NELSON QMATHS

GENERAL MATHEMATICS

YEAR 11

Ross Brodie
Regina Edwards
Stephen Swift

4TH EDITION

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Regina Edwards
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For the most current syllabus versions and curriculum information please refer to the QCAA website https://www.qcaa.qld.edu.au/.

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PREFACE

Nelson QMaths, Queensland's longest-running senior mathematics series, has been rewritten for the new syllabuses and assessment procedures for implementation from 2019. Based on the Australian Curriculum, four new senior mathematics courses have been introduced into Queensland schools.

- Essential Mathematics
- General Mathematics
- Mathematical Methods
- Specialist Mathematics

With the introduction of external QCE examinations, *Nelson QMaths* will have a renewed focus on assessment, and will include features such as topic tests, practice examinations, *ExamView* question banks and chapter quizzes, video tutorials and worked solutions to all exercise questions. In this book, teachers will find familiar features such as clear worked examples, graded exercises, strong syllabus coverage, Investigations, Technology, chapter summaries, chapter reviews and a glossary/index. We wish all teachers and students using this book every success in embracing the new mathematics courses.

ABOUT THE AUTHORS

Ross Brodie has taught mathematics and been Head of Mathematics at a variety of regional and urban schools in Queensland. He is an experienced teacher and author, having co-written *Nelson QMaths 7–10*, *New QMaths 11–12* and *Nelson Senior Maths 11–12* (Australian Curriculum).

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Sharon Kirkby (Moreton Bay College) wrote many of the *NelsonNet* worksheets

John Drake, **Katie Jackson** and **Joanne Magner** created the video tutorials.

Trisha Goss wrote the topic tests.

Steven Purse and **Greg Neal** wrote the worked solutions to all exercise sets.

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SYLLABUS REFERENCE GRID

Topics and subtopics	*Nelson QMaths 11 General Mathematics chapter*
UNIT 1: MONEY, MEASUREMENT AND RELATIONS	
Consumer arithmetic	
Applications of rates, percentages and use of spreadsheets	1 Incomes and budgets 4 Prices and interest
Shape and measurement	
Pythagoras' theorem	2 Length, area and volume
Mensuration	2 Length, area and volume 5 Surface area and similar figures
Similar figures and scale factors	5 Surface area and similar figures
Linear equations and their graphs	
Linear equations	3 Linear equations and graphs
Straight-line graphs and their applications	3 Linear equations and graphs
Simultaneous linear equations and their applications	6 Simultaneous equations and piece-wise graphs
Piece-wise linear graphs and step graphs	6 Simultaneous equations and piece-wise graphs
UNIT 2: APPLIED TRIGONOMETRY, ALGEBRA, MATRICES AND UNIVARIATE DATA	
Applications of trigonometry	
Applications of trigonometry	8 Trigonometry of right-angled triangles 11 Trigonometry of general triangles
Algebra and matrices	
Linear and non-linear relationships	7 Linear and non-linear expressions
Matrices and matrix arithmetic	10 Matrices
Univariate data analysis	
Making sense of data relating to a single statistical variable	9 Data and statistics
Comparing data for a numerical variable across two or more groups	12 Comparing data

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ABOUT THIS BOOK

AT THE BEGINNING OF EACH CHAPTER

- Each chapter begins on a double-page spread showing the **Chapter contents** and a list of syllabus subject matter (© Queensland Curriculum and Assessment Authority 2017)

CONSUMER ARITHMETIC

1.

INCOMES AND BUDGETS

'You can't buy happiness', but there are lots of things you do need to buy. Getting money for ordinary living is important. Spending your money on what you really want is just as important. You need to plan what you do with your money to get the things you want. Otherwise you may fritter it away and have nothing to show for it. In this chapter you will look at earning and spending.

1.01 Percentages
1.02 Calculation of percentages
1.03 Calculation with percentages
1.04 Wages and salaries
1.05 Overtime and penalty rates
1.06 Commission and piecework
1.07 Allowances and pensions
1.08 Personal budgets
Chapter summary
Chapter review

SYLLABUS SUBJECT MATTER

Application of rates and percentages

- review definitions of rates and percentages
- calculate weekly or monthly wage from an annual salary, wages from an hourly rate including situations involving overtime and other allowances and earnings based on commission or piecework
- calculate payments based on government allowances and pensions, e.g. various youth allowances for unemployment, disability and study
- prepare a personal budget for a given income taking into account fixed and discretionary spending
- use a spreadsheet to display examples of the above computations when multiple or repeated computations are required e.g. preparing a wagesheet displaying the weekly earnings of workers in a fast food store where hours of employment and hourly rates of pay may differ, preparing a budget, or investigating the potential cost of owning and operating a car over a year

Real learning

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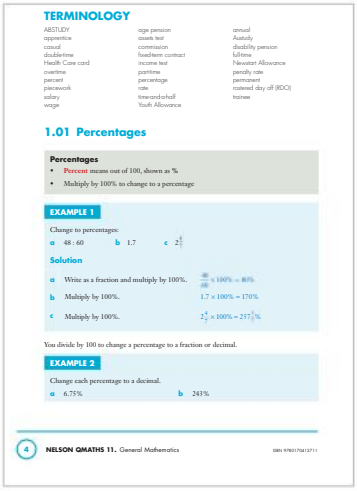
- **Terminology** previews the key words and phrases from within the chapter.



TERMINOLOGY

ABSTUDY
apprentice
casual
doubletime
Health Care card
overtime
percent
piecework
salary
wage
age pension
assets test
commission
fixedterm contract
income test
parttime
percentage
rate
timeandahalf
Youth Allowance
annual
Austudy
disability pension
fulltime
Newstart Allowance
penalty rate
permanent
rostered day off (RDO)
trainee

1.01 Percentages

Percentages

- Percent means out of 100, shown as %
- Multiply by 100% to change to a percentage

EXAMPLE 1

Change to percentages:

a $48 : 60$ b 1.7 c $2\frac{1}{2}$

Solution

a Write as a fraction and multiply by 100%. $\frac{48}{60} \times 100\% = 80\%$

b Multiply by 100%. $1.7 \times 100\% = 170\%$

c Multiply by 100%. $2\frac{1}{2} \times 100\% = 257\frac{1}{2}\%$

You divide by 100 to change a percentage to a fraction or decimal.

EXAMPLE 2

Change each percentage to a decimal.

a 6.75% b 243%

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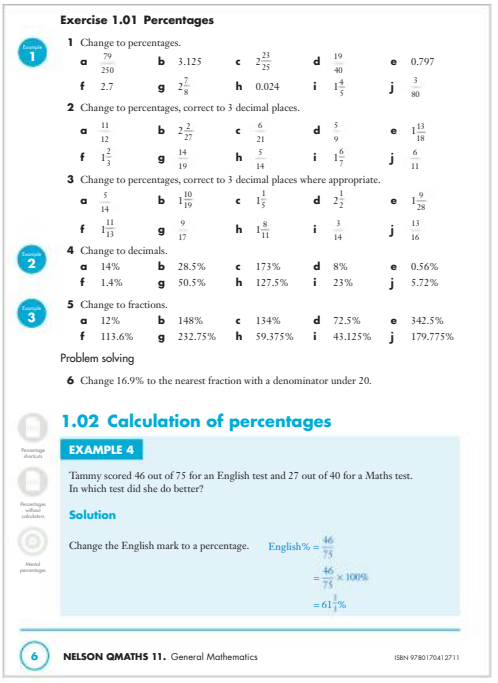
IN EACH CHAPTER

- Worked examples are explained clearly step-by-step, with the mathematical working shown on the right-hand side.
- Important facts and formulas are highlighted in a shaded box.
- Important words and phrases are printed in red and listed in the glossary at the back of the book.
- Graded exercises include **Problem solving** questions, are linked to worked examples and include exam-style problems and realistic applications.
- Worked solutions to all exercise questions are provided on the *NelsonNet* teacher website.
- **Investigations** explore the syllabus in more detail, providing ideas for modelling activities and assessment tasks.
- **Technology** promotes ICT in the classroom, using spreadsheets and the Internet.



Exercise 1.01 Percentages

Example 1

1 Change to percentages.

a $\frac{79}{250}$ **b** 3.125 **c** $2\frac{23}{25}$ **d** $\frac{19}{40}$ **e** 0.797

f 2.7 **g** $2\frac{7}{8}$ **h** 0.024 **i** $1\frac{4}{5}$ **j** $\frac{3}{80}$

2 Change to percentages, correct to 3 decimal places.

a $\frac{11}{12}$ **b** $2\frac{2}{27}$ **c** $\frac{6}{21}$ **d** $\frac{5}{9}$ **e** $1\frac{13}{18}$

f $1\frac{2}{3}$ **g** $\frac{14}{19}$ **h** $\frac{5}{14}$ **i** $1\frac{6}{7}$ **j** $\frac{6}{11}$

3 Change to percentages, correct to 3 decimal places where appropriate.

a $\frac{5}{14}$ **b** $1\frac{10}{19}$ **c** $1\frac{1}{5}$ **d** $2\frac{1}{2}$ **e** $1\frac{9}{28}$

f $1\frac{11}{13}$ **g** $\frac{9}{17}$ **h** $1\frac{8}{11}$ **i** $\frac{3}{14}$ **j** $\frac{13}{16}$

Example 2

4 Change to decimals.

a 14% **b** 28.5% **c** 173% **d** 8% **e** 0.56%

f 1.4% **g** 50.5% **h** 127.5% **i** 23% **j** 5.72%

Example 3

5 Change to fractions.

a 12% **b** 148% **c** 134% **d** 72.5% **e** 342.5%

f 113.6% **g** 232.75% **h** 59.375% **i** 43.125% **j** 179.775%

Problem solving

6 Change 16.9% to the nearest fraction with a denominator under 20.

1.02 Calculation of percentages

Percentage shortcuts

Percentages without calculators

Mental percentages

EXAMPLE 4

Tammy scored 46 out of 75 for an English test and 27 out of 40 for a Maths test. In which test did she do better?

Solution

Change the English mark to a percentage.

$$\text{English\%} = \frac{46}{75}$$
$$= \frac{46}{75} \times 100\%$$
$$= 61\tfrac{1}{3}\%$$

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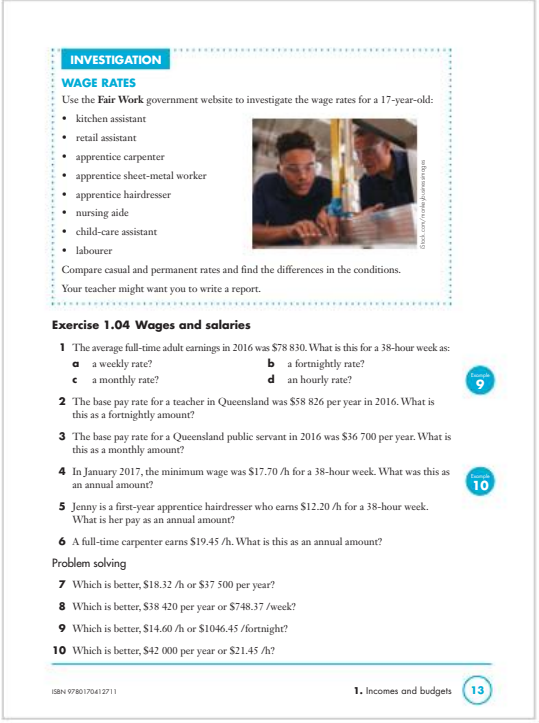


INVESTIGATION

WAGE RATES

Use the **Fair Work** government website to investigate the wage rates for a 17-year-old:

- kitchen assistant
- retail assistant
- apprentice carpenter
- apprentice sheet-metal worker
- apprentice hairdresser
- nursing aide
- child-care assistant
- labourer

Compare casual and permanent rates and find the differences in the conditions.

Your teacher might want you to write a report.

Exercise 1.04 Wages and salaries

Example 9

1 The average full-time adult earnings in 2016 was $78 830. What is this for a 38-hour week as:

a a weekly rate? **b** a fortnightly rate?

c a monthly rate? **d** an hourly rate?

2 The base pay rate for a teacher in Queensland was $58 826 per year in 2016. What is this as a fortnightly amount?

3 The base pay rate for a Queensland public servant in 2016 was $36 700 per year. What is this as a monthly amount?

Example 10

4 In January 2017, the minimum wage was $17.70 /h for a 38-hour week. What was this as an annual amount?

5 Jenny is a first-year apprentice hairdresser who earns $12.20 /h for a 38-hour week. What is her pay as an annual amount?

6 A full-time carpenter earns $19.45 /h. What is this as an annual amount?

Problem solving

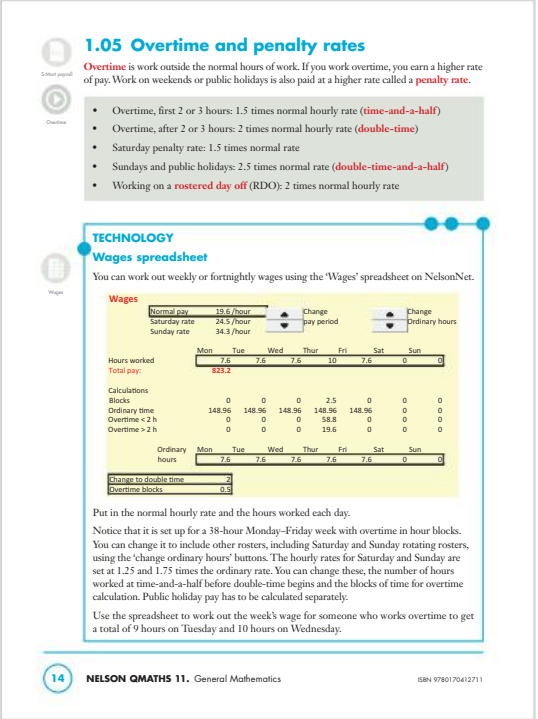
7 Which is better, $18.32 /h or $37 500 per year?

8 Which is better, $38 420 per year or $748.37 /week?

9 Which is better, $14.60 /h or $1046.45 /fortnight?

10 Which is better, $42 000 per year or $21.45 /h?

ISBN 9780170412711 **1.** Incomes and budgets 13



1.05 Overtime and penalty rates

S-Mart payroll

Overtime

Overtime is work outside the normal hours of work. If you work overtime, you earn a higher rate of pay. Work on weekends or public holidays is also paid at a higher rate called a **penalty rate**.

- Overtime, first 2 or 3 hours: 1.5 times normal hourly rate (**time-and-a-half**)
- Overtime, after 2 or 3 hours: 2 times normal hourly rate (**double-time**)
- Saturday penalty rate: 1.5 times normal rate
- Sundays and public holidays: 2.5 times normal rate (**double-time-and-a-half**)
- Working on a **rostered day off** (RDO): 2 times normal hourly rate

TECHNOLOGY

Wages spreadsheet

Wages

You can work out weekly or fortnightly wages using the 'Wages' spreadsheet on NelsonNet.

Wages

Normal pay	19.6 /hour
Saturday rate	24.5 /hour
Sunday rate	34.3 /hour

Change pay period

Change Ordinary hours

	Mon	Tue	Wed	Thur	Fri	Sat	Sun
Hours worked	7.6	7.6	7.6	10	7.6	0	0
Total pay:	823.2						

Calculations

Blocks	0	0	0	2.5	0	0	0
Ordinary time	148.96	148.96	148.96	148.96	148.96	0	0
Overtime < 2 h	0	0	0	58.8	0	0	0
Overtime > 2 h	0	0	0	19.6	0	0	0

	Mon	Tue	Wed	Thur	Fri	Sat	Sun
Ordinary hours	7.6	7.6	7.6	7.6	7.6	0	0

Change to double time	2
Overtime blocks	0.5

Put in the normal hourly rate and the hours worked each day.

Notice that it is set up for a 38-hour Monday–Friday week with overtime in hour blocks. You can change it to include other rosters, including Saturday and Sunday rotating rosters, using the 'change ordinary hours' buttons. The hourly rates for Saturday and Sunday are set at 1.25 and 1.75 times the ordinary rate. You can change these, the number of hours worked at time-and-a-half before double-time begins and the blocks of time for overtime calculation. Public holiday pay has to be calculated separately.

Use the spreadsheet to work out the week's wage for someone who works overtime to get a total of 9 hours on Tuesday and 10 hours on Wednesday.

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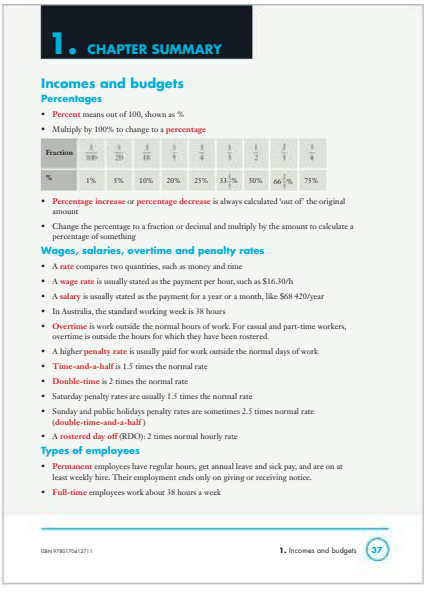
AT THE END OF EACH CHAPTER

- **Chapter summary** summarises the key concepts, skills and formulas of the chapter.
- **Chapter review** contains revision exercises and are linked to worked examples.
- **Practice examinations** after every three chapters revise the skills and knowledge of those chapters.



1. CHAPTER SUMMARY

Incomes and budgets

Percentages

- **Percent** means out of 100, shown as %
- Multiply by 100% to change to a **percentage**

Fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
%	1%	5%	10%	20%	25%	$33\frac{1}{3}$%	50%	$66\frac{2}{3}$%	75%

- **Percentage increase** or **percentage decrease** is always calculated 'out of' the original amount
- Change the percentage to a fraction or decimal and multiply by the amount to calculate a percentage of something

Wages, salaries, overtime and penalty rates

- A **rate** compares two quantities, such as money and time
- A **wage rate** is usually stated as the payment per hour, such as $16.30/h
- A **salary** is usually stated as the payment for a year or a month, like $68 420/year
- In Australia, the standard working week is 38 hours
- **Overtime** is work outside the normal hours of work. For casual and part-time workers, overtime is outside the hours for which they have been rostered.
- A higher **penalty rate** is usually paid for work outside the normal days of work
- **Time-and-a-half** is 1.5 times the normal rate
- **Double-time** is 2 times the normal rate
- Saturday penalty rates are usually 1.5 times the normal rate
- Sunday and public holidays penalty rates are sometimes 2.5 times normal rate (**double-time-and-a-half**)
- A **rostered day off** (RDO): 2 times normal hourly rate

Types of employees

- **Permanent** employees have regular hours, get annual leave and sick pay, and are on at least weekly hire. Their employment ends only on giving or receiving notice.
- **Full-time** employees work about 38 hours a week

ISBN 9780170412711 1. Incomes and budgets 37

AT THE END OF THE BOOK

- **Glossary and index** includes a comprehensive dictionary of course terminology.
- **Answers** (with worked solutions on the teacher website).

NELSONNET STUDENT WEBSITE

Margin icons link to print (PDF) and multimedia resources found on the *NelsonNet* student website, **www.nelsonnet.com.au**. These include:

- **Worksheets** and **puzzle sheets** that are write-in enabled PDFs
- **Prior learning** exercises that revise prerequisite skills for the chapter
- **Skillsheets** of examples and exercises of prerequisite skills and knowledge
- **Video tutorials**: worked examples explained by 'flipped classroom' teachers
- **Spreadsheets**: *Excel* files
- ***ExamView* quizzes**: interactive and self-marking

NELSONNET TEACHER WEBSITE

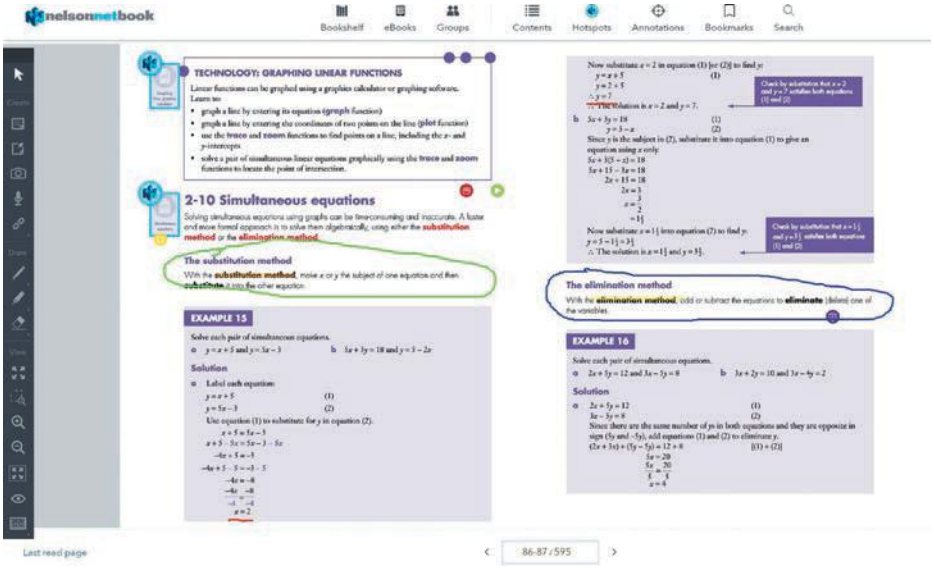
The *NelsonNet* teacher website, also at **www.nelsonnet.com.au**, contains:

- a **teaching plan**, in Microsoft Word and PDF formats
- **Topic tests**, in Microsoft Word and PDF formats
- **Worked solutions** to each exercise set
- **Chapter PDFs** of the textbook
- **ExamView** exam-writing software and question banks
- **Resource Finder**: search engine for *NelsonNet* resources.

NELSONNETBOOK

NelsonNetBook is the web-based interactive version of this book found on *NelsonNet*.

- To each page of **NelsonNetBook** you can add notes, voice and sound bites, highlighting, weblinks and bookmarks
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students



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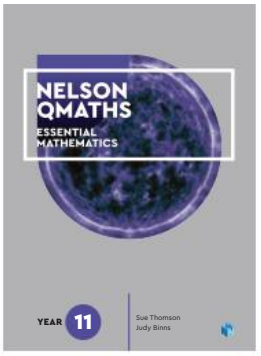

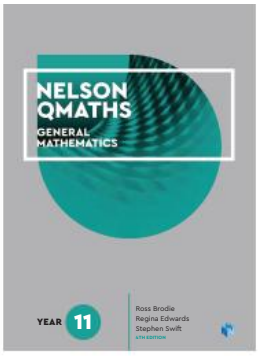

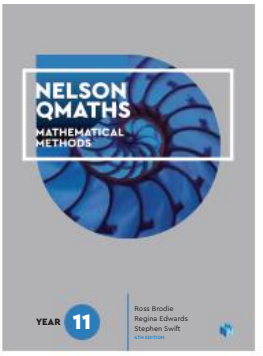

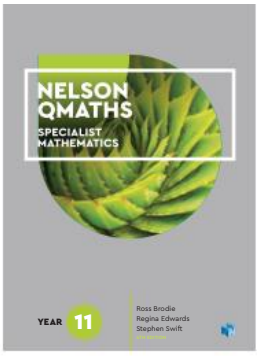

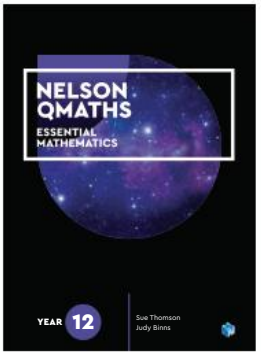

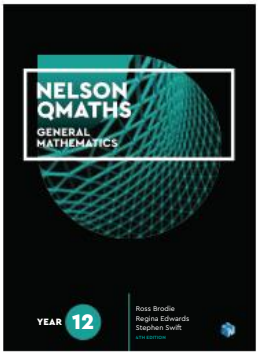

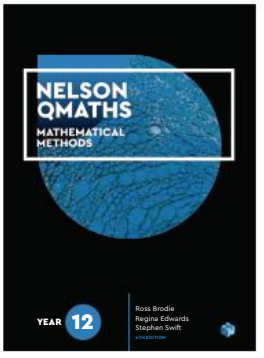

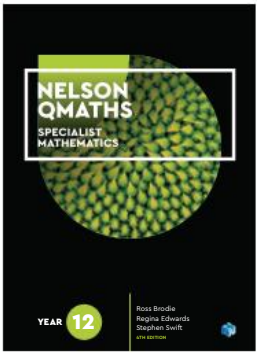

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LIST OF SYMBOLS AND ABBREVIATIONS

$=$	is equal to
$\neq$	is not equal to
$\approx$	is approximately equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$\pm$	plus or minus
π	pi ≈ 3.14
$\equiv$	is congruent to
$\parallel\!\mid$	is similar to
…	and so on
$^\circ$	degree
$\angle$	angle
$\triangle$	triangle
$\parallel$	is parallel to
$\perp$	is perpendicular to
$\therefore$	therefore
$\sqrt{\ }$	square root, radical sign
SA	surface area
CSA	curved surface area
S 37° W	a compass bearing
217°T	a true bearing
AUD	Australian dollar
GST	goods and services tax
ATSI	Aboriginal and Torres Strait Islander
CPI	consumer price index
PE	price-earnings ratio
RDO	rostered day off
RRP	recommended retail price
$\overline{A}, A'$	not A, the complement of A
S	sample space
$n(A)$	the number of elements in A
$P(A)$	the probability of A
LHS	left-hand side
RHS	right-hand side
m	gradient of a straight line
c	y-intercept of a straight line
(x, y)	Cartesian coordinates, ordered pair
AA, AAA	angle-angle test
RHS	right-angle hypotenuse side test
SAS	side angle side test
SSS	side side side test
%	percentage
$\mathbf{A}$, $\underset{\sim}{A}$	matrix A
a_{34}	element in 3rd row and 4th column of matrix $\mathbf{A}$
$\mathbf{I}$	identity matrix
cos	cosine ratio
sin	sine ratio
tan	tangent ratio
$\bar{x}$	mean
Σ	the sum of
Q_0	minimum value
Q_1	first quartile or lower quartile
Q_2	median (second quartile)
Q_3	third quartile or upper quartile
Q_4	maximum value
IQR	interquartile range
SD, s	standard deviation
σ	standard deviation of a population
CV	coefficient of variation

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LIST OF MATHEMATICAL VERBS

A glossary of 'doing words' commonly found in mathematics problems

analyse: study and state in detail the relationship of parts of a situation

apply: use knowledge or a procedure in a given situation

calculate: find a numerical value

comment: express an opinion or judgement about a statement or calculation

communicate: transmit information to others

compare: state similarities and differences and their significance

consider: take into account

construct: draw an accurate diagram or logically arrange items or ideas

convert: change from one form to another

define: give the meaning of or identify in exact terms

demonstrate: show to be correct

describe: state the features of a situation, object, pattern, event, etc.

determine: find the answer or make a decision using evidence

discuss: give reasons based on evidence for and against a position or proposal

establish: introduce and develop a result

evaluate: find the value of or state the application, strengths and limitations of a solution

examine: state the details and assumptions of a situation

expand: remove brackets or change a product to an extended sum of terms

explain: state the meaning in logical detail

explore: examine or state the details and assumptions of a situation

factorise: convert to factors or change a sum of terms to a product of factors

formulate: give mathematical expression to an idea or situation

hence find/prove: find an answer or prove a result using previous answers or information supplied

identify: state the type, name or distinguishing feature of an item or situation

interpret: state a conclusion or trend from given information

investigate: establish facts, trends or conclusions from collected information

isolate a variable: express a formula or equation with one variable only, usually on the left-hand side (LHS)

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justify: give reasons or evidence for an answer or conclusion

modify: change to accommodate different information

obtain: find an answer or conclusion

prove: use logical steps to establish the truth of

recall: remember (and state)

recognise: use knowledge to identify features of a situation

show that: in questions where the answer is given, to use mathematical reasoning to prove that the answer is true

simplify: reduce the size of numbers in a fraction, or reduce the size of an algebraic expression

sketch: draw a diagram that shows the general shape and includes relevant features

solve: find the answer or explanation for a problem, particularly the values of variables

substitute: replace a variable by a number to calculate an answer

translate (to mathematical form): express a situation as mathematical relationships

verify: check a solution or result, usually referring to the given situation

write/state: give the answer, formula or result without showing any working or explanation (This usually means that the answer can be found mentally, or in one step)

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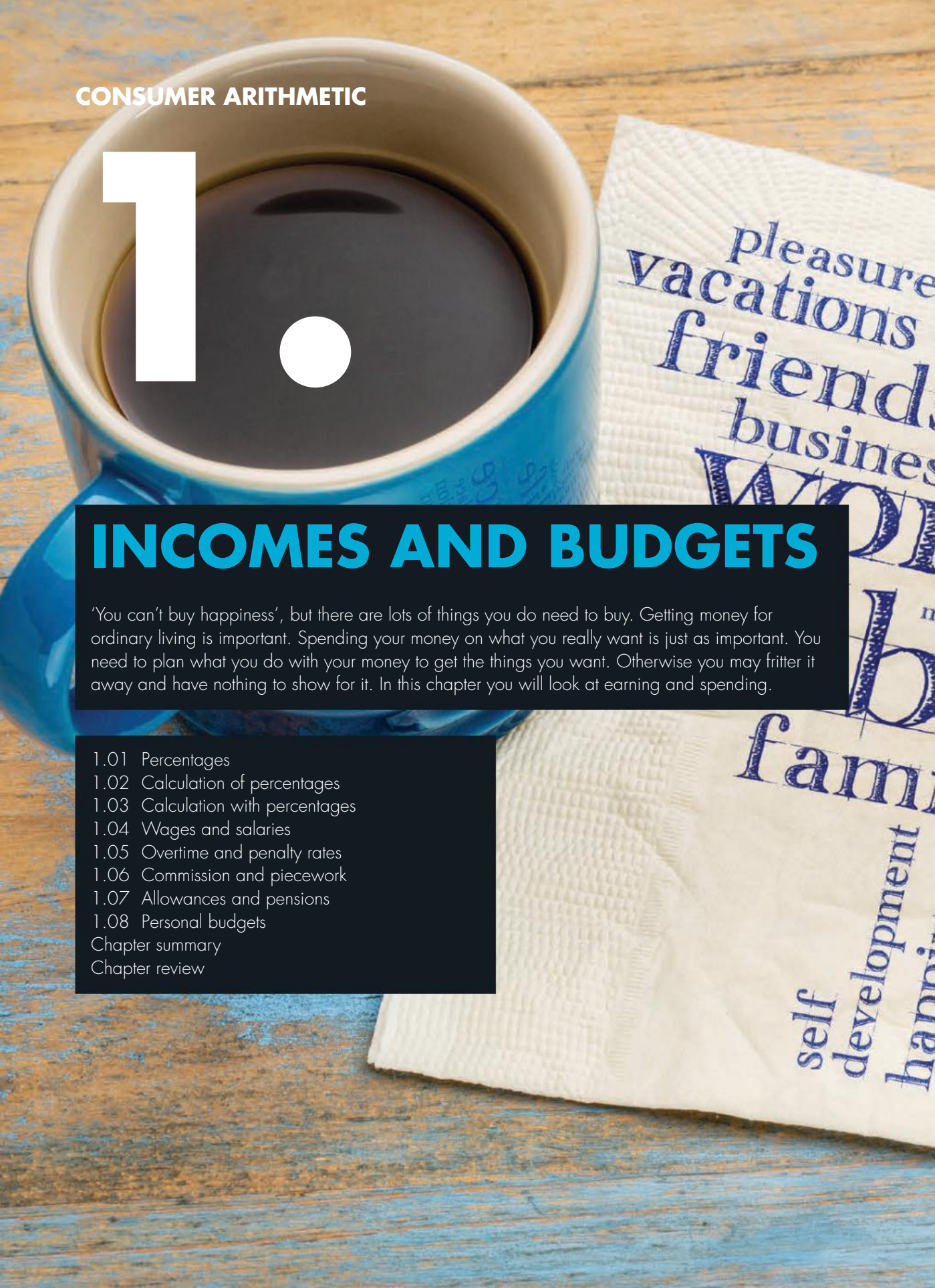

CONSUMER ARITHMETIC

1.

INCOMES AND BUDGETS

'You can't buy happiness', but there are lots of things you do need to buy. Getting money for ordinary living is important. Spending your money on what you really want is just as important. You need to plan what you do with your money to get the things you want. Otherwise you may fritter it away and have nothing to show for it. In this chapter you will look at earning and spending.

1.01 Percentages
1.02 Calculation of percentages
1.03 Calculation with percentages
1.04 Wages and salaries
1.05 Overtime and penalty rates
1.06 Commission and piecework
1.07 Allowances and pensions
1.08 Personal budgets
Chapter summary
Chapter review

SYLLABUS SUBJECT MATTER

Application of rates and percentages

- review definitions of rates and percentages
- calculate weekly or monthly wage from an annual salary, wages from an hourly rate including situations involving overtime and other allowances and earnings based on commission or piecework
- calculate payments based on government allowances and pensions, e.g. various youth allowances for unemployment, disability and study
- prepare a personal budget for a given income taking into account fixed and discretionary spending
- use a spreadsheet to display examples of the above computations when multiple or repeated computations are required e.g. preparing a wage-sheet displaying the weekly earnings of workers in a fast food store where hours of employment and hourly rates of pay may differ, preparing a budget, or investigating the potential cost of owning and operating a car over a year

Prior learning

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TERMINOLOGY

ABSTUDY
apprentice
casual
double-time
Health Care card
overtime
percent
piecework
salary
wage
age pension
assets test
commission
fixed-term contract
income test
part-time
percentage
rate
time-and-a-half
Youth Allowance
annual
Austudy
disability pension
full-time
Newstart Allowance
penalty rate
permanent
rostered day off (RDO)
trainee

1.01 Percentages

Percentages

- **Percent** means out of 100, shown as %
- Multiply by 100% to change to a percentage

EXAMPLE 1

Change to percentages:

a 48 : 60 **b** 1.7 **c** $2\frac{4}{7}$

Solution

a Write as a fraction and multiply by 100%. $\frac{48}{60} \times 100\% = 80\%$

b Multiply by 100%. $1.7 \times 100\% = 170\%$

c Multiply by 100%. $2\frac{4}{7} \times 100\% = 257\frac{1}{7}\%$

You divide by 100 to change a percentage to a fraction or decimal.

EXAMPLE 2

Change each percentage to a decimal.

a 6.75% **b** 243%

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Solution

a Divide by 100. $6.75\% = 6.75 \div 100$

Move the decimal point back 2 places. $= 0.0675$

b Divide by 100. $243\% = 243 \div 100$

Move the decimal point back 2 places. $= 2.43$

EXAMPLE 3

Change each percentage to a fraction.

a 165% **b** $5\frac{1}{4}\%$

Solution

a Divide by 100. $165\% = \frac{165}{100}$

Cancel if possible. $= 1\frac{13}{20}$

b Change to an improper fraction and divide by 100. $5\frac{1}{4}\% = \frac{21}{4} \div 100$

Change to multiplication. $= \frac{21}{4} \times \frac{1}{100}$

Can't cancel, so just multiply. $= \frac{21}{400}$

Common percentages

Fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
Percentage	1%	5%	10%	20%	25%	$33\frac{1}{3}\%$	50%	$66\frac{2}{3}\%$	75%

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Exercise 1.01 Percentages

Example 1

1 Change to percentages.

a $\frac{79}{250}$ **b** 3.125 **c** $2\frac{23}{25}$ **d** $\frac{19}{40}$ **e** 0.797

f 2.7 **g** $2\frac{7}{8}$ **h** 0.024 **i** $1\frac{4}{5}$ **j** $\frac{3}{80}$

2 Change to percentages, correct to 3 decimal places.

a $\frac{11}{12}$ **b** $2\frac{2}{27}$ **c** $\frac{6}{21}$ **d** $\frac{5}{9}$ **e** $1\frac{13}{18}$

f $1\frac{2}{3}$ **g** $\frac{14}{19}$ **h** $\frac{5}{14}$ **i** $1\frac{6}{7}$ **j** $\frac{6}{11}$

3 Change to percentages, correct to 3 decimal places where appropriate.

a $\frac{5}{14}$ **b** $1\frac{10}{19}$ **c** $1\frac{1}{5}$ **d** $2\frac{1}{2}$ **e** $1\frac{9}{28}$

f $1\frac{11}{13}$ **g** $\frac{9}{17}$ **h** $1\frac{8}{11}$ **i** $\frac{3}{14}$ **j** $\frac{13}{16}$

Example 2

4 Change to decimals.

a 14% **b** 28.5% **c** 173% **d** 8% **e** 0.56%

f 1.4% **g** 50.5% **h** 127.5% **i** 23% **j** 5.72%

Example 3

5 Change to fractions.

a 12% **b** 148% **c** 134% **d** 72.5% **e** 342.5%

f 113.6% **g** 232.75% **h** 59.375% **i** 43.125% **j** 179.775%

Problem solving

6 Change 16.9% to the nearest fraction with a denominator under 20.

1.02 Calculation of percentages

Percentage shortcuts

Percentages without calculators

Mental percentages

EXAMPLE 4

Tammy scored 46 out of 75 for an English test and 27 out of 40 for a Maths test. In which test did she do better?

Solution

Change the English mark to a percentage.

$$\text{English\%} = \frac{46}{75}$$
$$= \frac{46}{75} \times 100\%$$
$$= 61\tfrac{1}{3}\%$$

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Change the Maths mark to a percentage.	$\text{Maths\%} = \frac{27}{40}$ $= \frac{27}{40} \times 100\%$ $= 67\frac{1}{2}\%$
Compare the answers.	Assuming the tests were equally difficult, she did better in Maths.

A **percentage increase** or **decrease** is usually calculated 'out of' the original amount.

EXAMPLE 5

Find the percentage increase correct to 2 decimal places if necessary.

a \$80 increases to \$95

b 40 mm more gives 165 mm

Solution

a	Find the increase.	Increase = \$95 – \$80 = \$15
	Multiply 'increase : original' by 100%.	$\text{Percentage increase} = \frac{15}{80} \times 100\%$
	Calculate.	$= 18.75\%$
	Alternative method	
	Multiply 'new : original' by 100%.	$\text{New amount\%} = \frac{95}{80} \times 100\%$ $= 118.75\%$
	Subtract 100% to find the percentage increase.	Percentage increase = 118.75% – 100% = 18.75%
b	Find the original.	165 – 40 mm = 125 mm
	Multiply 'increase : original' by 100%.	$\text{Percentage increase} = \frac{40}{125} \times 100\%$
	Calculate.	$= 32\%$

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EXAMPLE 6

Find the percentage decrease, correct to 2 decimal places if necessary.

a 250 g becomes 234 g

b After a 10 kg decrease, the mass was 73 kg

Solution

a Find the decrease. Decrease = 250 – 234 g
= 16 g

Multiply 'decrease : original' by 100%. Percentage decrease $= \frac{16}{250} \times 100\%$

Calculate. $= 6.4\%$

Alternative method

Multiply 'new : original' by 100%. New amount% $= \frac{234}{250} \times 100\%$
$= 93.6\%$

Subtract from 100% to find the percentage decrease. Percentage decrease = 100% – 93.6%
$= 6.4\%$

b Find the original. 73 + 10 kg = 83 kg

Multiply 'decrease : original' by 100%. Percentage decrease $= \frac{10}{83} \times 100\%$

Calculate and round off. $= 12.05\%$

Exercise 1.02 Calculation of percentages

1 Use percentages to compare 16 out of 25 to 25 out of 35.

2 Which is better, 32 out of 45 or 38 out of 55?

Example 4

Example 5

3 Find the increase of the first to the second of each of the following as a percentage. Round to two decimal places where necessary.

a \$84 to \$98 **b** 128 mL to 200 mL **c** 62 g to 69 g

d 110 mm to 240 mm **e** 18 to 31

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4 Find each increase as a percentage, correct to 2 decimal places if necessary.

a 14 L extra gives 128 L

b 23 more gives 47

c 53 is 6 more

d an increase of 27 gives 35

e 79 more produces a total of 120

5 Find by what percentage the second value is reduced from the first. Round to two decimal places where necessary.

Example 6

a 16 c to 13 c

b \$28 to \$25

c \$135 to \$115

d 63 L to 58 L

e 230 to 100

6 Find the decrease as a percentage. Round to two decimal places if necessary.

a 28 less gives 142

b \$170 is \$20 less

c 75 kg is 20 kg less

d a reduction of 17 gives 63

e 415 mg is a 35 mg reduction

Problem solving

7 An increase of \$16 is followed by a decrease of \$16. Show that the percentage changes are different.

8 A shop decreases prices by \$30 on all pairs of jeans at a sale. Find an expression for the percentage increase that is necessary to return a pair of jeans to the original price of \$$x$.

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Working with percentages

Percentage calculations

1.03 Calculation with percentages

To calculate a percentage of an amount:

- change the percentage to a fraction or decimal
- multiply it by the amount

EXAMPLE 7

Find 6.3% of \$483.

Solution

Write the percentage as a decimal.	$6.3\% = 0.063$
Multiply by the amount.	6.3% of \$483 $= 0.063 \times \$483$
	$= \$30.429$
Round appropriately.	$\approx \$30.43$

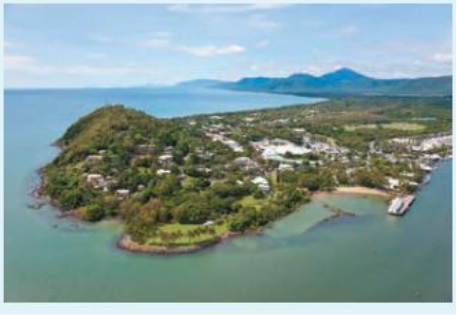
You might be given a percentage that is part of a given amount.

To calculate something that is part of a percentage:

- write what the percentage is out of as 100%
- write the given amount as a percentage
- write the amount you want as a percentage
- calculate the desired amount using the ratio of percentages

EXAMPLE 8

The population of a town decreased by 30% to 5320 over 3 years. What was the decrease?



Getty Images/Peter Adams

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Solution

Write what the percentage is out of.	Original population = 100%
Write the given amount as a percentage	5320 = 70% of original
Write the decrease as a percentage.	Decrease = 30% of original
Use the ratio of percentages.	$= \frac{30}{70} \times 5320$ $= 2280$
Write the answer.	The population decreased by 2280.

You can verify your steps in a problem like Example 8 by working out the given percentage. A decrease of 2280 gives an original population of 5320 + 2280 = 7600.

Check that 2280 out of 7600 works out to 30%.

Exercise 1.03 Working with percentages

1 Evaluate:

a	32% of \$560	**b**	14.5% of 48 L	**c**	16% of 63.45
d	210% of \$88.47	**e**	11% of \$620	**f**	5.4% of 248 g
g	128% of \$17.50	**h**	19% of 740 kg	**i**	21.5% of \$784
j	3.43% of \$6420	**k**	37% of \$293	**l**	7.08% of \$2400

Example 7

2 What increase of 15% gives a final amount of \$6256?

Example 8

3 What decrease of 20% gives \$516?

4 What original amount gives \$490 after an increase of 25%?

5 \$420 was left after a decrease of 14%. What was the original amount?

Problem solving

6 After 12% evaporation, there was 308 mL of solvent left. How much was there originally?

7 The weight of the clothes was increased by 24% by the water they retained from the washing machine. Before hanging them out to dry, they weighed 13 kg. What would they weigh when they were dry?

8 Adding 270 g of dye to the molten mixture increased the weight of recycled plastic by 4.5%. What was the final weight?

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Wages and salaries

Wage sheet

Wage sheet

1.04 Wages and salaries

Wages and salaries

- A **rate** compares two quantities, such as money and time
- A **wage rate** is usually stated as the payment per hour, such as \$16.30/h
- A **salary** is usually stated as the payment for a year or a month, like \$68 420/year

EXAMPLE 9

Change an annual salary of \$68 420 to an hourly rate, assuming a 38-hour week.

In Australia, the standard working week is 38 hours.

Solution

Find the number of working hours in a year.

$$1 \text{ year} = 52 \times 38 \text{ h}$$
$$= 1976 \text{ h}$$

Divide to find the hourly rate.

$$\text{Hourly rate} = \frac{\$68\,420}{1976} \text{ /h}$$

Using your calculator, round to the nearest cent.

$$\approx \$34.63 \text{ /h}$$

EXAMPLE 10

Change a wage of \$21.43 /h for a 38-hour week to an annual amount.

Solution

Find the working hours in a year.

$$1 \text{ year} = 52 \times 38 \text{ h}$$
$$= 1976 \text{ h}$$

Multiply to find the annual rate.

$$\text{Annual rate} = \$21.43 \times 1976$$

Using your calculator, round to the nearest dollar.

$$\approx \$42\,346$$

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INVESTIGATION

WAGE RATES

Use the **Fair Work** government website to investigate the wage rates for a 17-year-old:

- kitchen assistant
- retail assistant
- apprentice carpenter
- apprentice sheet-metal worker
- apprentice hairdresser
- nursing aide
- child-care assistant
- labourer

iStock.com/monkeybusinessimages

Compare casual and permanent rates and find the differences in the conditions.

Your teacher might want you to write a report.

Exercise 1.04 Wages and salaries

1 The average full-time adult earnings in 2016 was $78 830. What is this for a 38-hour week as:

a a weekly rate? **b** a fortnightly rate?

c a monthly rate? **d** an hourly rate?

Example 9

2 The base pay rate for a teacher in Queensland was $58 826 per year in 2016. What is this as a fortnightly amount?

3 The base pay rate for a Queensland public servant in 2016 was $36 700 per year. What is this as a monthly amount?

4 In January 2017, the minimum wage was $17.70 /h for a 38-hour week. What was this as an annual amount?

Example 10

5 Jenny is a first-year apprentice hairdresser who earns $12.20 /h for a 38-hour week. What is her pay as an annual amount?

6 A full-time carpenter earns $19.45 /h. What is this as an annual amount?

Problem solving

7 Which is better, $18.32 /h or $37 500 per year?

8 Which is better, $38 420 per year or $748.37 /week?

9 Which is better, $14.60 /h or $1046.45 /fortnight?

10 Which is better, $42 000 per year or $21.45 /h?

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S-Mart payroll

Overtime

1.05 Overtime and penalty rates

Overtime is work outside the normal hours of work. If you work overtime, you earn a higher rate of pay. Work on weekends or public holidays is also paid at a higher rate called a **penalty rate**.

- Overtime, first 2 or 3 hours: 1.5 times normal hourly rate (**time-and-a-half**)
- Overtime, after 2 or 3 hours: 2 times normal hourly rate (**double-time**)
- Saturday penalty rate: 1.5 times normal rate
- Sundays and public holidays: 2.5 times normal rate (**double-time-and-a-half**)
- Working on a **rostered day off** (RDO): 2 times normal hourly rate

Wages

TECHNOLOGY

Wages spreadsheet

You can work out weekly or fortnightly wages using the 'Wages' spreadsheet on NelsonNet.

Wages

Normal pay	19.6 /hour
Saturday rate	24.5 /hour
Sunday rate	34.3 /hour

Change pay period

Change Ordinary hours

	Mon	Tue	Wed	Thur	Fri	Sat	Sun
Hours worked	7.6	7.6	7.6	10	7.6	0	0
Total pay:	823.2						
Calculations							
Blocks	0	0	0	2.5	0	0	0
Ordinary time	148.96	148.96	148.96	148.96	148.96	0	0
Overtime < 2 h	0	0	0	58.8	0	0	0
Overtime > 2 h	0	0	0	19.6	0	0	0

Ordinary hours	Mon	Tue	Wed	Thur	Fri	Sat	Sun
	7.6	7.6	7.6	7.6	7.6	0	0

Change to double time	2
Overtime blocks	0.5

Put in the normal hourly rate and the hours worked each day.

Notice that it is set up for a 38-hour Monday–Friday week with overtime in hour blocks. You can change it to include other rosters, including Saturday and Sunday rotating rosters, using the 'change ordinary hours' buttons. The hourly rates for Saturday and Sunday are set at 1.25 and 1.75 times the ordinary rate. You can change these, the number of hours worked at time-and-a-half before double-time begins and the blocks of time for overtime calculation. Public holiday pay has to be calculated separately.

Use the spreadsheet to work out the week's wage for someone who works overtime to get a total of 9 hours on Tuesday and 10 hours on Wednesday.

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Overtime usually counts in hour, half-hour or quarter-hour blocks. If you work for an extra hour and 5 minutes, you get payment for $1\frac{1}{2}$ hours for half-hour blocks.

EXAMPLE 11

Jack is a senior waiter who earns \$783.30 for a 38-hour week. Last week, his rostered time was Wed 6 h, Thur 6 h, Fri 8 h, Sat 10 h, and Sun 8 h. Rostered work on Saturday is paid at 1.25 times the normal rate and on Sunday at 1.75 times. A rostered day off is paid at double time. He gets time-and-a-half for the first 2 hours of overtime and double time thereafter in half-hour blocks. His actual hours were:

Day	M	T	W	Th	F	S	S
Time	0	4.3	6	10.6	10	10.5	8

Work out his pay for the week.

Solution

Saturday and Sunday rates are worked out *before* overtime calculations.

Work out his hourly rate. $\text{Hourly rate} = \frac{\$783.30}{38} = \$20.61$

Tuesday work (RDO). 4.5 h at 2×

Thursday overtime. 2 h at 1.5×, 3 h at 2×

Friday overtime. 2 h at 1.5×

Saturday rate. $1.25 \times \$20.61 = \$25.76$

Saturday overtime. 0.5 h at 1.5×

Sunday rate. $1.75 \times \$20.61 = \$36.07$

Work out ordinary pay.

$$\begin{aligned}\text{Ordinary pay} &= (6 + 6 + 8) \times \$20.61 + 10 \times \$25.76 \\ &\quad + 8 \times \$36.07 \\ &= \$958.36\end{aligned}$$

Work out RDO pay.

$$\begin{aligned}\text{RDO pay} &= 4.5 \times 2 \times \$20.61 \\ &= \$185.49\end{aligned}$$

Work out overtime pay.

$$\begin{aligned}\text{Overtime pay} &= 4 \times 1.5 \times \$20.61 + 3 \times 2 \times \$20.61 \\ &\quad + 0.5 \times 1.5 \times \$25.76 \\ &= \$266.64\end{aligned}$$

Work out total pay.

$$\begin{aligned}\text{Total pay} &= \$958.36 + \$185.49 + \$266.64 \\ &= \$1410.49\end{aligned}$$

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Rates of pay for Saturday and Sunday work vary according to the industry. Industries where weekend work is common normally have rosters that include weekends. In these, rostered Saturday work is often paid at 1.25 times the normal rate and rostered Sunday work at 1.75 times. For other industries **penalty rates** for weekend work are usually 1.5 and 2 times the normal rate. People on salaries, like teachers, do not get paid for extra work.

Types of employment

- **Permanent** employees have regular hours, receive annual leave and sick pay, and are on at least weekly hire. Their employment ends only on giving or receiving notice.
- **Full-time** employees work about 38 hours a week
- **Part-time** employees work fewer than 38 hours a week. They get the same hourly rate as full-time employees.
- **Fixed term contract** employees are employed for a specific time or to complete a specific task. They usually get the same hourly rate as full-time employees
- **Casual** employees have no guaranteed hours of work, annual leave or sick pay, work irregular hours and their employment can usually end without notice
- Casual employees earn a higher hourly pay rate, normally about 25% of the base rate higher than full-time or part-time employees. They are usually paid for a minimum of 4 hours work.

INVESTIGATION

CASUALISATION OF THE AUSTRALIAN WORKFORCE

It is said that part-time and casual work has increased in Australia.

Find out how the Australian Bureau of Statistics (ABS) defines casual work.

- How does the ABS decide whether someone is employed or unemployed?
- How does the ABS define part-time and full-time work?
- What proportion of workers are currently casual?
- What proportion are currently part-time?
- What is under-employment?
- What proportion of people are underemployed?
- Are the claims of newspapers and internet news sites about part-time and causal work supported by ABS figures?
- Can you find information about part-time and casual work in your local area?

Your teacher might want you to write a report.

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EXAMPLE 12

Dreamstime.com/Wavebreakmedia Ltd

Sharon drives a van for deliveries and is paid \$712.50 /week, with penalty rates of 1.5× for Saturday, 2× for Sunday work, 2.5× for public holidays and 3× for Good Friday or Christmas Day. A casual driver is paid an extra 25% of the base rate for ordinary or penalty rates. A permanent driver is paid at 1.5× for the first 2 hours of overtime and 2× thereafter. A casual worker gets an extra 10% of the base rate for overtime.

a What pay rate does a permanent driver get for overtime?

b What pay rate does a permanent driver get for working on Anzac Day?

c What is the ordinary pay rate for a casual driver?

d What pay rate does a casual driver get for overtime?

Solution

a Work out the hourly rate.

$$\text{Hourly rate} = \frac{\$712.50}{38}$$
$$= \$18.75\ /\text{h}$$

Work out the overtime rates.

$$\text{Overtime rate up to 2 h} = 1.5 \times \$18.75\ /\text{h}$$
$$= \$28.13\ /\text{h}$$
$$\text{Overtime rate after 2 h} = 2 \times \$18.75\ /\text{h}$$
$$= \$37.50\ /\text{h}$$

Write the answer.

The overtime rate is \$28.13 /h up to 2 h and \$37.50 /h thereafter.

b Work out the rate for Anzac Day.

$$\text{Public holiday rate} = 2.5 \times \$18.75\ /\text{h}$$
$$= \$46.88\ /\text{h}$$

c	Work out 25% extra.	Casual rate = 1.25 × \$18.75 /h = 23.44 /h
d	Add 10% to overtime rates. A casual normally gets their hours for a week in advance. If they have to work outside the notified time, they get overtime.	Casual overtime rate up to 2 h = 1.6 × \$18.75 /h = \$30 /h Casual overtime rate after 2 h = 2.1 × \$18.75 /h = \$39.38 /h
	Write the answer.	The overtime rate is \$30 /h up to 2 h and \$39.38 /h thereafter.

INVESTIGATION

JUNIOR AND APPRENTICE RATES

It is usual to pay junior staff or apprentices at a lower rate to reflect their inexperience.

The rates are based on age or experience, so older staff or apprentices in later years of training get a higher rate.

Investigate the rates of pay as proportions of adult pay for junior shop assistants, junior fast food assistants, junior labourers, apprentice hairdressers, apprentice carpenters and apprentice cabinet makers.

What do you find?

Your teacher might want you to do a report.

If you start work straight after leaving school, you will be paid at a lower rate than experienced workers. Apprentices get a rate based on the year of their apprenticeship. Juniors get a rate based on their age.

Apprentice and **junior** pay is a proportion of the adult (qualified) rate:

- **4-year apprentices**
  - First year 40%
  - Second year 55%
  - Third year 75%
  - Fourth year 90%

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- **3-year apprentices**
  - First year 40%
  - Second year 55%
  - Third year 75%
- **Juniors**
  - Under 16 years old 45%
  - 16 years old 50%
  - 17 years old 60%
  - 18 years old 70%
  - 19 years old 80%
  - 20 years old 90% (often only for the first 6 months, then adult)
- **Trainees** 40%

The rates shown above are lower if you haven't completed Year 12.

EXAMPLE 13

The basic adult rate of pay for a shop assistant is \$19.44.

a Mohammed is 17 years old. What will he get per week as a full-time assistant?

b Elena is 18. What will she get as a part-time assistant working 25 hours a week?

Solution

a Work out the pay rate.

Hourly rate = 60% of \$19.44
= \$11.66

Work out the pay for 38 hours.

Weekly pay = 38 × \$11.66
= \$443.08

b Work out the pay rate.

Hourly rate = 70% of \$19.44
= \$13.61

Work out the pay for 28 hours.

Weekly pay = 25 × \$13.61
= \$340.25

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TECHNOLOGY

Weekly wages spreadsheet

You can make a spreadsheet for the payment of wages to the employees of a business like a fast food outlet.

Open a blank spreadsheet and put 'Weekly wages sheet' and the business name in cells A1 and A2.

Put 'Time and half up to' and 2 in cells A4 and B4. If Time-and-a-half is after 3 hours, put 3 instead of 2.

Put 'Name' and 'Rate' in cells A5 and B5.

Put 'Hours worked' in cell C4.

Put M, T, W, Th, F, Sa, S in cells C5–I5 and make the widths of these columns 3.

Put 'Weeks' and 'wages' in cells J4 and J5.

Put 'Normal hours' in cell K4.

Put M, T, W, Th, F, Sa, S in cells K5–Q5 and make the widths of these columns 3.

Put 'Ordinary hours worked' in cell R4.

Put M, T, W, Th, F, Sa, S in cells R5–X5 and make the widths of these columns 3.

Put 'Time and half' in cell Y4.

Put M, T, W, Th, F, Sa, S in cells Y5–AE5 and make the widths of these columns 3.

Put 'Double Time' in cell AF4.

Put M, T, W, Th, F, Sa, S in cells AF5–AL5 and make the widths of these columns 3.

Type employees' names in column A, their hourly wage rates in column B and their normal hours in columns K–Q from line 6 down.

Initially, put zeros in columns C–I from row 6 down.

Put the formula '=MIN(C6,K6)' in cell R6 and copy down columns R–X.

Put the formula '=IF(C6>K6,MIN(C6-K6,$B$4),0)' in cell Y6 and copy down columns Y–AE

Put the formula '=IF(C6>K6+$B$4,C6-K6-$B$4,0)' in cell AF6 and copy down columns AF–AL.

Put the formula '=B6*(SUM(R6:X6)+1.5*SUM(Y6:AE6)+2*SUM(AF6:AL6))' in cell J6 and copy down column J.

Make the format of columns B and J from line 6 down.

Put 'Wages bill' in cell G2.

Type the formula '=SUM(J6:J26)', say, in cell J2

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Now type the actual hours worked in a week into the Hours worked columns to calculate each employee's wages and the total wages bill for the week.

You can use **Format cells**, **Borders** and **Fill** to make the spreadsheet look attractive.

Weekly wagesheet
Hansons Fish and Chips

Wages bill **$3,259.78**

Time and half up to		2	Hours worked							Weeks	Normal hours							Ordin	
Name	Rate		M	T	W	Th	F	Sa	Su	wage	M	T	W	Th	F	Sa	Su	M	T
Tillie Ringer	$14.45		6	9	10	11	8	0	0	$686.38	6	8	8	8	8	0	0	6	
Chip Fryer	$15.30		7	8	8	8	8	0	0	$604.35	6	8	8	8	8	0	0	6	
Hank Fish	$16.80		0	6	8	8	8	8	0	$638.40	0	6	8	8	8	8	0	0	
Betty Batter	$13.90		0	0	8	8	8	8	8	$569.90	0	0	6	8	8	8	8	0	
Lottie Rappa	$17.90		0	0	6	8	9	10	8	$760.75	0	0	6	8	8	8	8	0	

Exercise 1.05 Overtime and penalty rates

1 Tamsin is the head chef in a Cairns restaurant and gets $854.60 a week. Work out her pay rates for overtime and public holiday work.

2 Jason is a security officer who gets $777.40 for a 38-hour week (M–F). Find his overtime rates and his pay rate for a public holiday. Example 11

3 Barry works as a casual bouncer on the door at a night-club in Surfers Paradise. This week he worked 6 hours each night from Tuesday to Friday and 8 hours on Saturday. The award rate for a permanent bouncer is $755.60. What is Barry's pay rate and what did he earn for the week? Example 12

4 Kerry is a permanent qualified sheet metal worker on $782.18 a week. What are the casual pay and casual overtime rates?

5 A welder with a trades certificate gets $783.30 a week. Work out the hourly pay rates for 3-year apprentices for each year of their apprenticeship. Example 13

6 An adult warehouse worker gets $756.40 per week. Work out the hourly pay rates for junior staff in the warehouse.

Problem solving

7 Samantha works in the kitchen at a Townsville hotel and normally gets $718.60 a week, working 5 days a week on a rotating roster of 4 days with 8 hours work and 1 day with 6 hours. Work out her pay for the week below. She was supposed to be off on Saturday.

Day	Sun	Mon	Tue	Wed	Thur	Fri	Sat
Hours worked	6	7	8	9	10	0	5

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8 William is an 18-year-old junior clerical assistant on the front desk of the same hotel. The adult wage is $743.30 a week. Work out William's pay for a week when he does the following hours.

Day	Sun	Mon	Tue	Wed	Thur	Fri	Sat
Hours worked	0	6	8	10	11	11	0

9 One week, Tom got $45.49 extra for 1.5 hours overtime. What was his total pay?

10 Tammy got $166.53 for working 4 hours overtime on Thursday. She didn't do any other overtime that week. What was her total pay?

11 A 17-year-old casual junior staff member was paid $303.24 for working 20 hours. What is the full-time adult wage?

12 A third-year apprentice shopfitter doing a 3-year apprenticeship got $750.88 for a week when she worked 3 hours overtime on each of 2 days. What would a casual adult shopfitter get for a 30-hour week?

Commission and piecework

Income – commission and piecework

Payday

1.06 Commission and piecework

Commission and piecework

- **Commission** workers earn a proportion of the value of sales or services. It is usually a percentage of the value
- Some commission workers also receive a small **retainer**, which is paid regularly regardless of sales so they keep working
- **Piecework** payments are made for each item made or handled
- **Outworkers** work from home and are often employed on a piecework basis
- Some businesses pay a piecework **bonus** or **incentive** in addition to wages

EXAMPLE 14

Serena works for a real estate agency that charges 2.5% commission. She receives 65% of this for sales she makes, less an administration fee of $939. How much does she make for selling a house for $720 000?

Solution

Calculate the agency's commission.

Agency commission $= 0.025 \times \$720\,000$

$= \$18\,000$

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Calculate Serena's commission.	Serena's commission $= 0.65 \times \$18\,000$ $= \$11\,700$
Subtract the administration fee.	Payment $= \$11\,700 - \$939$ $= \$10\,761$
Write the answer.	Serena receives \$10 761.

EXAMPLE 15

Charles is paid \$1.35 per parcel delivered for Australia Post as a subcontractor. He starts at 4:30 a.m., sorting and packing his van to start delivering at about 7 a.m. and usually finishes about 2:30 p.m. He usually delivers an average of 145 parcels a day.

a How much is he paid for a week?

He estimates his van costs \$180 a week to run and \$300 a week to pay off.

b How much does he actually make, allowing for GST $\left(\frac{1}{11}\right)$ as well?

c What is Charles's average hourly rate of pay?

Solution

a	Work out the total number of parcels.	Number delivered $= 145 \times 5$ $= 725$
	Work out the total payment.	Payment $= 725 \times \$1.35$ $= \$978.35$
b	Work out the cost of the van.	Van expenses $= \$480$
	Work out the GST.	$\text{GST} = \frac{\$978.35}{11}$ $= \$88.94$
	Calculate actual earnings.	Amount earned $= \$978.35 - (\$480 + \$88.94)$ $= \$409.41$
c	Calculate hours.	Total hours $= 10 \times 5$ $= 50$ h
	Calculate rate of pay.	$\text{Hourly rate} = \frac{\$409.41}{50}$ $\approx \$8.19$ /h

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Piecework is often poorly-paid when calculated on an hourly basis.

INVESTIGATION

COMMISSION AND PIECEWORK

Collect information about commission and piecework.

Find out percentages paid as commissions and piecework rates per item.

Is there any relationship between the values and the commissions or piecework rates?

Find out whether or not a retainer is paid for each situation. If a retainer is paid, relate this to the level of commission paid.

iStock.com/sturti

You could try to contact people employed as:

- new car salespeople
- real-estate agents
- taxi-drivers
- door-to-door salespeople
- delivery drivers
- used car salespeople
- insurance salespeople
- party-plan sales organisers
- telemarketers
- clothing outworkers

Exercise 1.06 Commission and piecework

Unless otherwise stated, ignore GST in this exercise.

Example 14

1 Shirley works on 4.5% commission in a department store. After working an 8-hour shift on a Saturday, she sold $2825 worth of stock.

a What does she earn for the shift?

b What is her hourly rate?

c Would you expect her to make this rate throughout the week?

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2 David gets 45% of the takings for driving a taxi. The owner pays for fuel. He took $640 in fares on Wednesday, starting at 6 a.m. and finishing at 6 p.m.

a How much did he make?

b What was his hourly rate?

c What could he expect to make in a 5-day week?

3 A driver for a different owner gets 50% commission but has to pay for fuel. He worked the same shift as David and took $720 in fares. Fuel cost him $93.

a How much did he make?

b What was his hourly rate?

c What could he expect to make in a 5-day week?

4 Ahmed was paid $6 per delivery for pizzas, but had to pay his own car expenses. He estimated his car cost about 55c/km to run, including fuel, services and other costs. Ahmed made about 20 deliveries in 6 hours and drove about 140 km in that time.

Example **15**

a How much was he paid?

b How much did it cost to run his car?

c How much did he actually make?

d What was his hourly rate?

e Explain if he would be better off taking a payment of $9.30 /h + $2 per delivery?

5 Georgina got a job picking cherry tomatoes near Bundaberg and was paid $1.80 per bucket. When she started, she could only do about 35 buckets per day, but after a week she was doing 80 buckets a day for a 10-hour day. Georgina had to pay $140 a week accommodation in a dormitory at the farm.

a What did she earn in a day at the start?

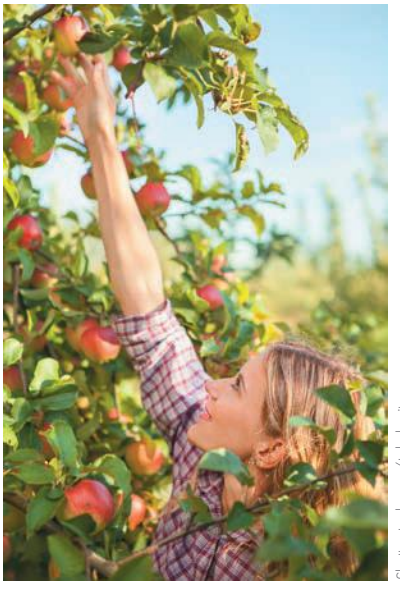
b What did she earn in a day after the first week?

c Assuming she improved steadily over the first week, what would she have been paid in the first week for a 6-day week, after accommodation was deducted?

d What was she paid per week after the first week?

e What was her average hourly rate in the first week?

f What was her average hourly rate after the first week?



Shutterstock.com/takaburito

Problem solving

6 Ainslie got a job picking apples in Stanthorpe in February. She was paid $20 a bin and after a while was able to pick about 5 bins in a 10-hour day, 5 days a week. She did not have to pay accommodation. Calculate her hourly rate and her earnings for 6 weeks work.

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7 A real-estate agent works on a commission-only basis. For each property, the commission is separately calculated as 0.45% of the value of the property up to $400 000 and 0.15% of the remainder. One month, she sold properties for $555 000, $478 000 and $688 000. What is her income for that month?

8 A seamstress working from home made $4.80 for every shirt sewn. One week she was paid $432, working for 30 hours over the week. What was her hourly rate and how many shirts did she do?

9 Jake sells whitegoods. He has the option of working for a straight commission of 7% or for a commission of 5.5% and a retainer of $170 a week. What value of whitegoods would he need to sell in a week to be better off on straight commission?

10 Aziz sells videos at a weekend market. He pays $5 per video for stock and sells them for $12 each. He has to pay a fee of $80 per day for a stall at the market. The market opens at 8 a.m. and closes at 3 p.m. It takes Jake half an hour to set up before the market opens and half an hour to pack up afterwards. In an average week, he sells 75 videos over the weekend.

Calculate how much he money he takes, how much he gets after expenses and his hourly rate.

11 Candace works on a commission of 12% plus retainer of $100 a week selling cosmetics. One week, she worked for 45 hours and sold $5940 worth of cosmetics.

a Calculate her earnings and hourly rate for the week.

b Calculate the value of cosmetics she sold in a week when she earned $940.

12 A pizza parlour in Toowoomba pays $6.20 per delivery. One night, Peter delivered 24 pizzas between 5:30 p.m. and 11:30 p.m., but also covered 240 km in his car.

a How much did he get from the pizza parlour?

b His car uses 11.2 L/100 km and petrol cost him 120.9 c/L. How much did he pay for petrol?

c What were Peter's real earnings and hourly rate? Explain your reasoning.

Earning money

1.07 Allowances and pensions

The federal government pays people who are too old to work, cannot work, cannot find work, cannot properly support their family, or who are studying. In each case, the payments you receive depend on your family situation, income and assets.

Age and disability pensions

- Single age or disability over 21: $877.10 /fortnight
- Age or disabled couple over 21 living together: $661.20 /fortnight each
- Disabled under 18 living at home: $360.60 /fortnight
- Disabled 18–20 living at home: $408.70 /fortnight

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- Disabled couple or independent under 21: \$556.70 /fortnight (each)
- The **income test** reduces payments by \$0.50 /dollar/fortnight for income over \$164 for singles and over \$292 combined for couples
- Pensioners get a **health care card** which entitles them to cheaper medicines

Source: https://www.humanservices.gov.au/customer/services/centrelink/disability-support-pension

The payments shown include some special payments. At present, you have to be aged at least 65 to get the age pension. This will gradually increase over the next few years.

The disabled payment assistance is also changing over the next few years.

The payments change each year according to changes in the cost of living.

There is also an **assets test** that reduces payments based on the value of things that you own.

EXAMPLE 16

How much do an aged couple living together get altogether in a year if they have a combined income from investments of \$700 /fortnight?

Solution

Find the income over \$292 (each).

Amount over \$292 = \$350 – \$292
= \$58 /fortnight

Find the reduction.

Reduction = 0.5 × \$58
= \$29 /fortnight

Find the pension paid.

Pension = \$661.20 – \$29
= \$632.20 /fortnight

Find the total pension for a year.

Total for year = 26 × \$632.20
= \$16 437.20

Find the total investment income.

Investment income = 26 × \$350
= \$9100

Find the total.

Total income = \$16 437.20 + \$9100
= \$25 537.20 each per year

Find their combined income.

Combined income for the year = 2 × \$25 537.20
= \$51 074.40

There is a tax offset that means you do not pay tax on a pension. You might pay tax on other income.

If you are an Australian apprentice or studying full-time, you are probably eligible for one of the following. Aboriginal or Torres Strait Islanders (ATSI) have some different conditions.

Youth allowance, Austudy and ABSTUDY

Youth Allowance is paid to those aged 16 to 24, Austudy to those aged 25 or more, while ABSTUDY is paid to Indigenous (ATSI) students.

Age	ATSI age	Status	Fortnightly payment
16 and over	16–21	Single with children	$567.70
16 and over	16–21	Partnered with children	$475.70
16 and over	16–21	No children	$433.20
18–24	18–21	Living at home	$285.20
16–17	16–17	Living at home	$232.10
	Under 16	Living at home	$32.20
	Over 21	Single with children	$571.90
	Over 21	Single, no children	$528.70
	Over 21	Partnered	$477.40

The **income test** reduces fortnightly payments by $0.50 for every dollar of income over $433 and $0.60 for every dollar of income over $519. There are also reductions based on your parents' income and assets.

Note that single ATSI students over 21 with children receive an extra $4.20 /fortnight to help overcome disadvantage. ATSI students are also entitled to some assistance in purchasing materials.

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EXAMPLE 17

Danica is aged 20 and lives away from home to attend a TAFE college full-time. She works part-time in a takeaway, earning $265/week. How much Youth Allowance does Danica get?

Solution

Work out her fortnightly income.	Income = 2 × $265 = $530
Work out her income over $519/fortnight.	Amount over $519 = $530 – $519 = $11
Work out the reduction in allowance.	Reduction = 0.5 × ($519 – $433) + 0.6 × $11 = $49.60
Work out the allowance.	Youth Allowance = $433.20 – $49.60 = $383.60

For people over 21, the **Newstart Allowance** could be paid while you look for work. There are many conditions. Breaching the conditions could mean you have to wait up to 8 weeks to receive any further payment.

Exercise 1.07 Allowances and pensions

1 Merv is on a single disability pension. He also gets $230 a week income from a trust fund. How much does he get from his pension and what is his total income for the year?

2 Coola is 70 years old and is on the pension. She gets an income of $250 a week from the widows' benefit of her husband's superannuation. How much does she get from the pension and how much does she get altogether in a year? Example 16

3 Carole is 18 years old and living at home while she goes to uni. She works 4 nights a week at a local pizza shop and makes $66 a night. How much Youth Allowance does she get? Example 17

4 Tony has an Aboriginal background and is doing the same course as Carole. He works as a bike mechanic on weekends and gets $120 a day. How much ABSTUDY allowance does he get?

5 Sue-Ellen is in her last year of a certificate III course in office management, and works part-time as a receptionist. She gets $490 a fortnight for the work. She is single and has just turned 26. How much does she get in Austudy?

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6 Annelise is in the same share-house as Sue-Ellen and is doing the same course. She comes from Thursday Island and is 27 years old. Annelise works at a local shop on the weekend and gets $720 a fortnight. How much does she get in ABSTUDY?

Problem solving

7 Danielle and Sam are both in second year at Uni when they start living together. They are both 22 and have incomes of $210 and $240 a week respectively from part-time jobs.

a How much did they get in Youth Allowance before they started living together?

b How much will they get now?

8 Compare the amounts that 18-year-old high school students living at home get in Youth Allowance if they don't work and if they get $400 a week from part-time work.

9 What would Aboriginal or Torres Strait Islander students get in the same situation as the students in question **8**?

10 Cyndi is a 17-year-old student who has to live away from home to complete Year 12. She gets an allowance from her parents but also gets $233.20 /fortnight Youth Allowance. How much a week do her parents give her?

11 Colin manages to arrange his investments so that he gets $10 a fortnight disability pension, so he also gets the health care card. How much do his investments earn in a year?

12 Anastasia is a 27-year-old single mother attending university. She works part-time and gets $320.70 /fortnight Austudy.

a How much a week does she earn from her part time job?

b She regards the money she loses from Austudy because of her part-time job as tax. What is the effective tax-rate?

13 Gerald is an aged pensioner who acts as a caretaker for the block of flats where he lives in exchange for free rent. Centrelink assess the rent on his flat as being worth $425 a week. What is his effective tax rate from caretaking?

14 Andrew is a 70-year-old widower who gets part of the age pension. He gets only $210 a week.

a What is his other income?

b Andrew is going to marry Susan, who is 68 and has no income apart from the age pension. What happens to their pensions when they get married?

c Compare their total incomes after they got married to their incomes before they got married.

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1.08 Personal budgets

Personal budgets

Budget grid

Budgeting scenarios

Managing your spending is one of the most important life skills. Do it well and you will have the money you need to pay for things. There are some simple things you can do to improve your budgeting:

- Check what you spent money on in the last month
- Write down the things you must pay every day, week or month, like rent, transport, groceries, electricity: these are **fixed expenses**
- Write down what you would like to buy or spend money on, like nights out or entertainment: these are **discretionary expenses**
- Write down some **goals** that you want to save for, like buying a car or a holiday
- Write down the money that you get: this is your **income**
- Look for ways of saving money: Can you eat more cheaply? How often do you go out or eat takeaway? Do you buy lunch or take lunch with you?
- Be honest with yourself about your weaknesses and avoid situations where you will spend money unwisely

Make a **budget**: complete lists of income and expenses over the same period. At first, use the period that you get paid. Then try to move to a longer period. Try to leave some money over to save in an investment account. *Make a list* before you go shopping, otherwise you may buy things on impulse. A **balanced budget** has equal income and expenses. You can make budgets for special purposes, as well as for normal living.

EXAMPLE 18

Antonia is going for a 7-day sailing trip around the Whitsundays with 4 friends. The yacht costs \$700 a day to hire, and she estimates it will use about 100 L of diesel at \$1.30/L. Food for the week will probably be about \$900 and it will cost about \$50 each to get to the port. Draw up a budget and find the cost per person.

Solution

Add up the expenses.

Hire cost:	$7 \times \$700 = \$\ 4900$
Food:	\$ 900
Fuel:	$100 \times \$1.30 = \$\ 130$
Transport:	$5 \times \$50 = \$\ 250$
Total	\$ 6180

Divide by the number of people.

Cost/person = \$6180 ÷ 5

= \$1236

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Make an allowance for problems.	$1300 – $1236 = $64 $64 × 5 = $320
Write the answer.	Allowing $320 for contingencies, the cost per person is $1300.

Contingencies are costs that arise unexpectedly.

When you do a budget, allow 4 weeks in a month, 12 weeks in a quarter and 50 weeks in a year when you divide expenses to make a weekly budget. This gives you a margin for price changes.

EXAMPLE 19

Karen earns $546 a week after tax. She rents a furnished unit with a friend and her share of the rent is $350 a fortnight. She spends about $170 a week on food, household items and toiletries, including takeaways. She catches the train to work each weekday and pays $4.50 a day. The power (electricity and gas) bills total about $280 a quarter and she pays half of this. Karen estimates that she spends $210 a month on clothes. She spends about $120 a month on her prepaid mobile and $30 a week on nights out and entertainment.

a Do a weekly budget and work out how much she could save in a year.

b Suggest where she could make savings.

Solution

a Work out her weekly expenses.

Rent: $350 ÷ 2	$ 175
Food, etc:	$ 170
Train: $4.50 × 5	$ 22.50
Power: $280 ÷ 2 ÷ 12	$ 11.67
Clothes: $210 ÷ 4	$ 52.50
Mobile: $120 ÷ 4	$ 30
Entertainment:	$ 30
Total	$ 491.67

Calculate her weekly savings.

Savings/week = $546 – $491.67 = $54.33

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	Calculate her savings/year.	Savings/year = 50 × \$52.33 = \$2716.50
	Write the answer.	If Karen's estimates are right, she could save \$52.33 a week, which is \$2716.50 per year.
b	Consider her expenses.	She might be able to cut spending on food by buying more generic items. She could cut her mobile use by topping up smaller amounts and rationing her use.

Many people get a takeaway coffee on their way to work or at lunchtime. A cup of coffee a day at even \$4 a cup comes to about \$1000 a year. When you save some money, you should keep some in reserve for emergencies. When you make a budget, be realistic about entertainment expenses and how much it costs to go out with your mates.

TECHNOLOGY

Budget spreadsheet

Personal budget

You can work out weekly, fortnightly or monthly budgets using the 'Personal budget' spreadsheet on NelsonNet.

The 'spinners' change between weekday, weekly, fortnightly, monthly, quarterly and annual for the budget, income and expenses.

Put in the information and the spreadsheet automatically works out the budget.

The snapshot shown is Karen's weekly budget from Example 19.

Personal budget

Weekly budget

Change budget period

Income:					per week
\$ 546.00	per	week	=	\$	546.00
	Total	income	=	\$	546.00

Savings = \$ 54.33 per week

= \$ 2,716.50 per year

Change periods

Expenses					per week
\$ 350.00	per	fortnight	=	\$	175.00
\$ 170.00	per	week	=	\$	170.00
\$ 4.50	per	week-day	=	\$	22.50
\$ 140.00	per	quarter	=	\$	11.67
\$ 210.00	per	month	=	\$	52.50
\$ 120.00	per	month	=	\$	30.00
\$ 30.00	per	week	=	\$	30.00
		Total expenses	=	\$	491.67

Change periods

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INVESTIGATION

LEAVING HOME

Can you afford to move away from home when you start work?

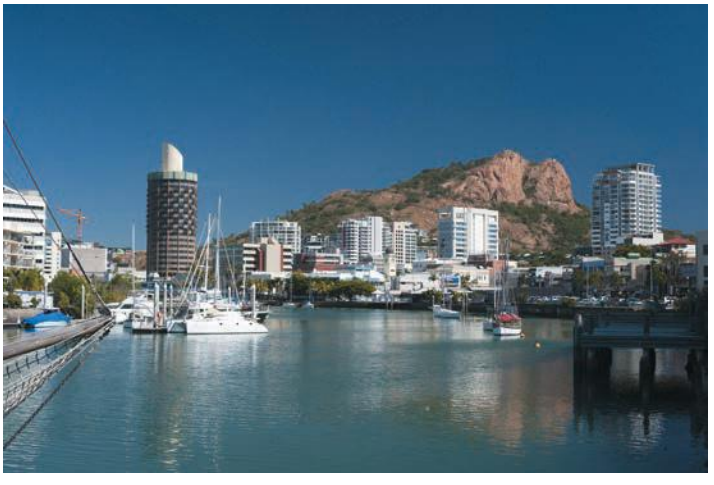
- Use advertisements to find how much it costs to rent a flat or part of a share-house
- Write down lists of what you spend money on, including haircuts, mobile charges, soap, deodorant, cosmetics, train and bus fares, snacks, shoes, sports fees, home entertainment and movies
- Use the lists to estimate your living costs
- Estimate the cost of your food, including breakfast cereal, fruit, vegetables, butter and cooking ingredients as well as main meals. Don't forget takeaway food and drinks
- Talk to other people to investigate the costs of power bills and other house costs like cleaning materials
- Convert the amounts to weekly or fortnightly amounts and work out a budget

Check the income you can get and decide if you could afford to move out.

Exercise 1.08 Personal budgets

Example 18

1 A couple plan to fly to Townsville from Brisbane for a 7-day holiday on Magnetic Island. The return airfare is $449 per person, and airport transfers to and from Townsville and Magnetic Island are $165 each. The tariff for their room is $285 a night, which includes breakfast. They think that lunch and dinner will be a total of about $65 a day each. They think they will spend about $25 each a day on other purchases. Work out a budget for the couple's trip.



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2 A club is selling hot dogs to raise funds. The treasurer gets the following prices from a wholesaler:

Wieners:	\$60/100
Bread rolls:	\$55/100
Tomato sauce:	\$10.80 for a 4 L bottle

The treasurer thinks that most people will have 20 mL of sauce on their hot dog.

Work out the cost of 1000 hot dogs and the profit made if they sell 950 at \$4 each.

3 Aya is sharing a small furnished flat in a big house with her best friend. She earns \$510 a week after tax and the flat costs them \$410 a fortnight, but that includes power. Aya hardly ever eats out, so she spends only \$95 a week on food and household necessities. She pays \$24 a week for the bus to and from work. She spends \$200 a month on her mobile and \$105 a week on nights out and entertainment. Aya spends about \$1900 a year on clothes and is trying to save up for an overseas trip. Work out her budget and how much she could save in a year.

Example 19

Problem solving

4 Three friends plan a cycling tour of the Brisbane Valley. They have worked out that they will stay overnight at Fernvale, Esk, Kilcoy and Caboolture and that bed and breakfast at hotels or motels will cost about \$80 each for each night. They will buy takeaways for lunch and have a good meal at night. They estimate that takeaway snacks and drinks at lunchtime will be about \$15 each and evening meals about \$45 each. They will be away for 5 full days but will eat at home for breakfast on the first day and dinner on the last day. They buy maps and repair materials for their bikes, totalling \$96. Work out a budget for the trip.

5 The Year 11 and 12 theatre students in a school are going on an excursion to see a play. The entry cost is \$10.50 for each student, and buses cost \$275 for a 25-seater and \$482 for a 46-seater. There are 110 students and 5 teachers going on the excursion. Draw up a budget to work out the cost for each student for the excursion, allowing for contingencies.

6 A 5-day Year 8 camp is planned. It is anticipated that 50 girls and 41 boys will attend. The camp will be held at a campsite that costs \$100 per day for each cabin. The five teachers will be accommodated in one cabin, and the students will use separate cabins for boys and girls. There are six bunks in each cabin. The catering is \$122 per person. Buses seating 55 people will be hired for the trip, at a cost of \$482 each way for each bus.

a Draw up a budget to work out the total cost of the camp.

b The total cost is to be paid by the students attending the camp, rounded up to the nearest \$5. What is the cost per student?

c Is rounding up enough for contingencies?

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7 A nursery grows 900 punnets of tomato seedlings a week for retail nurseries. They plant 20 seeds in each punnet and pinch out the weak ones. Each plastic punnet costs 12 cents and the seeds are about $1.20 for 100. Seed-raising mixture costs $25 for enough to fill 300 punnets. The supplier estimates that one person takes about an hour altogether to prepare 150 punnets of seedlings for sale. With on-costs, labour is about $30 an hour. 20-punnet trays cost $0.50 each.

a What is the total cost of 3000 punnets?

b How much should they charge for a tray of 20 punnets of seedlings?

8 Sandy shares a furnished house with two of her friends. She earns $1360 a fortnight after tax and the house costs them $870 a fortnight, which includes electricity charges. All three are vegetarians who mostly eat at home, so they only spend about $90 each a week on food and household items. Sandy spends about $180 a month on her mobile and allows herself $80 a week for entertainment and incidentals. Sandy walks to the bus stop and pays $32 a week on fares to and from work. She knows from experience that she spends about $2100 a year on clothes and shoes. Sandy is trying to save up for a car. Work out her budget and how much she could save in a year.

9 Macka earns $536 a week after tax. He shares a house with three other people. The house costs $350 a week and the shared power bill is usually about $420 for a month. Macka buys lots of takeaways and spends about $220 a week on food and other household items. He is paying off his car at $160 a month and spends about $75 a fortnight to run it. Macka spends about $1000 a year on clothes and $150 a month on his mobile. He finds it impossible to save any money. He does go out a lot at the weekend. Work out Macka's budget and suggest how he could save money for an overseas trip.

10 Justin earns $1380 a fortnight after tax. He shares a flat with his best mate, Hao. The flat costs $450 a week and the shared power bill is usually about $120 for a month. Justin and Hao share the cost of groceries and other household items. This usually comes to about $190 a week. Justin likes to eat takeaway food; he also likes to party. He estimates that he spends about $150 a week on takeaway food and entertainment.

Justin is paying off his motorbike at $180 a month and spends about $30 a fortnight to run it. He estimates that he spends about $1400 a year on clothes and $100 a month on his mobile. Work out Justin's budget and how he could save for the deposit on a car.

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1. CHAPTER SUMMARY

Incomes and budgets

Percentages

- **Percent** means out of 100, shown as %
- Multiply by 100% to change to a **percentage**

Fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$
%	1%	5%	10%	20%	25%	$33\frac{1}{3}$%	50%	$66\frac{2}{3}$%	75%

- **Percentage increase** or **percentage decrease** is always calculated 'out of' the original amount
- Change the percentage to a fraction or decimal and multiply by the amount to calculate a percentage of something

Wages, salaries, overtime and penalty rates

- A **rate** compares two quantities, such as money and time
- A **wage rate** is usually stated as the payment per hour, such as $16.30/h
- A **salary** is usually stated as the payment for a year or a month, like $68 420/year
- In Australia, the standard working week is 38 hours
- **Overtime** is work outside the normal hours of work. For casual and part-time workers, overtime is outside the hours for which they have been rostered.
- A higher **penalty rate** is usually paid for work outside the normal days of work
- **Time-and-a-half** is 1.5 times the normal rate
- **Double-time** is 2 times the normal rate
- Saturday penalty rates are usually 1.5 times the normal rate
- Sunday and public holidays penalty rates are sometimes 2.5 times normal rate (**double-time-and-a-half**)
- A **rostered day off** (RDO): 2 times normal hourly rate

Types of employees

- **Permanent** employees have regular hours, get annual leave and sick pay, and are on at least weekly hire. Their employment ends only on giving or receiving notice.
- **Full-time** employees work about 38 hours a week

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- **Part-time** employees work less than 38 hours a week. They get the same hourly rate as full-time employees
- **Fixed-term contract** employees are employed for a specific time or to complete a specific task. They usually get the same hourly rate as full-time employees.
- **Casual** employees have no guaranteed hours of work, annual leave or sick pay, work irregular hours and their employment can usually end without notice
- Casual employees get a higher hourly pay rate, normally about 25% of the base rate higher than full-time or part-time employees. They are usually paid for a minimum of 4 hours work.
- **Apprentices** and **trainees** are learning a trade or skill
- **Juniors** are under 21 years old
- **Apprentices, juniors** and **trainees** are paid a proportion of the adult (qualified) rate
- **Commission** workers get a proportion of the value of sales or services. It is usually a percentage of the value.
- Some commission workers also get a small **retainer**, which is paid regularly regardless of sales so they keep working
- **Piecework** payments are made for each item made or handled
- **Outworkers** work from home and are often employed on a piecework basis
- Some businesses pay a piecework **bonus** or **incentive** in addition to wages

Pensions and allowances

- **Age** and **disability pensions** are paid to eligible people who are too old to work or unable to work because of a disability
- **Youth allowance**, **ABSTUDY** and **Austudy** are paid to eligible Australian apprentices or full-time students
- An **income test** reduces pensions or allowances because of other income
- An assets test reduces pensions or allowances because of the value of things owned
- The **Newstart Allowance** may be paid to people looking for work

Budgets

- **Income** is money received
- **Expenses** are money spent
- **Fixed expenses** are items you must pay
- **Discretionary expenses** are things you choose to pay
- A **budget** is a complete list of income and expenses over the same period
- A **balanced budget** has equal income and expenses
- Contingencies are costs that arise unexpectedly

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1. CHAPTER REVIEW

Incomes and budgets

1 Change to percentages, correct to 3 decimal places where appropriate. Example 1

a $1\frac{1}{4}$ **b** 0.053 **c** 4.6 **d** $\frac{5}{7}$ **e** $2\frac{7}{13}$

2 Change to decimals. Example 2

a 13% **b** 0.05% **c** 135%

3 Change to fractions. Example 3

a 16% **b** 142.5% **c** 2.55% **d** 168% **e** 17.5%

4 Kandie got 17 out of 25 for her English essay and 27 out of 35 for her Maths assignment. Find which is the better mark using percentages. Example 4

5 Find the percentage increase from Eric's first javelin throw of 24.78 m to his second of 30.24 m. Example 5

6 Marcia's white blood cell count was 13 700 when she started antibiotics and 6300 when she finished the course. What was the percentage decrease? Example 6

7 What is 19.5% of $407.60? Example 7

8 The mercury level in a lake went down by 18% to 467.4 ppm over a month. What was the original level? Example 8

9 Change an annual salary of $72 438 to an hourly rate. Example 9

10 Change $29.58/h to an annual wage. Example 10

11 Change $784.32 per week to an hourly rate. Example 11

12 Simone's normal hours are 8:30 a.m. to 5:06 p.m., Monday–Friday with an hour for lunch. She gets $1618.80 a fortnight. During stocktake week she finished late as follows. Example 11

Mon	Tue	Wed	Thurs	Fri
6.06 p.m.	5:38 p.m.	7 p.m.	8:44 p.m.	5:30 p.m.

She is paid time-and-a-half for the first 2 hours overtime and double-time thereafter in half-hour blocks. The second half of the fortnight she only worked late on Monday, to 6 p.m. What was her pay for that fortnight?

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Example 12

13 Paul and Callum are both brickies labourers. Paul is casual and Callum is permanent. Callum gets $974.50 a week. What does Paul get for a week where he works 6 a.m. to 1 p.m. Monday–Friday, with only breaks for drinks?

Example 13

14 After leaving school at 18, Bronwen starts work as a part-time junior in a shop at 70% of the adult wage. She works for 30 hours a week and the adult wage is $845.60 /week. What is Bronwen's weekly wage?

Example 14

15 Suzanne is a manufacturer's representative who works on a 15% commission with a retainer of $250 a week. One week she persuades 40 shops to buy a complete display of hand tools at $145 each. How much does she earn?

Example 15

16 Lee does quality control for an importer of radio-controlled toy cars. He gets paid $3.90 for every car he inspects. One day he checked 43 cars in 6 hours. How much did he get, and what was this as an hourly rate?

Example 16

17 Guiseppe is on the age pension of $877.10 /fortnight, but also earns $300 /fortnight for part-time work. His pension is reduced by 50c /dollar for every dollar over $164 that he earns. What is his pension, and what is his annual income?

Example 17

18 Kylie is a 22-year old single mother going to TAFE to do a certificate III course. She is entitled to a $567.70 /fortnight Youth Allowance and also gets $280 a week working part-time at a supermarket checkout. Her mum looks after her daughter for much of the week and often puts her to bed before Kylie gets home. Her allowance is reduced by 50 cents in the dollar for every dollar earned over $433 a fortnight and 60 cents in the dollar for every dollar over $519 a fortnight. What is her actual allowance?

Example 18

19 Four friends plan a trip to a rock concert in Sydney. They are driving down in a car that uses 15 L/100 km and the trip will be about 1500 km altogether. Petrol costs $1.32 a litre. Tickets for the concert are $150 each and they are staying overnight in an on-site cabin that costs $225 for up to six people. They think that food will cost them about $110 each for the trip. Draw up a budget and work out the cost for each person for the trip.

Example 19

20 Phan has just moved into her own flat. She earns $880 a week after tax and pays $380 a fortnight for her flat. Her food and other day-to-day expenses come to $320 a week, and transport costs her $70 a week. She is paying off a wide-screen TV at $112 a month and also spends about $380 a month on clothes. Her aerobics class costs $18 a week and her mobile costs $120 a month. Draw up a budget and work out how much she could save in a year.

Problem solving

21 Change 8.54% to the nearest fraction with a denominator under 30.

22 What percentage increase will cancel out a 20% decrease?

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23 The concentration of 2.5 L of sugar solution increased by 15% to 471.5 g/L. How much extra sugar had been dissolved?

24 Which is better, \$21.42 /h, \$1620.92 /fortnight or \$43 300 a year?

25 A casual shop assistant got \$554.50 for a 20-hour week. What is the normal permanent wage?

26 Julie got \$136.01 extra in a week when she worked 4 hours overtime on Thursday, but no other overtime. She got time-and-a-half for the first 2 hours and double-time thereafter for overtime. What was her total pay for the week?

27 Nina is a representative for a gourmet food company selling food to delicatessens all over the city. She works for a commission of 19% on all sales, but also has the option to be paid a retainer. If she chose a retainer of \$250 /week, her commission rate would drop to 13%. What value of sales does she need to make each week to be better off on a straight commission basis?

28 Roger works as a fruit-picker getting \$35 a bin. On his first day he manages to do only one bin, but by the end of the first week he is doing a bin every $2\frac{1}{2}$ hours. He maintains this rate for the rest of the 6 weeks that he works. He works for 10 hours a day, 5 days a week. Assuming he improved steadily in the first week, what did he earn altogether? Apart from the first week, what was his hourly rate of pay?

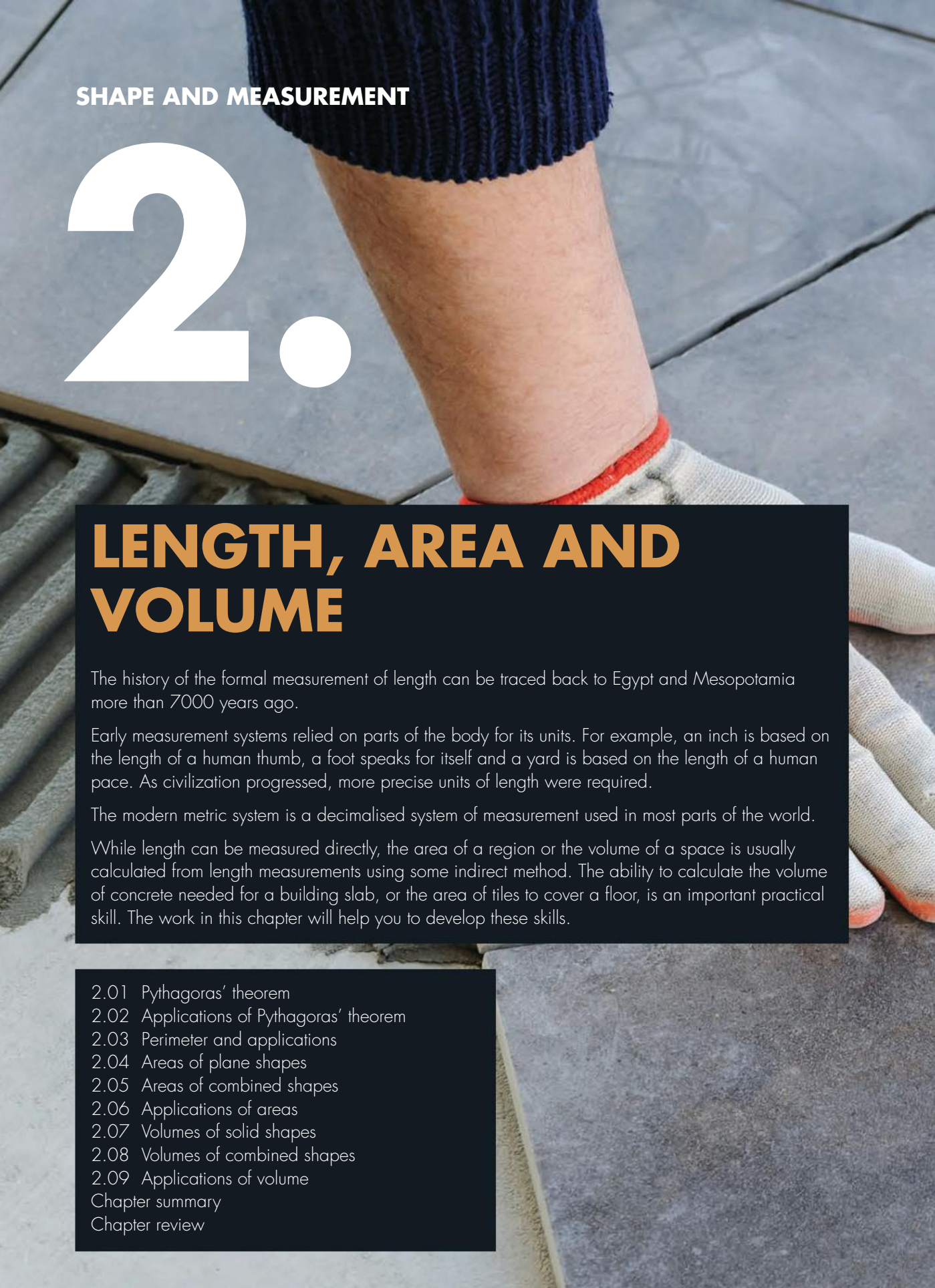
29 Cassie works part-time as well as attending university full-time to help pay for her accommodation and living expenses. She is 19, single and gets \$178.70 ABSTUDY a fortnight. The ABSTUDY is reduced from the full amount of \$433.20/fortnight by 50c for each dollar over \$433/fortnight and 60c for each dollar over \$519/fortnight that she earns.

a What are her weekly earnings?

b What is her total annual income?

30 Petra earns \$624/week after tax. She is thinking of moving into a flat that costs \$350/fortnight with power included. She estimates that she currently spends \$4800 a year on clothes and \$300 a week on entertainment. Her parents say that it costs them \$320 a week for food and groceries for themselves, Petra and her 15-year-old brother. Petra also spends \$380 a month on her mobile and \$4.60 each week-day to get to and from work. Draw up a budget and advise Petra about moving out of home.

Practice quiz



SHAPE AND MEASUREMENT

2.

LENGTH, AREA AND VOLUME

The history of the formal measurement of length can be traced back to Egypt and Mesopotamia more than 7000 years ago.

Early measurement systems relied on parts of the body for its units. For example, an inch is based on the length of a human thumb, a foot speaks for itself and a yard is based on the length of a human pace. As civilization progressed, more precise units of length were required.

The modern metric system is a decimalised system of measurement used in most parts of the world.

While length can be measured directly, the area of a region or the volume of a space is usually calculated from length measurements using some indirect method. The ability to calculate the volume of concrete needed for a building slab, or the area of tiles to cover a floor, is an important practical skill. The work in this chapter will help you to develop these skills.

2.01 Pythagoras' theorem
2.02 Applications of Pythagoras' theorem
2.03 Perimeter and applications
2.04 Areas of plane shapes
2.05 Areas of combined shapes
2.06 Applications of areas
2.07 Volumes of solid shapes
2.08 Volumes of combined shapes
2.09 Applications of volume
Chapter summary
Chapter review

SYLLABUS SUBJECT MATTER

Pythagoras' theorem

- review Pythagoras' theorem and use it to solve practical problems in two dimensions and for simple applications in three dimensions

Mensuration

- solve practical problems requiring the calculation of perimeters and areas of circles, sectors of circles, triangles, rectangles, trapeziums, parallelograms and composites
- calculate the volumes and capacities of standard three-dimensional objects, for example, spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations such as the volume of water contained in a swimming pool

Prior learning

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TERMINOLOGY

acute-angled
area
centi-
combined shape
cylinder
hemisphere
kite
obtuse-angled
prism
Pythagorean triples
rectangular prism
sector
square metre
triangle
apex
base
circle
cone
diameter
hypotenuse
litre
parallelogram
pyramid
radius
rhombus
semicircle
three-dimensional (3D) shapes
two-dimensional (2D) shapes
arc length
capacity
circumference
cubic metre
hectare
kilo-
milli-
perimeter
Pythagoras' theorem
rectangle
right-angled
sphere
trapezium
volume

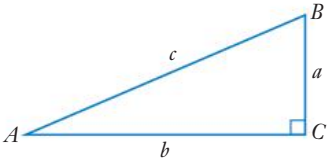
Pythagoras' theorem

Pythagoras' theorem time trial

Pythagorean two-step problems

2.01 Pythagoras' theorem

The naming convention for triangles is that the vertices (corners) have capital letters and the opposite sides have the same lower-case letter. In $\triangle ABC$ shown here, $AC = b$, $AB = c$ and $BC = a$. The right angle is shown by a small square in the corner.



In a right-angled triangle, the longest side is opposite the right angle and is called the **hypotenuse**.

Pythagoras' theorem

The square of the hypotenuse is equal to the sum of the squares of the other two sides.

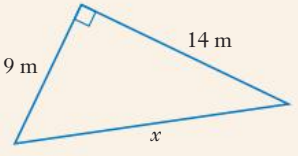
$$c^2 = a^2 + b^2$$

where c is the hypotenuse and the other two sides are a and b (see diagram above).

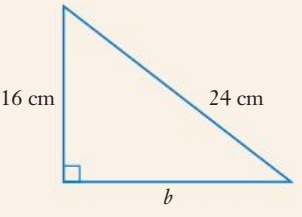
EXAMPLE 1

Find the unknown length in each triangle.

a



b



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Solution

a	Write Pythagoras' theorem.	$c^2 = a^2 + b^2$
	x is the hypotenuse.	$x^2 = 9^2 + 14^2$
	Work out the squares.	$= 81 + 196$
	Calculate the sum.	$= 277$
	Find the square root.	$x = \sqrt{277}$
	Use $\sqrt{}$ on your calculator.	$= 16.6433\ldots$
	Round off.	≈ 16.6
	Write the answer in a sentence.	The unknown length is about 16.6 m.
b	Write Pythagoras' theorem.	$c^2 = a^2 + b^2$
	24 is the hypotenuse.	$24^2 = 16^2 + b^2$
	Work out the squares.	$576 = 256 + b^2$
	Rearrange and evaluate.	$b^2 = 576 - 256$
		$= 320$
	Find the square root.	$b = \sqrt{320}$
	Use $\sqrt{}$ on your calculator.	$= 17.8885\ldots$
	Round off.	≈ 17.9 cm
	Write the answer.	The third side is about 17.9 cm.

INVESTIGATION

PYTHAGOREAN TRIPLES

Pythagoras' theorem can be used to find some standard right-angled triangles where the values of the sides are whole numbers. These are called **Pythagorean triples**. The 3, 4, 5 and 5, 12, 13 triples and their multiples are the most common, but there are many others.

1 Work in pairs or groups of three to find which of the following are Pythagorean triples.

5, 6, 7	6, 8, 10	9, 12, 15	12, 16, 20
10, 24, 26	8, 15, 17	10, 14, 18	7, 24, 25
9, 40, 41	12, 35, 37	16, 63, 65	20, 24, 32

2 Can you find others?

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Pythagorean triads

Pythagorean triples (or triads)

Common Pythagorean triples include:

3, 4, 5 5, 12, 13 8, 15, 17 7, 24, 25

Any multiple of these Pythagorean triples will also be a Pythagorean triple.

You can use Pythagorean triples to save time if you recognise them.

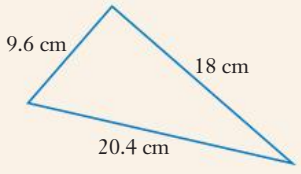
You can also use Pythagoras' theorem to work out if a triangle is right-angled.

The reverse of Pythagoras' theorem says that, if the sides of a triangle satisfy $c^2 = a^2 + b^2$, where c is the longest side, then the angle opposite side c is a right angle. If $c^2 < a^2 + b^2$, the triangle is acute-angled; if $c^2 > a^2 + b^2$, it is obtuse-angled.

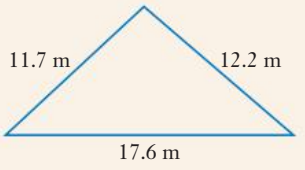
EXAMPLE 2

Determine whether each triangle is right-angled.

a



b



Solution

a If the triangle is right-angled, Pythagoras' theorem will apply, so $24.4^2 = 9.6^2 + 18^2$.

Find the sum of the squares of the shorter sides.	$9.6^2 + 18^2 = 92.16 + 324$ $= 416.16$
Find the square of the longest side.	$20.4^2 = 416.16$
Compare the results.	$20.4^2 = 9.6^2 + 18^2$
This satisfies Pythagoras' theorem.	The triangle is right-angled.

b

Find the sum of the squares of the shorter sides.	$11.7^2 + 12.2^2 = 136.89 + 148.84$ $= 285.73$
Find the square of the longest side.	$17.6^2 = 309.76$
Compare the results.	$17.6^2 > 11.7^2 + 12.2^2$
Write the answer.	The triangle must be obtuse-angled.

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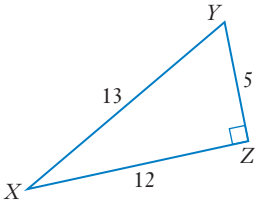
Exercise 2.01 Pythagoras' theorem

1 Which of the following is correct for $\triangle XYZ$?



A $x^2 = y^2 + z^2$

B $13^2 = y^2 + x^2$

C $y^2 = x^2 + z^2$

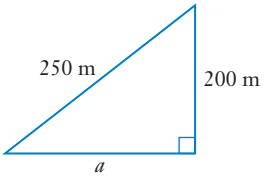
D $12^2 = 5^2 + z^2$

E $13^2 = 12^2 - x^2$

2 Find the value of the unknown side in each triangle.

Example 1

a



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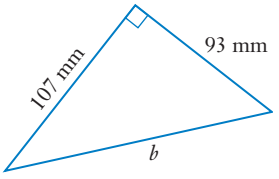

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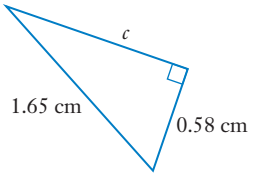

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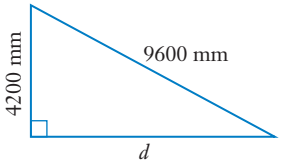

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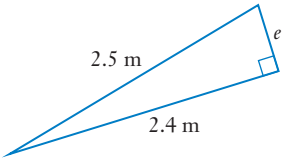

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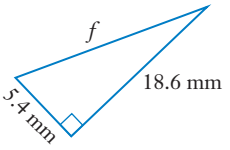
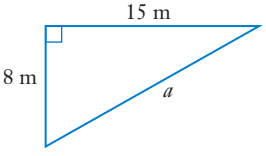


3 Use Pythagoras' theorem to find the unknown side in each triangle.

a



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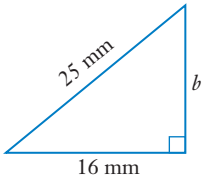

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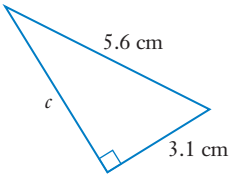

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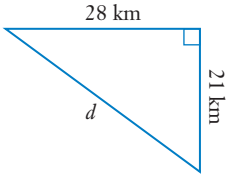

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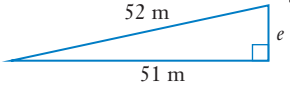

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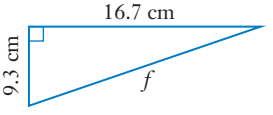
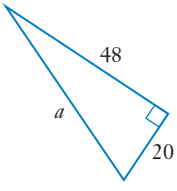


4 Use Pythagorean triples to find the missing side in each triangle.

a



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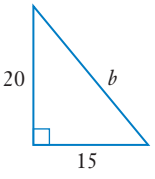

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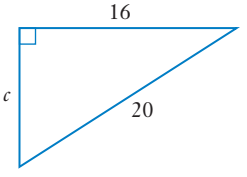

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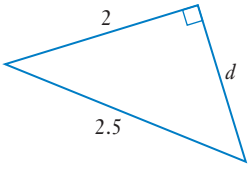

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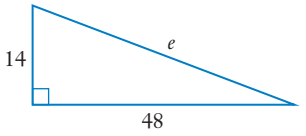
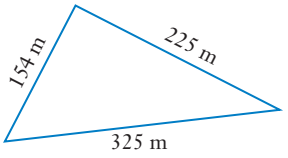


Example 2

5 Determine whether each triangle is right-angled.

a



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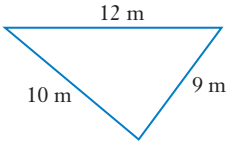

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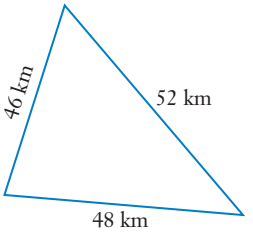

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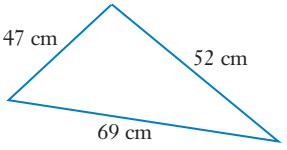

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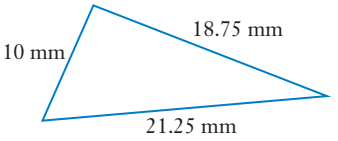
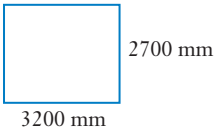


Problem solving

6 Find the length of the diagonal in each rectangle.

a



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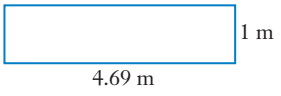

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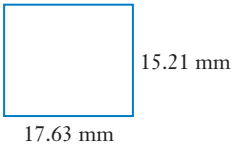
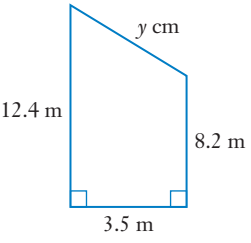


7 Calculate the unknown side in each figure.

a



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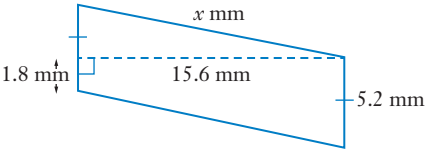

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2.02 Applications of Pythagoras' theorem

Applications of Pythagoras' theorem

Pythagoras' problems

EXAMPLE 3

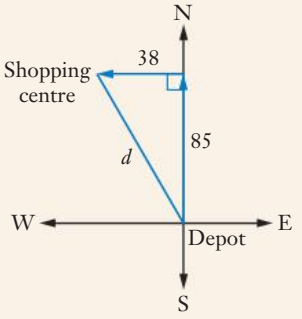
A truck leaves a country depot and travels directly north for a distance of 85 km. The driver then turns due west and travels for another 38 km to a shopping centre where a delivery is made. How far is the shopping centre from the depot?

Solution

Draw a diagram to represent this information.

Draw the journey north first then draw the journey due west.

Join the positions of the depot and shopping centre with a straight line and label it d.



Apply Pythagoras' theorem. $d^2 = 85^2 + 38^2$

Evaluate. $= 8669$

Take square roots of both sides. $d = \sqrt{8669}$

Evaluate. $= 93.107\ldots$

Round off and state the result. The shopping centre is about 93.1 km from the depot.

Pythagoras' theorem is widely used in building and construction.

EXAMPLE 4

The front of this machinery shed is 5.8 m wide. The sides are 3.4 m high and the highest point of the roof is 6.2 m above its base. The roof of the shed is made from sheets of iron that overhang by 10 cm on both sides.

How long are the sheets of iron?

Shutterstock.com/Eric Backman

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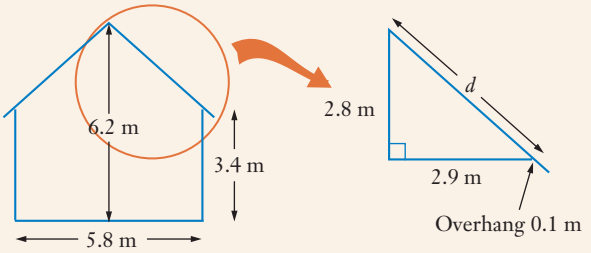
Solution

Draw a diagram and add the dimensions.

Change the overhang to the same units as the other dimensions (m).



Apply Pythagoras' theorem to the triangle. $d^2 = 2.9^2 + 2.8^2$

Evaluate. $= 8.41 + 7.84$

$= 16.25$

Take the square root of both sides. $d = 4.0311\ldots$

The overhang needs to be added. Sheet length $= 4.0311\ldots + 0.1$ m

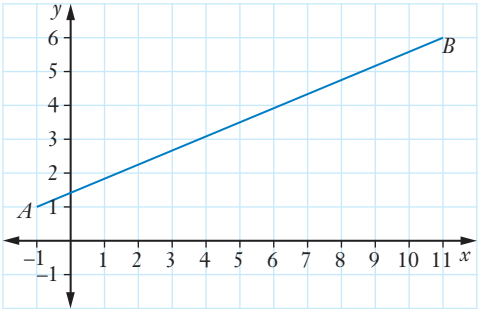
$= 4.1311\ldots$

Round off and state the result. The roof sheets are about 4.1 m long.

Exercise 2.02 Applications of Pythagoras' theorem

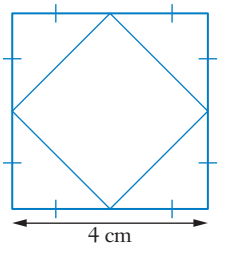
1 The length of AB is:

A 5
B 10
C 11
D 13
E 15



2 A smaller square is drawn inside a larger square as shown in the diagram.

Use Pythagoras' theorem to determine the side length of the smaller square and hence calculate the area of the smaller square.



Example 3

3 Melinda leaves home on her morning jog. She travels for 850 m due north from her home and then jogs due east for 620 m before turning south and continuing for 1.2 km. How far, to the nearest metre, is Melinda from her home?

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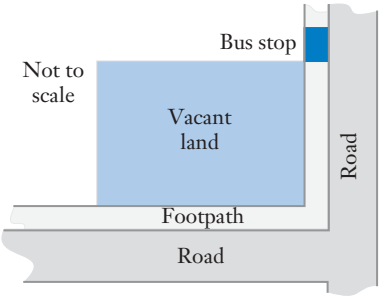
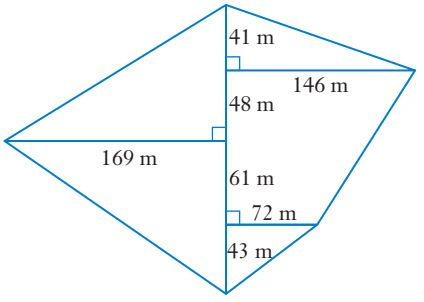
Example 4

4 A vacant corner block of land measures 32 m by 22 m. Instead of going around the footpath, many people walk diagonally across the unfenced block to reach a bus stop. How much further will they need to walk when the block is fenced?



Problem solving

5 A block of ground is being surveyed prior to subdivision. Calculate the perimeter of the block.

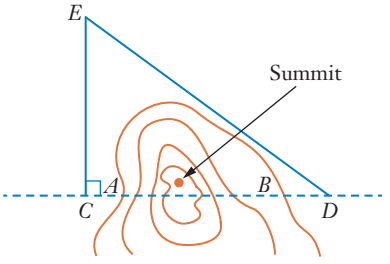


6 A survey cannot measure directly over a small hill, so the following measurements are made between the points shown in the diagram, where A and B are on the same contour line:

$AC = 45$ m $\quad BD = 55$ m

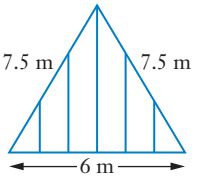
$CE = 248$ m $\quad DE = 427$ m

Find the horizontal distance AB.



7 A rectangular room measuring 4 m by 8 m is to be used for a home theatre. The walls of the room are 3 m high. The projector is to be mounted on the ceiling halfway between the longer side walls and facing one of the 4 m wide walls where the screen will be mounted. The screen measures 1.6 m by 0.9 m and is set up with its centre in the centre of the 4 m wide wall. If the optimum distance for the projector is 7 m from the centre of the screen, how far along the ceiling from the wall on which the screen is mounted should the projector be installed?

8 Amira, the carpenter, is building a roof for a new garage. The roof has a gable end in the form of an isosceles triangle, with a base of 6 m and sloping sides of 7.5 m. She decides to place 5 evenly-spaced vertical pieces of timber as supports for the gable as shown. How many metres of the support timber is needed?



9 A wall frame 3500 long and 2100 high is to have a diagonal brace fitted. How long must the brace be?

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2.03 Perimeter and applications

Measurements of length in the metric system are based on the standard unit: the metre (m). Other length units are derived from the metre using prefixes.

Metric units of length

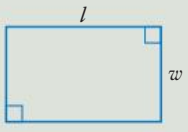
Unit	Relationship	
millimetre (mm)		1 mm = 0.001 m = 0.1 cm
centimetre (cm)	1 cm = 10 mm	1 cm = 0.01 m
metre (m)	1 m = 100 cm = 1000 mm	1 m = 0.001 km
kilometre (km)	1 km = 1000 m	

The **perimeter** of a flat shape is the length around the outside. The perimeter of a circle is called its **circumference** (C).

Perimeter formulas

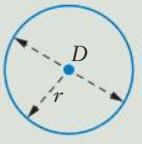
Rectangle

$P = 2(l + w)$



Circle

$C = 2\pi r = \pi D$

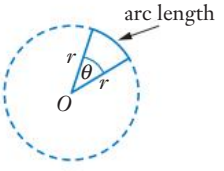


A page of circular shapes

Perimeter and area of a sector

Perimeter and area of a sector

The circle with centre O, on the right shows a piece of the circle called a **sector**.



The **arc length** of part of a circle depends on the sector angle (θ).

The arc length for a circle with angle θ is $\frac{\theta}{360°}$ of the circumference.

Arc length of a sector

$$\text{Arc length} = \frac{\theta}{360°} 2\pi r$$

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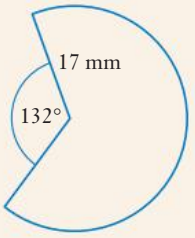
EXAMPLE 5

Calculate the perimeter of this sector.



Solution

Calculate the sector angle.	$\theta = 360° - 132°$
	$= 228°$
Calculate the arc length.	Arc length $= \dfrac{\theta}{360°} 2\pi r$
Substitute for known values.	$= \dfrac{228°}{360°} \times 2 \times \pi \times 17$
Evaluate using the π key on your calculator.	$= 67.6489\ldots$
Find the perimeter of the sector.	$P =$ arc length $+ 2r$
Substitute.	$= 67.6489\ldots + 2 \times 17$
Round off and state the result.	The perimeter is about 101.7 mm.

Perimeter calculations have many real life applications, such as working out the cost of fencing an area.

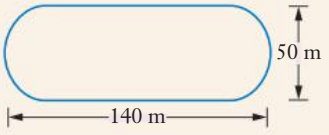
EXAMPLE 6

A rectangular sports-ground 140 m long and 50 m wide has semicircular ends. A reinforced fence is to be erected all the way around, with four gates, each 2 m wide. The gates cost \$250 each and the fencing costs \$85/m. Find the cost of fencing the ground.

Solution

Sketch the sportsground.

Show the measurements.



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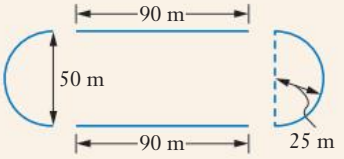
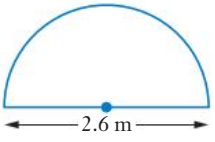
Divide the perimeter into sections.	
Calculate the circular length at each end.	End $= \frac{1}{2}\pi D$ $= \frac{1}{2}\pi \times 50$ $= 78.5398\ldots$
Calculate the perimeter.	Perimeter $= 90 + 78.5398\ldots + 90 + 78.5398\ldots$ $= 337.0796\ldots$
Subtract the 4 gate lengths of 2 m each.	Fence length $= 337.0796\ldots - 4 \times 2$ m $= 329.0796\ldots$ m
Find the total cost.	Total cost $= 329.0796\ldots \times \$85 + 4 \times \$250$ $= \$27\ 971.768\ldots + \$1000$ $= \$28\ 971.768\ldots$
Round off and write the answer.	The cost of fencing the ground is about \$28 972.

Exercise 2.03 Perimeter and applications

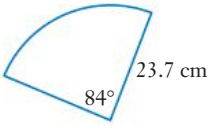
Example 5

1 Find the perimeter of each shape.

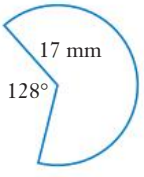
a



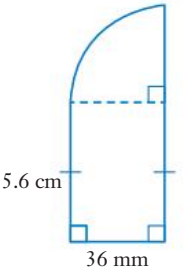
b



c



d



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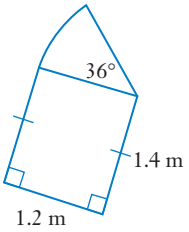
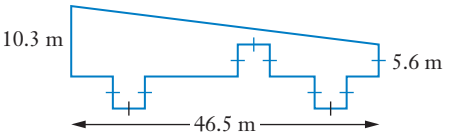
e

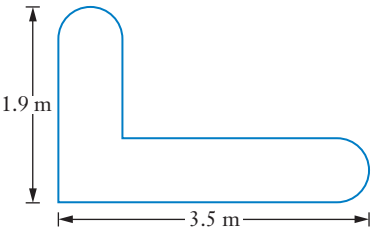


f



Example 6

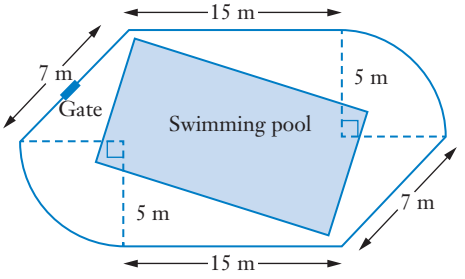
2 The L-shaped kitchen bench with semicircular ends shown here has been made from chipboard veneer. The bench is 600 mm wide. It must have an edging strip of veneer attached all the way along the edge to finish the job. What length of edging veneer is required?



Problem solving

3 A fence is to be erected around a block of land that has a slanting front boundary and parallel side boundaries of 27 m and 42 m. The rear boundary is 20 m long and is at right angles to the side boundaries. The posts are to be a maximum of 2 m apart, there are two railings, and 100 mm palings are to be spaced 20 mm apart. The posts cost \$12.50 each, the railings cost \$5.40/m and the palings cost \$1.40 each. A 2.5 m gate costing \$130 is to be placed at the front. Find the cost of materials to complete the job.

4 A swimming pool is set in a wooden deck as shown. A pool fence is to be erected around the border, with a 1200 mm wide gate. The cost of materials for a 1200 mm high fence is \$42/m, and the gate costs \$120. Installation costs are \$10/m for the fence and gate.



a Calculate the cost of materials to complete the job.

b Find the cost of installing the fence and gate.

5 A circular horse-racing track is 700 m across the inside and is 15 m wide. The track has to be re-fenced on both sides with post-and-rail fencing at a cost of \$34/m for materials and labour. Find the cost of erecting the fences.

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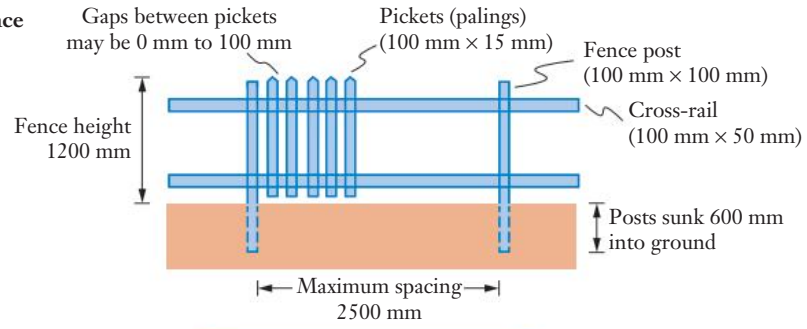
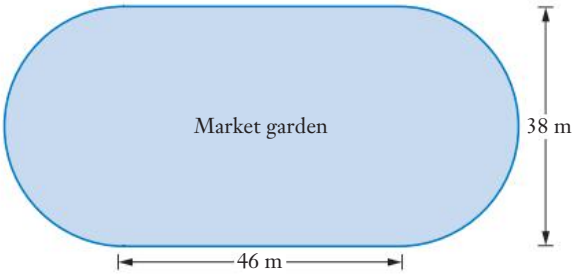
6 The market garden plot shown below is to be enclosed with the wooden post-and-rail fence as illustrated.

Post-and-rail fence



Market garden plot



a Determine the number of posts and the length of cross-rails needed to complete the task.

b Calculate the minimum number of 100 mm × 15 mm palings required if the maximum spacing desired is 75 mm.

c Work out the cost of the materials if posts cost \$14.50 each, rails cost \$3.40/m and palings cost \$2.10 each.

2.04 Areas of plane shapes

Area ID

A page of circular shapes

The **area** of a flat shape measures the size of the region enclosed by the boundary of the shape.

One square metre is 100 cm long and 100 cm wide, so

$$1\ \text{m}^2 = 100\ \text{cm} \times 100\ \text{cm} = 10\ 000\ \text{cm}^2$$

and

$$1\ \text{m}^2 = 1000\ \text{mm} \times 1000\ \text{mm} = 1\ 000\ 000\ \text{mm}^2.$$

One hectare is 100 m long and 100 m wide, so:

$$1\ \text{ha} = 100\ \text{m} \times 100\ \text{m} = 10\ 000\ \text{m}^2$$

One square kilometre is 1000 m long and 1000 m wide, so:

$$1\ \text{km}^2 = 1000\ \text{m} \times 1000\ \text{m} = 1\ 000\ 000\ \text{m}^2 = 100\ \text{ha}$$

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EXAMPLE 7

Convert:

a 8.6 cm^2 to mm^2 **b** 120 000 cm^2 to m^2 **c** 2.34 km^2 to ha.

Solution

a Write the conversion.

$1\ cm^2 = 10\ mm \times 10\ mm$

$= 100\ mm^2$

Larger unit → smaller unit, so multiply by 100 (move decimal point 2 places to right).

$8.6\ cm^2 = 8.6 \times 100\ cm^2$

$= 860\ mm^2$

b Write the conversion.

$1\ m^2 = 100\ cm \times 100\ cm$

$= 10\ 000\ cm^2$

Smaller unit → larger unit, so divide by 10 000 (move decimal point 4 places to left).

$120\ 000\ cm^2 = 120\ 000 \div 10\ 000\ m^2$

$= 12\ m^2$

c Write the conversion.

$1\ km^2 = 100\ ha$

Larger unit → smaller unit, so multiply by 100 (move decimal point 2 places to right).

$2.34\ km^2 = 2.34 \times 100\ ha$

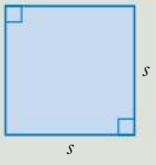
$= 234\ ha$

Length, area and volume conversions

In most situations, an area can be determined using the areas of basic shapes. The formulas for the areas of the common shapes are shown below.

Area formulas

Square

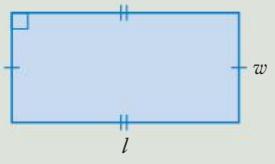


A = side × side

$= s \times s$

$= s^2$

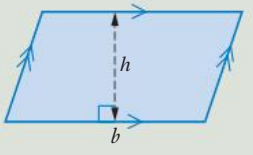
Rectangle



A = length × width

$= lw$

Parallelogram



A = base × perpendicular height

$= bh$

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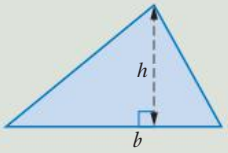
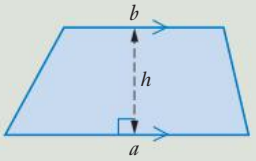
Triangle



$A = \frac{1}{2} \times$ base $\times$ perpendicular height

$= \frac{1}{2}bh$

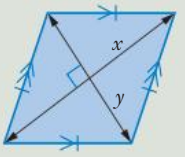
Trapezium



$A = \frac{1}{2} \times$ sum of parallel sides $\times$ perpendicular height

$= \frac{1}{2}(a + b)h$

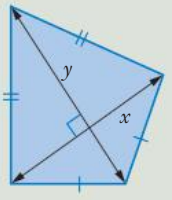
Rhombus



$A = \frac{1}{2} \times$ product of diagonals

$= \frac{1}{2}xy$

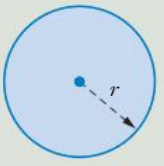
Kite



$A = \frac{1}{2} \times$ product of diagonals

$= \frac{1}{2}xy$

Circle



$A = \pi \times (\text{radius})^2$

$= \pi r^2$

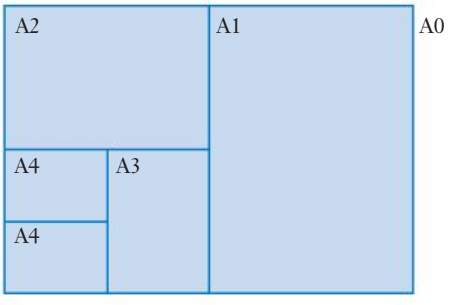
INVESTIGATION

PAPER SIZES

Paper comes in sizes A0, A1, A2, A3, A4 and so on. All sizes are generated from the A0 size, which is a rectangular shape with an area of 1 m^2.

The sides of the rectangle are in the ratio $\sqrt{2} : 1$.

The sizes A1, A2 and so on are formed by halving the longer side.



1 Determine the dimensions of a sheet of A0 paper.

2 Now use the dimensions that you have just worked out to calculate the dimensions of a sheet of A4 paper. Measure a sheet of A4 and compare its dimensions with those you have calculated.

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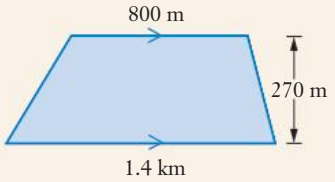
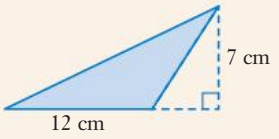
EXAMPLE 8

Calculate the area of each figure, writing your answers in the larger unit.

a



b



Solution

a Put measurements in the same units. $800 \text{ m} = 0.8 \text{ km}$
$270 \text{ m} = 0.27 \text{ km}$

Write the formula. $A = \frac{1}{2}(a + b)h$

Write $\frac{1}{2}$ as $\div 2$. $= (1.4 + 0.8) \times 0.27 \div 2$

Do brackets first. $= 2.2 \times 0.27 \div 2$

Calculate answer. $= 0.297 \text{ km}^2$

b Write the formula. $A = \frac{1}{2}bh$

Substitute for known values. $= \frac{1}{2} \times 12 \times 7$

Evaluate. $= 42$

State the result. $= 42 \text{ cm}^2$

Exercise 2.04 Areas of plane shapes

1 To convert 320.8 mm^2 to cm^2, you need to:

A multiply by 100
B move the decimal place two places to the right
C divide by 10
D move the decimal place one place to the left
E divide by 100

2 To convert 469.7 ha to km^2 you need to:

A divide by 10
B move the decimal place one place to the left
C move the decimal place one place to the right
D divide by 100
E multiply by 10

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Example 7

3 Change each measurement to the square unit indicated.

a 250 000 $cm^2 \rightarrow m^2$ **b** 0.000 24 $m^2 \rightarrow mm^2$ **c** 5800 $m^2 \rightarrow km^2$

d 258 500 $m^2 \rightarrow$ ha **e** 0.036 $km^2 \rightarrow m^2$ **f** 0.0021 ha $\rightarrow m^2$

g 6 720 000 $mm^2 \rightarrow m^2$ **h** 56.8 $mm^2 \rightarrow cm^2$

Example 8

4 Find the area of each object described.

a Triangular scarf with base 1.5 m and height 0.8 m

b Circular lid of radius 25 mm

c Triangular wall tile with base 25 cm and height 12 cm

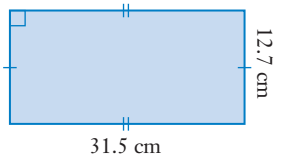
d Trapezoidal table top with parallel sides 1.2 m and 1.8 m and height 80 cm

e Compact disc of diameter 12 cm

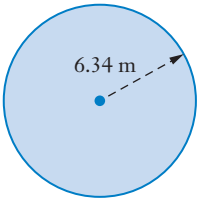
f Circular tablecloth of radius 1.5 m

5 Find the area of each figure.

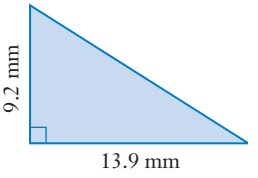
a



b

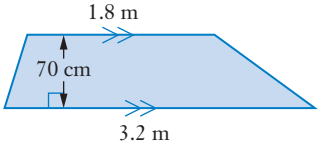


c



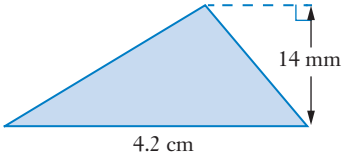
6 Find the area of each shape.

a



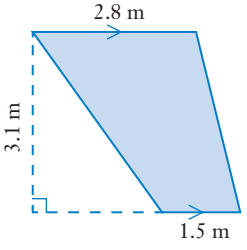
(Answer in m^2.)

b

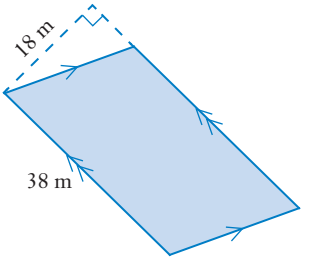


(Answer in cm^2.)

c



d



Problem solving

7 A circular plate has an area of 707 cm^2. What is its radius?

8 A square piece of metal has an area of 110.25 cm^2. What are its dimensions?

9 A triangle has an area of 208.32 cm^2. If the base of the triangle is 9.3 cm long, what is its perpendicular height?

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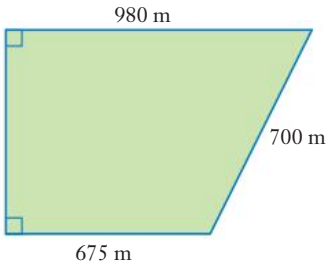
10 The floor of a room measures 3540 mm by 5650 mm. How many square metres of carpet are required to cover the floor? (Give your answer correct to one decimal place.)

11 A farmer has a large field in the shape of a trapezium as shown here. The farmer is preparing the field for a crop of maize and must use 43 kg of fertiliser per ha.

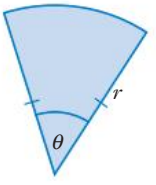
How much fertiliser is required?



2.05 Areas of combined shapes

The fraction of a circle that a sector represents is given by:

$$\text{Fraction} = \frac{\theta}{360°}, \text{ where } \theta \text{ is the sector angle}$$



Area of a sector

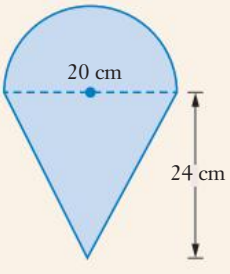
$$\text{Area of sector} = \frac{\theta}{360°}\pi r^2$$

To find the area of a **combined shape**, calculate the areas of each part and then add them together.

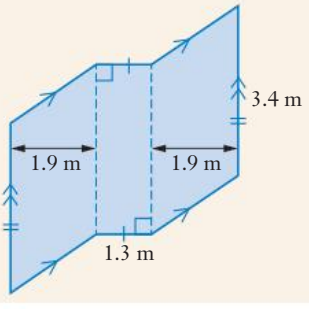
EXAMPLE 9

Find the area of each shape.

a



b



Composite areas

Areas of composite shapes

A page of circular shapes

Perimeter and area of a sector

Perimeter and area of a sector

ISBN 9780170412711

Solution

a The shape is a semicircle joined to a triangle. A semicircle is half a circle.

Find the area of the semicircle. Area of semicircle $= \frac{1}{2}\pi r^2$

The radius is 10 cm. $= \frac{1}{2} \times \pi \times 10^2$

Put this value into memory. $= 157.0796\ldots$

Now calculate the area of the triangle. Area of triangle $= \frac{1}{2}bh$

Substitute for known values and evaluate. $= 0.5 \times 20 \times 24$

$= 240$

Calculate the combined shape area. Area of shape $= 157.0796\ldots + 240$

$= 397.0796\ldots$

Round off and state the answer. The area of the shape is about 397 cm^2.

b This shape has a rectangle and two equal parallelograms.

Area of rectangle. $A = lw$

Substitute for known values. $= 1.3 \times 3.4$

$= 4.42$

Area of parallelogram. $A = bh$

Substitute for known values. $= 3.4 \times 1.9$

$= 6.46$

Calculate the area of the combined shape. Total area $= 4.42 + 2 \times 6.46$

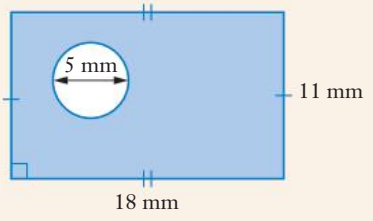
$= 17.34$

Round off and state the result. The area of the shape is about 17 m^2.

Some combined shapes are formed by removing a piece away from another shape.

EXAMPLE 10

Find the area of the shaded region.



ISBN 9780170412711

Solution

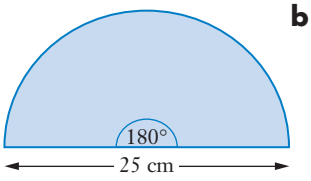
The shaded region is a rectangle with a circular unshaded region.

Begin with the rectangle.	Rectangle area $= lw$
Substitute for l and w.	$= 18 \times 11$
Evaluate.	$= 198$
Write the area rule for a circle.	Circle area $= \pi r^2$
Use $D = 2r$ to substitute for r.	$= \pi \times 2.5^2$
Evaluate.	$= 19.6349\ldots$
Find the shaded area.	Shaded area = rectangle area – circle area
Substitute.	$=198 - 19.6349\ldots$
Evaluate	$= 178.3650\ldots$
Round off and state the result	The shaded region is about 178.4 mm^2.

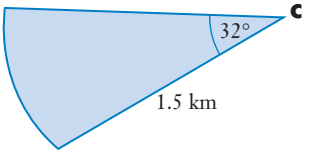
Exercise 2.05 Areas of combined shapes

1 Calculate the area of each sector, correct to one decimal place.

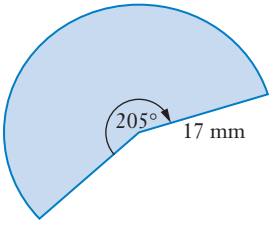
a



b



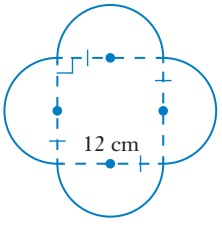
c



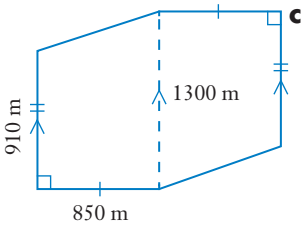
2 Find the area of each combined shape.

Example 9

a



b



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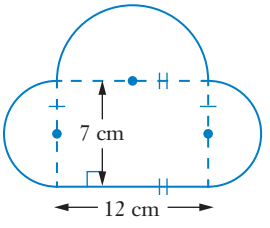

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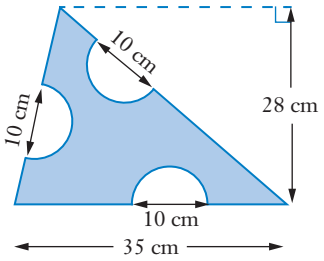
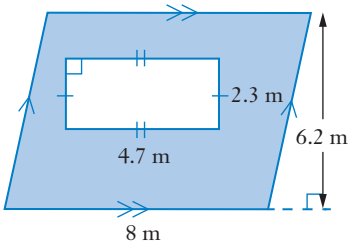
Example 10

3 Find the area of each shaded region.

a

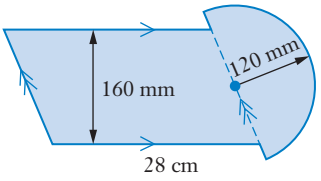


b



4 Find the area of each shape.

a



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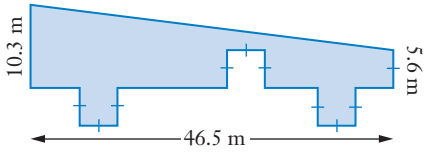
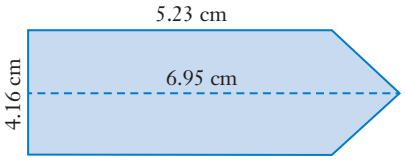
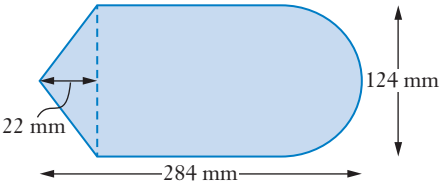


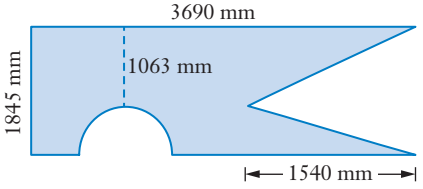
c



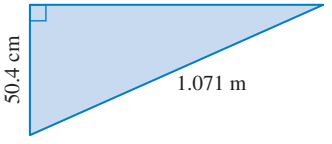
d



e



f

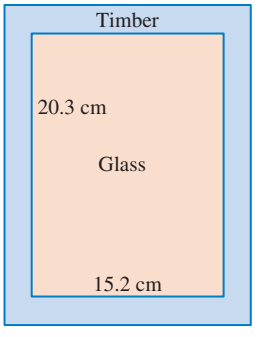


Problem solving

5 Five squares of side 32 cm are cut from a sheet of vinyl 1.02 m by 67 cm. What area of vinyl remains?

6 A photo frame consists of a rectangular piece of glass surrounded by a timber edge. The glass is 15.2 cm by 20.3 cm and the frame is 1.8.cm wide.

What is the area of the timber frame?



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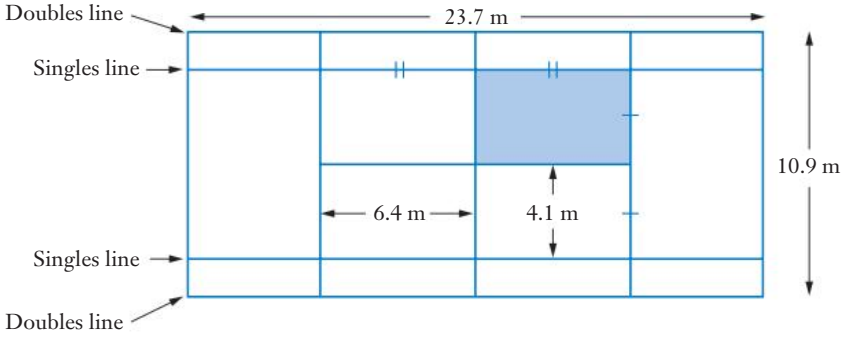
7 The largest possible circular disc is cut from a rectangular sheet of clear perspex measuring 35 cm by 50 cm. Find the area of the perspex that remains, correct to 1 decimal place.

8 Here is a diagram of a full-sized tennis court.



a What area (shaded) does a player have to serve into?

b Find the total area for play in a singles match.

c Find the extra area that is available for a doubles match.

9 The minute hand of a clock is 10 cm long. It is 3 cm longer than the hour hand. In one full revolution, how much more of the face is covered by the minute hand than by the hour hand?

2.06 Applications of areas

Applications of area

Measurement in the home

Areas are important for many jobs that need to be done in or around the house. In the building industry, measurements are usually stated in millimetres.

When you calculate quantities for a job always round up to the nearest sensible amount, like a half-tile. Tiles have an allowance for grout. Small tiles (less than 100 mm) normally 3 mm, but larger tiles have 5 mm.

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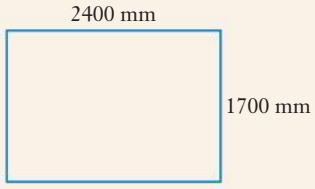
EXAMPLE 11

Calculate the number of 200 mm × 200 mm tiles needed to tile the floor of a laundry measuring 2400 mm by 1700 mm.

Solution

Draw a floor plan.



Work out the tile size, with 5 mm of grout. Tile width = 200 + 5

= 205 mm

Work out the number for the length. Number for 2400 mm wall = 2400 ÷ 205

= 11.7073…

More than a half, so round up. ≈ 12 tiles

Work out number for the width. Number for 1700 mm wall = 1700 ÷ 205

= 8.2926…

Less than a half, so make $\frac{1}{2}$. ≈ 8.5 tiles

Multiply to get total. Number of tiles = 12 × 8.5

= 102 tiles

Add 5% wastage. Total with wastage = 105% of 102 tiles

= 1.05 × 102

= 107.1 tiles

Round off and write the answer. 107 tiles are needed for the laundry.

When estimating the area of walls for painting, openings such as windows and doors are usually ignored. House paint is normally available in 1 L and 4 L tins.

In practical applications, allow about 5% for waste.

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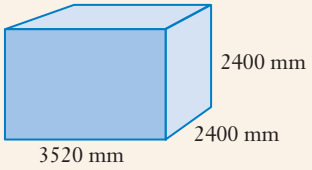
EXAMPLE 12

a How many litres of paint are needed to give the walls of a bedroom 2400 mm by 3520 mm two coats of paint covering 14 m^2/L, assuming the ceiling height is 2400 mm?

b How much ceiling white is needed at the same coverage?

Solution

Draw a diagram, ignoring doors and windows.



a Calculate total wall length in metres.

Wall length = 2 × 2.4 + 2 × 3.52
= 11.84 m

Calculate wall area.

Wall area = 11.84 × 2.4
= 28.416 m^2

Allow for 2 coats.

Area to paint = 2 × 28.416
= 56.832 m^2

Calculate quantity of paint.

Paint needed = 56.832 ÷ 14 L
= 4.0594 … L

Add 5% wastage.

Wastage = 5% of 4.0594… L
= 0.2029… L

Calculate the total required.

Total needed = 4.0594… + 0.2029…
= 4.2624… L

Write the answer.

A 4 L tin and a 1 L tin are needed.

b Calculate ceiling area.

Ceiling area = 2.4 × 3.52
= 8.448 m^2

Allow for 2 coats.

Area to paint = 2 × 8.448
= 16.896 m^2

Calculate quantity of paint.

Paint needed = 16.896 ÷ 14 L
= 1.2068… L

Add 5% wastage.

Wastage = 5% of 1.2068… L
= 0.0603… L

Calculate the total required.

Total needed = 1.2068… + 0.0603…
= 1.2672… L

Write the answer.

Two 1 L tins of ceiling white are needed.

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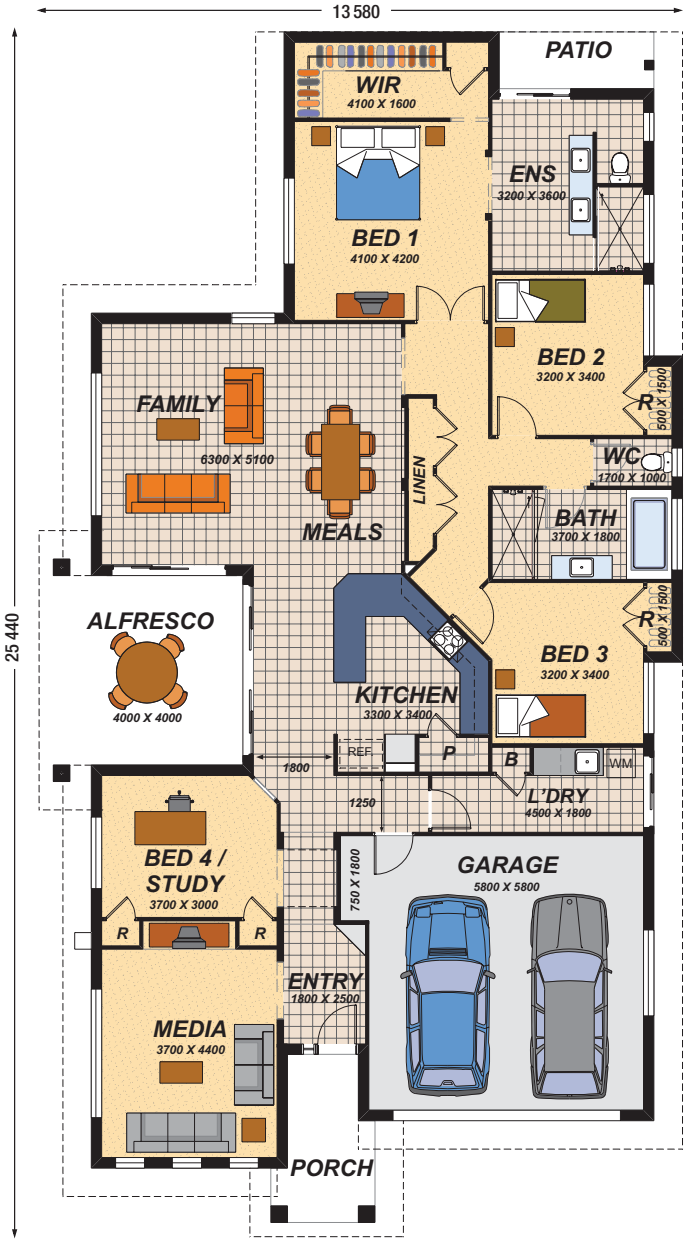
Exercise 2.06 Applications of areas

Questions **1** to **5** refer to the 'Sandalwood' house design.

Sandalwood



Metro Homes Australia Pty Ltd

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1 The ensuite (ENS) floor is to be tiled with ceramic tiles 230 mm square. Allow for 5 mm grout and wastage of 5%. The tiles cost \$4.80 each, glue costs \$40/L and grout costs \$44.50 a bag. A litre of glue will cover 2 m^2, and a bag of grout is enough for 5 m^2 of tiles.

Example 11

a How many tiles are needed?

b How much glue is needed?

c How much grout is needed?

d What is the total cost?

2 The four bedrooms are to be carpeted with carpet costing \$85/m^2, including the cost of laying. The floors of the wardrobes (WIR and R) are not to be carpeted. What will be the carpeting cost?

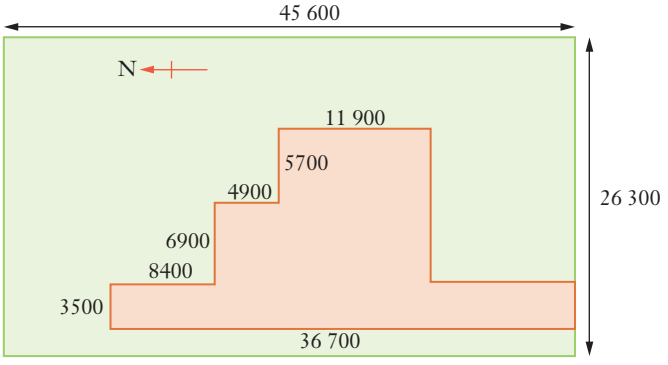
3 Assuming the walls are 2400 mm high, how much paint is needed to do two coats on the bedroom walls, if the paint covers 12 m^2/L?

Example 12

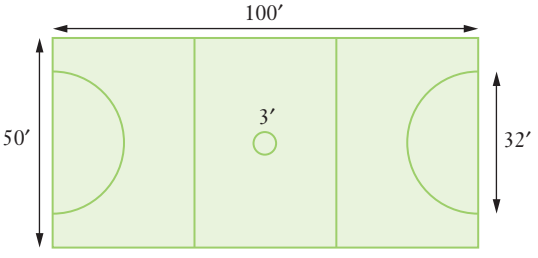
4 Estimate how much ceiling white would be needed for two coats of the whole house (not including the garage or the alfresco area), if coverage is 10 m^2/L.

5 Assuming that the bath (850 mm wide) is not tiled, how many 100 mm square tiles with 3 mm grout are needed for the bathroom and toilet floors?

6 If A-grade turf costs \$12.00/m^2, find the cost of turfing the yard shown below, assuming that the green-shaded area is to be turfed.



7 A netball court is shown here, with measurements in feet. The court is divided into three congruent sections, which are referred to as 'thirds'.



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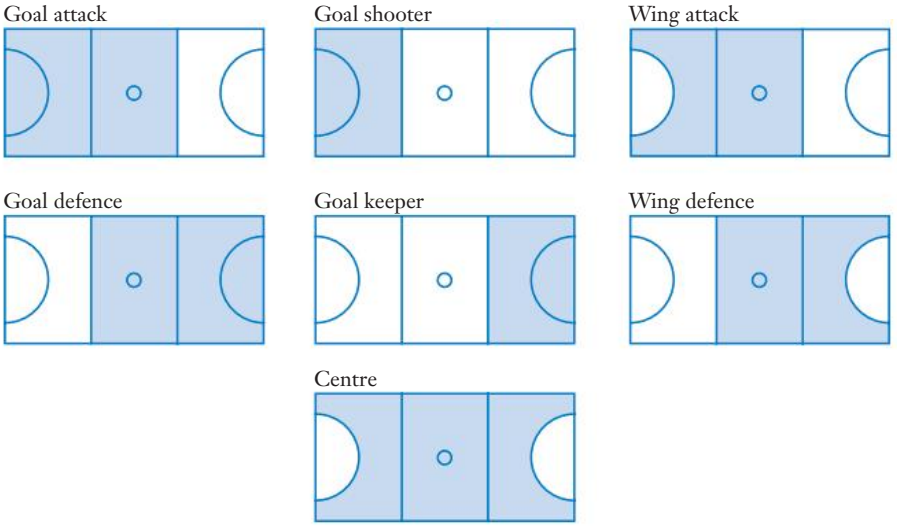
a Convert the dimensions to metric measurements using 1 foot (1′) ≈ 0.3048 m.

b Each player may enter only certain areas, as shown below. Calculate the area that each player may enter and express it as a percentage of the whole court area.



8 A netball is 230 mm in diameter and the hoop is 380 mm in diameter. What percentage of the hoop is occupied when a goal is scored?

Estimating area and volume

Volumes of water

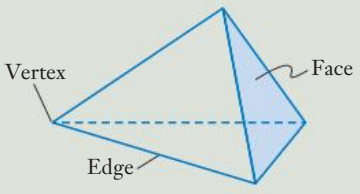
2.07 Volumes of solid shapes

Three-dimensional (3D) shapes occupy space and are **solid** shapes that have length, width and depth. The cross-section and/or the faces are used to name 3D shapes.

