PEARSON

Mathematics

STUDENT BOOK | 3RD EDITION

7

AUSTRALIAN CURRICULUM

Pearson

Pearson Mathematics 7 3rd Edition

Student Book

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We pay our respects to Elders, past and present.

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Winyama: p. 455.

Contents

1 Number properties 1

Recall 2

1.1 Understand and calculate squares and square roots 3

1.2 Understand and use exponent notation to represent numbers 9

1.3 Represent numbers in prime factor form 12

1.4 Understand and identify common factors and multiples 19

1.5 Understand and use expanded notation to represent numbers 25

Topic review 29

2 Operations with decimals 32

Recall 33

2.1 Round decimals for different purposes 34

2.2 Add and subtract decimals 41

2.3 Multiply decimals 47

2.4 Divide decimals 52

Topic review 57

3 Fractions, decimals and percentages 61

Recall 62

3.1 Understand and create equivalent fractions 63

3.2 The lowest common denominator and comparing fractions 68

3.3 Simplify fractions 71

3.4 Understand the relationship between decimals, percentages and fractions 75

3.5 Locate fractions and decimals on a number line 80

3.6 Compare fractions and decimals on a number line 83

Topic review 86

4 Operations with fractions 91

Recall 92

4.1 Understand how to add and subtract fractions 93

4.2 Understand mixed numbers and improper fractions 97

4.3 Find the fraction of an amount 101

4.4 Understand how to multiply fractions 105

4.5 Understand how to divide fractions 108

Topic review 114

5 Modelling with fractions, decimals and percentages 118

Recall 119

5.1 Choose an appropriate representation to solve problems 120

5.2 Determine the proportion of a quantity 124

5.3 Understand percentage as applied to taxation 129

5.4 Use fractions, decimals and percentages to solve financial problems 133

5.5 Use, interpret and calculate discounts and mark-ups 137

Topic review 142

6 Integers 147

Recall 148

6.1 Understand integers and operations 149

6.2 Add and subtract integers 154

6.3 Model and solve practical problems using integers 161

Topic review 170

7 Understanding ratios 173

Recall 174

7.1 Write ratios 175

7.2 Understand equivalent ratios 180

7.3 Understand the connection between fractions and ratios 185

Topic review 189

Contents

8 Algebra 193

Recall 194
8.1 Understand variables in algebra 195
8.2 Identify and simplify like terms 199
8.3 Calculate the value of algebraic expressions 202
8.4 Write algebraic expressions in context 208
Topic review 210

9 Linear relationships 215

Recall 216
9.1 Understand the components of an equation and how they are combined 217
9.2 Generate a table of values using a linear equation 220
9.3 Recognise the connection between points on a Cartesian plane, a table of values and a linear pattern 229
9.4 Model linear patterns using manipulatives, diagrams and graphs 239
Topic review 244

10 Making and reading linear graphs 250

Recall 251
10.1 Graph linear relationships from a rule 252
10.2 Interpret the gradient of a linear graph 257
10.3 Interpret, discuss and analyse relationships in graphs 264
Topic review 273

11 Solving linear equations 277

Recall 278
11.1 Write linear equations to represent simple word problems 279
11.2 Write linear equations to represent diagrams and patterns 282
11.3 Use flow charts to determine the value of an expression 289
11.4 Use flow charts to solve linear equations 293
11.5 Use the balance method to solve linear equations 299
Topic review 305

12 Area and volume 310

Recall 311
12.1 Understand and convert between units of area 312
12.2 Calculate the area of a parallelogram 317
12.3 Calculate the area of a triangle 324
12.4 Understand volume measured in cubic units 334
12.5 Establish the volume formula and units for prisms 339
12.6 Explore the connection between volume and capacity 346
Topic review 353

13 Circles (features and circumference) 359

Recall 360
13.1 Recognise circle features 361
13.2 Understand how circumference is related to radius and diameter 368
13.3 Solve length problems involving circles 373
Topic review 381

14 Angles and transversals 384

Recall 385
14.1 Construct perpendicular and parallel lines 387
14.2 Identify corresponding, alternate and co-interior angles 391
14.3 Determine the exterior angle of a triangle 400
14.4 Calculate the internal angle sum of a triangle 404
Topic review 409

15 Ratios and measurement 415

Recall 416

15.1 Solve practical problems involving ratios of length 417

15.2 Explore ratios in measured quantities 425

Topic review 432

16 Visualising 3D 435

Recall 436

16.1 Visualise and apply nets 437

16.2 Visualise and draw the perspective views of a 3D object 443

16.3 Draw and build objects made from cubes 448

16.4 Draw and use objects represented in art form 453

Topic review 458

17 Triangles and quadrilaterals 463

Recall 464

17.1 Construct triangles and quadrilaterals 465

17.2 Classify types of triangles by their side and angle properties 473

17.3 Classify types of quadrilaterals by their side and angle properties 482

17.4 Use parallel side lengths of quadrilaterals to construct rectangles of equivalent area 491

Topic review 499

18 Translations, reflections, rotations 504

Recall 505

18.1 Translate and reflect points and shapes on a Cartesian plane 506

18.2 Rotate shapes on a Cartesian plane 514

Topic review 525

19 Data and measures of central tendency 536

Recall 537

19.1 Understand different data types 538

19.2 Calculate the mean and range of a set of data 549

19.3 Calculate the mean and range for a set of grouped data 556

19.4 Determine the median of a data set 564

19.5 Determine an appropriate measure of central tendency 570

Topic review 576

20 Displaying and understanding data 580

Recall 581

20.1 Understand stem-and-leaf plots 582

20.2 Create and interpret statistical plots 588

20.3 Determine statistical measures of centre from data displays 596

20.4 Interpret and describe numerical data displays 607

Topic review 615

21 Probability (simulations and sample space) 621

Recall 622

21.1 List sample spaces and calculate the probability of single-step events 623

21.2 Record outcomes and run trials of chance experiments 632

21.3 Compare theoretical and experimental probability 641

Topic review 647

Glossary 651

Answers 667

How to use Pearson Mathematics

The components of Pearson Mathematics

This *Student Book* is a complementary resource that offers a print medium for corresponding lessons in *Pearson Secondary Teaching Hub* and the *Student Companion*. It is designed on the principles of current best practice and provides students with an offline version to access the theory, worked examples and practice questions from the digital lessons in the *Hub*, to give the perfect blend of online and offline learning.

The *Student Book* (also available as an eBook) gives you an offline version of the theory, worked examples and independent practice questions. The *Student Book* has been supplied to minimise digital distraction and screen fatigue, supporting teachers and students to find the perfect balance of online and offline learning.

The *Hub* is a one-of-a-kind digital product designed to simplify teaching and energise learning, with high-quality content created by experienced Australian educators who know how to engage students.

The *Student Companion* is a write-in workbook that has been designed to support the guided practice phase of the lesson, giving students scaffolded instruction as an introduction to the skill. The *Student Companion* also supports the creation of a bound reference or portfolio of learning over the year.

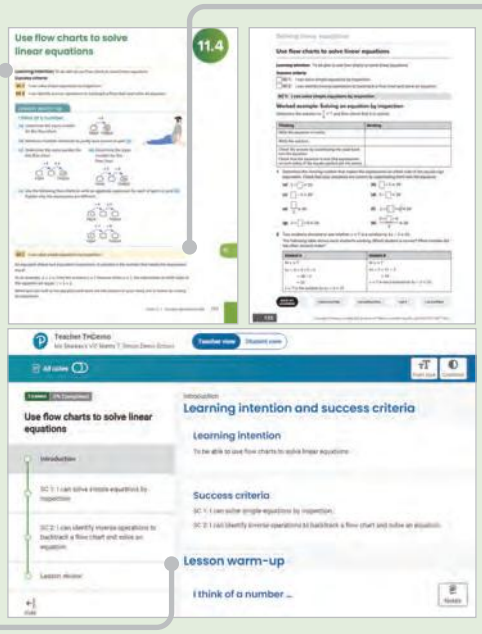
Chunking with learning intentions and success criteria

The structure and relationship between each of the three components in the Pearson Mathematics series (*Hub*, *Student Book* and *Student Companion*) has been designed to manage the cognitive load of our students. Chunking has been used to break down complex concepts and strategies into smaller, more manageable sections, defined by the success criteria.

In the *Hub* and the *Student Companion*, the chunking of content directly supports success criteria, while the *Student Book* is chunked by learning intention, to give both teachers and students autonomy in delivery.

Learning intentions are statements aligned to the relevant curriculum. Each of the lessons in the *Hub* have been developed around a single purpose for learning. The learning intentions are communicated using student-friendly language and share the learning goals or objectives from the corresponding digital lesson in the *Hub* and pages of the *Student Companion*. They describe what learners should know, understand or be able to do by the end of the lesson.

Every lesson starts with a **Lesson warm-up**, a 3- to 5-minute activity designed to activate prior knowledge and prime students for learning. In the *Hub*, teachers are able to access teaching notes that contain sample solutions, different ways to deliver the activity, and enabling and extending prompts to ensure all learners have an entry point and can be extended.



Success criteria clarify expectations and describe what success looks like. They are used to determine how well a student has met the learning intention. The Pearson Mathematics lesson design is based on cognitive load principles and, as such, each lesson has between one and four success criteria. The success criteria are specific, concrete and measurable, so learners can actively engage with and reflect on their evidence of learning within each lesson. The success criteria form the basis of all three components — the *Hub*, the *Student Book* (and eBook) and the *Student Companion* — to ensure they are referred to throughout the lesson and provide the basis for feedback.

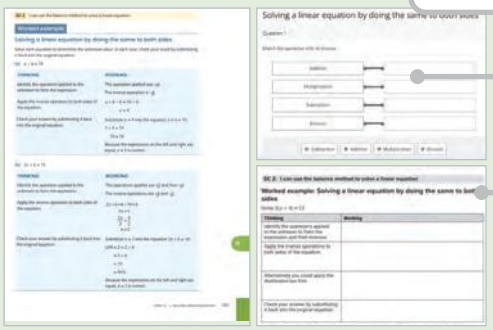
Reducing cognitive load with worked examples

Worked examples provide students with a step-by-step solution to a problem.

The worked examples in the *Student Book* correspond to those in the digital lesson, and are provided for each skill to:

- scaffold learning
- support skill acquisition
- reduce the cognitive load.

In the *Hub*, each worked example is presented as a video walkthrough, and has a 'Check your understanding' section with one to three autocorrecting questions. This gives students and teachers immediate feedback on student understanding of the required skills to succeed, before moving on to the independent practice phase of the lesson.



The *Student Companion* supports the gradual release of responsibility, supporting the guided practice phase of the lesson, by giving students access to a similar problem presented as a 'try yourself' worked example and one to three additional scaffolded practice questions. The *Student Companion* also provides students with the opportunity to develop a learning portfolio, and can be used in a flipped environment, in classroom learning, as homework, or in preparation to create a bound reference or summary.

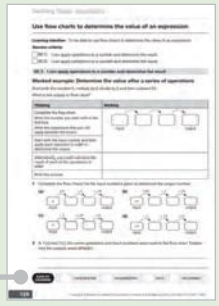
Gradually releasing responsibility with print and digital

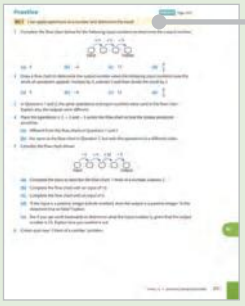
I do: Students and teachers are supported by new information being demonstrated and modelled with worked examples and the worked example videos. The 'Check your understanding' questions provide immediate feedback to both students and teachers on the student's level of understanding of each new concept.

We do: The *Student Companion* offers a 'try yourself' version of the worked example and a small set of scaffolded questions designed to facilitate the guided practice phase of the lesson.

You do: The success criteria each have a set of 'Practice' questions available to access in both the *Hub* and the *Student Book*, and are designed to be used in the independent practice phase of the lesson.

Each lesson in the *Student Companion* contains a space for students to reflect on their understanding. The simple and intuitive design of the **lesson reflection tool** allows students to scale their confidence, reflect on their learning, and identify areas in which they need additional support.





The **Practice** section in the *Student Book* contains the independent practice questions for the lesson, still broken down by success criteria. These questions are provided in both the *Hub* and the *Student Book* to give teachers and students the choice of digital or print for the independent practice phase of the lesson. These questions are designed to ensure learners build confidence and demonstrate proficiency.

Each lesson in the *Hub* contains an additional six to eight auto-correcting questions. These questions provide a summative assessment of the lesson and utilise the skills and knowledge obtained throughout the entire lesson. These assessments give students and teachers immediate feedback on student readiness to move forwards in the topic sequence.

Pearson Mathematics 3E

Simplified teaching, energised learning for the new Australian Curriculum Mathematics Year levels 7–10

Powered by the Pearson Secondary Teaching Hub

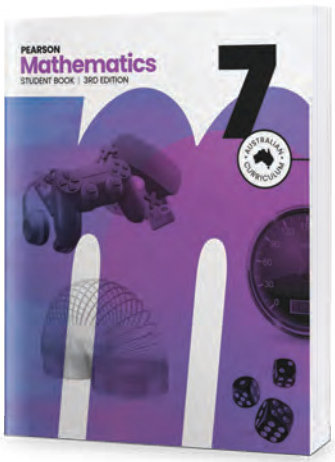
How have we simplified teaching?

In consultation with Australian teachers and students, we've built a simple-to-use hub of lesson content for the new Australian Curriculum Mathematics.

How have we energised learning?

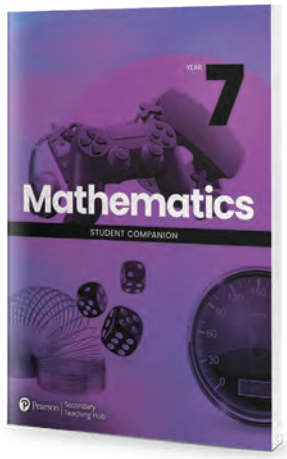
Engaging videos, worksheets, auto-correcting questions, worked examples and more, with the perfect mix of online and offline resources

Student Books



Teaching Hub with eBooks

Student Companions



Student Books link to the *Hub* for flexible teaching

Giving you the autonomy to teach online or offline to meet the needs of your students

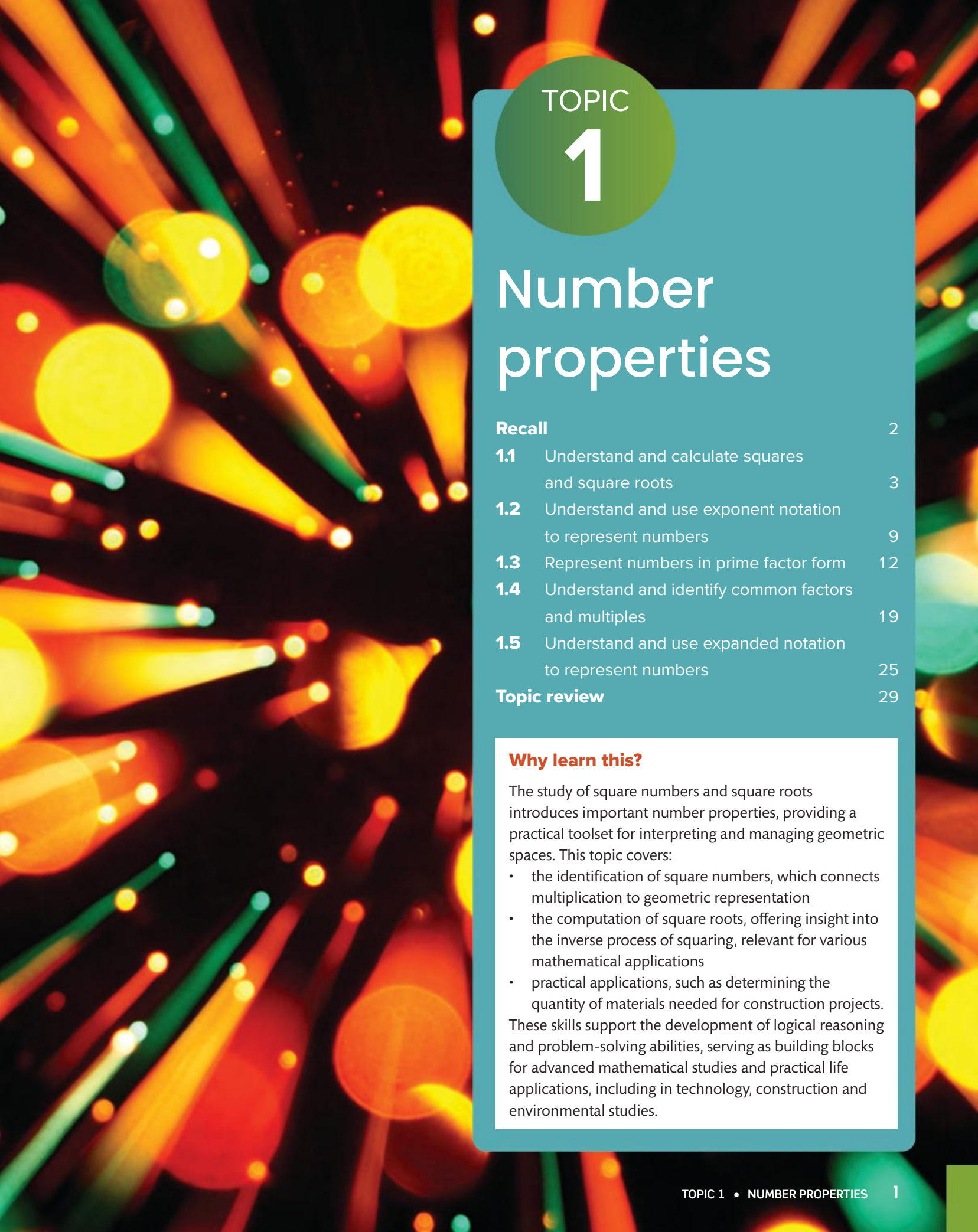
- Developed using the latest teaching and learning research
- Access to all 7-10 content in the *Hub* for students that need revision, or those that need extension
- eBook versions available in the *Hub*

Student Companions provide write-in practice books

Accompanying the Teaching Hub, with 'try yourself' worked examples and exercise questions to support guided practice

- 1000s of practice questions across the series
- Use as a homework activity program, guided practice in class, or to support multiple exposures

Discover Pearson Mathematics at **pearson.com.au/schools/maths-AC**



TOPIC
1

Number properties

Recall 2

1.1 Understand and calculate squares and square roots 3

1.2 Understand and use exponent notation to represent numbers 9

1.3 Represent numbers in prime factor form 12

1.4 Understand and identify common factors and multiples 19

1.5 Understand and use expanded notation to represent numbers 25

Topic review 29

Why learn this?

The study of square numbers and square roots introduces important number properties, providing a practical toolset for interpreting and managing geometric spaces. This topic covers:

- the identification of square numbers, which connects multiplication to geometric representation
- the computation of square roots, offering insight into the inverse process of squaring, relevant for various mathematical applications
- practical applications, such as determining the quantity of materials needed for construction projects.

These skills support the development of logical reasoning and problem-solving abilities, serving as building blocks for advanced mathematical studies and practical life applications, including in technology, construction and environmental studies.

RECALL

ANSWERS Page 667

I can identify the place value of numbers

1 Create two charts like the one shown below and write the digits of each number in their correct place value on each chart. State the place value of the underlined digit.

(a) $\underline{2}408091$ **(b)** $610\,9\underline{3}1$

millions	hundred thousands	ten thousands	thousands	hundreds	tens	units

I can arrange numbers into ascending or descending order

1 Arrange the following groups of numbers in ascending order:

(a) 8, 86, 6, 826, 628, 286 **(b)** 3000, 3003, 30, 033, 33, 30

2 Arrange the following groups of numbers in descending order:

(a) 768, 678, 867, 876, 687 **(b)** 158, 851, 1058, 805, 815

I can add numbers of different place value together

1 Simplify the following:

(a) 70 000 + 3000 + 200 + 90 + 1 **(b)** 2 000 000 + 200 000 + 20 000 + 2000 + 2

I can round numbers to the nearest 10, 100 or 1000

1 Round the following numbers to the nearest:

(i) 10 **(ii)** 100 **(iii)** 1000

(a) 3785 **(b)** 12 438

I can multiply more than two numbers together

1 Find each of the following:

(a) $4 \times 10 \times 9$ **(b)** $6 \times 3 \times 3 \times 2$ **(c)** $3 \times 3 \times 5 \times 5$ **(d)** $4 \times 4 \times 4 \times 3$

I can perform simple addition and subtraction

1 Calculate the value of the following:

(a) $67 + 9 + 310$ **(b)** $821 - 342$ **(c)** $2600 - 189$ **(d)** $8127 + 26 + 3 + 298$

I can perform simple multiplication and division without the aid of a calculator

1 Calculate the value of the following:

(a) 145×3 **(b)** 57×19 **(c)** $315 \div 5$ **(d)** $2709 \div 9$

Understand and calculate squares and square roots

1.1

Learning intention: To understand and calculate squares and square roots

Success criteria:

SC 1 I can identify square numbers.

SC 2 I can determine the square root of a square number.

SC 3 I can place the square root of any number between its two closest natural numbers.

SC 4 I can apply squares and square roots to real-life situations.

Lesson warm-up

Is it square?

On grid paper, draw a square that has an area of 15 cm^2. You can shade only full squares. Is this possible? Explain why or why not.

SC 1 I can identify square numbers

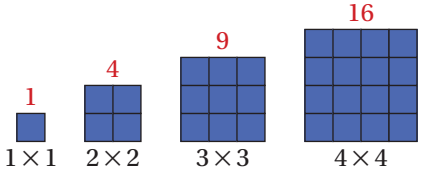
A square number is formed by multiplying a whole number by itself.

The shorthand symbol for this is a superscript 2.

For example, $1 \times 1 = 1^2 = 1$, $5 \times 5 = 5^2 = 25$ and $x \times x = x^2$

$1^2 = 1$	$6^2 = 36$	$11^2 = 121$	$16^2 = 256$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$	$17^2 = 289$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$	$18^2 = 324$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$	$19^2 = 361$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$	$20^2 = 400$

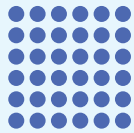
Square numbers also can be represented as the area of squares.



Worked example

Identifying common square numbers

(a) Is 36 a square number?

THINKING	WORKING
Recall the factors of 36. Can 36 be written as the product of a number multiplied by itself?	$2 \times 18 = 36$ $3 \times 12 = 36$ $4 \times 9 = 36$ $6 \times 6 = 36$ 
Write the answer.	Yes, 36 is a square number.

(b) Is 12 a square number?

THINKING	WORKING
Recall the factors of 12. Can 12 be written as the product of a number multiplied by itself?	$2 \times 6 = 12$ $3 \times 4 = 12$ No whole number multiplies by itself to equal 12. The diagrams of 2×6 and 3×4 do not create squares.
Write the answer.	No, 12 is not a square number.

SC 2 I can determine the square root of a square number

The opposite of squaring a number is taking the square root of a number.

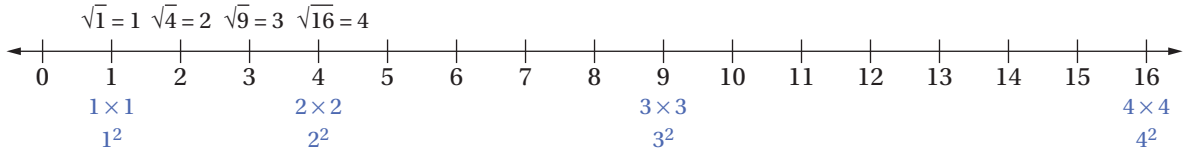
The symbol for square root is $\sqrt{\ }$. It is a number that can be multiplied by itself to produce the original number.

For example, $\sqrt{4} \times \sqrt{4} = 2 \times 2 = 4$

Squaring a number is the opposite, or inverse, of taking the square root of a number.

$3^2 = 9, \sqrt{9} = 3$

All positive numbers have a square root, but these may not be whole numbers. Only square numbers have square roots that are whole numbers.



Worked example

Calculating the square root of a square number

Determine the square root of the square number 25.

THINKING	WORKING
Determine the number that when multiplied by itself gives the square number.	$5 \times 5 = 25$ so $\sqrt{25} = 5$
Write the answer.	The square root of 25 is 5.

SC 3 I can place the square root of any number between its two closest natural numbers

Worked example

Estimating the value of the square root of a number

The square root of 80 is between which two whole numbers?

THINKING	WORKING
Recall the square numbers above and below 80.	$8^2 = 64$ and $9^2 = 81$
Write the square root for each number.	$\sqrt{64} = 8$ and $\sqrt{81} = 9$
Write the answer.	The square root of 80 is between 8 and 9.

SC 4 I can apply squares and square roots to real-life situations

Worked example

Applying squares and square roots

A carpenter will place floor tiles in a square room measuring 4 m on each side. Each tile is a square measuring 50 × 50 cm. How many tiles are required?

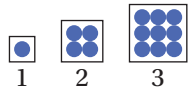
THINKING	WORKING
Calculate the number of tiles along one wall.	4 m = 400 cm $\frac{400}{50} = 8$
Since the room is square, the number of tiles is found by squaring 8.	$8^2 = 8 \times 8$ $= 64$
Write the answer.	64 tiles are needed to cover the whole floor.

Practice

ANSWERS Page 667

SC 1 I can identify square numbers

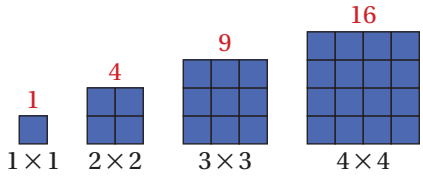
1 Some square numbers are represented in the diagrams below.



(a) The diagrams show the first three square numbers 1, 4 and 9. Explain what these numbers represent.

(b) How many counters would you need to make the fourth square number?

You can also create squares with arrays.



(c) Use graphing software or grid paper to draw squares with side lengths 4 cm, 5 cm and 6 cm.

(d) Determine the area of each of the squares you drew in part **(c)**.

(e) Explain how you can calculate the value of a square number.

(f) How would you work out the area of a square with side lengths of 10 cm?

2 Copy and complete this table of the first 20 square numbers.

$1^2 = 1$	$6^2 =$	$11^2 =$	$16^2 =$
$2^2 = 4$	$7^2 =$	$12^2 =$	$17^2 =$
$3^2 = 9$	$8^2 =$	$13^2 =$	$18^2 =$
$4^2 =$	$9^2 =$	$14^2 =$	$19^2 =$
$5^2 =$	$10^2 =$	$15^2 =$	$20^2 =$

3 Which of the following numbers are square numbers?

A 10 **B** 25 **C** 100 **D** 200

4 Write the square numbers between 10 and 40.

5 Copy and complete the following sentences.

(a) There are ___ square numbers less than 99.

(b) There are ___ square numbers between 100 and 199.

(c) There are ___ square numbers between 200 and 299.

(d) There are ___ square numbers between 300 and 399.

6 How would you explain what a square number is to your friend?

7 Describe how you know that 18 is not a square number.

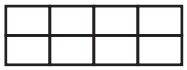
8 The following answers were given by students in response to the question "What is a square number?"

Which answer is correct and why are the others incorrect?

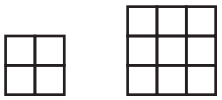
Student A says that 8 is a square number because it can be found by multiplying 2 by itself 3 times.

$2 \times 2 \times 2 = 8$

Student B says that 8 is a square number because it can be drawn as a 2×4 rectangle as shown.



Student C says that 8 is not a square number and offers these two diagrams as evidence.



$2^2 = 4$ $3^2 = 9$

9 You can square two-digit numbers by using the multiplication shown above, but here is a quick trick you might like to try.

For example, calculate 53^2.

Write down 5^2 followed by 3^2.	2509	Write both as two-digit numbers.
Multiply 5×3 and double it.	30	Note this is written under the middle two digits.
Add the numbers.	2809	

Try a few for yourself.

(a) 15^2 **(b)** 64^2 **(c)** 83^2 **(d)** 92^2

SC 2 I can determine the square root of a square number

1 Determine the square root of the following square numbers.

(a) 4 **(b)** 9 **(c)** 25

(d) 49 **(e)** 169 **(f)** 324

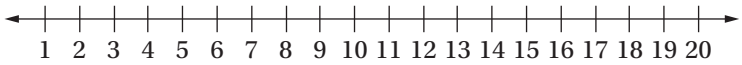
SC 3 I can place the square root of any number between its two closest natural numbers

1 The square root of 10 is between which two whole numbers?

2 Determine the whole number above and below the square root of:

(a) 20 **(b)** 30 **(c)** 120 **(d)** 270

3 Place the following square roots on the number line shown.



(a) $\sqrt{8}$ **(b)** $\sqrt{14}$ **(c)** $\sqrt{56}$ **(d)** $\sqrt{200}$

4 Explain to your friend how to determine the approximate square root of a number.

5 Describe how you know that the square root of 18 is between 4 and 5.

SC 4 I can apply squares and square roots to real-life situations

1 A carpenter lays floor tiles in a square room measuring 3 m on each side. Each tile is a square measuring 30 × 30 cm. How many tiles are required?

2 A carpenter lays floor tiles in a square room measuring 3 m on each side. Each tile is a square measuring 50 × 50 cm. How many tiles are required?

3 A carpenter lays floor tiles in a square room measuring 4.2 m on each side. Each tile is a square measuring 30 × 30 cm. How many tiles are required?

4 A couple who are unknowingly infected with a virus visit a large city for a two-week vacation. Each day, an infected person will pass the virus on to one other person. Assume that the large city was originally virus-free.

(a) Copy and complete this table of infections for the first 5 days.

Day	New infections	Total infections
1	–	2
2	2	4
3		
4		
5		

(b) What do you notice about the numbers in the columns?

After day 5, people are no longer infectious.

(c) What will be the total number of infections on day 6?

(d) Copy and complete this table of infections for days 6 to 10.

Day	New infections	Recovered patients	Total infections
6		2	
7			
8			
9			
10			

Understand and use exponent notation to represent numbers

1.2

Learning intention: To understand and be able to use exponent notation to represent numbers

Success criteria:

SC 1 I can correctly use the terms 'base' and 'exponent'.

SC 2 I can express repeated multiplication by using exponent notation.

Lesson warm-up

To the power of...

Multiplication is the operation used to write repeated addition, for example $2 + 2 + 2 = 3 \times 2$. But how is repeated multiplication written?

Using your calculator, complete the multiplications below:

$3 \times 3 =$

$3 \times 3 \times 3 =$

$3 \times 3 \times 3 \times 3 =$

$3 \times 3 \times 3 \times 3 \times 3 =$

Using the exponent function on your calculator (some calculators use x^y or ^), complete the exponent expressions below:

$3^2 =$

$3^3 =$

$3^4 =$

$3^5 =$

What conclusion can you make?

SC 1 I can correctly use the terms 'base' and 'exponent'

Base → 7^3 ← Exponent (Power or Index)

Base → a^x ← Exponent (Power or Index)

$7^2 = 7 \times 7$

$7^3 = 7 \times 7 \times 7$

The 7 is the base and the superscripted figure is called the exponent. It tells you the number of times to multiply the base by itself. The exponent is also known as the power or index.

Note: 7^3 is called the exponent form, and $7 \times 7 \times 7$ is the expanded form.

Worked example

Identifying the base and exponent

Identify the base and exponent in 4^3.

THINKING	WORKING
Identify the base. The base is the large number at the bottom.	The 4 is the base.
Identify the exponent. The exponent (or power) is the superscripted number.	The exponent is 3.

SC 2 I can express repeated multiplication by using exponent notation

Worked example

Understanding the link between exponent and expanded form

Write the expression $12 \times 12 \times 12$ in exponent form.

THINKING	WORKING
Identify the base.	Base = 12
Identify the exponent by counting the number of times the base occurs in the expression.	Exponent = 3
Write the answer.	$12 \times 12 \times 12 = 12^3$

Practice

ANSWERS Page 668

SC 1 I can correctly use the terms 'base' and 'exponent'

1 Identify the base and exponent in 8^3.

2 Identify the base and exponent in:

(a) 5^2 **(b)** 5^8 **(c)** x^2 **(d)** x^y

3 You can calculate the value of expressions written in exponent form.

For example, $2^5 = x$.

Since $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, $x = 32$.

Calculate the value of x in these equations.

(a) $2^3 = x$ **(b)** $2^4 = x$ **(c)** $3^2 = x$ **(d)** $3^4 = x$

4 You can determine the value of an unknown exponent.

For example, $2^x = 8$.

Since $2 \times 2 \times 2 = 2^3 = 8$, the exponent, x is equal to 3.

Determine the value of the unknown exponents in these equations.

(a) $2^x = 16$ **(b)** $2^x = 32$ **(c)** $3^x = 9$ **(d)** $3^x = 27$

5 You can determine the value of an unknown base.

For example, $x^2 = 9$.

Since $3 \times 3 = 3^2$, the base, x is equal to 3.

Determine the missing base in these equations.

(a) $x^4 = 16$ **(b)** $x^5 = 32$ **(c)** $x^2 = 25$ **(d)** $x^3 = 125$

6 There are 100 insects in a swamp, but this number doubles every week over the next three months.

(a) How many insects are present after three weeks? Express your answer as both a number and in the form 100×2^n.

(b) How many insects are present after 12 weeks? Express your answer as both a number and in exponent form.

(c) Why is it simpler to write your answer in exponent form, especially for larger numbers?

SC 2 I can express repeated multiplication by using exponent notation

1 Write the following in exponent form.

(a) $4 \times 4 \times 4 \times 4 \times 4 \times 4$ **(b)** $4 \times 4 \times 4 \times 4$

(c) $4 \times 4 \times 4$ **(d)** 4×4

2 Write the following expressions in exponent form.

(a) $2 \times 2 \times 2$ **(b)** $3 \times 3 \times 3$ **(c)** $50 \times 50 \times 50$ **(d)** $a \times a \times a$

3 When two or more factors are involved, they can be simplified by writing them in exponent form.
For example, $2 \times 2 \times 2 \times 5 \times 5 = 2^3 \times 5^2$

Write the following expressions in exponent form.

(a) $5 \times 5 \times 6 \times 6 \times 6 \times 6$ **(b)** $2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

(c) $2 \times 2 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ **(d)** $2 \times 2 \times 2 \times 3 \times 3 \times 4 \times 4 \times 4 \times 4$

4 A mixture of numerals and variables can be represented in exponent form.
For example, $2 \times 2 \times 2 \times a \times a = 2^3a^2$. Notice the $\times$ sign can be left out when multiplying numerals and variables.

Write the following expressions in exponent form.

(a) $5 \times 5 \times x \times x \times x \times x$ **(b)** $2 \times 2 \times 2 \times 2 \times 2 \times y \times y \times y$

(c) $2 \times 2 \times b \times b \times b \times b \times b \times b$ **(d)** $2 \times 2 \times 2 \times c \times c \times d \times d \times d \times d$

5 Jezza expressed 5^2 as $2 \times 2 \times 2 \times 2 \times 2$. What mistake was made? Use mathematical terms in your explanation.

6 Joffa answered that $5^3 = 15$. What mistake was made? Use mathematical terms in your explanation.

1.3 Represent numbers in prime factor form

1

Learning intention: To be able to represent numbers in prime factor form

Success criteria:

SC 1 I can determine the prime factors of a number.

SC 2 I can use a factor ladder to determine the prime factors of a number.

SC 3 I can use a factor tree to determine the prime factors of a number.

SC 4 I can write a number as a product of its prime factors.

Lesson warm-up

Sieve of Eratosthenes

The sieve of Eratosthenes is used to identify prime numbers. Use the algorithm below to find all the prime numbers less than 100.

1. Cross out 1, as this is not a prime or a composite.
2. Highlight/circle the next uncovered number. (2)
3. Cross out all the other multiples of 2. (4, 6, 8, 10, ...)
4. Highlight/circle the next uncovered number. (3)
5. Cross out all the other multiples of 3. (6, 9, 12, ...)
6. Highlight/circle the next uncovered number. (5)
7. Cross out all the other multiples of 5. (5, 10, 15, ...)
8. Complete this pattern until you have highlighted all the prime numbers.

SC 1 I can determine the prime factors of a number

A factor is an integer (whole number) that divides into another number, leaving no remainder.

For example, 6 divides into 42, so 6 is a factor of 42.

A prime number is a number with only two factors: itself and 1. (Note that 1 is not a prime number as it has only one factor.)

A prime factor is both a prime number and a factor.

Worked example

Determining prime factors

Determine the prime factors of 36 using trial and error.

THINKING	WORKING
Write the first factor pair as a product of 1 and itself.	1×36
Try 2, 3, 4, 5, 6 until there is no difference between the two factors, or the factors start repeating.	2×18 3×12 4×9 5 is not a factor 6×6
List the factors.	1, 2, 3, 4, 6, 9, 12, 18, 36
Highlight the factors that are prime numbers.	1, 2, 3, 4, 6, 9, 12, 18, 36
Write the answer.	2 and 3 are the prime factors of 36.

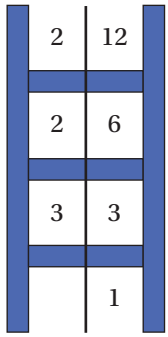
SC 2 I can use a factor ladder to determine the prime factors of a number

Another method for determining prime factors is to use a factor ladder.

This involves first knowing the prime numbers, 2, 3, 5, 7, 11, 13, ...

You begin by dividing the number you are factoring by the lowest possible prime number. This means you first try to divide by 2. If this does not work, then divide by the next prime number 3. If this does not work, then divide by the next prime number 5. You continue this process until the number cannot be factored further.

For example, this is the factor ladder for the number 12.



The number 12 appears at the top right and the prime factors are found in the first column. You have identified all of the prime factors when the second column reaches 1. The diagram looks a little like a ladder, hence the name.

The prime factors of 12 are 2 and 3.

Worked example

Using a factor ladder to determine the prime factors of a number

(a) Determine the prime factors of 12 using a factor ladder.

THINKING	WORKING
Recall the smallest prime number.	The smallest prime number is 2.
Divide the number by the smallest prime number until it no longer divides evenly or until the final division gives a result of 1.	Does 2 divide evenly into 12? Yes $12 \div 2 = 6$ Write the answer below the 12. 2 | 12 | 6 Does 2 divide evenly into 6? Yes $6 \div 2 = 3$ Write the answer below the 6. 2 | 12 2 | 6 | 3 Does 2 divide evenly into 3? No.
Recall the next prime number.	The next prime number is 3.
Divide the result in the ladder by the next prime number until it no longer divides evenly or until the final division gives a result of 1.	Does 3 divide evenly into 3? Yes $3 \div 3 = 1$ Write the answer below the 3. 2 | 12 2 | 6 3 | 3 | 1
Write the answer.	The prime factors of 12 are 2 and 3.

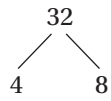
(b) Determine the prime factors of 36 using a factor ladder.

THINKING	WORKING
Recall the smallest prime number.	The smallest prime number is 2.
Divide the number by the smallest prime number until it no longer divides evenly or until the final division gives a result of 1.	Does 2 divide evenly into 36? Yes $36 \div 2 = 18$ Write the answer below 36. 2 | 36 | 18 Does 2 divide evenly into 18? Yes $18 \div 2 = 9$ Write the answer below 18. 2 | 36 2 | 18 | 9 Does 2 divide evenly into 9? No
Recall the next prime number.	The next prime number is 3.
Divide the result in the ladder by the next prime number until it no longer divides evenly or until the final division gives a result of 1. Try dividing by the next prime number 3.	Does 3 divide evenly into 9? Yes $9 \div 3 = 3$ Write the answer below 9. 2 | 36 2 | 18 3 | 9 | 3 Does 3 divide evenly into 3? Yes $3 \div 3 = 1$ Write the answer below 3. 2 | 36 2 | 18 3 | 9 3 | 3 | 1
Write the answer.	The prime factors of 36 are 2 and 3.

SC 3 I can use a factor tree to determine the prime factors of a number

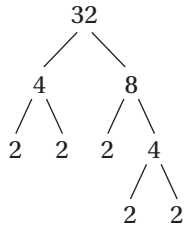
A factor tree can be used to determine prime factors. This involves writing the number at the top of the page, finding any factor pair, and writing them below the number.

For example,



Repeat the process with any factors that are not prime numbers until you only have prime factors left.

For example,

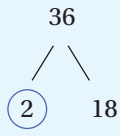
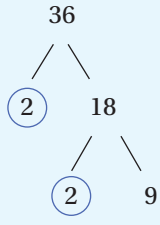


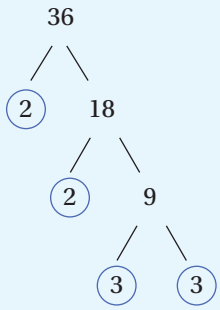
The diagram gets its name because it looks a little like a tree.

Worked example

Using a factor tree to determine the prime factors of a number

Use a factor tree to determine the prime factors of 36.

THINKING	WORKING
Recall any factor pair that does not include 1.	$36 = 2 \times 18$ 
Recall a factor pair for any of the non-prime factors that does not include 1. Continue until all factors listed are prime.	2 is a prime number. $18 = 2 \times 9$ 

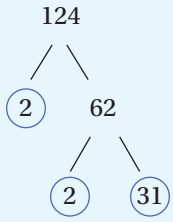
	2 is a prime number. $9 = 3 \times 3$ 
Write the answer.	The prime factors of 36 are 2 and 3.

SC 4 I can write a number as a product of its prime factors

Worked example

Writing the prime factorisation of a number in exponent form

Write 124 as a product of its prime factors. Express your answer in exponent form.

THINKING	WORKING
Use a factor ladder or tree to determine the prime factors of 124.	
Write the prime factors as a product.	$124 = 2 \times 2 \times 31$
Express your answer in exponent form.	$124 = 2^2 \times 31$

Practice

ANSWERS Page 668

SC 1 I can determine the prime factors of a number

1 List the factors of the following numbers.

(a) 10 **(b)** 12 **(c)** 14

(d) 20 **(e)** 24 **(f)** 36

2 Identify the prime factors from the list of factors given in Question **1**.

SC 2 I can use a factor ladder to determine the prime factors of a number

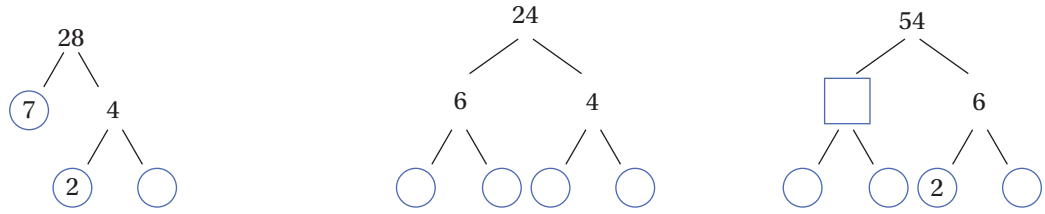
1 Use a factor ladder to determine the prime factors of:

(a) 16 **(b)** 28 **(c)** 50

SC 3 I can use a factor tree to determine the prime factors of a number

1 Copy and complete the following factor trees to determine the prime factors of the number given.

(a) 28 **(b)** 24 **(c)** 54



2 Use a tree diagram to determine the prime factors of:

(a) 16 **(b)** 28 **(c)** 50

3 Is there only one tree diagram that correctly finds the prime factors of every number? Use diagrams to explain your answer.

SC 4 I can write a number as a product of its prime factors

1 Express the following numbers as a product of their prime factors, then write them in exponent form.

(a) 16 **(b)** 28 **(c)** 50

(d) 75 **(e)** 196

2 In expanded form, Shayne wrote the prime factors of 32 as $2 \times 2 \times 2 \times 2 \times 2$. Shayne then tried to simplify this by writing it in exponent form as $32 = 2 \times 5$. What mistake has Shayne made?

3 Answer true or false for each of the statements below.

(a) The factors of 7 are 1 and 7.

(b) The prime factors of 7 are 1 and 7.

(c) The prime factors of 22 are 1, 2 and 11.

(d) 28 written as a product of its prime factors is $2 \times 2 \times 7$.

4 Detective Maths

Determine the number/s from these clues.

(a) I am less than 80 and the product of 3 different prime numbers. (Find all 3 possibilities.)

(b) I am greater than 10, but less than 150 and the product of one prime factor raised to the power of 3. (Find both possibilities.)

(c) I am less than 50 and the product of two prime factors, each raised to the power of 2.

Understand and identify common factors and multiples

1.4

Learning intention: To understand and be able to identify common factors and multiples

Success criteria:

SC 1 I can determine the highest common factor (HCF) of a pair of numbers.

SC 2 I can determine the lowest common multiple (LCM) of a pair of numbers.

SC 3 I can solve problems involving highest common factors and lowest common multiples.

Lesson warm-up

What's the meaning?

In pairs, brainstorm a definition and example of the following terms:

- Multiple
- Factor
- Common factor
- Common multiple

Give an example to help show your definition.

(a) multiples of 6

(b) factors of 20

(c) common factors of 6 and 20

(d) two common multiples of 6 and 20.

SC 1 I can determine the highest common factor (HCF) of a pair of numbers

Factors are numbers you can multiply together to get another number. A common factor is a number that divides exactly into two or more numbers.

The HCF, Highest Common Factor, is the largest common factor (the largest number common to both numbers).

For example,

the factors of 12 are 1, 2, 3, 4, 6, 12

the factors of 32 are 1, 2, 4, 8, 16, 32

The common factors are 1, 2 and 4.

The HCF is 4.

Worked example

Finding the highest common factor (HCF)

Determine the highest common factor (HCF) of 24 and 64.

THINKING	WORKING
List the factors of each number.	The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24. The factors of 64 are 1, 2, 4, 8, 16, 32, 64.
From the list of factors, identify the highest factor common to both lists.	24 → 1, 2, 3, 4, 6, 8, 12, 24 64 → 1, 2, 4, 8, 16, 32, 64
Write the answer.	The highest common factor (HCF) is 8.

SC 2 I can determine the lowest common multiple (LCM) of a pair of numbers

A multiple is a number that results from multiplying a certain integer (whole number) by another whole number.

For example, the multiples of 5 are 5, 10, 15, 20, 25, 30, ...

A common multiple is a multiple that two or more numbers share.

The Lowest Common Multiple (LCM) is the smallest number that is a multiple of two or more numbers.

Worked example

Finding the lowest common multiple (LCM)

Determine the lowest common multiple of 12 and 18.

THINKING	WORKING
List the first five multiples for each.	The first five multiples of 12 are 12, 24, 36, 48, 60, ... The first five multiples of 18 are 18, 36, 54, 72, 90, ...
From the list of multiples, identify the lowest multiple that is common to both lists.	12 → 12, 24, 36, 48, 60, ... 18 → 18, 36, 54, 72, 90, ...
Write the answer.	The lowest common multiple (LCM) is 36.

SC 3 I can solve problems involving highest common factors and lowest common multiples

Worked example

Solving problems using the LCM

Determine the smallest whole number which when divided by 2, 3, 4, 5 and 6 leaves a remainder of 1 each time.

THINKING	WORKING
Describe the steps needed to solve the problem.	Determine the lowest common multiple (LCM) of 2, 3, 4, 5 and 6, then add 1 to the result.
Determine the lowest common multiple of 2, 3, 4, 5 and 6.	Instead of listing multiples of each number, you can use your understanding of related multiples. For example, the lowest common multiple of 2, 3, 4 and 6 is 12. Then determine the lowest common multiple of 12 and 5. $12 \times 5 = 60$
State the lowest common multiple (LCM).	The LCM of 2, 3, 4, 5 and 6 is 60.
Add 1 to the result.	$60 + 1 = 61$
Check the reasonableness of your answer.	$61 \div 2 = 30$ r1 $\left(\text{or } 30\frac{1}{2}\right)$ $61 \div 3 = 20$ r1 $\left(\text{or } 20\frac{1}{3}\right)$ $61 \div 4 = 15$ r1 $\left(\text{or } 15\frac{1}{4}\right)$ $61 \div 5 = 12$ r1 $\left(\text{or } 12\frac{1}{5}\right)$ $61 \div 6 = 10$ r1 $\left(\text{or } 10\frac{1}{6}\right)$
Write the answer.	The smallest whole number which when divided by 2, 3, 4, 5 and 6 leaves a remainder of 1 each time is 61.

Worked example

Solving problems using the HCF

A gardener has 18 bean plants and 24 tomato plants. The plants will be planted in rows containing the same number of bean plants and tomato plants, using all the plants. How many rows will the gardener need to create and how many of each plant will be in a row?

THINKING	WORKING
Describe the steps needed to solve the problem.	Determine the highest common factor (HCF) of 18 and 24. This will give the number of rows. Then divide the number of plants equally into the rows.
List the factors of each number and identify the highest factor common to both lists.	18 → 1, 2, 3, 6, 9, 18 24 → 1, 2, 3, 4, 6, 8, 12, 24
Interpret the highest common factor (HCF).	HCF = 6 The number of rows is 6.
Determine the number of plants in each row.	The number of bean plants in each of the six rows is 18 ÷ 6 = 3. The number of tomato plants in each of the six rows is 24 ÷ 6 = 4.
Write the answer.	The gardener will need 6 rows with 3 bean plants and 4 tomato plants in each.

Practice

ANSWERS Page 669

SC 1 I can determine the highest common factor (HCF) of a pair of numbers

1 Determine the highest common factor (HCF) of 16 and 24.

- **(a)** List the factors of 16.
- **(b)** List the factors of 24.
- **(c)** List the common factors of 16 and 24.
- **(d)** Identify the highest common factor (HCF).

2 Determine the highest common factor (HCF) of 21 and 45.

- **(a)** List the factors of 21.
- **(b)** List the factors of 45.
- **(c)** List the common factors of 21 and 45.
- **(d)** Identify the highest common factor (HCF).

3 Determine the highest common factor (HCF) of 32, 48 and 56.

(a) List the factors of 32.

(b) List the factors of 48.

(c) List the factors of 56.

(d) List the common factors of 32, 48 and 56.

(e) Identify the highest common factor (HCF).

4 Determine the highest common factor (HCF) of the following numbers.

(a) 16 and 30 **(b)** 16 and 36

(c) 16, 24 and 96 **(d)** 18, 36 and 96

5 Determine the highest common factor (HCF) of 180 and 225.

(a) List the factors of 180.

(b) List the factors of 225.

(c) List the common factors of 180 and 225.

(d) Identify the highest common factor (HCF).

6 Two pipes are 12 m and 16 m long, respectively. They are to be cut into smaller pieces of equal length. Determine the maximum length of each piece.

7 Eggs from three different birds are collected before placing them into cartons for sale by type. The numbers of each egg type collected is 72, 108 and 132, respectively. What is the largest number of eggs that can be placed into each carton so that all cartons hold the same number of eggs?

SC 2 I can determine the lowest common multiple (LCM) of a pair of numbers

1 Determine the lowest common multiple (LCM) of 8 and 12.

(a) List the first 5 multiples of 8.

(b) List the first 5 multiples of 12.

(c) Identify the lowest common multiple (LCM).

2 Determine the lowest common multiple (LCM) of 6 and 5.

(a) List the first 6 multiples of 6.

(b) List the first 6 multiples of 5.

(c) Identify the lowest common multiple (LCM).

3 Determine the lowest common multiple (LCM) of 4, 6 and 8.

(a) List the first 12 multiples of 4.

(b) List the first 6 multiples of 8.

(c) List the first 8 multiples of 6.

(d) Identify the lowest common multiple (LCM).

4 Determine the lowest common multiple (LCM) of the following pairs of numbers.

(a) 8 and 10 **(b)** 6 and 15 **(c)** 10 and 12 **(d)** 9 and 12

5 A military sergeant tries to line up his troops for a parade in even rows of 2, 3, 5 and 6, but finds there is always one soldier left over. What is the smallest number of troops that could be on parade that day?

1

SC 3 I can solve problems involving highest common factors and lowest common multiples

1 Four bells are set to ring at intervals of 4, 7, 12 and 84 seconds. If they all ring at 1 pm, when will they next all ring at the same time?

2 Decorative plates are sold in packets of 10 and paper cups in packets of 25. You want to have the same number of plates and cups for a party. What is the minimum number of each packet that you need to buy?

3 From a standing start, three friends can jump 2 m, 3 m and 4 m respectively. After the first jump, the person who has travelled the least distance takes another jump. Again, the person who has travelled the least takes another jump while the others stay where they are. This pattern continues until all three friends have jumped the same distance.

How far are they from the starting place the next time they are together?

4 A gardener has 33 carrot plants and 18 tomato plants. She wishes to create rows containing the same number of carrot plants and tomato plants. She wishes to use all the plants. How many rows will she need and how many of each plant will there be in a row?

5 Three neighbours went on a picnic together with their families. They took a total of 60 sandwiches, 30 burgers and 40 cakes. Every person had an equal share of everything. How many people were at the picnic?

6 Determine the largest number that divides into 157 and 87, leaving 3 as a remainder each time.

7 A farmer has a rectangular paddock measuring 180 m by 150 m. The farmer wants to plant evenly spaced trees around all four sides of the paddock, including the corners.

(a) Calculate the greatest distance possible between the trees.

(b) Determine the minimum number of trees to be planted.

Exploration Task

Prime life cycles

Cicadas are insects known for their unique life cycles. Periodical cicadas emerge from the ground every 13 or 17 years.

Determine the main predators of cicadas.

Investigate why prime numbered cycles help cicadas survive.

What would happen if cicadas emerged every 12 years?

Understand and use expanded notation to represent numbers

1.5

Learning intention: To understand and be able to use expanded notation to represent numbers

Success criteria:

SC 1 I can write large powers of 10 in both expanded form and exponent form.

SC 2 I can write large numbers in expanded notation.

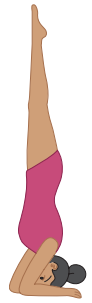
Lesson warm-up

Classroom jokes

Two members of the class are chatting and decide it would be funny to whisper a joke. This is the first round of the joke. The pair each tell two other people the joke. This is now the second round of the joke. If the people who were told the joke in the second round each tell two other people the joke, how many rounds would it be until the whole class knows?

How many people in total would know the joke at the tenth round?

Assume that no one is told the joke twice.



SC 1 I can write large powers of 10 in both expanded form and exponent form

Worked example

Writing large powers of 10 in expanded form and exponent form

(a) Write the number 100 in both expanded form and exponent form.

THINKING	WORKING
Identify the base number.	Base number: 10
Write the number in expanded form. Expanded form shows the base number multiplied by itself.	$10 \times 10 = 100$ 100 written in expanded form is: 10×10
Identify the number of times the base number appears in the product.	Two lots of 10 multiply to give 100.
Write the number in exponent form. The exponent shows the number of times the base number appears in the product. This is written as a superscript.	Base number: 10 Exponent (number of bases in the product): 2 Exponent form: 10^2
Write the answer.	Expanded form: 10×10 Exponent form: 10^2

(b) Write the number 100 000 in both expanded form and exponent form.

THINKING	WORKING
Identify the base number.	Base number: 10
Write the number in expanded form. Expanded form shows the base number multiplied by itself.	10 $10 \times 10 = 100$ $10 \times 10 \times 10 = 1000$ $10 \times 10 \times 10 \times 10 = 10\,000$ $10 \times 10 \times 10 \times 10 \times 10 = 100\,000$ 100 000 written in expanded form is: $10 \times 10 \times 10 \times 10 \times 10$
Identify the number of times the base number appears in the product.	10 is multiplied 5 times to give 100 000.
Write the number in exponent form. The exponent shows the number of times the base number appears in the product. This is written as a superscript.	Base number: 10 Exponent (number of bases in the product): 5 Exponent form: 10^5
Write the answer.	Expanded form: $10 \times 10 \times 10 \times 10 \times 10$ Exponent form: 10^5

SC 2 I can write large numbers in expanded notation

The value of each digit in a number depends on its place within the number.

For example, the digits 4, 7 and 0 can be used to create six different numbers, three of which are shown in the table below.

		Place value		
Number	**Expanded form**	**Hundreds**	**Tens**	**Ones**
470	400 + 70	4	7	0
407	400 + 7	4	0	7
047	40 + 7	0	4	7

Writing a number in expanded form shows the place value of each digit.

Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Ones
100 000	10 000	1000	100	10	1
10^5	10^4	10^3	10^2	10^1	10^0
$10 \times 10 \times 10 \times 10 \times 10$	$10 \times 10 \times 10 \times 10$	$10 \times 10 \times 10$	10×10	10	1

Worked example

Writing in expanded form using powers of 10

Write 5078 in expanded form using exponent notation.

THINKING	WORKING
Write the number in expanded form.	$5078 = 5000 + 70 + 8$
Rewrite the expanded form by multiplying each digit by a power of 10.	$5078 = 5 \times 1000 + 7 \times 10 + 8 \times 1$
Write each power of ten in exponent form. Recall that $1000 = 10^3$, $100 = 10^2$, $10 = 10^1$ and $1 = 10^0$.	$5078 = 5 \times (10 \times 10 \times 10) + 7 \times (10) + 8 \times (1)$ $= 5 \times 10^3 + 7 \times 10^1 + 8 \times 10^0$
Write the answer.	$5078 = 5 \times 10^3 + 7 \times 10^1 + 8 \times 10^0$

Practice

ANSWERS Page 669

SC 1 I can write large powers of ten in both expanded form and exponent form

1 Answer the following questions.

(a) Copy and complete this table.

Number	Expanded form	Exponent form
100 000	$10 \times 10 \times 10 \times 10 \times 10$	
10 000	$10 \times 10 \times 10 \times 10$	10^4
1000		
100		
10		

(b) Copy and complete this sentence.

The exponent is the same as the number of _____ in the number.

(c) The number of zeros decreases by one as you go down the column. Describe the pattern in the value of the exponent going down the column.

2 Write the following in exponent form with a base of 10.

(a) 1 000 000 **(b)** 10 000 000 **(c)** 1 000 000 000

3 Identify the mistakes made by students in answering these two questions.

(a) $10^4 = 4 \times 10 = 40$ **(b)** $500 = 5 \times 100 = 5 \times 10^2 = 50^2$

4 Prefixes are another way to express exponential numbers. For example, kilo is used to mean 1000 times. Copy and complete this table.

Prefix	Meaning	Expanded form	Exponent form
kilo	× 1000	10 × 10 × 10	10^3
mega	× 1 000 000		
giga	× 1 000 000 000		
tera	× 1 000 000 000 000		
peta	× 1 000 000 000 000 000		

5 The prefix exa means 10^{18}. How many zeros are there after the 1?

SC 2 I can write large numbers in expanded notation

1 Place value is shown in the table below. Copy and complete the table with numbers of different forms, using 10 as a base.

	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Ones
General form		10 000	1000			1
Exponent form	10^5				10^1	10^0
Expanded form				10 × 10	10	1

2 The number 879 is eight hundreds, seven tens and nine ones.

This can be written as $(8 \times 100) + (7 \times 10) + (9 \times 1)$ or in expanded form using exponent notation as $8 \times 10^2 + 7 \times 10^1 + 9 \times 10^0$.

Write the following numbers in expanded form using exponent notation.

(a) 82 **(b)** 826 **(c)** 8026 **(d)** 8206 **(e)** 8260

3 Write these numbers given in expanded notation in general form (as numbers).

(a) $3 \times 10^2 + 5 \times 10^1 + 2 \times 1$ **(b)** $3 \times 10^2 + 5 \times 10^1$

(c) $3 \times 10^2 + 2 \times 1$ **(d)** $4 \times 10^3 + 3 \times 10^2 + 5 \times 10^1 + 2 \times 1$

(e) $4 \times 10^3 + 5 \times 10^1 + 2 \times 1$ **(f)** $4 \times 10^3 + 3 \times 10^2 + 2 \times 1$

(g) $4 \times 10^3 + 2 \times 1$

4 What mistakes have been made in these answers?

(a) $4 \times 10^3 + 3 \times 10^2 + 2 \times 1 = 431$

(b) $3 \times 10^2 + 5 \times 10^1 = 35$

(c) $4 \times 10^3 + 5 \times 10^1 + 2 \times 1 = 4520$

5 Write these numbers in expanded form using exponent notation.

(a) four hundred and twenty-six

(b) five thousand two hundred and three

(c) fifteen thousand two hundred and seventeen

TOPIC REVIEW

Multiple choice

ANSWERS Page 735

1 Which of the following numbers are square numbers?

4, 8, 24, 36, 50

A 4, 8 and 36 **B** 4 and 36 **C** 8, 24 and 50 **D** All of them

2 Choose the number that completes this sentence.

There are _____ square numbers between 10 and 50.

A 1 **B** 2 **C** 3 **D** 4

3 Calculate the value of $\sqrt{16}$.

A 2 **B** 3 **C** 4 **D** 8

4 Calculate the value of $\sqrt{49}$.

A 5 **B** 6 **C** 7 **D** 8

5 $\sqrt{70}$ is between:

A 7 and 9 **B** 8 and 9 **C** 9 and 10 **D** 10 and 11

6 The number with a base of 3 and index of 2 is equal to:

A 7 **B** 8 **C** 9 **D** 10

7 $2 \times 2 \times 2 \times 10 \times 10 \times 10 \times 10 \times 10$, written in exponent notation is:

A 20^8 **B** $2^3 \times 10^5$ **C** $2^3 \times 10^4$ **D** 6×10^5

8 The common factors of 12 and 56 are:

A 2 and 4 **B** 1, 2 and 4 **C** 4 **D** 1, 2, 3 and 4

9 The lowest common multiple of 5 and 15 is:

A 3 **B** 5 **C** 15 **D** 30

10 The prime factors of 24 are:

A 2, 3, 4, 6, 12 and 24 **B** 3, 4, 6, 12 and 24 **C** 2, 3 and 4 **D** 2 and 3

11 $5 \times 10^3 =$

A 50 **B** 500 **C** 5000 **D** 50^3

12 $7^3 =$

A 21 **B** 3^7 **C** 249 **D** 343

Short answer

1 Show why 9 is considered a square number by using a diagram.

2 Explain why 10 is not a square number.

3 Write the square numbers between 20 and 101.

4 Calculate the following square roots.

(a) $\sqrt{4}$ **(b)** $\sqrt{64}$ **(c)** $\sqrt{81}$

(d) $\sqrt{144}$ **(e)** $\sqrt{289}$

5 Write the following in expanded form.

(a) 4^3 **(b)** y^4

6 Determine the lowest common multiple of 8 and 12.

7 Determine the highest common factors of:

(a) 14 and 35 **(b)** 24 and 33

(c) 16 and 64 **(d)** 8, 28 and 52

8 Determine the prime factors of the following numbers.

(a) 14 **(b)** 24

(c) 63 **(d)** 84

9 Write the following in exponent form with a base of 10.

(a) 1000 **(b)** 1 000 000 **(c)** 100

10 Write these expanded notations as numbers.

(a) $5 \times 10^2 + 2 \times 10^1 + 4 \times 1$

(b) $6 \times 10^2 + 5 \times 10^1$

(c) $7 \times 10^2 + 4 \times 1$

(d) $6 \times 10^3 + 3 \times 10^2 + 5 \times 10^1 + 2 \times 1$

(e) $6 \times 10^3 + 4 \times 10^1 + 5 \times 1$

(f) $6 \times 10^3 + 3 \times 10^2 + 5 \times 1$

(g) $6 \times 10^3 + 5 \times 1$

Extended response

1 A music class is divided into 3 groups.

Group A clap on the beat, but skip every 3rd beat.

Group B clap on the beat, but skip every 4th beat.

Group C clap on the beat, but skip every 5th beat.

Every beat takes a second.

(a) How many times during the first 19 beats does group A not clap?

(b) How many times during the first 19 beats does group B not clap?

(c) How many times during the first 19 beats does group C not clap?

(d) How many times is only one group clapping in the first 19 seconds?

(e) When is the first time all three groups miss at the same time? It is longer than 19 seconds. (You and two friends could try this out, with your teacher's permission of course.)

2 Write the number 64 as a product of:

(a) 2 factors (this can be done 4 different ways, find all of them)

(b) 3 factors greater than 1

(c) 3 perfect squares

(d) 6 prime numbers

3 The numbers 72, 24, 10, 5, 36 and 15 are grouped into pairs that all have the same product.

Which number is paired with 15?

4 Jimmy is very superstitious and will not buy a home where the house number is divisible by the numbers 2, 3 or 5. His wife Jemima does not want to live in a house with a 1, 3 or 5 in the address.

The real estate agent wants to show them houses numbered from 20 to 120 inclusive.

(a) How many houses can Jimmy choose from?

(b) How many houses can Jemima choose from?

(c) How many houses are available to both Jimmy and Jemima to choose from?

5 Using the digits 1, 3 and 5 once each, how many 3-digit prime numbers can you create? Can you explain why this is the case?

TOPIC

2

Operations with decimals

Recall 33

2.1 Round decimals for different purposes 34

2.2 Add and subtract decimals 41

2.3 Multiply decimals 47

2.4 Divide decimals 52

Topic review 57

Why learn this?

Rounding decimals is a practical skill that simplifies numbers, making them easier to understand and apply. This topic provides:

- methods to round decimal values to a specified number of decimal places, which helps to make measurements and financial calculations more precise
- techniques for visualising decimal rounding to support other skills such as estimating
- strategies to determine the appropriate level of accuracy for various contexts, such as scientific data, currency transactions and everyday measurements.

These skills help us to make quick, informed decisions in areas like shopping, budgeting and scientific experiments, where exact figures are not always practical. The ability to round effectively is also useful in more complex mathematical operations and problem-solving scenarios found in various fields, from engineering to economics.

RECALL

ANSWERS Page 670

I can write decimals given their place value

1 Write the following as decimal numbers:

(a) five hundredths **(b)** three thousandths

(c) fifteen tenths **(d)** twenty-five thousandths

I can partition decimals to describe them using place values

1 Partition these decimals into sums of their place values.

(a) 1.47 **(b)** 3.02 **(c)** 2.495 **(d)** 4.602

I can perform operations with whole numbers

1 Calculate the following additions.

(a) 15 + 9 **(b)** 17 + 12 **(c)** 26 + 37

(d) 128 + 93 **(e)** 235 + 827 **(f)** 1268 + 3254

2 Calculate the following subtractions.

(a) 15 – 9 **(b)** 17 – 12 **(c)** 37 – 26

(d) 128 – 93 **(e)** 827 – 235 **(f)** 3254 – 1268

3 Calculate the following multiplications.

(a) 2×8 **(b)** 3×12 **(c)** 10×14

(d) 15×16 **(e)** 20×13 **(f)** 25×9

4 Calculate the following divisions.

(a) $18 \div 6$ **(b)** $66 \div 11$ **(c)** $50 \div 10$

(d) $144 \div 6$ **(e)** $515 \div 5$ **(f)** $3004 \div 4$

I can round whole numbers to the nearest 10, 100 and 1000

1 Round the following numbers to the nearest 10.

(a) 76 **(b)** 34 **(c)** 55

2 Round the following numbers to the nearest 100.

(a) 104 **(b)** 188 **(c)** 150

3 Round the following numbers to the nearest 1000.

(a) 5623 **(b)** 13 348 **(c)** 1772

2.1 Round decimals for different purposes

Learning intention: To be able to round decimals for different purposes

Success criteria:

SC 1 I can round decimal values to any stated number of decimal places.

SC 2 I can round decimals on a number line.

SC 3 I can choose an appropriate level of accuracy in practical situations.

Lesson warm-up

Counting to one million

Estimate the time it would take you to count to 1 million by counting by ones out loud.

Explain the reasoning behind your estimate.

SC 1 I can round decimal values to any stated number of decimal places

Rounding decimals

A digit in the range 0–4: the number is rounded down, which means the digits after that place are removed.

A digit in the range 5–9: the number is increased by one, and the rest of the digits are removed.

For example, cutting a piece of wood to 2.3457 m long requires knowing how to measure with accuracy. In cases like this, the number is rounded off to some specified level of accuracy.

For example, rounding 2.3457 to each of the following:

Problem	Thinking	Rounded number
3 decimal places (or correct to 3 decimal places)	The fourth digit after the decimal place is in the range 5–9, round up.	2. 3 4 5 $\underline{7}$ = 2. 3 4 6 (3 d.p.)
2 decimal places	The third digit after the decimal place is in the range 5–9, round up.	2. 3 4 $\underline{5}$ 7 = 2. 3 5 (2 d.p.)
1 decimal place	The second digit after the decimal point is in the range 0–4, round down.	2.3 $\underline{4}$ 5 7 = 2. 3 (1 d.p.)
To the nearest whole number	The first digit after the decimal place is in the range 0–4, round down.	2. $\underline{3}$ 4 5 7 ≈ 2 (to the nearest whole number)

Worked example

Rounding to a stated number of decimal places

Round each of the following values to the number of decimal places indicated in brackets.

(a) 16.237 99 (1)

THINKING	WORKING
Recall the rule for rounding to the required number of decimal places.	16.237 99 is being rounded to 1 decimal place. To do this, examine the second digit from the decimal point.
Identify the value immediately to the right of the decimal place of interest.	16.2<u>3</u>7 99 The second digit from the decimal point is 3.
Recall the rule when examining the digit to the right and determine whether the number is rounded up or rounded down.	The second digit is in the range 0–4, which means the number is rounded down.
Write the answer.	16.237 99 = 16.2 (1 d.p.)

(b) 16.237 99 (2)

THINKING	WORKING
Recall the rule for rounding to the required number of decimal places.	16.237 99 is being rounded to 2 decimal places. To do this, examine the third digit from the decimal point.
Identify the value immediately to the right of the decimal place of interest.	16.23<u>7</u> 99 The third digit from the decimal point is 7.
Recall the rule when examining the digit to the right and determine whether the number is rounded up or rounded down.	The third digit is in the range 5–9, which means the number is rounded up.
Write the answer.	16.237 99 = 16.24 (2 d.p.)

(c) 16.237 99 (4)

THINKING	WORKING
Recall the rule for rounding to the required number of decimal places.	16.237 99 is being rounded to 4 decimal places. To do this, examine the fifth digit from the decimal point.
Identify the value immediately to the right of the decimal place of interest.	16.237 99 The fifth digit from the decimal point is 9.
Recall the rule when examining the digit to the right and determine whether the number is rounded up or rounded down.	The third digit is in the range 5–9, which means the number is rounded up.
To round a digit of 9 up, add a digit of 1 to the 9 digit.	16.2379 + 0.0001 = 16.2380
Write the answer.	16.237 99 = 16.2380 (4 d.p.)

SC 2 I can round decimals on a number line

The diagram below represents the decimal number 3156.2748.

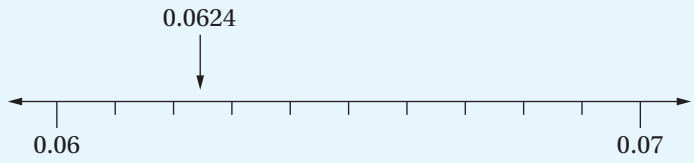
Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten-thousandths
1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$
3	1	5	6	2	7	4	8

The place value table serves as a reminder of the names used for each place in a decimal number. Sometimes, the rounding instructions are given as something like 'round to the nearest thousandth'. Rounding to the nearest thousandth is the same as rounding to three decimal places.

Worked example

Rounding decimals on a number line

Round 0.0624 to three decimal places and position it on a number line.

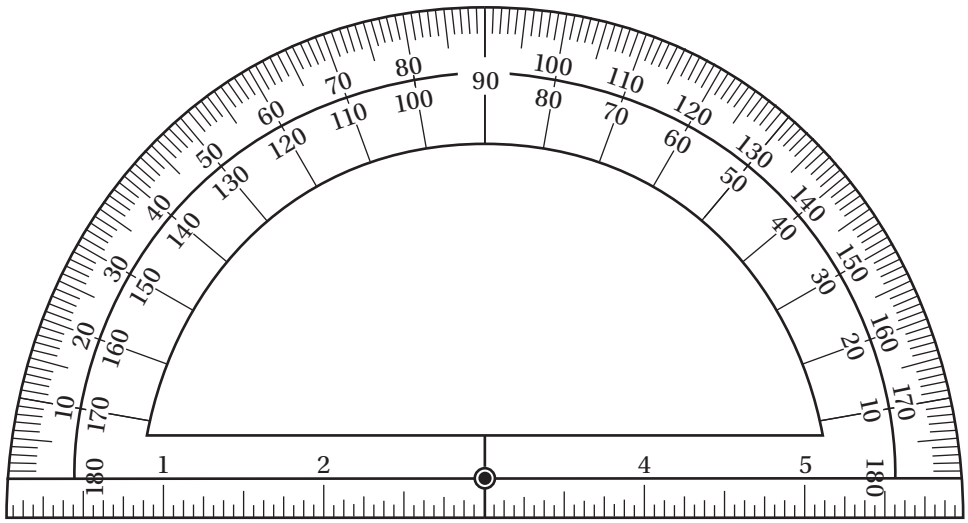
THINKING	WORKING
Determine the interval in which the number is located.	The number is located between 0.062 and 0.063.
Construct the number line between the interval and position the number.	
Determine which end of the interval the number is closer to.	0.0624 is closer to 0.062 than to 0.063, so 0.0624 = 0.062 (3 d.p.).

SC 3 I can choose an appropriate level of accuracy in practical situations

Worked example

Finding the accuracy of a measuring device

Consider the protractor shown below.



(a) What is the gap between labelled values on this protractor?

THINKING	WORKING
Use the protractor and read off consecutive marked values to determine the gap between them.	There is a 10° gap between marked values.

(b) What does each small mark between the labelled points represent?

THINKING	WORKING
Identify how many marks there are between the labelled points.	There are 9 small marks between the marked values, so each mark represents 1°.

(c) To what accuracy could you measure an angle using this protractor?

THINKING	WORKING
Determine whether an estimate of any angle between the small marks would be accurate.	It is not realistic to accurately estimate anything between the small marks, so the protractor can only measure angles correct to the nearest degree.

Practice

ANSWERS Page 670

SC 1 I can round decimal values to any stated number of decimal places

1 Round each of the following numbers to the number of decimal places indicated in brackets.

(a) 4.1025 (2) **(b)** 6.247 38 (3) **(c)** 11.667 (1) **(d)** 15.989 27 (4)

2 Round each of the following numbers to the number of decimal places indicated in brackets.

(a) 14.3065 (1) **(b)** 21.4035 (2) **(c)** 121.9892 (1) **(d)** 15.999 (2)

3 In each instance, determine which student has answered the question correctly when rounding the number to the number of decimal places in brackets.

(a) 5.2998 (1)

Student A	Student B
5.2998 (1) The first digit after the decimal point is in the range 0–4, so the number is rounded down. 5.2998 = 5.2 (1 d.p.)	5.2998 (1) The second digit after the decimal point is in the range 5–9, so the number is rounded up. 5.2998 = 5.3 (1 d.p.)

(b) 5.2998 (2)

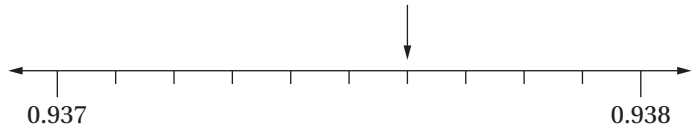
Student A	Student B
5.2998 (2) The third digit after the decimal point is in the range 5–9, so the number is rounded up. 5.2998 = 5.210 (3 d.p.) = 5.21 (2 d.p.)	5.2998 (2) The third digit after the decimal point is in the range 5–9, so the number is rounded up. 5.2998 = 5.30 (2 d.p.)

SC 2 I can round decimals on a number line

1 Draw separate number lines to show the location of the following decimals, rounded to the number of decimal places shown in brackets.

(a) 0.10853 (3) **(b)** 0.230076 (4)

2 **(a)** Which option best represents the value indicated on the number line shown?



A 0.9367 **B** 0.9376 **C** 0.9386 **D** 0.9368

(b) Which option best describes the degree of accuracy shown for the value?

A Hundredths **B** Thousandths

C Ten-thousandths **D** Hundred-thousandths

3 Round 23.109 082 to the following degrees of accuracy.

(a) The nearest hundredth **(b)** The nearest ten-thousandth

(c) The nearest whole number

4 Round each of the following numbers to the nearest hundredth.

(a) 1.247 **(b)** 2.0138 **(c)** 0.0989 **(d)** 8.004 999

5 Minjee is training for a long-distance run. Over the course of a seven-day training period she ran a total of 146 km.

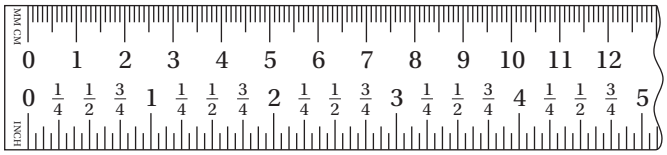
(a) Divide the total distance by 7 to calculate the average number of kilometres Minjee ran each day. What is it?

(b) Round this value to two decimal places. What is it?

(c) Is this now recorded as the nearest metre, tens of metres or hundreds of metres?

SC 3 I can choose an appropriate level of accuracy in practical situations

1 Consider the ruler shown in the image below.



(a) What is the smallest length that can measured on the metric side of this ruler?

(b) What is the smallest length that can be measured on the imperial (lower) side of this ruler?

(c) What would be the closest you could measure something that was actually 9.83 cm in length?

(d) What would be the closest you could measure something that was actually $3\frac{3}{32}$ inches in length?

2 There are many units that can be used to measure time. Choose the accuracy that is most likely used in each of the following situations.

(a) The time taken for a lunar mission to reach the moon

A Hours **B** Days **C** Weeks **D** Years

(b) The time taken to win the 100 m sprint at the Olympic Games

A Hundredths of seconds **B** Tenths of seconds

C Seconds **D** Minutes

(c) The time taken to complete the marathon at the Olympic Games

A Seconds **B** Minutes **C** Hours **D** Days

(d) The time taken for the Earth to complete one revolution of the Sun

A Weeks **B** Days **C** Years **D** Decades

3 Consider the image below taken from Google Maps.

8 hr 52 min (878.6 km) via National Highway M31 and M31

(a) To what level of accuracy has the expected time for this journey been measured?

(b) To what level of accuracy has the distance for this journey been measured?

(c) Do you think these estimates are reasonable? Write a paragraph explaining your point of view.

4 The following table shows the return on an investment.

Amount at end of half-year	Interest for the half-year
\$5000	\$375
\$5375	\$403.125
\$5778.125	\$433.359 375
\$6211.484 375	\$465.861 328 1
\$6677.345 703	

(a) How much was in the account at the start?

(b) How much was in the account at the start of the second year?

(c) How long, to the half-year, did it take to have at least \$6300 in the account?

(d) Did you round up or round down to find this last answer? Explain why this is a little unusual.

5 Babar was out to dinner with two friends. They decided to split the bill evenly. Babar calculated that each share of the bill would be \$84.333 333 3.

(a) Given that Babar was using cash, how much does he need to contribute?

(b) To what level of accuracy is it rounded to find Babar's payment?

(c) Is there a problem with rounding the bill like this? Explain in a sentence what you are thinking.

6 Write three different numbers that would round to each of the following numbers.

(a) 1.6

(b) 1.64

(c) 1.647

Add and subtract decimals

2.2

Learning intention: To be able to add and subtract decimals

Success criteria:

SC 1 I can model decimal addition and subtraction.

SC 2 I can add and subtract decimals with the same number of decimal places.

SC 3 I can add and subtract decimals with different numbers of decimal places.

Lesson warm-up

A decimat can be used to model a decimal number.

The rectangular decimat represents a single unit.

The decimat is divided into ten smaller rectangles; each rectangle represents a tenth.

The top left rectangle is divided into ten small squares; each small square represents a hundredth.

The image below shows the decimal 0.23; it is the shaded portion of the decimal. It doesn't matter which of the rectangles are shaded, so long as two of the larger ones and three of the smaller ones are shaded.

Using a piece of grid paper, draw decimats to represent the decimal numbers:

(a) 0.25 **(b)** 0.57

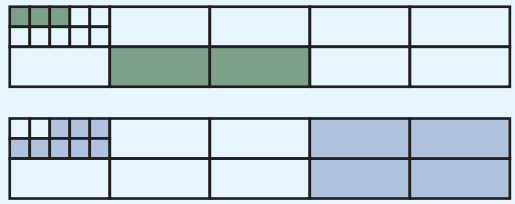
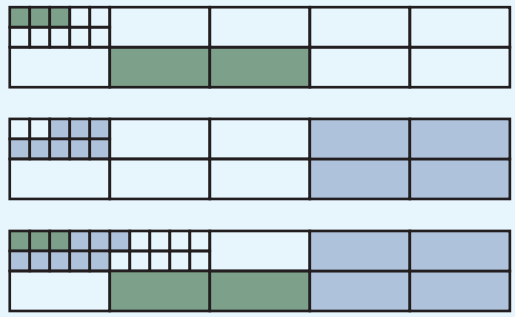
SC 1 I can model decimal addition and subtraction

When adding decimals on decimats, you might find that you shade 10 smaller rectangles, which is equivalent to a larger rectangle. With subtraction, you need to have enough of the smallest pieces on the top decimat so that the number of smaller pieces in the lower decimat can be subtracted. To do this, you will need to convert one of the larger rectangles into the smaller size.

Worked example

Using a decimat to model addition of decimals

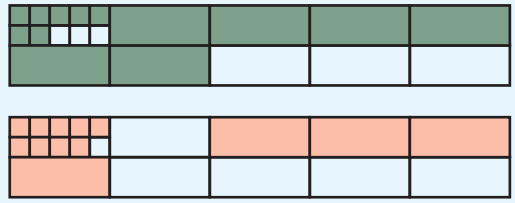
Use decimats to model 0.23 + 0.48.

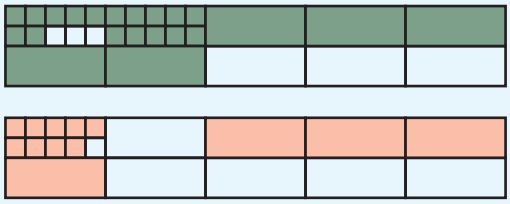
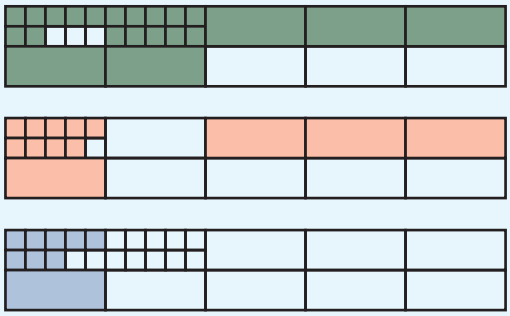
THINKING	WORKING
Create two decimats showing each of the fractions. Put them one above the other.	
When the two images are added together, how many little rectangles would need to be shaded in total?	There are 11 little rectangles.
Can this number be shaded inside one of the bigger rectangles? If not, what can be done?	No, only 10 little rectangles fit inside the bigger rectangle. This means another of the larger rectangles needs to be divided into 10 equal pieces.
Demonstrate the addition and its result on a new decimat.	
Determine the result by counting the number of rectangles that are shaded.	0.23 + 0.48 = 0.71

Worked example

Using a decimat to model subtraction of decimals

Use decimats to model 0.67 – 0.49.

THINKING	WORKING
Use two decimats, each showing one of the decimals. Put them one above the other.	

Decide whether there are a sufficient number of smaller shaded squares in the top decimat to enable the subtraction to take place. If not, convert one of the larger rectangles.	There are not enough smaller rectangles, so convert one of the larger rectangles. 
Solve the subtraction and show its result on a new grid.	
Calculate the result of the subtraction by counting the number of larger rectangles that are shaded and the number of smaller rectangles.	$0.67 - 0.49 = 0.18$

SC 2 I can add and subtract decimals with the same number of decimal places

When the number of decimal places is the same, addition and subtraction are exactly the same as the addition and subtraction of whole numbers. Line up the decimal points and perform the operation. When subtracting, remember it is the top digit minus the bottom digit, no matter which digit is larger. Borrowing may be required to complete the operation.

Worked example

Adding decimals with the same number of decimal places

Calculate $3.67 + 2.55 + 12.89$.

THINKING	WORKING
Represent the calculation in a vertical format.	$\begin{array}{r} 3.67 \\ + \quad 2.55 \\ + \quad 12.89 \end{array}$
Calculate the addition, carrying as appropriate.	$\begin{array}{r} {\scriptstyle 2\ 2\ \ } \\ 3.67 \\ + \ 2.55 \\ + 12.89 \\ \hline 19.11 \end{array}$
Check that the answer is reasonable by rounding each decimal to the nearest whole number and adding them.	$3.67 + 2.55 + 12.89 \approx 4 + 3 + 13 = 20$, so the answer is reasonable.

SC 3 I can add and subtract decimals with different numbers of decimal places

The key to successfully adding decimal values is to make sure the decimal points are aligned. Then fill in any spaces with zeros and add as you would in a whole number calculation. The decimal point in the answer will be aligned with the other decimal points.

Worked example

Adding decimals with different numbers of decimal places

Calculate 2.312 + 14.23 + 16.9.

THINKING	WORKING
Represent the calculation in a vertical format and fill in any gaps with zeros.	$\begin{array}{r} 02.312 \\ +\ 14.230 \\ +\ \underline{16.900} \end{array}$
Calculate the addition, carrying as appropriate.	$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ \ \ } \\ 02.312 \\ +\ 14.230 \\ +\ \underline{16.900} \\ 33.442 \end{array}$
Check that the answer is reasonable by rounding each decimal to the nearest whole number and adding them.	$2.312 + 14.23 + 16.9 \approx 2 + 14 + 17$ $= 33$ The answer is reasonable.

Worked example

Subtracting decimals with different numbers of decimal places

To subtract decimals where the number of decimal places is different, fill out any missing digits with zeros and then subtract as you would with whole numbers. The decimal point stays aligned with its current position.

Calculate 5.34 – 2.067.

THINKING	WORKING
Represent the calculation in a vertical format and fill in any gaps with zeros.	$\begin{array}{r} 5.340 \\ -\ \underline{2.067} \end{array}$
Calculate the addition, carrying as appropriate.	$\begin{array}{r} {\scriptstyle 2\ 3\ \ 1\ \ } \\ 5.\not{3}\not{4}0 \\ -\ \underline{2.067} \\ 3.273 \end{array}$
Check that the answer is reasonable by rounding to one decimal place.	$5.34 - 2.067 \approx 5.3 - 2.1$ $= 3.2$ The answer is reasonable.

Practice

ANSWERS Page 671

SC 1 I can model decimal addition and subtraction

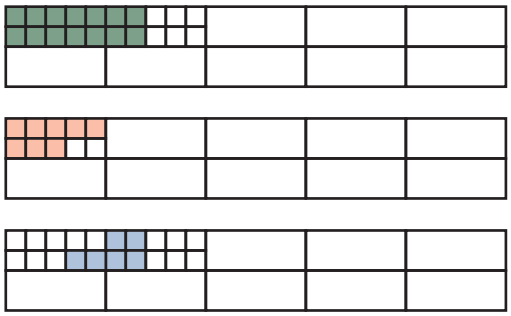
1 Use decimats to model the following additions.

(a) 0.35 + 0.47 **(b)** 0.49 + 0.38

2 Use decimats to model the following subtractions.

(a) 0.95 – 0.43 **(b)** 0.86 – 0.54 **(c)** 0.43 – 0.26 **(d)** 0.55 – 0.47

3 Choose the best option for the following.
What calculation is shown in the following decimat?



(a) 0.1 – 0.08 = 0.02 **(b)** 0.04 – 0.00 = 0.04

(c) 0.14 – 0.08 = 0.06 **(d)** 1.4 – 0.08 = 0.06

SC 2 I can add and subtract decimals with the same number of decimal places

1 Calculate the following.

(a) 9.5 + 10.3 + 8.6 **(b)** 8.08 + 12.07 + 3.09

(c) 3.023 + 14.987 + 21.008 + 16.901 **(d)** 42.1234 + 21.5678 + 33.0009 + 15.0608

2 Calculate the following.

(a) 9.89 – 4.63 **(b)** 9.04 – 7.02 **(c)** 15.68 – 4.36 **(d)** 22.93 – 11.81

3 Calculate the following.

(a) 10.04 – 7.23 **(b)** 8.06 – 4.29 **(c)** 12.37 – 8.58 **(d)** 15.003 – 7.996

4 Choose the best option for each of the following.

(a) 6.5 + 8.6

A 2.1 **B** 14.1 **C** 14.11 **D** 15.1

(b) 7.9 – 6.1

A 1.8 **B** 8.1 **C** 8.3 **D** 17.0

(c) 12.3 – 9.6

A 3.3 **B** 2.7 **C** 3.9 **D** 21.9

(d) 4.6 – 2.3 + 6.1

A –3.8 **B** 7.7 **C** 8.4 **D** 13.0

SC 3 I can add and subtract decimals with different numbers of decimal places

1 Calculate the following.

(a) 31.24 + 6.093 + 0.5 **(b)** 6.04 + 12.7 + 0.267

(c) 11.24 + 12.7 + 15.909 **(d)** 26.1 + 15.09 + 3.214

2 Calculate the following.

(a) 9.203 – 1.86 **(b)** 10.24 – 3.719

(c) 2.004 – 1.23 **(d)** 6.35 – 4.292

3 Calculate the following.

(a) 4.0003 – 2.7 **(b)** 8.06 – 4.099

(c) 11.703 – 7.9986 **(d)** 15.1 – 7.8964

4 Choose the best option for each of the following.

(a) 8.634 – 2.95

A 6.324 **B** 5.684 **C** 6.384 **D** 6.676

(b) 600 – 243.809

A 357.809 **B** 443.809 **C** 356.191 **D** 443.191

(c) 3.63 – 1.00987

A 2.62013 **B** 2.63987 **C** 2.63 **D** 4.63987

5 Complete the appropriate decimal calculations to answer each of the following.

(a) During the first week of March 2022, Ballina received a total of 171.2 mm of rain. The individual totals for the first six days were 60.4, 2.0, 5.0, 3.0, 31.0 and 3.2 mm. What was the amount of rainfall on the seventh day?

(b) Write any three decimal numbers, each with three decimal places, that add together to give 6.84.

(c) In the Olympic Games, the marathon is run over a distance of 42.195 km. How far does Dimitri still have to run after he has completed 28.24 km?

6 One of the questions on the Decimals topic test was 2.046 – 1.957.

(a) Estelle gave the answer as 1.911. What do you think she did to get this answer?

(b) How could you use rounding to convince Estelle that she was incorrect?

(c) What is the correct answer?

Multiply decimals

2.3

Learning intention: To be able to multiply decimals

Success criteria:

SC 1 I can model multiplication of decimals by integers on a number line.

SC 2 I can multiply decimals by integers.

SC 3 I can multiply decimals by powers of 10.

SC 4 I can multiply decimals by decimals.

Lesson warm-up

Multiply some of these numbers together to try to get an answer of 2.4.
Is there more than one way to do this? Try multiplying without a calculator.

0.3	0.8	1.5	6	0.2	1.6	0.5
8	3	0.4	0.6	4.8	10	0.24

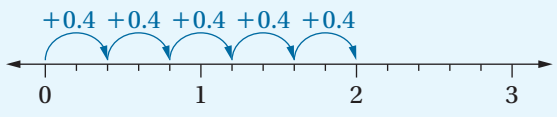
SC 1 I can model multiplication of decimals by integers on a number line

As with the multiplication of whole numbers, multiplication of a decimal by an integer can be modelled using the idea of repeated addition. The multiplication 0.4×5 can be thought of as 'five lots of 0.4' or $0.4 + 0.4 + 0.4 + 0.4 + 0.4$.

Worked example

Using a number line to model multiplication of a decimal by an integer

Use a number line to model 0.4×5 and then state the answer.

THINKING	WORKING
Construct a number line and show the repeated addition that matches the question.	$0.4 \times 5 = 0.4 + 0.4 + 0.4 + 0.4 + 0.4$ 
Write the answer.	$0.4 \times 5 = 0.4 + 0.4 + 0.4 + 0.4 + 0.4$ $= 2.0$

SC 2 I can multiply decimals by integers

Worked example

Multiplying a decimal by an integer

Calculate 2.7×4.

THINKING	WORKING
Estimate the value of the calculation.	The answer will be bigger than $2 \times 4 = 8$, but smaller than $3 \times 4 = 12$.
Perform the calculation as you would for whole number values.	$\begin{array}{r} {}^{2}27 \\ \times\ 4 \\ \hline 108 \end{array}$
Count the number of digits there are in total after the decimal point in the question.	There is 1 digit after the decimal point in the question.
Write the answer. Check the reasonableness of the answer.	$27 \times 4 = 108$ so $2.7 \times 4 = 10.8$. The answer is reasonable because it is greater than 8 and less than 12.

SC 3 I can multiply decimals by powers of 10

One of the simplest and most useful skills in decimal multiplication is multiplying by powers of 10.

Consider the following calculations:

$2.1345 \times 10 = 21.2345$, $2.12345 \times 100 = 212.345$ and $2.12345 \times 1000 = 2123.45$

The results all contain the same digits but the digits have different place values.

Worked example

Multiplying a decimal by powers of 10

Calculate 26.14×60.

THINKING	WORKING
Represent the calculation as a multiplication by a power of 10.	$26.14 \times 60 = 26.14 \times 6 \times 10$
Calculate the first multiplication.	$= 156.84 \times 10$
Multiply by the power of 10.	$= 1568.4$
Check the reasonableness of the answer.	$26.14 \times 60 \approx 26 \times 60$ $= 26 \times 6 \times 10$ $= 156 \times 10$ $= 1560$ The answer seems reasonable.

SC 4 I can multiply decimals by decimals

Consider the following calculations:

$12 \times 9 = 108$
$12 \times 0.9 = 10.8$

Based on these calculations, what is the result for 1.2×0.9?

Using strategies from previous sections, a simple approximation for 1.2×0.9 is $1.2 \times 1 = 1.2$ and the digits in the answer should be the same as those in the results above. So it is reasonable to suggest that $1.2 \times 0.9 = 1.08$.

Worked example

Multiplying a decimal by a decimal

Calculate the value of 3.5×1.4.

THINKING	WORKING
Estimate the answer.	The answer will be greater than $3.5 \times 1 = 3.5$, but less than $3.5 \times 2 = 7$.
Calculate as you would for whole number values.	$\begin{array}{r} 35 \\ \times\ 14 \\ \hline 140 \\ +350 \\ \hline 490 \end{array}$
Count the number of digits there are in total after the decimal point in the question.	There are $1 + 1 = 2$ digits after the decimal points.
Write the answer. The answer should contain the same number of digits after the decimal place as the question. Check the reasonableness of the answer against your estimate.	$3.5 \times 1.4 = 4.90$ This could be written as $3.5 \times 1.4 = 4.9$ because the trailing zero can be removed. The answer is reasonable because it is greater than 3.5 and less than 7.

Practice

ANSWERS Page 671

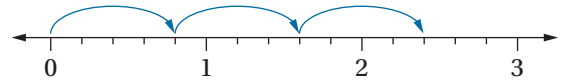
SC 1 I can model multiplication of decimals by integers on a number line

1 Use number lines to model each of the following multiplications and state the result.

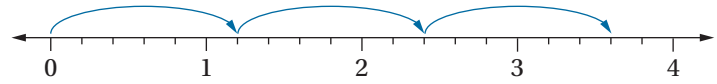
(a) 0.7×3 **(b)** 0.5×8 **(c)** 0.2×7 **(d)** 0.3×6

2 For each of the following, state the multiplication that is shown, and the result of that multiplication.

(a)



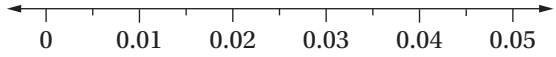
(b)



3 Use separate number lines to model each of the following multiplications and state the result.

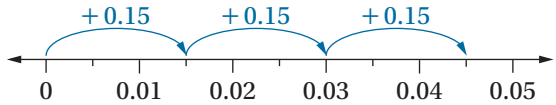
(a) 0.25×3 **(b)** 0.35×4

4 Consider the following number line.



(a) What value do each of the smallest intervals represent?

(b) What multiplication is represented by the following number line, and what is its result?



SC 2 I can multiply decimals by integers

1 For each of the following multiplications, identify the range within which the answer must occur and give the actual result.

(a) 3.6×5 **(b)** 2.95×11 **(c)** 3.615×3 **(d)** 8.1111×10

2 Determine the answer to each of the following.

(a) Apples cost \$3.25/kg. Determine the cost of 6 kg.

(b) A taxi fare consists of a fixed cost of \$3.90 plus \$2.12/km. What is the total fare for a journey of 25 km?

(c) Jorge can run 100 m in 10.35 s. At this rate, how long would it take him to run 1500 m?

(d) In one hour, 13.3 mm of rain fell. This rate continued for 12 hours. How much rain fell in total?

SC 3 I can multiply decimals by powers of 10

1 Evaluate each of the following.

(a) 26.2482×1000 **(b)** 108.63×70

(c) 1.2069×3000 **(d)** $0.482\,03 \times 500$

2 The decimal 14.037 is multiplied by what integer to give a result of 9825.9?

3 Identify three different integers that when multiplied by 9.27 give an answer between 30 000 and 35 000.

SC 4 I can multiply decimals by decimals

1 Evaluate each of the following.

(a) 6.2×1.7 **(b)** 8.5×0.6 **(c)** 2.4×8.1 **(d)** 7.3×7.3

2 Evaluate each of the following.

(a) 3.2×1.04 (b) 2.31×1.01 **(c)** 4.2346×1.2 **(d)** 8.6303×1.9

3 Identify the number of decimal places in the answers for each of the following decimal multiplications.

(a) 0.034×0.26 **(b)** 12.2006×3.95

(c) 11.6204×9.0005 **(d)** 108.26×109.04

4 One of the multiple-choice questions on the Decimals test was "Evaluate 2.605×11.44" and the options provided were:

(a) 29.8012 **(b)** 29.80123 **(c)** 29.801 **(d)** 29.8

Derek chose **(b)** 29.80123. It was the only option that had $3 + 2 = 5$ digits after the decimal point. Derek was surprised to discover his answer was wrong. Explain why he was wrong and state the correct answer.

5 Daria went to the petrol station and found that ULP was \$1.99/litre. Daria needed 62.4 L to fill the tank.

(a) How much would the petrol cost Daria? Give an exact answer.

(b) Daria paid using cash. How much was she charged?

(c) How is the cash price rounded?

6 At the supermarket's delicatessen counter, Padraig's favourite cheese costs \$32.55/kg.

(a) Padraig indicates to the server the size of the piece he wants and when put on the scales, it shows as 223 g. How much will this piece of cheese cost Padraig? Give your answer to the nearest 5 cents.

(b) Padraig was prepared to pay up to \$10 for the cheese. What is the greatest whole number of grams he could buy for this price?

2.4 Divide decimals

Learning intention: To be able to divide decimals

Success criteria:

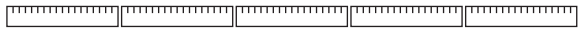
SC 1 I can model division of decimals by integers on a number line.

SC 2 I can divide decimals by integers.

SC 3 I can divide decimals by powers of 10.

SC 4 I can divide decimals by decimals.

Lesson warm-up



Estimate how many 30 cm rulers you could fit across the room, end to end.

Measure the width of the room in metres using a long measuring tape.

You are going to calculate: How many lots of 30 cm are there in ____ metres?

Rewrite this question using a division symbol (÷).

Calculate the number of rulers.

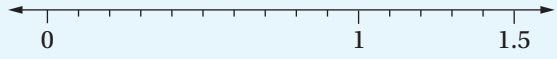
SC 1 I can model division of decimals by integers on a number line

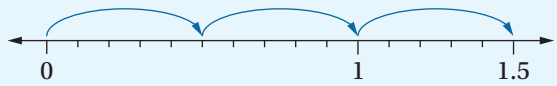
Worked example

Using a number line to model the division of a decimal by an integer

A number line can be used to model the division of a decimal by a whole number.

Use a number line to model 1.5 ÷ 3 and state the result.

THINKING	WORKING
Construct a number line showing scale markings at 0.1 intervals with the number being divided (the dividend) clearly marked.	
Count the number of scale marks between 0 and the end point.	There are 15 scale marks.

Calculate by dividing the number of scale marks by the number on the right of the division (the divisor).	$15 \div 3 = 5$
This value tells you how big each jump needs to be. Demonstrate this on your number line.	
Write the answer.	$1.5 \div 3 = 0.5$

SC 2 I can divide decimals by integers

To divide a decimal by an integer, use the same method as for dividing a whole number by a whole number. The only difference is that there is a decimal point in the question.

Worked example

Using a numerical process to divide a decimal by an integer

Calculate the value of $12.06 \div 3$.

THINKING	WORKING
Write the division as you would with whole numbers, include the decimal point.	$3\overline{)12.06}$
Calculate as normal for a division, lining up the decimal in the answer (the quotient) with the decimal point in the divisor.	$\begin{array}{r} 4.02 \\ 3\overline{)12.06} \end{array}$
Write the answer.	$12.06 \div 3 = 4.02$

SC 3 I can divide decimals by powers of 10

As with multiplication, a simple and useful skill in decimal division is dividing by powers of 10. These are the numbers 10, 100, 1000, 10 000 and so on.

Consider the following calculations:

$2123.45 \div 10 = 212.345$
$2123.45 \div 100 = 21.2345$
$2123.45 \div 1000 = 2.12345$

The results all contain the same digits but the digits have different place values.

This can be extended to multiples of 10 or multiples of other powers of 10. To calculate $25.25 \div 50$, consider the calculation as $25.25 \div 5 \div 10$. So, divide by a single-digit integer and then adjust the decimal point.

Worked example

Dividing a decimal by powers of 10

Calculate the value of 25.25 ÷ 50.

THINKING	WORKING
Represent the calculation with a power of 10.	$25.25 \div 50 = 25.25 \div 5 \div 10$
Calculate the first division.	$= 5.05 \div 10$
Recognise the division by the power of 10.	$= 0.505$
Check the reasonableness of the answer.	$25.25 \approx 25$ $25 \div 50 = 0.5$ The answer seems reasonable.

SC 4 I can divide decimals by decimals

Division is made simpler by multiplying both numbers in the division so that the number being divided is a whole number (integer).

$$1.044 \div 0.24 = \frac{1.044}{0.24} = \frac{1.044 \times 100}{0.24 \times 100} = \frac{104.4}{24} = 4.35$$

$$24\overline{)104.^{8}4^{12}0} \quad 4.3\ 5$$

Worked example

Dividing a decimal by a decimal

(a) Calculate the value of 76.23 ÷ 0.003.

THINKING	WORKING
Decide which power of 10 is needed to multiply the divisor to make it an integer.	Need to multiply by 1000.
Rewrite the division by multiplying both the dividend and divisor by this power of 10 and complete the calculation.	$76.23 \div 0.003 = \frac{76.23}{0.003} = \frac{76.23 \times 1000}{0.003 \times 1000} = \frac{76\,230}{3}$
Calculate the answer using short division.	$3\overline{)7^{1}6^{1}230} \quad 2\ 5\ 410$
Write the answer.	76.23 ÷ 0.003 = 25 410

(b) Calculate 6.324 ÷ 0.6.

THINKING	WORKING
Decide which power of 10 is needed to multiply the divisor to make it an integer.	Need to multiply by 10.
Rewrite the question by multiplying both the dividend and divisor by this power of 10 and complete the calculation.	$6.324 \div 0.6 = \frac{6.324}{0.6}$ $= \frac{6.324 \times 10}{0.6 \times 10}$ $= \frac{63.24}{6}$
Calculate the answer using short division. You might need to write more zeros to continue the division.	$6\overline{)63.^{3}2^{2}4}$ $10.\ 5\ 4$
Write the answer	6.324 ÷ 0.6 = 10.54

Practice

ANSWERS Page 672

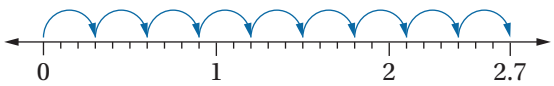
SC 1 I can model division of decimals by integers on a number line

1 Use separate number lines to model each of the following divisions and then state the result.

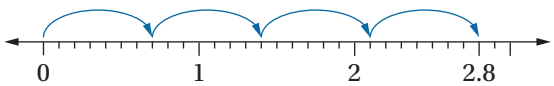
(a) 1.8 ÷ 3 **(b)** 2.4 ÷ 6 **(c)** 2.5 ÷ 5 **(d)** 1.6 ÷ 2

2 Identify the decimal division that is modelled on each of the following number lines.

(a)



(b)



SC 2 I can divide decimals by integers

1 Evaluate each of the following.

(a) 14.24 ÷ 4 **(b)** 205.15 ÷ 5 **(c)** 242.34 ÷ 6 **(d)** 121.22 ÷ 11

2 Evaluate each of the following.

(a) 81.31 ÷ 2 **(b)** 18.623 ÷ 5 **(c)** 226.14 ÷ 8 **(d)** 99.386 ÷ 4

SC 3 I can divide decimals by powers of 10

1 Calculate the value of each of the following.

(a) 846.297 ÷ 100 **(b)** 48.648 ÷ 4000 **(c)** 168.035 ÷ 200 **(d)** 0.8064 ÷ 40

2 A formal birthday party was attended by 200 people. The total cost of the party was $7100. How much did the party cost per person?

3 **(a)** Write three numbers, each with two decimal places, that when divided by 9000 give an answer between 5 and 6.

(b) Describe the process you followed to find these values.

SC 4 I can divide decimals by decimals

1 Calculate the value of each of the following.

(a) 26.24 ÷ 0.4 **(b)** 4.362 ÷ 0.06 **(c)** 5.5555 ÷ 0.005 **(d)** 0.181 818 ÷ 0.0009

2 Calculate the value of each of the following.

(a) 4620.5 ÷ 0.005 **(b)** 3.936 45 ÷ 0.07 **(c)** 2025.5 ÷ 0.25 **(d)** 154.62 ÷ 0.4

3 Use separate number lines to model each of the following divisions and then state the result.

(a) 1.8 ÷ 0.2 **(b)** 2.4 ÷ 0.3

4 Complete the following statements.

(a) When a decimal is divided by a decimal greater than 1, the result (quotient) is larger/smaller than the original number (dividend).

(b) When a decimal is divided by a decimal smaller than 1, the result (quotient) is larger/smaller than the original number (dividend).

5 In each case, choose the calculation that should be used to evaluate the question stated.

(a) 0.2864 ÷ 8.67

A $867\overline{)286.4}$

B $867\overline{)2864}$

C $867\overline{)28.64}$

D $867\overline{)28640}$

(b) 22.306 ÷ 0.015

A $15\overline{)22306}$

B $15\overline{)2230.6}$

C $15\overline{)223.06}$

D $15\overline{)223060}$

6 Jai wants to cut a 10 m long piece of timber into three equal pieces.

(a) Esther tells Jai that he will have trouble doing this. Why do you think that is the case?

(b) How accurately do you think Jai will be able to measure his cuts?

(c) How long will each piece be at this accuracy?

7 An 18 km taxi trip costs Veronika $44.78. This cost includes a flag fare of $4.10.

(a) What is the cost of the actual driving part of the journey?

(b) What is the rate per kilometre?

8 Choose the best option for each of the following.

(a) 15.037 ÷ 0.5 is closest to:

A 3 **B** 30 **C** 300 **D** 3000

(b) 0.0075 ÷ 0.03 is closest to:

A 0.25 **B** 2.5 **C** 25 **D** 250

9 18.0246 ÷ 0.08 = 225.3075. Based on this, write three other questions that give this exact answer.

TOPIC REVIEW

Multiple choice

ANSWERS Page 737

1 Calculate the value of 9.99 + 1.03.

A 8.96 **B** 10.02 **C** 11.02 **D** 11.12

2 Calculate the value of 8.69 − 4.35.

A 4.04 **B** 4.34 **C** 12.04 **D** 13.04

3 Calculate the value of 10.34 − 6.58.

A 3.24 **B** 3.76 **C** 4.24 **D** 16.92

4 How many decimal places will there be in the result of the decimal multiplication 5.5×2.21?

A One **B** Two **C** Three **D** Four

5 When you multiply a decimal value, A, between 0 and 1, by a decimal value, B, greater than 1, the result will be:

A greater than A but less than B. **B** less than A.

C dependent on the values of A and B. **D** less than A, but greater than B.

6 When you multiply a decimal value, A, greater than 1, by a decimal value, B, also greater than 1, where $A < B$, the result will be:

A greater than A but less than B. **B** less than A.

C greater than B. **D** dependent on the values of A and B.

7 Choose all the options that are the same as $65 \div 0.1$.

A $65 \div \frac{1}{10}$ **B** 65×10 **C** 650 **D** $\frac{65}{10}$

8 $0.3 \div 0.8 =$

A 3.8 **B** 0.0375 **C** 2.67 **D** 0.375

9 $8037.9 \div 0.24 =$

A 334.9125 **B** 3349.125 **C** 33 491.25 **D** 334 912.5

10 $0.618 \div 0.6 =$

A 0.103 **B** 1.03 **C** 10.3 **D** 103

11 Jan measures her four attempts at long jump. They were 4.2 m, 4.38 m, 4.65 m and 4.069 m. The difference between her shortest jump and her longest jump is:

A 0.45 m **B** 0.131 m **C** 0.581 m **D** 0.619 m

12 5.396 rounded to 2 decimal places is:

A 5.3 **B** 5.4 **C** 5.39 **D** 5.40

13 Which of the following cannot be rounded to 3.68? There may be more than one answer.

A 3.684 **B** 3.688 **C** 3.679 **D** 3.675

14 $2.4 \times 0.05 =$

A 0.012 **B** 0.12 **C** 1.2 **D** 12

15 $2.34 \times 1.6 =$

A 0.03744 **B** 0.3744 **C** 3.774 **D** 37.44

Short answer

1 Calculate each of the following.

(a) $6 + 3.45 + 1.26$ **(b)** $23.4 - 6.75$

2 Calculate each of the following.

(a) 0.125×300 **(b)** 0.56×0.03

3 Calculate each of the following.

(a) $0.0566 \div 4$ **(b)** $0.045 \div 0.9$

4 State the number of decimal places you would expect in the answer for each of the following calculations.

(a) 28.4×2.89 **(b)** 1.485×0.77 **(c)** 4.3×20.1

(d) 9.75×5.25 **(e)** 0.471×50.3 **(f)** 0.64×100.6

5 Answer the following questions.

(a) Given that one tin of tomatoes has a mass of 0.475 kg, calculate the total mass of 2.5 tins.

(b) Do you think this is a reasonable degree of accuracy for the answer?

6 Each number in the following pattern is found by multiplying the previous number by 0.25: 64, 16, 4, 1. Determine the next three numbers in the pattern.

7 For each of the following, first estimate the answers by rounding to the nearest whole number, then calculate the exact result.

(a) 4.3×7.4 **(b)** 10.1×2.6 **(c)** 4.62×3.8 **(d)** 9.03×1.06

8 Bronte uses three and a half timber sleepers to build her vegetable garden. Each timber sleeper is 2.4 m in length. Calculate the total length of timber that Bronte uses to build her garden.

9 Lin buys 5.8 L of lawnmower fuel at $1.98/L.

(a) Calculate the amount that Lin would pay in cash.

(b) Calculate the amount that Lin would pay via EFT.

10 During the school holidays, Ashton works 6 hours 15 mins per day and is paid $18.25/hour.

(a) Calculate how much he is paid in cash for a five-day working week.

(b) If his pay is directly credited to his bank account, how much would that be?

11 A tap is left dripping. Every hour, 67.5 mL of water drips into a previously empty fish tank.

(a) Calculate how much water is in the tank after 8.5 hours.

(b) The following image shows a medicine measuring cup.

Do you think your answer to **(a)** is stated to a reasonable degree of accuracy? How do you think the answer should be written?

12 Calculate each of the following.

(a) $18.72 \div 0.9$ **(b)** $21 \div 1.4$

(c) $70.2 \div 0.06$ **(d)** $70.7 \div 0.008$

13 Luey has earned $139.95 for four and a half hours of work. Determine how much Luey earns for each hour of work.

14 The height of a toy brick is 0.96 cm. Determine the smallest number of toy bricks required to build a tower 150 cm tall.

15 Calculate the following.

(a) 55.22×300 **(b)** 0.0626×5000

16 Calculate the following.

(a) $0.0462 \div 0.002$ **(b)** $810.45 \div 500$

17 Ethan bought a packet of chips with a marked price of $1.86 and paid cash.

(a) How much would Ethan actually have to pay for the chips?

(b) How much change would Ethan get from $5?

Extended response

1 One of the questions on the Decimals test was to evaluate 87.26×5.461. Esther gave an answer of 746.52686.

(a) Round each value down to the closest whole number and multiply them to get an estimate of the answer.

(b) Round each value up to the closest whole number and multiply them to get an estimate of the answer.

(c) How can you use these estimates to convince Esther that her answer is incorrect?

(d) Evaluate the exact answer.

(e) How do you think Esther got her answer?

2 While preparing for a party, Saad finds he has 1.8 kg of flour. He is using the flour to make some rolls. Each batch of the recipe makes 12 rolls and uses 0.25 kg (2 cups) of flour.

(a) How many whole batches of rolls can Saad make?

(b) How many rolls can Saad make?

(c) Given that Saad can use the left-over flour to make part of a batch, how many additional rolls can he make?

(d) How much flour will still be unused? If necessary, round your answer to four decimal places.

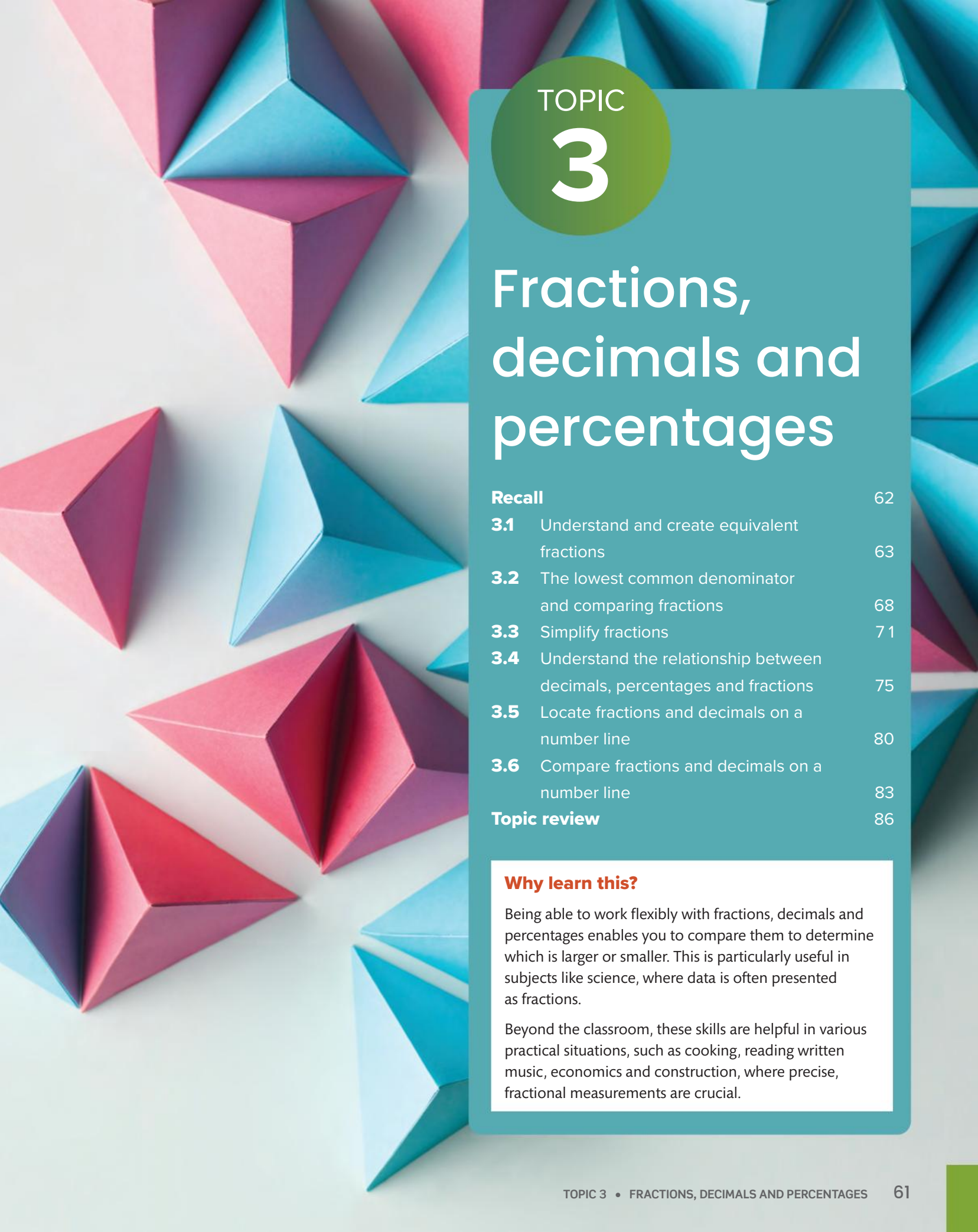
3 In Olympic diving competitions, there are seven judges who each give a score from 0 to 10. The top two scores and the bottom two scores are not used, while the remaining three scores are added together and multiplied by the degree of difficulty of the dive. This is a number associated with the dive attempted.

(a) Greg attempted a forward $3\frac{1}{2}$ somersault in the pike position that had a degree of difficulty of 3.0. The judges scored him 8.3, 9.0, 8.8, 9.1, 8.5, 8.7, 8.4. What score would Greg get for his dive?

(b) If Greg had instead attempted a forward $4\frac{1}{2}$ somersault in the pike position, the degree of difficulty would have been 4.3. If Greg did this and the judges gave the same scores, what would be the difference in his score?

(c) If Greg attempted a simple forward somersault with a degree of difficulty of 1.6, what would be the highest score that Greg could get for this dive?

(d) Greg has the final dive in the competition and knows he only needs 42.6 points to win. He decides on the relatively easy back somersault dive with a degree of difficulty of 1.7. What is the minimum number of points he needs from the middle three judges to win?



TOPIC 3

Fractions, decimals and percentages

Recall 62

3.1 Understand and create equivalent fractions 63

3.2 The lowest common denominator and comparing fractions 68

3.3 Simplify fractions 71

3.4 Understand the relationship between decimals, percentages and fractions 75

3.5 Locate fractions and decimals on a number line 80

3.6 Compare fractions and decimals on a number line 83

Topic review 86

Why learn this?

Being able to work flexibly with fractions, decimals and percentages enables you to compare them to determine which is larger or smaller. This is particularly useful in subjects like science, where data is often presented as fractions.

Beyond the classroom, these skills are helpful in various practical situations, such as cooking, reading written music, economics and construction, where precise, fractional measurements are crucial.

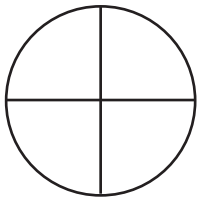
RECALL

ANSWERS Page 673

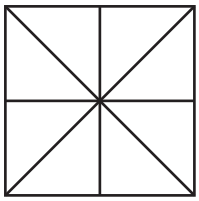
I can identify fractions of shapes

1 Copy each shape and then colour in three equal parts out of four equal parts.

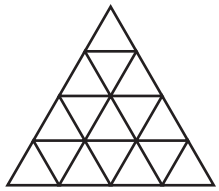
(a)



(b)

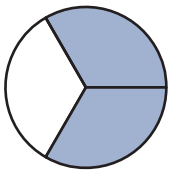


(c)



2 Which of these shapes show two-thirds of the whole shape?

A



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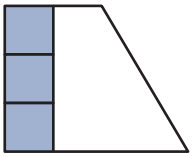

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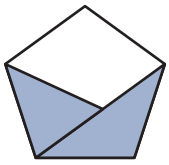

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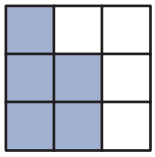

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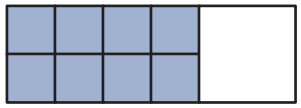


I can describe the fraction the division of a quantity represents

1 Identify the fraction represented by each statement.

(a) Dividing a quantity by 2

A $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{1}{4}$ **D** $\frac{1}{5}$

(b) Three lots of a quantity divided by 4

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** $\frac{3}{4}$ **D** $\frac{3}{5}$

I can compare and order simple fractions

1 Identify which of the three symbols, $<$, $>$ or $=$, should go in between the following fractions.

(a) $\frac{3}{8} __ \frac{1}{2}$ **(b)** $\frac{1}{4} __ \frac{1}{8}$ **(c)** $\frac{2}{4} __ \frac{3}{9}$ **(d)** $\frac{2}{8} __ \frac{3}{12}$

I can multiply and divide decimals by powers of 10

1 Calculate the following multiplications.

(a) 0.8×10 **(b)** 1.32×100 **(c)** 1.6×1000

(d) 3.67×10 **(e)** 4.672×100 **(f)** 7.323×1000

2 Calculate the following divisions.

(a) $12.4 \div 10$ **(b)** $263.6 \div 100$ **(c)** $692.1 \div 1000$

(d) $20.35 \div 10$ **(e)** $1004.6 \div 100$ **(f)** $9080 \div 1000$

Understand and create equivalent fractions

3.1

Learning intention: To understand and create equivalent fractions

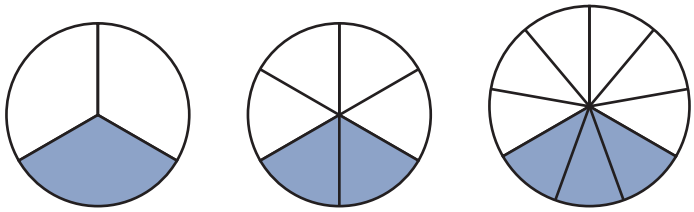
Success criteria:

SC 1 I can use a model to create equivalent fractions.

SC 2 I can write a string of equivalent fractions.

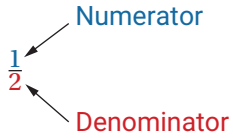
Lesson warm-up

Write down the fractions represented by the three diagrams. Do they represent the same area of each circle? Are these fractions equal?



SC 1 I can use a model to create equivalent fractions

In a fraction, the value on the top (numerator) tells you how many equal parts are of interest and the bottom number tells you how many equal parts there are in total.

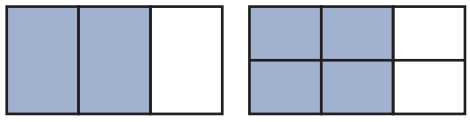


So, the fraction $\frac{3}{5}$ tells you that you are interested in three of the five equal parts. However, a fraction can be written in a number of equivalent ways.

Equivalent fractions

Equivalent fractions are fractions that have different denominators and numerators, but represent the same value or proportion.

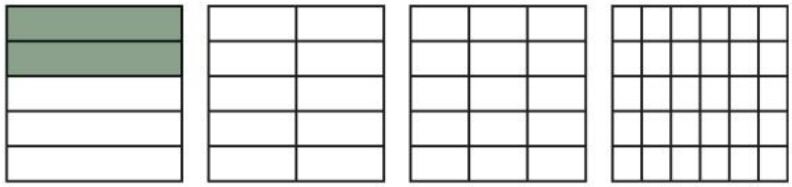
For example, the first diagram of a rectangle shows 1 whole divided into 3 equal parts, with 2 parts shaded. This shows the fraction $\frac{2}{3}$. In the second diagram, the same proportion of the rectangle is shaded but there are twice as many parts, so it shows the fraction $\frac{4}{6}$. These two fractions are equivalent.

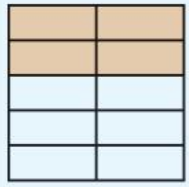
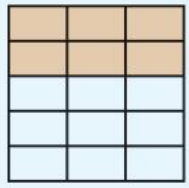
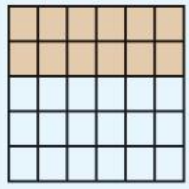


Worked example

Using an area model to find equivalent fractions

Use the diagrams below to create three equivalent fractions for $\frac{2}{5}$.



THINKING	WORKING
Count the number of parts in the first unshaded rectangle.	The whole is divided into 10 equal parts.
When the same part of the rectangle is shaded, determine how many parts will be shaded.	 4 parts will be shaded.
Represent this as a fraction.	$\frac{4}{10}$
Repeat this process for the second unshaded rectangle.	The whole is divided into 15 equal parts,  6 of which are shaded: $\frac{6}{15}$.
Repeat this process for the third unshaded rectangle.	The whole is divided into 25 equal parts,  10 of which are shaded: $\frac{10}{25}$.
Write the list of equivalent fractions formed.	$\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{10}{25}$

SC 2 I can write a string of equivalent fractions

One way to write an equivalent fraction is to multiply both the numerator and the denominator by the same value.

$$\frac{1}{2} \overset{\times 2}{=} \frac{2}{4} \overset{\times 2}{=} \frac{4}{8}$$

$\frac{1}{2}, \frac{2}{4}, \frac{4}{8}$ are equivalent fractions.

Worked example

Writing an equivalent fraction

Create an equivalent fraction for $\frac{3}{7}$ that has a denominator of 42.

THINKING	WORKING
Determine what you need to multiply the original denominator by to get the new denominator.	$7 \times 6 = 42$
Calculate by multiplying the numerator and the denominator to create an equivalent fraction.	$\frac{3}{7} = \frac{3 \times 6}{7 \times 6} = \frac{18}{42}$
Write the answer.	$\frac{3}{7} = \frac{18}{42}$

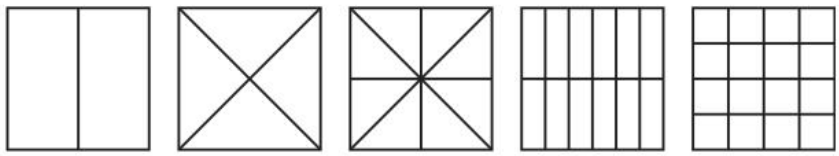
Practice

ANSWERS Page 673

SC 1 I can use a model to create equivalent fractions

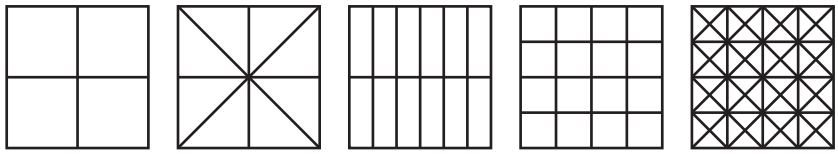
1 Draw each of the five squares shown, then shade the fraction given and write the equivalent fractions.

(a) $\frac{1}{2}$



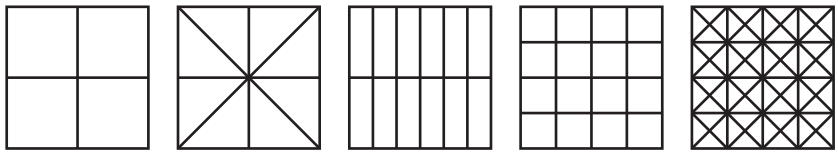
$\frac{1}{2} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

(b) $\frac{1}{4}$



$\frac{1}{4} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

(c) $\frac{3}{4}$



$\frac{3}{4} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

2 Use an area model to show that the following pairs of fractions are equivalent.

(a) $\frac{4}{7} = \frac{12}{21}$ **(b)** $\frac{3}{8} = \frac{6}{16}$ **(c)** $\frac{2}{3} = \frac{8}{12}$

3 A table of multiples can be used to identify equivalent fractions.

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

As an example, using the yellow numbers with the blue 2 row gives:

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16} = \frac{9}{18} = \frac{10}{20}$

(a) Write the string of equivalent fractions using the yellow numbers and the blue 4 row.

(b) Write the string of equivalent fractions using the yellow numbers and the blue 7 row.

(c) Write the string of equivalent fractions using the yellow numbers and the blue 10 row.

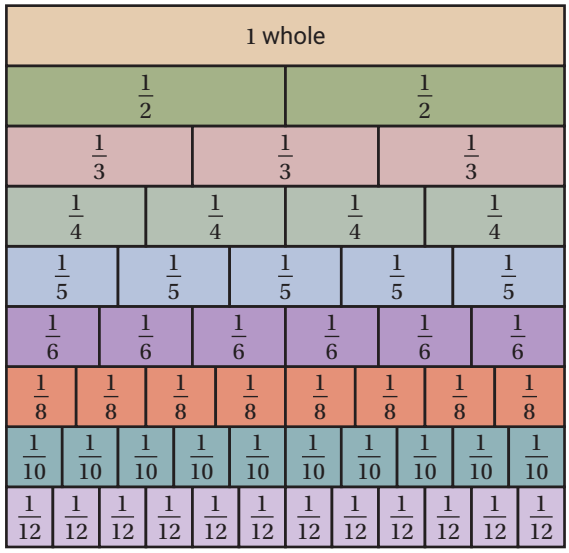
4 Use the fraction wall to write:

(a) five fractions equivalent to $\frac{1}{2}$

(b) two fractions equivalent to $\frac{1}{3}$

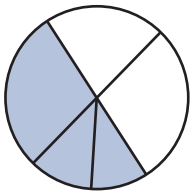
(c) two fractions equivalent to $\frac{3}{4}$

(d) a fraction equivalent to $\frac{5}{6}$.

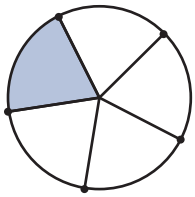


5 Which of the following diagrams shows a fraction equivalent to $\frac{3}{5}$?

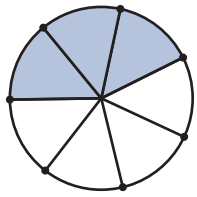
A



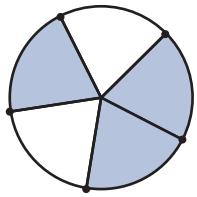
B



C

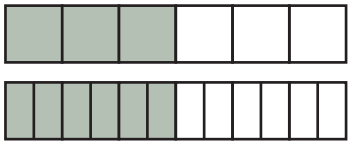


D

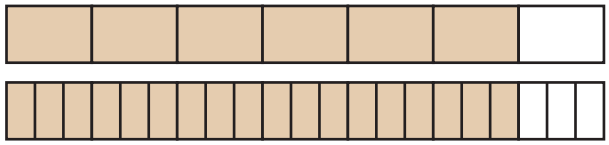


6 What pair of equivalent fractions is shown by each of the following fraction strip diagrams?

(a)



(b)



SC 2 I can write a string of equivalent fractions

1 Copy and complete the following strings of equivalent fractions.

(a) $\frac{2}{5}=\frac{\square}{15}=\frac{\square}{25}=\frac{\square}{40}$ **(b)** $\frac{3}{7}=\frac{\square}{14}=\frac{\square}{49}=\frac{\square}{63}$ **(c)** $\frac{3}{11}=\frac{\square}{33}=\frac{\square}{55}=\frac{\square}{110}$ **(d)** $\frac{8}{13}=\frac{\square}{39}=\frac{\square}{78}=\frac{\square}{130}$

2 Copy and complete the following strings of equivalent fractions.

(a) $\frac{4}{9}=\frac{16}{\square}=\frac{36}{\square}=\frac{44}{\square}$ **(b)** $\frac{5}{8}=\frac{25}{\square}=\frac{60}{\square}=\frac{100}{\square}$

(c) $\frac{2}{15}=\frac{6}{\square}=\frac{14}{\square}=\frac{22}{\square}$ **(d)** $\frac{15}{17}=\frac{45}{\square}=\frac{90}{\square}=\frac{300}{\square}$

3 Copy and complete the following strings of equivalent fractions.

(a) $\frac{\square}{6}=\frac{4}{\square}=\frac{21}{42}=\frac{\square}{54}$ **(b)** $\frac{2}{\square}=\frac{18}{54}=\frac{\square}{60}=\frac{22}{\square}$ **(c)** $\frac{11}{\square}=\frac{\square}{28}=\frac{55}{\square}=\frac{110}{140}$

4 Are the following pairs of fractions equivalent? Answer yes or no.

(a) $\frac{12}{56}$ and $\frac{15}{70}$ **(b)** $\frac{90}{115}$ and $\frac{18}{46}$ **(c)** $\frac{42}{63}$ and $\frac{12}{8}$

3.2 The lowest common denominator and comparing fractions

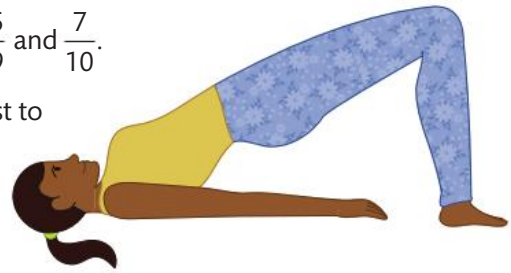
Learning intention: To understand the lowest common denominator and use it to compare fractions

Success criteria:

SC 1 I can determine the lowest common denominator and use it to compare fractions.

Lesson warm-up

Create a fraction wall to show the fractions $\frac{3}{8}$, $\frac{4}{7}$, $\frac{2}{3}$, $\frac{5}{9}$ and $\frac{7}{10}$.



Can you put any of the fractions in order (from smallest to largest) before drawing them?

After completing the fraction wall, write all the fractions in order from smallest to largest (ascending order). Can you put them in order by comparing fractions to the nearest half? Do you need to make equivalent fractions, using a common denominator?

SC 1 I can determine the lowest common denominator and use it to compare fractions

Given two different fractions that have the same denominator, it is easy to determine the larger fraction. The fraction with the greater numerator will be larger: $\frac{3}{5} > \frac{2}{5}$.

Given two different fractions that have the same numerator, the fraction with the smaller denominator will be larger: $\frac{3}{5} > \frac{3}{7}$.

The task is a little more complicated if the fractions share neither numerator nor denominator. In these cases, it is usual to find equivalent fractions with the same denominator. To do this, you should find the lowest common denominator and convert both fractions to this denominator. To find the lowest common denominator, make lists of the multiples of each denominator and choose the smallest number common to both lists.

Worked example

Identify the lowest common multiple

Determine the lowest common multiple of 8 and 12.

THINKING	WORKING
List the first 6 multiples of the first value.	Multiples of 8: 8, 16, 24, 32, 40, 48, ...
List the first 4 multiples of the second value. You can stop writing multiples when you have a multiple in common with the first set.	Multiples of 12: 12, 24, 36, 48, ...
Identify the lowest common multiple.	The lowest common multiple is 24.

Worked example

Using the lowest common denominator to compare fractions

Insert the symbol $>$ or $<$ to make the following statement true: $\frac{7}{8}$ ___ $\frac{11}{12}$.

THINKING	WORKING
Identify the lowest common denominator. This is the lowest common multiple of the fraction denominators.	The lowest common denominator is 24.
Represent the first fraction as an equivalent fraction using this denominator.	$\frac{7}{8} \times \frac{3}{3} = \frac{21}{24}$
Represent the second fraction as an equivalent fraction using this denominator.	$\frac{11}{12} \times \frac{2}{2} = \frac{22}{24}$
Write the answer.	$\frac{21}{24} < \frac{22}{24}$ so $\frac{7}{8} < \frac{11}{12}$.

Practice

ANSWERS Page 673

SC 1 I can determine the lowest common denominator and use it to compare fractions

1 Identify the lowest common multiple for the following pairs and triples of numbers.

(a) 14 and 21 **(b)** 24 and 30 **(c)** 3, 4 and 5 **(d)** 2, 5 and 12

2 Insert the symbol > or < to make the following statements true.

(a) $\frac{3}{5} __ \frac{3}{4}$ **(b)** $\frac{5}{8} __ \frac{4}{7}$ **(c)** $\frac{3}{8} __ \frac{5}{12}$

(d) $\frac{4}{11} __ \frac{3}{10}$ **(e)** $\frac{13}{14} __ \frac{8}{9}$ **(f)** $\frac{5}{11} __ \frac{6}{13}$

3 Identify three fractions that lie between each pair of fractions.

(a) $\frac{3}{5}$ and $\frac{4}{5}$ **(b)** $\frac{4}{9}$ and $\frac{5}{9}$ **(c)** $\frac{3}{10}$ and $\frac{4}{10}$

4 Lidia said that she could tell which fraction was larger in a pair of fractions if they were both very close to $\frac{1}{2}$. She used this as her example: $\frac{11}{21} > \frac{13}{25}$.

(a) How much larger than $\frac{1}{2}$ is $\frac{11}{21}$? Express this as a fraction using whole numbers.

(b) How much larger than $\frac{1}{2}$ is $\frac{13}{25}$? Express this as a fraction using whole numbers.

(c) How can you tell which of these two fractions is larger? What do they have in common?

(d) Try to explain Lidia's method. Does it make a difference if the fractions are just a little smaller than one-half?

Exploration Task

Glass half full

Arrange the glasses in order from having the least amount of water to the most amount of water.

Describe the fraction of water in each glass.

Estimate how many full glasses of water there are.

Simplify fractions

3.3

Learning intention: To understand how to simplify fractions

Success criteria:

SC 1 I can determine the highest common factor for a string of values.

SC 2 I can simplify fractions.

Lesson warm-up

Consider the fractions

$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$.

Can you identify any patterns or relationships between the numerators and denominators?

SC 1 I can determine the highest common factor for a string of values

To determine the highest common factor for a string of values, write out the factor list for each value and identify the highest value common to all lists. Every string of values must have at least one common factor, namely 1, as it is a factor of every number.

Worked example

Determine the highest common factor

Determine the highest common factor for 12, 30 and 42.

THINKING	WORKING
List the factors of each number, in ascending order, for each of the values.	12: 1, 2, 3, 4, 6, 12 30: 1, 2, 3, 5, 6, 10, 15, 30 42: 1, 2, 3, 6, 7, 14, 21, 42
Identify the highest value that is common to all lists. This is the highest common factor.	The highest common factor is 6.

SC 2 I can simplify fractions

A fraction written in simplest form does not have any common factors in the numerator and the denominator. This applies for all the ways fractions can be written.

Worked example

Simplifying a fraction using common factors

Write the following fractions in simplest form.

(a) $\frac{16}{24}$

THINKING	WORKING
Determine the highest common factor for the numerator and the denominator.	The highest common factor is 8.
Represent the fraction using this common denominator.	$\frac{16}{24}=\frac{2\times 8}{3\times 8}$
Represent the fraction in the simplest form by cancelling the common factor.	$=\frac{2\times \cancel{8}}{3\times \cancel{8}}$ $=\frac{2}{3}$

(b) $\frac{26}{8}$

THINKING	WORKING
Determine the highest common factor for the numerator and the denominator.	The highest common factor is 2.
Represent the fraction using this common denominator.	$\frac{26}{8}=\frac{13\times 2}{4\times 2}$
Represent the fraction in the simplest form by cancelling the common factor. (The question was written in improper fraction form, so the answer should be in the same form.)	$=\frac{13\times \cancel{2}}{4\times \cancel{2}}$ $=\frac{13}{4}$

(c) $1\frac{4}{12}$

THINKING	WORKING
Determine the highest common factor for the numerator and the denominator of the fraction part.	The highest common factor is 4.
Represent the fraction part using this common denominator.	$\frac{4}{12}=\frac{1\times 4}{3\times 4}$
Represent the fraction in the simplest form by cancelling the common factor.	$=\frac{1\times \cancel{4}}{3\times \cancel{4}}$ $=\frac{1}{3}$
Represent the mixed number in simplest form.	$1\frac{4}{12}=1\frac{1}{3}$

Practice

ANSWERS Page 674

SC 1 I can determine the highest common factor for a string of values

1 Determine the highest common factor of the following values.

(a) 10 and 15 **(b)** 12 and 16 **(c)** 14, 20 and 36 **(d)** 7, 14 and 48

2 One way of writing the factors of a number is to write the factors in pairs that multiply to give the number. Write the factors for each of the following numbers, using the pairing method.

(a) 40 **(b)** 48 **(c)** 99

(d) How do you know when you have found all of the factors?

3 Use the pairing method to find the factors of the following numbers.

(a) 100 **(b)** 81 **(c)** 121

(d) What do you notice about the factor pairs for these numbers? What type of numbers are they?

SC 2 I can simplify fractions

1 Represent the following fractions in simplest form.

(a) $\frac{18}{42}$ **(b)** $\frac{24}{50}$ **(c)** $\frac{66}{36}$

(d) $\frac{110}{45}$ **(e)** $3\frac{14}{35}$ **(f)** $4\frac{33}{120}$

2 Choose the best option for each of the following.

(a) Which of the following is the simplest form of $\frac{210}{330}$?

A $\frac{21}{33}$ **B** $\frac{7}{11}$ **C** $\frac{42}{66}$ **D** $\frac{3}{11}$

(b) Which of the following is the simplest form of $-4\frac{48}{60}$?

A $-\frac{288}{60}$ **B** $-4\frac{4}{5}$ **C** $\frac{24}{5}$ **D** $-\frac{4}{5}$

3 Lucinda was helping her younger brother, Oscar, with his homework. One of the questions was to write $\frac{1200}{720}$ in simplest form. Oscar said the numbers were too big for him to be able to easily identify the highest common factor. Lucinda said that was fine, it could be done in parts.

(a) This is what Lucinda showed Oscar to do. Copy and complete the working.

$$\frac{1200}{720} = \frac{120\times10}{72\times\square}$$
$$= \frac{120}{72}$$
$$= \frac{60\times\square}{36\times\square}$$
$$= \frac{60}{36}$$
$$= \frac{10\times6}{\square\times6}$$
$$= \frac{10}{6}$$
$$= \frac{\square\times2}{\square\times2}$$
$$= \frac{5}{3}$$

(b) Use Lucinda's method to write $\frac{166\,320}{997\,920}$ in simplest form.

4 For each of the following mixed numbers, write three different improper fractions that simplify to the stated value.

(a) $2\frac{3}{5}$ **(b)** $-5\frac{1}{4}$

Understand the relationship between decimals, percentages and fractions

3.4

Learning intention: To understand the relationship between decimals, percentages and fractions

Success criteria:

SC 1 I understand the connection between place value, decimals and fractions.

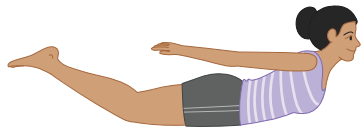
SC 2 I can write a decimal as a fraction.

SC 3 I can write a decimal as a percentage.

SC 4 I can convert between fractions, decimals and percentages.

Lesson warm-up

Converting between number forms



The same number can be expressed in different ways. For example, 50%, 0.5 and $\frac{1}{2}$ are all the same number, expressed differently as a percentage, a decimal and a fraction.

Create a fraction wall that shows commonly used fractions, decimals and percentages.

SC 1 I understand the connection between place value, decimals and fractions

Numbers that are not whole numbers can be written in a variety of formats. For example, fractions such as $\frac{2}{5}$ and $3\frac{3}{7}$ can be used, or decimals such as 0.6 or 2.564.

A place value chart gives you a clearer understanding of what a decimal number is.

For example, the decimal 213.47 can be written on the place value chart by adding each digit to the correct place value.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
							$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
				2	1	3	4	7	

The decimal 213.47 can be written in expanded form as $200+10+3+\frac{4}{10}+\frac{7}{100}$.

Worked example

Writing a decimal in expanded form

Write the following numbers in expanded form.

(a) 19.87

THINKING	WORKING
Use the place value chart to help with this task (you can also write the decimal value on a copy of the chart).	$19.87 = 10 + 9 + \frac{8}{10} + \frac{7}{100}$

(b) 1987.652

THINKING	WORKING
Use the place value chart to help with this task (you can also write the decimal value on a copy of the chart).	$1987.652 = 1000 + 900 + 80 + 7 + \frac{6}{10} + \frac{5}{100} + \frac{2}{1000}$

SC 2 I can write a decimal as a fraction

To convert a decimal value to fraction form, imagine the value written on the place value chart. So, 0.63 would have a 6 in the tenths column and a 3 in the hundredths column. To write this as a single fraction, use the values written after the decimal point as the numerator and the value of the column furthest to the right that has been used as the denominator. This gives $0.63 = \frac{63}{100}$. Any values prior to the decimal point remain as they are. So, $24.63 = 24\frac{63}{100}$.

Worked example

Converting a decimal to fraction form

Convert 193.287 to fraction form.

THINKING	WORKING
Determine in which column the last digit after the decimal point would be located.	It would be located in the thousandths column.
Recognise that this is the denominator of the fraction part; the numerator is the string of digits after the decimal point.	The fraction part is $\frac{287}{1000}$.
Write the answer.	$193.287 = 193\frac{287}{1000}$.

Another way of approaching this is to write the digits following the decimal point as the numerator and for the denominator, write a 1 followed by as many zeros as there are digits after the decimal point.

SC 3 I can write a decimal as a percentage

A percentage is an amount out of 100.

82% is the same as $\frac{82}{100}$. So, $0.82 = \frac{82}{100} = 82\%$.

To write a decimal in percentage form, multiply by 100%, so 0.78 = 78%.

For example, one whole is 100%, so twelve groups of one whole will be 1200%.

Worked example

Converting a decimal to percentage form

Convert the following decimals to percentage form.

(a) 0.367 **(b)** 0.0367 **(c)** 3.67

THINKING	WORKING
Represent the decimal in percentage form by multiplying by 100%.	**(a)** $0.367 \times 100\% = 36.7\%$ **(b)** $0.0367 \times 100\% = 3.67\%$ **(c)** $3.67 \times 100\% = 367\%$

SC 4 I can convert between fractions, decimals and percentages

Fractions with a denominator of 100 are the simplest to convert to decimal or percentage form.

For example: $\frac{9}{100} = 0.09 = 9\%$ or $\frac{7}{50} = \frac{14}{100} = 0.14 = 14\%$

Worked example

Converting a fraction to a decimal and a percentage

Convert $3\frac{3}{4}$ to percentage form and then to decimal form.

THINKING	WORKING
Represent the mixed number as an improper fraction.	$3\frac{3}{4} = \frac{15}{4}$
Create an equivalent fraction that has a denominator of 100.	$= \frac{15}{4} \times \frac{25}{25}$ $= \frac{375}{100}$
Represent as a percentage.	$= 375\%$
Represent as a decimal.	$= 3.75$

Worked example

Converting a percentage to a fraction

To convert a percentage to a fraction, use the percentage value as the numerator and use 100 for the denominator. Don't forget to write the fraction in the simplest form.

Convert 95% to a fraction written in simplest form.

THINKING	WORKING
Use the percentage value as the numerator and use 100 as the denominator.	$95\% = \frac{95}{100}$
Simplify the fraction by identifying a common factor.	$= \frac{5 \times 19}{5 \times 20}$ $= \frac{19}{20}$

Practice

ANSWERS Page 674

SC 1 I understand the connection between place value, decimals and fractions

1 Write the following in expanded fraction form.

(a) 1.2 **(b)** 12.5 **(c)** 12.56 **(d)** 125.687

(e) 1256.8793 **(f)** 28.985 **(g)** 1267.9 **(h)** 12 356.987

2 Write the following expanded expressions in simplest decimal form.

(a) $300 + 10 + 8 + \frac{3}{10} + \frac{1}{100} + \frac{8}{1000}$ **(b)** $20 + 6 + \frac{7}{10} + \frac{2}{100} + \frac{1}{1000}$

(c) $1000 + 60 + 7 + \frac{8}{10} + \frac{2}{100}$ **(d)** $1000 + 400 + 60 + 3 + \frac{7}{1000}$

3 Choose the best answer for each of the following.

(a) The value of the 3 in 2031.42 is:

A 300 **B** 30 **C** 3 **D** $\frac{3}{10}$

(b) The value of the 5 in 1234.0056 is:

A 5 **B** $\frac{5}{10}$ **C** $\frac{5}{100}$ **D** $\frac{5}{1000}$

4 Write the following as decimal values.

(a) Six tenths **(b)** Nine hundredths

(c) Eight tenths and six thousandths **(d)** Seven units, two tenths and six hundredths

SC 2 I can write a decimal as a fraction

1 Convert the following decimals to fraction form.

(a) 0.1 **(b)** 0.12 **(c)** 0.123

(d) 0.1234 **(e)** 0.12345 **(f)** 12.9

(g) 132.861 **(h)** 2564.227 **(i)** 106.107

SC 3 I can write a decimal as a percentage

1 Convert the following decimal values to percentage form.

(a) 0.015 87 **(b)** 0.1587 **(c)** 1.587 **(d)** 15.87

(e) 0.222 897 **(f)** 2.228 97 **(g)** 22.2897 **(h)** 222.897

(i) 6.008 **(j)** 90.2013

SC 4 I can convert between fractions, decimals and percentages

1 Convert the following mixed numbers to percentage form and then to decimal form.

(a) $2\frac{2}{5}$ **(b)** $8\frac{3}{10}$ **(c)** $5\frac{1}{4}$ **(d)** $6\frac{7}{10}$

2 Convert the following percentages to fractions written in simplest form. Use mixed numbers if necessary.

(a) 88% **(b)** 72% **(c)** 25% **(d)** 100%

3 Convert the following percentages to fractions written in simplest form. Use mixed numbers if necessary.

(a) 125% **(b)** 230% **(c)** 850% **(d)** 892%

4 Convert the following percentages to decimals.

(a) 963% **(b)** 298.65%

5 Half of a quarter is an eighth. $\frac{1}{4} = 0.25$, so $\frac{1}{8} = 0.125$. Use this relationship to write the following fractions as decimals.

(a) $\frac{3}{8}$ **(b)** $\frac{5}{8}$ **(c)** $\frac{7}{8}$ **(d)** $1\frac{5}{8}$

6 Recall the same reasoning as in the previous question and write the following fractions as percentages.

(a) $\frac{1}{16}$ **(b)** $\frac{5}{16}$ **(c)** $\frac{11}{16}$ **(d)** $\frac{13}{16}$

3.5 Locate fractions and decimals on a number line

Learning intention: To be able to locate fractions and decimals on a number line

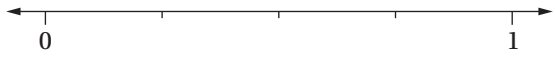
Success criteria:

SC 1 I can divide 1 into equal parts to locate fractions between 0 and 1.

SC 2 I can divide 1 into equal parts to locate decimals between 0 and 1.

Lesson warm-up

1. Draw a number line from 0 to 1, with tick marks at 0, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.
2. Identify the fraction that falls halfway between 0 and $\frac{1}{4}$ on the number line.
3. Next, identify the fraction that falls three-quarters of the way between 0 and 1 on the number line.
4. Finally, identify the fraction that falls halfway between $\frac{1}{2}$ and $\frac{3}{4}$ on the number line.



SC 1 I can divide 1 into equal parts to locate fractions between 0 and 1

The space between any two whole numbers on a number line can be divided into any number of equal parts to help locate a fraction. The number of equal parts will be the same as the denominator of the fraction.

Worked example

Locating a fraction on a number line

Draw a number line and place $\frac{3}{5}$ on it.

THINKING	WORKING
Draw a number line (it only needs to show the space between 0 and 1).	
Divide the space into a number of equal intervals to match the required denominator (in this case, that is 5). Count across from the left to locate the desired fraction.	

SC 2 I can divide 1 into equal parts to locate decimals between 0 and 1

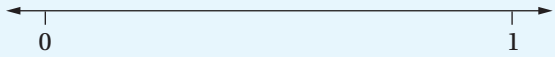
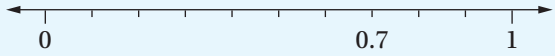
Decimals can also be placed on a number line, although they are usually shown using a part of the number line that is equally divided into 10 intervals.

For example, a decimal such as 0.3 will be shown located between 0 and 1, whereas a decimal such as 0.36 will be shown located between 0.3 and 0.4.

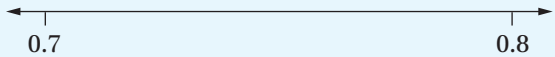
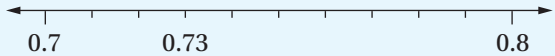
Worked example

Locating a decimal on a number line

(a) Draw a number line and plot 0.7 on it.

THINKING	WORKING
Draw a number line (it only needs to show the space between 0 and 1).	
Divide the space equally into 10 intervals and count across from the left to locate the desired decimal.	

(b) Draw a number line and plot 0.73 on it.

THINKING	WORKING
Draw a number line (in this case it needs to show the space between 0.7 and 0.8).	
Divide the space equally into 10 intervals and count across from the left to locate the desired decimal.	

Practice

ANSWERS Page 675

SC 1 I can divide 1 into equal parts to locate fractions between 0 and 1

1 Plot the following fractions on number lines.

(a) $\frac{1}{4}$ **(b)** $\frac{3}{7}$ **(c)** $\frac{6}{11}$ **(d)** $\frac{7}{12}$

2 The point marked *A* is closest to:

A $\frac{7}{12}$ **B** $\frac{2}{3}$ **C** $\frac{3}{4}$ **D** $\frac{1}{2}$

3

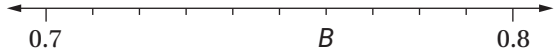
SC 2 I can divide 1 into equal parts to locate decimals between 0 and 1

1 Plot the following decimals on a number line.

(a) 0.8 **(b)** 0.55 **(c)** 0.126

2 Choose the best response for each of the following.

(a) The point marked *B* is closest to:



A 0.77 **B** 0.766 **C** 0.76 **D** 0.6

(b) The point marked *C* is closest to:

A 0.187 **B** 0.88 **C** 0.192 **D** 0.188

3 Applying what you know, state the two values that would be the end points on a number line for each of the following decimals.

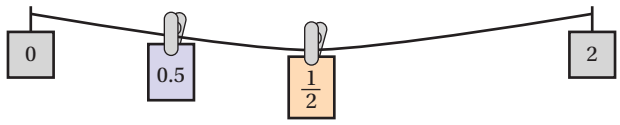
(a) 0.8 **(b)** 0.87 **(c)** 0.876 **(d)** 0.8762

(e) 0.3 **(f)** 0.31 **(g)** 0.317 **(h)** 0.3174

Exploration Task

Number or operation?

Two students were asked to peg the numbers 0.5 and $\frac{1}{2}$ on a string labelled with endpoints 0 and 2.



Which number is placed correctly?

Describe the misconception that led to the placement of the card that is incorrect.

Redraw the activity with the correct placement of the numbers.

Compare fractions and decimals on a number line

3.6

Learning intention: To be able to compare fractions and decimals on a number line

Success criteria:

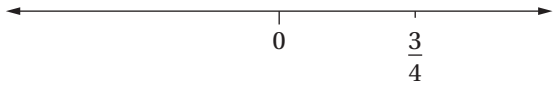
SC 1 I can locate and compare decimals and fractions on a number line without scale marks.

SC 2 I can locate and compare positive and negative numbers on a number line.

Lesson warm-up

Fractions on number lines

Show the correct positions of $-\frac{2}{5}$, $-\frac{7}{8}$ and $\frac{1}{3}$ on the number line and explain your answer.



SC 1 I can locate and compare decimals and fractions on a number line without scale marks

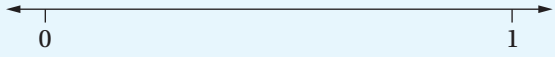
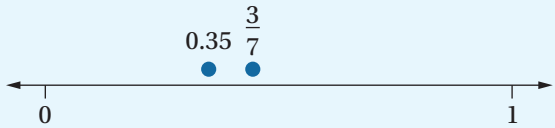
At times, number lines are drawn without scale marks placed on them. In these instances, you need to make the best estimate of the location that you can.

As with whole numbers, the further you are to the right on a number line, the greater the value.

Worked example

Using a number line to compare a fraction and a decimal

Plot both $\frac{3}{7}$ and 0.35 on the same number line and determine which is larger.

THINKING	WORKING
Draw a number line showing 0 to 1.	
Determine an estimated location of the fraction by mentally dividing the line into the required number of intervals (in this case, it is 7). Do the same for the decimal (in this case, the dot will be halfway between 0.3 and 0.4).	
Write the answer.	$\frac{3}{7} > 0.35$

SC 2 I can locate and compare positive and negative numbers on a number line

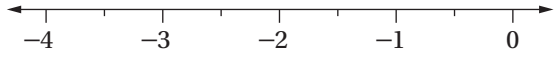
Whether dealing with whole numbers, fractions or decimals, the further to the right the value is located on a number line, the greater the value is.

When placing negative fractions or decimals on a number line, the starting point is still zero, but the movement is to the left.

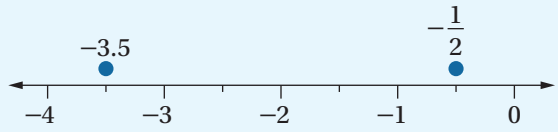
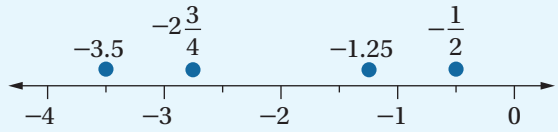
Worked example

Locating negative values on a number line

Plot the following values on the number line shown.



$-3.5, -\frac{1}{2}, -1.25$ and $-2\frac{3}{4}$

THINKING	WORKING
Identify the placement of the scale marks on the number line.	The scale marks appear at intervals of 0.5.
Plot the first two points, because they match the scale marks.	
Determine the location of the other two points.	−1.25 will be halfway between −1 and −1.5, while $-2\frac{3}{4}$ will be halfway between −2.5 and −3.
Plot the points on the number line.	

Practice

ANSWERS Page 675

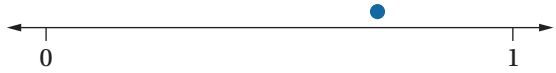
SC 1 I can locate and compare decimals and fractions on a number line without scale marks

1 Place each pair of values on a number line and determine which value is larger.

(a) $\frac{5}{6}$ and 0.8 **(b)** $\frac{5}{9}$ and 0.6 **(c)** $\frac{1}{7}$ and 0.2 **(d)** $\frac{3}{11}$ and 0.25

2 Choose the best option in each of the following.

(a) The point marked on the following number line is closest to which of the stated values?



A $\frac{2}{3}$ **B** $\frac{5}{7}$ **C** $\frac{4}{5}$ **D** $\frac{14}{15}$

(b) The point marked on the following number line is closest to which of the stated values?

A 0.35 **B** 0.4 **C** 0.45 **D** 0.5

3 A number line showing 0 to 1 is 5 cm in length. How far from 0 would you find each of the following? Give your answers correct to one decimal place.

(a) 0.3 **(b)** $\frac{4}{9}$ **(c)** $\frac{5}{7}$ **(d)** 0.75

4 **(a)** Rachel needed to draw a number line to show $\frac{2}{7}$ and 0.3. Which of the following lengths for a number line showing 0 to 1 would make it easiest for her to show these values?

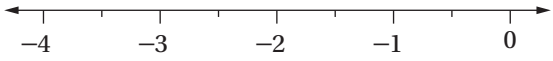
A 5 cm **B** 7 cm **C** 10 cm **D** 12 cm

(b) Give a brief reason for your choice.

SC 2 I can locate and compare positive and negative numbers on a number line

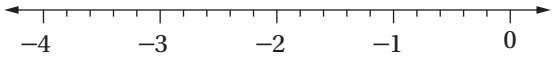
1 Locate the following values on the number lines shown.

(a)



$-2.5, -1\frac{1}{2}, -3.75, -\frac{1}{4}$

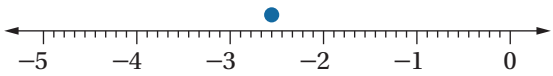
(b)



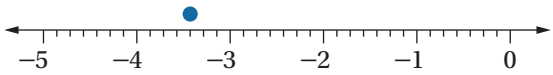
$-0.4, -2\frac{4}{5}, -1.7, -3\frac{1}{2}$

2 Determine the value of the fraction indicated on each of the following number lines.

(a)

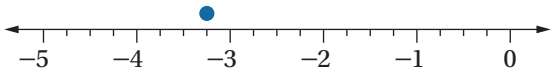


(b)

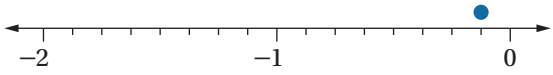


3 Determine the value of the decimal indicated on each of the following number lines.

(a)



(b)



4 Locate each of the following fractions on the same number line and then write the fractions in order from smallest to largest.

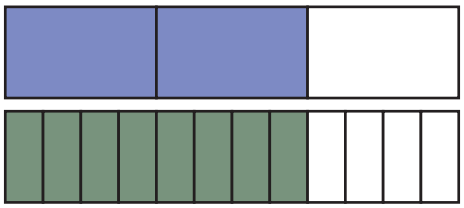
$-\frac{12}{7}, 1\frac{5}{7}, -2\frac{4}{7}, \frac{10}{7}, -\frac{6}{7}$

TOPIC REVIEW

Multiple choice

ANSWERS Page 737

1 Determine the equivalent fraction for $\frac{2}{3}$ as shown by the shaded parts of the fraction strips.



A $\frac{8}{3}$ **B** $\frac{2}{8}$ **C** $\frac{8}{12}$ **D** $\frac{12}{8}$

2 Which fraction is not equivalent to $\frac{3}{8}$?

A $\frac{6}{16}$ **B** $\frac{5}{10}$ **C** $\frac{9}{24}$ **D** $\frac{12}{32}$

3 What is $\frac{36}{48}$ in simplest form?

A $\frac{3}{4}$ **B** $\frac{6}{8}$ **C** $\frac{9}{12}$ **D** $\frac{12}{16}$

4 Jeremiah says he will eat $\frac{1}{4}$ of a large pizza. How many slices is this, given that the pizza is cut into 16 slices?

A 2 **B** 4 **C** 8 **D** 12

5 Darpan correctly answers $\frac{21}{35}$ of the questions on a test. To help convert this to a percentage, what is Darpan's equivalent score out of 100?

A $\frac{60}{100}$ **B** $\frac{63}{100}$ **C** $\frac{72}{100}$ **D** $\frac{86}{100}$

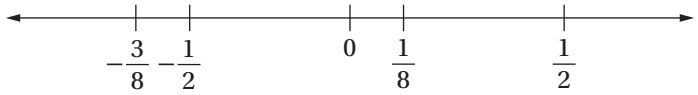
6 The chance of a train arriving on time is 92% . Given that there are 250 trains, how many should arrive on time?

A 23 **B** 92 **C** 225 **D** 230

7 Which list of fractions is in descending order (from largest to smallest)?

A $\frac{5}{8}, -\frac{4}{8}, \frac{3}{8}$ **B** $\frac{1}{4}, -\frac{3}{5}, -\frac{7}{10}$ **C** $\frac{3}{10}, \frac{2}{5}, \frac{1}{2}$ **D** $\frac{1}{8}, \frac{1}{7}, \frac{1}{6}$

8 One of the values on this number line is placed incorrectly. Which is it?



A $-\frac{3}{8}$ **B** $-\frac{1}{2}$ **C** $\frac{1}{8}$ **D** $\frac{1}{2}$

9 What is the best way to compare the fractions $-\frac{4}{7}$ and $-\frac{3}{8}$?

A Compare the denominators. The fraction with the smaller denominator is the larger number.

B Round the fractions to the nearest half. The fraction furthest from 0 on the number line is the larger number.

C Make equivalent fractions with a common denominator. The equivalent fraction furthest to the right on the number line is the larger number.

D Compare the numerators. The fraction with the larger numerator is the larger number.

10 Order the fractions $-\frac{1}{3}, -\frac{4}{5}, -\frac{3}{6}$ in order from smallest to largest (ascending order).

A $-\frac{1}{3}, -\frac{4}{5}, -\frac{3}{6}$ **B** $-\frac{1}{3}, -\frac{3}{6}, -\frac{4}{5}$ **C** $-\frac{3}{6}, -\frac{4}{5}, -\frac{1}{3}$ **D** $-\frac{4}{5}, -\frac{3}{6}, -\frac{1}{3}$

11 Which of the following improper fractions is equivalent to the whole number −7?

A $\frac{-7}{7}$ **B** $\frac{-14}{-7}$ **C** $\frac{-49}{7}$ **D** $\frac{70}{-7}$

12 Select the correct option. The value of the 3 in 124.003 is:

A 3 **B** $\frac{3}{10}$ **C** $\frac{3}{100}$ **D** $\frac{3}{1000}$

13 The highest common factor for 540 and 660 is:

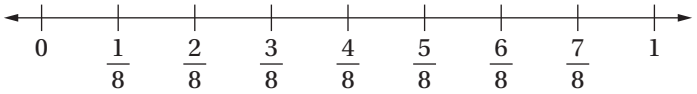
A 12 **B** 15 **C** 20 **D** 60

14 In simplest mixed number form, $\frac{82}{7}$ is:

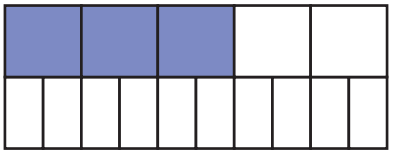
A $11\frac{5}{7}$ **B** $11\frac{2}{7}$ **C** $8\frac{2}{7}$ **D** $7\frac{1}{4}$

Short answer

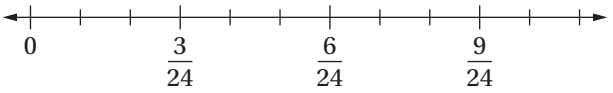
1 Place $\frac{6}{24}$ on the number line to show its equivalent fraction.



2 Use this fraction wall to state an equivalent fraction for $\frac{3}{5}$.



3 Place the fractions $\frac{7}{24}, \frac{5}{12}, \frac{1}{3}, \frac{1}{12}$ on the number line.



4 Place the fractions $\frac{7}{10}, \frac{5}{6}, \frac{2}{3}, \frac{13}{15}, \frac{4}{5}, \frac{23}{30}$ in order from smallest to largest.

5 Copy and complete the following to show equivalent fractions.

$\frac{\square}{5} = \frac{\square}{10} = \frac{6}{15} = \frac{\square}{100}$

6 Tilly records the time she spends doing different activities on a typical day. Copy and complete the following table, showing fractions in simplest form.

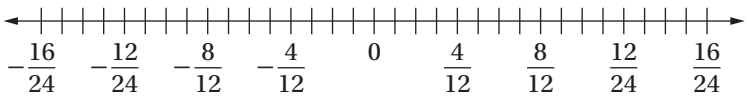
Activity	Time spent (hours)	Fraction of day (24 hours)
Sleeping	8	
School	6	
Sport	1	
Work	3	
Other (eating, travelling, etc.)		

7 In a basketball game, the following statistics for 'free-throw efficiency' were recorded.

- Durant made 9 free throws from 10 attempts
- Lillard made 11 free throws from 12 attempts
- Rubio made 5 free throws from 6 attempts

(a) For each player, write the 'free-throw efficiency' as a fraction with a common denominator.

(b) Now arrange these players in order from best to worst in 'free-throw efficiency'.

8 

(a) Place the fractions $-\frac{5}{8}, \frac{5}{8}, -\frac{7}{12}, \frac{7}{12}$ on the same number line.

(b) List the fractions in order from smallest to largest.

9 **(a)** Complete a fraction wall to show the fractions $\frac{3}{8}, \frac{4}{7}, \frac{2}{3}, \frac{5}{9}, \frac{7}{10}$.

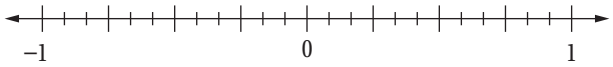
(b) Arrange the fractions in order from smallest to largest.

10 On a horizontal number line, which fraction is further to the left: $-\frac{2}{5}$ or $-\frac{2}{10}$?

11 Select the correct symbol $(<, >, =)$ to make each expression true.

(a) $\frac{1}{3} \square \frac{5}{9}$ **(b)** $\frac{2}{5} \square \frac{2}{3}$ **(c)** $-\frac{3}{5} \square -\frac{3}{4}$ **(d)** $-\frac{1}{2} \square \frac{2}{4}$

12 Place $-\frac{2}{3}, \frac{1}{6}, -\frac{5}{12}, \frac{1}{2}$ on a copy of the following number line.



13 Arrange the fractions $-\frac{3}{4}, -\frac{7}{9}, \frac{4}{5}, -\frac{5}{12}, \frac{2}{3}$ in order from largest to smallest (descending order.)

14 Find equivalent fractions by copying and completing the following.

(a) $\frac{3}{8} = \frac{60}{\square}$ **(b)** $\frac{150}{100} = \frac{\square}{4}$ **(c)** $\frac{180}{24} = \frac{\square}{2}$ **(d)** $-\frac{36}{48} = \frac{3}{\square}$

15 Between which two whole numbers will each of the following be located?

(a) $3\frac{1}{9}$ **(b)** $-\frac{2}{3}$ **(c)** $\frac{21}{6}$ **(d)** $-\frac{15}{8}$

16 State, as mixed numbers, the value of each of the points plotted on the number lines below.

(a)

(b)

17 Write the following decimals in extended fraction form.

(a) 0.23 **(b)** 0.125 **(c)** 0.0056

Extended response

1 Steffi needs to mark the following fractions and mixed numbers on a number line: $\frac{3}{10}, -\frac{2}{5}, 1\frac{1}{2}, 1\frac{1}{10}, \frac{9}{5}$.

(a) Determine the smallest range of whole numbers that Steffi can use on her number line and be able to fit all the values on it.

(b) What scale division should Steffi use on her number line to make this task easier?

(c) Draw the number line and mark each of the values on it.

(d) Which of the values placed on the number line is closest to $\frac{9}{8}$?

(e) Imagine you could extend the number line to the right. How many whole numbers would there be between $\frac{13}{10}$ and $\frac{43}{5}$?

2 The decimal 0.03 can be written in single fraction form as $\frac{3}{100}$.

(a) Write the following decimals as single fractions in simplest form.

(i) 0.35

(ii) 0.57

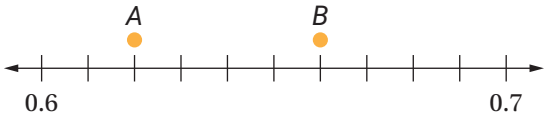
(iii) 0.206

(b) Between which two tenths does 0.35 lie on a number line?

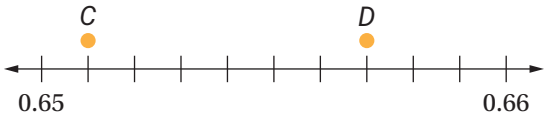
(c) Between which two hundredths does 0.206 lie on a number line?

(d) For each of the points marked on the following number lines, write the value as a decimal and as a fraction in simplest form.

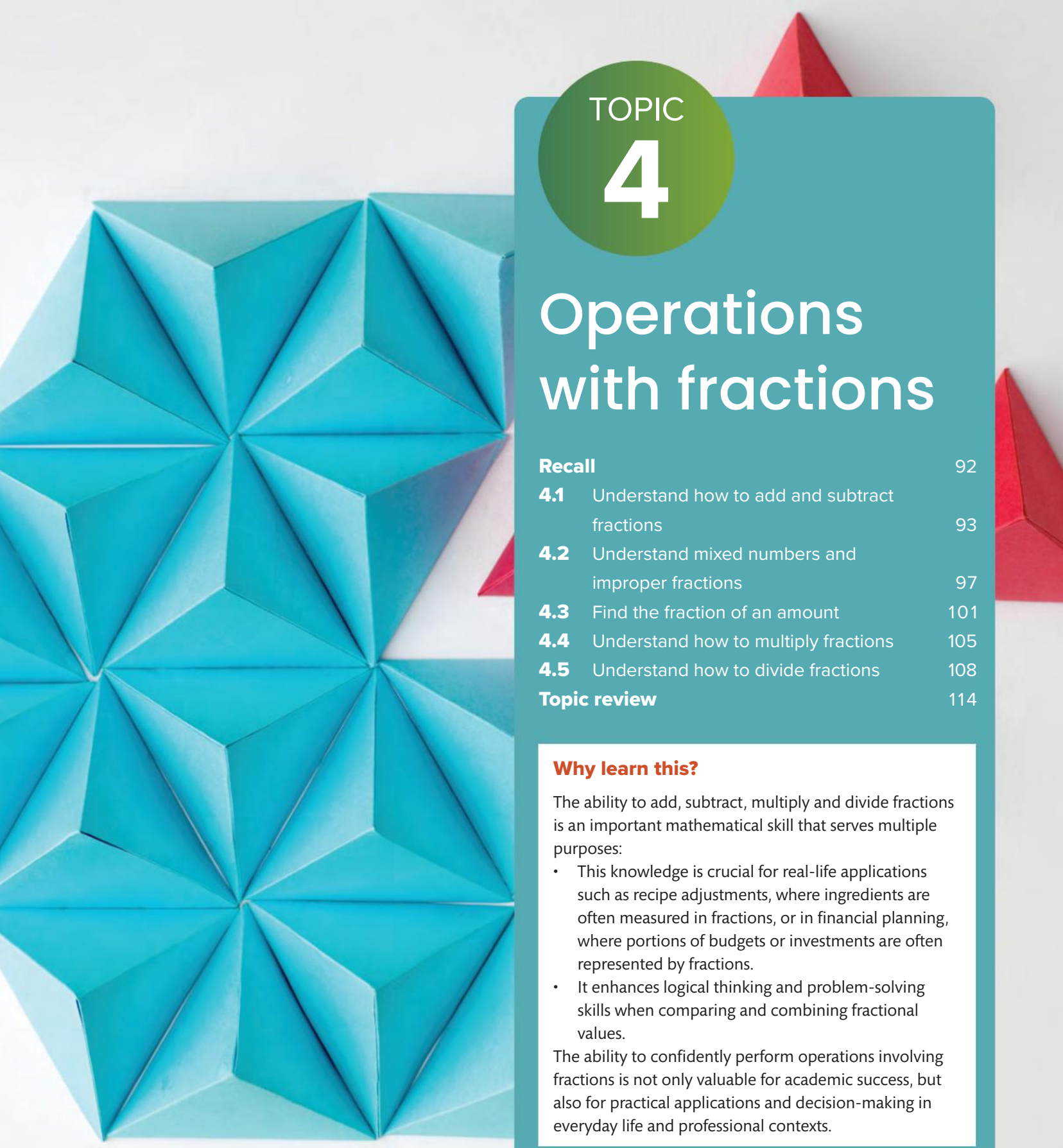
(i)



(ii)



(e) Draw a similar number line to show the location of 0.045.



TOPIC
4

Operations with fractions

Recall 92

4.1 Understand how to add and subtract fractions 93

4.2 Understand mixed numbers and improper fractions 97

4.3 Find the fraction of an amount 101

4.4 Understand how to multiply fractions 105

4.5 Understand how to divide fractions 108

Topic review 114

Why learn this?

The ability to add, subtract, multiply and divide fractions is an important mathematical skill that serves multiple purposes:

- This knowledge is crucial for real-life applications such as recipe adjustments, where ingredients are often measured in fractions, or in financial planning, where portions of budgets or investments are often represented by fractions.
- It enhances logical thinking and problem-solving skills when comparing and combining fractional values.

The ability to confidently perform operations involving fractions is not only valuable for academic success, but also for practical applications and decision-making in everyday life and professional contexts.

RECALL

ANSWERS Page 676

I can identify and compare multiples of numbers

1 List the first seven multiples of each of these numbers.

(a) 4 **(b)** 8 **(c)** 12

2 Identify any numbers that appear in all three lists for Question **1**, and state the name given to these numbers.

I can find the highest common factor for numbers

1 Write the largest number that divides into the following number groups with no remainder.

(a) 8, 6, 12 **(b)** 27, 36, 72 **(c)** 14, 28, 56 **(d)** 104, 39, 78

2 Identify the highest common factor for the numbers 18, 27 and 36 after first listing their factors.

I can multiply and divide decimals by powers of 10

1 Calculate the following.

(a) $1.6 \times 1000 =$ _____ **(b)** $1.32 \times 100 =$ _____ **(c)** $0.8 \times 10 =$ _____

(d) $3.67 \times 10 =$ _____ **(e)** $4.672 \times 1000 =$ _____ **(f)** $4.32 \times 10 =$ _____

2 Calculate the following.

(a) $4.32 \div 10 =$ _____ **(b)** $4.672 \div 1000 =$ _____ **(c)** $3.67 \div 10 =$ _____

(d) $0.8 \div 10 =$ _____ **(e)** $1.32 \div 100 =$ _____ **(f)** $1.6 \div 1000 =$ _____

I can apply factors and multiples to fraction denominators to determine equivalent representations

1 Find the missing values to form equivalent fractions.

(a) $\frac{1}{2} = \frac{_}{4} = \frac{_}{8} = \frac{_}{12}$ **(b)** $\frac{2}{3} = \frac{_}{6} = \frac{_}{9} = \frac{_}{12}$ **(c)** $\frac{_}{4} = \frac{_}{8} = \frac{9}{12}$ **(d)** $\frac{_}{3} = \frac{_}{6} = \frac{3}{9} = \frac{_}{12}$

Understand how to add and subtract fractions

4.1

Learning intention: To understand how to add and subtract fractions

Success criteria:

SC 1 I can add and subtract fractions with the same denominator.

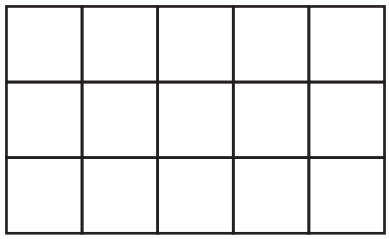
SC 2 I can add and subtract fractions with related denominators.

SC 3 I can add and subtract fractions with unrelated denominators.

Lesson warm-up

Grids for fractions

Use a grid like the one below to demonstrate the fraction addition $\frac{1}{3}+\frac{2}{5}$.



SC 1 I can add and subtract fractions with the same denominator

Just as two plus three equals five, so two-sevenths plus three-sevenths gives five-sevenths.

For two fractions that have the same denominator, adding the two fractions can be done by adding the numerators together.

For example, $\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$.

Subtraction is dealt with in the same way, except the numerator of the fraction on the right is subtracted from the numerator of the fraction on the left.

$\frac{5}{7}-\frac{3}{7}=\frac{2}{7}$

Worked example

Adding fractions with the same denominator

Evaluate $\frac{6}{11}+\frac{4}{11}$.

THINKING	WORKING
Determine whether the denominators are the same.	Yes, the denominators are the same.
For denominators that are the same, perform the calculation by writing the denominator unchanged and add the numerators.	$\frac{6}{11}+\frac{4}{11}=\frac{6+4}{11}$ $=\frac{10}{11}$

SC 2 I can add and subtract fractions with related denominators

Sometimes the denominators are not the same, but one is a multiple of the other. In cases like this, change the fraction with the smaller denominator to an equivalent fraction with a denominator that matches the larger value. Then proceed with addition or subtraction.

Worked example

Adding and subtracting fractions with related denominators

Evaluate $\frac{5}{6}-\frac{1}{12}$.

THINKING	WORKING
Rewrite the expression so that the denominators are the same (in this case, this will be the larger of the two denominators).	$\frac{5}{6}-\frac{1}{12}=\frac{5\times2}{6\times2}-\frac{1}{12}$ $=\frac{10}{12}-\frac{1}{12}$
Now complete the calculation.	$=\frac{9}{12}$
Simplify the fraction by dividing both the numerator and denominator by a common factor.	$=\frac{\cancel{9}^{3}}{\cancel{12}^{4}}$ $=\frac{3}{4}$

SC 3 I can add and subtract fractions with unrelated denominators

Most fraction calculations do not involve denominators that are related. However, the process to add or subtract them remains basically the same. Determine the lowest common denominator and write the fractions as equivalent fractions using this lowest common denominator.

