\text{-intercept is } (1\frac{1}{2}, 0) \text{ and the } y\text{-intercept is } (0, -2).$$

Example 18

Worksheet

More sketch graphs

MAT09NAWK00061

Use the intercepts to sketch the graph of $y = 3x + 6$.**Solution**Find the x -intercept.

$$0 = 3x + 6$$

Solve for x .

$$x = -2, \text{ so the } x\text{-intercept is } -2$$

Compare with $y = mx + c$.

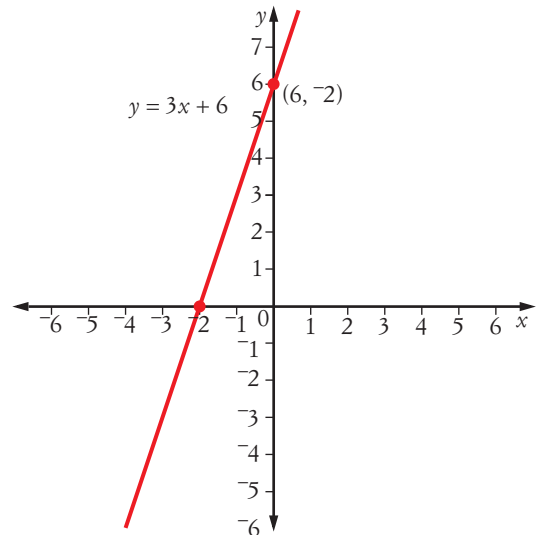
$$y\text{-intercept} = c = 6$$

Draw the axes.

Mark the intercepts at their approximate positions.

Rule a line through the points.

Write the equation next to the line.



If the intercepts could be fractions or they are very close together, it may be easier to use points that are well separated, whose coordinates are whole numbers.

Example 19

Use convenient points to sketch $3x + 2y = 8$.

Solution

Even values of x will give whole numbers for y .

Substitute into the equation.

Simplify.

Solve each equation.

State the points.

Draw the axes.

Mark the points at their approximate positions.

Rule a line through the points.

Write the equation next to the line.

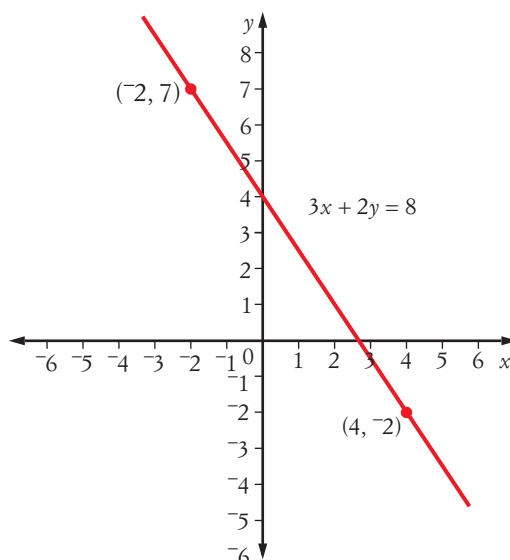
Choose $x = -2$ and $x = 4$.

$$3 \times -2 + 2y = 8 \text{ and } 3 \times 4 + 2y = 8$$

$$-6 + 2y = 8 \text{ and } 12 + 2y = 8$$

$$y = 7 \text{ and } y = -2$$

$(-2, 7)$ and $(4, -2)$ are on the line.



You can use a linear graph to solve related equations. Sometimes this is as simple as looking up a value on the graph.

Example 20

- a Sketch the graph of $y = 2x - 3$.
- b Use your graph to solve the equations
 - i $2x - 3 = 7$,
 - ii $2x - 3 = -7$.

Solution

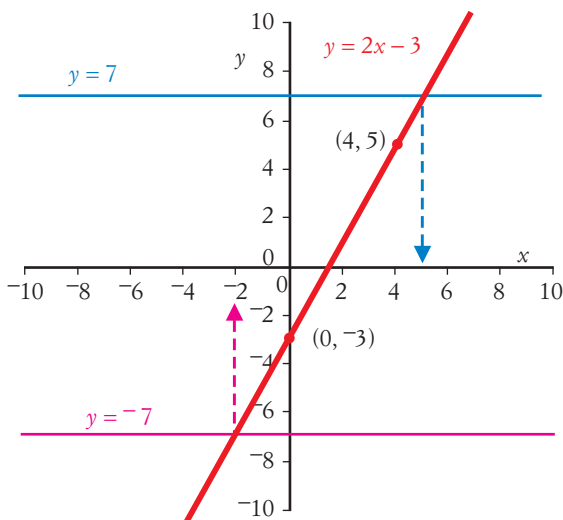
- a Compare with $y = mx + c$.
Choose another point.

The y -intercept $(0, -3)$ is on the graph.

For $x = 4$, $y = 2 \times 4 - 3 = 5$, so $(4, 5)$ is also a point.

Sketch the graph.

Sketch the lines $y = 7$ and $y = -7$.



- b** Find the x values for the intersections.

i $x = 5$ and **ii** $x = -2$

Last year, you may have looked at graphs of quadratic functions like $y = x^2 + x - 6$. These have a shape called a **parabola**. You can use graphs of quadratic functions to solve **quadratic equations** in the same way as you can use linear graphs to solve linear equations.

Example 21

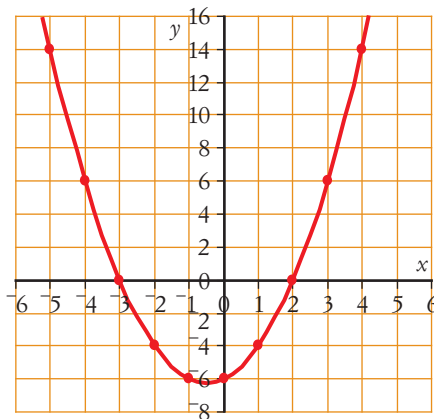
- a** Plot points to draw the graph of $y = x^2 + x - 6$ and describe the graph.
b Use the graph to solve the equation $x^2 + x = 6$.
c Solve the equation $x^2 + x = 20$.

Solution

- a** Find values for $-5 \leq x \leq 5$.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	14	6	0	-4	-6	-6	-4	0	6	14	24

Plot points and join with a smooth line.



Find the value at $x = -\frac{1}{2}$

$$y = \left(-\frac{1}{2}\right)^2 + \frac{1}{2} - 6 = -6\frac{1}{4}$$

Describe the graph.

The graph is a parabola symmetrical about the line $x = -\frac{1}{2}$ with y -intercept $(0, -6)$, x -intercepts $(-3, 0)$ and $(2, 0)$ and turning point $-\frac{1}{2}, -6\frac{1}{4}$

b Write the equation.

$$x^2 + x = 6$$

Rewrite using the

equation $y = x^2 + x - 6$.

$$x^2 + x - 6 = 0$$

Describe it.

$x^2 + x - 6 = 0$ is the equation for $y = 0$, the x -intercepts

Write the answer.

$x^2 + x - 6 = 0$ has the solutions $x = -3$ or $x = 2$

c Write the equation.

$$x^2 + x = 20$$

Rewrite using the

equation $y = x^2 + x - 6$.

$$x^2 + x - 6 = 14$$

Describe it.

$x^2 + x - 6 = 14$ is the equation for $y = 14$

Use the graph.

$y = 14$ at the points $(-5, 14)$ or $(4, 14)$

Write the answer.

$x^2 + x = 20$ has the solutions $x = -5$ or $x = 4$

Investigate: Drawing parabolas

Use values of x between -6 and 6 to draw each of the following quadratic functions.

1 $y = x^2$

2 $y = -x^2$

3 $y = x^2 + 3$

4 $y = -x^2 + 5$

5 $y = x^2 - 4$

6 $y = -x^2 - 2$

7 $y = (x + 1)^2 - 3$

8 $y = -(x - 2)^2 + 4$

Use your graphs to answer the following questions.

- What shape is the graph of a parabola with a positive x^2 term?
- What shape is the graph of a parabola with a negative x^2 term?
- Where is the axis of symmetry in $y = (x - a)^2$?
- Where is the turning point in $y = (x - a)^2 + b$?

You can use the distance formula developed in the last section to work out the equation of a circle.

Consider the point (x, y) on the circumference of a circle of radius 4 with centre at the origin. Since the circle has radius 4, the distance between the centre at $(0, 0)$ and the point (x, y) is 4.

From the distance formula, this is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

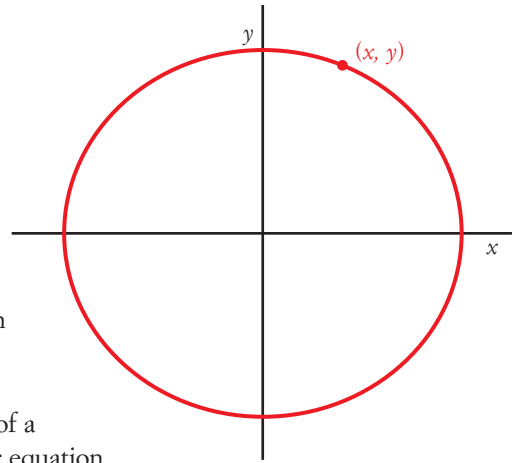
Substituting the values gives

$$\sqrt{(x - 0)^2 + (y - 0)^2} = 4.$$

Simplifying and squaring both sides gives the equation $x^2 + y^2 = 4^2$.

No assumptions were made about the exact position of (x, y) except that it was on the circumference.

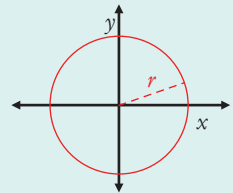
The equation must apply to *any* point on the circumference. Furthermore, it is clear that instead of a radius of 4, you could use any radius to get a similar equation.



Important!

Equation of a circle

The equation of a circle with centre at the origin and radius r is given by $x^2 + y^2 = r^2$.



Example 22

Draw a graph of $x^2 + y^2 = 25$

Solution

Write the equation.

$$x^2 + y^2 = 25$$

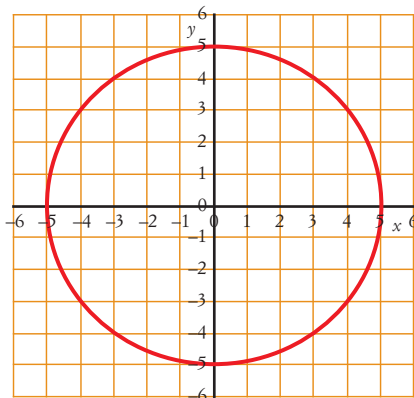
Write in the form $x^2 + y^2 = r^2$.

$$x^2 + y^2 = 5^2$$

Describe the relation.

It is a circle of radius 5 with centre at the origin.

Draw the graph.



CAS TI-Nspire exercise

Coordinates and graphs

MAT09NATI00010

CAS ClassPad exercise

Coordinates and graphs

MAT09NACP00010

TLF Learning object

Grapher (L3531)

MAT09NAIN00010

Technology

Excel worksheet:
Curve sketcher

MAT09NACT00028

Exercise 10.3 Drawing lines and graphs

1 Find the intercepts of each of the following.

a $3x + 4y = 12$

b $4x - y = 8$

c $y = x - 3$

d $x + 2y = 5$

e $3x + 2y + 5 = 0$

Understanding

Extra questions

Exercise 10.3

MAT09NAEQ00035

See Example 17

2 Use the intercepts to sketch the graph of $y = 2x - 4$.

3 Use the intercepts to sketch the graph of $y = -4x + 12$.

4 Use the intercepts to sketch the graph of $2y - 3x = 12$.

5 Use convenient points to sketch the graph of $4x - 3y = 5$.

6 Use convenient points to sketch the graph of $5x + 2y = 8$.

7 Use convenient points to sketch the graph of $3x - 5y + 7 = 0$.

8 Draw the parabola with the equation $y = -x^2 + 2$ from $x = -4$ to 4.

9 Draw the parabola with the equation $y = x^2 + 2$ from $x = -4$ to 4.

10 Draw the parabola with the equation $y = -x^2 + 4$ from $x = -4$ to 4.

11 Draw the parabola with the equation $y = x^2 - 3$ from $x = -4$ to 4.

12 Draw the circle with the equation $x^2 + y^2 = 4$.

13 Draw the circle with the equation $x^2 + y^2 = 64$.

14 Draw the circle with the equation $x^2 + y^2 = 49$.

Fluency

See Example 18

See Example 19

See Example 21

See Example 22

15 Sketch the graph of $y = 2x - 5$ and use it to solve $2x - 5 = 7$.

16 Sketch the graph of $y = 4 - 3x$ and use it to solve $3x - 4 = 11$.

17 Sketch the graph of $y = 4x + 1$ and use it to solve $4x + 1 = 9$.

18 Draw the graph of $y = x^2 - x - 3$ from $x = -4$ to 4 and use it to solve $x^2 - x - 3 = 0$.

19 Draw the graph of $y = x^2 + 3x - 4$ from $x = -5$ to 5 and use it to solve $x^2 + 3x = 4$.

20 Draw the graph of $y = x^2 + 3x + 2$ from $x = -3$ to 3 and use it to solve $x^2 + 3x + 2 = 0$.

21 Draw the graph of $y = x^2 - x - 12$ from $x = -5$ to 5 and use it to solve $x^2 = x + 12$.

22 Draw the graph of $y = x^2 + 3x - 10$ from $x = -6$ to 6 and use it to solve $x^2 + 3x = 10$.

23 Draw the graph of $y = 2x - x^2 + 8$ from $x = -5$ to 5 and use it to solve $x^2 = 2x + 8$.

24 Draw the graph of $y = 5x - x^2 - 6$ from $x = -4$ to 4 and use it to solve $5x = x^2 + 6$.

25 Draw the circles with equations $x^2 + y^2 = 16$ and $x^2 + y^2 = 36$ on the same axes.

26 Draw the circle with the equation $x^2 + y^2 = 10$.

27 Explain how many times $x^2 + y^2 = 4$ and $y = x^2$ intersect.

28 Explain how many times $y = x^2 - 2$ and $x^2 + y^2 = 4$ intersect.

Problem solving

See Example 20

Worked solutions

Exercise 10.3

MAT09NAWS00035

See Example 21

Worked solutions

Exercise 10.3

MAT09NAWS00035

Worked solutions

Exercise 10.3

MAT09NAWS00035

Reasoning

10.4 Parallel and perpendicular lines

Technology Straight lines with the same gradient

Sketch each of the following pairs of straight lines with the same gradient on the same axes.

1 $y = 2x + 3$ $y = 2x - 1$

2 $2x + 3y = 6$ $2x + 3y = 12$

3 $y = 5 - \frac{1}{2}x$ $y = -2 - \frac{1}{2}x$

What do you find?

Important!

Parallel lines

Parallel lines have the same gradient. $m_1 = m_2$.

Example 23

Are any of the following lines parallel?

L1: $y = 3x + 5$

L2: $6x + 2y = 5$

L3: $3x = y + 4$

Solution

State the gradient of L1: $y = 3x + 5$.

Rearrange the equation of L2.

Divide by 2

State the gradient.

Rearrange the equation of L3.

State the gradient.

Compare the gradients to write the answer.

The line L1 has a gradient $m_1 = 3$.

$$6x + 2y = 5$$

$$2y = -6x + 5$$

$$y = -3x + 2\frac{1}{2}$$

L2: $6x + 2y = 5$ has a gradient $m_2 = -3$.

$$3x = y + 4$$

$$y = 3x - 4$$

L3: $3x = y + 4$ has a gradient $m_3 = 3$.

L1 and L3 have the same gradient, so they are parallel.

Investigate: Gradients of perpendicular lines

Use grid paper for this investigation and mark axes from -10 to 10 on the paper at the same scale.

Plot each of the following pairs of lines on your grid paper.

1 $y = 2x - 2$, $y = -\frac{1}{2}x + 3$

2 $y = x + 2$, $y = 3 - x$

3 $3x + 2y + 6 = 0$, $2x - 3y + 6 = 0$

4 $2x + 6y - 9 = 0$, $3x - y + 9 = 0$

For each pair of lines, do the following.

- Measure the angle between the lines.
- Find the gradient of each line.
- Calculate the product of the gradients of the lines.

What do you find?

Important!

Perpendicular lines

The product of the gradients of **perpendicular lines** is -1 .

$$m_1 \times m_2 = -1 \text{ or } m_2 = \frac{-1}{m_1}$$

Example 24

Are any of the following lines perpendicular?

L1: $y = 4x + 2$

L2: $x + 4y - 6 = 0$

L3: $4x - 2y + 3 = 0$

Solution

State the gradient of L1: $y = 4x + 2$.

Rearrange the equation of L2.

Divide by 4

State the gradient of L2: $x + 4y - 6 = 0$.

Rearrange the equation of L3.

Divide by 2.

The line L1 has a gradient $m_1 = 4$.

$$x + 4y - 6 = 0$$

$$4y = -x + 6$$

$$y = -\frac{1}{4}x + 1\frac{1}{2}$$

The line L2 has a gradient $m_2 = -\frac{1}{4}$.

$$2y = -4x + 3$$

$$y = -2x + 1\frac{1}{2}$$

State the gradient of L3: $3x = y + 4$.

Try multiplying the gradients.

Write your conclusion.

L3 has a gradient $m_3 = -2$.

$$m_1 \times m_2 = -1, m_1 \times m_3 = -8, m_2 \times m_3 = -\frac{1}{2}$$

L1: $y = 4x + 2$ and L2: $x + 4y - 6 = 0$ are perpendicular.

A line parallel to the x -axis is perpendicular to a line parallel to the y -axis.

Example 25

Puzzle sheet

Find the gradient 2

MAT09NAPS00048

Find the equation of a line passing through $(2, 5)$

a parallel to $x = 4$

b perpendicular to $x = 4$.

Solution

a $x = 4$ is parallel to the y -axis.

The line is of the form $x = a$.

b $x = 4$ is parallel to the y -axis.

The line is of the form $y = b$.

The required line is parallel to the y -axis.

The line is $x = 2$.

The required line is parallel to the x -axis.

The line is $y = 5$.

Example 26

Find the equation of a line passing through $(0, 4)$ that is parallel to $3x + y = 7$.

Solution

Find the gradient of the line given.

$$3x + y = 7$$

Rearrange the equation.

$$y = -3x + 7$$

State the gradient.

The gradient of the given line is $m_1 = -3$.

Give the requirements for the new line.

For the new line $m_2 = -3$ and $c = 4$.

Use the format $y = mx + c$.

The required line is $y = -3x + 4$ or $3x + y - 4 = 0$.

Example 27

Animated example

Parallel and perpendicular lines

MAT09NAAE00010

Find the equation of a line with y -intercept -2 that is perpendicular to $2x - 3y = 1$.

Solution

Find the gradient of the line given.

$$2x - 3y = 1$$

Rearrange the equation.

$$3y = 2x - 1$$

Divide by 3.

$$y = \frac{2}{3}x - \frac{1}{3}$$

State the gradient.

The gradient of the given line is $m_1 = \frac{2}{3}$.

Find the gradient of the new line.

$$m_2 = \frac{-1}{m_1} = \frac{-3}{2}$$

Give the requirements for the new line.

For the new line $m_2 = \frac{-3}{2}$ and $c = -2$.

Use the format $y = mx + c$.

The required line is $y = \frac{-3}{2}x - 2$ or $3x + 2y + 4 = 0$.

Exercise 10.4 Parallel and perpendicular lines

- 1 Determine if any of the following lines are parallel or perpendicular.

L1: $2x - y - 2 = 0$

L2: $2x - 3y - 9 = 0$

L3: $2x - y - 3 = 0$

L4: $2x - 3y - 6 = 0$

L5: $x - 2y + 10 = 0$

L6: $2x + 3y - 12 = 0$

- 2 Determine if any of the following lines are parallel or perpendicular.

L1: $4x - 3y - 6 = 0$

L2: $3x + 4y + 12 = 0$

L3: $3x + 4y + 16 = 0$

L4: $2x + 3y + 9 = 0$

L5: $2x - 3y - 6 = 0$

L6: $3x - 4y - 12 = 0$

- 3 Find the equation of a line passing through $(5, -3)$

a parallel to $x = -3$

b perpendicular to $x = -3$.

- 4 Find the equation of a line passing through $(-3, 6)$

a parallel to $y = 2$

b parallel to $x = 1$.

- 5 Find the equation of a line passing through $(-1, -7)$

a perpendicular to $x = -6$

b perpendicular to $y = -2$.

- 6 Find the equation of a line with y -intercept 3 that is parallel to $3x + 2y + 2 = 0$.

- 7 Find the equation of a line passing through $(0, -2)$ that is parallel to $2x - 5y = 3$.

- 8 Find the equation of a line with y -intercept -1 that is perpendicular to $2x - y - 5 = 0$.

- 9 Find the equation of a line passing through $(0, 4)$ that is perpendicular to $3x + 6y + 4 = 0$.

- 10 Find the equations of lines parallel and perpendicular to $2x + y = 8$ that pass through $(2, 6)$.

- 11 Find the equations of lines parallel and perpendicular to $3x - 4y + 5 = 0$ that pass through $(-3, 3)$.

- 12 Find the equations of lines parallel and perpendicular to $4x + 5y - 3 = 0$ that pass through $(5, -2)$.

- 13 Show that the points $A(3, 6)$, $B(2, -2)$ and $C(-1, 4)$ form a right-angled triangle.

- 14 Show that the points $A(2, 1)$, $B(6, 6)$, $C(8, 0)$ and $D(4, -5)$ form a parallelogram.

- 15 Show that the points $A(-1, 8)$, $B(7, 2)$, $C(4, -2)$ and $D(-4, 4)$ form a rectangle and find its area.

Understanding

Extra questions

Exercise 10.4

MAT09NAEQ00036

See Example 24

See Example 25

Fluency

See Example 26

See Example 27

Problem solving

Worked solutions

Exercise 10.4

MAT09NAWS00036

Reasoning

Worked solutions

Exercise 10.4

MAT09NAWS00036

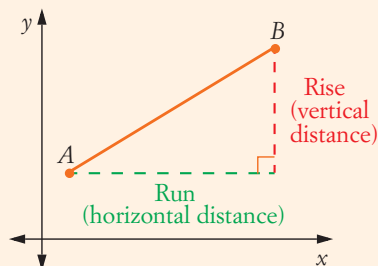
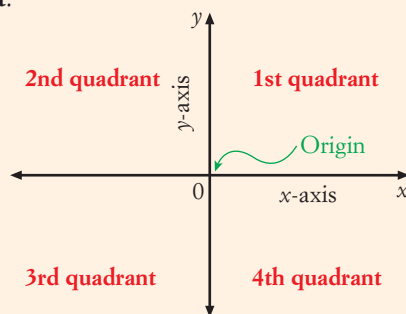
Chapter 10 summary

Quiz

Coordinates and graphs

MAT09NAQZ00010

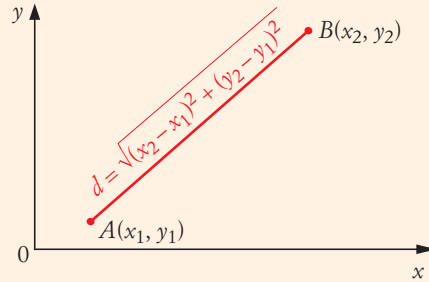
- The **Cartesian plane** has two axes at right angles. It is also called a **number lattice**.
- The **first axis** is horizontal and is called the **x-axis**. The **second axis** is vertical and is known as the **y-axis**. The axes cross at the **origin**.
- Every point on the Cartesian plane is opposite a number on each axis. The points are named using numbers called **coordinates**.
- Coordinates are shown in parentheses (round brackets) with a comma between them. The origin is the point (0, 0). The **x-coordinate** is always shown *first* and the **y-coordinate** *last*.
- Each quarter of the Cartesian plane is called a **quadrant**.
- You can plot points using a **table of values**. A rule that produces a straight line is called a **linear function** and the graph is called a **linear graph**.
- Linear functions are always of the form $y = ax + b$, where a and b are constants. The equation of a straight line is often referred to as the **equation of the line**.
- If the coordinates of a point make the equation of a line true, you say that the point **satisfies** the equation. In this case the point must be on the line.
- For two points on a straight line, the **rise** is the vertical distance between the points and the **run** is the horizontal distance between the points. The **gradient (slope)** of a straight line is calculated as $\text{gradient} = \frac{\text{rise}}{\text{run}}$ for any two points on the line. The usual symbol for gradient is m .
- A **rising line** slopes upwards from left to right and has a positive gradient.
- A **falling line** slopes downwards from left to right and has a negative gradient.
- The **x-intercept** of a straight line is the part of the x-axis from the origin to the point where the line crosses the axis. The x-coordinate of the intersection or the point itself is usually taken as the intercept.
- The **y-intercept** is the y-coordinate of the intersection (or the point) of the line and the y-axis.
- The **gradient-intercept (or slope-intercept) form** of the equation of a straight line is $y = mx + c$, where m is the gradient and c is the y-intercept.
- The **standard (or general) form** of the equation of a straight line is written with no fractions and all the terms on one side, as $ax + by + c = 0$, where a , b and c are constants (numbers).
- The **line parallel to the x-axis** through (a, b) has the equation $y = b$.
- The **line parallel to the y-axis** through (a, b) has the equation $x = a$.
- The values of the coordinates of the **midpoint of a line segment** between two points are the averages of the coordinates of the points.
- The midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.



- The **distance** between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

- The **equation of a circle** of radius r centred at the origin is given by $x^2 + y^2 = r^2$.
- The **equation of a (simple) parabola** symmetrical about the y -axis is given by either $y = x^2 + a$ or $y = -x^2 + a$, where $(0, a)$ is the **turning point (vertex)** of the parabola. A parabola with a positive x^2 term is cup-shaped and one with a negative x^2 term is hill-shaped.



- Parallel lines** have the same gradient: $m_1 = m_2$.
- The product of the gradients of **perpendicular lines** is -1 : $m_1 \times m_2 = -1$ or $m_2 = \frac{-1}{m_1}$

Chapter 10 review

Understanding

Worksheet

Coordinates and graphs review

MAT09NAWK00062

See Example 3

See Example 4

See Example 8

See Examples 12, 15

See Example 17

See Example 24

See Example 25

- For the line $4x - 3y + 7 = 0$, state if each of the given points is on the line.
 - $(-4, 3)$
 - $(2, 3)$
 - $(3, 4)$
 - $(5, 9)$
 - $(-7, -7)$
- Find the gradients of the line segments through each of the following pairs of points.
 - $(4, 3)$ and $(6, 8)$
 - $(-4, 5)$ and $(2, -7)$
- Find the equations of lines passing through $(7, 3)$
 - parallel to the x -axis
 - parallel to the y -axis
- State the midpoint and distance formulas for the points (c, d) and (g, h) .
- Find the intercepts of each of the following.
 - $2x - 3y = 12$
 - $3x - 2y + 7 = 0$
- Determine if any of the following lines are parallel or perpendicular.

L1: $4x + 5y - 3 = 0$	L2: $x - 3y + 4 = 0$
L3: $5x - 4y + 2 = 0$	L4: $3x - y - 2 = 0$
L5: $8x + 10y + 5 = 0$	L6: $15x - 12y + 13 = 0$
- Find the equation of a line passing through $(1, -5)$
 - parallel to $x = 4$
 - perpendicular to $y = -2$.

Fluency

See Example 1

See Example 2

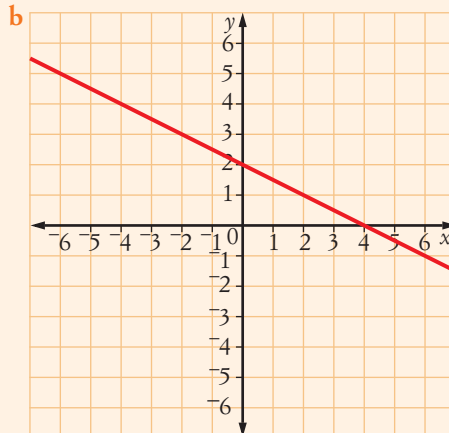
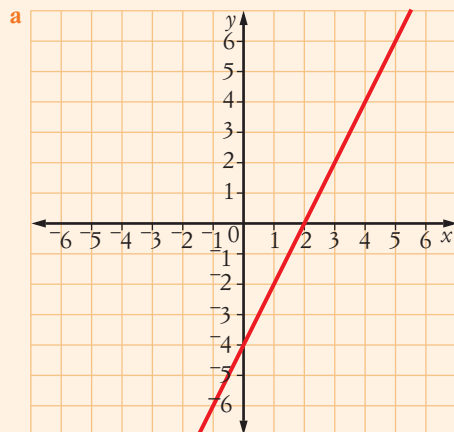
See Example 5

See Example 7

- Plot the following set of points and describe the relationship.

x	-4	-2	0	2	4	6
y	6	$4\frac{1}{2}$	3	$1\frac{1}{2}$	0	$-1\frac{1}{2}$

- Use tables of values to plot each of the following functions for the stated values of x .
 - $y = 3x - 2$ for $-2 \leq x \leq 3$
 - $y = -2x + 5$ for $-3 \leq x \leq 2$
- Draw graphs of each of the following lines for suitable values of x . Choose two points to find the gradient and then find the intercepts.
 - $y = -3x + 5$
 - $2x - 3y + 9 = 0$
- Find the equations of each of the lines shown below.



- 12** Write the equations of the lines with the properties below in gradient-intercept form.
a gradient = -1 and y -intercept = 6 **b** gradient = $\frac{3}{4}$ and y -intercept = $(0, -3)$
- 13** Find the midpoint M and distance d between the points $(4, -5)$, and $(1, 7)$. See Examples 13, 16
- 14** Use the intercepts to sketch the graph of $y = 3x + 9$. See Example 18
- 15** Use convenient points to sketch the graph of $3x + 4y - 4 = 0$. See Example 19
- 16** Draw the circle with the equation $x^2 + y^2 = 36$. See Example 22
- 17** Draw the parabola with the equation $y = -x^2 + 5$. See Example 21
- 18** Write the equations of the lines with the properties below in standard form. See Example 10
a gradient = 2 and y -intercept = -5
b gradient = $-\frac{3}{5}$ and y -intercept = $(0, -3)$
- 19** Find the gradient and y -intercept of each of the lines below. See Example 11
a $y = 3x + 4$ **b** $3x - 2y + 6 = 0$ **c** $2x + 4y - 7 = 0$
- 20** Find the equation of a line with y -intercept $(0, 4)$ that is parallel to $2x + 4y - 3 = 0$. See Example 26
- 21** Find the equation of a line with y -intercept $(0, -4)$ that is perpendicular to $3x - 4y - 3 = 0$. See Example 27
- 22** Check whether or not a function passing through the points $A(-3, 7)$, $B(0, 5)$, $C(9, -1)$, $D(15, -5)$ is linear, and if it is, find its equation. **Problem solving**
See Example 6
- 23** It costs \$55 an hour plus \$250 delivery to hire a cherry-picker. Write the cost C for hiring for h hours as a function and find the cost of hiring for 4 hours to trim some trees. See Example 9
- 24** The midpoint of EF is $(-1, 4)$. If E is $(3, 9)$, find F . See Example 14
- 25** Sketch the graph of $y = 3x - 2$ and use it to solve $3x - 2 = -8$. See Example 20
- 26** Draw the graph of $y = x^2 - 3x - 10$ from $x = -6$ to 6 and use it to solve $x^2 + 3x + 10$. See Example 21
- 27** Sketch the circle with the equation $x^2 + y^2 = 8$.
- 28** Find the equation, in both gradient-intercept and standard forms, of a line with gradient 3 passing through the point $(-1, 3)$.
- 29** Find the equations of lines parallel and perpendicular to $4x - 5y + 2 = 0$ that pass through $(4, -1)$.
- 30** Show that $\triangle ABC$, for the points $A(-1, 3)$, $B(1, -1)$ and $C(3, 1)$ is isosceles. **Reasoning**
- 31** Show that no matter what order (x_1, y_1) and (x_2, y_2) are chosen for two given points, the answer for the distance is the same.
- 32** Find a formula for the equation of a line with gradient m passing through the point (a, b) .
- 33** Explain how many times $y = x^2 + a$ and $y = -x^2 + a$ intersect.
- 34** Show that the points $A(5, 3)$, $B(9, 1)$, $C(6, -5)$ and $D(2, -3)$ form a rectangle.



Measurement and geometry

11

Shapes and angles



Contents

11.1 Angle relationships

11.2 Angles of shapes

11.3 Geometric proof

Chapter summary

Chapter review

Prior learning

Chapter 11

MAT09MGPL00011

Parent guide

Chapter 11

MAT09MGPG00011

Curriculum guide

Chapter 11

MAT09MGCU00011

Australian Curriculum statements

Geometric reasoning

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning. (Year 8)

Formulate proofs involving congruent triangles and angle properties. (Year 10)

Video tutorial

Shapes and angles

MAT09MGVT00011

Weblink

Maths overflow

MAT09MGWB00011

Even in ancient times people needed to be able to measure lengths, angles and areas. When the pharaohs ruled Egypt thousands of years ago, there were surveyors who could accurately calculate and document land areas. This was important because the annual flooding of the Nile meant that the land available for crops changed and so needed to be remeasured every year. The work of these surveyors was the beginning of the field of mathematics called geometry – the study of the properties of points, lines, surfaces and solids.

The study of geometry flourished in ancient Greece. The most outstanding of these scholars were Pythagoras, Euclid, Archimedes and Apollonius. It is therefore not surprising that the term ‘geometry’ comes from the Greek roots *geo* (earth) and *metria* (measure) and it literally means ‘measurement of the Earth’.

Mathematical literacy

Maths dictionary

MAT09ASDI00001

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

adjacent angles	corresponding	kite	square
alternate angles	angles	parallel	supplementary
angles at a point	diagonally opposite	parallelogram	angles
base	exterior angle	plane	T-chart
co-interior angles	geometric proof	quadrilateral	transversal
complementary	intersect	rectangle	trapezium
angles	interior angle	required to prove	vertically opposite
congruent	isosceles triangle	rhombus	angles

11.1 Angle relationships

Technology

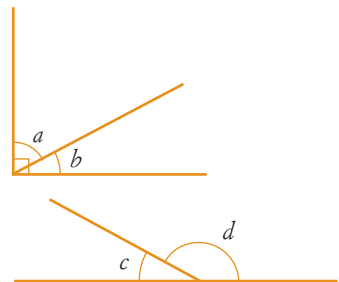
Geogebra:

Complementary angles

MAT09MGTC00009

A right angle measures 90° . Angles that add together to make 90° are called **complementary angles**.

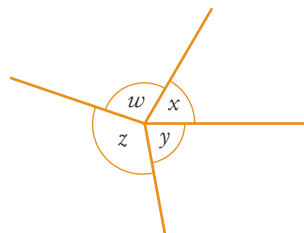
In the diagram on the right, $a + b = 90^\circ$ so a and b are complementary angles.



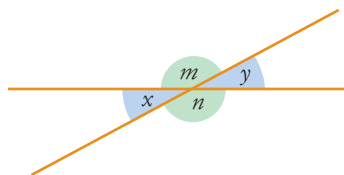
You know that a straight angle measures 180° . Angles that add to 180° are called **supplementary angles**. Angles that have the same vertex and are next to each other are called **adjacent angles**. This means that adjacent angles on a straight line must be supplementary.

In the diagram on the right, $c + d = 180^\circ$ so c and d are supplementary angles.

The angles at a point all add together to make 360° . So in the diagram on the right, $w + x + y + z = 360^\circ$



If two lines are drawn that cross each other (**intersect**) as shown here, they form two pairs of angles. Angles x and y are opposite each other. So are angles m and n .



Pairs of angles formed in this way are called **vertically opposite angles**. If you measure the angles, you will see that $x = y$ and $m = n$. Vertically opposite angles formed when two lines intersect will always be equal.

Important!

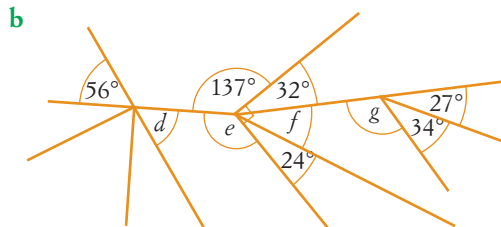
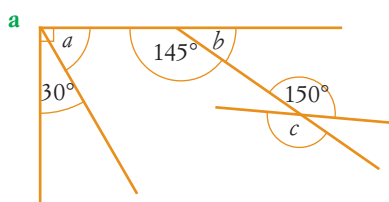
Angle relationships at a point and on lines

- **Adjacent angles** have the same vertex and are next to each other.
- **Complementary angles** (right angle symbol) add up to 90° .
- **Supplementary angles** (straight line symbol) add up to 180° .
- **Angles at a point** (full circle symbol) that are all adjacent add up to 360° .
- **Vertically opposite angles** (X symbol) are formed by the intersection of lines. They are equal in size.

You can see in the definition box above that there are symbols for some of the angle relationships. These symbols can help you to remember the relationships. They are also 'shorthand' ways of writing the relationships.

Example 1

Find the angles indicated by unknowns in the figures below. Give a reason in each case.



Solution

a a is complementary with 30° .

b is supplementary with 145° .

c is vertically opposite 150° .

b d is vertically opposite 56° .

e is an angle at a point with 137° and 90° .

f is complementary with 32° and 24° .

g is supplementary with 27° and 34° .

$$a = 60^\circ$$

$$b = 35^\circ$$

$$c = 150^\circ$$

$$d = 56^\circ$$

$$e = 133^\circ$$

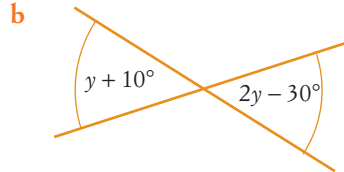
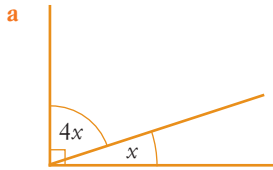
$$f = 34^\circ$$

$$g = 119^\circ$$

Some angle problems can involve algebra.

Example 2

Find x and y in the following figures. Give a reason in each case.



Solution

- a** x and $4x$ are complementary.

Simplify the LHS.

Use inverse operations to solve.

Simplify.

- b** $(y + 10^\circ)$ and $(2y - 30^\circ)$ are vertically opposite.

Group the variables.

Simplify.

Use inverse operations to solve.

Simplify.

State the result.

$$x + 4x = 90^\circ$$

$$5x = 90^\circ$$

$$\frac{5x}{5} = \frac{90^\circ}{5}$$

$$x = 18^\circ$$

$$y + 10^\circ = 2y - 30^\circ$$

$$y + 10^\circ - y = 2y - 30^\circ - y$$

$$10^\circ = y - 30^\circ$$

$$10^\circ + 30^\circ = y - 30^\circ + 30^\circ$$

$$40^\circ = y$$

$$y = 40^\circ$$

We show that lines are **parallel** by putting matching arrows on them. You can think of parallel lines as lines in the same **plane** that never meet. This is the usual assumption for *Euclidean geometry*—the form of geometry that we are dealing with in this chapter.

In Euclidean geometry, a **transversal** is a line that crosses any two other lines. When a pair of lines is crossed by a transversal, angles are formed. These angles are given special names.

Important!

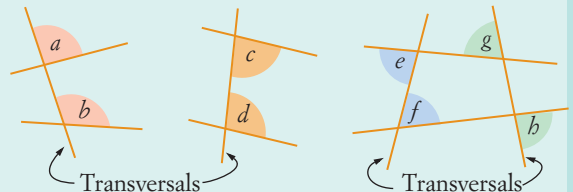
Angles formed by a transversal

a and b are corresponding angles.

c and d are co-interior angles.

e and f are alternate angles.

g and h are also considered alternate angles by many people. In this book, we will also consider them as alternate.



Co-interior angles

Lines in the plane that are not parallel will eventually meet. The angles in the triangle formed when a transversal crosses non-parallel lines add up to 180° . Therefore, the **co-interior angles** on that side must be less than 180° altogether.

As the lines become more parallel, the sum of the co-interior angles increases.

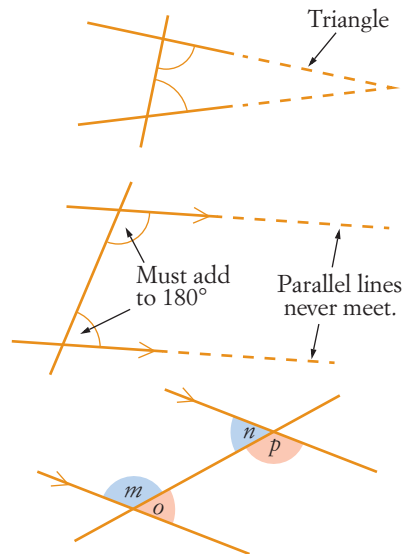
Parallel lines do not meet. This means that the co-interior angles must add up to 180° by themselves, because there is no third angle.

So, for the diagram on the right,

$$m + n = 180^\circ$$

$$\text{and } o + p = 180^\circ$$

Co-interior angles on parallel lines are supplementary.



Corresponding angles

a and b are **corresponding angles** on parallel lines.

You can see that

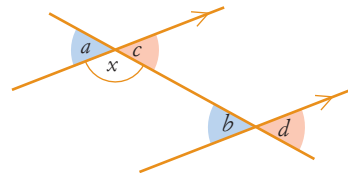
$$a + x = 180^\circ$$

$$\text{and } b + x = 180^\circ \text{ (co-interior)}$$

$$\text{So } a = b$$

In the same way we can show that $c = d$.

Corresponding angles on parallel lines are always equal.



Alternate angles

h and i are **alternate angles** on parallel lines. You can see that

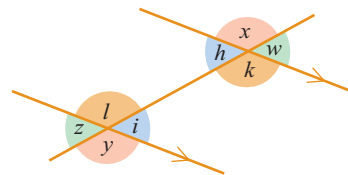
$$i + k = 180^\circ \text{ (co-interior)}$$

$$\text{and } h + k = 180^\circ$$

$$\text{So } i = h$$

We can also show that $l = k$, $y = x$ and $z = w$.

Alternate angles on parallel lines are always equal.



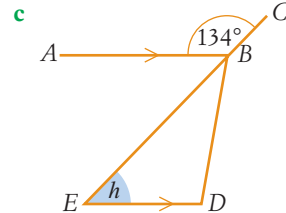
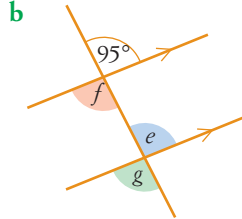
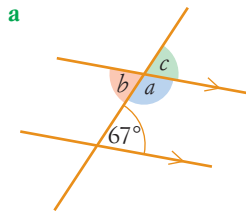
Important!

Angle relationships on parallel lines

- **Co-interior angles** are supplementary ($\angle$).
- **Corresponding angles** are equal ($\angle$).
- **Alternate angles** are equal ($\angle$).

Example 3

Find the angles indicated by unknowns in the figures below. Give a reason in each case.



Solution

- a** a and 67° are co-interior.

Solve.

b and 67° are alternate.

c and 67° are corresponding.

- b** e and 95° are corresponding.

f and 95° are vertically opposite.

g and 95° are alternate.

- c** $\angle ABE$ and 134° are supplementary.

Solve.

h and $\angle ABE$ are alternate.

$$a + 67^\circ = 180^\circ \quad \text{↗ ↘}$$

$$a = 113^\circ$$

$$b = 67^\circ \quad \text{↗ ↘}$$

$$c = 67^\circ \quad \text{↗ ↘}$$

$$e = 95^\circ \quad \text{↗ ↘}$$

$$f = 95^\circ \quad \text{↗ ↘}$$

$$g = 95^\circ \quad \text{↗ ↘}$$

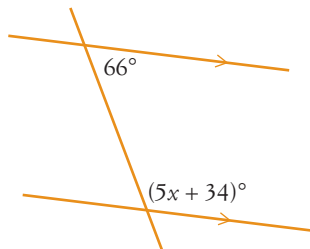
$$\angle ABE + 134^\circ = 180^\circ \quad \text{↗ ↘}$$

$$\angle ABE = 46^\circ$$

$$h = 46^\circ \quad \text{↗ ↘}$$

Example 4

Find x in the following figure. Give a reason for your answer.



Solution

$(5x + 34)^\circ$ and 66° are co-interior.

Simplify the LHS.

Use inverse operations to solve.

Use inverse operations to solve.

Simplify.

$$(5x + 34)^\circ + 66^\circ = 180^\circ \quad \text{↗ ↘}$$

$$5x + 100^\circ = 180^\circ$$

$$5x + 100^\circ - 100^\circ = 180^\circ - 100^\circ$$

$$5x = 80^\circ$$

$$\frac{5x}{5} = \frac{80^\circ}{5}$$

$$x = 16^\circ$$

Exercise 11.1 Angle relationships

Understanding

Extra questions

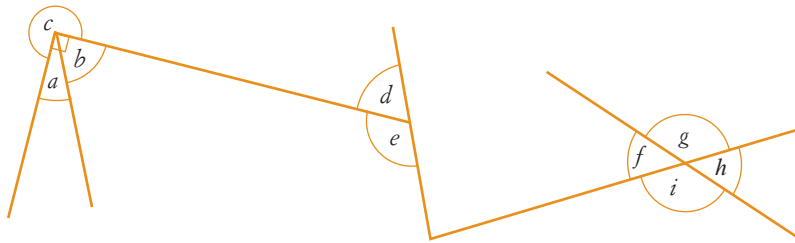
Exercise 11.1

MAT09MGEQ00037

- 1 Use the list of terms and the diagram below to describe the relationship between the stated angles in each case.

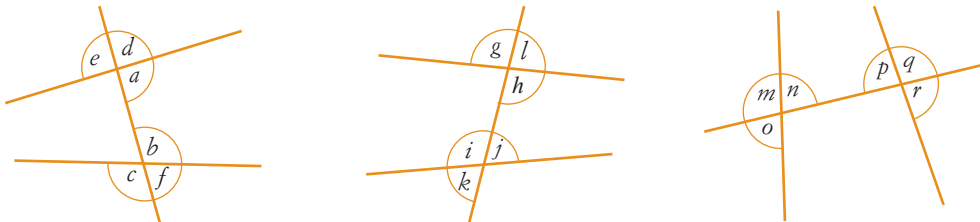
supplementary angles angles at a point
vertically opposite angles complementary angles

- a** a and b **b** d and e **c** f and h **d** a, b and c
e f and g **f** g and i **g** f, g, h and i **h** g and h

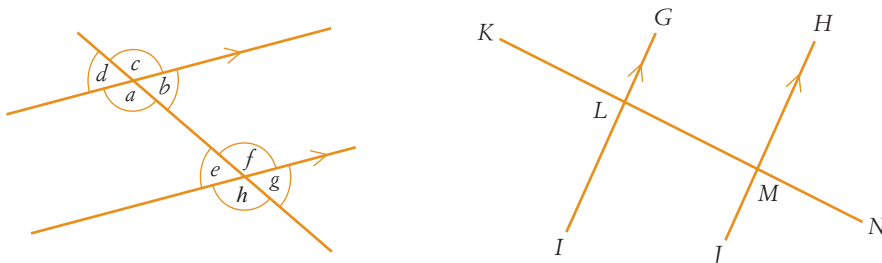


- 2 What word would you use to describe the relationship between each of the pairs of angles shown below?

- a** a and b **b** c and d **c** a and f **d** e and f **e** h and i
f j and l **g** l and k **h** m and p **i** n and p **j** q and o



- 3 Use the diagrams below to answer the following questions.



- a** For a , write down an angle that is:
i corresponding **ii** alternate **iii** co-interior.
b For b , write down an angle that is:
i corresponding **ii** alternate **iii** co-interior.
c For $\angle GLM$, write down an angle that is:
i corresponding **ii** alternate **iii** co-interior.
d For $\angle ILK$, write down an angle that is:
i corresponding **ii** alternate **iii** co-interior.

4 Use the diagrams from question 3 to state the relationship between:

a a and e

b b and e

c c and f

d d and e

e d and g

f a and h

g c and h

h $\angle KLI$ and $\angle LMJ$

i $\angle GLM$ and $\angle LMJ$

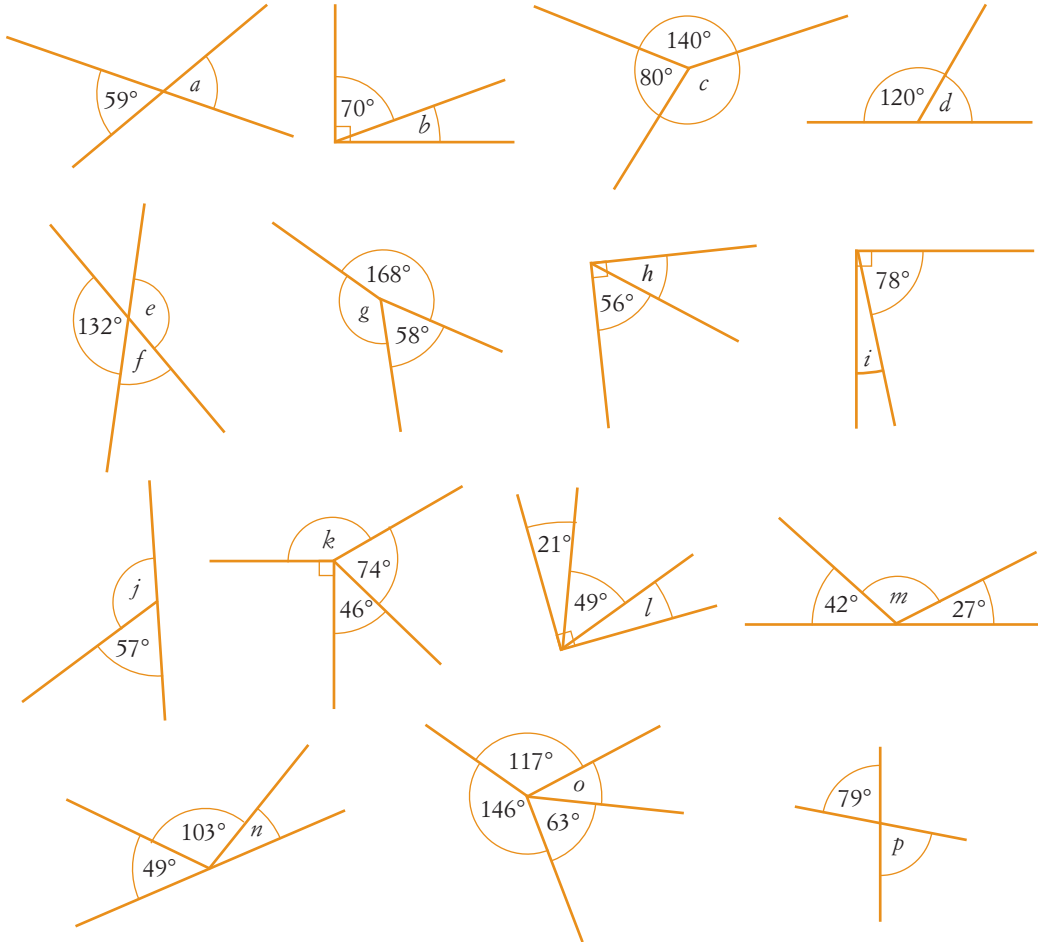
j $\angle HMN$ and $\angle GLM$

k $\angle GLK$ and $\angle MLI$

l $\angle KLG$ and $\angle JMN$.