3D shapes

The **faces** of a 3D shape separate the inside from the outside.

The lines where faces meet are called **edges**.
The corners where edges meet are called **vertices** (the singular form is **vertex**).



Classifying 3D shapes

A **prism** has constant polygonal cross-sections in one direction. All the cross-sections parallel to one face (the **base**) are the same shape and size as the base.

A **pyramid** has a base that is a polygon. The other faces are triangles that all meet at a **vertex** called the **apex**. A pyramid does not have a constant cross-section, so it is not a prism.

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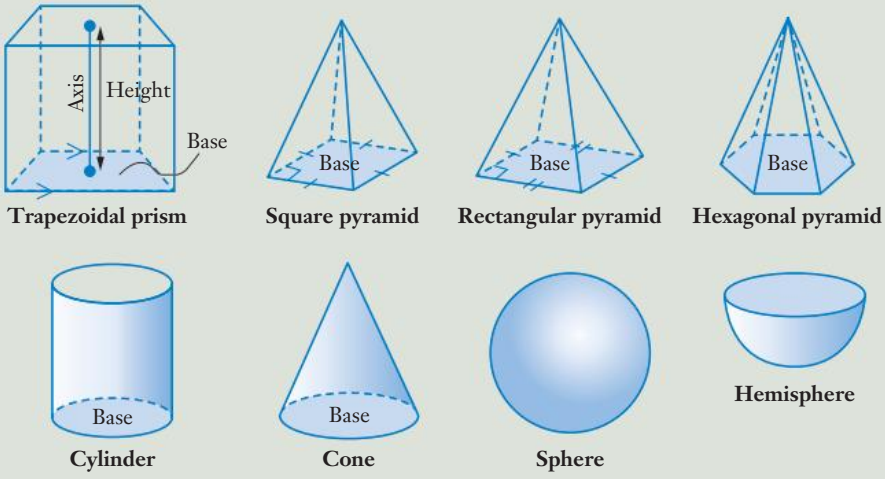
The **axis** of a prism or pyramid is a line passing through the centres of all the cross-sections parallel to its base. The height of a prism or pyramid is measured at right angles (90°) to the base.

A **cylinder** is like a prism with a circular base. A **cone** is like a pyramid with a circular base.

A **sphere** is a ball shape. A **hemisphere** is half a sphere.



The **volume** of a solid shape measures the size of the space occupied by the solid. It is calculated as the number of cubes that will fit into the space, so is given in cubic units. Capacity is another term for volume, but is usually used for volumes of containers for liquids or gases. The common units and conversions are shown below.

Volume and capacity units

Length, area and volume conversions

Unit	Abbreviation	Examples
Cubic centimetre	cm^3	Volumes of small objects
Cubic metre	m^3	Volumes of sand, gravel, soil
Millilitre	mL	Capacities of spoons, glasses, small cans
Litre	L	Capacities of large containers, drums, tanks
Kilolitre	kL	Capacities of swimming pools, farm dams
Megalitre	ML	Capacities of reservoirs

$1\ m^3 = 1\ 000\ 000\ cm^3$ $\quad$ $1\ L = 1000\ mL$ $\quad$ $1\ ML = 1\ 000\ 000\ L = 1000\ m^3$

$1\ m^3 = 1\ kL = 1000\ L$ $\quad$ $1\ mL = 1\ cm^3$

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Volume and capacity units

EXAMPLE 13

Convert:

a 120 000 cm^3 to m^3

b 2.34 m^3 to L

c 83 cm^3 to mL

d 3.6 L to cm^3

Solution

a Write the conversion. $1\ m^3 = 1\ 000\ 000\ cm^3$

Smaller unit → larger unit, so divide. $120\ 000\ cm^3 = 120\ 000 \div 1\ 000\ 000\ m^3$

$= 0.12\ m^3$

b Write the conversion. $1\ m^3 = 1000\ L$

Larger unit → smaller unit, so multiply. $2.34\ m^3 = 2.34 \times 1000\ L$

$= 2340\ L$

c Write the conversion. $1\ mL = 1\ cm^3$

The values are the same. $83\ cm^3 = 83\ mL$

d Write the conversion. $1\ L = 1000\ mL$

Larger unit → smaller unit, so multiply. $3.6\ L = 3.6 \times 1000\ mL$

$= 3600\ mL$

Use the fact that $1\ mL = 1\ cm^3$. $3600\ mL = 3600\ cm^3$

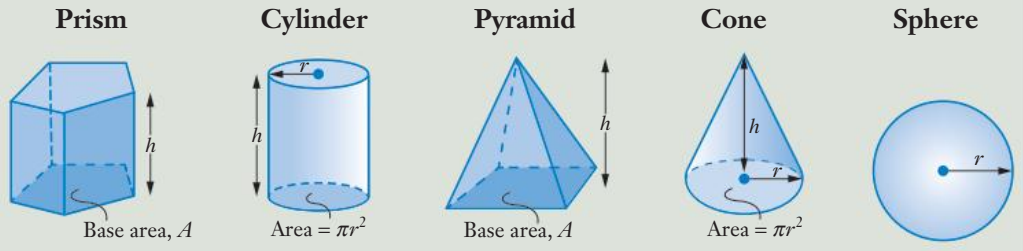
State the result. $3.6\ L = 3600\ cm^3$

The most common formulas for volume are shown below.

Volumes of prisms and cylinders

Solid shapes

Volumes of common 3D shapes



Prism	Cylinder	Pyramid	Cone	Sphere
$V = Ah$	$V = \pi r^2 h$	$V = \frac{1}{3} \times Ah$	$V = \frac{1}{3}\pi r^2 h$	$V = \frac{4}{3}\pi r^3$

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EXAMPLE 14

Find the volume of the can of pineapple shown.

Shutterstock.com/Sheila Fitzgerald

Solution

A cylinder is like a prism.

The base is a circle.

Radius $= \frac{1}{2} \times 7.4 = 3.7$ cm.

$V = Ah$

$= \pi r^2 \times h$

$= \pi \times 3.7^2 \times 10.8$

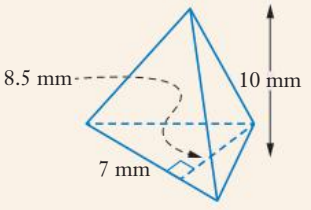
$= 464.4907 \ldots$

Round and write the answer.

The volume is about 464 cm^3.

EXAMPLE 15

Find the volume of this triangular pyramid.



Solution

Find the area of the base.

$A = \frac{1}{2} bh$

Substitute for known values.

$= \frac{1}{2} \times 7 \times 8.5$

$= 29.75$

Now find the volume of the pyramid.

$V = \frac{1}{3} Ah$

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Write $\frac{1}{3}$ as $\div 3$.	$= 29.75 \times 10 \div 3$
Evaluate.	$= 99.1666 \ldots$
Round off and write the answer.	The volume is about 99 mm^3.

Exercise 2.07 Volumes of solid shapes

1 The most appropriate unit for the volume of a backyard swimming pool is:

A cm^3 **B** m^3 **C** mL **D** mm^3 **E** ML

2 5.73 L is the same as:

A 5730 mm^3 **B** 573 mL **C** mL **D** 0.0573 m^3 **E** 0.0573 kL

3 What would be the most appropriate unit for the volume of each of these objects?

a a can of softdrink **b** a tablespoon **c** a shipping container
d Wivenhoe dam **e** an engine's cylinders **f** a laundry tub
g a dessertspoon **h** a bath **i** a house
j a car tyre

Example 13

4 Convert each measurement to the unit indicated.

a 26 $m^3 \rightarrow cm^3$ **b** 320 000 $cm^3 \rightarrow m^3$ **c** 2.8×10^{11} $cm^3 \rightarrow$ ML
d 800 mL $\rightarrow$ L **e** 56 kL $\rightarrow m^3$

5 Convert each measurement to the unit indicated.

a 2400 mL to L **b** 3.2 L to cm^3 **c** 7.5 L to mL
d 730 mL to L **e** 41 cm^3 to mL **f** 8.2 m^3 to L
g 432 mL to cm^3 **h** 3.24 L to m^3 **i** 2320 cm^3 to L

Example 14

6 Find the volume of a cylinder with:

a radius 17 cm, height 22 cm **b** diameter 20 mm, height 15 mm
c diameter 2.4 m, height 1.9 m **d** radius 8.3 cm, height 5.2 cm

7 Find the volume of:

a a conical pile of sugar with a height of 3.4 m and a radius of 4.8 m
b a square-based pyramid with a base measuring 13 cm by 13 cm and a height of 11 cm
c a rectangular-based pyramid with a base measuring 2.1 m by 1.7 m and a height of 3.3 m
d a conical funnel with a height of 26 cm and a diameter of 18 cm
e a rectangular-based pyramid with a base measuring 42 mm by 21 mm and a height of 11 mm

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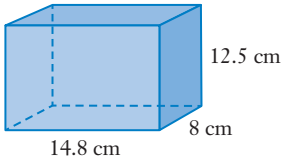
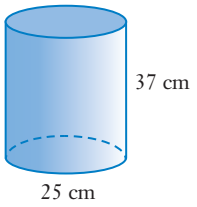
Example 15

8 Find the volume of each solid.

a



b



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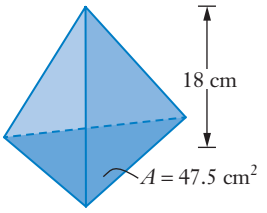

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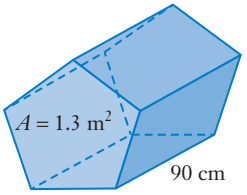

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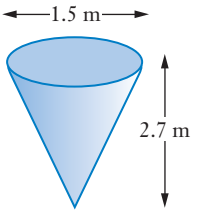

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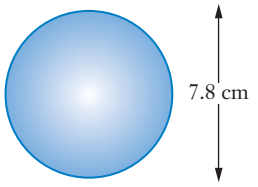

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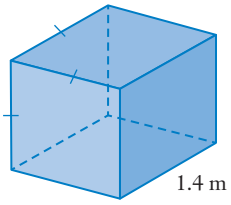

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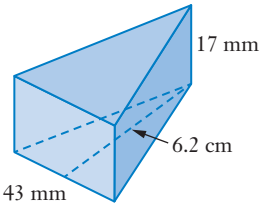

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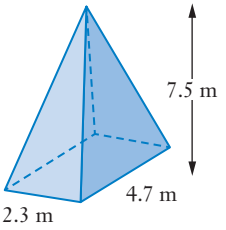


Problem solving

9 A house brick measures 230 mm × 110 mm × 76 mm. What is its volume in:

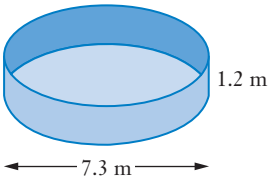
a cubic millimetres? **b** cubic centimetres?

10 Some swimming pools are drawn below. For each pool, find:

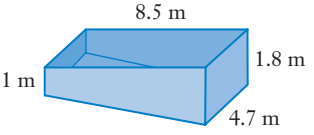
i the capacity in litres

ii the time taken to fill the pool at a rate of 8 L/minute.

a



b



11 An engineering company wants to build a truck body that will hold 18 m^3 of material. The truck body will be in the shape of a rectangular prism. If the dimensions of the base are 1.8 m by 4.8 m, how high will the sides need to be?

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A page of solid shapes

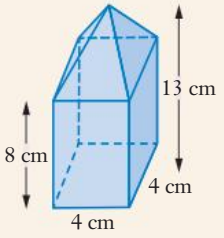
Volumes of solids

2.08 Volumes of combined shapes

Combined shapes are formed using two or more standard solids, including prisms, cylinders, pyramids, cones and spheres.

EXAMPLE 16

Calculate the volume of this 3D shape.



The shape can be considered as a rectangular prism with a square-based pyramid on top.

Solution

Calculate the volume of the prism.	$V = \text{Area of base} \times h$
The base is a square and $h = 8$ cm.	$= 4 \times 4 \times 8$
Evaluate.	$= 128 \text{ cm}^3$
Next, calculate the volume of the pyramid.	
Find the height.	Height of pyramid $= 13 - 8 = 5$ cm
Find the area of the base.	Area of base $= 4 \times 4 = 16 \text{ cm}^2$
Calculate the volume.	$V = \frac{1}{3}Ah$
Substitute for known values.	$= \frac{1}{3} \times 16 \times 5$
Evaluate.	$= 26.6666 \ldots \text{ cm}^3$
Add to find the combined volume.	Volume of shape $= 128 + 26.6666 \ldots$
	$= 154.6666 \ldots$
Round and write the answer.	The volume of the 3D shape is about 155 cm^3.

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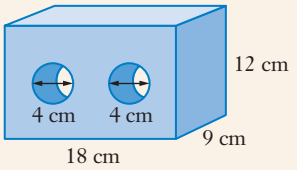
Some solid shapes are formed by removing parts of standard shapes.

EXAMPLE 17

Find the volume of clay needed to make this brick.



Solution

The shape is a rectangular prism with two cylinders taken out.

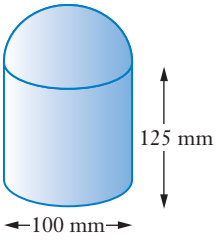
Find the volume of the prism.	$V = A \times h$
Evaluate.	$= 18 \times 9 \times 12$
	$= 1944 \text{ cm}^3$
Next, find the volume of a cylinder.	$V = \pi r^2 h$
The radius is 2 cm.	$= \pi \times 2^2 \times 9$
	$= 113.0973\ldots \text{ cm}^3$
Subtract to find the combined volume.	Volume of clay $= 1944 - 2 \times 113.0973\ldots$
Evaluate.	$= 1717.8053\ldots \text{ cm}^3$
Round and write the answer.	The volume of clay needed is about 1720 cm^3.

Exercise 2.08 Volumes of combined shapes

Example 16

1 Find the volume of each 3D shape.

a



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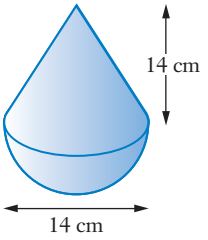

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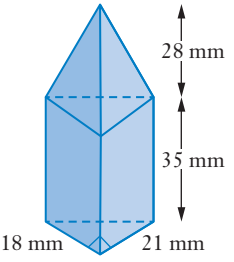
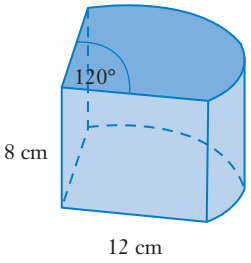


2 Find the volume of each solid.

a



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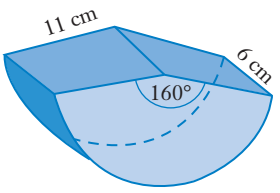

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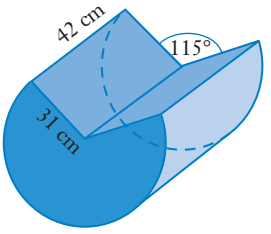
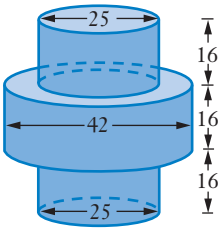


Example 17

3 Each object below is made of plastic. Calculate the volume of plastic in each object. All dimensions shown are in centimetres.

a



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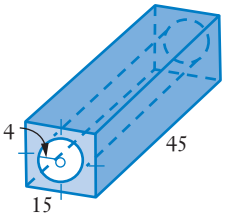

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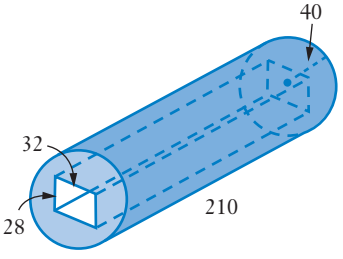

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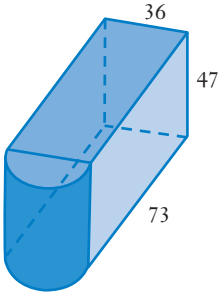
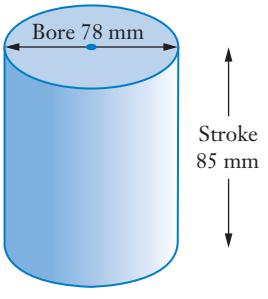


Problem solving

4 The engine capacity of a motor vehicle is the total capacity of the cylinders in the engine. It is usually stated in cubic centimetres or litres.

Calculate the engine capacity of a four-cylinder car if each cylinder has a bore (diameter) of 78 mm and a stroke (length) of 85 mm. Answer in litres.



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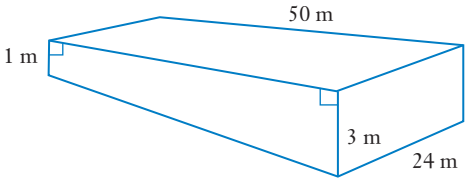
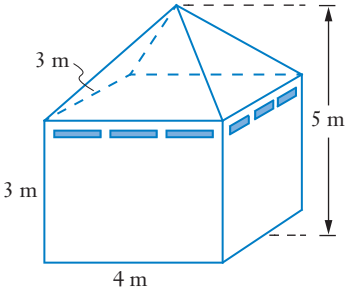
5 **a** Find the volume of this swimming pool in cubic metres.



b What is the capacity of the pool in kilolitres?

c The pool is filled using a pipe with a cross-sectional area of 0.02 m^2. How long will this take if the water flows through the pipe at 2 m per second?

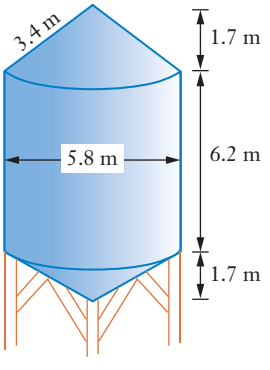
6 This is a tent used for the fumigation of imported goods suspected of carrying pests from overseas.



a Calculate the volume of the tent.

b The gas used for fumigation is dense and it pushes out the air through special vents when the tent is prepared for use. If the gas can be delivered at the rate of 0.035 m^3/s, how long will the tent take to fill?

7 This grain silo is made up of a cylinder and two identical shallow cones.



a How many cubic metres of barley can this silo hold?

b Some barley is removed from the full silo and the level of grain drops by 1 m. What percentage of the grain has been removed?

c A farmer wishes to build a silo to these dimensions. How many square metres of sheet metal are required if 5% must be allowed for overlaps and wastage?

8 The Great Pyramid in Egypt has been reduced in size from its original dimensions by weather and a number of other causes. It is known that it originally had a volume of 2.57 million m^3 and a square base of side length 230 m.

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a What was its original height?

b Its volume has been reduced by about 6% over the years. Assuming that the base is still 230 m square, what is its height now?

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2. CHAPTER SUMMARY

Length, area and volume

Length, area and volume find-a-word

- The naming convention for triangles is that the vertices (corners) are named using capital letters and the opposite sides have the same lower-case letter.
- **Pythagoras' theorem** states that in a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. The **hypotenuse** is the side opposite the right angle. $c^2 = a^2 + b^2$, where c is the hypotenuse and a and b are the other two sides.
- **Pythagorean triples** (triads) are whole numbers that satisfy Pythagoras' theorem, such as 3, 4, 5 and 5, 12, 13.
- The **perimeter** of a shape is the distance around the outside.
- The **arc length** of a circle, $l = \frac{\theta}{360°}\pi D$.
- The **circumference** of a circle is given by $C = \pi D$.
- Common metric measures of length are the millimetre (mm), centimetre (cm), metre (m) and kilometre (km).

  1 mm = 0.001 m

  1 cm = 0.01 m

  1 km = 1000 m
- Building measurements are normally given in millimetres.
- The **area** of a flat shape measures the size of the region enclosed by the boundary of the shape.
- The common metric units of area are the square centimetre (cm^2), square metre (m^2), hectare (ha) and square kilometre (km^2).

  1 km^2 = 100 ha = 1 000 000 m^2

  1 ha = 10 000 m^2

  1 m^2 = 10 000 cm^2
- The formulas for the areas of common shapes are:

  Rectangle: $A = lw$

  Parallelogram: $A = bh$

  Trapezium: $A = \frac{1}{2}(a + b)h$

  Triangle: $A = \frac{1}{2}bh$

  Circle: $A = \pi r^2$

  Sector: $A = \frac{\theta}{360°}\pi r^2$

  Rhombus: $A = \frac{1}{2}xy$

  Kite: $A = \frac{1}{2}xy$

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- The volume of a solid shape measures the size of the space occupied by the solid.
- Capacity is used to indicate the volume of a container for liquids.
- The common units of volume and capacity are the cubic centimetre (cm^3), cubic metre (m^3), litre (L), millilitre (mL), kilolitre (kL) and megalitre (ML).

  $1\ m^3 = 1\ 000\ 000\ cm^3$

  $1\ cm^3 = 1$ mL and $1\ m^3 = 1$ kL = 1000 L

  1 L = 1000 mL, 1 kL = 1000 L and 1 ML = 1 000 000 L
- A **prism** is a 3D shape with a constant polygonal cross-section, usually called the base.
- A **pyramid** is a 3D shape that tapers from the polygonal base to a point.
- The formulas for the volumes of some common 3D shapes are:

  Prism: $V = Ah$ Cylinder: $V = \pi r^2 h$ Pyramid: $V = \frac{1}{3}Ah$

  Cone: $V = \frac{1}{3}\pi r^2 h$ Sphere: $V = \frac{4}{3}\pi r^3$

2. CHAPTER REVIEW

Length, area and volume

Example 6

1 The circumference of a circle is given by:

A $C = \pi r$ **B** $C = 2\pi D$ **C** $C = \pi r^2$ **D** $C = \pi D$ **E** $C = \pi D^2$

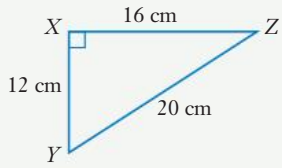
Example 3

2 Which equation is correct for $\triangle XYZ$?

A $x^2 + y^2 = z^2$ **B** $y^2 = x^2 + z^2$

C $16^2 = 12^2 + x^2$ **D** $20^2 = z^2 + y^2$

E $x^2 = z^2 - y^2$



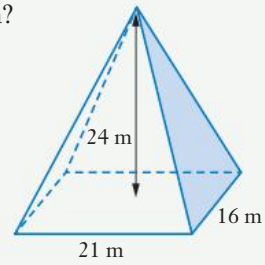
Example 15

3 The volume of this pyramid can be found using which calculation?

A $\frac{1}{2}(21 + 16) \times 24$ **B** $21 \times 16 \times 24$

C $\frac{1}{3} \times 21^2 \times 24$ **D** $\frac{1}{3} \times 21 \times 16 \times 24$

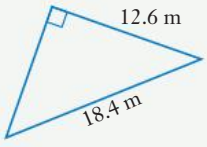
E $\frac{1}{3}(21 + 24) \times 16$



Example 1

4 Use Pythagoras' theorem to find the missing side in each triangle.

a



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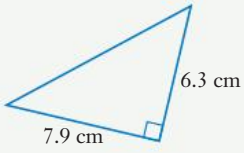

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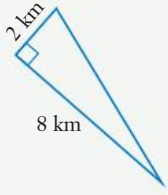

Example 2

5 Use Pythagoras' theorem to work out whether a triangle with the following dimensions is right-angled.

a 3, 5, 7 **b** 8, 10, 12 **c** 8, 15, 17 **d** 13, 15, 17

Example 3

6 A triangular course for a yacht race has a north and an east leg, and the final leg returns to the start. The final leg is 7 km long and the first leg is 4.5 km long. How long is the second leg?

7 Find the length of steel bracing needed to fit a diagonal brace to a wall frame 6.2 m long and 2.4 m high.

Example 5

8 Calculate the circumference of a circle with a diameter of 15.8 cm. Give your answer correct to 1 decimal place.

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9 Change each measurement to the unit of area indicated. Example 5

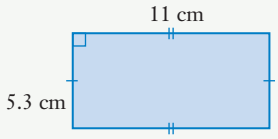
a 38 000 $cm^2 \rightarrow m^2$ **b** 0.0086 $m^2 \rightarrow mm^2$ **c** 26 000 $m^2 \rightarrow km^2$

d 6700 $m^2 \rightarrow$ ha **e** 0.0036 $km^2 \rightarrow m^2$ **f** 0.056 ha $\rightarrow m^2$

g 28 700 000 $mm^2 \rightarrow m^2$ **h** 126.7 $mm^2 \rightarrow cm^2$

10 Find the area of each figure. Example 8

a



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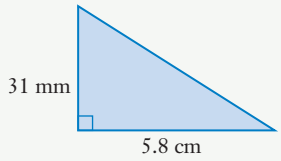

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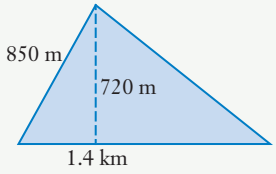

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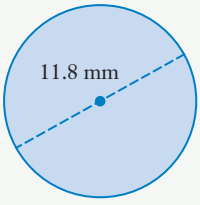

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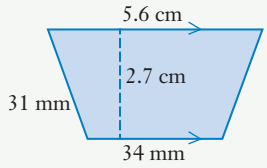

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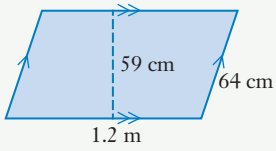
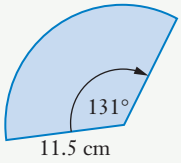
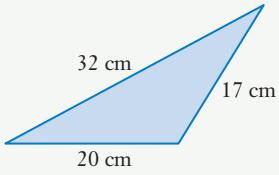


11 Calculate the perimeter and area of this sector, correct to one decimal place. Example 9

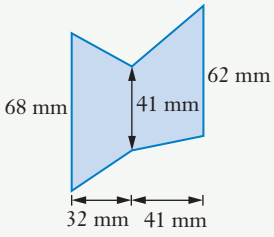


12 Find the area of each shape. Example 9

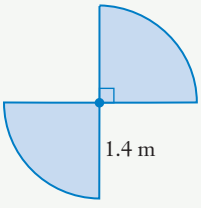
a



b

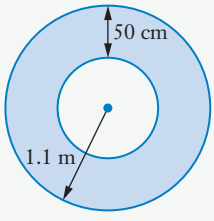


c

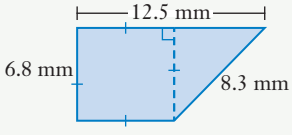


13 Calculate the area of each shape, correct to one decimal place. Example 10

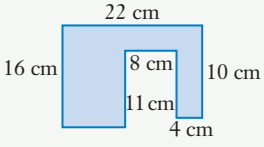
a



b



c



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Example 11

14 Calculate the number of 300 mm by 300 mm tiles needed to tile a floor measuring 4800 mm by 2800 mm.

Example 12

15 A masonry wall 20.5 m long and 2.8 m high is to be coated with paint that has a coverage of 8 L/m^2. How many tins of paint are needed?

Example 13

16 Convert:

a 15.3 cm^3 to mm^3 **b** 2 180 000 cm^3 to m^3

Example 13

17 Convert:

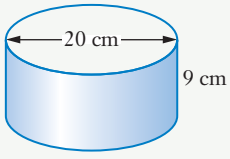
a 15 m^3 to L **b** 280 mL to L

c 570 cm^3 to mL **d** 570 cm^3 to m^3

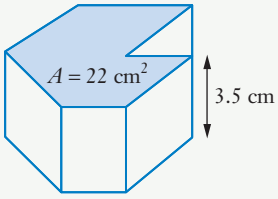
Example 14

18 Calculate the volume of each object.

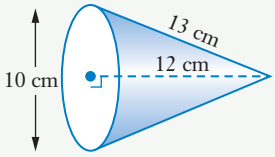
a



b



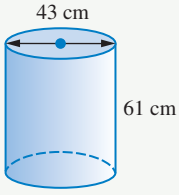
c



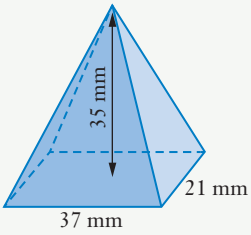
Example 15

19 Find the volume of each shape.

a



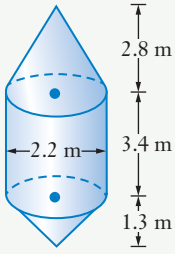
b



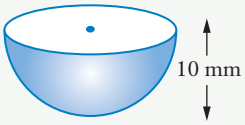
Examples 16–17

20 Find the volume of each shape.

a



b



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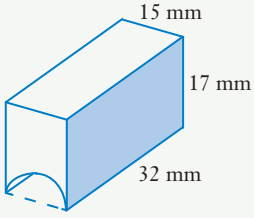

Problem solving

21 A circular running track is to have a length of 400 m. What diameter should it be?

22 A circle has a diameter of 16 m. What is the perimeter of a 60° sector?

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Use the site plan below to answer questions **23** to **25**.

Example 4

23 Calculate the area of:

a the patio

b the office

c the house

d the path and the driveway.

24 The back yard from the property line to the two fences is to be turfed. It turf costs \$9.50/m^2, find the cost of buying the turf needed.

25 The roof of the garage is flat. It is to be covered with metal sheets measuring 2400 mm × 750 mm with a 50 mm overlap. Find the number of sheets required.

26 A cannery has ordered tomato juice cans that are 20 cm high and 8 cm in diameter. The labels are to be printed and cut from 50 m rolls of 40 cm wide paper. Each label requires a 1 cm overlap when placed on the can. How many labels can be cut from each roll?

27 A duct is 20 m long. It has a 10 m-long central rectangular section that is 600 mm by 800 mm, a 5 m-long section changing to an 800 mm round section at one end, and a 5 m-long section changing to a 700 mm square section at the other end. Find the volume of gas in the duct.

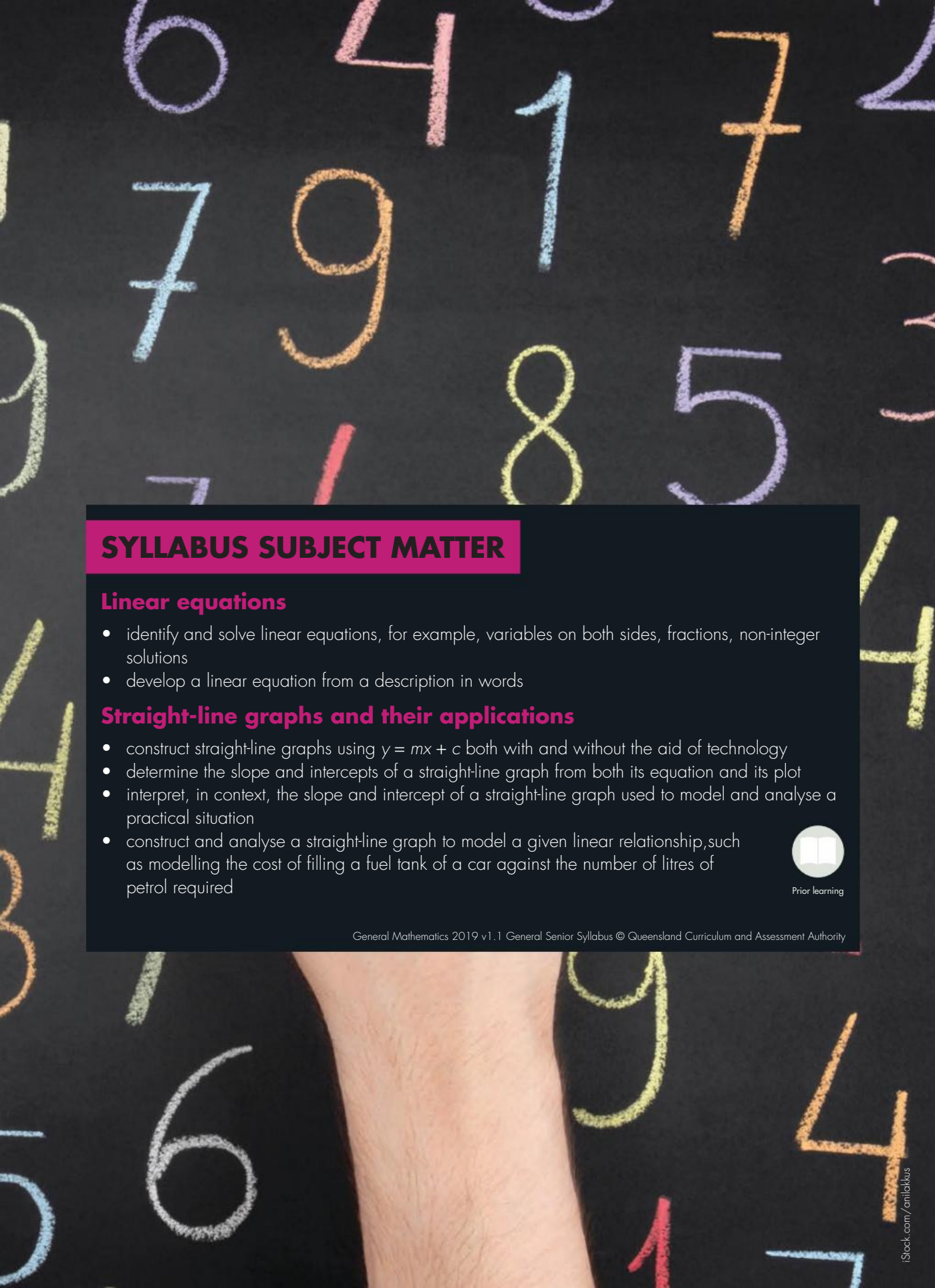
Practice quiz

3.

LINEAR EQUATIONS AND GRAPHS

Looking for patterns and relationships helps us to understand the world around us. A mathematical model is a mathematical representation of a real-world situation and such models usually involve one or more equations. In this chapter you will review the solution of linear equations, then examine and graph a range of linear equations. You will use these skills to model and interpret linear relationships.

3.01 Simple linear equations
3.02 Equations with variables on both sides
3.03 Equations involving fractions
3.04 Using equations to solve problems
3.05 Linear functions
3.06 The slope of a line
3.07 The intercepts of a line
3.08 Applications of linear models
Chapter summary
Chapter review



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SYLLABUS SUBJECT MATTER

Linear equations

- identify and solve linear equations, for example, variables on both sides, fractions, non-integer solutions
- develop a linear equation from a description in words

Straight-line graphs and their applications

- construct straight-line graphs using $y = mx + c$ both with and without the aid of technology
- determine the slope and intercepts of a straight-line graph from both its equation and its plot
- interpret, in context, the slope and intercept of a straight-line graph used to model and analyse a practical situation
- construct and analyse a straight-line graph to model a given linear relationship,such as modelling the cost of filling a fuel tank of a car against the number of litres of petrol required

Prior learning

General Mathematics 2019 v1.1 General Senior Syllabus © Queensland Curriculum and Assessment Authority

TERMINOLOGY

balance
equation
gradient
intercept
linear model
output
range
root
slope
standard form
x-coordinate
y-coordinate
coordinates
falling
gradient–intercept form
inverse operation
ordered pair
parallel lines
rise
run
solution
transpose
x-intercept
y-intercept
domain
function
input
linear equation
origin
quadrants
rising
satisfy
solve
x-axis
y-axis

One and two-step equations

Solving equations by balancing

Solving equations using diagrams

Solving equations by backtracking

Substitution code puzzle

3.01 Simple linear equations

An **equation** has an 'equals' sign to show the left-hand side (LHS) is equal to the right-hand side (RHS).

A **linear equation** *in one variable* is an equation that can be written in the form $ax + b = c$ where a, b, and c are real numbers (constants) and x is a variable (unknown). **Solving** an equation means finding a value for the variable that makes the statement true.

A value that makes the equation true is called a **solution** or **root** of the equation. The solution is said to **satisfy** the equation.

EXAMPLE 1

For each equation, determine whether $x = 8$ is a solution.

a $\dfrac{x-2}{3} = 3$ **b** $5x + 8 = x^2 - 2x$

Solution

a Substitute 8 for x in the LHS. $\text{LHS} = \dfrac{x-2}{3}$

$= \dfrac{8-2}{3}$

Evaluate. $= 2$

Write the RHS. $\text{RHS} = 3$

Compare the LHS and RHS. $\text{LHS} \neq \text{RHS}.$

State the result. $x = 8$ is not a solution.

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b Substitute 8 for x in the LHS — $\text{LHS} = 5x + 8$

$= 5 \times 8 + 8$

Evaluate. — $= 48$

Substitute 8 for x in the RHS. — $\text{RHS} = x^2 - 2x$

$= 8^2 - 2 \times 8$

Evaluate. — $= 64 - 16$

$= 48$

Compare the LHS and RHS. — LHS = RHS.

State the result. — $x = 8$ is a solution.

To solve an equation, you need to keep the equation **balanced**. You solve equations with **inverse operations** to **transpose** the equation.

Inverse operations

An inverse operation has the effect of undoing the original operation.

Whatever you do to one side of the formula must be done to the other side. Work in the reverse order to the usual order of operations.

Operation	Inverse operation
+	–
–	+
×	÷
÷	×
square (^{2})	$\sqrt{\ }$
$\sqrt{\ }$	square (^{2})

EXAMPLE 2

Solve each equation.

a $m - 53 = 46$ **b** $q + 18 = 78$

c $7b = 98$ **d** $\frac{g}{9} = 15$

Solution

a 53 is subtracted from m to give 46. — $m - 53 = 46$

Undo '– 53' using the inverse operation. — $m - 53 + 53 = 46 + 53$

Evaluate. — $m = 99$

b 18 is added to q to give 78. — $q + 18 = 78$

Undo '+ 18' using the inverse operation. — $q + 18 - 18 = 78 - 18$

Evaluate. — $q = 60$

c	7 is multiplied by b to give 98.	$7b = 98$
	Undo '× 7' using the inverse operation.	$\frac{7b}{7} = \frac{98}{7}$
	Evaluate.	$b = 14$
d	g is divided by 9 to give 15.	$\frac{g}{9} = 15$
	Undo '÷ 9' using the inverse operation.	$\frac{g \times 9}{9} = 15 \times 9$
	Evaluate.	$g = 135$

When more than one operation is performed on the variable, you need to use inverse operations in the *reverse order*.

EXAMPLE 3

Solve the following equations.

a $2d + 7 = 13$ **b** $16 - 3n = 7$ **c** $\frac{y}{4} - 3 = 5$ **d** $\frac{4m}{3} = 16$

Solution

a	Write the equation.	$2d + 7 = 13$
	Undo the '+ 7' first.	$2d + 7 - 7 = 13 - 7$
	Evaluate.	$2d = 6$
	Undo the '× 2' next.	$\frac{2d}{2} = 6 \div 2$
	Evaluate.	$d = 3$
b	Write the equation.	$16 - 3n = 7$
	Rewrite the equation.	$-3n + 16 = 7$
	Undo the '+ 16' first.	$-3n + 16 - 16 = 7 - 16$
	Evaluate.	$-3n = -9$
	Undo the '× (–3)' next.	$\frac{-3n}{-3} = \frac{-9}{-3}$
	Evaluate.	$n = 3$

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c Write the equation. $\frac{y}{4} - 3 = 5$

Undo the '– 3' first. $\frac{y}{4} - 3 + 3 = 5 + 3$

Evaluate. $\frac{y}{4} = 8$

Undo the '÷ 4' next. $\frac{y \times 4}{4} = 8 \times 4$

Evaluate. $y = 32$

d Write the equation. $\frac{4m}{3} = 16$

Undo the '÷ 3' first. $\frac{4m \times 3}{3} = 16 \times 3$

Evaluate. $4m = 48$

Undo the '× 4' next. $\frac{4m}{4} = \frac{48}{4}$

Evaluate. $m = 12$

Exercise 3.01 Simple linear equations

1 For each equation, determine whether $x = 12$ is a solution.

a $x + 8 = 20$ **b** $4x - 11 = 37$ **c** $x^2 - 21 = 119$

d $4x - x = 36$ **e** $3(x - 5) = 24$ **f** $9 - \frac{2x}{3} = 1$

g $x^2 - 5x = 82$ **h** $2(x + 2) = 4(x - 8)$ **i** $x^2 - 3x = 8x + 14$

j $5x = (x - 7)^2 + 35$ **k** $(x - 9)^2 = 2x - 8$ **l** $7x + 34 = x^2 - 26$

Example 1

2 a The solution to the equation $38 - x = 56$ is:

A $x = 18$ **B** $x = -94$ **C** $x = -18$ **D** $x = 94$ **E** $x = -24$

b What is the solution to the equation $-\frac{2x}{3} + 11 = 14$?

A $x = -4.5$ **B** $x = 37.5$ **C** $x = -37.5$ **D** $x = -9$ **E** $x = 4.5$

c What is the solution to the equation $\frac{2 - x}{3} = 4.8$?

A $x = 7.2$ **B** $x = 12.4$ **C** $x = -7.2$ **D** $x = -12.4$ **E** $x = 14.4$

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Example 2

3 Solve each linear equation.

a $x - 21 = 13$ **b** $a + 22 = 46$ **c** $5u = 45$

d $\frac{d}{7} = 12$ **e** $f - 28 = 13$ **f** $k + 46 = 17$

g $-8e = 112$ **h** $\frac{y}{11} = 13$ **i** $h - 17 = -28$

j $b + 36 = 19$ **k** $6w = -84$ **l** $-\frac{r}{8} = 12$

4 Solve each equation.

a $8 - k = 8$ **b** $10.5 = 8.4 - x$ **c** $6 - t = 0$

d $9 - g = 6$ **e** $5 - p = 9$ **f** $121 - u = 98$

g $-y + 4 = 7$ **h** $-22 - n = -8$ **i** $3 - a = 1.7$

j $-4 = -7 - z$ **k** $3.9 = 6.7 - e$ **l** $-2r - 3 = 0$

Example 3

5 Solve each equation.

a $3z - 5 = 16$ **b** $5x + 22 = 7$ **c** $4y - 9 = 7$

d $8a + 17 = 57$ **e** $13 + 5b = 28$ **f** $17 + 9c = 8$

g $3 = 6w - 15$ **h** $-7 = 2u + 5$ **i** $7v - 12 = 30$

j $3.2p - 9.3 = 13.1$ **k** $0.8q - 1.7 = 3.1$ **l** $30r - 25 = 65$

6 Solve each equation.

a $\frac{d}{3} + 2 = 4$ **b** $\frac{m}{4} - 6 = -3$ **c** $\frac{t}{7} = \frac{12}{28}$

d $-\frac{p}{5} + 1 = 7$ **e** $8 - \frac{y}{3} = -2$ **f** $5 - \frac{x}{6} = 7$

g $\frac{r}{7} - 4 = 1$ **h** $\frac{2y}{3} = -4$ **i** $-\frac{4x}{5} = -12$

j $-\frac{2h}{7} = 8$ **k** $\frac{3q}{8} = -15$ **l** $-\frac{3g}{11} = -\frac{15}{2}$

Problem solving

7 For each word statement, write a mathematical equation using a variable and then solve the equation.

a Twelve is added to a number and the result is 31.

b A number is divided by 7 and the result is 5.

c Eight is subtracted from a number and the result is 25.

d The product of a number and 3 is added to 7 and the result is 22.

e Seventeen is subtracted from the product of 5 and a number and the result is 18.

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8 The meter on a taxi calculates the fare using the following information. The flag fall (the initial amount charged regardless of distance travelled) is \$4.30 and the distance charge is \$2.17 per kilometre. How far did a person travel if the fare charged for a trip is \$41.19?

Shutterstock.com/ChameleonsEye

9 The diagram below shows a pattern of triangles.

a Write an equation for the matches (m) required to form t triangles.

b How many matches would be needed to form 125 triangles?

3.02 Equations with variables on both sides

Equations riddle

If a variable appears on both sides of an equation, you must first gather the variable on one side of the equation. It is still important to remember that whatever is done to one side of an equation must be done to the other side.

EXAMPLE 4

Solve each equation.

a $8m = 5m + 21$ **b** $11 - 6u = 3u - 16$

Solution

a Write the equation. $8m = 5m + 21$

Gather variables on the LHS by subtracting $5m$ from both sides. $8m - 5m = 5m + 21 - 5m$

Evaluate. $3m = 21$

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	Undo the '× 3'.	$\frac{3m}{3} = \frac{21}{3}$
	Evaluate.	$m = 7$
b	Write the equation.	$11 - 6u = 3u - 16$
	Gather variables on the RHS by adding $6u$ to both sides.	$11 - 6u + 6u = 3u - 16 + 6u$
	Evaluate.	$11 = 9u - 16$
	Reverse the equation.	$9u - 16 = 11$
	Undo the '– 16'.	$9u - 16 + 16 = 11 + 16$
	Evaluate.	$9u = 27$
	Undo the '× 9'.	$\frac{9u}{9} = \frac{27}{9}$
	Evaluate.	$u = 3$

You should be expand brackets before solving an equation.

EXAMPLE 5

Solve each equation.

a $4(p + 3) = 15 - 3p$

b $3(y + 4) = 2(3y - 4)$

Solution

a	Write the equation.	$4(p + 3) = 15 - 3p$
	Expand the brackets.	$4p + 12 = 15 - 3p$
	Gather variables on the LHS by adding $3p$ to both sides.	$4p + 12 + 3p = 15 - 3p + 3p$
	Evaluate.	$7p + 12 = 15$
	Undo the '+ 12'.	$7p + 12 - 12 = 15 - 12$
	Evaluate.	$7p = 3$
	Undo the '× 7'.	$\frac{7p}{7} = \frac{3}{7}$
	Evaluate.	$p = \frac{3}{7}$

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b	Write the equation.	$3(y+4)=2(3y-4)$
	Expand the brackets.	$3y+12=6y-8$
	Gather variables on the RHS by subtracting $3y$ from both sides.	$3y+12-3y=6y-8-3y$
	Evaluate.	$12=3y-8$
	Reverse the equation.	$3y-8=12$
	Undo the '– 8'.	$3y-8+8=12+8$
	Evaluate.	$3y=20$
	Undo the '× 3'.	$\frac{3y}{3}=\frac{20}{3}$
	Evaluate.	$y=6\frac{2}{3}$

Exercise 3.02 Equations with variables on both sides

1 a To solve the equation $7x+3=-9-4x$, the first step is to:

A add $7x$ to both sides **B** add 3 to both sides
C add $4x$ to both sides **D** subtract $4x$ from both sides
E subtract 9 from both sides

b To solve the equation $9x-5=7x+6$, the first step is to:

A subtract $7x$ from both sides **B** add $7x$ to both sides
C subtract 5 from both sides **D** add 6 to both sides
E add $9x$ to both sides

2 Solve each linear equation.

Example 4

a $7g=4g-9$ **b** $7x=-x+16$ **c** $12s=7s-25$
d $21+5u=-2u$ **e** $5w=2w-36$ **f** $13q-28=-q$
g $9x=-2x-33$ **h** $17y=12y+20$ **i** $5r-15=2r$
j $18-3m=-5m$ **k** $32-3z=5z$ **l** $13c=5c-24$

3 Solve each equation.

a $3b+4=9-5b$ **b** $5x+13=3-2x$ **c** $3w-7=4-5w$
d $7a-6=4a+9$ **e** $6y-11=5y+8$ **f** $5n+2=6n+1$
g $6h+5=h-8$ **h** $4u+7=11u+12$ **i** $3d-15=d-8$
j $9k-4=-3k+7$ **k** $8r+4=-3r+15$ **l** $10v+7=4v-9$

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4 Solve each equation.

a $d - 6 = 7d + 10$ **b** $x - 7 = 9x - 2$ **c** $3y + 11 = 8y - 3$

d $11 - 8p = 6 + 5p$ **e** $-8w + 3 = -4w - 13$ **f** $12 - 7u = 13 - 4u$

g $-4z + 5 = 6z + 15$ **h** $4g + 9 = 6g + 7$ **i** $6b + 5 = 13b + 9$

Example 5

5 Solve each equation.

a $5(b - 2) = 3b + 7$ **b** $5(y + 1) = y - 13$ **c** $3(m - 8) = 5m$

d $4(r + 7) = r$ **e** $3(z - 5) = 9 - 2z$ **f** $6u - 7 = 3(2 - u)$

g $-3(x - 4) = 7x$ **h** $-4(3d - 1) = 5d$ **i** $2m - 3 = 5(m - 3) - 4m$

6 Solve each equation.

a $3(e + 2) = 4(e - 2)$ **b** $6(y - 1) = 4(y + 5)$ **c** $5(p - 3) = 2(p - 3)$

d $5(x + 6) = 4(2 - 3x)$ **e** $2(n - 12) = 3(2n - 3)$ **f** $4(k + 5) = 2(3k + 7)$

g $2(3w + 7) = 4(w + 2)$ **h** $5(q - 2) = 3(2q + 5) - 5$ **i** $2 - (3t + 1) = 6(t + 2)$

Problem solving

7 True or false?

a $w = 8$ is a solution of $3w - 6 = 2(w + 1)$

b $x = -2$ is a solution of $-2(2x - 3) = 3(x + 5)$

c $y = 3$ is a solution of $\frac{4y}{3} - 5 = 3y$

d $z = -4$ is a solution of $\frac{5z}{4} - 3 = 4 + 3z$

8 A student's attempt to solve an equation is shown below.

$3(x - 4) = 7x + 4$

$3x - 12 = 7x + 4$ Line 1

$4x - 12 = 4$ Line 2

$4x = 16$ Line 3

$x = 4$ Line 4

a Determine if the stated solution is true.

b If the solution is not true, identify the line where an error has been made.

9 A student's attempt to solve an equation is shown below.

$2(x + 3) = 7 - 3(x - 2)$

$2x + 6 = 7 - 3x - 6$ Line 1

$2x = -3x - 5$ Line 2

$5x = -5$ Line 3

$x = -1$ Line 4

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a Determine if the stated solution is true.

b If the solution is not true, identify the line where an error has been made.

10 For each worded statement, write a mathematical equation using a variable and then solve the equation.

a Twice the sum of a number and 5 equals the sum of the number and 7.

b The sum of 4 and three times a number equals 4 times the sum of the number and 3.

c The product of 3 and 4 less than a number is equal to twice the sum of the number and 5.

3.03 Equations involving fractions

For equations involving fractions, you need to multiply both sides by the denominator to remove the fractions.

More complex equations

Equations code puzzle

EXAMPLE 6

Solve each linear equation.

a $\frac{3w}{7} = 12$ **b** $5 - \frac{2a}{3} = 6$

Solution

a Write the equation. $\frac{3w}{7} = 12$

Undo the '÷ 7'. $\frac{3w \times 7}{7} = 12 \times 7$

Simplify both sides. $3w = 84$

Undo the '× 3'. $\frac{3w}{3} = \frac{84}{3}$

Simplify both sides. $w = 28$

b Write the equation. $5 - \frac{2a}{3} = 6$

Undo the '+ 5'. $5 - \frac{2a}{3} - 5 = 6 - 5$

Simplify both sides. $-\frac{2a}{3} = 1$

Undo the '÷ 3'. $-\frac{2a \times 3}{3} = 1 \times 3$

Simplify both sides.	$-2a = 3$
Undo the '× (–2)'.	$\frac{-2a}{-2} = \frac{3}{-2}$
Simplify both sides.	$a = -1\frac{1}{2}$

EXAMPLE 7

Solve each equation.

a $\frac{5}{m} = \frac{7}{8}$ **b** $\frac{7}{2r} = -6$ **c** $\frac{3}{1-2x} = \frac{5}{2-x}$

Solution

a

Write the equation.	$\frac{5}{m} = \frac{7}{8}$
Invert both sides.	$\frac{m}{5} = \frac{8}{7}$
Undo the '÷ 5'.	$\frac{m}{5} \times 5 = \frac{8}{7} \times 5$
Simplify both sides.	$m = \frac{40}{7}$
Write as a mixed number.	$m = 5\frac{5}{7}$

b

Write the equation.	$\frac{7}{2r} = -6$
Invert both sides.	$\frac{2r}{7} = -\frac{1}{6}$
Undo the '÷ 7'.	$\frac{2r}{7} \times 7 = -\frac{1}{6} \times 7$
Simplify both sides.	$2r = -\frac{7}{6}$
Undo the '× 2'.	$\frac{2r}{2} = -\frac{7}{6 \times 2}$
Simplify both sides.	$r = -\frac{7}{12}$

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c	Write the equation.	$\frac{3}{1-2x}=\frac{5}{2-x}$
	Invert both sides.	$\frac{1-2x}{3}=\frac{2-x}{5}$
	Undo the '÷ 3'.	$\frac{1-2x}{3}\times 3=\frac{2-x}{5}\times 3$
	Simplify both sides.	$1-2x=\frac{3(2-x)}{5}$
	Undo the '÷ 5'.	$(1-2x)\times 5=\frac{3(2-x)}{5}\times 5$
	Simplify and expand both sides.	$5(1-2x)=3(2-x)$
		$5-10x=6-3x$
	Gather the variables on the LHS.	$5-10x+3x=6-3x+3x$
		$5-7x=6$
	Undo the '+ 5'.	$5-7x-5=6-5$
	Simplify both sides.	$7x=-1$
	Undo the '× 7'.	$\frac{7x}{7}=\frac{-1}{7}$
	Simplify both sides.	$x=-\frac{1}{7}$

In the solution for Example **7c**, you could have undone the '÷ 3' and the '÷ 5' in one step by multiplying both sides of the equation by $3 \times 5 = 15$.

Exercise 3.03 Equations involving fractions

1 a The solution to the equation $14-\frac{2x}{3}=6$ is:

A $x=27$ **B** $x=-12$ **C** $x=-18$ **D** $x=12$ **E** $x=-27$

b What is the solution to the equation $\frac{6-x}{2}=4.5$?

A $x=-3$ **B** $x=8$ **C** $x=-2$ **D** $x=4$ **E** $x=3$

c What is the solution to the equation $\frac{5}{m}=\frac{7}{8}$?

A $x=12$ **B** $x=5\frac{5}{7}$ **C** $x=5.5$ **D** $x=-5\frac{1}{3}$ **E** $x=-12$

Example 6

2 Solve each linear equation.

a $\frac{m}{4} = 15$ **b** $-\frac{y}{8} = 11$ **c** $\frac{4x}{3} = 12$

d $\frac{2n}{5} = 8$ **e** $-\frac{3p}{8} = -6$ **f** $\frac{7v}{3} = 35$

g $\frac{z}{6} + 5 = 9$ **h** $-\frac{a}{4} + 6 = 15$ **i** $\frac{4g}{7} - 8 = 16$

j $7 - \frac{8h}{3} = 11$ **k** $\frac{5k}{9} - 3 = 12$ **l** $-\frac{4h}{5} - 3 = 13$

3 Solve each equation.

a $\frac{g+2}{3} = 7$ **b** $\frac{y-6}{5} = 6$ **c** $\frac{c-11}{4} = -2$

d $\frac{x+12}{9} = -6$ **e** $\frac{7-y}{11} = 2$ **f** $\frac{-u+6}{7} = -8$

g $\frac{h+3.7}{2.6} = 5$ **h** $\frac{4.6-t}{7.3} = 4.1$ **i** $\frac{-p-8.2}{5.7} = 10$

j $\frac{120-v}{134} = 0$ **k** $\frac{125-j}{111} = 1$ **l** $\frac{-x-8.6}{7.2} = 11.4$

4 Solve each equation.

a $\frac{2x+1}{4} = 5$ **b** $\frac{3g-1}{2} = 11$ **c** $\frac{5y-4}{3} = -4$

d $\frac{2m+7}{5} = -4$ **e** $\frac{4-2e}{7} = 3$ **f** $\frac{-4r+5}{3} = -6$

g $\frac{-2k-3}{3} = 7$ **h** $\frac{8.3-2t}{1.2} = 5.5$ **i** $\frac{9s-14}{3.5} = 20$

j $\frac{5-3x}{8} = \frac{1}{2}$ **k** $\frac{9-4b}{12} = \frac{1}{3}$ **l** $\frac{-2n-5}{10} = \frac{3}{4}$

5 Solve each equation.

a $\frac{m}{3} - 2 = 4m$ **b** $\frac{w}{5} + 3 = 3w$ **c** $\frac{3x}{4} - 9 = 3x$

d $\frac{3x}{2} + 3 = 5x + 7$ **e** $\frac{3g}{5} + 4 = 2 - 4g$ **f** $\frac{7z}{3} - 4 = 5 + 3z$

g $5 - \frac{2n}{3} = 4 - 3n$ **h** $6 - \frac{3r}{4} = 7 + 5r$

6 Solve each equation.

a $\frac{x}{2} - 5 = \frac{3x}{2} + 7$ **b** $\frac{5m}{4} + 6 = 3 - \frac{m}{4}$ **c** $\frac{2z}{3} - 1 = 3 - \frac{z}{6}$

d $\frac{3n}{4} + 2 = \frac{m}{8} - 1$ **e** $\frac{2u}{5} + 3 = 4 - \frac{3u}{10}$ **f** $\frac{4e}{3} - 7 = 4 + \frac{5e}{12}$

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Example 7

7 Solve each equation.

a $\frac{3}{n}=\frac{1}{2}$ **b** $\frac{4}{p}=-5$ **c** $\frac{2}{g}=\frac{3}{4}$ **d** $\frac{5}{m}=-\frac{10}{3}$

e $\frac{4.5}{y}=\frac{1.8}{10}$ **f** $\frac{6}{5k}=\frac{3}{2}$ **g** $-\frac{7}{3h}=\frac{4}{9}$ **h** $\frac{6}{5c}=-\frac{2}{15}$

i $\frac{8}{3w}=\frac{2}{5}$ **j** $\frac{5}{6y}=-\frac{3}{7}$ **k** $\frac{8}{5x}=\frac{25}{-12}$ **l** $\frac{4}{9d}=\frac{-5}{12}$

8 Solve the following equations. You need to multiply by the *common denominator*.

a $\frac{1}{2x}+\frac{3}{4x}=6$ **b** $\frac{1}{x-3}=\frac{4}{3-2x}$ **c** $\frac{x+1}{1-2x}=4$

d $\frac{3x+2}{4x-3}=5$ **e** $\frac{2}{3-x}=\frac{5}{4-3x}$ **f** $\frac{x+2}{3}=\frac{2x-7}{4}$

g $\frac{3x+1}{1-4x}=\frac{3}{4}$ **h** $\frac{1}{2x-3}=\frac{4}{3x+2}$ **i** $\frac{3x+4}{8}-\frac{2x-1}{4}=x$

Problem solving

9 A student's attempt to solve an equation is shown below.

$5-\frac{2x}{3}=12$

$\frac{2x}{3}=12-5$ Line 1

$\frac{2x}{3}=7$ Line 2

$2x=21$ Line 3

$x=\frac{21}{2}$ Line 4

$x=10\frac{1}{2}$ Line 5

a Determine if the stated solution is true.

b If the solution is not true, identify the line in which an error has been made.

10 A student's attempt to solve an equation is shown below.

$\frac{8}{x}-\frac{7}{6x}=3$

$16-7=18x$ Line 1

$18x=9$ Line 2

$x=\frac{1}{2}$ Line 3

a Determine if the stated solution is true.

b If the solution is not true, identify the line where an error has been made.

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11 Solve the linear equation $mx + n = 0$ for x.

12 Solve the linear equation $\frac{1}{px+q} - \frac{1}{rx+s} = 0$ for x.

Using equations to solve problems

Word problems with equations

3.04 Using equations to solve problems

The following table shows some of the typical words and phrases that indicate the mathematical operations of +, –, × and ÷.

Operation	Word/phrase
Addition	Add, added to, the sum of, more than, increased by, the total of, in all, exceeds by, plus
Subtraction	Subtract, subtract from, difference, less, less than, fewer than, decreased by, diminished by, take away, reduced by, exceeds, minus
Multiplication	Multiply, times, the product of, multiplied by, times as much, of
Division	Divide, divides, divided by, the ratio of, the quotient of, equal shares of, per

EXAMPLE 8

Write an equation for each statement, using x to represent the unknown.

a When 12 is added to a number, the result is 21.

b Five times the sum of a number and three is reduced by 4 and the result is 17.

c When two times a number is divided by 15, the result is 3 more than the number.

Solution

a 'Added to' means '+'. $x + 12 = 21$

b Begin with the operation most closely associated with the variable. 'The sum of a number and three' means '$x + 3$'. $x + 3$

'Five times' means '× 5'. $5(x + 3)$

'Reduced by 4' means '– 4'. $5(x + 3) - 4$

'Result is 17' means '= 17'. $5(x + 3) - 4 = 17$

c Begin with the operation most closely associated with the variable. 'Two times' means '× 2'. $2x$

'Is divided by 15' means '÷ 15'. $\frac{2x}{15}$

'Three more than the number' means '$x + 3$'. $\frac{2x}{15} = x + 3$

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The following points may be helpful when solving word problems.

1. Read the problem carefully.
2. Identify the unknown quantity and give it an appropriate variable name.
3. Pick out key words and phrases and determine their equivalent mathematical meanings.
4. Draw a diagram if you think it will help you to understand the problem.
5. Form an equation, using the information given in the problem.
6. You don't have to use all the information. Some information may be unnecessary or even misleading.
7. Solve the problem and state the solution in words.
8. Check the solution in the original problem to make sure that it works.

EXAMPLE 9

The product of 6 and 5 more than a number is 18. What is the number?

Solution

Let n be the number.	n = the unknown number
'Product' means '×'; 'more than' means '+'.	$6(n + 5) = 18$
Undo the '× 6'.	$\frac{6(n+5)}{6} = \frac{18}{6}$
Simplify.	$n + 5 = 3$
Undo the '+ 5'.	$n + 5 - 5 = 3 - 5$
Simplify.	$n = -2$
Check the result.	$6(-2 + 5) = 6 \times 3 = 18$ ✓
State the result.	The number is –2.

EXAMPLE 10

A rectangular shed is 7 m longer than it is wide. Its perimeter is 58 m. What is its width?

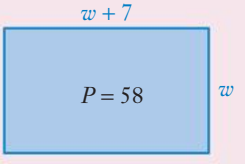
Solution

Draw a diagram.

The perimeter is 58 m.

Let the width be w.

The length must be $w + 7$.



Write the formula for the perimeter of a rectangle.	$P = 2(l + w)$
Substitute the values shown on the diagram.	$58 = 2[(w + 7) + w]$
Simplify.	$58 = 2(2w + 7)$
Expand using the distributive law.	$58 = 4w + 14$
Subtract 14 from both sides.	$44 = 4w$
Divide both sides by 14.	$w = 11$
Check the result.	$P = 2[(11 + 7) + 11]$
	$= 2 \times 29 = 58$ ✓
	$w = 11$ is the solution.
Write the result.	The width of the rectangle is 11 m.

Exercise 3.04 Using equations to solve problems

1 Which equation best matches each statement?

a Half a particular number is 17.

A $x = \frac{17}{2}$ **B** $2x = 17$ **C** $\frac{x}{2} = \frac{17}{2}$

D $\frac{x}{2} = 17$ **E** $x = 2 \times 17$

b A particular number is divided by 3 and the result is decreased by 5 to give 12.

A $\frac{x}{3} - 5 = 12$ **B** $3x + 5 = 12$ **C** $5 - \frac{x}{3} = 12$

D $\frac{x}{3} + 5 = 12$ **E** $x + 5 = 3 \times 12$

c When the sum of a number and three is multiplied by 5, the result is seven more than twice the number.

A $5x + 3 = 7 + 2x$ **B** $5(x + 3) = 7 + 2x$ **C** $x + 3 \times 5 = 7 + 2x$

D $5(x - 3) = 2x - 7$ **E** $5x + 3 = 7 - 2x$

d Three times the difference between a number and 2 is the same as four times the difference between twice the number and 7.

A $3(2 + x) = 4(7 + 2x)$ **B** $3 \times x - 2 = 4(2x + 7)$ **C** $3x - 2 = 4 \times 2x - 7$

D $4x - 2 = 3 \times 2x - 7$ **E** $3(x - 2) = 4(2x - 7)$

Example 8

2 Write a linear equation for each statement, using n to represent the unknown.

a The product of 10 and 12 more than a number is equal to 180.

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b The quotient of 14 and 7 less than a number is 2.

c When the sum of a number and 15 is halved and is then decreased by 5, the result is 3.

d If 4 times a number is increased by 17 and then doubled, the answer is –6.

e Three more than the product of 7 and 2 more than a number is 11.

f A number is increased by a third of itself and then reduced by 14, giving 9.

g When ten times a number is reduced by 7 and then the result is doubled, the answer is –12.

h If two-thirds of a number is reduced by the product of 7 and the number, the result is 10.

3 Write an expression for the perimeter of each rectangle in terms of its width (w).

a The length is twice the width.

b The length is 4 cm more than the width.

c The width is 3 m less than the length.

d The width is one-third of the length.

e The length is 4 mm longer than twice the width.

f The length is 5 cm shorter than three times the width.

g The width is 7 m shorter than one-quarter of the length.

h The length decreased by 12 mm is equal to the width.

4 For each situation:

i write an equation

ii find the solution of the equation.

Example 9

a If 7 is added to y, the result is 4.

b If 9 is added to m, the result is 2.

c If 31 is added to a, the result is 13.

d If 12 is added to x, the result is 8.

e If 3 is added to d, the result is –13.

f If y is subtracted from 26, the result is 39.

g If g is added to –12, the result is 4.

h If w is subtracted from 7, the result is –4.

5 Bosco scored 25 marks more in his History exam than he did in his Science exam. If he scored a total of 145 marks in both exams, how many marks did he score in the History exam?

Example 10

Problem solving

6 Maxine is four times as old as her son Juan, and the sum of their ages is 50. How old is Juan?

7 Two consecutive numbers add to give a total of 31. What are the numbers?

8 When 5 is subtracted from a number and the result is multiplied by 4, the answer is the same as when double the number is added to 10. Find the number.

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9 Amari hired a car for a fee of \$130 plus \$28 per day. His friend Sinead hired a car from a different company and the rate was \$160 plus \$25 per day. If their final cost was the same, how long was the rental period?

10 A business owner wants to have some flyers delivered for his new coffee shop. He contracts a local delivery service that charges a \$120 establishment fee plus 80 cents per flyer. If the bill amounts to \$724, how many flyers were delivered?

11 Ewan is a tyre fitter. He is paid \$12.50 per hour, plus \$3 for each tyre that he fits. If he earned \$208 for an 8-hour shift, how many tyres did he fit?

12 A square swimming pool is surrounded by a paved area that is 2 metres wide. If the area of the paving is 80 m^2, what is the length of the pool?

13 A laboratory needs 10 litres of a 15% acid solution for a certain procedure. The supplier will only deliver a 10% solution and a 30% solution. How many litres of 10% solution and 30% solution should they order?

14 In four more years, Yuan's grandfather will be seven times as old as Yuan was last year. The sum of the present ages of Yuan and his grandfather is 77. How old is Yuan now?

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3.05 Linear functions

A **function** shows the relationship between two sets of numbers – **input** numbers and **output** numbers – according to a rule. An input number is substituted into the rule and the value that results is called the output. Consider the function:

$$f(x) = 2x + 3$$

Where the input number is 7, the output number is:

$$f(7) = 2 \times 7 + 3 = 17$$

For any function, each input value has a *unique* corresponding output value. This means that any input number cannot result in 2 or more different output numbers. The two values form an **ordered pair**. For the function $f(x) = 2x + 3$, (7, 17) is an ordered pair for the function.

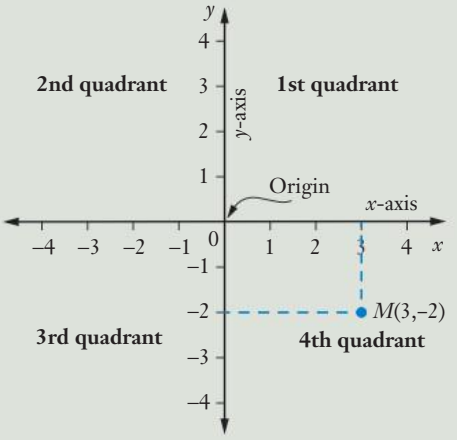
The order of the values is important. Input values are always given first.

A Cartesian plane has a numbered horizontal ***x*-axis** and a numbered vertical ***y*-axis**.

The axes divide the plane into four **quadrants**.

Any point in the plane has a value on each axis. These values are called **coordinates**. They are written in round brackets with the ***x*-coordinate** first. For example, point M in the diagram is named by the coordinates (3, –2).

The **origin** is the point (0, 0).



The input values for a function are also called the **domain** of the function, while the corresponding output values are called the **range** of the function.

When you draw the graph of a function, the domain values are the x-coordinates and the range values (outputs) are the y-coordinates.

On a graph, you often replace $f(x)$ by y. So $f(x) = 2x + 3$ is the same as $y = 2x + 3$.

When you draw the graph of a function, you should state the domain (input) values. For a graph of all the x values from –2 to 2 (inclusive), you write $-2 \le x \le 2$.

Graphing linear functions

Linear graphs

A page of number planes

Graphing linear functions

Graphing linear equations

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EXAMPLE 11

Draw a graph of the function $y = 3x - 5$ using the domain $0 \le x \le 4$.

Solution

You need at least three points to draw the graph of a straight line. Choose any three values for x from 0 to 4.

Always choose 0 as one point if it is in the domain.

Draw up a table of x and y values.

x	0	2	4
y	–5	1	7

Calculate the y values for the x values you chose.

When $x = 0$, $y = 3 \times 0 - 5 = -5$

When $x = 2$, $y = 3 \times 2 - 5 = 1$

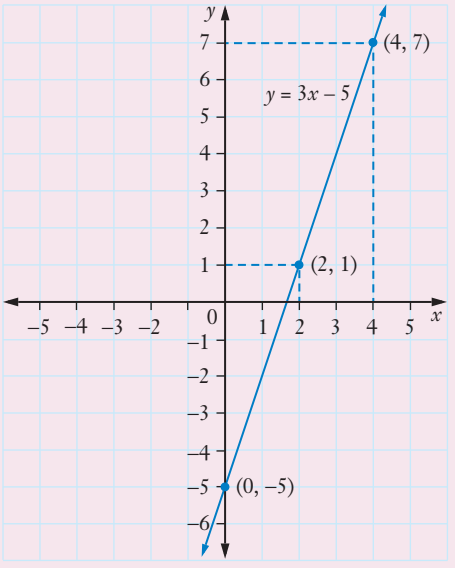
When $x = 4$, $y = 3 \times 7 - 5 = 7$

Write the ordered pairs for the function. (0, –5), (2, 1) and (4, 7)

Draw up a suitable set of x- and y-axes.

They have to show x values from 0 to 4 and y values from –5 to 7.

Mark in the ordered pairs and join them up.



Functions like the one in the above example are called **linear functions** because their graphs are straight lines. The ordered pairs (2, 1), (4, 7) and (0, –5) **satisfy** the function $y = 3x - 5$.

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TECHNOLOGY

Graphing linear functions

Use Excel to draw the graph of the linear function $y = 4x - 7$ for $-3 \le x \le 6$.

Type 'x' in cell A1 and 'y' in cell A2.

Enter the domain (x) values from –3 to 6 into cells B1, C1, D1, …, K1.

Enter the rule '=4*B1–7' into cell B2.

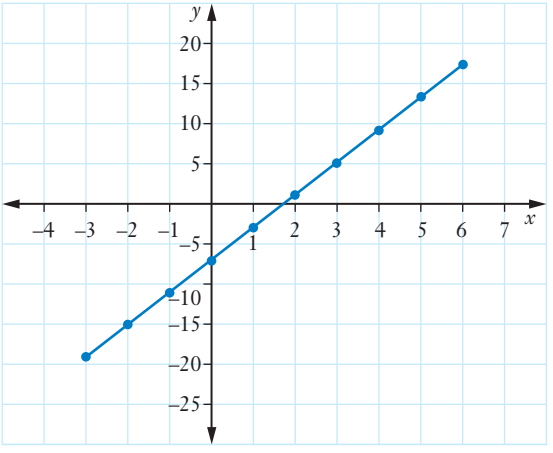
Copy the formula across cells C2 to K2.

Select the values in the table, i.e. the region A1:K2.

Open the **Chart** menu and select **Scatter** and then **Straight Marked Scatter**.

You can change the colour and the thickness of the lines and markers, add gridlines and so on to suit your needs.

The graph will look something like the one below, depending on the settings you have chosen.



Once you have set up this spreadsheet, it can be used again to draw graphs of different functions. All you need to do is change the x values and change the rule.

Use a spreadsheet to graph each function below.

1 $y = 4x + 7$ for $-5 \le x \le 4$

2 $y = 7x - 12$ for $-3 \le x \le 6$

3 $y = -5x + 8$ for $-3 \le x \le 6$

4 $y = 10 - 9x$ for $-4 \le x \le 5$

Sometimes you have to rearrange a function before you graph it. When you do a table of values for a function, it is easiest to use the form $y = mx + c$.

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EXAMPLE 12

Graph the function $2y + 3x - 12 = 0$ for $-2 \le x \le 4$.

Solution

Rearrange the equation so that y is isolated on the LHS.

Write the function. $2y + 3x - 12 = 0$

Undo the '– 12'. $2y + 3x - 12 + 12 = 0 + 12$

Simplify. $2y + 3x = 12$

Undo the '+ 3x'. $2y + 3x - 3x = 12 - 3x$

Simplify. $2y = 12 - 3x$

Rearrange the RHS. $2y = -3x + 12$

Undo the '× 2'. $\frac{2y}{2} = \frac{-3x + 12}{2}$

Simplify and write the RHS as two separate fractions.

$$y = \frac{-3x}{2} + \frac{12}{2}$$
$$= -\frac{3x}{2} + 6$$

Use x values of –2, 0 and 4 to draw up a table of x and y values.

x	–2	0	4
y			

Calculate the y values that correspond to the chosen x values.

$x = -2$: $y = -\frac{3 \times (-2)}{2} + 6 = 3 + 6 = 9$

$x = 0$: $y = -\frac{3 \times 0}{2} + 6 = 0 + 6 = 6$

$x = 4$: $y = -\frac{3 \times 4}{2} + 6 = -6 + 6 = 0$

x	–2	0	4
y	9	6	0

Write the ordered pairs for the function. (–2, 9), (0, 6) and (4, 0)

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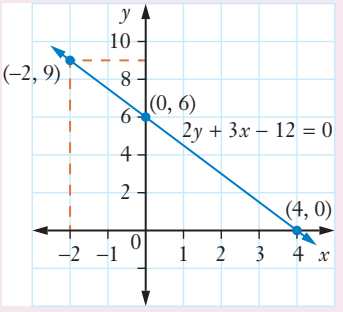
Draw up a suitable set of x- and y-axes.

You need to be able to show x values from -2 to 4 and y values from 0 to 9. Mark in the ordered pairs and join them up.



If an ordered pair satisfies a function, the point represented by the ordered pair must lie on the graph of the function.

EXAMPLE 13

Determine if the point $A(-1, 2)$ lies on

a $y = 4x - 5$ **b** $3x - 2y + 7 = 0$

Solution

a Write the function. $y = 4x - 5$

$(-1, 2)$ so $x = -1$ and $y = 2$.

Substitute for x in the RHS. $\text{RHS} = 4 \times (-1) - 5$

Evaluate. $= -9$

Substitute for y in the LHS. $\text{LHS} = 2$

Compare with the LHS. $\text{LHS} \neq \text{RHS}$

The y value that corresponds to $x = -1$ is 2. $(-1, 2)$ does not satisfy $y = 4x - 5$.

State the result. $(-1, 2)$ does not lie on $y = 4x - 5$.

b Write the function. $3x - 2y + 7 = 0$

Substitute for x and y in the LHS. $\text{LHS} = 3 \times (-1) - 2 \times 2 + 7$

Evaluate. $= 0$

This is the same as the RHS $(-1, 2)$ satisfies $3x - 2y + 7 = 0$.

State the result. $(-1, 2)$ lies on $3x - 2y + 7 = 0$.

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Exercise 3.05 Linear functions

1 Complete the table of values for each linear function.

a $y = -3x + 2$

x	0	2	5
y			

b $y = 4x + 1$

x	–3	0	3
y			

c $y = 5x - 12$

x	–2	0	2
y			

d $y = 7 - 3x$

x	–1	0	3
y			

Example 11

2 Complete the table for each function, then plot their graphs on separate Cartesian planes.

a $y = 2x + 5$

x	–3	0	3
y			

b $y = 4x - 11$

x	–1	2	4
y			

c $y = 8 - 5x$

x	–1	0	4
y			

d $y = -3x + 8$

x	–2	0	2
y			

3 Plot the graphs of each function. (Remember to use any three x values within the domain shown in each case.)

a $y = 2x - 3$ $(-2 \le x \le 2)$
b $y = 4 - 3x$ $(-3 \le x \le 3)$
c $y = -4x + 8$ $(-2 \le x \le 4)$
d $y = 10 - 5x$ $(-3 \le x \le 2)$
e $y = -3x + 11$ $(-1 \le x \le 5)$
f $y = 9 - 4x$ $(-1 \le x \le 3)$
g $y = 6x - 5$ $(-2 \le x \le 3)$
h $y = -7x + 4$ $(-1 \le x \le 3)$

Example 12

4 Graph each linear function over the domain values indicated.

a $2x + 5y = 5$ $(-2 \le x \le 2)$
b $3x + 2y = 8$ $(-2 \le x \le 3)$
c $4x - 3y = 9$ $(-3 \le x \le 3)$
d $5x - 2y = 8$ $(-4 \le x \le 4)$
e $3x + 4y = 8$ $(-4 \le x \le 4)$
f $3y + 4x + 12 = 0$ $(-6 \le x \le 1)$
g $5y - 8x - 10 = 0$ $(-5 \le x \le 5)$
h $7x + 4y - 10 = 0$ $(-4 \le x \le 4)$
i $12 + 4y - 5x = 0$ $(-4 \le x \le 8)$
j $3y - 7x - 15 = 0$ $(-6 \le x \le 3)$

5 Which of the points lies on the line $y = 5 - 3x$?

A (2, 1) **B** (–1, 2) **C** (5, 10) **D** (–2, 11) **E** (1, –2)

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6 Which of the points lies on the line $y - 4x = 7$?

A $(-1, 3)$ **B** $(2, 1)$ **C** $(3, -22)$ **D** $(-1, -3)$ **E** $(-2, 1)$

7 Which of the points lies on the line $2y - 3x + 5 = 0$?

A $(5, 10)$ **B** $(-3, -7)$ **C** $(1, 1)$ **D** $(-5, 10)$ **E** $(-1, 4)$

8 For each linear function, work out which of the points lies on its graph.

Example 13

a	$3x + 2y + 8 = 0$	$(-4, 2)$	$(3, 4)$	$(-2, 1)$	$(2, -7)$	$(4, 0)$
b	$y = x - 3$	$(-1, 2)$	$(1, -2)$	$(3, 0)$	$(-2, 5)$	$(-3, -6)$
c	$y = 2x + 6$	$(0, -6)$	$(2, 8)$	$(1, 14)$	$(-2, 2)$	$(3, 12)$
d	$y = -3x + 2$	$(1, 1)$	$(-1, 3)$	$(3, -7)$	$(0, -2)$	$(-4, 14)$
e	$3x - y - 6 = 0$	$(-4, 6)$	$(0, -3)$	$(2, 0)$	$(10, -12)$	$(-4, -18)$

Problem solving

9 **a** On the same set of axes, plot the graphs of $y = x - 2$ and $y = -3x + 6$ for $0 \le x \le 3$.

b What are the coordinates of the point where the two graphs cross?

c Check whether this ordered pair satisfies $y = x - 2$.

d Check whether this ordered pair satisfies $y = -3x + 6$.

10 **a** Plot the graphs of $2x + y = 8$ and $3x - y = 7$ on the same set of axes for $0 \le x \le 5$.

b Use your graphs to find the coordinates of the point that satisfies both functions.

11 Lisa and Giang look through separate telescopes at the same object in a roughly north-westerly direction. Lisa looks along the line $y + 2x = 6$, and Giang looks along the line $y + 4x = -3$. Use a Cartesian plane to find the distance between the telescopes.

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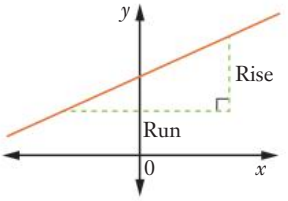
Drawing gradients

3.06 The slope of a line

The **slope** or **gradient** of a line measures how much the line goes up vertically for each horizontal step.

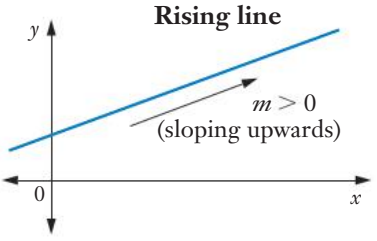


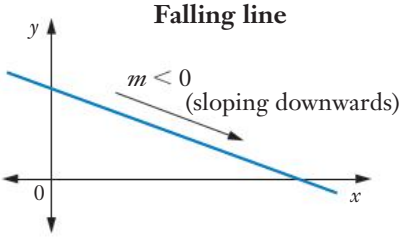
The gradient is represented by the symbol m.

The gradient is calculated using the formula:

$$\text{Gradient} = m = \frac{\text{rise}}{\text{run}}$$

Lines that slope upwards (rise) from left to right are described as **rising** and have a positive gradient ($m > 0$). **Falling** lines slope downwards (fall) from left to right and have a negative gradient ($m < 0$).

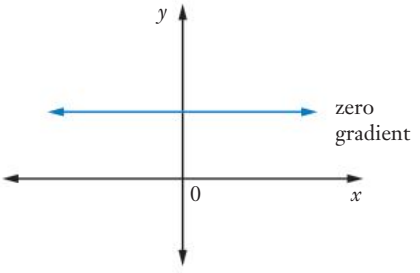




For a horizontal or flat line, the vertical rise is 0.

$$\text{So } m = \frac{0}{\text{run}} = 0.$$

This means a horizontal line has a **zero gradient**.

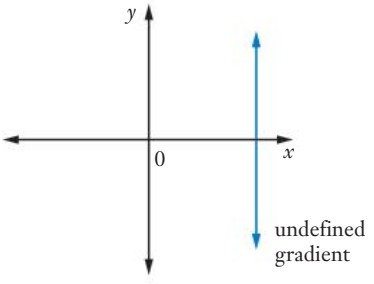


For a vertical line, the vertical run is 0.

$$\text{So } m = \frac{\text{rise}}{0}.$$

But division by 0 is undefined, so the gradient is undefined.

A vertical line has an **undefined gradient**.



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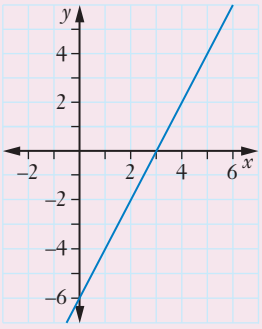
EXAMPLE 14

For each line below:

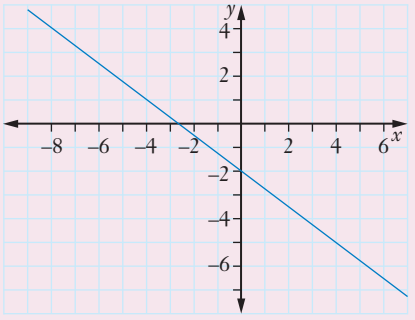
i state whether the gradient is positive or negative

ii calculate the gradient

a



b

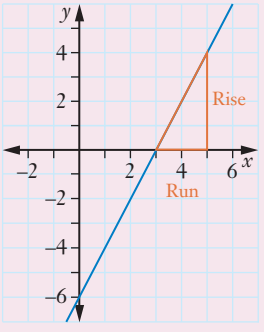


Solution

a **i** The line slopes upwards from left to right.

The gradient is positive.

ii Choose two convenient points on the line and draw a right-angled triangle to show the rise and run.



Write the formula. $m = \frac{\text{rise}}{\text{run}}$

Substitute the values for rise and run from the diagram. $= \frac{4}{2}$

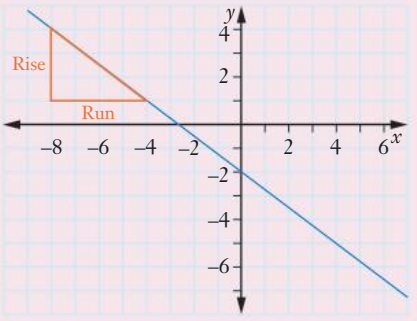
Simplify. $= 2$

State the result. The gradient of the line is 2.

b i The line slopes downwards from left to right.

The gradient is negative.

Choose two convenient points on the line and draw a right-angled triangle to show the rise and run.



Write the formula.

$$m = \frac{\text{rise}}{\text{run}}$$

Substitute the values for rise and run from the diagram.

$$= \frac{-3}{4}$$

State the result.

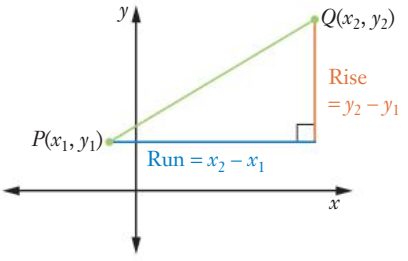
The gradient of the line is $-\frac{3}{4}$.

You can find the gradient using the coordinates of any two points on the line.

If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points on a straight line:

- the rise from P to Q is the y-distance $y_2 - y_1$
- the run from P to Q is the x-distance $x_2 - x_1$

The gradient of PQ has the symbol m_{PQ}.



$$\text{Gradient of } PQ = m_{PQ} = \frac{\text{rise}}{\text{run}}$$

Finding the gradient between two points on a line

If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points on a straight line:

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$

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EXAMPLE 15

Calculate the gradient of the straight line that passes through the points (–2, 5) and (1, –7)

Solution

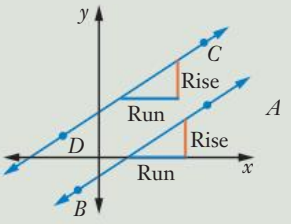
Identify the points.	Let $(x_1, y_1) = (-2, 5)$ and $(x_2, y_2) = (1, -7)$.
Write the formula for gradient.	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
Substitute for known values.	$= \dfrac{-7-5}{1-(-2)}$
Evaluate.	$= \dfrac{-12}{3}$ $= -4$
State the result.	The gradient of the line is –4.

Lines that are parallel never cross. This means they have the same gradient.

Parallel lines

If $AB \parallel CD$, $m_{AB} = m_{CD}$.

The gradients of **parallel lines** are equal.



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EXAMPLE 16

Show that the points $A(1, 2)$, $B(-2, 4)$ and $C(-5, 6)$ are collinear.

Solution

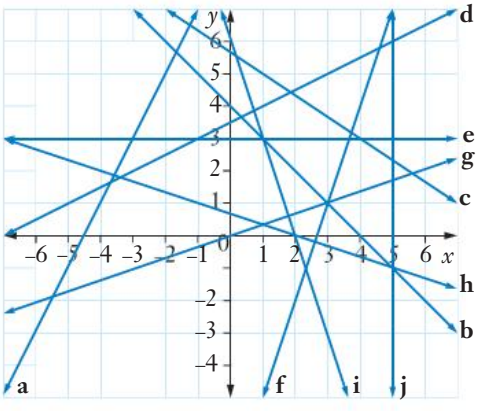
If the three points are collinear, they must lie on the same line. This means that the gradients of AB and AC will be equal.

Calculate the gradient of AB.	$m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$
Substitute values.	$= \dfrac{4-2}{-2-1}$
Evaluate.	$= \dfrac{-2}{3}$
Calculate the gradient of AC.	$m_{AC} = \dfrac{y_2 - y_1}{x_2 - x_1}$
Substitute values.	$= \dfrac{6-2}{-5-(1)}$
Evaluate.	$= -\dfrac{4}{6} = -\dfrac{2}{3}$
State the result.	$A(1, 2)$, $B(-2, 4)$ and $C(-5, 6)$ are collinear, since $m_{AB} = m_{AC}$.

Exercise 3.06 The slope of a line

Example 14

1 For each line shown, state whether the gradient is positive, negative or neither.



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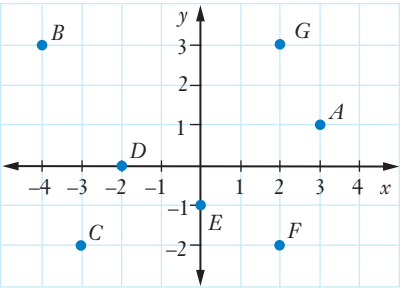
2 Calculate the gradient of each of the lines in question **1**.

3 Use the diagram to calculate the gradient of the line that passes through:

a A and B **b** B and D
c D and C **d** G and F
e D and E **f** E and F
g D and G **h** C and F
i A and D **j** C and A



4 Calculate the gradient of the straight line that passes through each pair of points. Example 15

a (3, 5) and (5, 11) **b** (1, 6) and (4, 0) **c** (–1, –3) and (2, 3)
d (3, –6) and (4, 1) **e** (4, 2) and (–4, 1) **f** (0, 3) and (5, –7)
g (–3, 4) and (3, –2) **h** (6, 6) and (7, 7) **i** (–2, 3) and (–6, 3)
j (–4, 0) and (0, –3) **k** (–4, –5) and (–4, 6) **l** (2, 4) and (4, 1)

5 Determine whether each set of points are collinear. Example 16

a $A(5, 6)$, $B(-4, -6)$ and $C(-1, -2)$ **b** $X(-1, 1)$, $Y(-3, -3)$ and $Z(-6, -9)$
c $P(-5, -1)$, $Q(10, 4)$ and $R(-2, 0)$ **d** $J(12, 9)$, $K(0, -1)$ and $L(-6, -6)$
e $W(3, 1)$, $R(0, -2)$ and $S(-3, -5)$ **f** $F(-2, 6)$, $G(2, 0)$ and $H(4, -3)$

6 One line passes through $M(-2, -6)$ and $N(4, 6)$ while another line passes through $C(3, -2)$ and $D(8, 8)$. Determine if MN and CD are parallel.

7 One line passes through $X(-3, 8)$ and $Y(8, -8)$ while another line passes through $W(-6, 7)$ and $R(5, -6)$. Determine if XY and WR are parallel.

Problem solving

8 Find b given the line joining:

a $D(-2, -1)$ and $F(3, b)$ is parallel to a line with gradient $\frac{4}{5}$.

b $Q(-4, b)$ and $T(b, 3)$ is parallel to a line with gradient $\frac{1}{6}$.

9 Show that the points (–2, 0), (2, 2), (3, –1) and (–1, –3) are the vertices of a parallelogram.

10 Find two points which, when joined, will result in a straight line with a gradient of –2.

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Equations of a line

Gradient and y-intercept

$y = mx + c$

Finding the equation of a line

General equation of a straight line

Linear functions code puzzle

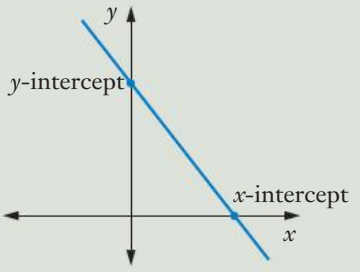
Graphing $y = mx + c$

3.07 The intercepts of a line

Intercepts

The **intercepts** of a straight line are the values where it crosses the axes of the Cartesian plane.

The ***x*-intercept** is where the line crosses the x-axis and the ***y*-intercept** is where it crosses the y-axis.



The x-intercept is the value of the function where $y = 0$ and the y-intercept is the value when $x = 0$.

EXAMPLE 17

a Draw the graph of $y = 3x - 4$ for $-2 \le x \le 5$ and use the graph to find the intercepts.

b Find the intercepts of $2x + 3y - 8 = 0$.

Solution

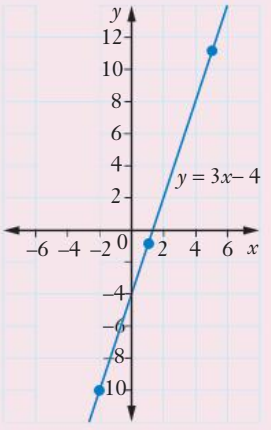
a Construct a table of x and y values using any three x values form -2 to 5.

$y = 3x - 4$

x	-2	1	5
y	-10	-1	11

Plot the points and draw the straight line.

Find where the line cuts the axes.



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	State the result.	The x-intercept is about 1.3. The y-intercept is –4.
b	Let $y = 0$.	$2x + 3 \times 0 - 8 = 0$
	Solve for x.	$2x = 8$ $x = 4$
	State the result.	The x-intercept is 4.
	Let $x = 0$.	$2 \times 0 + 3y - 8 = 0$
	Solve for y.	$3y = 8$ $\frac{3y}{3} = \frac{8}{3}$ $y = 2\frac{2}{3}$
	State the result.	The y-intercept is $y = 2\frac{2}{3}$.

INVESTIGATION

EQUATIONS OF LINES

1 Draw graphs of the following lines.

a $y = 2x + 5$ **b** $y = 2x + 7$ **c** $y = 2x + 9$

d $y = 3x + 5$ **e** $y = 0.5x + 5$ **f** $y = 4x + 6$

2 Work out the gradient and y-intercept for each.

3 In each case, compare the gradient and y-intercept you found with the equation of the line.

4 What do you see? Does this always work?

5 Write a short statement about how the values of a and b for the equation of the line $y = ax + b$ are related to the gradient and y-intercept of the line.

Forms of the equation of a line

The **gradient–intercept form** of the equation of a line is $y = mx + c$, where

- gradient $= m$
- y-intercept $= c$

The **standard form** of the equation of a line is $ax + by + c = 0$ where a, b and c are constants.

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EXAMPLE 18

a Write the equation of the line with gradient $-\frac{3}{4}$ and y-intercept 5 in standard form.

b What is the gradient and y-intercept of $3x + 2y - 6 = 0$?

Solution

a Write in the gradient–intercept form. $y = mx + c$

Substitute values. $y = -\frac{3}{4}x + 5$

Undo the '÷ 4'. $y \times 4 = -\frac{3}{4}x \times 4 + 5 \times 4$

Simplify. $4y = -3x + 20$

Undo the '– $3x$'. $4y + 3x = -3x + 20 + 3x$

Simplify. $4y + 3x = 20$

Undo the '+ 20'. $4y + 3x - 20 = 20 - 20$

Simplify. $4y + 3x - 20 = 0$

b Write the equation. $3x + 2y - 6 = 0$

Change to the form $y = mx + c$. $3x + 2y - 6 - 3x + 6 = 0 - 3x + 6$

Simplify. $2y = -3x + 6$

Undo the '× 2'. $\frac{2y}{2} = \frac{-3x}{2} + \frac{6}{2}$

Simplify. $y = -\frac{3x}{2} + 3$

Compare with the gradient–intercept form. $y = mx + c$

State the result. Gradient $m = -\frac{3}{2}$, y-intercept $c = 3$

You can draw the graph of a linear function by plotting the y-intercept on the y-axis and using the gradient to locate a second point on the graph.

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EXAMPLE 19

Gradient and y-intercept of a line

Draw a sketch of each of the following lines using the gradient and y-intercept.

a $y = 5 - \frac{3}{5}x$ **b** $3x - 2y + 4 = 0$

Solution

a Write the equation. $y = 5 - \frac{3}{5}x$

Rearrange equation to match $y = mx + c$. $y = -\frac{3}{5}x + 5$

Identify the gradient and y-intercept. Gradient $= m = -\frac{3}{5}$

y-intercept $= c = 5$

Draw a set of axes.

The y-intercept is 5, so plot a point at (0, 5).

Gradient $= \frac{\text{rise}}{\text{run}} = \frac{-3}{5}$.

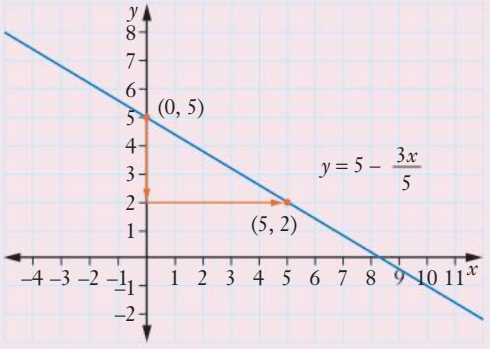
The gradient is negative, so the graph is falling.

Start at (0, 5) and move 3 units *down* and 5 *across* to the right.

Plot the point at (5, 2).

Join the points with a straight line.

Label the graph.



b Write the equation. $3x - 2y + 4 = 0$

Undo the '$-2y$' and then simplify. $3x - 2y + 4 + 2y = 0 + 2y$

$3x + 4 = 2y$

Reverse the equation. $2y = 3x + 4$

Undo the '$\times 2$'. $\frac{2y}{2} = \frac{3x}{2} + \frac{4}{2}$

Simplify. $y = \frac{3}{2}x + 2$

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Compare with $y = mx + c$ to identify the gradient and y-intercept.

Gradient $= m = \frac{3}{2}$

y-intercept $= c = 2$

Draw a set of axes.

The y-intercept is 2, so plot a point at (0, 2).

Gradient $= \frac{\text{rise}}{\text{run}} = \frac{3}{2}$.

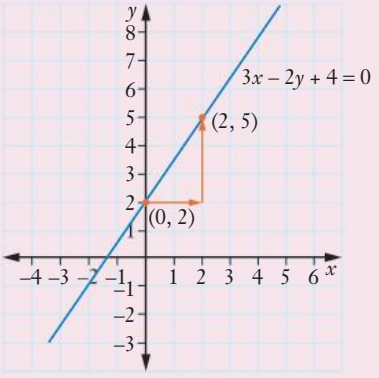
The gradient is positive, so the graph is rising.

Start at (0, 2) and move 3 units *up* and 2 *across* to the right.

Plot the point at (2, 5).

Join the points with a straight line.

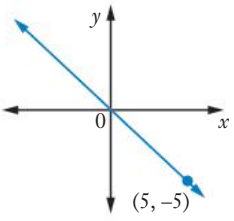
Label the graph.



Exercise 3.07 The intercepts of a line

1 Match each graph to its linear equation next page.

a



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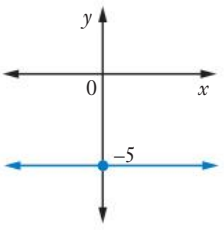

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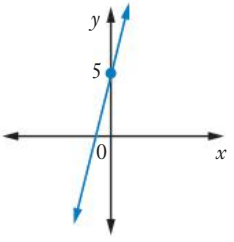

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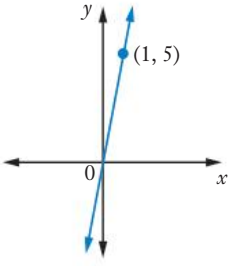

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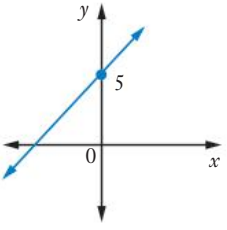

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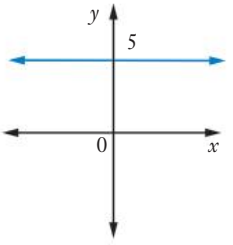

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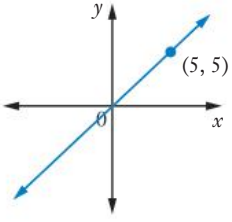

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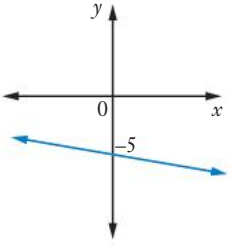

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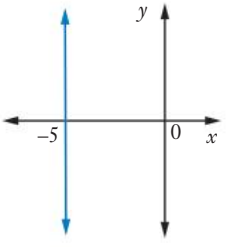

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i $y = x + 5$ **ii** $y = 5$ **iii** $x = 5$

iv $y = 5 - x$ **v** $y = 5x$ **vi** $y = x - 5$

vii $y = -5$ **viii** $y = 5x + 5$ **ix** $y = x$

x $x = -5$ **xi** $y = -x$ **xii** $y = -\frac{1}{2}x - 5$

2 Graph each linear equation and state the intercepts from the graph if possible. Example 17

a $y = 2x - 3$ for $-3 \le x \le 6$ **b** $y = -3$ for $-4 \le x \le 4$

c $x = 1$ for $-3 \le x \le 3$ **d** $y = 2x + 1$ for $-5 \le x \le 5$

e $y = -x + 2$ for $-6 \le x \le 6$ **f** $y = -2x - 3$ for $-6 \le x \le 3$

g $y = 3x - 1$ for $-2 \le x \le 3$ **h** $y = -3$ for $-3 \le x \le 3$

i $y = 3x + 4$ for $-4 \le x \le 2$ **j** $y = -x - 3$ for $-6 \le x \le 6$

3 Write the equation of each line in gradient–intercept form.

a gradient = 2 and y-intercept = 4 **b** slope = –3 and y-intercept = 1

c y-intercept = –6 and slope = 5 **d** gradient = 3 and y-intercept = –2

e y-intercept = –3 and slope = 0 **f** slope = –4 and y-intercept = 0

4 Write the equation of each of the following lines in standard form. Example 18

a gradient = 4 and y-intercept = –1 **b** gradient = $\frac{2}{3}$ and y-intercept = –2

c y-intercept = –1 and gradient = $-\frac{4}{3}$ **d** y-intercept = 2 and gradient = $\frac{5}{2}$

e gradient = $3\frac{1}{3}$ and y-intercept = –4 **f** y-intercept and slope = $-1\frac{3}{4}$

5 Without drawing a graph, what is the gradient and y-intercept of each line?

a $y = 2x + 5$ **b** $y = 5 - 3x$ **c** $y = -x + 1$

d $y = 4 - 7x$ **e** $y = x + 2$ **f** $y = -8x$

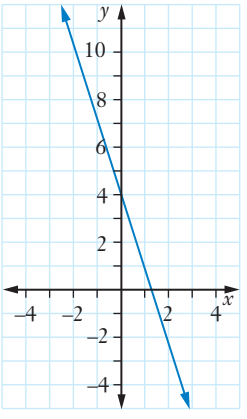
g $x - 2y + 1 = 0$ **h** $7x + y - 14 = 0$ **i** $x - 3y + 6 = 0$

j $3y - 8 = -4x$ **k** $7x = 3y$

6 The gradient of this line is:

A 2 **B** –2 **C** 3

D –3 **E** 0



7 The y-intercept of this line is:

A –1 **B** 4 **C** 1.3

D 1.5 **E** –4

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8 The y-intercept of this line with equation $3x - 2y = 8$ is

A -4 **B** 8 **C** 3 **D** $\frac{3}{2}$ **E** 4

Example 19

9 Draw a sketch of each line using the gradient and y-intercept.

a $y = x + 3$ **b** $y = \frac{1}{2}x - 3$ **c** $y = 3x - 5$

d $y = 4 - x$ **e** $y = 5 - 2x$ **f** $y = 3 - \frac{2}{3}x$

10 Draw a sketch of each line using the gradient and y-intercept.

a $2x + y = 3$ **b** $3x - y = 5$ **c** $6x - 2y + 10 = 0$

d $2x + 4y - 12 = 0$ **e** $3x - 2y + 6 = 0$ **f** $5x + 3y - 9 = 0$

Problem solving

11 Without drawing graphs, work out whether each pair of lines is parallel.

a $2x + 3y + 8 = 0$ and $6y = 5 - 4x$ **b** $2x - y - 4 = 0$ and $y = 2x + 6$

c $5y - 2x = 3$ and $2y - 5x + 6 = 0$ **d** $3y = 4x - 9$ and $y - \frac{4}{3}x + 4 = 10$

e $y = 4 - 2\frac{1}{2}x$ and $2y + 5x - 2 = 0$ **f** $3y + 5x = 9$ and $5y - 3x + 6 = 0$

12 Work out the standard and gradient–intercept forms of the equation of the line with:

a x-intercept 2 and y-intercept 1 **b** x-intercept -3 and y-intercept 4

c x-intercept 4 and y-intercept -2 **d** x-intercept -1 and y-intercept 4

13 Work out the standard and gradient–intercept forms of the equation of the line that passes through the points:

a $(3, 0)$ and $(0, -8)$ **b** $(-5, 0)$ and $(0, -3)$

c $(0, -1)$ and $(5, 0)$ **d** $(0, 6)$ and $(-4, 0)$

Practical applications

3.08 Applications of linear models

Relationships that can be represented by functions of the form $y = mx + c$ are called **linear models**.

The variables could be different, like, C is for cost, t for time and d for distance.

The independent variable is on the horizontal axis and the dependent variable is on the vertical axis.

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EXAMPLE 20

The height, h cm, of a seedling is given by $h = 2d + 10$, where d is the number of days since it was planted.

a Complete a table of values for the function for $0 \le d \le 8$.

b Use the table to draw a graph of the height function.

c What type of function is the height function?

d Compare the height function with the gradient–intercept form of a linear function ($y = mx + c$). What do the values 2 and 10 in the height function represent?

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Solution

a Draw up a table of values for d and h using any three values of d from 0 to 8.

$h = 2d + 10$

d	1	5	8
h			

Calculate the corresponding values of h.

When $d = 1$: $h = 2 \times 1 + 10 = 12$

When $d = 5$: $h = 2 \times 5 + 10 = 20$

When $d = 8$: $h = 2 \times 8 + 10 = 26$

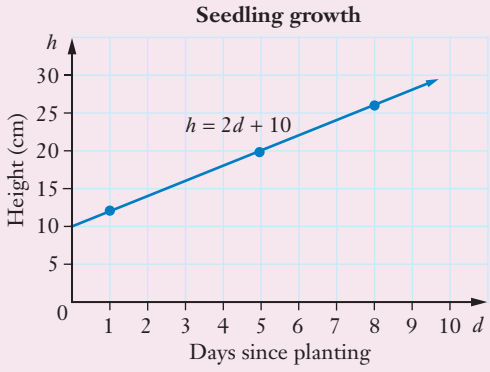
d	1	5	8
h	12	20	26

State the ordered pairs.

(1, 12), (5, 20) and (8, 26)

b Plot the ordered pairs and join them with a straight line.

Label the axes and add a title to the graph.



c	The graph is a straight line.	The function $h = 2d + 10$ is linear.
d	Write the height function.	$h = 2d + 10$
	Write the gradient–intercept form of a linear function.	$y = mx + c$
	Compare the functions.	$y \leftrightarrow h$ and $x \leftrightarrow d$
	For the height function, d is the independent variable. The height (h) depends on d and so h is the independent variable.	The horizontal axis shows the independent variable (d) and the vertical axis shows the dependent variable (h).
	Compare the coefficients of the variables and the constant terms in the two functions.	$m \leftrightarrow 2$ and $c \leftrightarrow 10$ In the height function, 2 is the gradient of the graph and 10 is the intercept on the vertical axis, i.e., the height when the seedling was planted ($d = 0$).

You should note in the previous example that because the slope of the graph is positive, the function is rising and the height increases as the number of days increases.

EXAMPLE 21

A small business buys \$20 000 worth of computer equipment. For tax purposes, the value, \$$V$, of the equipment decreases (depreciates) over time according to the equation

$$V = 20\,000 - 4000t$$

where t is the number of years since purchase.

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a Complete a table of values for the value function for $0 \le t \le 6$.

b Use the table to draw a graph of the value function.

c When will the computer equipment be worthless for taxation purposes?

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Solution

a Draw up a table of values for t and V using any three values of t from 0 to 6.

$V = 20\,000 - 4000t$

t	1	3	6
V			

Calculate the corresponding values of V.

When $t = 1$: $V = 20\,000 - 4000 \times 1$

$= 16\,000$

When $t = 3$: $V = 20\,000 - 4000 \times 3$

$= 8000$

When $t = 6$: $V = 20\,000 - 4000 \times 6$

$= -4000$

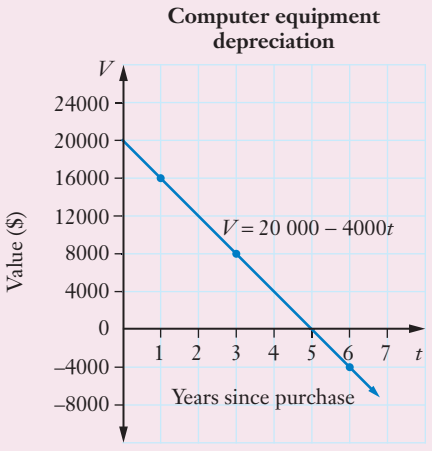
t	1	3	6
V	16 000	8000	–4000

State the ordered pairs.

(1, 16 000), (3, 8000) and (6, –4000).

b Plot the ordered pairs and join them with a straight line.

Label the axes and add a title to the graph.



c Find the value of t when $V = 0$.

$t = 5$

State the result.

The computer equipment will be worthless for tax purposes after 5 years.

Exercise 3.08 Applications of linear models

Practical applications

1 The total cost (C) of producing a number of items (n) is given by the linear function:

$$C = 15n + 25$$

a Which variable is the independent variable?

b If the function is graphed, what is the intercept on the vertical axis?

2 The total weight (W) of a pallet holding a number of boxes (b) is given by the linear function:

$$W = 7.8b + 15.5$$

a Which variable is the dependent variable?

b If the function is graphed, what is the intercept on the vertical axis?

3 The cost (C) of running a function for a number of people (n) is given by the rule $C = 32n + 650$. The gradient of the graph of the function is:

A 650 **B** 32 **C** 0 **D** –650 **E** –32

Example 20

4 The volume, V (in litres), of petrol remaining in a portable generator is given by:

$$V = 8 - \frac{3}{2}h$$

where h is the number of hours after the generator starts with a full tank.

a Construct a table of values for the function for $0 \le h \le 6$.

b Use the table to draw a graph of the function.

c What type of function is it?

d Use the graph to find when there is 3 L of petrol left in the generator.

e How many litres of petrol are left in the generator after 2 hours?

f What does the value of the function at $h = 0$ represent?

g What does the value of the function where the graph crosses the h-axis represent?

5 The cost, C (in dollars), of hiring a clown for a birthday party is given by:

$$C = 50 + 40h$$

where h is the number of hours for which the clown is hired.

a Draw a graph of the function for $0 \le h \le 8$.

b What type of function is it?

c Use the graph to find the cost of hiring the clown for 3 hours.

d For how long could the clown be hired with \$200 if the time may be charged in fractions of an hour?

e What does the value 50 in the cost function represent?

f What does the value 40 in the cost function represent?

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6 A coffee wholesaler spends a maximum of \$40 000 a week on coffee. He can buy coffee from Africa for \$15/kg or from Papua New Guinea (PNG) for \$9/kg.

Example 21

- **a** Write an equation for the number of kilograms of coffee from Africa (a) and PNG (p) that he can buy for \$20 000.
- **b** Construct a table of values for a and p and use it to draw the graph of the equation.
- **c** If the supply of PNG coffee beans in one week is restricted to 800 kg, use the graph to work out how many kilograms of African coffee beans the wholesaler can buy.

Problem solving

7 In question **1**, you examined the cost function $C = 15n + 25$. This function represents the total cost (C) of producing a number of items (n).

What could the value 25 in this function represent?

8 In question **2**, you examined the weight function $W = 7.8b + 15.5$. This function represents the total weight (W) of a pallet holding a number of boxes (b).

What could the value 15.5 in this function represent?

9 A minibus can carry 20 passengers and a coach can carry 70 passengers. A total of 250 pupils are to be taken on an excursion. Write the number of minibuses used as x and the number of coaches as y.

- **a** How many pupils can be carried by x minibuses and y coaches?
- **b** Write an equation for the number of minibuses and the number of coaches that could be used to carry 250 pupils.
- **c** Draw the graph for this equation.
- **d** Use the graph to find the whole number values of x and y that can be used for 250 pupils.
- **e** On the same graph, graph the equations for transporting 150 students and 400 pupils.
- **f** What do you notice about the graphs?

10 A theatre is filled to capacity with 600 people. When the show finishes, people start leaving at the rate of 80 people per minute.

- **a** Express P, the number of people in the theatre, as a function of t, the number of minutes after the show ends.
- **b** Draw a graph of the function.
- **c** Use the graph to find out how long after the show ends the theatre will still be half full.

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3. CHAPTER SUMMARY

Linear equations and graphs

Equations

- An **equation** is a mathematical statement that 2 things are equal.
- A **linear equation** in one variable is an equation that can be written in the form $ax + b = c$, where a, b, and c are constants and x is a variable.
- The **standard** (or **general**) **form** of a linear equation in two variables is $ax + by + c = 0$.
- Equations that are not linear are called **non-linear equations**.
- **Solving** an equation means finding a value for the variable that makes the statement true.
- The solution **satisfies** the equation and is also called a **root** of the equation.
- To solve an equation, you need to keep the equation **balanced** by doing the same thing to both sides of the equation.
- An **inverse operation** has the effect of undoing the original operation.

Operation	Inverse operation
$+$	$-$
$-$	$+$
$\times$	$\div$
$\div$	$\times$
square (^{2})	$\sqrt{\ }$
$\sqrt{\ }$	square (^{2})

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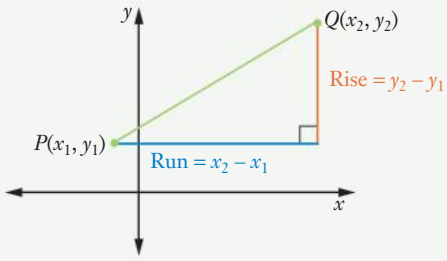
Linear graphs

- A **function** shows the relationship between **input** numbers and **output** numbers according to a rule.
- Input values make up the **domain** of the function and output values are called the **range**.
- An input number and the corresponding output number form an **ordered pair**.
- The Cartesian plane consists of a horizontal ***x*-axis** and a vertical ***y*-axis** that divide the plane into four **quadrants**. Any point in the plane is identified using its **coordinates**. The ***x*-coordinate** is always stated first. The **origin** is the point (0, 0).
- The slope of a line is called its **gradient** (m).
- Gradient $= m = \dfrac{\text{rise}}{\text{run}}$
- If $P(x_1, y_1)$ and $Q(x_2, y_2)$ lie on a straight line:

  $$\text{Gradient of } PQ = m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$



- **Rising** lines slope upwards from left to right and have a positive gradient ($m > 0$). **Falling** lines slope downwards from left to right and have a negative gradient ($m < 0$).
- The gradient of a horizontal line is 0. The gradient of a vertical line is **undefined**.
- The gradients of parallel lines are equal.
- The **intercepts** of a straight line are the points where it crosses the axes of the Cartesian plane. The ***x*-intercept** is where $y = 0$ and the ***y*-intercept** is where $x = 0$.
- The **gradient–intercept form** of the equation of a line is $y = mx + c$, where m = gradient and $c = y$-intercept.
- The **standard form** of the equation of a line is $ax + by + c = 0$, where a, b and c are constants.
- Relationships that can be represented by functions of the form $y = mx + c$ are called **linear models**.

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3. CHAPTER REVIEW

Linear equations and graphs

Example 1

1 Determine whether $x = -2$ is a solution to each equation.

a $\frac{2x-5}{3} = -3$ **b** $4x - 9 = 3 - 2x$

Example 2

2 Solve each linear equation.

a $q - 12 = 15$ **b** $m + 9 = 25$ **c** $8a = -112$ **d** $\frac{r}{7} = 12$

Example 3

3 Solve each linear equation.

a $2h + 15 = 3$ **b** $11 - 4p = 3$ **c** $\frac{x}{3} - 4 = 1$ **d** $\frac{2y}{5} = 8$

Example 3

4 What is the solution to the equation $3(21 - 4x) = 15$?

A -4 **B** 4 **C** 2 **D** 4 **E** -1

Examples 4,5

5 Solve each linear equation.

a $3g = 11g + 24$ **b** $2s - 5 = 23 - 5s$

c $3(u + 9) = -5 - 5u$ **d** $4(k + 3) = 3(2k - 5)$

Example 6

6 Solve each linear equation.

a $\frac{y}{3} - 2 = 8$ **b** $6 - \frac{3z}{4} = 2$ **c** $\frac{x-4}{2} = 9$ **d** $\frac{3h}{5} - 4 = 3h - 8$

Example 7

7 Solve each equation.

a $\frac{3}{x} = \frac{9}{17}$ **b** $\frac{5}{3g} = -10$ **c** $\frac{3}{1-2x} = \frac{5}{2-x}$

Examples 8,9

8 5 less than 3 times a number is 7.

a Write a linear equation for this statement, using x to represent the unknown number.

b What is the number?

Example 10

9 A rectangular garden bed is 3 m shorter than it is long. Its perimeter is 22 m. What is its length?

Example 11

10 Draw a graph of the function $y = 3x - 4$ using the domain $-2 \le x \le 2$.

Example 12

11 Graph the function $4x + 3y - 12 = 0$ for $-2 \le x \le 6$.

Example 13

12 Determine if the point $A(5, 6)$ lies on $y = 2x - 3$.

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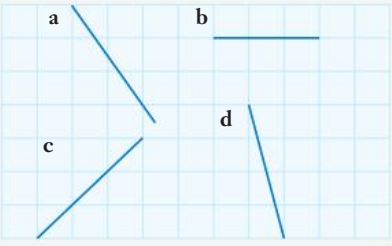
13 Which one of the following points does not lie on the line with the equation $y = 5 - 3x$? Example 13

A $(-1, 8)$ **B** $(3, -12)$ **C** $(1, 2)$ **D** $(-4, 17)$ **E** $(5, -10)$

14 A line passes through the origin and the point $(3, -2)$. The gradient of this line is: Example 14

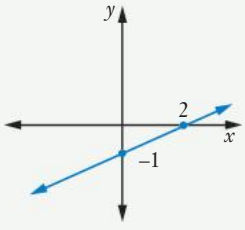
A 3 **B** $-\frac{2}{3}$ **C** $-\frac{3}{2}$ **D** -3 **E** -2

15 Use the graphs shown to calculate the gradient of each of the segments. Example 14



16 Match each graph to its linear equation. Example 18

a



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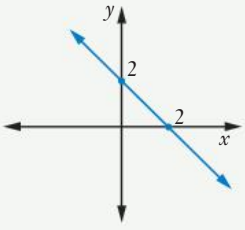

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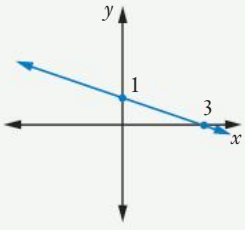

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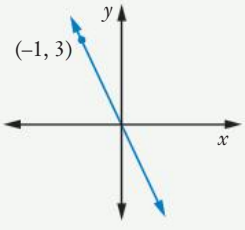


A $y = -\frac{1}{3}x + 1$ **B** $y = -3x$ **C** $y = \frac{1}{2}x - 1$ **D** $y = -x + 2$

17 For this graph of $y = mx + c$: Example 17

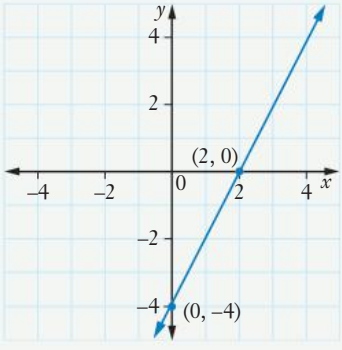
A $m = 2$ and $c = 4$

B $m = 1$ and $c = -4$

C $m = \frac{1}{2}$ and $c = -4$

D $m = 2$ and $c = -4$

E $m = -2$ and $c = -4$



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Example 15

18 Calculate the gradient of the straight line that passes through the points (–2, 4) and (1, –2).

Example 16

19 Show that the points $A(-3, 10)$, $B(-1, 6)$ and $C(4, -4)$ are collinear.

Example 17

20 Draw the graph of $y = 2x - 3$ for $-3 \leq x \leq 3$ and use the graph to find the intercepts.

Example 18

21 Write in standard form the equation of the line with a gradient of 3 and a y-intercept of –1.

Example 18

22 What are the gradient and y-intercept of the line $x - 2y + 4 = 0$?

Example 19

23 Draw a sketch of $y = -2x + 3$ using the gradient and y-intercept.

Example 20

24 The speed of a car after passing a checkpoint is given by $S = 10 + 3T$, where S is the speed in m/s and T is the time in seconds after passing the checkpoint.

a Draw a graph of the function for $0 \leq T \leq 10$.

b Use the graph to find when the speed of the car will reach 30 m/s.

c Use the graph to find the speed of the car after 4 seconds.

d What does the value 10 in the speed function represent?

Example 21

25 The owner of a fast-food stall at a market has a total of \$200 to spend on hamburger patties and frankfurters for hot dogs each day. Each hamburger patty costs \$1.60 while each frankfurter costs 60c.

a Write an equation for the number of hamburger patties (h) and frankfurters (f) that the owner could buy.

b Complete a table of values for h and f and use it to draw a graph of the equation.

c If the owner knows that 70 hamburgers are usually sold each day, how many frankfurters should be bought?

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Problem solving

26 Mark and Izzy decide to buy a second-hand NS14 dinghy to race together. Mark already has some sails, so he puts in \$900 less than Izzy. After negotiating a discount of 10% of the original price, Izzy paid 55% of the original price as his share. What was the original price, and how much did each person pay?

27 David is now 5 years older than his sister Mary. In 8 years he will be one and a half times her age. How old are they now?

28 Without drawing a graph, determine whether $3x - 4y + 2 = 0$ and $8x + 6y + 1 = 0$ are parallel.

29 Show that the points (5, –4), (–4, –1), (–6, 6) and (3, 3) are the vertices of a parallelogram.

Practice quiz

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Practice examination 1

Time: 90 minutes
Perusal time: 5 minutes
Marks: 50

Instructions

- Students are permitted to bring or use: pens, pencils, highlighters, erasers, sharpeners, rules and an approved graphics calculator.
- Students must show appropriate working and justification to gain full marks.
- A formula sheet is provided.
- Unless otherwise stated, numerical answers should be exact.
- Unless otherwise indicated, no diagrams in this examination are drawn to scale.
- All written responses must be in English.
- Answer all questions.
- **Students are NOT permitted to bring or use notes of any kind, correction fluid/tape, mobile phones and/or any other unauthorised electronic devices.**

Year 11 formulas

Question 1 (2 marks)

Anya achieved 15 out of 22 for her English test and 40 out of 57 for her Science test. If both subjects were equally difficult, in which did she do better?

Question 2 (2 marks)

a Change $19.45/hour full-time to an annual amount.

b Change $695.78/week to an hourly rate.

Question 3 (2 marks)

Joy is a part-time worker who usually works 5 hours a day Thursday to Sunday inclusive for $356/week. What does she receive in a week where she works 6 hours on Friday and 8 hours on Saturday, assuming she earns time-and-a-half for the first two hours overtime and double time thereafter?

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Question 4 (2 marks)

The full-time rate for a petrol station cashier is \$22.40/hour. What does the cashier earn for a 22-hour week?

Question 5 (1 mark)

Allie works on 17% commission. What does she earn for a week when she sells \$5320 worth of goods?

Question 6 (2 marks)

Draw a graph of the function $y = x + 3$ for the domain $-6 \le x \le 3$.

Question 7 (3 marks)

Solve each linear equation.

a $3m - 11 = 7$

b $5 - \frac{2w}{7} = 3$

c $\frac{8}{3b} = \frac{4}{9}$

Question 8 (2 marks)

Graph the function $2y - x + 2 = 0$ for $-6 \le x \le 6$.

Question 9 (2 marks)

Determine if the point $P(-3, 1)$ lies on $y = -2x - 5$.

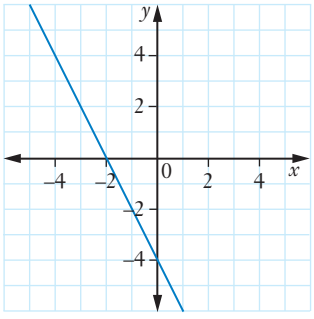
Question 10 (3 marks)

For this line:

a state whether the gradient is positive or negative

b calculate the gradient

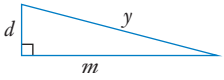
c state the y-intercept



Question 11 (2 marks)

In the triangle, $m = 15$ and $y = 18$.

Find d.



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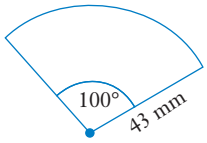
Question 12 (1 mark)

Convert 25 000 cm^2 to m^2.

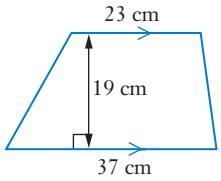
Question 13 (2 marks)

Find the perimeter of this sector.



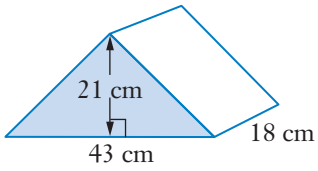
Question 14 (2 marks)

Find the area of this shape.



Question 15 (2 marks)

Find the volume of this solid.



Question 16 (3 marks)

David sells safety equipment to tradespeople. He receives \$200 a week as a retainer and 5% commission on sales. Over the last 2 months, he has made an average of \$620 a week. What commission would he have to get to give up the retainer?

Question 17 (4 marks)

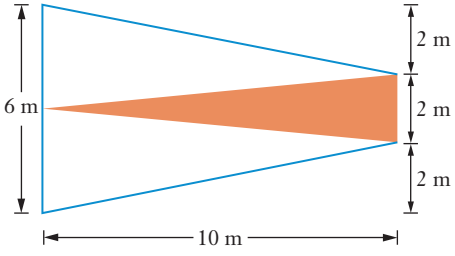
Determine if the points (–4, –5), (–2, 3), (4, 1) and (1, –6) are the vertices of a parallelogram.

Question 18 (3 marks)

A banner is made of different coloured fabrics, as shown here.

a Find the area of the orange part of the banner.

b What fraction of the total area does it form?



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Question 19 (3 marks)

Danii is a 19-year old apprentice hairdresser who lives at home. She earns \$860.60 a fortnight. Her fortnightly youth allowance is \$285.20, reduced by 50 cents for every dollar earned over \$433 (per fortnight) and 50 cents for every dollar earned over \$519 (per fortnight).

a How much youth allowance does she get?

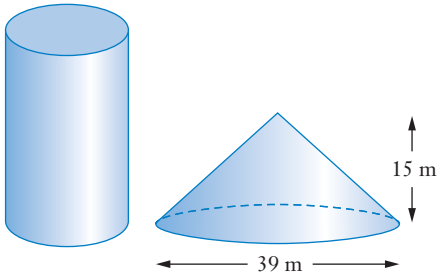
b What reasons would she have to work overtime or refuse overtime?

Question 20 (3 marks)

When this silo filled with sugar was emptied, the sugar formed a conical pile 15 m high with a base diameter of 39 m.

a What volume of sugar was removed from the silo?

b Find the height of the silo if it has a radius of 8.5 m.



Question 21 (4 marks)

A triangle is formed by the graphs of $y = 5x + 1$, $y + 5 = 2x$ and $2y = x + 2$.
Find the coordinates of the vertices of the triangle.

END OF EXAMINATION

CONSUMER ARITHMETIC

4.

PRICES AND INTEREST

You want to get the best you can for your money. When you buy groceries, you want to get the best deal. When you save money, you want the best interest you can get. If you go to another country, you want the best exchange rate possible. If you invest money, you want to make sure it is a good investment. The work in this chapter will help you with these aims.

4.01 Comparing prices
4.02 Mark-ups, discounts, profit and loss
4.03 Simple interest
4.04 Compound interest and inflation
4.05 Currency exchange
4.06 Shares and dividends
Chapter summary
Chapter review

SYLLABUS SUBJECT MATTER

Application of rates and percentages

- compare prices and values using the unit cost method
- apply percentage increase or decrease in various contexts, e.g. determining the impact of inflation on costs and wages over time, calculating percentage mark-ups and discounts, calculating GST, calculating profit or loss in absolute and percentage terms, and calculating simple and compound interest
- use currency exchange rates to determine the cost in Australian dollars of purchasing a given amount of a foreign currency, such as US$1500, or the value of a given amount of foreign currency when converted to Australian dollars, such as the value of €2050 in Australian dollars
- calculate the dividend paid on a portfolio of shares, given the percentage dividend or dividend paid per share, for each share; and compare share values by calculating a price-to-earnings ratio
- use a spreadsheet to display examples of the above computations when multiple or repeated computations are required, e.g. preparing a wage-sheet displaying the weekly earnings of workers in a fast food store where hours of employment and hourly rates of pay may differ, preparing a budget, or investigating the potential cost of owning and operating a car over a year

Prior learning

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TERMINOLOGY

buying price
consumer price index (CPI)
deflation
dividend percentage
exchange
goods and services tax (GST)
interest
marked price
mid-rate
nominal interest rate
operating margin
percentage
profit
selling price
simple interest
term
buying rate
cost price
discount
dividend yield
face value
gross profit margin
interest rate
market price
net (nett)
official
paper loss
price-to-earnings (PE) ratio
recommended retail price (RRP)
selling rate
speculation
trade discount
compound interest
currency
dividend
earnings per share (EPS)
flat-rate interest
inflation
loss
mark-up
net profit margin
operating expense ratio
par value
principal
rest (or rest period)
share
stock market
unit price

Best buys riddle

4.01 Comparing prices

Alamy Stock Photo/RosalreneBetancourt 9

Which is better value: 200 g for \$2.40 or 2 kg for \$22?

200 g for \$2.40 = 1 kg for \$12 and 2 kg for \$22 = 1 kg for \$11.

It appears that 2 kg for \$22 is better value.

But if you only wanted 400 g altogether, it would be better to get 2 @ \$2.40.

Compare prices by considering:

- **Unit price**: the cost of 1 item, 1 kg, 1 litre, etc.
- Quality: is the quality acceptable?
- Quantity required: how much is needed?

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If you buy too much of something at a low unit price, it may be *false economy*. If you buy twice as much as you need at a slightly lower unit price, you don't save money.

Low-quality purchases can also waste money. If you buy cheap bananas but throw them away because they are too squishy, you have wasted money.

EXAMPLE 1

You need 1.3 kg of self-raising flour to make a cake. You can buy premium flour in packets of 500 g for \$1.93 or 2 kg for \$5.50. What should you get?

Solution

Consider the unit prices.	500 g for \$1.93 = \$1.93 × 2 = \$3.86 /kg 2 kg for \$5.50 = \$5.50 ÷ 2 = \$2.75 /kg
Check the quantity needed.	For 1.3 kg, you need 3 × 500 g packets or 1 × 2 kg packet.
Check the prices.	3 × \$1.93 = \$5.79 and 1 × \$5.50 = \$5.50
Write the answer.	Assuming the flour is of acceptable quality, it is cheaper to buy the 2 kg packet.

Perishable items might go off before you can use a large quantity

EXAMPLE 2

Julie lives by herself and uses milk only in her tea. She can purchase 3 L of milk for \$4.89 or 600 mL for \$1.10. Which should she buy?

Solution

How long does milk last?	Milk goes off 4–5 days after opening.
Calculate the unit price.	3 L for \$6.40 = $\frac{\$4.89}{3}$ = \$1.63 /L 600 mL for \$1.10 = $\frac{\$1.10}{0.6}$ ≈ \$1.83/L
Work out how much she would use each day.	4 cups × 60 mL/cup = 240 mL/day
Consider the costs.	3 L would go off before she used it all, so would cost \$4.89 for 4–5 days. 1 carton of 600 mL would last $2\frac{1}{2}$ days so she could buy 2 to last 5 days for \$2.20.
Write the answer.	Julie should buy 600 mL cartons.

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TECHNOLOGY

Best buy

You should use unit costs as part of comparison shopping.

Food items in Australia should have unit costs displayed. Supermarkets have it in quite small print on the shelf label.

You only need to do a simple division or multiplication to calculate unit costs.

You can use the calculator on your smartphone.

Exercise 4.01 Comparing prices

Example 1

1 Collette needs 3 tablespoons (90 g) of tomato paste for a Madras curry. She can buy 200 g sachets for $2.00 each, 140 g tubs for $1.80 each or a 500 g bottle for $3.25. She uses tomato paste about once a week. What should she buy?

2 Collette also needs 3 tsp (15 g) of ground coriander for her curry. She can purchase 25 g for $1.76 or 250 g for $2.80. Which should she buy?

3 Brown onions are available in a 1 kg pack for $1.50 or loose at $3.00 per kg. Onions vary in size but average about 140 g each.

a Which is the better buy for a family?

b Which is the better buy for a single pensioner?

4 A family is buying milk. Which is the better buy: 3 L for $4.89 or 600 mL for $1.10?

Example 2

5 Children's toothpaste costs $1.71 for 45 g or $2.49 for 110 g. Which is the better buy?

6 Caster sugar is $1.86 for 500 g or $2.64 for 1 kg. Which is the better buy?

7 Gary lives on his own. He can buy 1.1 kg of diced beef for $17.60, 750 g of round steak for $11.24, 900 g of chuck steak for $12.60 or 550 g of gravy beef for $7.98. All are suitable for the stew he wants to make. Which is the best buy?

8 You can purchase 1 kg of chicken breast for $16, 700 g with the skin off for $12.60, 500 g of stir-fry pieces for $10.50, breast fillet with the skin off at 500 g for $15.25 or 600 g for $18.84. Which should you choose for a Thai green curry for 2 people?

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4.02 Mark-ups, discounts, profit and loss

Shops sell items at a price higher than what it cost them to buy from the distributor or manufacturer. Sometimes they sell things at a lower price at a sale. They try to make a profit on the money they use to run the shop.

Repeated percentage change

Profit and loss

- **Cost price** (or **buying price**) is the amount of money paid for an item by a shop
- **Mark-up** is the amount added to the cost price of an item for sale
- **Marked price** (or **selling price**) = cost price + mark-up
- A **profit** is made when items are sold for more than they cost

  Profit = selling price – cost price
- A **loss** is made when goods are sold for less than what they cost.

  Loss = cost price – selling price
- A **discount** is an amount taken off the marked price
- A **trade discount** is the discount of the **recommended retail price** (**RRP**) that is given to retailers by distributors or manufacturers so that they can make a profit when selling at the RRP
- **Percentage** mark-ups, discounts, profits and losses are usually 'out of' the cost price
- In Australia, the **Goods and Services Tax** (**GST**) is added to the price of an item or service and paid to the government by the seller. It is set at 10% of the value and must be included in retail prices by shops.

EXAMPLE 3

A shoe shop marked up items by 145% to cover staff costs and other overheads and make a reasonable profit. The shop buys cross trainers from a distributor for $30.80 a pair.

a What is the marked price?

b The shop sells sandals for $48.78 a pair. What do they pay for them?

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Solution

a Find the marked price as a percentage of the cost price.

Marked price = 245% of cost price

$= 2.45 \times \$30.80$

$= \$75.46$

Write the answer.

The marked price is \$75.46.

b Write the marked price as a percentage of the cost price.

Marked price = 245% of cost price

$\$47.78 = 2.45 \times$ cost price

Solve the equation.

$$\text{Cost price} = \frac{\$47.78}{2.45}$$

$$\approx \$19.50$$

Write the answer.

The shop paid \$19.50 a pair for the sandals.

GST included in a price = Price ÷ 11

Including GST in Example 3 makes a considerable difference. The \$75.46 would be the ex-GST price, so the price paid by the customer would be 110% of that, which would be would be \$83.01, including \$7.55 GST. The shop would have paid $\frac{\$30.80}{11} = \$2.80$ in GST, so would have to send $\$7.55 - \$2.80 = \$4.75$ to the ATO.

INVESTIGATION

MARK-UPS IN DIFFERENT BUSINESSES

Investigate the mark-ups used in different types of shops and businesses, such as

- restaurants
- fast-food outlets
- supermarkets
- corner shops
- clothes shops
- shoe shops
- electrical goods shops
- furniture shops
- car parts
- used car yards
- new car dealers
- petrol stations
- other businesses such as plumbers, electricians, etc.

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EXAMPLE 4

At a sale, a department store discounts electrical goods by 25%. A refrigerator has a marked price of \$682.

a What is the discounted price?

b What is the change in the amount of GST included in the price?

c What is the marked price of a toaster sold at a discounted price of \$78.75?

Solution

a Find the new price as a percentage of the old price.

$$\text{Discounted price} = 75\% \text{ of marked price}$$
$$= 0.75 \times \$682$$
$$= \$511.50$$

b Find the GST in the marked price.

$$\text{GST in marked price} = \frac{1}{11} \times \$682$$
$$= \frac{\$682}{11}$$
$$= \$62$$

Find the GST in the discounted price.

$$\text{GST in discounted price} = \frac{1}{11} \times \$511.50$$
$$= \frac{\$511.50}{11}$$
$$= \$46.50$$

Calculate the change.

$$\text{Change in GST} = \$62 - \$46.5$$
$$= \$15.50$$

Write the answer.

The GST included in the price decreases by \$15.50.

c Write the new price as a percentage of the old price.

$$\text{Discounted price} = 75\% \text{ of marked price}$$

Write as an equation and solve for the marked price.

$$\$78.75 = 0.75 \times \text{marked price}$$
$$\text{Marked price} = \frac{\$78.75}{0.75}$$
$$= \$105$$

Write the answer.

The marked price of the toaster was \$105.

Businesses usually express their expenses, profit or loss as a percentage of sales. This makes it easier to keep track of their business and calculate taxes.

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Business margins

Business margins are expressed as a percentage of sales (**turnover**).

- **Operating margin** $= \frac{\text{profit}}{\text{sales}} \times 100\%$, where profit = sales – costs and interest and taxes are not included in the costs. Also called the **gross profit margin**.
- **Net profit margin** $= \frac{\text{net profit}}{\text{sales}} \times 100\%$, where net profit = sales – costs and interest and taxes are included in the costs.
- **Operating expense ratio** $= \frac{\text{expenses}}{\text{sales}} \times 100\%$, where expenses include all costs except interest and taxes.

EXAMPLE 5

A market stall holder bought 120 new DVDs for \$666. She sold them for \$12 each, but at 2 p.m. she still had 30 left. She dropped the price to \$5 each and sold the rest before the market closed at 3 p.m. She did not have to pay GST.

a What percentage profit did she make on the DVDs she sold for \$12 each?

b What percentage loss did she make on the DVDs she sold for \$5 each?

c The stall cost her \$50 for the day. What percentage profit did she make overall?

d What was her profit margin?

Solution

a Find the cost of a DVD.

$$\text{Cost of DVD} = \frac{\$666}{120} = \$5.55$$

Find the profit.

$$\text{Profit} = \$12 - \$5.55 = \$6.45$$

Find the percentage profit.

$$\text{Percentage profit} = \frac{\$6.45}{\$5.55} \times 100\% \approx 116.2\%$$

b Calculate the loss.

$$\text{Loss} = \$5.55 - \$5 = \$0.55$$

Find the percentage loss.

$$\text{Percentage loss} = \frac{\$0.55}{\$5.55} \times 100\% \approx 9.9\%$$

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c Find the total sales.

$$\text{Total sales} = 90 \times \$12 + 30 \times \$5$$
$$= \$1230$$

Find the total cost.

$$\text{Total cost} = \$666 + \$50$$
$$= \$716$$

Find the net profit.

$$\text{Net profit} = \$1230 - \$716$$
$$= \$514$$

Net (or nett) means with all costs.

Find the net profit percentage.

$$\text{Net profit percentage} = \frac{\$514}{\$716} \times 100\%$$
$$\approx 71.8\%$$

Write the answer.

Overall net profit is 71.8%.

d Find profit as a percentage of sales.

$$\text{Profit margin} = \frac{\$514}{\$1230} \times 100\%$$
$$\approx 41.8\%$$

Exercise 4.02 Mark-ups, discounts, profit and loss

Ignore GST in these questions unless otherwise stated.

1 Copy and complete the following table.

Example 3

	Item	Cost price	Percentage mark-up	Marked price
a	DVD player	$250	40%	
b	Table	800	75%	
c	Lamp		85%	$210
d	Software bundle		90%	$570
e	Bottle of soft drink	$3.00		$4.50
f	Magazine	$5.00		$7.50
g	Travel bag	$30	110%	
h	Wide-screen TV	$1400		$2660

2 A hardware shop works on a 65% markup. What would the shop charge for:

a spades with a cost price of $12?

b drills that cost the shop $140?

c packs of ceiling insulation that cost $60 each?

d tubes of silicon costing $3 each?

e a chainsaw with a cost price of $280?

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3 A furniture store sells a three-piece lounge suite for \$5000, or the chairs and sofa separately at \$1400 for each chair and \$2800 for the sofa. The cost price of the chairs was \$760 and the cost price of the sofa was \$1240. Calculate the percentage mark-up on:

a the lounge suite as a set

b the sofa sold separately

c a chair sold separately.

4 A takeaway works on a mark-up of 180%. What is the cost price of each item?

a Spring roll sold for \$3.20

b Cod sold for \$6.20 a piece

c Mini dim-sims sold for \$1.10 each

d Seafood snacks sold for \$2.85 each

Example 4

5 After a fire, smoke-damaged goods are sold at a discount of 15% to clear stock.

a What is the discount price of a tracksuit with an original price of \$175?

b What would a dress that originally cost \$240 sell for?

c A jacket is marked down to \$238. What was the original price?

d What was the original price of a top that sold for \$68?

6 A hardware store has a '$12\frac{1}{2}$% off' day where everything in the shop is discounted by $12\frac{1}{2}$%. The garden section is discounting wheelbarrows by \$10 on top of the $12\frac{1}{2}$%.

a What are the final price and discount on a wheelbarrow marked at \$72?

b What is the overall percentage discount?

7 A new car distributor gets a trade discount of 20% of the recommended retail price (RRP) of a new car (without on-road costs). The distributor also adds \$1500 'delivery costs' for detailing the car. The RRP of the car, without delivery and on-road costs, is \$39 800, and registration and insurance cost \$1750.

a What does the dealer make on selling the car?

b What is the final price of the car when someone buys it?

8 A sweets manufacturer offers packs of sweets at a RRP of \$2 to organisations for use in fund-raising. The trade discount offered is 60%. One school buys 5000 packets and sells all of them.

a How much does the manufacturer get?

b How much does the school make?

Another school buys 7000 packets but can only sell 5000 at the full price. It sells off the rest at \$1 a packet.

c How much does the manufacturer get from this school?

d How much does the school make?

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9 Calculators with a RRP of $45 are sold to shops at a trade discount of 40%. What does the distributor get for 3500 calculators?

10 An electrical store sells fridges for $850. It buys them for $500.

Example 5

a What is the percentage profit?

b What is the profit margin?

11 A new car dealer values a trade-in at $8000 to help clinch the sale of a new car, but can only get $7500 for it from a wholesale dealer. What percentage loss does the car dealer make on the trade-in?

12 A local convenience store has a turnover of $315 000. The owner's gross profit is $87 000, but overheads, including wages, taxes, power, accounting fees, advertising, rent and other charges, come to $54 600. What is the net profit margin?

13 An agricultural supplier works on a percentage profit of 28%. Drums of pesticide are sold for $582.40 each. What was the cost price?

14 A second-hand dealer bought the following items: fishing reel ($15), video camera ($220), jewellery box ($35), filing cabinet ($80) and printer ($340). The fishing reel sold the next day for $20, the video camera sold for $290 and the filing cabinet sold for $130. The jewellery box and printer were in the window for several weeks, and eventually sold for $25 and $255 respectively. Calculate the percentage profit or loss on each item.

15 A sales representative bought a new car for $32 500 and after using it for 2 years sold it for $18 600. What was the percentage loss?

16 A snack bar in a shopping centre paid $15 000 a month rent over the previous year. The snack bar's payroll, insurance and other wages costs were $95 000 a year. Its power, light and cleaning costs were $28 000 a year. Its turnover for the year was $1 210 000, and its suppliers charged a total of $540 000 for the year. They paid $89 910 tax.

a What was the percentage profit on the snacks, without taking costs into account?

b What was the operating margin?

c What was the net profit margin?

17 A shop paid $120 for a coffee table and sold it at a mark-up of 65%.

a How much GST does the customer pay?

b What is the final price of the coffee table?

c How much GST was included in the price paid by the shop?

d How much GST does the shop have to send to the ATO for the coffee table?

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Problem solving

18 Discounts of 10% and 20% are applied successively. Explain why the total discount is not 30%.

19 A bookshop receives a trade discount of 40% on popular fiction. What mark-up should they apply to sell at the recommended retail price?

20 Mead Murphy, an antique dealer, made a profit of \$171 when he sold an antique bisque doll. His percentage profit was 57%. What did he pay for the doll? What was the sale price?

21 Peter made 15% profit when he sold some pine bookcases he had made, taking all costs into account. If he considered only the cost of the wood, it would have been a 55% profit. The sale price was \$372 each. How much did the wood for a bookcase cost?

4.03 Simple interest

Simple interest 1

Simple interest 2

Simple interest – Buying on terms

Compounding periods: Spreadsheet

When you use credit or borrow money you have to pay extra money back. This is called **interest**. If you invest money, you are paid interest for the use of your money.

Simple interest

- The **term** is the time of a loan or investment
- The **principal** is the amount borrowed or invested
- The **interest rate** is the cost of a loan for each year of the term. It is written as a proportion of the principal as a decimal or percentage
- **Simple interest** is the product of the principal, interest rate and term in years. Simple interest is also called **flat-rate** interest
- $I = Pin = \frac{Pnr}{100}$ and $A = P + I$, where I = total interest, P = principal, i = interest rate as a decimal, r = interest rate as a percentage, n = term in years, A = amount to be repaid

Personal loans, hire-purchase and other loans with terms under about 5 years are calculated using simple interest. Investments that pay a regular return also use simple interest.

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EXAMPLE 6

Annabelle received payments of \$38 a quarter from \$2500 worth of Australian Government Bonds. What is the interest rate?

Solution

Work out the payment for one year.

$$\text{Interest for 1 year} = 4 \times \$38$$
$$= \$152$$

Calculate the interest rate.

$$\text{Interest rate} = \frac{\$152}{\$2500} \times 100\%$$
$$= 6.08\%$$

Write the answer.

The interest rate is 6.08%.

The interest rate for 1 year is sometimes called the **nominal interest rate**.

You usually pay back a hire-purchase loan in regular equal instalments. The instalments are usually *rounded up* so the last payment may be a little less.

EXAMPLE 7

Darryl bought a car for \$24 320 including on-road costs on \$500 deposit. He looked at the contract after a few weeks and found that he was being charged \$300 a year loan insurance and a \$400 establishment fee, incorporated into the loan. The interest rate was 22% flat-rate over 5 years with monthly instalments.

iStock.com/alempkht

a How much were the monthly instalments?

Twelve months later, he could not afford to pay the insurance and registration so the car was repossessed and sold at auction for \$14 500.

b How much did he have left to pay after losing the car?

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Solution

a Calculate the total principal.

$$\text{Principal} = \$24\,320 + \$400 + 5 \times \$300 - \$500$$
$$= \$25\,630$$

Calculate the total interest.

$$I = Pin$$
$$= \$25\,630 \times 0.22 \times 5$$
$$= \$28\,193$$

Calculate the total to be repaid.

$$A = \$25\,630 + \$28\,193$$
$$= \$53\,823$$

Calculate the monthly instalment.

$$\text{Monthly instalment} = \frac{\$53\,823}{5 \times 12}$$
$$= \$897.05$$

b Calculate the amount paid.

$$\text{Amount paid} = 12 \times \$897.05$$
$$= \$10\,764.60$$

Calculate the amount still owed.

$$\text{Amount owed} = \$53\,823 - \$10\,764.60 - \$14\,500$$
$$= \$28\,558.40$$

Write the answer.

Darryl has a debt of $28 558.40 he still has to pay.

Take as much time as you need to understand a contract *before* you sign it.

EXAMPLE 8

A fixed-deposit income-producing account has 4.2% interest and pays interest every fortnight. How much do you have to invest to get $150 a fortnight?

Solution

Write down the values for the formula.

$$I = 150,\ i = 0.042,\ n = \frac{1}{26}$$

Write the formula, substitute values and solve to find the principal.

$$I = Pin$$
$$150 = P \times 0.042 \times \frac{1}{26}$$
$$P = \frac{26 \times 150}{0.042}$$
$$\approx \$92\,857.14$$

Write the answer.

You need to invest about $92 858.

The amount in Example 8 was rounded *up* to get at least $150 a fortnight.

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TECHNOLOGY

Simple interest spreadsheet

You can make a spreadsheet for simple interest calculations.

Put the following headings in the cells indicated.

A3: Principal

A4: Interest rate%

A5: Term

A6: Payments/year

D3: Loan

D4: Total

D5: Instalment

D7: Investment

D8: Lump sum

D9: Regular payment

You can use cell formatting, colours and column width to make it look professional.

Put in, say, 20000 for the principal, 6.8 for the interest rate, 4 for the term and 12 for the payments/year in cells B3 to B6.

The total to pay back for a loan is the principal + interest, $P + Prn \div 100$.

This translates to the formula =B3+B3*B4*B5/100. Put this formula in cell E4.

Don't forget the equals sign to make it a formula.

To get the instalments you divide by the total number of payments, B5*B6.

Type the formula =E4/(B5*B6) in cell E5.

You need the brackets to make sure the multiplication is done first.

An investment can give either a lump sum at the end or a regular payment.

For the lump sum, put the formula =B3+B3*B4*B5/100 in cell E8.

For the regular payment, put the formula =B3*B4/(100*B6) in cell E9.

Try your spreadsheet with different problems.

A completed 'Simple interest' spreadsheet can be downloaded from NelsonNet.

Simple interest

Simple interest

Principal	20000
Interest rate%	6.8
Term	4
Payments/year	12

Loan	
Total	25440
Instalment	530

Investment	
Lump sum	25440
Regular payment	113.3333333

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Exercise 4.03 Simple interest

1 David has some bonds worth $6000 paying interest of $82.50 every quarter. What interest rate are the bonds earning?

2 Peter invests $12 000 in bonds paying 5.7% interest with monthly payments. How much does he get each month?

Example 6

3 For each investment and final amount, calculate the simple interest rate.

- **a** $5000 amounted to $5750 after 2 years.
- **b** $800 amounted to $1008 after 4 years.
- **c** $9000 amounted to $10 312.50 after 2 years and 4 months.
- **d** Thalia got $5000 back after investing $2000 for 3 years.

4 Find the quarterly interest paid when Hassan invests $16 000 in bonds paying 7.5% interest.

5 The simple interest on a loan of $500 over 3 years is $225. What is the interest rate?

6 Gabriella invested $75 000 in 30-day bonds at 6.4% simple interest. What did she get back?

7 Rajendra bought some $100 Australian Government Bonds paying 6.2% interest. He bought 180 bonds and receives interest each quarter. How much does he get each quarter?

Example 7

8 Julia borrowed $7500 at 8.3% simple interest for 5 years. Calculate the interest and fortnightly payment.

9 What is the simple interest charged on each loan?

- **a** $500 at 12% for 3 years
- **b** $3200 at 15% for 8 months
- **c** $4500 at 8% for 4 years
- **d** $800 at 25% for 3 months

10 Elena borrowed $8000 at 13.8% simple interest over 3 years. How much does she pay back each fortnight?

11 Hamish received $500 credit for 2 weeks at 45% simple interest from a *payday lender*. He also had to pay a $100 establishment fee. How much did Hamish have to pay back at the end of the 5 weeks? What is the effective interest rate?

Example 8

12 Phan can invest in a fixed-interest account that earns 6.4% interest. How much must she invest to get $200 a week?

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13 What is the final amount for each investment at simple interest?

a \$2100 at 17.5% for 9 months

b \$7000 at 11% for $2\frac{1}{2}$ years

c \$9000 at 13.9% for 4 years

d \$6100 at 13.5% for 2 years and 10 months

14 How much would you have to put in an investment account to get \$300 a month interest at 5.6%?

15 An executive on a salary of \$2400 a week wants to make sure that she gets at least 60% of her salary on retirement. She can earn 6% p.a. interest. How much must she have invested to reach her goal?

Problem solving

16 A company borrowed \$200 000 on the short-term money market for 60 days. The company had to repay \$207 000. What interest rate did the company pay?

17 Carmen can invest \$6000 in two ways. She can invest it at 12% simple interest for 2 years, or she can invest \$2000 at 13% and the rest at 11.6%. Calculate the return from each method and state which investment would give the greater return.

18 John bought 750 shares for \$3450. He got half-year dividends of 11 cents a share. What is the equivalent interest rate?

4.04 Compound interest and inflation

Consider investing \$100 at simple interest of 10% p.a. for 3 years.

You would get \$130 at the end.

What if you reinvested the money plus interest every year?

After one year you would get \$110.

Investing this for another year would give you \$121.

Investing this for another year would give you \$133.10.

Compound interest

- Compound interest is worked out as if the money was regularly re-invested
- The time between calculations is called the **rest period** (or **rest**)

Compound interest

Simple and compound interest

Simple vs compound interest: Spreadsheet

Spreadsheets - Simple and compound interest

Compounding periods: Spreadsheet

Compound interest table

EXAMPLE 9

How much would you get back if you invested \$7000 at 5.8% compound interest for 2 years with 6-monthly rests?

Solution

Calculate the amount after 6 months.	Amount after 1 rest = \$7000 + 2.7% of \$7000 = 102.7% of \$7000 = 1.027 × \$7000 = \$7189
Enter 7000 on your calculator and press = .	7000
Now enter × 1.027 = in your calculator.	7189
Calculate the amount after a year.	Amount after 2 rests = 1.027 × \$7189 = \$7383.103
Just press = now.	7383.103
Calculate the amount after 18 months.	Amount after 3 rests = 1.027 × \$7383.103 = \$7582.44…
Press = again.	7582.446781
Calculate the amount after 2 years.	Amount after 4 rests = 1.027 × \$7582.44… = \$7787.17…
Press = again.	7787.172844
Round and write the answer.	You would have \$7787.17 after 2 years.

Pressing = repeats the previous multiplication because each answer (ANS) is being multiplied by 1.027. This is called the *constant multiplication*.

You can see from Example 9 that $(1 + i) = \left(1+\frac{r}{100}\right)$ is multiplied by the principal for each rest, where i or r is the interest rate divided by the number of rests in a year.

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Compound interest formula

- $A = P(1+i)^n = P\left(1+\frac{r}{100}\right)^n$ and $I = A - P$, where A = amount after n years, P = principal, i = interest rate, r = interest rate as a percentage and I = total interest
- $A = P\left(1+\frac{i}{k}\right)^{kn} = P\left(1+\frac{r}{100k}\right)^{kn}$ and $I = A - P$, where A = amount after n years, P = principal, i = interest rate, r = interest rate as a percentage, k is the number of rests in a year and I = total interest

Compound interest

Comparing interest rates

Comparing interest rates

Interest calculator

Compound interest graph

EXAMPLE 10

a Gerry invests \$6200 at 7.5% compound interest with monthly rests for 4 years. How much interest does she get?

b Anthony can invest in a 5-year fixed-term account at 9% with monthly rests. How much should he invest to get \$10 000 after 5 years?

c Fatima invested \$8300 in an account with quarterly rests. After 2 years it had grown to \$9500. What was the average interest rate?

Solution

a Write the values of the variables. $P = \$6200$, $i = 0.075$, $k = 12$ and $n = 4$

Write the formula and substitute values.

$$A = P\left(1+\frac{i}{k}\right)^{kn}$$

$$= 6200\left(1+\frac{0.075}{12}\right)^{12\times 4}$$

Simplify. $= 6200 \times 1.00625^{48}$

Put 6200 × 1.00625 ^ 48 = on your calculator.