Worked example

Adding and subtracting fractions with unrelated denominators

Calculate $\frac{4}{7}+\frac{3}{11}$.

THINKING	WORKING
Write the multiples of each denominator and identify the lowest common denominator.	Multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77 Multiples of 11: 11, 22, 33, 44, 55, 66, 77 LCD: 77
Rewrite the calculation using equivalent fractions with this common denominator.	$\frac{4}{7}+\frac{3}{11}=\frac{4\times11}{7\times11}+\frac{3\times7}{11\times7}$ $=\frac{44}{77}+\frac{21}{77}$
Write the answer, simplifying when possible.	$=\frac{65}{77}$

Practice

ANSWERS Page 676

SC 1 I can add and subtract fractions with the same denominator

1 Calculate the following fraction additions.

(a) $\frac{6}{13}+\frac{4}{13}$ **(b)** $\frac{9}{17}+\frac{3}{17}$ **(c)** $\frac{4}{15}+\frac{3}{15}$ **(d)** $\frac{11}{19}+\frac{3}{19}$

2 Calculate the following fraction subtractions.

(a) $\frac{5}{13}-\frac{2}{13}$ **(b)** $\frac{8}{17}-\frac{3}{17}$ **(c)** $\frac{7}{15}-\frac{6}{15}$ **(d)** $\frac{13}{19}-\frac{5}{19}$

(e) $\frac{3}{5}-\frac{4}{5}$ **(f)** $\frac{4}{9}-\frac{7}{9}$ **(g)** $\frac{4}{7}-\frac{6}{7}$ **(h)** $\frac{8}{15}-\frac{11}{15}$

SC 2 I can add and subtract fractions with related denominators

1 Calculate the following fraction subtractions.

(a) $\frac{3}{4}-\frac{1}{8}$ **(b)** $\frac{5}{14}-\frac{2}{7}$ **(c)** $\frac{4}{9}-\frac{5}{18}$ **(d)** $\frac{4}{5}-\frac{3}{10}$

2 Calculate the following fraction additions.

(a) $\frac{3}{5}+\frac{2}{10}$ **(b)** $\frac{5}{12}+\frac{5}{6}$ **c)** $\frac{8}{15}+\frac{4}{5}$ **(d)** $\frac{3}{8}+\frac{3}{4}$

3 Calculate the following.

(a) $\frac{4}{5}+\frac{4}{15}$ **(b)** $\frac{3}{7}-\frac{9}{14}$ **(c)** $\frac{6}{7}+\frac{4}{21}$ **(d)** $\frac{2}{9}-\frac{13}{36}$

4 Calculate the following.

(a) $\frac{2}{9}-\frac{5}{18}+\frac{7}{27}$ **(b)** $\frac{3}{8}+\frac{1}{16}-\frac{47}{48}$ **(c)** $\frac{6}{25}-\frac{4}{5}-\frac{3}{50}$ **(d)** $\frac{13}{24}-\frac{5}{8}-\frac{11}{96}$

5 Answer the following fraction problems.

(a) Addison ate $\frac{5}{14}$ of a large pizza and Ethan ate $\frac{1}{2}$ of it. How much was leftover?

(b) Audrey, Evelyn and Henry shared a bag of lollies. Audrey ate $\frac{2}{5}$ of them, while Evelyn ate $\frac{3}{10}$ of them. What fraction was left for Henry?

SC 3 I can add and subtract fractions with unrelated denominators

1 Calculate the following fraction additions.

(a) $\frac{3}{5}+\frac{5}{6}$ **(b)** $\frac{2}{9}+\frac{7}{11}$ **(c)** $\frac{4}{7}+\frac{8}{9}$ **(d)** $\frac{3}{4}+\frac{5}{9}$

2 Calculate the following fraction subtractions.

(a) $\frac{4}{7}-\frac{2}{5}$ **(b)** $\frac{10}{11}-\frac{2}{3}$ **(c)** $\frac{4}{9}-\frac{1}{5}$ **(d)** $\frac{13}{20}-\frac{7}{30}$

3 Calculate the following fraction calculations.

(a) $\frac{2}{5}+\frac{3}{4}-\frac{5}{6}$ **(b)** $\frac{3}{8}-\frac{5}{16}+\frac{1}{3}$ **(c)** $\frac{9}{11}+\frac{2}{3}-\frac{3}{4}$ **(d)** $\frac{8}{9}-\frac{5}{12}+\frac{5}{6}$

4 For each calculation, determine which student is correct and describe the common mistake made by the student who got the incorrect answer.

(a) $\frac{2}{3}+\frac{3}{7}=$

Student A	Student B
$\frac{2}{3}+\frac{3}{7}=\frac{2+3}{3+7}$ $=\frac{5}{21}$	$\frac{2}{3}+\frac{3}{7}=\frac{2\times7}{3\times7}+\frac{3\times3}{7\times3}$ $=\frac{14}{21}+\frac{9}{21}$ $=\frac{23}{21}$

(b) $\frac{7}{8}-\frac{1}{6}=$

Student A	Student B
$\frac{7}{8}-\frac{1}{6}=\frac{7\times3}{8\times3}-\frac{1\times4}{6\times4}$ $=\frac{21}{24}-\frac{4}{24}$ $=\frac{17}{24}$	$\frac{7}{8}-\frac{1}{6}=\frac{7-1}{8-6}$ $=\frac{6}{2}$ $=3$

Understand mixed numbers and improper fractions

4.2

Learning intention: To understand mixed numbers and improper fractions

Success criteria:

SC 1 I can convert between improper fractions and mixed numbers.

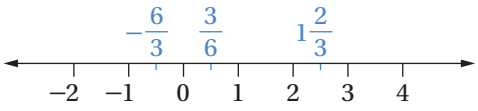
SC 2 I can add and subtract mixed numbers.

Lesson warm-up

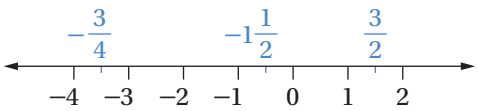
Correct position on a number line

Some of the numbers have been placed incorrectly on each number line.
Plot each of the points correctly on a number line.

(a)



(b)



SC 1 I can convert between improper fractions and mixed numbers

A simple fraction, or proper fraction, is one where the numerator is less than the denominator. For positive fractions, an example of a simple fraction between 0 and 1 is $\frac{2}{3}$. Positive fractions that are greater than 1 can be written in two ways: as a mixed number such as $1\frac{2}{5}$ or the equivalent improper fraction $\frac{7}{5}$.

Improper fractions and mixed numbers

An improper fraction has a numerator larger than its denominator. In the diagram, there are 19 quarters shaded, which is $\frac{19}{4}$ as an improper fraction.

Mixed numbers are written in two parts: one part is a whole number and the other is a proper fraction. Another way to describe the number of shaded parts in the diagram is by using a mixed number. There are 4 wholes and $\frac{3}{4}$ shaded, or $4\frac{3}{4}$. So, the improper fraction $\frac{19}{4}$ is the same as $4\frac{3}{4}$.

Worked example

Converting a mixed number to an improper fraction

Convert $3\frac{2}{5}$ to improper fraction form.

THINKING	WORKING
Write the whole number as a fraction using the denominator given.	$3=\frac{3}{1}$ $=\frac{3}{1}\times\frac{5}{5}$ $=\frac{15}{5}$
Add the fraction part to the newly found equivalent for the whole number part to complete the conversion.	$3\frac{2}{5}=\frac{15}{5}+\frac{2}{5}$ $=\frac{17}{5}$
Alternatively, multiply the whole number by the denominator, then add the numerator, putting the value over the existing denominator.	$3\frac{2}{5}=\frac{3\times5+2}{5}$ $=\frac{15+2}{5}$ $=\frac{17}{5}$

Worked example

Converting an improper fraction to a mixed number

Convert $\frac{22}{6}$ to a mixed number.

THINKING	WORKING
A fraction represents the numerator divided by the denominator.	$22 \div 6$
Divide the numerator by the denominator. If there is a remainder, write it down.	$6\overline{)22}$ $\begin{matrix}3\\ \\ \end{matrix}$ remainder 4
The mixed number is formed combining the whole and the remainder written as a fraction of the denominator.	3 wholes and $\frac{4}{6}$
Write the answer.	$\frac{22}{6}=3\frac{4}{6}=3\frac{2}{3}$

SC 2 I can add and subtract mixed numbers

Worked example

Adding and subtracting mixed numbers

Calculate the value of $2\frac{2}{3}-1\frac{1}{4}$.

Method 1: Units and fractions separately

THINKING	WORKING
Rewrite the calculation in expanded form.	$2\frac{2}{3}-1\frac{1}{4}=2+\frac{2}{3}-\left(1+\frac{1}{4}\right)$ $=2+\frac{2}{3}-1-\frac{1}{4}$ $=2-1+\frac{2}{3}-\frac{1}{4}$
Write the fractions with the same denominator.	$=1+\frac{2\times4}{3\times4}-\frac{1\times3}{4\times3}$ $=1+\frac{8}{12}-\frac{3}{12}$
Write the answer.	$=1+\frac{5}{12}$ $=1\frac{5}{12}$

Method 2: Use improper fractions

THINKING	WORKING
Convert the mixed numbers to improper fractions.	$2\frac{2}{3}-1\frac{1}{4}=\frac{8}{3}-\frac{5}{4}$
Make equivalent fractions that have the lowest common denominator (LCD).	$=\frac{8\times4}{3\times4}-\frac{5\times3}{4\times3}$ $=\frac{32}{12}-\frac{15}{12}$
Subtract fractions with the same denominator by subtracting the numerators.	$=\frac{32}{12}-\frac{15}{12}$ $=\frac{17}{12}$
Convert the improper fraction to a mixed number.	$=1\frac{5}{12}$

Practice

ANSWERS Page 676

SC 1 I can convert between improper fractions and mixed numbers

1 Convert the following mixed numbers into improper fractions.

(a) $2\frac{2}{7}$ **(b)** $3\frac{4}{11}$ **(c)** $4\frac{5}{9}$ **(d)** $12\frac{1}{6}$

2 Convert the following improper fractions to mixed number form.

(a) $\frac{29}{5}$ **(b)** $\frac{47}{6}$ **(c)** $\frac{81}{11}$ **(d)** $\frac{65}{12}$

3 Negative fractions are dealt with in exactly the same way, except a negative sign is involved. Convert the following mixed numbers to improper fraction form.

(a) $-5\frac{5}{6}$ **(b)** $-4\frac{7}{11}$ **(c)** $-10\frac{1}{9}$ **(d)** $-12\frac{2}{3}$

4 Convert the following improper fractions to mixed number form.

(a) $-\frac{58}{9}$ **(b)** $-\frac{49}{8}$ **(c)** $-\frac{103}{10}$ **(d)** $-\frac{79}{11}$

5 When adding fractions, the result is sometimes an improper fraction. It is usual to convert these improper fractions to mixed number form. The fraction part should also be simplified if possible. Solve the following fraction calculations.

(a) $\frac{2}{5}+\frac{4}{5}$ **(b)** $\frac{5}{9}+\frac{7}{9}$ **(c)** $\frac{3}{8}+\frac{5}{8}$ **(d)** $\frac{4}{7}+\frac{6}{7}$

SC 2 I can add and subtract mixed numbers

1 Calculate the addition of the following mixed numbers.

(a) $1\frac{1}{4}+2\frac{3}{5}$ **(b)** $4\frac{2}{9}+7\frac{5}{18}$ **(c)** $8\frac{3}{5}+10\frac{2}{3}$ **(d)** $11\frac{5}{7}+21\frac{5}{14}$

2 Calculate the following mixed number subtractions.

(a) $4\frac{1}{3}-1\frac{5}{6}$ **(b)** $6\frac{3}{13}-2\frac{4}{9}$ **(c)** $8\frac{1}{4}-5\frac{4}{7}$ **(d)** $11\frac{5}{11}-4\frac{1}{2}$

3 Solve the following word questions using fractions.

(a) A 50 m rope has two pieces cut from it that are $10\frac{1}{4}$ m and $21\frac{2}{3}$ m respectively. What length of rope remains?

(b) Olivia has a part-time job at the local bakery. One week she worked three days, with her hours being $6\frac{1}{3}$, $5\frac{3}{4}$ and $4\frac{5}{6}$ hours respectively. For how many hours did she work this week?

4 Given $a=2\frac{1}{3}$, $b=3\frac{2}{5}$ and $c=1\frac{7}{15}$, calculate the value of each of the following expressions.

(a) $a+b-c$ **(b)** $b-(a+c)$ **(c)** $c+b-a$ **(d)** $a-b-c$

Find the fraction of an amount

4.3

Learning intention: To be able to find the fraction of an amount

Success criteria:

SC 1 I can multiply fractions by whole numbers.

SC 2 I can find a fraction of an amount.

Lesson warm-up

Lollipop fractions

Use the diagram of twelve lollipops to show that $\frac{5}{6}$ of $\frac{1}{2}$ is $\frac{5}{12}$.

SC 1 I can multiply fractions by whole numbers

For solving calculations such as $5 \times \frac{2}{3}$, it is helpful to think about the multiplication symbol being the same as the expression 'lots of'. So, this calculation can be thought of as 5 lots of $\frac{2}{3}$. This is the same as $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$, which is equal to $\frac{10}{3}$ or $3\frac{1}{3}$.

There is a shortcut that can be taken: multiply the whole number by the numerator of the fraction part and put it over the denominator. Then simplify the answer or put it into mixed number form.

Worked example

Multiplying a whole number by a fraction

Calculate the value of $6 \times \frac{4}{5}$.

THINKING	WORKING
Multiply the whole number by the numerator and put it over the denominator.	$6 \times \frac{4}{5} = \frac{6 \times 4}{5}$ $= \frac{24}{5}$
Convert to mixed number form.	$= 4\frac{4}{5}$

Worked example

Multiplying a whole number by a mixed number

Calculate the value of $6 \times 1\frac{4}{5}$.

THINKING	WORKING
Write the mixed number as an improper fraction.	$6 \times 1\frac{4}{5} = 6 \times \frac{9}{5}$
Multiply the whole number by the numerator and put it over the denominator.	$= \frac{6 \times 9}{5}$ $= \frac{54}{5}$
Convert to mixed number form.	$= 10\frac{4}{5}$

SC 2 I can find a fraction of an amount

To calculate $\frac{1}{3}$ of 28, for instance, rewrite the question as a multiplication.

This gives $\frac{1}{3} \times 28$.

Recall that $2 \times 3 = 3 \times 2$, so $\frac{1}{3} \times 28 = 28 \times \frac{1}{3}$. Then proceed in exactly the same way as in the previous section.

Worked example

Finding a fraction of a quantity

Calculate $\frac{3}{4}$ of \$36.

THINKING	WORKING
Write as a fraction multiplication and put the whole number over a denominator of 1.	$\frac{3}{4} \times \frac{36}{1}$
Simplify by dividing by common factors when possible.	$\frac{3}{4} \times \frac{36}{1} = \frac{3}{\cancel{4}^{1}} \times \frac{\cancel{36}^{9}}{1}$
Complete the calculation.	$= \frac{27}{1}$
Write the answer.	$\frac{3}{4}$ of \$36 is \$27.

Practice

ANSWERS Page 676

SC 1 I can multiply fractions by whole numbers

1 Calculate the value of the following products.

(a) $3\times\frac{2}{5}$ **(b)** $8\times\frac{6}{7}$ **(c)** $10\times\frac{1}{5}$ **(d)** $12\times\frac{2}{3}$

2 Calculate the value of the following products.

(a) $7\times4\frac{1}{5}$ **(b)** $8\times3\frac{5}{6}$

3 Copy and complete part **(a)** to establish a method for estimating fractional expressions to the nearest whole number. Use the same process to determine the nearest whole number in parts **(b)** to **(d)**.

(a) $8\times2\frac{1}{9}=8\times\left(2+\frac{1}{9}\right)$

$=8\times2+8\times\frac{1}{9}$

$=\square+\frac{8}{9}$

$=\square\frac{\square}{\square}$

The nearest whole number is $\square$.

(b) $4\times6\frac{2}{3}$ **(c)** $12\times3\frac{1}{5}$ **(d)** $21\times5\frac{6}{7}$

4 Solve the following problems.

(a) A piece of rope was cut into 5 equal pieces, each $2\frac{2}{3}$ m in length, with no rope left over. How long was the rope originally?

(b) A different piece of rope was cut into 6 equal pieces, each $3\frac{1}{5}$ m in length, with $1\frac{1}{4}$ m of rope left over. How long was the rope originally?

SC 2 I can find a fraction of an amount

1 Calculate the following.

(a) $\frac{2}{3}$ of \$51 **(b)** $\frac{3}{5}$ of 25 m **(c)** $\frac{4}{7}$ of 24 minutes **(d)** $\frac{8}{11}$ of 460 L

2 Draw the objects and then circle the fraction of the following quantities.

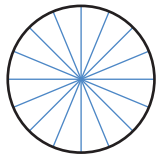
(a) $\frac{1}{3}$ of the counters

(b) $\frac{1}{5}$ of the stars

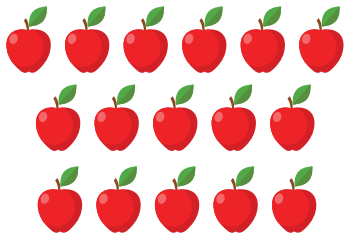
(c) $\frac{1}{6}$ of the counters

(d) $\frac{1}{3}$ of the rope

(e) $\frac{1}{8}$ of the circle



(f) $\frac{1}{4}$ of the apples



3 Calculate the following products.

(a) $\frac{3}{5}$ of $2\frac{1}{4}$ cups **(b)** $\frac{2}{9}$ of $3\frac{2}{3}$ L **(c)** $\frac{2}{3}$ of $8\frac{6}{7}$ kg **(d)** $\frac{8}{11}$ of $4\frac{6}{7}$ m

4 Calculate the following fractions of fractions of quantities.

(a) $\frac{1}{3}$ of $\frac{1}{3}$ of 9 **(b)** $\frac{1}{4}$ of $\frac{3}{4}$ of 64 **(c)** $\frac{1}{4}$ of $\frac{1}{5}$ of 120 **(d)** $\frac{1}{9}$ of $\frac{3}{5}$ of 900

5 A box has 18 chocolates.

(a) How many chocolates are in $\frac{1}{2}$ of the box? **(b)** How many chocolates are in $\frac{1}{3}$ of the box?

(c) How many chocolates are in $\frac{1}{6}$ of the box? **(d)** How many chocolates are in $\frac{1}{9}$ of the box?

(e) How many chocolates are in $\frac{1}{18}$ of the box?

Understand how to multiply fractions

4.4

Learning intention: To understand how to multiply fractions

Success criteria:

SC 1 I can multiply fractions using an area model.

SC 2 I can multiply fractions using numerical methods.

Lesson warm-up

Greater or less?

Decide whether the following statement is sometimes true, always true or never true. Give examples to explain your answer.

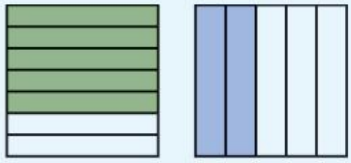
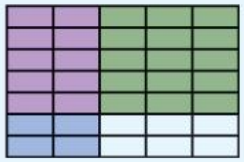
"When you multiply a number by another number, the result is always larger."

SC 1 I can multiply fractions using an area model

Worked example

Multiplying fractions using an area model

Calculate $\frac{5}{7} \times \frac{2}{5}$ using an area model.

THINKING	WORKING
Draw two squares the same size. Divide the first square into rows, shaded to represent the first fraction. Divide the second square into columns, shaded to represent the second fraction.	First square: 7 rows with 5 shaded, $\left(\frac{5}{7}\right)$. Second square: 5 columns with 2 shaded, $\left(\frac{2}{5}\right)$. 
Draw the two squares overlapped to represent one fraction multiplying the fractions. The number of cells overlapped (with both colours) represents the numerator of the product. The total number of cells in the whole square represents the denominator.	Overlapping squares: 7 rows (5 shaded) with 5 columns (2 shaded).  Total overlapping cells (numerator): $5 \times 2 = 10$ Total cells in square (denominator): $7 \times 5 = 35$

Write the result with the new numerator and denominator. You can simplify the result by cancelling common factors.	$\frac{10}{35}=\frac{2\times5}{7\times5}=\frac{2}{7}$

SC 2 I can multiply fractions using numerical methods

Worked example

Multiply fractions using numerical methods

Use a numerical approach to find $\frac{2}{3}$ of $\frac{9}{11}$.

THINKING	WORKING
Write the calculation as a multiplication and simplify when possible.	$\frac{2}{3}\times\frac{9}{11}=\frac{2}{\cancel{3}^{1}}\times\frac{\cancel{9}^{3}}{11}$
Simplify the calculation.	$=\frac{2\times3}{1\times11}$ $=\frac{6}{11}$

Worked example

Finding a fraction of a mixed number

Use a numerical approach to determine $\frac{3}{5}$ of $1\frac{7}{8}$.

THINKING	WORKING
Write the calculation as a multiplication of the fraction and an improper fraction and simplify when possible.	$\frac{3}{5}\times1\frac{7}{8}=\frac{3}{5}\times\frac{15}{8}$ $=\frac{3}{\cancel{5}^{1}}\times\frac{\cancel{15}^{3}}{8}$
Complete the calculation, writing the result as a mixed number.	$=\frac{3\times3}{1\times8}$ $=\frac{9}{8}$ $=1\frac{1}{8}$

Practice

ANSWERS Page 677

SC 1 I can multiply fractions using an area model

1 Investigate the following model.

(a) Use a copy of the array to find $\frac{1}{3}$ of $\frac{1}{2}$.

(b) Use another copy of the array to find $\frac{1}{3}$ of $\frac{1}{4}$.

(c) Use another copy of the array to find $\frac{3}{4}$ of $\frac{5}{6}$.

SC 2 I can multiply fractions using numerical methods

1 Complete the following calculations.

(a) $\frac{1}{3}\times\frac{1}{7}$ **(b)** $\frac{2}{5}\times\frac{1}{8}$ **(c)** $\frac{3}{7}\times\frac{5}{12}$

(d) $\frac{4}{9}\times\frac{18}{23}$ **(e)** $\frac{5}{11}\times\frac{22}{25}$ **(f)** $\frac{6}{25}\times\frac{15}{36}$

2 Complete the following calculations and write your answers in the simplest form, using mixed numbers when possible.

(a) $\frac{4}{5}\times 2\frac{7}{9}$ **(b)** $\frac{2}{5}\times 1\frac{9}{16}$ **(c)** $\frac{3}{7}\times 1\frac{2}{19}$ **(d)** $\frac{4}{9}\times 5\frac{1}{7}$

3 Complete the following calculations and write your answers in the simplest form, using mixed numbers if necessary.

(a) $1\frac{2}{5}\times 2\frac{3}{4}$ **(b)** $4\frac{1}{2}\times 1\frac{3}{5}$ **(c)** $2\frac{2}{3}\times 4\frac{5}{7}$ **(d)** $5\frac{5}{6}\times 3\frac{1}{5}$

4 A rope is damaged and frayed (becoming untwisted) at one end.

Jacinda finds that $\frac{1}{10}$ of the rope is frayed, leaving $\frac{9}{10}$ undamaged. Jacinda cuts off $\frac{1}{3}$ of the undamaged section of the rope.

(a) What fraction of the whole rope does Jacinda now have?

(b) The rope was originally 25 m in length. What length of rope does Jacinda have?

4.5 Understand how to divide fractions

Learning intention: To understand how to divide fractions

Success criteria:

SC 1 I can divide an integer by a fraction using a diagram.

SC 2 I can divide an integer by a fraction by multiplying by the reciprocal.

SC 3 I can divide a fraction by an integer.

SC 4 I can divide fractions.

Lesson warm-up

Sharing again and again?

What is the result when a fraction is divided by a fraction? Consider the following statement:

When a fraction is divided by another fraction, the result is always less than the original quantity.

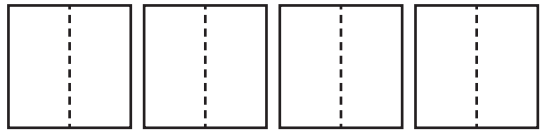
Do you agree with the statement? Investigate divisions of fractions to decide.

SC 1 I can divide an integer by a fraction using a diagram

Divisions can be thought of as 'how many' questions. Using a whole number example, $24 \div 6$ can be thought of as 'twenty-four, how many sixes?' or 'how many sixes are there in twenty-four?' In either case, the answer is 4.

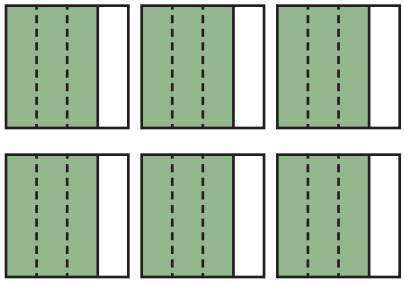
So, $4 \div \frac{1}{2}$ can be thought of as 'how many halves are there in four?'.

This can be demonstrated using a diagram of 4 wholes, each divided in half.



There are 8 halves in total: $4 \div \frac{1}{2} = 8$.

Another example is the following diagram, which can be used to determine the answer to $6 \div \frac{3}{4}$.



How many lots of $\frac{3}{4}$ are in 6? There are 6 lots shaded. The three unshaded pieces on the top row make another lot of $\frac{3}{4}$ as do the three unshaded pieces in the bottom row.

In total, there are 8 lots of $\frac{3}{4}$, $6 \div \frac{3}{4} = 8$.

Worked example

Dividing an integer by a fraction using a diagram

Use a diagram to calculate $3 \div \frac{1}{5}$.

THINKING	WORKING
Draw the appropriate number of wholes (in this case 3) and divide each into the number of pieces indicated by the denominator of the fraction (in this case 5).	
Count the number of pieces.	There are 15 equal pieces.
Write the solution.	$3 \div \frac{1}{5} = 15$

SC 2 I can divide an integer by a fraction by multiplying by the reciprocal

Another way of looking at the question $6 \div \frac{3}{4}$ is to ask 'how many lots of $\frac{1}{4}$ are there in 6 wholes?'. The answer is 24.

Since $\frac{3}{4}$ is 3 lots of $\frac{1}{4}$, there will only be $\frac{1}{3}$ the number of lots of $\frac{3}{4}$.

Putting this together gives:

$$\begin{aligned} 6 \div \frac{3}{4} &= 6 \times \frac{4}{1} \times \frac{1}{3} \\ &= \frac{6^{2} \times 4}{\cancel{3}^{1}} \\ &= \frac{2 \times 4}{1} \\ &= 8 \end{aligned}$$

This shows that $6 \div \frac{3}{4} = 6 \times \frac{4}{3}$. The two fractions $\frac{3}{4}$ and $\frac{4}{3}$ are known as the reciprocals of each other. The numerator and denominator are flipped.

Worked example

Dividing by fractions by multiplying by the reciprocal

Calculate $4 \div \frac{2}{5}$.

THINKING	WORKING
Rewrite the question by changing the division to multiplication and using the reciprocal of the right-hand value.	$4 \div \frac{2}{5} = 4 \times \frac{5}{2}$
Simplify by dividing by any common factors when possible and then evaluate the answer.	$= \frac{\cancel{4}^{2}}{1} \times \frac{5}{\cancel{2}^{1}}$ $= \frac{2 \times 5}{1 \times 1}$ $= 10$

SC 3 I can divide a fraction by an integer

To divide a fraction by an integer, apply the process covered in the last section of turning a division into a multiplication by the reciprocal.

The reciprocal of 2 is $\frac{1}{2}$, since 2 can be written as $\frac{2}{1}$.

Worked example

Dividing a fraction by an integer

Evaluate $\frac{3}{4} \div 2$.

THINKING	WORKING
Rewrite the question so that you are multiplying by the reciprocal.	$\frac{3}{4} \div 2 = \frac{3}{4} \div \frac{2}{1}$ $= \frac{3}{4} \times \frac{1}{2}$
Simplify when possible and write the answer.	$= \frac{3 \times 1}{4 \times 2}$ $= \frac{3}{8}$

SC 4 I can divide fractions

To divide a fraction by a fraction, including mixed numbers in either or both positions, the process is the same as for simpler division:

- change the operation to multiplication
- multiply by the reciprocal of the right-hand fraction.

It is important that you do not use the reciprocal of both fractions and remember to change mixed numbers to improper fractions before you start.

Worked example

Dividing a fraction by a fraction

Calculate the value of $\frac{2}{3} \div \frac{3}{4}$.

THINKING	WORKING
Rewrite the question so that you are multiplying by the reciprocal.	$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3}$
Simplify when possible and write the answer.	$= \frac{2 \times 4}{3 \times 3}$ $= \frac{8}{9}$

Worked example

Dividing a mixed number by a mixed number

Calculate the value of $3\frac{2}{3} \div 1\frac{2}{5}$.

THINKING	WORKING
Write the question using improper fractions.	$3\frac{2}{3} \div 1\frac{2}{5} = \frac{11}{3} \div \frac{7}{5}$
Rewrite the question so that you are multiplying by the reciprocal.	$= \frac{11}{3} \times \frac{5}{7}$
Simplify when possible and write the answer.	$= \frac{55}{21}$ $= 2\frac{13}{21}$

Practice

ANSWERS Page 677

SC 1 I can divide an integer by a fraction using a diagram

1 Use a diagram to calculate each of the following.

(a) $5 \div \frac{1}{4}$ **(b)** $2 \div \frac{1}{6}$

2 Four quarters of a tomato is the same amount as one whole tomato.

Write each of the following questions as a whole number divided by a fraction, and calculate the answer.

(a) How many quarter tomatoes are there in 2 whole tomatoes?

(b) How many quarter tomatoes are there in 3 whole tomatoes?

(c) How many half tomatoes are there in 3 whole tomatoes?

(d) How many groups of $\frac{3}{4}$ can be made from 3 whole tomatoes?

(e) The tomatoes are instead cut into 5 equal pieces. Given that you have 4 whole tomatoes, how many salads can be made using $\frac{2}{5}$ of a tomato each?

SC 2 I can divide an integer by a fraction by multiplying by the reciprocal

1 Calculate each of the following by multiplying by the reciprocal.

(a) $10 \div \frac{2}{3}$ **(b)** $8 \div \frac{4}{5}$ **(c)** $12 \div \frac{6}{7}$ **(d)** $15 \div \frac{5}{9}$

2 Calculate each of the following by multiplying by the reciprocal. Write the answers as mixed numbers.

(a) $6 \div \frac{4}{5}$ **(b)** $9 \div \frac{5}{7}$

3 Calculate each of the following by multiplying by the reciprocal. As a first step, convert the mixed number to an improper fraction.

(a) $6 \div 2\frac{2}{3}$ **(b)** $9 \div 1\frac{3}{8}$

SC 3 I can divide a fraction by an integer

1 Calculate the value of each of the following.

(a) $\frac{4}{5} \div 3$ **(b)** $\frac{2}{9} \div 4$ **(c)** $\frac{3}{7} \div 2$ **(d)** $\frac{5}{11} \div 6$

2 Calculate the value of each of the following.

(a) $1\frac{2}{5} \div 6$ **(b)** $2\frac{2}{9} \div 3$ **(c)** $4\frac{1}{3} \div 3$ **(d)** $6\frac{2}{5} \div 4$

3 Determine which student is correct and explain the error made by the student who answered incorrectly.

(a) $\frac{10}{11} \div 2 =$

Student A	Student B
$\frac{10}{11} \div 2 = \frac{10}{11} \times \frac{2}{1}$ $= \frac{20}{11}$ $= 1\frac{9}{11}$	$\frac{10}{11} \div 2 = \frac{10}{11} \times \frac{1}{2}$ $= \frac{10}{22}$ $= \frac{5}{11}$

(b) $10 \div \frac{5}{8} =$

Student A	Student B
$10 \div \frac{5}{8} = 10 \times \frac{8}{5}$ $= (5 \times 2) \times \frac{8}{5 \times 1}$ $= 2 \times \frac{8}{1}$ $= 16$	$10 \div \frac{5}{8} = (5 \times 2) \div \frac{5}{4 \times 2}$ $= 5 \times \frac{5}{4}$ $= \frac{25}{4}$

4 Calculate the value of each of the following.

(a) $2\frac{3}{4} \div 6 \times \frac{2}{3}$ **(b)** $5 \div 3\frac{3}{5} \times \frac{5}{8}$ **(c)** $3\frac{1}{2} \times 1\frac{1}{4} \div 7$

SC 4 I can divide fractions

1 Calculate the value of each of the following fraction divisions.

(a) $\frac{3}{5} \div \frac{2}{7}$ **(b)** $\frac{4}{7} \div \frac{8}{9}$ **(c)** $\frac{3}{11} \div \frac{9}{16}$ **(d)** $\frac{7}{9} \div \frac{14}{81}$

2 Calculate the value of each of the following fraction divisions.

(a) $1\frac{1}{4} \div 2\frac{1}{5}$ **(b)** $3\frac{2}{3} \div 1\frac{4}{5}$ **(c)** $6\frac{2}{3} \div 1\frac{5}{6}$ **(d)** $3\frac{1}{5} \div 4\frac{2}{3}$

3 $\frac{4}{5}$ of a number is 200.

(a) What is $\frac{1}{5}$ of the number? **(b)** What is the number?

4 Given $A = 1\frac{2}{3}$, $B = 2\frac{1}{4}$, $C = 3\frac{5}{6}$, evaluate each of the following expressions.

(a) $A + B - C$ **(b)** $A \times B \div C$ **(c)** $A \times B + C$

(d) $A \times (B + C)$ **(e)** $B \div (A + C)$ **(f)** $A - C \div B$

TOPIC REVIEW

Multiple choice

ANSWERS Page 739

1 $\frac{1}{9}+\frac{5}{6}=$

A $\frac{6}{15}$ **B** $\frac{6}{9}$ **C** $\frac{17}{18}$ **D** 1

2 $\frac{12}{7}-1\frac{5}{14}=$

A $-\frac{3}{14}$ **B** $-\frac{1}{2}$ **C** $\frac{5}{14}$ **D** $1\frac{1}{14}$

3 $2\frac{2}{3}-\frac{3}{4}=$

A 5 **B** $\frac{41}{12}$ **C** $1\frac{11}{12}$ **D** $2\frac{5}{7}$

4 $7-\frac{3}{80}=$

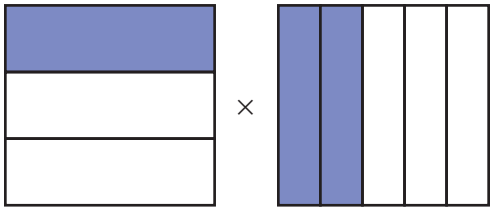
A $4\frac{7}{80}$ **B** $6\frac{3}{80}$ **C** $6\frac{7}{80}$ **D** $6\frac{77}{80}$

5 A unit fraction is a fraction that has 1 as its numerator, such as $\frac{1}{2}$ or $\frac{1}{7}$. Any unit fraction can be written as the sum of unit fractions.

Which of the following are correct? (There may be more than one correct answer.)

A $\frac{1}{2}=\frac{1}{3}+\frac{1}{6}$ **B** $\frac{1}{3}=\frac{1}{4}+\frac{1}{12}$ **C** $\frac{1}{4}=\frac{1}{5}+\frac{1}{6}$ **D** $\frac{1}{5}=\frac{1}{6}+\frac{1}{30}$

6 What multiplication is shown in the following area diagram?



A $\frac{1}{3}\times\frac{2}{4}$ **B** $\frac{2}{3}\times\frac{2}{5}$ **C** $\frac{3}{1}\times\frac{5}{2}$ **D** $\frac{1}{3}\times\frac{2}{5}$

7 Which of the following shows the result for $\frac{1}{3}\times\frac{9}{11}$ expressed in simplest form?

A $\frac{9}{3}$ **B** $\frac{3}{11}$ **C** $\frac{3}{33}$ **D** $\frac{5}{7}$

8 A proper fraction is multiplied by another proper fraction. What is the result?

A A mixed number

B An improper fraction

C A fraction that is bigger than either of the original fractions

D A fraction that is smaller than either of the original fractions

9 $\frac{4}{5} \div \frac{1}{3} =$

A $1\frac{1}{3}$ **B** $\frac{3}{5}$ **C** $\frac{5}{12}$ **D** $2\frac{2}{5}$

10 $\frac{1}{5} \div 1\frac{2}{3} =$

A $\frac{3}{25}$ **B** $\frac{3}{5}$ **C** $\frac{1}{3}$ **D** $8\frac{1}{3}$

11 Select the correct method for calculating $\frac{2}{7} \div \frac{7}{3}$.

A Cancel the 7 in the numerator with the 7 in the denominator

B Change the improper fraction into a mixed number

C Instead of dividing, multiply by the reciprocal of the second fraction

D Write the reciprocal of the first fraction and multiply

12 Omar has 35 biscuits and eats 7. He then gives away one-quarter of the remaining biscuits and keeps the rest to eat later. How many biscuits did Omar give away?

A 28 **B** 21 **C** 14 **D** 7

13 A quarter of a cake is to be shared equally between three people. Which of the following shows a correct calculation for the fraction of the original cake that is given to each person?

A $3 \div 4$ **B** $3 \div \frac{1}{4}$ **C** $\frac{1}{4} \div \frac{1}{3}$ **D** $\frac{1}{4} \div 3$

14 Nick shared a punnet of strawberries with his dad: $\frac{2}{5}$ for his dad and $\frac{3}{5}$ for Nick. Given that his dad ate all 10 of the strawberries he was given, how many strawberries were in the full punnet?

A 15 **B** 20 **C** 25 **D** 30

Short answer

1 Calculate:

(a) $\frac{4}{5} - \frac{6}{11}$ **(b)** $\frac{3}{16} + \frac{11}{24}$

2 Calculate:

(a) $3\frac{2}{5} - 2 + 1\frac{1}{4}$ **(b)** $3\frac{1}{4} - 4\frac{3}{7} + 2\frac{1}{2}$

3 Calculate $\frac{4}{5} - 2\frac{1}{4}$.

4 Use subtraction to determine the missing fraction: $\frac{28}{30} = \frac{1}{2} + \frac{1}{3} + \frac{\square}{\square}$.

5 Calculate each of the following, expressing your answers in simplest form.

(a) $1\frac{1}{3} \times \frac{3}{16}$ **(b)** $\frac{4}{5} \times \frac{15}{16} \times 8$

6 There are 36 pencils in a box, but $\frac{2}{3}$ of them fall out. How many pencils are left in the box?

7 On her computer, Mali draws a hand that is $\frac{2}{3}$ as long as her real hand. She then shrinks this image to $\frac{1}{10}$ of its original length. How long is the final image as a fraction of Mali's real hand length? Give your answer in simplest form.

8 In a class of 30 students, $\frac{2}{3}$ play a team sport. Of those players, $\frac{3}{5}$ play soccer. How many students play a team sport and how many play soccer?

9 Your four-year-old cousin wants a nice chair that looks exactly like their favourite doll's chair. The doll's chair is $13\frac{1}{2}$ cm high, but your cousin is $5\frac{1}{9}$ times as tall as the doll. To fit your cousin, how high should you make the chair?

10 Complete the following calculations, expressing your answers in simplest form.

(a) $\frac{1}{6} \div \frac{5}{6}$ **(b)** $5 \div \frac{7}{8}$ **(c)** $5 \div 1\frac{1}{4}$ **(d)** $2\frac{2}{3} \div 3$

11 State whether the following statements are True or False.

(a) $\frac{3}{5} \times \frac{4}{9} > \frac{1}{2}$ **(b)** $\frac{3}{4} \times \frac{3}{5} < \frac{1}{2}$ **(c)** $2\frac{2}{5} \div 1\frac{9}{10} > 1$ **(d)** $\frac{16}{5} \div \frac{12}{7} < 1$

12 Calculate the value of each of the following.

(a) $\frac{8}{9} - \frac{3}{4} \times \frac{1}{3}$ **(b)** $\frac{3}{5} \div \frac{1}{5} - \frac{4}{5}$ **(c)** $\frac{31}{50} + \frac{1}{5} \times \frac{2}{5}$

13 Arrange the following calculations in order from smallest to largest (ascending order).

$1\frac{1}{7} \times 21$; $\frac{3}{5} \times \left(2 - \frac{7}{3}\right)$; $9 \div \frac{1}{3}$; $20 \times \frac{4}{5}$; $\frac{7}{16} - \frac{1}{4}$

14 Over the course of a basketball tournament, five players equally share $8\frac{1}{3}$ navel oranges, $2\frac{1}{4}$ blood oranges and $4\frac{5}{8}$ Valencia oranges. How many oranges did each player consume?

15 A pet shop purchases cat food in containers with $40\frac{1}{2}$ kg of the food. The store repackages the cat food into packets containing $2\frac{1}{4}$ kg of food. Given that the pet shop purchases 5 containers of cat food, how many smaller packages will it be able to make?

16 Calculate each of the following, giving your answers in simplest form, including mixed numbers if appropriate.

(a) $\frac{12}{11}$ of \$55 **(b)** $2\frac{1}{7}\times 5\frac{2}{3}$ **(c)** $3\frac{4}{5}\div\frac{3}{10}$ **(d)** $4\frac{2}{3}\div 14$

17 Simplify the following by using a correct order of operations. Write your answers in simplest form, and as mixed numbers if appropriate.

(a) $\frac{4}{7}\times\frac{5}{8}+7$ **(b)** $\frac{6}{7}\div 3\times\frac{7}{8}$ **(c)** $\frac{2}{3}-\frac{4}{5}\times\frac{3}{4}$ **(d)** $\frac{6}{11}+\frac{8}{5}\div\frac{13}{5}$

Extended response

1 Mitchell is making banana bread. The recipe says to use:

- $1\frac{1}{4}$ cups of mashed banana
- $\frac{1}{3}$ cup of melted butter
- one egg $\left(\frac{1}{6}\text{ cup}\right)$
- $\frac{1}{2}$ cup of sugar
- $1\frac{3}{4}$ cups of flour

Calculate the total number of cups of ingredients in this recipe to help Mitchell choose a large enough bowl in which to mix the ingredients.

2 Samira is creating clay flowerpots. She has $1\frac{7}{9}$ packets of clay. Each clay pot uses $\frac{2}{5}$ of a packet of clay.

(a) How many whole flowerpots can Samira create?

(b) What fraction of a flowerpot could Samira create with the remaining clay?

(c) What fraction of a packet of clay does Samira have left over?

3 George has a business baking and delivering cakes. His recipe for chocolate cake says he needs $2\frac{1}{3}$ cups of plain flour for each cake.

(a) George has one full packet of flour plus another packet that is $\frac{2}{3}$ full, and another packet that is $\frac{3}{4}$ full.

He combined the open packets with the full packet. How many packets of flour does George have? Write your answer as a mixed number in simplest form.

(b) Each packet contains the equivalent of 8 cups of flour. Use your answer from **(a)** to calculate how many cups of flour George has altogether.

(c) George wants to make 4 cakes. Will one full packet of flour be enough for this? Explain how you know this.

(d) How many whole cakes can George make with the flour he has available?

TOPIC

5

Modelling with fractions, decimals and percentages

Recall 119

5.1 Choose an appropriate representation to solve problems 120

5.2 Determine the proportion of a quantity 124

5.3 Understand percentage as applied to taxation 129

5.4 Use fractions, decimals and percentages to solve financial problems 133

5.5 Use, interpret and calculate discounts and mark-ups 137

Topic review 142

Why learn this?

The ability to choose an appropriate representation of fractional amounts provides students with the required tools to interpret, model and solve problems involving everyday transactions.

RECALL

ANSWERS Page 677

I can convert between percentage and decimal values

1 Write the following percentages as decimals.

(a) 15% **(b)** 30% **(c)** 45% **(d)** 72%

2 Write the following decimals as a percentage.

(a) 0.10 **(b)** 0.25 **(c)** 0.65 **(d)** 0.8

I can write simple percentages as fractions

1 Answer true (T) or false (F) for the following statements.

(a) 25% as a fraction is $\frac{1}{4}$. **(b)** 50% as a fraction is $\frac{1}{5}$.

(c) $\frac{2}{10}$ as a percentage is 2%. **(d)** 75% as a fraction is $\frac{3}{4}$.

(e) $\frac{4}{5}$ as a percentage is 80%. **(f)** $\frac{1}{8}$ as a percentage is 8%.

I can simplify fractions using common factors

1 List the factors of the following numbers.

(a) 12 **(b)** 15 **(c)** 24

(d) 30 **(e)** 35 **(f)** 60

2 Determine the highest common factor of the following pairs of numbers.

(a) 12 and 15 **(b)** 15 and 24 **(c)** 24 and 30

(d) 30 and 35 **(e)** 35 and 60 **(f)** 12 and 60

3 Simplify the following fractions.

(a) $\frac{12}{15}$ **(b)** $\frac{15}{24}$ **(c)** $\frac{24}{30}$

(d) $\frac{30}{35}$ **(e)** $\frac{35}{60}$ **(f)** $\frac{12}{60}$

I can write equivalent fractions with a denominator of 100

1 Determine an equivalent fraction with a denominator of 100.

(a) $\frac{3}{50} = \frac{\quad}{100}$ **(b)** $\frac{9}{25} = \frac{\quad}{100}$ **(c)** $\frac{17}{20} = \frac{\quad}{100}$

(d) $\frac{7}{10} = \frac{\quad}{100}$ **(e)** $\frac{3}{5} = \frac{\quad}{100}$ **(f)** $\frac{3}{4} = \frac{\quad}{100}$

5.1 Choose an appropriate representation to solve problems

Learning intention: To be able to choose an appropriate representation to solve problems

Success criteria:

SC 1 I can identify common equivalent forms for fractions, decimals and percentages.

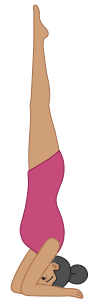
SC 2 I can choose the best representation to solve problems.

Lesson warm-up

Choosing a representation

How do you represent numbers in a real-life context? Do you choose a ratio, fraction, percentage or decimal?

Match the real-life contexts (1–6) with the appropriate representations (A–F). There may be multiple ways of representing each context.



Contexts	Representations
1 Comparing the heights of students in a class.	A Decimals
2 Expressing the probability of an event occurring.	B Fractions
3 Converting a large population count into a more manageable form.	C Percentages
4 Describing the concentration of a solution in a chemistry lab.	D Ratios
5 Calculating the discount on a sale item.	E Scientific notation
6 Representing measurements on a ruler.	F Whole numbers

SC 1 I can identify common equivalent forms for fractions, decimals and percentages

The form in which a number is written does not affect its size. So, 20% of 100 is the same as $\frac{1}{5}$ of 100 and the same as 0.2×100. All three are equal to 20.

Worked example

Writing equivalent forms

Express $\frac{1}{8}$ as a decimal and as a percentage.

THINKING	WORKING
To convert a fraction to a decimal, divide the denominator into the numerator.	$8\overline{)1.000}$ 0.125
Represent $\frac{1}{8}$ as a decimal.	$\frac{1}{8} = 0.125$
To convert a fraction to a percentage, multiply the fraction by $\frac{100}{1}$.	$\frac{1}{8} = \frac{1}{8^2} \times \frac{\cancel{100}^{25}}{1}\%$ $= \frac{25}{2}\%$ $= 12\frac{1}{2}\%$

Note that the percentage form can be calculated from the decimal form by multiplying it by 100. That is, $0.125 \times 100\% = 12.5\%$.

SC 2 I can choose the best representation to solve problems

Although $12.5\% = 0.125 = \frac{1}{8}$, the degree of difficulty in some calculations changes depending on the format chosen. As an example, calculating 12.5% of 80 is much easier using the fraction equivalent because 8 and 80 have a common factor that will make cancelling easier.

Worked example

Choosing a representation

Convert 37.5% to both fraction and decimal formats and then state which would be the best representation to use for calculating 37.5% of 56 kg.

THINKING	WORKING
Convert to decimal format.	$37.5\% = 0.375$
Convert to simplest fraction format.	$37.5\% = \frac{37.5}{100}$ $= \frac{375}{1000}$ $= \frac{3}{8}$
Select the best representation.	Since 56 is a multiple of 8, the fraction representation would be best.

Practice

ANSWERS Page 678

SC 1 I can identify common equivalent forms for fractions, decimals and percentages

1 Represent each of the following fractions as both a decimal and a percentage.

(a) $\frac{3}{5}$ **(b)** $\frac{7}{8}$

2 Represent each of the following decimals as a fraction in simplest form and a percentage.

(a) 0.65 **(b)** 0.3

3 Represent each of the following percentages as both a fraction in simplest form and a decimal.

(a) 92% **(b)** 28.5%

4 Choose the best option for each of the following.

(a) $\frac{1}{5}$% expressed as a fraction in simplest form is:

A $\frac{1}{5}$ **B** $\frac{1}{50}$ **C** $\frac{1}{500}$ **D** 5

(b) The decimal equivalent of 0.0035% is:

A 0.000 035 **B** 0.035 **C** 0.35 **D** 3.5

(c) Which of the following is equivalent to $\frac{47}{400}$?

A 4.7% **B** 11.75% **C** 23.5% **D** 47%

5 Answer each of the following.

(a) As part of his annual employment review, Eric was offered a $3\frac{1}{2}$% wage rise. Represent Eric's wage rise as a fraction in simplest form.

(b) Celeste wanted to travel to a football match and worked out she needed to catch a train, then a tram. Travelling by train was 88% of the journey. What is the decimal equivalent of this?

(c) During February, the price of unleaded petrol rose by $\frac{59}{200}$ of its starting price. Represent this as both a decimal and a percentage.

6 Arrange each of the following sets of values in order from smallest to largest.

(a) $\frac{2}{3}, 75\%, \frac{6}{11}, \frac{4}{17}, 0.3$ **(b)** $33\%, \frac{2}{3}, \frac{1}{3}, 0.3, \frac{69}{200}$

SC 2 I can choose the best representation to solve problems

1 Convert each of the following values to the other two representations. Then decide which format would be the best form for the calculation stated. There is no need to complete the calculation.

(a) $\frac{5}{8}$ of 98 L **(b)** $\frac{7}{8}$ of 160 kg **(c)** 25% of \$160

(d) 75% of 2000 m **(e)** 10% of 233 m **(f)** 0.001 × 15 seconds

2 Copy and complete the following table of equivalent values.

Fraction	Decimal	Percentage
$\frac{1}{4}$		
	0.5	
		75%
	0.125	
$\frac{3}{8}$		
	0.625	
		87.5%
	$0.\dot{3}$	
$\frac{2}{3}$		

3 Choose the best response for each of the following.

(a) Which of the following shows how to calculate $9\frac{1}{5}\%$ of 515?

A $\frac{91}{5}\times\frac{515}{1}$ **B** $\frac{46}{5}\times\frac{515}{1}$ **C** $\frac{46}{500}\times\frac{515}{1}$ **D** $\frac{9.2}{515}\times\frac{100}{1}$

(b) Which of the following shows how to calculate $0.\dot{6}$ of 321?

A $\frac{6}{10}\times\frac{321}{1}$ **B** $\frac{2}{3}\times\frac{321}{1}$ **C** 0.66 × 321 **D** 0.6667 × 321

4 It is known that $\frac{1}{9}=0.\dot{1}$.

(a) What is the decimal equivalent of $\frac{2}{9}$?

(b) What is the fractional equivalent of $44\frac{4}{9}\%$?

(c) Evaluate $0.\dot{7}\times 9999$.

5.2 Determine the proportion of a quantity

Learning intention: To be able to determine the proportion of a quantity

Success criteria:

SC 1 I can determine the stated proportion, expressed as a decimal, fraction or percentage, of a real number.

SC 2 I can determine the proportion, expressed as a decimal, fraction or percentage, that one quantity represents of another.

Lesson warm-up

Proportions and multiplication

Match the statements from Column 1 with statements from Column 2 and Column 3.

Column 1	Column 2	Column 3
$\frac{1}{4}$ of 100	6.25% of 100	0.125×100
$\frac{1}{8}$ of 100	50% of 100	0.0625×100
$\frac{1}{16} \times 100$	$\frac{125}{1000} \times 100$	0.25×100
$\frac{1}{2} \times 100$	$\frac{25}{100} \times 100$	0.5×100

SC 1 I can determine the stated proportion, expressed as a decimal, fraction or percentage, of a real number

A proportion of a quantity is just another way of saying a part of something.

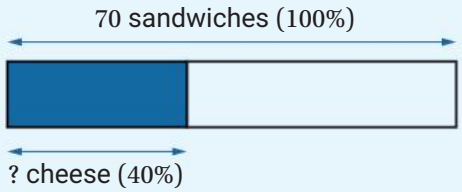
For example, the question 'What proportion of students in the class have brown eyes?' is another way of asking 'How many students out of the total number of students have brown eyes?'

A proportion is often expressed as a percentage, although fractions and decimals can also be used.

Worked example

Finding a proportion of a known quantity using a diagram

The school canteen sold 70 sandwiches yesterday. Cheese sandwiches accounted for 40% of the sandwiches. How many cheese sandwiches were sold yesterday?

THINKING	WORKING
Create a bar model diagram to help you understand the question. The total number of sandwiches sold is 70 (this is the 'whole'). Some of these were cheese sandwiches (this is the 'part').	
Represent the question as a mathematical expression. Use fractions (or decimals) to calculate the answer.	$40\% \times 70 = \frac{40}{100} \times 70$ $= \frac{4}{10} \times 70$ $= 28$
Write your answer.	The canteen sold 28 cheese sandwiches yesterday.

Worked example

Finding a proportion of a known quantity

Jaime spent 20% of her saved pocket money at the fun park. Given that she had $120 at the start, how much did she spend and how much did she have left?

THINKING	WORKING
Represent the question in mathematical format, using the most convenient format for the proportion and complete the calculation.	$20\% \text{ of } 120 = \frac{20}{100} \times \frac{120}{1}$ $= 24$ 20% of $120 = $24
Represent the answer, including any unit required.	She spent $24 and had $120 − $24 = $96 left.

SC 2 I can determine the proportion, expressed as a decimal, fraction or percentage, that one quantity represents of another

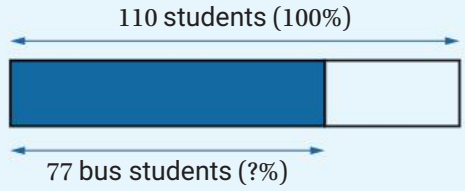
To calculate the proportion that one quantity is of another, it is important that the two quantities are stated using the same unit. For units that are different, convert one quantity so they both have the same unit.

Worked example

Finding what proportion one quantity is of another using a diagram

There are 110 students in Year 7 at a school. Seventy-seven of them travel to school by bus.

What percentage of Year 7 students travel to school by bus?

THINKING	WORKING
Create a bar model diagram. The total number of students is 110. Some of these students (77) travel to school by bus.	
Estimate your answer.	Half of 110 is 55. Seventy-seven is larger than this, so the answer should be bigger than 50%.
Represent the number of students who travel to school by bus as a fraction of the students in the whole year level.	$\frac{77}{110}$
Convert this fraction to a percentage.	$\frac{77}{110} = \frac{77}{110} \times \frac{100}{1}\%$ $= \frac{770}{11}\%$ $= 70\%$
Represent your answer.	70% of Year 7 students travel to school by bus.

Worked example

Finding what proportion one quantity is of another

Determine the proportion that 30 seconds is of 5 minutes. State the proportion as a fraction.

THINKING	WORKING
Determine whether the units are the same. If not, convert the larger unit to the smaller.	Minutes and seconds are not the same unit. 5 min = 5×60 = 300 seconds
Represent the proportion as a fraction.	$\frac{30}{300}$
Now simplify the fraction.	$\frac{30}{300} = \frac{1}{10}$
Write the answer.	30 seconds is $\frac{1}{10}$ of 5 minutes.

Practice

ANSWERS Page 678

SC 1 I can determine the stated proportion, expressed as a decimal, fraction or percentage, of a real number

1 Evaluate the following proportions.

(a) $\frac{5}{8}$ of 96 L **(b)** $\frac{3}{4}$ of 160 kg **(c)** 25% of \$160

(d) 75% of 2000 m **(e)** 10% of 233 m **(f)** 0.001×15 seconds

2 Calculate the following.

(a) On a test with 66 questions, Erica answered $66\frac{2}{3}$% of them correctly. How many questions did Erica answer correctly?

(b) A 10 L bucket is filled to a proportion of 0.825. How many litres of water are in the bucket?

(c) On a certain date, the unemployment rate in Standardtown was reported as 4.5%. Given that the employable population of Standardtown was 124 000, how many people were unemployed?

3 Convert the following mixed number percentages to simplest fraction form and use that to evaluate the answers.

(a) A water tank, with a capacity of 3300 L is $12\frac{2}{3}$% full. How much water is in the tank?

(b) Of the 1400 students at the school, $7\frac{1}{2}$% were absent one day. How many students were absent?

(c) A family has a weekly food budget of \$360. The amount spent on fruit and vegetables represents $36\frac{5}{12}$% of the total. How much is this?

4 Complete the following to find a different way of evaluating proportions.

(a) What is 10% of 2200?

(b) So, what is 5% of 2200?

(c) Use these two values to find $12\frac{1}{2}\%$ of 2200.

(d) Use a similar technique to find $22\frac{1}{2}\%$ of 1800.

5 Choose the best estimate for each of the following.

(a) $47\frac{1}{2}\%$ of 9616 is closest to:

A 240 **B** 960 **C** 4570 **D** 4800

(b) $13\frac{1}{3}\%$ of 6000 is closest to:

A 60 **B** 180 **C** 600 **D** 800

SC 2 I can determine the proportion, expressed as a decimal, fraction or percentage, that one quantity represents of another

5

1 Evaluate the following proportions, giving your answers in simplest fraction form.

(a) 3 kg of 10 kg **(b)** 400 mL of 3 L

(c) 4 hours of a day **(d)** 55 cents of $4

2 Estelle has completed three different mathematics tests this semester. On the first test, she got 25 out of 30, on the second 35 out of 45 and on the third 45 out of 55. Represent each as a percentage, correct to one decimal place, and order the results from worst to best.

3 Choose the best answer for each of the following.

(a) Which of the following shows the correct way to calculate 25 cm as a percentage of 1.2 m?

A $\frac{25}{100}\times\frac{1.2}{1}\%$ **B** $\frac{12}{1000}\times\frac{25}{1}\%$ **C** $\frac{25}{120}\times\frac{100}{1}\%$ **D** $\frac{25}{1200}\times\frac{100}{1}\%$

(b) Which of the following shows the correct way to calculate 3 days as a percentage of 2 weeks?

A $\frac{3}{7}\times\frac{100}{1}\%$ **B** $\frac{3}{100}\times\frac{7}{1}\%$ **C** $\frac{14}{100}\times\frac{3}{1}\%$ **D** $\frac{3}{14}\times\frac{100}{1}\%$

4 Rashid had his hourly pay rate increased from $17 hour to $17.75 hour. Correct to one decimal place, what percentage increase does this represent?

Understand percentage as applied to taxation

5.3

Learning intention: To be able to understand percentage as applied to taxation

Success criteria:

SC 1 I can determine the GST payable.

SC 2 I can interpret income tax tables and use them to determine tax payable.

Lesson warm-up

What is the GST?

1. What is the GST?
2. How does the Australian GST work?
3. Are there any GST exemptions? Give some examples.

SC 1 I can determine the GST payable

What is GST?

The Federal Government collects money (taxation) from its citizens to pay for services such as health, education, welfare, military, transport, etc. The Goods and Services Tax (GST) in Australia is a tax of 10% on the price of things or services that you buy (such as a bicycle, a chocolate bar or a guitar lesson). Businesses collect GST from customers and send the money to the government.

- Prices in stores usually have GST included. These are called GST-inclusive prices.
- Some prices don't include GST, so GST has to be added at the checkout. These can be called pre-GST prices or ex-GST prices.
- Some important items are not taxed and do not ever have GST added to them (for example, most fresh food).

This diagram shows the GST for an item with a pre-GST price of $100. You can see that the GST is 10% of the pre-GST price. The GST is also one eleventh of the GST-inclusive price.

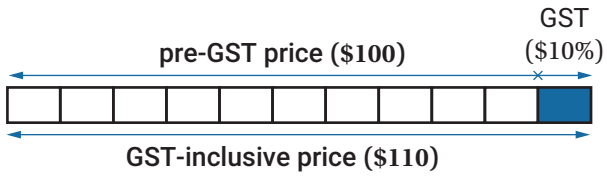
Calculating GST

GST = pre-GST price × 0.1

GST-inclusive price = pre-GST price × 1.1

pre-GST price = GST-inclusive price ÷ 1.1

GST = GST-inclusive price ÷ 11



Worked example

Calculating GST

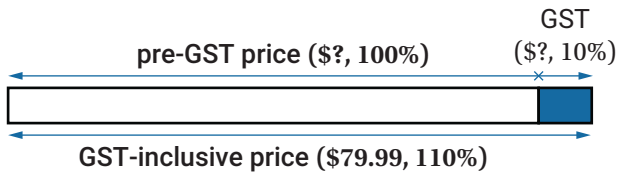
An online store sells a bicycle for $325 ex-GST (pre-GST). Calculate the GST-inclusive price.

THINKING	WORKING
Recall that adding GST means adding 10%. Calculate 110% of the pre-GST price.	$325 \times 110\% = 325 \times 1.1$ $= \$357.50$
Write your answer.	The GST-inclusive price is $357.50.

Worked example

Calculating the pre-GST price

A store is selling an item for $79.99 including GST. Calculate the pre-GST price and the GST amount.



Method 1

THINKING	WORKING
Recall that the GST-inclusive price is 110% of the pre-GST price. Divide the GST-inclusive price by 1.1.	$79.99 \div 1.1 = \$72.72$ (to nearest cent)
Recall that the GST amount is the GST-inclusive price minus the pre-GST price.	$79.99 - 72.72 = \$7.27$
Write your answer.	The pre-GST price is $72.72 and the GST amount is $7.27.

Method 2

THINKING	WORKING
Recall that the GST amount is one eleventh of the GST-inclusive price. Divide the GST-inclusive price by 11.	$79.99 \div 11 = \$7.27$ (to nearest cent)
Recall that the pre-GST price is the GST-inclusive price minus the GST amount.	$79.99 - 7.27 = \$72.72$
Write your answer.	The pre-GST price is $72.72 and the GST amount is $7.27.

SC 2 I can interpret income tax tables and use them to determine tax payable

Income tax is the most important source of income for the Federal Government. The money raised is used to provide funding for many things, including defence, roads, hospitals and schools.

The Australian taxation system is a progressive system. This means that the more money you earn, the greater the tax rate applied to your earnings.

The following table shows the income tax rates for the financial year 2023–2024.

Income	Rate	Tax payable
$0–$18 200	0%	Nil
$18 201–$45 000	19%	19c for each $1 over $18 200
$45 001–$120 000	32.5%	$5092 plus 32.5c for each $1 over $45 000
$120 001–$180 000	37%	$29 467 plus 37c for each $1 over $120 000
$180 000 and over	45%	$51 667 plus 45c for each $1 over $180 000

Based on data from © Australian Taxation Office for the Commonwealth of Australia.

The rates shown are known as the marginal tax rates. So, a person earning $95 000 has a marginal tax rate of 32.5%.

Worked example

Calculating income tax using the table

Calculate the income tax payable on an annual income of $75 600.

THINKING	WORKING
Identify the marginal tax rate.	The marginal tax rate is 32.5%.
Identify the starting amount of tax for this marginal tax rate.	For 32.5% the starting amount of tax is $5092.
Calculate the amount of income in that bracket.	$(75 600 – 45 000) = $30 600
Calculate the additional amount of tax by multiplying this value by the tax rate.	32.5% of $30 600 = $9945
Calculate the full tax payable by adding this to the starting amount of tax.	$(5092 + 9945) = $15 037

Practice

ANSWERS Page 679

SC 1 I can determine the GST payable

1 Calculate the GST payable and the final cost for each of the following purchases. They do not yet include the GST.

(a) $990 for legal services

(b) $19 990 for the purchase of a new car

(c) $1255 for paint for a new house

(d) $175.60 on non-exempt groceries

2 Calculate and solve the following.

(a) Calculate 10% of 1000.

(b) What is the total if you add this value to the original?

(c) Express the final value as a decimal proportion of the original.

(d) By what decimal can you multiply an amount to calculate the final amount including GST?

3 Calculate the total, after GST, for each of the following purchases.

(a) A fishing boat for $26 500

(b) A caravan for $47 899

(c) A garden shed for $773

(d) A block of chocolate for $2.95

4 Calculate the amount of GST included in each of the following prices.

(a) $220 **(b)** $1595 **(c)** $328.90 **(d)** $621.50

5 A book sells for $39.05, including GST. What is the pre-GST price of the book?

SC 2 I can interpret income tax tables and use them to determine tax payable

1 Calculate the tax payable on each of the following annual incomes.

(a) $19 850 **(b)** $42 750 **(c)** $82 900 **(d)** $128 000

2 Determine the percentage of total income paid in income tax by a person who earns each of the following annual incomes. Give your answers correct to one decimal place.

(a) $33 280 **(b)** $88 765 **(c)** $156 745 **(d)** $220 345

3 Some people think it would be fairer to have a flat rate of taxation for everybody. Calculate the amount of tax that would need to be paid on each of the following annual incomes with a 17.5% flat rate of taxation.

(a) $33 280 **(b)** $88 765 **(c)** $156 745 **(d)** $220 345

(e) Which end of the income scale benefits from such a scheme?

4 A family has two working-age members.

(a) Calculate the income tax payable, given that only one of the members works and has an annual income of $160 000.

(b) Calculate the income tax payable, given that both members work and each earns $80 000.

(c) Why is there a difference in these two amounts of tax, even though the annual income is the same?

Use fractions, decimals and percentages to solve financial problems

5.4

Learning intention: To be able to use fractions, decimals and percentages to solve financial problems

Success criteria:

SC 1 I can define, and use in calculations, the terms profit, loss, cost price and selling price.

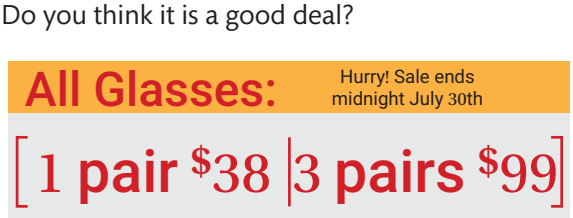
SC 2 I can calculate the profit or loss on an item and express this as a percentage of the cost price or the selling price.

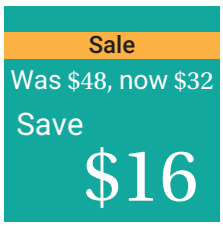
Lesson warm-up

Is it a good deal?

Look at these three advertisements.

Write a sentence or two explaining the purpose of each advertisement.
Do you think it is a good deal?





SC 1 I can define, and use in calculations, the terms profit, loss, cost price and selling price

Cost price (CP): The price paid to buy an item.

Selling price (SP): The amount received when an item is sold.

Profit: The difference between the selling price of an item and its cost price when the selling price is greater than the cost price.

Loss: The difference between the selling price of an item and its cost price when the selling price is less than the cost price.

Worked example

Calculating profit

Rochelle paid $12 500 for a painting and sold it for $18 000. How much profit did she make?

THINKING	WORKING
To calculate the profit, subtract the cost price from the selling price.	Profit = $(18 000 – 12 500) = $5500

SC 2 I can calculate the profit or loss on an item and express this as a percentage of the cost price or the selling price

Converting the profit or loss to a percentage of the cost price or the selling price is the same as finding one value as a percentage of another value.

Worked example

Expressing the loss as a percentage of the cost price

Calculate the percentage loss, based on the cost price, when a car purchased for $42 999 is sold for $18 500. State the answer correct to one decimal place.

THINKING	WORKING
Calculate the loss.	Loss = $(42 999 – 18 500) = $24 499
Calculate the percentage loss by putting the loss over the cost price and multiplying by 100.	$\frac{24\,499}{42\,999} \times \frac{100}{1}\% = 56.975\ldots\%$
Round the answer to the stated degree of accuracy.	The loss is 57.0% of the cost price.

Worked example

Expressing the loss as a percentage of the selling price

Calculate the percentage loss, based on the selling price, when a car purchased for $42 999 is sold for $18 500. State the answer correct to one decimal place.

THINKING	WORKING
Calculate the loss.	Loss = $(42 999 – 18 500) = $24 499
Calculate the percentage loss by putting the loss over the selling price and multiplying by 100.	$\frac{24\,499}{18\,500} \times \frac{100}{1}\% = 132.427\ldots\%$
Round the answer to the stated degree of accuracy.	The loss is 132.4% of the selling price.

As shown in the worked example, it is possible for the percentage loss to be greater than 100% when based on the selling price. It is not possible for the percentage loss to be greater than 100% when based on the cost price. Can you explain why?

Practice

ANSWERS Page 679

SC 1 I can define, and use in calculations, the terms profit, loss, cost price and selling price

1 Calculate the profit on each of the following transactions.

(a) An item purchased for $44 500 and later sold for $62 300.

(b) An item purchased for $123.50 and later sold for $200.99.

(c) An item purchased for $876.25 and later sold for $999.

(d) A house purchased for $789 600 and later sold for $925 000.

2 Calculate the loss on each of the following transactions.

(a) A car purchased for $56 780 and later sold for $35 300.

(b) An item purchased for $29.99 and later sold for $8.50.

(c) An item purchased for $123 985 and later sold for $119 267.

(d) An item purchased for $678 and later sold for $599.

3 Calculate the cost price for each of the following transactions.

(a) An item sold for $567 on which a profit of $235 is made.

(b) An item sold for $8979 on which a loss of $2312 is made.

(c) An item sold for $12.78 on which a profit of $2.05 is made.

(d) An item sold for $231 678 on which a loss of $53 489 is made.

4 Calculate the selling price for each of the following transactions.

(a) An item with a cost price of $321.50 on which a profit of $99.75 is made.

(b) An item with a cost price of $2389 on which a loss of $789 is made.

(c) An item with a cost price of $223 799 on which a profit of $98 755 is made.

(d) An item with a cost price of $15.79 on which a loss of $2.99 is made.

5 Complete the following statements.
An item purchased for $\$A$ is sold for $\$B$.

(a) If $A > B$, then a profit/loss of $? is made.

(b) If $A < B$, then a profit/loss of $? is made.

SC 2 I can calculate the profit or loss on an item and express this as a percentage of the cost price or the selling price

1 Calculate the percentage loss, based on the cost price, for each of the following situations. State the answers correct to one decimal place.

(a) An item purchased for $3655 is sold for $2955.

(b) An item purchased for $545 690 is sold for $495 990.

2 Calculate the percentage profit, based on the cost price, for each of the following situations. State the answers correct to one decimal place.

(a) An item purchased for $12.50 is sold for $15.

(b) An item purchased for $5698 is sold for $8210.

3 Calculate the percentage loss, based on the selling price, for each of the following situations. State the answers correct to one decimal place.

(a) An item purchased for $23 560 is sold for $18 760.

(b) An item purchased for $19.50 is sold for $18.

4 Calculate the percentage profit, based on the selling price, for each of the following situations. State the answers correct to one decimal place.

(a) An item purchased for $309.95 is sold for $415.

(b) An item purchased for $106 450 is sold for $120 000.

5 Evaluate each of the following.

(a) What is the minimum price that an item purchased for $350 can be sold at to ensure a profit, based on cost price, of at least $12\frac{1}{2}\%$?

(b) What is the minimum price that an item purchased for $238 765 can be sold at to ensure a profit of at least 23%.

6 Solve the following steps to determine the minimum price that an item purchased for $1250 can be sold at to ensure a profit, based on the selling price, of at least 15%.

(a) Let the profit be $\$x$. Write an expression for the selling price.

(b) Write an equation that can now be used to calculate the profit. Write the percentage profit in decimal form.

(c) Solve this equation to calculate the profit.

(d) Now state the minimum selling price required.

(e) Use a similar method to determine the minimum price that an item purchased for $45 899 can be sold for to ensure a profit, based on the selling price, of at least 10%.

Use, interpret and calculate discounts and mark-ups

5.5

Learning intention: To be able to use, interpret and calculate discounts and mark-ups

Success criteria:

SC 1 I can calculate a percentage discount.

SC 2 I can calculate a percentage mark-up.

Lesson warm-up

Discounts!

Evaluate these two advertisements.

Write a sentence or two explaining the purpose of each advertisement. Do you think it is a good deal?

SC 1 I can calculate a percentage discount

When a business lowers the price of an item, it is called a discount. The lower price is called the discounted price.

Discounts are often written as percentages. For example, a store might advertise a discount on all of its items.

Goods are often available at a discounted price. The amount of the discount can be advertised in several ways, including:

- up to 50% off
- a minimum of 20% discount
- buy two, get one free
- buy two, get a third at half-price.

Worked example

Calculating the percentage discount

Calculate the percentage discount, based on the original price, for the following two items. Give your answers correct to one decimal place.

(a) $4.50ea

(b) $5.50pk

THINKING	WORKING
For item **(a)**, calculate the original price by adding the discount to the sale price.	$(4.50 + 1) = $5.50
Calculate the discount percentage.	$\text{Discount} = \frac{1}{5.5} \times \frac{100}{1}\%$ $= 18.1818\ldots\%$
State the answer to the required degree of accuracy.	There is a 18.2% discount.
For item **(b)**, calculate the original price by adding the discount to the sale price.	$(5.50 + 1) = $6.50
Calculate the discount percentage.	$\text{Discount} = \frac{1}{6.5} \times \frac{100}{1}\%$ $= 15.3846\ldots\%$
State the answer to the required degree of accuracy.	There is a 15.4% discount.

SC 2 I can calculate a percentage mark-up

A business makes money by selling items for more than they cost to make. The increase in price is called a mark-up and is often a percentage of the cost price. The increased price is called the selling price.

Worked example

Calculating the percentage mark-up

A grocery store sells mushrooms for $18.99 per kilogram. The store bought the mushrooms for a cost price of $12.30 per kilogram. Calculate the percentage mark-up.

THINKING	WORKING
Method 1	
Calculate the mark-up amount.	18.99 – 12.30 = $6.69 The cost price was marked up (increased) by $6.69.
Calculate the mark-up as a percentage of the cost price.	$\frac{6.69}{12.30} \times 100\% = 54\%$ (to 1 d.p.)
Write your answer.	The mark-up from $12.30 to $18.99 is 54%.
Method 2	
Calculate the selling price as a percentage of the cost price.	$\frac{18.99}{12.30} \times 100\% = 154\%$ (to 1 d.p.)
Subtract 100% from the percentage to calculate the percentage increase.	154% – 100% = 54%

Practice

ANSWERS Page 679

SC 1 I can calculate a percentage discount

1 Calculate the percentage discount, based on the original price, for the following four items. Give your answers correct to one decimal place.

(a) $11/kg

(b) $18.50/kg

(c) $13/ea

(d) $26/kg

2 The following appeared in a sale catalogue under the heading:

For question parts **(a)** to **(c)**, calculate the actual discount, based on the original price.

(a) $15 ea
Save $5
Race car toys

(b) $27
Save $10
Racing car track

(c) $20 set
PRICE AFTER PROMOTION $25
Race car collector set

(d) What do you think the ^ means in the heading for the page?

3 The following questions refer to bed linen.

(a) What is the actual discount for the item shown?

(b) What is the actual discount for the item shown?

(c) Why do you think there is this slight difference from the advertised percentage discount?

4 Answer the following questions about sale promotions.

(a) A sale promotion is 'buy one, get one free'. What is the discount percentage?

(b) A sale promotion is 'buy one, get one half-price'. What is the discount percentage?

SC 2 I can calculate a percentage mark-up

Casmir manufactures his own brand of laundry powder. He sells it in two sizes: a 1 kg box and a 4 kg box.

The manufacturing cost is $2.85 for each kilogram of laundry powder.

The packaging cost is $0.30 for each 1 kg cardboard box and $0.37 for each 4 kg cardboard box.

For the 1 kg box, he uses a mark-up of 100% on the cost price to get the selling price.

For the 4 kg box, he uses a mark-up of 40% on the cost price to get the selling price.

After the mark-up, Casmir then adds 10% GST.

1 Using a table like the one shown below, calculate the following for each size of laundry powder.

(a) Total cost of manufacturing and packaging per box

(b) Selling price per box (after the mark-up has been added)

(c) GST amount

(d) GST-inclusive price (after GST has been added).

Box size	Total cost price per box	Selling price per box	GST amount	GST-inclusive price
1 kg				
4 kg				

2 Casmir decides to discount the 4 kg boxes by 15%. Calculate the following.

(a) New GST-inclusive price

(b) New selling price per box

(c) New GST amount

(d) New mark-up percentage

TOPIC REVIEW

Multiple choice

ANSWERS Page 739

1 0.06 written as a percentage is:

A $\frac{6}{100}\%$ **B** 6% **C** 60% **D** $\frac{3}{50}$

2 Which of the following is the missing value in $0.36 = \frac{\square}{100}$?

A 2 **B** 3 **C** 3.6 **D** 36

3 Which of the following represents $\frac{2}{3}$ as a decimal, rounded to 2 decimal places?

A 0.23 **B** 0.66 **C** 0.67 **D** 0.7

4 Amanda has collected eight of the 24 dolls in a set. What percentage of dolls does she need to complete the set? Round your answer to one decimal place.

A 6.7% **B** 33.3% **C** 3.3% **D** 66.7%

5 0.32 written as a percentage is:

A $\frac{32}{100}$ **B** 32% **C** 3.2% **D** $\frac{8}{25}$

6 The decimal 8.85 written as a fraction in simplest form is:

A $88\frac{5}{100}$ **B** $8\frac{17}{20}$ **C** $8\frac{1}{2}$ **D** $8\frac{7}{20}$

7 7.2 expressed in simplest fraction form is:

A $\frac{7}{2}$ **B** $7\frac{2}{10}$ **C** $7\frac{1}{5}$ **D** $\frac{72}{100}$

8 $1500 increased by 28.2% is:

A $1528.20 **B** $1923 **C** $1900 **D** $1282

9 Which of the following is equivalent to 128.5%?

A $\frac{57}{200}$ **B** $1\frac{57}{200}$ **C** 12.85 **D** 12850

10 An electrician produced a bill for $635 for work completed during some home renovations. This did not include the GST of 10% . Which of the following represents the actual amount paid by the householder?

A $63.50 **B** $571.50 **C** $577.27 **D** $698.50

11 A painting purchased for $11 500 is later sold for $15 000. The percentage profit, based on the original cost price, is closest to:

A 23.33% **B** 30.43% **C** 76.67% **D** 130.43%

12 A dining table and chairs set is originally advertised for $1499. During a sale, it is reduced by 15% and then, near the end of the sale, a further 10% is taken from the price. The final selling price is closest to:

A $1124 **B** $1147 **C** $1274 **D** $1349

13 A sound bar has an advertised price of $528, including GST. The amount of GST included in this price is:

A $48 **B** $52.80 **C** $475.20 **D** $480

14 At the supermarket, 2 kg of sugar can be purchased for $2.80, while 3 kg of the same brand costs $4.10. Which of the following statements is correct?

A The 2 kg bag is better value by $1.30.

B The 3 kg bag is better value by $1.50.

C The 2 kg bag is better value by 40 cents/kg .

D The 3 kg bag is better value by about 3 cents/kg .

Short answer

1 Write the following lists in ascending (smallest to largest) order.

(a) 3.21, 3.12, 3.021

(b) 0.35, $\frac{1}{3}$, 0.3, 33%

(c) $\frac{1}{5}$, 19% , 0.20001, $\frac{9}{40}$

2 A trader bought goods for $45 000 and later sold them for $63 000.

(a) What profit did the trader make?

(b) Express the percentage profit in terms of the cost price.

3 Complete the following calculations.

(a) A T-shirt is made using 95% cotton. Each T-shirt has a mass of 0.138 kg. Calculate the amount of cotton in 5000 of these T-shirts.

(b) A suburb has 15 000 residents. The local council knows that 16% of the residents attend school, 4% attend university and 3% are involved in other educational activities. Calculate how many residents attend some sort of educational institution.

4 Daniel bought four toothbrushes at $1.67 each and paid in cash.

(a) Determine the amount that he would have to pay for the toothbrushes.

(b) How much change would he get from $10?

5 Calculate each of the following.

(a) 68.5% of 1480

(b) 15% of 76

(c) 93% of 82

6 Copy and complete the table.

Fraction (simplest form)	Decimal	Percentage
$\frac{1}{10}$	0.1	
	0.333...	
		36%
$1\frac{3}{5}$		
	0.125	
		7.5%
$\frac{17}{40}$		

7 Complete each of the following calculations. Where necessary, express your answers correct to 2 decimal places.

(a) 25% of 36% of 294

(b) 30% of 72% of 1630

(c) 19% of 10% of 120

8 Write the following as percentages.

(a) 9.006 **(b)** 0.007

9 Express the following as decimals.

(a) 23.002% **(b)** 0.0004%

10 Express the following in percentage form.

(a) 9 as a percentage of 90

(b) 4.5 as a percentage of 90

(c) 18 as a percentage of 90

(d) 13.5 as a percentage of 432

11 Write the following as fractions in simplest form.

(a) 2150% **(b)** 10.85%

12 The GST rate is 10% . Calculate the amount of GST that needs to be added to the following pre-GST costs.

(a) A plumber's bill for $350

(b) A block of chocolate for $3.70

(c) A car costing $29 990

13 Bella and Sophie compete in a 100 m race. When they compared their results, Bella ran faster and her time was 80% of Sophie's time. Given that Bella completed the race in 12 seconds, how long did Sophie take to complete the race?

14 The marked price of a jacket is $280, but it is sold at a 35% discount.

(a) Calculate the amount of discount offered on the jacket.

(b) What is the final selling price of the jacket?

15 Complete the following calculations.

(a) The recommended daily intake of calcium for teenagers is 1300 mg . Karina has consumed 185% of her recommended daily intake. Calculate how much calcium Karina has consumed.

(b) There are 1650 students at a school. There are 290 Year 7 students. Calculate the percentage of students at the school who are in Year 7. Give your answer correct to two decimal places.

16 The following prices already include GST. Determine the pre-GST cost.

(a) A book with a sale price of $47.85

(b) A drink with a sale price of $4.07

(c) A camper trailer with a sale price of $27 995

17 The following appeared in a bed linen catalogue.

Washed Linen valance by M.U.S.E.
Belgian flax linen

DB ~~$109.95~~ **$76 ea** QB ~~$119.95~~ **$83 ea**
KB ~~$129.95~~ **$89 ea** SKB ~~$139.95~~ **$97 ea**

NB: Also available in white

(a) Calculate the actual discount for each of the items advertised.

(b) How do you think this sale may have been advertised?

18 A house uses 18.3 kWh of electricity in one day. The house has solar panels that contribute 8 kWh of electricity to the house per day. The remaining electricity is from the power company. The power company produces 14% of its power from its own solar panels.

Calculate the total amount of electricity from solar panels used by the house each day. Give your answer correct to three decimal places.

Extended response

1 The following table shows the applicable tax rates for the financial year 2023–2024.

Income	Rate	Tax payable
$0 – $18 200	0%	Nil
$18 201 – $45 000	19%	19 c for each $1 over $18 200
$45 001 – $120 000	32.5%	$5092 plus 32.5 c for each $1 over $45 000
$120 001 – $180 000	37%	$29 467 plus 37 c for each $1 over $120 000
$180 001 and over	45%	$51 667 plus 45 c for each $1 over $180 000

Based on data from © Australian Taxation Office for the Commonwealth of Australia.

(a) Calculate the tax payable on an income of exactly $45 000.

(b) Explain how you can state the tax payable on an income of $120 000 without the need for any calculation.

(c) Calculate the percentage of tax paid on an annual income of $100 000. State the answer correct to 2 decimal places.

(d) What is the weekly income of Erica who earns $118 000 annually? Round this to the nearest dollar.

(e) Compare the tax payable on Erica's current salary to that payable after she receives a pay increase of $75/week.

(f) Why do you think a taxation system like this is called a 'progressive' tax system?

2 While shopping for paint online, Dave saw an offer of 22 litres of paint for a cost of $11.50/litre.

(a) How much will this paint cost?

(b) There was another offer for a similar paint of 25 litres for $275. How much will this paint cost per litre?

(c) Which is the better buy? Give reasons for your answer.

3 Use your understanding of percentages to answer the following questions.

(a) In a group of Year 5 students, some walk to school and some don't. On average, 3 students walk to school for every 2 who don't walk to school. What percentage of the students don't walk to school?

(b) Given that there are 60 students in the group, how many walk to school?

(c) Of the Year 6 students, 10 students walk to school out of 44 students. What percentage is this, expressed in decimal form correct to 2 decimal places?

(d) When the Year 5 and Year 6 students are put together, what percentage of the students walk to school? Express this answer in exact fraction form, and in decimal form correct to 2 decimal places.

(e) The school has a total of 480 students. It is expected that this will grow to 520 students in the next year. What is the expected percentage increase, expressed in fraction form?

4 A holiday cruise has a price of $4320/person.

(a) A 5% discount is offered to members of a club. What is the price of the cruise for them?

(b) Last year, the full price of the same cruise was $3675. What is the percentage increase in the price of the cruise compared to the price the previous year? Express your answer in decimal form, correct to 2 decimal places.

(c) Next year, the price is expected to rise a further 10%. To the nearest $10, what will the price be then?

(d) In a booking of 60 people, 15% were under 20 years of age, 65% were adults, and the rest were pensioners. People under 20 pay only half the full fare, while pensioners pay one quarter of the full fare. What was the total amount paid by this group for the current year?

TOPIC 6

Integers

Recall 148
6.1 Understand integers and operations 149
6.2 Add and subtract integers 154
6.3 Model and solve practical problems using integers 161
Topic review 170

Why learn this?

Learning to work with integers and their operations, such as plotting on a number line, comparing, ordering, and recognising that they have both magnitude and direction, is foundational for mathematical literacy and problem solving. This topic introduces:

- a concrete representation of positive and negative values, critical for visualising and solving real-world problems in many pursuits including hospitality, science, transportation and economics
- evaluating the relative position and magnitude of integers on a number line, necessary for statistical analysis and logical reasoning
- skills identifying the need for and application of negative values as a basis for understanding concepts including displacement, temperature changes, account balances, and altitude in fields of study such as physics, budgeting in finance, geography and transportation
- preparation for advanced mathematical concepts, including simplifying algebraic expressions and equations, which are critical in fields like engineering, computer science and architecture.

These competencies are essential for academic success and practical application in a wide range of future career paths.

RECALL

ANSWERS Page 680

I can identify odd and even numbers

1 Write all the even numbers between 9 and 19.

2 Write all the odd numbers between 195 and 212.

3 Write the next four consecutive even numbers for the following.

(a) 38, 40, 42 **(b)** 88, 86, 84

4 Write the next four consecutive odd numbers for the following.

(a) 59, 61, 63 **(b)** 107, 105, 103

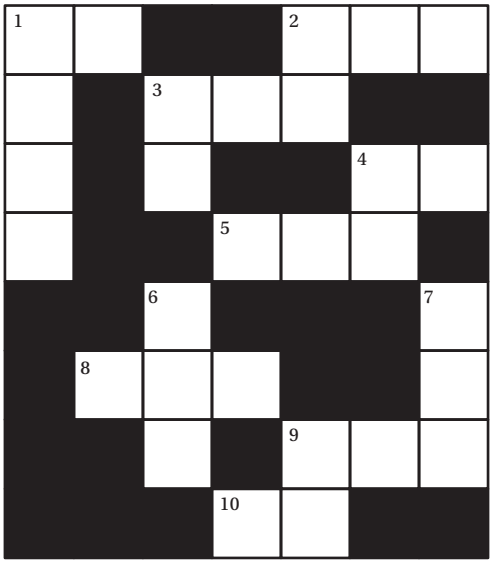
I can compare positive numbers

1 Which symbol, $<$, $>$ or $=$, makes a true statement when placed between the following pairs of values?

(a) 1105 ____ 1110 **(b)** 840 ____ 807 **(c)** 1405 ____ 1054

I can add and subtract whole numbers

1



Across

1 $12 + 16 - 8$

2 $172 - 60 + 42$

3 $36 + 44 + 20$

4 $18 + 62 - 28$

5 $31 + 16 + 61$

8 $318 - 79 - 15$

9 $36 + 55 + 36 - 18$

10 $92 - 32 - 35 + 50$

Down

1 $545 + 678 + 858$

2 $87 - 64 - 13$

3 $98 - 54 - 32$

4 $105 - 60 + 13$

6 $67 + 39 + 16$

7 $89 + 75 - 25$

9 $82 - 16 - 25 - 26$

2 Calculate the following:

(a) $17 + 12 + 32 - 40$ **(b)** $52 - 13 - 7 + 32$ **(c)** $120 - 72 + 48 - 96$ **(d)** $327 + 89 - 67 - 82$

I can order positive and negative temperatures

1 Order the following temperatures from coldest to warmest.

10 °C, −8 °C, −10 °C , −14 °C

2 Order the following temperatures from warmest to coldest.

−16 °C, −12 °C, −10 °C, −14 °C

Understand integers and operations

6.1

Learning intention: To understand integers and operations

Success criteria:

SC 1 I can plot integers on a number line.

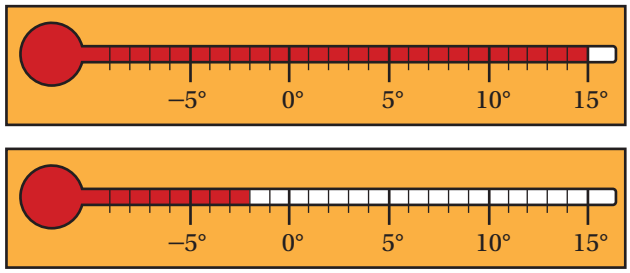
SC 2 I can compare and order integers.

SC 3 I can determine the magnitude and direction of integers.

Lesson warm-up

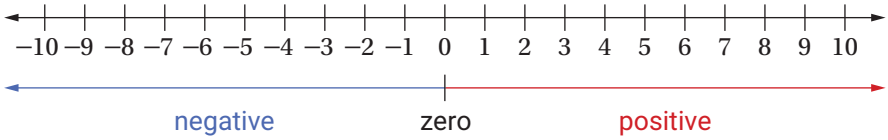
Reading thermometers

What temperatures are shown by these thermometers?



SC 1 I can plot integers on a number line

Integers are whole numbers that can be positive (greater than zero), negative (less than zero) or neither (zero). A number line can be used to represent all integers.



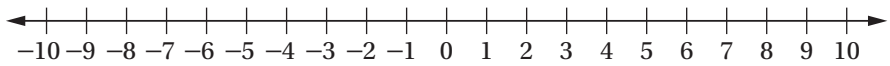
Values lower than 0 are named and described using the term 'negative' (i.e., −4 is read as negative four).

Values higher than 0 are named and described using the term 'positive' or just the number (i.e., +4 is read as positive four, or just four).

Worked example

Locating and plotting integers on a number line

Show the position of the integers −9, −5, −3, 0, 2, and 8 on the number line.

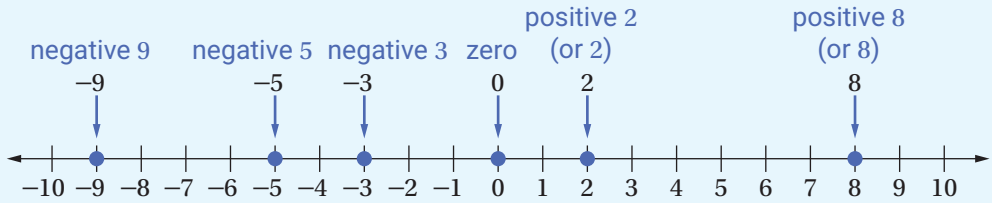


THINKING

Plot the negative numbers to the left of zero and positive numbers to the right of zero.

Use a small dot to show where each value is positioned on the number line.

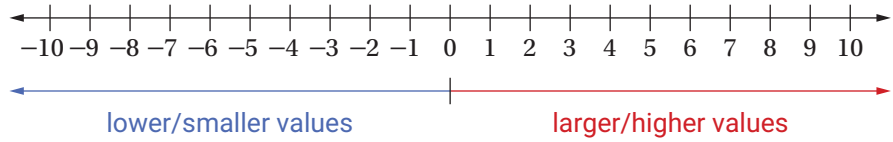
WORKING



SC 2 I can compare and order integers

Moving away from zero to the left on the number line, negative numbers represent smaller or lower values.

Moving away from zero to the right on the number line, positive numbers represent larger or higher values.



A quick rule with all values on a number line is that numbers towards the left are smaller or lower than numbers towards the right.

For example:

On the number line, you can see that (−5) is further to the left than 5, therefore (−5) is less than 5: $(-5) < 5$.

Alternatively, 5 is further to the right than (−5), therefore 5 is greater than (−5): $5 > (-5)$.

Worked example

Comparing integers

Use the integers −15, 7, 20, −9, −2, 4 and −7 to answer the following questions.

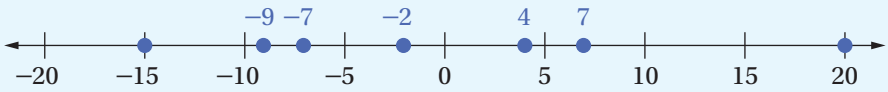
(a) Plot the integers on a number line.

THINKING

Plot the negative numbers to the left of zero and the positive numbers to the right of zero.

Use a small dot to show where each value is positioned on the number line.

WORKING



(b) Arrange the integers in ascending order (smallest to largest).

THINKING	WORKING
List the numbers, as they appear on the number line, from smallest (left) to largest (right).	−15, −9, −7, −2, 4, 7, 20

(c) Arrange the integers in descending order (largest to smallest).

THINKING	WORKING
List the numbers, as they appear on the number line, from largest (right) to smallest (left).	20, 7, 4, −2, −7, −9, −15

SC 3 I can determine the magnitude and direction of integers

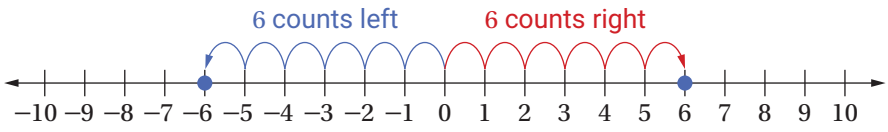
Positive and negative integers are also called directed numbers because positive and negative values have both size and direction.

The size of a number is called the magnitude of that number.

(−6) means 6 counts to the left, from 0

(+6) or 6, means 6 counts to the right, from 0

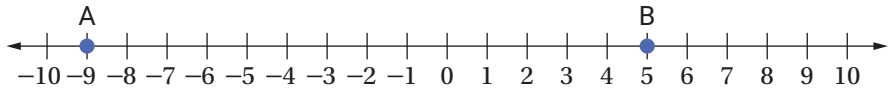
The integers (−6) and 6 both have a magnitude of 6. They are both 6 counts away from zero.



Worked example

Determining the magnitude of integers

Determine the magnitude of the values plotted on the number line.



THINKING	WORKING
Read the position of each integer on the number line.	A = −9 and B = 5
Determine the magnitude or size of each plotted integer, ignoring direction.	The magnitude of A is 9. The magnitude of B is 5. Even though A = −9 is less than B = 5, A has a greater magnitude than B (because it is further from zero).

Practice

ANSWERS Page 680

SC 1 I can plot integers on a number line

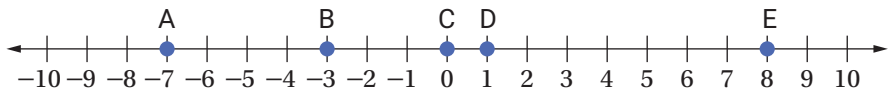
1 For each set of integers, draw a number line from −8 to 8 and show their position on it.

(a) −6, −4, −2, 3 and 5

(b) −7, −4, −2, −1, 0, 3, 6 and 8

(c) −4, −2, −1, 2, 4 and 7

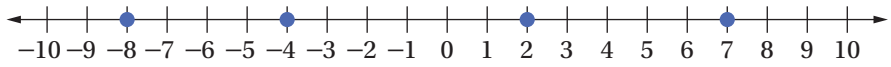
2 Label each of the integers plotted on the number line using both numbers and words.



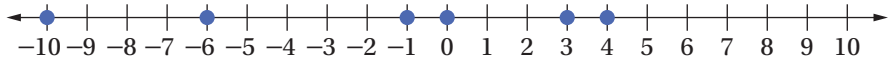
SC 2 I can compare and order integers

1 List the integers shown on each number line in ascending order (smallest to largest).

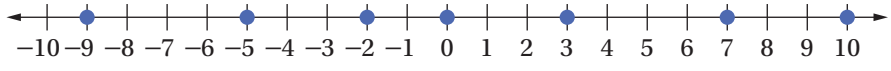
(a)



(b)



(c)



2 Arrange each set of integers in ascending order (from smallest to largest) and descending order (largest to smallest).

(a) −3, −8, 10, −9, 3, 4 and −6 **(b)** −25, 18, 30, −19, −5, 16 and −1

(c) −15, 8, 20, −9, −2, 4 and −7 **(d)** −125, 58, 204, −90, −50, 64 and −72

3 Describe your strategy to compare and order integers.

4 Determine which symbol belongs between each pair of numbers.
(< is read as 'is less than' and > is read as 'is greater than').

(a) (−3) __ 4 **(b)** (−1) __ (−3) **(c)** (−9) __ 5 **(d)** (−10) __ (−5)

5 **(a)** List all the integers greater than −4 and less than 3.

(b) List all the integers greater than −12 and less than −6.

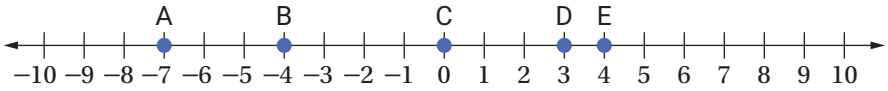
(c) List all the integers greater than −1 and less than 5.

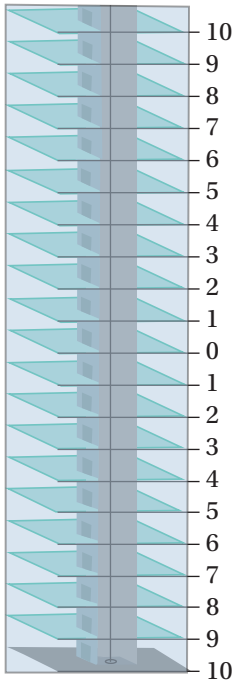
6 **(a)** The sequence −6, −4, −2, ... is increasing by 2. List the next three terms of the sequence.

(b) The sequence 15, 10, 5, ... is decreasing by 5. Write the next four terms of the sequence.

SC 3 I can determine the magnitude and direction of integers

1 Determine the location and the magnitude of the values plotted on the number line.





2 Describe what the magnitude of an integer represents.

3 Consider the set of integers below, each representing elevator floor levels, where 0 is the ground floor.
A = −2, B = −8, C = 10, D = −9, E = 3, F = 4 and G = −6

(a) List the integers (elevator levels) in ascending order.

(b) Explain what your answer to part **(a)** represents.

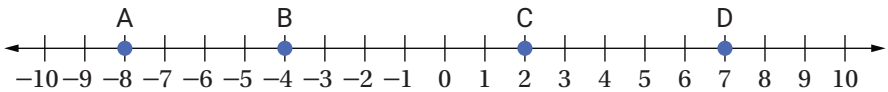
(c) Determine the magnitude of each integer.

(d) List the magnitude of the elevator levels A–G in ascending order.

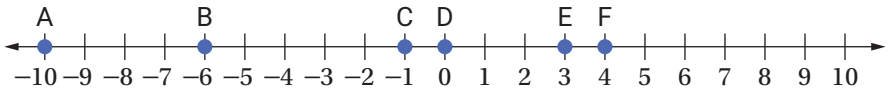
(e) Explain what your answer to part **(c)** represents.
Why is it different from the answer you gave in part **(a)**?

4 Write the magnitude of the integers shown on each number line in order from smallest to largest. Using an elevator as an example (where 0 represents ground level), describe the position and direction from the ground floor of each position. List the magnitude in ascending order.

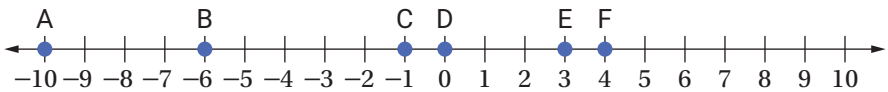
(a)



(b)



(c)



6.2 Add and subtract integers

Learning intention: To be able to add and subtract integers

Success criteria:

SC 1 I can add integers.

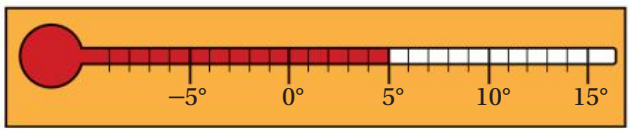
SC 2 I can subtract integers.

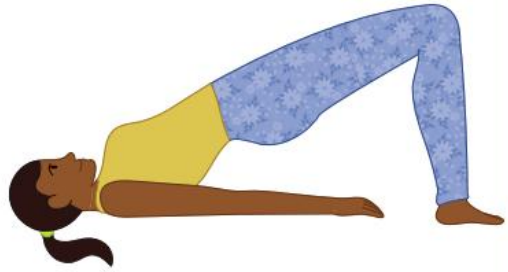
SC 3 I can perform addition and subtraction of integers using different strategies.

Lesson warm-up

Reading thermometers

A thermometer measures the temperature of water to be 5 °C.





What happens to the temperature of the water after you add ice cubes?

What happens to the temperature of the water after you add hot water?

Consider the statement:

'Starting with 5, I can add a number to produce a number smaller than 5, 5 exactly, or a number larger than 5.'

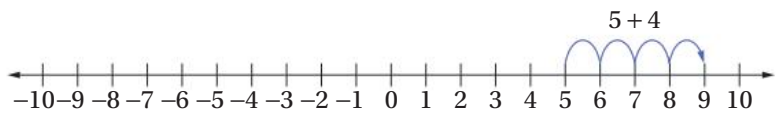
Is the statement true or false?

SC 1 I can add integers

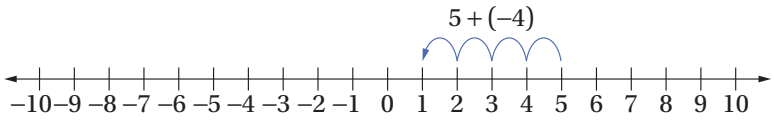
Any number will remain the same value when adding zero. Starting with a number and adding a non-zero integer can either increase the value or decrease the value.

Consider each of these scenarios.

A glass of water had a starting temperature of 5 °C. Warm water was added so that the temperature of the water increased by 4 °C. The final temperature of the water is 9 °C.

Number sentence	Number line
5 + 4 = 9	

A glass of water had a starting temperature of 5 °C. Ice cubes were added so that the temperature of the water decreased by 4 °C. The final temperature of the water was 1 °C.

Number sentence	Number line
$5 + (-4) = 1$	

Using the number lines given, you can create a general rule for adding integers.

Start at the first number and face right.

When the number being added is positive, step forward. When the number being added is negative, step backward.

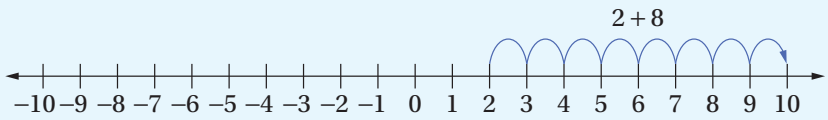
You can remember that when the signs are the same (you add a positive number), the value will get bigger; when the two signs are different (you add a negative number), the value will get smaller.

Worked example

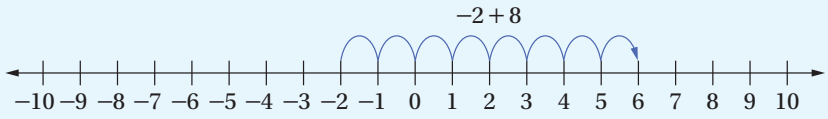
Adding integers

For each of the following, complete the calculation and interpret the calculation.

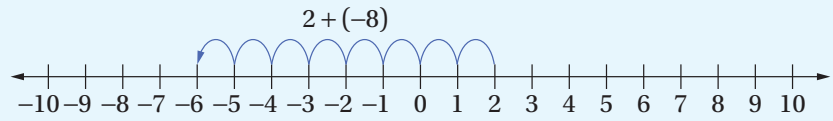
(a) I had \$2 and earned \$8 more, 2 + 8.

THINKING	WORKING
Draw the addition on the number line. To add a positive number, face right and step forward.	Start at 2. Move 8 steps to the right. 
Perform the addition.	$2 + 8 = 10$
Interpret the answer.	I had \$2 and earned \$8 more. I now have \$10.

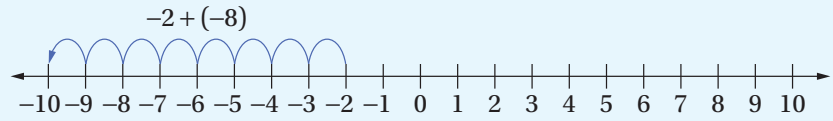
(b) I owed \$2 and earned \$8, −2 + 8.

THINKING	WORKING
Draw the addition on the number line. To add a positive number, face right and step forward.	Start at −2. Move 8 steps to the right. 
Perform the addition.	$-2 + 8 = 6$
Interpret the solution.	I owe \$2 and earned \$8. I now have \$6.

(c) I had $2 and bought an item worth $8, $2 + (-8)$.

THINKING	WORKING
Draw the addition on the number line. To add a negative number, face right and step backward.	Start at 2. Move 8 steps to the left. 
Perform the addition.	$2 + (-8) = -6$
Interpret the solution.	I had $2 and bought an item worth $8. I now owe $6.

(d) I owed $2 and bought an item worth $8, $-2 + (-8)$.

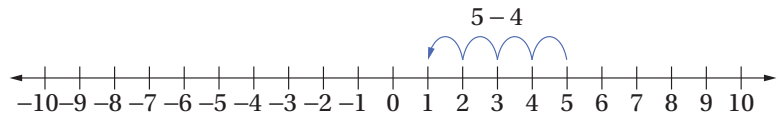
THINKING	WORKING
Draw the addition on the number line. To add a negative number, face right and step backward.	Start at −2. Move 8 steps to the left. 
Perform the addition.	$-2 + (-8) = -10$
Interpret the solution.	I owed $2 and bought an item worth $8. I now owe $10.

SC 2 I can subtract integers

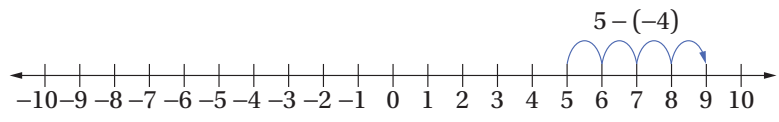
Any number will remain the same value when subtracting zero. Starting with a number and subtracting a non-zero integer can either increase the value or decrease the value.

Consider each of these scenarios.

A glass of water had a starting temperature of 5 °C. A heat source is removed so that the temperature of the water decreases by 4 °C. The final temperature of the water is 1 °C.

Number sentence	Number line
$5 - 4 = 1$	

A glass of water had a starting temperature of 5 °C. Ice cubes are removed so that the temperature of the water increases by 4 °C. The final temperature of the water is 9 °C.

Number sentence	Number line
$5 - (-4) = 9$	

Using the number lines given, you can create a general rule for subtracting integers.

Start at the first number and face left.

When the number being subtracted is positive, step forward. When the number being subtracted is negative, step backward.

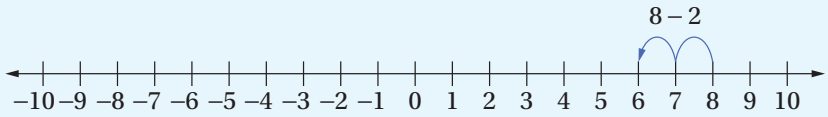
You can remember that when the signs are the same (you subtract a negative number), the value will get bigger; when the two signs are different (you subtract a positive number), the value will get smaller.

Worked example

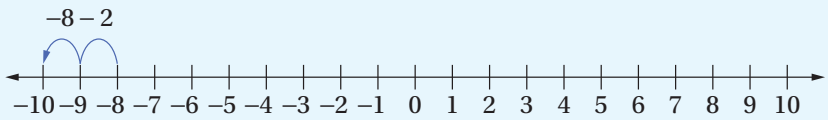
Subtracting integers

For each of the following, complete the calculation and interpret the calculation.

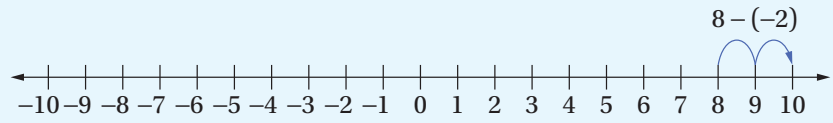
(a) I had \$8 and spent \$2, $8 - 2$.

THINKING	WORKING
Draw the subtraction on the number line. To subtract a positive number, face left and step forward.	Start at 8. Move 2 steps to the left. 
Perform the subtraction.	$8 - 2 = 6$
Interpret the answer.	I had \$8 and spent \$2. I have \$6.

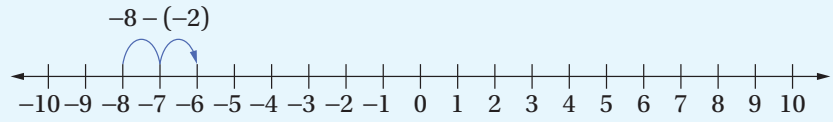
(b) I owed \$8 and spent another \$2, $-8 - 2$.

THINKING	WORKING
Draw the subtraction on the number line. To subtract a positive number, face left and step forward.	Start at −8. Move 2 steps to the left. 
Perform the subtraction.	$-8 - 2 = -10$
Interpret the solution.	I owed \$8 and spent another \$2. I owe \$10.

(c) I had $8 and removed a debt of $2 after I received a refund, $8 - (-2)$.

THINKING	WORKING
Draw the subtraction on the number line. To subtract a negative number, face left and step backward.	Start at 8. Move 2 steps to the right. 
Perform the subtraction.	$8 - (-2) = 10$
Interpret the solution.	I had $8 and removed a debt of $2 after I received a refund. I now have $10.

(d) I owed $8 and removed a debt of $2 after I received a refund, $-8 - (-2)$.

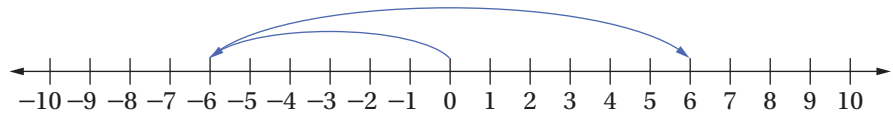
THINKING	WORKING
Draw the subtraction on the number line. To subtract a negative number, face left and step backward.	Start at –8. Move 2 steps to the right. 
Perform the subtraction.	$-8 - (-2) = -6$
Interpret the solution.	I owed $8 and removed a debt of $2 after I received a refund. I now owe $6.

SC 3 I can perform addition and subtraction of integers using different strategies

Worked example

Adding and subtracting integers

Write four different addition/subtraction expressions to represent the calculation shown on the number line.



THINKING	WORKING
Identify the starting location.	Start at 0.
Identify the direction and magnitude of the first operation. List both ways to achieve the result.	The first operation is 6 steps to the left. This can be achieved by adding −6, or subtracting 6.
Identify the direction and magnitude of the second operation. List both ways to achieve the result.	The second operation is 12 steps to the right. This can be achieved by adding 12 or subtracting −12.
Answer the question by writing the expressions using combinations of the operations identified.	$0 - 6 + 12$ $0 - 6 - (-12)$ $0 + (-6) + 12$ $0 + (-6) - (-12)$

Practice

ANSWERS Page 681

SC 1 I can add integers

1 Draw the following additions on a number line.

(a) $3 + 5$ **(b)** $-3 + 5$ **(c)** $3 + (-5)$ **(d)** $-3 + (-5)$

2 Explain how to add two integer values.

3 Calculate:

(a) $6 + 8$ **(b)** $6 + 7$ **(c)** $6 + 6$

(d) $6 + 5$ **(e)** $6 + 4$ **(f)** $6 + 3$

4 Calculate:

(a) $-6 + 8$ **(b)** $-6 + 7$ **(c)** $-6 + 6$

(d) $-6 + 5$ **(e)** $-6 + 4$ **(f)** $-6 + 3$

5 Calculate:

(a) $4 + (-1)$ **(b)** $4 + (-2)$ **(c)** $4 + (-3)$

(d) $4 + (-4)$ **(e)** $4 + (-5)$ **(f)** $4 + (-6)$

6 Calculate:

(a) $-4 + (-1)$ **(b)** $-4 + (-2)$ **(c)** $-4 + (-3)$

(d) $-4 + (-4)$ **(e)** $-4 + (-5)$ **(f)** $-4 + (-6)$

SC 2 I can subtract integers

1 Draw the following subtractions on a number line.

(a) $5 - 3$ **(b)** $-5 - 3$ **(c)** $5 - (-3)$ **(d)** $-5 - (-3)$

2 Explain how to subtract one integer from another.

3 Calculate:

(a) $4 - 1$ **(b)** $4 - 2$ **(c)** $4 - 3$

(d) $4 - 4$ **(e)** $4 - 5$ **(f)** $4 - 6$

4 Calculate:

(a) $-4 - 1$ **(b)** $-4 - 2$ **(c)** $-4 - 3$

(d) $-4 - 4$ **(e)** $-4 - 5$ **(f)** $-4 - 6$

5 Calculate:

(a) $2 - (-1)$ **(b)** $2 - (-2)$ **(c)** $2 - (-3)$

(d) $2 - (-4)$ **(e)** $2 - (-5)$ **(f)** $2 - (-6)$

6 Calculate:

(a) $-2 - (-1)$ **(b)** $-2 - (-2)$ **(c)** $-2 - (-3)$

(d) $-2 - (-4)$ **(e)** $-2 - (-5)$ **(f)** $-2 - (-6)$

SC 3 I can perform addition and subtraction of integers using different strategies

1 Calculate:

(a) $2 - 11$ **(b)** $-2 - 6$ **(c)** $-2 - (-3)$ **(d)** $4 - 3$

(e) $-5 - (-3)$ **(f)** $8 - (-4)$ **(g)** $-2 + 6$ **(h)** $5 - 9$

(i) $-5 - 3$ **(j)** $-2 + (-3)$

2 Calculate:

(a) $25 - (-10)$ **(b)** $-52 - 20$ **(c)** $202 - (-3)$

(d) $45 - (-30)$ **(e)** $75 + (-32)$ **(f)** $-62 - (-30)$

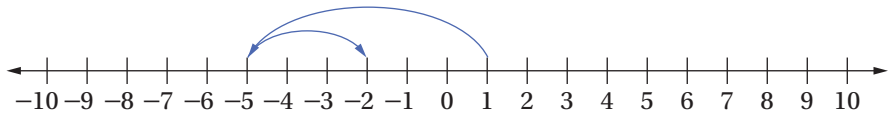
3 Calculate:

(a) $5 - (-2) + 3$ **(b)** $-2 - 2 + 8$ **(c)** $20 - (-4) + 6$

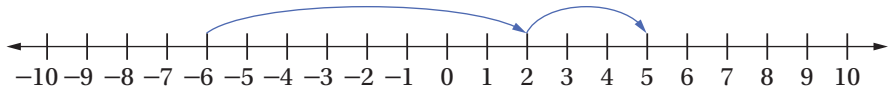
(d) $-45 - (-10) + 20$ **(e)** $55 + (-42) + 7$ **(f)** $-42 + (-30) - 22$

4 Write four different addition/subtraction expressions using integers that each display could represent.

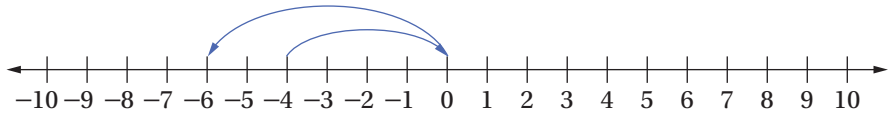
(a)



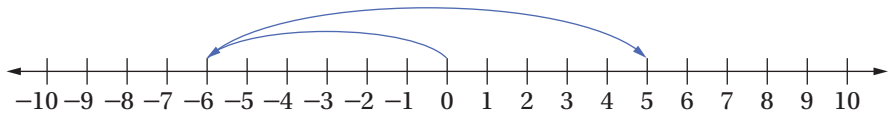
(b)



(c)



(d)



Model and solve practical problems using integers

6.3

Learning intention: To be able to model and solve practical problems using integers

Success criteria:

SC 1 I can apply integer operations to word problems.

SC 2 I can represent profit and loss as positive and negative values.

SC 3 I can use integers to model and solve practical problems.

Lesson warm-up

Troublesome elevator

A building has 3 basement levels, a ground floor, and 20 floors above the ground that all can be accessed using an elevator.

Travelling to upper levels is gaining height above ground (positive direction).

Travelling to lower levels is losing height above ground (negative direction).

One day, the elevator is not working properly and will travel up only 3 floors at a time or down only 5 floors at a time.

A person who can't use the stairs is on the 3rd floor. How can that person travel to the 11th floor using the elevator?

How would you communicate your solution to a friend?

SC 1 I can apply integer operations to word problems

Worked example

Adding and subtracting integers in word problems

Calculate the following.

(a) Ming has $4500 in savings and borrows money to spend $6250 on a new entertainment system. What is Ming's debt?

THINKING	WORKING
Write the word problem as an expression.	$4500 − $6250
Calculate the value of the expression.	$4500 − $6250 = −$1750
Write your answer.	Ming has a debt of $1750.

(b) A survey ship detects a shipwreck −350 m from the ocean surface. A crane lowers a cable, starting from 25 m above the ocean surface, to lift items found on the shipwreck. How far does the cable stretch from the crane to the shipwreck?

THINKING	WORKING
Interpret the problem. Note that the direction of the numbers (positive or negative) does not matter as the value calculated is a distance.	You need to determine the total distance from the shipwreck (350 m below the surface) to the crane (25 m above the surface).
Write the word problem as an expression.	350 + 25
Calculate the value of the expression.	350 + 25 = 375
Write the answer.	The cable stretches 375 m from the crane to the shipwreck.

SC 2 I can represent profit and loss as positive and negative values

Businesses sell items, produce goods or provide services to make money. They also have expenses that cost money, such as materials, staff wages and bills.

When the money coming in is greater than the money going out, the business makes money (a profit).

When the money coming in is less than the money going out, the business doesn't make money (a loss).

Directed numbers can be used to record money coming into or out of a business.

Worked example

Understanding profit and loss

Loni runs a seasonal vegetable stall at a weekend market. The table shows the income (money coming into the business from sales) and expenses (money going out of the business) for one month.

A ledger is used to record income as a positive value and an expense as a negative value.

Item	Income	Expense	Ledger
Carrots	$275		
Pumpkins	$235		
Paper bags		$45	
Onions	$238		
Mushrooms	$245		
Potatoes	$330		
Electricity bill		$275	
Rent		$380	
Peanuts	$250		
Total			

Complete the ledger column and determine the balance.

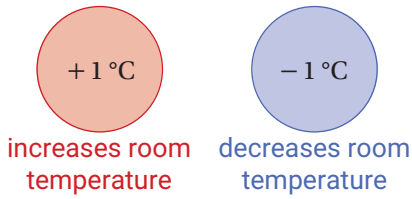
THINKING	WORKING
Record each value in the income column as a positive value in the ledger column. Record each value in the expense column as a negative value in the ledger column.	<table><tr><th>Item</th><th>Income</th><th>Expense</th><th>Ledger</th></tr><tr><td>Carrots</td><td>$275</td><td></td><td>$275</td></tr><tr><td>Pumpkins</td><td>$235</td><td></td><td>$235</td></tr><tr><td>Paper bags</td><td></td><td>$45</td><td>−$45</td></tr><tr><td>Onions</td><td>$238</td><td></td><td>$238</td></tr><tr><td>Mushrooms</td><td>$245</td><td></td><td>$245</td></tr><tr><td>Potatoes</td><td>$330</td><td></td><td>$330</td></tr><tr><td>Electricity bill</td><td></td><td>$275</td><td>−$275</td></tr><tr><td>Rent</td><td></td><td>$380</td><td>−$380</td></tr><tr><td>Peanuts</td><td>$250</td><td></td><td>$250</td></tr><tr><td>Total</td><td></td><td></td><td></td></tr></table>
Calculate and record the sum of each column. Note: the sum of the ledger is called the balance.	Income: $\$275 + \$235 + \$238 + \$245 + \$330 + \$250 = \$1573$ Expenses: $\$45 + \$275 + \$380 = \$700$ Balance: $\$275 + \$235 + (-\$45) + \$238 + \$245 + \$330 + (-\$275) + (-\$380) + \$250$ $= \$873$
Check the reasonableness of the balance (sum of the ledger).	Balance = Income − Expenses $= \$1573 - \$700$ $= \$873$
Interpret the answer.	Loni's vegetable stall made a profit of $873.

SC 3 I can use integers to model and solve practical problems

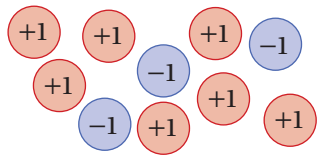
Worked example

Adding negative one and positive one

Temperature discs increase or decrease the temperature by 1 °C.



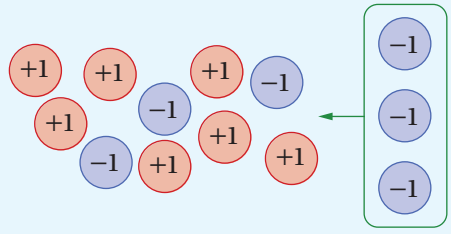
Consider the following temperature change.



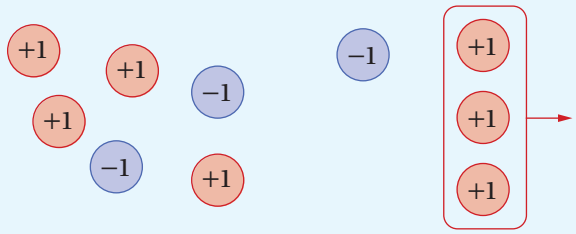
(a) Determine the change in the room temperature.

THINKING	WORKING
Count the number of positive and negative discs in the room.	Number of positive discs: 7 Number of negative discs: 3
Calculate the change in room temperature	7 + (−3) = 4
Interpret the answer.	The change in room temperature is 4 °C. The temperature would increase by 4 °C.

(b) Determine the number and type of discs that would need to be added, given the desired increase in temperature was 1 °C.

THINKING	WORKING
Calculate the change in temperature required to reach the goal temperature.	4 °C − 3 °C = 1 °C The temperature needs to decrease by 3 °C.
Identify what discs must be added to the room to make that change.	Adding 3 negative discs to the room will reduce the temperature by 3 °C.
Model the addition.	 7 + (−6) = 1
Interpret the answer.	The room temperature can be increased by 1 °C by adding 3 negative discs.

(c) Determine the number and type of discs that would need to be taken out, given the desired increase in temperature was 1 °C.

THINKING	WORKING
Calculate the change in temperature required to reach the goal temperature.	4 °C − 3 °C = 1 °C The temperature needs to decrease by 3 °C.
Identify what discs must be taken from the room to make that change.	Subtracting 3 positive discs from the room will reduce the temperature by 3 °C.
Model the answer.	 7 + (−3) − 3 = 1
Interpret the answer.	The room temperature can be increased by 1 °C by subtracting 3 positive discs.

Worked example

Understanding profit and loss

Mason purchased 25 surfboards for $180 each at the start of summer. In the lead up to the summer holidays, Mason sold 15 of the surfboards for $250 each.

The rest of the surfboards were sold for the reduced price of $99 each.

(a) Calculate the profit Mason made on the 15 surfboards sold in the lead up to the summer holidays.

THINKING	WORKING
Identify the selling price and the purchase price of each item.	Mason purchased the surfboards for $180. Mason sold the surfboards for $250.
Calculate the difference between the selling price and the purchase price of each item.	$250 − $180 = $70 Mason makes a profit of $70 per surfboard sold in the lead up to the summer holidays.
Calculate the total profit by multiplying the profit per item by the number of items sold.	Mason sold 15 surfboards, each making a profit of $70 for a total profit of 15 × $70 = $1050.

(b) Calculate the loss Mason made on the surfboards that were sold at a reduced price.

THINKING	WORKING
Identify the selling price and the purchase price of each item.	Mason purchased the surfboards for \$180. Mason sold the surfboards for \$99.
Calculate the difference between the selling price and the purchase price of each item.	\$99 − \$180 = −\$81 Mason makes a loss of \$81 per surfboard sold at a reduced price.
Calculate the total loss by multiplying the loss per item by the number of items sold.	Mason sold 25 − 15 = 10 surfboards, each making a loss of \$81 for a total loss of $10 \times \$81 = \$810$.

(c) Determine whether Mason made a profit or a loss overall.

THINKING	WORKING
Identify the profit from each of the two sale types. A loss can be written as negative profit.	Profit (leading up to the summer holidays): \$1050 Profit (reduced price): −\$810
Calculate the total profit.	\$1050 + (−\$810) = \$240
Interpret your answer.	Mason made a profit of \$240.

Practice

ANSWERS Page 681

SC 1 I can apply integer operations to word problems

1 The temperature in Bright increased from −4 °C overnight to 6 °C during the day. How much did the temperature rise?

2 Kristina set the room thermostat to 20 °C. Outside the house, it is −6 °C. Determine the temperature difference between the room and outside the house.

3 The table below shows the average minimum and average maximum temperatures for five Asian cities.

(a) Copy and complete the table by calculating the difference between the maximum and minimum average temperatures for each city.

City	Minimum	Maximum	Difference
Kabul (Afghanistan)	−2 °C	25 °C	
Dhaka (Bangladesh)	19 °C	29 °C	
Beijing (China)	−3 °C	27 °C	
Hong Kong (China)	16 °C	29 °C	
Pyongyang (North Korea)	−6 °C	25 °C	

(b) Which city has the smallest difference between the average minimum and average maximum temperature?

(c) Which city has the greatest difference between the average minimum and average maximum temperature?

4 A cup contains boiling water with a temperature of 100 °C. It is left to cool and the temperature change is recorded as −15 °C. Determine the new water temperature.

5 The highest point on Earth is Mt Everest at 8848 m. The lowest point is in the Mariana Trench at −11 034 m. Determine the difference in height between these two points.

6 A submarine sits at a depth of −280 m before descending 70 m more. Determine the new depth of the submarine.

7 Jao had a credit card balance of −$85 and used the card to purchase $175 worth of clothes. After receiving $1450 and paying off the credit card, how much money does Jao now have?

8 22 people boarded a bus service at the start of its route. The change in passenger numbers after each of the first four stops was +9, −15, +7 and −11.

(a) How many passengers were on the bus after the second stop?

(b) How many passengers were on the bus after the fourth stop?

(c) What was the overall change in passenger numbers after the third stop?

(d) What was the overall change in passenger numbers after the fourth stop?

(e) Write a story describing the number of passengers on the bus.

9 A glider was initially travelling above the ground at an elevation of 350 m. Its elevation changed during the 30 minutes of its flight by −25 m, +56 m, −38 m and +36 m.

(a) What was the highest elevation of the glider during this time?

(b) What was the overall change in elevation during the 30 minutes?

(c) What was the greatest change in elevation during the 30 minutes?

SC 2 I can represent profit and loss as positive and negative values

1 Calculate the total profit/loss at the end of the month for each balance sheet.

(a)

Date	Ledger
2/2/23	−$130
12/2/23	$155
21/2/23	−$215
28/2/23	−$45

(b)

Date	Ledger
2/2/23	−$135
12/2/23	$155
21/2/23	$215
28/2/23	−$65

2 Myra owns a bookstore and, near the end of June, held a one-day sale to clear out old stock. Sales on that day are shown in the table.

Title	Cost price	Sale price	Profit/Loss Sale price − Cost price
Great Gardens	$29	$20	
Creative Cooks	$18	$21	
Devilish Designs	$22	$22	
Eastern Elegance	$39	$30	

(a) Copy and complete the table by calculating the profit or loss on each book.

(b) Which book did Myra make a profit on?

(c) Which book does Myra make the smallest loss on?

3 Jacinda bought a car for $35 000 and sold it three years later for $22 500.

(a) Express the difference between the purchase price and the sale price as a profit.

(b) Express the difference between the purchase price and the sale price as a loss.

SC 3 I can use integers to model and solve practical problems

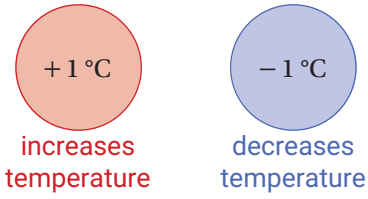
1 A hot air balloon rises and falls, depending on the temperature of the air inside it. One burst of hot air from the burner will raise the balloon's height by 10 m, and a gust of cool air will lower its height by 10 m.
Calculate the new height of a hot-air balloon travelling 120 m above the ground after:

(a) 2 bursts of hot air and 3 gusts of cool air

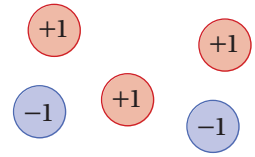
(b) 4 bursts of hot air and 3 gusts of cool air

(c) 3 bursts of hot air and 5 gusts of cool air.

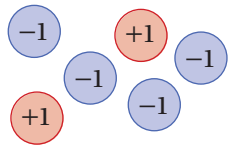
2 Mixtures of temperature discs are used to increase or decrease the temperature of a vat of liquid by 1 °C. In each mixture below, determine what needs to be added or removed to create the required change in temperature.



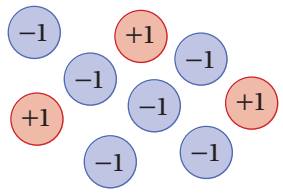
(a) What must be added to this mixture for a +5 °C change in temperature?



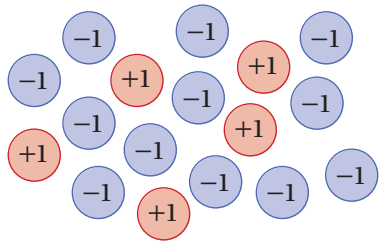
(b) What must be removed from this mixture for a −3 °C change in temperature?



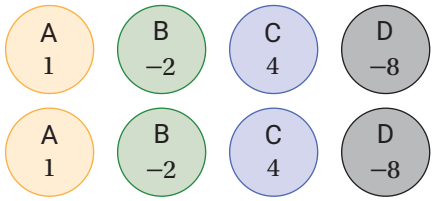
(c) What must be added to this mixture for a −6 °C change in temperature?



(d) What must be removed from this mixture for a 0 °C change in temperature?



3 The four pairs of counters pictured are placed in a bag. A counting game involves drawing three of these counters at random from the bag and adding the values.



(a) What is the largest possible total for each game?

(b) What is the smallest possible total for each game?

(c) How many different values between the smallest and largest totals can you form by drawing three counters?

4 A buyer for a clothing store purchases 40 dresses for $1800. These dresses are sold in the store for an average price of $42 per dress. Determine the profit or loss made on these 40 dresses.

5 Basil started the day with $480 and had only $260 at the end of the day, after making three purchases. Lunch cost $50 and Basil had receipts for two other purchases: a movie and a new pair of shoes. Given that the movie cost less than lunch, what integer prices could be written on the two receipts?

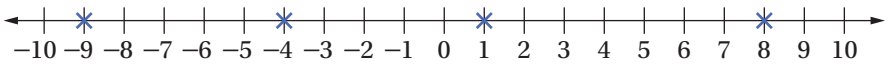
6 A health store paid $140 for two 10 kg bags of yeast to sell to its customers. The store sold 5 kg of the yeast for $11 per kilogram and sold the remaining yeast during a sale for $90. Determine the profit or loss made on the sale of this yeast.

TOPIC REVIEW

Multiple choice

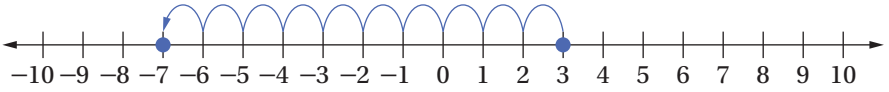
ANSWERS Page 740

1 Choose the correct answer. In order from smallest to largest, the numbers shown on this number line are:



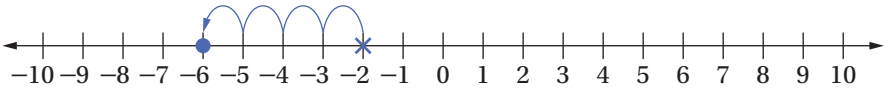
A −9, −4, 1, 8 **B** 1, −4, −9, 8 **C** 1, −4, 8, −9 **D** 8, 1, −4, −9

2 The expression shown on this number line is:



A 3 + (−7) **B** 3 + (−7) = −4 **C** 3 − 10 = −7 **D** −3 + 10 = 3

3 The expression shown on this number line is:



A −2 + (−4) = −6 **B** 2 + (−6) = −4 **C** 2 + (−8) = −6 **D** −2 −6 = −8

4 Which of these groups of integers are ranked from smallest to largest?

A −15, 8, 20, −9, −6 **B** −4, −6, 8, −12, 20 **C** −15, −9, −6, 8, 20 **D** 0, −1, 2, 3, 4, 5

5 −6 + 4 =

A −2 **B** −10 **C** 2 **D** 10

6 −5 − 3 =

A 8 **B** 2 **C** −8 **D** −2

7 −5 − (−4) =

A −9 **B** −1 **C** 1 **D** 9

8 55 − (−20) =

A 75 **B** 35 **C** −35 **D** −75

9 5 − (−2) + 3 =

A 6 **B** 10 **C** −1 **D** 4

10 −3 − 3 + 6 =

A −12 **B** −6 **C** 0 **D** 6

11 −15 − (−10) + 30 =

A −5 **B** −35 **C** 35 **D** 25

12 155 − (−10) − 90 =

A 55 **B** 65 **C** 75 **D** −55

Short answer

1 List the integers:

(a) greater than −2 and less than 4

(b) greater than −9 and less than −6

(c) greater than −4 and less than 0.

2 **(a)** The sequence −7, −5, −3, ... is increasing by 2. Write the next three terms.

(b) The sequence −75, −65, −55, ... is increasing by 10. Write the next three terms.

(c) The sequence −2, −6, −10, ... is decreasing by 4. Write the next three terms.

3 Jason has a debt of $1640. He gets paid a $2500 bonus at work and pays off the debt. How much money does Jason have after paying his debt?

4 The temperature on Saturn varies between −140 °C and 21 °C, depending on season, time of day and location. Calculate the difference in the temperatures.

5 Use this balance sheet to calculate the debt at the end of the month.

Date	Amount
2/6/22	−235.20
13/6/22	450.00
21/6/22	−115.40
29/6/26	−225.40

6 On 5 August 2022, the maximum temperature recorded at the Kuwait airport was 46 °C and the minimum temperature at the Amundsen-Scott South Pole Station (Antarctica) was −69 °C . Calculate the difference in the temperatures.

7 A submarine is sitting at a depth of 310 m.

(a) Calculate the new depth when it drops another 17 m.

(b) The captain wishes to use the periscope, so the submarine must rise from 310 m below the surface to a depth of 6 m.

(i) How much does the submarine rise?

(ii) The 8 m periscope is then raised. What is the height of the periscope above sea-level?

8 Josiah had $600 in his wallet when he left for work, but when he got home, he only had $150. He remembered 3 purchases during the day, but could not remember what they were or the amounts. Create a set of 3 receipts that could explain the remaining money in his wallet.

9 Trinh, Mai and Gayle were playing a board game. In the first round, Mai won 10 tokens from Trinh, but in the second round, she lost 25 tokens to Gayle. Given that she started with 8 tokens, how many tokens does she have now?

10 Asif is transferring from his job in Cairns, where the average maximum temperature currently is 31 °C, to a base in Antarctica, where the average daily temperature is −21 °C . What temperature change will he experience?

11 On a clear day in outback Australia, the overnight minimum was −2 °C and the maximum daytime temperature was 13 °C. By how much did the temperature rise on that day?

12 Li had a balance of −$13.50 on her credit account and spent another $55.50 on credit before she made a payment of $90. Calculate the new balance on her credit account.

13 Calculate the following.

(a) $-3 + -6$ **(b)** $-74 - 8 + 19$ **(c)** $8 - (-5)$

14 You start with −4, add −2, then subtract −3. What number do you finish with?

Extended response

1 Mel owns a bookstore and towards the end of the month she held a one-day "$19.99 sale" to clear out some of the old stock to make room for new titles. Her sales on the day of the sale are shown in the table below.

Title	Cost	Sale price	Profit/loss
Fake Fashions	$14.50	$19.99	
Majestic Mountains	$17.50	$19.99	
Killer Kingdoms	$23.50	$19.99	
Curios Quizzing	$22.50	$19.99	

(a) Complete the table by calculating the profit or loss made on each book.

(b) Which book/s does she make a profit on?

(c) The sales for the day were as shown. Copy and complete the table.

Title	Number sold	Profit/loss per book	Profit/loss
Fake Fashions	5		
Majestic Mountains	6		
Killer Kingdoms	4		
Curios Quizzing	3		

(d) What was her overall profit/loss on the sale items?

2 Here is a list of places on Earth with extreme temperatures. Find two cities with a difference in temperature of:

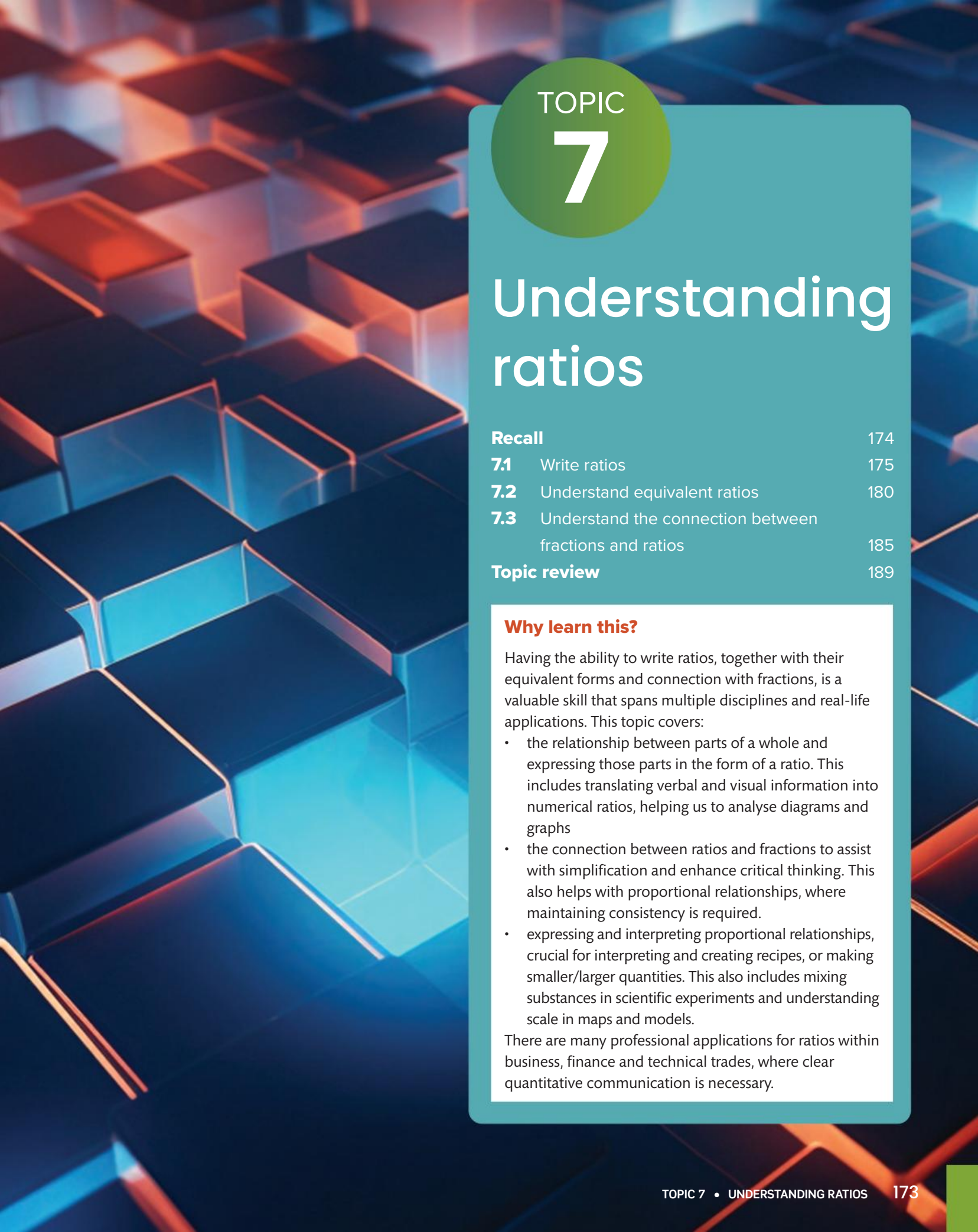
(a) 116 °C

(b) 104 °C

(c) 127 °C

(d) 115 °C

City	Temperature
Verkhoyansk (Russia)	−70 °C
Denali (USA)	−74 °C
Snag (Canada)	−63 °C
Oymyakon (Russia)	−66 °C
Churu (India)	42 °C
Pilani (India)	41 °C
Death Valley (USA)	57 °C
Dallol (Ethiopia)	49 °C



TOPIC 7

Understanding ratios

Recall 174
7.1 Write ratios 175
7.2 Understand equivalent ratios 180
7.3 Understand the connection between fractions and ratios 185
Topic review 189

Why learn this?

Having the ability to write ratios, together with their equivalent forms and connection with fractions, is a valuable skill that spans multiple disciplines and real-life applications. This topic covers:

- the relationship between parts of a whole and expressing those parts in the form of a ratio. This includes translating verbal and visual information into numerical ratios, helping us to analyse diagrams and graphs
- the connection between ratios and fractions to assist with simplification and enhance critical thinking. This also helps with proportional relationships, where maintaining consistency is required.
- expressing and interpreting proportional relationships, crucial for interpreting and creating recipes, or making smaller/larger quantities. This also includes mixing substances in scientific experiments and understanding scale in maps and models.

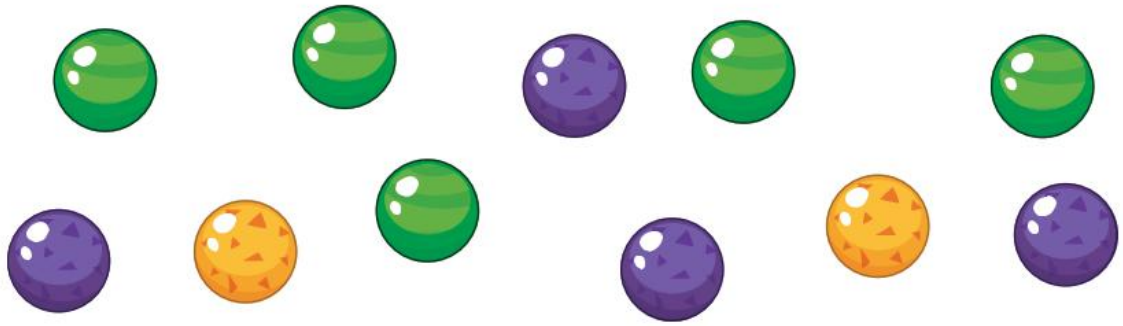
There are many professional applications for ratios within business, finance and technical trades, where clear quantitative communication is necessary.

RECALL

ANSWERS Page 682

I can group and count items by common attributes

1 Copy and complete the table to identify the number of each type of marble shown.



Yellow speckled	Purple speckled	Green striped

2 For the list of numbers shown, identify how many odd numbers there are compared to even numbers.

11, 2, 3, 9, 7, 6, 8, 12, 1, 13, 4

I can multiply decimals by powers of 10

1 Identify the smallest power of 10 needed to multiply each decimal to make it a whole number.

(a) 0.7 **(b)** 0.25 **(c)** 0.099 **(d)** 1.5 **(e)** 15.25

2 Multiply 3.45 by the following numbers.

(a) 0.1 **(b)** 1 **(c)** 10 **(d)** 100 **(e)** 1000

3 Describe what happens to the value of the 3 digit in Question **2**, when the number is multiplied by a power of 10.

I can convert decimals to fractions

1 Write the following decimals as a fraction with a denominator of a power of 10.

(a) 0.5 **(b)** 0.25 **(c)** 0.8 **(d)** 0.75 **(e)** 0.24

2 Write each of these decimals as a fraction in simplest form.

(a) 0.5 **(b)** 0.25 **(c)** 0.8 **(d)** 0.75 **(e)** 0.24

I can simplify fractions

1 Write each of these fractions in simplest form.

(a) $\frac{25}{30}$ **(b)** $\frac{36}{40}$ **(c)** $\frac{15}{35}$

Write ratios

7.1

Learning intention: To be able to write ratios

Success criteria:

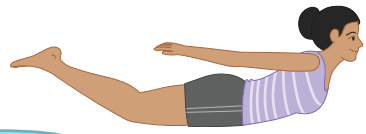
SC 1 I can write ratios to represent a diagram.

SC 2 I can use ratios to translate simple statements.

Lesson warm-up

The perfect cordial?

Cordial is a fruit syrup that is served with water.



The more cordial that is added to the mix, the sweeter the drink.

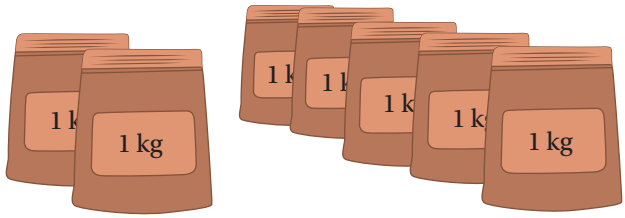
The manufacturer of a brand of cordial gave the instructions '1 : 4' or '1 to 4'.

Describe how to make your perfect cup of cordial.

SC 1 I can write ratios represented in a diagram

A ratio is one way to compare amounts of the same type of quantity. A ratio has no units.

2 kg compared with 5 kg gives the amounts in the ratio 2 : 5.



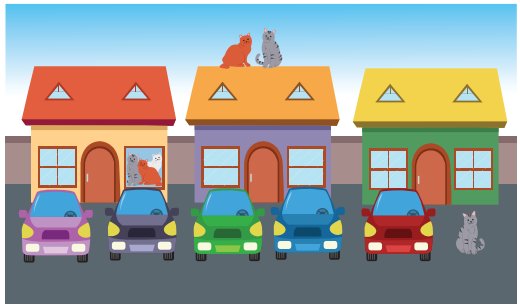
So 2 lots of 1 kg compared with 5 lots of 1 kg.

6 kg compared with 15 kg gives the amounts in the ratio 6 : 15 or, more simply 2 : 5, since 6 kg is two lots of 3 kg and 15 kg is five lots of 3 kg.

Worked example

Writing ratios from a diagram

Write a ratio to compare each of the following:



(a) the number of cats at the house on the right compared to the middle house

THINKING	WORKING
Identify the number of items. List them in the order given.	Cats on the right: 1 Cats in the middle: 2
Write the ratio.	Cats on the right compared to the middle 1 : 2

(b) the number of cars in front of the first two houses from the left

THINKING	WORKING
Identify the number of items. List them in the order given.	Cars on the left: 2 Cars in the middle: 2
Write the ratio, simplifying where necessary.	Cars on the left compared to the middle 2 : 2 or more simply 1 : 1

(c) the number of cars in front of all three houses from left to right.

THINKING	WORKING
Identify the number of items. List them in the order given.	Cars on the left: 2 Cars in the middle: 2 Cars on the right: 1
Write the ratio.	Cars on the left compared to the middle and the right 2 : 2 : 1

SC 2 I can use ratios to translate simple statements

Worked example

Writing ratios to interpret statements

A bouquet of flowers has three roses, ten carnations, one stem of fern leaves and two stems of eucalyptus leaves. Write a ratio that compares:

(a) the number of carnations to roses

THINKING	WORKING
Identify the number of items. List them in the order given.	Carnations: 10 Roses: 3
Write the ratio.	Carnations to roses 10 : 3

(b) the number of stems of greenery to flowers

THINKING	WORKING
Identify the number of items. List them in the order given.	Greenery: 1 + 2 = 3 Flowers: 3 + 10 = 13
Write the ratio.	Greenery to flowers 3 : 13

(c) the number of stems of roses to carnations to greenery.

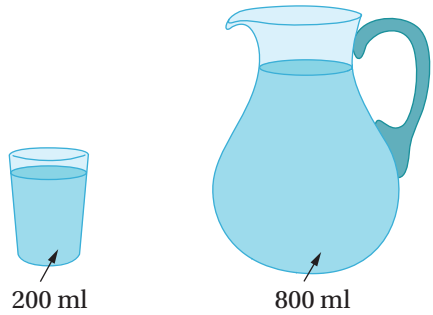
THINKING	WORKING
Identify the number of items. List them in the order given.	Roses: 3 Carnations: 10 Greenery: 3
Write the ratio.	Roses to carnations to greenery 3 : 10 : 3

Practice

ANSWERS Page 682

SC 1 I can write ratios represented in a diagram

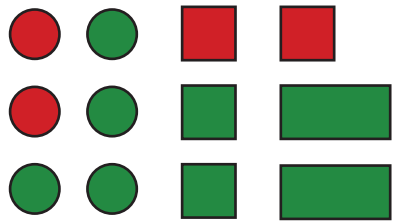
1 Write a ratio, in simplest form, to compare each of the following:



(a) the volume of water in the glass to the jug

(b) the volume of water in the jug to the glass.

2 Write a ratio, in simplest form, to compare each of the following:

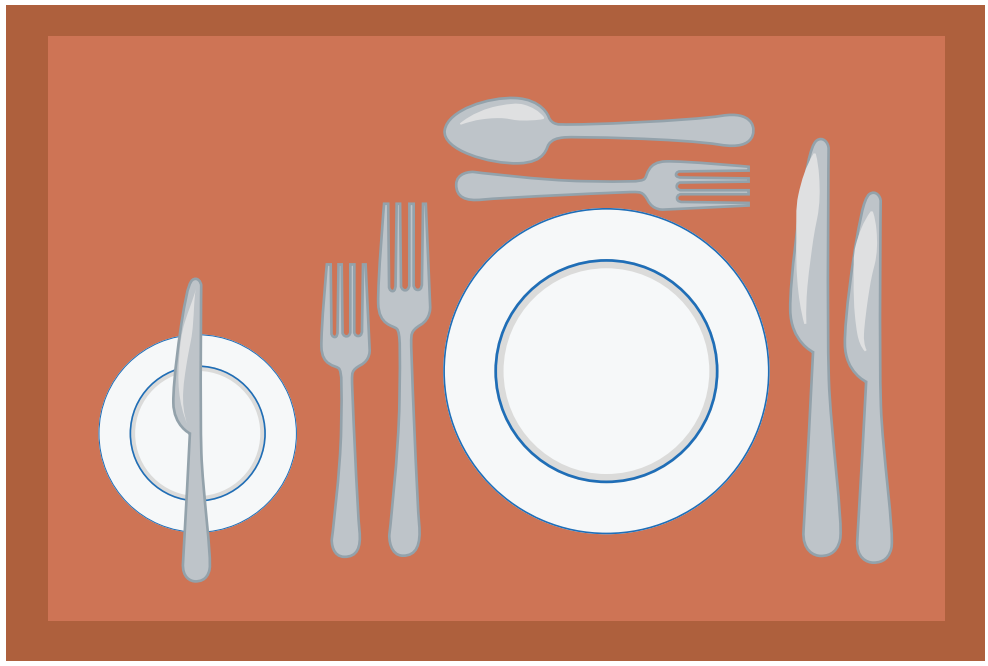


(a) the number of circles to squares

(b) the number of squares to rectangles

(c) the number of circles to squares to rectangles.

3 Write a ratio, in simplest form, to compare each of the following:



(a) the number of knives to forks

(b) the number of knives to forks to spoons

(c) the number of spoons to plates.

4 Name the point that divides the number line into the given ratio for length of left part to right part.

(a) 5 : 7 **(b)** 7 : 5 **(c)** 1 : 1 **(d)** 5 : 1 **(e)** 2 : 1

SC 2 I can use ratios to translate simple statements

1 A family has 3 cats, 2 dogs and 5 goldfish. Write each of the following comparisons as ratios.

(a) Number of cats to dogs

(b) Number of goldfish to total pets

(c) Number of dogs to cats to goldfish

(d) Number of cats to total pets

2 A weekly grocery shop for a family includes 2 packs of chops, 1 pack of chicken, 2 packs of sausages, 2 packs of mince, 2 cartons of milk, 2 cartons of eggs and 4 loaves of bread. Write each of the following comparisons as ratios.

(a) Cartons of milk to eggs

(b) Packs of meat to cartons of milk

(c) Cartons of milk to loaves of bread

(d) Packs of chops to chicken to sausages to mince

3 Write each of the following comparisons as the required ratio.

(a) Double the number of cars as houses: car to house ratio

(b) Twice as many boys as girls: girl to boy ratio

(c) Three times as many birds as kangaroos: bird to kangaroo ratio

(d) Six times as many flowers as bees: bee to flower ratio

4 In a box of red, green, orange and white snake lollies, there are twice as many red as green, three times as many orange as green, and six times as many red as white. Write the following comparisons as ratios.

(a) Number of red to green

(b) Number of orange to green

(c) Number of white to red

(d) Number of red to green to orange to white

7.2 Understand equivalent ratios

Learning intention: To understand equivalent ratios

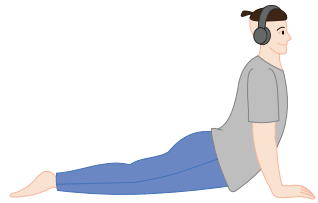
Success criteria:

SC 1 I can write ratios in simplest form.

SC 2 I can compare quantities using ratios.

Lesson warm-up

What is your preference?



What is your preferred ratio for each of these?

- Vegemite on toast
- Cordial in water
- Tomato sauce on hot chips
- Fruit and nut mix

SC 1 I can write ratios in simplest form

Fractions are equal if the numerator and denominator of one fraction can be multiplied or divided by the same number to obtain the other fraction.

For example, $\frac{3}{4} \times \frac{5}{5} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

So, $\frac{3}{4} = \frac{15}{20}$.

Similarly, ratios are equivalent if each part of one ratio can be multiplied or divided by the same number to obtain the other ratio.

For example, multiply each part in the ratio 3 : 4 : 7 by the number 5

$3 \times 5 : 4 \times 5 : 7 \times 5 = 15 : 20 : 35$

So, 3 : 4 : 7 = 15 : 20 : 35.

A ratio is in simplest form when each part is a whole number and the highest factor common to all parts is 1.

Worked example

Simplifying ratios

Simplify each ratio.

(a) 8 : 10 : 12

THINKING	WORKING
Determine the highest common factor (HCF).	The HCF of 8, 10 and 12 is 2.
Divide each part by the HCF.	$\frac{8}{2} : \frac{10}{2} : \frac{12}{2} = 4 : 5 : 6$
Write the answer.	8 : 10 : 12 written in simplest form is 4 : 5 : 6.
The answer to **(a)** gives the number of 2s in each part.	

(b) 4.5 : 3.25

THINKING	WORKING
Multiply each part by the same multiple of 10 (in this case 100) to obtain whole numbers.	4.5 : 3.25 = 450 : 325
Determine the HCF.	The HCF of 450 and 325 is 25.
Divide each part by the HCF.	Dividing both parts by 25, $\frac{450}{25} : \frac{325}{25} = 18 : 13$
Write the answer.	4.5 : 3.25 written in simplest form is 18 : 13.
The answer to **(b)** gives the number of quarters (or lots of 0.25) in each part.	

(c) $2\frac{1}{2}:3$

THINKING	WORKING
Write both parts as fractions with a common denominator.	$2\frac{1}{2}:3=\frac{5}{2}:\frac{6}{2}$
Compare just the numerators.	$\frac{5}{2}:\frac{6}{2}=5:6$
Determine the HCF.	The HCF of 5 and 6 is 1, so the ratio is in its simplest form.
Write the answer	$2\frac{1}{2}:3$ written in simplest form is 5 : 6.
The answer to **(c)** gives the number of halves in each part.	

For **(b)** and **(c)**, since each ratio has only two parts, you can check your answer on the calculator by dividing the first part by the second, then putting the answer in fraction (or improper fraction) form, so $4.5 \div 3.25 = \frac{18}{13}$ and $2\frac{1}{2} \div 3 = \frac{5}{6}$.

SC 2 I can compare quantities using ratios

When comparing quantities of the same unit, the units cancel out as part of the simplification process. So when comparing 4 kg to 10 kg, the simplified ratio is 2 : 5.

It may be appropriate to say how many of the smaller quantity make up the larger quantity, or what portion of the larger quantity the smaller represents.

4 kg is 0.4 $\left(\text{or } 40\% \text{ or } \frac{2}{5}\right)$ of 10 kg; or 10 kg is 2.5 $\left(\text{or } \frac{5}{2}\right)$ times more than 4 kg.

Worked example

Comparing quantities using ratios

Compare 80 cm to 1.2 m in each of the following ways:

(a) as a ratio

THINKING	WORKING
Write the quantities, in the order given, as a ratio with the same units.	80 cm : 1.2 m = 80 cm : 120 cm
Remove units from both sides and simplify the ratio.	80 : 120 = 2 : 3

(b) writing what fraction of the larger quantity the smaller one represents

THINKING	WORKING
Write the ratio as a fraction, expressing the comparison in words.	80 cm is $\frac{2}{3}$ of 1.2 m.

(c) writing how many of the smaller quantity make up the larger.

THINKING	WORKING
Divide the larger quantity by the smaller, rounding appropriately.	$\frac{120\text{ cm}}{80\text{ cm}} = \frac{3}{2} = 1.5$
Express the comparison in words.	1.2 m is 1.5 times as long as 80 cm.

Practice

ANSWERS Page 683

SC 1 I can write ratios in simplest form

1 Create equivalent ratios by multiplying or dividing both parts of each ratio as instructed.

(a) 8 : 16; (÷ 2) **(b)** 1 : 2; (× 6) **(c)** 5 : 4; (× 5) **(d)** 3 : 70; (÷ 10)

(e) 9 : 33; (÷ 3) **(f)** 17 : 24; (÷ 2) **(g)** 3 : 1; (× 100) **(h)** 2 : 15; (× 4)

2 Write each ratio in simplest form.

(a) 8 : 20 **(b)** 55 : 66 **(c)** 50 : 400 **(d)** 42 : 32

(e) 24 : 18 : 12 **(f)** 30 : 60 : 20 **(g)** 15 : 18 : 24 **(h)** 50 : 25 : 75

3 Write each ratio in simplest form.

(a) 3 : 7.1 **(b)** 5 : 6.2 **(c)** 2.11 : 3.4

(d) 2.25 : 6 **(e)** 7.7 : 5.72 **(f)** 40 : 50.2

4 Write each ratio in simplest form.

(a) $4\frac{1}{2} : 6$ **(b)** $2 : 1\frac{1}{3}$ **(c)** $5\frac{2}{3} : 6\frac{1}{3}$ **(d)** $7\frac{1}{2} : 9\frac{1}{2}$

(e) $\frac{2}{5} : 2\frac{1}{5}$ **(f)** $3\frac{1}{2} : 4\frac{1}{3}$ **(g)** $3\frac{3}{5} : 2\frac{1}{7}$ **(h)** $\frac{3}{5} : 4\frac{1}{3}$

5 There must be at least 1 supervisor for every 6 participants in an orienteering course.

(a) Write the comparison of participants to instructors as a ratio.

(b) 24 people wish to participate in the course. Use an equivalent ratio to determine the number of supervisors needed.

(c) Given that 5 supervisors are available, use an equivalent ratio to determine the maximum number of participants.

SC 2 I can compare quantities using ratios

1 Compare the quantities by writing the simplest ratio.

(a) 6.4 m to 8 m

(b) 5 weeks to $1\frac{1}{2}$ fortnights

(c) 5 L to 300 mL

(d) 10 months to 4 years

(e) 800 m to 1500 m to 10 km

(f) 55 cents to 80 cents to $3.20

2 For each of the following mammals, compare the mass of each mother to its newborn baby as a decimal, correct to 2 decimal places where necessary. Write your answers in a sentence.

(a) Jersey cattle: mother, 350 kg; newborn calf, 40 kg

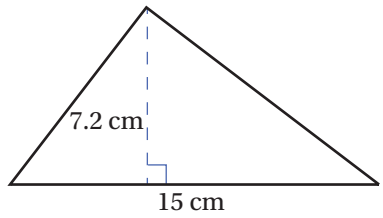
(b) Kangaroo: mother, 40 kg; newborn joey, 0.85 g

(c) Orca whale: mother, 10 tonnes; newborn calf, 150 kg

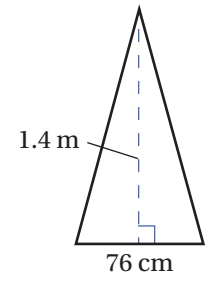
3 Four members of a family give their heights as follows: mother 165 cm; husband 1.8 m; daughter 148 cm; son 94 cm. Compare the height of each of the other family members with the mother's height in decimal form, correct to 2 decimal places. Write your answers in a sentence.

4 For each of the triangles, compare the height with the base length in decimal form, correct to 2 decimal places where necessary. Write your answers to complete the sentence: The height is _____ of/times the base length.

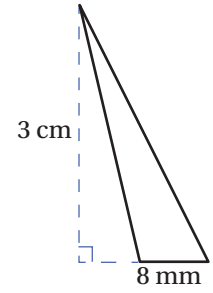
(a)



(b)



(c)



5 The tallest mountain in Australia is in NSW. Mount Kosciuszko is about 2228 m high. Compare the height of Mount Kosciuszko to the highest mountain in each of the other states and territories in decimal form, correct to 2 decimal places.

(a) Queensland, Mount Bartle Frere is about 1622 m.

(b) ACT, Mount Bimberi is about 1913 m.

(c) Victoria, Mount Bogong is about 1986 m.

(d) Tasmania, Mount Ossa is about 1617 m.

(e) NT, Mount Zeil is about 1531 m.

(f) SA, Mount Woodroffe is about 1435 m.

(g) WA, Mount Meharry is about 1249 m.

6 For each of the following pairs of quantities, write the fraction that shows the proportion of the smaller to the larger.

(a) Cartons of milk: 600 mL, 2 L

(b) Tins of tuna: 125 g, 425 g

(c) Lengths of rulers: 15 cm, 1 m

Understand the connection between fractions and ratios

7.3

Learning intention: To understand the connection between fractions and ratios

Success criteria:

SC 1 I can convert ratios to describe the relationship between parts and the whole.

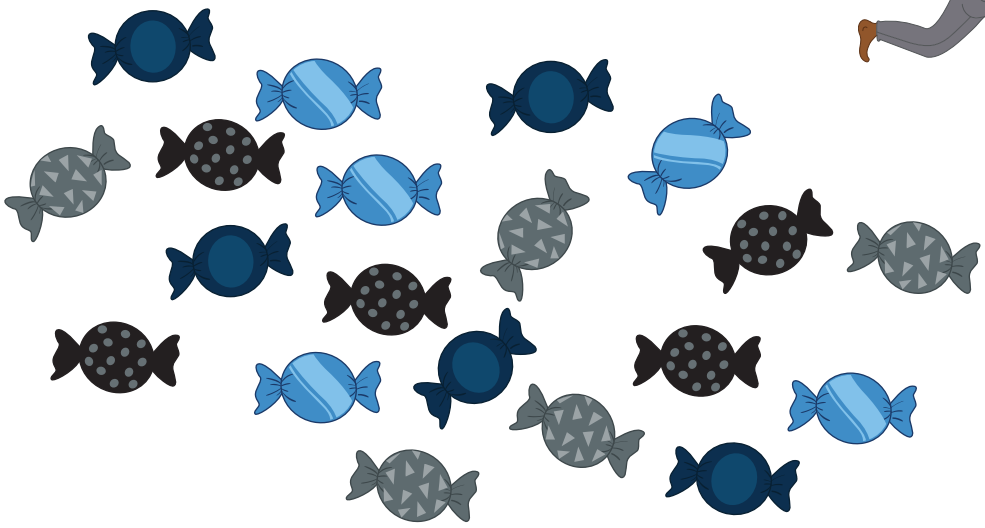
SC 2 I can convert fractions of the whole to part : part ratios.

SC 3 I can share quantities into given ratios.

Lesson warm-up

What is your preference?

Marissa and Haneef buy a bag of 20 lollies together. Marissa paid $3 and Haneef paid $2. How do you think they should divide the lollies between them?



SC 1 I can convert ratios to describe the relationship between parts and the whole

The strip of coloured squares, 2 yellow, 4 red and 3 blue, allows many comparisons of amount.

Yellow to red is 1 : 2.

Blue to yellow is 3 : 2.

Red to the whole piece is 4 : 9.

When a ratio is written in simplest form, think of each number in the ratio as the number of parts. The total number of parts is found by adding the individual numbers from the ratio.

For the ratio 2 : 3 there are 2 + 3 = 5 parts.

Worked example

Converting a ratio to describe fractions of the whole

The boy to girl ratio in a class is 5 : 6. What fraction of the class are boys?

THINKING	WORKING
Calculate the total number of parts in the ratio.	Total parts: 5 + 6 = 11
State the number of parts representing boys as a fraction of the total number of parts (representing boys and girls).	Fraction of class that are boys: $\frac{5}{11}$

SC 2 I can convert fractions of the whole to part : part ratios

Worked example

Converting fractions of the whole to part : part ratios

In the giraffe and zebra section of a zoo, $\frac{5}{12}$ of the animals are giraffes. Calculate the giraffe to zebra ratio for the section.

THINKING	WORKING
Convert the fraction to a part : whole ratio.	$\frac{5}{12}$ of the whole are giraffes, so the giraffe to whole ratio is 5 : 12.
Subtract to determine the number of parts for zebras.	Parts in the ratio for zebras: 12 − 5 = 7.
State the part : part ratio.	Giraffe to zebra ratio is 5 : 7.

SC 3 I can share quantities into given ratios

A table is a good way to sort out complicated problems involving ratios.

	A	B	C	Total
Parts	3	5	4	12
Quantities	60 kg	100 kg		240 kg

Notice that each number in the top row must be multiplied by the same number to obtain the quantity in the second row.

A part corresponds to a quantity of 20 kg in this case.

Worked example

Sharing quantities into given ratios

A carton containing 24 boxes of sultanas is to be shared between Emily, Cooper and David in the ratio 3 : 4 : 1. Determine the number that each person will receive.

THINKING	WORKING
Calculate the number of parts.	Number of parts: $3 + 4 + 1 = 8$
Express each person's portion as a fraction of the whole carton.	Emily will receive $\frac{3}{8}$ of the carton. Cooper will receive $\frac{4}{8}$ or $\frac{1}{2}$ of the carton. David will receive $\frac{1}{8}$ of the carton.
Multiply each fraction by the whole amount.	$\frac{3}{8} \times 24 = 9; \frac{1}{2} \times 24 = 12; \frac{1}{8} \times 24 = 3$
State the answer in words, checking that the total of the answers makes sense.	Emily will receive 9 boxes. Cooper will receive 12 boxes. David will receive 3 boxes. Note: 9 + 12 + 3 = 24 total boxes.

Practice

ANSWERS Page 683

SC 1 I can convert ratios to describe the relationship between parts and the whole

1 For each of the following ratios, calculate what fraction of the whole is taken up by the quantity represented by the larger part of the ratio.

(a) 1 : 3 **(b)** 5 : 7 **(c)** 8 : 5 **(d)** 6 : 1 **(e)** 2 : 3 : 5

2 10 litres of white paint has 500 mL of blue and 1 litre of red added to make a particular shade of mauve.

(a) Determine the ratio of white to the whole mixture.

(b) Calculate the fraction of the mixture that is red.

3 At a cricket game in Brisbane, there are 3 Queensland supporters for every Victoria supporter. Assume that every person in the crowd supports one of the two states playing.

(a) Determine the ratio of Queensland supporters to the whole crowd.

(b) Calculate the fraction of the crowd that are Victoria supporters.

4 A deciduous tree in autumn has red, yellow and green leaves in the ratio 2 : 3 : ?. Given that $\frac{1}{4}$ of the leaves are red, determine the fraction that are green.

5 A shopping basket contains 12 strawberries, 10 apples and 8 kiwifruit.

(a) Compare the number of each fruit as a ratio in the order given.

(b) Calculate the total number of parts for the ratio.

(c) Determine the fraction of all the fruit that are:

(i) strawberries

(ii) apples

(iii) kiwifruit.

SC 2 I can convert fractions of the whole to part : part ratios

1 Each of the following refers to the fraction of oranges in a bag containing just oranges and apples. Calculate the orange to apple ratio for each bag.

(a) $\frac{3}{4}$ **(b)** $\frac{5}{16}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{7}{20}$

2 A bag of lollies is 8% chocolates and the rest are gummy candies. Calculate the chocolate to gummy candy ratio.

3 A student spends 0.4 of the week sleeping and 0.12 of the week studying. Calculate the student's:

(a) sleep to whole week ratio

(b) study to whole week ratio

(c) sleep to study ratio.

4 A wallpaper pattern is $\frac{1}{3}$ green, $\frac{2}{5}$ yellow and the rest is white. Calculate the wallpaper's:

(a) green to whole surface ratio

(b) green to yellow ratio

(c) white to whole surface ratio.

SC 3 I can share quantities into given ratios

1 Share 45 chocolate frogs between Frankie and Gertrude respectively in the ratio 2 : 3.

2 Eighty-four cupcakes are to be shared for afternoon teas over three days, in the ratio 3 : 7 : 2. Determine the number of cupcakes for each day.

3 After 24 matches, the win to loss to draw ratio for a soccer team is 1 : 5 : 2. Determine the number of wins and losses.

4 The oil to petrol ratio for a lawn mower is 1 : 24. Determine how much oil and how much petrol are needed to fill a 6 L tank.

5 The profits for a small business are to be divided between the four investors in the same ratio as their initial investments of $5000, $300, $1600 and $2500 respectively. Determine the amount for each investor, given that the profit was $2540. Round your answers to the nearest dollar.

TOPIC REVIEW

Multiple choice

ANSWERS Page 741

1 There are 26 shrubs in a garden: 15 are grevillea and the rest are melaleuca (tea tree). The ratio of melaleuca to grevillea is:

A 11 : 26 **B** 15 : 11 **C** 15 : 26 **D** 11 : 15

2 A Year 7 class has 13 girls and 10 boys. The boy to girl ratio is:

A 13 : 10 **B** 10 : 13 **C** 13 : 23 **D** 10 : 23

3 A Year 7 class has 13 girls and 10 boys. The fraction of the class that are boys is:

A $\frac{10}{23}$ **B** $\frac{10}{13}$ **C** $\frac{13}{23}$ **D** $\frac{13}{10}$

Use the following image to answer Questions **4**, **5** and **6**.

4 Determine the ratio of dogs to hens in the illustration.

A 3 : 2 **B** 2 : 5 **C** 2 : 3 **D** 3 : 5

5 Determine the ratio of people to hens in the illustration.

A 3 : 10 **B** 1 : 3 **C** 2 : 5 **D** 3 : 8

6 Determine the ratio of people to dogs in the illustration.

A 1 : 1 **B** 1 : 2 **C** 2 : 1 **D** 3 : 2

7 The simplest form of 12 : 66 is:

A 11 : 2 **B** 2 : 11 **C** 1 : 5.5 **D** 0.2 : 1

8 The ratio $3\frac{1}{2} : 2\frac{1}{4}$ simplifies to:

A 9 : 7 **B** 2 : 1 **C** 3 : 2 **D** 14 : 9

9 The ratio $2 : 3\frac{1}{3}$ simplifies to:

A 3 : 5 **B** 3 : 10 **C** 1 : 5 **D** 5 : 6

10 The ratio 15 : 20 : 4 has how many parts?

A 25 **B** 11 **C** 8 **D** 39

11 Each netball team has a centre who can go anywhere except the goal circles, a wing and wing defence who can go on two thirds of the court except the goal circles, a goal attack and defence who can go on two thirds of the court including the goal circles, and a goal shooter and defence who can go on one third of the court including the goal circles. What fraction of the team can go into a goal circle?

A $\frac{2}{5}$ **B** $\frac{2}{3}$ **C** $\frac{4}{7}$ **D** $\frac{2}{7}$

12 A class of 20 students has a girl to boy ratio of 2 : 3. Four boys and two girls join the class. What is the new girl to boy ratio?

A 3 : 4 **B** 5 : 8 **C** 4 : 7 **D** 6 : 5

13 Alma, Benji and Calvin have invested in a company in the ratio 5 : 2 : 3 respectively. If a profit of $8000 is to be divided accordingly, how much will Benji receive?

A $800 **B** $3200 **C** $1600 **D** $2000

Short answer

1 A vet sees 2 horses, 5 dogs and 6 cats in a day. Calculate the following ratios.

(a) Number of cats to dogs

(b) Number of dogs to horses

(c) Number of cats to horses

(d) Number of horses to total number of animals seen

2 A piece of wood is cut into three lengths of 35 cm, 80 cm and 1.2 m respectively. Write the ratio of the lengths in the order given.

3 Cola comes in cans and bottles in sizes of 375 mL, 1 L and 1.5 L. Write the ratio of the volumes from smallest to largest.

4 There are 64 Cavendish and 56 ladyfinger bananas available from a grocery store.

(a) Write the ratio of Cavendish to ladyfingers.

(b) Write the ratio of Cavendish to the total number of bananas.

(c) Write the fraction of bananas that are ladyfingers.

5 Simplify each of the following ratios.

(a) 15 : 33 **(b)** 80 : 16 : 32 **(c)** 30 mm : 240 mm

6 Simplify each of the following ratios.

(a) $3 : 6\frac{1}{2}$ **(b)** $2\frac{1}{3} : 1\frac{2}{3}$ **(c)** $1\frac{1}{4} : 2\frac{1}{3}$

7 Simplify each of the following ratios.

(a) 40 seconds to 5 minutes

(b) 5 hours to 20 minutes

(c) 8 weeks to 3 fortnights

8 Calculate the ratio of the following ages.

(a) $3\frac{1}{4}$ years to 10 months

(b) 12 years to 7 years 5 months

9 A salad dressing is to be made by mixing oil and vinegar in the ratio 1 : 3 respectively.

(a) How much vinegar should be mixed with 75 mL of oil?

(b) How much oil should be mixed with 75 mL of vinegar?

10 The highest mountain in the world is Mount Everest at 8849 m. New Zealand's highest mountain is Mount Cook at 3724 m.

(a) How many times as high as Mount Cook is Mount Everest (correct to 2 decimal places)?

(b) Given that Mount Cook is 1.67 times as high as Mount Kosciuszko, calculate the height of Mount Kosciuszko (to the nearest 10 m).

11 Share each quantity in the given ratio.

(a) \$7200; 7 : 5

(b) 32 kg; 3 : 5

12 Jesse and Henry invest \$6500 and \$3500 respectively in a small business. How much of the \$850 profit from the first week should Henry receive?

13 A 40 m^2 vegetable garden is devoted to lettuce and cucumber in the ratio 5 : 3. What area has been allowed for lettuce?

14 The life expectancy in Australia, China and Japan is 83.8 years, 78.2 years and 84.9 years respectively. Write each ratio as a decimal (correct to 2 decimal places) comparing Australia's life expectancy to:

(a) China

(b) Japan.

15 The adult–student ratio for an excursion must be at least 1 : 5.

(a) Given that there are 26 students participating, determine the minimum number of adults required.

(b) Given that the minimum number of adults participate, calculate the actual adult–student ratio.

16 After a day, the contents of a 2 L milk container has gone down to 375 mL.

(a) Determine the ratio of milk consumed to milk remaining.

(b) Calculate the fraction of the milk that has been consumed.

Extended response

1 Lyndall has a daughter and a son. Her daughter Penny has two sons and a daughter. Lyndall's son Allister has two daughters and a son.

(a) Write the son to daughter ratios for Lyndall, Penny and Allister.

(b) Write the grandson to granddaughter ratio for Lyndall.

(c) Lyndall, Penny and Allister each have partners of the opposite gender. When all of the family get together, what will be the male to female ratio?

2 A 40-year-old woman and her 10-year-old son are 172 cm and 140 cm tall respectively.

(a) Determine:

(i) the height ratio of the woman to her son

(ii) the age ratio of the woman to her son.

(iii) Express the answers to **(i)** and **(ii)** as single decimal values (rounded to 2 decimal places, if necessary) to complete the sentence: The woman is ____ times as ____ as her son.

(b) The son grows 5 cm every year for the next 5 years. Determine the new:

(i) height ratio of the woman to her son

(ii) age ratio of the woman to her son.

(iii) Express the answers to **(i)** and **(ii)** as single decimal values to complete the sentence: The woman is now ____ times as ____ as her son.

3 Copy and complete the proportion table, then use the entries to answer the questions that follow.

	A	B	C	Total
	$50		$70	$210
Parts				
				$42
	$100			

Determine:

(a) the amount for A compared to the amount for B as a ratio

(b) the fraction A is of C

(c) the fraction that C is of the total

(d) the amount B has when the total is $42

(e) the amount C has when A has $100.

TOPIC
8

Algebra

Recall 194
8.1 Understand variables in algebra 195
8.2 Identify and simplify like terms 199
8.3 Calculate the value of algebraic expressions 202
8.4 Write algebraic expressions in context 208
Topic review 210

Why learn this?

Mastering the concept of variables, like terms and substitution into algebraic expressions is essential groundwork for progression into further topics including equations, graphing equations and much more. It enables the translation of real-world scenarios into mathematical expressions, making them easier to analyse and solve. These skills are particularly important in many fields of study including computer science for writing software algorithms; in physics for solving equations and evaluating formulas; in economics for analysing financial market trends; and in engineering for designing structures.

Understanding algebra is a crucial step for progressing to higher-level mathematics in further education. It also aids in developing logical thinking and problem-solving skills, which are valuable in many aspects of life.

RECALL

ANSWERS Page 684

I can find the unknown value in a number sentence

1 Find the missing number that makes each of the following true.

(a) 12 + ___ = 81 **(b)** 400 – ___ = 29 **(c)** 15 × ___ = 90 **(d)** ___ ÷ 2 = 111

2 Find the missing number that makes each of the following true.

(a) 85 + ___ = 20 × 8 **(b)** 100 – 75 = 5 × ___ **(c)** 720 ÷ 6 = 12 × ___ **(d)** 63 ÷ 3 = 1000 – ___

I can perform several operations on a term

1 10 is multiplied by 2 and 6 is added to the result, (10 × 2 + 6). What is the answer?