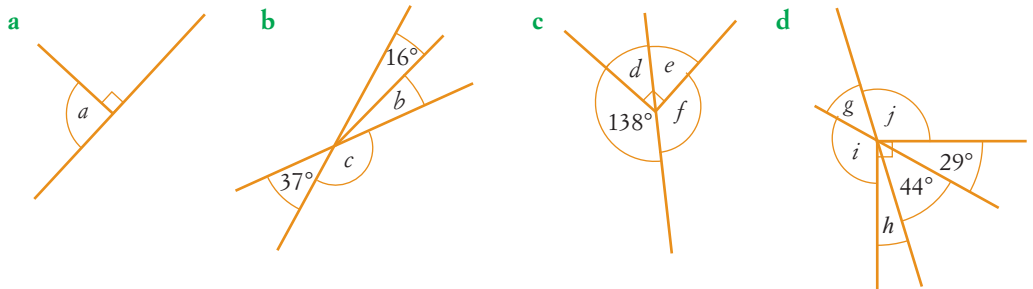
Fluency

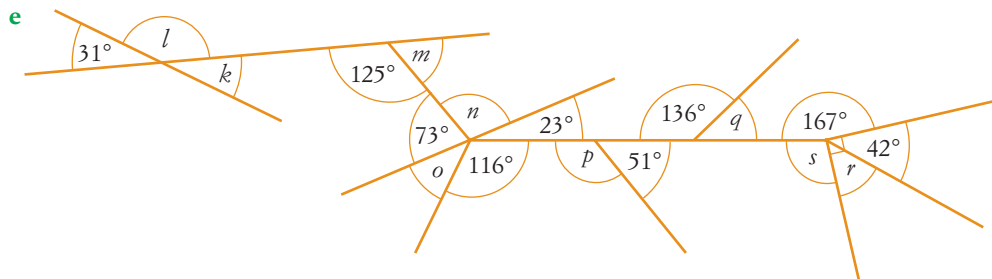
5 Find the values of the unknown angles in the following diagrams. In each case, give a reason for your answer in words or by using a symbol.



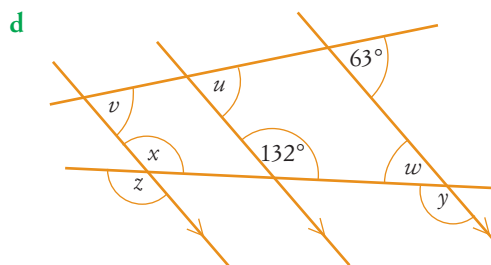
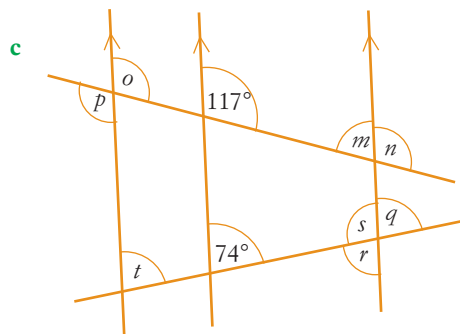
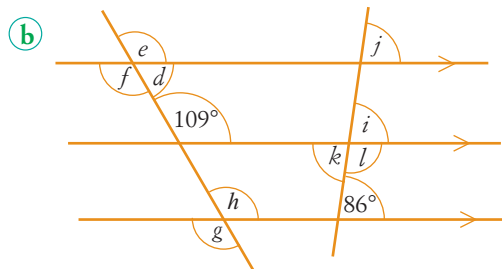
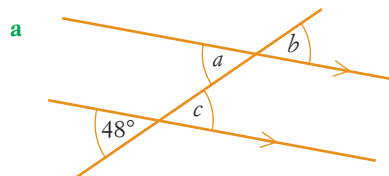
See Example 1

6 Find the values of the unknown angles in the following diagrams. In each case, give a reason for your answer in words or by using a symbol.





7 Find the values of the unknown angles in the following diagrams. Give a reason in each case.



Worked solutions

Exercise 11.1

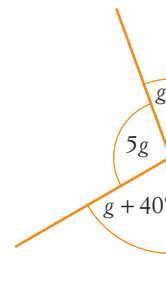
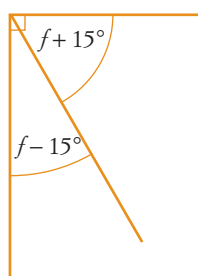
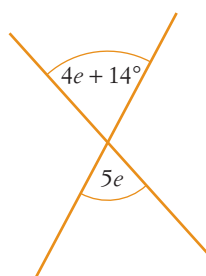
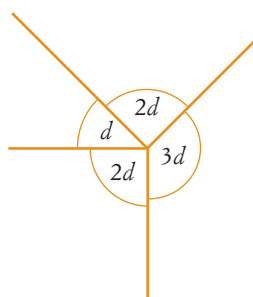
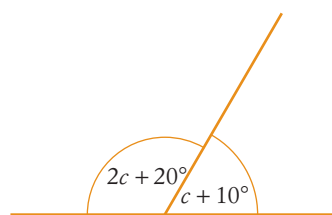
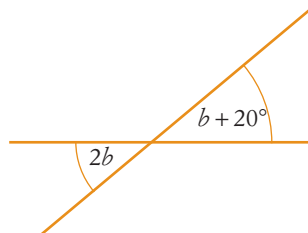
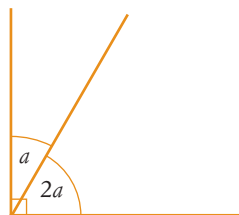
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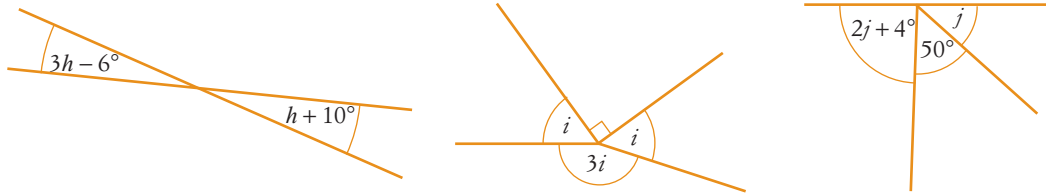
See Example 3

8 Work out the values of the pronumerals in the following diagrams. In each case, give a reason for your answer in words or by using a symbol.

Problem solving

See Example 2





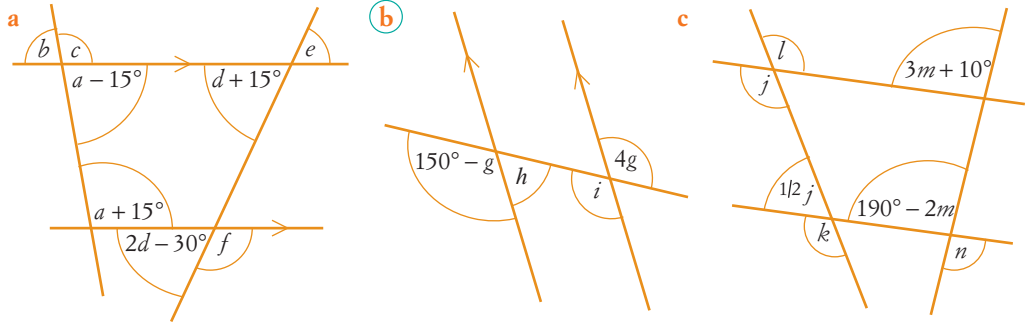
Worked solutions

Exercise 11.1

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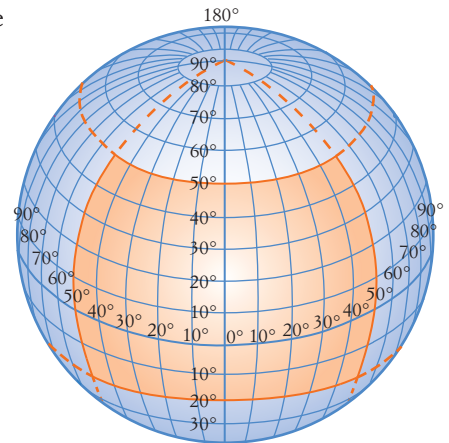
See Example 4

- 9 Find the value of the unknowns in the figures below. Give a reason in each case.



Reasoning

- 10 It is possible to have 'parallel' lines on a sphere. Unlike parallel lines in Euclidian geometry, 'parallel' lines on a sphere may meet. For example, lines of longitude meet at the Earth's poles. Would co-interior angles on parallel lines drawn on a sphere always add to 180° ? Experiment by drawing parallel lines on a spherical balloon.



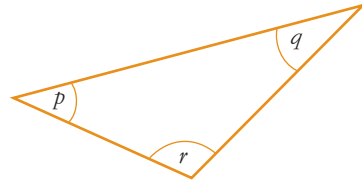
11.2 Angles of shapes

Triangles

The **interior angles** of a triangle are those within it.

The sum of the **interior angles** of a triangle is 180° .

For the diagram on the right, this means that $p + q + r = 180^\circ$.



If one of the sides of a triangle is extended, an **exterior angle** is formed.

Extending the side JH in $\triangle HIJ$ as shown here forms the exterior angle $\angle GHI = a$.

An exterior angle of a triangle is related to the interior angles in a special way.

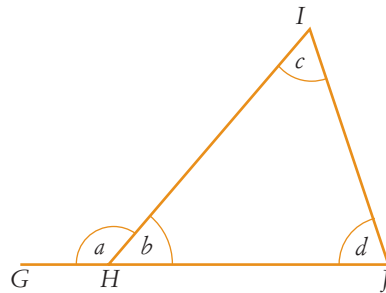
For $\triangle HIJ$,

$$a + b = 180^\circ$$

and $b + c + d = 180^\circ$

So $a + b = b + c + d$

or $a = c + d$



This means that an exterior angle of a triangle is equal to the sum of the two interior angles that are opposite it.

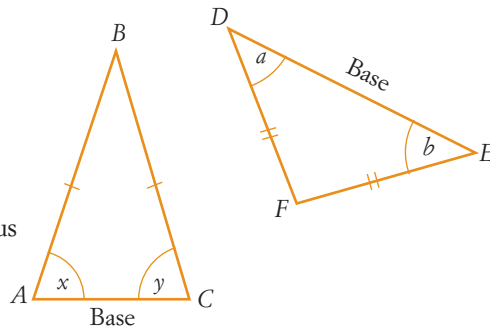
An **isosceles triangle** is special because it has two sides that are equal. The other side of an isosceles triangle is called the **base**.

The angles at the base of an isosceles triangle are equal to each other.

So in $\triangle ABC$ $x = y$

and in $\triangle DEF$ $a = b$.

As you saw with the angle relationships in the previous section, symbols may be helpful when using and remembering angle relationships in triangles.



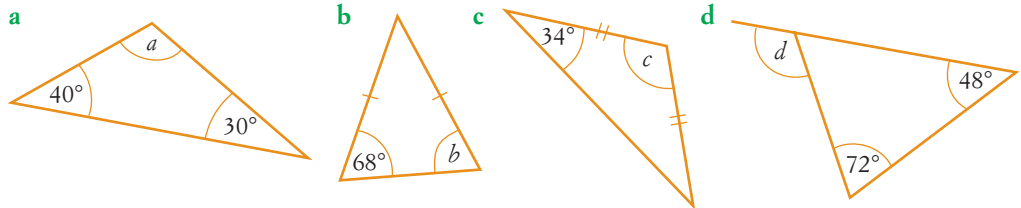
Important!

Angle relationships in triangles

- The **sum of angles in a triangle** is always 180° ().
- The **exterior angle of a triangle** is equal to the sum of the interior opposite angles ().
- The **angles at the base of an isosceles triangle** are always equal ().

Example 5

Find the angles indicated by unknowns in the figures below. Give a reason in each case.



Solution

- a** The sum of the angles is 180° .
Simplify.
Use inverse operations.
- b** The triangle is isosceles, so the angles at the base are equal.
- c** The angles at the base are equal and the sum of the angles is 180° .
Simplify.
Use inverse operations.
- d** The exterior angle equals the sum of the interior opposite angles.

$$\begin{aligned} a + 40^\circ + 30^\circ &= 180^\circ \quad \triangle \\ a + 70^\circ &= 180^\circ \\ a &= 110^\circ \end{aligned}$$

$$b = 68^\circ \quad \triangle$$

$$\begin{aligned} c + 34^\circ + 34^\circ &= 180^\circ \quad \triangle \triangle \\ c + 68^\circ &= 180^\circ \\ c &= 112^\circ \end{aligned}$$

$$\begin{aligned} d &= 72^\circ + 48^\circ \quad \triangle \\ d &= 120^\circ \end{aligned}$$

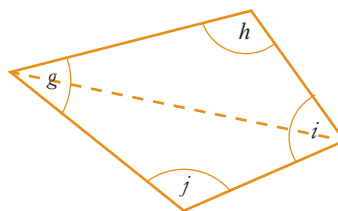


Quadrilaterals

It is possible to divide any **quadrilateral** into exactly two triangles. This means that the sum of the interior angles of a quadrilateral must be twice the sum of the angles in a triangle.

So, for the diagram on the right, we have

$$g + h + i + j = 360^\circ$$



Technology

Geogebra: Angle sum of a quadrilateral

MAT09MGTC00010

A **parallelogram** has opposite sides parallel and equal in length, as seen here for $KLMN$.

When a diagonal is drawn as shown, it divides the parallelogram into two triangles ($\triangle NKL$ and $\triangle LMN$).

Looking at $\triangle NKL$ and $\triangle LMN$, we can see that

$$NK = LM$$

$$KL = MN$$

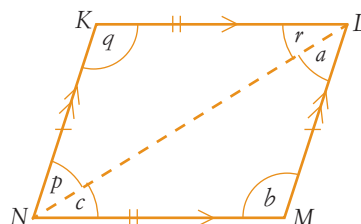
and NL is a common side.

This means that $\triangle NKL$ and $\triangle LMN$ are **congruent**, i.e. $\triangle NKL \equiv \triangle LMN$.

Because $\triangle NKL \equiv \triangle LMN$, we can say that $b = q$. We can also say that $a = p$ and $c = r$.

This means that $p + c = r + a$.

So the angles that are **diagonally opposite** in a parallelogram must be equal.



Important!

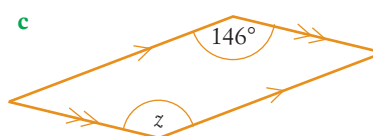
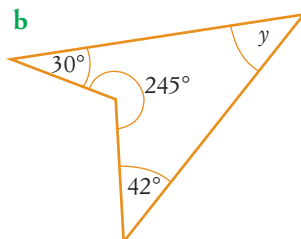
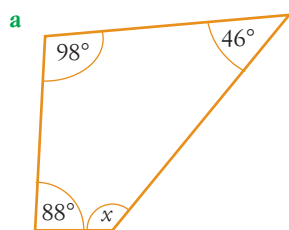
Angle relationships in quadrilaterals

The **sum of angles in a quadrilateral** is always 360° (□).

The **diagonally opposite angles of a parallelogram** are equal (▭).

Example 6

Find the unknown angles in the following figures. Give a reason in each case.



Solution

a The sum of the angles is 360° .

Simplify.

Use inverse operations.

$$x + 88^\circ + 98^\circ + 46^\circ = 360^\circ$$

$$x + 232^\circ = 360^\circ$$

$$x = 128^\circ$$

- b** The relationship holds for concave quadrilaterals also.

The sum of the angles is 360° .

Simplify.

Use inverse operations.

- c** Diagonally opposite angles of a parallelogram are equal.

$$y + 30^\circ + 245^\circ + 42^\circ = 360^\circ$$

$$y + 317^\circ = 360^\circ$$

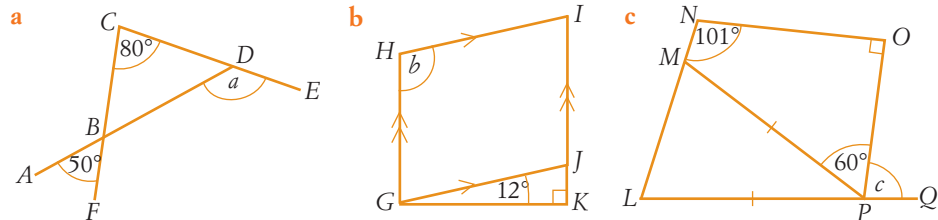
$$y = 43^\circ$$

$$z = 146^\circ$$

Sometimes you will need to use a number of different angle relationships to work out the value of a required angle.

Example 7

Find the angles indicated by unknowns in the following figures. Give a reason for each step.



Solution

- a** Use vertically opposite angles first.

$$\angle CBD = 50^\circ$$

Now use the exterior angle of the triangle.

$$\begin{aligned} a &= \angle CBD + 80^\circ \\ &= 50^\circ + 80^\circ \\ &= 130^\circ \end{aligned}$$

- b** $\angle IJG$ and b are diagonally opposite in parallelogram $GHIJ$.

$$\angle IJG = b$$

Now use the exterior angle of the triangle.

$$\begin{aligned} \angle IJG &= 12^\circ + 90^\circ \\ \angle IJG &= 102^\circ \\ b &= 102^\circ \end{aligned}$$

Simplify.

State the answer.

- c** Use sum of angles in quadrilateral $MNOP$.

$$\angle NMP + 101^\circ + 90^\circ + 60^\circ = 360^\circ$$

Simplify.

$$\angle NMP + 251^\circ = 360^\circ$$

Use inverse operations.

$$\angle NMP = 109^\circ$$

$\angle NMP$ and $\angle LMP$ are adjacent on LN .

$$\angle NMP + \angle LMP = 180^\circ$$

Substitute for $\angle NMP$.

$$109^\circ + \angle LMP = 180^\circ$$

Use inverse operations.

$$\angle LMP = 71^\circ$$

Animated example

Angles and shapes

MAT09MGAE00011

Worksheet

Equal angles

MAT09MGWK00063

Puzzle sheet

Triangles and quadrilaterals

MAT09MGPS00052

Puzzle sheet

Mixed angle problems

MAT09MGPS00053

Use angles at the base of an isosceles triangle.

Use the exterior angle of a triangle.

Substitute for $\angle LMP$ and $\angle MLP$.

Simplify.

Use inverse operations.

$$\angle LMP = \angle MLP = 71^\circ$$

$$c + 60^\circ = \angle LMP + \angle M$$

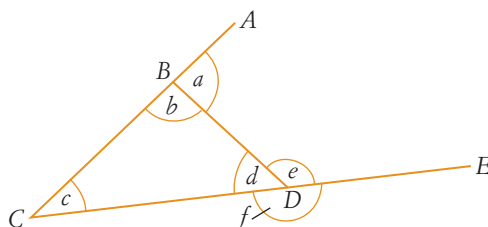
$$= 71^\circ + 71^\circ$$

$$= 142^\circ$$

$$c = 82^\circ$$

Exercise 11.2 Angles of shapes

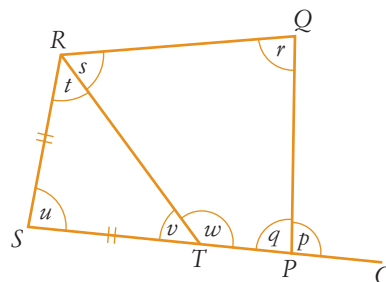
- 1 Use the marked angles in the diagram below to answer the following questions.



- Name any exterior angles for $\triangle CBD$.
- Name any angles adjacent to d .
- Name any angles that are supplementary.
- Complete: $\dots + \dots + \dots = 360^\circ$ (three angles)
- Complete: $\dots = \dots + \dots$ (two answers)
- Complete: $\dots + \dots + \dots = 180^\circ$ (three angles)
- Complete: $\dots + \dots = 180^\circ$ (two angles)

- 2 Use the marked angles in the diagram on the right to answer the following questions.

- Name any exterior angles for $\triangle RTS$.
- Name any pairs of adjacent angles.
- Name any angles that are supplementary.
- Complete: $\dots + \dots + \dots + \dots + \dots = 360^\circ$ (five angles)
- Complete: $\dots + \dots + \dots + \dots = 360^\circ$ (four angles)
- Complete: $\dots = \dots$ (two angles)
- Complete: $\dots + \dots = 180^\circ$ (two angles)
- Complete: $\dots = \dots + \dots$ (three angles)
- Complete: $\dots + \dots + \dots = 180^\circ$ (three angles)



Understanding

Extra questions

Exercise 11.2

MAT09MGEQ00038

Worksheet

Geometry summary

MAT09MGWK00064

Fluency

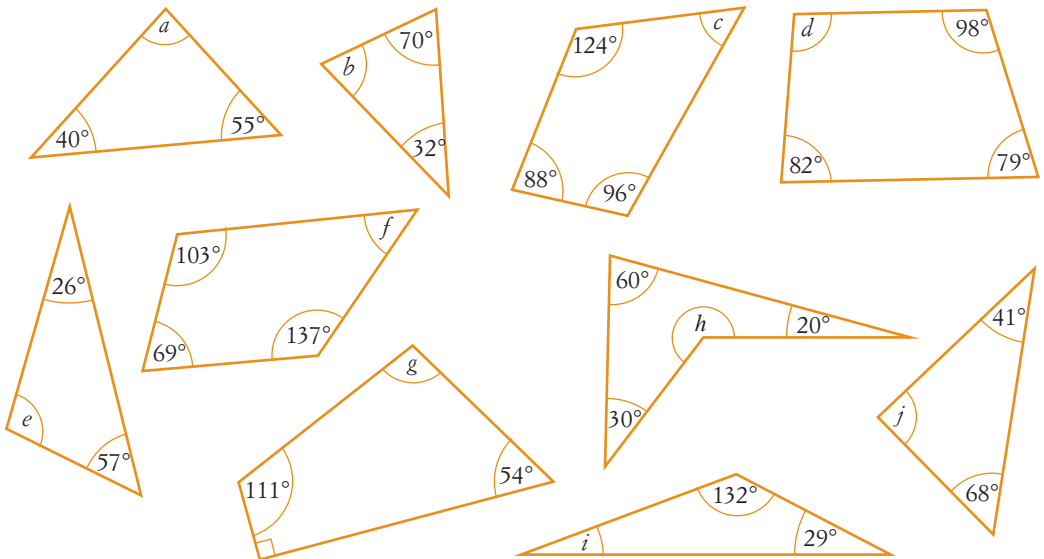
Worked solution

Exercise 11.2

MAT09MGWS00038

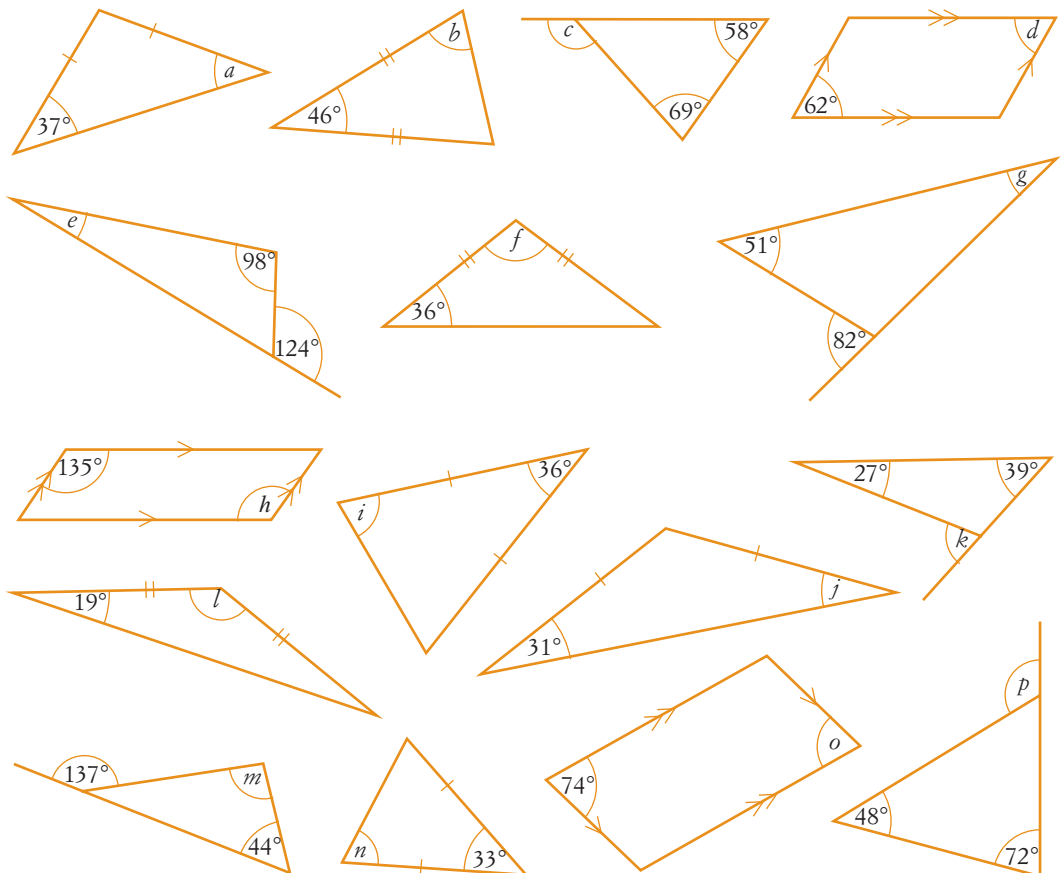
See Example 5

- 3 Find the unknown angles in the following diagrams. In each case, give a reason for your answer in words or by using a symbol.



See Example 6

- 4 Find the value of the unknown angles in the following diagrams. In each case, give a reason for your answer in words or by using a symbol.



Problem solving

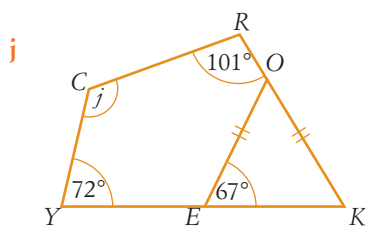
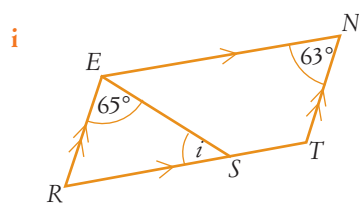
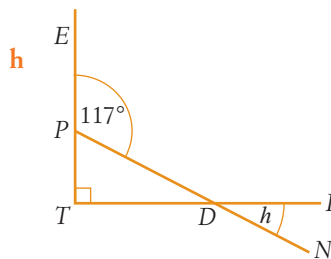
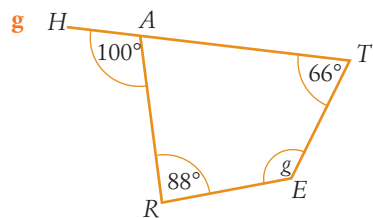
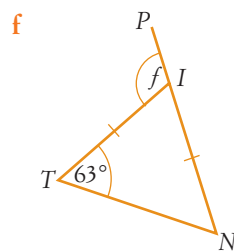
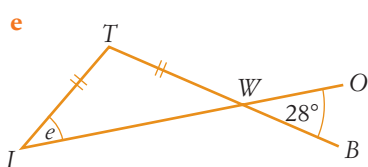
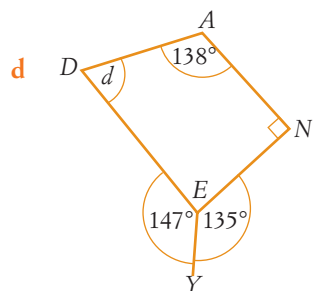
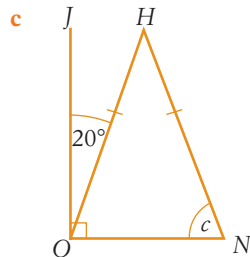
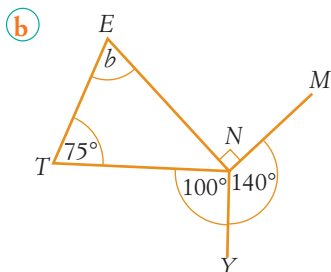
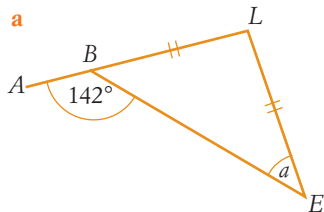
Worked solution

Exercise 11.2

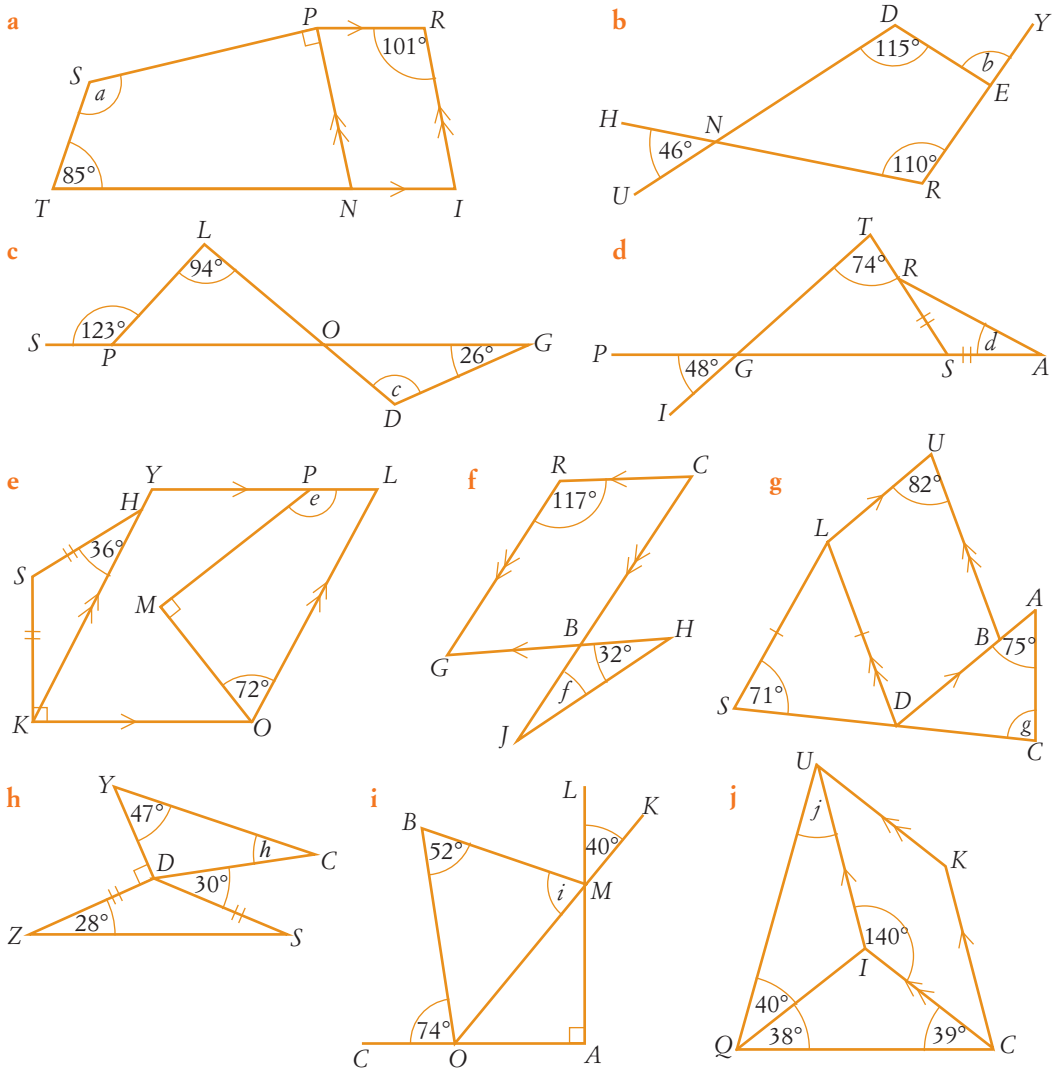
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See Example 7

- 5 Find the value of the unknown angles in the following diagrams. In each case, give a reason for your answer in words or by using a symbol.



- 6 Find the value of the unknown angles in the following diagrams. In each case, give a reason for your answer in words or by using a symbol.

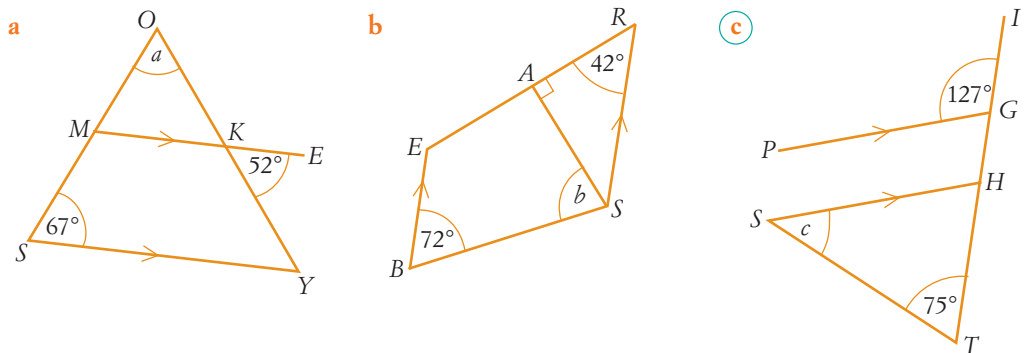


Worked solution

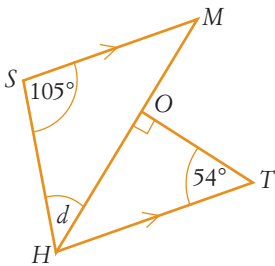
Exercise 11.2

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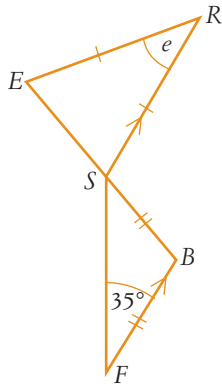
- 7 Find the value of the unknown angles in the following diagrams. Give a reason in each case.



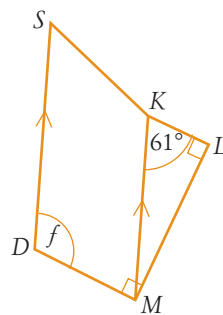
d



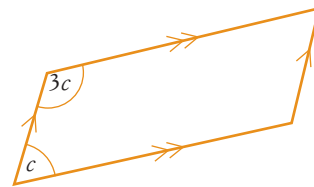
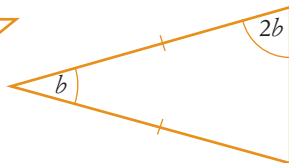
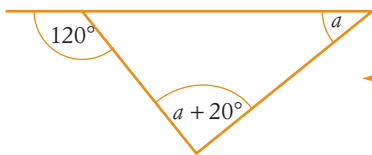
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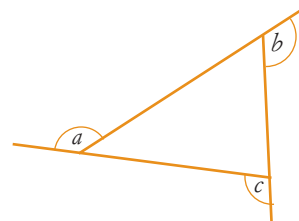
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- 8** Work out the values of the pronumerals in the following diagrams. In each case, give a reason for your answer in words or by using a symbol.



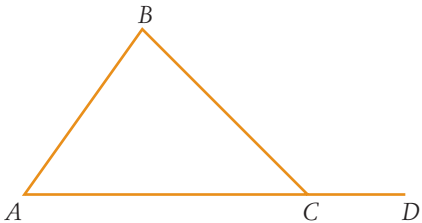
- 9** Use the diagram on the right to answer these questions.
- What is the sum of the three exterior angles?
 - Would this relationship hold for all triangles? Explain.



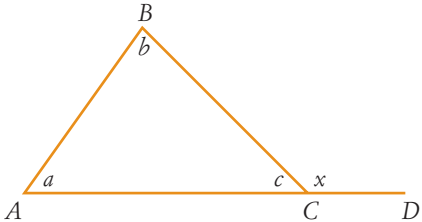
11.3 Geometric proof

So far we have used the rules of geometry to find the values of angles and sides in given situations. We can also use these rules to prove that certain properties of shapes are always true. There are a number of ways that **geometric proofs** are set out. For example, proofs usually have numbered steps and reasons are given for each step. We will see how to set out a proof by looking at an angle rule that we have already used in this chapter. Consider the rule that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

It is often helpful to draw a diagram that you can refer to. It is important to state what it is that needs to be proved. This can be stated as **required to prove (RTP)**. In this case, we would write:
Required to prove: $\angle BCD = \angle BAC + \angle CBA$



In many cases, it is easier to name the angles using variables as shown on the right. What is required to prove would then be:
RTP: $x = a + b$



A **T-Chart** like the one below can be helpful in organising the steps of the proof. The T-Chart has the ‘required to prove’ statement at the top and has the headings of ‘statement’ and ‘reason’ at the top of the left and right columns respectively. The proof is complete when you have proven what was required. It is usual to end a proof with the letters ‘QED’ which stands for the Latin phrase *quod erat demonstrandum*, which literally means ‘what was to be demonstrated’.

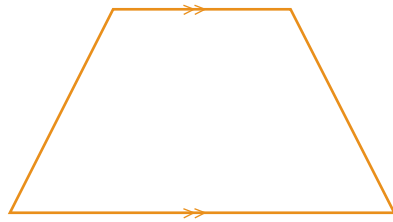
RTP: $x = a + b$

	Statement	Reason
x and c are adjacent angles on a line.	1. $x + c = 180^\circ$	1.
The angles of the triangle add to 180° .	2. $a + b + c = 180^\circ$	2.
The LHS of statements 1 and 2 both equal 180° .	3. $x + c = a + b + c$	3. Equate statements 1 and 2.
c can be removed from both sides of the equation.	4. $x = a + b$	4. Subtract c from both sides of the equation.
Statement 4 is what was required to be proved.	QED	

Geometric proofs often involve the properties of quadrilaterals, so it is worth looking at these to refresh your memory. The properties of these quadrilaterals can be summarised as follows:

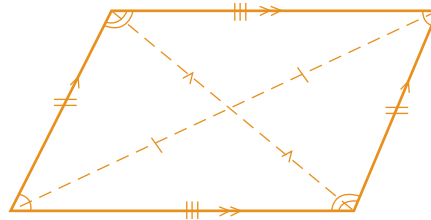
Trapezium

- one pair of opposite sides parallel



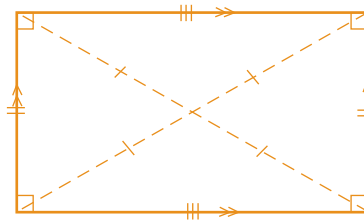
Parallelogram

- opposite sides parallel
- opposite sides equal
- opposite angles equal
- diagonals bisect each other



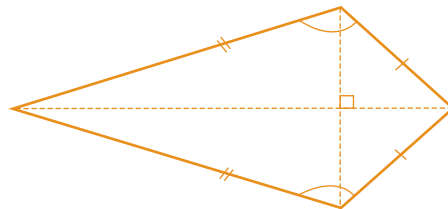
Rectangle

- opposite sides parallel
- opposite sides equal
- all angles are right angles
- diagonals are equal in length
- diagonals bisect each other



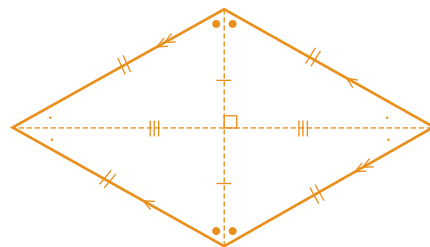
Kite

- two pairs of equal adjacent sides
- one pair of opposite angles equal
- diagonals intersect at right angles



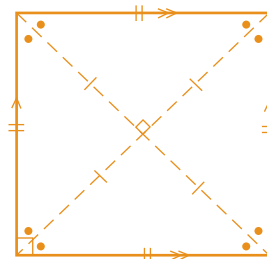
Rhombus

- opposite sides parallel
- all sides equal
- opposite angles equal
- diagonals bisect each other at right angles
- diagonals bisect the angles of the rhombus



Square

- opposite sides parallel
- all sides equal
- all angles are right angles
- diagonals equal in length
- diagonals bisect each other at right angles
- diagonals bisect the angles of the square



Worksheet

Classifying
quadrilaterals

MAT09MGWK00066

Worksheet

Diagonal properties
of quadrilaterals

MAT09MGWK00067

TLF Learning object

Exploring quadrilaterals
(L6562)

MAT09MGIN00011

As you can see above, each type of quadrilateral has a number of features. If you are proving that a four-sided figure is a particular type of quadrilateral, it is only necessary to show that some of these exist. The following is a summary of these properties.

Important!

A **trapezium** is a quadrilateral with at least one pair of opposite sides parallel.

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

A **rectangle** is a parallelogram with right angles.

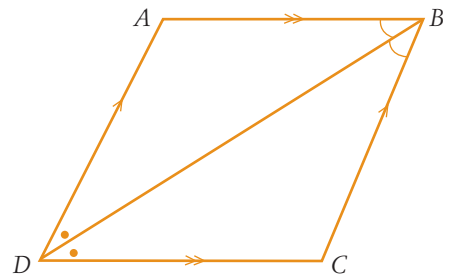
A **square** is a rectangle with two adjacent sides equal in length.

A **rhombus** is a parallelogram with two adjacent sides equal in length.

A **kite** is a convex quadrilateral with two pairs of equal adjacent sides.

Example 8

Prove that the parallelogram $ABCD$ shown here is a rhombus.

**Solution**

First write down what is required to be proved. A rhombus is a parallelogram with adjacent sides equal. Any pair of adjacent sides can be selected.

Draw up a T-chart.

RTP: $AD = CD$

	Statement	Reason
Consider $\triangle DAB$ and $\triangle DCB$.	1. $\angle ADB = \angle CDB$	1. Given
	2. $\angle ABD = \angle CBD$	2. Given
	3. DB is common	3. Given
Refer to the tests for congruent triangles.	4. $\triangle DAB \equiv \triangle DCB$	4. ASA
Look at corresponding sides.	5. $AD = CD$	5. Corresponding sides of congruent triangles.
	QED	

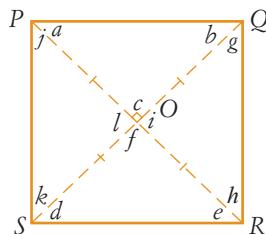
In some cases you will need to draw a sketch to help you develop a proof.

Example 9

Prove that a quadrilateral with equal-length diagonals bisecting at right angles is a square.

Solution

First draw a diagram that represents this situation, i.e., a quadrilateral with equal-length diagonals bisecting at right angles. Name the vertices of the quadrilateral and angles for future reference.



First write down what is required to be proved. A square is a rectangle with equal adjacent sides.

RTP:

$PQ = QR$ (or any other pair of adjacent sides equal)

$\angle SPQ = 90^\circ$ (or any other vertex equals 90°)

$PQ \parallel SR$ and $PS \parallel QR$

Statement	Reason
1. $PO = RO$	1. Given
2. QO is common	2. Given
3. $\angle POQ = \angle QOR$	3. Both equal 90°
4. $\triangle PQO \equiv \triangle QRO$	4. SAS
5. $\triangle QRO \equiv \triangle RSO$	5. SAS
6. $\triangle RSO \equiv \triangle SPO$	6. SAS
7. $\triangle PQO \equiv \triangle QRO \equiv \triangle RSO \equiv \triangle SPO$	7. Equate statements 4, 5 and 6
8. $PQ = QR = RS = SP$	8. Corresponding sides of congruent triangles.
9. $a + b + c = 180^\circ$	9.
10. $a + b + 90^\circ = 180^\circ$	10. $c = 90^\circ$
11. $a + b = 90^\circ$	11.
12. $a + a = 90^\circ$	12. $a = b$
13. $a = b = 45^\circ$	13. $2a = 90^\circ$ and $a = b$
14. $a = b = g = h = e = d = k = j = 45^\circ$	14. Corresponding angles of congruent triangles.
15. $\angle SPQ = 90^\circ$	15. $j = a = 45^\circ$ so $j + a = 90^\circ$
16. $a = b$	16. Both equal 45° .
17. $PQ \parallel SR$	17.
18. $g = k$	18. Both equal 45° .
19. $PS \parallel QR$	19.
QED	Proven by statements 8, 15, 17 and 19.

Exercise 11.3 Geometric proof

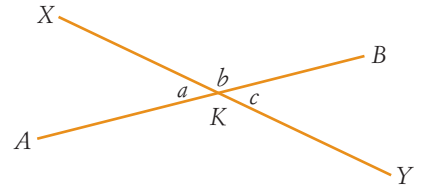
Understanding

Extra questions

Exercise 11.3

MAT09MGEQ00039

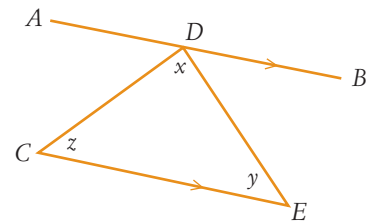
- 1 A proof of the rule ‘Vertically opposite angles are equal’ using this diagram is shown below. Complete the parts of the proof (**a** through **d**) that are missing.



RTP: $a = c$

Statement	Reason
1. $a + b = 180^\circ$	1. a
2. $a = 180^\circ - b$	2. Rearrange statement 1.
3. b	3.
4. c	4. Rearrange statement 3.
5. d	5. Equate statements 1 and 3.
QED	

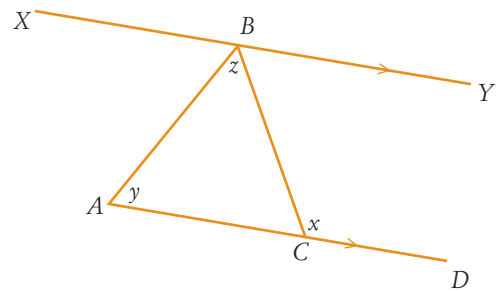
- 2 A proof of the rule ‘The sum of the angles in a triangle is 180° ’ using this diagram is shown below. Complete the parts of the proof (**a** through **d**) that are missing.



RTP: $x + y + z = 180^\circ$

Statement	Reason
1. $\angle ADC = z$	1.
2. $\angle BDE = y$	2. a
3. b	3.
4. $x + y + z = 180^\circ$	4. c
d	

- 3 A proof of the rule ‘The exterior angle of a triangle is equal to the sum of the interior opposite angles’ using this diagram is shown below. Complete the parts of the proof (**a** through **d**) that are missing.



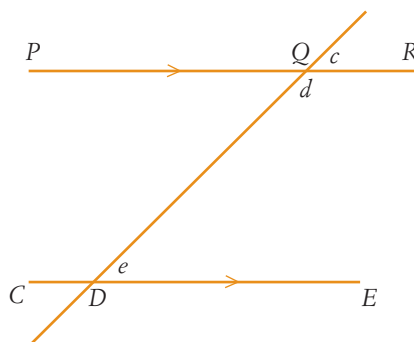
RTP: **a**

Statement	Reason
1. b	1.
2. $x = \angle ABX + z$	2. c
3. d	3. $\angle XBA = y$
QED	

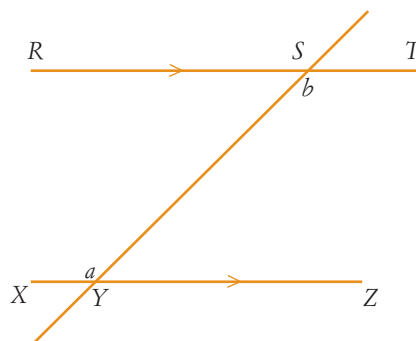
- 4 A proof of the rule 'Co-interior angles on parallel lines are supplementary' using this diagram is shown below. Complete the parts of the proof (a through d) that are missing.

RTP: a

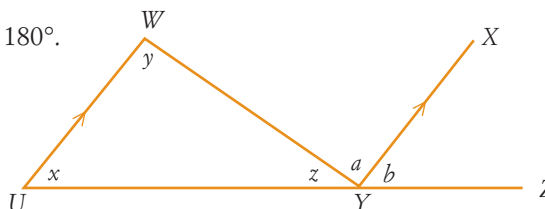
Statement	Reason
1. b	1.
2. $c = e$	2. c
3. d	3. Statement 2.
QED	



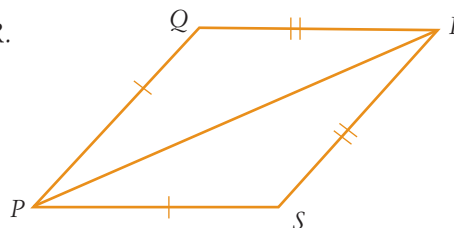
- 5 In the diagram on the right, $RT \parallel XZ$.
Prove that $\angle XYS = \angle YST$.



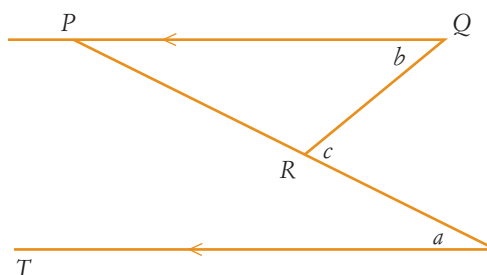
- 6 In the diagram on the right, $UW \parallel XY$.
Prove that $\angle UWY + \angle WYU + \angle YUW = 180^\circ$.