Your calculator might use x^y, y^x or x^n instead of =.

8361.314738

Find the interest. Interest = \$8361.31… – \$6200

= \$2161.31

Round and write the answer. Gerry gets \$2161.31 interest.

b Write the values of the variables. $n = 5$, $i = 0.09$, $k = 12$ and $A = 10\,000$

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Write the formula and substitute values.

$$A = P\left(1+\frac{i}{k}\right)^{kn}$$

$$10\,000 = P\left(1+\frac{0.09}{12}\right)^{12\times 5}$$

Isolate P.

$$P = 10\,000 \div \left(1+\frac{0.09}{12}\right)^{12\times 5}$$

Put (1 + 0.09 ÷ 12) ^ 60 = on your calculator.

1.565681027

Put 10 000 ÷ ANS = on your calculator.

6386.996986

Round and write the answer.

Anthony has to invest \$6387.

c Write the values of the variables.

$P = 8300$, $k = 4$, $n = 2$, $A = 9500$

Write the formula and substitute values.

$$A = P\left(1+\frac{i}{k}\right)^{kn}$$

$$9500 = 8300\left(1+\frac{i}{4}\right)^{4\times 2}$$

Divide by 6200.

$$\left(1+\frac{i}{4}\right)^{8} = \frac{9500}{8300}$$

Do the eighth root.

$$1+\frac{i}{4} = \sqrt[8]{\frac{9500}{8300}}$$

Put 8 $\sqrt[x]{y}$ (9500 ÷ 8300) = on your calculator.

1.0170228

Your calculator might use $x^{\frac{1}{y}}$ instead of $\sqrt[x]{y}$.

Solve the equation.

$$1+\frac{i}{4} = 1.017\ldots$$

$$\frac{i}{4} = 1.017\ldots - 1$$

$$i = 4\times(1.017\ldots - 1)$$

Put − 1 = and then × 4 = on your calculator.

0.0170228

0.068091199

Round.

$i = 0.0681$

Write the answer.

The interest rate is about 6.81%.

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TECHNOLOGY

Compound interest spreadsheet

You can do your own spreadsheet of compound interest calculations.

Put the headings 'Interest rate percentage', 'Principal', and 'Rests per year' in cells B2–B4.

Put the test values 8, 1000 and 12 in cells C2–C4.

Put the headings 'Year', 'Start amount', 'End amount', 'Interest' and 'Total interest' in cells A6–E6. Put the value 1 in cell A7.

Now put these formulas into the cells shown. Don't forget the equal signs.

B7: = C3

C7: = B7*(1+$C$2/(100*$C$4))^$C$4

D7: = C7–D7

E7: = D7

A8: = A7+1

B8: = C7

Copy C7–D7 down to C8–D8.

E8: = E7+D8

Now copy cells A8–E8 down the spreadsheet as far as you want.

You can make the spreadsheet prettier using format and colours.

Try changing the interest rate, principal and rest values one at a time to see what happens.

Try making the rests, 1, 4, 12, 52 and 365 and look at the changes.

Try making the interest rate 2, 3, 4, 5, 6, 7, 8, 9 and 10 and look at the changes.

A completed 'Compound interest' spreadsheet can be downloaded from NelsonNet.

Compound interest

Interest rate%	8
Principal	$1,000.00
Rests per year	12

Year	Amount at start	Amount at end	Interest	Total Interest
1	1,000.00	1,083.00	83.00	83.00
2	1,083.00	1,172.89	89.89	172.89
3	1,172.89	1,270.24	97.35	270.24
4	1,270.24	1,375.67	105.43	375.67
5	1,375.67	1,489.85	114.18	489.85

Compound interest

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Your parents have probably told you things were cheaper when they were young. It is true.

The value of money over the last 40 years has decreased by a factor of about 6. That means that you could buy about 6 times as much in the 1970s as you can now for $20. This is called **inflation**.

Courtesy ACT Heritage Library with permission Fairfax Syndication

Inflation, deflation and the Consumer Price Index (CPI)

- The **Consumer Price Index (CPI)** measures the cost of a standard 'basket' every quarter. The 'basket' used by the Australian Bureau of Statistics (ABS) is typical of the things Australians buy.
- **Inflation** occurs as the CPI increases and money loses value
- **Deflation** occurs as the CPI decreases and money gains value

The 'basket' of goods used for the CPI has been changed by the ABS over the years. It includes rent, petrol, groceries, fares and accommodation. The ABS does surveys so they know what Australians buy. The Australian Reserve Bank tries to keep inflation at an average of about 2.5% per year.

INVESTIGATION

INFLATION

Use the ABS site to find:

- inflation over the last 5 years
- inflation over the last 10 years
- inflation over the last 20 years
- the current CPI basket
- previous CPI baskets

What do the changes to the CPI baskets tell you about Australia?

Your teacher might want you to write a report.

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You use the compound interest formula to calculate the average rate of inflation between two years, because the percentage increase applies to each year in succession, just like interest.

EXAMPLE 11

a The average rate of inflation from 2010 to 2016 was 2.1%. What would a cabbage costing \$2.50 in 2010 be likely to have cost in 2016?

b The CPI rose from 70.2 in 2000 to 107.5 in 2015. What was the average rate of inflation?

Solution

a

Write the values of the variables.	$P = 2.5, i = 0.021, n = 6$
Write the formula and substitute values.	$A = P(1 + i)^n$
	$= 2.5 \times 1.021^6$
Calculate.	$= 2.8320\ldots$
Round and write the answer.	The cabbage would probably have cost about \$2.83 in 2016.

b

Write the values of the variables.	$A = 107.5, P = 70.2, n = 15$
Write the formula and substitute values.	$A = P(1 + i)^n$
	$107.5 = 70.2 \times (1 + i)^{15}$
Solve the equation.	$(1 + i)^{15} = \frac{107.5}{70.2}$
	$1 + i = \sqrt[15]{\frac{107.5}{70.2}}$
	$i = \sqrt[15]{\frac{107.5}{70.2}} - 1$
Use your calculator.	$= 0.0288\ldots$
Write the answer.	The average rate of inflation from 2000 to 2015 was about 2.9%.

You sometimes see news articles that talk about real increases in funding. This means that the increase takes inflation into account.

A real increase from 2010 to 2016 has to be more than 1.021^6 times the 2010 amount.

Exercise 4.04 Compound interest and inflation

Simple and compound interest

Example 9

When you use constant multiplication on your calculator, be very careful to count the number of times carefully.

1 Use the simple interest formula to find the final amount and interest earned for each investment at compound interest.

a \$20 000 at 8% with quarterly rests for 2 years

b \$8000 at 10% with weekly rests for 3 months (13 weeks)

c \$15 000 at 7.5% with monthly rests for 5 months

d \$7800 at 8.6% with 6-monthly rests for 3 years

2 Find the final amount and interest for:

a \$45 000 at 12% with weekly rests for 6 months

b \$120 000 at 14.5% with 6-monthly rests for $2\frac{1}{2}$ years

c \$76 000 at 9.6% with fortnightly rests for 20 weeks

d \$138 000 at 11.2% with monthly rests for $1\frac{1}{2}$ years

e \$225 500 at 10.3% with weekly rests for 3 months

Example 10

3 Find the final amount and interest gained for each investment at compound interest.

a \$2000 at 12% for 5 years

b \$8000 at 13.5% for 4 years

c \$7000 at 15.7% for 8 years

d \$500 at 18.1% for 9 years

e \$2140 at 11.25% for 7 years

f \$60 at 10% for 50 years

4 Find the final amount and interest gained for each investment at compound interest.

a \$1500 at 12% for 2 years with monthly rests

b \$2840 at 18% for 4 years with monthly rests

c \$9000 at 16.4% for 8 years with quarterly rests

d \$800 at 13.75% for 4 years with weekly rests

e \$5000 at 17% for 2 years with daily rests

f \$7500 at 16% for $1\frac{1}{2}$ years with monthly rests

g \$7500 at 16% for $1\frac{1}{2}$ years with quarterly rests

h \$7500 at 16% for $1\frac{1}{2}$ years with annual rests

i \$7500 at 16% for $1\frac{1}{2}$ years with daily rests

5 What compound interest rate would double \$2000 in 4 years?

6 After 8 years, \$1000 accumulates to \$3000. What was the interest rate?

7 How long will it take \$9000 to accumulate to at least \$15 000 at 16% compound interest?

8 About how long would it take for \$100 to accumulate to \$1000 at 15% compound interest?

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9 Assuming that inflation in Australia averaged 2.5%, find the price in 2018 of:

Example 11

- **a** a calculator that cost $12 in 1980
- **b** a refrigerator that cost $300 in 2000
- **c** a trailer that cost $800 in 2005
- **d** a set of speakers that cost $280 in 1995

10 The CPI rose from 7.6 in 1960 to 26.2 in 1980. What was the average inflation rate?

11 Hyper-inflation means very high inflation. Brazil had an inflation rate of about 2% a day for about 3 months at the start of 1990.

- **a** What was the inflation rate for a fortnight?
- **b** How long did it take for prices to double?

Problem solving

12 How long will it take to double your money at 8% compound interest (calculated annually)? You can work this out using an amount like $10 000.

13 Nikita had $5000 in an investment and after 4 years with monthly rests it had grown to $6300. What interest rate was Nikita getting?

14 How much should Ranjan invest in an account paying 6% interest calculated semi-annually to have $12 500 in 7 years time?

4.05 Currency exchange

When you buy things in other countries, you use their **currency** (money). This means that you have to change Australian money to their money. You do this at a money exchange to receive foreign cash. If you buy online or buy mail order from another country, you do this through a financial institution. You have to pay a charge for changing money.

WS Currency exchange

WS Currency rate exchange

WS Currency conversion graph

Currency exchange

- Each currency has a three letter code, like AUD for Australian dollars
- The **exchange rate** is stated from the viewpoint of the institution
- The **buying rate** is the amount in AUD they buy other currency for
- The **selling rate** is the price they sell other currency for in AUD
- The **mid-rate** is the average of the buying rate and the selling rate
- The **official rate** is an average mid-rate worked out by the Reserve Bank
- The mid-rate/official rate is used to find the relative values of currency

Money exchangers make money from the difference between the buying and selling rates. The selling rate is lower than the buying rate, so they give you less in AUD for other currency than you have to pay for other currency. The buying rate is usually given first.

The *B*uying rate is the *B*ig number, and the *S*elling rate is the *S*mall number.

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Large institutions trade currencies in the millions, so they get better rates than ordinary people.

The exchange rate for cash is usually different from the exchange rate for transferring money overseas to pay for something. When you work out a mid-rate, use the transfer rates.

Shutterstock.com/TK Kurikawa

EXAMPLE 12

The exchange rates for transferring money between Australian dollars and Japanese yen are JPY 90.32 (buying rate), 82.16 (selling rate).

a What is the mid-rate?

b What is AUD 250 worth in Japanese yen?

c What is JPY 10 000 worth in Australian dollars?

Solution

a Find the average.

$$\text{Mid-rate} = \frac{90.32 + 82.16}{2}$$
$$= \text{JPY } 86.24$$

b Write the mid-rate.

$$\text{AUD } 1 = \text{JPY } 86.24$$

Multiply to find the value.

$$\text{AUD } 1 \times 250 = \text{JPY } 86.24 \times 250$$
$$= \text{JPY } 21\,560$$

Write the answer.

AUD 250 is worth JPY 21 560.

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c	Write the mid-rate.	AUD 1 = JPY 86.24
	Find the value of JPY 1.	JPY 1 = AUD 1 ÷ 86.24
	Multiply to find the value.	JPY 1 × 10 000 = AUD 1 × 10 000 ÷ 86.24
		= AUD 10 000 ÷ 86.24
		≈ AUD 115.96
	Write the answer.	JPY 10 000 is worth AUD 115.96.

Australian currency × exchange rate → foreign currency
Foreign currency ÷ exchange rate → Australian currency

When you exchange money, it can be actual money or a transfer between accounts. A transfer costs less than actual money because the financial institution does not have to keep foreign cash on hand.

EXAMPLE 13

Andrew is buying a new muffler on his credit card from Britain for GBP 225 (pounds) including postage. His bank quotes rates of GBP 0.6176, 0.5572 for international money transfer with Britain. How much does the muffler cost him in AUD?

Solution

Andrew is buying GBP, so the bank is selling.	AUD 1 = GBP 0.5572
Find the cost of GBP 1 in AUD.	GBP 1 = AUD $\frac{1}{0.5572}$
Find the cost of GBP 225.	GBP 225 = AUD $\frac{225}{0.5572}$
Calculate the amount.	≈ AUD 403.81
Write the answer.	The muffler costs AUD 403.81.

Credit card purchases usually take 2 business days to process, so the exchange rate could change.

EXAMPLE 14

Kandy has come back from Thailand and still has 1500 baht left when she arrives at Brisbane Airport. The money exchange at the airport has rates of THB 27.78, 24.48 for AUD. How much will she get if she changes her money at the airport?

Solution

Kandy is selling THB, so they are buying.	AUD 1 = THB 27.78
Find the amount in AUD for 1 BHT.	$1 \text{ BHT} = \text{AUD } \frac{1}{27.78}$
Find the amount for THB 1500.	$1500 \text{ BHT} = \text{AUD } \frac{1500}{27.78}$
Calculate the amount.	$\approx$ AUD 54.00

Remember that money exchangers want to make money, so the rate used gives you less money than the mid-rate.

Traveller's cheques and money cards are considered safer than carrying cash. They are traded at the cash rate.

INVESTIGATION

Currency exchange rates

- Choose 4 different foreign currencies
- Check the exchange rates on the Reserve Bank site and a commercial site
- Work out the percentage differences between the official rate and the buying and selling rates
- Calculate how much the commercial site would make on a day when they bought and sold AUD 100 000.
- Does it make any difference which currency you look at?
- Do the same thing a week later.
- Why does the exchange rate change?

Your teacher might want you to write a report.

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Exercise 4.05 Currency exchange

Unless otherwise stated, use the exchange rates below for this exercise.

Country/Region	Currency unit	Code	Transfer	Cash
Canada	Dollar ($)	CAD	1.0221, 0.9307	1.0230, 0.9298
China	Renminbi Yuan (¥)	CNY	5.3574, 4.8486	5.4347, 4.7713
European Union	Euro (€)	EUR	0.7392, 0.6647	0.7400, 0.6641
Fiji	Dollar ($)	FJD	1.6193, 1.4616	1.6249, 1.4602
Hong Kong	Dollar ($)	HKD	5.9206, 5.4187	5.9271, 5.4051
India	Rupee	INR	50.880, 47.390	55.8820, 43.9590
Indonesia	Rupiah	IDR	11 011, 8820	11 078, 8753
Japan	Yen (¥)	JPY	90.32, 82.16	90.40, 82.07
Malaysia	Ringgit	MYR	3.5440, 3.0234	3.6106, 2.9568
New Zealand	Dollar ($)	NZD	1.0762, 0.9969	1.0812, 0.9960
Papua New Guinea	Kina	GPK	2.6068, 1.9874	2.6088, 1.9854
Philippines	Peso	PHP	38.700, 34.740	38.750, 34.670
Singapore	Dollar ($)	SGD	1.0986, 1.0079	1.1109, 1.0069
South Africa	Rand	ZAR	10.6715, 9.7172	10.7987, 9.7060
Sri Lanka	Rupee	LKR	116.4, 102.21	116.4, 97.74
Thailand	Baht	THB	27.42, 24.50	27.78, 24.48
Great Britain (UK)	Pound (£)	GBP	0.6176, 0.5572	0.6187, 0.5565
USA	Dollar ($)	USD	0.7708, 0.6939	0.7724, 0.6932

1 **a** What is the mid-rate for the Papua New Guinea Kina?

b What is AUD 900 worth in GPK?

c What is GPK 300 worth in AUD?

Example 12

2 **a** What is the mid-rate for the Fijian dollar?

b What is AUD 5000 worth in FJD?

c What is FJD 800 worth in AUD?

3 The exchange rate for transferring money to or from the Philippines is 38.700, 34.740.

a How much would be deposited in a Philippines account when you sent AUD 2000?

b How much would a Filipina have to send to buy an item from Sydney costing AUD 509?

Example 13

4 The exchange rate for a bank transfer to or from France is 0.7392, 0.6647.

a What would you have to send to Bordeaux for a bottle of wine costing EUR 68, including postage?

b What would a French company have to send to Australia for a vat costing AUD 3500?

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Example 14

5 The exchange rate for cash with the Indian rupee is 55.8820, 43.9590.

a How much would it cost you for INR 5000 to take with you to Mumbai?

b How much would you get for INR 1250 when you came back?

6 The exchange rate for cash with the Pound is 0.6187, 0.5565.

a How much would an English backpacker get for GBP 500 when they arrived in Australia?

b How much would you get in English pounds for AUD 1200 before going to Edinburgh for the comedy festival?

Problem solving

7 Angelique was changing 400 AUD to Fiji dollars. How much would she get?

8 When Yoshi came back from Japan he had 40 000 yen left. How much would he get for it in AUD?

9 How much would 500 USD be worth in Japanese yen (using the transfer rate)?

10 Alin was on exchange to Australia, but was paid in USD through electronic funds transfer. His salary was 1200 USD a fortnight after tax. How much in AUD did he get when he drew this out at an automatic teller machine in Australia?

11 When Hannah was travelling overseas, she wanted to get some travellers' cheques in US dollars, some in euros and some in Thai baht. If she split 6000 AUD evenly between the currencies, how much of each did she get?

12 Cara thought that a sewing machine advertised in a British magazine was especially cheap at £800. What would she pay in Australian currency?

4.06 Shares and dividends

WS Shares and dividends

WS Share tables

WS Share graphs

A company can raise funds by selling some of the business to other people. The business is divided into **shares**. The original owner will keep some and float (sell) the others on the **stock exchange** for a price like $20 a share. The profits are divided between the people who have shares, usually twice a year. The amount paid for each share is called a **dividend**. As time goes on, some of the people with shares decide to sell them. Shares are traded on the stock exchange and they keep a record of the prices paid. The value the shares were first sold for is called the **face value** of the share. The values of the shares will go up and down depending on the profits and losses of the company.

Some people try to make money by buying shares when they are cheap and selling if they go up in value. This is called **speculating** and is essentially the same as gambling. You are not allowed to use special knowledge in speculating. People who use 'inside' knowledge to make money by trading shares can be put in prison or heavily fined for insider trading.

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- **Market price**: the price that shares are sold for
- **Face value** (**par value**): the original price of a share
- **Dividend**: the amount paid for each share from company earnings. The amount paid is decided by the directors of the company. Some earnings are usually kept.
- **Dividend percentage** $= \dfrac{\text{Dividend per share}}{\text{Face value of share}} \times 100\%$
- **Dividend yield** $= \dfrac{\text{Dividend per share}}{\text{Market value of share}} \times 100\%$
- **Price-to-earnings** (**PE**) ratio $= \dfrac{\text{Market value of share}}{\text{Earnings per share}}$
- **Earnings per share** (**EPS**) $= \dfrac{\text{Net company income for year}}{\text{Number of shares}}$

When you buy and sell shares you have to pay fees and taxes, so speculators have to make enough money to cover these as well to make a profit.

Earnings per share is based on the last four quarters. *Estimated* EPS is based on projections for the next four quarters.

EXAMPLE 15

Alyssa bought 6900 shares in Symbio for \$5.75 per share a few weeks before the dividends were paid. 74% of the company profits were distributed to shareholders to give a dividend of 38.5 cents per share. After the dividend, the share price went down to \$5.21.

a How much did Alyssa receive in dividends?

b What was the dividend yield?

c How much did Alyssa appear to lose when the shares went down?

d What is the new PE ratio?

e What does the PE ratio mean?

Solution

a Find the total she received.

Alyssa's total dividend $= 6900 \times \$0.385$

$= \$2656.50$

Write the answer.

Alyssa got dividends of \$2656.50.

b Use the formula.

Dividend yield $= \dfrac{0.385}{5.75} \times 100\%$

$\approx 6.70\%$

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c	Find the amount she paid.	Total paid = \$5.75 × 6900 = \$39 675
	Find the current value.	Current value = \$5.21 × 6900 = \$35 549
	Find the apparent loss.	Apparent loss = \$39 675 – \$35 549 = \$3726
d	Find the earnings for 1 share.	38.5 cents = 0.74 × earnings per share Earnings per share = $\frac{38.5\text{c}}{0.74}$ ≈ 52 cents per share
	Find the PE ratio on the current value.	PE ratio = $\frac{5.21}{0.52}$ ≈ 10
e	State the meaning.	It would take 10 years at the current earnings rate for the company to earn the value of all the shares.

Alyssa's loss of \$3726 is called a **paper loss** because she hasn't lost any actual money. If she sells the shares at the current price she will make an actual loss.

The PE ratio measures the current share price as a multiple of income. High PE ratios show that investors think a company's income will grow. A low PE ratio shows the opposite but could indicate a share is undervalued. Across all shares, a PE ratio of about 15 is normal. However, it varies between sectors so you should only compare PE ratios of companies in similar industries.

EXAMPLE 16

The PE ratios of four Australian companies are 17, 22, 14 and 8. What can you say about the values of these shares, based only on the PE ratios?

Solution

State the meanings of the PE ratios for the share values.	The companies would take 8, 14, 17 and 22 years to earn the current share value.
Express the PE ratios as interest rates.	PE of 8 = $\frac{1}{8}$ = 0.125 equivalent interest rate PE of 14 = $\frac{1}{14}$ ≈ 0.071 equivalent interest rate PE of 17 = $\frac{1}{17}$ ≈ 0.059 equivalent interest rate PE of 22 = $\frac{1}{22}$ ≈ 0.045 equivalent interest rate

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State investor sentiment.	Investors are pessimistic about the stock with the PE ratio of 8, a little pessimistic about the 14, a little optimistic about the 17 and quite optimistic about the prospects of the company with a PE ratio of 22.

The PE ratio is *one* measure of share value. Investment decisions depend on many factors.

Your school might be a participant in the Australian Stock Exchange Sharemarket Game. It is an educational tool that helps you learn about the stock market.

Exercise 4.06 Shares and dividends

Example 15

1 Christine bought 2000 $1.00 shares for $6.75 each. The directors of the company decided to pay 70% of the company profits to shareholders as a dividend. The dividend was 28 cents per share. Calculate:

a the dividend percentage
b the dividend yield
c the price–earnings ratio
d her total dividend

2 Miharu had 2500 50-cent shares in a company paying a dividend of 23 cents. The company returned 40% of its profits to shareholders as a dividend. The market price of the shares was $4.37. Calculate:

a the dividend percentage
b the dividend yield
c the price–earnings ratio
d Miharu's dividend

3 Angelo bought some $2 shares with a dividend yield of 8%. The company returned 55% of its profits to shareholders as a dividend. He paid $1550 for 200 shares. Calculate:

a the market price
b the dividend in cents
c the dividend percentage
d the price–earnings ratio
e his total dividend

4 Julia bought 5000 50-cent shares for $2.40 each. The directors of the company decided to pay 50% of the company profits to shareholders as a dividend. The dividend was 10 cents per share. Calculate:

a the dividend percentage
b the dividend yield
c the price–earnings ratio
d her total dividend

5 Iain had 700 $5 shares in a company paying a dividend of 60 cents. The company returned 80% of its profits to shareholders as a dividend. The share market price was $8.50. Calculate:

a the dividend percentage
b the dividend yield
c the price–earnings ratio
d his total dividend

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Example 16

6 The PE ratios of four Australian companies are 48, 13, 16 and 7. What can you say about the values of these shares, based only on the PE ratios?

Problem solving

7 Kayla had some money to invest in shares. She wanted to choose the best investment from the following:

A \$2 shares paying a 15% dividend at a market price of \$3.20

B \$1 shares paying a 20% dividend at a market price of \$2.50

C \$5 shares paying a 10% dividend at a market price of \$4.50

Calculate the dividend yield for each and decide which would be the best investment.

Shutterstock.com/Stephen Coburn

8 Jasmin had some money to invest in shares. She wanted to choose the best investment from the following:

A \$1 shares paying an 18% dividend at a market price of \$4.45

B 50-cent shares paying a 38% dividend at a market price of \$1.97

C \$2 shares paying a 12% dividend at a market price of \$5.62

Work out the dividend percentage for each and decide the best investment.

9 Evan has 2000 shares in an oil exploration company with a market value of \$3.40 each. The face value of the shares is \$2 and they are paying a dividend of 9%. Evan can get 5% interest from his credit union. Would he do better to keep the shares or sell them and put his money in the credit union?

10 In 2016, Luis bought shares in two mining companies. He bought 30 000 shares at 55 cents in West Minerals and 20 000 shares at 83 cents in Prospect. Neither company paid dividends in 2016, 2017 or 2018, when he decided to sell both lots of shares. West Minerals had fallen to 15 cents, but there was rumour Prospect had found copper and its price has risen to \$1.48. Find:

a the total amount paid for the shares.

b the total amount obtained from the sale.

c the profit or loss as a percentage of the purchase price.

d Luis paid brokerage of \$100 + 0.5% for each parcel of shares bought or sold. Recalculate the percentage profit or loss including brokerage.

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4. CHAPTER SUMMARY

Investing money crossword

Prices and interest

Prices, mark-ups, discounts, profit, loss and margins

- **Unit price** is the cost of 1 item, 1 kg, 1 litre, etc. An item of acceptable quality at the lowest unit price is the best value to a shopper.
- **Cost price** (or **buying price**) is the amount of money paid for an item by a shop.
- **Mark-up** is the amount added to the cost price of an item for sale.
- **Marked price** (or **selling price**) = cost price + mark-up
- A **profit** is made when items are sold for more than they cost.
- Profit = selling price – cost price
- A **loss** is made when goods are sold for less than what they cost.
- Loss = cost price – selling price
- A **discount** is an amount taken off the marked price.
- A **trade discount** is the discount of the **recommended retail price** (**RRP**) that is given to retailers by distributers or manufacturers so that they can make a profit when selling at the RRP.
- **Percentage** mark-ups, discounts, profits and losses are usually 'out of' the cost price
- In Australia, the **Goods and Services Tax** (**GST**) is added to the price of an item or service and paid to the government by the seller. It is set at 10% of the value and must be included in retail prices by shops.
- **Operating margin** $= \frac{\text{profit}}{\text{sales}} \times 100\%$, where profit = sales – costs and interest and taxes are not included in the costs. Also called the **gross profit margin**.
- **Net profit margin** $= \frac{\text{net profit}}{\text{sales}} \times 100\%$, where net profit = sales – costs and interest and taxes are included in the costs.
- **Operating expense ratio** $= \frac{\text{expenses}}{\text{sales}} \times 100\%$, where expenses include all costs except interest and taxes.
- **Net** (or nett) means with all costs.

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Interest

- The **term** is the time of a loan or investment.
- The **principal** is the amount borrowed or invested.
- The **interest rate** is the cost of a loan for each year of the term. It is written as a proportion of the principal as a decimal or percentage.
- **Simple interest** is the product of the principal, interest rate and term in years. Simple interest is also called **flat-rate** interest
- For simple interest, $I = Pin = \frac{Pnr}{100}$ and $A = P + I$, where I = total interest, P = principal, i = interest rate as a decimal, r = interest rate as a percentage, n = term in years, A = amount to be repaid.
- The interest rate for one year is sometimes called the **nominal interest rate**.
- **Compound interest** is worked out as if the money was regularly re-invested.
- The time between compound interest calculations is called the **rest period** (or **rest**).
- For compound interest with annual rests, $A = P(1 + i)^n = P\left(1+\frac{r}{100}\right)^n$ and $I = A - P$, where A = amount after n years, P = principal, i = interest rate, r = interest rate as a percentage and I = total interest
- For compound interest with non-annual rests, $A = P\left(1+\frac{i}{k}\right)^{kn} = P\left(1+\frac{r}{100k}\right)^{kn}$ and $I = A - P$, where A = amount after n years, P = principal, i = interest rate, r = interest rate as a percentage, k is the number of rests in a year and I = total interest

Inflation

- The **Consumer Price Index (CPI)** measures the cost of a standard 'basket' every quarter. The 'basket' used by the ABS is typical of the things Australians buy.
- **Inflation** occurs as the CPI increases and money loses value.
- **Deflation** occurs as the CPI decreases and money gains value.

Currency exchange

- The money of a country is called its **currency**.
- The **exchange rate** is stated from the viewpoint of the institution.
- The **buying rate** is the amount in AUD they buy other currencies for.
- The **selling rate** is the price they sell other currencies for in AUD.
- The **mid-rate** is the average of the buying rate and the selling rate.
- The **official rate** is an average mid-rate worked out by the Reserve Bank.
- The mid-rate/official rate is used to find the relative values of currency.

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Shares and dividends

- **Share**: part of the ownership of a company.
- **Stock market**: the institution where shares are bought and sold.
- **Market price**: the price that shares are sold for.
- **Face value** (**par value**): the original price of a share.
- **Dividend**: the amount paid for each share from company earnings. The amount paid is decided by the directors of the company. Some earnings are usually kept.
- **Dividend percentage** $= \dfrac{\text{Dividend per share}}{\text{Face value of share}} \times 100\%$
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- **Earnings per share** (**EPS**) $= \dfrac{\text{Net company income for year}}{\text{Number of shares}}$

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4. CHAPTER REVIEW

Prices and interest

Example 1

1 Adam is very health-conscious. Which is the better buy for him to put on his bread?

A Unsalted butter $4.00 for 500 g

B Organic spreadable butter $6.00 for 200 g

C Normal butter $1.85 for 250 g

D Reduced cholesterol margarine $7.99 for 500 g

E Olive oil spread $3.50 for 500 g

Example 2

2 Which is the best buy of Vegemite for a family?

A 560 g jar for $8.80

B 145 g tube for $4.46

C 380 g jar for $6.60

D 150 g jar for $3.58

E 220 g for $4.23

Example 3

3 A tyre fitter marks up tyres by 35%. What price do you pay for tyres that cost them $145 each?

Example 3

4 A furniture shop marks up beds by 110%. What do they pay for beds they mark at $968?

Example 4

5 A fish and chips shop marks up fish by 140%.

a For what price do they sell 150 g fillets of fish that cost them $24/kg?

b What price do they pay for fillets that cost the customer $7.20 each?

Example 4

6 **a** What is the GST included in the price of LED light bulbs that cost a supermarket $3.63 each?

b The supermarket works on a mark-up of 40% on hardware items. What price does the customer pay for the light bulbs?

c How much of the amount the customer pays for light-bulbs does the supermarket send to the ATO?

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7 A corner shop works on a profit margin of 42% on confectionery. They sell 500 g chocolate Easter eggs for $34.36 each. Example 5

a How much of this is GST?

b How much is their mark-up?

c What do they pay for the Easter eggs?

d How much of the GST do they send to the ATO?

e What is their percentage profit on confectionery?

8 What interest rate is being paid on an investment of $9000 that pays $54 a month interest? Example 6

9 Teresa got a 2-year loan at 27% flat-rate interest through the shop where she bought a $1200 surf-board. She had no deposit, so she also had to pay additional fees of $300 incorporated into the loan. What was the total cost of her surfboard and what monthly repayments did she have to make? Example 7

10 What do you need to invest at 9% interest to get a payment of $120 a fortnight? Example 8

11 Jo invests $5000 for 3 years at 12% p.a. compounding interest every 6 months. Make a table showing the amount and compound interest every 6 months. Example 9

12 How much does $4000 accumulate to after 3 years at 6.7% compound interest with monthly rests? Example 10

13 What does Harry need to invest at 8% compound interest with weekly rests to get $6000 after 5 years? Example 10

14 Celia's bank account has grown from $500 to $520 in 4 years at compound interest with annual calculations. What is the average interest rate? Example 10

15 A heater costs $95. What would you expect it to cost 15 years later at an average rate of inflation of 2.8%? Example 11

16 Which is correct? The CPI is calculated using: Example 11

A the prices of groceries

B the prices of a basket of groceries bought by average Australians

C the prices of a group of selected goods and services across Australia

D the average amount spent by Australians on rent and transport

E the changes in the official exchange rate of the Australian dollar

17 The CPI rose from 30.8 to 103.7 over a 20-year period. What was the average rate of inflation? Example 11

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Examples 12-14

18 The exchange rate of Australian dollars and Vietnamese dong were given by a cash money exchange as AUD 1 to VND 15 217, 17 863.

a How many Vietnamese dong would you get for AUD 800?

b How many Australian dollars would you get for VND 120 000?

c How much is AUD 1250 worth in Vietnamese currency?

Examples 12-14

19 Chloe's bank's exchange rate of Australian Dollars and Euro was AUD 1 to EUR 0.7242, 0.6814. What would Chloe's bank account be debited if she bought a German amplifier on the internet for EUR 428 including shipping and postage?

Example 15

20 Caitlin had 3000 shares in a small mining company with a share price of $5.63 and a face value of $10. The dividend for 2018 was 43 cents. The directors had distributed 80% of the company earnings for the year. Calculate:

a the dividend percentage

b the dividend yield

c the earnings per share

d the PE ratio

Example 16

21 **a** What can you say about companies with PE ratios of 9, 14, 19 and 32?

b Which would you invest in if you were looking for a possible bargain?

c Which would you invest in if you were looking for growth, but didn't want to take a high risk?

Problem solving

22 Staff at a clothes shop get a discount of 10% on clothes they buy from the shop. A shop-soiled dress is marked down by 20%. Julie works in the shop and buys the dress because she knows how to get the stain out. She gets her staff discount on top of the other discount. The shop works on a mark-up of 120% and paid $88 for the dress. What does Julie pay and what profit or loss does the shop make on the dress?

23 What percentage profit is the same as a profit margin of 60%?

24 Cassie bought a large-screen TV for $2300 on a no-interest for 2 years deal. She had to pay a minimum of $70 a month for the TV, which she did. After 2 years and 1 day, she thought she should have finished paying for it, but discovered that she still owed $2000. It was explained to her that since she didn't pay it off in 24 months, she was charged interest from the date of purchase. The $2000 was the difference between the amount she had paid and the cost with interest for 2 years. What is the interest rate?

25 Pedro invested $3000 at 7.5% interest compounded every quarter. How long would it take to reach at least $6600? Explain your reasoning.

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26 Emily put $1000 in an investment account paying 8% compound interest at the beginning of each year for 4 years. How much would she have in the account after 4 years?

27 The cash rate of exchange for the South African Rand and the Australian dollar was 1 AUD to ZAR 10.92, 9.52 at a money exchange booth. What was the percentage being charged by the exchange booth?

28 A company with 2700 million shares has a current share price of $22.80 for $5 shares. The directors decided to distribute 70% of the company earnings and paid a dividend of $1.85. Calculate the net company earnings.

Practice quiz

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SHAPE AND MEASUREMENT

5.

SURFACE AREA AND SIMILAR FIGURES

Humans have always measured things. You have already seen how to calculate the area of a range of plane shapes. Measurement of real objects can be difficult at times. When objects are very small or very large, representing them at their actual size is often not possible. People use scale drawings to overcome this issue. The drawing and the object being represented are mathematically similar. The scale used for the drawing represents the relationship between the measurements of the drawing and the measurements of the real object. In this chapter you will calculate surface area and use scales in a range of real-world applications.

5.01 Surface area
5.02 Curved surface areas
5.03 Similar figures
5.04 Similar triangles
5.05 Applications of similar triangles
5.06 Scale drawings
5.07 Areas and volumes of similar figures
Chapter summary
Chapter review

SYLLABUS SUBJECT MATTER

Mensuration

- calculate the surface areas of standard three-dimensional objects including spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations, such as the surface area of a cylindrical food container

Similar figures and scale factors

- review the conditions for similarity of two-dimensional figures including similar triangles
- use the scale factor for two similar figures to solve linear scaling problems
- obtain measurements from scale drawings, such as maps or building plans, to solve problems
- obtain a scale factor and use it to solve scaling problems involving the calculation of the areas of similar figures, e.g. shadow sticks, calculating height of trees, use of clinometer
- obtain a scale factor and use it to solve scaling problems involving the calculation of surface areas and volumes of similar solids

Prior learning

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TERMINOLOGY

apex
curved surface area (CSA)
edges
image
plane shape
scale factor
slant height
three-dimensional
vertex
cone
cylinder
face
net
prism
similar
sphere
two-dimensional
congruent
dilation
hemisphere
object
pyramid
similarity
surface area
vertices

5.01 Surface area of flat-sided shapes

Surface area of a prism

Solid shapes

Nets of solids

The **surface area** of a three-dimensional (3D) shape is the area of the faces of the shape.

You can use the **net** of a 3D shape to help calculate the surface area of the shape.

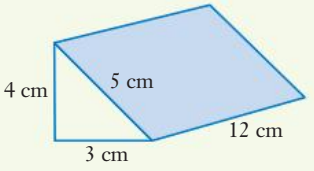
EXAMPLE 1

Find the surface area of a triangular prism of length 12 cm and base with sides 3, 4 and 5 cm.

Solution

Draw a sketch.

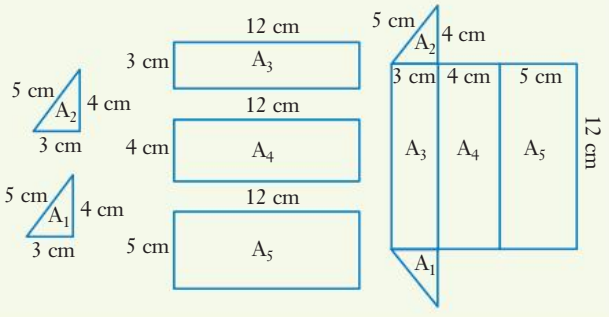
Label the sides.



Then draw each face flat (or draw flat as a net).

The triangles are right-angled (3–4–5).

Label the areas.



You put the faces together to make the net.

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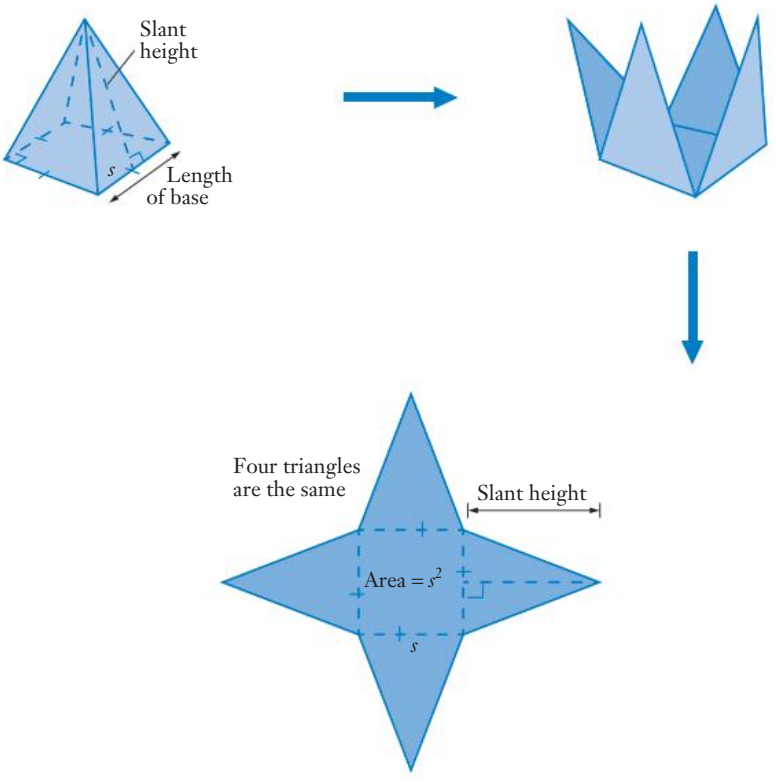
Calculate areas.	$A_1 = A_2 = \frac{1}{2}bh$ $= 0.5 \times 3 \times 4$ $= 6\ \text{cm}^2$ $A_3 = 12 \times 3$ $\quad$ $A_4 = 12 \times 4$ $= 36\ \text{cm}^2$ $\quad$ $= 48\ \text{cm}^2$ $A_5 = 12 \times 5$ $= 60\ \text{cm}^2$
Work out the total.	SA = 6 + 6 + 36 + 48 + 60 = 156
Write the answer.	The surface area is 156 cm^2.

A **pyramid** is a solid made up of a base and triangular faces that meet at an **apex**.
A **cone** is a solid with a circular base and an apex.

The height, h, of a pyramid is measured perpendicular to the base and the **slant height** is measured perpendicular from the edge of the base to the apex.

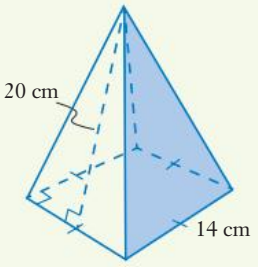
The sketch below shows how to draw the net of a pyramid.



Surface area of a pyramid

EXAMPLE 2

Find the surface area of this pyramid.



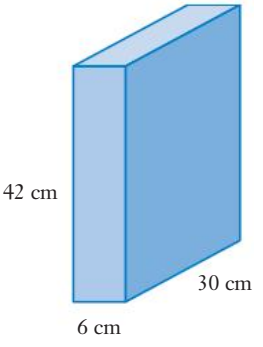
Solution

The base is a square.	Area of base $= 14 \times 14$
Evaluate.	$= 196 \text{ cm}^2$
Each side is a triangle.	Area of side $= \frac{1}{2}bh$
Substitute for b and h.	$= \frac{1}{2} \times 14 \times 20$
Evaluate.	$= 140 \text{ cm}^2$
Find the total.	$SA = 196 + 4 \times 140$
Evaluate.	$= 756 \text{ cm}^2$

Exercise 5.01 Surface area of flat-sided shapes

1 The surface area of this rectangular prism is closest to:

A 1250 cm^2
B 3400 cm^2
C 1700 cm^2
D 2400 cm^2
E 340 cm^2



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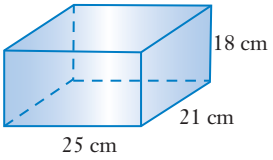
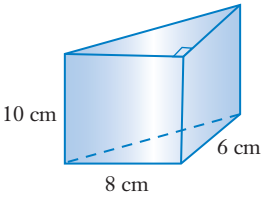
2 For each shape, draw a net including all necessary measurements and use it to calculate the surface area of the shape.

Example 1

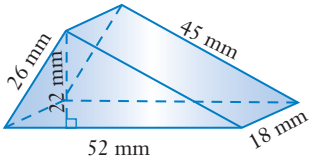
a



b

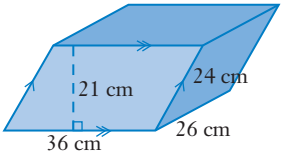


c

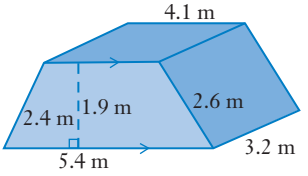


3 Calculate the surface area of each solid.

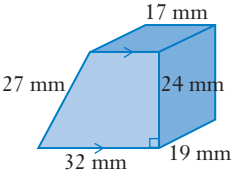
a



b



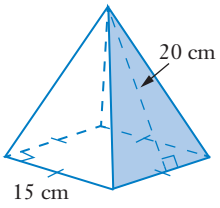
c



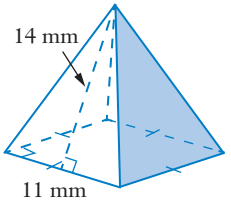
4 Calculate the surface area of each pyramid.

Example 2

a



b



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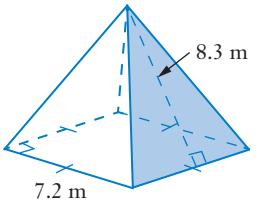
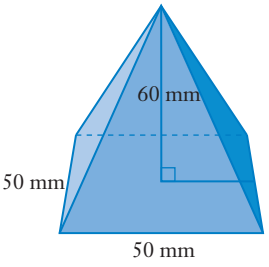
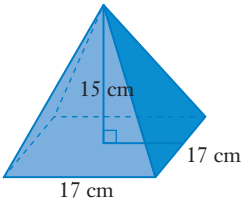


5 Calculate the surface area of each pyramid, using Pythagoras' theorem to find the slant height.

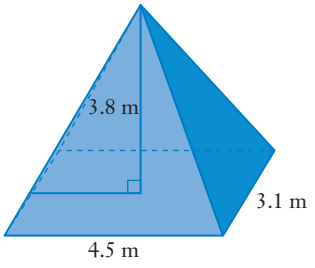
a



b



c



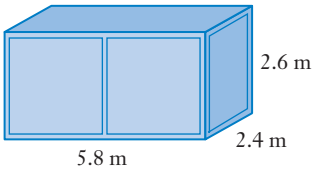
ISBN 9780170412711

Problem solving

6 This shipping container needs to be painted all over.

a Find the total area to be painted.

b Find the cost of painting the container if the paint comes only in 4 L tins that cost \$52, each litre of paint covers 8 m^2, and labour costs are \$7.50/m^2.

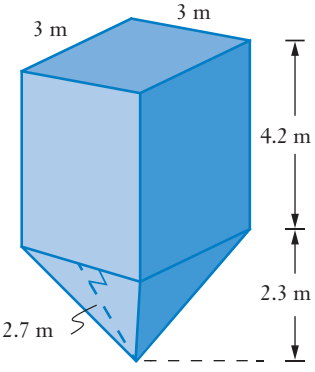


7 A cereal box is 8 cm wide, 25 cm long and 20 cm high. What area of cardboard is needed to make 5000 boxes, allowing 15% extra for wastage and joins?

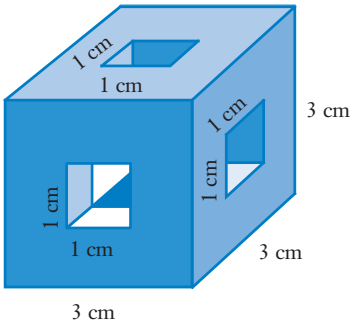
8 A tent with a floor is in the shape of a triangular prism 3 m long, 1.8 m high and 2.1 m wide. What area of cloth is needed to make the tent if no allowance is made for window and doorflaps and joins?

Shutterstock.com/Molarjung

9 This grain hopper stores wheat. The outside of the hopper has to be painted with two coats of paint. How much will the painting cost if a contractor will do the job for \$34/m^2?



10 This cubic block has holes that pass all the way through it. Each hole is 1 cm × 1 cm × 3 cm. Find the total surface area (internal and external) of the block.



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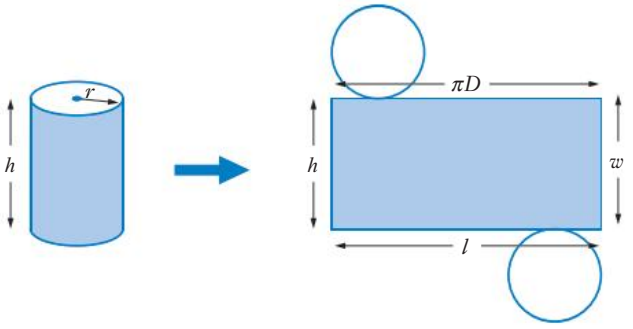
5.02 Curved surface areas

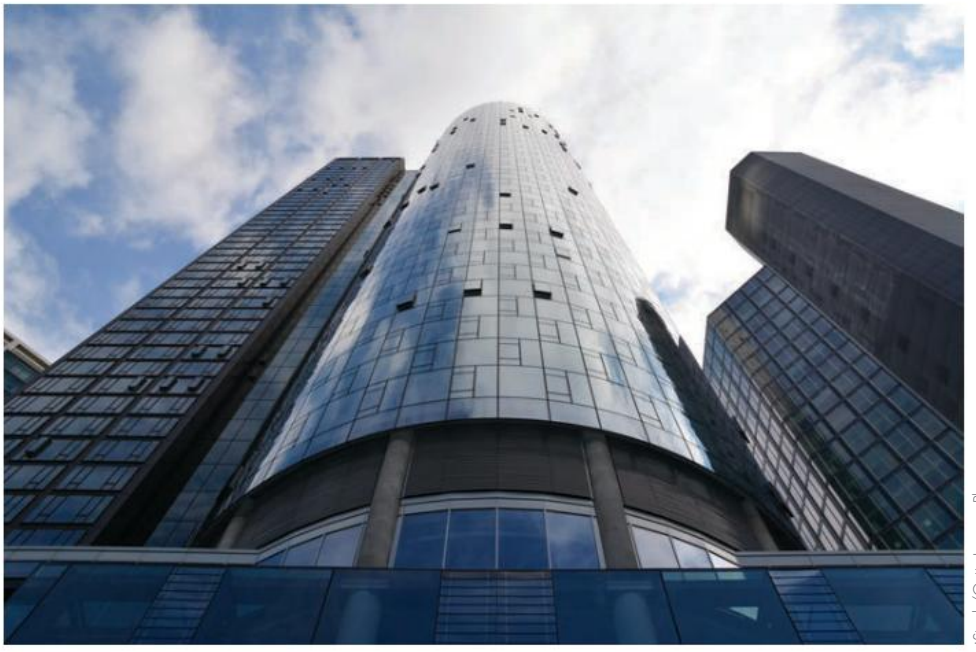
You might have to use Pythagoras' theorem to work out the slant height of a pyramid or cone.

The curved area of a **cylinder**, without the top and bottom, is called the **curved surface area** (**CSA**).

You can see that the curved surface area is calculated as follows:

$$\begin{aligned} \text{CSA} &= l \times w \\ &= \pi D \times h \\ &= \pi \times 2r \times h \\ &= 2\pi rh \end{aligned}$$





iStock/Getty Images Plus

Surface area riddle

A page of solid shapes

Surface area of a cylinder

Surface area of solids

Surface area

Formula matching game

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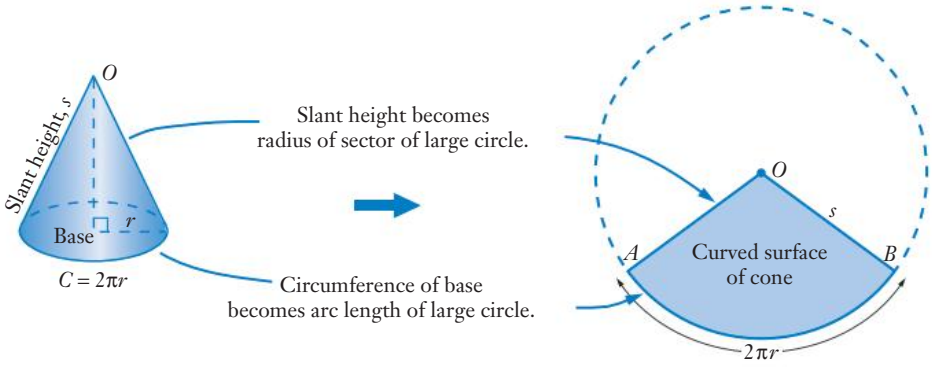
The net of a cone consists of the curved surface and the circular base as shown here. The base is a circle with radius r. The curved surface is the sector of a circle with radius, s, and arc length AB. The length of AB must be the same as the circumference of the base, i.e. $2\pi r$.



$$\frac{\text{Area of sector } AOB}{\text{Area of large circle}} = \frac{\text{arc length } AB}{\text{circumference of circle}}$$

$$\frac{\text{Area of sector } AOB}{\pi s^2} = \frac{2\pi r}{2\pi s} = \frac{r}{s}$$

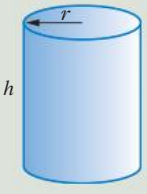
$$\text{Area of sector } AOB = \frac{r}{s} \times \pi s^2$$
$$= \pi r s$$

You have to add the areas of the circles of closed cones or cylinders.

The surface area of a sphere is all curved.

Curved surface area

Cylinder



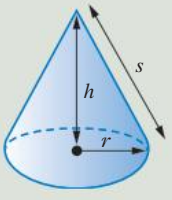
CSA $= 2\pi rh$

Top $= \pi r^2$

Bottom $= \pi r^2$

Total $SA = 2\pi r^2 + 2\pi rh$

Cone



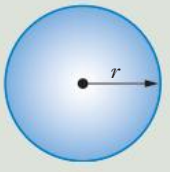
CSA $= \pi rs$

where $s = \sqrt{r^2 + h^2}$

Bottom $= \pi r^2$

Total $SA = \pi r^2 + \pi rs$

Sphere



CSA $= 4\pi r^2$

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EXAMPLE 3

Surface area of a cone and sphere

Find the surface area of:

a a cylinder with no top, a diameter of 16 cm and a height of 12 cm

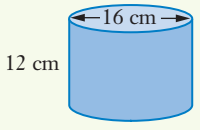
b a closed cone with a radius of 7.8 cm and a height of 10.4 cm

c a sphere with a radius of 17 cm

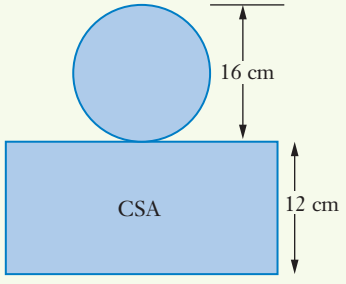
d a closed hemisphere with a diameter of 42 cm

Solution

a Draw a sketch.



Draw a net.



Work out side area (use $r = 8$ cm). $\text{CSA} = 2\pi rh$

$= 2 \times \pi \times 8 \times 12$

Keep on calculator. $= 603.1857\ldots \text{ cm}^2$

Work out area of base. $\text{Area of bottom} = \pi r^2$

$= \pi \times 8^2$

$= 201.0619\ldots \text{ cm}^2$

Work out the total. $\text{Total area} = 603.1857\ldots + 201.0619\ldots$

Use the values on the calculator. $= 804.2477\ldots$

Round off and write the answer. The surface area is about 804 cm^2.

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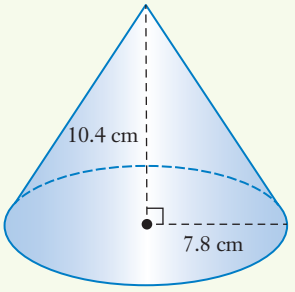
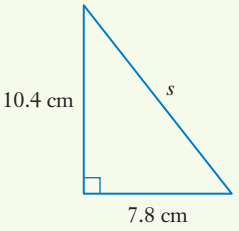
b Draw a sketch.



Make a sketch of the triangle formed by the height of the cone and the slant height.

s = slant height of the cone



Calculate the slant height. $s^2 = 10.4^2 + 7.8^2$

Evaluate. $= 169$

Take square roots of both sides. $s = 13$ cm

Use the rule for the area of the cone. SA of cone $= \pi r^2 + \pi rs$

Substitute for known values. $= \pi \times 7.8^2 + \pi \times 7.8 \times 13$

Evaluate. $= 509.691\ldots$

Round off and state the result. The surface area is about 510 cm^2.

c Write the rule for the SA of a sphere. SA $= 4\pi r^2$

Substitute for r. $= 4 \times \pi \times 17^2$

Evaluate. $= 3631.68\ldots$

Round off. ≈ 3632 cm^2

d Find the half sphere + circle. SA $= 2\pi r^2 + \pi r^2$

Use half the diameter for r. $= 3 \times \pi \times 21^2$

Evaluate. $= 4156.32\ldots$

Round off. ≈ 4156 cm^2

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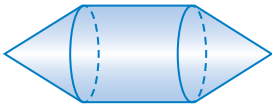
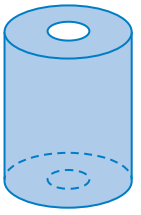
Exercise 5.02 Curved surface areas

1 How many surfaces does this solid have?



A 2
B 3
C 4
D 5
E 6

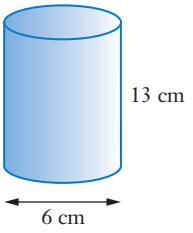
2 How many surfaces does this solid have?



A 2
B 3
C 4
D 5
E 6

3 The surface area of this closed cylinder is closest to:

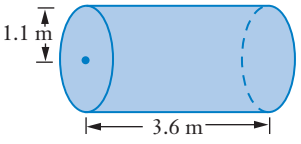
Example 3



A 245 cm^2
B 57 cm^2
C 150 cm^2
D 300 cm^2
E 350 cm^2

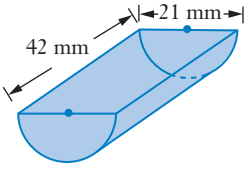
4 For each shape, draw a net including all necessary measurements and use it to calculate the surface area of the shape.

a



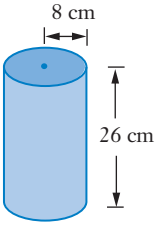
Closed cylinder

b



Open half-cylinder

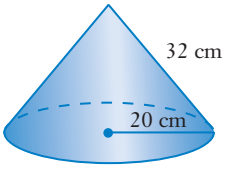
c



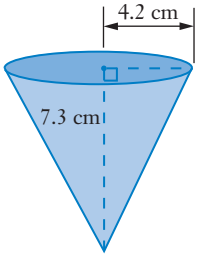
Cylinder (open both ends)

5 Find the surface area of each cone.

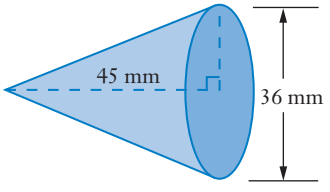
a



b



c



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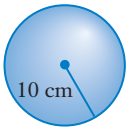
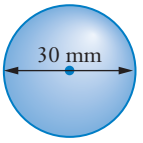
6 Find the surface area of each of each sphere.

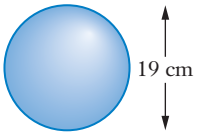
a



b

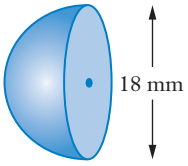


c



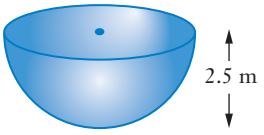
7 Find the surface area of each hemisphere.

a



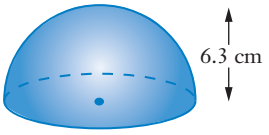
Closed

b



Open

c



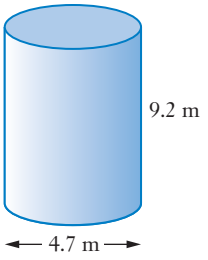
Closed

Problem solving

8 This silo is constructed from an aluminium alloy. It has a top and a base. The material costs \$8.50/m^2 and 5% needs to be allowed for wastage. Labour costs for the construction of the silo are \$45.50/h.

a How much metal is needed?

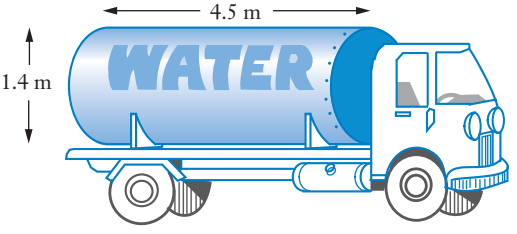
b What is the total material and labour cost of constructing the silo if it takes 20 hours?



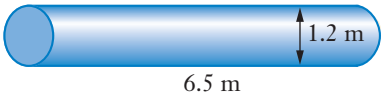
9 The water tank on this truck needs repainting.

a Find the area to be painted (including the ends).

b Find the number of litres of paint needed if the coverage is 9 m^2/L.



10 A rectangular sheet of steel has to be bent and welded to form this pipe.



a Find the dimensions of the smallest sheet of steel that could be used.

b Find the cost of constructing the pipe if 1.35 m^2 must be allowed for overlap and materials and labour cost \$75/m^2.

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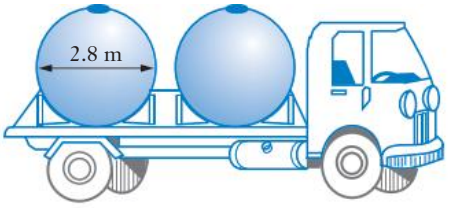
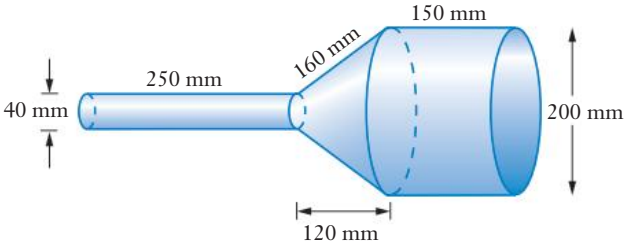
11 This truck carries liquid oxygen in two identical spherical containers. Find the total surface area of the spheres.



12 This water funnel is to be constructed of copper sheeting. What area of sheeting will be required, if 5% must be allowed for seams and overlap?



5.03 Similar figures

When an object is dilated, the image is the same shape but a different size.

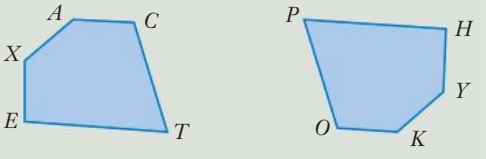
Finding sides in similar triangles

Finding an unknown side in similar figures

Congruent figures are exact copies of each other.

The figures are exactly the same shape and size.

The symbol for congruency is ≡.

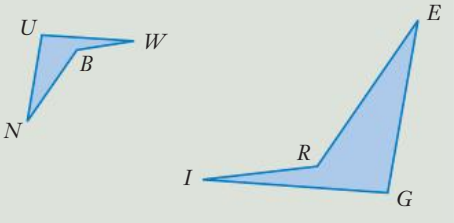


ACTEX ≡ *KOPHY*

Similar figures are exactly the same shape but not necessarily the same size.

The symbol for similarity is |||.

When naming similar or congruent figures, corresponding vertices are named in the same order (as above).



UWBN ||| *GIRE*

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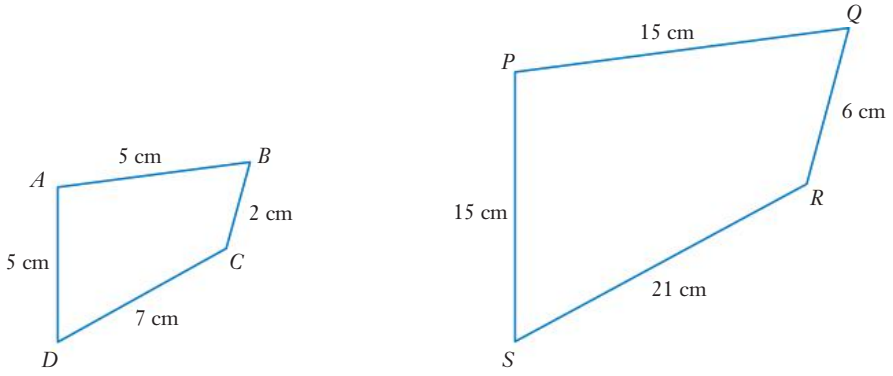
If the quadrilaterals below are similar and $ABCD \parallel\!| PQRS$, then corresponding angles are equal, so $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and $\angle D = \angle S$.



The sides opposite corresponding angles are called corresponding sides.

$$\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RS}{CD} = \frac{SP}{DA} = 3$$

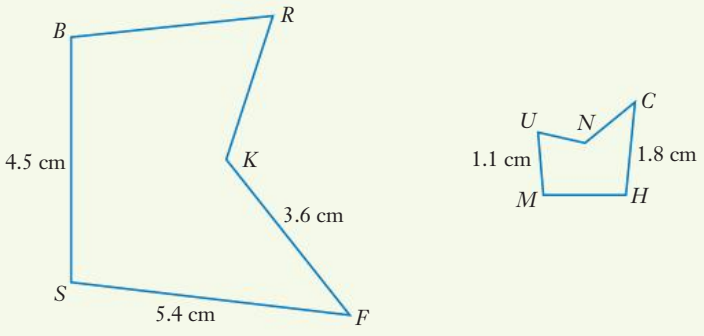
The ratios of corresponding sides are equal. This ratio is the **scale factor**.

Properties of similar figures

- Corresponding angles are equal
- Ratios of corresponding sides are equal
- Scale factor = ratio of corresponding sides = $\dfrac{\text{image length}}{\text{object length}}$

EXAMPLE 4

a Write a statement of similarity for these similar figures.



b Calculate the scale factor.

c Calculate the values of BR and NC.

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Solution

a Identify corresponding vertices. $B \leftrightarrow M$, $R \leftrightarrow U$, $K \leftrightarrow N$, $F \leftrightarrow C$ and $S \leftrightarrow H$.

Write the statement of similarity. $BRKFS \parallel\!| MUNCH$

b Identify corresponding sides where lengths are known. $SF \leftrightarrow HC$

Use the rule for scale factor. $\text{Scale factor} = \dfrac{SF}{HC}$

Substitute $= \dfrac{1.8}{5.4}$

$= \dfrac{1}{3}$

State the result. The scale factor is $\frac{1}{3}$.

c Write the scale factor rule for BR. $\dfrac{MU}{BR} = \dfrac{1}{3}$

Solve for BR. $BR = 3 \times MU$

$= 3 \times 1.1$

$= 3.3$

Write the scale factor rule for NC. $\dfrac{NC}{KF} = \dfrac{1}{3}$

Solve for NC. $NC = \dfrac{1}{3} \times KF$

$= \dfrac{1}{3} \times 3.6$

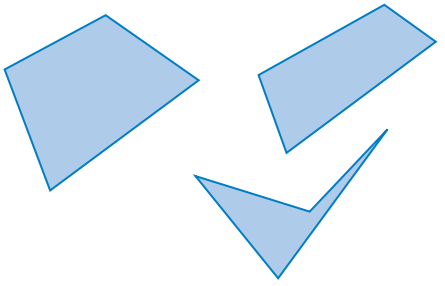
$= 1.2$

Write the answer. $BR = 3.3$ cm and $NC = 1.2$ cm

INVESTIGATION

ANGLES AND SIDES

Of these three quadrilaterals, two have the same sides and two have the same angles, but none are congruent or similar.



Draw each of the following pairs so that they are neither congruent nor similar, if you can.

1 Two convex quadrilaterals with the same sides in the same order. (A convex quadrilateral has no inward pointing corners.)

2 Two pentagons with the same sides in the same order.

3 Two pentagons with the same angles in the same order.

4 Two triangles with the same sides.

5 Two triangles with the same angles.

Discuss your findings as a class group. Are there any general rules?

Exercise 5.03 Similar shapes

1 These two shapes are similar.
The statement of similarity is:

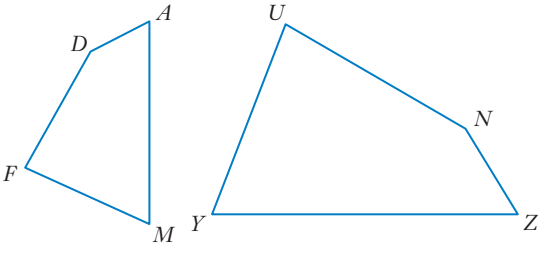
A *FDAM* ||| *YZNU*

B *DAMF* ||| *NZYU*

C *MDFA* ||| *NZYU*

D *DAMF* ||| *YNUZ*

E *MFAD* ||| *YNUZ*



2 These two shapes are similar. The statement of similarity is:

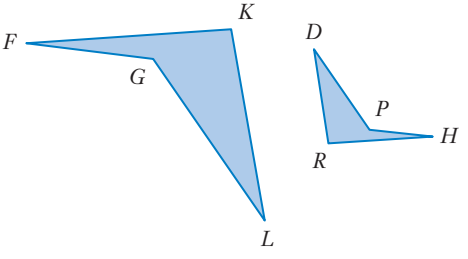
A *GFKL* ||| *RHDP*

B *KLFG* ||| *RDPH*

C *GLKF* ||| *PHRD*

D *FKLG* ||| *HRDP*

E *FKGL* ||| *RHDP*



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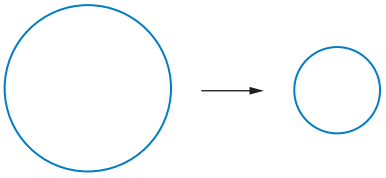
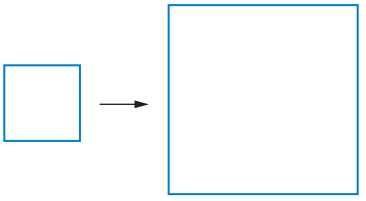
3 Each pair of shapes below are similar. Use a ruler to find the scale factor.

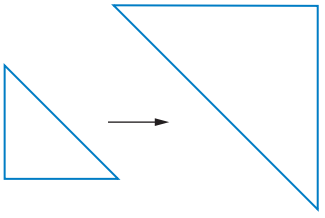
a



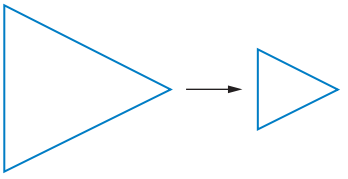
b



c



d



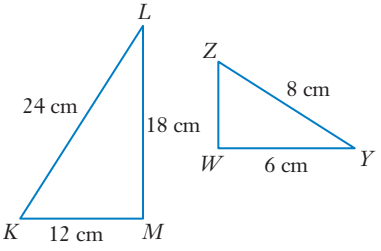
4 Each pair of shapes is similar. For each pair:

Example 4

i write a statement of similarity

ii calculate the scale factor.

a



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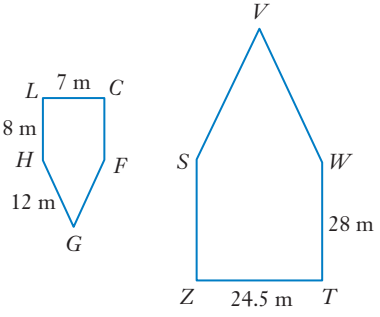
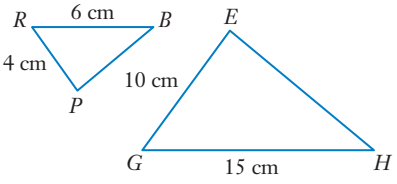


c



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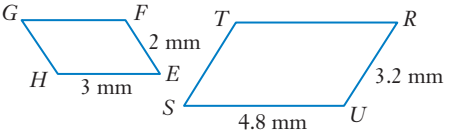

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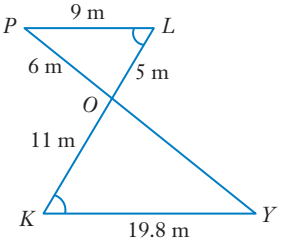

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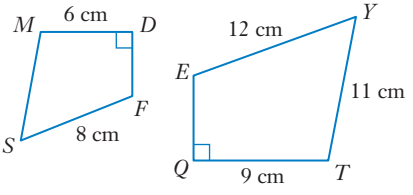

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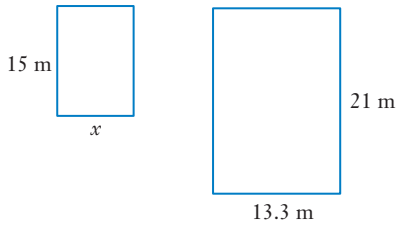
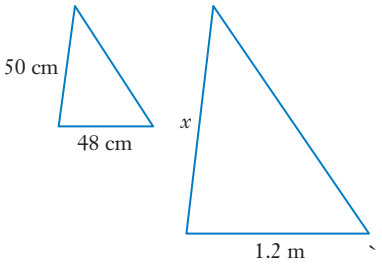
5 If each pair of shapes are similar, calculate the value of x.

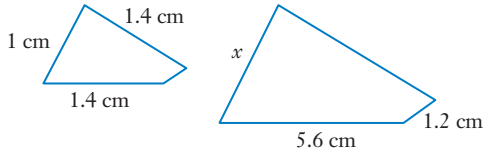
a



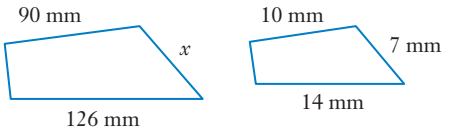
b



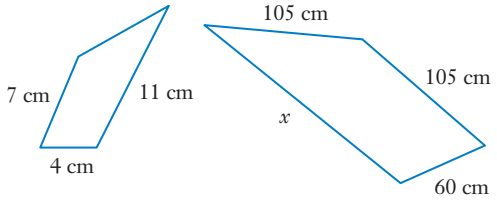
c



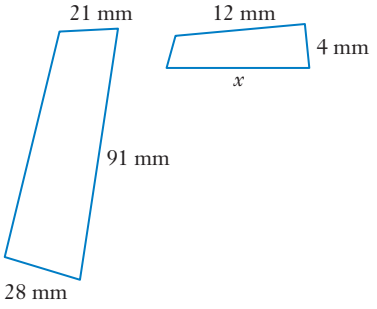
d



e



f



Problem solving

6 Is each statement true (T) or false (F)?

a All squares are similar.

b Corresponding sides of similar figures are in the same ratio.

c All rhombuses are similar.

d All circles are similar.

e Corresponding angles of similar figures are equal.

f All parallelograms are similar.

g Any two isosceles triangles are always equal.

h All equilateral triangles are similar.

i All hexagons are similar.

j Since the angles of two rectangles are equal, they must be similar.

k All right-angled triangles are similar.

l All regular octagons are similar.

m If two triangles have two pairs of corresponding angles equal, they must be similar.

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7 A photograph measures 105 mm wide by 60 mm high. An enlargement has a height of 20 cm. What is the width?

8 **a** Explain why two parallelograms with corresponding angles that are equal are not necessarily similar.

b Draw a pair of parallelograms with equal corresponding angles that are similar.

c Draw a pair of parallelograms with equal corresponding angles that are not similar.

5.04 Similar triangles

Finding sides in similar triangles

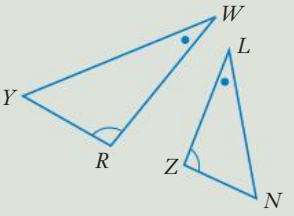
Tests for similar triangles

Angle/angle (AA)

Two angles of the triangles are the same size.

Two pairs of angles equal makes the third pair equal as well.

So $\triangle YWR \equiv \triangle NLZ$ by AAA.

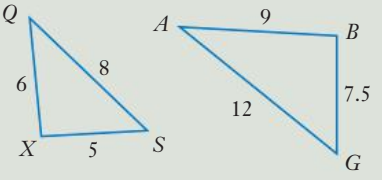


Side/side/side (SSS)

All corresponding sides of the triangles are in the same ratio.

$$\frac{XS}{BG} = \frac{SQ}{GA} = \frac{QX}{AB} = 1.5$$

So we say $\triangle QSX \equiv \triangle AGB$ by SSS.

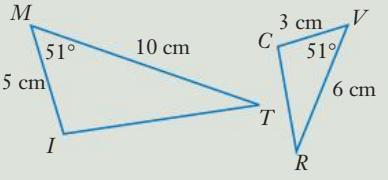


Side/angle/side (SAS)

Two corresponding sides of the triangles are in the same ratio and the enclosed angles are equal.

$$\frac{CV}{IM} = \frac{VR}{MT} = 0.6 \text{ and } \angle M = \angle V$$

So $\triangle QSX \equiv \triangle AGB$ by SAS.

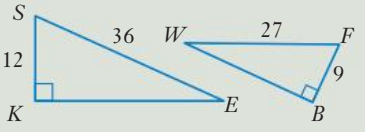


Right angle/hypotenuse/side (RHS)

The hypotenuses and one pair of corresponding sides are in the same ratio in two right-angled triangles.

$$\frac{FW}{SE} = \frac{BF}{KS} = 0.75 \text{ and } \angle K = \angle B = 90°$$

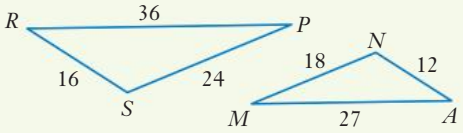
So $\triangle SKE \equiv \triangle FBW$ by RHS.



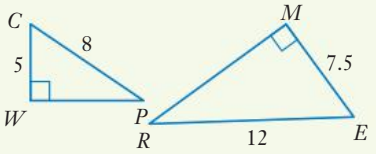
EXAMPLE 5

Determine if each pair of triangles are similar.

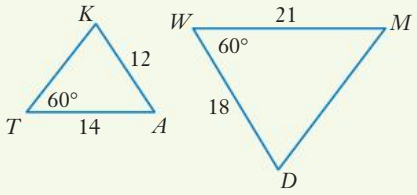
a



b



c



Solution

a Three sides are known, so the SSS test can be used.

Find ratio of corresponding sides.

$$\frac{MA}{PR} = \frac{27}{36} = \frac{3}{4}$$

$$\frac{AN}{RS} = \frac{12}{16} = \frac{3}{4}$$

$$\frac{MN}{PS} = \frac{18}{24} = \frac{3}{4}$$

The ratios of corresponding sides are equal. State the result.

$\triangle RPS \parallel\!| \triangle AMN$ (SSS).

b The RHS test could be used.

$\angle M = \angle W = 90°$

Find the ratio of the hypotenuses.

$$\frac{RE}{PC} = \frac{12}{8} = \frac{3}{2}$$

Find the ratio of the corresponding sides.

$$\frac{ME}{WC} = \frac{7.5}{5} = \frac{3}{2}$$

The hypotenuses and a corresponding side are in the same ratio and the triangles are right-angled.

$\triangle CPW \parallel\!| \triangle ERM$ (SSS).

c Two sides and an angle in each triangle are known so the SAS test could be used. No other test for similar triangles can be used.

$\angle T = \angle W$, but $\angle T$ is not enclosed between the two known sides. So $\angle T$ and $\angle W$ are not in corresponding positions.

State the result.

The two triangles are not necessarily similar as they do not satisfy any test for similar triangles.

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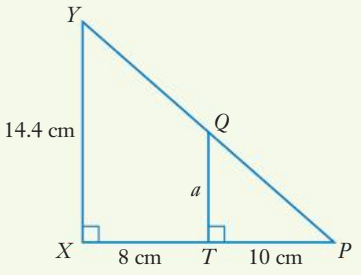
EXAMPLE 6

Similar triangles

Calculate the value of a in the diagram.



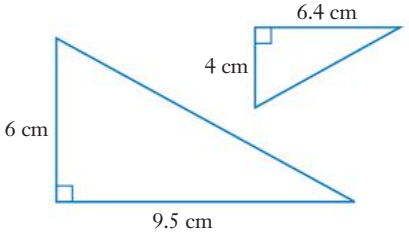
Solution

Identify the triangles.	The triangles are $\triangle YPX$ and $\triangle QPT$.
The RHS rule cannot be used. Use the AAA rule.	$\angle X = \angle T = 90°$
	$\angle P$ is common to both triangles.
	So $\angle Y = \angle Q$ (angle sums)
State the result.	$\triangle YPX \parallel\!\!\mid \triangle QPT$ (AAA).
Since the triangles are similar, the ratio of corresponding sides is equal.	$\frac{QT}{YX} = \frac{PT}{PX}$
Substitute for known values.	$\frac{a}{14.4} = \frac{10}{10+8}$
Evaluate.	$a = \frac{10 \times 14.4}{18}$ $= 8$

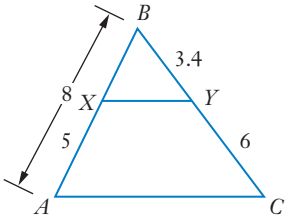
Exercise 5.04 Similar triangles

1 Which triangle similarity test should be used to show that these triangles are similar?

A AAA
B RHS
C SAS
D SSS
E ASA



2 Which triangle similarity test should be used to show that $\triangle XYB$ and $\triangle BAC$ are similar?



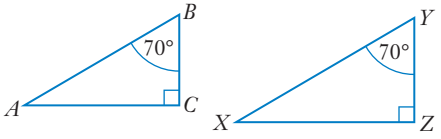
A ASA
B SSS
C RHS
D AAA
E SAS

3 **a** Draw $\triangle DAF$ with $\angle D = 70°$, $\angle F = 50°$ and $DF = 60$ mm.
b Now draw $\triangle KUT$ with $\angle K = 70°$, $\angle T = 50°$ and $KT = 90$ mm.
c Is $\triangle DAF$ similar to $\triangle KUT$? Why?

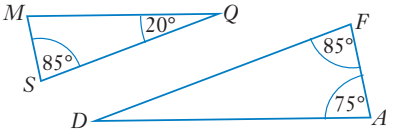
Example 5

4 Test each pair of triangles for similarity. If they are similar, write a statement of similarity and state the test used.

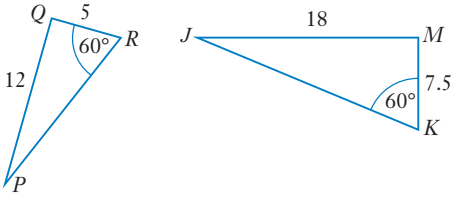
a



b



c



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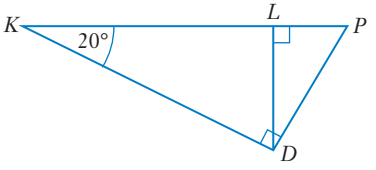
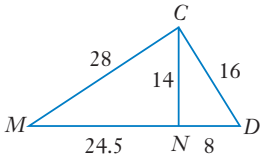
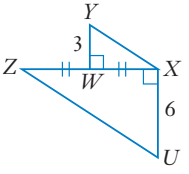


e



f



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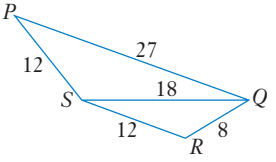
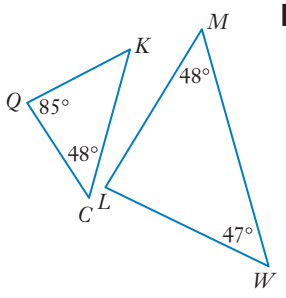
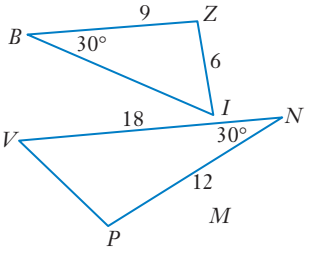


5 Which pair of triangles are similar? If they are similar, write a statement of similarity and state the test used.

a



b



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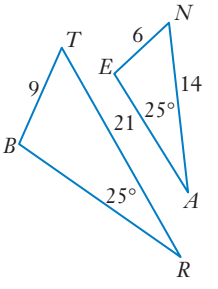

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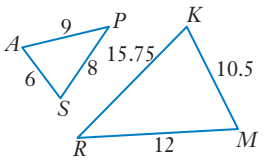

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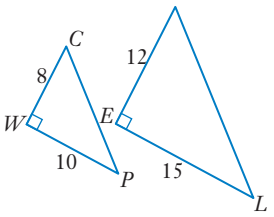

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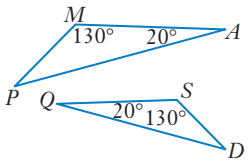

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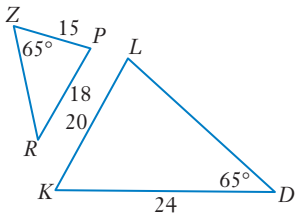

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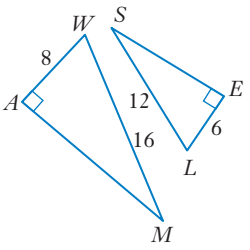

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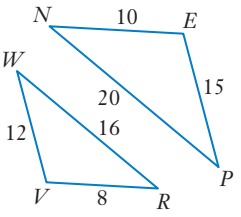
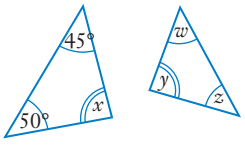


6 The following pairs of triangles are similar. Find the value of the unknowns in each case.

Example 6

a



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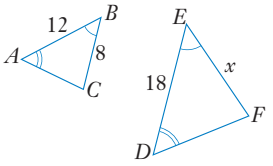

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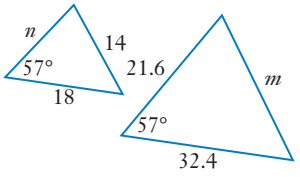

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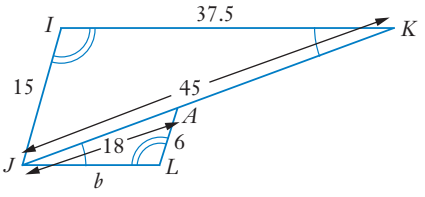

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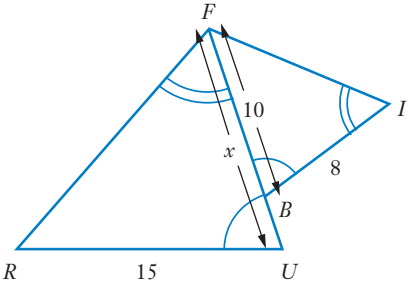

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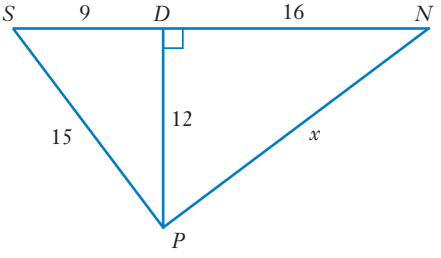

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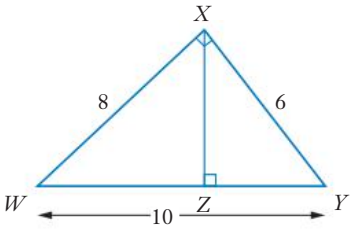
Problem solving

7 A right-angled triangle has sides that measure 14 cm, 48 cm and 50 cm. A similar triangle has a hypotenuse that measures 20 cm. Find the length of the shortest side.

8 Show that triangles *WXZ*, *WXY* and *XYZ* are all similar.

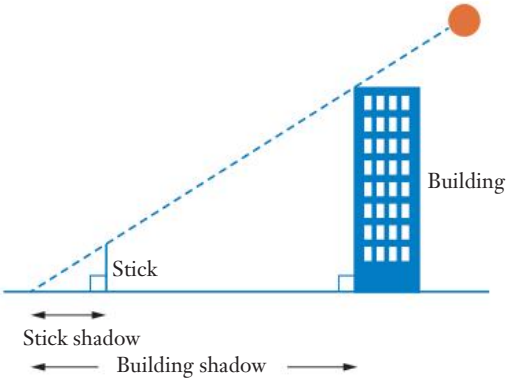


Applying similar triangles

5.05 Applications of similar triangles

You can use similar triangles of shadows cast by sticks to calculate the height of a tree or building. In 240 BCE, the Greek astronomer Eratosthenes used shadows to measure the size of Earth.

The shadows must be measured at the same time to make the triangles similar.



EXAMPLE 7

A stick of height 1.5 m casts a shadow of length 3.8 m. At the same time, a tree casts a shadow 31 m long. What is the height of the tree?

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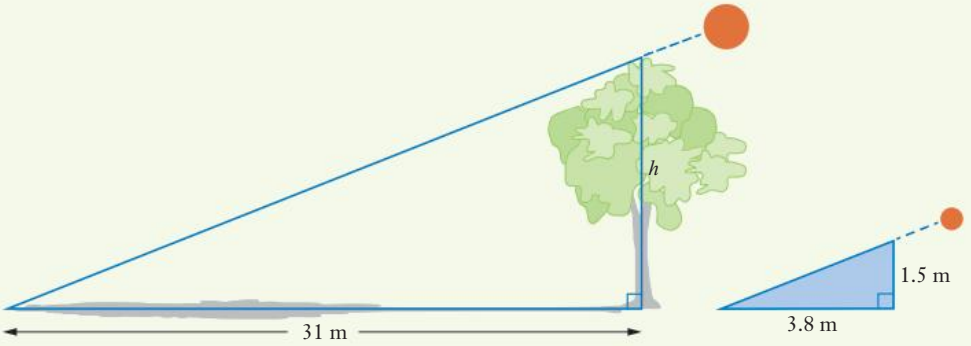
Solution

Draw a diagram to show the information.



The triangles are similar.	The ratio of corresponding sides is equal.
Identify corresponding sides.	$\frac{h}{1.5} = \frac{31}{3.8}$
Solve for h.	$h = \frac{31}{3.8} \times 1.5$
Evaluate.	$= 12.2368\ldots$
Round and write the answer.	The tree is about 12.2 m high.

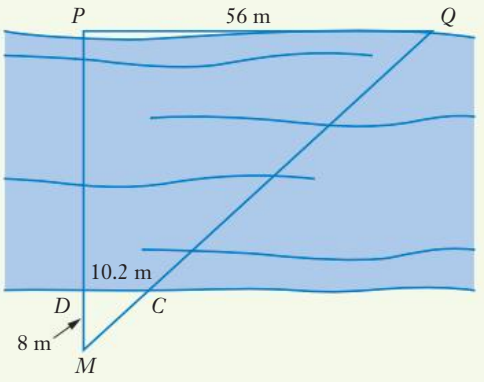
Similar triangles can also be used in surveying to find distances that cannot be measured exactly. This is called **triangulation**.

EXAMPLE 8

A surveyor makes a sketch of a river at the site where a bridge is to be constructed. The width of a river needs to be known. A surveyor places two pegs (P and Q) 56 m apart on one side of the river as shown.

Another peg is placed perpendicular to P on the other side of the river at M.

A line of sight is taken from M to Q and a peg is placed at C. A final peg is placed at D so that $DC \parallel PQ$.



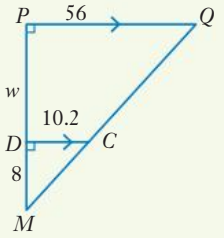
DC and DM are measured to be 10.2 m and 8 m respectively.

Calculate the width of the river.

Solution

Draw a diagram using the information provided.

Let the width of the river be w.



Use the AAA test. $\triangle MDC \parallel\!| \triangle MPQ$ (AAA)

The ratio of corresponding sides is equal. $\dfrac{PM}{DM} = \dfrac{QM}{CM}$

Substitute for known values. $\dfrac{w+8}{8} = \dfrac{56}{10.2}$

Rearrange. $w + 8 = \dfrac{56}{10.2} \times 8$

Evaluate. $w = 43.921\ldots$

Round off and state the result. The river is about 43.9 m wide.

INVESTIGATION

SHADOW STICKS

A metre ruler makes an excellent shadow stick because the calculations are simplified by having the stick exactly 1 m high. Use a metre ruler and a long tape in groups of 2 or 3 to measure the heights of buildings and trees around the school.

You will need to make sure the metre ruler is vertical when you are measuring its shadow. This can be done using a spirit level. If you don't have a spirit level, you can check that the stick is vertical by line of sight against something known to be vertical, such as the corner of a building or a pole.

Check your measurements by comparing the results of your group with the results of other groups.

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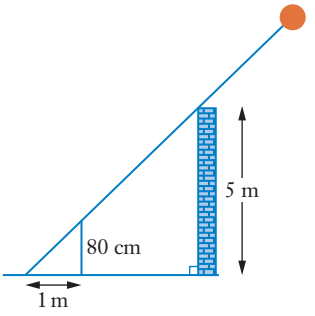
Exercise 5.05 Applications of similar triangles

Example 7

1 An 80 cm long stick is placed vertically so that the end of its shadow coincides with the shadow cast by a wall 5 m high. The base of the stick is 1 m away from the end of its shadow.

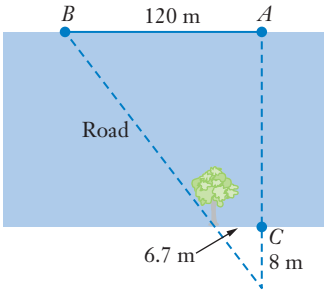
Calculate the length of the shadow of the wall.



Example 8

2 The distance across a road cutting is to be calculated using triangulation. Sightings are taken of two points A and B on the opposite side of the road that are known to be 120 m apart.

The sides of the road cutting are parallel, and point C on the surveyor's side is directly opposite point A. From a point 8 m away from the road edge opposite C, the surveyor notices that a small tree on the edge of the road just obscures point B. The tree is found to be 6.7 m from C measured parallel to the road. What is the width of the road cutting?



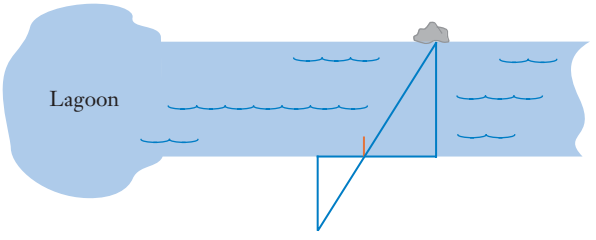
Problem solving

3 A group of students used a straight stick 1.4 m long as a shadow stick to help calculate the height of a tree on the edge of the school oval. The shadow of the tree was 44.1 m long, and at the same time, the shadow of the stick was 1.8 m long. How high is the tree?

4 The facade of a historic building was preserved when the building was demolished for a new shopping centre. In the morning, the sun shone through the facade, casting a shadow across the street. There were bright patches in the shadow from the windows of the facade. If the whole shadow was 25 m long and the bright patch from a 1.8 m-high window was 2.4 m long, how high was the facade?

5 Water is rushing out the long, narrow entrance to a lagoon at low tide. There is a large rock on the opposite bank of the entrance. A fisherman is directly opposite the rock. He walks 4 m along the bank of the entrance and puts a stick in the sand. After walking a further 2 m along the bank, he then has to move 4.5 m away from the bank in order to line up the stick and rock.

How wide is the lagoon entrance?



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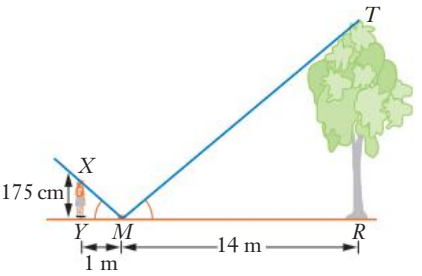
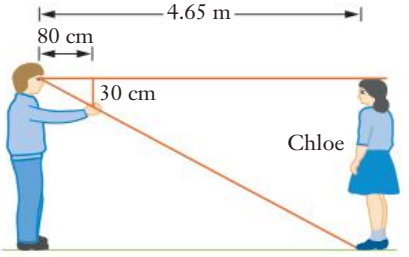
6 Madison measures the height of a tree (TR) by placing a mirror (M) on level ground and moving backwards in line with the tree until she can see the top of the tree in the mirror. Madison is standing at Y and her eyes are 175 cm above the ground. The mirror is 1 m from her and 14 m away from the tree. How high is the tree?



7 Ryan is holding a 30 cm ruler 80 cm from his eyes as shown. He finds that when Chloe is standing 4.65 m away from him, the top of her head lines up with the top of the ruler and her feet line up with the bottom. How tall is Chloe?



5.06 Scale drawings

Scales and scale diagrams

Interpreting an office plan

A **scale drawing** is a diagram that represents a real object with a similar figure. Building plans and maps are examples of scale drawings.

Scale

Scale = drawing length : real length

EXAMPLE 9

a Change a scale of 1 cm : 200 m to a ratio.

b Complete the following: 1 : 500 is the same as 1 cm : … m.

Solution

a Make the units the same using 1 m = 100 cm. 1 cm : 200 m = 1 cm : 200 × 100 cm

= 1 cm : 20 000 cm

Remove the units. = 1 : 20 000

b The real object is 500 times bigger. Real size for 1 cm = 1 × 500 cm

= 500 cm

Change to metres. = 5 m

Write the answer. 1 : 500 is the same as 1 cm : 5 m.

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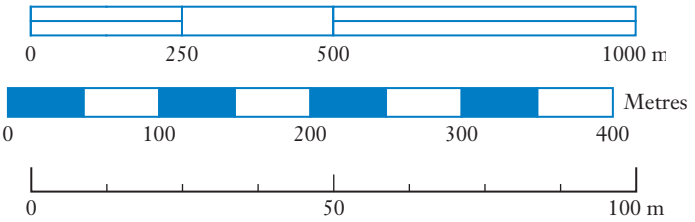
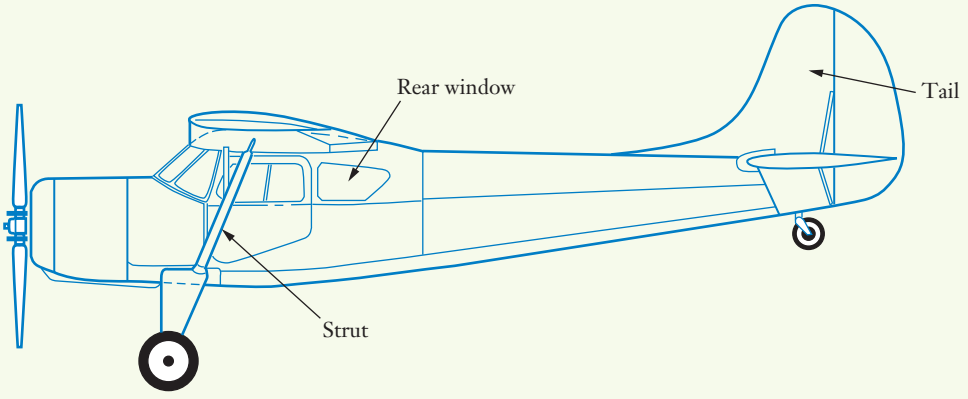
Sometimes a scale is shown like these:



Scale drawings

EXAMPLE 10

The drawing below shows a fixed wing aircraft drawn to a scale of 1 : 125.



a What is the length of the propeller?

b A line is drawn between the centres of the front and rear wheels on the drawing. If the actual distance is 10.88 m, how long should the line on the drawing be?

Solution

a Measure the propeller length on the drawing. — Drawing length = 32 mm

Write the scale. — Scale = 1 : 125

Every 1 mm on the drawing is the same as 125 mm in actual measurement.

Multiply by the scale factor, 125. — Real length = 32 × 125 mm
= 4000 mm

Write the answer in metres. — The length of the propeller is 4 m.

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b	Write the scale.	Scale = 1 : 125
	Name the unknown quantity.	Drawing length = x
	Write an equivalent ratio to the scale using the actual measurement and x.	1 : 125 = x : 10.88 m
	Convert to cm.	1 : 125 = x : 1088 cm
	Write the ratios as fractions.	$\frac{1}{125} = \frac{x}{1088}$
	Reverse the equation and solve for x.	$x = \frac{1}{125} \times 1088$
	Evaluate.	$= 8.704$
	Round off and write the answer, including units.	The line on the drawing should be 8.7 cm long.

Exercise 5.06 Scale drawings

1 A scale model of the Eiffel Tower is 6 cm tall. The Eiffel Tower is 300 m tall. What is the scale of the model?

A 1 : 50 **B** 1 : 250 **C** 1 : 500 **D** 1 : 5000 **E** 1 cm : 5 m

2 A map has a scale of 1 : 250 000. The distance between two cities on the map is 18 cm. What is the actual distance between the cities?

A 7.2 km **B** 4.5 km **C** 45 km **D** 72 km **E** 450 km

3 An architect's blueprint for a new house shows the back deck being 7 cm long. The scale used to create the blueprint is 0.5 cm : 1 m. What is the actual length of the deck?

A 35 m **B** 14 m **C** 3.5 m **D** 140 m **E** 1.4 m

Example 9

4 Write each scale in ratio form.

a 1 cm : 1 m **b** 1 cm = 1 km **c** 1 mm = 1 m
d 1 mm : 10 m **e** 2 cm : 1 m **f** 5 mm = 1 km

5 Find each missing value.

a 1 : 100 is the same as 1 cm : … m **b** 1 : 1000 is the same as 1 mm = … m
c 1 : 500 is the same as 1 cm = … m **d** 1 : 10 000 is the same as 1 cm : … m
e 1 : 650 000 is the same as 1 cm = … km **f** 1 : 50 000 is the same as 1 mm : … m

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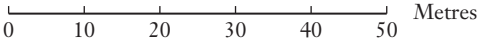
6 Write each scale as a ratio in simplest form.

a

b

c



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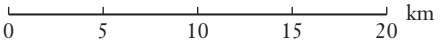

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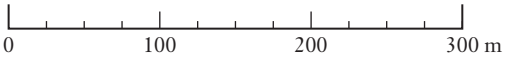


7 Use the aircraft plan in Example 10 to find the real measurements of:

- **a** the overall length of the aircraft
- **b** the length of the strut joining the wing and the forward wheel housing
- **c** the diameter of the forward wheel
- **d** the distance from the top of the tail to the bottom of the rear wheel
- **e** the width and height of the rear window.

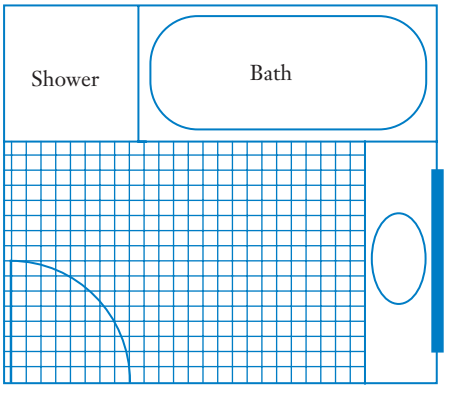
8 The distance between two points on a map is 275 mm. If the scale is 1 : 25 000, what is the true distance between the points?

9 The distance between Brisbane and Mackay is 800 km. What distance would this be on a map drawn to a scale of 1 : 5 000 000?

Problem solving

10 Use the bathroom plan to find the real measurement (in mm) for:

- **a** the length of the bath
- **b** the width of the shower
- **c** the width of the door
- **d** the width of the window
- **e** the length of the vanity unit
- **f** the width of the vanity unit.



Scale 1 : 50

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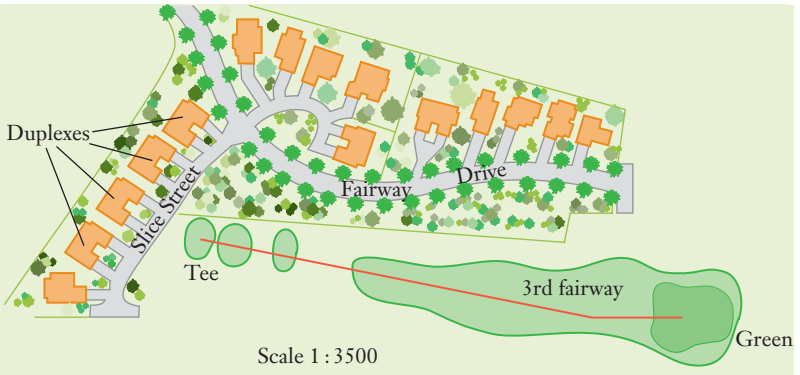
11 Use this scale drawing of part of a golf course housing development to answer the questions on next page.

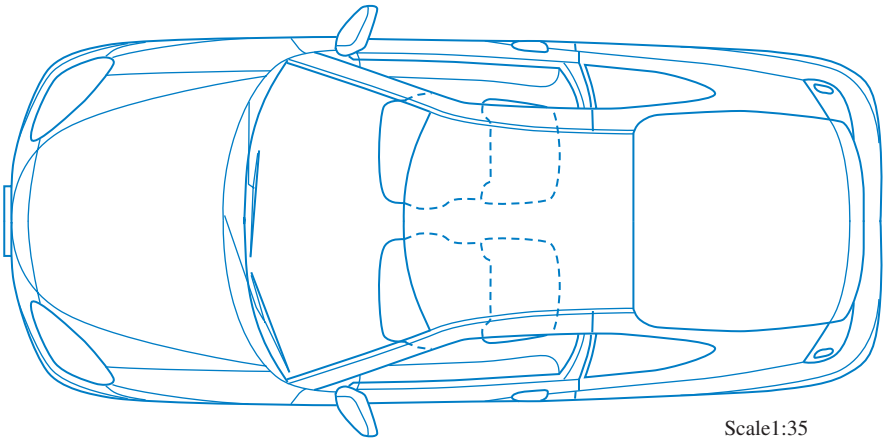


a How wide is Fairway Drive?

b The path of a golf ball hit from the tee to the green of the third hole is shown in red. How far did each of the two shots travel?

c What is the overall length of each duplex on Slice Street (measured parallel to the street)?

12 This is a scale drawing of the top of a car.



a How long is the car?

b How wide is the car body?

c How wide is the front car seat?

d What is the smallest distance between the seats?

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13 Use the map of the Hervey Bay area below to find each actual length.

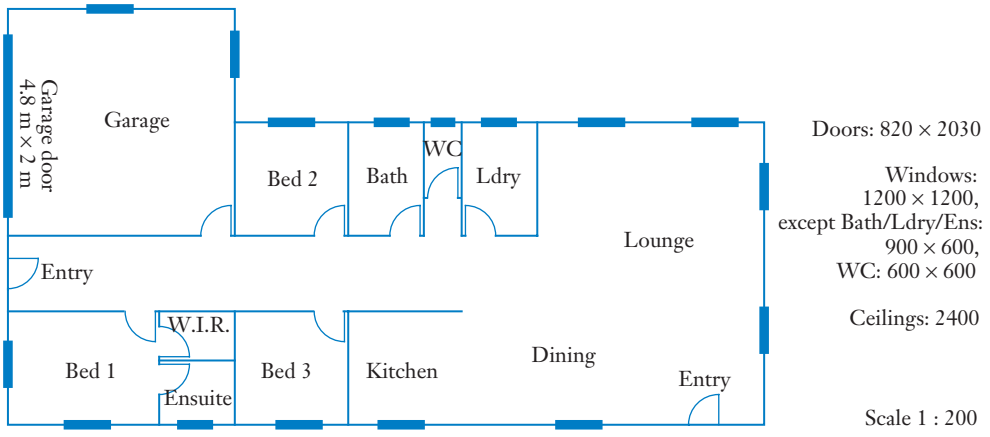
a The distance of the route shown between River Heads and Rooney Point

b The distance of the route shown between Boonooroo and Waddy Point

c The distance between Boonooroo and River Heads

d The greatest width of Fraser Island, measured east–west.

Questions **14** to **16** refer to the house plan shown below.



14 Find the cost of painting the ceilings of the house and garage with two coats, given that the paint covers 11 m^2/L and only comes in 4 L tins costing \$78.60 each. Fully explain your reasoning.

15 **a** Find the dimensions of the garage.

b How much will it cost to coat the floor of the garage with a seamless epoxy resin coating if materials and labour have been quoted at \$88.70/m^2?

16 How much will it cost to carpet the lounge/dining/entry floor if the cost of materials and labour is \$82.50/m^2?

17 A flat roof is 6.9 m wide and 10 m long.

a Find the number of sheets of corrugated iron needed to cover the roof if the sheets are 2 m by 900 mm and must be overlapped by 8 cm at their sides and ends.

b Find the number of concrete tiles needed for the same roof if they are 30 cm wide and 50 cm long and must be overlapped by 6 cm in each direction.

18 A car park is to be marked out on a vacant block of land that is 40 m deep and has a 50 m frontage. Car spaces of 2.6 m by 5 m and turning circles of 8 m are to be allowed. Two 3 m-wide entrances are needed also. Use a scale drawing to work out a plan for the car park, and hence find the number of cars that can be accommodated when it is full. Explain your reasoning and justify your decisions.

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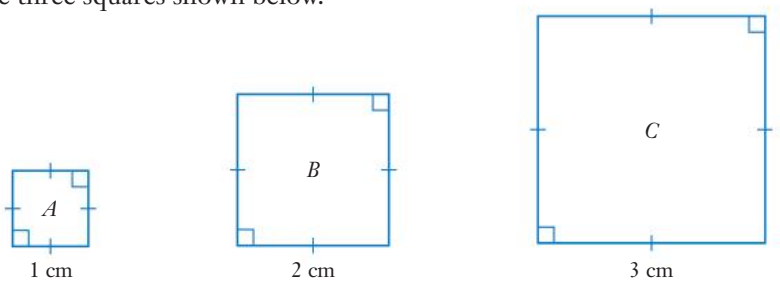
5.07 Areas and volumes of similar figures

Areas and volumes of similar figures

Areas of similar figures

Consider the three squares shown below.



Squares B and C are enlargements of square A, so the figures are similar.

When the linear scale factor is 2, the area scale factor $(B : A) = \frac{4}{1} = 4$

When the linear scale factor is 3, the area scale factor $(C : A) = \frac{9}{1} = 9$

Linear and area scale factors

If the linear scale factor is k, the area scale factor is k^2.

EXAMPLE 11

A photo has a length of 6 cm and an enlargement has a length of 15 cm.

a Calculate the area scale factor.

b If the smaller photo has an area of 24 cm^2, what is the area of the larger photo?

Alamy Stock Photo/Jean-Paul Ferrero/AUSCAPE

Solution

a Calculate the linear scale factor.

$$\text{Linear scale factor} = \frac{15}{6} = \frac{5}{2}$$

Calculate the area scale factor.

$$\text{Area scale factor} = (\text{linear scale factor})^2$$

Substitute.

$$= \left(\frac{5}{2}\right)^2$$

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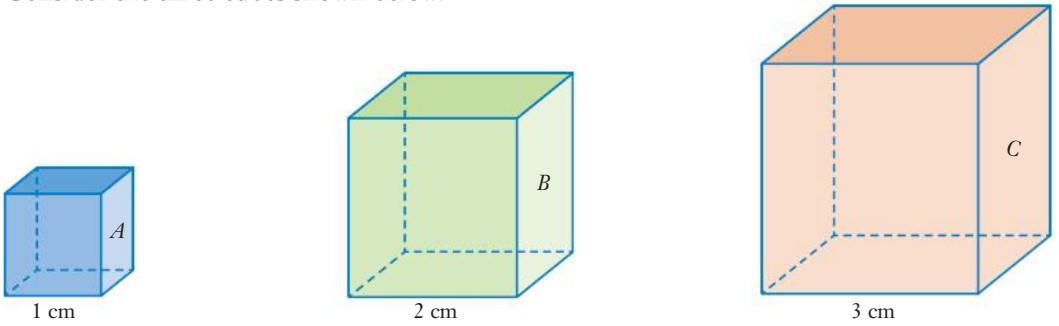
	Evaluate.	$= \frac{25}{4}$
	State the result.	The area scale factor $= \frac{25}{4}$
b	Calculate the area of the larger photo.	Area of larger photo $= \frac{25}{4} \times 24$
	Evaluate.	$= 150$
	State the result.	The area of the enlargement is 150 cm^2.

Consider the three cubes shown below.



Cubes B and C are enlargements of cube A, so the shapes are similar. The volume of a cube is $V = s^3$, where s is the side length.

When the linear scale factor is 2, the volume scale factor $(B : A) = \frac{8}{1} = 8$

When the linear scale factor is 3, the volume scale factor $(C : A) = \frac{27}{1} = 27$

Linear and volume scale factors

If the linear scale factor is k, the volume scale factor is k^3.

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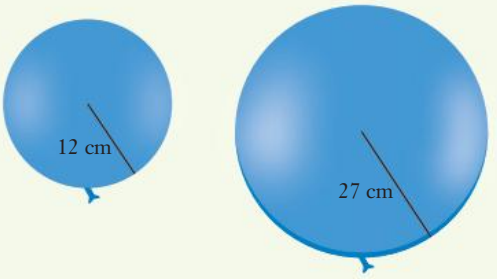
EXAMPLE 12

A spherical balloon has a radius of 12 cm. It is inflated until the radius reaches 27 cm.



a Calculate the linear scale factor.

b Calculate the volume scale factor.

c If the capacity of the larger balloon is 82.5 L, what is the capacity of the smaller one?

Solution

a Calculate the linear scale factor.

Linear scale factor $= \frac{27}{12}$

$= \frac{9}{4}$

b Calculate the volume scale factor.

Volume scale factor = (linear scale factor)^3

Substitute.

$= \left(\frac{9}{4}\right)^3$

$= \frac{729}{64}$

c Volume and capacity are the same.

Volume of larger balloon
= volume of smaller balloon
× volume scale factor

Let the volume of the small balloon = x.

$82.5 = x \times \frac{729}{64}$

Solve for x.

$x = \frac{82.5 \times 64}{729}$

Evaluate.

$= 7.2427\ldots$

State the result.

The capacity (volume) of the smaller balloon is about 7.2 L.

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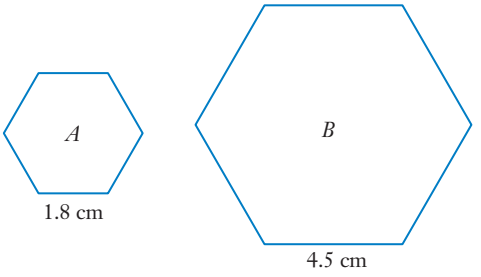
Exercise 5.07 Areas and volumes of similar figures

1 If a rectangle has a length of 4 cm and a mathematically similar rectangle has a length of 14 cm, then the area scale factor is:

A 3.5 **B** 20.25 **C** 15.2 **D** 12.25 **E** 16.5

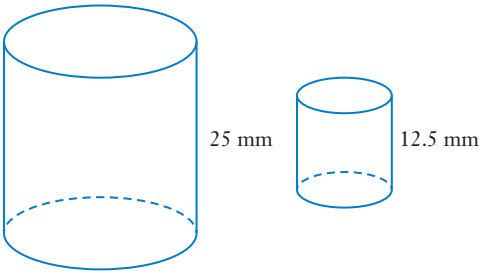
2 Shapes A and B are similar. If the area of A is 17 cm^2, then the area of B is:

A 42.5 cm^2
B 106.25 cm^2
C 125.25 cm^2
D 68 cm^2
E 85.5 cm^2



3 The two cylinders shown here are similar. The heights of the cylinders are shown. If the volume of the larger cylinder is 8 cm^3, then the volume of the smaller cylinder is:

A 4 cm^3
B 10 cm^3
C 1 cm^3
D 2 cm^3
E 64 cm^3

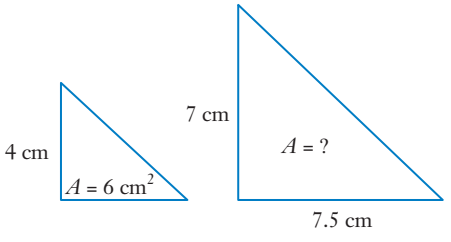


Example 11

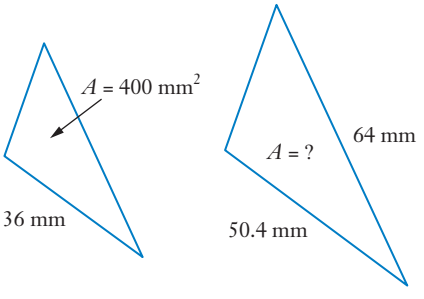
4 For each pair of similar shapes, calculate:

i the linear scale factor
ii the area scale factor
iii the unknown area

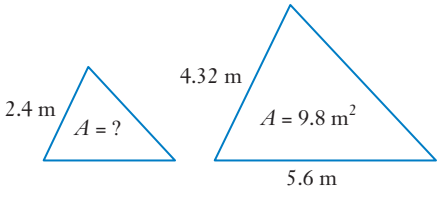
a



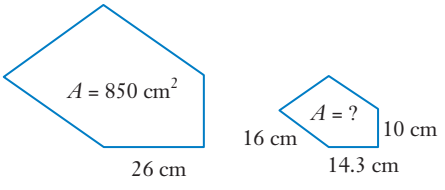
b



c



d



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5 The architect's plans for a building extension show a rectangular verandah with a length of 7 cm and a width of 3.5 cm. The scale on the plans is shown as 1 : 250.

a Calculate the area scale factor.

b Calculate the area of the verandah.

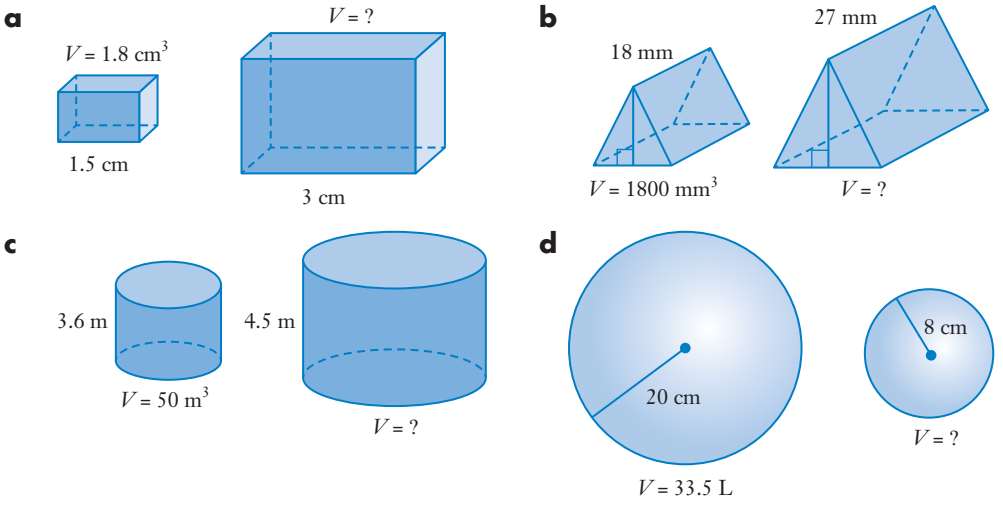
6 For each pair of similar 3D shapes, calculate:

Example 12

i the linear scale factor

ii the volume scale factor

iii the unknown volume indicated



Problem solving

7 A frozen yoghurt bar is in the shape of a rectangle. The maker of the yoghurt bars wants to improve cash flow by reducing the size of the bars. Each bar has a weight of 375 g. If the dimensions are reduced by 15%, what will the weight of the new bar be?

8 Two similar-shaped drink bottles are made by a manufacturer. One has a volume of 600 mL and the other has a volume of 1.25 L. Calculate the linear scale factor between two bottles.

9 A model of an SUV has a scale of 1 : 15.

a The surface area of the body of the SUV that is spray-painted is 6.74 m^2. What is the equivalent painted area on the model?

b The luggage capacity of the SUV is 2.42 m^3. What is the equivalent volume in the replica?

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5. CHAPTER SUMMARY

Surface area and similar figures

- The **surface area** of a three-dimensional (3D) shape is the area of all the faces.
- The **net** of a 3D shape is the 2D shape that can be folded up to make it.
- A **pyramid** is a solid made up of a base and triangular faces that meet at an **apex**.
- A **cone** is a solid with a circular base and an apex.
- The formulas for the **curved surface areas** (CSA) of some common shapes are:

  Cylinder: CSA = $2\pi rh$

  Cone: CSA = πrs, where $s = \sqrt{r^2 + h^2}$

  Sphere: CSA = $4\pi r^2$
- The magnification of a dilation is called the **scale factor**.
- **Congruent** figures are exactly the same shape and size. The symbol for congruency is $\equiv$.
- **Similar** figures are exactly the same shape but not necessarily the same size. The symbol for similarity is $|||$.
- In similar figures:
  - corresponding angles are equal
  - the ratio of corresponding sides is equal
  - scale factor = ratio of corresponding sides = $\dfrac{\text{image length}}{\text{object length}}$
- The tests for similar triangles are:
  - **Angle/angle** (**AAA**) Two angles of the triangles are the same size.
  - **Side/side/side** (**SSS**) All corresponding sides of the triangles are in the same ratio.
  - **Side/angle/side** (**SAS**) Two corresponding sides of the triangles are in the same ratio and the enclosed angles are equal.
  - **Right angle/hypotenuse/side** (**RHS**) The hypotenuses and one pair of corresponding sides are in the same ratio in two right-angled triangles.
- A **scale drawing** is one that represents a real object. The **scale** of a drawing is written as a ratio, drawing length : real length.
- If the linear scale factor is k, the **area scale factor** is k^2 and the **volume scale factor** is k^3.

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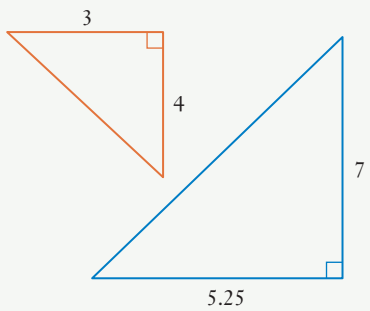
5. CHAPTER REVIEW

Surface area and similar figures

1 The surface area of a sphere with a diameter of 25 cm is closest to: Example 4

A 3926 cm^2 **B** 7850 cm^2 **C** 392.6 cm^2
D 785 cm^2 **E** 1963 cm^2

2 Which triangle similarity test should be used to prove that these triangles are similar? Example 5



A ASA
B AAA
C RHS
D SAS
E SSS

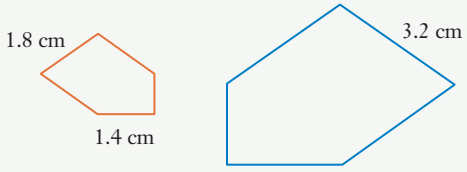
3 The height of a model of the 74 m high Story Bridge is 29.6 cm. The scale of the model is: Example 10

A 1 : 25 **C** 1 : 2500 **B** 1 cm : 25 m **D** 1 : 250 **E** 1 : 500

4 A map has a scale of 1 : 125 000. If the distance between two places on the map is 12 cm, the actual distance between the cities is: Example 10

A 7.5 km **B** 150 km **C** 1500 m **D** 75 km **E** 15 km

5 The two shapes shown here are similar. If the area of the smaller shape is 25 cm^2, then the area of the larger one is closest to: Example 11



A 50 cm^2
B 88 cm^2
C 79 cm^2
D 105 cm^2
E 150 cm^2

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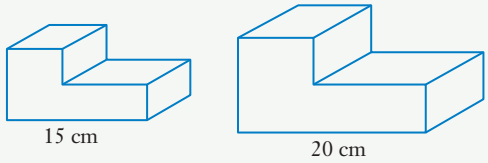
Example 12

6 The two solids shown here are similar. If the volume of the smaller object is 150 cm^3, then the volume of the larger one is closest to:



A 210 cm^3
B 360 cm^3
C 200 cm^3
D 250 cm^3
E 180 cm^3

Example 1

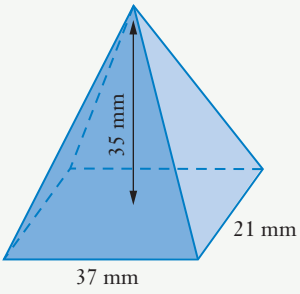
7 Find the surface area of:

a a box with a rectangular base measuring 42 cm by 30 cm and a height of 6 cm.

b a closed cylinder with a diameter of 66 cm and a height of 13 cm.

Example 2

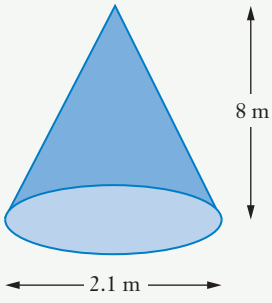
8 Find the surface area of this pyramid.



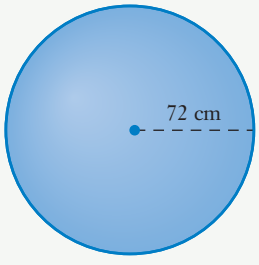
Example 3

9 Calculate the surface area of each solid shape.

a



b

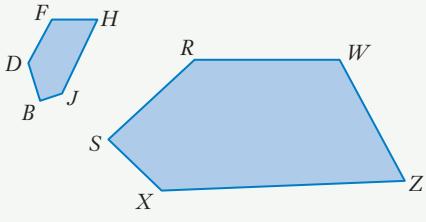


Example 4

10 The following shapes are similar.

a Write a statement of similarity.

b Use direct measurement to calculate the scale factor.



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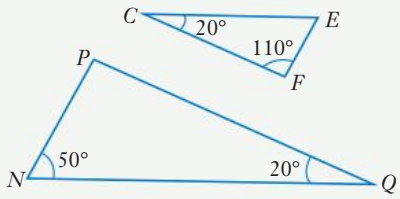
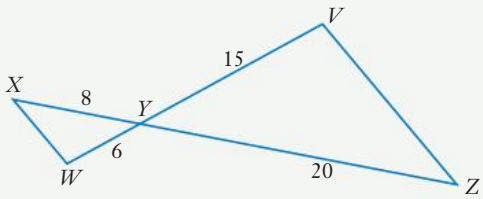
11 Determine if each pair of triangles are similar, giving a test for similarity if they are.

Example 5

a

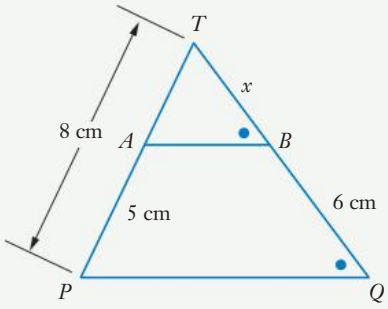


b



12 Calculate the value of x in the diagram.

Example 6



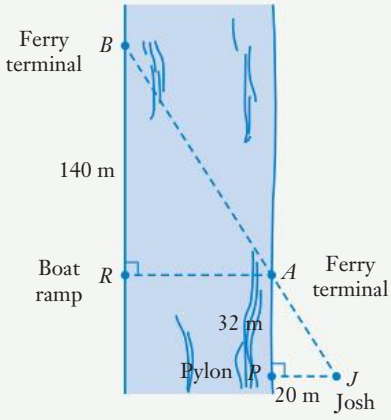
13 A tree casts a shadow 28 m long at the same time that a $4\frac{1}{2}$ m long stick casts a 6 m shadow. How tall is the tree?

Example 7

14 Josh is standing by a river at a point where two ferry terminals are in a direct line of view.

Josh is standing 20 m from a pylon on the bank of the river. The pylon is 32 m from the ferry terminal on the same side of the river. This terminal is directly opposite a boat ramp on the other bank of the river. If the ramp is 140 m away from the ferry terminal on the same side of the river, how wide is the river at this point?

Example 8



15 **a** Change a scale of 1 mm : 50 m to a ratio.

b Complete the following: 1 : 1250 is the same as 1 cm : … m.

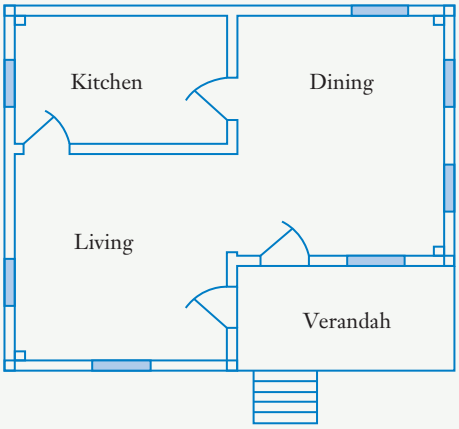
Example 9

Example 10

16 The drawing of the floor plan of a holiday cabin is drawn to a scale of 1 : 250.

a What is the overall length of the cabin?

b A rectangular dining table measuring 2.4 m by 1.4 m is to be placed in the dining area. What dimensions should be used to represent the table on the drawing?



Example 11

17 A photo has a length of 8 cm and an enlargement has a length of 22 cm.

a Calculate the area scale factor.

b If the smaller photo has an area of 36 cm^2, what is the area of the larger photo?

Example 12

18 A manufacturer makes two cylindrical containers that are mathematically similar. The radius of the smaller container is 3.8 cm and the diameter of the larger container is 17.1 cm.

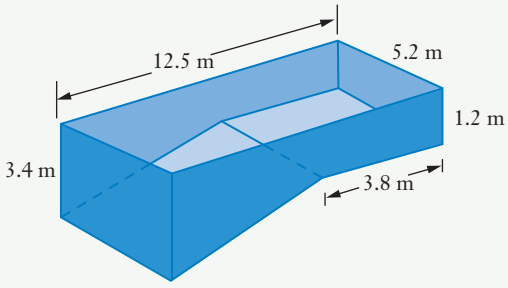
a Calculate the linear scale factor.

b Calculate the volume scale factor.

c If the capacity of the smaller container is 400 mL, what is the capacity of the larger one?

Problem solving

19 A newly-constructed swimming pool is shown. The interior surface requires one coat of special sealant that costs \$78.50/L. If each litre covers 8 m^2, calculate the cost of sealant required.



20 A cannery has ordered tomato juice cans that are 20 cm high and 8 cm in diameter. The labels are to be printed and cut from 50 m rolls of 40 cm wide paper. Each label requires a 1 cm overlap when placed on the can. How many labels can be cut from each roll?

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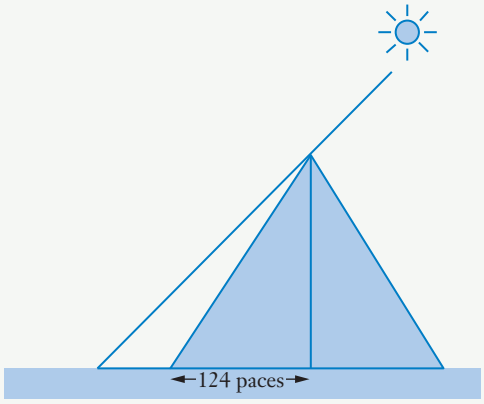
21 The Greek mathematician Thales of Miletus (about 600 BC) was the first to measure the height of the Great Pyramid by using geometry. He measured the distance from the centre of the pyramid to its edge to be 124 paces. The distance from that point to the end of the shadow of the pyramid was found to be 143 paces.

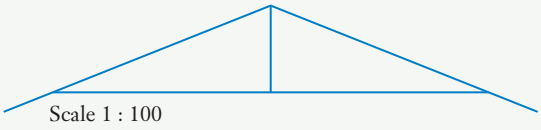


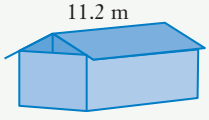
At the same time, a staff that was 2 paces long had a shadow that was 3 paces long.

A pace is about 0.9 m.

How tall was the pyramid at that time?

22 The scale drawing below shows the end of a gable roof for a shed.





It is to be covered with sheets of aluminium that can be supplied cut to any length up to 5 m. The sheets need to project 300 mm over the front and back of the shed. The sheets are 1.2 m wide and must be overlapped by 10 cm.

How many sheets are needed for the job? Explain your reasoning.

23 A company produces canned fruit in two different-sized cans. For each can, the diameter is four-fifths of the height. The larger can is 1.5 times larger than the smaller can. If the volume of the smaller can is 375 mL, what is the volume of the larger can?

Practice quiz

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6.

SIMULTANEOUS EQUATIONS AND PIECE-WISE GRAPHS

There are times when the solution to a problem involves more than one equation. Simultaneous equations are sets of equations that are true at the same time.

Imagine that a manufacturer produces a product using a particular machine. There are plans to buy some replacement machines and there are two types of machines that could be purchased. One type of machine is more expensive than the other one being considered. It is more expensive, takes up more room and needs more staff to operate it, but it can produce many more items and is more efficient than the other machine. Which type of machine should be purchased?

Solving simultaneous equations can help to work out feasible solutions to questions like this.

6.01 Solving simultaneous equations graphically
6.02 Solving simultaneous equations algebraically
6.03 Applications of simultaneous equations
6.04 Piece-wise linear graphs
6.05 Step graphs
6.06 Using piece-wise and step graphs
Chapter summary
Chapter review

SYLLABUS SUBJECT MATTER

Simultaneous linear equations and their applications

- solve a pair of simultaneous linear equations in the format $y = mx + c$, using technology when appropriate. Solve algebraically, graphical and by substitution, not using the elimination method.
- solve practical problems that involve finding the point of intersection of two straight-line graphs, for example, determining the break-even point where cost and revenue are represented by linear equations

Piece-wise linear graphs and step graphs

- sketch piece-wise linear graphs and step graphs, using technology when appropriate
- interpret piece-wise linear and step graphs used to model practical situations

Prior learning

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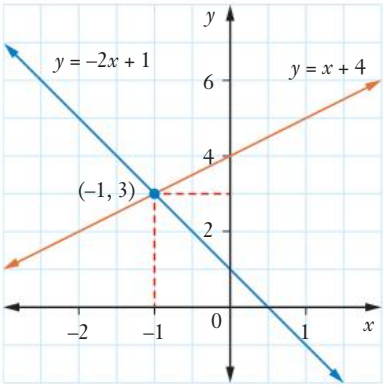
TERMINOLOGY

continuous
piece-wise linear function
simultaneous linear equations
step graph
discontinuous
piece-wise linear graph
simultaneous solution
substitution method
parallel lines
point of intersection
step function

Sketching simultaneous equations

Intersection of lines

6.01 Solving simultaneous equations graphically



Look at the two linear functions that are graphed here. The graphs cross at the point (–1, 3). This is called the **point of intersection**. Because the point of intersection lies on both the lines, (–1, 3) should satisfy both functions.

Let's check:

(–1, 3) satisfies $y = x + 4$ because, when $x = -1$:

$$y = -1 + 4 = 3$$

(–1, 3) also satisfies $y = -2x + 1$ because, when $x = -1$:

$$y = -2 \times (-1) + 1 = 2 + 1 = 3$$

The equations of the lines are called **simultaneous equations**. A **system** of equations is a set of two or more equations with the same variables. So $y = x + 4$ and $y = -2x + 1$ form a system of simultaneous equations.

The solution $x = -1$ and $y = 3$ is called a **simultaneous solution** to the equations because the ordered pair (–1, 3) satisfies both equations *at the same time*.

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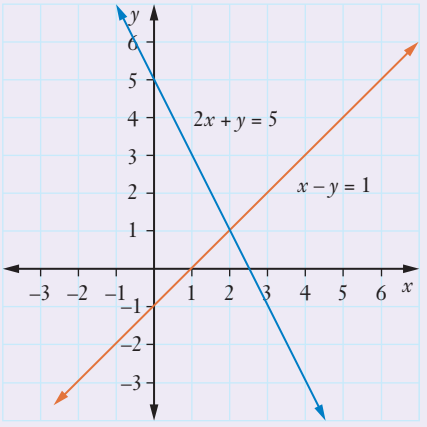
EXAMPLE 1

Use this graph to determine the simultaneous solution of the equations:

$$x - y = 1$$

$$2x + y = 5$$



Solution

Write the equations and number them.

$x - y = 1$ [1]

$2x + y = 5$ [2]

Locate the point of intersection of the two lines. This gives the solution.

Point of intersection = (2, 1)

So $x = 2$ and $y = 1$.

Check the solution by substituting $x = 2$ and $y = 1$ into the given equations.

Check equation [1]:

LHS = $x - y$ RHS = 1

$= 2 + 1$

$= 1$

LHS = RHS

Check equation [2]:

LHS = $2x + y$ RHS = 5

$= 2 \times 2 = 1$

$= 5$

LHS = RHS

Comment on the result.

In both cases, LHS = RHS, therefore (2, 1) satisfies both lines and $x = 2$ and $y = 1$ is the simultaneous solution.

EXAMPLE 2

Check whether the point (2, –1) satisfies the following system of equations.

$$2x - 3y = 11$$
$$4y + x = -2$$

Solution

Write the equations and number them.	$2x - 3y = 11$ [1] $4y + x = -2$ [2]
Substitute $x = 2$ and $y = -1$ into each equation.	Check equation [1]: LHS $= 2x - 3y$ RHS $= 11$ $= 2 \times 2 - 3 \times (-1)$ $= 4 + 1$ $= 5$ LHS = RHS Check equation [2]: LHS $= 4y + x$ RHS $= -2$ $= 4 \times (-1) + 2$ $= -4 + 2$ $= -2$ LHS = RHS
State the result.	In both cases LHS = RHS, therefore (2, –1) satisfies both lines and $x = 2$ and $y = 1$ is the simultaneous solution.

The solution to a pair of simultaneous equations can be found by graphing the two equations and identifying the coordinates of the point of intersection. You need to draw an accurate graph or use graphing software to solve simultaneous equations.

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EXAMPLE 3

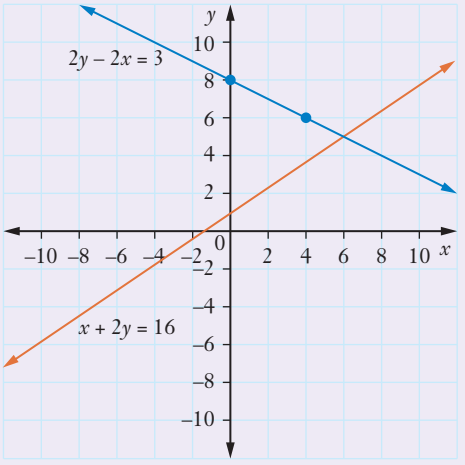
Find the solution of the following simultaneous equations by graphing.

$$3y - 2x = 3$$
$$x + 2y = 16$$

Solution

Write the equations and number them.	$3y - 2x = 3$ [1] $x + 2y = 16$ [2]
First find two points on the line.	Let $x = 0$:
Choose a convenient x value for [1] and solve for y.	$3y - 2 \times 0 = 3$ $3y = 3$ $y = 1$ $(0, 1)$ lies on $3y - 2x = 3$.
Choose another x value for [1] and solve for y.	Let $x = -6$: $3y - 2 \times (-6) = 3$ $3y + 12 = 3$ $3y = -9$ $y = -3$ $(-6, -3)$ lies on $3y - 2x = 3$.
Choose a convenient x value for [2] and solve for y.	Let $x = 0$: $0 + 2y = 16$ $2y = 16$ $y = 8$ $(0, 8)$ lies on $x + 2y = 16$.
Choose another x value for [2] and solve for y.	Let $x = 4$: $4 + 2y = 16$ $2y = 12$ $y = 6$ $(4, 6)$ lies on $x + 2y = 16$.

Use the points to plot both lines on graph paper.



Use the graph to state the point of intersection.

The graphs intersect at (6, 5).

State the result.

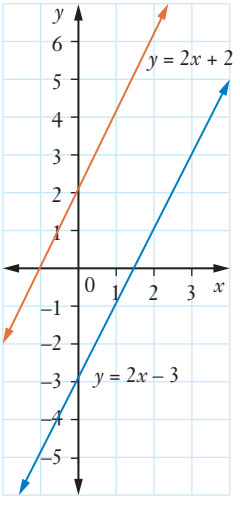
(6, 5) satisfies both graphs, so $x = 6$ and $y = 5$ is the simultaneous solution.

Parallel lines have the same gradient but a different y-intercept.

For example, $y = 2x + 2$ and $y = 2x - 3$ are parallel.

The graphs of $y = 2x + 2$ and $y = 2x - 3$ are shown here.

When parallel lines are graphed, they do not intersect, so there is no simultaneous solution to the equations.



Parallel lines

A system of equations represented by parallel lines has no simultaneous solution.

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TECHNOLOGY

Graphing simultaneous equations using a spreadsheet

You have seen how to use a spreadsheet like MS Excel to draw the graph of a linear equation. You can also use a spreadsheet to draw the graph of a system of linear equations.

Consider the system of equations:

$$y = 5 - 3x$$

$$y = 4x - 2$$

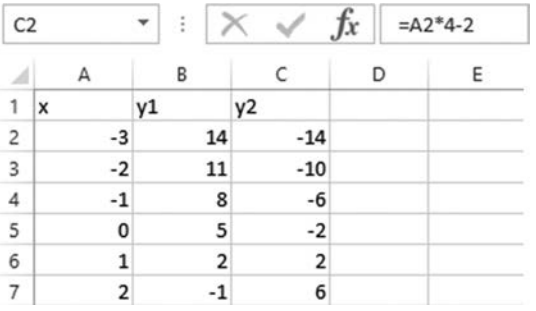
Open a spreadsheet workbook and enter 'x' in A1, '$y1$' in B1 and '$y2$' in C1.

x-values

Enter the x values $-3, -2, -1, \ldots, 4$ in cells A2 to A9.

y-values

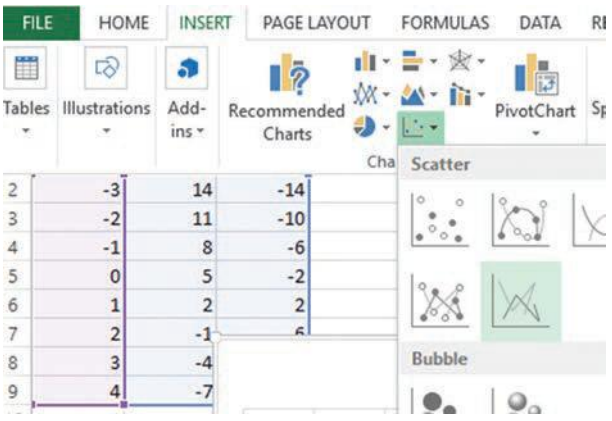
Begin with $y = 5 - 3x$. Enter the formula '=5-A2*3' in cell B2 and then copy the formula down to B9.



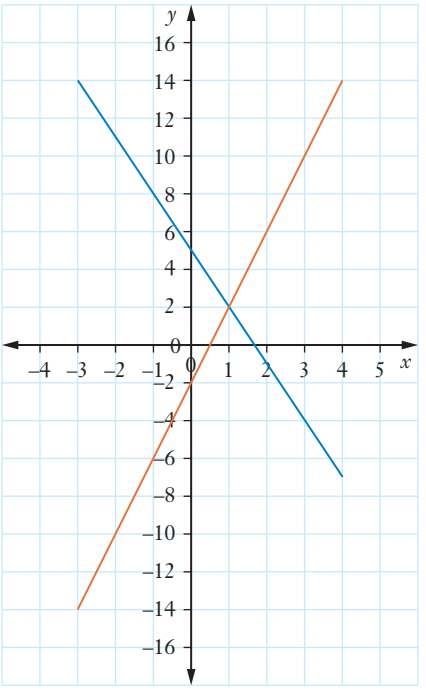
	A	B	C	D	E
1	x	y1	y2		
2	-3	14	-14		
3	-2	11	-10		
4	-1	8	-6		
5	0	5	-2		
6	1	2	2		
7	2	-1	6		

Now do $y = 4x - 2$. Enter the formula '=4*A2–2' in cell B2 and then copy the formula down to C9.

Highlight the cells A2:C9 and then click on the **INSERT** tab. Select the straight line scatter chart. The graph will then appear in the workbook.



You can adjust the look of your graph by right-clicking on each of the graph, axes, lines and titles and choosing to format them.



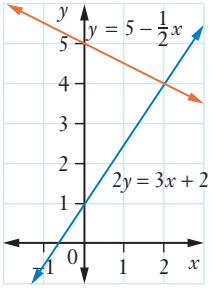
In this case, the simultaneous solution for $y = 5 - 3x$ and $y = 4x - 2$ is $x = 1$ and $y = 2$.

Exercise 6.01 Solving simultaneous equations graphically

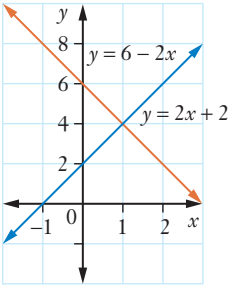
Example 1

1 Use each graph to find the simultaneous solutions to the pair of equations given.

a $y = 5 - \frac{1}{2}x$ and $2y = 3x + 2$



b $y = 6 - 2x$ and $y = 2x + 2$



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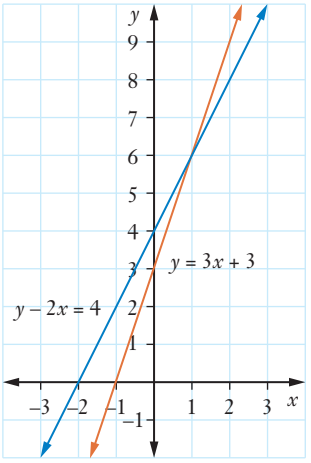
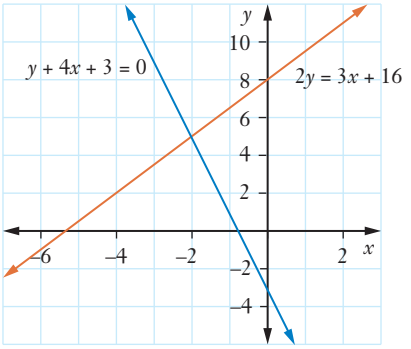
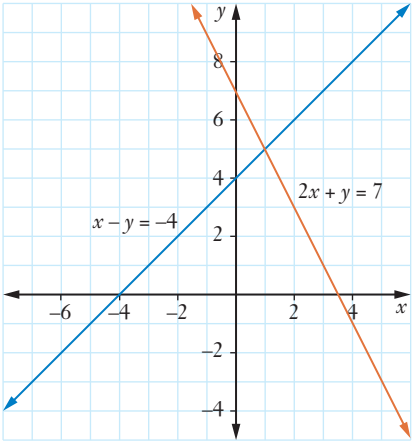
c $y = 3x + 3$ and $y - 2x = 4$



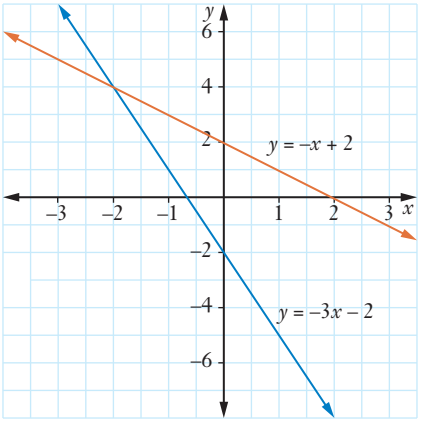
d $2y = 3x + 16$ and $y + 4x + 3 = 0$



e $x - y = -4$ and $2x + y = 7$



f $y = -x + 2$ and $y = -3x - 2$



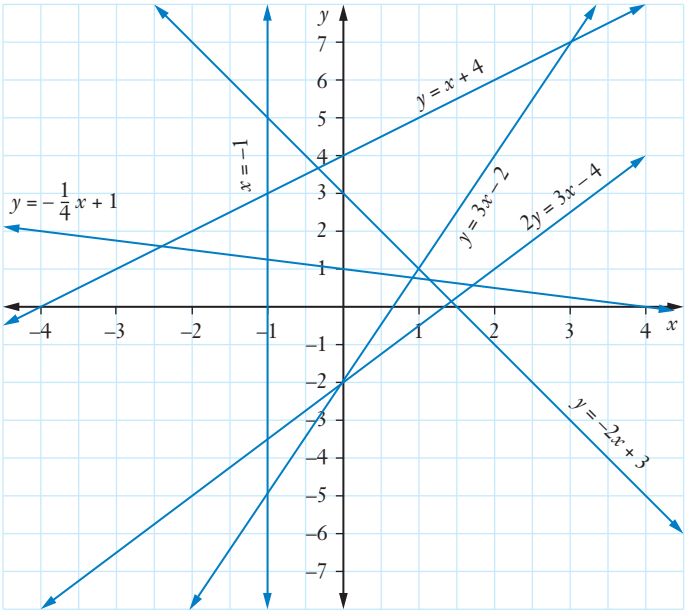
2 Use the graph to find the simultaneous solution to:

a $x = -1$ and $y = -2x + 3$

b $y = -2x + 3$ and $y = 3x - 2$

c $y = 3x - 2$ and $y = x + 4$

d $2y = 3x - 4$ and $y = 3x - 2$



3 Which of the following is the simultaneous solution for $x - y = 5$ and $2x + y = -8$?

A $(-1, 6)$ **B** $(1, 6)$ **C** $(1, 5)$ **D** $(-1, -6)$ **E** $(1, -6)$

4 Which of the following is the simultaneous solution for $3x + 4y = 27$ and $x + 2y = 1$?

A $(-5, 3)$ **B** $(5, 3)$ **C** $(-1, 6)$ **D** $(-5, -3)$ **E** $(5, -3)$

Example 2

5 For each system of simultaneous equations, determine if the given pair of coordinates is a simultaneous solution.

a $(-3, -1)$ $y = x + 2$ and $2y = 3x + 7$

b $(5, 1)$ $y = 5x - 24$ and $3y + 4x - 23 = 0$

c $(4, -2)$ $y = 6 - 2x$ and $x = 3y + 8$

d $(7, 5)$ $2y = 31 - 3x$ and $2x + 3y = 29$

e $(-2, -5)$ $3x - 2y - 4 = 0$ and $2x = 3y + 11$

f $(3, 7)$ $y = x + 4$ and $x = 16 - 2y$

g $(4, -3)$ $3x = y + 15$ and $4x + 7y + 5 = 0$

h $(6, -2)$ $x = 2 - 2y$ and $y = 16 - 3x$

i $(9, 1)$ $3y = 12 - x$ and $5x = 43 - 2y$

j $(2, 5)$ $x = 7 - y$ and $2x + 3y - 19 = 0$

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Example 3

6 Graph each pair of equations on the same set of axes, then use the graph to find the simultaneous solution.

a $y = x$ and $y = 2x + 1$ $(-3 \le x \le 3)$

b $y = x + 3$ and $y = 2x + 4$ $(-3 \le x \le 3)$

c $y = -x + 2$ and $y = 2x - 1$ $(-3 \le x \le 3)$

d $y = 2x - 7$ and $y = -x - 1$ $(-6 \le x \le 6)$

e $y = -2x + 2$ and $y = 3x + 7$ $(-4 \le x \le 2)$

7 First graph each pair of functions on the same set of axes. Then use the graph to find the simultaneous solution.

a $2x + y = 4$ and $x - y = 2$ $(-1 \le x \le 5)$

b $3x - y = 7$ and $2x + y = 3$ $(-1 \le x \le 5)$

c $10x - 5y = 50$ and $x + y = 2$ $(0 \le x \le 6)$

d $3x + 4y = -2$ and $x - 4y = -6$ $(-4 \le x \le 2)$

e $4x + y = 23$ and $x + y = 8$ $(0 \le x \le 10)$

8 Solve each pair of simultaneous equations using a graphical method.

a $2x + 3y = 6$ and $2x - y = -10$

b $3x + y = 11$ and $4x - y = 3$

c $4x - 2y = -5$ and $x + 3y = 4$

d $2x + y = 7$ and $x - y + 4 = 0$

e $2y - 4x = 5$ and $4y + 2x = 5$

Problem solving

9 The heights of water (h cm) in two containers after a certain time (t seconds) are given by the following equations:

Container A: $h = 2t$

Container B: $h = 3t - 20$

Use a graphical method to find the time when the two containers have the same height of water.

10 Line A is parallel to the line with equation $y + 2x = 5$ and passes through the point (2, 3). Line B is parallel to the line with equation $x - y = 2$ and passes through the point (–2, 2).

Draw the graphs of lines A and B and find their simultaneous solution.

11 Solve this system of 3 simultaneous equations graphically.

$$2y = 3x + 16$$
$$y + 4x + 3 = 0$$
$$y + 2x = 1$$

Substitution

Simultaneous equations puzzle

Solving simultaneous equations

Simultaneous equations order activity

Simultaneous equations

Simultaneous equations

6.02 Solving simultaneous equations algebraically

It may take a long time to use graphs to solve simultaneous equations. You may find it difficult to locate an exact point of intersection, so it is not always accurate. It is quicker and more accurate to use an *algebraic method* of solution called the **substitution method**.

This method requires one of the equations to be substituted into the other by replacing one of the variables. You should number the equations to make your steps clear.

EXAMPLE 4

Solve these simultaneous equations using the method of substitution:

$x + 2y = 23$ [1]

$y = x + 4$ [2]

Solution

Write [1].	$x + 2y = 23$ [1]
Write [2].	$y = x + 4$ [2]
[2] is ready for y to be substituted into [1].	
Substitute for y from [2] into [1].	$x + 2(x + 4) = 23$
Expand and solve for x.	$x + 2x + 8 = 23$
	$3x + 8 = 23$
	$3x = 15$
	$x = 5$
We can now find y by substituting for x in either [1] or [2]. Use [2] as it gives y straight away.	$y = x + 4$ [2]
Substitute 5 for x.	$y = 5 + 4$
	$y = 9$

Now check the result by substituting $x = 5$ and $y = 9$ into both [1] and [2].

Check in [1].	LHS $= 5 + 2 \times 9 = 5 + 18 = 23$
	$=$ RHS✓
Check in [2].	LHS $= 9$.
	RHS $= 5 + 4 = 9 =$ LHS✓
State the result.	The simultaneous solution is
	$x = 5$ and $y = 9$.

It is important to check that the simultaneous solution satisfies both of the original equations.

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Substitution method for simultaneous equations

- Use one equation to express one variable in terms of the other.
- Substitute this into the other equation.
- Solve the resulting equation.
- Use the solution to find the value of the other variable.
- Check the simultaneous solution.

EXAMPLE 5

Solve each system of simultaneous equations using the method of substitution:

a $2b - a = 3$
$a + 3b = 7$

b $4x + 3y = 8$
$3x + 2y = 7$

Solution

a Write the equations and number them.

$2b - a = 3$ [1]

$a + 3b = 7$ [2]

Subtract $3b$ from both sides of [2] to make a the subject.

$a = 7 - 3b$

Substitute for a in [1]. Use brackets to avoid mistakes.

$2b - (7 - 3b) = 3$

Expand (be careful with negatives).

$2b - 7 + 3b = 3$

Gather like terms.

$5b - 7 = 3$

Undo the '– 7' by adding 7 to both sides.

$5b = 10$

Undo the '× 5' by dividing both sides by 5.

$b = 2$

Substitute for b into [2].

$a + 3 \times 2 = 7$

$a + 6 = 7$

$a = 1$

Check in [1].

$\text{LHS} = 2 \times 2 - 1 = 4 - 1 = 3$

$= \text{RHS}$ ✓

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	Check in [2].	$\text{LHS} = 1 + 3 \times 2 = 1 + 6 = 7$ $= \text{RHS}\checkmark$
	State the result.	$a = 1$ and $b = 2$ is the simultaneous solution.
b	Write the equations and number them.	$4x + 3y = 8$ [1] $3x + 2y = 7$ [2]
	Rearrange [2] to isolate y, the variable with the smallest coefficient.	$2y = 7 - 3x$ $y = \frac{7-3x}{2}$
	Substitute for y in [1].	$4x + 3 \times \left(\frac{7-3x}{2}\right) = 8$
	Multiply by 2 to get rid of the fraction,	$4x \times 2 + 3 \times \left(\frac{7-3x}{2}\right) \times 2 = 8 \times 2$
	Expand the brackets.	$8x + 3(7 - 3x) = 16$
	Simplify and solve.	$8x + 21 - 9x = 16$ $21 - x = 16$ $-x = 16 - 21$ $-x = -5$ $x = 5$
	Substitute for x in [2].	$3 \times 5 + 2y = 7$ $15 + 2y = 7$ $2y = -8$ $y = -4$
	Check in [1].	$\text{LHS} = 4 \times 5 + 3 \times (-4) = 20 - 12 = 8$ $= \text{RHS}\checkmark$
	Check in [2].	$\text{LHS} = 3 \times 5 + 2 \times (-4) = 15 - 8 = 7$ $= \text{RHS}\checkmark$
	State the result.	$x = 5$ and $y = -4$ is the simultaneous solution.