2 20 is divided by 4 and the result is added to 12. What is the answer?

3 9 is subtracted from 5 times 7, (5 × 7 – 9). What is the result?

I can change word sentences to number sentences

1 Copy and complete each of the following sentences.

(a) The sum is the result of an _____ calculation.

(b) The difference is the result of a _____ calculation.

(c) The product is the result of a _____ calculation.

(d) The quotient is the result of a _____ calculation.

2 Translate the following sentences into numbers and mathematical symbols.

(a) The sum of seven and one equals eight.

(b) The product of three and five equals fifteen.

(c) Divide one hundred by four and the answer equals twenty-five.

(d) The quotient of thirty-two and eight equals four.

(e) Ten is subtracted from the product of two and seven, giving an answer of four.

I can change word sentences to number sentences and calculate the answer

Translate each of the following sentences in words into a number sentence. Then find the answer.

1 The sum of eight and twenty-eight.

2 The difference between one hundred and sixty-four.

3 Subtract two away from twenty-two then divide the result by five.

4 Multiply the sum of four and fifty by three.

5 The difference between forty and thirty is divided by ten.

Understand variables in algebra

8.1

Learning intention: To be able to understand variables in algebra

Success criteria:

SC 1 I can create algebraic expressions using variables.

SC 2 I can write algebraic expressions involving all four operations.

Lesson warm-up

Your hero

Write down the name of a person you consider to be a hero.

1 Copy and complete the table.

The number of letters in the first name (x)	The number of letters in the surname (y)

2 Calculate the value of the algebraic expressions and describe what the expressions represent.

(a) $x + y$ **(b)** $x - y$

SC 1 I can create algebraic expressions using variables

Mathematical expressions may contain unknown numbers.

An example is 'I think of a number and subtract 6'.

This could be written as an expression such as $\square - 6$ or $x - 6$.

When a letter is used to represent an unknown number, it is called a *variable*.

A *term* describes a number, a variable, or both, such as $2x$.

The term $2x$ is made up of a variable (or unknown number), x and a coefficient, 2.

When a number is multiplied by a variable, the number is written first and the multiplication symbol does not need to be written.

Worked example

Writing a simple algebraic expression

Write 'I think of a number, double it, then subtract six from the result' as an algebraic expression.

THINKING	WORKING
Assign a variable to represent the unknown number.	Let x represent the unknown number.
Write an algebraic expression to represent the first operation.	'I think of a number and double it'. Double means multiply by 2. The algebraic expression is $2 \times x$ or $2x$.
Write the algebraic expression by applying the second operation.	'I think of a number, double it, then subtract six from the result': $2x - 6$

SC 2 I can write algebraic expressions involving all four operations

The order in which the operations are applied can change the output or result of an expression.

The order of operations applied when reading an algebraic expression are:
brackets then multiplication and division, then addition and subtraction.

For example, consider the similarities and differences in the way these two algebraic expressions read:

I think of a number, double it, then subtract six from the result: $2x - 6$

I think of a number, subtract six from it, then double the result: $2(x - 6)$

Worked example

Writing a more complex algebraic expression

Write 'I think of a number, double it, reduce it by six, divide it by five, then add three to the result' as an algebraic expression.

THINKING	WORKING
Assign a variable to represent the unknown number.	Choose x as the variable. Let x represent the unknown number.
Write an algebraic expression to represent the first operation.	'I think of a number and double it'. Double means multiply by 2: $2 \times x$ or $2x$
Continue building the algebraic expression by applying the second operation.	'I think of a number, double it and reduce it by six'. Reduce a number by 6 means to subtract 6 from the number: $2x - 6$

Continue building the algebraic expression by applying the third operation. Note that when dividing an expression by a number, you can either put the expression inside brackets or, more commonly, write the expression as the numerator of a fraction.	'I think of a number, double it, reduce it by six and divide it by five'. $(2x - 6) \div 5$ is generally written as $\frac{2x - 6}{5}$
Write the algebraic expression by applying all four operations.	'I think of a number, double it, reduce it by six, divide it by five, then add three to the result'. $\frac{2x - 6}{5} + 3$

Practice

ANSWERS Page 684

SC 1 I can create algebraic expressions using variables

1 Use the variable x to represent the unknown number and write each of the following as an algebraic expression.

(a) I think of a number, then subtract four from it.

(b) I think of a number, then add four to it.

(c) I think of a number, then multiply it by four.

(d) I think of a number and divide it by four.

(e) I think of a number, triple it, then add four.

(f) I think of a number, divide it by three, then subtract four.

2 Write each algebraic expression as a sentence using words. In each algebraic expression, the variable x represents an unknown number.

(a) $5 - x$ **(b)** $x - 5$ **(c)** $5x$ **(d)** $\frac{5}{x}$ **(e)** $\frac{x}{5}$

3 Write each algebraic expression as an 'I think of a number' problem.

(a) $10m$ **(b)** $-10m$ **(c)** $10m - 7$ **(d)** $-10m + 7$ **(e)** $7 - 10m$

4 Write each algebraic expression as an 'I think of a number' problem.

(a) $x - 7$ **(b)** $2x - 7$ **(c)** $2(x - 7)$

5 Determine the outputs for each problem.

(i) Determine the output number if the unknown number is 10.

(ii) Determine the output number if the unknown number is 15.

(iii) Determine the output expression if the unknown number is represented by the variable x.

(a) I think of a number, multiply it by four, divide it by five, then subtract ten from the result.

(b) I think of a number, divide it by five, subtract ten, then multiply the result by four.

6 A family went on holiday visiting various towns on the coast. The variable x represents the distance, in kilometres, the family travelled on the first day. Describe the distances travelled represented in each algebraic expression.

(a) $x + 2$ **b)** $x - 2$ **(c)** $2 - x$ **(d)** $2x$ **(e)** $\frac{x}{2}$

SC 2 I can write algebraic expressions involving all four operations

1 Use the variable x to represent the unknown number and write each of the following as an algebraic expression.

(a) I think of a number, multiply it by two, divide it by three and add four to the result.

(b) I think of a number, add four, divide it by three and multiply the result by two.

(c) I think of a number, multiply it by two, add four, then divide the result by three.

(d) I think of a number, divide the result by three, add four, then multiply the result by two.

(e) Describe the similarities and differences in the expressions created in parts **(a)** through **(d)**. Do you think any of the expressions are equivalent?

2 In Question **3** below, all expressions given contain two subtraction symbols. When describing the operations applied, do all the subtraction symbols have the same meaning? Explain.

3 Write each algebraic expression as a sentence using words. In each expression, the variable x represents an unknown number.

(a) $2\left(\frac{5x}{4}-6\right)-3$ **(b)** $2\left(6-\frac{5x}{4}\right)-3$ **(c)** $\frac{2(6-5x)}{4}-3$

(d) $2\left(\frac{6-5x}{4}-3\right)$ **(e)** $\frac{2(5x-6)}{4}-3$ **(f)** $2\left(\frac{5x-6}{4}\right)-3$

4 A family went on holiday visiting various towns on the coast. Use x to represent the distance the family travelled on the first day. Write an algebraic expression for each of the following situations.

(a) On the second day, the family travelled twice as far as they did on the first day.

(b) On the third day, the family travelled one third of the first day's distance plus 50 km.

(c) On the fourth day, the family travelled 100 km more than they did on the third day.

(d) On the fifth day, the family travelled 20 km less than double the distance travelled on the second day.

5 Write each sentence as an algebraic expression. Use x to represent the first unknown and y to represent the second unknown.

(a) I think of a number, add another number to it, then multiply the result by five.

(b) I think of a number, multiply it by six, reduce this by five times another number and finally divide the result by four.

(c) I think of a number and divide it by another number, after which the result is multiplied by six and has nine added to it.

(d) I multiply two numbers together, multiply the result by eight, then subtract six from the result and finally divide by three.

Identify and simplify like terms

8.2

Learning intention: To be able to identify and simplify like terms

Success criteria:

SC 1 I can identify like terms.

SC 2 I can simplify expressions by adding and subtracting like terms.

Lesson warm-up

How many sheep?

In a paddock containing both sheep and kangaroos, a farmer counts 40 heads and 120 legs. Which of the following is the best guess as to how many sheep there are?

Be prepared to explain your answer.

A 5 sheep **B** 10 sheep

C 20 sheep **D** 25 sheep

E 30 sheep **F** 40 sheep

SC 1 I can identify like terms

In algebra, terms are said to be like terms if the variable part is the same. So, for the term $3ab$, some like terms are $4ab$, $6ba$ and ba, whereas some unlike terms are $4a$, $6b$ and $3abc$. The order of the variables is not important.

8

Worked example

Identifying like terms

From the list of terms that follow, state which ones are like terms for $3a^2b$:

$5ab$, $6ba^2$, $-2aab$, $-3a^2b$, $-2ab^2$, $2aba$

THINKING	WORKING
State the variable part of the original term.	The variable part is a^2b.
Identify the like terms. Like terms have the same variable part. Be sure to include terms with the variable part written in different forms.	The like terms are: $6ba^2$, $-2aab$, $-3a^2b$ and $2aba$

SC 2 I can simplify expressions by adding and subtracting like terms

Variables are placeholders for numbers. Different variables can represent different numbers.

For example:

$5a$ represents 5 groups of an unknown number, a.

If a is a placeholder for the number 9, then $5a$ can be read as 5 groups of 9.

$5a + 3a$ could be read as 5 groups of a plus 3 groups of a, or 8 groups of a (e.g. 8 groups of 9).

However, if you add a term with a different variable, the expression cannot be simplified.

$5a + 3b$ represents 5 groups of a plus 3 groups of another unknown number, b.

Worked example

Simplifying an expression by adding like terms

Simplify the following expressions.

(a) $3a + 6a$

THINKING	WORKING
Identify the variables in each term.	The variable in each term is a.
Determine whether the terms can be added.	The variable is the same, so the terms can be added.
Simplify the expression by collecting the like terms.	$3a + 6a = 9a$

(b) $3a + 6b$

THINKING	WORKING
Identify the variables in each term.	The variable in the first term is a, and the second term is b.
Determine whether the terms can be added.	The variables are not the same, so the terms cannot be added. The simplest form is $3a + 6b$.

(c) $3ab + 6ab$

THINKING	WORKING
Identify the variables in each term.	The variable in each term is ab.
Determine whether the terms can be added.	The variable in each term is the same, so the terms can be added.
Simplify the expression by collecting the like terms.	$3ab + 6ab = 9ab$

Practice

ANSWERS Page 685

SC 1 I can identify like terms

1 For each of the following, state which of the given terms are like terms for the original term.

(a) $4x$: $5x, \frac{5}{x}, \frac{x}{5}, 5x^2, 2\sqrt{x}$

(b) $\frac{2x}{y}$: $3xy, \frac{x}{y}, \frac{2y}{x}, 2x^y$

(c) $4abc$: $6ab, 6ac, 6bca, 6cba, 6bc$

(d) $3a^3b^2$: $3a^2b^3, 3a^2b^2, 2a^3b^2, b^2a^3, \frac{6a^3}{b^2}$

2 Tania was asked to write a like term for $7abc$. She wrote abc, but her friend Ivor told her she was wrong as her term did not contain a coefficient. Explain who is correct.

3 Jackson was asked to write a like term for $2aeiou$. Jackson wrote 3 vowels. Explain whether this is correct.

4 Given the term $-5bh$, write three like terms and describe what each has in common.

SC 2 I can simplify expressions by adding and subtracting like terms

1 Simplify the following additions.

(a) $4a + 7a + 3a$ **(b)** $8ab + 5ab$ **(c)** $7ab + 3ba + 6ab$

(d) $4abc + 3acb + 6bac$ **(e)** $2ac + 2ac + 2ac$ **(f)** $5ac + 3ac + 4a$

2 Simplify the following differences.

(a) $5ab - 3ab$ **(b)** $10cb - 9bc$ **(c)** $13b - 7b - 4b$ **(d)** $9abc - 2abc$

3 Write an expression that would simplify to each of the following.

(a) $8a$ **(b)** $8b$ **(c)** $8ab$ **(d)** $8a + 8b$ **(e)** $8a - 8b$

4 Simplify the following expressions.

(a) $2(3ab + 5b) + 3(ab - 2b)$

(b) $3(2a - 4b) - 2(3a + b)$

(c) $7(a + 3b) - 2(2a - 5b)$

(d) $11(2a - 6b) - 5(4a + 7b)$

8.3 Calculate the value of algebraic expressions

Learning intention: To be able to calculate the value of algebraic expressions

Success criteria:

SC 1 I can substitute values to evaluate algebraic expressions.

SC 2 I can create number patterns by substituting values into algebraic expressions.

SC 3 I can substitute values in practical situations.

Lesson warm-up

A new taste sensation – the Aporice

A lolly shop creates a new sweet called an Aporice.

To make one packet of Aporice, combine six licorice straps and three dried apricot halves.

Yum!!

SC 1 I can substitute values to evaluate algebraic expressions

To evaluate an algebraic expression given a particular value for the variable, just replace the variable with the given value and operate as normal.

Worked example

Evaluating an algebraic expression

Evaluate $3a - 4b$, given:

(a) $a = 5$ and $b = 4$

THINKING	WORKING
Substitute the values for the variables.	For $a = 5$ and $b = 4$: $3a - 4b = 3 \times 5 - 4 \times 4$
Simplify the expression.	$= 15 - 4 \times 4$ $= 15 - 16$ $= -1$
Write the answer.	The value of the expression $3a - 4b$ for $a = 5$ and $b = 4$ is -1.

(b) $a = -5$ and $b = 4$

THINKING	WORKING
Substitute the values for the variables.	For $a = -5$ and $b = 4$: $3a - 4b = 3 \times (-5) - 4 \times 4$
Simplify the expression.	$= -15 - 4 \times 4$ $= -15 - 16$ $= -31$
Write the answer.	The value of the expression $3a - 4b$ for $a = -5$ and $b = 4$ is -31.

(c) $a = -5$ and $b = -4$

THINKING	WORKING
Substitute the values for the variables.	For $a = -5$ and $b = -4$: $3a - 4b = 3 \times (-5) - 4 \times (-4)$
Simplify the expression.	$= -15 - 4 \times (-4)$ $= -15 + 16$ $= 1$
Write the answer.	The value of the expression $3a - 4b$ for $a = -5$ and $b = -4$ is 1.

SC 2 I can create number patterns by substituting values into algebraic expressions

A number pattern is formed by following a rule of some kind. The number pattern 1, 2, 3, ... is found by adding 1 to the previous value. The number pattern 1, 2, 4, 8, 16, ... is found by multiplying the previous value by 2. You should note that in each case, a starting value needs to be supplied.

Another way of creating a number pattern is to start with an algebraic expression and then substitute consecutive whole numbers.

Worked example

Creating a number pattern

Identify the first six terms of the number pattern formed by substituting consecutive whole numbers, starting at 1, into the expression $4a - 3$.

THINKING	WORKING
Substitute the first whole number value into the expression.	For $a = 1$, $4a - 3 = 4 \times 1 - 3$ $= 4 - 3$ $= 1$
Substitute the second whole number value into the expression.	For $a = 2$, $4a - 3 = 4 \times 2 - 3$ $= 8 - 3$ $= 5$
Substitute the remaining values.	$a = 3$: $4 \times 3 - 3 = 12 - 3$ $= 9$ $a = 4$: $4 \times 4 - 3 = 16 - 3$ $= 13$ $a = 5$: $4 \times 5 - 3 = 20 - 3$ $= 17$ $a = 6$: $4 \times 6 - 3 = 24 - 3$ $= 21$
Write the number pattern followed by ... to indicate that the pattern continues.	1, 5, 9, 13, 17, 21, ...

SC 3 I can substitute values in practical situations

In some cases, an expression is written in words rather than as an algebraic expression. This often occurs when some real-life element is involved.

Worked example

Substituting values in practical situations

Sam has twice as many siblings as Flavia.

(a) How many siblings does Sam have, given that Flavia has three siblings?

THINKING	WORKING
Write an expression to represent the unknown.	Sam has twice as many siblings as Flavia. Define the number of siblings Flavia has with the variable f. The number of siblings Sam has is $2f$.
Substitute the known value into the expression.	Flavia has 3 siblings, $f = 3$: $2f = 2 \times 3$ $= 6$
Write the answer.	Given that Flavia has three siblings, Sam has six siblings.

(b) How many siblings does Flavia have, given that Sam has two siblings?

THINKING	WORKING
Write an expression to represent the unknown.	Sam has twice as many siblings as Flavia. Define the number of siblings Sam has with the variable n. The number of siblings Flavia has is $\frac{n}{2}$.
Substitute the known value into the expression.	Sam has 2 siblings, $n = 2$ $\frac{n}{2} = \frac{2}{2} = 1$
Write the answer.	Given that Sam has two siblings, Flavia has one sibling.

Practice

ANSWERS Page 685

SC 1 I can substitute values to evaluate algebraic expressions

1 Calculate the value of each expression by substituting the values given.

(a) $4a + 2b$; $a = 1$, $b = 2$

(b) $6a - 4b$; $a = 3$, $b = 2$

(c) $7a + 3b$; $a = 3$, $b = 8$

(d) $3a - 2b$; $a = -2$, $b = -1$

2 For each of the following, determine which student completed the calculation correctly and state the mistake made by the student who answered incorrectly.

(a) $3a - 2b$ when $a = 4$, $b = -2$

Student A	Student B
$3a - 2b = 34 + 22$ $= 56$	$3a - 2b = 3 \times 4 + 2 \times (-2)$ $= 12 - 4$ $= 8$

(b) The value of $5b - 3a$ when $a = -1$, $b = 2$ is:

Student A	Student B
$5b - 3a = 5 \times 2 - 3 \times (-1)$ $= 10 - 3 \times (-1)$ $= 7 \times (-1)$ $= -7$	$5b - 3a = 5 \times 2 - 3 \times (-1)$ $= 10 - 3 \times (-1)$ $= 10 + 3$ $= 13$

3 Evaluate each expression using the values given.

(a) $4(2a - 3b)$; $a = 2$, $b = -1$

(b) $-5(3a + 2b)$; $a = -1$, $b = -2$

(c) $a(2b - 4)$; $a = -1$, $b = 6$

(d) $3a(2b + 2)$; $a = 3$, $b = -1$

4 Evaluate each expression using the values given.

(a) $\frac{3(6a+2b)}{4}$; $a = -2$, $b = 2$

(b) $\frac{3a}{5} - \frac{5b}{4}$; $a = 5$, $b = -8$

(c) $-2\left(\frac{5a}{7} - 4b\right)$; $a = 35$, $b = -2$

(d) $-3a\left(5 - \frac{4b}{3}\right)$; $a = -2$, $b = 9$

5 Evaluate each expression by substituting $a = 3$, $b = -2$, and $c = 5$.

(a) $ab + ac + bc$ **(b)** $3a + 2b - c$ **(c)** $a \times b \times c$ **(d)** $2a(3b - 4c)$

SC 2 I can create number patterns by substituting values into algebraic expressions

1 Identify the first six terms of the number pattern formed by substituting consecutive whole numbers, starting at $a = 1$, into each expression given.

(a) $2a + 2$ **(b)** $3a - 4$ **(c)** $5 - 2a$ **(d)** $4 - 3a$

2 Consider your answers to the previous question to answer the following.

(a) For a variable with a positive coefficient, does the number pattern increase or decrease?

(b) For a variable with a negative coefficient, does the number pattern increase or decrease?

(c) The difference between consecutive terms in the number pattern is equal to what?

3 Copy and complete each number pattern.

(a) $2a - 6$: -4, ___, ___, 2, ___, ___

(b) $5 - 3a$: ___, -1, -4, ___, ___, ___

4 State the first whole number substituted in each number pattern.

(a) $4a + 5$: 21, 25, 29, ...

(b) $6 - 3a$: -12, -15, -18, ...

SC 3 I can substitute values in practical situations

1 A recipe requires 3 eggs to make 12 biscuits.

(a) How many eggs are needed to make 36 biscuits?

(b) How many eggs are needed to make 120 biscuits?

(c) How many biscuits can be made with 12 eggs?

(d) How many biscuits can be made with 18 eggs?

2 Jasper is training for a long-distance run. He has decided to increase the distance he runs by 1.5 km/day.

(a) Given that Jasper runs 2 km on day one, how far will he run on each of days two through five?

(b) Given that Jasper runs 1.5 km on day one, how far will he run on each of days two through five?

(c) Given that Jasper ran 4 km on day three, how far did he run on day one?

(d) Given that Jasper ran 10 km on day five, how far did he run on day one?

3 Passenger buses can carry 52 passengers.

(a) How many passengers will fill 3 buses?

(b) How many buses are required for 208 passengers?

(c) A school has 542 pupils. How many buses will be required for the school excursion?

(d) Given that all but the last bus is full, how many seats are vacant on the last bus?

4 Eric completes 10 push-ups on day one and increases this by 2 push-ups each day.

(a) Write an expression that could be used to describe this situation. Make sure to define any variable you use.

(b) How many push-ups will Eric complete on day 10?

(c) Why will Eric never complete an odd number of push-ups on any given day?

8.4 Write algebraic expressions in context

Learning intention: To be able to write algebraic expressions in context

Success criteria:

SC 1 I can use mathematical variables and relationships to explore real-life processes.

Lesson warm-up

Multiply by 11 — it's simple

Want to know how to do this the easy way?

Just watch your teacher.

No speaking.

$$11 \times 53$$

SC 1 I can use mathematical variables and relationships to explore real-life processes

Worked example

Creating an expression from a real-life situation

Trinity needs to read three novels over the course of a 40-day holiday. To make her task easier, she has decided to read the same number of pages each day.

(a) Write an expression for the number of pages Trinity needs to read each day.

THINKING	WORKING
Identify the unknown quantity.	The total number of pages is unknown.
Define a variable to represent the unknown value.	Let t represent the total number of pages.
Use this variable to write an expression.	$\frac{t}{40}$ represents the number of pages per day.

(b) The novels have a combined total of 840 pages. How many pages does Trinity need to read each day?

THINKING	WORKING
Complete the calculation by substituting the known value.	$\frac{840}{40} = 21$ pages per day
Write the answer.	Trinity needs to read 21 pages per day.

Practice

ANSWERS Page 685

SC 1 I can use mathematical variables and relationships to explore real-life processes

1 An orchard contains a large number of orange trees. Basil can pick the oranges from two trees per hour. How long will it take Basil to finish picking all of the oranges?

- **(a)** What information does Basil need before he can work out how many hours it will take?
- **(b)** Define a variable to represent this unknown quantity.
- **(c)** Write an expression to represent the number of hours Basil must work to complete the task.
- **(d)** How many hours does Basil need to work, given that the orchard contains 76 orange trees?

2 A baker uses 4 cups of flour for each loaf of bread.

- **(a)** Let a represent the number of loaves of bread. Write an expression for the number of cups of flour required.
- **(b)** How many cups of flour are required for the baker to make 150 loaves of bread?
- **(c)** If the baker uses 116 cups of flour, how many loaves of bread are made?

3 At the local café, a hamburger costs twice as much as a sausage roll.

- **(a)** Define a variable and use this single variable to write an expression for the cost of a hamburger and a sausage roll.
- **(b)** Determine the cost of a hamburger and a sausage roll, given that a sausage roll costs $4.20.
- **(c)** Determine the cost of a hamburger and a sausage roll, given that a hamburger costs $9.50.
- **(d)** Write an expression that could be used to determine the number of each item that could be purchased for a variable amount of money, m.

4 Victor is a middle-distance runner. While training, he tries to keep his running speed constant.

- **(a)** Given that Victor knows the distance, d km, he has run, what else does he need to know to be able to work out his speed?
- **(b)** Define a variable for this other value and use the two variables to write an expression for his speed.
- **(c)** Write an expression for Victor's speed, given that he ran 3000 m.
- **(d)** Calculate Victor's speed, given that he ran this distance in 8 minutes.

TOPIC REVIEW

Multiple choice

ANSWERS Page 741

1 Sara is buying juices that cost \$5 each. The total cost of the juices is $5j$. In Sara's expression, which of the following does j stand for?

A Dollars **B** The cost of one juice

C The number of juices **D** One juice

2 For the start of school, Chin-Mae bought p pencils and m markers. Each pencil cost \$0.50 and each marker cost \$2. Which expression gives the total cost of Chin-Mae's pencils and markers?

A $0.5p + 2m$ **B** $0.5 + p + 2 + m$ **C** $p + m$ **D** $50p + 2m$

3 For the expression $x + y$, which values of x and y make the expression equal to 23?

A $x = 8$ and $y = 17$ **B** $x = 7$ and $y = 14$ **C** $x = 12$ and $y = 11$ **D** $x = 10$ and $y = 12$

4 Define c as the current amount of money in Ziggy's bank account and define d as the amount of money owed for a telephone bill. Which of the following is an expression for the amount of money in Ziggy's account after he pays the bill?

A $d - c$ **B** $c - d$ **C** $c + d$ **D** $c \times d$

5 You start with an unknown number, multiply it by 3 and then subtract 7. Which expression shows this?

A $3 + m - 7$ **B** $3 \times m + 7$ **C** $3m - 7$ **D** $3m - 21$

6 Mal exercises by running upstairs and doing push-ups. He takes 12 seconds to run up one flight of stairs and 7 seconds to do one set of push-ups. Define f as the number of flights of stairs and define p as the number of sets of push-ups. Which expression gives the total amount of time that Mal exercises?

A $(12f + 7p) \div 60$ seconds **B** $12 + f + 7 + p$ seconds

C $7f - 12p$ seconds **D** $12f + 7p$ seconds

7 Ariana has a bag containing 32 balls for various sports. An unknown number of tennis balls are taken out of the bag, then two basketballs are removed. She writes the expression $32 - x - 2$ to give the number of balls left in the bag. In this expression, what does x represent?

A The number of balls left in the bag

B The number of tennis balls remaining in the bag

C The number of tennis balls taken out of the bag

D The number of tennis balls and basketballs combined

8 Which of the following best represents $2s - p + 5 + 8p + 4$ in simplest form?

A $9ps + 9$ **B** $2s + 7p + 9$ **C** $-19ps$ **D** $2s - 7p + 1$

9 Which of the following best represents $7x + 2y - x - 5y$ in simplest form?

A $3xy$ **B** $-3xy$ **C** $6x - 3y$ **D** $-8x - 7y$

10 Which of the following is the result of substituting $x = 5$ and $y = 2$ into $6x + 2y$?

A 8 **B** 22 **C** 34 **D** 40

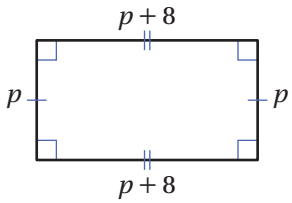
11 What is the result of substituting $n = \frac{1}{4}$ and $p = \frac{2}{5}$ into the expression $np + \frac{n}{p}$?

A $\frac{1}{10}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{29}{40}$

12 The number pattern 2, 5, 8, 11, 14, ... can be made by substituting the values $x = 0, 1, 2, 3, 4, \ldots$ into which expression?

A $2x$ **B** $2x + 5$ **C** $2x + 2$ **D** $3x + 2$

13 The rectangle shown has a length 8 mm longer than its width. The width, in mm, is represented by p.



Which expression gives the perimeter of the rectangle, in millimetres?

A $p^2 + 64$ **B** $2(p^2 + 8p)$

C $4p + 16$ **D** $2p + 16$

14 Lilly has x bags of lollies, with p lollies in each bag. She gives a total of q lollies to her friends. The total number of lollies that Lilly has left can be written as which of the following?

A $x + p - q$ **B** $x + p + q$ **C** $px - q$ **D** $\frac{px}{q}$

15 The number pattern 10, 9, 6, 1, –6, ... can be made by substituting the values $x = 0, 1, 2, 3, 4, \ldots$ into which rule?

A $10x$ **B** $x^2 + 10$ **C** $10 - x^2$ **D** $10 - x$

Short answer

1 Decide whether each pair of algebraic expressions have the same value 'sometimes', 'always' or 'never'.

(a) ab and ba **(b)** $a + 1$ and $b + 1$ **(c)** $b - 3$ and $b - 10$ **(d)** $5a$ and a^2

2 Write algebraic expressions for each of the following.

(a) An unknown number, n, is multiplied by 5 and then 4 is added to the result.

(b) A sandwich shop charges \$7 for each sandwich and \$2 for a can of drink. Erica does the lunch run for her office and buys m sandwiches and d drinks. What is the total cost?

3 Audrey is getting a new internet connection for her business. The cost for the technician to set it up is \$399, the cost for a new modem is \$226 and the cost for the internet connection is \$90/month. Write an expression for the total cost after a period of n months.

4 Lidia earns \$10 for every hour she works.

(a) Using H as the number of hours she works, write an expression for the amount of money Lidia earns.

(b) Use this expression to calculate the amount Lidia earns for 9.5 hours of work.

5 Which of the following are like terms for $2m$?

$-4n, 2, -2m, \frac{1}{5m}, 2n, 7mn, \frac{4m}{5}, -m$

6 Write each of the following in simplest form.

(a) $8b - 2a + 4b - 3a - 2 + a$

(b) $-4a + 3b + 9b - 10a - 3a$

(c) $-12b - 2b + 9 - 2 + 8b - a + 7a$

7 Substitute the stated value into the given expression.

(a) $\sqrt{a}; a = 4$

(b) $e + e + 19; e = 3$

(c) $3(j + 2); j = 7$

(d) $1 - p; p = \frac{2}{3}$

8 List the number pattern created by substituting the whole numbers 0 to 4 for f into the expression $9f - 7$.

9 Determine the following values.

(a) $3 - \frac{m+6}{v}$ for $m = 4$ and $v = 5$

(b) $\sqrt{s^2 - p} - z$ for $s = 9$, $p = 17$ and $z = 9$

10 Explain why m and $\frac{m}{3}$ are like terms, but m and $\frac{3}{m}$ are unlike terms.

11 Substitute $x = 3$ into each expression and calculate the result.

(a) $4 + x$

(b) $x - 5$

(c) $9(2 - x)$

(d) $\frac{5x}{15}$

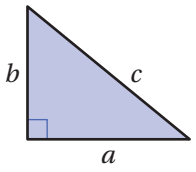
12 List the number pattern created by substituting the whole numbers 0 to 4 for k into the expression $\frac{k+1}{5}$.

13 The variables m, u, p, s and z are related by the rules $m = 2u + s^2$ and $p = m - (z + s)$.

(a) Substitute the values $u = \frac{1}{2}$ and $s = 3$ to calculate the value of m.

(b) Now use this to calculate the value of p, given that $z = 4$.

14 The formula $c = \sqrt{a^2 + b^2}$ can be used to calculate the unknown length of a right-angled triangle, given the other side lengths.



(a) Calculate the side length, c, given the other two sides are 5 cm and 12 cm.

(b) Calculate the side length, c, given the other two sides are both 10 cm. Give your answer correct to 2 decimal places.

(c) The value of c is 10 cm. The other two side lengths are equal to each other. What is this value, correct to 2 decimal places?

15 Write the following in simplest form using algebra.

(a) a is multiplied by itself, b is multiplied by itself, then the two results are added.

(b) The square root of x is halved and then has 3 added to it.

16 Sandra has a apples. Mindy has six more apples than Sandra. Tara has twice as many apples as Sandra.

(a) Write an expression for the number of apples Mindy has.

(b) Write an expression for the number of apples Tara has.

(c) Write an expression, in simplest form, for the total number of apples they have together.

(d) How many apples do each of the other girls have, given that Sandra has 5 apples?

(e) How many apples do they have together, given that Tara has 20 apples?

17 Use algebra to complete the following.

(a) Choose a number, square it, then choose another number and divide it by three, then multiply the sum of the two results by two.

(b) What value is produced, given that the first number is 5 and the second is 12?

18 Answer true (T) or false (F) for each of the following.

(a) For $a = 3$, $2(a + 4) = 14$

(b) For $t = 5$, $7t - 20 = 55$

(c) For $s = 3$, $s^2 - 2s + 5 = 10$

(d) For $k = -2$, $-3(2k + 4) = 0$

Extended response

1 Imagine you are planning a party, but you don't know how many people will come. You know that food will cost $10 per person, plus one birthday cake will cost $25.

(a) Given that there are 16 people, what will be the total cost for food and cake?

(b) Given that there are 34 people, what will be the total cost for food and cake?

(c) Write a rule in words for calculating the total cost of food and cake for any number of people at the party.

(d) Write your rule using a variable, defining n as the number of people at the party.

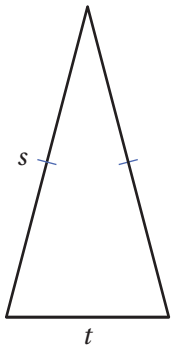
2 An isosceles triangle has two equal sides and one other side.

(a) Using the variables defined on the diagram, write an expression for the total length of all the sides (perimeter).

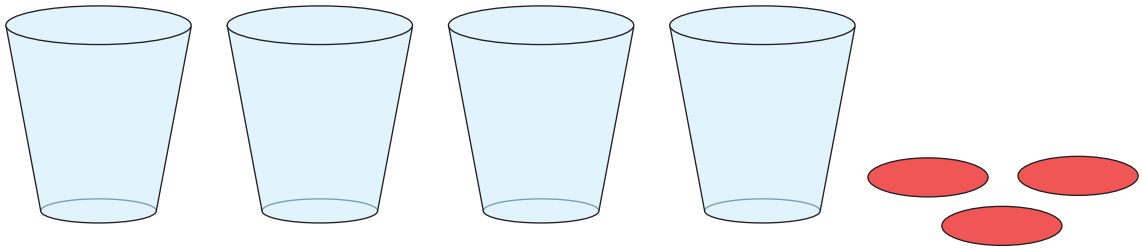
(b) You are further told that $s = 2t$. Rewrite the expression using just the variable t.

(c) What is the perimeter for $t = 5$ cm?

(d) What is the perimeter for $s = 12$ cm?



3 In the image, each cup contains the same number of counters, and there are some spare counters shown as well.



(a) Using the variable n to represent the number of counters in each cup, write an expression for the total number of counters in the image.

(b) How many counters are there in total, given that each cup contains:

(i) 1 counter **(ii)** 5 counters **(iii)** 11 counters?

(c) Given that there are 63 counters in total, how many are there in each cup?

4 Ivan is planning the prize money for a raffle. The second prize needs to be \$100 less than the first prize, while the third prize should be half of the second prize. The fourth prize should be \$10 less than the third prize.

(a) Write an expression in simplest form for the total value of the four prizes, where p represents the value in dollars of the first prize.

(b) Write the value of each of the four prizes, given that the first prize is \$500.

(c) What is the total value of the prizes, given that the first prize is \$150?

(d) If $p = \$110$, what would the values of the four prizes be?

(e) How does this match with the simplified expression for the sum of the prizes?

(f) What does this tell you about the minimum value of the first prize?

5 I open a bank account. In the first week, I deposit $\$x$. Each week after that, I deposit \$2 more than the previous week.

(a) Write an expression, in simplest form, for the amount of the fifth deposit.

(b) Write an expression to represent the total amount deposited in the first 5 weeks.

(c) What was the initial deposit, given that there is \$80 in the account after 5 weeks?

TOPIC
9

Linear relationships

Recall 216

9.1 Understand the components of an equation and how they are combined 217

9.2 Generate a table of values using a linear equation 220

9.3 Recognise the connection between points on a Cartesian plane, a table of values and a linear pattern 229

9.4 Model linear patterns using manipulatives, diagrams and graphs 239

Topic review 244

Why learn this?

Linear relationships can be represented as a straight line when graphed on a Cartesian plane. Linear equations provide practical benefits in various contexts, showing a consistent relationship exists between two variables.

Being able to model linear relationships has many important environmental, science and financial applications, where data collected involving two variables requires quick visual analysis to determine whether a straight-line relationship between them exists.

RECALL

ANSWERS Page 686

I can equate number sentences

1 Indicate which equations given are true and which are false.

(a) $2 + 10 = 6 + 6$ **(b)** $2 \times 6 = 10 + 4$ **(c)** $12 \times 4 = 60 - 15$

(d) $20 \times 3 - 10 = 10 \times 5$ **(e)** $4 \times 8 + 9 = 20 - 17 + 29$

I can complete a table of values using a rule

1 Multiply each value in the table by 2, then subtract 4.

1	2	3	4	5

2 Divide each value in the table by 3, then add 5.

3	6	9	12	15

I can interpret and identify parts of an algebraic expression

1 Copy and complete each of the following sentences.

(a) $5x + y - 3a$ is an expression with _____ terms.

(b) There are _____ terms in the expression $5a - 2b + 1$

(c) $3 \times x$ can be rewritten as _____

(d) To simplify $6 \times a \times b$, the _____ symbols can be removed so $6 \times a \times b =$ _____

(e) Another way of writing $\frac{a}{2}$ is _____ ÷ _____

(f) The coefficient of a term is the number in front of the pronumeral. So, the coefficient of m in the term $4m$ is _____

I can substitute numbers in to find the value of an algebraic expression

1 If $x = 4$, then what is the value of $5x + 9$?

2 If $a = 21$, then what is the value of $\frac{a-3}{2}$?

3 What is the value of $3a + b$, when $a = 2$ and $b = 9$?

4 What is the value of $7h - 4k$, when $h = 2$ and $k = 1$?

Understand the components of an equation and how they are combined

9.1

Learning intention: To understand the components of an equation and how they are combined

Success criteria:

SC 1 I can explain the difference between an expression and an equation.

Lesson warm-up

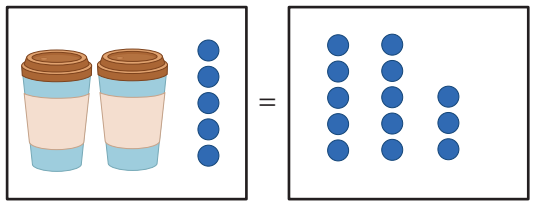
Expressions or equations

The expression $2p + 5$ is read as '2 groups of p, which is an unknown number of counters in cups, plus 5 more counters'.

1 How many counters are in each cup?

2 How many counters in total are shown? Explain.

The equation $2p + 5 = 13$ is shown using an equation mat.



3 In what ways is it similar to the expression?

4 How is it different from the expression?

5 How many counters are in each cup?

SC 1 I can explain the difference between an expression and an equation

An **expression** is made up of numbers and variables that are connected by operations (e.g. +, −, ×, ÷).

An expression that has both numbers and variables is called an algebraic expression.

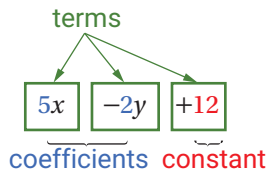
Examples of expressions are: 6, x, $5a - 1$, $7x^2 + 3$

An **equation** contains an equals symbol between two expressions.

Examples of equations are: $2x + 3x = 5x$, $5a - 1 = 9$

A number on its own in an algebraic expression is called a **constant**.

A number that is multiplied by a variable is called a **coefficient**.



The parts of an algebraic expression that are separated by operations are called **terms**.

Worked example

Writing expressions and equations

Darius runs the same number of laps of his local oval every day as part of his training program.

(a) Write an expression for the number of laps he will run in a week.

THINKING	WORKING
Define a variable for the number of laps run each day.	Let t be the number of laps run each day.
Decide what is needed to convert daily amounts to weekly amounts.	Multiply by 7, so $t \times 7$
Simplify the expression.	$7t$

(b) Write an equation that shows Darius running a total of 84 laps in a week.

THINKING	WORKING
The variable expression is equivalent to the given total, so write an equals symbol between them.	$7t = 84$

Practice

ANSWERS Page 686

SC 1 I can explain the difference between an expression and an equation

1 Darius runs the same number of laps of his local oval every day as part of his training program. Let t be the number of laps run each day.

(a) Write expressions for the number of laps he will run:

(i) in a fortnight **(ii)** in a month (30 days) **(iii)** in a year (365 days).

(b) Write an equation expressing that Darius ran 600 laps in a month.

2 Isaac starts a collection with 25 trading cards.

(a) Letting the unknown number be x each time, write an expression for the number of cards Isaac will have, given that:

(i) he gives some cards to his friend Andy

(ii) Andy gives him some cards.

(b) Write an equation expressing that Isaac has 19 cards left after giving some to Andy.

3 A packet contains an unknown number of lollies. Let the unknown number be k.

(a) Write an expression for the number of lollies each person will get, given that they are shared equally between:

(i) 6 friends **(ii)** 4 friends.

(b) Write an equation to express the situation in which each of 6 friends receives 8 lollies.

4 Write equations to represent each situation.

(a) Each student in the class writes and illustrates a picture book containing the same number of pages. The total number of pages is 1200. Let a be the number of pages written by each student and b be the number of students in the class.

(b) On a school camp, Eric and Tan each eat a number of pieces of fruit. The total number of pieces eaten is 6. Let a and b be the number of pieces eaten by Eric and Tan, respectively.

(c) Meilin has some action figures, but decides to give some of them to charity. She has 15 left. Let a be the number of action figures she has originally and b be the number of action figures she gives away.

5 In 2 years' time, Logan will be one third of his mother's age.

(a) Given that Logan's current age in years is l, write an expression for his age in 2 years' time.

(b) Given that Logan's mother's current age in years is m, write an expression for her age in 2 years' time.

(c) Write an expression for one third of the mother's current age.

(d) Write an expression for one third of the mother's age in 2 years' time.

(e) Write the equation for the information given in the question.

6 Catherine is training for a race. In the first week of training, she runs x km. In the second week, she runs half as much again as the first week. In the third week, she runs twice as far as the first week.

(a) Write an expression for the number of kilometres run during the second week.

(b) Write an expression for the number of kilometres run during the third week.

(c) Write an expression for the total number of kilometres run during all three weeks.

(d) Write an equation to show that the total distance Catherine ran in three weeks was 45 km.

7 A lolly jar has a mass of 160 g. The jar contains 60 lollies of equal mass, r grams. Write an equation to show that the total mass of the jar of lollies is 200 g.

9.2 Generate a table of values using a linear equation

Learning intention: To be able to generate a table of values using a linear equation

Success criteria:

SC 1 I can substitute numbers into an equation to determine the value of a variable.

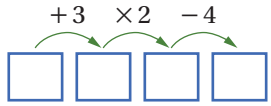
SC 2 I can generate a table of values from a linear equation.

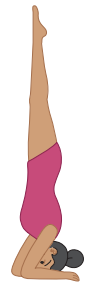
SC 3 I can describe the rule connecting variables in a table of values.

Lesson warm-up

I think of a number

I think of a number, add three, double it and then subtract four.

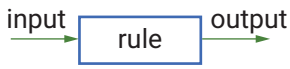




Determine the output number when the number in the first box is:

(a) 1 **(b)** 2 **(c)** 5 **(d)** 10

SC 1 I can substitute numbers into an equation to determine the value of a variable



When given a rule, or formula, where one variable is equal to an expression of the other, a value for one of the variables can be substituted to calculate the value of the other variable.

The input variable is said to be independent.

The output variable is dependent on the value of the input variable.

Paired input/output values can be written as ordered pairs:

(input, output) or (independent variable, dependent variable).

Worked example

Substituting a value into an equation

Substitute the value $x = 3$ into each of the equations to determine the value of y. Write your answers as ordered pairs in the form (x, y).

(a) $y = x + 5$

THINKING	WORKING
Write the equation and the value to be substituted.	$y = x + 5$, for $x = 3$
Substitute the independent variable and calculate the value of the dependent variable.	$y = 3 + 5$ $= 8$
Write the answer.	For $x = 3$, $y = 8$: (3, 8)

(b) $y = 5x$

THINKING	WORKING
Write the equation and the value to be substituted.	$y = 5x$, for $x = 3$
Substitute the independent variable and calculate the value of the dependent variable.	$y = 5 \times 3$ $= 15$
Write the answer.	For $x = 3$, $y = 15$: (3, 15)

(c) $y = 5 - x$

THINKING	WORKING
Write the equation and the value to be substituted.	$y = 5 - x$, for $x = 3$
Substitute the independent variable and calculate the value of the dependent variable.	$y = 5 - 3$ $= 2$
Write the answer.	For $x = 3$, $y = 2$: (3, 2)

(d) $y = 4x - 6$

THINKING	WORKING
Write the equation and the value to be substituted.	$y = 4x - 6$, for $x = 3$
Substitute the independent variable and calculate the value of the dependent variable.	$y = 4 \times 3 - 6$ $= 6$
Write the answer.	For $x = 3$, $y = 6$: (3, 6)

SC 2 I can generate a table of values from a linear equation

In equations where y is equal to an expression in x, a table of pairs of values (x, y) can be generated. Given the required values of the independent variable, x, each of the corresponding values of the dependent variable, y, can be calculated from the equation.

Worked example

Generating a table of values

Generate a table of values for the x-values 1, 2, 3, 4, 5 for the following equations:

(a) $y = 3x$

THINKING	WORKING
Set up the table of values with the independent variable first. Arrange the values in ascending order.	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td></tr></table>
Substitute each x-value into the equation to calculate the y-value.	$y = 3x$ <table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$y = 3x$</td><td>3×1 $= 3$</td><td>3×2 $= 6$</td><td>3×3 $= 9$</td><td>3×4 $= 12$</td><td>3×5 $= 15$</td></tr></table>
Write the answer.	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td></tr></table>

(b) $y = 3x - 4$

THINKING	WORKING
Set up the table of values with the independent variable first.	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td></tr></table>
Substitute each x-value into the equation to calculate the y-value.	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>$3 \times 1 - 4$ $= 3 - 4$ $= -1$</td><td>$3 \times 2 - 4$ $= 6 - 4$ $= 2$</td><td>$3 \times 3 - 4$ $= 9 - 4$ $= 5$</td><td>$3 \times 4 - 4$ $= 12 - 4$ $= 8$</td><td>$3 \times 5 - 4$ $= 15 - 4$ $= 11$</td></tr></table>
Write the answer.	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>-1</td><td>2</td><td>5</td><td>8</td><td>11</td></tr></table>

SC 3 I can describe the rule connecting variables in a table of values

In a table of (x, y) values for linear relationships, when the x-values increase by a regular amount, the y-values will always show a regular pattern of increase (or decrease).

To determine the rule for a linear equation, decide whether the x-values were multiplied by a number, then see whether any adjustment is needed by adding or subtracting a number.

- If the y-values are increasing 3 times as much as the x-values, the rule will begin $y = 3x$ ...
- If the y-values are decreasing, the number used to multiply the x-values must be negative.
- If the y-values are decreasing 3 times as much as the x-values are increasing, the rule will begin $y = -3x$ ...

Division is treated like multiplication because, for example, dividing by three ($\div 3$) is the same as multiplying by one third $\left(\times\frac{1}{3}\right)$.

- If the y-values are increasing only $\frac{1}{3}$ as much as the x-values, the rule will begin $y = \frac{1}{3}x\ldots$ or $y = \frac{x}{3}\cdots$

Worked example

Determining the rule for a linear relationship

Determine the rule that was used to generate each of the following tables of values.

(a)

x	1	2	3	4	5
y	4	5	6	7	8

THINKING	WORKING
Write the consistent change in the independent variable, moving left to right.	The independent variable increases by 1.
Write the consistent change in the dependent variable.	The dependent variable increases by 1. There is no multiplication required in the rule. The rule begins $y = x$ ...
Determine the adjustment needed by addition or subtraction.	(1, 4): $1 + 3 = 4$ (2, 5): $2 + 3 = 5$ Add 3 to each x-value to get the y-value.
Write the answer.	The rule is: $y = x + 3$

(b)

x	4	5	6	7
y	−8	−10	−12	−14

THINKING	WORKING
Write the consistent change in the independent variable, moving left to right.	The independent variable increases by 1.
Write the consistent change in the dependent variable.	The dependent variable decreases by 2. Multiply each x-value by −2. The rule begins $y = -2x$ ...
Determine any adjustment needed after the multiplication.	(4, −8): $4 \times (-2) = -8$ (5, −10): $5 \times (-2) = -10$ No adjustment is needed.
Write the answer.	The rule is: $y = -2x$

(c)

x	2	4	6
y	9	19	29

THINKING	WORKING
Write the consistent change in the independent variable, moving left to right.	The independent variable increases by 2.
Write the consistent change in the dependent variable.	The dependent variable increases by 10. Multiply each x-value by $10 \div 2 = 5$. The rule begins $y = 5x$ ...
Determine any adjustment needed after the multiplication.	(2, 9): $2 \times 5 = 10$; $10 - 1 = 9$ (4, 19): $4 \times 5 = 20$; $20 - 1 = 19$ (6, 29): $6 \times 5 = 30$; $30 - 1 = 29$ Subtract 1 from each value of $5x$ to get the y-value.
Write the answer.	The rule is: $y = 5x - 1$

Practice

ANSWERS Page 686

SC 1 I can substitute numbers into an equation to determine the value of a variable

1 Substitute the value $x = 8$ into each equation to determine the value of y. Write your answer as an ordered pair in the form (x, y).

(a) $y = x + 5$ **(b)** $y = x - 5$ **(c)** $y = x + 1$ **(d)** $y = x - 12$

2 Substitute the value $x = -3$ into each equation to determine the value of y. Write your answer as an ordered pair in the form (x, y).

(a) $y = 2x$ **(b)** $y = -2x$ **(c)** $y = 6x$ **(d)** $y = -6x$

3 Substitute the value $x = -3$ into each equation to determine the value of y. Write your answer as an ordered pair in the form (x, y).

(a) $y = 10 - x$ **(b)** $y = 6 - x$ **(c)** $y = -1 - x$ **(d)** $y = -5 - x$

4 Substitute the value $x = 5$ into each formula to determine the value of y. Write your answer as an ordered pair in the form (x, y).

(a) $y = 4x - 6$ **(b)** $y = 6 - 4x$ **(c)** $y = 2x + 2$ **(d)** $y = 5x + 1$

5 A rule has the following steps: Add 3 to the input number, divide the result by 6, and finally subtract 1. Determine the output or end value for each of the following input values.

(a) 9 **(b)** 21 **(c)** −3 **(d)** −15

6 The diagram shows what a number machine does to an input value as it travels through. Determine the output value for each of the given input values.

(a) 8 **(b)** 16 **(c)** 22 **(d)** 27

7 For the number machine in the previous question, determine the input number for the given output numbers.

(a) 15 **(b)** 29 **(c)** 12 **(d)** $10\frac{1}{2}$

SC 2 I can generate a table of values from a linear equation

1 Generate a table of values for the x-values 1, 2, 3, 4, 5 for the following equations.

(a) $y = 5x$ **(b)** $y = 5x - 1$

2 Generate a table of values for the x-values 1, 2, 3, 4, 5 for the following equations.

(a) $y = 4x$ **(b)** $y = 4x + 3$

3 Copy and complete the following tables of values.

(a) $y = 2x + 5$

x	1	2	3	4	5
y					

(b) $b = 6a - 5$

a	1	2	3	4	5
b					

(c) $d = 2(c - 3)$

c	1	2	3	4	5
d					

(d) $f = \frac{2e}{5}$ (Use fractions where necessary.)

e	1	2	3	4	5
f					

4 Complete the following tables of values.

(a) $b = 3a - 2$

a	−2	1	6	10	15
b					

(b) $d = \frac{4c}{5} - 2$ (Use fractions where necessary.)

c	−5	0	2	6	10
d					

(c) $f = -2(e + 4)$

e	−3	−1	4	6	9
f					

(d) $h = \frac{5 - 2g}{3}$ (Use fractions where necessary.)

g	−3	0	1	5	7
h					

5 Consider the following table of values:

$y = 5x - 1$

x	0	1	2	3	4
y	−1	4	9	14	19

(a) Determine the y-value increase as the x-value increases:

(i) from 0 to 1 **(ii)** from 1 to 2 **(iii)** from 2 to 3 **(iv)** from 3 to 4

(b) Describe where the number representing the regular increase appears in the equation.

6 Consider the following table of values.

$y = 2(1 - 5x)$

x	−2	−1	0	1	2
y	22	12	2	−8	−18

(a) Describe the change in the y-value as the x-value increases:

(i) from −2 to −1 **(ii)** from −1 to 0 **(iii)** from 0 to 1 **(iv)** from 1 to 2

(b) Write the consistent change in x as a single number.

(c) Expand the right-hand side of the equation. Describe where the number representing the regular change appears in the expanded expression.

SC 3 I can describe the rule connecting variables in a table of values

1 Determine the rule that was used to generate each of the following tables of values.

(a)

x	1	2	3	4	5
y	8	9	10	11	12

(b)

x	3	4	5	6	7
y	0	1	2	3	4

2 Determine the rule that was used to generate each of the following tables of values.

(a)

x	1	2	3	4	5
y	6	12	18	24	30

(b)

x	4	5	6	7	8
y	12	15	18	21	24

(c)

x	1	2	3	4	5
y	−4	−8	−12	−16	−20

(d)

x	4	8	12	16	20
y	1	2	3	4	5

3 Determine the rule that was used to generate each of the following tables of values.

(a)

x	1	2	3	4
y	3	5	7	9

(b)

x	2	3	4	5
y	1	4	7	10

(c)

x	2	4	6
y	14	22	30

(d)

x	1	2	3	4
y	1	6	11	16

4 Determine the rule that was used to generate each of the following tables of values.

(a)

x	1	2	3	4	5
y	9	8	7	6	5

(b)

x	1	2	3	4	5
y	1	0	−1	−2	−3

(c)

x	1	2	3	4	5
y	−5	−7	−9	−11	−13

(d)

x	1	2	3	4	5
y	1	−2	−5	−8	−11

5 Determine the rule that was used to generate each of the following tables of values.

(a)

x	2	4	6	8
y	4	5	6	7

(b)

x	3	6	9	12
y	−1	0	1	2

6 For each table of values:

(i) assuming a linear relationship, fill in the missing values

(ii) determine the rule for the relationship.

(a)

x	15	17	19	21	23
y	3				43

(b)

x	7	8	9	10
y	25			10

7 When a washing machine needs repair, the householder is charged a callout fee on top of the charge for the time taken for the repairs. The cost for replacement parts is charged separately. A 2-hour repair costs $155 and a 5-hour repair costs $335.

(a) Copy and complete the table, showing the cost for any repair up to 5 hours.

Number of hours, t	1	2	3	4	5
Total cost in dollars, C					

(b) Write a rule for cost in terms of time, in the form of an equation.

(c) Determine the cost of the callout.

(d) Determine the hourly rate.

Recognise the connection between points on a Cartesian plane, a table of values and a linear pattern

9.3

Learning intention: To be able to recognise the connection between points on a Cartesian plane, a table of values and a linear pattern

Success criteria:

SC 1 I can represent a table of coordinate values on a Cartesian plane.

SC 2 I can recognise a linear pattern from a set of points on a Cartesian plane and create a table of values.

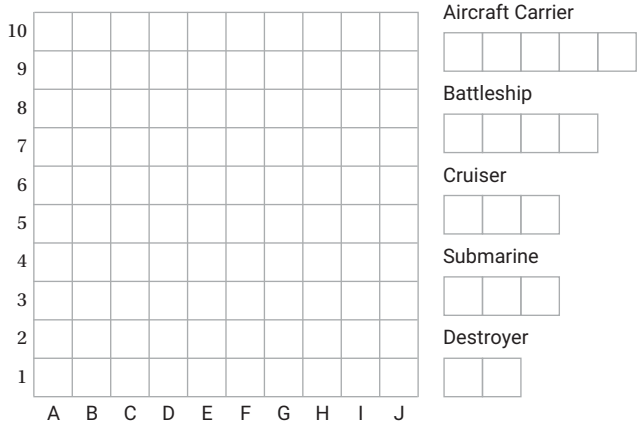
SC 3 I can recognise and describe consistent changes in the values of coordinates and link them to a linear pattern.

Lesson warm-up

Battleships

Battleships is a game played in pairs.

1. Colour in the ships on your grid. Ships can only be drawn vertically or horizontally. They cannot overlap.
2. Players take turns guessing the coordinate location of the other player's ships. Use coordinates, e.g. (A5). If a player guesses the location of your ship, you say 'hit', otherwise say 'miss'.
3. Ships are sunk when all parts of the ship are 'hit'.
4. The first player to sink all five of the other player's ships wins the game.



Aircraft Carrier

Battleship

Cruiser

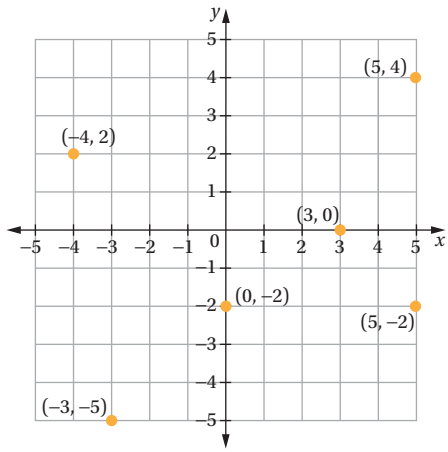
Submarine

Destroyer

9

SC 1 I can represent a table of coordinate values on a Cartesian plane

The following graph shows a number of points labelled on a set of Cartesian axes, also known as the Cartesian plane.



Every point on a Cartesian plane is represented by a pair of values, called the coordinates of the point. The structure of this pair of points is (horizontal value, vertical value) or (independent variable, dependent variable) or simply (x, y).

The points in a linear relationship form a straight line on the Cartesian plane.

Worked example

Plotting a table of values on a Cartesian plane

Complete the following table of values and plot the coordinate pairs on the Cartesian plane.

$y = 3x - 2$

x	−4	−1	1	3	5
y					
(x, y)					

THINKING

Substitute the x-values into the equation to calculate the y-values.

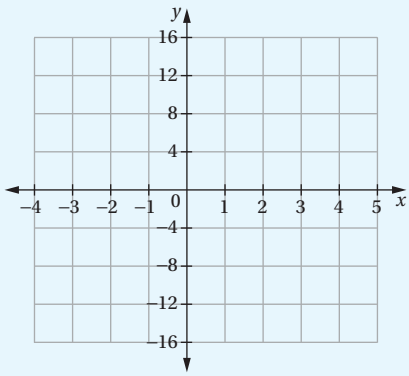
WORKING

x	−4	−1	1	3	5
y	$3 \times (-4) - 2$ $= -12 - 2$ $= -14$	$3 \times (-1) - 2$ $= -3 - 2$ $= -5$	$3 \times (1) - 2$ $= 3 - 2$ $= 1$	$3 \times (3) - 2$ $= 9 - 2$ $= 7$	$3 \times (5) - 2$ $= 15 - 2$ $= 13$

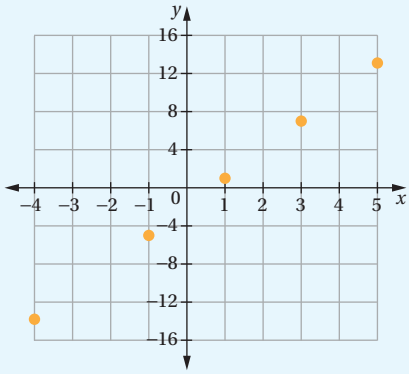
List the x- and y-values as coordinate pairs (x, y).

x	-4	-1	1	3	5
y	-14	-5	1	7	13
(x, y)	$(-4, -14)$	$(-1, -5)$	$(1, 1)$	$(3, 7)$	$(5, 13)$

Sketch the Cartesian plane, making sure the axes are long enough to cover the range of values needed.



Plot each of the points on the Cartesian plane. Start at the origin, (0, 0) and then move horizontally (the x-direction) and vertically (the y-direction).



SC 2 I can recognise a linear pattern from a set of points on a Cartesian plane and create a table of values

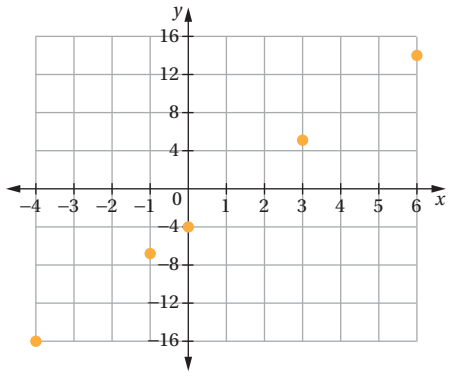
A linear pattern is easy to see on an accurately drawn graph because the points will line up.

The table of values can be read off the graph. It is important that coordinate pairs are written using the format (x, y).

Worked example

Using a graph to make a table of values

Construct the table of values for the linear graph shown.



THINKING	WORKING
Starting with the point furthest to the left on the graph, write the coordinate pair for each point.	(x, y): (−4, −16), (−1, −7), (0, −4), (3, 5), (6, 14)
Write each pair of values as separate values of x and y in the table, placing the x-values in the first row.	

x	−4	−1	0	3	6
y	−16	−7	−4	5	14

SC 3 I can recognise and describe consistent changes in the values of coordinates and link them to a linear pattern

Worked example

Recognising and determining the rule of a linear pattern

For each of the following tables of values:

(i) describe the change in the y-coordinates in terms of the increase in the x-coordinates

(ii) write the rule for the relationship.

(a)

x	1	3	6	8	9
y	−1	5	14	20	23

THINKING	WORKING
Describe the change in the x-coordinates, then the y-coordinates for the first two points.	x: 1 to 3; the increase is $3 - 1 = 2$ y: -1 to 5; the change is $5 - (-1) = +6$
Moving from left to right across the table, describe the change in the x-coordinates, then the y-coordinates for each two sets of points.	**Increase in x** / **Change in y**: 2 / +6; 3 / +9; 2 / +6; 1 / +3
Describe the pattern in terms of the simplest change of the x-values.	The y-values increase by 3 times as much as the x-values.
Use the described pattern to set the coefficient for x.	The rule begins: $y = 3x$ ...
Use any point to determine the constant term in the rule.	Using (3, 5): $3x = 9$ $9 - 4 = 5$, so the adjustment is to subtract 4. The rule is $y = 3x - 4$.
Use any other point to check that your answer is correct.	Check using (6, 14): $3x - 4 = 3 \times 6 - 4$ $= 14$ This confirms the rule.

(b)

x	2	5	7	11	16
y	16	10	6	-2	-12

THINKING	WORKING
Describe the change in the x-coordinates, then the y-coordinates for the first two points.	x: 2 to 5; the increase is $5 - 2 = 3$ y: 16 to 10; the change is $10 - 16 = -6$
Moving from left to right across the table, describe the change in the x-coordinates, then the y-coordinates for each two sets of points.	**Increase in x** / **Change in y**: 3 / -6; 2 / -4; 4 / -8; 5 / -10
Describe the pattern in terms of the simplest change of the x-values.	The y-values decrease by 2 times as much as the increase in the x-values.

Use the described pattern to set the coefficient for x.	The rule begins: $y = -2x$ ...
Use any point to determine the constant term in the rule.	Using (2, 16): $-2x = -4$ $-4 + 20 = 16$, so the adjustment is to add 20. The rule is $y = -2x + 20$.
Use any other point to check that your answer is correct.	Check using (5, 10): $-2x + 20 = -2 \times 5 + 20$ $= 10$ This confirms the rule.

Practice

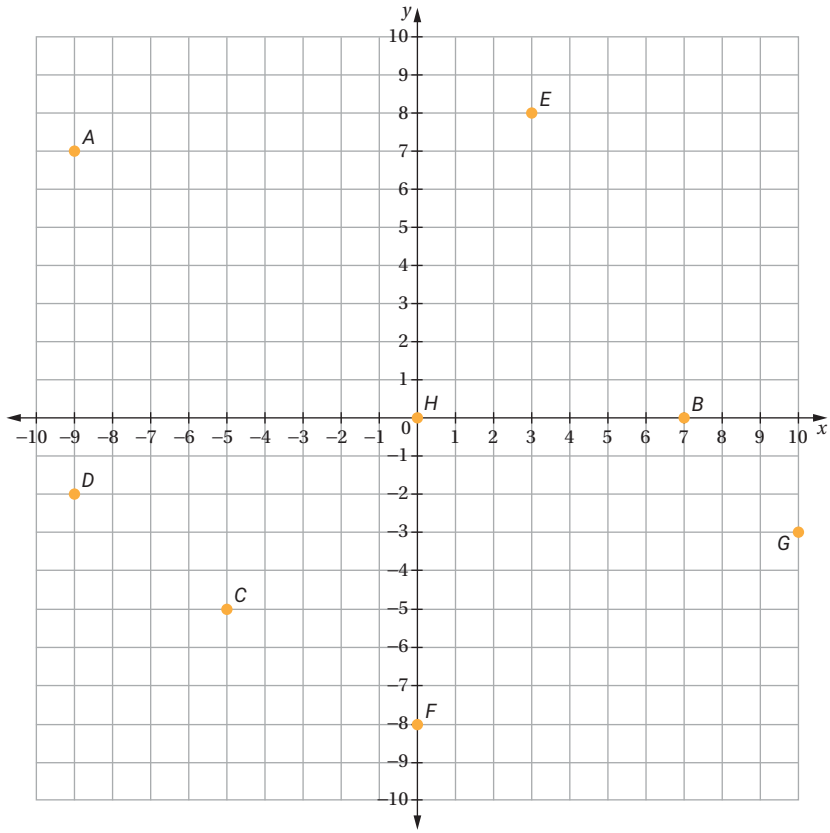
ANSWERS Page 687

SC 1 I can represent a table of coordinate values on a Cartesian plane

1 Complete the coordinates row of the linear relationship in the table.

x	−1	0	1	2	3
y	20	14	8	2	−4
(x, y)	(−1, 20)				

2 Write the coordinates of each of the labelled points *A* through *H* graphed on the Cartesian axes.



3 Plot the given points for the linear relationship $y = 2x - 5$ onto a Cartesian plane.

x	-1	0	1	2	3	4
y	-7	-5	-3	-1	1	3
(x, y)	$(-1, -7)$	$(0, -5)$	$(1, -3)$	$(2, -1)$	$(3, 1)$	$(4, 3)$

4 Complete the following tables of values and plot the coordinate pairs on the Cartesian plane.

(a) $y = 2x + 3$

x	-4	-1	1	3	4
y	5				
(x, y)	$(-4, 5)$				

(b) $y = -3x + 4$

x	-2	-1	1	3	5
y	10				
(x, y)	$(-2, 10)$				

5 Complete the following tables of values and plot the coordinate pairs on the Cartesian plane.

(a) $y = 4 - 3x$

x	-6	-2	1	4	7
y	22				
(x, y)	$(-6, 22)$				

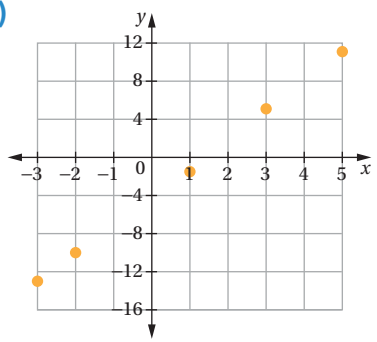
(b) $y = \dfrac{3(x - 1)}{4}$

x	-7	-3	1	5	9
y	-6				
(x, y)	$(-7, -6)$				

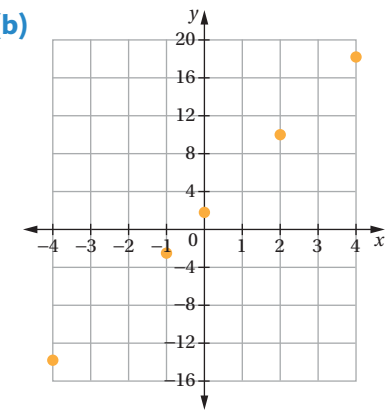
SC 2 I can recognise a linear pattern from a set of points on a Cartesian plane and create a table of values

1 Construct a table of values from each of the following linear graphs.

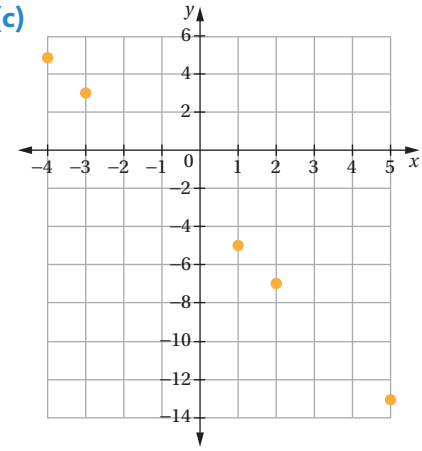
(a)



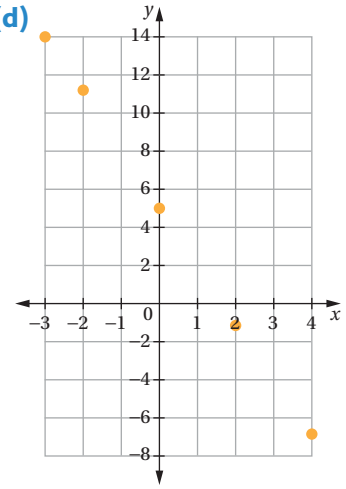
(b)



(c)



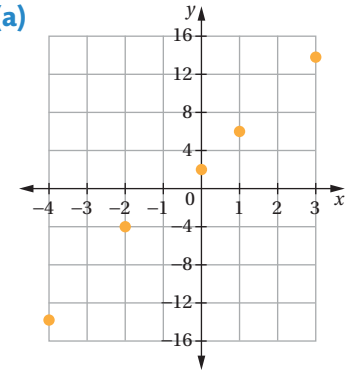
(d)



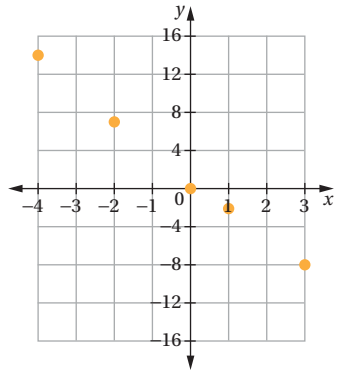
2 Use any pair of points to write the rule for each graph in the previous question.

3 In each of the following graphs, one of the points does not line up with the others. Identify the coordinates of that point, then alter its vertical coordinate to make the points linear.

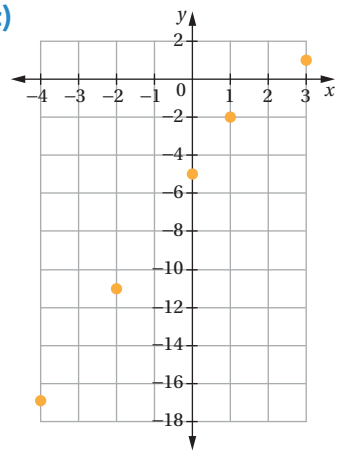
(a)



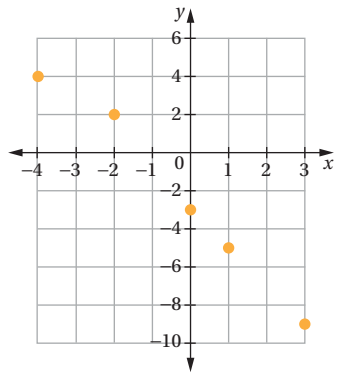
(b)



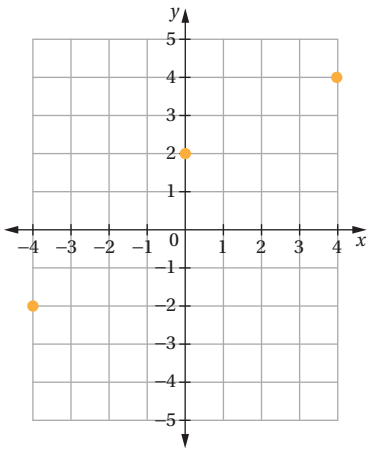
(c)



(d)



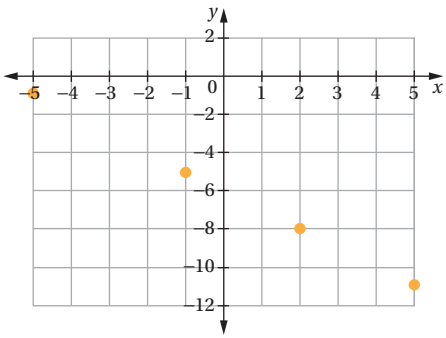
4 The three points on the Cartesian plane below do not line up.



(a) Beginning with the point on the far left, for each point in turn, write an alternative point with the same x-coordinate that would line up with the other two points.

(b) Write the rule that goes with each new set of three points.

5 For the linear graph below:



(a) write the rule for the graphed points

(b) determine the coordinates of at least three other points that satisfy the rule and lie within the graphed points. Choose points with integral coordinates.

SC 3 I can recognise and describe consistent changes in the values of coordinates and link them to a linear pattern

1 Each of the following tables of values represents a linear relationship. Describe the change in the y-coordinates in terms of the increase in the x-coordinates.

(a)

x	1	2	3	4	5
y	3	8	13	18	23

(b)

x	−2	−1	0	1	2
y	−2	1	4	7	10

(c)

x	1	2	3	4	5
y	2	0	−2	−4	−6

(d)

x	−2	−1	0	1	2
y	10	4	−2	−8	−14

2 Each of the following tables of values represents a linear relationship. Describe the change in the y-values in terms of the increase in the x-values.

(a)

x	1	4	6	8	11
y	7	22	32	42	57

(b)

x	−4	−2	1	5	9
y	3	5	8	12	16

(c)

x	1	4	6	8	11
y	3	0	−2	−4	−7

(d)

x	−4	−2	1	5	9
y	4	0	−6	−14	−22

3 Use the pattern of change in the y-values compared to the x-values to determine the rule for the following tables of values.

(a)

x	−4	−2	3	7	9
y	−5	1	16	28	34

(b)

x	−5	−1	1	6	10
y	16	8	4	−6	−14

(c)

x	−6	−3	1	5	10
y	22	13	1	−11	−26

(d)

x	−6	0	2	8	16
y	0	3	4	7	11

4 For each table of values, determine the coordinate pair that does not fit the linear pattern. In each case, alter the y-coordinate so that the point fits the pattern.

(a)

x	−3	−1	4	10	13
y	−17	−9	11	32	47

(b)

x	1	3	8	11	15
y	10	13	23	29	37

(c)

x	−2	1	4	6	12
y	12	9	7	4	−2

(d)

x	−5	−4	−2	1	5
y	13	10	7	1	−7

5 The three coordinate pairs in the table of values below do not represent a linear relationship.

x	−3	1	5
y	2	−2	10

(a) Taking each pair in turn, write an alternative pair of coordinates with the same x-value that would form a linear relationship with the other two pairs.

(b) Write the rule that goes with each new set of three pairs of coordinates.

Model linear patterns using manipulatives, diagrams and graphs

9.4

Learning intention: To be able to model linear patterns using manipulatives, diagrams and graphs

Success criteria:

SC 1 I can use manipulatives or draw diagrams to model a linear pattern.

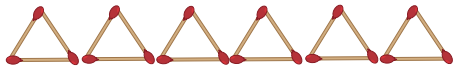
Lesson warm-up

Patterns with triangles

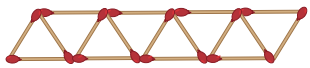
Describe each of the patterns in terms of the number of triangles and the number of matchsticks used.

Which patterns are linear? Explain.

(a)

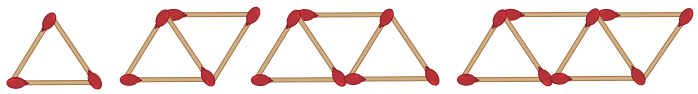


(b)



SC 1 I can use manipulatives or draw diagrams to model a linear pattern

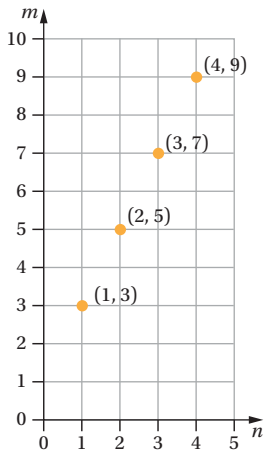
The number of matchsticks needed to build 1, 2, 3 or more equilateral triangles in the style below depends on the number of triangles in each diagram.



1 triangle 2 triangles 3 triangles 4 triangles

In the pattern shown, 2 extra matches are added each time to build each extra triangle.

The number of matches is 3, 5, 7 and so on. As a table of values:



Number of triangles, n	Number of matchsticks, m	Coordinate pair (n, m)
1	3	(1, 3)
2	5	(2, 5)
3	7	(3, 7)
4	9	(4, 9)

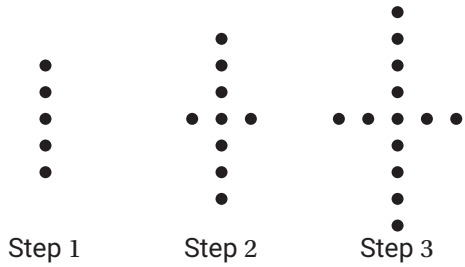
Written as a rule: The number of matchsticks increases by 2 for each triangle added, $m = 2n + 1$.

The pattern also forms a linear graph.

Worked example

Recognising a linear pattern from a diagram

Consider the dot-pattern diagram below.



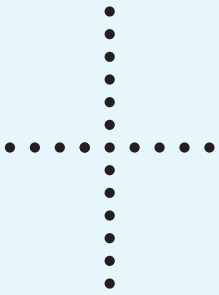
(a) Generate a table of values for the three steps shown.

THINKING	WORKING
Define the variables.	Define the step number as n. Define the number of dots as d.
Construct a table of values and then count the dots in each step.	**Step number, n** | 1 | 2 | 3 **Number of dots, d** | 5 | 9 | 13 **Ordered pair (n, d)** | (1, 5) | (2, 9) | (3, 13)

(b) Write a rule for the number of dots in terms of the step number.

THINKING	WORKING
Determine the change in the number of dots for a step increase of 1.	**Step number, n** | 1 | 2 | 3 **Number of dots, d** | 5 | 5 + 4 = 9 | 9 + 4 = 13 As the step increases by 1, the number of dots increases by 4.
Use the rate of change to add a row in the table and determine the difference between the calculated and given values.	**Step number, n** | 1 | 2 | 3 **$4n$** | $4 \times 1 = 4$ | $4 \times 2 = 8$ | $4 \times 3 = 12$ **Number of dots, d** | $4 + 1 = 5$ | $8 + 1 = 9$ | $12 + 1 = 13$
Write the rule.	The number of dots is 1 more than 4 times the step number. $d = 4n + 1$

(c) Use your rule to determine the number of dots in Step 5.

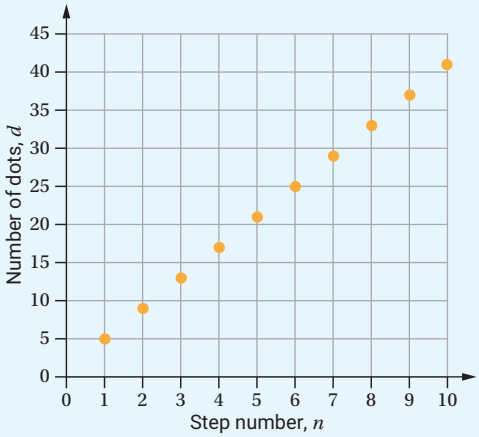
THINKING	WORKING
Substitute the value into the formula and calculate the value of d.	The step number, $n = 5$ $d = 4n + 1$ $= 4 \times 5 + 1$ $= 20 + 1$ $= 21$
Check the reasonableness of your answer by drawing a diagram.	 Step 5

(d) Construct a graph of the pattern for the first 10 steps.

THINKING	WORKING
Generate a table of values and coordinate pairs.	

Step number, n	1	2	3	4	5	6	7	8	9	10
Number of dots, d	5	9	13	17	21	25	29	33	37	41
Ordered pair (n, d)	(1, 5)	(2, 9)	(3, 13)	(4, 17)	(5, 21)	(6, 25)	(7, 29)	(8, 33)	(9, 37)	(10, 41)

Draw a Cartesian plane and plot the points.

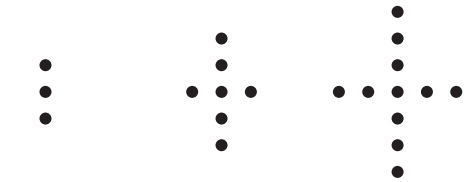


Practice

ANSWERS Page 689

SC 1 I can use manipulatives or draw diagrams to model a linear pattern

1 Consider the dot-pattern diagram below.



(a) Complete the table of values.

Step number, x	1	2	3	4	5
Number of dots, y					
Coordinate pairs (x, y)					

(b) To check your work, sketch the diagram for Step 5.

(c) Write a rule, in words, for the number of dots in terms of the step number.

(d) Model the rule as an equation for y in terms of x.

(e) Predict the number of dots for the 10th step of the pattern.

2 For each of the following patterns made from regular polygons:

(i) complete a table of values, as shown, for the number of toothpicks required to build a model containing 1, 2, 3 and so on, of each polygon.

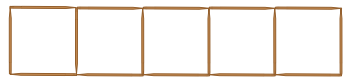
Number of polygons	1	2	3	4	5
Number of toothpicks					

(ii) write a formula, in words, for the relationship beginning: Number of toothpicks =

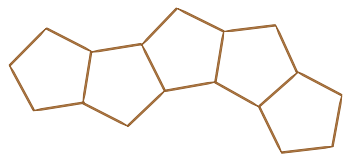
(iii) predict the number of toothpicks needed to make a model with 10 polygons

(iv) plot the graph of the pattern for the first 10 polygons.

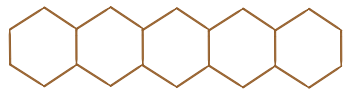
(a)



(b)



(c)



3 Consider the following diagram showing three steps of a toothpick model.

(a) Determine the number of toothpicks used when there is:

(i) 1 square **(ii)** 2 squares **(iii)** 3 squares.

(b) Write a formula, in words, for the number of toothpicks in terms of the number of squares.

(c) Predict the number of toothpicks needed for a model containing 50 squares.

4 A fence is made using vertical and horizontal poles, all the same length. A 1-section fence has 5 horizontal poles and 2 vertical poles. The diagram shows the first three sizes for this type of fence.

(a) Create a table of values for fences from 1-section to 6-sections, with the number of sections in the first row and the number of poles in the second row.

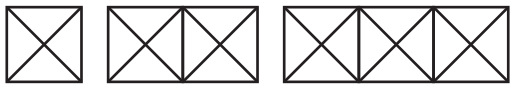
(b) Write a rule to help calculate y, the number of poles needed, from x, the number of sections.

(c) Plot the points on a Cartesian plane with x from 1 to 20 and y from 1 to 100.

(d) Using your results from parts **(b)** and **(c)**, determine the number of poles needed for a 10-section fence.

(e) Using your results from parts **(b)** and **(c)**, determine the number of sections in a fence that uses 91 poles.

5 A pattern is made using squares with two diagonals drawn, so the first square consists of 6 lines. The diagram shows the first three sizes for this pattern.



(a) Create a table of values for patterns from 1 square to 4 squares. Write the number of squares in the first row and the number of lines in the second.

(b) Write a rule connecting x, the number of squares, and y, the number of lines.

(c) Plot the points on a Cartesian plane with x from 1 to 10 and y from 1 to 50.

(d) Using your results from parts **(b)** and **(c)**, determine the number of lines used in a 6-square pattern.

(e) Using your results from parts **(b)** and **(c)**, determine the number of squares in a pattern that has 46 lines.

(f) The diagonal lines in the pattern are longer than the sides of the squares. Let d be the number of diagonal lines and s be the number of side lines. Write rules for:

(i) d in terms of x **(ii)** s in terms of x **(iii)** y in terms of d and s.

TOPIC REVIEW

Multiple choice

ANSWERS Page 742

1 For the expression $x + y$, which values of x and y make the expression equal to 23?

A $x = 8$ and $y = 15$

B $x = 7$ and $y = 14$

C $x = 10$ and $y = 12$

D $x = 12$ and $y = 11$

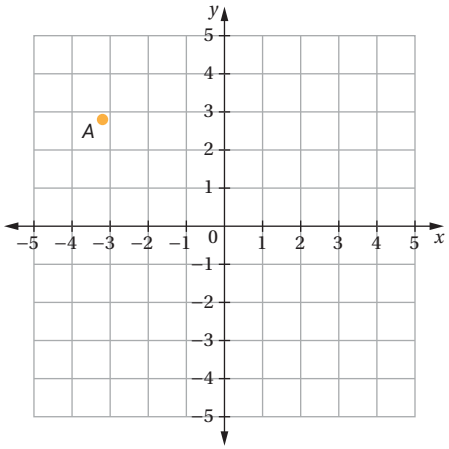
2 Substitute $b = 2$ and $c = 3$ into the expression $3b - 7c$. What is the result?

A 27 **B** 15 **C** −15 **D** −53

3 Substitute $m = -5$ and $u = 3$ into the expression $u(9 - m) + 3$. What is the result?

A −39 **B** −29 **C** 15 **D** 45

4 Where is the point *A* located? What are its coordinates?



A (2.8, −3.2) **B** (−3.2, 2.8) **C** (−2.8, 3.2) **D** (−3.8, 2.2)

5 Which of the following statements about linear patterns is true?

A Any set of points makes a linear pattern on a Cartesian plane.

B If points make a linear pattern, then one straight line can go through all the points.

C A linear pattern is the only pattern made by a set of points.

D There is only one linear pattern that can be made on a Cartesian plane.

6 Which set of points on the Cartesian plane forms a linear pattern?

A (−4,0), (−2,1), (−1,3) **B** (−3, −2), (−2,1), (−1,4)

C (−4,0), (−1, −2), (1, −3) **D** (−4,0), (−2,1), (2,5)

7 A rule has the following steps: subtract 6 from the input number, multiply the result by 4, and finally divide by 2. What is the result for an input of −3?

A 18 **B** 6 **C** −6 **D** −18

8 Consider the following table of values. As the x-value increases by 1, how does the y-value change?

x	-6	-2	0	3	11
y	12	$5\frac{1}{3}$	-2	-3	$-16\frac{1}{3}$

A Increases by $1\frac{2}{3}$ **B** Increases by $3\frac{1}{3}$ **C** Decreases by $6\frac{2}{3}$ **D** Decreases by $1\frac{2}{3}$

9 The rule used to generate this table of values is:

x	1	2	3	4	5
y	3	8	13	18	23

A $y = 3x$ **B** $y = 5x - 2$ **C** $y = x + 2$ **D** $y = 4x - 1$

10 A jar is filled with 125 identical marbles. Let t represent the mass of each marble. The mass of the jar filled with marbles is 295 g. Given that the jar by itself has a mass of 160 g, which of the following expressions could be used to find the mass of each marble?

A $160t + 125 = 295$ **B** $125t = 455$ **C** $125t + 160 = 295$ **D** $160t = 455$

Short answer

1 Substitute $a = 7$ into each expression and calculate the result.

(a) $a + 3$ **(b)** $a - 2$ **(c)** $4a + 3$ **(d)** $3(a + 6)$

2 A number pattern follows the rule $3x + 6$. Complete the table to calculate the next three numbers in the pattern.

Input: x	0	1	2	3
Output: $3x + 6$	6			

3 Substitute $p = -1$ into each expression and calculate the result.

(a) $3 + p$ **(b)** $p - 2$ **(c)** $4(p + 2)$ **(d)** $\sqrt{24 - p}$

4 Draw a triangle with the vertices (0, –4), (1, –1.5), (–3, 1). Label the coordinates of each vertex.

5 Determine whether or not each table of values forms a linear pattern and explain why.

(a)

x	-2	-1	0	1	2
y	-1	-5	-9	-12	-16

(b)

x	-6	-5	-4	-3	-2
y	-3	-14	-25	-36	-47

(c)

x	0	1	2	3	4
y	-2	4	10	16	22

6 Determine the missing value in each of the following tables below to make each a linear pattern.

(a)

x	−3	−1	1	3	5
y	−5	−1	3		11

(b)

x	−5	−1	0	2	7
y	−17	−5		4	19

(c)

x	−3	−2	−1	0	1
y	$-3\frac{1}{2}$		$-\frac{1}{2}$	1	$2\frac{1}{2}$

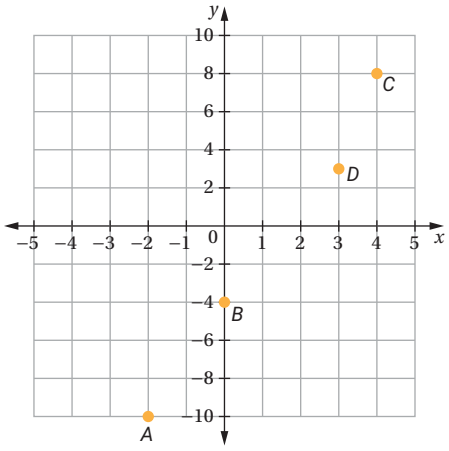
7 On a Cartesian plane, plot the values from the table. Do the points make a linear pattern?

x	−5	−4	0	2
y	4	3	0	−2

8 Determine whether the following statements are true or false.

	True	**False**
Any three points on a Cartesian plane will make a linear pattern.		
Any two points of a Cartesian plane can be joined by a straight line.		

9 Look at the following graph.



(a) Which point does not fit the linear pattern?

(b) Move this point vertically so it fits the pattern. What are its new coordinates?

(c) If you move the point horizontally to fit the pattern, the new coordinate pair will not both be whole numbers. What would be a reasonable estimate for this new coordinate pair?

10 Complete the following tables of values, using fractions where necessary.

(a) $y = \frac{3x}{2} - 1$

x	−3	−2	−1	0	1
y					

(b) $y = \frac{-2(4-x)}{3}$

x	−1	0	2	5	7
y					

11 Find the rule for each of the following tables of values.

(a)

x	−3	−1	2	6	15
y	11	5	−4	−16	−43

(b)

x	−5	0	5	10	15
y	5	4	3	2	1

12 Copy and complete the following to make each a true equation.

(a) $6x - 5 = __ + 3x - 5$

(b) $2x + 8 = 3x + 7 + __$

(c) $5(x-2) = 3x + 9 + ____$

(d) $\frac{4x}{3} + 5 = x - 4 + ____$

13 Write three expressions that are equivalent to each of the following.

(a) $6x + 1$ **(b)** $4 - 3x$ **(c)** $2(x + 1)$ **(d)** $3(5 - 2x)$

14 Write each of the following expressions in simplest form.

(a) $2x + 3x + 4 - 2$

(b) $7ab - 4 - 3b + 6ba + 9b$

(c) $8a + 9b - 6ab + 4$

(d) $2(3x - 1) + 5(2x - 3)$

15 Complete the following to make correct equations.

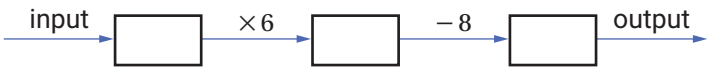
(a) $12ab - 8ac = __(3b_____)$

(b) $24 - 12a = __(a_____)$

(c) $30a - 10 - 8a^2 + 4ab = __(_____ - 1) - 4a(_____)$

(d) $12 - 6a - 30b + 5c = __(2_____) - 5(_____)$

16 A number machine is shown below.

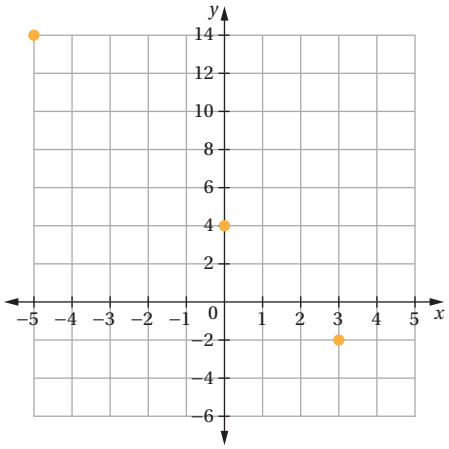


(a) Given that the output value is 34, what was the input value?

(b) Given that the output value is −26, what was the input value?

(c) Given that the output value is 0, what was the input value?

17 Some points have been plotted on a Cartesian plane, as shown.



(a) Complete the following table of values for the linear relationship shown by the points plotted.

x	−5	−1	0	3	5
y					

(b) Write a rule to represent this linear relationship.

18 Given the following are true equations, state the value of x in each example.

(a) $x + 4 = 9$ **(b)** $6 - x = 10$

(c) $2x = 12$ **(d)** $2x - 3 = 5$

Extended response

1 This input–output machine uses an algebraic rule to make a number pattern. The machine takes in 'input' values that are consecutive integers (numbers that go up by 1 each time). The machine then substitutes each input value into the algebraic rule to calculate each 'output' value.

(a) When the machine uses the rule $3x - 2$, an input of 0 gives an output of −2. Use substitution to calculate the next three outputs in the number pattern. Describe the pattern.

(b) When the machine's rule is changed to $x^2 + 1$, an input of 0 gives an output of 1. Use substitution to calculate the next three outputs in the number pattern. Describe the pattern.

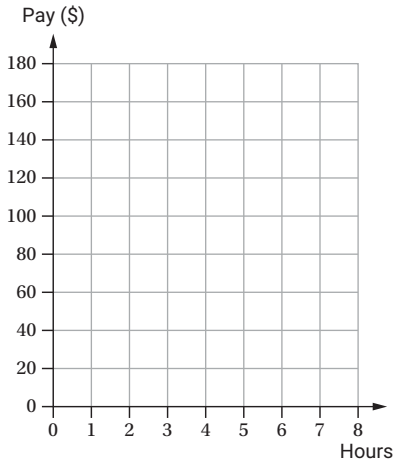
(c) When the machine's rule is changed to $\frac{1}{2+x}$, an input of 0 gives an output of $\frac{1}{2}$. Use substitution to calculate the next three outputs in the number pattern. Describe the pattern.

2 A shop assistant earns $20 per hour.

(a) Copy and complete the table of values showing the earnings for a day of up to 8 hours.

Hours	2	3	5	8
Pay ($)				

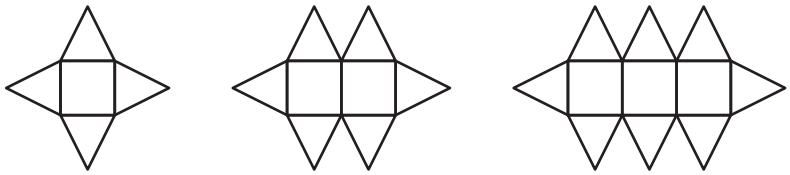
(b) Plot these points on a Cartesian plane like the following.



(c) How much would the shop assistant earn for a 7-hour day?

(d) If you had to plot a point to represent 0 hours of work, where would it go? Explain why this is a reasonable place to put this point.

3 Consider the following diagrams.



(a) How many toothpicks are used when there is just one square in the pattern?

(b) How many toothpicks are used when there are two squares in the pattern?

(c) How many toothpicks are added each time a square is added to the pattern?

(d) Write a rule for the number of toothpicks needed for a given number of squares.

(e) Use the pattern with three squares to check that your rule is correct.

TOPIC

10

Making and reading linear graphs

Recall 251

10.1 Graph linear relationships from a rule 252

10.2 Interpret the gradient of a linear graph 257

10.3 Interpret, discuss and analyse relationships in graphs 264

Topic review 273

Why learn this?

Graphing linear relationships from a rule provides a visual representation of how two variable quantities are directly related to each other.

Predicting future outcomes by extending linear graphs based on existing patterns is particularly useful in fields like finance and science where identifying trends is crucial for decision-making purposes.

Having the ability to graph linear relationships is not only important for academic success in mathematics, but also for applying mathematical reasoning in everyday life and various professional contexts.

RECALL

ANSWERS Page 690

I can generate a table of values using an equation

1 Use the equation given to complete each table.

(a) $b = 2a$

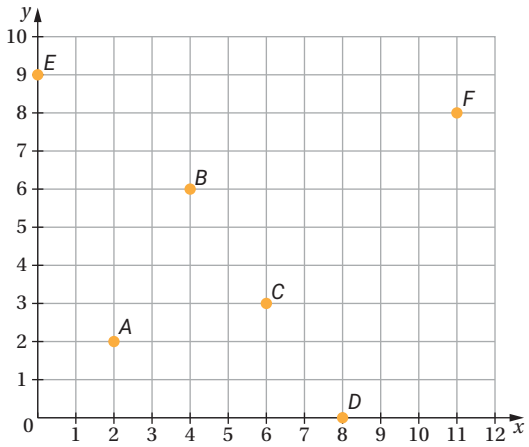
a	2	3	4	5
b				

(b) $y = 4x + 1$

x	1	2	3	4
y				

I can read and plot points on a Cartesian plane

1 Write the coordinates for each point plotted on the number plane shown.



2 Plot and label the following points on the Cartesian plane.

(a) A(2, 2) **(b)** B(−4, 5) **(c)** C(8, −3)

(d) D(5, −2) **(e)** E(4, 0) **(f)** F(0, −6)

10.1 Graph linear relationships from a rule

Learning intention: To be able to graph linear relationships from a rule

Success criteria:

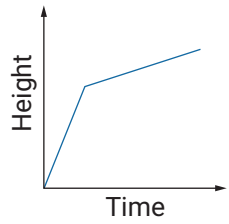
SC 1 I can graph a linear relationship from a rule.

SC 2 I can graph a linear relationship using the y-intercept.

Lesson warm-up

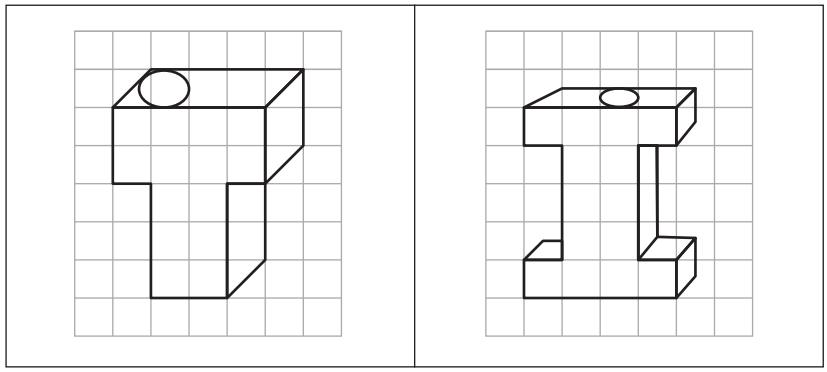
Vase filling

(a) The height of water in a vase being filled at a constant rate is represented by the graph below.



(b) Describe the change in height over the time the vase is filled.

Water is then poured into each of the two containers shown.



(c) Which container being filled would match the graph above?

(d) Draw the graph of the height of the water in the other container.

SC 1 I can graph a linear relationship from a rule

A linear relationship can be drawn using any two points. When graphing relationships by hand, it is wise:

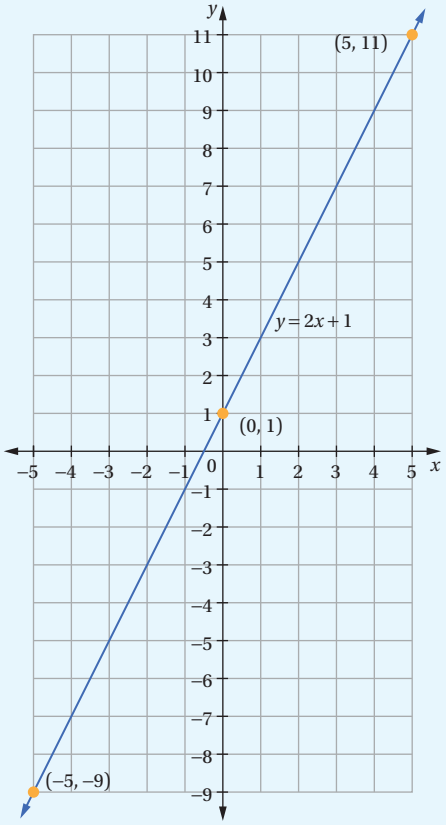
- to plot well-spaced points to see where the graph goes over a range of values
- to plot at least three points to pick up errors in calculation or plotting
- to include a negative value of x, a positive value and zero, if appropriate.

Linear graphs with no specified domain (the range of x-values) should have arrows at either end to show that the relationship continues infinitely. If a domain is specified, it is usual to write the coordinates of the endpoints on the graph.

Worked example

Graphing a linear relationship

Graph $y = 2x + 1$ on a set of Cartesian axes by determining three points on the line.

THINKING	WORKING
Create a table of values.	x | -5 | 0 | 5 y | | |
Substitute the x-values from the table to calculate the y-value for each point.	$y = 2x + 1$ For $x = -5$: $y = 2 \times (-5) + 1$ $= -10 + 1$ $= -9$ For $x = 0$: $y = 2 \times 0 + 1$ $= 0 + 1$ $= 1$ For $x = 5$: $y = 2 \times 5 + 1$ $= 10 + 1$ $= 11$
Complete the table of values.	x | -5 | 0 | 5 y | -9 | 1 | 11 (x, y) | $(-5, -9)$ | $(0, 1)$ | $(5, 11)$
Construct a Cartesian plane on grid paper, noting the extent of values required for each axis. Plot the points on the axes. Join the points with a straight line, extended and arrowed at each end. Label the graph with its rule.	x-values from -5 to 5. y-values from -9 to 11. 

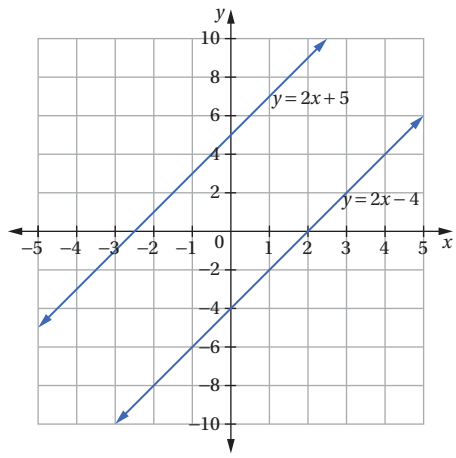
SC 2 I can graph a linear relationship using the y-intercept

The following Cartesian axes show the graphs of $y = 2x + 5$ and $y = 2x - 4$.

Notice that each graph intersects the vertical axis at the value of the constant term in the rule.

The y-intercept for $y = 2x + 5$ is 5, with coordinates (0, 5).

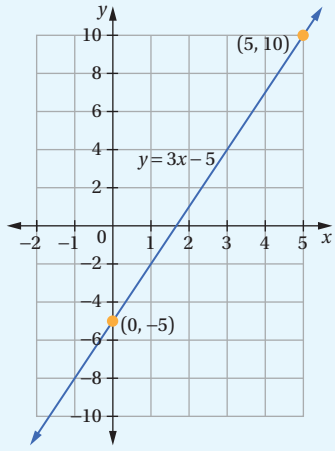
The y-intercept for $y = 2x - 4$ is -4, with coordinates (0, -4).



Worked example

Using the y-intercept to graph a linear relationship

Create the graph of $y = 3x - 5$ by plotting the y-intercept and one other point.

THINKING	WORKING
Determine the coordinates of the y-intercept ($x = 0$).	For $x = 0$, $y = 3x - 5$ $= 3 \times 0 - 5$ $= 0 - 5$ $= -5$ The y-intercept is -5. The coordinate pair for the y-intercept is (0, -5).
Determine the coordinates of a second point by substituting any x-value (other than 0).	For $x = 5$, $y = 3 \times 5 - 5$ $= 15 - 5$ $= 10$ The coordinates of a second point is (5, 10).
Create a set of Cartesian axes. Plot the two points on the axes and then join them with a straight line. Extend the line at both ends and add arrows.	

Practice

ANSWERS Page 690

SC 1 I can graph a linear relationship from a rule

1 Copy and complete the table of values.

x	-5	0	5
$y = 2x$			
$y = -2x$			
$y = -2x + 1$			

2 For each rule, complete the table of values, then graph the line on a Cartesian plane.

x	-2	0	2
y			
(x, y)			

(a) $y = x + 3$ **(b)** $y = x - 3$ **(c)** $y = 2x + 3$

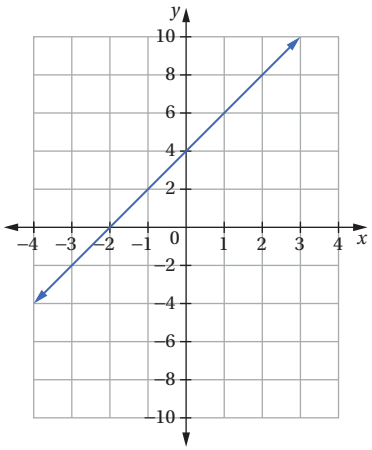
(d) $y = 2x - 3$ **(e)** $y = -2x + 3$ **(f)** $y = -2x - 3$

3 Graph each of the following linear relationships on a set of Cartesian axes for x-values from -5 to 5. Label the endpoints of each line segment with its coordinates.

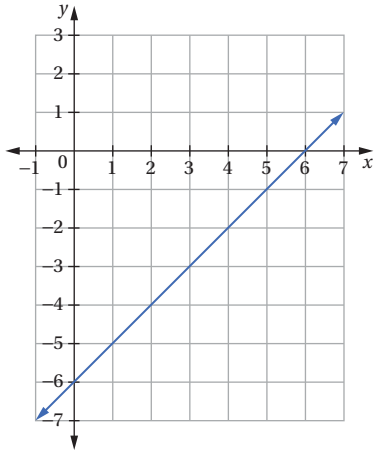
(a) $y = x + 5$ **(b)** $y = 3x - 3$ **(c)** $y = -x + 4$ **(d)** $y = -3x + 3$

4 For each of the following graphs, read off three coordinate pairs that lie on the line and then determine the rule for the graph.

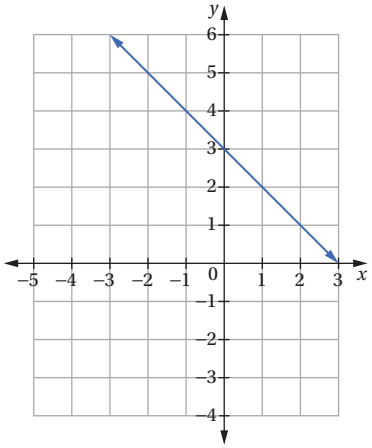
(a)



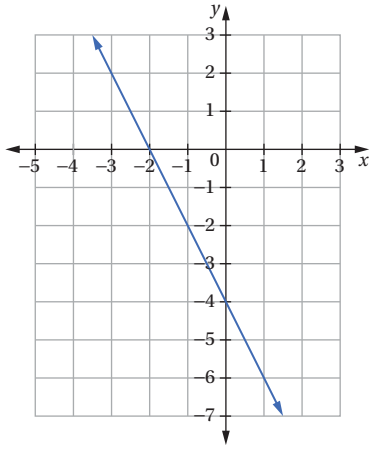
(b)



(c)



(d)



5 Determine the coordinates of a point that lies on a line through (0, –2) and (4, 18), given that the x-coordinate is –1.

SC 2 I can graph a linear relationship using the y-intercept

1 Graph the equation $y = 3x + 5$ by plotting the y-intercept and the point where $x = 2$.

2 Determine the coordinates of the y-intercept for each of the following.

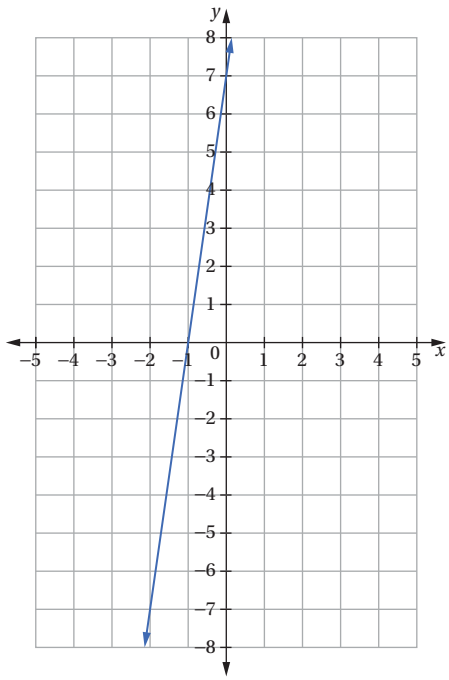
(a) $y = x + 4$ **(b)** $y = x + 5$ **(c)** $y = x - 4$ **(d)** $y = x - 5$

(e) $y = 2x + 4$ **(f)** $y = 2x - 4$ **(g)** $y = 4 - 2x$

3 Graph the following relationships, marking the y-intercept as a coordinate pair.

(a) $y = 5x - 2$ **(b)** $y = -3x + 6$

4 Determine the rule for the graph shown.



5 Determine the rule for the linear graphs with the following tables of values.

(a)

x	0	4
y	2	10

(b)

x	0	5
y	–1	14

(c)

x	–4	0
y	13	1

(d)

x	–5	0
y	–7	–2

6 Determine the rule for the graphs passing through each of the following pair of points.

(a) (0, 6) and (3, 9) **(b)** (0, –8) and (2, 2)

(c) (–2, 11) and (0, 3) **(d)** (–4, –2) and (0, –6)

Interpret the gradient of a linear graph

10.2

Learning intention: To be able to interpret the gradient of a linear graph

Success criteria:

SC 1 I can classify the slope of a line as increasing, decreasing or zero.

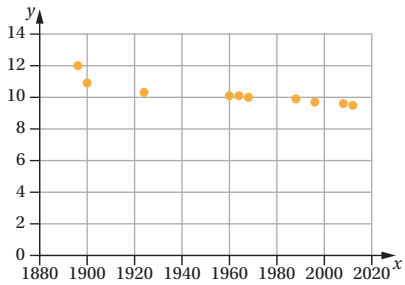
SC 2 I can interpret linear conversion graphs.

Lesson warm-up

Olympic times

(a) Which Olympic event do you think might be shown by the graph?

(b) What information is needed for the graph to be simpler to read?



SC 1 I can classify the slope of a line as increasing, decreasing or zero

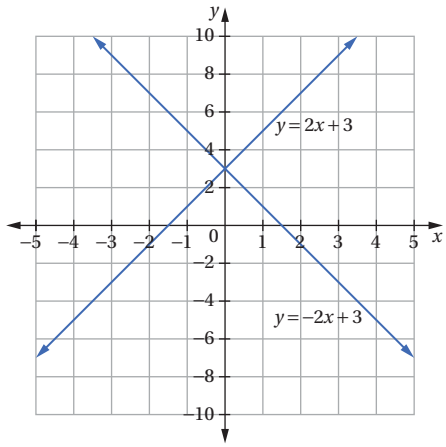
The gradient or slope of a graph is read from left to right, in the positive direction of the x-axis.

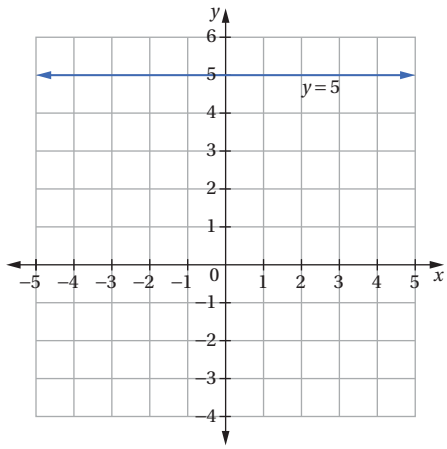
The two graphs on the first set of axes shown at right have the same y-intercept, but $y = 2x + 3$ slopes upwards from this point, a positive slope, while $y = -2x + 3$ slopes downwards, a negative slope.

- For a positive coefficient of x, the graph has a positive gradient. A positive gradient means that as one variable increases, so does the other one.
- For a negative coefficient of x, the graph has a negative gradient. A negative gradient means that as one variable increases, the other decreases.

The line $y = 5$ is shown on the second set of axes. In this relationship, the y-value is a constant, not dependent on the x-values.

This graph is parallel to the x-axis. Its gradient is zero. As the x-values increase, the y-value remains the same. The gradient of a vertical line is undefined.

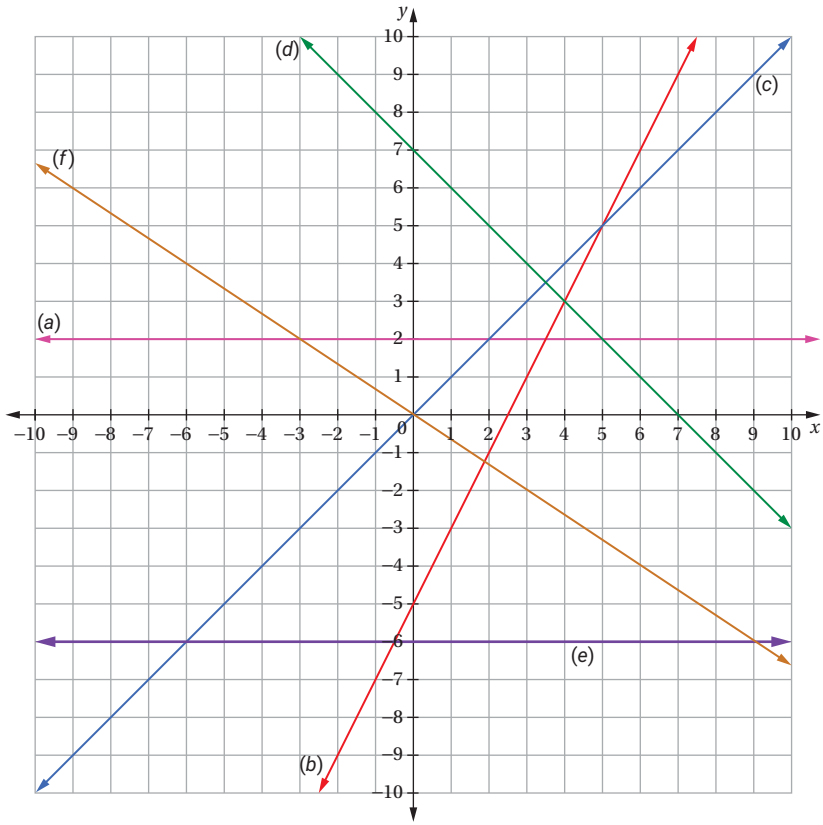




Worked example

Determining the nature of the gradient of a linear graph

For each of the linear graphs shown on the Cartesian axes, determine whether its gradient is positive, negative or zero.



THINKING	WORKING
Identify the linear graphs with a gradient of zero. Linear graphs with a gradient of zero are horizontal, the x-values increase and the y-values remain unchanged.	The graphs with a zero gradient are **(a)** and **(e)**.
Identify the linear graphs with a positive gradient. Linear graphs with a positive gradient increase from left to right. The y-values increase as the x-values increase.	The graphs with a positive gradient are **(b)** and **(c)**.
Identify the linear graphs with a negative gradient. Linear graphs with a negative gradient decrease from left to right. The y-values decrease as the x-values increase.	The graphs with a negative gradient are **(d)** and **(f)**.

SC 2 I can interpret linear conversion graphs

A conversion graph converts from one unit to another. There is not necessarily an independent variable, so the choice of axes for the variables may be arbitrary.

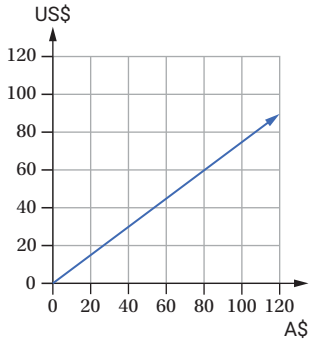
Most (although not all) conversion graphs pass through the origin because zero of one quantity usually means zero of the other. Most conversion graphs are drawn in the first quadrant as only positive quantities make sense. An exception to both of these usual scenarios is a temperature conversion graph, where negative values do make sense and the freezing point of water is not zero in temperature scales other than degrees Celsius or centigrade.

Unless a digital graph is being used, an estimate of the value may be necessary, as a graph is not the most precise way of doing conversions.

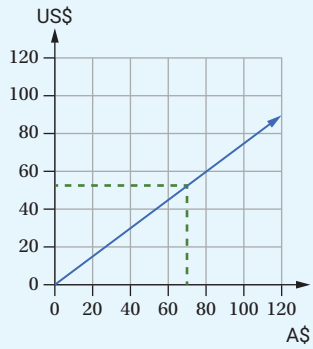
Worked example

Reading values from a conversion graph

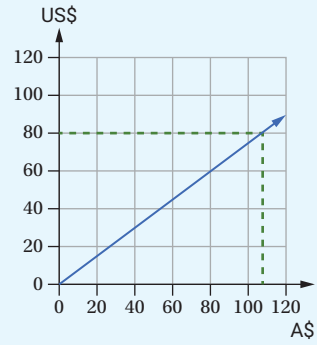
The following graph can be used to convert Australian dollars (A$ or AUD) to United States dollars (US$ or USD) for a particular day. Use the conversion graph shown to determine each of the following values, for the day shown.



(a) The value in US$ of A$70.

THINKING	WORKING
Draw a vertical line from 70 on the A$ axis until it touches the conversion line. Then draw a horizontal line across to the US$ axis and read off the value. An approximation will be required.	 A$70 ≈ US$52

(b) The value in A$ of US$80.

THINKING	WORKING
Draw a horizontal line from 80 on the US$ axis until it touches the conversion line. Then draw a vertical line down to the A$ axis and read off the value. An approximation will be required.	 US$80 ≈ A$107

(c) The value in A$ of US$8000.

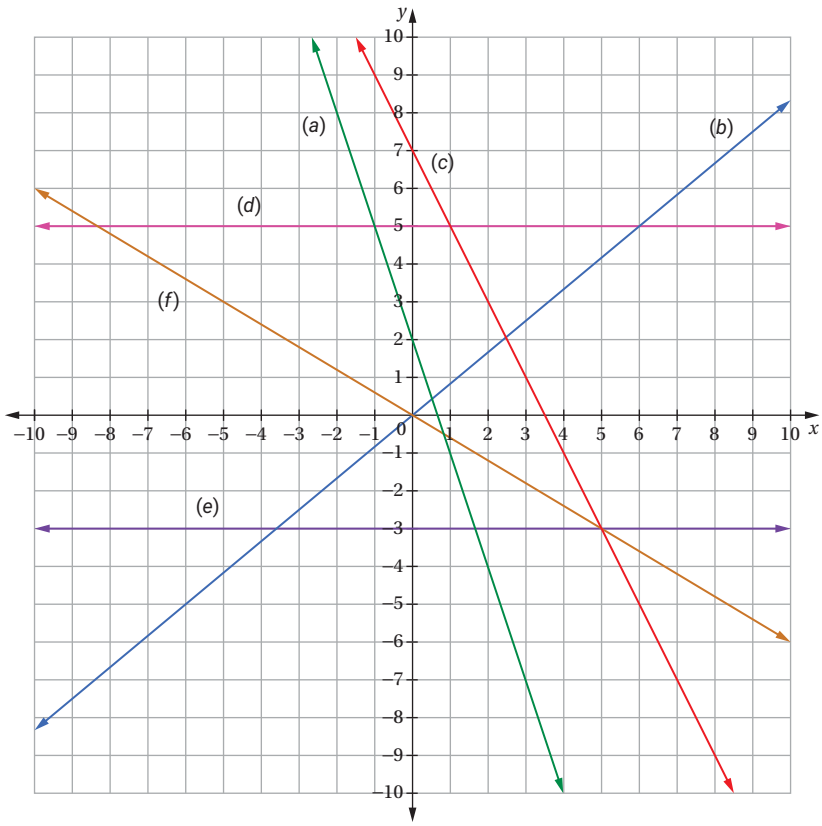
THINKING	WORKING
Use proportion to estimate the value from the previous answer.	$80 \times 100 = 8000$ Since the graph passes through the origin: US$8000 ≈ A$107 × 100 = A$10 700

Practice

ANSWERS Page 693

SC 1 I can classify the slope of a line as increasing, decreasing or zero

1 For each of the straight lines shown on the Cartesian axes, determine whether its gradient is positive, negative or zero.



2 For each of the following linear relationships, determine whether the graph would have a positive, negative or zero gradient.

(a) $y = 3x + 6$ **(b)** $y = 6 - 2x$ **(c)** $y = 2 + 5x$

(d) $y = -9$ **(e)** $y = -3 + 5x$ **(f)** $y = \frac{2x}{7}$

3 For each of the following pairs of points, determine whether the line segment joining the points would have a positive, negative or zero gradient.

(a) (0, 0) and (2, 9) **(b)** (4, 3) and (8, 2) **(c)** (−3, 5) and (1, 5)

(d) (−2, −2) and (0, 4) **(e)** (0, 6) and (−4, 2) **(f)** (−2, −3) and (−5, −5)

(g) (3, 1) and (1, 3) **(h)** (4, 4) and (0, 1)

4 For each of the following tables of values, determine whether the graph would have a positive, negative or zero gradient and then write a rule for the linear relationship.

(a)

x	−3	−1	0	5	10
y	10	6	4	−6	16

(b)

x	−4	0	1	5	7
y	−18	−6	−3	9	15

(c)

x	−7	−1	3	12	29
y	−6	−6	−6	−6	−6

(d)

x	−12	−6	0	6	12
y	2	1	0	−1	−2

5 For each of the following relationships, determine whether the linear graph for the relationship would have a positive or negative gradient.

(a) The amount of water in a bath as it is filled from a tap in terms of the time since the tap was turned on.

(b) The amount of water in a bath after the plug is removed in terms of the time since the plug was removed.

(c) The amount of flour needed for a cake in terms of the number of eggs available for a particular recipe.

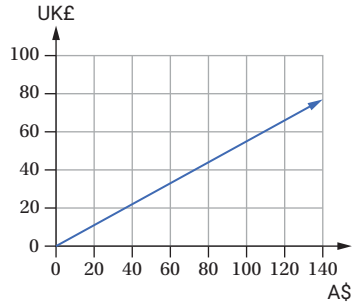
(d) The distance driven in terms of the period of time for a particular speed.

(e) The number of oranges left in a bowl in terms of the number of oranges taken out from the bowl.

6 Explain how to determine whether a gradient is positive, negative or zero.

SC 2 I can interpret linear conversion graphs

1 The graph shown at right allows A$ (Australian Dollars) and UK£ (UK Pounds) to be converted from one to the other. Use the graph to perform the conversions stated.



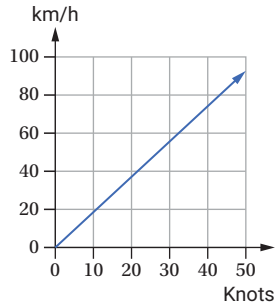
(a) A$120 to UK£ **(b)** A$12000 to UK£

(c) A$50 to UK£ **(d)** A$500 to UK£

(e) UK£75 to A$ **(f)** UK£750 to A$

(g) UK£20 to A$ **(h)** UK£2000 to A$

2 The speed of boats and ships can be measured using km/h or knots. A knot is a speed used in sailing and represents the number of nautical miles per hour. Use the graph shown at right to convert the speed given to the other scale.

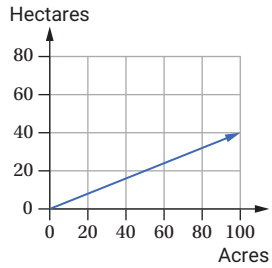


(a) 25 knots **(b)** 400 knots (aircraft speed)

(c) 5 knots (marina speed) **(d)** 60 km/h

(e) 100 km/h **(f)** 40 km/h

3 The graph shown at right allows the user to convert between acres and hectares, which are both units of area.



(a) Use the graph to convert the area given to the other unit.

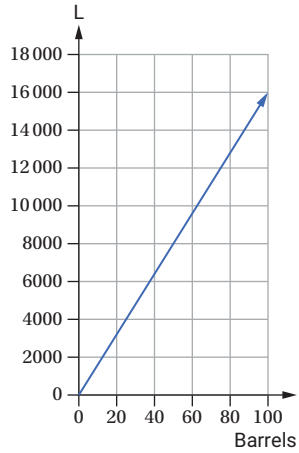
(i) 100 acres **(ii)** 80 acres

(iii) 20 ha **(iv)** 100 ha

(b) Complete each of the following:

(i) 1 ha = ____ acres **(ii)** 1 acre = ____ ha

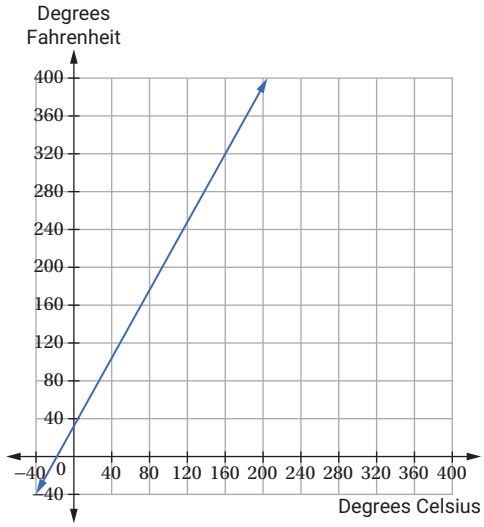
4 The amount of oil produced is measured in a unit called a barrel. The following graph allows conversion between barrels of oil and litres.



(a) Use the graph to determine the approximate number of litres in a barrel.

(b) Explain how you obtained your answer.

5 Temperatures in Australia are measured using the Celsius scale. On this scale, water freezes at 0 °C and boils at 100 °C. Previously (and still in the USA), temperatures were measured using the Fahrenheit scale where water freezes at 32 °F and boils at 212 °F.



(a) Use the graph to convert each given temperature to the other scale.

(i) 60 °C **(ii)** 120 °F

(iii) –40 °C **(iv)** 100 °F

(b) 80 °F is about the same temperature as 27 °C, so the Celsius temperature is about a third of the Fahrenheit temperature in this case. Explain why 30 °F is not about the same temperature as 10 °C.

(c) When Australia changed from the Fahrenheit scale to the Celsius scale, many people still had ovens with temperatures in degrees Fahrenheit. The 'rule of thumb' for oven temperatures was to 'double' the degrees in Fahrenheit to get the degrees in Celsius. A moderate oven is 180 °C. What is the temperature in degrees Fahrenheit:

(i) using the 'rule of thumb'

(ii) using the conversion graph?

10.3 Interpret, discuss and analyse relationships in graphs

Learning intention: To be able to interpret, discuss and analyse relationships in graphs

Success criteria:

SC 1 I can use graphs to interpret simple rates.

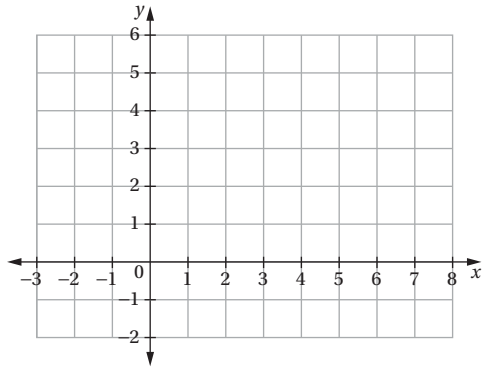
SC 2 I can interpret and analyse travel graphs.

SC 3 I can interpret and analyse time graphs.

Lesson warm-up

Enclosed area

Determine the area between the lines $y = x - 2$, $x = 7$ and the x-axis.



SC 1 I can use graphs to interpret simple rates

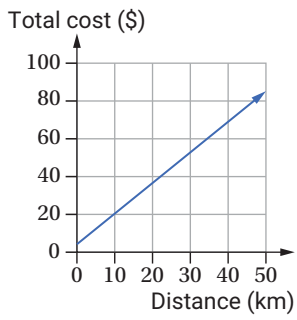
Linear graphs through the origin, with one quantity graphed against another, allow you to calculate the constant rate at which one quantity increases compared to the other.

Linear graphs that cut the vertical axis above the origin represent relationships where a fixed starting amount is involved as well as an amount that increases at a constant rate.

Worked example

Interpreting a graph to determine the rate

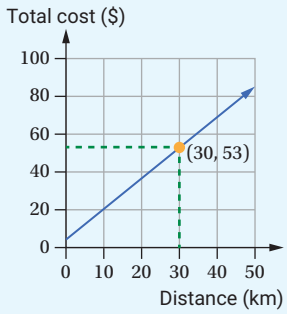
The graph at right shows the cost of a trip in a taxi where there is a fee called the flag fall, which is a fixed amount for every journey, and a set rate per kilometre.



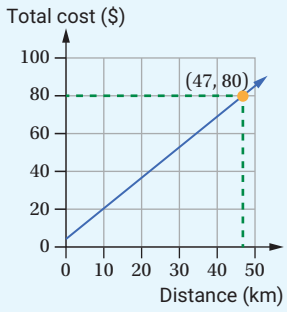
(a) Determine the flag fall amount.

THINKING	WORKING
Identify the coordinates of the y-intercept.	The coordinates of the y-intercept are (0, 5).
Write the answer.	The flag fall (the fee before travel) is $5.

(b) Determine the cost for a trip of 30 km.

THINKING	WORKING
Construct a line from the distance axis on the horizontal scale up to the conversion line and then read across to the value on the cost axis.	
Write the answer.	The trip of 30 km costs $53.

(c) Identify the distance that can be travelled for $80.

THINKING	WORKING
Construct a line from the cost axis on the vertical scale across to the conversion line and then read down to the value on the distance axis.	
Write the answer.	An $80 trip will be a distance of about 47 km.

(d) Determine the charge rate (cost per distance travelled).

THINKING	WORKING
Identify two known points.	A 30 km trip costs $53: (30, 53) A 47 km trip costs $80: (47, 80)
Calculate the difference in cost.	$80 – $53 = $27
Calculate the difference in distance travelled.	47 – 30 = 17 km

Interpret the differences.	An extra 17 km travelled costs an extra $27.
Divide the extra cost by the extra distance.	$\frac{\$27}{17\text{ km}} \approx \$1.588\ldots/\text{km}$
Write the answer.	The charge rate is approximately $1.59/km.

SC 2 I can interpret and analyse travel graphs

A travel graph represents a journey. The horizontal axis usually represents time, and the vertical axis represents the distance from a designated place, sometimes referred to as home.

A travel graph is often made up of several straight-line segments.

- A positive slope usually means that the object is moving away from home.
- A horizontal line (zero gradient) usually means the object is stationary.
- A negative slope usually means the object is moving back towards home.

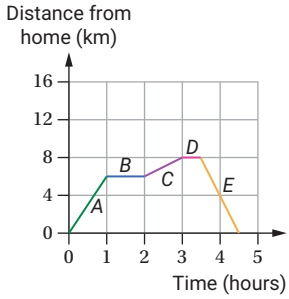
Whenever the travel graph meets the horizontal axis, it means the object is at home.

The speed at which the person or object is travelling can be calculated using the formula $\text{speed} = \frac{\text{distance}}{\text{time}}$.

Worked example

Reading a travel graph

Consider the following travel graph that shows Lee's journey, starting from home.



(a) Determine the amount of time Lee travelled before resting for the first time.

THINKING	WORKING
Recall the meaning of the slope of the line.	A period of resting is shown by a horizontal line on a travel graph; the time increases but the distance from home does not change.
Locate the section of the graph.	The sections where Lee is resting are B and D.
Answer the question.	Section B shows Lee resting for the first time, Lee travelled for 1 hour before resting.

(b) Determine the total time Lee spent resting.

THINKING	WORKING
Identify the sections.	The sections where Lee is resting are B and D.
Determine the time spent in each section of the graph.	Section B: 1 hour Section D: $\frac{1}{2}$ hour
Write the answer.	Lee spent $1\frac{1}{2}$ hours resting.

(c) Determine the period of time during which Lee travelled the slowest.

THINKING	WORKING
Recall the meaning of the slope of the line.	Slow periods are shown by flatter slopes or gradients; over a period of time, less distance is travelled.
Identify the section.	Section C is the flattest.
Write the answer.	Lee travelled the slowest in Section C, from 2 hours to 3 hours, or during the third hour.

(d) Calculate the speed at which Lee returned home.

THINKING	WORKING
Determine the distance travelled and time taken.	Section E shows the return home. Distance travelled: 8 km Time taken: 1 hour
Recall the formula for speed.	$\text{speed} = \frac{\text{distance}}{\text{time}}$
Substitute the values to calculate speed.	$\text{speed} = \frac{8\text{ km}}{1\text{ h}}$ $= 8\text{ km/h}$
Write the answer.	Lee returned home at a speed of 8 km/h.

(e) Determine the distance Lee travelled in total.

THINKING	WORKING
Describe the travel away from home.	Lee started at home (0 km) and travelled a total of 8 km from home. Lee goes 8 km away from home and he gets back home again, so he travelled a total of 16 km.
Describe the travel towards home.	On the return journey, Lee started 8 km from home and travelled all the way home.
Write the answer.	Lee travelled a total of 8 + 8 = 16 km.

SC 3 I can interpret and analyse time graphs

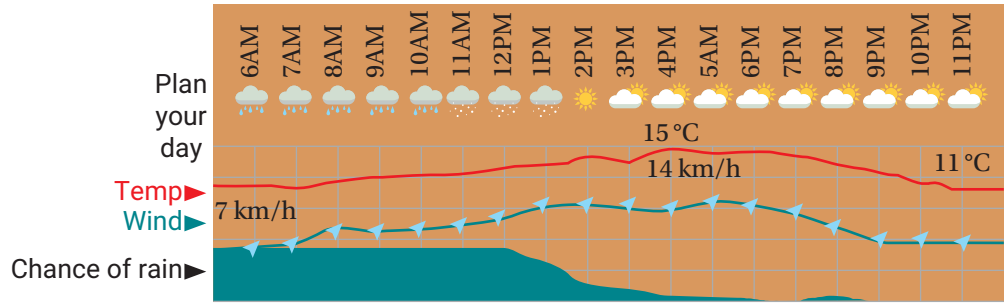
Weather graphs show variations in temperature, rainfall or other variables over the course of a day, a week, a month or a year. They are useful for making predictions.

Line graphs generally are used to draw comparisons and identify trends.

Worked example

Analysing time graphs

The following graph shows some measures of daily weather varying over the course of most of a day.



Describe what the graph shows.

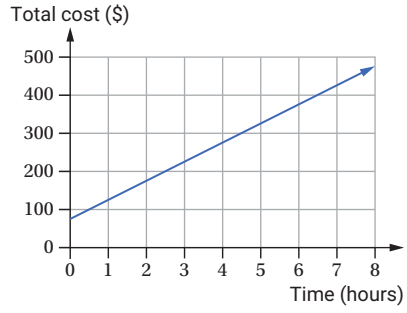
THINKING	WORKING
Identify the data shown on the graph.	The graph shows the temperature, wind speed and chance of rain.
Determine the minimum and maximum points.	The minimum temperature is 11 °C at 11 pm. The maximum temperature is expected to be 15 °C at 4 pm. The minimum wind speed is 7 km/h at 6 am, and the maximum wind speed is 14 km/h at 5 pm. The chance of rain is highest until 12 pm, then the chance of rain decreases.

Practice

ANSWERS Page 693

SC 1 I can use graphs to interpret simple rates

1 An electrician charges a fixed callout fee plus a set hourly rate for the time worked. The time is rounded up to the nearest number of whole hours.



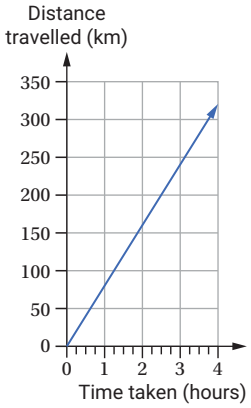
(a) Determine the fixed callout fee.

(b) Use the graph to determine the cost for a 7-hour job.

(c) Determine the hours charged for, given that the total cost is $375.

(d) Determine the hourly rate of the electrician.

2 The graph below shows the distance travelled by a car over a period of time.



(a) Determine the time period represented by the gap between each of the small marks on the horizontal axis.

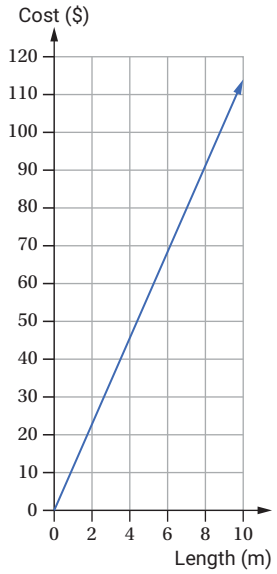
(b) Use the graph to determine the distance the car travelled in 3 hours.

(c) Determine the distance the car travelled in 1 hour and 45 minutes.

(d) Calculate the speed at which the car was travelling.

3 In some linear graphs involving rate, all of the points along the line are valid within a certain range of values. For others, individual points are valid, but not the points in between. Determine which of the graphs in the previous two questions are only valid at certain points. Explain your choice.

4 The graph at right shows the cost for fabric that can be purchased by the metre. Lengths of over a metre may be purchased in increments of 0.1 m.



(a) Determine the cost for each of the following lengths of material, to the nearest dollar.

(i) 2 m **(ii)** 9 m

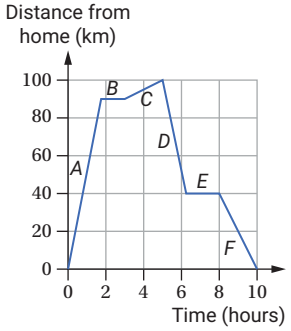
(b) For the following amounts of money, determine the length of material purchased to the nearest 0.1 m.

(i) $75 **(ii)** $30

(c) Determine the cost of the fabric per metre.

SC 2 I can interpret and analyse travel graphs

1 Consider the following travel graph that shows Dylan's journey.



(a) Determine the amount of time Dylan travelled before stopping for the first time.

(b) Determine the total time Dylan spent stationary.

(c) Determine the period of time during which Dylan travelled the slowest.

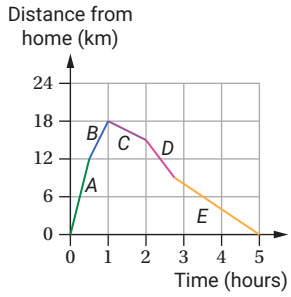
(d) Calculate the speed at which Dylan:

(i) first headed for home

(ii) travelled the last leg to arrive home.

(e) Determine the distance Dylan travelled in total.

2 Consider the following travel graph, which shows Minjee's journey.



(a) Determine the time taken for Minjee to complete her journey.

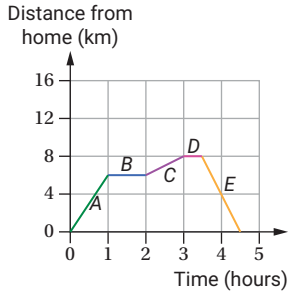
(b) Determine the period of time during which Minjee was travelling the fastest.

(c) Calculate the distance Minjee travelled in total.

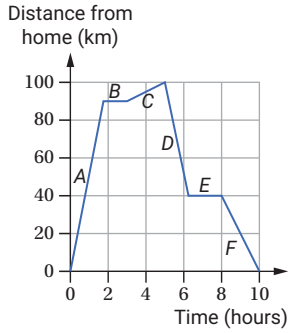
(d) Explain how you know that Minjee did not rest during her journey.

3 Calculate Minjee's speed for each section of her journey in the previous question.

4 Write a story to explain Lee's journey shown in the graph at right. Choose a mode of transport that suits the speed indicated. Make sure your story involves the direction and relative speeds and duration of time for each section.

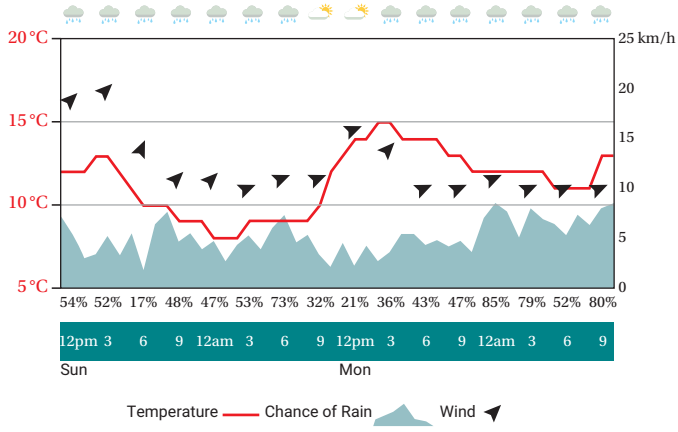


5 Write a story to explain Dylan's journey shown in the graph at right. Choose a mode of transport that suits the speed indicated. Make sure your story involves the direction and relative speeds and duration of time for each section.



SC 3 I can interpret and analyse time graphs

1 The following graph shows some measures of daily weather, varying over the course of 48 hours.



(a) Determine the expected highest temperature over the 48 hours and state the time and day it occurred.

(b) Determine the expected maximum wind speed and state the time and day it occurred.

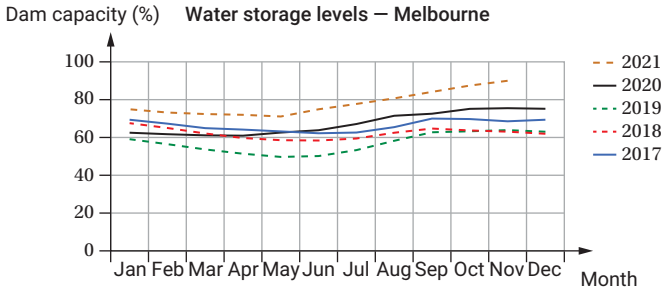
(c) Explain the scale for the temperature graph.

(d) Explain the scale for the wind speed graph.

(e) In your own words, describe the expected chance of rain at 6 pm on Sunday evening.

(f) Write two different times on Sunday when it could be said that there is an even chance of rain.

2 The following graph shows the average fill percentage for the dams servicing the Melbourne metropolitan area for the years from 2017 to 2021.



Based on data from Melbourne Water

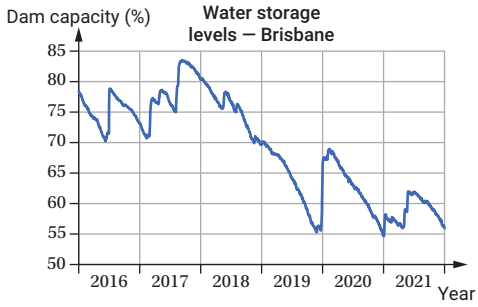
(a) Determine the year in which the dams contained the most water.

(b) Determine the year in which the dams contained the least water.

(c) Determine the season of the year when the water is at its lowest levels.

(d) Determine which year saw the biggest increase in levels between the start of the year and the end of the year.

3 The following graph shows the average fill percentage for the dams in South-East Queensland.



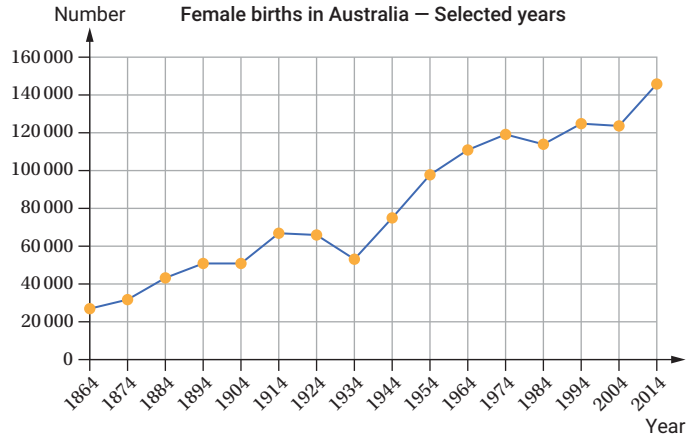
Based on data from Seqwater

(a) Describe the overall trend in the fill levels of the dams over the range of years shown.

(b) Determine which year had drought conditions in South-East Queensland.

(c) Describe which part of the year is the wettest in South-East Queensland.

4 The following graph shows the number of female births recorded in Australia at ten-year intervals.



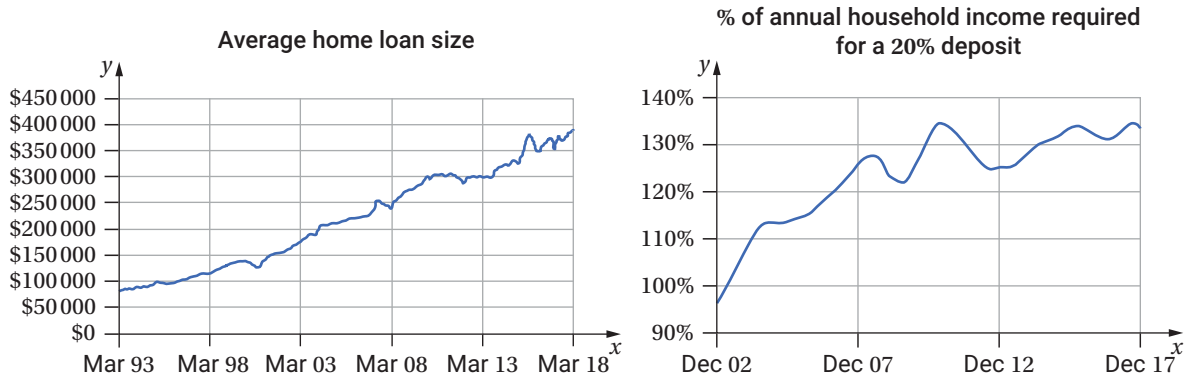
Based on data from Australian Bureau of Statistics © Commonwealth of Australia, ABS cat. no. 3105.0.65.001 Australian Historical Population Statistics, 2019. Licensed under CC BY 4.0, https://creativecommons.org/licenss/by/4.0

(a) Describe the general trend in the graph, including the main exception to the trend.

(b) State the ten-year period in which the number of female births had its greatest decrease. Suggest a possible reason for this.

(c) State the ten-year period in which the number of female births had its greatest increase. Explain how the graph helps to answer this question.

5 The following graphs are related. One shows the average size of home loans in Australia during 1993–2018 and the second shows the percentage of the average household income that was needed for a 20% deposit during 2002–2017.



Based on data from Corelogic, ANU and Australian Bureau of Statistics © Commonwealth of Australia, ABS cat. no. 3105.0.65.001 Australian Historical Population Statistics, 2019.

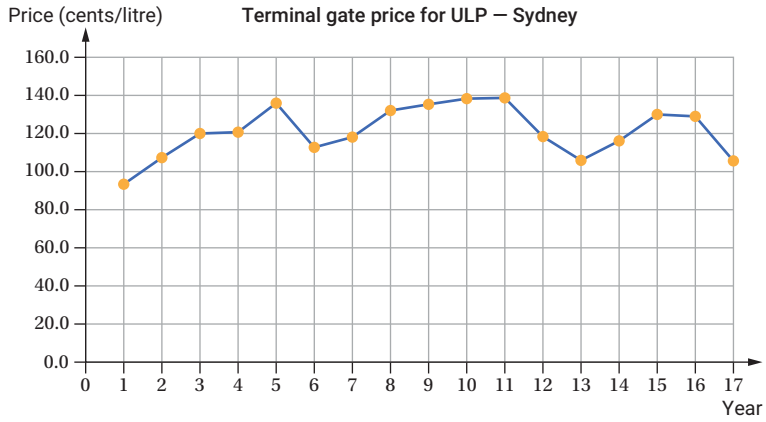
(a) Describe the change in the size of the average home loan.

(b) Describe an important piece of information that would have to be known in order to be able to comment on the size of the increased prices.

(c) Describe how the second graph justifies the conclusion that it has become more difficult to afford a home.

(d) What does the second graph tell us about the size of the average home loan?

6 The following graph shows the average yearly price (cents/litre) of unleaded petrol (ULP) at the terminal gate in Sydney.



Based on data from Australian Institute of Petroleum

(a) Describe what you see happening to this price over the years as shown in the graph.

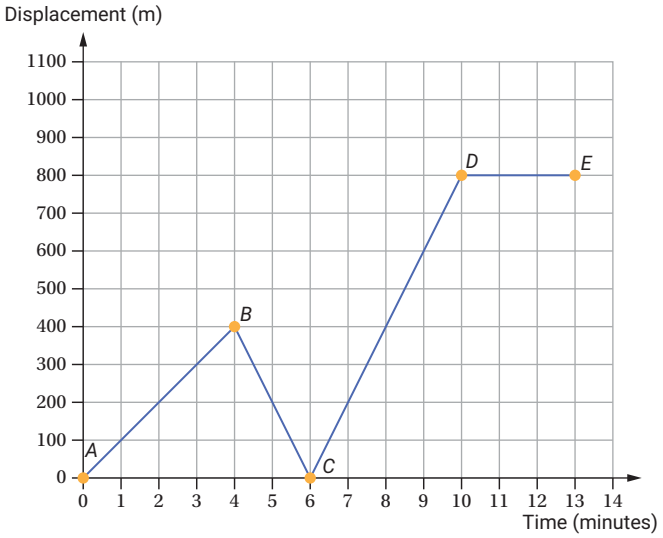
(b) What do you predict will happen to this price over the next few years?

TOPIC REVIEW

Multiple choice

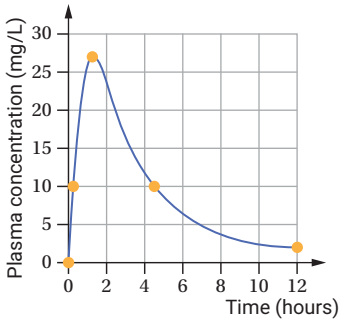
ANSWERS Page 744

1 Which segment of the travel graph demonstrates that an object is stationary?



A A to B **B** B to C **C** C to D **D** D to E

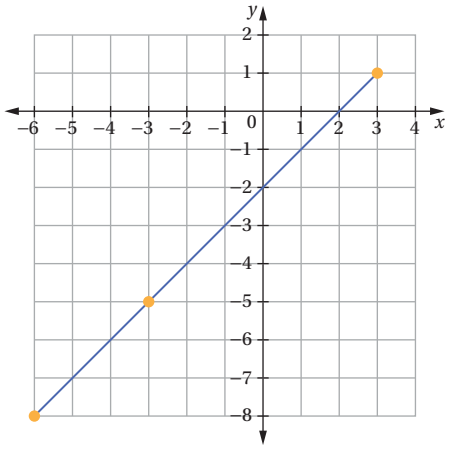
2 The concentration of a drug in our system after taking a dose is shown in this graph.



To be effective, there must be at least 10 mg/L in the body. For approximately how long is this drug effective?

A 15 minutes **B** 1 hour **C** 4 hours 15 minutes **D** 4 hours 30 minutes

3 Which table does not match the graph?



A

x	−8	−5	−2	1
y	−10	−7	−4	−1

B

x	−6	−3	0	1
y	−8	−5	−2	−1

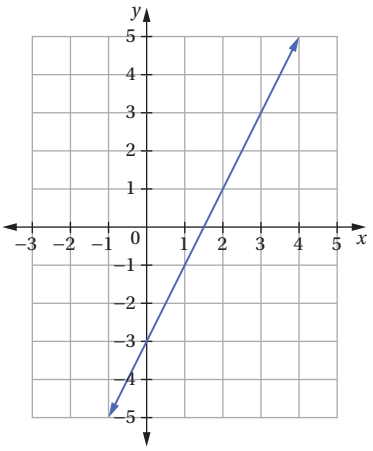
C

x	1	2	5	8
y	2	4	3	6

D

x	−4	0	3	7
y	−6	−2	1	5

4 What are the coordinates of the x-intercept and the y-intercept of this graph?



A (0, 1) and (0, 3)

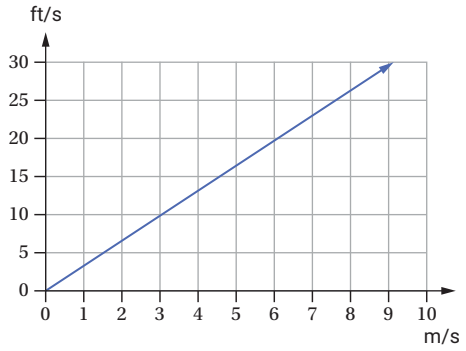
B (1, 1) and (0, 3)

C (1.5, 0) and (0, 3)

D (1.5, 0) and (0, −3)

5 The graph at right can be used to convert between metres/second (m/s) and feet/second (ft/s).

4 m/s is closest to:



A 1.25 ft/s

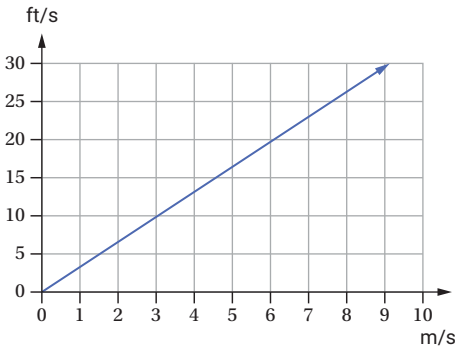
B 10.3 ft/s

C 13 ft/s

D 20 ft/s

6 The graph at right can be used to convert between metres/second (m/s) and feet/second (ft/s).

20 ft/s is closest to:



A 5.9 m/s

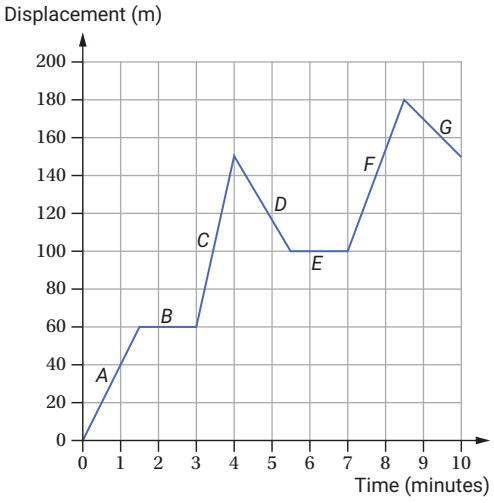
B 6.1 m/s

C 60 m/s

D 66 m/s

Short answer

1 Consider the following travel graph.



(a) For which parts of the graph is the object moving away from the origin?

(b) For which parts of the graph is the object moving towards the origin?

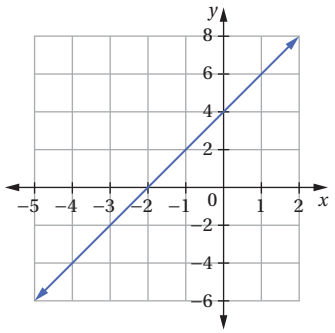
(c) For which parts of the graph is the object stationary?

(d) Where is the object after 10 minutes?

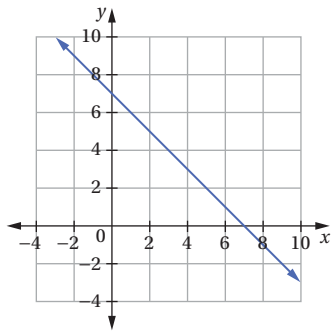
(e) What unit would be used to measure the speed of the object?

2 Write an equation for each of the following graphs.

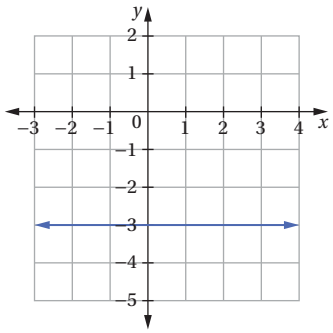
(a)



(b)

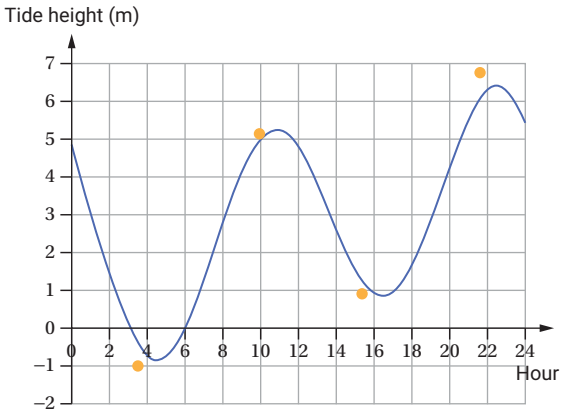


(c)



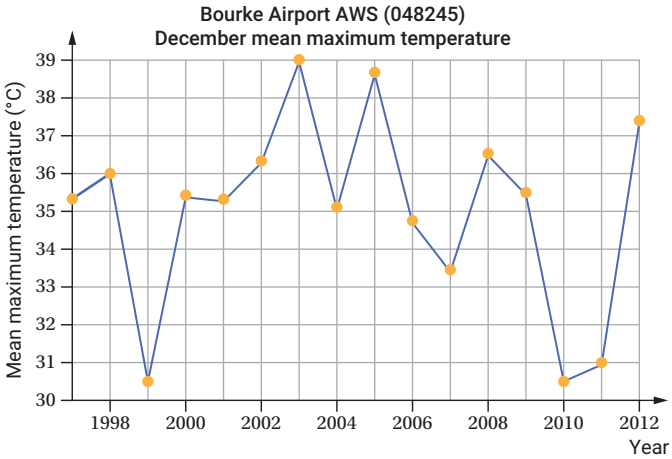
3 The water level at the head of a river varies due to the tide in the bay that the river empties into. This graph shows the expected tide height for the next day starting at midnight.

For a safe journey up the river, the water height must be at least 2 m for the entire 30 minute journey. During what times would it be safe to start a journey up the river?



4 Determine the y-intercept and gradient for the line $y = \frac{1}{5} - \frac{2x}{5}$.

5 The following graph shows the mean (average) daily maximum temperature for the month of December in Bourke.



Based on data from the Bureau of Meteorology

(a) In which years was the average daily maximum temperature less than 32 °C?

(b) What time period is shown on the graph?

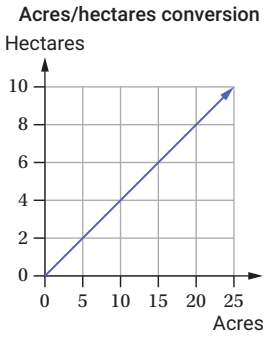
6 The graph at right can be used to convert acres and hectares.

Use the conversion graph to convert:

(a) 10 acres into hectares

(b) 7 hectares into acres

(c) 30 acres into hectares.



7 For each of the following rules for linear relationships, state whether the gradient of the line will be positive, zero, negative or undefined.

(a) $y = \frac{3(x-4)}{2}$ **(b)** $y = 6 - \frac{3x}{5}$ **(c)** $y = 0.25$

8 The following graph represents the charge for an electrician to visit your house to complete some work.

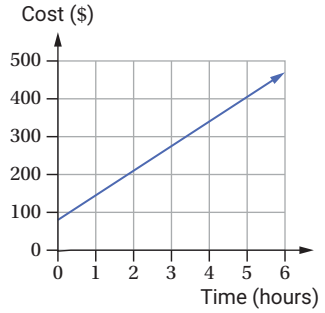
(a) How much is the callout fee?

(b) How much is it for a job that takes 3 hours?

(c) How much is it for a job that takes 5 hours?

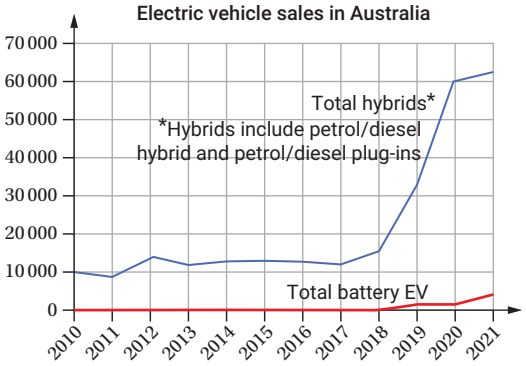
(d) How many hours did it take to complete a job that cost $340?

(e) Write a rule that represents the charges made by the electrician.

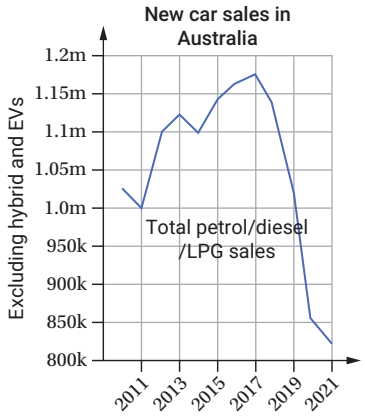


Extended response

1 The two graphs below show some information about the sales of electric vehicles and other vehicles in Australia.



Based on data from VFACTS by the Federal Chamber of Automotive Industries



Based on data from VFACTS by the Federal Chamber of Automotive Industries

(a) Which type of vehicle has the least number of sales?

(b) Which type of vehicle shows the biggest decrease in sales over the time period shown?

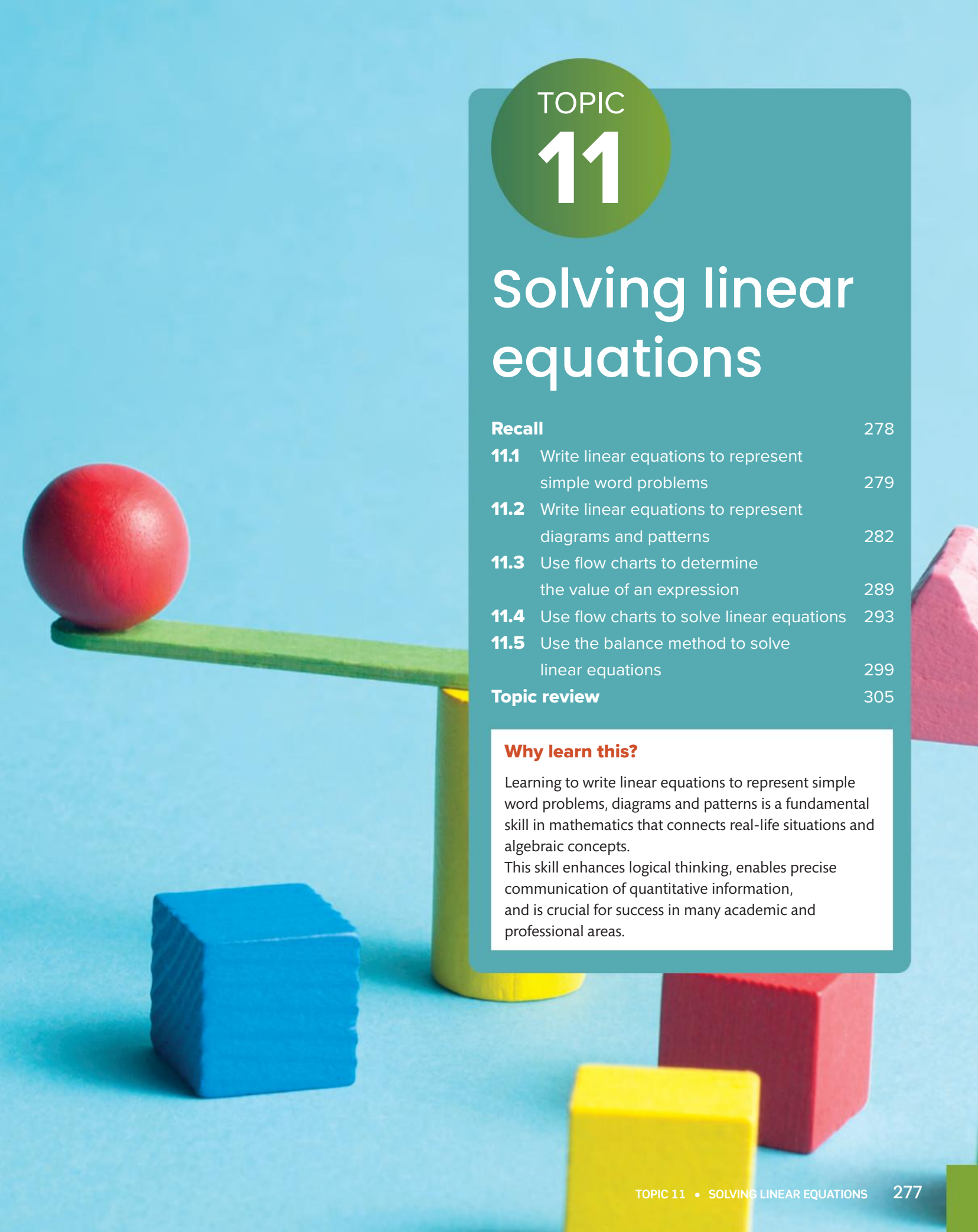
(c) Which type of vehicle shows the greatest increase in sales over the time period shown?

(d) What about the vertical axis is different in the second graph from the first graph?

(e) About how many vehicles in total were sold in 2011?

(f) About how many vehicles in total were sold in 2020?

(g) What do you think will be the future sales structure for vehicles in Australia?



TOPIC
11

Solving linear equations

Recall 278

11.1 Write linear equations to represent simple word problems 279

11.2 Write linear equations to represent diagrams and patterns 282

11.3 Use flow charts to determine the value of an expression 289

11.4 Use flow charts to solve linear equations 293

11.5 Use the balance method to solve linear equations 299

Topic review 305

Why learn this?

Learning to write linear equations to represent simple word problems, diagrams and patterns is a fundamental skill in mathematics that connects real-life situations and algebraic concepts.
This skill enhances logical thinking, enables precise communication of quantitative information, and is crucial for success in many academic and professional areas.

RECALL

ANSWERS Page 694

I can rewrite word sentences as algebraic sentences

1 Copy and complete each of the following sentences.

(a) $3x + 2y$ is an _____ with ___ terms.

(b) $x + 3 = 8$ is an _____ and the value of x is ___, because ___ + 3 = 8

(c) $7a = 42$ is an _____ and the value of a is ___, because 7 × ___ = 42

2 Rewrite each of the following sentences using mathematical symbols.

(a) Five is subtracted from 100 and the result is ninety-five. 100 − ___ = ___

(b) Eight plus y is equal to nine. ___ + y = ___

(c) y is equal to x plus nineteen. ___ + ___ = ___

(d) Four times n is equal to sixty. ___ n = ___

I can apply a series of operations to a given number

1 Calculate each of the following.

(a) Start with the number 5. Add 1, then add another 7 to the result.

(b) Start with the number 12. Multiply this by 2 and then subtract 4 from the result.

2 Calculate the final result after the following operations are performed each time on the number 20.

(a) × 2 then + 5

(b) + 13 then ÷ 11

(c) − 18 then ÷ 2

(d) + 20 then − 5, and then × 2

(e) ÷ 4 then + 17, and then ÷ 11

I can substitute numbers into algebraic expressions

1 Determine the value of the expression $2a + 5$ for the following values of a.

(a) 0

(b) 1

(c) 5

(d) 10

(e) −10

(f) 25

2 Complete the table of values.

x	−3	−2	−1	0	1	2	3
$4x - 1$							

3 Determine the value of the following expressions when $p = -10$ is substituted.

(a) $p - 5$

(b) $5 - p$

(c) $5p$

(d) $-5p$

(e) $\frac{p}{5}$

(f) $\frac{5}{p}$

Write linear equations to represent simple word problems

11.1

Learning intention: To be able to write linear equations to represent simple word problems

Success criteria:

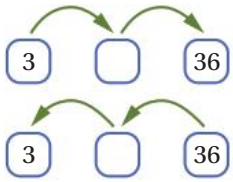
SC 1 I understand the connection between number sentences and word problems.

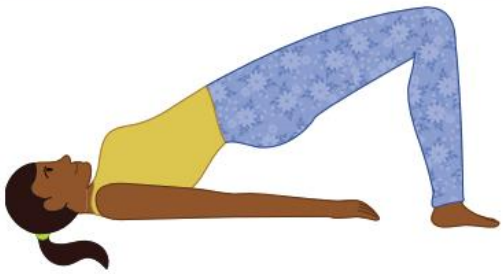
SC 2 I understand the connection between linear equations and word problems.

Lesson warm-up

Back and forth

How can you get from 3 to 36 in two steps? And how can you get back from 36 to 3 in two steps?





SC 1 I understand the connection between number sentences and word problems

Worked example

Writing a number sentence using words

Write 8 – 3 = 5 as a sentence using words.

THINKING	WORKING
Identify the operations or symbols used.	The operation – is subtraction. The symbol = means 'equals'.
List words that can be used to describe the operations and symbols.	subtract, difference, less than, decrease equal, the same as
Write the sentence using words.	Examples: Subtracting 3 from 8 is the same as 5. 3 less than 8 is equal to 5. The difference between 8 and 3 is 5.

SC 2 I understand the connection between linear equations and word problems

Sometimes a number sentence includes an unknown value or variable. This can be represented by a pronumeral such as x.

Worked example

Writing an equation with an unknown in words

Write the equation $x - 3 = 8$ in words.

THINKING	WORKING
Identify the operations in the equation.	The operation $-$ is subtraction. The symbol $=$ means 'equals'.
Identify the unknown value or variable.	The unknown value or variable is x.
List words that can be used to describe the operations and symbols.	subtract, difference, take away, less than, decrease equal, the same as
Write the equation using words.	Examples: Subtracting 3 from an unknown number x is the same as 8. 3 less than an unknown number x is equal to 8. The difference between an unknown number x and 3 is 8.

Practice

ANSWERS Page 694

SC 1 I understand the connection between number sentences and word problems

1 Write each number sentence using words.

(a) $12 + 3 = 15$ **(b)** $15 - 3 = 12$ **(c)** $3 \times 5 = 15$ **(d)** $15 \div 3 = 5$

2 Brainstorm words or phrases that can describe each operation or symbol.

+	−	×	÷	=
add sum plus more than increase				

3 Explain what is the same and what is different about the expressions $3 \times 4 + 2$ and $3 \times (4 + 2)$. Are the expressions equal?

4 Write the following using only numerals and mathematical symbols.

(a) Multiply the number 3 by 2 to give the answer 6.

(b) Multiply 3 by 2 and then add 4 to give a result of 10.

(c) Calculate the sum of 4 and 3, then multiply the result by 2 to give 14.

SC 2 I understand the connection between linear equations and word problems

1 Write each equation using words.

(a) $x + 3 = 12$ **(b)** $x - 3 = 12$ **(c)** $3x = 12$ **(d)** $\frac{x}{3} = 12$

2 The equations $3x + 2 = 12$ and $3(x + 2) = 12$ have the same numbers and operations. Will the value of x be the same in both equations? Explain.

3 Decide whether each pair of expressions are always equal, never equal or sometimes equal. Justify your answer with calculations.

(a) $m + 3$ and $3 + m$ **(b)** $t - 3$ and $3 - t$ **(c)** $3 \times p$ and $p \times 3$ **(d)** $d \div 3$ and $3 \div d$

4 Write the following using only numerals and mathematical symbols.

(a) Multiply an unknown number n by 2 to give the answer 11.

(b) Add 7 to a number to give the answer 11.

(c) Multiply a number by 2, then add 7 to give the answer 11.

(d) Add 7 to a number, then multiply the result by 2 to give the answer 11.

5 Paulo is 8 years old.

(a) Write a number sentence to show Jane's age J, given that Jane is the following:

(i) 3 years younger than Paulo

(ii) 1 year older than Paulo

(iii) three times Paulo's age

(iv) half of Paulo's age.

(b) Assume that Paulo is 12 years old. How do the equations in part **(a)** change?

(c) Assume that Paulo's age is unknown and write it as P. How do the equations in part **(a)** change?

6 Sally is 42 years older than her daughter Kyan and 2 years younger than her partner Alex. Their three ages together add to 116 years.

(a) Define s as Sally's current age. Write an expression in terms of s for Kyan's age (k) and an expression for Alex's age (a).

(b) Write an equation in terms of s for the sum of their ages (do not simplify the equation).

7 Which expression in each pair represents the larger number? Is one expression always larger than the other, or only sometimes larger? Justify your answer for each pair of expressions with calculations.

(a) $x + 4$ and $x - 3$

(b) $2p$ and $3p$

(c) $-2y$ and $2y$

(d) $5m + 2$ and $5m + 1$

11.2 Write linear equations to represent diagrams and patterns

Learning intention: To be able to write linear equations to represent diagrams and patterns

Success criteria:

SC 1 I can formulate information in a diagram as a linear equation.

SC 2 I can formulate information in a pattern as a linear equation.

Lesson warm-up

Make that pattern

A Sierpinski triangle is a fractal image. It is made in stages by repeatedly removing the 'middle' triangle from each solid triangle in the image.

Look carefully to see how the pattern changes at each stage.

Stage 0 Stage 1 Stage 2 Stage 3

? ?

Stage 4 Stage 5

How many solid triangles are there in Stage 3, Stage 4 and Stage 5?

Complete the table.

Stage	Triangles
0	1
1	3
2	9
3	
4	
5	

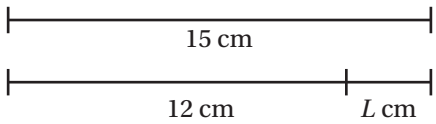
SC 1 I can formulate information in a diagram as a linear equation

When finding the perimeter from a diagram, make sure you include all the side lengths, not just the side lengths written on the diagram.

Worked example

Writing an equation from a length diagram

Write an equation to find the length L shown in the following diagram.

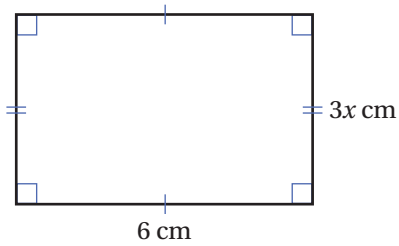


THINKING	WORKING
Determine how the unknown length is related to the known lengths.	The unknown length and the 12 cm length together are the same as the 15 cm length.
Write an equation that shows this relation.	$L + 12 = 15$
Solve the equation to find the unknown length.	$L = 15 - 12$ $= 3$ So the length of L is 3 cm.

Worked example

Writing an equation for perimeter and area

Write equations for the perimeter and area of the rectangle shown.



(a) Perimeter of the rectangle

THINKING	WORKING
The perimeter P is the sum of all the side lengths.	$P = 6 + 6 + 3x + 3x$
Simplify the expression and state the answer.	$P = 12 + 6x$ The perimeter of the rectangle is given by $P = 12 + 6x$ cm.

(b) Area of the rectangle

THINKING	WORKING
The area A is the amount of space inside the shape. Recall the formula. The area of a rectangle is calculated by multiplying the length and the width.	$A = L \times W$
Define the variables.	Define length $L = 6$ and the width $W = 3x$.
Substitute the given lengths into the formula and simplify the expression.	$A = 6 \times 3x$ $= 18x$
Write the answer. Remember to include the correct units for area.	The area of the rectangle is given by $A = 18x$ cm^2.

SC 2 I can formulate information in a pattern as a linear equation

Worked example

Writing an equation to represent a pattern

Write a rule for this pattern that connects the number of matchsticks n to the number of triangles t.

Number of triangles t	1	2	3	4	5
Number of matchsticks n	3	5	7	9	11

THINKING	WORKING
Describe the relationship between the number of matchsticks and the number of triangles.	Each triangle is made by adding 2 more matchsticks to the pattern. The pattern begins with 1 extra matchstick to start the first triangle. So, there are 2 matchsticks for every triangle, plus 1 extra matchstick.
Write the relationship as a rule, connecting the number of matchsticks to the number of triangles.	The number of matchsticks is 2 times the number of triangles, plus 1.
Write this rule as an equation.	$n = 2t + 1$

Substitute values to check that the equation works.	For 1 triangle: $n = 2 \times 1 + 1$ $= 3$ 3 matchsticks are needed. For 2 triangles: $n = 2 \times 2 + 1$ $= 5$ 5 matchsticks are needed. For 5 triangles: $n = 2 \times 5 + 1$ $= 11$ 11 matchsticks are needed.

Worked example

Identifying a more complex pattern from a table of values

Determine the equation that connects the two variables in the table of values.

a	1	2	3	4	5
b	1	4	7	10	13

THINKING	WORKING
Describe how one variable changes as the other variable changes.	At each step, the value of a increases by 1 while b increases by 3.
Write an equation that shows how the changes in each variable are related. This will be the start of the equation that connects the two variables.	At each step: (change in b) = 3 × (change in a) So: $b = 3a + ?$
Determine the values for the expression and find the difference between this expression and the other variable.	Values for $3a$: a: 1, 2, 3, 4, 5 $3a$: 3, 6, 9, 12, 15 b: 1, 4, 7, 10, 13 At each step, the value of b is 2 less than $3a$.
Write the full equation that connects the two variables.	$b = 3a - 2$

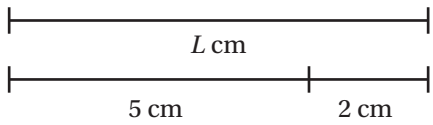
Practice

ANSWERS Page 695

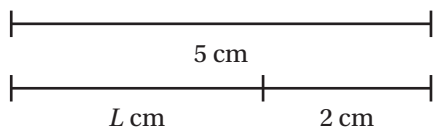
SC 1 I can formulate information in a diagram as a linear equation

1 Write an equation to find the unknown length L shown in each diagram.

(a)



(b)



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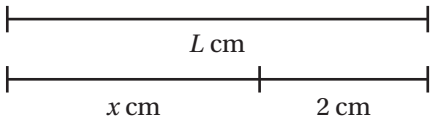
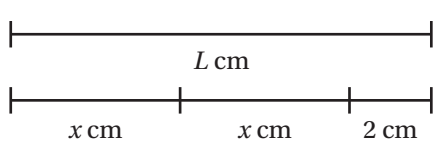


(d)



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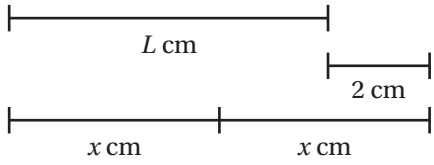
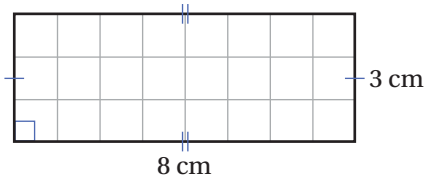
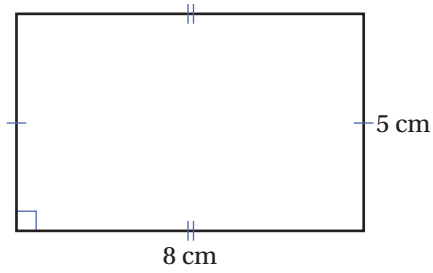


2 Write equations for the perimeter P and area A of each rectangle.

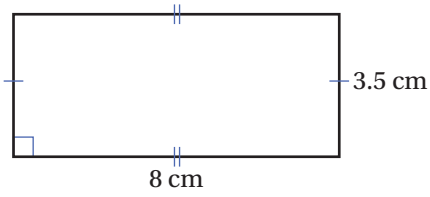
(a)



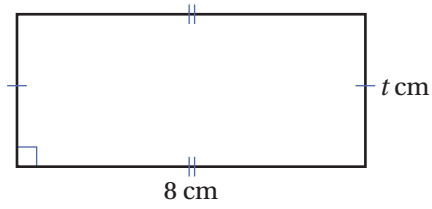
(b)



(c)



(d)



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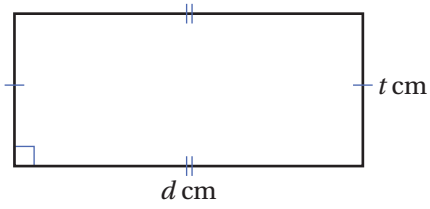

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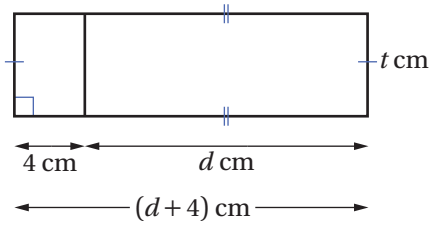
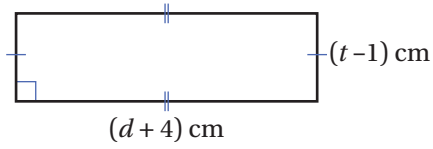
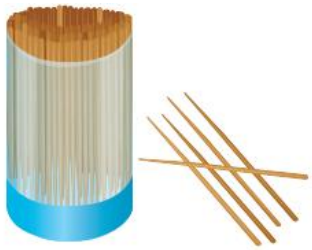


(g)



3 There are a number of toothpicks in a jar and also 5 extra toothpicks.



(a) Write a number sentence for the total number of toothpicks, given that there are 50 toothpicks in the jar.