- 7 In the diagram on the right, $PQ = PS$ and $QR = SR$.
Prove that PR bisects $\angle QPS$.



- 8 In the diagram on the right, $PQ \parallel TS$.
Prove that $\angle QRS = \angle QPR + \angle PST$.



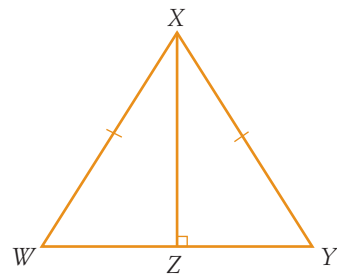
Fluency

Worked solutions

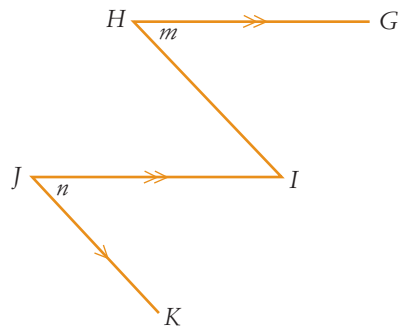
Exercise 11.3

MAT09MGWS00039

- 9 Given that $\triangle WXY$ has $WX = YX$ and $XZ \perp WY$, prove that Z is the midpoint of WY .



- 10 In the diagram on the right, $HG \parallel JI$ and $HI \parallel JK$. Prove that $\angle GHI = \angle IJK$.



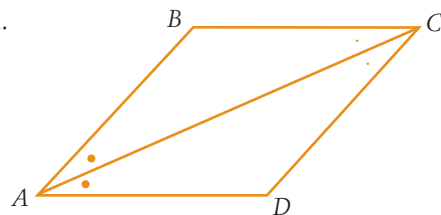
Worked solutions

Exercise 11.3

MAT09MGWS00039

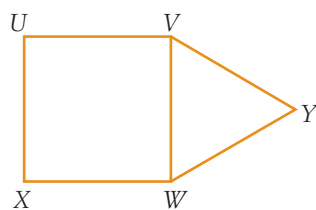
See Example 8

- 11 In the diagram on the right, $ABCD$ is a parallelogram. Prove that $ABCD$ is a rhombus.



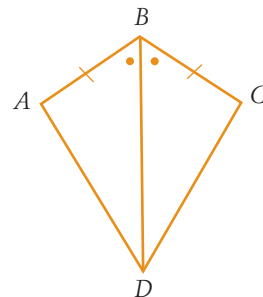
Problem solving

- 12 In the diagram on the right, $UVWX$ is a square and $\triangle VYW$ is isosceles. Prove that $UY = XY$.

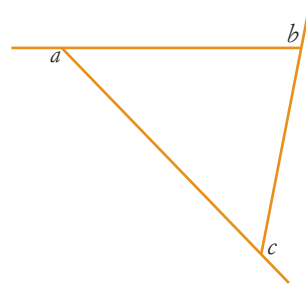


See Example 8

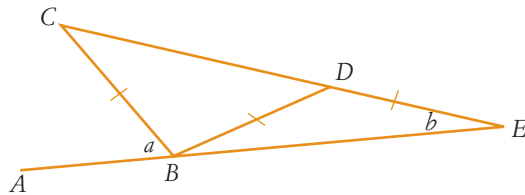
- 13 In the diagram on the right, BD bisects $\angle ABC$ and $AB = BC$. Prove that $ABCD$ is a kite.



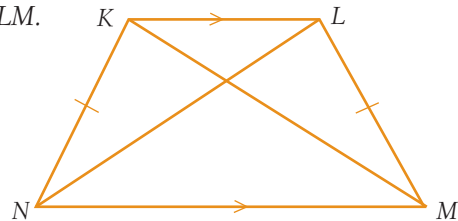
- 14 The diagram on the right is formed from three straight lines.
Prove that $a + b + c = 360^\circ$.



- 15 Given that $BC = BD = DE$ in the diagram shown on the right, prove that $a = 3b$.



- 16 In the trapezium $KLMN$ shown on the right, $KN = LM$.
Prove that $NL = MK$.

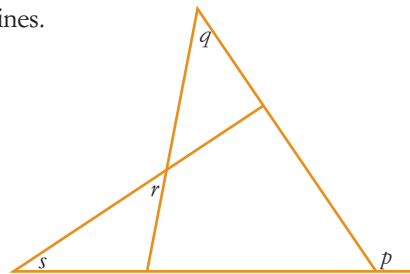


Worked solutions

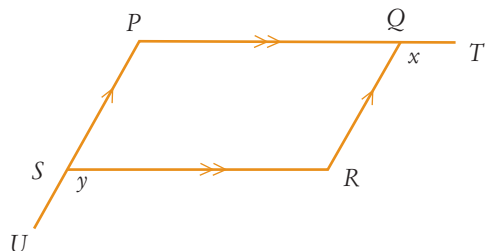
Exercise 11.3

MAT09MGWS00039

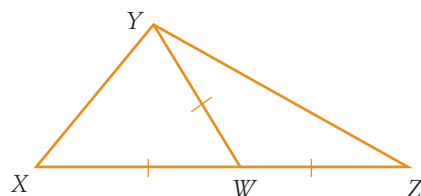
- 17 The diagram on the right is formed from four straight lines.
Prove that $p = q + r + s$.



- 18 $PQRS$ shown on the right is a parallelogram.
Prove that $x = y$.



- 19 In the diagram on the right, $XW = WY = WZ$.
Prove that $\angle XYZ = 90^\circ$.



- 20 Prove that a quadrilateral with congruent opposite sides is a parallelogram.

See Example 9

Chapter 11 summary

Quiz

Shapes and angles

MAT09MGQZ00011

- **Adjacent angles** are next to each other.
- **Complementary angles** add up to 90° (.
- **Supplementary angles** add up to 180° (.
- **Angles at a point** add up to 360° (.
- Two intersecting lines form **vertically opposite angles** that are equal (.
- We show that lines are **parallel** by putting matching arrows on them.
- In Euclidean geometry, parallel lines in the same **plane** never meet.
- A **transversal** is a line that crosses any two other lines.
- When a **transversal** crosses other lines, it makes **corresponding**, **co-interior** and **alternate angles**.
- For **parallel** lines, corresponding angles are equal (, alternate angles are equal (, and co-interior angles are supplementary (.
- The **interior angles** of a triangle are those within it.
- If one of the sides of a triangle is extended, an **exterior angle** is formed.
- The **sum of angles in a triangle** is 180° (.
- The **exterior angle of a triangle** is equal to the sum of the opposite angles inside the triangle (.
- An **isosceles triangle** is special because it has two sides that are equal. The other side of an isosceles triangle is called the **base**.
- The **angles at the base of an isosceles triangle** are equal (.
- The **angle sum of a quadrilateral** is 360° (.
- **Diagonally opposite angles of a parallelogram** are equal (.
- A geometric proof is set out with numbered steps and reasons are given for each step. What needs to be proved (**required to prove – RTP**) is usually stated at the start of the proof. A T-Chart can be helpful for organising the steps of the proof. The proof is complete when you have proven what was required. It is usual to end a proof with the letters 'QED'.
- A **trapezium** is a quadrilateral with at least one pair of opposite sides parallel.
- A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.
- A **rectangle** is a parallelogram with right angles.
- A **square** is a rectangle with two adjacent sides equal in length.
- A **rhombus** is a parallelogram with two adjacent sides equal in length.
- A **kite** is a convex quadrilateral with two pairs of equal adjacent sides.

Understanding

Worksheet

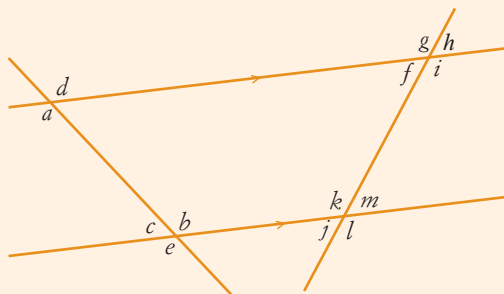
Shapes and angles
review

MAT09MGWK00068

- 1 Draw the symbol codes for these angle relationships.

a Supplementary angles c Complementary angles	b Vertically opposite angles d Angles at a point
- 2 Draw the symbol codes for these angle relationships.

a Co-interior angles on parallel lines b Corresponding angles on parallel lines c Alternate angles on parallel lines	
- 3 Use the diagram below to name an angle that is:



a alternate to b d supplementary with k	b co-interior with m e vertically opposite f	c corresponding to d f alternate to g
- 4 Use the diagram from question 3 and the list of words below to state the relationship between the stated angles in each case.

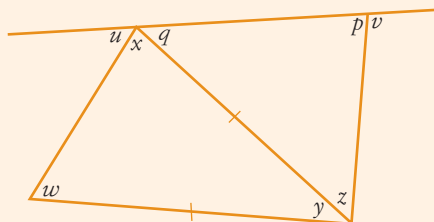
corresponding	co-interior	alternate
supplementary	vertically opposite	angles at a point

a d and b d f and m	b c and b e k, m, j and l	c k and l f j and f
 - 5 Draw the symbol codes for these angle relationships.

a Angles at the base of an isosceles triangle b Sum of the interior angles of a triangle c Sum of the interior angles of a quadrilateral d Exterior angle of a triangle	

Chapter 11 review

- 6 Use the marked angles in the diagram below to complete the following.

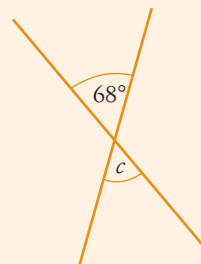
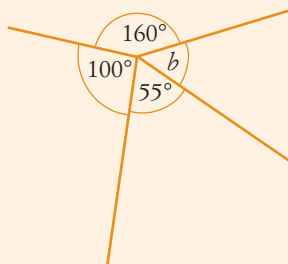
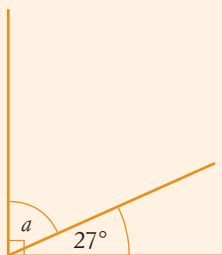


- a ... + ... + ... = 180°
- b ... + ... + ... = 180° (two answers)
- c ... = ...
- d ... = ... + ...
- e ... + ... + ... + ... + ... + ... = 360°
- f ... + ... = 180°
- g ... + ... = ... + ...

Fluency

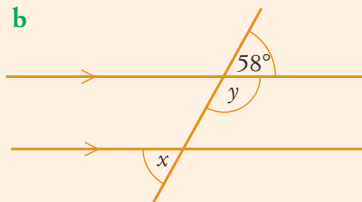
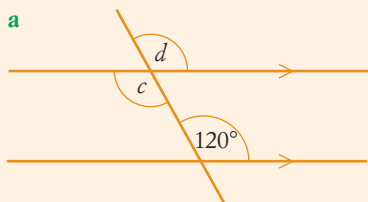
See Example 1

- 7 Work out the values of the unknown angles below. Give reasons for your answers in words and symbols.



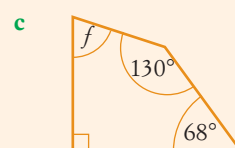
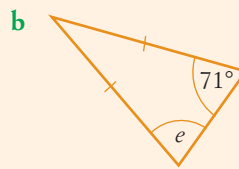
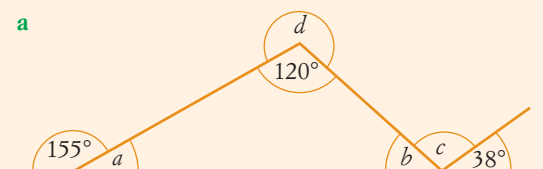
See Example 3

- 8 Work out the values of the unknown angles below. Give reasons for your answers in words and symbols.



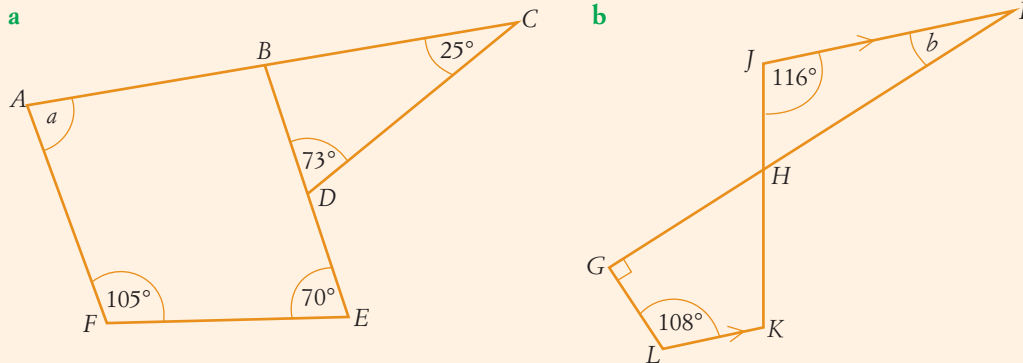
See Example 5

- 9 Work out the values of the unknown angles below. Give reasons for your answers in words and symbols.

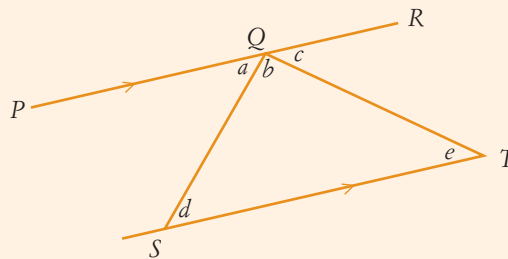


- 10** Work out the values of the unknown angles below. Give reasons for each step in words and symbols.

See Examples 6, 7



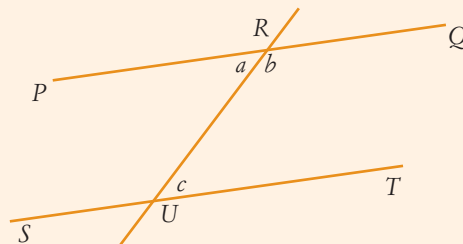
- 11** A proof of the rule ‘The sum of the angles in a triangle is 180° ’ using this diagram is shown below. Complete the parts of the proof (**a** through **e**) that are missing.



RTP: a

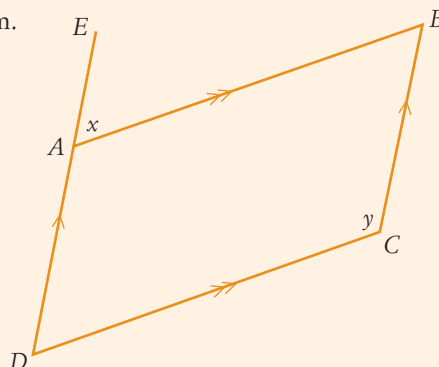
Statement	Reason
1. b	1.
2. $c = e$	2. c
3. d	3.
4. e	4. Combine statements 1, 2 and 3
QED	

- 12** In the diagram on the right, $PQ \parallel ST$.
Prove that $\angle QRU + \angle TUR = 180^\circ$.



Chapter 11 review

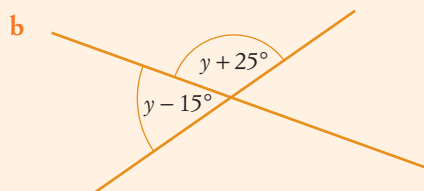
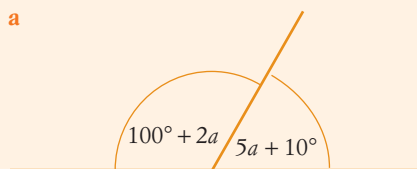
- 13** In the diagram on the right, $ABCD$ is a parallelogram. Prove that $\angle EAB + \angle BCD = 180^\circ$.



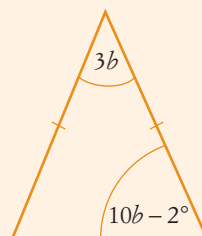
Problem solving

See Example 2

- 14** Work out the values of the pronumerals in these diagrams. Give reasons for your answers in words and symbols.

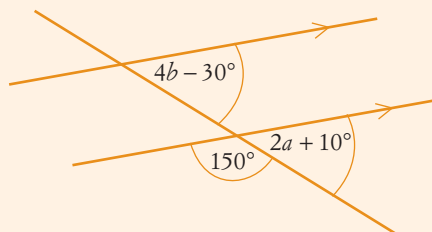


- 15** Find the value of the pronumeral in this diagram. Give reasons for your answer in words and symbols.



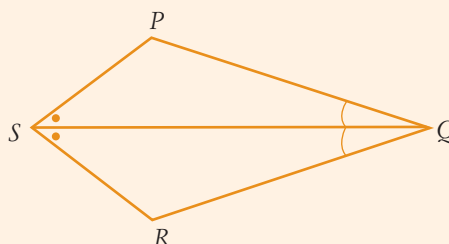
See Example 4

- 16** Find the values of the pronumerals in the diagram on the right. Give reasons for your answers in words and symbols.

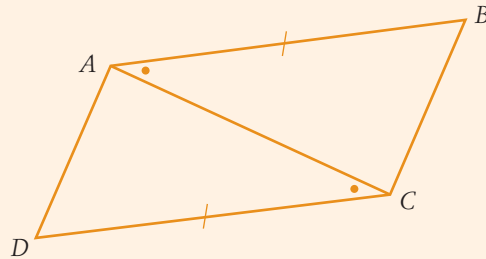


See Example 8

- 17** In the diagram on the right, SQ bisects $\angle PQR$ and $\angle PSR$. Prove that $PQRS$ is a kite.

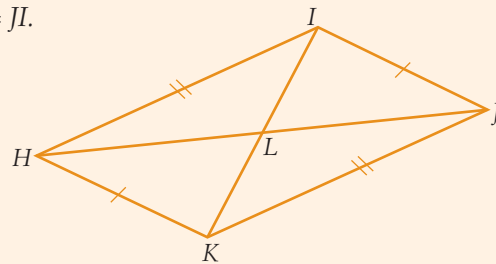


- 18** In the diagram on the right, $AB = CD$ and $\angle BAC = \angle ACD$.
Prove that $ABCD$ is a parallelogram.



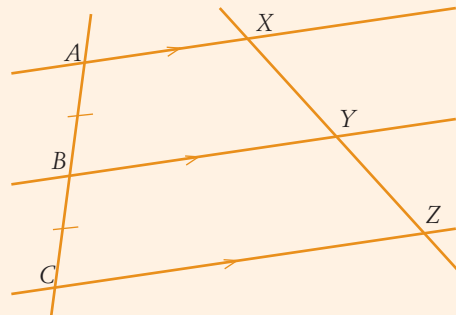
See Example 8

- 19** In the diagram on the right, $HI = JK$ and $HK = JL$.
Prove that $HIJK$ is a parallelogram.



See Example 8

- 20** In the diagram on the right, $AX \parallel BY \parallel CZ$ and $AB = BC$.
Prove that $XY = YZ$.



Reasoning



Number and algebra

12

**Money and
business**



Contents

- 12.1 Percentages
- 12.2 Business percentages
- 12.3 Simple and compound interest
- Chapter summary
- Chapter review

Prior learning

Chapter 12

MAT09NAPL00012

Parent guide

Chapter 12

MAT09NAPG00012

Curriculum guide

Chapter 12

MAT09NACU00012

Australian Curriculum statements

Money and financial mathematics

Solve problems involving simple interest.

Video tutorial

Money and business

MAT09NAVT00012

Weblink

Maths careers

MAT09NAWB00012

Over the next few years, you may buy a car, leave home and make important financial decisions. You may already have a part-time job, mobile phone and a credit, debit or EFTPOS card. Managing your money effectively involves budgeting and knowledge of how loans, interest and investments work.

The work in this chapter will help you to understand how personal finance and business operate. This means that you will be in a better position to make good use of your assets and income.

Mathematical literacy

Maths dictionary

MAT09ASDI00001

Weblink

Money stuff!

MAT09NAWB00012

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

commission	Goods and Services	mark-up	repayment
compound interest	Tax (GST)	mortgage	rest
cost price	instalment	percentage decrease	retainer
credit card	interest	percentage increase	secured
debit card	interest rate	principal	selling price
discount	interest-free period	profit	simple interest
EFTPOS	loss	ready reckoner	term
flat-rate interest	marked price	reducing interest	transaction fee

12.1 Percentages

Percentages are used extensively in society to make comparisons and to state government and business rates. Sometimes we are interested in a **percentage increase** or **decrease**.

Important!

Percentages

A **percentage** is a fraction with a denominator of 100, but the /100 is replaced by the % sign.

For example, $\frac{9}{100} = 9\%$.

A common fraction or decimal can be changed to a percentage by multiplying by 100%.

To find the percentage of a quantity, you should change the percentage to a fraction or decimal and multiply by the quantity.

The calculation of a percentage increase or percentage decrease is based on the original.

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original amount}} \times 100\%$$

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{original amount}} \times 100\%$$

Example 1

Express the following as percentages.

a $2\frac{4}{5}$

b 0.347

c 6 out of 17

Solution

a Write the fraction.

Convert to an improper fraction and multiply by 100%.

$$\begin{aligned} 2\frac{4}{5} &= \frac{14}{5} \times \frac{100\%}{1} \\ &= 280\% \end{aligned}$$

b Write the decimal.

Multiply by 100%.

$$\begin{aligned} 0.347 &= 0.347 \times 100\% \\ &= 34.7\% \end{aligned}$$

c Write the fraction.

Multiply by 100%.

Use your calculator.

$$\begin{aligned} \frac{6}{17} &= \frac{6}{17} \times \frac{100\%}{1} \\ &= 35\frac{5}{17} \approx 35.29\% \end{aligned}$$

CAS TI-Nspire exercise

Money and business

MAT09NATI00012

CAS ClassPad exercise

Money and business

MAT09NACP00012

Puzzle sheet

Fractions to percentages

MAT09NAPS00054

Example 2

a Change 4.8% to a decimal.

c Change $21\frac{2}{3}\%$ to a fraction.

b Change 145% to a mixed number.

Solution

a Write the percentage.

Write % as hundredths.

Treat as division by 100.

$$\begin{aligned} 4.8\% &= \frac{4.8}{100} \\ &= 0.048 \end{aligned}$$

b Write the percentage.

Write % as hundredths.

Cancel down.

$$\begin{aligned} 145\% &= \frac{145}{100} \\ &= 1\frac{9}{20} \end{aligned}$$

c Write the percentage.

Write as an improper fraction.

Write % as hundredths.

Cancel down.

$$\begin{aligned} &= 21\frac{2}{3}\% \\ &= \frac{65}{3}\% \\ &= \frac{65}{3 \times 100} \\ &= \frac{13}{60} \end{aligned}$$

Puzzle sheet

Percentages to fractions

MAT09NAPS00055

Technology

Excel worksheet:
Percentages, fractions
and decimals

MAT09NACT00029

Example 3

Andrew scored 15 out of 20 for English and 19 out of 26 for Science. Which is the better score?

Solution

Change the English score to a percentage.

$$\frac{15}{20} = \frac{15}{20} \times \frac{100\%}{1} = 75\%$$

Change the Science score to a percentage

$$\frac{19}{26} = \frac{19}{26} \times \frac{100\%}{1} = 73.08\%$$

Choose the larger percentage.

15 out of 20 for English is the better score.

CAS TI-Nspire exercise

Money and business

MAT09NATI00012

CAS ClassPad exercise

Money and business

MAT09NACP00012

Worksheet

Working with percentages

MAT09NAWK00070

Puzzle sheet

Percentages of an amount

MAT09NAPS00056

Example 4

Methylated spirits contains 1.2% pyridine to make it taste awful. How much pyridine is there in 40 kg of methylated spirits?

Solution

Write the problem.

1.2% of 40 kg

Write in decimal form.

= 0.012 × 40

Enter as 0.012 × 40 = .

0.012×40 0.48

Write the answer.

40 kg of methylated spirits contains 0.48 kg (480 g) of pyridine.

Using a diagram can make many percentage calculations easier.

Example 5

Oranges are shipped from California in cartons. On the journey, about 15% of the oranges go rotten or become otherwise unsuitable for sale. At a fruit shop, 148 Californian oranges are thrown away. How many oranges were sold?

Solution

Show the percentages on the top scale of a 'ruler' and the oranges on the bottom scale.



Find the percentage of good oranges.

$$\text{Good oranges percentage} = 100\% - 15\% = 85\%$$

Write as a ratio problem.

$$\frac{\text{Good oranges}}{85\%} = \frac{148}{15\%} \text{ or } \frac{\text{Good oranges}}{148} = \frac{85\%}{15\%}$$

Solve for the number of good oranges. $\text{Good oranges} = \frac{85}{15} \times 148$
Do the calculation. ≈ 838.67
Round and write the answer. **About 839 oranges were sold.**

Example 6

The enrolment at a school in a mining town increased from 745 to 800. What was the percentage increase?



Solution

Find the increase in enrolment.

$$\text{Increase} = 800 - 745 = 55$$

Write the formula.

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original amount}} \times 100\%$$

Substitute values.

$$= \frac{55}{745} \times 100\% \\ \approx 7.38\%$$

Round and write the answer.

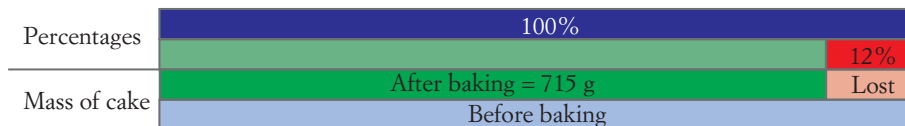
The enrolment increased by about 7.4%.

Example 7

During cooking, the mass of a cake decreased by 12%. The baked cake has a mass of 715 g. What was its mass before baking?

Solution

Show the percentages on the top scale of a 'ruler' and the mass of cake on the bottom scale.



Find the percentage after baking.

$$\text{After baking percentage} = 100\% - 12\% = 88\%$$

Write as a ratio problem.

$$\frac{\text{Before baking}}{100\%} = \frac{715\text{g}}{88\%} \text{ or } \frac{\text{Before baking}}{715\text{g}} = \frac{100\%}{88\%}$$

Solve for the mass before baking.

$$\text{Before baking} = \frac{100}{88} \times 715\text{g} \\ \approx 812.5\text{ g}$$

Do the calculation.

Round and write the answer.

The cake had a mass of about 813 g before baking.

Exercise 12.1 Percentages

Understanding

Extra questions

Exercise 12.1

MAT09NAEQ00040

See Example 1

1 Express as percentages.

a $2\frac{1}{4}$

b 39 out of 60

c 0.423

d 42 out of 96

e 111 out of 127

f $\frac{3}{11}$

g $\frac{5}{8}$

h 16 out of 19

i $\frac{57}{24}$

j 7 out of 13

k 5.36

l $4\frac{3}{4}$

2 Change to fractions or mixed numbers.

a 225%

b 150%

c 320%

d 165%

e 130%

f 22.6%

g 37.5%

h 243%

3 Change to decimals.

a 3.6%

b 24.8%

c 0.3%

d 220%

e 128%

f 1.46%

g 43.8%

h 15.9%

4 Calculate each of the following.

a 16.2% of \$500

b 3.8% of 45 L

c 79.7% of 340 kg

d 34.6% of \$88

e 17.5% of \$48.74

f 23.1% of 5.67 kg

g 224% of 5.8 km

h 152% of 540 s

Fluency

See Example 2

5 Change to fractions or mixed numbers.

a $66\frac{2}{3}\%$

b $27\frac{1}{2}\%$

c $94\frac{1}{2}\%$

d $45\frac{1}{5}\%$

e $41\frac{3}{4}\%$

f $22\frac{3}{5}\%$

g $241\frac{2}{3}\%$

h $16\frac{3}{4}\%$

6 Calculate each of the following.

a $12\frac{1}{2}\%$ of 480 g

b $15\frac{2}{3}\%$ of \$9000

c $8\frac{1}{3}\%$ of \$5400

d $4\frac{3}{5}\%$ of 20 kg

e $4\frac{1}{3}\%$ of 800 L

f $7\frac{4}{5}\%$ of 2300 t

g $5\frac{1}{4}\%$ of \$440

h $11\frac{1}{4}\%$ of 240 m

7 Calculate the overall percentage change in each of these cases.

a Increase of 50%, followed by a decrease of 50%

b Decrease of 50%, followed by an increase of 50%

c Increase of 30%, followed by a decrease of 30%

d Decrease of 30%, followed by an increase of 30%

Problem solving

8 Debbie bought some potatoes for her fruit shop. She got Kennebecs for 80c/kg, Brownells for \$1.20/kg and Pink-eyes for \$1.75/kg. She sold the Kennebecs for \$2.10/kg, the Brownells for \$2.60/kg and the Pink-eyes for \$4.20/kg.

a What was the profit per kg for each variety?**b** What was the percentage profit for each?**c** Which variety gave Deb the best profit?

See Example 3

9 David scored 14 out of 20 for Maths and 9 out of 15 for English. Use percentages to work out the better result.

10 Tran and Andrew have different bank accounts. Tran received \$4 interest and Andrew received \$5. They had \$85 and \$110 in the bank respectively, before interest. Who got the better percentage interest?

Worked solutions

Exercise 12.1

MAT09NAWS00040

- 11** A hardware store gives 5% off for immediate cash payment, and has an extra charge of 5% a month for credit. What would be the amount paid for each of these items?
- a** A sink marked at \$185, paid immediately
 - b** A front door listed at \$680, paid in 2 months time
 - c** The same door, paid immediately
 - d** A deadlock at \$69.40, on 3 months credit



- 12** Rhonda sells houses. She earns 2.6% of the sale price on any house she sells. How much does she receive for selling houses at the following prices?

See Example 4

- a** \$472 900 **b** \$558 990 **c** \$632 000 **d** \$1 289 500

- 13** A grocery store sells things for 15% more than it pays for them. What are the selling prices of these items?

See Example 5

- a** Weetbix bought for \$3.25 **b** Honey (they paid \$2.34)
- c** Juice packs for which they paid 58c **d** Butter bought at \$2.59

- 14** Tass's Newsagency bought 50 pens for \$15.00 at a fire sale and sold them for 50c each. Unfortunately, they had to give refunds for 5 that wouldn't write.

- a** How much money did they make?
- b** What was their percentage profit?
- c** What would their percentage profit have been if all the pens had worked?

- 15** Donna is an importer. She bought cane chairs for \$45 each and sold them to a wholesaler for \$65 each, but had to pay \$10 each in freight and duty. The wholesaler sold them for \$80 each and the final customer paid \$120 each for them.

- a** Work out the profits made by the importer, wholesaler and retailer.
- b** Work out their percentage profits.
- c** What is the total of their percentage profits?
- d** How much extra did the customer pay, compared with Donna?
- e** What is this extra as a percentage? Compare with part **c**.

- 16** A manufacturer adds 25% to the cost of materials to cover costs and profit, a wholesaler adds 15% to cost, and the retailer adds 30%. Skis cost a manufacturer \$140 for materials.

- a** Work out the cost to the wholesaler.
- b** Work out the cost to the retailer.
- c** Work out the cost to the customer.
- d** What is the total extra percentage that the customer pays?

Worked solutions

Exercise 12.1

MAT09NAWS00040

- 17 A surf shop offers a discount of 20% to members of a lifesaving club and a 15% discount for cash payment. If Giorgio is a member and pays cash for a \$500 board, does it make any difference how the total discount is worked out?



See Example 6

- 18 The estimated population of a local country region over 6 years is shown in the table. Work out the percentage increase for each year.

Year	Population
2006	146 065
2007	148 799
2008	151 592
2009	154 390
2010	157 188
2011	159 949

- 19 The consumer price index (CPI) measures the increase in the prices of items that an average Australian buys. The CPI increase in 2011 was 2.9%. If the prices below are for the end of 2011, work out the expected prices at the end of 2012.

- a Refrigerator, \$920 b Rent for 2-bedroom unit, \$280 per week
c Train fare to work, \$5.60 d Takeaway fish and chips, \$8.30

- 20 Vida is a grape grower. The yields from her vineyard were as follows.

Year	2008	2009	2010	2011
Yield (t)	257	350	450	350

- a Calculate the percentage increase or decrease for each year.
b The increase in tonnage for 2009–10 was the same as the decrease for 2010–11, but the percentage changes are different. Explain why the percentages are different.

Reasoning

Worked solutions

Exercise 12.1

MAT09NAWS00040

- 21 In one year, 53 000 t of coal is mined from an Australian site. The next year, production increases by 40%, and the following year it decreases by 40%. Find the overall percentage change.
- 22 Andrew is a florist. One week he sells 500 bouquets. The next week his sales decrease by 30%, and the following week they increase by 30%. What is the overall percentage change?

12.2 Business percentages

Worksheet

Earning money

MAT09NAWK00069

Investigate: Business percentages

You can use business and accounting calculators to work out business percentage problems.

There is a spreadsheet on the NelsonNet website that you can use to automatically calculate business percentage problems.

This spreadsheet also shows you the ruler model of business percentages, so you can check your method and calculate the percentages yourself before getting the answers.

	A	B	C	D	E	F	G	H	I	J	K	L												
1																								
2		Business percentages spreadsheet																						
3						Profit																		
4		Instructions					Amounts																	
5		Press the button for the kind of problem					<table border="1"> <tr> <td>Buying price</td> <td>28</td> </tr> <tr> <td>Selling price</td> <td></td> </tr> <tr> <td>Profit</td> <td></td> </tr> </table>							Buying price	28	Selling price		Profit						
Buying price	28																							
Selling price																								
Profit																								
6						Loss																		
7		Type in two things you know					Percentages																	
8		Press return after each one					<table border="1"> <tr> <td>Selling price as a %</td> <td></td> </tr> <tr> <td>Profit as a %</td> <td>10</td> </tr> </table>							Selling price as a %		Profit as a %	10							
Selling price as a %																								
Profit as a %	10																							
9		One of them <i>must be an amount</i>																						
10						Discount																		
11		Then press the model button																						
12		When you're ready, press the Answers button					<table border="1"> <tr> <td>Model</td> <td>Answers</td> <td>Clear</td> </tr> </table>							Model	Answers	Clear								
Model	Answers	Clear																						
13		Finally press the clear button to start again																						
14																								
15		<table border="1"> <tr> <td></td> <td colspan="11">\$28.</td> </tr> </table>												\$28.										
	\$28.																							
16																								
17		Amounts																						
18																								
19																								
20		Percentages	100%																					
21			10%																					
22																								
23																								
24																								

Your teacher may already have loaded the spreadsheet onto the school intranet.

Otherwise, go to the website and try out the automatic Business Percentage calculator.

In business and government, many quantities are expressed as percentages to make comparisons easier. You should know all the terms in the following box.

Important!**Business percentages**

The **cost price (buying price)** of an item is the price that was originally paid by the seller. The **selling price** is the price it is then sold for. The normal selling price (before any discount) is also called the **marked price**.

A **profit** is made when goods are sold for *more* than what they cost.

$$\text{Profit} = \text{selling price} - \text{cost price}$$

A **loss** is made when goods are sold for *less* than what they cost.

$$\text{Loss} = \text{cost price} - \text{selling price}$$

A **mark-up** is added to the cost price to get the **marked price** (normal selling price).

$$\text{Marked price} = \text{cost price} + \text{mark-up}$$

A **discount** is an amount taken off a normal price (the marked price).

$$\text{Discount} = \text{normal marked price} - \text{price actually paid}$$

A **commission** is money paid for a service that a person has carried out. It is usually expressed as a percentage of the value of the service, such as the value of goods sold by a sales representative. A fixed **retainer** may also be paid, even if no sales are made.

When discount, mark-up, profit, loss or commission is calculated as a percentage, it is usually considered to be 'out of' the original price. You can think of this as:

$$\text{Percentage amount} = \frac{\text{amount}}{\text{original price}} \times 100\%$$

In Australia, the **Goods and Services Tax (GST)** is paid to the government whenever items are sold or a service is provided. It is set at 10%.

Example 8

A used-car dealer bought a car for \$3400 and sold it at a profit of 24%. What was the selling price?

Solution

Calculate the profit.

$$\text{Profit} = 24\% \text{ of } \$3400$$

Write the percentage as a decimal.

$$= 0.24 \times \$3400$$

$$= \$816$$

Find the selling price.

$$\text{Selling price} = \text{cost price} + \text{profit}$$

$$= \$3400 + \$816 = \$4216$$

Write the answer.

$$\text{The selling price was } \$4216.$$

Example 9

At a liquidation sale, chairs that cost \$150 were sold for \$120. What was the percentage loss?

Solution

Calculate the loss.

$$\text{Loss} = \$150 - \$120 = \$30$$

Use the original amount to calculate the percentage loss.

$$\begin{aligned} \% \text{ loss} &= \frac{30}{150} \times 100\% \\ &= 20\% \end{aligned}$$

Write the answer.

There was a 20% loss.

Example 10

A pair of jeans marked at \$95 is discounted by 20% during a sale. What is the new price?

Solution

Show the percentages on the top scale of a 'ruler' and the prices on the bottom scale.



Find the new price percentage.

$$\begin{aligned} \text{New price percentage} &= 100\% - 20\% \\ &= 80\% \end{aligned}$$

Write as a ratio problem.

$$\frac{\text{New price}}{80\%} = \frac{\$95}{100\%} \quad \text{or} \quad \frac{\text{New price}}{\$95} = \frac{80\%}{100\%}$$

Solve for the new price.

$$\begin{aligned} \text{New price} &= \frac{80}{100} \times \$95 \\ &= \$76 \end{aligned}$$

Write the answer.

The new price is \$76.

Puzzle sheet

Percentage profit or
loss

MAT09NAPS00057

Example 11

A store works on a mark-up of 30%. The wholesale price of a mixer is \$80. What should the selling price be?

Solution

Show the percentages on the top scale of a 'ruler' and the prices on the bottom scale.

Percentages	100%	30%
Prices	Wholesale price = \$80	Mark-up
	Selling price	

Find the selling price percentage.

$$\text{Selling price percentage} = 100\% + 30\% = 130\%$$

Write as a ratio problem.

$$\frac{\text{Selling price}}{130\%} = \frac{\$80}{100\%} \quad \text{or} \quad \frac{\text{Selling price}}{\$80} = \frac{130\%}{100\%}$$

Solve for the selling price.

$$\begin{aligned} \text{Selling price} &= \frac{130}{100} \times \$80 \\ &= \$104 \end{aligned}$$

Write the answer.

The selling price should be \$104.

Example 12

A sales representative works on 10% commission with a weekly retainer of \$350. How much does she earn in a week when she sells \$7300 worth of furnishings?

**Solution**

Calculate the commission.

$$\begin{aligned} \text{Commission} &= 10\% \text{ of } \$7300 \\ &= \$730 \end{aligned}$$

Calculate earnings.

$$\begin{aligned} \text{Earnings} &= \text{retainer} + \text{commission} \\ &= \$350 + \$730 = \$1080 \end{aligned}$$

Write the answer.

She earns \$1080.

Example 13

A jacket sold at a discount of 15% cost the customer \$119. What was the original price?

Solution

Show the percentages on the top scale of a 'ruler' and the prices on the bottom scale.



Find the discount price percentage.

$$\text{Discount price percentage} = 100\% - 15\% = 85\%$$

Write as a ratio problem.

$$\frac{\text{Original price}}{100\%} = \frac{\$119}{85\%} \text{ or } \frac{\text{Original price}}{\$119} = \frac{100\%}{85\%}$$

Solve for the original price.

$$\begin{aligned} \text{Original price} &= \frac{100}{85} \times \$119 \\ &= \$140 \end{aligned}$$

Write the answer.

The original price was \$140.

Example 14

A real estate agent working on a retainer of \$900/month and commission of 2.5% made \$7640 in January. What was the value of his sales?

Solution

Write the relationship.

$$\text{Earnings} = \text{retainer} + \text{commission}$$

Substitute values.

$$\$7640 = \$900 + \text{commission}$$

Calculate the commission.

$$\begin{aligned} \text{Commission} &= \$7640 - \$900 \\ &= \$6740 \end{aligned}$$

Write the percentage calculation.

$$\text{Commission} = 2.5\% \text{ of sales}$$

Substitute values and use a decimal.

$$\$6740 = 0.025 \times \text{sales}$$

Solve for the value of sales.

$$\begin{aligned} \text{Sales} &= \frac{\$6740}{0.025} \\ &= \$269\,600 \end{aligned}$$

Write the answer.

The value of his sales was \$269 600.

Exercise 12.2 Business percentages

Understanding

Extra questions

Exercise 12.2

MAT09NAEQ00041

See Example 8

- 1 A car saleyard works on a profit margin of 28%. For what price does it sell cars that cost:

a \$9100 **b** \$3780 **c** \$18 000 **d** \$7590
e \$4900 **f** \$24 500 **g** \$6250 **h** \$6700

Fluency

See Example 9

- 2 Work out the percentage profit or loss for each of the following items.

a Dictionary, bought for \$45 and sold for \$54
b Bicycle, bought for \$240 and sold for \$300
c Foot pump, bought for \$5 and sold for \$4.70
d Tent, bought for \$800 and sold for \$710
e Flyscreen doors, bought for \$70 and sold for \$85

See Example 11

- 3 A clothing store works on a mark-up of 45%. Work out its selling price for:

a blouses that cost \$30 **b** nighties that cost \$12
c trousers that cost \$98 **d** socks that cost \$3
e jeans that cost \$75 **f** shirts that cost \$36
g jackets that cost \$140 **h** belts that cost \$25

See Example 10

- 4 At a sale, Delphine's Discount Electrical Goods marked the percentage discount on the tickets of items. What were the actual prices for these items?

a Toasters at \$28, discount 30% **b** Electric frypans at \$96, discount 15%
c Jaffle makers at \$55, discount 20% **d** Microwave ovens at \$240, discount 12%
e Steam-and-dry irons at \$85, discount 25%

See Example 12

- 5 Pam is a travelling sales representative for a confectionery distributor. She works on a retainer of \$150 a week plus 15% commission.

a How much does she earn if she sells \$2000 worth of confectionery?
b How much does she earn in a week when she sells \$5000 worth?

See Example 14

- 6 What sales did Pam in question 5 make when she earned these weekly amounts?

a \$600 **b** \$1000

Problem solving

Worked solutions

Exercise 12.2

MAT09NAWS00041

- 7 The wholesale price of a portable air-conditioner was \$450. Bargain Blowers sold it for \$630. What was the percentage mark-up?

- 8 For imported cars there is a special tax called import duty which has to be paid to the Federal Government. If the declared value of an imported car was \$28 500, and the import duty was \$17 100, what would the percentage import duty be?

- 9 A furniture store has worked out a price of \$520 for a table, but has not included GST. What is the final price including GST?

- 10 A shop discounts all electrical goods by the same percentage. A fridge normally priced at \$1400 is sold for \$1120, and a digital video recorder is sold for \$600.

a What is the percentage discount?
b What was the original price of the digital video recorder?

Worked solutions

Exercise 12.2

MAT09NAWS00041

- 11 A shop that works on a mark-up of 35% sells guitars for \$94.50 and baby grands for \$10 800. What was the wholesale price of each?
- 12 Federico bought an electric guitar for \$638, including GST. How much GST was included?

Reasoning

See Example 13

Worked solutions

Exercise 12.2

MAT09NAWS00041

12.3 Simple and compound interest

When you want to buy large items, you can either save for the item or borrow money to buy the item immediately and pay it back over a period of time. When you pay money back, you normally have to pay more than the loan amount. The extra amount paid is called **interest**. If you deposit money with a bank, credit union or building society, it pays *you* interest.

Some shops will put an item aside while you pay for it over a period. This is called lay-by. You normally have to pay a deposit and make regular payments until the item is completely paid off. If you do not complete payment within a particular time, usually 3 months, you lose the item and the money you have already paid.

Important!

Interest

Interest is the charge on a loan. It is normally worked out as a percentage, called the **interest rate**, of the total amount loaned.

The amount lent or borrowed is called the **principal**.

The amount that has to be paid back each week or month for a loan on **terms** is called the **instalment** or **repayment**.

Simple interest is calculated using the whole loan for the whole time it is borrowed. It is also called **flat-rate interest**.

Compound interest is calculated at regular intervals called **rests**, and added to the loan, with the result that the principal increases at each rest.

Example 15

Shanaleigh bought a mountain bike at a sale on terms of \$7 a week for 2 years. The sale price was \$448.

- a What is the total amount paid? b What is the total interest?
- c What is the interest rate?

Solution

- a Calculate the total for the 2 years (104 weeks).

$$\begin{aligned}\text{Total paid} &= 104 \times \$7 \\ &= \$728\end{aligned}$$

- b Find the extra amount paid.

$$\begin{aligned}\text{Total interest} &= \$728 - \$448 \\ &= \$280\end{aligned}$$

- c Work out the interest for 1 year.

Calculate the interest as a percentage of the principal.

$$\begin{aligned}\text{Annual interest} &= \$280 \div 2 \\ &= \$140\end{aligned}$$

$$\begin{aligned}\text{Interest rate} &= \frac{\text{annual interest}}{\text{principal}} \times 100\% \\ &= \frac{\$140}{\$448} \times 100\% = 31.25\%\end{aligned}$$

Example 16

Paul paid a deposit of \$800 for a new boat priced at \$6500 and signed a contract to pay it off over 3 years at 15% flat-rate interest.

- a What is the amount borrowed?
b Work out his monthly repayments for the loan.

Solution

- a Calculate the amount borrowed (the principal).

Write the answer.

- b Calculate the interest for 1 year.

Calculate the total interest.

Find the amount to pay back.

Find the monthly repayment over 3 years (36 months).

Write the answer.

$$\begin{aligned}\text{Amount borrowed} &= \$6500 - \$800 \\ &= \$5700\end{aligned}$$

Amount borrowed is \$5700.

$$\begin{aligned}\text{Interest for 1 year} &= 15\% \text{ of } \$5700 \\ &= 0.15 \times \$5700 \\ &= \$855\end{aligned}$$

$$\begin{aligned}\text{Interest for 3 years} &= 3 \times \$855 \\ &= \$2565\end{aligned}$$

$$\begin{aligned}\text{Total to repay} &= \text{principal} + \text{interest} \\ &= \$5700 + \$2565 \\ &= \$8265\end{aligned}$$

$$\begin{aligned}\text{Repayment} &= \frac{\text{total to repay}}{\text{number of payments}} \\ &= \frac{\$8265}{36} \approx \$229.58\end{aligned}$$

Paul has to pay approximately \$229.58 each month.

Example 17

Kellie put a washing machine costing \$940 on lay-by. A 15% deposit and weekly payments of at least \$10 are required. The whole amount must be paid within 3 months.

- a How much deposit is required?
b What regular weekly payment is needed to pay it off?
c The same washing machine can be bought for the same deposit on terms of 24% interest for 3 months. What weekly payment would be required if Kellie took the washing machine straight away?
d What do you recommend?



Solution

- a** Calculate 15% for the deposit.

$$\begin{aligned}\text{Deposit} &= 15\% \text{ of } \$940 \\ &= 0.15 \times \$940 \\ &= \$141\end{aligned}$$

- b** Calculate the amount left to pay.

$$\begin{aligned}\text{Amount left} &= \$940 - \$141 \\ &= \$799\end{aligned}$$

There are 13 weeks in 3 months.

$$\begin{aligned}\text{Weekly payment} &= \$799 \div 13 \\ &\approx \$61.46\end{aligned}$$

Write the answer.

Regular weekly payment is \$61.46.

- c** Calculate the interest for 1 year.

$$\begin{aligned}\text{Interest for 1 year} &= 24\% \text{ of } \$799 \\ &= 0.24 \times \$799 \\ &= \$191.76\end{aligned}$$

Calculate the interest for a quarter of a year.

$$\begin{aligned}\text{Interest for 3 months} &= \$191.76 \div 4 \\ &= \$47.94\end{aligned}$$

Find the amount to pay back.

$$\begin{aligned}\text{Total to repay} &= \text{principal} + \text{interest} \\ &= \$799 + \$47.94 \\ &= \$846.94\end{aligned}$$

The principal is \$799.

Find the weekly repayment.

$$\begin{aligned}\text{Repayment} &= \frac{\text{total to repay}}{\text{number of payments}} \\ &= \frac{\$846.94}{13} \\ &\approx \$65.15\end{aligned}$$

Write the answer.

Weekly payment is \$65.15.

- d** Write your recommendation.

If Kellie needs the washing machine immediately, it will cost her \$3.69 extra each week. If she can wait, she will save \$47.94. She should wait if possible.

Example 18

Peter invested \$4000 for 2 years at 14% compound interest with 6-monthly rests. What are the final amount and total interest?

Solution

Find the interest % for a 6-month rest.

$$\begin{aligned}\text{Interest \% for a rest} &= 14\% \div 2 = 7\% \\ &= 0.07\end{aligned}$$

Change to a decimal.

Calculate the interest for the first 6 months.

$$\begin{aligned}\text{Interest for 1st rest} &= 0.07 \times \$4000 \\ &= \$280\end{aligned}$$

Add to find the total.

$$\text{Principal after 1st rest} = \$4000 + \$280 = \$4280$$

Calculate the interest for the next 6 months.

$$\begin{aligned}\text{Interest for 2nd rest} &= 0.07 \times \$4280 \\ &= \$299.60\end{aligned}$$

Worksheet

Compound interest

MAT09NAWK00071

Technology

Excel worksheet:
Comparing investments

MAT09NACT00030

Add to find the total.

$$\begin{aligned}\text{Principal after 2nd rest} &= \$4280 + \$299.60 \\ &\approx \$4579.60\end{aligned}$$

Calculate the interest for the next 6 months.

$$\begin{aligned}\text{Interest for 3rd rest} &= 0.07 \times \$4579.60 \\ &\approx \$320.57\end{aligned}$$

Add to find the total.

$$\begin{aligned}\text{Principal after 3rd rest} &= \$4579.60 + \$320.57 \\ &= \$4900.17\end{aligned}$$

Calculate the interest for the next 6 months.

$$\begin{aligned}\text{Interest for 4th rest} &= 0.07 \times \$4900.17 \\ &\approx \$343.01\end{aligned}$$

Add to find the total.

$$\begin{aligned}\text{Principal after 4th rest} &= \$4900.17 + \$343.01 \\ &= \$5243.18\end{aligned}$$

Find the increase over 2 years.

$$\begin{aligned}\text{Total interest} &= \$5243.18 - \$4000 \\ &= \$1243.18\end{aligned}$$

Write the answer.