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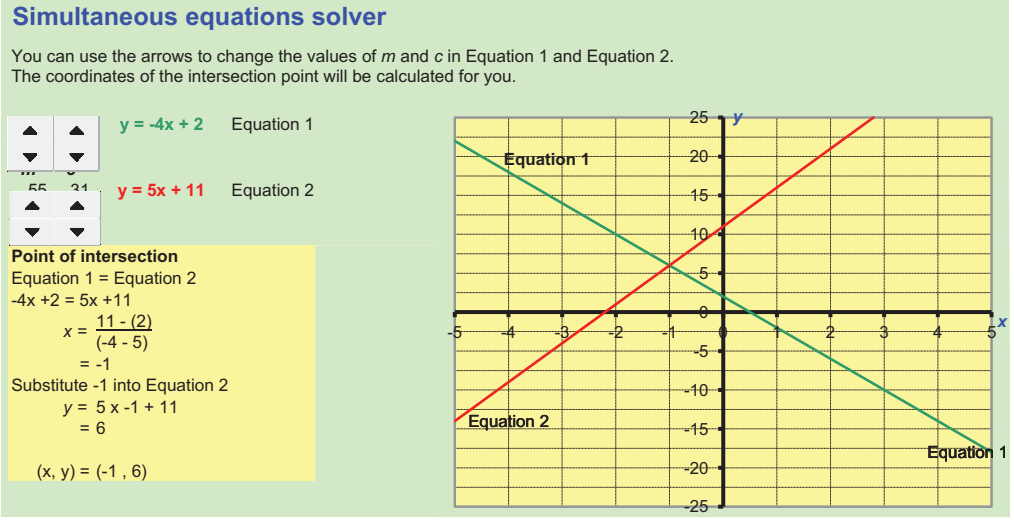
TECHNOLOGY

Solving simultaneous equations using substitution

Simultaneous equations solver

On the NelsonNet website you will find a spreadsheet called 'Simultaneous equations solver' that finds the simultaneous solution for two intersecting lines and displays the applicable steps for the substitution method.



Use the spreadsheet to solve each pair of simultaneous linear equations.

a $y = -7x - 7$ and $y = 5x + 5$

b $y = 2x + 3$ and $y = 3x - 2$

c $y = x - 4$ and $y = 2x + 1$

d $y = -2x + 6$ and $y = 4x - 3$

e $y = -5x - 3$ and $y = 3x$

f $y = 12x + 9$ and $y = -3x + 12$

g $y = -4x - 7$ and $y = 14x + 3$

h $y = 4x + 5$ and $y = 4x - 2$

i $y = -2x + 5$ and $y = -4x + 10$

j $2x + y = 8$ and $6x - 2y = 12$

Exercise 6.02 Solving simultaneous equations algebraically

1 The simultaneous equations $y = 3x - 1$ and $y = x$ are solved using the substitution method. The solution is:

A $x = 1$ and $y = 1$ **B** $x = -1$ and $y = -1$ **C** $x = -\frac{1}{2}$ and $y = -\frac{1}{2}$

D $x = \frac{1}{2}$ and $y = \frac{1}{2}$ **E** $x = 2$ and $y = -2$

2 The simultaneous equations $y - 4x = 10$ and $x + y = 0$ are solved using the substitution method. The solution is:

A $x = 1$ and $y = 1$ **B** $x = -2$ and $y = 2$ **C** $x = -\frac{1}{2}$ and $y = -\frac{1}{2}$

D $x = \frac{1}{2}$ and $y = \frac{1}{2}$ **E** $x = -2$ and $y = -2$

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3 Given the system of equations:

$$p - 5q + 8 = 0 \qquad [1]$$

$$p + 3q = 16 \qquad [2]$$

a make p the subject of equation [1].

b use your answer to part **a** to find the simultaneous solution using the method of substitution.

Example 4

4 Use the method of substitution to find simultaneous solutions for each system of equations.

a $x = y + 3$, $x + 2y = 9$

b $y = 3x$, $2x - y = -6$

c $b = 2a - 3$, $a + 3b = 5$

d $3a + b = 8$, $b = a - 12$

e $x + 2y = 6$, $x = 3y - 9$

f $c = 3d - 1$, $5c - 7d = 19$

g $c = d + 1$, $2c + d = 14$

h $c = 7 - 2d$, $3c - 2d = 9$

i $x = 1 - y$, $3 = y + 2x$

Example 5

5 Find the simultaneous solutions for each system of equations.

a $x + y = 5$, $x - y = 1$

b $2x + y = 12$, $3x - y = 13$

c $5m - 2n = 13$, $m + n = 4$

d $4a - b = 10$, $a + 3b = 9$

e $4m - n = 6$, $3m + 2n = -1$

f $5x + y = 12$, $3x + 2y = 3$

g $3x + 2y = 6$, $2x - 3y = 17$

h $5a + 2b = -8$, $a - 3b = -5$

i $2m + 3n = -1$, $m + 4n = 2$

Problem solving

6 Use the method of substitution to find a simultaneous solution to:

$$ax + y = b$$

$$y = ax$$

7 The length of a rectangular room is 2 m longer than its width. The perimeter of the room is 68 m.

a Write an equation for the relationship between the length (l) and the width (w) of the room and another equation for the perimeter of the room.

b Use the method of substitution to find the length and width of the room.

8 A particular DVD costs \$6 less than a certain Blu-ray disc.

2 of the DVDs and 3 of the Blu-ray discs cost a total of \$118.

Write 2 equations for this situation and find the simultaneous solution using the substitution method.

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6.03 Applications of simultaneous equations

Simultaneous equations problems

EXAMPLE 6

A manufacturer sells shirts for \$26. The amount received for selling n shirts is given by the equation $A = 26n$. The amount it costs to make n shirts is given by the equation $A = 1100 + 4n$. The break-even point for the manufacturer is the value of n when the cost of production is the same as the revenue. The maximum production run is 300 shirts.

a Draw graphs of the equations on the same set of axes.

b Use the graph to find the break-even point.

c Determine the profit or loss, in dollars, for the following shirt orders:

i 30 shirts **ii** 110 shirts

Solution

a Write the equations and number them.

$A = 26n$ [1]

$A = 1100 + 4n$ [2]

To find two points to graph [1], first choose some convenient n value for [1].

Let $n = 0$:

$A = 26 \times 0 = 0$

(0, 0) lies on $A = 26n$.

Select another n value for [1].

Let $n = 100$:

$A = 26 \times 100 = 2600$

(100, 2600) lies on $A = 26n$.

To find two points to graph [2], first choose some convenient n value for [2].

Let $n = 0$:

$A = 1100 + 4 \times 0 = 1100$

(0, 1100) lies on $A = 1100 + 4n$.

Select another n value for [2].

Let $n = 100$:

$A = 1100 + 4 \times 100 = 1500$

(100, 1500) lies on $A = 1100 + 4n$

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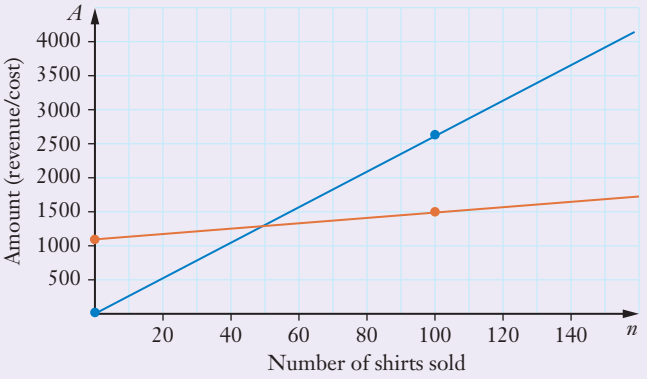
Plot the points and join each pair with a straight line.



b The break-even point is where the graphs intersect.

Point of intersection = (50, 1300)

State the result.

The break-even point is when 50 shirts are sold. The revenue and income are both \$1300 when 50 shirts are sold.

c **i** Find the revenue when $n = 30$.

$A = 26 \times 30 = 780$

Find the production cost when $n = 30$.

$A = 1100 + 4 \times 30 = 1220$

Find the difference between revenue and production cost.

Revenue – production cost = 780 – 1220

= –420

The negative sign means that revenue is less than costs, that is, a loss is made.

When 30 shirts are sold, the manufacturer makes a loss of \$420.

ii Find the revenue when $n = 110$.

$A = 26 \times 110 = 2860$

Find the production cost when $n = 110$.

$A = 1100 + 4 \times 110 = 1540$

Find the difference between revenue and production cost.

Revenue – production cost = 2860 – 1540

= 1320

The positive value means that revenue is more than costs, i.e. a profit is made.

When 110 shirts are sold, the manufacturer makes a profit of \$1320.

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Solving problems with simultaneous equations

1. Read the information carefully
2. Choose variable names for the unknown quantities
3. Use the information in the problem to form equations in terms of the variables
4. Solve the equations using the most convenient method
5. Check the solution
6. Write the solution in the same terms as the stated problem

EXAMPLE 7

At a school play, adults were charged \$5 admission and children \$2. A total of 400 people attended the play and the receipts were \$1100.

a Develop two equations that represent this situation.

b Find the simultaneous solution to the equations developed.

c How many adults attended the play?

Solution

a	Assign the variables.	Let the number of adults = a.
		Let the number of children = c.
	There were 400 people altogether.	$a + c = 400$ [1]
	There was \$1100 received.	$5a + 2c = 1100$ [2]
b	Isolate a in [1].	$a = 400 - c$
	Substitute for a into [2].	$5(400 - c) + 2c = 1100$
	Expand and solve.	$2000 - 5c + 2c = 1100$
		$-3c = -900$
		$c = 300$
	Substitute 300 for c in [1].	$a + 300 = 400$
		$a = 100$
	Check in [1].	LHS = 100 + 300 = 400 = RHS

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	Check in [2].	LHS $= 5 \times 100 + 2 \times 300$
		$= 500 + 600 = 1100$
		$=$ RHS
	State the solution.	The simultaneous solution is $a = 100$ and $c = 300$.
c	State the result.	There were 100 adults at the play.

EXAMPLE 8

The length of a rectangle is 7 less than twice its width. The perimeter of the rectangle is 52 m. Find the length of the rectangle.

Solution

Assign the variables.	Let the length $= l$
	Let the width $= w$
The length is 7 less than $2w$.	$l = 2w - 7$ [1]
The perimeter is 52 m.	$52 = 2(l + w)$ [2]
Substitute for l into [2]. Remember to use brackets.	$52 = 2[(2w - 7) + w]$
Expand brackets and solve.	$52 = 2(3w - 7)$
	$52 = 6w - 14$
	$66 = 6w$
	$11 = w$
	$w = 11$
Substitute 11 for w in [1].	$l = 2 \times 11 - 7$
	$= 15$
Check in [1].	LHS $= 15$
	RHS $= 2 \times 11 - 7 = 15 =$ LHS
Check in [2].	LHS $= 52$
	RHS $= 2(15 + 11) = 2 \times 26 = 52 =$ LHS
State the solution.	The simultaneous solution is $w = 11$ and $l = 15$.
State the result.	The length of the rectangle is 15 m.

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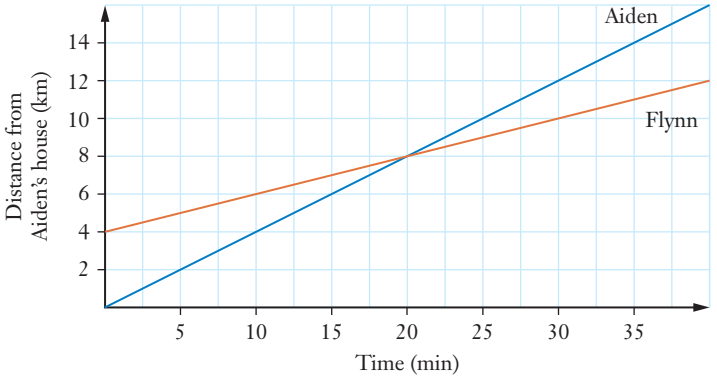
Exercise 6.03 Applications of simultaneous equations

1 Aiden set off from his home and rode his bike at 24 km/h towards the sports ground. Flynn started running to the same sports ground at 12 km/h. Flynn began his journey from a place that was 4 km closer to the sports ground than Aiden's place. The travel graph below shows their journeys.

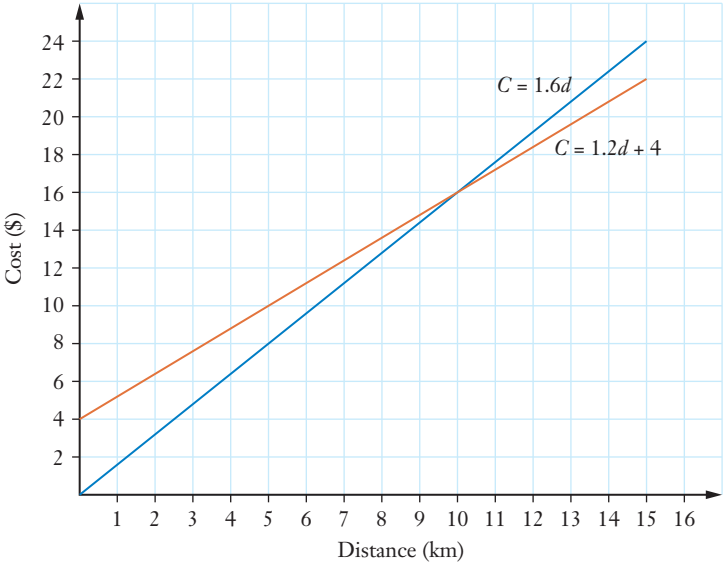


a What are their relative positions 5 minutes after they start?

b What are their relative positions after 30 minutes?

c What is the significance of the point where the graphs intersect?

The Red Top taxi company charges a flag fall of \$4 plus \$1.20 per km, this can be modelled by the linear equation: $C = 1.2d + 4$, where C is the cost to travel d kilometres. The Metro taxi company has a charging structure which gives the equation $C = 1.6d$. The graph of these equations is shown below. Use this information to answer questions **2** and **3**.



2 For a journey of 15 km, which taxi company is cheaper and by how much?

A Red Top by \$4 **B** Metro by \$2 **C** Metro by \$4
D Red Top by \$1 **E** Red Top by \$2

3 What are the coordinates of the point where the cost of the journey is the same for both taxi companies?

A (15, 10) **B** (10, 16) **C** (0, 0)
D (10, 15) **E** (10, 10)

Example 6

4 Stella sells handmade scented candles at a market for \$12.50. The amount she earns, A, for selling n candles is represented by the equation $A = 12.5n$. The amount (A) it costs to make n candles is represented by the equation $A = 84 + 5.5n$.

a Draw graphs of the equations on the same set of axes.

b Use the graph to find the break-even point.

c In the context of this problem, what does the break-even point mean?

d Determine the profit or loss, in dollars, if Stella sells 16 candles.

5 A number of adults (a) and children (c) attend a school fundraising concert. The number of people attending the concert is given by $a + c = 350$. The funds raised by the concert is given by the equation $25a + 15c = 6450$

a Draw graphs of the equations on the same set of axes.

b Use the graph to find the total number of adults and children who attended the concert.

Example 7

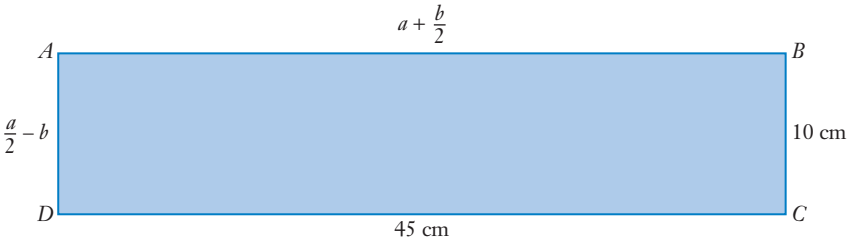
6 Mila bought 4 bagels and 3 croissants for \$21, and Ariel bought 2 bagels and 4 croissants for \$19.80.

a Letting b represent the cost of a bagel and c represent the cost of a croissant, write a pair of simultaneous equations to represent this information.

b Find the solution to the simultaneous equations developed in part **a**.

Example 8

7 $ABCD$ is a rectangle. Use this diagram to find the values of a and b.



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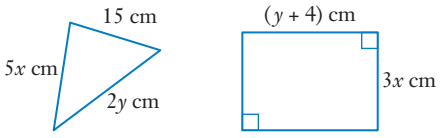
Problem solving

8 Two workers laid 250 bricks. If Sam had laid twice as many as he did and Peta had laid half as many as she did, there would have been 50 bricks left over. How many bricks did each lay?

9 I bought 3 trays of strawberries and 4 trays of cherries for \$36 at the local fruit shop. My friend bought 4 trays of strawberries and 3 trays of cherries for \$34. How much did we pay for each tray of strawberries and cherries?

10 The rectangle and triangle shown here have the same perimeter of 96 cm.



Work out the dimensions of the rectangle.

11 The total cost of tickets for 2 adults and 3 children to see a show was \$20. It cost \$31 for the tickets for a family group with 3 adults and 5 children. Find the cost of each type of ticket.

12 Ross, Trish and Helen are comparing the number of watches that they own. Ross and Trish own the same number of watches but Helen has fewer than either of them. Ross has five less than six times the number of Helen's watches, while Trish has one more than three times Helen's number of watches. How many watches does Helen have?

13 A fast-food stall owner at a football game knows that hot chips are three times as popular as meat pies and four times as popular as Chiko rolls. During half-time in the game, the fast-food outlet served 209 people who each bought one item. How many serves of each food item were sold during this period?

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6.04 Piece-wise linear graphs

Most of the functions you have seen are defined by simple formulas. Some functions have separate segments or *pieces*.

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For example, a plumber charges \$100 for a house call, including the first hour of labour and then an additional hourly rate of \$60/h. This means the plumber charges you \$100 to come to your house and work for an hour and then an extra \$60 an hour for every hour after that.

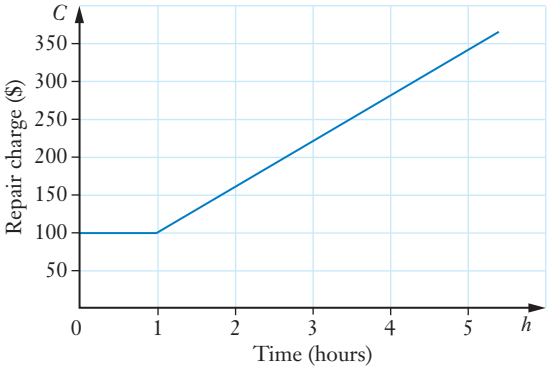
You can write the cost function, C, as:

$$C = \begin{cases} 100 & \text{if} \quad 0 < h \leq 1 \\ 100 + 60(h-1) & \text{if} \quad h > 1, \text{where } h = \text{hours worked.} \end{cases}$$

$0 < h \leq 1$ means all values of h greater than zero up to *and including* 1.

The graph is shown on the right.

Functions like this are called **piece-wise** linear functions or **line segment** functions.



Piece-wise linear functions

A piece-wise linear function is defined using two or more linear functions.

The graph of a piece-wise function consists of two or more non-overlapping line segments.

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You know only two points are required to draw the graph of a linear function. This means you can draw each segment of a piece-wise function using two points. Use the endpoints of each segment.

EXAMPLE 9

Draw a graph of the piece-wise function

$$y = \begin{cases} 3x - 4 & \text{if} \quad -1 \le x < 3 \\ x + 2 & \text{if} \quad 3 \le x \le 6 \end{cases}$$

Solution

Find the endpoints of the first segment.

$y(-1) = 3 \times (-1) - 4 = -7$

$y(3) = 3 \times 3 - 4 = 5$

State the coordinates of the endpoints.

Endpoints = $(-1, -7)$ and $(3, 5)$.

Find the endpoints of the second segment.

$y(3) = 3 + 2 = 5$

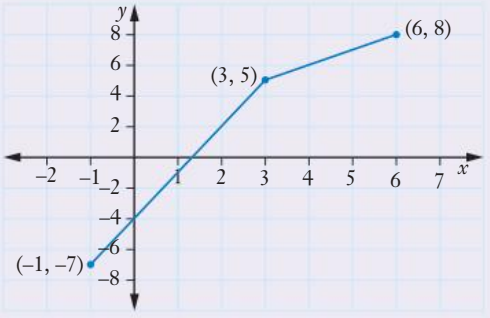
$y(6) = 6 + 2 = 8$

State the coordinates of the endpoints.

Endpoints = $(3, 5)$ and $(6, 8)$

Plot the endpoints and join them with a straight line.

$(3, 5)$ is included as it is in the second segment.



The previous example is called a **continuous** piece-wise function because the graph has no holes or jumps.

The next example is a **discontinuous** piece-wise function.

EXAMPLE 10

Draw a graph of the piece-wise function

$$y = \begin{cases} 3-4x & \text{if} \quad x<2 \\ x-3 & \text{if} \quad 2 \le x \le 6 \end{cases}$$

Solution

Choose a value of $x < 2$ for the first point at $x = 2$.

When $x = -1$,

$y(-1) = 3 - 4 \times (-1) = 7$

Now do the upper boundary.

$y(2) = 3 - 4 \times 2 = -5$

State the coordinates of the two points.

Endpoints = (–1, 7) and (2, –5), but (2, –5) is not included.

$y = x - 3 \qquad 2 \le x \le 6$

Find the endpoints of the second segment.

$y(2) = 2 - 3 = -1$

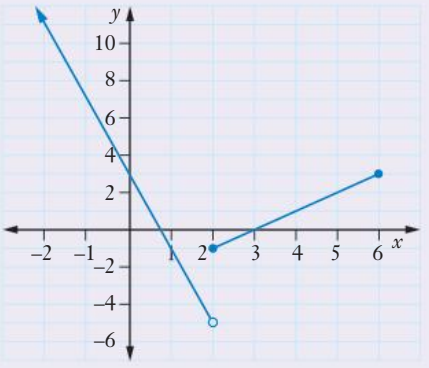
$y(6) = 6 - 3 = 3$

State the coordinates of the endpoints.

Endpoints = (2, –1) and (6, 3)

Plot the endpoints and join them with a straight line.

Insert closed circles at (2, –1) and (6, 3) to indicate these points are included in the segment. Use an open circle at (2, –5) to show it is not included.



You can see that even though the piece-wise function

$$y = \begin{cases} 3-4x & \text{if} \quad x<2 \\ x-3 & \text{if} \quad 2 \le x \le 6 \end{cases}$$

is discontinuous, the line segments do not overlap.

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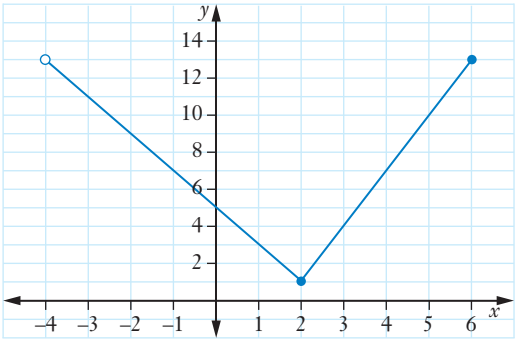
Exercise 6.04 Piece-wise linear graphs

Use the following piece-wise function and its graph to answer questions **1** and **2**.



$$y = \begin{cases} 5 - 2x \text{ if } -4 < x \le 2 \\ 3x - 2 \text{ if } 2 < x \le 6 \end{cases}$$

1 For the segment $y = 5 - 2x$, which of the following is true?

A The point (–4, 13) is included, but the point (2, 1) is not included.

B Neither the point (–4, 13) nor the point (2, 1) are included.

C Both the point (–4, 13) and the point (2, 1) are included.

D The point (–4, 13) is not included, but the point (2, 1) is included.

E None of the above.

2 For the segment $y = 3x - 2$, which of the following is true?

A Both the point (2, 1) and the point (6, 13) are included.

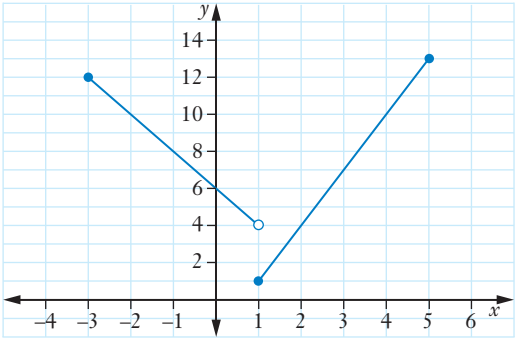
B The point (2, 1) is not included but the point (6, 13) is included.

C The point (2, 1) is included but the point (6, 13) is not included.

D Neither the point (2, 1) nor the point (6, 13) are included.

E None of the above.

3 The graph of a piece-wise function is shown on the right. Which of the following correctly defines the function?



A $y = \begin{cases} 6 - 2x & \text{if } -3 \le x < 1 \\ 3x - 2 & \text{if } 1 \le x \le 5 \end{cases}$

B $y = \begin{cases} 6 - 2x & \text{if } -3 < x < 1 \\ 3x - 2 & \text{if } 1 \le x \le 5 \end{cases}$

C $y = \begin{cases} 6 - 2x & \text{if } -3 \le x \le 1 \\ 3x - 2 & \text{if } 1 \le x \le 5 \end{cases}$

D $y = \begin{cases} 6 - 2x & \text{if } -3 \le x < 1 \\ 3x - 2 & \text{if } 1 \le x < 5 \end{cases}$

E $y = \begin{cases} 6 - 2x & \text{if } -3 \le x < 1 \\ 3x - 2 & \text{if } 1 < x \le 5 \end{cases}$

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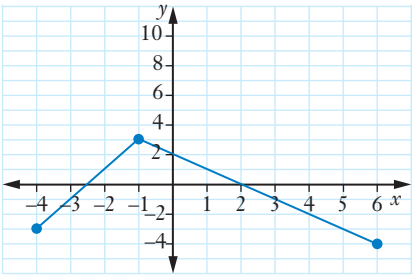
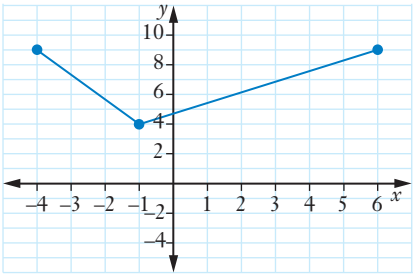
4 Which graph best represents the piece-wise linear function?

$$y = \begin{cases} 2x + 5 & \text{if } -4 \le x \le -1 \\ x + 4 & \text{if } -1 < x \le 6 \end{cases}$$

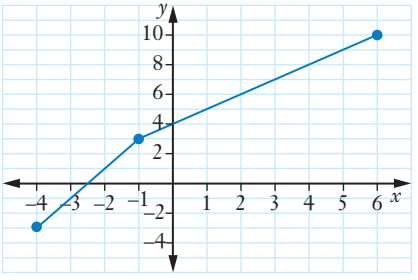
A



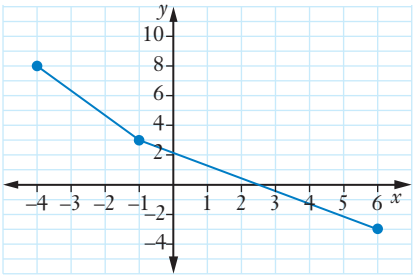
B



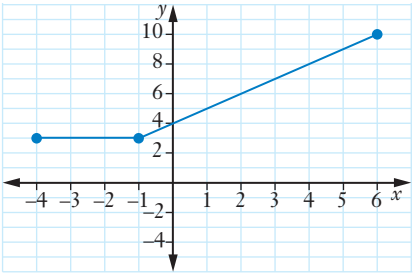
C



D



E



5 Draw a graph of each piece-wise linear function.

a $y = \begin{cases} x & \text{if } x < 3 \\ 3 & \text{if } x \ge 3 \end{cases}$

b $y = \begin{cases} -x & \text{if } x \le 1 \\ -1 & \text{if } x > 1 \end{cases}$

c $y = \begin{cases} x + 5 & \text{if } x < -2 \\ 3 & \text{if } x \ge -2 \end{cases}$

d $y = \begin{cases} 1 - x & \text{if } x \le 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

e $y = \begin{cases} x + 2 & \text{if } x < 2 \\ 6 - x & \text{if } x \ge 2 \end{cases}$

f $y = \begin{cases} 2x + 1 & \text{if } x \le -2 \\ x - 1 & \text{if } x > -2 \end{cases}$

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6 Draw a graph of each piece-wise linear function.

Example 9

a $y = \begin{cases} 2x-1 & \text{if } -4 \le x < 1 \\ 3x-2 & \text{if } 1 \le x \le 5 \end{cases}$

b $y = \begin{cases} \frac{1}{2}x + \frac{3}{2} & \text{if } -4 \le x \le 1 \\ 3-x & \text{if } 1 < x \le 5 \end{cases}$

c $y = \begin{cases} 2x+1 & \text{if } -3 \le x < 1 \\ 4-x & \text{if } 1 \le x \le 4 \end{cases}$

d $y = \begin{cases} 5x-1 & \text{if } -4 \le x \le -2 \\ x-9 & \text{if } -2 < x \le 5 \end{cases}$

e $y = \begin{cases} 2x & \text{if } -3 \le x \le 1 \\ 3-x & \text{if } 1 < x \le 5 \end{cases}$

f $y = \begin{cases} -x & \text{if } -2 \le x < 2 \\ x-4 & \text{if } 2 \le x \le 6 \end{cases}$

7 Draw a graph of each piece-wise linear function.

Example 10

a $y = \begin{cases} x+2 & \text{if } -2 \le x \le 2 \\ 2x+1 & \text{if } 2 < x \le 5 \end{cases}$

b $y = \begin{cases} x+3 & \text{if } -4 \le x < 0 \\ x & \text{if } 0 \le x \le 4 \end{cases}$

c $y = \begin{cases} x-8 & \text{if } -3 \le x \le 3 \\ \frac{1}{3}x-2 & \text{if } 3 < x \le 9 \end{cases}$

d $y = \begin{cases} x+6 & \text{if } -6 \le x < -3 \\ -\frac{2}{3}x-3 & \text{if } -3 \le x \le 6 \end{cases}$

e $y = \begin{cases} 2x+5 & \text{if } -3 \le x \le 1 \\ x+1 & \text{if } 1 < x \le 5 \end{cases}$

f $y = \begin{cases} \frac{1}{2}x-5 & \text{if } -4 \le x < 2 \\ -x-1 & \text{if } 2 \le x \le 6 \end{cases}$

Problem solving

8 Draw a graph of this piece-wise linear function.

$$y = \begin{cases} -x-1 & \text{if } -5 \le x \le -2 \\ x+1 & \text{if } -2 < x \le 1 \\ 1-x & \text{if } 1 \le x < 4 \end{cases}$$

9 For the graph of this piece-wise function:

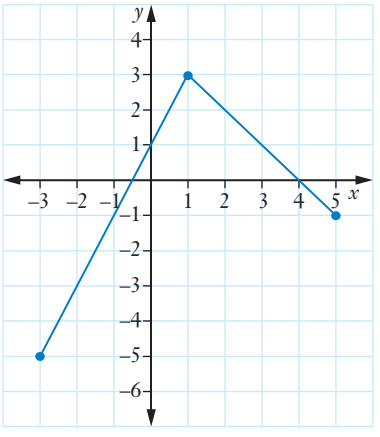
a state the domain for each segment

b calculate the gradient and y-intercept of the segment on the left

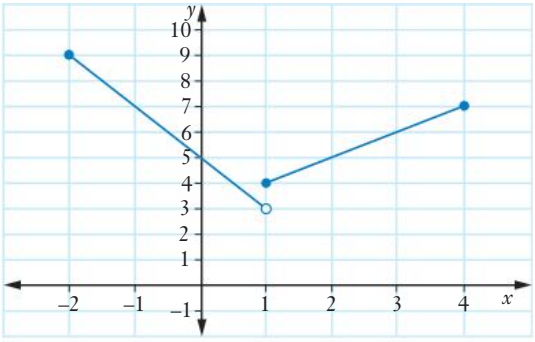
c write the equation of the segment on the left

d write the equation of the segment on the right

e write the equation for the piece-wise function.



10 Write the equation for this piece-wise function.



Step graphs

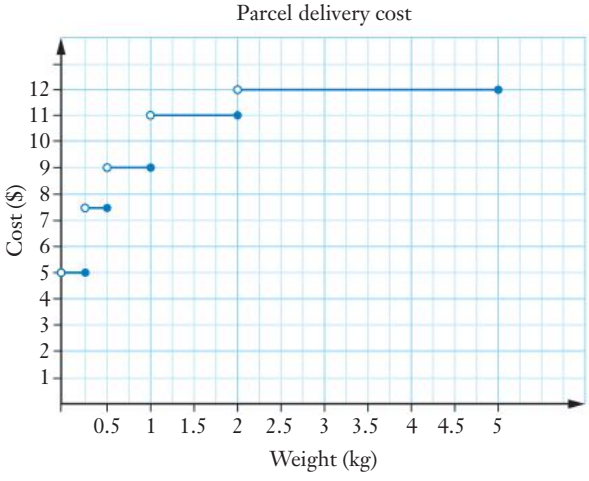
6.05 Step graphs

A small courier specialises in delivering parcels up to 5 kg in weight to buildings in the Brisbane central business district (CBD). Its charges are as follows:

Up to and including 250 g	$5.00
More than 250g and up to and including 500 g	$7.50
More than 500g and up to and including 1 kg	$9.00
More than 1 kg and up to and including 2 kg	$11.00
More than 2 kg and up to and including 5 kg	$12.00

Dreamstime.com/Marklamoyne

These charges are shown here.



This is a special type of piece-wise graph called a **step graph** because it looks like steps.

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EXAMPLE 11

Use the step graph on the previous page to answer the following questions.

a How much would it cost to deliver a 4 kg parcel to the CBD?

b How much would it cost to deliver a 2 kg parcel?

c How much would it cost to deliver a 500 g parcel?

d How much would it cost to deliver a 251 g parcel?

e If you only have $9 to spend, what are the range of weights the parcel could be?

Solution

a Draw a line vertically up from 4 kg until it crosses a step.

Read the cost from the vertical axis.

State the result. A 4 kg parcel would cost $12 to be delivered.

b Draw a line vertically up from 2 kg until it crosses a step.

Read the cost from the vertical axis.

A 2 kg parcel would cost $11 to be delivered.

c Draw a line vertically up from 500 g until it crosses a step.

A 500 g parcel would cost $7.50 to be delivered.

d Draw a line vertically up from 251 g until it crosses a step.

A 251 g parcel would cost $7.50 to be delivered.

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e Locate \$9 on the vertical axis.

Locate the step that corresponds to \$9.

State the result.

For \$9, parcels greater than 500 g and less than or equal to 1 kg can be delivered.

A step graph can be defined by equations like piece-wise functions.

EXAMPLE 12

Graph the step function defined by the equation:

$$y = \begin{cases} 2 & \text{if } -1 < x \leq 1 \\ 4 & \text{if } 1 < x \leq 3 \\ 6 & \text{if } 3 < x \leq 5 \\ 8 & \text{if } 5 < x \leq 7 \end{cases}$$

Solution

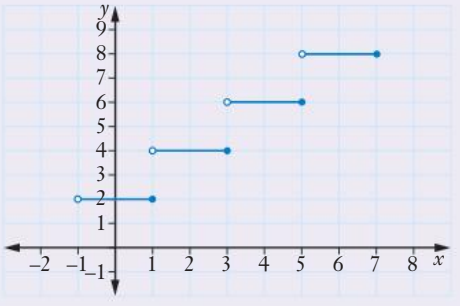
The first step has $y = 2$ for x values between –1 and 1.

Draw a horizontal line from –1 to 1 at $y = 2$.

$x = -1$ is not included, so draw an open circle at (–1, 2).

$x = 1$ is included, so draw a closed circle at (1, 2).

Repeat the process for the other steps to complete the graph.



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INVESTIGATION

FLOOR AND CEILING FUNCTIONS

Two types of step functions are the floor and ceiling functions.

The 'floor' symbol is written as $\lfloor\ \rfloor$ and is defined as follows:

$\lfloor x \rfloor$ is the greatest integer less than or equal to x.

The 'ceiling' symbol is written as $\lceil\ \rceil$ and is defined as follows:

$\lceil x \rceil$ is the least integer greater than or equal to x.

1 The floor function is $f(x) = \lfloor x \rfloor$ for all numbers x.

a Complete the following table of values for the floor function.

x	–3.5	–2.1	–1.8	–1	1.6	3.7	5.9	7.01	7.99	8.3	8.37
$\lfloor x \rfloor$											

b Draw the graph of $f(x) = \lfloor x \rfloor$ for $-4 \le x < 4$.

2 The ceiling function is $f(x) = \lceil x \rceil$ for all numbers x.

a Complete the following table of values for the ceiling function.

x	–3.5	–2.1	–1.8	–1	1.6	3.7	5.9	7.01	7.99	8.3	8.37
$\lceil x \rceil$											

b Draw the graph of $f(x) = \lfloor x \rfloor$ for $-4 \le x < 4$.

3 The floor function is also known as the greatest-integer function or the rounding-down function. The ceiling function is also called the rounding-up function.

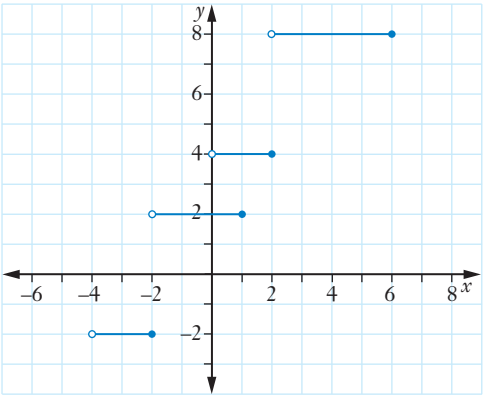
a Find out where floor functions and ceiling functions are used.

b Make a list of applications where these functions could be used.

Exercise 6.05 Step graphs

1 A student is supplied with a correct equation for a particular step function and asked to draw its graph. The graph produced by the student is shown below.

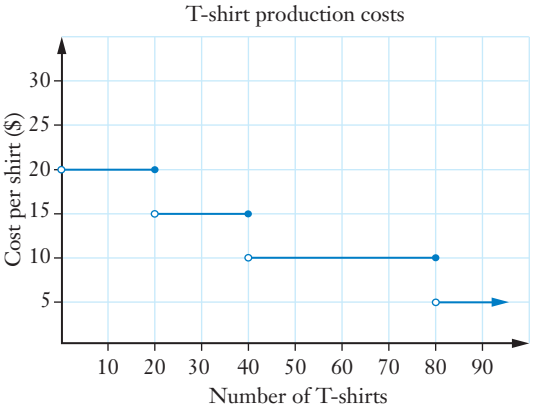
List any errors that you think the student has made.



Example 11

2 A company makes T-shirts for small and large retailers. The cost charged to manufacture a T-shirt varies depending on how many T-shirts are ordered.

The graph on the right shows the cost per T-shirt paid by retailers.



a How much does each T-shirt cost if:

- **i** 15 are ordered?
- **ii** 20 are ordered?
- **iii** 21 are ordered?
- **iv** 62 are ordered?

b What is the total cost of two separate orders for 52 T-shirts and 25 T-shirts?

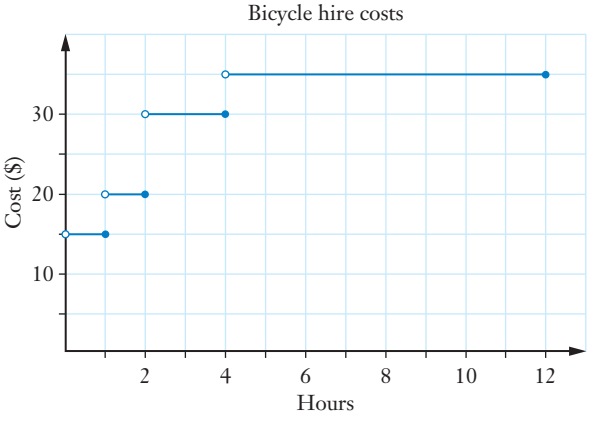
c How does the unit cost of a T-shirt change as the number of T-shirts manufactured increases? Why?

d How much does each T-shirt cost if 200 are ordered?

3 The cost of hiring a bicycle is shown in the step graph at right.

a What is the cost per hour when a bike is hired for 7 hours?

b What is the cost per hour when a bike is hired for 2 hours?



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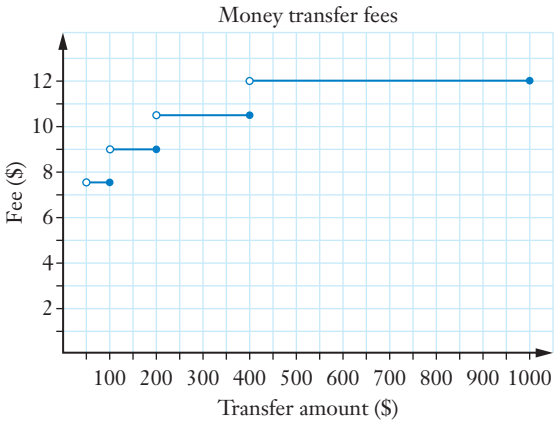
c What is the longest period a bicycle can be hired for $30?

d Three friends hire a bicycle each. One rides for half an hour, the second rides for 8 hours and the third for 4 hours. What is the total cost for the group to hire their bicycles?

4 The various fees charged to transfer money overseas are shown in the graph at right.



a How much does it cost to transfer $150?

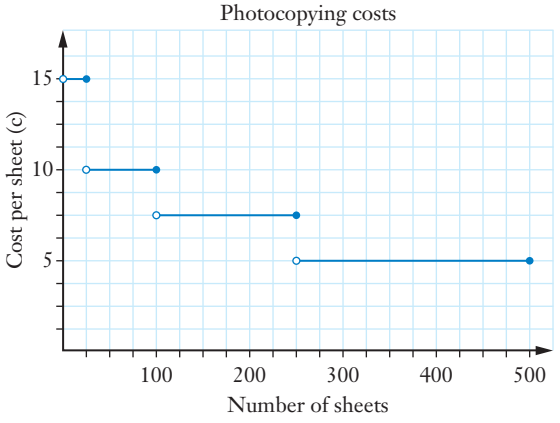
b How much does it cost to transfer $526?

c How much does it cost to transfer $201?

d How much does it cost to transfer $398?

e What is the maximum amount of money that can be transferred for $7.50?

5 The cost per sheet for photocopying charged by an office supplies company is shown in the graph at right.



a How much does each copy cost if:

i 75 are printed?

ii 10 are printed?

iii 421 are printed?

iv 250 are printed?

b What is the total cost of two separate print runs of 280 copies and 95 copies?

c How does the unit cost of copying change as the number of copies increases?

Example 12

6 Graph each step function.

a $y = \begin{cases} -2 & \text{if } -3 < x \le -1 \\ 1 & \text{if } -1 < x \le 5 \\ 4 & \text{if } 5 < x \le 9 \end{cases}$

b $y = \begin{cases} 2 & \text{if } 0 < x \le 10 \\ 4 & \text{if } 10 < x \le 20 \\ 8 & \text{if } 20 < x \le 30 \\ 16 & \text{if } x > 30 \end{cases}$

c $C = \begin{cases} \$12 & \text{if } 0 < x \le 8 \\ \$16 & \text{if } 8 < x \le 20 \\ \$20 & \text{if } 20 < x \le 32 \\ \$30 & \text{if } 32 < x \le 50 \end{cases}$

d $y = \begin{cases} -3 & \text{if } -3 \le x < -2 \\ -2 & \text{if } -2 \le x \le -1 \\ -1 & \text{if } -1 \le x \le 0 \\ 0 & \text{if } 0 \le x \le 1 \\ 1 & \text{if } 1 \le x \le 2 \\ 2 & \text{if } 2 \le x \le 3 \\ 3 & \text{if } 3 \le x \le 4 \end{cases}$

Problem solving

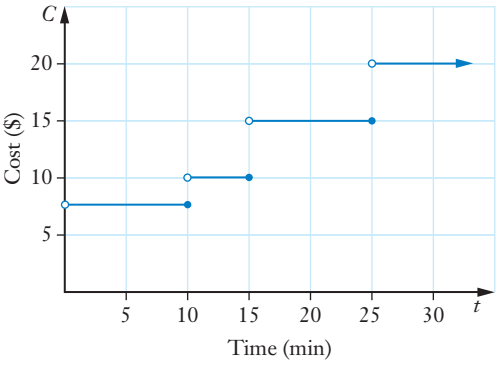
7 A mobile gardener charges a call-out fee of \$70 including the first hour of work. Labour is then charged at the rate of \$30 per hour or part thereof for the next 2 hours and \$25 per hour or part thereof for any additional hours.

a Use the information to complete the following table.

Hours (h)	0	0.5	1	1.5	2	2.5	3	3.5	4	5	6
Cost (C)	0										

b Use the completed table and the information provided to draw a step graph representing the fees charged by the gardener.

8 Use the step graph shown at right to develop an equation for the step function it represents.



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6.06 Using piece-wise and step graphs

There are many real-life situations where piece-wise functions and step functions are used. Examples include:

- wage rates
- taxation rates
- electricity rates
- taxi fares
- commissions, and
- parking fees.

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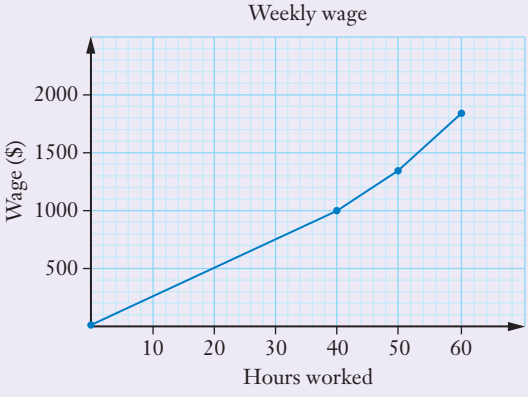
EXAMPLE 13

Monique is a chef who works a normal 40-hour week. She is paid a penalty rate when she works more than 40 but less than 50 hours a week and an even higher rate of pay when she works more than 50 but less than 60 hours a week.

The piece-wise graph below shows Monique's weekly wages for various hours worked.



a Describe the graph and what it represents.

b Use the graph to find the wages earned when Monique works for 53 hours.

c Use the graph to find how many hours Monique worked if she earned $600 for the week.

d Calculate Monique's hourly rate of pay if she works less than 40 hours in a week.

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Solution

a Study the graph and comment on your observations.

There are three line segments in this piece-wise graph.

If h is the hours worked in a week, the first segment covers $0 < h \leq 40$, the second segment covers $40 < h \leq 50$ and the third segment covers $50 < h \leq 60$.

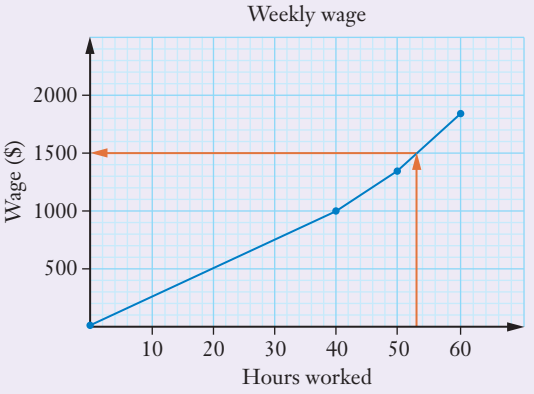
The gradients of the segments increase as the hourly rate of pay increases.

The gradient of the segment represents the hourly rate of pay for the number of hours covered by the segment.

b Locate 53 hours on the horizontal axis.

Draw a line vertically until it reaches the segment.

Draw a line horizontally across to the vertical axis and read the value.



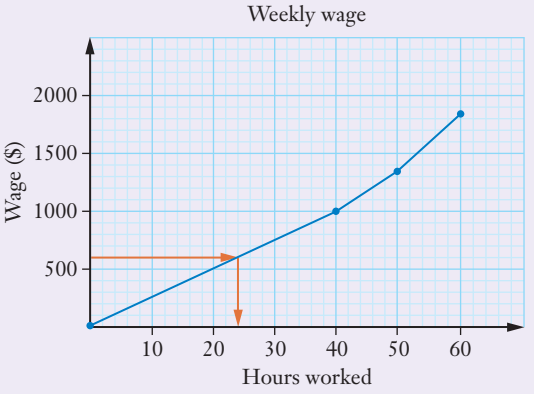
State the result.

Monique earned $1500 for the week.

c Locate $600 on the vertical axis.

Draw a line horizontally until it reaches the segment.

Draw a line vertically down to the horizontal axis and read the value.



State the result.

Monique worked 24 hours in the week.

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d The hourly rate of pay is the same as the gradient for the segment. Calculate the gradient of the first segment by choosing any convenient point, e.g. (500, 20).

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{500}{20} = 25$$

State the result.

The hourly rate of pay for working less than 40 hours a week is \$25/hour.

There are some situations where you will need to draw a piece-wise or step graph from the information provided.

EXAMPLE 14

A security system salesman works for commission only. He earns 8% commission for weekly sales up to and including \$6000, 12% for sales greater than \$6000 up to and including \$15 000 and 18% on all sales above \$15 000.

a Use this information to complete the following table.

Weekly sales (\$$S$)	0	6000	15 000	20 000
Commission (\$$C$)				

b Use the completed table to draw a piece-wise graph for the commission (C) earned by the salesman for weekly sales (S) up to \$20 000.

Solution

a State the different rates of commission.

Commission rates are:

8% when $0 < S \leq 6000$
12% when $6000 < S \leq 15\,000$
18% when $15\,000 < S \leq 20\,000$

When there are no sales, no commission is paid.

When $S = \$0$, $C = \$0$.

Calculate the commission on sales of \$6000.

$C = 8\%$ of \$6000

$8\% = \frac{8}{100} = 0.08$

$= 0.8 \times 6000$

$= \$480$

State the result.

When $S = \$6000$, $C = \$480$

For sales of \$15 000, commission of 8% is paid on the first \$6000 and 12% on the remainder, i.e. \$15 000 – \$6000 = \$9000.

Calculate the commission on sales of \$15 000.

Commission on \$15 000

= commission on \$6000 + commission on \$9000

Commission on \$6000 has already been calculated.

= \$480 + 12% of \$9000

= 480 + 0.12 × 9000

= 480 + 1080

= \$1560

State the result.

When S = \$15 000, C = \$1560

For sales of \$20 000, commission has already been paid on sales up to \$15 000. Calculate the remainder.

Remainder = \$20 000 – \$15 000

= \$5000

Commission on the first \$15 000 has already been calculated.

Commission on \$20 000

= \$1560 + 18% of \$5000

= 1560 + 0.18 × 5000

= 1560 + 900

= \$2460

State the result.

When S = \$20 000, C = \$2460

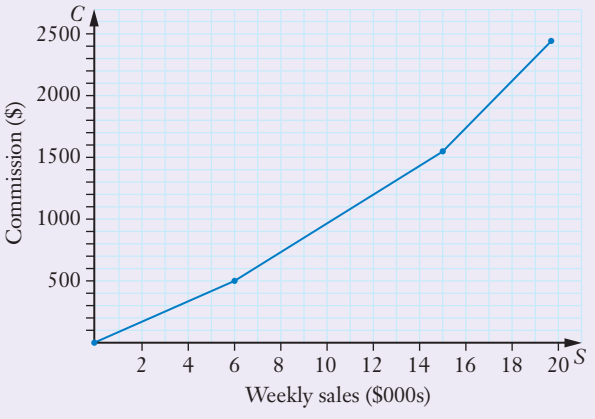
Use the calculated values to complete the table.

***S* (\$)**	0	6000	15 000	20 000
***C* (\$)**	0	480	1560	2460

b Draw up a set of axes. Label the horizontal axis 'Weekly sales (\$$S$)'. Label the vertical axis 'Commission (\$$C$)'.

Plot the points from the table of values.

Joint the end points of each segment



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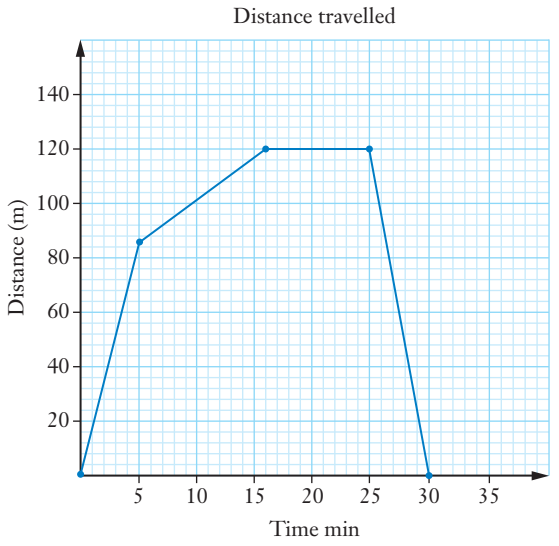
Exercise 6.06 Using piece-wise and step graphs

This piece-wise graph shows the distance travelled by an object from its starting point. Use the graph to answer questions **1**–**4**.



1 The object is at rest after

A 5 min

B 28 min

C 30 min

D 16 min

E 10 min

2 The object returns to its starting point after

A 20 min **B** 30 min **C** 15 min **D** 5 min **E** 25 min

3 The object changes speed

A 2 times **B** 0 times **C** 4 times **D** 3 times **E** 1 time

4 The total distance travelled by the object is:

A 0 m **B** 120 m **C** 30 m **D** 300 m **E** 240 m

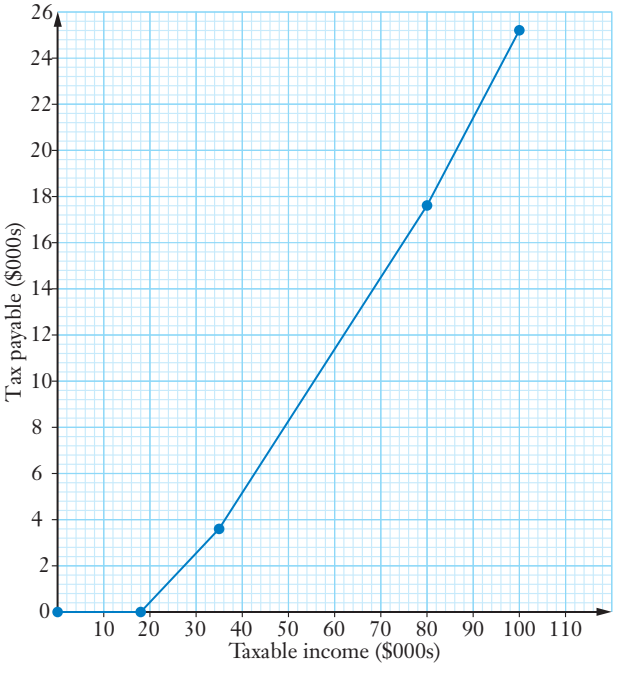
Example **13**

5 This piece-wise linear graph shows the income tax payable on taxable incomes up to $100 000 in Australia.

a Describe the major features of the graph.

b Which incomes attract the highest rate of taxation?

c Which incomes attract the lowest rate of taxation?



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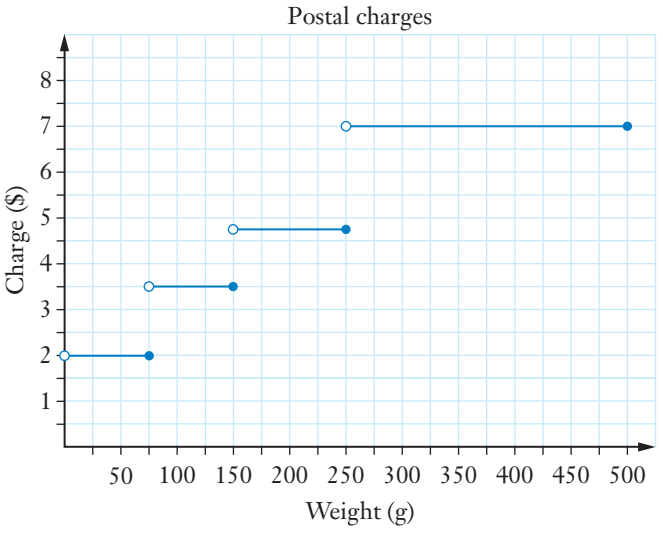
d Use the graph to find the tax payable on a taxable income of:

i \$15 000 **ii** \$85 000 **iii** \$60 000 **iv** \$48 000

e What is the approximate rate of taxation for incomes between \$37 000 and \$80 000?

6 The step graph shown below shows the postal charges for non-standard items up to 500 g in weight.



a How much does it cost to post an item that weighs:

i 110 g? **ii** 250 g? **iii** 300 g? **iv** 25 g? **v** 252 g?

b If the postal charge is C and the weight of the item is w, complete the following equation for the postal cost step function.

$$C = \begin{cases} \$2 & \text{if } \ldots < w \leq \ldots \\ \$\ldots & \text{if } \ldots < w \leq \ldots \\ \ldots & \text{if } \ldots < w \leq \ldots \\ \ldots & \text{if } \ldots < w \leq \ldots \end{cases}$$

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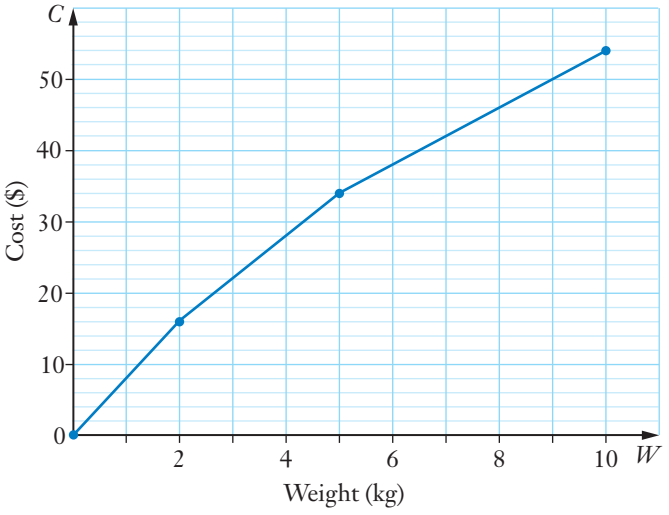
7 The piece-wise graph below shows the cost of purchasing almonds from a wholesaler.



Use the graph to answer the questions that follow.

a How much does it cost to purchase 3 kg of almonds?

b How much does it cost to purchase 8 kg of almonds?

c How much does it cost to purchase 6.5 kg of almonds?

d What is the price per kilogram charged for more than 2kg but less than 5 kg of almonds?

e Comment on how the price per kilogram charged for the almonds changes as the weight of almonds purchased increases.

8 An online retailer charges different fees for packaging and shipping orders depending on the value of the item purchased. The fees are shown below:

Example 14

$$P = \begin{cases} \$3.50 & \text{if } 0 < c \leq 25 \\ \$6.50 & \text{if } 25 < c \leq 75 \\ \$8.25 & \text{if } 75 < c \leq 125 \\ \$10 & \text{if } c > 125 \end{cases}$$

where P is the packaging and shipping fee and c is the cost of the order.

a What type of function is described?

b Use this information to draw a graph for the packaging and shipping fees for orders up to $150.

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Problem solving

9 A wholesaler sells mobile phone screen covers online. The wholesaler charges a flat delivery fee of $10 regardless of the size of the order. The cost of the screen covers is:

- $2.50 each for orders up to and including 20
- $2 each for orders of more than 20 and up to and including 50
- $1.80 for orders over 50

a Use this information to copy and complete this table.

Number ordered (n)	0	20	50	100
Cost ($\$C$)				

b Use the completed table to draw a piece-wise graph for the cost of orders up to 100 screen covers.

10 A mobile phone carrier has a plan that includes unlimited talk and text for $30 a month. The data charges are $4/month for up to 500 MB, $10/month for more than 500MB and up to (and including) 2GB, $15/month for more than 2 GB and up to (and including) 5 GB and $20/month for more than 5 GB and up to (and including) 10 GB.

a Draw a step graph to represent this data.

b Use the graph to calculate the monthly charge if 4.3 GB of data is used.

c If C is the monthly cost of the plan and n is the number of GB used, write an equation to define this function.

11 The Hawks netball team decides to order custom-made T-shirts for its grand final win. If the Hawks order 20 or fewer T-shirts, the cost is $15 per shirt. If they order more than 20 but less than or equal to 50, the cost is $12 per shirt. If they order more than 50, the cost is $10 per shirt.

a Use this information to copy and complete the following table.

T-shirts ordered (n)	0	20	50	70
Cost ($\$C$)				

b Use the completed table to draw a piece-wise graph for the cost of orders up to 70 T-shirts.

c Use the graph find the cost if :

i 45 T-shirts are ordered **ii** 62 T-shirts are ordered

d If C is the total cost of the order and n is the number of T-shirts ordered, write an equation to define this function.

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12 The table shows the personal income tax rates on the annual taxable incomes of Australian citizens.

Taxable Income	Tax on this Income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$87 000	$3572 plus 32.5c for each $1 over $37 000
$87 001–$180 000	$19 822 plus 37c for each $1 over $87 000
$180 001 and over	$54 232 plus 45c for each $1 over $180 000

Source: https://www.ato.gov.au/Rates/Individual-income-tax-rates/

a How many rates of income tax are possible?

b What is the highest rate of income tax?

c Use the data in the table to construct a linear piece-wise graph that shows the tax payable on annual taxable incomes up to $200 000.

d Use the graph to find the tax paid by someone who earns $50 000 p.a.

e Use the graph to find the tax paid by someone who earns $150 000 p.a.

13 The fares of an inner city taxi company are defined as:

$$T = \begin{cases} 6 & \text{if } 0 < x \leq 2 \\ 2.5(x-2)+6 & \text{if } 3 < x \leq 8 \\ 4(x-8)+21 & \text{if } 8 < x \leq 20 \end{cases}$$

where T is the fare ($) and x is the distance travelled (km).

a What type of function is described here?

b Use this information to draw a graph for taxi fares for distances up to 20 km.

c How much would you expect to pay if you travelled a distance of 15 km?

6. CHAPTER SUMMARY

Simultaneous equations and piece-wise graphs

- **Simultaneous equations** are equations that can be solved at the same time. Any solution of these equations is known as a **simultaneous solution**.
- Simultaneous equations may be solved graphically or algebraically. The point where two graphs intersect is called the **point of intersection**. The point of intersection indicates the simultaneous solution of the equations. Simultaneous equations whose graphs are parallel have no simultaneous solution.
- One algebraic method for the solution of simultaneous linear equations is known as **substitution**. The substitution method follows these steps:
  - Use one equation to express one variable in terms of the other.
  - Substitute this into the other equation.
  - Solve the resulting equation.
  - Use the solution to find the value of the other variable.
  - Check the simultaneous solution.
- The procedure for solving word problems using simultaneous equations is:
  1. Read the information carefully
  2. Choose variable names for the unknown quantities.
  3. Use the information in the problem to form equations in terms of the variables
  4. Solve the equations using the most convenient method
  5. Check the solution
  6. Write the solution in the same terms as the stated problem
- A **piece-wise linear function** is defined using two or more linear functions. The graph of a piece-wise function consists of two or more non-overlapping line segments. The ends of each segment in the graph of a piece-wise function are labelled with a circle. An open circle indicates that the point is not included in the segment. A closed circle is used to indicate that the point is included in the segment.
- The graph of a **continuous** piece-wise function has no holes or jumps. In the graph of a **discontinuous** piece-wise function, the segments do not connect.
- A **step graph** is a special type of piece-wise graph made up of horizontal segments. The ends of the segments have either open or closed circles. The graph looks like the steps of a staircase.

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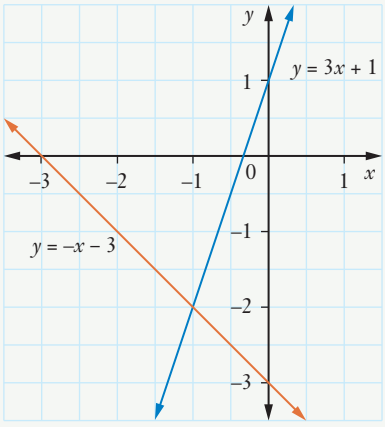
6. CHAPTER REVIEW

Simultaneous equations and piece-wise graphs

1 Use this graph to determine the simultaneous solution of the equations:

$y = 3x + 1$

$y = -x - 3$



Example 1

2 Check whether the point (1, 2), satisfies the following system of equations.

$$7x + y = 9$$

$$2y - 3x = 1$$

Example 2

3 Find the solution of the following simultaneous equations by graphing.

$$y + 2x = 10$$

$$2y - 3x + 8 = 0$$

Example 3

4 What is the simultaneous solution for:

$$3x + 5y = 31 \text{ and } 2x + 3y = 20$$

A (–2, 7) **B** (1, 3) **C** (7, 2) **D** (–1, –3) **E** (0, 6)

Example 2

5 What is the simultaneous solution for:

$$y - 3x = 18 \text{ and } 7x - 3y = -44$$

A (5, –3) **B** (1, 21) **C** (2, 24) **D** (–5, 3) **E** (–2, –8)

Example 2

6 Solve these simultaneous equations.

$$3a - 2b = 19$$

$$a = -3 - 4b$$

Example 4

7 Solve each pair of simultaneous equations.

a $4a + b = 6$
$a - b = 4$

b $a + 7b = 19$
$2a - 3b = -13$

Example 5

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8 The simultaneous equations $y = 3 - 2x$ and $3x - y = 2$ have the solution

A $x = 1$ and $y = 1$ **B** $x = -1$ and $y = -1$ **C** $x = -\frac{1}{2}$ and $y = -\frac{1}{2}$

D $x = \frac{1}{2}$ and $y = \frac{1}{2}$ **E** $x = 1$ and $y = -1$

Example 6

9 At Burleigh Heads beach, it is possible to hire a surfboard by the hour in two different locations. At the Ride High surf hire shop the cost is \$20 plus \$12 per hour, while at the Ripper Tube surf hire shop the cost is \$8 plus \$18 per hour. The surfboards can be rented for up to 8 hours.

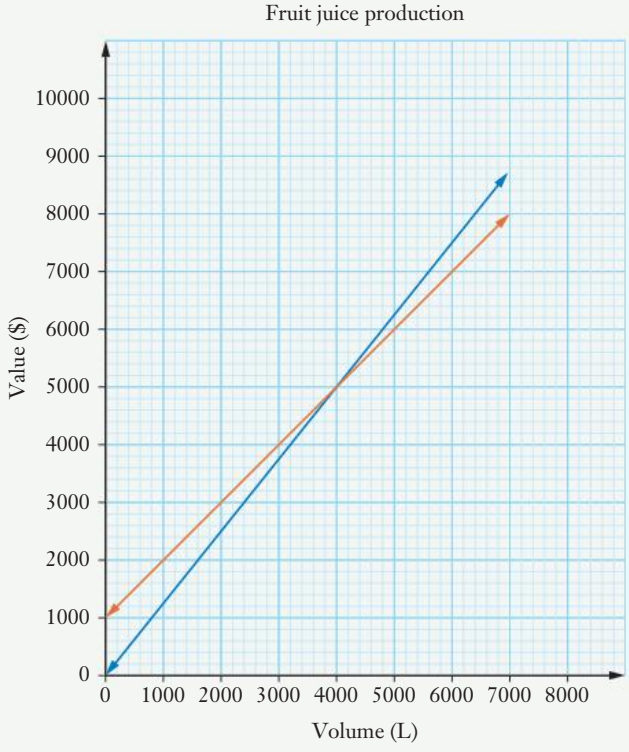
a Write the rules relating cost to the length of rental.

b On the same set of axes, sketch a graph of cost (C) (y-axis) against hours of rental (h) (x-axis) for 0 to 8 hours.

c Use your graph to find for what rental times, if any, the Ride High rental is cheaper than the Ripper Tube rental.

d For what length of rental time are the two rental schemes identical in cost?

The graph below shows the production cost (red line) and the revenue (blue line) for the production of fruit juice at the Berry Good Juice Company. Use this information to answer questions **12** and **13**.



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10 Joel needs to buy some flash drives and memory sticks to back up a large amount of data that has been generated by an accounting firm. He buys 6 flash drives and 3 memory sticks for \$96. He later realises these are not sufficient and so buys another 5 flash drives and 4 memory sticks for \$116. (Assume the same rate per item was charged for each visit.) Example 7

a Develop two equations that represent this situation.

b How much did each flash drive and each memory stick cost?

11 The width of a rectangle is 5 more than half its length. The perimeter of the rectangle is 64 m. Find the length of the rectangle. Example 8

12 If 1600 L of juice is produced, the result for the company is: Example 6

A a loss of \$2600 **B** a profit of \$600 **C** a profit of \$2600
D a loss of \$600 **E** a loss of \$300

13 What are the coordinates of the point where the cost of production is the same as the revenue? Example 6

A (0, 0) **B** (4000, 5000) **C** (3000, 3000)
D (5000, 4000) **E** (6000, 4000)

14 A piece-wise function is defined as: Example 9

$$y = \begin{cases} -x-2 & \text{if } x < 2 \\ x-6 & \text{if } x \geq 2 \end{cases}$$

Draw the graph of this function.

15 Draw a graph of the piece-wise function Example 10

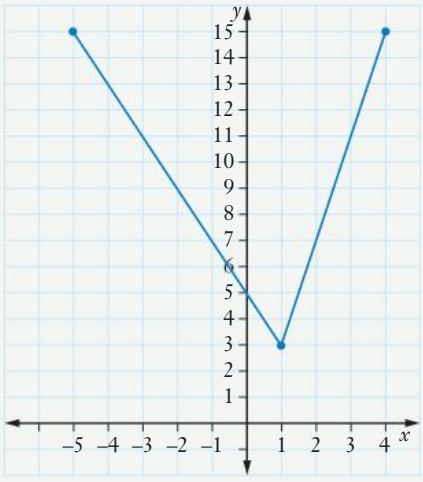
$$y = \begin{cases} x-2 & \text{if } -5 \leq x < 2 \\ 4-2x & \text{if } 2 \leq x \leq 6 \end{cases}$$

16 Draw a graph of the piece-wise function Example 10

$$y = \begin{cases} 3-x & \text{if } -4 < x \leq 2 \\ 2x+1 & \text{if } 2 < x \leq 7 \end{cases}$$

Use the following piece-wise function and its graph to answer questions **17** and **18**.

$$y = \begin{cases} 5-2x & \text{if } -5 \leq x < 1 \\ 4x-1 & \text{if } 1 \leq x \leq 4 \end{cases}$$



17 For the segment $y = 5 - 2x$, which of the following is true? Example 11

A Neither the point (–5, 15) nor the point (1, 3) are included.

B The point (–5, 15) is included but the point (1, 3) is not included.

C Both the point (–5, 15) and the point (1, 3) are included.

D The point (–5, 15) is not included but the point (1, 3) is included.

E None of the above.

Example 11

18 For the segment $y = 4x - 1$, which of the following is true?

A The point (1, 3) is included but the point (4, 15) is not included.

B The point (1, 3) is not included but the point (4, 15) is included.

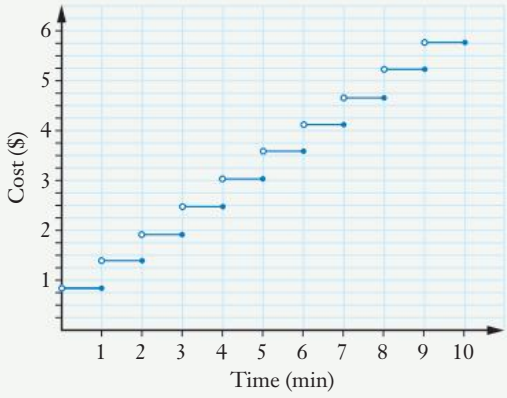
C Both the point (1, 3) and the point (4, 15) are included.

D Neither the point (1, 3) nor the point (4, 15) are included.

E None of the above.

Example 11

19 Ethan is travelling overseas. He pays 25c for making a call and an additional fee of 55c/minute or part thereof with the phone card he is using. The step graph shows the charges for calls up to 10 minutes in duration.



a One call is charged at $1.95. What is the longest period for which that call could have been made?

b Another call is charged at $4.10. For what range of times could that call have lasted?

c How much will a call lasting 5 seconds cost?

d If there is only $3 left on his phone card, what is the longest call that Ethan can make?

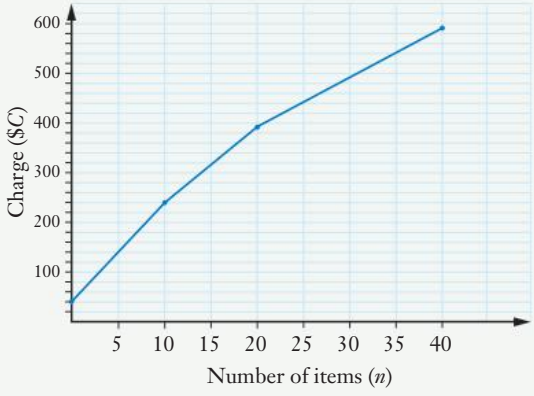
Example 12

20 Graph the step function defined by the equation:

$$C = \begin{cases} 6 & \text{if } 0 < t \le \frac{1}{2} \\ 10 & \text{if } \frac{1}{2} < t \le 1 \\ 16 & \text{if } 1 < t \le 2 \\ 24 & \text{if } t > 2 \end{cases}$$

Example 13

21 A silk-screen shop specialises in printing images on T-shirts. The owner of the shop charges an initial fee to create the silk screen and then an additional charge to print the T-shirts. The graph represents the silk-screen shop's charges.



a What is the fee to set up the silk screen?

b How much would it cost for an order of 25 T-shirts?

c What is the cost per item for an order of 10 T-shirts?

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22 The fines for speeding in Queensland are shown in the table.

Example 14

Speed over the speed limit	Fine
Less than 13 km/h	\$ 162
At least 13 km/h but not more than 20 km/h	\$ 243
More than 20 km/h but not more than 30 km/h	\$ 406
More than 30 km/h but not more than 40 km/h	\$ 568
More than 40 km/h over the speed limit	\$1137

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Use this information to draw a step graph to represent the data.

Problem solving

23 Solve the following simultaneous equations.

$$\frac{1}{m}+\frac{3}{n}=10 \text{ and } \frac{2}{m}+\frac{4}{n}=16$$

24 A rectangular garden bed is 0.5 m longer than it is wide. Find its dimensions if its perimeter is 9 m.

25 Aria is three times as old as her daughter, Isla, but in 10 years she will only be twice as old as her daughter. What are their present ages?

26 A cyclist rides away from her home at 10 km/h for 30 minutes. She then meets a friend and stops for coffee for $1\frac{1}{2}$ hours. She then continues her journey away from her home for an hour at 15 km/h, after which she returns home in 1 hour.

a Use this information to copy and complete the following table.

Time (hours)	0	0.5	1	1.5	2	2.5	3	3.5	4
Distance from home (km)									

b Use the completed table to draw a piece-wise graph for the distance from home (d) after t hours for the cyclist.

Practice quiz

Practice examination 2

Time: 90 minutes
Perusal time: 5 minutes
Marks: 50

Instructions

- Students are permitted to bring or use: pens, pencils, highlighters, erasers, sharpeners, rules and an approved graphics calculator.
- Students must show appropriate working and justification to gain full marks.
- A formula sheet is provided.
- Unless otherwise stated, numerical answers should be exact.
- Unless otherwise indicated, no diagrams in this examination are drawn to scale.
- All written responses must be in English.
- Answer all questions.
- **Students are NOT permitted to bring or use notes of any kind, correction fluid/tape, mobile phones and/or any other unauthorised electronic devices.**

WS
Year 11 formulas

Question 1 (1 mark)

Which is the best buy for someone who lives alone, given that apples weigh about 150 g each?

A 5 kg of apples for $13

B 2 kg of apples for $9

C 5 apples for $4

Question 2 (2 marks)

After a discount of 20%, Andrea paid $384 for a power saw. What was the original price of the saw?

Question 3 (2 marks)

An electric piano costs $460 in cash. Max paid $42.50 a fortnight for 6 months instead, with no deposit. What interest rate was he charged?

Question 4 (3 marks)

Paula had 2400 $2 shares in a company with a share price of $3.50. The company returned 85% of its profits to the shareholders, paying a dividend of 42 cents.

a How much did Paula get?

b What was the price/earnings ratio?

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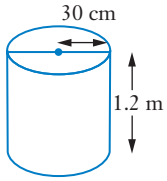
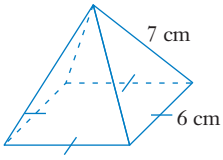
Question 5 (4 marks)

Calculate the surface area of each solid shape, correct to 3 significant figures.

a



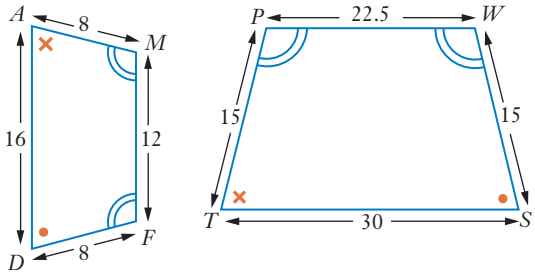
b



Question 6 (3 marks)

a Show that these shapes are similar.

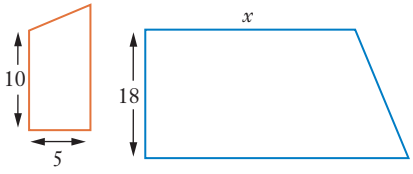
b Calculate the scale factor.



Question 7 (2 marks)

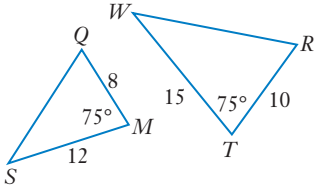
These shapes are similar.

Calculate the value of x.



Question 8 (2 marks)

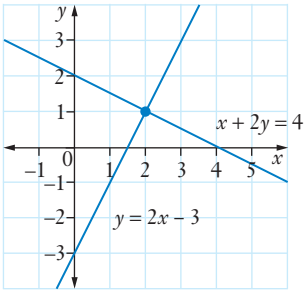
Determine if these triangles are similar. If they are, write a statement of similarity and the test for similarity that was used.



Question 9 (1 mark)

Use this graph to determine the simultaneous solution of the equations:

$$y = 2x - 3$$
$$x + 2y = 4$$



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Question 10 (2 marks)

Find the solution of these simultaneous equations by graphing.

$$x + y = 6$$
$$2x + 4y = 20$$

Question 11 (1 mark)

The simultaneous solution for the system of equations:

$$y + 2x = 3 \text{ and } x + 2y = 0$$

is:

A (–2, 1) **B** (2, 1) **C** (–2, –1) **D** (2, –1) **E** (1, –2)

Question 12 (2 marks)

Solve these simultaneous equations.

$$3x + 4y = 29$$
$$y = 2x - 1$$

Question 13 (2 marks)

Solve these simultaneous equations.

$$5a - b = 8$$
$$3a + b = 16$$

Question 14 (2 marks)

Draw a graph of the function:

$$y = \begin{cases} x - 5 & \text{if } x > 0 \\ -2x - 5 & \text{if } -5 \le x \le 0 \end{cases}$$

Question 15 (2 marks)

Graph the function defined by:

$$y = \begin{cases} -8 & \text{if } -3 \le x < -2 \\ -6 & \text{if } -2 \le x < -1 \\ -2 & \text{if } 0 \le x < 1 \\ 2 & \text{if } 2 \le x < 3 \\ 6 & \text{if } 4 \le x < 5 \end{cases}$$

Question 16 (3 marks)

The exchange rate for US dollars and Australian dollars for cash is AUD 1 to USD 0.79. What is AUD 400 worth in US dollars?

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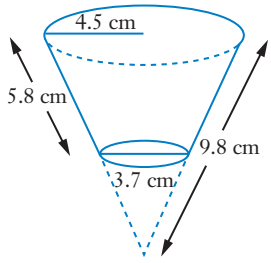
Question 17 (2 marks)

This truncated cone is constructed using sheets of aluminium metal.

What area of aluminium sheeting is required?



Question 18 (1 mark)

A photograph measures 120 mm wide by 90 mm high. When an image of the photograph is projected onto a screen, the height of the image is 1.98 m. What is the width of the image?

Question 19 (4 marks)

The total cost of tickets for 3 adults and 7 children to see a show was \$168. If a child's ticket is \$6 cheaper than an adult's ticket, find the cost of each type of ticket.

Question 20 (3 marks)

Peter can invest \$3000 for 4 years at 8% interest compounding monthly.

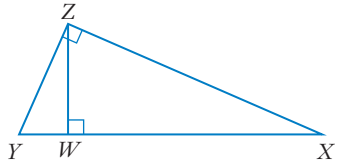
What flat-rate interest rate would give him the same amount after 4 years?

Question 21 (2 marks)

The altitude ZW is drawn from the right angle Z in $\triangle XYZ$ to form two new triangles.

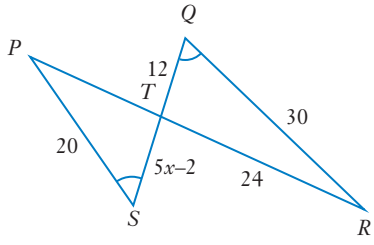
The two triangles formed are similar.

Why are the triangles similar to each other?



Question 22 (1 mark)

Calculate the value of x.



Question 23 (4 marks)

Solve these simultaneous equations:

$$\frac{y-x}{2}-\frac{x+y}{5}=\frac{1}{4}$$

$$\frac{x}{3}+\frac{y}{2}=\frac{1}{6}$$

END OF EXAMINATION

ALGEBRA AND MATRICES

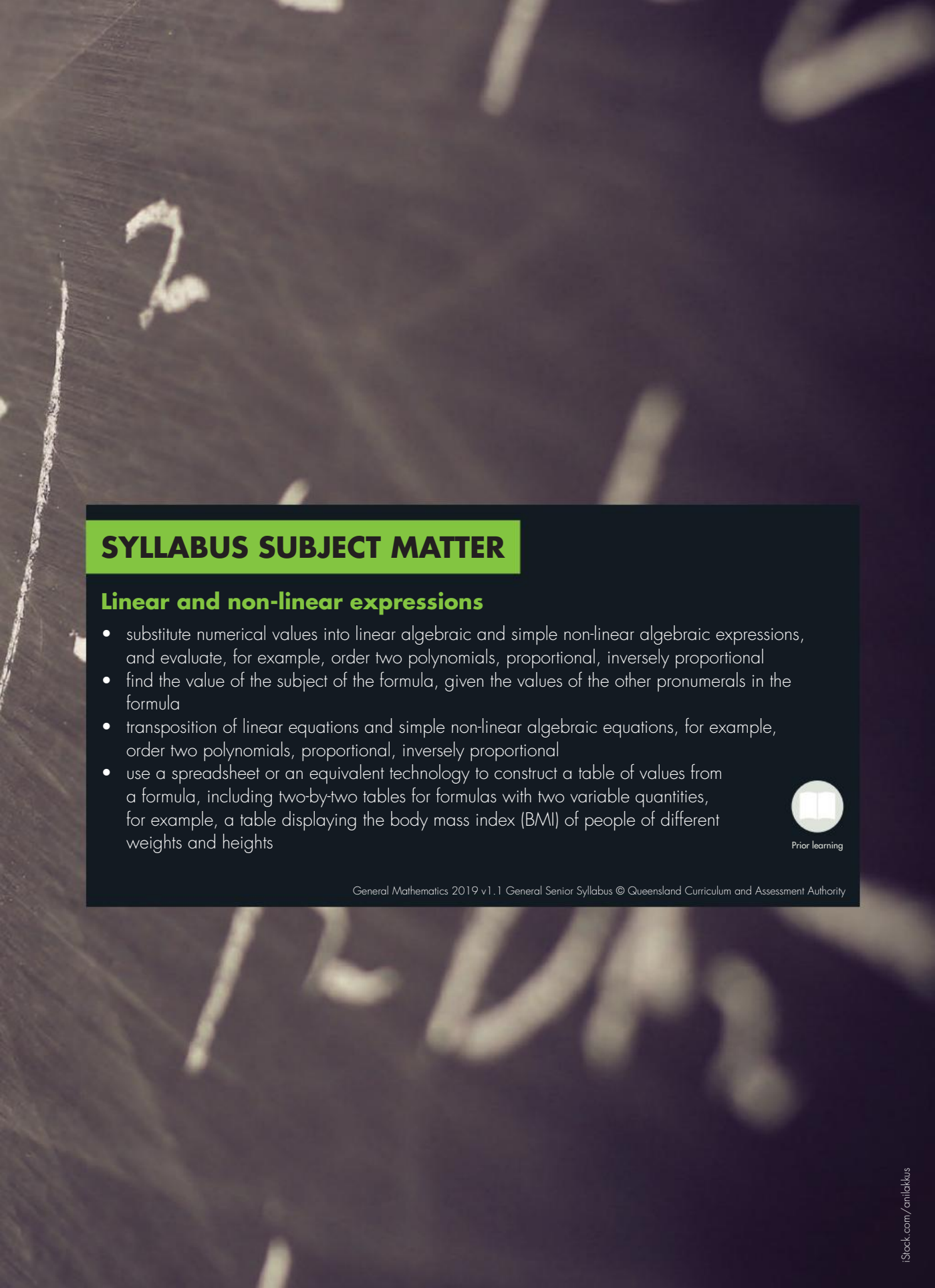
7.

LINEAR AND NON-LINEAR EXPRESSIONS

The Babylonians developed a form of algebra more than 4000 years ago. The evidence of equations is recorded on clay tablets. It shows that the Babylonians used algebra to solve problems. In algebra, unknown quantities are represented by variables. Variables are used in rules or equations that can then be used to model problems.

In this chapter you will identify different types of algebraic expressions and equations. You will also manipulate these expressions and equations using the rules of algebra.

7.01 Linear and non-linear expressions
7.02 Using formulas
7.03 Transposing equations
7.04 Tables of values
Chapter summary
Chapter review



SYLLABUS SUBJECT MATTER

Linear and non-linear expressions

- substitute numerical values into linear algebraic and simple non-linear algebraic expressions, and evaluate, for example, order two polynomials, proportional, inversely proportional
- find the value of the subject of the formula, given the values of the other pronumerals in the formula
- transposition of linear equations and simple non-linear algebraic equations, for example, order two polynomials, proportional, inversely proportional
- use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities, for example, a table displaying the body mass index (BMI) of people of different weights and heights

Prior learning

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TERMINOLOGY

algebraic expression
dependent variable
formula
isolate
linear expression
subject
coefficient
equation
independent variable
like terms
non-linear expression
transpose
constant term
expression
inverse operation
linear equation
operation

Algebra using diagrams

Expanding algebra

Algebra review

Collecting like terms

7.01 Linear and non-linear expressions

An **algebraic expression** is a collection of numbers and symbols connected by **operations** such as +, –, × and ÷. The parts of an algebraic expression separated by + or – signs are called **terms**. For example, $12 - 5y$ contains the terms 12 and $5y$. Terms that contain only numbers (12 in the above example) are called **constant terms**.

A constant that is multiplied by a variable is called the **coefficient** of the variable. The algebraic expression $5a + 4b(c - 7) - 8d$ has:

- three terms: $5a$, $4b(c - 7)$ and $8d$
- four variables: a, b, c and d
- three coefficients: 5, 4 and 8.

Like terms have identical variable parts. For example, x and $4x$ are like terms and they can be added to give $5x$, but x and x^2 are unlike terms because the variable parts are not identical.

EXAMPLE 1

Simplify each of the following

a $8a + 4b - 5a + 7b + c$

b $3x^2 + 4xy - 7xx + 6$

c $2m \times 3n \times 6p$

d $\dfrac{14cd^3}{-5cd \times 4ce}$

Solution

a Group like terms. $8a + 4b - 5a + 7b + c = 8a - 5a + 4b + 7b + c$

Add/subtract like terms. $= 3a + 11b + c$

b Group like terms. $3x^2 + 4xy - 7xx + 6 = 3x^2 - 7x^2 + 4xy + 6$

Add/subtract like terms. $= -4x^2 + 4xy + 6$

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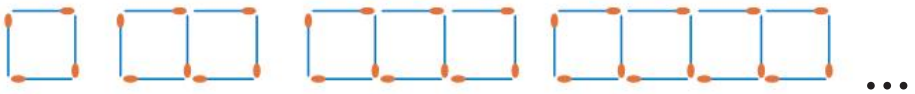
c	Multiply variables and coefficients separately.	$2m \times 3n \times 6p = 2 \times 3 \times 6 \times m \times n \times p$
	Simplify.	$= 36mnp$
d	Cancel and simplify.	$\dfrac{14cd^3}{-5cd \times 4ce} = \dfrac{7d^2}{10ce}$

Consider the sequence of squares formed using matches shown below.

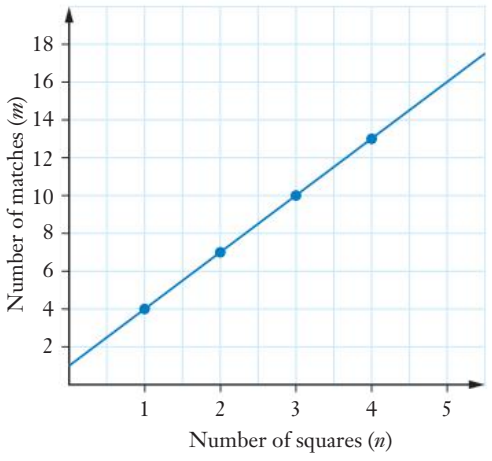


1 Square 2 Squares 3 Squares 4 Squares ...

The number of matches m needed to make n squares is given by $m = 3n + 1$.

n is the **independent variable** and m is the **dependent variable** because its value depends on the value of n.

The graph has the independent variable on the horizontal axis and the dependent variable on the vertical axis.



All the points are on a straight line.

Linear expression

A **linear expression** is a relationship between two variables that when plotted gives a straight line. In a linear expression, the highest power of the variable is 1.

Expressions that are not linear are called **non-linear expressions**.

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EXAMPLE 2

Identify the following expressions as linear or non-linear.

a $4k + 7$ **b** $x^2 - 5x + 9$ **c** $3\sqrt{a}$

d $4(b - 6)$ **e** $\dfrac{3u-7}{5}$ **f** $\dfrac{9-2x}{x}$

Solution

a	Identify the variable.	k is the variable.
	Write the power of the variable.	k has the power 1.
	Check against the definition.	The expression is linear.
b	Identify the variable.	x is the variable.
	Write the power of the variable.	x has the powers 2 and 1.
	Check against the definition.	The expression is non-linear.
c	Write the expression using powers.	$3\sqrt{a} = 3a^{\frac{1}{2}}$
	Write the power of the variable.	a has the power $\frac{1}{2}$.
	Check against the definition.	The expression is non-linear.
d	Expand the expression.	$4(b - 6) = 4b - 24$
	Write the power of the variable.	b has the power 1.
	Check against the definition.	The expression is linear.
e	Write the expression as a difference of terms.	$\dfrac{3u-7}{5} = \dfrac{3}{5}u - \dfrac{7}{5}$
	Write the power of the variable.	u has the power 1.
	Check against the definition.	The expression is linear.
f	Write the expression as a difference of terms.	$\dfrac{9-2x}{x} = \dfrac{9}{x} - \dfrac{2x}{x}$
	Write the expression using powers.	$= 9x^{-1} - 2$
	Write the power of the variable.	x has the power -1.
	Check against the definition.	The expression is non-linear.

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EXAMPLE 3

Use tables to draw graphs of the following expressions to classify them as linear or non-linear.

a $3x - 5$

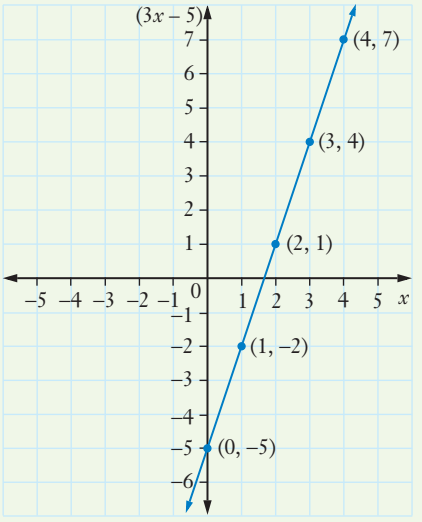
b $x^2 - 5$

Solution

a Draw up a table of x and $3x - 5$.

x	0	1	2	3	4
$3x - 5$	–5	–2	1	4	7

Plot the points and join them.



Identify the expression.

The graph is a straight line, so the expression is linear.

b Construct a table for the values of x and $x^2 - 5$.

x	–4	–3	–2	–1	0	1	2	3	4
$x^2 - 5$	11	4	–1	–4	–5	–4	–1	4	11

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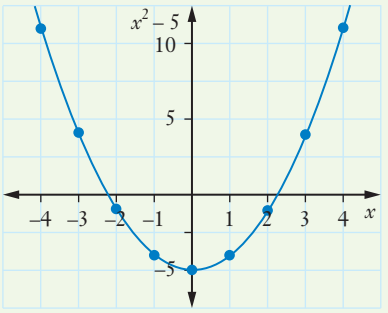
Plot the points and join them.



Identify the expression.

The graph of the expression is not a straight line, so $x^2 - 5$ is a non-linear expression.

In the previous example $3x - 5$ increases by the same amount each time x goes up by 1.

This is true for all linear expressions.

You can use this to determine if a function is linear.

TECHNOLOGY

Non-linear expressions

The age and length of dugongs in Moreton Bay are measured in the field. The results are shown here.

Age (years)	Length (m)
1.0	1.8
1.5	1.9
2.5	2.0
4.0	2.3
5.0	2.3
7.0	2.4
8.0	2.5
9.5	2.4
12.0	2.5
13.0	2.5
13.0	2.5
14.5	2.6
15.5	2.7
16.5	2.6
20.0	2.7
22.5	2.7
29.0	2.7
31.5	2.8

1. Open a new spreadsheet and enter the headings 'Age' and 'Length' in cells A1 and B1 respectively.
2. Enter the ages shown here into cells A2–A19 and then enter the lengths into cells B2–B19.
3. Select the area A1:B19, click on the **Insert** tab, and then click on the '**Charts**' menu.
4. Select '**Marked Scatter**' to draw a scatter plot of the data.
5. Next add a line of best fit for the data. The line of best fit is also called a trendline. Right click on any data point and select '**Add Trendline…**'. The Linear trendline will be selected, so press '**OK**'.
6. Try different types of trendlines until you find one that best fits the data.
7. Is the data linear or non-linear?

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Exercise 7.01 Linear and non-linear expressions

Example 1

1 Simplify each expression.

a $10g - 8g + 2h + 10$

b $8m - 4 - 2m + 5n$

c $3(p - 5) + p - 4$

d $2x^2 + 10 - 7x^2 - 8y + 3xy - 7$

e $-4 + 7q^2 + 3pq - 2q^2$

f $a - 5 + a + 10b + 4a - 3ab$

g $8m(2m + 1) - 2m(5m + 2 + 2m - 5)$

h $3(-7y + 5 + 8y) - 4(3 + y + 5)$

i $7y(y - 1) - 3(2y + 3) + 2(3y^2 + 1)$

j $2(7p + 1) + p(3 + 2p - 1) + 5p(p - 10)$

2 Simplify each expression.

a $a \times 4b^2 \times 2c$

b $3x^2 \times (-y^3) \times 4xy$

c $3d \times d^2g^2 \times (-5)$

d $7y \times 2y^2 \times (-x^4) \times (-3xy)$

e $5rs \times (-2r^2s^3) \times 3r$

f $4p \times 6q^4 \times 2p \times p^3q$

3 Simplify each expression.

a $24xy \div 16x^2$

b $36ab^2 \div (-12a^3b)$

c $\dfrac{34x^4y^3}{28x^2y^2z}$

d $\dfrac{38h^5j^3k^2}{42h^2jk^3}$

Example 2

4 Classify of the following expressions as linear or non-linear.

a $8a + 2$

b $3y - 2y + 4$

c $2w - w^2$

d $5x(2x - 7)$

e $\dfrac{2m - 4}{3}$

f $\dfrac{8y - 2y^2}{6}$

g $4m - \dfrac{m}{3}$

h $7c - 2\sqrt{c}$

i $\dfrac{2m - 4m^3}{3m}$

j $4g + 3g - 9$

k $4b^2 - 5b + 9b^3$

l $7 + \sqrt{y^3} - 5y$

m $(4 + 3n)(n - 1)$

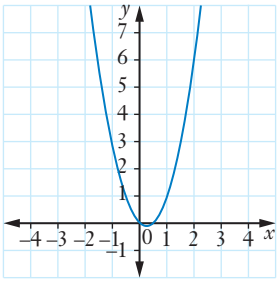
n $\dfrac{1}{3}(2n - 7)$

o $\dfrac{5s - 3t}{t^2}$

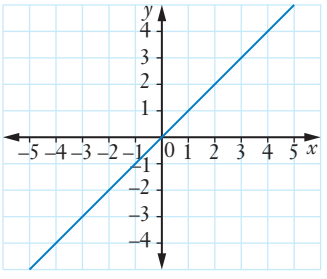
Example 3

5 The graphs of a number of different expressions are shown below. The values of the independent variable are shown on the horizontal axis and the values of the expression are shown on the vertical axis. Use the graphs to identify any non-linear expressions.

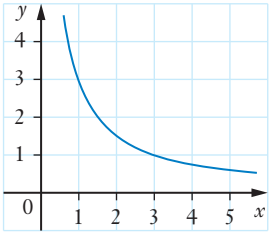
a



b



c



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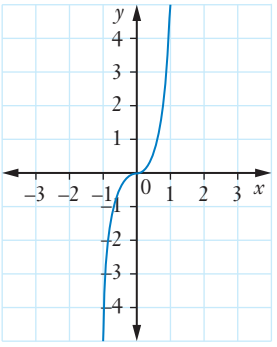
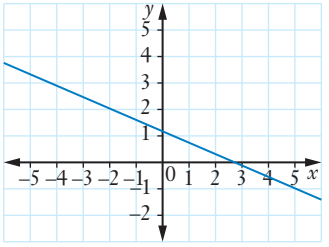
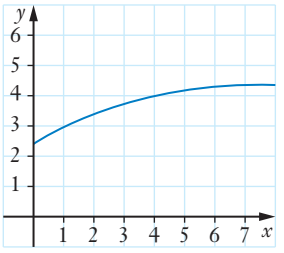


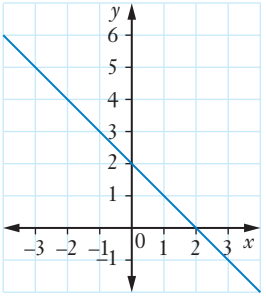
e



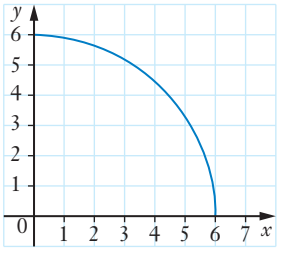
f



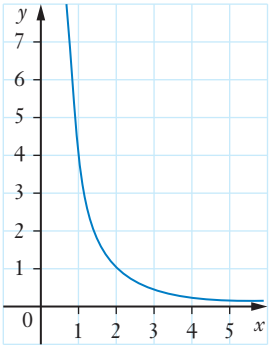
g



h



i



6 Draw a graph of each expression for the domain indicated and use the graph to determine if the expression is linear or non-linear.

a $7 - 2x$ for $-2 \le x \le 5$

b $9 - x^2$ for $-4 \le x \le 4$

c $x^3 + 2$ for $-3 \le x \le 2$

d $\frac{5x}{4} + 3$ for $-4 \le x \le 3$

e $x(x + 4)$ for $-6 \le x \le 2$

f $\frac{3}{5}(3x - 8)$ for $-2 \le x \le 4$

g $x + \frac{1}{x}$ for $0 \le x \le 5$

h $3(2x - 1)$ for $-2 \le x \le 3$

i $2 + \sqrt{x}$ for $0 \le x \le 4$

7 The values of a variable and the corresponding values of an expression using the variable are shown in the tables below. Determine if each expression is linear or non-linear.

a

Variable	1	2	3	4
Expression	3	13	23	33

b

Variable	0	1	4	9
Expression	0	2	4	6

c

Variable	–3	–2	–1	0
Expression	9	4	1	0

d

Variable	5	6	7	8
Expression	1	2	3	4

e

Variable	1	2	3	4
Expression	0.25	0.5	0.75	1

f

Variable	1	4	7	10
Expression	1	16	49	100

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g

Variable	2	4	6	8
Expression	50	35	20	5

h

Variable	0.5	1	1.5	2
Expression	13	26	39	52

i

Variable	0	5	10	15
Expression	20	16	14	6

j

Variable	0	2	4	6
Expression	0	2	8	16

Problem solving

8 The data here shows the writing score and maths score for a group of students.

Writing score	50	59	78	60	93	55	90	65	57	81	48	54	56	43
Maths score	52	56	71	61	90	60	86	63	56	82	50	53	52	40

a Enter the data in a spreadsheet and add a trendline to determine if there is a linear relationship between the two scores.

b Predict the maths score for a student whose writing score is 70.

9 A wildlife park has a breeding program for lions. The staff involved in the program know that the minimum daily calorie intake for lion cubs changes as they get older. The table below shows how this changes.

Age (weeks)	Calorie intake
1	750
2	1100
3	1250
4	1450
5	1500

Use a spreadsheet to determine if the relationship between minimum daily calorie intake and the age of the lion cubs is linear or non-linear.

10 Some physically-disabled people are helped by trained monkeys that can perform household tasks like switching things on and off.

Years	6.0	0.6	8.0	2.0	7.0	9.0	5.0	1.5	10.0	0.5	6.5	0.4	0.3
Tasks	28	18	28	22	27	29	27	21	28	17	28	20	15

This data gives the number of tasks selected monkeys can perform along with the number of years the monkeys have been working with disabled people.

Use a spreadsheet to determine if the relationship between the number of years a monkey works with disabled people and the number of tasks performed is linear or non-linear.

11 The term life premium rates for a sum insured of $50 000 for a male smoker are given in the table. Use a spreadsheet to determine if the relationship between the premium rates and the age of the person insured is linear or non-linear.

Age	33	35	37	39	42	44	47	49	52	55
Male smoker	130	140	155	170	210	250	325	395	535	715

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7.02 Using formulas

Working with formulas

Formulas

A **formula** is an algebraic rule to show the relationship between quantities.

The **subject** of a formula is the variable by itself on the left-hand side.

EXAMPLE 4

The formula for the displacement (s) of an object is given by:

$$s = ut + \frac{1}{2}at^2$$

The subject of this formula is s.

where u is the initial velocity, a is the acceleration and t is the time.

Find the value of s when $u = 26$, $a = -5$ and $t = 4$.

Solution

Write the formula. $s = ut + \frac{1}{2}at^2$

Substitute for the values of the variables. $= 26 \times 4 + \frac{1}{2} \times (-5) \times 4^2$

Evaluate. $= 104 - 40$

$= 64$

You may need to **transpose** a formula to make a different variable the subject. You do this using **inverse operations**. An inverse operation 'undoes' another operation.

Formulas and equations

Inverse operations

An inverse operation has the effect of undoing the original operation.

Operation	Inverse operation
+	−
−	+
×	÷
÷	×
square (^{2})	$\sqrt{\ }$
$\sqrt{\ }$	square (^{2})

Whatever you do to one side of the formula must be done to the other side. Work in the reverse order to the usual order of operations when you transpose a formula.

Formulas and equations contain an equals sign. The abbreviation LHS is used to refer to the left-hand side of a formula or equation and RHS is used to refer to the right-hand side.

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EXAMPLE 5

Make a the subject of each formula.

a $ad + c = p$ **b** $a(p + q) = k$ **c** $5a - m = 2a + 3n$

Solution

a Write the formula. $ad + c = p$

Undo the $+ c$ first. Subtract c from both sides. $ad + c - c = p - c$

Simplify. $ad = p - c$

Undo the $\times d$. Divide both sides by d. $\frac{ad}{d} = \frac{p-c}{d}$

Simplify the LHS. $a = \frac{p-c}{d}$

b Write the formula. $a(p + q) = k$

Undo the $\times (p + q)$. Divide both sides by $(p + q)$. $\frac{a(p+q)}{(p+q)} = \frac{k}{(p+q)}$

Simplify the LHS. $a = \frac{k}{p+q}$

c Write the formula. $5a - m = 2a + 3n$

Group the terms containing a on the LHS. $5a - m - 2a = 2a + 3n - 2a$

Simplify both sides. $3a - m = 3n$

Undo the $- m$. Add m to both sides. $3a - m + m = 3n + m$

Simplify both sides. $3a = 3n + m$

Undo the $\times 3$. Divide both sides by 3. $\frac{3a}{3} = \frac{3n-m}{3}$

Simplify. $a = \frac{3n-m}{3}$

EXAMPLE 6

Make m the subject of each formula and calculate the value of m when $n = 8$ and $p = 12$.

a $m^2 + n = p$ **b** $\frac{2n}{3m} - 4 = p$

Changing the subject of a formula

Changing the subject of a formula

Subject of a formula

Changing the subject of a formula

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Solution

a Write the formula. $m^2 + n = p$

Undo the $+ n$. $m^2 + n - n = p - n$

Simplify both sides. $m^2 = p - n$

Now undo the square by taking the positive and negative square root of both sides. $\sqrt{m^2} = \sqrt{p-n}$

Simplify both sides. $m = \pm\sqrt{p-n}$

Substitute for $n = 8$ and $p = 12$. $m = \pm\sqrt{12-8}$

Simplify the RHS. $= \pm\sqrt{4}$

$= \pm 2$

b Write the formula. $\frac{2n}{3m} - 4 = p$

Undo the -4 first. $\frac{2n}{3m} - 4 + 4 = p + 4$

Simplify both sides. $\frac{2n}{3m} = p + 4$

Invert both sides. Remember, $a = \frac{a}{1}$. $\frac{3m}{2n} = \frac{1}{p+4}$

Undo the $\div 2n$. $\frac{3m}{2n} \times 2n = \frac{1}{p+4} \times 2n$

Simplify both sides. $3m = \frac{2n}{p+4}$

Undo the $\times 3$. $\frac{3m}{3} = \frac{2n}{3(p+4)}$

Simplify the LHS. $m = \frac{2n}{3(p+4)}$

Substitute for $n = 8$ and $p = 12$. $m = \frac{2 \times 8}{3(12+4)}$

Simplify the RHS. $= \frac{16}{3 \times 16}$

$= \frac{1}{3}$

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Exercise 7.02 Using formulas

1 Find the value of A in each formula if $x = 3$, $y = -2$ and $z = 5$. Example 4

a $A = 2x + 3y$ **b** $A = 3x - 4y$ **c** $A = 5(x + 2y)$

d $A = 3(-2x + z)$ **e** $A = y^2 - x^2$ **f** $A = y^2 - z^2$

g $A = 3(x + y)^2$ **h** $A = x(y + z)^2$ **i** $A = 3xz - 2y$

2 If $y = mx + c$, find y when:

a $m = -1, x = 4, c = 3$ **b** $m = 1, x = -5, c = -2$ **c** $m = 2, x = -3, c = -5$

d $m = -2, x = -3, c = 4$ **e** $m = 0, x = 9, c = -3$ **f** $m = -8, x = 0, c = 14$

g $m = -4, x = -1, c = -2$ **h** $m = -1.5, x = -4, c = 5$

3 If $a = 2.4$, $b = -3.5$ and $c = 1.2$, find Y when:

a $Y = a - b + c$ **b** $Y = -abc$ **c** $Y = -2a + 3b$ **d** $Y = 4a - 2b$

e $Y = \dfrac{a + 6b}{c}$ **f** $Y = ab - 5c$ **g** $Y = a^2 + b^2$ **h** $Y = a(2b - 3c)$

i $Y = 4ac - 3b^2$ **j** $Y = \dfrac{7(3a - 4bc)}{12b}$

4 Transpose each formula so that the variable shown in brackets is the subject. Example 5

a $v = u(2 - 3at)$ (t) **b** $G = k(I - ct)$ (c) **c** $m = \dfrac{b - a}{c - a}$ (b)

d $a = \dfrac{u + at}{s}$ (u) **e** $\dfrac{b}{a} = \dfrac{c}{d}$ (d) **f** $kn = d - h$ (k)

5 Transpose each formula so that the variable shown in brackets is the subject.

a $(R - a)^2 + c = d^2$ (R) **b** $v = \sqrt{\dfrac{a - b}{c}}$ (a)

c $a^2 = b^2 + c^2$ (b) **d** $d = \sqrt{bc} - 5$ (c)

e $m = \dfrac{a}{b} - \dfrac{1}{c}$ (c) **f** $\dfrac{1}{f} = \dfrac{1}{u} - \dfrac{1}{v}$ (v)

g $v = a\left(\dfrac{1}{b} - \dfrac{1}{c}\right)$ (b) **h** $v^2 = u^2 + 2as$ (u)

i $y = 4ab^2c^2$ (b) **j** $x + y = \dfrac{r}{3 + r}$ (r)

6 Transpose each formula so that A is the subject and then find the value of A if $x = 3$, $y = 2$ and $z = 5$. Example 6

a $x = A + z$ **b** $15x = Az^2$ **c** $6y = xA$

d $y = \sqrt{3x} - A$ **e** $z = \sqrt{6xy} + 3A$

7 Transpose each formula as required and then find the value of:

a u if $v = u + at$ and $v = 12$, $a = 2$ and $t = 3$

b m if $k = m(1 - ct)$ and $k = 28$, $c = 4$ and $t = 2$

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c E if $I = \dfrac{E-V}{R}$ and $I = 4.2$, $V = 7$ and $R = 2.5$

d h if $d = 8\sqrt{\dfrac{h}{a}}$ and $d = 24$ and $a = 5$

e t if $s = \dfrac{(u+v)t}{2}$ and $s = 15$, $u = 3$ and $v = 5$

f c if $a^2 = b^2 + c^2$ and $a = 24$ and $b = 10$

g r if $p = ghr + B$ and $p = 980$, $g = 9.8$, $h = 15$ and $B = 24.5$

8 The compound interest formula is $FV = PV\left(1+\dfrac{r}{n}\right)^{nt}$, where:

FV = the future value of the investment or loan plus interest

PV = the initial amount invested or initial loan amount

r = the annual interest rate expressed as a decimal

n = is the number of times that interest is compounded per year

t = the number of years the amount is invested or borrowed

Use the formula to calculate the amount to which:

a \$6000 will grow if it is invested at 5% pa for 7 years compounded annually

b \$10 000 will grow if it is invested at 7% pa for 9 years compounded monthly

c \$8000 will grow if it is invested at 8% pa for 5 years compounded daily

Problem solving

9 The density of a body, D g/cm^3, is related to its volume, V cm^3, and its mass, M g, by the formula

$$D = \frac{M}{V}$$

a Find the density of steel if a 26 g block has a volume of 4.8 cm^3.

b Find the mass of a 645 cm^3 block of iron if iron has a density of 7.5 g/cm^3.

c Find the volume of a block of steel with density 9.6 g/cm^3 and mass 2.4 kg.

10 The time, T s, for a pendulum to complete one swing is given by the formula

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where $g = 9.8$ m/s^2 and l = length of the pendulum in metres.

a Express l as the subject of the formula.

b Find the length of a pendulum (nearest cm) that will make the time for a swing 4.5 s.

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11 Dan jumped out of an aircraft to begin skydiving. Without a parachute, it would take him 24.74 s to hit the ground, accelerating at $g = 9.8$ m/s^2. Use the formula $s = gt^2$ to determine from what height, s, Dan fell if t is the time it would take him to reach the ground under gravity.

Shutterstock.com/2happy

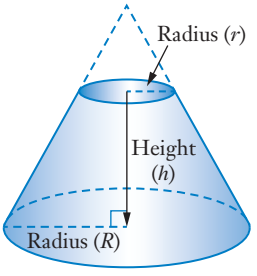
12 The area, A, of a triangle drawn on the surface of a balloon is given by

$$A = \pi r^2$$

where r is the radius of the balloon. A second balloon is inflated to twice the diameter of the first balloon and another triangle is drawn. If the new triangle has an area three times the area of the original triangle, how does the first value of k compare with the second value?

13 A frustum is the part of a cone that remains when it is cut parallel to its base. The volume of the frustum is one-third of the product of:

- the vertical distance, h, between the base and the cut end, and
- the sum of the flat circular areas and the square root of the product of these areas.



Find a formula for this volume involving r, R and h.

7.03 Transposing equations

An **equation** has an 'equals' sign to show the left-hand side (LHS) is equal to the right-hand side (RHS).

A **linear equation** *in one variable* is an equation that can be written in the form

$$ax + b = c$$

where a, b, and c are real numbers (constants) and x is a variable (unknown).

A linear equation *in two variables* can be written in the form

$$ax + by = c$$

where a, b, and c are constants ($a, b \neq 0$) and x and y are the variables.

You can transpose equations just like formulas.

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EXAMPLE 7

Rearrange each of the following to isolate x on the LHS of the equation.

a $y = kx + m$ **b** $6(y + 1) = 7(x - 2)$

Solution

a Write the equation. $y = kx + m$

Reverse the equation. $kx + m = y$

Undo the $+ m$. $kx + m - m = y - m$

Simplify both sides. $kx = y - m$

Undo the $\times k$. $\frac{kx}{k} = \frac{y - m}{k}$

Simplify the LHS. $x = \frac{y - m}{k}$

b Write the equation. $6(y + 1) = 7(x - 2)$

Reverse the equation. $7(x - 2) = 6(y + 1)$

Undo the $\times 7$. $\frac{7(x - 2)}{7} = \frac{6(y + 1)}{7}$

Simplify the LHS. $x - 2 = \frac{6(y + 1)}{7}$

Undo the $- 2$. $x = \frac{6(y + 1)}{7} - 2$

You can also transpose a **non-linear equation**.

EXAMPLE 8

In each of the following, isolate the variable indicated in brackets.

a $y = 2x^2 + 4y - 7$ (x) **b** $3a = \sqrt{7b + 5}$ (b) **c** $2x + \frac{1}{y} = 4$ (y)

Solution

a Write the equation. $y = 2x^2 + 4y - 7$

Group like terms on the RHS. $0 = 2x^2 + 4y - y - 7$

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	Simplify.	$0 = 2x^2 + 3y - 7$
	Reverse the equation.	$2x^2 + 3y - 7 = 0$
	Undo the $+ 3y$ and the $- 7$.	$2x^2 = 7 - 3y$
	Undo the $\times 2$.	$\frac{2x^2}{2} = \frac{7-3y}{2}$
	Simplify.	$x^2 = \frac{7-3y}{2}$
	Take the square root of both sides.	$x = \pm\sqrt{\frac{7-3y}{2}}$
b	Write the equation.	$3a = \sqrt{7b+5}$
	Reverse the equation.	$\sqrt{7b+5} = 3a$
	Square both sides.	$7b + 5 = (3a)^2$
	Evaluate.	$7b + 5 = 9a^2$
	Undo the $+ 5$.	$7b = 9a^2 - 5$
	Undo the $\times 7$.	$\frac{7b}{7} = \frac{9a^2-5}{7}$
	Simplify.	$b = \frac{9a^2-5}{7}$
c	Write the equation.	$2x + \frac{1}{y} = 4$
	Undo the $+ 2x$.	$\frac{1}{y} = 4 - 2x$
	Invert both sides. (Remember $a = \frac{a}{1}$).	$y = \frac{1}{4-2x}$

Exercise 7.03 Transposing equations

Example 7

1 Transpose each equation to isolate x on the left-hand side.

a $y = 3x$ **b** $y = 2x - 7$ **c** $y = 7x + 8$

d $y + 5 = 3x - 8$ **e** $4(y + 3) = 2(5 - x)$ **f** $6(x - 2) = y$

g $7(x - 2) = 4y + 1$ **h** $8x - 5y = 11$ **i** $3(x + 6) = 7(2x - y)$

j $7(x - 5) = 2x + 3y$ **k** $3(y - 2x) = 4(x + 1)$ **l** $3(7x - y) + 8 = 6(2y - 5x)$

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2 Isolate x on the LHS of each equation.

a $2x + 9y + 3 = 0$ **b** $5x - 8y = 2$ **c** $4x + 7y = 9$

d $4x - 12y - 8 = 0$ **e** $7x - 3y = 4$ **f** $6x + 3y + 2 = 0$

g $3x + 6y - 12 = 0$ **h** $8x - 4y - 7 = 0$ **i** $9x - 2y + 6 = 0$

3 Isolate y on the LHS of each equation.

a $4x - 3y = 1$ **b** $7x - 2y + 3 = 0$ **c** $8x - 4y = -6$

d $2x - 4y + 6 = 0$ **e** $6x + 5y - 9 = 0$ **f** $-5x + 3y + 7 = 0$

g $-5x + 10y + 4 = 0$ **h** $-9x - 6y - 12 = 0$ **i** $4x + 9y - 2 = 0$

Example 8

4 Transpose each equation to isolate y on the LHS.

a $x = 3y - 6x^2 + 8$ **b** $5x = y^2 - 12$

c $5x - \sqrt{2y} - 3 = 0$ **d** $3xy - 7 = 0$

e $6x(x + 7) = 2(5x - 8y)$ **f** $6(y - 2) = 3x - 8$

g $7x + 2 = \sqrt{3 - 2y}$ **h** $2x - \dfrac{3}{y} = 5$

i $\dfrac{2}{x} + \dfrac{4}{y} = 8$ **j** $7y(x - 2) = 3x^2 + 2x$

5 In each equation isolate the variable indicated in brackets.

a $V = \dfrac{4}{3}\pi r^3$ (r) **b** $a^2 + b^2 = 25$ (b)

c $\dfrac{1}{v} + \dfrac{1}{u} = 5$ (u) **d** $3 = \sqrt{(x-2)^2 + (y-1)^2}$ (y)

7.04 Tables of values

Tables are used to show the values of an equation (output values) for different values of a variable (input values). You can use a table of values to see the relationship between input values and output values.

Spreadsheets are particularly suited to constructing tables of values.

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EXAMPLE 9

The volume of a sphere is given by the equation

$$V = \frac{4}{3}\pi r^3$$

a Use a spreadsheet to construct a table to show the volumes of spheres where $r = 1, 2, 3, \ldots, 10$.

b What is the volume of a sphere with a radius of 15 cm?

Solution

a Type 'Radius' in cell A1 and 'Volume' in cell B1.

Enter '1' into A2 and '2' into A3.

Block A2:A3 and drag the bottom right-hand corner of the block down to A11. This fills cells A4 to A11 with values that increase by 1 each time.

Enter the equation for the volume of a sphere into cell B2 as follows:

'=(4/3)*PI()*POWER(A2,3)'

This equation uses the value in cell A2 raised to the power of 3.

Hold the bottom right-hand corner of B2 and drag the block down to B11.

This copies the equation into each cell.

	A	B
1	Radius	Volume
2	1	4.19
3	2	33.51
4	3	113.10
5	4	268.08
6	5	523.60
7	6	904.78
8	7	1436.76
9	8	2144.66
10	9	3053.63
11	10	4188.79
12		
13		

b Enter '15' into A12.

Click on B11 and then drag the bottom right-hand corner to B12.

	A	B
1	Radius	Volume
2	1	4.19
3	2	33.51
4	3	113.10
5	4	268.08
6	5	523.60
7	6	904.78
8	7	1436.76
9	8	2144.66
10	9	3053.63
11	10	4188.79
12	15	14137.17
13		

State the result.

The volume of a sphere with a radius of 15 cm is about 14 137.17 cm^3.

ISBN 9780170412711

INVESTIGATION

BODY MASS INDEX (BMI)

The body mass index (BMI) is a method used to estimate your total amount of body fat. It is calculated by dividing your mass (m) in kilograms by your height (h) in metres squared. In practice, a person's weight is used rather than their mass.

The equation for BMI is:

$$B = \frac{m}{h^2}$$

iStock.com/michaeljung

BMI is a simple indicator of body fat rather than a true measure. It is a useful indicator because it relies solely on height and weight and does not require any other equipment. This means that people can have their BMI routinely measured and calculated with reasonable accuracy.

Studies have shown that BMI levels are good indicators of future health risks. This means that BMI is a screening tool to decide if your weight might be putting you at risk for a variety of health problems.

There are limitations to the usefulness of BMI because it is a measure of excess weight rather than excess body fat. So BMI does not distinguish between excess fat, muscle, or bone.

Differences in the values of BMI between people of the same age and sex are attributed to body fat. BMI does not take into account the facts that older adults, women and body builders tend to have more body fat than other people for an equivalent BMI.

BMI	Weight status
Below 18.5	Underweight
18.5 to 24.9	Normal
25.0 to 29.9	Overweight
30 and above	Obese

The table shows how BMI can be interpreted.

1 Construct a spreadsheet to calculate the BMI of Renata who is 168 cm tall and weighs 64 kg.

Type 'm' in cell A1, 'h' in cell B1 and 'B' in cell C1.

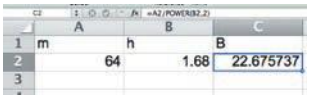
Enter '64' into A2 and '1.68' into A3.

Enter the equation for BMI into cell C2 as follows:

'=A2/POWER(B2,2)'

This equation uses the value in cell B2 raised to the power of 2.

Renata has a BMI of 22.7.



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2 Extend the spreadsheet to calculate the BMI for each of the following people:

Person	Weight (kg)	Height (cm)
Adel	73	157
Shani	69	169
Lindell	52	149
Kai	77	171
Riley	54	174
Lucca	68	162
Madison	75	168
Lola	77	153
Matteo	81	181
Mia	92	177

3 Use the BMI measure to determine the weight status (underweight, normal, overweight or obese) of each of the people in the table above.

EXAMPLE 10

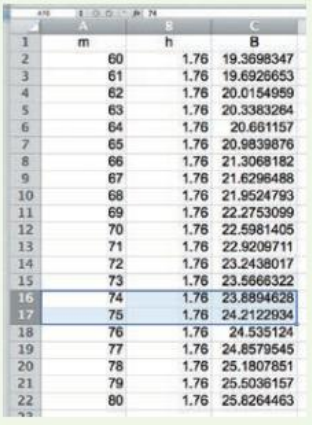
The equation used to calculate the body mass index (BMI) of a person h metres tall weighing m kg is:

$$B = \frac{m}{h^2}$$

A 176 cm tall person wishes to achieve a BMI of 24. Use a spreadsheet to determine the weight (correct to 1 decimal place) the person should reach.

Solution

Construct a spreadsheet to calculate BMI values from 60 to 80.



	A	B	C
1	m	h	B
2	60	1.76	19.3698347
3	61	1.76	19.6926653
4	62	1.76	20.0154959
5	63	1.76	20.3383264
6	64	1.76	20.661157
7	65	1.76	20.9839876
8	66	1.76	21.3068182
9	67	1.76	21.6296488
10	68	1.76	21.9524793
11	69	1.76	22.2753099
12	70	1.76	22.5981405
13	71	1.76	22.9209711
14	72	1.76	23.2438017
15	73	1.76	23.5666322
16	74	1.76	23.8894628
17	75	1.76	24.2122934
18	76	1.76	24.535124
19	77	1.76	24.8579545
20	78	1.76	25.1807851
21	79	1.76	25.5036157
22	80	1.76	25.8264463

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Examine the values in the table.

The weight required is between 74 kg and 75 kg.

Enter 74 in A24 and 74.1 in A25. Block A24:A25 and then place the cursor on the bottom right-hand corner of the block and drag the cursor down to cell A34. This gives a range of weights from 74 kg to 75 kg.

	A	B	C
21	79	1.76	25.5036157
22	80	1.76	25.8264463
23			
24	74	1.76	23.8894628
25	74.1	1.76	23.9217459
26	74.2	1.76	23.9540289
27	74.3	1.76	23.986312
28	74.4	1.76	24.018595
29	74.5	1.76	24.0508781
30	74.6	1.76	24.0831612
31	74.7	1.76	24.1154442
32	74.8	1.76	24.1477273
33	74.9	1.76	24.1800103
34	75	1.76	24.2122934

Examine the values in the table.

The weight required is between 74.3 kg and 74.4 kg.

Enter 74.31 in A36 and 74.32 in A37. Block A36:A37 and then place the cursor on the bottom right-hand corner of the block and drag the cursor down to cell A45. This gives a range of weights from 74.31 kg to 74.4 kg.

	A	B	C
32	74.8	1.76	24.1477273
33	74.9	1.76	24.1800103
34	75	1.76	24.2122934
35			
36	74.31	1.76	23.9895403
37	74.32	1.76	23.9927686
38	74.33	1.76	23.9959969
39	74.34	1.76	23.9992252
40	74.35	1.76	24.0024535
41	74.36	1.76	24.0056818
42	74.37	1.76	24.0089101
43	74.38	1.76	24.0121384
44	74.39	1.76	24.0153667
45	74.4	1.76	24.018595

Examine the values in the table.

The weight required is closer to 74.3 kg than 74.4 kg.

State the result.

A 176 cm tall person should weigh approximately 74.3 kg to achieve a BMI of 24.

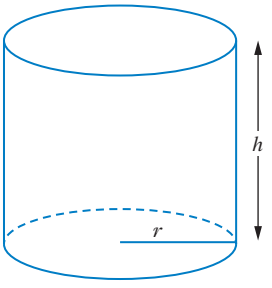
Exercise 7.04 Tables of values

Examples 9, 10

1 The curved surface area of a cylinder is given by the equation

$$SA = 2\pi rh$$

where r is the radius and h is the height of the cylinder.



a Use a spreadsheet to construct a table to show the curved surface areas of cylinders that are 10 cm high for $r = 1, 2, 3, \ldots, 15$.

b Construct a table to show the curved surface areas of cylinders with a radius of 10 cm for $h = 10, 11, 12, \ldots, 20$.

c Use the spreadsheet to find the curved surface area for a cylinder that is 17 cm high with a radius of 7 cm.

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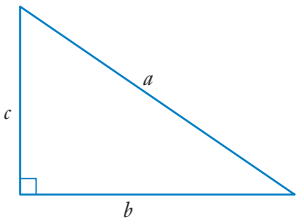
2 The length of the hypotenuse (a) of a right-angled triangle is given by:



$$a^2 = b^2 + c^2$$

where b and c are the other two sides.

a Write the equation in terms of a.

b Construct a spreadsheet that can be used to find the length of the hypotenuse of a right-angled triangle where the other two sides are known.

c Use the spreadsheet to find the length of the hypotenuse of the triangles where:

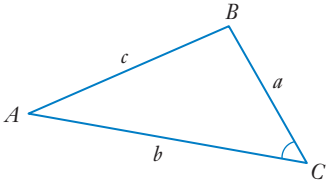
i $b = 7$ and $c = 12$

ii $b = 15$ and $c = 26$

iii $b = 42$ and $c = 17$

iv $b = 29$ and $c = 31$

3 The area of a triangle is given by



$$A = \frac{1}{2}ab \sin(C)$$

where a and b are adjacent sides of the triangle and C is the angle between them.

Use a spreadsheet to calculate the areas of the following triangles.

a $a = 10$ cm, $b = 15$ cm, $C = 28°$

b $a = 12$ mm, $b = 19$ mm, $C = 67°$

c $a = 1.5$ m, $b = 2.3$ m, $C = 98°$

d $a = 32$ cm, $b = 17$ cm, $C = 112°$

e $a = 56$ mm, $b = 41$ mm, $C = 77°$

4 The simple interest (I) earned by an amount invested (P) over t years with an annual interest rate (r) is given by the equation

$$I = Prt$$

where r is expressed as a decimal.

a Use a spreadsheet to construct a table to show the amounts to which \$1000 will grow at 6.2% pa for $t = 1, 2, 3, \ldots, 10$.

b How many years will it take for \$1000 invested at 8.5% to reach \$1500?

c Use the table to calculate the interest that would be earned by \$8000 invested at 4.3% for 15 years.

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5 The volume of a cylinder is given by the equation

$$V = \pi r^2 h$$

where r is the radius and h is the height of the cylinder.

a Use a spreadsheet to construct a table to show the volumes of cylinders with a radius of 8 cm and $h = 10, 11, 12, \ldots, 20$.

b Use the table to find the volumes of the following cylinders:

i $r = 10$ cm and $h = 15$ cm

ii $r = 0.4$ m and $h = 2.1$ m

iii $r = 48$ mm and $h = 56$ mm

iv $r = 18$ cm and $h = 52$ cm

6 An arithmetic progression (AP) is a sequence of numbers in which each successive term differs from the term before by the same amount. For example:

$$3, 7, 11, 15, 19, \ldots$$

The first term is known as a and the difference between terms is d. So for the AP shown above, $a = 3$ and $d = 7 - 3 = 4$.

The general form of an AP is:

$$a, a + d, a + 2d, a + 3d, \ldots$$

The number of terms in an AP is n, so the last term (l) of an AP is:

$$l = a + (n - 1)d$$

Use a spreadsheet to calculate the last term of each of the following APs.

a 5, 11, 17, 23, … to 17 terms

b 111, 102, 93, 84, … to 10 terms

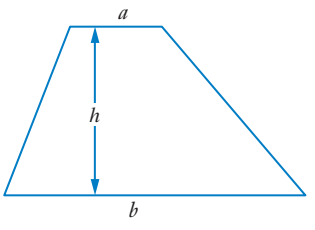
c 24, 32, 40, 48, … to 25 terms

d 73, 78, 83, 88, … to 42 terms

e 207, 196, 185, 174, … to 14 terms

7 The area of a trapezium is given by

$$A = \frac{1}{2}(a + b)h$$



Use a spreadsheet to calculate the area of a trapezium in which:

a $a = 12$ cm, $b = 21$ cm and $h = 34$ cm

b $a = 2.5$ m, $b = 3.7$ m and $h = 1.9$ m

c $a = 53$ mm, $b = 28$ mm and $h = 27$ mm

d $a = 27$ cm, $b = 68$ cm and $h = 52$ cm

e $a = 4.7$ m, $b = 2.9$ m and $h = 2.2$ m

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Problem solving

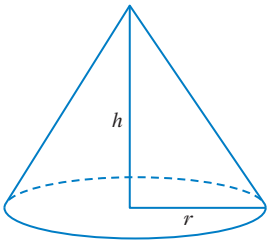
8 The equation used to calculate the body mass index (BMI) of a person h metres tall weighing m kg is:

$$B = \frac{m}{h^2}$$

Use a spreadsheet to determine the height in centimetres (correct to 1 decimal place) of a person who weighs 80 kg who has a BMI of 30.

9 The volume (V) of a cone is given by:

$$V = \frac{1}{3}\pi r^2 h$$



a Use a spreadsheet to construct a table to calculate the volume of a cone with radius 10 cm and $h = 10, 11, 12, \ldots, 20$ cm.

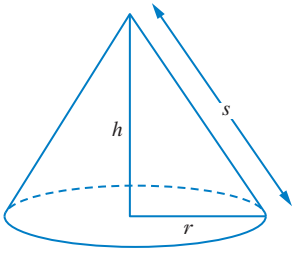
b Use the table of values to calculate the height (correct to 1 decimal place) of a cone with a radius of 10 cm and volume of 1500 cm^3.

10 **a** Use a spreadsheet to construct a table to calculate the volume of a cone with a height of 20 cm and $r = 5, 6, 7, \ldots, 15$ cm.

b Use the table of values to calculate the radius (correct to 1 decimal place) of a cone with height 20 cm and volume of 3200 cm^3.

11 The surface area of a cone with a base is given by

$$SA = \pi r^2 + \pi rs$$



a Use a spreadsheet to construct a table to calculate the surface area of a cone with radius 1.2 m, and $s = 1.0, 1.1, 1.2, \ldots, 2$ m.

b Use the table of values to calculate the height (correct to 2 decimal places) of a cone with radius 1.2 m and surface area of 1500 cm^3.

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7. CHAPTER SUMMARY

Linear and non-linear expressions

- An **algebraic expression** is a collection of numbers and symbols connected by **operations** including +, –, × and ÷.
- **Constant terms** contain only numbers.
- A constant that is multiplied by a variable is called the **coefficient** of the variable.
- **Like terms** have identical variable parts. Only like terms may be added or subtracted.
- A linear expression can be identified in a number of ways.
  - the graph is a straight line
  - the highest power of the variable is 1
  - the rate of change is constant
- Expressions that are not linear are called **non-linear expressions**.
- A **formula** is a rule that uses algebra to show the relationship between quantities.
- The **subject** of a formula is the variable by itself on the left-hand side.
- A formula can be manipulated or **transposed** using **inverse operations**.

  An inverse operation has the effect of undoing the original operation.

Operation	Inverse operation
+	–
–	+
×	÷
÷	×
square (^{2})	$\sqrt{\ }$
$\sqrt{\ }$	square (^{2})

- An **equation** is a mathematical statement that two things are equal.
- When graphing, the **independent variable** is represented on the horizontal axis and the **dependent variable** is represented on the vertical axis.
- A **linear equation** in one variable is an equation that can be written in the form $ax + b = c$, where a, b, and c are constants and x is a variable.
- The **standard** (or **general**) **form** of a linear equation in two variables is $ax + by + c = 0$.
- Equations that are not linear are called **non-linear equations**.
- Expressing an equation in terms of one of the variables is called **isolating** the variable.
- Tables are used to present data because it makes it easier to see trends in data when they are presented in a table.

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7. CHAPTER REVIEW

Linear and non-linear expressions

1 Simplify each expression. Example 1

a $12x - 7x + 5y + 8$ **b** $2(3m - 7) + 8m - 9$

c $3w^2 + 9 - 7w^2 - 5u + 6wu - 6$ **d** $2d(3d + 5) - 3d(4d + 6 + 2de)$

e $5x(4x - 7) - 2(5x + 2) + 3(5x^2 - 8)$ **f** $7(-2z + 3 + 5z^2) - 3z(4 + 6z)$

2 Simplify each expression. Example 1

a $3x^4 \times 8y^2 \times 2x^3y$ **b** $2m^3 \times 5n^5 \times 3m \times m^2n$

c $27a^5 \div 6a^3b^2$ **d** $\dfrac{48p^6q^2r^4}{12p^5qr^2}$

3 State whether each of the following expressions are linear or non-linear. Example 2

a $5d + 7$ **b** $3y(2y - 5)$ **c** $4x - xy + 6$

d $7a - 4b + 6$ **e** $3x - 3\sqrt{x}$ **f** $\dfrac{5y^3 - 2y + 9}{6y}$

g $\dfrac{4r - 7}{2}$ **h** $8q + \dfrac{2q}{5}$ **i** $y + 6x = 7$

4 Draw a graph of each of the following expressions for the domain indicated and use the graph to determine if the expression is linear or non-linear. Example 3

a $9 - 4x$ for $-1 \le x \le 5$ **b** $4 - x^2$ for $-4 \le x \le 4$ **c** $2(3x - 1)$ for $-2 \le x \le 3$

5 The values of a variable and the corresponding values of an expression using the variable are shown in the tables below. Determine if each expression is linear or non-linear. Example 3

a

Variable	1	2	3	4
Expression	5	13	21	29

b

Variable	2	4	6	8
Expression	6	11	16	21

c

Variable	–3	–2	–1	0
Expression	–17	–13	–9	–5

d

Variable	1	3	6	8
Expression	4	16	34	46

e

Variable	1	4	7	10
Expression	7	18	32	41

6 Find the value of m in each equation if $a = 2$, $b = -3$ and $c = 5$. Example 4

a $m = 2a + 3b$ **b** $m = b^2 - a^2$ **c** $m = 3ac - 2b$

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Example 5

7 Transpose each formula so that the variable shown in brackets is the subject.

a $2a = b(4 - 3cd)$ (d)

b $\frac{g}{2h} = \frac{3j}{k}$ (k)

c $M = u(A - vw)$ (v)

d $q = \frac{x - y}{z - y}$ (z)

Examples 6, 7

8 Transpose $z = 4A - \sqrt{6xy}$ so that A is the subject and then find the value of A if $x = -3$, $y = -2$ and $z = 10$.

Example 8

9 Transpose each equation to isolate y on the left-hand side.

a $3x(x + 5) = 6(3x - 2y)$

b $3x - \sqrt{5y} + 2 = 0$

c $\frac{4}{y} - 3x = 7$

d $5(y - 1) = 4x - 10$

e $4 + 5xy = 0$

f $\sqrt{4 - 3y} = 5x + 3$

g $2y(3x - 1) = 2x^2 + 5x$

h $\frac{3}{x} + \frac{5}{y} = 15$

i $y^2 - 8 = 7x$

j $2y - 5x^2 + 9 = 4x$

Example 9

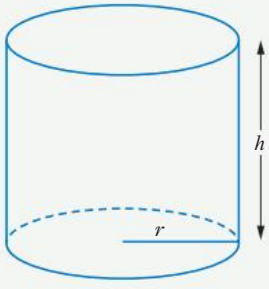
10 The volume of a cylinder is given by the equation

$$V = \pi r^2 h$$

where r is the radius and h is the height of the cylinder.

a Use a spreadsheet to construct a table to show the volumes of cylinders that are 20 cm high for $r = 1, 2, 3, \ldots, 15$.

b Use the spreadsheet to find the volume of a cylinder that is 25 cm high with a radius of 28 cm.



Problem solving

11 The volume of water in a tank that is being drained is given in the table.

a Use a spreadsheet to determine if the relationship between volume and time is linear or non-linear.

b Use the data to estimate when the volume is 60 L.

Time (min)	Volume (L)
1	100
2	83
3	71
4	63
5	56
6	50
7	45
8	42
9	38
10	36

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12 A number of people are studied as part of an investigation into the effects of smoking. The table shows the heights (cm) and forced expiratory volume (lung capacity, L) of 20 people in the investigation.

Height (cm)	152	155	155	156	157	158	159	160	161	163	165	170	171	173	174	175	177	179	180	183
Volume (L)	2.0	2.1	2.3	2.4	2.2	2.4	2.6	2.9	2.7	2.8	3.1	3.5	3.7	4.0	4.1	4.1	4.2	4.4	4.5	4.8

a Enter the data in a spreadsheet and add a trendline to determine if there is a linear relationship between the two measurements.

b Use the data to predict the lung capacity of a person who is 168 cm tall.

c Predict the height of a person whose lung capacity is 2.5 L.

13 The equation used to calculate the potential energy (in joules) stored when a spring is compressed is:

$$PE = \frac{1}{2}kx^2$$

where k is the spring constant in N/m (Newtons per metre) and x is the distance in metres the spring is compressed.

A spring with a spring constant of 50 000 N/m is compressed. Use a spreadsheet to determine the distance in centimetres (correct to 1 decimal place) that a spring must be compressed to store 400 J of energy.

Practice quiz

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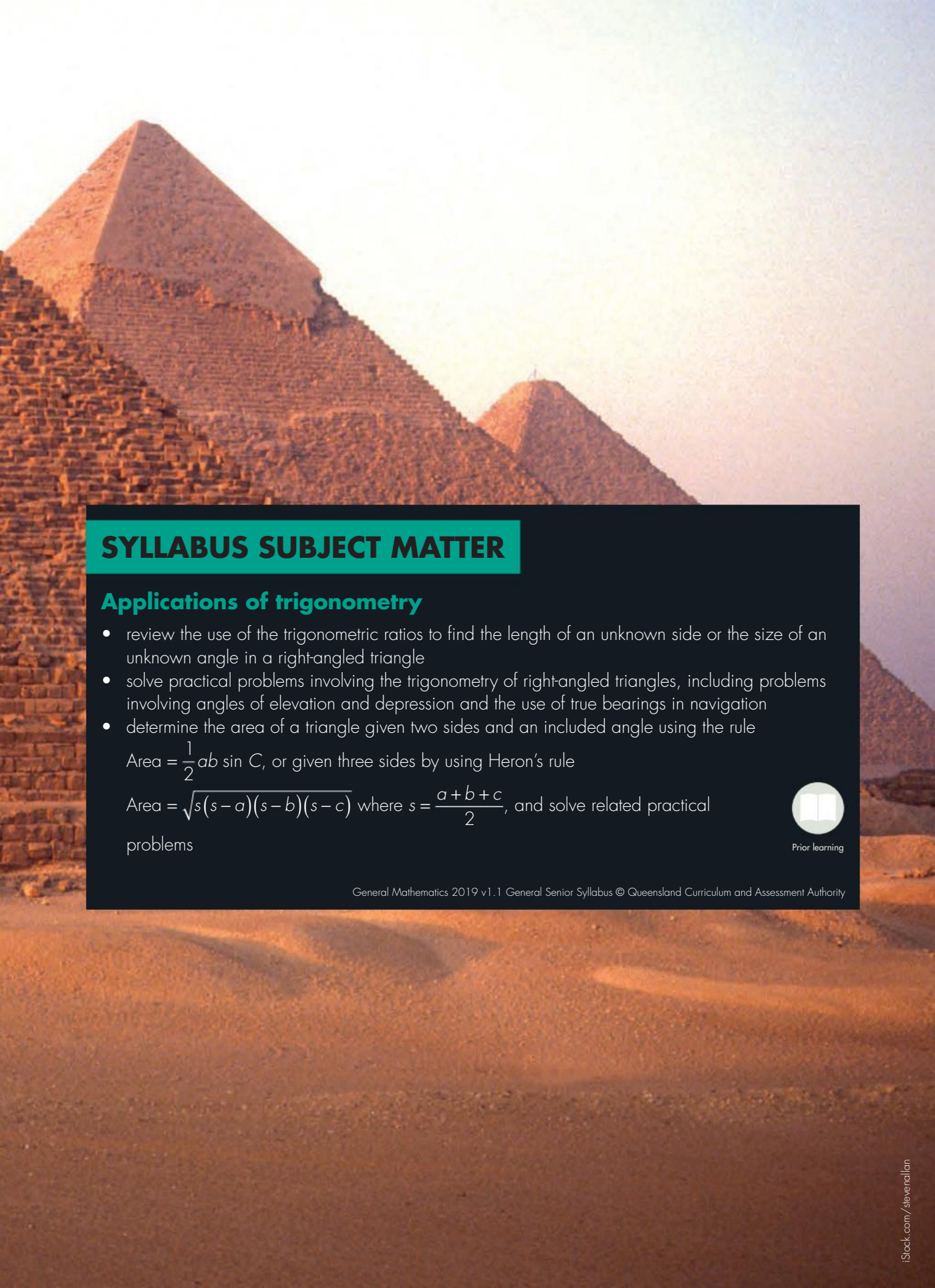
8.

TRIGONOMETRY OF RIGHT-ANGLED TRIANGLES

Trigonometry is the study of the sides and the angles of triangles. Long before trigonometry developed as a formal branch of mathematics, the ancient Babylonians used trigonometric functions in their study of astronomy and the ancient Egyptians used trigonometry to assist in building the pyramids and surveying land. Since that time, trigonometry has found many practical applications, including in navigation, building, surveying and computer graphics.

8.01 The trigonometric ratios
8.02 Finding unknown sides
8.03 Finding unknown angles
8.04 Angles of elevation and depression
8.05 Bearings and distances
8.06 The area of a triangle
Chapter summary
Chapter review



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SYLLABUS SUBJECT MATTER

Applications of trigonometry

- review the use of the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle
- solve practical problems involving the trigonometry of right-angled triangles, including problems involving angles of elevation and depression and the use of true bearings in navigation
- determine the area of a triangle given two sides and an included angle using the rule

  Area $= \frac{1}{2}ab \sin C$, or given three sides by using Heron's rule

  Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$, and solve related practical problems

Prior learning

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TERMINOLOGY

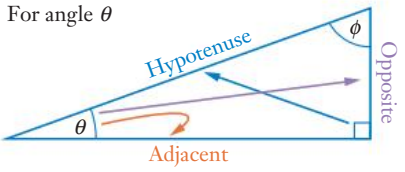
adjacent
compass bearing
opposite
angle of depression
degree mode
right-angled triangle
angle of elevation
hypotenuse
true bearing

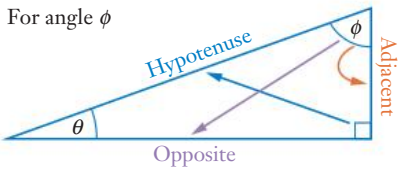
8.01 The trigonometric ratios

Trigonometric ratios

Identifying the correct trigonometric ratio

The trigonometric ratios relate the *sides* to the *angles* of right-angled triangles. The three sides of the triangle are named in relation to the angle in which you are interested.





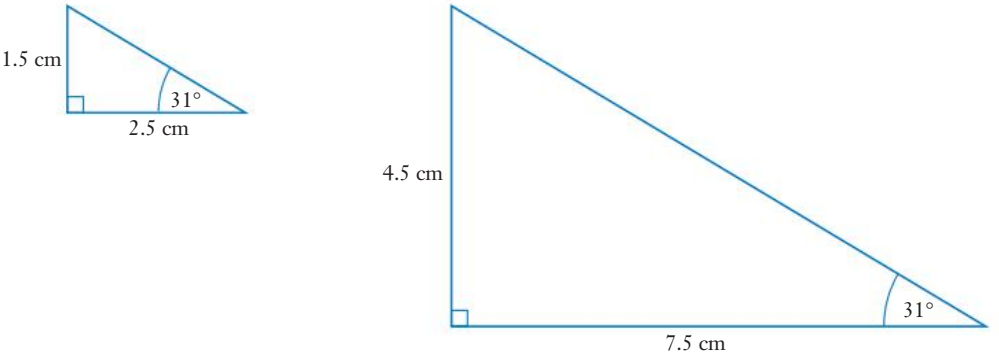
The longest side is opposite the right angle and is called the **hypotenuse**.

The **opposite** side is furthest away from the angle. It is diagonally opposite the angle.

The **adjacent** side is next to the angle and makes one 'arm' of the angle. (The hypotenuse is the other 'arm'.)

The two right-angled triangles below have a corner angle of 31° and are similar triangles with a scale factor of 3.

Two right-angled triangles with the same corner angle always have the same shape and are **similar** (by the AAA test).



Also, the opposite and adjacent sides in each triangle divide to give the same value.

$$\frac{1.5}{2.5} = 0.6 \quad \text{and} \quad \frac{4.5}{7.5} = 0.6$$

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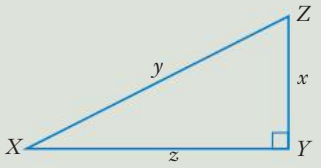
Every right-angled triangle with a 31° corner angle has the same ratio of opposite : adjacent = 0.6. Similarly, the ratio of opposite : hypotenuse is the same for every right-angled triangle with a 31° corner angle. The same applies for the ratios of adjacent : hypotenuse. Each ratio has a special name:

$$\text{Sine of angle } X = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{x}{y}$$

$$\text{Cosine of angle } X = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{z}{y}$$

$$\text{Tangent of angle } X = \frac{\text{opposite}}{\text{adjacent}} = \frac{x}{z}$$



These **trigonometric ratios** are abbreviated as:

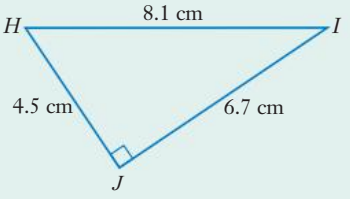
sin (X), **cos** (X) and **tan** (X)

A useful mnemonic to recall the ratios and sides is **S**ome **O**ld **H**edgehogs **C**an't **A**lways **H**ide **T**heir **O**wn **A**bdomens.

EXAMPLE 1

For this triangle find, correct to two decimal places, the trigonometric ratio:

a tan (H) **b** cos (H) **c** sin (H)



Solution

a Tangent uses opposite and adjacent.

The opposite side is 6.7 and the adjacent side is 4.5.

$$\tan(H) = \frac{\text{Opp}}{\text{Adj}} = \frac{6.7}{4.5} \approx 1.49$$

b Cosine uses adjacent and hypotenuse.

The adjacent side is 4.5 and the hypotenuse side is 8.1.

$$\cos(H) = \frac{\text{Adj}}{\text{Hyp}} = \frac{4.5}{8.1} \approx 0.56$$

c Sine uses opposite and hypotenuse.

The opposite side is 6.7 and the hypotenuse side is 8.1.

$$\sin(H) = \frac{\text{Opp}}{\text{Hyp}} = \frac{6.7}{8.1} \approx 0.83$$

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When using a calculator for trigonometry, make sure it is set to **degree mode**. Most scientific calculators will show a small DEG or D on the display if they are set in this mode. If the display shows RAD or GRAD, it will not give the correct answers.

EXAMPLE 2

Use your calculator to find, correct to 4 decimal places:

a tan (34.8°) **b** H such that sin (H) = 0.89

Solution

a Use the **TAN** key. tan 34.8° = 0.695 018...

Round and write the answer. tan (34.8°) ≈ 0.6950.

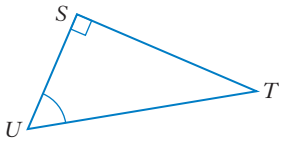
b Use the **SIN**$^{-1}$ key (2nd function **SIN**). sin^{-1} 0.89 = 62.873 246...

Round and write the answer. $H \approx 62.8732°$.

Exercise 8.01 The trigonometric ratios

1 Name the opposite, adjacent and hypotenuse for each marked angle below.

a



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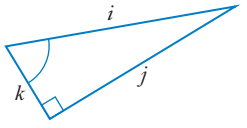

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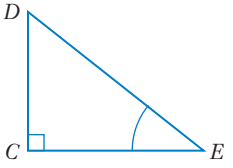

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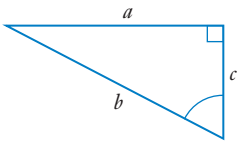

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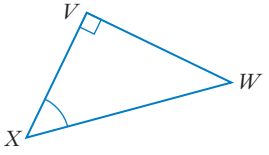
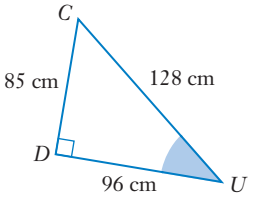


Example 1

2 Find the value of sine, cosine and tangent of each shaded angle below.

a



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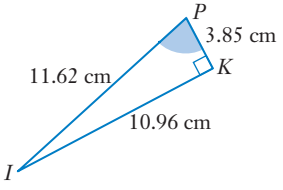

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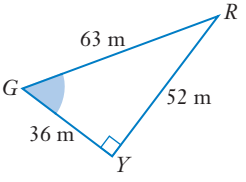

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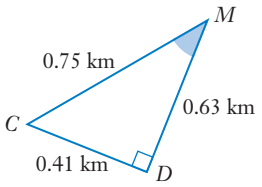

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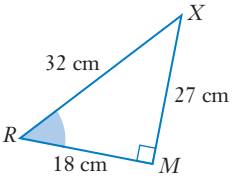

Example 2

3 Calculate correct to 4 decimal places:

a	sin (76.8°)	**b**	sin (5°)	**c**	sin (32°)
d	sin (84.6°)	**e**	sin (17.2°)	**f**	cos (76.8°)
g	cos (5°)	**h**	cos (32°)	**i**	cos (84.6°)
j	cos (17.2°)	**k**	tan (1.4°)	**l**	tan (24.2°)
m	tan (41.3°)	**n**	tan (62.9°)	**o**	tan (89.2°)

4 Calculate correct to 4 decimal places:

a	$\sin^{-1}(0.85)$	**b**	$\sin^{-1}(0.371)$	**c**	$\sin^{-1}(0.99)$
d	$\sin^{-1}(0.31415)$	**e**	$\sin^{-1}(0.5)$	**f**	$\cos^{-1}(0.85)$
g	$\cos^{-1}(0.371)$	**h**	$\cos^{-1}(0.99)$	**i**	$\cos^{-1}(0.31415)$
j	$\cos^{-1}(0.5)$	**k**	$\tan^{-1}(0.0036)$	**l**	$\tan^{-1}(0.15)$
m	$\tan^{-1}(1)$	**n**	$\tan^{-1}(4.6)$	**o**	$\tan^{-1}(784)$

Problem solving

5 In question **3**, what do you notice about all the values of *sine*? Can you explain your answer?

6 In question **3**, what do you notice about all the values of *cosine*? Can you explain your answer?

7 Is the pattern observed for the values of *sine* and *cosine* also applicable to the *tangent* ratio? Hint: Your answers from question **3** could help you explain this.

8 Use your calculator to try to find the following:

a **i** $\sin^{-1}(2)$ **ii** $\cos^{-1}(2)$ **iii** $\tan^{-1}(2)$

b What do you notice in each case? Can you explain why? (Hint: refer to your answers for questions **5**, **6** and **7**).

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Finding an unknown side

Elevations and bearings

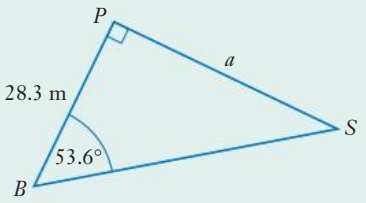
8.02 Finding unknown sides

You can use the trigonometric ratios to calculate unknown lengths of sides in right-angled triangles if at least one angle and one side are known.

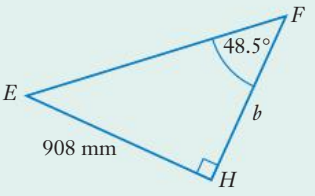
EXAMPLE 3

Find the unknown side in each triangle using the tangent ratio.

a



b



Solution

a The unknown side is opposite the known angle B. $\tan(B) = \frac{\text{Opp}}{\text{Adj}}$

The adjacent is known. The opposite is required. $\tan(53.6°) = \frac{a}{28.3}$

Multiply to get a. $a = 28.3 \times \tan(53.6°)$

Use your calculator, keeping the exact number. $= 28.3 \times 1.3563 \ldots$

Round and write the answer. a is about 38.4 m.

b The missing side is not opposite the known angle. The angle opposite the required side can be worked out using the sum of angles in a triangle. $E = 180° - 90° - 48.5°$

$= 41.5°$

Use the tangent ratio. $\tan(E) = \frac{\text{Opp}}{\text{Adj}}$

$\tan(41.5°) = \frac{b}{908}$

Multiply to get b. $b = 908 \times \tan(41.5°)$

Use your calculator, keeping the exact number. $= 908 \times 0.8847 \ldots$

$= 803.33 \ldots$ mm

Round and write the answer. b is about 803 mm.

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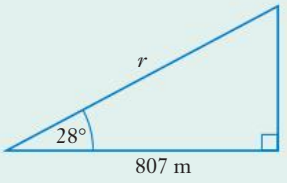
EXAMPLE 4

The slope of a suburban road up a steep hill is 28°. The survey map shows that the road covers a horizontal distance of 807 m to reach the top of the hill. What is the actual length of road surface that must be used for calculations of guttering and sealing requirements?

Solution

Draw a diagram, put in the information and mark the wanted side with a letter.



Adjacent and hypotenuse are involved, so use cosine.

$$\cos(28°) = \frac{807}{r}$$

Multiply by r.

$$r \times \cos(28°) = 807$$

Now divide by cos (28°) to get r.

$$r = \frac{807}{\cos(28°)}$$

Use your calculator, keeping accuracy.

$$= \frac{807}{0.8829}$$

You may want to use the memory.

$$= 913.9840\ldots$$

Round to the accuracy of the question.

$$\approx 914 \text{ m}$$

Write the answer.

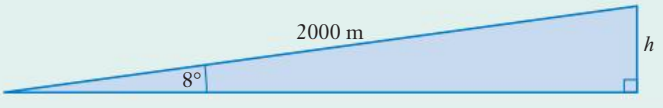
The actual length of road surface is about 914 m.

EXAMPLE 5

A train travels 2 km along a sloping track. The track is at an angle of 8°. Through what height does the train rise?

Solution

Draw a diagram, put in the information and mark the required side with a letter.