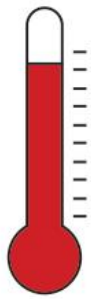
(b) Let k be the number of toothpicks in the jar. Write a sentence to describe the situation for each equation:

(i) $k - 2 = 64$ **(ii)** $2k = 64$

(iii) $k \div 2 = 64$ **(iv)** $k + 2 = 64$

4 Complete the table by writing an equation for each situation and starting temperature, using F to represent the final temperature.

Starting temperature	6 °C	−6 °C	p °C
The starting temperature value is doubled.			
The starting temperature falls by 15 °C.			
The final temperature is two thirds of the starting temperature value.			
The final temperature is 5 °C higher than half of the starting temperature value.			



5 A rectangular pool is 38 m longer than it is wide. The perimeter is 124 m. Write an equation for the perimeter in terms of the pool length L, in metres.

SC 2 I can formulate information in a pattern as a linear equation

1 The table of values shows the relationship between the triangles and matchsticks.

a	1	2	3	4	5
b	3	6	9	12	15

(a) From the diagram, what do the variables a and b represent?

(b) Describe the relationship between the number of triangles and the number of matchsticks.

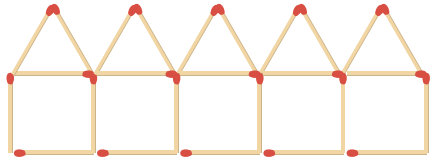
(c) Write a rule for this pattern using the variables a and b.

2 Write the rule that connects the variables for each table of values.

(a)

Number of squares s	1	2	3	4	5
Number of matchsticks m	4	8	12	16	20

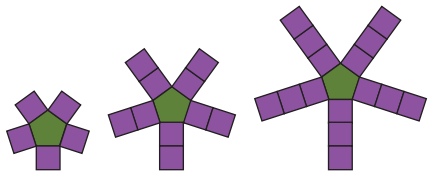
(b)



Number of house shapes h	1	2	3	4	5
Number of matchsticks m	6	11	16	21	26

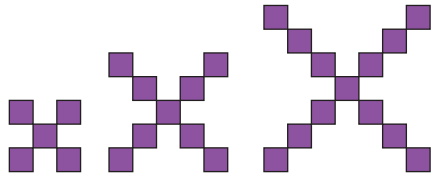
3 For each pattern, write an equation that connects the term number t to the number of squares s.

(a)



Term number t	1	2	3
Number of squares s	5	10	15

(b)



Term number t	1	2	3
Number of squares s	5	9	13

4 Determine the equation that connects the two variables in each table of values.

(a)

m	1	2	3	4	5
p	5	6	7	8	9

(b)

t	1	2	3	4	5
q	−5	−10	−15	−20	−25

(c)

d	1	2	3	4	5
g	0.5	1	1.5	2	2.5

(d)

a	1	2	3	4	5
b	4	9	14	19	24

(e)

q	1	2	3	4	5
p	5	3	1	−1	−3

(f)

x	1	2	3	4	5
y	1.25	1.5	1.75	2	2.25

5 Determine the equation that connects the two variables in each table of values.

(a)

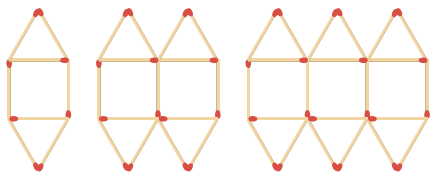
a	1	3	6	7
b	7	15	27	31

(b)

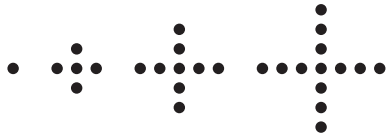
a	1	4	6	10
b	−4	−13	−19	−31

6 Write an equation that connects the step number s with the number of matchsticks or dots n.

(a) Steps 1, 2, 3:



(b) Steps 1, 2, 3, 4:



Use flow charts to determine the value of an expression

11.3

Learning intention: To be able to use flow charts to determine the value of an expression

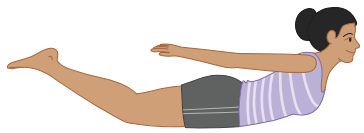
Success criteria:

SC 1 I can apply operations to a number and determine the result.

SC 2 I can apply operations to a variable and determine the result.

Lesson warm-up

I think of a number...



(a) Start with the input number 3, add 2 and then multiply the result by 4. What is the output number?

(b) Add another step to the process in part **(a)** so that the output number is 15. Can you think of more than one way to do this?

(c) Start with an unknown input number, apply the same operations in part **(a)** and end with an output number 48. What was the input number?

(d) Create your own 'Think of a number' problem.

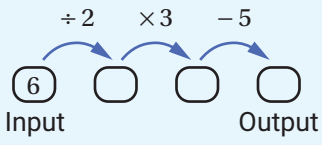
SC 1 I can apply operations to a number and determine the result

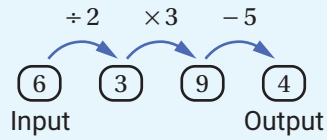
You can use a flow chart to determine the final value after a series of operations are performed on a number.

Worked example

Determine the value after a series of operations

Start with the number 6, divide it by 2, multiply it by 3 and then subtract 5. What is the output or final value?

THINKING	WORKING
Draw a flow chart. The number you start with is written in the first box. The operations that you will apply are written between the boxes.	

Start with the input number and then apply each operation in order to determine the output.	
Alternatively, you could calculate the result of each of the operations in order.	$6 \div 2 = 3$ $3 \times 3 = 9$ $9 - 5 = 4$
Answer the question.	Start with the number 6, divide it by 2, multiply it by 3, then subtract 5. The output is 4.

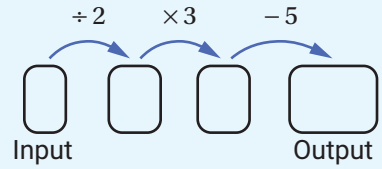
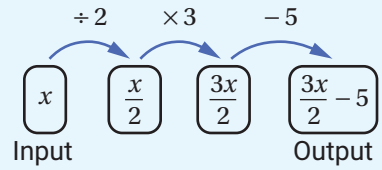
SC 2 I can apply operations to a variable and determine the result

You can use a flow chart to write an algebraic expression even when the initial number is unknown.

Worked example

Determine the expression after a series of operations

Start with an unknown number x, divide it by 2, multiply it by 3 and then subtract 5. What is the final output expression?

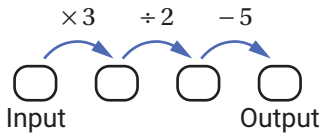
THINKING	WORKING
Create a flow chart using the operations in order.	
Enter the variable x in the first box and complete the operations from left to right to determine the output expression.	
Write the answer.	Start with the number x, divide it by 2, multiply it by 3 and then subtract 5. The output expression is $\frac{3x}{2}-5$.

Practice

ANSWERS Page 696

SC 1 I can apply operations to a number and determine the result

1 Complete the flow chart below for the following input numbers to determine the output number.



(a) 4 **(b)** −4 **(c)** 12 **(d)** $\frac{4}{3}$

2 Draw a flow chart to determine the output number when the following input numbers have this series of operations applied: multiply by 3, subtract 5 and then divide the result by 2.

(a) 4 **(b)** −4 **(c)** 12 **(d)** $\frac{4}{3}$

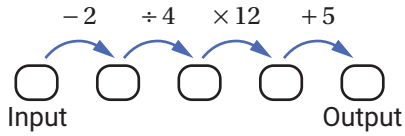
3 In Questions **1** and **2**, the same operations and input numbers were used in the flow chart. Explain why the outputs were different.

4 Place the operations × 3, ÷ 2 and − 5 across the flow chart so that the output produced would be:

(a) different from the flow charts in Questions **1** and **2**

(b) the same as the flow chart in Question **1**, but with the operations in a different order.

5 Consider the flow chart shown.



(a) Complete the story to describe the flow chart. 'I think of a number, subtract 2, ...'

(b) Complete the flow chart with an input of 10.

(c) Complete the flow chart with an input of 5.

(d) 'If the input is a positive integer (whole number), then the output is a positive integer.' Is this statement true or false? Explain.

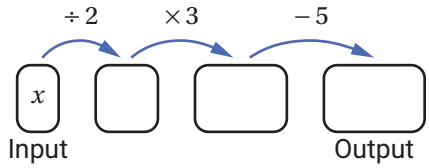
(e) See if you can work backwards to determine what the input number is, given that the output number is 53. Explain how you worked it out.

6 Create your own 'I think of a number' problem.

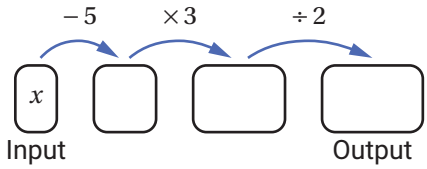
SC 2 I can apply operations to a variable and determine the result

1 Complete the following flow charts for an input of x.

(a)

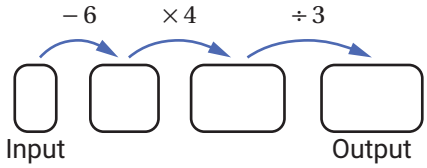


(b)



(c) What is similar and what is different about the output expressions in the flow charts in parts **(a)** and **(b)**?

2 Consider the flow chart shown.



(a) Write an 'I think of a number' story to describe the flow chart.

(b) Determine the output, given that the input is 0.

(c) Complete the flow chart for an input of t.

(d) How would the expression from part **(c)** change if the input was z?

3 Complete the following questions.

(a) Draw and complete flow charts to determine the output expression for each of the following.

(i) Start with an unknown number x, multiply it by 3, subtract 8 and then divide the result by 5.

(ii) Start with an unknown number x, subtract 8, multiply it by 3 and then divide the result by 5.

(iii) Start with an unknown number x, divide by 5, subtract 8 and then multiply the result by 3.

(b) Substitute a number for the input number x to demonstrate that the output expressions in part **(a)** are not equivalent.

4 Write a number story and draw a flow chart to represent each of the following expressions.

(a) $\frac{2p}{7} - 5$ **(b)** $\frac{2p-5}{7}$ **(c)** $\frac{2(p-5)}{7}$ **(d)** $5 - \frac{2p}{7}$

5 Write your own 'think of a number' problem. Make sure you can draw a flow chart and work out your output number.

Use flow charts to solve linear equations

11.4

Learning intention: To be able to use flow charts to solve linear equations

Success criteria:

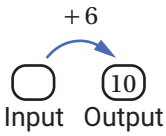
SC 1 I can solve simple equations by inspection.

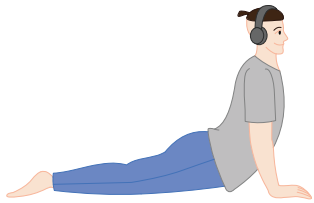
SC 2 I can identify inverse operations to backtrack a flow chart and solve an equation.

Lesson warm-up

I think of a number ...

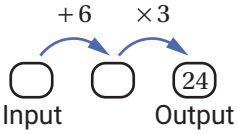
(a) Determine the input number for this flow chart.



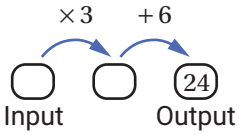


(b) Write two number sentences to justify your answer in part **(a)**.

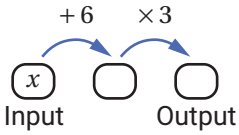
(c) Determine the input number for this flow chart.

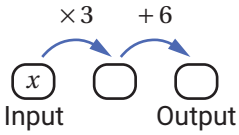


(d) Determine the input number for this flow chart.



(e) Use the following flow charts to write an algebraic expression for each of parts **(c)** and **(d)**. Explain why the expressions are different.





SC 1 I can solve simple equations by inspection

An equation shows two equivalent expressions. A solution is the number that makes the expressions equal.

As an example, $x + 2 = 3$ has the solution $x = 1$ because when $x = 1$, the expressions on both sides of the equation are equal: $1 + 2 = 3$.

When you can look at the equation and work out the solution in your head, this is known as solving by inspection.

Worked example

Solving an equation by inspection

Determine the solution to $\frac{x}{4} = 3$ and then check that it is correct.

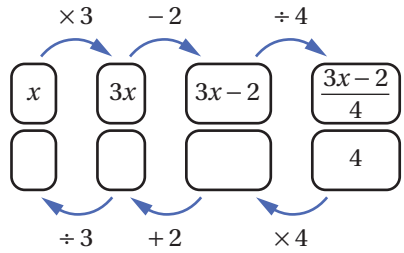
THINKING	WORKING
Write the equation in words.	$\frac{x}{4} = 3$ is the same as $\square \div 4 = 3$, except the unknown number is defined with the variable x. Divide a number by four and the answer is three.
Write the solution.	$x = 12$
Check the answer by substituting the value back into the equation. Check that the equation is true (the expressions on both sides of the equals symbol are the same).	$12 \div 4 = 3$ The expressions on both sides of the equals symbol are equal, so the equation is true when $x = 12$.

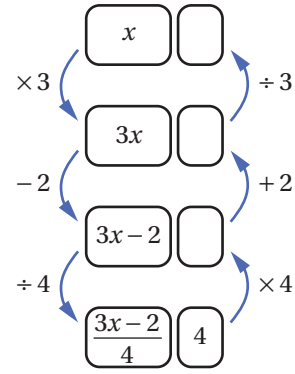
SC 2 I can identify inverse operations to backtrack a flow chart and solve an equation

The expression $\frac{3x-2}{4}$ can be written as an equation by making it equal to another expression; for example, 4. The equation becomes $\frac{3x-2}{4} = 4$.

A flow chart can be extended to show the result attached to the final step of the expression building process. Inverse operations are then used to work backwards and determine the solution (or the value of the unknown number that makes the equation true).

This can be shown as either a horizontal or vertical flow chart.





The inverse operation for + is −

The inverse operation for − is +

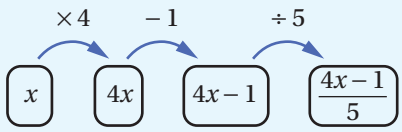
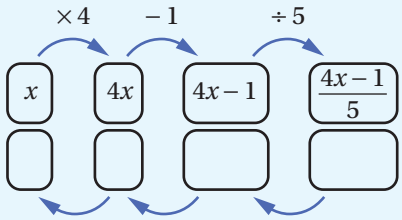
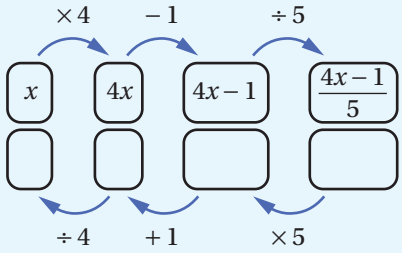
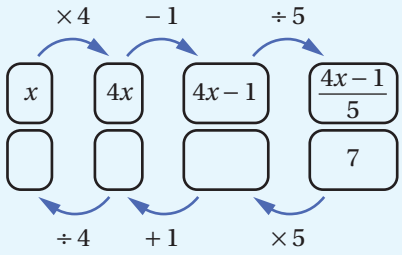
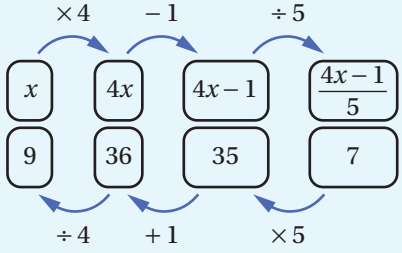
The inverse operation for × is ÷

The inverse operation for ÷ is ×

Worked example

Solving an equation by backtracking a flow chart

Solve $\frac{4x-1}{5}=7$.

THINKING	WORKING
Build the flow chart for the expression.	
Add boxes under each expression in the flow chart to add the equivalent expression. Draw arrows pointing in the opposite direction.	
Write the inverse operations on the arrows.	
Create the equation by aligning the equivalent expressions.	The equation given is $\frac{4x-1}{5}=7$. 
Complete the flow chart by applying the inverse operations to the value in the RHS box.	
Write the solution. The solution is the value underneath the initial variable.	$x=9$

Check that your solution is correct. Substitute the solution into the expression to check that the LHS is equal to the RHS.	$\text{LHS} = \frac{4x-1}{5}$ $= \frac{4\times 9-1}{5}$ $= \frac{36-1}{5}$ $= \frac{35}{5}$ $= 7$ $\text{LHS} = \text{RHS}$ The left-hand side expression is equal to the right-hand side expression when $x = 9$.

Practice

ANSWERS Page 698

SC 1 I can solve simple equations by inspection

1 Determine the missing number that makes each of the equations true. Remember to check that your solutions are correct by substituting them into the equation.

(a) $4 \times \square = 12$ **(b)** $\square + 4 = 12$ **(c)** $\square - 4 = 12$ **(d)** $4 - \square = 12$

(e) $\frac{\square}{4} = 12$ **(f)** $2 \times (\square + 4) = 12$ **(g)** $2 \times \square + 4 = 12$ **(h)** $\frac{2 \times \square + 4}{3} = 12$

2 Two students checked to see whether $x = 9$ is a solution to $3x - 1 = 26$. The following table shows each student's working. Which student is correct? What mistake was made by the student who did not do it correctly?

Student A	Student B
At $x = 9$ $3x - 1 = 39 - 1$ $= 38$ $x = 9$ is not a solution to $3x - 1 = 26$.	At $x = 9$ $3x - 1 = 3 \times 9 - 1$ $= 27 - 1$ $= 26$ $x = 9$ is the solution to $3x - 1 = 26$.

3 For each of the following equations, determine whether the solution is $x = 1$, $x = 2$ or $x = 3$.

(a) $x + 7 = 10$ **(b)** $3x + 7 = 13$ **(c)** $3(x + 7) = 24$ **(d)** $\frac{x}{3} + 7 = 8$

(e) $\frac{7x}{3} - 1 = 6$ **(f)** $\frac{7x+1}{3} = 5$ **(g)** $\frac{5}{x+3} - 7 = -6$

4 Complete the following questions.

(a) Copy and fill in the table of values to determine the values of the expression $2x + 4$.

x	1	2	3	4	5
$2x + 4$					
$3x - 1$					

(b) Which value of x makes the equation $2x + 4 = 6$ true?

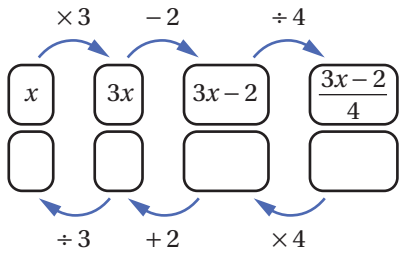
(c) Determine the solution to the equation $3x - 1 = 8$.

(d) Determine the solution to the equation $2x + 4 = 3x - 1$. Explain.

SC 2 I can identify inverse operations to backtrack a flow chart and solve an equation

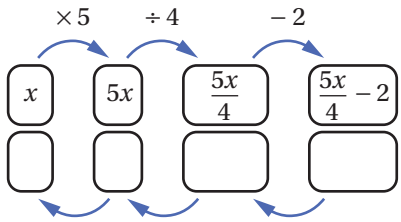
1 Copy and complete the flow chart to determine the solution to each of the following equations.

(a) $\frac{3x-2}{4} = 7$ **(b)** $\frac{3x-2}{4} = 10$



2 Complete the following questions.

(a) Copy and complete the appropriate sections of the flow chart to determine the solution to the following equations.



(i) $5x = 40$ **(ii)** $\frac{5x}{4} = 10$ **(iii)** $\frac{5x}{4} - 2 = 8$

(b) Explain what is similar and what is different about the three equations in part **(a)**.

3 Draw flow charts to determine the solution to the following equations.

(a) $x + 2 = 9$ **(b)** $3x + 2 = 9$ **(c)** $3(x + 2) = 9$ **(d)** $\frac{3(x+2)}{4} = 9$

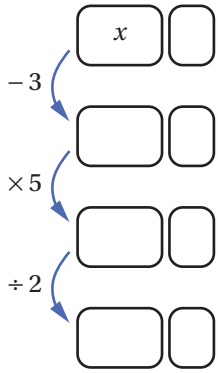
4 Complete the following questions.

(a) Copy and fill in the appropriate sections of the vertical flow chart to determine the solution to each of the following equations.

(i) $x - 3 = 4$

(ii) $5(x - 3) = 20$

(iii) $\frac{5(x-3)}{2} = 10$



(b) What can be said about the pairs of expressions in a flow chart?

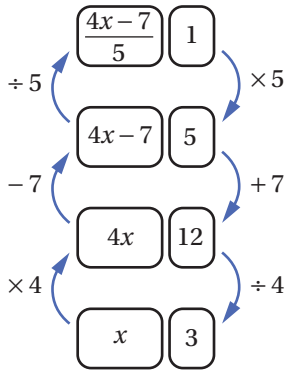
5 Complete the following questions.

(a) Create and backtrack a flow chart of your choice to determine the solution to the following equations.

(i) $\frac{x}{3} = 5$ **(ii)** $\frac{x}{3} + 1 = 5$ **(iii)** $-5\left(\frac{x}{3} + 1\right) = 5$ **(iv)** $10 - 5\left(\frac{x}{3} + 1\right) = 5$

(b) Describe the similarities and differences in each flow chart.

6 Equations can also be solved without drawing a flow chart. Start with the equation, in this case $\frac{4x-7}{5} = 1$, and then solve it in a similar way as a vertical flow chart as shown, but without drawing a flow chart. Describe the similarities and differences in the approaches shown.



$\frac{4x-7}{5} = 1$

$4x - 7 = 5$

$4x = 12$

$x = 3$

7 For each of the following, write an equation and then solve it using a method of your choice. Remember to define a variable for each equation.

(a) I think of a number, double it, add 6 and then divide by 3. The result is 4.

(b) I think of a number, divide it by 3, subtract 2 and then multiply by 4. The result is 4.

(c) I think of a number, multiply it by −1, add 6 and then divide by 4. The result is 1.

(d) I think of a number, divide it by −5, subtract 3 and then multiply by 4. The result is −20.

8 Describe the advantages and disadvantages of using flow charts to solve equations.

Use the balance method to solve linear equations

11.5

Learning intention: To be able to use the balance method to solve linear equations

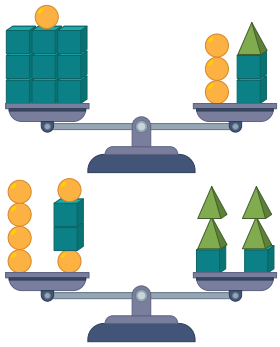
Success criteria:

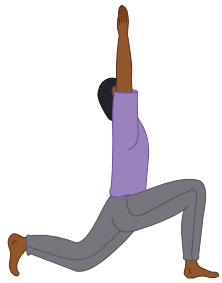
SC 1 I understand how to keep an equation balanced.

SC 2 I can use the balance method to solve a linear equation.

Lesson warm-up

Balancing scales

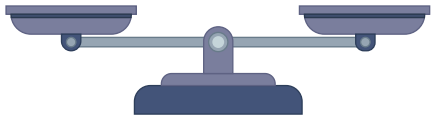




The mass of the cube is 6 g. Determine the mass of the sphere and cone.

SC 1 I understand how to keep an equation balanced

The balance method of solving equations can be demonstrated by looking at a balance scale. For the scale to stay balanced, both plates must have the same weight. If you add any amount to (or subtract any amount from) one of the plates, then you must also add to (or subtract from) the other plate to keep the scale balanced.

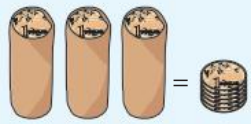
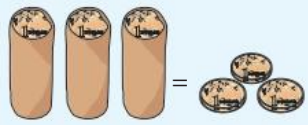


This idea is also true for equations. Both sides of the equation must stay equal, just like both sides of the scales must stay balanced. So, if you do any operation to one side of the equation, then you must do that same operation to the other side. This is sometimes known as 'doing the same to both sides'.

Worked example

Keeping an equation balanced

The following diagram shows $11 on each side of the equation. On one side there are 3 bank rolls (bundles of coins) and five $1 coins. On the other there are eleven $1 coins. Use a balance method to determine the amount of money in each of the bank rolls.

THINKING	WORKING
Define the unknown with a variable.	Let x represent the unknown amount of money in each bank roll.
Write an equation to represent the problem.	$3x + 5 = 11$
Keep the scale balanced by doing the same to both sides to isolate the unknown on one side.	$3x + 5 - 1 = 11 - 1$ $3x + 4 = 10$ $3x + 4 - 1 = 10 - 1$ $3x + 3 = 9$ $3x + 3 - 3 = 9 - 3$ $3x = 6$ 
Determine the value of one of the unknowns.	$\frac{3x}{3} = \frac{6}{3}$ $x = 2$ 
Write the answer.	Each bank roll contains $2.

11

SC 2 I can use the balance method to solve a linear equation

Worked example

Solving a linear equation by doing the same to both sides

Solve each equation to determine the unknown value. In each case, check your result by substituting it back into the original equation.

(a) $x + 6 = 10$

THINKING	WORKING
Identify the operation applied to the unknown to form the expression.	The operation applied was + 6. The inverse operation is – 6.
Apply the inverse operation to both sides of the equation.	$x + 6 - 6 = 10 - 6$ $x = 4$
Check your answer by substituting it back into the original equation.	Substitute $x = 4$ into the equation $x + 6 = 10$: $4 + 6 = 10$ $10 = 10$ Because the expressions on the left and right are equal, $x = 4$ is correct.

(b) $2x + 6 = 10$

THINKING	WORKING
Identify the operation applied to the unknown to form the expression.	The operations applied are × 2 and then + 6. The inverse operations are – 6 and ÷ 2.
Apply the inverse operation to both sides of the equation.	$2x + 6 - 6 = 10 - 6$ $2x = 4$ $\frac{2x}{2} = \frac{4}{2}$ $x = 2$
Check your answer by substituting it back into the original equation.	Substitute $x = 2$ into the equation $2x + 6 = 10$: LHS $= 2 \times 2 + 6$ $= 4 + 6$ $= 10$ $=$ RHS Because the expressions on the left and right are equal, $x = 2$ is correct.

(c) $2(x + 6) = 10$

THINKING	WORKING
Identify the operation applied to the unknown to form the expression.	The operations applied are + 6 and then × 2. The inverse operations are ÷ 2 and − 6.
Apply the inverse operation to both sides of the equation.	$\frac{2(x+6)}{2} = \frac{10}{2}$ $x + 6 = 5$ $x + 6 - 6 = 5 - 6$ $x = -1$
Alternatively, you could apply the distributive law first.	$2(x + 6) = 10$ $2x + 12 = 10$ $2x + 12 - 12 = 10 - 12$ $2x = -2$ $\frac{2x}{2} = \frac{-2}{2}$ $x = -1$
Check your answer by substituting it back into the original equation.	Substitute $x = -1$ into the equation $2(x + 6) = 10$: LHS = $2(-1 + 6)$ $= 2 \times 5$ $= 10$ $=$ RHS Because the expressions on the left and right are equal, $x = -1$ is correct.

Practice

ANSWERS Page 700

SC 1 I understand how to keep an equation balanced

1 Write a balanced number sentence by performing the following operations to both sides of the equals symbol in: $5 + 3 = 2 \times 4$.

(a) add 7 **(b)** subtract 7 **(c)** multiply by 7

2 Write a balanced equation by performing the following operations to the expressions on both sides of the equals symbol: $x + 3 = 2x$.

(a) add 7 **(b)** subtract 7 **(c)** multiply by 7

3 Complete the final line in each of the following equations to keep the equations balanced.

(a)
$5x - 6 = 12$
$5x - 6 + 2 = 12 + 2$
$5x - 4 =$

(b)
$5x = 15$
$\frac{5x}{5} = \frac{15}{5}$
$x =$

(c) $\dfrac{2(x-4)}{3}=4$

$\dfrac{2(x-4)}{3}\times 3=4\times 3$

$2(x-4)=$

(d) $6-\dfrac{3x}{2}=10$

$6-\dfrac{3x}{2}-6=10-6$

$-\dfrac{3x}{2}=$

4 Complete the final line in each of the following equations to keep the equations balanced.

(a) $5x-6=12$

$5x-7=$

(b) $5x=15$

$10x=$

(c) $\dfrac{2(x-4)}{3}=4$

$8(x-4)=$

(d) $6-\dfrac{3x}{2}=10$

$16-\dfrac{3x}{2}=$

5 Determine whether the following pairs of equations are equivalent or not.

(a) $3x-6=4$ and $3x-4=2$

(b) $\dfrac{3x}{2}=5$ and $3x=10$

(c) $3(x-6)=15$ and $x-6=12$

(d) $4-\dfrac{3x}{2}=10$ and $-\dfrac{3x}{2}=6$

6 Create two equivalent equations that could be found using the balance method on $\dfrac{2(x+4)}{5}=6$.

7 Consider the equation $3(x-4)=6$.

(a) Describe the order of operations applied to the variable x in the expression $3(x-4)$.

(b) List the order of operations to undo the operations given in part **(a)**.

8 Two students were asked to keep the equation $3x+9=15$ balanced to determine the value of x that makes the statement true. Which student is correct and why? What could the student who is incorrect do to make their answer correct?

Student A	Student B
$3x+9=15$	$3x+9=15$
$3x+9-9=15-9$	$\dfrac{3x}{3}+9=\dfrac{15}{3}$
$3x=6$	$x+9=5$
$\dfrac{3x}{3}=\dfrac{6}{3}$	$x+9-9=5-9$
$x=2$	$x=-4$

SC 2 I can use the balance method to solve a linear equation

1 Solve the following equations using the balance method (doing the same to both sides).

(a) $x+3=12$ **(b)** $x-3=12$ **(c)** $3x=12$ **(d)** $\dfrac{x}{3}=12$

(e) $x+4=20$ **(f)** $x-4=20$ **(g)** $4x=20$ **(h)** $\dfrac{x}{4}=20$

(i) $x+5=1$ **(j)** $x-5=1$ **(k)** $5x=1$ **(l)** $\dfrac{x}{5}=1$

2 Solve the following equations using the balance method (doing the same to both sides).

(a) $2x + 1 = 7$ **(b)** $2x - 1 = 7$ **(c)** $\frac{x}{2} + 1 = 7$ **(d)** $\frac{x}{2} - 1 = 7$

(e) $\frac{x+1}{2} = 7$ **(f)** $\frac{x-1}{2} = 7$ **(g)** $6x + 3 = 15$ **(h)** $6x - 3 = 15$

(i) $\frac{x}{6} + 3 = 15$ **(j)** $\frac{x}{6} - 3 = 15$ **(k)** $\frac{x+3}{6} = 15$ **(l)** $\frac{x-3}{6} = 15$

3 Fabian is checking a solution to an algebraic equation, but finds that it is not correct. Explain where a mistake has been made and correct it.

$$\frac{x+3}{5} = 6$$
$$\frac{x+3}{5} - 3 = 6 - 3$$
$$\frac{x}{5} = 3$$
$$\frac{x}{5} \times 5 = 3 \times 5$$
$$x = 15$$

4 Consider the equation $-2(x + 7) - 3 = -11$

(a) List the operations in the order they were applied to the variable x to form the expression $-2(x + 7) - 3$.

(b) List the inverse operations.

(c) Solve the equation by applying the inverse operations to both sides of the equation.

5 Solve the following equations.

(a) $\frac{2x}{3} - 7 = 3$ **(b)** $4(2x + 4) = 48$ **(c)** $\frac{4x-1}{7} = 5$ **(d)** $\frac{4+8(3-x)}{6} = 6$

6 Solve the following equations, writing your answer as a mixed number.

(a) $4(2x + 1) = 48$ **(b)** $3 + \frac{2x}{5} = 4$ **(c)** $4 - \frac{3x}{7} = 6$ **(d)** $\frac{5-2x}{4} - 3 = 7$

7 Solve the following.

(a) $\frac{3(2-x)}{7} + 6 = 3$ **(b)** $\frac{10-4(x+9)}{6} = -5$ **(c)** $40 = \frac{60(x-8)}{3}$

8 Let L represent Lily's age. Write an equation to help you solve this problem.

Emma is 4 years older than her sister Lily. If Emma is 18 years old, how old is Lily?

9 Let J represent the amount of money that John has now. John needs $44 to reach his goal of $250. How much money does John have now?

Write an equation to help you solve this problem.

10 Let c represent the cost of 1 ticket. You spend $245 to buy 5 concert tickets and a T-shirt. Given that the T-shirt costs $15, determine the cost for 1 ticket.

Write an equation to help you solve this problem.

11 Let r represent the amount of money Ria earns each hour. Ria earns $176 for 8 hours of work. Ria is paid an hourly wage plus a $20 bonus. Calculate Ria's hourly wage.

Write an equation to help you solve this problem.

TOPIC REVIEW

Multiple choice

ANSWERS Page 744

1 Select the equation that represents the product of an unknown number and 4 that gives a result of 12.

A $x + 4 = 12$ **B** $4x = 12$ **C** $\frac{x}{4} = 12$ **D** $\frac{12}{4} = x$

2 Which of the following is the solution to $x + 7 = 10$?

A $x = 3$ **B** $x = 4$ **C** $x = 10$ **D** $x = 17$

3 Which of the following is the solution to $\frac{a}{2} + 3 = 13$?

A $a = 8$ **B** $a = 20$ **C** $a = 23$ **D** $a = 26$

4 What is the next step in solving the equation $-3(2 - x) + 10 = -2$?

A $-3(2 - x) = 8$ **B** $-3(2 - x) = -12$ **C** $2 - x = -4$ **D** $x = 1$

5 Which of the following shows the correct solution to the equation $2x + 3 = 11$?

A
$2x + 3 = 11$
$-3 \mid -3$
$2x = 8$
$\times 2 \mid \times 2$
$x = 16$

B
$2x + 3 = 11$
$-2 \mid -2$
$x + 3 = 9$
$-3 \mid -3$
$x = 6$

C
$2x + 3 = 11$
$\div 2 \mid \div 2$
$x + 3 = 5.5$
$-3 \mid -3$
$x = 2.5$

D
$2x + 3 = 11$
$-3 \mid -3$
$2x = 8$
$\div 2 \mid \div 2$
$x = 4$

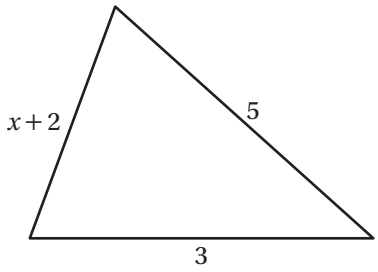
6 Which equation describes the perimeter P of a square with side length x?

A $P = x^2$ **B** $P = 4x$ **C** $P = 2x$ **D** $P = 2x^2$

7 A taxi charges \$3.50 for each trip, plus \$1.50 for every minute of the trip. Which equation describes the total cost C of a trip, in terms of the time t (in minutes)?

A $C = 3.50t + 1.50t$ **B** $C = 3.50t + 1.50$ **C** $C = 1.50t + 3.50$ **D** $C = 1.50t + 3.50t^2$

8 Which equation describes the perimeter of the triangle shown?



A $P = 10 + x$ **B** $P = 5 + 3 + 2$ **C** $P = x + 2$ **D** $P = 10x$

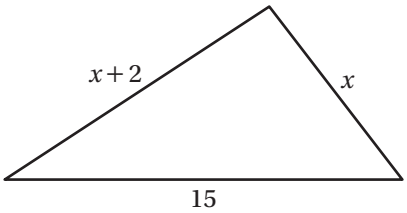
9 Janelle needs \$55 to reach her bank balance target of \$300. Which equation best describes her current bank balance B?

A $B = 55 + 300$ **B** $55 + B = 300$ **C** $300 + B = 55$ **D** $B = 55 - 300$

10 Which of the following expressions describes doubling a number, then subtracting three?

A $2x + 3$ **B** $x + 3 \times 2$ **C** $2x - 3$ **D** $3 - 2x$

11 The three sides of a triangle are given in terms of x, as shown. Given that the perimeter of the triangle is 20 units, which equation describes the perimeter of the triangle?



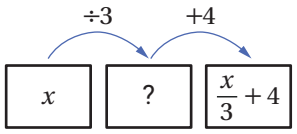
A $20 = 17 + 2$

B $20 = 15 + x + 2 + x$

C $20 = x^2 + 2 + 15$

D $20 = x + 2 + x$

12 The expression missing from the box below is:



A $x + 3$ **B** $3x$ **C** $\frac{x}{3}$ **D** $4x$

13 Using backtracking, the first step in solving $\frac{x+6}{4} + 11 = 15$ is to:

A multiply both sides by 4.

B add 11 to both sides.

C divide both sides by 4.

D subtract 11 from both sides.

14 The equation $\frac{4(x-2)}{3} = 12$ has the solution:

A $x = 7$ **B** $x = 11$ **C** $x = 12$ **D** $x = 36$

Short answer

1 Write a word problem that describes each of the following equations.

(a) $p + 5 = 20$ **(b)** $p - 5 = 20$ **(c)** $5p = 20$ **(d)** $\frac{p}{5} = 20$

2 Create an equation from the steps below.

Start with $x = 10$.

Subtract 5.

Multiply by 12.

Add 32.

3 Luke has $780 and needs to buy two monitors and a new graphics card for his computer. The store offers a combo sale so that if you buy a new monitor, you can buy a graphics card for only $240 more than the cost of the monitor. Given that Luke spends all of his $780, how much does he spend on the graphics card?

4 Apply the steps in order to each side of the starting equation to arrive at the final equation.

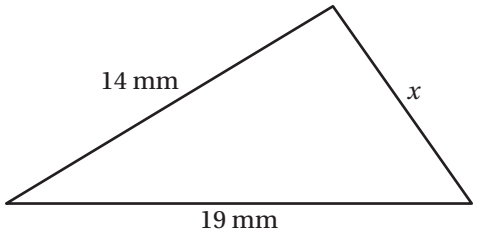
Start with $x = -6$.

Multiply by -4.

Add 15.

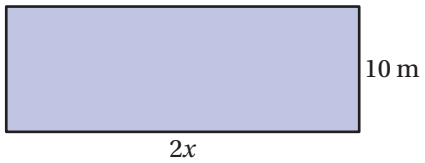
Divide by 3.

5 The triangle shown has a perimeter of 45 mm. Write an equation that describes the perimeter in terms of the side lengths shown.



6 Sarah has 50 candles and 5 rooms in her house. Write an equation to describe how many candles are in each room, given that each room has the same number of candles (c).

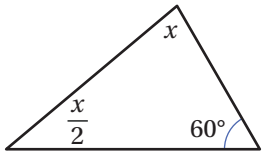
7 The rectangle shown has a total perimeter of 92 m. Write an equation that describes the perimeter in terms of the side lengths shown.



8 An unknown value m is subtracted from 8 and the result is 9. Write an equation to describe this.

9 An unknown number x is tripled, then 4 is added to this. The result is -15. Write an equation to describe this.

10 The angles in a triangle add to make 180°. Write an equation to describe the sum of the angles in terms of the angles shown.



11 6 is subtracted from $\frac{5}{8}$ of a. The result is $-\frac{2}{3}$. Write an equation to describe this. (You don't need to solve the equation.)

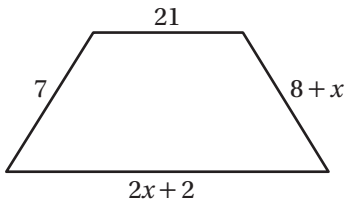
12 A retail assistant earns $150 total for a 6-hour shift. The retail assistant is paid an hourly wage of h dollars per hour, plus an extra $35 sales bonus for meeting a sales target. Write an equation to describe this situation.

13 A farmer has a rectangular paddock with a length that is twice the width. Write an equation to describe this situation, where the length of the fence all around the paddock is 1500 m and the paddock's width is x.

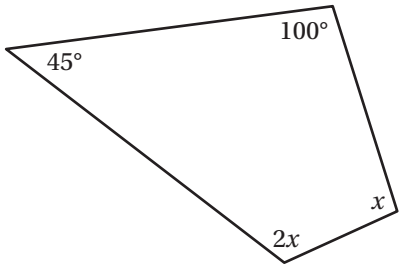
14 Caitlyn buys three lollipops and two cupcakes. A lollipop is half the price of a cupcake, and altogether Caitlyn spends \$7. Write an equation to describe this situation, where the cost of a cupcake is c.

15 Double x is added to 17 and the result is -6. Write an equation to describe this situation.

16 The shape shown has a perimeter of 111 m. All the labelled side lengths are given in metres. Write an equation to describe this situation, simplifying the sides containing the variable as much as you can.



17 The angles in the quadrilateral shown add together to make 360°. Write an equation to describe this situation, simplifying the angles containing the variable as much as possible. (Do not use any degree ° symbols in the working out.)



18 State true (T) or false (F) for each number sentence.

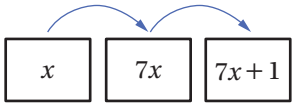
(a) $\frac{6+8+6}{2} = \frac{18+4}{2}$ **(b)** $28 - 6 \times 2 = 4 + 18 - 6$ **(c)** $5 \times 3 = \frac{45}{3}$

19 Write each of the following equations in words.

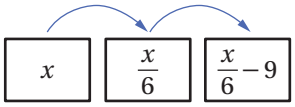
(a) $y + 2 = 8$ **(b)** $\frac{a}{5} = 2$ **(c)** $6x = 36$

20 Write, in order, the missing operations needed to complete each of the following flow charts.

(a)



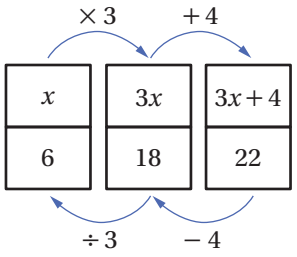
(b)



21 The following flow chart represents an equation.

(a) What is the equation to be solved?

(b) What is the solution to the equation?

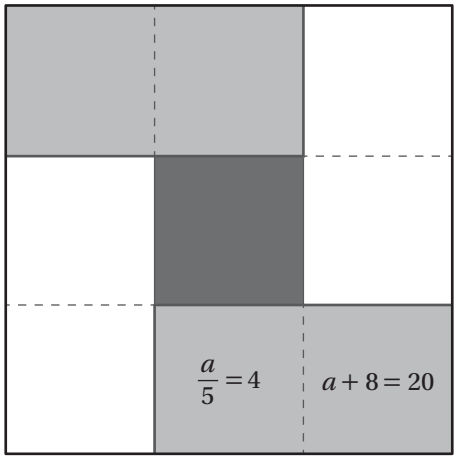


Extended response

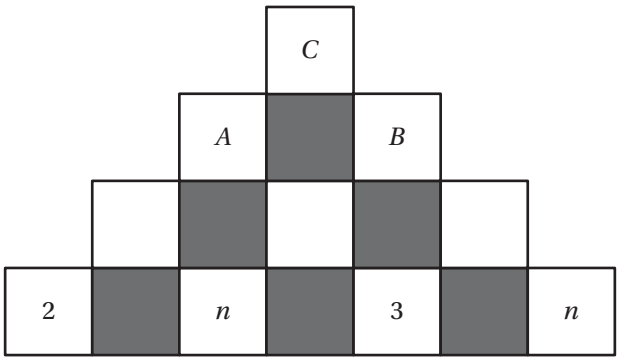
1 Mr Ross has invented a domino algebra game. He places the following dominoes on the board and asks the class to arrange them in the shape below. Which three dominoes can be used to form a square so that the puzzle is complete? Write them down. (In dominoes you can join two ends of the same value.)

Domino 1		Domino 2		Domino 3	
$4a + 6 = 22$	$a - 3 = 4$	$a + 6 = 10$	$a - 13 = 7$	$\frac{a}{10} = 2$	$3a + 4 = 37$

Domino 4		Domino 5		Domino 6	
$2a + 1 = 15$	$\frac{a}{2} = 6$	$\frac{a}{5} = 4$	$a + 8 = 20$	$4(a + 1) = 32$	$a + 15 = 17$



2 In number pyramids, a white square on top of a black square contains the sum of the two white squares on either side of the black square.



(a) Use the letter n to write expressions for the sums at positions A, B and C in the pyramid. (Write your answers in simplest form.)

(b) You are told that the top number is 15. Write an equation using n.

(c) Solve the equation from **(b)** for n. Check your answer in the pyramid.

TOPIC
12

Area and volume

Recall 311
12.1 Understand and convert between units of area 312
12.2 Calculate the area of a parallelogram 317
12.3 Calculate the area of a triangle 324
12.4 Understand volume measured in cubic units 334
12.5 Establish the volume formula and units for prisms 339
12.6 Explore the connection between volume and capacity 346
Topic review 353

Why learn this?

Learning about volume and capacity provides an understanding of how to measure and quantify the space inside a three-dimensional object. This knowledge helps you to:

- determine the amount of space an object occupies, useful in packing, storage and space management
- calculate the amount a container can hold, which is relevant in cooking, chemical mixing or filling a water tank
- apply these concepts to real-world scenarios, such as estimating the capacity of a suitcase or the volume of a room
- develop spatial awareness and problem-solving skills
- lay a foundation for more complex geometry and algebraic problems in higher-level mathematics
- support practical applications in various fields, such as engineering, architecture, interior design and logistics.

RECALL

ANSWERS Page 701

I can calculate with decimals

1 Calculate the following decimal additions and subtractions.

(a) 6.31 + 7.14 + 5.36 **(b)** 17.34 + 21.2 + 0.43

(c) 6.72 – 3.41 **(d)** 8 – 6.15

2 Calculate the following decimal multiplications.

(a) 6.73 × 21.4 **(b)** 0.016 × 19.2 **(c)** 78 × 0.36 **(d)** 2.62 × 0.34

3 Calculate the following decimal multiplications and division by powers of 10.

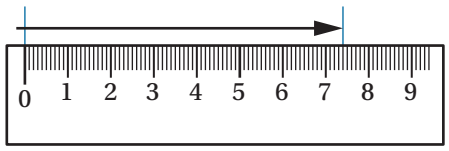
(a) 5.9 × 100 **(b)** 741.6 ÷ 100 **(c)** 84.63 × 1000 **(d)** 9530 ÷ 1000

I can read lengths from rulers

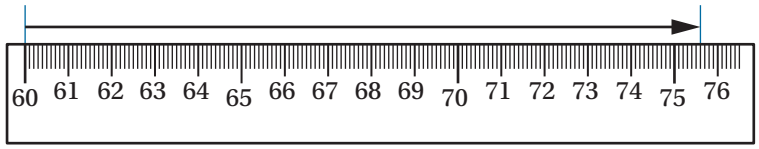
1 State the length of the arrow shown on each ruler (the numbers on the ruler are in centimetres):

(i) in centimetres **(ii)** in millimetres.

(a)



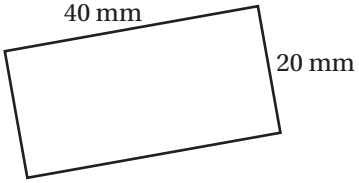
(b)



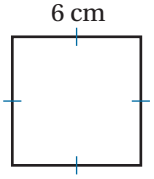
I can calculate the perimeter and area of plane shapes

1 Calculate the perimeter and area for the following rectangles and square.

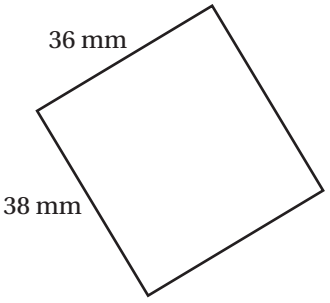
(a)



(b)



(c)



I can convert between metric units of measurement

1 Complete the following unit conversions.

(a) 5 m to cm **(b)** 20 cm to mm **(c)** 907 cm to m

(d) 5000 km to m **(e)** 6.5 m to mm

12.1 Understand and convert between units of area

Learning intention: To be able to understand and convert between units of area

Success criteria:

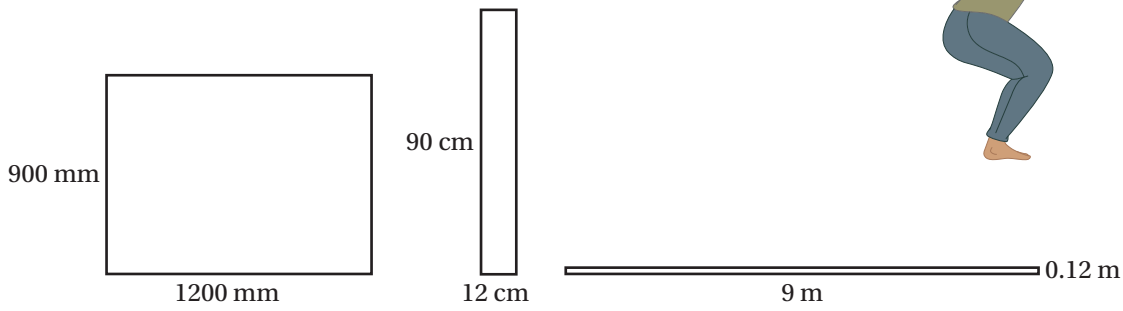
SC 1 I can understand units of area.

SC 2 I can convert between units of area.

Lesson warm-up

Which rectangle takes up the most space?

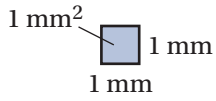
For each of the rectangles shown, convert the lengths to the same unit of measurement and then calculate and compare the areas by multiplying the dimensions of the rectangle.



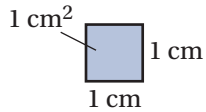
SC 1 I understand units of area

Area units are paired with units of length in the following way:

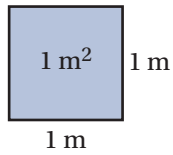
One square millimetre (mm^2)



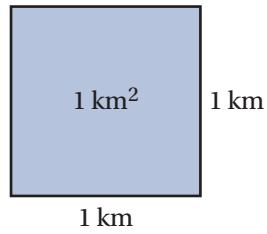
One square centimetre (cm^2)



One square metre (m^2)



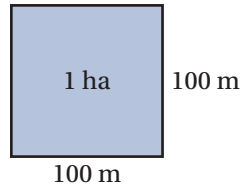
One square kilometre (km^2)



Hectares

A unit between the square metre and the square kilometre is the hectare (ha).

A hectare, when drawn as a square, has a side length of 100 m, so a hectare is the size of a fairly large playing field.

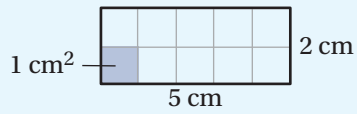


Worked example

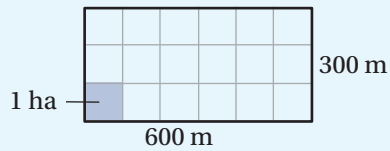
Understanding units of area

Determine the number of:

(a) square centimetres in a rectangle 5 cm by 2 cm

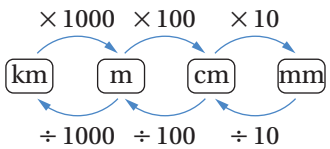
THINKING	WORKING
Create the rectangle, breaking the surface into 1 cm^2 squares.	
Calculate the number of unit squares by multiplying the number of columns by the number of rows.	$5 \times 2 = 10$
Write the answer.	The number of square centimetres is 10, i.e. the area is 10 cm^2.

(b) hectares in a rectangular field 600 m by 300 m.

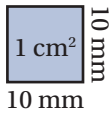
THINKING	WORKING
Create the rectangle, breaking the surface into 100 m × 100 m squares.	
Calculate the number of 100 m × 100 m squares by multiplying the number of columns by the number of rows.	$6 \times 3 = 18$
Write the answer.	The number of hectares is 18. The area is 18 ha.

SC 2 I can convert between units of area

Conversion chart for units of length



The linear conversion factors must be squared before using them as conversion factors for area.

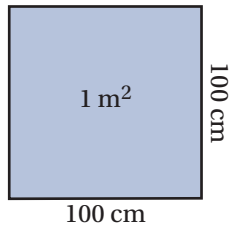


Length: 1 cm = 10 mm

Area:

$1\ cm^2 = 10\ mm \times 10\ mm$

$= 100\ mm^2$

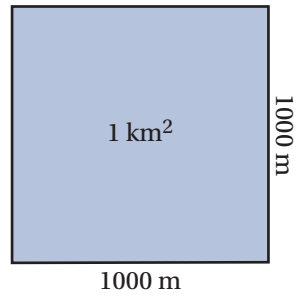


Length: 1 m = 100 cm

Area:

$1\ m^2 = 100\ cm \times 100\ cm$

$= 10\ 000\ cm^2$



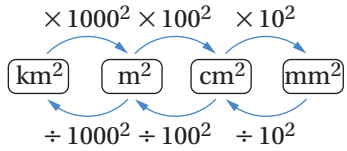
Length: 1 km = 1000 m

Area:

$1\ km^2 = 1000\ m \times 1000\ m$

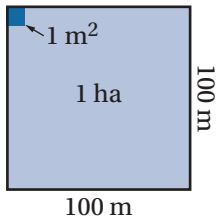
$= 1\ 000\ 000\ m^2$

Conversion chart for units of area



Hectares

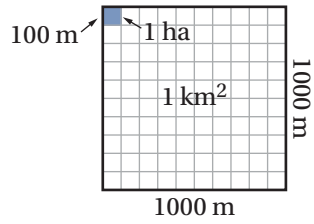
There is such a large number of square metres in a square kilometre that it is useful to have a unit of area in between.



Area:

$1\ ha = 100\ m \times 100\ m$

$= 10\ 000\ m^2$



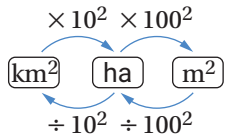
$1\ km^2 = (10 \times 100\ m) \times (10 \times 100\ m)$

$= 100 \times (100 \times 100)\ m^2$

$= 100 \times 1\ ha$

$= 100\ ha$

Conversion chart for hectares



Worked example

Converting units of area

Perform the following area conversions.

(a) 2.5 m^2 to mm^2

THINKING	WORKING
Recall the required area conversion factor.	1 m = 1000 mm, so 1 m^2 = 1000^2 mm^2
Convert to the smaller unit (the number increases), so multiply by the conversion factor.	2.5 m^2 = (2.5 × 1000^2) mm^2 = 2 500 000 mm^2

(b) 8200 cm^2 to m^2

THINKING	WORKING
Recall the required area conversion factor.	1 m = 100 cm, so 1 m^2 = 100^2 cm^2
Convert to the larger unit (the number decreases), so divide by the conversion factor.	8200 cm^2 = (8200 ÷ 100^2) m^2 = 0.82 m^2

(c) 0.25 ha to m^2

THINKING	WORKING
Recall the required area conversion factor.	1 ha = 100^2 m^2
Convert to the smaller unit (the number increases), so multiply by the conversion factor.	0.25 ha = (0.25 × 10 000) m^2 = 2500 m^2

(d) 5100 ha to km^2

THINKING	WORKING
Recall the required area conversion factor.	1 km^2 = 100 ha
Convert to a larger unit (the number decreases), so divide by the conversion factor.	5100 ha = (5100 ÷ 100) km^2 = 51 km^2

Practice

ANSWERS Page 702

SC 1 I understand units of area

1 Calculate the number of square centimetres in a rectangle:

(a) 8 cm by 4 cm **(b)** 7 cm by 3 cm **(c)** 20 cm by 15 cm

2 Calculate the area of a rectangle:

(a) 11 mm by 4 mm **(b)** 12 m by 5 m **(c)** 9 km by 8 km

3 Calculate the number of hectares in a rectangular field:

(a) 400 m by 100 m **(b)** 700 m by 200 m **(c)** 800 m by 50 m

4 Calculate the area of a rectangle:

(a) 5.3 cm by 3.1 cm **(b)** 3.28 m by 2.4 m **(c)** 7.4 km by 5 km

5 Calculate the number of hectares in a rectangular field:

(a) 620 m by 430 m **(b)** 120 m by 80 m

6 Calculate the number of hectares in a rectangular field:

(a) 1.3 km by 800 m **(b)** 2.5 km by 200 m

SC 2 I can convert between units of area

1 Perform the following units of area conversion.

(a) 3 m^2 to cm^2 **(b)** 15 cm^2 to mm^2 **(c)** 18 200 cm^2 to m^2

(d) 1 700 000 m^2 to km^2 **(e)** 712 cm^2 to m^2 **(f)** 0.0011 m^2 to cm^2

(g) 3.27 cm^2 to mm^2 **(h)** 8230 mm^2 to cm^2

2 Explain the common errors in the following calculations. Write the corrected final answer in each case.

(a) $A = lw$
$= 12 \times 40$
$= 480\ cm^2$
$= 4.8\ m^2$

(b) $A = \frac{bh}{2}$
$= \frac{1.2 \times 1.5}{2}$
$= 0.9\ m^2$
$= 90\ cm^2$

(c) $A = \frac{a+b}{2} \times h$
$= \frac{60+90}{2} \times 40$
$= 3000\ mm^2$
$= 3\ m^2$

(d) $A = bh$
$= 500 \times 150$
$= 75\ 000\ m^2$
$= 75\ km^2$

3 Determine the area of each rectangle in square centimetres.

(a) 2.5 m by 1.8 m **(b)** 36 mm by 14 mm

(c) 3.5 m by 78 cm **(d)** 2.7 cm by 9 mm

4 Perform the following units of area conversion.

(a) 23 ha to m^2 **(b)** 30 700 m^2 to ha **(c)** 23 ha to km^2

(d) 1 700 000 m^2 to ha **(e)** 7.03 ha to m^2 **(f)** 4.25 km^2 to ha

(g) 1820 ha to km^2 **(h)** 0.75 km^2 to ha

5 A beachside town has houses along 4.6 km of beachfront, occupying land up to 200 m inland from the beach. Determine the area occupied by the town in:

(a) m^2 **(b)** ha **(c)** km^2

6 Determine the area of each rectangle in hectares.

(a) 1.5 km by 1.4 km **(b)** 420 m by 67 m **(c)** 3.8 km by 460 m

Calculate the area of a parallelogram

12.2

Learning intention: To be able to calculate the area of a parallelogram

Success criteria:

SC 1 I can determine the area of a parallelogram by drawing a rectangle with the same area.

SC 2 I can calculate the area of a parallelogram using a formula.

Lesson warm-up

Matching parallelograms with rectangles of the same area

Draw two copies of a parallelogram on grid paper.

Shade both diagrams the same way.

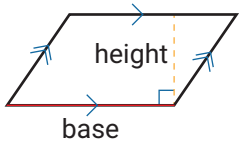
Cut out one parallelogram, then cut a triangle by cutting from a vertex through to the opposite parallel side.

Form a rectangle by sliding the triangle to the opposite side from which it was cut.

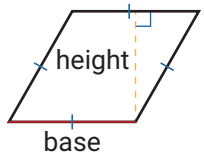
Complete the sentence: A parallelogram with a base of 5 cm and a height of 3 cm has the same area as a rectangle with a length of _____ and a width of _____

SC 1 I can determine the area of a parallelogram by drawing a rectangle with the same area

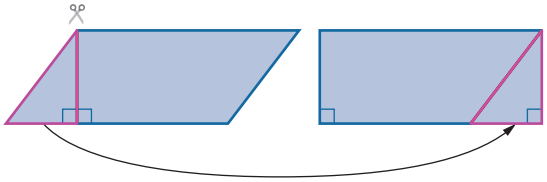
A parallelogram is a quadrilateral that has equal and parallel opposite sides.



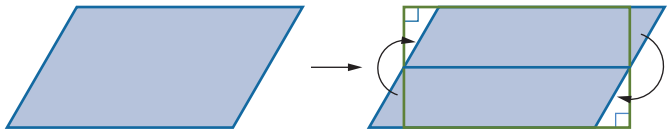
A rhombus is a special parallelogram with all sides equal.



A parallelogram can be transformed into a rectangle of the same area by cutting from a vertex, at right angles to the base, and sliding the removed right-angled triangle to the other end of the separated parallelogram.

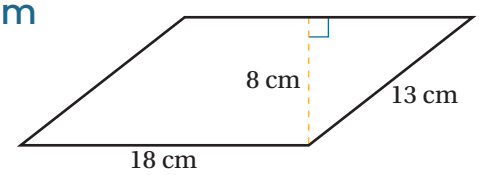


The midpoint method can also be used to transform a parallelogram to a rectangle of equal area.

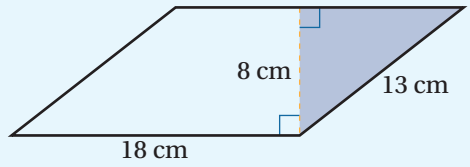
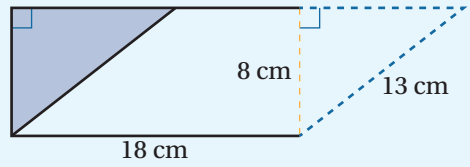
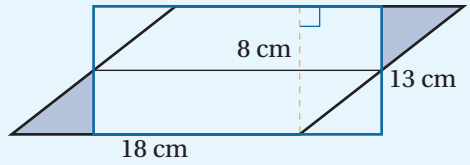
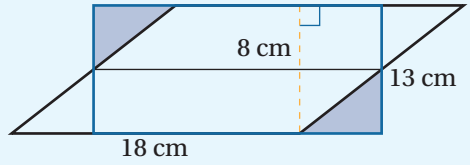


Worked example

Determining the area of a parallelogram by reshaping it into a rectangle



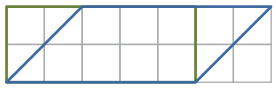
(a) Determine the dimensions of a rectangle that would have the same area as the parallelogram shown.

THINKING	WORKING
Truncated triangle method Identify the right-angled triangle in the parallelogram.	
Imagine sliding the right-angled triangle to the opposite end of the parallelogram. Shade the relocated triangle within the rectangle.	
Midpoint method In the parallelogram, construct a line parallel to the base that passes through the midpoints of each slanting side. Create a rectangle, using vertical lines through the midpoints as well as the base line and its opposite side.	
Shade the relocated triangles within the rectangles.	
Write the dimensions of the rectangle formed.	The dimensions of the rectangle formed are length 18 cm and width (or height) 8 cm.

(b) Calculate the area of the parallelogram.

THINKING	WORKING
Calculate the area of the rectangle using the dimensions found. The area of the parallelogram is the same.	$18 \times 8 = 144$
Write the answer.	The area of the parallelogram is 144 cm^2.

SC 2 I can calculate the area of a parallelogram using a formula



The area of a rectangle is calculated by multiplying the length by the width: $A = lw$.

Since the two dimensions are at right angles, the area corresponds to the total grid squares within the boundary: $5 \times 2 = 10$ square units.

In the diagram:

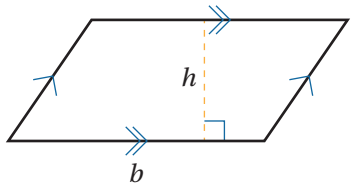
- the length of the rectangle becomes the **base** of the parallelogram and
- the width of the rectangle becomes the **perpendicular height** of the parallelogram.

Note: The perpendicular height is measured from the base.

The area of both shapes is the same, so the area of the parallelogram remains as $5 \times 2 = 10$ square units.

The formula for the area of a parallelogram is $A = bh$, where b is the length of the base and h is the perpendicular height of the parallelogram.

To use the formula, the base and height must be in the same units of length (mm, cm, m), then the area will be in the corresponding square units (mm^2, cm^2, m^2).

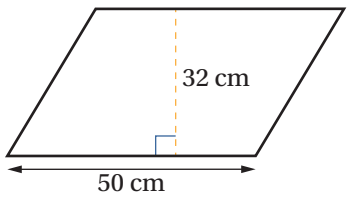


Worked example

Calculating the area of a parallelogram using a formula

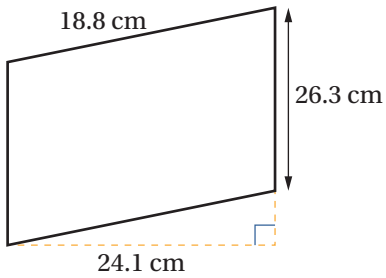
Use a formula to calculate the area of each parallelogram.

(a)



THINKING	WORKING
Identify the required dimensions.	Base: $b = 50$ cm Perpendicular height: $h = 32$ cm
Recall the formula for the area of a parallelogram.	$A = bh$
Substitute the values into the formula and calculate.	$A = 50 \times 32$ $= 1600$
Write the area with square units.	The area of the parallelogram is 1600 cm^2.

(b)



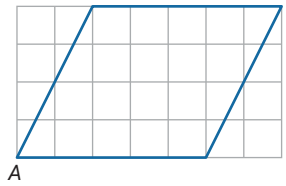
THINKING	WORKING
Identify the required dimensions.	Base: $b = 26.3$ cm Perpendicular height: $h = 24.1$ cm
Recall the formula for the area of a parallelogram.	$A = bh$
Substitute the values into the formula and calculate.	$A = 26.3 \times 24.1$ $= 633.83$
Write the area with square units.	The area of the parallelogram is 633.83 cm^2.

Practice

ANSWERS Page 702

SC 1 I can determine the area of a parallelogram by drawing a rectangle with the same area

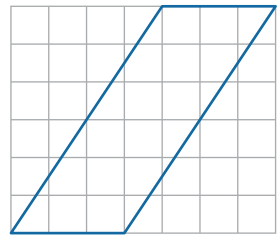
1 Draw a rectangle from point A that has the same area as the parallelogram. Explain how you know that the rectangle and parallelogram have the same area.



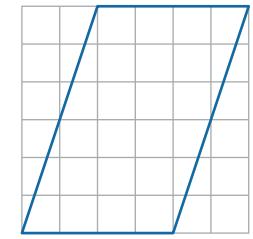
2 For each of the parallelograms below:

- use the midpoint method to form rectangles of the same area
- shade triangles so that equal areas are easily seen
- describe how the dimensions of the rectangles relate to the original shapes.

(a)

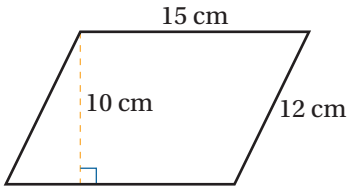


(b)

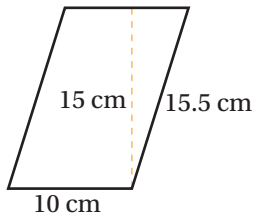


3 Determine the dimensions of a rectangle that would have the same area as the parallelogram shown. Write the area of the rectangle and hence the parallelogram each time.

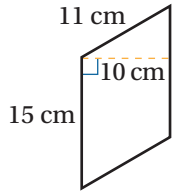
(a)



(b)

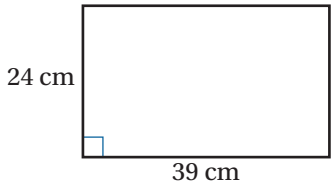


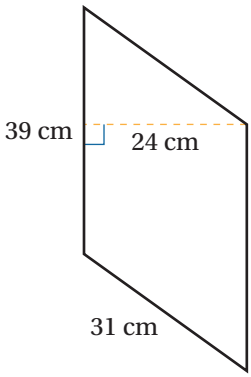
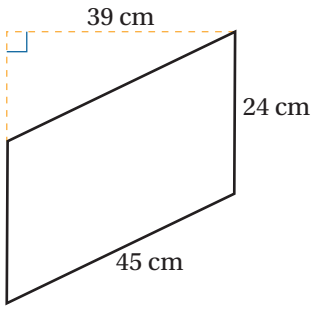
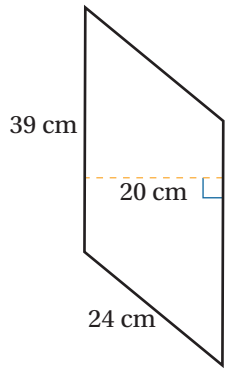
(c)



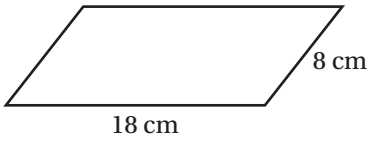
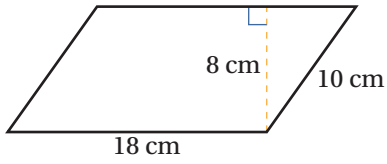
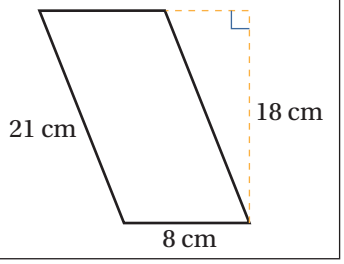
4 Determine which parallelograms have the same area as the given rectangle.

(a) Rectangle

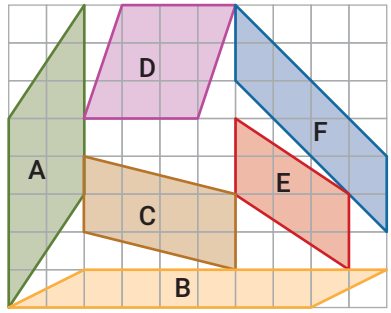


Parallelogram I	Parallelogram II	Parallelogram III
		

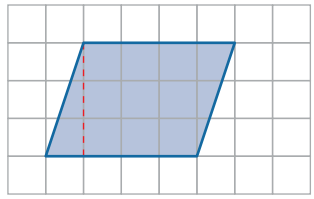
(b) Rectangle: Length: 18 cm; width: 8 cm

Parallelogram I	Parallelogram II	Parallelogram III
		

5 Which of the parallelograms shown on the grid have the same area as each other?



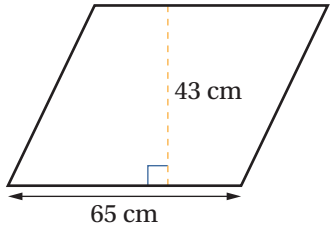
6 Determine the area of the parallelogram on the grid in square units.



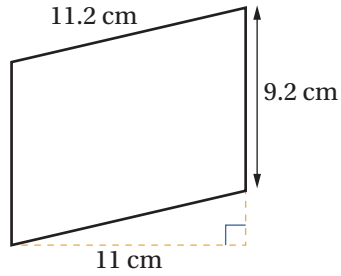
SC 2 I can calculate the area of a parallelogram using a formula

1 Calculate the area of each parallelogram.

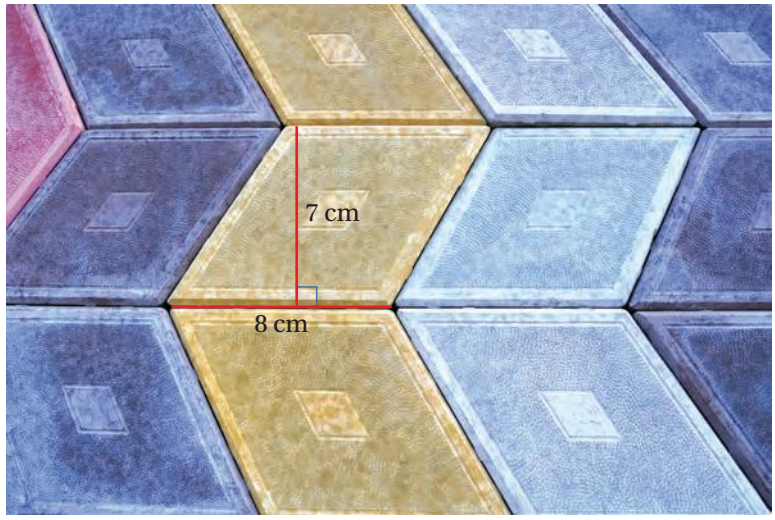
(a)



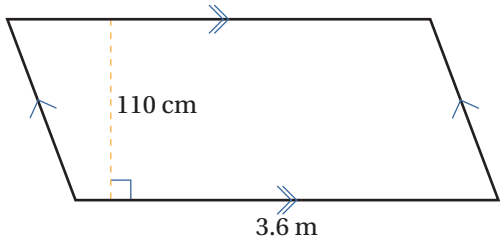
(b)



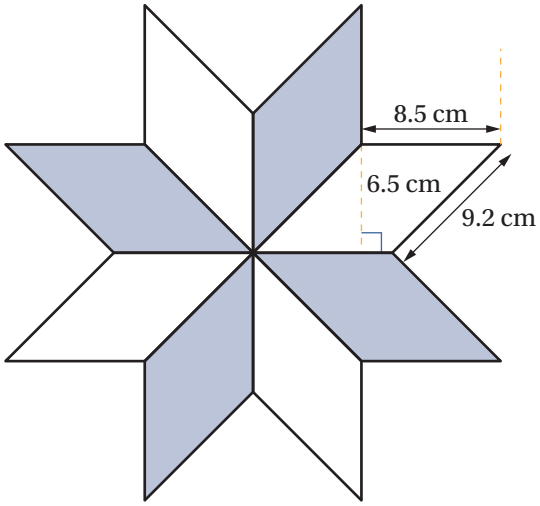
2 Parallelogram-shaped tiles can be arranged to form many interesting patterns. Calculate the area of the tile shown.



3 Determine the area of the parallelogram in each of the given units.

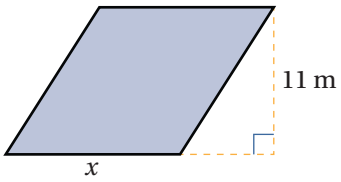


4 This design is made from 8 identical parallelograms. Determine the total area of the design.

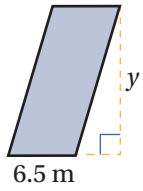


5 Calculate the unknown dimension for each parallelogram, given an area of 143 m^2 for each.

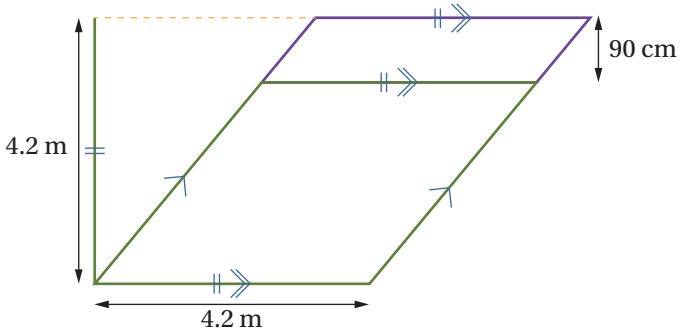
(a)



(b)



6 Nita has drawn this plan for some paving and a flower bed. Calculate the area of the green flower bed.



12.3 Calculate the area of a triangle

Learning intention: To be able to calculate the area of a triangle

Success criteria:

SC 1 I can determine the area of a triangle as half of a rectangle.

SC 2 I can determine the area of a triangle as half of a parallelogram.

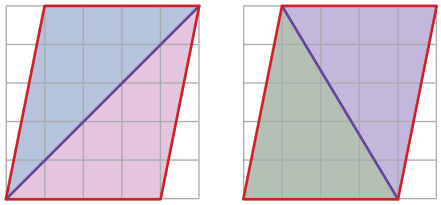
SC 3 I can calculate the area of a triangle using a formula.

Lesson warm-up

Making pairs of congruent triangles from parallelograms

Use grid paper to draw two copies of the same parallelogram of any dimensions you choose. Make your dimensions different from the example provided and different from the students sitting near you.

Draw the two different diagonals, one on each parallelogram, like the example below.

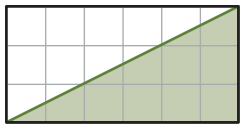


Copy down the two sentences and fill in the missing words.

Complete the sentences: Each of the four triangles has the _____ base and the _____ height as the parallelogram. Each parallelogram is broken into a pair of congruent triangles, where the area of each triangle is _____ the area of the parallelogram.

SC 1 I can determine the area of a triangle as half of a rectangle

You can see that a right-angled triangle takes up half the area of a rectangle that shares two perpendicular sides.

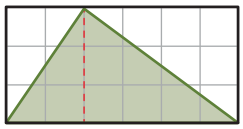


Given the perpendicular sides of the triangle, the area of the rectangle is:

$6 \times 3 = 18$ square units.

So, the area of the triangle is half of that: $\frac{18}{2} = 9$ square units.

Rectangles can also be used to determine the area of triangles that do not have perpendicular sides, as long as the base length and perpendicular height are known.



Recall that the area of the rectangle is $6 \times 3 = 18$ square units.

Also recall that the area of the triangle is half of that: $\frac{18}{2} = 9$ units.

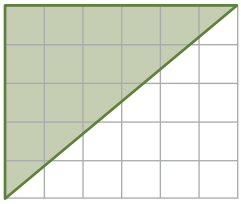
Worked example

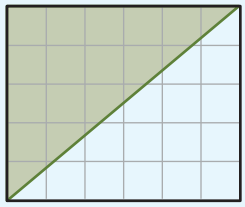
Deducing the area of a triangle from the area of a rectangle

For each of the triangles:

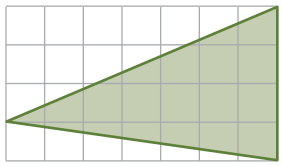
- construct a related rectangle with double the area of the triangle
- calculate the area of the rectangle
- calculate the area of the triangle.

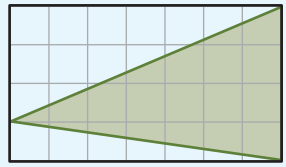
(a)



THINKING	WORKING
Complete the rectangle.	
Calculate the area of the rectangle.	$A = lw$ $= 6 \times 5$ $= 30$ The area of the rectangle is 30 square units.
Calculate the area of the triangle.	$\frac{30}{2} = 15$ The area of the triangle is 15 square units.

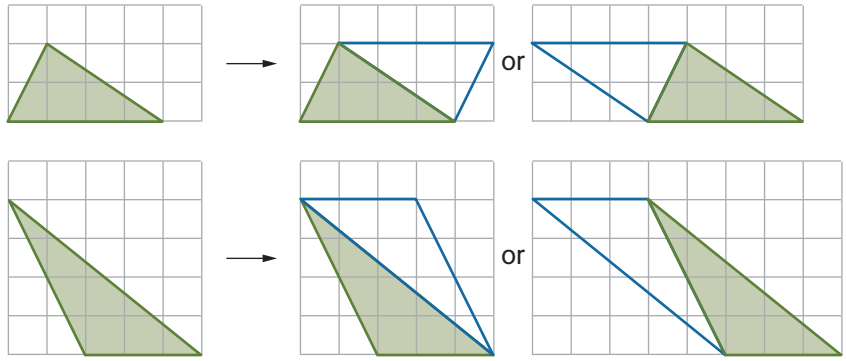
(b)



THINKING	WORKING
Complete the rectangle by constructing a related rectangle with double the area of the triangle.	
Calculate the area of the rectangle.	$A = lw$ $= 7 \times 4$ $= 28$ The area of the rectangle is 28 square units.
Calculate the area of the triangle.	$\frac{28}{2} = 14$ The area of the triangle is 14 square units.

SC 2 I can determine the area of a triangle as half of a parallelogram

Parallelograms can be constructed from triangles. A line equal in length and parallel to the base is drawn from the opposite vertex. The end of the line is then connected to the closest vertex at the base.



In each case:

- the base of the triangle is the same as the base of the parallelogram
- the perpendicular height of the triangle is the same as the perpendicular height of the parallelogram
- the area of the triangle is exactly half that of the parallelogram.

Worked example

Deducing the area of a triangle from the area of a parallelogram

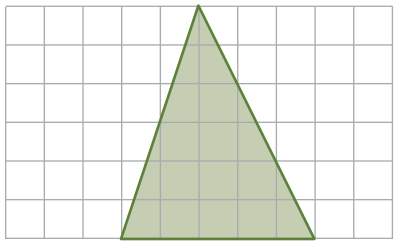
For each of the triangles:

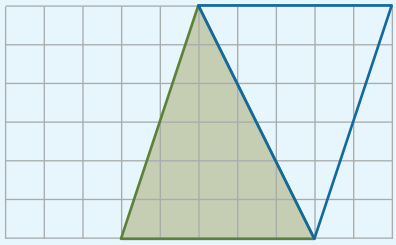
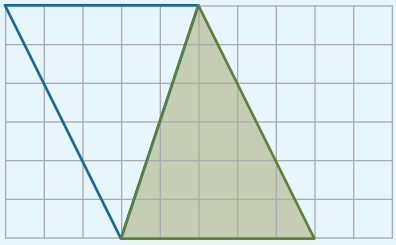
(i) construct a parallelogram with double the area of the triangle

(ii) calculate the area of the parallelogram

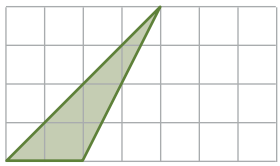
(iii) calculate the area of the triangle.

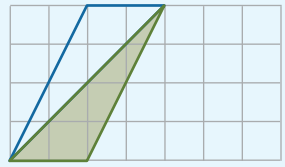
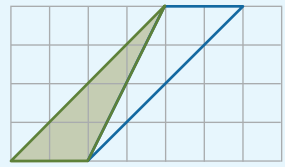
(a)



THINKING	WORKING
Construct a line equal and parallel to the base from the opposite vertex. Complete the parallelogram.	
Alternately, construct a line to the left of the vertex. Complete the parallelogram.	
Calculate the area of the parallelogram.	$A = bh$ $= 5 \times 6$ $= 30$ The area of the parallelogram is 30 square units.
Calculate the area of the triangle.	$\frac{30}{2} = 15$ The area of the triangle is 15 square units.

(b)



THINKING	WORKING
Construct a line equal and parallel to the base from the opposite vertex. Complete the parallelogram.	
Alternately, draw a line to the right of the vertex. Complete the parallelogram.	
Calculate the area of the parallelogram.	$A = bh$ $= 2 \times 4$ $= 8$ The area of the parallelogram is 8 square units.
Calculate the area of the triangle.	$\frac{8}{2} = 4$ The area of the triangle is 4 square units.

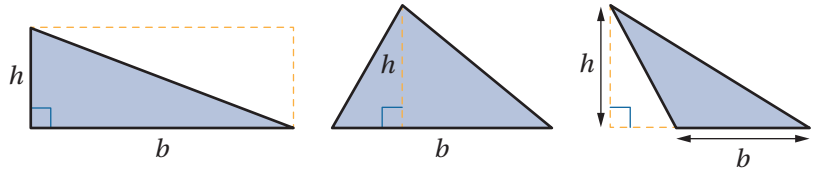
SC 3 I can calculate the area of a triangle using a formula

Multiplying the base by the height gives the area of a rectangle or parallelogram. Halving this area gives the area of the triangle.

The area of a triangle can be calculated by multiplying the length of its base with its perpendicular height, then halving the result.

The base can be any side of the triangle.

The perpendicular height is measured from the chosen base to the opposite vertex.



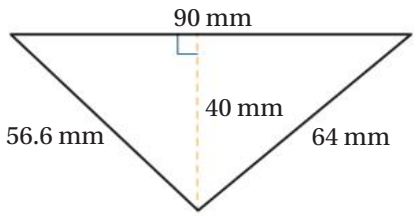
Area of a triangle: $\frac{1}{2}(\text{base} \times \text{height})$ so $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

Worked example

Calculating the area of a triangle from a formula

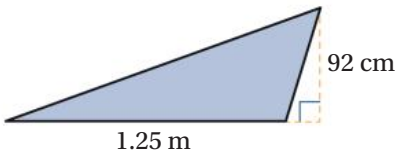
Use a formula to calculate the area of each triangle.

(a)



THINKING	WORKING
Identify a base and a perpendicular height.	Base: $b = 90$ mm Perpendicular height: $h = 40$ mm
Recall the formula for the area of a triangle.	$A = \frac{1}{2}bh$
Substitute into the triangle area formula and calculate.	$A = \frac{1}{2} \times 90 \times 40$ $= 1800$
Write the answer.	The area of the triangle is 1800 mm^2.

(b)



THINKING	WORKING
Convert lengths to be in the same unit of measurement.	Base: 1.25 m = 125 cm
Recall the formula for the area of a triangle.	$A = \frac{1}{2}bh$
Substitute into the triangle area formula and calculate.	$A = \frac{1}{2} \times 125 \times 92$ $= 5750$ cm^2
Write the answer.	The area of the triangle is 5750 cm^2 or 0.575 m^2.

Practice

ANSWERS Page 702

SC 1 I can determine the area of a triangle as half of a rectangle

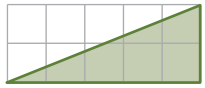
1 For each right-angled triangle below:

(i) copy the triangle onto your own grid and construct a related rectangle with double the area of the triangle

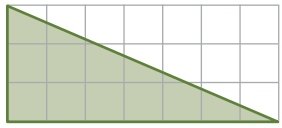
(ii) calculate the area of the rectangle

(iii) calculate the area of the triangle.

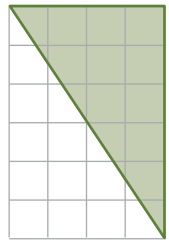
(a)



(b)



(c)



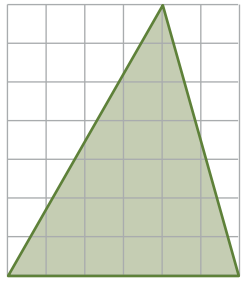
2 For each triangle below:

(i) copy the triangle onto your own grid and construct a related rectangle with double the area of the triangle

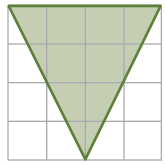
(ii) calculate the area of the rectangle

(iii) calculate the area of the triangle.

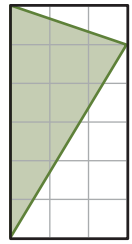
(a)



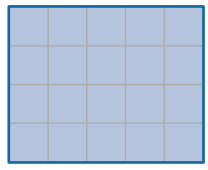
(b)



(c)



3 Here is a rectangle with a length of 5 cm and a width of 4 cm.



(a) Inside the rectangle, draw a triangle with a base of 5 cm and a height of 4 cm:

(i) using one line and shading

(ii) using two lines and shading.

(b) Calculate the area of the rectangle.

(c) Calculate the area of each shaded triangle.

4 The three side lengths of some right-angled triangles are given below. For each right-angled triangle:

(i) determine the dimensions of the perpendicular sides

(ii) calculate the area of the triangle.

Hint: Before beginning, think about which side in a right-angled triangle is the longest.

(a) 5 cm, 12 cm, 13 cm

(b) 60 mm, 80 mm, 100 mm

(c) 2 m, 2.1 m, 2.9 m

5 The base and height of some triangles are given. For each triangle:

(i) determine the area of a rectangle sharing the same base and height

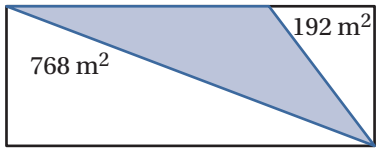
(ii) calculate the area of the triangle.

(a) Base: 12 cm; height: 8 cm

(b) Base: 27 mm; height: 42 mm

(c) Base: 1.8 m; height: 70 cm

6 Determine the shaded area.



SC 2 I can determine the area of a triangle as half of a parallelogram

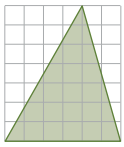
1 For each triangle below:

(i) copy the triangle onto your own grid, then use two lines to complete a parallelogram with double the area of the triangle

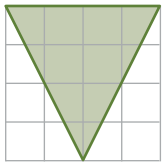
(ii) calculate the area of the parallelogram

(iii) calculate the area of the triangle.

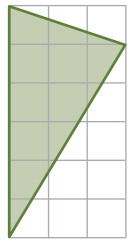
(a)



(b)



(c)



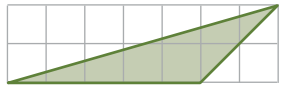
2 For each triangle below:

(i) copy the triangle onto your own grid, then use two lines to complete a parallelogram with double the area of the triangle

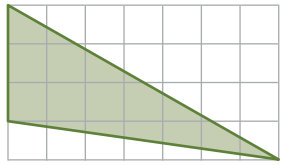
(ii) calculate the area of the parallelogram

(iii) calculate the area of the triangle.

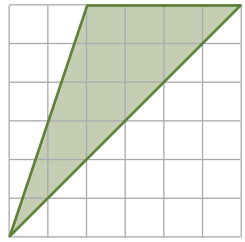
(a)



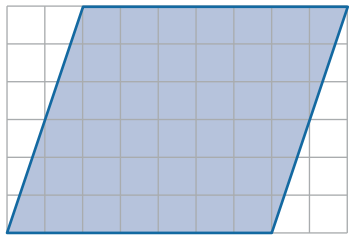
(b)



(c)



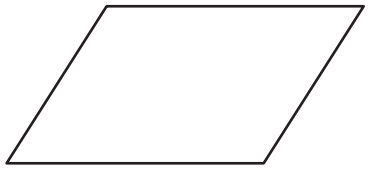
3 Copy the parallelogram diagram twice.



(a) On each copy:
- draw a diagonal (different each time)
- shade one of the triangles
- mark the base and height of the shaded triangle.

(b) Complete the conclusion: A triangle with the same _____ and _____ as a parallelogram has _____ the area of the parallelogram.

4 Copy the parallelogram diagram twice.



On each copy:
- draw a diagonal (different each time)
- shade one of the triangles
- label the base and height of the shaded triangle.

5 The base and height of some triangles are given below. For each triangle:

(i) determine the area of a parallelogram with the same base and height as the triangle

(ii) calculate the area of the triangle.

(a) Base: 25 cm; height: 16 cm

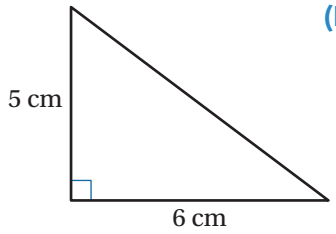
(b) Base: 40 mm; height: 33 mm

(c) Base: 50 cm; height: 2.1 m

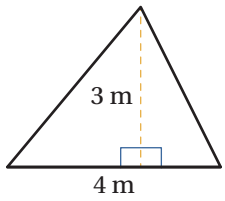
SC 3 I can calculate the area of a triangle using a formula

1 Calculate the area of each triangle.

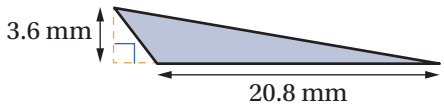
(a)



(b)

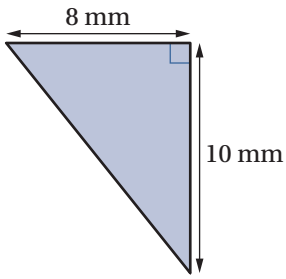


(c)

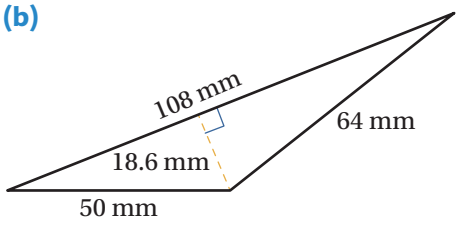


2 Calculate the area of each triangle.

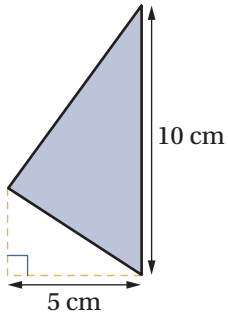
(a)



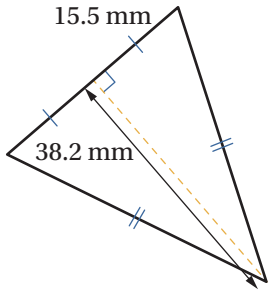
(b)



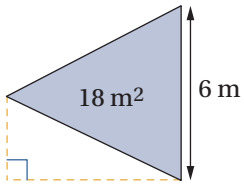
(c)



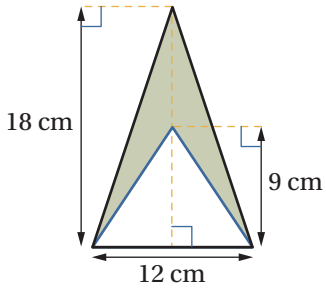
3 Determine the area of the isosceles triangle.



4 Determine the perpendicular height of the triangle. The base is already shown on the diagram.



5 Determine the shaded area.



6 The perimeter of a right-angled triangle is 1.2 m and two of its sides are 48 cm and 52 cm. Determine the area of the triangle.

12.4 Understand volume measured in cubic units

Learning intention: To be able to understand volume measured in cubic units

Success criteria:

SC 1 I can determine the volume of a solid by counting and visualising unit cubes.

SC 2 I can create rectangular prisms with unit cubes.

SC 3 I can determine the volume of rectangular prisms.

Lesson warm-up

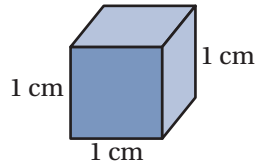
Make that object

Using 20 cubes, create as many rectangular prisms as possible. Take note of the length, width and height of the rectangular prism made.

SC 1 I can determine the volume of a solid by counting and visualising unit cubes

Volume is a three-dimensional measure of the space taken up by an object.

Volume is measured in cubic units. The volume of an object in cubic centimetres gives the number of unit cubes of edge length 1 cm needed to produce an equivalent volume.

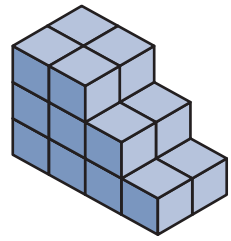


The volume of an object made of unit cubes can be found by counting the cubes.

Worked example

Counting cubes to determine the volume of a prism

Determine the volume of the prism, given that the edge of each unit cube is 1 cm.



THINKING	WORKING
Count the cubes in the front layer.	Front: 9 unit cubes
State the number of identical layers.	Layers: 2
Calculate the total number of unit cubes.	Total: $9 \times 2 = 18$
Write the volume in cubic units.	The volume is 18 cm^3.

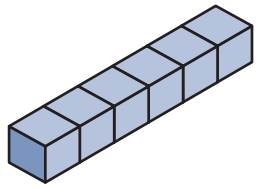
SC 2 I can create rectangular prisms with unit cubes

A rectangular prism with whole-number edges can be created by placing cubes in rows side-by-side to form a rectangular layer, then adding identical layers vertically.

Worked example

Creating a rectangular prism

A square-based prism is made from 6 unit cubes of edge length 1 cm.



(a) Four more identical square-based prisms are placed beside the first one, forming a rectangular layer. How many unit cubes are there in the layer?

THINKING	WORKING
State the number of rows and columns of cubes.	Rectangular layer: 6 by (1 + 4 = 5)
Calculate the number of cubes.	$6 \times 5 = 30$ There are 30 unit cubes.

(b) Three more identical layers are placed above the first one, forming a rectangular prism. What are the dimensions of the prism?

THINKING	WORKING
State the number of layers.	$1 + 3 = 4$ There are 4 layers.
State the dimensions of the prism.	Dimensions: 6 cm by 5 cm by 4 cm

(c) How many unit cubes are there altogether?

THINKING	WORKING
Calculate the number of cubes.	$30 \times 4 = 120$ There are 120 unit cubes.

SC 3 I can determine the volume of rectangular prisms

The volume of a rectangular prism made from unit cubes can be calculated by:

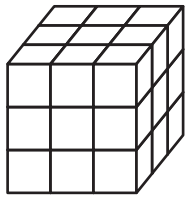
- determining the number of unit cubes, then
- the volume is that same number of cubic units.

Worked example

Determining the volume of a rectangular prism by layering unit cubes

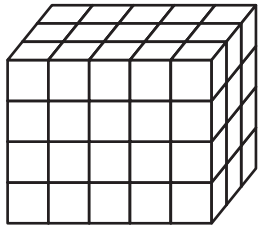
Determine the volume of each rectangular prism made up of unit cubes of edge length 1 cm.

(a)



THINKING	WORKING
Determine the number of unit cubes in each layer.	Each layer: $3 \times 3 = 9$
Calculate the total number of unit cubes by multiplying by the number of layers.	Total: $9 \times 3 = 27$
Write the answer.	Volume: 27 cm^3

(b)



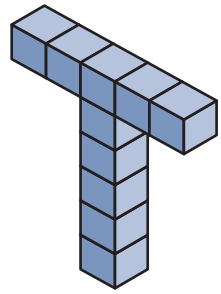
THINKING	WORKING
Determine the number of unit cubes in each layer.	Each layer: $5 \times 3 = 15$
Calculate the total number of unit cubes by multiplying by the number of layers.	Total: $15 \times 4 = 60$
Write the answer.	Volume: 60 cm^3

Practice

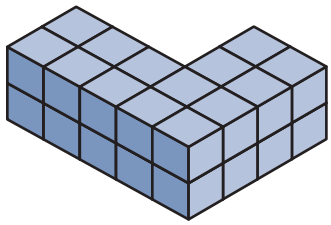
ANSWERS Page 704

SC 1 I can determine the volume of a solid by counting and visualising unit cubes

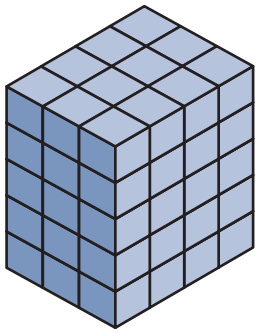
1 Determine the volume of the solid, given that the edge of each unit cube is 1 cm.



2 Determine the volume of the solid, given that the edge of each cube is 1 cm.

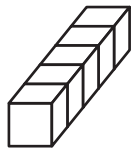


3 Determine the volume of the solid, given that the edge of each cube is 1 cm.



SC 2 I can create rectangular prisms with unit cubes

1 A square-based prism is made from 5 unit cubes of edge length 1 cm.

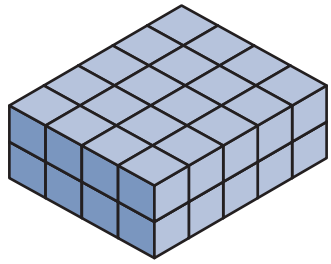


(a) Three more identical square-based prisms are placed beside the first one, forming a rectangular layer. How many unit cubes are there in the layer?

(b) Four more identical layers are placed above the first one, forming a rectangular prism. What are the dimensions of the prism?

(c) How many unit cubes are there altogether?

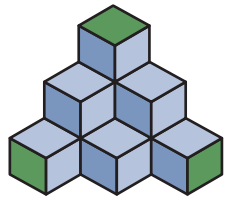
2 A prism is made from unit cubes of edge length 1 cm.



(a) An identical prism is placed on top of this one to form a larger rectangular prism. What are the dimensions of the new prism?

(b) How many unit cubes will there be altogether?

3 Unit cubes, some with one, two or three faces painted green, are being used to build a cube with all exterior faces green.

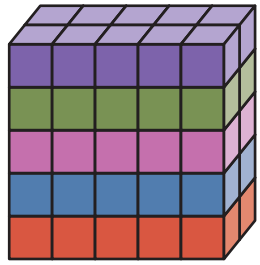


(a) How many additional cubes are needed?

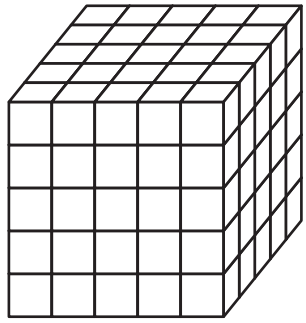
(b) How many of the additional cubes have at least one green face?

SC 3 I can determine the volume of rectangular prisms

1 Calculate the volume of the prism, given that each unit cube shown is a centimetre cube.

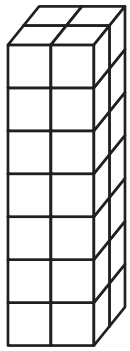


2 Determine the volume of the cube made up of unit cubes of edge length 1 cm.

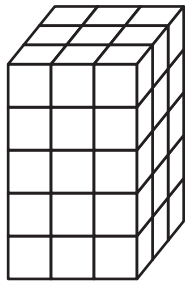


3 Determine the volume of each prism made up of unit cubes of edge length 1 cm.

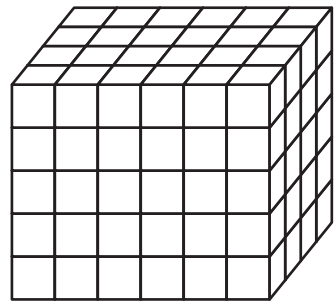
(a)



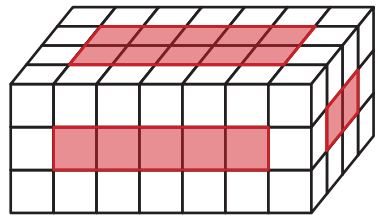
(b)



(c)



4 A solid rectangular prism (7 cm × 4 cm × 3 cm) has the middle sections of cubes removed by cutting through from one face to the opposite (parallel) face. Determine the volume of the remaining solid.



Establish the volume formula and units for prisms

12.5

Learning intention: To be able to establish the volume formula and units for prisms

Success criteria:

SC 1 I can apply the rectangular prism volume formula.

SC 2 I can determine the volume of any solid of constant cross-sectional area.

SC 3 I can determine the volume of a slanted prism.

Lesson warm-up

Shapes and cubes

Use unit cubes to approximate the volume of prisms in your classroom. Describe the calculation required to determine the volume in each case.

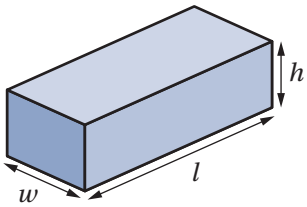
SC 1 I can apply the rectangular prism volume formula

When the edge lengths of a rectangular prism are whole numbers, it is possible to visualise unit cubes in rows, columns and layers to determine the volume of the prism.

The number of unit cubes is found by multiplying the number in each row by the number in each column on the top (to get the number in that layer), then multiplying by the number of layers.

Total number of cubes = number of columns × number of rows × number of layers

The volume of any rectangular prism can be found when all three dimensions are known.



Volume = length × width × height or $V = lwh$, where:

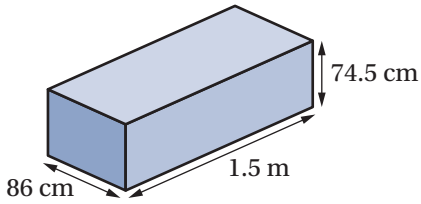
- length, width and height must be in the same units and
- volume will be in the corresponding cubic units.

Note: the product of three numbers is the same no matter the order of multiplication, hence it does not matter which edge is allocated the labels of length, width and height.

Worked example

Calculating the volume of a rectangular prism using a formula

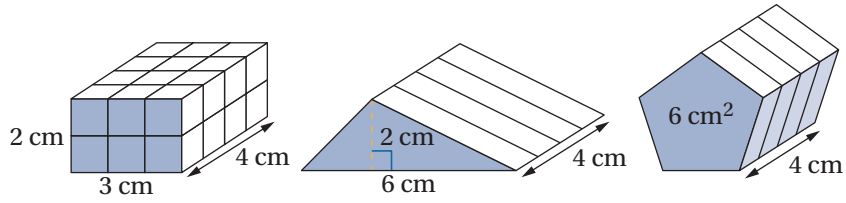
Use a formula to calculate the volume of the rectangular prism.



THINKING	WORKING
Write equivalent lengths so that all units are the same.	1.5 m = 150 cm
Allocate values for length, width and height.	$l = 150$ cm, $w = 86$ cm, $h = 74.5$ cm
Write the formula for volume. Substitute the values. Calculate.	$V = lwh$ $= 150 \times 86 \times 74.5$ $= 961\,050$
Write the volume with cubic units.	The volume is 961 050 cm^3.

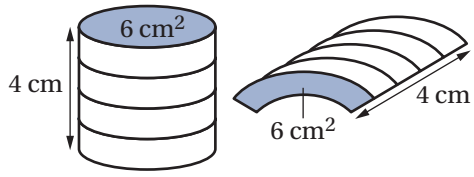
SC 2 I can determine the volume of any solid of constant cross-sectional area

The three shapes below are all right prisms with the same constant cross-sectional area and height.



Hence, these prisms will have the same volume.

The two shapes below are not prisms (because of the curved bases) yet have the same constant cross-sectional area and height.

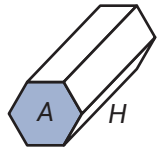


Hence, these solids will have the same volume as the three prisms.

To calculate the volume of shapes with constant cross-sectional (base) area, multiply the area of the base by the height of the prism.

Volume = area of base × height or $V = AH$, where:

- area and height must be in related units (such as cm^2 and cm) and
- volume will be in the corresponding units (cm^3).

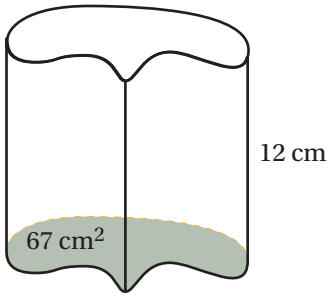


Worked example

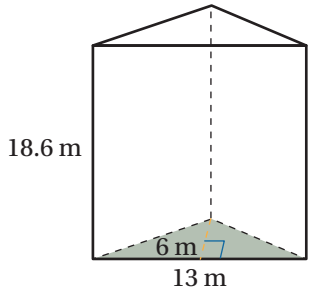
Calculating the volume of any solid of constant cross-sectional area

Calculate the volume of the solids.

(a)



(b)



(a)

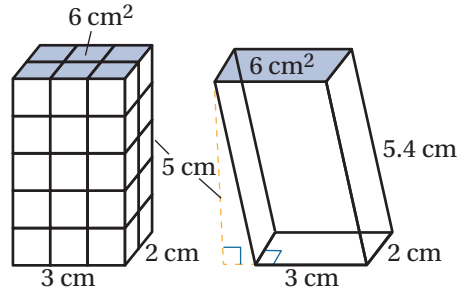
THINKING	WORKING
Identify the given information. Write using pronumerals.	Base area: $A = 67$ cm^2 Height: $H = 12$ cm
Write the formula for volume. Substitute the values. Calculate.	$V = AH$ $= 67 \times 12$ $= 804$
Write the volume with correct units.	The volume is 804 cm^3.

(b)

THINKING	WORKING
Identify the given information for the base. Write using pronumerals.	Triangle: Base: $b = 13$ m; Height: $h = 6$ m
Write the formula for the area of the base. Substitute the values. Calculate the area including units.	$A = \frac{bh}{2}$ $= \frac{13 \times 6}{2}$ $= 39$ m^2
Identify the height of the prism. Write using pronumerals.	Height: $H = 18.6$ m
Write the formula for volume. Substitute the values. Calculate.	$V = AH$ $= 39 \times 18.6$ $= 729.4$
Write the volume with correct units.	The volume is 729.4 m^3.

SC 3 I can determine the volume of a slanted prism

It is usual to assume that a given rectangular prism is a 'right' rectangular prism with right angles at all corners.



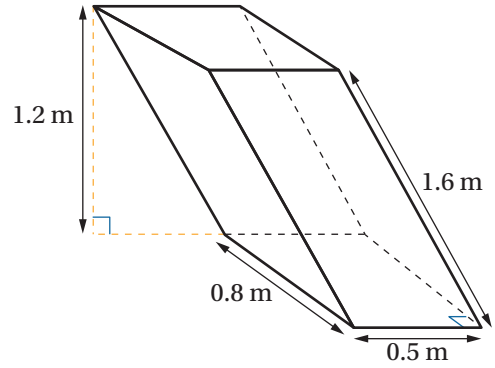
The slanted figure above is technically a rectangular prism because its base is a rectangle and there is a constant cross-sectional area. Its volume can still be found using $V = lwh$, but the altitude (or perpendicular height) would be 5 cm, not 5.4 cm.

Note: Both figures have the volume $3 \times 2 \times 5 = 30$ cm^3

Worked example

Calculating the volume of a slanted prism of constant cross-sectional area

Calculate the volume of the rectangular prism.



THINKING	WORKING
Identify the relevant information. Write using pronumerals.	Base length: $l = 0.8$ m Base width: $w = 0.5$ m Height of prism: $h = 1.2$ m
Recall the formula for the volume of a prism.	$V = lwh$
Substitute the values into the formula and calculate the volume.	$V = lwh$ $= 0.8 \times 0.5 \times 1.2$ $= 0.48$
Write the answer.	The volume is 0.48 m^3.

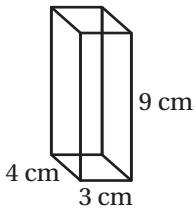
Practice

ANSWERS Page 704

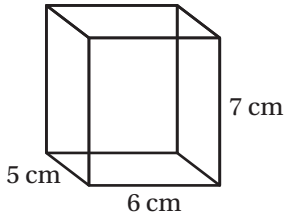
SC 1 I can apply the rectangular prism volume formula

1 Determine the volume of each prism by visualising unit cubes.

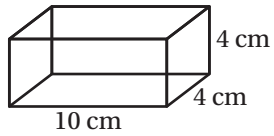
(a)



(b)



(c)

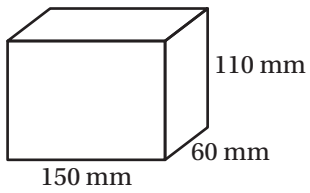


2 The dimensions of a rectangular prism are 1.5 m by 65 cm by 40 cm. Determine the volume of the prism in cubic centimetres.

3 A serving tray is in the shape of a rectangular prism and has a volume of 180 cm^3. The base has dimensions of 20 cm by 6 cm. Determine the height of the serving tray.

4 A carton contains 18 boxes of soap.

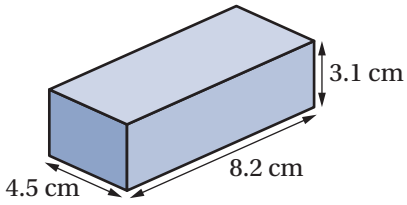
(a) Given that the dimensions of each soap box are 150 mm by 60 mm by 110 mm, what is the volume of the carton?



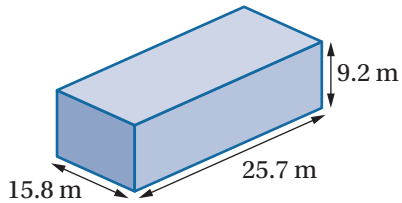
(b) The soap boxes are stacked in two layers in the carton. What are the most likely dimensions of the carton (in millimetres)?

5 Calculate the volume of each rectangular prism.

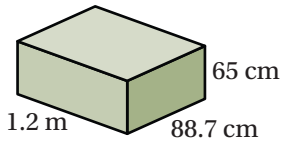
(a)



(b)

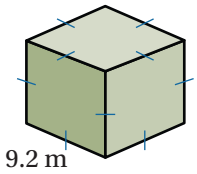


(c)

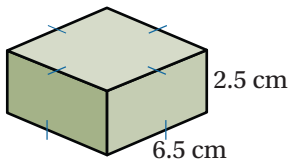


6 Calculate the volume of each special rectangular prism.

(a)



(b)



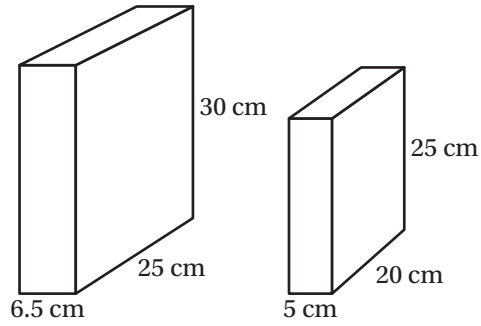
7 The smallest postal box available at the post office has dimensions 22 cm by 16 cm by 7.7 cm. The next size up is 31 cm by 22.5 cm by 10.2 cm. What is the difference in volume between the two postal boxes?

8 A solid concrete bath has outer dimensions as shown. The volume of the inner hollow section of the bath is 0.7161 m^3. What is the volume of concrete needed to make the bath?

9 A packaging company needs new cartons that are rectangular prisms with a volume of 162 cm^3. The base of each carton must be a square with a side length of 6 cm. Determine the height of the new carton.

10 A cubic centimetre of gold has a mass of 19.3 grams. This can be described as a density of 19.3 g/cm^3. Given that gold costs approximately $42 per gram, calculate the cost of a gold bar in the shape of a rectangular prism 4 cm high, 6 cm wide and 26.75 cm long.

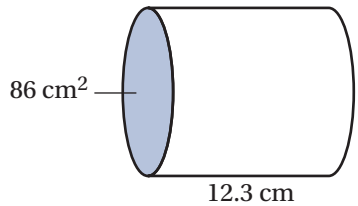
11 Two cereal boxes are rectangular prisms and have the dimensions shown. The larger box costs $9.75. Both boxes have the same cost per cubic centimetre of volume. Determine the cost of the smaller box.



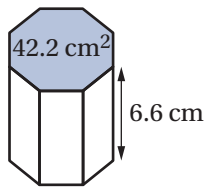
SC 2 I can determine the volume of any solid of constant cross-sectional area

1 Calculate the volume of each solid.

(a)

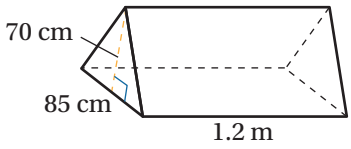


(b)

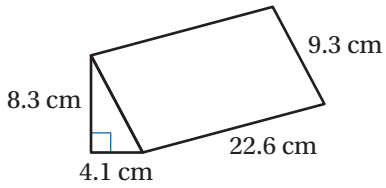


2 Calculate the volume of the triangular prism.

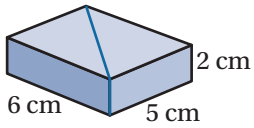
(a)



(b)

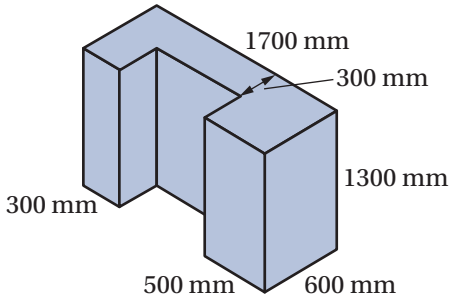


3 A rectangular prism has been cut in half as shown.



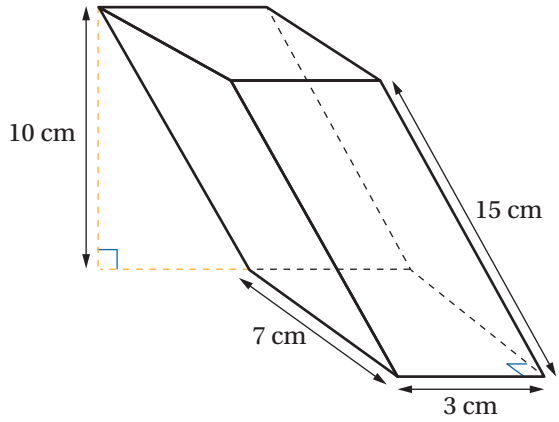
Describe the shape of one part and determine its volume.

4 Calculate the amount of concrete, in cubic metres, needed to build the barbecue.

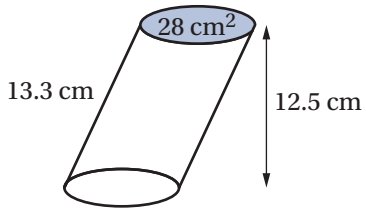


SC 3 I can determine the volume of a slanted prism

1 Calculate the volume of the rectangular prism.



2 Calculate the volume of the slanted cylinder.



12.6 Explore the connection between volume and capacity

Learning intention: To explore the connection between volume and capacity

Success criteria:

SC 1 I can determine the most suitable units of measurement for given containers.

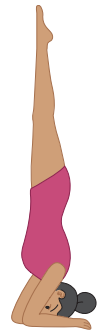
SC 2 I can estimate volume and capacity.

SC 3 I can determine the capacity of containers.

Lesson warm-up

Milk as a unit of measurement

Measure or estimate the height, length and width of a 1 litre milk carton and determine the volume of the milk carton. Comparing the volume and capacity (1 L), what conclusion can you make?

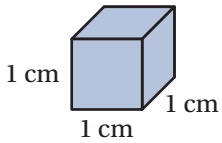


SC 1 I can determine the most suitable units of measurement for given containers

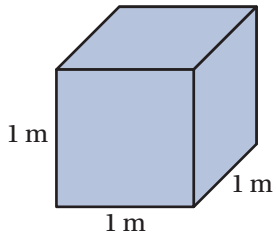
The space inside a container can be given as a volume, usually in cm^3 for containers in a household, or m^3 for tankers, swimming pools or reservoirs.

The inside volume of a container is its capacity and the units of millilitres, mL; litres, L; and kilolitres, kL are used (and megalitres, ML, for large reservoirs).

Two of the units for capacity match the volume of a unit cube.

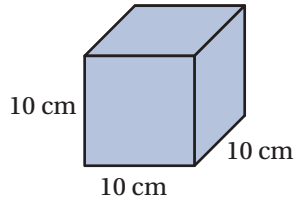


$1\ cm^3$ is equivalent to 1 mL.



$1\ m^3$ is equivalent to 1 kL.

Because there are a million millilitres in a kilolitre, it makes sense that there has to be a useful unit in between. Although it is not a unit cube, a cube of side 10 cm has a volume equivalent to a capacity of 1 L.



$1000\ cm^3$ is equivalent to 1 L.

To better understand the units of volume, it is necessary to have benchmarks or a visual idea of what common units look like.

The smallest commonly used litre-based unit is the millilitre (mL).	All units are based on litre (L).	1000 L = 1 kL
About 20 drops of liquid make 1 mL. 	Milk is often purchased in a 1 L carton or a 1 L bottle. 	The recycling bin provided to all homes by the council holds about 240 L. So, 1 kL is just a bit more than four of these bins.

Worked example

Choosing units of capacity

State the most suitable unit of capacity for each situation: mL, L, kL or ML.

(a) coffee mug

THINKING	WORKING
Compare to known benchmarks.	Drops of liquid (mL), carton of milk (L),
Choose an appropriate unit.	Coffee mug: millilitres (mL)

(b) Sydney Harbour

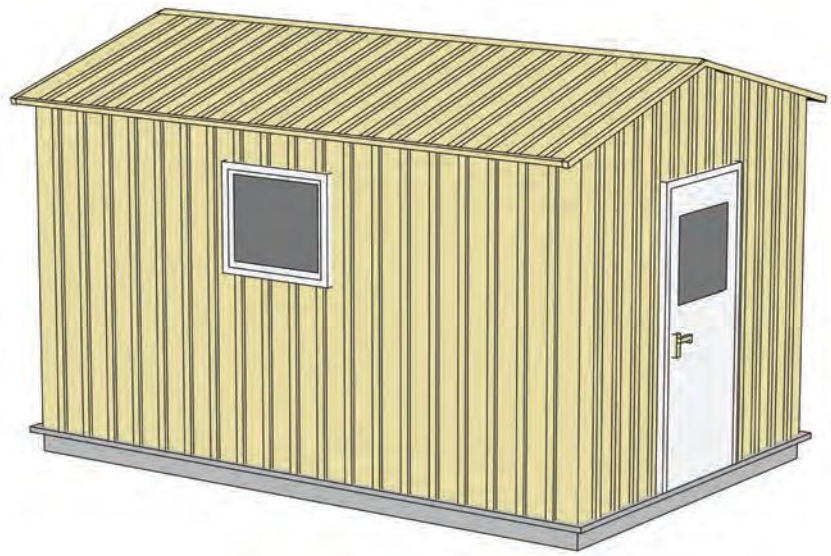
THINKING	WORKING
Use a benchmark to estimate the capacity.	Swimming pool (kL), large body of water (ML)
Choose an appropriate unit.	Sydney Harbour: megalitres (ML)

SC 2 I can estimate volume and capacity

Worked example

Estimating volume and capacity

(a) Estimate the volume of the shed.



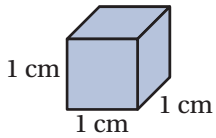
THINKING	WORKING
Estimate length, width and height of an equivalent box shape, all in the same units.	Length: 3 m Width: 2 m Height: 2 m
Recall the formula to calculate the volume of a rectangular prism.	$V = lwh$
Substitute the values and calculate volume using the estimated dimensions.	$V = lwh$ $= 3 \times 2 \times 2$ $= 12$
Write the answer.	The volume is roughly 12 m^3.

(b) A can of soft drink holds 375 mL. Estimate the capacity of a small kitchen glass.

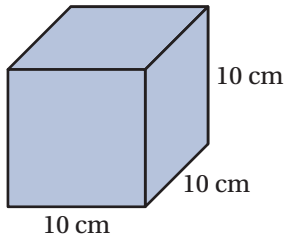
THINKING	WORKING
Compare the known and unknown capacities.	The glass will hold about half a can of soft drink.
Calculate using the estimated comparison.	$\frac{375}{2} = 187.5$
Write the answer.	The capacity of the glass is about 200 mL.

SC 3 I can determine the capacity of containers

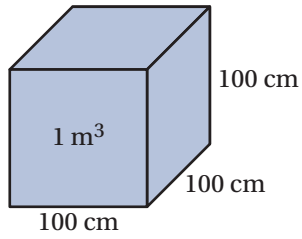
Given that the internal volume of space inside a container can be calculated, the capacity is found by a simple conversion.



$1\ cm^3 = 1\ mL$



$1000\ cm^3 = 1\ L$



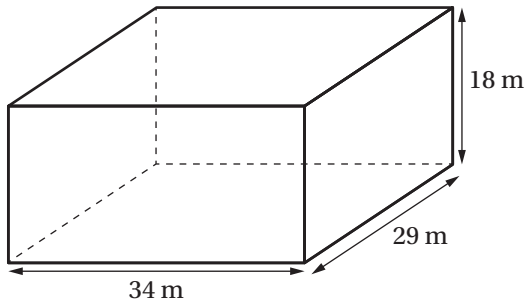
$1\ m^3 = 1\ kL$

Worked example

Calculating capacity

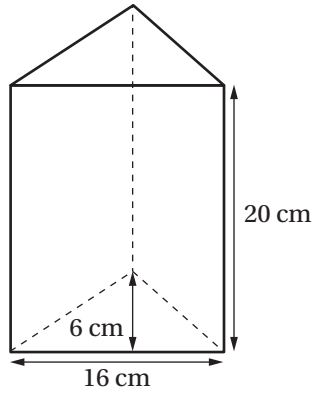
Calculate the capacity of each container.

(a)



THINKING	WORKING
Calculate the volume.	$l = 34$ m, $w = 29$ m, $h = 18$ m $V = lwh$ $= 34 \times 29 \times 18$ $= 17\ 748\ m^3$
Choose a suitable conversion to capacity.	$1\ m^3 = 1\ kL$
Convert to capacity units.	The capacity is 17 748 kL.

(b)



THINKING	WORKING
Calculate the area of the base.	$b = 16$ cm, $h = 6$ cm $A = \frac{bh}{2}$ $= \frac{16 \times 6}{2}$ $= 48 \text{ cm}^2$
Calculate the volume.	$H = 20$ cm $V = AH$ $= 48 \times 20$ $= 960 \text{ cm}^3$
Choose a suitable conversion to capacity.	$1 \text{ cm}^3 = 1$ mL and $1000 \text{ cm}^3 = 1$ L Just under a litre, so either unit is suitable.
Convert to capacity units.	The capacity is 960 mL or 0.96 L.

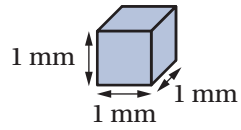
Practice

ANSWERS Page 704

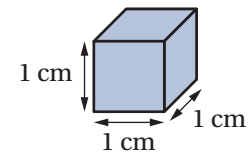
SC 1 I can determine the most suitable units of measurement for given containers

1 State the volume of each of the following unit cubes.

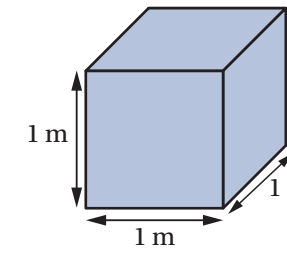
(a)



(b)



(c)



2 For each object below, state the most suitable unit of volume.

(a)

(b)

(c)

3 Around the home, some other measures of capacity have become common and standardised to metric measures.

A metric cup is 250 mL.

A metric tablespoon is 20 mL.

A metric teaspoon is 5 mL.

A standard eyedropper dispenses 0.05 mL per drop, so there are 20 drops in a millilitre.

Calculate the volume in millilitres for each of the following.

(a) $\frac{1}{2}$ a cup of sugar **(b)** 3 tablespoons of butter

(c) 6 cups of vegetable stock **(d)** 10 drops of vanilla essence

SC 2 I can estimate volume and capacity

1 The volume of metal pinheads range from about 0.5 mm^3 to 2 mm^3. Estimate the volume of a regular-sized matchstick head.

2 The volume of a regular dishwasher is about 0.3 m^3. Estimate the volume of a regular-sized microwave oven.

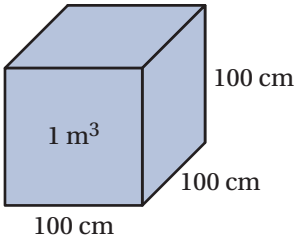
3 A small saucepan can hold a maximum of 1.5 L of water. Estimate the capacity of a large saucepan.

4 A standard laundry bucket holds 10 L. Estimate the capacity of a child's beach sand bucket.

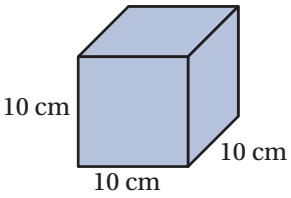
SC 3 I can determine the capacity of containers

1 Calculate the capacity of each container.

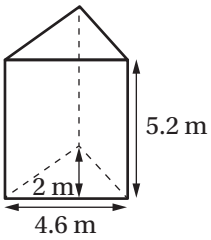
(a)



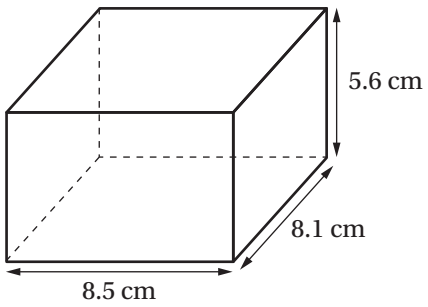
(b)



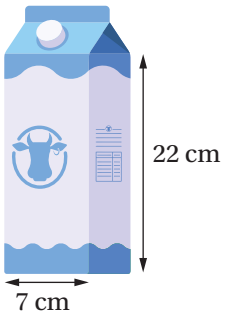
(c)



(d)



2 A carton of milk has a square base of 7 cm and a maximum milk height of 22 cm.



(a) Calculate the capacity of the carton in millilitres and litres.

(b) A 125 mL carton of cream is made with the same base. What is the height of the cream in a full carton, to the nearest millimetre?

3 A fish tank is a rectangular prism with a base that is 62.5 cm long and 50 cm wide. The tank is filled with 100 L of water. What height will the water level reach?

4 Jenya has a vase that is a square-based prism. The vase is made from glass that is 1 cm thick at each side. The outside dimensions of the vase are 10 cm × 10 cm × 33 cm. Calculate the capacity of Jenya's vase.

TOPIC REVIEW

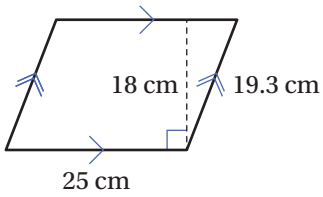
Multiple choice

ANSWERS Page 745

1 A parallelogram of area 24 cm^2 and height 12 cm has a base length of:

A 2 cm **B** 4 cm **C** 8 cm **D** 6 cm

2 The area of the parallelogram shown is:



A 450 cm^2 **B** 62.3 cm^2 **C** 86 cm^2 **D** 482.5 cm^2

3 A parallelogram with a base of 13 cm, and a height of 3 cm has an area of:

A 32 cm^2 **B** 39 cm^2 **C** 3.2 cm^2 **D** 26 cm^2

4 What is the area of a triangle with a base of 8 cm and a height of 3 cm?

A 24 cm^2 **B** 12 cm^2 **C** 12 cm **D** 11 cm

5 Which of the following is NOT a correct formula for the area of a triangle?

A $A = \frac{1}{2}bh$

B $\text{Area} = \frac{\text{base} \times \text{height}}{2}$

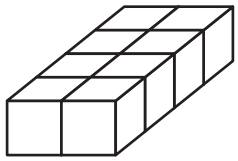
C Area = 0.5 × base × height

D Area = base × height

6 A right-angled triangle has area 24 m^2 and base length 4 m. Its height is:

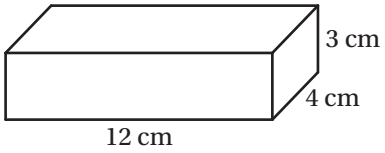
A 3 m **B** 6 m **C** 12 m **D** 30 m

7 Assuming that each cube represents 1 cm^3, the volume of the following object is:



A 8 cm^3 **B** 4 cm^3 **C** 10 cm^3 **D** 12 cm^3

8 The volume of the following rectangular prism is:



A 19 cm^3 **B** 144 cm^3 **C** 24 cm^3 **D** 144 cm^2

9 A rectangular prism has a volume of 420 cm^3. Given that its height is 5 cm and its width is 7 cm, what is its length?

A 4 cm **B** 6 cm **C** 10 cm **D** 12 cm

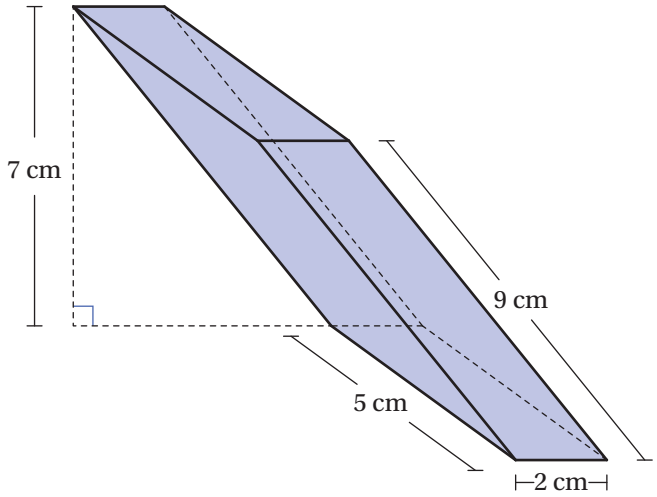
10 The volume of a rectangular prism with a width of 3.8 cm, a height of 4 cm and a depth of 8 cm is:

A 364.8 cm^3 **B** 121.6 cm^3 **C** 115.5 cm^3 **D** 23.2 cm^3

11 The volume of a rectangular prism with a height of 2 cm, a width of 3 cm and a length of 4 cm is:

A 8 cm^3 **B** 16 cm^3 **C** 24 cm^3 **D** 48 cm^3

12 Choose the correct calculation for the volume of this rectangular prism in cubic centimetres.



A Volume = $2 \times 5 \times 7$ **B** Volume = $2 \times 5 \times 9$

C Volume = $2 \times 5 \times 9 \times 7$ **D** Volume = $5^2 \times 7$

13 When a cube with edge length 10 cm is filled with water it will hold:

A 10 mL **B** 10 L **C** 1 L **D** 40 L

Short answer

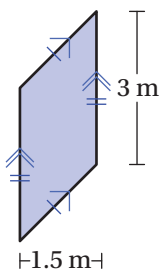
1 Calculate the area of parallelograms that have:

(a) a base length of 22 cm and a height of 14 cm

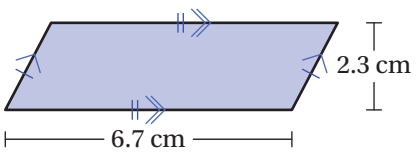
(b) a base length of $10\frac{1}{4}$ cm and a height of $6\frac{3}{8}$ cm.

2 Calculate the area of the parallelograms.

(a)

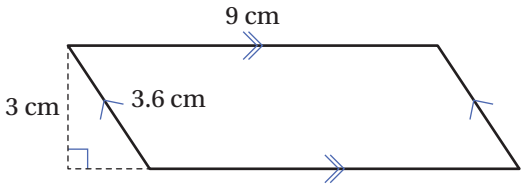


(b)

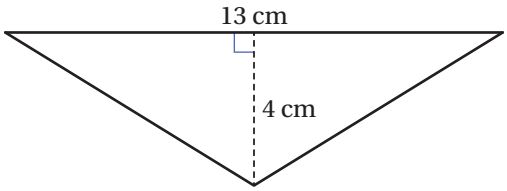


3 A parallelogram has a base length of 8 cm and an area of 72 cm^2. What is the height of the parallelogram?

4 Calculate the area of the parallelogram.



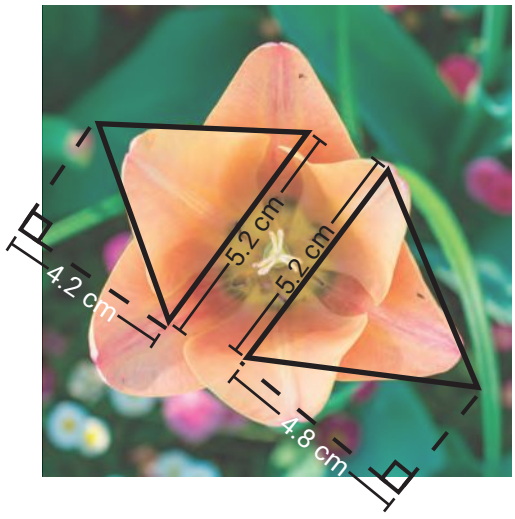
5 Calculate the area of the triangle.



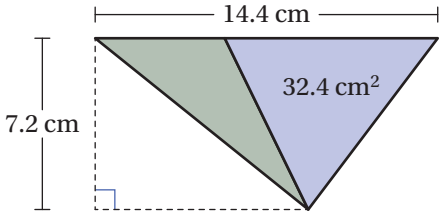
6 A triangle has a base of 7 cm and a height of 3.5 cm. What is the area of the triangle?

7 A beach umbrella is made of 8 triangular pieces of fabric, arranged to form a shady octagon. Each triangular piece has a base of 1.1 m and a height of 1.4 m. Determine the area of fabric needed to make the beach umbrella, assuming there is 20% wastage, correct to 1 decimal place.

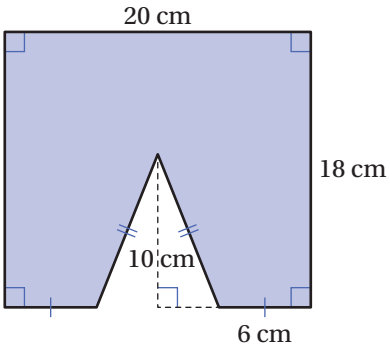
8 Some tulip flowers have petals that are approximately triangular. By treating the tulip petals in the picture as triangles, calculate the approximate difference in area between the inner and outer petals.



9 Calculate the area of the green triangle on the left.



10 Calculate the shaded area of the given shape.

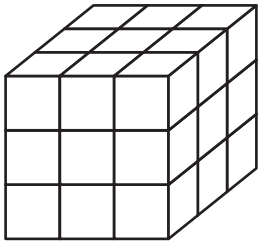


11 When the wind hits the sails of a yacht, the boat is pushed along the water. Triangles have been drawn on this image of a yacht to estimate the area of the two sails.

(a) Calculate the total area of the sails.

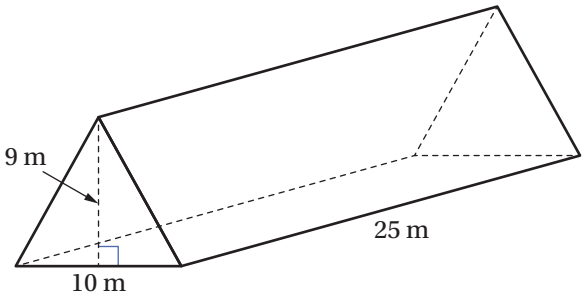
(b) Calculate the cost of making the sails, given the rate is \$98/m^2.

12 Determine the volume of the rectangular prism. (Each small cube represents 1 cm^3.)



13 A rectangular prism has a volume of 105 cm^3. Given that its length is 7 cm and its width is 5 cm, what is its height?

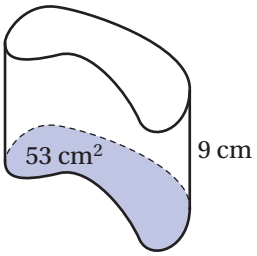
14 Calculate the volume of the triangle-based prism.



15 The capacity of a baking dish determines how much food can fit in the dish. Olly's baking dish has an approximately square base with side lengths of 22 cm and a depth of 8 cm. Calculate the capacity of Olly's baking dish in millilitres.

16 Calculate the capacity, in litres, of a rectangular prism packing box with side lengths of 52 cm, 38 cm and 56 cm.

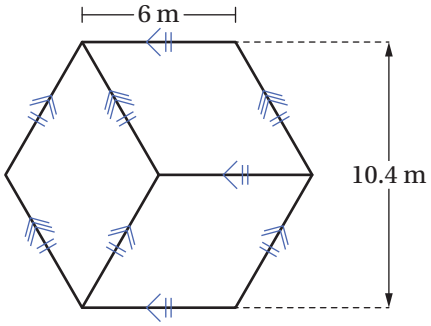
17 Calculate the capacity of the container.



18 Ahmad has a square-based jar that is 18 cm high. The base has a side length of 7 cm. Tamara has a rectangular prism-shaped jar that is 12 cm high. The base has a width of 10 cm and a depth of 3 cm. The glass of both jars is 5 mm thick. Which jar holds more, and by how much?

Extended response

1 This hexagon has all sides the same length. It is made from three identical parallelograms.



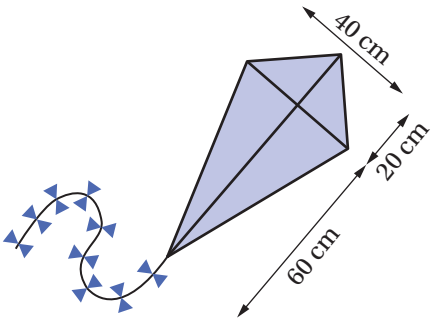
(a) Calculate the area of the hexagon.

(b) If a rectangle of the same height and area was placed beside the hexagon, how wide would the rectangle be?

(c) If a parallelogram of the same height and area was placed beside the hexagon, how wide would the parallelogram be?

(d) If a triangle of the same height and area was placed beside the hexagon, how wide would the triangle be?

2 A kite made of fabric is shown, with 9 bows attached to the tail.



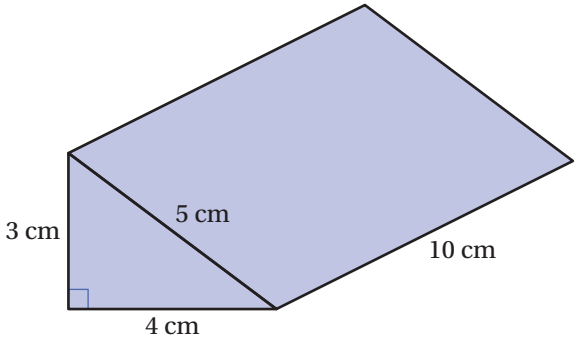
(a) Calculate the area of fabric in the kite in m^2.

(b) Calculate the cost of fabric at \$35/m^2, assuming there is no wastage.

(c) Each bow uses approximately 12 cm of ribbon. Calculate the cost of the required ribbon at \$6.20/m.

(d) Calculate the cost of ribbon at \$6.20/m.

3 Children's plastic shapes are designed as right-angled-triangular prisms as shown.



(a) How many shapes will fit into a box of (interior) edge lengths 30 cm, 40 cm and 50 cm?

(b) Given that the shapes are sold in packs of 10 for \$3.20, what is the value of a box of the shapes?

(c) Given that each shape has a mass of 5.6 g, determine the mass of the full box, in kilograms, including 150 g for the box itself.

(d) How many boxes of shapes are needed to create a volume of 1 m^3? Round your answer up to the nearest whole number.

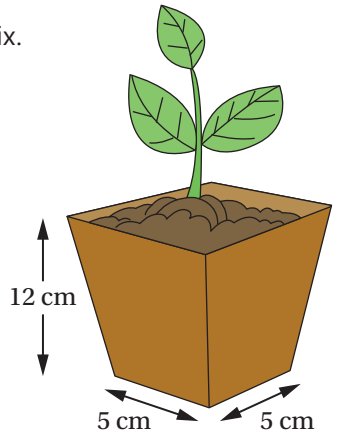
4 Seedling containers are to be filled to around 2 cm of the top with potting mix.

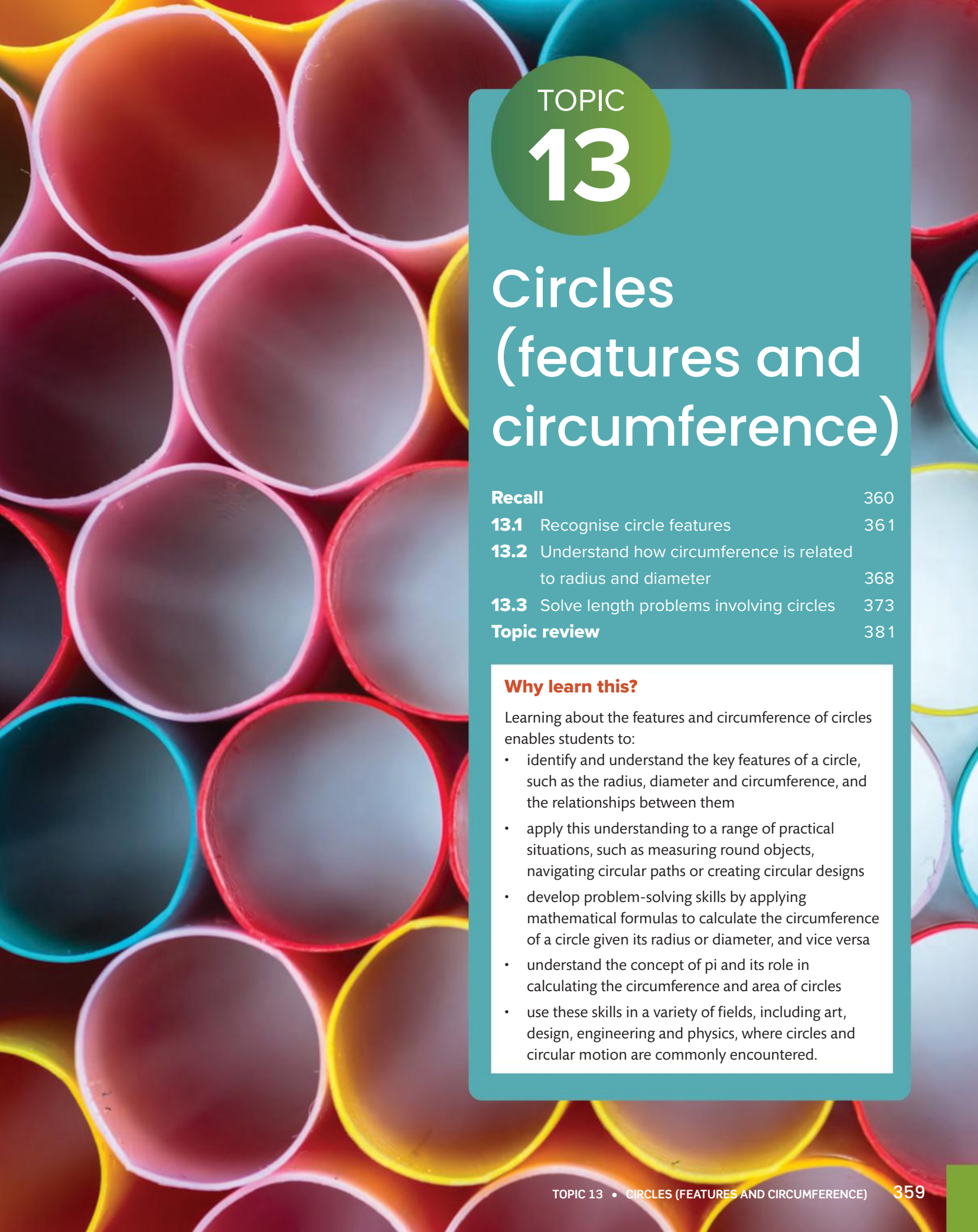
(a) Estimate the volume of potting mix needed for each container.

(b) Bags of potting mix containing 65 L are purchased for \$19.80. Determine the cost of potting mix per container.

(c) Given that seedling containers are sold at 10 for \$15.30, determine the total cost of setting up 100 seedlings prior to planting.

(d) Given that \$28 worth of seeds gives about 100 viable plants, determine a fair price per potted seedling, allowing 80% profit margin for the producer.





TOPIC
13

Circles (features and circumference)

Recall 360
13.1 Recognise circle features 361
13.2 Understand how circumference is related to radius and diameter 368
13.3 Solve length problems involving circles 373
Topic review 381

Why learn this?

Learning about the features and circumference of circles enables students to:

- identify and understand the key features of a circle, such as the radius, diameter and circumference, and the relationships between them
- apply this understanding to a range of practical situations, such as measuring round objects, navigating circular paths or creating circular designs
- develop problem-solving skills by applying mathematical formulas to calculate the circumference of a circle given its radius or diameter, and vice versa
- understand the concept of pi and its role in calculating the circumference and area of circles
- use these skills in a variety of fields, including art, design, engineering and physics, where circles and circular motion are commonly encountered.

RECALL

ANSWERS Page 704

I can round decimals to the required place value

1 Write each of the following numbers to 2 decimal places.

(a) 67.3415 **(b)** 4.8196 **(c)** 2.2645

(d) 9.0073 **(e)** 4.308

I can perform calculations involving decimals

1 Calculate:

(a) 2.6×3.7 **(b)** 7.3×0.7 **(c)** 7.03×8.1 **(d)** 0.04×6.2

I can square integers and decimal numbers

1 Calculate:

(a) 3^2 **(b)** 5^2 **(c)** 11^2

(d) 0.2^2 **(e)** 0.5^2 **(f)** 1.1^2

I can convert between metric units of measurement

1 Complete the following unit conversions:

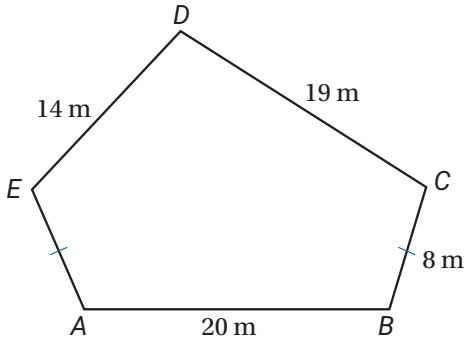
(a) 2.5 cm to mm **(b)** 1.25 m to cm **(c)** 305 mm to cm

(d) 25.5 cm to m **(e)** 5.25 m to mm

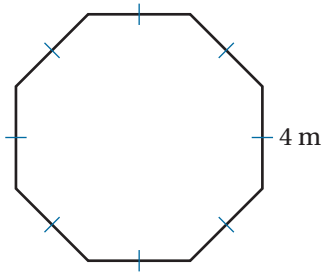
I can calculate the perimeter of shapes

1 Calculate the perimeter of these shapes.

(a)



(b)



I can draw circles of any radius using a compass

1 Briefly describe how you would use a compass to draw a circle of:

(a) radius 4 cm

(b) diameter 12 cm.

Recognise circle features

13.1

Learning intention: To be able to recognise circle features

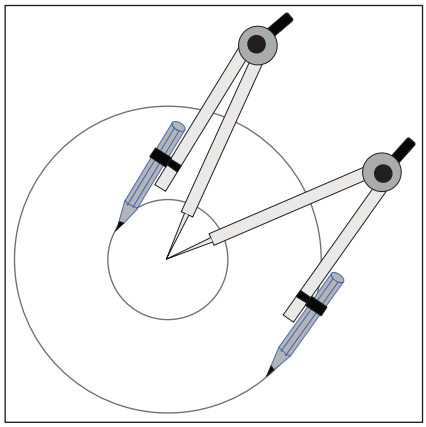
Success criteria:

SC 1 I can identify and define circles and their features.

SC 2 I understand the relationship between the radius and diameter of a circle.

Lesson warm-up

How does a compass draw a circle?



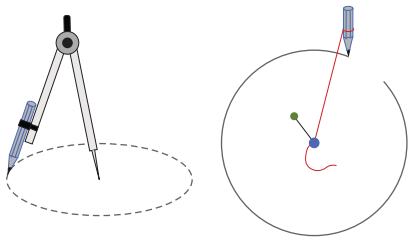
1 Write down how the compasses have been used to draw the circles.

2 Write down similarities and differences between the construction of the two circles.

3 List the features you need to name to accurately compare the circles.

SC 1 I can identify and define circles and their features

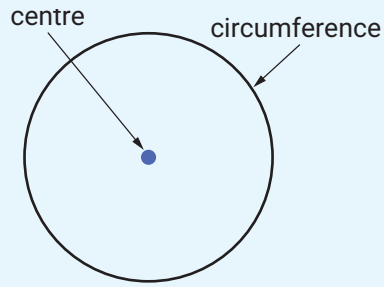
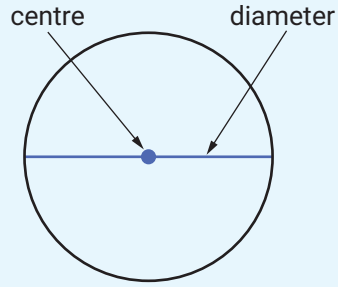
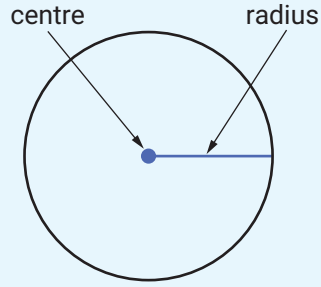
A circle is a two-dimensional (flat) shape, with the property that every point on the circle is the same distance from the centre.



Worked example

Identifying circle features

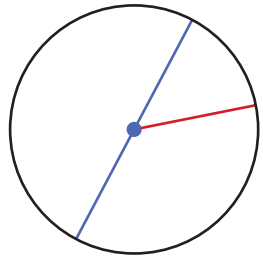
Draw and label the following features of a circle: circumference, radius and diameter.

THINKING	WORKING
Draw and label the circumference. The circumference is the perimeter of a circle.	
Draw and label the diameter. A diameter of a circle is a line segment that touches two points on the circle, passing through the centre. A diameter divides a circle into two semicircles (half-circles).	
Draw and label the radius. A radius is a straight line connecting the centre to any point on the circle. The plural of radius is radii.	

SC 2 I understand the relationship between the radius and diameter of a circle

The distance from the centre of a circle to the circumference is called the radius. The distance across a circle, through the centre, is the diameter.

By definition, every point on the circumference of a circle is the same distance from the centre. This means that every radius of a circle is the same length.



Worked example

Relating radius and diameter

(a) Given that the radius is 12 cm, calculate the length of the diameter.

THINKING	WORKING
Determine the relationship between the diameter and radius. A radius is the length from the centre to the circumference. A diameter is the length joining two points on the circumference through the centre.	The length of the diameter is two times the length of the radius. diameter = 2 × radius
Substitute the length of the radius to calculate the length of the diameter.	diameter = 2 × radius = 2 × 12 cm = 24 cm
Interpret the answer.	A circle with a radius of 12 cm has a diameter of 24 cm.

(b) Given that the diameter is 12 cm, calculate the length of the radius.

THINKING	WORKING
Determine the relationship between the diameter and the radius. A radius is the length from the centre to the circumference. A diameter is the length joining two points on the circumference through the centre.	The length of the radius is half the length of the diameter. $\text{radius} = \frac{1}{2}\ \text{diameter}$
Substitute the length of the diameter to calculate the length of the radius.	$\text{radius} = \frac{1}{2} \times \text{diameter}$ $= \frac{1}{2} \times 12\ \text{cm}$ $= 6\ \text{cm}$
Interpret the answer.	A circle with a diameter of 12 cm has a radius of 6 cm.

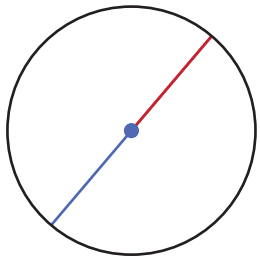
Practice

ANSWERS Page 705

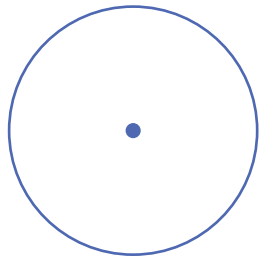
SC 1 I can identify and define circles and their features

1 Name and define the feature highlighted blue in each circle.

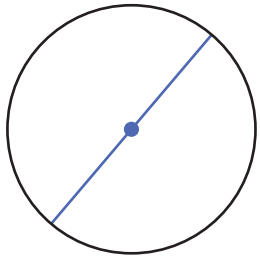
(a)



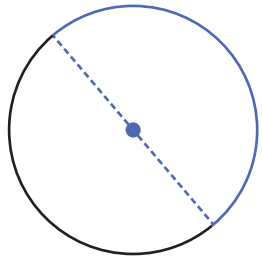
(b)



(c)



(d)



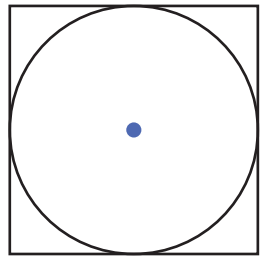
2 Draw and label the following circle features.

(a) A diameter

(b) A radius

(c) A semicircle

3 A circle is drawn inside a square, with the sides just touching. Name the feature of the circle that is the same length as the square's side length.



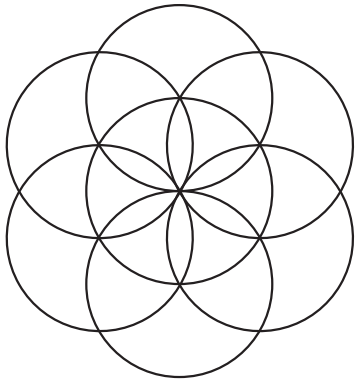
4 Is it possible to draw a straight line segment connecting two points on a circle that is:

(a) shorter than the radius

(b) longer than the diameter?

Explain each answer using a diagram.

5 Overlapping circles can be used to draw flower-like patterns such as the following.



(a) When only one circle is drawn, there is no visible centre point. Explain how you could locate the centre using paper folding.

(b) Describe how to place the circle template to draw a second circle.

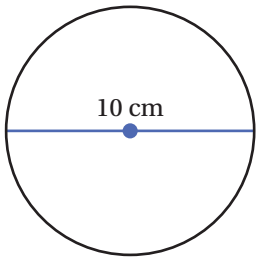
(c) Describe how to place the third circle.

(d) Explain why there are six outer circles.

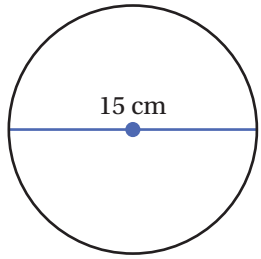
SC 2 I understand the relationship between the radius and diameter of a circle

1 Given the length of the diameter, determine the length of the radius for each circle.

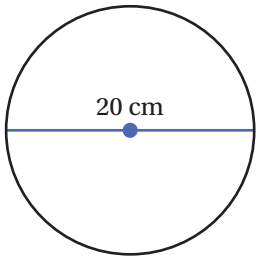
(a)



(b)



(c)



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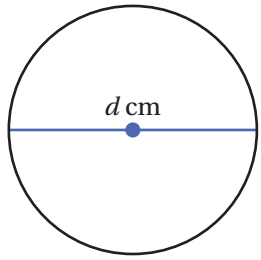
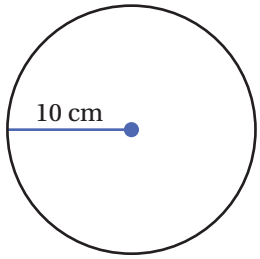


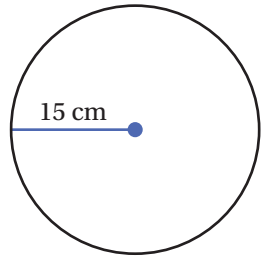
(e) Given the length of the diameter, describe how to calculate the length of the radius.

2 Given the length of the radius, determine the length of the diameter for each circle.

(a)



(b)



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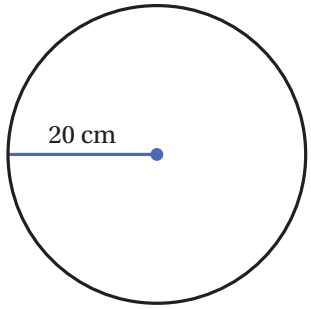
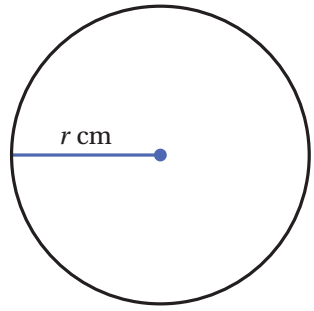
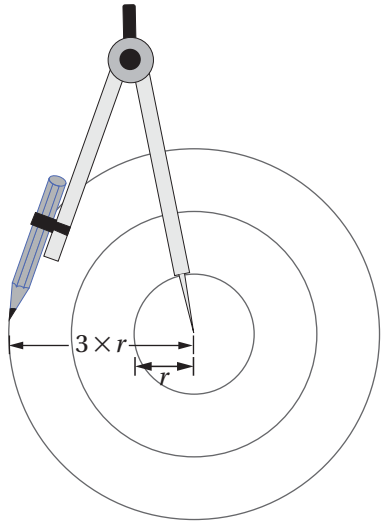


(d)



(e) Given the length of the radius, describe how to calculate the length of the diameter.

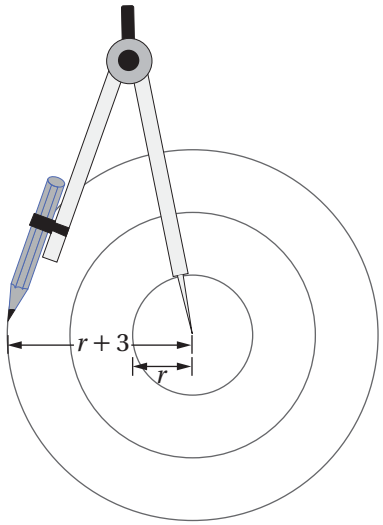
3 A compass setting is changed to draw a circle with a radius three times larger than the radius of the smaller circle. Two students describe how they think the diameter will change.



Student A	Student B
The diameter of the bigger circle will be three times as long as the diameter of the smaller circle.	The diameter of the bigger circle will be six times as long as the diameter of the smaller circle.

Which student is correct? Explain.

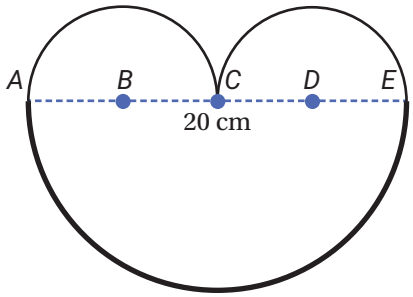
4 A compass setting is changed to draw a circle with a radius 3 cm longer than the radius of the smaller circle. Two students describe how they think the diameter will change.



Student A	Student B
The diameter of the bigger circle will be 3 cm longer than the diameter of the smaller circle.	The diameter of the bigger circle will be 6 cm longer than the diameter of the smaller circle.

Which student is correct? Explain.

5 Three semicircles are connected so that the width of the shape is 20 cm as shown.



(a) Determine the length of the diameter of the large semicircle, AE.

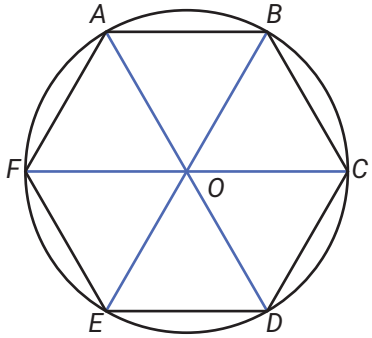
(b) Label and determine the length of the radius of the large semicircle.

(c) Label and determine the length of the diameter of the small semicircles.

(d) Label and determine the length of the radii of the small semicircles.

6 A hexagon is drawn within a circle as shown.

Use the letter labels of each point to name the line segments shown.



(a) Name each line segment that is a radius.

(b) Name each line segment that is a diameter.

13.2 Understand how circumference is related to radius and diameter

Learning intention: To understand how circumference is related to radius and diameter

Success criteria:

SC 1 I understand the relationship between the circumference, the radius and the diameter.

SC 2 I can approximate the value of pi.

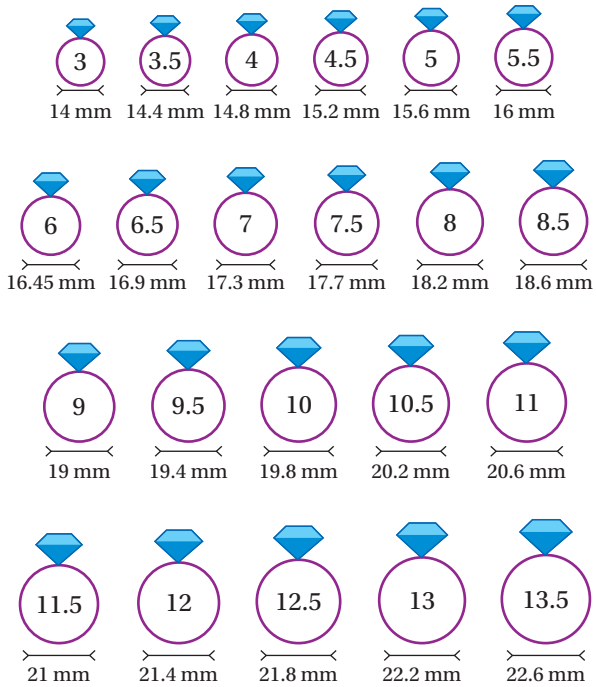
Lesson warm-up

Circles: when and how are they measured?

What are some examples of circumferences that we measure? What is the most practical way to measure or estimate the circumference of circles? Discuss.

What kind of measures can be used to work out how big a circle is?

What are some examples of circumferences that we measure? What is the most practical way to measure or estimate the circumference of circles? Discuss.

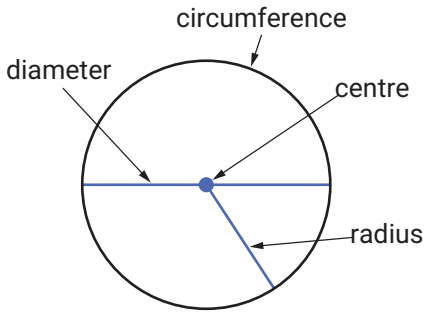


SC 1 I understand the relationship between the circumference, the radius and the diameter

The perimeter of a circle is called the circumference.

The distance from the centre to the circumference is called the radius.

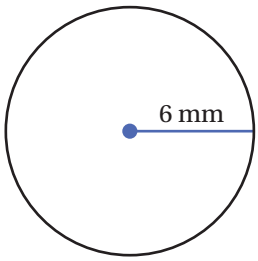
A diameter of a circle is a straight line segment connecting two points on the circle and passing through the centre.



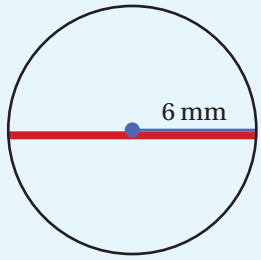
Worked example

Comparing lengths of circumference, radius and diameter

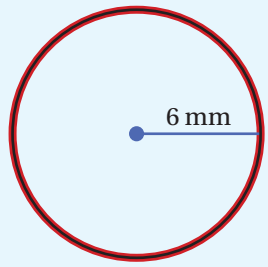
A circle has a radius of 6 mm.



(a) Calculate the length of the diameter.

THINKING	WORKING
Draw the diameter. The diameter is a straight line segment connecting two points on a circle and passing through the centre.	
Calculate the length of the diameter as two times the radius length.	$2 \times 6 = 12$ The diameter is 12 mm.

(b) Estimate the length of the circumference, given that it is known to be between 3 and 4 times the length of the diameter.

THINKING	WORKING
Draw the circumference. The circumference is the perimeter of the circle.	
Calculate the estimations for the circumference.	The diameter of the circle is 12 mm. The circumference is longer than $3 \times 12 = 36$ mm and shorter than $4 \times 12 = 48$ mm
Interpret the answer.	The circumference of the circle is between 36 mm and 48 mm.

SC 2 I can approximate the value of pi

The exact value of the ratio of a circle's circumference to its diameter $\frac{C}{d}$ (where C is circumference and d is diameter) is given the Greek letter pi (π): $\frac{C}{d} = \pi$.

Worked example

Approximating the value of π

Calculate an approximate value for π, to 2 decimal places, given the measurements: radius 14.32 cm, circumference 90 cm.

THINKING	WORKING
Write the given measurements using symbols.	$r = 14.32$ cm, $C = 90$ cm
Calculate the diameter from the radius.	$d = 2r$ $= 2 \times 14.32$ $= 28.64$
Substitute values into the formula for π. Use the 'approximately equals' symbol $\approx$ to show that the measured values are approximate.	$\pi = \frac{C}{d}$ $\approx \frac{90}{28.64}$ $\approx 3.142\ldots$
Round the result to 2 decimal places.	$\pi = 3.14$ (2 d.p.)

Practice

ANSWERS Page 706

SC 1 I understand the relationship between the circumference, the radius and the diameter

1 Draw a circle with each of the radii below. Measure the circumference of each circle to the nearest millimetre.

(a) 2 cm **(b)** 3 cm **(c)** 4 cm **(d)** 6 cm

(e) Explain what happens to the length of the circumference when the length of the radius doubles.

(f) Explain what happens to the length of the circumference when the length of the radius is increased by 1 unit.

2 Use your answers from Question **1** to:

(a) copy and complete the table

Radius	Diameter	Circumference	$\frac{C}{d}$	$\frac{C}{r}$
2 cm				
3 cm				
4 cm				
6 cm				

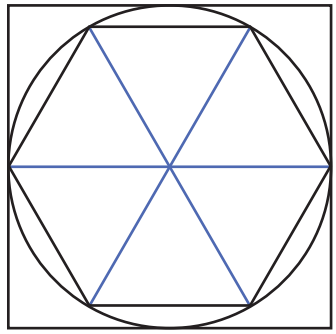
(b) complete the statements.

(i) The circumference is about _____ times as long as the diameter.

(ii) The circumference is about _____ times as long as the radius.

3 A circle fits exactly inside a square as shown.

A regular hexagon is made of six equilateral triangles. If the hexagon is inside a circle (as shown), then the hexagon sides are each the same length as the circle radius.



(a) Describe the side length of the square in terms of the radius of the circle.

(b) Describe the side length of the hexagon in terms the radius of the circle.

(c) Use your answers to parts **(a)** and **(b)** to determine the perimeter of each of the shapes and list them in order from longest to shortest.

4 Estimate the length of the circumference of the following circles and explain your answer.

(a) The length of the radius is 6 m. **(b)** The length of the radius 6.5 m.

(c) The length of the diameter is 6 m. **(d)** The length of the diameter is 6.5 m.

5 Estimate the length of the diameter of the following circles and explain your answer.

(a) Circumference of 30 cm **(b)** Circumference of 35 cm **(c)** Circumference of 35.5 cm

SC 2 I can approximate the value of pi

1 $\pi \approx 3.141\ 592\ 654...$

Write the value of π correct to the following number of decimal places:

(a) 1 decimal place **(b)** 8 decimal places.

2 Determine an approximation for $\pi \approx 3.141\ 592\ 654...$ using $\frac{C}{d}$, given the following circle measurements.

(a) Diameter 12 mm, circumference 38 mm

(b) Diameter 12.1 mm, circumference 38.1 mm

(c) Diameter 12.136, circumference 38.126

(d) What happens to the ratio $\frac{C}{d}$ as the accuracy of the measurements of C and d increases?

3 Describe the relationship between the radius and diameter of a circle.

4 Determine an approximation for $\pi \approx 3.141\ 592\ 654...$ using $\frac{C}{d}$, given the following circle measurements.

(a) Radius 2.5 m, circumference 16 m

(b) Radius 2.55 m, circumference 16.02 m

(c) Radius 2.549 m, circumference 16.016 m

(d) What happens to the ratio $\frac{C}{d}$ as the accuracy of the measurements C and r increases?

5 Before scientific calculators, people often used the fraction $\frac{22}{7}$ as a convenient approximation for π.

(a) Compare the calculations for the following circles and write each answer correct to six significant figures (in this case, six digits).

Diameter	$C = \pi \times d$	$C = \frac{22}{7} \times d$
1 m		
10 m		
100 m		
1000 m		
10 000 m		

(b) Compare the calculations. Do you think the fraction $\frac{22}{7}$ is a good approximation for π? Explain.

Solve length problems involving circles

13.3

Learning intention: To be able to solve length problems involving circles

Success criteria:

SC 1 I can calculate circumference given the radius or diameter.

SC 2 I can determine the length of a radius and diameter from a circumference.

SC 3 I can calculate the perimeter of shapes involving circles.

Lesson warm-up

What is a yarning circle and how much space do you need to create one?

A yarning circle is an important part of Aboriginal and Torres Strait Islander culture. It is used to learn, to build respectful relationships and to ensure that cultural knowledge is preserved.

How to participate in a yarning circle

1. Everyone sits in a circle with their legs crossed.
2. Everyone introduces themselves.
3. The host introduces the purpose of the yarning circle with a focus question.
4. Participants take turns to share ideas.
5. Participants take time to reflect.

Task

With the students in your class, what is the diameter of space you would need to create a yarning circle?

SC 1 I can calculate circumference given the radius or diameter

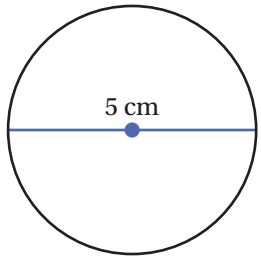
For every circle: $\frac{\text{circumference}}{\text{diameter}} = \frac{C}{d} = \pi$, where $\pi = 3.141\ 59...$

This means the length of the circumference can be calculated if the diameter or radius is known, using the formula: $C = \pi d$ or $C = 2\pi r$.

Worked example

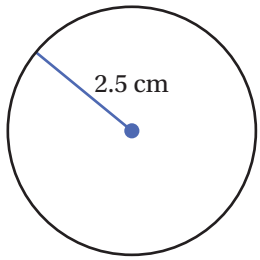
Calculating circumference

(a) Calculate the circumference, correct to 2 decimal places.



THINKING	WORKING
Write the given information using variables.	Diameter: $d = 5$ cm
Choose the circumference formula that includes the given information (diameter or radius).	Circumference: $C = \pi d$
Substitute the values into the formula and calculate the result.	$C = \pi \times 5$ $= 15.707...$
Round the result to the required number of decimal places.	$C = 15.71$ cm (2 d.p.)
Interpret the answer.	A circle with a diameter of 5 cm has a circumference of 15.71 cm.

(b) Using the radius, calculate the circumference of the same circle, correct to two decimal places.



THINKING	WORKING
Write the given information using variables.	Radius: $r = 2.5$ cm
Choose the circumference formula that includes the given information (diameter or radius).	Circumference: $C = 2\pi r$
Substitute the values into the formula and calculate the result.	$C = 2 \times \pi \times 2.5$ $= 15.707\ldots$
Round the result to the required number of decimal places.	$C = 15.71$ cm (2 d.p.)
Interpret the answer.	A circle with a radius of 2.5 cm has a circumference of 15.71 cm.

SC 2 I can determine the length of a radius and diameter from a circumference

Given that the relationship between the circumference and diameter is known to be:

$\frac{\text{circumference}}{\text{diameter}} = \frac{C}{d} = \pi$, where $\pi = 3.141\ 59\ldots$

then given the circumference, the radius and diameter can be determined by changing the subject of the formulas:

$C = \pi d$ or $C = 2\pi r$

$d = \frac{C}{\pi}$ or $r = \frac{C}{2\pi}$

Worked example

Given the circumference, determine the length of the radius and diameter

(a) Determine the length of the diameter, correct to 2 decimal places, of the circle with a circumference of 22 mm.

THINKING	WORKING
Write the given information using variables.	Circumference: $C = 22$ mm
Choose the circumference formula that includes the given information (diameter or radius).	Circumference: $C = \pi d$
Substitute the values into the formula and calculate the result.	$22 = \pi \times d$ $d = \frac{22}{\pi}$ $= 7.0028\ldots$
Round the result to the required number of decimal places.	$d = 7.00$ mm (2 d.p.)
Interpret the answer.	A circle with a circumference of 22 mm has a diameter of 7.00 mm.

(b) Calculate the length of the radius, correct to 2 decimal places, of the circle with a circumference of 22 mm.

THINKING	WORKING
Write the given information using variables.	Circumference: $C = 22$ mm
Choose the circumference formula that includes the given information (diameter or radius).	Circumference: $C = 2\pi r$
Substitute the values into the formula and calculate the result.	$22 = 2 \times \pi \times r$ $r = \frac{22}{2\pi}$ $= 3.501\ldots$
Round the result to the required number of decimal places.	$r = 3.50$ mm (2 d.p.)
Interpret the answer.	A circle with a circumference of 22 mm has a radius of 3.50 mm.

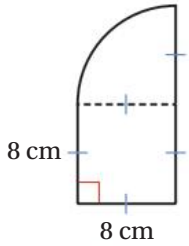
SC 3 I can calculate the perimeter of shapes involving circles

The perimeter is the distance around a shape.

Worked example

Calculating the perimeter of shapes involving circles

Determine the perimeter of this shape, correct to 2 decimal places.



THINKING	WORKING
Identify the sections of the perimeter.	There are three sides of a square; each is 8 cm. One circle radius that is 8 cm. One quarter of a circumference.
Write an equation to determine the length of the curved edge.	Quarter of the circumference of circle: $\frac{1}{4} \times C = \frac{1}{4} \times 2\pi r$ radius, $r = 8$ cm
Write the perimeter formula for the complete shape.	$P =$ 3 sides of square + radius + quarter circle $= (8+8+8)+8+\left(\frac{1}{4} \times 2\pi r\right)$
Substitute the values into the formula and calculate the result.	$P = (8+8+8)+8+\left(\frac{1}{4} \times 2 \times \pi \times 8\right)$ $= 24+8+12.5663\ldots$ $= 44.5663\ldots$
Round the result to the required number of decimal places.	$P = 44.57$ cm (2 d.p.)
Interpret the answer.	The perimeter of the shape, correct to 2 decimal places is 44.57 cm.

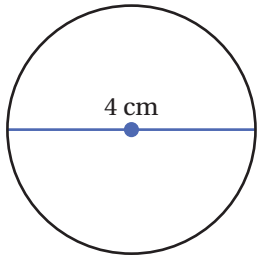
Practice

ANSWERS Page 707

SC 1 I can calculate circumference given the radius or diameter

1 Calculate the circumference of each circle, correct to 2 decimal places.

(a)



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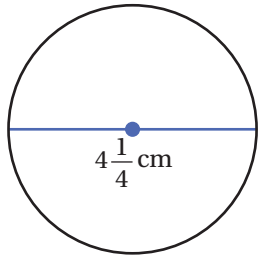

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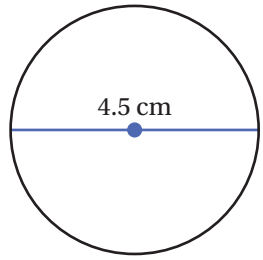
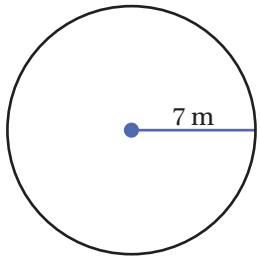
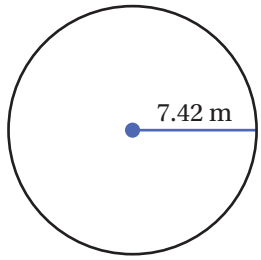


2 Calculate the circumference of each circle, correct to 2 decimal places.

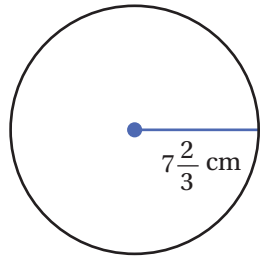
(a)



(b)



(c)



3 Calculate the diameter d and circumference C of each circle, correct to 2 decimal places, given the following radii.

(a) $r = 25$ mm **(b)** $r = 30$ mm **(c)** $r = 35$ mm

4 Calculate the radius r and circumference C of each circle, correct to 2 decimal places, given the following diameters.

(a) $d = 15$ m **(b)** $d = 18$ m **(c)** $d = 21$ m

5 When calculating the circumference, do you have a preferred method or strategy? Explain.

SC 2 I can determine the length of a radius and diameter from a circumference

1 For circles with the following circumferences, use the formula $C = \pi d$ to:

(i) determine the length of the diameter d

(ii) then halve part **(i)** to determine the length of the radius r.

Round your answers to 2 decimal places.

(a) 2.5 cm **(b)** 25 cm **(c)** 250 cm

2 For circles with the following circumferences, use the formula $C = 2\pi r$ to:

(i) determine the length of the radius r

(ii) then double part **(i)** to determine the length of the diameter d of the same circle.

Round your answers to 2 decimal places.

(a) 2.5 cm **(b)** 25 cm **(c)** 250 cm

3 Given a circumference, explain when you might use each of the formulas $C = 2\pi r$ and $C = \pi d$.

4 Given the following circumferences, calculate the diameter of each circle, correct to 2 decimal places.

(a) 25 mm **(b)** 30 mm **(c)** 35 mm

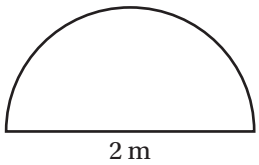
5 Given the following circumferences, calculate the radius of each circle, correct to 2 decimal places.