The final amount is \$5243.18 and the total compound interest is \$1243.18.

Most people have **credit cards** or **debit cards** or both. Although these cards look the same, the way they work is different.

When you use a credit card, you are *borrowing money* and in most cases you will have to pay interest. Some credit cards have an **interest-free period**. This means that, if you pay the account in full by the end of this period, you don't actually pay any interest. The interest on credit card debts is about 2% on the *monthly* account.

When you use a debit card, the money is taken immediately from your account (**linked to your account**), so you *must have the money* in your account to use this type of card.

Credit and debit cards used in **EFTPOS** (Electronic Funds Transfer at Point of Sale) machines are issued by banks, credit unions, building societies and other organisations.

You may be charged a **transaction fee** every time you use a credit or debit card or there is other activity on your account. This charge is often about 20 cents for every transaction.

Example 19

Stephen uses his credit card to buy a new set of tyres for his car. The tyres cost \$520 altogether. The first monthly account is interest-free if it is paid in full by the due date; after that, he pays 2% monthly on the outstanding amount. Transaction fees of 20 cents are charged. If he leaves the cost of the tyres for 2 months before paying it off, how much will he pay for them?

Solution

Calculate the interest at 2%.

$$\begin{aligned}\text{Interest for 1st month} &= 0.02 \times \$520 \\ &= \$10.40\end{aligned}$$

Calculate the amount owing.

$$\begin{aligned}\text{Amount after 1st month} &= \$520 + \$10.40 \\ &= \$530.40\end{aligned}$$

Calculate the next month's interest.

$$\begin{aligned}\text{Interest for 2nd month} &= 0.02 \times \$530.40 \\ &= \$10.61\end{aligned}$$

Animated example

Simple and compound interest

MAT09NAAE00012

Calculate the amount owing.

$$\begin{aligned}\text{Amount after 2nd month} &= \$530.40 + \$10.61 \\ &= \$541.01\end{aligned}$$

Work out the transaction fees, including two interest calculations.

$$\begin{aligned}\text{Transaction fees} &= 3 \times \$0.20 \\ &= \$0.60\end{aligned}$$

Add the fees to the amount owing.

$$\begin{aligned}\text{Total owing} &= \$541.01 + \$0.60 \\ &= \$541.61\end{aligned}$$

Write the answer.

He will pay \$541.61 for the tyres.

Example 20

Peta has \$425 in her bank account. Her debit card is linked to the account. If she uses it to buy \$35 worth of petrol and then buys a pair of shorts for \$56.50, how much will she have left in her account? (Assume that she pays 20 cents per transaction.)

Solution

Work out the total cost.

$$\begin{aligned}\text{Total} &= \$35 + \$56.50 + 2 \times \$0.20 \\ &= \$91.90\end{aligned}$$

Calculate the remainder.

$$\begin{aligned}\text{Amount left} &= \$425 - \$91.90 \\ &= \$333.10\end{aligned}$$

Write the answer.

Peta has \$333.10 left in her account.

When you borrow a large amount of money, the calculation of interest is done every time you make a (monthly) payment. This means that you are really paying compound interest. However, the payments are set so that you gradually pay off the loan, so the amount of interest gets less as you pay off the loan. This is called **reducing interest** and is used for loans like **mortgages** for houses. A mortgage loan is **secured** against the property—if you don't pay, the lender takes the property instead. The calculation of mortgage payments is very complicated, so we usually use a **ready reckoner** table such as the one below. Ready reckoners are also available at many financial institution sites on the Internet.

Monthly payments for mortgages on \$1000

Years	Interest rate						
	6%	7%	8%	9%	10%	11%	12%
5	19.333	19.801	20.276	20.758	21.247	21.742	22.244
10	11.102	11.611	12.133	12.668	13.215	13.775	14.347
15	8.439	8.988	9.557	10.143	10.746	11.366	12.002
20	7.164	7.753	8.364	8.997	9.650	10.322	11.011
25	6.443	7.068	7.718	8.392	9.087	9.801	10.532
30	5.996	6.653	7.338	8.046	8.776	9.523	10.286

TLF Learning object

Maths and the car: loan calculator (L1449)

MAT09NAIN00012

Puzzle sheet

Mortgage
ready-reckoner

MAT09NAPS00059

Example 21

- a** What would a mortgage of \$340 000 over 15 years at 8% cost each month?
b What is the total amount paid?

Solution

- a** Use the 8% column for 15 years.
 Multiply by 340 to get the payment for \$340 000.

Write the answer.

- b** Calculate the number of payments in 15 years.

Find the total paid for the loan.

Write the answer.

$$\text{Payment for \$1000} = \$9.557 \text{ per month}$$

$$\begin{aligned} \text{Payment for loan} &= \$9.557 \times 340 \\ &= \$3249.38 \text{ per month} \end{aligned}$$

The mortgage would cost \$3249.38 per month.

$$\begin{aligned} \text{Number of payments} &= 12 \times 15 \\ &= 180 \end{aligned}$$

$$\begin{aligned} \text{Total paid} &= 180 \times \$3249.38 \\ &= \$584\,888.40 \end{aligned}$$

The total amount paid is \$584 888.40.

Financial institutions such as banks, credit unions and building societies impose conditions on loans such as requiring mortgage insurance to ensure they are paid even if you lose your job. A good 'rule-of-thumb' for the amount you can afford to repay is to calculate 25% of your income, after allowing for existing payments.

Example 22

Dave and Sarah have a combined monthly income of \$7990 after tax. They are paying off a car at \$350 a month and have saved a deposit of \$72 000 towards buying a house or unit.

- a** What is the biggest loan they can afford if the bank they go to has 30-year loans at 9%?
b What is the maximum purchase price they can afford?

Solution

- a** Find the affordable monthly amount.

Subtract existing commitments.

In the table above, use the 9% column for 30 years.

Divide to find the thousands.

Round *down* for safety.

Write the answer.

- b** Find the total.

Allow \$7000 for legal and other fees.

Write the answer.

$$\text{Payments total} = \$7990 \div 4$$

$$= \$1997.50 \text{ per month}$$

$$\begin{aligned} \text{Mortgage payment} &= \$1997.50 - \$350 \\ &= \$1647.50 \text{ per month} \end{aligned}$$

$$\text{Payment for \$1000} = \$8.046 \text{ per month}$$

$$\begin{aligned} \text{Affordable loan} &= \$1647.50 \div \$8.046 \times \$1000 \\ &\approx \$200\,000 \end{aligned}$$

They can afford a loan of \$200 000.

$$\begin{aligned} \text{Loan plus deposit} &= \$200\,000 + \$72\,000 \\ &= \$272\,000 \end{aligned}$$

$$\begin{aligned} \text{Affordable purchase} &= \$272\,000 - \$7000 \\ &= \$265\,000 \end{aligned}$$

They can buy a house or unit for up to \$265 000.

Exercise 12.3 Simple and compound interest

- ① Calculate the monthly repayments on the following loans at flat-rate interest.

a \$5000 at 16% for 4 years	b \$3500 at 19% for 3 years
c \$2500 at 18% for 5 years	d \$7500 at 14% for 2 years
e \$6200 at 17.5% for 4 years	f \$3978 at 13.25% for 5 years
- 2 The following investments are calculated using flat-rate interest. Work out the interest and the total amount for each investment.

a \$3000 at 6% for 4 years	b \$8600 at 8% for 6 years
c \$7000 at 5.5% for 5 years	d \$1200 at 6.7% for 3 years
e \$2460 at 9% for 4 years	f \$3600 at 7.8% for 2 years
- ③ The following investments are worked out using compound interest. Work out the final amount and the total interest paid in each case.
 - a \$4000 at 7.5% over 3 years with yearly rests
 - b \$5000 at 9% over 2 years with 6-monthly rests
 - c \$2400 at 6% over 6 months with monthly rests
 - d \$1900 at 7% over 3 years with quarterly rests
 - e \$8000 at 5.5% over 5 years with 6-monthly rests
- 4 Work out the monthly payments for the following mortgages.

a \$70 000 at 8% for 15 years	b \$90 000 at 10% for 20 years
c \$80 000 at 11% for 25 years	d \$120 000 at 8% for 30 years
e \$200 000 at 12% for 10 years	f \$350 000 at 6% for 25 years
- 5 Work out the biggest loan that can be afforded at the given rates over the specified periods on the following incomes and commitments.
 - a \$1700 a month, no commitments, at 8% over 20 years
 - b \$2400 a month, no commitments, at 7% over 15 years
 - c \$4000 a month with \$700 a month committed, at 11% over 25 years
 - d \$5700 a month with \$200 a month committed, at 10% over 30 years
 - e \$4800 a month, no commitments, at 9% over 25 years.
- 6 Find the final amount and compound interest for each of the following investments.
 - a \$3000 at 12% compound interest for 3 years with annual rests
 - b \$5000 at 16% compound interest for 3 years with annual rests
 - c \$2500 at 15% compound interest for 2 years with annual rests
 - d \$1800 at 12% compound interest for 18 months with half-yearly rests
 - e \$6000 at 17% compound interest for 15 months with quarterly rests

Understanding

Extra questions

Exercise 12.3

MAT09NAEQ00042

Worked solutions

Exercise 12.3

MAT09NAWS00042

Fluency

Worked solutions

Exercise 12.3

MAT09NAWS00042

See Example 18

Problem solving

See Example 15

- 7 Anton wants to buy a surf ski costing \$1200. He can save \$80 a week until he has enough, or buy it on instalment terms of \$240 a month for 6 months.
- How long would it take him to save up for the surf ski?
 - How much would he pay on terms?
 - How much interest would he pay on terms?
 - What is the interest rate?



See Example 15

- 8 A furniture company offers a sofa at \$1599 cash, or on terms of 20% deposit and \$20 a week for 2 years.
- How much is the deposit?
 - How much is paid altogether on terms?
 - How much interest is paid on terms?
 - What is the interest rate?

See Example 16

- 9 Carmen bought a used car for \$7000 and is paying it off at 16% flat-rate interest over 3 years. What is the monthly payment?

See Example 16

- 10 James paid a deposit of \$1500 on a new home entertainment system priced at \$7600. He agrees to pay it off at 15% simple interest over the next 2 years.
- How much is he borrowing?
 - What is his monthly repayment?

Worked solutions

Exercise 12.3

MAT09NAWS00042

- 11 A customer puts a waterbed worth \$1800 on lay-by (minimum deposit 10%, payment within 3 months) and pays the minimum deposit.
- What is the minimum deposit?
 - How much must then be paid each week?
- 12 Jon puts an iPod priced at \$85 on lay-by (minimum deposit \$10, payment within 3 months).
- What is the minimum weekly payment?
 - When will the iPod be paid off if Jon makes a weekly payment of \$10?
- 13 Carolyn puts a rolltop desk on lay-by (minimum deposit 10%, payment within 3 months). The desk is priced at \$790 and she pays the minimum deposit.
- What is the minimum deposit?
 - How much must be paid each week?
- 14 David uses his credit card to buy a pair of jeans for \$75.90. He pays this as soon as he receives the account. What does he actually pay for the jeans if he has an interest-free period of 14 days and pays 20c transaction fees?
- 15 Samantha uses a credit card to pay for a sprinkler system worth \$965, but doesn't pay the account for 2 months. What does she pay altogether? She has no interest-free period, pays 20c transaction fees and 2% interest per month.
- 16 Andrew and Juanita fly to Sydney for a weekend, and pay for the tickets with a credit card. The tickets cost \$408.60. They have an interest-free period of 14 days, pay 20c transaction fees and have a monthly interest rate of 2.1%. Work out how much the tickets cost them if they pay the account:
- after 1 month
 - after 2 months
 - after 5 months.
- 17 Julie has \$270 in her credit-union account and her Starcard is linked to the account. If she uses the card to buy some groceries for \$15.70 and a T-shirt for \$12, how much will she have left in her account?

- 18** Renato has \$425 in his bank account. He deposits another \$140 at an ATM and uses his Mastercharge card to pay the plumber for some work on his house. The plumber charges \$98. How much does Renato have left in his account?
- 19** Robyn has \$150 deducted from her pay each fortnight and this is paid directly into her credit-union account on the Thursday she is paid. She is trying to save up for a holiday in Fiji. Her Ezeecard is linked to her account. At the beginning of January she has \$270 in her account. She uses the card in the middle of February to buy a new swimsuit and beach coat from separate stores for a total of \$98. How much does she have in her account at the end of February? (Use this year's calendar.)

- 20** Heidi and Franz want a new washing machine. They can't afford to pay cash, but could pay a deposit of up to \$200. The Lighthouse Appliance Store offers them a no-deposit deal of \$60 a month for 2 years, and they get to keep the \$200. Discount Washstop offers them the same machine for \$150 deposit and \$70 a month for 15 months. They see the same machine at another shop for \$875 cash.
- a** How much would they pay if they bought from Lighthouse?
 - b** How much would they pay by buying from Discount Washstop?
 - c** How much interest would be paid to each shop?
 - d** Which of the three deals appears to be the best?

- 21** A digital camera is sold for \$480 cash. It is also available on lay-by (minimum deposit 10%, payment by 3 months), or on no-deposit terms of \$50 a week for 3 months.
- a** What is the minimum deposit on lay-by?
 - b** What weekly payment would be required on lay-by?
 - c** How much would be paid on terms?
 - d** How much interest would be paid on terms?



- 22** Lyle puts a new fishing rod and reel on lay-by for \$220 (minimum deposit 10%, payment within 3 months). Unfortunately, he makes only 3 payments after paying the deposit. How much money does he lose?
- 23** Kamilah has a Bankcard and a Mastercharge card. The Bankcard is a credit card and the Mastercharge card is a debit card. She buys a canoe for \$840 using her Bankcard, and 2 months later she uses her Mastercharge to transfer the money from her bank account to pay the Bankcard account. The Bankcard has no interest-free period and charges 20c transaction fees and 2% interest per month. How much does the canoe actually cost her?
- 24** If a couple have a combined income of \$3500 a month, and the interest rate is 10%, work out the biggest loan they can afford over a period of:
- a** 10 years **b** 15 years **c** 20 years **d** 25 years.
- 25** Work out what happens to the amounts that the couple in question 23 can afford if interest rates go down to 8%.

Reasoning

See Example 19

See Example 22

- 26 a** For a loan of \$80 000, work out the total amount repaid at 10% over a period of:
- i** 10 years **ii** 15 years **iii** 20 years **iv** 25 years.
 - b** Comment on your findings.

See Example 21

Chapter 12 summary

Quiz

Money and business

MAT09NAQZ00012

- A **percentage** is a number written as a fraction with a denominator of 100, but the /100 is replaced by the % sign.
- A fraction or decimal can be changed to a percentage by multiplying by 100%.
- To find a percentage of an amount, the percentage is changed to a fraction or decimal and multiplied by the amount.
- Calculation of a **percentage increase** or **percentage decrease** is based on the original amount.

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original amount}} \times 100\%$$

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{original amount}} \times 100\%$$

- The **cost price (buying price)** of an item is the price paid by the person selling the item.
- The **selling price** is the price it is then sold for.
- A **discount** is an amount taken off a normal price.

$$\text{Discount} = \text{marked price} - \text{price paid}$$

- Shops add a **mark-up** to the cost price to get the **marked price** of goods they sell.

$$\text{Marked price} = \text{cost price} + \text{mark-up}$$

- A **profit** is made when goods are sold for *more* than their original cost.

$$\text{Profit} = \text{selling price} - \text{cost price}$$

- A **loss** is made when goods are sold for *less* than their original cost.

$$\text{Loss} = \text{cost price} - \text{selling price}$$

- A **commission** is money paid for a service carried out, particularly when the amount paid is calculated as a percentage. A **retainer** is paid in addition to a commission.
- When discount, profit, loss, mark-up or commission is calculated as a percentage, it is usually considered to be 'out of' the original price.

$$\text{Percentage amount} = \frac{\text{amount}}{\text{original price}} \times 100\%$$

- The Goods and Services Tax (GST) is currently set at 10% in Australia. It is collected by the seller or service provider and paid to the government.
- **Interest** is the charge on a loan. It is normally worked out as a percentage, called the **interest rate**. The amount lent or borrowed is called the **principal**.
- The amount that has to be paid back each week or month for a loan is called the **instalment** or **repayment**.
- **Simple interest** is calculated from the entire loan for the whole time it is borrowed. It is also called **flat-rate interest**.
- **Compound interest** is calculated at regular intervals called **rests** and added to the loan, so that the principal increases at each rest.
- A **credit card** is used to *borrow money* and in most cases you will have to pay interest. Some credit cards have an **interest-free period**.
- A **debit card** is used to take money immediately from your account (**linked to your account**), so you *must have the money* in your account.

- Credit and debit cards used in **EFTPOS** (Electronic Funds Transfer at Point of Sale) machines are issued by banks, credit unions, building societies and other organisations. You are usually charged a **transaction fee** every time your card is used.
- **Reducing interest** is used for very large loans like **mortgages** for houses. A mortgage loan is **secured** against the property. Calculation of interest is done each time a payment is made.
- A **ready reckoner** can be used to calculate mortgage payments.
Banks usually say that you can afford about a quarter of your after-tax income for mortgage and other loan repayments.

Chapter 12 review

Understanding

See Example 1

- 1 Express the following as percentages.

a 11 out of 14

b $4\frac{2}{5}$

Worksheet

- 2 Change 230% to a decimal and a mixed number.

- 3 Calculate 42.5% of \$320.

- 4 A boat yard bought a boat for \$2500 and sold it at a loss of 10%. How much did the boat yard lose?

Money and financial
mathematics review

MAT09NAWK00072

See Example 8

Fluency

See Example 2

- 5 Change $4\frac{3}{4}\%$ to a fraction.

- 6 Calculate $11\frac{2}{3}\%$ of 960 kg.

See Example 12

- 7 Janeen works on a commission of 20% and a retainer of \$120 a week. One week she makes sales of \$3200. How much does she earn?

See Example 16

- 8 A loan of \$5400 is taken at 20% flat-rate interest over 3 years. What are the monthly repayments?

- 9 An investment of \$1200 is made at 7% flat-rate interest for 3 years. What are the interest and final amount?

- 10 How much will \$6000 amount to after 2 years at 8% compound interest with quarterly rests?

- 11 What is the overall percentage change when a decrease of 60% is followed by an increase of 60%?

See Example 14

- 12 Janeen (from question 7) was paid \$780 in one week. What was the value of her sales?

- 13 Use the table on page 439 to find the monthly cost of a mortgage for \$85 000 at 11% over 20 years.

Problem solving

See Example 3

- 14 Andrew got $\frac{25}{34}$ for Geography and $\frac{15}{22}$ for History. In which subject did he perform better?

- 15 A fridge bought for \$890 was sold for \$1240. What was the percentage mark-up?

- 16 After making a loss of 16%, a shop got only \$210 each for some MP3 players. How much did the shop pay for them?

- 17 A shop works on a mark-up of 30%. It buys filing cabinets for \$120. At a sale it offers a discount of 20%.

a What is the marked price of the filing cabinets?

b What is the discount price?

c What is the actual percentage profit?

- 18 Alanna buys a PVR on lay-by for \$855. She pays a 20% deposit and makes weekly payments over the next 3 months.

a How much deposit does she pay?

b What is the weekly payment?

- 19** John's credit card has an interest rate of 2.3% a month and charges 30 cents per transaction. If he bought a pair of joggers for \$80 and a tennis racquet for \$145 with his credit card, how much would he pay after 2 months?
- 20** Carol made a total profit of \$740 one week from her flower business. The following week, her sales went down by 35%, but the week after they went up by 40%. What was the overall percentage change from the first week to the third week?
- 21** The owner of a clothes shop finds that discounts increase her sales. She has worked out that every 5% discount she gives increases the amount she sells by 30%. If she gives a 10% discount, her sales go up by 60%; and if she gives a 25% discount, her sales go up by 150%.
- a** She presently has some very profitable denim jackets priced at \$180. She makes \$84 for every jacket sold. Work out what discount she should put on the jackets to make the greatest profit.
 - b** Work out the discount she should offer on \$150 jeans if she makes \$20 per sale.
- 22** A couple have a combined income of \$4245 per month and have commitments of \$300 per month. What is the largest loan they can afford at 9% over 25 years?

Reasoning

See Example 22

Chapter 1

Exercise 1.1

- 1 a Population b Sample c Sample
d Population e Sample
- 2 a Nominal b Continuous c Ordinal
d Discrete (only 500 g, 600 g, 700 g, 800 g and 860 g)
e Retail prices are discrete but wholesale prices are continuous
- 3 a Numerical b Categorical
c Categorical (they are not really measurements)
d Numeric e Categorical
- 4 a 27, 35, 43, 51, 58, 66, 74
b 41, 2, 12, 21, 31
c 50, 62, 74, 86, 98, 111, 2, 14, 26, 38
d 3, 20, 37, 55, 72
e 38, 56, 74, 92, 110, 129, 147, 2, 20
- 5 a Use a sample such as: every 10th customer on a couple of days each week, making sure you spread them across the days of the week.
b Ask students to complete a form instead of using nurses, and use a smaller sample such as a selection of students from a selection of schools.
c Use a smaller sample and phone instead of using the electoral rolls.
d Use a sample such as some registered owners from major centres such as Brisbane, Gold Coast, Sunshine Coast, Townsville, etc.
e Use a smaller sample from selected high schools. The descriptions could also be improved to be less emotive.
- 6 a Ask a wider range of people: the group chosen is biased towards a Yes answer.
b Use a more typical sample by asking a smaller number of students from a variety of schools.
c SMS gives a bias towards younger people, and there is a bias towards the group that watch the program. Choose a more typical sample.
d Choose a more typical sample and avoid the emotive term 'deserving' sports. For example, give a list of sports that could be given (more) money.

7

Height (cm)	Gender	
	F	M
141–150	2	0
151–160	13	3
161–170	5	5
171–180	2	7
181–190	0	3

There were fewer boys than girls in the group, but they were generally taller.

- 8 a Fair, because the population is used.
b Biased; the 14-year-olds in Perth might not be typical of *all* Australians. For example, there is a greater proportion of people of English origin in Perth than in many other parts of Australia.
c Fair; the group is representative of the voters in the state.
d Fair, because the population is used.
e Biased; the phrasing is likely to lead students to describe themselves as healthy.
- 9 **For:** it is difficult to include all kinds in such a small sample. **Against:** It depends on how the sample is chosen and on the size of the population. For a population of 20 it would be good.
- 10 **For:** any sample could miss out unknown characteristics that could affect the result.
Against: A well-chosen sample should cover all or nearly all cases.
- 11 Not necessarily, as the sample could unknowingly miss some important cases. It just means that the survey has a good chance of being within 3%. In any case, comparison of different survey results shows that it cannot be the case.
- 12 a Get 5 random numbers from the calculator, multiply each by 101, discard the decimal part and add 200.
b Get 10 random numbers from the calculator, multiply each by 101, discard the decimal part and add 100.
c Count the number of pages in the White Pages by checking the page numbers and count the (maximum) number of names on a page. Get 2 random numbers from your calculator. Multiply the first by the number of pages and the second by the number of names. Discard the decimal part and add 1 to each. Choose the page and the name on the page given by this process (as long as there are enough names on the page). Repeat until you get 4 different names and their phone numbers.

- d Number all the students in the school from 1 to the number of students in the school. Get 10 random numbers from your calculator, multiply each by the number of students, discard the decimal part and add 1 to find which students to choose. Choose these students.

Exercise 1.2

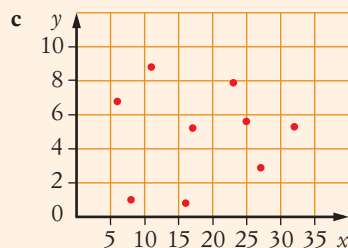
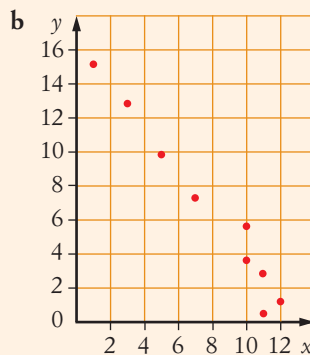
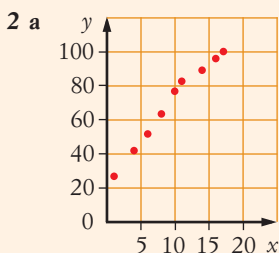
- 1 a Mean = 10.45, median = 10, mode = 8, range = 9
 b Mean $\approx$ 5.61, median = 7, mode = 8, range = 9
 c Mean = 22.28, median = 22, mode = 26, range = 20
 d Mean $\approx$ 50.14, median = 50.5, mode = 54, range = 21
 e Mean = 12.125, median = 12.5, mode = 16, range = 14
- 2 a median, mode b mode
 c mean, median, mode d median, mode
 e mean, median, mode (although some mathematicians say this is ordinal data)
- 3 a $Q1 = 10$, $Q3 = 16$, IQR = 6
 b $Q1 = 5$, $Q3 = 8$, IQR = 3
 c $Q1 = 3$, $Q3 = 7$, IQR = 4
 d $Q1 = 3$, $Q3 = 7$, IQR = 4
 e $Q1 = 7$, $Q3 = 9$, IQR = 2
- 4 a Mean $\approx$ 4.89, median = 5, mode = 6
 b Mean $\approx$ 25.55, median = 25.5, mode = 25
 c Mean = 9.5, median = 10, mode = 10
 d Mean $\approx$ 12.43, median = 14, mode = 13
 e Mean $\approx$ 32.44, median = 33, mode = 35
- 5 a OK b ★★★★★ c B
 d N e above average
- 6 a 1 : 3.2 b 1 : 3.5 c 1 : 2.5
 d 1 : 1.67 e 1 : 4.11
- 7 a Median = 69, IQR = 29, outliers 7 and 146
 b Median = 11.5, IQR = 6, no outliers
 c Median = 30, IQR = 10, outlier 2
- 8 240 altogether and the typical number is 4 per box. A few boxes had many more.
- 9 The mean would have been about $35\frac{1}{2}$, and the median would be the same (32 minutes). It would probably not be a good idea to keep in the whole class, as half would then be sitting idle at least 13 minutes after they finished.
- 10 The increase was probably done on the mean, so the previous mean would have been about \$3.90/kg, as

a 250% increase would mean the new price was about 3.5 times the old price. According to the information, the previous median would also have been about \$3.90 and the spread of prices was very small.

- 11 Safety ratings are nominal data, so using the mean is poor practice. More importantly, only about 16 vehicles altogether were tested each year, so the number of small and large cars tested would each be quite small. However, it is obvious that the median for the small cars must also have increased, and this is significant.
- 12 Yes, if the distribution had most results at the extremes, such as 3, 3, 3, 3, 4, 5, 6, 7, 8, 8, 8, 8. This has both a range and interquartile range of 5.

Exercise 1.3

- 1 a Symmetrical b Skewed right
 c Skewed right d Skewed left
 e Bimodal f Bimodal
 g Symmetrical h Skewed left



Answers

3 Girls: Mean ≈ 6.7 h, median = 4 h, mode = 2 h, IQR = 8 h, range = 29 h; **Boys:** Mean = 7.5 h, median = 5 h, mode = 5 h, IQR = 6 h, range = 26 h; the boys generally spend more time on sport, and the girls vary more on the time they spend on sport.

4 North: Mean ≈ 19.2 , median = 20, mode = 21, IQR = 4, range = 12; **South:** Mean ≈ 17.0 , median = 15, mode = 14, IQR = 6, range = 14; Vehicles heading south typically encountered about 5 fewer red lights, although the number they encountered was more spread out. Perhaps the lights were synchronised for southbound travel.

5 a Marks in two subjects: $3|4 = 43 = 4|3$

Maths		Science
6 5	1	2 2 8 8
7 3	2	0 8 9
5 1 0	3	2 2 3 8
7 4 4 1 0	4	0 2 3 5 5 5 7
8 7 6 6 4	5	0 3 6 7 7 7 9 9
9 9 9 8 7 7 5 1 1 1	6	0 3 6 6 6
7 7 4 4 3 3 2 0	7	1 2 3 4 8
8 2 1 0	8	3 4
	9	0 0

b Students generally scored higher in Maths than Science.

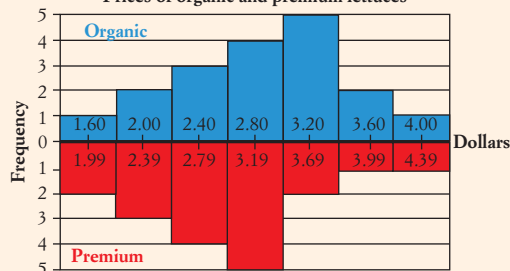
6 a Potatoes supplied $4|25 = 25.4\text{kg} = 25|4$

Potato wizards		Spud kings
2	22	
7 6 4 3 2 2	23	3 8 9 9
9 6 6	24	0 1 2 3 4 6 7 8 9
9 8 7 5 3 2 2 1 0	25	0 0 1 1 1 2 3 6 7 9
8 5 4	26	1
3 2	27	
0	28	

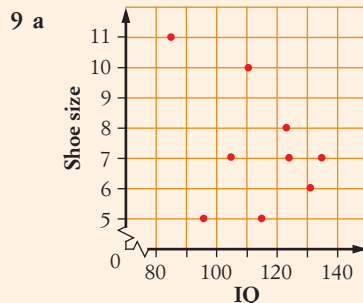
b Both suppliers have a significant number of underweight bags, but the Spud kings have more consistent weights.

7 The fertilised plants have a much higher yield and are slightly more consistent.

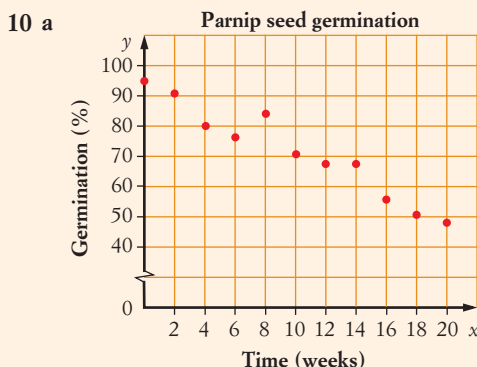
8 a Prices of organic and premium lettuces



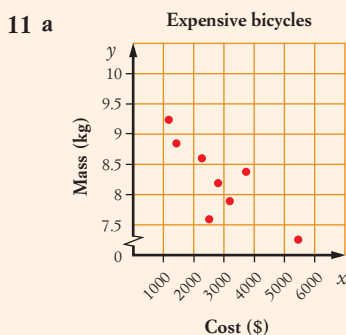
b The organic lettuces have a higher price than the premium lettuces.



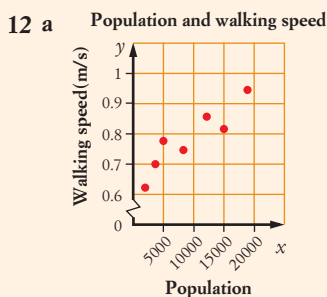
b There does not seem to be any pattern.



b The germination rate decreases with age.



b The higher priced bicycles are generally lighter.



b People seem to walk faster in larger towns.

13 For the data set 2, 2, 9, 9, 10, 10 that is skewed to the left, the mean is 8.4 and the median is 9. For the data set 2, 2, 4, 4, 10, 10 that is skewed to the right, the mean is 6.4 and the median is 4. When the data is skewed to the left, the mean is to the left of (lower than) the median. When the data is skewed to the right, the mean is to the right of (higher than) the median.

- 14 a** The mean and median will be about the same because the data is symmetrical.
b The mean will be higher than the median because the data is skewed right.
c The mean will be higher than the median because the data is skewed right.
d The mean will be lower than the median because the data is skewed left.
e The mean and median will be about the same because the data is symmetrical, although bimodal.
f The mean and median will be about the same because the data is symmetrical, although bimodal.
g The mean and median will be about the same because the data is symmetrical.
h The mean will be lower than the median because the data is skewed left.

Chapter review

- 1 a** Population **b** Sample
c Sample, because it affects other drivers as well.
- 2 a** Continuous **b** Ordinal
c Discrete (only to cents)
- 3 a** Mean = 11.24, median = 11, mode = 10, range = 12
b Mean $\approx$ 95.2, median = 95, mode = 111, range = 44
c Mean $\approx$ 16.9, median = 18, mode = 19, range = 12
- 4 a** Mean, median, mode **b** Median, mode
c Mean, median, mode **d** Mean, median, mode
- 5 a** Skewed right **b** Symmetrical
c bimodal
- 6 a** $Q_1 = 26$, $Q_3 = 37.5$, IQR = 11.5
b $Q_1 = 14$, $Q_3 = 20$, IQR = 6
c $Q_1 = 15$, $Q_3 = 21$, IQR = 6
- 7 a** Numerical **b** Categorical **c** Numerical
- 8** 127, 152, 177, 202, 228, 3, 28, 53, 78, 103

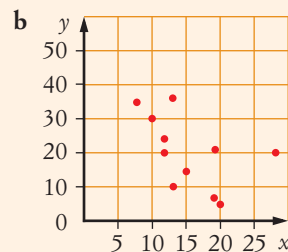
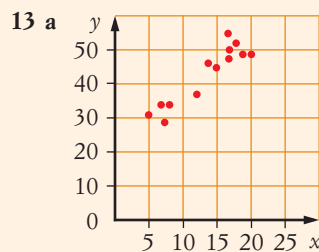
- 9 a** Mean $\approx$ 5.1, median = 5, mode = 5
b Mean $\approx$ 13.9, median = 14, mode = 14
c Mean $\approx$ 24.8, median = 25, mode = 27

- 10 a** ★★ ★ **b** C
c N (although there are only 6/26 D or SD)

		Age		
		14	15	16
Siblings	0	0	4	2
	1	1	5	4
	2	4	4	4
	3	5	4	3
	4	0	2	1
	5	0	1	1

The number of siblings does not seem related to age.

- 12 a** 27 is an outlier **b** No outliers
c 20 is an outlier



- 14 a** This is biased towards those committed to sport, so ask a more representative group.
b The question is phrased in favour of public transport, so make it more open.
- 15** The new median cannot be found.
 Total = $23 \times 172 = 3956$, New total = 4460, so the new mean $\approx$ 171.5 cm. Since one more is added to the frequency of the mode, it must stay at 169 cm.
- 16** There is no apparent difference between the men's and women's scores.
- 17** For the Grandmasters, mean = 4, median = 4, range = $4\frac{1}{2}$ and IQR = $1\frac{1}{2}$, with the winner's score being $\frac{6}{8}$. For the amateurs, mean = 4,

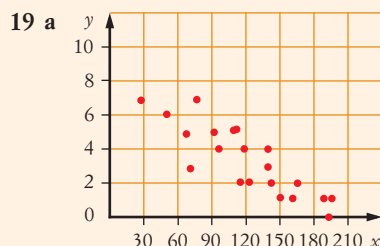
Answers

median = $4\frac{1}{2}$, range = $6\frac{1}{2}$ and IQR = 7, with the winner's score being $7\frac{1}{2}/8$. The Grandmasters are more evenly matched and must have more draws than the amateurs.

- 18 a Cooking class results. Key $4|3 = 34 = 3|4$

Girls		Boys
4 0	2	0
1 0	3	8
8	4	
2 2 1	5	
9 6 0 0	6	5
	7	0 1 2 2 5
7 4 1	8	5

- b Apart from a couple of boys who performed quite badly, the boys in the class did better than the girls, even though there were less of them.



- b Generally, those who took more time made less mistakes.
- 20 a The sample is fair but the question is biased towards 'Yes'. The last part of the question should be omitted.
- b It is a fair question, but the sample is biased because people from only one street are asked.
- c The sample is fair for the suburb, but not for drivers who might go into the suburb and the question is biased because it asks people if they have 'strong objections'.
- 21 Large samples are more likely to be typical of the population than small samples, so they are likely to be fairer than small samples. If the samples are chosen in the same way, you can be surer about the results.
- 22 You would need to know how many parents actually responded. If the scale was only 1, 2, 3, 4 or 5, then the change is unlikely to be significant because this is a nominal scale and the median should be used. This is likely to be 3 for both anyway.
- 23 a Get a random number, multiply by 833, add 8 and ignore the decimal part. Repeat this 6 more times ignoring duplicates.
- b If there are n patients in the hospital, number them from 1 to n . Then get a random number, multiply by $n + 1$, add 1 and ignore the decimal part. Repeat this 5 more times ignoring duplicates.
- 24 Yes: the first could be 6, 10, 13, 24, 24, 25, 29, 30 and the second 10, 11, 21, 28, 28, 29, 30, 30.
- 25 a The median is below the mean.
- b The median and the mean are probably very similar.
- c The median and the mean are very similar (actually 54 and 53.9).

Chapter 2

Exercise 2.1

- 1 a 13 b 25 c 11 d 3
e 6 f 22 g 38 h 113
i 58 j 8 k 99 l 10
- 2 a 612 b -257 c -132 d -57
e -405 f 57 g -76 h -105
i 86 j -51 k -156 l -98
m 52 528 n 44 o 4 p -240
- 3 a 46.2 b 0.405 c 0.0868
d 3.32 e 0.0398 f 23.7
g 1.372 h 12.57 i 0.004 92
j 121.4 k 1.307 l 58.7
- 4 a 2980.9338 b 659.0529
c 193 207.3149 d 1207.0251
e 2855.136 766 f 516.147 76
- 5 a 1.52 b 3.087 c 0.1296
d 125.4 e 4.187 f 0.232
g 0.2475 h 0.002415
- 6 a -366.49 b -2.9375
c $-0.028\ 62$ d $0.04\overline{7}$
e $-60.83\overline{07692}$ f 0.000 733 6
g -54 h -0.02448
i 19.912 j $-0.123\ 24$
k 1360 l $-0.00320\overline{5}$
- 7 a 0.269 437 2 b -4.6763
c $-0.034\ 52$ d $-0.001\ 256\ 877$
e 30.3995 f $-3766.780\ 16$
g -0.3270 h 20.061 195

- 8 a $5\frac{1}{3}$ b $2\frac{2}{3}$ c $7\frac{1}{3}$ d $8\frac{1}{2}$
 e 2 f $8\frac{2}{3}$ g 4 h $3\frac{1}{2}$
 i $\frac{4}{5}$ j $7\frac{1}{3}$ k $8\frac{1}{3}$ l $\frac{1}{10}$
- 9 a $10\frac{7}{12}$ b $\frac{13}{20}$ c $1\frac{7}{10}$ d $1\frac{1}{6}$
 e $\frac{11}{24}$ f $\frac{2}{7}$ g 2 h 2
 i $5\frac{1}{6}$ j $15\frac{1}{2}$ k $7\frac{29}{36}$ l $5\frac{5}{8}$
- 10 a $\frac{-4}{21}$ b $4\frac{8}{15}$ c $-1\frac{11}{27}$ d $\frac{3}{16}$
 e $1\frac{17}{28}$ f $\frac{26}{55}$ g $20\frac{5}{6}$ h $14\frac{11}{15}$
 i $\frac{-15}{76}$ j $2\frac{4}{7}$ k $-20\frac{5}{6}$ l $\frac{-39}{64}$
- 11 a 24 b 41 c 36 d 810
 e 200 f 57 g 30 h 49.5
 i 16.5 j 25
- 12 a 1 b 3 c -23 d 11
 e 240 f 10 g -12 h -11
 i 17 j 10 k 77 l 22
 m 36 n 21 o 50 p -66
- 13 a 12 382 m b 15 742 m c 10 098 m
- 14 1754 mangoes 15 23 642 L
- 16 a 9285 people b 2385 people c 745 people
 d 6900 people e 10 030 people
- 17 11 278 kg 18 11 km/h 19 285 mm
- 20 227 tiles (one cut) 21 \$32 294
- 22 146 loads (one not full) 23 9000 matches
- 24 10 h 28 min 25 34°C 26 \$3528
- 27 About 11 months
- 28 a Rianna b Stephen
- 29 a Shipwreck b Sand dunes
 c Shipwreck, rock, reef, coral island, sand dunes
- 30 a 38°C b -10°C
- 31 a 59, 119, 179, ... b 60, 120, 180, ...
- 32 35 plants 33 1200 hard drives
- 34 290 km, 72.5 km/h
- 35 a \$1800 b \$3300 c \$100
 d \$4600 e \$6240 f \$40
- 36 a -1 m b -41 m c 9 m
 d 12 m e 4 m f -46 m

- 37 890 years 38 53°C 39 12 floors
- 40 \$24 overdrawn (\$24)
- 41 a $-71\text{ }\mu\text{C}$ b $-12\text{ }\mu\text{C}$ c $-15\text{ }\mu\text{C}$
- 42 a 33 km/h b 17 km/h
- 43 $1400 \times \$900 - 50 \times \$1200 = \$1\,200\,000$
- 44 18 sparks 45 a $\frac{11}{12}$ b $\frac{1}{12}$
- 46 a Mary $\frac{2}{5}$, Peter $\frac{5}{8}$, Theresa $\frac{7}{10}$
 b \$3.06 c About \$2.65
- 47 $\frac{5}{12}$ 48 $\frac{43}{60}$ 49 $\frac{89}{180}$ 50 $\frac{4}{15}$
- 51 a $\frac{49}{60}$ b $\frac{11}{60}$
- 52 60 doses 53 \$187.92
- 54 a 870 m b 28 min 44 s
- 55 a About $\frac{5}{24}$ b About $\frac{25}{219}$
 c About 12 000 hours d About 93 120 hours
- 56 a $\frac{1}{12}$ b $\frac{1}{4}$
- 57 a \$42.05 b 2.506 kg
- 58 151 blinds 59 9 pages
- 60 100 bricks 61 3 pears and 2 apples
- 62 a 76 m b 3 joins, 85 m
- 63 24, 12, 8, 6, 4 or 3 (not 2, because that would be only one friend)
- 64 7 tables and 3 stools, or 4 tables and 7 stools, or 1 table and 11 stools
- 65 2 km/h towards the spaceship 66 16 600 m
- 67 a $\frac{1}{28}$ per hour b $\frac{1}{8}$ per hour
 c 9:26 p.m. d 9:07 a.m.

Exercise 2.2

- 1 a 125 b 16 807 c 243
 d 64 e 4096 f 1024
 g 1024 h 169 i 625
- 2 a 50 625 b 2197 c 676
 d 6859 e 331 776 f 1 889 568
 g 1156 h 79 507 i 65 536
- 3 a 8 b 5 c 10
 d 20 e 12 f 9
 g 15 h 100 i 40

Answers

4 a 7.071 b 15.811 c 12.166
d 9.220 e 2.646 f 4.899
g 6.481 h 31.623 i 648.074

5 a 729 b $\frac{1}{243}$ c 1 d $\frac{1}{81}$ e $\frac{1}{11}$
f $\frac{1}{216}$ g $\frac{1}{196}$ h 1 i $\frac{1}{625}$

6 a 4^{10} b 7^7 c 12^6 d 6^9 e 10^{16}
f 9^9 g 14^{12} h 3^{12} i 25^{10}

7 a 8^2 b 3^4 c 7^4 d 11 e 6^6
f 15^5 g 14^6 h 17^{10} i 12

8 a $7^4 \times 5^4$ b $6^6 \times 3^6$ c $9^4 \times 11^4$
d $6^7 \times 8^7$ e $12^8 \times 13^8$ f $2^{12} \times 7^{12}$

9 a 4^{15} b 8^{10} c 5^{28}
d 12^8 e 9^{24} f 3^9

10 a 5 b 13 c 2
d 4 e 2 f 3
g 10 h 15 i 20

11 a 0.008 b 6.928 c 2.632
d 61.4656 e 5.450 f 6.281
g 0.159 h 0.002 07 i 1.118

12 a 12.126 b 7.225 c 4
d 13.133 e 2.398 f 225.698

13 a 15^{10} b 3^{11} c 19^{10} d 17^{11} e 11^8
f 3^{14} g 7^{10} h 14^{17} i 4^{14}

14 a 15^6 b 6^4 c 19^2
d 4^4 e 7^{-6} f 13^4
g 11^{-4} h 6^{11} i 11^{-7}

15 a $4^6 \times 13^6$ b $14^9 \times 19^9$ c $7^9 \times 3^9$
d $11^2 \times 19^2$ e $13^2 \times 7^2$ f $17^6 \times 14^6$

16 a 4^{15} b 8^{10} c 5^{28}
d 12^8 e 9^{24} f 3^9

17 a 8^{35} b $\frac{1}{6^4}$ c 1 d $\frac{1}{4^{15}}$ e 7^6
f $\frac{4^{18}}{5^9}$ g $\frac{6^5}{8^{20}}$ h $\frac{11^6}{7^3}$ i 8^5

18 a $\sqrt{36} = 6$, $\sqrt{9} = 3$, $\sqrt{4} = 2$, $\sqrt{36} = \sqrt{9} \times \sqrt{4}$
b $\sqrt{100} = 10$, $\sqrt{25} = 5$, $\sqrt{4} = 2$, $\sqrt{100} = \sqrt{25} \times \sqrt{4}$
c $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

19 a 4, 2.520, 2, 1.741, 1.587
b 0.25, 0.397, 0.5, 0.574, 0.630
c They get closer to 1.

Exercise 2.3

1 a 6.3×10^3 b 1.42×10^8 c 7.5×10^{-3}
d 9.38×10^{-7} e 5×10^7 f 6×10^{-4}
g 2.1×10^2 h 7.9×10^{-1} i 3.109×10^7
j 4.062×10^{-2} k 9.623×10^6 l 2×10^{-9}

2 a 430 000 b 0.0021 c 9 840 000
d 0.319 e 0.000 000 006 f 20 000
g 42 h 0.058 i 1.62
j 0.000 095 k 464 l 2100

3 a 1.472×10^2 b 7.314×10^{10}
c 2.200×10^{19} d 2.494×10^{-14}
e 2.244×10^{13} f 2.633×10^{-18}

4 a 3 b 2 c 1 d 1 e 1
f 3 g 4 h 5 i 2

5 a 475 b 103 c 5.047
d 1 057 000 e 247.6 f 0.000 000 644
g 100.5 h 0.06047

6 a 2.716×10^{17} b 2.976×10^{-6}
c 6.541×10^8 d 1.969×10^{21}
e 1.987×10^{-8} f 3.111×10^{-3}

Exercise 2.4

1 a $\frac{4^6}{5^6}$ b $\frac{2^8}{3^4}$ c $\frac{3^{18}}{5^6}$
d $\frac{1}{3^{10}}$ e $\frac{5^{11}}{2^{15}}$ f $\frac{3^4}{7^{14}}$
g $\frac{1}{3 \times 5^4}$ h $7^8 \times 2^5$ i 1

2 a $\frac{2^5}{3^2}$ b $\frac{3^3}{2^2 \times 7^2}$ c $\frac{3^4}{2^2}$
d $\frac{7^{11}}{2^4}$ e $\frac{5^{15}}{3^7}$ f $2^{10} \times 11^{12}$

3 a $x^6 y^9$ b $\frac{x^4}{y^8}$ c $x^6 y^3$
d b e 1 f $\frac{1}{c^2}$
g $\frac{n^3}{2m}$ h $\frac{q^4}{5^2}$ i $\frac{6rt^2}{s^2}$

4 a 1.961×10^{-27} kg b 3.836×10^{-27} kg
c 1.246×10^{-23} kg

5 1.9817×10^{20} N

6 a 1.0868×10^{21} m³ b 1.4123×10^{27} m³
c 5495 kg/m³ d 1408 kg/m³
e The Sun is about 3.33×10^5 times heavier than the Earth.