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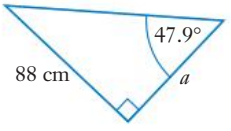
Opposite and hypotenuse are involved, so use sine.	$\sin(8°) = \dfrac{h}{2000}$
Multiply to get h.	$h = 2000 \times \sin(8°)$
	$= 2000 \times 0.1391$
Use your calculator, keeping accuracy.	$= 278.3462\ldots$
Round to the accuracy of the question.	≈ 300 m
Write the answer.	The train rise is approximately 300 m.

Exercise 8.02 Finding unknown sides

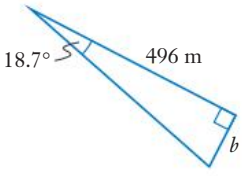
Example 3

1 Find the unknown side in each triangle using the tangent ratio.

a



b



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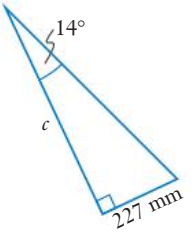

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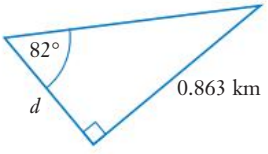
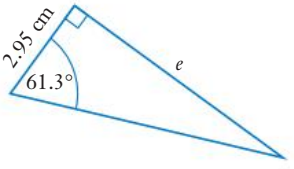


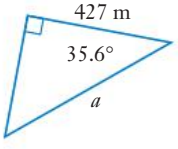
e



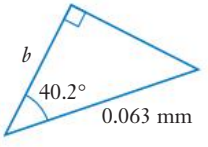
Example 4

2 Find the unknown side in each triangle using the cosine ratio.

a



b



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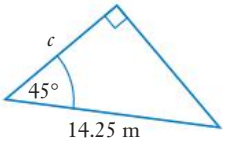

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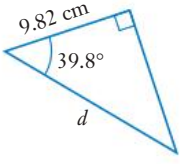

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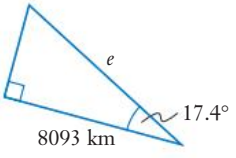

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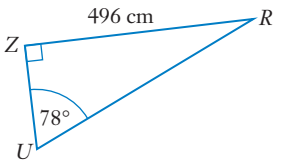
Example 5

3 Find an unknown side in each triangle using the sine ratio.

a



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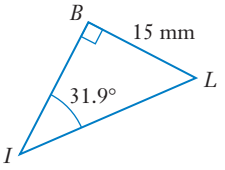

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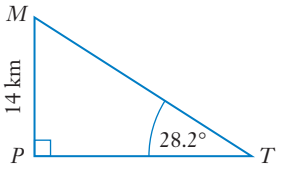

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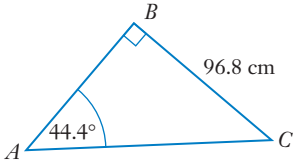

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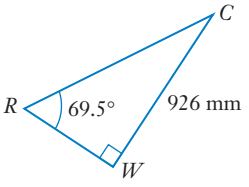
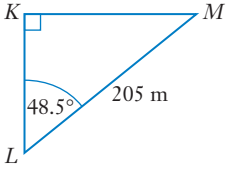


4 Find all the unknown sides in each triangle.

a



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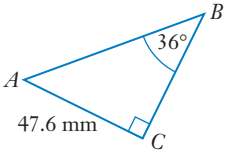

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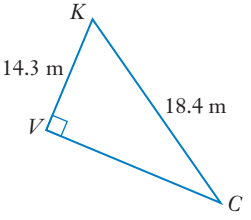

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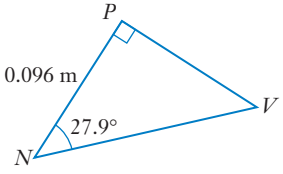

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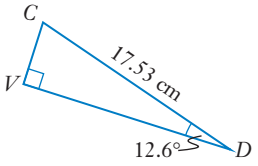
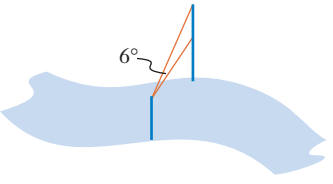


Problem solving

5 A surveyor's assistant has a 3.5 m-high stick on the other side of a stream. The surveyor finds that he has to turn his theodolite up an angle of 6° to see the top of the stick. The theodolite is 1 m off the ground. How wide is the stream?



6 A long slope for a train line is inclined at an angle of 8.4°. To lay the track for the slope, 248 sections of 20 m track were used. What horizontal distance does the train travel in going up the slope?

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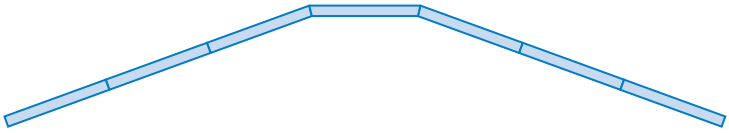
7 A slot-car enthusiast has 7 straight sections of track to make a bridge with. She plans to use 3 to slope up and 3 to slope back down, with one flat in the middle. The bridge can have a maximum slope of 15°, and the track sections are each 30 cm long. What is the maximum height of the bridge?



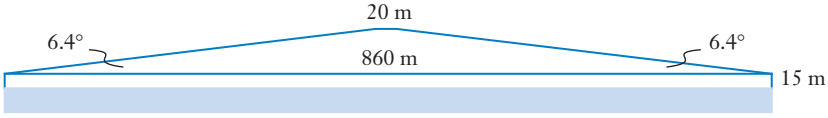
8 Aluminium planks 6 m long are used on a building site. The maximum safe slope to run wheelbarrows of concrete along from one floor to another is 12°. Each floor is 2.7 m higher than the previous one.

a What is the maximum height rise for one plank?

b What is the horizontal distance covered when the slope is 12°?

c How many planks must be used to go up three floors?

9 A bridge slopes up from each end at an angle of 6.4° and has a flat section 20 m long in the middle. If the river it crosses is 860 m wide and the ends are 15 m above the water, how high is the middle section above the water?



10 The mast of a yacht is 5.4 m from the stern. A taut line from the top of the mast to the stern makes an angle of 74.8° with the deck. How long is the line and how high is the mast?

iStock.com/olgalings

11 A tent rope is looped around a peg that is 4825 mm away from the tent pole. When stretched tight from the peg to the top of the pole, it makes an angle of 27.3° with the ground. The rope is 6 m long, and after looping around the peg the rope goes to a cleat partway up the tight rope. Calculate how far up the rope the cleat is from the peg.

12 A wire 30 m long is used to hold a radio mast in place. The wire is at an angle of 72° to the ground and is tied 2 m from the top of the mast. How high is the radio mast?

13 When it is standing, the legs of an ironing board make an angle of 42.5° with the floor. The crossed legs just fit under the board, which is 1.2 m long. How high is the ironing board?

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14 A fire hose is tied to the top of a pole to dry. It is pulled 12.5 m away from the pole so that none of it lies on the ground, and it makes an angle of 65.4° with the ground. How long is the fire hose?

15 A jet climbs at an angle of 26° to the ground. The cruising height for the jet is 8000 m, the climbing speed is 300 km/h and the cruising speed is 450 km/h.

a How far does the jet travel horizontally to reach its cruising height?

b How far is it from the airport 5 minutes after taking off?

16 At a scout rally, a flying fox is set up to travel between a treehouse and a raised platform. The platform is 2 m off the ground, and the treehouse is 6.5 m from the ground. To work properly, the rope must be at an angle of at least 8°. What are the maximum length of the rope and the maximum distance of the platform from the treehouse?

8.03 Finding unknown angles

If the ratio of two sides of a right-angled triangle is known, you can use a calculator to find the angle corresponding to that ratio.

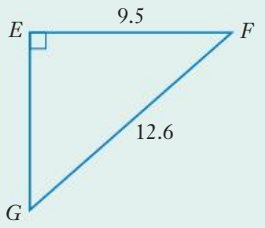
Finding an unknown angle

Calculating lengths and angles

Trigonometry problems

EXAMPLE 6

Find, correct to one decimal place, the unknown angles in this triangle.

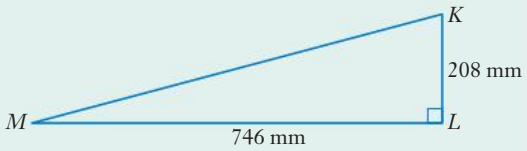


Solution

Consider angle G. The sides are opposite and hypotenuse.	$\sin(G) = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$
Opposite is 9.5 and hypotenuse is 12.6.	$= \dfrac{9.5}{12.6}$
Keep the exact number in your calculator.	$= 0.7539\ldots$
Use $\sin^{-1}$ to find G, then round.	$G \approx 48.9°$
Use the sum of angles in a triangle to find F.	$F \approx 180° - 90° - 48.9°$
	$= 41.1°$
Write the answers.	The angles are about 48.9° and 41.1°.

EXAMPLE 7

Find the unknown angles in this triangle.



Solution

Consider the sides in relation to angle M.

$$\tan(M) = \frac{\text{Opposite}}{\text{Adjacent}}$$

Opposite is 208 and adjacent is 746.

$$= \frac{208}{746}$$

Keep the exact number on your calculator.

$$= 0.2788\ \ldots$$

Use $\tan^{-1}$ to find M, then round.

$$M \approx 15.6°$$

Use the sum of angles in a triangle to find k.

$$K \approx 180° - 90° - 15.6°$$

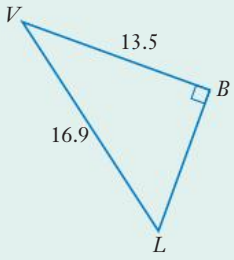
$$\approx 74.4°$$

Write the answers.

The angles are about 15.6° and 74.4°.

EXAMPLE 8

Use cosine to find the angles in this triangle.



Solution

Consider angle V. The sides are adjacent and hypotenuse.

$$\sin(V) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$= \frac{13.5}{16.9}$$

Keep the exact value in your calculator.

$$= 0.7988\ \ldots$$

Use $\cos^{-1}$ to find V and round.

$$V \approx 37.0°$$

Use the sum of angles in a triangle to find L.

Thus $L \approx 180° - 90° - 37.0°$

$$= 53.0°$$

Write the answers, showing correct accuracy.

The angles are about 37.0° and 53.0°.

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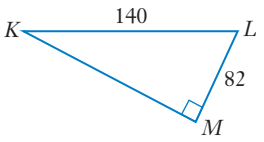
Exercise 8.03 Finding unknown angles

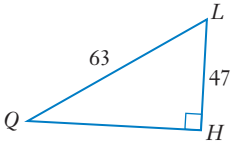
1 Find the unknown angles in each triangle.

Example 6

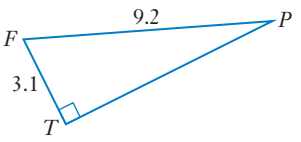
a



b



c



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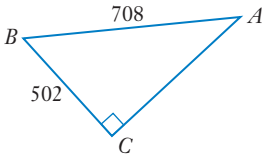
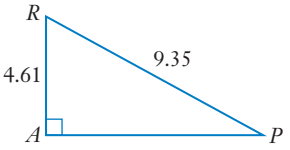


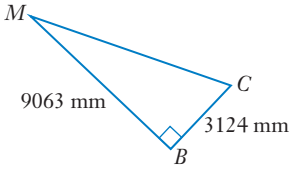
e



2 Find the unknown angles in each triangle.

Example 7

a



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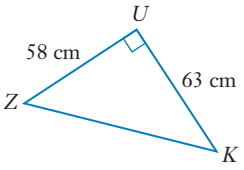

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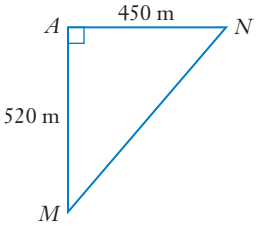

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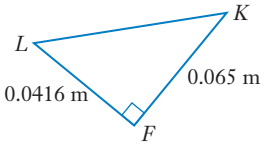

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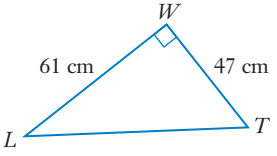
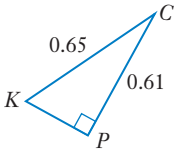


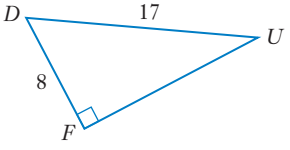
3 Find the unknown angles in each triangle.

Example 8

a



b



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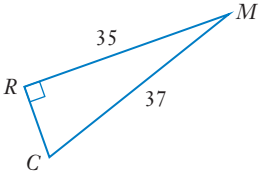

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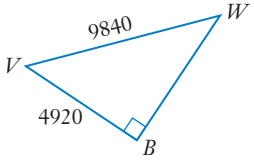

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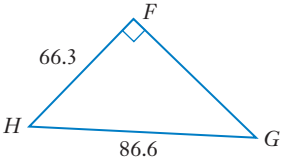

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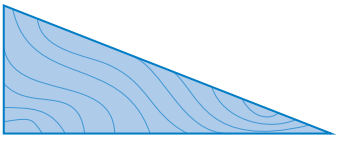
Problem solving

4 A carpenter needs to cut a right-angled wedge out of timber. The wedge has to be 25 cm long, and the sloping edge must be 28 cm. Find the angle the carpenter should use to cut the timber, and explain your reasoning.



5 A wheelchair ramp up to an entrance has a horizontal length of 7555 mm and a sloping length of 7580 mm. The maximum angle allowed for such ramps by Australian Standards is 4.76°.

Dreamstime.com/Baloncici

Source: http://www.edgekz.com/kazakhstan-making-strides-in-disability-access-acceptance/

a What is the angle of the ramp?

b Does the ramp meet the standard?

6 A ladder 5 m long has its end just resting on the top of a fence 3.7 m high. What angle does the ladder make with the ground?

7 Two slides, one metal and one plastic, are to be built in a playground. The top of the metal slide is 2.5 m high and the bottom of the slide is 5.2 m from the point directly below the top of the slide. The plastic slide is 2.2 m high with the end 4.6 m from the point below the top of the slide.

a Find the angles that each slide makes with the ground, correct to two decimal places.

b Which slide is steeper?

8 An escalator ramp is 29 m long and covers a horizontal distance of 25 m. What is the angle of inclination?

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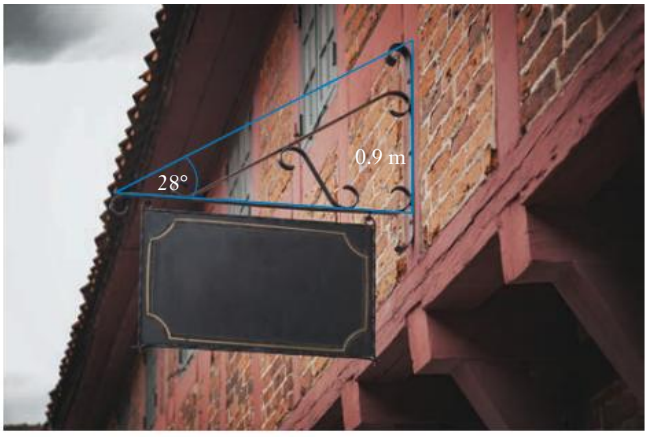
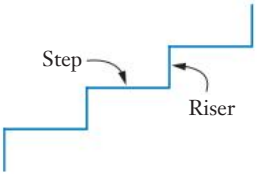
9 An overhead sign is positioned 0.9 m from the top of a steel framework and is supported by a wire. If the angle between the wire and the sign is 28°, how much wire is needed?



Shutterstock.com/Sophie McAulay

10 A cyclist cycles up a steady slope for 3.2 km and ends up 72 m higher than he started. What was the angle of inclination of the hill?

11 A staircase has 16 steps that are 265 mm wide and risers that are 165 mm high. The maximum angle of a wheelchair ramp is 7°.



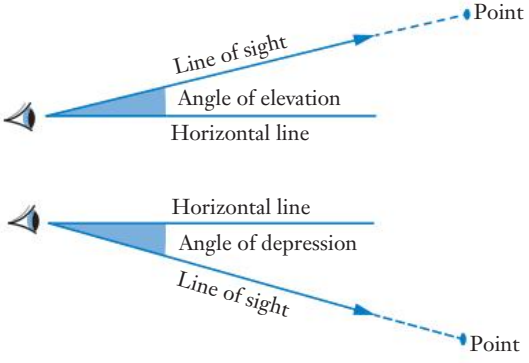
a What will be the angle of a ramp placed directly on the stairs?

b How can a wheelchair ramp of the correct slope be fitted?

8.04 Angles of elevation and depression

An **angle of elevation** is formed when you have to look up from a horizontal line to see a point.

An **angle of depression** is formed when you have to look down from a horizontal line to see a point.



Angles of elevation and depression

Angles of elevation and depression

- The angle you have to *look up* is the angle of elevation.
- The angle you have to *look down* is the angle of depression.

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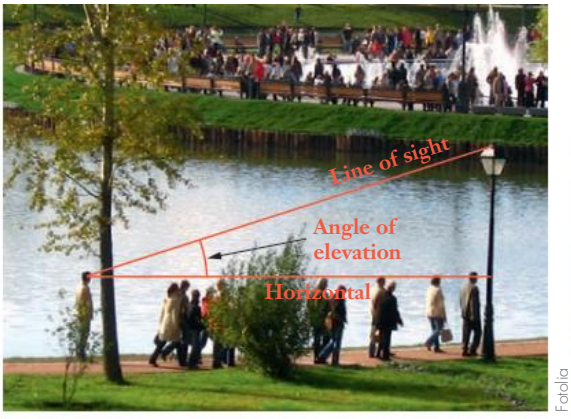
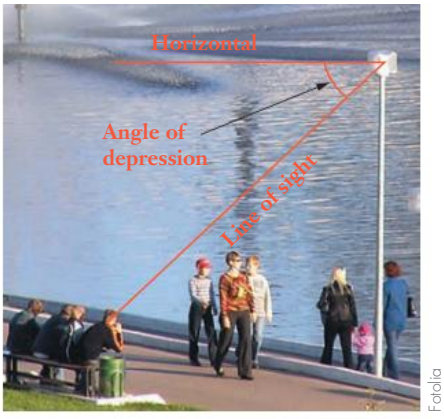


Fotolia



Fotolia

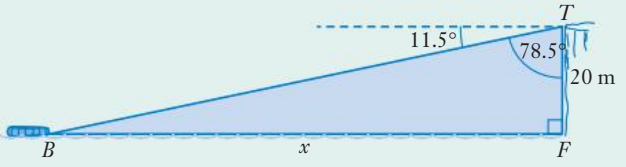
Angle of depression

EXAMPLE 9

From the top of a cliff 20 m high, the angle of depression of a surf-boat is 11.5°. How far is the boat from the foot of the cliff?

Solution

Draw a diagram, label points and show the information.



Find an angle in the triangle.

$$\angle FTB = 90° - 11.5°$$
$$= 78.5°$$

Use tangent since opposite and adjacent are involved.

$$\tan(78.5°) = \frac{\text{Opp}}{\text{Adj}}$$

Put in values.

$$= \frac{x}{20}$$

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Multiply by 20 to find x.

$$x = 20 \times \tan(78.5°)$$
$$= 20 \times 4.9151 \ldots$$
$$= 98.301 \ldots$$

Round to the accuracy of the question.

$$\approx 98 \text{ m}$$

Write the answer in terms of the question.

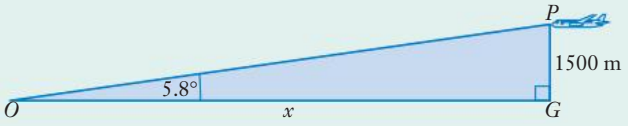
The boat is about 98 m from the cliff.

EXAMPLE 10

An observer spots a plane flying at a height of 1500 m. He notes that the plane has an angle of elevation of 5.8°. How far away (on the ground) is the plane?

Solution

Draw a diagram, label points and show the information.



Use tangent since opposite and adjacent are involved.

$$\tan(5.8) = \frac{\text{Opp}}{\text{Adj}}$$

Put in values.

$$= \frac{1500}{x}$$

Multiply by x.

$$x \times \tan(5.8°) = 1500$$

Divide by tan 5.8° to find x.

$$x = \frac{1500}{\tan(5.8°)}$$
$$= \frac{1500}{0.1015\ldots}$$
$$= 14\,767.2248 \ldots$$

Round to accuracy of question.

15 000 m or 15 km

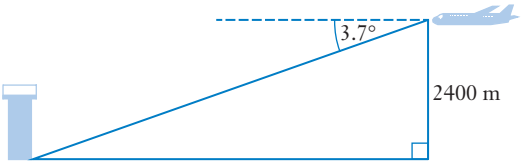
Write the answer.

The plane is about 15 km away.

Exercise 8.04 Angles of elevation and depression

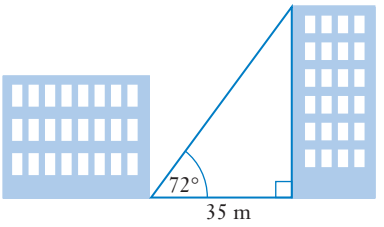
Example 9

1 The pilot of a plane observes that the angle of depression of the control tower at the airport is 3.7°. If the altimeter shows the plane is at a height of 2400 m, find the horizontal distance to the control tower.



Example 10

2 From the other side of the street, the angle of elevation of the top of a building is 72°. If the street is 35 m wide, find the height of the building.



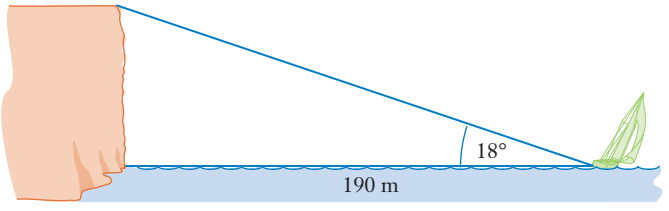
Problem solving

3 From the top of a lighthouse 47 m high, the depression angle of a boat at sea is 4.25°. At sea-level, how far from the lighthouse is the boat?

4 Sharon knows that she takes 12 paces every 10 m. She paces out a distance of 60 paces from the foot of a tree and looks up at the top. Its elevation angle is then 38°. Sharon's eyes are 1.5 m from the ground. What is the height of the tree?

5 The height of a mountain is shown on a map as 1700 m. From the top of a hill 370 m high, the angle of elevation to the top of the mountain is 6.7°. Find the horizontal distance between the hill and the mountain, in kilometres.

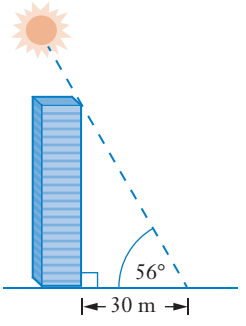
6 The elevation angle from a yacht to the top of a cliff is 18°. If the yacht is 190 m from the base of the cliff, find how high the cliff is.



7 The shadow of a building is 30 m long when the elevation angle of the sun is 56°.

a Calculate the height of the building.

b How long will the shadow be when the angle of elevation of the sun becomes 32°?



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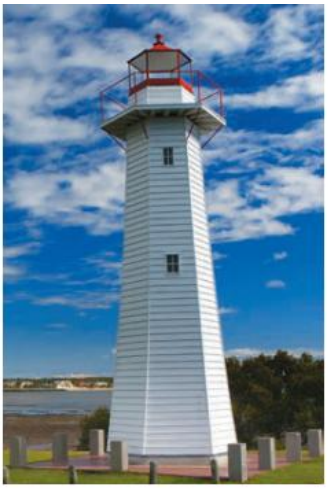
8 An observer in a lighthouse, 19 m high, sights a boat at sea at an angle of depression of 18°. If the base of the lighthouse is at sea-level, how far is the boat from the base of the lighthouse?



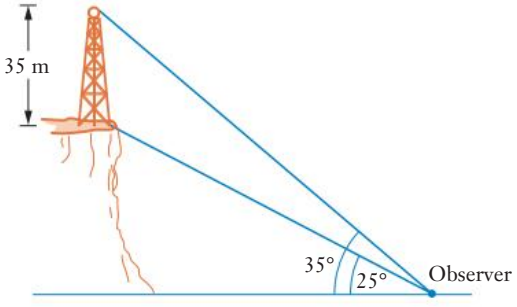
Shutterstock.com/Shaun Robinson

9 A helicopter flying at a height of 1800 m is directly overhead. After 15 minutes, it is still 1800 m above the ground, but its elevation angle is 8°. Calculate:

a the distance travelled in that time, to the nearest 100 m

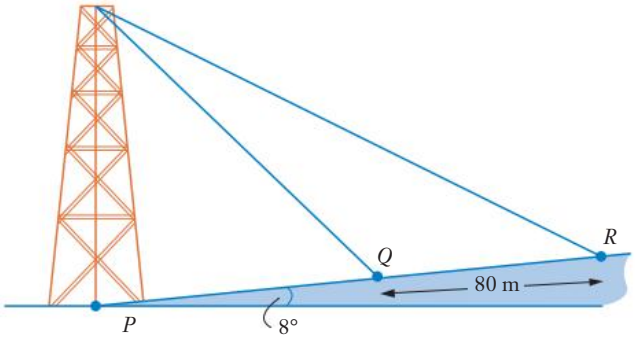
b the speed of the helicopter in km/h.

10 A beacon is mounted on a tower that sits on a vertical cliff. The beacon is 35 m higher than the top of the cliff. To an observer some distance from the base of the cliff, the beacon and the top of the cliff appear to be at angles of elevation of 35° and 25° respectively. How far is the observer from the base of the cliff?

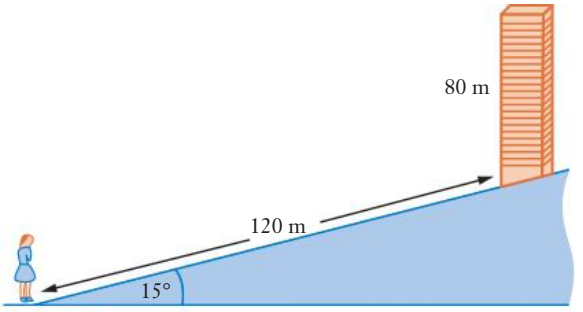


11 A vertical structure, 80 m tall, stands on top of a hill. From a point some distance from the base of the hill, the angle of elevation to the top of the structure is 11.4°. From the same point, the elevation angle to the bottom of the structure is 8.2°. Find the height of the hill.

12 A tower is erected at the base of a road that is inclined at a constant angle of 8° to the horizontal. Angles of elevation to the top of the tower are measured at two points that are 80 m apart, some distance up the slope away from the tower. The angles are found to be 25° and 8° respectively. What is the height of the tower?



13 Melissa's eye level is 1.8 m from the ground. She stands at the base of a road that is at a constant incline of 15° to the horizontal. There is a building 80 m tall at a distance of 120 m from the base of the road where she is standing, as measured along the road.



a Find the vertical distance that the base of the building is above the ground where Melissa is standing.

b Calculate the angle of elevation of the top of the building from her eyes.

c Calculate the angle subtended by the top and bottom of the building at her eye level.

Bearings

Identifying bearings

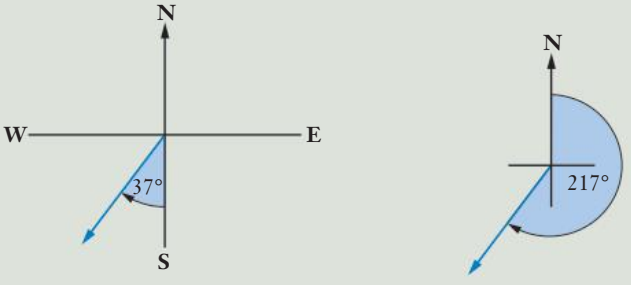
Bearings

Elevations and bearings

8.05 Bearings and distances

Directions on the Earth are given in terms of the basic compass points north, south, east and west.

- A **bearing** is the direction of a fixed point, or the path of an object, from the point of observation.
- **Compass bearings** are specified as angles either side of north or south. For example, a compass bearing of S 37° W is found by facing south and moving through an angle of 37° to the west.
- **True (or three-figure) bearings** are measured in degrees clockwise from the north line. Three digits are used to specify the direction. For example, the compass bearing S 37° W as a true bearing would be 217° or 217° T.



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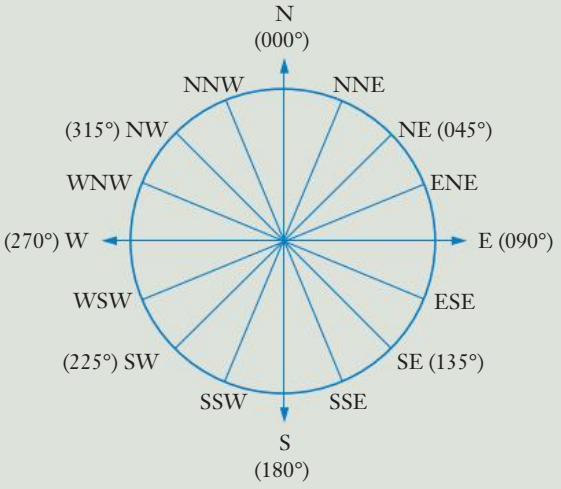
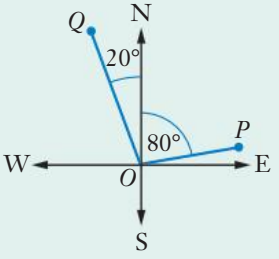
The diagram below shows the bearings of the major compass points.

16 points of the compass



EXAMPLE 11

State the compass bearing and true bearing of P and Q from O as shown in the diagram.



Solution

a P is 80° east of north. — The compass bearing of P is N 80° E.

P is 80° clockwise from north. — The true bearing of P is 080°.

b Q is 20° west of north. — The compass bearing Q is N 20° W.

Q is 340° clockwise from north. — The true bearing of Q is 340°.

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EXAMPLE 12

Sketch the bearing 306°.

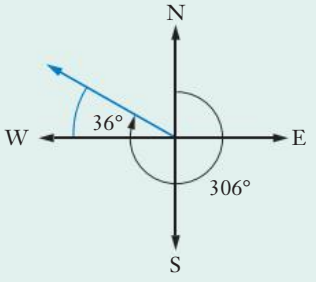
Solution

Sketch a simplified compass rose.

$$306° = 270° + 36°$$

Draw in an arrow from the origin at an angle of 36° clockwise from west.

A simplified compass rose like the one shown here is usually drawn when working with bearings.



EXAMPLE 13

True bearings

Bearings

A plane leaves a runway and remains on a bearing of 122° for 260 km.

a How far south of the runway is the plane?

b What is the bearing of the runway from the plane?

Solution

a Mark in the origin (O) and the position of the plane (P) on the sketch.

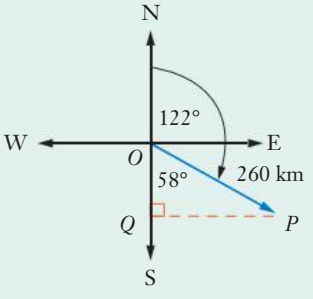
Draw in PQ to form the right-angled triangle OPQ.

We need to know the distance OQ.

We know $OP = 260$ km.

$\angle POQ$ and 122° are supplementary, so $\angle POQ = 58°$.

For ΔPOQ, adjacent and hypotenuse are involved. So use the cosine ratio.



$$\cos(58°) = \frac{OQ}{260}$$

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Multiply by 260 to find OQ.

$OQ = 260 \times \cos(58°)$

$= 137.7790087$

Round.

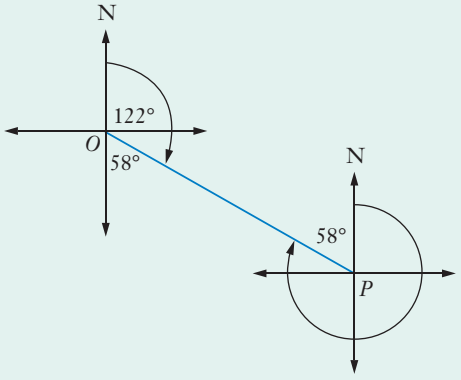
$= 137.8$

Write the answer as a sentence.

The plane is about 137.8 km south of the runway.

b Draw a compass rose with the centre at P.

The angle between OP and due north at P must be 58°, since it is an alternate angle, with the angle between OP and due south at O.



Calculate the bearing.

Bearing of the runway from the plane

$= 360° - 58°$

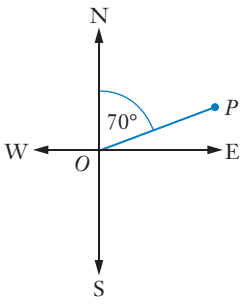
$= 302°$

Exercise 8.05 Bearings and distances

Example 11

1 For each diagram, state the compass bearing and true bearing of P from O.

a



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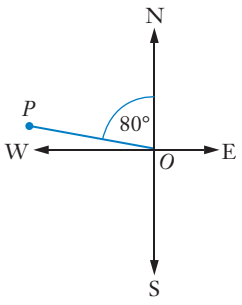

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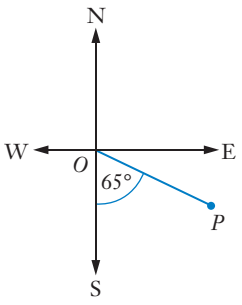

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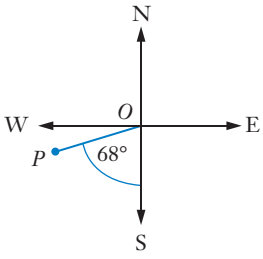
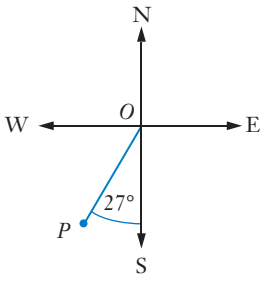
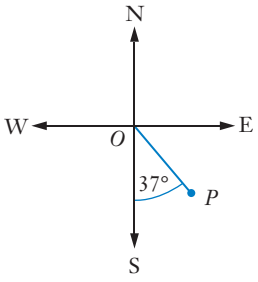


e



f



Example 12

2 For each compass bearing:

i sketch on a compass rose

ii convert to a true bearing

a N 80° E	**b** SW	**c** SE	**d** N
e S 10° E	**f** NW	**g** S 20° W	**h** N 30° W

3 For each true bearing:

i sketch on a compass rose

ii convert to a compass bearing

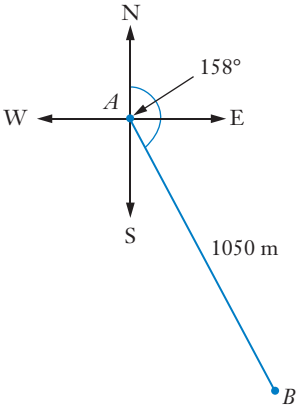
a 120°	**b** 080°	**c** 205°	**d** 315°
e 330°	**f** 210°	**g** 320°	**h** 045°

Example 13

4 A hiker leaves a base camp (A) and remains on a bearing of 158° for 1050 m.

a How far east of the base camp is the hiker?

b What is the bearing of the base camp from the hiker?



Problem solving

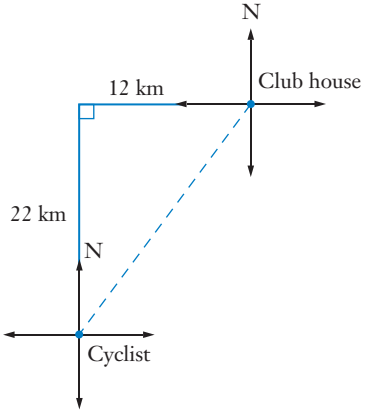
5 A ship travels 150 km on a bearing of 134°. How far south and east does it travel?

6 After travelling for 2 hours in a direction S 36° E, an aeroplane is 600 km south of its starting point. Find the actual distance travelled and its speed.

7 A cyclist begins from a club house and travels due west for 12 km, then due south for 22 km.

a How far is the cyclist from the club house?

b What is the bearing of the cyclist from the club house?



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8 A train is travelling at 60 km/h on a track running from west to east. Brandon is south of the track and notes that the train is directly north of him. Half an hour later the train is at a bearing of 020°. What is the distance of Brandon south of the track?

9 Luke is travelling on a road running north–south and notices that a hill is directly west of him. After cycling at 20 km/h for a further 2 hours, he notices that the direction of the hill is now N 75° W. How far is the hill from the road, and how far is it from Luke's new position?

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10 One town is directly north of another. From a point 20 km to the west of the road joining them, the towns are at bearings of 050° and 120°. How far apart are the towns?

11 Two ships leave the same port. The first travels for 220 km on a bearing of 168° while the second ship travels for 186 km on a bearing of 256°.

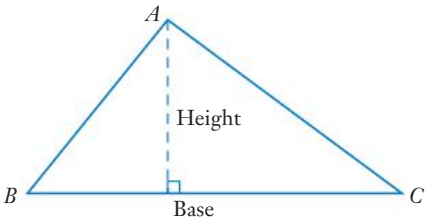
a How far apart are the two ships?

b Calculate the bearing of the second ship from the first.

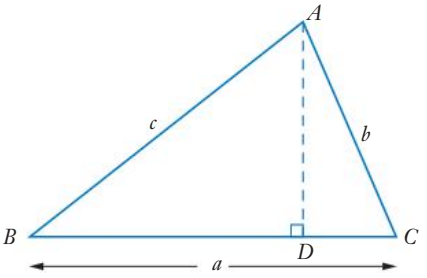
8.06 The area of a triangle

Area of $\triangle ABC = \frac{1}{2}$ base $\times$ height

We can make another rule for the area using the sine ratio.



Consider the diagram below. Draw the perpendicular height from A down to D.



Then in $\triangle ADC$, $\sin(C) = \frac{AD}{b}$

So $AD = b \sin(C)$

Now area of the triangle $\triangle ABC = \frac{1}{2}$ base $\times$ height

$$= \frac{1}{2} a \times AD$$

$$= \frac{1}{2} a \times b \sin(C)$$

$$\text{Area of } \triangle ABC = \frac{1}{2} ab \sin(C)$$

It doesn't matter which way around we draw the triangle, or whether the angle used is acute or obtuse.

Area of a triangle

- Area of $\triangle ABC = \frac{1}{2} bc \sin(A)$
- Area of $\triangle ABC = \frac{1}{2} ac \sin(B)$
- Area of $\triangle ABC = \frac{1}{2} ab \sin(C)$

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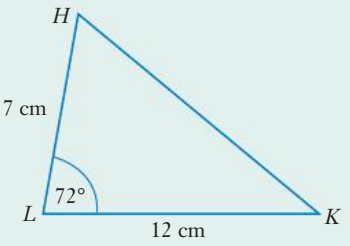
EXAMPLE 14

For $\triangle HKL$, $h = 12$ cm, $k = 7$ cm and $\angle HLK = 72°$. Calculate the area of $\triangle HKL$.

Solution

Draw a diagram and show the information.



The rule for the area of a triangle is:

$$\triangle ABC = \frac{1}{2}ab\sin(C)$$

In this case:

$$A \leftrightarrow H \text{ and } a \leftrightarrow h$$

$$B \leftrightarrow K \text{ and } b \leftrightarrow k$$

$$C \leftrightarrow L \text{ and } c \leftrightarrow l$$

Write the rule.

$$\text{Area of } \triangle HKL = \frac{1}{2}hk\sin(L)$$

Substitute relevant values.

$$= \frac{1}{2}12 \times 7 \times \sin(72°)$$

$$\approx 39.9$$

Write the answer.

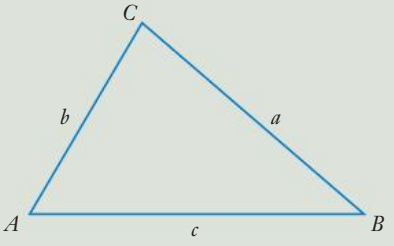
The area of $\triangle HKL$ is about 39.9 cm^2.

If all three sides of a triangle are known, Heron's formula can be used. Heron's formula follows from Pythagoras' rule but is not proven here.

Heron's formula

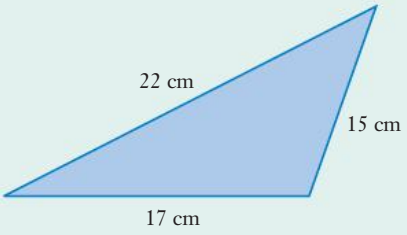
Area $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{1}{2}(a+b+c)$ is the **semiperimeter**.



EXAMPLE 15

Calculate the area of this triangle.



Solution

Calculate the semiperimeter.

$$s = \frac{1}{2}(a + b + c)$$

$$= (22 + 17 + 15) \div 2$$

$$= 27 \text{ cm}$$

Write the formula.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Put in values.

$$= \sqrt{27(27-22)(27-17)(27-15)}$$

$$= \sqrt{27 \times 5 \times 10 \times 12}$$

$$= \sqrt{16200}$$

$$= 127.2792 \ldots$$

Round the answer.

$$\approx 130 \text{ cm}^2$$

Exercise 8.06 The area of a triangle

Example 14

1 Find the area of each triangle described.

a $\triangle ABC$ where $a = 5$, $b = 6$ and $C = 38°$

b $\triangle XYZ$ where $x = 9$, $y = 5$ and $Z = 60°$

c $\triangle PQR$ where $r = 15$, $q = 9$ and $P = 27.7°$

d $\triangle MNO$ where $n = 17$, $m = 8$ and $O = 114°$

Example 15

2 Find the area of each triangle described.

a $\triangle ABC$ where $a = 3$, $b = 5$ and $c = 7$

b $\triangle XYZ$ where $x = 2$, $y = 4$ and $z = 5$

c $\triangle PQR$ where $p = 12$, $q = 8$ and $r = 10$

d $\triangle MNO$ where $m = 6$, $n = 6$ and $o = 6$

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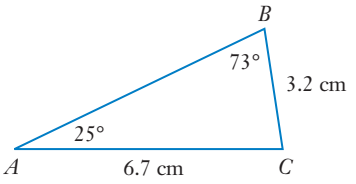
Problem solving

3 Two sides of a triangle are 4.5 cm and 6 cm and the area is 11.25 cm^2. Find the angle included by the sides.

4 Calculate the area of this triangle, correct to the nearest cm^2.



5 Find the area of a parallelogram with sides 5.5 cm and 8.9 cm and one interior angle 57°.

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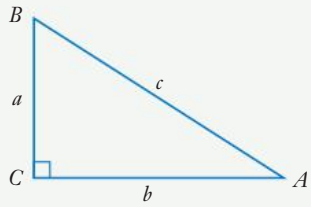
Trigonometry crossword

8. CHAPTER SUMMARY

Trigonometry of right-angled triangles



- In any right-angled triangle, the **tangent, sine** and **cosine ratios** of an angle are given by

$$\tan(A) = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

- An **angle of elevation** is formed by looking up from a horizontal line to a point.
- An **angle of depression** is formed by looking down from a horizontal line to a point.
- A **compass bearing** is given as the angle from north or south towards either east or west.
- The **true bearing** of the object is given as the 3-digit clockwise angle the observer turns from north to face the object.
- The area of a triangle can be calculated by using two lengths of the triangle and their included angle. $\triangle ABC = \frac{1}{2}ab\sin(C)$
- If the sides of a triangle are known, **Heron's formula** can be used to calculate the area.

$$\text{Area } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c).$$

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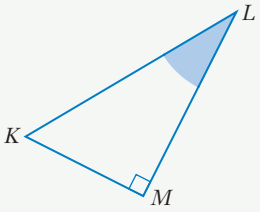
8. CHAPTER REVIEW

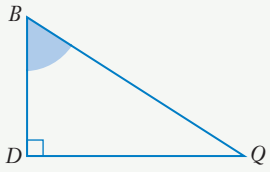
Trigonometry of right-angled triangles

1 Name the opposite, adjacent and hypotenuse for the angle marked in each triangle. Example 1

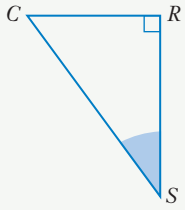
a



b

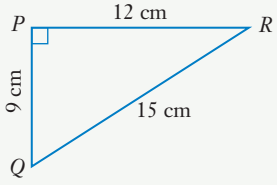


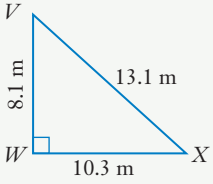
c

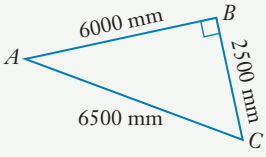


2 Use the diagrams below to find: Example 1

a sin (Q) **b** sin (V) **c** sin (A) **d** sin (C) **e** cos (Q)
f cos (R) **g** cos (X) **h** cos (C) **i** tan (Q) **j** tan (A)







3 Use your calculator to find, correct to 4 decimal places: Example 2

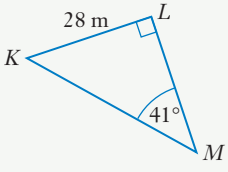
a sin (47°) **b** cos (3.8°) **c** tan (85°) **d** tan (45°)
e cos (72°) **f** sin (54.9°) **g** cos (19.6°) **h** sin (45°)

4 Find the value (correct to 1 decimal place) of the angle for which: Example 2

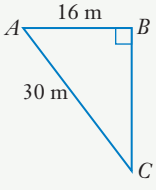
a sin (T) = 0.55 **b** cos (R) = 0.247 **c** tan (E) = 3.5
d tan (K) = 0.34 **e** cos (P) = 0.8023 **f** sin (W) = 0.2

5 Find all unknown sides in each triangle. Example 3

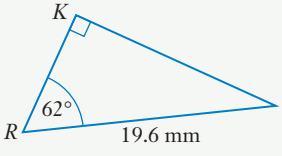
a



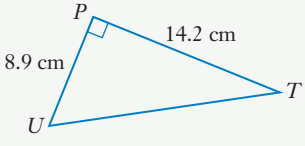
b



c



d



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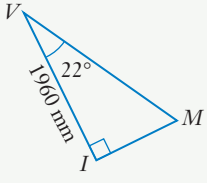

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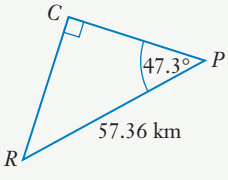

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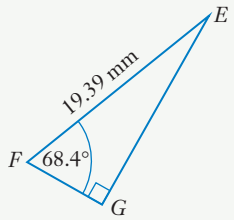

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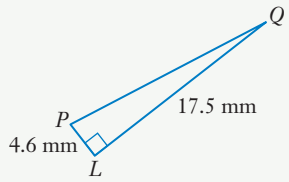

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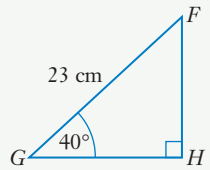
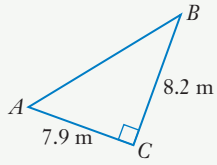


Examples 6-8

6 Find all unknown angles in each triangle.

a



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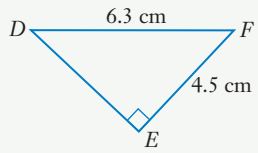

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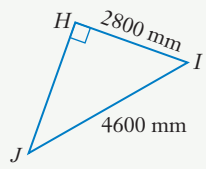

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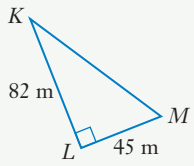

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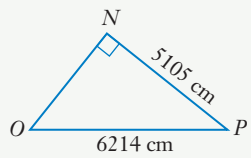

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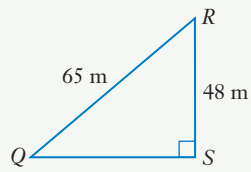
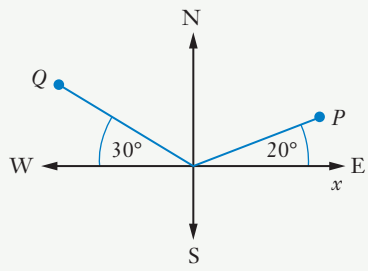


Examples 11,12

7 For the following diagram, find:

a the compass bearings of *P* and *Q*.

b the true bearings of *P* and *Q*.



Example 9

8 Anne is flying from Perth to Geraldton at a speed of 150 km/h. She thinks her altimeter, which shows a height of 15 000 feet (4600 m), is faulty. Looking ahead, she sees the top of a hill marked as 405 m on her map at an angle of depression of 13°. Two minutes later, the angle of depression has increased to 20.6°.

a Find her height above the hill and thus her altitude.

b What should the air traffic controller tell her about her altimeter?

Problem solving

9 A cabinet maker has splayed the legs of a table out at an angle of 15° to the vertical. If the height of the table surface is 850 mm and the table-top is 45 mm thick, find the length of the legs.

10 From a point 100 m from the cliff base, the angle of elevation of the cliff top is 16°. The elevation angle of the top of a lighthouse built on the cliff is 30°. Find the height of the cliff and the height of the lighthouse.

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11 A kite line is let out 70 m while a kite rises 45 m. If the line is tight, find the elevation angle of the kite from the boy holding the string.

12 An aircraft is 370 km north of its original position. Its bearing is 342° from the original position. Find the distance actually travelled.

13 Two sides of a triangle are 4.8 m and 7 m and the area is 8.3 m^2. Find the angle included by the sides.

14 Find the area of a parallelogram with sides 4.2 mm and 6.3 mm and one interior angle 38°.

Practice quiz

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UNIVARIATE DATA ANALYSIS

9. DATA AND STATISTICS

The Australian Bureau of Statistics (ABS) conducts a census of the Australian population every 5 years. It collects and analyses data about the Australian economy, industries, environment, people, their health and working conditions. Governments and businesses use it for planning and policy decisions. The ABS calculates many indexes that show changes in Australian conditions. Market research companies and polling organisations also collect statistical information for businesses. In this chapter you will learn how to interpret basic statistical data.

9.01 Classifying data and variables
9.02 Displaying categorical variables
9.03 Displaying discrete variables
9.04 Displaying continuous variables
9.05 Measures of centre
9.06 The range and interquartile range
9.07 The standard deviation
9.08 The shape of a distribution
9.09 Statistical investigations
Chapter summary
Chapter review

SYLLABUS SUBJECT MATTER

The statistical investigation process

- review statistical investigation as a process, for example, identifying a problem and posing a statistical question, collecting or obtaining data, displaying and analysing the data, interpreting and communicating the results
- define univariate data

Making sense of data relating to a single statistical variable

- classifying statistical variables as categorical or numerical
- classify a categorical variable as ordinal or nominal and use tables and pie, bar and column charts to organise and display the data, for example, ordinal — income level (high, medium, low), or nominal — place of birth (Australia, overseas)
- classify a numerical variable as discrete or continuous, for example, discrete — the number of rooms in a house, or continuous— the temperature in degrees Celsius
- select and justify an appropriate graphical display to describe the distribution of a numerical data set including dot plot, stem plot, column chart or histogram
- describe the graphical displays in terms of the number of modes, shape (symmetric versus positively or negatively skewed), measures of centre and spread, outliers and interpret this information in the context of the data
- determine the mean and standard deviation (using technology) of a data set and use these statistics as measures of location and spread of a data distribution, being aware of significance of the size of the standard deviation

Prior learning

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TERMINOLOGY

bar graph
bimodal
bimodal distribution
categorical
class centre (class midpoint)
class interval
class width
coefficient of variation
column graph
continuous
discrete
dispersion
dot plot
first quartile
frequency polygon
histogram
interquartile range (IQR)
Likert scale
linear interpolation
lower class limit
lower quartile
mean
measure of central tendency
measure of spread
median
modal class
mode
negative skew
nominal
numerical
ordinal
outlier
picture graph
pie chart
plateau distribution
population
positive skew
range
relative standard deviation (RSD)
skewed to the left
skewed to the right
standard deviation
stated class limits
statistical investigation process
stem and leaf
symmetrical distribution
third quartile
true class limits
univariate data
upper class limit
upper quartile

Statistical data match-up

9.01 Classifying data and variables

Statistical data are the properties or attributes of people or items. You can observe different values for each property, like type, age, colour, size, preference, etc. Each separate property is called a **variable**, because the values vary. The kind of values of a variable determine what you can do with it.

Types of data

- **Univariate data** has only one variable
- **Categorical data** has categories that can be sorted into groups, but cannot be measured
  - **Nominal** data can be counted, but cannot be put in order
  - **Ordinal** data can be put in order, or have a rating scale
- **Numerical data** has values that can be measured
  - **Discrete** data can take only particular values
  - **Continuous** data can have any values; the domain can be finite or infinite

In this chapter, you will study only univariate data.

It makes sense to add numerical data. You can add ages, heights, number of children, etc. You cannot add countries of birth, preferred flavours of ice-cream, clothes sizes, etc.

If you can add the data values, it is numerical. If you can't, it is categorical.

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EXAMPLE 1

Jessica is 16 years old, 169 cm high, has fair hair, 3 living grandparents and rates rock music as 'like' on a scale from 'dislike strongly' to 'like strongly'.

Classify each of her stated attributes as nominal, ordinal, discrete or continuous.

Solution

She is 16, rounded down to the nearest year.	Age is a continuous numerical variable, but age to the nearest year is discrete.
Height can be measured with a ruler or tape.	Height is a continuous numerical variable.
Hair colours are categories that don't have an order.	Hair colour is a nominal categorical variable.
She can only have 0, 1, 2, 3 or 4 living grandparents.	Number of living grandparents is a discrete numerical variable.
The 'dislike-like' scale has an order.	Music preference is an ordinal categorical variable.

You should check data to see whether it is reasonable. If someone said their height was 16 cm, you would be wise to discard this data.

A rating scale such as '1: dislike strongly, 2: dislike, 3: don't care, 4: like, 5: like strongly' is called a **Likert scale** after its inventor. It is ordinal categorical, but is sometimes treated as discrete numerical.

EXAMPLE 2

What is the expected range for:

a an adult's height?

b a school student's weekly earnings from part-time work?

Solution

a The shortest person in the world was 55 cm and the tallest was 272 cm.	The expected range of adult height is about 55–270 cm.
b Only some school students do part-time work.	Minimum value is \$0.
School-aged students are not allowed to work more than 12 hours in a school week. In a non-school week, the maximum is 38 hours.	Even at a pay rate of \$20/h, the maximum earnings would be \$240 in a school week and \$760 in a non-school week.
State the answer.	The expected range would be \$0–\$760.

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Exercise 9.01 Classifying data and variables

Example 1

1 Classify each data item.

- **a** memory sizes of computers at your school
- **b** overall performances of different tyres
- **c** distances to shops from your school
- **d** costs of restaurant meals
- **e** types of trees native to your area
- **f** placings in the final of the 100 m race
- **g** eye colours of members of your family
- **h** pulse rates of athletes after an event

2 Classify the following data.

- **a** teams in the National Basketball League
- **b** ages of girls in a volleyball team
- **c** birth weights of babies born at Wesley Hospital
- **d** shoe sizes of Year 11 students at Ipswich High
- **e** hair colours of students in your class
- **f** weekly incomes of doctors in Rockhampton
- **g** numbers of tries scored by sides in the Rugby Union World Cup

3 Identify the variable type for each question on the following form.

What is your age?
What is your height?
What is your weight?
Do you use a gym for exercise?
How many times do you exercise each week?
How do you travel to school/work?

4 Identify the variable type for each question on the following questionnaire.

What is your name?
What is your age?
For how long have you been out of work (months)?
How many job advertisements did you reply to last week?
How many interviews did you attend last week?
How far (km) from home are you prepared to travel for work?

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Example 2

5 Give the expected data range for each variable.

a number of students in a secondary school

b mass of a passenger car

c price of a new car

d value of a current Australian coin

e value of an old Australian coin

6 Give the expected data range for each variable.

a mass of an adult (in kilograms)

b mass of a newborn baby (in kilograms)

c weekly pocket money received by a Year 11 student

d shoe size of a Year 11 girl

e total when two normal dice are thrown

f number of points scored by an individual in a basketball game

7 Some schools allow adult students to enrol in Years 11 and 12 and attend normal classes.

a Does your school seem to allow this?

b What is the expected range for student age in Year 11 in *your* school?

c What is the expected range for student age in Year 11 in *Queensland*?

Problem solving

8 A fruit shop has three different grades of tomatoes for sale at prices of \$2.99/kg, \$4.99/kg and \$6.99/kg. Peter said that the average price of tomatoes sold must be \$4.99/kg. State whether you agree or disagree with Peter and give reasons for your answer.

9 Semi-trailers are sometimes classified by the number of wheels they have, including both the prime mover and the trailer(s). For example, a prime mover could have a pair of front wheels and 2 pairs of double wheels on the back. If it was pulling a trailer with 3 pairs of double wheels on the back, the whole rig would be a 22-wheeler. Estimate the range of 'wheelers' for semi-trailers in Australia.

10 The travel destinations of people leaving Brisbane International Airport were coded as follows.

New Zealand = 1, *Asia* = 2, *Pacific Islands* = 3, *America* = 4, *Africa* = 5, *Europe* = 6

The average for 50 people was worked out to be 2.2, and this was given as evidence that the most common destination was Asia. Explain what is wrong with this conclusion.

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11 In a survey, people were asked to respond to the following questions on Likert scales.

- *Violent criminals should not be allowed any privileges such as library access in gaol.*
- *Capital punishment should be brought back for murders involving torture.*

The responses were then averaged, giving results of 3.4 and 3.6 respectively. Explain why it is incorrect to say that this means that people are more in favour of capital punishment than of withdrawal of privileges for violent criminals.

12 Jan said that the distance for a javelin throw is a discrete variable because it is always measured to the nearest 2 cm. Deirdre disagreed with Jan, saying that the distance for a throw is continuous. Explain who you think is correct, carefully justifying your answer.

13 The table below shows the ages of students in a Year 11 class.

Age	15	16	17	18
Number students	4	15	3	1

a What kind of variable is student age? Give reasons for your answer.

b What kind of variable is the number of students? Justify your answer.

14 The prices of fruit and vegetables vary during the year according to availability. Oranges sold in Australia in winter are often imported from California. Give an estimate of the price range of oranges (per kilogram) during the year, justifying your answer.

9.02 Displaying categorical variables

Every picture tells a story

Frequency tables

Frequency distribution tables

Australian statistics

You can use a **frequency table** to organise any categorical data. The order of nominal data doesn't matter, but you should put ordinal values in order. Make sure you check data to ensure that it is valid.

EXAMPLE 3

Some high school students completed a survey with the following questions by circling their responses.

Q1	What year are you in?	7 8 9 10 11 12
Q2	Are you male or female?	M F
Q3	Would like to start and finish school earlier in the day?	Y N
Q4	If you answer yes to Question 3, how much earlier?	$\frac{1}{2}$ 1 $1\frac{1}{2}$ 2 hours

The responses were as follows.

Respondent	Q1	Q2	Q3	Q4
1	11	M	Y	1
2	10	F	N	

Compile frequency tables for Year and gender.

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Respondent	Q1	Q2	Q3	Q4
3	12	M	N	1
4	11	F	N	
5		F	N	
6	10	M	N	
7	7	M	Y	0.5
8	9	F	N	
9	9	F	Y	0.5
10	7	M	Y	1.5
11	11	M		
12	11	M	N	
13	11	M	Y	1
14	6	F	N	
15	12	F	Y	2
16	8	F	Y	1.5
17	9	M	Y	0.5
18	11	M	N	
19	10	F	Y	1.5
20	9	F	Y	0.5

Solution

Check data.

If some of a response is invalid, you should discard the *entire* response.

Response 3 has contradictory responses.

Responses 5 and 11 are incomplete.

Response 13 has an invalid year.

Compile the remaining data for Year in order.

Year	Frequency
7	2
8	1
9	4
10	3
11	5
12	1
Total	**16**

Compile the remaining data for gender.

Gender	Frequency
F	8
M	8
Total	**16**

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Pie charts and **picture graphs** are more suitable for nominal data than ordinal data. You can use **bar** and **column** graphs for either.

EXAMPLE 4

Draw a pie chart of the following sales of novels.

Type	Horror	Romance	Historical	Mystery	Crime	Other
Sales	4097	14 875	6172	8942	7241	12 320

Solution

Find the total sales.

Total = 53 647

Find the scale for 1 degree.

$$\text{Number of 1 degree} = \frac{53\,647}{360}$$

$$\approx 149 \text{ novels/degree}$$

Find the angle for each type of novel.

$\frac{4097}{149} \approx 27$, etc.

Type	Angle
Horror	27°
Romance	100°
Historical	41°
Mystery	60°
Crime	49°
Other	83°

Draw the graph.

Show the total represented in the title of a pie chart.

Label each sector.

Novels sold

Total = 53 647

Bar graphs and **column graphs** have a natural order, so are suitable for ordinal data. Before drawing a column or bar graph, decide what scale you will use.

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EXAMPLE 5

40 people on Simpson St gave the following ratings of radio station 4LM in Mt Isa. Draw a column graph to show the ratings.

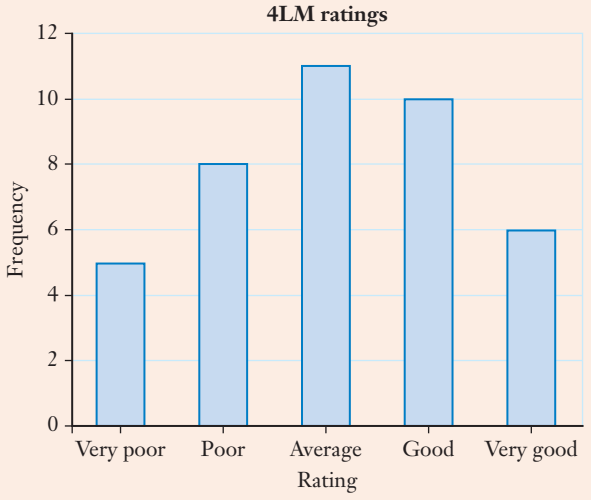
Rating	Very poor	Poor	Average	Good	Very good
Number of people	5	8	11	10	6

Solution

Decide on the scales.

Make the height 2 people/cm and the width of each column 1 cm. Leave a gap of 0.5 cm between columns.

Draw the graph.



Exercise 9.02 Displaying categorical variables

1 The table below shows the responses of students on a survey of their grades. Compile frequency tables for

a Year **b** As **c** Passes (Grades A-C).

Example 3

Student	Year	As	Bs	Cs	Ds	Es
1	11	1	0	4	1	0
2	11	0	0	2	1	3
3	12	0	1	3	1	1
4	11	0	0	1	4	1
5	10	2	3	2	1	0
6	11	1	1	3	1	0
7	7	4	3	1	0	0
8	12	2	3	1	0	0
9	9	0	1	4	1	2

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Student	Year	As	Bs	Cs	Ds	Es
10	8	1	2	4	0	1
11	12	0	1	3	2	0
12	8	0	1	2	3	2
13	11	1	2	3	0	0
14	8	3	2	2	1	0
15	10	0	2	5	1	0
16	11	0	1	2	3	0
17	8	1	2	3	2	0
18	10	0	1	7	0	0
19	9	2	1	4	1	0
20	8	1	0	4	2	1
21	11	1	1	3	1	0
22	9	2	2	4	0	0
23	11	1	1	2	2	0
24	11	2	0	4	0	0
25	7	2	5	1	0	0
26	10	1	0	6	1	0
27	8	1	1	5	1	0
28	10	1	3	3	1	0
29	7	1	2	4	1	0
30	10	0	3	3	1	1

2 People rated some sweeteners in order. The ratings of each sweetener were as follows from 1 as first preference to 5 as last.

Aspartame: 1, 4, 1, 1, 1, 5, 2, 3, 3, 4, 3, 1, 2, 3, 3, 4, 3, 1, 2, 1,
1, 4, 1, 1, 1, 1, 1, 4, 1, 1, 1, 5, 2, 3, 3, 4, 3, 1, 2, 1

Sucralose: 2, 1, 2, 5, 3, 2, 1, 2, 5, 3, 2, 4, 1, 2, 5, 3, 2, 4, 1, 2,
2, 1, 2, 5, 3, 2, 2, 1, 2, 5, 3, 2, 1, 2, 5, 3, 2, 4, 1, 2

Stevia: 3, 5, 5, 3, 4, 3, 4, 5, 1, 1, 5, 5, 5, 5, 1, 1, 5, 5, 5, 4,
3, 5, 5, 3, 4, 4, 3, 5, 5, 3, 4, 3, 4, 5, 1, 1, 5, 5, 5, 4

Saccharine: 4, 2, 4, 4, 5, 4, 5, 4, 4, 2, 1, 3, 4, 4, 4, 2, 1, 3, 4, 5,
4, 2, 4, 4, 5, 5, 4, 2, 4, 4, 5, 4, 5, 4, 4, 2, 1, 3, 4, 5

Acesulfame K: 5, 3, 3, 2, 2, 1, 3, 1, 2, 5, 4, 2, 3, 1, 2, 5, 4, 2, 3, 3,
5, 3, 3, 2, 2, 3, 5, 3, 3, 2, 2, 1, 3, 1, 2, 5, 4, 2, 3, 3

a Complete a frequency table of first preferences for each sweetener

b Complete a table of first or second preferences for each sweetener.

c Complete a frequency table for preferences for Acesufame K.

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3 Jeremy counted the types of vehicle that went past the school over a short period.

Cars	Motorbikes	Trucks	Taxis	Other
卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 III	卌 IIII	卌 卌 卌 卌 卌 卌	卌 卌	卌 卌 卌 卌 卌 卌 卌 卌 II

Complete a frequency table of the observations.

4 Draw a pie chart of the results of question **3**.

Example 4

5 Draw a pie chart showing the origins of the overseas-born Australians below from 2015:

United Kingdom:	1 207 050
Other Europe:	1 125 560
New Zealand:	611 400
China:	481 800
India:	432 700
Other Asia:	1 557 070
Other:	1 206 320

Source: ABS 3412.0 - Migration, Australia, 2014–15, Dataset: Estimated Resident Population by Country of birth, 30 June 1992 to 2015

6 Draw a picture graph to show the preferences of people for the milkshake flavours shown below, using symbols like .

Strawberry:	24
Chocolate:	38
Lime:	15
Banana:	22
Other:	19

7 The average monthly rainfall and average number of clear days in Townsville is shown in the table below.

Example 5

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Rainfall	270.1	298.7	192.4	66.4	31.7	21.2	14.9	16.1	10.4	23.4	58.4	127.7
Clear days	4.6	3.2	6.4	8.9	10.1	12.8	15.4	16.4	14.8	12.5	9.1	6.6

Source: Bureau of Meteorology

a Draw a column graph of the rainfall.

b Draw a bar graph of the average number of clear days.

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8 Draw a column graph of the ratings of people for Sucralose:

Rating	1	2	3	4	5
Frequency	8	17	6	3	6

9 Draw a column graph of the accommodation ratings shown below.

1 star: 5 2 star: 15 3 star: 55 4 star: 48 5 star: 32

Problem solving

10 Four different kinds of cream were tasted by a class of Year 11 students. The creams were re-packaged and labelled as A, B, C and D. The students' orders of preference for the creams were as follows, with the best cream given first in each case:

B-A-C-D	A-D-B-C	B-A-D-C	D-C-A-B	D-B-C-A
A-D-B-C	B-D-C-A	B-D-C-A	D-C-A-B	B-C-D-A
C-D-A-B	C-B-A-D	B-D-C-A	A-B-D-C	D-C-A-B
B-A-C-D	C-B-A-D	D-B-A-C	B-C-D-A	A-D-C-B
D-A-B-C	D-A-B-C	D-B-C-A	C-D-B-A	C-B-A-D

a Compile a frequency table of first preferences for each cream.

b Devise a method to use the first two preferences of each student to determine the most preferred cream. Justify your method.

11 Likert scales, with 1 being the most favoured and 5 the least, were used for 5 questions about the attitudes of Year 11 students to social media. The responses were as follows:

Q1: 4, 2, 1, 5, 1, 1, 2, 1, 2, 2, 2, 1, 5, 4, 2, 1, 2, 2, 1, 1, 3, 2, 1, 4, 1, 2, 1, 4, 3, 4, 3,
4, 1, 2, 2, 2, 2, 4, 1, 2, 2, 2, 3, 4, 2, 1, 2, 2, 1, 2, 1, 2, 1, 2, 1, 2, 4, 3, 1, 5, 3, 3, 2

Q2: 3, 1, 5, 5, 5, 4, 5, 2, 5, 5, 2, 5, 4, 2, 5, 4, 4, 4, 2, 2, 2, 4, 4, 4, 4, 5, 4, 4, 4, 5, 5,
5, 4, 4, 4, 3, 4, 4, 3, 5, 3, 5, 3, 3, 5, 3, 1, 3, 5, 4, 4, 4, 5, 4, 3, 3, 1, 5, 4, 4, 5, 1, 4

Q3: 2, 3, 3, 3, 5, 3, 2, 2, 3, 4, 3, 3, 4, 4, 5, 1, 3, 5, 3, 3, 1, 2, 3, 3, 3, 3, 3, 2, 2, 3, 3,
5, 4, 2, 2, 2, 2, 3, 3, 5, 3, 3, 3, 3, 5, 2, 3, 3, 2, 3, 4, 2, 3, 3, 1, 4, 3, 3, 2, 1, 2, 4, 4

Q4: 5, 1, 1, 1, 1, 2, 3, 5, 1, 5, 1, 1, 1, 5, 2, 4, 5, 3, 3, 5, 1, 3, 5, 4, 1, 5, 1, 1, 5, 1, 2,
1, 2, 5, 5, 5, 2, 5, 2, 5, 3, 4, 4, 1, 4, 3, 4, 2, 5, 2, 5, 1, 1, 1, 5, 1, 1, 1, 1, 3, 2, 5, 5

Q5: 2, 1, 3, 3, 4, 2, 3, 5, 1, 5, 1, 1, 1, 1, 5, 5, 4, 4, 2, 3, 4, 3, 4, 1, 2, 5, 4, 2, 1, 1, 1,
2, 4, 3, 4, 3, 1, 4, 3, 5, 3, 4, 4, 1, 5, 5, 1, 3, 4, 4, 2, 4, 4, 5, 2, 1, 3, 4, 2, 2, 1, 5, 5

Complete frequency tables for each question and a combined column graph for the responses. Compare the responses to each question.

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9.03 Displaying discrete variables

The number of children in a family and the sizes of screws are both discrete variables.

Individual discrete data may be shown by **dot plots**, column graphs and **stem-and-leaf** plots.

It is unusual to use pie charts or bar graphs for discrete data.

Shutterstock.com/Robert Kneschke

EXAMPLE 6

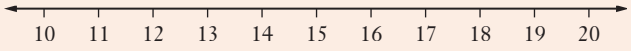
The following shows the sizes, in mm, of mixed O-rings. Make a dot plot of the sizes.

18	16	20	15	16	10	15	16	10	20
18	18	14	10	15	12	12	15	10	20
16	20	10	20	14	15	14	15	12	20
15	15	18	10	12	20	12	12	10	12

Solution

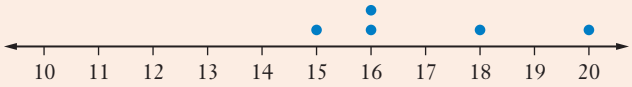
Draw the scale.

The lowest value is 10 and the highest is 20.



Put a dot above the first values: 18, 16, 20 and 15.

Put an evenly spaced dot over the next number, 16.



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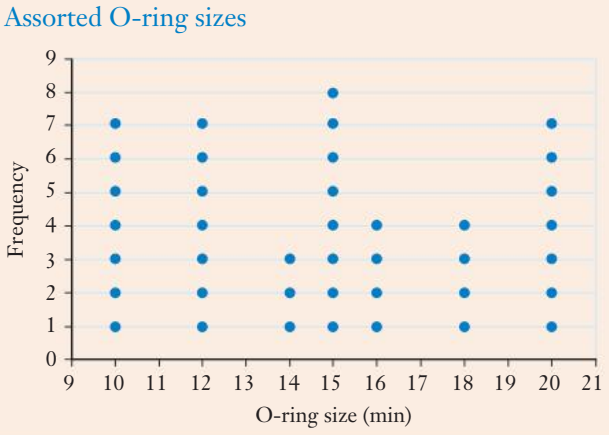
Continue with the rest of the numbers to complete the dot plot.

Add a frequency scale, title and scale names.

You can use a dot plot to work out frequencies and the shape at the same time.



Stem-and-leaf plots

EXAMPLE 7

Complete a stem-and-leaf plot of these ages of the mothers of some Year 11 students.

38	43	35	52	55	57	47	49	39	44	46	43
48	44	40	51	53	36	42	49	52	39	44	

Solution

The first digits are 3, 4 and 5.

Put these in the stem on the left.

Then put the second digits of the first few scores.

Stem	Leaf
3	8 5
4	3
5	2

Continue with the rest of the scores, leaving spaces between the leaves.

Stem	Leaf
3	8 5 9 6 9
4	3 7 9 4 6 3 8 4 0 2 9 4
5	2 5 7 1 3 2

When you have done all the data, rearrange the leaves in order.

Add a title and key.

Ages of mothers of Year 11 students

Stem	Leaf
3	5 6 8 9 9
4	0 2 3 3 4 4 4 6 7 8 9 9
5	1 2 2 3 5 7

Key: 4 | 3 = 43

You should group data when the values have a large domain. The groups are called **classes**. The distance from the beginning of one class interval to the next is called the **class width**. Choose no more than about 10 classes. Avoid class widths of 3, 7 or 9 to make calculations easier.

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You display grouped data using a column graph without gaps between the columns. This is often referred to as a **histogram**, even though histograms are really used for continuous data. The columns should start and finish halfway between the values. Unless the lowest class starts from 0, it is normal to leave a gap before the first and after the last column.

EXAMPLE 8

Complete a frequency table and column graph for these ages of fathers of some Year 11 students.

49	59	50	60	48	58	39	60	37	52	50	49
53	40	61	50	51	49	51	50	55	49	60	48
53	49	55	44	60	48	50	48	50	46	54	35
53	47	51	44	51							

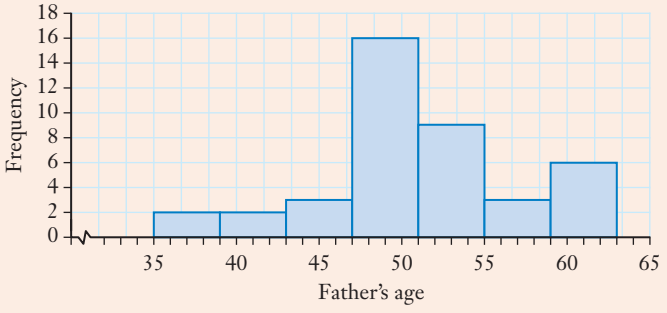
Solution

The lowest age is 35 and the highest is 61. Class intervals of 4 will be appropriate.

Complete the frequency table.

Father's age	Frequency
35–38	2
39–42	2
43–46	3
47–50	16
51–54	9
55–58	3
59–62	6
Total	41

Draw the column graph.



Exercise 9.03 Organising and displaying discrete variables

Example 6

1 Make a dot plot of this set of data.

6	9	4	9	8	11	6	9	8	9	7
9	8	9	7	9	7	11	5	9	4	8

2 Make a dot plot of this set of data.

21	28	21	20	21	26	18	25
21	22	27	27	21	24	24	

3 Make a dot plot of this set of data.

82	86	84	86	83	86	83	86	83	87	82
87	82	85	84	85	85	85	84	87	81	86
84	86	79	85	83	88	84	87	82	86	81
88	82	88	84	88	83	86	83			

Example 7

4 Make a stem-and-leaf plot of this set of data.

12	34	12	50	27	32	32	44	24	54	20	63
31	46	30	57	26	44	24	41	27	48	19	

5 Make a stem-and-leaf plot of this set of data.

39	61	36	57	41	48	36	56	40	54	42
69	41	43	41	50	38	55	38	57	37	53
37	85	36	63	42	49	41	61	33		

6 Make a stem-and-leaf plot of this set of data.

111	156	108	138	112	134	105	155	108	138	107
127	108	136	97	159	112	148	101	158	115	126
111	149	101	144	115	141	110	130	115		

Example 8

7 Make a histogram of this set of grouped data.

Values	5–9	10–14	15–19	20–24	25–29	30–34
Frequency	3	7	13	10	7	5

8 Make a histogram of this set of grouped data.

Values	30–33	34–37	38–41	42–45	46–49	50–53	54–57
Frequency	3	8	12	18	11	7	3

9 Make a histogram of this set of grouped data.

Values	0–99	100–199	200–299	300–399	400–499	500–599
Frequency	5	15	22	21	14	4

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Problem solving

10 **a** Make a dot plot of these ages of people at a party.

17	17	16	20	14	18	15	18	16	18	17	18	15
17	15	18	17	19	16	17	15	18	14	18	16	

b What is the most common age?

11 **a** Make a dot plot of these numbers of people in the families of some Year 11 students.

5	5	4	7	2	7	4	6	2	7	4	5
4	5	4	10	2	8	4	5	4	6	4	5
4	8	4	6	3	9	5	8	4	6	4	8
4	7	4	7	3	7	4	6	5			

b What is the highest frequency?

12 **a** Make a stem-and-leaf plot of these ages of living grandparents of some Year 11 students.

58	84	71	75	71	84	63	81	68	77	59
91	63	96	59	88	67	84	61	82	71	79
67	80	69	73	68	76	70	90	67	90	58
93	57	73	63	77	69	92	70	75	71	74
70	81	71	79	63	83	67				

b In which decade are the greatest number of grandparents?

13 The numbers of driving lessons that some students had before obtaining their driving licences are shown below. Choose suitable classes, complete a frequency table and draw a histogram. Comment on your results.

17	26	18	24	11	24	22	39	9	26
13	34	14	45	20	34	20	25	17	26
20	23	11	24	13	43	19	37	14	31
19	29	18	25	19	23	11	12	14	12

14 The numbers of songs that some Year 11 students had on their personal music players (including mobiles) were as follows. Choose classes, make a frequency table and draw a histogram. Comment on your results.

75	403	131	320	194	202	154	306	152	226	173	320
185	317	53	224	101	280	173	321	158	371	114	282
115	214	163	497	173	259	136	283	135	291	193	208
130	217	119	494	93	223	173	328	170	263	156	420

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Histograms

9.04 Displaying continuous variables

A person's BMI (body-mass index) may be stated to the nearest whole number. A stated BMI of 22 actually means a BMI in the class interval 21.5–22.5.

Continuous variable classes

- Continuous variable values are collected in **class intervals** like 49.5–54.5
- The start of a class interval is the **lower class limit**, and the top is the **upper class limit**
- **True class limits** have no gaps between them. **Stated class limits** may have gaps between them.
- The **class width** is the difference between the true class limits
- The **class centre** (**class midpoint**) $= \dfrac{\text{upper + lower class limits}}{2}$

Measurements are at best accurate to the smallest division on the instrument. As a result, most continuous class intervals do not start at an exact number.

EXAMPLE 9

Frequency histogram

The speeds (in km/h) of motorists in an 80 km/h zone were recorded by police as follows.

74.6	78.6	67.8	80.1	69.6	95.4	72.3	82.9	69.5	83.7	72.7
84	74.3	89.9	71.9	76.5	71.2	79.8	63.5	78.1	68.6	79.7
72.3	75.5	72.4	78.6	61.1	75.5	71.8	81.1	69	81.7	74.7
76.8	67	80.9	65.6	81.8	70.1	83	64.6	87.7	66.9	

Make a frequency table of the speeds and draw a histogram

Solution

The lowest speed was 61.1 km/h and the highest was 95.4 km/h. Classes of width 5 give a reasonable number of classes.

The maximum legal speed is 80 km/h, so make the start of one class 81.

Compile the table.

Class interval	Frequency
60.5–65.5	3
65.5–70.5	10
70.5–75.5	9
75.5–80.5	10
80.5–85.5	8
85.5–90.5	2
90.5–95.5	1
Total	43

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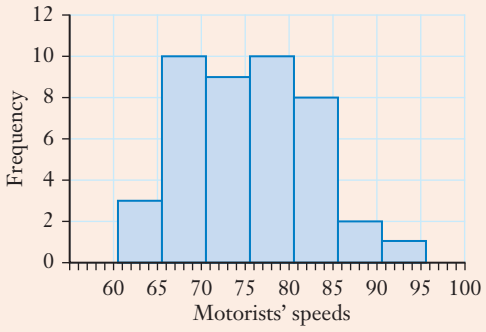
Draw the histogram, making sure you put the edges of the columns at the true class limits.

Leave spaces at the ends.

Use the *true class limits* at the edges of columns of histograms.



iStock.com/Mateo_Pearson

You can also use a **frequency polygon** to show continuous data.

A frequency polygon:

- is a line segment graph
- has each point at a class centre and frequency height
- begins and ends on the horizontal axis

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EXAMPLE 10

Draw a frequency polygon to show the following data.

Value	20–24	25–29	30–34	35–39	40–44	45–49
Frequency	2	5	9	7	4	1

Solution

State the true class limits.

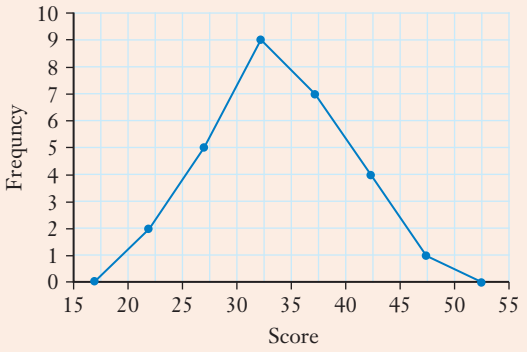
The true class limits are 19.5–24.5, 24.5–29.5, etc.

State the class centres.

The class midpoints are (17), 22, 27, 32, 37, 42, 47, (52)

Put points at (17, 0), (22, 2), (27, 5), etc.

Join the points with straight lines to make the polygon.



TECHNOLOGY

Spreadsheet graphs

You can use a spreadsheet to draw many kinds of graphs. Your teacher may want you to work in pairs.

1 Enter the following data in a spreadsheet and highlight the data. Then use the **insert graph** facility to do a pie graph. Refine the graph by clicking on different parts.

Activity	School	Sleep	Eating, etc	Transport	Friends	Other
Time	6	8	2	1	5	2

2 Enter the following data in a spreadsheet and draw a column graph. Refine the graph.

Age	13	14	15	16	17	18	19	20
Number	2	8	9	12	15	6	2	1

3 Enter the data from Example 11 and draw a frequency polygon.

4 Try to do a histogram with the data from Example 10. How can you do the horizontal scale?

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INVESTIGATION

CLASS DATA

As a group, collect some or all of the following data from your class.

Heights, number of siblings, number of bedrooms in house, engine size of family car, hand span, eye colour, hair colour, number of languages spoken, state/country of parent's birth.

You teacher might want you to collect other data, or to use data from the Census At School website.

Draw tables and individually draw appropriate graphs for the data.

Exercise 9.04 Displaying continuous variables

Example 9

1 Use classes of 5–10, 11–14, … to complete a frequency table for the following continuous data.

20	27	21	27	23	24	23	31	23	29	27	31
24	24	21	29	23	15	17	17	15	27	27	22
22	33	26	36	25	18	24	29	25	14	19	12
26	20	23	28	9	30	16	24	25	27	31	9
24	22	31	24	26	19	19	34	15			

2 Use classes of 10–19, 20–29, … to complete a frequency table for the following data.

63.8	90.8	35.4	54	61.5	92.3	50	66.1	35	32.1
54.2	61.6	47.3	60.4	66.2	79.8	55.1	74.3	68.4	51.3
45.4	55	70	53.4	50.7	67	28.4	66	67	29.8
43.3	60.6	47.1	51.8	43.6	107	56.4	23	47.7	74.4
5.4	68.7	63.6	57.6	74	44.8				

3 Draw a frequency histogram of the following data.

Score	19.5–24.5	24.5–29.5	29.5–34.5	34.5–39.5	39.5–44.5	44.5–49.5
Frequency	2	8	12	17	9	4

4 Draw a frequency histogram of the following data.

Score	11–20	21–30	31–40	41–50	51–60	61–70	71–80	81–90
Frequency	1	8	12	16	10	6	3	1

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Example 10

5 Draw a frequency polygon for the following continuous data.

Score	239.5–249.5	249.5–259.5	259.5–269.5	269.5–279.5	279.5–289.5	289.5–299.5
Frequency	3	9	15	16	9	4

6 Draw a frequency polygon for the following continuous data.

Score	40–44	45–49	50–54	55–59	60–64	65–69
Frequency	2	9	13	16	8	3

Problem solving

7 The weights to the nearest kg of 40 bungee jumpers were recorded as follows:

41 58 63 37 49 58 71 33 85 58
73 81 46 55 38 80 48 50 62 59
63 44 77 62 58 73 62 75 52 60
69 61 55 47 76 42 66 70

iStockphoto/Ron Sumners

a Complete a frequency table and draw a frequency histogram.

b From 200 bungee jumpers, how many would you expect to weigh 70 kg or more? Justify your answer.

8 A mobile mechanic used the following amounts of oil (to the nearest litre) on consecutive working days, starting on a Monday.

179 157 171 192 206 116 179 157 186 190
209 106 183 171 163 193 227 93 180 159
172 201 204 119 160 154 173 219 212 103

a Construct a frequency table and polygon.

b How many days a week did he work? Justify your answer.

c How much oil would you expect him to use each week? Explain your answer.

9 A couple make specialty soaps to sell at a local craft market. The masses of 50 cakes of their soap were as follows.

Mass (g)	Frequency
255 to 259	2
260 to 264	12
265 to 269	15
270 to 274	15
275 to 279	4
280 to 284	2

a Construct a frequency histogram.

b What percentage of the soaps had a mass less than 270 g?

c From 800 cakes of soap, how many would you expect to have a mass less than 265 g? Justify your answer.

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10 The heights to the nearest cm of the 170 Year 11 students at a school were as follows.

169	155	169	161	167	166	170	159	165	159	175
171	172	186	152	186	157	164	185	179	163	166
179	173	168	151	156	163	161	159	166	167	167
154	151	159	167	171	168	171	171	161	160	155
170	169	171	163	174	157	161	164	163	178	163
149	155	165	158	166	161	140	168	171	160	171
156	179	146	164	161	162	175	168	151	161	165
167	171	168	165	158	161	162	156	169	141	167
166	175	164	183	165	160	159	151	156	172	153
163	164	159	165	159	180	180	163	169	172	178
159	159	172	148	179	165	165	170	175	150	165
151	166	178	163	161	175	167	167	148	165	164
171	173	170	162	169	163	165	166	163	163	179
169	174	169	167	167	160	167	165	172	168	166
161	161	163	176	183	152	173	169	169	158	166
158	151	170	163	145						

a Construct a frequency table and polygon of their heights.

b What can you say about the school?

9.05 Measures of centre

You often want a 'typical' value. The three typical values are called **measures of central tendency**.

Measures of central tendency

- The **mode** is the most common score. It has the highest frequency. For grouped data, the **modal class** occurs most often.
- The **mean** is the calculated average of the scores and has the symbol $\bar{x}$. You add all the scores and divide by the total number of scores. For grouped data, you use the class centre as the score for a group. $\bar{x} = \frac{\sum x}{n}$ for individual scores or $\bar{x} = \frac{\sum fx}{\sum f}$, where $\sum$ means 'the sum of', x is the score, f is its frequency and n is the total number of scores.
- The **median** is the middle score when all scores are written in order from smallest to largest. If there is an even number of scores, then it is the average of the middle two. In general, the median is the $\left(\frac{n+1}{2}\right)$th score, provided the scores are in order.

Statistical measures

Statistical match-up

Mean, median, mode and range

Mode, median and mean

Measures of central tendency

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EXAMPLE 11

Find the mode, mean and median of the following scores.

9 6 7 14 3 5 14 5 8 5 12

Solution

There are more 5s than any other score.	The mode is 5.
For the mean, find the sum of the scores and divide by the number of scores.	$\Sigma x = 9 + 6 + 7 + 14 + 3 + 5 + 14 + 5 + 8 + 5 + 12$ $= 88$ $\bar{x} = \frac{\sum x}{n} = \frac{88}{11} = 8$
For the median, arrange the scores in order and choose the middle score.	3, 5, 5, 5, 6, 7, 8, 9, 12, 14, 14
Write the answers.	The mode is 5, mean is 8 and median is 7.

In the last example, $n = 11$ so $\frac{n+1}{2} = 6$. The median is the 6th score in order. The mode, mean and median are not usually so different as above. Which you should use depends on the circumstances.

Choosing the typical value

- The mode is the *most likely* to occur. It is the only measure of central tendency you can use for nominal data.
- The mean is most useful for continuous data, when numerical data is *evenly spread* or when the value is used in calculations.

The mean is often a fraction.

- The median is the *halfway* value. It is most useful for numerical data that is *unevenly spread* to one side or for ordinal data.

Statistics from a frequency table

For frequency tables, use cumulative frequency to find the median and add a 'score × frequency' column for the mean.

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EXAMPLE 12

A sample of people at a club in the city on Saturday night gave their ages as follows.

Age	18	19	20	21	22	23	24	25	26
Frequency	15	18	19	15	10	10	8	7	2

a Find the mode, mean and median.

b State the typical age.

Solution

a Redraw as a vertical table with cumulative frequency and score × frequency (fx) columns.

Age x	f	Cumulative f	fx
18	15	15	270
19	18	33	342
20	19	52	380
21	15	67	315
22	10	77	220
23	10	87	230
24	8	95	192
25	7	102	175
26	2	104	52
Total	104		2176

Find the mode. The most common age was 20.

Find the mean. $\bar{x} = \dfrac{\sum fx}{\sum f} = \dfrac{2176}{104} \approx 20.92$

Find the value of $\dfrac{n+1}{2}$. $\dfrac{n+1}{2} = \dfrac{105}{2} = 52.5.$

Find the median. Scores 34–52 are 20s and scores 53–67 are 21s.

$$\text{Median} = \frac{52\text{nd} + 53\text{rd scores}}{2}$$
$$= \frac{20+21}{2}$$
$$= 20.5.$$

State the answers. The mode was 20, mean was 20.0 and median was 20.5.

b State the typical age, with a reason. The data is spread unevenly, so the median, 20.5 is the typical age.

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You should use **linear interpolation** to find the median for grouped data.

Linear interpolation for the median

- Use cumulative frequency to find the class containing the median.
- Suppose:
  - the median is the mth score
  - the cumulative frequency of the previous class is F
  - the frequency of the medial class is f
  - the true lower class limit is L.
  - the width of the class is w
- then median $= L + \dfrac{m - F}{f} \times w$

EXAMPLE 13

The prices, in thousands of dollars, for houses sold in Cairns over a month are shown.

Price	200–299	300–399	400–499	500–599	600–699	700–799	800–899	2000
Number	13	36	13	10	3	3	1	1

a Find the mean and median prices.

b What was the typical price?

Solution

a Redraw as a vertical table with cumulative frequency, class centre x, and score × frequency (fx) columns.

Price	x	f	Cumulative f	fx
200–299	249.5	13	13	3243.5
300–399	349.5	36	49	12582
400–499	449.5	13	62	5843.5
500–599	549.5	10	72	5495
600–699	649.5	3	75	1948.5
700–799	749.5	3	78	2248.5
800–899	849.5	1	79	849.5
2000	2000	1	80	2000
Total		80		34 210.5

Find the mean.

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{34\ 210.5}{80} \approx 427.6$$

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	Find the value of $\frac{n+1}{2}$.	$\frac{n+1}{2} = \frac{81}{2} = 41.5$.
	State the position of the median.	The 13th value is between 199.5 and 299.5. The next 36, including the 41.5th are between 299.5 and 399.5.
	State the position in the class.	The median is the $41.5 - 13 = 28.5$th score in the class. It is 28.5 thirty-sixths of the way from 299.5 to 399.5
	Use interpolation.	$\text{Median} = 299.5 + \frac{28.5}{36} \times 100$ ≈ 378.7
	State the answers.	The mean is \$427 600 and the median is \$387 700.
b	State the typical price, with a reason.	The typical price of houses sold in Cairns in the month was \$387 700. The prices are unevenly spread, so the median is more typical than the mean.

The sale of a house for \$2 million in Cairns considerably raised the mean.

TECHNOLOGY

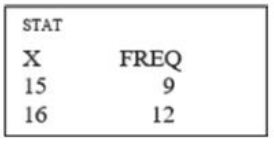
Statistical calculations

You can use scientific calculators to calculate the mean and median from a frequency table. Consider the following data.

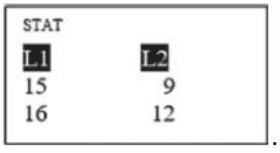
x	13	14	15	16	17	18	19
f	2	8	9	12	15	6	2

Use the STAT mode of your calculator. You might have to use **SET UP** or **MODE** to make sure both score and frequency columns are available.

Enter the data like



or



.

Then use the **1-Var Stats** to calculate the mean, $\bar{x}$, and the median, med.

Your calculator might display them after you choose **1-Var Stats** or you may have to get each one separately. Check your manual to see or ask your teacher.

Your calculator will probably also show you the minimum **minx**, maximum **maxx**, total frequency n and other values dealt with later in this chapter.

For the data above, your calculator should give $n = 54$, $\bar{x} = 16.03\ldots$ and med = 16.

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Exercise 9.05 Measures of centre

Example 11

1 Find the mean, median and mode of each set of data.

a 28, 25, 37, 24, 33, 28, 24, 28, 35, 25, 21, 27, 22

b 3, 6, 8, 10, 5, 4, 6, 8, 8, 3, 4, 6

c 6, 4, 7, 5, 9, 6, 8, 6, 7

2 Find the mean, median and mode of each set of data.

a Shoe sizes of the Brisbane Bullets basketball players:

11 15 14 12 14 13 14

b Average speed (km/h) for the lead car in the first 10 laps of a Formula One Grand Prix:

205 212 208 212 217 220 215 225 210 217

c Number of computers used in a classroom:

6	10	7	15	6	14	5	9	5	8	6	8	6
10	4	13	7	7	3	10	6	10	7	12	5	11
5	10	4	11	6	10	6	9	6	10	7	15	6
7	3	14	5	7	6							

Example 12

3 Find the mean, median and mode of each set of data.

a

x	3	4	5	6	7	8	9
f	5	7	6	3	2	0	1

b

x	5	6	7	8	9	10	11	12	13	14
f	2	4	5	7	8	6	3	2	2	1

c

x	0	1	2	3	4	5	6	7	8	9	10
f	4	6	5	10	12	13	6	5	3	0	2

4 Find the mean, median and mode of each set of data.

a

Adult teeth lost	3	4	5	6	7	8	9	10	11	12	13
Frequency	2	4	6	10	12	8	6	5	2	1	3

b

Age for first job	15	16	17	18	19	20	21	22	23
Number	5	9	8	12	15	9	6	4	1

c

Dress size	6	8	10	12	14	16	18	20	22	24
Number	1	2	5	10	12	15	16	13	9	5

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5 Find the mean, median and modal class of each set of data.

Example 13

a

Class	0–4	5–9	10–14	15–19	20–24
Frequency	1	4	9	14	2

b

Class	40–49	50–59	60–69	70–79	80–89	90–99	100–109	110–119
Frequency	4	8	12	23	9	5	3	1

c

Class	70–109	110–149	150–189	190–229	230–269	270–309
Frequency	4	7	11	18	37	22

6 Find the mean, median and modal class of each set of data.

a Year 11 student heights

Height (cm)	140–144	145–149	150–154	155–159	160–164	165–169
Frequency	1	3	3	5	12	10
Height (cm)	170–174	175–179	180–184	185–189	190–194	
Frequency	12	8	11	5	3	

b

Mine production	50–54.9	55–59.9	60–64.9	65–69.9	70–74.9	75–79.9	80–84.9	85–89.9
Years	1	4	3	4	5	5	2	1

c

Room width (m)	1.5–1.9	2–2.4	2.5–2.9	3–3.4	3.5–3.9	4–4.4	4.5–4.9	5–5.4	5.5–5.9	6–6.4
Number	2	1	3	8	6	1	2	4	5	2

Problem solving

7 The shoe sizes of a group of Year 11 students were:

8 9 5 6 8 7 6 8 12 9 8 7 6
10 8 6 7 5 8 11 6 8 9 10 8

a Find the mean, median and modal shoe sizes.

b What is the typical shoe size of this group?

8 Thirty employees of the 150 employed by a local engineering firm were absent for the following numbers of days last year.

Days absent	0	1	2	3	4	5	6	7	8	9	10
Frequency	4	3	6	6	3	1	3	2	1	0	1

a Find the mean, mode and median for this data.

b How many days of absence would you expect altogether?

c How many days absence did the 'average' employee have last year?

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9 The number of cars parked on a side street to a major shopping centre was recorded every hour for 8 trading hours over 5 weekdays.

15	20	18	19	20	19	18	15	18	20
12	18	21	17	19	14	16	20	17	18
17	19	15	20	16	19	18	20	19	17
20	18	20	15	18	12	19	18	17	19

a Find the mode, median and mean of this data.

b What was the typical number of cars parked on the side street over this period?

10 The monthly incomes of the employees of a manufacturer are shown in the table.

a Find the mean, median and modal class for the employees' incomes.

b What is the typical employee's income? Explain your answer.

Income ($)	Number
2500–2999	12
3000–3499	29
3500–3999	44
4000–4499	39
4500–4999	38
5000–5499	25
5500–5999	17
6000–6499	9
6500–6999	5
7000–7499	2
7500–7999	1

11 The masses, in kg, of frozen chickens at a supermarket are:

1.6	1.7	1.2	1.4	1.5	1.4	2.0	1.1	1.8	1.9	1.7	1.6
2.2	1.6	1.4	1.3	1.5	1.5	1.7	1.5	1.3	1.6	1.8	1.1
1.7	1.4	1.2	1.9	2.1	1.4	1.6	2.2	1.5	1.6	1.9	1.5
1.4	2.2	1.5	1.9	1.6	1.5	1.8	1.5	1.6	2.4	1.7	1.5
1.8	2.3	1.7	1.6	2.2	1.7	1.6	1.7	1.4	1.6	1.7	1.8
1.6	1.0	1.4	1.6	1.5	2.2	1.7	1.8	1.6	1.9	1.7	1.6
1.5	2.3	1.8	2.0	1.9	1.5	1.2	2.5				

a Choose appropriate class intervals and arrange this data into a frequency distribution table.

b What are the mean, median and mode of the sizes?

c What is the typical size?

12 The table below shows the masses of a group of students.

Mass (kg)	40–49	50–59	60–69	70–79	80–89	90–99	100–109	110–119
Number	6	13	17	19	12	8	5	2

What is the typical mass? Explain your answer.

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9.06 The range and interquartile range

Quartiles and ranges

- The values Q_0, Q_1, Q_2, Q_3 and Q_4 divide ordered data into quarters.
- Q_0 is the **minimum** value.
- Q_1 is the value a quarter of the way through, called the **first** or **lower quartile**.
- Q_2 is the value halfway through. It is the **median**.
- Q_3 is the value three-quarters of the way through, called the **third** or **upper quartile**.
- Q_4 is the **maximum** value.
- The **range** $= Q_4 - Q_0 =$ maximum $-$ minimum.
- The **interquartile range** (**IQR**) $= Q_3 - Q_1$.
- Half the data is within the interquartile range.

Interquartile range

Statistical measures puzzle

Interquartile range

Interquartile range

Statistics mode: Graphics calculator

EXAMPLE 14

Find the range and interquartile range of

12 17 6 9 3 3 7 8 10 19 10 12 11

Solution

Find the range.

Range = maximum − minimum

$= 19 - 3$

$= 13$

Arrange in order.

3, 3, 6, 7, 8, 9, 10, 10, 11, 12, 12, 17, 19

Find the median (middle value).

3, 3, 6, 7, 8, 9, 10, 10, 11, 12, 12, 17, 19

There are 6 values before and 6 values after 10.

Find the middle of each half.

3, 3, 6 | 7, 8, 9, 10, 10, 11, 12 | 12, 17, 19

If a quartile is between two values, you take their average like you do for the median.

$Q_1 = \frac{6+7}{2} = 6.5$, $Q_3 = \frac{12+12}{2} = 12$

Find the interquartile range.

IQR $= Q_3 - Q_1$

$= 12 - 6.5$

$= 5.5$

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There are many ways of finding the positions of the first and third quartiles, but the following agrees with the values you get using the method above.

Quartile positions

- For n odd, Q_1 is the $\left(\frac{n+1}{4}\right)$th value and Q_3 is the $\left(\frac{3n+3}{4}\right)$th value
- For n even, Q_1 is the $\left(\frac{n+2}{4}\right)$th value and Q_3 is the $\left(\frac{3n+2}{4}\right)$th value

EXAMPLE 15

Find the range and interquartile range for each set of data.

a

x	7	8	9	10	11	12	13	14
f	3	5	7	9	14	12	5	2

b

Class	55–59	60–64	65–69	70–74	75–79	80–84	85–89	90–94
f	5	7	14	8	6	6	4	2

Solution

a Find the range.

Range = maximum − minimum

$= 14 - 7$

$= 7$

Find n.

$\Sigma f = 57$, which is odd.

Find the positions of Q_1 and Q_3.

Position of $Q_1 = \frac{n+1}{4} = \frac{58}{4} = 14.5$

Position of $Q_3 = \frac{3n+3}{4} = \frac{174}{4} = 43.5$

Find Q_1 and Q_3.

The 14th and 15th are both 9s, so $Q_1 = 9$.

The 43rd and 44th are both 12s, so $Q_3 = 12$.

Find the IQR.

Interquartile range $= Q_3 - Q_1$

$= 12 - 9$

$= 3$

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b State the true classes.

The true classes are 54.4–59.5, 59.5–64.5, etc.

Find the range.

Range = maximum − minimum

$= 94.5 - 54.4$

$= 40$

Find n.

$\Sigma f = 52$, which is even.

Find the positions of Q_1 and Q_3.

Position of $Q_1 = \frac{n+2}{4} = \frac{54}{4} = 13.5$

Position of $Q_3 = \frac{3n+2}{4} = \frac{158}{4} = 39.5$

Find Q_1.

The 12th value is between 59.5 and 64.5.

The next 14 are between 64.5 and 69.5.

Thus, the 13.5th is 1.5 fourteenths of the way from 64.5 to 69.5.

$Q_1 = 64.5 + \frac{1.5}{14} \times 5$

≈ 65.0

Find Q_3.

The 34th value is between 69.5 and 74.5.

The next 6 are between 74.5 and 79.5.

Thus, the 39.5th is 5.5 sixths of the way from 74.5 to 79.5.

$Q_3 = 74.5 + \frac{5.5}{6} \times 5$

≈ 79.1

Find the IQR.

Interquartile range $= Q_3 - Q_1$

$\approx 79.1 - 65.0$

$= 14.1$

Modern scientific calculators calculate quartiles as well as the mean and median.

Exercise 9.06 The range and interquartile range

Example 14

1 Find the range and interquartile range for each set of data.

a 4, 2, 5, 2, 7, 5, 8, 8, 3, 5, 8, 3, 5, 9

b 18, 15, 14, 12, 13, 17, 18, 16, 14, 16, 17, 19, 12, 16, 16, 19

c 24, 26, 29, 33, 39, 42, 41, 27, 36, 34, 28, 27, 24, 27, 44

2 Find the range and interquartile range for each set of data.

a 28, 25, 37, 24, 33, 28, 24, 28, 35, 25, 21, 27, 22

b 3, 6, 8, 10, 5, 4, 6, 8, 8, 3, 4, 6

c 6, 4, 7, 5, 9, 6, 8, 6, 7

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Example 15

3 Find the range and interquartile range for each set of data.

a

Score	0	1	2	3	4	5	6	7	8	9
Frequency	2	5	5	7	7	6	6	3	2	1

b

Score	8	9	10	11	12	13	14	15	16	17	18
Frequency	3	7	6	10	12	13	6	5	3	0	2

c

Score	25	26	27	28	29	30	31	32	33
Frequency	3	6	10	16	22	20	17	6	4

4 Find the range and interquartile range for each set of data.

a

Class	105–109	110–114	115–119	120–124	125–129
Frequency	28	43	17	5	2

b

Class	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	2	5	7	11	10	8	4	1

c

Class	35–39	40–44	45–49	50–54	55–59	60–64	65–69	70–74	75–79	80–84	85–89
Frequency	2	8	12	15	13	12	10	7	5	3	2

Problem solving

5 The times (seconds) taken for 25 job applicants to complete a manual dexterity test are shown below.

14	11	10	12	19	14	5	17	12	10	11	19	15
19	10	21	14	27	16	23	15	14	18	20	17	

a Within how many seconds would you expect half of the applicants to complete the test?

b Within how many seconds would the middle half of the applicants complete the test?

6 Tanya and Juan are employed as quality controllers at a factory where computer motherboards are assembled. The numbers of motherboards that they each inspect each hour over a 9-hour period are shown below.

Tanya: 30, 31, 29, 30, 29, 23, 33, 35, 30

Juan: 28, 30, 26, 12, 29, 31, 35, 48, 31

Who is the more consistent worker? Explain your reasoning.

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ISBN 9780170412711

7 The air pollution levels in Brisbane measured over a short period are shown below.

Pollution level	0–19	20–39	40–59	60–79	80–99	100–119	120–139	140–159
Frequency	30	21	15	6	6	5	1	4

What can you say about the consistency of the observations?

9.07 The standard deviation

The range and interquartile range use only two data items each to measure spread. It is better to use all the data to measure the spread.

Standard deviation

Statistical calculations

Statistics review

Calculating and interpreting summary statistics

INVESTIGATION

MEASURING DATA SPREAD

Your teacher might give small groups in your class different data sets for this investigation.

Look first at this set of data.

8	7	5	8	6	7	7	8	6	8
11	10	11	12	10	8	12	9	6	11

- Find the mean $\bar{x}$.
- For every score x, find the **deviation** $x - \bar{x}$ from the mean. Remember that some are negative!
- Find the total of the deviations. What do you find?
- Now, square each deviation instead. What sign will they be?
- Find the mean of the square deviations.
- Find the square root of the mean square deviation.

If you have your own set of data, do it with your own set.

Discuss the results.

You should have found that the *root mean square* deviation of the data above from the mean was about 2.086. This is actually called the **standard deviation** and is the most commonly used measure of spread. The root mean square is used to measure other quantities as well.

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Standard deviation, s or σ

- The standard deviation is the root mean square deviation of a set of data from its mean
- For individual data, $s = \sqrt{\dfrac{\sum (x-\bar{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - (\bar{x})^2}$
- For grouped data, $s = \sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - (\bar{x})^2}$
- s = standard deviation of a set of data
- σ = standard deviation of a population

The second formula for standard deviation is sometimes called the *calculation formula*.

It gives the same answer as the first, but is quicker to use.

EXAMPLE 16

Find the mean and standard deviation of the following data:

9 8 9 8 8 4 3 4 8 5 7 5

Solution

Add the scores.	$9+8+9+8+8+4+3+4+8+5+7+5=78$
Find the mean.	$\bar{x} = \dfrac{\sum x}{n} = \dfrac{78}{12} = 6.5$
Add the squares of the scores.	$9^2+8^2+9^2+8^2+8^2+4^2+3^2+4^2+8^2+5^2+7^2+5^2$ $=81+64+81+64+64+16+9+16+64+25+49+25$ $=558$
Find the standard deviation.	$s = \sqrt{\dfrac{\sum x^2}{n} - (\bar{x})^2}$ $= \sqrt{\dfrac{558}{12} - (6.5)^2}$ $= \sqrt{4.25}$ ≈ 2.06
Write the answers.	The mean is 6.5 and the standard deviation is about 2.06.

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You could use $\frac{\sum fx^2}{\sum f} = \frac{2\times81+4\times64+49+2\times25+2\times16+9}{12}$ above.

Data that is concentrated around the mean has a small standard deviation. The standard deviation measures the spread of a data set. You can use a modern scientific calculator to find the standard deviation as well as the mean.

EXAMPLE 17

The shoe sizes of men's shoes sold one Friday in two different stores are shown below:

City store: 8, 9, 10, 11, 8, 9, 10, 13, 10, 12, 10, 9, 8, 6, 8, 9, 10, 10

Suburban store: 6, 8, 10, 12, 10, 8, 9, 9, 9, 6, 8, 10, 12, 11

Which store has the greater concentration of sizes sold?

Solution

Enter the *City store* data in the STAT mode of your calculator.

or

Then find the mean and SD. *City*: Mean ≈ 9.44, standard deviation ≈ 1.57

Do the same for the other store. *Suburban*: Mean ≈ 9.14, standard deviation ≈ 1.81

Write the answer. The sizes sold in the city were more concentrated.

For grouped data, use the class centre in calculations of the mean and standard deviation.

In Example 17, the means are similar, so the standard deviation alone is sufficient to tell which set of scores has greater variation.

Coefficient of variation (CV)

- **Coefficient of variation** $= \frac{s}{\bar{x}}$ or $\frac{\sigma}{\mu}$. It is usually multiplied by 100 to express as a percentage.
- The coefficient of variation measures the variation of a set of data.
- It is also called the **relative standard deviation** (**RSD**).

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EXAMPLE 18

The numbers of rooms in some Australian dwellings were as follows:

3, 13, 4, 6, 4, 11, 5, 12, 2, 8, 4, 9, 4, 9, 6

The times in minutes taken to complete a Centrelink form were as follows:

24, 18, 17, 43, 46, 11, 32, 33, 19, 34, 17, 30

Which set of data has the greater variation?

Solution

Use your calculator for the number of rooms.	Mean rooms number ≈ 6.67 Standard deviation ≈ 3.34
Find the coefficient of variation.	CV rooms $= \frac{3.34}{6.67} \times 100 \approx 50\%$
Use your calculator for the form time.	Mean form time = 27 min Standard deviation ≈ 10.59 min
Find the coefficient of variation.	CV forms $= \frac{10.59}{27} \times 100 \approx 39\%$
State the answer.	There is more variation in the number of rooms than in the time to complete the form.

Dreamstime.com/Gogilk83

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Exercise 9.07 The standard deviation

Your teacher will tell you for which problems you should use the STAT mode on your calculator.

1 Find, correct to two decimal places, the mean and standard deviation of each set of data. Example 16

a 11, 6, 6, 11, 7, 8, 9, 10, 7, 12

b 8, 6, 3, 4, 7, 7, 6, 3, 0, 10, 7, 5

c 16, 16, 13, 18, 14, 12, 12, 14, 16, 17, 12, 12, 17, 16, 16, 13

2 Find, correct to two decimal places, the mean and standard deviation of each set of data.

a 12, 12, 9, 8, 15, 8, 11, 12, 10, 12, 8, 15

b 10, 7, 6, 13, 6, 10, 5, 13, 6, 16, 7, 14, 8, 14, 6

c 20.8, 25.4, 23.1, 24.4, 18.9, 20.3, 18.5, 23.8, 20.6, 22.6

3 Determine which set of data is more consistent. Example 17, 18

A: 5, 9, 13, 9, 13, 13, 0, 10, 2, 7

B: 9, 9, 4, 7, 6, 11, 9, 14, 13, 7, 11, 2

4 Determine which set of data is more consistent.

A: 9, 29, 12, 15, 14, 12, 13, 11, 10, 2, 13, 22

B: 11, 8, 9, 20, 11, 3, 14, 14, 13, 17

5 Determine which set of data is more varied.

A: 4, 4, 7, 6, 8, 8, 2, 11, 8, 12

B: 6, 11, 1, 12, 10, 9, 6, 11, 6

6 Determine which set of data is more varied.

A: 13, 47, 27, 23, 24, 28, 26, 13, 11, 23

B: 15, 26, 21, 25, 25, 22, 22, 27, 26, 28, 23, 25

Problem solving

7 The prices of potatoes during the year from a local fruit and vegetable shop are shown.

Price per kg	Weeks at that price
\$2.00–\$2.49	2
\$2.50–\$2.99	5
\$3.00–\$3.49	10
\$3.50–\$3.99	11
\$4.00–\$4.49	11
\$4.50–\$4.99	8
\$5.00–\$5.49	2
\$5.50–\$5.59	3

What can you say about potato prices during the year?

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Shapes of distributions

The shape of a frequency distribution

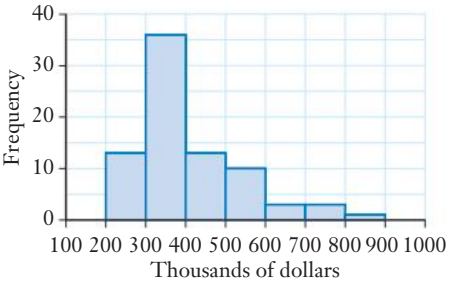
9.08 The shape of a distribution

A histogram of Cairns house prices, excluding million dollar sales, is graphed here.

There is a long 'tail' of values stretching out to the positive (right) end.

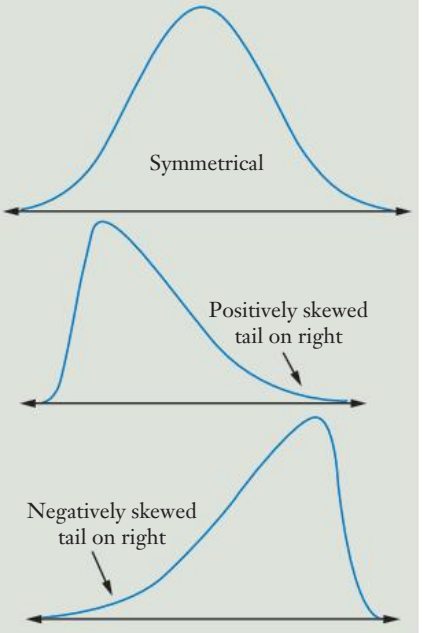
The distribution is **positively skewed**.

This pattern is typical of many kinds of data.



Symmetrical and skewed distributions

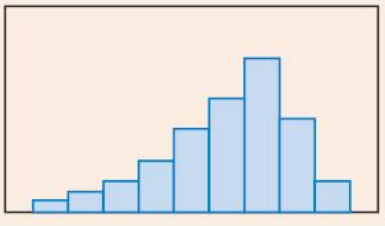
- A **symmetrical** distribution with one mode is roughly the same to either side of the highest frequency
- A **positively skewed** distribution with one mode has more data on the positive side of the highest frequency. It is also described as **skewed to the right** and has a long tail on the right-hand (positive) side.
- A **negatively skewed** distribution with one mode has more data on the negative side of the highest frequency. It is also described as **skewed to the left** and has a long tail on the left-hand (negative) side.



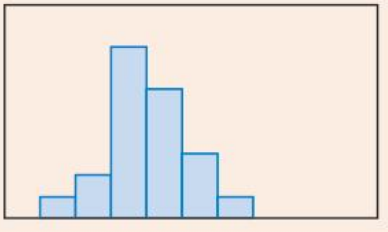
EXAMPLE 19

Describe the shape of each distribution shown.

a



b



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Solution

a The graph has a long tail on the left. The distribution is negatively skewed.

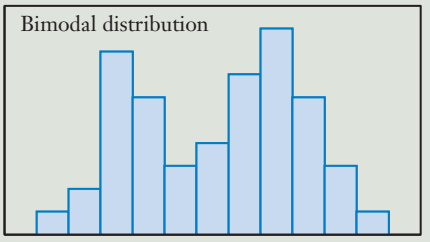
b The graph is balanced to each side. The distribution is symmetrical.

Distributions can have more than one peak, or can have a largely flat central section.

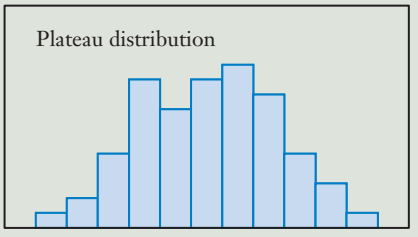
Bimodal and plateau distributions

- A **bimodal distribution** has two distinct peaks. It could be a combination of two distributions with different centres.

A bimodal distribution does not always have peaks of the same height.



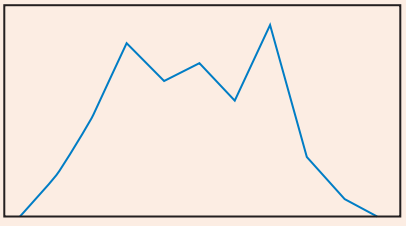
- A **plateau distribution** has similar frequencies for all except the beginning and end.



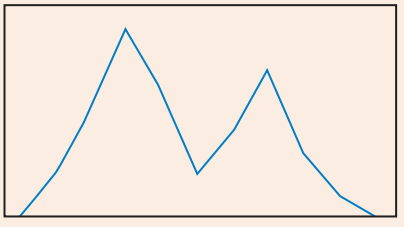
EXAMPLE 20

Describe the shape of each of the distributions shown below.

a



b



Solution

a The graph is tapered at the ends only. It is a plateau distribution.

b The graph has two distinct peaks. It is a bimodal distribution.

In some numerical data collections, a single value or a group of values may be quite distant from the rest of the values. You can test the interquartile range or standard deviation to test values like this.

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Outliers

Outliers

- An **outlier** is a value numerically distant from most of the values in a distribution
- **Interquartile range test:** if $Q_1 - 1.5 \times \text{IQR} < x < Q_3 + 1.5 \times \text{IQR}$, then x is not an outlier
- **Standard deviation test:** if $\bar{x} - 2.5 < x < \bar{x} + 2.5 \times \text{SD}$, then x is not an outlier

The interquartile range test is appropriate for discrete or skewed data. The values used to check for outliers are sometimes called **fences**.

You should check outliers. They could be values that have a significantly different origin, they could just be extreme normal values, or they could be errors in data collection.

For example, a 1.5 V battery that lasts much longer than others could be a different type.

EXAMPLE 21

Use appropriate tests to find possible outliers in each set of data.

a 26, 21, 19, 23, 18, 8, 16, 20, 20, 25, 23, 20

b 20, 34.5, 23.3, 26.2, 21, 29.4, 25, 27.5, 21.8, 30.9, 20, 37.3, 23.8, 25.4, 23.5

c 16.5, 18, 17.4, 25.7, 17.9, 19.3, 17.5, 32.8, 16.1, 44.7

Solution

a	Discrete data, so arrange in ordered quarters.	8, 16, 18 | 19, 20, 20 | 20, 21, 23 | 23, 25, 26
	State the quartiles.	$Q_1 = 18.5$, $Q_3 = 23$
	Find the interquartile range.	$\text{IQR} = 23 - 18.5 = 5.5$
	Find the 'fences'.	$Q_1 - 1.5 \times \text{IQR} = 10.25$
		$Q_3 + 1.5 \times \text{IQR} = 31.25$
	State the answer.	The value 8 is outside the test range, so it is a possible outlier.
b	Use your calculator to find the mean and standard deviation.	$\bar{x} \approx 25.97$, $s \approx 4.98$
	Find the outside values (fences).	$\bar{x} - 2.5 \times s \approx 13.52$
		$\bar{x} + 2.5 \times s \approx 38.42$
	State the answer.	There are no values outside the test range, so there are no likely outliers.
c	Use your calculator to find the mean and standard deviation.	$\bar{x} \approx 22.12$, $s \approx 7.75$

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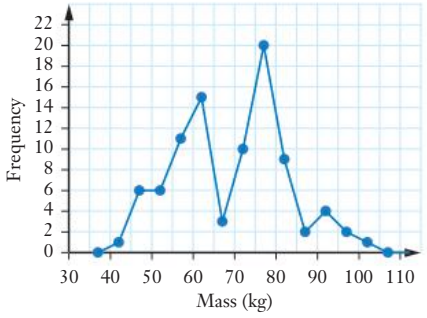
Find the 'fences'.	$\bar{x} - 2.5 \times s \approx 2.75$
	$\bar{x} + 2.5 \times s \approx 41.5$
State the answer.	The value 44.7 is outside the test range, so it is a possible outlier.

Exercise 9.08 The shape of a distribution

Example 19, 20

1 Describe the shape of each distribution shown.

a



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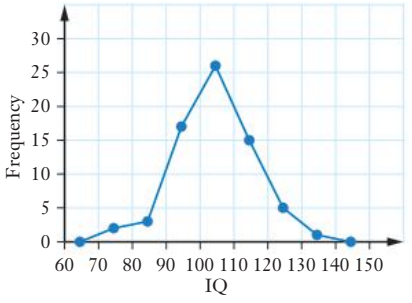

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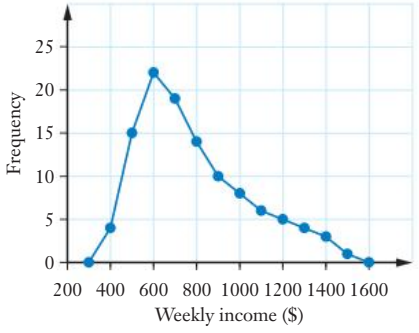
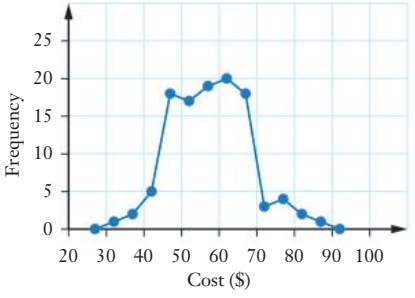
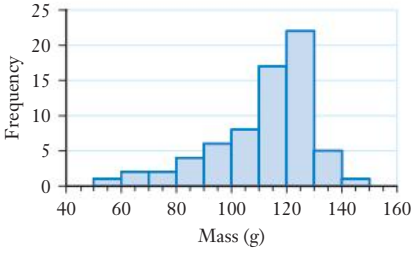


d



2 Describe the shape of each distribution shown.

a



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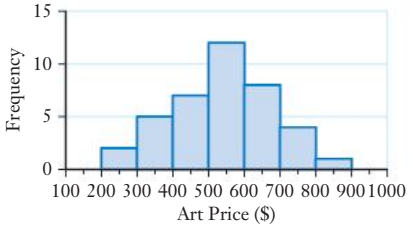

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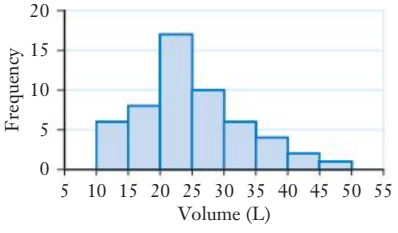

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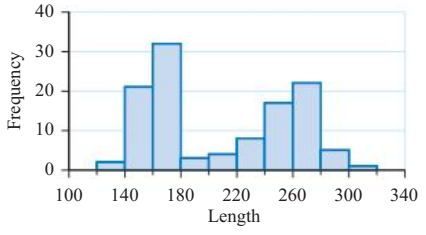

Example 21

3 Use appropriate tests to find possible outliers in each data set.

a 13, 31, 10, 48, 12, 17, 14, 16, 13, 26

b 34.4, 48.1, 43.4, 42.6, 42.5, 6.1, 61.3, 31.6, 39.5, 56.1, 43.7, 37.2, 39.5

c 12.25, 17.91, 24.81, 1.42, 14.3, 22.24, 10.02, 29.03, 16.93, 16.1, 18.31

d 30, 52, 21, 53, 47, 51, 37, 54, 45, 53, 44, 50

e 8, 13, 10, 15, 7, 11, 10, 13, 8, 12, 10, 15, 9, 12

f 22.7, 21.5, 27.6, 25.9, 19, 21.7, 31.9, 38.2, 27, 27.5, 31.2, 63.2, 35.6, 8.5, 40.3

4 Use an appropriate test to decide if the stated value is an outlier of each data set.

a $Q_1 = 24$, $Q_3 = 32$, value 53

b $\bar{x} \approx 67.5$, $s \approx 8.3$, value 39.6

c $\bar{x} \approx 4.8$, $s \approx 1.6$, value 7.3

d $Q_1 = 19$, $Q_3 = 24.5$, value 9.1

e $\bar{x} \approx 14.1$, $s \approx 3.6$, values 2.8, 5.3

f $Q_1 = 11$, $Q_3 = 17.4$, values 4.7, 22.9

Problem solving

5 A student wrote down the ages of residents in an aged care facility as follows.

81	92	78	93	84	98	79	90	79	98	81	94
83	89	72	93	76	42	93	81	103	82	89	78
90	72	93	80	87	76	86	72	88	75	91	80
92	79	35	100	80	99	77	95	78			

Make a frequency table and find appropriate measures.
Comment on any unusual features.

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9.09 Statistical investigations

Student survey form

Census questions

The Australian Bureau of Statistics collects data to help governments make decisions. It also analyses and reformats data for organisations as a paid service.

You need to collect the right data in the right way to answer statistical questions.

The **statistical investigation process** has the following steps.

- Refine the problem to pose a question
- Determine the population
- Decide relevant variables
- Decide how data will be collected
- Collect data
- Display and analyse data
- Interpret and communicate results

You need to know how to analyse data before you can properly design an investigation. Understanding and analysing variables is essential.

EXAMPLE 22

Donna and Simon are both qualified in human movement and want to start a suburban gym in Townsville.

a What do they need to know about different Townsville suburbs?

b What questions should they ask to get the right information?

c What variables should they investigate about possible locations?

Solution

a What kinds of things do they need to know about choosing a good location?

They need to know about existing gyms and how many people in an area would use a new gym. They need to know whether they could make a profit.

b They need to determine the likely number of people who would use a new gym.

How close is the nearest gym?

How full are nearby gyms?

How many people are in the area nearby?

What kind of gym would they go to?

How much would they pay?

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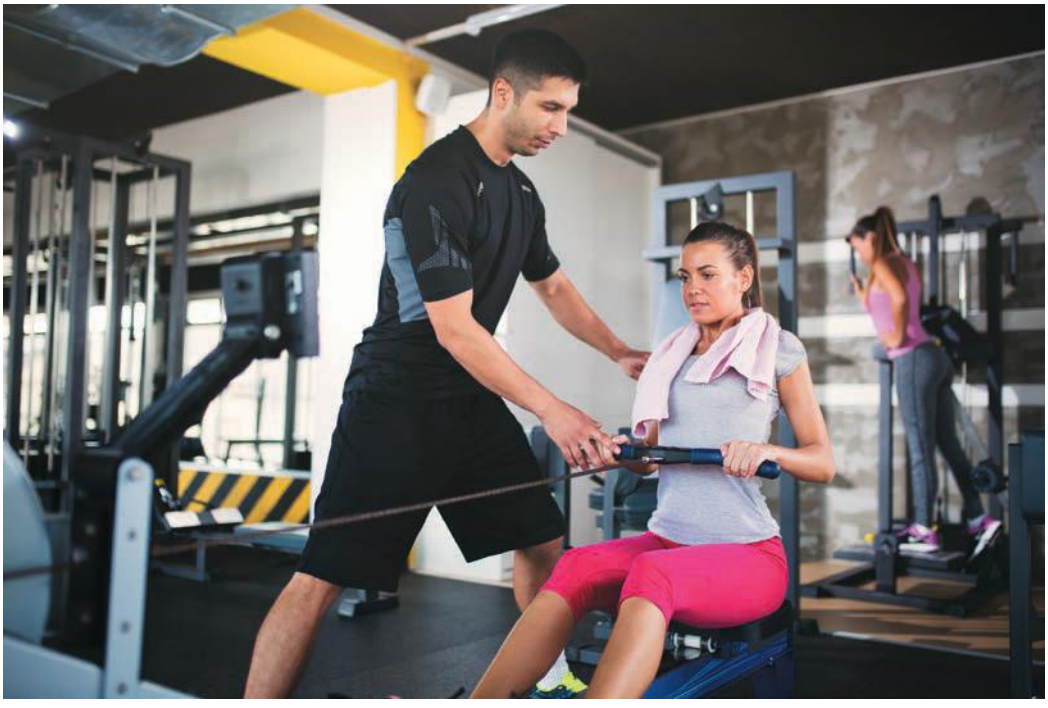
c	The variables should be questions that are relevant to each location.	Distances to other gyms. Percentage use of other gyms. Population that would use a gym. Types of exercise and training people want. Cost of existing gyms. Likely turnover, costs and profit.

Some information can be found by direct research. You can find information on the Internet about existing gyms. You could find possible locations by looking for gaps. You might have to find information about the popularity of gyms by using a survey. For a survey on possible gym use, you need to ask gym users. You need to decide what kind of survey to use: phone, postal, door-to-door, etc. Postal is the cheapest, but the least reliable, door-to-door is the most reliable, but the most expensive to set up.

People will not answer long surveys, so you need to focus your questions. You need to make them easy to answer with a number or single word answer if possible.

Many people will not answer detailed questions about their specific income or age, so you should make these category questions.



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EXAMPLE 23

Design questions for the survey of a suburb for a possible gym location.

Solution

Your questions need to be simple and direct.

1. Do you currently use a gym?
2. If you do, how far away is it?
3. How much would you pay to use a local gym?
4. How many times a week would you use a gym?
5. What equipment would you use in a gym?
6. What times would you want a local gym to be open?

INVESTIGATION

REFINING SURVEY QUESTIONS

Your teacher might want you to work in small groups for this investigation.

Choose one of these to work out what data is needed and the questions to ask.

- Setting up a coffee shop
- Starting a lawn-mowing service
- Starting a laundry service
- Running a baby-sitting service
- Starting a child-minding centre
- Starting a new high school
- Starting a new primary school
- Starting a hair-dressing salon
- Starting a local football team
- Starting a local netball team

Your teacher might want you to write a report or survey form.

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EXAMPLE 24

Rewrite each survey question so it will give better results.

a 'How much pocket money does your father give you?'

b 'What sport do you like?'

Solution

a The question does not give a time frame, doesn't say what it means by pocket money and assumes that students get money from their father.

Pocket money is money you get from your parents or guardians to spend on yourself, not including things like lunch money, bus money, etc.

Circle the amount of pocket money you receive each week:

$0	less than $10	$10–$20
$21–$30	$31–40	More than $40

b The question doesn't specify whether it means to play, go to a match, watch on TV, etc.

Do you play a sport?

If you do, what sport(s) do you play?

What sport(s) do you go to see played?

Exercise 9.09 Statistical investigations

Rewrite each survey question so it will give better results.

Example 24

1 'What is your income?'

2 'How old are you?'

3 'What qualifications do you have?'

4 'How much do you spend on food?'

5 'How much exercise do you do?'

6 'How big is your family?'

7 'How many holidays have you had this year?'

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Problem solving

8 **a** What information would you need to decide where to set up a childcare centre?

b What questions would you need to answer?

c What are some variables?

d Design 2 suitable questions.

9 **a** What information would you need to decide whether to start a local under-18 soccer team?

b What questions would you need to answer?

c What are some variables?

d Design 2 suitable questions.

10 **a** What information would you need to set up a fast-food outlet?

b What questions would you need to answer?

c What are some variables?

d Design 2 suitable questions.

11 **a** What information would an insurance manager need to decide where to send a sales team for home insurance?

b What questions would you need to answer?

c What are some variables?

d Design 2 suitable questions.

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9. CHAPTER SUMMARY

Data and statistics

Data types

- **Categorical data** has values that can be sorted into groups, but cannot be measured
  - **Nominal** data can be counted, but cannot be put in order
  - **Ordinal** data can be put in order, or have a rating scale
- **Numerical data** has values that can be measured
  - **Discrete** data can take only particular values
  - **Continuous** data can have any values; the domain can be finite or infinite

Organising and displaying data

- **Pie charts** and **picture graphs** are more suitable for nominal data than ordinal data. You can use **bar** and **column** graphs for either.
- **Bar graphs** and **column graphs** are suitable for ordinal data
- **Dot plots**, column graphs, **stem-and-leaf** plots are suitable for individual discrete data
- Use a column graph without gaps between the columns. This is often referred to as a **histogram**, even though histograms are really used for continuous data.
- The values of continuous variables are collected in **class intervals** like 49.5–54.5
- The start of a class interval is the **lower class limit** and the top is the **upper class limit**
- **True class limits** have no gaps between them. **Stated class limits** may have gaps between them.
- The **class width** is the difference between the true class limits
- The **class centre** (**class midpoint**) $= \dfrac{\text{upper + lower class limits}}{2}$
- A **frequency polygon** is a line segment graph with each point at the class centre and frequency. It begins and ends on the horizontal axis.

Measures of central tendency

- The **mode** is the most common score. For grouped data, the **modal class** occurs most often.
- The **mean** is the calculated average of the scores and has symbol $\bar{x}$. You add all the scores and divide by the total number of scores. $\bar{x} = \dfrac{\sum x}{n}$ or $\bar{x} = \dfrac{\sum fx}{\sum f}$, where Σ means 'the sum of', x is the score, f is its frequency and n is the total number of scores.

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- The **median** is the middle score when all scores are written in order from smallest to largest. If there is an even number of scores, then it is the average of the middle two. The median is the $\left(\frac{n+1}{2}\right)$th score, provided the scores are in order.
- The mode is the *most likely* to occur. It is the only measure of central tendency you can use for nominal data.
- The mean is most useful for continuous data, when numerical data is *evenly spread* or when the value is used in calculations.
- The median is the *halfway* value. It is most useful for numerical data that is *unevenly spread* to one side or for ordinal data.
- For grouped data, **linear interpolation** is used to approximate the position of the median in the medial class
- The formula is Median $= L + \frac{m-F}{f} \times w$, where L is the true lower limit, $m = \frac{n+1}{2}$, F is the cumulative frequency of the previous class, w is the true class width, and f is the frequency of the medial class

Measures of spread (dispersion)

- The values Q_0, Q_1, Q_2, Q_3 and Q_4 divide ordered data into quarters, where Q_0 is the **minimum** value, Q_2 is the **median** and Q_4 is the **maximum** value
- Q_1 is the value a quarter of the way through, called the **first** or **lower quartile**.
- Q_3 is the value three-quarters of the way through, called the **third** or **upper quartile**.
- The **range** $= Q_4 - Q_0 =$ maximum $-$ minimum
- The **interquartile range** (**IQR**) $= Q_3 - Q_1$
- Half the data is within the interquartile range
- For n odd, Q_1 is the $\left(\frac{n+1}{4}\right)$th value and Q_3 is the $\left(\frac{3n+3}{4}\right)$th value
- For n even, Q_1 is the $\left(\frac{n+2}{4}\right)$th value and Q_3 is the $\left(\frac{3n+2}{4}\right)$th value
- The **standard deviation** is

  for individual data, $s = \sqrt{\frac{\sum (x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$

  for grouped data, $s = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$
- $s =$ standard deviation of a set of data
- $\sigma =$ standard deviation of a population

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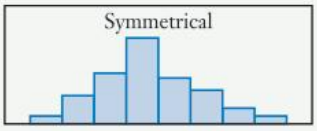
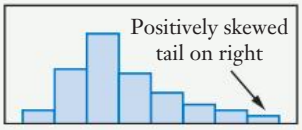
- **Coefficient of variation** $= \frac{s}{\bar{x}}$ or $\frac{\sigma}{\mu}$. It is usually expressed as a percentage. The coefficient of variation (**relative standard deviation**, **RSD**) measures the variation of a set of data

The shape of a distribution

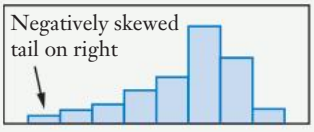
- A **symmetrical** distribution with one mode is roughly the same to either side of the highest frequency



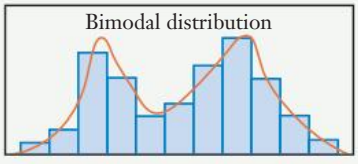
- A **positively skewed** distribution with one mode has more data on the positive side of the highest frequency. It is also described as **skewed to the right** and has a long tail on the right-hand (positive) side.



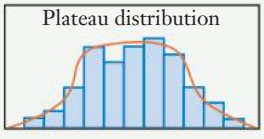
- A **negatively skewed** distribution with one mode has more data on the negative side of the highest frequency. It is also described as **skewed to the left** and has a long tail on the left-hand (negative) side.



- A **bimodal distribution** has two distinct peaks. It could be a combination of two distributions with different centres.



- A **plateau distribution** has similar frequencies for all except the beginning and end.



Outliers

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- **Standard deviation test**: if $\bar{x} - 2.5 < x < \bar{x} + 2.5 \times \text{SD}$, then x is not an outlier

The statistical investigation process

- Refine the problem to pose a question
- Determine the population
- Decide relevant variables
- Decide how data will be collected
- Collect data
- Display and analyse data
- Interpret and communicate results

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9. CHAPTER REVIEW

Data and statistics crossword

Data and statistics

1 Classify each data item. Example 1

a Prices of chocolates **b** Diameters of trees **c** Makes of cars

2 What is the expected data range for each variable? Example 2

a Diameter of a mature tree **b** Price of a new mattress

c Age of a primary school child

3 Responses to a survey on favoured prison terms (in years) for a variety of crimes are shown below. The respondents gave their sex (Q1) and their opinions on fair prison terms for assault with a weapon (Q2) and actual bodily harm (Q3). Compile frequency tables for prison terms for Assault with a weapon and Actual bodily harm. Example 3

Q1	F	M	F	F	F	M	M	M	M	F	M	M	M	F	M	M	M	M	F	F
Q2	11	6	11	3	7	2	2	6	8	6	11	7	8	6	4	10	9	10	5	4
Q3	11	11	14	7	7	4	3	6	8	10	15	9	11	7	9	15	11	11	9	6

4 Draw a pie chart showing where a group of council workers live. Example 4

Suburb	Helensvale	Arundel	Ashmore	Labrador	Gaven
Number	18	13	11	7	5

5 Draw a column graph to show the highest monthly temperatures in Mackay one year. Example 5

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
T °C	34.4	35.6	32.6	31.0	31.4	29.1	28.5	29.8	31.2	31.1	35.5	33.5

6 Do a dot plot to show the following results. Example 6

5 8 5 7 6 8 5 10 5 8 6 8 7

11 3 10 5 10 5 9 5 8 4 8 4

7 Draw a stem-and-leaf plot to show the following response times in seconds to calls for emergency telephone operators. Example 7

12 75 45 52 61 49 53 45 64 22 54

15 23 45 68 51 40 39 60 61 55 58

41 69 66 52 38 79 18 54 22 31 47

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Example 8

8 Draw a frequency histogram of this grouped data.

Scores	0–3	4–7	9–11	12–15	16–19	20–23
Frequency	3	5	9	10	4	2

Example 9

9 The heights of some tree seedlings after 1 year are shown below. Use classes of 70–74, 75–79, … to make a frequency table and histogram.

78.1	104.1	84.4	112.1	94.5	105.5	75	103.6	74.4	108
81.7	103.5	63.6	105	93.2	107.9	69.5	107.3	84.5	117.7
76.5	112.1	97.1	106	78.1	102.9	82.6	114.7	97.5	104.7
91.6	102.7	99.9	102.1	86	107.5	88.7	108.1	74	103.3
81	106.6	87.6	107.8	94.3	119.6	93	108.4	90.8	

Example 10

10 Draw a frequency polygon to show the following data.

Class	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	1	6	9	21	16	12	6	2

Examples 11-13

11 Find the mean, median and mode (or modal class) of each set of data.

a 18, 19, 15, 10, 13, 9, 14, 21, 17, 19, 26, 19, 23, 23

b

Score	2–5	6–9	10–13	14–17	18–21	22–25
Frequency	2	3	5	4	2	1

c

Class	14.5–19.5	19.5–24.5	24.5–29.5	29.5–34.5	34.5–39.5	39.5–44.5	44.5–49.5	49.5–54.5
Frequency	1	7	11	9	6	4	3	1

Examples 14-17

12 Find the range, interquartile range and standard deviation of each set of data.

a 23, 28, 29, 25, 26, 25, 29, 28, 22, 24, 21, 31, 32, 24, 27, 24, 26

b

x	12	13	14	15	16	17	18	19	20
f	3	4	7	9	6	5	4	3	1

c

Class	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59
f	3	6	12	18	19	23	21	17	14	8	4

Example 18

13 Which set of data is the more consistent?

A: 3, 7, 4, 6, 8, 5, 9, 7, 2, 2, 3, 7

B: 22, 24, 26, 21, 20, 29, 31, 27, 26, 24

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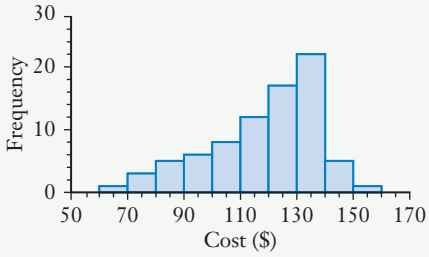
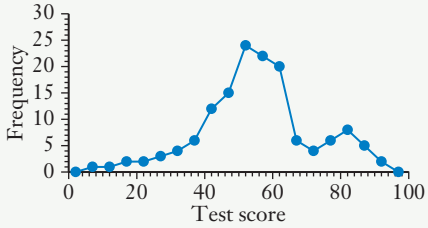
14 Describe the shape of each distribution.

Example 19,20

a



b



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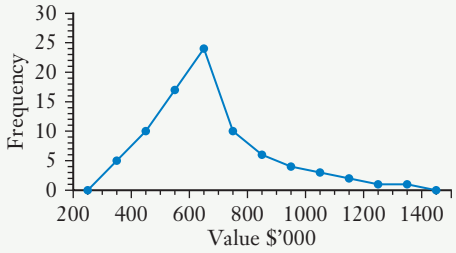
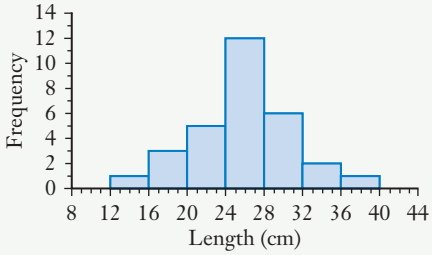


d



15 Use appropriate tests to find possible outliers in each data set.

Example 21

a 20, 22, 22, 22, 20, 22, 17, 24, 22, 26

b 31.69, 34.77, 37.24, 28.14, 31.3, 24.76, 47.19, 21.42, 37.53, 41.22, 29.1, 20.34, 62.31, 41.57, 21.3, 53.58

16 Rewrite each question so it will give better results.

Example 24

a 'Are you wealthy?' **b** 'Do you watch much TV?'

Problem solving

17 The following were the preferences of 33 Year 11 students for different flavours of chips. Each student rated the chips from most to least preferred.

The codes are Bacon = Ba, BBQ = Bb, Cheese = Ch, Plain = Pl, Salt and vinegar = Sv.

Pl-Ch-Sv-Ba-Bb	Pl-Bb-Sv-Ch-Ba	Pl-Sv-Bb-Ch-Ba	Pl-Ch-Bb-Ba-Sv
Ba-Pl-Sv-Bb-Ch	Pl-Sv-Ch-Ba-Bb	Sv-Pl-Ch-Bb-Ba	Pl-Ba-Sv-Ch-Bb
Bb-Pl-Ba-Sv-Ch	Ch-Ba-Pl-Sv-Bb	Pl-Bb-Sv-Ba-Ch	Ba-Pl-Ch-Sv-Bb
Bb-Pl-Sv-Ba-Ch	Pl-Sv-Bb-Ba-Ch	Pl-Ba-Bb-Sv-Ch	Pl-Ch-Sv-Ba-Bb
Bb-Pl-Sv-Ba-Ch	Pl-Sv-Bb-Ch-Ba	Bb-Sv-Ba-Ch-Pl	Pl-Sv-Bb-Ba-Ch
Ba-Sv-Ch-Pl-Bb	Ba-Sv-Bb-Pl-Ch	Sv-Ba-Pl-Ch-Bb	Pl-Sv-Bb-Ba-Ch
Bb-Pl-Sv-Ch-Ba	Sv-Pl-Ba-Bb-Ch	Pl-Sv-Ba-Bb-Ch	Pl-Bb-Sv-Ba-Ch
Pl-Ba-Bb-Ch-Sv	Pl-Sv-Ch-Bb-Ba	Sv-Pl-Bb-Ch-Ba	Sv-Pl-Ba-Bb-Ch
Bb-Ba-Sv-Pl-Ch			

a Give the order of popularity by first preference alone.

b Give the order of popularity using another system, explaining your reasoning.

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18 The weight loss (in kg) by new members of a popular diet program are given here (negative numbers show a weight gain).

1.2	4.9	–2.4	8.6	4.7	15.9	3.4	7.5	7.2	4	6.5
11.9	10.7	2.1	11.1	5.4	–2	10.6	10.6	6.9	12.1	–0.1
4.1	14.2	14.9	16.3	20.7	6.6	6.5	4.3	3	16	0.7
6.7	6.2	7.9	6.5	0.9	15.4	5.3	10.7			

Draw a histogram and comment on the effectiveness of the program. Explain your answer.

19 The ages of spectators at a hockey match are given in the table. What is the typical age? Justify your answer.

Age	15–24	25–34	35–44	45–54	55–64	65–74
Number	57	188	221	75	117	51

20 To obtain a learner's licence in Queensland, you need to score at least 27 out of 30 on a test on road rules. The following scores were obtained by people sitting the test.

21	29	25	29	25	30	27	29	17	30
28	29	26	30	29	29	28	29	29	30
26	30	25	30	18	29	25	30	26	

a What is the typical score? Comment on your finding.

b Between what scores are the middle 50% of results?

c What is the pass rate?

21 The ages of people at a birthday party were as follows.

16	18	51	17	16	17	15	17	45	18
17	18	78	17	16	21	16	21	16	

What can you say about the people at the party? Explain your answer.

Practice quiz

22 Delia has decided to start an ironing service in the Bundaberg area. What information will she need and what variables does she need to investigate? Write 2 questions she might use in a questionnaire.

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Practice examination 3

Time: 90 minutes
Perusal time: 5 minutes
Marks: 50

Instructions

- Students are permitted to bring or use: pens, pencils, highlighters, erasers, sharpeners, rules and an approved graphics calculator.
- Students must show appropriate working and justification to gain full marks.
- A formula sheet is provided.
- Unless otherwise stated, numerical answers should be exact.
- Unless otherwise indicated, no diagrams in this examination are drawn to scale.
- All written responses must be in English.
- Answer all questions.
- **Students are NOT permitted to bring or use notes of any kind, correction fluid/tape, mobile phones and/or any other unauthorised electronic devices.**

Year 11 formulas

Question 1 (2 marks)

Which of the following expressions are linear?

A $3x - \frac{5x}{3}$ **B** $2a(3a + 1)$ **C** $2y - 5x + 9$

D $6p + 2\sqrt{p}$ **E** $\frac{6m + 5}{4}$ **F** $\frac{x^3 + 5x - 7}{2x}$

Question 2 (1 mark)

The values of a variable and corresponding values of an expression are shown in the table. Determine if the expression is linear or non-linear.

Variable	–2	0	3	4
Expression	8	2	–7	–10

Question 3 (2 marks)

Transpose each formula so that the pronumeral shown in brackets is the subject.

a $w(2 - 5xy) = 4u$ (y) **b** $\frac{3m}{5n} = \frac{4p}{r}$ (r)

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Question 4 (2 marks)

Draw a graph of $3 - 2xy = 0$ for $1 \le x \le 6$ and use the graph to determine if the equation is linear.

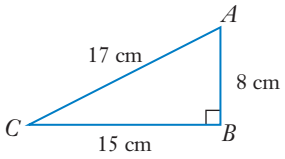
Question 5 (2 marks)

The table below shows values of values for an equation in the variables x and y. Determine if the equation is linear or non-linear.

x	–3	–1	1	4
y	24	16	8	–4

Question 6 (1 mark)

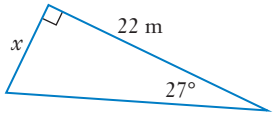
Find cos (A) for this triangle.



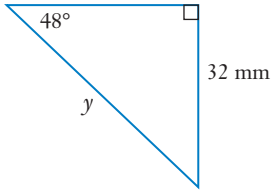
Question 7 (4 marks)

Find the value of each pronumeral, correct to 1 decimal place.

a

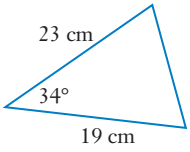


b



Question 8 (1 mark)

Find the area of this triangle, correct to one decimal place.



Question 9 (2 marks)

Classify each data item.

a The reaction times of drivers.

b The RACQ ratings of vehicle safety: from Excellent to Very Poor.

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Question 10 (3 marks)

This table shows the number of web-connectible devices owned by a group of 35 people.

Number	2	3	4	5	6	7	8	9	10
Frequency	2	9	9	6	4	1	2	1	1

a What is the typical number of devices owned?

b What is the interquartile range?

Question 11 (3 marks)

Find the mean and standard deviation of these heights of seedlings 5 weeks after germination.

Height (cm)	15–19	20–24	25–29	30–34	35–39	40–44	45–49
Frequency	5	31	33	39	13	3	2

Question 12 (2 marks)

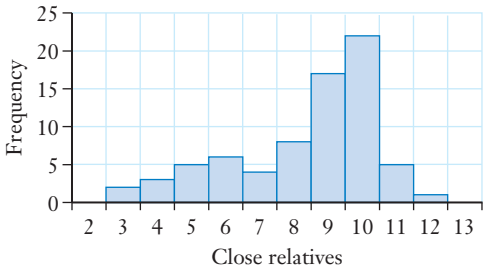
Use an appropriate test to decide if there are any possible outliers in this discrete data.

19, 21, 18, 19, 19, 28, 19, 21, 17, 23, 16, 23, 17, 20, 18, 20, 18, 23, 19, 20, 19, 19

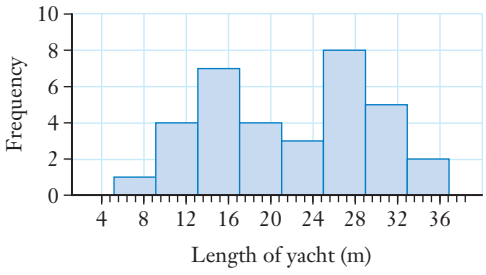
Question 13 (2 marks)

Describe the shape of each distribution.

a



b



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Question 14 (1 mark)

The staff at a zoo are hand-rearing a lion cub abandoned by its mother. The minimum daily calorie intake required for normal development of a lion cub is shown in the table below.

Age (weeks)	1	2	3	4	5
Daily calorie intake	825	1000	1185	1320	1420

Is the minimum daily calorie intake of a lion cub a linear function of its age in weeks?

Question 15 (2 marks)

Make x the subject of the formula $2z = \sqrt{x + y}$.

Question 16 (2 marks)

A train is travelling at 20 m/s on a straight line in the direction N 35° E.
How much further north will it travel in 3 minutes?

Question 17 (5 marks)

The students in Year 11 were asked to write their weights in kg on pieces of paper.

53	76	38	69	52	76	55	76	55	128	56	79	117
79	44	73	45	83	51	79	57	72	57	85	56	86
62	76	68	79	52	67	50	77	62	75	52	85	49
76	47	65	55	78	55	90	46	82	54	80	44	85
62	69	52	78	53	65	50	73	46	90	55	87	46
79	58	83	60	87	57	81	52	64	54	94	40	78
51	66	52	81	49	78	50	87	55	73.			

Compile a table and find the typical weight.

Question 18 (1 mark)

Determine if the area of a circle is a linear or non-linear function of its radius.
Explain your reasoning.

Question 19 (2 marks)

Make R the subject of the formula $S = 4\pi r\sqrt{\dfrac{R^2 + r^2}{2}}$.

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Question 20 (2 marks)

A paddle boarder who was taken out in a rip estimates the top of a cliff on the headland is at an angle of elevation of 20°. She knows that the cliff is 25 m high and that it has a narrow beach 2 m wide at its base. How far does she have to paddle to reach the beach?

Question 21 (5 marks)

The ages of people watching a football match were as follows.

19	12	15	81	8	56	11	51	17	38	15	47	21
42	38	39	71	13	15	51	45	15	19	14	15	42
43	82	35	37	15	13	9	49	71	52	37	15	11
13	53	15	15	74	15	15	16	11	40	43	15	

Describe the distribution and make any conclusions you can about the people. Carefully justify your answer.

END OF EXAMINATION

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ALGEBRA AND MATRICES

10.

MATRICES

You use tables for organising information in many ways. In mathematics, rectangular tables of numbers can be used for processing data. Such tables can be added, subtracted and multiplied. They are very useful in a range of applications, and they have a special name: **matrices**. They are used in probability, physics, economics, computing and ecology.

10.01 Matrices
10.02 Types of matrices
10.03 Operations with matrices
10.04 Matrix multiplication
10.05 Powers of matrices
10.06 Applications of matrices
Chapter summary
Chapter review

SYLLABUS SUBJECT MATTER

Matrices and matrix arithmetic

- use matrices for storing and displaying information that can be presented in rows and columns, for example, tables, databases, links in social or road networks
- recognise different types of matrices (row, column, square, zero, identity) and determine their size
- perform matrix addition, subtraction, and multiplication by a scalar
- perform matrix multiplication (manually up to a 3×3 but not limited to square matrices, using technology for larger matrices)
- determining the power of a matrix using technology with matrix arithmetic capabilities when appropriate
- use matrices, including matrix products and powers of matrices, to model and solve problems, e.g. costing or pricing problems, squaring a matrix to determine the number of ways pairs of people in a communication network can communicate with each other via a third person

Prior learning

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TERMINOLOGY

adjacency matrix
conformable
element
multiplication by a scalar
power of a matrix
scalar multiple
transition matrix
column matrix
cryptography
identity matrix
network
product of matrices
size
zero matrix
communications network
dimension
matrix (matrices)
order
row matrix
square matrix

10.01 Matrices

This table shows the results of the Group B qualifying rounds for the 2014 FIFA (soccer) World Cup held in Brazil.

'Home' team	'Away' team	Result
Spain	The Netherlands	1–5
Chile	Australia	3–1
Australia	The Netherlands	2–3
Spain	Chile	0–2
Australia	Spain	0–3
The Netherlands	Chile	2–0

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These sports results can be shown in a **matrix**.

A matrix (plural **matrices**) is a table of numbers written in parentheses ( ) or brackets [ ]. The symbol name for a matrix is a bold capital letter like **A**. In handwriting, a matrix name has a wavy underline like $\underset{\sim}{A}$.

Matrices

The **size** (or **dimension** or **order**) of a matrix is written as the number of rows × the number of columns.

For example, $\mathbf{A} = \begin{bmatrix} 2.4 & 0 & 6 & \frac{1}{4} \\ 0 & -5.3 & 4 & -5 \end{bmatrix}$ is a 2×4 ('2 by 4') matrix.

Each number in a matrix is called an **element**. Elements are identified by a small letter with the row and column as a subscript, for example, for matrix **A** above, $a_{24} = -5$ and $a_{13} = 6$.

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EXAMPLE 1

a Show the Group B results as a matrix **R** in the order Australia, Chile, The Netherlands, Spain.

b What do the 0s in the diagonal show?

c What do r_{13} and r_{31} show?

Solution

a It is usual to put the goals scored in the rows with the opponents in the columns. Row 1 shows that Australia scored 1 goal against Chile, 2 against The Netherlands and 0 against Spain.

$$\mathbf{R} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 3 & 0 & 0 & 0 \\ 3 & 2 & 0 & 5 \\ 3 & 0 & 1 & 0 \end{bmatrix}$$

A matrix score table like this often has the first letters of the team names across the top and down the side.

b The diagonals have the same country in the row and the column.