(a) 15 m **(b)** 18 m **(c)** 21 m

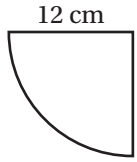
SC 3 I can calculate the perimeter of shapes involving circles

1 Calculate the perimeter of each shape, correct to 2 decimal places. Each curved side is part of a circle.

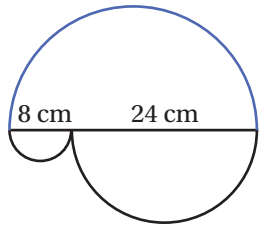
(a)



(b)

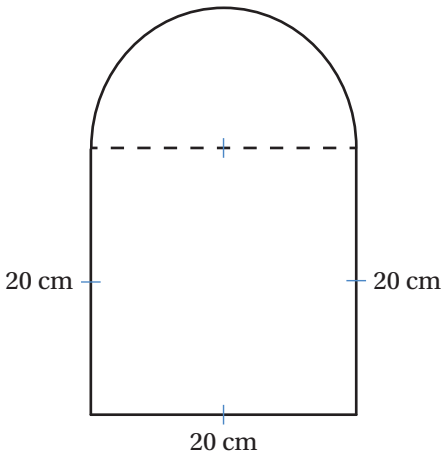


2 Calculate the perimeter of this shape, correct to 2 decimal places. Each curved side is part of a different circle.

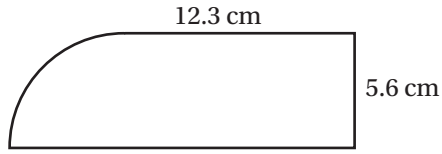


3 Calculate the perimeter of each shape, correct to 2 decimal places. Each curved side is part of a circle.

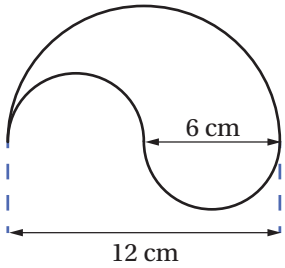
(a)



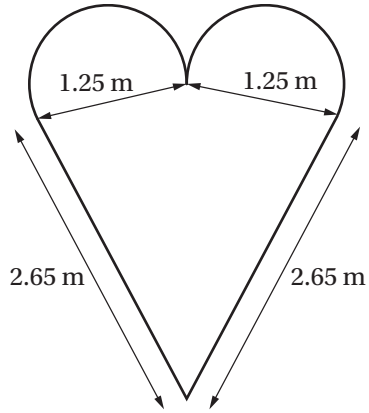
(b)



(c)

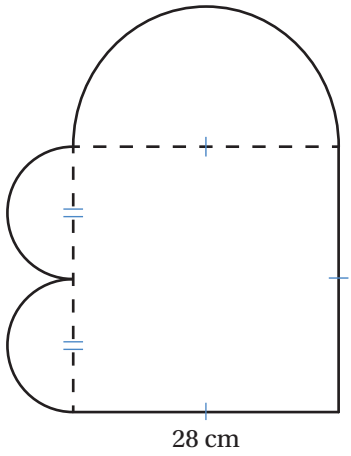


(d)

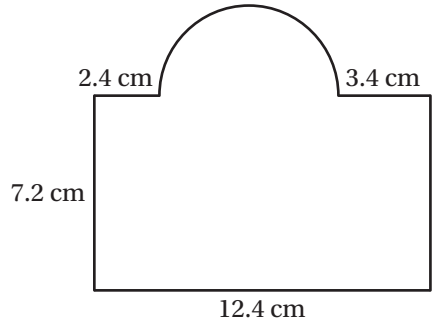


4 Calculate the perimeter of each shape, correct to 2 decimal places. Each curved side is part of a circle.

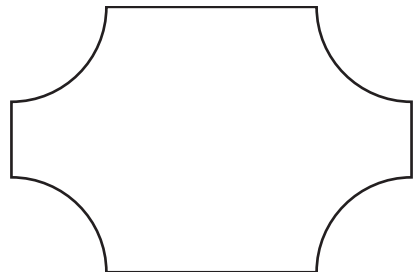
(a)



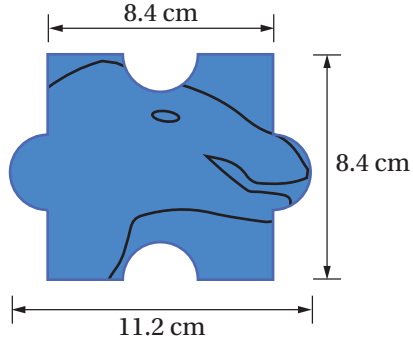
(b)



5 A rectangle measuring 128 mm by 82 mm has quarter circles of radius 30 mm cut from each corner. Determine the perimeter of the resulting shape, correct to the nearest millimetre.



6 A jigsaw puzzle piece is shaped like a square with identical semicircles added or removed at each side, as shown. Determine its perimeter, correct to 1 decimal place (in centimetres).



TOPIC REVIEW

Multiple choice

ANSWERS Page 746

1 Which of the following representations of pi, the ratio between circumference and diameter, is incorrectly rounded?

A 3.14 **B** 3.141 **C** 3.1416 **D** 3.1

2 The lengths of diameter, radius and circumference are very roughly in the ratio:

A 1 : 2 : 3 **B** 1 : 2 : 6 **C** 2 : 1 : 3 **D** 2 : 1 : 6

3 A circle of diameter $20\frac{1}{8}$ cm has a circumference, correct to 2 decimal places, of:

A 63.22 cm **B** 126.44 cm **C** 63.23 cm **D** 126.45 cm

4 A circle of radius 16 cm has a circumference, correct to 2 decimal places, of:

A 100.54 cm **B** 50.27 cm **C** 100.53 cm **D** 50.26 cm

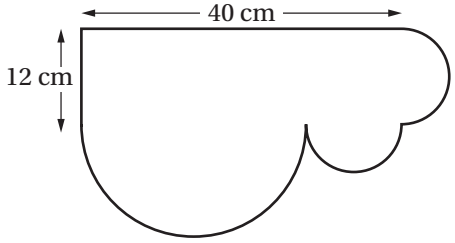
5 A semicircle of radius 10.5 cm has a perimeter, correct to 2 decimal places, of:

A 53.99 cm **B** 32.99 cm **C** 107.97 cm **D** 65.98 cm

Short answer

1 Draw a circle showing two chords, one shorter than the radius and one longer than the radius.

2 The two small semicircles attached to the rectangle are the same size.



(a) Determine the lengths of the diameters of the three semicircles.

(b) Determine the lengths of the radii of the three semicircles.

3 Use a pair of compasses to draw a circle of radius 7.2 cm.

(a) What is the length of the diameter?

(b) Use a strip of paper to determine the length of the circumference to the nearest millimetre.

(c) Determine the values of the ratios $\frac{C}{d}$ and $\frac{C}{r}$ correct to 2 decimal places.

4 The circumference of a circle is 30 cm. Complete the statement by choosing one of each alternative in each case. The radius of the circle will a bit be more/less than 5 cm/10 cm/60 cm.

5 The circumference of a circle is 85 cm. Determine the length of the radius to the nearest centimetre.

6 Approximate a value for π, correct to 2 decimal places, given:

(a) radius of 2.4 m, circumference of 15.1 m

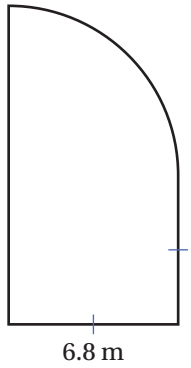
(b) diameter of 37 cm, circumference of 1.2 m.

7 Use $\pi \approx \frac{22}{7}$ to calculate the circumference, given:

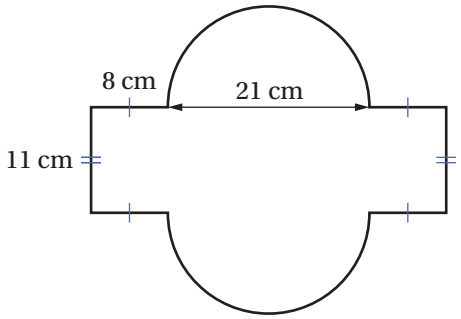
(a) diameter of 35 m

(b) radius of 77 cm.

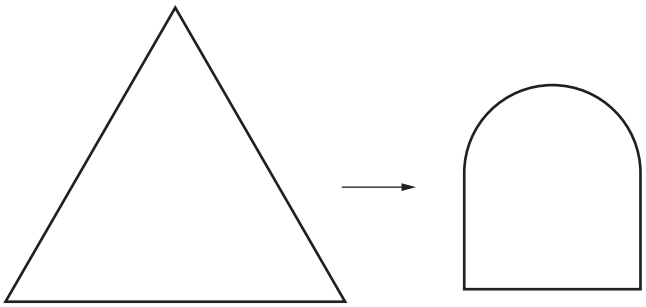
8 Calculate the perimeter of the shape, correct to 2 decimal places.



9 Use $\pi \approx \frac{22}{7}$ to calculate the perimeter of the shape.



10 A loop of string forms an equilateral triangle of side length 84 cm. The string is then used to form the shape of a semicircle on a rectangle.

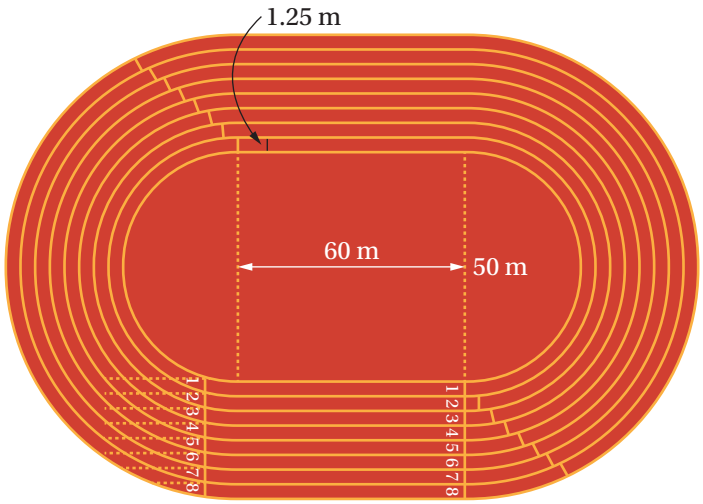


(a) Determine the length of the other two sides, correct to the nearest millimetre, given the base is 50 cm.

(b) Use guess and check to determine the length of the base of the shape, to the nearest centimetre, given the two vertical sides are 50 cm.

Extended response

1 The middle section of a running track used for athletics training is in the shape of a rectangle 60 m by 50 m, with semicircles on either end. Each lane is 1.25 m wide. The distance travelled in each lane assumes the runner hugs (stays very close to or follows) the inner line marking. Round answers to the nearest centimetre.



(a) Determine the distance travelled over one lap in Lane 1.

(b) How much further does a person travel for each lap in Lane 2 than in Lane 1?

(c) What distance does a person in Lane 8 cover in 10 laps?

TOPIC

14

Angles and transversals

Recall 385

14.1 Construct perpendicular and parallel lines 387

14.2 Identify corresponding, alternate and co-interior angles 391

14.3 Determine the exterior angle of a triangle 400

14.4 Calculate the internal angle sum of a triangle 404

Topic review 409

Why learn this?

Understanding how to construct perpendicular and parallel lines, as well as identifying angles on parallel lines, offers a solid foundation for the study of geometry. This knowledge can be applied in a variety of real-world scenarios, such as designing architectural plans, creating art with accurate perspective, or even understanding the layout of roads in a city.

The skills developed in this topic:

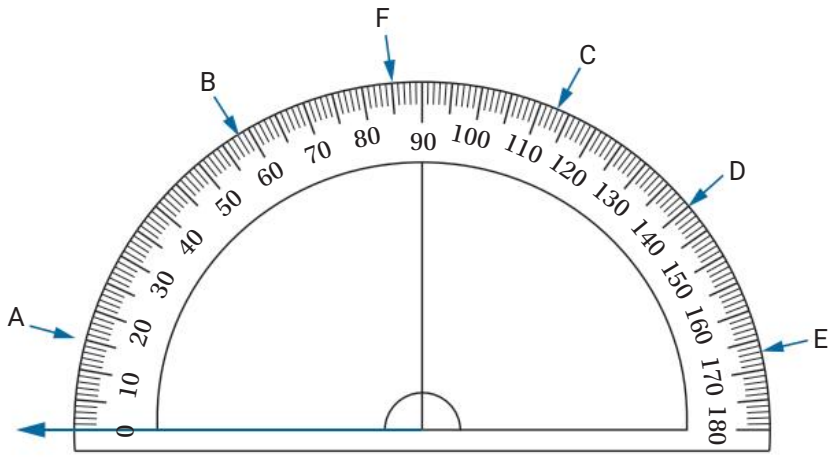
- serve as a stepping stone to more complex mathematical concepts taught in higher levels; for example, the ability to determine the size of angles in triangles is essential in the study of trigonometry
- are beneficial in a range of career paths, including engineering, architecture and design.

RECALL

ANSWERS Page 707

I can use a protractor to measure the size of angles

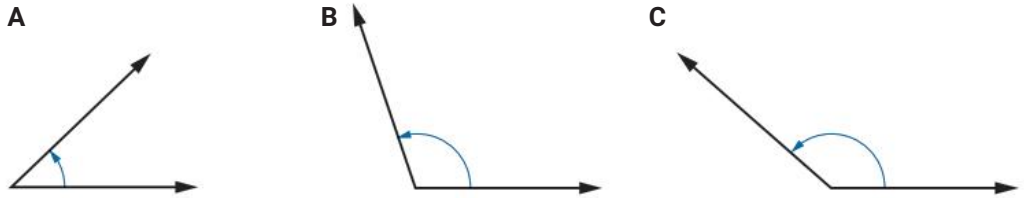
1 State the value indicated by each arrow in the diagram from the line drawn to each letter around the protractor.



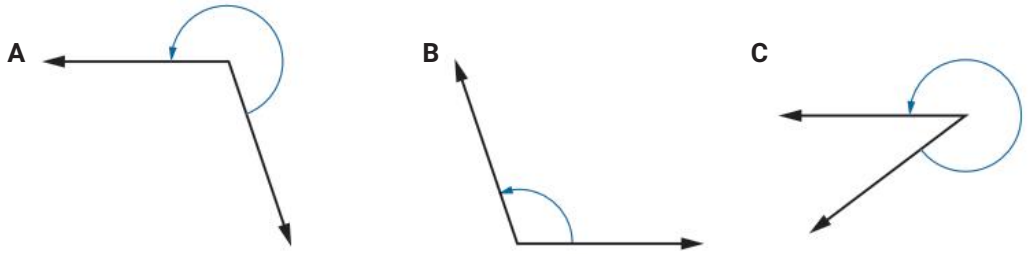
I can identify angles by their size

1 In each of the following examples, write which angle (A, B or C) is the largest angle and which is the smallest angle.

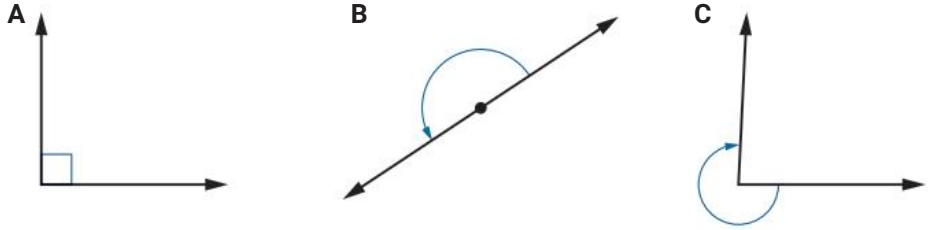
(a)



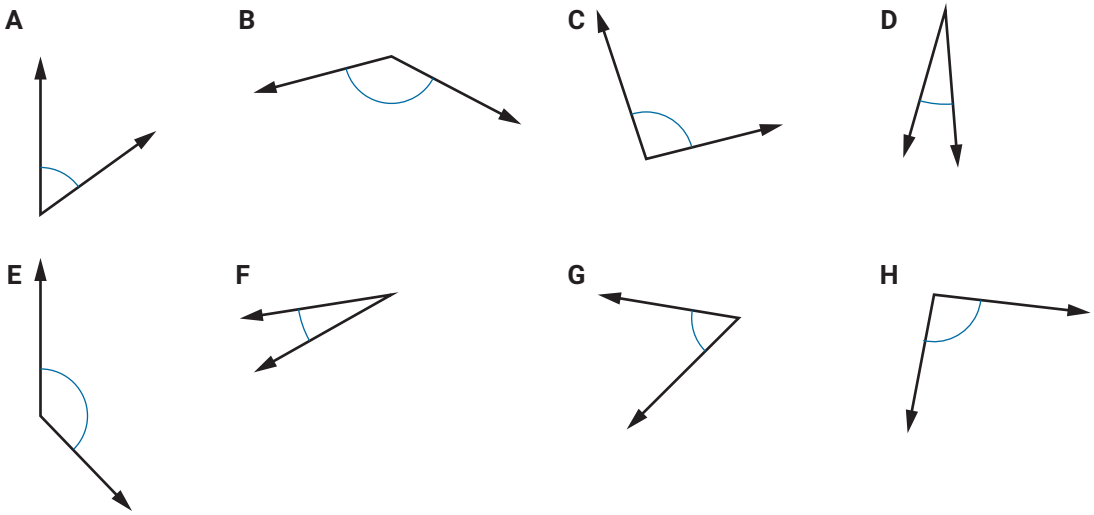
(b)



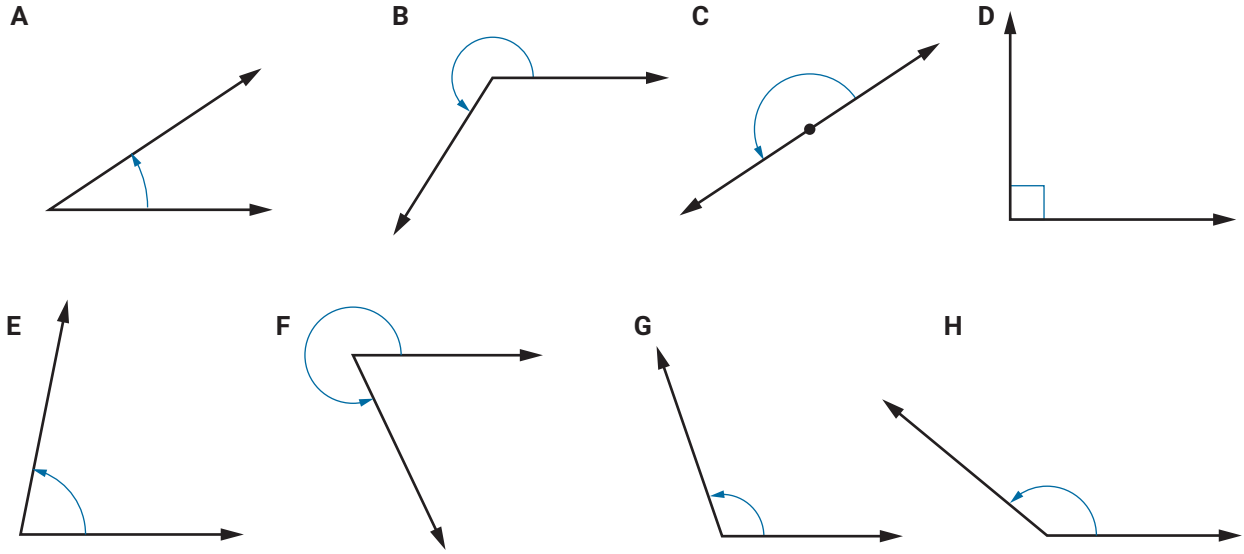
(c)



2 Name each pair of angles that are the same size.

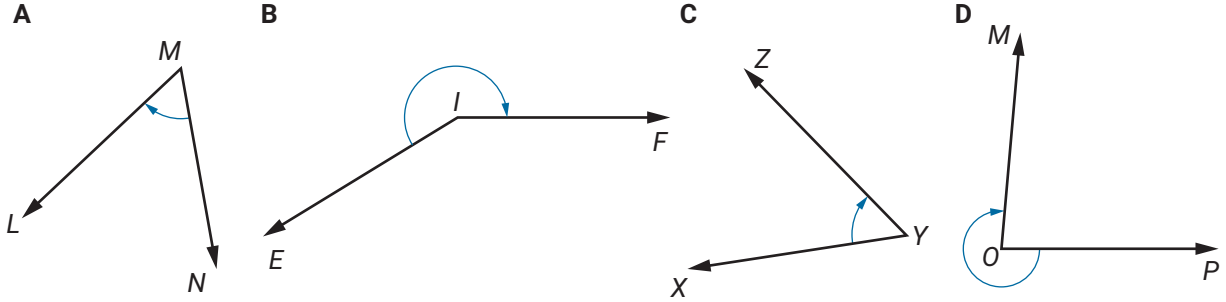


3 Identify whether the following angles are acute, right, obtuse, straight or reflex angles.



I can name angles

1 Name the following angles.



Construct perpendicular and parallel lines

14.1

Learning intention: To be able to construct perpendicular and parallel lines

Success criteria:

SC 1 I can construct perpendicular lines.

SC 2 I can construct parallel lines.

SC 3 I can construct shapes that involve parallel sides or perpendicular diagonals.

Lesson warm-up

Parallel and perpendicular

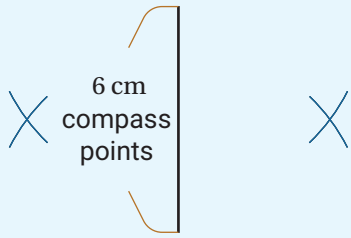
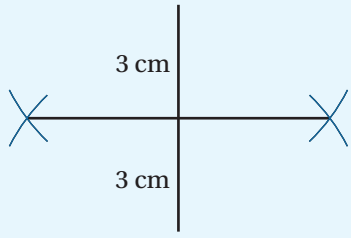
Find examples of parallel lines and perpendicular lines in the world. Look around your classroom or yard. List as many examples as you can and explain how you know they are parallel or perpendicular.

SC 1 I can construct perpendicular lines

Worked example

Constructing perpendicular lines using a pair of compasses and a ruler

Draw a line 6 cm long and construct a perpendicular line through its middle. (This will be the perpendicular bisector of the line.)

THINKING	WORKING
Draw the line using a ruler, to the given length. Open a pair of compasses to more than half of the length of the line. With the point on one end of the line, make arcs at either side of the line. With the point on the other end of the line, make arcs at either side of the line to intersect the original arcs.	
Join the intersections of the arcs with a straight line.	

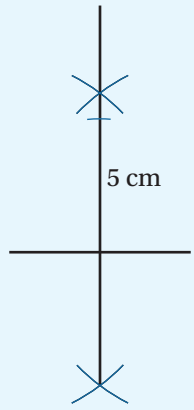
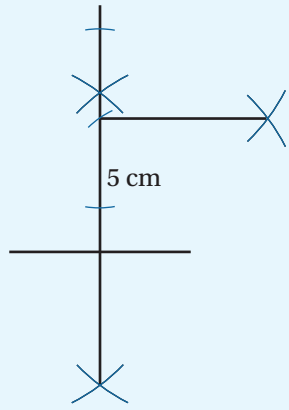
SC 2 I can construct parallel lines

Two lines that are perpendicular to the same line will be parallel to each other.

Worked example

Constructing parallel lines at a fixed distance

Construct a line parallel to the given line, 5 cm above the line.

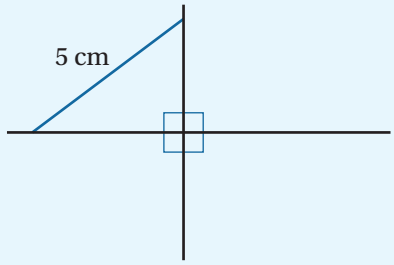
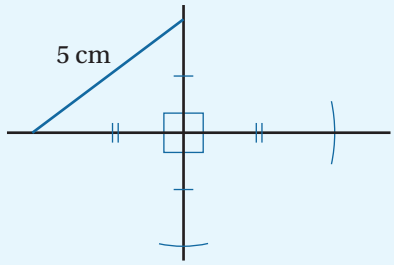
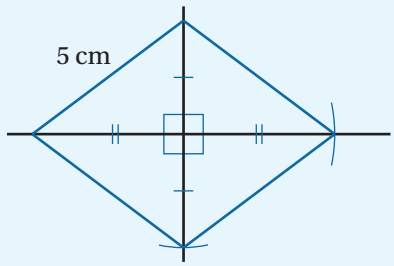
THINKING	WORKING
Draw the line. Construct the perpendicular bisector of the line and extend the perpendicular well above the line. Mark off a point 5 cm above the line.	
Construct a perpendicular to the mark above the line. Mark arcs on the vertical line above and below the 5 cm mark. Using these points as centres, make intersecting arcs to the right. Join the 5 cm mark to the intersection of the arcs.	
State what has been achieved.	The second perpendicular is parallel to the original line and 5 cm above it.

SC 3 I can construct shapes that involve parallel sides or perpendicular diagonals

Worked example

Constructing a rhombus from its diagonals

Draw a pair of perpendicular lines using a ruler and set square. Using the perpendicular lines to position the diagonals, construct a rhombus of side length 5 cm.

THINKING	WORKING
Draw the pair of perpendicular lines. Use a ruler to mark off a side length of 5 cm by joining points on different perpendicular lines.	
Since the diagonals of a rhombus bisect each other at right angles, use a pair of compasses to mark the positions of the other vertices.	
Join the vertices to complete the rhombus.	

Practice

ANSWERS Page 708

SC 1 I can construct perpendicular lines

1. Draw a line 10 cm long, then construct the perpendicular bisector of the line using a pair of compasses and a ruler.
2. Draw a line 10 cm long, then construct a line at right angles 4 cm from one end using a pair of compasses and a ruler.
3. Draw a line 10 cm long and put a dot above the line. Construct a line through the dot at right angles to the original line using a pair of compasses and a ruler.
4. Draw a pair of lines that intersect to form an acute angle and an obtuse angle. Construct right angles to each line at the point of intersection.

SC 2 I can construct parallel lines

1. Construct a line parallel to an oblique line 6 cm away from the line. Justify your diagram.
2. Draw a line 5 cm long, then use the properties of a rhombus to construct a parallel line of the same length. Justify your diagram.
3. Draw a line 8 cm long, then use the properties of a parallelogram to construct a parallel line of the same length. Justify your diagram.
4. Draw a line 7 cm long, then use the properties of a rectangle to construct a parallel line of the same length. Justify your diagram.

SC 3 I can construct shapes that involve parallel sides or perpendicular diagonals

1. Construct a rhombus of side length 10 cm using the properties of the diagonals to assist. Justify your steps.
2. Construct a square using the properties of the diagonals to assist. Justify your steps.
3. Using a ruler and set square, draw a set of parallel lines 3 cm apart. Use the lines to construct a parallelogram of side lengths 8 cm and 4 cm.
4. Draw a pair of parallel lines using a ruler and set square. Using the lines to position the parallel sides, construct three trapeziums of different shapes, all of which have parallel side lengths 8 cm and 5 cm.

Identify corresponding, alternate and co-interior angles

14.2

Learning intention: To be able to identify corresponding, alternate and co-interior angles

Success criteria:

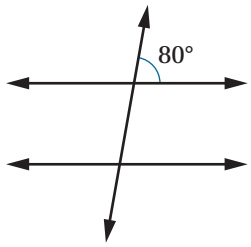
SC 1 I can identify angle relationships around a transversal.

SC 2 I can determine the size of unknown angles involving parallel lines.

Lesson warm-up

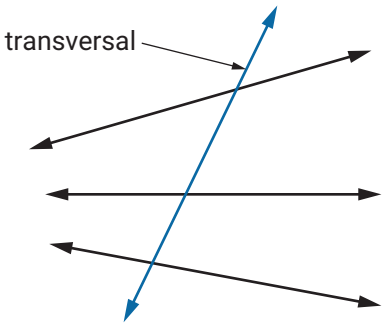
Guess the angle

Write the sizes of as many angles in this diagram as you can.
Explain your reasoning.



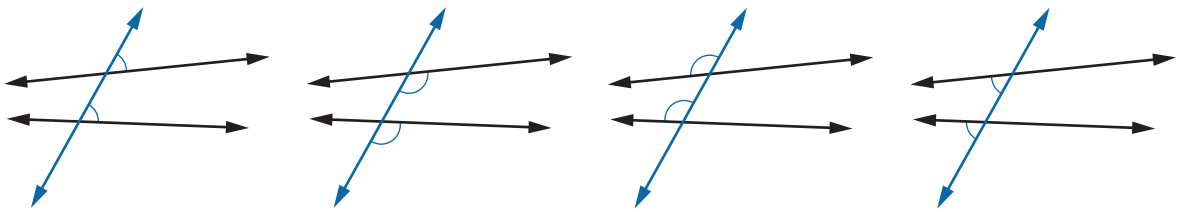
SC 1 I can identify angle relationships around a transversal

When two or more lines are cut or intersected by another line, this other line is called a transversal.

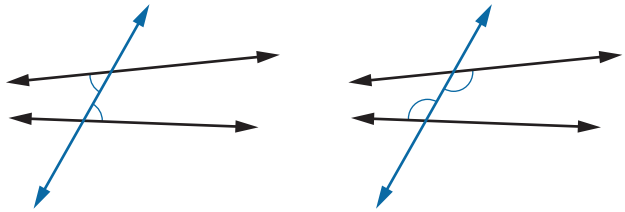


Angles around a transversal and the intersecting lines have names related to their positions.

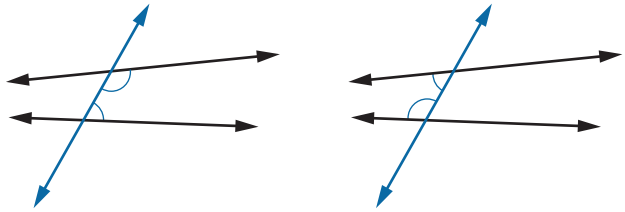
Corresponding angles are on the same side of the transversal and the same side of the intersecting line as each other. All four possible pairs of corresponding angles for a particular set of lines are shown. Notice the F shape (sometimes upside down or back to front) in the line segments that form the angle.



Alternate angles refer to angles between the intersecting lines only. They are on different sides of the transversal and different sides of the intersecting lines as each other. Both possible pairs of alternate angles for a particular set of lines are shown. Notice the Z shape (sometimes back to front) in the line segments that form the angle.



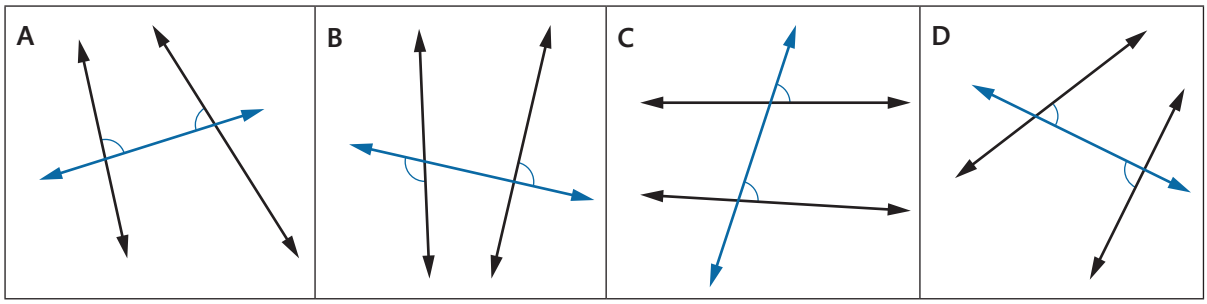
Co-interior angles refer to angles between the intersecting lines only. They are on the same side of the transversal. Both possible pairs of co-interior angles for a particular set of lines are shown. Notice the U or C shape (sometimes back to front) in the line segments that form the angle.



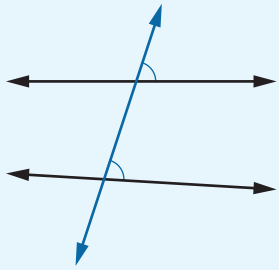
Worked example

Identifying corresponding, alternate and co-interior angle relationships

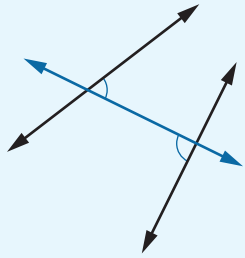
Consider the following pairs of angles.



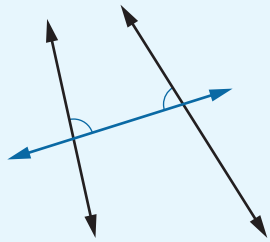
(a) Identify a pair of corresponding angles.

THINKING	WORKING
Recall the requirements for corresponding angles.	Corresponding angles are on the same side of the transversal and the same side of the intersecting lines as each other.
Choose the correct alternative.	A corresponding relationship is shown in **C**. 

(b) Identify a pair of alternate angles.

THINKING	WORKING
Recall the requirements for alternate angles.	Alternate angles refer to angles between the intersecting lines only. They are on different sides of the transversal and different sides of the intersecting lines as each other.
Choose the correct alternative.	An alternate relationship is shown in **D**. 

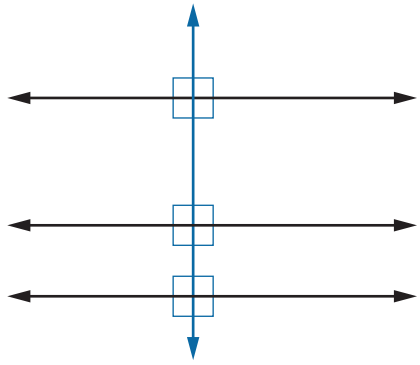
(c) Identify a pair of co-interior angles.

THINKING	WORKING
Recall the requirements for co-interior angles.	Co-interior angles refer to angles between the intersecting lines only. They are on the same side of the transversal.
Choose the correct alternative.	A co-interior relationship is shown in **A**. 

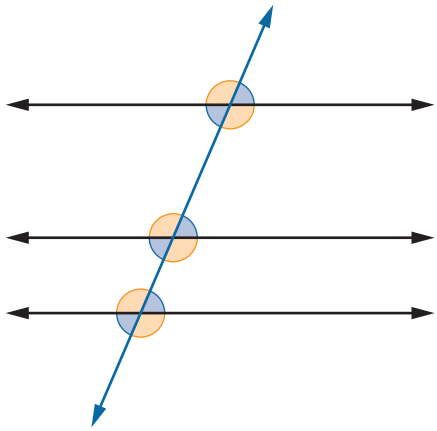
SC 2 I can determine the size of unknown angles involving parallel lines

When a transversal cuts a set of parallel lines, one of two things can happen:

If the transversal is perpendicular to the parallel lines, all angles will be right angles.



If the transversal is not perpendicular to the parallel lines, all angles will be either acute or obtuse angles.

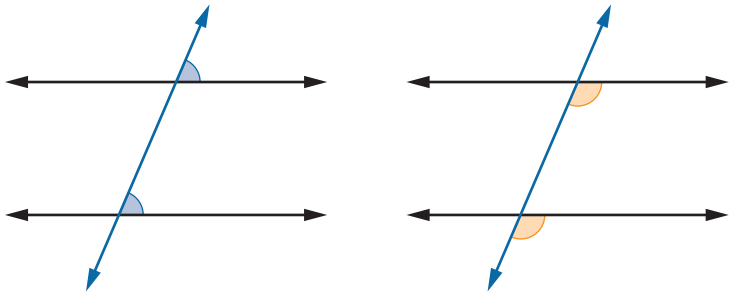


In this case,

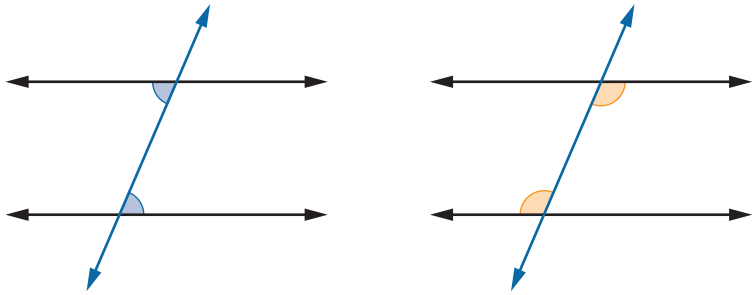
- all the acute angles are equal to each other
- all the obtuse angles are equal to each other
- any pair of acute and obtuse angles are supplementary (add to 180°)
- vertically opposite angles are equal.

Picking out pairs of corresponding, alternate and co-interior angles, it is clear that:

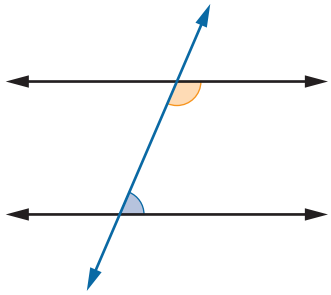
- any pair of corresponding angles on parallel lines are equal (either both acute or both obtuse, unless all the angles are right angles)



- any pair of alternate angles on parallel lines are equal (either both acute or both obtuse, unless all the angles are right angles)



- any pair of co-interior angles on parallel lines are supplementary (with one acute and one obtuse, unless all the angles are right angles).

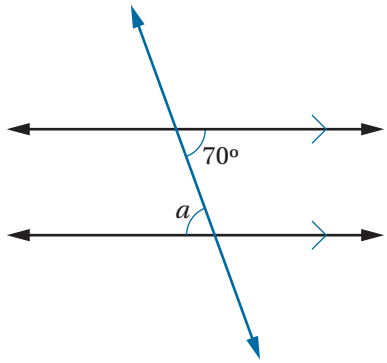


Worked example

Solving angles on parallel lines

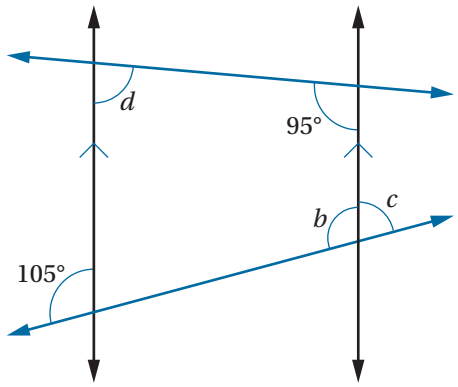
Determine the size of each angle marked with a variable. Justify your answers.

(a)



THINKING	WORKING
Recall the relationship between the unknown angle and the given angle.	The angles are alternate.
Determine whether the angles are equal or supplementary.	$a = 70°$
Write the justification.	Alternate angles on parallel lines are equal.

(b)



THINKING	WORKING
Recall the relationship between the first unknown angle and the given angle. Determine whether the angles are equal or supplementary.	Angle b corresponds to the 105° angle. $b = 105°$
Write the justification.	Corresponding angles on parallel lines are equal.

Recall the relationship between the second unknown angle and any known angle. Determine whether the angles are equal or supplementary.	Angles b and c are supplementary (adjacent on a straight angle). $b + c = 180°$, so $c = 180° - 105°$ $= 75°$
Write the justification.	Adjacent angles on a straight line are supplementary.
Decide on the relationship between the next unknown angle and any known angle. Determine whether the angles are equal or supplementary.	Angles d and 95° are co-interior. $d + 95° = 180°$ $d = 180° - 95°$ $= 85°$
Write the justification.	Co-interior angles on parallel lines are supplementary.

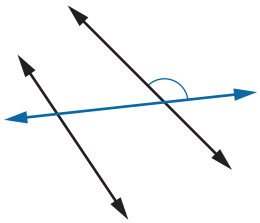
Practice

ANSWERS Page 709

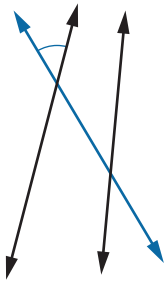
SC 1 I can identify angle relationships around a transversal

1 Copy each diagram and mark in the angle corresponding to the given angle.

(a)

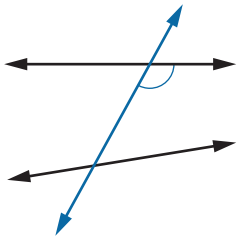


(b)

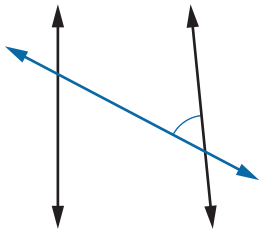


2 Copy each diagram and mark in the angle alternate to the given angle.

(a)

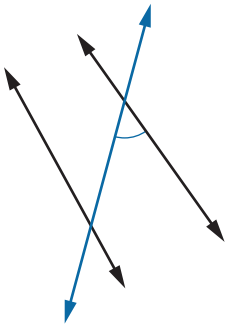


(b)

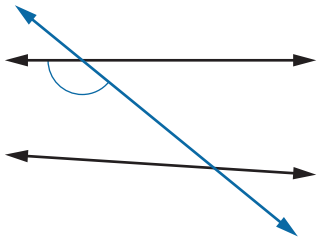


3 Copy each diagram and mark in the angle co-interior to the given angle.

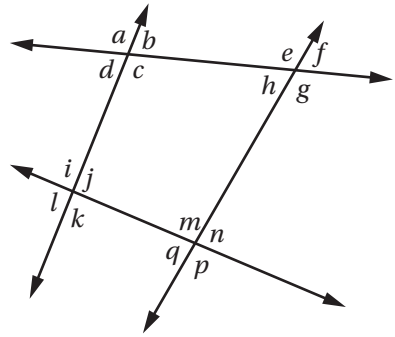
(a)



(b)



4 The diagram shows a pair of lines cutting another pair of lines, so any of the four lines could be a transversal for two other lines.



(a) List all the angles that are corresponding with a.

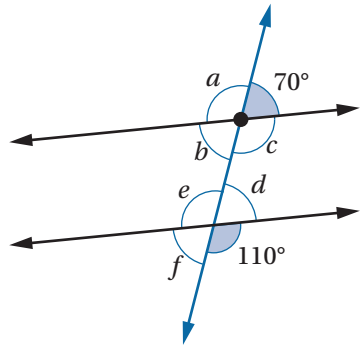
(b) List all the angles that are corresponding with b.

(c) List all the angles that are alternate with c.

(d) List all the angles that are co-interior with c.

SC 2 I can determine the size of unknown angles involving parallel lines

1 Copy and complete the following sentences for this diagram.



(a) Angle b is alternate to angle _____

(b) Angle d is vertically opposite to angle _____

(c) Angle e is co-interior to angle _____

(d) Angle b is corresponding to angle _____

2 Using the diagram from Question **1**, determine the size of each angle and write an explanation.

3 Decide whether the following pairs of angles are equal or supplementary (add up to 180°).

(a) Corresponding angles on parallel lines

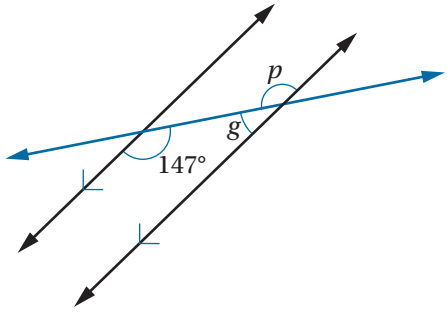
(b) Co-interior angles on parallel lines

(c) Alternate angles on parallel lines

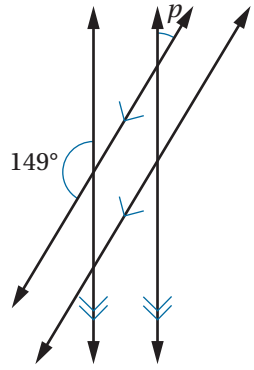
(d) Vertically opposite angles

(e) Adjacent angles on a straight line

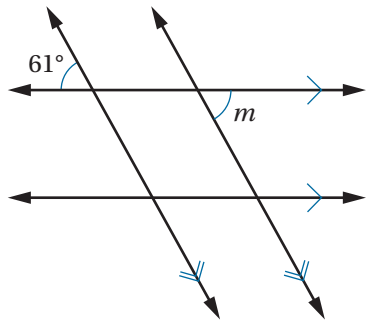
4 Determine the sizes of angle p and angle g.



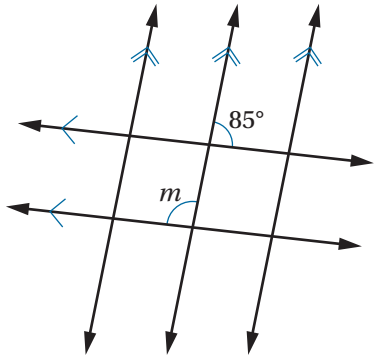
5 This diagram has two pairs of parallel lines. What is the size of angle p?



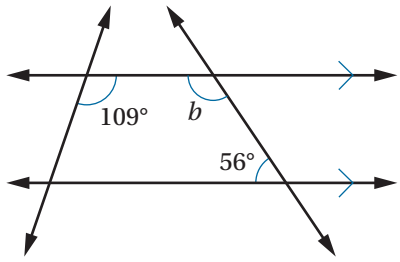
6 This diagram has two pairs of parallel lines. Determine the size of angle m.



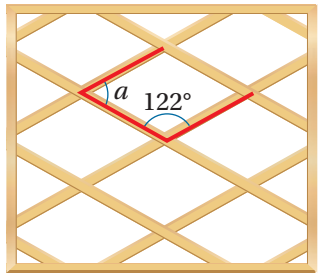
7 The diagram below has two sets of parallel lines. Determine the size of angle m.



8 Determine the size of angle b.



9 This wooden lattice decorates the top of a fence. It is made of straight parallel lines.



Determine the size of the unknown acute angle (shown as a) in the wooden lattice.

14.3 Determine the exterior angle of a triangle

Learning intention: To be able to determine the exterior angle of a triangle

Success criteria:

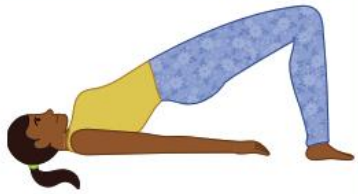
SC 1 I can use parallel lines to deduce the exterior angle property for triangles.

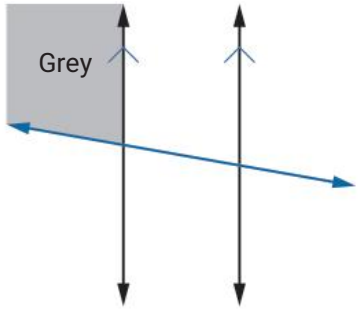
SC 2 I can determine the size of unknown angles involving the exterior angle rule.

Lesson warm-up

Coloured paddocks

The diagram shows a top view of some paddocks. Two parallel fences are crossed by a third transversal fence. The top left paddock is grey. Use the clues below to colour the other paddocks.





The purple paddock has two co-interior angles.

The grey paddock has an angle corresponding to an angle in the purple paddock.

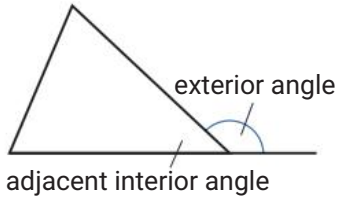
The orange paddock has an angle adjacent to angles in both the black and purple paddocks.

The blue paddock has an angle that is alternate to an angle in the purple paddock, and vertically opposite to an angle in the orange paddock.

The red paddock has an angle that is vertically opposite the purple paddock.

SC 1 I can use parallel lines to deduce the exterior angle property for triangles

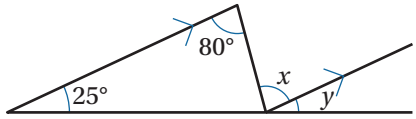
An exterior angle of a triangle is formed when one side of the triangle is extended. The exterior angle forms a straight angle with the interior angle adjacent to it.



Worked example

Deducing the exterior angle of a triangle rule

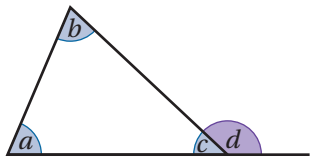
Solve for each variable and hence deduce the exterior angle of a triangle rule.



THINKING	WORKING
Use the parallel lines to solve for the angles marked with variables.	$x = 80°$, alternate angles on parallel lines are equal. $y = 25°$, corresponding angles on parallel lines are equal.
Determine the size of the exterior angle.	Exterior angle: $x + y$ The exterior angle is $25° + 80° = 105°$
State how the exterior angle is obtained from two of the angles in the triangle.	The exterior angle of a triangle is equal to the sum of the interior opposite angles.

SC 2 I can determine the size of unknown angles involving the exterior angle rule

The exterior angle rule states that the size of any exterior angle of a triangle is equal to the sum of the two interior opposite angles. This was deduced from a parallel line splitting the exterior angle into two angles that matched the interior opposite angles.



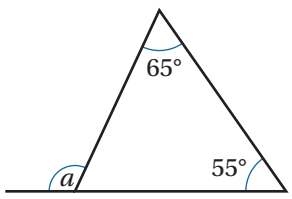
Algebraically: $d = a + b$

Worked example

Solving angles using the exterior angle rule

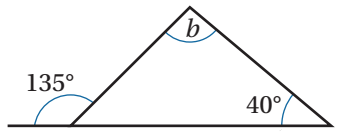
Use the exterior angle rule to calculate the size of the angle marked with a variable.

(a)



THINKING	WORKING
Apply the exterior angle rule.	$a = 65° + 55°$, exterior angle of a triangle
Solve the angle by adding.	$a = 120°$

(b)



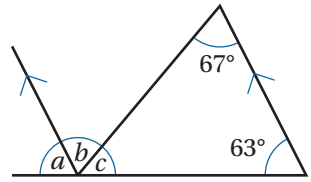
THINKING	WORKING
Apply the exterior angle rule.	$135° = b + 40°$, exterior angle of a triangle
Solve the angle by subtracting.	$b = 135° - 40°$ $= 95°$

Practice

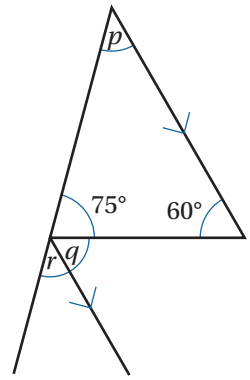
ANSWERS Page 710

SC 1 I can use parallel lines to deduce the exterior angle property for triangles

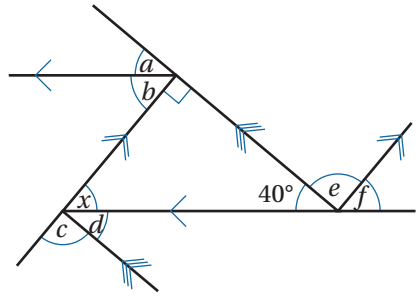
1 Solve for each variable and hence deduce the exterior angle of a triangle rule.



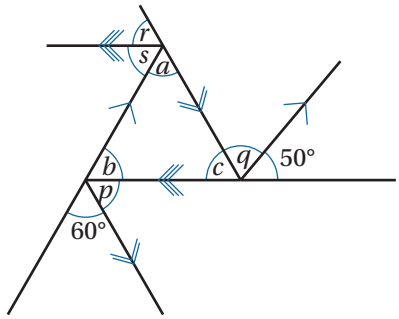
2 Solve for each variable and hence deduce the exterior angle of a triangle rule.



3 Solve for each variable and hence deduce the exterior angle of a triangle rule.



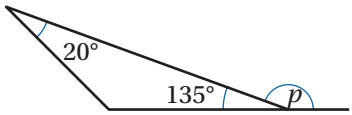
4 Solve for each variable and hence deduce the exterior angle of a triangle rule.



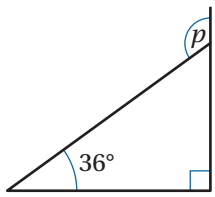
SC 2 I can determine the size of unknown angles involving the exterior angle rule

1 Use the exterior angle rule to calculate the value of p.

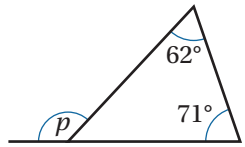
(a)



(b)

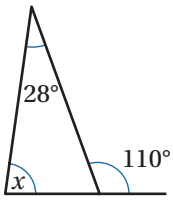


(c)

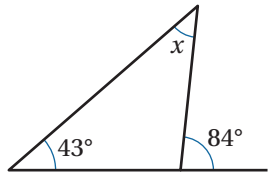


2 Use the exterior angle rule to calculate the value of x.

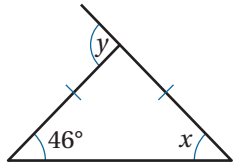
(a)



(b)

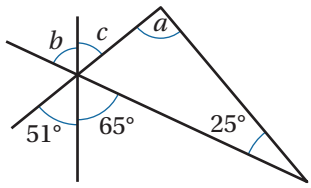


3 Use angle rules to calculate the size of the angles marked with variables.

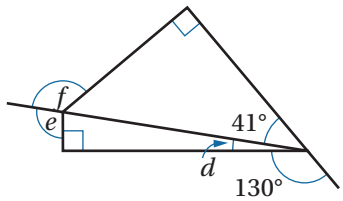


4 Use angle rules to calculate the size of angles marked with variables.

(a)



(b)



14.4 Calculate the internal angle sum of a triangle

Learning intention: To be able to determine the internal angle sum of a triangle

Success criteria:

SC 1 I can use parallel lines to deduce the angle sum of a triangle.

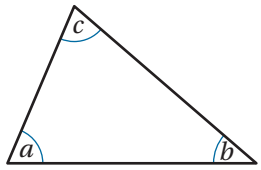
SC 2 I can determine the angle sum of any polygon.

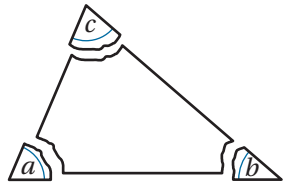
SC 3 I can determine unknown angles involving the angle sum rules.

Lesson warm-up

Angles and triangles.

- Draw a triangle on a sheet of paper. Make sure your lines are straight.
- Measure your triangle's interior angles with a protractor. Write the size of each angle on the triangle.
- Cut out your triangle.
- Tear the three corners off the triangle. (Make the torn pieces as large as you can.)
- Arrange the torn corners along a straight line



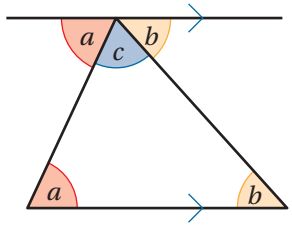


1. Add the sizes of the three angles together. What is the sum?
2. Discuss your answer with a partner. Did you expect to get this answer? Why or why not?
3. Write some other angle totals found by other students in your class.
4. What is one thing you notice about the answers from your class?

SC 1 I can use parallel lines to deduce the angle sum of a triangle

$a + b + c = 180°$

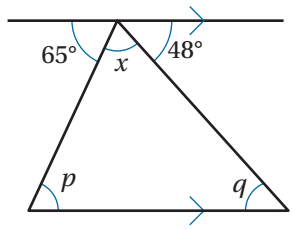
The angle sum of a triangle is 180°.



Worked example

Deducing the angle sum of a triangle using parallel line relationships

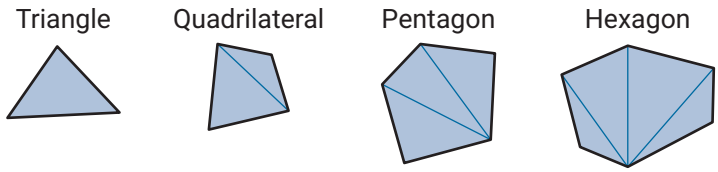
Determine the size of each angle marked with a variable, and hence deduce the angle sum of a triangle rule.



THINKING	WORKING
Calculate the unknown angle forming a straight line.	$65° + x + 48° = 180°$, a straight angle is 180°. $x = 180° - 65° - 48°$ $= 67°$
Calculate the other unknown angles using parallel line relationships.	$p = 65°$ and $q = 48°$, alternate angles on parallel lines are equal.
Make a conclusion about the angle sum in terms of the straight angle.	The three angles of the triangle are the same as the three angles forming a straight line. Hence the angle sum of a triangle is 180°.

SC 2 I can determine the angle sum of any polygon

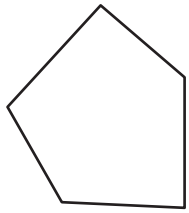
The angle sum of any polygon can be determined by dividing the polygon into triangles using diagonals that do not overlap.

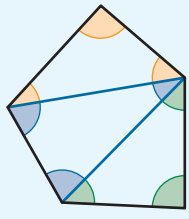


Worked example

Using triangles to calculate the angle sum of a polygon

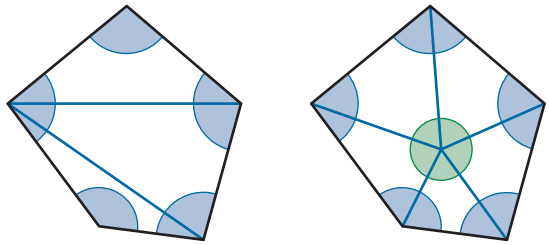
Determine the angle sum of the pentagon.



THINKING	WORKING
Divide the pentagon into triangles using diagonals. Ensure that none of the diagonals cross each other. The angles within each triangle have been coloured for effect.	
Count the number of triangles.	There are 3 triangles, each with an angle sum of 180°.
Determine the angle sum of the pentagon.	The angle sum of the pentagon is: $180° \times 3 = 540°$

SC 3 I can determine unknown angles involving the angle sum rules

The angle sum of an n-sided polygon is $180° \times (n - 2)$ or $180° \times n - 360°$



Worked example

Solving angles of polygons using the angle sum rule

A hexagon has internal angles of 100°, 95°, 135°, 87° and 132°. Determine the unknown angle.

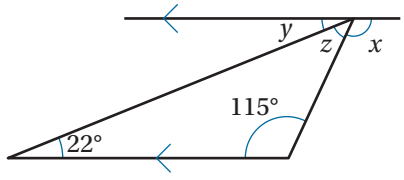
THINKING	WORKING
State the number of sides as a value of n.	For a hexagon, $n = 6$
Use a formula to calculate the angle sum.	$180° \times (n - 2) = 180° \times (6 - 2)$ $= 180° \times 4$ $= 720°$ The angle sum of a hexagon is 720°.
Use subtraction to calculate the unknown angle.	$720 - 100 - 95 - 135 - 87 - 132 = 171°$
Write the answer.	The unknown angle is 171°.

Practice

ANSWERS Page 710

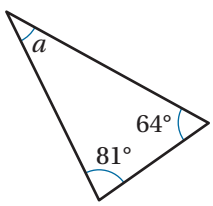
SC 1 I can use parallel lines to deduce the angle sum of a triangle

1 Determine the size of each angle marked with a variable, and hence deduce the angle sum of a triangle rule.



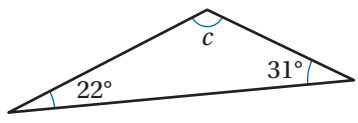
2 Use the angle sum of a triangle rule to determine the size of each angle marked with a variable.

(a)



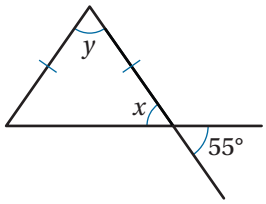
(b)

(c)

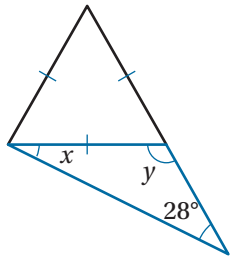


3 Determine the size of each angle marked with a variable. Justify your working with reasons.

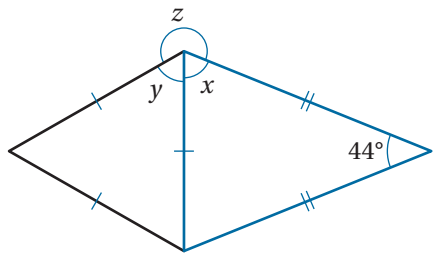
(a)



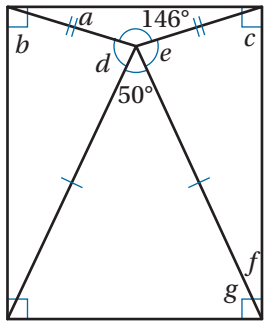
(b)



(c)



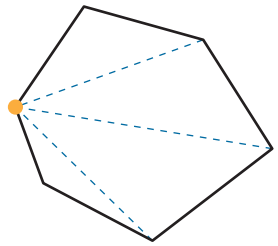
4 Determine the size of each angle marked with a variable. Justify your working with reasons.



14

SC 2 I can determine the angle sum of any polygon

1 Using a method of drawing all diagonals from a given point:



(a) copy the diagram and draw the diagonals

(b) copy and complete the following:

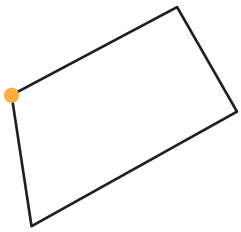
Name of polygon:

Number of sides (or angles):

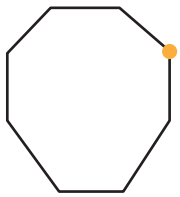
Number of triangles:

(c) make a conclusion about the relationship between the number of sides and the number of triangles.

(i)



(ii)



(iii)

2 Draw three versions of a convex heptagon.

(a) Divide each heptagon into triangles, with as much variation as you can.

(b) Count the number of triangles for each of your diagrams.

(c) Determine the angle sum of a heptagon.

3 Use a template to draw a regular octagon. Place a point near the centre.

(a) Join each vertex to the centre point.

(b) Count the number of triangles you have produced.

(c) Determine the total number of degrees within these triangles that are not part of the interior angles of the octagon. Justify your answer with a reason.

(d) Hence determine the angle sum of an octagon.

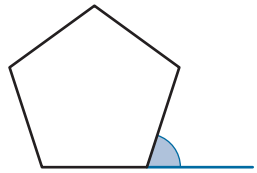
(e) Determine the interior angle size of the regular octagon.

4 For a regular pentagon:

(a) use the triangle method (using diagonals that do not overlap) to calculate the angle sum

(b) determine the size of the internal angles

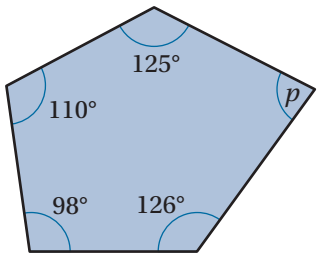
(c) determine the size of any exterior angle drawn.



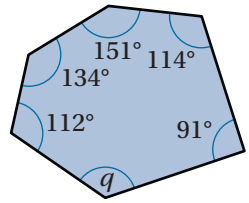
SC 3 I can determine unknown angles involving the angle sum rules

1 Determine the size of the unknown interior angle.

(a)

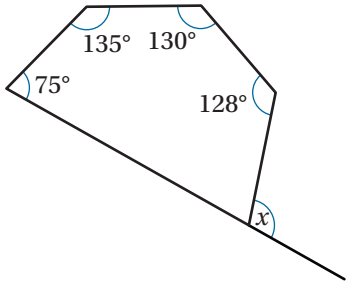


(b)

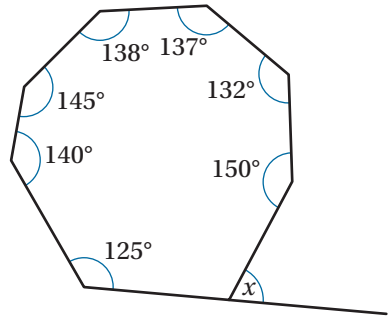


2 Determine the size of the exterior angle marked x.

(a)



(b)



3 Use the angle sum formula to determine the size of the interior angle for each shape.

(a) Regular decagon **(b)** Regular heptagon **(c)** Regular quadrilateral

4 For regular polygons, the size of the exterior angle is a constant value.
Use the angle sum formula to determine the size of the exterior angle for each shape.

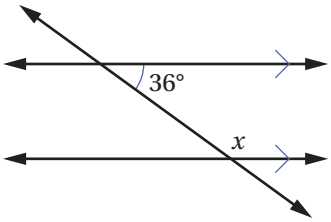
(a) Regular hexagon **(b)** Regular octagon **(c)** Regular quadrilateral

TOPIC REVIEW

Multiple choice

ANSWERS Page 746

1 Choose the alternative with the correct answer and reason.



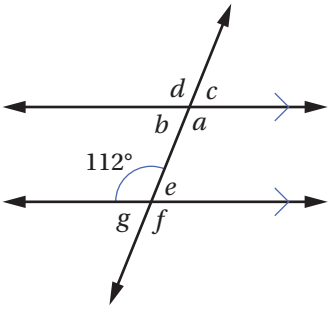
A $x = 36°$, corresponding angles are equal.

B $x = 144°$, co-interior angles are supplementary.

C $x = 36°$, co-interior angles are equal.

D $x = 144°$, corresponding angles are supplementary.

2 Choose the alternative with the correct answer and reason.



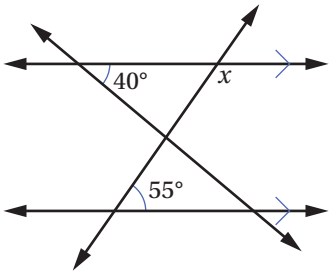
A $b = 112°$, corresponding angles are equal.

B $d = 112°$, co-interior angles are equal.

C $c = 68°$, corresponding angles are supplementary.

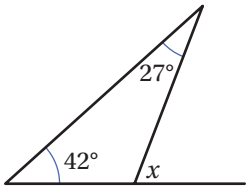
D $a = 112°$, alternate angles are equal.

3 Choose the correct value of x.



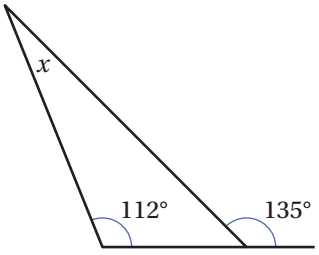
A 140° **B** 125° **C** 95° **D** 85°

4 Choose the correct value of x.



A 15° **B** 63° **C** 69° **D** 111°

5 Choose the correct value of x.



A 23° **B** 42° **C** 36° **D** 68°

6 Below are the instructions for constructing perpendicular lines.

I From a point on the line, mark equal distances on the line to the right and left.

II Draw a horizontal line.

III With your compass point on the point to the right, make an arc on either side of the line.

IV Open a pair of compasses to more than half the distance between the right and left points.

V Join the intersection points

VI With your compass point on the point to the left, make an arc on either side of the line to intersect the first two arcs.

Choose the option that shows the correct order of the instructions.

A I, II, V, VI, IV, III

B II, I, V, VI, III, IV

C II, I, V, III, IV, VI

D II, I, IV, III, VI, V

7 A set of instructions for constructing parallel lines begins as follows.

- Draw a line.
- Use a set square to draw a line at right angles to the first line.

The next instruction could be:

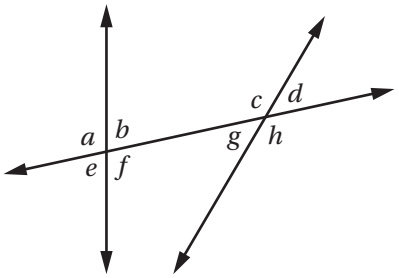
A Use a set square to draw another line at right angles to the first line.

B Use a set square to draw a line at right angles to the second line.

C With your compass point on the intersection, make arcs to the left and right of the intersection.

D With your compass point on the second line, make two arcs on the first line.

8 What is the relationship between the pair of angles a and c in the diagram below?



A Corresponding **B** Alternate **C** Co-interior **D** None of the above

9 Choose the true statement regarding angle relationships around a transversal.

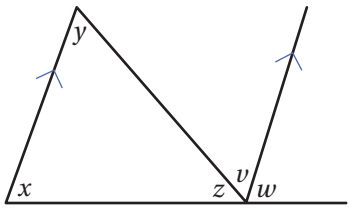
A Corresponding angles are equal whether or not the other lines are parallel.

B Co-interior angles on parallel lines are equal.

C Alternate angles form a Z shape whether or not the other lines are parallel.

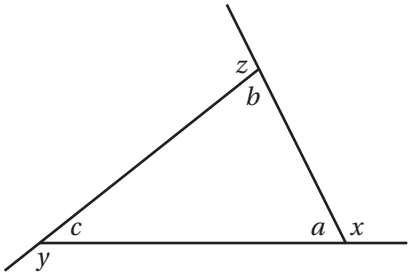
D It is possible for a pair of corresponding angles to be supplementary when neither of them is a right angle.

10 Choose the correct equation.



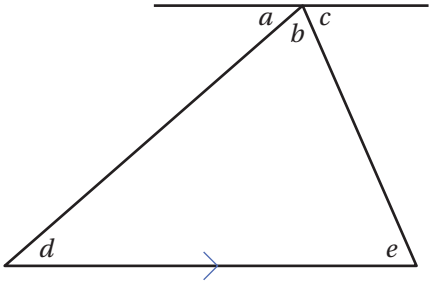
A $x = w$ **B** $y = w$ **C** $w = z$ **D** $x + y = z$

11 Choose the correct equation.



A $x + z = y$ **B** $c + a = x$ **C** $c + b = z$ **D** $a + b = y$

12 For the diagram below, choose the incorrect equation.



A $c = e$

B $b = d$

C $a + c = d + e$

D $a + b + c = b + d + e$

13 For the diagram shown in Question **12**, choose the correct statement.

A a is co-interior with e.

B c is corresponding with d.

C b is supplementary to a.

D a is alternate with d.

14 The number of triangles that a hexagon can be divided into, using diagonals that do not overlap, is:

A 3 **B** 4 **C** 5 **D** 6

15 The angle sum of an octagon can be found by calculating:

A $180° \times 6$ **B** $180° \times 7$ **C** $180° \times 8$ **D** $180° \times 10$

16 The incorrect expression for calculating the angle sum of an n-sided polygon is:

A $180° \times (n - 2)$

B $180° \times n - 360°$

C $180° \times n - 180° \times 2$

D $180° \times n + 360°$

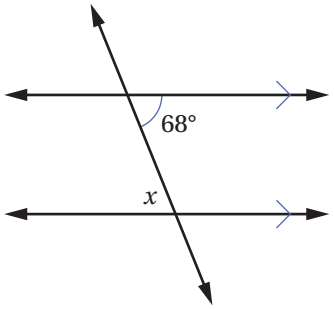
17 An expression to calculate the interior angle of a regular decagon could be:

A $\frac{180° \times 10}{8}$ **B** $\frac{180° \times 10}{12}$ **C** $\frac{180° \times 8}{10}$ **D** $\frac{180° \times 12}{10}$

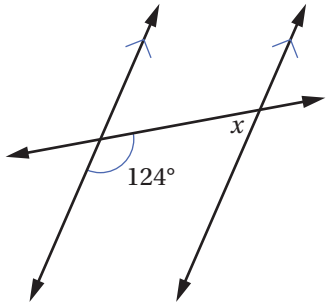
Short answer

1 Calculate the size of each angle marked with a variable. Justify your answers with reasons.

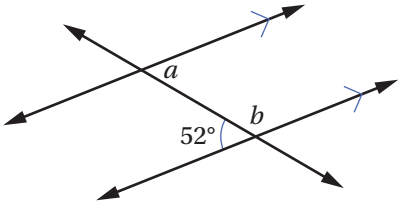
(a)



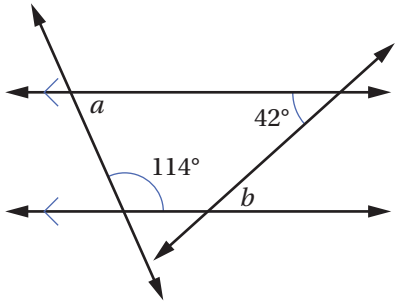
(b)



(c)

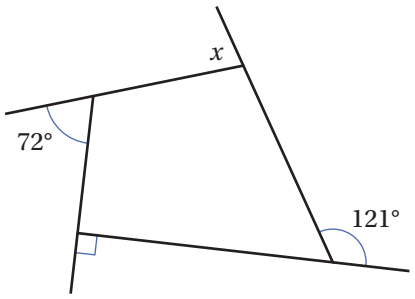


2 Calculate the size of each angle marked with a variable. Justify your answers with reasons.

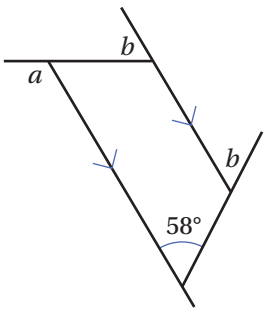


3 Calculate the size of each angle marked with a variable. Justify your answers with reasons.

(a)

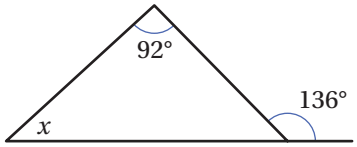


(b)

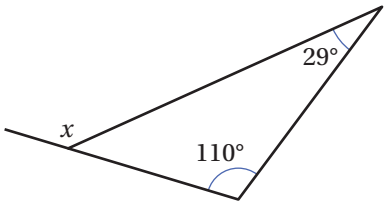


4 Use the exterior angle rule to calculate the value of x.

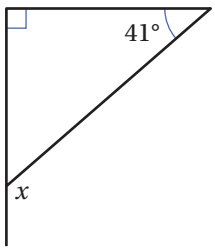
(a)



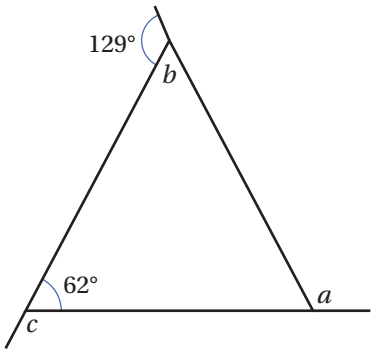
(b)



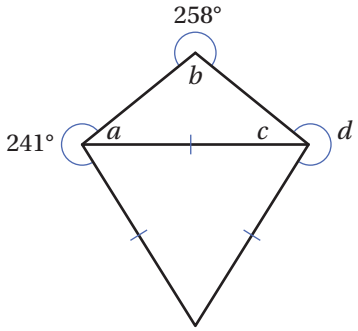
(c)



5 Calculate the size of each angle marked with a variable. Justify your answers with reasons.

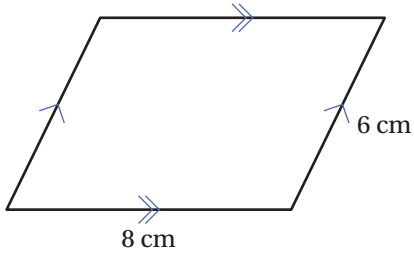


6 Calculate the size of each angle marked with a variable. Justify your answers with reasons.

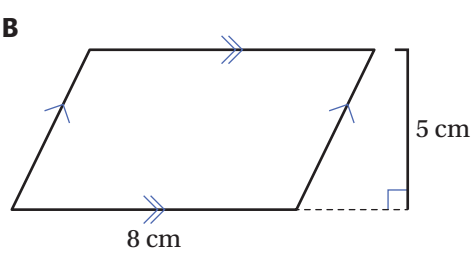


7 Below are two parallelograms with some dimensions given. Determine which is suitable for construction with a set square and ruler, and which is easier to construct with a ruler and pair of compasses.

A



B



8 Calculate the angle sum for:

(a) a heptagon

(b) a dodecagon (12-sided polygon).

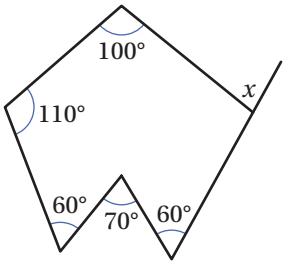
9 Calculate the size of the internal angles of a regular:

(a) hexagon

(b) octagon.

10 Four of the interior angles of a pentagon are 100°. Determine the size of the other angle.

11 Determine the size of the exterior angle.

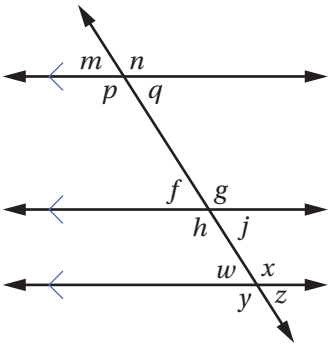


12 Calculate the size of the external angles of a regular:

(a) pentagon **(b)** hexagon.

Extended response

1 Determine which angles in the diagram are:



(a) corresponding to p

(b) alternate to j

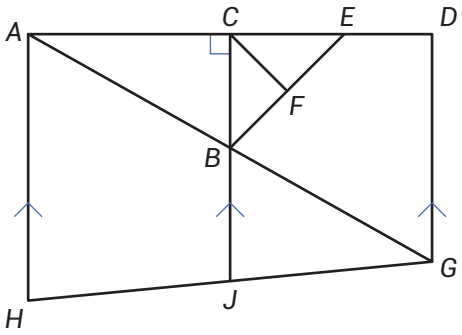
(c) co-interior with w

(d) co-interior with g

(e) corresponding to w

(f) alternate to p.

2 Use names such as ABC to name each described angle.



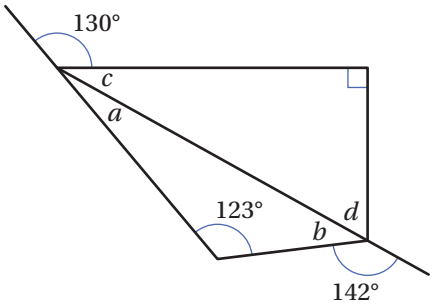
(a) An angle corresponding to angle AHJ

(b) An angle alternate to angle AGH

(c) An angle supplementary to angle EBG

(d) An angle complementary to angle BCF

3 Calculate the size of each angle marked with a variable. Justify your answers with reasons.



TOPIC
15

Ratios and measurement

Recall 416

15.1 Solve practical problems involving ratios of length 417

15.2 Explore ratios in measured quantities 425

Topic review 432

Why learn this?

Solving practical problems involving ratios of length prepares learners for real-world applications where proportional reasoning is key. This topic offers:

- skills to describe and calculate changes in dimensions due to enlargement, relevant to creating scale models or adjusting designs
- the ability to estimate ratios in circular measures, which is important in fields such as construction and manufacturing
- proficiency in solving ratio problems, vital for tasks that range from crafting to understanding geometric properties in nature.

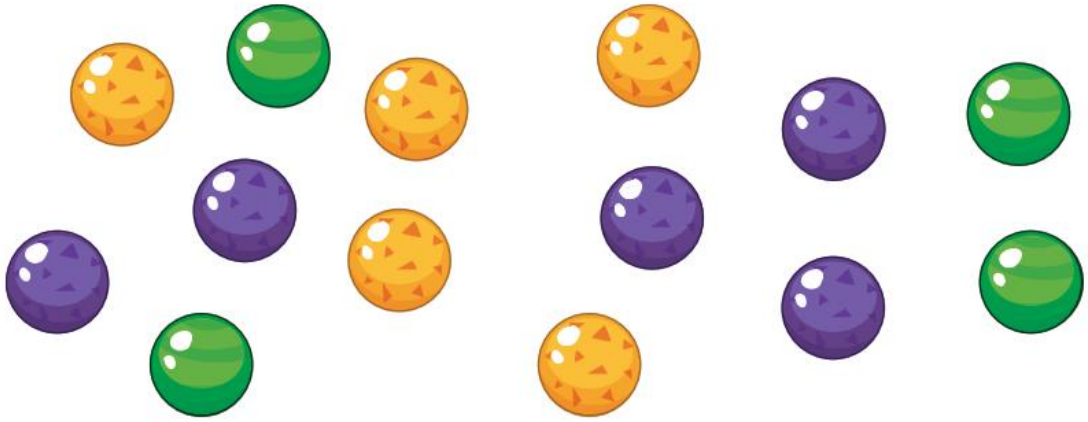
The knowledge and skills gained are not only crucial for academic success in mathematics, but also for practical tasks in daily life and various professional fields, including architecture, surveying and graphic design.

RECALL

ANSWERS Page 711

I can group and count items by common attributes

1 Complete the table to identify the number of each type of marble shown:



Yellow speckled	Purple speckled	Green striped

I can identify the highest common factor for a set of numbers

1 Write the HCF for each set of numbers.

(a) 8, 28, 4, 12 **(b)** 6, 24, 15, 18 **(c)** 16, 48, 32, 52

I can use the HCF to simplify fractions

1 Simplify each of the following fractions.

(a) $\frac{9}{15}$ **(b)** $\frac{18}{24}$ **(c)** $\frac{18}{15}$ **(d)** $\frac{32}{20}$

I can write equivalent fractions

1 Identify the missing number in each of the following fractions.

(a) $\frac{24}{\square} = \frac{3}{13}$ **(b)** $\frac{7}{9} = \frac{\square}{18}$ **(c)** $\frac{\square}{3} = \frac{22}{33}$ **(d)** $\frac{16}{28} = \frac{4}{\square}$

I can express mixed numbers as improper fractions

1 Write the following mixed numbers as equivalent improper fractions.

(a) $2\frac{1}{3}$ **(b)** $5\frac{3}{4}$ **(c)** $4\frac{2}{7}$ **(d)** $8\frac{5}{6}$

Solve practical problems involving ratios of length

15.1

Learning intention: To be able to solve practical problems involving ratios of length

Success criteria:

SC 1 I can describe the change in length with enlargement.

SC 2 I can determine an approximation for the ratio between measures of a circle.

SC 3 I can solve practical problems involving ratios of length.

Lesson warm-up

Scale models and measures

Describe the ratio of the scale model shown and the real house.

SC 1 I can describe the change in length with enlargement

To describe a change in length with an enlargement, the enlarged value is written first in the ratio

scale diagram : original

A scale factor is a unit ratio written as a fraction:

$$\text{scale factor} = \frac{\text{scale diagram}}{\text{original}}$$

For example,

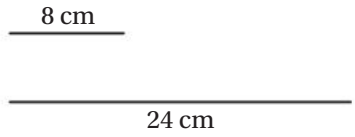
Problem	**Ratio** (scale diagram : original)	**Scale factor** (enlargement/reduction factor)
A length of 10 cm enlarged to 12 cm: $\frac{\text{10 cm}}{\text{12 cm}}$	12 cm : 10 cm or 1.2 : 1 written as a unit ratio.	The ratio 1.2 : 1 can be written as $\frac{1.2}{1} = 1.2$.
A length of 12 cm reduced to 10 cm: $\frac{\text{12 cm}}{\text{10 cm}}$	10 cm : 12 cm or 0.83 : 1 (2 d.p.) written as a unit ratio.	The ratio 0.83 : 1 can be written as $\frac{0.83}{1} = 0.83$.

Worked example

Describing enlargements using a single number

Describe the change in length when:

(a) 8 cm is enlarged to 24 cm using an enlargement factor



THINKING	WORKING
Estimate the scale factor.	$8 \times 3 = 24$ The enlarged line is 3 times the length of the original.
Recall the ratio formula.	scale diagram : original
Write the ratio of the enlarged length to the original length.	24 cm : 8 cm
Divide both parts of the ratio by the original length to write as a unit ratio.	$\frac{24}{8} : \frac{8}{8}$ $= 3 : 1$ As a fraction, 3 : 1 can be written as an enlargement factor of $\frac{3}{1} = 3$.
Alternatively, you can use the formula for the enlargement factor.	$\text{enlargement factor} = \frac{\text{scale diagram}}{\text{original}}$ $= \frac{24}{8}$ $= 3$ The enlargement factor is 3.
Write the answer. Check the reasonableness of your answer against your original estimation.	24 cm is 3 times as long as 8 cm.

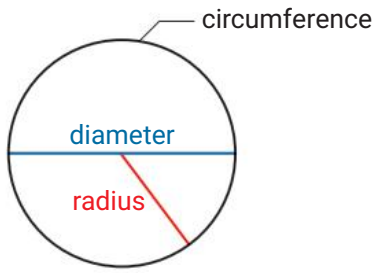
(b) 85 cm is enlarged to 1.3 m using an enlargement factor.

85 cm

1.3 m

THINKING	WORKING
Estimate the scale factor.	$85 \times 2 = 170$ The scale factor is between 1 and 2.
Recall the ratio formula.	scale diagram : original
Write the ratio of the enlarged length to the original length, in the same units.	1.3 m : 85 cm = 130 cm : 85 cm
Divide both parts of the ratio by the original length to write as a unit ratio.	$= \frac{130}{85} : \frac{85}{85}$ = 1.53 : 1 (2 d.p.)
Alternatively you can use the formula for the enlargement factor.	$\text{Enlargement factor} = \frac{\text{scale diagram}}{\text{original}}$ $= \frac{1.53}{1}$ $= 1.53$ The enlargement factor is 1.53.
Write the answer. Check the reasonableness of your answer against your original estimation.	1.3 m is 1.53 times as long as 85 cm.

SC 2 I can determine an approximation for the ratio between measures of a circle



Circle measurements are often rounded, so calculations from these measurements give approximate, rather than exact, values.

Ratios can be written as fractions, so 3 : 4 is the same as $\frac{3}{4}$ and unit ratios can be written as single numbers, so $5.27 : 1 = \frac{5.27}{1} = 5.27$.

Worked example

Calculating approximations for the ratio between circumference and diameter

The diameter, D, of a circle is 15.3 cm and its circumference, C, is 48.1 cm. Determine, correct to 2 decimal places:

(a) the ratio of circumference to diameter as a single number

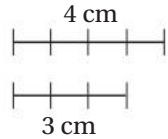
THINKING	WORKING
Divide the circumference by the diameter.	$\frac{C}{D} = \frac{48.1}{15.3}$ $= 3.143\ldots$
Write the answer.	The ratio of circumference to diameter is approximately 3.14.

(b) the ratio of circumference to radius, r, as a single number.

THINKING	WORKING
Calculate the radius by halving the diameter.	$r = \frac{D}{2}$ $= \frac{15.3}{2}$ $= 7.65$ The length of the radius is 7.65 cm.
Divide the circumference by the radius.	$\frac{C}{r} = \frac{48.1}{7.65}$ $= 6.287\ldots$
Write the answer.	The ratio of circumference to radius is approximately 6.29.

SC 3 I can solve practical problems involving ratios of length

If lengths are in the ratio 3 : 4,

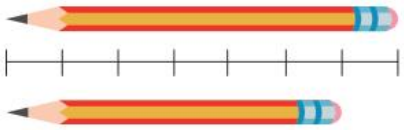


- multiplying the larger length by $\frac{3}{4}$ will reduce the value to the smaller one (this is the same as dividing by four and multiplying by three)
- multiplying the smaller value by $\frac{4}{3}$ will increase it to the larger one (this is the same as dividing by three and multiplying by four).

Worked example

Calculating lengths from given ratios

The lengths of two pencils are in the ratio 6 : 7.



To the nearest millimetre, determine the length of:

(a) the longer pencil, given that the shorter pencil length is 12 cm

THINKING	WORKING
Interpret the ratio.	For every six parts of shorter pencil, there are seven parts of the longer pencil.
Determine the unit length and calculate the longer length.	Shorter pencil: 12 cm Every unit is worth $12 \div 6 = 2$ cm. The longer pencil is $2 \times 7 = 14$ cm.
Alternatively, multiply by the fraction that will increase the length.	Longer pencil: $12 \times \frac{7}{6} = 14$ cm
Write the answer.	The longer pencil is 14 cm.

(b) the shorter pencil, given that the longer pencil length is 10.5 cm.

THINKING	WORKING
Interpret the ratio.	For every seven parts of longer pencil, there are six parts of the shorter pencil.
Determine the unit length and calculate the shorter length.	Longer pencil: 10.5 cm Every unit is worth $10.5 \div 7 = 1.5$ cm. The shorter pencil is $1.5 \times 6 = 9$ cm.
Alternatively, multiply by the fraction that will decrease the length.	Shorter pencil: $10.5 \times \frac{6}{7} = 9$
Write the answer.	The shorter pencil length is 9 cm.

Practice

ANSWERS Page 711

SC 1 I can describe the change in length with enlargement

1 Determine the enlargement factor when a length of 10 cm is increased to:

(a) 20 cm **(b)** 30 cm **(c)** 50 cm **(d)** 100 cm

2 Determine the enlargement factor when a length of 10 cm is increased to:

(a) 11 cm **(b)** 12 cm **(c)** 15 cm **(d)** 19 cm

3 Determine the reduction factor when a length of 10 cm is decreased to:

(a) 5 cm **(b)** 1 cm **(c)** 2 cm **(d)** 2.5 cm

4 Each of the following ratios is written in the form scale length : original length. For each case, describe the scale factor.

(a) 25 cm : 50 cm **(b)** 25 cm : 100 cm **(c)** 50 cm : 25 cm

(d) 100 cm : 25 cm **(e)** 50 cm : 100 cm **(f)** 100 cm : 50 cm

5 Compare the length of the longest river in Western Australia, the Gasgoyne, at 834 km with Victoria's longest river, the Goulburn, at 645 km:

(a) as a unit ratio of Gasgoyne : Goulburn, rounding appropriately

(b) in a sentence using the enlargement factor.

6 The Murray River is Australia's longest river; 1808 km of the river lies in New South Wales and 700 km of the river lies in South Australia.

(a) Write NSW length : South Australia length as a unit ratio.

(b) Compare the lengths of the river in the two states in a sentence using the enlargement factor.

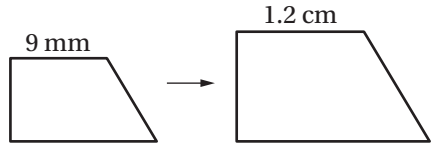
7 Consider the ratios 2558 : 2640 and 3.71 : 1.6.

(a) Choose the ratio that represents an enlargement and write the equivalent unit ratio, correct to 2 decimal places.

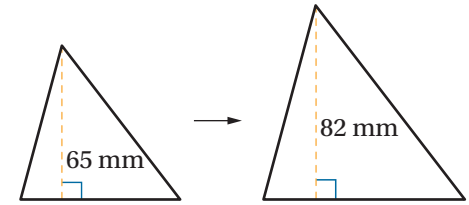
(b) Choose the ratio that represents a reduction and write the equivalent unit ratio, correct to 2 decimal places.

8 Calculate the enlargement factor for each of the following transformations, correct to 2 decimal places.

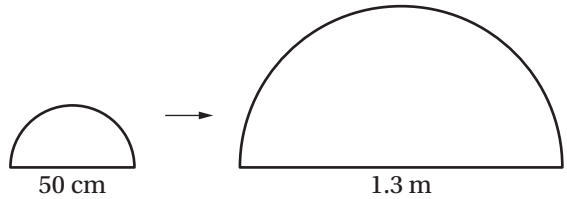
(a)



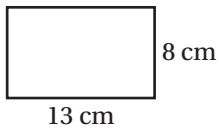
(b)



(c)



9 The rectangle shown is to be enlarged so that the perimeter is 1 m. Determine the enlargement factor, correct to 2 decimal places.



SC 2 I can determine an approximation for the ratio between measures of a circle

1 Determine the ratio of circumference, C, to diameter, D, as a single number, correct to the required number of decimal places.

(a) $D = 8.4$ m, $C = 26.4$ m (2 decimal places)

(b) $D = 4.313$ cm, $C = 13.55$ cm (3 decimal places)

(c) $D = 76$ mm, $C = 24$ cm (1 decimal place)

2 Determine the ratio of circumference, C, to radius, r, as a single number, correct to the required number of decimal places.

(a) $r = 24$ mm, $C = 15.1$ cm (1 decimal place)

(b) $r = 5$ m, $C = 31.4$ m (2 decimal places)

(c) $r = 7.34$ cm, $C = 46.12$ cm (3 decimal places)

3 Compare the circumference of a circle, $C = 2.64$ m, to its diameter, $D = 84$ cm:

(a) in its simplest form **(b)** as an improper fraction.

4 The ratio of circumference to diameter for every circle is approximately 3.14. Determine the length of the circumference from the given diameter and radius lengths.

(a) $D = 20$ cm **(b)** $D = 2.5$ m **(c)** $r = 13$ cm **(d)** $r = 23$ mm

5 Calculate the circumference of each circle, given that the ratio of circumference to diameter can be estimated as $\frac{22}{7}$.

(a) $D = 70$ mm **(b)** $D = 5.6$ cm **(c)** $r = 105$ mm **(d)** $r = 2.1$ m

6 Given that the ratio between any circle is approximately 3.1416, determine the values of the diameter, D, and radius, r, for each of the given circumferences. Write answers in the same units, correct to 2 decimal places.

(a) $C = 50$ cm **(b)** $C = 140$ mm **(c)** $C = 2.5$ m

SC 3 I can solve practical problems involving ratios of length

1 In a human body, the ratio of femur length to body height is approximately 1 : 4.

(a) Estimate the height of a person who has a femur length of:

(i) 20 cm **(ii)** 30 cm **(iii)** 40 cm **(iv)** 50 cm

(b) Estimate the femur length for people with the following heights:

(i) 150 cm **(ii)** 160 cm **(iii)** 170 cm **(iv)** 180 cm

(c) Use your answers to part **(a)** and **(b)** to describe what you notice about the change in one length as the other increases by 10 cm.

2 The longer to shorter ratio of two steel rods is 2.31 : 1, or written simply as 2.31.

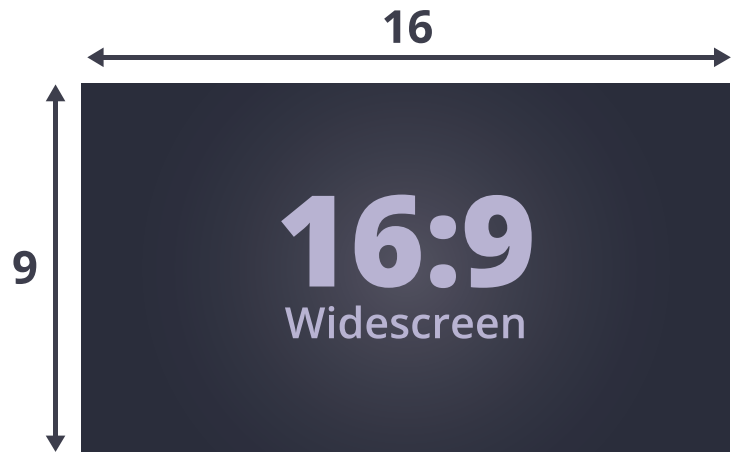
(a) Calculate the length of the longer rod, given that the shorter one is:

(i) 20 cm **(ii)** 59 mm **(iii)** 1.6 m

(b) Calculate the length of the shorter rod in the same units, correct to 2 decimal places, given that the longer one is:

(i) 3.2 m **(ii)** 36 cm **(iii)** 300 mm

3 TV screens in Australia have an aspect ratio of 16 : 9, referring to the width and height. To the nearest centimetre, determine:



(a) the height of a TV that has a width of:

(i) 45 cm **(ii)** 55 cm

(b) the width of a TV that has a height of:

(i) 85 cm **(ii)** 100 cm

4 The width and height of a picture frame have a ratio of 1.6. Given that the width of the frame is 98 cm, how tall is the frame, to the nearest centimetre, for the following the picture orientations?

(a) landscape **(b)** portrait

5 Copy and complete each of the following proportion tables, keeping the lengths in each row in the same units. Round correct to 2 decimal places, where necessary.

(a)

Scale diagram	Original
5	8
15 cm	
	30 cm

(b)

Scale diagram	Original
20	11
	6 m
12 cm	

Explore ratios in measured quantities

15.2

Learning intention: To be able to explore ratios in measured quantities

Success criteria:

SC 1 I can make connections between length, area and volume ratios after scaling transformations.

SC 2 I can apply and solve ratio problems involving mixtures.

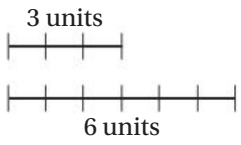
Lesson warm-up

Creating with coffee

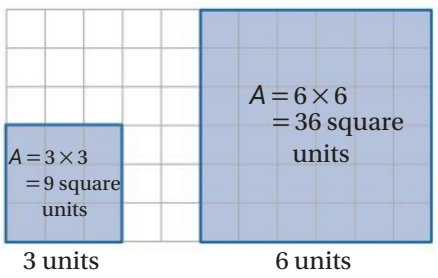
1. Which drink has the strongest coffee flavour? Explain.
2. Which drink has the weakest coffee flavour?
3. Describe the difference between a latte and a double latte. Why do you think it is called a double latte?
4. What do you think the words espresso, doppio and lungo mean?

SC 1 I can make connections between length, area and volume ratios after scaling transformations

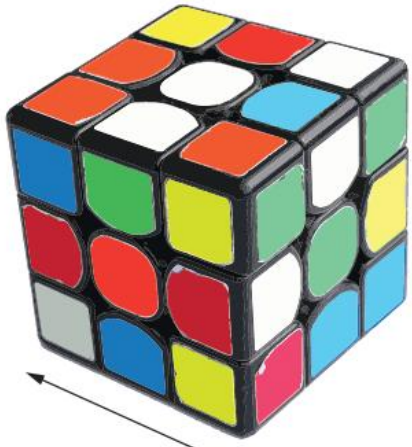
The enlargement factor used to increase the size of a two-dimensional figure refers to the ratio between each of the new lengths and the corresponding original length, including heights of triangles, diameters of circles and perimeters.



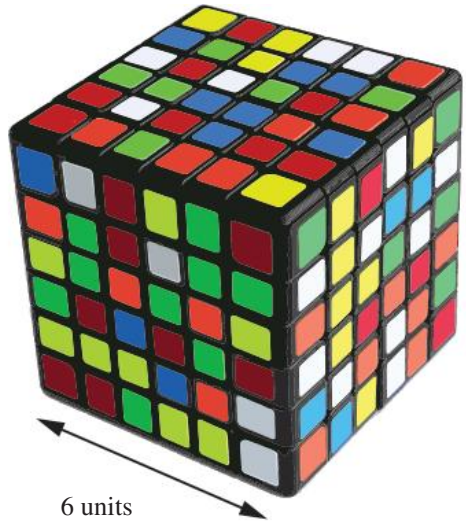
Formulas for areas involve multiplying two lengths together, so the enlargement factor for the lengths must be squared to give the ratio comparing the new area with the original area.



Similarly, since formulas for volume involve multiplying three lengths together, the enlargement factor for the lengths must be cubed to give the ratio comparing the new volume with the original volume.



$V = 3 \times 3 \times 3$
$= 27$ cubic units

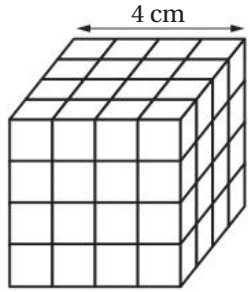


$V = 6 \times 6 \times 6$
$= 216$ cubic units

Worked example

Connecting the enlargement factor to area and volume

The cube has been enlarged using the ratio 1.5.



(a) Determine the dimensions of the enlarged cube.

THINKING	WORKING
Interpret the ratio.	The side length of the enlarged cube is 1.5 times the original cube.
Determine the dimensions of the enlarged cube by multiplying the length by the scale factor.	$L_1 = 4$ cm $L_2 = 4 \times 1.5 = 6$ cm
Write the answer.	The enlarged cube has a side length of 6 cm.

(b) Calculate and compare the areas of the square face of the original and enlarged cube.

THINKING	WORKING
Determine the area of the original square by multiplying the dimensions.	$A_1 = 4 \times 4$ $= 16 \text{ cm}^2$
Determine the area of the enlarged square.	$A_2 = 6 \times 6$ $= 36 \text{ cm}^2$
Compare the calculations.	$A_2 = 6 \times 6$ $= (1.5 \times 4) \times (1.5 \times 4)$ $= (1.5)^2 \times (4 \times 4)$ $= (1.5)^2 \times A_1$
Write the answer.	The area of enlarged square is found by squaring the scale factor, 1.5 and multiplying the result by the area of the original square.

(c) Calculate and compare the volumes of the original and enlarged cube.

THINKING	WORKING
Determine the volume of the original cube by multiplying the dimensions.	$V_1 = 4 \times 4 \times 4$ $= 64 \text{ cm}^3$
Determine the area of the enlarged square.	$V_2 = 6 \times 6 \times 6$ $= 216 \text{ cm}^3$
Compare the calculations.	$V_2 = 6 \times 6 \times 6$ $= (1.5 \times 4) \times (1.5 \times 4) \times (1.5 \times 4)$ $= (1.5)^3 \times (4 \times 4 \times 4)$ $= (1.5)^3 \times V_1$
Write the answer.	The volume of the enlarged cube is found by cubing the scale factor, 1.5 and multiplying the result by the volume of the original cube.

SC 2 I can apply and solve ratio problems involving mixtures

Problem solving with mixtures involves considering the number of parts for each ingredient of the mixture.

Unit ratios are used when considering the strength of the mixture in terms of a particular ingredient.

Worked example

Using ratios to alter the strength of a mixture

Two different shades of pink paint have been produced.

Paint A	Paint B
3 cups of red paint with 4 cups of white paint	5 cups of red paint with 6 cups of white paint.

(a) Determine which paint has the darker shade of pink.

THINKING	WORKING
Identify the colour that contributes the darker shade.	Red contributes the darker shade.
Determine the number of parts in the whole mixture.	Paint A: 3 + 4 = 7 parts Paint B: 5 + 6 = 11 parts
Determine the fraction of the darker shade in each mixture.	Paint A fraction of red: $\frac{3}{7} = 0.43$ (2 d.p.) Paint B fraction of red: $\frac{5}{11} = 0.45$ (2 d.p.)
Write the answer.	Paint B is darker; it is made with approximately 45% red paint.

(b) Given that the darker shade is preferred, how much red paint should be added to the other mixture to make it the same shade as the preferred mixture?

THINKING	WORKING
Write the red to white ratio for the preferred shade.	The preferred red to white ratio is 5 : 6.
Calculate the total amount of red required for the white that is already in the mixture.	Total red needed in first mixture $4 \text{ cups} \times \frac{5}{6} = 3\frac{1}{3} \text{ cups}$
Calculate the amount of extra red paint to be added by subtracting the amount already in the mixture.	$3\frac{1}{3}$ cups required – 3 cups in the mixture = $\frac{1}{3}$ cup of red paint required
Write the answer.	To make the same shade of pink, one third of a cup of red paint should be added.

Practice

ANSWERS Page 712

SC 1 I can make connections between length, area and volume ratios after scaling transformations

1 A prism is enlarged in a ratio of 3 : 1.

Describe:

(a) the side lengths of the enlarged prism

(b) the area of each face of the enlarged prism

(c) the volume of the enlarged prism.

2 A prism is reduced in a ratio of 1 : 3.

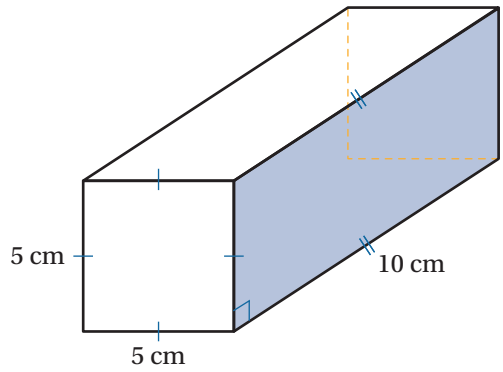
Describe:

(a) the side lengths of the reduced prism

(b) the area of each face of the reduced prism

(c) the volume of the reduced prism.

Questions **3** and **4** relate to a rectangular prism with side lengths 5 cm × 5 cm × 10 cm shown below.



3 The rectangular prism with side lengths 5 cm × 5 cm × 10 cm is enlarged 6 : 5.

(a) Describe the scale factor.

(b) Determine the side lengths of the enlarged prism.

(c) Determine the area of the shaded face in the original prism.

(d) Determine the area of the shaded face in the enlarged prism.

(e) Determine the volume of the original prism.

(f) Determine the volume of the enlarged prism.

4 The rectangular prism with side lengths 5 cm × 5 cm × 10 cm is reduced 4 : 5.

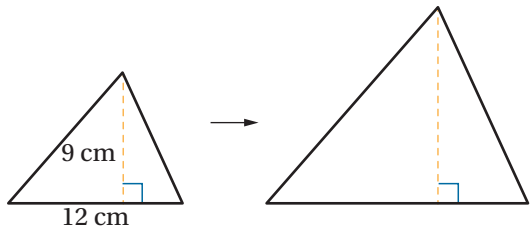
(a) Describe the scale factor.

(b) Determine the side lengths of the reduced prism.

(c) Determine the area of the shaded face in the reduced prism.

(d) Determine the volume of the enlarged prism.

5 The triangle below has been enlarged in the ratio 1.4.



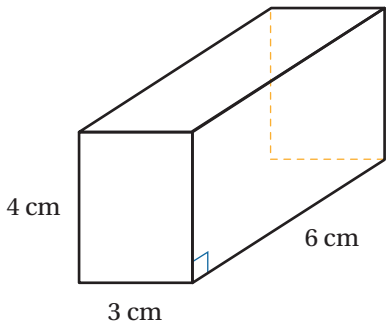
(a) Determine the dimensions of the enlarged triangle.

(b) Calculate the areas of the original and enlarged triangles.

(c) Calculate the ratio of the enlarged area to the original area as a single number.

(d) Describe the relationship between the ratios for length and area.

6 A rectangular prism, 3 cm × 4 cm × 6 cm, is enlarged by a factor of 1.6.



(a) Determine the dimensions of the enlarged prism.

(b) Determine the surface area of the prism (the total area of all six faces).

(c) Determine the surface area of the enlarged prism.

(d) Calculate the ratio comparing the new surface area with the original surface area.

(e) Calculate the volume of the original prism.

(f) Calculate the volume of the enlarged prism.

(g) Calculate the ratio comparing the new volume with the original volume.

(h) Describe the relationship between the ratios for length and surface area as well as for length and volume.

SC 2 I can apply and solve ratio problems involving mixtures

1 Cordial is made by adding 400 mL of water to 100 mL of concentrate. The mixture is not strong enough, so 50 mL of concentrate is added.

(a) Given that the mixture is now too strong, calculate how much added water would take it back to the original strength.

(b) Suggest an amount of water to add to make the taste just right.

2 Two different shades of purple paint have been produced: the first by mixing $4\frac{1}{2}$ cups of blue paint with $2\frac{1}{2}$ cups of red; the second by mixing 3 cups of blue paint with 2 cups of red.

(a) Determine which paint is more maroon (higher portion of red).

(b) Given that less maroon is preferred, how much blue paint should be added to the more maroon mixture to make it the same shade as the preferred mixture?

3 A dirt bike rider fills his tank with 25 L of petrol at the bowser on top of a litre of oil. The rider wants to use up the 10 L of petrol he has at home.

(a) What fraction of his usual amount of petrol is the rider using?

(b) How much oil should he put in first?

(c) The rider finds that he has only 300 mL of oil. How much of the petrol should he add?

4 A basic scone recipe uses $2\frac{1}{2}$ cups of self-raising flour, 60 g of butter and $\frac{3}{4}$ of a cup of milk. The recipe makes 12 scones.

(a) Determine the amount of each ingredient needed to make 30 scones.

(b) The baker has 3 cups of flour left in the pantry to use.

(i) How much butter and milk should be used?

(ii) How many scones should the dough be divided into (to the nearest whole number)?

TOPIC REVIEW

Multiple choice

ANSWERS Page 747

1 The Murrumbidgee River, passing through NSW and ACT, is 1488 km long. Its tributary, the Lachlan River, is 1450 km. The Murrumbidgee is how many times as long as the Lachlan?

A 1.03 **B** 1.02 **C** 1.76 **D** 0.97

2 Circles have a circumference to diameter ratio of approximately $\frac{22}{7}$. Given that a diameter is 77 mm, the circumference is nearest to:

A 121 mm **B** 484 mm **C** 242 mm **D** 24.5 mm

3 Circles have a circumference to diameter ratio of approximately $\frac{22}{7}$. Given that a circumference is 44 cm, the radius is nearest to:

A 138.3 cm **B** 69.1 cm **C** 14 cm **D** 7 cm

4 Given that the circumference of a circle is 67 cm and the diameter is 21 cm, the ratio of circumference to diameter is nearest to:

A 3.1 **B** 3.13 **C** 3.15 **D** 3.2

Questions **5** to **7** relate to the information shown in the table below.

5	8	10
(a)		15 cm
	60 mm	**(b)**
2.6 m	**(c)**	

5 The missing length in the proportion table for **(a)** is:

A 30 cm **B** 10 cm **C** 7.5 cm **D** 5 cm

6 The missing length in the proportion table for **(b)** is:

A 48 mm **B** 75 mm **C** 62 mm **D** 72 mm

7 The missing length in the proportion table for **(c)** is:

A 0.52 m **B** 4.16 m **C** 5.6 m **D** 5.2 m

8 A two-dimensional figure is enlarged by a ratio of 2.9. The enlargement ratio for the area is nearest to:

A 8.4 **B** 5.8 **C** 4.9 **D** 4.6

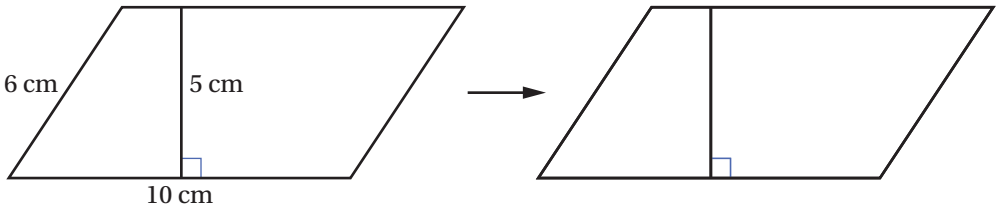
9 A three-dimensional figure is enlarged by a ratio of 1.8. The enlargement ratio for the volume is closest to:

A 8 **B** 4.8 **C** 5.8 **D** 5.4

10 When a polygon is enlarged so that all lengths increase by 35%, the increase in its area will be closest to:

A 82% **B** 70% **C** 52% **D** 40%

11 The parallelogram has all dimensions increased in the ratio 1.2. The area of the transformed parallelogram is:



A 72 cm^2 **B** 30 cm^2 **C** 18 cm^2 **D** 36 cm^2

12 A recipe for apple and cinnamon muffins requires 200 g of cooked apple pieces to make 8 muffins. What mass of cooked apple pieces is required to make 20 muffins?

A 212 g **B** 500 g **C** 300 g **D** 2000 g

13 The instructions next to the fuel cap on a lawn mower advise that 200 mL of oil should be added for each 5 L of petrol. The amount of oil to add for 8 L of petrol is:

A 125 mL **B** 203 mL **C** 320 mL **D** 3200 mL

Short answer

1 Express the following comparisons of length as unit ratios, correct to 2 decimal places.

(a) 3.2 m to 85 cm **(b)** 18 mm to 26 mm **(c)** 82 cm to 63 cm

2 A newborn baby blue whale is 8.4 m long, while its mother is 18 m long. Determine the single number ratio that compares the mother's length to the baby's, correct to 2 decimal places.

3 The circumference to diameter ratio for any circle is approximately 3.14. Determine the length of the circumference for a diameter of 12.5 cm.

4 Using a circumference to diameter ratio of $\frac{22}{7}$, determine the length of:

(a) the circumference, given a diameter of 24.5 cm

(b) the circumference, given a radius of 43.4 cm

(c) the diameter, given a circumference of 16.5 cm

(d) the radius, given a circumference of 92.4 cm.

5 A 1 m ruler is marked in centimetres. To the nearest millimetre, at what measurements should the ruler be cut so that the lengths are in the ratio 2 : 3 : 4 from the zero end of the ruler?

6 Copy and complete the proportion table, keeping lengths in the same rows in the same units.

2	4	5
		22 cm
1.5 m		
	42 mm	

7 A solid rectangular prism 15 cm × 25 cm × 35 cm is to be built from 1 cm cubes.

(a) How many cubes are needed?

(b) The entire shape is transformed by enlarging the side of each cube in the ratio 1.6.

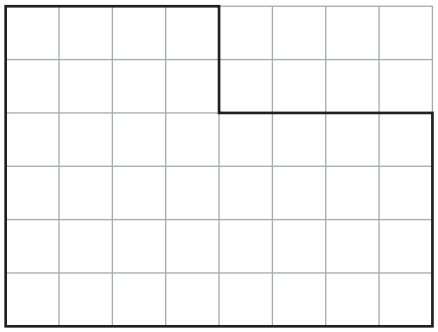
(i) Determine the volume of each cube.

(ii) Determine the volume of the transformed shape.

(c) Determine the dimensions of the new prism.

(d) How many 1 cm cubes would be needed to build the transformed shape?

8 The shape shown at right has been drawn on a grid of unit squares.



(a) Determine the number of unit squares covered by the shape.

(b) The shape is enlarged by a factor of 1.5 with the bottom left corner in the same position. How many extra unit squares will be covered by the enlarged shape?

9 A vegetable soup recipe requires 4 medium potatoes and 250 g of butternut pumpkin, skin left on. The soup will feed 8 people as an entree course. Determine the amount of pumpkin (to the nearest 50 g) and potato that are needed for a dinner party of 15 guests.

10 A fruit punch is to be made with the volumes of chopped fruit to juice in the ratio 3 : 8.

(a) Given that 6 L of juice are available, how many cups of chopped fruit are needed? (Note: 1 L = 4 cups)

(b) If two extra cups of fruit are chopped up by mistake, what is the actual fruit-to-juice ratio?

Extended response

1 In an art class, each student is given a baby-blue paint made from 5 cups of navy blue and 10 cups of white paint.

(a) Write the navy blue to white ratio as a single decimal.

(b) Paint is added to three of the students' allocations to make three new shades. By calculating the navy blue to white ratios, rounded correct to 2 decimal places where necessary, describe the shades below.

(i) Shade I: 1 cup of navy blue added

(ii) Shade II: 1 cup of white added

(iii) Shade III: 3 cups of navy blue added

(c) If equal amounts of the paint of all three students were to be mixed, describe its shade in terms of baby-blue, shade I, shade II and shade III.

(d) The three students decide to return to the original baby-blue colour. Determine how much of a single colour each must add to their mixture.

TOPIC
16

Visualising 3D

Recall 436

16.1 Visualise and apply nets 437

16.2 Visualise and draw the perspective views of a 3D object 443

16.3 Draw and build objects made from cubes 448

16.4 Draw and use objects represented in art form 453

Topic review 458

Why learn this?

Visualising and applying nets is a process that transforms flat, two-dimensional patterns into three-dimensional objects. This skill offers multiple benefits.

- It enhances spatial reasoning, allowing for a deeper understanding of how different shapes can be assembled to form complex structures.
- The ability to visualise and manipulate shapes in three dimensions prepares learners for future studies in geometry and related disciplines, and it provides a foundation for careers that require the visualisation of spatial relationships.

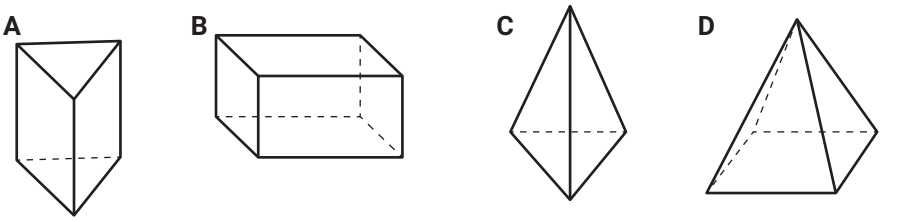
Mastering the visualisation and application of nets is a support for solving real-world problems that involve three-dimensional shapes.

RECALL

ANSWERS Page 712

I can identify faces, vertices and edges of solids

1 Write the number of faces, edges and vertices each of these solids have.



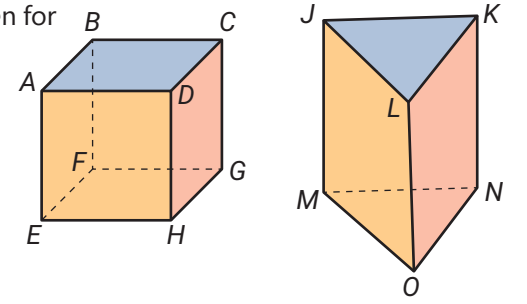
(a) Faces: ____ Edges: ____ Vertices: ____

(b) Faces: ____ Edges: ____ Vertices: ____

(c) Faces: ____ Edges: ____ Vertices: ____

(d) Faces: ____ Edges: ____ Vertices: ____

2 Name the common edge for each pair of faces given for these prisms:



(a) ABCD and DCGH

(b) JLOM and JKL

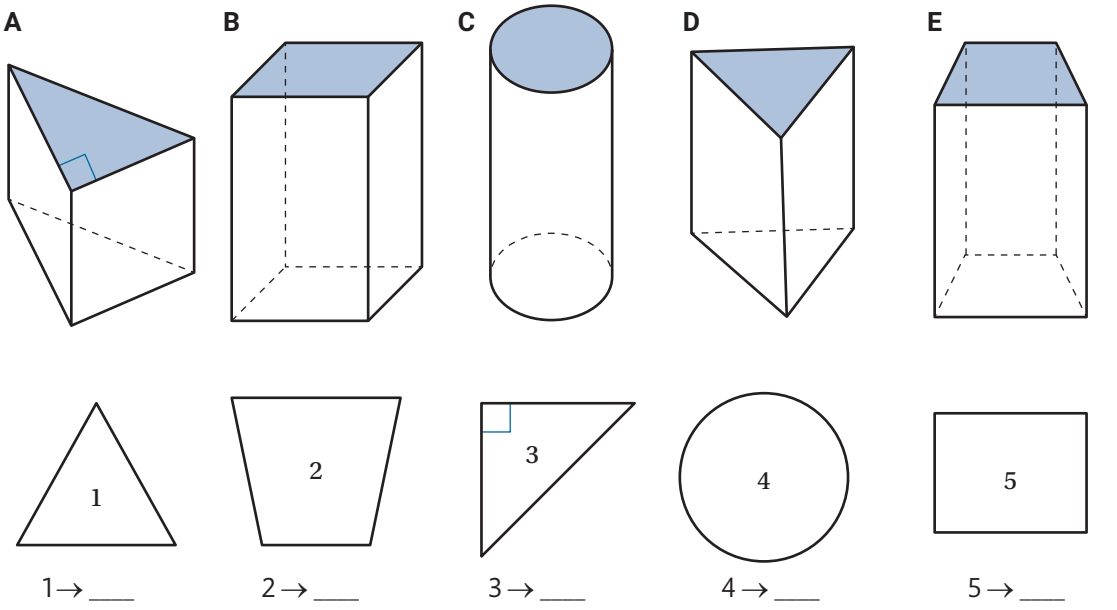
(c) KLON and JLOM

(d) ABFE and EFGH

(e) MNO and JKNM

I can connect the cross-section shape to its matching right-prism

1 Match the number of each cross-section shape with the correct right-solid.



1→ ___ 2→ ___ 3→ ___ 4→ ___ 5→ ___

Visualise and apply nets

16.1

Learning intention: To be able to visualise and apply nets

Success criteria:

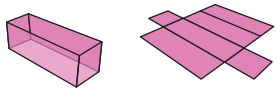
SC 1 I can identify common shapes from nets.

SC 2 I can draw a net for any prism or pyramid.

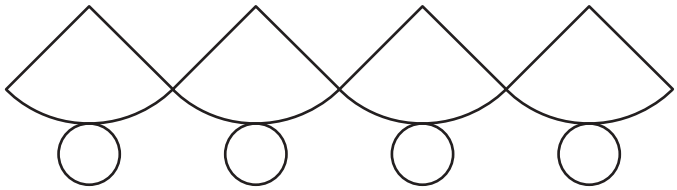
Lesson warm-up

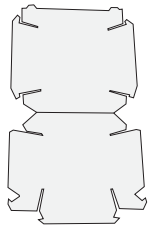
What does this fold into?

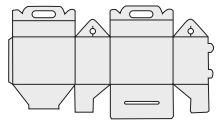
The flat shape can be folded to form the closed rectangular prism as shown.



What do these shapes fold into? Explain.



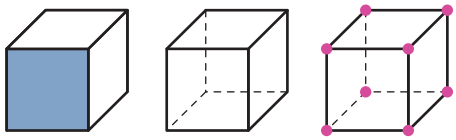




SC 1 I can identify common shapes from nets

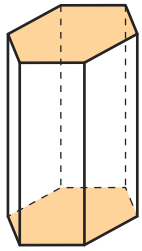
A solid is a three-dimensional (3D) shape or object. A solid whose faces are polygons is known as a polyhedron. The sides of each face of the polygons are edges of the polyhedron.

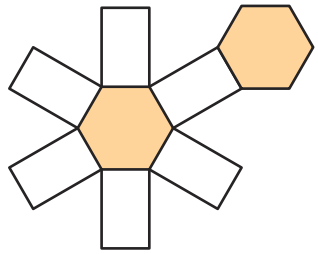
A cube has 6 faces, 12 edges and 8 vertices.



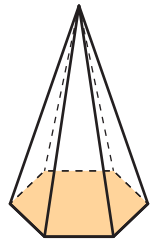
The net of a polyhedron is a flat (2D) representation showing all faces joined at the edges to make a single piece. The net can be imagined to be folded along the line segments and manipulated to form the 3D shape.

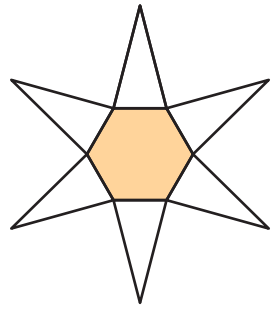
The net of a prism can be identified by two representations of the cross-sectional shape with rectangles to join the edges of the ends.





The net of a pyramid can be identified by just one representation of the base shape, with triangles to join the edges of the ends. The top vertex of the triangles all meet at the top vertex of the pyramid.



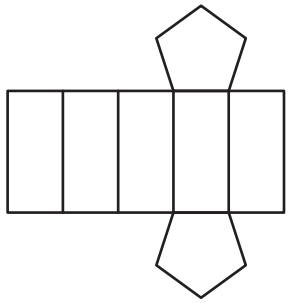


Worked example

Identifying prisms and pyramids from their nets

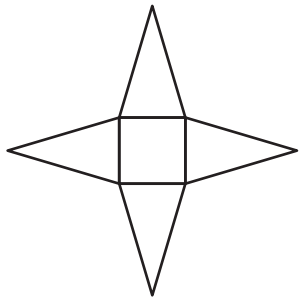
Identify the common shape from its net.

(a)



THINKING	WORKING
Describe the base shape and identify how many there are in the net.	There are two pentagons.
Identify and count the other shapes.	There are 5 rectangles.
After visualising the rectangles wrapping around to fit the folded-up bases, write your answer.	This is the net for a pentagonal prism.

(b)



THINKING	WORKING
Describe the base shape and identify how many there are in the net.	There is one square.
Identify and count the other shapes.	There are 4 triangles.
After visualising the triangles lifting up to meet a single point, write your answer.	This is the net for a square-based pyramid.

SC 2 I can draw a net for any prism or pyramid

To draw the net for a pyramid:

- start by drawing the base
- estimate the height
- create an isosceles triangle on every side of the base.

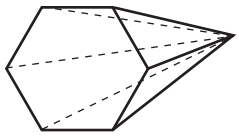
To draw the net for a prism:

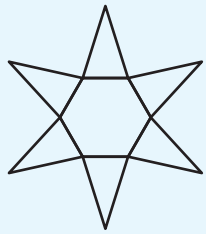
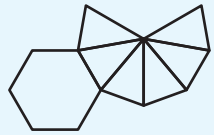
- start by drawing the base
- estimate the height of the prism
- draw a rectangle from one side, then attach another base shape
- extend the width of the rectangle on each side and mark off rectangles until there are enough to wrap around the base.

Worked example

Drawing the net of a pyramid

Sketch the net of this hexagonal pyramid.

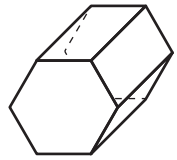


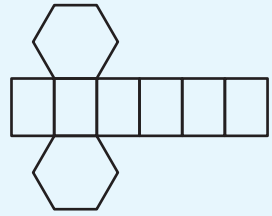
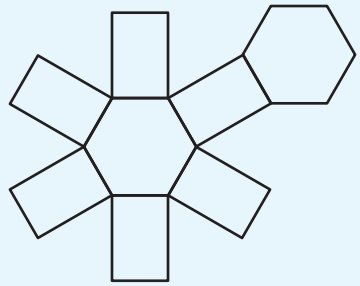
THINKING	WORKING
Draw the base shape (hexagon). Estimate the height of the pyramid and draw identical isosceles triangles from each side.	
Alternatively, draw the base shape, estimate the height of the pyramid and draw identical isosceles triangles.	

Worked example

Drawing the net of a prism

Sketch the net of this hexagonal prism.



THINKING	WORKING
Draw the base shape (hexagon). Estimate the height of the prism and draw a rectangle from one side, then attach another base shape. Extend the width of the rectangle on each side and mark off rectangles until there are enough to wrap around the base.	
Alternatively, draw the base shape, joined by a rectangle, with the side lengths representing the height of the prism, and connect it to the other base shape. Then draw the remaining rectangles around the base shape.	

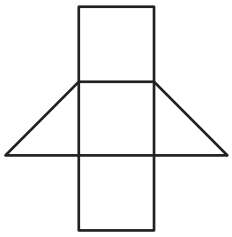
Practice

ANSWERS Page 713

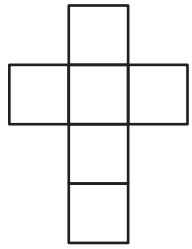
SC 1 I can identify common shapes from nets

1 Identify the shape from its net.

(a)

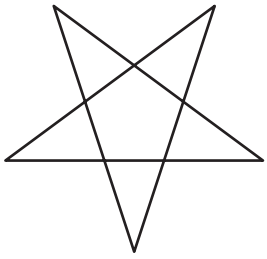


(b)

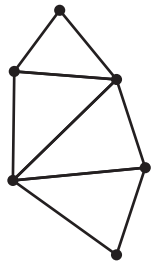


2 Identify the shape from its net.

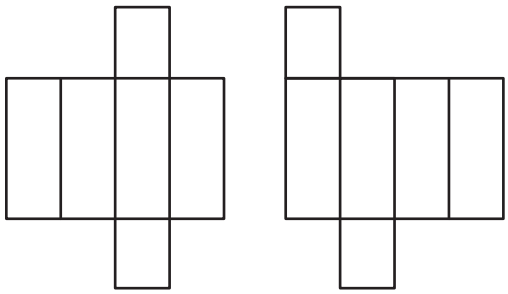
(a)



(b)

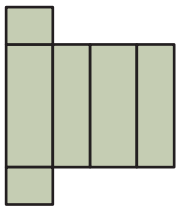


3 Explain whether or not these two nets will form the same shaped solid.

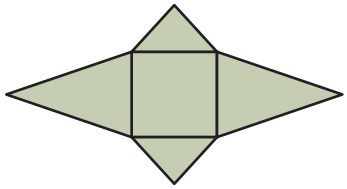


4 Explain why each of the following cannot be nets for a solid shape.

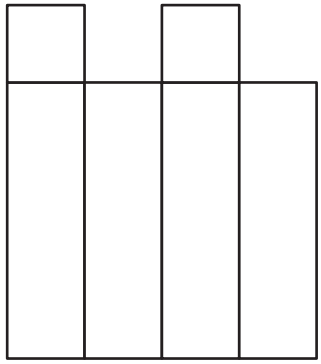
(a)



(b)

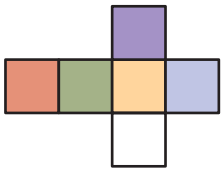


(c)

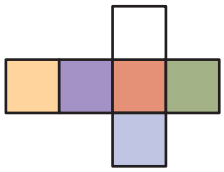


5 A cube has 6 identical square faces. The faces of the nets for a cube have different patterns on them, as shown.

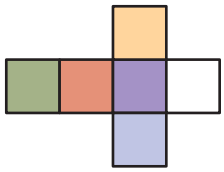
Net 1



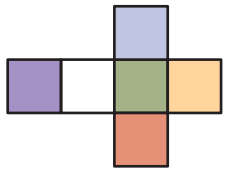
Net 2



Net 3



Net 4



Choose the net that can produce each of the solid cubes below.
(Copying each net to make a cube is a good idea.)

(a)

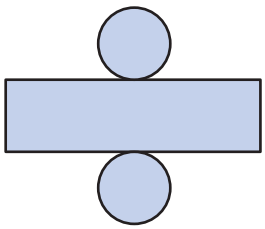
(b)

(c)

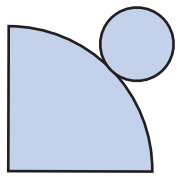
(d)

6 The following diagrams are not technically nets because the curves touch at a point instead of an edge. Use visualisation to help name the common shapes that each will form.

(a)

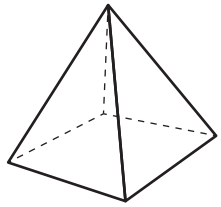


(b)



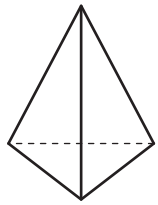
SC 2 I can draw a net for any prism or pyramid

1 Sketch the net of a square-based pyramid.

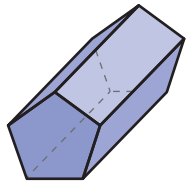


2 How would the net be different, given that the square-based pyramid was taller than the one in Question **1**?

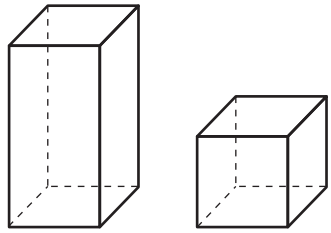
3 Sketch the net of a triangular pyramid.



4 Sketch a net for a pentagonal prism.



5 Draw the nets for these rectangular prisms of different heights.



6 Draw three different nets that can make a cube. As an extension, see how many different nets you can draw.

Visualise and draw the perspective views of a 3D object

16.2

Learning intention: To be able to identify footprint and side views of a 3D object

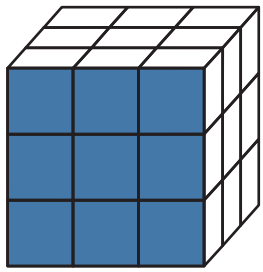
Success criteria:

SC 1 I can draw the perspective views of a 3D object.

SC 2 I can identify prisms within structures.

Lesson warm-up

How many cubes?



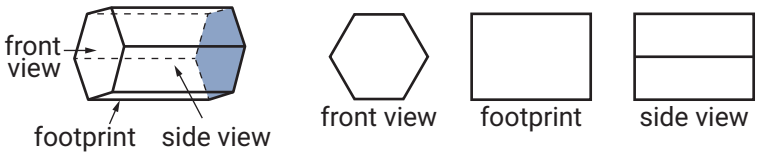
How many cubes are in this solid object? Is there only one answer?

SC 1 I can draw the perspective views of a 3D object

Taking a 3D object and pressing it into sand will make a 2D shape called a footprint.

Any side can be the footprint, depending upon the orientation.

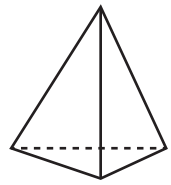
Three views of a hexagonal prism are given below.



The front view is a hexagon, the footprint is the bottom horizontal surface and the side view shows two rectangles joining.

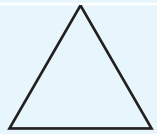
Worked example

Drawing the perspective views of a pyramid

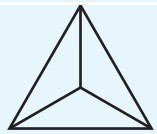


Draw the following views of this triangular pyramid.

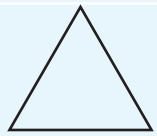
(a) The side view

THINKING	WORKING
Imagine looking directly at one face of the 3D object. Draw the shape you see.	

(b) The top view

THINKING	WORKING
Imagine looking directly down at the 3D object from above. Draw the shape you see.	

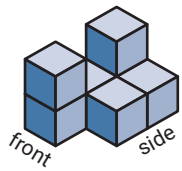
(c) The footprint

THINKING	WORKING
The object is pressed in sand. What 2D shape does the base leave? Draw the shape.	

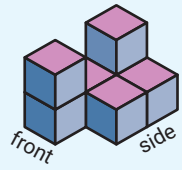
Worked example

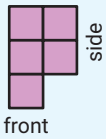
Drawing the perspective views of an object made of cubes

Draw the following views of the solid shown.

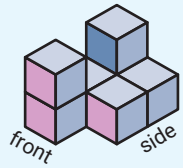
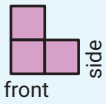


(a) The top view

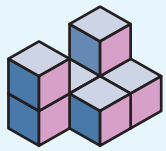
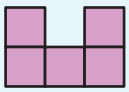
THINKING	WORKING
Highlight the faces seen from the top.	Top view: 

Draw squares to show the shape seen from the top.	In this case, the top view is the same as the footprint. 

(b) The front view

THINKING	WORKING
Highlight the faces seen from the front.	Front view: 
Draw squares to show the shape seen from the front.	

(c) The side view

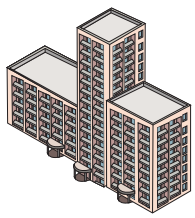
THINKING	WORKING
Highlight the faces seen from the side.	Side view: 
Draw the squares to show the shape seen from the side.	

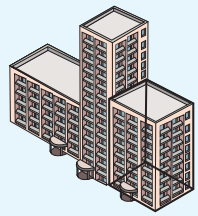
SC 2 I can identify prisms within structures

Worked example

Identifying prisms to approximate structures

Describe simple prisms to approximate the structure.



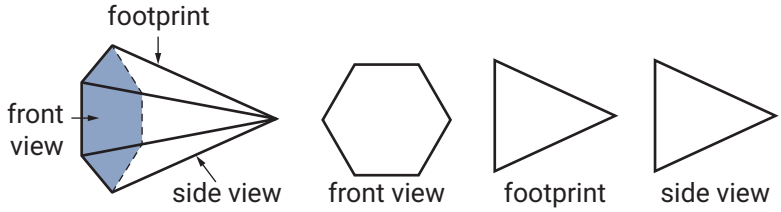
THINKING	WORKING
Imagine or draw around the solids that make up the structure.	
Describe the solids you see.	The three buildings are similar in shape to rectangular prisms.
Describe each part, noting which dimensions are the same and which are different.	The prism on the left is 5 windows long and 5 windows high. The middle prism is 14 windows high and 2 windows wide. The prism on the right is 8 windows high and 3 windows wide. All buildings appear to be approximately the same depth.

Practice

ANSWERS Page 713

SC 1 I can draw the perspective views of a 3D object

1 Three views of a hexagonal pyramid are given below. Describe the front view, footprint and side view.



2 Describe the front view, footprint and side view for each of the following.

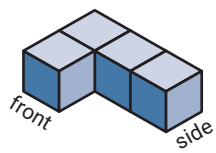
(a) Equilateral triangle-based prism with its base on the ground.

(b) A cylinder with its base on the ground.

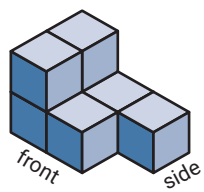
3 An aerial photo of The Pentagon in Washington DC is shown here. Note that the middle section is gardens. For the actual building, draw the footprint and one possible side view.

4 For each of the following solids, use grid paper to draw and label three perspective views.

(a)

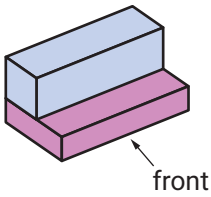


(b)

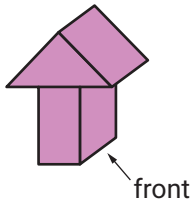


5 The following structures are made from coloured blocks. Draw the front view, as indicated, then imagine moving around to the right to draw the side view.

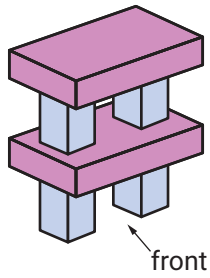
(a)



(b)



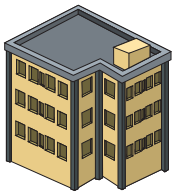
(c)



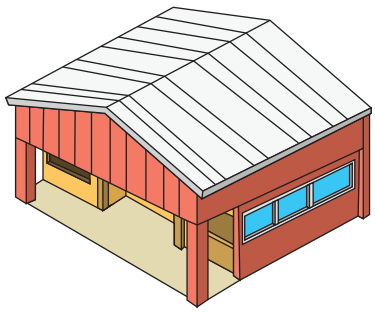
SC 2 I can identify prisms within structures

1 Describe simple prisms to approximate each structure.

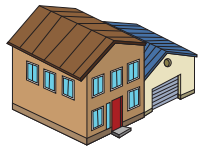
(a)



(b)



(c)

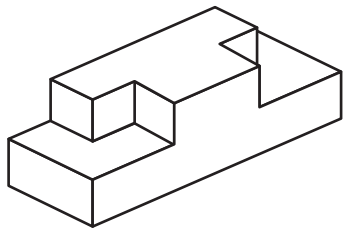


2 Describe the following in terms of simple prisms.

(a) Elephant Tower in Bangkok

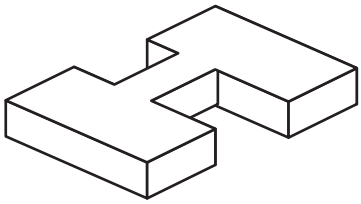
(b) Cape Cod house

3 Give dimensions for three rectangular prisms that together form the shape below.

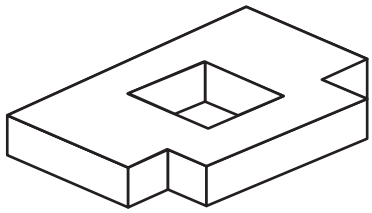


4 Describe each shape using the smallest number of rectangular prisms.

(a)



(b)



16.3 Draw and build objects made from cubes

Learning intention: To be able to build objects from isometric drawings

Success criteria:

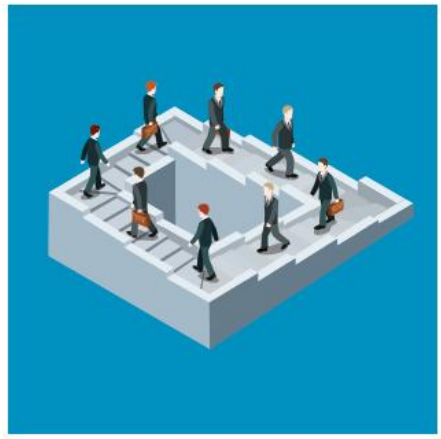
SC 1 I can build objects with cubes from isometric drawings and perspective drawings.

SC 2 I can draw a 2D representation of a 3D object using an isometric grid.

Lesson warm-up

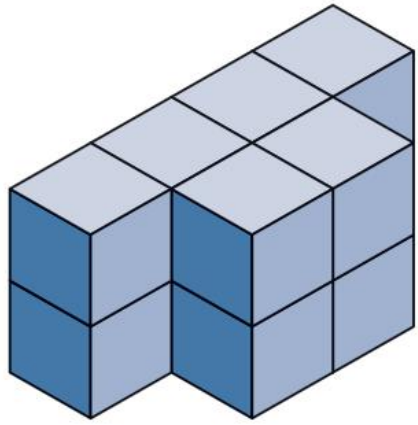
Perseverance

What do you notice about this drawing? Why might it be used to show perseverance?



SC 1 I can build objects with cubes from isometric drawings and perspective drawings

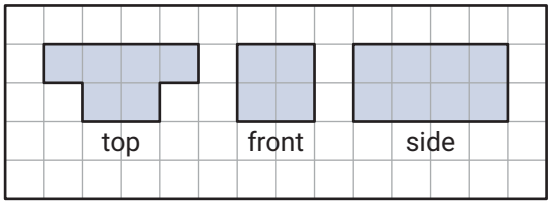
An isometric drawing is a representation of a 3D object in 2D. Vertical lines are maintained, but other lines are at 45°. Shading all the faces pointing in one direction helps give the diagram a 3D look, or appearance.

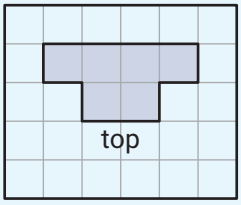
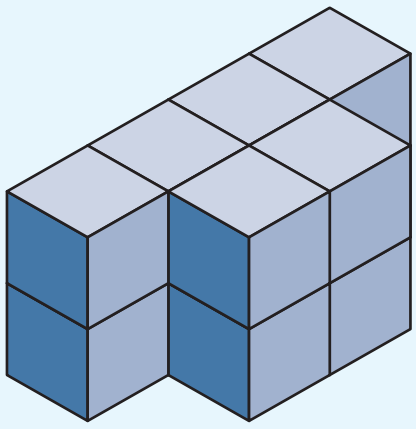


Worked example

Constructing objects from perspective drawings

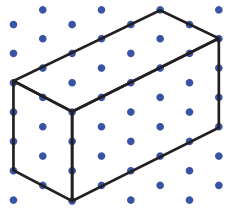
Use 12 unit squares to construct a shape from the grid paper perspective drawings.



THINKING	WORKING
Build and describe the bottom layer to match the top view, assuming there are no holes or gaps.	Bottom layer:  A row of 4 cubes with 2 cubes in front, in the middle.
Because both the front and side have a height of 2 units, build a second layer that is the same as the first.	Second layer: Same as the first layer.
Check that all perspectives are correct. Check that the correct number of cubes were used. Note that all sides facing to the left have been shaded to highlight the 3D nature of the shape.	

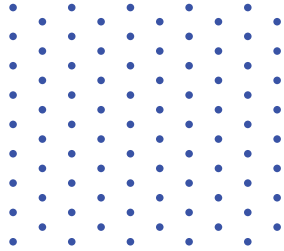
SC 2 I can draw a 2D representation of a 3D object using an isometric grid

Isometric dot or grid paper is useful in drawing 3D shapes whose edges are whole-number lengths and whose faces are at right angles.

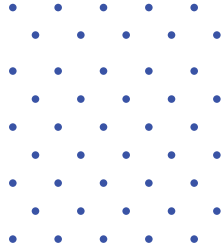


Note that orienting the dot paper in a particular way can make drawing easier.

recommended



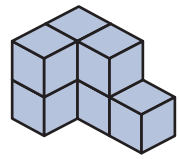
not recommended

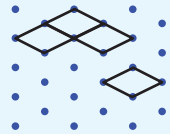
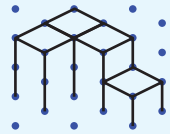
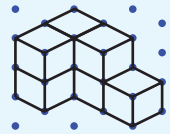
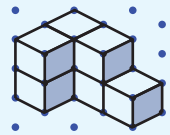


Worked example

Drawing solids on an isometric grid

Use isometric dot paper to copy the shape.



THINKING	WORKING
Draw all top faces as a rhombus. (Note the isometric grid distorts a square by pushing the top sideways, so it looks like a rhombus.)	
Draw all vertical lines from the top face.	
Complete the diagram with all remaining lines representing horizontal edges.	
Create a 3D effect by shading all faces of a particular direction (top view, front view or one of the side views).	

Practice

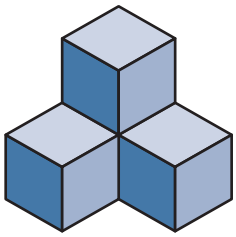
ANSWERS Page 714

SC 1 I can build objects with cubes from isometric drawings and perspective drawings

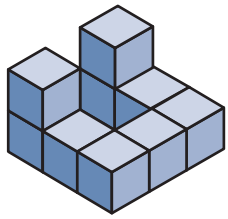
Use unit cubes to help answer the questions.

1 In these isometric diagrams, the front of each shape is shaded. Use unit cubes to build each shape.

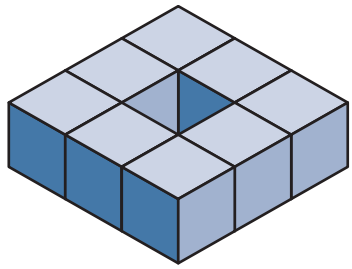
(a)



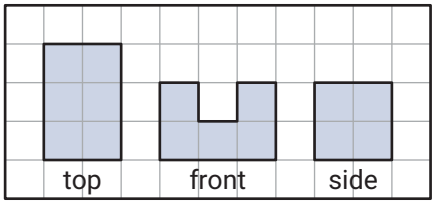
(b)



(c)

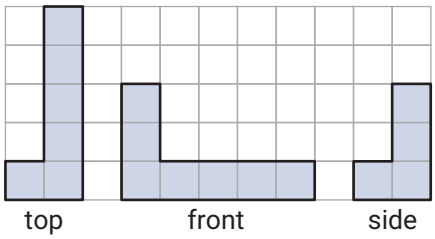


2 Use 10 unit cubes to construct a shape from the grid paper perspective drawings.

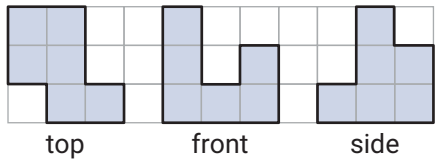


3 Use the number of unit cubes indicated to construct a shape from the grid paper perspective drawings.

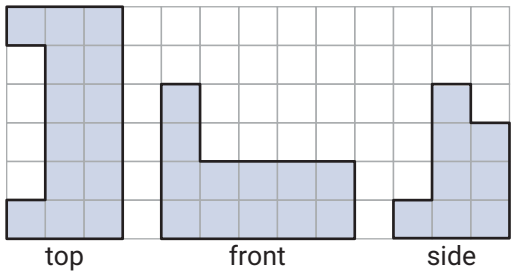
(a) 8 unit cubes



(b) 9 unit cubes



4 Work in pairs with 21 unit cubes to build the shape whose perspective views are given below.



SC 2 I can draw a 2D representation of a 3D object using an isometric grid

1 Use isometric dot paper to draw the following cubes.

(a) 3 units × 3 units × 3 units

(b) 5 units × 5 units × 5 units

2 Use isometric dot paper to draw the following rectangular- or square-based prisms. (Any orientation of length, width and height is allowed.)

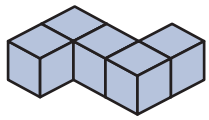
(a) 2 units × 2 units × 6 units

(b) 4 units × 4 units × 1 unit

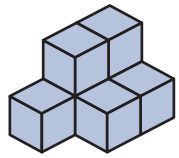
(c) 3 units × 4 units × 5 units

3 Use isometric dot paper to copy each shape.

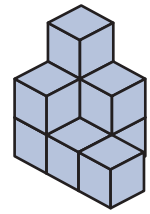
(a)



(b)

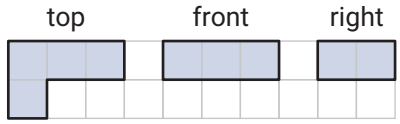


(c)

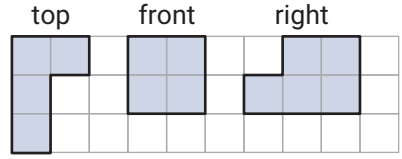


4 Build the shape from the perspective views, then use isometric dot paper to draw the diagram.

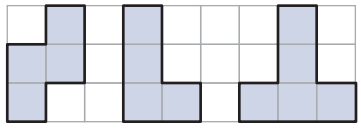
(a)



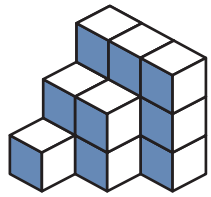
(b)



(c)

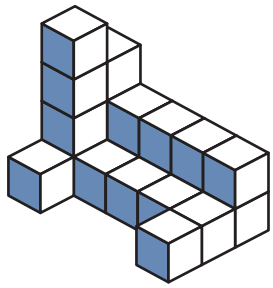


5 A mat plan of a shape made from unit cubes shows the top view as a grid, with the number of cubes above each position written in the grid.



3	3	3
2	2	
1		

(a) Draw the mat plan for the shape below.



(b) Build the shape below from the mat plan, then use isometric dot paper to draw the shape.

3	1	1
3	1	
2	1	

Draw and use objects represented in art form

16.4

Learning intention: To be able to draw and use objects represented in art form

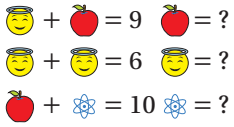
Success criteria:

SC 1 I can represent objects in art form.

SC 2 I can use maps represented in art form.

Lesson warm-up

Solve



SC 1 I can represent objects in art form

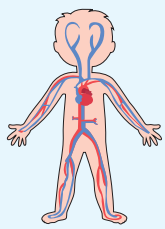
Perspective views and isometric drawings are good ways to represent 3D objects when accuracy of size and shape are important. Artistic views can be effective in featuring aspects of 3D objects and maps.

The internal organs or spines of animals or fish are common in Aboriginal and Torres Strait Islander peoples' works of art.

Worked example

Representing internal structures in art form

The circulatory system of humans features blood flowing through arteries and veins. Oxygenated blood is red, whereas non-oxygenated blood is more like the colour of the veins you see at your wrists. Show the circulatory system in an effective, unlabelled diagram.

THINKING	WORKING
Draw the outline of a human body. Show blood vessels in two colours representing both oxygenated and non-oxygenated blood. Draw in the heart, which pumps the blood, and the lungs, which oxygenate it.	

SC 2 I can use maps represented in art form

Maps can be accurate plan views of a landscape, using symbols to show rivers, roads or railway lines.

Worked example

Understanding a map represented in art form

For the children's tourist map, describe where each of the following is used and why.

(a) Mixture of plan and front views

THINKING	WORKING
Identify the part of the map drawn in plan view.	The layout of roads, rail and the river is plan view.
Identify the part of the map drawn in front view.	The attractions are drawn in front view.
Explain why you think this is.	The layout of roads and the river is plan view to help with navigation. Each building or attraction is front view so you can see what the attractions look like.

(b) Mixture of scales

THINKING	WORKING
Describe the scale used.	There is one scale for the layout (roads and the river), and another scale for the attractions. The attractions seem to be scaled differently.
Explain why you think this is.	The roads, river and rail include only enough detail to support navigation. The objects are shown on different scales for each one so that they are able to be identified.

(c) Use of symbols in shapes and colour

THINKING	WORKING
List and describe the symbols used.	Blue river Green parks
Explain why you think this is.	Symbols, shapes and colour are used to add detail without needing words.

Practice

ANSWERS Page 715

SC 1 I can represent objects in art form

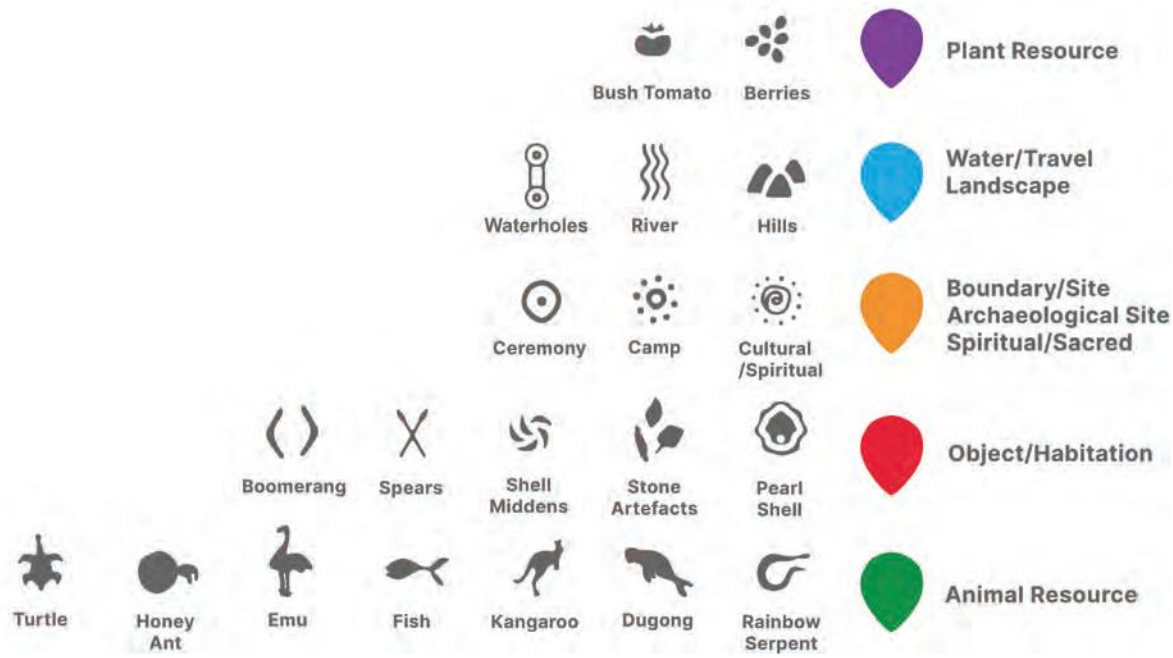
1 The skeletal system of a mammal features a spine and rib cage as well as jointed bones to the limbs. Sketch the skeletal system for each of the following mammals in an effective, unlabelled diagram.

(a) A human **(b)** A cow

2 Create an artwork of a family setting off on a trip in a car with luggage in the boot. Show the car as simply an outline and the people and luggage as symbolic shapes.

3 Create an artwork of a top view of your living room, showing the layout of furniture and rugs as outline shapes only.

4 Aboriginal and Torres Strait Islander artists use many symbols in their work. Some symbols have several different meanings and can be completely different from place to place. Use the symbols to create your own story about the Country you live on.



SC 2 I can use maps represented in art form

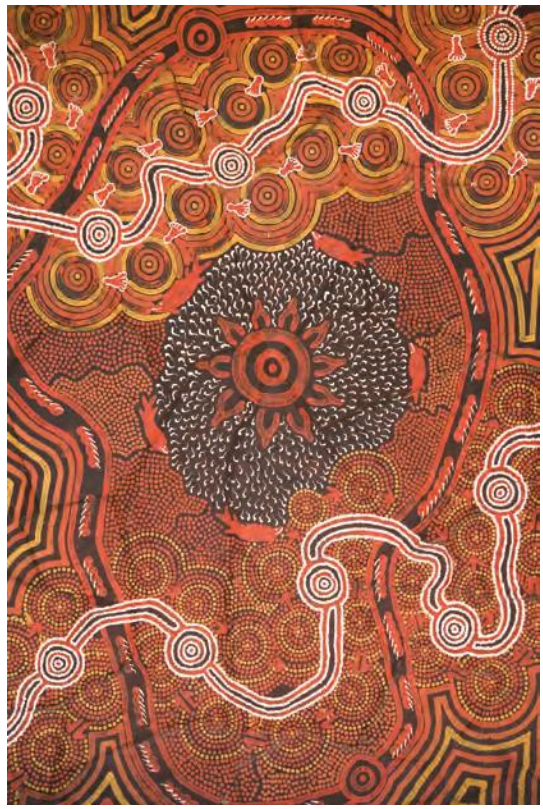
1 For the children's zoo map, describe where each of the following is used and why.

(a) Mixture of plan and front views

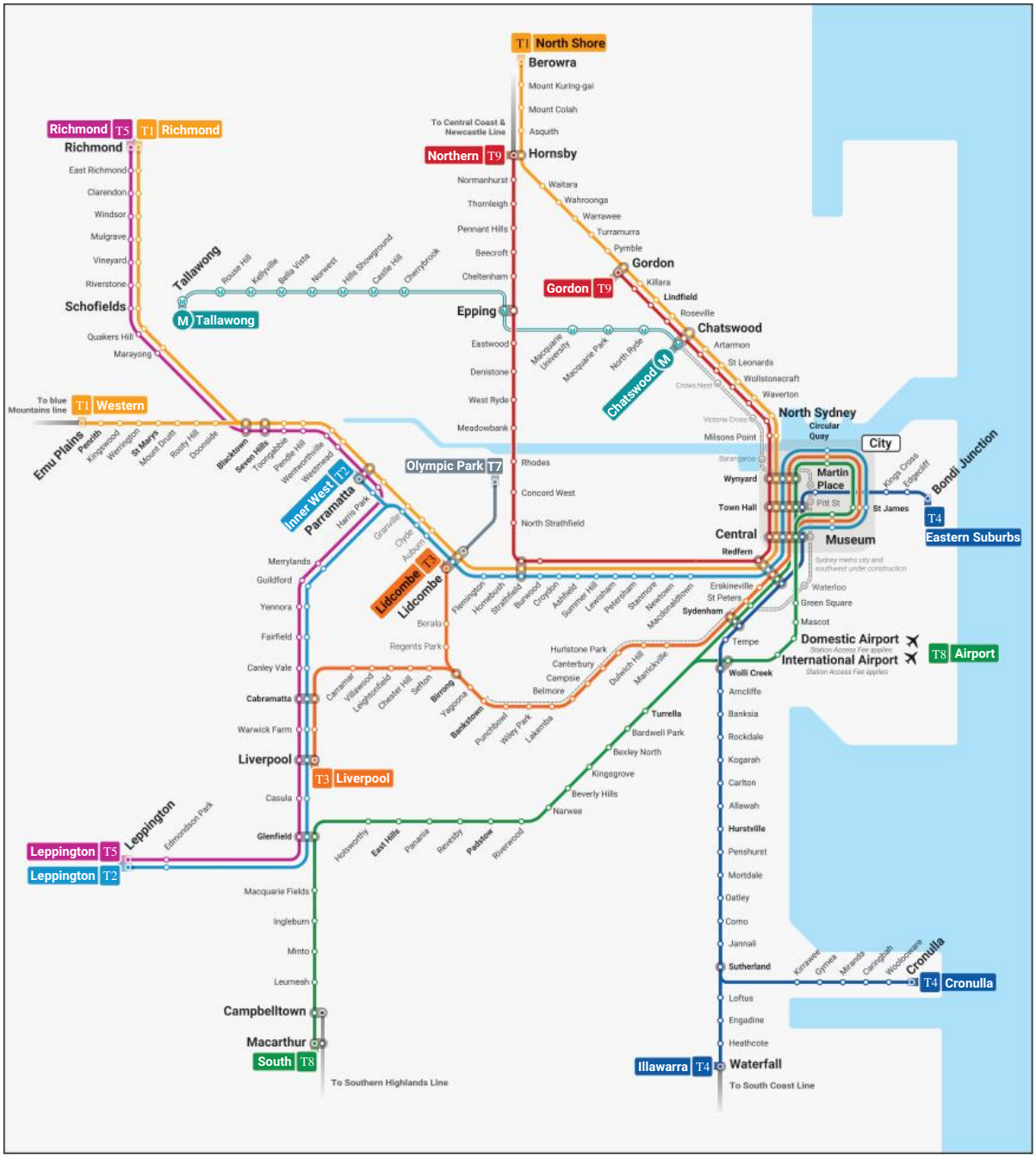
(b) Mixture of scales

(c) Use of symbols

2 Aboriginal and Torres Strait Islander artists often represent Country in their artwork and may include movement of animals and people. Using the symbols shown in SC1 Question **4** as a guide, describe what you think the artist is showing in this artwork.



3 Use the Sydney metro map to answer the questions.



(a) Which stations outside the city loop serve exactly three lines?

(b) How would you go from St James to North Sydney?

(c) Which line goes under three other lines at once? Where?

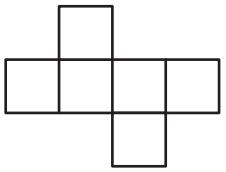
TOPIC REVIEW

Multiple choice

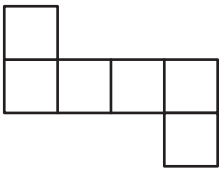
ANSWERS Page 747

1 Which of the following could not be the net for a cube?

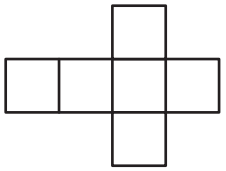
A



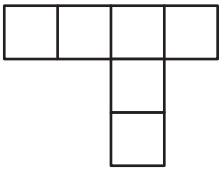
B



C

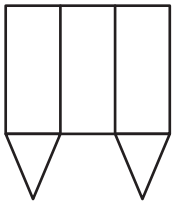


D

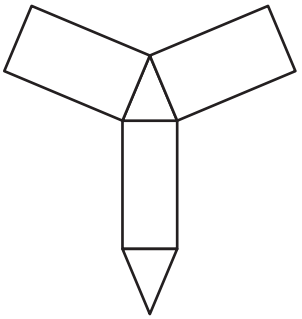


2 Which of the nets cannot be folded to make a triangular prism?

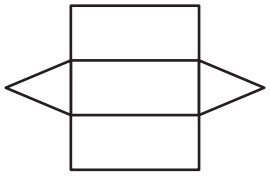
A



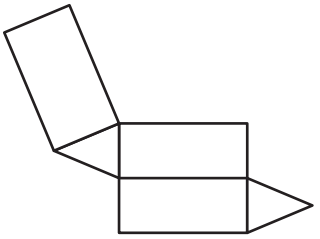
B



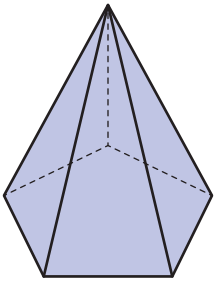
C



D



3 Which description matches the pentagonal pyramid?



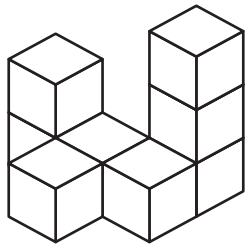
A 5 faces, 10 edges, 5 vertices

B 6 faces, 10 edges, 6 vertices

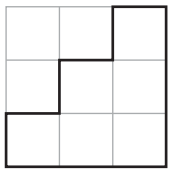
C 10 faces, 10 edges, 6 vertices

D 6 faces, 9 edges, 6 vertices

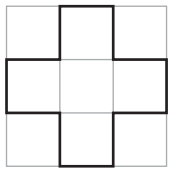
4 Which is a correct top view for the isometric diagram?



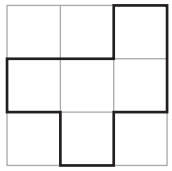
A



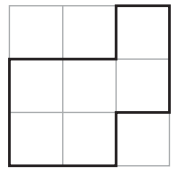
B



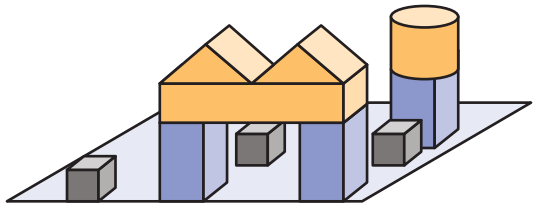
C



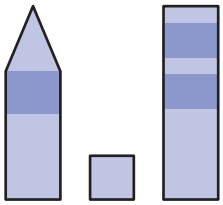
D



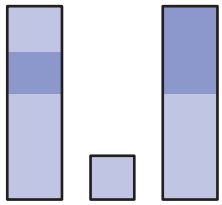
5 Which diagram is a possible side view for the blocks that are sitting on a mat?



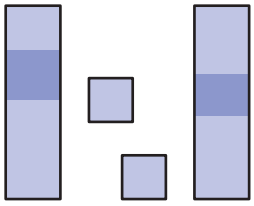
A



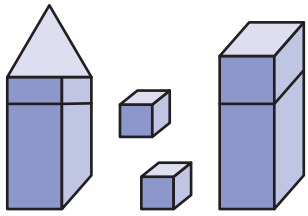
B



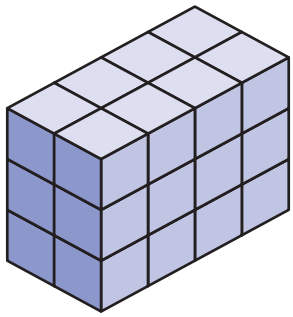
C



D

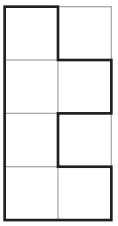


6 Twenty-four wooden cubes have been glued together and the external surfaces have been painted. How many of the cubes have paint on just two faces?

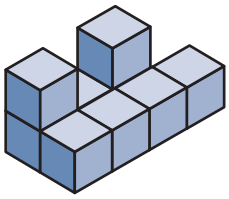


A 4 **B** 8 **C** 12 **D** 16

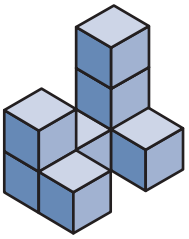
7 Which shape has the top view shown?



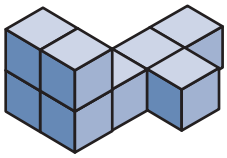
A



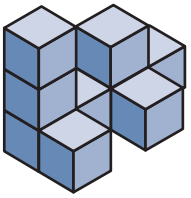
B



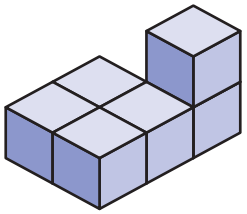
C



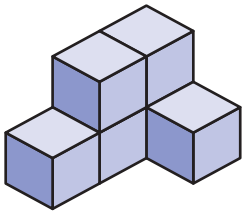
D



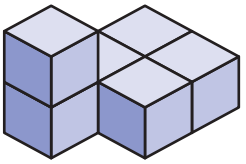
8 Which object is the same as the one shown?



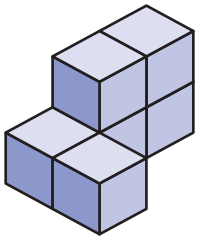
A



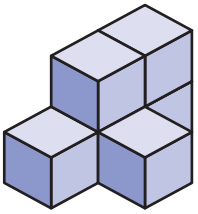
B



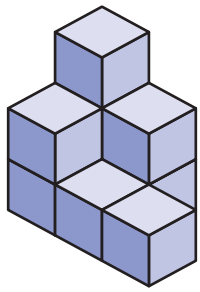
C



D

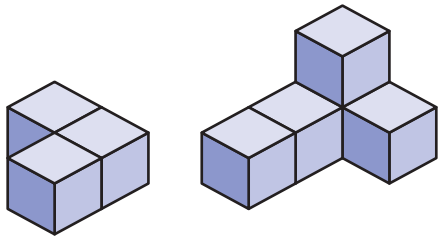


9 This solid has been built from cubes. The number of cubes is:

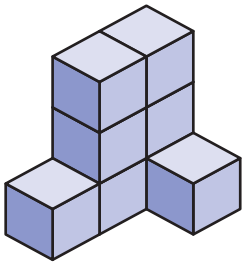


A 6 **B** 7 **C** 8 **D** 9

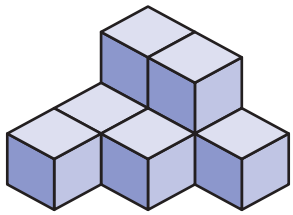
10 Which of the shapes can be made by joining the two shapes below?



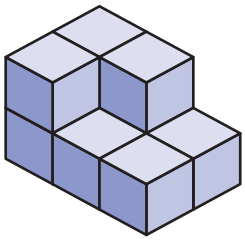
A



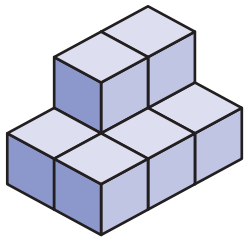
B



C



D



Short answer

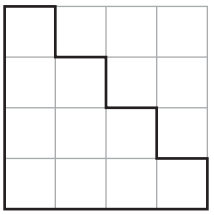
1 Draw nets for an octagonal prism and an octagonal pyramid such that the similarities and differences between the two are obvious.

2 Draw the nets for a triangular pyramid and a rectangular pyramid such that the similarities and differences between the two are obvious.

3 Draw the net of a trapezium-based pyramid. Mark the equal sides of your diagram.

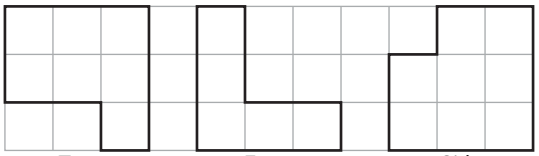
4 Draw a scalene-right-angled-triangle-based prism.

5 A solid is made of unit cubes. The top, front and side views all look the same, as shown.

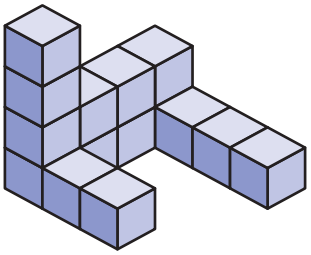


How many cubes make up the solid?

6 Use 12 unit cubes to construct a shape from the grid paper perspective drawings.



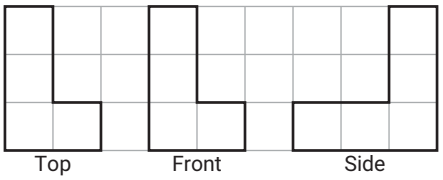
7 The solid shown here is made from unit cubes.



(a) Use isometric dot paper to copy the shape.

(b) On grid paper, draw the top, front and side views.

8 Three perspectives of a solid are drawn on grid paper. Make the solid from unit cubes, then draw its shape on isometric dot paper.



9 Investigate the organs that make up the human digestive system. Draw an unlabelled artist's sketch of the human digestive system.

Extended response

1 A compound shape is to be built of a square-based prism in the middle with square-based pyramids on either end. One of the pyramids is to be much taller than the other.

(a) Draw possible versions of the three shapes.

(b) Draw nets for the three shapes, then construct the shapes.

(c) After joining the shapes, draw plan, side and front views of the compound shape.

TOPIC
17

Triangles and quadrilaterals

Recall 464

17.1 Construct triangles and quadrilaterals 465

17.2 Classify types of triangles by their side and angle properties 473

17.3 Classify types of quadrilaterals by their side and angle properties 482

17.4 Use parallel side lengths of quadrilaterals to construct rectangles of equivalent area 491

Topic review 499

Why learn this?

Constructing triangles and quadrilaterals equips learners with an understanding of geometric principles that are useful in various real-life contexts. This topic offers:

- insight into geometric conditions necessary to form stable structures, supporting knowledge used in fields like architecture and engineering
- skills to construct accurate representations of triangles from given dimensions, aiding in tasks such as technical drawing and design
- an understanding of the variability of quadrilaterals with fixed side lengths, highlighting the importance of additional constraints to determine shape, relevant in manufacturing and graphic design.

These skills are essential for academic success in geometry and provide practical benefits for activities such as crafting, model building and spatial problem-solving. They help us to interpret, create and manipulate shapes, both at school and university, and in the workplace.

RECALL

ANSWERS Page 716

I can identify triangles and quadrilaterals

1 Copy and complete the following sentences.

(a) A triangle is a _____ shape that has _____ sides and _____ internal angles.

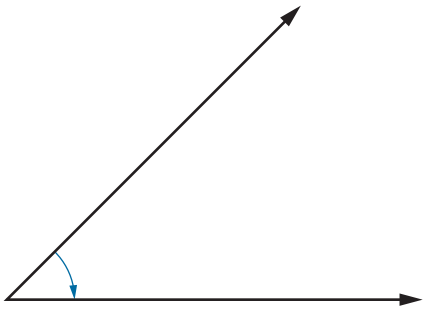
(b) A quadrilateral is a _____ shape that has _____ sides and _____ internal angles.

(c) Squares and rectangles are special types of _____.

I can use a protractor to measure and name angles

1 Using a protractor, measure each angle and then name the type of angle.

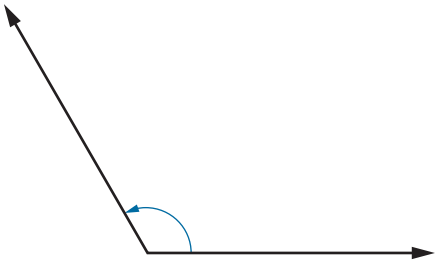
(a)



Angle size = _____°

Name of angle = _____

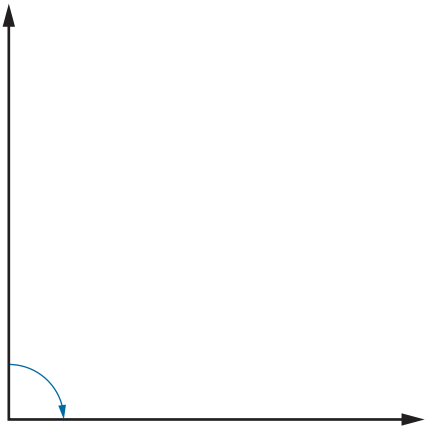
(b)



Angle size = _____°

Name of angle = _____

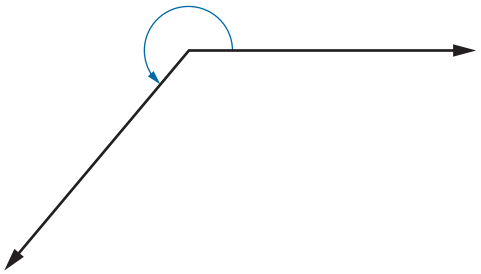
(c)



Angle size = _____°

Name of angle = _____

(d)



Angle size = _____°

Name of angle = _____

Construct triangles and quadrilaterals

17.1

Learning intention: To be able to construct triangles and quadrilaterals

Success criteria:

SC 1 I can determine whether or not a triangle can be formed from given side lengths.

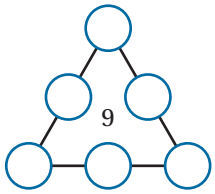
SC 2 I can construct a triangle given the three side lengths.

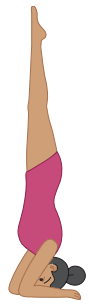
SC 3 I can demonstrate that side lengths alone do not determine the shape of quadrilaterals.

Lesson warm-up

Number triangles

Place the numbers 1 to 6 in the circles so that each side adds to 9.





17

SC 1 I can determine whether or not a triangle can be formed from given side lengths

A triangle is formed from three lengths of structural material that has a rigid (or fixed) shape, so the triangle cannot flex to give other angles. This makes triangles a common feature of most bridge trusses, since the bridge will be braced against flex.

Worked example

Forming triangles from given sides

Determine whether or not a triangle could be formed with the given side lengths.

(a) 15 cm, 18 cm, 25 cm

THINKING	WORKING
Calculate the total of the two 'smaller' sides.	15 + 18 = 33 cm
Compare the total with the longest side.	33 cm > 25 cm
A triangle can be formed if the total is greater than the longest side.	A triangle can be formed.

(b) 7 cm, 8 cm, 15 cm

THINKING	WORKING
Calculate the total of the two 'smaller' sides.	7 + 8 = 15 cm
A triangle can be formed if the total is greater than the longest side.	A triangle cannot be formed.

(c) 4 cm, 11 cm, 11 cm

THINKING	WORKING
Calculate the total of the two 'smaller' sides.	4 + 11 = 15 cm
Compare the total with the longest side.	15 cm > 11 cm
A triangle can be formed if the total is greater than the longest side.	A triangle can be formed.

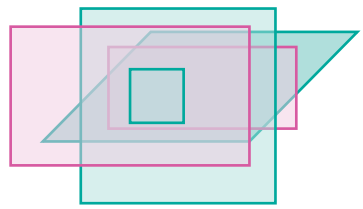
Exploration Task

Properties of shapes

Consider the following statements.

- All squares are rectangles.
- Only some rectangles are parallelograms.
- Only some squares are parallelograms.

Do you agree? Explore the properties of different shapes to understand this.



SC 2 I can construct a triangle given the three side lengths

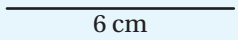
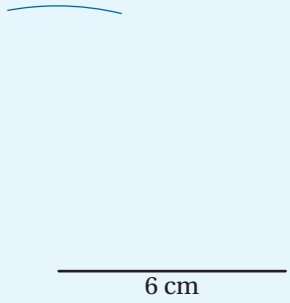
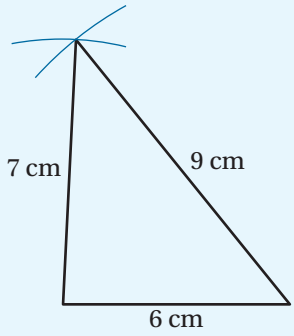
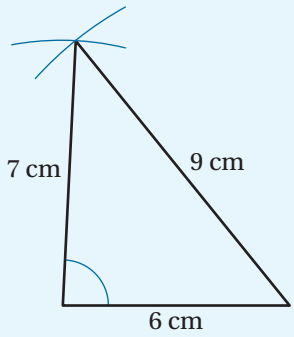
Given the three side lengths of a triangle, all triangles constructed will be the same shape. This is evidence that the triangle cannot flex because the angle sizes are fixed.

Worked example

Constructing triangles from given side lengths

Use a ruler and compass to construct a triangle of side lengths 6 cm, 7 cm and 9 cm.

As a check of accuracy, use a protractor to measure the largest angle (the angle opposite the largest side).

THINKING	WORKING
Use the ruler to draw any side. Label the side with the given length.	
Open the compass to the length of one of the other sides. With the compass point on one end of the first line, make an arc.	
Open the compass to the length of the third side. With the compass point on the other end of the first line, make a second arc overlapping the first arc. Join each end of the first line to the intersection and label the sides.	
Use a protractor to measure the largest angle (opposite the largest side).	 The largest angle is approximately 87°.

SC 3 I can demonstrate that side lengths alone do not determine the shape of quadrilaterals

Quadrilaterals of fixed side length can take on an infinite variety of shapes. Hence structures of such shapes must be braced to stop them from flexing.

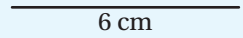
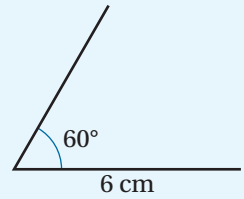
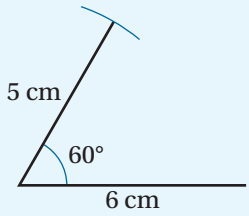
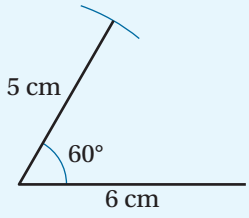
Worked example

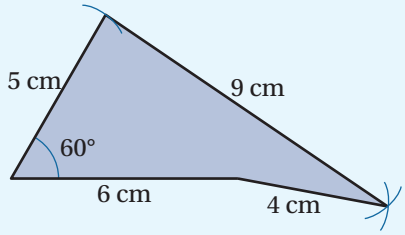
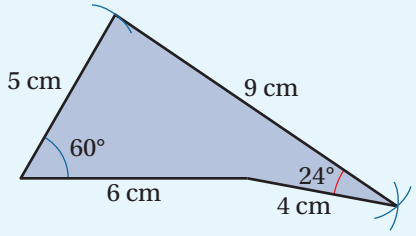
Constructing quadrilaterals with given sides and one angle

Use a ruler, compass and protractor to construct a quadrilateral of side lengths 6 cm, 5 cm, 9 cm and 4 cm, in order, with each of the given angles between the first two sides.

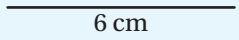
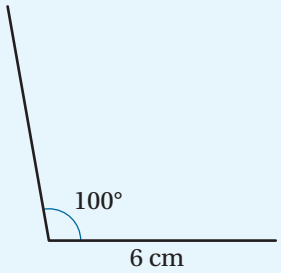
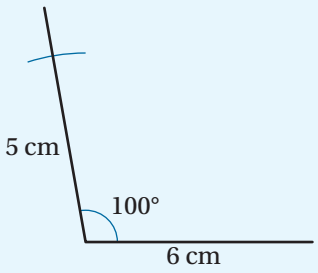
As a check of accuracy, measure the angle opposite the given angle.

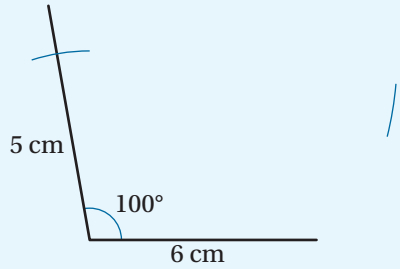
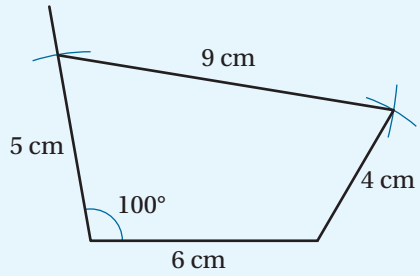
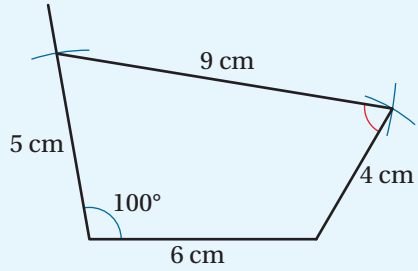
(a) 60°

THINKING	WORKING
Use a ruler to draw the first side. Label the side with the given length.	
Use the protractor to form the given angle at one end of the line.	
Open the compass to the length of the second side. With the compass point at the vertex of the given angle, make an arc to intersect the second arm of the angle.	
Open the compass to the length of the third side. With the compass point on the intersection, make an arc.	

THINKING	WORKING
Open the compass to the length of the fourth side. With the compass point on the other end of the first side, make an arc to intersect the previous arc. Join the intersections at the end of the first line and label the sides.	
Draw and measure the angle opposite the given angle.	 The opposite angle is approximately 24°.

(b) 100°

THINKING	WORKING
Use a ruler to draw the first side. Label the side with the given length.	
Use the protractor to form the given angle at one end of the line.	
Open the compass to the length of the second side. With the compass point at the vertex of the given angle, make an arc to intersect the second arm of the angle.	