7 It might be possible to replace approximately 25 of the 500 MW/s power stations or 14 of the 900 MW/s power stations

Chapter review

- 1 a 8 b 56 c 7
- 2 a 45.2 b 0.362
c $1\frac{1}{2}$ d $\frac{1}{2}$
- 3 a 649.4558 b 354.773 c 162.755
d $8\frac{5}{12}$ e $3\frac{1}{2}$ f $3\frac{11}{15}$
- 4 a 2945.2307 b $12\frac{3}{8}$
- 5 a 0.1475 b 0.0163 c $\frac{2}{15}$ d 16
- 6 a 265.182 b 28.25 c $\frac{3}{32}$ d $1\frac{5}{9}$
- 7 a $9\frac{3}{5}$ b $\frac{6}{7}$
- 8 a 64 b 81 c 343 d 7
e 11 f 30
- 9 a 3375 b 441 c 1 336 336
d 6.928 e 14.595 f 483.735
- 10 a $4^6 \times 5^{-6}$ b $\frac{2^3 \times 3^4}{5^4}$ c 3^{15}
- 11 a 2×5 b 3
- 12 a 31.8072 b 415.673
c $\frac{1}{10}$ d $-1\frac{7}{15}$
- 13 a -340.67 b 5.009 c $9\frac{7}{40}$
d $5\frac{1}{18}$ e $10\frac{3}{10}$
- 14 a $\frac{-61}{72}$ b $11\frac{13}{36}$ c $-3\frac{41}{45}$
- 15 a 8.5 b 55.38 c 6 d $1\frac{2}{3}$
- 16 a 3.429 24 b -13.2 c $-15\frac{2}{5}$ d $4\frac{4}{9}$
- 17 a $-35.387\ 752$ b -0.1069 c $\frac{25}{252}$
- 18 a 4 b 57 c 50
- 19 a 130 b 336 c -400 d 10
e 22 f 11
- 20 a 2744 b 1 c $\frac{1}{16}$ d 6
e 3 f 500
- 21 a 14^{15} b 20^8 c $3^6 \times 7^6$
d 5^9 e 14^4 f $13^8 \times 11^8$

- 22 a 0.004 44 b 2.080 c 110.592
d 3.736 e 3.353 f 0.004 06
- 23 a 14.697 b 119.428 c 0.003 (0.003 235)
- 24 a 14^{15} b 20^8 c $3^6 \times 7^6$
d 5^9 e 14^4 f $13^8 \times 11^8$
g 5^{18} h 13^{-8} i 9^{15}
- 25 a 6×10^{-5} b 2.37×10^8 c 5.342×10^{-3}
- 26 a 680 000 000 b 0.000 004 35 c 712.5
- 27 a 2.4×10^{14} b 3.484×10^{11} c 8.411×10^{-16}
- 28 a 3 b 2 c 5
- 29 435 200
- 30 a 1.004×10^{29} b 1.619×10^{-13} c 9.033×10^{11}
- 31 a x^3y^6 b $\frac{1}{a^3}$ c $\frac{3}{mn^3}$
- 32 5973 bricks 33 377 kg 34 414 000 km
- 35 40 years 36 12°C
- 37 $-5700 + 1600 = -4100$. The miner is at a depth of 4100 m.
- 38 7 strikes 39 a $1\frac{7}{30}$ b $1.8\bar{6}$ kg
- 40 27.5 kg (27.499 kg)
- 41 a 9 leads b $1\frac{3}{4}$ m
- 42 1.638×10^{-15} J
- 43 a First hose $\frac{1}{20}$, second hose $\frac{1}{12}$ b $7\frac{1}{2}$ hours

Chapter 3

Exercise 3.1

- 1 a 5 m^2 b 600 mm^2 c $25\ 000\text{ cm}^2$
d 1.8 cm^2 e 0.6 m^2 f 0.4 m^2
g 2500 mm^2 h $70\ 000\ 000\text{ cm}^2$ i 0.0032 m^2
- 2 a 9 cm^2 b 15 cm^2 c 6.6 cm^2
d 15.7 cm^2
- 3 a 10 cm^2 b 14 cm^2 c 13 cm^2
- 4 a About 8 cm^2 b About 7 cm^2
c About 8 cm^2

Answers

- 5 a 60 000 m² b 15 000 000 m²
 c 0.28 km² d 1.5 ha
 e 0.5 km² f 42 000 000 m²
 g 3 ha h 2500 m²
 i 0.007 km²
- 6 a 144 cm² b 63 m² c 13.23 m²
 d 10.24 km² e 0.16 m² f 9.2 km²
- 7 a 96 cm² b 225 m² c 91 mm²
 d 154 km² e 7.6 cm² f 1.4 km²
- 8 a 729 cm² b 165 cm² c 680 mm²
 d 13.2 cm² e 0.78 km²
- 9 a 20 cm² b 25.8 m² c 21 km²
 d 135 mm² e 1.52 m² f 0.69 cm²
- 10 a 1256.6 mm² b 113.1 cm² c 346.4 cm²
 d 66.5 m² e 118.8 cm² f 1.247 km²
- 11 a 78 cm² b 69 mm² c 136 m²
 d 8 km² e 16.05 cm² f 0.87 m²
- 12 a 54 m² b 2850 cm² or 0.285 m²
 c 628.3 mm² d 650 m²
 e 1134.1 km² f 228 cm²
 g 0.925 m²
- 13 a 1.32 m² b 448 cm² c 96 m²
 d 360 cm² e 7.2 m² f 574 mm²
- 14 a m² b mm² or cm² c km²
 d cm² or m² e m² f cm²
 g ha or km² h cm² i m²
 j km²
- 15 234 ha 16 7348 m²
- 17 a 2 kg b 14 kg c 7050 kg
 d 56 000 kg e 2 240 000 peas
- 18 a 175 cm² b 210.8 m² c 917.1 mm²
 d 158 m² e 858 cm² f 50 m²
- 19 a 1 700 000 km² b 230 000 km²
 c 2 530 000 km²
- 20 50 m 21 About 78 kg
- 22 a 2.25 m by 2.25 m b 40.5 m²
- 23 a 75 477 m² b 8 ha
 c 13.7 h (13 h 43 min)
- 24 a \$194 580 b \$240 000 c \$311 100
- 25 \$2768

Exercise 3.2

- 1 a 168 cm² b 139.52 m² c 132.54 mm²
 d 392 cm² e 407.62 mm²
- 2 a 92.5 m² b 829.4 cm² c 301.6 m²
 d 912.6 cm² e 273.7 cm² f 600 mm²
- 3 a 502.65 cm² b 3958.41 mm²
 c 33.93 m² d 2804.44 cm²
- 4 a 1570.8 cm² b 89.5 m² c 21 167.1 mm²
 d 8191.2 cm²
- 5 a 40.84 m² b 4533.32 cm²
 c 6069.56 mm² d 5971.54 cm²
- 6 a 3434.24 cm² b 3894.0 cm²
 c 4791.2 cm² d 5368.3 cm²
- 7 744 cm² 8 1210.4 cm²
- 9 a 3520 cm² b 3872 cm² c 7744 m²
- 10 587 m² 11 \$1182.72 12 \$2100
- 13 a 5.3 m by 9.4 m b 49.95 m²
- 14 a About 1668 m² b About 334 L
- 15 a 454 cm²
 b 105 labels. (You get 110 if you divide the area of the sheet by the area of a label.)
- 16 a 254 cm² b 292.1 cm² c \$817.88
- 17 a 3381 m² b 211.3 L
- 18 About 19 757 cm²

Exercise 3.3

- 1 a 36 cm³ b 21 cm³ c 80 cm³
 d 26 cm³ e 27 cm³ f 23 cm³
- 2 a 160 cm³ b 210 cm³ c 108 cm³
 d 144 cm³ e 336 cm³ f 125 cm³
- 3 a 0.5 L b 1270 cm³ c 78 mL
 d 4.3 m³ e 24 L f 8000 mL
- 4 a 4000 cm³ b 1.2 kL c 1050 L
 d 45.3 kL e 446 000 m³ f 0.0064 ML
- 5 a 114.3 m³ b 669.6 cm³ c 293.7 cm³
 d 4.067 m³ e 0.408 12 m³ f 101.32 m³

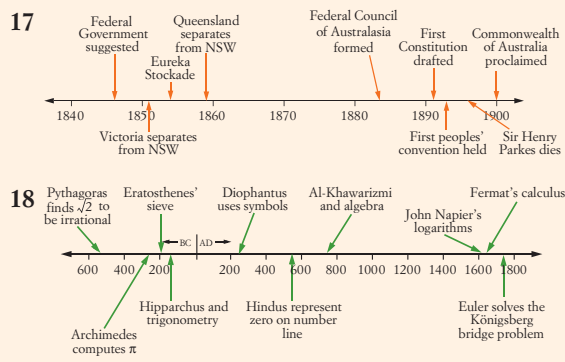
- 6 a 64 cm^3 b 252 mm^3
c 2.24 m^3 d 702 cm^3
- 7 a 1407.4 cm^3 b 22.7 m^3
c 16.69 m^3 d 66.8 m^3
- 8 a 42 cm^3 b 78 mm^3
c 2.16 m^3 d 3864 m^3
- 9 a 2262 m^3 b 42.22 cm^3
c 18.56 m^3 d 4948 cm^3
- 10 a 256 cm^3 b 9236 mm^3 c 12.48 m^3
d 6157.5 cm^3 e 143 cm^3 f 4.86 m^3
g 88.04 cm^3 h 573.5 m^3 i 2714 cm^3
- 11 a 1350 cm^3 b $22\,176 \text{ cm}^3$
- 12 a 8580 cm^3 b 13.78 m^3 c 8764 mm^3
- 13 (All answers approximate)
a $10\,819 \text{ cm}^3$ b 3.41 m^3 c 9693.4 cm^3
- 14 About $42\,965 \text{ cm}^3$
- 15 a L b kL c mL
d mL e L f L
g kL or ML h L i L
- 16 $48\,000 \text{ cm}^3$
- 17 a $258\,000 \text{ cm}^3$ b 0.258 m^3
- 18 a 3.2 kL b 3200 L
- 19 a 1890 cm^3 b 461.8 cm^3 c 126 cm^3
- 20 a 9 L b 729
- 21 a 18 m^3 b $\$121.50$ c $\$1215$
- 22 a 0.125 m^3 b 306 m^3 c 2448 bales
- 23 a 2.36 m^3 b $\$729.24$
- 24 a 13.2 m^3 b $13\,200 \text{ L}$
- 25 a 132 m^3 b 79.2 m^3
c $79\,200 \text{ L}$ d 79.2 t
- 26 a 25 m^3 b 30 wagons
- 27 $263\,900 \text{ kg}$ or 263.9 t
- 2 a 120 min b 720 min c 2 min
d 7 min e 150 min f $\frac{1}{2} \text{ min}$
g 2160 min h $140\,760 \text{ min}$
- 3 a $9 \text{ h } 5 \text{ min}$ b $40 \text{ h } 2 \text{ min}$ c $43 \text{ h } 55 \text{ min}$
d $68 \text{ h } 8 \text{ min}$ e $73 \text{ h } 30 \text{ min}$
- 4 a 11 December b 13 January c 3 April
d 3 December e 20 May f 3 December
- 5 a 0730 h b 1712 h c 1150 h
d 2350 h e 2230 h f 0545 h
g 1830 h h 0750 h i 2212 h
j 0134 h
- 6 a $6:30 \text{ a.m.}$ b $2:40 \text{ p.m.}$ c $10:35 \text{ a.m.}$
d $7:20 \text{ p.m.}$ e $11:10 \text{ a.m.}$ f $9:10 \text{ a.m.}$
g $11:24 \text{ p.m.}$ h $5:20 \text{ a.m.}$ i $2:00 \text{ p.m.}$
j $9:12 \text{ p.m.}$ k $9:56 \text{ a.m.}$ l $11:23 \text{ a.m.}$
- 7 a 840 h b 1095 d
c 50 y d 12.29 wk
e 10.74 centuries f $432\,000 \text{ s}$
g 32.5 decades h $3\frac{1}{2} \text{ wk}$
i $4\frac{1}{4} \text{ h}$
- 8 a 129 d b 196 d
c 56 d d $34 \text{ y } 277 \text{ d}$
e $7 \text{ y } 8 \text{ months}$ f 1699 y
g 526 y h $6 \text{ y } 7 \text{ months}$
i $6 \text{ y } 69 \text{ d}$
- 9 a 5th BC b 19th c 7th BCE
d 16th BC e 21st f 15th
- 10 a $3 \text{ h } 30 \text{ min}$ b $6 \text{ h } 15 \text{ min}$
c $33 \text{ min } 12 \text{ s}$ d $6 \text{ d } 9 \text{ h } 36 \text{ min}$
e $5 \text{ min } 27 \text{ s}$ f $22 \text{ decades } 4 \text{ y}$
g $9 \text{ h } 36 \text{ min}$ h $5 \text{ h } 20 \text{ min } 24 \text{ s}$
i $5 \text{ d } 3 \text{ h } 36 \text{ min}$
- 11 a $10:30 \text{ a.m.}$ b $10:00 \text{ a.m.}$ c $8:30 \text{ a.m.}$
d $10:00 \text{ a.m.}$ e $10:30 \text{ a.m.}$ f $10:30 \text{ a.m.}$
- 12 a $7:45 \text{ p.m.}$ b $8:15 \text{ p.m.}$ c $5:45 \text{ p.m.}$
d $7:15 \text{ p.m.}$ e $8:45 \text{ p.m.}$ f $8:45 \text{ p.m.}$
- 13 a $2:00 \text{ a.m.}$ b Noon c 1500 h
d $10:30 \text{ p.m.}$ e $6:30 \text{ p.m.}$ f 0510 h
g $6:00 \text{ p.m.}$
- 14 a $4:00 \text{ a.m.}$ b $10:00 \text{ p.m. the previous day}$
c $1:00 \text{ a.m.}$ d $8:15 \text{ a.m.}$
- 15 a $3:00 \text{ a.m. the next day}$ b Midnight
c $8:00 \text{ a.m. the next day}$ d $10:30 \text{ a.m.}$

Exercise 3.4

- 1 a 120 s b 600 s c $10\,800 \text{ s}$ d 270 s
e 75 s f 225 s g $15\,300 \text{ s}$ h $86\,400 \text{ s}$

Answers

16	Time taken for:	Suitable unit	Reasonable time taken
	A direct flight from Brisbane to Melbourne	h	About 3 h
	An Olympic 100 m dash	s	About 9.9 s
	You to walk 4 km	h	About 1 h
	A drive from Brisbane to Rockhampton	h	About 8 h
	A typist to type 500 words	min	About 7 min
	A glacier to move 10 m	days	About 15 d
	A hailstone to fall 1000 m	s	About 30 s
	A cricket ball to reach a batsman	s	About 0.5 s
	The erosion of the Grand Canyon	y	Millions of years
	A student to clean a blackboard	s	About 30 s
	You to ride to school on a bike	min	Time will vary.



- 19 About 1×10^{-4} s and 2×10^{-5} s
- 20 **a** No, the only factors of 85 are 1, 5, 17, 85.
b Answers will vary according to the year, e.g. 2/2/04, 1/5/05, 2/3/06, 7/1/07, 4/2/08.
- 21 Days are longer in summer and shorter in winter. Only southern states such as Victoria and Tasmania have twilight hours.
- 22 **a** 2 h **b** 2 h 50 min **c** 2 h 25 min
d Forward duration 8 h 30 min, return duration 7 h 35 min.
e Forward flying time 4 h 50 min, return flying time 5 h 40 min.

23 Perth 12:15 p.m., Alice Springs 1:45 p.m., Melbourne 2:15 p.m., Hobart 3:15 p.m.

24 June 3:00 p.m., December 4:00 p.m.

Chapter review

- 1 **a** 0.6 m^2 **b** 1600 mm^2 **c** $50\,000 \text{ cm}^2$
d 12 cm^2
- 2 **a** 60 ha **b** 420 ha **c** $50\,000 \text{ m}^2$
d 24 cm^3 **e** 80 cm^3
- 3 **a** 0.6 m^3 **b** 4200 mL **c** 5000 L
d $66\,000\,000 \text{ cm}^3$ **e** 0.25 ML
f 0.073 kL
- 4 **a** 2 min 30 sec **b** 19 d **c** 7.5 decades
d 0815 h **e** 2132 h
- 5 **a** 8:20 p.m. **b** 7:43 a.m. **c** 10:43 a.m.
d 33.28 m^2 **e** 89.76 m^2 **f** 39.69 cm^2
- 6 **a** 18.2 m^2 **b** 172.03 cm^2
c 1290.5 cm^2 **d** 121.6 mm^2
- 7 **a** 4.2 m^2 **b** 394 cm^2
- 8 **a** 76.48 m^2 **b** 98.02 m^2
- 9 **a** 32.52 m^2 **b** 1474 cm^2 **c** 3066 mm^2
d 362.7 cm^3 **e** $69.858\,516 \text{ m}^3$
- 10 **a** 150 cm^3 **b** 27.2 m^3 **c** 142.0 m^3
d 320.9 cm^3 **e** 182.0475 cm^3 **f** 0.4324 m^3
g 588.4 cm^3
- 11 **a** 31 h 45 min **b** 50 d **c** 90 d **d** 292 y
- 12 4:35 p.m. 20 **a** m^2 **b** cm^2
- 13 123.62 cm^2 22 1570.8 cm^2 23 12.364 m^2
- 14 $\text{SA} = 46.64 \text{ m}^2$ which is slightly less than the SA of the original prism of 47.12 m^2 .
- 15 **a** 3690.2 cm^3 **b** 39 m^3
- 16 **a** mL or cm^3 **b** L , kL or m^3

- 27 69 h 20 min 28 25 cm
 29 3600 kg or 3.6 t 30 12 h 50 min
 31 a 222 cm² b \$5550
 32 35 700 L, requiring 9 (8.925) cylinders of gas
 33 1.4625 kg

Chapter 4

Exercise 4.1

- 1 a 4c b $-7mn$ and $4nm$
 c $-2r^4$ and $25r^4$ d $5q^3p$
 e $6ghg$ and $3hg^2$ f ead^3
- 2 a $3a^2, a^2, 4a^2$ b $9y^6, -5y^6, 3y^2y^4$
 c $-8w^4, wwvw$ d $-5g^3, 2g^2g$
 e $-5j^2a, 7jaj$ f $(bc)^2, -cb^2c$
- 3 a 10a b 16q c 7mn
 d 12rt e $2s^2u$ f $2b^2d$
 g 21acg h $3pq^2t$ i $6jk^2m$
- 4 a 8en b 18dy c 12adf
 d 20bqt e $21u^2xy$ f $4cv^2t$
 g $24bj^2k$ h $33eh^2t$ i $15hf^2n^2$
- 5 a 6g b 6ab c 2gh
 d 3er e 7j f 11r
 g 5m h $\frac{12h}{5}$ i $\frac{4p}{3t}$
- 6 a $7m^2$ b $12f^3r$ c $8mg^2$
 d $9h^2$ e $8cd^3$ f $9m^5$
 g 0 h $3x^3 + 7x$ i $6f^2 + 3f^3$
 j $9r^2 - 7r$ k $5k^2 + 4ck$ l $-5n^3$
- 7 a $4x^2y + 5xy$ b $9m^3 - 7mb^2$
 c $10 + 4c^2q^2 - 2c^2q^4$ d $4d + 8d^3$
 e $2f^3r + 9fr$ f $6gy + 6y^3 - 8y$
 g $17 - 16n + 5nj^2$ h $5ew^3 - 4e^2w^3$
- 8 a $-6at$ b 20hqu c $-24pr$
 d 30bmt e $-84hjk^2$ f $-45e^2fg^2r$
 g $-9mn^2p^2q$ h $28g^2x^3y^2$ i $48g^4h^3$
 j $24h^3w^5$ k $27d^3e^5f$ l $-28f^6r^3$
- 9 a 4v b $-5hk$ c 3g
 d 5d e 2n f $-4pr$
 g 3f h $\frac{-7}{4}$ i $\frac{8d}{3}$
- 10 a 2d b $\frac{-3}{2}$ c 2pqr
 d $-2c$ e $9q^2$ f $-3y$
 g $-21u^6$ h $4h^2$ i $\frac{4n^2}{r}$
 j $\frac{12m^3n^2}{5}$ k $\frac{-4x^5y^2}{z}$ l $\frac{-6a^5b}{c}$
- 11 a 22a b 17g c 6p d 5cd
 e 34hj f 8xy g 23xyz h abh
 i $-16q$ j $-12mn$ k $-5rt$ l $-4cd$
- 12 a $15m^8h^{10}$ b $2k^{12}p^5$
 c $7g^{10}a^9$ d $27t^{11}q^8$
 e $5h^{10}d^{14}$ f $3q^{14}h^{10}$
 g $8d^9m^7$ h $18u^{10}p^{10}$
 i $14d^{10}g^9$ j $7w^6r^9$
 k $36a^{10}x^7$ l $49h^5d^4$
 m $6p^6x^7$ n $35p^6w^5$
 o $32k^{12}h^8$ p $9r^8x^{12}$
 q $6u^{11}y^9$ r $35z^{11}h^7$
 s $16b^{13}m^5$ t $54k^{11}m^6$
- 13 a $\frac{5n}{c^4}$ b $\frac{t}{4m}$ c $\frac{7t^2}{6x}$ d $\frac{6}{23h^3y^3}$
 e $\frac{2t^5z}{3}$ f $\frac{9}{2q^2z^2}$ g $\frac{4c^4}{11}$ h $\frac{11w^2}{2b^2}$
 i $\frac{b^2z^3}{2}$ j $\frac{4a^3}{t^2}$ k $\frac{1}{3v^2y^3}$ l $\frac{7}{3a^2d^2}$
 m $\frac{9x^2}{4}$ n $\frac{5}{3gb}$ o $\frac{2a^2}{13}$ p $\frac{w^5}{11z^3}$
 q $\frac{4k}{5h^5}$ r $\frac{7n^2}{5c^2}$ s $\frac{q^3w^2}{3}$ t $\frac{2k^2}{3h^5}$
- 14 a $\frac{2b}{a}$ b $\frac{10ax}{z}$ c $\frac{21n^4}{m}$ d $\frac{p}{q^2}$ e $\frac{32pq}{21v}$
- 15 About 131 N, compared to 784 N, so the force on the Moon is about $\frac{1}{6}$ that on the Earth.
- 16 18%
- 17 a 14m b 32p
 c $9x + 8$ d $8a + 6b + 12$
 e $26q - 8r + 24$ f $14m - 2n + 54$
- 18 a $15x^2y$ b $49q^2$ c $48m^2$
 d $24a^3b^2$ e $12p^2q^4$ f $\frac{21}{2}m^3n^4$

Answers

Exercise 4.2

- 1 a $3(m+1) = 3 \times m + 3 \times 1 = 3m + 3$
 b $2(d-5) = 2 \times d - 2 \times 5 = 2d - 10$
 c $5(6+q) = 5 \times 6 + 5 \times q = 30 + 5q$
 d $4(6+y) = 4 \times 6 + 4 \times y = 24 + 4y$
 e $2(5r-7) = 2 \times 5r - 2 \times 7 = 10r - 14$
 f $7(3-4q) = 7 \times 3 - 7 \times 4q = 21 - 28q$
 g $d(3p+5) = d \times 3p + d \times 5 = 3dp + 5d$
 h $(8b+6)a = 8b \times a + 6 \times a = 8ab + 6a$

- 2 a $6+2a$ b $b-2$ c $3+3h$
 d $4a+20$ e $14-8a$ f $12d+30$
 g $36-9e$ h $44+36h$ i $6-12u$
 j $20g+35$ k $16y-80$ l $42-21k$

- 3 a f^2+2f b $y-y^2$ c z^2+4z
 d $9p+p^2$ e $gr+g$ f $mp-mz$
 g $3e+eu$ h $qr+r$ i $sw+7s$
 j $5y-y^2$ k a^2-4a l $7p-p^2$

- 4 a $5g+1$ b $7c+11$ c $7q-4$
 d $10k-6$ e $5b+13$ f $9y-22$

- 5 a $8v^2-6v-5$ b $3m^2+4m-7$
 c $10q^2+41q-18$ d $3e^2-26e-9$
 e $10n^2-29n+10$ f $15m^2+17m-4$
 g $3c^2+14c-5$ h $8d^2-2d-45$
 i $16m^2-8m-3$ j $25r^2-25r+4$
 k $8r^2+14r+3$ l $9r^2+45r+56$
 m $4m^2-27m+18$ n $6c^2-31c+35$
 o $20k^2+59k+42$ p $8t^2-22t-21$
 q $12x^2+37x+21$ r $10p^2-11p-8$
 s $6c^2+19c-36$ t $20u^2-33u+7$

- 6 a $2a^2+6a$ b $5h-5h^2$ c $4c^2+12c$
 d $9k-6kw$ e $24g+12g^2$ f $6m-18m^2$
 g $-2e+6e^2$ h $4bj+28aj$ i $-18t+12rt$
 j $20z+15z^2$ k $4d^2+28d$ l $6f^2-27f$

- 7 a $6d^2+2d$ b $8a^2+20a$
 c $8r^2+6r$ d $10a^2+6a$
 e $6g^2-18g$ f $15s^2-10s$
 g $-21g^2-7g$ h $-18k+45k^2$
 i $24q^2+42pq$ j $8mn-12n^2$
 k $6x^2+15xy$ l $-18xy+30x^2$

- 8 a $48r-8c-64m$ b $-72q+48r+80k$
 c $18t+3z-12x$ d $-21q+7a+49m$
 e $18c-66g-12h$ f $18v+6q+78q$
 g $25a+35e+25m$ h $-72k+72r-56t$
 i $24x-15c+18h$ j $-14a-35u+21b$
 k $-32q+64y+48n$ l $6u-10y-16r$
 m $15h-35w-40g$ n $-9g-54u-81r$
 o $72b-9n-54u$ p $-8g+72w+24h$
 q $-8r-32k-4d$ r $9z+18p+12p$
 s $5y-65v-35y$ t $16g-28p-20t$

- 9 a $72a^5r+72a^3r^3$ b $12n^5v^2-10n^3v^4$
 c $12y^3n^2+40y$ d $32he-20e^2$
 e $21pg^3+7p^4g^3$ f $24p^4v-6p^4v^4$
 g $63w^4+9wm$ h $28a^2c^2+12ac$
 i $8p^4w^4-16pw^2$ j $14c^5v^3+6v^4$
 k $14e^2m^3-6e^4m^4$ l $32z^4m^3-32z^4m^2$
 m $7uw^2-105u^3v^2$ n $24h^4w^4-32h$
 o $15g^3n^2-3g^3n^4$ p $21m^2-70my$
 q $28p^4u^3-35$ r $12e^4+4e^5c^2$
 s $6d^5p^4-15d^3p^4$ t $9q^5+27qq$

- 10 a $-32b^8n^3-8n^7$ b $-25n^8y^4+10n^4y^4$
 c $16y^7g^5-12y^6g^7$ d $50k^3g^8-120k^6g^5$
 e $-80y^3n+80y^4n^2$ f $-36u^2z^5-4u^5z^2$
 g $18c^3a^5+24c^8a^8$ h $27n^8u^3+3n^4u^3$
 i $-28h^6e^7+21h^4e$ j $-30z^6+5e^4z^5$
 k $30k^6+72u^4k^4$ l $-20k^5y^4+12k^8$
 m $80k^5u^8+10k^5u^2$ n $6k^3+4k^4g^3$
 o $-55r^7h^5+132r^2h^7$ p $18h-20hx^8$
 q $-6b^6z^5+12b^8z$ r $35e^6v^6+7v^8$
 s $42b^8u^2+78u^3$ t $21c^5+39v^2c^4$

- 11 a $8w^2+6w^3+14wx$ b $3m+9m^2p-18m^3$
 c $10a-30a^2m+5a^3m$ d $6h^2-12h^3-15h^2j$
 e $8d^2+4d^3g+12d^2g$ f $-8tu^2+6u^4+12tu$
 g $-6x^2y-9x^3+15xy$ h $20d-8d^2e^2-28d^2e$
 i $-8y^2z-10yz^2-6yz$ j $10r^2+15r^3-5j^2r^2$

- 12 a $6p+2$ b $10n-1$ c $8s-16$
 d $8e-27$ e $7z-35$ f $24x-42$

- 13 a $5x+15$ b $6q-3$
 c $7d+3$ d $5-10j$
 e $9-12a$ f $5+6e$
 g $5r+1$ h $-18n+12n^2$
 i $21z+2$ j $23g+4$

- k** $17 - 8u$ **l** 23
m $6a^2 + 13a + 16$ **n** $-12a^2 + 22a - 32$
o $5b + 3$ **p** $-6h^2 + 28h + 10$
- 14 a** $15z^3 - 51mz - 47mzc + 15z$
b $40hw + 96uh + 47uh^3w - 10u^2h^3w^3 - 25u^3h^3w$
c $81z^2g^3q^4 - 9g^2q^2 - 35gq^4 + 66zg^2q^3 + 44g^3q^2$
d $12m^3z^3 + 70u^2m^2z^3 - 28z^2 - 12u^2 + 12u^2z^2$
e $72p + 65c^3pe^2 - 45p^2e^4 - 35c^2e^3 - 14cp^3e$
f $45za^2 - 18z^3a^2 + y^2z^2a^2 + 4y^2z - 24z^2y^3a^2$
g $10ay^2n^2 + 40ay^2n - 53a^2y^2n^2 - 27a^3 - 117yn$
h $104b^2a^2c^4 - 120b^2a^3 + 80a^3c^2 + 6a + 48b^2ac^3$
i $20y^2v^3 + 16d - 16d^2v + 18y^2d^3v^3 + 16y^3v^4$
j $20h - 48a - 6avh - 45v^2h^2 + 20vh^4$
- 15 a** $81q^2 - 81q + 14$ **b** $54y^2 - 51y + 5$
c $56q^2 + 97q + 42$ **d** $40v^2 + 51v - 7$
e $64m^2 + 64m - 9$ **f** $32g^2 - 44g + 9$
g $9x^2 + 11x + 2$ **h** $2y^2 + 5y - 25$
i $54u^2 + 21u - 20$ **j** $16d^2 + 58d + 45$
k $64d^2 + 16d - 63$ **l** $24t^2 - 41t - 35$
m $5e^2 + 29e - 6$ **n** $2g^2 + 21g + 49$
o $4u^2 - 3u - 1$ **p** $63p^2 - 65p + 8$
q $45u^2 + 121u + 72$ **r** $2q^2 + 11q + 9$
s $20x^2 - 63x + 49$ **t** $49m^2 - 1$
- 16 a** $15k^2 + 14kr + 3r^2$ **b** $40v^2 + 53vy + 9y^2$
c $40g^2 - gc - 6c^2$ **d** $32u^2 + 92uh + 63h^2$
e $24x^2 + 37xb - 5b^2$ **f** $32e^2 - 44er - 63r^2$
g $49r^2 + 42rv + 9v^2$ **h** $25t^2 - 10te - 63e^2$
i $72g^2 - 7ge - 49e^2$ **j** $7w^2 + 36we + 5e^2$
k $48n^2 + 26ne + 3e^2$ **l** $45t^2 - 62ta - 16a^2$
m $32t^2 + 44tw - 21w^2$ **n** $6r^2 - 5rd - 4d^2$
o $20v^2 + 27vd + 9d^2$ **p** $36w^2 - 73wn - 18n^2$
q $6m^2 - 5me + e^2$ **r** $28n^2 - 13nc - 63c^2$
s $9x^2 + 76xn - 45n^2$ **t** $36g^2 - 71gn + 14n^2$
- 17 a** $39q^2 - 51qt + 27t^2$ **b** $70e^2 - 38ve - 51v^2$
c $40y^2 + 28yk + 10k^2$ **d** $144g^2 + 20gt - 36t^2$
e $33k^2 - 15ka - 69a^2$ **f** $76c^2 + 32cn - 43n^2$
g $37m^2 - 46r^2 - 44rm$ **h** $71y^2 - 50yh - 39h^2$
i $79xz - 8x^2 - 19z^2$ **j** $70g^2 - 110gu + 90u^2$
k $12m^2 - 36mg - 35g^2$ **l** $24y^2 + 147yg - g^2$
- m** $21m^2 - 19mx - 7x^2$ **n** $71hu - 2h^2 - 75u^2$
o $58d^2 + 30dx - 4x^2$ **p** $20x^2 + 22xu - 94u^2$
q $8k^2 + 46ke - 23e^2$ **r** $15c^2 + 26ce - 10e^2$
s $34q^2 - 79qe + 34e^2$ **t** $-1x^2 - 97xm - 60m^2$
- 18 a** $10p + 30 \text{ cm}^2$ **b** $12x - 24 \text{ m}^2$
c $55z + 33 \text{ cm}^2$
- 19 a** $d^2 + d - 6 \text{ m}^2$ **b** $3w^2 + 13w + 4 \text{ cm}^2$
c $9k^2 - 4.5k - 2 \text{ or } \frac{1}{2}(18k^2 - 9k - 2) \text{ mm}^2$
- 20 a** $2x^2 + 5xy - 3y^2 \text{ m}^2$
b $10x^2 + 6.5xy + n^2 \text{ or } \frac{1}{2}(20x^2 + 13xy + 2n^2) \text{ mm}^2$
c $8v^2 + 14w - 4w^2 \text{ cm}^2$
- 21 a** $a^2 - 1, m^2 - 64, 16p^2 - 25$
b $(x + y)(x - y) = x^2 - y^2$ (use expansion)
- 22 a** $z^2 + 2z + 1, 25w^2 + 10w + 1, 9y^2 + 42y + 49$
b $(x + y)^2 = x^2 + 2xy + y^2$ (use expansion)
- 23 a** $m^2 - 16m + 64, 9q^2 - 24q + 16, 16k^2 - 40k + 25$
b $(x - y)^2 = x^2 - 2xy + y^2$ (use expansion)

Exercise 4.3

- 1 a** $8(a - 2)$ **b** $6(2g + 3)$ **c** $7(8m - 5)$
d $6(7q - 3)$ **e** $8(5m - 1)$ **f** $8(2e + 3)$
g $4(7m + 1)$ **h** $5(5y + 3)$ **i** $9(7p - 1)$
j $8(6k - 5)$ **k** $3(5h - 2)$ **l** $3(6k + 5)$
m $3(4g + 5)$ **n** $7(5y + 12)$ **o** $9(2c - 5)$
p $7(m + 3)$ **q** $4(5e + 3)$ **r** $3(9u + 1)$
s $3(9e + 2)$ **t** $5(9k - 1)$
- 2 a** $6(6t - n)$ **b** $3(p + 8m)$ **c** $4(2u + d)$
d $-7(d - 2y)$ **e** $9(4a - u)$ **f** $3(4x + h)$
g $2(8p + w)$ **h** $7(5z - 4q)$ **i** $9(9g - 2g)$
j $9(2c + u)$ **k** $-6(3k - 7n)$ **l** $-9(6b + 5t)$
m $8(8y - g)$ **n** $7(2r - 3n)$ **o** $-8(8c - 3w)$
p $-9(2n + h)$ **q** $-7(2d + 7d)$ **r** $2(3d + p)$
s $8(y + 3n)$ **t** $-8(9n + e)$
- 3 a** $8g(2g - h)$ **b** $7x(5x + 2e)$
c $7b(2b + x)$ **d** $4q(q + 7a)$
e $5h(5h - 8)$ **f** $9d(4d - 7q)$
g $3y(y + 6d)$ **h** $9n(5n + 8q)$
i $3x(8x + 8a)$ **j** $5w(w + e)$
k $9r^2(2r + 8h)$ **l** $-2m^2(7m + 5q)$
m $6q(7q - 13v)$ **n** $-2q^2(7q^2 - 2z)$
o $5z(8z^2 - 5t)$ **p** $-4w^2(7w + 13x)$
q $-4y(3y - 2b)$ **r** $7u(4u^2 + 3n)$
s $-9v(7v - 4r)$ **t** $-6m^2(2m^2 + 3n)$

Answers

- 4 a** $10(9v + 2)$ **b** $9(7y + 13)$ **c** $15(3u - 10)$
d $9(3y - 10)$ **e** $15(5v - 2)$ **f** $2(8x + 1)$
g $4(2p + 7)$ **h** $3(2h + 8)$ **i** $5(9t + 4)$
j $14(5k - 12)$ **k** $8(6p + 5)$ **l** $6(k + 6)$
m $12(3u - 1)$ **n** $15(4v - 7)$ **o** $9(2d - 3)$
p $11(6x + 1)$ **q** $4(5u + 12)$ **r** $4(7p + 4)$
s $7(3v + 7)$ **t** $13(6r - 1)$
- 5 a** $7(7u - r)$ **b** $-10(7x - 3z)$
c $-3(3q - 7n)$ **d** $15(5k - z)$
e $14(4g - 3d)$ **f** $5(6c - 5n)$
g $9(8q - y)$ **h** $4(9m + 10r)$
i $-10(2h - 8c)$ **j** $11(7e - 6p)$
k $5(5e + 2a)$ **l** $9(2n + y)$
m $-2(3z + c)$ **n** $13(6k - y)$
o $4(5u + 3b)$ **p** $-5(6d - 5t)$
q $15(5w + 8g)$ **r** $-15(9w - 4q)$
s $-3(q + 5w)$ **t** $-10(8g - 3m)$
- 6 a** $9(8h + 5q)$ **b** $-14(8y + 7r)$
c $15(7p - 8v)$ **d** $-3(3d + 13k)$
e $-9(9z + 2a)$ **f** $4(2g + 11p)$
g $-112(b - w)$ **h** $5(6g - 7u)$
i $9(d + 12q)$ **j** $3(5c + 12w)$
k $4(y - 11h)$ **l** $-3(6x + 11u)$
m $5(9d - 13m)$ **n** $-14(u + 4g)$
o $9(6e - 5g)$ **p** $-13(6a - 11k)$
q $9(2t - 3m)$ **r** $8(9r - 8b)$
s $-3(6x - 5z)$ **t** $-3(8c - 12m)$
- 7 a** $14(10g - 7y)$ **b** $12(13b + u)$
c $-7(15e - x)$ **d** $9(k + 9n)$
e $-2(13t + 12z)$ **f** $4(14w + 5e)$
g $-8(2g - 13k)$ **h** $-5(11r + 13c)$
i $-10(3q - 7x)$ **j** $-2(2g - 5u)$
k $2(6w - f)$ **l** $10(13h - 10p)$
m $11(2q - 8t)$ **n** $-7(9b - 13r)$
o $-13(2q + 7u)$ **p** $13(8w - r)$
q $5(5p - 8c)$ **r** $-12(15p - 8a)$
s $9(8g + 12r)$ **t** $-9(5e - 11f)$
- 8 a** $4(6a - x + 8)$ **b** $7(9q - 3t + 4)$
c $8(9n - 4w + 1)$ **d** $5(8c - 5k - 7)$
e $9(5w - 7b - 3)$ **f** $9(2q - 2y - 5)$
g $2(c - 7m + 5)$ **h** $4(9g + v - 7)$
i $6(3h - 10z - 3)$ **j** $4(3w + 3k - 5)$
k $4(2h - 5p + 6)$ **l** $3(2p - 5c - 7)$
m $7(d - y - 4)$ **n** $6(6v - 2q - 5)$
o $32(e - 2t + 1)$ **p** $3(2c + 3w - 7)$
q $8(9v + g + 6)$ **r** $8(7w + 5w - 2)$
s $3(7c + 13w - 6)$ **t** $5(8x + 9a + 7)$
- 9 a** $3(r + 12y - 2u)$ **b** $6(2z - 7t - 11p)$
c $8(8k - 11b - 6e)$ **d** $6(2r + 2e - 5r)$
e $-2(7p - 4p - 8g)$ **f** $4(6b - z - 5m)$
g $-5(3k + 10y - 10v)$ **h** $-7(5h + z + 12r)$
i $16(2q - 4e + 5p)$ **j** $2(9g - 8x - 10u)$
k $-4(3u - 5d + 12v)$ **l** $-9(2y + 6g + 3b)$
m $-6(8a - 3c - 7u)$ **n** $9(5m + 11n + 4r)$
o $8(4v + 13m - z)$ **p** $-9(5n + 6b + 12z)$
q $4(5z - 13e + 6w)$ **r** $3(4w - 7d - 13t)$
s $5(8t + 3x - 11g)$ **t** $9(8e + t + 8y)$
- 10 a** $7p^2(6p - g)$ **b** $2g^2(7g^4 - 13v)$
c $6m^4(7m^2 - 5h)$ **d** $9c(4c^2 - 8v)$
e $5b^2(3b^4 + 10d)$ **f** $-8x(4x^4 - 5y)$
g $2z^2(8z^2 - 8k)$ **h** $9x^4(3x - 10v)$
i $-4y^2(y^2 + 3g)$ **j** $4y^3(7y^2 - 2a)$
k $4g^3(6g^4 + 5a)$ **l** $2m(7m^4 - 13w)$
m $4a(9a^2 + t)$ **n** $2b(2b + 7r)$
o $6d(2d^2 + u)$ **p** $-9b^2(9b^4 - 7z)$
q $-8g^2(9g + 5z)$ **r** $-4n^4(9n^3 + 2h)$
s $-2u^3(2u^2 + 7e)$ **t** $9x^2(6x^4 + 5x)$
- 11 a** $12x^3(4r + k^4r - 4)$
b $-5r^3t^4y^2(4t - 5r^2y^3 - 9ty^3)$
c $5c^2d(8c^3d^2 + 8n^5 - 5n^2d)$
d $-3(6yn^4 - 5 - 9b^2y^3)$
e $8ud(9m^5u - 3m^4u^4d + 10d)$
f $8q^2h(7h^2 - eq^3 - 2e^3q^3h^3)$
g $-8v(d + 3vy - d^3v^5y^4)$
h $8x^3m^3(u^3x^2 + 3u^5x^2m + 8)$
i $-9n^3(2t - 4r^4t^2 + 3r^4)$
j $12b^3d^2a^2(b - 4a - 4b^2d^3a)$
k $2v(3nm^4 - 8v^2n^3 - v^3m)$
l $-4ry^3(9 - ry^2n^4 + 3n)$
m $-72q^2(1 + b^3w^4 - bqmw)$
n $7wy(u^5y^4 - 2w^4y - 5w^2u^2)$
o $9q^3(7e^5q^2 - 3u^2 + 2q)$
p $4d(9q + 4m^2 + 6q^2dm^3)$
q $8q^4r(9qr^4b^3 + 5q + 4rb^5)$
r $-3n(9ndt^5 + 4n^3t^5 - 3d^5)$
s $3a^3x^2(4ta^2x^2 + 8 - 3x^3)$
t $3w^3(6n^5w^2 - 5t^2w^2 - 3t^2)$
- 12** Area of each end = πr^2 , curved surface area = $2\pi r \times h$, Total = $2 \times \pi r^2 + 2\pi rh = 2\pi r(r + h)$.

Exercise 4.4

$$\begin{array}{lll} 1 \text{ a } \frac{18x-23}{24} & \text{b } \frac{16m+11}{9} & \text{c } \frac{11b-39}{14} \\ \text{d } \frac{26w-65}{40} & \text{e } \frac{41k+18}{35} & \text{f } \frac{44p-25}{40} \\ \text{g } \frac{7c+17}{12} & \text{h } \frac{15d+18}{14} \end{array}$$

$$\begin{array}{lll} 2 \text{ a } \frac{-10a-23}{10} & \text{b } \frac{5n+2}{6} & \text{c } \frac{-13f-13}{24} \\ \text{d } \frac{13-q}{20} & \text{e } \frac{13t-14}{12} & \text{f } \frac{15-2b}{10} \\ \text{g } \frac{-13e-21}{20} & \text{h } \frac{-20s-5}{36} \end{array}$$

$$\begin{array}{lll} 3 \text{ a } \frac{32v+41}{20} & \text{b } \frac{19-25c}{42} & \text{c } \frac{17y-79}{42} \\ \text{d } \frac{11g-9}{6} & \text{e } \frac{9u+8}{20} & \text{f } \frac{33z+1}{18} \\ \text{g } \frac{-k-15}{36} & \text{h } \frac{j}{2} & \text{i } \frac{11-12w}{30} \\ \text{j } \frac{-6a-11}{36} & \text{k } \frac{-m-21}{30} & \text{l } \frac{9b-11}{12} \end{array}$$

$$\begin{array}{ll} 4 \text{ a } \frac{4n^2-8n-5}{16} & \text{b } \frac{x^2-4}{12} \\ \text{c } \frac{7p^2+21p+14}{15} & \text{d } \frac{7t^2+9t+2}{4} \\ \text{e } \frac{15q^2+34q+15}{15} & \text{f } \frac{2s^2-s-6}{8} \\ \text{g } \frac{28y^2-21y-49}{12} & \text{h } \frac{7b^2+14b+7}{9} \end{array}$$

$$\begin{array}{ll} 5 \text{ a } \frac{52x-19}{15} & \text{b } \frac{60x-10}{21} \\ \text{c } \frac{14x+4+\pi x+5\pi}{12} & \text{or } \frac{x(\pi+14)+5\pi+4}{12} \end{array}$$

$$6 \frac{14x^2-x-4}{30}$$

$$\begin{aligned} 7 \text{ For } 5\text{b } \frac{3x+1}{3} - \frac{13x+2}{21} &= \frac{8x+5}{21}, \\ \frac{8x+5}{21} \times \frac{8x+5}{21} &= \frac{64x^2+80x+25}{441}, \\ \frac{13x+2}{21} \times \frac{3x-4}{7} &= \frac{39x^2-46x-8}{147}, \end{aligned}$$

$$\begin{aligned} \text{so Area} &= \frac{64x^2+80x+25}{441} + \frac{39x^2-46x-8}{147} \\ &= \frac{181x^2-58x+1}{441} \end{aligned}$$

For 5c, $r = \frac{x+5}{12}$, so the semicircle area

$$= \pi \times \frac{x+5}{12} \times \frac{x+5}{12} = \frac{\pi(x^2+10x+25)}{144} \text{ and the}$$

$$\text{area of the rectangle} = \frac{2x-1}{4} \times \frac{x+5}{6}$$

$$= \frac{2x^2+9x-5}{24}, \text{ so the whole area is}$$

$$\frac{\pi(x^2+10x+25) + 12x^2+54x-30}{144} \text{ or}$$

$$\frac{x^2(\pi+12) + x(10\pi+54) + 25\pi-30}{144}.$$

Chapter review

$$1 \text{ a } 5n^2, 4nn \quad \text{b } 7nm, -3mn \text{ and } 6n^2m, -4nm^2$$

$$2 \text{ a } 8z \quad \text{b } -3t \quad \text{c } -5b^2 \quad \text{d } 2gk$$

$$\begin{array}{lll} 3 \text{ a } 20xp & \text{b } 63mn^2p & \text{c } 6t \\ \text{d } \frac{5p}{4t} & \text{e } 7m^2 & \text{f } 8ay^2+4ay \end{array}$$

$$\begin{array}{ll} 4 \text{ a } 5g+20 & \text{b } 42-24r \\ \text{c } h^2+4h & \text{d } 8m-m^2 \end{array}$$

$$\begin{array}{ll} 5 \text{ a } 5(v+3) & \text{b } 6(c-2) \\ \text{c } 3(w+15) & \text{d } 9(d-4) \end{array}$$

$$6 \text{ a } 7e+5 \quad \text{b } 12p+43$$

$$\begin{array}{ll} 7 \text{ a } 6y^2+37y+56 & \text{b } 8w^2+30w-27 \\ \text{c } 3w^2-22w-16 & \text{d } 6n^2-23n+7 \end{array}$$

$$\begin{array}{lll} 8 \text{ a } 4(6t+5r) & \text{b } 2(2v+7z) & \text{c } -5(5c-2p) \\ \text{d } -2(9b+2m) \end{array}$$

$$\begin{array}{ll} 9 \text{ a } -5t(2e^2+3t) & \text{b } -6ga^3(2+3g^2) \\ \text{c } 6k^3(9b+2) & \text{d } -2p^2(w^2-2p) \end{array}$$

$$\begin{array}{ll} 10 \text{ a } 3h^2y+5hy+3hy^2 & \text{b } 8m^3-6m^2n^2 \\ \text{c } 24-4uv+9v \end{array}$$

$$11 \text{ a } -35pq \quad \text{b } 48ek^2 \quad \text{c } 6y \quad \text{d } \frac{-4d}{5}$$

Answers

- 12 a $2g^2 + 14g$ b $3t^2 - 21t$
 c $8em - 20m^2$ d $5r - 15p - 20q$
- 13 a $2(6w + 5)$ b $5(3k + 2)$ c $6(3v - 7)$
 d $4(5y + 4)$
- 14 a $12t^3z^4 - 15tz^4$ b $-11x^2b^2 + 33x^4b^3$
 c $4c^3by^2 + 4b^3y^2 + 32b^3y^3$
 d $-99r^2q + 99hrq^2 - 110h^2r^2q^2$

- 15 a $-3r + 51$ b $5q + 9$
 c $20mc^3 - 13mc^4n + 14mc^3n^2 - 6m^2c^2n^2$

- 16 a $5v^2 - 36v + 7$ b $27p^2 + 57p + 28$

- 17 a $8(5t + 6d)$ b $-28(2k - 3q)$
 c $8(11a + 13b)$ d $-5(2y + 13m)$

- 18 a $63a^{11}n^7$ b $2x^3u$ c $\frac{9m^2}{b^2}$
 d $\frac{3}{4p^9}$ e $\frac{12t^2}{m}$

- 19 a $21r^2 - 31rq + 4q^2$ b $7m^2 - 51mc - 40c^2$

- 20 a $48g^2 - 6gd - 27d^2$ b $-6r^2 + 48rm - 46m^2$

- 21 a $-3n^3p^2(5p - n^3)$ b $11n^4m(7n^2 - 3m^4)$
 c $14ny^4(ny + 4)$ d $-9a^4n(7n^2 + 3a^2)$

- 22 a $8w^5(5m^2 + 12m^4w^3 - 7)$
 b $11r^4(9r + 3v^3h^2 - rv^5)$
 c $-10g^2t^2h(5g^3h^3 + g^2t^2 - t^2h)$
 d $4c^4a^2(7c^2a^4 - 5g^5a^3 + g^5c)$

- 23 a $\frac{48m - 46}{45}$ b $\frac{21 - 5q}{18}$
 c $\frac{15y^2 - 7y - 2}{24}$

- 24 \$308

- 25 a $2(6x + 4)$ or $12x + 8$ b $12d + 4$

- 26 $7m^3n$

- 27 a $32m + 24cm^2$ b $\frac{1}{2}(4v^2 - 17v - 15)mm^2$
 c $\frac{3}{4}(5t^2 + 16tv + 3v^2)$

- 28 $\frac{92x + 15}{12}$

- 29 Proof

$$30 \quad (a - 7)(a^2 + 7a + 49) = (a - 7)(a^2 + 7a + 49) \\
(a - 7)(a^2 + 7a + 49) \\
= a^3 - 343$$

In general, it follows that $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

- 31 The height of the triangle at the top is

$$\frac{5x - 2}{2} - \frac{2x + 1}{3} = \frac{11x - 8}{6}, \text{ so its area is given by}$$

$$\frac{11x - 8}{6} \times \frac{10x + 3}{4} = \frac{110x^2 - 47x - 24}{24}. \text{ The area of}$$

$$\text{the rectangle is } \frac{2x + 1}{3} \times \frac{10x + 3}{4} = \frac{20x^2 + 16x + 3}{12},$$

so the whole area is

$$\frac{110x^2 - 47x - 24}{24} + \frac{20x^2 + 16x + 3}{12} = \frac{50x^2 - 5x - 6}{8}.$$

Chapter 5

Exercise 5.1

- 1 a \$1.30/kg b \$3.75/m
 c 12.5 L/100 km d About 18.1 m/s
 e 2.5°C/min f \$320/kg
 g \$37/head h 32 runs/wicket
 i 40 g/m² j 4.8 runs/over

- 2 a 72 km/h b $4\bar{4}$ m/s c 72 kL/h
 d 25.92 km/h e $14\bar{4}$ L/s f $1388\bar{4}$ L/min
 g 2.8 cents/g h \$150/kg i $26\bar{6}$ m/s

- 3 a 140 kL b About 21 h 26 min
 c About 1.7 minutes

- 4 a \$2.49/kg b \$5.23

- 5 a 4.17 m/s b 15 km/h

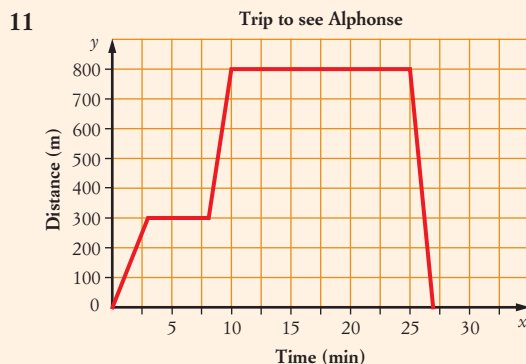
- 6 a \$941 b \$707 c \$446.50

- 7 a 43.2 L b \$15.84

- 8 a 200 g for \$11.60 b 1.6 kg for \$11.60
 c 240 g for \$1.12 d 2.5 kg for \$17.50

- 9 a 100 m/min b 1 minute
 c 600 m d 300 m/min
 e She ran, it was fast. Her return journey speed was far greater than her arrival speed.