The 0s in the diagonal show that countries did not play each other.

c The positions are for Australia vs The Netherlands and The Netherlands vs Australia.

$r_{13} = 2$ shows Australia scored 2 goals and $r_{31} = 3$ shows The Netherlands scored 3 goals in their match, so The Netherlands won.

EXAMPLE 2

$$\mathbf{G} = \begin{bmatrix} 2 & 3 & 11 & -3 & 9 \\ 5 & 0 & -4 & 8 & -1 \\ 7 & 1 & 6 & -5 & 4 \end{bmatrix}$$

a What is g_{23}?

b What is g_{35}?

c What is g_{42}?

Solution

a It is in the 2nd row and 3rd column. $g_{23} = -4$

b It is in the 3rd row and 5th column. $g_{35} = 4$

c It is in the 4th row and 2nd column. There is no 4th row so g_{42} does not exist.

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Exercise 10.01 Matrices

Example 1

1 Which of the following are matrices?

a $\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & & \\ & 1 & \\ & & 3 \end{bmatrix}$ **c** $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

d $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & -2 \\ & 1 & 3 \end{bmatrix}$ **e** $\begin{bmatrix} \sqrt{3} & \frac{5}{6} \\ -2 & 7 \end{bmatrix}$ **f** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Example 2

2 $\mathbf{A} = \begin{bmatrix} 1 & 4 & 3 & -2 \\ 9 & 0 & 6 & -1 \\ 7 & 2 & 8 & 5 \end{bmatrix}$. What is the size of **A**?

If possible, write down the value of:

a a_{12} **b** a_{21} **c** a_{34}

d a_{22} **e** a_{32} **f** a_{43}.

Problem solving

3 A manufacturer builds desktop computers with a variety of central processor speeds and hard drive capacities. The result of a stock inventory is shown in the table below.

	Processor (GHz)								
Hard drive (GB)	**1.86**	**2.0**	**3.8**	**2.33**	**2.67**	**2.8**	**3.0**	**3.2**	**3.5**
160	3	2	4	1	5	0	3	6	1
250	1	8	1	7	1	8	0	5	8
320	4	0	5	6	4	5	9	1	7
500	7	9	2	0	1	1	10	9	1

a Write the information in the table as a 4×9 matrix **M**.

b Write the value of

i m_{32}

ii m_{21}

iii m_{27}

c What is the size of **M**?

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4 This table shows the prices of timber from four different hardware suppliers. All prices listed are per metre or per square metre (sheet).

Product	Jones & Co.	Brown Bros	Pine Hut	Allgoods
Hardwood framing	\$4.20	\$4.30	Cannot supply	\$4.40
Chipboard flooring	\$9.10	\$8.80	\$8.70	\$9.00
Structural plywood	\$6.20	\$6.40	\$6.50	\$6.10
Pine chamfer	\$3.80	\$3.90	\$3.60	\$3.70

a This table cannot be changed to a matrix. Why not?

b Change the table so that it can be converted to a square matrix.

c Write the matrix **T**.

d Write the value of t_{24}.

5 On a particular trading day, the exchange rates for Australian dollars (AUD), United States dollars (USD), Great Britain pounds (GBP) and Japanese yen (JPY) were as follows:

1 AUD = 0.7324 USD, 0.5874 GBP, 86.24 JPY.

1 USD = 1.3654 AUD, 0.8020 GBP, 117.75 JPY.

1 GBP = 1.7024 AUD, 1.2469 USD, 146.82 JPY.

1 JPY = 0.011 59 AUD, 0.008 93 USD, 0.006 811 GBP.

a What is the equivalent of 1 AUD in USD?

b Put the information in a 4×4 matrix **E** in the order AUD, USD, GPB and JPY.

c What is the value of e_{23}?

d What is the value of e_{32}?

e In this matrix, what is the relationship between e_{ij} and e_{ji}, where $i \neq j$?

6 A manufacturer of marbelite bathroomware uses the following quantities of materials (in kilograms) to make handbasins, shower bases, baths and spas.

Material	Handbasin	Shower base	Bath	Spa
Plaster filler	4	9	20	40
Epoxy resin	1	2	4	9
Catalyst	0.2	0.3	0.5	1

a Write the information as a matrix **B** with the materials in rows.

b What is the value of b_{23}?

c What is the value of b_{32}?

d What does the value of b_{22} represent?

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7 An ecologist studied the diets of water beetles, mosquito larvae, fingerlings and adult fish in a freshwater pool. She found that mosquito larvae and fingerlings both ate microscopic algae and microscopic animals in the water, but that fingerlings also ate approximately 3 mosquito larvae each day. Water beetles ate about 5 mosquito larvae and 2 fingerlings each day, and adult fish ate about 6 fingerlings and 4 water beetles each day.

iStock.com/Byronsdad

a Write the information as a 4×4 matrix **P**, showing the predation of species on each other. Write it in the order: mosquito larvae, fingerlings, water beetles and adult fish.

b What is the significance of the row of 0s for mosquito larvae?

c Why does p_{33} have that value?

10.02 Types of matrices

Type of matrix	Example
A **row matrix** has only one row.	$\begin{bmatrix} 2 & 4 & 5 & 0 \end{bmatrix}$
A **column matrix** has only one column.	$\begin{bmatrix} 5 \\ -2 \\ 0.4 \end{bmatrix}$
A **square matrix** has the same number of rows and columns. The size of a square matrix is given as its order.	$\begin{bmatrix} 1 & -4 \\ 6 & 2 \end{bmatrix}$ This matrix has order 2.
A **zero matrix** 0 has all elements equal to 0.	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
An **identity matrix** **I** is a square matrix with every element 0 except for the diagonal values, which are all 1s.	$\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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EXAMPLE 3

Describe the type and size of each matrix.

a $\begin{bmatrix} -2 & 0.5 & 0 \\ 3 & 1 & 0 \\ 0 & 8 & 2 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution

a The matrix has 3 rows and 3 columns. It is a square matrix of order 3.

b Every element is zero. It is a 2×3 zero matrix.

EXAMPLE 4

Construct the identity matrix of order 4.

Solution

It has 1s in the diagonal and 0s everywhere else.

$$\mathbf{I}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exercise 10.02 Types of matrices

1 Describe the type and size of each matrix. Example 3

a $\begin{bmatrix} 3 & 4 & 0 & 2 & 5 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ **c** $\begin{bmatrix} 2 \\ 3 \\ 0 \\ -4 \end{bmatrix}$

d $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ **e** $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **f** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

g $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ **h** $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ **i** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

2 Construct the identity matrix $\mathbf{I}_5$. Example 4

3 Construct the zero column matrix of order 3.

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4 Construct the 5×2 zero matrix.

5 Construct the 2×7 zero matrix.

6 Construct a square matrix of order 3 with the row elements equal to the row number.

Problem solving

7 Construct the square matrix **Q** of order 3 with elements given by $q_{ij} = i + j$.

8 Construct the row matrix **P** of order 6 with elements given by $p_{ij} = 7 - ij$.

Addition and subtraction of matrices

10.03 Operations with matrices

Suppose a milk processor produced full cream milk, skim milk, reduced fat milk, high calcium milk and lactose-free milk in 600 mL, 1 L, 2 L and 3L containers. The milk sold by a shop on Monday could be $\mathbf{M} = \begin{bmatrix} 10 & 5 & 4 & 6 & 4 \\ 15 & 7 & 9 & 8 & 2 \\ 25 & 10 & 6 & 10 & 3 \\ 10 & 3 & 2 & 5 & 1 \end{bmatrix}$, where the rows are for different sizes and the columns are for different types in the order shown above. The amounts sold on Tuesday and Wednesday could be **T** and **W** set up the same way.

How would you get the total for the three days? You would just add the corresponding elements of the matrices. It makes sense to say this is the sum of the matrices, $\mathbf{M} + \mathbf{T} + \mathbf{W}$.

What if the amount sold on Saturday was double the amount sold on Monday? You would just double each element of **M** to get the matrix **S** for Saturday. You would say that $\mathbf{S} = 2\mathbf{M}$.

You could work out how much more was sold on Saturday than Tuesday by subtracting Tuesday's elements from Saturday's. It would be sensible to write this as $\mathbf{S} - \mathbf{T}$.

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Operations with matrices

- Matrices must be the same size to be added or subtracted
- To **add matrices**, add the corresponding elements
- To **subtract matrices**, subtract the corresponding elements
- To multiply a matrix by a number, multiply every element by the number. This is called **multiplication by a scalar** or a **scalar multiple**
- In formal terms, $\mathbf{A} + \mathbf{B} = (a_{ij} + b_{ij})$, $\mathbf{A} - \mathbf{B} = (a_{ij} - b_{ij})$ and $c\mathbf{A} = (ca_{ij})$ for a number c
- Two matrices are **equal** if and only if they are the same size and corresponding elements are all the same

EXAMPLE 5

Given $\mathbf{A} = \begin{bmatrix} -2 & 3 \\ -1 & 1 \\ 3 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 & 3 \\ 0 & -2 \\ 1 & -3 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 6 & -4 \\ 2 & -3 \\ 3 & 2 \end{bmatrix}$, find:

a $\mathbf{A} + \mathbf{C}$

b $\mathbf{B} - \mathbf{C}$

Solution

a Add corresponding elements.

$$\mathbf{A} + \mathbf{C} = \begin{bmatrix} -2 & 3 \\ -1 & 1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 6 & -4 \\ 2 & -3 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -1 \\ 1 & -2 \\ 6 & 2 \end{bmatrix}$$

b Subtract corresponding elements.

$$\mathbf{B} - \mathbf{C} = \begin{bmatrix} 2 & 3 \\ 0 & -2 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} 6 & -4 \\ 2 & -3 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 7 \\ -2 & 1 \\ -2 & -5 \end{bmatrix}$$

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EXAMPLE 6

Given $\mathbf{D} = \begin{bmatrix} 3 & -3 & 2 & -3 \\ -2 & 5 & -5 & -4 \end{bmatrix}$ and $\mathbf{E} = \begin{bmatrix} 1 & -6 & 4 & 6 \\ -2 & 5 & 4 & -8 \end{bmatrix}$, find:

a $-3\mathbf{E}$

b $2\mathbf{E} - 4\mathbf{D}$

Solution

a Multiply every element by -3.

$$-3\mathbf{E} = -3\begin{bmatrix} 1 & -6 & 4 & 6 \\ -2 & 5 & 4 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 18 & -12 & -18 \\ 6 & -15 & -12 & 24 \end{bmatrix}$$

b Do the scalar multiples.

$$2\mathbf{E} - 4\mathbf{D} = 2\begin{bmatrix} 1 & -6 & 4 & 6 \\ -2 & 5 & 4 & -8 \end{bmatrix} - 4\begin{bmatrix} 3 & -3 & 2 & -3 \\ -2 & 5 & -5 & -4 \end{bmatrix}$$

Changing a subtraction to addition of the opposite helps avoid some errors.

$$= \begin{bmatrix} 2 & -12 & 8 & 12 \\ -4 & 10 & 8 & -16 \end{bmatrix} + \begin{bmatrix} -12 & 12 & -8 & 12 \\ 8 & -20 & 20 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 0 & 0 & 24 \\ 4 & -10 & 28 & 0 \end{bmatrix}$$

TECHNOLOGY

Matrices on a graphics calculator 1

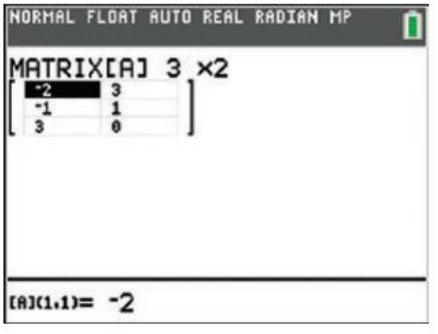
You can do the matrix calculations from Examples 5 and 6 on a graphics calculator.

TI 84 Plus CE

Press [2nd][x^{-1}] (**matrix**), move the cursor across to **EDIT** and press [enter].

Now press 3 [enter], 2 [enter] for the size and put in the elements of **A**.

Press [2nd][mode] to quit and then put in **B** and **C**. Use the cursor to go down to **EDIT** each one. Make sure you use the [(-)] button for the negatives.



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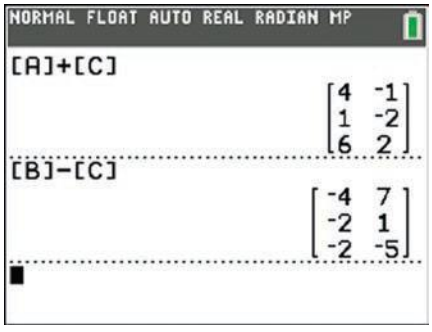
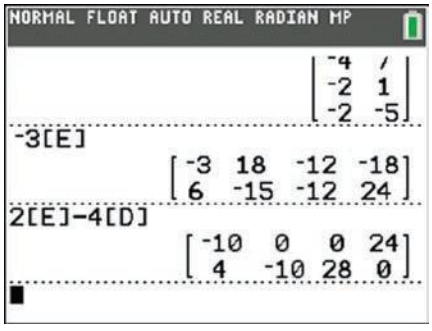
For **A** + **B**, press [2nd][x⁻¹] and press [enter] for **A**. Then press [+], [2nd][x⁻¹], use the cursor to go down to **C** in **NAMES** and press [enter] again. Do **B** – **C** similarly.



Now put in **D** and **E** with the correct sizes and do the operations from Example 6.



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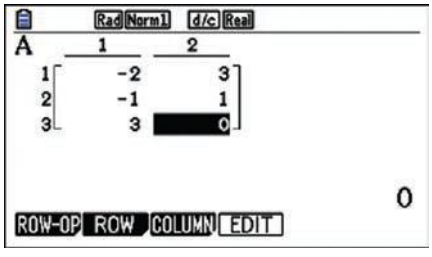
Choose **1 Run-Matrix** and press [F3] (▶**MAT**).

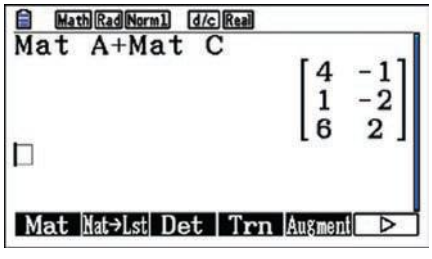
Press [F3] again (**DIM**), 3 [EXE] and 2 [EXE] for the size and put in the elements of **A**.

Press [EXIT], move down and do **B** and **C** in the same way. Make sure you use the [(−)] button for the negatives.



Press [EXIT] to leave matrix entry. For **A** + **B**, press [OPTN] [F2] (**MAT**), then [ALPHA] A.

Press [+] and do MAT C as for MAT A and press [EXE].



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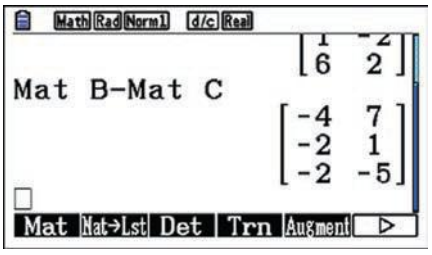
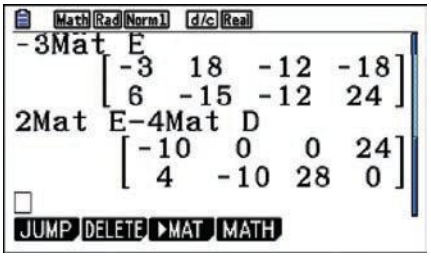
Do **B** – **C** similarly.



Now EXIT twice and put in **D** and **E** with the correct sizes.

Do the operations from Example 6.



Exercise 10.03 Operations with matrices

Example 5,6

1 Given $\mathbf{A} = \begin{bmatrix} 2 & 0 \\ -1 & -2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & 6 \\ 9 & -3 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} -6 & 4 \\ 8 & 10 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 5 & -7 \\ -3 & 1 \end{bmatrix}$, find:

a	$\mathbf{A}+\mathbf{B}$	**b**	$\mathbf{C}+\mathbf{B}$	**c**	$\mathbf{B}+\mathbf{D}$	**d**	$\mathbf{C}+\mathbf{D}$
e	$-\mathbf{B}$	**f**	$\mathbf{C}-\mathbf{A}$	**g**	$\mathbf{B}-\mathbf{D}$	**h**	$\mathbf{C}-\mathbf{B}$
i	$\mathbf{A}+\mathbf{A}+\mathbf{A}$	**j**	$3\mathbf{A}$	**k**	$5\mathbf{B}$	**l**	$4\mathbf{C}$
m	$\frac{1}{2}\mathbf{C}$	**n**	$\frac{1}{3}\mathbf{B}$	**o**	$0.4\mathbf{B}$	**p**	$5\mathbf{A}-\mathbf{B}$
q	$3\mathbf{B}-2\mathbf{A}$	**r**	$\mathbf{C}+2\mathbf{D}-\mathbf{A}$	**s**	$3\mathbf{B}+5\mathbf{A}-\mathbf{C}$	**t**	$2\mathbf{C}-(-4\mathbf{A})+2\mathbf{B}$

2 Given $\mathbf{A} = \begin{bmatrix} 4 & 3 & 1 \\ 0 & 3 & 6 \\ 5 & 0 & -1 \\ 0 & 5 & -2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} -1 & -4 & -3 \\ 2 & -3 & 1 \\ 4 & 2 & 4 \\ -4 & -3 & -2 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 1 & 5 & 8 \\ 0 & 2 & 4 \\ 0 & 6 & 2 \\ 7 & 0 & 0 \end{bmatrix}$, find:

a	$\mathbf{A}+\mathbf{B}$	**b**	$\mathbf{A}+\mathbf{C}$	**c**	$\mathbf{C}+\mathbf{B}$	**d**	$\mathbf{B}+\mathbf{A}$
e	$-\mathbf{B}$	**f**	$\mathbf{C}-\mathbf{A}$	**g**	$\mathbf{A}-\mathbf{C}$	**h**	$\mathbf{B}-\mathbf{C}$
i	$\mathbf{B}-\mathbf{A}$	**j**	$\mathbf{A}-\mathbf{B}$	**k**	$4\mathbf{A}$	**l**	$3\mathbf{B}$
m	$-2\mathbf{C}$	**n**	$0.5\mathbf{A}$	**o**	$-\frac{3}{4}\mathbf{B}$	**p**	$4\mathbf{C}-\mathbf{A}$
q	$3\mathbf{C}-\mathbf{B}$	**r**	$2\mathbf{C}-3\mathbf{A}+2\mathbf{B}$	**s**	$4\mathbf{A}-4\mathbf{B}+4\mathbf{C}$	**t**	$\mathbf{A}-\mathbf{B}+\mathbf{C}$

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3 Given $\mathbf{X} = \begin{bmatrix} 3 & 0 & -1 & 2 \\ 5 & -4 & 3 & 1 \\ -1 & 2 & 0 & 4 \end{bmatrix}$, $\mathbf{Y} = \begin{bmatrix} 1 & 4 & 3 & -5 \\ -3 & 2 & -1 & -6 \\ 0 & 5 & -2 & 3 \end{bmatrix}$, and $\mathbf{Z} = \begin{bmatrix} 6 & -5 & 4 & -3 \\ -1 & 8 & -9 & 2 \\ 4 & 7 & -2 & 5 \end{bmatrix}$,

find:

a	$\mathbf{X}+\mathbf{Y}$	**b**	$\mathbf{Z}+\mathbf{Y}$	**c**	$\mathbf{Y}+\mathbf{Y}$	**d**	$2\mathbf{Y}$
e	$-\mathbf{Z}$	**f**	$-\mathbf{Y}+\mathbf{X}$	**g**	$\mathbf{X}-\mathbf{Y}$	**h**	$\mathbf{Z}-\mathbf{Y}$
i	$5\mathbf{X}$	**j**	$-3\mathbf{Z}$	**k**	$4\mathbf{Y}$	**l**	$-5\mathbf{X}$
m	$\frac{1}{2}\mathbf{Z}$	**n**	$0.1\mathbf{Y}$	**o**	$\frac{1}{3}\mathbf{Z}$	**p**	$4\mathbf{X}-3\mathbf{Z}$
q	$4\mathbf{Z}-5\mathbf{X}$	**r**	$2\mathbf{X}+5\mathbf{Z}-\mathbf{Y}$	**s**	$2\mathbf{Y}-3\mathbf{X}-6\mathbf{Z}$	**t**	$3\mathbf{Z}-6\mathbf{Y}-5\mathbf{X}$

Problem solving

4 Find x in each equation.

a $\begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix} + \begin{bmatrix} 3 & x \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -1 & x \end{bmatrix}$

b $\begin{bmatrix} 4 & 3 & 4 \\ -2 & 0 & x \end{bmatrix} + \begin{bmatrix} -1 & x^2 & -4 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 0 \\ -1 & 0 & -1 \end{bmatrix}$

c $\begin{bmatrix} x^2 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -x^3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -8 \end{bmatrix}$

5 Find $\mathbf{A}$ if $\begin{bmatrix} -1 & 2 \\ -4 & 7 \end{bmatrix} + \mathbf{A} = \begin{bmatrix} 2 & 4 \\ -1 & 7 \end{bmatrix}$

6 Find $\mathbf{M}$ if $\mathbf{M} + \begin{bmatrix} 5 & -2 \\ 10 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ 2 & -1 \end{bmatrix}$

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7 A bank has three branches in Cairns: City, Earlville and Stratford. There are four types of accounts: Low fee, Business, Savings and Student/Pensioner. The numbers of each type of account at the beginning of a year are shown in the table.

Branch	Low fee	Business	Savings	Student/Pensioner
City	300	400	200	150
Earlville	400	220	300	200
Stratford	250	150	400	150

The numbers of closures of accounts in that year were:

Branch	Low fee	Business	Savings	Student/Pensioner
City	30	70	20	20
Earlville	20	30	40	30
Stratford	40	10	50	10

The numbers of new accounts opened in that year were:

Branch	Low fee	Business	Savings	Student/Pensioner
City	50	100	10	15
Earlville	15	50	30	10
Stratford	30	25	20	40

Use a 3×4 matrix to show:

a the number of accounts at the beginning of the year

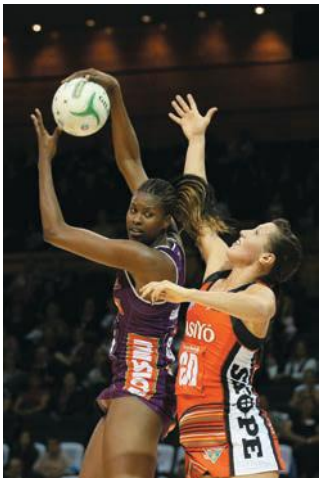
b the number of closures in the year

c the number of accounts opened in the year

d an evaluated matrix expression for the number of accounts at the end of the year.

8 The results of the first 5 rounds, middle 5 rounds and last 5 rounds of a netball competition are shown by the following 3 matrices. Each row represents the results of a team. The first column is the number of games won, the second is drawn games and the third is games lost.

$$\mathbf{F} = \begin{bmatrix} 4 & 0 & 1 \\ 3 & 0 & 2 \\ 1 & 1 & 3 \\ 2 & 1 & 2 \\ 1 & 0 & 4 \\ 3 & 0 & 2 \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 4 \\ 1 & 2 & 2 \\ 3 & 0 & 2 \\ 2 & 1 & 2 \\ 3 & 0 & 2 \end{bmatrix} \quad \mathbf{L} = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & 3 \\ 2 & 0 & 3 \\ 3 & 1 & 1 \\ 1 & 1 & 3 \\ 3 & 0 & 2 \end{bmatrix}$$



Getty Images/Chris Hyde

a Write a matrix expression for the total results.

b Evaluate your expression to obtain the final results.

c Which team (1st to 6th) is the best team?

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9 The wholesale prices (in \$) of some caulking compounds are shown by this matrix. The first column represents the prices of 100 mL tubes, the second the prices of 250 mL tubes and the third the prices of 400 mL mastic gun cartridges. The first row is for *bathroom sealant*, the second for *gutter and downpipe sealant*, the third for *masonry sealant* and the fourth for *flexible gap filler*.

$$\mathbf{W} = \begin{bmatrix} 2.40 & 4.90 & 8.40 \\ 4.20 & 7.90 & 11.80 \\ 3.80 & 8.40 & 11.00 \\ 5.90 & 11.80 & 14.40 \end{bmatrix}$$

a The retailer applies a markup of 70%. Calculate the matrix for the retail prices.

b The GST is 10%. Find the matrix for the prices actually paid by the customer.

10.04 Matrix multiplication

Multiplying matrices

Matrices can be used to show purchases from suppliers in two dimensions. For example, a bulk meat supplier might make up packs of meat as shown below so that caterers, hotels and hostels can order meat in packs to suit their requirements.

Pack	Sausages	Minced beef	Chops	Stewing steak	BBQ steak	Prime steak
BBQ	20	10	15	0	25	0
Basic	15	15	10	20	5	5
Quality	5	20	20	10	0	15
Deluxe	5	15	15	5	0	30

You can write this as the matrix $\mathbf{B} = \begin{bmatrix} 20 & 10 & 15 & 0 & 25 & 0 \\ 15 & 15 & 10 & 20 & 5 & 5 \\ 5 & 20 & 20 & 10 & 0 & 15 \\ 5 & 15 & 15 & 5 & 0 & 30 \end{bmatrix}$.

A caterer specialising in home parties ordered the following packs.

Month	BBQ	Basic	Quality	Deluxe
July	2	3	1	0
August	3	2	2	1
September	4	1	1	3

You can write this as $\mathbf{C} = \begin{bmatrix} 2 & 3 & 1 & 0 \\ 3 & 2 & 2 & 1 \\ 4 & 1 & 1 & 3 \end{bmatrix}$.

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You can combine the information to work out the amount of each meat each month as follows:

For Sausages in July: 2 BBQ × 20 + 3 Basic × 15 + 1 Quality × 5 + 0 Deluxe × 5 kg

$= 2 \times 20 + 3 \times 15 + 1 \times 5 + 0 \times 5 = 90$ kg

For Chops in August: 3 BBQ × 15 + 2 Basic × 10 + 2 Quality × 20 + 1 Deluxe × 15 kg

$= 3 \times 15 + 2 \times 10 + 2 \times 20 + 1 \times 15 = 120$ kg

Completing each month for each kind of meat gives the table below.

Month	Sausages	Minced beef	Chops	Stewing steak	BBQ steak	Prime steak
July	90	85	80	70	65	30
August	105	115	120	65	85	70
September	115	120	135	45	105	110

This table has the same number of *rows* as the caterer's orders and the same number of *columns* as the meats in each pack. There are 4 *columns* in the caterer's orders and 4 *rows* in the supplier's packs.

Every number in the new table is the result of multiplying the numbers in a row of the caterer's order by the numbers in a column of the bulk meat packs.

This is how **matrix multiplication** is defined.

$$\begin{bmatrix} 2 & 3 & 1 & 0 \\ 3 & 2 & 2 & 1 \\ 4 & 1 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 20 & 10 & 15 & 0 & 25 & 0 \\ 15 & 15 & 10 & 20 & 5 & 5 \\ 5 & 20 & 20 & 10 & 0 & 15 \\ 5 & 15 & 15 & 5 & 0 & 30 \end{bmatrix} = \begin{bmatrix} 90 & 85 & 80 & 70 & 65 & 30 \\ 105 & 115 & 120 & 65 & 85 & 70 \\ 115 & 120 & 135 & 45 & 105 & 110 \end{bmatrix}$$

Chops in August was the second row by the third column: answer goes in row 2, column 3.

The first matrix must have the same number of columns as the second matrix has rows.

Matrix multiplication

- Matrices **A** and **B** are **conformable** if **A** has m columns and **B** has m rows and they can be multiplied together to give **AB**
- The **product** of an $n \times m$ matrix **A** and an $m \times p$ matrix **B** is an $n \times p$ matrix **C**
- The elements c_{ij} of **C** are the sums of the products of the ith row of **A** and the jth column of **B**: $c_{ij} = \Sigma a_{ik} \times b_{kj}$.

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EXAMPLE 7

$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 4 & 5 \\ -2 & 2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & 1 & -3 & 4 \\ 9 & 4 & 1 & 0 \end{bmatrix}$. Find $\mathbf{D} = \mathbf{AB}$.

Solution

$d_{11} = 1 \times 2 + 3 \times 9 = 29,$

$d_{12} = 1 \times 1 + 3 \times 4 = 13,$

$d_{13} = 1 \times (-3) + 3 \times 0 = -3,$ etc.

$$\mathbf{D} = \begin{bmatrix} 1 & 3 \\ 4 & 5 \\ -2 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & -3 & 4 \\ 9 & 4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & 13 & 0 & 4 \\ 53 & 24 & -7 & 16 \\ 14 & 6 & 8 & -8 \end{bmatrix}$$

EXAMPLE 8

Matrix multiplication

$\mathbf{B} = \begin{bmatrix} 2 & 1 & -3 & 0 \\ 9 & 4 & 1 & 4 \\ -3 & 5 & 6 & 0 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 3 & 1 \\ 0 & 4 \\ -2 & -4 \\ 3 & 2 \end{bmatrix}$. Find $\mathbf{BC}$.

Solution

$2 \times 3 + 1 \times 0 + (-3) \times (-2) + 0 \times 3 = 12,$

$2 \times 1 + 1 \times 4 + (-3) \times (-4) + 0 \times 2 = 18,$

$9 \times 3 + 4 \times 0 + 1 \times (-2) + 4 \times 3 = 37,$ etc.

$$\mathbf{BC} = \begin{bmatrix} 2 & 1 & -3 & 0 \\ 9 & 4 & 1 & 4 \\ -3 & 5 & 6 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 0 & 4 \\ -2 & -4 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 18 \\ 37 & 29 \\ -27 & -7 \end{bmatrix}$$

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EXAMPLE 9

$$\mathbf{P} = \begin{bmatrix} -1 & -2 \\ 0 & 3 \\ -6 & 5 \end{bmatrix} \text{ and } \mathbf{Q} = \begin{bmatrix} 3 & 1 & 0 & 2 \\ 0 & 4 & 1 & -3 \\ -2 & -4 & -1 & 4 \\ 3 & 2 & 2 & 0 \end{bmatrix}.$$ Explain why neither **PQ** nor **QP** exist.

Solution

Row × column does not work either way around.

PQ does not exist because **P** has 2 columns and **Q** has 4 rows so they are not conformable.

QP does not exist because **Q** has 4 columns and **P** has 3 rows so they are not conformable.

TECHNOLOGY

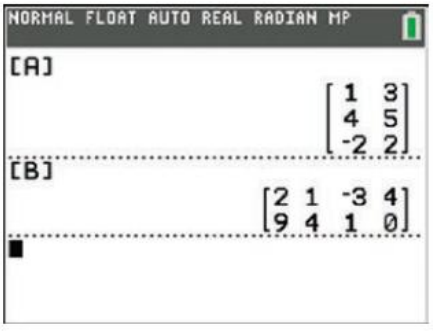
Matrices on a graphics calculator 2

You can do the matrix multiplications from Examples 7 and 8 on a graphics calculator.

TI 84 Plus CE

Put in the matrices $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 4 & 5 \\ -2 & 2 \end{bmatrix}$ and

$\mathbf{B} = \begin{bmatrix} 2 & 1 & -3 & 4 \\ 9 & 4 & 1 & 0 \end{bmatrix}$ in the way shown on page 429. You can check whether they are correct by using the **NAMES** part of the **matrix** menu.



Now press [2nd][x⁻¹] and [enter] to get **A**.

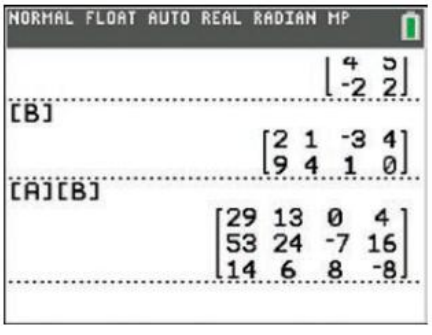
You can put [×] if you want to.

Press [2nd][x⁻¹], go down one and press [enter] to get **B**.

Press [enter] to get the product.

Enter the matrices and do the product from Example 8 to check your understanding.

Your teacher may have other examples for you to practise on.



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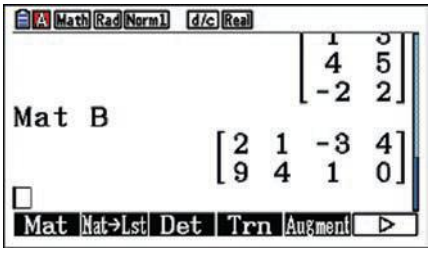
Put in the matrices $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 4 & 5 \\ -2 & 2 \end{bmatrix}$ and

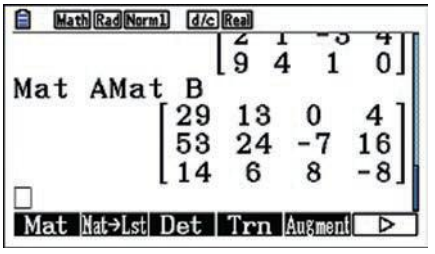
$\mathbf{B} = \begin{bmatrix} 2 & 1 & -3 & 4 \\ 9 & 4 & 1 & 0 \end{bmatrix}$ in the way shown on page 429. You can check whether they are correct by using [OPTN] [F2] [ALPHA] A to get Mat A and press [EXE] to show the matrix. You can do Mat B similarly.



Now press [OPTN] [F2] [ALPHA] A to get Mat A.

You can put [×] if you want to.

Now do Mat B and press [EXE] to get the product.



Enter the matrices and do the product from Example 8 to check your understanding.

Your teacher may have other examples for you to practise on.

There are many online matrix calculators available. Your teacher may have put one on the school intranet for you to use instead of graphics calculators.

INVESTIGATION

MATRIX MULTIPLICATION ORDER

- For $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -1 & 2 & 1 \\ 3 & -3 & 4 \end{bmatrix}$, calculate the product **AB**.
- Can you calculate the product **BA**?
- Now for $\mathbf{D} = \begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix}$ and $\mathbf{E} = \begin{bmatrix} 0 & 3 \\ -2 & 2 \end{bmatrix}$, calculate the product **DE**.
- Now calculate the product **ED**.
- What can you say about the order of a matrix product?
- Can you find two matrices that have products that are 2×2 in one order and 3×3 in the reverse order?

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Exercise 10.04 Matrix multiplication

Your teacher might want you to do some of this exercise using technology.

Example 7,8

1 Calculate each product.

a $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & -3 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$ **c** $\begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}\begin{bmatrix} 3 & -1 & 2 \end{bmatrix}$

d $\begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & -3 \end{bmatrix}$ **e** $\begin{bmatrix} 5 & 1 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ -2 & 4 \end{bmatrix}$ **f** $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$

g $\begin{bmatrix} 3 & 4 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & -3 \end{bmatrix}$ **h** $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ **i** $\begin{bmatrix} 5 & -1 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$

j $\begin{bmatrix} \frac{1}{2} & 3 \\ 2 & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} -2 & 4 \\ 0 & 1 \end{bmatrix}$ **k** $\begin{bmatrix} \frac{1}{3} & 1 \\ 1 & -\frac{2}{3} \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 1 & 3 \end{bmatrix}$ **l** $\begin{bmatrix} 6 & 12 \\ 6 & 18 \end{bmatrix}\begin{bmatrix} \frac{1}{2} & -\frac{1}{3} \\ -\frac{1}{6} & \frac{1}{6} \end{bmatrix}$

2 Calculate each product.

a $\begin{bmatrix} 3 & 4 \\ -2 & 0 \\ 4 & -3 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 & -2 \\ -3 & 4 & 1 & 3 \end{bmatrix}$ **b** $\begin{bmatrix} 3 & 4 & -2 \\ -2 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & 0 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & -4 \\ 0 & 4 & 1 \end{bmatrix}$

3 Calculate $\begin{bmatrix} 3 & 6 \\ 10 & 5 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 3 & 6 \\ 10 & 5 \end{bmatrix}$.

What is unusual about these products?

4 Calculate $\begin{bmatrix} 3 & 1 & 3 \\ 1 & 11 & 1 \\ 3 & 1 & 3 \end{bmatrix}\begin{bmatrix} -2 & 1 & -2 \\ 1 & 1 & 1 \\ -2 & 1 & -2 \end{bmatrix}$ and $\begin{bmatrix} -2 & 1 & -2 \\ 1 & 1 & 1 \\ -2 & 1 & -2 \end{bmatrix}\begin{bmatrix} 3 & 1 & 3 \\ 1 & 11 & 1 \\ 3 & 1 & 3 \end{bmatrix}$.

What is unusual about these products?

5 Calculate $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$. Is this what you expected?

Example 9

6 Explain why the product $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & -1 \end{bmatrix}\begin{bmatrix} 2 & -2 & 0 \\ -3 & 1 & 3 \end{bmatrix}$ does not exist. Does the order make any difference?

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Problem solving

7 Multiply $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ by some other 2×2 matrices. What do you find?

8 Multiply $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ by some 2×4 matrices. What do you find?

9 Multiply some 2×2 matrices by $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. What do you find?

10 Multiply some 3×2 matrices by $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. What do you find?

11 Find pairs of non-zero 2×2 matrices whose products are zero matrices.

12 A kitchenware chain has five stores ordering stock from a central warehouse. The orders of cutlery sets for a catalogue special are as follows.

Setting	Brisbane	Logan	Chermside	Cairns	Gold Coast
6 place	12	15	10	12	16
8 place	10	8	10	12	6
10 place	8	4	10	4	12

a Write the orders as a 5×3 matrix.

b The prices of the sets are \$117.80, \$155 and \$190 respectively. Write the prices as a column matrix.

c Multiply the matrices to obtain the invoice amounts for each store.

13 An international airline packages tours to sell through its own and other agents. Tours are priced according to the principal destination, number of nights accommodation and number of stopovers. Agents add their own commission onto the total package prices, but the airline's agents work on a commission of 35%. The packages most commonly sold are:

- British Explorer—includes 2 stopovers and 14 days accommodation
- European Wanderer—includes 4 stopovers and 21 days accommodation
- East Coast US Explorer—includes 2 stopovers and 14 days accommodation
- West Coast US Explorer—includes 3 stopovers and 14 days accommodation
- East Coast US Wanderer—includes 5 stopovers and 28 days accommodation
- West Coast US Wanderer—includes 4 stopovers and 21 days accommodation

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Principal destination	Accommodation	Stopovers
London $3000	$240/day	Singapore, New Delhi, Rome, Frankfurt, Athens, Paris $400 each
Los Angeles $2800	$220/day	Honolulu, San Francisco, Las Vegas, Seattle, Vancouver $440 each
New York $3200	$210/day	Honolulu, Los Angeles, Chicago, St Louis, Kansas City, Toronto, Quebec $500 each

a Calculate the cost of each tour and write it as a column matrix.

b Use a scalar multiple to find the commission for each tour charged by the airline's agents.

c Use matrix addition or another scalar multiple to find the cost to the customer of each tour.

Three agents forward the following bookings to the airline.

Agent	BE	EW	ECE	WCE	ECW	WCW
A	2	1	0	3	2	1
B	4	2	2	4	1	2
C	3	1	5	3	3	2

d Write the bookings as a 3×6 matrix.

e Use matrix multiplication to find the amounts that each agent will forward to the airline.

f Use matrix multiplication to find the amount of commission that each agent will receive.

14 The results of the first 5 rounds, middle 5 rounds and last 5 rounds of a netball competition are shown by the following 3 matrices. Each row represents the results of a team. The first column is the number of games won, the second is drawn games and the third is games lost.

$$\mathbf{F} = \begin{bmatrix} 4 & 0 & 1 \\ 3 & 0 & 2 \\ 1 & 1 & 3 \\ 2 & 1 & 2 \\ 1 & 0 & 4 \\ 3 & 0 & 2 \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 4 \\ 1 & 2 & 2 \\ 3 & 0 & 2 \\ 2 & 1 & 2 \\ 3 & 0 & 2 \end{bmatrix} \quad \mathbf{L} = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & 3 \\ 2 & 0 & 3 \\ 3 & 1 & 1 \\ 1 & 1 & 3 \\ 3 & 0 & 2 \end{bmatrix}$$

Team scores are calculated by allotting 3 points for a win, 1 for a draw and 0 for a loss.

a Write the points as a column matrix **P**.

b Use matrix multiplication to find the team points for each group of five rounds.

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10.05 Powers of matrices

5^3 means $5 \times 5 \times 5$. You can do this with any number. You can do repeated multiplication for only some matrices.

Powers of matrices

- For the square matrix **A**, $\mathbf{A}^n$ for a positive integer is called the nth **power** of **A**
- $\mathbf{A}^2$ and $\mathbf{A}^3$ are called the square and cube of **A** respectively
- Only square matrices can have powers

EXAMPLE 10

Given $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$, find:

a $\mathbf{A}^2$ **b** $\mathbf{A}^3$

Solution

a Write as a multiplication. $\mathbf{A}^2 = \mathbf{A} \times \mathbf{A}$

Substitute **A**. $= \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$

Do the multiplication. $= \begin{bmatrix} 1 & -7 \\ 21 & 22 \end{bmatrix}$

b Write as a multiplication. $\mathbf{A}^3 = \mathbf{A} \times \mathbf{A} \times \mathbf{A}$

Substitute **A** and $\mathbf{A} \times \mathbf{A}$. $= \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & -7 \\ 21 & 22 \end{bmatrix}$

Do the multiplication. $= \begin{bmatrix} -19 & -36 \\ 108 & 89 \end{bmatrix}$

Powers of 3×3 matrices are done the same way, but there is more work involved.

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EXAMPLE 11

For $\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix}$, find $\mathbf{A}^2$.

Solution

Write as a multiplication. $\mathbf{A}^2 = \mathbf{A} \times \mathbf{A}$

Substitute **A**. $= \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{bmatrix}$

Do the multiplication. $= \begin{bmatrix} 1 & -4 & -6 \\ 6 & 1 & 6 \\ 4 & -5 & 1 \end{bmatrix}$

INVESTIGATION

POWERS THAT GIVE ZERO MATRICES

The matrix $\begin{bmatrix} -4 & 2 \\ -8 & 4 \end{bmatrix}$ to any power will always give the 2×2 zero matrix.

Is it possible to have a power of a non-zero matrix that gives the 3×3 zero matrix?

Can you find at least one example?

What about 4×4 matrices?

Why is it not possible to have a power of a matrix that gives the 2×3 zero matrix?

Exercise 10.05 Powers of matrices

Example 10

1 Find the square of each matrix.

a $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ **b** $\begin{bmatrix} 3 & 1 \\ 3 & 1 \end{bmatrix}$ **c** $\begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$

d $\begin{bmatrix} -4 & 3 \\ 2 & -2 \end{bmatrix}$ **e** $\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$

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2 Find the cube of each matrix.

a $\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & 4 \\ -3 & -1 \end{bmatrix}$ **c** $\begin{bmatrix} -3 & 6 \\ 1 & 4 \end{bmatrix}$

d $\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$ **e** $\begin{bmatrix} 1 & 2 \\ -6 & -6 \end{bmatrix}$

3 Find the fourth power of each matrix.

a $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **c** $\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$

d $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}$ **e** $\begin{bmatrix} 6 & -7 \\ -5 & 2 \end{bmatrix}$

4 For $\mathbf{A} = \begin{bmatrix} 12 & -9 \\ 16 & -12 \end{bmatrix}$, find:

a $\mathbf{A}^2$ **b** $\mathbf{A}^3$ **c** $\mathbf{A}^4$ **d** $\mathbf{A}^5$

e Comment on your answers.

5 Find the square of each matrix.

Example 11

a $\begin{bmatrix} 3 & 0 & -3 \\ 5 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix}$ **b** $\begin{bmatrix} 9 & -3 & 0 \\ 6 & 6 & 0 \\ 3 & -9 & 0 \end{bmatrix}$ **c** $\begin{bmatrix} 3 & -2 & 1 \\ 3 & -2 & 1 \\ 3 & -2 & 1 \end{bmatrix}$ **d** $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

6 Find the cube of each matrix.

a $\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **c** $\begin{bmatrix} 0 & 3 & 1 \\ 2 & 0 & 1 \\ 3 & 2 & 0 \end{bmatrix}$ **d** $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

7 Find the fourth power of each matrix.

a $\begin{bmatrix} 3 & -4 & 0 \\ 2 & -2 & 6 \\ 5 & 0 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \end{bmatrix}$

c $\begin{bmatrix} 2 & -3 & 0 \\ 0 & 2 & 3 \\ -2 & -1 & 4 \end{bmatrix}$ **d** $\begin{bmatrix} 6 & -4 & -2 \\ 0 & 3 & 0 \\ -4 & 0 & 1 \end{bmatrix}$

Problem solving

8 Find a simple rule for writing non-zero matrices whose powers are the 2×2 zero matrix.

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9 Find the square, cube and fourth powers of each matrix.

a $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **c** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d Comment on your results.

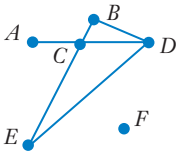
10 a Find the square, cube and fourth powers of the matrix $\mathbf{M} = \begin{bmatrix} 9 & -4 \\ 18 & -8 \end{bmatrix}$.

b Comment on your results.

10.06 Applications of matrices

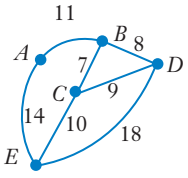
A **network** shows connections between points. You can represent it by a matrix–with 1s showing connections and 0 where there are no connections. Alternatively, you can show the distances between connections instead. Two different kinds of networks shown below are represented by the matrices on their right.

Connection network



$$\mathbf{N} = \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

'Road network'



$$\mathbf{R} = \begin{bmatrix} 0 & 11 & 0 & 0 & 14 \\ 11 & 0 & 7 & 8 & 0 \\ 0 & 7 & 0 & 9 & 10 \\ 0 & 8 & 9 & 0 & 18 \\ 14 & 0 & 10 & 18 & 0 \end{bmatrix}$$

These are called **adjacency matrices**. They show the points that are next to each other (adjacent) in the network.

A **communications network** shows communication links.

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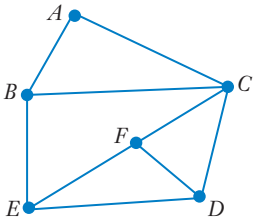
The network below shows the official communications in an office between the manager, Alice, the deputy managers Callum and Bella and the sales reps Derek, Ellie and Felizia. It is represented by the adjacency matrix on its right.



$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

The square of a communications network shows the number of ways that each person can communicate to another via a third person.

EXAMPLE 12

Find the number of ways that Derek can communicate with each of Bella and Ellie via a third person in the office communications network shown above.

Solution

Square the adjacency matrix.

$$\mathbf{V} = \mathbf{C}^2 = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 2 & 0 & 2 \\ 1 & 1 & 4 & 1 & 3 & 1 \\ 1 & 2 & 1 & 3 & 1 & 2 \\ 1 & 0 & 3 & 1 & 3 & 1 \\ 1 & 2 & 1 & 2 & 1 & 3 \end{bmatrix}$$

Find the values of v_{42} and v_{45}. $v_{42} = 2$ and $v_{45} = 1$

Write the answer. Derek can communicate with Bella via a third person in 2 ways, and with Ellie in only one way.

You can use a matrix to code a message to numbers. You can use a second matrix to decode it. Complex methods of doing this are used in cryptography.

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Matrix coding

- Make a coding 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $ad - bc \neq 0$
- Change the letters of the message to numbers, with space = 0, A = 1, B = 2, …, Z = 26.
- Write the numbers in a matrix with 2 rows. If there are an odd number of numbers, just put 0 in the last place.
- Multiply the coding matrix by the message matrix
- Write the numbers as a list
- To decode the numbers, rewrite the list as a matrix of numbers
- Multiply the decoding matrix $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ by the coded message matrix and divide by $(ad - bc)$
- Change the numbers back to letters

The decoding number $(ad - bc)$ is called the **determinant** of the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

EXAMPLE 13

a Use the matrix $\mathbf{C} = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ to code the message 'CODING IS EASY'.

b Write down the decoding matrix **D** and the determinant $e = ad - bc$.

c Decode the message.

Solution

a Change the message to numbers.

CODING IS EASY → 3, 15, 4, 9, 14, 7, 0, 9, 19, 0, 5, 1, 19, 25

Write as a matrix with pairs of numbers in columns.

$$\mathbf{M} = \begin{bmatrix} 3 & 4 & 14 & 0 & 19 & 5 & 19 \\ 15 & 9 & 7 & 9 & 0 & 1 & 25 \end{bmatrix}$$

Multiply $\mathbf{C} \times \mathbf{M}$.

$$\mathbf{CM} = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 3 & 4 & 14 & 0 & 19 & 5 & 19 \\ 15 & 9 & 7 & 9 & 0 & 1 & 25 \end{bmatrix}$$

$$= \begin{bmatrix} 84 & 57 & 77 & 45 & 57 & 20 & 182 \\ 66 & 44 & 56 & 36 & 38 & 14 & 138 \end{bmatrix}$$

Write the coded message.

84, 66, 57, 44, 77, 56, 45, 36, 57, 38, 20, 14, 182, 138

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b Work out **D** (the decoding matrix) and $|\mathbf{C}|$ (the determinant).

$$\mathbf{D} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$$

$$|\mathbf{C}| = ad - bc = 3 \times 4 - 5 \times 2 = 2$$

c Write the coded message as a matrix.

$$\begin{bmatrix} 84 & 57 & 77 & 45 & 57 & 20 & 182 \\ 66 & 44 & 56 & 36 & 38 & 14 & 138 \end{bmatrix}$$

Multiply by **D**.

$$\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 84 & 57 & 77 & 45 & 57 & 20 & 182 \\ 66 & 44 & 56 & 36 & 38 & 14 & 138 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 8 & 28 & 0 & 38 & 10 & 38 \\ 30 & 18 & 14 & 18 & 0 & 2 & 50 \end{bmatrix}$$

Divide by $|\mathbf{C}|$.

$$\frac{1}{2}\begin{bmatrix} 6 & 8 & 28 & 0 & 38 & 10 & 38 \\ 30 & 18 & 14 & 18 & 0 & 2 & 50 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & 14 & 0 & 19 & 5 & 19 \\ 15 & 9 & 7 & 9 & 0 & 1 & 25 \end{bmatrix}$$

Change back to a list. 3, 15, 4, 9, 14, 7, 0, 9, 19, 0, 5, 1, 19, 25

Change to letters. CODING IS EASY

You can make the decoding easier if you choose a matrix so that $ad - bc = 1$.

INVESTIGATION

CODING WITH MATRICES

- Investigate more complicated methods of coding with matrices.
- Find out about the Hill cypher method.
- Why are large prime numbers important in modern cryptography?
- What is modern cryptography used for in commercial applications?

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Transition matrices

If it rains today, it is more likely to rain tomorrow than if it is fine or cloudy today.

If it is cloudy today, it is more likely to be cloudy tomorrow than if it is rainy or fine today.

If it is fine today, it is more likely to be fine tomorrow than if it is rainy or cloudy today.

You can make a 3×3 matrix of probabilities in the order fine, cloudy, rain.

$$\mathbf{P} = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.3 \\ 0.3 & 0.35 & 0.35 \end{bmatrix}$$ stands for the following.

Fine today, tomorrow's probabilities are: 0.5 fine, 0.3 cloudy, 0.2 rain.

Cloudy today, tomorrow's probabilities are: 0.4 fine, 0.3 cloudy, 0.3 rain.

Rain today, tomorrow's probabilities are: 0.3 fine, 0.35 cloudy, 0.35 rain.

$\mathbf{P}^2$ gives the probabilities for 2 days time, $\mathbf{P}^3$ for 3 days time, and so on.

You can write today being cloudy as $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$.

If today is cloudy, then the probabilities for 2 days time are $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\mathbf{P}^2$.

EXAMPLE 14

Use the weather matrix above to find the probability that it will be fine in 2 days time if it is cloudy today.

Solution

Work out $\mathbf{P}^2$ for two days.

$$\mathbf{P}^2 = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.3 \\ 0.3 & 0.35 & 0.35 \end{bmatrix}\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.3 \\ 0.3 & 0.35 & 0.35 \end{bmatrix}$$

$$= \begin{bmatrix} 0.43 & 0.31 & 0.26 \\ 0.41 & 0.315 & 0.275 \\ 0.395 & 0.3175 & 0.2875 \end{bmatrix}$$

Now work out the result for today being cloudy.

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0.43 & 0.31 & 0.26 \\ 0.41 & 0.315 & 0.275 \\ 0.395 & 0.3175 & 0.2875 \end{bmatrix}$$

$$= \begin{bmatrix} 0.41 & 0.315 & 0.275 \end{bmatrix}$$

Write the answer. There is a 41% chance that it will be fine in 2 days time.

Matrices like the one above are called **transition matrices**.

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Exercise 10.06 Applications of matrices

1 The communications matrix for Pranav, Quentin, Robert, Sally, Toni, Ursula and Vila is given by $\mathbf{C} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$. Example 12

Find the number of ways that:

a Quentin can communicate with Robert via a third person

b Pranav can communicate with Ursula via a third person

c Vila can communicate with Sally via a third person.

2 Use the matrix $\begin{bmatrix} 2 & 4 \\ 4 & 7 \end{bmatrix}$ to code the message 'THE SUN HAS GOT HIS HAT ON'. Example 13

3 The matrix $\begin{bmatrix} 2 & 4 \\ 4 & 7 \end{bmatrix}$ was used to code a message and it gave the code 58, 102, 8, 16, 36, 67, 38, 76, 66, 117, 62, 115, 56, 105, 60, 105, 122, 224, 80, 140, 46, 88, 102, 179. What was the message?

4 The probabilities of a taxi ending up in North, South or West Townsville if it starts a trip in North Townsville are 0.2, 0.5 and 0.3. If it starts in South Townsville, they are 0.3, 0.3 and 0.4. If it starts in West Townsville, they are 0.4, 0.5 and 0.1. Example 14

a Write a transition matrix for the taxi.

b It starts in South Townsville. What is the probability that it will end up in South Townsville after 2 trips?

c What is the probability of ending up in West Townsville after 3 trips if it starts there?

5 The shares of a bank can rise, remain steady or fall each day. Their probabilities of changing are given by:

Rise today, tomorrow's probabilities are:	rise 0.2, steady 0.2, fall 0.6
Steady today, tomorrow's probabilities are:	rise 0.5, steady 0.3, fall 0.2
Fall today, tomorrow's probabilities are:	rise 0.7, steady 0.1, fall 0.2

a Write a transition matrix for the shares.

b What is the probability of a fall in 2 days time if they fall today?

c What is the probability of a rise in 3 days time if they rise today?

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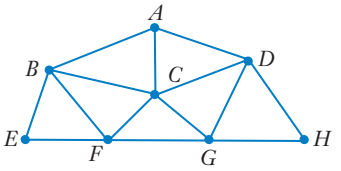
Problem solving

6 The communication network between people in a factory is shown.



a Write the adjacency matrix for the network.

b Find the number of ways that B can communicate with G via a third person.

7 The communication matrix between towns in a telephone network is

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$, where the towns are in the order A, B, C, D, E, F.

a Draw a network showing the telephone lines between the towns.

b In how many ways can town D be connected to town F via a third town?

8 The matrix $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 3 & 2 & 0 \end{bmatrix}$ is used to code the message 'THE BEST IN THE WORLD'.

What is the coded message?

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10. CHAPTER SUMMARY

Matrices

Matrices

- A **matrix** (plural **matrices**) is a table of numbers in parentheses ( ) or brackets [ ].
- The **size** (or **dimension** or **order**) of a matrix is written as the number of rows × the number of columns.

  For example, $\mathbf{A} = \begin{bmatrix} 2.4 & 0 & 6 & \frac{1}{4} \\ 0 & -5.3 & 4 & -5 \end{bmatrix}$ is a 2×4 ('2 by 4') matrix.
- The **elements** of a matrix are identified by a small letter with the row and column as a subscript, for example, for matrix **A** above, $a_{24} = -5$ and $a_{13} = 6$.

Type of matrix	Example
A **row matrix** has only one row.	$\begin{bmatrix} 2 & 4 & 5 & 0 \end{bmatrix}$
A **column matrix** has only one column.	$\begin{bmatrix} 5 \\ -2 \\ 0.4 \end{bmatrix}$
A **square matrix** has the same number of rows and columns. The size of a square matrix is given as its order.	$\begin{bmatrix} 1 & -4 \\ 6 & 2 \end{bmatrix}$ This matrix has order 2.
A **zero matrix** **0** has all elements equal to 0.	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
An **identity matrix** **I** is a square matrix with every element 0 except for the diagonal values, which are all 1s.	$\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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Operations with matrices

- Matrices must be the same size to be added or subtracted
- To **add matrices**, add the corresponding elements
- To **subtract matrices**, subtract the corresponding elements
- To multiply a matrix by a number, multiply every element by the number. This is called **multiplication by a scalar** or a **scalar multiple**
- In formal terms, $\mathbf{A} + \mathbf{B} = (a_{ij} + b_{ij})$, $\mathbf{A} - \mathbf{B} = (a_{ij} - b_{ij})$ and $c\mathbf{A} = (ca_{ij})$ for a number c
- Two matrices are **equal** if and only if they are the same size and corresponding elements are all the same
- Matrices **A** and **B** are **conformable** if **A** has m columns and **B** has m rows and they can be multiplied together to give **AB**
- The **product** of an $n \times m$ matrix **A** and an $m \times p$ matrix **B** is an $n \times p$ matrix **C**
- The elements c_{ij} of **C** are the sums of the products of the ith row of **A** and the jth column of **B**: $c_{ij} = \Sigma a_{ik} \times b_{kj}$
- For the square matrix **A**, $\mathbf{A}^n$ for a positive integer is called the nth **power** of **A**
- $\mathbf{A}^2$ and $\mathbf{A}^3$ are called the square and cube of **A** respectively
- Only square matrices can have powers

Matrix applications

- A **network** shows connections between points
- An **adjacency matrix** shows a network with 1s for connections and 0s for no connection. They can have numbers to represent distances instead of 1s.
- A **communications network** shows communication links. It can be shown by an adjacency matrix. The square of a communications network shows the number of ways that each person can communicate to another via a third person
- A **transition matrix** shows the probabilities of changing from one condition to another

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Matrix coding

- Make a coding 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $ad - bc \neq 0$
- Change the letters of the message to numbers, and write the numbers in a matrix with 2 rows.
- Multiply the coding matrix by the message matrix
- Multiply the decoding matrix $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ by the coded message matrix and divide by $(ad - bc)$
- Change the numbers back to letters

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10. CHAPTER REVIEW

Matrices

Example 1

1 Which of the following are matrices?

a $\begin{bmatrix} 6 & 5 \\ & 3 \end{bmatrix}$ **b** $\begin{bmatrix} 4 & 5 & 3 \\ 7 & 1 & 2 \\ 0 & -3 & 3 \end{bmatrix}$ **c** $\begin{bmatrix} 1 & 4 & \\ 3 & 2 & 5 \end{bmatrix}$

Example 2

2 $\mathbf{A} = \begin{bmatrix} 2 & 3 & 10 & 7 \\ -2 & 0 & 6 & -6 \\ 1 & -5 & 5 & 4 \end{bmatrix}$. If possible, write down the value of:

a a_{21} **b** a_{43} **c** a_{34} **d** a_{23}

Example 3

3 Describe the type and size of each matrix.

a $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 3 & -2 & 4 & 0 \end{bmatrix}$ **c** $\begin{bmatrix} 2 \\ -2 \end{bmatrix}$

Example 4

4 Construct the 2×3 zero matrix.

Examples 5,6

5 Given $\mathbf{A} = \begin{bmatrix} 1 & 3 & -2 & 5 \\ 0 & 4 & -4 & 1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 5 & -3 & 6 & -9 \\ 3 & 0 & -4 & 4 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 1 & -3 & 0 & -2 \\ -4 & 5 & 5 & 0 \end{bmatrix}$, find:

a $\mathbf{A} + \mathbf{B}$ **b** $\mathbf{C} + \mathbf{B}$ **c** $-\mathbf{B} + \mathbf{A}$
d $-\mathbf{A} + \mathbf{C}$ **e** $\mathbf{C} - \mathbf{B}$ **f** $4\mathbf{A}$
g $-3\mathbf{B}$ **h** $-\mathbf{C}$ **i** $-0.5\mathbf{C}$
j $4\mathbf{B} - \mathbf{A} + 3\mathbf{C}$

Example 7

6 Which of the following phrases best describes **conformable matrices**?

A matrices with the same number of rows and columns
B matrices that have all zeros
C $n \times p$ and $m \times p$ matrices
D $n \times p$ and $p \times m$ matrices
E matrices with 1s in the diagonal

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7 Find each product if possible. If it is not possible, explain why.

Examples 7-9

a $\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 1 & -2 & -3 \\ 4 & 3 & -2 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & -2 & -3 \\ 4 & 3 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}$

c $\begin{bmatrix} 2 & -3 & 5 \\ -1 & 2 & 4 \\ 3 & 4 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$

8 Find each power, if possible. If not possible, explain why.

Examples 10,11

a $\begin{bmatrix} 2 & -2 & 3 \\ 0 & 1 & -3 \end{bmatrix}^3$ **b** $\begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix}^3$ **c** $\begin{bmatrix} 2 & 2 & 1 \\ -1 & 0 & 1 \\ 3 & 4 & 2 \end{bmatrix}^2$

9 The matrix of a communication network is given by

Example 12

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \text{ for A-G in order.}$$

In how many ways can B communicate with E via a third person?

10 Use the matrix $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ to encode the message 'MATHS IS EASY'.

Example 13

11 A transition matrix is given by $\begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.5 & 0.2 & 0.3 \\ 0.4 & 0.4 & 0.2 \end{bmatrix}$ for **P**, **Q** and **R**.

Example 14

a What is the probability of transition from **Q** to **R** in 2 steps?

b Starting from **Q**, what is the probability of **Q** after 3 steps?

Problem solving

12 Given $3\begin{bmatrix} x & 4 \\ 1 & -2 \end{bmatrix} + 2\begin{bmatrix} 6 & -3 \\ 2 & y \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 7 & -5 \end{bmatrix}$, find x and y.

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13 An ecologist investigating in a rainforest is studying caterpillars, adult butterflies and moths, sap-sucking bugs, insect-eating beetles, wasps and their larvae, spiders and insectivorous birds. This table shows the average daily diets of the non-herbivorous animals.

Animal	Daily diet
Beetle	1 caterpillar/butterfly/moth and 5 bugs
Wasp	1 caterpillar/butterfly/moth, 1 beetle and 3 bugs
Spider	1 caterpillar/butterfly/moth, 2 beetles and 1 wasp
Bird	3 caterpillars/butterflies/moths, 5 beetles, 1 wasp and 3 spiders

a Place the information in a 6×6 matrix **A** with each row showing the diet of each animal. Use the order caterpillar/butterfly/moth, bug, beetle, wasp, spider and bird.

b What is the value of a_{34}?

c What is the significance of the column of 0s?

d In one treetop area, it is estimated that there are 25 000 caterpillars/butterflies/moths; 70 000 bugs, 4000 beetles, 500 wasps, 100 spiders and 6 birds. Write these numbers as a row matrix.

e In another treetop area, there are three times as many animals. Use scalar multiplication to find the numbers in the second area.

f Use matrix multiplication to find the number of each animal eaten in one day in each area.

14 The costs of trucking small parcels between Brisbane, Mt Isa, Rockhampton, Sydney and Toowoomba are shown by the following matrix, where the rows give the costs from each place in alphabetical order:

$$\begin{bmatrix} 0 & 10.80 & 9.80 & 6.40 & 5.00 \\ 10.80 & 0 & 7.20 & 16.40 & 15.40 \\ 9.80 & 7.20 & 0 & 16.20 & 14.40 \\ 6.40 & 16.40 & 16.20 & 0 & 11.00 \\ 5.00 & 15.40 & 14.40 & 11.00 & 0 \end{bmatrix}$$

a Why does the diagonal have 0s?

b The handling costs for parcel delivery are a flat \$4. Write a matrix **H** to give the handling costs and calculate matrix **T** to give the total delivery costs for small parcels.

c Medium-sized parcels cost $1\frac{1}{2}$ times as much for trucking but the handling costs are the same. Write a matrix expression for the delivery costs for medium-sized parcels.

d Large parcels cost twice as much for handling and three times as much for trucking as small parcels. Write a matrix expression for the delivery costs of large parcels.

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15 A smallgoods manufacturer produces three types of salami—Italian, Hungarian and Austrian—in mild and hot versions. The prices of each type are shown in the table.

Type of salami	Italian	Hungarian	Austrian
Mild	\$12.50	\$15.00	\$17.50
Hot	\$13.20	\$16.40	\$18.00

The orders from three delicatessens are as follows:

George's: 3 MI, 1 HI, 2 MH, 3 HH, 1 MA, 0 HA

Frank's: 2 MI, 2 HI, 3 MH, 1 HH, 3 MA, 2 MA

Maria's: 4 MI, 5 HI, 0 MH, 1 HH, 0 MA, 2 HA

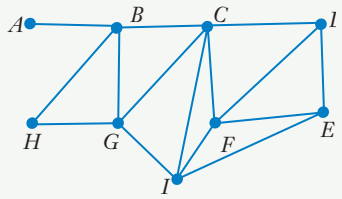
a Write the prices as a 1×6 matrix.

b Write the orders as a 6×3 matrix.

c Use matrix multiplication to find the price of each deli's order.

16 Construct a 3×4 matrix **H** with elements $h_{ij} = ij + 3$

17 A communications network is shown.



a Construct the adjacency matrix for the network.

b In how many ways can C communicate with E via a third person?

Practice quiz

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APPLICATIONS OF TRIGONOMETRY

11.

TRIGONOMETRY OF GENERAL TRIANGLES

The word 'trigonometry' is Greek for 'triangle measuring'. Trigonometry started with the astronomy of ancient Greeks and the buildings of ancient Egyptians. Early sea explorers used trigonometry to navigate their way. Nowadays trigonometry is used in many practical applications including building, surveying, navigation and engineering.

Your previous experience with trigonometry was restricted to right-angled triangles. In this chapter, you will use trigonometry for non-right-angled triangles.

11.01 The sine rule
11.02 The cosine rule
11.03 Applications of the sine and cosine rules
11.04 Solving two-dimensional problems
11.05 Solving three-dimensional problems
Chapter summary
Chapter review

SYLLABUS SUBJECT MATTER

Applications of trigonometry

- solve two-dimensional problems involving non-right-angled triangles using the sine rule (ambiguous case excluded) and the cosine rule
- solve two-dimensional practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression and the use of true bearings

Prior learning

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TERMINOLOGY

bearing
cosine rule
included angle
non-included angle
non-right-angled triangle
right-angled triangle
sine rule

Trigonometric calculations

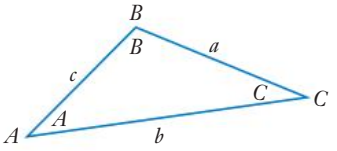
The sine rule - Finding lengths of sides

The sine rule - Finding angles

The sine rule

11.01 The sine rule

The sides in a triangle are named by the lower-case letter of the opposite angle.



$\angle A$ is at vertex A, which is opposite side a.

$\angle B$ is at vertex B, which is opposite side b.

$\angle C$ is at vertex C, which is opposite side c.

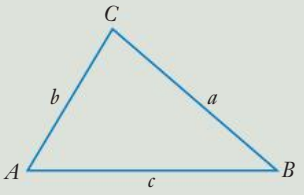
You calculated sides and angles for right-angled triangles in Chapter 8. For non-right-angled triangles, you can use the **sine rule**.

Sine rule

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

or

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$



You saw in Chapter 8 that the area of any $\triangle ABC$ is given by:

$$\frac{1}{2}ab\sin(C) = \frac{1}{2}bc\sin(A) = \frac{1}{2}ca\sin(B)$$

Multiplying by 2 gives: $ab\sin(C) = bc\sin(A) = ca\sin(B)$

Dividing by abc: $\dfrac{\sin(C)}{c} = \dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b}$

Inverting and rearranging gives: $\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$

You can use the sine rule when you know either of the following:

- two angles and one side
- two sides and an opposite angle.

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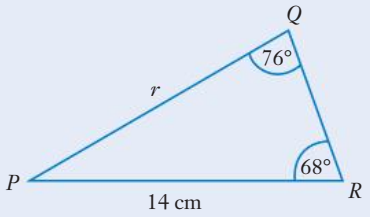
EXAMPLE 1

Calculate the value of r in the triangle shown.



Solution

Write the sine rule for r, R, q and Q.

$$\frac{r}{\sin(R)} = \frac{q}{\sin(Q)}$$

Substitute.

$$\frac{r}{\sin(68°)} = \frac{14}{\sin(76°)}$$

Multiply both sides by sin (68°).

$$\frac{r}{\sin(68°)} \times \sin(68°) = \frac{14}{\sin(76°)} \times \sin(68°)$$

Simplify and evaluate.

$$r = \frac{14\sin(68°)}{\sin(76°)}$$

$$= 13.3779\ldots$$

Make sure your calculator is in degree mode.

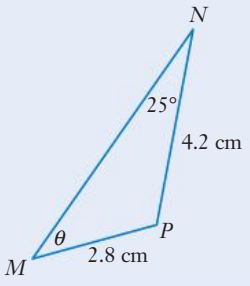
Round off and state the result.

r is about 13.4 cm long.

Use the second form of the sine rule to find angles.

EXAMPLE 2

Given $\triangle MPN$ as shown on the right, calculate the value of θ correct to the nearest degree.



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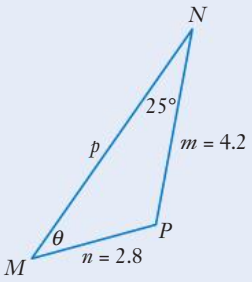
Solution

Label the sides of the triangle.



Write the sine rule using the unknown and known sides.

$$\frac{\sin(M)}{m} = \frac{\sin(N)}{n}$$

Substitute.

$$\frac{\sin(\theta)}{4.2} = \frac{\sin(25°)}{2.8}$$

Multiply both sides by 4.2.

$$\frac{\sin(\theta)}{4.2} \times 4.2 = \frac{\sin(25°)}{2.8} \times 4.2$$

Simplify.

$$\sin(\theta) = \frac{4.2\sin(25°)}{2.8}$$

Use the inverse sine function ($\sin^{-1}$) on your calculator.

$$\theta = \sin^{-1}\left[\frac{4.2\sin(25°)}{2.8}\right]$$

Evaluate.

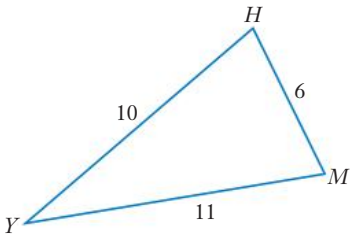
$$= 39.3404\ldots$$

State the result.

$$\theta \approx 39°$$

Exercise 11.01 The sine rule

1 Which rule is true for $\triangle HMY$?



A $\dfrac{11}{\sin(Y)} = \dfrac{10}{\sin(M)}$

B $\dfrac{6}{\sin(Y)} = \dfrac{10}{\sin(M)}$

C $\dfrac{6}{\sin(Y)} = \dfrac{11}{\sin(M)}$

D $\dfrac{11}{\sin(Y)} = \dfrac{10}{\sin(M)}$

E $\dfrac{6}{\sin(M)} = \dfrac{10}{\sin(Y)}$

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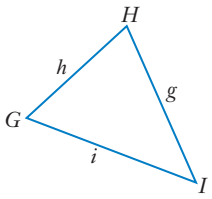
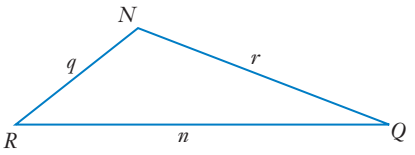
2 Write the sine rule for each triangle.

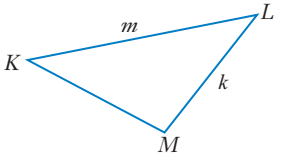
a



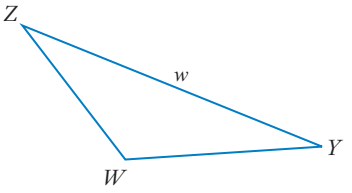
b



c



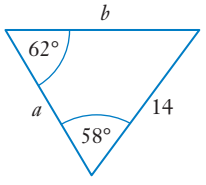
d



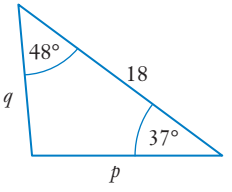
3 Calculate the lengths of the unknown sides in each triangle, correct to one decimal place.

Example 1

a



b



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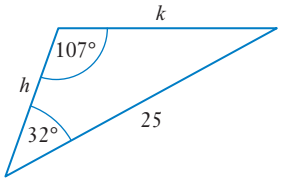

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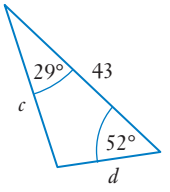


4 In $\triangle MPR$, $M = 38°$, $R = 44°$ and $MP = 12$ cm. Find the length of PR.

5 In $\triangle FAM$, $F = 18°$, $M = 32°$ and $m = 2.4$ cm. Find the length of a.

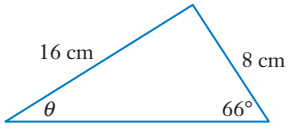
6 In $\triangle HKG$, $H = 56°$, $G = 50°$ and $k = 1.4$ m. Find the length of h.

7 Find the length of the longest side of $\triangle PQR$ if $\angle P = 31°42'$, $\angle Q = 28°50'$ and $PR = 42$ mm.

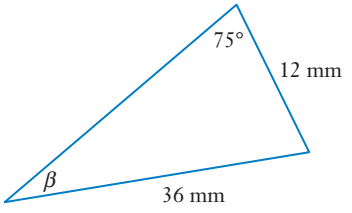
8 In each triangle, find the unknown angle correct to the nearest degree.

Example 2

a



b



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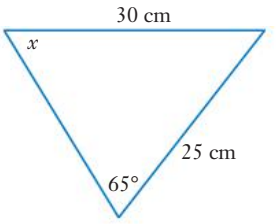
c



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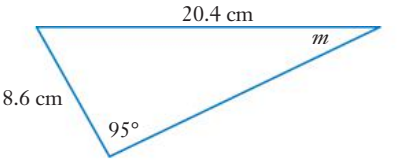


9 In $\triangle ACD$, $CD = 4$ cm, $AC = 8$ cm and $\angle D = 70°$. Calculate the value of $\angle A$ correct to the nearest degree.

10 In $\triangle PSQ$, $PS = 5.4$ cm, $PQ = 8.2$ cm and $\angle S = 125°$. Calculate the value of $\angle Q$ correct to the nearest degree.

11 In $\triangle MTV$, $t = 3.7$ mm, $v = 5.2$ mm and $\angle T = 40°$. Calculate the value of $\angle V$ correct to the nearest degree.

12 Find the size of the largest angle of $\triangle ABC$ if $\angle C = 28°$, $c = 6.6$ cm and $AC = 12$ cm.

The cosine rule - Angles and sides

Finding an unknown side

Finding an unknown angle

11.02 The cosine rule

The **cosine rule** is used in triangles where you can't use the sine rule.

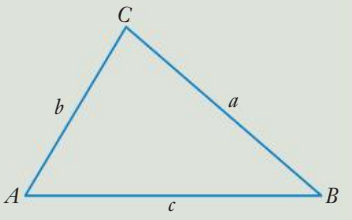
The cosine rule

In any $\triangle ABC$

$$a^2 = b^2 + c^2 - 2bc\cos(A) \qquad \cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

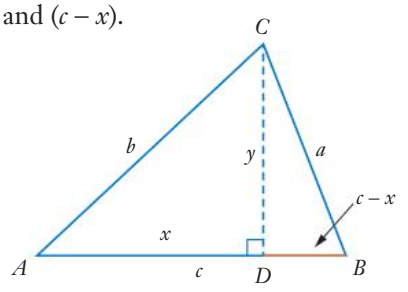
$$b^2 = a^2 + c^2 - 2ac\cos(B) \quad \text{or} \quad \cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos(C) \qquad \cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$



You can show the cosine rule as follows.

Divide $\triangle ABC$ into two right-angled triangles with bases x and $(c - x)$.



Using Pythagoras' theorem, you get:

In $\triangle ACD$, $\quad b^2 = y^2 + x^2$

So, $\quad y^2 = b^2 - x^2$

and in $\triangle BDC$, $\quad a^2 = y^2 + (c - x)^2$

$\quad y^2 = a^2 - (c - x)^2$

Equating expressions for y^2:

$$b^2 - x^2 = a^2 - (c - x)^2$$

Rearranging gives $\quad a^2 = b^2 - x^2 + (c - x)^2$

$$= b^2 - x^2 + c^2 - 2cx + x^2$$

So $\quad a^2 = b^2 + c^2 - 2cx \qquad [1]$

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In $\triangle ACD$, $\cos(A) = \dfrac{x}{b}$

Or, $x = b\cos(A)$

Substitute for x in [1]. $a^2 = b^2 + c^2 - 2bc\cos(A)$

You can rearrange to get $2bc\cos(A) = b^2 + c^2 - a^2$

Divide both sides by $2bc$. $\dfrac{2bc\cos(A)}{2bc} = \dfrac{b^2 + c^2 - a^2}{2bc}$

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

You can choose the altitude AD or BD to get the other forms of the rule. You could also just re-name the triangle to do the same thing.

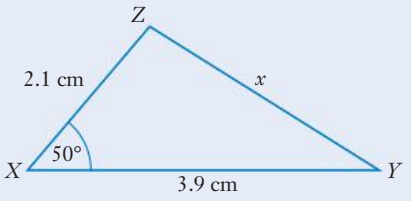
You should use the cosine rule when you know either of the following.

- two sides and the included angle are known; or
- three sides are known.

EXAMPLE 3

The cosine rule

Calculate x correct to two decimal places in the triangle shown.



Solution

Write the cosine rule for $\triangle XYZ$. $x^2 = y^2 + z^2 - 2yz\cos(X)$

Substitute values. $= 2.1^2 + 3.9^2 - 2 \times 2.1 \times 3.9 \times \cos(50°)$

Evaluate. $= 9.0911\ldots$

Take square roots of both sides. $x = \sqrt{9.0911\ldots}$

Only the positive square root is possible as side length can't be negative.

$= \pm 3.0151\ldots$

Round and write the positive answer. x is approximately 3.02 cm.

In any triangle, the longest side is always opposite the largest angle and the smallest side is always opposite the shortest side.

The cosine rule for angles

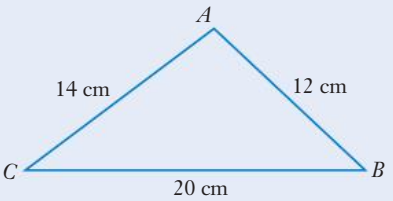
EXAMPLE 4

Calculate, correct to one decimal place, the largest angle in the triangle with sides 12 cm, 14 cm and 20 cm.

Solution

Make a rough sketch.

The largest angle, A, is opposite the longest side, 20 cm.



Write the cosine rule. $\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$

Substitute for known values. $= \dfrac{14^2 + 12^2 - 20^2}{2 \times 14 \times 12}$

Keep the exact number on your calculator. $= -0.1785\ldots$

Because $\cos(A)$ is negative, A is obtuse.

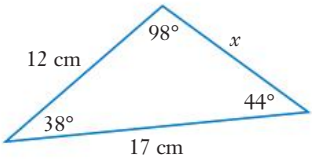
Use the $\cos^{-1}$ function on your calculator. $A = \cos^{-1}(-0.1785\ldots)$

Evaluate. $= 100.2865\ldots$

Write the answer in words. The largest angle is about 100.3°.

Exercise 11.02 The cosine rule

1 Which rule is true for the triangle shown here?



A $x^2 = 12^2 + 17^2 - 2 \times 12 \times 17 \times \cos(44°)$

B $17^2 = 12^2 + x^2 - 24x \times \cos(98°)$

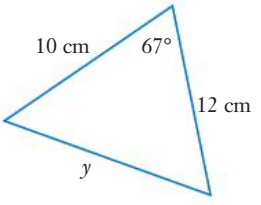
C $x^2 = 12^2 + 17^2 - 2 \times 12 \times 17 \times \cos(98°)$

D $12^2 = x^2 + 17^2 - 34x \times \cos(38°)$

E $17^2 = 12^2 + x^2 - 24x \times \cos(44°)$

Example 3

2 The value of y in this triangle is closest to:



A 15.2 cm

B 13.1 cm

C 12.3 cm

D 14.6 cm

E 9.8 cm

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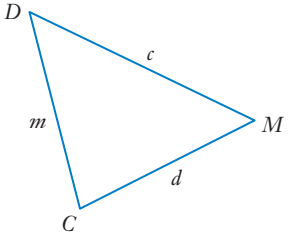
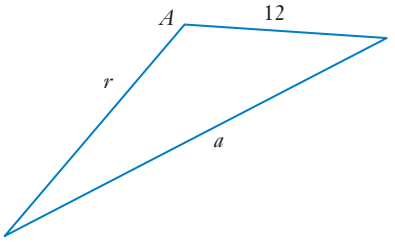
3 Write the cosine rule for each triangle.

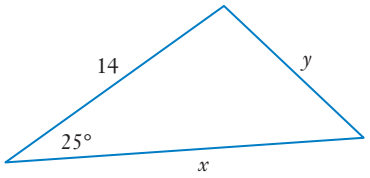
a



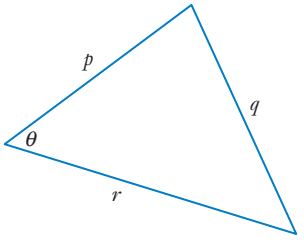
b



c

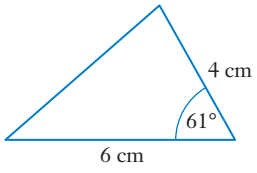


d

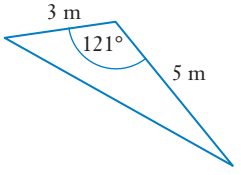


4 Calculate the unknown side length in each triangle.

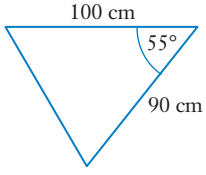
a



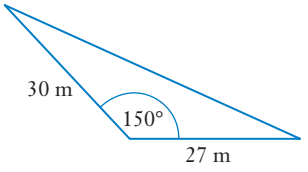
b



c



d



5 Given $\triangle MNO$, where $M = 41.1°$, $n = 27.8$ and $o = 39.2$, calculate m.

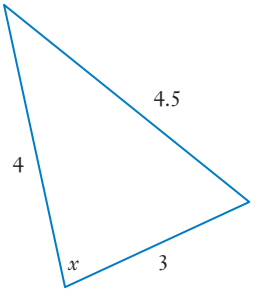
6 Find the third side of $\triangle GHI$, if $G = 67.3°$, $h = 37.9$ and $i = 40.8$.

7 In $\triangle STU$, $S = 112.8°$, $t = 62.8$ and $u = 122$. Calculate the third side.

8 The value of x in this triangle is closest to:

Example **4**

A 85°
B 79°
C 63°
D 72°
E 81°



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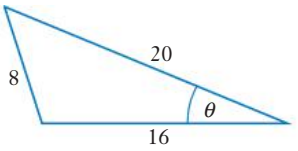
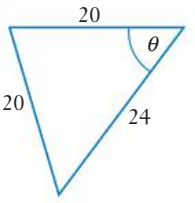
9 Find θ in each triangle.

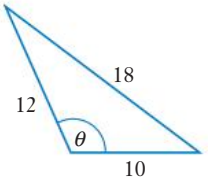
a



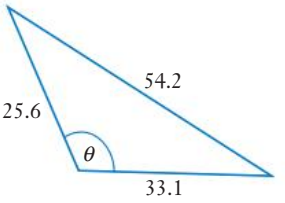
b



c



d



10 In $\triangle DAG$, $\angle D = 121°$, $a = 3$ m and $g = 5$ m. Find all unknown angles and sides.

11 Solve $\triangle UVW$, where $\angle W = 55°$, $u = 45$ cm and $v = 50$ cm.

12 Calculate the angles of $\triangle XYZ$, if $x = 7.2$ m, $y = 12.5$ m and $z = 8.3$ m.

13 In $\triangle ABC$, $a = 12$, $b = 9$ and $c = 8$. Calculate the greatest angle.

14 In $\triangle XYZ$, $x = 8.8$, $y = 11$ and $z = 17.6$. Calculate the smallest angle.

Sine rule problems

Cosine rule problems

11.03 Applications of the sine and cosine rules

Always draw a sketch when the information does not include a diagram.

EXAMPLE 5

Jermaine sees that the top of a sand dune at the end of a beach is at an elevation angle of 16°. On walking 40 m closer, she finds that the angle of elevation increases to 22°. What is the height of the sand dune, correct to one decimal place?

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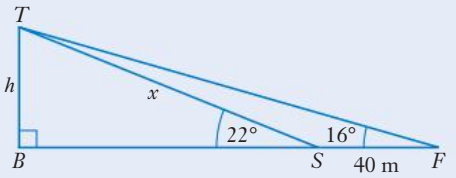
Solution

Make a sketch of the information.

The height, h, is side TB.

Side TS needs to be found first to find h. Call it x.



Use the sine rule in $\triangle SFT$ to find x.

$22° = 16° + \angle STF$ (exterior angle of $\triangle SFT$)

To find $\angle STF$ first, use the exterior angle of $\triangle SFT$.

$\angle STF = 6°$

Use the sine rule in $\triangle BFT$

$$\frac{x}{\sin(16°)} = \frac{40}{\sin(6°)}$$

Don't bother to calculate x yet.

$$x = \frac{40\sin(16°)}{\sin(6°)}$$

Now use $\triangle BST$ and the sine ratio to find h.

$$\sin(22°) = \frac{h}{x}$$

Isolate h.

$$h = x\sin(22°)$$

Evaluate using the uncalculated value of x.

$$= \frac{40\sin(16°)}{\sin(6°)} \times \sin(22°)$$

$$= \frac{40\sin(16°)\sin(22°)}{\sin(6°)}$$

$$= 39.5129\ldots$$

Round and write the answer.

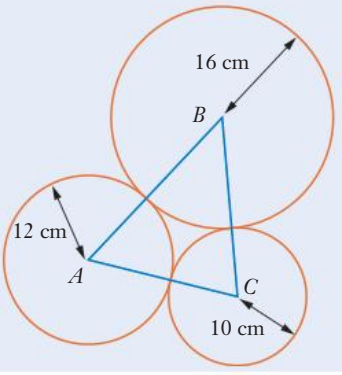
The dune is about 39.5 m high.

EXAMPLE 6

Three circles of radii 10 cm, 12 cm and 16 cm are arranged so that they just touch one another as shown in the diagram on the right.

The centres of the circles form the vertices of the triangle $\triangle ABC$.

Find, correct to one decimal place, the largest angle in the triangle.



Solution

Each side of $\triangle ABC$ is made up of two radii.	$AB = 12\text{ cm} + 16\text{ cm} = 28\text{ cm}$
	$BC = 16\text{ cm} + 10\text{ cm} = 26\text{ cm}$
	$AC = 10\text{ cm} + 12\text{ cm} = 22\text{ cm}$
The largest angle is opposite the longest side.	$\angle C$ is the largest angle.
Write the cosine rule for $\angle C$.	$\cos(C) = \dfrac{AC^2 + BC^2 - AB^2}{2 \times AC \times BC}$
Substitute for known values.	$= \dfrac{22^2 + 26^2 - 28^2}{2 \times 22 \times 26}$
Evaluate.	$= 0.3286\ldots$
Use the $\cos^{-1}$ function on your calculator.	$C = \cos^{-1}(0.3286\ldots)$
Evaluate.	$= 70.8118\ldots$
Write the answer in words.	Largest angle is about 70.8°.

The sine and cosine rules are often used in problems involving bearings and navigation.

EXAMPLE 7

A tower in a direction N 52° E is found to be at a range of 4.86 km. The range finder shows that another tower in a direction 106°T is at a range of 7.96 km. Find, correct to two decimal places, the distance between the towers.

Solution

Sketch a diagram and label the points.

Name the observation point P.

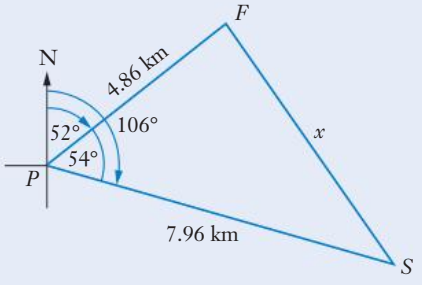
Name the tower positions F and S.

Draw in the bearings.

The angle between the towers must be 54°.

The distance between the towers, FS, needs to be calculated.

Label FS as x.



Write the cosine rule for x. $FS^2 = PF^2 + PS^2 - 2 \times PF \times PS \times \cos(P)$

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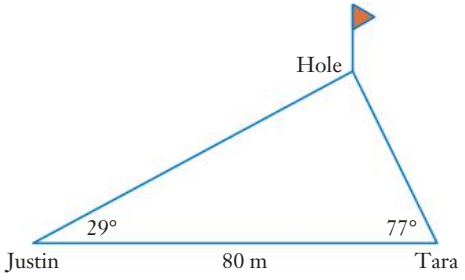
Substitute.	$= 4.86^2 + 7.96^2 - 2 \times 4.86 \times 7.96 \times \cos(54°)$
Keep the exact value on you calculator.	$= 41.5035\ldots$
Take the square root of both sides.	$FS = \sqrt{41.5035\ldots}$
Only the positive value makes sense.	$= 6.4423\ldots$
Round off and write the answer.	The distance between the towers is about 6.44 km.

Exercise 11.03 Applications of the sine and cosine rules

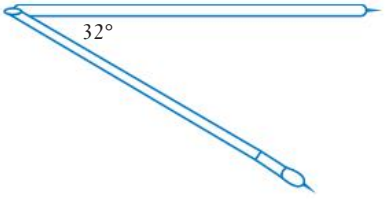
1 Tara and Justin are on the fairway of a golf course standing 80 m apart. The angle of each golfer to the hole are shown on the diagram.

What distance will each golfer have to hit the ball in order to reach the hole?



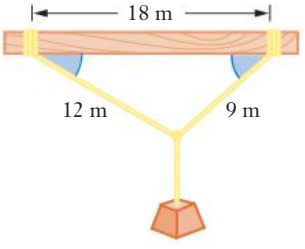
Example 5

2 You use a set of compasses to draw circles. The set shown here have been opened so that the angle between the arms is set at 32°. If the arms of the compass are each 12 cm long, calculate the diameter of the circle that could be drawn with the arms in this position.



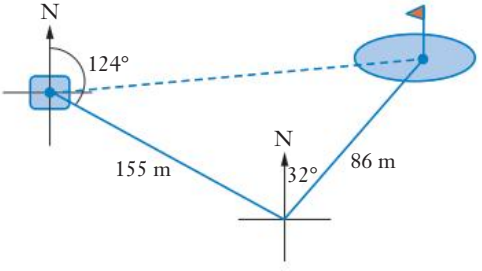
Example 5

3 A weight is supported by cables attached to a horizontal beam as shown. Find the angle that each cable makes with the beam.



Example 6

4 A golfer plays a wayward shot from the tee. She hits the ball 155 m away from the tee at a bearing of 124°T. The golfer's recovery shot is much better. She hits the ball 86 m on a bearing of N 32° E and it ends up in the hole! How much further was the path taken by the golfer compared with the direct distance between the tee and the hole?

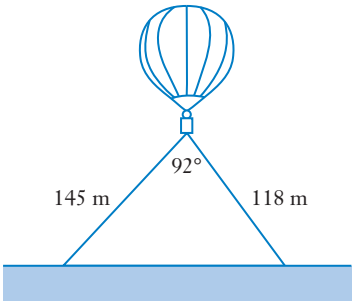


Example 7

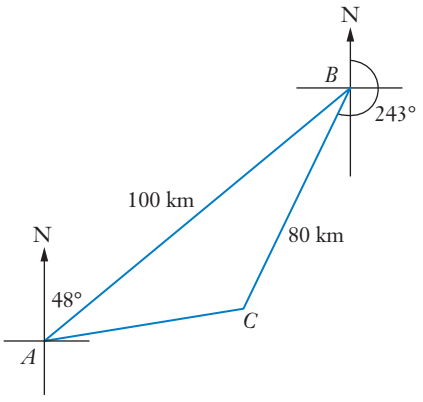
Problem solving

5 A hot air balloon is fixed to the ground by 2 ropes that are 145 m and 118 m long.

If the angle between the ropes is 92°, how far apart are the points at which the ropes are fixed to the ground?

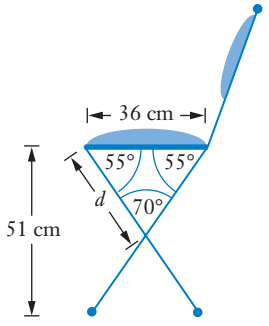


6 The location of 3 towns, A, B and D is shown in the diagram. The bearing from A to B is N 48° E, while the bearing from B to C is 243°T. The towns at A and B are 100 km apart, while the towns at B and C are separated by 80 km. How far apart are the towns A and C?



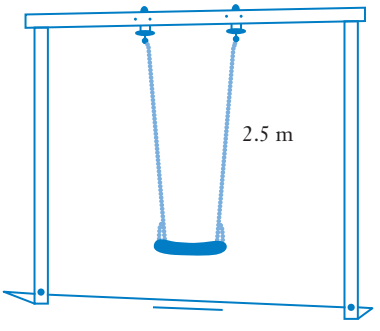
7 The folding chair shown has a seat that is 36 cm deep. The legs of the chair are inclined at an angle of 55° to the seat and join at an angle of 70°. The seat is 51 cm above the floor.

Find the distance from the seat, d, at which the legs are joined.



8 A playground swing is 2.5 m long. The swing makes an angle of 68° at its swing point, in one complete swing.

Calculate the horizontal distance between the extreme positions of the swing seat.



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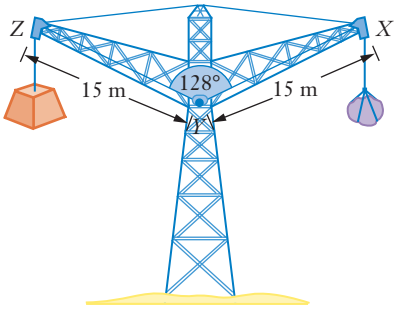
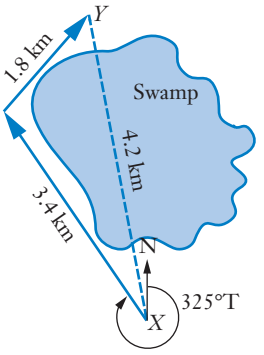
9 The diagram shows a crane with two arms, each 15 m long. The arms of the crane are separated by an angle of 128°. The load is attached at point X and the counterweight of equal mass is attached at point Z. Find XZ.

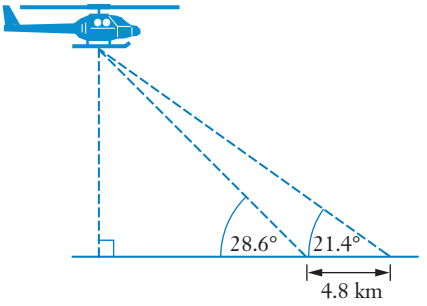


10 A hiker encounters a swamp at point X as shown. To travel around the swamp, the hiker walks 3.4 km from X on a bearing of 325°T and then walks 1.8 km to point Y on the other side of the swamp. Find the bearing of X from Y.



11 A helicopter is sighted at the same time by two ground observers who are 4.8 km apart, as shown in the diagram. The angles from the observers to the helicopter (measured to the horizontal) are 21.4° and 28.6°.

How high is the helicopter?



12 To calculate the height of a building, Madison measures the angle of elevation to the top as 46°. She then walks 20 m closer to the building and measures the angle of elevation as 68°. How high is the building?

13 Toby is flying from Brisbane to Dalby at a speed of 150 km/h. He thinks his altimeter, which shows a height of 15 000 feet (4600 m), is faulty. Looking ahead, he sees the top of a hill marked as 405 m on his map at an angle of depression of 13°. Two minutes later, the angle of depression has increased to 20.6°.

a Find Toby's height above the hill and thus his altitude.

b What should the radio operator tell Toby about his altimeter?

14 A national park ranger is located in an observation tower. The ranger sees a small bushfire at a bearing of N 31° W at a distance of 12 km. The fire response team is currently located at a bearing of 106°T and 15 km from the tower. How far is it from the fire response team to the bushfire?

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15 Olivia is bushwalking. She sees two mountain peaks. On her map, the distances to the peaks are 8 km and 12.5 km. She estimates that the angle between her lines of sight to the peaks is 105°. How far apart are the peaks?

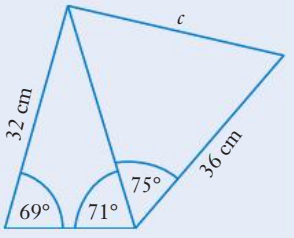
11.04 Solving two-dimensional problems

When solving triangles:

- look for solutions using right-angled triangles first
- use the sine rule when you know:
  - two angles and an opposite side; or
  - two sides and a non-included angle.
- use the cosine rule when you know:
  - two sides and an included angle; or
  - three sides.
- after using the cosine rule once, the sine rule may be useful for other angles and sides.
- if the known side is in a different triangle from an unknown side, the side common to the triangles should usually be found as an intermediate step.

EXAMPLE 8

Find, correct to one decimal place, the value of c in this triangle.



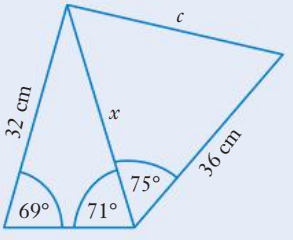
Solution

The triangle on the left has three things known.

c is in the triangle on the right.

The common side can be found.

Label this side as x.



Use the sine rule for x.

$$\frac{x}{\sin(69°)} = \frac{32}{\sin(71°)}$$

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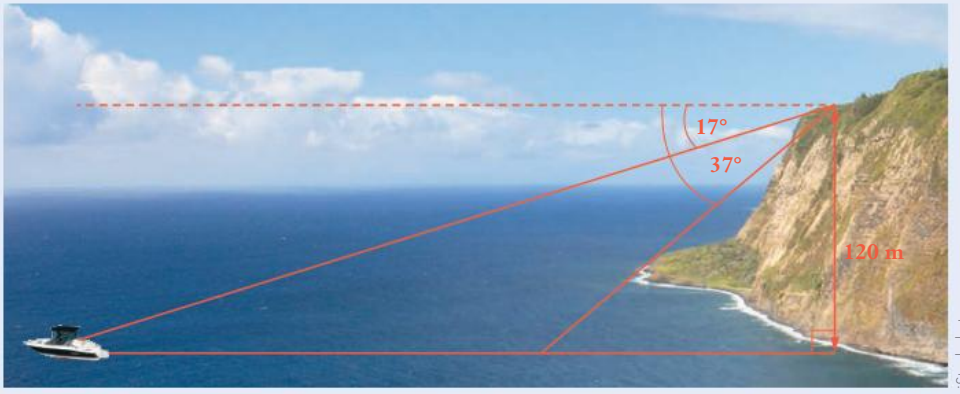
Multiply both sides by sin (69°).	$x = \dfrac{32\sin(69°)}{\sin(71°)}$
Keep the exact answer on your calculator.	$= 31.5959\ldots$
Now use the cosine rule for c.	$c^2 = x^2 + 36^2 - 2 \times x \times 36 \times \cos(75°)$
Substitute for x and evaluate. Keep the exact answer on your calculator.	$= 1705.5151\ldots$
Take the square root of both sides.	$c = \sqrt{1705.5151\ldots}$
	$= 41.2978\ldots$ (c must be positive)
Round and write answer.	c is about 41.3 cm.

EXAMPLE 9

To an observer on top of a cliff that is 120 m above sea level, a boat appears to be at an angle of depression of 17°. After the boat sails directly towards the cliff, the angle of depression increases to 37°. How far did the boat sail, to the nearest metre?



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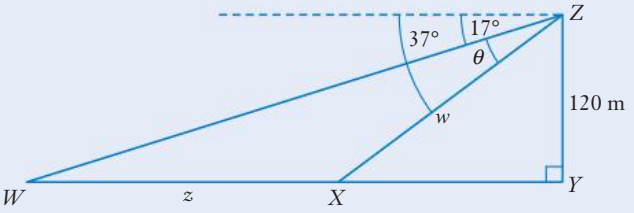
Solution

Draw a diagram with the information provided.

The distance to be calculated is z.



Give the intermediate side a name. Let $XZ = w$

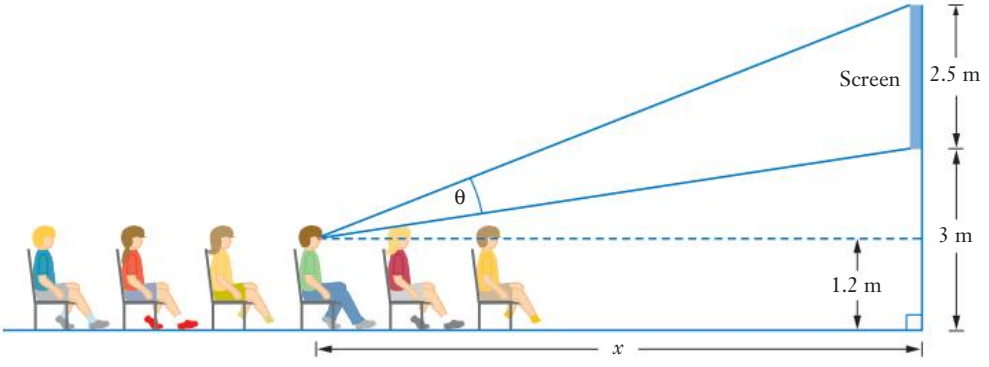
Find the angles.	$\angle ZXY = 37°$ (Alternate angles on parallel lines.)
	$\angle ZWX = 17°$ (Alternate angles on parallel lines.)
Use the diagram.	$\theta = 37° - 17° = 20°$ (Exterior angle of $\triangle WXZ$.)
Use the sine rule with $\triangle WZX$.	$\frac{z}{\sin(\theta)} = \frac{w}{\sin(W)}$
Now calculate w using $\sin(X)$ in $\triangle XYZ$.	$\sin(37°) = \frac{120}{w}$
Rearrange to isolate w.	$w = \frac{120}{\sin(37°)}$
Evaluate and retain this value.	$= 199.3968\ldots$
Substitute values.	$\frac{z}{\sin(20°)} = \frac{199.3978\ldots}{\sin(17°)}$
Multiply both sides by $\sin(20°)$.	$z = \frac{199.3978\ldots \times \sin(20°)}{\sin(17°)}$
Evaluate.	$= 233.2569\ldots$
Round and write the answer in words.	The boat sailed approximately 233 m.

INVESTIGATION

VIEWING DISTANCES AND ANGLES

This diagram shows a cinema. The screen is 2.5 m high and is mounted so that the bottom of the screen is 3 m from the floor.

Eye level for the audience is approximately 120 cm.



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Work in groups to determine the distance x (to the nearest 10 cm) that will give the viewer the maximum viewing angle θ. Use a table of values of x such as that below, or a spreadsheet, to find the best distance.

x (m)	1	2	3	4	5	6	7	8
θ								

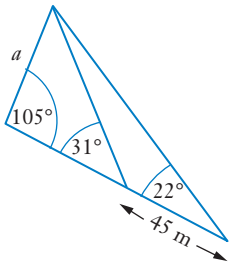
Modern cinemas have a sloping floor to make it easier for viewers to see the screen. Keep the same dimensions for the screen, but change the seating so that the floor slopes up towards the back. Make another table or spreadsheet to do your investigations. Look at the changes of angle with flat or sloping floors of different angles.

Exercise 11.04 Solving two-dimensional problems

Example 8

1 Find the marked side in each triangle.

a



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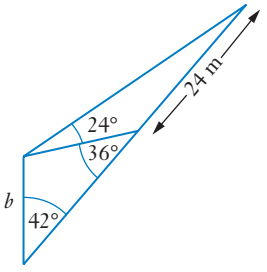
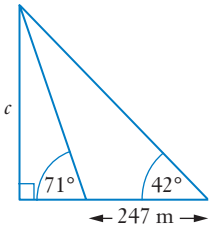
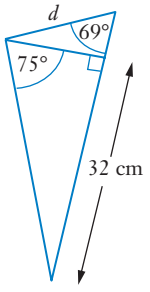


c

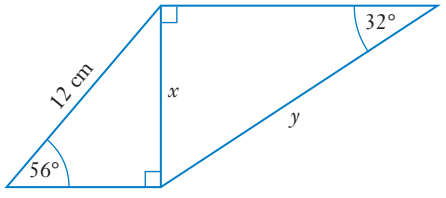


d



2 In each diagram, find x and y, correct to 1 decimal place.

a



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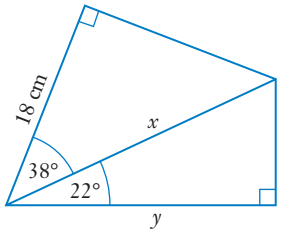

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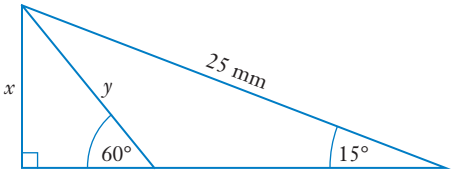
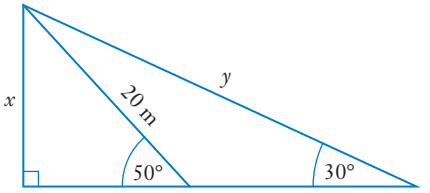
c



d



Example 9

3 An observer in a lighthouse, 120 m above sea level, is watching a fishing trawler move toward the lighthouse. The angle of depression of the trawler from the observer is 18°.

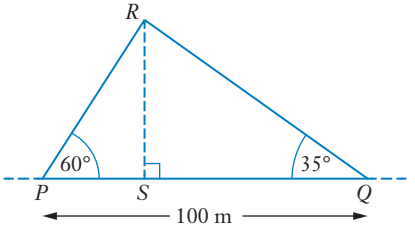
a How far is the trawler from the lighthouse?

b Some time later, the angle of depression is measured as 28°. How far has the trawler travelled in this time?

Problem solving

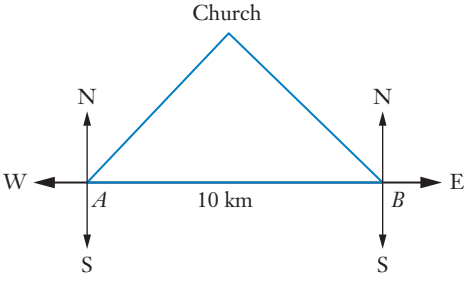
4 Two points, P and Q, are 100 m apart on level ground. A vertical tower stands somewhere between P and Q, as shown in the diagram on the right. The angles of elevation to the top of the tower from P and Q are 60° and 35° respectively.

How high is the tower?



5 Dafina is walking on a straight road running east–west between two towns located at A and B, 10 km apart. On her walk, she sees a church some distance away in a northerly direction. The bearing from A to the church is N 48° E and from B, the church is at a bearing of N 67° W.

Find the distance from A to the church.



6 The top of a lighthouse is at an elevation angle of 35° from an observer some distance from a point on its base, as shown.

The top of the lighthouse is 38 m above the point on the base. This point appears to be at an angle of elevation of 8° from the observer. How far is the observer from the base of the lighthouse?

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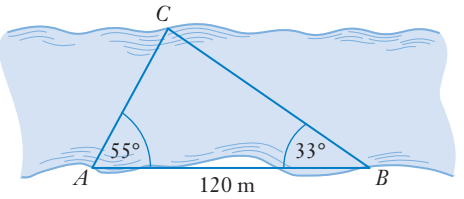
7 Two points, A and B, on the same bank of a river are 120 m apart. Point C is on the opposite bank. $\angle CAB$ and $\angle CBA$ are measured to be 55° and 33° respectively. Find the width of the river.



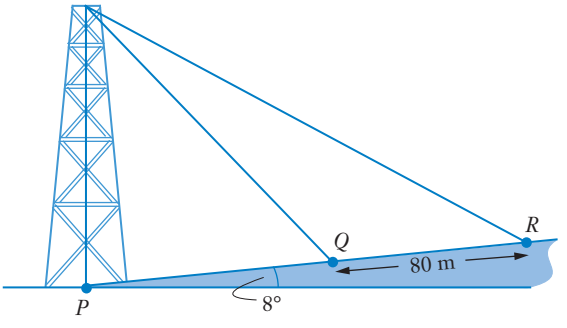
8 Sophie and Will set out from point A at the same time. Sophie travels at 30 km/h along a straight road in the direction 042°T. Will travels at 24 km/h along another straight road in the direction 142°T. Find their distance apart after 4 hours.

9 An aircraft flies on a course of 322°T for 250 km and then changes course to 244°T to an airstrip where it lands. If the airstrip is 480 km from the starting position, calculate:

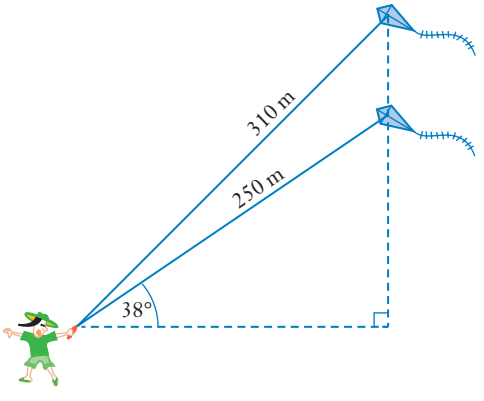
a the distance travelled from the time that the course was changed until the plane landed

b the bearing from the airstrip to the starting position.

10 A tower is erected at the base of a road that is inclined at a constant angle of 8° to the horizontal. Angles of elevation to the top of the tower are measured at two points that are 80 m apart, some distance up the slope away from the tower. The angles are found to be 25° and 8° respectively. What is the height of the tower?



11 Felicity is flying a kite and has extended 250 m of string when an updraft makes the kite rise suddenly and she has to let out another 60 m of string. If the angle of elevation to the kite was 38° before the updraft, how far has the kite been driven vertically?



11.05 Solving three-dimensional problems

Solution of 3D problems

- A line box drawing may help to visualise the problem.
- Make a 3D drawing including triangles and all relevant information.
- Label points on the drawing.
- 'Unfold' the diagram and draw the triangles flat.
- Label the unknowns and intermediate sides and angles.
- Solve the problem and write the answer.

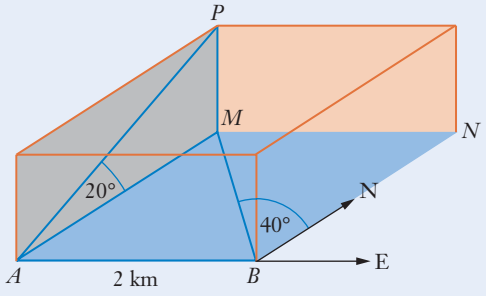
EXAMPLE 10

From a certain point, A, a mountain peak, P, is due north and has an elevation angle of 20°, as shown in the diagram on the right. From another point, B, 2 km east of A and on the same level as A, the bearing of the peak is N 40° W.

Find, correct to the nearest metre, the height of the peak above the level of A and B.

Solution

Make a box drawing.



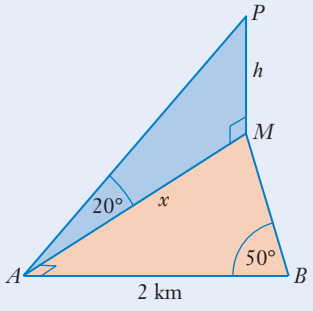
Find the value of $\angle ABM$.

$\angle ABM = 50°$

Redraw the diagram with the sides and angles marked.

Ignore the parts of the box that you don't need.

Call the height of the mountain h. Call the distance from A to the foot of the mountain x.



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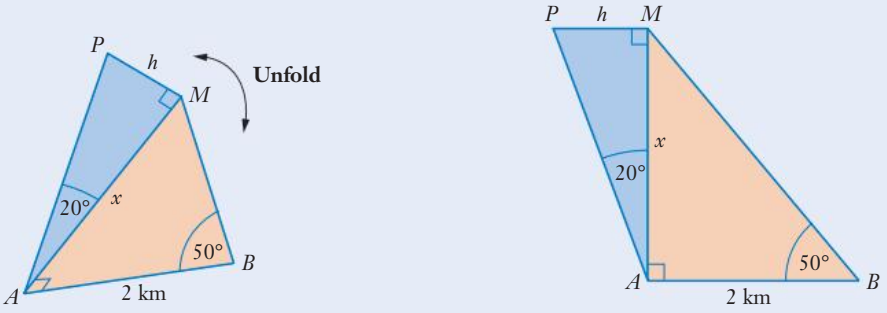
Now unfold the diagram to make the triangles lie in the same plane. In this case, they are joined along AM.



Use tan in $\triangle MAB$ to find x.	$\tan(50°) = \frac{x}{2}$
Rearrange, but don't calculate.	$x = 2\tan(50°)$
Now use tan in $\triangle PAM$ to find h.	$\tan(20°) = \frac{h}{x}$
Rearrange.	$h = x\tan(20°)$
Substitute for x.	$= 2\tan(50°) \times \tan(20°)$
Calculate the answer.	$= 0.8675\ldots$ km
Round and write the answer in appropriate form.	The peak is about 868 m high.

EXAMPLE 11

Hassan observes that the top of a transmission tower at a bearing of 038°T is at an angle of elevation of 12°. Fatima is 575 m due east of Hassan, and she says the bearing of the tower is 295°T.

Find the height of the tower to the nearest metre.

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Solution

Draw a box and then sketch inside the box.

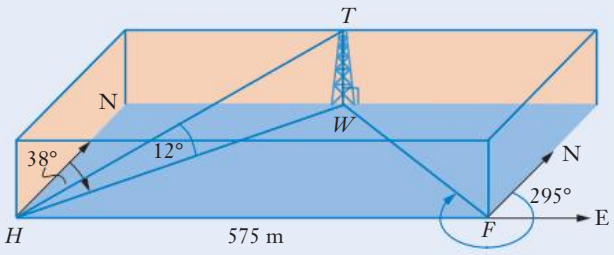
Call Hassan's position H.

Call Fatima's position F.

Show north and east.

Call the bottom of the tower W.

Call the top of the tower T.



Calculate $\angle FHW$.

$$\angle FHW + 38° = 90°$$
$$\angle FHW = 52°$$

Calculate $\angle WFH$.

$$\angle WFH = 295° - 270°$$
$$= 25°$$

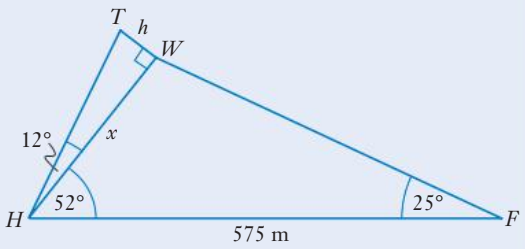
TW is perpendicular to the plane containing $\triangle WFH$.

$$\angle TWH = 90°$$

Unfold and draw the triangles flat.

Call the height h.

Call the common side x.



The angles in $\triangle WFH$ add up to 180°.

$$\angle HWF + 25° + 52° = 180°$$
$$\angle HWF = 103°$$

Use the sine rule in $\triangle WHF$ to find x.

$$\frac{x}{\sin(25°)} = \frac{575}{\sin(103°)}$$

Rearrange but don't calculate.

$$x = \frac{575 \sin(25°)}{\sin(103°)}$$

Now use tan in $\triangle THW$ to find h.

$$\tan(12°) = \frac{h}{x}$$

Rearrange.

$$h = x \tan(12°)$$

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Substitute for x and calculate.

$$= \frac{575 \sin(25°)}{\sin(103°)} \times \tan(12°)$$

$$= \frac{575 \sin(25°) \tan(12°)}{\sin(103°)}$$

$$= 53.0110\ldots$$

Round and write the answer. The transmission tower is about 53 m high.

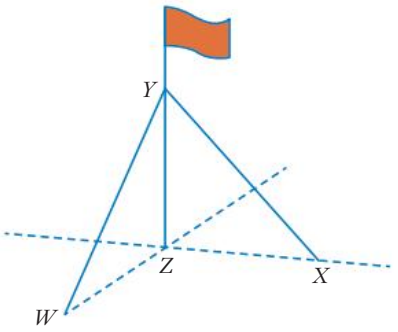
Exercise 11.05 Solving three-dimensional problems

1 A vertical flagpole is supported by two wires WY and XY, as shown in the diagram on the right.

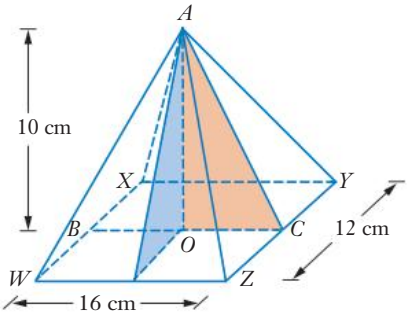
WY is 6.4 metres long, XY is 5.8 metres long, Y is 4.6 metres above ground level and $\angle YZX$ is a right angle.

a Find the distance from W to X.

b Calculate the angle between WY and XY.



2 A rectangular-based pyramid with a base measuring 16 cm by 12 cm and a height of 10 cm is constructed using perspex sheeting. Calculate the angle that the triangular faces AYZ and AZW make with the horizontal base.



3 A flagpole 12 m high is temporarily erected for an Australia Day ceremony. It is held in place by two wires: one 15 m long fastened due west of the pole and the other 17 m long fastened due south of the pole. Both supporting wires are attached to the top of the flagpole.

Calculate:

a the distance that each wire is from the pole at the point where it meets the ground

b the angle of inclination that each wire makes with the horizontal

c the distance between the points where the wires meet the ground

d the angle between the wires.

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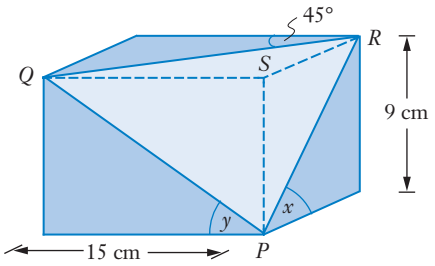
4 A sculptor has removed a section from a copper block with dimensions as shown in the diagram on the right. Calculate:

a QR

b x

c y

d the angle that the triangular face PQR makes with the horizontal base.

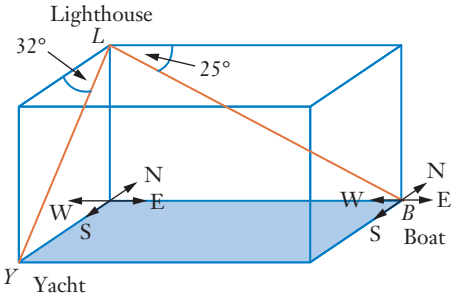


Examples 10,11

5 From a lighthouse 150 m above sea level, a lighthouse keeper observes a boat due east at an angle of depression of 25° and a yacht due south at an angle of depression of 32°. Find:

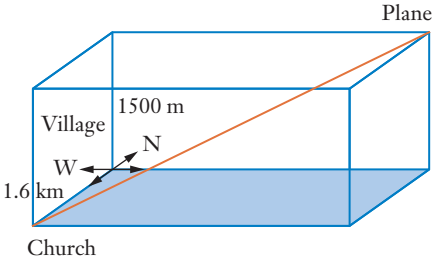
a the distance that separates the boats

b the bearing of the boat to the yacht.



Problem solving

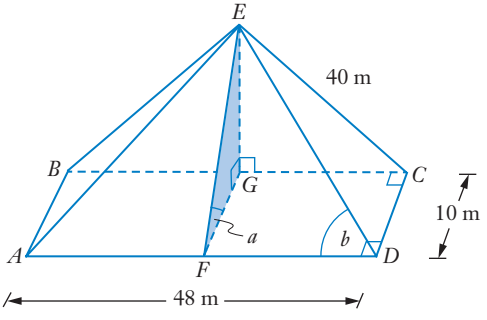
6 An aircraft flying due east at 385 km/h at a constant height of 1500 m passes over a village at a particular instant. One minute later, a person who is in a church 1.6 km south of the village sees the plane. In what direction and at what angle of elevation does the person see the plane?



7 A large tent is constructed in the shape of a rectangular-based pyramid $ABCDE$ with the vertex E vertically above G, the midpoint of BC, as shown in the diagram on the right. Calculate:

a the angle that the triangular face AED makes with the base of the pyramid

b the angle between the lines DE and AD.



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8 Two groups of people set off to spotlight possums at night. One group spots a possum. The tree that has the possum is at a bearing of N 28° W. The angle of elevation of their spotlight beam is 38°. At this instant, the other group, which is 100 m west of the first group, sights the same tree at a bearing of N 29° E. How high up the tree is the possum?

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9 Elena sees that the angle of elevation to the top of a tower, 80 m high, which is due south of her position is 37°. The top of another tower, 60 m high, which is due west of her is at an angle of elevation of 28°. Calculate the horizontal distance between the two towers.

10 Miguel is 90 m due west of Consuella on a level road. He sees that the angle of elevation of a kite flown by their son is 30°, while Consuella sees that it is due north at an angle of elevation of 38°. What is the height of the kite?

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11. CHAPTER SUMMARY

Trigonometry of general triangles

- The **sine rule** may be used to solve non-right-angled triangles where a side and an opposite angle are known.

  In any $\triangle ABC$, $\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$ or $\dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b} = \dfrac{\sin(C)}{c}$

- The **cosine rule** may be used to solve triangles when all three sides are known, or in cases where two sides and the enclosed angle are known.

  In any $\triangle ABC$, $a^2 = b^2 + c^2 - 2bc\cos(A)$ or $\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$

- The **angle of elevation** of an object is the angle an observer must look upwards from the horizontal to see the object.
- The **angle of depression** of an object is measured downwards from the horizontal.
- When solving triangles:
  - look for solutions using right-angled triangles first.
  - use the sine rule when you know two angles and an opposite side or two sides and a non-included angle
  - use the cosine rule when you know two sides an included angle, or three sides
  - after using the cosine rule once, the sine rule may be useful for other angles and sides
  - if the known side is in a different triangle from an unknown side, the side common to the triangles should usually be found as an intermediate step.
- To solve three-dimensional problems:

  1 A line box drawing may help to visualise the problem.

  2 Make a 3D drawing including triangles and all relevant information.

  3 Label points on the drawing.

  4 'Unfold' the diagram and draw the triangles flat.

  5 Label the unknowns and intermediate sides and angles.

  6 Use the methods of right-angled triangles, sine and cosine rules to find the unknowns.

  7 Write the solution to the problem in an appropriate form.

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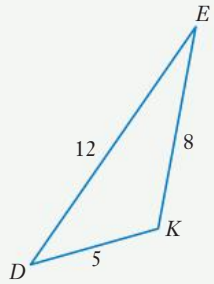
11. CHAPTER REVIEW

Trigonometry of general triangles

1 Which rule is true for $\triangle DEK$?



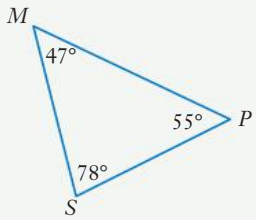
Example 1

A $\dfrac{5}{\sin(E)} = \dfrac{12}{\sin(D)}$ **B** $\dfrac{8}{\sin(D)} = \dfrac{12}{\sin(K)}$

C $\dfrac{12}{\sin(K)} = \dfrac{8}{\sin(E)}$ **D** $\dfrac{8}{\sin(E)} = \dfrac{5}{\sin(K)}$

E $\dfrac{8}{\sin(E)} = \dfrac{12}{\sin(K)}$

2 Which rule is true for $\triangle MPS$?



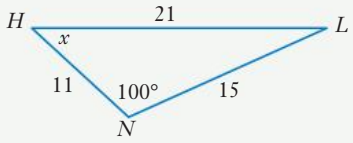
Example 1

A $\dfrac{p}{\sin(55°)} = \dfrac{s}{\sin(47°)}$ **B** $\dfrac{m}{\sin(47°)} = \dfrac{p}{\sin(78°)}$

C $\dfrac{s}{\sin(78°)} = \dfrac{m}{\sin(47°)}$ **D** $\dfrac{m}{\sin(55°)} = \dfrac{s}{\sin(78°)}$

E $\dfrac{p}{\sin(47°)} = \dfrac{m}{\sin(55°)}$

3 The expression for finding x in $\triangle HLN$ is:



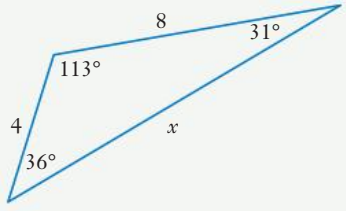
Example 2

A $\sin^{-1}\left[\dfrac{5\sin(100°)}{7}\right]$ **B** $\sin^{-1}\left[\dfrac{11\sin(100°)}{15}\right]$

C $\sin^{-1}\left[\dfrac{21\sin(100°)}{11}\right]$ **D** $\sin^{-1}\left[\dfrac{15\sin(100°)}{11}\right]$

E $\sin^{-1}\left[\dfrac{21\sin(100°)}{11}\right]$

4 Which rule is true for the triangle shown here?



Example 3

A $x^2 = 4^2 + 8^2 - 2 \times 4 \times 8 \times \cos(31°)$

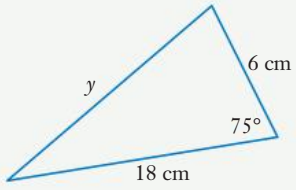
B $8^2 = 4^2 + x^2 - 4x \times \cos(36°)$

C $x^2 = 80 - 32\cos(113°)$

D $4^2 = x^2 + 8^2 - 16x \times \cos(113°)$

E $8^2 = 64 + x^2 - 8x \times \cos(31°)$

5 The value of y in this triangle is closest to:



Example 3

A 18.2 cm **B** 15.1 cm

C 17.8 cm **D** 16.7 cm

E 17.4 cm

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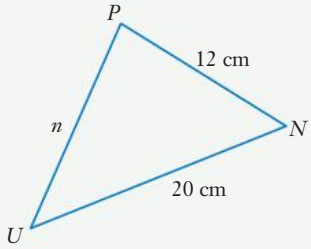
Example 4

6 Which rule is true for the triangle shown here?



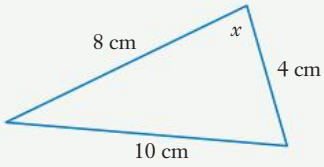
A $\cos(N) = \dfrac{445 - n^2}{2 \times 12 \times 20}$ **B** $\cos(U) = \dfrac{n^2 + 256}{12n}$

C $\cos(P) = \dfrac{n^2 - 256}{24n}$ **D** $\cos(N) = \dfrac{544 + n^2}{240}$

E $\cos(U) = \dfrac{12^2 + n^2 - 20^2}{24n}$

Example 4

7 The value of x in this triangle is closest to:



A 90° **B** 86°

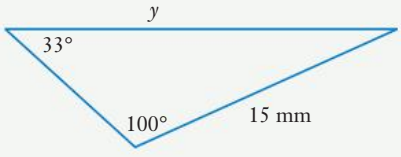
C 95° **D** 108°

E 110°

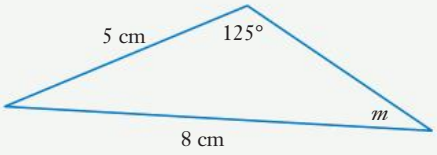
Example 2

8 Use the sine rule to find the unknown in each triangle.

a



b



Example 2

9 Solve each triangle.

a $\triangle ABC$, $\angle A = 42°$, $b = 36$ m, $a = 28$ m

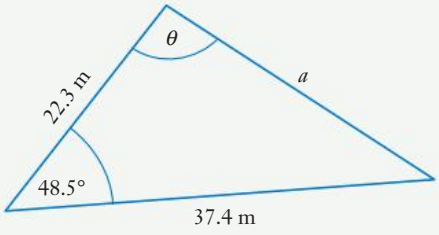
b $\triangle FEG$, $\angle F = 58.2°$, $f = 42.5$ cm, $\angle E = 28.4°$

c $\triangle KMN$, $\angle M = 35°$, $m = 29$ m, $n = 19$ m

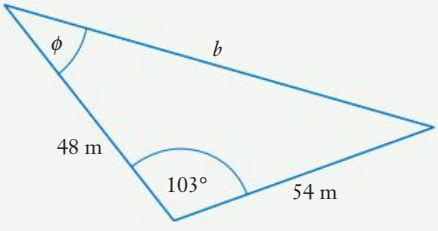
Example 3

10 Find the unknown angles and sides marked in each triangle.

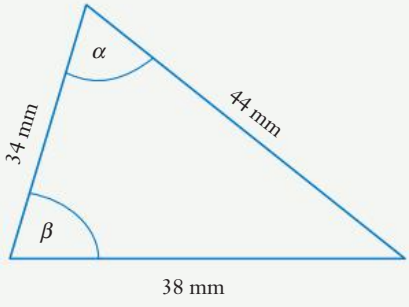
a



b



c



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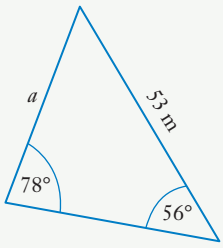
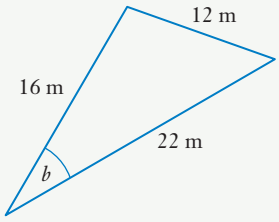
11 Find the unknown angle or side marked in each triangle.

Examples 2, 3

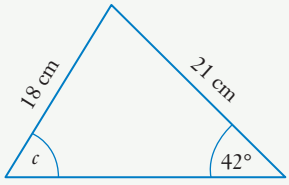
a



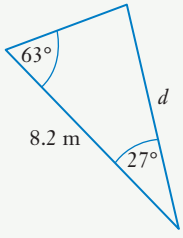
b



c



d



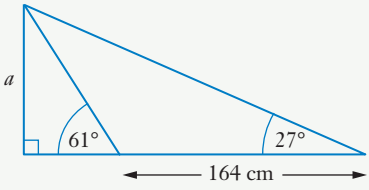
12 Calculate the largest angle in the triangle with sides 7 mm, 6 mm and 3 mm.

Example 4

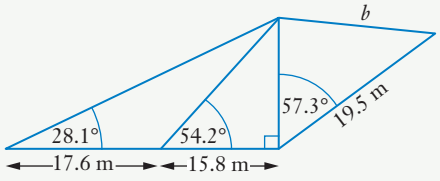
13 Find the marked side in each diagram.

Example 5

a



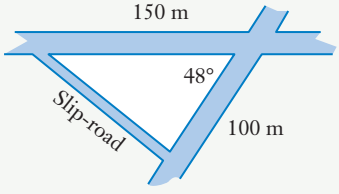
b



14 A fence is to be built on the perimeter of a triangular island where two roads and a slip-road meet. The angle between the two roads is 48°, and the distances along each road to the slip-road are 100 m and 150 m.

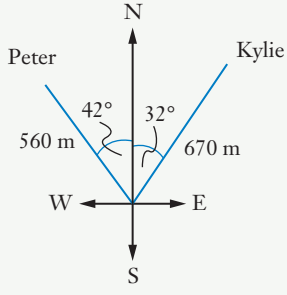
Find the length of the fence.

Example 6



15 After a chat, Peter and Kylie walk away from each other. Peter walks 560 m in the direction N 42° W while Kylie walks 670 m in the direction N 32° E. Find their distance apart.

Example 7



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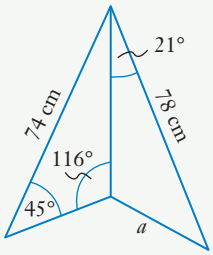
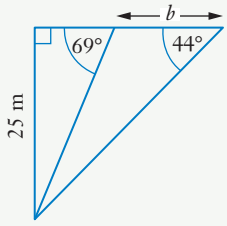
Example 8

16 Find the marked side in each diagram.

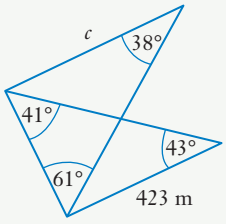
a



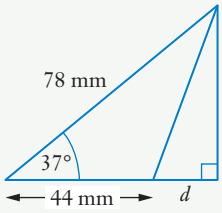
b



c

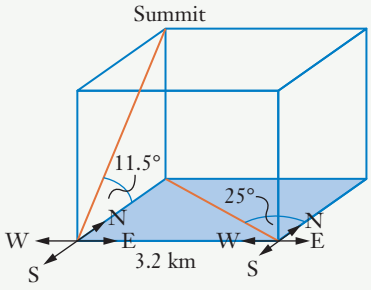


d



Example 10

17 From a point on a level straight road running due east, an observer finds that the summit of a mountain bears due north and that the angle of elevation is 11.5°. From a second point on the road 3.2 km due east of the first point, the observer finds that the bearing of the summit is N25°W. Find the height of the mountain summit above the level of the road.



Example 11

18 Two identical buildings 40 m tall are erected on a straight level road running east–west. From a point due south of one of the buildings, the angles of elevation to the building tops are 35° and 47° respectively. Find:

a the distance from the point to the base of each building

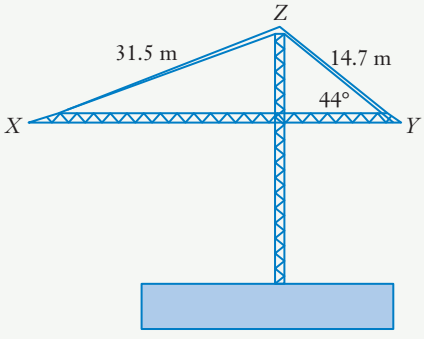
b the distance between the buildings.

Problem solving

19 A large crane is being used on a construction site. The crossbeam (XY) is supported by two metal stays, XZ and ZY, which are 31.5 m and 14.7 m long respectively.

XY makes an angle of 44° with the crossbeam.

Calculate the size of $\angle ZXY$ and the length of the crossbeam XY.



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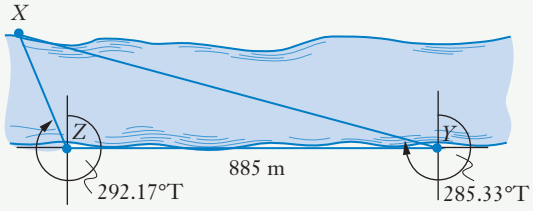
20 To calculate the height of a building, Eva measures the angle of elevation to the top as 46°. She then walks 21 m closer to the building and measures the angle of elevation as 62°. How high is the building?

21 A person standing at point Z on the bank of a river observes a tree at point X on the opposite bank, at a bearing of 292.17°T. The person then walks 885 m down the bank to point Y and notes that the tree is now at a bearing of 285.33°T.

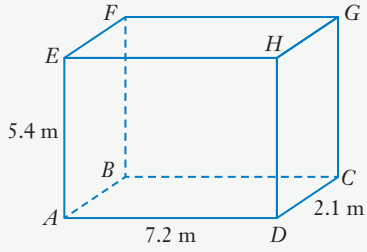


Find the distance between the tree and the initial position, Z, of the observer.

22 The top of a hill is at an angle of elevation of 7.5°. A hundred metres closer to the hill, the angle of elevation is 8°. What is the height of the hill?

23 The object shown is a rectangular prism. Use the information provided to calculate:

- **a** the angle between EC and the base of the prism
- **b** the angle between the planes $BCHE$ and $ADHE$
- **c** the angle between the lines AG and GC.



Practice quiz

UNIVARIATE DATA ANALYSIS

12.

COMPARING DATA

The average height of 16-year-old indigenous Inuit youth in northern Canada does not show if they are tall. You have to compare it to the average heights of other groups of 16-year-old youths. Comparing two or more data sets is often more important than studying one set of data by itself.

12.01 The five-number summary
12.02 Constructing box plots
12.03 Using parallel box plots to compare data sets
12.04 Comparing data sets
12.05 Comparing the shapes of data sets
12.06 Comparing data and the statistical investigation process
Chapter summary
Chapter review

SYLLABUS SUBJECT MATTER

The statistical investigation process

- review statistical investigation as a process, for example, identifying a problem and posing a statistical question, collecting or obtaining data, displaying and analysing the data, interpreting and communicating the results
- define univariate data

Comparing data for a numerical variable across two or more groups

- construct and use parallel box plots (including the use of the 'Q1 – 1.5 × IQR' and 'Q3 + 1.5 × IQR' criteria for identifying possible outliers) to compare data sets in terms of median, spread (IQR and range) and outliers to interpret and communicate the differences observed in the context of the data
- compare data sets using medians, means, IQRs, ranges or standard deviations for a single numerical variable; interpret the differences observed in the context of the data; and report the findings in a systematic and concise manner
- implement the statistical investigation process to answer questions that involve comparing the data for a numerical variable across two or more data sets, for example, are Year 11 students the fittest in the school?

Prior learning

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TERMINOLOGY

back-to-back stem plot	bimodal	box-and-whisker plot
box plot	first quartile	five-number summary
histogram	interquartile range	location
median	negatively skewed	outlier
plateau	population pyramid	positively skewed
quartile	side-by-side	spread
standard deviation	statistical investigation process	stem-and-leaf plot
symmetrical	third quartile	

Five-number summaries

Five-number summaries

12.01 The five-number summary

Five-number summary

- Q_0 is the **minimum** value
- Q_1, is the **first quartile**, the $\left(\frac{n+1}{4}\right)$th or $\left(\frac{n+2}{4}\right)$th score for n odd and even respectively
- Q_2, is the **median**, the $\left(\frac{n+1}{2}\right)$th score in the ordered data of n values
- Q_3, is the **third quartile**, the $\left(\frac{3n+3}{4}\right)$th or $\left(\frac{3n+2}{4}\right)$th score for n odd and even respectively
- Q_4 is the **maximum** value
- The **five-number summary** is Q_0, Q_1, Q_2, Q_3, Q_4

EXAMPLE 1

Find the five-number summary for the scores 5, 8, 9, 5, 7, 3, 6, 8, 6, 5, 5, 2, 9, 5, 6, 7, 4.

Solution

Arrange the scores in order.	2, 3, 4, 5, 5, 5, 5, 5, 6, 6, 6, 7, 7, 8, 8, 9, 9
Count the scores.	$n = 17$
Choose the middle one.	$\text{Median} = \frac{17+1}{2} = 9\text{th score} = 6$
Find the first and third quartiles.	$Q_1 = \frac{17+1}{4} = 4\frac{1}{2}\text{th score} = \frac{5+5}{2} = 5$
	$Q_3 = \frac{3\times 17+3}{4} = 13\frac{1}{2}\text{th score} = \frac{7+8}{2} = 7.5$
Write the minimum and maximum.	$Q_0 = 2, Q_4 = 9$
State the answer.	The five-number summary is 2, 5, 6, 7.5, 9.

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For the data in Example 1:

- 25% of the scores are from 2 to 5
- 50% are from 6 to 7.5
- 25% are from 7.5 to 9

so the scores are more concentrated at the top end.

In small data sets like Example 1, you can just divide the ordered data into quarters to find the median and quartiles.

For grouped data, you have to use the class limits for the minimum and maximum, Q_0 and Q_4. You have to interpolate to find the other points. Always use cumulative frequency.

EXAMPLE 2

Find the five-number summary for these heights of Year 11 students.

Height (cm)	140–144	145–149	150–154	155–159	160–164	165–169
Number	2	3	2	8	12	10
Height (cm)	170–174	175–179	180–184	185–189	190–194	195–199
Number	17	9	7	1	1	2

Solution

Redraw with true class limits and a cumulative frequency column.

Height (cm)	f	Cumulative f
139.5–144.5	2	2
144.5–149.5	3	5
149.5–154.5	2	7
154.5–159.5	8	15
159.5–164.5	12	27
164.5–169.5	10	37
169.5–174.5	17	54
174.5–179.5	9	63
179.5–184.5	7	70
184.5–189.5	1	71
189.5–194.5	1	72
194.5–199.5	2	74

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State the minimum and maximum.	$Q_0 = 139.5$, $Q_4 = 199.5$
Find the median position.	$Q_2 = \frac{74+1}{2} = 37.5$th score.
Use interpolation to find the median.	Median is the 0.5th score in 169.5–174.5, so Median $= 169.5 + \frac{0.5}{17} \times 5 \approx 169.6$ cm
Find the positions of the first and third quartiles.	Q_1 is the $\frac{74+2}{4} = 19$th score Q_2 is the $\frac{3 \times 74+2}{4} = 56$th score
Use interpolation to find the Q_1.	Q_1 is the 4th score in 159.5–164.5, so $Q_1 = 159.5 + \frac{4}{12} \times 5 \approx 161.2$ cm
Use interpolation to find the Q_3.	Q_3 is the 2nd score in 174.5–179.5, so $Q_1 = 174.5 + \frac{2}{9} \times 5 \approx 174.9$ cm
Write the answer.	The five-number summary is 139.5, 161.2, 169.6, 174.9, 199.5.

When you examine data, you should always check for possible **outliers**. For the five-number summary, you should use the **IQR test**.

Interquartile range outlier test

- If $x < Q_1 - 1.5 \times \text{IQR}$ or $x > Q_3 + 1.5 \times \text{IQR}$, then x is a possible outlier
- Invalid outliers should be excluded from a data set

An invalid outlier is one that is found to be either erroneous or from outside the target population. As you will rarely be able to exclude a possible outlier, it is prudent to provide five-number summaries with and without possible outliers.

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EXAMPLE 3

Give the five-number summary for this data:

28, 51, 36, 54, 41, 75, 46, 52, 39, 51, 47, 53, 45, 51, 40, 51, 40, 56.

Check for possible outliers.

Solution

Put the data in order.	28, 36, 39, 40, 40, 41, 45, 46, 47, 51, 51, 51, 51, 52, 53, 54, 56, 75
Divide in quarters.	28, 36, 39, 40, 40, 41, 45, 46, 47 | 51, 51, 51, 51, 52, 53, 54, 56, 75
State the summary.	The five-number summary is 28, 40, 49, 52, 75.
Find the IQR.	Interquartile range = 52 − 40 = 12
Find the expected boundaries of the data.	Lower boundary = 40 − 1.5 × 12 = 22 Upper boundary = 52 + 1.5 × 12 = 70
State any possible outliers.	75 is a possible outlier, but there is no known reason to exclude it.
Try exclusion.	28, 36, 39, 40 | 40, 41, 45, 46, 47, 51, 51, 51, 51 | 52, 53, 54, 56
State the new summary.	The five-number summary, without 75, is 28, 40, 47, 51.5, 56
State the answer.	Including all data, the five-number summary is 28, 40, 49, 52, 75. Excluding a possible outlier, it is 28, 40, 47, 51.5, 56.

Exercise 12.01 The five-number summary

Example 1

1 Find the five-number summary for each data set.

a 26, 27, 23, 24, 27, 25, 23, 19, 25, 21, 22, 20, 27, 21

b 2, 6, 5, 9, 2, 6, 6, 9, 7, 5, 4, 2, 3, 7, 8, 6, 6, 7, 4, 2

c 36, 39, 43, 43, 38, 37, 33, 37, 49, 42, 31, 37, 46

d 10, 16, 8, 18, 12, 16, 8, 18, 14, 16, 14, 18, 14, 15, 15, 17, 12, 17, 10, 19, 8, 16

2 Find the five-number summary for each data set.

a

Score	3	4	5	6	7	8	9	10	11	12
Frequency	2	3	5	6	7	6	3	3	3	2

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b

Score	22	23	24	25	26	27	28	29	30	31	32
Frequency	3	7	4	11	13	10	6	5	3	2	2

c

Score	104	105	106	107	108	109	110	111	112
Frequency	7	9	11	17	21	25	16	9	5

Example 2

3 Find the five-number summary for each data set.

a

Class	Frequency
5–9	2
10–14	6
15–19	9
20–24	8
25–29	4

b

Class	Frequency
0–9	8
10–19	16
20–29	32
30–39	20
40–49	5

c

Class	Frequency
11–15	13
16–20	28
21–25	57
26–30	46
31–35	32
36–40	11

d

Class	Frequency
0–99	3
100–199	7
200–299	13
300–399	10
400–499	4
500–599	2

Example 2

4 Find the five-number summary for each data set, checking for outliers.

a 12, 19, 14, 27, 14, 23, 14, 15, 14, 21, 15, 23, 13, 40, 14, 16, 15, 23, 14

b 10, 16, 12, 12, 7, 13, 10, 13, 8, 14, 9, 13, 11, 13, 10, 13, 12, 13, 3, 14, 10, 13

c 29, 19, 24, 25, 52, 28, 44, 33, 24, 30, 27, 18, 8, 28, 44, 29, 24, 44, 23, 25

d 13, 31, 15, 31, 21, 26, 18, 30, 11, 28, 23, 27, 14, 26, 19, 39, 19, 34, 18, 36, 17, 28, 22

Problem solving

5 The number of hours a group of students spent using a computer during the week are shown in the table. Find the typical time and the five-number summary.

Time (h)	2–4	5–7	8–10	11–13	14–16	17–19	20–22	23–25
Number	2	4	6	15	14	8	6	4

6 The self-reported number of computers that some 5-year-olds said were in their house were as follows:

2, 2, 2, 3, 1, 4, 0, 4, 2, 3, 2, 2, 2, 3, 1, 3, 1, 200, 2, 3, 2, 3, 1, 2, 2, 2, 1, 3, 2, 4

Find the typical number of computers and find a five-number summary.

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12.02 Constructing box plots

The five-number summary is useful, but a visual representation is easier to understand.

Box plots

Box plots 1

Box plots 2

Box-and-whisker plots

Box plots: Graphics calculator

Box plots

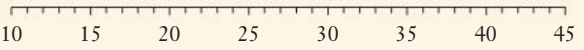
- A **box plot** is also called a **box-and-whisker plot**
- Draw a scale that covers all the data
- Draw a rectangle above the scale from Q_1 to Q_3, the first and third quartiles
- Draw a vertical line that divides the rectangle at Q_2, the median
- Draw horizontal lines ('**whiskers**') from the ends of the rectangle out to Q_0 and Q_4, the minimum and maximum. You can put stop lines at the ends
- Excluded outliers can be shown by crosses or dots at the level of the whiskers

EXAMPLE 4

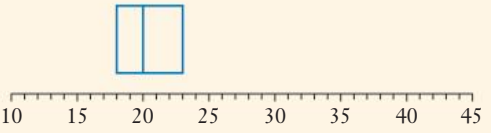
Draw a box plot for the five-number summary 14, 18, 20, 23, 28 with excluded outlier 40.

Solution

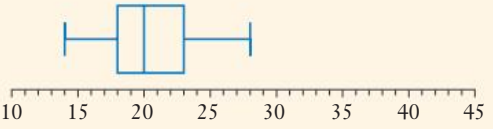
Draw the scale from 10 to 45.



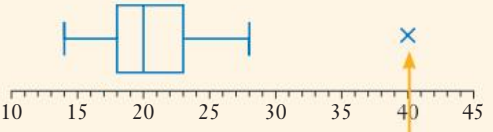
Draw the box from 20 to 23 and divide at 20.



Draw the whiskers out to 14 and 28.



Add the outlier.



When you draw by hand, a cross is easier to see than a dot.

You usually have to find the five-number summary before you draw a box plot.

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Box plots and outliers

EXAMPLE 5

Draw box plots for each data set, showing possible outliers as appropriate.

a 18, 51, 35, 66, 28, 6, 11, 20, 28, 15, 30, 30, 29, 16, 34, 28, 31, 18, 12, 24, 11, 20, 19, 33

b

Score	0–4	5–9	10–14	15–19	20–24
Frequency	6	8	25	52	55
Score	25–29	30–34	35–39	40–44	45–49
Frequency	46	31	10	2	1

Solution

a Arrange in order.

6, 11, 11, 12, 15, 16, 18, 18, 19, 20, 20, 24, 28, 28, 28, 29, 30, 30, 31, 33, 34, 35, 51, 66

Find the quartiles and interquartile range.

$Q_1 = 17$, $Q_3 = 30.5$

IQR = 13.5

Find the boundaries of 'normal' data.

$Q_1 - 1.5 \times \text{IQR} = 17 - 1.5 \times 13.5 = -3.75$

$Q_3 + 1.5 \times \text{IQR} = 17 + 1.5 \times 13.5 = 37.25$

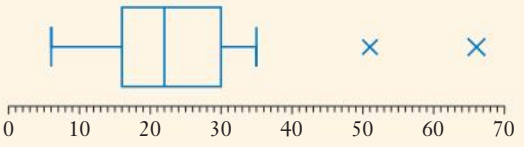
State possible outliers.

51 and 66 are possible outliers.

Do a five-number summary, excluding possible outliers.

Five-number summary, excluding 51 and 66, is 6, 16, 22, 30, 35.

Draw the box plot, showing possible outliers.



b Rearrange the table vertically and add a column for cumulative frequency.

Score	Frequency	Cumulative frequency
0–4	6	6
5–9	8	14
10–14	25	39
15–19	52	91
20–24	55	146
25–29	46	192
30–34	31	223
35–39	10	233
40–44	2	235
45–49	1	236

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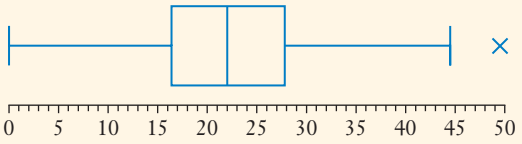
Find the quartiles and interquartile range.	$Q_1 = 16.5$, $Q_3 = 27.9$ IQR = 11.4
Find the boundaries of 'normal' data.	$Q_1 - 1.5 \times \text{IQR} = 16.5 - 1.5 \times 11.4 = -0.6$ $Q_3 + 1.5 \times \text{IQR} = 27.9 + 1.5 \times 11.4 = 45.2$
State possible outliers.	The last possible score, 49.5 is a possible outlier, so exclude score group 45–49.
Find the five-number summary.	The five-number summary, with 45–49 excluded, is 0, 16.4, 22.0, 27.8, 44.5.
Draw the box plot, showing the possible outlier.	

Exercise 12.02 Constructing box plots

1 Draw a box plot for the five-number summary 3, 5, 6, 8.5, 11 with excluded outlier 18. Example 4

2 Draw a box plot for the five-number summary 24, 36.5, 40.5, 45, 60.

3 Draw a box plot for the five-number summary 9, 17, 21, 23, 27 with outliers 2 and 38.

4 Draw a box plot for the following data, showing possible outliers as appropriate: Example 5

21 23 21 23 21 23 22 25 19

22 20 24 21 22 19 24 9

5 Draw a box plot for the following data, showing possible outliers as appropriate.

Score	10–19	20–29	30–39	40–49	50–59	60–69	70–79
Frequency	3	8	12	17	16	8	5

6 Draw a box plot for the following data, showing possible outliers as appropriate.

104 113 110 113 109 115 105 111 110

115 108 112 108 113 109 112 104 116

108 114 110 116 109 116 109 113 108

7 Draw a box plot for the following data, showing possible outliers as appropriate.

Score	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59
Frequency	1	0	3	7	12	24	19	8	5

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Problem solving

8 The number of errors in a page of typing for some business college students were recorded as follows.

4	1	0	1	2	9	4	5	7	2
0	1	9	5	32	9	1	0	2	1
4	7	6	3	5	4	8	0		

Draw a box plot of the data and state the typical number of errors.

9 At the start of a weight-watchers' course, all participants took part in a weigh-in. Their masses are summarised in the table below. Draw a box plot of the results and state the typical starting mass.

Mass (kg)	70–74	75–79	80–84	85–89	90–94	95–99	100–104
Frequency	4	8	10	14	11	9	4

10 The masses, in grams, of a hoard of denarius coins found in Gaul (modern France) from Roman times were checked and found to be as follows.

2.34	2.73	2.66	3.03	2.76	1.89	2.67	2.71	2.67	2.66
2.34	2.7	2.88	2.71	2.71	2.78	2.96	2.49	2.93	2.55
2.86	2.81	2.63	2.78	2.76	2.4	2.57	2.4	2.5	2.55
2.58	2.71	2.77	2.72	2.91	2.51	2.79	2.68	2.6	2.71
2.72	2.96	2.94	2.94	2.65	2.6	2.66			

Draw a box plot and state a typical mass.

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12.03 Using parallel box plots to compare data sets

Box-and-whisker plots

Double box plots

Box plots allow you to examine the relative spread of data easily. When you compare 2 or more sets of data, you can use parallel box plots on the same scale to make immediate comparisons.

EXAMPLE 6

Construct parallel box plots of these data sets to compare them.

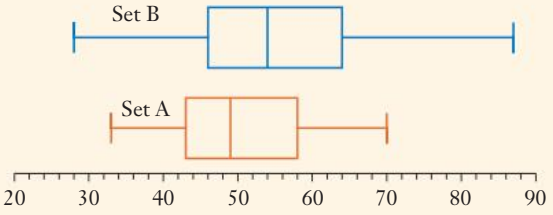
Set A:	33	58	49	70	39	60	43	50	43	70		
	43	58	34	58	44	62	41	50	47			
Set B:	49	87	54	70	42	61	39	64	28	70	44	64
	53	62	33	64	50	60	46	85	49	63	49	

Solution

Find A's five-number summary. — The five-number summary for A is 33, 43, 49, 58, 70.

Find B's five-number summary. — The five-number summary for B is 28, 46, 54, 64, 87.

Draw the box plots on the same scale.



Compare the data sets. — Set B is generally higher than A, and is also spread out further. At least one value in set B is lower than all in set A.

There may be obvious reasons why one set of data is different to another, but you should be careful of overstating conclusions. Parallel box plots are particularly useful for *relatively small data sets* or *discrete data sets*.

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EXAMPLE 7

The data below shows the times (in seconds) taken by a group of 14-year-old students to run 50 metres. Draw parallel box plots to compare the times of boys and girls.

Boys:	8.3	8.1	8.1	7.8	8.8	8.1	7.4	8.0	7.8	9.1
	8.2	8.6	8.1	8.2	7.6	8.0	7.6	7.9	8.8	8.5
	8.3	8.3	8.3	8.5	8.2	8.5	8.0	8.0		
Girls:	8.4	8.3	8.8	8.2	8.9	9.7	8.5	8.4	8.8	9.0
	8.7	8.3	8.1	8.4	9.0	8.6	8.9	8.4	9.3	9.1
	8.5	8.4	9.5	8.5	8.5	8.5	9.7	8.9	8.6	8.4

Solution

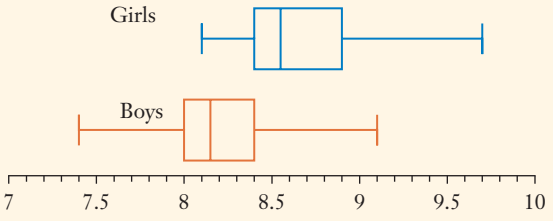
Find the five-number summary for the boys.