THINKING	WORKING
Open the compass to the length of the third side. With the compass point on the intersection, make an arc.	
Open the compass to the length of the fourth side. With the compass point on the other end of the first side, make an arc to intersect the previous arc. Join the intersections at the end of the first line and label the sides.	
Draw and measure the angle opposite the given angle.	 The opposite angle is about 69°.

17

Practice

ANSWERS Page 716

SC 1 I can determine whether or not a triangle can be formed from given side lengths

1 Working in pairs, cut some strips of paper in the following lengths: three each of 5 cm; one each of 2 cm, 3 cm, 6 cm, 7 cm, 8 cm and 10 cm.
Form triangles with each of the following combinations of lengths, where possible.
Comment on the shape and rigidity of the formed triangles or explain why the lengths could not form a triangle.

(a) 5 cm, 5 cm, 5 cm **(b)** 3 cm, 7 cm, 10 cm **(c)** 5 cm, 5 cm, 8 cm

(d) 2 cm, 3 cm, 8 cm **(e)** 6 cm, 8 cm, 10 cm **(f)** 2 cm, 5 cm, 5 cm

2 Explain what is special about the sets of side lengths that can form a triangle.

3 Determine whether a triangle could be formed with the given side lengths.

(a) 15 cm, 16 cm, 17 cm **(b)** 8 cm, 9 cm, 20 cm

(c) 10 cm, 40 cm, 45 cm **(d)** 16.2 cm, 18.3 cm, 35.5 cm

(e) 27.6 m, 37.8 m, 55.9 m

4 Determine which of the following triples (sets of three numbers with the same units) can be used to make a triangle.

(a) (1, 1, 1) **(b)** (1, 1, 2) **(c)** (1, 2, 2) **(d)** (1, 1, 3)

(e) (1, 2, 3) **(f)** (2, 2, 3) **(g)** (2, 3, 3)

SC 2 I can construct a triangle given the three side lengths

1 Use a ruler and compass to construct a triangle of the given side lengths.
As a check of accuracy, measure the largest angle (the angle opposite the longest side).

(a) 5 cm, 7 cm, 8 cm **(b)** 6 cm, 8 cm, 10 cm **(c)** 5 cm, 7 cm, 9 cm

(d) 7 cm, 7 cm, 7 cm **(e)** 6 cm, 9 cm, 9 cm

2 Use a ruler and compass to construct a scale model of a triangle of side lengths 25 m, 35 m and 45 m. As a check of accuracy, use a protractor to measure all the angles.

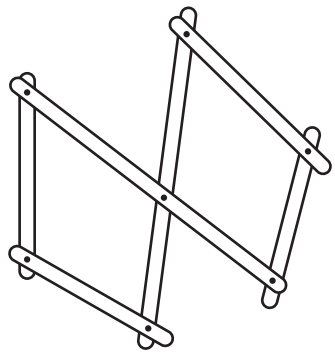
3 Use a ruler and compass to construct three triangles of side lengths:

- 3.6 cm, 4.8 cm, 5 cm
- 3.6 cm, 4.8 cm, 6 cm
- 3.6 cm, 4.8 cm, 7 cm.

(a) Use a protractor to measure the largest angle of each triangle.

(b) Describe the relationship between the three triangles and the effect of changing the longest side.

4 A rectangular wooden farm gate, 2.8 m by 1.3 m, is braced by a pair of diagonal struts. Use a scale drawing and protractor to measure the angle at which the struts intersect.



SC 3 I can demonstrate that side lengths alone do not determine the shape of quadrilaterals

1 Working in pairs, cut some strips of paper in the following lengths: four each of 5 cm; two each of 3 cm and 8 cm; one each of 7 cm, 6 cm and 16 cm.
Form quadrilaterals with each of the following combinations of lengths, where possible, in the order given. Comment on the shape and rigidity of the formed quadrilaterals or explain why the lengths could not form a quadrilateral.

(a) 5 cm, 5 cm, 5 cm, 5 cm **(b)** 3 cm, 5 cm, 7 cm, 16 cm

(c) 5 cm, 8 cm, 5 cm, 8 cm **(d)** 3 cm, 3 cm, 8 cm, 8 cm

(e) 6 cm, 5 cm, 8 cm, 5 cm

2 For each of the quadrilaterals that could be formed in the previous question, what happens when you fix the angle between the first two sides at 60° or 90° or 100°?

3 Use a ruler, compass and protractor to construct a quadrilateral of side lengths 8 cm, 4 cm, 7 cm and 10 cm, in order, with each of the given angles between the first two sides.
As a check of accuracy, measure the angle opposite the given angle.

(a) 50° **(b)** 90° **(c)** 120°

4 Use a ruler and compass construction to answer the following question.
A quadrilateral of side lengths 5 cm, 8 cm, 4 cm and 7 cm, in order, is formed, then braced for rigidity with a 6 cm diagonal from the starting point to the intersection between the 5 cm and 7 cm sides.
Measure the length of the other diagonal.

5 Use a ruler and protractor construction to answer the following question.
A parallelogram of equal side lengths 4 cm and 11 cm is braced to form a rectangle.
Determine the length of the (equal) diagonals.

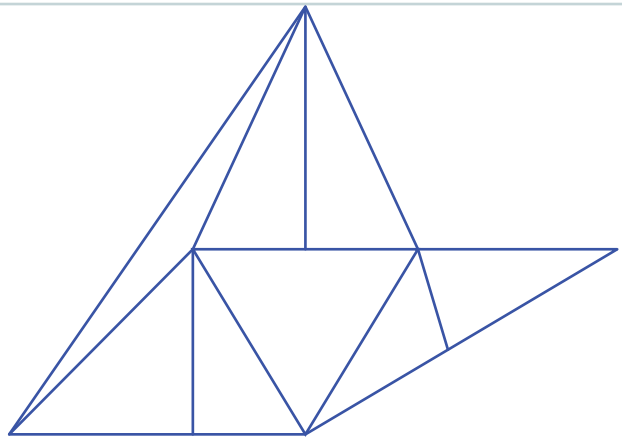
6 Use a ruler and compass construction to answer the following question.
A kite of equal side lengths 50 cm and 80 cm is braced for rigidity with a shorter diagonal of 60 cm.
Determine the length of the longer diagonal, to the nearest centimetre.

Exploration Task

17

Properties of triangles

Investigate the triangles in this diagram.
How many different triangles can you identify?
What makes the triangles different?



Classify types of triangles by their side and angle properties

17.2

Learning intention: To be able to classify types of triangles by their side and angle properties

Success criteria:

SC 1 I can identify and sketch triangles according to their side properties.

SC 2 I can identify and sketch triangles according to their angle properties.

SC 3 I can categorise triangles using side lengths and angles.

Lesson warm-up

Drawing a 'Drong'

Mika was asked to define his own shape, which he called a Drong.

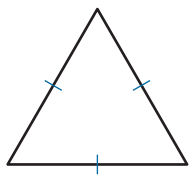
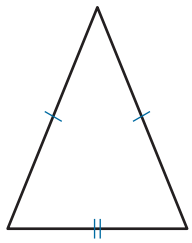
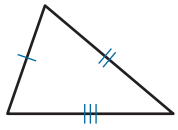
He defined a Drong as a flat shape that has one right angle, a pair of parallel sides and at least one curved line.

Draw a Drong.

SC 1 I can identify and sketch triangles according to their side properties

The side names for triangles are based on the number of sides that are the same length.

Note that identical markings on sides indicate that the sides are equal length. Sometimes different markings are used to emphasise different lengths.

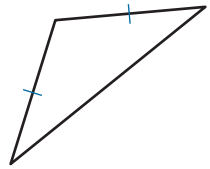
Property	Triangle
Equilateral triangles have all sides of equal length.	
Isosceles triangles have exactly two sides of equal length.	
Scalene triangles have all sides of different length.	

Worked example

Identifying triangles by side properties

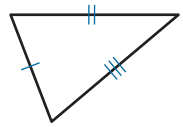
Identify each of the following triangles, giving its side name.

(a)



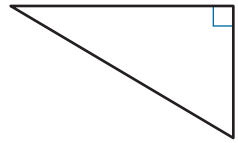
THINKING	WORKING
Give the side name according to the number of equal sides.	Two sides are equal, so this is an isosceles triangle.

(b)



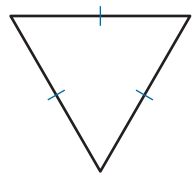
THINKING	WORKING
Give the side name according to the number of equal sides.	No sides are equal, so this is a scalene triangle.

(c)



THINKING	WORKING
Give the side name according to the number of equal sides.	No sides are equal, so this is a scalene triangle.

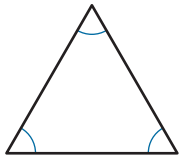
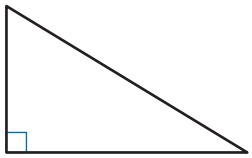
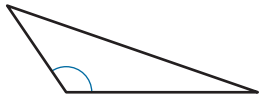
(d)



THINKING	WORKING
Give the side name according to the number of equal sides.	All sides are equal, so this is an equilateral triangle.

SC 2 I can identify and sketch triangles according to their angle properties

At least two angles of a triangle are acute (less than 90°). The largest angle may be acute, a right angle or obtuse. The angle name for a triangle refers to the largest angle of the triangle.

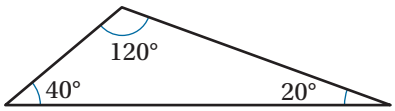
Acute-angled triangles have all three angles less than 90°. (The angles may or may not be equal.)	**Right-angled triangles** have one angle of 90°.	**Obtuse-angled triangles** have one angle that is greater than 90°.
		

Worked example

Identifying triangles by angle properties

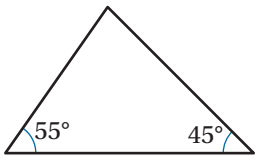
Identify each of the following triangles, giving its angle name.

(a)



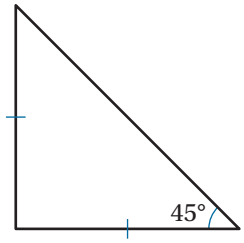
THINKING	WORKING
Name the triangle based on the largest angle.	The largest angle is 120°. This is an obtuse-angled triangle.

(b)



THINKING	WORKING
Solve for the unknown angle, in case it is the largest angle.	Unknown angle: $180° - 55° - 45° = 80°$
Name the triangle based on the largest angle.	The largest angle is 80°. This is an acute-angled triangle.

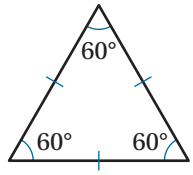
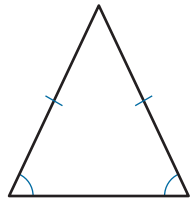
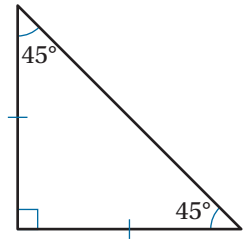
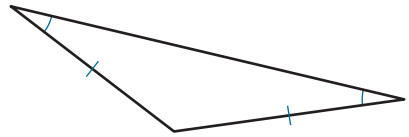
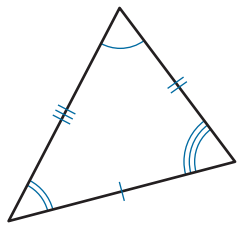
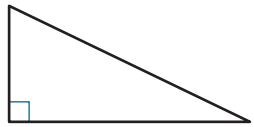
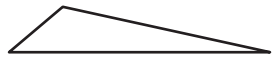
(c)



THINKING	WORKING
Solve for the unknown angles, in case one is the largest angle.	The angles opposite the equal sides are both 45°. Unknown angle: $180° - 2 \times 45° = 90°$
Name the triangle based on the largest angle.	The largest angle is 90°. This is a right-angled triangle.

SC 3 I can categorise triangles using side lengths and angles

Triangles can be classified using both side and angle properties.

	Acute-angled	Right-angled	Obtuse-angled
Equilateral		Not possible	Not possible
Isosceles			
Scalene			

Since equilateral triangles are always acute-angled, the one word is sufficient.

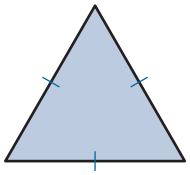
Angles opposite equal sides are equal, and angles opposite unequal sides are unequal.

Worked example

Classifying triangles

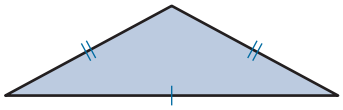
Considering side and angle properties, classify each triangle.

(a)



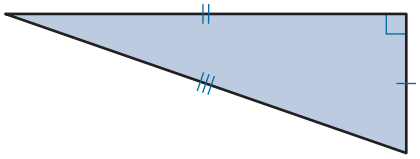
THINKING	WORKING
Consider how many sides are equal in length.	Three sides are equal. This is an equilateral triangle.

(b)



THINKING	WORKING
Consider how many sides are equal in length.	Two sides are equal. This is an isosceles triangle.
Consider the size of the largest angle.	The largest angle is obtuse. This is an obtuse-angled triangle.
Classify the triangle by side and angle.	This is an isosceles obtuse-angled triangle.

(c)



THINKING	WORKING
Consider how many sides are equal in length.	No sides are equal. This is a scalene triangle.
Consider the size of the largest angle.	The largest angle is a right angle. This is a right-angled triangle.
Classify the triangle by side and angle.	This is a scalene right-angled triangle.

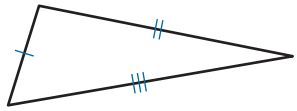
Practice

ANSWERS Page 717

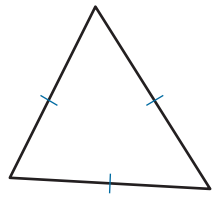
SC 1 I can identify and sketch triangles according to their side properties

1 Identify each of the following triangles, giving its side name.

(a)



(b)



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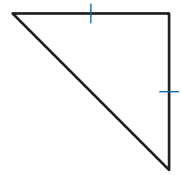
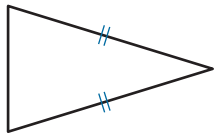
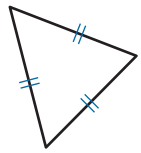


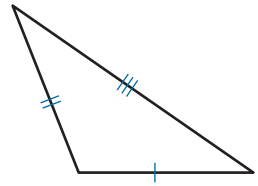
(d)



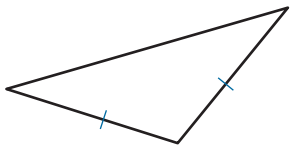
(e)



(f)



(g)



2 Sketch each of the following from the description to assist in identifying the type of triangle in terms of its side lengths.

(a) All side lengths of 5 cm

(b) Side lengths of 6 cm, 7 cm and 8 cm

(c) Two side lengths of 4 cm and one of 7 cm

(d) Right-angled triangle with shorter lengths of 5 cm and 7 cm

(e) Right-angled triangle with both shorter lengths of 8 cm

(f) Obtuse-angled triangle with a longest side of 12 cm (give all possible types)

(g) Acute-angled triangle (give all possible types)

(h) Right-angled triangle (give all possible types)

3 When triangles are drawn, the longest side is always opposite the largest angle and the shortest side is opposite the smallest angle, with equal sides opposite equal angles.
Identify the type of each triangle based on the angle information below. (You can sketch each one based on the description to assist in identifying the triangle type in terms of its side lengths.)

(a) Angles of 50° and 70°

(b) Two angles of 50°

(c) A right-angle and an angle of 75°

(d) Angles of 10° and 20°

(e) Two angles of 60°

(f) A right-angle and an angle of 45°

4 Identify the type of each triangle, based on the description below. (You can sketch each one to assist in identifying the triangle type in terms of its side lengths.)

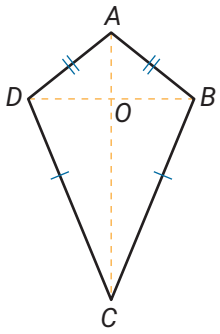
(a) Triangle formed by a square with one diagonal

(b) Triangle formed by a rectangle with one diagonal

(c) Triangle formed by a square with two diagonals

(d) Triangle formed by a rectangle with two diagonals

5 For kite $ABCD$ with diagonals intersecting at O, identify each of the listed triangles by type in terms of side lengths.



(a) ΔABC **(b)** ΔACD **(c)** ΔAOD **(d)** ΔDBC

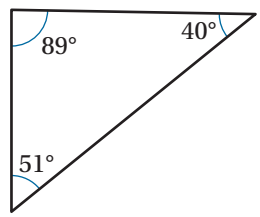
6 A rhombus $ABCD$ has diagonals intersecting at O. Identify each of the listed triangles by type in terms of its side lengths.

(a) ΔABC **(b)** ΔAOD **(c)** ΔDBC

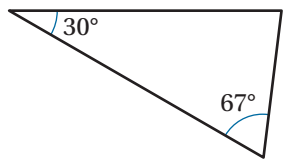
SC 2 I can identify and sketch triangles according to their angle properties

1 Identify each of the following triangles, giving its angle name.

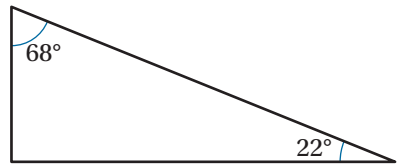
(a)



(b)



(c)



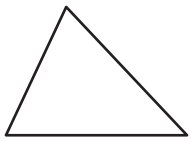
(d) A triangle where two of the angles are 70°

(e) A triangle where two of the angles are 38°

(f) A triangle where two of the angles are 25° and 55°, respectively

2 Identify each of the following triangles, giving its angle name. (You can measure, if needed.)

(a)



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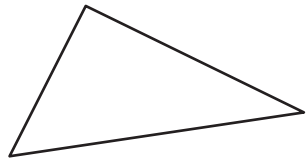

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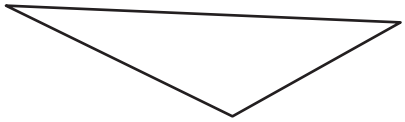
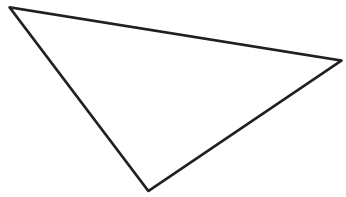
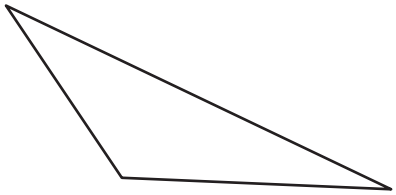


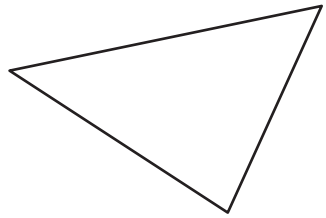
(d)



(e)



(f)



3 For each of the described triangles, identify the type, giving its angle name.

(a) A rectangle divided into two triangles by a single diagonal

(b) A rectangle divided into four triangles by two diagonals

4 A kite is divided into triangles as below. For each of the triangles formed, identify the type, giving its angle name.

(a) A single diagonal giving two identical triangles

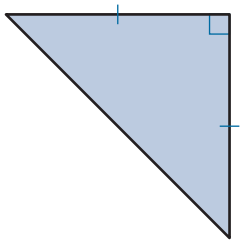
(b) A single diagonal giving two different triangles

(c) Two diagonals giving four triangles

SC 3 I can categorise triangles using side lengths and angles

1 Considering side and angle properties, classify each triangle.

(a)



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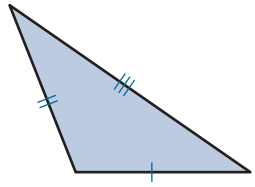

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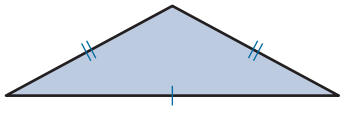
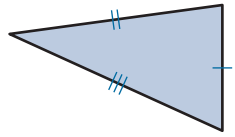
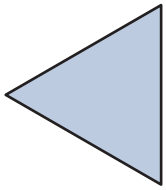


(d)



2 Measuring sides or angles, where necessary, classify each triangle.

(a)



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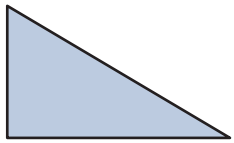

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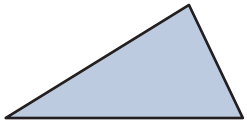
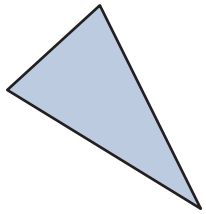


(d)



3 Calculate the third angle to classify each triangle.

(a) A triangle with angles of 31° and 59°

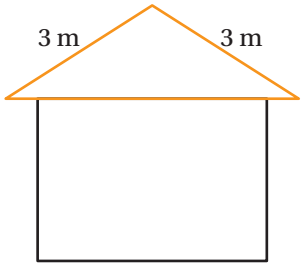
(b) A triangle with angles of 26° and 128°

(c) A triangle with two angles of 70°

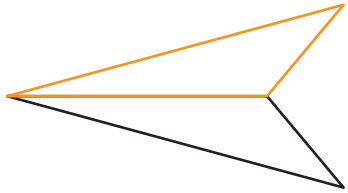
(d) A triangle with angles of 29° and 71°

4 Classify each of the featured triangles.

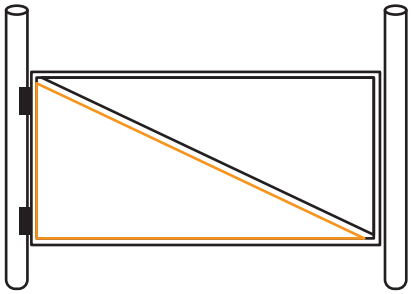
(a)



(b)



(c)



5 Classify each of the described triangles.

(a) A square divided into two triangles by a single diagonal

(b) A square divided into four triangles by two diagonals

6 Two equilateral triangles of the same size form a quadrilateral, with the joining sides forming one diagonal. Classify each of the described triangles.

(a) The two triangles formed by the other diagonal

(b) The four triangles formed by the two diagonals

7 Use a ruler, compass and protractor to construct each triangle. Measure sides or angles to classify each triangle.

(a) A triangle with side lengths 4 cm, 5 cm and 8 cm

(b) A triangle with side lengths 6 cm and 8 cm with an angle of 50° between them

17.3 Classify types of quadrilaterals by their side and angle properties

Learning intention: To be able to classify types of quadrilaterals by their side and angle properties

Success criteria:

SC 1 I can identify and describe the side and angle properties of quadrilaterals.

SC 2 I can identify a hierarchy within types of quadrilaterals.

SC 3 I can use a flow chart for polygons to categorise triangles and quadrilaterals.

Lesson warm-up

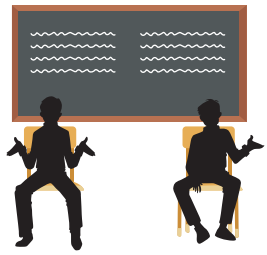
Guess the name

Two students volunteer to sit out the front on chairs with their backs against the board, facing the rest of the class.

The teacher draws a shape on the board behind each of the students, so they cannot see it.

Asking questions that the rest of the class answers with only 'yes' or 'no', they must try to guess what is drawn behind them.

The winner is the first to guess the name of the shape.

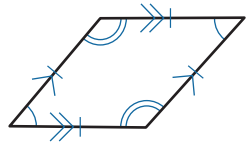


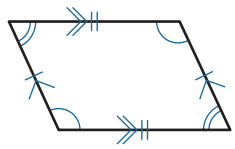
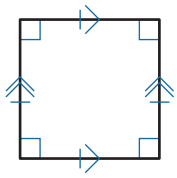
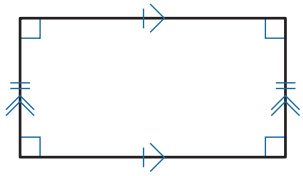
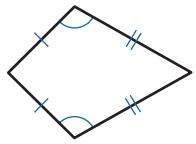
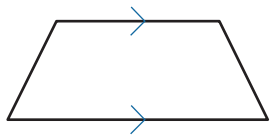
SC 1 I can identify and describe the side and angle properties of quadrilaterals

Quadrilaterals are four-sided plane figures.

Some quadrilaterals with special properties have common names, although a quadrilateral can have special properties without having a special name.

All of the following special quadrilaterals are convex, having no reflex angles.

Properties	Quadrilateral
A rhombus has: • all four sides equal • both pairs of opposite sides parallel • both pairs of opposite angles equal.	

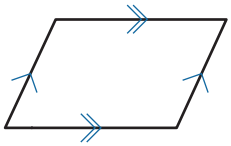
Properties	Quadrilateral
A parallelogram has: • both pairs of opposite sides equal and parallel • both pairs of opposite angles equal.	
A square has: • all four sides equal • both pairs of opposite sides parallel • all angles equal (right angles).	
A rectangle has: • both pairs of opposite sides equal and parallel • all angles equal (right angles).	
A kite has: • two pairs of adjacent (beside each other) sides equal • one pair of opposite angles equal.	
A trapezium has: • one pair of opposite sides parallel.	

17

Worked example

Naming special quadrilaterals

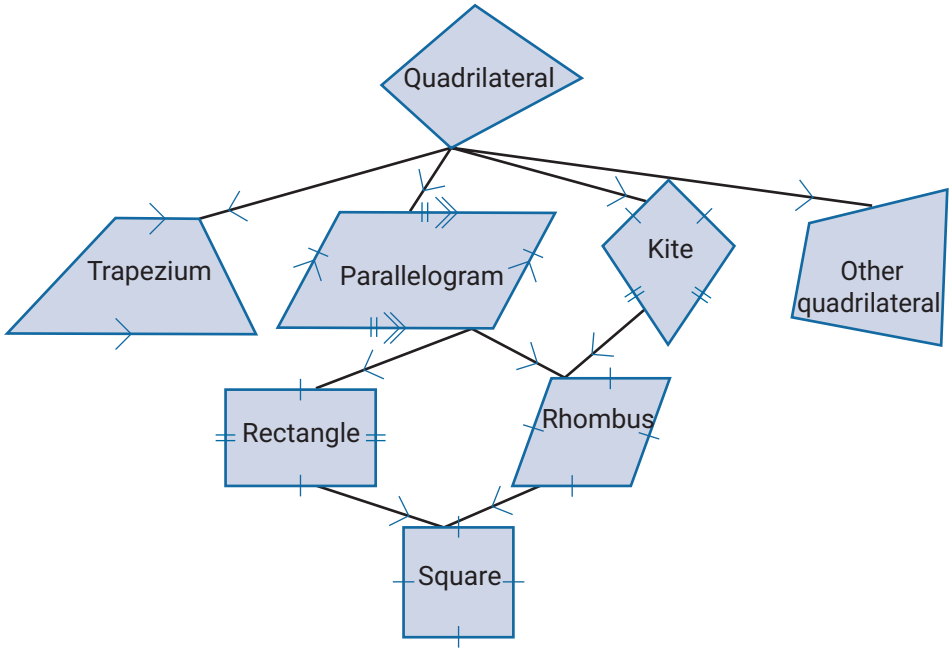
Identify the quadrilateral shown.



THINKING	WORKING
List the special features.	A quadrilateral with: • both pairs of opposite sides equal and parallel • both pairs of opposite angles equal.
Identify the quadrilateral.	This quadrilateral is a parallelogram.

SC 2 I can identify a hierarchy within types of quadrilaterals

Some quadrilaterals can be described as a special case of another quadrilateral because they have all of their properties and more.



In the hierarchy, the square is the most special, having all of the properties of the others.

A rhombus and a rectangle are special parallelograms. In the same way, a square can be described as a special rectangle or rhombus.

Worked example

Comparing shared properties of special quadrilaterals

Consider a square and a rhombus.

(a) Identify the property common to just these two special quadrilaterals.

THINKING	WORKING
Choose the property shared by squares and rhombuses, but not parallelograms.	Squares and rhombuses have all sides equal.

(b) Identify the property that distinguishes the two special quadrilaterals from each other.

THINKING	WORKING
Choose the property of a square that is not shared by a rhombus.	Squares have all angles right angles, or Squares have all angles equal, or Squares have adjacent angles equal.

(c) Copy and complete the statement: A _____ is a _____ with one angle a _____ angle.

THINKING	WORKING
If any (one) angle of a rhombus is a right angle, then all angles are equal.	A square is a rhombus with one angle a right angle.

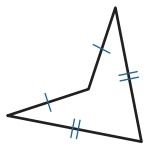
SC 3 I can use a flow chart for polygons to categorise triangles and quadrilaterals

A polygon (meaning 'many angles') is any plane shape made from straight sides.

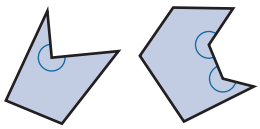
Every polygon has the same number of sides as angles, so their names can correspond to either the number of sides or the number of angles.

Number of sides	Polygon name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

A regular polygon has all sides of equal length and all angles of equal size. All regular polygons are convex.

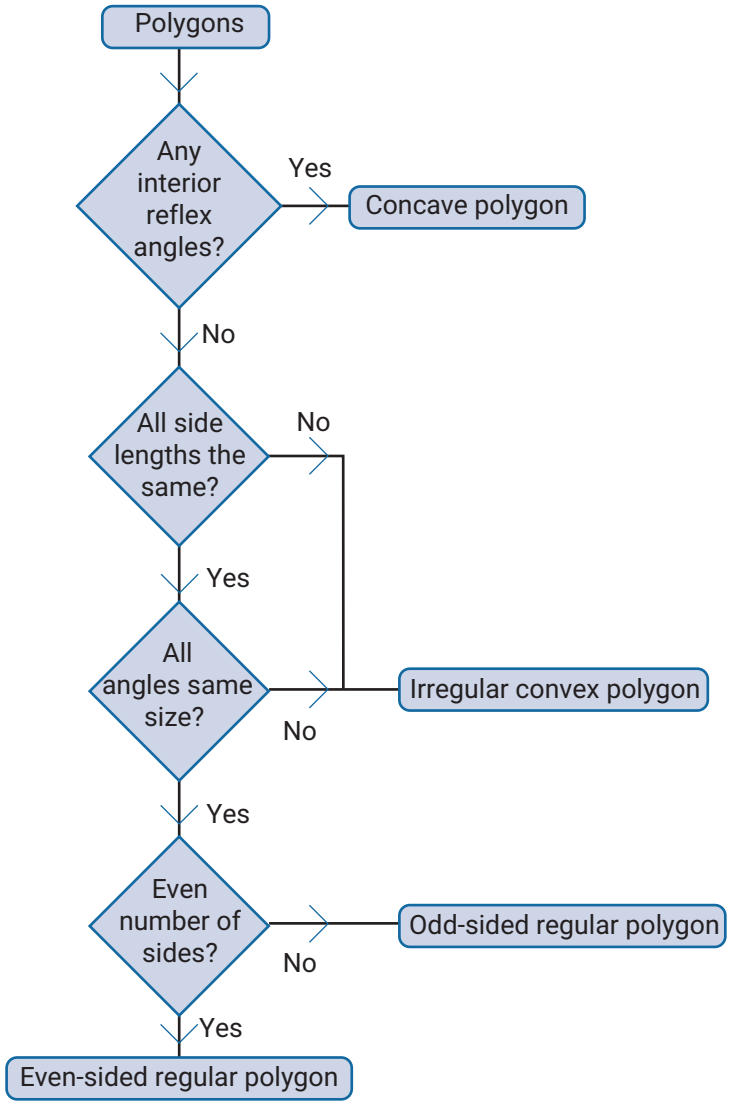


A concave polygon is any polygon with at least one interior reflex angle.



Triangles cannot be concave, since the total of all their angles is 180°.

The flow chart shows one method for classifying polygons.



Worked example

Classifying polygons for triangles and quadrilaterals

Use the polygon classification flow chart to describe each of the shapes.

(a) Isosceles triangle

THINKING	WORKING
Trace a path from the top of the flow chart (polygon) to an endpoint.	Polygon Interior reflex angles? No All sides same length? No Irregular convex polygon
Draw a conclusion.	An isosceles triangle is an irregular convex polygon.

(b) Parallelogram

THINKING	WORKING
Trace a path from the top of the flow chart (polygon) to an endpoint.	Polygon Interior reflex angles? No All sides same length? No Irregular convex polygon
Draw a conclusion.	A parallelogram is an irregular convex polygon.

(c) Square

THINKING	WORKING
Trace a path from the top of the flow chart (polygon) to an endpoint.	Polygon Interior reflex angles? No All sides same length? Yes All angles same size? Yes Even number of sides? Yes Even-sided regular polygon
Draw a conclusion.	A square is an even-sided regular polygon.

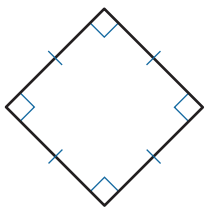
Practice

ANSWERS Page 717

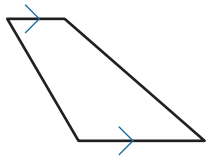
SC 1 I can identify and describe the side and angle properties of quadrilaterals

1 Identify each of the following quadrilaterals.

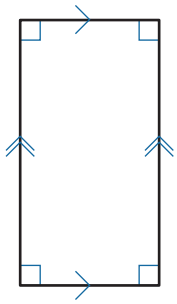
(a)



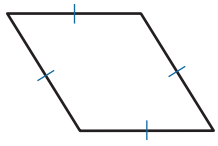
(b)



(c)



(d)



2 Determine the shape described in each of the following. Sketch the shape, if needed.

(a) A four-sided figure with one pair of opposite sides equal and parallel

(b) A quadrilateral with all sides of 5 cm and an angle of 70°

(c) A four-sided figure with two adjacent sides of 8 cm and the other two sides 10 cm. Assume that the shape is convex (no indents).

3 Below is a list of rectangle properties. For each property, name a special quadrilateral with the same property.

(a) Two pairs of opposite sides equal

(b) Two pairs of opposite sides parallel

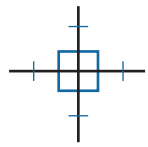
(c) All angles being right angles

(d) Exactly two lines of symmetry

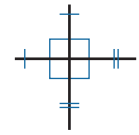
(e) Rotational symmetry of order 2 (so a 180° rotation maps it to the original shape)

4 The diagrams below show the diagonals of some quadrilaterals intersecting at right angles. Where possible, name the shape of the special quadrilaterals they come from.

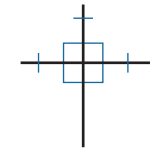
(a)



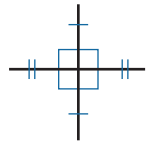
(b)



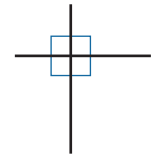
(c)



(d)



(e)



17

SC 2 I can identify a hierarchy within types of quadrilaterals

1 Consider a rectangle and a parallelogram.

(a) Identify a property common to these two special quadrilaterals.

(b) Identify the property that distinguishes the two special quadrilaterals from each other.

(c) Copy and complete the statement: A _____ is a _____ with one angle a _____ angle.

2 All rhombuses are parallelograms, but not all parallelograms are rhombuses. Explain.

3 Is a parallelogram a type of trapezium? Explain.

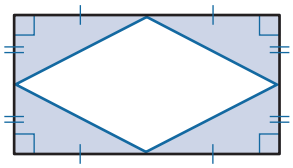
4 Both a rhombus and a rectangle are parallelograms, but a rectangle is not necessarily a rhombus. Explain.

5 The diagonals of a quadrilateral bisect each other.

(a) Name all of the special quadrilaterals this could be. (Sketch a diagram, where needed.)

(b) What is the least special name for quadrilaterals with this property?

6 A rectangle has the midpoints of each side joined to make another quadrilateral.

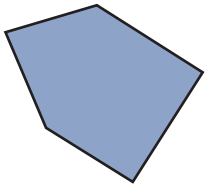


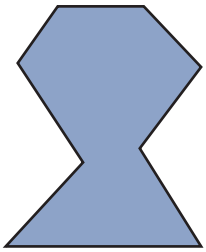
(a) Name the special quadrilateral formed.

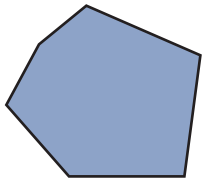
(b) If the original rectangle was a square, what would the special quadrilateral in the middle be?

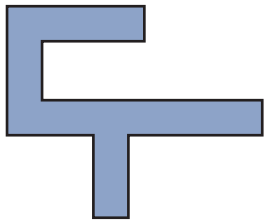
SC 3 I can use a flow chart for polygons to categorise triangles and quadrilaterals

1 Name each of the polygons below and state whether each is concave or convex.

(a) 

(b) 

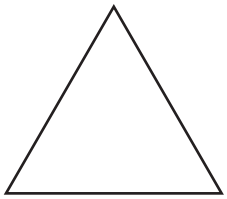
(c) 

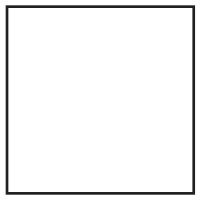
((d) 

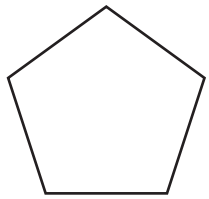
2 For each of the regular polygons up to 8 sides:

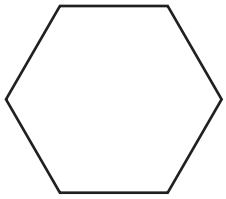
(a) state the number of pairs of parallel sides

(b) write a rule for the number of pairs of parallel sides in a regular polygon.

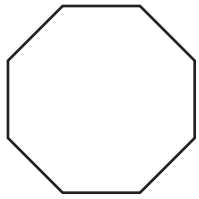

A regular triangle


A regular quadrilateral


A regular pentagon


A regular hexagon

A regular heptagon


A regular octagon

(c) How many pairs of parallel sides are in:

(i) a regular nonagon? **(ii)** a regular dodecagon?

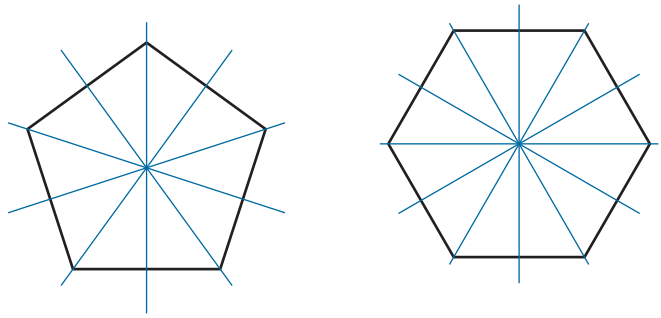
3 Use the polygon classification flow chart to describe each of the shapes below.

(a) Rectangle **(b)** Obtuse-angled triangle

(c) Trapezium **(d)** Quadrilateral with three angles of 50°

4 Explain why an equilateral triangle is the only equilateral polygon that is certain to be a regular polygon. Use quadrilaterals as an example to assist your explanation.

5 Regular shapes have many special properties due to symmetry. All regular polygons have the same number of lines of symmetry as the number of side lengths.

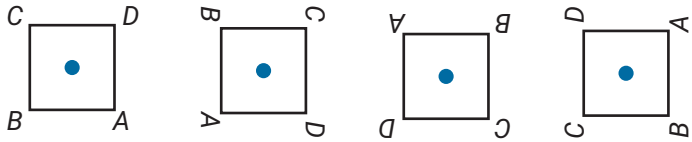


Describe, in as much detail as possible, how lines of symmetry are drawn:

(a) for odd-sided regular polygons

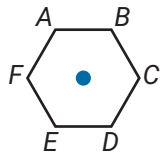
(b) for even-sided regular polygons.

6 All regular polygons have the same order of rotational symmetry as the number of side lengths.



(a) What is the order of rotational symmetry for a regular hexagon?

(b) Show the named vertices of the hexagon below as it rotates through each matching position.

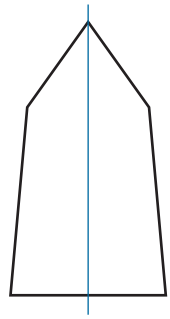


7 Sketch an example of each of these irregular shapes.

(a) A hexagon with exactly 2 lines of symmetry

(b) An octagon with exactly 4 lines of symmetry

8 Sketch any pentagon. Draw the minimum number of diagonals to form triangles inside the pentagon.



(a) Determine the sum of the interior angles for any pentagon.

(b) Determine the size of the equal angles of a regular pentagon.

(c) Draw a concave pentagon and measure its interior angles. (Note: the angles should have the same sum as found in **(a)**.)

Use parallel side lengths of quadrilaterals to construct rectangles of equivalent area

17.4

Learning intention: To be able to use parallel side lengths of quadrilaterals to construct rectangles of equivalent area

Success criteria:

SC 1 I can form quadrilaterals with parallel sides from appropriate pairs of triangles.

SC 2 I can form a rectangle (or square) of equivalent area from any quadrilateral that has a pair of parallel sides.

SC 3 I can calculate the area of trapeziums using the average of parallel sides.

Lesson warm-up

The biggest paddock

A farmer has 100 m of portable fencing. What is the biggest straight-sided paddock that can be made?

SC 1 I can form quadrilaterals with parallel sides from appropriate pairs of triangles

All quadrilaterals can be split into two triangles by drawing a diagonal.

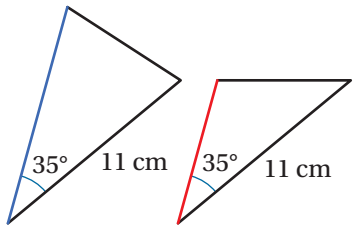
Given that two triangles have a side that is equal in length, they can be combined to form a convex quadrilateral.

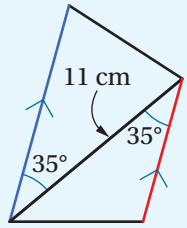
Worked example

Forming quadrilaterals with parallel sides from triangles

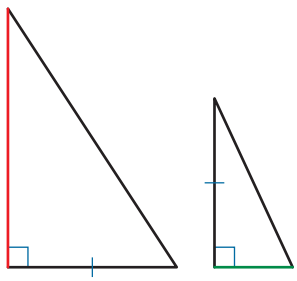
Fit each pair of triangles together to form a quadrilateral with a pair of parallel sides. Name the quadrilateral.

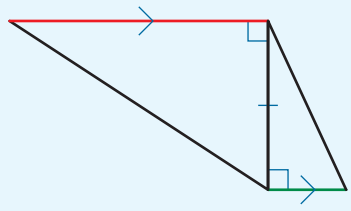
(a)



THINKING	WORKING
Place the triangles together along the equal sides. Make sure the equal angles form a Z shape (alternate angles). Mark in the parallel sides.	
List the special properties.	The quadrilateral has one pair of parallel sides.
Identify the quadrilateral.	The quadrilateral is a trapezium.

(b)



THINKING	WORKING
Place the triangles together along the equal sides. Make sure the equal angles form a Z shape (alternate angles). Mark in the parallel sides.	
List the special properties.	The quadrilateral has one pair of parallel sides.
Identify the quadrilateral.	The quadrilateral is a trapezium.

SC 2 I can form a rectangle (or square) of equivalent area from any quadrilateral that has a pair of parallel sides

Worked example

Forming rectangles of equivalent area

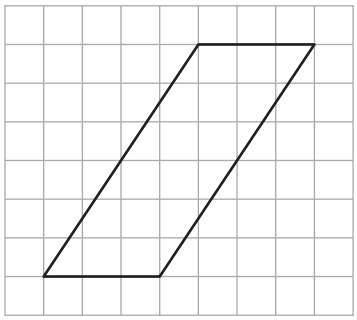
From each quadrilateral with parallel sides:

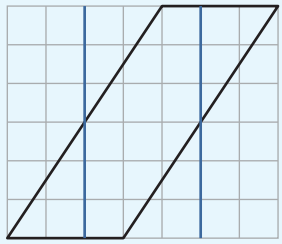
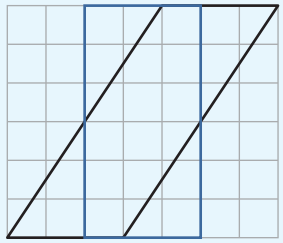
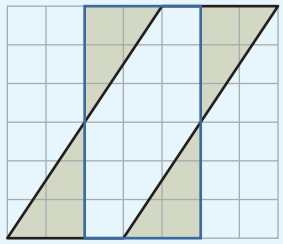
(i) use the midpoints of the sides to form rectangles of equivalent area

(ii) shade triangles so that equal areas are easily seen

(iii) describe how the dimensions of the rectangles relate to the original shapes.

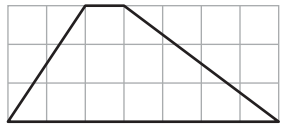
(a)

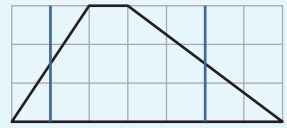
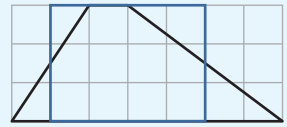
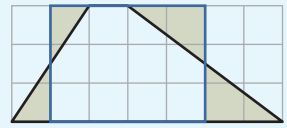


THINKING	WORKING
Draw vertical lines through the midpoints of the oblique sides.	
Use the vertical sides to complete the rectangle.	
Shade the four triangles to show that equal areas have been added and subtracted.	

THINKING	WORKING
Describe the horizontal dimension (width) of the rectangle in terms of the horizontal dimensions of the original shape.	The width of the rectangle is the same as the base length of the parallelogram.
Describe the vertical dimension (height) of the rectangle in terms of the vertical dimension of the original shape.	The height of the rectangle is the same as the height of the parallelogram.

(b)



THINKING	WORKING
Draw vertical lines through the midpoints of the oblique sides.	
Use the vertical sides to complete the rectangle.	
Shade the four triangles to show that equal areas have been added and subtracted.	
Describe the horizontal dimension (width) of the rectangle in terms of the horizontal dimensions of the original shape.	The width of the rectangle (4 units) is the average of the parallel side lengths (1 unit and 7 units) of the trapezium. $\frac{1+7}{2}=4$
Describe the vertical dimension (height) of the rectangle in terms of the vertical dimension of the original shape.	The height of the rectangle is the same as the height of the trapezium.

SC 3 I can calculate the area of trapeziums using the average of parallel sides

The area of a rectangle is length × width or base × height.

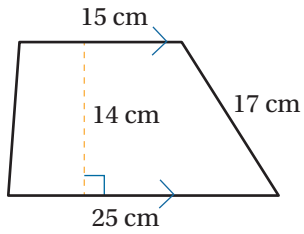
In the same way, the area of a trapezium can be found by multiplying the average of the parallel sides by the distance between them.

Area = average of parallel sides × distance between them

Worked example

Calculating the area of a trapezium

Use the average of parallel sides to determine the area of the trapezium.



THINKING	WORKING
Identify the parallel sides.	Parallel sides: 15 cm, 25 cm
Calculate the average.	Average: $\frac{15+25}{2} = 20$ cm
Identify the height, the distance between the parallel sides.	Height: 14 cm
Calculate the area. Remember that area is expressed in square units.	Area = average of parallel sides × distance between them $A = 20 \times 14$ $= 280\,\text{cm}^2$

Practice

ANSWERS Page 718

SC 1 I can form quadrilaterals with parallel sides from appropriate pairs of triangles

1 Which pair of the following triangles could form a quadrilateral? Describe the orientation of the sides for each possible quadrilateral.

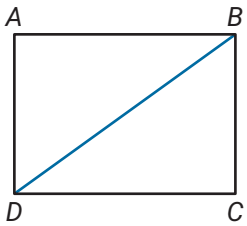
(a) Side lengths 3 cm, 4 cm, 5 cm

(b) Side lengths 5 cm, 7 cm, 8 cm

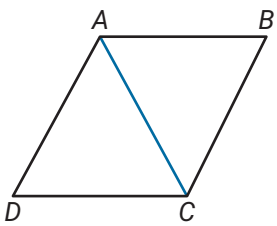
(c) Side lengths 6 cm, 9 cm, 10 cm

2 In each of the quadrilaterals shown, which pair of angles must be equal so that AB is parallel to DC ($AB \parallel DC$)? Use three letters to name the angles.

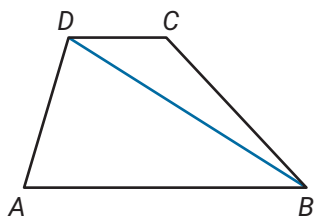
(a)



(b)

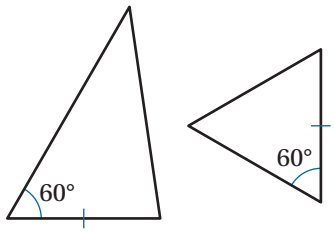


(c)

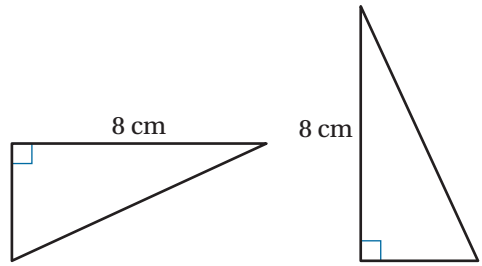


3 Fit each pair of triangles together to form a quadrilateral with a pair of parallel sides. Name the quadrilateral.

(a)



(b)



4 Two right-angled isosceles triangles are placed together to form a quadrilateral. What shape is the quadrilateral if:

(a) the triangles are congruent (same size and shape)? Give both possibilities.

(b) the triangles are similar (same shape but not the same size)?

5 Two congruent (same size and shape), scalene right-angled triangles are placed together to form a quadrilateral. What shape is the quadrilateral? Give all possibilities.

6 Two congruent triangles of side lengths 6 cm, 7 cm and 8 cm are placed together in six different ways to form quadrilaterals.

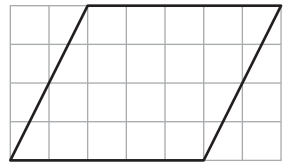
(a) What shape are the ones with parallel sides? Explain how you thought through your answer.

(b) What shape are the ones with no parallel sides? Explain how you thought through your answer.

17

SC 2 I can form a rectangle (or square) of equivalent area from any quadrilateral that has a pair of parallel sides

1 Draw a rectangle from point A that has the same area as the parallelogram.



Explain how you know that the rectangle and parallelogram have the same area.

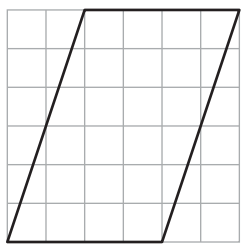
2 For each quadrilateral with parallel sides:

(i) use the midpoints of the sides to form rectangles of equivalent area

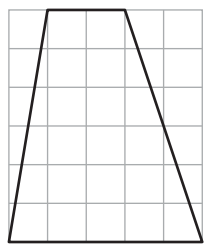
(ii) shade triangles so that equal areas are easily seen

(iii) describe how the dimensions of the rectangles relate to the original shapes.

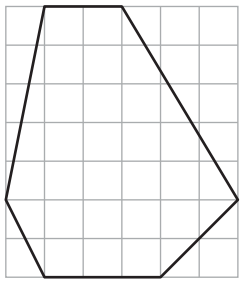
(a)



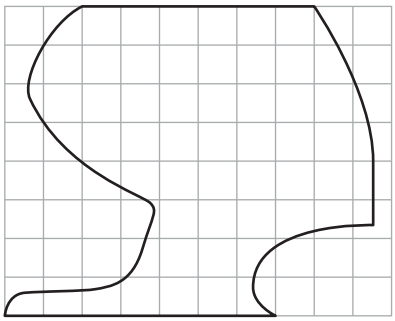
(b)



3 Give the dimensions of two rectangles that could be used to calculate the area of the hexagon.



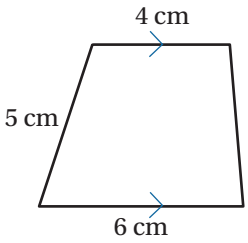
4 By finding the average length of the horizontal grid lines within the shape, give the dimensions of a rectangle that could be used to estimate the area of the shape.



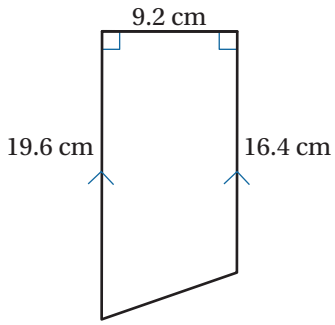
SC 3 I can calculate the area of trapeziums using the average of parallel sides

1 Calculate the average of the parallel sides for each trapezium.

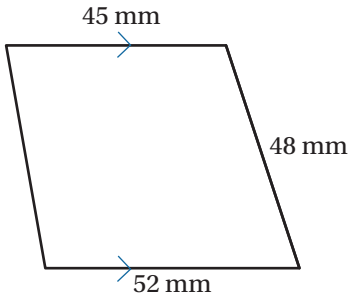
(a)



(b)

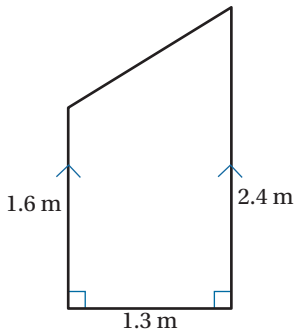


(c)

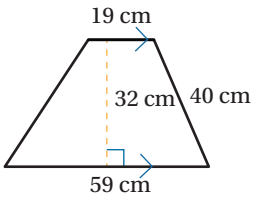


2 Determine the dimensions of a rectangle, the same height as each trapezium, that will have the same area as the trapezium.

(a)



(b)



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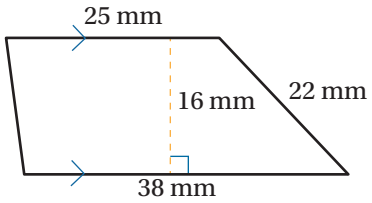
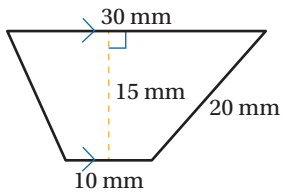
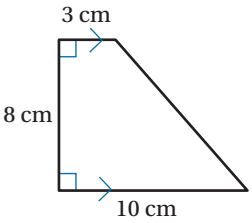


3 Use the average of parallel sides to determine the area of each trapezium.

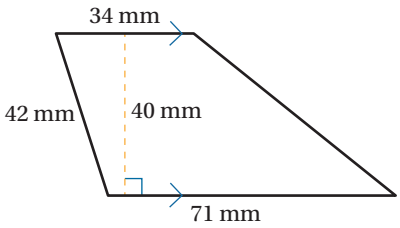
(a)



(b)



(c)

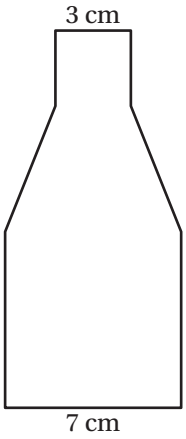


4 A regular hexagon has a side length of 20 cm and an overall height of 34.6 cm.

(a) Determine the area of the hexagon.

(b) Determine the dimensions of a rectangle with the same height and area of the regular hexagon.

5 The compound shape below is made up of a trapezium between two squares. The shape has an overall height of 15 cm. Determine the area of the compound shape.

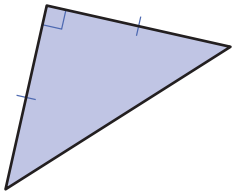


TOPIC REVIEW

Multiple choice

ANSWERS Page 749

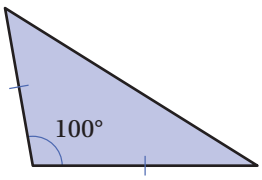
1 The triangle shown is best described as:



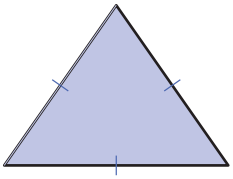
A isosceles, acute-angled
B isosceles, right-angled
C scalene, acute-angled
D scalene, right-angled

2 Which one of these is a right-angled isosceles triangle?

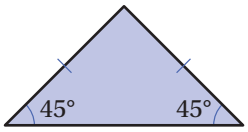
A



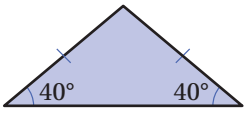
B



C



D



3 Select the correct property of a rectangle.

A Two pairs of adjacent sides equal
B One pair of opposite sides parallel
C Opposite sides equal in length, all angles 90°
D Two sides equal in length

4 Select the correct property of a kite.

A Two pairs of adjacent sides equal
B One pair of opposite sides parallel
C Opposite sides equal in length, all angles 90°
D Two sides equal in length

5 Select the correct property of an isosceles triangle.

A Two pairs of adjacent sides equal
B One pair of opposite sides parallel
C Opposite sides equal in length, all angles 90°
D Two sides equal in length

6 Select the correct property of a trapezium.

A Two pairs of adjacent sides equal
B One pair of opposite sides parallel
C Opposite sides equal in length, all angles 90°
D Two sides equal in length

7 A quadrilateral has both pairs of opposite sides equal and parallel.
You can be sure the quadrilateral is a:

A parallelogram **B** rhombus **C** rectangle **D** square

8 Which statement is true?

A A triangle can have two obtuse angles.

B A square has all the properties of a rhombus.

C A kite has at least one pair of parallel sides.

D The diagonals of a rectangle bisect the angles of the rectangle.

9 Which statement is false?

A A kite has one pair of opposite angles equal.

B A trapezium is not a special rectangle.

C A rhombus is a special rectangle.

D A rhombus is a special parallelogram.

10 Which statement is true?

A All quadrilaterals have at least one pair of parallel sides.

B All trapeziums have a pair of sides of equal length.

C All quadrilaterals have a right angle.

D All kites have a pair of equal angles.

11 Which of these are not always parallel?

A The opposite sides of a square

B The opposite sides of a parallelogram

C The opposite sides of a rhombus

D The opposite sides of a kite

12 Which statement is true?

A A trapezium has one pair of opposite sides equal and parallel.

B A square is a special rectangle.

C The diagonals of a rhombus are equal.

D The diagonals of a kite bisect each other at right angles.

13 Which statement is true?

A A kite has one pair of adjacent angles equal.

B A rectangle is a special kite.

C A rhombus is a special rectangle.

D A rhombus is a special trapezium.

14 Which statement is false?

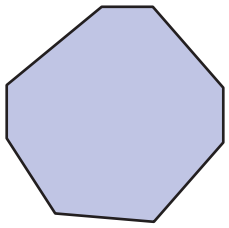
A All parallelograms have two pairs of parallel sides.

B All trapeziums have one pair of parallel sides equal in length.

C All kites have a pair of equal angles.

D All rectangles have a right angle.

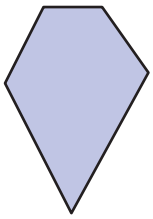
15 The name of the polygon is:



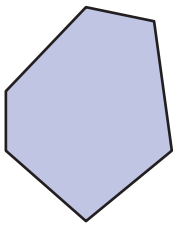
A A hexagon **B** A heptagon **C** An octagon **D** A nonagon

16 Which of the polygons is a heptagon?

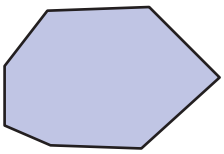
A



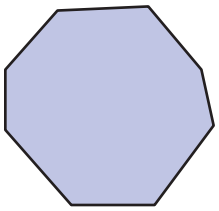
B



C



D



17 Three triangles have side lengths of:

- 6 cm, 8 cm and 8 cm
- 6 cm, 9 cm and 10 cm
- 5 cm, 5 cm and 8 cm respectively.

Which pairs of triangles could not form a quadrilateral?

A The first and second only

B The first and third only

C The second and third only

D All pairs

18 Two congruent scalene right-angled triangles are joined to form a quadrilateral. The quadrilateral could be:

A a parallelogram or a rhombus

B a rhombus or a rectangle

C an isosceles triangle or a rectangle

D a parallelogram or a rectangle

Short answer

1 Decide whether or not the three given lengths can form a triangle. If not, explain why that is.

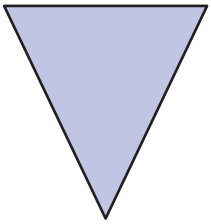
(a) 16 cm, 18 cm, 35 cm **(b)** 6 cm, 6 cm, 12 cm **(c)** 18 cm, 20 cm, 36 cm

2 Construct a triangle with side lengths 8 cm, 8 cm and 11 cm. As a check for accuracy, measure the largest angle.

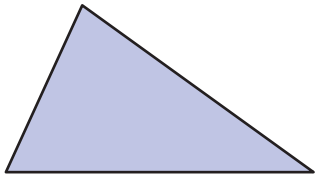
3 Construct a quadrilateral with side lengths 10 cm, 6 cm, 5 cm and 7 cm, in the order given, with an angle of 70° between the first two sides. As a check for accuracy, measure the largest angle.

4 Give the side-length name for each of the triangles. Use a ruler to measure the lengths, if necessary.

(a)



(b)



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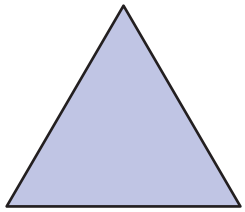
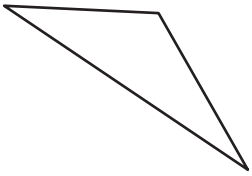
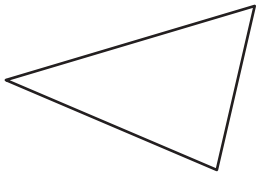


5 Give the angle name for each of the triangles.

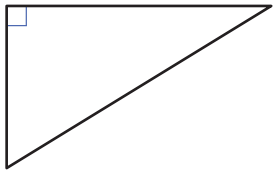
(a)



(b)

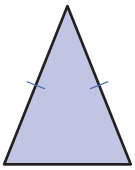


(c)

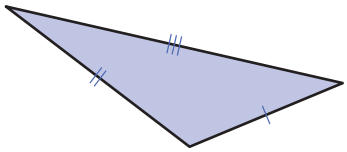


6 Describe each of the triangles with a side and angle name.

(a)



(b)



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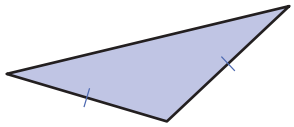

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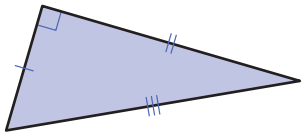
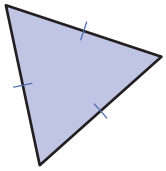
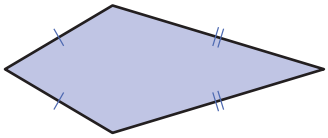


(e)

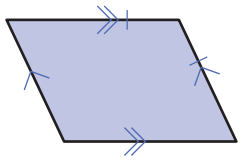


7 Correctly name each of the quadrilaterals.

(a)



(b)



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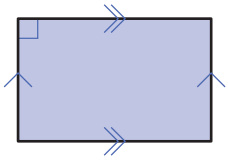
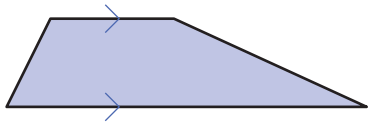


(d)



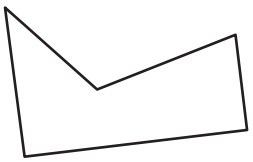
8 What properties of a square make it also classifiable as:

(a) a rectangle **(b)** a rhombus **(c)** a parallelogram?

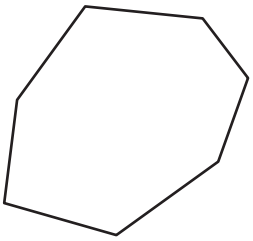
9 Explain why a rhombus is not a regular quadrilateral.

10 Name each of the polygons and say whether each is regular or irregular, concave or convex.

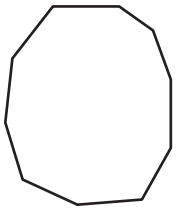
(a)



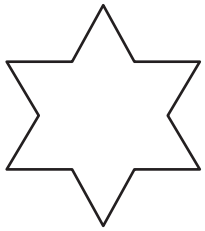
(b)



(c)



(d)



11 A trapezium has parallel sides of lengths 15 cm and 34 cm, with 12 cm between them.

(a) Give the dimensions of a rectangle with the same height and an equal area to the trapezium.

(b) Calculate the area of the trapezium.

Extended response

1 Consider the three lengths 18 cm, 20 cm and 39 cm.

(a) Write the new lengths and angle names of the triangle which, after adding 2 cm to one value, the lengths make:

(i) an isosceles triangle **(ii)** a scalene triangle.

(b) Subtract 15 cm from one value so that the lengths make an acute-angled triangle, and write the side name for the triangle.

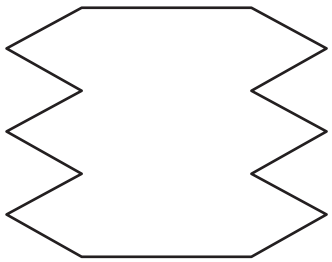
(c) Subtract 12 cm from one value and add 12 cm to another so that the lengths make a triangle. Describe the different triangles that can be formed in this way.

2 A parallelogram is divided into triangles as given below. For each of the triangles formed, identify the type of triangle, giving its angle name.

(a) A single diagonal giving two identical triangles (Give both possibilities.)

(b) Two diagonals giving four triangles

3 The concave tetra-decagon is made of six identical trapeziums, or three identical hexagons. The height of the tetra-decagon is 30 cm and its width varies from 20 cm to 40 cm.



(a) Give the dimensions of a rectangle that is the same height as a hexagon, with the same area as the hexagon.

(b) Calculate the area of one hexagon.

(c) Calculate the area of the tetra-decagon.

(d) Give the dimensions of a rectangle that is the same height as the tetra-decagon, with the same area as the tetra-decagon.

TOPIC
18

Translations, reflections, rotations

Recall 505

18.1 Translate and reflect points and shapes on a Cartesian plane 506

18.2 Rotate shapes on a Cartesian plane 514

Topic review 525

Why learn this?

Learning to translate and reflect points and shapes on a Cartesian plane provides a comprehensive understanding of how objects can be manipulated within a coordinate system. This topic teaches students to:

- perform and describe translations in shifting objects in space, which is useful in computer graphics and animation
- master reflections used in optics and physics, where understanding the behaviour of light or sound waves against surfaces is essential
- recognise and describe the properties of these transformations, which is critical in problem-solving across mathematics, particularly in geometry.

These skills also form the basis for practical applications in technology, engineering and the visual arts, where spatial transformations are frequently used.

RECALL

ANSWERS Page 719

I can identify transformations of plane shapes

1 Copy and complete each of these sentences with the correct term for the described transformation.

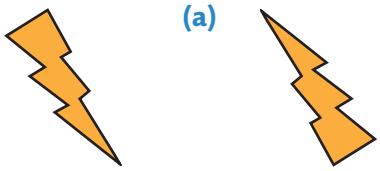
(a) A shape that moves two units to the right has undergone a _____

(b) A shape that has spun on the spot 180° has undergone a _____

(c) A shape that has flipped about a straight axis has undergone a _____

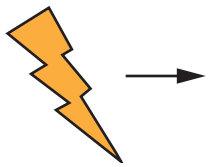
2 Describe the transformation of this shape in each diagram.

(a)



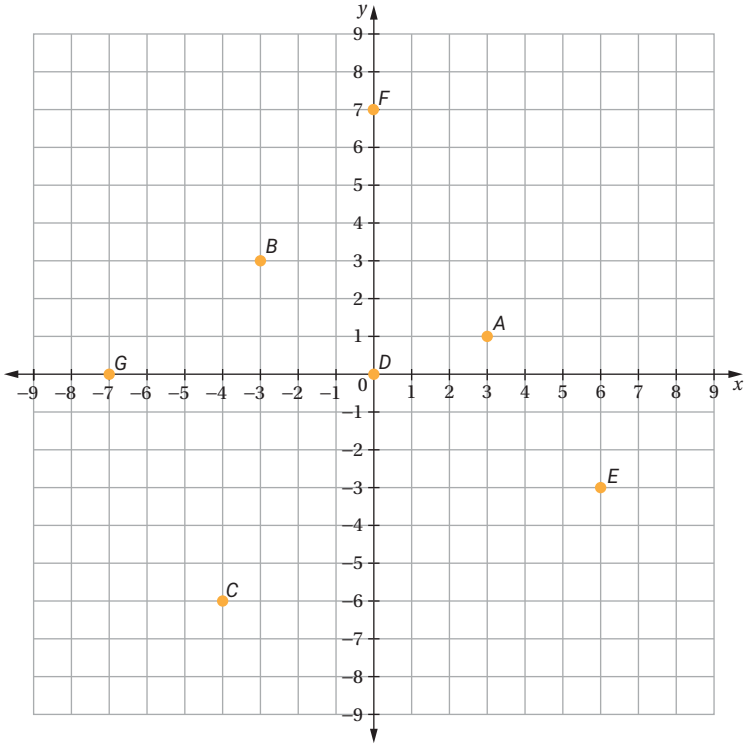
(b)

(c)



I can read and plot points on a Cartesian plane

1 Write the coordinates to identify the position of each capital letter.



2 Plot and label the following points on a Cartesian plane.

(a) $A(1, 6)$ **(b)** $B(-5, 0)$ **(c)** $C(3, -8)$

(d) $D(-2, 5)$ **(e)** $E(4, 0)$ **(f)** $F(-3, -2)$

18.1 Translate and reflect points and shapes on a Cartesian plane

Learning intention: To be able to translate and reflect points and shapes on a Cartesian plane and describe these transformations

Success criteria:

SC 1 I can perform translations on a Cartesian plane.

SC 2 I can perform reflections in an axis.

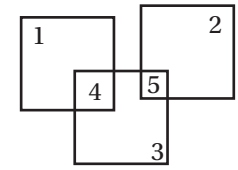
SC 3 I can identify and describe translations and reflections.

Lesson warm-up

How many squares?

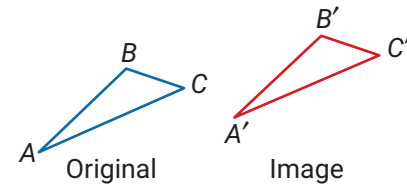
Draw 3 overlapping squares of the same size and see how many squares you can create.

For example, this arrangement has five squares: three large, one medium and one small.



SC 1 I can perform translations on a Cartesian plane

A translation is a simple slide movement of a figure in a particular direction. The figure does not change size, shape or orientation.



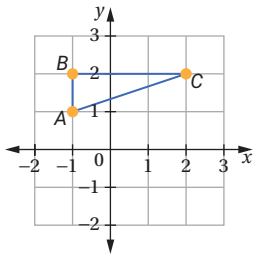
The image points are labelled with a 'dash' or 'prime', so ABC becomes $A'B'C'$.

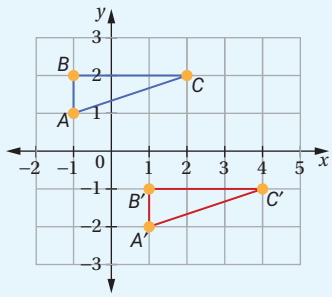
In coordinate geometry, the direction is split into the x-coordinate (left or right) and the y-coordinate (up or down). This translation can be written in the form $[x, y]$. The square brackets are used to distinguish the translation from the coordinates of any points involved.

Worked example

Translating a shape on a Cartesian plane

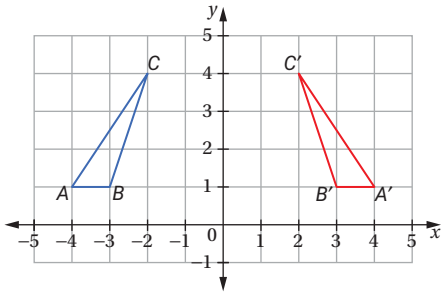
Translate the given triangle [2, –3] and write the coordinates of the image.

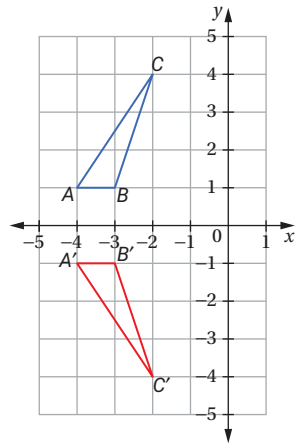


THINKING	WORKING
Interpret the translation.	$[x, y] \Rightarrow [2, -3]$ The translation is: add 2 units to each x-coordinate and subtract 3 from each y-coordinate.
Use each value in the translation to state the new coordinates.	$A(-1, 1) \rightarrow A'(-1 + 2, 1 - 3) = A'(1, -2)$ $B(-1, 2) \rightarrow B'(-1 + 2, 2 - 3) = B'(1, -1)$ $C(2, 2) \rightarrow C'(4, -1)$
Make a comment about the result of the translation.	Geometrically, the triangle will be 2 to the right and 3 down from its original position. 

SC 2 I can perform reflections in an axis

A reflection in either a horizontal or vertical axis keeps a figure the same size and shape, but changes the orientation, so that if ABC is labelled as anticlockwise, then $A'B'C'$ will be labelled as clockwise.

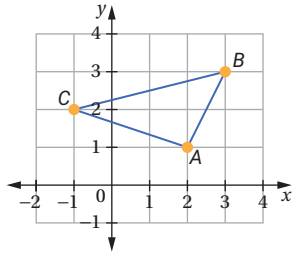




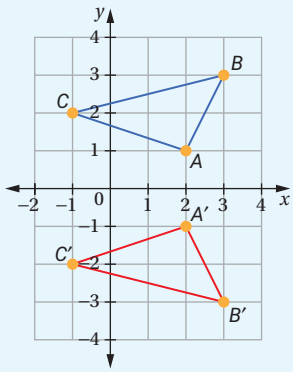
Worked example

Reflecting a shape in an axis

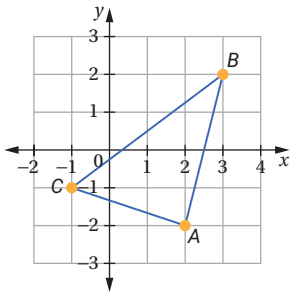
(a) Reflect the given triangle in the x-axis and write the coordinates of the image.

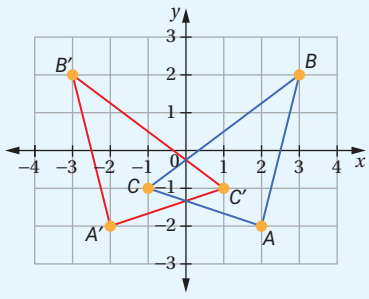


THINKING	WORKING
Determine the coordinates after a reflection in the x-axis. The x-coordinate is unchanged. The y-coordinate is multiplied by -1 to change the sign.	$A(2, 1) \rightarrow A'(2, 1 \times (-1)) \rightarrow A'(2, -1)$ $B(3, 3) \rightarrow B'(3, 3 \times (-1)) \rightarrow B'(3, -3)$ $C(-1, 2) \rightarrow C'(-1, 2 \times (-1)) \rightarrow C'(-1, -2)$

Plot the points to form the reflected image.	
Describe the reflection.	The shape is reflected through the horizontal axis.

(b) Reflect the given triangle in the y-axis and write the coordinates of the image.



THINKING	WORKING
Determine the coordinates after a reflection in the y-axis. The y-coordinate is unchanged. The x-coordinate is multiplied by -1 to change the sign.	$A(2, -2) \rightarrow A'(2 \times (-1), -2) \rightarrow A'(-2, -2)$ $B(3, 2) \rightarrow B'(3 \times (-1), 2) \rightarrow B'(-3, 2)$ $C(-1, -1) \rightarrow C'(-1 \times (-1), -1) \rightarrow C'(1, -1)$
Plot the points to form the reflected image.	
Describe the reflection.	The shape is reflected through the vertical axis.

SC 3 I can identify and describe translations and reflections

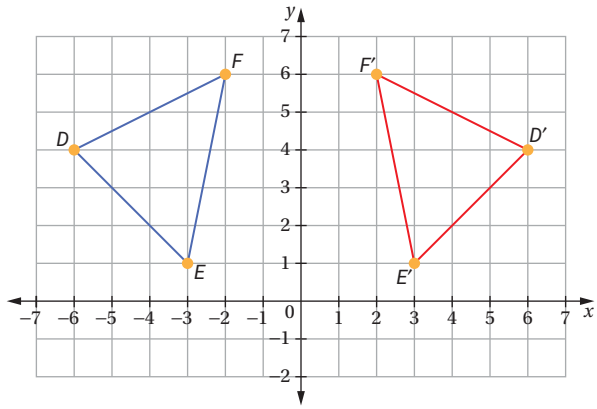
A translation has taken place if a flat 2D shape maintains its shape but has been moved in any direction. A reflection is how the shape will appear in a mirror. Unless the shape is regular, a reflected shape cannot be translated to be the same as the original.

Worked example

Identifying and describing translations and reflections

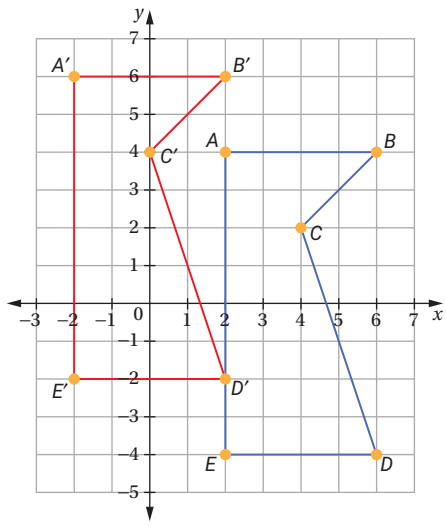
Identify and describe each of the transformations.

(a)



THINKING	WORKING
Describe the transformation.	The triangle has been flipped over across the y-axis.
Locate the corresponding coordinates.	$D(-6, 4) \rightarrow D'(6, 4)$ The y-value stays the same, while the x-value changes sign.
Describe the transformation.	$DEF \rightarrow D'E'F'$ shows a reflection in the y-axis.

(b)



THINKING	WORKING
Describe the transformation.	The concave pentagon has moved to the left and up.
Locate the corresponding coordinates.	$A(2, 4) \rightarrow A'(-2, 6)$ The x-value decreased by 4; the y-value increased by 2.
Describe the transformation.	$ABCDE \rightarrow A'B'C'D'E'$ shows a translation of $[-4, 2]$.

Practice

ANSWERS Page 720

SC 1 I can perform translations on a Cartesian plane

1 Describe the following transformations.

(a) $[1, 1]$ **(b)** $[1, -1]$ **(c)** $[-1, -1]$ **(d)** $[-1, 1]$

2 Use the notation $[x, y]$ for the following transformations.

(a) 2 units right and 3 units up **(b)** 2 units right and 3 units down

(c) 2 units left and 3 units up **(d)** 2 units left and 3 units down

3 Determine the coordinates of the image of the point $P(1, 5)$ under the following transformations:

(a) $[4, 2]$ **(b)** $[4, -2]$ **(c)** $[-4, 2]$ **(d)** $[-4, -2]$

4 Determine the coordinates of the image of each given point under the given translation.

(a) $A(2, 3)$: $[4, 2]$ **(b)** $B(-3, 2)$: $[2, -3]$ **(c)** $C(-1, 8)$: $[0, 3]$

(d) $D(0, 6)$: $[-1, -4]$ **(e)** $E(-4, 0)$: $[3, 1]$ **(f)** $F(-5, -5)$: $[-2, 5]$

5 A line AB with $A(4, 2)$ and $B(6, -1)$ is translated $[-2, 2]$.

(a) Determine the coordinates of the image of each given point.

(b) Show AB and its image $A'B'$ on the same Cartesian plane.

6 A triangle ABC with $A(-1, 4)$, $B(2, 3)$ and $C(0, 2)$ is translated $[2, -1]$.

(a) Determine the coordinates of the image of each given point.

(b) Show ΔABC and its image $\Delta A'B'C'$ on the same Cartesian plane.

7 A quadrilateral $ABCD$ with $A(1, 3)$, $B(0, 4)$, $C(-2, 3)$ and $D(-1, 1)$ is translated $[2, -3]$.

(a) Determine the coordinates of the image of each given point.

(b) Show $ABCD$ and its image $A'B'C'D'$ on the same Cartesian plane.

SC 2 I can perform reflections in an axis

1 Describe what happens to coordinates when they are reflected through the x-axis.

2 Describe what happens to coordinates when they are reflected through the y-axis.

3 Describe what happens to the coordinates of the image $P(4, 5)$ when reflected in:

(a) the x-axis **(b)** the y-axis.

4 Determine the coordinates of the image of each given point reflected in the given axis.

(a) $A(6, 4)$: x-axis **(b)** $B(-1, 1)$: x-axis **(c)** $C(-2, 3)$: y-axis

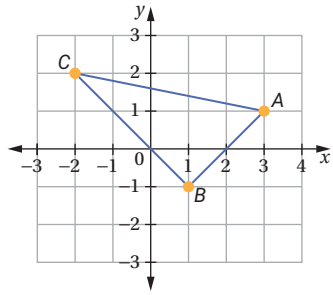
(d) $D(5, 0)$: y-axis **(e)** $E(-1, -3)$: y-axis **(f)** $F(-7, 0)$: x-axis

5 A quadrilateral $ABCD$ with $A(1, 3)$, $B(0, 4)$, $C(-2, 3)$ and $D(-1, 1)$ is reflected in the x-axis.

(a) Determine the coordinates of the image of each vertex.

(b) Show $ABCD$ and its image $A'B'C'D'$ on the same Cartesian plane.

6 ΔABC is to be reflected in the y-axis.



(a) Determine the coordinates of each vertex of the original triangle and its image.

(b) Show ΔABC and its image $\Delta A'B'C'$ on the same Cartesian plane.

7 A triangle ABC with $A(-1, 5)$, $B(3, 1)$ and $C(2, -1)$ is reflected first in the x-axis and then in the y-axis.

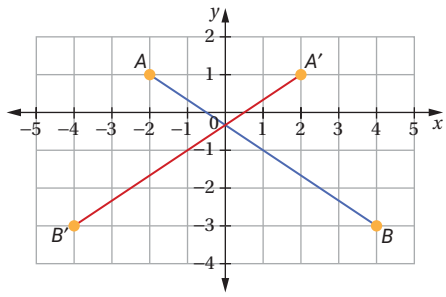
(a) Determine the coordinates of the vertices of the first image, $A'B'C'$.

(b) Determine the coordinates of the vertices of the second image, $A''B''C''$.

(c) Show triangle ABC and the second image $A''B''C''$ on the same Cartesian plane.

SC 3 I can identify and describe translations and reflections

1 Identify and describe the transformation that has taken line AB to line $A'B'$.



2 Each triangle has been translated, with just one pair of corresponding points given each time.

(i) Describe the translation.

(ii) Determine the coordinates of the points in the original or image that have not been given.

(a) $\Delta ABC \rightarrow \Delta A'B'C'$, with $A(4, 2)$, $B(6, -1)$, $C(-3, 0)$, $A'(-3, 0)$.

(b) $\Delta DEF \rightarrow \Delta D'E'F'$, with $D(1, -5)$, $D'(4, -6)$, $E'(5, -2)$, $F'(1, 1)$.

3 Each triangle has been reflected in an axis, with just one pair of corresponding points given each time.

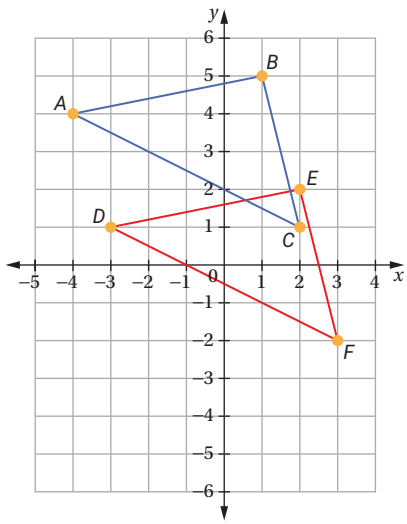
(i) Determine which axis has been used.

(ii) Determine the coordinates of the points in the original or image that have not been given.

(a) $\Delta ABC \rightarrow \Delta A'B'C'$, with $A(3, 3)$, $B(-4, 2)$, $C(-1, 5)$, $B'(-4, -2)$.

(b) $\Delta DEF \rightarrow \Delta D'E'F'$, with $F(2, 2)$, $D'(-3, -1)$, $E'(1, -3)$, $F'(-2, 2)$.

4 For the pair of triangles, identify and describe each of the following transformations.



(a) ΔABC moves to ΔDEF

(b) ΔDEF moves to ΔABC

18.2 Rotate shapes on a Cartesian plane

Learning intention: To be able to rotate shapes on a Cartesian plane

Success criteria:

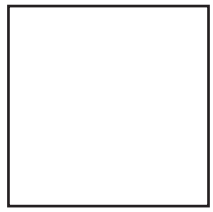
SC 1 I can describe and perform rotations of shapes on a Cartesian plane.

SC 2 I can identify and describe combined transformations.

Lesson warm-up

Where does the mirror go?

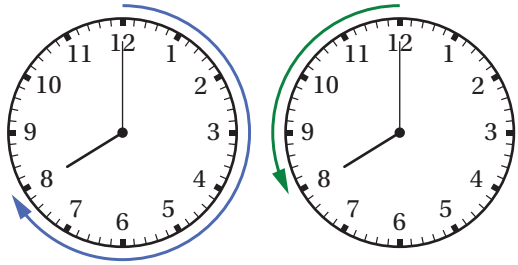
How many places can you put a mirror across this square to reflect a square?



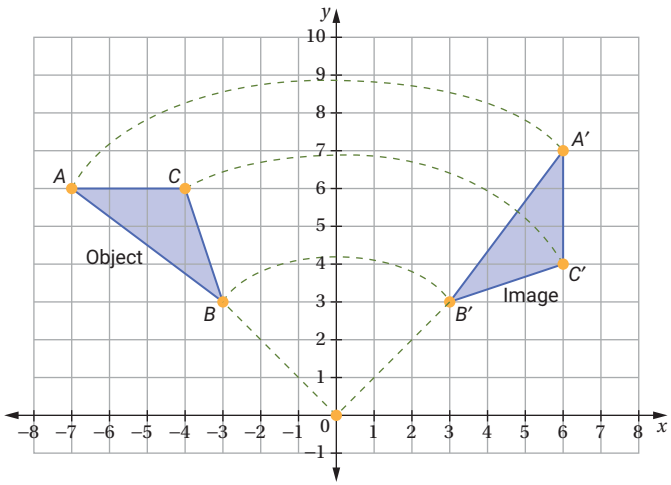
SC 1 I can describe and perform rotations of shapes on a Cartesian plane

For a rotation, you need a point about which the rotation occurs, the angle of the rotation and its direction.

Clockwise rotation follows the same direction as a clock's hands moving around an analogue clock face. Anticlockwise (or 'counter-clockwise') is the opposite direction to clockwise.

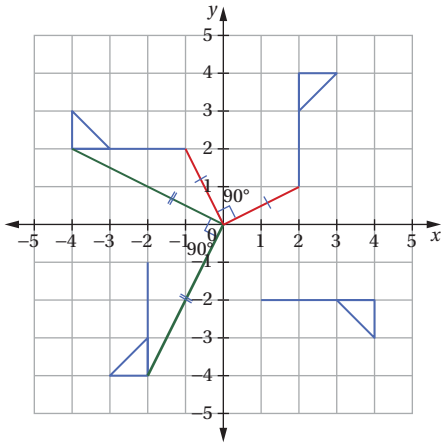


Rotations of 90°, 180° or 270° around the origin, (0, 0), are easily understood on the Cartesian plane. Here the ΔABC has been rotated about the origin to form $\Delta A'B'C'$.



An anticlockwise rotation of 90° gives the same result as a clockwise rotation of 270°.

A rotation of 180° in either direction results in the same image.

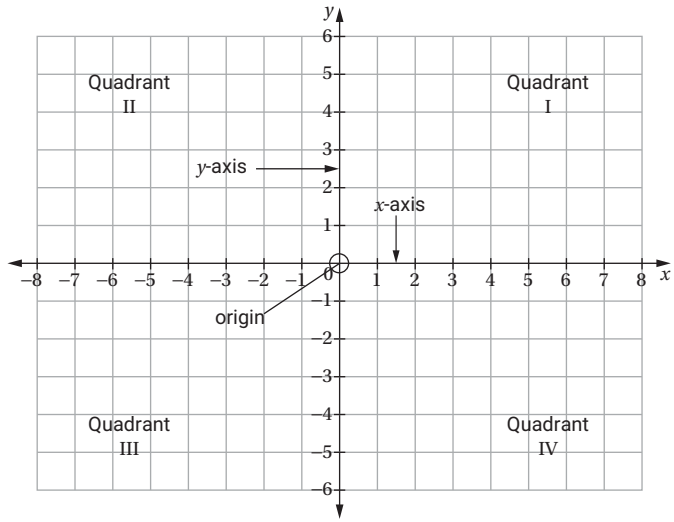


The red rotation shown is 90° clockwise; the point (–1, 2) becomes (2, 1), or generally, (x, y) becomes $(y, -x)$.

The green rotation shown is 90° anticlockwise; the point (–4, 2) becomes (–2, –4), or generally, (x, y) becomes $(-y, x)$.

Note that the distance from the point of rotation remains the same after rotation.

A Cartesian plane is divided into 4 quadrants (quarters) by the x- and y-axes. They are numbered as shown in this diagram.

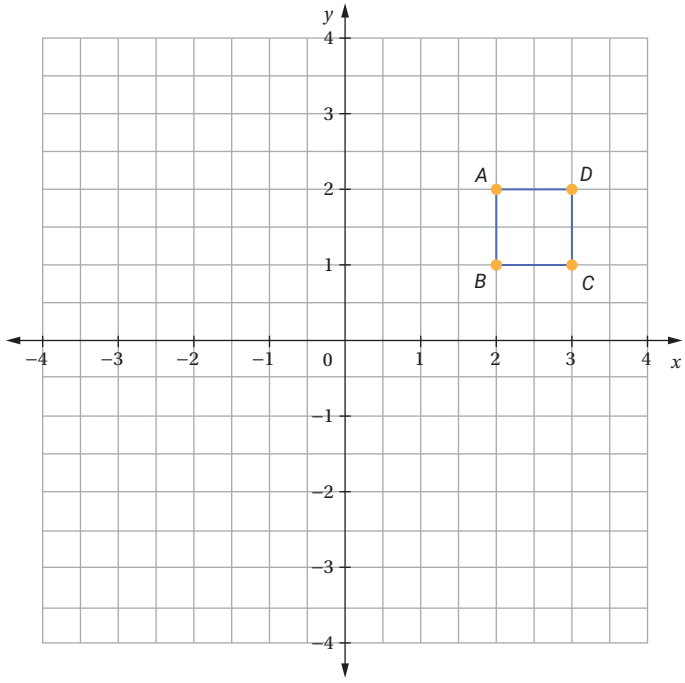


In general, the signs of a reflection can be worked out by checking which quadrant the point falls into.

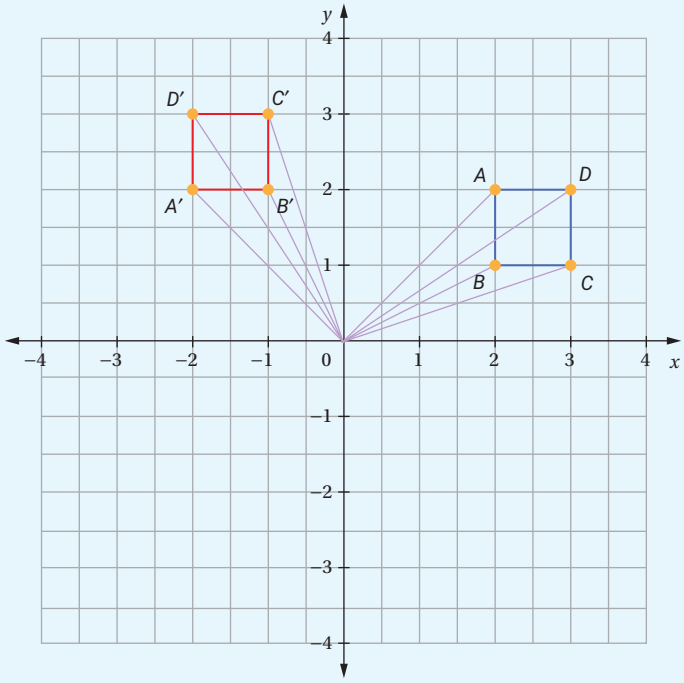
Worked example

Rotating shapes on a Cartesian plane

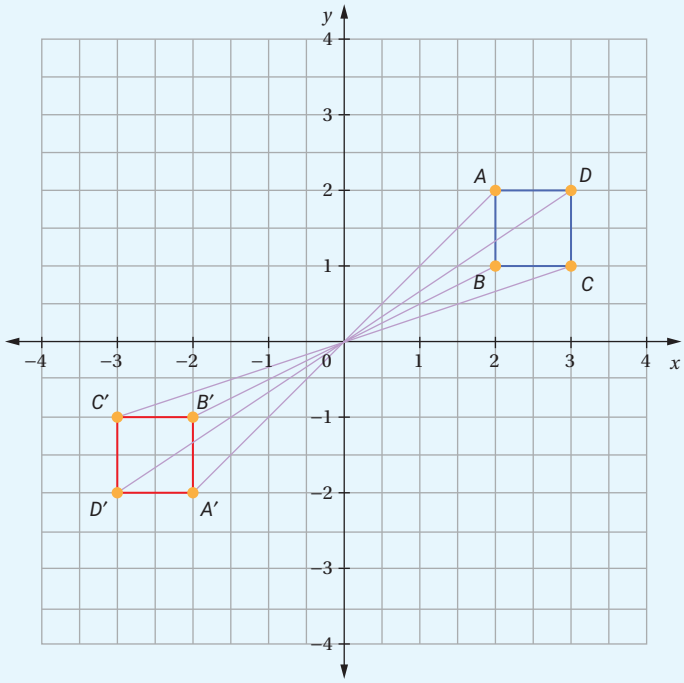
Rotate the square around the origin, O, as instructed, noting the coordinates of the vertices of each image point.



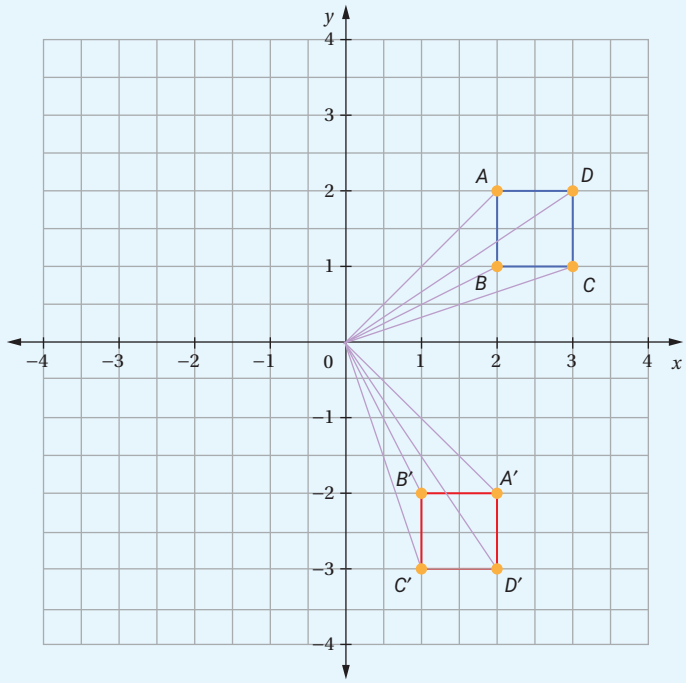
(a) 90° anticlockwise (or 270° clockwise)

THINKING	WORKING
Describe the position of the original shape.	The original square is in the first quadrant.
Determine the position of the rotated image.	90° anticlockwise is in the second quadrant.
Recall the form of coordinates in the new quadrant.	Points in the second quadrant have the form $(-x, y)$.
Determine the position of the image points. A 90° rotation results in the magnitude of the x- and y-coordinates switching places.	$A(2, 2) \rightarrow A'(-2, 2)$ $B(2, 1) \rightarrow B'(-1, 2)$ $C(3, 1) \rightarrow C'(-1, 3)$ $D(3, 2) \rightarrow D'(-2, 3)$
Plot each point to check that the line extending from the origin to a point, and the origin to its image forms a right-angle.	

(b) 180°

THINKING	WORKING
Determine the position of the rotated image.	The original square is in the first quadrant, so rotate to the third quadrant.
Recall the form of coordinates in the new quadrant.	Points in the third quadrant have the form $(-x, -y)$.
Determine the position of the image points. The x- and y-coordinates both change signs.	$A(2, 2) \rightarrow A'(-2, -2)$ $B(2, 1) \rightarrow B'(-2, -1)$ $C(3, 1) \rightarrow C'(-3, -1)$ $D(3, 2) \rightarrow D'(-3, -2)$
Plot each point to check that the line extending from the origin to a point, and the origin to its image forms a straight-angle.	

(c) 270° anticlockwise

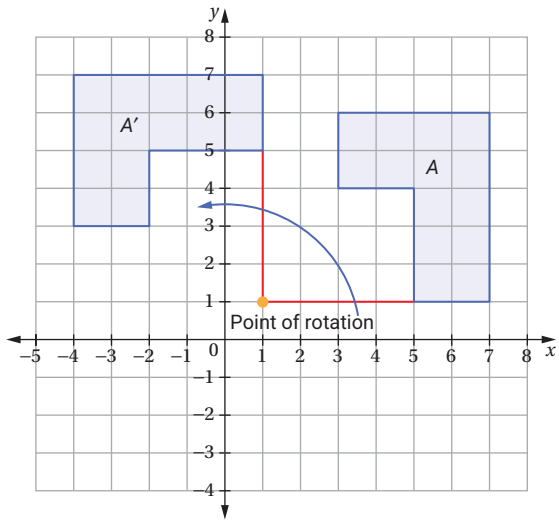
THINKING	WORKING
Determine the position of the rotated image.	270° anticlockwise is the same as 90° clockwise. The original square is in the first quadrant, so rotate to the fourth quadrant.
Recall the form of coordinates in the new quadrant.	Points in the fourth quadrant have the form $(x, -y)$.
Determine the position of the image points. A 90° rotation results in the magnitude of the x- and y-coordinates switching places.	$A(2, 2) \rightarrow A'(2, -2)$ $B(2, 1) \rightarrow B'(1, -2)$ $C(3, 1) \rightarrow C'(1, -3)$ $D(3, 2) \rightarrow D'(2, -3)$
Plot each point to check that the line extending from the origin to a point and the origin to its image forms a right-angle.	

SC 2 I can identify and describe combined transformations

Two or more individual transformations can be applied to the same object.

These transformations may be any combination of translation, reflection or rotation.

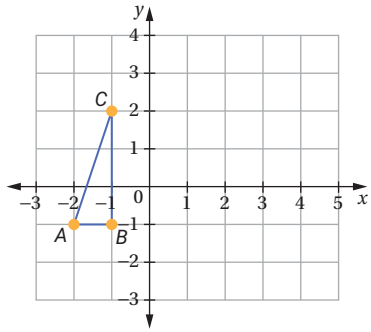
A rotation of an object around a point other than the origin uses a combination of translation and rotation.



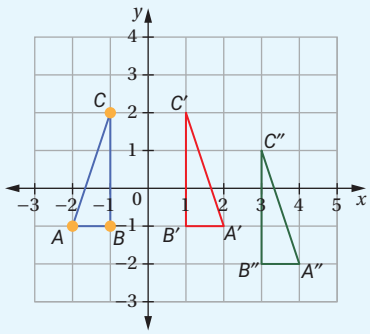
Worked example

Performing and describing combined transformations of shapes

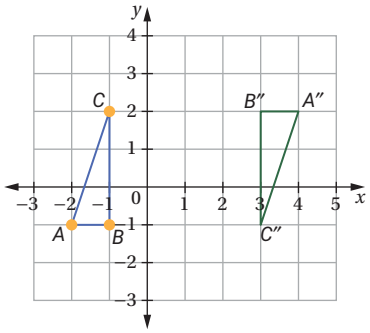
The triangle ABC is given below.

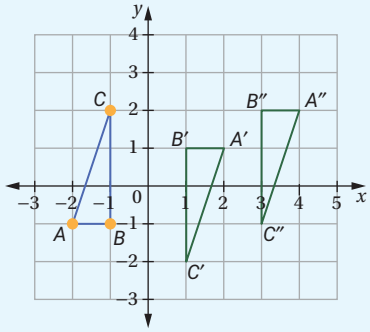


(a) Perform a reflection in the y-axis followed by a translation of $[2, -1]$ on ABC.

THINKING	WORKING
Reflection in the y-axis: move each point horizontally across the mirror line, labelling the result $A'B'C'$. Translation of $[2, -1]$: move each point on the first image 2 units to the right and 1 unit down, labelling the result $A''B''C''$.	

(b) ABC has been rotated about the origin, then translated to give the final result shown. Describe both of the transformations.

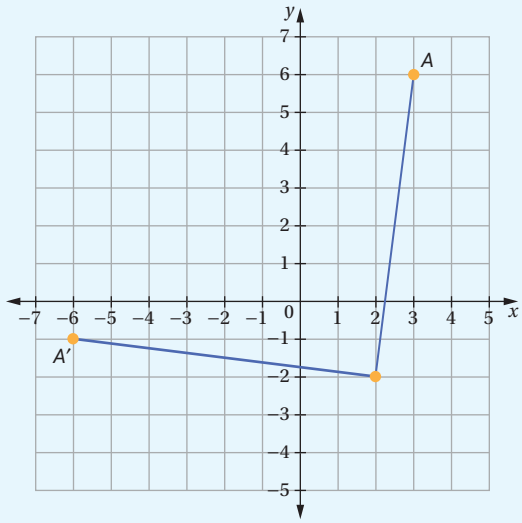


THINKING	WORKING
Use horizontal or vertical lines to determine the amount of the rotation.	BC and $B''C''$ are both vertical, so the rotation was 180° about the origin.
Sketch $A'B'C'$ using B' as a starting point.	
Use $B' \rightarrow B''$ to describe the translation.	The first image moves right 2 units and up 1 unit to reach the second image, so the translation is $[2, 1]$.

Worked example

Rotating shapes on a Cartesian plane about a point other than the origin

Rotate the point $A(3, 6)$ 90° anticlockwise about the point (2, –2).

THINKING	WORKING
What translation is required to move the point of rotation to the origin?	To translate (2, –2) to the origin, a translation of [–2, 2] is required.
Translate the point A in the same way.	A becomes (1, 8).
Perform the required rotation on this new point.	The point becomes (–8, 1).
Now undo the translation that was done in the first step.	The translation required is [2, –2]. The point becomes (–6, –1).
Show this on a set of Cartesian axes.	

Practice

ANSWERS Page 721

SC 1 I can describe and perform rotations of shapes on a Cartesian plane

1 Plot and rotate each point about the origin:

(i) 90° anticlockwise

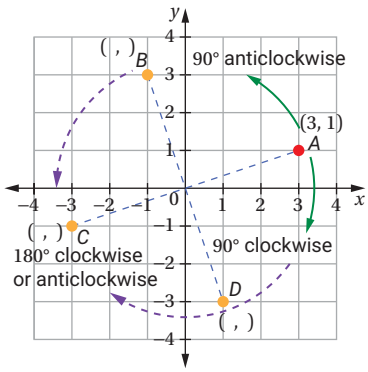
(ii) 180°

(iii) 270° anticlockwise.

(a) $A(6, -3)$ **(b)** $B(-4, 1)$ **(c)** $C(-5, -2)$ **(d)** $D(3, 0)$ **(e)** $E(0, -4)$

2 Plot and rotate the line AB, with $A(4, 2)$ and $B(3, -2)$, 180° about the origin.

3 Point A is rotated 90° anticlockwise around the origin to give the image point B, rotated 90° clockwise to give the image point D and rotated 180° to point C. Determine the coordinates of points B, C and D.



4 How do the coordinates of the points change when rotated? How does the rotation change the positions in the different quadrants on the plane? Complete the following sentences to explain.

(a) When a point is rotated 90° clockwise around the origin, the point is transformed to _____

(b) When a point is rotated 90° anticlockwise around the origin, the point is transformed to _____

(c) When a point is rotated 180° around the origin in either direction, the point is transformed to _____

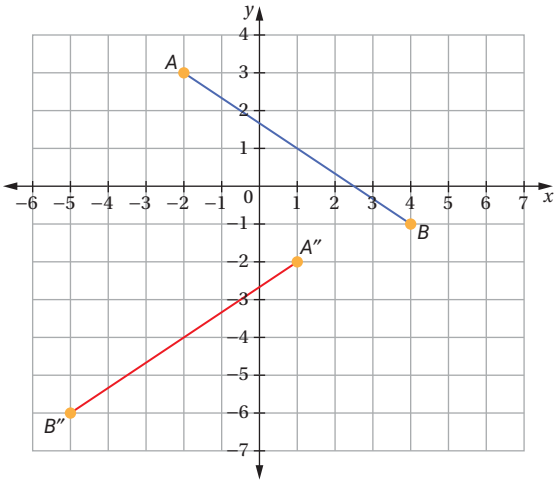
5 Rotate the point $A(3, 5)$ as directed.

(a) 90° clockwise about (0, 0) **(b)** 180° about (0, 0)

SC 2 I can identify and describe combined transformations

1 Triangle ABC has $A(-1, 2)$, $B(2, 3)$, $C(3, -1)$.
Sketch the triangle on a Cartesian plane, translate the figure [3, −1], then rotate this image 90° clockwise about the origin. Label all three triangles appropriately.

2 Line AB has been translated to $A'B'$, then this image has been reflected in an axis to $A''B''$.

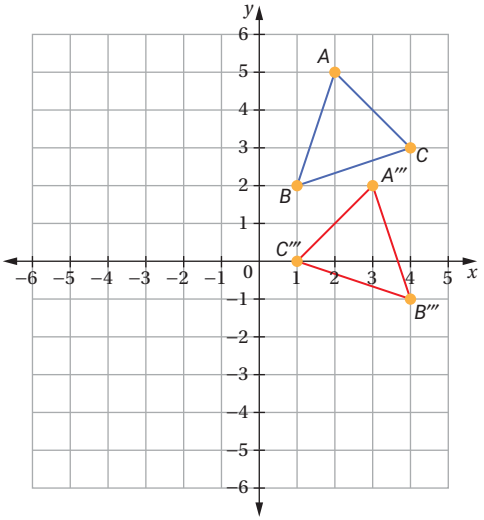


(a) Add the first image, $A'B'$, to the Cartesian plane.

(b) Describe the translation and reflection.

3 Triangle ABC has $A(-1, 1)$, $B(-1, -2)$, $C(0, 0)$.
Sketch the triangle on a Cartesian plane, rotate the triangle 90° anticlockwise about (−1, −2), reflect this image in the x-axis, then translate the previous image [4, −4]. Label all four triangles appropriately.

4 Triangle ABC has been reflected in the x-axis, translated [−5, 3] and then rotated about the origin to end up as $A'''B'''C'''$.



(a) Copy the image onto a Cartesian plane and add the first and second images.

(b) Determine the centre, amount and direction of the rotation.

TOPIC REVIEW

Multiple choice

ANSWERS Page 749

1 Describe the translation given by [4, –3].

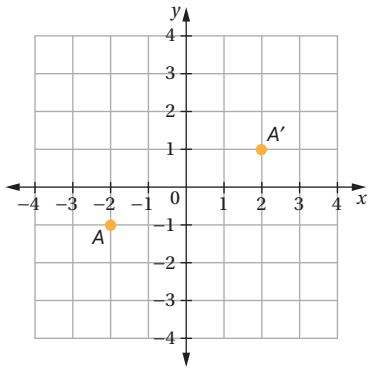
A 4 units left, 3 units down

B 4 units left, 3 units up

C 4 units right, 3 units down

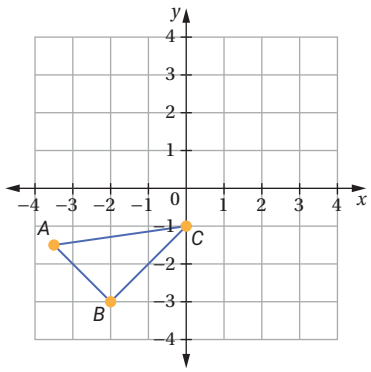
D 4 units right, 3 units up

2 Choose the correct translation when point A is translated to the image A'.



A [4, 2] **B** [4, –2] **C** [2, 4] **D** [–2, –4]

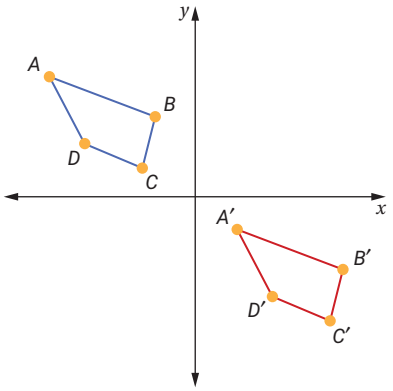
3 The shape shown is reflected in the x-axis. Determine the coordinates of point B'.



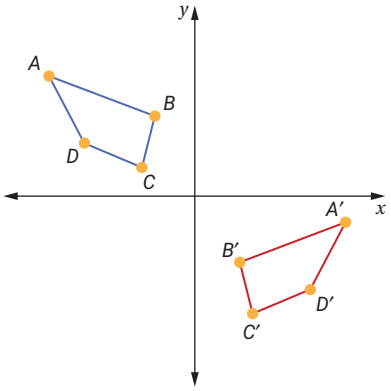
A (–2, –3) **B** (2, –3) **C** (2, 3) **D** (–2, 3)

4 Select the diagram that correctly shows the quadrilateral $ABCD$ reflected in both the x- and y-axes to produce the image $A'B'C'D'$.

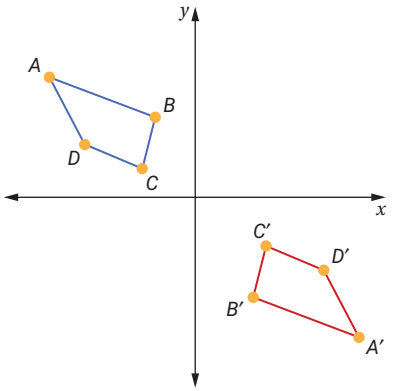
A



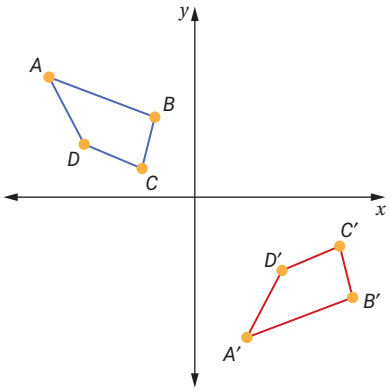
B



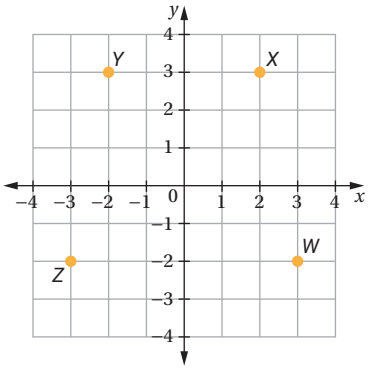
C



D



5 Choose both of the options that are correct for this diagram. All rotations are about the origin.



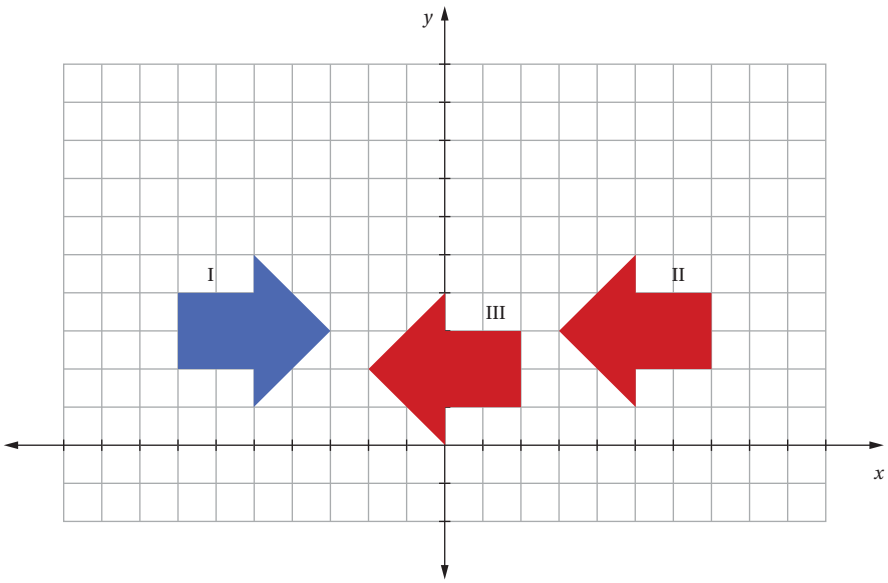
A Y is rotated 90° clockwise to give an image X.

B Y is rotated 90° anticlockwise to give an image Z.

C W is rotated 180° to give an image Y.

D X is rotated 90° clockwise to give an image W.

6 The arrow images II and III have been transformed from the original arrow I. Which of the following statements could be true?



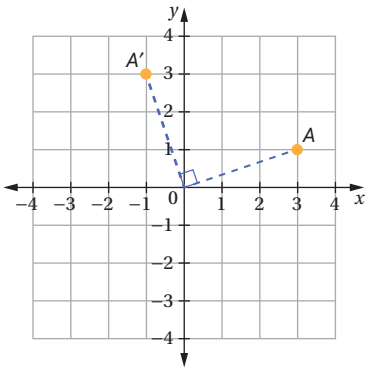
A II is a reflection of I in the y-axis.

B II is a reflection of I in the x-axis.

C III is a translation of I.

D III has been made by reflecting I in the x-axis and then translating II.

7 Point A is transformed to the image point A'. More than one transformation could have been used. Which transformation does NOT move A to A'?



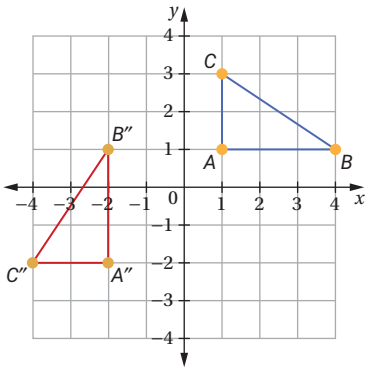
A Translate [−4, 2]

B Rotate 90° clockwise around the origin

C Rotate 90° anticlockwise around the origin

D Rotate 270° clockwise around the origin

8 Which sequence of transformations will change ΔABC to $\Delta A''B''C''$?



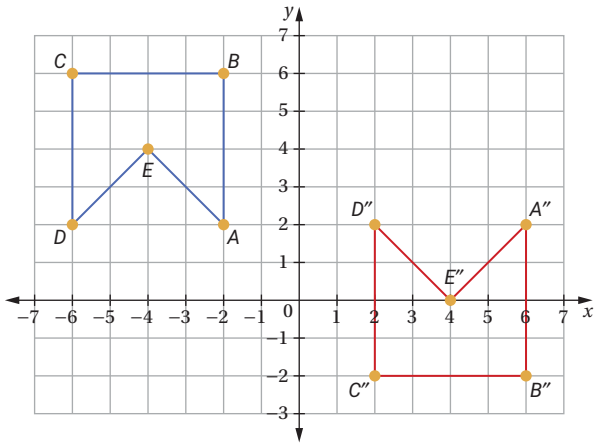
A Reflection in the y-axis, then the translation [–1, –3]

B Rotation of 90° anticlockwise about (1, 1), then the translation [–3, –3]

C Rotation of 90° clockwise about the origin, then a reflection in the x-axis

D Reflection in the x-axis, then the translation [–3, –1]

9 Which sequence of transformations will change $ABCDE$ to $A''B''C''D''E''$?



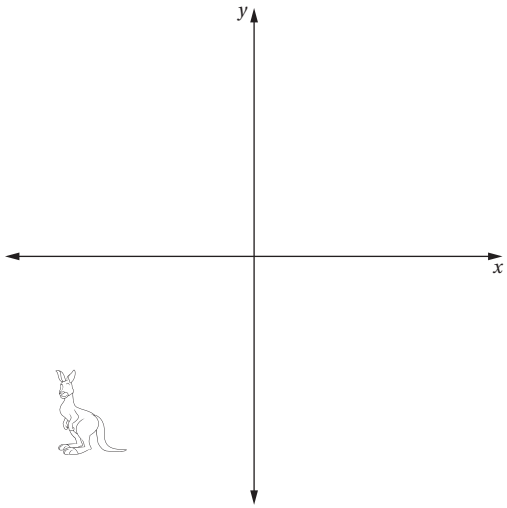
A Reflection in the y-axis, then the translation [0, 2]

B Rotation of 90° clockwise about (–2, 2), then the translation [2, 0]

C Rotation of 180° about the origin, then the translation [0, 2]

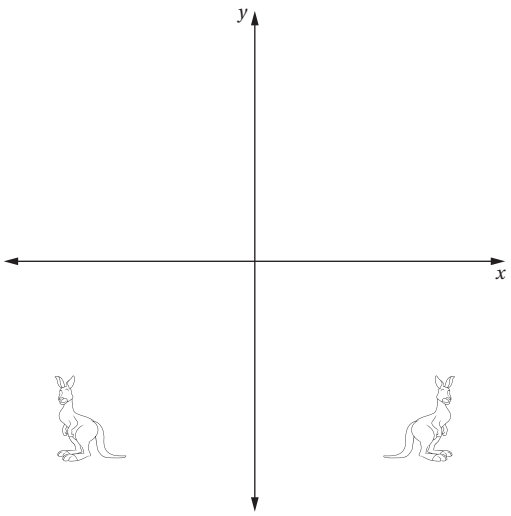
D Reflection in the x-axis, then the translation [8, 4]

10 A kangaroo is located at a point in the third quadrant as shown.

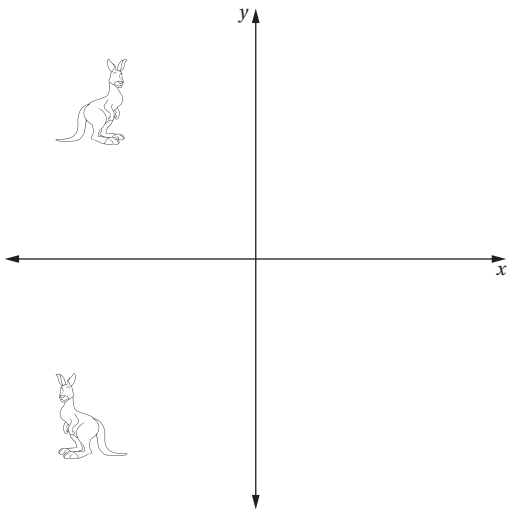


Which diagram correctly shows the kangaroo when reflected in the horizontal axis?

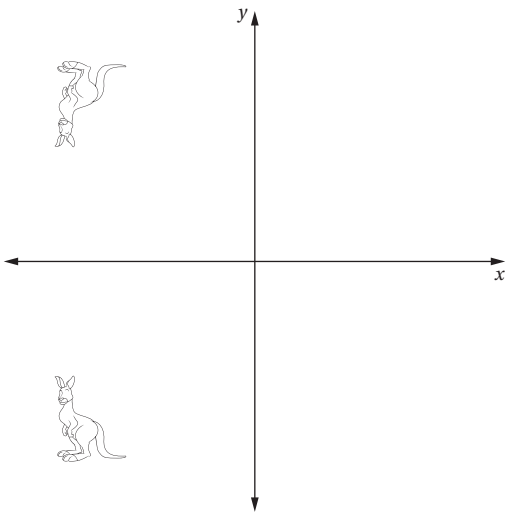
A



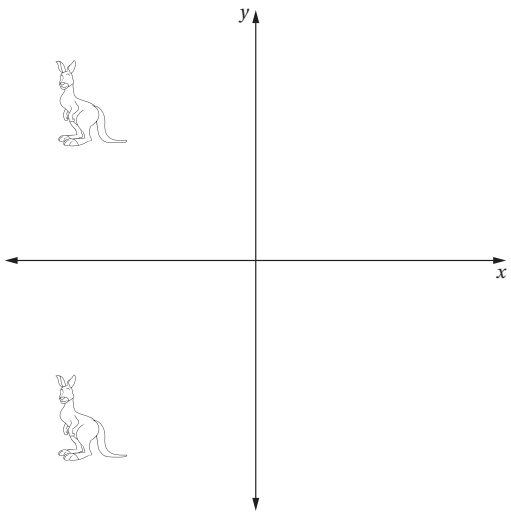
B



C



D



Short answer

1 Determine the image of the given points after the given translations.

(a) Point $M(-3, -1)$ after $[5, -1]$

(b) Point $A(3, -1)$ after $[-4, 3]$

(c) Point $P(2, -5)$ after moving 8 units left and 5 units up

2 Determine the image of the given points after the given rotations around the origin.

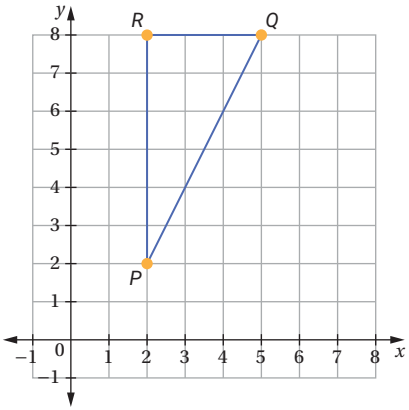
(a) Point $G(-2, 4)$ after a 90° anticlockwise rotation

(b) Point $M(4, -3)$ after a 90° clockwise rotation

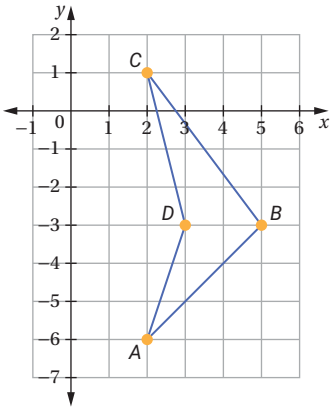
(c) Point $F(3, 7)$ after a 180° rotation

3 Plot the figure and its image after reflection in the given axis.

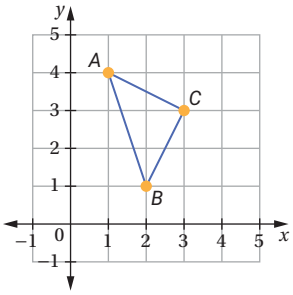
(a) The horizontal axis



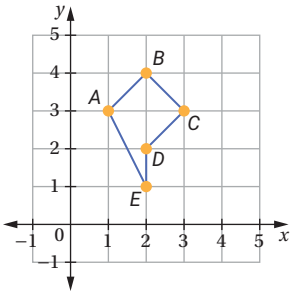
(b) The vertical axis



4 Plot the figure and its image after a translation of [−4, −3].



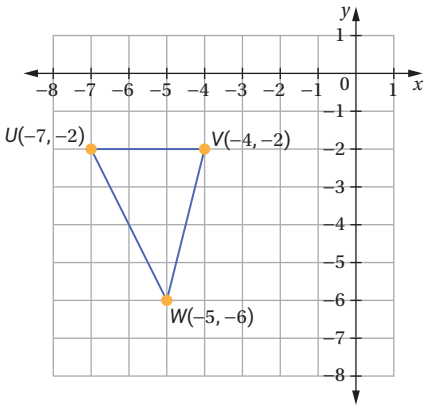
5 Reflect concave pentagon $ABCDE$:



(a) in the x-axis

(b) in the y-axis.

6 Determine the vertices of the images of triangle UVW after each of the following rotations.

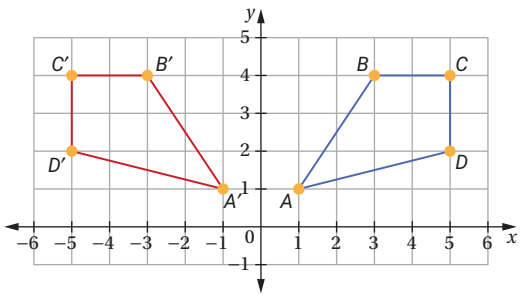


(a) 180° around the origin

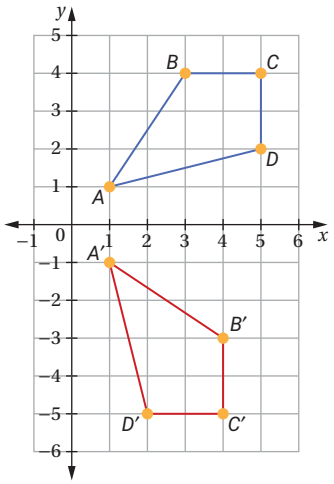
(b) 90° clockwise around the origin

7 For each of the following diagrams, describe the transformation that has been performed on quadrilateral $ABCD$ to produce the image $A'B'C'D'$.

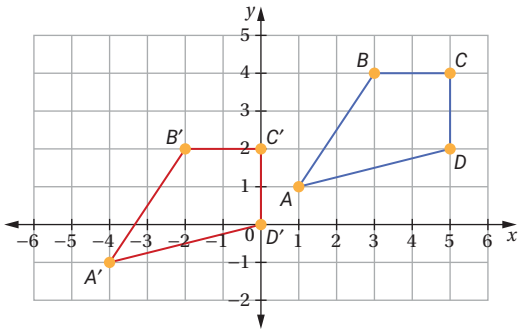
(a)



(b)

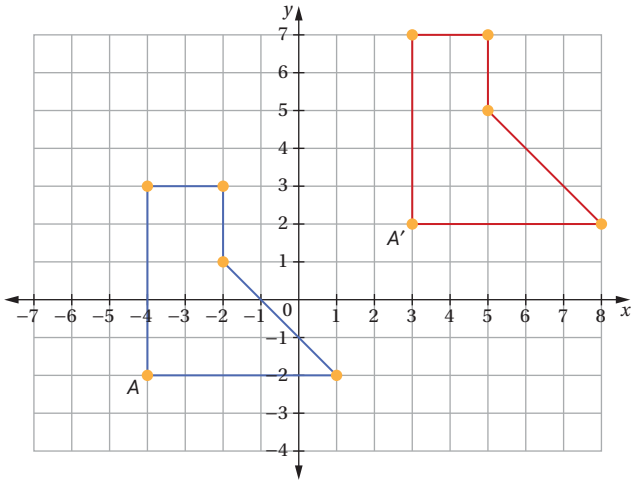


(c)

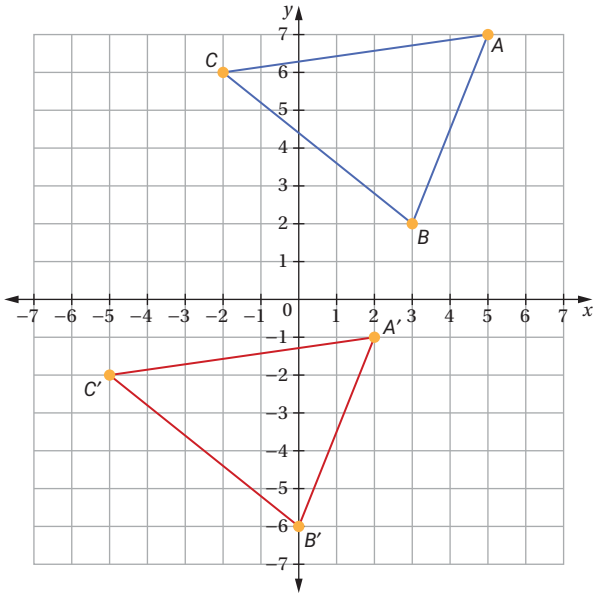


8 Describe each of the following translations using ordered pair notation.

(a)

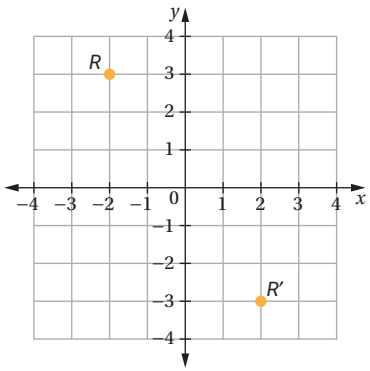


(b)

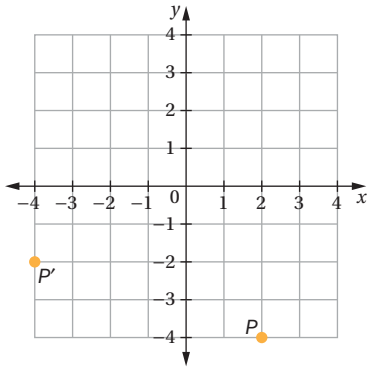


9 Describe each rotation about the origin in terms of amount and direction.

(a)



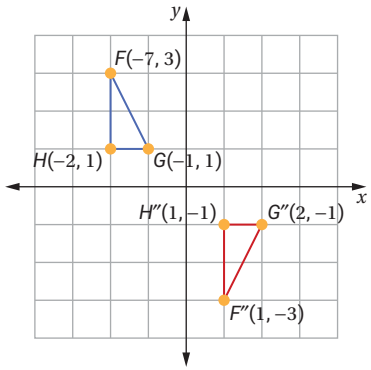
(b)



10 Point $X(4, 7)$ is reflected over the horizontal axis to give X', then X' is rotated 90° clockwise to give X''. Determine the coordinates of X''.

11 Point $R(3, 3)$ is translated $[0, -3]$ to give R', then R' is reflected in the y-axis to give R''. Determine the coordinates of R''.

12 Triangle FGH is translated to triangle $F'G'H'$, then the image is reflected in the x-axis to give triangle $F''G''H''$.



Determine the details of the translation, expressing the answer as a coordinate pair.

13 A quadrilateral $QRST$ with vertex points $Q(-2, 3)$, $R(-1, 2)$, $S(-2, 0)$, $T(-3, 2)$ is reflected in an axis, then its image is translated $[1, 4]$, so that the final image of Q is $Q''(-1, 1)$.

(a) Determine in which axis the reflection occurred.

(b) Determine the coordinates of all vertices of the first image after the reflection.

(c) Determine the coordinates of R'', S'' and T''.

Extended response

1 For the quadrilateral $ABCD$ with $A(-2, 5)$, $B(1, 4)$, $C(1, 3)$, $D(-1, 3)$ and each of the following transformations:

(i) write the coordinates of each vertex of $A'B'C'D'$

(ii) map $ABCD$ to $A'B'C'D'$ on a Cartesian plane

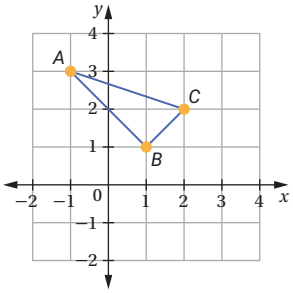
(iii) describe the transformation that would map $A'B'C'D'$ back to $ABCD$.

(a) The translation $[5, -2]$

(b) A reflection in the x-axis

(c) A clockwise rotation of 90° about the point $(-1, 1)$

2 Triangle ABC is drawn on a Cartesian plane. Copy the figure onto a Cartesian plane that goes from -2 to 4 along the horizontal axis and from -3 to 4 along the vertical axis.



(a) Perform all transformations on the same Cartesian plane.

(i) Reflect each vertex in the x-axis to form the image triangle $A'B'C'$.

(ii) Translate the image $A'B'C'$ 2 units right and 3 units up to form triangle $A''B''C''$.

(iii) Rotate $A''B''C''$ 90° anticlockwise about the origin to form triangle $A'''B'''C'''$.

(b) Write a sequence of transformations that would map $A'''B'''C'''$ back to ABC.

TOPIC
19

Data and measures of central tendency

Recall 537
19.1 Understand different data types 538
19.2 Calculate the mean and range of a set of data 549
19.3 Calculate the mean and range for a set of grouped data 556
19.4 Determine the median of a data set 564
19.5 Determine an appropriate measure of central tendency 570
Topic review 576

Why learn this?

Understanding different types of data is pivotal in today's information-driven environment. This topic provides:

- an understanding of different data types, which is essential in research and data collection processes and critical for fields such as marketing, psychology and health sciences
- the ability to interpret and calculate with data, which is crucial in sectors like finance and engineering.

These skills are essential when interpreting and evaluating data, supporting informed decision-making in academic research, business strategies and everyday problem-solving. They also lay the groundwork for further study in statistics and data science.

RECALL

ANSWERS Page 724

I can order numbers by size

1 Arrange each set of numbers in order from smallest to largest.

(a) 9, 12, 3, 18, 6

(b) 2.2, 1.4, 1.8, 1.2, 2.0, 2.4

(c) $4\frac{1}{2}, 2\frac{1}{2}, 1, 3, 5\frac{1}{2}, 1\frac{3}{4}$

(d) $5, 3\frac{1}{2}, 2.5, 0.5, 1\frac{2}{3}, 6.5, 3, 3\frac{1}{4}, 1$

I can determine the range of scores in a data set

1 Determine the range of the values in each set.

(a) 9, 12, 3, 18, 6

(b) 2.2, 1.4, 1.8, 1.2, 2.0, 2.4

(c) $4\frac{1}{2}, 2\frac{1}{2}, 1, 3, 5\frac{1}{2}, 1\frac{3}{4}$

(d) $5, 3\frac{1}{2}, 2.5, 0.5, 1\frac{2}{3}, 6.5, 3, 3\frac{1}{4}, 1$

I can determine the score with the highest frequency (mode) in a set of data

1 Find the mode of each of the following sets of numbers

(a) 1, 1, 1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4, 6, 6, 7, 8

(b) 3, 4, 2, 1, 3, 6, 7, 3, 3, 2, 1, 4, 3, 5, 8, 9, 8, 8, 8

(c) 3, 2, 4, 6, 8, 5, 3, 2, 8, 6, 5, 4, 8, 5, 8

Exploration Task

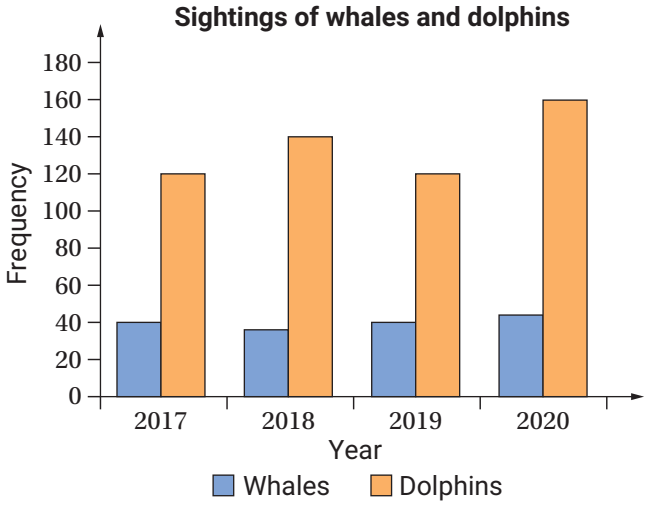
Whales and dolphins

Off the coast of New South Wales, companies operate whale-watching tours. Sightings for whales and dolphins were reported over the years 2017 to 2020 using a grouped bar graph.

(a) How many whales were sighted in 2018?

(b) How many dolphins were sighted in 2019?

(c) How many aquatic mammals (whales and dolphins combined) were sighted in 2020? Explain how you worked this out.



19.1 Understand different data types

Learning intention: To be able to understand different data types

Success criteria:

SC 1 I can classify data as primary or secondary.

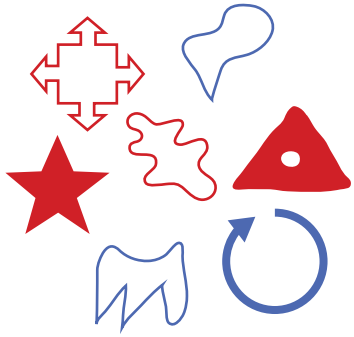
SC 2 I can classify data as categorical or numerical.

SC 3 I can classify numerical data as either continuous or discrete.

Lesson warm-up

Sorting images

Sort these images into the categories given.



Images generated by computer	Images drawn by hand

Are there other ways you could have sorted the images?

SC 1 I can classify data as primary or secondary

Data can be obtained in number of different ways and can be classified into one of two different source types: primary or secondary.

Primary data

Primary data is data collected by yourself or another researcher for a specific purpose via questionnaires, observations, surveys etc.

The data/information you gather comes directly from the sample/population being studied.

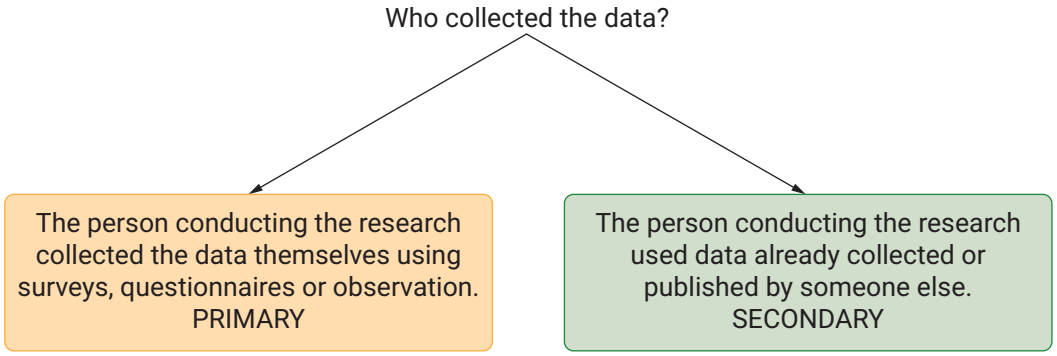
For example, you want to study the impressions of Youth Group members about a recent camp. Since the population is small, available and convenient, conducting a census is the most effective method. As this is data that you collect yourself, it is classified as primary data.

Secondary data

Secondary data is data someone else has collected for a specific purpose; for example, population data presented by the Australian Bureau of Statistics (ABS) or online news sites.

The data/information was not collected directly by you, but was collected or published by someone else.

For example, you may wish to investigate the average income of working-age people in countries across the world. This data is much more difficult to collect by yourself. The population is not available to you, and it is not practical to conduct this survey yourself. You would therefore need to use secondary data, gathered by other researchers. Provided this secondary data comes from reliable sources, it presents a much more realistic option.



Primary data may be more relevant, but can be impractical or impossible to collect.

For example, determining Australian citizens' average income, or investigating the average temperature in a region over a ten-year period may cost too much or take too long to do yourself.

Not all secondary data sources (such as websites) include reliable information, so it is important to check that these data sources can be verified.

Worked example

Classifying data as primary or secondary

Would you use primary or secondary data to research the following?

(a) Voting on ideas for your class's fundraising activity

THINKING	WORKING
Can you collect the data yourself easily?	Yes. The population is small, convenient and you can collect the data yourself.
Recall the definition of primary data and secondary data.	Primary data is data you collect yourself.
Answer the question.	To research voting on ideas for your class's fundraising activity, you would use primary data.

(b) The remaining numbers of endangered species

THINKING	WORKING
Can you collect the data yourself easily?	No. You cannot collect the data yourself.
Recall the definition of secondary data and primary data.	Secondary data is data already collected/ published by another person or team that will help you with your research.
Answer the question.	To research the remaining numbers of endangered species, you would use secondary data.

(c) The number of road accidents last month in Washington

THINKING	WORKING
Can you collect the data yourself easily?	No. You cannot collect the data yourself.
Recall the definition of secondary data and primary data.	Secondary data is data already collected/ published by another person or team that will help you with your research.
Answer the question.	To research the number of road accidents last month in Washington, you would use secondary data.

(d) The opinions of students in your school about new uniform options

THINKING	WORKING
Can you collect the data yourself easily?	The population is convenient, and it is unlikely the data already exists, so you can collect the data yourself. Therefore, you would use primary data.
Recall the definition of primary data and secondary data.	Primary data is data you collect yourself.
Answer the question.	To research the opinions of students in your school about new uniform options, you would use primary data.

SC 2 I can classify data as categorical or numerical

Sourced data can be further classified into one of two types: categorical or numerical.

Categorical data

The data collected is usually able to be given in words. The frequency or number of times the response is given can be counted, then sorted into groups (or categories) with a name or label. Each bit of data sourced or counted can only be placed into one category because they do not overlap.

For example, "Which is your preferred social media platform?"

The votes for each different platform in the survey are tallied/counted, and the results for each platform (or category) are presented/displayed.

Categorical data can be displayed using bar charts, pie charts, column graphs or frequency tables.

Summary statistics, such as calculating the average, are not used for categorical data.

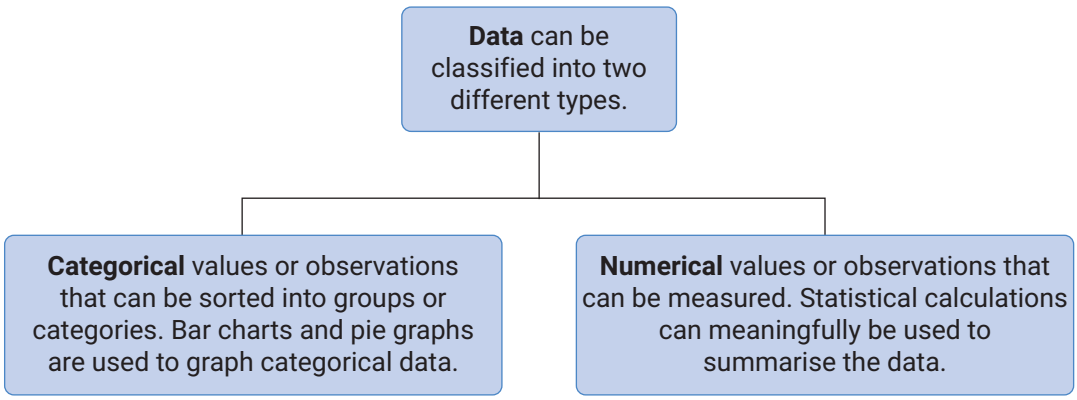
Numerical data

The data collected is given in numbers. Numerical data can be whole numbers, decimals or fractions. Values can overlap and be organised into groups called a range. Numerical data is data collected by counting or measuring.

For example, "Measure the heights of a class of students" or "State the number of family members that you live with."

The height measurements for each student are collected and the results are displayed using line graphs, stem-and-leaf plots, box-and-whisker plots or frequency tables.

Summary statistics, such as finding the average, middle value, range, etc., are useful for interpreting numerical data.



Worked example

Classifying data as categorical or numerical

Classify each of the following as categorical or numerical data.

(a) Eye colour

THINKING	WORKING
Describe the data collected.	Eye colours are represented by categories such as blue, brown, grey, hazel and green.
Recall the definition for categorical and numerical data.	Categorical data can be organised into categories or groups.
Answer the question.	Eye colour is categorical data.

(b) Brand of mobile phone

THINKING	WORKING
Describe the data collected.	There are many brands of mobile phones, with each brand name representing a category.
Recall the definition for categorical and numerical data.	Categorical data can be organised into categories or groups.
Answer the question.	Brand of mobile phone is categorical data.

(c) Daily road accident figures for the previous month

THINKING	WORKING
Describe the data collected.	The data is measured, and both the data and frequency are numbers.
Recall the definition for numerical and categorical data.	Numerical data is data given as numbers.
Answer the question.	Daily road accident figures for the previous month are numerical data.

(d) Petal lengths of flowers on a tree

THINKING	WORKING
Describe the data collected.	The data is measured, and both the data and frequency are numbers.
Recall the definition for numerical and/or categorical data.	Numerical data is data given as numbers.
Answer the question.	Petal lengths of flowers on a tree are numerical data.

SC 3 I can classify numerical data as either continuous or discrete

Numerical data is data in the form of numbers.

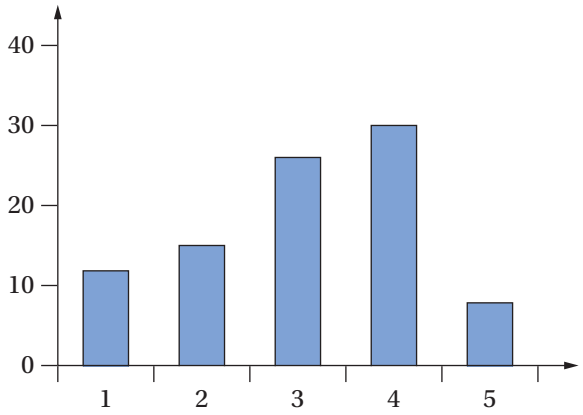
This data can be measured and it can be further split into two types: discrete and continuous.

Discrete numerical data

This is numerical data where there are a finite number of possible values over a given range. Generally, discrete data is collected by counting.

Example: The number of children in a family. This data is discrete because you can have 2 children or 3 children – but you cannot have 2.3 children, only whole numbers.

This column chart is useful for displaying discrete numerical data.



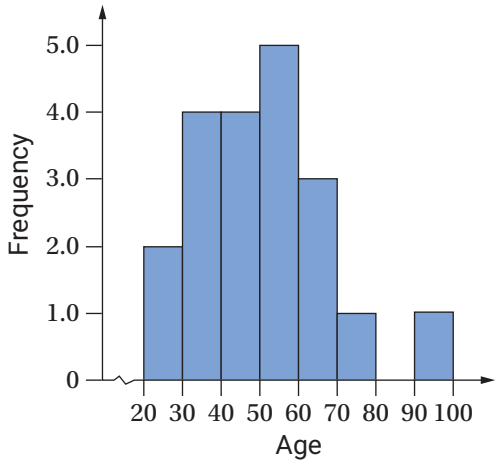
This data is discrete and there is space between the vertical columns. The values for this data can only be 1, 2, 3, 4 or 5; there are no decimal or fractional values in between. These charts can also be used for categorical data.

Continuous numerical data

This is numerical data where all measured values are possible.

Example: the weight of packets of flour. This data is continuous and each bag can have slightly different decimal weights.

This histogram is useful for displaying continuous numerical data.



This data is continuous, and any value is possible within the range represented by each column. The age can be any fraction or decimal value.

The frequency represents how many people are counted within each ten-year age-group range.

Worked example

Classifying numerical data as discrete or continuous

For each of the following examples, classify the numerical data as discrete or continuous.

(a)

Score (x)	Frequency (f)
5	3
10	12
15	8
20	2

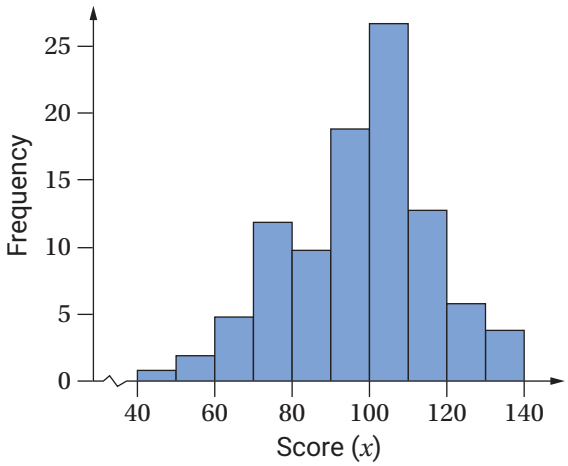
THINKING	WORKING
Describe the data.	The data is restricted to the integer scores 5, 10, 15, 20.
Recall the definition of discrete and continuous data.	Discrete data has a defined number of integer outcomes.
Write the answer.	The data is discrete.

(b)

Score (x)	Frequency (f)
5–<10	3
10–<15	12
15–<20	8
20–<25	2

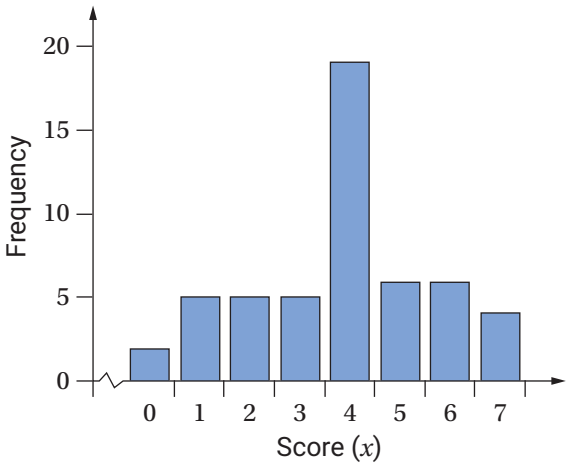
THINKING	WORKING
Describe the data.	The data values can be any fraction or decimal within each given range of scores.
Recall the definition of continuous and discrete data.	Continuous data can have an unlimited number of possible outcomes.
Write the answer.	The data is continuous.

(c)



THINKING	WORKING
Describe the data.	There are no gaps between the columns that each represent a range of data values, including fractions or decimals.
Recall the definition of continuous and discrete data.	Continuous data can have an unlimited number of possible outcomes.
Write the answer.	The data is continuous.

(d)



THINKING	WORKING
Describe the data.	The separate columns indicate that they represent only the data values 0, 1, 2, 3, 4, 5, 6, and 7 respectively.
Recall the definition of discrete and continuous data.	Discrete data has a defined number of integer outcomes.
Write the answer.	The data is discrete.

Practice

ANSWERS Page 724

SC 1 I can classify data as primary or secondary

1 Raj plans to holiday in Timbuktu, Mali, and searches the internet for historical data about the average maximum daily temperatures there in January. Is this collected data from a primary or secondary source?

2 Students in Adit's class are planning the end-of-year school celebration. They research what type of celebration could be successful.
State whether each of the items below represent primary or secondary data collection.

(a) Conduct a survey of the students in the same year level.

(b) Research types of celebrations and their level of success on the internet.

(c) Survey parents and teachers about what they think is a good alternative.

(d) Conduct a social media poll asking "What type of celebration would work?"

3 The Conservation Department wants to know how many bilbies are living in a particular national park. Should they look for primary or secondary data? Explain your choice.

4 Jeri is negotiating a new employment contract. It would be useful to know the contracts of other employees in a similar role before beginning negotiations.

(a) Give an example of primary data that Jeri could collect and discuss any potential problems with collecting this type of data.

(b) Give an example of secondary data that Jeri could use and discuss any potential problems with collecting this type of data.

5 Research on the number of frogs living in a large lake is conducted. Since it is not possible to count every frog, the researchers count the number of frogs in various areas.
An average of 22.3 frogs were counted across all the areas. This is lower than the average of 76.5 frogs from a similar survey of the same areas from a few years ago.
Identify which data type each frog count represents.

6 Explain the difference between primary and secondary data, giving an example of each type.

SC 2 I can classify data as categorical or numerical

1 Sofi is planning a skiing trip and uses the resort's website to collect the daily snow-depth data for the previous two years. Is the data Sofi collected categorical or numerical?

2 Traci's store sells shoes, handbags and T-shirts. To find the best-selling item in the store, Traci conducted a survey over one day.
The survey results are summarised in this frequency table.

Line	Sales frequency (f)
Shoes	5
Handbags	3
T-shirts	8

(a) Is the data collected categorical or numerical? Explain your choice.

(b) How could Traci visually display the data?

3 Consider this frequency table showing the total number of each colour chosen in a survey.

Colour chosen	Frequency (f)
Red	18
Blue	21
White	38
Black	36

(a) Maxine argues that because there are numbers, the data is numerical. Is this correct? Why or why not?

(b) The mean of the frequencies is 28.25. Does this help us understand the data better? Why or why not?

4 Identify the type of data (categorical or numerical) each of the following will collect.

(a) A survey of Year 8 students to determine the mean height

(b) A survey of Year 8 students to identify their favourite sports team

(c) The daily pH level of a swimming pool

(d) Using the Australian Census data to determine the average income for Australian workers

(e) Using Bureau of Meteorology data to determine the average minimum temperature in March

(f) Counting the type of cars parked in the school car park

5 Explain the difference between categorical and numerical data, providing an example for each.

SC 3 I can classify numerical data as either continuous or discrete

1 Gemma's store collects data to help make decisions about future stock purchases and improve efficiency in operations. Classify the type of numerical data (discrete or continuous) each of these surveys will collect.

(a) The number of ice creams sold

(b) The electricity bills each month

(c) The number of sales during one hour

(d) The length of time a salesperson spends with each customer

(e) The volume of music played in the store

2 A six-sided die was rolled 150 times and the results were recorded in this table.

Number rolled (x)	Frequency (f)
1	18
2	20
3	32
4	26
5	24
6	30

(a) Is the data collected numerical data? Explain your answer.

(b) Is the data collected discrete or continuous? Explain your choice.

3 Are the following statements true (T) or false (F)? Provide an example to support your answer.

(a) Data can be both primary and categorical.

(b) Data can be both secondary and discrete.

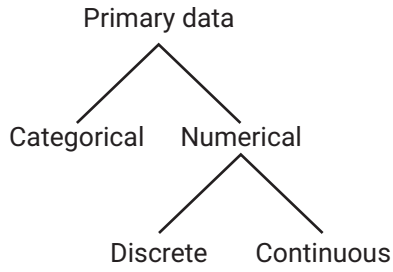
(c) Data cannot be both numerical and discrete.

(d) Numerical data must be discrete.

(e) Categorical data cannot be continuous.

(f) Secondary data can be continuous.

4 Consider the following flow chart.



(a) How does this flow chart help describe the relationship between data types for primary data?

(b) Would this diagram look the same for secondary data?

5 Match each data description to the most accurate category.

(a) Popular shirt sizes

(b) Weight of each pineapple in a box

(c) Number of fish in a fish tank

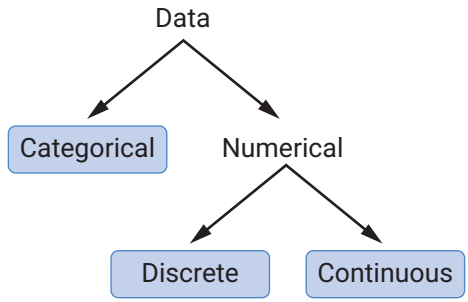
(d) Time for race cars to complete a lap

(e) Quantity of various types of rubbish removed from a river

(f) Number of each peg colour in a basket

(g) Vote results for most popular school canteen item

(h) Dorsal-fin lengths of a pod of dolphins



Calculate the mean and range of a set of data

19.2

Learning intention: To be able to calculate the mean and range of a set of data

Success criteria:

SC 1 I can calculate the range.

SC 2 I can calculate the mean.

SC 3 I understand the impact on mean and range with extreme values.

Lesson warm-up

Why is it funny?

An English lecturer told this joke at the opening of a Maths Teachers Conference held at a university.

"It's lovely to see so many maths teachers here on campus. It certainly lifts the average IQ of the University...

Of course, without your presence in schools the average IQ has lifted there too."

Why is the joke funny? What was the lecturer implying?

SC 1 I can calculate the range

In statistics, it is useful to know how spread out collected values are for a set of raw data.

Calculating the range is one method used to determine that spread.

Range = highest value (maximum) – lowest value (minimum)

Consider these two sets of data.

Set A: 4, 3, 4, 8, 7, 10

Set B: –4, –2, –3, –6, –7, –38

Arrange the data values in ascending order.

Set A: 3, 4, 4, 7, 8, 10
The lowest value (minimum) is 3 and the highest value (maximum) is 10. Therefore, the range of values in Set A is $10 - 3 = 7$

Set B: –38, –7, –6, –4, –3, –2
The lowest value (minimum) is –38 and the highest value (maximum) is –2. Therefore, the range of values in Set B is $-2 - (-38) = 36$

Comparing the ranges, Set B > Set A.

This means the raw data in Set B is more spread out than the raw data in Set A.

Worked example

Calculating the range

Determine the range of these data sets.

(a) 4, 8, 2, 15, 22, 31, 17, 17, 8, 4

THINKING	WORKING
Identify the highest (maximum) and lowest (minimum) values.	The highest value is 31. The lowest value is 4.
Calculate the range by subtracting the lowest from the highest.	31 – 4 = 27
Answer the question.	The range of the data set is 27.

(b) 12.4, 9.5, –4.5, –6.2, 3.8, 7.2, 0, 15.7, 3, 1.5

THINKING	WORKING
Identify the highest (maximum) and lowest (minimum) values.	The highest value is 15.7. The lowest value is –6.2.
Calculate the range by subtracting the lowest value from the highest value.	15.7 – (–6.2) = 21.9
Write the answer.	The range of the data set is 21.9.

SC 2 I can calculate the mean

There are three calculations that measure the centre for a set of raw data: mean, median and mode.

In this section, you will be learning about the mean.

The mean score requires a calculation that is like finding the average. It is denoted using the symbol $\bar{x}$.

The mean $(\bar{x})$ is the sum of n values or scores in a data set, divided by the number of values (n) in the data set.

$$\text{Mean}(\bar{x}) = \frac{\text{sum of } n \text{ values}}{n}$$

For example, consider the mean of the data values or scores 5, 10 and 24. There are 3 scores, so $n = 3$.

Therefore, the mean is:

$$\bar{x} = \frac{5+10+24}{3}$$
$$= \frac{39}{3}$$
$$= 13$$

This is equivalent to sharing the sum of 39 equally between the 3 groups or pieces of data.

Worked example

Calculating the mean

Jayde recorded these scores playing a favourite online game.

450 520 540 610 640

What is Jayde's mean score?

THINKING	WORKING
Recall the formula used to calculate mean.	$\text{Mean}(\bar{x}) = \frac{\text{sum of } n \text{ values}}{n}$
Substitute the data values (scores) and the number of data values into the formula to calculate the mean.	$\bar{x} = \frac{450+520+540+610+640}{5}$ $= \frac{2760}{5}$ $= 552$
Write the answer.	Jayde's mean score is 552.

Worked example

Determining a missing data value when the mean is known

The number of seeds in each handful drawn from a packet were counted.

23 18 20 24 22 19

How many seeds need to be in the next handful for the mean number of seeds to be 22?

THINKING	WORKING
Identify the values given in the question.	Mean, $\bar{x} = 22$ seeds The number of data values, $n = 6 + 1 = 7$ The number of seeds collected in the seventh handful is unknown, a.
Substitute the data values into the formula.	$\text{Mean}(\bar{x}) = \frac{\text{sum of } n \text{ values}}{n}$ $22 = \frac{23+18+20+24+22+19+a}{7}$ $22 = \frac{126+a}{7}$ $22 \times 7 = 126 + a$ $154 = 126 + a$

Calculate the value of the unknown.	154 seeds need to be collected in total; 126 seeds were collected in the first six handfuls. Difference: 154 – 126 = 28 seeds
Write the answer.	To ensure a mean count of 22 per handful, the handful must contain 28 seeds.

SC 3 I understand the impact on mean and range with extreme values

Consider these two sets of data.

Set A: 1, 1, 1, 1, 2, 2

Set B: 1, 1, 1, 1, 2, 400

Set	Mean $(\bar{x})$	Range
Set A 1, 1, 1, 1, 2, 2	$\bar{x} = \frac{\text{sum of } n \text{ values}}{n}$ $= \frac{1+1+1+1+2+2}{6}$ $= 1.33$	2 – 1 = 1
Set B 1, 1, 1, 1, 2, 400	$\bar{x} = \frac{\text{sum of } n \text{ values}}{n}$ $= \frac{1+1+1+1+2+400}{6}$ $= 67.67$	400 – 1 = 399

The only difference between the scores in Set A and Set B is the extreme value 400.

Extreme values such as this are called 'outliers'.

This large value of 400 had an extreme impact on the calculation of the mean and range values.

For this reason, the mean and range are not useful for determining a data set's central value and spread when extreme values (outliers) are present.

Worked example

Understanding the impact of extreme values (outliers)

Calculate the mean and range for Set A and Set B. Explain why the mean is not a useful representation of the centre score for both sets of data.

Set A: 11, 11, 12, 15, 16

Set B: –99, 11, 12, 15, 16

THINKING	WORKING
Calculate the mean of both data sets using the formula.	Set A: $\bar{x} = \frac{\text{sum of } n \text{ values}}{n}$ $= \frac{11+11+12+15+16}{5}$ $= \frac{65}{5}$ $= 13$ Set B: $\bar{x} = \frac{-99+11+12+15+16}{5}$ $= \frac{-45}{5}$ $= -9$
Calculate the range of both data sets.	Set A: Range = 16 – 11 = 5 Set B: Range = 11 – (–99) = 110
Compare the mean and range for each data set and comment on why they are different.	Set A has similar values and no extreme values. So the mean and range are accurate representations of the data. Set B has an extreme value that had a big impact on the calculation of the mean and range. So the mean and range are not accurate representations of the data.

Practice

ANSWERS Page 725

SC 1 I can calculate the range

1 **(a)** Calculate the range for each set of data.

(i) Set A: 1, 2, 3, 4, 5, 6, 7, 8, 9

(ii) Set B: 3, 6, 9, 2, 4, 10, 15, 8, 2

(iii) Set C: –3, 9, –6, 24, 64, –31, 14

(b) Which set has the greatest spread? How do you know?

2 **(a)** Calculate the range for each set of data.

(i) Set A: 2.5, 4.5, 3.0, 0.5, 1.5, 2.0

(ii) Set B: $\frac{1}{2}, \frac{3}{4}, 1\frac{1}{2}, 1\frac{1}{4}, 3\frac{1}{4}, 2\frac{1}{2}$

(iii) Set C: −3.5, 2.5, −2, 3, 0, 4.5, 1, −1.5

(b) Which set has the least-spread scores?

3 Create a data set of five scores with a highest value of 42 and a range of 27.

4 A negative score is added to the set of data shown to give it a range of 18.
6, −1.5, −6, 7.5, 2, 5.5, −4
What is the score?

5 The raw data shown here is missing a score.
8, −3, 6, 11, 4, −1, 0, 5, 10
Given that the range of the scores is 15, what two values could the missing score be?

SC 2 I can calculate the mean

1 Calculate the mean score for each of these data sets.

(a) 1, 2, 3, 4, 5

(b) 1, 2, 3, 4, 5, 6, 7

(c) −10, −12, −13, −15

(d) 8, 12, 17, 9, 5, 15

(e) 102, 120, 240, 300, 450

(f) 1.5, 4.2, 6.3

(g) $3\frac{1}{4}, 4\frac{2}{5}, 5\frac{3}{5}, 6\frac{1}{4}$

2 A packet of ten matchboxes claims that there are 50 matchsticks in each box.
Counting the matches results in a total of 554 matchsticks.
What is the actual mean number of matchsticks in a box for this packet?

3 Byron has shot a long-term mean of 16.4 points per basketball game. Over the last five games, Byron has scored 14, 20, 18, 22 and 12 points. Is this scoring rate higher or lower than expected?

4 Twelve people have a total height of 20.16 m. What is their mean height?

5 A group of people have a total age of 178 years. Their mean age is 8.9 years. How many people are in the group?

6 The lengths (cm) of six fish during a capture-and-release survey were measured.
32.5, 28.2, 30.4, 34.0, 31.5, 29.4
What length would the seventh fish need to be for the mean measurement of all the fish to be 32 cm?

SC 3 I understand the impact on mean and range with extreme values

1 **(a)** Jenna decides that it is not appropriate to calculate the mean and range for this set of data.
5, 6, 8, 4, 14, 210, 10
Explain why you do or do not agree with Jenna.

(b) Jamila decides it is not appropriate to calculate the mean and range for this set of data.
15, 6, 8, 10, 4, 14, 20
Explain why you do or do not agree with Jamila.

2 The following experimental data was measured and recorded at various stages of a chemical reaction.

Temperature	70	72	71	22	74	73

(a) Calculate the mean temperature.

(b) Calculate the range of temperatures.

(c) After checking the results, the measurement of 22 was recorded incorrectly. It was actually 72.

(i) Calculate the new mean temperature.

(ii) Calculate the new range of temperatures.

(d) Comment upon the difference in the two results.

3 Determine whether the statements below are true (T) or false (F), then give an example to show why.

(a) An extreme value has no effect on the mean of an otherwise closely-grouped data set.

(b) The range is more affected than the mean by an extreme value in the data set.

(c) An extreme value must be much greater than the rest of the data.

(d) When the data is ranked (put in order from lowest to highest) the extreme value can be either the highest or lowest value.

Exploration Task

Problems with problems

Some of these problems have more than one correct answer. How can you work with statistics problems that have more than one possible answer? How can the answers be checked? What do the possible answers for each question have in common?

- The mean of five data values is 14, but none of the values is 14. What could the data values be?
- The mean of five values is 10 and the range is 6. What could the values be?
- The mean of four values is 5, the mode (most common value) is 5, and the range is 2. What could the values be?

19.3 Calculate the mean and range for a set of grouped data

Learning intention: To be able to calculate the mean and range for a set of grouped data

Success criteria:

SC 1 I can determine the mean and range from a frequency table.

SC 2 I can determine the mean and range from grouped data.

Lesson warm-up

Library audit

How could you arrange the books on the shelf to display, organise and count them easily?

SC 1 I can determine the mean and range from a frequency table

When a data set contains a large number of scores, a frequency table can be used to summarise the data.

The scores (x) are recorded in one column and the frequency (f) of each score is placed in the column next to it.

This frequency table is a summarised representation of the data set:

10, 10, 20, 20, 20, 20, 30, 30, 30, 40

x	f
10	2
20	4
30	3
40	1
Total	10

The score '**10**' occurs twice, so it has a frequency of '**2**'.

The score '20' occurs four times, so it has a frequency of '4' and so on.

Range = highest value − lowest value

$$\text{Mean } (\bar{x}) = \frac{\text{sum of } n \text{ values}}{n}$$

The total number of data values of scores (n) in the set of data is the sum of the values in the frequency column (10).

Each score is first multiplied by the frequency ($x \times f$), then added together to determine the total sum of n data values.

$$\bar{x} = \frac{10 \times 2 + 20 \times 4 + 30 \times 3 + 40 \times 1}{10}$$
$$= \frac{230}{10}$$
$$= 23$$

Worked example

Determining range from a frequency table

Determine the range of this data set.

Score (x)	Frequency (f)
2	18
7	41
12	34
17	19
Total	112

THINKING	WORKING
Describe what the data represents.	There are 112 data values or scores. The frequency shows the number of times each score appears. For example, there are eighteen 2s in the data set.
Identify the highest (maximum) and lowest (minimum) data values or scores.	The highest value is 17. The lowest value is 2.
Calculate the range by subtracting the lowest value from the highest value.	Range = 17 – 2 = 15
Answer the question.	The set of data has a range of 15.

Worked example

Determining mean from a frequency table

Calculate the mean for this set of data.

Score (x)	Frequency (f)
2	18
7	41
12	34
17	19
Total	112

THINKING	WORKING
Describe what the data represents.	There are 112 data values or scores. The frequency shows the number of times each score appears. For example, there are eighteen 2s in the data set.
Recall the formula used to calculate the mean.	$\text{Mean } (\bar{x}) = \frac{\text{sum of } n \text{ values}}{n}$
Substitute the data values (scores) and the number of data values into the formula to calculate the mean.	$\bar{x} = \frac{(2\times18)+(7\times41)+(12\times34)+(17\times19)}{112}$ $= \frac{1054}{112}$ $= 9.41 \text{ (2 d.p.)}$
Answer the question.	The set of data has a mean score of 9.41.

SC 2 I can determine the mean and range from grouped data

Grouping data

Continuous data with many different values is grouped into class intervals, with the frequency of each class interval recorded using a tally.

Each class interval must be the same size and care needs to be taken when tallying the end values of each group.

This raw data is recorded in the frequency table using class intervals of 10.

12 32 41 47 33 18 23 46 30 8

24 34 31 19 28 36 34 37 20 43

Class	Tally	Frequency (f)
0–9	|	1
10–19	||	2
20–29	||||	4
30–39	𝍸 |||	8
40–49	𝍸 |	6

Values are crossed off, tallied, and then counted to obtain the frequency of scores in each group.

Determining the mean for grouped data

The data in each class/group can have a wide range of values, and a single score representing each group is needed to calculate the mean. The single score used is the mean of the end values of each group.

Class	Score (x)	Frequency (f)
0–9	$\frac{0+9}{2}=4.5$	1
10–19	$\frac{10+19}{2}=14.5$	2
20–29	$\frac{20+29}{2}=24.5$	4
30–39	$\frac{30+39}{2}=34.5$	8
40–49	$\frac{40+49}{2}=44.5$	6

These scores representing each class are then used to calculate the mean score.

$$\begin{aligned}\text{Mean } (\bar{x}) &= \frac{4.5\times1+14.5\times2+24.5\times4+34.5\times8+44.5\times6}{1+2+4+8+6}\\ &= \frac{674.5}{21}\\ &\approx 32.1\end{aligned}$$

The mean calculated in this way is an approximate value because the actual recorded data values were not used.

The range can be estimated using grouped data as the difference between the maximum value of the highest interval and the minimum value of the lowest interval.

$$\begin{aligned}\text{Range} &= \text{max} - \text{min}\\ &= 49 - 0\\ &= 49\end{aligned}$$

Worked example

19

Recording grouped data into a frequency table

Tally this set of measurements in the frequency table and complete the frequency column.

1.5, 2.3, 4.1, 0.7, 1.3,

1.8, 3.2, 4.6, 3.0, 2.8,

2.7, 3.4, 1.3, 1.9, 2.0,

3.6, 1.4, 3.7, 0.4, 4.3.

Class	Tally	Frequency (f)
0.0–0.9		
1.0–1.9		
2.0–2.9		
3.0–3.9		
4.0–4.9		

THINKING	WORKING
Tally scores in the correct class interval as they are crossed off.	1.5, 2.3, 4.1, 0.7, 1.3, 1.8, 3.2, 4.6, 3.0, 2.8, 2.7, 3.4, 1.3, 1.9, 2.0, 3.6, 1.4, 3.7, 0.4, 4.3.

Class	Tally	Frequency (f)
0.0–0.9	𝍸	
1.0–1.9	𝍸 |	
2.0–2.9	𝍸 𝍸	
3.0–3.9	𝍸	
4.0–4.9	𝍸	

Count each tally and write the value into the frequency column.

Class	Tally	Frequency (f)
0.0–0.9	𝍸	2
1.0–1.9	𝍸 |	6
2.0–2.9	𝍸 𝍸	4
3.0–3.9	𝍸	5
4.0–4.9	𝍸 𝍸	3

Worked example

19

Determining the mean and range of grouped data

Calculate the mean and range of this data.

Class	Frequency (f)
0–4	1
5–9	4
10–14	4
15–19	9
20–24	3

THINKING	WORKING
Replace the class interval with the mean value for each group.	

Class	Score (x)	Frequency (f)
0–4	$\frac{0+4}{2}=2$	1
5–9	$\frac{5+9}{2}=7$	4
10–14	$\frac{10+14}{2}=12$	4
15–19	$\frac{15+19}{2}=17$	9
20–24	$\frac{20+24}{2}=22$	3
Total	n	21

Substitute into the formula for mean.	$\bar{x}=\frac{2\times1+7\times4+12\times4+17\times9+22\times3}{21}$ $=\frac{297}{21}$ $=14.1$
Identify the minimum value in the lowest class interval and the maximum value in the highest class interval.	The minimum or lowest possible value is 0. The maximum or highest possible value is 24.
Estimate the range by subtracting the lowest possible score from the highest possible score.	Range = 24 – 0 = 24
Write the answer.	The range of the data is 24 and the mean score is 14.1.

Practice

ANSWERS Page 725

SC 1 I can determine the mean and range from a frequency table

1 **(a)** Calculate the range of each data set.

(i) Set A

x	f
10	15
20	34
30	14
40	17
10	15

(ii) Set B

x	f
5	23
10	41
15	25
20	29
25	32

(b) Which set has the greatest spread? How do you know?

2 Calculate the mean and range $(\bar{x})$ for each displayed data set.

(a)

x	f
1	12
2	14
3	15
4	9
Total	50

(b)

x	f
0	8
2	14
4	12
6	6
Total	40

(c)

x	f
10	16
20	24
30	15
40	9

3 The total number of cars in a school car park over a 30-day period is recorded in this table.

x	f
0	8
5	1
6	4
7	11
8	5
12	1

(a) Calculate the range in the number of cars in the school car park.

(b) Calculate the mean number of cars parked over the 30-day period.

(c) What would be the main reason for there being 8 days with no cars?

4 **(a)** Multiply each score by its frequency to complete the $(x \times f)$ column and totals. Use this column to calculate the mean score $(\bar{x})$.

x	f	$x \times f$
3	6	
5	4	
7	8	
9	6	
11	5	
13	4	
Total		

(b) Describe how the values in the third column help with calculating the mean score.

5 A data set contains 48 scores and has a range of 7.
Copy and complete the table with a possible set of scores and frequencies to match this information.

x	f
Total	

SC 2 I can determine the mean and range from grouped data

1 **(a)** Copy and complete the table. The score is determined by calculating the mean of each class interval.

(b) Calculate the range.

Class	Score (x)	Frequency (f)
0–4		10
5–9		14
10–14		7
15–19		9
20–24		3

2 **(a)** Copy and complete the table.

Class	Score (x)	Frequency (f)
0–9		10
10–19		14
20–29		7
30–39		9
40–49		3

(b) Calculate the total number of scores.

(c) Calculate the mean score of the grouped data.

3 **(a)** Copy and complete the table.

(b) Calculate the mean of this grouped data.

Class	Score (x)	Tally	Frequency (f)
0.0–0.4		𝍸	
0.5–0.9		𝍷	
1.0–1.4		𝍸	
1.5–1.9		𝍷	
2.0–2.4		𝍸	

4 **(a)** Copy and complete the frequency table for this set of data:
9, 47, 6, 18, 32, 48, 45, 16, 35, 46, 14, 23, 36, 27, 28, 36, 41

Class	Score (x)	Tally	Frequency (f)
0–9			
10–19			
20–29			
30–39			
40–49			

(b) Calculate the mean using the grouped data in the table.

(c) Calculate the mean from the raw data.

(d) Comment upon the difference between your answers to **(b)** and **(c)**.

5 A brand of coffee comes in 200 g, 500 g and 750 g jars. Records are kept of the sales for each month. Does this data need to be grouped? Explain your answer.

19.4 Determine the median of a data set

Learning intention: To be able to calculate the median of a data set

Success criteria:

SC 1 I can determine the median of a set of raw data.

SC 2 I can determine the median from a frequency table.

Lesson warm-up

Interpreting data

1. How might you determine which bottle would be a good measure of centre?
2. What would happen to the measure of centre if one of the end bottles was removed from the group?
3. How would the measure of centre change if a water tank was added to the group?

SC 1 I can determine the median of a set of raw data

Extreme values (called outliers) can greatly influence the accuracy of the mean score of a data set. When there are outliers, other measures of central tendency that are not affected by extreme values are used. The first of these is the median value.

The median is the middle value after the data scores have been arranged in ascending order (lowest to highest).

The median value splits the data set into two halves.

The total number of scores (n) in a set of data may be odd or even. Each case is treated differently.

If n is odd, the median value is the score located in the $\frac{n+1}{2}$th position.

For example, these 7 scores have been arranged in ascending order: 4, 6, 8, 9, 12, 14, 15.

$n = 7$ (odd), so the median value is the score located in the $\left(\frac{7+1}{2}\right) = 4$, fourth position.

The score 9 is in the fourth position of 4, 6, 8, 9, 12, 14, 15 and is therefore the median value.

If n is even, the median value is the mean of the two scores located on either side of the $\frac{n+1}{2}$th position.

For example, these 8 scores have been arranged in ascending order: 4, 6, 8, 9, 12, 14, 15, 20.

$n = 8$ (even), so the median value is the mean of the scores located on either side of the $\frac{8+1}{2} = 4.5$th position. Therefore, it is the mean of the fourth and fifth scores.

The scores 9 and 12 are the fourth and fifth scores of 4, 6, 8, 9, 12, 14, 15, 20.

The mean of 9 and 12 is $\frac{9+12}{2} = 10.5$, therefore 10.5 is the median value.

Worked example

Determining the median value for raw data

Determine the median value for each of these data sets.

(a) 5, 8, 2, 14, 9

THINKING	WORKING
Arrange all the scores in ascending order (smallest to largest).	2, 5, 8, 9, 14
Count the number of data values. Is this an odd or even number?	There are an odd number of data values. $n = 5$
For an odd number of data values, identify the score in the $\frac{n+1}{2}$th position.	$\frac{n+1}{2} = \frac{5+1}{2} = 3$ Identify the data value in the middle (third) position. 2, 5, 8, 9, 14
Write the answer.	The median value is 8.

(b) 5, 8, 2, 14, 9, 16

THINKING	WORKING
Arrange all the scores in ascending order (smallest to largest).	2, 5, 8, 9, 14, 16
Count the number of data values. Is this an odd or even number?	There are an even number of data values. $n = 6$
For an even number of data values, identify the two scores on either side of the $\frac{n+1}{2}$th position.	$\frac{6+1}{2} = 3.5$ Identify the third and fourth data value. 2, 5, 8, 9, 14, 16
Calculate the mean of the two scores.	$\frac{8+9}{2} = 8.5$
Write the answer.	The median value is 8.5.

SC 2 I can determine the median from a frequency table

Determining the median value for data in a frequency table

When there are many scores, a frequency table is used to order and summarise the data set.

To simplify the count of the total scores and determine the position of the median value, a cumulative frequency (*c.f.*) column can be completed next to the frequency column. It is a running total of the frequency column.

For example:

Score (x)	Frequency (f)	Cumulative frequency ($c.f.$)
1	10	10
2	12	22
3	9	31
4	14	45

The last value in the cumulative frequency column (45) represents the total number of scores (n).

Worked example

Calculating the median value for data in a frequency table.

Determine the median value for the data displayed in each of the frequency tables.

(a)

x	f
1	2
2	4
3	3
4	6

THINKING	WORKING
Complete a cumulative frequency column for the table data.	x | f | $c.f.$ 1 | 2 | 2 2 | 4 | 6 3 | 3 | 9 4 | 6 | 15
Count the number of data values. Is the number odd or even?	$n = 15$ (odd)

For an odd number of data values, identify the score in the $\frac{n+1}{2}$th position.	$\frac{15+1}{2}=8$ 8th position
Locate the position (or higher) in the cumulative frequency table.	x | f | $c.f.$ 1 | 2 | 2 2 | 4 | 6 3 | 3 | 9 4 | 6 | 15 The first six data values are two 1s, followed by four 2s. The next three data values are 3s. The eighth data value will be a 3.
Write the answer.	The median value is 3.

(b)

x	f
4	33
6	10
8	18
10	25

THINKING	WORKING
Complete a cumulative frequency column for the table data.	x | f | $c.f.$ 4 | 33 | 33 6 | 10 | 43 8 | 18 | 61 10 | 25 | 86
Count the number of data values. Is the number odd or even?	$n = 86$ (even)
For an even number of data values, identify the two scores on either side of the $\frac{n+1}{2}$th position.	$\frac{86+1}{2}=43.5$ The median value is between the 43rd and 44th data values.

Locate the position (or higher) in the cumulative frequency table.	x | f | $c.f.$ 4 | 33 | 33 6 | 10 | 43 8 | 18 | 61 10 | 25 | 86 The 43rd score is 6. The 44th score is 8.
Calculate the mean of the two scores.	$\frac{6+8}{2}=7$
Write the answer	The median value is 7.