- 10 a 20 km b 1 hour
 c 48 km/h d 80 km/h
 e He stopped. f 40 km/h (including the stop)



Exercise 5.2

- 1 a 1 : 3 b 1 : 2 c 3 : 2
d 5 : 3 e 2 : 3
- 2 a 5 : 2 b 1 : 2 c 40 : 3 d 5 : 3
e 2 : 3 f 22 : 15 g 21 : 25 h 7 : 2
i 5 : 3 j 3 : 4 k 4 : 5 l 3 : 1
m 6 : 25 n 25 : 8 o 2 : 7 p 5 : 1
q 14 : 9 r 3 : 7 s 3 : 1 t 7 : 2

- 3 a \$50 : \$30 b 10L : 5L
c $100 \text{ cm}^3 : 400 \text{ cm}^3$ d \$70.40 : \$25.60
e 10 kg : 45 kg f 48 m : 80 m
g \$50 : \$40 h \$55 : \$65
i \$28 : \$32

4 10 : 5 : 4

- 5 a 3 : 5 : 6 b 8 : 12 : 5 c 5 : 30 : 14
d 13 : 6 : 9 e 34 : 47 f 38 : 44 : 23
g 74 : 31 : 12 h 28 : 5 : 10 i 9 : 13 : 5

- 6 a 7 : 16 b 43 : 89
c 18 412 : 379 d 56 : 80

- 7 a 2 : 5 b 2 : 5 c 4 : 25

	Number of teeth		Ratio of teeth	Number of full turns	
	Cog A	Cog B		Cog A	Cog B
a	20	40	1 : 2	2	1
b	10	30	1 : 3	6	2
c	9	36	1 : 4	20	5
d	12	60	1 : 5	35	7
e	32	8	4 : 1	10	40
f	21	7	3 : 1	11	33
g	14	56	1 : 4	8	2

9 800 g (0.8 kg)

10 63 and 18

	Number of teeth			Ratio of teeth	Number of full turns		
	Cog A	Cog B	Cog C		Cog A	Cog B	Cog C
a	30	20	10	3 : 2 : 1	2	3	6
b	5	10	20	1 : 2 : 4	12	6	3
c	9	24	12	3 : 8 : 4	24	9	18
d	16	40	24	2 : 5 : 3	15	6	10
e	8	24	16	1 : 3 : 2	6	2	3
f	90	27	30	30 : 9 : 10	3	10	9
g	35	10	25	7 : 2 : 5	10	35	14

12 400 kg of cement, 1000 kg of sand, 1600 kg of gravel

13 50 g of mixed peel, 200 g of dates

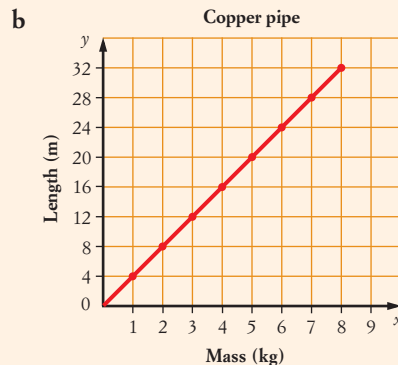
- 14 a 1 : 75.09, 1 : 40.89, 1 : 84 respectively
b 1 : 0.68, 1 : 1.08, 1 : 0.63 respectively
c Ireland

Exercise 5.3

- 1 a Yes b Yes c No d Yes
e No f Yes

- 2 a Yes b Yes c No d Yes
e No f Yes

3 a	M (kg)	1	2	3	4	5	6	7	8
	L (m)	4	8	12	16	20	24	28	32



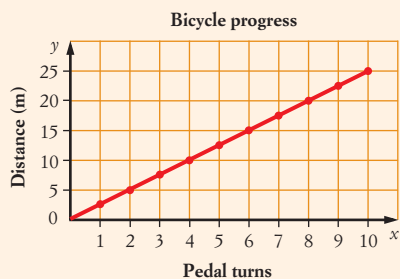
c Yes, they give a linear graph that passes through the origin.

d 5.5 kg e 3.75 kg f 18 m g 25 m

4 a	Number of pedal turns, n	1	2	3	4	5
	Distance, d (m)	2.5	5	7.5	10	12.5

	Number of pedal turns, n	6	7	8	9	10
	Distance, d (m)	15	17.5	20	22.5	25

b



c Yes, they give a linear graph that passes through the origin,

d 17.5 m

e About 23.8 m

f 8 turns

g Almost 6 turns

5 a $b = 0.9$ b $m = 1.75$ c $x = 2, y = 24$

d $m = 3$ e $x = 9$ f $x = 6, y = 12$

6 a $k = 3$

b $k = 2.5$

c $k = 6$

d $k = 4.8$

e $k = 3$

f $k = -1.6$

7 a 168 children

b 2 km

c 375 g

d 156 reds

e 144 females

8 a 30 mL

b 12.5 mL

c 23.5 mL

9 204 L

10 a 165 wings

b 35 wings

c 390 wings

11 a \$1458.33

b \$10 000

12 About 17 cm (16.7 cm)

13 About 26 s (26.25 s)

14 a 30 m

b 60 m

c 15 m

15 6.75 s

16 2.58 m

17 Another 3 h 4 min to finish

18 550 people

19 2029.4 km

20 216 m^2

21 64 km/h

22 142.8 m^3

23 a $m = kv$

b 270 g

24 a 38.4 m

b 72 m

25 a 4 min 5 s

b About 85.7 L

26 a 686 kPa

b 306.1 m

27 62.5 miles

28 0.92 m by 1.38 m

Exercise 5.4

1 a Yes

b No

c Yes

d Yes

e No

f No

2 a Yes, $k = 40$ b Yes, $k = 12$ c No

d Yes, $k = 60$ e Yes, $k = 6$ f Yes, $k = \frac{1}{2}$

3 a $d \propto t, d = kt$

c $A \propto h, A = kh$

e $t \propto \frac{1}{s}, t = \frac{k}{s}$

g $I \propto \frac{1}{d^2}, I = \frac{k}{d^2}$

b $C \propto r, C = kr$

d $P \propto s, P = ks$

f $V \propto \frac{1}{P}, V = \frac{k}{P}$

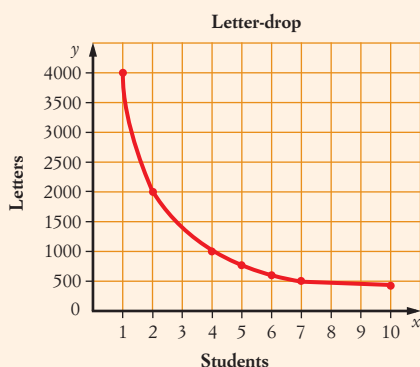
h $F \propto \frac{1}{t^2}, F = \frac{k}{t^2}$

4 a

Number of students	1	2	4	5
Number of letters	4000	2000	1000	800

Number of students	6	7	10
Number of letters	666.7	571.4	400

b



c Inverse proportion

d About 1333 letters

e 8 students

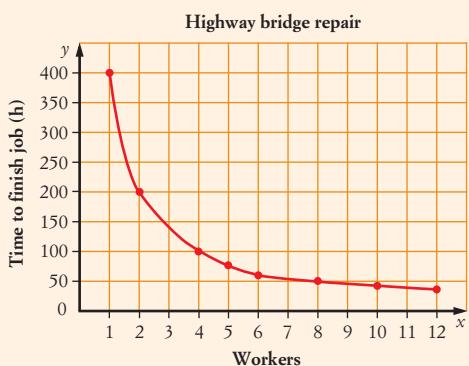
f 4 students

5 a

Number of workers	1	2	4	5
Time to finish job (h)	400	200	100	80

Number of workers	6	8	10	12
Time to finish job (h)	66.7	50	40	33.3

b



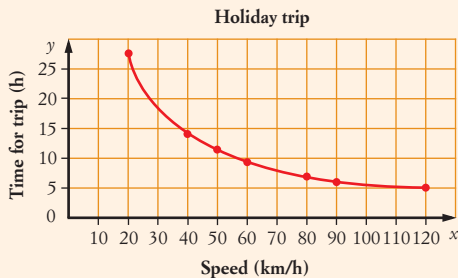
- c Inverse proportion d 50 h
e About 57 h f 9 workers
g No, because 17 workers would be needed and only 12 are available
h 50 h

6 a

Average speed (km/h)	120	90	80	60
Time for trip (h)	4.5	6	6.75	9

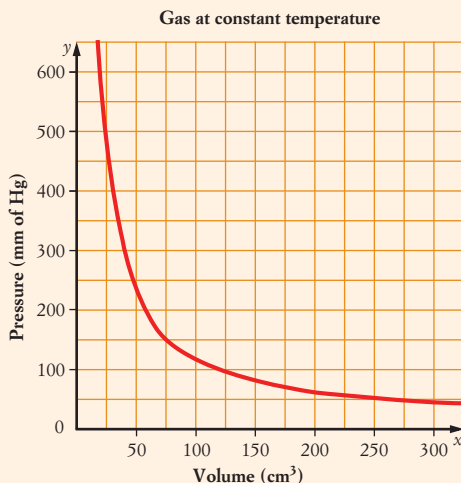
Average speed (km/h)	50	40	20
Time for trip (h)	10.8	13.5	27

b



- c Inverse proportion d 18 h e 5.4 h
f 54 km/h g 90 km/h

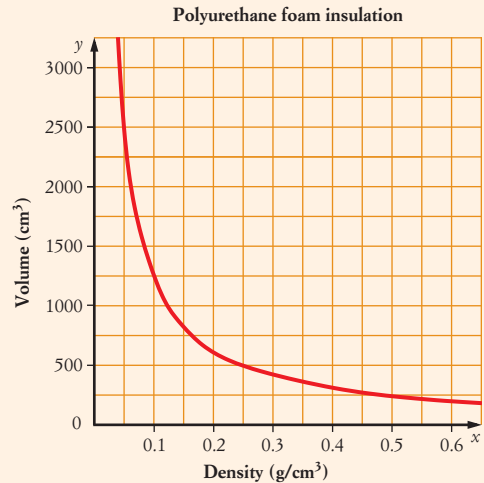
7 a



The graph is a hyperbola.

- b i 300 mm of Hg ii 90 mm of Hg
iii 65 mm of Hg iv 100 mm of Hg
c i 45 cm³ ii 55 cm³
iii 190 cm³ iv 90 cm³

8 a



The graph is a hyperbola.

- b i 300 cm³ ii 480 cm³
iii 1500 cm³ iv 340 cm³
c i 0.4 g/cm³ ii 0.24 g/cm³
iii 0.12 g/cm³ iv 0.06 g/cm³

9 3 h 12 min

- 10 a i 132.48 cycles/s ii 552 cycles/s
b 0.552 m

11 63 km/h 12 6 hours 13 14 400 mAh

14 6 h 51 min 15 360 000 km

16 7 h 12 min 17 2816 J

18 The black pipe is 3.125 mm longer.

Chapter review

1 32 runs/wicket

- 2 a 7 : 10 b 6 : 5
c 24 : 7 d 9 : 16

3 \$35 : \$21 4 Yes 5 $4.1\bar{6}$ m/s 6 15 : 24

7 a Yes b No

8 a $m = 2$ b $x = 12, y = 12$

9 $k = 4.8$

10 a Yes b No c No

11 a No b Yes, $k = 36$

12 a $E \propto m, E = km$ b $t \propto \frac{1}{T}, t = \frac{k}{T}$

c $E \propto d^2, E = kd^2$

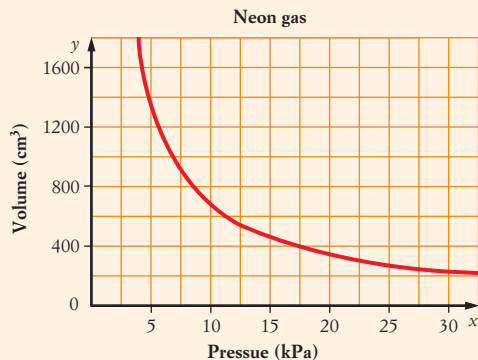
Answers

13 \$120 14 \$270

15 750 g for \$9.20 (just)

16 a 36 m b 333.3 g

17 a



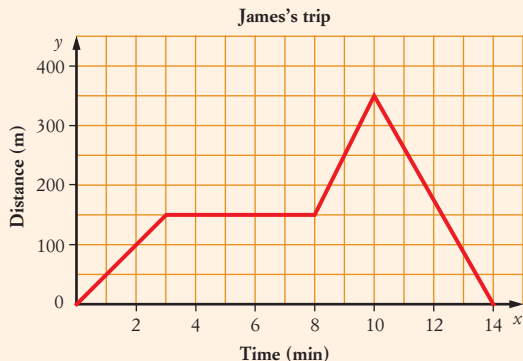
It is a hyperbola.

b 600 cm³ c 8 kPa

18 About 267 vibrations/s

19 50 clerks 20 20.25 m

21 a



b 100 m/min

Chapter 6

Exercise 6.1

- 1 a i $\angle A \leftrightarrow \angle Y$, $\angle B \leftrightarrow \angle X$, $\angle C \leftrightarrow \angle Z$
 ii $AB \leftrightarrow YX$, $BC \leftrightarrow XZ$, $CA \leftrightarrow ZY$
 iii $\triangle ABC \equiv \triangle YXZ$
 b i $\angle D \leftrightarrow \angle N$, $\angle G \leftrightarrow \angle O$, $\angle H \leftrightarrow \angle P$,
 $\angle K \leftrightarrow \angle M$
 ii $DG \leftrightarrow NO$, $GH \leftrightarrow OP$, $HK \leftrightarrow PM$,
 $KD \leftrightarrow MN$
 iii $\triangle DGHK \equiv \triangle NOPM$

c i $\angle W \leftrightarrow \angle V$, $\angle Y \leftrightarrow \angle P$, $\angle X \leftrightarrow \angle Q$,
 $\angle Z \leftrightarrow \angle U$

ii $WY \leftrightarrow VP$, $YX \leftrightarrow PQ$, $XZ \leftrightarrow QU$, $ZW \leftrightarrow UV$

iii $\triangle ABC \equiv \triangle YXZ$

d i $\angle T \leftrightarrow \angle J$, $\angle S \leftrightarrow \angle L$, $\angle R \leftrightarrow \angle K$

ii $TS \leftrightarrow JL$, $SR \leftrightarrow LK$, $RT \leftrightarrow KJ$

iii $\triangle TSR \equiv \triangle JLK$

2 $HERSMILB \equiv SHEONLUT$, $OEKAM \equiv WETNU$,
 $FIRL \equiv NOEC$, $NAO \equiv TIM$, $NIFSWOT \equiv AGNIDRB$,
 $LIVED \equiv OLANI$, $OAT \equiv NGB$

3 $AEDNH \equiv OMEPS$, $ULPRF \equiv BAITU$,
 $IDEF \equiv EHRQ$, $OBCEQTH \equiv LIJKEFT$,
 $WOULDS \equiv SAILOR$, $EUP \equiv ONC$,
 $ABCD \equiv FOGY$

4 a The shapes have the same shape and size.

b i Each is about 104 mm.

ii Each is 30 square units.

c The perimeters and areas are equal.

5 a $W(1, 2) \rightarrow W'(3, 1)$, $E(3, 3) \rightarrow E'(5, 2)$,
 $D(4, 3) \rightarrow D'(6, 2)$, $G(4, 2) \rightarrow G'(6, 1)$

b $W(1, 2) \rightarrow W'(0, 4)$, $E(3, 3) \rightarrow E'(2, 5)$,
 $D(4, 3) \rightarrow D'(3, 5)$, $G(4, 2) \rightarrow G'(3, 4)$

c $W(1, 2) \rightarrow W'(4, 5)$, $E(3, 3) \rightarrow E'(5, 3)$,
 $D(4, 3) \rightarrow D'(5, 2)$, $G(4, 2) \rightarrow G'(4, 2)$

d $W(1, 2) \rightarrow W'(2, 1)$, $E(3, 3) \rightarrow E'(1, 3)$,
 $D(4, 3) \rightarrow D'(1, 4)$, $G(4, 2) \rightarrow G'(2, 4)$

e $W(1, 2) \rightarrow W'(1, 2)$, $E(3, 3) \rightarrow E'(3, 1)$,
 $D(4, 3) \rightarrow D'(4, 1)$, $G(4, 2) \rightarrow G'(4, 2)$

f $W(1, 2) \rightarrow W'(5, 2)$, $E(3, 3) \rightarrow E'(3, 3)$,
 $D(4, 3) \rightarrow D'(2, 3)$, $G(4, 2) \rightarrow G'(2, 2)$

g $W(1, 2) \rightarrow W'(0, 2)$, $E(3, 3) \rightarrow E'(4, 4)$,
 $D(4, 3) \rightarrow D'(6, 4)$, $G(4, 2) \rightarrow G'(6, 2)$

h $W(1, 2) \rightarrow W'(-5, -1)$, $E(3, 3) \rightarrow E'(3, 3)$,
 $D(4, 3) \rightarrow D'(7, 3)$, $G(4, 2) \rightarrow G'(7, -1)$

i $W(1, 2) \rightarrow W'(1, 2)$, $E(3, 3) \rightarrow E'(2, 2.5)$,
 $D(4, 3) \rightarrow D'(2.5, 2.5)$, $G(4, 2) \rightarrow G'(2.5, 2)$

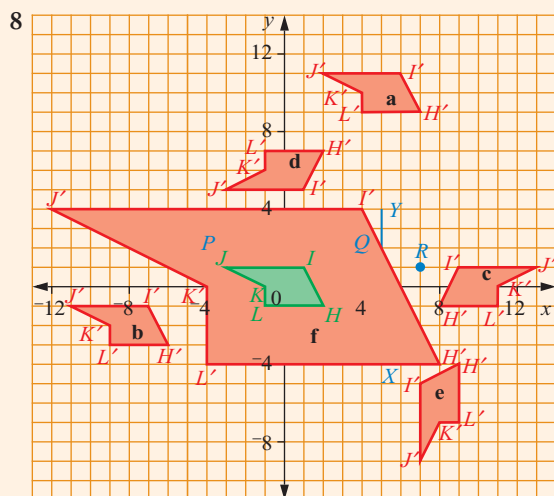
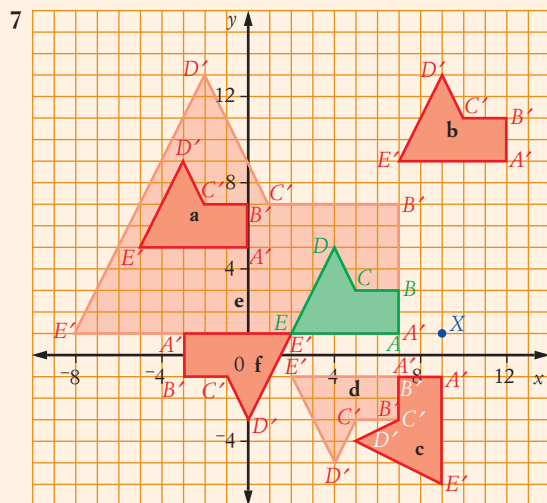
6 a Reflection in the y-axis

b Reflection in the x-axis

c Translation, 13 right, 14 up

d Rotation of 90° clockwise with centre of rotation A

e Enlargement, magnification of 2 with centre (0, 0)



9 a 2 b $\frac{2}{3}$

- 10 a i 2 ii 12 cm, 24 cm iii 6 cm^2 , 24 cm^2
 b i 2.5 ii 4.8 m, 12 m iii 1.44 m^2 , 9 m^2
 c i 1.5 ii about 119.4 mm, about 179.1 mm
 iii about 1134.1 mm^2 , 2551.8 mm^2
 d i 1.8
 ii 160 cm, 288 cm
 iii 1375 cm^2 , 4455 cm^2

- 11 a Translation b Reflection or rotation
 c Rotation d Translation

12 Answers other than those stated here are also possible.

- a Translation 2 right, 1 down.
 b Translation 1 left, 2 up.
 c Rotation 90° clockwise around G.
 d Rotation 90° anticlockwise around (2, 2).
 e Reflection in the line through W and G.
 f Reflection in the vertical line through E ($x = 3$).
 g Dilation with magnification of 2 and centre (2, 2).
 h Dilation with magnification of 4 and centre E.
 i Dilation with magnification $\frac{1}{2}$ and centre W.

13 Yes

14 a Perimeter of image = scale factor $\times$ perimeter of object

b Area of image = (scale factor) $^2 \times$ area of object

Other answers are possible for question 15.

- 15 a Rotation of 270° clockwise around C and then a translation of 4 units down.
 b Reflection in the line parallel to and 1 unit below PQ then a translation of 5 units to the right.
 c Reflection in CD then a translation of 4 units up and 2 units to the right.
 d Reflection in the horizontal line through Y then a rotation of 270° clockwise around X.

Exercise 6.2

- 1 a $\triangle ABC \equiv \triangle RQP$ b $\triangle XZY \equiv \triangle HGK$
 c $\triangle GDF \equiv \triangle PNM$ d $\triangle IJK \equiv \triangle WVU$
 e $\triangle RST \equiv \triangle NKB$ f $\triangle FPW \equiv \triangle RQX$

- 2 a RHS b SSS c SAS
 d SSS e SAS f ASA

- 3 a Not congruent b Congruent, SSS
 c Not congruent d Not congruent
 e Not congruent f Congruent, ASA
 g Congruent, SAS h Congruent, SSS
 i Not congruent j Congruent, RHS

- 4 a $B \equiv C$, SSS b $A \equiv B$, ASA c $A \equiv B$, RHS
 d $A \equiv C$, SAS e $B \equiv C$, ASA

- 5 a $A \equiv B$, ASA b $A \equiv C$, SAS c $A \equiv B$, RHS
 d $A \equiv C$, ASA e $A \equiv B$, SSS f $B \equiv C$, SAS

- 6 $A \equiv F$, SSS; $B \equiv P$, SAS; $C \equiv R$, RHS;
 $E \equiv H$, ASA; $I \equiv O$, ASA; $J \equiv N$, RHS;
 $K \equiv L$, SSS; $M \equiv S$, SAS

7 Only **a** and **d** are sufficient.

- 8 **a** $z = 68^\circ$ (SSS) **b** $p = 22$ mm (ASA)
c $m = 56^\circ, k = 26$ mm **d** $y = 52^\circ$ (SSS)
e $h = 46^\circ, g = 17$ mm (SAS)
f $c = 68^\circ, d = 8$ cm (SAS)

9 Proofs

Exercise 6.3

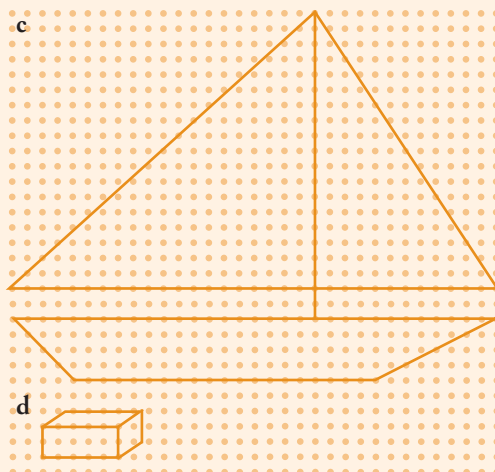
- 1 **a** $\triangle TUY \parallel \triangle KLM$ **b** $\triangle BSG \parallel \triangle PTM$
c $\triangle WVM \parallel \triangle JED$ **d** $\triangle DUH \parallel \triangle RNF$
e $\triangle QSX \parallel \triangle AGB$ **f** $\triangle YWR \parallel \triangle NJZ$
- 2 **a** SSA **b** SSS **c** AAA
d SSA **e** SSS **f** AAA
- 3 **a** SSA **b** SSS **c** AAA
d AAA **e** SSS **f** SSA
- 4 **a** $y = 63^\circ, x = 12$ cm **b** $a = 24$ cm, $b = 20$ cm
c $m = 64^\circ, n = 44^\circ$ **d** $s = 56$ cm, $r = 30$ cm,
 $w = 62^\circ$
e $h = 16$ cm, $g = 90^\circ$ **f** $u = 32^\circ, k = 36$ cm
- 5 C 6 B 7 $x = 72$ cm, $y = 96$ cm
8 84 cm 9 50.4 mm 10 30 cm

- 11 **a** **i** $\triangle PQR \parallel \triangle TQS$ (AAA)
ii $k = 15$ cm, $h = 9$ cm
a **i** $\triangle WXY \parallel \triangle ZXU$ (AAA)
ii $a = 22$ cm, $b = 18$ cm
a **i** $\triangle HIL \parallel \triangle JKL$ (AAA)
ii $m = 12$ cm, $n = 6$ cm

12 Proofs

Exercise 6.4

- 1 **a** 1 : 100 **b** 1 : 10
c 1 : 1 000 000 **d** 10 : 1
e 1 : 10 000 **f** 100 : 1
g 1 : 10 **h** 10 000 : 1



- 3 **a** $x = 11$ **b** $k = 6$ **c** $p = 4.5$
d $m = 3$ **e** $h = 1.2$ **f** $d = 7.2$

4 **b** and **c**

- 5 **a** 56 cm **b** 1.33 m **c** 1.61 m **d** 32 cm

- 6 **a** 2.0 mm **b** 2.7 mm **c** 0.9 mm **d** 1.4 mm

- 7 **a** 0.02 mm **b** About 0.06 mm **c** 0.17 mm

- 8 **a** $r = 12, q = 14$ **b** $p = 18, q = 15$
c $a = 10, c = 8$ **d** $a = 15, b = 12$

- 9 **a** Similar **b** Similar **c** Not similar

- 10 **a** $a = 18, b = 21$ **b** $h = 2, k = 5.5$
c $x = 4, y = 9$

11 1 : 20

12 Answers to the nearest km.

- a** 38 km **b** 50 km
c 72 km (Answers will vary depending on the route taken.)
d 130 km² **e** 280 km²

- 13 a $x = 30, y = 33$
 b $s = 9, t = 12$
 c $x = 37.5, y = 62.5$

- 14 i Proofs
 ii a $x = 25$ b $y = 10$
 15 2.8 m 16 52.5 m

Chapter review

- 1 a Similar b Congruent c Neither
 d Similar e Neither f Congruent
 g Similar h Congruent

- 2 a True b False c True d True

- 3 a $\triangle ABC \equiv \triangle GJH$ b $\triangle QWR \equiv \triangle LMP$
 c $\triangle DEF \equiv \triangle SRT$ d $\triangle XYZ \equiv \triangle PKO$

- 4 a SSS b RHS c ASA d SAS

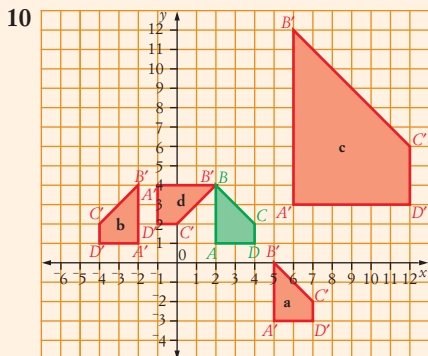
- 5 a $\triangle ABC \parallel \triangle YRT$ b $\triangle XYZ \parallel \triangle QRT$
 c $\triangle LMN \parallel \triangle FED$ d $\triangle UVW \parallel \triangle NML$

- 6 a SSS b SSA c AAA d SSA

- 7 a 1 : 10 000 b 1 : 100
 c 100 : 1 d 1 : 1 000 000

- 8 a $s = 3$ b $x = 14$ c $m = 7.5$

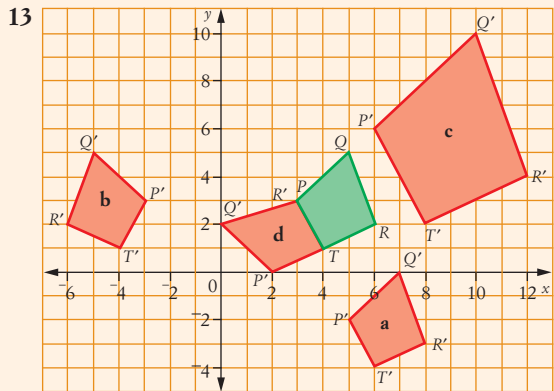
- 9 a Reflection in the x -axis
 b Translation 7 left, 3 down



- 11 a i Translation 2 down and 4 left
 ii $P(3, 3) \rightarrow P'(-1, 1), Q(5, 5) \rightarrow Q'(1, 3),$
 $R(6, 2) \rightarrow R'(2, 0), T(4, 1) \rightarrow T'(0, -1)$

- b i Reflection in x -axis
 ii $P(3, 3) \rightarrow P'(3, -3), Q(5, 5) \rightarrow Q'(5, -5),$
 $R(6, 2) \rightarrow R'(6, -2), T(4, 1) \rightarrow T'(4, -1)$

- 12 a i 3
 ii about 25.1 cm, about 75.4 cm
 iii about 50.3 cm², about 452.4 cm²
 b i $\frac{5}{9}$
 ii 180 mm, 100 mm
 iii 1458 mm², 450 mm²



- 14 $B \equiv E$, ASA; $C \equiv F$, SAS
 15 $A \equiv E$, RHS and $B \equiv D$, SAS; but C is not necessarily congruent to any others.
 16 a $\triangle GAJ \parallel \triangle QPM$ (SSA)
 b $\triangle XVW \parallel \triangle PLK$ (SSS)
 c $\triangle DHP \parallel \triangle RTF$ (AAA)
 d $\triangle OTH \parallel \triangle APS$ (SSS)

- 17 A 18 D

- 19 a $x = 45^\circ, y = 19.2$

- 20 a $x = 16, y = 14$ b $m = 28, n = 33$

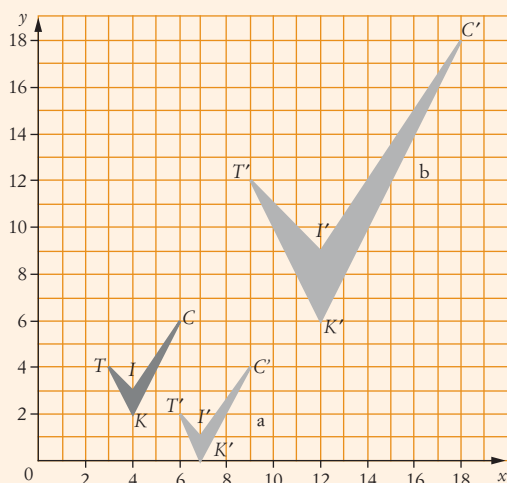
- 21 a $x = 5.7, y = 7$

- 22 a 20 000
 b $AB = 1300$ m, $CD = 1000$ m

- 23 a $y = 21, z = 36$ b $a = 54, z = 12$

- 24 a Translation 3 units right and 2 units down

b Magnification of 3 with centre (0, 0)



c $T'(7, -1)$, $I'(8, -2)$, $C'(10, 1)$, $K'(8, -3)$
 d $T'(15, 20)$, $I'(20, 15)$, $C'(30, 30)$, $K'(20, 10)$

- 25 a $A(-1, 7)$, $B(0, 8)$, $C(2, 3)$
 b $A(3, -5)$, $B(4, -6)$, $C(6, -1)$
 c $A(9, 15)$, $B(12, 18)$, $C(18, 3)$
 d $A(5, -3)$, $B(6, -4)$, $C(1, -6)$

- 26 a Rotation of 90° clockwise around X followed by a translation of 8 units to the left.
 b Reflection in the horizontal line 1 unit below FE followed by a translation of 4 units to the right and 3 units up.

27 Proof 28 Proof

29 a $y = 14$ b $k = 16$

30 Proof 31 $z = 4.5$ 32 3.2 m

Chapter 7

Exercise 7.1

1 a

Length of nail (mm)	Frequency
7	1
8	5
9	4
10	3
11	4
12	2
13	4
14	2
15	1
16	3
Total	29

b 11 mm c 8 mm d 13 mm

e

Length of nail (mm)	Relative frequency
7	0.03
8	0.17
9	0.14
10	0.10
11	0.14
12	0.07
13	0.14
14	0.07
15	0.03
16	0.10

f 69 students g 172 students

- 2 a On their way somewhere else
 b Buying food
 c Clothes: 0.12, passing through: 0.14, food: 0.14, CDs/DVDs: 0.08
 d Clothes: 84 shoppers, passing through: 100 shoppers, food: 101 shoppers, CDs/DVDs: 55 shoppers
 e Food f CDs/DVDs
- 3 a 0.04 b 511 chocolates
- 4 a 0.03 b 0.04
 c 163 biscuits d 179 biscuits
- 5 a Yellow b Primary colour
 c Red $\frac{4}{21}$, yellow $\frac{5}{21}$, blue $\frac{2}{21}$, black $\frac{1}{7}$, white $\frac{1}{3}$
 d 48 times

- 6 a i $\frac{1}{4}$ (0.25) ii $\frac{5}{12}$ (0.42) iii $\frac{1}{3}$ (0.33)
 b i 20 greens ii 33 reds iii 27 yellows

- 7 a i $\frac{1}{4}$ or 0.25 ii $\frac{5}{12}$ or 0.42 iii $\frac{1}{3}$ or 0.33
 b i $\frac{1}{4} \times 50 \approx 13$ ii $\frac{5}{12} \times 50 \approx 21$ iii $\frac{1}{3} \times 50 \approx 17$

8 15%

9 A match with a win/loss result, 98%.

10 a

Minutes out	Frequency	Relative frequency
-3	3	0.11
-2	4	0.14
-1	4	0.14
0	9	0.32
+1	3	0.11
+2	3	0.11
+3	0	0
+4	1	0.04
+5	1	0.04

b 311 watches
d 381 watches

c 277 watches
e 796 watches

11 a Fine

b $\frac{7}{30}$

12 a $\frac{3}{13}$ (0.23) **b** $\frac{1}{13}$ (0.077) **c** $\frac{3}{13}$ (0.23)

d $\frac{4}{13}$ (0.31) **e** $\frac{2}{13}$ (0.15) **f** 0

g 1 **h** $\frac{6}{13}$ (0.46) **i** $\frac{3}{13}$ (0.23)

j $\frac{1}{13}$ (0.077)

13 a $\frac{1}{3}$ (0.33) **b** $\frac{1}{6}$ (0.17) **c** $\frac{1}{2}$ (0.5)

14 73 students **15** 826 shoppers

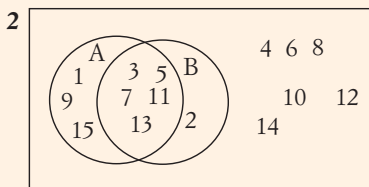
16 200 students

Exercise 7.2

1

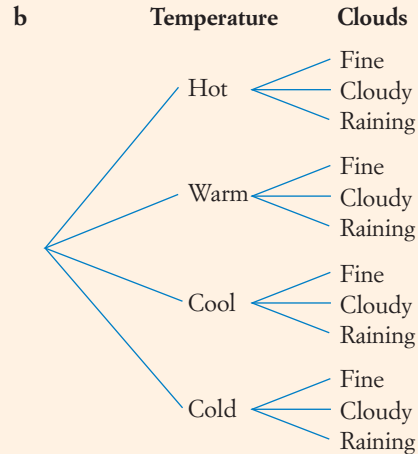
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a $\frac{5}{12}$ **b** $\frac{5}{9}$ **c** $\frac{7}{18}$ **d** $\frac{7}{12}$ **e** $\frac{4}{9}$



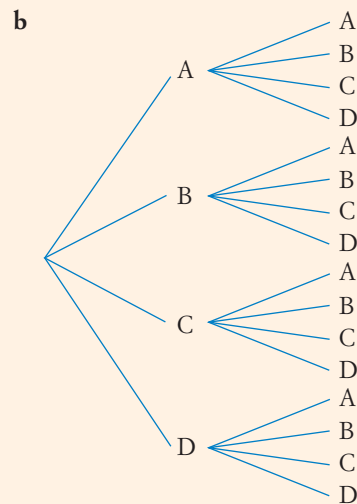
3 a

	Hot	Warm	Cool	Cold
Fine	Hot and fine	Warm and fine	Cool and fine	Cold and fine
Cloudy	Hot and cloudy	Warm and cloudy	Cool and cloudy	Cold and cloudy
Rain	Hot and raining	Warm and raining	Cool and raining	Cold and raining

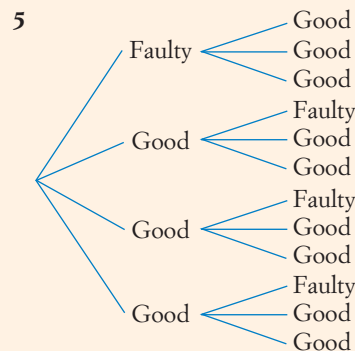


4 a

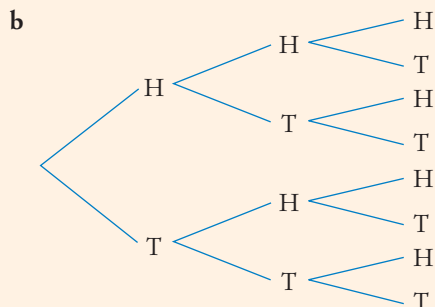
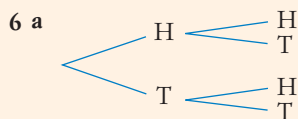
	A	B	C	D
A	AA	AB	AC	AD
B	BA	BB	BC	BD
C	CA	CB	CC	CD
D	DA	DB	DC	DD



c 16 elements



Answers

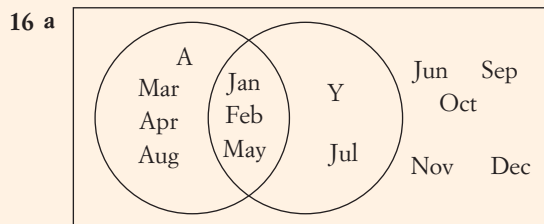


7 a $\frac{11}{36} \approx 0.306 = 30.6\%$ b $\frac{1}{9} \approx 0.111 = 11.1\%$
 c $\frac{1}{3} \approx 0.333 = 33.3\%$ d $\frac{7}{12} \approx 0.583 = 58.3\%$
 e $\frac{7}{18} \approx 0.389 = 38.9\%$

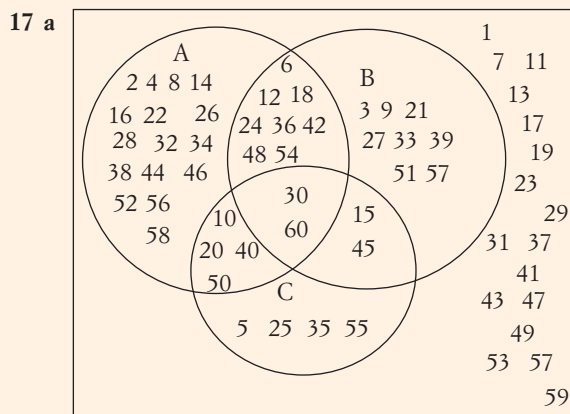
8 a

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

10 a $\frac{1}{9} \approx 0.111 = 11.1\%$ b $\frac{4}{9} \approx 0.444 = 44.4\%$
 11 a $\frac{1}{25} = 0.04 = 4\%$ b $\frac{16}{25} = 0.64 = 64\%$
 12 a $\frac{1}{21} \approx 0.0476 = 4.76\%$ b $\frac{1}{7} \approx 0.143 = 14.3\%$
 c $\frac{2}{7} \approx 0.286 = 28.6\%$ d $\frac{4}{21} \approx 0.190 = 19\%$
 13 a $\frac{1}{4} = 0.25 = 25\%$ b $\frac{1}{2} = 0.5 = 50\%$
 c $\frac{1}{4} = 0.25 = 25\%$
 d The probabilities of the same colour drop to $\frac{1}{6}$ (about 16.7%) and of different colours rises to $\frac{2}{3}$.
 14 a $\frac{1}{8} = 0.125 = 12.5\%$ b $\frac{1}{2} = 0.5 = 50\%$
 c $\frac{3}{16} = 0.1875 = 18.75\%$ d $\frac{1}{4} = 0.25 = 25\%$
 e $\frac{3}{4} = 0.75 = 75\%$ f $\frac{1}{2} = 0.5 = 50\%$
 g $\frac{1}{16} = 0.0625 = 6.25\%$ h $\frac{11}{16} = 0.6875 = 68.75\%$
 15 a $\frac{1}{3} \approx 0.333 = 33.3\%$ b $\frac{1}{15} \approx 0.0667 = 6.67\%$
 c $\frac{4}{5} = 0.8 = 80\%$



b $\frac{1}{4} = 0.25 = 25\%$ c $\frac{7}{12} \approx 0.583 = 58.3\%$
 d $\frac{1}{3} \approx 0.333 = 33.3\%$ e $\frac{5}{12} \approx 0.417 = 41.7\%$
 f $\frac{1}{4} = 0.25 = 25\%$ g $\frac{1}{15} \approx 0.0667 = 6.67\%$

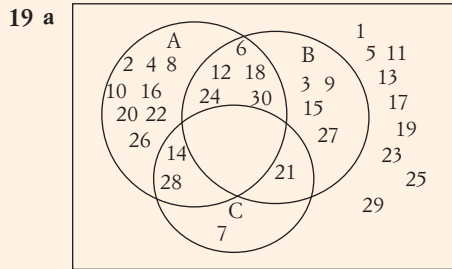


b $\frac{2}{3} \approx 0.667 = 66.7\%$ c $\frac{1}{6} \approx 0.167 = 16.7\%$
 d $\frac{1}{2} = 0.5 = 50\%$ e $\frac{3}{5} = 0.6 = 60\%$
 f $\frac{1}{10} = 0.1 = 10\%$ g $\frac{1}{2} = 0.5 = 50\%$
 h $\frac{7}{15} \approx 0.467 = 46.7\%$ i $\frac{1}{15} \approx 0.0667 = 6.67\%$
 j $\frac{2}{5} = 0.4 = 40\%$ k $\frac{11}{15} \approx 0.733 = 73.3\%$
 l $\frac{1}{30} \approx 0.0333 = 3.33\%$

18 a

	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14

b $\frac{2}{13} \approx 0.154 = 15.4\%$ c $\frac{1}{3} \approx 0.333 = 33.3\%$
 d $\frac{2}{13} \approx 0.154 = 15.4\%$ e $\frac{2}{5} = 0.4 = 40\%$
 f $\frac{1}{13} \approx 0.0769 = 7.69\%$ g $\frac{1}{5} = 0.2 = 20\%$
 h 0 i 1



- b** $\frac{1}{3} \approx 0.333 = 33.3\%$ **c** $\frac{1}{2} = 0.5 = 50\%$
d $\frac{2}{15} \approx 0.133 = 13.3\%$ **e** $\frac{1}{2} = 0.5 = 50\%$
f $\frac{1}{10} = 0.1 = 10\%$ **g** $\frac{1}{4} = 0.25 = 25\%$
- 20 a** $\frac{1}{5} = 0.2 = 20\%$ **b** $\frac{1}{3} \approx 0.333 = 33.3\%$
c $\frac{7}{20} = 0.35 = 35\%$ **d** $\frac{7}{8} = 0.875 = 87.5\%$
e $\frac{13}{20} = 0.65 = 65\%$
- 21 a** $\frac{11}{62} \approx 0.177 = 17.7\%$ **b** $\frac{11}{434} \approx 0.0253 = 2.53\%$
c $\frac{11}{64} \approx 0.172 = 17.2\%$ **d** $\frac{33}{185} \approx 0.178 = 17.8\%$
e $\frac{3}{8} = 0.375 = 37.5\%$ **f** $\frac{61}{185} \approx 0.330 = 33\%$
g 18.1% and 20.7%, so 2.6% more of the TV watchers apparently have fast reaction times.
h The differences are very small in this sample, so no real conclusions can be made.
- 22 a** $\frac{1}{8} = 0.125 = 12.5\%$ **b** $\frac{7}{8} = 0.875 = 87.5\%$
c $\frac{1}{24} \approx 0.0417 = 4.17\%$ **d** $\frac{1}{24} \approx 0.0417 = 4.17\%$
e $\frac{3}{4} = 0.75 = 75\%$ **f** $\frac{3}{4} = 0.75 = 75\%$
g $\frac{3}{8} = 0.375 = 37.5\%$ **h** $\frac{1}{4} = 0.25 = 25\%$
i $\frac{1}{2} = 0.5 = 50\%$ **j** $\frac{1}{2} = 0.5 = 50\%$

23 The student forgot that some have both, so calculated 26 out of 40 instead of 19 out of 40, giving 47.5%.

24 a $P(\text{FH}) = 0.4$, $P(\text{FS}) = 0.25$, $P(\text{FH and FS}) = 0.175$ and $P(\text{FH or FS}) = 0.475$.

b $P(\text{FH and FS}) = P(\text{FH}) + P(\text{FS}) - P(\text{FH or FS})$ or $P(\text{FH or FS}) = P(\text{FH}) + P(\text{FS}) - P(\text{FH and FS})$

25 a $P(A) = \frac{n}{u}$, $P(B) = \frac{m}{u}$,

$$P(A \text{ and } B) = \frac{n + m - p}{u},$$

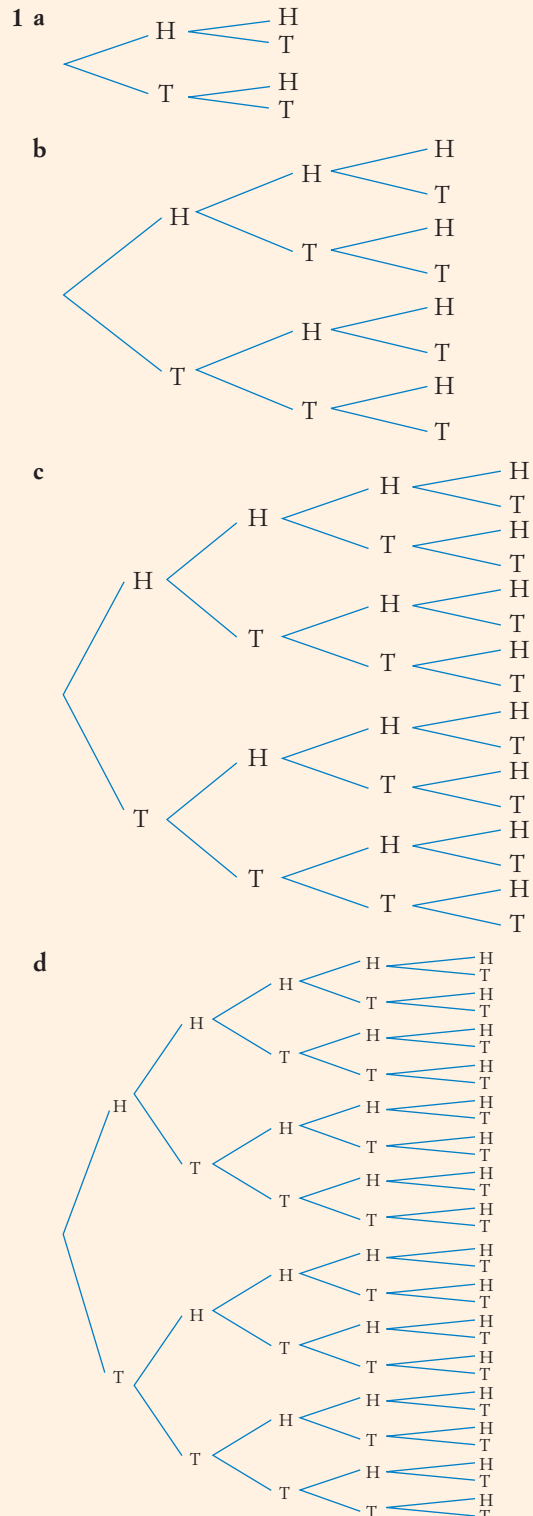
$$P(A \text{ or } B) = \frac{p}{u}$$

b $P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$ or $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

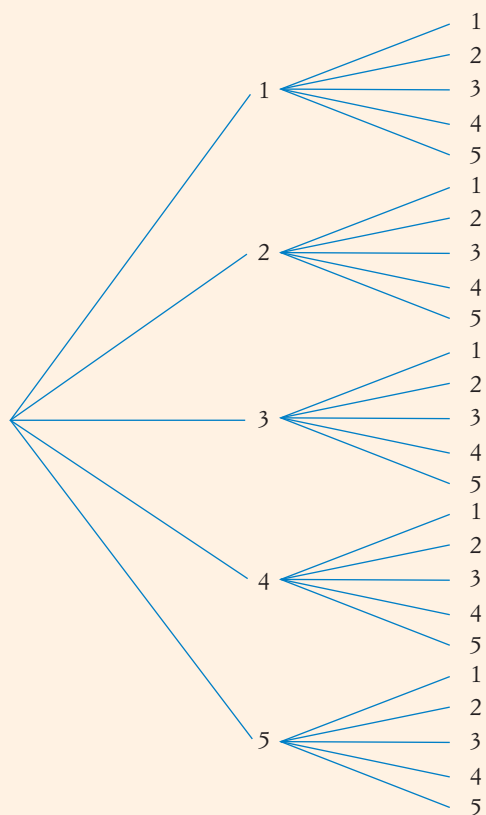
c Yes, since nothing was assumed about the sets, A and B could be any events within the set of simple outcomes in the Universal set, so it must always be true for events A and B.

26 $P(A \text{ or } B \text{ but not both}) = P(A \text{ or } B) - P(A \text{ and } B)$.