The boys' five-number summary is 7.4, 8.0, 8.15, 8.4, 9.1.

Find the five-number summary for the girls.

The girls' five-number summary is 8.1, 8.4, 8.55, 8.9, 9.7.

Draw the parallel box plot.



Compare the times.

These boys were generally faster (had lower times) than the girls, although there were some girls who were faster than most of the boys.

Example 6,7

Exercise 12.03 Using parallel box plots to compare data sets

1 Construct parallel box plots of these data sets to compare them.

Set A:	10	14	14	39	47	22	46	25	33	33	38
	13	33	40	34	19	14	36	57	38	37	27
Set B:	40	38	25	26	43	28	28	12	30	29	
	21	34	33	37	20	13	7	55	31		

2 Construct parallel box plots of these data sets to compare them.

Set A:	15	32	13	28	14	23	20	29	15		
	26	18	26	11	28	19	24	15	30		
Set B:	15	25	11	24	17	22	16	19	11	23	15
	17	16	21	17	19	9	19	14	20	16	

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3 Construct parallel box plots of these data sets to compare them.

Set A:	8	27	24	26	21	23	28	21	21	17	15	27	24	10
	20	24	18	30	23	26	29	19	28	16	22	23	27	
Set B:	31	27	24	23	30	13	25	28	15	11	21	30	33	33
	22	28	25	10	15	24	27	30	7	32				

4 Construct parallel box plots of these data sets to compare them.

Set A:	50	76	49	58	39	69	53	33	46	64	36	55
	49	80	48	75	40	60	67	69	58	79	56	
Set B:	40	69	50	58	79	65	67	54	37	69		
	56	60	65	78	57	50	46	71	57			

Problem solving

5 The relative humidity (%) in two towns at 3 p.m. each day is recorded over 2 weeks.

Town A: 33, 35, 67, 45, 48, 67, 84, 56, 58, 57, 45, 48, 68, 56

Town B: 45, 48, 67, 78, 79, 84, 65, 58, 43, 59, 69, 89, 78, 69

Draw parallel boxplots to compare the likely storm activity of these towns.

6 The daily maximum temperatures (°C) in two Queensland towns over a fortnight were:

Town A: 22, 24, 28, 32, 35, 27, 24, 29, 28, 29, 27, 31, 23, 30

Town B: 25, 26, 26, 24, 26, 28, 28, 26, 29, 30, 27, 24, 29, 31

Draw parallel boxplots to find which town is more likely to be near the sea. Explain your answer.

7 Golf balls have a nominal diameter of 4.3 cm. Samples of 20 balls from two makers had the following diameters (in cm):

Beaut Ball Company:	4.29	4.30	4.29	4.30
	4.34	4.30	4.32	4.30
	4.29	4.29	4.28	4.31
	4.31	4.27	4.28	4.30
	4.29	4.30	4.28	4.28
E-Zee Ball Company:	4.27	4.36	4.31	4.30
	4.30	4.29	4.28	4.30
	4.37	4.31	4.30	4.37
	4.28	4.30	4.26	4.34
	4.31	4.35	4.34	4.39

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Use parallel boxplots to decide which company has better production processes. Justify your answer.

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8 Twenty-five smokers who entered a new program to stop smoking were surveyed on the number of cigarettes that each smoked per day. Two months after completing the program, the same people were surveyed again.

Cigarettes per day before program:

9	30	8	32	10	36	18	20	10	27	17	24	18
11	12	24	17	21	15	23	15	22	18	13	22	

Cigarettes per day after program:

8	34	9	21	11	23	8	23	12	9	24	19	25
17	23	16	22	17	27	16	24	12	10	21	9	

Use side-by-side box plots to determine the success of the program and justify your answer.

12.04 Comparing data sets

Comparing city temperatures

Parallel box plots are very good for comparing small sets of data.

You should use the centre and spread to compare large data sets.

Comparing word lengths

Comparing sports scores

Comparing group measures

Investigating young drivers

Comparing measures of central tendency and spread

- Use median and interquartile range for discrete data
- Use mean and standard deviation for continuous data
- The range is less important than the other measures of spread. It is used only if they are equal.
- Discard invalid outliers and recalculate the measures before comparing
- Use the IQR test for discrete data and the standard deviation test for continuous data to find possible outliers
- The data set with the greater mean/median is above the other group
- The data set with the greater IQR/SD has the greater spread
- Compare individual scores from related data sets using the number of standard deviations from the mean. The number of IQRs from the median should only be used for discrete data sets with a small range

The range is determined by only the minimum and maximum values, so it is generally considered to be only a crude measure of spread. Although the interquartile range is also determined by only two values, their positions are affected by the rest of the data set.

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EXAMPLE 8

Compare the following sets of discrete data.

Set A:	2	13	6	16	7	14	6	15	12	16	9	13
	8	13	8	14	2	14	5	15	6	12	10	
Set B:	6	14	9	11	8	14	9	14	6	12	9	
	16	10	11	8	22	6	11	8	15	8	10	

Solution

Arrange set A in order.	2, 2, 5, 6, 6, 6, 7, 8, 8, 9, 10, 12, 12, 13, 13, 13, 14, 14, 14, 15, 15, 16, 16
State the median and IQR.	Median = 12, and interquartile range = 8
Arrange set B in order.	6, 6, 6, 8, 8, 8, 8, 9, 9, 9, 10, 10, 11, 11, 11, 12, 14, 14, 14, 15, 16, 22
State the median and IQR.	Median = 10 and interquartile range is 6
Make the comparison.	Set A is the higher set, but B has the lower spread.

Even though the range of set B is 16 and the range of set A is 14, we still say that set B has the lower spread because its interquartile range is smaller than set A. The range would only be considered if the interquartile ranges were equal.

For **continuous data**, you should use the **standard deviation test** to determine any possible outliers. Remember that if $x < \bar{x} - 2.5 \times \text{SD}$ or $x > \bar{x} + 2.5 \times \text{SD}$, then x is a possible outlier.

EXAMPLE 9

Compare the following sets of continuous data.

Set A:	17.3	16.8	18.1	15.4	16.9	16	16.7	15.6	17.8	17.5
	16.2	16	16.3	15.8	17	19.4	18.2	17.2	18	18.5
	15	16.4	19.7	17.6	18.6	17	16.2	17.9		
Set B:	18.6	18.4	20.1	20	19.1	21.3	17.4	19	19.7	
	16.7	21.6	18.5	17	18.6	20.9	19.1	21.5	21.2	
	18.3	17.2	16.9	20.2	19.4	19.4	20			

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Solution

Use your calculator or other technology to find measures for set A.	For set A, $\bar{x} \approx 17.1$ and $s \approx 1.16$
Do set B.	For set B, $\bar{x} \approx 19.2$ and $s \approx 1.44$
Make the comparison.	Set B is higher and more spread than set A.

EXAMPLE 10

In English, Aya scored 28 out of 40 and in Maths she scored 17 out of 25. The mean and standard deviation for English were 24 and 6 respectively, while for Maths they were 15 and 5. In which subject did she do better?

Solution

Find the English result in SDs.	English result $= \dfrac{28-24}{6} \approx 0.67$ SDs above mean
Find the Maths result in SDs.	Maths result $= \dfrac{17-15}{5} = 0.4$ SDs above mean
Make the comparison.	Aya did better in English than Maths as her result was more standard deviations above the mean.

INVESTIGATION

RESULTS IN YOUR CLASSES

Your teacher should be able to obtain the results in some of the subjects taken by students in your maths class last term or last semester.

- Calculate the means and standard deviations of the subjects.
- Check your calculations with other students in your class.
- Compare your own results in different subjects.
- Do the results of your calculations agree with your own feelings about which subjects your best ones?

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Exercise 12.04 Comparing data sets

Example 8

1 Compare the following sets of discrete data.

Set A: 18, 20, 17, 21, 20, 22, 19, 21, 18, 20, 19, 24, 18, 23, 18, 20, 18, 21, 20

Set B: 20, 25, 20, 23, 19, 25, 18, 23, 19, 22, 21, 25, 20, 26, 17, 23, 18, 23, 20, 23, 22

2 Compare the following sets of discrete data, excluding possible outliers.

Set A: 25, 27, 24, 31, 21, 26, 24, 27, 23, 37, 24, 29, 21, 30, 24, 29, 21, 32

Set B: 27, 29, 26, 28, 22, 32, 19, 30, 26, 29, 22, 28, 26, 34, 28, 30, 26, 29, 27, 29

3 Compare the following sets of discrete data, excluding possible outliers.

Set A: 22, 28, 22, 32, 19, 29, 24, 33, 23, 31, 8, 29, 1, 28, 21

Set B: 22, 29, 23, 26, 19, 26, 17, 24, 20, 26, 17, 33, 20, 25, 19, 26, 20, 24

Example 9

4 Compare the following sets of continuous data.

Set A:	11	13	9	15	21	18	16	13	8			
	7	17	13	14	17	20	15	17	14			
Set B:	14	15	19	21	13	15	20	17	17	18	17	17
	17	14	22	15	15	21	22	18	18	17	18	20

5 Compare the following sets of continuous data, excluding possible outliers.

Set A:	48	48	37	48	33	35	72	31	42	32	41	43
	44	55	15	38	40	44	58	45	53	44	46	
Set B:	35	32	57	41	36	38	37	42	34	50	64	36
	36	48	37	33	45	28	58	35	42	41	31	

6 Compare the following sets of continuous data, excluding possible outliers.

Set A:	27	40	24	26	25	28	24	27	22	
	25	23	32	27	26	23	26	21	39	
	26	19	24	30	20	22	22	29	17	
Set B:	37	9	34	42	27	37	33	37	36	26
	19	46	37	33	30	25	32	21	37	43
	28	42	20	29	40	33	28	30	31	38

Example 10

7 Which is the more unusual of these?

A: A score of 20 in a population with a mean of 25 and a standard deviation of 2

B: A score of 33 in a population with a mean of 40 and a standard deviation of 5

8 Which is the higher of these?

A: A score of 16 in a population with a mean of 12 and a standard deviation of 3

B: A score of 14 in a population with a mean of 10 and a standard deviation of 2

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9 Which is the lower anxiety level?

A: An anxiety score of 12 on a psychological questionnaire with a mean of 16 and a standard deviation of 5

B: An anxiety score of 16 on a psychological questionnaire with a mean of 24 and a standard deviation of 6

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Problem solving

10 The following scores were obtained by students from the same class.

English:	13	14	16	12	8	6	15	18	12
	14	13	11	10	9	7	9	12	8
	9	7	10	10	9	11	13		
Maths:	5	2	9	7	9	12	8	9	7
	10	10	9	11	18	11	14	16	17
	8	6	20	18	12	4	6		

Compare the results.

11 Compare these results of 15 students who study both Art and Music.

Art: 38, 28, 56, 35, 40, 17, 53, 29, 80, 60, 40, 32, 53, 41, 47

Music: 63, 52, 64, 58, 48, 56, 34, 60, 48, 55, 61, 46, 49, 53, 43

12 Julia is 17 and thinks that she is overweight. She is 168 cm tall and weighs 60 kg. The mean weight for Australian women aged 17 or over is 67 kg with a standard deviation of 15 kg. The mean height is 161 cm with a standard deviation of 7 cm. Is she really fatter or thinner than average?

13 The lengths of words in typical definitions in two biology textbooks were counted:

Text 1: 5, 8, 10, 6, 9, 6, 10, 8, 5, 6, 8, 10, 8, 7, 9, 7, 8, 10

Text 2: 12, 8, 5, 24, 7, 9, 5, 5, 22, 5, 13, 9, 10, 9, 5, 17, 14, 8, 14, 7

Compare the difficulty of the texts, assuming that longer words make a text harder to read.

14 The pH of the water in a swimming pool should be close to 7.6. The pH in two pools with different automatic dosing systems was measured at 3 p.m. each day for a month with the following results. You can adjust the set pH level on each system. They were both set to 7.6 for the trial. Which dosing system would you recommend? Explain your answer.

Easy as ABC:	7.2	7.5	7.5	7.1	7.6	7.4	7.1	7.5	7.4	7.3
	7.3	7.5	7.2	7.5	7.6	7.4	7.5	7.3	7.2	7.5
	7.6	7.2	7.4	7.2	7.5	7.5	7.5	5.6	7.6	7.3

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Setnforget:	7.7	7.7	7.2	7.4	7.6	7.5	7.5	7.3	7.5	7.5
	7.4	7.6	7.5	7.6	7.5	7.3	7.6	7.5	7.6	7.7
	7.5	7.5	7.6	7.5	7.1	7.3	7.5	5.8	7.3	7.5

15 A local cricket club has narrowed the choice of the last batsman for a match down to two candidates. Their scores in their last 12 matches are as follows.

'Basher' Smith: 6, 27, 34, 22, 31, 21, 0, 32, 21, 21, 37, 21

'Slogger' Jones: 18, 25, 0, 17, 27, 12, 24, 25, 42, 25, 28, 27

Which batsman should the club choose, and why?

16 A large catering company buys its vegetables direct from producers. It has samples of brown onions from two farmers, with the masses of the onions (in grams) shown below.

Farmer Hoe:	201	212	263	253	73	292	190	198	200	289
	321	122	348	374	129	143	96	208	187	206
Farmer Joe:	189	282	162	165	152	218	161	312	122	192
	199	190	127	204	195	140	156	205	133	198

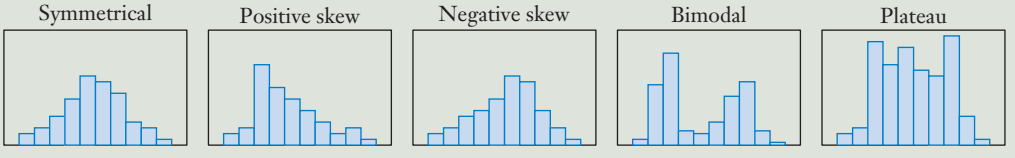
Which producer should the company choose? Carefully justify your choice.

12.05 Comparing the shapes of data sets

In Chapter 9, you saw that the shape of a set of data can be described from a histogram or frequency polygon.

The shape of a data set

- **Symmetrical** data sets spread evenly to each side of a central peak
- **Positively skewed** data sets spread more on the right side of the peak
- **Negatively skewed** data sets spread more on the left side of the peak
- **Bimodal** distributions data sets have two distinct peaks
- **Plateau** distributions have similar frequencies for most of the range



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EXAMPLE 11

Compare the following sets of discrete data.

Set A

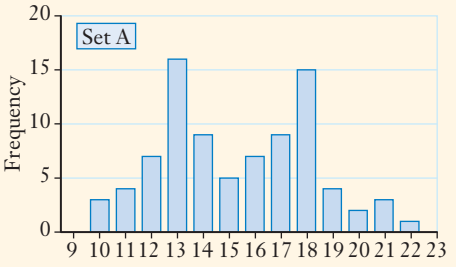
x	10	11	12	13	14	15	16	17	18	19	20	21	22
f	3	4	7	16	9	5	7	9	15	4	2	3	1

Set B

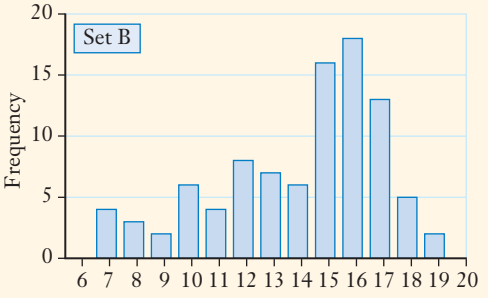
x	7	8	9	10	11	12	13	14	15	16	17	18	19
f	4	3	1	6	4	8	7	6	16	18	13	5	2

Solution

Draw a column graph of set A.



Draw a column graph of set B.



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Draw cumulative frequency tables to find the medians and interquartile ranges.

x	f	cf
10	3	3
11	4	7
12	7	14
13	16	30
14	9	39
15	5	44
16	7	51
17	9	60
18	15	75
19	4	79
20	2	81
21	3	84
22	1	85

x	f	cf
7	4	4
8	3	7
9	2	9
10	6	15
11	4	19
12	8	27
13	7	34
14	6	40
15	16	56
16	18	74
17	13	87
18	5	92
19	2	94

Find the medians and interquartile ranges.

Set A: Median = 15, IQR = 5

Set B: Median = 15, IQR = 4

Make the comparison.

The data sets have the same median, and set B is a little less spread than set A.

Set A is bimodal and set B is negatively skewed.

EXAMPLE 12

Compare the following sets of continuous data.

Set A

x	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54
f	6	19	34	25	12	10	16	6	5	7

Set B

x	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54
f	8	3	13	13	15	25	31	16	8	3

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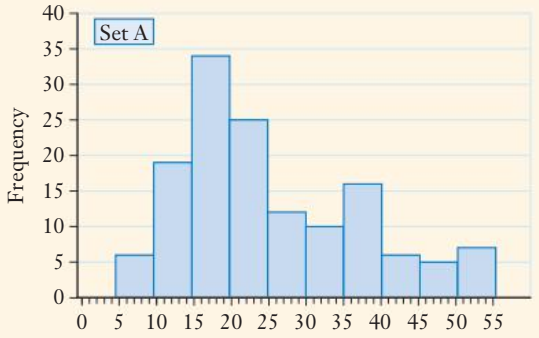
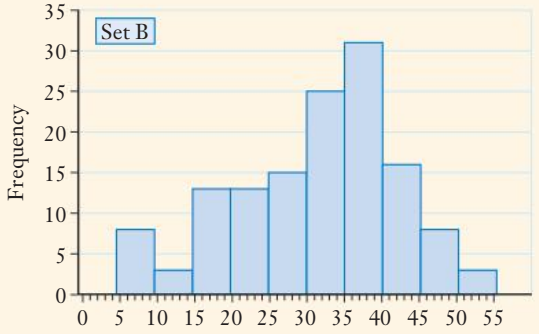
Solution

Draw a histogram of set A.



Draw a histogram of set B



Add fx and fx^2 columns to the tables to find the means and standard deviations.

x	f	fx	fx^2
7	6	42	294
12	19	228	2736
17	34	578	9826
22	25	550	12 100
27	12	324	8748
32	10	320	10 240
37	16	592	21 904
42	6	252	10 584
47	5	235	11 045
52	7	364	18 928
$\sum$	140	3485	106 405

x	f	fx	fx^2
7	8	56	392
12	3	36	432
17	13	221	3757
22	13	286	6292
27	15	405	10 935
32	25	800	25 600
37	31	1147	42 439
42	16	672	28 224
47	8	376	17 672
52	3	156	8112
$\sum$	135	4155	143 855

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Find the means and standard deviations.	Set A: $\bar{x} = \frac{3485}{140} = 24.89\ldots, s = \sqrt{\frac{106\,405}{140} - 24.89\ldots^2} = 11.848\ldots$ Set B: $\bar{x} = \frac{4135}{135} = 30.77\ldots, s = \sqrt{\frac{143\,855}{135} - 30.77\ldots^2} = 10.877\ldots$
Make the comparison.	Set B is higher than set A with a mean of about 30.8 compared to 24.9. It is a little less spread out. Set B is negatively skewed and set A is positively skewed.

You can use your calculator to find the mean and median, as shown in Chapter 9 on page 377.

You can use combined graphs and plots to compare data sets directly.

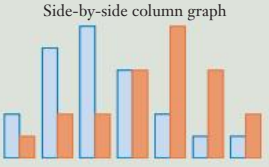
Comparing the shapes of data sets

- **Back-to-back stem-and-leaf plots** have leaves from each set on different sides
- **Side-by-side** column graphs use different-coloured columns for each data set
- **Side-by-side histograms** have columns for each data set opposite each other, either vertically or horizontally
- **Population pyramids** are side-by-side histograms for males and females in each age group, with the lowest age group at the bottom

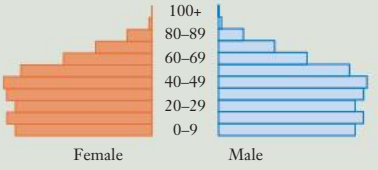
Back-to-back stem-and-leaf plot

Leaves	Stem	Leaves
9	2	7 8
2	3	3 4 7 9
6 5 4	4	3 3 5
8 4 2 2	5	8 9
2 1	6	1
1	8	

Side-by-side column graph



Population pyramid
Australia census



EXAMPLE 13

Use a back-to-back stem-and-leaf plot to compare the ages of Year 11 students' mothers and fathers shown below.

Mothers' ages:

38	43	35	52	55	57	47	49	39	44	46	43
48	44	40	51	53	36	42	49	52	39	44	

Fathers' ages:

41	46	38	55	58	57	48	48	40	49	45	49
52	48	43	57	61	38	42	51	54	41	44	

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Solution

The ages range from 53 to 61, so the stems go from 3 to 6.

Put the mothers leaves on the right and the fathers on the left.

Ages of parents of Year 11 students

Father: Leaf	Stem	Mother: Leaf
8 8	3	5 6 8 9 9
9 9 8 8 8 6 5 4 3 2 1 1 0	4	0 2 3 3 4 4 4 6 7 8 9 9
8 7 7 5 4 2 1	5	1 2 2 3 5 7
1	6	

Key: 8|3 = 38 = 3|8

Make the comparison.

The fathers of Year 11 students are in general a little older than the mothers.

Example 11

Exercise 12.05 Comparing the shapes of data sets

1 Compare these sets of discrete data.

Set A

x	20	21	22	23	24	25	26	27	28	29	30
f	2	8	10	13	11	18	16	9	8	4	2

Set B

x	20	21	22	23	24	25	26	27	28	29	30
f	1	7	6	10	15	13	16	11	12	2	3

2 Compare these sets of discrete data.

Set A

x	6	7	8	9	10	11	12	13	14	15	16	17	18	19
f	5	6	6	9	10	8	18	18	20	10	12	8	7	3

Set B

x	6	7	8	9	10	11	12	13	14	15	16	17	18	19
f	4	9	12	18	18	21	20	14	8	12	7	4	1	3

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3 Compare these sets of discrete data.

Set A

x	44	45	46	47	48	49	50	51	52	53	54	55	56
f	4	6	5	9	9	8	10	12	11	6	8	6	6

Set B

x	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58
f	2	1	7	10	9	16	17	15	11	18	9	5	8	1	1	2

4 Compare these sets of continuous data.

Example 12

Set A

x	30–34	35–39	40–44	45–49	50–54	55–59	60–64	65–69	70–74
f	4	7	21	39	40	24	8	6	1

Set B

x	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	65–69	70–74	75–79	80–84
f	4	9	20	16	20	17	22	12	8	9	4	4

5 Compare these sets of continuous data.

Set A

x	70–79	80–89	90–99	100–109	110–119	120–129	130–139
f	2	12	25	28	14	11	1

Set B

x	70–79	80–89	90–99	100–109	110–119	120–129	130–139	140–149
f	1	13	27	31	25	16	5	2

6 Compare these sets of continuous data.

Set A

x	1–5	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
f	3	8	15	17	23	16	15	24	12	4

Set B

x	1–5	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
f	7	7	7	12	18	23	17	18	16	9

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Example 13

7 Use a back-to-back stem-and-leaf plot to compare the annual incomes of samples of plumbers and teachers.

Plumbers ($'000):	61	73	57	78	63	106	48	76
	57	73	64	65	58	73	54	81
	44	80	53	76	56	97	59	89
Teachers ($'000):	52	94	61	109	74	103	66	117
	68	87	74	127	74	92	66	118
	69	82	56	103	58	118	74	109

8 Use a side-by-side column graph to compare the age samples of students working part-time at a takeaway and a large hardware store.

Takeaway: 15, 19, 16, 16, 15, 19, 15, 17, 16, 16, 15, 16, 16, 17

Hardware: 18, 21, 17, 19, 19, 20, 17, 20, 17, 20, 17, 22, 19, 20

Problem solving

9 The populations by age of three countries are shown below.

Japan		Canada		Mexico	
Age	Thousands	Age	Thousands	Age	Thousands
0–4	4982	0–4	1961	0–4	11 156
5–9	5318	5–9	1985	5–9	11 276
10–14	5548	10–14	1886	10–14	11 350
15–19	6062	15–19	2065	15–19	11 100
20–24	6141	20–24	2469	20–24	10 225
25–29	6427	25–29	2517	25–29	9411
30–34	7298	30–34	2530	30–34	8940
35–39	8188	35–39	2456	35–39	8359
40–44	9761	40–44	2345	40–44	5046
45–49	9108	45–49	2415	45–64	20 937
50–54	7945	50–54	2711	65+	7880
55–59	7547	55–59	2653		
60–64	8234	60–64	2300		
65–69	10 272	65–69	1976		
70–74	7409	70–74	1439		
75–79	6447	75–79	1034		
80–84	5153	80–84	753		
85–89	3247	85–89	493		
90–94	1460	90+	294		
95–99	380				
100+	66				

Change the classes if necessary to compare the populations. What can you conclude about these countries? Justify your answers.

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12.06 Comparing data and the statistical investigation process

You had a brief introduction to the statistical investigation process in Chapter 9. The approach is much more useful when comparing groups.

The **statistical investigation process** has the following steps:

- Refine the problem to pose a question
- Determine the population
- Decide relevant variables
- Decide how data will be collected
- Collect data
- Display and analyse data
- Interpret and communicate results

You might be interested in the best use of students' time to improve school results. There are many different aspects to this problem.

EXAMPLE 14

Refine the problem about use of student time to pose some questions and determine the population.

Solution

To refine the problem, you need to consider some possible uses of student time that might affect student performance in tests and exams.	Some possible factors could be school attendance, completion of classwork, completion of homework, regular time spent on homework, physical fitness, study before tests, revision program and so on.
Refine some of these into questions.	Does school attendance affect results? Does completion of classwork affect results? Does homework completion affect results? Does homework time affect results? Does playing sport affect results? Does regular exercise affect results? Does study in the prior week affect results? Does study the prior night affect results?
What is the population?	The population is school students.

It is not usually possible to investigate all questions.

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Once you have decided on some questions to investigate, you need to decide on some variables that you can investigate and decide how data will be collected.

EXAMPLE 15

Choose some variables to investigate the link between time spent by students and their results. How can data be collected?

Solution

What variables could you investigate about homework?	The amount of time spent each week on set homework, t is a possible variable. The amount of set homework completed, h as a proportion is a possible variable.
What variables could you investigate about results?	The results in each subject as percentages or as grades, g, is a possible variable.
What variables could you investigate about exercise?	The time spent on exercise each week, e, is a possible variable.
How could the data be collected?	Survey students in different subjects, as any possible relationships could depend on the subject.

Once you have collected data, you need to examine the data to determine the results.

EXAMPLE 16

The data sets below were obtained by dividing Year 11 students doing 2 or more Science subjects into those who obtained at least a C in each subject and those who got less than a C in at least one subject. The results of the question, how much time did you spend on Science homework last week were compile for each groups. Compare the groups.

C+ group

Time (hours)	0–1	2–3	4–5	6–7	8–9	10+
Number	3	6	21	22	10	5

C– group

Time (hours)	0–1	2–3	4–5	6–7	8–9	10+
Number	11	8	6	5	1	0

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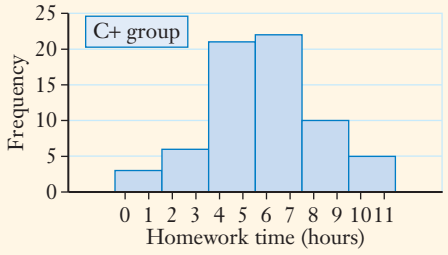
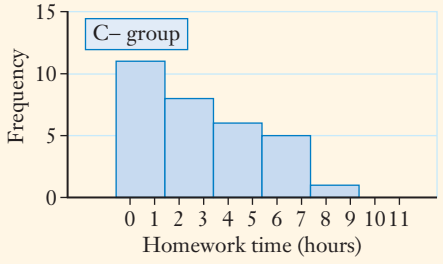
Solution

Draw a histogram for the C+ group.



Draw a histogram for the C– group.



Find the mean and standard deviation for each group.

C+ group mean ≈ 5.8, SD ≈ 2.38

C– group mean ≈ 3.0, SD ≈ 2.38

Make the comparison.

The groups are equally spread, but the C+ students with better results did about 2.8 hours more homework. Their homework hours formed a symmetrical distribution, but the C– students who did less well had a positively skewed distribution.

These conclusions may seem obvious. However, without proper investigation you cannot be sure that 'obvious' answers are actually true.

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INVESTIGATION

SCHOOL ISSUES

In small groups, investigate a school issue in which you are interested Your teacher will need to approve your investigation and the proposed methods.

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You might choose one of the following, or some other issue.

- Classwork and results
- Exercise and results
- Number of siblings and results
- Sport participation
- TV habits
- Time spent travelling
- Homework and results
- Parents' education and results
- Part-time work
- Gaming
- Hobbies
- Sleep habits

Exercise 12.06 Comparing data and the statistical investigation process

Example 14

1 Refine the following problems to decide some questions for statistical investigation.

a TV viewing habits among high school students.

b Computer use outside school.

c Instruments played by high school students.

Example 15

2 Choose some variables to investigate the following.

a Sports participation

b Part-time work

c Car ownership

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Example 16

3 The results below show the amounts of time students reported doing exercise in a week in two groups: those who got at least 5 Cs and those that got fewer than 5 Cs last semester. Compare the groups.

5 C+ group

Hours	0–1	2–3	4–5	6–7	8–9	10–11	12–13	14–15
Number	4	10	14	7	6	0	2	2

Under 5 C group

Hours	0–1	2–3	4–5	6–7	8–9	10–11	12–13	14–15	16–17	18–19
Number	8	9	2	2	2	1	3	1	1	1

4 The results below show the amounts of time students reported working part-time per week and their sports participation. Compare the sports players with the non-sports players.

Sports players

Hours	0–1	2–3	4–5	6–7	8–9	10–11
Number	25	5	7	3	1	2

Non-sports players

Hours	0–1	2–3	4–5	6–7	8–9	10–11	12–13	14–15
Number	22	3	4	1	3	5	3	2

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12. CHAPTER SUMMARY

Comparing data

Five-number summary

- Q_0 is the **minimum** value
- Q_1 is the **first quartile**
- Q_2 is the **median**
- Q_3 is the **third quartile**
- Q_4 is the **maximum** value

Interquartile range outlier test

- If $x < Q_1 - 1.5 \times \text{IQR}$ or $x > Q_3 + 1.5 \times \text{IQR}$, then x is a possible outlier
- Invalid outliers should be excluded from a data set

Box plots

- Draw a rectangle above the scale from Q_1 to Q_3, the first and third quartiles
- Draw a vertical line that divides the rectangle at Q_2, the median
- Draw horizontal lines ('*whiskers*') from the ends of the rectangle out to Q_0 and Q_4, the minimum and maximum. You can put stop lines at the ends
- Excluded outliers can be shown by crosses or dots at the level of the whiskers

Standard deviation outlier test

- If $x < \bar{x} - 2.5 \times \text{SD}$ or $x > \bar{x} + 2.5 \times \text{SD}$, then x is a possible outlier

Comparing measures of central tendency and spread

- Use median and interquartile range for discrete data
- Use mean and standard deviation for continuous data
- The range is less important than the other measures of spread. Use only if they are equal
- Discard invalid outliers and recalculate the measures before comparing
- Use the IQR test for discrete data and the standard deviation test for continuous data to find possible outliers
- The data set with the greater mean/median is the higher group
- The data set with the greater IQR/SD has the greater spread
- Compare individual scores from related data sets using the number of standard deviations from the mean. The number of IQRs from the median should only be used for discrete data sets with a small range

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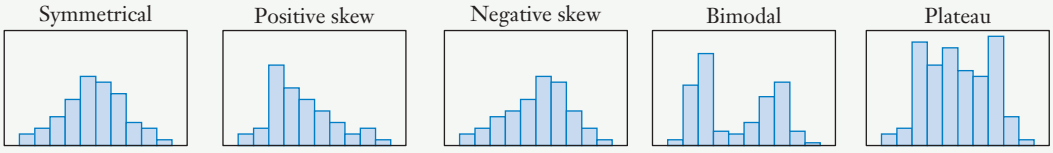
The shape of a data set

- **Symmetrical** data sets spread evenly to each side of a central peak
- **Positively skewed** data sets spread more on the right side of the peak
- **Negatively skewed** data sets spread more on the left side of the peak
- **Bimodal** distribution data sets have two distinct peaks
- **Plateau** distributions have similar frequencies for most of the range

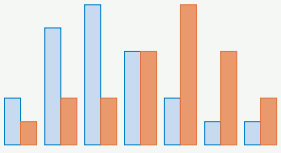


Comparing the shapes of data sets

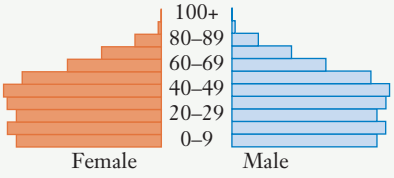
Back-to-back stem-and-leaf plot

Leaves	Stem	Leaves
9	2	7 8
2	3	3 4 7 9
6 5 4	4	3 3 5
8 4 2 2	5	8 9
2 1	6	1
1	8	

Side-by-side column graph



Population pyramid



The statistical investigation process has the following steps:

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- Determine the population
- Decide relevant variables
- Decide how data will be collected
- Collect data
- Display and analyse data
- Interpret and communicate results

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12. CHAPTER REVIEW

Comparing data

Example 1

1 Find the five-number summary for these scores.

38 41 32 52 37 44 31 42 31 43 31 48 37 45
38 42 34 40 32 44 34 43 33 44 38 41 33

Example 2

2 Find the five-number summary for each data set.

a

x	5	6	7	8	9	10	11	12	13	14	15
f	1	4	1	3	9	6	3	20	11	6	6

b

x	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89
f	17	36	101	44	37	30	21	18	16

Example 3

3 Find the five-number summary for each data set and check for outliers.

a 5, 11, 3, 10, 5, 16, 3, 20, 7, 12, 3, 26, 5, 15, 6, 9, 6, 10

b 10, 20, 11, 18, 11, 16, 12, 18, 9, 20, 10, 12, 10, 14, 10, 16, 11, 17, 10, 13, 8, 13

c 39, 45, 12, 49, 29, 41, 28, 43, 39, 43, 34, 46, 32, 40, 33, 42, 38, 44, 35

Example 4

4 Draw a box plot for the five-number summary 20, 25, 31, 33, 42 with the excluded outliers 5, 57, 62.

Example 5

5 Draw a box plot for each data set, showing possible outliers as appropriate.

a 0 11 7 14 4 11 6 9 4 13 6 19 0 11 3 17
3 29 5 11 1 12 4 12 6 15 5 12 5 9

b

x	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
y	1	0	1	3	7	15	13	8	3	2

Examples 6,7

6 Construct parallel boxplots of the following data sets to compare them.

Set A: 18 19 14 16 24 12 23 16 20 32 27 14
24 20 20 11 23 16 14 25 18 33

Set B: 16 29 20 22 14 24 25 14 17 25 21 25 20 21
27 27 27 18 16 28 25 27 22 20 26 29 22

Examples 8,9

7 Compare the following sets of continuous data.

Set A: 30.6 43.4 43.6 49.1 50.3 24.3 33 45.2 20.5 40.7
17.2 33 34.8 32.2 71.2 44.1 37.9 42.7 26.3

Set B: 41.2 27.6 41.2 24.7 28.7 30.1 27.1 23.9 47.9 44 24.9
24.2 18.3 20.1 51.4 30.9 34 38.1 26.9 22.8 22.6

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8 Campbell scored 16 out of 30 for a test in History and 15 out of 24 for a test in Geography. The mean and standard deviations of the marks for the tests were 14.2 and 6.4 for History, and 13.1 and 4.8 for Geography. In which subject did he do better? Example 10

9 Compare the following sets of discrete data. Example 11

Set A

x	5	6	7	8	9	10	11	12	13
f	1	2	4	5	6	12	15	6	2

Set B

x	5	6	7	8	9	10	11	12	13	14
f	1	4	8	12	10	11	10	9	3	2

10 Compare the following sets of continuous data. Example 12

Set A

x	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49
f	2	4	7	11	5	4	10	6	2

Set B

x	0–4	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49
f	1	3	5	9	16	19	14	6	2	1

11 Use a back-to-back stem-and-leaf plot to compare the ages of teachers and firefighters. Example 13

Teachers:	36	23	61	50	52	40	65	62	48	59	57	58	59	54	28
	56	44	64	23	64	57	44	61	43	43	37	45	38	42	22
	50	55	43	45	49	23	54	55	52	41					
Firefighters:	47	34	43	38	37	21	38	25	47	40	38	30	44	43	
	36	63	42	42	29	33	41	53	34	50	43	38	37	35	
	34	50	36	55	49	28	27	47	44	34	52	53			

12 **a** Refine the investigation of gaming habits of Year 11 students to pose some questions for statistical investigation. Examples 14,15

b State some variables for the investigation.

Problem solving

13 The following are the self-reported amounts of pocket money that some Year 8 students said they received each week.

\$5	\$8	\$3	\$0	\$0	\$10	\$20	\$6	\$12	\$8	\$2	\$2	\$0	\$0
\$30	\$24	\$18	\$600	\$10	\$15	\$3	\$8	\$5	\$10	\$20			

Find the typical amount and draw a five-number summary. Comment on the results.

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14 These were the amounts of monthly weight loss in kg on a new diet, according to a report prepared by an advertising company.

4.2	27.8	4.2	12.3	4.4	63.2	3.2	65.9	2.4	22.5
4.4	67.2	0.3	7.6	0.3	26.5	3.2	23.8	2.8	26.2
4.4	12.5	4.2	8.3	5	18.3	2.5	28.6	4.4	14.3

State the typical amount lost, commenting as necessary.

15 The numbers of bookings each week for two restaurants were as follows.

Brodie's Brasserie:	75	73	72	88	88	102	61	84	96	81
	77	50	81	93	66	98	78	88	69	97
	56	75	97	89	67	85	72	76	49	98
	95	63	81	131	135	78	71	87	86	77
	80	83	102	45	50	83	89	92	79	87
Swift's Steakhouse:	36	56	53	1	57	53	65	36	57	104
	50	61	86	50	63	64	63	44	56	45
	73	59	64	41	67	41	72	61	62	65
	55	42	71	50	58	49	37	52	50	50
	44	41	36	55	55	79	59	37	51	75

Draw parallel box plots to compare the restaurants. State any conclusions you can make about the restaurants. Justify your conclusions.

16 The chlorine level in swimming pools should be close to 3 parts per million (ppm). Two different chlorinating systems were used in similar pools and their chlorine levels measured at midday over 30 days. The results were as follows.

System A:	3.2	3.6	2.2	3.7	3.5	2.5	2.4	3.1	2.8	3.3
	3.8	2.9	3.1	2.4	2.4	2.7	3.5	3.2	4.4	3.6
	3.4	2.9	3.8	3.3	3.6	3.8	2.8	3.3	2.7	2.6
System B:	2.9	3.8	2	3	3.5	3.2	2.8	3.1	2.4	3.3
	3.5	1.2	2.2	2.8	2.5	1.7	1.4	2.5	3.6	2.8
	2.5	4	3.4	2.2	4.2	2.2	3.5	3.4	3.3	3,

Compare the systems and decide which is better. Carefully explain your decision.

Practice quiz

17 The data below shows the proportions of employees who were members of a trade union. Use an appropriate display to compare the proportions.

Age group	15–19	20–24	25–34	35–44	45–54	55–59	60–64	65+
Males	27%	38%	47%	52%	53%	56%	51%	13%
Females	27%	32%	35%	35%	41%	43%	26%	3%

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Practice examination 4

Time: 90 minutes
Perusal time: 5 minutes
Marks: 50

Instructions

- Students are permitted to bring or use: pens, pencils, highlighters, erasers, sharpeners, rules and an approved graphics calculator.
- Students must show appropriate working and justification to gain full marks.
- A formula sheet is provided.
- Unless otherwise stated, numerical answers should be exact.
- Unless otherwise indicated, no diagrams in this examination are drawn to scale.
- All written responses must be in English.
- Answer all questions.
- **Students are NOT permitted to bring or use notes of any kind, correction fluid/tape, mobile phones and/or any other unauthorised electronic devices.**

Year 11 formulas

Question 1 (1 mark)

Which of the following are matrices?

X $\begin{bmatrix} \text{Yes} & \text{No} \\ \text{No} & \text{Yes} \end{bmatrix}$ **Y** $\begin{bmatrix} 5 & 3 & 2 \\ -2 & & 4 \end{bmatrix}$ **Z** $\begin{bmatrix} 2 & 3 \\ -0.4 & 2 \\ 0.4 & -1 \end{bmatrix}$

Question 2 (3 marks)

$\mathbf{A} = \begin{bmatrix} 4 & -2 \\ 3 & -1 \\ 2 & 5 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 4 & 3 & 1 & 0 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 1 & 0 & -3 \\ -3 & 0 & 4 \\ 2 & 2 & 2 \end{bmatrix}$.

a Describe what type of matrix each of **A**, **B** and **C** are.

b Find a_{23}, b_{13} and c_{23} if possible. If not, explain why not.

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Question 3 (3 marks)

$$\mathbf{A} = \begin{bmatrix} 3 & -4 \\ 2 & 0 \\ 4 & -5 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} -3 & 2 \\ -5 & 6 \\ 0 & 1 \end{bmatrix}$$

Find:

a $\mathbf{A} + \mathbf{B}$ **b** $3\mathbf{B}$ **c** $\mathbf{B} - \mathbf{A}$

Question 4 (3 marks)

Find each product if possible. If not, explain why not.

a $\begin{bmatrix} 2 & 0 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 5 & -1 \\ 3 & 0 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & 1 & -2 \\ 0 & 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & -3 & 0 \\ 6 & 0 & 4 \end{bmatrix}$

Question 5 (1 mark)

A communication matrix for A, B, C, D and E in order is given by

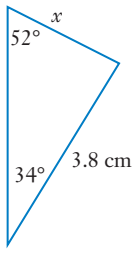
$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

With whom can C communicate directly?

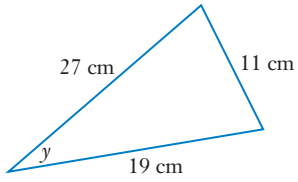
Question 6 (2 marks)

Calculate the value of the unknown in each triangle, correct to one decimal place.

a



b



Question 7 (2 marks)

In $\triangle ABC$, $\angle A = 44°$, $\angle B = 80°$ and $b = 12$ cm. Calculate a correct to one decimal place.

Question 8 (2 marks)

In $\triangle XYZ$, $x = 16$ cm, $y = 14$ cm and $z = 12$ cm. Calculate $\angle X$ correct to one decimal place.

Question 9 (2 marks)

Find the five-number summary for these scores.

16, 37, 22, 36, 31, 38, 26, 36, 26, 44, 26, 39, 15, 43, 29, 41, 27, 40, 25

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Question 10 (2 marks)

Draw a box plot for the five-number summary 3, 6, 9, 11, 13 with an outlier of 20.

Question 11 (3 marks)

Construct parallel box plots of the following data sets to compare them.

Set A: 7, 19, 12, 13, 5, 16, 11, 14, 11, 20, 12, 13, 8, 18, 10, 16, 11, 13, 5, 14, 9, 17, 10, 14

Set B: 8, 17, 10, 19, 6, 18, 9, 17, 6, 19, 11, 17, 9, 19, 10, 16, 4, 16, 9, 15, 5, 16, 13, 16, 11, 15, 12

Question 12 (4 marks)

Compare these two data sets, including their shape.

Set A:

Score	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49
Frequency	1	3	5	6	7	9	12	11	4

Set B:

Score	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49
Frequency	1	6	9	10	14	11	10	7	4

Question 13 (2 marks)

Use a back-to-back stem-and-leaf plot to compare the following sale prices of houses (in thousands of dollars) in Mt Isa and Toowoomba.

Mt Isa:	290	800	300	170	180	430	210	150	235	150	310	200	290
	310	330	420	290	190	180	160	320	290	210	320	210	
Toowoomba:	330	490	230	580	460	420	330	260	230	360	350	470	380
	350	340	340	330	290	270	260	260	230	390	160	500	320

Question 14 (4 marks)

A cold drinks stand at a theme park serves milk shakes and thick shakes in regular, large and giant sizes. The prices of each drink are shown in the table.

	Milk shake	Thick shake
Regular	$3.00	$3.50
Large	$4.00	$4.50
Giant	$5.00	$6.50

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a Construct a 6×1 matrix **P** of the prices.

b Three different customers place orders at the till and then move to the service area. Their orders are:

A: 2 regular milkshakes and 3 large thick shakes

B: 2 giant thick shakes and 2 large thick shakes

C: 2 large milkshakes and a giant thick shake.

Write their orders as the 3×6 matrix **D**.

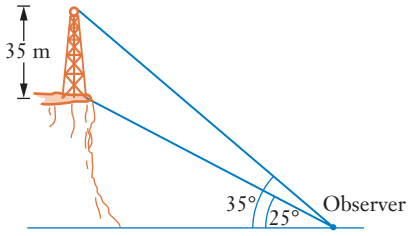
c Use matrix multiplication to find the cost of each order.

Question 15 (2 marks)

A beacon is mounted on a tower that sits on a vertical cliff. The beacon is 35 m higher than the top of the cliff.

To an observer some distance from the base of the cliff, the beacon and the top of the cliff appear to be at angles of elevation of 35° and 25° respectively.

How far is the observer from the base of the cliff?



Question 16 (4 marks)

Compare the masses in kilograms of Year 11 students from two different schools, using the shape of the data sets. Justify any conclusions.

School A:

Mass	40–44	45–49	50–54	55–59	60–64	65–69	70–74	75–79
Number	2	2	4	6	4	3	1	1

School B:

Mass	45–49	50–54	55–59	60–64	65–69	70–74	75–79	80–84	85–89	90–94	95–99	100–104
Number	1	2	2	5	4	1	1	3	4	1	0	1

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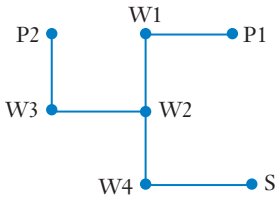
Question 17 (4 marks)

This diagram shows the network connections of an old office local area network.



The server is labelled S, the printers P and the workstations W.

P1 is a black-and-white printer and P2 is a colour printer.

a Construct a communication matrix **M** for the network in the order W1, W2, W3, W4, S, P1, P2.

b Find $\mathbf{D} = \mathbf{M}^2$ and explain the significance of m_{18} and d_{18}.

Question 18 (2 marks)

Two towns, Abbotsford and Bourkesville, are 65 km apart and are joined by a straight road. The bearing of Bourkesville from Abbotsford is 112.6°T. The bearings of another town, Calen, from Abbotsford and Bourkesville are 182.9°T and 224.3°T respectively. Find the shortest distance from Calen to the road that joins Abbotsford and Bourkesville.

Question 19 (4 marks)

Two groups of seedlings have been germinated in separated containers. Group B has been treated with a hormone. Set A is the control group, so has not been treated. The heights of the seedlings after 12 days are as follows.

Group A:	22	20	22	20	9	35	18	17	15	12
	12	20	17	17	12	26	18	17	8	20
	21	21	11	16	17	24	14	30	20	28
	13	24	14	15	14	16	15	14	16	17
	25	22	21	20	15	14	13	13	29	20
Group B:	25	22	26	19	29	22	13	9	25	12
	33	25	26	23	21	24	21	30	27	26
	28	25	29	2	17	16	26	16	15	28
	25	26	27	3	23	34	22	25	27	23
	18	10	25	27	26	30	19	22	37	8

Compare the seedlings using an appropriate method and justify any conclusions you make.

END OF EXAMINATION

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ANSWERS

Chapter 1

Exercise 1.01

1 **a** 31.6% **b** 312.5% **c** 292% **d** 47.5%
e 79.7% **f** 270% **g** 287.5% **h** 2.4%
i 180% **j** 3.75%

2 **a** 91.667% **b** 207.407% **c** 28.571%
d 55.556% **e** 172.222% **f** 166.667%
g 73.684% **h** 35.714% **i** 185.714%
j 54.545%

3 **a** 35.714% **b** 152.632% **c** 120%
d 250% **e** 132.143% **f** 184.615%
g 52.941% **h** 172.727% **i** 21.429%
j 81.25%

4 **a** 0.14 **b** 0.285 **c** 1.73 **d** 0.08
e 0.0056 **f** 0.014 **g** 0.505 **h** 1.275
i 0.23 **j** 0.0572

5 **a** $\frac{3}{25}$ **b** $1\frac{12}{25}$ **c** $1\frac{17}{50}$ **d** $\frac{29}{40}$
e $3\frac{17}{40}$ **f** $1\frac{17}{125}$ **g** $2\frac{131}{400}$ **h** $\frac{19}{32}$
i $\frac{69}{160}$ **j** $1\frac{3191}{4000}$

6 $\frac{1}{6}$

Exercise 1.02

1 $\frac{16}{25} = 64\%$, $\frac{25}{35} \approx 71.43\%$, so 25 out of 35 is greater.

2 $\frac{32}{45} \approx 71.11\%$, $\frac{38}{55} \approx 69.09$, so 32 out of 45 is greater.

3 **a** 16.67% **b** 56.25% **c** 11.29%
d 118.18% **e** 72.22%

4 **a** 12.28% **b** 95.83% **c** 12.77%
d 337.5% **e** 192.68%

5 **a** 18.75% **b** 10.71% **c** 14.81%
d 7.94% **e** 56.52%

6 **a** 16.47% **b** 10.53% **c** 21.05%
d 21.25% **e** 7.78%

7 Suppose the original price is x. The percentage increase is $\frac{1600}{x}\%$. The percentage decrease is $\frac{1600}{x+16}\%$.

8 Percentage increase $= \frac{3000}{x-30}\%$

Exercise 1.03

1 **a** \$179.20 **b** 6.96 L
c 10.152 **d** About \$185.79
e \$68.20 **f** 13.392 g
g \$22.40 **h** 140.6 kg
i \$168.56 **j** About \$220.21
k \$108.41 **l** \$169.92

2 \$5440
3 \$645
4 \$392
5 About 488.37
6 350 mL
7 About 10.5 kg
8 6.27 kg

Exercise 1.04

1 **a** \$1515.96 **b** \$3031.92
c \$6569.17 **d** \$39.89

2 \$2262.54
3 \$3058.33
4 \$34 975
5 \$24 107
6 \$38 433 for a 38 h week
7 \$37 500 per year
8 \$748.37 /week
9 \$14.60 /h

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10 The hourly rate is better for a 38 h week, but the annual is better for a 37 h week.

Exercise 1.05

1 Overtime: $33.73 /h up to 2 hours, $44.98 /h thereafter. Public holiday $56.22 /h

2 Overtime: $30.69 /h up to 2 hours, $40.92 /h thereafter. Public holiday $51.14 /h

3 $19.88 and $715.68

4 Casual overtime: $32.93 /h up to 2 hours, $43.22 /h thereafter

5 $8.24 /h, $11.34 /h and $15.46 /h for 1st, 2nd and 3rd years

6 $8.96 /h, $9.96 /h, $11.95 /h, $13.94, $15.93 and $17.92 for U16, 16, 17, 18, 19 and 20 years old

7 $992.80 (including 8 h on Monday)

8 $698.19

9 $813.86

10 $1070.55

11 $768.20

12 $782.17

Exercise 1.06

1 **a** $127.13 **b** $15.89

c No, there would probably be fewer customers on week days.

2 **a** $288 **b** $24/h **c** $1440

3 **a** $267 **b** $22.25 /h **c** $1335

4 **a** $120 **b** $77 **c** $43 **d** $7.17

e No, he would only make $18.80 after paying his car costs.

5 **a** $63 **b** $144 **c** $481

d $724 **e** $8.02 /h **f** $12.07 /h

6 $10 /h and $3000

7 $6181.50

8 $14.40 /h for 90 shirts

9 At least $11 333

10 He takes $900, making $365 at $22.81 /h

11 **a** $812.80 at $18.06 /h **b** $7000

12 **a** $148.80 **b** $32.50

c $116.30 after paying petrol and $19.38 /h, but the real cost of running his car is more than the petrol, so it is less than this.

Exercise 1.07

1 $729.10 /fortnight, $30 916.60

2 $709.10 /fortnight, $31 436.60

3 $236.8 /fortnight

4 $261.70 /fortnight

5 $404.70 /fortnight

6 $365.10 /fortnight

7 **a** Danielle $433.20 /fortnight, Sam $409.70/ fortnight

b $433.20 /fortnight each, the same

8 $285.20 /fortnight, $73.60 /fortnight

9 The same amounts

10 $780.67 /fortnight

11 $49 353.20

12 **a** $429.50 a week **b** 20.94%

13 24.8%

14 **a** $1498.20 /fortnight

b Susan gets $444.45 /fortnight less and Andrew gets $222.65/fortnight more

c Before married $67 217.80, after $61 451.00

Exercise 1.08

1

Air fares:	$ 898
Airport transfers	$ 330
Accommodation:	$1995
Meals:	$ 910
Other:	$ 350
Total	$4483

2

Wieners:	$ 600
Bread rolls:	$ 550
Tomato sauce:	$ 54
Total cost:	$1204
Income:	$3800
Profit:	$2596

3 Paid weekly, so budget weekly.

Flat:	$102.50
Food, etc.:	$ 95.00
Bus:	$ 24.00
Mobile:	$ 50.00
Entertainment:	$105.00
Clothes:	$ 38.00
Total expenses:	$414.50

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Savings: 95.50 /week, $4775 a year. She could save more by reducing her mobile and entertainment expenses.

4 B&B: $ 960
Lunches: $ 225
Dinners: $ 540
Other:: $ 96
Total cost: $1821 = $607 each

5 Buses for 115 people:

5 × 25 seaters	$1375
3 × 46 seaters	$1446
2 × 46 + 25 seater	$1239✓
Entry	$1155✓
Contingencies	$ 100✓
Total cost	$2494

Cost per student = $23 each (rounding up)

6 **a**

Girls cabins	$ 900
Boys cabins	$ 700
Teachers cabin	$ 100
Catering	$11 712
Buses	$ 1 928
Total	$15 340

b Cost per pupil = $170

c $130 for contingencies is probably enough

7 **a**

Punnets	$ 360.00
Trays	$ 75.00
Seeds	$ 720.00
Seed-raising mix	$ 250.00
Labour	$ 600.00
Cost/3000	$2005.00

b Cost/tray ≈ $13.37, so about $20 /tray for reasonable profit.

8 Paid fortnightly, so budget fortnightly.

House:	$290
Food etc:	$180
Mobile:	$ 90
Entertainment:	$160
Bus:	$ 64
Clothes:	$ 84
Total expenses:	$868

Savings: $492 fortnight, $12 300 a year.

She could pay cash for a new car after 2 years.

9 Paid weekly, so budget weekly.

House:	$ 87.50
Power:	$ 26.25
Food, etc:	$220.00
Car payments:	$ 40.00
Running costs:	$ 37.50
Clothes:	$ 20.00
Mobile:	$ 37.50
Total expenses:	$468.75

He must spend $67.25 going out.

He could save more by eating fewer takeaways, rationing his mobile and going out less.

10 Paid fortnightly, so budget fortnightly.

Flat:	$ 450
Power:	$ 30
Groceries:	$ 190
Entertainment:	$ 300
Bike payments:	$ 90
Running costs	$ 30
Clothes:	$ 56
Mobile:	$ 50
Total expenses:	$1196

Savings: $184 /fortnight, $4600 a year.

Get a cheaper flat and stop spending so much on takeaways and parties.

Chapter review

1 **a** 125% **b** 5.3% **c** 460% **d** 71.429% **e** 253.846%

2 **a** 0.13 **b** 0.0005 **c** 1.35

3 **a** $\frac{4}{25}$ **b** $1\frac{17}{40}$ **c** $\frac{51}{2000}$ **d** $1\frac{17}{25}$ **e** $\frac{7}{40}$

4 Maths (77.1%), English (68%)

5 About 22%

6 About 54%

7 $79.48

8 570 ppm

9 $36.66 /h

10 $58 450.08

11 $20.64 /h

12 $1943.63

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13 $1122.10
14 $467.40
15 $1120
16 $167.70, $27.95 /h
17 $809.10 /fortnight, $28 836.60
18 $500.10 /fortnight
19

Petrol	$ 297
Tickets	$ 600
On-site cabin	$ 225
Food	$ 440
Total	$1562, $390.50 each

20 Paid weekly, so budget weekly.

Flat:	$190
Living expenses:	$320
Transport:	$ 70
TV:	$ 28
Clothes:	$ 95
Aerobics:	$ 18
Mobile:	$ 30
Total expenses:	$751

Savings: $129 /week, $6450 a year

21 $\frac{1}{12}$
22 25%
23 153.75 g
24 $43 300 /year
25 $842.84 /week
26 $874.35
27 At least $4166.67
28 $3937.50, $14/h
29 $435.75 /w, $27 305.20
30 Paid weekly, so budget weekly.

Entertainment:	$300
Transport:	$ 23
Food and groceries:	$ 80
Mobile:	$ 95
Clothes:	$ 96
Total expenses:	$594

Another $175 /w for the flat would make her expenses $769 /w, so she couldn't move out without changing her spending She could easily spend less on entertainment, clothes and her mobile.

Chapter 2

Exercise 2.01

1 B
2 **a** $a = 150$ m **b** $b \approx 141.8$ mm **c** $c \approx 1.54$ cm **d** $d \approx 8630$ mm **e** $e = 0.7$ m **f** $f \approx 19.4$ mm
3 **a** $a = 17$ m **b** $b \approx 19.2$ mm **c** $c \approx 4.7$ cm **d** $d = 35$ km **e** $e \approx 10$ m **f** $f \approx 19.1$ cm
4 **a** $a = 52$ **b** $b = 25$ **c** $c = 12$ **d** $d = 1.5$ **e** $e = 50$
5 **a** Obtuse **b** Acute **c** Acute **d** Acute **e** Right-angled
6 **a** 4187 mm **b** 4.795 m **c** 23.28 mm
7 **a** 547 cm **b** 15.97 mm

Exercise 2.02

1 D
2 Area = 8 cm^2
3 712 m
4 15.2 m
5 756.7 m
6 About 248 m
7 6.8 m
8 About 20.6 m
9 4082 mm

Exercise 2.03

1 **a** 6.7 m **b** About 82.15 cm **c** About 102.84 mm **d** About 24.05 cm **e** About 5.95 m **f** 142.7 m
2 10.28 m ≈ 10.3 m
3 $2706 (for 58 posts, 111.5 m of rails, 892 full palings, gate)
4 **a** $2577.33 **b** $597.08 (including gate)
5 $152 744.23
6 **a** 85 posts, 423 m of rail
b 1208 palings
c $5208

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Exercise 2.04

1 E

2 D

3 **a** 25 m^2 **b** 240 mm^2 **c** 0.0058 km^2
d 25.85 ha **e** 36 000 m^2 **f** 21 m^2
g 6.72 m^2 **h** 0.568 cm^2

4 **a** 0.6 m^2 **b** 1963.5 mm^2 **c** 150 cm^2
d 1.2 m^2 **e** 113.1 cm^2 **f** 7.1 m^2

5 **a** 400.05 $cm^2 \approx 400$ cm^2
b 126.3 m^2
c 63.9 mm^2

6 **a** 1.75 m^2 **b** 2.94 cm^2
c 6.665 m^2 **d** 684 m^2

7 15.0 cm

8 10.5 cm by 10.5 cm

9 44.8 cm

10 20.0 m^2

11 About 2243 kg

Exercise 2.05

1 **a** 245.4 cm^2 **b** 0.6 km^2 **c** 517.0 mm^2

2 **a** 370.2 cm^2 **b** 1 878 500 m^2 (1.8785 km^2)
c 179.0 cm^2

3 **a** 372.2 cm^2 **b** 38.8 m^2

4 **a** 674.19 cm^2 **b** 401.035 $m^2 \approx 401$ m^2
c 25.33 cm^2 **d** 38 240 mm^2
e 4.427 m^2 **f** 0.238 14 m^2

5 1714 cm^2

6 140.76 cm^2

7 787.9 cm^2

8 **a** 26.24 m^2 **b** 194.34 m^2 **c** 63.99 m^2

9 160.2 cm^2

Exercise 2.06

When rounding occurs, it may affect the final answers in this exercise.

1 **a** 228 tiles **b** 5.76 L, so 6 L
c 3 bags **d** \$1467.90

2 \$4008

3 22.6 L with wastage

4 Ceiling area ≈ 171 m^2, so about 36 L with wastage.

5 About 681 tiles with wastage.

6 \$10 646

7 **a** Court 30.48 m × 15.24 m; centre circle 0.9144 m diameter; goal circles 9.7536 m diameter.
b Goal attack and goal defence 310 m^2, 66.7%; goal shooter and goal keeper 155 m^2, 33.3%; wing attack and wing defence 272 m^2, 58.6%; centre 390 m^2, 83.9%

8 36.6%

Exercise 2.07

1 B

2 D

3 **a** cm^3 (mL) **b** cm^3 (mL) **c** m^3 (kL)
d ML **e** cm^3 (mL) **f** L
g cm^3 (mL) **h** L **i** m^3
j cm^3 (mL)

4 **a** 26 000 000 cm^3 **b** 0.32 m^3
c 280 ML **d** 0.8 L
e 56 m^3

5 **a** 2.4 L **b** 3200 cm^3 **c** 7500 mL
d 0.73 L **e** 41 mL **f** 8200 L
g 432 cm^3 **h** 0.003 24 m^3 **i** 2.32 L

6 **a** 19 974.2 cm^3 **b** 4712.4 mm^3
c 8.6 m^3 **d** 1125.4 cm^3

7 **a** 82 m^3 **b** 619.7 cm^3 **c** 3.927 m^3
d 2205.3 cm^3 **e** 3234 mm^3

8 **a** 1480 cm^3 **b** 18 200 cm^3 **c** 285 cm^3
d 1.17 m^3 **e** 1.59 m^3 **f** 248 cm^3
g 2.744 m^3 **h** 22.6 cm^3 **i** 27 m^3

9 **a** 1 922 800 mm^3 **b** 1922.8 cm^3

10 **a** **i** 50 225 L **ii** About 105 h
b **i** 55 930 L **ii** About 117 h

11 2.083 m

Exercise 2.08

1 **a** 1 243 547 mm^3 **b** 1437 cm^3
c 8379 mm^3

2 **a** 1206.4 cm^3 **b** 1013.7 cm^3 **c** 86 295.1 cm^3

3 **a** 37 875 cm^3 **b** 7863.1 cm^3
c 867 415.1 cm^3 **d** 147 436.1 cm^3

4 1.624 L

5 **a** 2400 m^3 **b** 2400 kL **c** 17 hours

6 **a** 44 m^3 **b** About 21 min

7 **a** 193.8 m^3 **b** About 4.5% **c** 183.7 m^2

8 **a** 146 m **b** 137 m

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Chapter review

1 D
2 D
3 D
4 **a** 13.4 m **b** 10.1. cm **c** 8.2 km
5 **a** Obtuse-angled **b** Acute-angled
c Right-angled **d** Acute-angled
6 5.4 km
7 6.65 m
8 49.6 cm
9 **a** 3.8 m^2 **b** 8600 mm^2 **c** 0.026 km^2
d 0.67 ha **e** 3600 m^2 **f** 560 m^2
g 28.7 m^2 **h** 1.267 cm^2
10 **a** 58.3 cm^2 **b** 8.99 cm^2 **c** 0.504 m^2
d 109.4 mm^2 **e** 12.15 cm^2 **f** 0.708 m^2
11 49.3 cm, 151.2 cm^2
12 **a** 147.9 cm^2 **b** 3855.5 mm^2 **c** 3.08 m^2
13 **a** 2.7 m^2 **b** 65.6 cm^2 **c** 240 cm^2
14 160 tiles
15 2×4 L tins
16 **a** 15 300 mm^3 **b** 2.18 m^3
17 **a** 15 000 L **b** 0.28 L
c 570 mL **d** 0.000 57 m^3
18 **a** 2827.4 cm^3 **b** 77 cm^3 **c** 314.2 cm^3
19 **a** $V = 88\ 584.3$ cm^3 **b** $V = 9065$ mm^3
20 **a** 18.12 m^3 **b** 2094.4 mm^3
c 5332.6 mm^3
21 127.3 m
22 24.4 m
23 **a** 38.18 m^2 **b** 48.76 m^2 **c** 133.58 m^2
d 66.155 m^2 (assuming the office is halfway between the house and the footpath)
24 \$2857.22
25 31 sheets
26 382 labels (with some wastage at the end of the roll)
27 9.8 m^3

Chapter 3

Exercise 3.01

1 $x = 12$ is a solution for **a**, **b**, **d**, **f**, **j** and **l**.
2 **a** C **b** A **c** D
3 **a** $x = 34$ **b** $a = 24$ **c** $u = 9$
d $d = 84$ **e** $f = 41$ **f** $k = -29$
g $e = -14$ **h** $y = 143$ **i** $h = -11$
j $b = -17$ **k** $w = -14$ **l** $r = -96$
4 **a** $k = 0$ **b** $x = -2.1$ **c** $t = 6$
d $g = 3$ **e** $p = -4$ **f** $u = 23$
g $y = -3$ **h** $n = -14$ **i** $a = 1.3$
j $z = -3$ **k** $e = 2.8$ **l** $r = -1.5$
5 **a** $z = 7$ **b** $x = -3$ **c** $y = 4$
d $a = 5$ **e** $b = 3$ **f** $c = -1$
g $w = 3$ **h** $u = -6$ **i** $v = 6$
j $p = 7$ **k** $q = 6$ **l** $r = 3$
6 **a** $d = 6$ **b** $m = 12$ **c** $t = 3$
d $p = -30$ **e** $y = 30$ **f** $x = -12$
g $r = 35$ **h** $y = -6$ **i** $x = 15$
j $h = -28$ **k** $q = -40$ **l** $g = 27.5$
7 **a** $12 + x = 31, x = 19$ **b** $\frac{x}{7} = 5, x = 35$
c $x - 8 = 25, x = 33$ **d** $3x + 7 = 22, x = 5$
e $5x - 17 = 18, x = 7$
8 17 km
9 **a** $m = 2t + 1$ **b** 251

Exercise 3.02

1 **a** C **b** A
2 **a** $g = -3$ **b** $x = 2$ **c** $s = -5$ **d** $u = -3$
e $w = -12$ **f** $q = 2$ **g** $x = -3$ **h** $y = 4$
i $r = 5$ **j** $m = -9$ **k** $z = 4$ **l** $c = -3$
3 **a** $b = \frac{5}{8}$ **b** $x = -1\frac{3}{7}$ **c** $w = 1\frac{3}{8}$ **d** $a = 5$
e $y = 19$ **f** $n = 1$ **g** $h = -2\frac{3}{5}$ **h** $u = -\frac{5}{7}$
i $d = 3\frac{1}{2}$ **j** $k = \frac{11}{12}$ **k** $r = 1$ **l** $v = -2\frac{2}{3}$
4 **a** $d = -2\frac{2}{3}$ **b** $x = -\frac{5}{8}$ **c** $y = 2\frac{4}{5}$
d $p = \frac{5}{13}$ **e** $w = 4$ **f** $u = -\frac{1}{3}$
g $z = -1$ **h** $g = 1$ **i** $b = -\frac{4}{7}$
5 **a** $b = 8.5$ **b** $y = -4.5$ **c** $m = -12$
d $r = -9\frac{1}{3}$ **e** $z = 4\frac{4}{5}$ **f** $u = 1\frac{4}{9}$
g $x = 1\frac{1}{5}$ **h** $d = \frac{4}{17}$ **i** $m = -12$

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6 **a** $e = 14$ **b** $y = 13$ **c** $p = 3$
d $x = -1\frac{5}{17}$ **e** $n = -3\frac{3}{4}$ **f** $k = 3$
g $w = -3$ **h** $q = -20$ **i** $t = -1\frac{2}{9}$

7 **a** True **b** False **c** False **d** True

8 **a** Solution is not true. **b** Line 2 has an error

9 **a** Solution is not true. **b** Line 1 has an error

10 **a** $2(x + 5) = x + 7, x = -3$
b $3x + 4 = 4(x + 3), x = -8$
c $3(x - 4) = 2(x + 5), x = 22$

Exercise 3.03

1 **a** D **b** A **c** B

2 **a** $m = 60$ **b** $y = -88$ **c** $x = 9$
d $n = 20$ **e** $p = 16$ **f** $v = 15$
g $z = 24$ **h** $a = -36$ **i** $g = 42$
j $h = -1\frac{1}{2}$ **k** $k = 27$ **l** $h = -20$

3 **a** $g = 19$ **b** $y = 36$ **c** $c = 3$
d $x = -66$ **e** $y = -15$ **f** $u = 62$
g $h = 9.3$ **h** $t = -25.33$ **i** $p = -65.2$
j $v = 120$ **k** $j = 14$ **l** $x = -90.68$

4 **a** $x = 9.5$ **b** $g = 7\frac{2}{3}$ **c** $y = -1\frac{3}{5}$
d $m = -13.5$ **e** $e = -8.5$ **f** $r = 5\frac{3}{4}$
g $k = -12$ **h** $t = 0.85$ **i** $s = 9\frac{1}{3}$
j $x = \frac{1}{3}$ **k** $b = 1\frac{1}{4}$ **l** $n = -6\frac{1}{4}$

5 **a** $m = -\frac{6}{11}$ **b** $w = 1\frac{1}{14}$ **c** $x = -4$
d $x = -1\frac{1}{7}$ **e** $g = -\frac{10}{23}$ **f** $z = -13.5$
g $n = -\frac{3}{7}$ **h** $r = -\frac{4}{23}$

6 **a** $x = -12$ **b** $m = -2$ **c** $z = 4\frac{4}{5}$
d $n = -4\frac{4}{5}$ **e** $u = 1\frac{3}{7}$ **f** $e = 12$

7 **a** $n = 6$ **b** $p = -\frac{4}{5}$ **c** $g = 2\frac{2}{3}$
d $m = -1\frac{1}{2}$ **e** $y = 25$ **f** $k = \frac{4}{5}$
g $h = -5\frac{1}{4}$ **h** $c = -9$ **i** $w = 6\frac{2}{3}$
j $y = -1\frac{17}{18}$ **k** $x = -\frac{96}{125}$ **l** $d = -1\frac{1}{15}$

8 **a** $x = \frac{5}{24}$ **b** $x = 2\frac{1}{2}$ **c** $x = \frac{1}{3}$
d $x = 1$ **e** $x = -7$ **f** $x = 14.5$
g $x = -\frac{1}{24}$ **h** $x = 2\frac{4}{5}$ **i** $x = \frac{2}{3}$

9 **a** Solution is not true.
b Error made in Line 1

10 **a** Solution is not true.
b Error made in Line 1

11 $x = -\frac{n}{m}$

12 $x = \frac{q - s}{r - p}$

Exercise 3.04

1 **a** D **b** A **c** B **d** E

2 **a** $10(12 + n) = 180$ **b** $\frac{14}{n - 7} = 2$
c $\frac{1}{2}(n + 15) - 5 = 3$ **d** $2(4n + 17) = -6$
e $3 + 7(n + 2) = 11$ **f** $n + \frac{1}{3}n - 14 = 9$
g $2(10n - 7) = -12$ **h** $\frac{2}{3}n - 7n = 10$

3 **a** $P = 6w$ **b** $P = (4w + 8)$ cm
c $P = (4w + 6)$ m **d** $P = 8w$
e $P = (6w + 8)$ mm **f** $P = (8w - 10)$ cm
g $P = (10w + 56)$ m **h** $P = (4w + 24)$ mm

4 **a** **i** $7 + y = 4$ **ii** $y = -3$
b **i** $m + 9 = 2$ **ii** $m = -7$
c **i** $a + 31 = 13$ **ii** $a = -18$
d **i** $x + 12 = 8$ **ii** $x = -4$
e **i** $d + 3 = -13$ **ii** $d = -16$
f **i** $26 - y = 39$ **ii** $y = -13$
g **i** $-12 + g = 4$ **ii** $g = 16$
h **i** $7 - w = -4$ **ii** $w = 11$

5 85 marks

6 10 years old

7 15 and 16

8 15

9 10 days

10 755 flyers

11 36 tyres

12 8 m by 8 m

13 7.5 L

14 11 years old

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Exercise 3.05

1 **a** $y = -3x + 2$

x	0	2	5
y	2	−4	−13

b $y = 4x + 1$

x	−3	0	3
y	−11	1	13

c $y = 5x - 12$

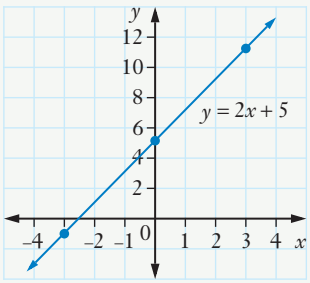
x	−2	0	2
y	−22	−12	−2

d $y = 7 - 3x$

x	−1	0	3
y	10	7	−2

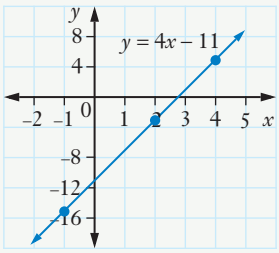
2 **a** $y = 2x + 5$

x	−3	0	3
y	−1	5	11



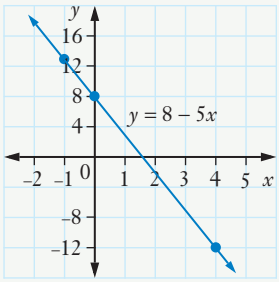
b $y = 4x - 11$

x	−1	2	4
y	−15	−3	5



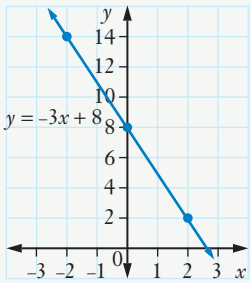
c $y = 8 - 5x$

x	−1	0	4
y	13	8	−12

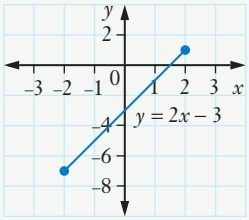


d $y = -3x + 8$

x	−2	0	2
y	14	8	2



3 **a**



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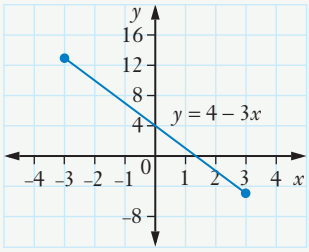

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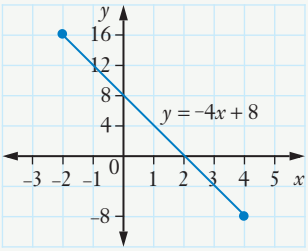

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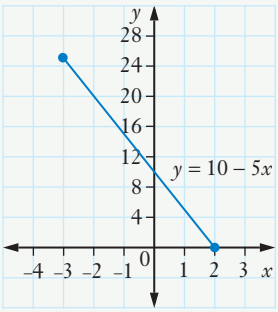

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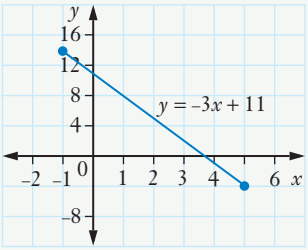

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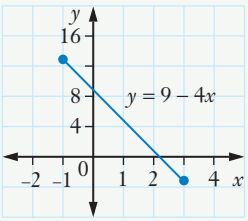

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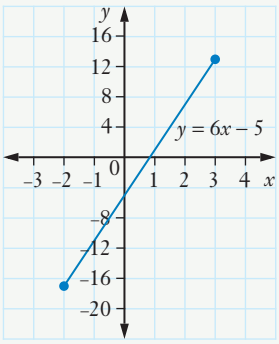

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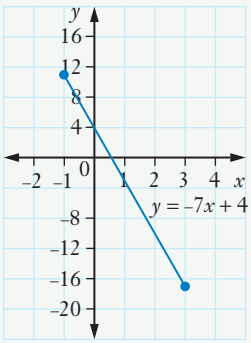

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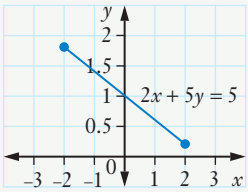

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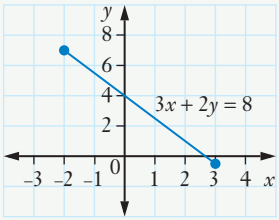

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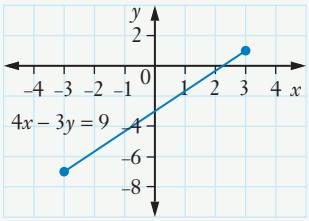

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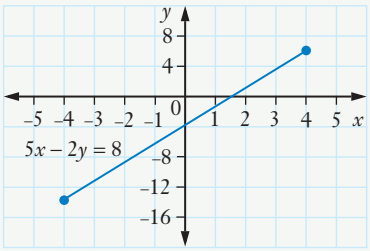

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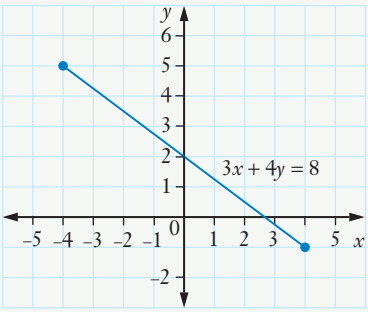

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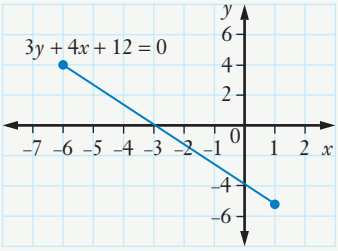

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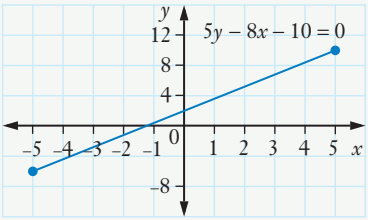

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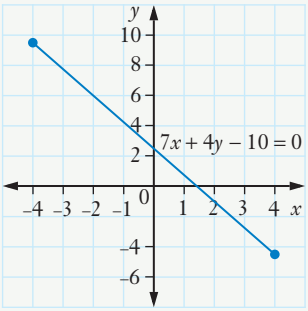

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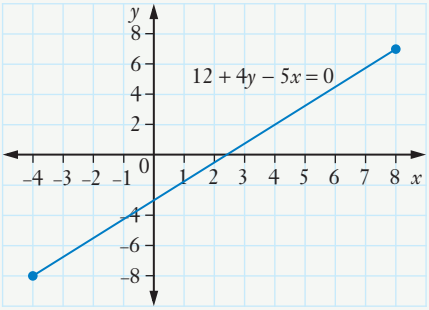

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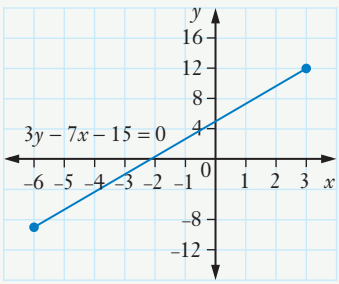


5 D

6 A

7 B

8 **a** (–4, 2) and (2, –7) lie on the line.

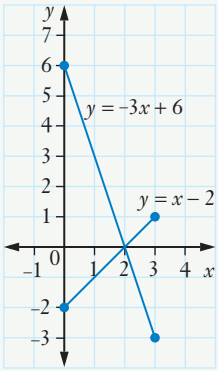
b (1, –2), (3, 0) and (–3, –6) lie on the line.

c (–2, 2) and (3, 12) lie on the line.

d (3, –7) and (–4, 14) lie on the line.

e (2, 0) and (–4, –18) lie on the line.

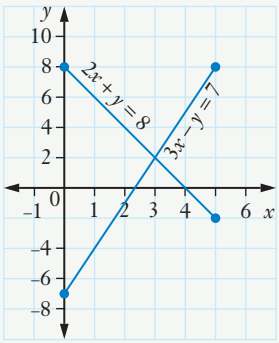
9 **a**



b (2, 0) **c** (2, 0) satisfies $y = x - 2$.

d (2, 0) satisfies $y = -3x + 6$.

10 **a**



b (3, 2) satisfies both functions.

11 9 units on the y-axis

Exercise 3.06

1 **a**, **d**, **f** and **g** are positive; **b**, **c**, **h** and **i** are negative; **e** and **j** are neither.

2 **a** 2 **b** –1 **c** $\frac{-2}{3}$ **d** $\frac{1}{2}$

e 0 **f** 3 **g** $\frac{1}{3}$ **h** $\frac{-1}{3}$

i –3 **j** Not defined

3 **a** $\frac{-2}{7}$ **b** $\frac{-3}{2}$ **c** 2

d Not defined **e** $\frac{-1}{2}$ **f** $\frac{-1}{2}$

g $\frac{3}{4}$ **h** 0 **i** $\frac{1}{5}$

j $\frac{1}{2}$

4 **a** 3 **b** –2 **c** 2 **d** 7

e $\frac{1}{8}$ **f** –2 **g** –1 **h** 1

i 0 **j** $-\frac{3}{4}$ **k** Not defined

l $-\frac{3}{2}$

5 **a**, **b**, **c**, **d**, **e** and **f** are collinear.

6 $MN \parallel CD$

7 XY and WR are not parallel.

8 **a** $b = 3$ **b** $b = 2$

9 Opposite sides are parallel.

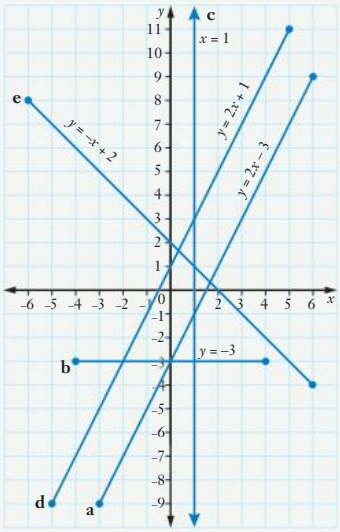
10 Many answers are possible, for example, (2, –4) and (–1, 2).

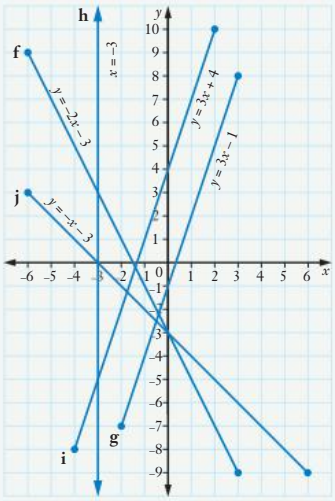
Exercise 3.07

1 **a** xi **b** vii **c** viii **d** v **e** i

f ii **g** ix **h** xii **i** x

2





Answers are in the order: x-intercept; y-intercept

a $(1\frac{1}{2}, 0)$; (0, –3)

b No x-intercept; (0, –3)

c (1, 0); no y-intercept

d $(-\frac{1}{2}, 0)$; (0, 1)

e (2, 0); (0, 2)

f $(-1\frac{1}{2}, 0)$; (0, –3)

g $(\frac{1}{3}, 0)$; (0, –1)

h (–3, 0); no y-intercept

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i $(-1\frac{1}{3}, 0)$; $(0, 4)$

j $(-3, 0)$; $(0, -3)$

3 **a** $y = 2x + 4$ **b** $y = -3x + 1$

c $y = 5x - 6$ **d** $y = 3x - 2$

e $y = -3$ **f** $y = -4x$

4 **a** $4x - y - 1 = 0$ **b** $2x - 3y - 6 = 0$

c $4x + 3y + 3 = 0$ **d** $5x - 2y + 4 = 0$

e $10x - 3y - 12 = 0$ **f** $7x + 4y + 7 = 0$

5 Answers are in the order: gradient, y-intercept.

a 2, 5 **b** −3, 5 **c** −1, 1 **d** −7, 4

e 1, 2 **f** −8, 0 **g** $\frac{1}{2}, \frac{1}{2}$ **h** −7, 14

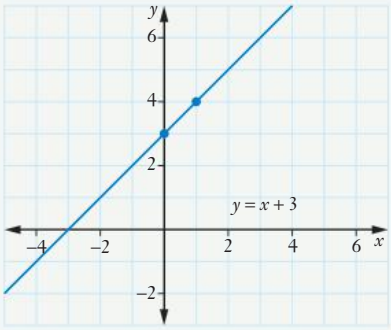
i $\frac{1}{3}, 2$ **j** $-\frac{4}{3}, 2\frac{2}{3}$ **k** $\frac{7}{3}, 0$

6 D

7 B

8 A

9 **a**



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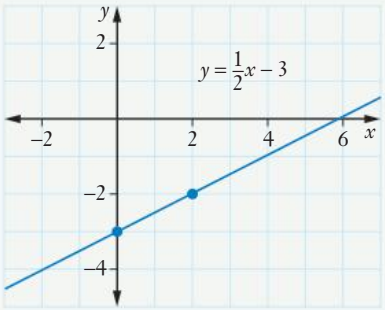

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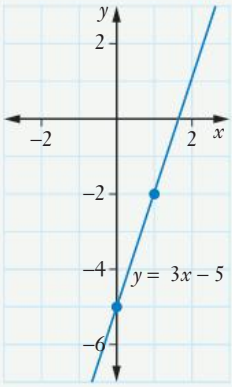
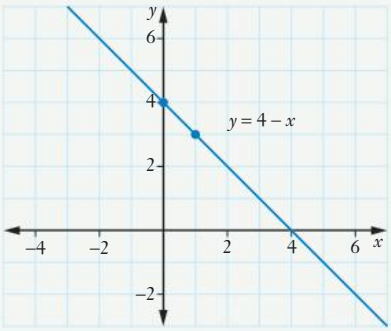
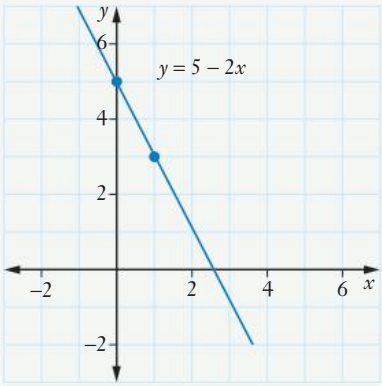


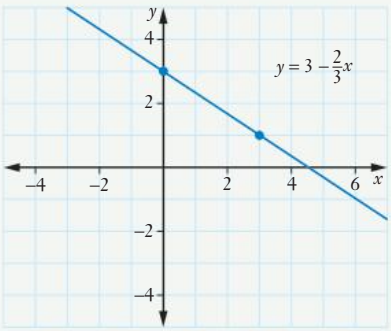
d



e



f



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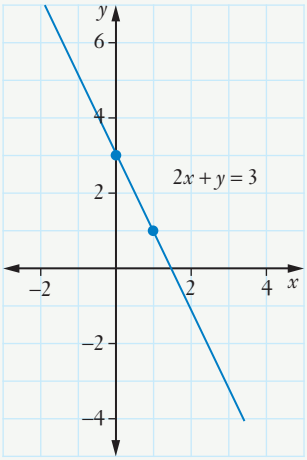
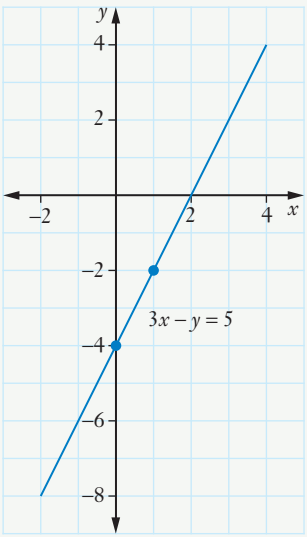
10 **a**



b



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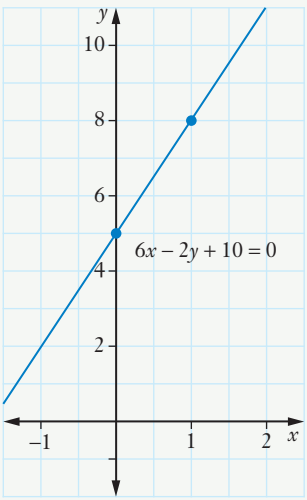

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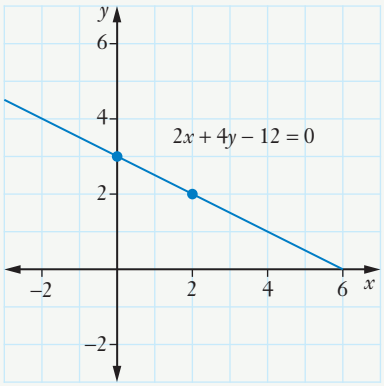

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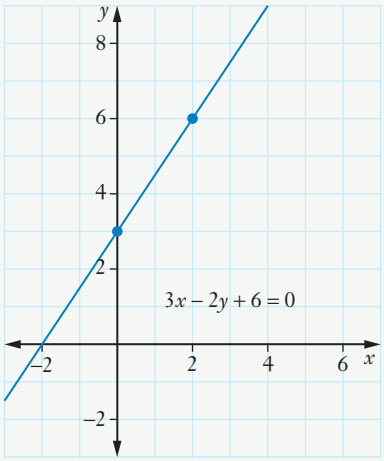
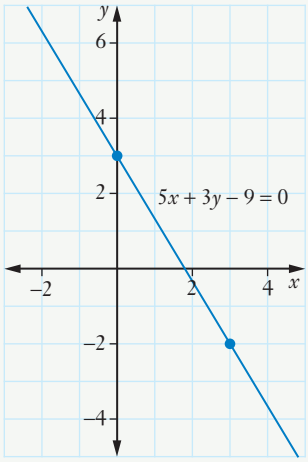


f



11 **a** Parallel **b** Parallel **c** Not parallel
d Parallel **e** Parallel **f** Not parallel

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12 **a** $x + 2y - 2 = 0, y = -\frac{1}{2}x + 1$

b $4x - 3y + 12 = 0, y = \frac{4}{3}x + 4$

c $x - 2y - 4 = 0, y = \frac{1}{2}x - 2$

d $4x - y + 4 = 0, y = 4x + 4$

13 **a** $8x - 3y - 24 = 0, y = \frac{8}{3}x - 8$

b $3x + 5y + 15 = 0, y = -\frac{3}{5}x - 3$

c $x - 5y - 5 = 0, y = \frac{1}{5}x - 1$

d $3x - 2y + 12 = 0, y = \frac{3}{2}x + 6$

Exercise 3.08

1 **a** n **b** 25

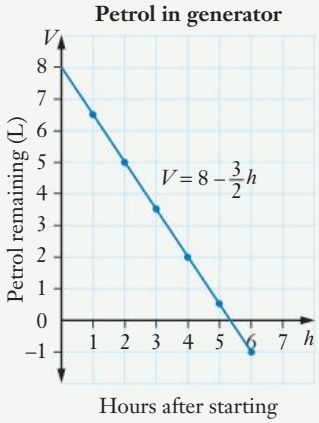
2 **a** W **b** 15.5

3 B

4 **a**

h	0	1	2	3	4	5	6
V	8	6.5	5	3.5	2	0.5	–1

b



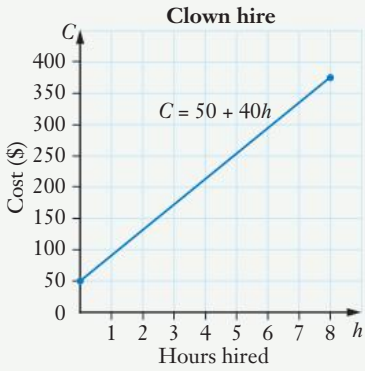
c Linear function **d** After 3.5 hours

e 5 L

f The volume of petrol in the generator before it started.

g The number of hours after starting when the generator runs out of petrol.

5 **a**



b Linear function **c** \$170

d 3.75 hours

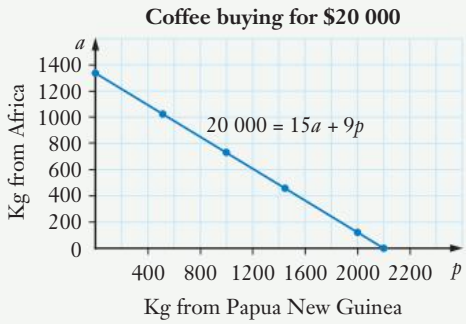
e The cost of hiring the clown regardless of how many hours he or she works. It is sometimes called a service fee or a call-out fee.

f The hourly rate of pay for the clown.

6 **a** $20\,000 = 15a + 9p$

b

p	0	500	1000	1500	2000
a	1333	1033	733	433	133



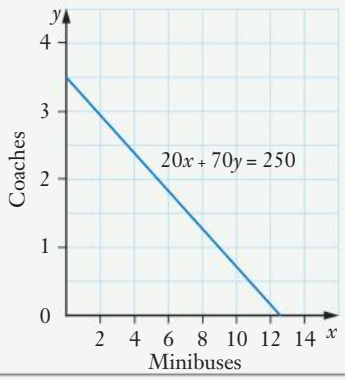
c About 850 kg of African coffee.

7 The cost incurred before any items are produced e.g. set up costs.

8 The weight of the empty pallet.

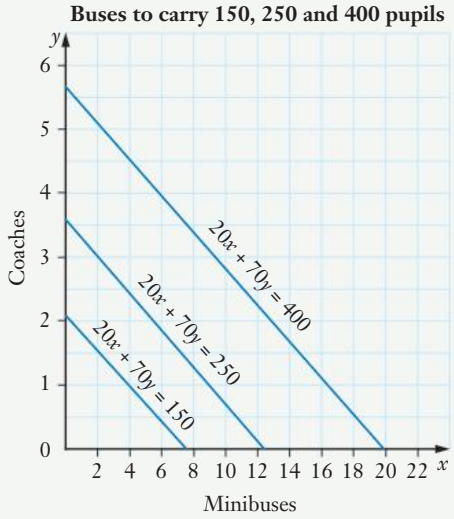
9 **a** $20x + 70y$ pupils **b** $20x + 70y = 250$

c



d 9 minibuses and 1 coach or 2 minibuses and 3 coaches.

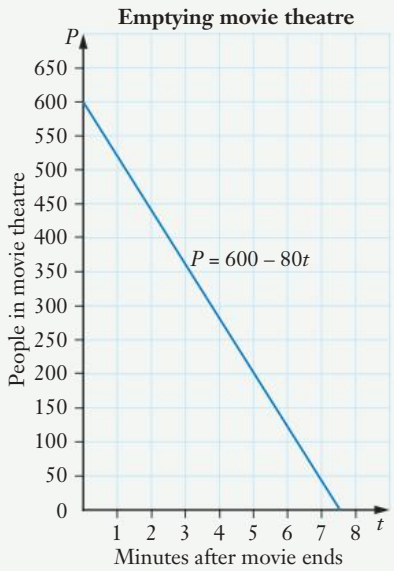
e



f The graphs are parallel.

10 a $P = 600 - 80t$

b



c About 3.8 minutes.

Chapter review

1 a Yes **b** No

2 a $q = 27$ **b** $m = 16$ **c** $a = -14$ **d** $r = 84$

3 a $h = -6$ **b** $p = 2$ **c** $x = 15$ **d** $y = 20$

4 D

5 a $g = -3$ **b** $s = 4$ **c** $u = -4$ **d** $k = 13.5$

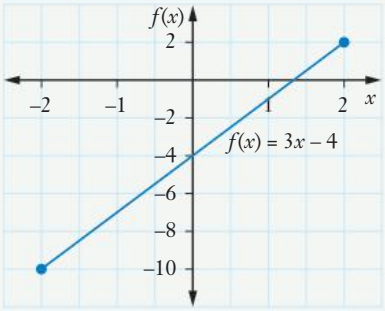
6 a $y = 30$ **b** $z = 5\frac{1}{3}$ **c** $x = 22$ **d** $h = 1\frac{2}{3}$

7 a $x = 5\frac{2}{3}$ **b** $g = -\frac{1}{6}$ **c** $x = -\frac{1}{7}$

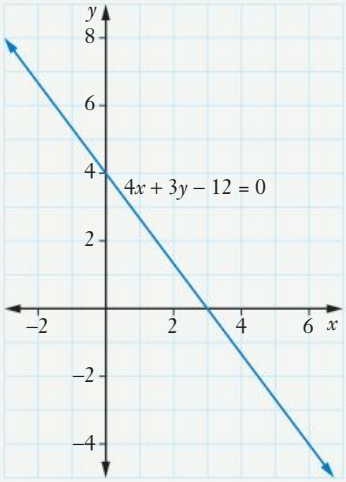
8 a $3x - 5 = 7$ **b** $x = 4$

9 Length = 7 m

10



11



12 Not on the line

13 B

14 B

15 a $-\frac{3}{2}$ **b** 0 **c** 1 **d** –4

16 a C **b** D **c** A **d** B

17 D

18 –2

19 $m_{AB} = m_{AC}$

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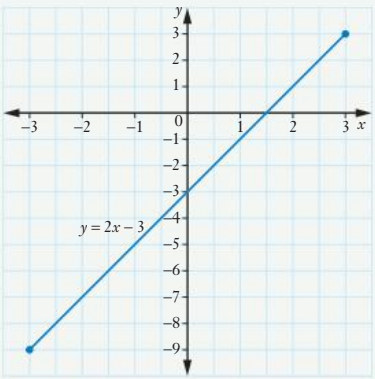
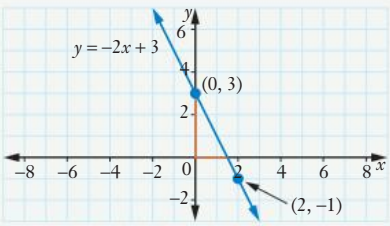
20 x-intercept = 1.5, y-intercept = –3



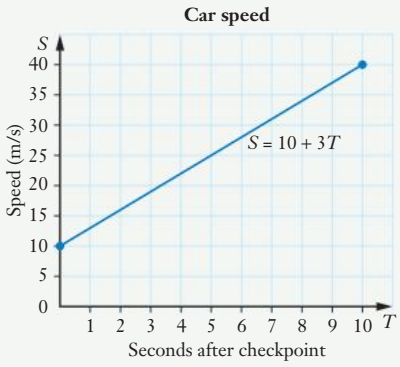
21 $3x - y - 1 = 0$

22 Gradient = $\frac{1}{2}$, y-intercept = 2

23



24 **a**



b After about 6.7 seconds

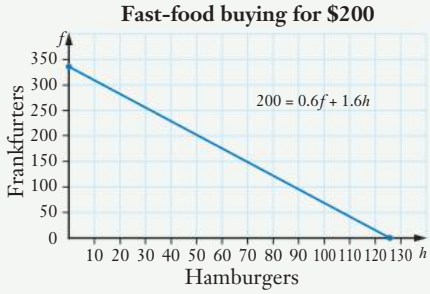
c About 22 m/s

d The speed of the car when it passes the checkpoint.

25 **a** $200 = 0.6f + 1.6h$

b

h	0	25	50	75	100	125
f	333	267	200	133	67	0



c 146 frankfurters

26 $4500, Izzy $2475, Mark $1575.

27 David is 7, Mary is 2.

28 Not parallel.

29 Opposite sides are parallel.

Practice Examination 1

1 Art is better, Art 70.2%, English 68.2% (1 mark each)

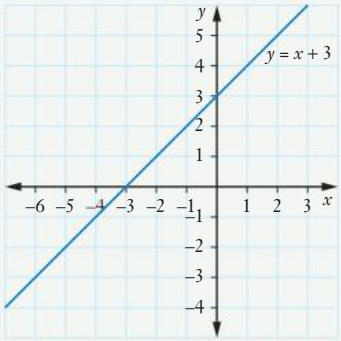
2 **a** $38 433.20 **b** $18.31/h (1 mark each)

3 $471.70 (1 mark for rates, 1 mark for answer)

4 $492.80 (1 mark for rate, 1 mark for earnings)

5 $904.40 (1 mark)

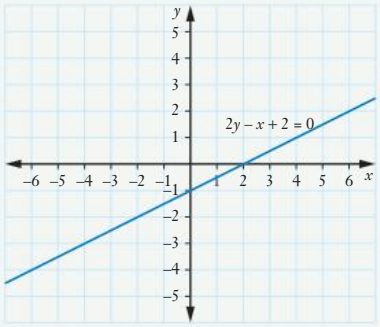
6 (2 marks)



7 **a** $m = 6$ **b** $w = 7$

c $b = 6$ (1 mark each)

8 (2 marks)



9 Yes, $-2 \times (-3) - 5 = 6 - 5 = 1 = y$ (1 mark working, 1 mark answer)

10 **a** Negative

b $m = -2$ **c** $(0, -4)$ (1 mark each)

11 About 9.9 (1 mark for working, 1 mark for answer)

12 2.5 m^2 (1 mark)

13 About 161 mm (1 mark for working, 1 mark for answer)

14 570 cm^2 (1 mark for working, 1 mark for answer)

15 8127 cm^3 (1 mark for working, 1 mark for answer)

16 At least 7.4% (1 mark for equation, 1 mark for manipulation, 1 mark for answer)

17 No – opposite sides not parallel OR diagonals not bisecting OR opposite sides not equal (working 4 marks)

18 **a** 10 m^2 (1 mark) **b** $\frac{1}{4}$ (2 marks)

19 **a** \$37/fn (1 mark)

b She would lose the Youth Allowance if she worked more than 3 hours overtime altogether in a fortnight. If she refused, she might endanger her employment. Overtime would give her more experience. (2 marks – 1 for each reason)

20 **a** 5973.0 m^3 (1 mark)

b 26.3 m (2 marks)

21 (–2, –9), (0, 1) and (4, 3) (1 mark for answers, 3 marks for working)

Chapter 4

Exercise 4.01

1 The leftover from a sachet is difficult to keep, so it is really 90 g for \$2. If she can keep the bottle in the fridge for at least 4 weeks after opening, it is cheapest. Otherwise, the tubs are best.

2 The coriander will last indefinitely, so the 250 g is the best.

3 **a** The 1 kg pack **b** Single onions

4 3 L

5 110 g

6 As long as you use it in reasonable time, 1 kg.

7 If he made 2 meals, the chuck steak is best, otherwise the gravy beef.

8 500 g is probably not quite enough chicken for 2 people, so the 700 g would be best.

Exercise 4.02

1

a	DVD player	\$250	40%	\$350
b	Table	800	75%	\$1400
c	Lamp	\$114	85%	\$210
d	Software bundle	\$300	90%	\$570
e	Bottle of soft drink	\$3.00	50%	\$4.50
f	Magazine	\$5.00	50%	\$7.50
g	Travel bag	\$30	110%	\$63
h	Widescreen TV	\$1400	90%	\$2,660

2 **a** \$19.80 **b** \$231 **c** \$99 **d** \$4.95 **e** \$462

3 **a** 81.2% **b** 125.8% **c** 84.2%

4 **a** \$1.14 **b** \$2.21 **c** 39 c **d** \$1.02

5 **a** \$148.75 **b** \$204.00 **c** \$280 **d** \$80

6 **a** \$53 and \$19 **b** 26.4%

7 **a** \$9640 **b** \$43 050

8 **a** \$4000 **b** \$6000 **c** \$5600 **d** \$6400

9 \$94 500

10 **a** 70% **b** 41.2%

11 6.25%

12 10.3%

13 \$455

14 Fishing reel 33.3% profit, video camera 31.8% profit, jewellery box 28.6% loss, filing cabinet 62.5% profit, printer 25% loss

15 42.8%

16 **a** 124.1% **b** 30.3% **c** 22.9%

17 **a** \$19.80 **b** \$217.80 **c** \$10.91 **d** \$8.89

18 Because $90\% \times 80\% = 72\%$, not 70%

19 $66\frac{2}{3}\%$

20 \$300, \$471

21 \$240

Exercise 4.03

1 5.5%

2 \$57

3 **a** 7.5% **b** 6.5% **c** 6.25% **d** 50%

4 \$300

5 15%

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6 $75 394.52
7 $279
8 $3112.50, $81.64
9 **a** $180 **b** $320 **c** $1440 **d** $50
10 $145.03
11 $608.66, 565%
12 $162 500
13 **a** $2375.63 **b** $8925 **c** $14 004
d $8433.25
14 $64 285.72
15 $1 248 000
16 21.3%
17 $7440, $7448, the second is better
18 4.78%

Exercise 4.04

1 **a** $23 433.19. $3433.19
b $8202.32, $202.32
c $15 474.65, $474.65
d $10 041.54, $2241.54
2 **a** $47 779.34, $2779.34
b $170 281.61, $50 281.61
c $78 853.24, $2853.24
d $163 118.11, $25 118.11
e $231 376.14, $5876.14
3 **a** $3524.68, $1524.68
b $13 276.19, $5276.19
c $22 478.37, $15 478.37
d $2234.70, $1734.70
e $4513.50, $2373.50
f $7043.45, $6983.45
4 **a** $1904.60, $404.60
b $5803.48, $2963.48
c $32 558.61, $23 558.61
d $1385.60, $585.60
e $7024.18, $2024.18
f $9519.26, $2019.26
g $9489.89, $1989.89
h $9370.19, $1870.19
i $9533.87, $2033.87
5 18.92%
6 14.72%
7 4 years
8 16 or 17 years
9 **a** $30.67 **b** $467.90 **c** $1102.81
d $494.09
10 About 6.4%
11 **a** 31.9% **b** 36 days
12 10 years
13 5.79%
14 $8263.97

Exercise 4.05

1 **a** 2.2971 **b** GPK 2067.39
c AUD 130.60
2 **a** 1.5405 **b** FJD 7702.50
c AUD 519.31
3 **a** PHP 69 480 **b** PHP 19 698.30
4 **a** AUD 102.30 **b** EUR 2587.20
5 **a** AUD 113.74 **b** AUD 22.37
6 **a** AUD 808.15 **b** GBP 667.80
7 FJD 584.08
8 AUD 442.48
9 JPY 53 295.55
10 AUD 1556.82
11 USD 1386.40, EUR 1328.20, THB 48 960
12 AUD 1437.56

Exercise 4.06

1 **a** 28% **b** 4.15% **c** 16.9 **d** $560
2 **a** 46% **b** 5.26% **c** 7.6 **d** $575
3 **a** $7.75 **b** 62 cents **c** 31% **d** 6.9
e $124
4 **a** 20.0% **b** 4.17% **c** 12.0 **d** $500
5 **a** 12.0% **b** 7.06% **c** 11.3 **d** $420
6 Their equivalent interest rates are 2.1%, 7.7%, 6.3% and 14.3% respectively. Investors are very optimistic about the first, a little pessimistic about the second, neither optimistic nor pessimistic about the third and quite pessimistic about the last.
7 A 9.38%, B 8%, C 11.11%, so C is best.
8 A 4.04%, B 9.64%, C 4.27%, so B is the best.
9 The dividend yield (5.29%) is little better than the credit union interest, if he thinks it is a safe investment.
10 **a** $33 100 **b** $34 100 **c** 3% profit
d 0.8%

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Chapter review

1 E
2 A
3 \$195.75
4 \$460.95
5 **a** \$8.64 **b** \$3
6 **a** \$0.33 **b** \$5.08 **c** \$0.13
7 **a** \$3.12 **b** \$9.24 **c** \$22
d \$1.12 **e** 42%
8 7.2%
9 \$2310, \$96.25
10 \$34 666.67
11

Time	Amount at end	Interest	Total Interest
6 m	5300.00	300.00	300.00
1 year	5618.00	318.00	618.00
1 y 6 m	5955.08	337.08	955.08
2 years	6312.38	357.30	1312.38
2 y 6 m	6691.13	378.74	1691.13
3 years	7092.60	401.47	2092.60

12 \$4887.77
13 \$4023.16
14 About 1% (0.985%)
15 \$143.75
16 C
17 About 6.3% (6.258)
18 **a** VND 12 173 600 **b** \$6.72
c VND 19 021 250
19 AUD 628.12
20 **a** 4.3% **b** 7.6% **c** \$0.54
d About 10.5
21 **a** The equivalent interest rates are 11.1%, 7.1%, 5.3% and 3.1% respectively. Investor are not confident about value of the first increasing, think the second is likely to stay about the same, have some confidence in the second and are very confident about the last.
b The first, with PE ratio 9.
c The third, with PE ratio 19.
22 \$139.39, 58.4% profit
23 150%
24 30%
25 Amount after 10 years 6 months

$$= 3000 \times \left(1+\frac{0.075}{4}\right)^{42} = \$6545.78$$

Amount after 10 years 9 months

$$= 3000 \times \left(1+\frac{0.075}{4}\right)^{43} = \$6668.51$$

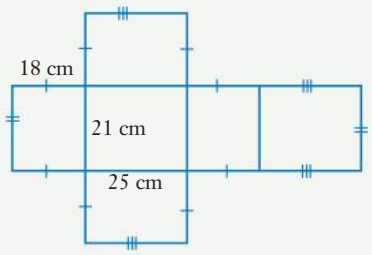
10 years and 9 months
26 \$4866.60
27 14.7% (AUD to ZAR)
28 About \$7135 million

Chapter 5

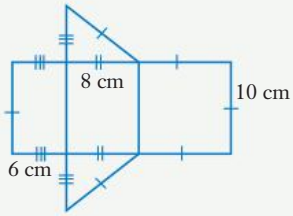
Exercise 5.01

1 B
2 **a** 2706 cm^2 **b** 288 cm^2 **c** 3358 mm^2

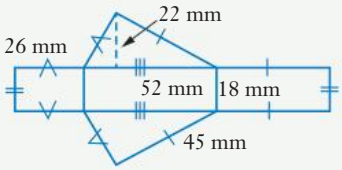
a



b



c



3 **a** 4632 cm^2 **b** 64.45 m^2 **c** 3076 mm^2
4 **a** 825 cm^2 **b** 429 mm^2 **c** 171.4 m^2
5 **a** 9000 mm^2 **b** 873.8 cm^2 **c** 46.46 m^2
6 **a** 70.48 m^2 **b** \$684.60
7 989 m^2
8 22.6 m^2
9 \$5140.80
10 72 cm^2

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Exercise 5.02

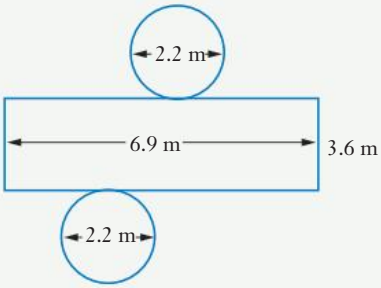
1 B

2 C

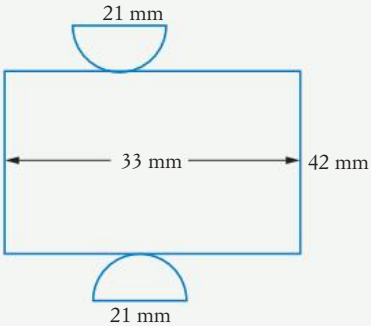
3 D

4 **a** 32.5 m^2 **b** 1731.8 mm^2
c 1306.9 cm^2

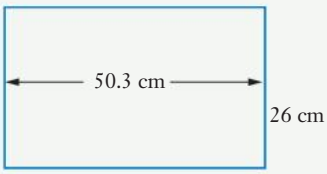
a



b



c



5 **a** 3267.3 cm^2 **b** 166.3 cm^2
c 3758.8 mm^2

6 **a** 1256.6 cm^2 **b** 2827.4 mm^2
c 1134.1 cm^2

7 **a** 763.4 mm^2 **b** 39.3 m^2
c 374.1 mm^2

8 **a** 179 m^2 **b** \$2431.50

9 **a** 22.9 m^2 **b** 2.5 L

10 **a** 3.8 m × 6.5 m **b** \$1953.75

11 49.3 m^2

12 194 241 mm^2 (1942.41 cm^2)

Exercise 5.03

1 B

2 D

3 **a** 0.5 **b** 2.5 **c** 1.8 **d** 0.5

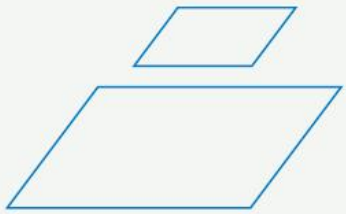
4 **a** **i** $\triangle KLM \parallel\!| \triangle ZYW$
ii scale factor = $\frac{1}{3}$
b **i** $LCFGH \parallel\!| ZTWVS$
ii scale factor = 3.5
c **i** $\triangle RBP \parallel\!| \triangle GHE$
ii scale factor = 2.5
d **i** $GFEH \parallel\!| TRUS$
ii scale factor = 1.6
e **i** $\triangle PLO \parallel\!| \triangle YKO$
ii scale factor = 2.2
f **i** $MDFS \parallel\!| QTYE$
ii scale factor = 1.5

5 **a** 9.5 m **b** 125 cm **c** 4 cm
d 63 mm **e** 165 cm **f** 13 mm

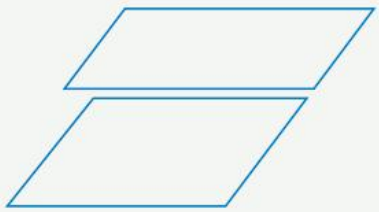
6 **a** T **b** T **c** F **d** T **e** T
f F **g** F **h** T **i** F **j** F
k F **l** T **m** T

7 35 cm

8 **a** The parallelograms are not necessarily similar as the ratio of corresponding sides may not be equal.
b Many answers are possible.



c Many answers are possible.



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Exercise 5.04

1 C

2 E

3 **a, b** Teacher to check.

c Yes, $\triangle DAF \parallel\!| \triangle KUT$ (AAA).

4 **a** $\triangle ABC \parallel\!| \triangle XYZ$ (AAA)

b $\triangle MSQ \parallel\!| \triangle AFD$ (AAA)

c No test can be applied.

d $\triangle KLD \parallel\!| \triangle DLP \parallel\!| \triangle KDP$ (AAA)

e $\triangle MCN \parallel\!| \triangle CDN$ (SSS)

f $\triangle XYW \parallel\!| \triangle ZUX$ (RHS)

g $\triangle PQS \parallel\!| \triangle QSR$ (SSS)

5 **a** $\triangle CKQ \parallel\!| \triangle MWL$ (AAA)

b $\triangle BIZ$ and $\triangle NPV$ are not necessarily similar.

c $\triangle BRT$ and $\triangle EAN$ are not necessarily similar.

d $\triangle APS \parallel\!| \triangle MKR$ (SSS)

e $\triangle CPW \parallel\!| \triangle MLE$ (SAS)

f $\triangle AMP \parallel\!| \triangle QSD$ (AAA)

g $\triangle PRZ$ and $\triangle DKL$ are not similar.

h $\triangle AMW \parallel\!| \triangle SEL$ (RHS)

i $\triangle WVR \parallel\!| \triangle PEN$ (SSS)

6 **a** $w = 50°, z = 45°, x = y = 85°$

b $x = 12$

c $n = 12, m = 25.2$

d $b = 15$ **e** $x = 12$ **f** $x = 20$

7 5.6 cm

8 Consider $\triangle WXZ$ and $\triangle XYZ$.

$\angle WXZ = 90° - \angle XWZ$

$\angle XYZ = 90° - \angle XWZ$

$\angle WZX = \angle XZY = 90°$

$\triangle WXZ \parallel\!| \triangle XYZ$ (AAA)

Use a similar approach to show that $\triangle WXZ \parallel\!| \triangle WXY$.

Hence $\triangle WXZ \parallel\!| \triangle XYZ \parallel\!| \triangle WXY$.

Exercise 5.05

1 6.25 m

2 143.3 – 8 = 135.3 m

3 34.3 m

4 18.75 m

5 9 m

6 24.5 m

7 174 cm

Exercise 5.06

1 D

2 C

3 B

4 **a** 1 : 100 **b** 1 : 100 000 **c** 1 : 1000
d 1 : 10 000 **e** 1 : 50 **f** 1 : 200 000

5 **a** 1 m **b** 1 m **c** 5 m
d 100 m **e** 6.5 km **f** 50 m

6 **a** 1 : 5000 **b** 1 : 10 000 000
c 1 : 1000 **d** 1 : 50
e 1 : 400 000 **f** 1 : 5000

7 **a** 15 m **b** 2.1 m **c** 1.0 m
d 4.0 m **e** 1.3 m, 0.8 m

8 6.875 km

9 16 cm

10 **a** 2000 mm **b** 900 mm **c** 800 mm
d 1200 mm **e** 1600 mm **f** 500 mm

11 **a** 10.5 m **b** About 231 m and 53 m
c 24.5 m each

12 **a** 4.1 m **b** 1.7 m **c** 53 cm
d 12 cm

13 **a** 83 km **b** 109 km **c** 34 km
d 30 km

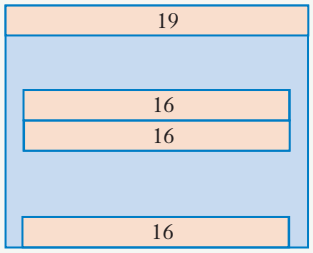
14 $1021.80

15 **a** 7 m by 7 m **b** $4346.30

16 $5775

17 **a** 56 sheets **b** 654 tiles

18 67 cars



Exercise 5.07

1 D

2 B

3 C

4 **a** **i** 1.75 **ii** 3.0625 **iii** About 18.4 cm^2
b **i** 1.4 **ii** 1.96 **iii** 784 mm^2

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c **i** 1.8 **ii** 3.24 **iii** About 3.03 m^2

d **i** 0.55 **ii** 0.3025 **iii** About 257.1 cm^2

5 **a** 62 500 **b** 153.1 m^2

6 **a** **i** 2 **ii** 8 **iii** 14.4 cm^3

b **i** 1.5 **ii** 3.375 **iii** 6075 mm^3

c **i** 1.25 **ii** 1.953 125 **iii** About 97.7 m^3

d **i** 0.4 **ii** 0.064 **iii** About 2.1 L

7 About 230 g

8 About 1.28

9 **a** About 300 cm^2 **b** About 717 cm^3

Chapter review

1 E

2 D

3 D

4 E

5 C

6 B

7 **a** 3384 cm^2 **b** 9537.88 cm^2

8 SA = 2960.37 mm^2

9 **a** 30.08 m^2 **b** 6.51 m^2

10 **a** *BDFHJ* ||| *SRWZX* **b** 3

11 **a** Similar, AAA **b** Similar, SAS

12 $x = 3.6$

13 21 m

14 87.5 m

15 **a** 1 : 50 000 **b** 1 cm : 12.5 m

16 **a** 17.5 m **b** 9.6 mm by 5.6 mm

17 **a** 2.75 **b** About 272 cm^2

18 **a** 2.25 **b** 11.390625

c About 4.6 L

19 about \$1370

20 382 labels (with some wastage at the end of the roll)

21 About 160 m

22 22 sheets each cut to 3.8 m long. Each side is 11.2 m long, but projections increase the length to 11.6 m. The effective width of the sheet is 1.1 m. 11.8 ÷ 1.1 ≈ 10.72 or 11 sheets. So 22 sheets are needed to cover both sides of the roof.

23 1.266 L

Chapter 6

Exercise 6.01

1 **a** (2, 4) **b** (1, 4) **c** (1, 6)

d (–2, 5) **e** (1, 5) **f** (–2, 4)

2 **a** (–1, 5) **b** (1, 1) **c** (3, 7)

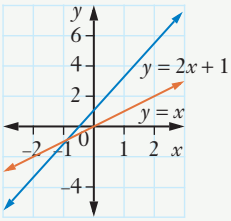
d (0, –2)

3 D

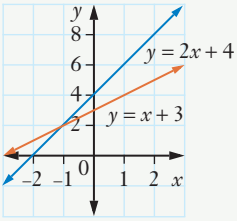
4 B

5 **a, b, d, e, g, h** and **j** are simultaneous solutions.

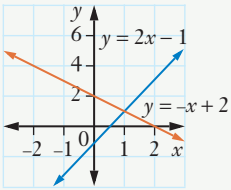
6 **a** Simultaneous solution $x = -1, y = -1$



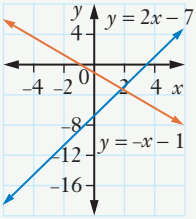
b Simultaneous solution $x = -1, y = 2$



c Simultaneous solution $x = 1, y = 1$



d Simultaneous solution $x = 2, y = -3$



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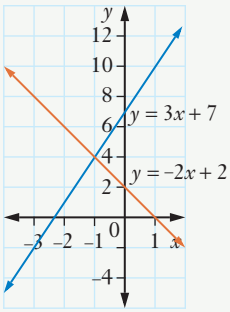
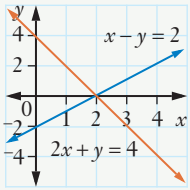
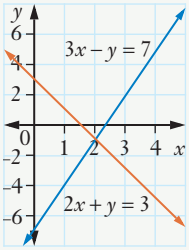
e Simultaneous solution $x = -1, y = 4$



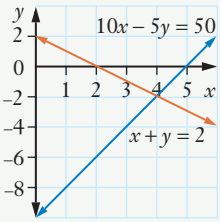
7 a Simultaneous solution $x = 2, y = 0$



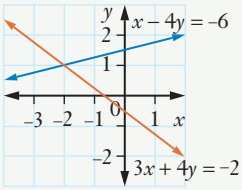
b Simultaneous solution $x = 2, y = -1$



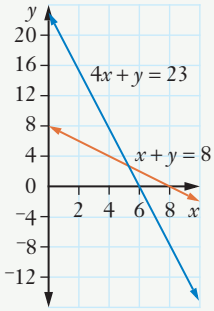
c Simultaneous solution $x = 4, y = -2$



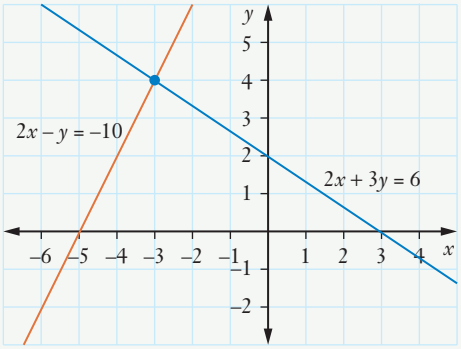
d Simultaneous solution $x = -2, y = 1$



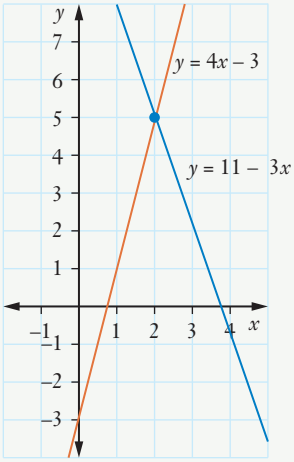
e Simultaneous solution $x = 5, y = 3$



8 a Simultaneous solution $x = -3, y = 4$



b Simultaneous solution $x = 2, y = 5$



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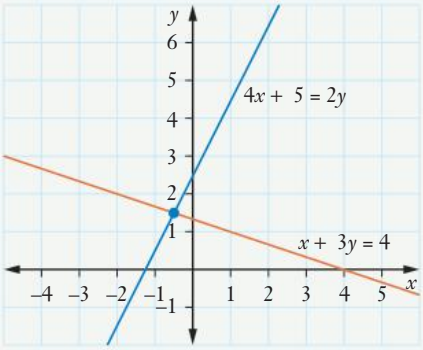
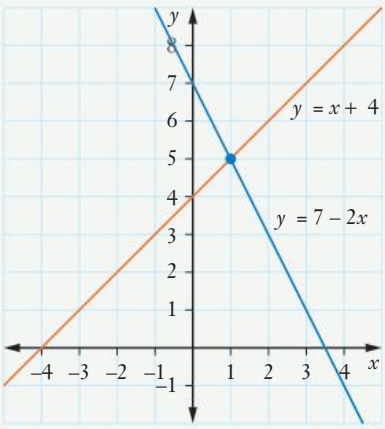
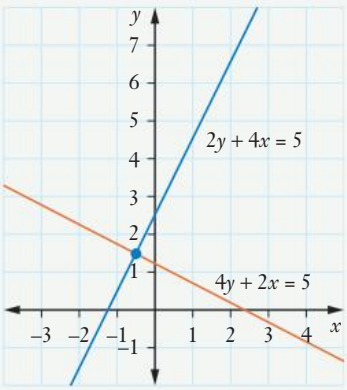
c Simultaneous solution $x = -\frac{1}{2}, y = 1\frac{1}{2}$



d Simultaneous solution $x = 1, y = 5$



e Simultaneous solution $x = -\frac{1}{2}, y = 1\frac{1}{2}$



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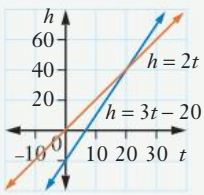
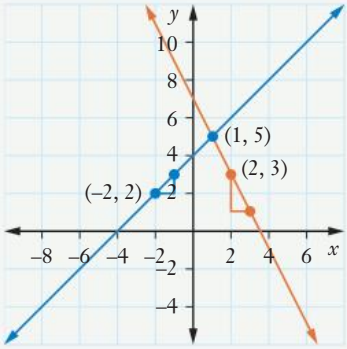
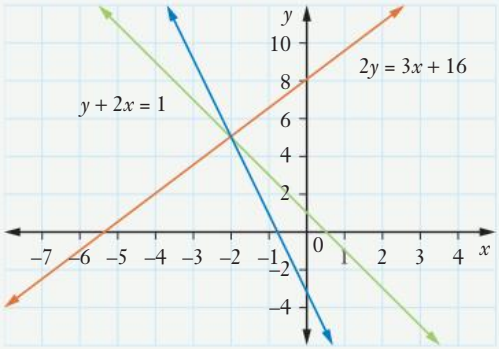


Same height after 20 seconds

10 Simultaneous solution is $x = 1$ and $y = 5$.



11 Simultaneous solution is $x = -2$ and $y = 5$.



$y + 4x + 3 = 0$

Exercise 6.02

1 D

2 B

3 **a** $p = 5q - 8$ **b** $p = 7, q = 3$

4 **a** $x = 5, y = 2$ **b** $x = 6, y = 18$ **c** $a = 2, b = 1$
d $a = 5, b = -7$ **e** $x = 0, y = 3$ **f** $c = 8, d = 3$
g $c = 5, d = 4$ **h** $c = 4, d = 1\frac{1}{2}$ **i** $x = 2, y = -1$

5 **a** $x = 3, y = 2$ **b** $x = 5, y = 2$ **c** $m = 3, n = 1$
d $a = 3, b = 2$ **e** $m = 1, n = -2$ **f** $x = 3, y = -3$
g $x = 4, y = -3$ **h** $a = -2, b = 1$ **i** $m = -2, n = 1$

6 $x = \frac{b}{2a}, y = \frac{b}{2}$

7 **a** $l = w + 2$ and $2(l + w) = 68$
b $l = 18, w = 16$

8 $d = b - 6$ and $2d + 3b = 118$; $d = 20$ and $b = 26$

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Exercise 6.03

1 **a** Flynn is 3 km ahead of Aiden.

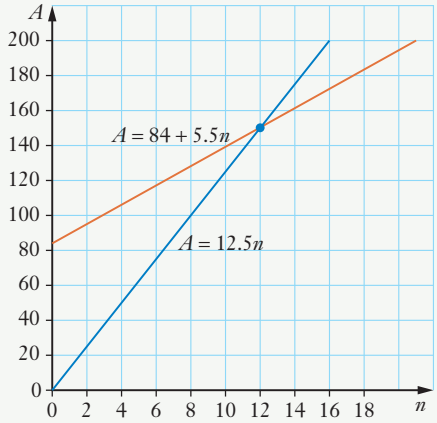
b Aiden is 2 km ahead of Flynn.

c This the time when they are the same distance from the sports ground.

2 E

3 B

4 **a**

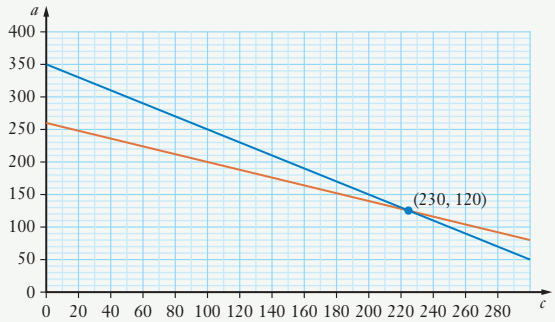


b Break-even point is when $n = 12$.

c The point where sales revenue equals production costs.

d Profit of \$28.

5 **a**



b 230 children and 120 adults attended the concert.

6 **a** $4b + 3c = 21$ and $2b + 4c = 19.8$

b A bagel costs \$2.46 and a croissant costs \$3.72.

7 $a = 40$ cm, $b = 10$ cm

8 Sam 50 bricks, Peta 200 bricks

9 Strawberries \$4/tray, cherries \$6/tray

10 Length = 27 cm and width = 21 cm

11 Adult's ticket \$7, child's ticket \$2

12 2 watches

13 132 serves of hot chips, 44 meat pies and 33 Chiko rolls.

Exercise 6.04

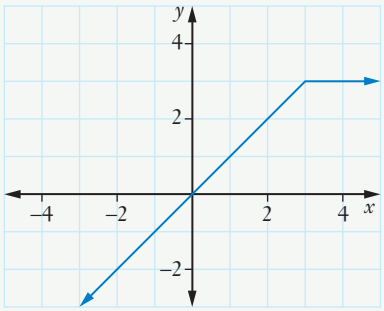
1 D

2 B

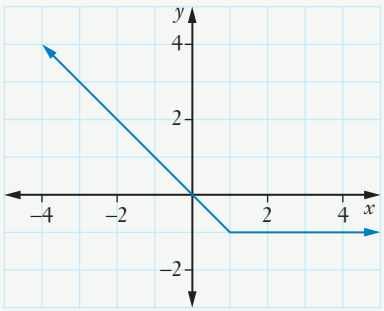
3 A

4 C

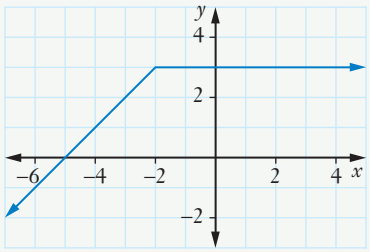
5 **a**



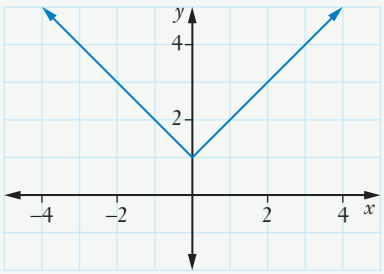
b



c



d



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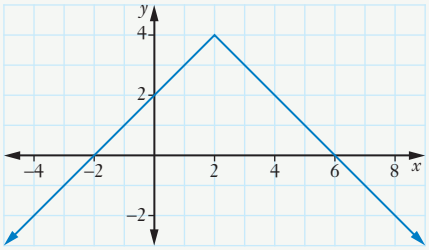

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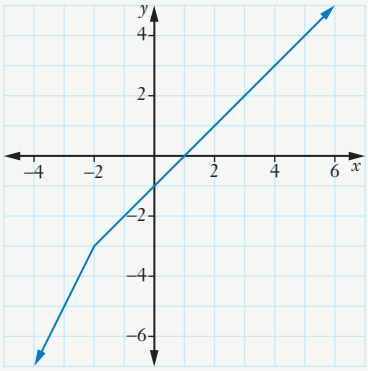
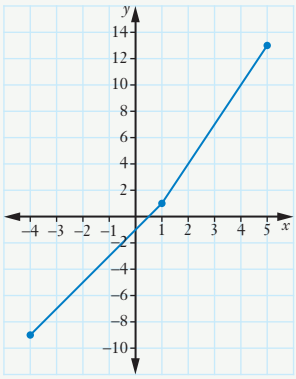


6 a



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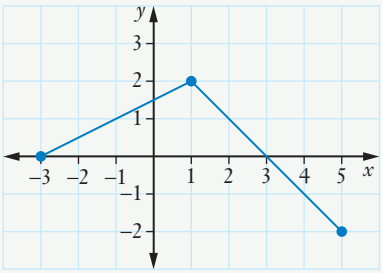

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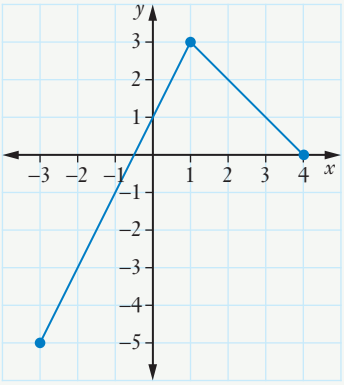

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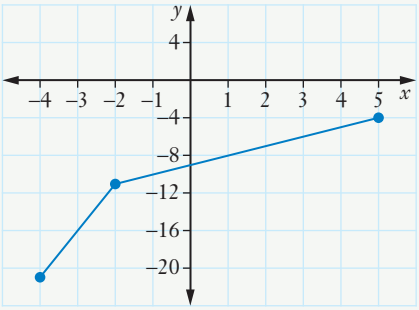

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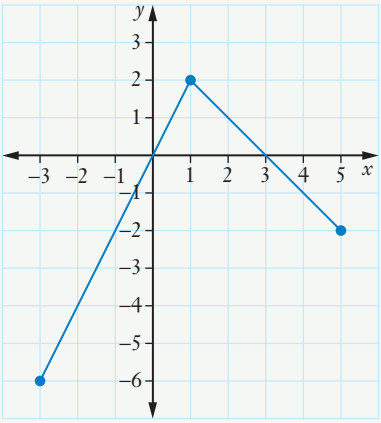

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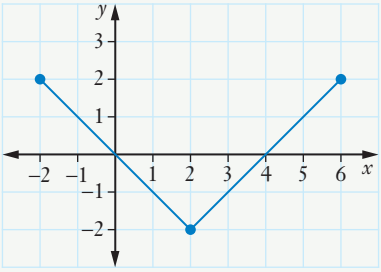

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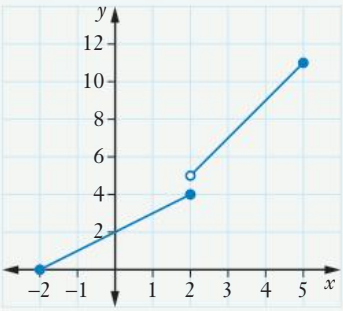
7 **a**



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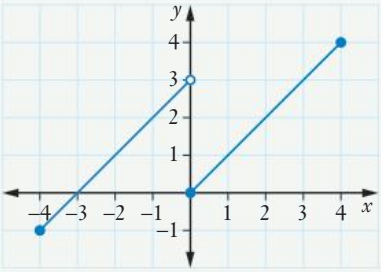

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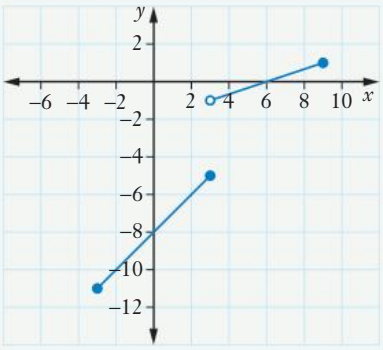

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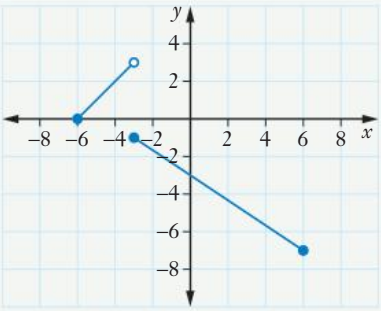

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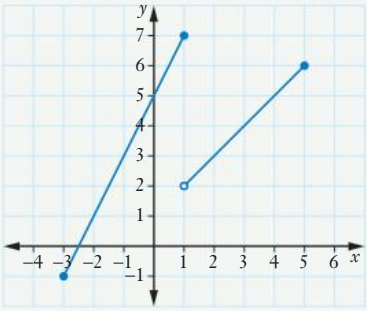

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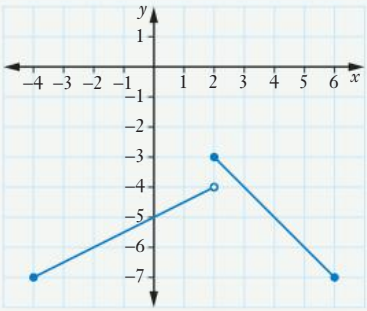

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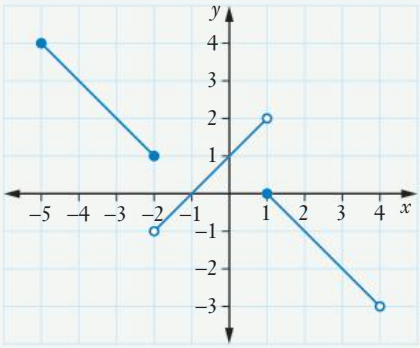

9 **a** $-3 \le x \le 1$ and $1 \le x \le 5$

b $m = 2$ and $c = 1$

c $y = 2x + 1$ **d** $y = -x + 4$

e $y = \begin{cases} 2x+1 & \text{if} \quad -3 \le x \le 1 \\ -x+4 & \text{if} \quad 1 < x \le 5 \end{cases}$

10 $y = \begin{cases} x+3 & \text{if} \quad -2 \le x < 1 \\ -2x+5 & \text{if} \quad 1 \le x \le 4 \end{cases}$

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Exercise 6.05

1 The segments cannot overlap. This means that the segments for $-2 \le x \le 1$ and $0 \le x \le 2$ have not been drawn correctly. It also means that either the right-hand end point of the segment for $0 \le x \le 2$ or the left-hand end point of the segment for $2 \le x \le 6$ is incorrect.

2 **a** **i** \$20 **ii** \$20 **iii** \$15 **iv** \$10

b \$895

c The cost/T-shirt decreases (in steps) as more are ordered. The set up and fixed production costs are distributed over a greater number of T-shirts and so the cost per T-shirt is less.

d \$5

3 **a** \$5 **b** \$10 **c** 4 hours **d** \$80

4 **a** \$9 **b** \$12 **c** \$10.50

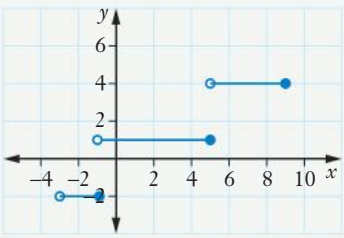
d \$10.50 **e** \$100

5 **a** **i** 10 cents **ii** 15 cents **iii** 5 cents **iv** 7 cents

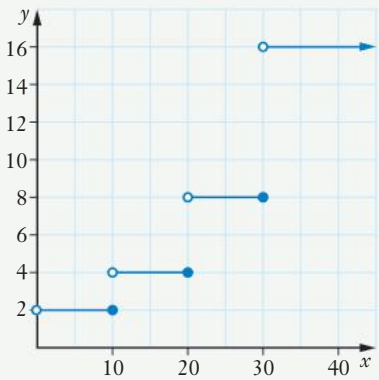
b \$23.50

c The cost/copy decreases (in steps) as more are printed.

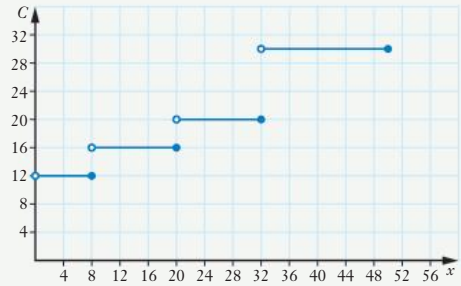
6 **a**



b



c



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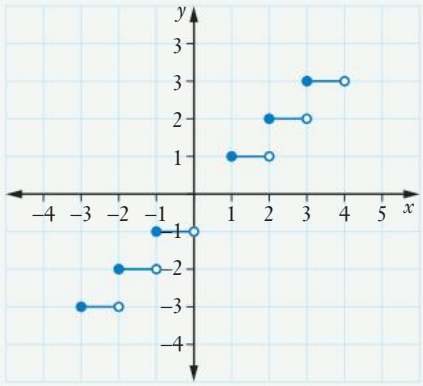
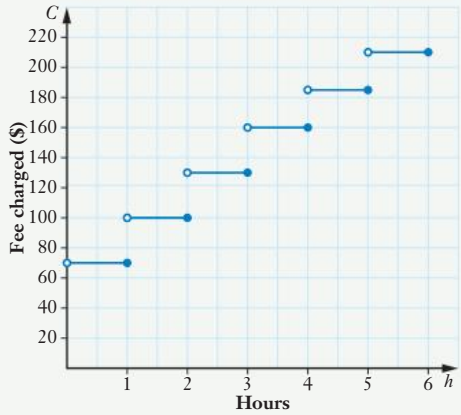


7 **a**

Hours (h)	0	0.5	1	1.5	2	2.5	3	3.5	4	5	6
Cost (C)	0	70	70	100	100	130	130	160	160	185	210

b



8 $C = \begin{cases} \$7.5 & \text{if } 0 < t \le 10 \\ \$10 & \text{if } 10 < t \le 15 \\ \$15 & \text{if } 15 < x \le 25 \\ \$20 & \text{if } t > 25 \end{cases}$

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Exercise 6.06

1 D

2 B

3 D

4 E

5 **a** The graph is made up of 4 line segments. The gradient of each line segment increases as the taxable income increases.

b The highest rate of taxation is paid for taxable incomes greater than $80 000.

c The lowest rate of taxation is paid on taxable income less than $18 000.

d **i** $0 **ii** About $19 400

iii About $11 050 **iv** About $7150

e $3600 + 33c per $1 over $37 000

6 **a** **i** $3.50 **ii** $4.75 **iii** $7.00

iv $2.00 **v** $7.00

b $C = \begin{cases} \$2 & \text{if } 0 < w \le 75 \\ \$3.50 & \text{if } 75 < w \le 150 \\ \$4.75 & \text{if } 150 < w \le 250 \\ \$7.00 & \text{if } 250 < w \le 500 \end{cases}$

7 **a** $22 **b** $46 **c** $40 **d** $6/kg

e The price per kilogram charged for the almonds decreases as the total weight of almonds purchased increases.

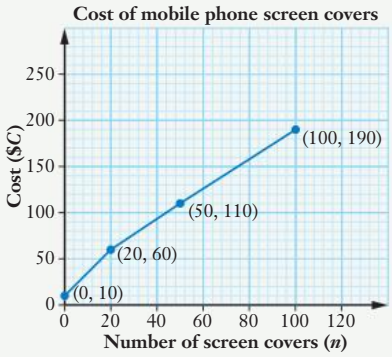
8 **a** This is a step function.

b

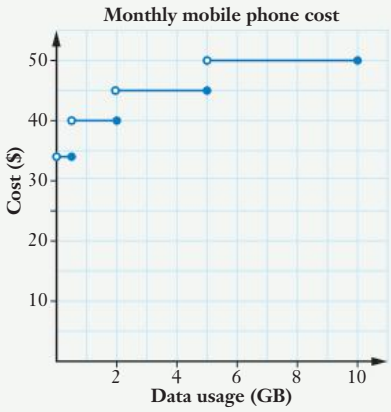
9 **a**

Number ordered (n)	0	20	50	100
Cost ($\$C$)	10	60	110	190

b



10 **a**



b $45

c $C = \begin{cases} \$34 & \text{if } 0 < n \le 0.5 \\ \$40 & \text{if } 0.5 < n \le 2 \\ \$45 & \text{if } 2 < n \le 5 \\ \$50 & \text{if } 5 < n \le 10 \end{cases}$

11 **a** Use this information to complete the following table.

T-shirts ordered (n)	0	20	50	70
Cost ($\$C$)	0	300	660	860

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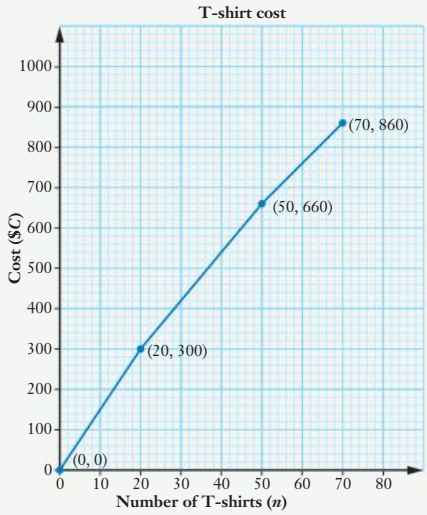
b

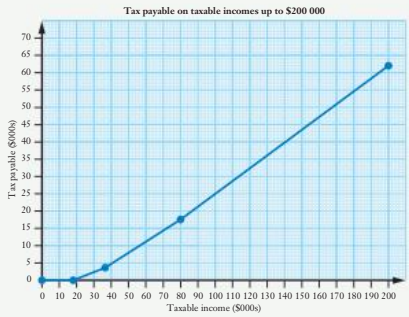


c **i** \$600 **ii** \$780

d $C = \begin{cases} 15n & \text{if } 0 < n \le 20 \\ 12(n-20)+300 & \text{if } 20 < n \le 50 \\ 10(n-50)+660 & \text{if } 50 < n \le 70 \end{cases}$

12 **a** 5 **b** 45%

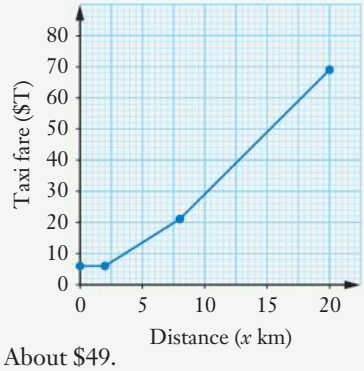
c



d About \$7800 **e** About \$43 500

13 **a** A piece-wise function

b **Taxi fares for distances up to 20 km**



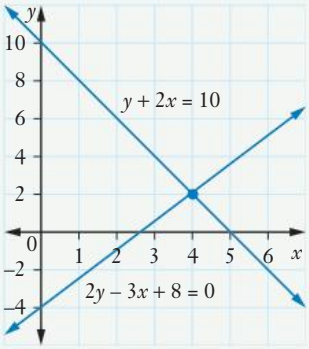
c About \$49.

Chapter review

1 Simultaneous solution is $x = -1$ and $y = -2$.

2 (1, 2) is a simultaneous solution.

3 Simultaneous solution is $x = 4$ and $y = 2$.



4 C

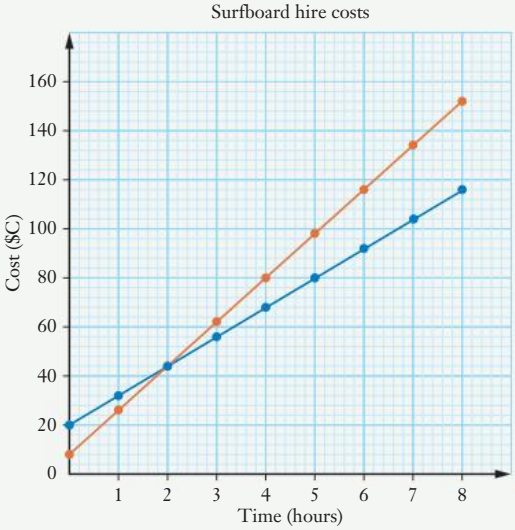
5 D

6 Simultaneous solution is $a = 5$ and $b = -2$.

7 **a** $a = 2, b = -2$ **b** $a = -2, b = 3$

8 A

9 **a** $C = 20 + 12h$ and $C = 8 + 18h$



c Ride High is cheaper after 2 hours ($2 < h \le 8$).

d 2 hours

10 **a** $6f + 3m = 96$ and $5f + 4m = 116$

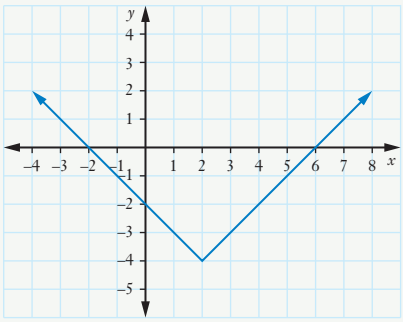
b Flash drive = \$4, memory stick = \$24

11 Length = 18 m

12 D

13 B

14



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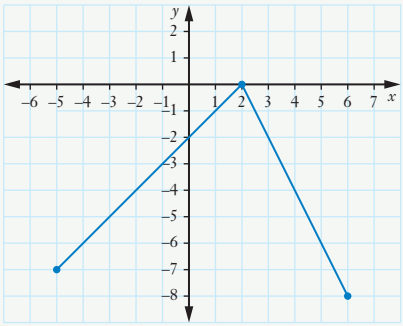

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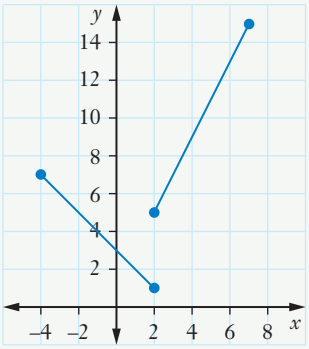


17 B

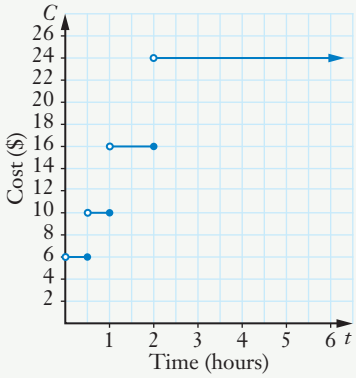
18 C

19 **a** 3 min

b More than 6 and up to and including 7 min.

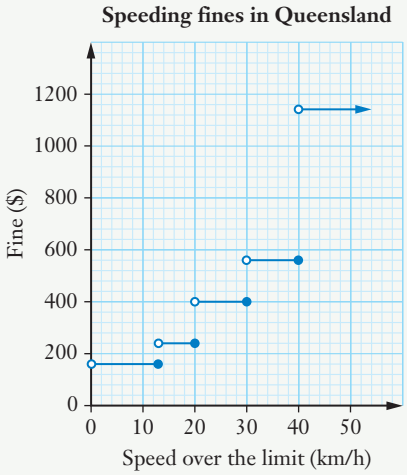
c 80 c **d** 5 minutes

20



21 **a** $40 **b** $440 **c** $20

22



23 $m = \frac{1}{4}$ and $n = \frac{1}{2}$

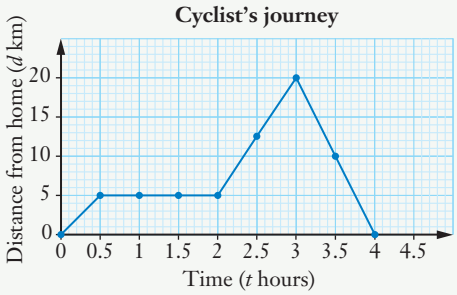
24 2.5 m by 2 m

25 Aria 30, Isla 10

26 **a**

Time (t hours)	0	0.5	1	1.5	2	2.5	3	3.5	4
Distance from home (d km)	0	5	5	5	5	12.5	20	10	0

b



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Practice Examination 2

1 B (1 mark)

2 \$480 (1 mark for working, 1 mark for answer)

3 About 40.2% (1 mark for working, 1 mark for answer)

4 **a** \$1008 **b** About 7.1 (1 mark each)

5 **a** 2.83 m^2 **b** 112 cm^2 (2 marks each)

6 **a** Ratio of corresponding sides are equal and corresponding angles are equal, so $AMFD \parallel\!| TPWS$. (2 marks)

b Scale factor = 1.875 (1 mark)

7 $x = 36$ (2 marks)

8 $\triangle MSQ \parallel\!| \triangle TWR$ (SAS) (2 marks)

9 $x = 2, y = 1$

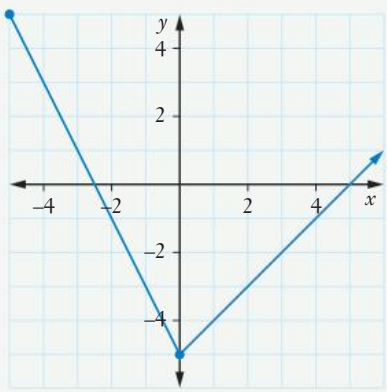
10 $x = 2, y = 4$

11 D

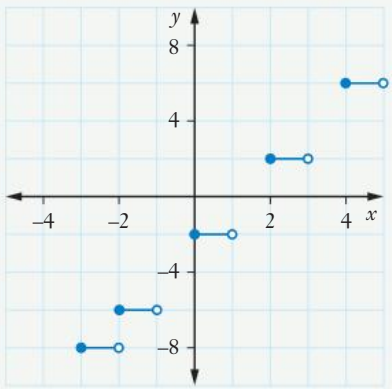
12 $x = 3, y = 5$

13 $x = 3, y = 7$

14 (2 marks)



15 (2 marks)



16 USD 316 (1 mark for working, 1 mark for answer)

17 179.0 cm^2 (2 marks)

18 2.64 m (2 marks)

19 Adult's ticket = \$21, child's ticket = \$15

20 about 9.4% (1 mark for working, 1 mark for answer)

21 Triangles are similar because corresponding angles are equal. (1 mark each)

22 $x = 2$ (1 mark for working, 1 mark for answer)

23 $x = -\frac{1}{6}, y = \frac{4}{9}$

Chapter 7

Exercise 7.01

1 **a** $2g + 2h + 10$ **b** $6m - 4 + 5n$
c $4p - 19$ **d** $3 - 5x^2 - 8y + 3xy$
e $5q^2 + 3pq - 4$ **f** $6a - 5 + 10b - 3ab$
g $2m^2 + 14m$ **h** $-y - 17$
i $13y^2 - 13y - 7$ **j** $7p^2 - 34p + 2$

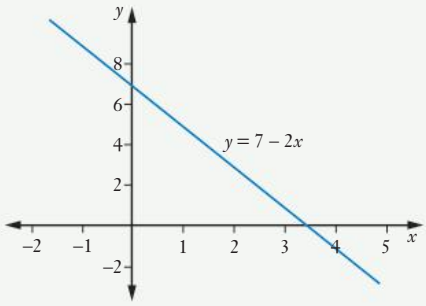
2 **a** $8ab^2c$ **b** $-12x^3y^4$ **c** $-15d^3g^2$
d $42y^4x^5$ **e** $-30r^4s^4$ **f** $48\,p^5q^5$

3 **a** $\frac{3y}{2x}$ **b** $-\frac{3b}{a^2}$
c $\frac{17x^2y}{14z}$ **d** $\frac{19h^3j^2}{21k}$

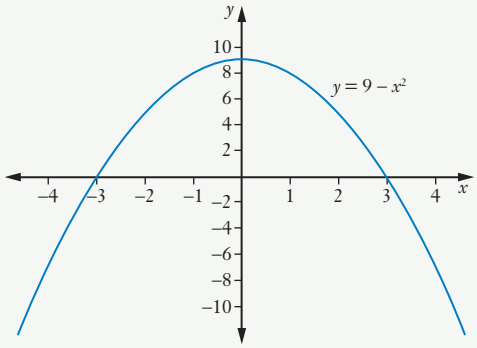
4 **a**, **b**, **e**, **g**, **j** and **n** are linear expressions.