Practice

ANSWERS Page 726

SC 1 I can determine the median of a set of raw data

1 Determine the median of each of these sets of scores.

(a) 1, 2, 3, 4, 5

(b) 1, 2, 3, 4, 5, 6, 7

(c) 10, 12, 13, 15, 16

(d) 8, 12, 17, 9, 5, 9, 15

(e) 102, 120, 240, 300, 450

(f) 1.5, 2.3, 4.2, 5.8, 6.3, 7.2, 10.3

(g) 6.2, 3.5, 4.8, 5.1, 6.8

2 Determine the median of each of these groups of numbers.

(a) 1, 2, 3, 4, 5, 6

(b) 1, 2, 3, 4, 5, 6, 7, 8

(c) 10, 12, 12, 13, 15, 16

(d) 8, 12, 17, 9, 5, 9, 15, 10

(e) 102, 120, 240, 300, 360, 450

(f) 1.5, 2.3, 4.2, 4.9, 5.8, 6.3, 7.2, 10.3

(g) 6.2, 6.4, 3.5, 4.8, 5.1, 6.8

3 For this data set 14, 15, 18, 19, 20, 24, 26, 29, 5000:

(a) calculate the mean score $\bar{x}$

(b) determine the median value

(c) identify the value that best represents the central tendency of the data and explain your choice.

4 A data set of 5 whole-numbered scores has a mean $(\bar{x})$ of 6, and a range of 9. Create a data set that matches these conditions and calculate the median value.

5 Suki is 7 years old and lives with two parents who are both 28 years old.

(a) What is the median of their ages?

(b) Suki's grandparent is 56 years old and pays a visit. What effect does this have on the median age of the four family members? Explain your answer.

(c) Suki's Uncle San also visits the same day, and the median age stays the same. Can the age of Uncle San be determined by this information? Explain your answer.

(d) Suki's Uncle Akira and Aunty Ado also arrive later with their three children. Their ages are 2, 5, 8, 27 and 32. Given that Uncle San is 58 years old, what is the median age of all ten family members present?

SC 2 I can determine the median from a frequency table

1 Complete the cumulative frequency columns for each table.

(a)

x	f	*c.f.*
5	21	
6	15	
7	18	
8	20	
9	11	

(b)

Class	x	f	*c.f.*
10–14	12	38	
15–19	17	47	
20–24	22	29	
25–29	27	35	

2 Determine the median value for each data set.

(a)

x	f	*c.f.*
1	11	11
2	14	25
3	15	40
4	9	49

(b)

Class	x	f	*c.f.*
0–4	2	8	8
5–9	7	13	21
10–14	12	15	36
15–19	17	14	50

3 Complete the cumulative frequency column and calculate the median value for each data set.

(a)

x	f	*c.f.*
10	16	
20	24	
30	15	
40	9	

(b)

x	f	*c.f.*
5	9	
6	8	
7	10	
8	23	
9	5	

19.5 Determine an appropriate measure of central tendency

Learning intention: To be able to determine an appropriate measure of central tendency

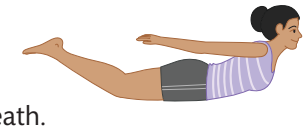
Success criteria:

SC 1 I can identify the mode of a set of raw data and data in a frequency table.

SC 2 I can choose which measure of central tendency is the most appropriate for a scenario.

Lesson warm-up

Ducks on a pond



The 'Ducks on a pond' carnival game involves selecting three ducks from the water and adding together the single-digit numbers written underneath.

If you are the person operating the game, how do you ensure that everyone gets a prize, but that players rarely win the big prizes?

SC 1 I can identify the mode of a set of raw data and data in a frequency table

The mode is the score that has the greatest frequency in a set of data.

If two scores have the same greatest frequency, the data set is called bimodal.

If more than two scores have the same greatest frequency, the data set has no mode.

Worked example

Identifying the mode

Identify the mode in each of these data sets.

(a) 2, 5, 6, 3, 9, 2, 4, 2, 4

THINKING	WORKING
Arrange the scores in ascending order (smallest to largest).	2, 2, 2, 3, 4, 4, 5, 6, 9
Identify the score or data value that has the highest frequency.	2 has the highest frequency. It occurs three times.
Write the answer.	The mode is 2.

(b) 2, 5, 6, 3, 9, 5, 2, 4, 2, 4, 5

THINKING	WORKING
Arrange the scores in ascending order.	2, 2, 2, 3, 4, 4, 5, 5, 5, 6, 9
Identify the score or data value that has the highest frequency.	2 and 5 have the same highest frequency. Both scores occur three times.
Write the answer.	The data set is bimodal. The two modes are 2 and 5.

(c)

Score (x)	Frequency (f)
2	1
7	4
12	4
17	9

THINKING	WORKING
Identify the score that has the highest frequency.	17 has the highest frequency of 9.
Write the answer.	The mode is 17.

(d)

Class	Frequency (f)
0–4	1
5–9	4
10–14	4
15–19	9
20–24	3

THINKING	WORKING
Identify the class that has the highest frequency.	The class 15–19 has the highest frequency of 9.
Write the answer.	The modal class is 15–19.

SC 2 I can choose which measure of central tendency is the most appropriate for a scenario

Scores in data sets can be continuous (such as length measurements), or discrete (such as counting items).

Each data set is unique, and the decision for which measure of centre is the most appropriate is made on a case-by-case basis.

Mean

- Most common measure of centre, especially for continuous data.
- If an extreme value (outlier) exists in the data, the mean's ability to accurately represent the centre of the data is affected.
- For discrete data, the mean can result in a decimal value and may not be relevant to the data.

Median

- Used instead of the mean when data contains an outlier.
- Calculating the middle value after ordering the scores in ascending order ensures that any outliers are excluded.
- For discrete data, the median can result in a decimal value and may not be relevant to the data.

Mode

- Used when the data is discrete and the most frequently occurring score is important to know.
- Specifically useful for determining the most popular size of items sold in a store for restocking purposes.

Range

- Used only when it is important to know the spread of the data.
- If an extreme value (outlier) exists in the data, the range's ability to accurately represent the typical spread of the data is affected.

Worked example

Choosing an appropriate measure of centre

For these data sets, determine whether the mean, median or mode is the best representation of the central tendency.

(a) 4, 8, 2, 15, 22, 31, 17, 17, 8, 4

THINKING	WORKING
Determine whether the scenario suits using the mean value.	There are no extreme values (outliers), so the mean can be used as a measure of centre.
Determine whether the scenario suits using the median value.	The median can be used as a measure of centre.

Determine whether the scenario suits using the modal value.	The data has four data values with the highest frequency, so the mode is not appropriate.
Write the answer.	The mean and median can be used to indicate centre.

(b) 4, 8, 2, 15, 22, 31, 17, 17, 8, 4, 487

THINKING	WORKING
Determine whether the scenario suits using the mean value.	487 is an outlier, so the mean is not appropriate.
Determine whether the scenario suits using the median value.	The median can be used as a measure of centre.
Determine whether the scenario suits using the modal value.	The data has four data values with the highest frequency, so the mode is not appropriate.
Write the answer.	The median is the best measure of centre.

(c) T-shirt sizes sold over the past week are recorded in the table to prepare for reordering.

T-shirt size (x)	Total sold (f)
8	18
10	42
12	34
14	27

THINKING	WORKING
Determine whether the scenario suits using the mean value.	There are no extreme values (outliers), so the mean can be used as a measure of centre. However, this is discrete data, so the mean can result in a decimal value and may not be relevant to the data.
Determine whether the scenario suits using the median value.	The median value will just provide the T-shirt size in the middle of the data and may not be useful for reorders.
Determine whether the scenario suits using the modal value.	The mode score identifies the most popular T-shirt sizes sold and will help with reordering. So, the mode is the best measure to use here.
Write the answer.	The mode gives the most commonly sold T-shirt size.

Practice

ANSWERS Page 727

SC 1 I can identify the mode of a set of raw data and data in a frequency table

1 Determine the mode of each of these groups of numbers.

(a) 1, 1, 3, 1, 5

(b) 5, 4, 3, 4, 5, 5, 7

(c) 10, 12, 12, 15, 12

(d) 8, 8.5, 8.7, 9, 9.1, 9.3, 15

(e) 102, 102, 240, 240, 240

2 Identify the modal score for each data set.

(a)

x	f
1	9
2	14
3	15
4	9

(b)

x	f
10	16
20	24
30	15
40	16

(c)

x	f
0–2	8
2–4	14
4–6	12
6–8	7

3 For the data set 4, 4, 4, 19, 20, 24, 26, 29, 40:

(a) calculate the mean score $(\bar{x})$

(b) identify the modal score

(c) identify the value that best represents the central tendency of the data and explain why.

4 Consider the data set 2, 3, 2, 5, 3, 7, 8, 94.

(a) Identify the mode of the data.

(b) Two values, 7 and 94, are added to the data set. What effect does this have on the mode?

5 A data set of 5 whole-numbered scores has a mean $(\bar{x})$ of 4, a range of 8, a median of 3 and a minimum score of 1. Create a data set that matches these conditions and does not have a mode.

6 A bag contains a mix of different whole-numbered tiles. What information about the tiles in the bag is needed for each of the following calculations?

(a) Range of the tile values

(b) Mean tile value

(c) Median value in the bag

(d) Modal tile

SC 2 I can choose which measure of central tendency is the most appropriate for a scenario

1 For each of these statements, decide whether they are always true, sometimes true or never true.

(a) The mean is the better choice for representing the central tendency of a data set.

(b) The mean is not an appropriate measure of central tendency when the data includes an extreme value.

(c) The median value will be less than the mode.

(d) The mode is the better choice for representing the central tendency of a data set.

(e) The median value is never affected by extreme values.

(f) The median is more representative of a data set than the mode.

(g) The median is the better choice for representing the central tendency of a data set.

2 A shoe store owner records the size of shoes sold on a particular day.
10 12 10 8 9 8 8 9 10 8

(a) Calculate the mean $(\bar{x})$.

(b) Calculate the median value.

(c) Identify the mode.

(d) Explain why the mode would be the better measure of central tendency.

3 The actual weight of ten packets of 250 g packets of rice are recorded in this table.

Weight class (g)	Class centre (x)	Frequency (f)
240–<250		1
250–<260		5
260–<270		2
270–<280		2

(a) Complete the class centre values column.

(b) Calculate the mean $\bar{x}$.

(c) Calculate the median value.

(d) Identify the mode.

(e) Which measure best represents the central tendency? Why?

4 The amount of money ($) counted in ten charity boxes after a day of collecting was recorded.

193.45, 161.75, 158.20, 152.00, 206.35, 174.55, 200.30, 381.95, 185.30, 190.80

Calculate the value that best represents the typical amount collected in the charity boxes.

5 The mode of a data set is 6 and the median is 8.
Can you use this information to determine the mean of the data set? Explain what you need to determine the mean if your answer is no.

6 Using four of these seven numbers:
3, 4, 5, 6, 6, 7, 8
create a data set that has the same value for the median and mode, and a range of 4.

7 Using seven of these ten scores,
10, 3, 6, 4, 14, 9, 8, 7, 3, 11
create a data set in which the median value is:

(a) greater than the mean value

(b) equal to the mean value.

TOPIC REVIEW

Multiple choice

ANSWERS Page 751

1 Which of the following is an example of secondary data?

- **A** Asking your fellow team members what they would like to do to celebrate the end of the season
- **B** A survey of Year 8 students at Dirtwater High School
- **C** Population data taken from the Australian Census website
- **D** A survey conducted on social media

2 Which of the following is an example of primary data?

- **A** Weather statistics found online
- **B** A survey taken by you of students in your class
- **C** Population data taken from the Australian Census website
- **D** Online data about income levels across suburbs in Melbourne

3 Which of the following groups contains only categorical data?

- **A** Eye colour, height, favourite TV series
- **B** Eye colour, weight, favourite colour
- **C** Eye colour, type of mobile phone, favourite celebrity
- **D** Eye colour, height, the size of classes in Year 7

4 Which of the following groups contains only numerical data?

- **A** Daily maximum temperature, height, favourite TV series
- **B** Daily maximum temperature, weight, favourite colour
- **C** Eye colour, type of mobile phone, daily minimum temperature on the Moon
- **D** Daily maximum temperature, height, the size of classes in Year 7

5 The frequency of cars passing through a busy intersection is recorded over several days. The data is most accurately described as:

- **A** categorical and discrete.
- **B** numerical and discrete.
- **C** categorical and continuous.
- **D** numerical and continuous.

6 The mean of these figures 4, 6, 12, 18 is:

- **A** 4
- **B** 10
- **C** 14
- **D** 18

7 The mean of this data is:

x	f
10	5
20	6
30	3
40	1
Total	15

A 15 **B** 20 **C** 75 **D** 300

8 The range of this data set 2, 4, 7, 19, 25, 46, 62 is:

A 62 **B** 60 **C** 23.6 **D** 19

9 The median of this data set 4, 6, 8, 9, 11, 13, 15 is:

A 4 **B** 9 **C** 9.4 **D** 11

10 The median of this data set 14, 6, 8, 19, 10, 13 is:

A 8 **B** 10.5 **C** 11.5 **D** 13

Use the following table to answer Questions **11** and **12**.

x	f
0–4	2
5–9	4
10–14	4
15–19	9
20–29	2

11 The mode is:

A 2 and 4 **B** 0–4 and 20–29 **C** 9 **D** 17

12 The median group is:

A 2 and 4 **B** 15–19 **C** 9 **D** 17

13 The median of the following data is:

x	f
10	5
20	6
30	3
40	1
Total	15

A 15 **B** 20 **C** 75 **D** 300

Short answer

1 Jed is planning a hiking trip and uses the park's website to collect monthly data on the number of visitors for the previous two years. Is the data collected categorical or numerical?

2 Max owns a hardware store and the top-selling items are nails, screws and screwdrivers.

(a) Is the collected data categorical or numerical?

This frequency table summarises the survey results.

Item	Sales
Boxes of nails	25
Boxes of screws	13
Screwdrivers	8

(b) Why are the mean, median and mode not useful here?

3 Explain the difference between categorical and numerical data, using an example of each.

4 Label each of these sets of data as categorical or numerical.

(a) Measuring the height of Year 7 students

(b) A survey of Year 7 students to find their favourite social media app

(c) Measuring the daily pH level of the drinking water in a dam

(d) Using the Australian Government Census data to find the average income in each Sydney suburb

(e) Using Bureau of Meteorology data to find the average minimum temperature in Kalgoorlie in April

(f) Recording the colour of cars parked in the school car park

5 Jayde shoots a long-term mean of 18.4 points per game of basketball. Over the last five games, Jayde has scored 18, 26, 28, 12 and 14 points. Is this scoring rate more or less than is expected?

6 Twenty people have a total height of 33.6 m. What is their mean height?

7 A group of people have a total age of 1167 years. Their mean age is 38.9 years. How many people are there?

8 Consider these two sets of data.

Set A: 4, 5, 5, 6, 6, 6, 8, 9, 10

Set B: 4, 5, 7, 9, 12, 24, 240

(a) Determine the mean and range of each set.

(b) Comment on whether the mean and range are suitable statistical measures for both sets.

9 Find the median of this set of grouped data.

x	f
4	20
6	24
8	26
10	19

10 (a) Calculate the range for each set of data.

(i) Set A: 2, 4, 6, 8, 19, 21 **(ii)** Set B: 7, 2, 5, –3, 14 **(iii)** Set C: –5, 6, 1, 0, –9, 12

(b) Which set has the greatest spread? How do you know?

11 Create a set of 5 scores with a highest value of 16 and a range of 5.

12 A negative score is added to the set of data shown below to give it a range of 12.

What is the score?

10, 9, 7, 4, 8, 2

Extended response

1 Cassie and Dina are two promising basketballers in the National Youth Team. Potential recruiters keep a close watch on them during the world championships and collect these sets of statistics.

Points per game

Cassie: 21, 14, 18, 24, 12, 15, 34, 16, 18, 17, 14 Dina: 6, 14, 18, 28, 12, 15, 23, 16, 16, 15, 14

Rebounds per game

Cassie: 6, 12, 13, 9, 10, 15, 16, 16, 10, 7, 14 Dina: 8, 9, 11, 24, 12, 11, 9, 6, 9, 10, 5

(a) Calculate the mean, median and mode for Cassie and Dina for:

(i) points per game,

(ii) rebounds per game.

(b) Describe the similarities and differences between the two sets of rebounding figures.

2 Answer the following questions.

(a) The mean of the set of numbers 12, 17, 14, 23, x, 31 is 20.

Determine the value of:

(i) x

(ii) the median

(iii) the mode.

(b) This data has a range of 64. The maximum value is 70 and the mean is 29. In order, the figures are p, 14, 18, 25, q, 70.

Determine the value of:

(i) the median

(ii) p

(iii) q.

3 Write down what you consider to be the five most important things you have learnt about statistics in this topic.

TOPIC
20

Displaying and understanding data

Recall 581
20.1 Understand stem-and-leaf plots 582
20.2 Create and interpret statistical plots 588
20.3 Determine statistical measures of centre from data displays 596
20.4 Interpret and describe numerical data displays 607
Topic review 615

Why learn this?

Data displays such as stem-and-leaf plots are essential tools for organising and visualising numerical data. These visualisation techniques are extremely useful in fields that involve statistical analysis, such as market research, environmental science and quality control. They help us to quickly identify data distribution, central tendency (median and mode) and potential outliers. Mastery of these lessons improves analytical skills, providing a foundation for more advanced statistical work and contributing to effective data-driven decision-making in the workplace.

RECALL

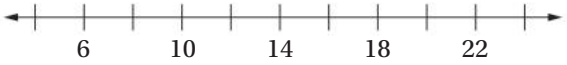
ANSWERS Page 728

I can identify the scale of number lines

1 Complete the scale on each of the following number lines.

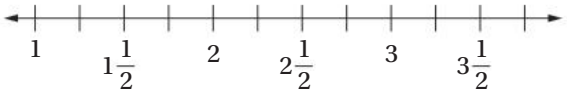
(a)

(b)



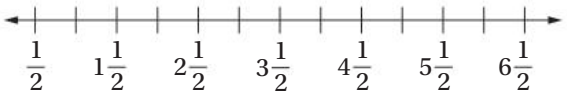
(c)

(d)



(e)

(f)



I can tally scores and complete a frequency table to display collected data

1 A student recorded all the vowels in one paragraph of a book as follows.
e, e, i, o, e, a, e, e, i, o, e, i, i, a, o, u, o, e, e, e, a, a, e, e, u, i, u, o, i, e, i, i, o, a, a, u, o, e, e

(a) Complete a tally, grouped by five, for each vowel recorded.

(b) Count the tally for each letter to determine the frequency of each vowel.

2 Complete a frequency table for this data showing the age of students in a group.
16, 15, 17, 16, 16, 15, 14, 17, 15, 16, 16, 15, 16, 15, 16

20.1 Understand stem-and-leaf plots

Learning intention: To understand stem-and-leaf plots

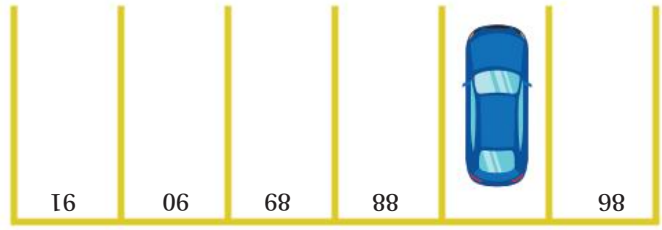
Success criteria:

SC 1 I can create and interpret a stem-and-leaf plot.

SC 2 I can create a split stem-and-leaf plot.

Lesson warm-up

Where is the car parked?



SC 1 I can create and interpret a stem-and-leaf plot

Data values can be arranged into a stem-and-leaf plot.

Just as a plant has many leaves growing from its stems, so do these types of data plots.

For example, the maximum temperatures (°C) recorded over a 10-day period were:

19, 22, 24, 25, 27, 27, 32, 33, 36, 40.

The stem-and-leaf plot for this data is:

	Stem	Leaf
19 shown as:	1	9
22, 24, 25, 27, 27 shown as:	2	2 4 5 7 7
32, 33, 36 shown as:	3	2 3 6
40 shown as:	4	0

Key $1 \mid 9 = 19$

The total number of leaves equals the number of scores recorded.

The leaf values for each stem are evenly spaced and arranged in ascending order, away from the stem. Never separate values in the leaf column with commas.

Creating a stem-and-leaf plot has many advantages for larger data sets.

The spread and shape of the distribution can be clearly seen.

This makes it easier to compare two or more data sets.

The data is ordered, so the mode and median are easier to determine.

The key for the plot shows that the stem is the leading digit (representing the tens value), and the leaves are the remaining digits (representing the ones value) of each recorded temperature.

Including a key is vital for ensuring that data is read accurately from the plot. Keys always contain two parts: the stem and the leaf, and can represent data values in many different ways.

For example:

Key	Interpreting 20|3
1|9 = 1.9	20.3
1|9 = 19	203
1|9 = 190	2030

Worked example

Creating a stem-and-leaf plot

26 patrons exiting a movie theatre were surveyed and their ages were recorded.

23 28 32 45 48 61 42 43 65 56 31 33 42

24 31 43 47 20 23 29 32 55 57 28 43 36

Create a stem-and-leaf plot using this data.

THINKING	WORKING
Identify the minimum and maximum values to determine a sensible stem.	Minimum value: 20 Maximum value: 65 Stems representing tens place values work.
Create a key to show how each data value will be represented.	The data being recorded is the ages of theatre patrons. The stem will represent the tens digit, and the leaf will represent the ones digit. Key 2|0 = 20
Create a table with the stems down the left-hand column, ranging from the minimum to the maximum tens value.	Stem | Leaf 2 3 4 5 6 Key 2 | 0 = 20
List each leaf value as the ones digit of each number. Note: The stem value represents the tens digit of each number.	Stem | Leaf 2 | 3 8 4 0 3 9 8 3 | 2 1 3 1 2 4 6 4 | 5 8 2 3 2 3 7 5 | 6 5 7 6 | 1 5 Key 2 | 0 = 20

Arrange the leaves in ascending order away from each stem.

Stem	Leaf
2	0 3 3 4 8 8 9
3	1 1 2 2 3 4 6
4	2 2 3 3 5 7 8
5	5 6 7
6	1 5

Key 2 | 0 = 20

SC 2 I can create a split stem-and-leaf plot

Large sets of data can have many leaf values attached to the same stem. This can make plots difficult to analyse properly. When this occurs, the stems can be split so that the leaves for each have a smaller range of values. These are called split stem-and-leaf plots.

The key remains the same, however, the leaf values for each stem are most commonly divided into the groups 0 to 4 and 5 to 9.

Stem-and-leaf plot

Stem	Leaf
0	3 4 4 4 5 5 7 8 9
1	1 1 3 4 6 7 8
2	3 3 4 4 5 5 5 6 9 9
3	5 6 7 7 8 9

Key 1 | 3 = 13

Split stem-and-leaf plot

Stem	Leaf
0	3 4 4 4
0	5 5 7 8 9
1	1 1 3 4
1	6 7 8
2	3 3 4 4
2	5 5 5 6 9 9
3	
3	5 6 7 7 8 9

Key 1 | 3 = 13

Note: Split stem-and-leaf plots can sometimes result in stems with no leaves, as in stem value 3 in the plot above.

Worked example

Creating a split stem-and-leaf plot

Present this displayed data as a split stem-and-leaf plot.

Stem	Leaf
2	3 3 4 5 5 7 7 7 8 8 9
3	1 1 1 3 4 5 6 6 6 7 7 8 9
4	2 2 3 3 4 4 5 5 6 6 7 9 9
5	0 1 2 4 5 6 6 7 7 8 9

Key 2 | 3 = 23

THINKING	WORKING
Split each stem into two parts.	Stem | Leaf 2 | 3 3 4 5 5 7 7 7 8 8 9 2 | 3 | 1 1 1 3 4 5 6 6 6 7 7 8 9 3 | 4 | 2 2 3 3 4 4 5 5 6 6 7 9 9 4 | 5 | 0 1 2 4 5 6 6 7 7 8 9 5 | Key 2 | 3 = 23
The leaves showing the digits 0–4 remain with the first part of the split stem. The leaves 5–9 are placed with the second part of the split stem. The key remains the same.	Stem | Leaf 2 | 3 3 4 2 | 5 5 7 7 7 8 8 9 3 | 1 1 1 3 4 3 | 5 6 6 6 7 7 8 9 4 | 2 2 3 3 4 4 4 | 5 5 6 6 7 9 9 5 | 0 1 2 4 5 | 5 6 6 7 7 8 9 Key 2 | 3 = 23

Practice

ANSWERS Page 728

SC 1 I can create and interpret a stem-and-leaf plot

1 Determine the values represented in this stem-and-leaf plot by the:

(a) square 1|4 **(b)** circle 3|0 **(c)** triangle 4|3.

Stem	Leaf
1	0 3 3 4 8 8 9
2	1 1 2 2 3 5 6 8 8
3	0 1 1 2 3 3 5 7 8
4	1 3 3 5 6 7 9
5	0 0 0 1 5

Key 1 | 1 = 11

2 Determine the values represented in this stem-and-leaf plot by the:

(a) square 4|8 **(b)** circle 5|5 **(c)** triangle 7|0.

Stem	Leaf
3	0 3 3 4 8 8 9
4	1 1 2 2 3 5 6 8 8
5	0 1 1 2 3 3 5 7 8
6	1 3 3 5 6 7 9
7	0 0 0 1 5

Key $1 \mid 1 = 1.1$

3 Each set of measurements has been recorded into a stem-and-leaf plot. Create a suitable key for each plot.

(a) 11, 12, 12, 15, 17, 23, 25, 27, 32

Stem	Leaf
1	1 2 2 5 7
2	3 5 7
3	2

(b) 1.1, 1.2, 1.3, 1.5, 1.9, 2.0, 2.1, 2.4, 2.8, 3.2, 3.5

Stem	Leaf
1	1 2 3 5 9
2	0 1 4 8
3	2 5

(c) 101, 102, 103, 105, 109, 110, 111, 114, 118, 122, 125

Stem	Leaf
10	1 2 3 5 9
11	0 1 4 8
12	2 5

4 Create a stem-and-leaf plot for each set of data. Include a key.

(a) 11, 12, 13, 14, 25, 27, 32, 34, 36

(b) 1.1, 1.2, 1.3, 1.4, 2.5, 2.7, 3.2, 3.4, 3.6

(c) 107, 120, 110, 122, 126, 135, 106, 127, 132, 134, 114

(d) 85, 92, 67, 79, 65, 89, 73, 90, 81, 68, 71, 81

5 Create a stem-and-leaf plot for this set of data. Include a key.

44, 67, 36, 46, 37, 62, 51, 38, 40, 39, 43, 53, 51, 56, 30, 34, 56, 36, 45, 57, 44, 33, 42, 54, 56

6 Explain why a stem-and-leaf plot would not be useful for this data.

20, 22, 23, 23, 24, 26, 27, 27, 27, 28, 29

SC 2 I can create a split stem-and-leaf plot

1 Determine the value of the highlighted numbers displayed in these split stem-and-leaf plots.

(a)

Stem	Leaf
4	0 2 3 3 4
4	5 5 8
5	1 2 2 3 4
5	5 6 7 8 8 8 9
6	0 1 3 3 3 3 4
6	7 **8**

Key $4 \mid 0 = 40$

(b)

Stem	Leaf
4	0 2 3 3 4
4	5 5 8
5	1 2 2 3 4
5	5 6 7 8 8 8 9
6	0 1 3 3 3 3 4
6	7 **8**

Key $4 \mid 0 = 4.0$

(c)

Stem	Leaf
4	0 2 3 3 4
4	5 5 8
5	1 2 2 3 4
5	5 6 7 8 8 8 9
6	0 1 3 3 3 3 4
6	7 **8**

Key $4 \mid 0 = 400$

2 Present this displayed data as a split stem-and-leaf plot.

Stem	Leaf
11	1 3 4 6 8 8
12	1 2 4 6 7 9 9
13	1 5
14	1 3 6 6

Key $10 \mid 2 = 10.2$

3 For this set of data, create a split stem-and-leaf plot.
4, 4, 4, 6, 7, 8, 10, 13, 17, 19, 20, 22, 22, 24, 25, 26, 28, 29

20.2 Create and interpret statistical plots

Learning intention: To be able to create and interpret statistical plots

Success criteria:

SC 1 I can construct and interpret dot plots.

SC 2 I can determine the range and mode of values from dot plots and stem-and-leaf plots.

Lesson warm-up

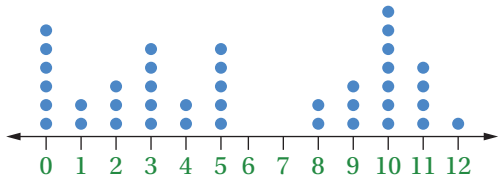
Laughing clowns

Look at the picture of a completed sideshow clown game and answer the following questions.

1. When playing this game using 6 balls, how could you quickly calculate the score achieved?
2. How could you score a total of 8 with all 6 balls?

SC 1 I can construct and interpret dot plots

A dot plot is a visual way of representing every data value in a set.



The horizontal axis shows the value, groups or interval recorded, and the frequency can be determined by counting the number of dots.

Dot plots show every data value, and the associated frequency can easily be counted.

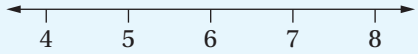
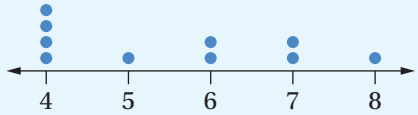
A dot plot also can be displayed as a stem-and-leaf plot to show the distribution of the values by group (stem). The reverse is not useful because the accuracy of measures such as the range of values and individual value frequencies become difficult to determine.

Worked example

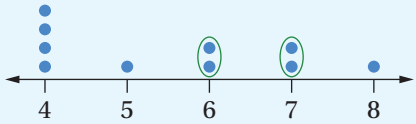
Constructing a dot plot

Consider the following data: 4, 8, 7, 6, 4, 6, 5, 7, 4, 4

(a) Create a dot plot of the data set.

THINKING	WORKING
Identify the lowest and highest values in the data.	4, 8, 7, 6, 4, 6, 5, 7, 4, 4 The lowest value is 4. The highest value is 8.
Draw a horizontal axis covering the range of values in the data.	The range is 4 to 8. 
Create a dot plot by adding one dot for each time the value occurs. Ensure the dots are spaced evenly above each value. Note: The number of dots plotted above each value will represent the frequency of that value.	

(b) Which data value/s had a frequency of 2?

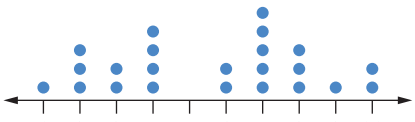
THINKING	WORKING
Identify the values in the plot that have two dots above them.	 The values 6 and 7 have two dots above them.
Write the answer.	The values 6 and 7 have a frequency of 2.

Worked example

Interpreting dot plots

The dot plot displays the chute number each 5 kg parcel was sent down in a sorting centre.

Dot plot showing chutes parcels were sorted into



(a) Which chute did not receive any parcels?

THINKING	WORKING
Identify the number with no dots.	Chute number 5 has no dots above it.
Write the answer.	Chute number 5 did not receive any parcels.

(b) How many chutes received three or more parcels?

THINKING	WORKING
Identify the numbers with three or more dots.	Chutes numbered 2, 4, 7 and 8 have three or more dots plotted above them.
Write the answer.	Chutes 2, 4, 7 and 8 received three or more parcels.

(c) Which chute received the most parcels?

THINKING	WORKING
Identify the number with the highest frequency.	Chute number 7 has the most dots above it.
Write the answer.	Chute 7 received the most parcels.

(d) How many parcels were sorted?

THINKING	WORKING
Count the total number of dots plotted.	There are 23 dots in total.
Write the answer.	23 parcels were sorted.

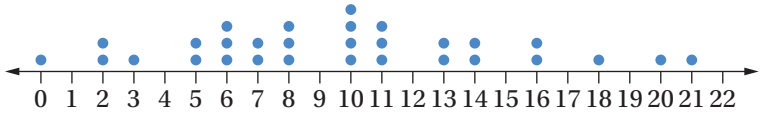
(e) Determine the total weight of parcels directed down chute 4.

THINKING	WORKING
Count the number of dots in the required category.	There are 4 dots in total above chute 4.
Multiply the parcel weight by the number of dots.	4×5 kg $= 20$ kg
Write the answer.	20 kg of parcels were directed down chute 4.

Worked example

Constructing a stem-and-leaf plot from a dot plot

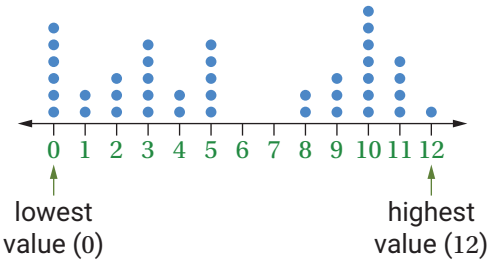
Construct a stem-and-leaf plot for the data displayed in the dot plot.



THINKING	WORKING
Identify the lowest and highest values in the data.	The lowest value is 0. The highest value is 21.
Determine a suitable key for the stem-and-leaf plot.	Key 1 | 5 = 15
Construct a stem-and-leaf plot and insert the tens digits as stems. Remember to include the key.	Stem | Leaf 0 1 2 Key 1 | 5 = 15
Reading left to right on the dot plot, add each leaf value to the matching stem	Stem | Leaf 0 | 0 2 2 3 5 5 6 6 6 7 7 8 8 8 1 | 0 0 0 0 1 1 1 3 3 4 4 6 6 8 2 | 0 1 Key 1 | 5 = 15
With the large number of values in each stem, the data also could be split as shown here.	Stem | Leaf 0 | 0 2 2 3 0 | 5 5 6 6 6 7 7 8 8 8 1 | 0 0 0 0 1 1 1 3 3 4 4 1 | 6 6 8 2 | 0 1 Key 1 | 5 = 15

SC 2 I can determine the range and mode of values from dot plots and stem-and-leaf plots

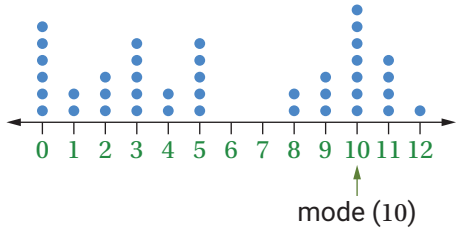
The range of a set of data is the difference between the highest and lowest values.



Stem	Leaf	
0	0 2 2 3 5 5 6 6 6 7 7 8 8 8	lowest value (0)
1	0 0 0 0 1 1 1 3 3 4 4 6 6 8	
2	0 1	highest value (21)

Key 1 | 5 = 15

The mode is the value that occurs the most often (has the highest frequency). For dot plots, this is the value with the most dots above it, while for stem-and-leaf plots, it is the value that has the most leaves.



Stem	Leaf	
0	0 2 2 3 5 5 6 6 6 7 7 8 8 8	
1	0 0 0 0 1 1 1 3 3 4 4 6 6 8	
2	0 1	mode (10)

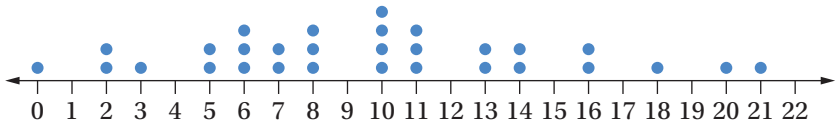
Key 1 | 5 = 15

For a data set that has two values with the same highest frequency, both values are the mode and the data is called 'bimodal'.

Worked example

Determining the range and mode from plots

(a) Determine the range and mode of the data displayed in the dot plot.

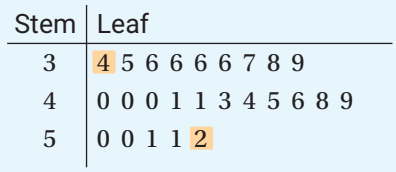
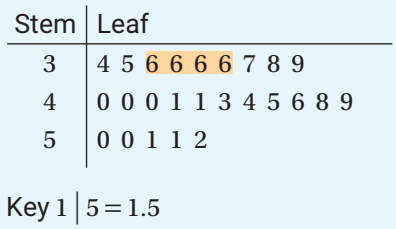


THINKING	WORKING
Identify the lowest and highest values in the data.	The lowest value is 0. The highest value is 21.
Calculate the range by finding the difference between the highest and lowest value.	Range = highest value − lowest value = 21 − 0 = 21
Identify the value with the highest frequency (most dots plotted above it).	The value with the highest frequency is 10.
Write the answer.	For the data in the dot plot, the range of scores is 21 and the mode is 10.

(b) Calculate the range and mode of the data displayed in the stem-and-leaf plot.

Stem	Leaf
3	4 5 6 6 6 6 7 8 9
4	0 0 0 1 1 3 4 5 6 8 9
5	0 0 1 1 2

Key 1 | 5 = 1.5

THINKING	WORKING
Identify the lowest and highest values in the data.	 The lowest value is 3.4. The highest value is 5.2.
Calculate the range by finding the difference between the highest and lowest value.	Range = highest value – lowest value = 5.2 – 3.4 = 1.8
Identify the value with the highest frequency (most leaves).	 The value with the highest frequency is 3.6.
Write the answer.	For the data in the stem-and-leaf plot, the range of values is 1.8 and the mode is 3.6.

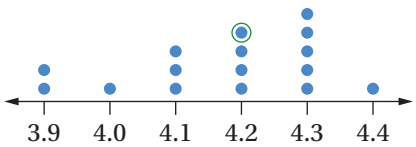
Practice

ANSWERS Page 729

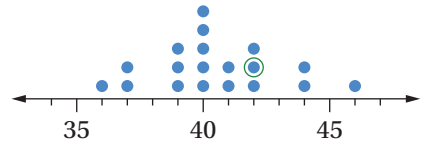
SC 1 I can construct and interpret dot plots

1 Identify the value represented by the circled dots on each plot.

(a)



(b)



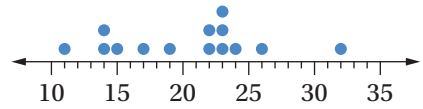
2 Create a dot plot for each of these data sets.

(a) 4, 2, 3, 1, 2, 4, 5, 3, 2, 3, 4, 2, 3, 2, 2

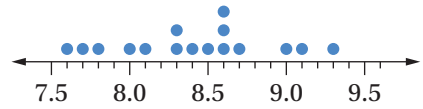
(b) 8.8, 8.6, 8.7, 8.9, 9.0, 8.7, 8.5, 8.6, 8.8, 8.6, 8.7, 8.6, 8.6, 8.8, 8.7, 9.0

3 Create a stem-and-leaf plot from each of these dot plots.

(a)



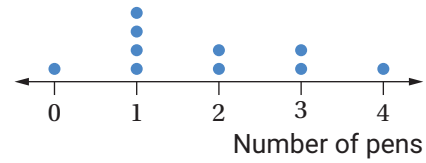
(b)



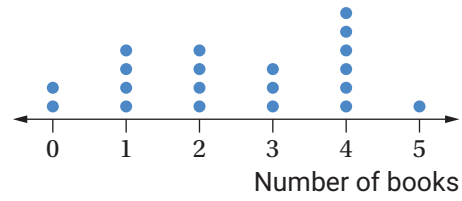
SC 2 I can determine the range and mode of values from dot plots and stem-and-leaf plots

1 Determine the range and mode for the data in each dot plot.

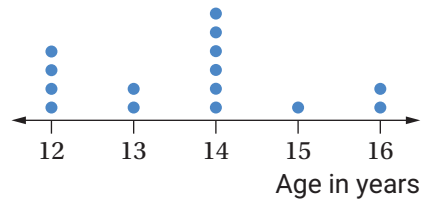
(a) **Number of pens each student possesses**



(b) **Number of books read in a month by students**



(c) **Age of students in Maths club**



2 Calculate the range and mode for the data displayed in each stem-and-leaf plot.

(a)

Stem	Leaf
1	1 4 4 5 7 9
2	2 2 3 3 3 4 6 8
3	2

Key $1 \mid 5 = 15$

(b)

Stem	Leaf
7	6 7 8
8	0 1 3 3 4 5 6 6 6 7
9	0 1 3

Key $1 \mid 5 = 1.5$

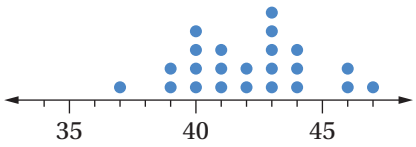
(c)

Stem	Leaf
3	
3	7 9 9
4	1 1 1 2 2 3 3 3 3 3 3 4 4 4
4	6 6 7

Key $3 \mid 5 = 3.5$

3 Identify the mode of each display, and then the value that will make the data set bimodal when added to the data set.

(a)

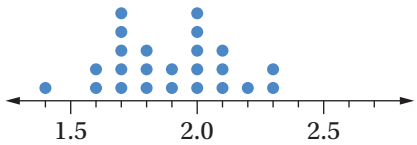


(b)

Stem	Leaf
8	4 4 5 6 6 6 7 7 8 9 9
9	0 0 1 1 1 1 2 2 2 3 4 5 6 6 6 7 9 9
10	0 0 1 2 2 3 3 3 3 3 4 4 5 7 8 8 9 9 9
11	0 1 1 1 2 3 3 4 5 6 6 6 7 9

Key $1 \mid 5 = 15$

4 Identify the largest value that can be added to the following dot plot to give the data set a range of 1.2.



20.3 Determine statistical measures of centre from data displays

Learning intention: To be able to determine statistical measures of centre from data displays

Success criteria:

- **SC 1** I can calculate statistical measures of centre from a dot plot.
- **SC 2** I can calculate statistical measures of centre from a stem-and-leaf plot.
- **SC 3** I can identify outliers in data displays and understand their effects on measures of centre.

Lesson warm-up

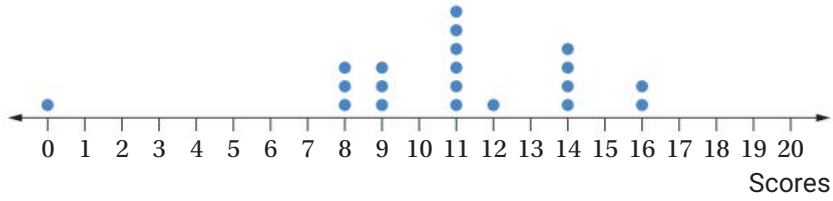
Dart scores

A darts player practised by picking a number on the dartboard and aiming to hit it as often as possible.

The outer black ring scores 0 points.

What can you deduce about the score values hit from this dot plot of the first 20 throws?

Scores for first 20 practice dart throws aiming for a particular value

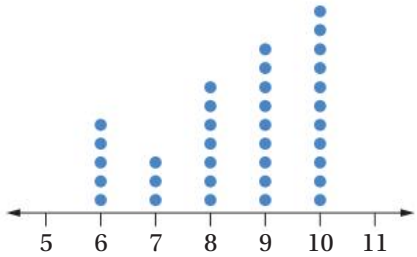


SC 1 I can calculate statistical measures of centre from a dot plot

Statistical data displays can make a data set easier to interpret quickly.

In this dot plot, you can quickly identify that the mode (most common data value) is 10, as this value has the most dots.

The range is 4. The range is the difference between the highest (10) and lowest (6) plotted values.

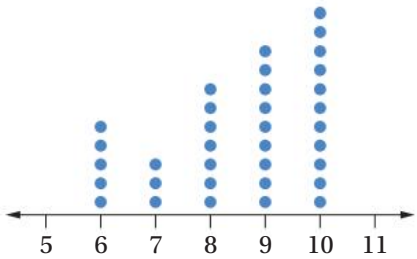


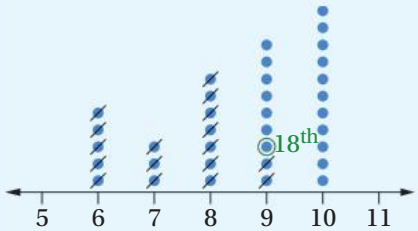
Identifying the other statistical measures of centre — mean and median — is not as easily done. Both will be covered in this section because they use similar methods.

Worked example

Determining median and mean from a dot plot

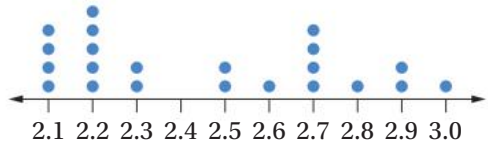
(a) Calculate the median and mean for the data displayed in this dot plot. Write the values accurate to 1 decimal place where required.

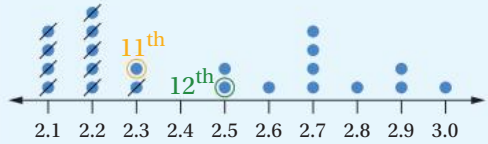


THINKING	WORKING
Determine the position of the median value. The median is the middle value when the data is arranged in ascending order.	There are 35 dots, so there are 35 individual data values. $n = 35$ Since n is odd, the median is the value located in the $\frac{n+1}{2}$ th position. $\frac{35+1}{2} = 18$ Therefore, the median is the data value in the 18th position.
Locate the median value by counting, starting from the bottom dot of the lowest value. Always start from the bottom of each adjacent column of dots.	Counting to the data value at position 18 in order:  the median value is 9.

Use the formula to calculate the mean of the data set.	To calculate the sum of all the data values, multiply each data value by the number of dots plotted above it and add them together. Sum of all values $= (5 \times 6) + (3 \times 7) + (7 \times 8) + (9 \times 9) + (11 \times 10)$ $= 30 + 21 + 56 + 81 + 110$ $= 298$ $\text{Mean } (\bar{x}) = \frac{\text{sum of all values}}{\text{number of values}}$ $= \frac{298}{35}$ $= 8.5 \text{ (1 d.p.)}$ The mean is 8.5, correct to 1 decimal place.
Write the answer.	For the data displayed in the dot plot, the median value is 9 and the mean value is 8.5, correct to 1 decimal place.

(b) Calculate the median and mean for the data displayed in this dot plot. Write the values accurate to 2 decimal places.



THINKING	WORKING
Determine the location of the median value. The median is the middle value when the data is arranged in ascending order.	There are 22 dots, so there are 22 individual data values. $n = 22$ Since n is even, the median is the average of the two values located on either side of the $\frac{n+1}{2}$ th position. $\frac{22+1}{2} = 11.5$ Therefore, the median is the average of the data values in the 11th and 12th positions.
Locate the median value by counting, starting from the bottom dot of the lowest value. Always start from the bottom of each adjacent column of dots.	Counting to the data values in the 11th and 12th position:  The median value is the average of 2.3 and 2.5.

Calculate the median value.	$\frac{2.3+2.5}{2}=\frac{4.8}{2}$ $=2.4$ The median value is 2.4.
Use the formula to calculate the mean of the data set.	To calculate the sum of all the data values, multiply each data value by the number of dots plotted above it and add them together. Sum of all values $=(4\times 2.1)+(5\times 2.2)+(2\times 2.3)+(2\times 2.5)$ $+(1\times 2.6)+(4\times 2.7)+(1\times 2.8)$ $+(2\times 2.9)+(1\times 3.0)$ $=8.4+11+4.6+5+2.6+10.8+2.8+5.8+3.0$ $=54$ $\text{Mean}(\bar{x})=\frac{\text{sum of all values}}{\text{number of values}}$ $=\frac{54}{22}$ $=2.45\ (2\text{ d.p.})$ The mean is 2.45, correct to 2 decimal places.
Write the answer.	For the data displayed in the dot plot, the median value is 2.4 and the mean value is 2.45, correct to 2 decimal places.

SC 2 I can calculate statistical measures of centre from a stem-and-leaf plot

Worked example

Determining the median and mean from a stem-and-leaf plot

(a) Calculate the median and mean values for the data in the stem-and-leaf plot.

Stem	Leaf
2	0 3 3 3 8 8 9
3	1 1 2 2 3 4 6
4	2 2 3 3 5 7 8
5	5 6 7 9
6	1 5

Key $2\,|\,0=20$

THINKING	WORKING
Determine the location of the median value. The median is the middle value when the data is arranged in ascending order.	There are 27 leaves, so there are 27 data values. $n = 27$ Since n is odd, the median is the value located in the $\frac{n+1}{2}$ th position. $\frac{27+1}{2} = 14$ Therefore, the median is the data value in the 14th position.
Locate the median value. Count starting from the lowest leaf on the lowest stem.	Counting to the data value at position 14 in order: Stem | Leaf 2 | 0 3 3 3 8 8 9 3 | 1 1 2 2 3 4 ⑥ 14th 4 | 2 2 3 3 5 7 8 5 | 5 6 7 9 6 | 1 5 Key $2 \mid 0 = 20$ The median value is 36.
Use the formula to calculate the mean of the data set. When calculating the sum of all the data, simplify by multiplying repeated values by the number of times they occur.	Sum of all values $= 20 + (3 \times 23) + (2 \times 28) + 29 + (2 \times 31) + (2 \times 32) + 33 + \ldots + 61 + 65$ $= 1066$ $\text{Mean}(\bar{x}) = \frac{\text{sum of all values}}{\text{number of values}} = \frac{1066}{27} = 39.5 \ (1 \text{ d.p.})$
Write the answer.	For the data displayed in the stem-and-leaf plot, the median value is 36 and the mean value is 39.5, correct to 1 decimal place.

(b) Calculate the mean and median for the data displayed in the split stem-and-leaf plot.

Stem	Leaf
10	0 5 5
10	8 8 8 8 9
11	0 0 1 1 1 2 2 3
11	6 8
12	0 1

Key $10 \mid 5 = 105$

THINKING	WORKING
Determine the location of the median value. The median is the middle value when the data is arranged in ascending order.	There are 20 leaves, so there are 20 data values. $n = 20$ Since n is even, the median is the average of the values located on either side of the $\frac{n+1}{2}$ th position. $\frac{20+1}{2} = 10.5$ Therefore, the median is the average of the data values in the 10th and 11th positions.
Locate the median value. Count starting from the lowest leaf on the lowest stem.	Counting to the data values in the 10th and 11th positions: Stem | Leaf 10 | 0 5 5 10 | 8 8 8 8 9 11 | 0 0 1 1 1 2 2 3 (10th, 11th) 11 | 6 8 12 | 0 1 Key $10 \mid 5 = 105$ The median value is the average of 110 and 111.
Calculate the median value.	$\frac{100+101}{2} = \frac{201}{2}$ $= 110.5$ The median value is 110.5.
Use the formula to calculate the mean of the data set. When calculating the sum of all the data, simplify by multiplying repeated values by the number of times they occur.	$\text{mean } (\bar{x})$ $= \frac{\text{sum of all values}}{\text{number of values}}$ $= \frac{100 + (2 \times 105) + (4 \times 108) + 109 + \ldots + 120 + 121}{20}$ $= \frac{2216}{20}$ $= 110.8$ The mean is 110.8.
Write the answer.	For the data displayed in the stem-and-leaf plot, the median value is 110.5 and the mean value is 110.8.

Worked example

Determining statistical measures from stem-and-leaf plots

This stem-and-leaf plot shows assignment marks out of 30 achieved by 18 students. Determine the mode, range, median and mean (to 1 decimal place) for this data.

Assignment marks out of 30

Stem	Leaf
1	3 5 6 6 6 9
2	0 0 0 1 3 5 5 6 8 9
3	0 0

Key 1 | 3 = 13

THINKING	WORKING
Determine the value of the mode. The mode is the most common value.	The data value 16 occurs 3 times, and the data value 20 also occurs 3 times, so the data set is bimodal. The modes are 16 and 20.
Calculate the range. The range is the difference between the highest and lowest data values.	Range = maximum value − minimum value = 30 − 13 = 17
Determine the location of the median value. The median is the middle value when the data is arranged in ascending order.	There are 18 leaves, so there are 18 data values. $n = 18$ Since n is even, the median is the average of the values located on either side of the $\frac{n+1}{2}$ th position. $\frac{18+1}{2} = 9.5$ Therefore, the median is the average of the data values in the 9th and 10th positions.
Locate the median by counting, starting from the lowest leaf on the lowest stem.	Counting to the data values in the 9th and 10th positions: **Assignment marks out of 30** Stem | Leaf 1 | 3 5 6 6 6 9 2 | 0 0 0 1 3 5 5 6 8 9 3 | 0 0 (9th, 10th) Key 1 | 3 = 13 The median value is the average of 20 and 21.

Calculate the median.	$\frac{20+21}{2}=\frac{41}{2}$ $=20.5$ The median value is 20.5.
Use the formula to calculate the mean of the data set. When calculating the sum of all the data, simplify by multiplying repeated values by the number of times they occur.	Sum of all values $= 13 + 15 + (3 \times 16) + 19 + (3 \times 20) + 21 + 23$ $+ (2 \times 25) + 26 + 28 + 29 + (2 \times 30)$ $= 392$ $\text{Mean}(\bar{x}) = \frac{\text{sum of all values}}{\text{number of values}}$ $= \frac{392}{18}$ $= 21.8 \ (1 \text{ d.p.})$ The mean is 21.8.
Write the answer.	The assignment marks data is bimodal, with modes of 16 and 20. The range of assignment marks is 17. The median assignment mark is 20.5, and the mean mark is 21.8.

SC 3 I can identify outliers in data displays and understand their effects on measures of centre

In a data set, sometimes there are values that stand out because they are much higher or much lower than the rest of the data. Values like this are called outliers.

For example, this stem-and-leaf plot displays amounts of money raised by 30 students in one day.

Stem	Leaf
2	0 0 2 2 5 5 6 9
3	0 3 3 5 5 5 7 8 8 8
4	0 1 2 5 5 5 5 5 9
5	0 0 5
6	
7	
8	
9	9
10	
11	

Key $2 \mid 6 = 26$

The value $99 is much higher than any of the other values in this data set. It is not typical of the other values and is called an outlier.

This outlier might be an error in the data (if the wrong number is recorded), or it might represent one student who made an exceptional fundraising effort.

Effects of outliers

Outliers can have a large effect on the range and mean of data, so it is important to check whether the outliers are errors.

Median and mode are less affected by outliers, so these measures of centre are more useful when analysing data sets containing outliers.

Worked example

Identifying outliers

This data represents the money withdrawn each day from Willa's bank account:

$9, $12, $0, $22, $0, $0, $0, $0, $449, $6, $5, $10, $10, $7, $5

Identify any outliers and suggest reasons for the outliers.

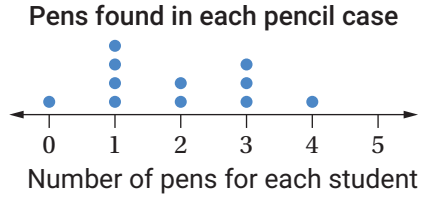
THINKING	WORKING
Look for values that are very high compared with the rest of the data.	The value $449 is much higher than the other values. The next highest value is $22. $449 is an outlier.
Also, look for values that are very low compared with the rest of the data.	The data values of $0 are low, but there are several of these values. They are also still close to other data values, such as $5 and $6. $0 is not an outlier.
Think about each outlier – could it be an error or is it correct?	Possible reasons for the outlier $449: Willa might have bought an expensive item that cost $449. Willa may have accidentally entered $449 instead of $49, which the bank then paid.

Practice

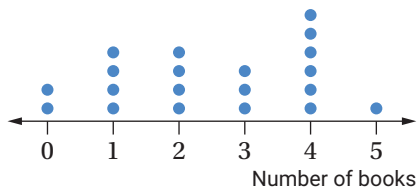
ANSWERS Page 729

SC 1 I can calculate statistical measures of centre from a dot plot

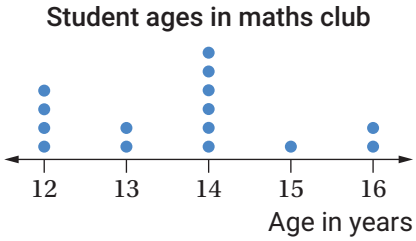
1 Determine the mode, range, median and mean for the number of pens found in each pencil case. Round your answers to 1 decimal place.



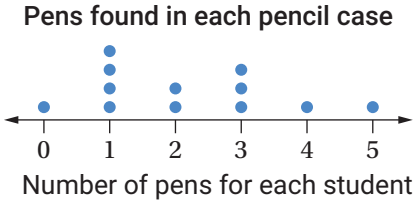
2 Determine the mode, range, median and mean for the number of books read in a month. Round your answers to 1 decimal place.



3 Determine the mode, range, median and mean for the ages of students in a maths club. Round your answers to 1 decimal place.



4 A new student joined the class from Question **1**. The class plans to add 'pens in pencil case' data for this student to the dot plot. There were 5 pens found in the pencil case. Which two measures of centre change when this value is added to the plot?



SC 2 I can calculate statistical measures of centre from a stem-and-leaf plot

1 Determine the median of these split stem-and-leaf plots.

(a)

Stem	Leaf
4	1 1 1 3 3 4
4	5 5 8
5	1 2 2 2 3 4
5	5 6 7 8 8 9
6	0 1 3 3 4
6	5 6

Key $4 \mid 0 = 40$

(b)

Stem	Leaf
4	0 2 3 3 4
4	5 5 8
5	1 2 2 3 4
5	5 6 8 8 8 8 9
6	0 1 2 2 3 3 4
6	6 6 7 7 8 8

Key $4 \mid 0 = 4.0$

2 Calculate the mean of these split stem-and-leaf plots.

(a)

Stem	Leaf
4	0 2 3 3 4
4	5 5 7
5	2 2 3 4
5	8 8 9

Key $4 \mid 0 = 40$

(b)

Stem	Leaf
5	2 2 3 3 4
5	5 6 7 8 8 8 9
6	0 3 3 4 4 4 4
6	8 8 8 9

Key $4 \mid 0 = 4.0$

3 Calculate the mode, range, median and mean for each of these stem-and-leaf plots.

(a)

Stem	Leaf
1	1 2 3 4 4 5 5 5
2	5 7 7
3	2 4 4 6 6

Key $1 \mid 1 = 11$

(b)

Stem	Leaf
1	1 2 3 4
2	5 5 5 7
3	0 2 4 6

Key $1 \mid 1 = 1.1$

(c)

Stem	Leaf
6	5 5 5 7 8
7	1 3 9
8	1 1 1 5 5 9
9	0 2 5

Key $6 \mid 5 = 65$

SC 3 I can identify outliers in data displays and understand their effects on measures of centre

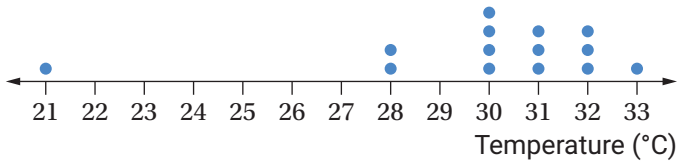
1 Identify any outlier data values in each of these data sets.

(a) Heights of students (in centimetres)

Stem	Leaf
13	2
14	5 8
15	1 2 9
16	3 7
17	4
18	
19	
20	
21	5

Key $14 \mid 5 = 145$

(b) Maximum temperature each day



2 Does it make sense that the outliers belong to the data sets in Question **1**, or do you believe the outliers are errors? Write one reason supporting whether each outlier might be a real data value or an error.

3 This data represents the money withdrawn each day from Nullah's bank account:
$9, $12, $0, $22, $0, $30, $0, $0, $449, $6, $5, $10, $10, $7, $5

(a) Write the data values in ascending order and mark the outlier.

(b) Calculate the mode, range, mean and median.

(c) Remove the outlier from the list and then recalculate the mode, range, mean and median.

(d) After removing the outlier, which summary statistics change the most?

(e) Remove a different value from the data instead of the outlier. Recalculate the mode, range, mean and median. Share your results with the class.

4 For this split stem-and-leaf plot, determine whether the median or mean best represents the statistical measure of centre (sometimes called the central tendency). Give reasons why.

Stem	Leaf
0	4 4 4
0	5 5 5 5 6 7
6	2

Key $0 \mid 4 = 4$

Interpret and describe numerical data displays

20.4

Learning intention: To be able to interpret and describe numerical data displays

Success criteria:

SC 1 I can interpret and choose appropriate data displays.

SC 2 I can describe the shape of a distribution.

Lesson warm-up

Choose me!

Instructions:

Which of these players would you select for your basketball team?
Be prepared to share your reason with others in the class.

Player A:
My mean score is 8 points, with a range of 4.

Player B:
My median score is 8 points, with a range of 10.

Player C:
I have a modal score of 10 points, with a range of 10.

20

SC 1 I can interpret and choose appropriate data displays

Graphing numerical data helps you to visualise the shape of the data set. Certain displays are better for particular types of data.

Discrete data

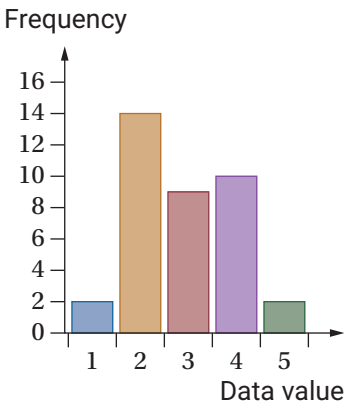
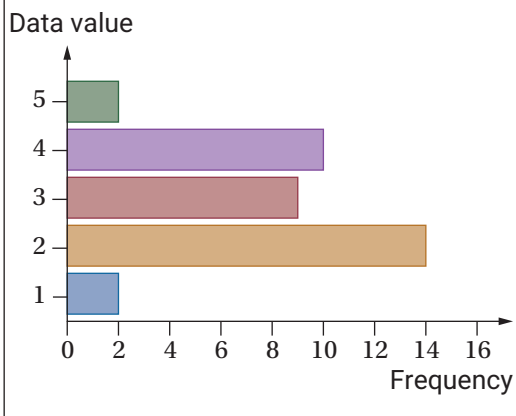
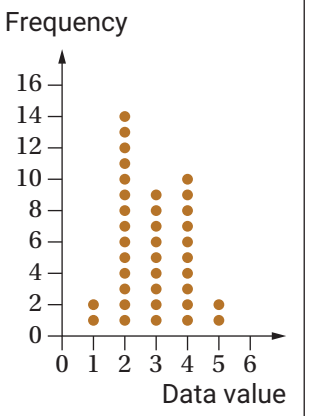
Discrete data can show only specific values; for example, the frequency of different data values. The height or length of bars represents frequency. Since the possible values are limited – usually to whole numbers or data collected by counting – the graph consists of a series of separate columns.

For discrete data, the frequency of data values can be recorded as a column graph, bar graph or dot plot.

Notice that each display has a gap between the data values to indicate that each value is discrete.

The disadvantage of dot plots is that every data value requires you to draw a dot, therefore dot plots are generally used only for small samples of data, approximately 30 data values or less.

Data value	Frequency
1	2
2	14
3	9
4	10
5	2

Column graph (bar graph with vertical bars)	Bar graph (horizontal bars)	Dot plot
		

Stem-and-leaf plots

A stem-and-leaf plot can also be used to record the frequency of discrete data displaying a greater range of data values.

Stem-and-leaf plots also have the benefit of being able to display small samples of continuous data, as well as showing the exact values recorded.

Stem	Leaf
1	1 3
2	0 0 1 1 1 1 4 5 5 6 7 7 8 9
3	0 1 1 2 3 4 4 5 8
4	0 1 1 2 2 3 4 5 6 9
5	8 9

Key $1 \mid 1 = 1.1$

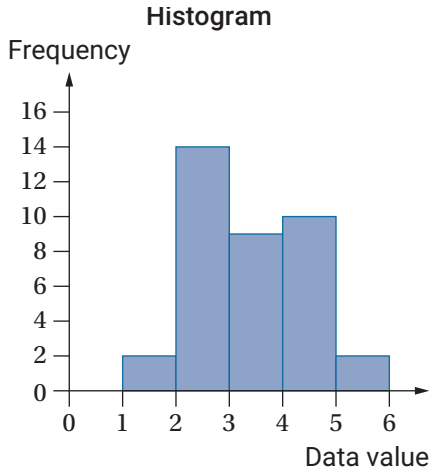
Continuous data

Continuous data represents varying values obtained through measuring. Because all values are possible, a histogram with the columns joined is used in place of a column graph. Each column in a histogram represents the frequency of a range of values.

In the histogram shown, the first column represents numbers from 1.0 and up to, but not including, 2.0. Only one of the end values for each range is included to avoid counting those values twice.

Notice that a histogram does not have gaps between the bars when displaying continuous data.

Data value	Frequency
1–<2	2
2–<3	14
3–<4	9
4–<5	10
5–<6	2



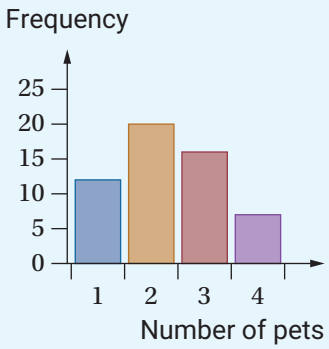
Worked example

Choosing and constructing data displays

Display the data in each of the following frequency tables using an appropriate graph.

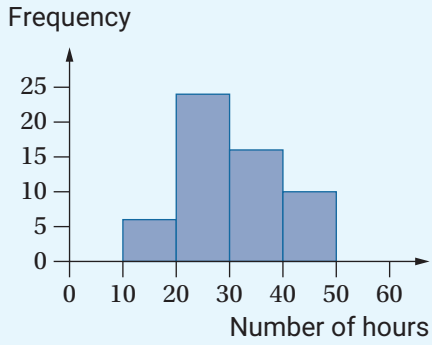
(a) The survey recorded the number of pets in each household.

Number of pets	Frequency
1	12
2	20
3	16
4	7

THINKING	WORKING
Determine the type of data recorded.	The data is numerical and discrete since the number of pets can only be a whole number and is determined by counting. There are 55 data values recorded.
Choose an appropriate data display.	A column graph or bar graph should be used.
Determine a suitable scale. Remember to extend the scale slightly beyond the highest frequency.	The maximum frequency is 20, so a scale from 0 to 25 is appropriate.
Construct a data display.	

(b) The survey recorded the number of hours spent on social media per week.

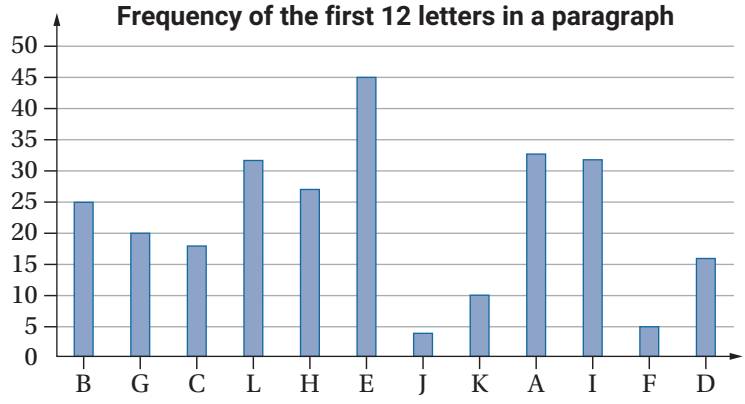
Number of hours	Frequency
10–<20	6
20–<30	24
30–<40	16
40–<50	10

THINKING	WORKING
Determine the type of data recorded.	The data is numerical and continuous since the data records a measured time.
Choose an appropriate data display.	The exact times are not recorded. Therefore, a histogram should be used.
Determine a suitable scale.	The data is grouped: 10 up to less than 20 20 up to less than 30, etc. Note: The first value of each group range is included. Each column needs to represent a range of 10. The maximum frequency is 24, so a scale from 0 to 25 is appropriate.
Construct a data display.	

Exploration Task

Interpreting a column graph

Suggest a way to make this graph easier to interpret.

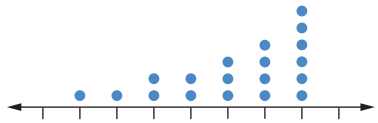
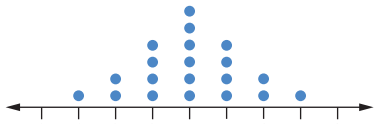
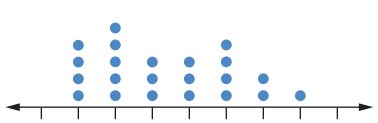


SC 2 I can describe the shape of a distribution

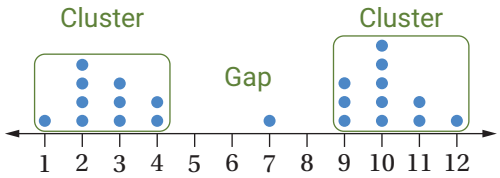
To describe the shape and spread of data presented in displays such as dot plots and stem-and-leaf plots, the following terms can be used:

- symmetrical
- skewed (positively or negatively)
- clusters
- gaps.

Consider the following displays.

Negatively skewed	Symmetrical	Positively skewed
		

Clusters and gaps



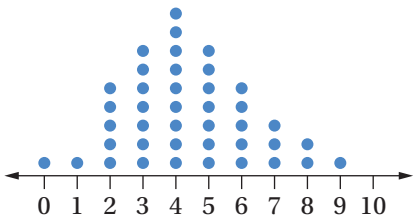
The data displayed in this dot plot has two clear data value groupings with a 'gap' in between. These groups of data are called 'clusters'.

Worked example

Describing the shape of data

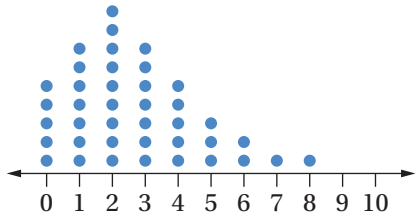
Describe the spread of the data in these displays.

(a)



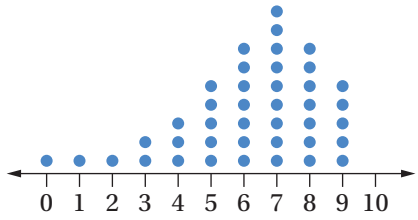
THINKING	WORKING
Determine whether the data is clustered. Clustered data has more than one peak.	The data has a single peak. The data is not clustered.
Determine whether the data is symmetrical or skewed to one side.	The data is symmetrical, evenly spread on either side of the peak.

(b)



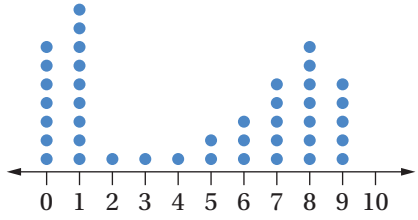
THINKING	WORKING
Determine whether the data is clustered. Clustered data has more than one peak.	The data has a single peak. The data is not clustered.
Determine whether the data is symmetrical or skewed to one side.	The data is not symmetrical.
Determine whether the data is positively skewed (trails to the right) or negatively skewed (trails to the left).	The data trails to the positive side (right side), so it is positively skewed.

(c)



THINKING	WORKING
Determine whether the data is clustered. Clustered data has more than one peak.	The data has a single peak. The data is not clustered.
Determine whether the data is symmetrical or skewed to one side.	The data is not symmetrical.
Determine whether the data is positively skewed (trails to the right) or negatively skewed (trails to the left).	The data trails to the negative side (left side), so it is negatively skewed.

(d)



THINKING	WORKING
Determine whether the data is clustered. Clustered data has more than one peak.	The data has more than one peak. The data is clustered.

Practice

ANSWERS Page 730

SC 1 I can interpret and choose appropriate data displays

1 Display each of the following frequency tables as a:

(i) column graph

(ii) bar graph

(iii) dot plot.

(a)

Category	Frequency
A	1
B	3
C	7
D	2
E	4

(b)

Category	Frequency
1	1
2	3
3	7
4	2
5	4

(c)

Category	Frequency
1	10
2	13
3	7
4	12
5	4

2 Describe when a bar or column graph or a dot plot would be a suitable display.

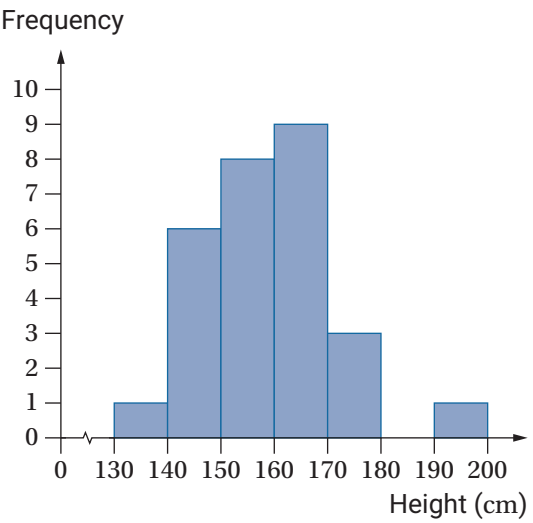
3 The following data represents the height of students in a maths class. Describe the similarities and differences between the displays of continuous numerical data.

Stem-and-leaf plot

Stem	Leaf
13	4
14	2 2 3 5 6 9
15	0 0 1 6 7 8 8 9
16	0 1 1 1 3 4 6 7 8
17	1 3 4
18	
19	5

Key 13 | 4 = 134 cm

Histogram



4 Display each of the following frequency tables as either a bar graph, dot plot or histogram and explain your choice of data display.

(a)

Data value	Frequency
1	4
2	8
3	3
4	1

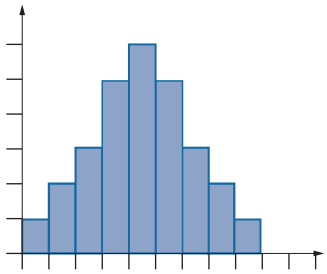
(b)

Data value	Frequency
10– < 20	7
20– < 30	15
30– < 40	24
40– < 50	12

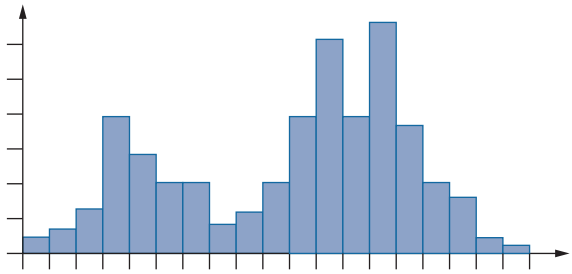
SC 2 I can describe the shape of a distribution

1 Describe the shape of the data as positively skewed, negatively skewed, symmetrical or clustered.

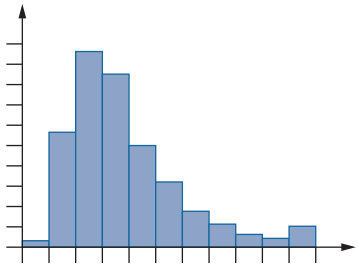
(a)



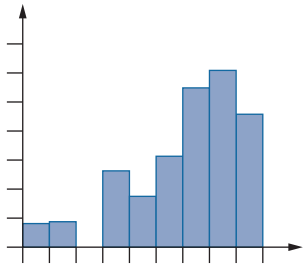
(b)



(c)



(d)



2 Draw a dot plot showing:

(a) a symmetrical distribution

(b) data that is positively skewed

(c) data that is negatively skewed

(d) data that is clustered.

3 Consider this data.

15, 16, 16, 17, 17, 17, 18, 18, 18, 18, 19, 19, 19, 20, 22

(a) Draw a dot plot to represent the data.

(b) Describe the shape of the data distribution.

4 Consider this data.

Stem	Leaf
1	2 4 6
2	1 1 3 4 5 6 7 7 9
3	2 5 7 8
4	3
5	4 7 8
6	2 2 3 3 3 4 5 6 6 7 8 8 9
7	1 3 4 4 5 6 7 7 8
8	1 5 7 9

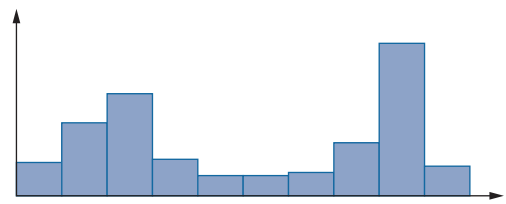
Key $1 \mid 2 = 12$

(a) Using interval sizes of 10, draw a histogram to represent the data.

(b) Describe the shape of the data distribution.

(c) Turn the stem-and-leaf plot on its side. What do you notice, compared with the histogram?

5 Why do we say that a graph like this is bimodal, even though the peaks of both clusters are different heights?



TOPIC REVIEW

Multiple choice

ANSWERS Page 752

1 This original data, 15, 18, 19, 21, 24, 27, 27, 28, 30, 32, has been placed into a stem-and-leaf plot as shown.

Stem	Leaf
1	5 8 9
2	1 4 7 7 8
3	0 2

The most appropriate key for the plot is:

A 1|5 = 1.5 **B** 1|5 = 15 **C** 1|5 = 150 **D** 2|2 = 2.2

2 This original data,1.5, 1.8, 1.8, 2.2, 2.4, 2.7, 2.7, 2.8, 3.4, 3.4, has been placed into a stem-and-leaf plot as shown.

Stem	Leaf
1	5 8 8
2	2 4 7 7 8
3	4 4

The most appropriate key for the plot is:

A 1|5 = 1.5 **B** 1|5 = 15 **C** 1|5 = 150 **D** 3|2 = 3.2

3 The value of the highlighted number in this split stem-and-leaf plot is:

Stem	Leaf
4	0 2 3 3 4
4	5 5 8
5	1 2 2 3 4
5	5 6 7 8 8 8 9
6	0 1 3 3 3 3 4

Key 4 | 0 = 4.0

A 4.0 **B** 5.6 **C** 5.56 **D** 56

4 A split stem-and-leaf plot is used to:

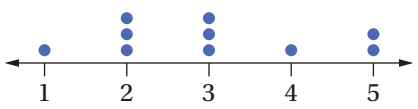
A spread the data out more

B cope with data with decimal places

C represent data with more than three figures

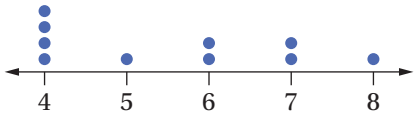
D show the spread of closely grouped data more clearly

5 This type of graph is known as a:



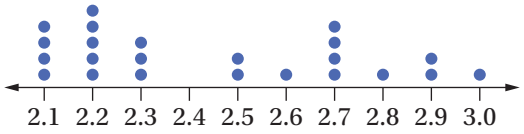
A bar graph **B** line graph **C** histogram **D** dot plot

6 In this diagram, the frequency of the number 7 is:



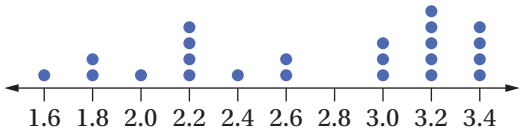
A 5 **B** 4 **C** 2 **D** 1

7 How many data values in the dot plot have a value of 2.4 or less?



A 0 **B** 9 **C** 11 **D** 12

8 Which stem-and-leaf plot is an accurate representation of this dot plot?



A

Stem	Leaf
1	6 8 8 0
2	2 2 2 2 4 6 6 0 0 0
3	2 2 2 2 2 4 4 4 4

Key 1 | 5 = 1.5

B

Stem	Leaf
1	6 8 8
2	0 2 2 2 2 4 6 6
3	0 0 0 2 2 2 2 2 4 4 4 4

Key 1 | 5 = 1.5

C

Stem	Leaf
1	6 8 8
2	0 0 0 2 2 2 2 4 6 6
3	2 2 2 2 2 4 4 4 4

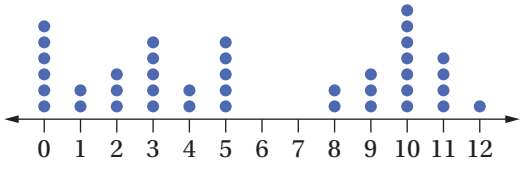
Key 1 | 5 = 15

D

Stem	Leaf
1	6 8 8
2	0 2 2 2 2 4 6 6
3	0 0 2 2 2 2 4 4 4 4

Key 1 | 5 = 15

Use the following dot plot to answer Questions **9** and **10**.



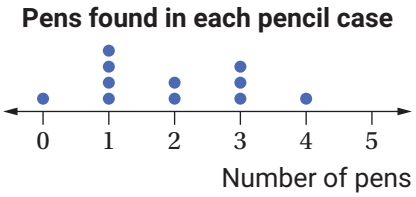
9 The range of this data is:

A 12 **B** 11 **C** 10 **D** 0

10 The mode of this data is:

A 12 **B** 11 **C** 10 **D** 0

Use the following dot plot to answer Questions **11** and **12**.



11 The median of this data is:

A 1 **B** 2 **C** 3 **D** 4

12 The mean of this data is:

A 11 **B** 5 **C** 4 **D** 1.9

Short answer

1 For the following stem-and-leaf plot, calculate:

Stem	Leaf
1	1 2 2
1	5 7
2	1 2 3 4
2	6 9

Key 1|1 = 11

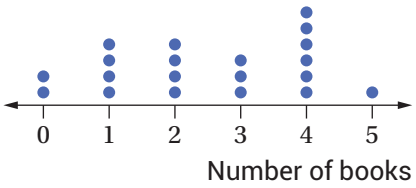
(a) the mean to one decimal place

(b) the median

(c) the mode

(d) the range.

2 For the following dot plot, calculate:



(a) the mean to one decimal place

(b) the median

(c) the mode

(d) the range.

3 Create a stem-and-leaf plot for this data.

23 28 32 45 48 61 42 43 65 56 31 33 42

24 31 43 47 20 23 29 32 55 57 28 34 36

4 Create a split stem-and-leaf plot from this stem-and-leaf plot.

Stem	Leaf
2	3 4 4 5 5 7 7 8 8 8 9
3	1 1 1 1 3 4 5 5 6 7 7 8 9
4	2 2 3 3 4 4 5 5 6 6 7 9 9
5	0 1 3 4 5 6 6 7 8 8 9

Key 2|3 = 23

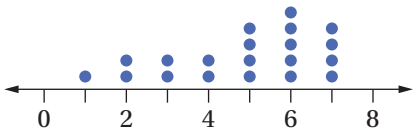
5 Create a dot plot from this stem-and-leaf plot displaying continuous data.

Stem	Leaf
3	2
6	3 3 8
7	5 6
8	1

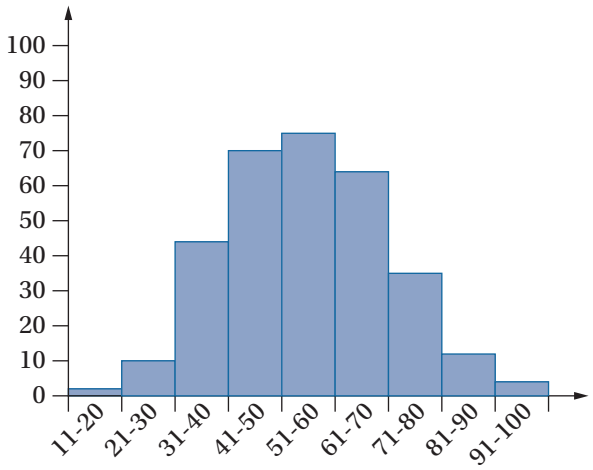
Key 3|2 = 3.2

6 Describe the skew of data in these displays.

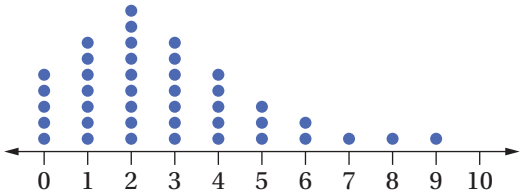
(a)



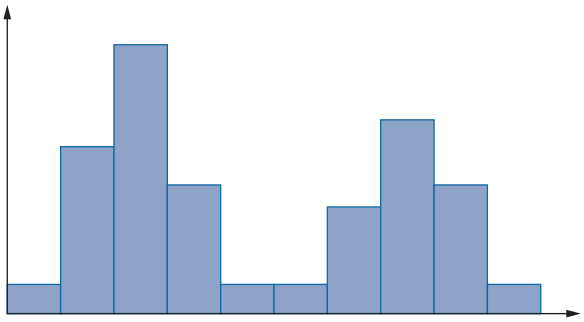
(b) Frequency



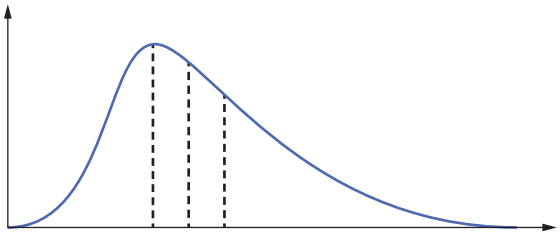
(c)



7 The following data is described as bimodal, even though the peak heights are different. Explain why.

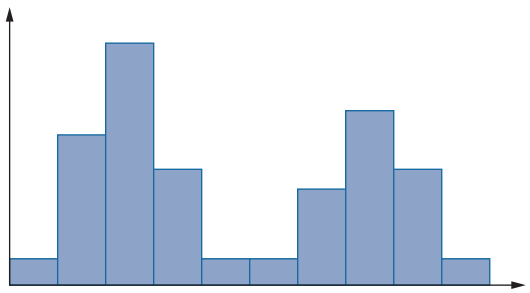


8 Copy this diagram and label an estimate of the mean, median and mode.

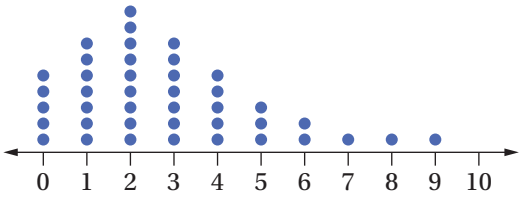


9 Is the data shown in each of these representations continuous or discrete? How do you know?

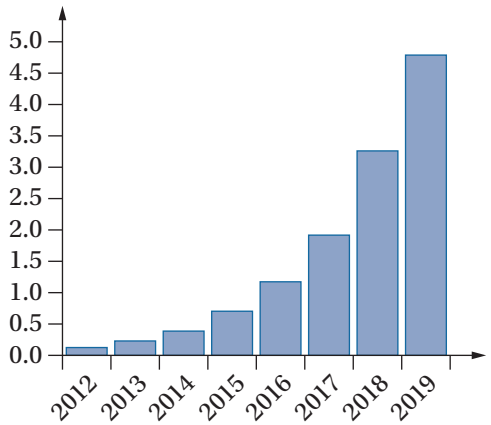
(a)



(b)



(c)



Extended response

1 The ages of the spectators at an U12 soccer match are shown in this split stem-and-leaf plot.

Stem	Leaf
1	1 2 4
1	5 7
2	1 2 3 4
2	7 9
3	1 2 3 3
3	4 9 9 9

Key 1|1 = 11

(a) Which figure is in the incorrect place? How do you know?

(b) Why was a split stem-and-leaf plot chosen?

(c) Calculate:

(i) the mode

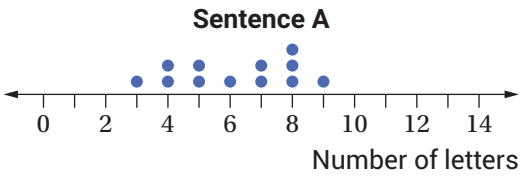
(ii) the median

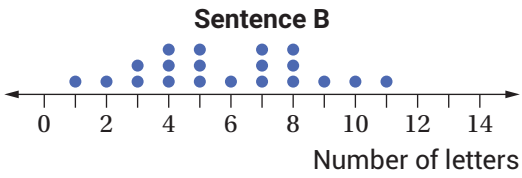
(iii) the mean.

(d) Which measure of central tendency is not appropriate here? Why?

(e) Calculate the range.

2 A single sentence is taken from two different texts. The number of characters per word in each sentence is recorded in these dot plots.





(a) Which sentence has the most words?

(b) Which sentence has the greater range?

(c) Which sentence has the greater median?

(d) Why isn't there a mode for Sentence B?

(e) Which sentence do you think would be easier to read? Explain your answer by talking about the measures of central tendency.

3 The age of parents attending a parent meeting at the school is recorded in this stem-and-leaf plot.

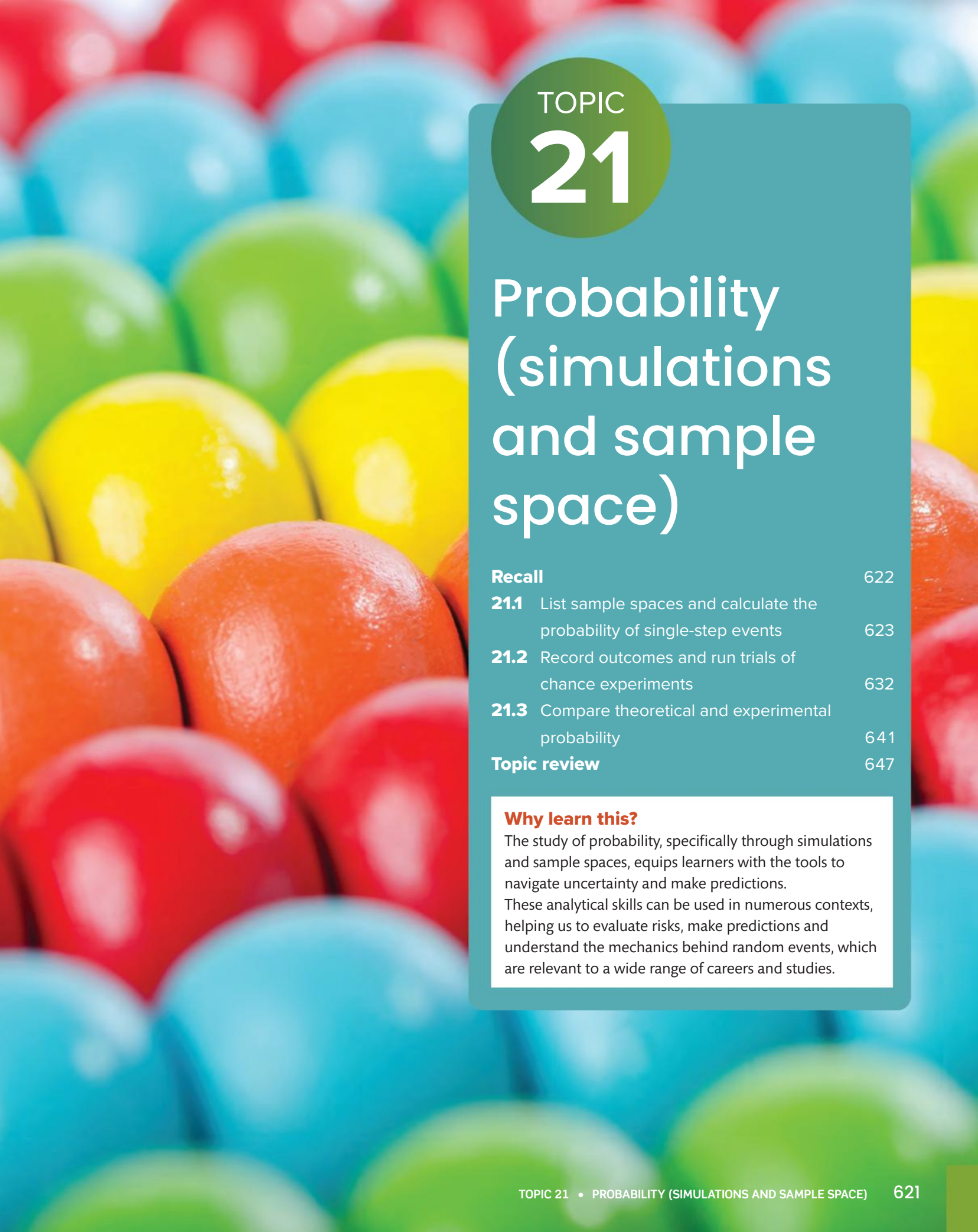
Stem	Leaf
2	7 8 8
3	3 3 4
3	5 6 6 6 8 9 9
4	0 2 2 3 3 3 4
4	5 5 5 7 7 8 9 9

(a) How many parents were less than 40?

(b) What was the median age of the parents?

(c) What was the mode?

4 Write down what you consider to be the five most important things you have learnt about statistics in this topic.



TOPIC
21

Probability (simulations and sample space)

Recall 622

21.1 List sample spaces and calculate the probability of single-step events 623

21.2 Record outcomes and run trials of chance experiments 632

21.3 Compare theoretical and experimental probability 641

Topic review 647

Why learn this?

The study of probability, specifically through simulations and sample spaces, equips learners with the tools to navigate uncertainty and make predictions.
These analytical skills can be used in numerous contexts, helping us to evaluate risks, make predictions and understand the mechanics behind random events, which are relevant to a wide range of careers and studies.

RECALL

ANSWERS Page 732

I can identify and complete patterns of numbers

1 Copy these sequences and fill in the missing terms, by finding whether you need to add, subtract, multiply or divide by a constant number.

(a) 11, 19, ___, ___, ___, 51, ___

(b) 23, ___, 47, ___, 71, 83, ___

(c) 80, ___, ___, ___, 5, ___

(d) 10, ___, 250, ___, ___, 31 250

I can write fractions in their simplest form

1 Simplify each of the following fractions into their simplest form.

(a) $\frac{24}{30}$ **(b)** $\frac{30}{45}$ **(c)** $\frac{42}{56}$ **(d)** $\frac{28}{98}$

I can use a scale to represent the chance of an event happening

1 Copy and complete each of these sentences about the chance of an event, using the following list of numbers/words (not all are used).

0, 25, 50, 75, 100, always, never, greater, lower, impossible, high, low

(a) An event that has no chance of happening has a ___% chance of occurring.

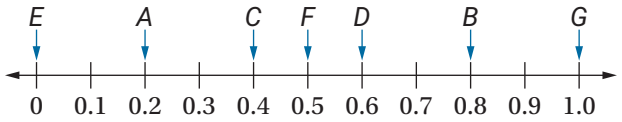
(b) An event that will _____ happen has a 100% chance of occurring.

(c) An event that has the same chance of failure or success has a _____% chance of succeeding.

(d) An event with a 75% chance of success has a _____ chance of succeeding than failing.

(e) An event with a 5% chance of success has a very _____ chance of failure.

I can use a scale of 0 to 1 to estimate the chances of events



1 Select the letter that best matches each chance description.

(a) A cyclist has a very good chance of finishing the race.

(b) The possibility of rolling a number 7 on a single six-sided die is impossible.

(c) The sun will always set in the West.

(d) There is a very small chance of staying dry when walking on a rainy day.

(e) The chance of flipping a head on a coin is the same as a tail.

(f) While guessing the outcome of a card colour in a deck, you were close to even, but more often wrong.

(g) Due to there being one more orange marble in a bag than there were grey marbles, the chance of drawing an orange marble was slightly better.

List sample spaces and calculate the probability of single-step events

21.1

Learning intention: To be able to list sample spaces and calculate the probability of single-step events

Success criteria:

SC 1 I can describe chance experiments using the language of probability.

SC 2 I can list sample spaces for single-step experiments.

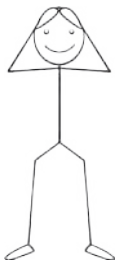
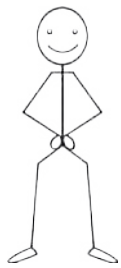
SC 3 I can calculate a theoretical probability using a sample space.

Lesson warm-up

Heads or tails?

Your teacher will toss a coin. To win this game, you need to be the last person standing.

Who thinks they are the luckiest person in the room?

Hands on head represents 'Heads'	Hands on hips represents 'Tails'
	

SC 1 I can describe chance experiments using the language of probability

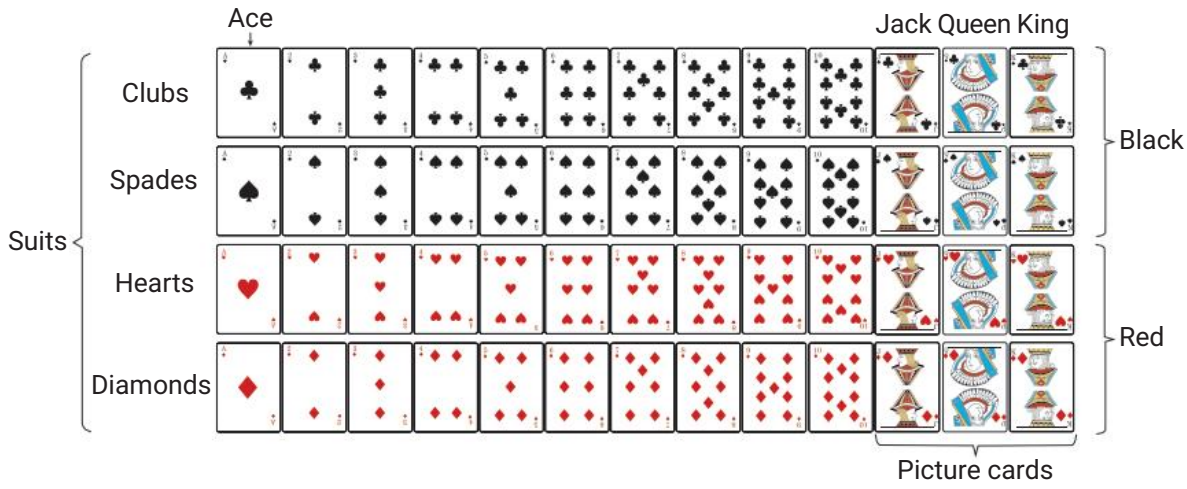
When learning about probability, it is helpful to familiarise yourself with some key terms that are commonly used. Here are a few to get you started.

Term	Definition
Probability	The chance of an event occurring. The probability of an event is given by a numerical value between 0 and 1. Probability is a scale, ranging from 0 being impossible to 1 being certain.
Experiment	The method of obtaining a possible result; for example, tossing a coin or rolling a die.
Outcome	The result of an experiment; for example, an outcome of flipping a coin could be heads.
Sample space	A list of all the possible outcomes of an experiment, written as a set S. We list every different outcome, but do not repeat outcomes; for example, the sample space for rolling a die is $S = \{1, 2, 3, 4, 5, 6\}$.
Event	A single result from the sample space; for example, rolling an even number on a die.

Worked example

Using the language of probability

An experiment is conducted by selecting a card from a standard deck of 52 playing cards.



(a) In an experiment, a card is selected at random, and the suit recorded. Identify the sample space S.

THINKING	WORKING
Recall the definition of a sample space.	A sample space is a list of all possible outcomes.
Identify all possible outcomes.	There are four suits: clubs, spades, hearts, diamonds.
Answer the question.	The sample space is $S = \{\text{clubs, spades, hearts, diamonds}\}$.

(b) Determine the number of outcomes for the event 'selecting a card and recording the colour'.

THINKING	WORKING
Recall the definition of an outcome.	The outcomes are the possible results of a chance experiment.
Describe the event.	Selecting a card from the pack, then recording the colour.
Determine the outcomes in the event.	There are only two card colours in a deck. The cards are either red (hearts and diamonds) or black (clubs and spades).
Answer the question by counting the number of outcomes.	There are two outcomes.

(c) A sample space contains 52 outcomes. Describe the event.

THINKING	WORKING
Recall the definition of an outcome.	The outcomes are the possible results of a chance experiment.
Consider the possible sample space that would have this number of outcomes.	In this experiment, there are 52 cards. This would correspond to a sample space with 52 outcomes.
Identify the event that would have this corresponding sample space.	Drawing a card at random and recording the outcome would have 52 different possibilities. This would be the event.

SC 2 I can list sample spaces for single-step experiments

Identify the set of all possible outcomes for a random experiment.

List all non-repeated outcomes for the experiment as the sample space S, with each outcome in braces.

For example, flipping a coin, the sample space is:

S = {Head, Tail}.

Flipping two coins simultaneously: S = {Head Head, Head Tail, Tail Tail}.

Worked example

Listing a sample space

Consider the following experiments.

(a) Describe an experiment that results in the sample space $S = \{1, 2, 3, 4, 5, 6\}$.

THINKING	WORKING
Recall the definition of a sample space.	A sample space is a list of all possible outcomes.
Describe what is shown in the sample space.	The possible outcomes are 1, 2, 3, 4, 5 and 6.
Describe an experiment with the possible outcomes listed.	Rolling a standard die or spinning a spinner numbered 1 to 6.

(b) List the sample space for the experiment 'flipping a coin twice'.

THINKING	WORKING
List each of the outcomes for a single toss of a coin.	When tossing a coin, the outcomes are head, H, and tail, T.
Systematically list all possible outcomes when tossing a coin twice.	Head, Head: HH Head, Tail: HT Tail, Head: TH Tail, Tail: TT
List the sample space by writing all outcomes in braces.	$S = \{HH, HT, TH, TT\}$

SC 3 I can calculate a theoretical probability using a sample space

To calculate the probability of a favourable outcome:

- identify the total number of possible outcomes for a given event
- identify the number of favourable outcomes for the event (the desired outcomes for the event that occur).

The theoretical probability is found by dividing the number of favourable outcomes by the total number of outcomes:

$$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

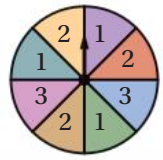
Probabilities are generally written as a fraction. Fractions are generally written in simplest form.

When comparing probabilities, you can write fractions with the same denominators.

Unless asked to do so, you should never change a probability to a decimal, because accuracy may be lost after rounding.

Worked example

Calculating the theoretical probability



The spinner shown above is spun one time. Calculate the probability of:

(a) spinning a 1

THINKING	WORKING
Count the possible outcomes.	There are 8 possible outcomes when spinning the spinner.
Count the favourable outcomes.	There are three 1s.
Substitute the values into the formula to calculate the probability.	$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ $= \frac{3}{8}$
Write the answer.	The probability of spinning a 1 is $\frac{3}{8}$.

(b) not spinning a 2

THINKING	WORKING
Count the possible outcomes.	There are 8 possible outcomes.
Count the favourable outcomes.	There are 5 outcomes that are 'not a 2'.
Substitute the values into the formula to calculate the probability.	$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ $= \frac{5}{8}$
Write the answer.	The probability of not spinning a 2 is $\frac{5}{8}$.

(c) spinning a 3

THINKING	WORKING
Count the possible outcomes.	There are 8 possible outcomes.
Count the favourable outcomes.	There are two 3s.
Substitute the values into the formula to calculate the probability and simplify the answer. Note: You can simplify the fraction if you wish, however the calculation might be more meaningful if it is not simplified.	$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ $= \frac{2}{8}$ $= \frac{1}{4}$
Write the answer.	The probability of spinning a 3 is $\frac{2}{8}$ or $\frac{1}{4}$.

(d) spinning a 4.

THINKING	WORKING
Count the possible outcomes.	There are 8 possible outcomes.
Count the favourable outcomes.	There are no 4s on the spinner, so there are 0 favourable outcomes.
Substitute the values into the formula to calculate the probability.	$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ $= \frac{0}{8}$
Write the answer.	It is impossible to spin a 4. The probability of spinning a 4 is 0.

Practice

ANSWERS Page 732

SC 1 I can describe chance experiments using the language of probability

1 Identify each of the following statements about rolling a standard 6-sided die, as an event, a sample space or an experiment. Explain your decision.

(a) Rolling an even number

(b) Rolling a single die 20 times

(c) {1, 2, 3, 4, 5, 6}

2 An experiment is conducted by tossing coins. Identify each of the following statements as an event, a sample space or an experiment. Explain your decision.

(a) Tossing a coin 10 times

(b) Obtaining two heads when tossing a coin twice

(c) Head-Head, Head-Tail, Tail-Head, Tail-Tail

3 The sample space for rolling a 6-sided die is {1, 2, 3, 4, 5, 6}.
Which of the following are possible events (yes or no)?

(a) Rolling a 4 **(b)** Rolling a 5 **(c)** Rolling a 7

(d) Rolling an even number **(e)** Rolling an odd number **(f)** Rolling a fraction

4 Marty writes the following statements about an experiment conducted by spinning the spinner. Choose from the words sample space, event and outcome to complete the sentences.

(a) The _____ was red, blue, green.

(b) The _____ was spinning green.

(c) The most likely _____ was red.

SC 2 I can list sample spaces for single-step experiments

1 List the sample space for:

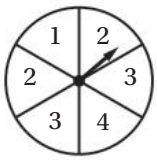
(a) choosing a King from a normal deck of cards

(b) tossing a coin

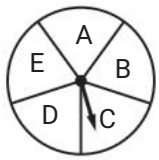
(c) rolling a 10-sided die.

2 List the sample space for each of these spinners.

(a)

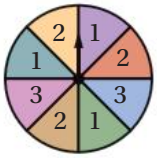


(b)



(c)

3 **(a)** List the sample space for this spinner.



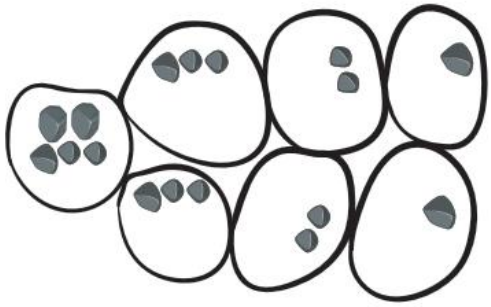
(b) Are all numbers equally likely to occur? Explain why or why not.

4 Given these sample spaces, provide a possible experiment for each.

(a) {Head, Tail} **(b)** {1, 2, 3, 4} **(c)** {W, X, Y, Z}

(d) {A, E, I, O, U} **(e)** {odd, even}

5 In some Aboriginal and Torres Strait Islander communities, children constructed 'spinners' out of leaves that floated in the wind. They created games, with points awarded for highest flight, longest flight, one-handed catches and behind-the-back catches. These games were called Koara, from the word 'play' in the language of the people who once inhabited Sunday Island off the coast of Western Australia.
One version of the game involved drawing circles on the ground, with points awarded when a 'spinner' landed in a circle.
Here is a set of possible targets. The points awarded for landing in a particular circle are designated by the number of rocks placed inside each circle.



(a) How many points are awarded if the spinner lands outside any of the circles?

(b) List the sample space for the points of this game.

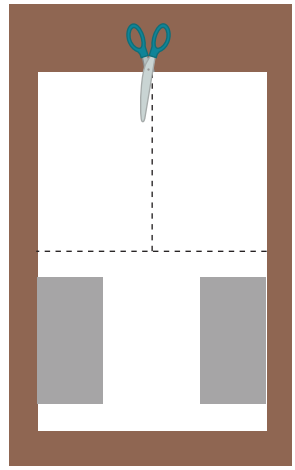
(c) Is each score equally likely? What factors would influence this?

(d) Create some rules and a winning score target so that you could explain the game to another person before you begin playing.

6 The game of Koara as described in Question **5** can be played by creating a spinner as follows.

- Cut the shape shown in the diagram from a piece of A4 paper. Fold and add the paperclip as shown.
- Draw circles on the ground and allocate points. The hardest target must have the most points, but any configuration of the circles can be used.
- Have a few practice rounds to establish reasonable positions to put the targets.
- Keep track of the results and use these to describe and compare the probabilities for each target being hit after you have played the game.

Write a paragraph describing the rules, the scoring system and what you found out about allocating points to each circle. What change would you make to the game to make it more fun to play?



SC 3 I can calculate a theoretical probability using a sample space

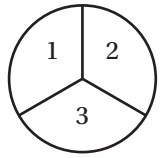
1 Consider the spinner shown at right.

(a) List the sample space.

(b) Calculate the probability of spinning a 2.

(c) Calculate the probability of not spinning a 2.

(d) Calculate the probability of spinning a 4.



2 For the spinner shown at right, determine the:

(a) sample space

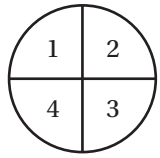
(b) probability of spinning a 1

(c) probability of spinning a 3

(d) probability of not spinning a 4

(e) probability of spinning an even number

(f) probability of spinning a 5.



3 A card is chosen from a standard deck of 52 playing cards. Determine the probability of drawing:

(a) an ace **(b)** not an ace **(c)** a 10

(d) a heart **(e)** a black card **(f)** the five of diamonds.

4 A single card is chosen from a standard deck of 52 playing cards.
Determine the probability of drawing:

(a) a red ace **(b)** a black picture card **(c)** an even-numbered diamond

(d) a red 4 or 5 **(e)** a black 6, 7 or 8.

5 Zach selects a pair of Kings and a pair of Queens from a standard deck of 52 playing cards, with one more card to be selected. Determine:

(a) the number of cards remaining in the deck

(b) the number of Kings remaining in the deck

(c) the probability of finishing with 3 Kings and 2 Queens

(d) the probability of finishing with 2 Kings and 3 Queens

(e) the probability of finishing with 2 Kings and 2 Queens and a card that is not a King or a Queen

(f) the probability of finishing with 3 of one type and 2 of the other.

6 Select a letter from the word 'maths' at random.

(a) List the sample space.

(b) Determine the probability of drawing an S.

(c) Determine the probability of not drawing an S.

(d) Determine the probability of drawing a vowel.

(e) Determine the probability of drawing a consonant.

7 Select a letter from the word 'mathematics' at random.

(a) List the sample space.

(b) Determine the probability of drawing an S.

(c) Determine the probability of not drawing an S.

(d) Determine the probability of drawing a vowel.

(e) Determine the probability of drawing a consonant.

8 Two friends cannot agree on a movie to see, so they go to an 8-cinema complex and buy tickets for a movie that will start about 15 minutes after they arrive. There are 3 action movies, 2 drama movies, 2 comedies and 1 horror movie.

(a) List the sample space for the genre of movies they could see.

(b) Determine which genre they are most likely to buy tickets for, given the random nature of the decision. Explain why.

(c) Determine the probability of seeing a drama movie.

(d) Determine the probability of seeing a horror movie.

21.2 Record outcomes and run trials of chance experiments

Learning Intention: To be able to record outcomes and run trials of chance experiments

Success criteria:

SC 1 I can run trials and record the outcomes of a single-step chance experiment.

SC 2 I can run large numbers of trials of a chance experiment.

Lesson warm-up

What happens next?

A coin is flipped five times and the results are listed as:
Tail, Tail, Tail, Tail and Tail.

What happens on the sixth flip of this coin? Explain.

SC 1 I can run trials and record the outcomes of a single-step chance experiment

Theoretical probability can be verified by running large numbers of trials of an appropriate experiment.

Worked example

Simulating an experiment

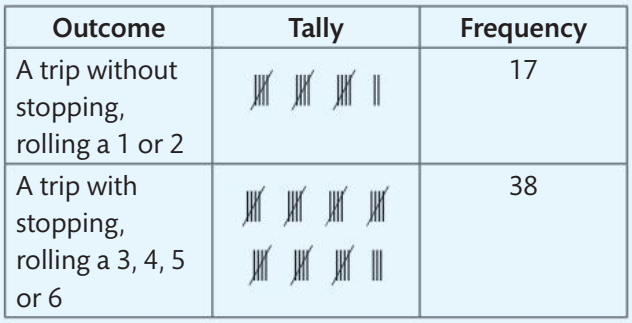
Maisie drives along the same route every day to university, passing through only one traffic light. The chance of the light being green and her being able to continue her trip without stopping is 1 in 3.

Maisie drives to university 55 times each semester.

(a) Estimate the number of times Maisie will stop at the traffic light.

THINKING	WORKING
Interpret the problem given.	The chance of Maisie reaching a green light is 1 in 3. Therefore, the chance of Maisie needing to stop is $\frac{2}{3}$.
Calculate the expected number by multiplying the probability of success with the number of trials.	$\frac{2}{3}$ of $55 = \frac{2}{3} \times 55$ $= 36.66\ldots$ ≈ 37
Write the answer.	During the 55 trips, Maisie can expect to stop approximately 37 times.

(b) Design an experiment and run a trial to determine the number of times Maisie would expect to stop at the traffic light.

THINKING	WORKING
Choose an experiment that has an event or outcome with the same chance of success as the given scenario.	Rolling a die has six equally likely outcomes $S = \{1, 2, 3, 4, 5, 6\}$ A trip without stopping has the likelihood of $\frac{1}{3}$. Choosing any 2 of the 6 outcomes would represent a trip without stopping.
Describe how the experiment will be conducted.	Roll a die 55 times. Rolling a 1 or 2 represents a trip without stopping. All other numbers represent stopping.
Conduct the experiment and record the results.	
Write the answer.	Based on the experiment, Masie could expect to be stopped at the traffic lights on 38 occasions of the 55 trips.

Outcome	Tally	Frequency
A trip without stopping, rolling a 1 or 2	𝍸 𝍸 𝍸 ||	17
A trip with stopping, rolling a 3, 4, 5 or 6	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	38

SC 2 I can run large numbers of trials of a chance experiment

A simulation is an experiment that is used to estimate the probability of an event. This is done by running a large number of trials.

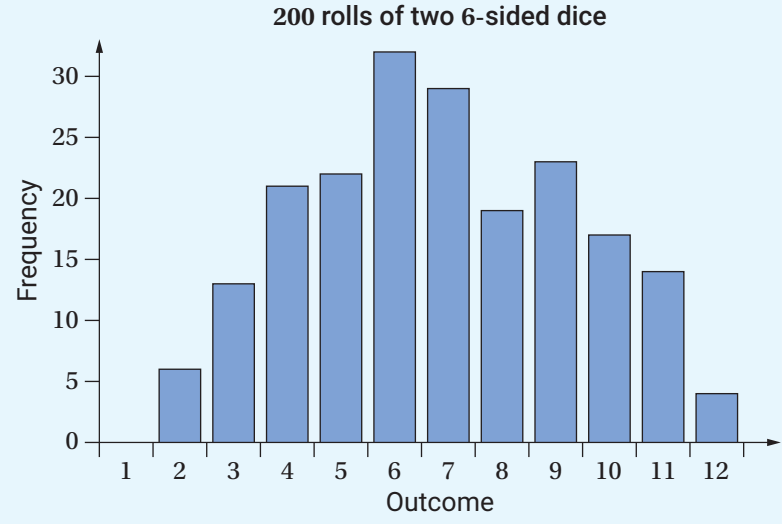
The simulation approximates a theoretical probability based on the large number of trials. This is because the experimental probability gets closer to the theoretical probability as the number of trials, n, increases.

21

Worked example

Comparing the results from simulations with a large number of trials

Is rolling a 12-sided die the same as rolling two 6-sided dice and calculating the sum of the two numbers that appear? Use technology to run an experiment with 200 trials. Give experimental evidence for your answer.

THINKING	WORKING
Determine the sample space for each situation.	The sample space for the sum of the two numbers is {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} The sample space for the 12-sided die is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} The sample space is not the same.
Use technology to run a large number of trials and record the results.	A simulator is used to run 200 trials of rolling a 12-sided die, presented as a bar chart, with the frequency of each of the 12 outcomes. An example could be:
Use technology to run a large number of trials and record the results.	A simulator is used to run 200 trials of rolling two 6-sided dice and adding the results. The data is presented as a bar chart with the frequency of each of the 12 outcomes. An example could be: 

Compare the results.	The two alternatives are not the same. There are many differences. A 1 cannot be rolled with 2 dice as the lowest total is 2. A clear pattern emerges for the two-dice scenario, with the sums in the middle being favoured over the smaller sums. The numbers are more symmetrically distributed for the 12-sided die.
Write the answer.	Rolling two 6-sided dice and calculating the sum of the two numbers that appear is not the same as rolling a 12-sided die.

Practice

ANSWERS Page 733

SC 1 I can run trials and record the outcomes of a single-step chance experiment

1 100 trials are conducted in an experiment that uses the following spinner.

The results are shown in this table.

Outcome	Blue	Red	Yellow
Frequency	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸

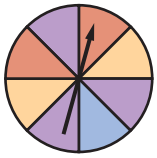
(a) How many times did the spinner land on blue?

(b) How many times did the spinner land on red?

(c) How many times did the spinner land on yellow?

(d) Are these the results what you would expect? Explain your answer.

2 100 trials are conducted in an experiment that uses this spinner.



The results are shown in this table.

Outcome	Blue	Red	Yellow	Purple
Frequency	𝍸 𝍸 |||	𝍸 𝍸 𝍸 𝍸 𝍸 |	𝍸 𝍸 𝍸 𝍸 𝍸	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸

(a) How many times did the spinner land on blue?

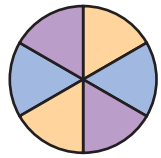
(b) How many times did the spinner land on red?

(c) How many times did the spinner land on yellow?

(d) How many times did the spinner land on purple?

(e) Why were the frequencies not more evenly distributed?

3 50 trials are conducted in an experiment that uses this spinner.



The results are shown in this table.

Outcome	Blue	Purple	Yellow
Frequency	𝍸 𝍸 𝍸 |	𝍸 𝍸 𝍸 𝍸 |||	𝍸 𝍸 𝍸

(a) How many times did the spinner land on blue?

(b) How many times did the spinner land on purple?

(c) How many times did the spinner land on yellow?

(d) The spinner above is swapped with the one below. Would you expect the results to be similar or different? Why?

4 Sometimes it is not straightforward to calculate the theoretical probability of an event, so a chance experiment can be used. Consider these two games involving rolling a pair of dice.

In the addition version of the game, the numbers rolled on the two dice are added together (the sum). If the total is even, Player 1 scores a point; if the total is odd, Player 2 scores a point. The winner is the first person to get to 10 points.

(a) Would you rather be Player 1 or 2, or does it not matter? Explain your reasoning.

(b) Play the game, recording the sum for each roll of the two dice in a table like the one shown below.

Total	2	3	4	5	6	7	8	9	10	11	12
Frequency											

(c) Based on your experiment, does either player seem to have an advantage? Why or why not?

In the multiplication version of the game, the numbers rolled on the two dice are multiplied together (the product). If the product is even, Player 1 scores a point; if the product is odd, Player 2 scores a point. Again, the winner is the first person to get to 10 points.

(d) This time, would you rather be Player 1 or 2, or does it not matter? Explain your thinking.

(e) Play the game, recording the product for each roll of the two dice in a table like the one shown below.

Product	1	2	3	4	5	6	8	9	10	12	15	16	18	20	24	25	30	36
Frequency																		

(f) From your experiment, does either player seem to have an advantage? Why/why not?

(g) Analyse and calculate the theoretical probability, by filling in the outcomes in a table like the one shown below for each game.

(i) Addition game

		Number on die 1					
	Sum	**1**	**2**	**3**	**4**	**5**	**6**
Number on die 2	**1**						
	2						
	3						
	4						
	5						
	6						

Number of even totals:

Number of odd totals:

Do both players have an equal chance? How does this compare with the results of your game?

(ii) Multiplication game

		Number on die 1					
	Product	**1**	**2**	**3**	**4**	**5**	**6**
Number on die 2	**1**						
	2						
	3						
	4						
	5						
	6						

Number of even totals:

Number of odd totals:

Do both players have an equal chance? How does this compare with the results of your game?

5 Mia's drive home each night from work takes 15 minutes plus 1 minute for every red traffic light she encounters. There are 4 traffic lights on her trip home, each with a 50% chance of being red when she arrives at the light.

(a) Design a trial that could be used to simulate the time it takes for Mia to get home each night.

(b) Conduct your trial 10 times. Based on your results, estimate the average time it will take Mia to get home each night.

(c) Would your answer be the same if you repeated the trial? Explain.

6 A lucky dip contains 200 prizes, with these values:

- 4 prizes worth $8
- 10 prizes worth $4
- 186 prizes worth 40 cents each.

(a) What does the cost of the prizes add to?

(b) Given that each dip costs $1, how much profit would the organiser make after selling all 200 dips?

(c) Discuss two different ways you could do an experiment on 50 dips.

(d) Determine the probability of winning a prize worth less than $1.

SC 2 I can run large numbers of trials of a chance experiment

1 Flip a coin 50 times (or use the simulator mentioned below).

(a) Record how many times the coin comes up as a head or a tail.

(b) Calculate the percentage of heads that have been tossed.

(c) Collect data from two other class members and add the results to your own. Recalculate the percentage of heads that have been tossed.

(d) Do you think a coin flip would be a reliable method of running a probability trial with two equally likely outcomes?

(e) Use a simulator to run a trial of 1000 tosses. Calculate the percentage of heads that have been tossed. Compare this with the results of your own experiment.

2 Put a symbol on one side of a bottle cap or lid that you can identify.

(a) Gently toss the bottle cap 40 times and record how many times the marked side is on the top. If the lid lands on its side rather than a flat side, ignore the result and toss again.

(b) Do you think your marked bottle cap would be a reliable method of running a probability trial with two equally likely outcomes?

3 You wish to run an experiment to simulate a situation where there are 3 equally likely outcomes.

(a) List two different ways this could be done.

(b) Why is tossing a coin not a suitable method?

(c) How could rolling a die be modified to be a suitable method?

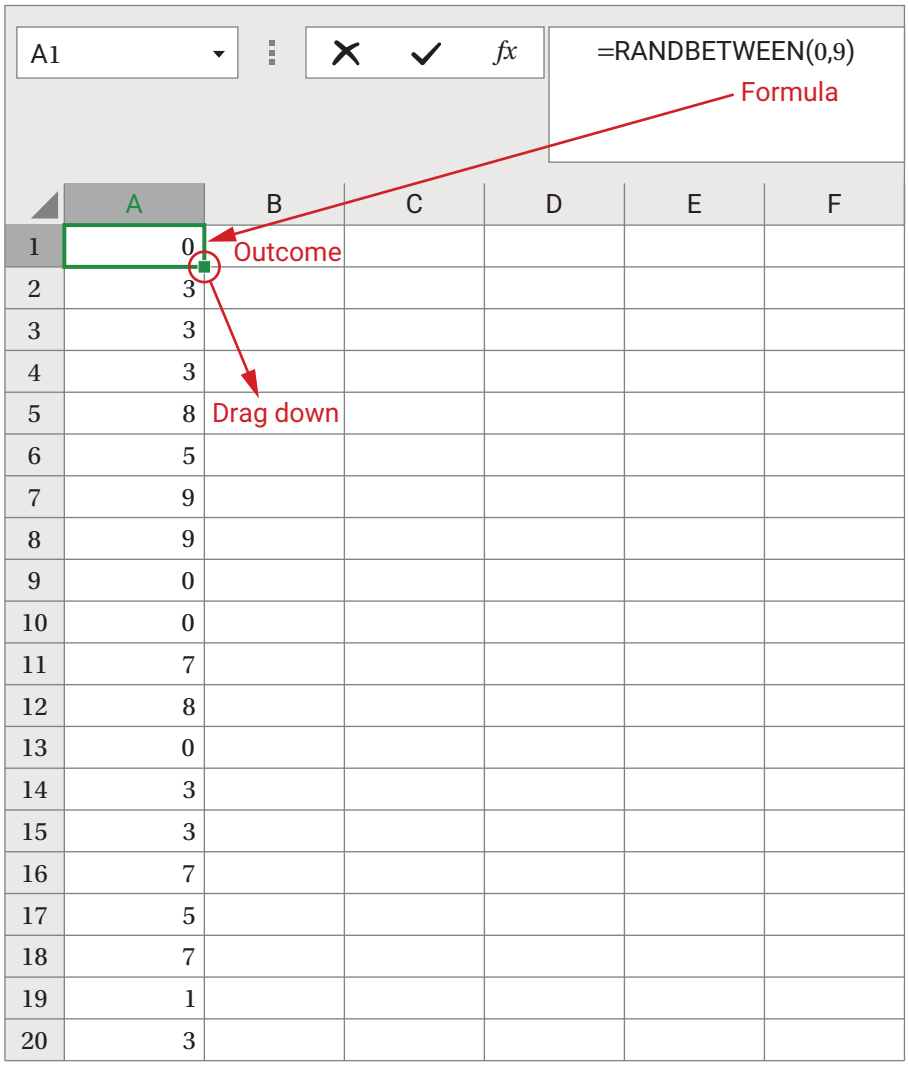
(d) Use a spinner simulation to create a spinner with three equal divisions and run 200 trials. Draw your spinner and report on the frequency of each outcome.

4 A spreadsheet can be used to generate random numbers.

Open the sheet and type =RANDBETWEEN(lower limit, upper limit) into cell A1.

In this example, you are looking for 20 random numbers between 0 and 9.

Grab the handle at the bottom right corner and drag down, holding the mouse to get as many numbers as you require.



	A	B	C	D	E	F
1	0					
2	3					
3	3					
4	3					
5	8					
6	5					
7	9					
8	9					
9	0					
10	0					
11	7					
12	8					
13	0					
14	3					
15	3					
16	7					
17	5					
18	7					
19	1					
20	3					

(a) What command would you type if you only wanted the numbers on a 6-sided die?

(b) How could you use this to simulate the tossing of a coin?

(c) Generate 100 single-digit random numbers.

(d) Complete this frequency table to confirm that the numbers are random.

Number	Tally	Frequency
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

5 Scientific calculators have an inbuilt random number generator RND. Instead of rolling a 10-sided die, you can press a button and it will give a random 3-digit number. For this experiment, treat each digit as a separate number. For example, if 379 shows, you have one 3, one 7 and one 9.

(a) Use your calculator to run 50 trials and record the frequency of each number occurring in a table like this.

Number	Tally	Frequency
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

(b) Do your results support the idea that the numbers from your calculator are random?

(c) Use the simulator to run 300 trials and comment on the results.

Compare theoretical and experimental probability

21.3

Learning intention: To be able to compare theoretical and experimental probability

Success criteria:

SC 1 I can predict the expected number of favourable outcomes for an event.

SC 2 I understand the effect of an increased number of trials.

Lesson warm-up

Buckley's chance

1 List the following words in order from impossible to certain.

Likely, absolutely, probably, fifty-fifty, unlikely, probably not, very unlikely, might

2 Place the phrase 'Buckley's chance' in your ordered list.

Two possible origins of the saying are listed below:

A possible origin of the saying is from William Buckley, a convict who escaped in Victoria in 1803. He lived among Aboriginal peoples for 32 years in the bush.

Before William Buckley, the survival of non-Indigenous Australians in the bush was 'no chance'.

Another possible origin of the saying stems from businessman Mars Buckley withdrawing gold from the Bank of Australasia during a financial crisis, meaning the banks could not pay out and they had 'Buckley's chance' (no chance) of getting the gold back.

SC 1 I can predict the expected number of favourable outcomes for an event

A favourable outcome is when the result you are interested in occurs. For example, if you were interested in the frequency with which a 6 is rolled on a die, then a favourable outcome would be rolling a 6.

The formula to calculate the expected number of successes (or favourable outcomes) in an experiment is:

$E(X) = n \times \Pr(x)$, where n = the number of trials and $\Pr(x)$ is the probability of success, x.

$E(X)$ is not a guarantee of what will happen, but the larger n becomes, the more likely it is to happen.

Worked example

Calculating the expected number of favourable outcomes

After many years of playing, Stefan Murray has a success rate of 4 out of 5 for his basketball free throws. In a particular match, he has 15 free throws.

(a) Determine the expected number of successful free throws.

THINKING	WORKING
Recall the formula to calculate the expected number of favourable outcomes.	$E(X) = n \times \Pr(x)$
Substitute the values into the formula and calculate the answer.	$E(X) = 15 \times \frac{4}{5}$ $= 12$
Write the answer.	Stefan Murray is expected to make 12 of the 15 free throws.

(b) Will he always get the expected number of free throws out of 15? Explain.

THINKING	WORKING
Recall the difference between a theoretical probability and an experimental result.	Theoretical probability represents the long-term average. Over a small number of trials, the experimental results can vary.
Write the answer.	No, he will not. Fifteen is a small number of trials and there will be variation in the number of successes.

SC 2 I understand the effect of an increased number of trials

Theoretical probability is calculated over the long term; that is, for a very large number of experimental trials.

For a smaller number of experimental trials, the experimental results will give an experimental probability, which often is different from the theoretical probability.

Recall the formula for the expected number of favourable outcomes, $E(X) = n \times \Pr(x)$, where n is the number of trials and $\Pr(x)$ is the probability of the event, x.

Worked example

Comparing theoretical and experimental results

Over his career, Stefan Murray has successfully made 1250 out of 1420 basketball free throws.

(a) Estimate the probability of Stefan being successful with a single free throw. Answer correctly to 2 decimal places.

THINKING	WORKING
Recall the formula to calculate the expected number of favourable outcomes.	$E(X) = n \times \Pr(x)$
Identify the known information.	$E(X) = 1250$ and $n = 1420$
Substitute the known values into the formula and solve for the unknown.	$E(X) = n \times \Pr(x)$ $1250 = 1420 \times \Pr(x)$ $\Pr(x) = \frac{1250}{1420}$ $= 0.88 \text{ (2 d.p.)}$
Write the answer.	The probability of being successful with a single free throw is approximately 0.88.

(b) In a particular match, he has 15 free throws and is successful with all 15. Comment on his performance in this game.

THINKING	WORKING
Determine the expected number of successes (favourable outcomes).	$E(X) = n \times \Pr(x)$ $= 15 \times 0.88$ $= 13.2$ The expected number of free throws is approximately 13 from 15.
Determine the difference between the expected number and the actual number.	$15 - 13 = 2$ Stefan scored 2 more free throws than expected.
Write the answer.	Stefan has exceeded expectations for this game; however, 15 free throws is a small number of trials, and a variation in performance is to be expected.

Practice

ANSWERS Page 734

SC 1 I can predict the expected number of favourable outcomes for an event

1 Over the course of her career, Serena Jones is successful with 7 out of 10 of her first serves in her tennis matches. In a particular match, she has 80 first serves.

(a) How many of her first serves are expected to go in?

(b) Will she always make this many successful first serves out of 80?

2 Over the course of her career, Sam Kurt is successful with 9 out of 10 of her penalty kicks during soccer matches. So far, she has taken 60 penalty kicks in her soccer career.

(a) How many penalty kicks is she expected to score?

(b) When Sam steps up to take a penalty kick, her coach says he has 100% confidence that she will score. Is his confidence supported by Sam's performance?

3 The long-term average for rolling a 4 on a standard 6-sided die is 1 in 6. In an experiment, you roll the die 120 times.

(a) How many 4s would you expect?

(b) Is the expected value a guarantee of how many 4s you will roll?

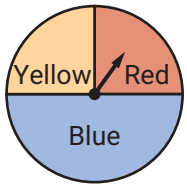
4 The long-term average for rolling each number on a standard 6-sided die is 1 in 6. In an experiment, you roll the die 160 times.

(a) How many even numbers would you expect?

(b) How many numbers greater than 3 would you expect?

(c) How many numbers less than 6 would you expect?

5 The following spinner is to be spun 240 times.



(a) How many times would you estimate the spinner to land on blue?

(b) How many times would you estimate the spinner to land on red?

(c) Explain how you arrived at your estimates for **(a)** and **(b)**.

6 Two sets of data are recorded for 'random' experiments.

Experiment 1: A nine-sided die is rolled 400 times.

Result	Frequency
1	55
2	48
3	39
4	45
5	46
6	45
7	48
8	32
9	42

Experiment 2: The first digit of 500 numbers in a financial magazine is recorded.

Result	Frequency
1	130
2	68
3	52
4	38
5	32
6	25
7	24
8	16
9	15

(a) Given that the 9-sided die was fair, how many favourable outcomes would you expect for a result out of 400 trials?

(b) What do you notice about the pattern of results in the digits from the financial magazine?

(c) Which of the two data sets is more likely to be the result of a random experiment? Explain.

SC 2 I understand the effect of an increased number of trials

1 **(a)** Flip a single coin 30 times. Record your results in the table as you go, completing all the questions across the row.

Number of tosses	Expected number of tails	Number of tails	% of tails
1			
5			
15			
21			
30			

(b) What conclusions can you make from the number of trials and the percentage of tails?

Use a coin-flipping simulator for 1000 trials.

(c) Calculate the percentage of tails.

(d) How does running a large number of trials affect the percentage of tails?

2 A spinner is spun, and the results are recorded below.

(a) After 20 trials, the results are as follows. Do you think the colours are evenly distributed on the spinner? Explain.

Colour	Frequency
Black	5
White	12
Red	3

(b) Design a spinner that, in theory, would give the results shown in part **(a)**.

(c) After 200 trials on the same spinner, the results are as follows. Do you think the colours are evenly distributed on the spinner? Explain.

Colour	Frequency
Black	62
White	80
Red	58

(d) Design a spinner that, in theory, would give the results shown in part **(c)**.

(e) Why might your answers to part **(a)** and **(b)** be different from the answers you gave in part **(c)** and **(d)**?

3 Generate random numbers between 0 and 9 using the random number capability on your scientific calculator. Record the results in a table like the one below.

(a) After 20 trials

Number	Frequency
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

(b) Do you think the numbers are evenly distributed from the calculator? Explain.

(c) Use a random number generator to simulate 1000 trials. Record the results in a table like the one below.

Number	Frequency
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

(d) Do you think the numbers from the simulator are random? Explain.

(e) Why might your answers be different for parts **(a)** and **(c)**?

TOPIC REVIEW

Multiple choice

ANSWERS Page 753

1 Which of the following is the sample space for drawing a letter from the word 'remarkable'?

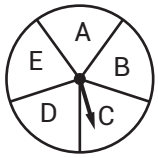
A R E M A R K A B L E **B** R E M A R K

C R E M A R K B L **D** R E M A K B L

2 When a coin is tossed 50 times, which of the following is called an event?

A H, T **B** Tossing a tail **C** Head **D** Tail

3 On the following spinner, which of the following is not a potential outcome?



A Spinning an A **B** Spinning a consonant

C Spinning a vowel **D** Spinning an F

4 Which of the following would be a suitable experiment to model the number of days it rains during the Phuket wet season, 240 days, given that it rains on average 5 out of 6 days?

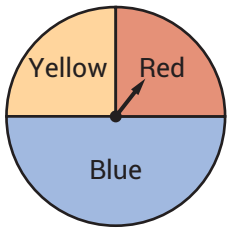
A Tossing a coin 240 times **B** Tossing a coin 6 times

C Rolling a die 240 times **D** Rolling a die 6 times

5 It rains 2 out of every 5 days over winter. Given that winter lasts for 91 days, how many days of rain would be expected?

A 36.4 **B** 16.2 **C** 54.6 **D** 12

Use the spinner shown below to answer Questions **6** and **7**.



6 This spinner is spun 120 times. The expected number of spins that will be yellow is:

A 25 **B** 30 **C** 40 **D** 45

7 This spinner is spun 203 times. The expected relative frequency of yellow is closest to:

A $\frac{16}{203}$ **B** $\frac{26}{203}$ **C** $\frac{56}{203}$ **D** $\frac{106}{203}$

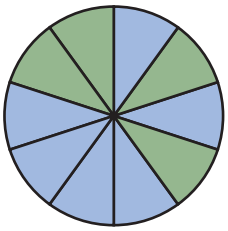
Short answer

1 List the sample space for the following.

(a) The suit drawn from a standard deck of 52 playing cards

(b) The colour drawn from a standard deck of 52 playing cards

2 Consider the following spinner.



(a) List the sample space.

(b) Is each outcome equally likely? Explain.

(c) If one of the green wedges was painted red, how would the expected frequency of the blue wedges be affected?

(d) If one of the green wedges was painted red, how would the frequency of the green wedges be affected?

(e) Which colour would you expect to have the highest relative frequency? Why?

3 An experiment needs to be conducted where there are 5 equally likely outcomes.

(a) List two different ways that this could be done.

(b) Why is tossing a coin not a suitable method?

(c) How could rolling a die be modified so that it is a suitable method?

4 Jack tosses a coin 50 times and obtains 30 tails and 20 heads.

(a) Do you think the coin is fair? Explain.

(b) The long-term average is 1 in 2 heads. Does this mean that Jack has an increased chance of tossing a head if he tosses the coin another 50 times?

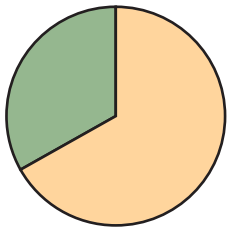
5 Milky either gets a ride to school with her mum, travels by train or waits for the school bus. Each method is equally likely.

(a) Describe an experiment that will model her transport method over her first 90 days of school.

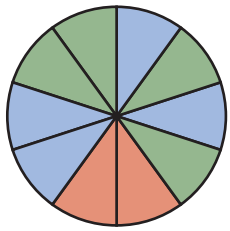
(b) What is the expected relative frequency of each outcome?

(c) Use a calculation to predict the expected number of times her mother will drive her to school.

6 Both spinners are to be spun 600 times.



Spinner 1



Spinner 2

(a) Estimate the expected number of yellow outcomes there will be for Spinner 1. Explain your thinking.

(b) Estimate the expected number of blue outcomes there will be for Spinner 2. Explain your thinking.

(c) On which spinner will there be fewer green wedges spun? Explain your thinking.

7 A bag containing 200 marbles, 50 red, 120 blue and 30 green, has one marble drawn at a time and replaced. This is repeated 200 times.

(a) Estimate the expected number of each colour that will be drawn.

(b) If the marble is not placed back in the bag after it is drawn, would the numbers of each colour drawn change? Explain your thinking.

8 Dai rolls a six-sided die 60 times. These are the results.

Result of die roll	1	2	3	4	5	6
Number of rolls (frequency)	8	6	12	9	15	10

(a) Based on this experiment, what is the probability of Dai rolling a 3?

(b) Write one possible reason why this is not the same as the theoretical probability, which is $\frac{1}{6}$.

9 For 48 school days, Jacinta records whether her bus is early, on time or late. Here are her results.

Bus is ...	**Number of days (frequency)**
Early	2
On time	36
Late	10

(a) Based on these results, calculate the probability of Jacinta's bus being on time next Monday.

(b) For the next 6 school days, Jacinta's bus is late every day. How would you calculate the probability of Jacinta's bus being late for the next school day after this?

Extended response

1 At a country show, one of the games involves buying 3 darts for $2 and throwing them at balloons pinned to a board and seeing how many balloons you can pop. The prize you win depends upon the number of balloons you can pop.

3 popped balloons wins a prize valued at $10.

2 popped balloons wins a prize valued at $5.

1 popped balloon wins a prize valued at $2.

0 popped balloons wins a prize valued at $1.

Long-term data for 1000 games is collected about how many balloons are popped. The results are shown below.

	Frequency	Prize for each pop
3 popped	20	$10
2 popped	50	$5
1 popped	200	$2
0 popped	730	$1
Total	1000	

Assume that the next 1000 games have the same results as the long-term data.

(a) What is the income from the 1000 games?

(b) What is the value of the prizes awarded?

(c) What profit does the operator make?

Design and conduct an experiment to run 100 trials.

(d) What is the income from the 100 games?

(e) What is the value of the prizes you awarded in your trial?

(f) What profit did you make? How does this compare to the percentage profit in the long-term data?

(g) If the gambling board announced that at least 90% of the money must be returned for the game to be legal, make a suggestion as to how the prizes could be altered to return more than 90%.

GLOSSARY

acquire data
The process of obtaining data by direct collection or from other sets of data so that it can be sorted, displayed and analysed.

acute angle
An angle that measures between 0 and 90 degrees, i.e. less than one right angle.

addition facts
The results associated with the sums of pairs of natural numbers from 0 to 9. They are foundational to arithmetic.

additive
A situation or relationship that involves addition, subtraction or both, e.g. giving change from a simple money transaction.

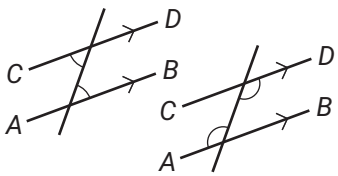
algebra
The process of manipulating variables and constants in a mathematical expression according to laws, properties or rules, e.g. simplifying an expression or solving an equation.

algebraic expression
A mathematical statement formed by combining numbers and algebraic symbols using arithmetic operations, e.g. $a^2 + 3ab - 2b^2$.

algebraically
Carrying out a process symbolically using variables, constants, operations and rules.

algorithm
A process that can be carried out systematically, using a well-defined set of instructions, to perform a particular task or solve a type of problem.

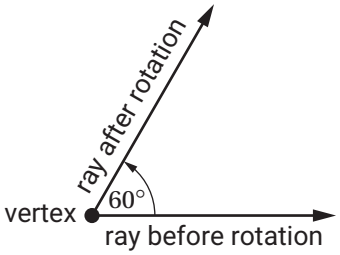
alternate angles
Angles on alternate sides of a transversal of a pair of parallel lines. If the lines AB and CD are parallel, then each pair of alternate angles are equal.



am
Any time before midday when using 12-hour time. An abbreviation of the Latin term *ante meridiem*.

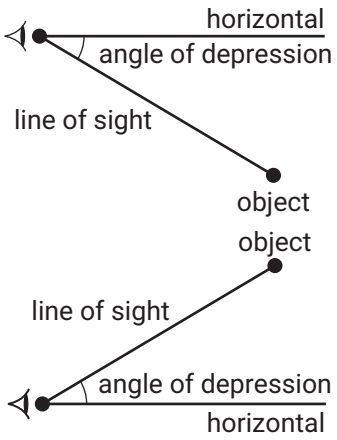
analog clock
A clock with hours marked from 1 to 12 arranged in a circle and moving hands to show you the time.

angle
The figure formed at the intersection of two rays in the plane with a common endpoint, called its vertex. It is typically indicated with an arc.

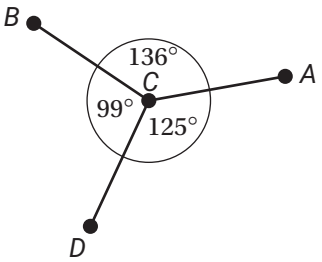


angle of depression
The angle between horizontal and the line of sight from an observer to an object that is lower than the observer.

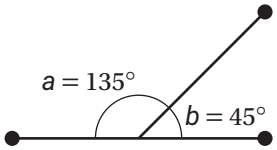
angle of elevation
The angle between horizontal and the line of sight from an observer to an object that is higher than the observer.



angles at a point
The sum of the angles with vertices at a common point in the plane is equivalent to four right angles or 360 degrees. This spatial fact can be applied to determine the size of missing angles.



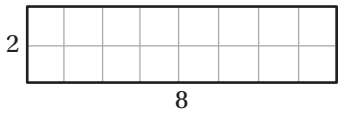
angles on a straight line
The sum of the angles that form a straight line is equivalent to two right angles or 180 degrees. This spatial fact can be applied to determine the size of missing angles.



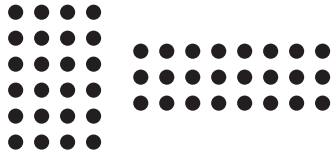
approximate
To obtain or state a value to a particular accuracy.

approximation
A result that is not exact, but is close enough for a given purpose, e.g. giving an approximation of the area of a complex shape by using a combination of basic shapes.

area
The amount of interior space bounded by the perimeter of a shape. It is usually measured in square units. For example, the area of this rectangle is given by $8 \times 2 = 16$ units.



array
An ordered collection of discrete objects or numbers, such as rectangular arrangements of rows and columns. For example, two arrays of dots (4×6 and 8×3) produce two different representations of the number 24.



Asia/Asian
Geographically, the largest continent, bounded by Europe and the Pacific, Arctic and Indian oceans. It can be described in cultural, religious, historical and language boundaries or commonalities.

assign probabilities
The process of providing a qualitative or quantitative value for the likelihood or chance of an event occurring. This may involve judgement based on experience, experiment, counting or measurement.

association(s)
Relationships between pairs of variables for different data types, e.g. in terms of strength, form and direction.

associative
Of or relating to an operation, which, that when applied to any three elements of an expression, is the same regardless of which pair of elements (without changing their order) is combined first.

associative law
In general, the addition or multiplication of a set of real numbers gives the same output, no matter how the numbers are grouped, e.g. $a + (b + c) = (a + b) + c$ and $a \times (b \times c) = (a \times b) \times c$.

attribute
A property of an object or event that can be measured and/or directly or indirectly compared.

back-to-back stem-and-leaf-plot
A method for comparing two data distributions by attaching two sets of 'leaves' to the same 'stem'.

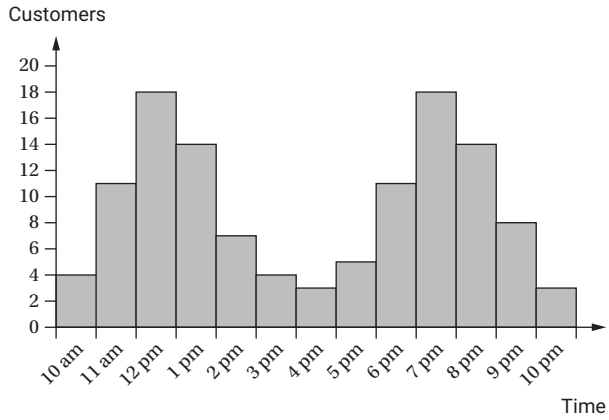
Pulse rate

Before		After
9 8 8 8	6	
8 6 6 4 1 1 0	7	
8 8 6 2	8	6 7 8 8
6 0	9	0 2 2 4 5 8 9 9
4	10	0 4 4
0	11	8
	12	4 4
	13	
	14	6

base-10
A number system which uses the digits 0–9 and the value of the digit is determined by its face value and its place value, e.g. $283 = 2 \times 100 + 8 \times 10 + 3 \times 1$ and $283 = 200 + 80 + 3$.

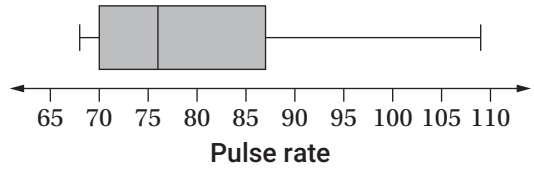
bias (statistics)
A measure being systematically moved away from the true value, leading to overestimating or underestimating that value.

bimodal distribution
The distribution of a set of data that has two modes. The term 'bimodal' is also used if the graph of the distribution has two distinct peak values, which may not be equal.



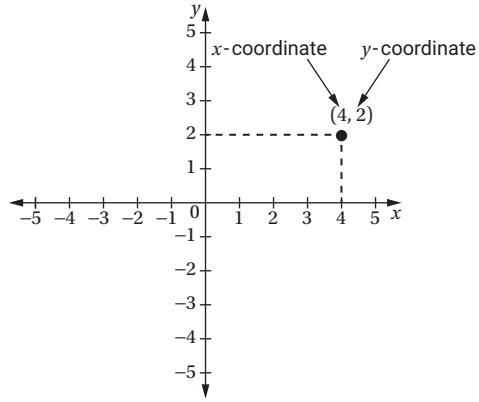
bivariate data
Data relating to measurement of two variables. It can be categorical data, numerical data or a combination of both.

boxplot
A graphical display of a 5-number summary of data. The 'box' covers the middle 50% of scores. The 'whiskers' extend to the maximum and minimum values in the data set.

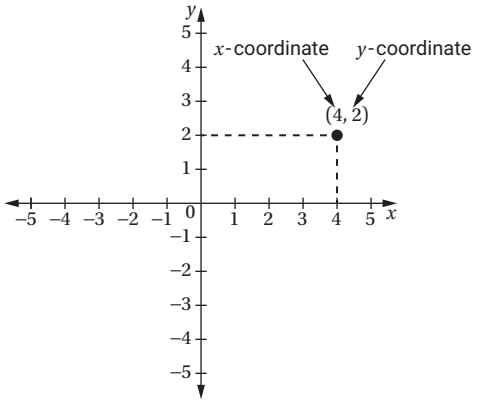


capacity
The amount a container will hold. It is often used in relation to the volume of fluids. Units of capacity (volume of fluids or gases) include litres (L) and millilitres (mL).

Cartesian coordinate system
The positioning system for any point on the Cartesian plane, represented by an ordered pair of real numbers (x, y), which are called the coordinates of the point relative to the origin at (0, 0).



Cartesian plane
A plane divided into 4 regions (quadrants) by 2 perpendicular axes, the x-axis and the y-axis, which intersect at a point called the origin.



categorical data
Data in discrete categories, e.g. data on blood groups with values of type A, B, AB or O.

categorical variable
A variable that takes values from sets of categories. There are two types of categorical variables: nominal and ordinal.

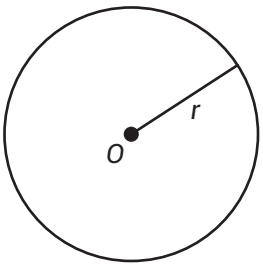
census
Collection of information about the whole population.

centre (space)
The middle point of a shape, object or space.

centre (statistics)
In a set of data, a single value that is used to represent or typify that data. Informally, it is a notion of average.

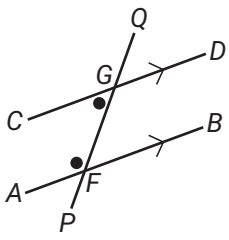
chance experiment (trial)
An investigation process that involves planning and conducting a repeatable experiment involving chance, where more than one possible outcome may occur. Experiments can be single or multi-step and results can vary across trials.

circle
The set of all points in the plane that are a fixed distance (the radius) from a given point (the centre of the circle).



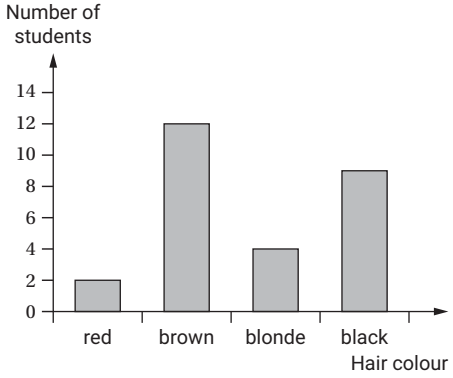
circumference
The boundary (perimeter) of a circle. The length of the circumference C is given by $C = \pi d$, where d is the diameter. Alternatively, it is given by $C = 2\pi r$ where r is the radius.

co-interior angles
A pair of angles that lie between a pair of parallel lines cut by a transversal and has a sum of 180 degrees or two right angles.



colonisation
The act of setting up a colony away from one's place of origin. With humans, it can be seen negatively because it tends to involve an invading culture taking control over an indigenous population.

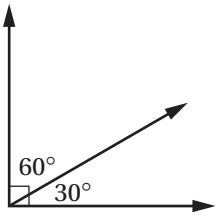
column graph
A graph for organising and displaying categorical data. Equal-width rectangular bars are constructed for each category with height equal to the observed frequency of the category. Two adjacent angles that form a right angle, i.e. the sum of the angles measured in degrees, is 90°.



commutative
An operation in which the result of applying it to any two elements of a set is the same, regardless of the order of the elements. Addition and multiplication are commutative on the set of natural numbers.

commutative property
In general, the commutative property of addition and multiplication of real numbers is that for all real numbers a and b, $a + b = b + a$ and $a \times b = b \times a$, respectively.

complementary angles
Two adjacent angles that form a right angle, i.e. the sum of the angles measured in degrees is 90°.

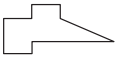


complementary events
Two mutually exclusive outcomes in a probability experiment, such that Pr(A) +Pr(B)=1, where Pr(A) is the probability of event A and Pr(B) is the probability of event B.

complex shapes and objects
A complex shape is a closed shape comprised of a number of plane shapes. Complex objects are comprised of simpler objects.

composite number
A non-zero natural number that has a factor other than 1 and itself.

composite shape
A shape that is formed by combining other plane shapes. Composite shapes are often described as 'complex' when they are made up of many and different shapes.



compound event
In probability, an event that consists of more than one event occurring at the same time, e.g. tossing a coin and selecting a card.

compound interest
The interest earned by investing a sum of money (the principal) when each successive interest payment is added to the principal for calculating the next interest payment.

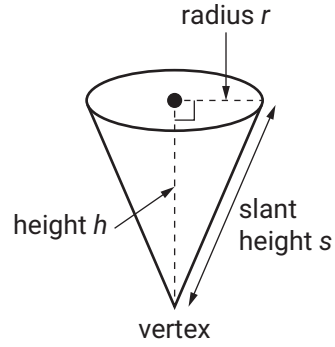
compounding error
The increasing level of error that arises when measurements with errors are used to calculate other quantities.

computation
The action of performing a mathematical calculation. It can be performed mentally or by using an algorithm or a digital tool, such as a calculator.

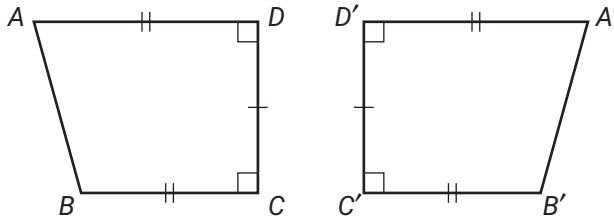
computational thinking
A way of thinking that helps to organise data logically by breaking down problems into parts; defining abstract concepts; and designing and using algorithms, patterns and models.

conditional probability
Consideration of whether the knowledge of the occurrence of one event, A, affects the probability of occurrence of another event, B.

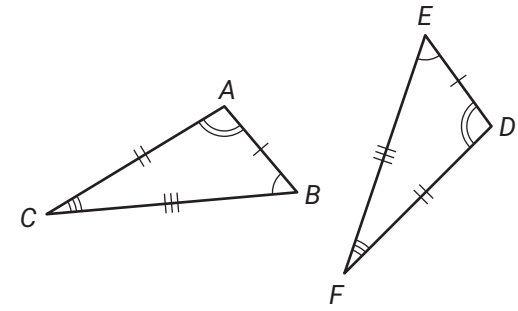
cone
An object that is formed by taking a circular base and a point not in the plane of this circle, called the vertex, and joining the vertex to each point on the circumference of the base.



congruent
An exact match between every part of one figure with the corresponding part of the other figure, i.e. congruent figures can be exactly superimposed on each other.



congruent triangles
Triangles that have the same size and shape. When not all measurements are known, the determination can be made using the four standard congruence tests for triangles (SSS, SAS, AAS, RHS).



conjecture
A statement whose truth or otherwise is not yet determined, but is open to further investigation, e.g. Goldbach's conjecture of 1742 has not yet been either proven to be true or shown to be false.

construct
To draw, make or build a mathematical object such as a shape, pattern, algorithm or proof.

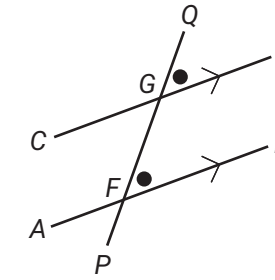
context
The situation or setting of an event, problem or application. Contexts can be real world or constructed.

continuous numerical variable
A numerical variable that can take any value that lies within an interval. In practice, the values taken are recorded to the accuracy of measurement of the instrument used to obtain these values.

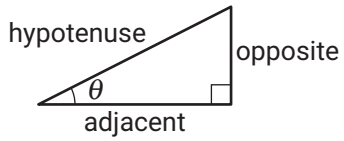
coordinate system
A system for showing a set of values that give an exact position, e.g. Cartesian coordinate system, grid map or a seating plan for a venue.

coordinates
A set of values that define the position of a point or an object in a space. The coordinate system being used will govern the nature of the coordinates.

corresponding angles
Angles that lie in the same relative position at the points of intersection, where a pair of parallel lines is cut by a transversal and which are equal.



cosine ratio
The ratio of the length of the base, b, of a right-angled triangle to the length of its hypotenuse, h, for a given base angle theta, θ. In any right-angled triangle,
$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$, where $0 < \theta < 90°$.

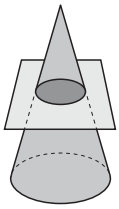


counting
The process of quantifying the number of objects in a set or collection.

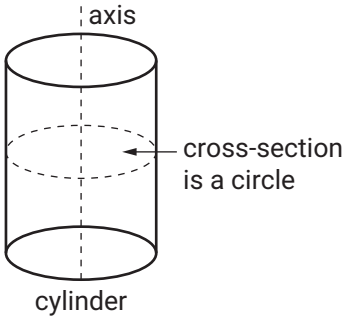
Country
A space mapped out that individuals or groups of First Peoples of Australia occupy and regard as their own and having varying degrees of spirituality. It includes lands, waters and sky.

Country/Place
Spaces mapped out that individuals or groups of Aboriginal and Torres Strait Islander Peoples occupy and regard as their own and that have varying degrees of spirituality. They include lands, waters and sky.

cross-section
The intersection of a solid object with a plane. It may differ for a given solid depending on the orientation of the slicing plane.



cylinder
An object that has parallel circular discs of equal radius at the ends, which are joined by a curved surface.



data
A general term for a set of observations or measurements collected during an investigation. Primary data is generated by the user; secondary data is collected from a secondary source, such as a journal, book or website.

data displays
Visual formats for organising, representing and summarising data. Also called visualisations.

data distributions
Patterns in the values of a set of data. The representation used to show the distribution depends on the type of data and how it is measured.

decay
Decrease in the value of the dependent variable of a function as the value of the independent variable increases.

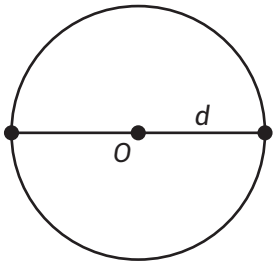
decimal
Used to describe aspects of the base-10 number system. The decimal point (. or ,) separates the whole number part of a number from its decimal part.

degrees
A unit of measurement for measuring an angle. Angles are measured as a proportion of a full turn, which is equivalent to 360 degrees, so that one degree is equal to $\frac{1}{360}$ of a full turn. Written as $\alpha°$.

denominator
In the fraction $\frac{a}{b}$, b is the denominator. It is the number of equal parts into which the whole is divided.

diagonal
A line segment joining two vertices of a polygon that do not share an edge.

diameter
A chord passing through the centre of a circle. Also describes the length of the diameter.



digit
A single symbol that is used to represent a number as a numeral. In the base–10 number system there are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

digital tools
Electronic devices, programs, websites or online resources that can be used to support the learning and doing of mathematics.

direct comparison
Comparing objects and events using a particular attribute, e.g. measuring the length of two pencils by placing them side by side or starting two events at the same time to compare duration.

direct proportion
The relationship between corresponding values of two variables in a fixed ratio relative to each other;
i.e. for variables x and y, $\frac{y}{x} = k$ and $y = kx$, where $k \neq 0$.

directional language
Words that communicate position, location and movement within a space.

discrete numerical data
Numerical data that can take only a countable number of values, e.g. the number of people in a car, or clothing sizes.

distributive
An operation that can take priority over the operation used for combination within brackets. Multiplication is distributive over addition, e.g. $8 \times 17 = 136$ as $8 \times 17 = 8 \times (10 + 7)$ and $8 \times (10 + 7) = 8 \times 10 + 8 \times 7$ and $8 \times 10 + 8 \times 7 = 80 + 56$, which equals 136.

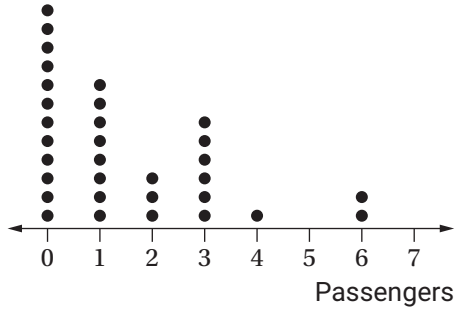
distributive property
In general, the distributive law (property) for multiplication over addition for real numbers states that for all real numbers a, b and c: $a(b + c) = a \times b + a \times c$.

divisible
Able to be divided exactly, i.e. without leaving a remainder.

division
For a finite set, the process of partitioning the set into subsets of equal size. For natural numbers, it expresses a given number as a multiple of a smaller number and any remainder.

division facts
Facts that draw on the inverse relationship between division and multiplication and are directly related to the multiplication facts, e.g. $2 \times 5 = 10$, so $\frac{10}{2} = 5$ and $\frac{10}{5} = 2$.

dot plot
A graph used to display numerical data. Using a number line, it displays a dot for each piece of data and, when there is more than one piece of data with the same value, the dots are stacked vertically.



duration
A measure of time that an event or activity takes to complete.

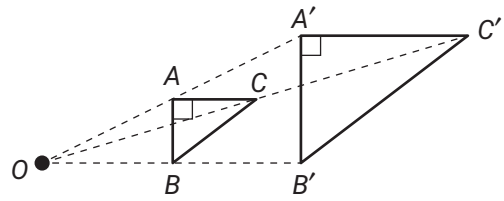
dynamic geometric software
Software that supports the representation, construction and manipulation of geometric shapes and objects, and related measures and other functions in an interactive and dynamic digital environment.

efficient strategies
The choice of suitable approaches (mental, written, using digital tools), and methods within these. The means used for calculation reflect the purpose and context.

element(s)
A member of a set, e.g. 3 is a member of the set of natural numbers $N = \{0, 1, 2, 3, ...\}$. This relation can be written more concisely as $3 \in N$ ('3 is an element of the set N').

enclosed spaces
Space that is enclosed by a boundary, such as the perimeter of a shape or the outer surface area of an object.

enlargement transformation
A scaling of a figure in which the corresponding lengths in the transformed figure are increased or decreased in proportion to the original figure. The two figures are similar.



equal sharing
Dividing a collection, shape or object into equal parts.

equally likely outcomes
An event for which all outcomes have the same probability of occurring.

equation
A statement that includes the '=' symbol. Equations are used to show the equality of two expressions.

equivalent
Equal in value or meaning, such as an expression or statement that is essentially the same, two or more sets that are capable of being mapped in a one-to-one relationship.

equivalent fractions
Fractions that are different representations of the same value, e.g. $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

error
The difference between an actual value and its measured or estimated value.

estimation
The skill of conceptualising and mentally manipulating numbers or measurements to find an approximate answer. The capacity to make reasonable adjustments to estimates is essential in estimating.

estimation strategies
Strategies used to estimate or find an approximate value for the amount, extent, duration, position or size of something.

Euler's formula
A relation between the number of faces, vertices and edges of any polyhedron. It is written $F + V = E + 2$, where F is the number of faces, V is the number of vertices and E is the number of edges.

even number
An integer that is divisible by 2. The even natural numbers are {0, 2, 4, 6, ...}

event
A subset of the sample space of a random experiment.

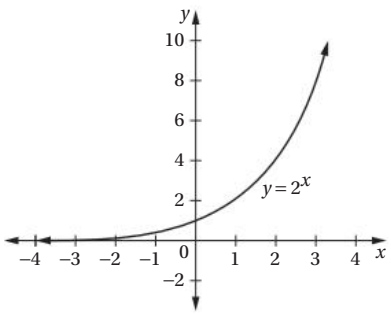
expanded notation
A way of representing a number, separating each digit and representing it in an additive number sentence to show each digit's place value, e.g. $2345 = 2 \times 1000 + 3 \times 100 + 4 \times 10 + 5$.

expected frequencies
The number of times that a particular event or outcome is predicted to occur using theoretical probability.

exponent
The power (or index) to which a number or algebraic expression is raised, shown using superscript, e.g. in the following expressions the exponent is 3: 2^3, $2x^3$, $2(x + 5)^3$.

exponent laws
Rules for manipulating exponents. Also referred to as index laws.

exponential functions
Functions comprising a positive constant raised to the power of the variable; i.e. $f(x) = ax$ where a is a constant and x is the variable.



expression
Two or more numbers or variables connected by operations.

factors
Let a, b and c be natural numbers such that $a \times b = c$, then a and b are factors (or divisors) of c, e.g. $3 \times 4 = 12$, so 3 and 4 are factors (divisors) of 12.

formulating expressions
The process of using mathematical notation, conventions and symbols to represent situations (including informal or worded statements) as mathematical expressions.

fraction
A number of the form $\frac{a}{b}$, where a is an integer and b is a non-zero integer, e.g. $\frac{4}{6}, \frac{11}{5}, -\frac{3}{7}$. A fraction is said to be in simplest form when a and b have no common divisor greater than 1.

frequency
The number of times that a particular value occurs in a data set. For grouped data, it is the number of observations that lie in that group or class interval.

frequency table
A table listing the frequency (number of occurrences) of observations, measurements or outcomes in different ranges, called class intervals.

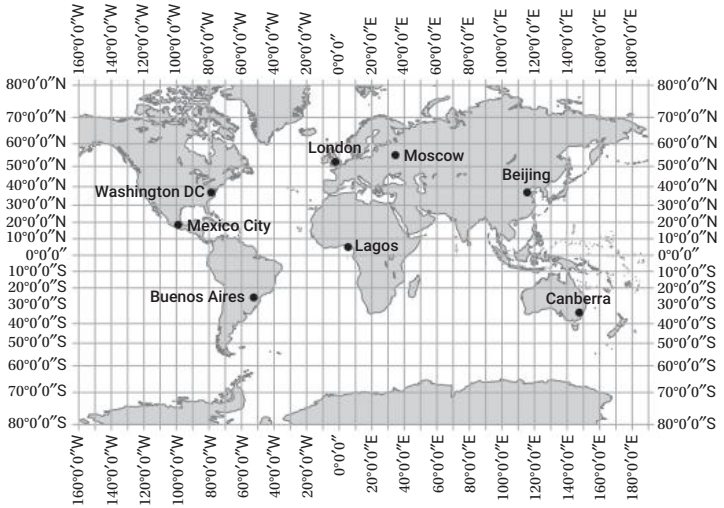
function
A correspondence (map or relation) between the elements of two sets where each element in the first set is mapped to exactly one corresponding element in the second set, either one-to-one or many-to-one.

function machine
An algorithmic process which takes an input, applies an operation (or operations) and results in an output.

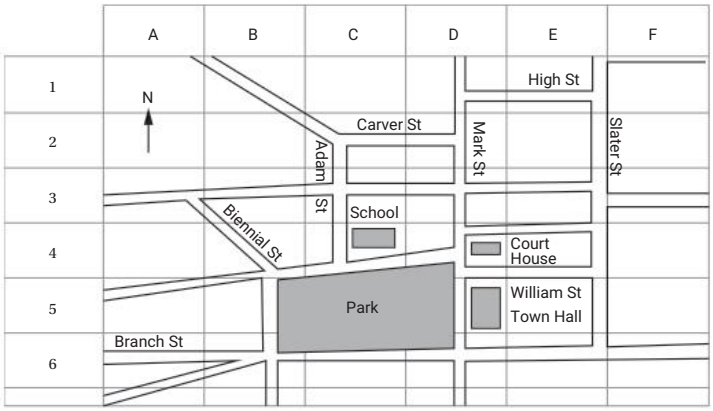
geometric patterns
Patterns formed by a series of shapes. They may be repeating or growing patterns which change in predicted ways that can be generalised. Tessellations are geometric patterns formed by transforming shapes.

gradient
The slope of a line. It is calculated as the gradient of a line segment it contains. The gradient (m) of the line containing $A(x_1, y_1)$ and $B(x_2, y_2)$ is calculated as $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$.

grid coordinate system
A system for referencing location within a space using labelled horizontal and vertical grid lines. Locations are defined using a set of coordinates which indicate points in the plane where two grid lines meet.



grid reference
A way of identifying a region on a map. Coordinates and grid lines are used to refer to specific features or locations, e.g. in the map shown, the school is located at the grid reference C4.



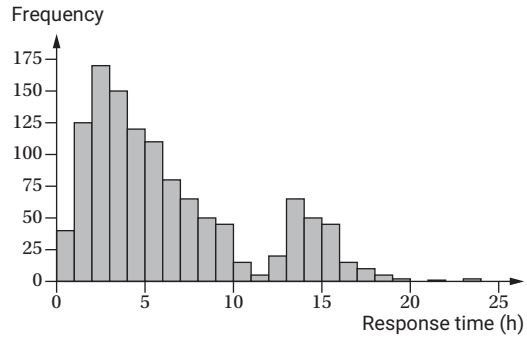
grouping
A process of partitioning a set into subsets according to a given criterion, e.g. a class of students according to eye colour. Equal grouping means each subset has the same number of elements.

growing pattern
A pattern where each term is either an increase or a decrease on the previous term. Change from term to term is predictable if the pattern is growing additively, multiplicatively or exponentially.

growth
A measure of the increase in value of the dependent variable of a function as the value of the independent variable increases, e.g. the length of a spring when weights are added.

guided statistical investigation
A teacher-led, scaffolded approach to the statistical investigation process, including collecting, representing and analysing data, and interpreting and communicating results.

histogram
A statistical graph for displaying the frequency distribution of continuous data. It is also a graphical representation of the information contained in a frequency table.



improper fraction
A fraction where the numerator is greater than or equal to the denominator, e.g. $\frac{7}{5}$ is an improper fraction, whereas $\frac{5}{7}$ is a proper fraction.

inequality
A mathematical expression containing the terms 'less than', 'less than or equal to', 'greater than', or 'greater than or equal to' and their respective symbolic representations $<$, $>$, $\leq$ and $\geq$.

inference (statistics)
The process for determining consequences or conclusions based on assumptions, evidence and reasoning.

informal units
Units whose values are decided on in a given context, e.g. the use of pace to measure distance (non-uniform unit); the use of paperclips to measure length (uniform unit).

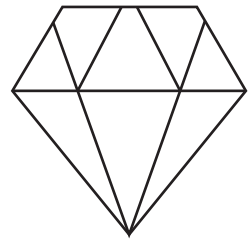
integer
An element of the infinite set of numbers $Z = \{... -3, -2, -1, 0, 1, 2, 3, ...\}$.

interval(s)
A continuous subset of the real number line, e.g. 'the set of all real numbers greater than or equal to 10'.

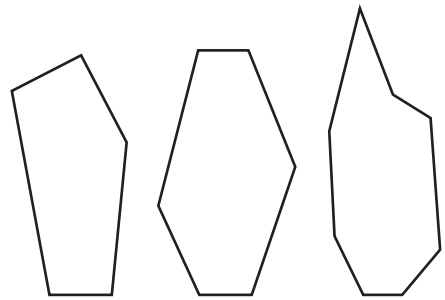
inverse operation
An operation in arithmetic which undoes the effect of another operation. Multiplication and division are inverse operations, as are addition and subtraction.

irrational number
A number that cannot be expressed as a fraction in the form $\frac{m}{n}$, where m and n are integers and n is non-zero. The decimal expansions of such numbers are infinite and non-recurring.

irregular object
An object with faces of varying shapes. Contrast with a regular object, such as a cube.

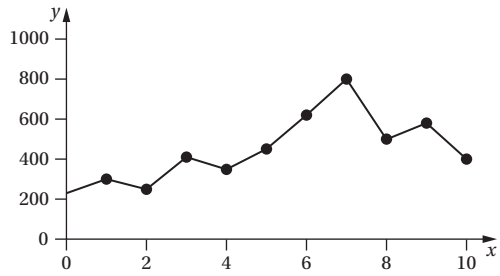

diamond

irregular shape
A shape with sides of varying lengths. Contrast with a regular shape, such as a rhombus or equilateral triangle.



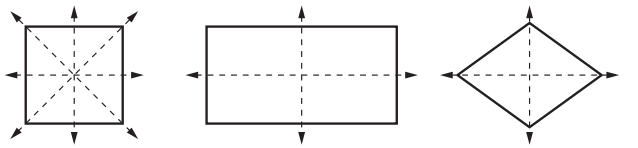
line
A model of what is perceived visually as straight. It has one dimension and extends indefinitely in the plane.

line graphs
Graphs that represent the changing relationship between two variables. They use lines to connect individual data points, and are generally used to show how something changes over time.



line segment (interval)
Let A and B be two distinct points on a line, then the part of the line between and including A and B is a line segment.

line symmetry
A line in which a shape is symmetrical, such that if you were to fold the shape about this line it would map onto itself exactly.



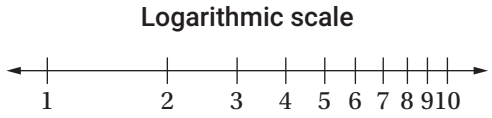
linear equations
Equations involving linear expressions. The general form of a linear equation in one variable is $ax + b = c$, where a, b and c are constants.

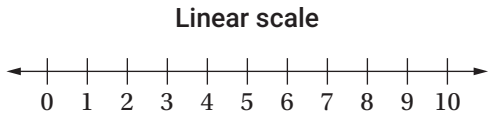
linear expression
An expression involving only variables to the power 1, and constants, e.g. $2x + 3$, $180(n - 2)$, $4x - 5y$.

linear functions
A function which when graphed on a Cartesian plane produces a straight line. It can be reduced to the general form $y = mx + b$.

linear inequalities
Inequalities that involve linear expressions and contain one of the inequality signs: either $<$, $>$, $\leq$ or $\geq$.

logarithmic scales
Scales where successive endpoint values of intervals increase by a constant factor (multiplicatively). Contrast with linear scales in which the increase is a constant amount.





mass
The measure for how much matter an object contains. The SI unit for mass is kilogram (kg).

mathematical modelling
A mathematical process involving the identification of a real-world problem, formulation and calculation of a solution to the problem mathematically using knowledge and understanding of mathematical concepts, structures and relationships, and the interpretation and communication of the mathematical solution(s) in terms of the situation. As students', mathematical modelling skills become more sophisticated, they begin to consider assumptions, choose variables, justify decisions and evaluate the effectiveness of the model.

mean
The sum of values in a data set divided by the total number of values in the data set. Also called the average.

measure
A record of the magnitude of an attribute (such as weight, length, duration and likelihood) associated with an object or event.

measure of central tendency
A statistic that is used to summarise a data set. There are three common measures of centre for a data set: mode, median and mean.

measure of turn
A measurement of rotation from an initial starting position. A turn can be a fraction of a circle (quarter turn) or complete rotation (revolution). It can also be directional (clockwise or anti-clockwise).

median
The value in a set of ordered data that divides the data into two equal parts. It is often called the 'middle value'.

mixed numeral
A form of representation of a rational number (also called a mixed fraction or mixed number) composed of an integer and a proper fraction, i.e. a fraction with a smaller numerator than denominator, e.g. $2\frac{3}{5}$.

mode
The most frequently occurring value in a set of data. There can be more than one mode. When there are two modes, the data set is said to be bimodal.

modelling
Using mathematical concepts, structures and relationships to describe and characterise, or model, a situation in a way that captures its essential features.

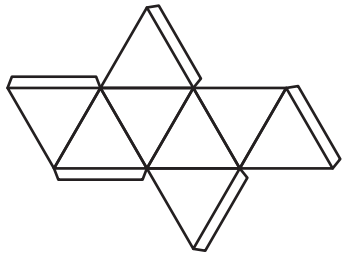
multiples
A multiple of a number is the product of that number and an integer. A multiple of a real number x is any number that is a product of x and an integer.

multiplication facts
The results associated with the products of pairs of natural numbers from 0 to 9, associated with reasons. They are foundational to arithmetic.

multiplicative
Problems or contexts that involve multiplication or division, e.g. calculating the number of seats in a theatre that has 30 rows of 24 seats.

natural numbers
The set $N = \{0, 1, 2, 3, ...\}$ or $N = \{1, 2, 3, ...\}$ depending on whether counting is started at 0 or 1. The elements of N are also called the counting numbers, used to count the number of elements in finite sets.

nets



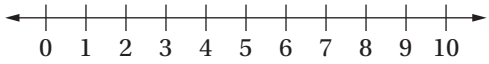
Plane figures that can be folded to form a polyhedron. More specifically, two-dimensional representations comprising joined shapes (the faces) that can be folded (along edges) to form the object.

network

A set of points (vertices or nodes), some of which are joined by lines or curves (edges) and sometimes enclose regions (faces), e.g. road networks, a family tree or the edges lining a tennis court.

number line

A line that gives a pictorial representation of real numbers.



number sentence

A statement of equality or inequality using numbers, operations and common symbols, e.g. $8 + 5 = 13$ and $16 - 6 = 10$.

numeral

The designation of a number in a given language, e.g. the number 'three' is designated by the Hindu-Arabic numeral 3, the Roman numeral III, and the Chinese numeral 三.

numerical data

Data associated with a numerical variable. It can be discrete or continuous, e.g. number of siblings in a family or heights of students in the class.

numerical variables

Variables with values that are numbers, and for which measuring and arithmetic processes, such as adding and subtracting or calculating an average, make sense.

object

A solid or three-dimensional (3D) skeletal representation composed of a number of faces, edges, vertices or closed surfaces, such as a prism, pyramid, cylinder or sphere. The term three-dimensional shape is also used.

obtuse angle

An angle that measures between 90 and 180 degrees, i.e. between one and two right angles.

odd number

An integer that is not divisible by 2. The odd numbers are $\{... -5, -3, -1, 1, 3, 5, ...\}$.

online tools

Digital hardware, software, platforms and resources that are connected and support communications between users. Examples of online tools include websites for online shopping and fitness apps.

operation

The process of combining numbers or expressions. Operations are arithmetic – addition, subtraction, multiplication and division – and also include exponentiation and substitution.

order of operations

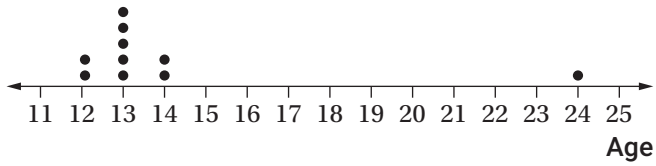
A set of conventions for evaluating expressions involving several operations. Operations in brackets are first, followed by exponents, multiplication/division, then addition/subtraction left to right.

ordinal categorical data

Categorical data that has a ranking or order, such as Likert scales, clothing sizes, placegetters in a race.

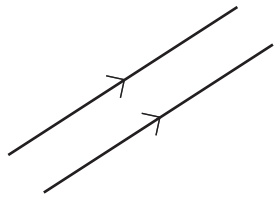
outlier

A data value that appears to stand out from the other members of the data set by being unusually high or low.



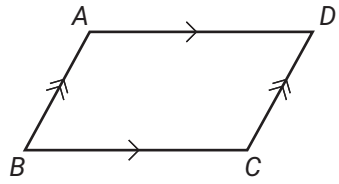
parallel lines

Two lines L and M in a plane that have no point of intersection in that plane, written $L \parallel M$.



parallelogram

A quadrilateral in which opposite sides are parallel and equal in length. The quadrilateral $ABCD$ shown is a parallelogram because $BA \parallel CD$ and $AD \parallel BC$.



parameters

A factor for which a specific value defines a condition or constraint for a function or relation.

partition numbers

Separating numbers additively or multiplicatively into two or more parts, e.g. 10 is $8 + 2$ and 8 is $3 + 3 + 2$. 12 can be divided into six equal parts of 2 (so $12 = 6 \times 2$ or $12 \div 6 = 2$), or into 3 equal parts of 4 (so $12 = 3 \times 4$ or $12 \div 3 = 4$).

partitioning

The ability to think about numbers as made up of two or more parts. Numbers can be partitioned into standard or non-standard place value partitions, such as 248 as $200 + 40 + 8$ or 62 as $50 + 10 + 2$.

part-part-whole reasoning

A model used to recognise relationships between a whole number and its parts, e.g. $7 = 5 + 2$ or $7 = 4 + 3$.

percentage

A ratio to 100 or a fraction whose denominator is 100.

perfect square
A number that is the square of an integer, e.g. 169 is a perfect square as $13^2 = 169$. Perfect squares can also be represented pictorially.

perimeter
The total length of the boundary of a plane figure or space.

perpendicular
Two lines, rays, line segments, vectors, planes or other objects that intersect at a 90° angle (a right angle) are considered perpendicular.

pi (π)
The ratio of the circumference of any circle to its diameter. It is denoted by the Greek letter π (pi). Approximate values are 3.14 or $\frac{22}{7}$.

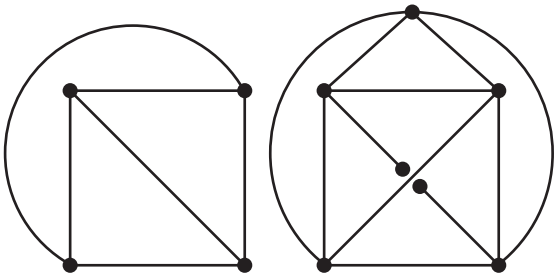
picture graph (pictographs)
A statistical graph for organising and displaying categorical data.