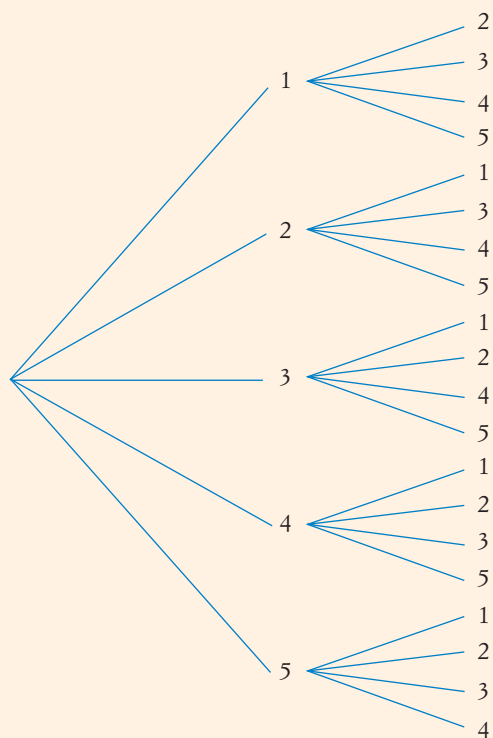
Exercise 7.3



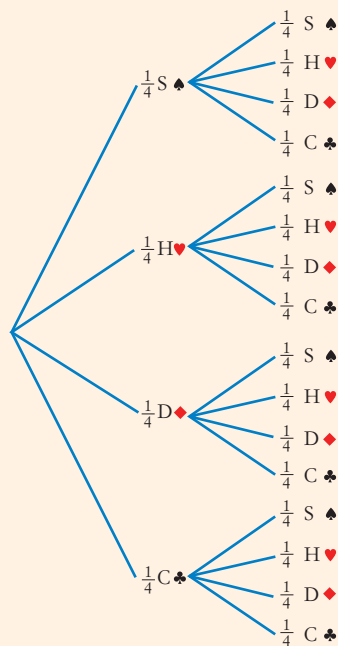
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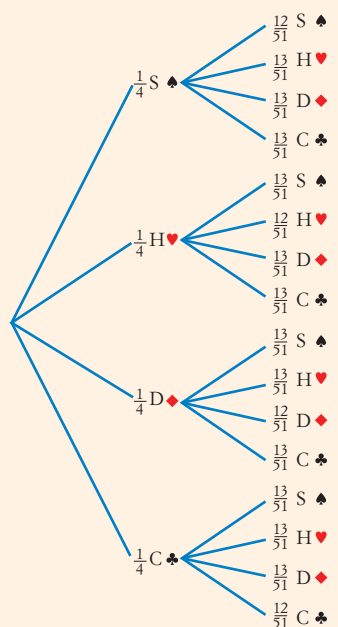
3



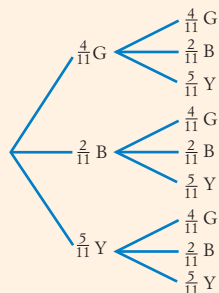
4 a



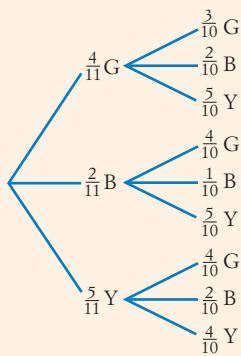
b



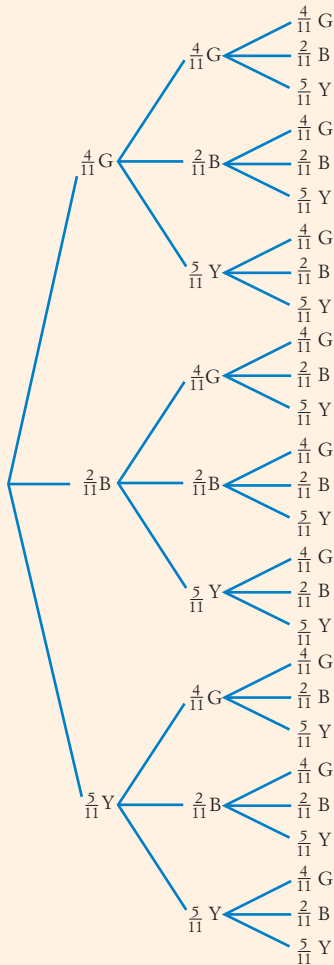
5 a



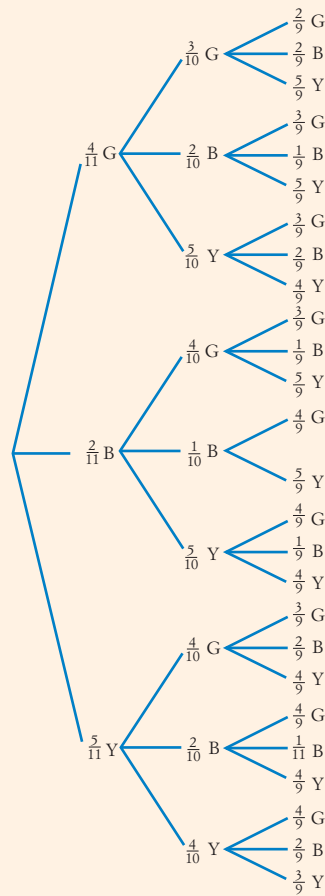
b



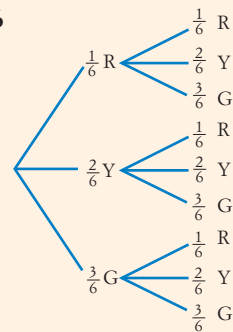
c



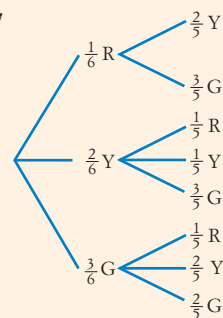
d



6



7



Answers

8 a $\frac{1}{4}$ b $\frac{1}{4}$ c $\frac{1}{4}$
d $\frac{1}{2}$ e 12 or 13 times

9 a $\frac{1}{8}$ b $\frac{1}{4}$ c $\frac{1}{8}$

10 a 2 heads and a tail b $\frac{1}{8}$
c $\frac{3}{8}$ d $\frac{1}{2}$ e $\frac{7}{8}$

11 a $\frac{1}{16}$ b $\frac{3}{8}$ c $\frac{1}{16}$ d $\frac{1}{4}$

12 a $\frac{1}{21}$ b $\frac{1}{7}$ c $\frac{2}{7}$ d $\frac{4}{21}$

13 a $\frac{4}{25}$ b $\frac{6}{25}$ c $\frac{12}{25}$ d $\frac{13}{25}$

14 a $\frac{1}{10}$ b $\frac{3}{10}$ c $\frac{3}{5}$ d $\frac{2}{5}$

15 a $\frac{1}{25}$ b $\frac{9}{100}$ c $\frac{1}{4}$ d $\frac{31}{50}$ e $\frac{19}{50}$

16 a $\frac{1}{45}$ b $\frac{1}{15}$ c $\frac{2}{9}$ d $\frac{31}{45}$ e $\frac{14}{45}$

17 a $\frac{9}{169}$ b $\frac{100}{169}$ c $\frac{30}{169}$

18 a $\frac{11}{221}$ b $\frac{10}{17}$ c $\frac{40}{221}$

19 a $\frac{27}{2197}$ b $\frac{1000}{2197}$ c $\frac{300}{2197}$
d $\frac{270}{2197}$ e $\frac{180}{2197}$ f $\frac{900}{2197}$

20 a $\frac{28}{1105}$ b $\frac{21}{65}$ c $\frac{168}{1105}$
d $\frac{144}{1105}$ e $\frac{96}{1105}$ f $\frac{504}{1105}$

21 a $\frac{1}{2}$ b $\frac{1}{2}$ c 100 people

22 a 2 red and 1 black b Red
c $\frac{2}{43}$ d $\frac{4}{43}$

23 a $\frac{32}{663}$ b About every 21 hands

24 a $\frac{9}{25}$ b $\frac{4}{25}$ c $\frac{6}{25}$

25 a $\frac{1}{3}$ b $\frac{2}{15}$ c $\frac{4}{15}$

26 $\frac{1}{270725}$

27 a H T; $\frac{1}{2}$
b HHH, HHT, HTH, THH, HTT, THT, TTH, TTT; $\frac{1}{8}$
c The sample spaces are different because in the first case we are only looking at tossing the coin once.

28 The simplest method is to draw a tree diagram (or part of one). The first stage has 5 branches with one for 'S'. At the second stage there are 4 branches from each, with one for 'P'. At the third stage there is one out of 3 branches for 'A' and at the fourth stage there are two branches with one for 'C'. There is only one letter left for each branch, so there are no further

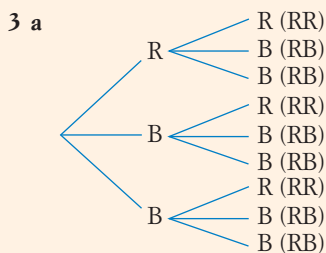
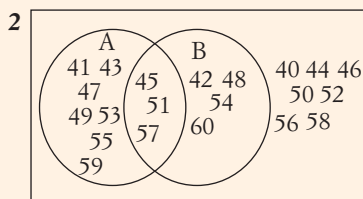
branches at the fifth letter. This means that there are $5 \times 4 \times 3 \times 2 = 120$ possible outcomes, and only one spells 'SPACE', so the probability that the letters spell space is $\frac{1}{120}$.

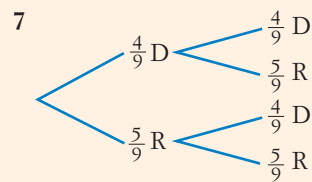
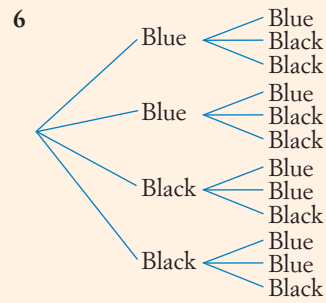
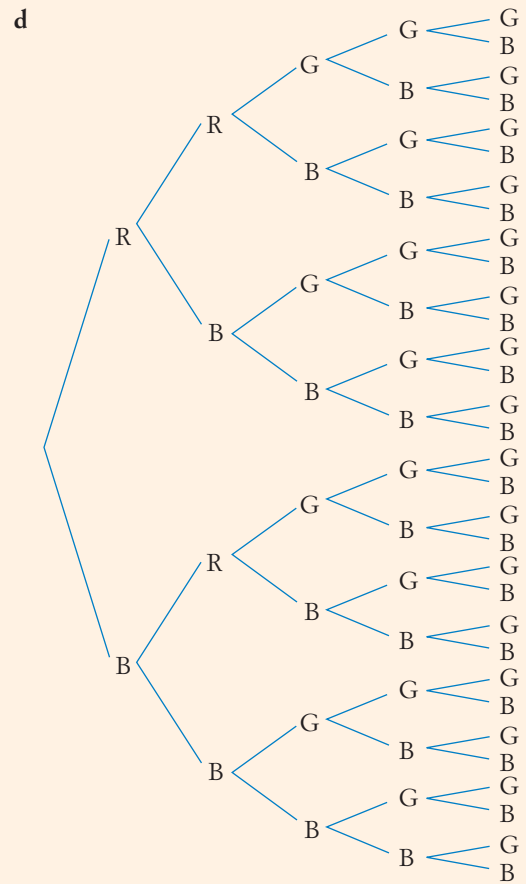
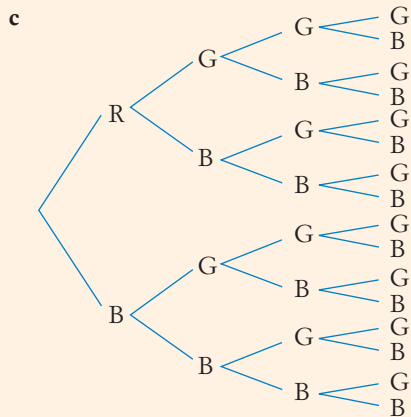
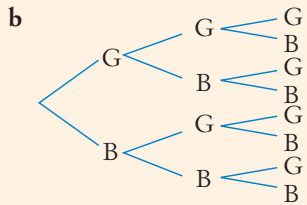
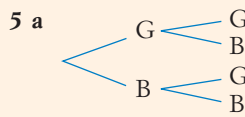
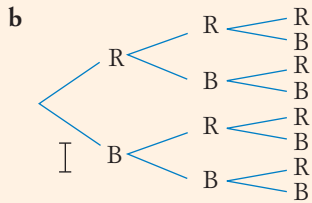
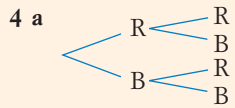
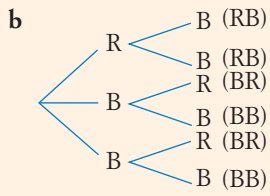
29 For an inside straight, there are 4 cards that will give you a straight, but for other straights there are 8 possible cards, so the chance for an inside straight ($\frac{4}{47}$) is only half that for other straights ($\frac{8}{47}$).

30 Consider 2000 people being tested, with the chances spread evenly. Then 10 of the people will have the disease, and for 9 of them, the test will be right, so they will get a true positive. The probability of a true positive is thus $\frac{9}{2000}$. From the 1990 people who don't have the disease, there will be 199 for whom the test gives the wrong result, so there will be 199 false positives. The probability of a false positive is thus $\frac{199}{2000}$. The fact that false positives are more likely than true positives for rare conditions is a problem because people think pathology tests are always accurate, but the doctor won't so will probably redo the test as well as using other symptoms.

Chapter review

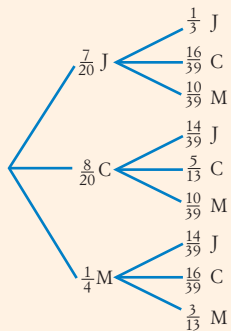
1	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36



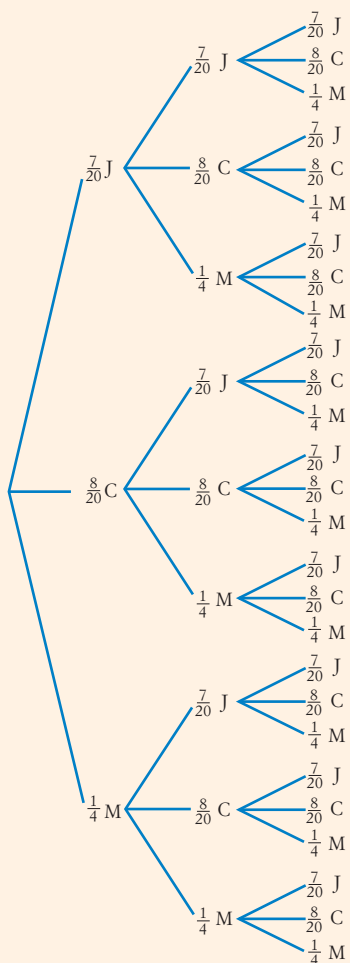


Answers

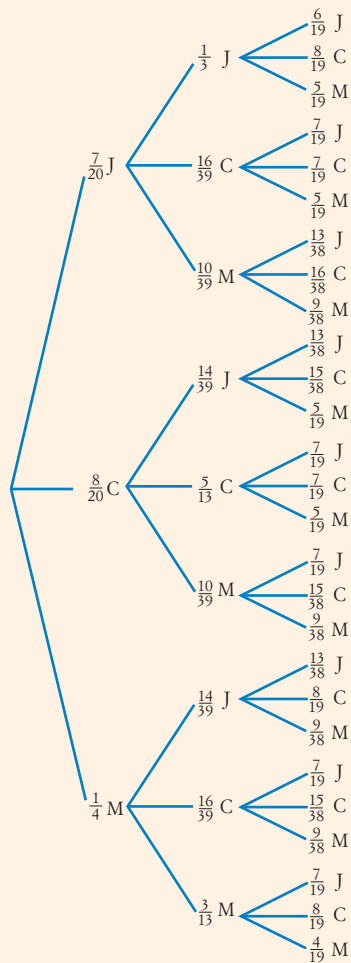
8 a



b



c



9 a Certain
d Unlikely

b Likely
e Impossible

c Likely

10 a

Width (m)	Frequency
5.6	1
5.7	0
5.8	4
5.9	3
6	4
6.1	3
6.2	7
6.3	3
Total	25

b 5.8 m

c 6.2 m

d

Width (m)	Frequency	Relative frequency
5.6	1	0.04
5.7	0	0.00
5.8	4	0.16
5.9	3	0.12
6	4	0.16
6.1	3	0.12
6.2	7	0.28
6.3	3	0.12
Total	25	1.00

e 84

f 160

11 a Pink

b i $\frac{2}{15}$

ii $\frac{7}{15}$

iii $\frac{2}{5}$

c i 16

ii 56

iii 48

12 45%

13 a $\frac{1}{6}$

b $\frac{8}{9}$

c $\frac{5}{18}$

d $\frac{3}{4}$

e $\frac{4}{9}$

14 a

	1	2	3	4	5	6	7	8
H	H1	H2	H3	H4	H5	H6	H7	H8
T	T1	T2	T3	T4	T5	T6	T7	T8

b $\frac{1}{16}$

c $\frac{9}{16}$

d $\frac{1}{2}$

e $\frac{7}{16}$

15 a 5

c $\frac{2}{9}$

b 0

d About 17 times ($16\frac{2}{3}$)

16 a $\frac{9}{25}$

c $\frac{4}{25}$

b $\frac{12}{25}$

d They become $\frac{1}{4}$, $\frac{17}{24}$ and $\frac{1}{24}$.

17 a $\frac{1}{7}$

b $\frac{2}{3}$

c $\frac{13}{21}$

d $\frac{1}{3}$

e $\frac{1}{3}$

f $\frac{4}{21}$

18 a

	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16

b $\frac{2}{15}$

c $\frac{1}{3}$

d $\frac{2}{15}$

e $\frac{2}{5}$

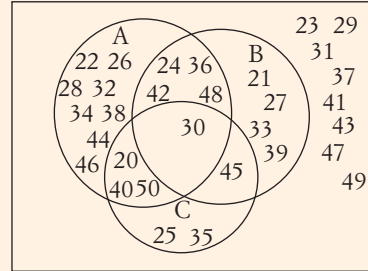
f $\frac{2}{15}$

g $\frac{1}{3}$

h $\frac{1}{6}$

i 1

19 a



b $\frac{5}{16}$

c $\frac{1}{2}$

d $\frac{1}{4}$

e $\frac{4}{7}$

f $\frac{1}{5}$

g $\frac{2}{7}$

20 a $\frac{1}{2}$

b $\frac{1}{2}$

c $\frac{1}{5}$

d $\frac{4}{5}$

21 a $\frac{25}{256}$

b $\frac{1}{16}$

c $\frac{83}{128}$

22 a $\frac{1}{12}$

b $\frac{1}{20}$

c $\frac{83}{120}$

23 a Liking

b $\frac{1}{5}$

c 100

d 535

24 a

Age	Tally	Frequency
14		1
15		2
16		7
17		3
18		0
19		0
20		1
21		2
22		2
23		0
24		1
25		1
26		0
27		1
28		1
29		0
30		0
31		0
32		1
Total		23

b There may have been an age/height/weight requirement that prevented 10-year-olds from riding.

c Under 20

25 a {O, O, O, R, R, C, C, C, C, C, L, L, L, L}

b Cola

c $\frac{2}{7}$

26 a $\frac{4}{25}$

b $\frac{2}{7}$

c $\frac{2}{5}$

d $\frac{3}{5}$

e $\frac{1}{5}$

f $\frac{1}{5}$

g $\frac{16}{25}$

h $\frac{1}{3}$

Answers

27 a $\frac{49}{110}$ b $\frac{16}{55}$ c $\frac{32}{49}$ d 1

e $\frac{47}{61}$ f $\frac{16}{27}$ g $\frac{28}{55}$

h English and History scores tend to be similar.

28 a $\frac{1}{4}$ b $\frac{5}{8}$ c $\frac{1}{24}$ d $\frac{1}{6}$ e $\frac{5}{11}$

f $\frac{2}{5}$ g $\frac{3}{10}$ h $\frac{3}{11}$ i $\frac{1}{5}$ j $\frac{4}{15}$

29 a $\frac{5}{24}$ b $\frac{1}{24}$

30 $\frac{33}{16\ 660}$

31 a $\frac{1}{17}$ b $\frac{24}{425}$ c $\frac{376}{4165}$

32 He usually leaves home just before the express bus is due, or the express bus usually comes just before the ordinary bus and there is a long wait for the next bus.

33 300 times 34 450 people

35 The student has calculated the probability the wrong way round. The probability of a student who likes chocolate also liking heavy metal is actually $\frac{6}{19} \approx 0.316$ while the probability of a student who likes heavy metal also liking chocolate is $\frac{3}{4} = 0.75$.

36 $P(\text{Liking heavy metal}) = \frac{16}{45}$, $P(\text{Liking heavy metal and chocolate}) = \frac{4}{15}$ and $P(\text{Liking chocolate, given heavy metal}) = \frac{3}{4}$.

37 $P(\text{Given HM, liking chocolate})$

$$= \frac{P(\text{Liking chocolate, given heavy metal})}{P(\text{Liking heavy metal})}$$

38 $P(\text{Pens or templates}) = \frac{7}{8}$, $P(\text{Pens and templates}) = \frac{1}{2}$ and $P(\text{Pens or templates but not both}) = \frac{3}{8}$.

39 $P(\text{Pens or templates but not both}) = P(\text{Pens or templates}) - P(\text{Pens and templates})$

40 Assuming the questions alternate between those you know and those you don't, for each two questions, your probability of getting them both right is $\frac{1}{3}$. Comparing with a tree diagram with branches for right and wrong for two questions, you could write the probabilities for right and wrong as $\frac{a}{b}$ and $1 - \frac{a}{b}$ respectively. From this diagram, the probability of getting two right would be $\left(\frac{a}{b}\right)^2$. This means that the probability of getting each questions right would be $\sqrt{\frac{1}{3}} \approx 0.5774$. The probability of getting 4 in a row right would be about $0.5774 \times 0.5774 \times 0.5774 \times 0.5774 \approx 0.111$. For 5 questions it would be about $0.5774 \times 0.5774 \times 0.5774 \times 0.5774 \times 0.5774 \approx 0.0642$.

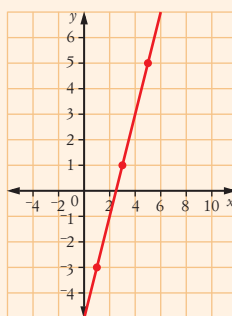
Chapter 8

Exercise 8.1

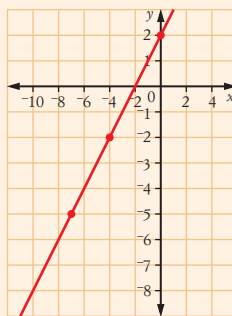
1 a No b Yes c Yes d No
 e Yes f No g No h No
 i Yes j No

2 a $x = 6$ b $m = 14$ c $h = 18$ d $g = 11$
 e $g = -5$ f $k = -6$ g $t = 4$ h $v = -12$
 i $p = 24$ j $m = -8$

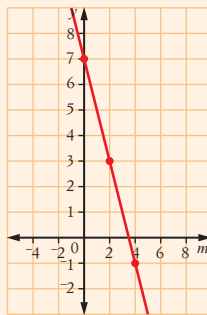
3 a $x = 4$



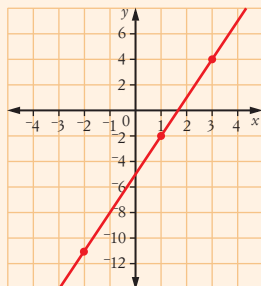
b $e = -5$



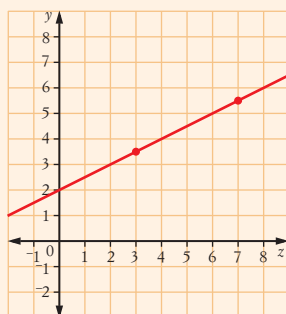
c $m = 3$



d $x = -1$



e $z = 6$



4 a $q \xrightarrow{-2} \square \xrightarrow{\times 3} \square \xrightarrow{+4} \boxed{13}$ $q = 5$

b $k \xrightarrow{+5} \square \xrightarrow{\div 3} \square \xrightarrow{-2} \boxed{4}$ $k = 13$

c $y \xrightarrow{\times 2} \square \xrightarrow{-9} \square \xrightarrow{\div 7} \square \xrightarrow{+3} \boxed{8}$ $y = 22$

d $h \xrightarrow{+3} \square \xrightarrow{\times 4} \square \xrightarrow{\div 5} \square \xrightarrow{-2} \boxed{10}$ $h = 12$

e $m \xrightarrow{\times 3} \square \xrightarrow{-7} \square \xrightarrow{\times 6} \square \xrightarrow{+3} \boxed{-21}$ $m = 1$

5 a $m = 8$ b $a = 24$ c $t = 7$ d $p = 10$

e $y = 2$ f $t = 6$ g $e = 5$ h $c = 6$

i $d = 6$ j $k = 36$

6 a $q = -9$ b $x = -12$ c $v = -10$ d $r = -10$

e $r = 6$ f $r = -5$ g $n = -5$ h $y = 9$

i $x = -11$ j $z = -3$

7 a $x = \frac{1}{4}$ b $r = 5\frac{1}{2}$ c $w = 1\frac{1}{5}$ d $m = 3\frac{1}{2}$

e $b = \frac{3}{4}$ f $h = \frac{2}{5}$ g $a = \frac{1}{3}$ h $d = 2\frac{1}{2}$

i $a = \frac{3}{4}$ j $w = \frac{1}{4}$

8 a $c = 7$ b $x = 9$ c $d = 8$ d $p = 8$

e $q = 3$ f $y = 8$ g $y = 12$ h $a = 7$

i $e = 10$ j $p = 12$

9 a $g = -12$ b $p = 8$ c $p = -8$ d $h = -4$

e $e = -7$ f $e = 6$ g $k = -4$ h $w = 5$

i $z = 8$ j $r = -7$

10 a $g = 7$ b $w = -7$ c $a = -9$ d $w = -7$

e $p = 9$ f $z = 2$ g $d = -5$ h $u = 8$

i $c = 12$ j $p = 3$

11 a $w = -3\frac{2}{3}$

b $a = -1\frac{3}{4}$

c $p = 2\frac{1}{2}$

d $w = 2\frac{1}{4}$

e $a = -9$

f $d = -2\frac{3}{4}$

g $z = -1\frac{1}{5}$

h $y = \frac{2}{5}$

i $n = 3\frac{1}{3}$

j $z = 3$

12 a $t = 1\frac{3}{5}$

b $y = -4\frac{1}{2}$

c $x = 12$

d $q = -3\frac{1}{3}$

e $w = 2\frac{3}{4}$

f $y = 2\frac{1}{2}$

g $n = -3\frac{2}{3}$

h $g = -5\frac{1}{2}$

i $z = -3$

j $t = -4\frac{1}{2}$

13 a $n = 128$ b $u = -6$ c $q = -6$ d $m = 4$

e $z = 4$ f $z = -3$ g $b = 10$ h $q = 3$

i $w = -7$ j $k = -8$

14 a $a = 18$

b $x = 12$

c $b = 6\frac{2}{3}$

d $m = 6\frac{6}{7}$

e $y = 6$

f $h = 60$

g $d = -6$

h $m = 15$

i $a = 10$

j $r = 4\frac{4}{5}$

k $d = -16$

l $b = 2$

15 a $x = -2\frac{2}{5}$

b $y = 14$

c $a = -\frac{8}{9}$

d $b = 1$

e $m = 46$

f $w = 17\frac{4}{5}$

g $x = 10\frac{5}{18}$

h $a = 44\frac{3}{4}$

i $d = 1$

j $a = -9$

16 a $4(3x+5) = 16x, x = 5$

b $x + 2(x+1) = 47, x = 15$

c $3(x-20) = 5x, x = -30$

d $5(3x-4) = x+8, x = 2$

e $5x-4 = 3(x+8), x = 14$

17 a A, $a = 42$ b C, $y = 52$ c C, $a = 12\frac{1}{3}$

d B, $x = 30$ e A, $m = 25$ f B, $a = 4$

Exercise 8.2

1 a $x = 60^\circ, y = 72^\circ$

b $x = 50^\circ$

c $x = 69^\circ, y = 42^\circ$

d $x = 73^\circ, y = 67^\circ$

2 19.5 by 14.5 cm

3 About 7.6 cm

4 About 9.5 cm, 480 cm³ (mL)

5 4

6 113 kg

7 Suzanna \$90, Karli \$60

8 \$180

9 \$6500

10 \$136

11 8 and 32

12 43 and 46

13 $r = e - \frac{cs}{100}$

14 $d = \frac{100(m-f)}{m}$

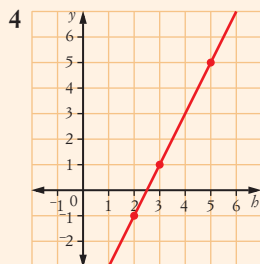
Exercise 8.3

- 1 a $m < 6$ b $r \geq 7$ c $q < 3$ d $r \leq 4$
 e $w \geq 8$ f $c < 1$ g $z \leq 9$ h $v \geq 9$
 i $m \leq 5$ j $r < 3$
- 2 a $y < 5$ b $p \leq 11$ c $m \geq 7$ d $r \leq 15$
 e $q < 13$ f $q < 13$ g $v \geq 15$ h $r > 18$
 i $n > 9$ j $t \geq 4$
- 3 a $d < 6$ b $g \geq 9$ c $q < 4$ d $p \leq 12$
 e $z \geq -9$ f $q < 10$ g $z \leq -3$ h $m \geq 12$
 i $m \leq -4$ j $z < 12$
- 4 a $a < 10$ b $x \leq 12$ c $x \geq 4$ d $x \leq -3$
 e $e < 6$ f $g < 1$ g $y \geq -3$ h $c < -4$
 i $b < 2$ j $w \leq -8$
- 5 a $k < 1$ b $w < 11$ c $z \geq 5$
 d $t \leq 2$ e $c < -3$ f $z \geq 6$
 g $c \geq 12$ h $y \leq 2$ i $w < -10$
 j $m \leq -1$

6–10 Proofs

Chapter review

- 1 a No b No c Yes
- 2 a $h \geq -13$ b $p < 19$ c $k \leq 8$ d $h < -4$
- 3 a $x = 6$ b $v = 23$ c $m = 28$ d $y = -6$



$$h = 4$$

- 5 $p \xrightarrow{\times 3} \square \xrightarrow{-4} \square \xrightarrow{\div 2} \square \xrightarrow{-3} 4$ $p = 6$
- 6 a $p = 6$ b $k = 21$ c $m = 26$ d $x = 10$
- 7 a $m = -1$ b $q = 5\frac{1}{2}$
- 8 a $t = 6$ b $p = -7$ c $d = 3\frac{3}{5}$ d $e = -11$
- 9 a $g = -5\frac{4}{5}$ b $w = -\frac{1}{4}$
- 10 a $m = 1$ b $c = -13\frac{3}{4}$

- 11 a $x < -4$ b $m \leq -19$
- 12 $\frac{n+11}{2} = 2n - 8, n = 9$ 13 $x = 95^\circ, y = 61^\circ$
- 14 1.75 cm 15 \$15 16 \$1450
- 17 2 and 14
- 18 a $B, f = 4$ b $B, x = 2\frac{4}{19}$
- 19 $h = \frac{A - 2\pi r^2}{2\pi r}$ 20 Proof

Chapter 9

Exercise 9.1

- 1 a 36 b 196 c 484
 d 27.04 e 14.44 f 50.2681
 g 185 h 39.27 i 61.8156
- 2 a 17 b 14 c 21
 d 18 e 24 f 37
- 3 a 3.74 b 12.49 c 18.08 d 20.02
 e 14.90 f 33.12 g 36.69 h 45.04
- 4 a 6.2 b 12.5 c 10.2
 d 10.5 e 8.8 f 19.1
- 5 a, c, d, f
- 6 a a b y c JM d AB e n
 f s g QR h YZ i d
- 7 D
- 8 a $MN^2 = MP^2 + NP^2$ b $JR^2 = KR^2 + KJ^2$
 c $MY^2 = QM^2 + QY^2$ d $m^2 = n^2 + r^2$
 e $u^2 = x^2 + z^2$ f $n^2 = m^2 + s^2$
 g $13^2 = 12^2 + 5^2$ h $15^2 = 12^2 + 9^2$
 i $50^2 = 14^2 + 48^2$

9 a, c, e, f, g, i

- 10 a 20 cm b 39 cm c 30 cm
 d 7.5 m e 11.3 cm f 44.7 mm
 g 4.5 km h 47.8 mm i 46.4 cm
- 11 a 15 cm b 2.4 m c 25 mm
 d 9.6 cm e 37.8 cm f 4.0 km
 g 23.3 cm h 54.8 mm i 3.3 m
- 12 a, e, f, h 13 125.9 cm 14 189.5 mm
- 15 832 cm 16 52 mm 17 141 cm
- 18 161.7 cm

19 Triangle can't be right-angled because $\sqrt{26^2} \neq \sqrt{15^2 + 21^2}$.

20 52.3 cm 21 1011.3 mm² 22 43.0 cm

23 63 cm 24 104.8 cm 25 Proof

Exercise 9.2

1 a i AC ii AB iii BC
 b i y ii z iii x
 c i PR ii PQ iii QR
 d i a ii c iii b
 e i ON ii OM iii MN
 f i p ii q iii r

2 a θ b θ
 c The right angle ($\angle XZY$) d α

3 See table below

4 a 22.88° b 82.27° c 5.97°
 d 51.17° e 44.80° f 77.3°
 g 13.79° h 78.16° i 67.39°

5 a 17°18' b 67°30' c 20°15'
 d 55°48' e 88°9' f 39°19'48"
 g 9°3'48" h 22°6'36" i 77°2'24"

6 a $\cos \theta$ b $\tan \theta$ c $\cos \theta$
 d $\cos \theta$ e $\sin \theta$ f $\tan \theta$

7 a $\sin x = 0.6$, $\cos x = 0.8$, $\tan x = 0.75$
 b $\sin x = 0.28$, $\cos x = 0.96$, $\tan x = 0.292$
 c $\sin x = 0.976$, $\cos x = 0.220$, $\tan x = 4.444$
 d $\sin x = 0.923$, $\cos x = 0.385$, $\tan x = 2.4$

e $\sin x = 0.8$, $\cos x = 0.6$, $\tan x = 1.333$

f $\sin x = 0.386$, $\cos x = 0.923$, $\tan x = 0.417$

8 a 0.176 b 0.940 c 0.643
 d 1.732 e 0.940 f 5.671
 g 0.978 h 0.407 i 9.514
 j 0.707 k 0.994 l 5.145

9 a 42.0° b 4.0° c 59.3°
 d 15.1° e 45.0° f 35.9°
 g 88.9° h 53.3° i 71.8°
 j 82.1° k 44.2° l 55.1°

10 a 36.9° b 67.4° c 45°
 d 16.3° e 60.0° f 28.1°

11 a 6.928 b 21.324 c 13.448
 d 2.544 e 113.341 f 30.991
 g 61.481 h 219.740 i 86.608
 j 60.768 k 3.426 l 32.453

12 a 43.6 b 53.7 c 13.9
 d 5.5 e 3.2 f 69.8
 g 12.9 h 51.5 i 22.2

13 a $x = 14.6$ m b $y = 30.0$ cm c $n = 18.4$ mm
 d $a = 14.9$ m e $f = 32.9$ cm f $d = 26.9$ mm
 g $h = 2.7$ km h $a = 17.7$ m i $b = 117.8$ cm

14 and

15 a $r = 176.2$ b $m = 16.0$ c $z = 25.6$
 d $q = 21.5$ e $h = 20.2$ f $k = 39.6$

16 a i 5 ii Teacher to check
 iii $\sin A = \frac{5}{13}$, $\tan A = \frac{5}{12}$

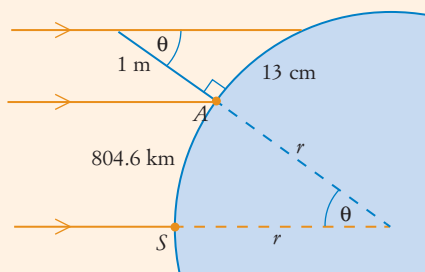
3	Triangle	Opposite side	Adjacent side	Hypotenuse	$\sin a$	$\cos a$	$\tan a$
a	XYZ	YZ	XY	XZ	$\frac{YZ}{XZ}$	$\frac{XY}{XZ}$	$\frac{YZ}{XY}$
b	PQR	PQ	QR	PR	$\frac{PQ}{PR}$	$\frac{QR}{PR}$	$\frac{PQ}{QR}$
c	ABC	BC	AB	AC	$\frac{BC}{AC}$	$\frac{AB}{AC}$	$\frac{BC}{AB}$
d	MNO	NO	MN	MO	$\frac{NO}{MO}$	$\frac{MN}{MO}$	$\frac{NO}{MN}$
e	RST	RS	ST	RT	$\frac{RS}{RT}$	$\frac{ST}{RT}$	$\frac{RS}{ST}$

- b** **i** 3 **ii** Teacher to check
iii $\tan B = \frac{4}{3}$, $\cos B = \frac{3}{5}$
c **i** 8 **ii** Teacher to check
iii $\tan C = \frac{8}{15}$, $\sin C = \frac{8}{17}$
d **i** 4 **ii** Teacher to check
iii $\tan B = \frac{3}{4}$, $\cos B = \frac{4}{5}$
e **i** $\sqrt{8}$ **ii** Teacher to check
iii $\tan C = \sqrt{8}$, $\sin C = \frac{\sqrt{8}}{3}$
f **i** $\sqrt{2}$ **ii** Teacher to check
iii $\cos A = \frac{1}{\sqrt{2}}$, $\sin A = \frac{1}{\sqrt{2}}$

- 17** $PR = 89.60$ m **18** $ZY = 199.3$ m
19 $AB = 295$ mm **20** $MO = 119$ m
21 $KL = 8.90$ m **22** $UV = 166.3$ cm

Exercise 9.3

- 1** **a** 236.1 m **b** About 82 m
2 17.7 m **3** 1058.9 m **4** 39.8 cm
5 5.2 m **6** 15.92 m
7 **a** 9.84 m **b** 10.95 m
8 1.9 m **9** 1.53 m **10** 2.2 m **11** 50.0 m
12 **a** 23.8 m **b** 62.6°
13 **a** 7.31 m **b** 67.3°
14 **a** 1.54 m **b** 1.95 m
15 **a** 2.66 m **b** 21.3°
16 **a** 68.9° **b** 4.66 m
17 **a** 23.9 m **b** 53.8°
18 **a** 59.2° **b** 24.4 m
19 5.4 m **20** 30.4 m
21 The information gives an answer of about 6200 km, which compares well with the true figure.



22–23 Proofs

Chapter review

- 1** **a** 256 **b** 185 **c** 39.27
d 9.17 **e** 20.25 **f** 14.28
2 **b**
3 **a** c **b** XY **c** GI
4 **a** $y^2 = x^2 + z^2$
b $PR^2 = PQ^2 + QR^2$
c $MS^2 = DM^2 + DS^2$
5 **c**
6 **a** **i** a **ii** c **iii** b
b **i** LM **ii** KM **iii** LK
c **i** EM **ii** PM **iii** EP
7 **a** 48.5° **b** 22.2° **c** 63.2°
8 **a** 81°24' **b** 12°42' **c** 52°19'48''
9 **a** $\sin \theta = 0.471$, $\cos \theta = 0.882$, $\tan \theta = 0.533$
b $\sin \theta = 0.810$, $\cos \theta = 0.571$, $\tan \theta = 1.417$
c $\sin \theta = 0.824$, $\cos \theta = 0.529$, $\tan \theta = 1.556$
10 **a** 0.414 **b** 0.141 **c** 0.304
11 **a** 31.9° **b** 30.8° **c** 44.7°
12 **a** 75.2 cm **b** 1.0 m (0.96 m) **c** 28.3 mm
13 **a** is right-angled
14 **a** 46.7° **b** 49.8° **c** 38.2°
15 **a** 19.583 **b** 15.120 **c** 37.554
d 64.720 **e** 55.511 **f** 87.175
16 **a** 23.8 **b** 41.6 **c** 39.4
17 **a** 18.9 cm **b** 40.5 mm **c** 2.3 m
18 Same as question 17 **19** 125 cm
20 The gate is not square.
21 No, it is not perpendicular. The wall leans away from the bricklayer.
22 393 cm **23** 2.57 m **24** 149.8 cm
25 66.0° **26** 5.5°
27 **a** 3.7° **b** 12.47 m
28 **a** 50.5° **b** 17.0 m
29 155.4 m²

Chapter 10

Exercise 10.1

- 1 a Yes No No Yes No
 b No Yes Yes No Yes
 c No No No Yes Yes
 d No No Yes No Yes
 e No No Yes No Yes

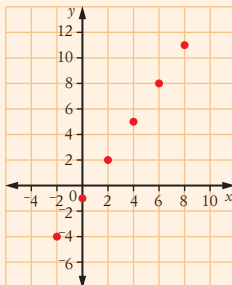
2 See table below

- 3 a $\frac{1}{3}$ b $\frac{4}{3}$ or $1\frac{1}{3}$ c $-\frac{4}{3}$ or $-1\frac{1}{3}$ d 1
 e $-\frac{5}{4}$ or $-1\frac{1}{4}$ f $\frac{2}{3}$ g $-\frac{3}{10}$ h $\frac{3}{11}$
 i $\frac{6}{5}$ or $1\frac{1}{5}$

4 The lines parallel to the x -axis are first.

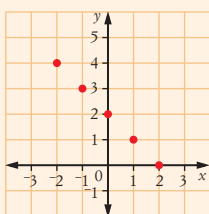
- a $y = -2, x = 3$ b $y = 5, x = -1$
 c $y = 6, x = 3$

5 a



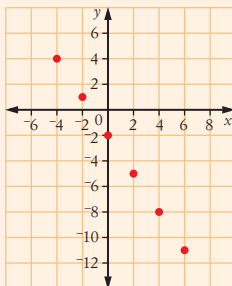
Increasing linear function

b



Decreasing linear function

c

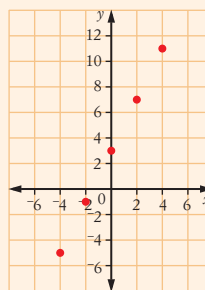


Decreasing linear function

2

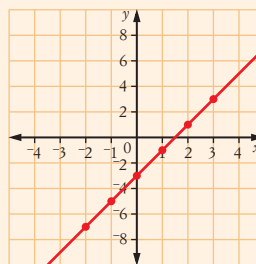
Line segment	AB	ST	GR	PK	PM	EF	FJ
Gradient	$\frac{3}{5}$	$-\frac{1}{5}$	3	-4	$\frac{5}{6}$	$\frac{7}{2}$ or $3\frac{1}{2}$	$-\frac{5}{12}$

d

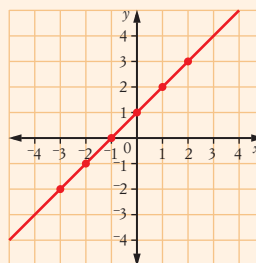


Increasing linear function

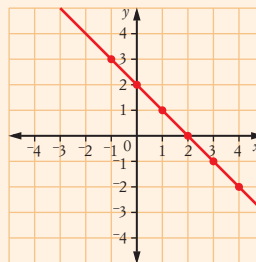
6 a



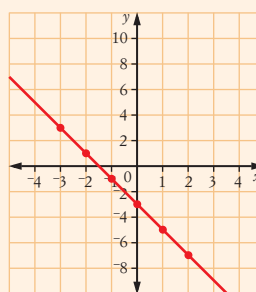
b



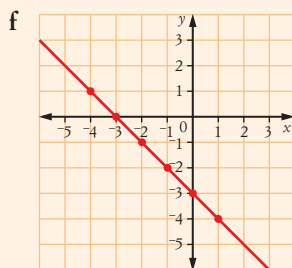
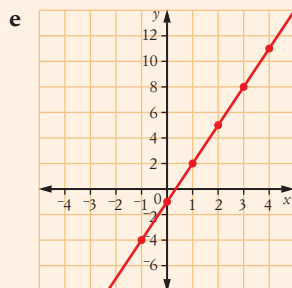
c



d



Answers



Increasing linear function

- 7 **a** $m = 3$, $(\frac{2}{3}, 0)$, $(0, -2)$ **b** $m = 1$, $(4, 0)$, $(0, 4)$
c $m = -2$, $(1\frac{1}{2}, 0)$, $(0, 3)$ **d** $m = -1$, $(-1, 0)$, $(0, -1)$
e $m = -\frac{1}{4}$, $(4, 0)$, $(0, 1)$
f $m = 1\frac{1}{2}$, $(1\frac{1}{3}, 0)$, $(0, -2)$
g $m = 2$, $(-\frac{1}{4}, 0)$, $(0, \frac{1}{2})$
h $m = -\frac{2}{3}$, $(-1\frac{1}{2}, 0)$, $(0, -1)$
i $m = -1\frac{2}{3}$, $(1\frac{1}{3}, 0)$, $(0, 1\frac{1}{3})$
j $m = 4$, $(2, 0)$, $(0, 8)$ **k** $m = -\frac{1}{3}$, $(4, 0)$, $(0, 1\frac{1}{3})$
l $m = 1\frac{1}{3}$, $(0, 0)$, $(0, 0)$

- 8 **a** $y = x + 3$ **b** $y = -\frac{1}{2}x + 2$ **c** $y = 2x - 3$
d $y = \frac{3}{2}x - 5$ **e** $y = -2x - 1$ **f** $y = -x + 2$

- 9 **a** $y = 2x + 4$ **b** $y = 3x + 1$ **c** $y = 5x - 6$
d $y = 3x - 2$ **e** $y = -3$ **f** $y = -4x$

- 10 **a** $4x - y - 1 = 0$ **b** $2x - 3y - 6 = 0$
c $4x + 3y + 3 = 0$ **d** $5x - 2y + 4 = 0$
e $10x - 3y - 12 = 0$ **f** $7x + 4y - 5 = 0$

11 Answers are in the order: gradient, y-intercept.

- a** 2, 5 **b** -3, 5 **c** -1, 1 **d** -7, 4
e 1, 2 **f** -8, 0 **g** $\frac{1}{2}$, $\frac{1}{2}$ **h** -7, 14
i $\frac{1}{3}$, 2 **j** $-1\frac{1}{3}$, $2\frac{2}{3}$ **k** $2\frac{1}{3}$, 0

- 12 **a** $y = -2x + 2$ **b** Not linear **c** $y = 2x - 1$
d $y = \frac{3}{2}x + 3$ **e** Not linear

- 13 **a** $C = 15h + 40$
b \$130
c 4 days.

14 **a** $P = 5h - 700$

b **i** They lose \$200 **ii** They make a profit of \$300

c 220

d \$665

15 $\frac{2}{35}$

16 **a** $y = 2x + 1$, $2x - y + 1 = 0$

b $y = -x + 3$, $x + y - 3 = 0$

c $y = x + 1$, $x - y + 1 = 0$

d $y = 4x - 8$, $4x - y - 8 = 0$

e $y = -4x + 6$, $3x + 4y - 24 = 0$

17 $m = \frac{d-b}{c-a}$ or $m = \frac{b-d}{a-c}$

Exercise 10.2

1 **a** $(\frac{v+x}{2}, \frac{w+y}{2})$ **b** $(\frac{p+q}{2}, \frac{r+s}{2})$

c $(\frac{k+n}{2}, \frac{m+e}{2})$ **d** $(\frac{r+t}{2}, \frac{s+u}{2})$

e $(\frac{d+g}{2}, \frac{a+f}{2})$

2 **a** $\sqrt{(v-x)^2 + (w-y)^2}$ **b** $\sqrt{(p-q)^2 + (r-s)^2}$

c $\sqrt{(k-n)^2 + (m-e)^2}$ **d** $\sqrt{(r-t)^2 + (s-u)^2}$

e $\sqrt{(d-g)^2 + (a-f)^2}$

3 **a** (5, 1) **b** $(-1, \frac{1}{2})$ **c** (0, 6) **d** $(\frac{1}{2}, \frac{1}{2})$

e (-5, 5) **f** $(\frac{1}{2}, 1\frac{1}{2})$ **g** $(-3\frac{1}{2}, 2\frac{1}{2})$

h $(4\frac{1}{2}, -2)$ **i** $(-4\frac{1}{2}, -8)$ **j** $(\frac{1}{2}, -3)$

4 **a** $\sqrt{8} \approx 2.83$

b $\sqrt{52} \approx 7.21$

c 13

d $\sqrt{274} \approx 16.55$

e $\sqrt{162} \approx 12.73$

f $\sqrt{162} \approx 12.73$

g $\sqrt{61} \approx 7.81$

h $\sqrt{148} \approx 12.17$

i $\sqrt{73} \approx 8.54$

j $\sqrt{233} \approx 15.26$

5 **B** is $(-12, -3)$

6 **Q** is (11, 4).

7 **R** is (2, -13).