5 **a**, **c**, **d**, **f**, **h** and **i** are non-linear.

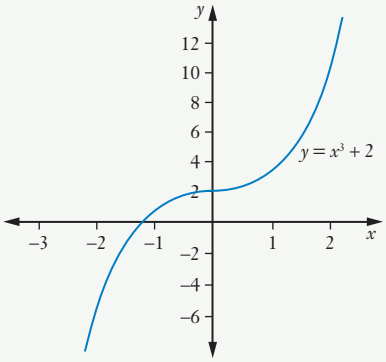
6 **a**



b



c



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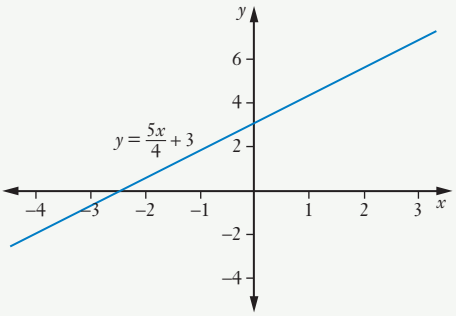
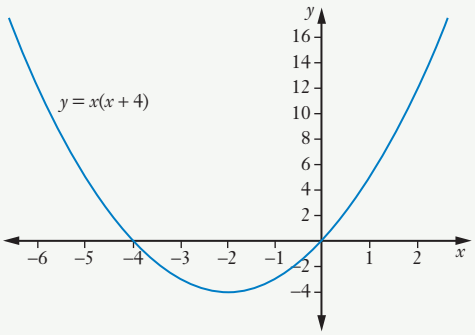
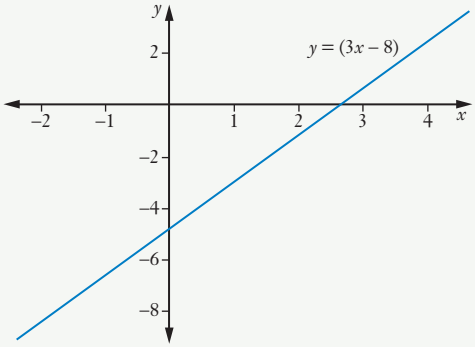


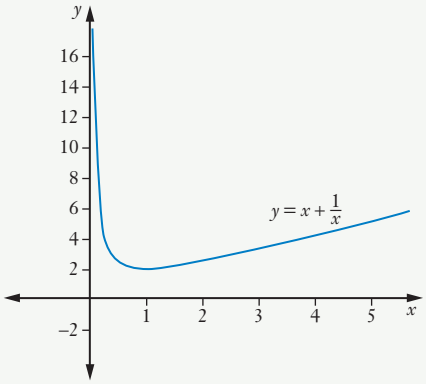
e



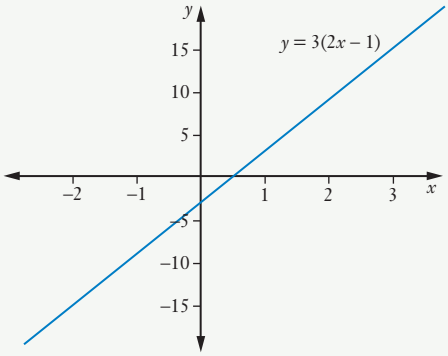
f



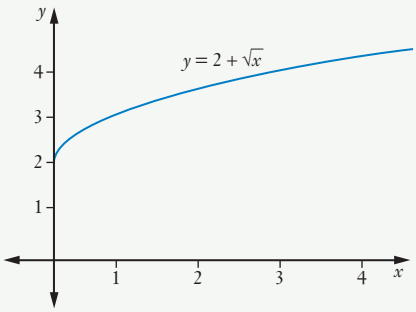
g



h



i



b, **c**, **e**, **g** and **i** are non-linear

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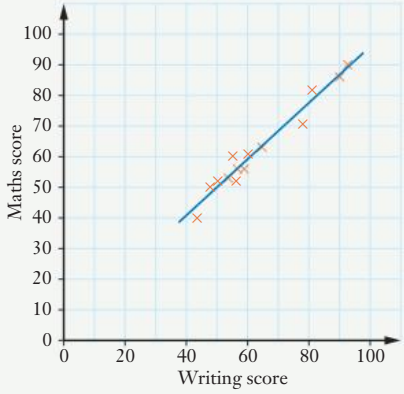
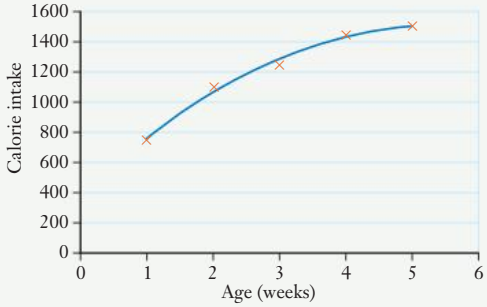
7 **b**, **c**, **f**, **i** and **j** are non-linear

8 **a** There is an approximately linear relationship.

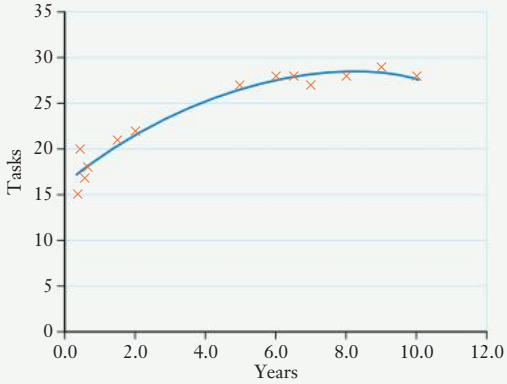


b About 68

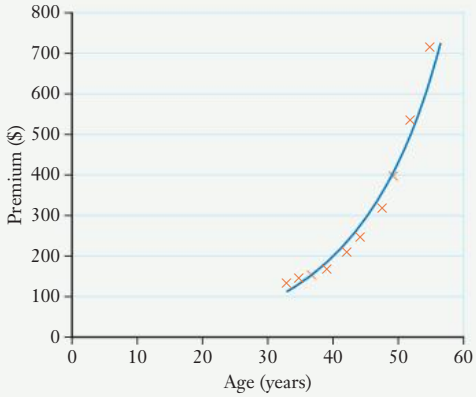
9 Non-linear



10 Non-linear



11 Non-linear



Exercise 7.02

1 **a** $A = 0$ **b** $A = 17$ **c** $A = -5$
d $A = -3$ **e** $A = -5$ **f** $A = -21$
g $A = 3$ **h** $A = 27$ **i** $A = 49$

2 **a** $y = -1$ **b** $y = -7$ **c** $y = -11$ **d** $y = 10$
e $y = -3$ **f** $y = 14$ **g** $y = 2$ **h** $y = 11$

3 **a** $Y = 7.1$ **b** $Y = 10.08$ **c** $Y = -15.3$
d $Y = 16.6$ **e** $Y = -15.5$ **f** $Y = -14.4$
g $Y = 18.01$ **h** $Y = -25.44$ **i** $Y = -25.23$
j $Y = -4$

4 **a** $t = \dfrac{2u - v}{3au}$ **b** $c = \dfrac{Ik - G}{kt}$
c $b = m(c - a) + a$ **d** $u = sa - at$ or $a(s - t)$
e $d = \dfrac{ac}{b}$ **f** $k = \dfrac{d - h}{n}$

5 **a** $R = a \pm \sqrt{d^2 - c}$ **b** $a = cv^2 + b$
c $b = \pm\sqrt{a^2 - c^2}$ **d** $c = \dfrac{(d+5)^2}{b}$
e $c = \dfrac{b}{a - bm}$ **f** $v = \dfrac{fu}{f - u}$
g $b = \dfrac{ac}{cv + a}$ **h** $u = \pm\sqrt{v^2 - 2as}$
i $b = \sqrt{\dfrac{y}{4ac^2}}$ **j** $r = \dfrac{3(x + y)}{1 - x - y}$

6 **a** $A = -2$ **b** $A = 1.8$ **c** $A = 4$
d $A = 1$ **e** $A = -\dfrac{1}{3}$

7 **a** $u = 6$ **b** $m = -4$ **c** $E = 17.5$
d $h = 45$ **e** $t = 3.75$ **f** $c = 2\sqrt{119}$
g $r = 6.5$

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8 **a** \$8442.60 **b** \$18 741.77 **c** \$11 934.07

9 **a** 5.4 g/cm^3 **b** 4837.5 g **c** 250 cm^3

10 **a** $l = g\left(\frac{T}{2\pi}\right)^2$ **b** 5.03 m

11 6000 m

12 $k_1 : k_2 = 4 : 3$

13 $V = \frac{\pi}{3}(R^2 + r^2 + Rr)h$

Exercise 7.03

1 **a** $x = \frac{1}{3}y$ **b** $x = \frac{1}{2}(y + 7)$

c $x = \frac{1}{7}(y - 8)$ **d** $x = \frac{1}{3}(y + 13)$

e $x = -2y - 1$ **f** $x = \frac{1}{6}y + 2$

g $x = \frac{1}{7}(4y + 15)$ **h** $x = \frac{1}{8}(5y + 11)$

i $x = \frac{1}{11}(7y + 18)$ **j** $x = \frac{3}{5}y + 7$

k $x = \frac{1}{10}(3y - 4)$ **l** $x = \frac{1}{51}(15y - 8)$

2 **a** $-\frac{1}{2}(9y + 3)$ **b** $\frac{2}{5}(4y + 1)$ **c** $\frac{1}{4}(9 - 7y)$

d $3y + 2$ **e** $\frac{1}{7}(3y + 4)$ **f** $-\frac{1}{6}(3y + 2)$

g $4 - 2y$ **h** $\frac{1}{8}(4y + 7)$ **i** $\frac{1}{9}(2y - 6)$

3 **a** $\frac{1}{3}(4x - 1)$ **b** $\frac{1}{2}(7x + 3)$ **c** $\frac{1}{2}(4x + 3)$

d $\frac{1}{2}(x + 3)$ **e** $\frac{1}{5}(9 - 6x)$ **f** $\frac{1}{3}(5x - 7)$

g $\frac{1}{10}(5x - 4)$ **h** $-\frac{1}{2}(3x + 4)$ **i** $\frac{1}{9}(2 - 4x)$

4 **a** $y = \frac{x + 6x^2 - 8}{3}$ **b** $y = \pm\sqrt{5x + 12}$

c $y = \frac{1}{2}(5x - 3)^2$ **d** $y = \frac{7}{3x}$

e $y = -\frac{x(3x + 16)}{8}$ **f** $y = \frac{x}{2} + \frac{2}{3}$

g $y = -\frac{49x^2 + 28x + 1}{2}$ **h** $y = -\frac{3}{5 - 2x}$

i $y = \frac{2x}{4x - 1}$ **j** $y = \frac{x(3x + 2)}{7(x - 2)}$

5 **a** $r = \sqrt[3]{\frac{3V}{4\pi}}$ **b** $b = \pm\sqrt{25 - a^2}$

c $u = \frac{v}{5v - 1}$ **d** $y = 1 \pm \sqrt{9 - (x - 2)^2}$

Exercise 7.04

1 **a**

	A	B	C
1	r	h	SA
2	1	10	62.83185
3	2	10	125.6637
4	3	10	188.4956
5	4	10	251.3274
6	5	10	314.1593
7	6	10	376.9911
8	7	10	439.823
9	8	10	502.6548
10	9	10	565.4867
11	10	10	628.3185
12	11	10	691.1504
13	12	10	753.9822
14	13	10	816.8141
15	14	10	879.6459
16	15	10	942.4778

b

	A	B	C
1	r	h	SA
2	10	10	628.3185
3	10	11	691.1504
4	10	12	753.9822
5	10	13	816.8141
6	10	14	879.6459
7	10	15	942.4778
8	10	16	1005.31
9	10	17	1068.142
10	10	18	1130.973
11	10	19	1193.805
12	10	20	1256.637

c About 747.7 cm^2

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2 **a** $a = \sqrt{b^2 + c^2}$

b

	A	B	C
1	b	c	a
2	7	12	13.89244
3	15	26	30.01666
4	42	17	45.31004
5	29	31	42.44997

c **i** 13.9 sq units **ii** 30 sq units
iii 45.3 sq units **iv** 42.4 sq units

3

	A	B	C	D
1	a	b	C	A
2	10	15	28	35.21037
3	12	19	67	104.9376
4	1.5	2.3	98	1.708212
5	32	17	112	252.194
6	56	41	77	1118.577

a 35.2 cm^2 **b** 104.9 mm^2 **c** 1.7 m^2
d 252.2 cm^2 **e** 1118.6 mm^2

4 **a**

	A	B	C	D
1	P	r	t	l
2	1000	6.2	1	62
3	1000	6.2	2	124
4	1000	6.2	3	186
5	1000	6.2	4	248
6	1000	6.2	5	310
7	1000	6.2	6	372
8	1000	6.2	7	434
9	1000	6.2	8	496
10	1000	6.2	9	558
11	1000	6.2	10	620
12	8000	4.3	15	5160

b The amount will be \$1500 after 5.88 years.
c \$5160

5 **a**

	A	B	C
1	r	h	V
2	8	10	2010.619
3	8	11	2211.681
4	8	12	2412.743
5	8	13	2613.805
6	8	14	2814.867
7	8	15	3015.929
8	8	16	3216.991
9	8	17	3418.053
10	8	18	3619.115
11	8	19	3820.177
12	8	20	4021.239

b

	A	B	C
1	r	h	V
2	10	15	4712.389
3	0.4	2.1	1.055575
4	48	56	405 340.85
5	18	52	52929.55

i 4712 cm^3 **ii** 1.06 m^3
iii 405.34 cm^3 **iv** 52 930 cm^3

6

	A	B	C	D
1	a	d	n	l
2	5	6	17	101
3	111	–9	10	30
4	24	8	25	216
5	73	5	42	278
6	207	–11	14	64

a 101 **b** 30 **c** 216
d 278 **e** 64

7

	A	B	C	D
1	a	d	h	A
2	12	21	34	561
3	2.5	3.7	1.9	5.89
4	53	28	27	1093.5
5	27	68	52	2470
6	4.7	2.9	2.2	8.36

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a 561 cm^2 **b** 5.89 m^2 **c** 1093.5 mm^2
d 2470 cm^2 **e** 8.36 m^2

8 163.3 cm

	A	B	C
1	B	m	h
2	30	80	163.2993

9 a

	A	B	C
1	r	h	V
2	10	10	1047.198
3	10	11	1151.917
4	10	12	1256.637
5	10	13	1361.357
6	10	14	1466.077
7	10	15	1570.796
8	10	16	1675.516
9	10	17	1780.236
10	10	18	1884.956
11	10	19	1989.675
12	10	20	2094.395

b 14.3 cm

	A	B	C
1	r	h	V
2	10	14	1466.077
3	10	14.1	1476.549
4	10	14.2	1487.021
5	10	14.3	1497.492
6	10	14.4	1507.964

10 a

	A	B	C
1	r	h	V
2	5	20	523.5988
3	6	20	753.9822
4	7	20	1026.254
5	8	20	1340.413
6	9	20	1696.46
7	10	20	2094.395
8	11	20	2534.218
9	12	20	3015.929
10	13	20	3539.528
11	14	20	4105.014
12	15	20	4712.389

b 12.4 cm

	A	B	C
1	r	h	V
2	12.3	20	3168.61
3	12.4	20	3220.342
4	12.5	20	3272.492

11 a

	A	B	C
1	r	s	SA
2	1.2	1	8.293805
3	1.2	1.1	8.670796
4	1.2	1.2	9.047787
5	1.2	1.3	9.424778
6	1.2	1.4	9.801769
7	1.2	1.5	10.17876
8	1.2	1.6	10.55575
9	1.2	1.7	10.93274
10	1.2	1.8	11.30973
11	1.2	1.9	11.68672
12	1.2	2	12.06372

b 2.51 m

Chapter review

1 a $5x + 5y + 8$ **b** $14m - 23$
c $3 - 4w^2 - 5u + 6wu$ **d** $-8d - 6d^2 - 6d^2e$
e $35x^2 - 45x - 28$ **f** $17z^2 - 26z + 21$

2 a $48x^7y^3$ **b** $30m^6n^6$
c $\frac{9a^2}{2b^2}$ **d** $4pqr^2$

3 **a**, **d**, **g**, **h** and **i** are linear.

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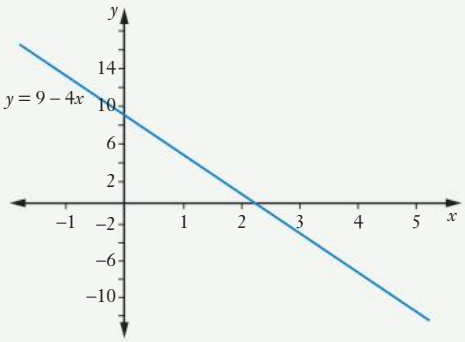
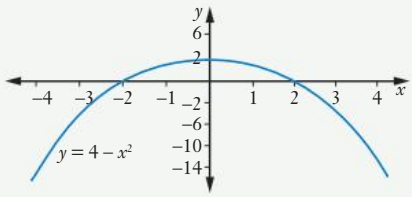
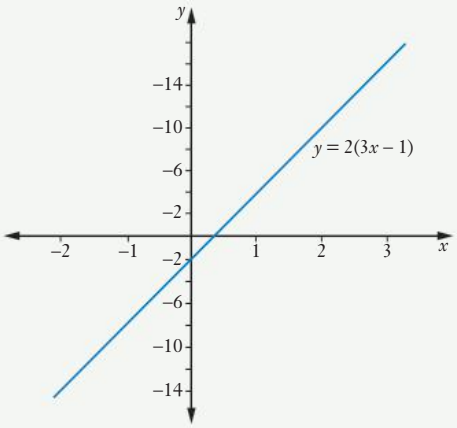
4 **a** Linear



b Non-linear



c Linear



5 **a**, **b**, **c** and **d** are linear.

6 **a** –5 **b** 5 **c** 36

7 **a** $d = \dfrac{2(2b-a)}{3cb}$ **b** $k = \dfrac{6hj}{g}$

c $v = \dfrac{uA-M}{uw}$ **d** $z = \dfrac{x-y}{q} - y$

8 $A = \dfrac{z+\sqrt{6xy}}{4}, A = 4$

9 **a** $y = \dfrac{1}{4}[x - x^2]$ **b** $y = \dfrac{1}{5}(3x+2)^2$

c $y = \dfrac{4}{7+3x}$ **d** $y = \dfrac{4}{5}x - 1$

e $y = -\dfrac{4}{5x}$ **f** $y = \dfrac{4-(5x+3)^2}{3}$

g $y = \dfrac{2x^2+5x}{2(3x-1)}$ **h** $y = \dfrac{5x}{15x-3}$

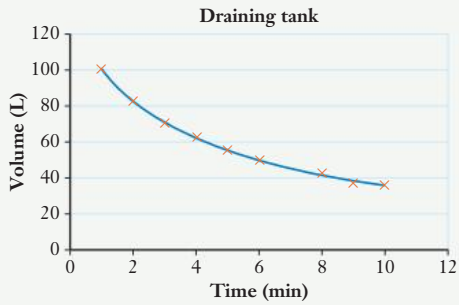
i $y = \pm\sqrt{7x+8}$ **j** $y = \dfrac{1}{2}(5x^2+4x-9)$

10 **a**

	A	B	C
1	r	h	V
2	1	20	62.8318531
3	2	20	251.327412
4	3	20	565.486678
5	4	20	1005.30965
6	5	20	1570.79633
7	6	20	2261.94671
8	7	20	3078.7608
9	8	20	4021.2386
10	9	20	5089.3801
11	10	20	6283.18531
12	11	20	7602.65422
13	12	20	9047.78684
14	13	20	10618.5832
15	14	20	12315.0432
16	15	20	14137.1669
17	28	25	61575.16

b About 61 575 cm^3

11 **a**



b About 4 min 20 sec.

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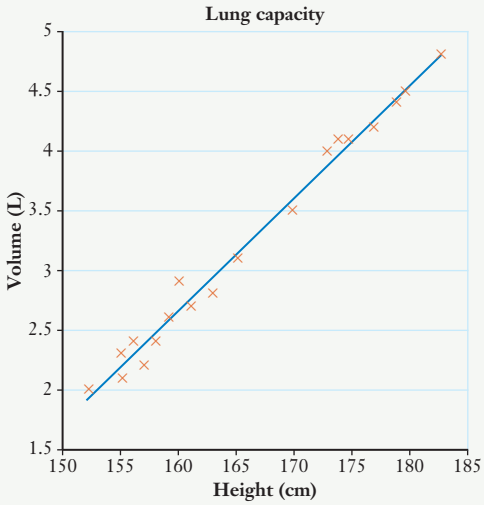
12 **a**



b About 3.4 L **c** About 158 cm

13 About 12.6 cm

Chapter 8

Exercise 8.01

1

	Opposite	Adjacent	Hypotenuse
a	*ST*	*SU*	*TU*
b	*j*	*k*	*i*
c	*CD*	*CE*	*DE*
d	*a*	*c*	*b*
e	*VW*	*VX*	*WX*

2

	Sine	Cosine	Tangent
a	≈ 0.67	≈ 0.75	≈ 0.89
b	≈ 0.94	≈ 0.33	≈ 2.85
c	≈ 0.83	≈ 0.57	≈ 1.44
d	≈ 0.55	≈ 0.84	≈ 0.65
e	≈ 0.84	≈ 0.56	≈ 1.5

3 **a** 0.9736 **b** 0.0872 **c** 0.5299
d 0.9956 **e** 0.2957 **f** 0.2284
g 0.9962 **h** 0.8480 **i** 0.0941
j 0.9553 **k** 0.0244 **l** 0.4494
m 0.8785 **n** 1.9542 **o** 71.6151

4 **a** 58.2117 **b** 21.7773 **c** 81.8904
d 18.3095 **e** 30.0000 **f** 31.7883
g 68.2227 **h** 8.1096 **i** 71.6905
j 60.0000 **k** 0.2063 **l** 8.5308
m 45.0000 **n** 77.7352 **o** 89.9269

5 All values of sine are between 0 and 1. Sine is the ratio of opposite over hypotenuse and the hypotenuse is always the longest side. Therefore, sine will always have a value less than 1.

6 All values of cosine are between 0 and 1. Similarly to sine (from question **5**), cosine is the ratio of adjacent over hypotenuse and the hypotenuse is always the longest side. Therefore, cosine will also always have a value less than 1.

7 No. The tangent ratio is opposite over adjacent. Therefore, the value will be less than 1 when opposite is smaller than adjacent, and a value more than 1 when opposite is greater than adjacent.

8 **a** **i** Cannot be found.
ii Cannot be found.
iii 63.4349°.

b **i** and **ii** Sine and cosine ratios will always be between 0 and 1. Therefore, attempting to find the angle corresponding to a ratio outside of this range will not work.
iii Tangent ratios can be greater than 1. Therefore, the angle can be found.

Exercise 8.02

1 **a** $a \approx 80$ cm **b** $b \approx 168$ m
c $c \approx 910$ mm **d** $d \approx 0.121$ km
e $e \approx 5.39$ cm

2 **a** $a \approx 525$ m **b** $b \approx 0.048$ mm
c $c \approx 10.08$ m **d** $d \approx 12.8$ cm
e $e \approx 8481$ km

3 **a** $RU \approx 507$ cm **b** $IL \approx 28$ mm
c $MT \approx 29.6$ km **d** $AC \approx 138.4$ cm
e $RC \approx 989$ mm

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4 **a** $KM \approx 154$ m, $LK \approx 136$ m
b $BC \approx 65.5$ mm, $AB \approx 81.0$ mm
c $CV \approx 11.6$ m
d $NV \approx 0.109$ m, $PV \approx 0.051$ m
e $DV \approx 17.11$ cm, $CV \approx 3.82$ cm
5 23.8 m
6 4907 m
7 23.3 cm
8 **a** 1.2 m **b** 5.9 m
c A single run up 3 floors would need 7 planks. For runs from floor to floor, 9 planks would be needed altogether.
9 62 m.
10 Line ≈ 20.6 m long, Mast ≈ 19.9 m high.
11 570 mm.
12 30.5 m
13 1.1 m
14 30.0 m
15 **a** 16.4 km **b** 26.5 km
16 Rope ≈ 32.3 m, Distance ≈ 32.0 m

Exercise 8.03

1 **a** $\angle K \approx 35.9°$ and $\angle L \approx 54.1°$
b $\angle Q \approx 48.2°$ and $\angle L \approx 41.8°$
c $\angle P \approx 19.7°$ and $\angle F \approx 70.3°$
d $\angle A \approx 45.2°$ and $\angle B \approx 44.8°$
e $\angle P \approx 29.5°$ and $\angle R \approx 60.5°$
2 **a** $\angle C \approx 71.0°$ and $\angle M \approx 19.0°$
b $\angle Z \approx 47.4°$ and $\angle K \approx 42.6°$
c $\angle M \approx 40.9°$ and $\angle N \approx 49.1°$
d $\angle L \approx 57.4°$ and $\angle K \approx 32.6°$
e $\angle L \approx 37.6°$ and $\angle T \approx 52.4°$
3 **a** $\angle C \approx 20.2°$ and $\angle K \approx 69.8°$
b $\angle D \approx 61.9°$ and $\angle U \approx 28.1°$
c $\angle M \approx 18.9°$ and $\angle C \approx 71.1°$
d $\angle V = 60°$ and $\angle W = 30°$
e $\angle H \approx 40.0°$ and $\angle G \approx 50.0°$
4 26.8°
5 **a** 4.65° **b** Yes.
6 47.73°
7 **a** Metal slide is 25.68°. Plastic slide is 25.56°.
b Metal slide is steeper.
8 30.45°
9 1.92 m of wire
10 1.29°
11 **a** 31.9° to horizontal.
b The ramp would need to rest on the top stair and begin 17.26 m from the bottom riser.

Exercise 8.04

1 37 km
2 108 m
3 632 m
4 40.6 m
5 11.3 km
6 61.7 m
7 **a** 44.5 m **b** 71.2 m
8 58.5 m
9 **a** 12.8 km **b** 51.2 km/h
10 149.6 m
11 200 m
12 41.5 m
13 **a** 31 m **b** 43.3° **c** 29.13°

Exercise 8.05

1 **a** N70°E, 070° **b** N80°W, 280°
c S65°E, 115° **d** S68°W, 248°
e S27°W, 207° **f** S37°E, 143°
2 **ii** **a** 080° **b** 225° **c** 135°
d 000° or 360° **e** 170° **f** 315°
g 200° **h** 330°
3 **ii** **a** S60°E **b** N80°E **c** S25°W
d NW **e** N30°W **f** S30°W
g N40°W **h** NE
4 **a** 393.3 m **b** 338°
5 104 km South, 108 km East
6 742 km, 371 km/h
7 **a** 25.1 km **b** 208.6°
8 82 km
9 149 km from road, 155 km from cyclist.
10 28 km
11 **a** 283.1 km **b** 307°

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Exercise 8.06

1 **a** 9.23 **b** 19.49 **c** 31.38 **d** 62.12
2 **a** 6.50 **b** 3.80 **c** 39.69 **d** 15.59
3 56.44°
4 10.62 cm^2
5 41.05 cm^2

Chapter review

1

	Opposite	Adjacent	Hypotenuse
a	*KM*	*LM*	*LK*
b	*DQ*	*BD*	*BQ*
c	*CR*	*SR*	*CS*

2 **a** 0.8 **b** 0.79 **c** 0.38 **d** 0.92
e 0.6 **f** 0.8 **g** 0.79 **h** 0.38
i 1.33 **j** 0.42
3 **a** 0.7314 **b** 0.9978 **c** 11.4301 **d** 1.0000
e 0.3090 **f** 0.8181 **g** 0.9421 **h** 0.7071
4 **a** 33.4 **b** 75.7 **c** 74.1
d 18.8 **e** 36.6 **f** 11.5
5 **a** $LM \approx 32.2$ m, $KM \approx 42.7$ m
b $BC \approx 25.4$ m
c $KR \approx 9.2$ mm, $KP \approx 17.3$ mm
d $UT \approx 16.8$ cm
e $IM \approx 791.9$ mm, $VM \approx 2113.9$ mm
f $CP \approx 38.9$ km, $CR \approx 42.2$ km
g $EG \approx 18.0$ mm, $FG \approx 7.1$ mm
h $PQ \approx 18.1$ mm
i $GH \approx 17.6$ cm, $HF \approx 14.8$ cm
6 **a** $\angle A \approx 46.1°$ and $\angle B \approx 43.9°$
b $\angle D \approx 45.6°$ and $\angle F \approx 44.4°$
c $\angle I \approx 52.5°$ and $\angle J \approx 37.5°$
d $\angle K \approx 28.8°$ and $\angle M \approx 61.2°$
e $\angle O \approx 55.2°$ and $\angle P \approx 34.8°$
f $\angle Q \approx 47.6°$ and $\angle R \approx 42.4°$
7 **a** *P* is N70°E, *Q* is N60°W
b *P* is 070°, *Q* is 300°
8 **a** Her height is 2992 m above the hill, so her altitude is really about 3397 m.
b The altimeter is reading high, i.e. she is flying lower than her altimeter suggests.
9 833.4 mm.
10 Cliff is 28.7 m high. Lighthouse is 29.1 m tall.
11 40.0°
12 389.0 km
13 29.61°
14 16.29 mm^2

Chapter 9

Exercise 9.01

1 **a** Discrete **b** Ordinal
c Continuous **d** Discrete
e Nominal **f** Ordinal
g Nominal **h** Continuous
2 **a** Nominal
b Discrete for age-classified teams
c Continuous **d** Ordinal **e** Nominal
f Continuous **g** Discrete
3 Nominal: 4, 6; Discrete: 1, 5; Continuous: 2, 3
4 Nominal: 1; Discrete: 2–5; Continuous: 6
5 **a** 50–2500 students **b** 400 kg–2500 kg
c \$10 000–\$200 000 **d** 5c–\$2
e 1c–\$1000
6 **a** 35–180 kg **b** 0.5–7 kg **c** \$0–\$30
d 4–9 **e** 2–12 **f** 0–60
7 **a** Depends on school **b** 15–20 or 15–45
c 15–45
8 Peter has just averaged the three prices without taking account of the sales of each grade of tomato. If most of the tomatoes sold are the cheap ones, the average will be closer to that price.
9 The smallest prime movers usually have 6 wheels and the biggest have 14 wheels. The smallest trailers have 8 wheels and the biggest have 12 wheels. Some cattle rigs used out west have four trailers, so the range would be from 14-wheelers to 62-wheelers.
10 The data is categorical, so it doesn't make sense to average the responses, even when they are coded as numbers.
11 The scales are ordinal and it doesn't make sense to average ordinal scales.
12 The distance is continuous, even though it is written discretely, so Deirdre is technically correct.
13 **a** Age is continuous, but it is normally rounded down to the nearest year for convenience.
b The number of students is discrete because it must be a whole number.

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14 The lowest price is about \$1/kg, direct from the orchard, but the cheapest shop price is about \$2/kg. The most expensive prices are about \$10/kg for high quality imported fruit in the off-season.

Exercise 9.02

1 a

Year	Frequency
7	3
8	6
9	3
10	6
11	9
12	3
Total	30

b

As	Frequency
0	10
1	12
2	6
3	1
4	1

c

Passes	Frequency
0	0
1	1
2	1
3	2
4	3
5	5
6	5
7	9
8	4
Total	30

2 a First preferences

Sweetener	Frequency
Aspartame	18
Sucralose	8
Stevia	6
Saccharine	3
Acesulfame K	5

b First or second preferences

Sweetener	Frequency
Aspartame	23
Sucralose	25
Stevia	6
Saccharine	9
Acesulfame K	17

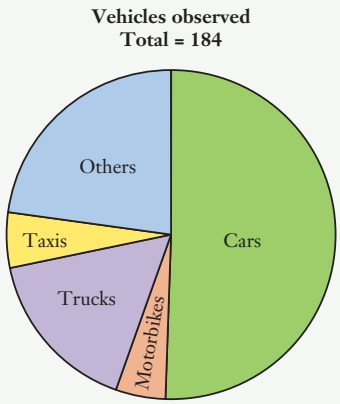
c

Rating	Frequency
1	5
2	12
3	14
4	3
5	6

3

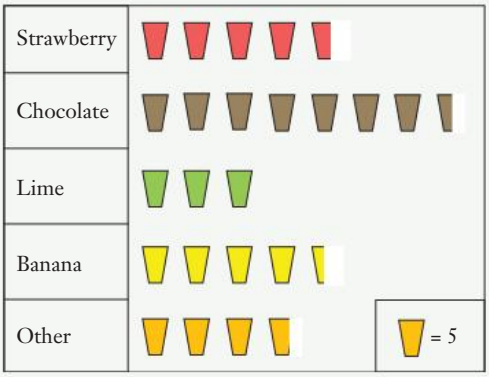
Vehicle	Frequency
Car	93
Motorbike	9
Truck	30
Taxi	10
Other	42
Total	**184**

4

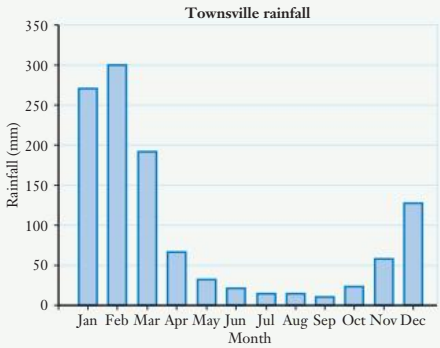


5 **Overseas born Australians 2015**
Total = 6 621 900

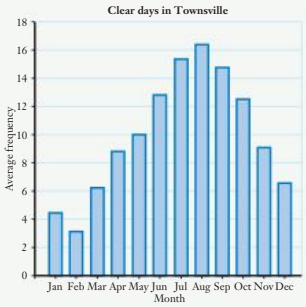
6 Milkshake preferences



7 **a**



b



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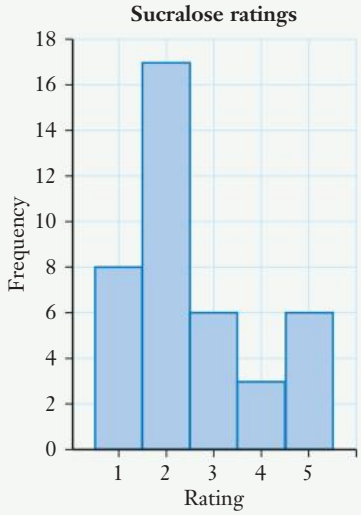

9

10 **a**

Cream	Frequency
A	4
B	8
C	5
D	8
Total	**25**

b If you distribute preferences, you eliminate A, then C and D wins 13 to 12.

If you multiply preferences by preference number D is still the most preferred.

11

Response	Q1 *f*	Q2 *f*	Q3 *f*	Q4 *f*	Q5 *f*
1	19	4	4	22	15
2	25	6	15	9	10
3	7	10	30	7	11
4	9	24	8	6	16
5	3	19	6	19	11

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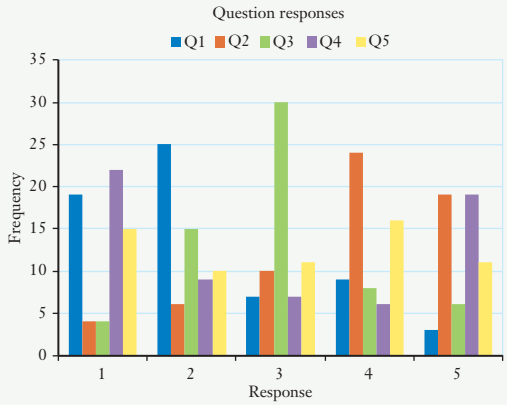
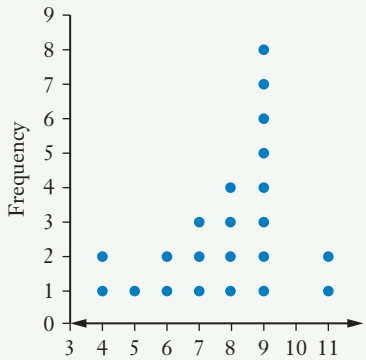


Year 11 students were much in favour of Q1, much against Q2, a bit in favour of Q3, polarised about Q4 and evenly divided about Q5.

Exercise 9.03

1



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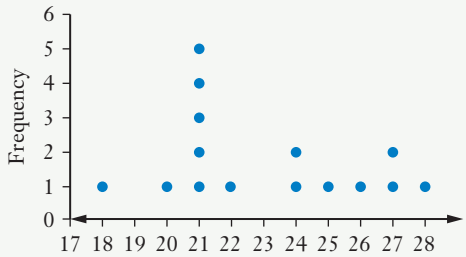

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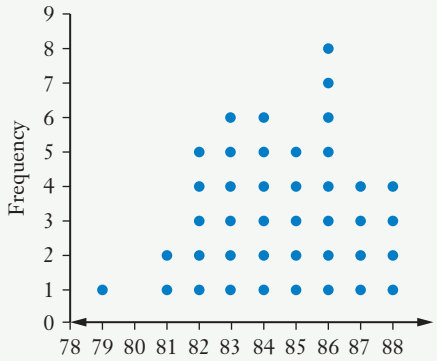


4

Stem	Leaf
1	2 2 9
2	0 4 4 6 7 7
3	0 1 2 2 4
4	1 4 4 6 8
5	0 4 7
6	3

key: 1|2 = 12

5

Stem	Leaf
3	3 6 6 6 7 7 8 8 9
4	0 1 1 1 1 2 2 3 8 9
5	0 3 4 5 6 7 7
6	1 1 3 9
7	
8	5

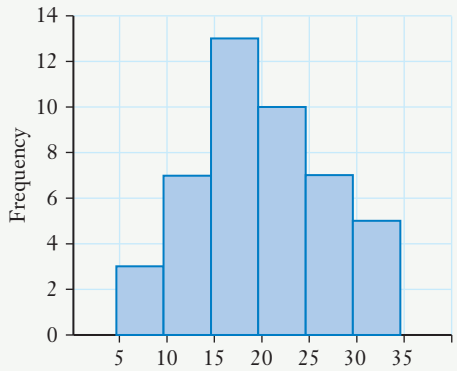
key: 3|3 = 33

6

Stem	Leaf
9	7
10	1 1 5 7 8 8 8
11	0 1 1 2 2 5 5 5
12	6 7
13	0 4 6 8 8
14	1 4 8 9
15	5 6 8 9

key: 9|7 = 97

7



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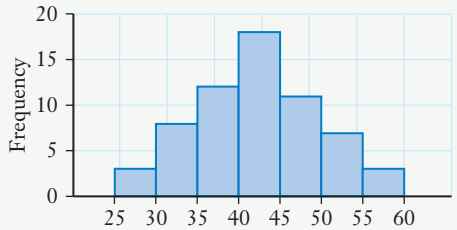

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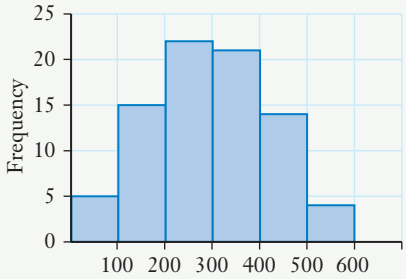
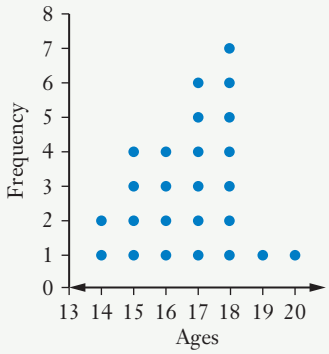
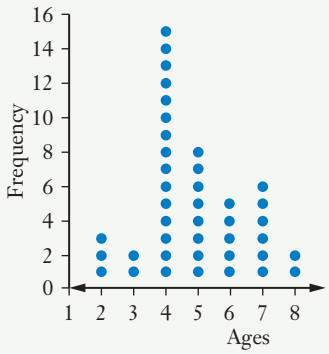


10 a



b 18

11 a



b 15

12 a Ages of living grandparents of Year 11 students

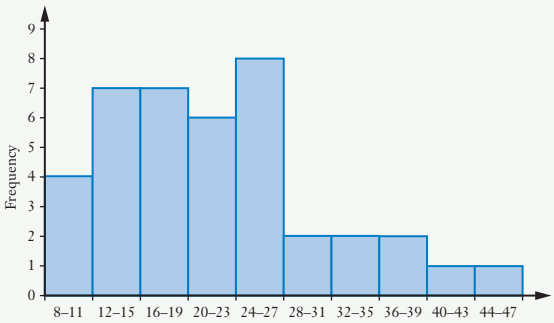
Stem	Leaf
5	7 8 8 9 9
6	1 3 3 3 3 7 7 7 7 8 8 9 9
7	0 0 0 1 1 1 1 1 3 3 4 5 5 6 7 7 9 9
8	0 1 1 2 3 4 4 4 8
9	0 0 1 2 3 6

Key: 6 | 3 = 63

b Their 70s.

13

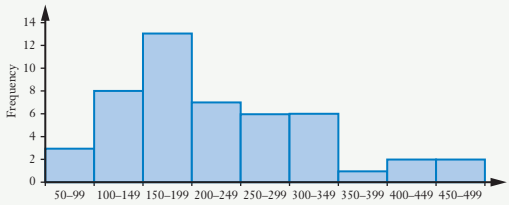
Driving lessons	Frequency
8–11	4
12–15	7
16–19	7
20–23	6
24–27	8
28–31	2
32–35	2
36–39	2
40–43	1
44–47	1
Total	40



Students were fairly evenly spread from about 9 to 26 lessons, then a few up to 45 lessons.

14

Driving lessons	Frequency
50–99	3
100–149	8
150–199	13
200–249	7
250–299	6
300–349	6
350–399	1
400–449	2
450–499	2
Total	48



There was a wide spread of numbers of songs, but most had between 100 and 350.

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Exercise 9.04

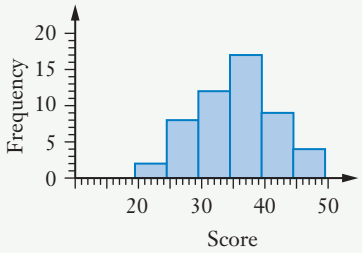
1

Class	Frequency
5–9	2
10–14	2
15–19	10
20–24	19
25–29	16
30–34	7
35–39	1

2

Class	Frequency
10–19	1
20–29	3
30–39	3
40–49	7
50–59	11
60–69	13
70–79	5
80–89	0
90–99	2
100–109	1

3



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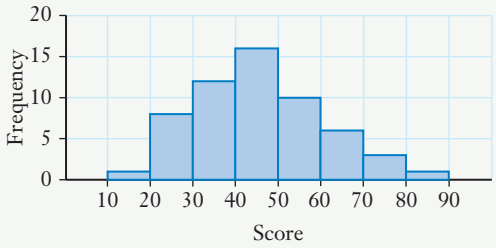

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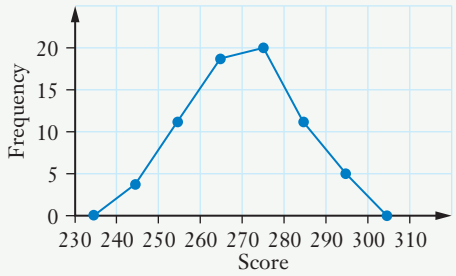

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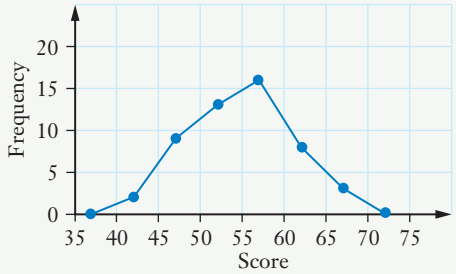
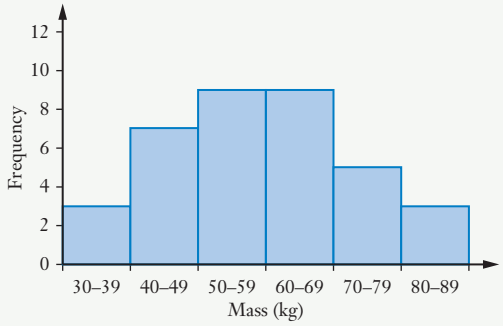


7 a

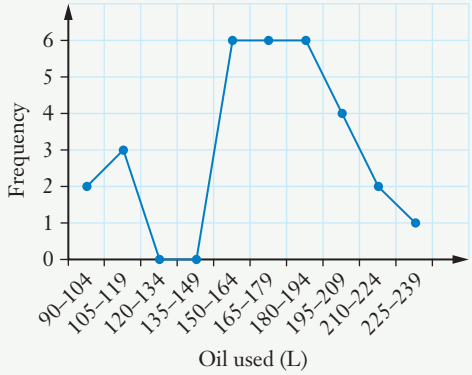
Weight (kg)	Frequency
30–39	3
40–49	7
50–59	9
60–69	9
70–79	7
80–89	3
Total	**38**



b 53

8 a

Oil (L)	Frequency
90–104	2
105–119	3
120–134	0
135–149	0
150–164	6
165–179	6
180–194	6
195–209	4
210–224	2
225–239	1

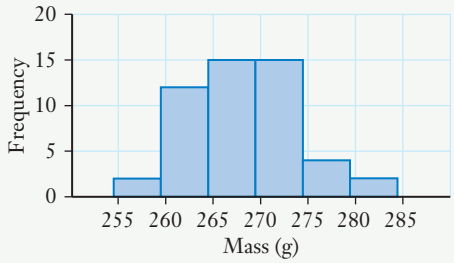


b Probably 6 days a week, including a half-day on Saturday.

There are 30 successive days, and every 6th has much smaller oil use.

c A total of 5134 litres of oil has been used over 5 weeks, so 5134/5 = 1026.8 about 1030 L/week.

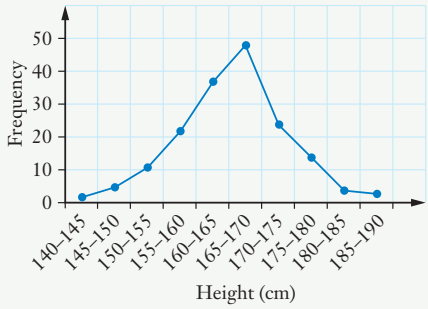
9 a



b 58% **c** About 224

10 a

Height	Frequency
140–145	2
145–150	5
150–155	11
155–160	22
160–165	37
165–170	48
170–175	24
175–180	14
180–185	4
185–190	3



b It may be an all-girls school. There is one peak at about 167 cm.

Exercise 9.05

1 a Mean ≈ 27.5, median = 27, mode = 28

b Mean ≈ 5.92, median = 6, modes = 6, 8

c Mean ≈ 6.44, median = 6, mode = 6

2 a Mean ≈ 13.3, median = 14, mode = 14

b Mean ≈ 214.1 km/h, median = 213.5 km/h, modes = 212, 217 km/h

c Mean ≈ 7.93, median = 7, mode = 6

3 a Mean ≈ 4.75, median = 4.5, mode = 4

b Mean = 8.85, median = 9, mode = 9

c Mean ≈ 4.15, median = 4, mode = 5

4 a Mean ≈ 7.41, median = 7, mode = 7

b Mean ≈ 18.45, median = 19, mode = 19

c Mean ≈ 16.66, median = 16, mode = 18

5 a Mean = 14, median ≈ 14.86, modal class = 15–19

b Mean ≈ 73.27, median ≈ 73.63, modal class = 70–79

c Mean ≈ 227.28, median ≈ 239.77, modal class = 230–269

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6 **a** Mean ≈ 170.29 cm, median = 170.54 cm, modal classes = 160–164, 170–174 cm

b Mean ≈ 68.65, median ≈ 70.45, modal classes = 70–74.9, 75–79.9

c Mean ≈ 4.02 m, median ≈ 3.7 m, modal class = 3–3.4 m

7 **a** Mean = 7.8 m, median = 8, mode = 8

b 8

8 **a** Mean ≈ 3.3 days, median = 3 days, modes = 2, 3 days

b 500 days **c** 3 days

9 **a** Mean = 17.75 cars, median = 18 cars, mode = 18 cars

b 18 cars

10 **a** Mean ≈ $4433, median ≈ $4326, modal class = $3500–$3999

b $4326 (median), because the mean is affected by a few very high incomes.

11 **a**

Mass (kg)	Frequency
1.0–1.1	3
1.2–1.3	5
1.4–1.5	21
1.6–1.7	26
1.8–1.9	13
2.0–2.1	3
2.2–2.3	7
2.4–2.5	2

b Mean ≈ 1.67 kg, modal class = 1.6–1.7 kg, median ≈ 1.63 kg

c About 1.63 kg. The distribution is slightly asymmetrical; the median is best.

12 Mean ≈ 73.3 kg, median ≈ 72.1 kg. The median is best as there are a few quite high masses.

Exercise 9.06

1 **a** Range = 7, interquartile range = 5

b Range = 7, interquartile range = 3.5

c Range = 20, interquartile range = 12

2 **a** Range = 16, interquartile range = 6.5

b Range = 7, interquartile range = 4

c Range = 5, interquartile range = 2

3 **a** Range = 9, interquartile range = 4

b Range = 10, interquartile range = 2

c Range = 8, interquartile range = 3

4 **a** range = 25, interquartile range ≈ 5.83

b range = 80, interquartile range ≈ 24.11

c range = 55, interquartile range ≈ 17.3

5 **a** 15 s, the median

b Between 11.5 and 19 seconds

6 Tanya, her range and IQR (12, 3) are both smaller than Juan's (36, 6).

7 Range is 159.5, but IQR is only 48.95, so results are fairly consistent.

Exercise 9.07

1 **a** Mean = 8.7, SD = 2.1

b Mean = 5.5, SD ≈ 2.57

c Mean = 14.625, SD ≈ 2.03

2 **a** Mean = 11, SD ≈ 2.38

b Mean = 9.4, SD ≈ 3.57

c Mean = 21.84, SD ≈ 2.24

3 A: Mean = 8.1, SD ≈ 4.37, RSD ≈ 54%, B: Mean = 8.5, SD ≈ 3.38, RSD ≈ 40%; B is more consistent.

4 A: Mean = 13.5, SD ≈ 6.40, RSD ≈ 47%, B: Mean = 12, SD ≈ 4.54, RSD ≈ 38%; B is more consistent.

5 A: Mean = 7, SD ≈ 2.97, RSD ≈ 42%, B: Mean = 8, SD ≈ 3.33, RSD ≈ 42%; they are equally varied.

6 A: Mean = 23.5, SD ≈ 9.84, RSD ≈ 42%, B: Mean = 23.75, SD ≈ 3.34, RSD ≈ 14%; A is more varied.

7 The price varied between $2.00 and $5.59, with a mean of $3.93 and standard deviation of about 85c. There were few prices below $3 or above $5 and a fairly even variation between $3 and $5 a kg.

Exercise 9.08

1 **a** Bimodal **b** Symmetrical

c Skewed positively **d** Plateau

2 **a** Skewed negatively **b** Symmetrical

c Skewed positively **d** Bimodal

3 **a** 48 is a possible outlier

b 6.1 is a possible outlier

c No possible outliers

d 21 is a possible outlier

e No possible outliers

f 63.2 is a possible outlier

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4 **a** 53 is a possible outlier

b 39.6 is a possible outlier

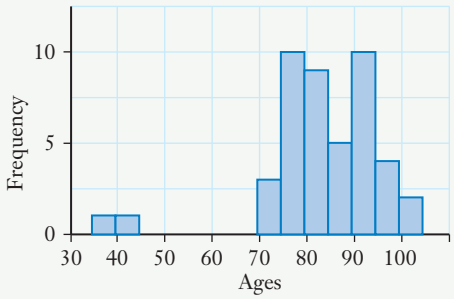
c 7.3 is not an outlier

d 9.1 is a possible outlier

e 2.8 is a possible outlier, 5.3 is not an outlier

f 4.7 and 22.9 are not outliers

5



The ages are bimodal, possibly because women live longer and the 35 and 42 year-olds are outliers.

They would be in an old-age care facility because nothing else suitable is available.

Exercise 9.09

Answers will vary, but should include main points.

1 Circle your average income per week.

under \$300, \$300-\$600, \$600-\$900, … over \$1500

2 Which age group are you in?

under 20, 20–29, 30–39, 40–49, …, 90 or more

3 Circle the highest qualification that you have:

Did not finish Year 12, Year 12, Level I certificate, Level 2 certificate, Level 3 certificate, Trade qualification, University degree, Higher degree

4 Estimate the amount of money you spend on food each week, including takeaways and snacks: Under \$50, \$51–\$100, \$101–\$150, …

5 How many hours do you spend exercising each week, including brisk walking, jogging, gym work, sports training, playing sport, etc:

Under 3 h, 4–5 h, 6–7 h, …, more than 20 h

6 How many people are in your immediate family, including living brothers and sisters, parents, spouse and children?

7 How many trips have you been on this year that have been for leisure (not work) and which include staying overnight away from home?

8 **a** Information about people living in the area, existing centres, suitable sites and availability of staff.

b How many children live in the area, how many need childcare, what people will pay, how much competition is there, is there room in the market?

c Number of likely clients, time required, staff needed, cost of centre, turnover.

d Would you use a local childcare centre?

Do you currently use childcare?

How much do you currently pay?

9 **a** What competitions are available, whether players would be available, whether there is already an existing local team, how many people would support you, where you could play.

b How many players would you get, would they be available for a season, how much would it cost to run, what sponsorship would be available?

c Number of players available, cost per week for competition, cost of training facilities and staff, amount of sponsorship.

d Do you play soccer?

How old are you?

Would you join a local team?

10 **a** What fast-food outlets are already available, availability and cost of site, staff availability, whether you could make a profit, what kind of fast food to offer.

b How often people in the area use fast food, what kind of fast food is there a gap in the market for, what would be the likely turnover, what would be the costs?

c Number of sales likely per week, return on sales, costs to set up and run, likely profit.

d What kind of fast food would you buy if it was available?

How many times do you have fast food each week?

Would you use a shop at a particular location?

11 **a** The kinds of homes, the risks in the area, the likelihood of making sales, the costs of sending a team.

b What are the likely sales, how likely are claims, what are the claims likely to be, what premiums should be charged to make a profit?

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c The likely number of policies, the returns for the policies, the likely costs of claims, the likely profit.

d Do you have home insurance?

What is your current premium?

Have you made a claim in the last 5 years?

Chapter review

1 **a** Discrete (multiples of 1 cent)

b Continuous **c** Nominal

2 **a** 15 cm to 10 m **b** $50 (cot) to $8000

c 4 (Prep) to 14 (repeat a year)

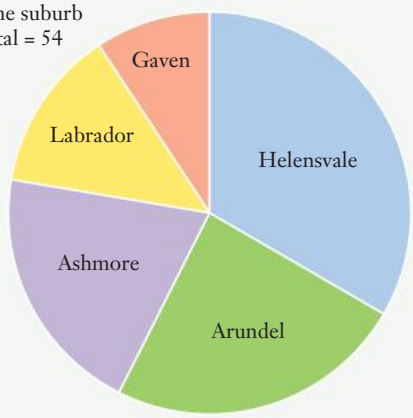
3 Assault with a weapon

Years	Frequency
2	2
3	1
4	2
5	1
6	4
7	2
8	2
9	1
10	2
11	3

Actual bodily harm

Years	Frequency
3	1
4	1
5	0
6	2
7	3
8	1
9	3
10	1
11	5
12	0
13	0
14	1
15	2

4 Home suburb
total = 54



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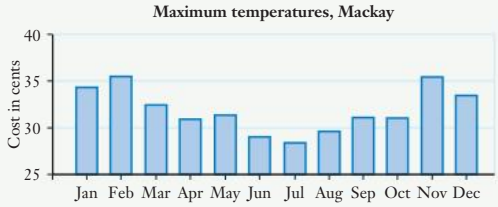

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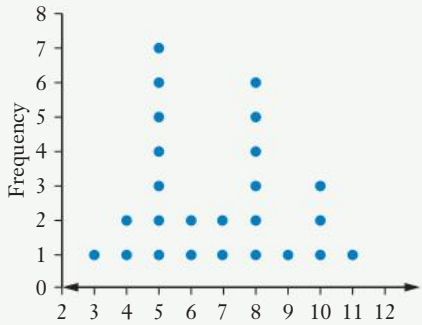
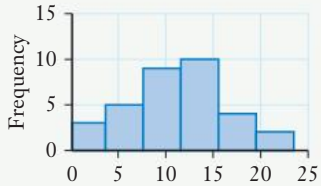


7 **Response times for 1000 calls (s)**

Stem	Leaf
1	2 5 8
2	2 2 3
3	1 8 9
4	0 1 5 5 5 7 9
5	1 2 2 3 4 4 5 8
6	0 1 1 4 6 8 9
7	5 9

key: 1|2 = 12

8



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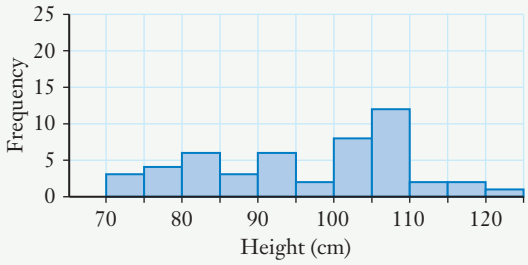
9

Height (cm)	frequency
70–74	3
75–79	4
80–84	6
85–89	3
90–94	6
95–99	2
100–104	8
105–109	12
110–114	2
115–119	2
120–124	1



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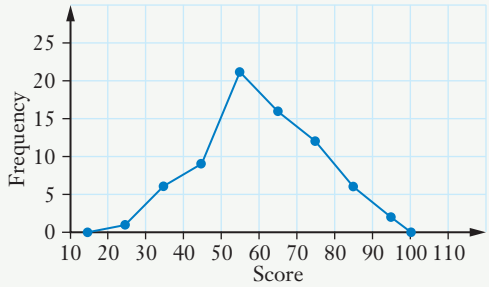


11 **a** Mean ≈ 17.6, median = 18.5, mode = 19

b Mean ≈12.44, median = 12.3, modal class = 10–13

c Mean ≈ 31.88, median ≈ 30.61, modal class = 24.5–29.5

12 **a** Range = 11, interquartile range = 4.5, SD ≈ 2.99

b Range = 8, interquartile range = 3, SD ≈ 2.04

c Range = 55, interquartile range ≈ 17.75, SD ≈ 11.82

13 B is the most consistent; coefficient of variation of 13%, compared to 44%.

14 **a** Skewed negatively **b** Bimodal

c Skewed positively **d** Symmetrical

15 **a** 17 and 26 are possible outliers

b None

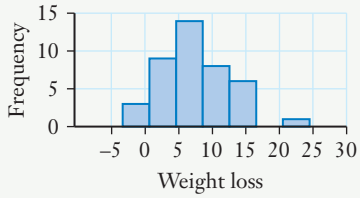
16 **a** Do you earn more than $150 000 a year? (or similar)

b Do you watch more than 30 hours of TV a week? (or similar)

17 **a** Plain 17, BBQ 6, Salt and vinegar 5, Bacon 4, Cheese 1

b Assigning 4, 3, 2, 1 and 0 points for preferences from 1st to last gives totals of Plain 105, Salt and vinegar 79, BBQ 59, Bacon 55, Cheese 32.

18



The program seems effective for most people. For some, it doesn't work or they don't follow it properly.

19 The typical age is about 42. The mean (41.76) is the best measure as the distribution is approximately symmetrical, even though it is bimodal. The median is about 39.5. Both are in the modal class.

20 **a** The typical score is 29, the median. The people who scored 17 and 18 probably didn't prepare properly for the test, but it is also negatively skewed.

b 26–29 inclusive (Q1 = 25.5, Q3 = 29.5)

c 65.5% of people passed.

21 The typical age is 17, most people are 16–21. There are 3 outliers at 45, 51 and 78. They are probably the parents and grandparent of the person whose birthday party it is.

22 She needs to know about other services, likely clients, costs and how much she could charge. She should also know how much time is involved. Some variables would be the number of likely clients, the distances they are from where she is, the rates for different items. Some questions might be:

- Do you already use an ironing service?
- Would you use an ironing service?
- Where do you live?
- Would you expect ironing to be picked up and dropped off?
- What kind and how many items would you want ironed?

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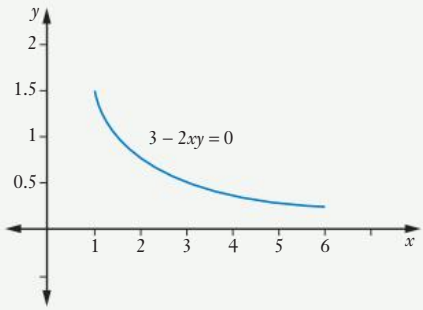
Practice Examination 3

1 A, C, E (2 marks)

2 Linear (1 mark)

3 **a** $y = \frac{2w - 4u}{5wx}$ **b** $r = \frac{20np}{3m}$ (1 mark each)

4



Non-linear (1 mark for graph, 1 mark for answer)

5 Non-linear (1 mark for working 1 mark for answer)

6 $\cos(A) = \frac{8}{17}$ (1 mark)

7 **a** $\tan(27°) = \frac{x}{22}$ (1 mark),

$x = 11.209\ldots \approx 11.2$ m (1 mark)

b $\sin(48°) = \frac{32}{y}$ (1 mark),

$y = 43.060\ldots \approx 43.1$ mm (1 mark)

8 $A = \frac{1}{2}ab\sin(C) = \frac{1}{2} \times 23 \times 19 \times \sin(34°)$

≈ 122.2 cm^2 (1 mark)

9 **a** Continuous numerical

b Ordinal categorical (1 mark each)

10 **a** Typical number is the median, 4 (1 mark)

b Interquartile range = 6 – 3 = 3 (1 mark)

11 Mean ≈ 28.6 cm (1 mark),

SD ≈ 6.1 cm mean (2 marks)

12 $Q_1 = 18$, $Q_3 = 21$, bounds 13.5, 25.5, so 28 is a possible outlier (1 for Q_1, Q_3, 1 for outlier)

13 **a** Negatively skewed

b Bimodal

14 No. The relationship is non-linear. (1 mark for working, 1 mark for answer)

15 $x = 4z^2 - y$ (1 mark for working, 1 mark for answer)

16 $3 \times 60 \times 20 \times \cos(35°)$ (1 mark)

$= 2948.94\ldots$ m ≈ 3 km (1 mark)

18 Non-linear. The graph of $A = \pi r^2$ is not linear. Could also use a table of values.

19 $R = \sqrt{\frac{S^2}{8\pi^2 r^2} - r^2}$ (1 mark for working, 1 mark for answer)

20 $\frac{25}{\tan(20°)} - 2$ (1 mark)

$= 66.686\ldots \approx 67$ m (1 mark)

17

Weight	30–39	40–49	50–59	60–69	70–79	80–89	90–99	100–109	110–119	120–129
Frequency	1	10	27	12	20	13	3	0	1	1

Table (2 marks)

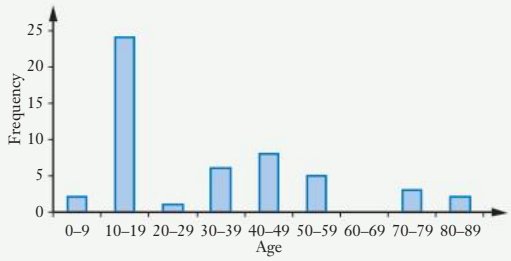
Bimodal (1 mark)

Typical (excluding 117, 128 outliers) = mean ≈ 65 kg (1 mark)

The modes are for girls and boys. (1 mark)

21

Age	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89
Frequency	2	24	1	6	8	5	0	3	2



The distribution is trimodal, with a lot of 13 people who are 15 years old, so it is probably a junior team, with siblings, friends, parents and grandparents attending. (2 marks for table, 1 mark for identification of type, 1 mark for age of team, 1 mark for others attending. Histogram not necessary)

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Chapter 10

Exercise 10.01

1 **a**, **c**, **e** and **f** represent matrices.

2 **a** 4 **b** 9 **c** 5
d 0 **e** 2 **f** There is no a_{43}.

3 **a** $\mathbf{M} =$

$$\begin{bmatrix} 3 & 2 & 4 & 1 & 5 & 0 & 3 & 6 & 1 \\ 1 & 8 & 1 & 7 & 1 & 8 & 0 & 5 & 8 \\ 4 & 0 & 5 & 6 & 4 & 5 & 9 & 1 & 7 \\ 7 & 9 & 2 & 0 & 1 & 1 & 10 & 9 & 1 \end{bmatrix}$$

b **i** 0 **ii** 1 **iii** 0 **c** 4×9

4 **a** Because there is no entry in one cell.
b Place a very high cost in that square to indicate secondary sourcing—say 1000.
c $\mathbf{T} = \begin{bmatrix} 4.20 & 4.30 & 1000 & 4.40 \\ 9.10 & 8.80 & 8.70 & 9.00 \\ 6.20 & 6.40 & 6.50 & 6.10 \\ 3.80 & 3.90 & 3.60 & 3.70 \end{bmatrix}$
d 9.00

5 **a** 0.7324 USD
b $\mathbf{E} =$

$$\begin{bmatrix} 1 & 0.7324 & 0.5874 & 86.24 \\ 1.3654 & 1 & 0.8020 & 117.75 \\ 1.7024 & 1.2469 & 1 & 146.82 \\ 0.011\,59 & 0.008\,493 & 0.006\,811 & 1 \end{bmatrix}$$

c 0.8020 **d** 1.2469 **e** $e_{ij} = \dfrac{1}{e_{ji}}$

6 **a** $\mathbf{B} = \begin{bmatrix} 4 & 9 & 20 & 40 \\ 1 & 2 & 4 & 9 \\ 0.2 & 0.3 & 0.5 & 1 \end{bmatrix}$
b 4 **c** 0.3
d It is the amount of epoxy needed to make a shower base.

7 **a** $\mathbf{P} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 \\ 0 & 6 & 4 & 0 \end{bmatrix}$
b Mosquito larvae do not eat the other animals.
c Water beetles do not eat each other.

Exercise 10.02

1 **a** A row matrix of order 5.
b A 3×4 matrix.
c A column matrix of order 4.
d A square matrix of order 3.
e The 3×4 zero matrix.
f The identity matrix $\mathbf{I}_3$.
g A 2×5 matrix where every element is 1.
h The zero row matrix of order 6.
i A square matrix of order 2.

2 $\mathbf{I}_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

3 $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

4 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

5 $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

6 $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

7 $\mathbf{Q} = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$

8 $\mathbf{P} = [6 \;\; 5 \;\; 4 \;\; 3 \;\; 2 \;\; 1]$

Exercise 10.03

1 **a** $\begin{bmatrix} 5 & 6 \\ 8 & -5 \end{bmatrix}$ **b** $\begin{bmatrix} -3 & 10 \\ 17 & -13 \end{bmatrix}$

c $\begin{bmatrix} 8 & -1 \\ 6 & -2 \end{bmatrix}$ **d** $\begin{bmatrix} -1 & -3 \\ 5 & -9 \end{bmatrix}$

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e $\begin{bmatrix} -3 & -6 \\ -9 & 3 \end{bmatrix}$ **f** $\begin{bmatrix} -8 & 4 \\ 9 & -8 \end{bmatrix}$

g $\begin{bmatrix} -2 & 13 \\ 12 & -4 \end{bmatrix}$ **h** $\begin{bmatrix} -9 & -2 \\ -1 & -7 \end{bmatrix}$

i $\begin{bmatrix} 6 & 0 \\ -3 & -6 \end{bmatrix}$ **j** $\begin{bmatrix} 6 & 0 \\ -3 & -6 \end{bmatrix}$

k $\begin{bmatrix} 15 & 30 \\ 45 & -15 \end{bmatrix}$ **l** $\begin{bmatrix} -24 & 16 \\ 32 & -40 \end{bmatrix}$

m $\begin{bmatrix} -3 & 2 \\ 4 & -5 \end{bmatrix}$ **n** $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$

o $\begin{bmatrix} 1.2 & 2.4 \\ 3.6 & -1.2 \end{bmatrix}$ **p** $\begin{bmatrix} 7 & -6 \\ -14 & -7 \end{bmatrix}$

q $\begin{bmatrix} 5 & 18 \\ 29 & -5 \end{bmatrix}$ **r** $\begin{bmatrix} 2 & -10 \\ 3 & -6 \end{bmatrix}$

s $\begin{bmatrix} 25 & 14 \\ 14 & -9 \end{bmatrix}$ **t** $\begin{bmatrix} 2 & 20 \\ 30 & -34 \end{bmatrix}$

2 **a** $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & 7 \\ 9 & 2 & 3 \\ -4 & 2 & -4 \end{bmatrix}$ **b** $\begin{bmatrix} 5 & 8 & 9 \\ 0 & 5 & 10 \\ 5 & 6 & 1 \\ 7 & 5 & -2 \end{bmatrix}$

c $\begin{bmatrix} 0 & 1 & 5 \\ 2 & -1 & 5 \\ 4 & 8 & 6 \\ 3 & -3 & -2 \end{bmatrix}$ **d** $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & 7 \\ 9 & 2 & 3 \\ -4 & 2 & -4 \end{bmatrix}$

e $\begin{bmatrix} 1 & 4 & 3 \\ -2 & 3 & -1 \\ -4 & -2 & -4 \\ 4 & 3 & 2 \end{bmatrix}$ **f** $\begin{bmatrix} -3 & 2 & 7 \\ 0 & -1 & -2 \\ -5 & 6 & 3 \\ 7 & -5 & 2 \end{bmatrix}$

g $\begin{bmatrix} 3 & -2 & -7 \\ 0 & 1 & 2 \\ 5 & -6 & -3 \\ -7 & 5 & -2 \end{bmatrix}$ **h** $\begin{bmatrix} -2 & -9 & -11 \\ 2 & -5 & -3 \\ 4 & -4 & 2 \\ -11 & -3 & -2 \end{bmatrix}$

i $\begin{bmatrix} -5 & -7 & -4 \\ 2 & -6 & -5 \\ -1 & 2 & 5 \\ -4 & -8 & 0 \end{bmatrix}$ **j** $\begin{bmatrix} 5 & 7 & 4 \\ -2 & 6 & 5 \\ 1 & -2 & -5 \\ 4 & 8 & 0 \end{bmatrix}$

k $\begin{bmatrix} 16 & 12 & 4 \\ 0 & 12 & 24 \\ 20 & 0 & -4 \\ 0 & 20 & -8 \end{bmatrix}$ **l** $\begin{bmatrix} -3 & -12 & -9 \\ 6 & -9 & 3 \\ 12 & 6 & 12 \\ -12 & -9 & -6 \end{bmatrix}$

m $\begin{bmatrix} -2 & -10 & -16 \\ 0 & -4 & -8 \\ 0 & -12 & -4 \\ -14 & 0 & 0 \end{bmatrix}$ **n** $\begin{bmatrix} 2 & 1\frac{1}{2} & \frac{1}{2} \\ 0 & 1\frac{1}{2} & 3 \\ 2\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 2\frac{1}{2} & -1 \end{bmatrix}$

o $\begin{bmatrix} \frac{3}{4} & 3 & 2\frac{1}{4} \\ -\frac{3}{2} & 2\frac{1}{4} & -\frac{3}{4} \\ -3 & -1\frac{1}{2} & -3 \\ 3 & 2\frac{1}{4} & 1\frac{1}{2} \end{bmatrix}$ **p** $\begin{bmatrix} 0 & 17 & 31 \\ 0 & 5 & 10 \\ -5 & 24 & 9 \\ 28 & -5 & 2 \end{bmatrix}$

q $\begin{bmatrix} 4 & 19 & 27 \\ -2 & 9 & 11 \\ -4 & 16 & 2 \\ 25 & 3 & 2 \end{bmatrix}$ **r** $\begin{bmatrix} -12 & -7 & 7 \\ 4 & -11 & -8 \\ -7 & 16 & 15 \\ 6 & -21 & 2 \end{bmatrix}$

s $\begin{bmatrix} 24 & 48 & 48 \\ -8 & 32 & 36 \\ 4 & 16 & -12 \\ 44 & 32 & 0 \end{bmatrix}$ **t** $\begin{bmatrix} 6 & 12 & 12 \\ -2 & 8 & 9 \\ 1 & 4 & -3 \\ 11 & 8 & 0 \end{bmatrix}$

3 **a** $\begin{bmatrix} 4 & 4 & 2 & -3 \\ 2 & -2 & 2 & -5 \\ -1 & 7 & -2 & 7 \end{bmatrix}$

b $\begin{bmatrix} 7 & -1 & 7 & -8 \\ -4 & 10 & -10 & -4 \\ 4 & 12 & -4 & 8 \end{bmatrix}$

c $\begin{bmatrix} 2 & 8 & 6 & -10 \\ -6 & 4 & -2 & -12 \\ 0 & 10 & -4 & 6 \end{bmatrix}$

d $\begin{bmatrix} 2 & 8 & 6 & -10 \\ -6 & 4 & -2 & -12 \\ 0 & 10 & -4 & 6 \end{bmatrix}$

e $\begin{bmatrix} -6 & 5 & -4 & 3 \\ 1 & -8 & 9 & -2 \\ -4 & -7 & 2 & -5 \end{bmatrix}$

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f $\begin{bmatrix} 2 & -4 & -4 & 7 \\ 8 & -6 & 4 & 7 \\ -1 & -3 & 2 & 1 \end{bmatrix}$

g $\begin{bmatrix} 2 & -4 & -4 & 7 \\ 8 & -6 & 4 & 7 \\ -1 & -3 & 2 & 1 \end{bmatrix}$

h $\begin{bmatrix} 5 & -9 & 1 & 2 \\ 2 & 6 & -8 & 8 \\ 4 & 2 & 0 & 2 \end{bmatrix}$

i $\begin{bmatrix} 15 & 0 & -5 & 10 \\ 25 & -20 & 15 & 5 \\ -5 & 10 & 0 & 20 \end{bmatrix}$

j $\begin{bmatrix} -18 & 15 & -12 & 9 \\ 3 & -24 & 27 & -6 \\ -12 & -21 & 6 & -15 \end{bmatrix}$

k $\begin{bmatrix} 4 & 16 & 12 & -20 \\ -12 & 8 & -4 & -24 \\ 0 & 20 & -8 & 12 \end{bmatrix}$

l $\begin{bmatrix} -15 & 0 & 5 & -10 \\ -25 & 20 & -15 & -5 \\ 5 & -10 & 0 & -20 \end{bmatrix}$

m $\begin{bmatrix} 3 & -2\frac{1}{2} & 2 & -1\frac{1}{2} \\ -\frac{1}{2} & 4 & -4\frac{1}{2} & 1 \\ 2 & 3\frac{1}{2} & -1 & 2\frac{1}{2} \end{bmatrix}$

n $\begin{bmatrix} 0.1 & 0.4 & 0.3 & -0.5 \\ -0.3 & 0.2 & -0.1 & -0.6 \\ 0 & 0.5 & -0.2 & 0.3 \end{bmatrix}$

o $\begin{bmatrix} 2 & -1\frac{2}{3} & 1\frac{1}{3} & -1 \\ -\frac{1}{3} & 2\frac{2}{3} & -3 & \frac{2}{3} \\ 1\frac{1}{3} & 2\frac{1}{3} & -\frac{2}{3} & 1\frac{2}{3} \end{bmatrix}$

p $\begin{bmatrix} -6 & 15 & -16 & 17 \\ 23 & -40 & 39 & -2 \\ -16 & -13 & 6 & 1 \end{bmatrix}$

q $\begin{bmatrix} 9 & -20 & 21 & -22 \\ -29 & 52 & -51 & 3 \\ 21 & 18 & -8 & 0 \end{bmatrix}$

r $\begin{bmatrix} 35 & -29 & 15 & -6 \\ 8 & 30 & -38 & 18 \\ 18 & 34 & -8 & 30 \end{bmatrix}$

s $\begin{bmatrix} -43 & 38 & -15 & 2 \\ -15 & -32 & 43 & -27 \\ -21 & -38 & 8 & -36 \end{bmatrix}$

t $\begin{bmatrix} -3 & -39 & -1 & 11 \\ -10 & 32 & -36 & 37 \\ 17 & -19 & 6 & -23 \end{bmatrix}$

4 **a** $x = -5$ **b** $x = -1$ **c** $x = 2$

5 $\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 3 & 0 \end{bmatrix}$

6 $\mathbf{M} = \begin{bmatrix} -8 & 5 \\ -8 & -5 \end{bmatrix}$

7 **a** $\begin{bmatrix} 300 & 400 & 200 & 150 \\ 400 & 220 & 300 & 200 \\ 250 & 150 & 400 & 150 \end{bmatrix}$

b $\begin{bmatrix} 30 & 70 & 20 & 20 \\ 20 & 30 & 40 & 30 \\ 40 & 10 & 50 & 10 \end{bmatrix}$

c $\begin{bmatrix} 50 & 100 & 10 & 15 \\ 15 & 50 & 30 & 10 \\ 30 & 25 & 20 & 40 \end{bmatrix}$

d $\mathbf{A} - \mathbf{B} + \mathbf{C} = \begin{bmatrix} 320 & 430 & 190 & 145 \\ 395 & 240 & 290 & 180 \\ 240 & 165 & 370 & 180 \end{bmatrix}$

8 **a** $\mathbf{F} + \mathbf{M} + \mathbf{L}$

b $\begin{bmatrix} 10 & 1 & 4 \\ 6 & 0 & 9 \\ 4 & 3 & 8 \\ 8 & 2 & 5 \\ 4 & 2 & 9 \\ 9 & 0 & 6 \end{bmatrix}$ **c** 1st team

9 **a** $\begin{bmatrix} 4.08 & 8.33 & 14.28 \\ 7.14 & 13.43 & 20.06 \\ 6.46 & 14.28 & 18.70 \\ 10.03 & 20.06 & 24.48 \end{bmatrix}$

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b $\begin{bmatrix} 4.50 & 9.15 & 15.70 \\ 7.85 & 14.80 & 22.05 \\ 7.10 & 15.70 & 20.05 \\ 11.05 & 22.05 & 26.95 \end{bmatrix}$

Customer prices are rounded to the nearest 5 cents.

Exercise 10.04

1 **a** $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ **b** $\begin{bmatrix} -7 & 4 \end{bmatrix}$

c $\begin{bmatrix} 6 & -2 & 4 \\ -9 & 3 & -6 \\ 12 & -4 & 8 \end{bmatrix}$ **d** $\begin{bmatrix} -2 & 1 \\ 6 & 3 \end{bmatrix}$

e $\begin{bmatrix} -17 & 9 \\ 8 & -16 \end{bmatrix}$ **f** $\begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix}$

g $\begin{bmatrix} 6 & -15 \\ -2 & -14 \end{bmatrix}$ **h** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

i $\begin{bmatrix} -12 & 6 \\ -8 & 4 \end{bmatrix}$ **j** $\begin{bmatrix} -1 & 5 \\ -4 & 7\frac{1}{2} \end{bmatrix}$

k $\begin{bmatrix} 0 & 3 \\ -3\frac{2}{3} & -2 \end{bmatrix}$ **l** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2 **a** $\begin{bmatrix} -6 & 19 & -4 & 6 \\ -4 & -2 & 0 & 4 \\ 17 & -8 & 3 & -17 \\ 0 & 0 & 0 & 0 \\ -1 & 5 & -1 & 1 \end{bmatrix}$

b $\begin{bmatrix} -5 & 10 & -18 \\ -2 & -8 & -1 \\ 7 & -7 & 12 \\ 4 & 24 & 4 \\ -2 & 11 & -2 \end{bmatrix}$

3 $\begin{bmatrix} 33 & 21 \\ 35 & 40 \end{bmatrix}$ and $\begin{bmatrix} 33 & 21 \\ 35 & 40 \end{bmatrix}$, the same answer in both.

4 $\begin{bmatrix} -11 & 7 & -11 \\ 7 & 13 & 7 \\ -11 & 7 & -11 \end{bmatrix}$ and $\begin{bmatrix} -11 & 7 & -11 \\ 7 & 13 & 7 \\ -11 & 7 & -11 \end{bmatrix}$, the same answer both ways.

5 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. The product is a zero matrix.

This doesn't happen for non-zero numbers.

6 They are not conformable. The number of columns in the first is not the same as the number of rows in the second. The same is true in the reverse order.

7 The answer is the same as the other matrix. It is like multiplying a number by 1.

8 The answer is the same as the other matrix. It is like multiplying a number by 1.

9 The answer is the same as the other matrix. It is like multiplying a number by 1.

10 The answer is the same as the other matrix. It is like multiplying a number by 1.

11 Any 2×2 matrices of the form $\begin{bmatrix} a & b \\ ac & bc \end{bmatrix}$ and $\begin{bmatrix} b & -bc \\ -a & ac \end{bmatrix}$.

12 **a** $\begin{bmatrix} 12 & 10 & 8 \\ 15 & 8 & 4 \\ 10 & 10 & 10 \\ 12 & 12 & 4 \\ 16 & 6 & 12 \end{bmatrix}$ **b** $\begin{bmatrix} 117.80 \\ 155 \\ 190 \end{bmatrix}$

c $\begin{array}{c} \begin{bmatrix} 4483.6 \\ 3767 \\ 4628 \\ 4033.6 \\ 5094.8 \end{bmatrix} \end{array} \begin{array}{l} B \\ L \\ Ch \\ C \\ GC \end{array}$

13 **a** $\begin{bmatrix} 7160 \\ 9640 \\ 7140 \\ 7200 \\ 11\,580 \\ 9180 \end{bmatrix}$ **b** $\begin{bmatrix} 2506 \\ 3374 \\ 2499 \\ 2520 \\ 4053 \\ 3213 \end{bmatrix}$

c $\begin{bmatrix} 9666 \\ 13\,014 \\ 9639 \\ 9720 \\ 15\,633 \\ 12\,393 \end{bmatrix}$ **d** $\begin{bmatrix} 2 & 1 & 0 & 3 & 2 & 1 \\ 4 & 2 & 2 & 4 & 1 & 2 \\ 3 & 1 & 5 & 3 & 3 & 2 \end{bmatrix}$

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e $\begin{bmatrix} 77\,900 \\ 120\,940 \\ 141\,520 \end{bmatrix}$ **f** $\begin{bmatrix} 27\,265 \\ 42\,329 \\ 49\,532 \end{bmatrix}$

14 a $\mathbf{P} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

b $\mathbf{FP} = \begin{bmatrix} 12 \\ 9 \\ 4 \\ 7 \\ 3 \\ 9 \end{bmatrix}, \mathbf{MP} = \begin{bmatrix} 10 \\ 3 \\ 5 \\ 9 \\ 7 \\ 9 \end{bmatrix}, \mathbf{LP} = \begin{bmatrix} 9 \\ 6 \\ 6 \\ 10 \\ 4 \\ 9 \end{bmatrix}$

Exercise 10.05

1 a $\begin{bmatrix} 3 & 3 \\ 6 & 6 \end{bmatrix}$ **b** $\begin{bmatrix} 12 & 4 \\ 12 & 4 \end{bmatrix}$

c $\begin{bmatrix} 7 & 3 \\ 2 & 6 \end{bmatrix}$ **d** $\begin{bmatrix} 22 & -18 \\ -12 & 10 \end{bmatrix}$

e $\begin{bmatrix} 25 & 0 \\ 0 & 4 \end{bmatrix}$

2 a $\begin{bmatrix} 27 & 0 \\ 0 & 64 \end{bmatrix}$ **b** $\begin{bmatrix} -28 & -36 \\ 27 & -1 \end{bmatrix}$

c $\begin{bmatrix} -39 & 114 \\ 19 & 94 \end{bmatrix}$ **d** $\begin{bmatrix} -68 & 76 \\ 57 & -49 \end{bmatrix}$

e $\begin{bmatrix} 49 & 38 \\ -114 & -84 \end{bmatrix}$

3 a $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}$

c $\begin{bmatrix} 5000 & 5000 \\ 5000 & 5000 \end{bmatrix}$ **d** $\begin{bmatrix} -107 & 84 \\ -252 & 61 \end{bmatrix}$

e $\begin{bmatrix} 7281 & -6160 \\ -4400 & 3761 \end{bmatrix}$

4 a $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **b** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

c $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **d** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

e All powers of **A** are the 2×2 zero matrix. This is different to numbers.

5 a $\begin{bmatrix} 3 & 0 & -21 \\ 25 & 4 & -15 \\ 14 & 0 & 10 \end{bmatrix}$ **b** $\begin{bmatrix} 63 & -45 & 0 \\ 90 & 18 & 0 \\ -27 & -63 & 0 \end{bmatrix}$

c $\begin{bmatrix} 6 & -4 & 2 \\ 6 & -4 & 2 \\ 6 & -4 & 2 \end{bmatrix}$ **d** $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

6 a $\begin{bmatrix} 1 & 75 & -15 \\ 0 & 1 & 0 \\ 0 & -15 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix}$

c $\begin{bmatrix} 13 & 33 & 11 \\ 22 & 13 & 11 \\ 33 & 22 & 13 \end{bmatrix}$ **d** $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

7 a $\begin{bmatrix} -607 & 492 & -24 \\ -216 & 8 & -750 \\ -600 & -20 & -359 \end{bmatrix}$

b $\begin{bmatrix} 6912 & 6912 & 6912 \\ 6912 & 6912 & 6912 \\ 6912 & 6912 & 6912 \end{bmatrix}$

c $\begin{bmatrix} 196 & -60 & -369 \\ -246 & 73 & 306 \\ -204 & 144 & 277 \end{bmatrix}$

d $\begin{bmatrix} 2328 & -2132 & -742 \\ 0 & 81 & 0 \\ -1484 & 1296 & 473 \end{bmatrix}$

8 $\begin{bmatrix} a & \dfrac{-a^2}{b} \\ ab & -a \end{bmatrix}$ or similar.

9 a $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d Powers of identity matrices are themselves.

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10 **a** $\begin{bmatrix} 9 & -4 \\ 18 & -8 \end{bmatrix}$.

b They are all the same. $\mathbf{M} = \mathbf{M}^2 = \mathbf{M}^3 = \ldots$, which is not like numbers.

Exercise 10.06

1 **a** 3 **b** none **c** 1

2 72, 136, 10, 20, 122, 223, 28, 56, 20, 39, 38, 76, 74, 133, 40, 80, 52, 95, 38, 76, 20, 39, 40, 80, 86, 158

3 AND HE'S COMING OUT TODAY

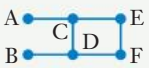
4 **a** $\begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.3 & 0.3 & 0.4 \\ 0.4 & 0.5 & 0.1 \end{bmatrix}$ **b** 0.44 **c** 0.274

5 **a** $\begin{bmatrix} 0.2 & 0.2 & 0.6 \\ 0.5 & 0.3 & 0.2 \\ 0.7 & 0.1 & 0.2 \end{bmatrix}$ **b** 0.48 **c** 0.388

6 **a** $\mathbf{C} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$

b 2

7 **a**



b 0

8 25, 53, 81, 5, 7, 9, 19, 58, 97, 9, 32, 55, 25, 53, 81, 15, 38, 61, 22, 52, 82

Chapter review

1 Only **b**.

2 **a** −2 **b** Does not exist

c 4 **d** 6

3 **a** The order 3 identity matrix $\mathbf{I}_3$.

b A 1×4 row matrix.

c A 2×1 column matrix.

4 $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5 **a** $\begin{bmatrix} 6 & 0 & 4 & -4 \\ 3 & 4 & -8 & 5 \end{bmatrix}$

b $\begin{bmatrix} 6 & -6 & 6 & -11 \\ -1 & 5 & 1 & 4 \end{bmatrix}$

c $\begin{bmatrix} -4 & 6 & -8 & 14 \\ -3 & 4 & 0 & -3 \end{bmatrix}$

d $\begin{bmatrix} 0 & -6 & 2 & -7 \\ -4 & 1 & 9 & -1 \end{bmatrix}$

e $\begin{bmatrix} -4 & 0 & -6 & 7 \\ -7 & 5 & 9 & -4 \end{bmatrix}$

f $\begin{bmatrix} 4 & 12 & -8 & 20 \\ 0 & 16 & -16 & 4 \end{bmatrix}$

g $\begin{bmatrix} -15 & 9 & -18 & 27 \\ -9 & 0 & 12 & -12 \end{bmatrix}$

h $\begin{bmatrix} -1 & 3 & 0 & 2 \\ 4 & -5 & -5 & 0 \end{bmatrix}$

i $\begin{bmatrix} -0.5 & 1.5 & 0 & 1 \\ 2 & -2.5 & -2.5 & 0 \end{bmatrix}$

j $\begin{bmatrix} 22 & -24 & 26 & -47 \\ 0 & 11 & 3 & 15 \end{bmatrix}$

6 D

7 **a** $\begin{bmatrix} 6 & -1 & -8 \\ -11 & 0 & 13 \end{bmatrix}$

b Not possible because the columns in the first do not match the rows of the second.

c $\begin{bmatrix} 31 \\ 9 \\ 3 \end{bmatrix}$

8 **a** Not possible, because it is not square.

b $\begin{bmatrix} 39 & 11 \\ 22 & 6 \end{bmatrix}$ **c** $\begin{bmatrix} 5 & 8 & 6 \\ 1 & 2 & 1 \\ 8 & 14 & 11 \end{bmatrix}$

9 1

10 27, 68, 48, 124, 38, 95, 37, 102, 5, 15, 21, 62, 50, 125

11 **a** 0.37 **b** 0.306

12 $x = -1, y = \frac{1}{2}$

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13 **a** $\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 5 & 0 & 0 & 0 & 0 \\ 1 & 3 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 1 & 0 & 0 \\ 3 & 0 & 5 & 1 & 3 & 0 \end{bmatrix}$

b $a_{34} = 0$

c Birds are not eaten by any of these.

d [25 000 70 000 4000 500 100 6]

e [75 000 210 000 12 000 1500 300 18]

f [4618 21500 730 106 18 0] and [13 854 64 500 2190 318 54 0]

14 **a** Because they do not truck parcels within a city.

b $\mathbf{H} = \begin{bmatrix} 0 & 4 & 4 & 4 & 4 \\ 4 & 0 & 4 & 4 & 4 \\ 4 & 4 & 0 & 4 & 4 \\ 4 & 4 & 4 & 0 & 4 \\ 4 & 4 & 4 & 4 & 0 \end{bmatrix}$

$\mathbf{T} = \begin{bmatrix} 0 & 14.80 & 13.80 & 10.40 & 9.00 \\ 14.80 & 0 & 11.20 & 20.40 & 19.40 \\ 13.80 & 11.20 & 0 & 20.20 & 18.40 \\ 10.40 & 20.40 & 20.20 & 0 & 15.00 \\ 9.00 & 19.40 & 18.40 & 15.00 & 0 \end{bmatrix}$

c $1.5\mathbf{C} + \mathbf{H}$

d $3\mathbf{C} + 2\mathbf{H}$

15 **a** [12.5 13.2 15 16.4 17.5 18]

b $\begin{bmatrix} 3 & 2 & 4 \\ 1 & 2 & 5 \\ 2 & 3 & 0 \\ 3 & 1 & 1 \\ 1 & 3 & 0 \\ 0 & 2 & 2 \end{bmatrix}$

c George \$147.40, Frank \$201.30, Maria \$168.40

16 $\mathbf{H} = \begin{bmatrix} 4 & 5 & 6 & 7 \\ 5 & 7 & 9 & 11 \\ 6 & 9 & 12 & 15 \end{bmatrix}$

17 **a** $\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$

b 3

Chapter 11

Exercise 11.01

1 B

2 **a** $\frac{g}{\sin(G)} = \frac{h}{\sin(H)} = \frac{i}{\sin(I)}$

b $\frac{n}{\sin(N)} = \frac{r}{\sin(R)} = \frac{q}{\sin(Q)}$

c $\frac{k}{\sin(K)} = \frac{l}{\sin(L)} = \frac{m}{\sin(M)}$

d $\frac{z}{\sin(Z)} = \frac{w}{\sin(W)} = \frac{y}{\sin(Y)}$

3 **a** $a = 13.7, b = 13.4$ **b** $p = 13.4, q = 10.9$

c $h = 17.2, k = 13.9$ **d** $c = 34.3, d = 21.1$

4 10.6 cm

5 3.5 cm

6 1.2 m

7 75.82 mm

8 **a** 27° **b** 19°

c 49° **d** 25°

9 28°

10 33°

11 115°

12 121°

Exercise 11.02

1 B

2 C

3 **a** $d^2 = m^2 + c^2 - 2mc\cos(D)$ or $m^2 = d^2 + c^2 - 2dc\cos(M)$ or $c^2 = m^2 + d^2 - 2md\cos(C)$

b $a^2 = r^2 + 144 - 24r\cos(A)$

c $y^2 = x^2 + 196 - 28x\cos(25°)$

d $q^2 = p^2 + r^2 - 2pr\cos(\theta)$

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4 **a** 5.4 cm **b** 7 m
c 88.2 cm **d** 55.1 m
5 $m = 26$
6 $g = 43.7$
7 $s = 157$
8 B
9 **a** 22.3° **b** 53.1°
c 109.5° **d** 134.4°
10 $\angle A = 21.4°$, $\angle G = 37.6°$, $d = 7$ m
11 $\angle U = 56.7°$, $\angle V = 68.3°$, $w = 44.1$ cm
12 $\angle X = 33.4°$, $\angle Y = 107.3°$, $\angle Z = 39.4°$
13 $\angle A = 89.6°$
14 $\angle X = 24.15°$

Exercise 11.03

1 Justin 81 m, Tara 40.3 m
2 About 13.2 cm
3 26.4° and 36.3°
4 About 66.4 m
5 About 190 m
6 About 30.7 km
7 31.4 cm
8 About 2.8 m
9 27 m
10 170°T or S10°E
11 6.69 km
12 About 35.6 m
13 **a** Her height is 2992 m above the hill, so her altitude is really about 3397 m.
b The altimeter is reading high.
14 About 25.1 km
15 About 16.5 km

Exercise 11.04

1 **a** 57.5 m **b** 10.8 m **c** 322 m **d** 9.2 cm
2 **a** $x \approx 9.9$ cm, $y \approx 18.8$ cm
b $x \approx 22.8$ cm, $y \approx 21.2$ cm
c $x \approx 6.5$ mm, $y \approx 7.5$ mm
d $x \approx 15.3$ m, $y \approx 30.6$ m
3 **a** 369.3 m **b** 143.6 m
4 The tower is approximately 50 m high.
5 4.31 km
6 68.6 m
7 54 m
8 166 km
9 **a** 361 km **b** 94.63°T
10 41.5 m
11 85 m

Exercise 11.05

1 **a** About 5.68 m **b** 55.23°
2 AYZ 51.34°, AZW 59.04°
3 **a** 12.04 m (south), 9 m (west)
b 44.9° (south), 53.13° (west)
c 15.03 m **d** 55.62°
4 **a** 21.21 cm **b** 30.97°
c 30.97° **d** 40.32°
5 **a** 401 m **b** 233.27°T or S53.27°W
6 76°T or N76°E at 12.78°
7 **a** 72.6° **b** 54.4°
8 81.48 m
9 155 m
10 77.1 m

Chapter review

1 B
2 C
3 A
4 D
5 E
6 C
7 D
8 **a** 27.1 mm **b** 30.8°
9 **a** $\angle B \approx 59.4°$, $\angle C \approx 78.6°$, $c \approx 41.0$ m
b $\angle G \approx 93.4°$, $e \approx 23.8$ cm $g \approx 49.9$ cm
c $\angle N \approx 22.1°$, $\angle K \approx 122.9°$, $k \approx 42.4$ m
10 **a** $a \approx 28.1$ m, $\theta \approx 95.1°$
b $b \approx 79.9$ m, $\phi \approx 41.2°$
c $\alpha \approx 56.6°$, $\beta \approx 75.1°$
11 **a** 45 mm **b** 32.2°
c 51.3° or 128.7° **d** 7.3 m
12 96.4°
13 **a** 116 cm **b** 20.0 m

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14 361.5 m

15 745 m

16 **a** 31.5 cm **b** 16.3 m
c 625 m **d** 18.3 mm

17 1396 m

18 **a** 37.3 m and 57.1 m **b** 43.3 m

19 $\angle ZXY \approx 18.9°$, $XY \approx 40.4$ m

20 48.4 m

21 1965 m

22 208.16 m

23 **a** 35.75° **b** 21.25° **c** 54.2°

Chapter 12

Exercise 12.01

1 **a** 19, 21, 23.5, 26, 27 **b** 2, 3.5, 6, 7, 9
c 31, 36.5, 38, 43, 49 **d** 8, 12, 15, 17, 19

2 **a** 3, 5.5, 7, 9, 12 **b** 22, 25, 26, 28, 32
c 104, 107, 108, 109.5, 112

3 **a** 4.5, 13.9, 18.1, 22.5, 29.5
b 0, 17.2, 24.7, 31.2, 49.5
c 10.5, 21.0, 25.1, 30.1, 40.5
d 0, 195.9, 272.6, 362, 599.5

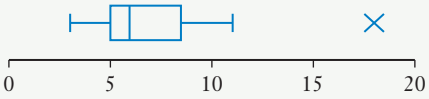
4 **a** 12, 14, 15, 23, 40, or with possible outlier 40 excluded 12, 14, 15, 21, 27.
b 3, 10, 12, 13, 16, or with possible outlier 3 excluded 7, 10, 12, 13, 16.
c 8, 24, 27.5, 31.5, 52, or with possible outliers 8, 44, 52 excluded 18, 24, 25, 29, 33.
d 11, 18, 23, 30, 39. No possible outliers

5 1.5, 11.1, 14.0, 17.7, 25.5 hours

6 Typical number is 2. Five-number summary with outliers 0 and 200 excluded is 1, 2, 2, 3, 4.

Exercise 12.02

1



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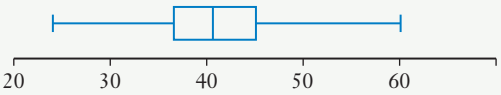

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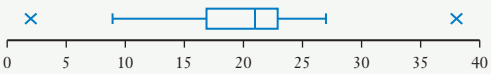

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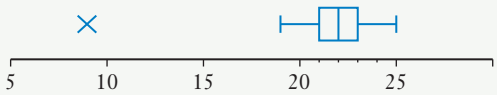

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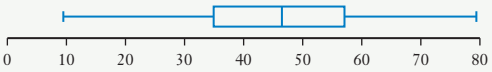

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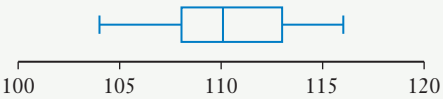

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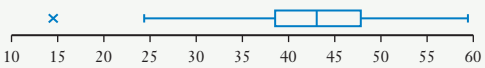

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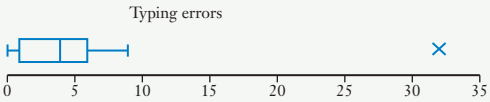
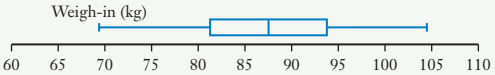


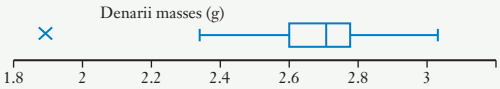
Students typically made 4 errors.

9



The typical weigh-in mass was 87.5 kg.

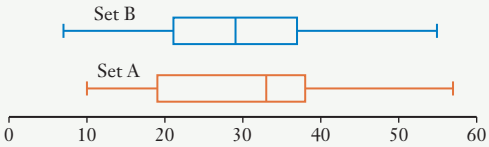
10



One coin was badly damaged. The typical mass was about 2.7 g.

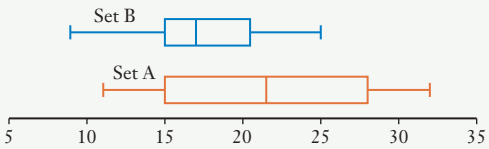
Exercise 12.03

1



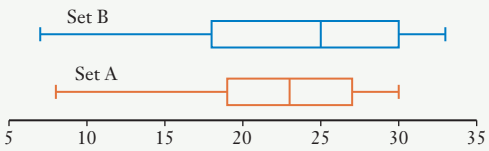
Typical data in set A is higher than B. The middle 50% of B is less spread out than in set A.

2



Most data in set B is much lower than in set A and set B is much less spread out.

3



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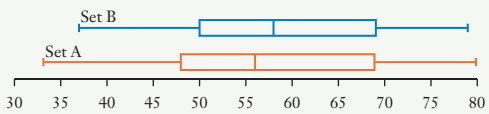
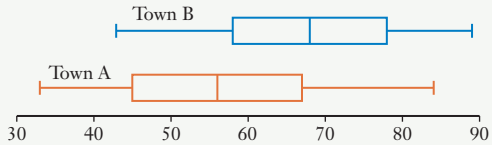
Data in set B is a little higher and more spread out than in set A.

4



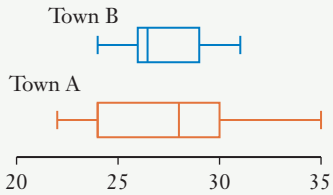
Data in set B is a little higher and little less spread out than in set A.

5



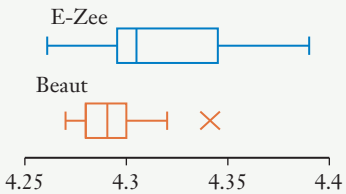
Town B is likely to have had more storm activity than town A because the humidity has generally been much higher.

6



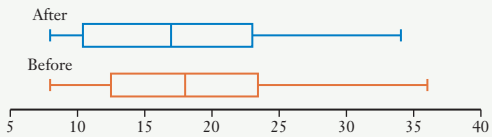
Town B is more likely to be closer to the sea than town A because the temperature is much less spread out.

7



The Beaut Balls are much more consistent in size.

8



The typical smoker has only reduced smoking by 1 cigarette per day, and no one has actually given up, so the program is a failure. It shows just how difficult it is to give up smoking.

Exercise 12.04

1 Set A: median = 20, IQR = 3; Set B: median = 22, IQR = 3.5

Set B is higher and spread out more than set A.

2 Set A: 37 excluded, median = 25, IQR = 5.5;

Set B: 19, 34 excluded, median = 28, IQR = 3.

Set B is higher and spread out less than set A.

3 Set A: 1, 8 excluded, median = 28, IQR = 8;

Set B: median = 23.5, IQR = 6.

Set B is lower than set A. Set A has the larger interquartile range, so set B is a little less spread out than set A.

4 Set A: mean ≈ 14.3, SD ≈ 3.76;

Set B: mean ≈ 17.5, SD ≈ 2.52.

Set B is higher and less spread out than set A.

5 Set A: 15, 72 excluded, mean ≈ 43.1, SD ≈ 7.11;

Set B: 64 excluded, mean ≈ 39.6, SD ≈ 7.70.

Set B is lower and a little less spread out than set A.

6 Set A: 39, 40 excluded, mean ≈ 24.6, SD ≈ 3.41;

Set B: 9 excluded, mean ≈ 32.8, SD ≈ 6.80.

Set B is higher and more spread out than set A.

7 A 2.5 SD, B 1.4 SD, A is more unusual.

8 A 1.33 SD, B 2 SD, B is higher.

9 A –0.8 SD, B –1.33 SD, B is lower.

10 English: mean ≈ 11.04, SD ≈ 2.95;

Maths: mean ≈ 10.3, SD ≈ 4.58.

The English results appear higher and less spread out than those of Maths.

11 Art: mean ≈ 43.3, SD ≈ 14.99;

Music: mean ≈ 52.7, SD ≈ 7.95.

Music is much higher and much less spread than Art.

12 Her height is 1 SD over average and her weight is 0.5 SD below average, so she is actually quite thin.

13 Text 1: The median length in text 2, 9, is higher than in text 1, where it is 8. However, since the IQR is much larger (7.5 compared to 3) there must be some very long words, making the text much harder.

14 Both systems have a very low outlier on the third last day, so there may have been a storm on that day. With that day excluded, the ABC has a mean of about 7.4 and SD of 0.15. The Setnforget has a mean of about 7.5 and a SD of 0.14. The Setnforget is better than the ABC because it is more consistent. They both need to have their level adjusted slightly.

15 Basher and Slogger have very similar IQRs, so they are about as consistent as each other, although Slogger has made a higher score. However, Slogger has a higher median, so he is the better batsman.

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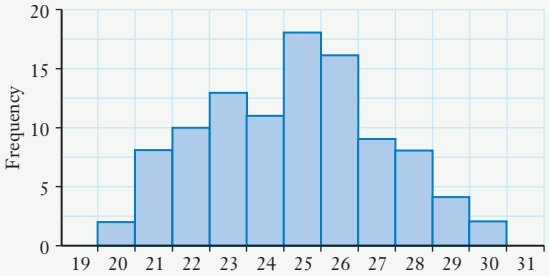
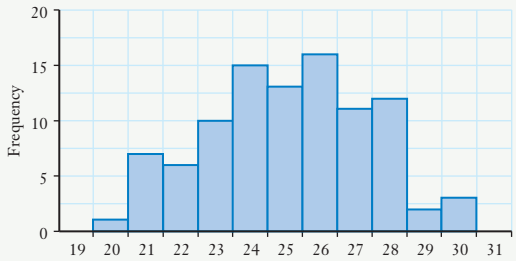
16 Hoe's onions are larger (215 g mean vs 185 g) and more varied in size (80 g vs 47 g), so they are more suitable for a variety of uses than Joe's. Excluding the outlier in Joe's sample makes the situation worse. Hoe's should be chosen unless the caterer wants consistently small onions.

Exercise 12.05

1 Set A: median 25, IQR 3

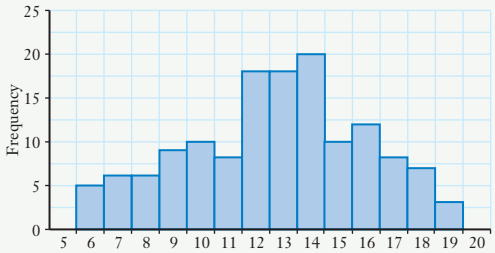


Set B: median 25, IQR 3.5

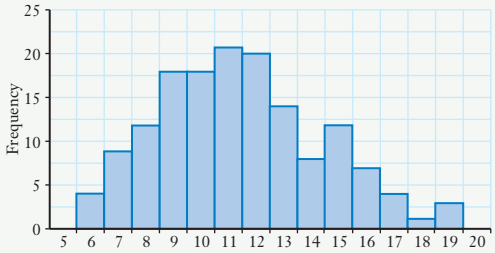


Sets A and B are about equal except that B is spread out a little more. Although they are both symmetrical, B tends more towards a plateau-like distribution.

2 Set A: median 13, IQR 5

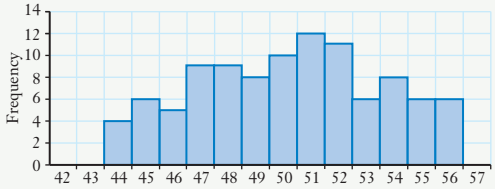


Set B: median 11, IQR 4

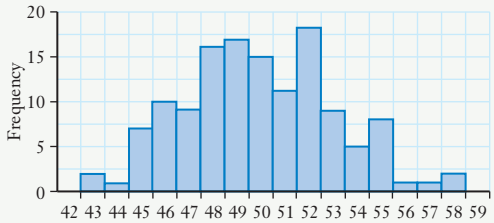


Set A is slightly negatively skewed and set B is positively skewed. Set A is higher than set B and a little more spread out.

3 Set A: median 50, IQR 5

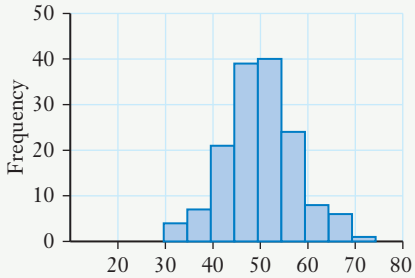


Set B: median 50, IQR 4



Set B is more symmetrical than set A. They are about the same, but set B has a greater range and a lower spread.

4 Set A: mean ≈ 50.1, SD ≈ 7.69



Set B: mean ≈ 51.4, SD ≈ 13.36

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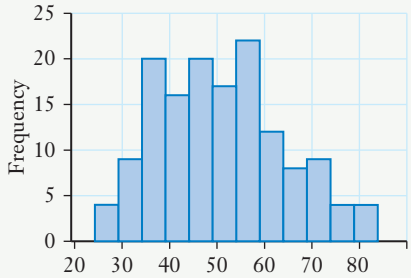
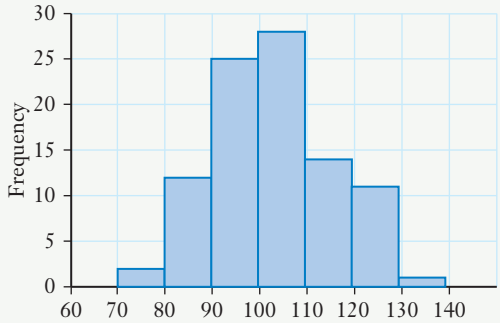
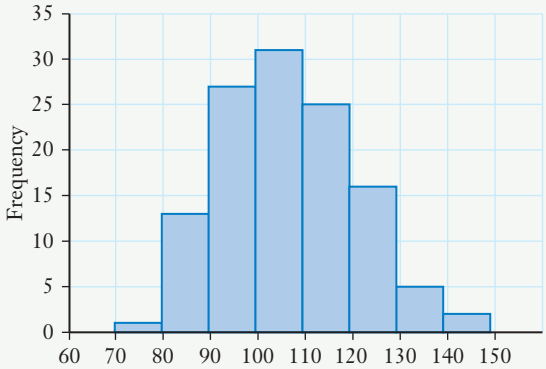


Set B is a little higher and much more spread out than set A.

5 Set A: mean ≈ 102.8, SD ≈ 12.92

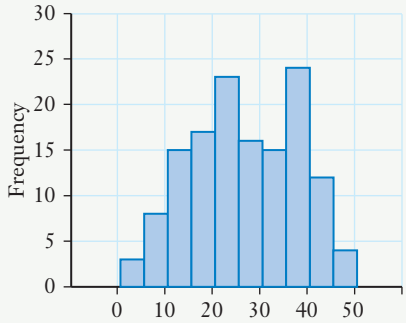


Set B: mean ≈ 106.5, SD ≈ 14.41

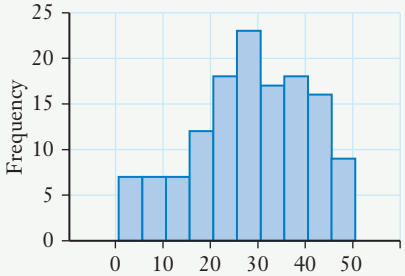


Set B is higher and a little more spread out than set A.

6 Set A: mean ≈ 26.8, SD ≈ 11.33



Set B: mean ≈ 28.4, SD ≈ 12.16



Set B is higher and slightly more spread out than set A, which is bimodal. Set B is slightly negatively skewed.

7

Teachers	Stem	Plumbers
	4	4 8
8 6 2	5	3 4 6 7 7 8 9
9 8 6 6 1	6	1 3 4 5
4 4 4 4	7	3 3 3 6 6 8
7 2	8	0 1 9
4 2	9	7
9 9 3 3	10	6
8 8 7	11	
7	12	

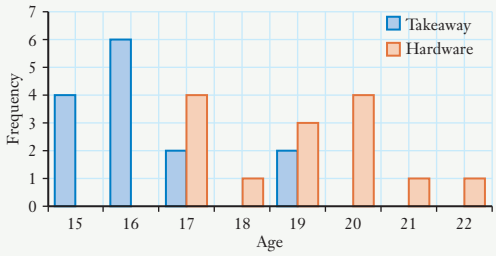
Key: 3 | 8 | = $38 000

Both have bimodal distributions, but the teachers generally earn more than the plumbers.

Teachers' mean ≈ $85 000 and SD ≈ $22 000;

Plumbers' mean ≈ $68 000and SD ≈ $15 000.

8



The hardware students are generally older. Median of takeaway workers is 16 and the median of the hardware workers is 19.

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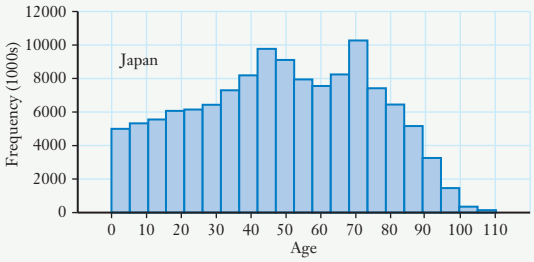
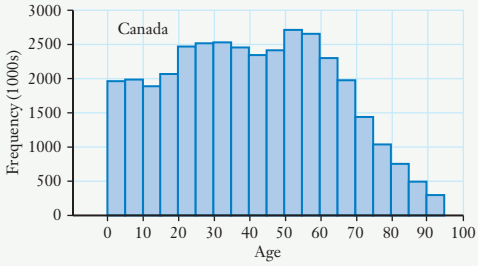
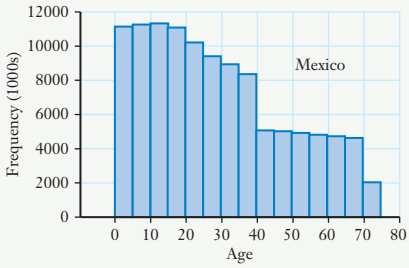
9 Japan: mean 46.6, SD 23.95, Canada: mean 40.8, SD 22.83, Mexico mean 29.9, SD 20.25.







Japan's people are much older than Canada's, who are older than Mexico's. Life expectancy in Japan is higher than in Canada, which is much higher than in Mexico. Japan's population is old, Canada's is aging and Mexico's is very young.

Exercise 12.06

1 **a** How often do you watch TV? How much time did you spend watching TV last week? Did you choose the programs? What channels did you watch? Did you do anything else at the same time?

b Whose computer do you use? What do you use the computer for? Where is the computer kept? How long did you spend using the computer?

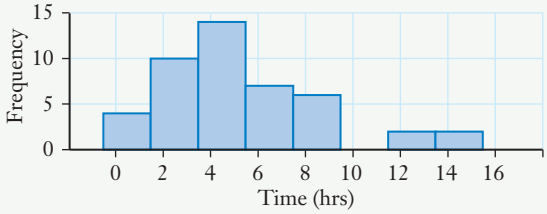
c Can you play an instrument? What instruments do you play? Do you play with other people? Where do you play? How much practice do you do? What music do you play? Do you have any qualifications?

2 **a** Number of sports played, time spent playing, time spent practicing, level played.

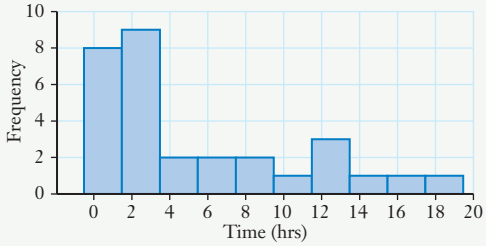
b Amount of work each week, length of shift, days of week, time of shift, amount paid each week.

c Whether a car is owned, distance driven each week, cost of running, standing costs, time owned, money owed, principle use.

3 5C+ Group: mean ≈ 5.3, SD ≈ 3.4

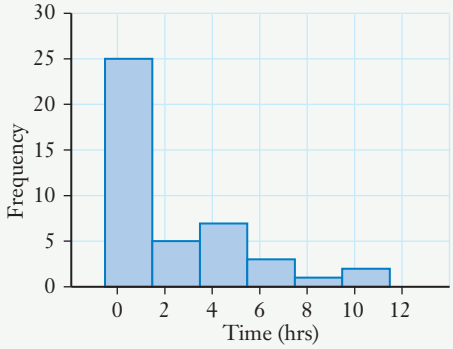


Under 5C Group: mean ≈ 5.4, SD ≈ 5.3



The amount of exercise that the students who got 5 or more Cs reported was symmetrically distributed. The majority of those who got less than 5 Cs did little exercise. There were some in this group who reported doing a great deal of exercise, so it was positively skewed. The average amount was the same for both groups, but those who got less than 5 Cs had a much wider spread.

4 Sports players: mean ≈ 2.5, SD ≈ 2.8



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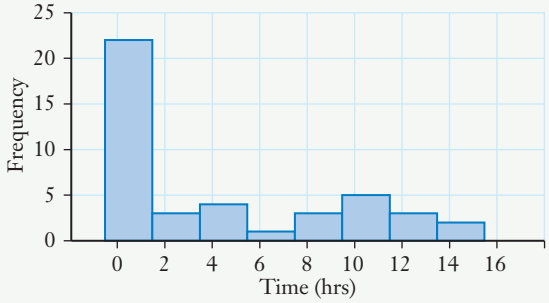
Non-sports players: mean ≈ 4.4, SD ≈ 4.8



The non-sports players generally report doing more part-time work than sports players and have a greater spread of hours worked, both in range and standard deviation. The majority of both do little part-time work.

Chapter review

1 31, 33, 38, 43, 52

2 **a** 5, 9, 12, 13, 15

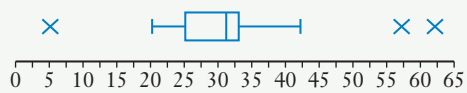
b 0, 22.2, 30.9, 51.2, 89.5

3 **a** 3, 5, 7, 11.5, 20 with possible outlier 26

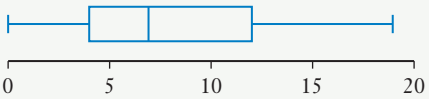
b 8, 10, 12, 16, 20, no possible outliers

c 28, 34, 39.5, 43, 49 with possible outlier 12

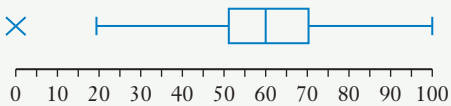
4



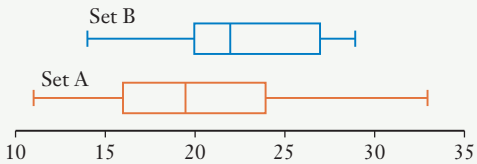
5 **a**



b



6



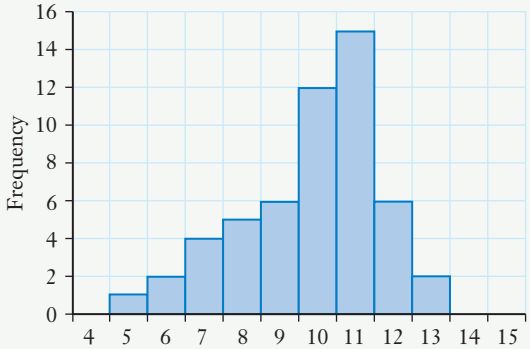
7 Set A: mean 36.1, SD 9.42, possible outlier 71.2

Set B: mean 31.0, SD 9.21, no possible outliers

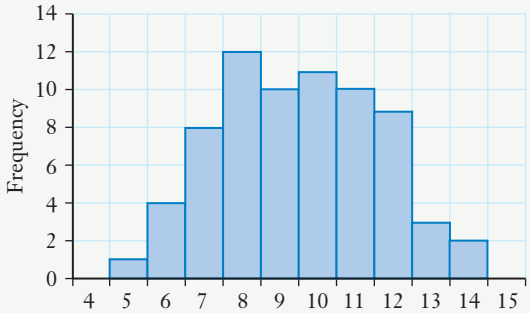
Set A is higher than set B, although they have similar spreads.

8 Campbell got about 0.40 SDs above the mean in Geography and about 0.28 SDs above the mean in History, so he did a little better in Geography.

9 Set A: median 10 and IQR 2

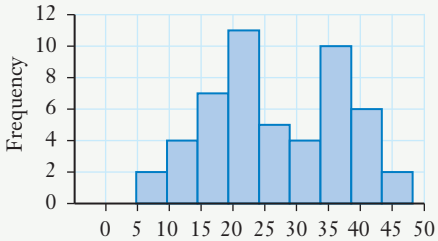


Set B: median 9.5 and IQR 3

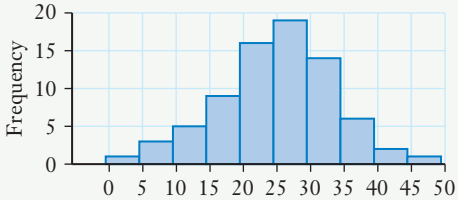


Set A is skewed negatively and set B has a plateau distribution. Set B is a little lower and is more spread out than set A

10 Set A: mean ≈ 27.5, SD ≈ 10.8



Set B: mean ≈ 25.0, SD ≈ 8.7



Set B is lower and less spread out than set A. Set A is bimodal and set B is symmetrical.

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11

Firefighters		Teachers
9 8 7 5 1	2	2 3 3 3 8
8 8 8 8 7 7 6 6 5 4 4 4 4 3 0	3	6 7 8
9 7 7 7 4 4 3 3 3 2 2 1 0	4	0 1 2 3 3 3 4 4 5 5 8 9
5 3 3 2 0 0	5	0 0 2 2 4 4 5 5 6 7 7 8 9 9
3	6	1 1 2 4 4 5

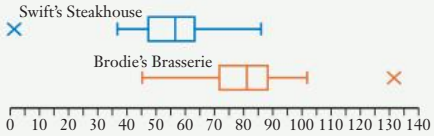
12 **a** What games do you play? Do you play at a commercial site or at home? How long did you spend playing last week? Do you play with friends or on the internet? How much did you spend on games last month?

b Length of time playing each week, weekly expenditure on playing, number of games owned.

13 The \$600 is an obvious outlier and should be discarded as untrue. The five-number summary is 0, 2.5, 8, 13.5, 30 and the typical amount is \$8 a week.

14 The typical amount lost was about 15.8 kg with a standard deviation of 18.8, even with the outliers of 63.2, 65.9, 67.2 kg excluded. The amounts lost were very widely spread.

15

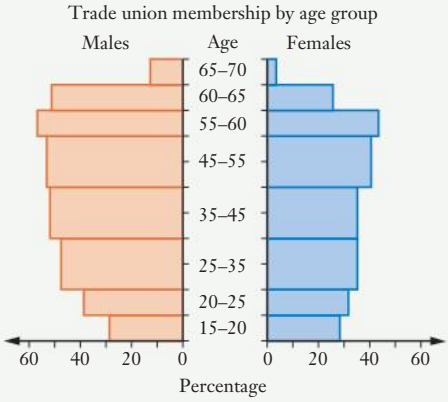


Brodie's Brasserie may seat more people than Swift's Steakhouse. Neither restaurant is doing particularly well.

16 A: mean 3.15, SD 0.52, B: mean 2.86, SD 0.73

System A is better than system B. They are both out by about the same amount, but system A is more consistent. System A is also high rather than low, and it is better for the chlorine level to be high rather than low.

17



Except for the 15–19 age group, a greater proportion of males than females were members of trade unions. Also, membership was greater among older age groups than younger.

Practice examination 4

1 **Z** only (1 mark)

2 **a** **A** is a rectangular matrix, **B** is a row matrix and **C** is a square matrix. (1 mark)

b a_{23} does not exist because there is no third column. (1 mark)

$b_{13} = 1, c_{23} = 4$ (1 mark)

3 **a** $\mathbf{A} + \mathbf{B} = \begin{bmatrix} 0 & -2 \\ -3 & 6 \\ 4 & -4 \end{bmatrix}$ **b** $3\,\mathbf{B} = \begin{bmatrix} -9 & 6 \\ -15 & 18 \\ 0 & 3 \end{bmatrix}$

c $\mathbf{B} - \mathbf{A} = \begin{bmatrix} -6 & 6 \\ -7 & 6 \\ -4 & 6 \end{bmatrix}$ (1 mark each)

4 **a** $\begin{bmatrix} 2 & 10 & -2 \\ 11 & -5 & 5 \end{bmatrix}$ (2 marks)

b Not possible because they are not conformable or the number of columns in the first does not match the number of rows in the second. (1 mark)

5 B, D and E (1 mark)

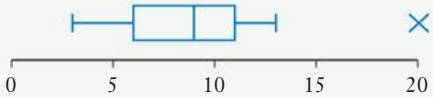
6 **a** 2.7 cm **b** 19.2°

7 $a = 8.5$ cm

8 $\angle X = 75.5°$

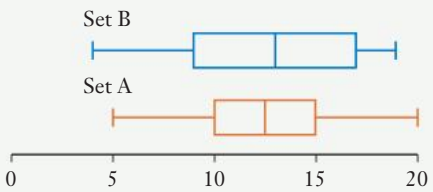
9 15, 26, 31, 39, 44 (2 marks)

10



(2 marks)

11



Set B is higher and more spread than set A.

(2 marks for box plot, 1 mark for comment)

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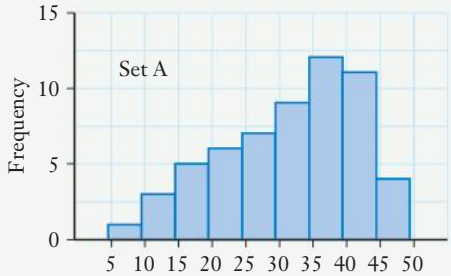
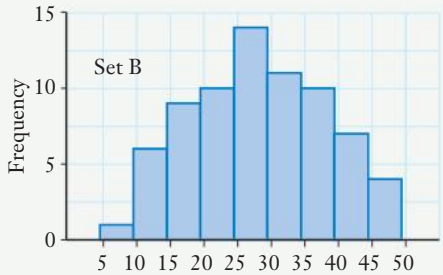
12





Set A: mean ≈ 31.6 and SD ≈ 10.2;
Set B: mean ≈ 28.3 and SD ≈ 10.0

Set A is skewed negatively and has a higher mean than set B, which is symmetrical. They are almost equally spread. (1 mark for graphs, 1 mark for mean, SD, 1 mark for skew/symmetrical, 1 mark for mean/spread)

13

House sale

Mt Isa		Toowoomba
9 8 8 7 6 5 5	1	6
9 9 9 9 3.5 1 1 1 0	2	3 3 3 6 6 6 7 9
3 2 2 1 1 9	3	2 3 3 3 4 4 5 5 6 8 9
3 2	4	2 6 7 9
	5	0 8
0	6	
0	7	
0	8	

Key: 3 | 7 | = $370 000

Mt Isa was mostly cheaper, but had a few quite expensive houses compared to Toowoomba.

(2 marks for plot, 1 mark for comment)

14 **a** $\mathbf{P} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 3.5 \\ 4.5 \\ 6.5 \end{bmatrix}$ (1 mark)

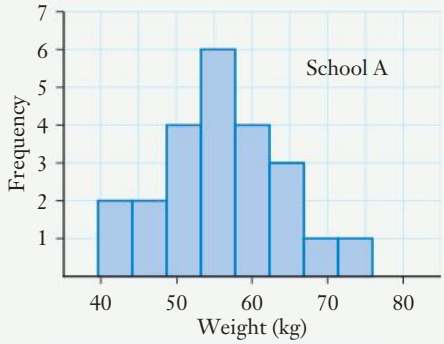
b $\mathbf{D} = \begin{bmatrix} 2 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 0 & 0 & 1 \end{bmatrix}$ (1 mark)

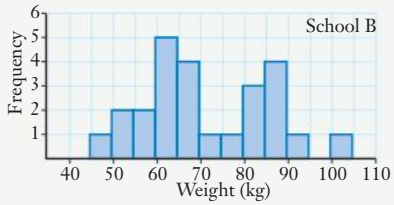
c $\mathbf{D} \times \mathbf{P} = \begin{bmatrix} 19.5 \\ 22 \\ 14.5 \end{bmatrix}$ (1 mark),

Cost of orders were A: $19.50, B: $22 and C: $14.50 (1 mark)

15 149.6 m

16





School A is symmetrical with a peak at about 60 kg. School B is bimodal with peaks at about 60 and 85 kg. It is likely that School A is girls only and school B is coeducational. Boys of this age are generally heavier than girls.

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17 **a** $\mathbf{M} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$ (2 marks)

b $\mathbf{D} = \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (1 mark),

$m_{18} = d_{18} = 0$, so W1 cannot communicate with the colour printer either directly or via 1 other entity in the network. (1 mark)

18 86 km

19 Set A: mean ≈ 18.2 cm, SD ≈ 5.5 cm

Height (cm)	5–9	10–14	15–19	20–24	25–29	30–34	35–39
Number	2	12	15	15	4	1	1

Set B: mean ≈ 22.3 cm, SD ≈ 7.3 cm, possible outliers 2, 3 cm

Height (cm)	0–4	5–9	10–14	15–19	20–24	25–29	30–34	35–39
Number	2	2	3	7	10	21	4	1

From the tables of values, both groups are fairly symmetrical, but even with the low outliers included, group B is higher than group A, so it appears that the hormone promotes better growth. The outliers suggest that there may be a small proportion of seeds that are badly affected by the hormone.

(2 marks for mean and SDs, 1 for outliers, 1 for comments)

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GLOSSARY AND INDEX

algebraic expression: A mathematical phrase that can contain constants, variables and operators (such as +, –, × and ÷). (p. 288, 312)

ABSTUDY: Government allowance paid to eligible Aboriginal apprentices and students. (p. 38)

adjacency matrix: A matrix that shows direct connections between points in a network. (p. 446)

adjacent side: The side that is next to a given angle in a right-angled triangle, which, with the hypotenuse, forms the arms of the angle. (p. 318)

age pension: A pension paid to eligible Australians who are aged over 66 and too old to work. (p. 38)

angle of depression: The angle formed by looking down from a horizontal line. (p. 331, 346, 480)

angle of elevation: The angle formed by looking up from a horizontal line. (p. 331, 346, 480)

annual: Each year, in a year. (p. 12)

apex: The vertex opposite the base of a three-dimensional shape, such as a cone or a pyramid. (p. 70, 187, 224)

apprentice: A person who is learning a recognised trade. (p. 18, 38). *See also* **trainee.**

arc length: The length of a part of a curve, measured along the curve. (p. 52, 80)

area: The size of a plane or curved surface, measured in square units. (p. 56, 80)

assets test: An assessment of the value of a person's possessions (for example, house, car), to work out the amount of an allowance or pension the person will be paid. (p. 27). *See also* **income test**.

Austudy: A government allowance paid to eligible apprentices and students. (p. 38)

axis: The perpendicular line from the base of a cone or pyramid to its apex. (p. 71)

back-to-back stem-and-leaf plot: Stem-and-leaf plots to the left and right of the same stem for two related sets of data, such as the ages of mothers and fathers of a group of students. (p. 509)

balanced equation: The way that the values of both sides of an equation are equal. An equation remains balanced if the same thing is done to both sides. (p. 89)

bar graph: A rectangular graph divided into vertical or horizontal bars with lengths or heights proportional to values. (p. 358). *See also* **pie chart**.

bearing: A compass direction stated as either a 3-digit angle clockwise from north, or as an angle east or west of north or south. (p. 336, 346). *See also* **compass bearing**, **true bearing**.

bimodal distribution: A frequency distribution with two distinct peaks, two values of high frequency. (p. 391, 402)

box-and-whisker plot, box plot: A display of the five-number summary of a set of data; the quartiles are shown by the left and right ends of a rectangle ('box'), divided by the position of the median; lines ('whiskers') extend from the ends of the rectangle to show the positions of the minimum and maximum values. (p. 493)

buying price: *See* **cost price**.

buying rate: The amount in AUD (Australian dollars) that a currency exchange pays for a unit of foreign currency. (p. 167). *See also* **selling rate**.

capacity: Another name for volume, used for the volumes of containers for liquids. (p. 71)

categorical data: A statistical variable with data that are categories (names), not numbers. (p. 352). *See also* **numerical variable**, **nominal variable**, **ordinal variable**.

circumference: The perimeter or boundary of a circle. (p. 52, 80)

class centre, class midpoint: The middle of a class. (p. 368, 400)

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class interval, class width: The difference between the lowest and highest values in a class. (p. 364, 368, 400)
class: A range of scores in a frequency table, like 5–9 or 9.5–19.5. (p. 364)
coefficient of variation, relative standard deviation (RSD): The standard deviation divided by the mean of a distribution, normally multiplied by 100 to express as a percentage. (p. 387)
coefficient: The number that precedes a variable in an algebraic term. The coefficient of $5x$ is 5. (p. 288, 312)
column graph: A graph with vertical bars with heights proportional to values. (p. 358)
column matrix: A matrix that has only one column. (p. 418, 445)
combined shape: A two- or three-dimensional shape that is formed by combining other shapes. (p. 61)
commission: Payment to workers such as salespeople as a proportion of the value of sales or services. (p. 22, 38)
communications network: A network that shows direct communication links. (p. 438, 446)
compass bearing: A compass direction such as N 18° E measured as an angle relative to north or south, in an easterly or westerly direction. (p. 346)
compound interest: Interest calculated where the money is regularly reinvested or reloaned, calculated on the changing current principal rather than the original principal only. (p. 178). *See also* **simple interest**.
conformable matrices: Matrices that may be multiplied; matrices **A** and **B** are conformable and may be multiplied to give **AB** if **A** is $m \times n$ and **B** is $n \times p$. (p. 428)
congruent figures: Figures with the same size and shape. (p. 197)
constant term: A term in an algebraic expression or equation that is a number. It has a fixed value. (p. 288, 312)
consumer price index (CPI): The price of a 'basket' of goods and services that average Australians buy, an indicator of the cost of living calculated quarterly by the Australian Bureau of Statistics. (p. 164, 178)
continuous data: A numerical variable such as height that can take on any possible value on a scale. (p. 352). *See also* **discrete variable**.
continuous function: A function whose graph has no breaks or holes. (p. 255, 276)
coordinates: An ordered pair of numbers used to identify a position. Cartesian coordinates have a horizontal x-coordinate and a vertical y-coordinate obtained from the x- and y-axes respectively. (p. 107, 132)
cosine rule: A rule relating the sides and angles of a triangle. For $\triangle ABC$, $a^2 = b^2 + c^2 - 2bc \cos(A)$ or $\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$. (p. 458, 480)
cosine: The trigonometric ratio given by $\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ in a right-angled triangle. (p. 346)
cost price, buying price: The amount of money paid for something by a shop. (p. 147, 177)
cryptography: The use of codes for secure transmission of messages. (p. 349)
currency: The money of a country, particularly one unit, for example, dollar, euro, yen. (p. 167, 178)
curved surface area (CSA): The area of the curved part of the surface of a cylinder, cone or sphere. (p. 191, 224)
cylinder: A can-shape with a uniform circular cross-section. (p. 71, 191)

deflation: A decrease of the consumer price index (CPI) and increase in the value of money over time. (p. 164, 178)
dependent variable: The variable that is calculated from the value of other variables in a function. (p. 289, 312). *See also* **independent variable**.
dilation: A transformation where an object is enlarged or reduced in size while maintaining its shape. (p. 224)
dimension, order (of a matrix): The size of a matrix; an $n \times m$ matrix has n rows and m columns. (p. 414, 445)
disability pension: A pension paid to eligible Australians who cannot work because of a medical condition. (p. 38)
discontinuous function: A function whose graph has a break or a hole. (p. 255, 276)
discount: The amount taken off the marked price of an item. (p. 147, 177)
discrete data: A numerical variable such as number of children that can only take on specific, distinct values, in this case, whole numbers. (p. 352). *See also* **continuous variable**.
dispersion: The spread of values in a statistical distribution. (p. 401)
dividend: The amount paid to shareholders for each share from company earnings. (p. 172, 173, 179)
dividend percentage: The dividend as a percentage of the face value of a share. (p. 173, 179)
dividend yield: The dividend as a percentage of the market value of a share. (p. 173, 179)
domain: The set of (possible) values of the independent variable for a function. (p. 107, 132)

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dot plot: A graph with each discrete value shown by a dot above a horizontal scale. (p. 363, 400)
double-time: Double the normal rate of pay for overtime or as a penalty rate. (p. 14, 37)

earnings per share (EPS): The net income of a company (per year) divided by the number of shares. (p. 173, 179)
edge: A line segment joining two vertices of a shape or network. The boundary where two faces of a 3D shape meet. (p. 70)
element of a matrix: One of the numbers in a matrix. (p. 414, 445)
equation: A mathematical sentence stating that two algebraic expressions are equal. (p. 88, 301, 312)
exchange rate: The rate at which another currency is exchanged for Australian dollars. (p. 167, 178). *See also* **buying rate**, **selling rate**, **mid-rate**, **official rate**.
expression: *see* **algebraic expression**.

face value, par value: The original price at which a share in a company was sold. (p. 172, 173)
face: Any flat surface of a 3D (solid) object. (p. 70)
falling (graph): Where the value of the function and graph decreases from left to right. (p. 114, 133)
first quartile, lower quartile (Q_1): The value below which a quarter of the values of a statistical distribution lie. (p. 381, 401, 488, 518)
fixed-term contract: Employment for a specified period or completion of a task. (p. 38)
flat-rate interest: *See* **simple interest**. (p. 154, 178)
formula: A mathematical equation that shows the relationship between different variables. (p. 296, 312)
frequency polygon: A graph with line segments between the values of a continuous frequency distribution, starting and ending on the horizontal axis. (p. 369, 400)
full-time worker: A person who is employed for at least 38 hours a week, employed on at least a weekly basis. (p. 16, 37). *See also* **part-time worker**.
function: A relation for which every element of the domain has at most one element in the range. (p. 107, 132)

goods and services tax (GST): A tax paid to the Australian government when goods or services are bought; currently 10% of the sale price. (p. 147, 177)
gradient: The mathematical measure of the steepness of a line, calculated using the formula $m = \frac{\text{rise}}{\text{run}}$. (p. 114, 133)
gradient–intercept form: The way of writing the equation of a line so that the gradient and y-intercept are immediately apparent, that is, $y = mx + c$, where m = gradient and $c = y$-intercept. (p. 133)
gross profit margin: *See* **operating margin**. (p. 150, 177)

health care card: A card that gives a discount on medical drugs. (p. 27)
hemisphere: Half a sphere. (p. 71)
Heron's formula: A formula used to calculate the area of a triangle using the lengths of the three sides. Area $= \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$. (p. 346)
histogram: A column graph of the frequency distribution of a continuous variable. There are no gaps between the columns. (p. 365, 400)
hypotenuse: The longest side in a right-angled triangle, opposite the right angle. (p. 44, 80, 318)

identity matrix: A square matrix **I** with every element 0 except for 1s in the diagonal $i_{11}, i_{22}, i_{33}, \ldots$ like $\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. (p. 418, 445)
image: An object that has undergone a transformation. (p. 197). *See also* **object**.
included angle: The angle between two particular sides of a triangle. (p. 346)
income test: An assessment of a person's regular income, to work out the amount of an allowance or pension a person will be paid. (p. 27, 28, 38). *See also* **assets test**.
independent variable: The variable that does not depend on the value of other variables in a function. (p. 289, 312). *See also* **dependent variable**.
inflation: An increase of the consumer price index (CPI) and decrease in the value of money over time. (p. 164, 178)
input (value): A value of the independent variable for a function. (p. 107, 132)
intercept: The value of a function where its graph cuts an axis (x-axis or y-axis). (p. 120) *See also* **x-intercept**, **y-intercept**.
interest rate, nominal interest rate: The cost of a loan or investment as a percentage of the loan/investment, usually stated for one year. (p. 154, 178)
interest: The cost of a loan or investment. (p. 154, 178)

interquartile range (IQR): A measure of spread that is the difference between the third and first quartiles. IQR = $Q_3 - Q_1$. (p. 381, 401, 402)
inverse operation: The operation that reverses the effect of another operation, for example, + and – are inverse operations of each other. (p. 296, 312)
isolate a variable: To express a formula or equation with one variable only, on the LHS. (p. 312)

like terms: Algebraic terms in an expression with identical variable parts, for example, $3x$ and $9x$. (p. 288, 312)
Likert scale: A scale of preference such as 'Strongly against, Against, Don't care, Prefer, Strongly prefer'. (p. 353)
line of best fit: The best approximation to a smooth or straight line passing through points plotted on a graph, the line that is closest to the plotted points. (p. 292)
linear equation: An algebraic equation in which the highest power of the variable is 1, such as $y = 2x + 7$ or $4x - 3 = 9$. The graph of a linear equation is a straight line. (p. 88, 301, 312)
linear expression: An algebraic expression in which the highest power of the variable is 1, such as $3x - 4$. (p. 289)
linear interpolation: Using a linear function to find an intermediate value; used to find the median and quartiles of a continuous variable. (p. 376, 401)
linear model: A model that involves a linear function (equation). (p. 126, 133)
loss: The amount lost by a business when items are sold for less than they cost; the cost price minus the selling price. (p. 147, 177). *See also* **profit**.
lower class limit: The bottom end of a class, such as 4.5 for 5–9. (p. 368, 400)
lower quartile: *See* **first quartile**.

marked price, selling price: The price for which an item is offered for sale by a shop. (p. 147, 177)
market price: The current price paid for a share on the stock exchange. (p. 173, 179)
mark-up: The amount added to the cost of an item for sale in a shop. (p. 147, 177)
matrix (plural: matrices): A table of numbers in parentheses ( ) or brackets [ ], like

$$\mathbf{A} = \begin{bmatrix} 1 & 0.5 & 4 & 3 \\ 0 & -2 & 7 & 3 \end{bmatrix}.$$ (p. 414, 445)

mean: A measure of central tendency that is the arithmetic average of the values of a distribution; the sum of all the values divided by the number of values. (p. 373, 400)
measure of central tendency, measure of location: A measure of the middle of statistical data or distributions. (p. 373). *See also* **mean**, **median**, **mode**.
measure of spread: a measure of the spread or dispersion of statistical data or distributions. (p. 385). *See also* **range**, **interquartile range**, **standard deviation**.
median, 2nd quartile (Q_2): A measure of central tendency; the value below which half of the values of a statistical distribution lie. (p. 373, 401, 488, 518)
mid-rate: The average of the **buying price** and **selling price** of a currency exchange rate. (p. 167, 178)
modal class: The class with the highest frequency in a statistical distribution. (p. 373, 400)
mode: The data value or category in a statistical distribution that has the highest frequency. (p. 373, 400)
multiplication (of a matrix) by a scalar: *See* **scalar multiple (of a matrix)**. (p. 421, 445)

negatively skewed, skewed to the left: Describes a statistical distribution with more distinct values on the negative or left side of the peak. (p. 390, 402, 505, 519)
net (of a 3D shape): The 2D shape that can be folded to make the solid shape. (p. 186, 224)
net profit margin: The profit of a company as a percentage of the sales, including loan interest and taxes. (p. 150, 177)
network: A diagram that shows connections between points, such as a road map. (p. 438, 446)
Newstart allowance: Government allowance paid to eligible unemployed Australians aged 21 and above looking for work. (p. 29, 38)
nominal data: A categorical variable with data that are names that cannot be ordered, such as the names of continents. (p. 352, 400). *See also* **ordinal variable**.
non-included angle: An angle that is not between two particular sides of a triangle. (p. 468). *See also* **included angle**.
non-linear equation: An equation that is not a **linear equation**. (p. 302)
non-linear expression: an expression that is not a **linear expression**. (p. 289, 312)
non-right-angled triangle: A triangle that does not have a right (90°) angle. (p. 454)

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numerical data: A statistical variable with data that are numbers, not categories. (p. 352, 400). *See also* **categorical variable**, **discrete variable**, **continuous variable**.

object: The original figure prior to undergoing a transformation. (p. 197). *See also* **image**.
operating expense ratio: The expenses of a company as a percentage of the sales, excluding loan interest and taxes. (p. 150, 177)
operating margin, gross profit margin: The profit of a company as a percentage of the sales, excluding loan interest or taxes. (p. 150, 177)
opposite side: The side opposite the given angle in a right-angled triangle. (p. 318)
order (of a matrix): *See* **dimension (of a matrix)**.
ordered pair: Two numbers placed in order inside parentheses, as x- and y-coordinates. (p. 107, 132).
ordinal data: A categorical variable that has data with a natural order, such as finishing position in a race. (p. 352). *See also* **nominal variable**.
origin: The point (0, 0) on the Cartesian plane where the x- and y-axes intersect, usually labelled with the letter O. (p. 107, 132)
outlier: A value that is well outside the range of the rest of the values in a set of data. (p. 392, 402)
output (value): A value of the dependent variable calculated from an input value for a function. (p. 107, 132)
overtime: Work outside the normal or agreed hours of work. (p. 14, 37)

par value: *See* **face value**.
parallel lines: Lines with the same slope or gradient, lines that point in the same direction and never meet. (p. 117)
part-time worker: A person who is employed for regular hours less than 38 hours a week, on at least a weekly basis. (p. 16, 38)
penalty rate: A higher than normal pay rate for work on a weekend, Sunday or public holiday. (p. 14, 37).
percent (%): Out of 100. (p. 4, 37)
percentage: A fraction with a denominator of 100, written as %. (p. 37, 147, 177)
perimeter: The length around the outside or boundary of a flat object. (p. 52, 80)
permanent worker: A person employed on at least a weekly basis. (p. 16, 37)
picture graph: A graph with pictures that show values, either by their number or size. (p. 358, 400)
pie chart: A graph that has sectors of a circle proportional in area and central angle to the values of a variable. (p. 358, 400)
piece-wise linear function: A linear function which is defined by two or more sub-functions, each applied to a particular interval of the domain of the function. (p. 254, 276)
piece-wise linear graph: The graph of a piece-wise function. (p. 254)
piecework: Payment for each item made or processed. (p. 22, 38)
plane shape: A two-dimensional shape that can be drawn on a flat surface. (p. 184)
plateau distribution: A frequency distribution with low frequencies for low and high values, but similar frequencies for the rest of the values. (p. 391, 402)
point of intersection: The point at which two or more graphs intersect. (p. 232, 276)
population pyramid: A side-by-side histogram for males and females in each age group, with the lowest age group at the bottom. (p. 509)
population: In statistics, the entire set of people or items that could be used for values of a statistical variable. (p. 395)
positively skewed, skewed to the right: Describes a statistical distribution with more distinct values on the positive or right side of the peak. (p. 390, 402)
power of a matrix: A matrix multiplied by itself repeatedly; $\mathbf{A}^n$ is the matrix product of n factors $\mathbf{A}$. (p. 446)
price-to-earnings (PE) ratio: The market price of a share divided by the earnings per share. (p. 173, 179)
principal: The initial amount of a loan or investment. (p. 154, 178)
prism: A three-dimensional shape with a uniform cross-section. (p. 70, 81)
product of matrices: The product of an $n \times m$ matrix $\mathbf{A}$ and an $m \times p$ matrix $\mathbf{B}$ is an $n \times p$ matrix $\mathbf{C}$ whose elements c_{ij} are the sums of the product of the ith row of $\mathbf{A}$ and the jth column of $\mathbf{B}$; $c_{ij} = \Sigma a_{ik} \times b_{kj}$. (p. 428)
profit: The amount made by a business when items are sold for more than they cost; the selling price minus the cost price. (p. 147, 177) *See also* **loss**.
pyramid: A three-dimensional shape with flat faces whose base is a polygon and whose sides are triangles meeting at a common vertex (apex). (p. 70, 81, 187, 224)
Pythagoras' theorem: A rule used for calculating the sides of right-angled triangles. For $\triangle ABC$ with hypotenuse a, $a^2 = b^2 + c^2$. (p. 80)
Pythagorean triple (triad): Any three whole numbers a, b, c such that $c^2 = a^2 + b^2$. (p. 45, 80)

quadrant: One of the quarters of the coordinate plane formed by the axes. (p. 107, 132)

range (of a set of data): The difference between the highest and lowest values of a statistical variable. (p. 381, 401)
range (of a function): The set of (possible) output values (of the dependent variable) for a function. (p. 107, 132)
rate: A comparison of two different quantities, such as money and hours in $/hour. (p. 12, 37, 178)
recommended retail price (RRP): the price that a distributor or manufacturer recommends to retailers for items to be sold to the public by shops. (p. 147, 177)
relative standard deviation (RSD): *See* **coefficient of variation**. (p. 387, 402)
rest (or rest period): The time between calculations of compound interest. (p. 159, 178)
right-angled triangle: A triangle with a 90° angle. (p. 318)
rise: The change in the y-values of a function measured from left to right. (p. 114, 133)
rising (graph): Where the value of the function and graph increases from left to right. (p. 114, 133)
root: One of the solutions of an equation. (p. 88)
rostered day off (RDO): Non-working day in a work roster. (p. 14, 37)
row matrix: A matrix that has only one row. (p. 418, 445)
run: The change in the x-values of a function measured from left to right. (p. 114, 133)

salary: Payment stated as an annual or monthly amount. (p. 12, 37)
scalar multiple (of a matrix), multiplication by a scalar: The product of a number and a matrix; every element of the matrix is multiplied by the number. (p. 421, 445)
scale drawing: A drawing that shows a real object with accurate sizes reduced or enlarged by a certain factor (called the scale). (p. 212, 224)
scale factor, scale: The fraction or ratio that relates the size of a drawing to the size of the actual object it represents. (p. 198, 224)
sector: A part of a circle that lies between two radii and the circumference. (p. 52)
selling price, marked price: The price for which a shop sells items. (p. 147, 177)
selling rate: The amount in AUD (Australian dollars) that a currency exchange sells a unit of foreign currency for. (p. 167). *See also* **buying rate**.
share: Part of the ownership of a company. (p. 172, 179)
side-by-side: Column graphs or histograms of related populations next to each other on the same scale. (p. 509)
similar figures: Shapes that have the same shape but not necessarily the same size. One is an enlargement of the other, with corresponding sides proportional and corresponding angles equal. (p. 197)
simple interest, flat-rate interest: Interest calculated on the original principal (only) of a loan or investment. (p. 154, 178) *See also* **compound interest**.
simultaneous equations: two or more equations (with several variables) that are considered together. (p. 232, 276)
simultaneous solution: The values that make all the equations in a system of simultaneous equations true. (p. 232)
sine rule: A rule relating the sides and angles of a triangle. For $\triangle ABC$, $\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$. (p. 454, 480)
sine: The trigonometric ratio given by $\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}}$ in a right-angled triangle. (p. 346).
skewed to the left: *See* **negatively skewed**. (p. 390, 402)
skewed to the right: *See* **positively skewed**. (p. 390, 402)
slant height: The perpendicular distance from the apex of a cone or a pyramid, along the face to the base. (p. 187)
solution: The value(s) of the variable(s) that makes an equation true, solves the equation. (p. 88)
solve: To find a solution, for example, a root of an equation. (p. 88, 132)
speculation: Trying to make a profit by buying and selling shares as the market price changes. (p. 172)
sphere: A ball shape. (p. 71)
square matrix: A matrix that has an equal number of rows and columns. (p. 418, 445)
standard deviation: A measure of spread that is the square root of the mean of the squares of deviation of the values from the mean. (p. 385, 401)
standard form of a linear equation: A standard way to write the equation of a straight line, $ax + by + c = 0$, where a, b and c are integers and a is positive. (p. 121, 132)
stated class limits: The stated ends of classes such as 5–9, 10–14 for true limits 4.5–9.5, 9.5–14.5 respectively. (p. 368, 400) *See also* **true class limits.**
statistical investigation process: The logical steps of investigation of a statistical problem. (p. 395, 513)
stem-and-leaf plot, stem plot: A display of data where the highest place values such as tens are shown in a column (the stems) and the lower place values such as units are shown horizontally across from their stems (the leaves). (p. 363, 400)

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step function: A piece-wise function in which each sub-function is a constant. (p. 262)
step graph: The graph of a step function consisting of a series of line segments. (p. 260, 276)
stock market: The institution where company shares are bought and sold. (p. 179)
subject of a formula: A single variable on the left-hand side of a formula (equation). (p. 296, 312)
substitution method: An algebraic method for solving simultaneous equations, in which one of the equations is written in terms of one of the variables and then substituted for that variable into the other equation. (p. 276)
surface area: The areas of the faces of a three-dimensional shape. (p. 186, 224)
symmetrical distribution: A statistical distribution that is roughly symmetrical on either side of the central peak. (p. 390, 402)

tangent: The trigonometric ratio given by

$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$ in a right-angled triangle. (p. 346)
term: The time of a loan or investment. (p. 154, 178)
third quartile, upper quartile (Q_3): The value below which three-quarters of the values of a statistical distribution lie. (p. 381, 401, 488, 518)
three-dimensional (3D): An object or solid that has three dimensions: length, width and height (or depth). (p. 70)
time-and-a-half: 1.5 times the normal rate of pay for overtime or as a penalty rate. (p. 37)
trade discount: The discount to the recommended retail price (RRP) that is given to retailers by distributors or manufacturers so they can make a profit when selling at the RRP. (p. 147, 177)
trainee: A person learning a skill other than a recognised trade. (p. 38). *See also* **apprentice.**
transition matrix: A matrix that shows probabilities of changes from one state to another, such as the probabilities of fine, cloudy or rainy days following fine, cloudy or rainy days. (p. 446)
transpose (a formula): To move a given term or variable from one side of a formula (or equation) to the other. (p. 296, 312)
trigonometric ratio: A ratios between the sides of a right-angled triangle, such as **sine**, **cosine**, **tangent**. (p. 318, 319)
true bearing: A compass direction such as 084° measured as an angle clockwise from north, from 000° to 360°. (p. 336, 346)
true class limits: The actual ends of classes such as 4.5–9.5, 9.5–14.5, for stated limits 4–9, 9–14 respectively. (p. 368, 400) *See also* **stated class limits**.
two-dimensional (2D): A shape that only has two dimensions, length and width, and no thickness (or height or depth). (p. 44)

undefined: Not having a mathematical meaning. (p. 133)
unit price: the cost of 1 item, 1 kg, 1 litre, etc. of goods, used to compare prices sold in different-sized packages. (p. 144, 177)
univariate data: Statistical data involving only one variable. (p. 352)
upper class limit: The top end of a class, like 9.5 for 5–9. (p. 368, 400)
upper quartile: *see* **third quartile**.

vertex: A corner or a point where lines or edges meet on a shape or network. (p. 70)
volume: A measure of the size of a three-dimensional shape using cubic units. (p. 71)

wage: A payment for work stated as an hourly rate. (p. 12, 37)

***x*-axis:** The horizontal axis on the Cartesian plane. (p. 107, 132)
***x*-coordinate:** The distance that a point is to the right (positive) or left (negative) of the origin. (p. 132)
***x*-intercept:** The value where the graph of the function cuts the *x*-axis. (p. 120, 133)

***y*-axis:** The vertical axis on the Cartesian plane. (p. 107)
***y*-coordinate:** The distance that a point is above (positive) or below (negative) the origin. (p. 132)
***y*-intercept:** The value where the graph of the function cuts the *y*-axis. (p. 120, 133)
youth allowance: Government allowance paid to eligible apprentices and students aged under 25 years of age. (p. 38)

zero matrix: A matrix with every element 0. (p. 418, 445)

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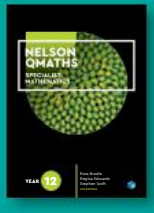

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