8–10 Proofs

Exercise 10.3

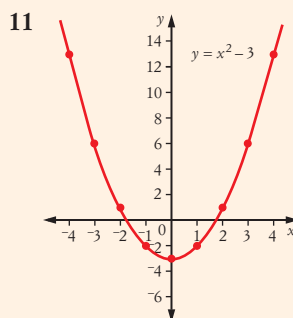
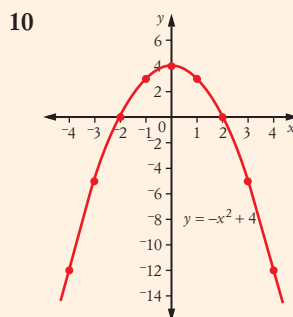
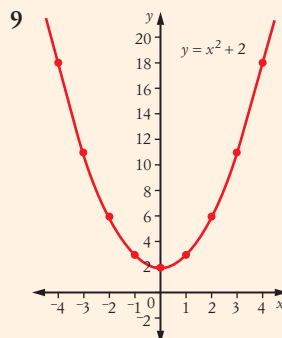
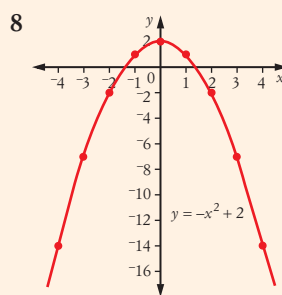
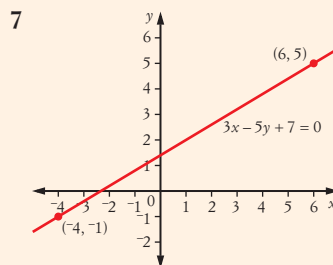
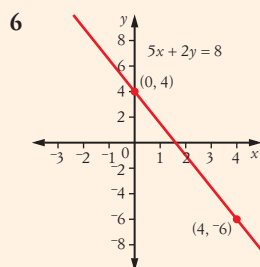
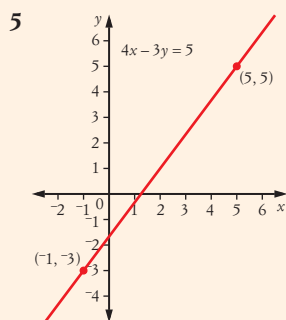
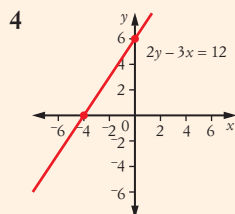
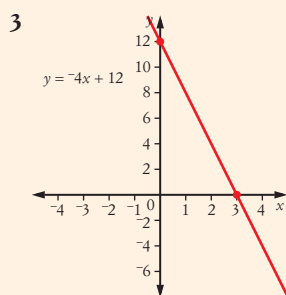
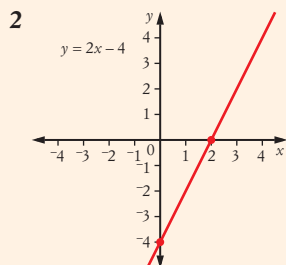
1 **a** (4, 0), (0, 3)

b (2, 0), (0, -8)

c (3, 0), (0, -3)

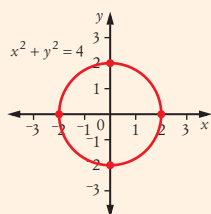
d (5, 0), (0, 2 $\frac{1}{2}$)

e $(-1\frac{2}{3}, 0)$, (0, -2 $\frac{1}{2}$)

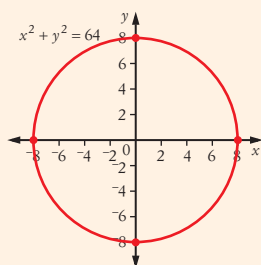


Answers

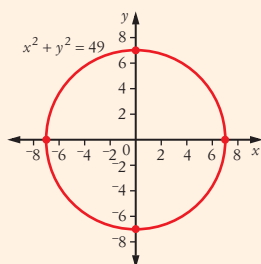
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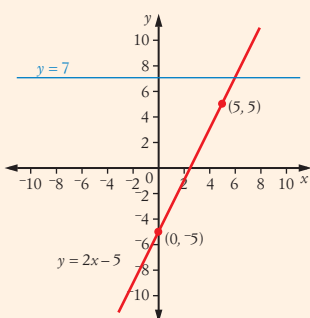
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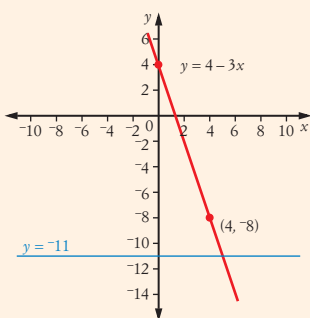
14



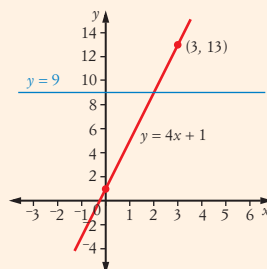
15 $x = 6$



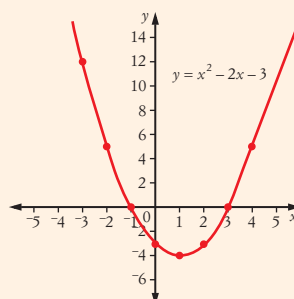
16 $x = 5$



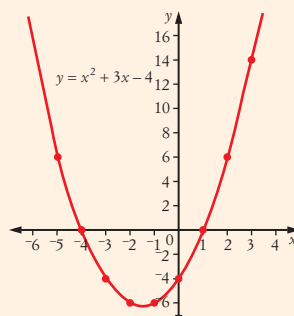
17 $x = 2$



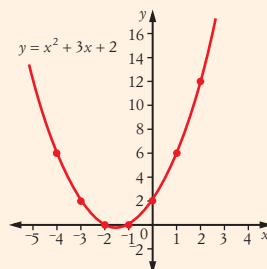
18 $x = -1$ or $x = 3$



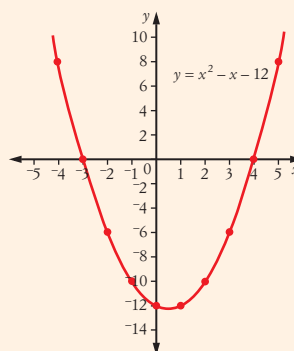
19 $x = -4$ or $x = 1$



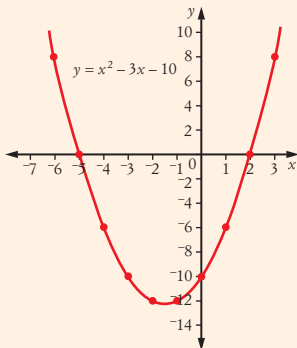
20 $x = -2$ or $x = -1$



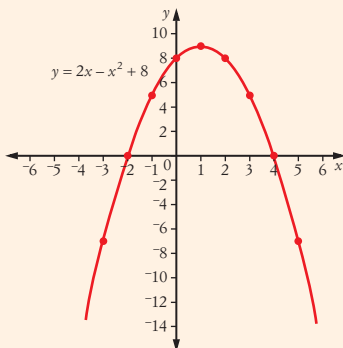
21 $x = -3$ or $x = 4$



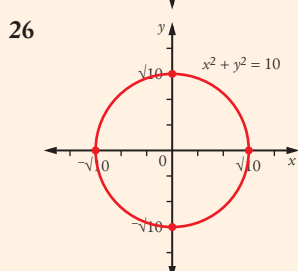
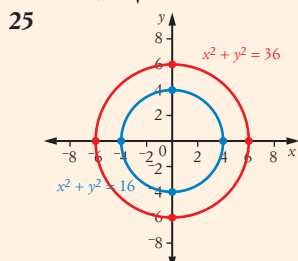
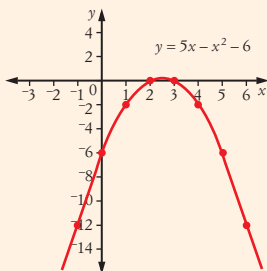
22 $x = -5$ or $x = 2$



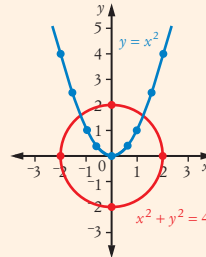
23 $x = -2$ or $x = 4$



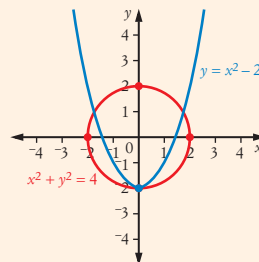
24 $x = 2$ or $x = 3$



27 Draw on the same axes to show 2 intersections.



28 Sketch both graphs on the same axes.



There are 3 intersections.

Exercise 10.4

1 L1 and L3 are parallel; L2 and L4 are parallel.

2 L2 and L3 are parallel and both are perpendicular to L1.

3 a $x = 5$

b $y = -3$

4 a $y = 6$

b $x = -3$

5 a $y = -7$

b $x = -1$

6 $3x + 2y - 6 = 0$

7 $2x - 5y - 10 = 0$

8 $x + 2y + 2 = 0$

9 $2x - y + 4 = 0$

10 Parallel: $2x + y - 10 = 0$,
perpendicular: $x - 2y + 10 = 0$

11 Parallel: $3x - 4y + 21 = 0$,
perpendicular: $4x + 3y + 3 = 0$

12 Parallel: $4x + 5y - 10 = 0$,
perpendicular: $5x - 4y - 33 = 0$

13–14 Proof

15 Proof, 50 square units.

Chapter review

- 1 **a** No **b** No **c** No
d Yes **e** Yes

2 **a** $2\frac{1}{2}$ **b** -2

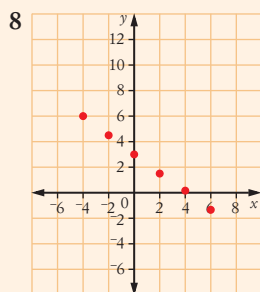
3 **a** $y = 3$ **b** $x = 7$

4 $M = \left(\frac{c+g}{2}, \frac{d+h}{2}\right)$, $d = \sqrt{(g-c)^2 + (h-d)^2}$

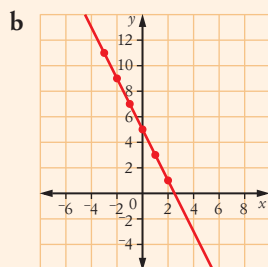
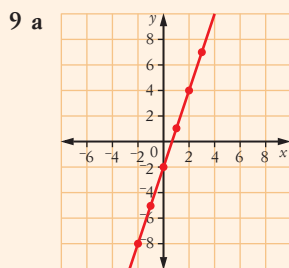
5 **a** $(6, 0)$ and $(0, -4)$ **b** $(-2\frac{1}{3}, 3\frac{1}{2})$

6 L1 and L5 are parallel and both are perpendicular to L3 and L6, which are themselves parallel.

7 **a** $x = 1$ **b** $x = 1$



Decreasing linear function



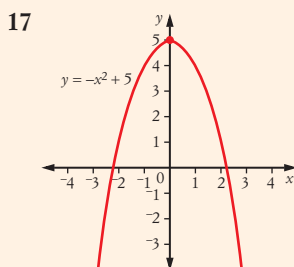
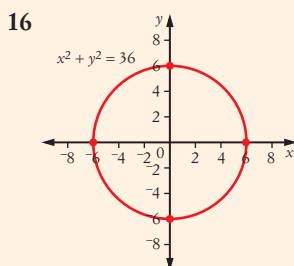
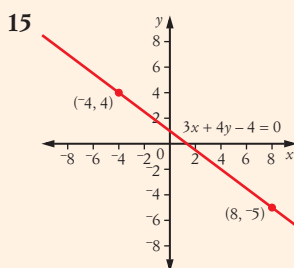
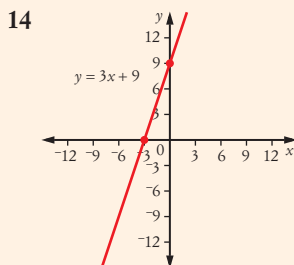
10 **a** $m = -3$, $c = 5$

b $m = \frac{2}{3}$, $c = 3$

11 **a** $y = 2x - 4$ **b** $y = -\frac{1}{2}x + 2$ **c** $y = \frac{4}{3}x - 1$

12 **a** $y = -x + 6$ **b** $y = \frac{3}{4}x - 3$

13 $M = (2\frac{1}{2}, 1)$, $d = \sqrt{153} \approx 12.37$



18 **a** $2x - y - 5 = 0$

b $3x + 5y + 15 = 0$

19 **a** $m = 3$, $c = 4$

b $m = 1\frac{1}{2}$, $c = 3$

c $m = -\frac{1}{2}$, $c = 1\frac{3}{4}$

20 $x + 2y - 8 = 0$

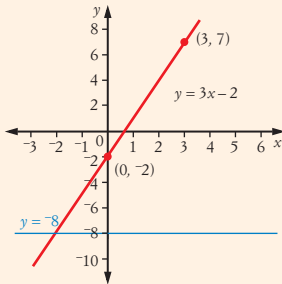
21 $4x + 3y + 12 = 0$

22 $2x + 3y - 15 = 0$

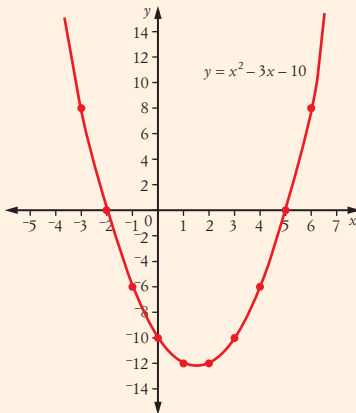
23 $C = 55h + 250$, \$470

24 $(-5, -1)$

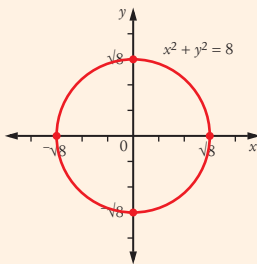
25 $x = -2$



26 $x = -2$ or $x = 5$



27



28 $y = 3x + 6$, $3x - y + 6 = 0$

29 Parallel: $4x - 5y - 21 = 0$,
perpendicular: $5x + 4y - 16 = 0$

30 Proof 31 Proof

32 $mx - y - ma + b = 0$

33 These are parabolas that have the same turning point but 'face' up and down, so they have one intersection, at $(0, a)$.

34 Proof

Chapter 11

Exercise 11.1

- 1 a Complementary angles
b Supplementary angles
c Vertically opposite angles
d Angles at a point
e Supplementary angles
f Vertically opposite angles
g Angles at a point
h Supplementary angles

- 2 a Co-interior b Alternate
c Corresponding d Alternate
e Alternate f Corresponding
g Alternate h Corresponding
i Co-interior j Alternate

- 3 a i h ii f iii e
b i g ii e iii f
c i $\angle HMN$ ii $\angle LMJ$ iii $\angle LMH$
d i $\angle JML$ ii $\angle HMN$ iii None

- 4 a Co-interior b Alternate
c Corresponding d Corresponding
e Alternate f Corresponding
g Alternate h Corresponding
i Alternate j Corresponding
k Vertically opposite l Alternate

- 5 $a = 59^\circ$, vertically opposite
 $b = 20^\circ$, complementary
 $c = 140^\circ$, angles at a point
 $d = 60^\circ$, supplementary
 $e = 132^\circ$, vertically opposite
 $f = 48^\circ$, supplementary
 $g = 134^\circ$, angles at a point
 $h = 34^\circ$, complementary
 $i = 12^\circ$, complementary
 $j = 123^\circ$, supplementary
 $k = 150^\circ$, angles at a point
 $l = 20^\circ$, complementary
 $m = 111^\circ$, supplementary
 $n = 28^\circ$, supplementary
 $o = 34^\circ$, angles at a point
 $p = 79^\circ$, vertically opposite

- 6 a $a = 90^\circ$, supplementary
b $b = 21^\circ$, vertically opposite
 $c = 143^\circ$, supplementary with 37° or $(b + 16^\circ)$

Answers

- c** $d = 42^\circ$, supplementary 
 $e = 48^\circ$, complementary with d 
 $f = 132^\circ$, angles at a point , or
 supplementary with e 
- d** $g = 44^\circ$, vertically opposite 
 $h = 17^\circ$, complementary 
 $i = 119^\circ$ supplementary with $(h + 44^\circ)$
 or $(g + h)$ 
 $j = 107^\circ$, supplementary , or angles at a point
 with $(g + i + 90^\circ)$ 
- e** $k = 31^\circ$, vertically opposite 
 $l = 149^\circ$, supplementary with 31° or k 
 $m = 55^\circ$, supplementary 
 $n = 107^\circ$, supplementary 
 $o = 41^\circ$, supplementary 
 $p = 129^\circ$, supplementary , or angles at a point
 with $(n + 212^\circ)$ 
 $q = 44^\circ$, supplementary 
 $r = 48^\circ$, complementary 
 $s = 103^\circ$, angles at a point 
- 7 a** $a = 48^\circ$, corresponding 
 $b = 48^\circ$, alternate 
 $c = 48^\circ$, vertically opposite 
- b** $d = 71^\circ$, co-interior 
 $e = 109^\circ$, corresponding 
 $f = 109^\circ$, alternate 
 $g = 109^\circ$, alternate 
 $h = 109^\circ$, corresponding 
 $i = 86^\circ$, corresponding 
 $j = 86^\circ$, corresponding 
 $k = 86^\circ$, alternate 
 $l = 94^\circ$, co-interior 
- c** $m = 63^\circ$, co-interior 
 $n = 117^\circ$, corresponding 
 $o = 117^\circ$, corresponding 
 $p = 117^\circ$, alternate 
 $q = 74^\circ$, corresponding 
 $r = 74^\circ$, alternate 
 $s = 106^\circ$, co-interior 
 $t = 74^\circ$, corresponding 
- d** $u = 63^\circ$, corresponding 
 $v = 63^\circ$, corresponding 
 $w = 48^\circ$, co-interior 
 $x = 132^\circ$, corresponding 
 $y = 132^\circ$, alternate 
 $z = 132^\circ$, alternate
- 8** $a = 30^\circ$, $a + 2a = 90^\circ$, complementary 
 $b = 20^\circ$, vertically opposite 
 $c = 50^\circ$, supplementary 
 $d = 45^\circ$, angles at a point 
 $e = 14^\circ$, vertically opposite 
 $f = 45^\circ$, complementary 
 $g = 20^\circ$, supplementary 
 $h = 8^\circ$, vertically opposite 
 $i = 54^\circ$, angles at a point 
 $j = 42^\circ$, supplementary 
- 9 a** $a = 90^\circ$, co-interior 
 $b = 75^\circ$, vertically opposite $(a - 15^\circ)$ 
 $c = 105^\circ$, supplementary with $(a - 15^\circ)$
 or b , or corresponding
 with $(a + 15^\circ)$ 
 $d = 45^\circ$, corresponding 
 $e = 60^\circ$, vertically opposite $(d + 15^\circ)$ , or
 alternate with $(2d - 30^\circ)$ 
 $f = 120^\circ$, supplementary with $(2d - 30^\circ)$ 
- b** $g = 30^\circ$, alternate 
 $h = 60^\circ$, supplementary 
 $i = 120^\circ$, corresponding with $(150^\circ - g)$ , or
 vertically opposite $4g$ , or
 co-interior with h 
- c** $j = 120^\circ$, co-interior 
 $k = 120^\circ$, corresponding with j , or
 supplementary with $\frac{1}{2}j$ 
 $l = 120^\circ$, vertically opposite j , or
 alternate with k 
 $m = 36^\circ$, corresponding 
 $n = 118^\circ$, vertically opposite $(190^\circ - 2m)$ ,
 or corresponding with $(3m + 10^\circ)$ 
- 10** The angle sum will not always be 180° .

Exercise 11.2

- 1 a** a and e **b** e and f
c a and b ; d and e
d $d + e + f = 360^\circ$ 
e $a = c + d$ ; $e = c + b$ 
f $c + b + d = 180^\circ$ 
g $a + b = 180^\circ$ ; $d + e = 180^\circ$ 
- 2 a** w
b v and w ; p and q ; s and t
c v and w ; p and q

- d** $u + t + s + r + q = 360^\circ$ 
- e** $s + r + q + w = 360^\circ$ 
- f** $t = v$ 
- g** $v + w = 180^\circ$ ; $p + q = 180^\circ$ 
- h** $w = t + u$ 
- i** $u + t + v = 180^\circ$ 
- 3** $a = 85^\circ$, angle sum of triangle 
 $b = 78^\circ$, angle sum of triangle 
 $c = 52^\circ$, angle sum of quadrilateral 
 $d = 101^\circ$, angle sum of quadrilateral 
 $e = 97^\circ$, angle sum of triangle 
 $f = 51^\circ$, angle sum of quadrilateral 
 $g = 105^\circ$, angle sum of quadrilateral 
 $h = 250^\circ$, angle sum of quadrilateral 
 $i = 19^\circ$, angle sum of triangle 
 $j = 71^\circ$, angle sum of triangle 
- 4** $a = 37^\circ$, isosceles triangle 
 $b = 67^\circ$, isosceles triangle  and angle sum of triangle 
 $c = 127^\circ$, exterior angle of triangle 
 $d = 62^\circ$, opposite angles of parallelogram  **A**
 $e = 26^\circ$, exterior angle of triangle 
 $f = 108^\circ$, isosceles triangle  and angle sum of triangle 
 $g = 31^\circ$, exterior angle of triangle 
 $h = 135^\circ$, opposite angles of parallelogram  **A**
 $i = 72^\circ$, isosceles triangle  and angle sum of triangle 
 $j = 31^\circ$, isosceles triangle 
 $k = 66^\circ$, exterior angle of triangle 
 $l = 142^\circ$, isosceles triangle  and angle sum of triangle 
 $m = 93^\circ$, exterior angle of triangle 
 $n = 73\frac{1}{2}$, isosceles triangle  and angle sum of triangle 
 $o = 74^\circ$, opposite angles of parallelogram  **A**
 $p = 120^\circ$, exterior angle of triangle 
- 5** **a** $\angle EBL = 38^\circ$, supplementary 
so $a = 38^\circ$, isosceles triangle 
b $\angle ENT = 30^\circ$, angles at a point 
so $b = 75^\circ$, angle sum of triangle 
c $\angle HON = 70^\circ$, complementary 
so $c = 70^\circ$, isosceles triangle 
d $\angle DEN = 78^\circ$, angles at a point 
so $d = 54^\circ$, angle sum of quadrilateral 
- e** $\angle TWI = 28^\circ$, vertically opposite 
so $e = 28^\circ$, isosceles triangle 
- f** $\angle N = 63^\circ$, isosceles triangle 
so $f = 126^\circ$, exterior angle of triangle 
- g** $\angle RAT = 80^\circ$, supplementary 
so $g = 126^\circ$, angle sum of quadrilateral 
- h** $\angle PDT = 27^\circ$, exterior angle of triangle 
so $h = 27^\circ$, vertically opposite 
- i** $\angle R = 63^\circ$, opposite angles of parallelogram  **A**
so $i = 52^\circ$, angle sum of triangle 
- j** $\angle K = 67^\circ$, isosceles triangle 
so $j = 120^\circ$, angle sum of quadrilateral 
- 6** **a** $\angle PNI = 101^\circ$, opposite angles of parallelogram  **A**
so $\angle PNI = 79^\circ$, supplementary 
so $a = 106^\circ$, angle sum of quadrilateral **SPNT** 
- b** $\angle DNR = 46^\circ$, vertically opposite 
so $\angle DER = 89^\circ$, angle sum of quadrilateral 
- c** $\angle LOP = 29^\circ$, exterior angle of triangle **LOP** 
so $\angle DOG = 29^\circ$, vertically opposite 
so $c = 125^\circ$, angle sum of triangle **DOG** 
- d** $\angle SGT = 48^\circ$, vertically opposite 
so $\angle ASR = 122^\circ$, exterior angle of triangle 
so $d = 29^\circ$, isosceles triangle  and angle sum of triangle 
- e** $\angle HKS = 36^\circ$, isosceles triangle 
so $\angle YKO = 54^\circ$, complementary 
so $\angle L = 54^\circ$, opposite angles of parallelogram  **A**
so $e = 144^\circ$, angle sum of quadrilateral **PLOM** 
- f** $\angle CBG = 117^\circ$, opposite angles of parallelogram  **A**
so $\angle HBJ = 117^\circ$, vertically opposite 
so $f = 31^\circ$, angle sum of triangle 
- g** $\angle LDS = 71^\circ$, isosceles triangle  and $\angle LDB = 82^\circ$, opposite angles of parallelogram  **A**
so $\angle BDC = 27^\circ$, supplementary 
so $g = 78^\circ$, angle sum of triangle 
- h** $\angle ZDS = 124^\circ$, isosceles triangle  and angle sum of triangle 
so $\angle YDC = 116^\circ$, angles at a point 
so $h = 17^\circ$, angle sum of triangle 

Answers

- i $\angle AMO = 40^\circ$, vertically opposite 
 so $\angle MOA = 50^\circ$, angle sum of triangle 
 so $\angle MOB = 56^\circ$, supplementary 
 so $i = 72^\circ$, angle sum of triangle 
- j $\angle QIC = 103^\circ$, angle sum of triangle 
 $\angle QIU = 117^\circ$, angles at a point 
 so $j = 23^\circ$, angle sum of triangle 
- 7 a $\angle Y = 52^\circ$, alternate 
 so $a = 61^\circ$, angle sum of triangle 
- b $\angle BEA = 138^\circ$, co-interior  and $\angle EAS = 90^\circ$,
 supplementary 
 so $b = 60^\circ$, angle sum of quadrilateral $BEAS$ 
- c $\angle GHS = 127^\circ$, corresponding 
 so $c = 52^\circ$, exterior angle of triangle 
- d $\angle OHT = 36^\circ$, angle sum of triangle HOT 
 so $\angle SMH = 36^\circ$, alternate 
 so $d = 39^\circ$, angle sum of triangle SMH 
Alternative method
 $OHT = 36^\circ$, angle sum of triangle HOT 
 so $d = 39^\circ$, co-interior angle 
- e $\angle SBF = 110^\circ$, isosceles triangle  and angle
 sum of triangle SBF 
 so $\angle RSB = 110^\circ$, alternate 
 so $\angle RSE = 70^\circ$, supplementary 
 so $e = 40^\circ$, isosceles triangle  and angle sum
 of triangle RSE 
- f $\angle KML = 29^\circ$, angle sum of triangle 
 so $\angle KMD = 61^\circ$, complementary 
 so $f = 119^\circ$, co-interior 
- 8 a 50° , exterior angle of triangle 
 $b = 36^\circ$, isosceles triangle  and angle sum of
 triangle 
 $c = 45^\circ$, opposite angles of parallelogram  and
 angle sum of quadrilateral 
- 9 a 360°
 b Yes, because the sum of the exterior and
 interior angles will always equal $3 \times 180^\circ$
 (three straight lines).

Exercise 11.3

- 1 a  b $b + c = 180^\circ$
 c $c = 180^\circ - b$ d $a = c$
- 2 a  b $\angle ADC + x + \angle BDE = 180^\circ$
 c $\angle ADC = z$ and $\angle BDE = y$ or Statements 1 and 2
 d QED

- 3 a $x = y + z$ b $\angle XBA = y$
 c  d $x = y + z$
- 4 a $d + e = 180^\circ$ b $d + c = 180^\circ$
 c  d $d + e = 180^\circ$

5–10 Proofs

Chapter review

- 1 a  b  c  d 
- 2 a  b  c 
- 3 a a b i c b
 d m or j e h f l
- 4 a corresponding b supplementary
 c vertically opposite d alternate
 e angles at a point f corresponding
- 5 a  b  c  d 
- 6 a $u + x + q = 180^\circ$
 b $w + x + y = 180^\circ$ or $p + z + q = 180^\circ$
 c $x = w$ d $v = p + z$
 e $w + x + q + p + z + y = 360^\circ$
 f $p + v = 180^\circ$
 g $u + x = p + z$
- 7 a 63° , complementary 
 $b = 45^\circ$, angles at a point 
 $c = 68^\circ$, vertically opposite 
- 8 a $c = 120^\circ$, alternate 
 $d = 120^\circ$, corresponding , or vertically
 opposite 
 b $x = 58^\circ$, alternate 
 $y = 122^\circ$, supplementary 
- 9 a $a = 25^\circ$, supplementary 
 $b = 35^\circ$, exterior angle of triangle , or angle
 sum of triangle with $(a + 120^\circ)$ 
 $c = 107^\circ$, supplementary with $(b + 38^\circ)$ 
 $d = 240^\circ$, angles at a point 
 b $e = 71^\circ$, isosceles triangle 
 c $f = 72^\circ$, angle sum of quadrilateral 
- 10 a $\angle ABE = 98^\circ$, exterior angle of triangle 
 so $a = 87^\circ$, angle sum of quadrilateral 
 b $\angle HKL = 116^\circ$, alternate 
 so $\angle GHK = 46^\circ$, angle sum of quadrilateral 
 so $\angle IHJ = 46^\circ$, vertically opposite 

so $b = 18^\circ$, angle sum of triangle 

Alternative method

$\angle HKL = 116^\circ$, alternate 

so $\angle GHK = 46^\circ$, angle sum of quadrilateral 

so $\angle JHG = 134^\circ$, supplementary 

so $b = 18^\circ$, exterior angle of triangle 

- 11 a $d + b + c = 180^\circ$ b $a = d$ c 
 d $a + b + c = 180^\circ$ e $d + b + e = 180^\circ$

12–13 Proofs

- 14 a $a = 10^\circ$, supplementary 
 b $y = 85^\circ$, supplementary 

- 15 $b = 8^\circ$, isosceles triangle  and angle sum of triangle 

- 16 $a = 10^\circ$, supplementary 
 $b = 15^\circ$, corresponding with $(2a + 10^\circ)$ 

17–20 Proofs

Chapter 12

Exercise 12.1

- 1 a 225% b 65% c 42.3%
 d 43.75% e 87.4% f 27.3%
 g 62.5% h 84.2% i 237.5%
 j 53.85% k 536% l 475%

- 2 a $2\frac{1}{4}$ b $1\frac{1}{2}$ c $3\frac{1}{5}$ d $1\frac{13}{20}$
 e $1\frac{3}{10}$ f $\frac{113}{500}$ g $\frac{3}{8}$ h $2\frac{43}{100}$

- 3 a 0.036 b 0.248 c 0.003 d 2.2
 e 1.28 f 0.0146 g 0.438 h 0.159

- 4 a \$81 b 1.71 L c 270.98 kg
 d \$30.45 e \$8.53 f 1.309 77 kg
 g 12.992 km h 820.8 seconds

- 5 a $\frac{2}{5}$ b $\frac{11}{40}$ c $\frac{189}{200}$ d $\frac{113}{250}$
 e $\frac{167}{400}$ f $\frac{113}{500}$ g $2\frac{5}{12}$ h $\frac{67}{400}$

- 6 a 60 g b \$1410 c \$450
 d 0.92 kg e 34.67 L f 179.4 t
 g \$23.10 h 27 m

- 7 a 25% decrease b 25% decrease
 c 9% decrease d 9% decrease

- 8 a Kennebecs \$1.30, Brownells \$1.40, Pink-eyes \$2.45
 b Kennebecs 162.5%, Brownells 116.7%, Pink-eyes 140%
 c Kennebecs (best percentage profit)

- 9 Maths (70%) is better than English (60%).

- 10 Tran (4.71%) got better interest than Andrew (4.55%).

- 11 a \$175.75 b \$749.70
 c \$646 d \$80.34

- 12 a \$12 295.40 b \$14 533.74
 c \$16 432 d \$33 527

- 13 a \$3.74 b \$2.69 c 67c d \$2.98

- 14 a \$7.50 b 50% c 66.7%

- 15 a Importer \$10, wholesaler \$15, retailer \$40
 b Importer 18.2%, wholesaler 23.1%, retailer 50%
 c 91.3% d \$65
 e 118.2%, which is more than the total percentage profit

- 16 a \$175 b \$201.25 c \$261.63 d 86.9%

- 17 If the discounts are added (35%), he pays \$325.
 If they are applied in (either) order, he pays \$340.

- 18 2006–07, 1.87%; 2007–08, 1.88%; 2008–09, 1.85%;
 2009–10, 1.81%; 2010–11, 1.76%

- 19 a \$946.68 b \$288.12 c \$5.76 d \$8.54

- 20 a 2008–09, up 36.2%; 2009–10, up 28.6%; 2010–11,
 down 22.2%

- b The percentage changes are out of different amounts.

- 21 Down 16% (to 44 520 t)

- 22 Down 9% (to 455 bouquets)

Exercise 12.2

- 1 a \$11 648 b \$4838.40 c \$23 040
 d \$9715.20 e \$6272 f \$31 360
 g \$8000 h \$8576

- 2 a Profit 20% b Profit 25% c Loss 6%
 d Loss 11.25% e Profit 21.4%

- 3 a \$43.50 b \$17.40 c \$142.10
 d \$4.35 e \$108.75 f \$52.20
 g \$203 h \$36.25

- 4 a \$19.60 b \$81.60 c \$44
 d \$211.20 e \$63.75

Answers

- 5 a \$450 b \$900
 6 a \$3000 b \$5666.67
 7 40% 8 60% 9 \$572
 10 a 20% b \$750
 11 Guitars \$70, pianos \$8000 12 \$58

Exercise 12.3

- 1 a \$170.83 b \$152.64 c \$79.17
 d \$400 e \$219.58 f \$110.22
 2 Order is: interest, amount.
 a \$720, \$3720 b \$4128, \$12 728
 c \$1925, \$8925 d \$241.20, \$1441.20
 e \$885.60, \$3345.60 f \$561.60, \$4161.60
 3 Order is: amount, interest.
 a \$4969.19, \$969.19 b \$5962.59, \$962.59
 c \$2472.91, \$72.91 d \$2339.73, \$439.73
 e \$10493.21, \$2493.21
 4 a \$668.99 b \$868.50 c \$784.08
 d \$880.56 e \$2869.40 f \$2255.05
 5 a \$50 000 b \$66 000 c \$30 000
 d \$139 000 e \$142 000
 6 a \$4214.78, \$1214.78 b \$7804.48, \$2804.48
 c \$3306.25, \$806.25 d \$2143.83, \$343.83
 e \$7388.08, \$1388.08
 7 a 15 weeks b \$1440 c \$240 d 40%
 8 a \$319.80 b \$2399.80
 c \$800.80 d 31.3%
 9 \$287.78
 10 a \$6100 b \$330.42
 11 a \$180 b \$124.62
 12 a \$5.77 b 8 weeks
 13 a \$79 b \$54.69

- 14 \$76.10 15 \$1004.59
 16 a \$834.76 b \$852.48 c \$907.89
 17 \$241.90 18 \$466.80 19 \$771.60
 20 a \$1440 b \$1200
 c Lighthouse \$565, Discount Washstop \$325
 d Cash

- 21 a \$48 b \$33.23 c \$600 d \$120

- 22 \$67.69 23 \$874.74

- 24 a \$66 000 b \$81 000
 c \$90 000 d \$96 000

- 25 a \$72 000 b \$91 000
 c \$104 000 d \$113 000

- 26 a i \$126 864 ii \$154 742.40
 iii \$185 280 iv \$218 088
 b The amount repaid is much greater than the loan. The amount is greater for longer loans.

Chapter review

- 1 a 78.6% b 440%
 2 2.3, $2\frac{3}{10}$ 3 \$136 4 \$250 5 $\frac{19}{400}$
 6 112 kg 7 \$760 8 \$240
 9 Interest \$252, final amount \$1452 10 \$7029.96
 11 36% decrease 12 \$3300 13 \$877.37
 14 Geography (73.5%) was better than History (68.2%).
 15 39.3% 16 \$250
 17 a \$156 b \$124.80 c 4%
 18 a \$171 b \$52.62
 19 \$236.10 20 9% decrease
 21 a 15% b No discount
 22 \$90 000

Word	Definition
adjacent (angle)	Angle sharing a common arm with another.
adjacent side	In a right-angled triangle, the side joining the right angle and nominated angle. <i>See also</i> hypotenuse, opposite.
allied (angles)	Inside angles on the same side of a transversal over two lines. <i>See also</i> co-interior angles.
alternate (angles)	Inside opposite (or outside opposite) angles on a transversal over two lines.
and	For statements P and Q in mathematical logic, P and Q means both P and Q . <i>See also</i> or.
back-to-back	Display for data for two groups, one on the left and the other on the right.
backtrack	Undo operations of an equation to find the solution.
base (power)	The bottom number of a power, such as the 5 in 5^4 .
base (shape)	The 'bottom' of a prism or pyramid.
bias	Data or method that is likely to give results generally different to the population. <i>See also</i> fair.
bimodal	Having two modes; two scores with equal highest frequency.
binomial products, binomial brackets	Product of two brackets, such as $(2x + 3)(4x - 1)$.
bivariate	Data set where each item has two measurement variables. <i>See also</i> univariate.
Cartesian plane	A flat surface with horizontal and vertical axes intersecting at 0.
categorical (data)	Data that can be put into categories, such as names of colours.
certain	Will always happen.
coefficient	Number in front of a variable; number that is multiplied by a variable.
co-interior (angles)	Inside angles on the same side of a transversal over two lines. <i>See also</i> allied angles.
collect (like terms)	Add or subtract like terms to simplify an expression.
commission	Money paid for a service such as selling goods.
common factor	Number or algebraic term that is a factor of each of some given numbers or terms.
common fraction	Fraction shown with a denominator and numerator, such as $\frac{5}{8}$. <i>See also</i> vulgar fraction.
commutative (law)	Addition (or multiplication) of two numbers is the same for either number order.
complementary (angles)	Adjacent angles that together form a right angle (90°).
complementary (events)	Two separate events that together make the entire sample space.

Word	Definition
compound interest	Interest calculated on the current balance of a loan or investment. <i>Also</i> reducing interest.
congruent (shapes), congruence	Shapes that are the same. They may be exactly superimposed, possibly by turning one over.
constant	A number, particularly a term of an expression that is just a number.
continuous (data)	Data that can have any values, such as heights.
converse	For logic or mathematical theorems, the reverse statement.
coordinates	The numbers showing position in a Cartesian plane; e.g. (3, 5).
corresponding (angles)	Angles on the same side of a transversal and the same sides of the lines it crosses.
cosine, cos	The adjacent side divided by the hypotenuse for a nominated angle in a right-angled triangle. <i>See also</i> sine, tangent.
cross-multiplication, cross-product	Multiply the first part of each ratio by the second of the other to determine equivalence.
cross-section	The 2D shape on a flat surface made by cutting through a solid (3D) shape.
cube (number)	The third power of a number, such as 5^3 .
cumulative frequency	The progressive total of frequencies of ordered numeric data.
curved surface area	The area of the curved face of a cone, cylinder, or sphere.
degree	Angle measure such that a complete revolution is 360 degrees, written as 360° . <i>See also</i> minute, second.
direct proportion	The variation of two variables where one is the product of a constant with the other. <i>See also</i> inverse proportion, joint proportion.
directed number	Any of zero, positive or negative whole numbers. <i>See also</i> integer.
discount	An amount taken off the usual price of an article.
discrete (data)	Data that can only have particular values, such as shoe sizes.
distributive (law)	The product of a number with the sum (or difference) of two others is the same as the sum (or difference) of the number with each of the other two; $a(b \pm c) = ab \pm ac$.
element	A basic probability outcome; '5' on a die. <i>See also</i> sample point.
equivalent ratios	Equal ratios.
event (probability)	An individual outcome or collection of outcomes.
expand	Multiply out brackets using the distributive law.
expected frequency	The number of times an event is expected from a particular number of trials.
experimental probability	The frequency of an outcome divided by the total frequency. <i>See also</i> relative frequency.

Word	Definition
exponent	The raised number of a power, such as the 4 in 5^4 ; plural <i>indices</i> . <i>See also</i> index.
expression	Variables and/or numbers connected by operations.
exterior (angle of a triangle)	Angle on the outside of a triangle made by extending one side.
factorise	Use the distributive law to change an expression into factors, usually including a bracketed factor.
fair	A sample/question/method that does not favour particular results and is truly representative. <i>See also</i> bias.
favourable outcome	The particular outcome being considered.
flat-rate interest	Interest calculated on the initial value of a loan or investment. <i>See also</i> simple interest.
formal solution	Show the steps of the solution of a equation using inverse operations.
formula	An equation that gives one quantity in terms of others. The quantity on its own is called the subject.
function	A rule that changes a number (input) to a new one (output).
given	For a statement P in mathematical logic, given P means ‘assuming that P is true’.
Goods and Services Tax (GST)	Tax paid on the selling price of goods or service; currently 10% in Australia.
gradient	Property of a straight line obtained by dividing the vertical distance between two points (rise) by the horizontal (run) distance. <i>See also</i> slope.
gradient-intercept form	The equation of a straight line in the form $y = mx + c$, where m is the gradient and c is the y -intercept of the line.
hectare	The area of a square 100 m by 100m; 10 000 m ² .
highest common factor (HCF)	The biggest number that is a factor of given numbers.
hypotenuse	The long side of a right-angled triangle (opposite the right-angle).
impossible	Will never happen.
index	The raised number of a power, such as the 4 in 5^4 ; plural <i>indices</i> . <i>See also</i> exponent.
inspection (solving by)	Solving an equation by (mentally) substituting values for the variable.
instalment	Equal payments used to repay a loan at fixed intervals, such as monthly instalments.
integer	Any of zero, positive or negative whole numbers. <i>See also</i> directed number.

Word	Definition
intercept	Position where a line intersects an axis.
interest rate	Interest stated as a yearly percentage of the amount borrowed.
interest	Charge made for the use of money (obtained as a loan). The amount of the loan is called the principal.
inverse operations	Operations that do the reverse of each other; $\times$ and $\div$, etc.
inverse proportion	The variation of two variables where the product of the variables is a constant. <i>See also</i> inverse proportion, joint proportion.
isosceles (triangle)	Triangle with two sides equal.
joint proportion	Variation where one variable is proportional to an algebraic expression involving the other variables. <i>See also</i> direct proportion, inverse proportion.
kite	Quadrilateral with two pairs of equal sides, each pair joined at one vertex.
like terms	Terms of an expression containing exactly the same variable parts.
line symmetry	Property of a shape with one or more lines of symmetry.
linear (rule, function)	A rule or function whose graph is a straight line.
loss	Difference between the cost price and selling price of an article when the selling price is lower than the cost price.
lowest common denominator (LCD)	The smallest number that is a denominator of all given fractions.
mantissa	The value part of a number in scientific notation. The mantissa of 3.52×10^4 is '3.52'.
mark-up	Difference between the price for which a retailer buys and sells an item.
mean	The arithmetic average, $\bar{x} = \text{total} \div \text{number}$; for 3, 6, 12, $\bar{x} = 21 \div 3 = 7$
median	The central value by count of ordered statistical data.
midpoint	The point halfway between two given points.
minute (angle)	$\frac{1}{60}$ of a degree. <i>See also</i> second.
mode	The value that occurs most often in statistical data; the score with the highest frequency.
nominal	Categorical data with no natural order, such as names of countries.
numeric, numerical (data)	Data that are numbers.
opposite side	In a right-angled triangle, the short side not joining the right angle, opposite the nominated angle. <i>See also</i> adjacent, hypotenuse.
or	For statements P and Q in mathematical logic, P or Q means <i>at least one of</i> P and Q . <i>See also</i> and.

Word	Definition
ordinal	Categorical data having a natural order, such as places in a race.
outcomes	The possible results of a trial.
outlier	A data value well outside the range of most of the numerical data in a set.
parabola	A mathematical relation such as $y = 3x^2 + 2$. Its graph is a symmetric U-shape with sides curving away from each other.
parallel	Lines or surfaces in the same direction that do not intersect.
parallelogram	A quadrilateral whose opposite sides are parallel.
percentage	A number written as a fraction out of 100, with the vinculum and 100 replaced by %; $\frac{2}{5} = 40\%$.
perpendicular	A line through another line or surface at an angle of 90° .
population	The entire group that could be relevant to a survey.
power	A number multiplied by itself, written as 5^4 , say, for $5 \times 5 \times 5 \times 5$.
principal	The initial amount of a loan.
prism	Polyhedron where every cross-section parallel to one face is identical to that face. <i>See also</i> base.
probability	The chance of an event; experimental or theoretical probability.
profit	The difference between selling price and buying price when the selling price is greater.
proof	A series of mathematical statements that logically show that a statement is true. <i>See also</i> QED, RTP, theorem.
pyramid	Polyhedron where cross-sections parallel to one face are similar and reduce evenly in size to a vertex called the apex opposite this face.
Pythagoras' theorem	Relationship between the sides of a right-angled triangle attributed to Pythagoras (569?–475? BCE).
Pythagorean triple	Three positive integers a, b, c such that $a^2 + b^2 = c^2$, that could be sides of a right-angled triangle.
QED	Quod erat demonstrandum; Latin phrase at the end of a formal proof to say it is finished. <i>See also</i> proof, RTP, theorem.
quadrilateral	Flat (2D) closed shape with four straight sides.
quartile	Any of 3 points that divide an ordered data set into 4 equal groups. The highest element may be called the 4th quartile.
random sample	Sample selected using a method that gives every element of a population an equal chance of being selected.
range	The difference between the highest and lowest scores of numeric data.
rate	The result of dividing one quantity by a different related quantity; \$2.40/kg.

Word	Definition
ratio	Comparison of quantities of the same kind.
ready reckoner	Table of numbers compiled to simplify calculations.
reducing interest	Interest calculated on the current balance of a loan. <i>See also</i> compound interest.
relative frequency	The frequency of an outcome divided by the total frequency. <i>See also</i> experimental probability.
rest	Time between regular calculations of compound interest.
retainer	When working on commission, an amount paid without consideration of sales.
rhombus	A quadrilateral with all sides the same.
rise	The vertical distance between two points on a graph. <i>See also</i> gradient.
root (indices)	Reverse of raising to a power. A square root, cube root, fourth root, etc. <i>See also</i> surd.
root (equation)	A solution of an equation.
rotational symmetry	When a shape can be rotated less than 360° to make the same shape.
RTP	Required to prove; statement of a theorem before its formal proof. <i>See also</i> proof, QED, theorem.
run	The horizontal distance between two points on a graph. <i>See also</i> gradient.
sample point	A basic probability outcome; '5' on a die. <i>See also</i> element.
sample space	List of all the simple outcomes of a situation. <i>See also</i> event space
sample	Part of a population (usually used for a survey).
satisfy	Make true; (3, 7) satisfies $y = 2x + 1$.
scale factor	The ratio of the sizes of an enlargement or reduction of a shape to the size of the original shape.
scatter plot	Graph with points showing bivariate data as coordinates.
scientific notation	Notation used to write a value as a number between 1 and 10, multiplied by a power of 10.
second (angle)	$\frac{1}{60}$ of a minute = $\frac{1}{3600}$ of a degree.
significant figures	The (number) of digits in the mantissa when a number is written in scientific notation.
similar	Shapes that are enlargements or reductions of each other.
similarity	The property of shapes being enlargements or reductions of each other.
simple interest	Interest calculated on the initial value of a loan or investment. <i>See also</i> flat-rate interest.
simple interest	Interest charged as a percentage of the initial loan, multiplied by the period of the loan.

Word	Definition
simplification, simplify	Multiply out brackets and/or collect like terms.
sine, sin	The opposite side divided by the hypotenuse for a nominated angle in a right-angled triangle. <i>See also</i> cosine, tangent.
skewed (distribution)	An asymmetrical set of scores with a unequal number of scores above and below the mean.
slope	Property of a straight line obtained by dividing the vertical distance between two points (rise) by the horizontal (run) distance. <i>See also</i> gradient.
solution	The answer to a problem, particularly an equation.
solve	Find the answer to a problem, particularly an equation.
spread	Measure of the differences between the values of some data.
square (number)	The product of a number with itself.
standard form	The equation of a straight line in the form $ax + by + c = 0$.
stem-and-leaf plot	Display with high place values listed vertically and the other places listed horizontally.
subject	The variable on its own (on the left) that is calculated from others in a formula.
supplementary (angles)	Adjacent angles that form a straight line; their sum is 180° .
surd (1)	The symbol $\sqrt{\quad}$ used to show a root. <i>See also</i> root (indices).
surd (2)	Root of a rational number (especially an integer) that is not rational.
surface area	The area of a surface, particularly the total area of the faces of a solid shape.
tangent, tan	The opposite side divided by the adjacent for a nominated angle in a right-angled triangle. <i>See also</i> cosine, sine.
T-Chart	Table used to organise the steps of a proof.
term (patterns)	One of the values of a number pattern; 6 is the second term of 3, 6, 9, 12, 15, . . .
term (algebra)	Part of an expression separated from the other parts by addition or subtraction.
theorem	Precise mathematical statement capable of formal proof consisting of the parts RTP, proof, QED.
theoretical probability	The number of simple outcomes in a probability event divided by the total number of simple outcomes.
time zone	A region of the Earth where all the clocks are set to the same time.
transformation	Change in a shape.
translation	Sliding movement of a shape; all points slid the same amount in the same direction.

Word	Definition
transversal	A (straight) line that crosses two or more other lines.
trapezium	A quadrilateral with one pair of opposite sides parallel.
tree diagram	Diagram with many branches showing the outcomes in a multi-stage situation.
trigonometric ratio	Commonly, any of $\cos \theta$, $\sin \theta$ or $\tan \theta$ for an angle θ . May also refer to $\operatorname{cosec} \theta$, $\cot \theta$ or $\sec \theta$.
turning point	Point on a curved line such as a parabola where it changes direction.
two-way table	A table that shows one set of data properties or outcomes across another set down so that each cell shows their intersection.
unitary method	Method that uses the value of a unit quantity to calculate a result.
univariate	Data set where each item has only one measurement variable. <i>See also</i> bivariate.
universal set	Set of all elements under consideration in a particular context. <i>See also</i> Venn diagram.
variable	A symbol, particularly a letter, that stands for a value.
variation	Differences in outcomes as a result of using different samples in probability or statistics.
Venn diagram	A diagram that shows the elements of events (or sets) in overlapping circles within a rectangle representing the universal set.
vertex	The intersection of two sides of a polygon or three or more faces of a solid shape; plural <i>vertices</i> .
vertically opposite (angles)	Opposite angles at the intersection of two lines; they are not adjacent.
<i>x</i>-axis	The horizontal axis of a Cartesian plane.
<i>x</i>-coordinate	The first coordinate of a point in the Cartesian plane; the 4 in (4, 3).
<i>y</i>-axis	The vertical axis of a Cartesian plane.
<i>y</i>-coordinate	The second coordinate of a point in the Cartesian plane; the 3 in (4, 3).

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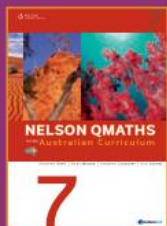
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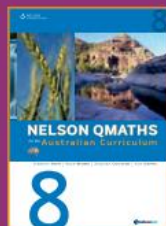
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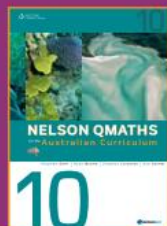
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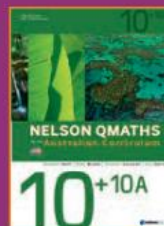
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