Ball sports played by students in Year 4	
Football	⚽⚽⚽⚽
Basketball	⚽⚽⚽
Netball	⚽⚽⚽⚽⚽
Soccer	⚽⚽
Rugby	⚽⚽⚽
Hockey	⚽⚽

Key ⚽ = 10 students

place value
The value of a digit as determined by its position in a number relative to the ones (or units) place. For integers, the ones place is occupied by the rightmost digit in the number.

planar graph
A connected graph that is drawn within a plane whose edges do not cross.



platonic solids
Three-dimensional, convex and regular objects that have polygonal faces. The five platonic solids are the tetrahedron, cube, octahedron, dodecahedron and icosahedron.

point
Representation of a location in space that has zero dimensions. Lines, curves, shapes, surfaces and objects are constructed from sets of points. Points on the Cartesian plane are specified by coordinates.

polyhedra
Three-dimensional objects whose faces are adjacent polygons. A convex polyhedron is a finite region bounded by planes, in the sense that the region lies entirely on one side of the plane.

population
The complete set of individuals, objects, places, etc. that we want information about. A census collects information about the whole population.

prime number
A natural number that is greater than 1 and its only factors are 1 and itself.

prism
A convex polyhedron that has two congruent and parallel polygonal faces; all its other faces are parallelograms. It is named according to these two congruent faces, e.g. a triangular prism.

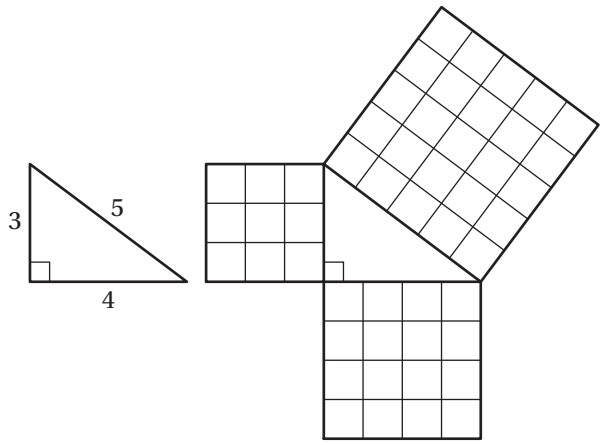
probability
The chance of something happening shown on a scale from 0 to 1 (inclusive), e.g. the probability that a fair coin toss will come up 'heads' is 0.5.

product
The result of multiplying together two or more numbers or algebraic expressions.

proof
A mathematical argument that demonstrates whether a proposition is true. A mathematical statement that has been proved is called a theorem.

proportion
A constant ratio between corresponding elements of two sets, e.g. the circumference and diameter of a circle are in proportion as their ratio is the constant π (pi).

Pythagoras' theorem
The square of the length of the hypotenuse, c, of a right-angled triangle equals the sum of the squares of the lengths of the other two sides, a and b, such that $c^2 = a^2 + b^2$.



quadratic equation
A quadratic expression that has a particular value. The general quadratic equation in one variable is $ax^2 + bx + c = 0$, for constants a, b, c and where $a \neq 0$.

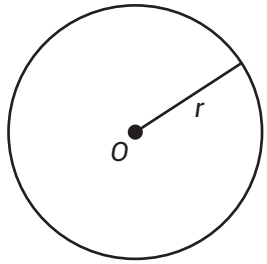
quadratic expression
An expression that contains one or more of the terms in which the variable is raised to the second power, but no variable is raised to a higher power. Its general form is $ax^2 + bx + c$, where a≠0.

quadratic function
A function in which the rule is a quadratic expression. When graphed on the Cartesian plane it produces a parabola and algebraically it can be represented in the form $y = ax^2 + bx + c$, where $a \neq 0$.

quadrilateral
A four-sided polygon. Examples include square (a regular quadrilateral, all sides and angles equal), rectangle, kite and trapezium.

quotient
The result of dividing one number or algebraic expression by another.

radius
The distance from the centre to the circumference of a circle.



random sample
A subset of the population chosen such that every element of the population has an equal chance of being selected; i.e. their selection is governed by chance, e.g. by using a random number generator.

random variable
A function that assigns real numbers to random events.

range
A set of different things of the same general type, e.g. different representations of the same fraction or a variety of strategies for solving problems involving the basic operations.

range (statistics)
The difference between the largest and smallest numerical values in a data set. It can be used as a measure of spread in a data set. It is sensitive to outliers and should be interpreted with care.

rate
A comparison of two quantities measured in different units, e.g. the rate of distance to time, known as speed, because distance and time are measured in different units (such as kilometres and hours).

ratio
A comparison of magnitudes of sets, quantities of the same kind or algebraic expressions. It is often used as a comparison of the size of two (or more) quantities relative to each other. and is written as $a{:}b$, $a{:}b{:}c$, etc .

rational number
An element of the infinite set of numbers $Q = \frac{m}{n}$, where m and n are integers and $n \neq 0$. It may be expressed in decimal form, e.g. $\frac{1}{8} = 0.125$ and $\frac{4}{9} = 0.444\ldots$

real number line
The representation of real numbers using the infinite set of points on a continuous line.

real numbers
The numbers generally used in mathematics, scientific work and everyday life. They are any of the elements of the infinite set R, which includes all the rational numbers and irrational numbers.

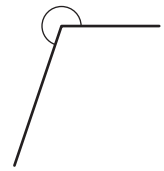
reasonableness (calculation)
Checking the results of calculations or operations to see if they are reasonable in the context, e.g. the sum of two odd natural numbers must be an even number.

rectangle
A quadrilateral in which all angles are right angles.

recurring decimal
A decimal expansion of a number that contains a finite sequence of digits that repeats indefinitely, such as 34.271271271271... The repeating sequence is often designated by a bar over it, such as $34.\overline{271}$.

reflection
A transformation where each point of a shape is mirrored to a position the same distance from an axis of symmetry.

reflex angle
An angle that measures between 180 and 360 degrees, i.e. between two and four right angles.



regular shape
A shape that has all sides of equal length, and all interior angles equal.

related denominators
One of the denominators is a multiple of the other denominator of a pair of fractions.

relation
A correspondence (map) between the elements of two sets, e.g. 'favourite colour' related to the set of students in a class (the domain) and the set of colours (the co-domain).

rename
To express a number according to the relationship between the place value powers of 10, e.g. 263 can be renamed 2 hundreds and 63 ones or 1 hundred, 16 tens and 3 ones.

repeated addition
Adding the same number several times, e.g. 3 + 3 + 3 + 3 = 4 × 3 = 12. A strategy sometimes used for multiplication.

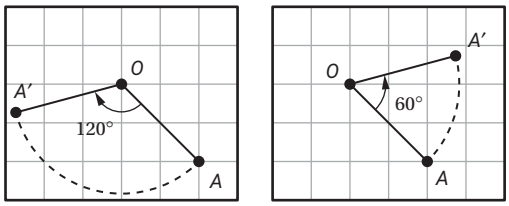
repeating pattern
A pattern where there is an identifiable repeating unit, such as beads arranged in a sequence of red, green, blue, red, green, blue ...; days of the week or months of the year.

revolution
A complete turn around a point.

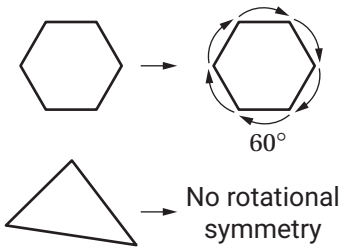
right angle
An angle formed by a quarter turn of a revolution. It is equal to 90 degrees. If two lines are at a right angle, they may also be referred to as being perpendicular.

right prism
Convex polyhedra that have two congruent and parallel faces and all remaining faces are congruent rectangles.

rotation
A transformation where each point in the plane is rotated through a given angle about a fixed point in the plane (the point or centre of rotation). For example, point A is rotated 120° clockwise and 60° anti-clockwise.



rotational symmetry
A quality of some shapes and objects to appear the same when turned around their centre for less than a full turn, e.g. a square has quarter-turn rotational symmetry.



rounding
The process for approximating a value that lies between two known values. It is used to specify a number correct to a given accuracy, e.g. 4.027 becomes 4.03 when rounded to 2 decimal places.

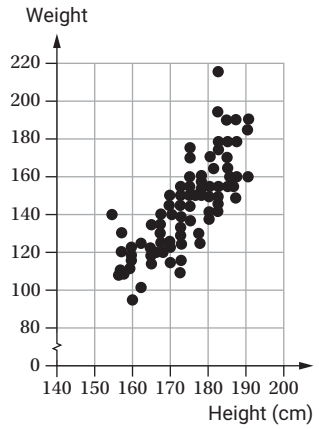
sample
A subset of a population used to estimate characteristics of the population, e.g. a randomly selected group of 8-year-olds (sample) selected to estimate the height of 8-year-olds in Australia (population).

sample space
The set of all possible outcomes of a chance experiment, e.g. the set of outcomes (also called sample points) from tossing two heads is {HH, HT, TH, TT}, where H represents a 'head' and T a 'tail'.

sampling
A process by which a subset of a population is selected for the purposes of data analysis. This subset is called a sample.

scale
Specification of proportion between two measures, e.g. a model of a house may be made on a 1:10 scale of length; on a graph, each interval of the vertical axis corresponds to a given value.

scatter plot
A visual representation of bivariate numerical data using the position of each dot on the horizontal and vertical axis to indicate the numerical values for an individual data point. Scatter plots are used to observe relationships between variables.



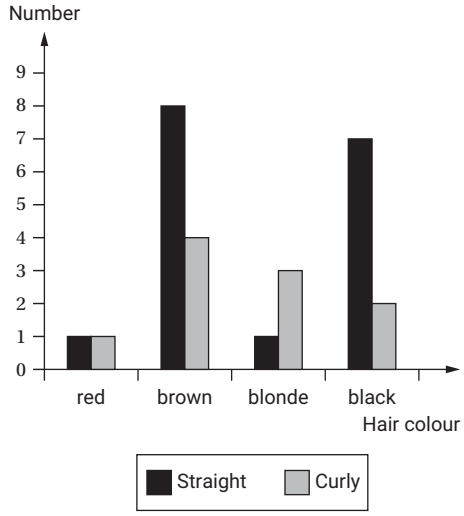
scientific notation
A way of writing numbers that are too big or too small to be written in an accessible way. It uses powers of 10, e.g. the scientific notation for 34 590 is 3.459 × 104.

set
A collection of objects or elements, usually specified by listing its elements (e.g. {1, 2, 3, 4}), by describing it in words (e.g. 'the set of primes') or by using a rule such as $\{(x, y): y = 2x + 1, x \in N\}$.

shape (space)
A geometric figure in 2-dimensional space, such as a freehand closed curve, a triangle or square. It can be regular or irregular, simple or complex.

shape (statistics)
The form of the visual representation of the distribution of numerical data, which may be symmetrical, skewed or multimodal.

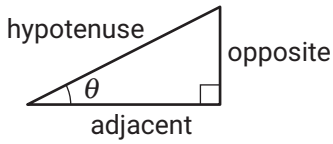
side-by-side column graph
A type of column graph that can be used to organise and display the data that arises when a group of individuals or things are categorised according to two or more criteria.



simulation
The process of simulating an event using technology or a device, e.g. if two players are equally likely to win a game on past performance, then games could be simulated by successive tossing of a fair coin.

simulations
A simulation is a re-creation of a random phenomena used to collect probability data using either physical or virtual manipulatives such as spinners, coins and cards or computer generated simulations, and which can be performed a large number of times.

sine ratio
In any right-angled triangle, $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$, where $0 < \theta < 90°$.



skip counting
Counting from a given starting value using multiples of a fixed natural number, e.g. {2, 4, 6, ...} or {7, 12, 17, ...}.

spread
A statistic that indicates how widely the values of a data set are distributed. Common measures of spread include range, interquartile range, quantiles and percentiles.

square number
The result of multiplying a natural number by itself, e.g. $16 = 4 \times 4 = 42$, and is an element of the set {0, 1, 4, 9, 16, 25}.

square root
A value which, when multiplied by itself gives the original number. The positive square root of a given real number x is the positive real number y, such that $y^2 = x$.

statistical investigation
An investigation process involving a chosen context, area of interest or an identified problem scenario, identifying the subjects and variables to be investigated, planning the approach, collecting data, representing, interpreting and analysing the data, making inferences and communicating the results of the investigation in terms of the context.

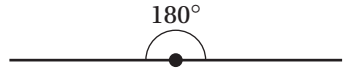
stem-and-leaf plot
A method of organising and displaying numerical data in which each data value is split into two parts: a 'stem' and a 'leaf'. Stem plots give a visual indication of spread.

Pulse rate

Stem	Leaves
6	8 8 8 9
7	0 1 1 4 6 6 8
8	2 6 8 8
9	0 6
10	4
11	0

key 6|8 = 68 bpm

straight angle
The angle formed by two opposite rays on a straight line from a common point. A straight angle is also formed by the half-turn of a ray about its endpoint. It measures 180 degrees.



subitising
The capacity to visually recognise the size of a small set of objects without counting.

supplementary angles
Two adjacent angles that form a straight angle and their sum is 180 degrees (a straight angle).

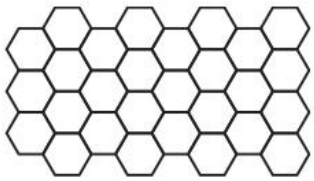
surface area
The measure of the total area of the surface(s) of a three-dimensional shape or object.

survey
A tool for collecting data that comprises a series of questions, which can be constructed to collect quantitative or qualitative data, numerical or categorical responses.

tangent ratio
The ratio of the length of the altitude, a, of a right-angled triangle to the length of its base, b, for a given base angle, theta, θ.

terminating decimal
A decimal expansion of a number which has a finite number of non-zero digits, e.g. 3.125.

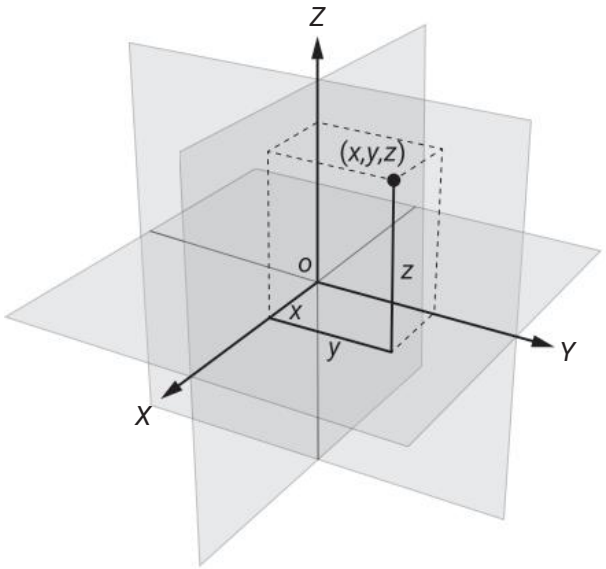
tessellation
A repeated pattern in the plane or on a surface where shapes completely fill all of the space around a given point where their boundaries meet, e.g. a honeycomb is a tessellation using hexagons.



theorem
A statement which has been derived from given statements using agreed rules of deduction (proof).

three-dimensional
An object that has three independent dimensions of length, width (breadth) and depth, or a space in which position is specified by three independent coordinates.

three-dimensional coordinate system
A coordinate system where any point in space is defined using three different dimensions, e.g. longitude, latitude and altitude for locating the position of a drone in the sky.



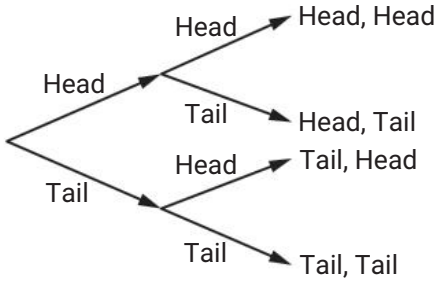
time zones
The 24 divisions of the globe, where each change of 15 degrees longitude corresponds to one hour. As time zones often align with national or regional boundaries, this is an approximate relationship.

transformation
A map of the plane onto itself. Transformations include dilation (enlargements or compressions), reflections, rotations and translations.

translation
Shifting a figure in the plane without turning it. Translations can be specified as a combination of a horizontal shift and a vertical shift.

transversal
A line that intersects two other lines obliquely.

tree diagram
A diagram consisting of line segments (edges) connected to points (vertices) like the branches of a tree. It shows the relationship between sets, events, or the set of outcomes of a multi-step random experiment.



trials
Any repeatable procedures with a well-defined sample space, e.g. the flipping of a coin.

trigonometric ratios
Relationships between the angles and sides of right triangles. The three basic trigonometric ratios are: sine (sin), cosine (cos) and tangent (tan).

two-dimensional
A shape that has two independent dimensions of length and width (breadth) is two-dimensional, as is a space in which position is specified by two independent coordinates, such as the plane.

two-way tables
A common way of displaying the two-way frequency distribution that arises when a group is categorised according to two criteria.

	Curly hair	Straight hair	Total
Red hair	1	1	2
Brown hair	8	4	12
Blonde hair	1	3	4
Black hair	7	2	9
Total	17	10	27

unit fraction
A fraction of the form $\frac{1}{n}$, where n is a non-zero natural number, such as $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

unit of repeat
In a repeating pattern, the pattern sequence that is repeated within the pattern, e.g. 1234 in the pattern 1234123412341234...

units
The basis for counting or measurement or both, e.g. the number 1 is the unit for counting; the metre is the standard unit for measurement of length in the metric system.

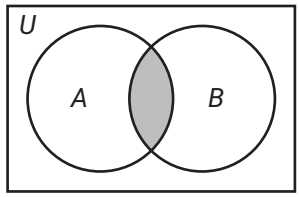
univariate data
Data relating to measurement of a single variable, e.g. shoe size.

validate (data)
The process of checking that data to be analysed is reasonable and fit for purpose. This can involve checking for accuracy, clarity, detail, consistency, correctness and completeness.

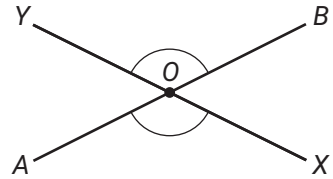
variable
Things that are measurable or observable and are expected to either change over time or between individual observations. They are often designated by symbols, such as n, x and y, to represent members of a set.

variation in data
The characteristic that elements of a data set differ, e.g. the maximum temperature recorded at different suburbs of a town on a given day.

Venn diagrams
Graphical representations, using several typically overlapping circles, showing elements of sets in relation to properties or attributes. They are drawn for some specified universal set.



vertically opposite angles
Two pairs of vertically opposite angles formed at the point of intersection of two lines in the plane. Vertically opposite angles are equal.



virtual materials
Dynamic digital representations of physical materials (sometimes called virtual manipulatives).

visualisations
Visual formats for organising, representing and summarising data. Also called data displays.

volume
The amount of space bounded by the surface of an object. It is usually measured in cubic units.

zero
The number represented by the numeral or the symbol 0, e.g. the origin on a number line. It also functions as an empty place-holder digit in the decimal expansion of numbers, e.g. 205.

Based on Australian Curriculum, Assessment and Reporting Authority (ACARA) materials

ANSWERS

Topic 1: Number properties

Recall

I can identify the place value of numbers

1 **(a)** millions **(b)** tens

I can arrange numbers in ascending or descending order

1 **(a)** 6, 8, 86, 286, 628, 826
(b) 30, 33, 3000, 3003, 30 033
2 **(a)** 876, 867, 768, 687, 678
(b) 1058, 851, 815, 805, 158

I can add numbers of different place value together

1 **(a)** 73 291 **(b)** 2 222 002

I can round numbers to the nearest 10, 100 or 1000

1 **(a)** **(i)** 3790 **(ii)** 3800 **(iii)** 4000
(b) **(i)** 12 440 **(ii)** 12 400 **(iii)** 12 000

I can multiply more than two numbers together

1 **(a)** 360 **(b)** 108 **(c)** 225 **(d)** 192

I can perform simple addition and subtraction

1 **(a)** 386 **(b)** 479 **(c)** 2411 **(d)** 8454

I can perform simple multiplication and division without the aid of a calculator

1 **(a)** 435 **(b)** 1083 **(c)** 63 **(d)** 301

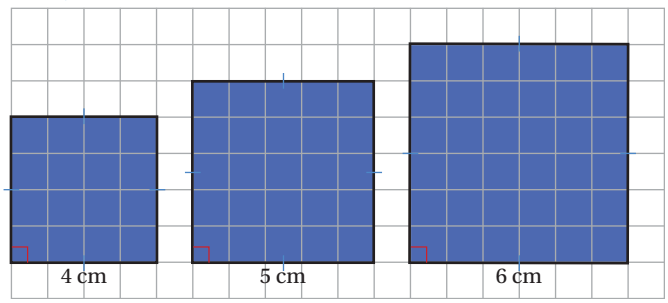
1.1 Understand and calculate squares and square roots

SC 1

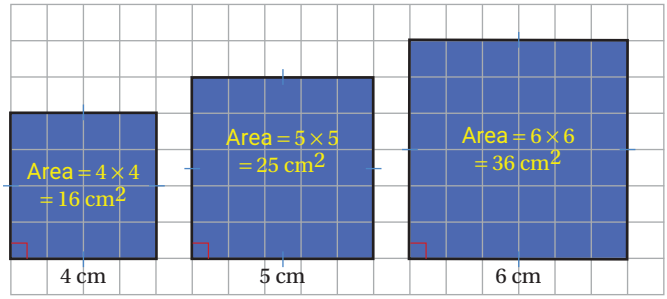
1 **(a)** The number of counters needed to make a square.
(b) 4 rows of 4 counters, 16 counters in total.

16

(c)



(d)



(e) A square number is the result of a whole number multiplied by itself.
A square number is the number of counters needed to make a square.
A square number is the area of a square with integer side lengths.

(f) Area = $10 \times 10 = 100\,cm^2$

2

$1^2 = 1$	$6^2 = 36$	$11^2 = 121$	$16^2 = 256$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$	$17^2 = 289$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$	$18^2 = 324$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$	$19^2 = 361$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$	$20^2 = 400$

3 **A** no **B** yes, 5×5
C yes, 10×10 **D** no

4 16, 25, 36

5 **(a)** 9 **(b)** 5 **(c)** 3 **(d)** 2

6 Answers will vary. Sample: A square number is formed by multiplying a whole number by itself.

7 $4 \times 4 = 16$ and $5 \times 5 = 25$.
18 is between 16 and 25. It cannot be a square number.

8 Student C's answer is correct, and the diagrams show that because 8 is between the square numbers 4 and 9, it cannot be a square number.
Student A's answer is not correct because a square has two dimensions. The $2 \times 2 \times 2$ represents a cube.
Student B's answer is not correct because a rectangle is not a square (a rectangle's four sides are not the same).

9 **(a)** $15^2 = 225$ **(b)** $64^2 = 4096$
(c) $83^2 = 6889$ **(d)** $92^2 = 8464$

SC 2

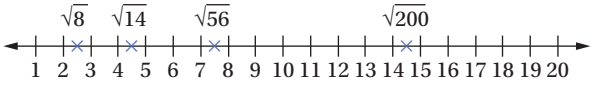
1 **(a)** 2 **(b)** 3 **(c)** 5
(d) 7 **(e)** 13 **(f)** 18

SC 3

1 3 and 4

2 **(a)** 4 and 5 **(b)** 5 and 6
(c) 10 and 11 **(d)** 16 and 17

3



4 Answers will vary.
Sample: Determine the square numbers above and below the number by trial and error. The square root Is between the square root of the two square numbers.

5 $4 \times 4 = 16$ and $5 \times 5 = 25$. Because 18 is between 16 and 25, the square root is between 4 and 5.

SC 4

1 100
2 36
3 196
4 **(a)**

Day	New infections	Total infections
1	–	2
2	2	4
3	4	8
4	8	16
5	16	32

(b) Each number doubles as it goes down the column.
(c) 32 new infections + 32 existing infections − 2 recovered = 62
(d)

Day	New infections	Recovered patients	Total infections
6	32	2	62
7	62	4	120
8	120	8	232
9	232	16	448
10	448	32	864

1.2 Understand and use exponent notation to represent numbers

SC 1

1 Base is 8, exponent is 3
2 **(a)** 5 is the base and 2 is the exponent
(b) 5 is the base and 8 is the exponent
(c) x is the base and 2 is the exponent
(d) x is the base and y is the exponent
3 **(a)** $x = 8$ **(b)** $x = 16$ **(c)** $x = 9$ **(d)** $x = 81$
4 **(a)** 4 **(b)** 5 **(c)** 2 **(d)** 3
5 **(a)** 2 **(b)** 2 **(c)** 5 **(d)** 5
6 **(a)** 800 or 100×2^3 **(b)** 409 600 or 100×2^{12}
(c) Exponent form requires less calculation and shows the number of weeks and scale factor clearly.

SC 2

1 **(a)** 4^6 **(b)** 4^4 **(c)** 4^3 **(d)** 4^2
2 **(a)** 2^3 **(b)** 3^3 **(c)** 50^3 **(d)** a^3
3 **(a)** $5^2 \times 6^4$ **(b)** $2^5 \times 5^3$
(c) $2^2 \times 10^6$ **(d)** $2^3 \times 3^2 \times 4^4$
4 **(a)** 5^2x^4 **(b)** 2^5y^3 **(c)** 2^2b^6 **(d)** $2^3c^2d^4$

5 Jezza has switched the base and exponent.
6 Joffa has multiplied 5 by 3, which is $5 + 5 + 5 = 15$, rather than raising it to the power of 3. The answer should be $5^3 = 5 \times 5 \times 5 = 125$.

1.3 Represent numbers in prime factor form

SC 1

1 **(a)** 1, 2, 5, 10 **(b)** 1, 2, 3, 4, 6, 12
(c) 1, 2, 7, 14 **(d)** 1, 2, 4, 5, 10, 20
(e) 1, 2, 3, 4, 6, 8, 12, 24
(f) 1, 2, 3, 4, 6, 9, 12, 18, 36
2 **(a)** 2, 5 **(b)** 2, 3 **(c)** 2, 7
(d) 2, 5 **(e)** 2, 3 **(f)** 2, 3

SC 2

1 **(a)**

2	16	Try 2
2	8	Try 2
2	4	Try 2
2	2	Try 2
	1	All the prime factors have been found.

The only prime factor of 16 is 2.

(b)

2	28	Try 2
2	14	Try 2
7	7	Try 2, then 3, then 5, then 7
	1	All the prime factors have been found.

The prime factors of 28 are 2 and 7.

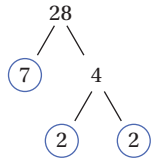
(c)

2	50	Try 2
5	25	Try 2, then 3, then 5
5	5	Try 5
	1	All the prime factors have been found.

The prime factors of 50 are 2 and 5.

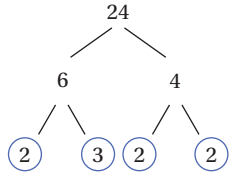
SC 3

1 **(a)**



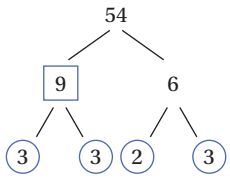
Prime factors of 28: 2 and 7

(b)



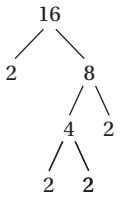
Prime factors of 24: 2 and 3

(c)



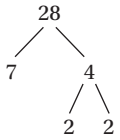
Prime factors of 54: 2 and 3

2 (a)



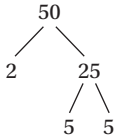
Prime factors of 16: 2

(b)



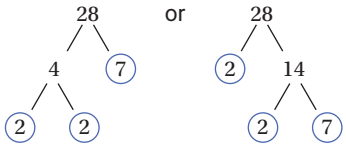
Prime factors of 28: 2 and 7

(c)



Prime factors of 50: 2 and 5

3 For numbers that are the square of a prime number (4, 9, 25, etc.), there is only one tree. However, for most numbers, the factors in the tree can be switched around, or different starting product pairs can be used.



SC 4

1 (a) $16 = 2 \times 2 \times 2 \times 2 = 2^4$
(b) $28 = 2 \times 2 \times 7 = 2^2 \times 7$
(c) $50 = 2 \times 5 \times 5 = 2 \times 5^2$
(d) $75 = 3 \times 5 \times 5 = 3 \times 5^2$
(e) $196 = 2 \times 2 \times 7 \times 7 = 2^2 \times 7^2$

2 The correct answer is $32 = 2^5$. Shayne has multiplied by 5 instead of raising to the power of 5.

3 (a) True **(b)** False, 1 is not a prime number.
(c) False, 1 is not a prime number. **(d)** True

4 (a) $2 \times 3 \times 5 = 30$, $2 \times 3 \times 7 = 42$ and $2 \times 5 \times 7 = 70$
(b) $3^3 = 27$ and $5^3 = 125$ **(c)** $2^2 \times 3^2 = 36$

1.4 Understand and identify common factors and multiples

SC 1

1 (a) 1, 2, 4, 8, 16 **(b)** 1, 2, 3, 4, 6, 8, 12, 24
(c) 1, 2, 4, 8 **(d)** 8

2 (a) 1, 3, 7, 21 **(b)** 1, 3, 5, 9, 15, 45
(c) 1, 3 **(d)** 3

3 (a) 1, 2, 4, 8, 16, 32
(b) 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
(c) 1, 2, 4, 7, 8, 14, 28, 56 **(d)** 1, 2, 4, 8 **(e)** 8

4 (a) 2 **(b)** 4 **(c)** 8 **(d)** 6

5 (a) 1, 2, 3, 4, 5, 6, 10, 12, 15, 18, 30, 36, 45, 60, 90, 180
(b) 1, 3, 5, 9, 15, 25, 45, 75, 225
(c) 1, 3, 5, 15, 45 **(d)** 45

6 4 m

7 The HCF for 72, 108 and 132 is 12. So there are 12 eggs in each carton.

SC 2

1 (a) 8, 16, 24, 32, 40 **(b)** 12, 24, 36, 48, 60
(c) 24

2 (a) 6, 12, 18, 24, 30, 36 **(b)** 5, 10, 15, 20, 25, 30
(c) 30

3 (a) 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48
(b) 8, 16, 24, 32, 40, 48
(c) 6, 12, 18, 24, 30, 36, 42, 48
(d) 48

4 (a) 40 **(b)** 30 **(c)** 60 **(d)** 36

5 The LCM is 30. Number of troops is 31.

SC 3

1 84 seconds after 1 pm or 1:01:24 pm

2 5 packets of plates and 2 packets of paper cups

3 12 m

4 3 rows, with 11 carrot and 6 tomato plants.

5 10 people

6 14

7 (a) 30 m **(b)** 22

Exploration Task: Prime life cycles

Periodic cicadas appear at the ground surface after feeding on shrubs and roots underground for a prime number of years: 7, 13 or 17 years. This is to avoid the life cycle of their predators, such as wasps, whose life cycles occur at different intervals such as every 6 or 12 years.

1.5 Understand and use expanded notation to represent numbers

SC 1

1 (a)

Number	Expanded form	Exponent form
100 000	$10 \times 10 \times 10 \times 10 \times 10$	10^5
10 000	$10 \times 10 \times 10 \times 10$	10^4
1000	$10 \times 10 \times 10$	10^3
100	10×10	10^2
10	10	10^1

(b) The exponent is the same as the number of zeros in the number.

(c) The index exponent decreases by 1 as we go down the column.

2 **(a)** 10^6 **(b)** 10^7 **(c)** 10^9

3 **(a)** The student has confused $10 \times 10 \times 10 \times 10 = 10\,000$ with $4 \times 10 = 40$

(b) $5 \times 10^2 \neq 50^2$ You cannot multiply the bases.

4

Prefix	Meaning	Expanded form	Exponent form
kilo	× 1000	$10 \times 10 \times 10$	10^3
mega	× 1 000 000	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	10^6
giga	× 1 000 000 000	$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	10^9
tera	× 1 000 000 000 000	$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	10^{12}
peta	× 1 000 000 000 000 000	$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	10^{15}

5 18

SC 2

1

	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Ones
General form	100 000	10 000	1000	100	10	1
Exponent form	10^5	10^4	10^3	10^2	10^1	10^0
Expanded form	$10 \times 10 \times 10 \times 10 \times 10$	$10 \times 10 \times 10 \times 10$	$10 \times 10 \times 10$	10×10	10	1

2 **(a)** $8 \times 10^1 + 2 \times 10^0$

(b) $8 \times 10^2 + 2 \times 10^1 + 6 \times 10^0$

(c) $8 \times 10^3 + 2 \times 10^1 + 6 \times 10^0$

(d) $8 \times 10^3 + 2 \times 10^2 + 6 \times 10^0$

(e) $8 \times 10^3 + 2 \times 10^2 + 6 \times 10^1$

3 **(a)** 352 **(b)** 350 **(c)** 302 **(d)** 4352 **(e)** 4052 **(f)** 4302 **(g)** 4002

4 **(a)** There should be a 2 in the ones place and a zero in the tens place, moving the first 2 digits to the left.

(b) The answer should be 350. The 0 in the ones column has been left out.

(c) The zero should be in the hundreds column. 4052

5 **(a)** $426 = 4 \times 10^2 + 2 \times 10^1 + 6 \times 10^0$

(b) $5203 = 5 \times 10^3 + 2 \times 10^2 + 3 \times 10^0$

(c) $152\,174 = 1 \times 10^5 + 5 \times 10^4 + 2 \times 10^3 + 1 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$

Topic 2: Operations with decimals

Recall

I can write decimals given their place value

1 **(a)** 0.05 **(b)** 0.003 **(c)** 1.5 **(d)** 0.025

I can partition decimals to describe them using place values

1 **(a)** 1.0 + 0.4 + 0.07 **(b)** 3.0 + 0.02

(c) 2.0 + 0.4 + 0.09 + 0.005

(d) 4.0 + 0.6 + 0.02

I can perform operations with whole numbers

1 **(a)** 24 **(b)** 29 **(c)** 63 **(d)** 221 **(e)** 1062 **(f)** 4522

2 **(a)** 6 **(b)** 5 **(c)** 11 **(d)** 35 **(e)** 592 **(f)** 1986

3 **(a)** 16 **(b)** 36 **(c)** 140 **(d)** 240 **(e)** 260 **(f)** 225

4 **(a)** 3 **(b)** 6 **(c)** 5 **(d)** 24 **(e)** 103 **(f)** 751

I can round whole numbers to the nearest 10, 100 and 1000

1 **(a)** 80 **(b)** 30 **(c)** 60

2 **(a)** 100 **(b)** 200 **(c)** 200

3 **(a)** 6000 **(b)** 13 000 **(c)** 2000

2.1 Round decimals for different purposes

SC 1

1 **(a)** 4.10 **(b)** 6.247 **(c)** 11.7 **(d)** 15.9893

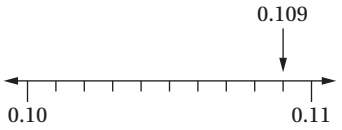
2 **(a)** 14.3 **(b)** 21.40 **(c)** 122.0 **(d)** 16.00

3 **(a)** Student B is correct. Student A has looked at the wrong digit.

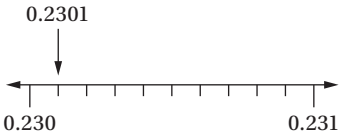
(b) Student B is correct. Student A has incorrectly changed the nine digit to a one.

SC 2

1 **(a)** The rounded value is 0.109.



(b) The rounded value is 0.2301.



2 **(a)** **B**, 0.9376 **(b)** **C**, ten-thousandths

3 **(a)** 23.11 **(b)** 23.1091 **(c)** 23

4 **(a)** 1.25 **(b)** 2.01 **(c)** 0.10 **(d)** 8.00

5 **(a)** 20.857 142 857... **(b)** 20.86 km

(c) This is the nearest tens of metres.

SC 3

1 **(a)** 1 mm **(b)** $\frac{1}{16}$ inch **(c)** 9.8 cm

(d) $3\frac{1}{16}$ or $3\frac{1}{8}$ inch (it is in the middle of these two scale markings)

2 **(a)** **B**, days **(b)** **A**, hundredths of seconds

(c) **B**, minutes **(d)** **C**, years

3 **(a)** It has been measured to the nearest minute.

(b) It has been measured to the nearest tenth of a kilometre.

(c) Neither of these seem likely estimates as the size of both Melbourne and Sydney could see 30–40 minutes and 20 or so kilometres added to just reach the nominal starting and finishing points. However, they give what might be described as a ballpark figure. So, about 9–10 hours would seem reasonable.

4 **(a)** $5000 **(b)** $5778.125

(c) It took 2 years.

(d) This value was rounded up, although it is closer to the previous half-year. However, the question required at least $6300, so the next half-year was required.

5 **(a)** $84.35 (the 5-cent coin is the smallest still in circulation)

(b) The nearest five-cent.

(c) The total bill is likely to have been $253. When each person contributes $84.33, the total will be one cent short.

6 Answers will vary.

(a) 1.58, 1.59, 1.62

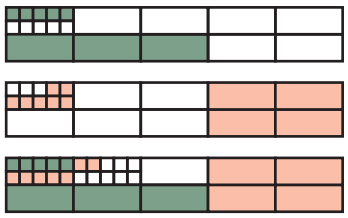
(b) 1.639, 1.641, 1.644

(c) 1.6468, 1.6471, 1.6473

2.2 Add and subtract decimals

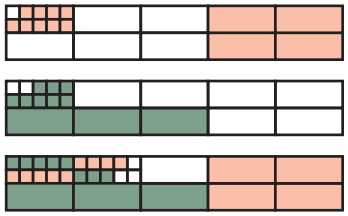
SC 1

1 **(a)**



0.35 + 0.47 = 0.82

(b)



0.49 + 0.38 = 0.87

2 **(a)** 0.95 − 0.43 = 0.52 **(b)** 0.86 − 0.54 = 0.32

(c) 0.43 − 0.26 = 0.17 **(d)** 0.55 − 0.47 = 0.08

3 **(a)** 0.14 − 0.08 = 0.06

SC 2

1 **(a)** 28.4 **(b)** 23.24 **(c)** 55.919 **(d)** 111.7529

2 **(a)** 5.26 **(b)** 2.02 **(c)** 11.32 **(d)** 11.12

3 **(a)** 2.81 **(b)** 3.77 **(c)** 3.79 **(d)** 7.007

4 **(a)** **D**, 15.1 **(b)** **A**, 1.8 **(c)** **B**, 2.7 **(d)** **C**, 8.4

SC 3

1 **(a)** 37.833 **(b)** 19.007 **(c)** 39.849 **(d)** 44.404

2 **(a)** 7.343 **(b)** 6.521 **(c)** 0.774 **(d)** 2.058

3 **(a)** 1.3003 **(b)** 3.961 **(c)** 3.7044 **(d)** 7.2036

4 **(a)** **B**, 5.684 **(b)** **C**, 356.191

(c) **A**, 2.62013

5 **(a)** 66.6 mm

(b) Any three numbers that work. For example, 2.226 + 2.224 + 2.39 = 6.84.

(c) 13.955 km

6 **(a)** It seems that Estelle has simply subtracted the smaller of two digits from the larger, irrespective of position in the calculation.

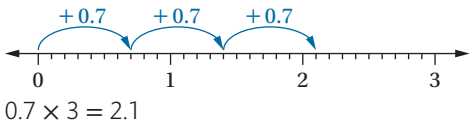
(b) The question involves a number that is just slightly more than 2, minus a number that is almost equal to 2. The answer should be close to zero.

(c) The correct answer is 0.089.

2.3 Multiply decimals

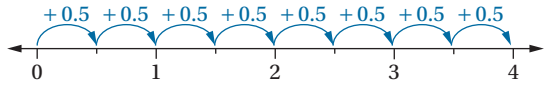
SC 1

1 **(a)**



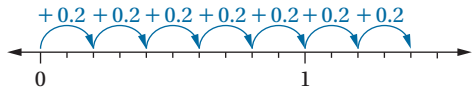
0.7 × 3 = 2.1

(b)



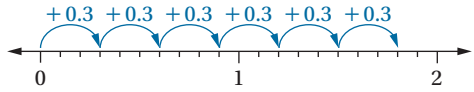
$0.5 \times 8 = 4$

(c)



$0.2 \times 7 = 1.4$

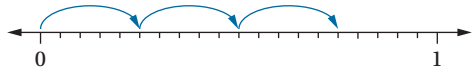
(d)



$0.3 \times 6 = 1.8$

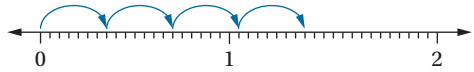
2 **(a)** $0.8 \times 4 = 3.2$ **(b)** $1.2 \times 3 = 3.6$

3 **(a)**



$0.25 \times 3 = 0.75$

(b)



$0.35 \times 4 = 1.4$

4 **(a)** 0.005 **(b)** $0.015 \times 3 = 0.045$

SC 2

1 **(a)** 15 and 20; $3.6 \times 5 = 18.0$

(b) 22 and 33; $2.95 \times 11 = 32.45$

(c) 9 and 12; $3.615 \times 3 = 10.845$

(d) 80 and 90; 81.111

2 **(a)** \$19.50 **(b)** \$56.90

(c) 155.25 s **(d)** 159.6 mm

SC 3

1 **(a)** 26 248.2 **(b)** 7604.1 **(c)** 3620.7 **(d)** 241.015

2 700

3 Sample answer: 3400, 3500, 3600

SC 4

1 **(a)** 10.54 **(b)** 5.1 **(c)** 19.44 **(d)** 53.29

2 **(a)** 3.328 **(b)** 2.3331 **(c)** 5.081 52 **(d)** 16.39757

3 **(a)** 5 **(b)** 6 **(c)** 8 **(d)** 4

4 When the calculation is done in full, the result is 29.80120, but the trailing zero does not have to be included. This is where Derek was tricked by the options provided.

5 **(a)** \$124.176 **(b)** \$124.20

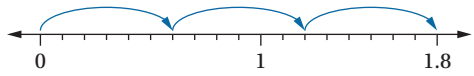
(c) The cash price is rounded to the nearest 5 cents.

6 **(a)** \$7.25 **(b)** 307 g

2.4 Divide decimals

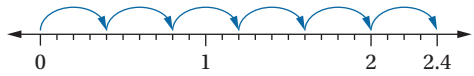
SC 1

1 **(a)**



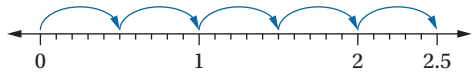
$1.8 \div 3 = 0.6$

(b)



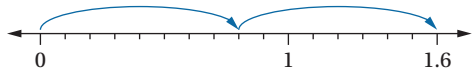
$2.4 \div 6 = 0.4$

(c)



$2.5 \div 5 = 0.5$

(d)



$1.6 \div 2 = 0.8$

2 **(a)** $2.7 \div 9 = 0.3$ **(b)** $2.8 \div 4 = 0.7$

SC 2

1 **(a)** 3.56 **(b)** 41.03 **(c)** 40.39 **(d)** 11.02

2 **(a)** 40.655 **(b)** 3.7246 **(c)** 28.2675 **(d)** 24.8465

SC 3

1 **(a)** 8.462 97 **(b)** 0.012 162

(c) 0.840 175 **(d)** 0.020 16

2 \$35.50 per person

3 **(a)** Sample answer: Any number between 45 000 and 54 000 will be correct.

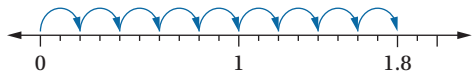
(b) The lowest value possible is $9000 \times 5 = 45\,000$ and the highest possible is $9000 \times 6 = 54\,000$.

SC 4

1 **(a)** 65.6 **(b)** 72.7 **(c)** 1111.1 **(d)** 202.02

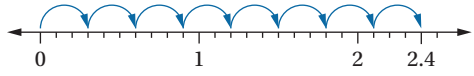
2 **(a)** 924 100 **(b)** 56.235 **(c)** 8102 **(d)** 386.55

3 **(a)**



$1.8 \div 0.2 = 9$

(b)



$2.4 \div 0.3 = 8$

4 **(a)** smaller **(b)** larger

5 **(a)** **C**, $867\overline{)28.64}$ **(b)** **A**, $15\overline{)22306}$

6 **(a)** $10 \div 3$ produces a non-terminating decimal so there is no accurate way to make the cuts.

(b) To the nearest centimetre.

(c) Each piece will be 3.33 m = 333 cm long.

7 **(a)** \$40.68 **(b)** \$2.26/km

8 **(a)** **B**, 30 **(b)** **A**, 0.25

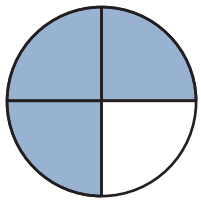
9 Sample answer: $1.802\,46 \div 0.008$; $0.180\,246 \div 0.0008$; $180.246 \div 0.8$

Topic 3: Fractions, decimals and percentages

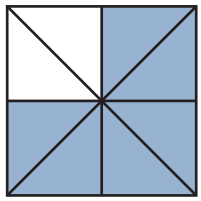
Recall

I can identify fractions of shapes

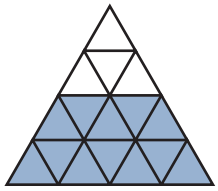
1 **(a)**



(b)



(c)



2 A, E

I can describe the fraction the division of a quantity represents

1 **(a)** **A**, $\frac{1}{2}$ **(b)** **C**, $\frac{3}{4}$

I can compare and order simple fractions

1 **(a)** $\frac{3}{8} > \frac{1}{2}$ **(b)** $\frac{1}{4} > \frac{1}{8}$ **(c)** $\frac{2}{4} > \frac{3}{9}$ **(d)** $\frac{2}{8} = \frac{3}{12}$

I can multiply and divide decimals by powers of 10

1 **(a)** 8 **(b)** 132 **(c)** 1600
(d) 36.7 **(e)** 467.2 **(f)** 7323

2 **(a)** 1.24 **(b)** 2.636 **(c)** 0.6921
(d) 2.035 **(e)** 10.046 **(f)** 9.080

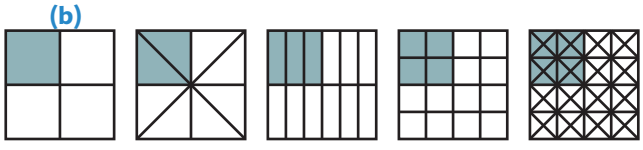
3.1 Understand and create equivalent fractions

SC 1

1 **(a)**

$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{6}{12} = \frac{8}{16}$

(b)

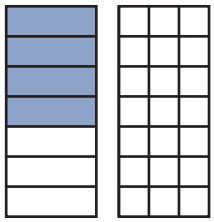


$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{16}{64}$

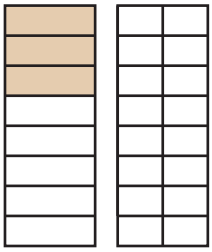
(c)

$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{48}{64}$

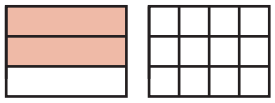
2 **(a)**



(b)



(c)



3 **(a)** $\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24} = \frac{7}{28} = \frac{8}{32} = \frac{9}{36} = \frac{10}{40}$

(b) $\frac{1}{7} = \frac{2}{14} = \frac{3}{21} = \frac{4}{28} = \frac{5}{35} = \frac{6}{42} = \frac{7}{49} = \frac{8}{56} = \frac{9}{63} = \frac{10}{70}$

(c) $\frac{1}{10} = \frac{2}{20} = \frac{3}{30} = \frac{4}{40} = \frac{5}{50} = \frac{6}{60} = \frac{7}{70} = \frac{8}{80} = \frac{9}{90} = \frac{10}{100}$

4 **(a)** $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$ **(b)** $\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$

(c) $\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$ **(d)** $\frac{5}{6} = \frac{10}{12}$

5 Option **D**

6 **(a)** $\frac{3}{6} = \frac{6}{12}$ **(b)** $\frac{6}{7} = \frac{18}{21}$

SC 2

1 **(a)** $\frac{2}{5} = \frac{6}{15} = \frac{10}{25} = \frac{16}{40}$ **(b)** $\frac{3}{7} = \frac{6}{14} = \frac{21}{49} = \frac{27}{63}$

(c) $\frac{3}{11} = \frac{9}{33} = \frac{15}{55} = \frac{30}{110}$ **(d)** $\frac{8}{13} = \frac{24}{39} = \frac{48}{78} = \frac{80}{130}$

2 **(a)** $\frac{4}{9} = \frac{16}{36} = \frac{36}{81} = \frac{44}{99}$ **(b)** $\frac{5}{8} = \frac{25}{40} = \frac{60}{96} = \frac{100}{160}$

(c) $\frac{2}{15} = \frac{6}{45} = \frac{14}{105} = \frac{22}{165}$ **(d)** $\frac{15}{17} = \frac{45}{51} = \frac{90}{102} = \frac{300}{340}$

3 **(a)** $\frac{3}{6} = \frac{4}{8} = \frac{21}{42} = \frac{27}{54}$ **(b)** $\frac{2}{6} = \frac{18}{54} = \frac{20}{60} = \frac{22}{66}$

(c) $\frac{11}{14} = \frac{22}{28} = \frac{55}{70} = \frac{110}{140}$

4 **(a)** Yes **(b)** No **(c)** No

3.2 The lowest common denominator and comparing fractions

SC 1

1 **(a)** 42 **(b)** 120 **(c)** 60 **(d)** 60

2 **(a)** $\frac{3}{5} < \frac{3}{4}$ **(b)** $\frac{5}{8} > \frac{4}{7}$ **(c)** $\frac{3}{8} < \frac{5}{12}$

(d) $\frac{4}{11} > \frac{3}{10}$ **(e)** $\frac{13}{14} > \frac{8}{9}$ **(f)** $\frac{5}{11} < \frac{6}{13}$

3 These are sample answers.

(a) $\frac{13}{20}, \frac{14}{20}, \frac{15}{20}$ **(b)** $\frac{17}{36}, \frac{18}{36}, \frac{19}{36}$ **(c)** $\frac{13}{40}, \frac{14}{40}, \frac{15}{40}$

4 **(a)** $\frac{0.5}{21} = \frac{1}{42}$ **(b)** $\frac{0.5}{25} = \frac{1}{50}$

(c) They are both unit fractions. $\frac{1}{42}$ must be greater than $\frac{1}{50}$ as there are only 42 equal pieces for a whole, compared to 50 equal pieces.

(d) When the denominators are both odd numbers and the fractions are both the smallest amount possible greater than $\frac{1}{2}$, then the fraction with the smaller denominator will always be greater. When the fractions are both the smallest amount possible less than $\frac{1}{2}$, then the fraction with the larger denominator will be greater. For example, using the same denominators as this question, $\frac{10}{21} < \frac{12}{25}$.

Exploration Task: Glass half full

$\frac{2}{7} + \frac{3}{7} + \frac{4}{7} + \frac{5}{7} + \frac{6}{7} = \frac{20}{7}$.

Approximately $2\frac{6}{7}$ glasses of water.

3.3 Simplify fractions

SC 1

1 **(a)** 5 **(b)** 4 **(c)** 2 **(d)** 1

2 **(a)** $1 \times 40, 2 \times 20, 4 \times 10, 5 \times 8$

(b) $1 \times 48, 2 \times 24, 3 \times 16, 4 \times 12, 6 \times 8$

(c) $1 \times 99, 3 \times 33, 9 \times 11$

(d) Assuming the pairs have been written in order, then the end is reached when there are no more factors between the last pair of numbers. As an example, for 40 there are no factors between 5 and 8.

3 **(a)** $1 \times 100, 2 \times 50, 4 \times 25, 5 \times 20, 10 \times 10$

(b) $1 \times 81, 3 \times 27, 9 \times 9$ **(c)** $1 \times 121, 11 \times 11$

(d) The last factor pair repeats the number; these are square numbers.

SC 2

1 **(a)** $\frac{3}{7}$ **(b)** $\frac{12}{25}$ **(c)** $\frac{11}{6}$

(d) $\frac{22}{9}$ **(e)** $3\frac{2}{7}$ **(f)** $4\frac{11}{40}$

2 **(a)** **B**, $\frac{7}{11}$ **(b)** **B**, $-4\frac{4}{5}$

3 **(a)** $\frac{1200}{720} = \frac{120 \times 10}{72 \times 10} = \frac{120}{72} = \frac{60 \times 2}{36 \times 2} = \frac{60}{36} = \frac{10 \times 6}{6 \times 6} = \frac{10}{6} = \frac{5 \times 2}{3 \times 2} = \frac{5}{3}$

(b) This is a sample answer. Students may identify different factors or use a different order.

$$\frac{166\,320}{997\,920} = \frac{16\,632 \times 10}{99\,792 \times 10} = \frac{16\,632}{99\,792} = \frac{4158 \times 4}{24\,948 \times 4} = \frac{4158}{24\,948} = \frac{2079 \times 2}{12\,474 \times 2} = \frac{2079}{12\,474} = \frac{693 \times 3}{4158 \times 3} = \frac{693}{4158} = \frac{231 \times 3}{1386 \times 3} = \frac{77 \times 3}{462 \times 3} = \frac{77}{462} = \frac{11 \times 7}{66 \times 7} = \frac{11}{66} = \frac{1 \times 11}{6 \times 11} = \frac{1}{6}$$

4 These are sample answers.

(a) $2\frac{3}{5} = 2\frac{6}{10} = 2\frac{9}{15} = 2\frac{12}{20}$

(b) $-5\frac{1}{4} = -5\frac{2}{8} = -5\frac{3}{12} = -5\frac{4}{16}$

3.4 Understand the relationship between decimals, percentages and fractions

SC 1

1 **(a)** $1.2 = 1 + \frac{2}{10}$ **(b)** $12.5 = 10 + 2 + \frac{5}{10}$

(c) $12.56 = 10 + 2 + \frac{5}{10} + \frac{6}{100}$

(d) $125.687 = 100 + 20 + 5 + \frac{6}{10} + \frac{8}{100} + \frac{7}{1000}$

(e) $1256.8793 = 1000 + 200 + 50 + 6 + \frac{8}{10} + \frac{7}{100} + \frac{9}{1000} + \frac{3}{10\,000}$

(f) $28.985 = 20 + 8 + \frac{9}{10} + \frac{8}{100} + \frac{5}{1000}$

(g) $1267.9 = 1000 + 200 + 60 + 7 + \frac{9}{10}$

(h) $12\,356.987 = 10\,000 + 2000 + 300 + 50 + 6 + \frac{9}{10} + \frac{8}{100} + \frac{7}{1000}$

2 **(a)** 318.318 **(b)** 26.721 **(c)** 1067.82 **(d)** 1463.007

3 **(a)** **B**, 30 **(b)** **D**, $\frac{5}{1000}$

4 **(a)** 0.6 **(b)** 0.09 **(c)** 0.806 **(d)** 7.26

SC 2

1 **(a)** $\frac{1}{10}$ **(b)** $\frac{12}{100}$ **(c)** $\frac{123}{1000}$

(d) $\frac{1234}{10\,000}$ **(e)** $\frac{12345}{100\,000}$ **(f)** $12\frac{9}{10}$

(g) $132\frac{861}{1000}$ **(h)** $2564\frac{227}{1000}$ **(i)** $106\frac{107}{1000}$

SC 3

1 **(a)** 1.587% **(b)** 15.87% **(c)** 158.7%
(d) 1587% **(e)** 22.2897% **(f)** 222.897%
(g) 2228.97% **(h)** 22 289.7%
(i) 600.8% **(j)** 9020.13%

SC 4

1 **(a)** 240% = 2.4 **(b)** 830% = 8.3
(c) 525% = 5.25 **(d)** 670% = 6.7

2 **(a)** $\frac{22}{25}$ **(b)** $\frac{18}{25}$ **(c)** $\frac{1}{4}$ **(d)** 1

3 **(a)** $1\frac{1}{4}$ **(b)** $2\frac{3}{10}$ **(c)** $8\frac{1}{2}$ **(d)** $8\frac{23}{25}$

4 **(a)** 9.63 **(b)** 2.9865

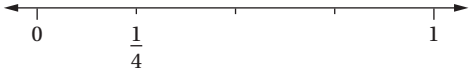
5 **(a)** 0.375 **(b)** 0.625 **(c)** 0.875 **(d)** 1.625

6 **(a)** 6.25% **(b)** 31.25% **(c)** 68.75% **(d)** 81.25%

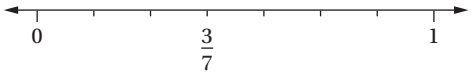
3.5 Locate fractions and decimals on a number line

SC 1

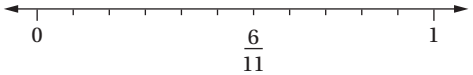
1 **(a)**



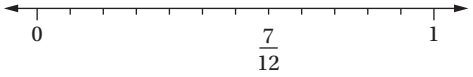
(b)



(c)



(d)

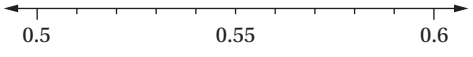


2 **(a)** **B**, $\frac{2}{3}$

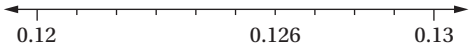
SC 2

1 **(a)**

(b)



(c)



2 **(a)** **C**, 0.76 **(b)** **D**, 0.188

3 **(a)** 0 and 1 **(b)** 0.8 and 0.9
(c) 0.87 and 0.88 **(d)** 0.876 and 0.877
(e) 0 and 1 **(f)** 0.3 and 0.4
(g) 0.31 and 0.32 **(h)** 0.317 and 0.318

Exploration Task: Number or operation?

The number 0.5 is placed correctly, $\frac{1}{2}$ is placed incorrectly.

The misconception that led to the misplaced card was that it should be placed halfway along the string.

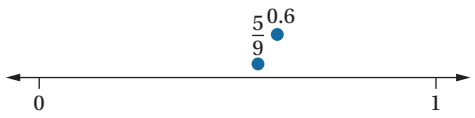
3.6 Compare fractions and decimals on a number line

SC 1

1 **(a)**

$\frac{5}{6} > 0.8$

(b)



$0.6 > \frac{5}{9}$

(c)

$0.2 > \frac{1}{7}$

(d)

$\frac{3}{11} > 0.25$

2 **(a)** **B**, $\frac{5}{7}$ **(b)** **C**, 0.45

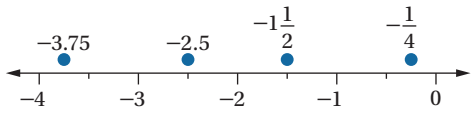
3 **(a)** 1.5 cm **(b)** 2.2 cm **(c)** 3.6 cm **(d)** 3.8 cm

4 **(a)** **B**, 7 cm

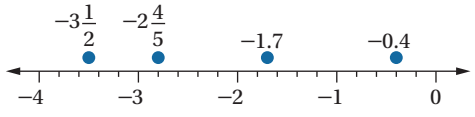
(b) Writing the values as fractions with a common denominator, they are $\frac{20}{70}$ and $\frac{21}{70}$, so using 7 cm = 70 mm makes the most sense.

SC 2

1 **(a)**



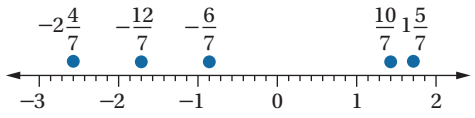
(b)



2 **(a)** $-2\frac{5}{9}$ **(b)** $-3\frac{3}{7}$

3 **(a)** −3.25 **(b)** −0.125

4



In ascending order: $-2\frac{4}{7}, -\frac{12}{7}, -\frac{6}{7}, \frac{10}{7}, 1\frac{5}{7}$

Topic 4: Operations with fractions

Recall

I can identify and compare multiples of numbers

1 **(a)** 4, 8, 12, 16, 20, 24, 28 **(b)** 8, 16, 24, 32, 40, 48, 56
(c) 12, 24, 36, 48, 60, 72, 84

2 Common multiple 24

I can find the highest common factor for numbers

1 **(a)** 2 **(b)** 9 **(c)** 14 **(d)** 13

2 HCF is 9
18 – 1, 2, 3, 6, 9, 18 27 – 1, 3, 9, 27
36 – 1, 2, 3, 4, 6, 9, 12, 18, 36

I can multiply and divide decimals by powers of 10

1 **(a)** 1600 **(b)** 132 **(c)** 8
(d) 36.7 **(e)** 4672 **(f)** 43.2

2 **(a)** 0.432 **(b)** 0.004672 **(c)** 0.367
(d) 0.08 **(e)** 0.0132 **(f)** 0.0016

I can apply factors and multiples to fraction denominators to determine equivalent representations

1 **(a)** $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\frac{6}{12}$ **(b)** $\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12}$
(c) $\frac{3}{4}=\frac{6}{8}=\frac{9}{12}$ **(d)** $\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}$

4.1 Understand how to add and subtract fractions

SC 1

1 **(a)** $\frac{10}{13}$ **(b)** $\frac{12}{17}$ **(c)** $\frac{7}{15}$ **(d)** $\frac{14}{19}$

2 **(a)** $\frac{3}{13}$ **(b)** $\frac{5}{17}$ **(c)** $\frac{1}{15}$ **(d)** $\frac{8}{19}$
(e) $-\frac{1}{5}$ **(f)** $-\frac{3}{9}=-\frac{1}{3}$ **(g)** $-\frac{2}{7}$ **(h)** $-\frac{3}{15}=-\frac{1}{5}$

SC 2

1 **(a)** $\frac{5}{8}$ **(b)** $\frac{1}{14}$ **(c)** $\frac{3}{18}=\frac{1}{6}$ **(d)** $\frac{5}{10}=\frac{1}{2}$

2 **(a)** $\frac{8}{10}=\frac{4}{5}$ **(b)** $\frac{15}{12}$ **(c)** $\frac{20}{15}$ **(d)** $\frac{9}{8}$

3 **(a)** $\frac{16}{15}$ **(b)** $-\frac{3}{14}$ **(c)** $\frac{22}{21}$ **(d)** $-\frac{5}{36}$

4 **(a)** $\frac{11}{54}$ **(b)** $-\frac{26}{48}=-\frac{13}{24}$ **(c)** $-\frac{31}{50}$ **(d)** $-\frac{19}{96}$

5 **(a)** $\frac{2}{14}=\frac{1}{7}$ **(b)** $\frac{3}{10}$

SC 3

1 **(a)** $\frac{43}{30}$ **(b)** $\frac{85}{99}$ **(c)** $\frac{92}{63}$ **(d)** $\frac{47}{36}$

2 **(a)** $\frac{6}{35}$ **(b)** $\frac{8}{33}$ **(c)** $\frac{11}{45}$ **(d)** $\frac{25}{60}=\frac{5}{12}$

3 **(a)** $\frac{19}{60}$ **(b)** $\frac{19}{48}$ **(c)** $\frac{97}{132}$ **(d)** $\frac{47}{36}$

4 **(a)** Student B is correct. Student A has not written the fractions with equivalent denominators.
(b) Student A is correct. Student B has not written the fractions with equivalent denominators.

4.2 Understand mixed numbers and improper fractions

SC 1

1 **(a)** $\frac{16}{7}$ **(b)** $\frac{37}{11}$ **(c)** $\frac{41}{9}$ **(d)** $\frac{73}{6}$

2 **(a)** $5\frac{4}{5}$ **(b)** $7\frac{5}{6}$ **(c)** $7\frac{4}{11}$ **(d)** $5\frac{5}{12}$

3 **(a)** $-\frac{35}{6}$ **(b)** $-\frac{51}{11}$ **(c)** $-\frac{91}{9}$ **(d)** $-\frac{38}{3}$

4 **(a)** $-6\frac{4}{9}$ **(b)** $-6\frac{1}{8}$ **(c)** $-10\frac{3}{10}$ **(d)** $-7\frac{2}{11}$

5 **(a)** $\frac{6}{5}=1\frac{1}{5}$ **(b)** $\frac{12}{9}=1\frac{3}{9}=1\frac{1}{3}$
(c) $\frac{8}{8}=1$ **(d)** $\frac{10}{7}=1\frac{3}{7}$

SC 2

1 **(a)** $3\frac{17}{20}$ **(b)** $11\frac{1}{2}$ **(c)** $19\frac{4}{15}$ **(d)** $33\frac{1}{14}$

2 **(a)** $2\frac{3}{6}=2\frac{1}{2}$ **(b)** $3\frac{92}{117}$ **(c)** $2\frac{19}{28}$ **(d)** $6\frac{21}{22}$

3 **(a)** $18\frac{1}{12}$ m **(b)** $16\frac{11}{12}$ hours

4 **(a)** $4\frac{4}{15}$ **(b)** $-\frac{6}{15}=-\frac{2}{5}$ **(c)** $2\frac{8}{15}$ **(d)** $-2\frac{8}{15}$

4.3 Find the fraction of an amount

SC 1

1 **(a)** $1\frac{1}{5}$ **(b)** $6\frac{6}{7}$ **(c)** 2 **(d)** 8

2 **(a)** $29\frac{2}{5}$ **(b)** $30\frac{2}{3}$

3 **(a)** $8\times2\frac{1}{9}=8\times\left(2+\frac{1}{9}\right)=8\times2+8\times\frac{1}{9}$
$=16+\frac{8}{9}=16\frac{8}{9}$
The nearest whole number is 17.
(b) 27 **(c)** 38 **(d)** 123

4 **(a)** $13\frac{1}{3}$ m **(b)** $20\frac{9}{20}$ m

SC 2

1 **(a)** \$34 **(b)** 15 m
(c) $13\frac{5}{7}$ minutes **(d)** $334\frac{6}{11}$ L

2 **(a)** Any 4 of the counters circled.
(b) Any 3 of the stars circled.
(c) Any 2 of the circles circled.
(d) One third of the rope sectioned off.
(e) Any 2 of the segments shaded.
(f) Any 4 of the apples circled.

3 **(a)** $1\frac{7}{20}$ cups **(b)** $\frac{22}{27}$ L
(c) $5\frac{19}{21}$ kg **(d)** $3\frac{41}{77}$ m

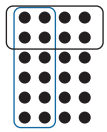
4 **(a)** 1 **(b)** 12 **(c)** 6 **(d)** 60

5 **(a)** 9 chocolates **(b)** 6 chocolates
(c) 3 chocolates **(d)** 2 chocolates
(e) 1 chocolate

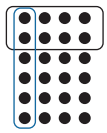
4.4 Understand how to multiply fractions

SC 1

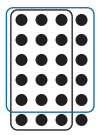
1 These are sample answers.

(a) 

$\frac{1}{3}$ of $\frac{1}{2} = \frac{4}{24} = \frac{1}{6}$

(b) 

$\frac{1}{3}$ of $\frac{1}{4} = \frac{2}{24} = \frac{1}{12}$

(c) 

$\frac{3}{4}$ of $\frac{5}{6} = \frac{15}{24}$

SC 2

1 **(a)** $\frac{1}{21}$ **(b)** $\frac{1}{20}$ **(c)** $\frac{5}{28}$
(d) $\frac{8}{23}$ **(e)** $\frac{2}{5}$ **(f)** $\frac{1}{10}$

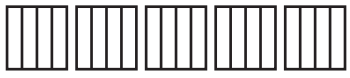
2 **(a)** $2\frac{2}{9}$ **(b)** $\frac{5}{8}$ **(c)** $\frac{9}{19}$ **(d)** $2\frac{2}{7}$

3 **(a)** $3\frac{17}{20}$ **(b)** $7\frac{1}{5}$ **(c)** $12\frac{4}{7}$ **(d)** $18\frac{2}{3}$

4 **(a)** $\frac{3}{10}$ **(b)** 7.5 m of rope

4.5 Understand how to divide fractions

SC 1

1 **(a)** 

$5 \div \frac{1}{4} = 20$

(b)

$2 \div \frac{1}{6} = 12$

2 **(a)** $2 \div \frac{1}{4} = 8$ **(b)** $3 \div \frac{1}{4} = 12$ **(c)** $3 \div \frac{1}{2} = 6$
(d) $3 \div \frac{3}{4} = 4$ **(e)** $4 \div \frac{2}{5} = 10$

SC 2

1 **(a)** 15 **(b)** 10 **(c)** 14 **(d)** 27

2 **(a)** $7\frac{1}{2}$ **(b)** $12\frac{3}{5}$

3 **(a)** $2\frac{1}{4}$ **(b)** $6\frac{6}{11}$

SC 3

1 **(a)** $\frac{4}{15}$ **(b)** $\frac{1}{18}$ **(c)** $\frac{3}{14}$ **(d)** $\frac{5}{66}$

2 **(a)** $\frac{7}{30}$ **(b)** $\frac{20}{27}$ **(c)** $1\frac{4}{9}$ **(d)** $1\frac{3}{5}$

3 **(a)** Student B is correct. Student A forgot to write the reciprocal.
(b) Student A is correct. Student B has incorrectly cancelled the factor of 2 from the numerator twice (rather than using $\frac{2}{2} = 1$).

4 **(a)** $\frac{11}{36}$ **(b)** $\frac{125}{144}$ **(c)** $\frac{5}{8}$

SC 4

1 **(a)** $2\frac{1}{10}$ **(b)** $\frac{9}{14}$ **(c)** $\frac{16}{33}$ **(d)** $4\frac{1}{2}$

2 **(a)** $\frac{25}{44}$ **(b)** $2\frac{1}{27}$ **(c)** $3\frac{7}{11}$ **(d)** $\frac{24}{35}$

3 **(a)** 50 **(b)** 250

4 **(a)** $\frac{1}{12}$ **(b)** $\frac{45}{46}$ **(c)** $7\frac{7}{12}$
(d) $10\frac{5}{36}$ **(e)** $\frac{9}{22}$ **(f)** $-\frac{1}{27}$

Topic 5: Modelling with fractions, decimals and percentages

Recall

I can convert between percentage and decimal values

1 **(a)** 0.15 **(b)** 0.30 **(c)** 0.45 **(d)** 0.72

2 **(a)** 10% **(b)** 25% **(c)** 65% **(d)** 80%

I can write simple percentages as fractions

1 **(a)** T **(b)** F **(c)** F
(d) T **(e)** T **(f)** F

I can simplify fractions using common factors

1 **(a)** 1, 2, 3, 4, 6, 12
(b) 1, 3, 5, 15
(c) 1, 2, 3, 4, 6, 8, 12, 24
(d) 1, 2, 3, 5, 6, 10, 15, 30
(e) 1, 5, 7, 35
(f) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

2 **(a)** 3 **(b)** 3 **(c)** 6 **(d)** 5 **(e)** 5 **(f)** 12

3 **(a)** $\frac{4}{5}$ **(b)** $\frac{5}{8}$ **(c)** $\frac{4}{5}$ **(d)** $\frac{6}{7}$ **(e)** $\frac{7}{12}$ **(f)** $\frac{1}{5}$

I can write equivalent fractions with a denominator of 100

1 **(a)** $\frac{3}{50}=\frac{6}{100}$ **(b)** $\frac{9}{25}=\frac{36}{100}$
(c) $\frac{17}{20}=\frac{85}{100}$ **(d)** $\frac{7}{10}=\frac{70}{100}$
(e) $\frac{3}{5}=\frac{60}{100}$ **(f)** $\frac{3}{4}=\frac{75}{100}$

5.1 Choose an appropriate representation to solve problems

SC 1

1 **(a)** $\frac{3}{5}=0.6=60\%$ **(b)** $\frac{7}{8}=0.825=82.5\%$

2 **(a)** $0.65=\frac{13}{20}=65\%$ **(b)** $0.3=\frac{3}{10}=30\%$

3 **(a)** $92\%=\frac{23}{25}=0.92$ **(b)** $28.5\%=\frac{57}{200}=0.285$

4 **(a)** **C**, $\frac{1}{500}$ **(b)** **A**, 0.000 035 **(c)** **B**, 11.75%

5 **(a)** $\frac{7}{200}$ **(b)** 0.88 **(c)** $0.295 = 29.5\%$

6 **(a)** $\frac{4}{17}, 0.3, \frac{6}{11}, \frac{2}{3}, 75\%$ **(b)** $0.3, 33\%, \frac{1}{3}, \frac{69}{200}, \frac{2}{3}$

SC 2

1 **(a)** $\frac{5}{8}=0.625=62.5\%$; fraction format would be best.
(b) $\frac{7}{8}=0.975=87.5\%$; fraction format would be best.
(c) $25\%=0.25=\frac{1}{4}$; fraction format would be best.
(d) $75\%=0.75=\frac{3}{4}$; fraction format would be best.
(e) $10\%=0.1=\frac{1}{10}$; decimal format would be best.
(f) $0.001=0.00001\%=\frac{1}{1000}$; decimal format would be best.

2

Fraction	Decimal	Percentage
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{1}{8}$	0.125	12.5%
$\frac{3}{8}$	0.375	37.5%
$\frac{5}{8}$	0.625	62.5%
$\frac{7}{8}$	0.875	87.5%
$\frac{1}{3}$	$0.\dot{3}$	$33\frac{1}{3}\%$
$\frac{2}{3}$	$0.\dot{6}$	$66\frac{2}{3}\%$

3 **(a)** **C**, $\frac{46}{500}\times\frac{515}{1}$ **(b)** **B**, $\frac{2}{3}\times\frac{321}{1}$

4 **(a)** $0.\dot{2}$ **(b)** $\frac{4}{9}$ **(c)** 7777

5.2 Determine the proportion of a quantity

SC 1

1 **(a)** 60 L **(b)** 120 kg **(c)** $40
(d) 1500 m **(e)** 23.3 m **(f)** 0.015 seconds

2 **(a)** 44 questions **(b)** 8.25 L
(c) 5580 people unemployed

3 **(a)** $\frac{19}{150}$; 418 L **(b)** $\frac{3}{40}$; 105 students
(c) $\frac{437}{1200}$; $131.10

4 **(a)** 220 **(b)** 110
(c) $220+\frac{110}{2}=220+55$
$=275$
(d) 10% = 180; 20% = 360; 5% = 90
$22\frac{1}{2}\%=360+\frac{90}{2}=360+45=405$

5 **(a)** **C**, 4570 **(b)** **D**, 800

SC 2

1 **(a)** $\frac{3}{10}$ **(b)** $\frac{2}{15}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{11}{80}$

2 Test 1: 83.3%, Test 2: 77.8%; Test 3: 81.8%
Test 2, Test 3, Test 1

3 **(a)** **C**, $\frac{25}{120}\times\frac{100}{1}\%$ **(b)** **D**, $\frac{3}{14}\times\frac{100}{1}\%$

4 4.4% increase

5.3 Understand percentage as applied to taxation

SC 1

1 **(a)** \$99; \$1089 **(b)** \$1999; \$21 989
(c) \$125.50; \$1380.50
(d) \$17.56; \$193.16

2 **(a)** 100 **(b)** 1100 **(c)** $\frac{1100}{1000} = 1.1$ **(d)** 1.1

3 **(a)** \$29 150 **(b)** \$52 688.90 **(c)** \$850.30
(d) \$3.25, rounded to the nearest 5 cents.

4 **(a)** \$20 **(b)** \$145
(c) \$29.90 **(d)** \$56.50

5 \$35.50

SC 2

1 **(a)** \$313.50 **(b)** \$4664.50
(c) \$17 409.50 **(d)** \$32 427

2 **(a)** 8.6% **(b)** 21.8% **(c)** 27.5% **(d)** 31.7%

3 **(a)** \$5824 **(b)** \$15 533.88
(c) \$27 430.38 **(d)** \$38 560.38
(e) The more your income increases, the more you save under this taxation arrangement.

4 **(a)** \$44 267
(b) \$(16 467 × 2) = \$32 934
(c) When the income is split, both people are entitled to the \$18 200 tax-free component, plus the marginal rate of taxation is lower, 32.5% compared to 37%.

5.4 Use fractions, decimals and percentages to solve financial problems

SC 1

1 **(a)** \$17 800 **(b)** \$77.49
(c) \$122.75 **(d)** \$135 400

2 **(a)** \$21 480 **(b)** \$21.49
(c) \$4718 **(d)** \$79

3 **(a)** \$332 **(b)** \$11 291
(c) \$10.73 **(d)** \$285 167

4 **(a)** \$421.25 **(b)** \$1600
(c) \$322 554 **(d)** \$12.80

5 **(a)** loss; \$$(A - B)$ **(b)** profit; \$$(B - A)$

SC 2

1 **(a)** 19.2% **(b)** 9.1%

2 **(a)** 20% **(b)** 44.1%

3 **(a)** 25.6% **(b)** 8.3%

4 **(a)** 25.3% **(b)** 11.3%

5 **(a)** \$393.75 **(b)** \$293 680.95

6 **(a)** \$$(1250 + x)$ **(b)** $\frac{x}{1250 + x} = 0.15$
(c) profit = \$220.59 **(d)** \$1470.59
(e) \$50 998.89

5.5 Use, interpret and calculate discounts and mark-ups

SC 1

1 **(a)** 15.4% **(b)** 7.5% **(c)** 17.2% **(d)** 17.5%

2 **(a)** 25% discount **(b)** 27% discount
(c) 20% discount
(d) The ^ probably means there is a minimum 20% discount.

3 **(a)** 40.6% **(b)** 41.4%
(c) The discount has been manipulated slightly so that the sale price ends in a 9. However, the advertising is truthful in that the discount is at least 40%.

4 **(a)** 50% discount **(b)** 25% discount

SC 2

1

Box size	**(a)** Total cost price per box	**(b)** Selling price per box	**(c)** GST amount	**(d)** GST-inclusive price
1 kg	2.85 + 0.30= \$3.15	3.15 × (1 + 1) = \$6.30	6.30 × 0.1 = \$0.63	6.30 × 1.1 = \$6.93
4 kg	2.85 × 4 + 0.37= \$11.77	11.77 × (1 + 0.4) = 16.478 = \$16.48	16.48 × 0.1 = 1.812 58 = \$1.81	16.48 × 1.1 = 18.1258 = \$18.13

2 **(a)** 18.13 × (1 − 0.15) = \$15.41
(b) Apply discount directly to selling price:
16.48 × (1 − 0.15) = \$14.01
Alternatively, use the answer from part **(a)** to calculate the pre-GST price:
15.41 ÷ 1.1 = \$14.01
(c) Apply discount directly to GST:
1.65 × (1 − 0.15) = \$1.40
Alternatively, subtract the answers from parts **(a)** and **(b)**:
15.41 − 14.01 = \$1.40
(d) $\frac{14.01 - 11.77}{11.77} \times 100\% = 19.0\%$

Topic 6: Integers

Recall

I can identify odd and even numbers

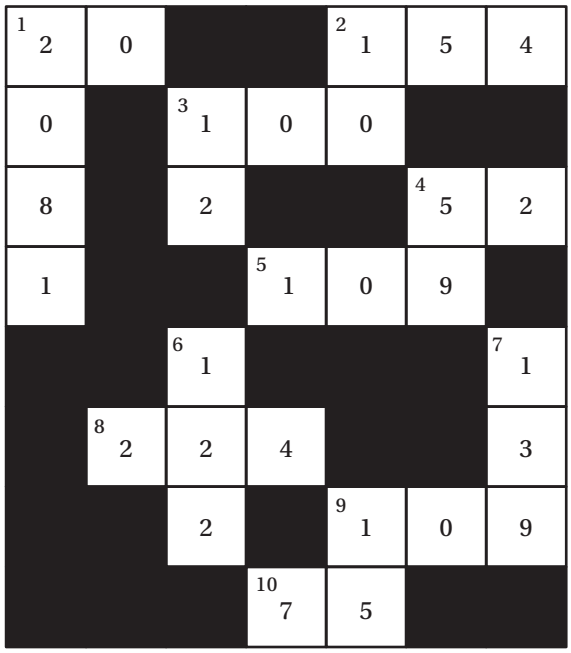
1 10, 12, 14, 16, 18
2 197, 199, 201, 203, 205, 207, 209, 211
3 **(a)** 44, 46, 48 **(b)** 82, 80, 78
4 **(a)** 65, 67, 69, 71 **(b)** 101, 99, 97, 95

I can compare positive numbers

1 **(a)** 1105 < 1110 **(b)** 840 > 807
(c) 1405 > 1054

I can add and subtract whole numbers

1



2 **(a)** 21 **(b)** 64 **(c)** 0 **(d)** 267

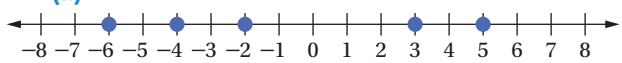
I can order positive and negative temperatures

1 –14°C, –10°C, –8°C, 10°C
2 14°C, 10°C, –12°C, –16°C

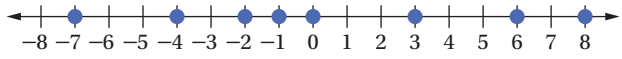
6.1 Understand integers and operations

SC 1

1 **(a)**



(b)



(c)

2 A = –7, negative seven
B = –3, negative three
C = 0, zero
D = 1, positive one
E = 8, positive eight

SC 2

1 **(a)** –8, –4, 2, 7 **(b)** –10, –6, –1, 0, 3, 4
(c) –9, –5, –2, 0, 3, 7, 10
2 **(a)** Ascending order: –9, –8, –6, –3, 3, 4, 10
Descending order: 10, 4, 3, –3, –6, –8, –9
(b) Ascending order: –25, –19, –5, –1, 16, 18, 30
Descending order: 30, 18, 16, –1, –5, –19, –25
(c) Ascending order: –15, –9, –7, –2, 4, 8, 20
Descending order: 20, 8, 4, –2, –7, –9, –15
(d) Ascending order: –125, –90, –72, –50, 58, 64, 204
Descending order: 204, 64, 58, –50, –72, –90, –125
3 Answers will vary. To arrange integers in ascending order, imagine the numbers on a number line and then list the numbers in order from smallest (left) to largest (right). To arrange the numbers in descending order, you can write them in reverse order.
4 **(a)** (–3) < 4 **(b)** (–1) > (–3)
(c) (–9) < 5 **(d)** (–10) < (–5)
5 **(a)** –3, –2, –1, 0, 1, 2 **(b)** –11, –10, –9, –8, –7
(c) 0, 1, 2, 3, 4
6 **(a)** 0, 2, 4 **(b)** 0, –5, –10, –15

SC 3

1 **(a)** A = –7. The magnitude of A is 7.
(b) B = –4. The magnitude of B is 4.
(c) C = 0. The magnitude of C is 0.
(d) D = 3. The magnitude of D is 3.
(e) E = 4. The magnitude of E is 4.
2 The magnitude represents the distance or count from zero.
3 **(a)** D = –9, B = –8, G = –6, A = –2, E = 3, F = 4, C = 10
(b) Part **(a)** lists the integers in order from the lowest level to the highest level. Negative numbers indicate floors below ground level and positive numbers indicate floors above ground level.
(c) A = –2. The magnitude is 2.
B = –8. The magnitude is 8.
C = 10. The magnitude is 10.
D = –9. The magnitude is 9.
E = 3. The magnitude is 3.
F = 4. The magnitude is 4.
G = –6. The magnitude is 6.
(d) A, E, F, G, B, D, C
(e) The magnitudes of the elevator levels given in part **(c)** represent the distance from the ground floor; hence all magnitudes are positive numbers. Part **(a)** lists the levels in order, so the negatives are included to indicate floors below ground level.

4 **(a)** C = 2 floors above ground level, B = 4 floors below ground level, D = 7 floors above ground level, A = 8 floors below ground level.

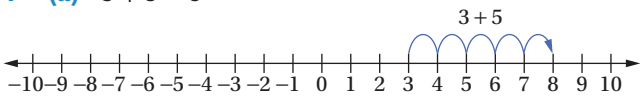
(b) D = 0 (ground floor), C = 1 floor below ground level, E = 3 floors above ground level, F = 4 floors above ground level, B = 6 floors below ground level, A = 10 floors below ground level.

(c) D = 0 (ground floor), C = 2 floors below ground level, E = 3 floors above ground level, B = 5 floors below ground level, F = 7 floors above ground level, A = 9 floors below ground level and G = 10 floors above ground level

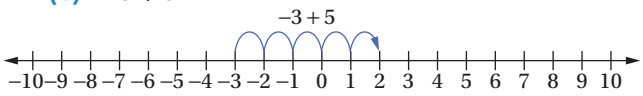
6.2 Add and subtract integers

SC 1

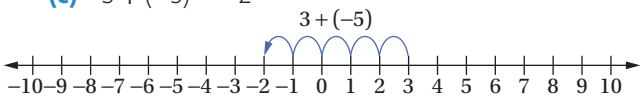
1 **(a)** $3 + 5 = 8$



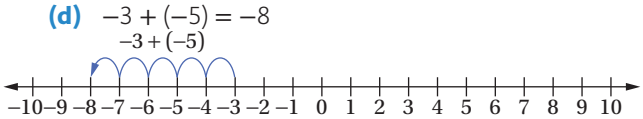
(b) $-3 + 5 = 2$



(c) $3 + (-5) = -2$



(d) $-3 + (-5) = -8$



2 Integers are whole numbers.
To add integer values on a number line, start on the first value and face to the right. When the number being added is positive, step forward. When the number being added is negative, step backward.

3 **(a)** 14 **(b)** 13 **(c)** 12
(d) 11 **(e)** 10 **(f)** 9

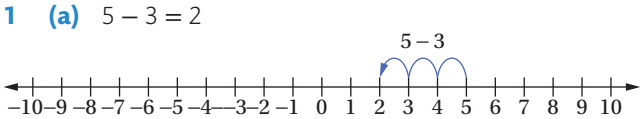
4 **(a)** 2 **(b)** 1 **(c)** 0
(d) −1 **(e)** −2 **(f)** −3

5 **(a)** 3 **(b)** 2 **(c)** 1
(d) 0 **(e)** −1 **(f)** −2

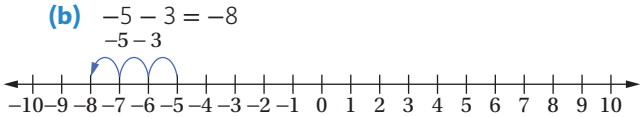
6 **(a)** −5 **(b)** −6 **(c)** −7
(d) −8 **(e)** −9 **(f)** −10

SC 2

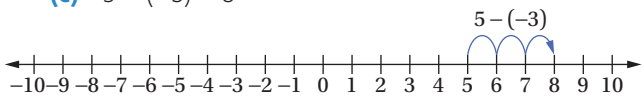
1 **(a)** $5 - 3 = 2$



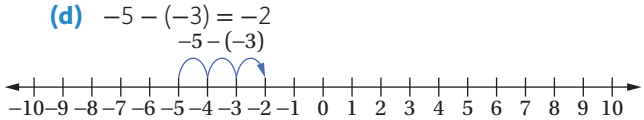
(b) $-5 - 3 = -8$



(c) $5 - (-3) = 8$



(d) $-5 - (-3) = -2$



2 Integers are whole numbers. To subtract an integer from another on a number line, start on the first value and face to the left. When the number being subtracted is positive, step forward; when the number being subtracted is negative, step backward.

3 **(a)** 3 **(b)** 2 **(c)** 1
(d) 0 **(e)** −1 **(f)** −2

4 **(a)** −5 **(b)** −6 **(c)** −7
(d) −8 **(e)** −9 **(f)** −10

5 **(a)** 3 **(b)** 4 **(c)** 5
(d) 6 **(e)** 7 **(f)** 8

6 **(a)** −1 **(b)** 0 **(c)** 1
(d) 2 **(e)** 3 **(f)** 4

SC 3

1 **(a)** −9 **(b)** −8 **(c)** 1 **(d)** −1 **(e)** −2
(f) 12 **(g)** 4 **(h)** −4 **(i)** −8 **(j)** −5

2 **(a)** 35 **(b)** −72 **(c)** 205
(d) 75 **(e)** 43 **(f)** −32

3 **(a)** 10 **(b)** 4 **(c)** 30
(d) −15 **(e)** 20 **(f)** −94

4 **(a)** $1 - 6 + 3 = -2$
$1 + (-6) + 3 = -2$
$1 - 6 - (-3) = -2$
$1 + (-6) - (-3) = -2$

(b) $-6 + 8 + 3 = 5$
$-6 + 8 - (-3) = 5$
$-6 - (-8) + 3 = 5$
$-6 - (-8) - (-3) = 5$

(c) $-4 + 4 - 6 = -6$
$-4 - (-4) - 6 = -6$
$-4 + 4 + (-6) = -6$
$-4 - (-4) + (-6) = -6$

(d) $0 - 6 + 11 = 5$
$0 + (-6) + 11 = 5$
$0 - 6 - (-11) = 5$
$0 + (-6) - (-11) = 5$

6.3 Model and solve practical problems using integers

SC 1

1 10 °C

2 26 °C

3 (a)

City	Minimum	Maximum	Difference
Kabul (Afghanistan)	−2 °C	25 °C	27 °C
Dhaka (Bangladesh)	19 °C	29 °C	10 °C
Beijing (China)	−3 °C	27 °C	30 °C
Hong Kong (China)	16 °C	29 °C	13 °C
Pyongyang (North Korea)	−6 °C	25 °C	31 °C

(b) Dhaka **(c)** Pyongyang

4 85 °C

5 19 882 m

6 −350 m

7 $1190

8 (a) 16 **(b)** 12 **(c)** +1 **(d)** −10

(e) At the start of the route, 22 people boarded the bus.
At the first stop, the number of passengers on the bus increased by 9, making a total of passengers.
At the second stop, the total number of passengers on the bus decreased by 15, leaving only 16 passengers travelling to the third stop.
At the third stop, the number of passengers increased by 7, making 23.
At the fourth stop, the number of passengers decreased by 11.
Only 12 passengers travelled on to the final stop.

9 (a) 381 m **(b)** + 29 m **(c)** + 56 m

SC 2

1 (a) Balance: A loss of $235 (or a profit of −$235)

(b) Balance: A profit of $170

2 (a)

Title	Cost	Sale Price	Profit/Loss made
Great Gardens	$29	$20	−$9 (loss)
Creative Cooks	$18	$21	$3 (profit)
Devilish Designs	$22	$22	$0 (even)
Eastern Elegance	$39	$30	−$9 (loss)

(b) Creative Cooks **(c)** Devilish Designs

3 (a) profit of (−$12 500) **(b)** loss of $12 500

SC 3

1 (a) 110 m **(b)** 130 m **(c)** 100 m

2 (a) 4 positive discs **(b)** 1 positive disc

(c) 3 negative discs **(d)** 7 negative discs

3 (a) 9 **(b)** −18

(c) 9 other possible values: −15, −12, −7, −6, −3, 0, 3, 6, 9.

4 A profit of $5

5 The two remaining receipts must add to $170, with the movie costing less than $50. One possible answer: Movie cost $30 and the new pair of shoes were $140.

6 Loss of $120

Topic 7: Understanding ratios

Recall

I can group and count items by common attributes

1

Yellow speckled	Purple speckled	Green striped
2	4	5

2 6 odds and 5 evens

I can multiply decimals by powers of 10

1 (a) 10 **(b)** $100 = 10^2$ **(c)** $1000 = 10^3$

(d) 10 **(e)** $100 = 10^2$

2 (a) 0.345 **(b)** 3.45 **(c)** 34.5 **(d)** 345 **(e)** 3450

3 The value of the 3 digit in the original number is 3 ones. When multiplied by a power of ten, the 3 digit takes on the place value of the number being multiplied, e.g. 3.45 × 10 = 34.5, the value of the 3 digit is 30 or 3 tens.

I can convert decimals to fractions

1 (a) $\frac{5}{10}$ **(b)** $\frac{25}{100}$ **(c)** $\frac{8}{10}$

(d) $\frac{75}{100}$ **(e)** $\frac{24}{100}$

2 (a) $\frac{1}{2}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{4}{5}$

(d) $\frac{3}{4}$ **(e)** $\frac{6}{25}$

I can simplify fractions

1 (a) $\frac{5}{6}$ **(b)** $\frac{9}{10}$ **(c)** $\frac{3}{7}$

7.1 Write ratios

SC 1

1 (a) 1 : 4 **(b)** 4 : 1

2 (a) 3 : 2 **(b)** 2 : 1 **(c)** 2 : 1 : 1

3 (a) 1 : 1 **(b)** 3 : 3 : 1 **(c)** 1 : 2

4 (a) F **(b)** H **(c)** G **(d)** K **(e)** I

SC 2

1 (a) 3 : 2 **(b)** 1 : 2 **(c)** 2 : 3 : 5 **(d)** 3 : 10

2 (a) 1 : 1 **(b)** 7 : 2 **(c)** 1 : 2 **(d)** 2 : 1 : 2 : 2

3 (a) 2 : 1 **(b)** 1 : 2 **(c)** 3 : 1 **(d)** 1 : 6

4 (a) 2 : 1 **(b)** 3 : 1 **(c)** 1 : 6 **(d)** 6 : 3 : 9 : 1

7.2 Understand equivalent ratios

SC 1

1 **(a)** 4 : 8 **(b)** 6 : 12 **(c)** 25 : 20
(d) 0.3 : 7 **(e)** 3 : 11 **(f)** 8.5 : 12
(g) 300 : 100 **(h)** 8 : 60

2 **(a)** 2 : 5 **(b)** 5 : 6 **(c)** 1 : 8 **(d)** 21 : 16
(e) 4 : 3 : 2 **(f)** 3 : 6 : 2 **(g)** 5 : 6 : 8 **(h)** 2 : 1 : 3

3 **(a)** 30 : 71 **(b)** 25 : 31 **(c)** 211 : 340
(d) 3 : 8 **(e)** 35 : 26 **(f)** 200 : 251

4 **(a)** 3 : 4 **(b)** 3 : 2 **(c)** 17 : 19 **(d)** 15 : 19
(e) 2 : 11 **(f)** 21 : 26 **(g)** 42 : 25 **(h)** 9 : 65

5 **(a)** 6 : 1
(b) 6 : 1 = 24 : 4, so 4 supervisors are needed.
(c) 6 : 1 = 30 : 5, so 30

SC 2

1 **(a)** 4 : 5 **(b)** 5 : 3 **(c)** 50 : 3
(d) 5 : 24 **(e)** 8 : 15 : 100 **(f)** 11 : 16 : 64

2 **(a)** The Jersey cow is 8.75 times the mass of her newborn calf.
(b) The kangaroo mother is 47 058.82 times the mass of her newborn joey.
(c) The orca whale mother is 66.67 times the mass of her newborn calf.

3 The husband is 1.09 times the height of his wife. The daughter's height is 0.90 of the height of her mother. The son's height is 0.57 of the height of his mother.

4 **(a)** The height is 0.48 of the base length.
(b) The height is 1.84 times the base length.
(c) The height is 3.75 times the base length.

5 **(a)** Queensland
$\frac{2228\text{ m}}{1622\text{ m}} \approx 1.37$
Mount Kosciuszko is 1.37 times higher than Mount Bartle Frere.
(b) ACT
$\frac{2228\text{ m}}{1913\text{ m}} \approx 1.16$
Mount Kosciuszko is 1.16 times higher than Mount Bimberi.
(c) Victoria
$\frac{2228\text{ m}}{1986\text{ m}} \approx 1.12$
Mount Kosciuszko is 1.12 times higher than Mount Bogong.
(d) Tasmania
$\frac{2228\text{ m}}{1617\text{ m}} \approx 1.38$
Mount Kosciuszko is 1.38 times higher than Mount Ossa.
(e) NT
$\frac{2228\text{ m}}{1531\text{ m}} \approx 1.46$
Mount Kosciuszko is times higher than Mount Zeil.
(f) SA
$\frac{2228\text{ m}}{1435\text{ m}} \approx 1.55$
Mount Kosciuszko is 1.55 times higher than Mount Woodroffe.
(g) WA
$\frac{2228\text{ m}}{1249\text{ m}} \approx 1.78$
Mount Kosciuszko is 1.78 times higher than Mount Meharry.

6 **(a)** Cartons of milk
$600\text{ mL} : 2\text{ L} = 600\text{ mL} : 2000\text{ mL}$
$= 600 : 2000$
$= 3 : 10$
The 600 mL carton of milk has $\frac{3}{10}$ the volume of the 2 L carton.
(b) Tins of tuna
$125\text{ g} : 425\text{ g} = 125 : 425$
$= 5 : 17$
The 125 g tin of tuna has $\frac{5}{17}$ the mass of the 425 g tin.
(c) Lengths of rulers
$15\text{ cm} : 1\text{ m} = 15\text{ cm} : 100\text{ cm}$
$= 15 : 100$
$= 3 : 20$
The 15 cm ruler has $\frac{3}{20}$ the length of the 1 m ruler.

7.3 Understand the connection between fractions and ratios

SC 1

1 **(a)** $\frac{3}{4}$ **(b)** $\frac{7}{12}$ **(c)** $\frac{8}{13}$ **(d)** $\frac{6}{7}$ **(e)** $\frac{1}{2}$

2 **(a)** 20 : 23 **(b)** $\frac{2}{23}$

3 **(a)** 3 : 4 **(b)** $\frac{1}{4}$

4 2 : 3 : 3 gives $\frac{1}{4}$ red, so $\frac{3}{8}$ are green.

5 **(a)** Strawberries to apples to kiwifruit ratio
12 : 10 : 8 = 6 : 5 : 4
(b) Total parts: 6 + 5 + 4 = 15
(c) **(i)** Fraction that are strawberries: $\frac{6}{15} = \frac{2}{5}$
(ii) Fraction that are apples: $\frac{5}{15} = \frac{1}{3}$
(iii) Fraction that are kiwifruit: $\frac{4}{15}$

SC 2

1 **(a)** 3 : 1 **(b)** 5 : 11 **(c)** 1 : 3 **(d)** 7 : 13
2 2 : 23
3 **(a)** 2 : 5 **(b)** 3 : 25 **(c)** 10 : 3
4 $\frac{1}{3}=\frac{5}{15}$ and $\frac{2}{5}=\frac{6}{15}$ so the green, yellow, white, and whole ratio is 5 : 6 : 4 : 15.
(a) 1 : 3 **(b)** 5 : 6 **(c)** 4 : 15

SC 3

1 Frankie receives 18 and Gertrude receives 27 chocolate frogs.
2 Day 1: 21; Day 2: 49; Day 3: 14
3 Wins: 3; losses: 15
4 Oil: 240 mL; petrol: 5.76 L
5 In the order given: $1351, $81, $432, $676

Topic 8: Algebra

Recall

I can find the unknown value in a number sentence

1 **(a)** 69 **(b)** 371 **(c)** 6 **(d)** 222
2 **(a)** 75 **(b)** 5 **(c)** 10 **(d)** 979

I can perform several operations on a term

1 26
2 17
3 26

I can change word sentences to number sentences

1 **(a)** Addition **(b)** Subtraction
(c) Multiplication **(d)** Division
2 **(a)** $7+1=8$ **(b)** $3\times5=15$
(c) $100\div4=25$ **(d)** $32\div8=4$
(e) $2\times7-10=4$

I can change word sentences to number sentences and calculate the answer

1 $8+28$
2 $100-64$
3 $(22-2)\div5$
4 $(4+50)\times3$
5 $(40-30)\div10$

8.1 Understand variables in algebra

SC 1

1 **(a)** $x-4$ **(b)** $x+4$ **(c)** $4x$
(d) $\frac{x}{4}$ **(e)** $3x+4$ **(f)** $\frac{x}{3}-4$

2 **(a)** An unknown number is subtracted from five.
(b) Five is subtracted from an unknown number.
(c) Five is multiplied by an unknown number.
(d) Five is divided by an unknown number.
(e) An unknown number is divided by five.
3 **(a)** I think of a number and multiply it by ten.
(b) I think of a number and multiply it by negative ten.
(c) I think of a number, multiply it by ten, then subtract seven.
(d) I think of a number, multiply it by negative ten and add seven.
(e) I think of a number, multiply it by negative ten and add seven.
4 **(a)** I think of a number and subtract seven from it.
(b) I think of a number, multiply it by two, then subtract seven.
(c) I think of a number, subtract seven and multiply it by two.
5 **(a)** **(i)** -2 **(ii)** 2 **(iii)** $\frac{4x}{5}-10$
(b) **(i)** -32 **(ii)** -28 **(iii)** $4\left(\frac{x}{5}-10\right)$
6 **(a)** 2 km farther than the distance travelled on the first day
(b) 2 km shorter than the distance travelled on the first day
(c) equal to the distance travelled on the first day subtracted from 2 km
(d) twice the distance travelled on the first day
(e) half the distance travelled on the first day

SC 2

1 **(a)** $\frac{2x}{3}+4$ **(b)** $2\left(\frac{x+4}{3}\right)$
(c) $\frac{2x+4}{3}$ **(d)** $2\left(\frac{x}{3}+4\right)$
(e) All expressions are created using the same operations and numbers, however none of the expressions are equivalent.
2 No. In parts **(c)** and **(d)**, the subtraction symbol ($-5x$) represents a multiplication by negative five. In part **(b)**, the subtraction symbol $\left(-\frac{5x}{4}\right)$ represents a multiplication of $-\frac{5}{4}$. All other subtraction symbols represent a subtraction.
3 **(a)** A number is multiplied by five and divided by four, subtract six from the result, then multiply the result by two and subtract three.
(b) A number is multiplied by negative five, divided by four, then six is added. Multiply the result by two, then subtract three.

(c) A number is multiplied by negative five, then added to six. The result is divided by four, multiplied by two, then three is subtracted.

(d) A number is multiplied by negative five, added to six, the result is divided by four, then three is subtracted. The result is multiplied by two.

(e) A number is multiplied by five, then subtract six. Multiply the result by two and divide by four. Subtract three from the result.

(f) A number is multiplied by five, then subtract six. Divide the result by four and multiple by two. Subtract three from the result.

4 **(a)** $2x$ **(b)** $\frac{x}{3}+50$

(c) $\frac{x}{3}+150$ **(d)** $2\times 2x-20$

5 **(a)** $5(x+y)$ **(b)** $\frac{6x-5y}{4}$

(c) $\frac{6x}{y}+9$ **(d)** $\frac{8xy-6}{3}$

8.2 Identify and simplify like terms

SC 1

1 **(a)** $5x, \frac{x}{5}$ **(b)** $\frac{x}{y}$

(c) $6bca, 6cba$ **(d)** $2a^3b^2, a^3b^2$

2 Tania is correct because abc could be written as $1abc$ if you really wanted to see the coefficient.

3 Jackson is incorrect because the variable part for a like term must contain the letters a, e, i, o, u in some order. It does not matter that they are vowels; that is irrelevant in this context.

4 Answers will vary. bh, $3hb$, $7bh$. Like terms can have any coefficient, including 1, but must contain the variable bh or hb.

SC 2

1 **(a)** $14a$ **(b)** $13ab$ **(c)** $16ab$

(d) $13abc$ **(e)** $6ac$ **(f)** $8ac+4a$

2 **(a)** $2ab$ **(b)** bc **(c)** $2b$ **(d)** $7abc$

3 Students' answers will vary.

(a) $3a+5a$ **(b)** $11b-3b$ **(c)** $ab+7ab$

(d) $3a+5a+11b-3b$ **(e)** $3a+5a+3b-11b$

4 **(a)** $9ab+4b$ **(b)** $-14b$

(c) $3a+31b$ **(d)** $2a-101b$

8.3 Calculate the value of algebraic expressions

SC 1

1 **(a)** 8 **(b)** 10 **(c)** 45 **(d)** −4

2 **(a)** Student B is correct. Student A has mistakenly replaced variables with digits.

(b) Student B is correct. Student A has mistakenly completed the order of operations in the incorrect order.

3 **(a)** 28 **(b)** 35 **(c)** −8 **(d)** 0

4 **(a)** −6 **(b)** 13 **(c)** −66 **(d)** −42

5 **(a)** −1 **(b)** 0 **(c)** −30 **(d)** −156

SC 2

1 **(a)** 4, 6, 8, 10, 12, 14

(b) −1, 2, 5, 8, 11, 14

(c) 3, 1, −1,−3, −5, −7

(d) 1, −2, −5, −8, −11, −14

2 **(a)** increases **(b)** decreases

(c) the coefficient of the variable

3 **(a)** −4, −2, 0, 2, 4, 6 **(b)** 2, −1, −4, −7, −10, −13

4 **(a)** $a=4$ **(b)** $a=6$

SC 3

1 **(a)** 9 eggs **(b)** 30 eggs

(c) 48 biscuits **(d)** 72 biscuits

2 **(a)** 3.5 km, 5 km, 6.5 km, 8 km

(b) 3 km, 4.5 km, 6 km, 7.5 km

(c) 1 km on day one **(d)** 4 km on day one

3 **(a)** 156 passengers **(b)** 4 buses

(c) 11 buses **(d)** 30 seats are vacant.

4 **(a)** $10+2(n-1)$, where n is the day number

(b) 28 push-ups

(c) The starting number is even, and each number that is added is even. Even plus even must result in even.

8.4 Write algebraic expressions in context

SC 1

1 **(a)** the number of orange trees

(b) Let t represent the number of orange trees.

(c) $\frac{t}{2}$

(d) It will take Basil 38 hours to finish the job.

2 **(a)** $4a$ **(b)** 600 cups of flour

(c) 29 loaves

3 **(a)** Let s be the cost of a sausage roll: $s+2s=3s$

(b) $12.60 **(c)** $14.25 **(d)** $\frac{m}{3s}$

4 **(a)** He needs to know the time taken.

(b) Let t be the time in minutes: $\frac{d}{t}$

(c) $\frac{3000}{t}$ m/min **(d)** 375 m/min

Topic 9: Linear relationships

Recall

I can equate number sentences

1 **(a)** T **(b)** F **(c)** F **(d)** T **(e)** F

I can complete a table of values using a rule

1

1	2	3	4	5
−2	0	2	4	6

2

3	6	9	12	15
6	7	8	9	10

I can interpret and identify parts of an algebraic expression

1 **(a)** 3 **(b)** 3 **(c)** $3x$
(d) $\times$, $6ab$ **(e)** $a \div 2$ **(f)** 4

I can substitute numbers in to find the value of an algebraic expression

1 29
2 9
3 15
4 10

9.1 Understand the components of an equation and how they are combined

SC 1

1 **(a)** **(i)** $14t$ **(ii)** $30t$ **(iii)** $365t$
(b) $30t = 600$
2 **(a)** **(i)** $25 - x$ **(ii)** $25 + x$
(b) $25 - x = 19$
3 **(a)** **(i)** $\frac{k}{6}$ **(ii)** $\frac{k}{4}$
(b) $\frac{k}{6} = 8$
4 **(a)** $ab = 1200$ or $ba = 1200$
(b) $a + b = 6$ or $b + a = 6$ **(c)** $a - b = 15$
5 **(a)** $l + 2$ **(b)** $m + 2$ **(c)** $\frac{m}{3}$
(d) $\frac{m+2}{3}$ **(e)** $l + 2 = \frac{m+2}{3}$
6 **(a)** $x + \frac{x}{2}$ or $\frac{3x}{2}$ or $1.5x$ **(b)** $2x$
(c) $x + 1.5x + 2x$ or $4.5x$
(d) $x + \frac{3x}{2} + 2x = 45$ or $4.5x = 45$
7 $60r + 160 = 200$

9.2 Generate a table of values using a linear equation

SC 1

1 **(a)** (8, 13) **(b)** (8, 3) **(c)** (8, 9) **(d)** (8, − 4)
2 **(a)** (−3, −6) **(b)** (−3, 6)
(c) (−3, −18) **(d)** (−3, 18)
3 **(a)** (−3, 13) **(b)** (−3, 9) **(c)** (−3, 2) **(d)** (−3, −2)
4 **(a)** (5, 14) **(b)** (5, −14) **(c)** (5, 12) **(d)** (5, 26)
5 **(a)** 1 **(b)** 3 **(c)** −1 **(d)** −3
6 **(a)** 13 **(b)** 17 **(c)** 20
(d) $22\frac{1}{2}$ or 22.5
7 **(a)** 12 **(b)** 40 **(c)** 6 **(d)** 3

SC 2

1 **(a)** $y = 5x$

x	1	2	3	4	5
y	5	10	15	20	25

(b) $y = 5x - 1$

x	1	2	3	4	5
y	4	9	14	19	24

2 **(a)** $y = 4x$

x	1	2	3	4	5
y	4	8	12	16	20

(b) $y = 4x + 3$

x	1	2	3	4	5
y	7	11	15	19	23

3 **(a)** $y = 2x + 5$

x	1	2	3	4	5
y	7	9	11	13	15

(b) $b = 6a - 5$

a	1	2	3	4	5
b	1	7	13	19	25

(c) $d = 2(c - 3)$

c	1	2	3	4	5
d	−4	−2	0	2	4

(d) $f = \frac{2e}{5}$

e	1	2	3	4	5
f	$\frac{2}{5}$	$\frac{4}{5}$	$1\frac{1}{5}$	$1\frac{3}{5}$	2

4 **(a)** $b = 3a - 2$

a	-2	1	6	10	15
b	-8	1	16	28	43

(b) $d = \frac{4c}{5} - 2$

c	-5	0	2	6	10
d	-6	-2	$-\frac{2}{5}$	$2\frac{4}{5}$	6

(c) $f = -2(e + 4)$

e	-3	-1	4	6	9
f	-2	-6	-16	-20	-26

(d) $h = \frac{5 - 2g}{3}$

g	-3	0	1	5	7
h	$3\frac{2}{3}$	$1\frac{2}{3}$	1	$-1\frac{2}{3}$	-3

5 **(a)** **(i)** 5 **(ii)** 5 **(iii)** 5 **(iv)** 5

(b) It is the coefficient of the x.

6 **(a)** **(i)** It decreases by 10.

(ii) It decreases by 10.

(iii) It decreases by 10.

(iv) It decreases by 10.

(b) -10

(c) $2(1 - 5x) = 2 - 10x$. The constant change, -10, is the coefficient of the x in the expanded expression.

SC 3

1 **(a)** $y = x + 7$ **(b)** $y = x - 3$

2 **(a)** $y = 6x$ **(b)** $y = 3x$

(c) $y = -4x$ **(d)** $y = \frac{x}{4}$

3 **(a)** $y = 2x + 1$ **(b)** $y = 3x - 5$

(c) $y = 4x + 6$ **(d)** $y = 5x - 4$

4 **(a)** $y = -x + 10$ or $y = 10 - x$ or $x + y = 10$

(b) $y = -x + 2$ or $y = 2 - x$ or $x + y = 2$

(c) $y = -2x - 3$

(d) $y = -3x + 4$ or $y = 4 - 3x$

5 **(a)** $y = \frac{x}{2} + 3$ **(b)** $y = \frac{x}{3} - 2$

6 **(a)** $y = 4x - 57$

x	15	17	19	21	23
y	3	11	19	27	35

(b) $y = -5x + 57$ or $y = 57 - 5x$

x	7	8	9	10
y	22	17	12	7

7 **(a)**

Number of hours, t	1	2	3	4	5
Total cost in dollars, C	95	155	215	275	335

(b) $C = 60t + 35$ **(c)** \$35 **(d)** \$60 per hour

9.3 Recognise the connection between points on a Cartesian plane, a table of values and a linear pattern

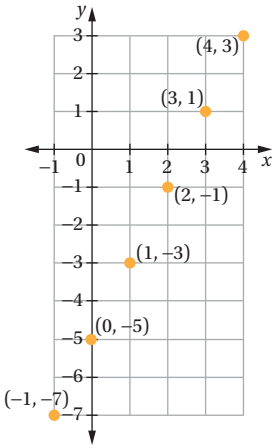
SC 1

1

x	-1	0	1	2	3
y	20	14	8	2	-4
(x, y)	$(-1, 20)$	$(0, 14)$	$(1, 8)$	$(2, 2)$	$(3, -4)$

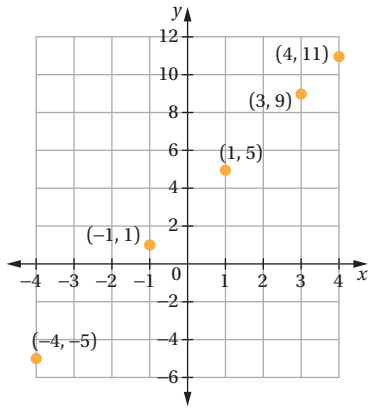
2 $A(-9, 7)$, $B(7, 0)$, $C(-5, -5)$, $D(-9, -2)$, $E(3, 8)$, $F(0, -8)$, $G(10, -3)$, $H(0, 0)$

3



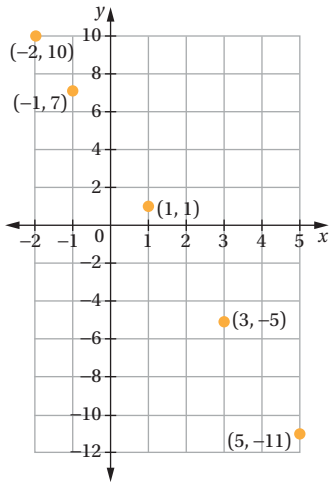
4 **(a)** $y = 2x + 3$

x	-4	-1	1	3	4
y	-5	1	5	9	11
(x, y)	$(-4, -5)$	$(-1, 1)$	$(1, 5)$	$(3, 9)$	$(4, 11)$



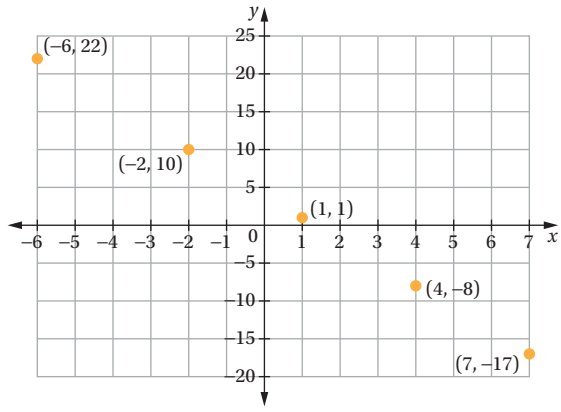
(b) $y = -3x + 4$

x	-2	-1	1	3	5
y	10	7	1	-5	-11
(x, y)	$(-2, 10)$	$(-1, 7)$	$(1, 1)$	$(3, -5)$	$(5, -11)$



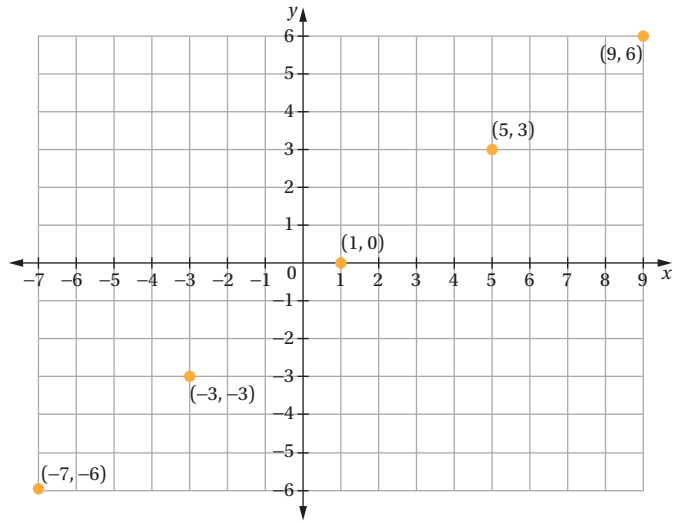
5 (a) $y = 4 - 3x$

x	-6	-2	1	4	7
y	22	10	1	-8	-17
(x, y)	$(-6, 22)$	$(-2, 10)$	$(1, 1)$	$(4, -8)$	$(7, -17)$



(b) $y = \dfrac{3(x - 1)}{4}$

x	-7	-3	1	5	9
y	-6	-3	0	3	6
(x, y)	$(-7, -6)$	$(-3, -3)$	$(1, 0)$	$(5, 3)$	$(9, 6)$



SC 2

1 (a)

x	-3	-2	1	3	5
y	-13	-10	-1	5	11

(b)

x	-4	-1	0	2	4
y	-14	-2	2	10	18

(c)

x	-4	-3	1	2	5
y	5	3	-5	-7	-13

(d)

x	-3	-2	0	2	4
y	14	11	5	-1	-7

2 (a) $y = 3x - 4$ **(b)** $y = 4x + 2$
(c) $y = -2x - 3$ **(d)** $y = -3x + 5$

3 (a) The point $(-2, -4)$ should be at $(-2, -6)$.
(b) The point $(0, 0)$ should be at $(0, 1)$.
(c) The point $(3, 1)$ should be at $(3, 4)$.
(d) The point $(-4, 4)$ should be at $(-4, 6)$.

4 (a) $(-4, -2)$ becomes $(-4, 0)$; $(0, 2)$ becomes $(0, 1)$; $(4, 4)$ becomes $(4, 6)$

(b) $(-4, 0)$ belongs to $y = \dfrac{x}{2} + 2$; $(0, 1)$ belongs to $y = \dfrac{3x}{4} + 1$; $(4, 6)$ belongs to $y = x + 2$

5 (a) $y = -x - 6$ or $x + y = -6$
(b) Any three of these coordinates: $(-4, -2)$, $(-3, -3)$, $(-2, -4)$, $(0, -6)$, $(1, -7)$, $(3, -9)$, $(4, -10)$

SC 3

1 (a) The y-values increased 5 times as much as the x-values.
(b) The y-values increased 3 times as much as the x-values.

(c) The y-values decreased 2 times as much as the x-values increased.

(d) The y-values decreased 6 times as much as the x-values increased.

2 (a) The y-values increased 5 times as much as the x-values.

(b) The y-values increased the same amount as the x-values.

(c) The y-values decreased the same amount as the x-values increased.

(d) The y-values decreased 2 times as much as the x-values increased.

3 (a) $y = 3x + 7$ **(b)** $y = -2x + 6$

(c) $y = -3x + 4$ **(d)** $y = \frac{x}{2} + 3$

4 (a) (10, 32) becomes (10, 35)

(b) (1, 10) becomes (1, 9)

(c) (4, 7) becomes (4, 6)

(d) (−4, 10) becomes (−4, 11)

5 (a) (−3, 2) becomes (−3, −14); (1, −2) becomes (1, 6); (5, 10) becomes (5, −6)

(b) (−3, −14) belongs to $y = 3x - 5$; (1, 6) belongs to $y = x + 5$; (5, −6) belongs to $y = -x - 1$

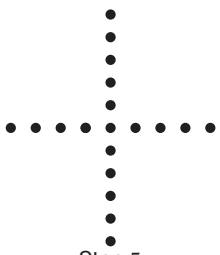
9.4 Model linear patterns using manipulatives, diagrams and graphs

SC 1

1 (a)

Step number, x	1	2	3	4	5
Number of dots, y	3	7	11	15	19
(x, y)	(1, 3)	(2, 7)	(3, 11)	(4, 15)	(5, 19)

(b)



Step 5

(c) Number of dots = step number × 4 − 1

(d) $y = 4x - 1$

(e) 39 dots

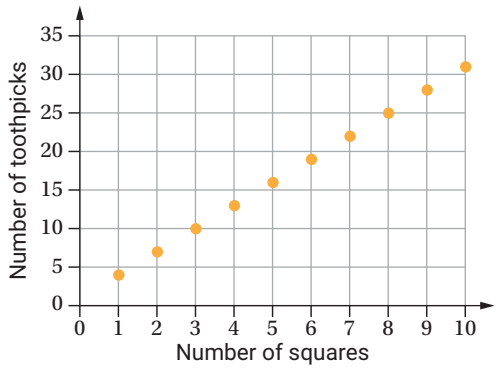
2 (a) (i)

Number of squares	1	2	3	4	5
Number of toothpicks	4	7	10	13	16

(ii) Number of toothpicks = number of squares × 3 + 1

(iii) 31 toothpicks

(iv)



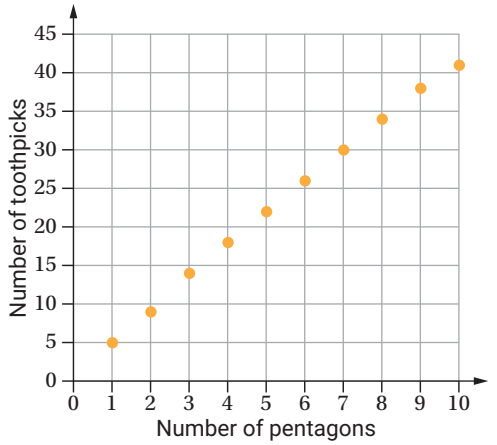
(b) (i)

Number of pentagons	1	2	3	4	5
Number of toothpicks	5	9	13	17	21

(ii) Number of toothpicks = number of pentagons × 4 + 1

(iii) 41 toothpicks

(iv)



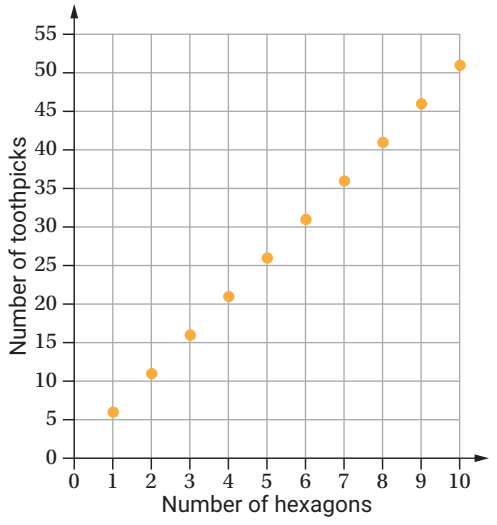
(c) (i)

Number of hexagons	1	2	3	4	5
Number of toothpicks	6	11	16	21	26

(ii) Number of toothpicks = number of hexagons × 5 + 1

(iii) 51 toothpicks

(iv)



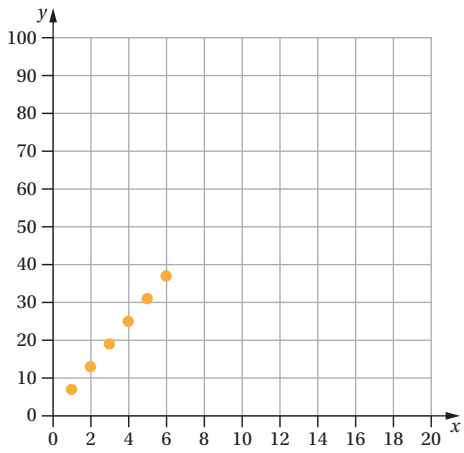
3 **(a)** **(i)** 8 toothpicks
(ii) 11 toothpicks
(iii) 14 toothpicks
(b) Number of toothpicks = number of squares × 3 + 5
(c) 155 toothpicks

4 **(a)**

Number of sections	1	2	3	4	5	6
Number of poles	7	13	19	25	31	37

(b) $y = 6x + 1$

(c)



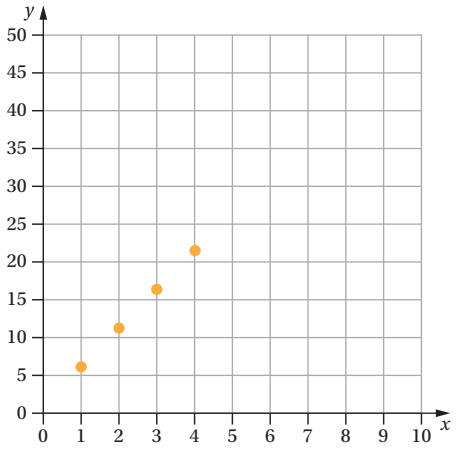
(d) 61 poles
(e) 15 sections

5 **(a)**

Number of squares	1	2	3	4
Number of lines	6	11	16	21

(b) $y = 5x + 1$

(c)



(d) 31 straight lines
(e) 9 sections
(f) **(i)** $d = 2x$ **(ii)** $s = 3x + 1$
(iii) $y = d + s$

Topic 10: Making and reading linear graphs

Recall

I can generate a table of values using an equation

1 **(a)**

a	2	3	4	5
b	4	6	8	10

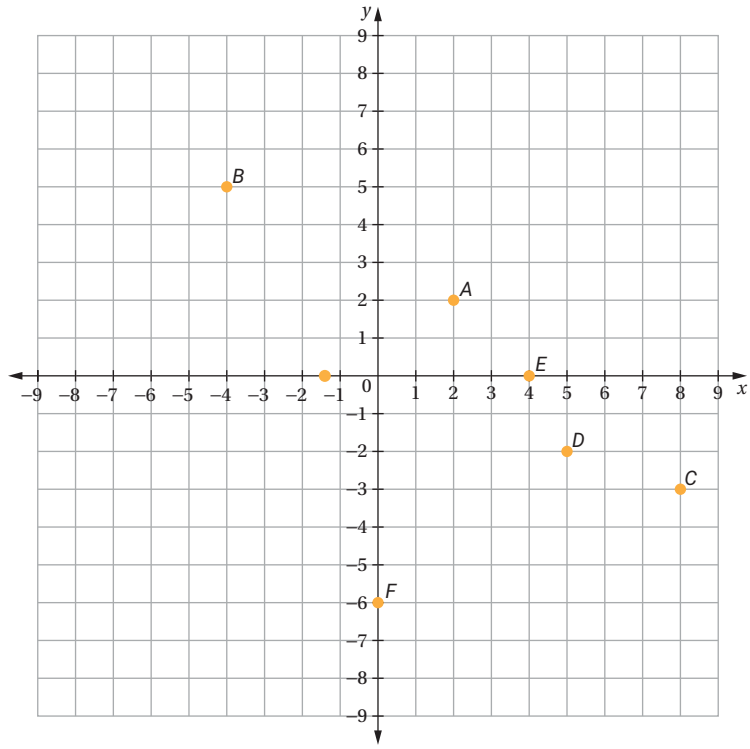
(b)

x	1	2	3	4
y	5	9	13	17

I can read and plot points on a Cartesian plane

1 $A(2, 2)$, $B(4, 6)$, $C(6, 3)$, $D(8, 0)$, $E(0, 9)$, $F(11, 8)$

2



(a) $A(2, 2)$ **(b)** $B(-4, 5)$ **(c)** $C(8, -3)$
(d) $D(5, -2)$ **(e)** $E(4, 0)$ **(f)** $F(0, -6)$

10.1 Graph linear relationships from a rule

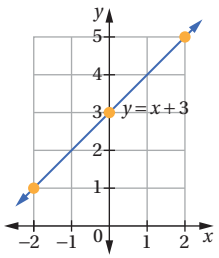
SC 1

1

x	−5	0	5
$y = 2x$	−10	0	10
$y = -2x$	10	0	−10
$y = -2x + 1$	11	1	−9

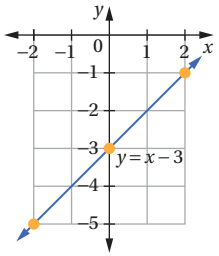
2 **(a)** $y = x + 3$

x	-2	0	2
y	1	3	5
(x, y)	$(-2, 1)$	$(0, 3)$	$(2, 5)$



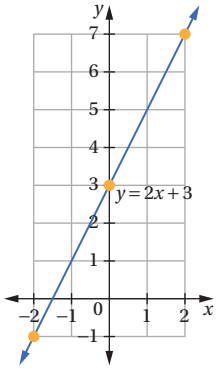
(b) $y = x - 3$

x	-2	0	2
y	-5	-3	-1
(x, y)	$(-2, -5)$	$(0, -3)$	$(2, -1)$



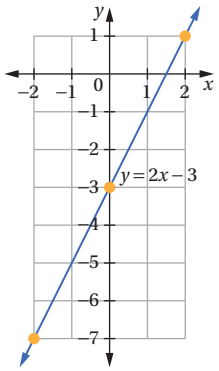
(c) $y = 2x + 3$

x	-2	0	2
y	-1	3	7
(x, y)	$(-2, -1)$	$(0, 3)$	$(2, 7)$



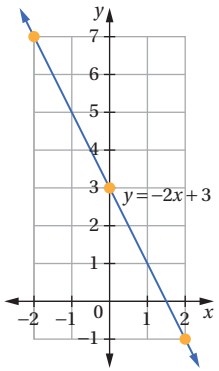
(d) $y = 2x - 3$

x	-2	0	2
y	-7	-3	1
(x, y)	$(-2, -7)$	$(0, -3)$	$(2, 1)$



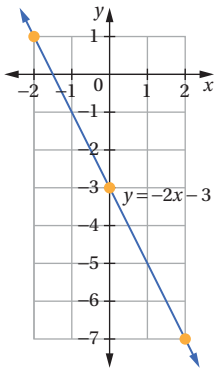
(e) $y = -2x + 3$

x	-2	0	2
y	7	3	-1
(x, y)	$(-2, 7)$	$(0, 3)$	$(2, -1)$

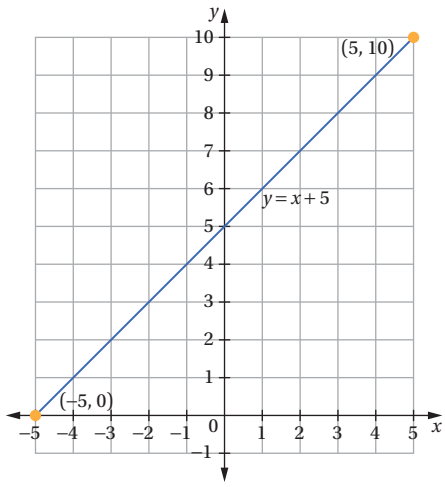


(f) $y = -2x - 3$

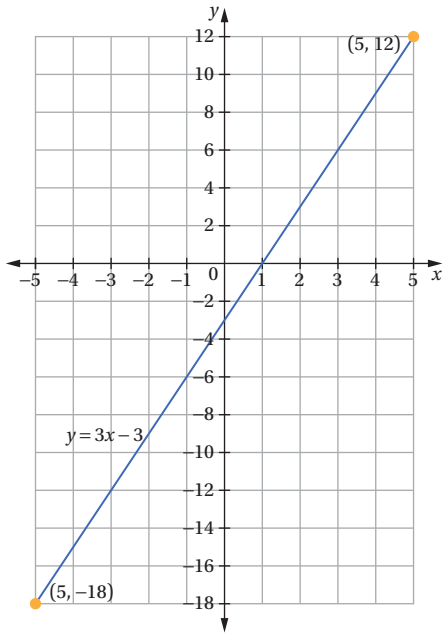
x	-2	0	2
y	1	-3	-7
(x, y)	$(-2, 1)$	$(0, -3)$	$(2, -7)$



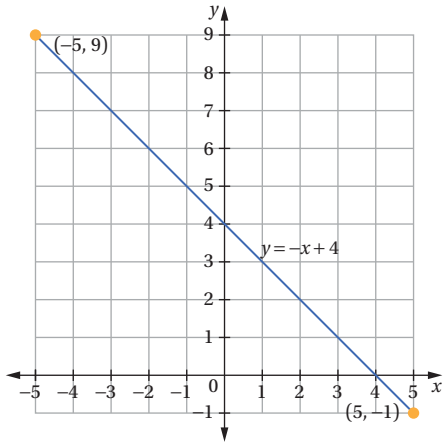
3 **(a)**



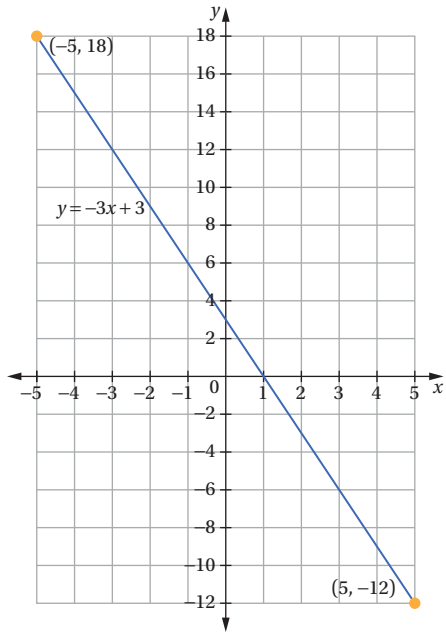
(b)



(c)



(d)



4 The coordinates given are sample answers. Other coordinate pairs are possible.

(a) (−4, −4), (0, 4), (3, 10); $y = 2x + 4$

(b) (−4, −10), (0, − 6), (6, 0); $y = x - 6$

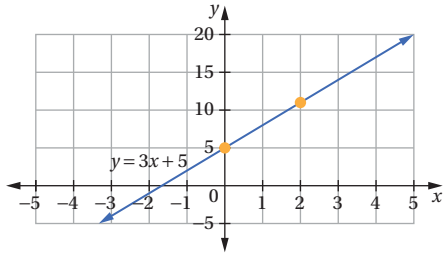
(c) (−6, 9), (0, 3), (4, −1); $y = -x + 3$

(d) (−6, 8), (0, −4), (2, − 8); $y = -2x - 4$

5 (−1, −7)

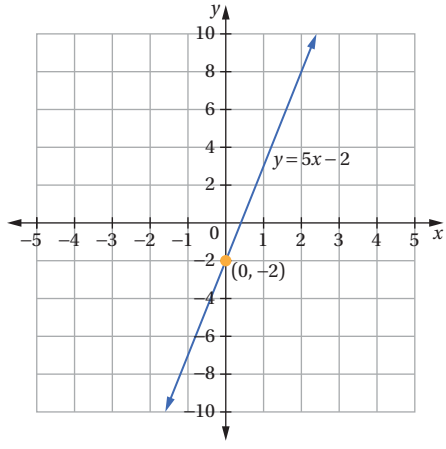
SC 2

1

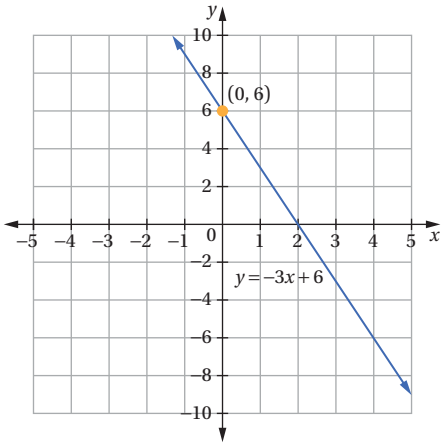


2 **(a)** (0, 4) **(b)** (0, 5) **(c)** (0, −4) **(d)** (0, −5) **(e)** (0, 4) **(f)** (0, −4) **(g)** (0, 4)

3 **(a)**



(b)



4 $y = 7x + 7$

5 **(a)** $y = 2x + 2$ **(b)** $y = 3x - 1$
(c) $y = -3x + 1$ **(d)** $y = x - 2$

6 **(a)** $y = x + 6$ **(b)** $y = 5x - 8$
(c) $y = -4x + 3$ **(d)** $y = -x - 6$

10.2 Interpret the gradient of a linear graph

SC 1

1 **(a)** negative **(b)** positive **(c)** negative
(d) zero **(e)** zero **(f)** negative

2 **(a)** positive **(b)** negative **(c)** positive
(d) zero **(e)** positive **(f)** positive

3 **(a)** positive **(b)** negative **(c)** zero
(d) positive **(e)** positive **(f)** positive
(g) negative **(h)** positive

4 **(a)** negative; $y = -2x + 4$ **(b)** positive; $y = 3x - 6$
(c) zero; $y = -6$ **(d)** negative; $y = \frac{-x}{6}$

5 **(a)** positive **(b)** negative **(c)** positive
(d) positive **(e)** negative

6 A positive gradient slopes up from left to right (the value of y increases with the value of x). A negative gradient slopes down from left to right (the value of y decreases as the value of x increases). A zero gradient is horizontal, (the y-value remains unchanged as the x-value changes).

SC 2

1 **(a)** UK£66 **(b)** UK£6600
(c) UK£27 **(d)** UK£270
(e) A$135 **(f)** A$1350
(g) A$36 **(h)** A$3600

2 **(a)** 46 km/h **(b)** 740 km/h
(c) 9 km/h **(d)** 32 knots
(e) 54 knots **(f)** 22 knots

3 **(a)** **(i)** 40 ha **(ii)** 32 ha
(iii) 50 acres **(iv)** 250 acres
(b) **(i)** 1 ha = 2.5 acres **(ii)** 1 acre = 0.4 ha

4 **(a)** 160 L
(b) 100 barrels is about 16 000 L. Because the graph passes through the origin, it is valid to use proportion. Divide both amounts by 100 to get each barrel at around 160 L.

5 **(a)** **(i)** 140 °F **(ii)** 50 °C
(iii) −40 °F **(iv)** 38 °C
(b) Because the graph does not pass through the origin, proportion cannot be used. Also, 30 °F is below freezing point and 10 °C is above freezing point.
(c) **(i)** 360 °F
(ii) 356 °F (or just a bit below 360 °F)

10.3 Interpret, discuss and analyse relationships in graphs

SC 1

1 **(a)** $75 **(b)** $425 **(c)** 6 hours **(d)** $50/h

2 **(a)** 15 minutes **(b)** 240 km
(c) 140 km **(d)** 80 km/h

3 In the first question, the graph is only valid for whole numbers of hours. These times are the rounded-up values of the actual time taken.

4 **(a)** **(i)** $23 **(ii)** $103
(b) **(i)** 6.6 m **(ii)** 2.6 m
(c) $11.40/m

SC 2

1 **(a)** $1\frac{1}{2}$ hours **(b)** 3 hours
(c) From 3 hours to 5 hours, so during the 4th and 5th hours (or section *C*)
(d) **(i)** Approximately 50 km/h **(ii)** 20 km/h
(e) 200 km

2 **(a)** 5 hours **(b)** The first $\frac{1}{2}$ hour **(c)** 36 km
(d) There are no sections with a zero gradient, so there are no rests along the way, although she must have stopped momentarily after 1 hour to turn around and head home.

3 **(a)** Section A: 24 km/h **(b)** Section B: 12 km/h
(c) Section C: 3 km/h **(d)** Section D: 8 km/h
(e) Section E: 4 km/h

4 Possible answer: Lee sets out from home to walk for an hour. She then sits down for a break to eat her sandwiches. After an hour's break, she continues the journey more slowly to her final destination and arrives an hour later. After a half hour's rest, her dad arrives with her bicycle, so she rides straight home.

5 Possible answer: Dylan left home in his SUV, travelling for an hour and a half. He stopped for breakfast at his favourite cafe, taking a proper break for an hour and a half. Dylan continued driving, more slowly this time because he became stuck in heavy traffic. After an hour of this, Dylan decided to go straight home. He performed a U-turn and headed back 60 km to a shopping centre. After an hour and a half of shopping and eating, he headed straight home, a little slower than before because he listened to his music.

SC 3

1 **(a)** 15 °C, 3 pm Monday
(b) 20 km/h, 3 pm Sunday
(c) The top line shows 20 °C with each line below representing 5 °C cooler.
(d) The top line shows 25 km/h with each number on the right-hand side of the graph below that representing 5 km/h slower.
(e) Rain is unlikely at 6 pm Sunday; in fact, there is a lower chance of rain at that time than at any time over the next 48 hours.
(f) Possible answer: 50% (or near) chance of rain at 3 pm and midnight

2 **(a)** 2021 **(b)** 2019
(c) Spring **(d)** 2020

3 **(a)** The fill level for the dams has generally decreased over the years shown.
(b) The most likely year for drought was 2019.
(c) The first month or two.

4 **(a)** The general trend is for the number of female births to increase, with the exception of the 1920s and 1930s.
(b) The ten-year period 1924–1934 saw the biggest decrease. The Great Depression occurred, following a huge number of deaths from a world war and the Spanish influenza.
(c) 1934–1944, 1944–1954, and 2004–2014 all appear to have the greatest increase. The gradient of the line segments is steepest for these periods.

5 **(a)** The size of the average home loan has increased about six times over the 25 years.
(b) The effect of inflation is not considered, so wages will have increased.
(c) This graph shows that almost double the proportion of wages are needed for 20% of the loan.
(d) The average home loan increased at about double the rate of wages.

6 **(a)** The price has fluctuated. The increase over 16 years was just a few cents.
(b) There is no distinct pattern other than a small increase over time.

Topic 11: Solving linear equations

Recall

I can rewrite word sentences as algebraic sentences

1 **(a)** $3x + 2y$ is an expression with 2 terms.
(b) $x + 3 = 8$ is an equation and the value of x is 5, because $5 + 3 = 8$
(c) $7a = 42$ is an equation and the value of a is 6, because $7 \times 6 = 42$

2 **(a)** $100 - 5 = 95$ **(b)** $8 + y = 9$
(c) $x + 19 = y$ **(d)** $4n = 60$

I can apply a series of operations to a given number

1 **(a)** 13 **(b)** 20

2 **(a)** 45 **(b)** 3 **(c)** 1
(d) 70 **(e)** 2

I can substitute numbers into algebraic expressions

1 **(a)** 5 **(b)** 7 **(c)** 15
(d) 25 **(e)** −15 **(f)** 55

2

x	−3	−2	−1	0	1	2	3
$4x - 1$	−13	−9	−5	−1	3	7	11

3 **(a)** −15 **(b)** 15 **(c)** −50
(d) 50 **(e)** −2 **(f)** $-\frac{1}{2}$

11.1 Write linear equations to represent simple word problems

SC 1

1 Answers may vary.
(a) The sum of 12 and 3 is 15.
(b) 3 less than 15 is 12.
(c) 3 groups of 5 is 15.
(d) 15 shared into 3 equal groups makes groups of 5.

2 Answers will vary.

+	−	×	÷	=
add sum plus more than increase	subtract take away difference less than decrease	multiply groups of times product	divide share equally how many quotient	equals is equal to result same as

3 The same operations and numbers are used. The difference is the order of operations, due to the brackets:
$3 \times 4 + 2 = 12 + 2 = 14$
$3 \times (4 + 2) = 3 \times 6 = 18$
The expressions are not equal.

4 **(a)** $3 \times 2 = 6$ **(b)** $3 \times 2 + 4 = 10$
(c) $2(4 + 3) = 14$

SC 2

1 Answers may vary.
(a) 3 more than an unknown number x is 12.
(b) Decreasing an unknown number x by 3 is equal to 12.
(c) The product of 3 and an unknown number x is 12.
(d) An unknown number x shared into groups of 3 makes 12 groups.

2 The difference is the brackets around the expression $x + 2$, which changes the order of operations:
$3x + 2$ means to multiply x by 3, then add 2.
$3(x + 2)$ means to add 2, then multiply the result by 3.
So the expressions are not equivalent and the value of x is not the same. This can also be shown by expanding the brackets:

$$3(x+2) = 3 \times x + 3 \times 2$$
$$= 3x + 6$$

$3x + 2 \neq 3x + 6$
The equations will have different solutions because the left-hand side is different in each equation.

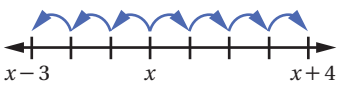
3 **(a)** Always equal. $m + 3 = 3 + m$
(b) Sometimes equal. $t - 3 = 3 - t$ only if $t = 3$, so that both expressions give $3 - 3 = 0$.
(c) Always equal. $3 \times p = p \times 3$
(d) Sometimes equal. $d \div 3 = 3 \div d$ only if $d = \pm 3$, so that both expressions give $3 \div 3 = 1$.

4 Define the unknown number as n.
(a) $2n = 11$ **(b)** $n + 7 = 11$
(c) $2n + 7 = 11$ **(d)** $2(n + 7) = 11$

5 **(a)** **(i)** $J = 8 - 3$ **(ii)** $J = 8 + 1$
(iii) $J = 8 \times 3$ **(iv)** $J = \frac{8}{2}$
(b) In each case, the 8 is replaced by 12.
For example, $J = 12 - 3$.
(c) In each case, the 8 is replaced by P.
For example, $J = P - 3$.

6 **(a)** $k = s - 42$ and $a = s + 2$
(b) $116 = s + (s - 42) + (s + 2)$

7 **(a)**



$x + 4$ is 4 greater than x. $x - 3$ is 3 less than x.
So $x + 4$ is always 7 greater than $x - 3$.

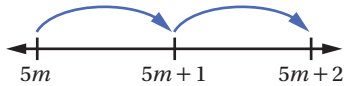
(b) $2p$ can be less than, greater than or equal to $3p$, depending on whether p is negative, zero or positive:

Negative numbers $p = -1$	$p = 0$	Positive numbers $p = 1$
$2p = 2 \times (-1) = -2$ $3p = 3 \times (-1) = -3$ $3p$ is less than $2p$.	$2p = 2 \times 0 = 0$ $3p = 3 \times 0 = 0$ $3p$ is equal to $2p$.	$2p = 2 \times 1 = 2$ $3p = 3 \times 1 = 1$ $3p$ is greater than $2p$.

(c) $-2y$ can be less than, greater than or equal to $2y$, depending on whether y is negative, zero or positive:

Negative numbers $y = -1$	$y = 0$	Positive numbers $y = 1$
$2y = 2 \times (-1) = -2$ $-2y = -2 \times (-1) = 2$ $-2y$ is greater than $2y$.	$2y = 2 \times 0 = 0$ $-2y = -2 \times 0 = 0$ $-2y$ is equal to $2y$.	$2y = 2 \times 1 = 2$ $-2y = -2 \times 1 = -2$ $-2y$ is less than $2y$.

(d)



$5m + 1$ is 1 more than $5m$. $5m + 2$ is 2 more than $5m$.
So $5m + 2$ is always 1 more than $5m + 1$.

11.2 Write linear equations to represent diagrams and patterns

SC 1

1 **(a)** $L = 5 + 2$, $L = 7$ cm
(b) $5 = L + 2$, $L = 5 - 2 = 3$ cm
(c) $L = x + 2$ cm
(d) $L = x + x + 2$, $L = 2x + 2$ cm
(e) $L + 2 = x + x$, $L = 2x - 2$ cm

2 **(a)** $P = 8 + 3 + 8 + 3 = 22$
The perimeter is 22 cm.
$A = 8 \times 3 = 24$
The area is 24 cm^2.

(b) $P = 8 + 5 + 8 + 5 = 26$
The perimeter is 26 cm.
$A = 8 \times 5 = 40$
The area is 40 cm^2.

(c) $P = 8 + 3.5 + 8 + 3.5 = 23$
The perimeter is 23 cm.
$A = 8 \times 3.5 = 28$
The area is 28 cm^2.

(d) $P = 8 + t + 8 + t = 16 + 2t$
The perimeter is $16 + 2t$ cm.
$A = 8 \times t = 8t$
The area is $8t$ cm^2.

(e) $P = d + t + d + t = 2d + 2t$
The perimeter is $2d + 2t$ cm.
$A = d \times t = dt$
The area is dt cm^2.

(f) $P = (d + 4) + t + (d + 4) + t = 2d + 2t + 8$
The perimeter is $2d + 2t + 8$ cm.
$A = (d + 4) \times t$
$= t(d + 4)$
$= t \times d + t \times 4$
$= dt + 4t$
The area is $dt + 4t$ cm^2.

(g) $P = (d + 4) + (t - 1) + (d + 4) + (t - 1)$
$= 2d + 2t + 6$
The perimeter is $2d + 2t + 6$ cm.
$A = (d + 4) \times (t - 1)$
$= d(t - 1) + 4(t - 1)$
$= d \times t - d \times 1 + 4 \times t - 4 \times 1$
$= dt - d + 4t - 4$
The area is $dt - d + 4t - 4$ cm^2.

3 **(a)** Define N as the total number of toothpicks.
$N = 50 + 5 = 55$
There are 55 toothpicks in total.

(b) Sample answers:
(i) Two less than the number of toothpicks in the jar is equal to sixty-four.
(ii) Double the number of toothpicks in the jar is equal to sixty-four.
(iii) Half the number of toothpicks in the jar is equal to sixty-four.
(iv) Two more than the number of toothpicks in the jar is equal to sixty-four.

4

Starting temperature	6 °C	−6 °C	p °C
The starting temperature value is doubled.	$F = 2 \times 6$ The final temperature is 12 °C.	$F = 2 \times (-6)$ The final temperature is −12 °C.	$F = 2 \times p$ The final temperature is $2p$ °C.
The starting temperature falls by 15 °C.	$F = 6 - 15$ The final temperature is −9 °C.	$F = -6 - 15$ The final temperature is −21 °C.	$F = p - 15$ The final temperature is $(p - 15)$ °C.
The final temperature is two thirds of the starting temperature value.	$F = \frac{2}{3} \times 6$ The final temperature is 4 °C.	$F = \frac{2}{3} \times (-6)$ The final temperature is −4 °C.	$F = \frac{2}{3} \times p$ The final temperature is $\frac{2p}{3}$ °C.
The final temperature is 5 °C higher than half of the starting temperature value.	$F = \frac{1}{2} \times 6 + 5$ The final temperature is 8 °C.	$F = \frac{1}{2} \times (-6) + 5$ The final temperature is 2 °C.	$F = \frac{1}{2} \times p + 5$ The final temperature is $\left(\frac{p}{2} + 5\right)$ °C.

5 Width is $L - 38$, so the perimeter P is:
$P = (L - 38) + L + (L - 38) + L$
$= 4L - 76$
Using the given value for the perimeter:
$124 = 4L - 76$ m

SC 2

1 **(a)** a represents the number of triangles, b represents the number of matchsticks.
(b) Every 3 matchsticks makes 1 more triangle.
The number of matchsticks is 3 times the number of triangles.
(c) $b = 3a$

2 **(a)** $m = 4s$ **(b)** $m = 5h + 1$

3 **(a)** $s = 5t$ **(b)** $s = 4t + 1$

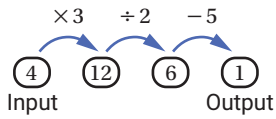
4 **(a)** $p = m + 4$ **(b)** $q = -5t$ **(c)** $g = \frac{d}{2}$
(d) $b = 5a - 1$ **(e)** $p = -2q + 7$ **(f)** $y = \frac{x}{4} + 1$

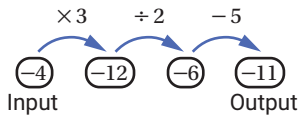
5 **(a)** $b = 4a + 3$ **(b)** $b = -3a - 1$

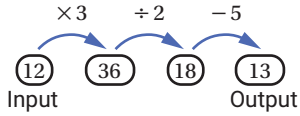
6 **(a)** $n = 7s + 1$ **(b)** $n = 4s - 3$

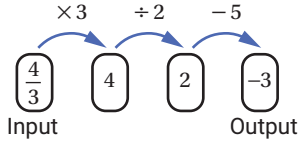
11.3 Use flow charts to determine the value of an expression

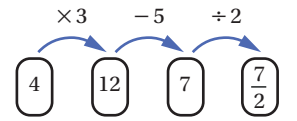
SC 1

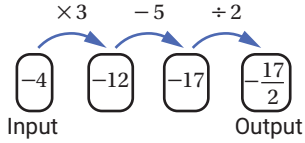
1 **(a)**


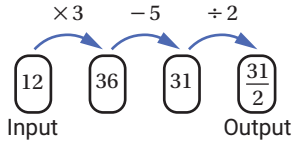
(b)


(c)


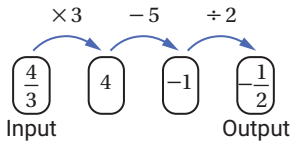
(d)


2 **(a)**


(b)


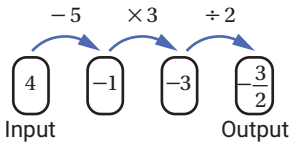
(c)


(d)

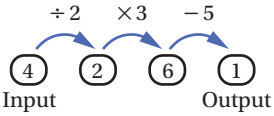


3 The order of operations is different.
In both questions, the first operation applied to the input number is multiplying by 3.
The only difference is that in the first question you divided that number by 2 and then subtracted 5.
However, in the second question, you subtracted 5 and then divided by 2.
The different order of operations are not equivalent.

4 **(a)** Sample answer. Check that the output is different for one of the inputs in Question 1.

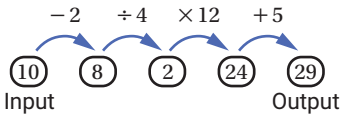


(b) Multiplying by 3 and then dividing by 2 is the same as multiplying by $\frac{3}{2}$, which is the same as dividing by 2 and multiplying by 3.

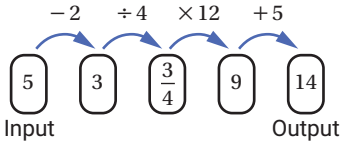


5 **(a)** 'I think of a number, subtract 2, divide by 4, multiply by 12, then add 5.'

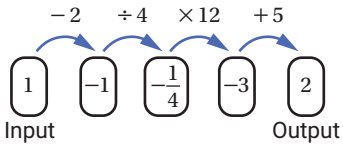
(b)



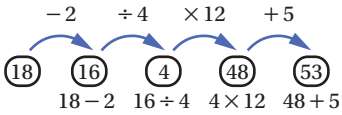
(c)



(d) For the output number to be positive, the second-last number in the flow chart has to be −4 or higher.
The two middle operations, dividing by 4 and then multiplying by 12, is the same as multiplying by 3.
The smallest number that can be in the second box in the flow chart is −1 because this will make the number in the second last box $-1 \times 3 = -3$.
Subtracting 2 from any positive whole input number will mean the second number will always be −1 or higher. Therefore the statement is true.
Alternatively, you could try different numbers as the input number to see whether you can find examples, counter-examples or patterns.



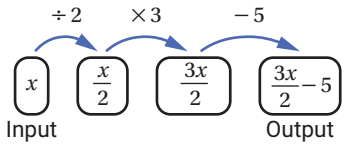
(e) Start from the output.
53 is the result of 48 + 5, so the number in the second-last box is 48.
48 is the result of 4 × 12, so the number in the middle box is 4.
4 is the result of 16 ÷ 4 = 4, so the number in the second box is 16.
16 is the result of 18 − 2, so the input number is 18.



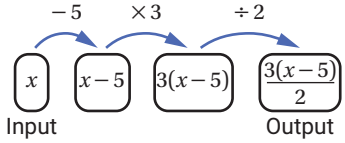
6 Answers will vary.

SC 2

1 **(a)**



(b)

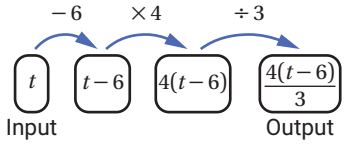


(c) The operations and input are the same for each, but the order of operations is different. The output expression is different in each flow chart because the order is different.

2 **(a)** I think of a number, subtract 6, multiply the result by 4 and then divide by 3.

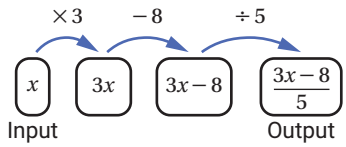
(b) $0 - 6 = -6$
$-6 \times 4 = -24$
$-24 \div 3 = -8$

(c)

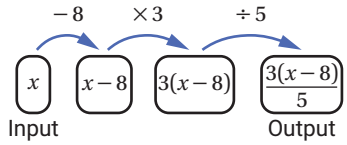


(d) The output expression becomes $\frac{4(z-6)}{3}$.

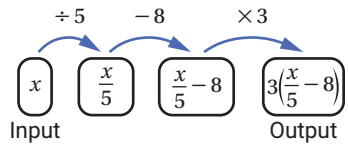
3 **(a)** **(i)**



(ii)



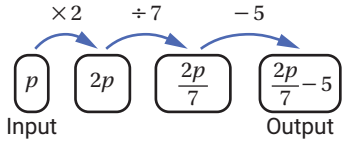
(iii)



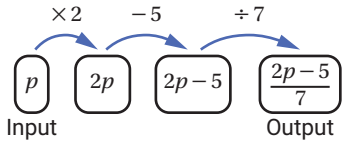
(b) If the input number is 1, or $x = 1$, the output numbers are different for each expression.

$$\frac{3x-8}{5} = \frac{3\times 1-8}{5}$$
$$= \frac{-5}{5}$$
$$= -1$$
$$\frac{3(x-8)}{5} = \frac{3(1-8)}{5}$$
$$= \frac{3\times(-7)}{5}$$
$$= -\frac{21}{5}$$
$$3\left(\frac{x}{5}-8\right) = 3\left(\frac{1}{5}-8\right)$$
$$= 3\times\left(-\frac{39}{5}\right)$$
$$= \frac{-117}{5}$$

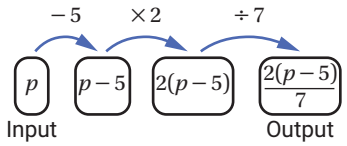
4 (a) I think of a number, multiply it by 2, divide it by 7, then subtract 5 from the result.



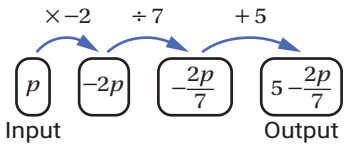
(b) I think of a number, multiply it by 2, subtract 5, then divide the result by 7.



(c) I think of a number, subtract 5, multiply it by 2, then divide the result by 7.



(d) I think of a number, multiply it by −2, divide it by 7, then add 5 to the result.



5 Answers will vary.

11.4 Use flow charts to solve linear equations

SC 1

1 **(a)** 3 **(b)** 8 **(c)** 16 **(d)** −8
(e) 48 **(f)** 2 **(g)** 4 **(h)** 16

2 Student B is correct. Student A substituted $x = 9$ as a digit, creating the number 39. They have not realised that $3x = 3 \times x$.

3

	$x = 1$	$x = 2$	$x = 3$
$x + 7 = 10$			X
$3x + 7 = 13$		X	
$3(x + 7) = 24$	X		
$\frac{x}{3} + 7 = 8$			X
$\frac{7x}{3} - 1 = 6$			X
$\frac{7x+1}{3} = 5$		X	
$\frac{5}{x+4} - 7 = -6$	X		

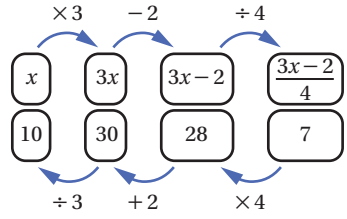
4 (a)

x	1	2	3	4	5
$2x + 4$	6	8	10	12	14
$3x - 1$	2	5	8	11	14

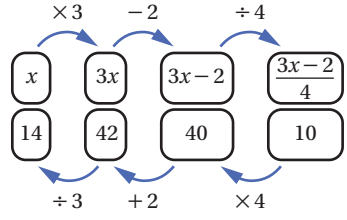
(b) $x = 1$
(c) $x = 3$
(d) $x = 5$ is the solution, which is the value of x that makes the expressions on either side of the equals symbol equal. For $x = 5$, each expression is equal to 14 and the equation is true.

SC 2

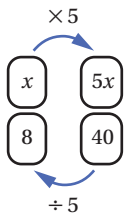
1 **(a)** $x = 10$



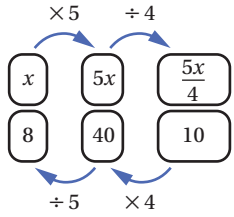
(b) $x = 14$



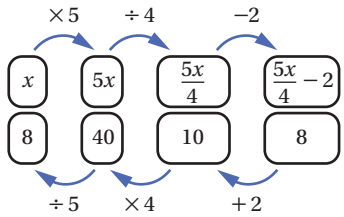
2 **(a)** **(i)**



(ii)

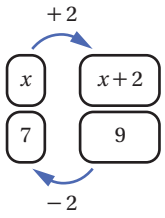


(iii)

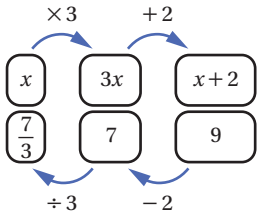


(b) Each equation has a solution of $x = 8$. The flow charts in part **(a)** show the first, second and third steps to write the expression $\frac{5x}{4} - 2$.

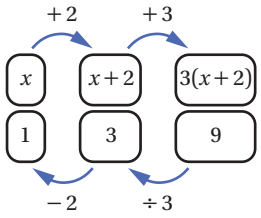
3 **(a)** $x = 7$



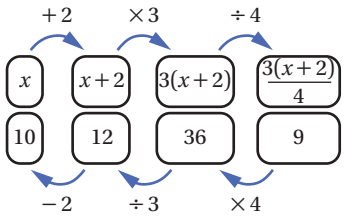
(b) $x = \frac{7}{3}$



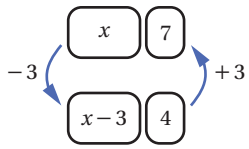
(c) $x = 1$



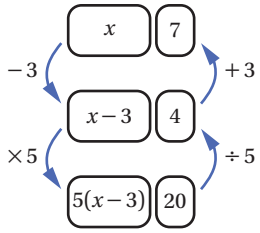
(d) $x = 10$



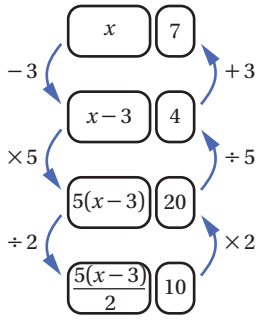
4 **(a)** **(i)** $x = 7$



(ii) $x = 7$



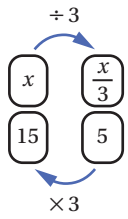
(iii) $x = 7$



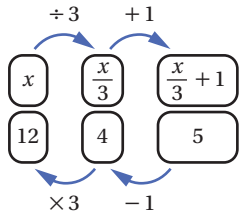
(b) The expressions are always equal.

5 **(a)** The following answers could also be drawn as vertical flow charts.

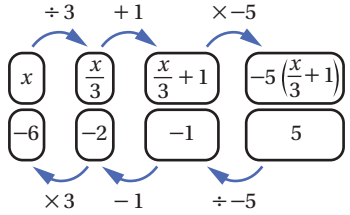
(i) $x = 15$



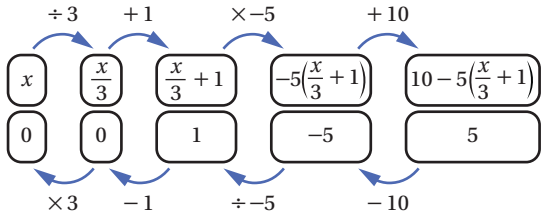
(ii) $x = 12$



(iii) $x = -6$



(iv) $x = 0$



(b) Each flow chart starts with the unknown number, marked with variable x. The value that is equal to the last expression in each flow chart is 5. The expression is built by following a set of instructions, and the solution is found by following the inverse operations in the opposite order.
Each set of boxes shows two equivalent expressions, so it is a true equation.

6 The similarities are that each step of the equation is built on the left and the numerical expression is shown on the right.
Each line of the flow chart and the equation shows a pair of equivalent expressions.
The differences are that the algebraic solution only contains the inverse operations. You have to know the correct order to apply them.
The algebraic solution shows the equals symbol between each pair of equal expressions, whereas in the flow chart these expressions sit in a pair of boxes.

7 **(a)** $\frac{2a+6}{3} = 4;\ a = 3$ **(b)** $4\left(\frac{b}{3}-2\right) = 4;\ b = 9$
(c) $\frac{6-c}{4} = 1;\ c = 2$
(d) $4\left(-\frac{d}{5}-3\right) = -20;\ d = 10$

8 Advantages: the order of operations is clear; the inverse operations are simple to follow.
Disadvantages: flow charts are time-consuming to create.

11.5 Use the balance method to solve linear equations

SC 1

1 **(a)** $5 + 3 + 7 = 2 \times 4 + 7$
(b) $5 + 3 - 7 = 2 \times 4 - 7$
(c) $7 \times (5 + 3) = 7 \times 2 \times 4$

2 **(a)** $x + 10 = 2x + 7$ **(b)** $x - 4 = 2x - 7$
(c) $7(x + 3) = 14x$

3 **(a)** $5x - 4 = 14$ **(b)** $x = 3$
(c) $2(x - 4) = 12$ **(d)** $-\frac{3x}{2} = 4$

4 **(a)** $5x - 7 = 11$ **(b)** $10x = 30$
(c) $8(x - 4) = 48$ **(d)** $16 - \frac{3x}{2} = 20$

5 **(a)** $3x - 6 = 4$
$3x - 6 + 2 = 4 + 2$
$3x - 4 = 6$
The equations are not equivalent.

(b) $\frac{3x}{2} = 5$
$\frac{3x}{2} \times 2 = 5 \times 2$
$3x = 10$
The equations are equivalent.

(c) $3(x-6) = 15$
$\frac{3(x-6)}{3} = \frac{15}{3}$
$x - 6 = 5$
The equations are not equivalent.

(d) $4 - \frac{3x}{2} = 10$
$4 - \frac{3x}{2} - 4 = 10 - 4$
$-\frac{3x}{2} = 6$
The equations are equivalent.

6 Sample answers.
$2(x + 4) = 30,\ x + 4 = 15$

7 **(a)** subtract 4, multiply by 3
(b) divide by 3 and then add 4

8 Student A is correct. Student B did not divide the entire expression on the left-hand side by 3.
Student B could have been correct if they had done the following:
$3x + 9 = 15$
$\frac{3x}{3} + \frac{9}{3} = \frac{15}{3}$
$x + 3 = 5$
$x + 3 - 3 = 5 - 3$
$x = 2$

SC 2

1 **(a)** $x = 9$ **(b)** $x = 15$ **(c)** $x = 4$ **(d)** $x = 36$
(e) $x = 16$ **(f)** $x = 24$ **(g)** $x = 5$ **(h)** $x = 80$
(i) $x = -4$ **(j)** $x = 6$ **(k)** $x = \frac{1}{5}$ **(l)** $x = 5$

2 **(a)** $2x + 1 - 1 = 7 - 1$
$2x = 6$
$\frac{2x}{2} = \frac{6}{2}$
$x = 3$

(b) $2x - 1 + 1 = 7 + 1$
$2x = 8$
$\frac{2x}{2} = \frac{8}{2}$
$x = 4$

(c) $\frac{x}{2} + 1 - 1 = 7 - 1$
$\frac{x}{2} = 6$
$\frac{2x}{2} = 2 \times 6$
$x = 12$

(d) $\frac{x}{2} - 1 + 1 = 7 + 1$
$\frac{x}{2} = 8$
$\frac{2x}{2} = 2 \times 8$
$x = 16$

(e) $\frac{x+1}{2}\times 2 = 7\times 2$

$x+1=14$

$x+1-1=14-1$

$x=13$

(f) $\frac{x-1}{2}\times 2 = 7\times 2$

$x-1=14$

$x-1+1=14+1$

$x=15$

(g) $6x+3-3=15-3$

$6x=12$

$\frac{6x}{6}=\frac{12}{6}$

$x=2$

(h) $6x-3+3=15+3$

$6x=18$

$\frac{6x}{6}=\frac{18}{6}$

$x=3$

(i) $\frac{x}{6}+3-3=15-3$

$\frac{x}{6}=12$

$\frac{x}{6}\times 6=12\times 6$

$x=72$

(j) $\frac{x}{6}-3+3=15+3$

$\frac{x}{6}=18$

$\frac{x}{6}\times 6=18\times 6$

$x=108$

(k) $\frac{x+3}{6}\times 6=15\times 6$

$x+3=90$

$x+3-3=90-3$

$x=87$

(l) $\frac{x-3}{6}\times 6=15\times 6$

$x-3=90$

$x-3+3=90+3$

$x=93$

3 The error occurs in line 2 where the wrong operation has been applied.

$\frac{x+3}{5}=6$

$\frac{x+3}{5}\times 5=6\times 5$

$x+3=30$

$x+3-3=30-3$

$x=27$

4 **(a)** add 7, multiply by −2, subtract 3

(b) subtract 7, divide by −2, add 3

(c) $-2(x+7)-3+3=-11+3$

$-2(x+7)=-8$

$\frac{-2(x+7)}{-2}=\frac{-8}{-2}$

$x+7=4$

$x+7-7=4-7$

$x=-3$

5 **(a)** $x=15$ **(b)** $x=4$ **(c)** $x=9$ **(d)** $x=-1$

6 **(a)** $x=5\frac{1}{2}$ **(b)** $x=2\frac{1}{2}$ **(c)** $x=-4\frac{2}{3}$ **(d)** $x=-17\frac{1}{2}$

7 **(a)** $\frac{3(2-x)}{7}+6=3$

$\frac{3(2-x)}{7}=-3$

$3(2-x)=-21$

$2-x=-7$

$-x=-9$

$x=9$

(b) $\frac{10-4(x+9)}{6}=-5$

$10-4(x+9)=-30$

$-4(x+9)=-40$

$x+9=10$

$x=1$

(c) $40=\frac{60(x-8)}{3}$

$120=60(x-8)$

$20=x-8$

$28=x$

$x=28$

8 $18=L+4$

$14=L$

Lily is 14 years old.

9 $J+44=250$

$J=206$

John currently has $206.

10 $5c+15=245$

$5c=230$

$c=46$

The concert tickets cost $46 each.

11 $8r+20=176$

$8r=156$

$r=19.5$

Ria earns $19.50 per hour.

Topic 12: Area and volume

Recall

I can calculate with decimals

1 **(a)** 18.81 **(b)** 38.97 **(c)** 3.31 **(d)** 1.85

2 **(a)** 144.022 **(b)** 0.3072

(c) 28.08 **(d)** 0.8908

3 **(a)** 590 **(b)** 7.416

(c) 84 630 **(d)** 9.530

I can read lengths from rulers

1 **(a)** **(i)** 7.4 cm **(ii)** 74 mm

(b) **(i)** 75.6 cm **(ii)** 756 mm

I can calculate the perimeter and area of plane shapes

1 **(a)** $P = 120$ mm, $A = 800$ mm^2

(b) $P = 24$ cm, $A = 36$ cm^2

(c) $P = 148$ mm, $A = 1368$ mm^2

I can convert between metric units of measurement

1 **(a)** 500 cm **(b)** 200 mm **(c)** 9.07 m

(d) 5 000 000 m **(e)** 6500 mm

12.1 Understand and convert between units of area

SC 1

1 **(a)** 32 cm^2 **(b)** 21 cm^2 **(c)** 300 cm^2
2 **(a)** 44 mm^2 **(b)** 60 m^2 **(c)** 72 km^2
3 **(a)** 4 ha **(b)** 14 ha **(c)** 4 ha
4 **(a)** 16.43 cm^2 **(b)** 7.872 m^2 **(c)** 37 km^2
5 **(a)** 26.66 ha **(b)** 0.96 ha
6 **(a)** 104 ha **(b)** 50 ha

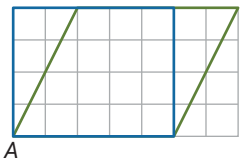
SC 2

1 **(a)** 30 000 cm^2 **(b)** 1500 mm^2 **(c)** 1.82 m^2
(d) 1.7 km^2 **(e)** 0.0712 m^2 **(f)** 11 cm^2
(g) 327 mm^2 **(h)** 82.3 cm^2
2 **(a)** The conversion factor from m to cm is 100^2, not 100; 0.048 m^2
(b) The conversion factor from m to cm is 100^2, not 100; 900 cm^2
(c) The conversion factor from m to mm is 1000^2, not 1000; 0.003 m^2
(d) The conversion factor from km to m is 1000^2, not 1000; 0.075 km^2
3 **(a)** 45 000 cm^2 **(b)** 5.04 cm^2
(c) 27 300 cm^2 **(d)** 2.43 cm^2
4 **(a)** 230 000 m^2 **(b)** 3.07 ha **(c)** 0.23 km^2
(d) 170 ha **(e)** 70 300 m^2 **(f)** 425 ha
(g) 18.2 km^2 **(h)** 75 ha
5 **(a)** 920 000 m^2 **(b)** 92 ha **(c)** 0.92 km^2
6 **(a)** 210 ha **(b)** 2.814 ha **(c)** 174.8 ha

12.2 Calculate the area of a parallelogram

SC 1

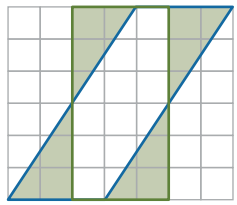
1



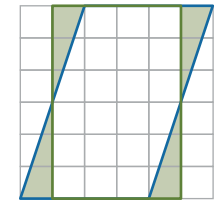
A

The non-included triangle to the right of the rectangle has been removed from the parallelogram and the new included triangle to the left of the parallelogram is the same size and shape, so they have the same area. The areas of the parallelogram and rectangle are the same because equal areas have been added and subtracted.

2 **(a)**



(b)



In each case the width of the rectangle is the same as the base length of the parallelogram; the height of the rectangle is the same as the height of the parallelogram.

3 **(a)** width: 10 cm, length: 15 cm; area: 150 cm^2
(b) width: 15 cm, length: 10 cm; area: 150 cm^2
(c) width: 10 cm, length: 15 cm; area: 150 cm^2
4 **(a)** I and II but not III **(b)** II and III but not I
5 B, C and F
6 12 square units

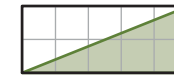
SC 2

1 **(a)** 2795 cm^2 **(b)** 101.2 cm^2
2 56 cm^2
3 39 600 cm^2, 3.96 m^2
4 442 cm^2
5 **(a)** $x = 13$ m **(b)** $y = 22$ m
6 13.86 m^2

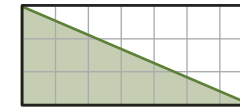
12.3 Calculate the area of a triangle

SC 1

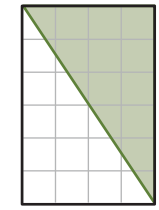
1 **(a)** **(i)**



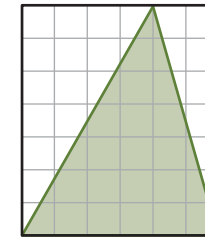
(ii) 10 square units
(iii) 5 square units
(b) **(i)**



(ii) 21 square units
(iii) 10.5 square units
(c) **(i)**



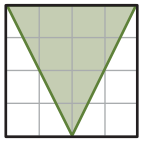
(ii) 24 square units
(iii) 12 square units
2 **(a)** **(i)**



(ii) 42 square units

(iii) 21 square units

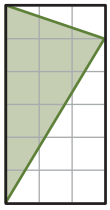
(b) **(i)**



(ii) 16 square units

(iii) 8 square units

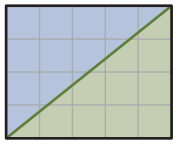
(c) **(i)**

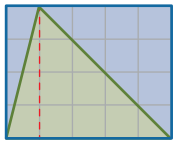


(ii) 18 square units

(iii) 9 square units

3 **(a)** Possible answers.





(b) 20 cm^2

(c) 10 cm^2

4 **(a)** **(i)** 5 cm, 12 cm **(ii)** 30 cm^2

(b) **(i)** 60 mm, 80 mm **(ii)** 2400 mm^2

(c) **(i)** 2 m, 2.1 m **(ii)** 2.1 m^2

5 **(a)** **(i)** 96 cm^2 **(ii)** 48 cm^2

(b) **(i)** 1134 mm^2 **(ii)** 567 mm^2

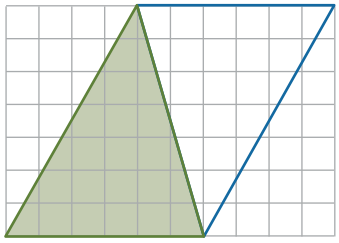
(c) **(i)** 1.26 m^2 or 12 600 cm^2

(ii) 0.63 m^2 or 6300 cm^2

6 576 m^2

SC 2

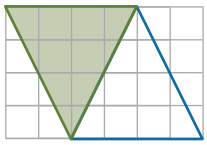
1 **(a)** **(i)** Possible diagram



(ii) 42 square units

(iii) 21 square units

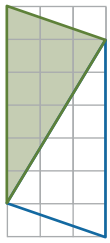
(b) **(i)** Possible diagram



(ii) 16 square units

(iii) 8 square units

(c) **(i)** Possible diagram



(ii) 18 square units

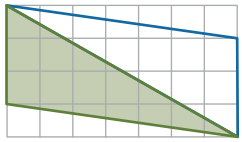
(iii) 9 square units

2 **(a)** **(i)** Possible diagram

(ii) 10 square units

(iii) 5 square units

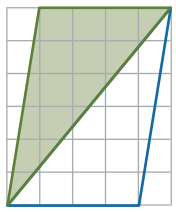
(b) **(i)** Possible diagram



(ii) 21 square units

(iii) 10.5 square units

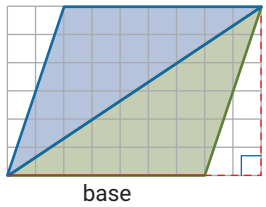
(c) **(i)** Possible diagram

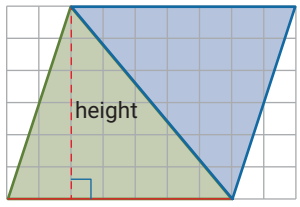


(ii) 24 square units

(iii) 12 square units

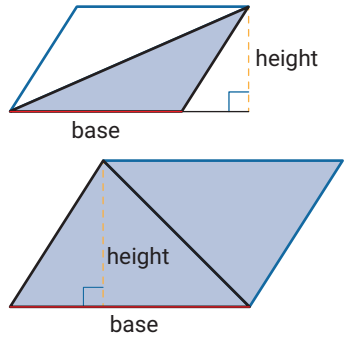
3





A triangle with the same base and height as a parallelogram has half the area of the parallelogram.

4



5 **(a)** **(i)** 400 cm^2 **(ii)** 200 cm^2
(b) **(i)** 1320 mm^2 **(ii)** 660 mm^2
(c) **(i)** 1.05 m^2 or 10 500 cm^2
(ii) 0.525 m^2 or 5250 cm^2

SC 3

1 **(a)** 15 cm^2 **(b)** 6 m^2 **(c)** 37.44 mm^2
2 **(a)** 40 mm^2 **(b)** 1004.4 mm^2 **(c)** 25 cm^2
3 592.1 mm^2
4 6 m
5 54 cm^2
6 480 cm^2

12.4 Understand volume measured in cubic units

SC 1

1 10 cm^3
2 28 cm^3
3 60 cm^3

SC 2

1 **(a)** 20 **(b)** 5 cm by 4 cm by 5 cm **(c)** 100
2 **(a)** 4 cm by 5 cm by 4 cm **(b)** 80
3 **(a)** 17 **(b)** 16

SC 3

1 50 cm^3
2 125 cm^3
3 **(a)** 28 cm^3 **(b)** 45 cm^3 **(c)** 120 cm^3
4 40 cm^3

12.5 Establish the volume formula and units for prisms

SC 1

1 **(a)** 108 cm^3 **(b)** 210 cm^3 **(c)** 160 cm^3
2 390 000 cm^3
3 1.5 cm
4 **(a)** 990 000 mm^3
(b) 450 mm by 180 mm by 220 mm
5 **(a)** 114.39 cm^3 **(b)** 3735.752 m^3
(c) 691 860 cm^3
6 **(a)** 778.688 cm^3 **(b)** 105.625 cm^3
7 4404.1 cm^3
8 0.3549 m^3
9 4.5 cm
10 $520 405.20
11 $5

SC 2

1 **(a)** 1057.8 cm^3 **(b)** 278.52 cm^3
2 **(a)** 357 000 cm^3 **(b)** 384.539 cm^3
3 Right-angled triangular-based prism of height 2 cm. The triangle has sides around the right angle of 6 cm and 5 cm. Volume: 30 cm^3
4 0.975 m^3

SC 3

1 210 cm^3
2 350 cm^3

12.6 Explore the connection between volume and capacity

SC 1

1 **(a)** 1 mm^3 **(b)** 1 cm^3 **(c)** 1 m^3
2 **(a)** cm^3 **(b)** m^3 **(c)** mm^3
3 **(a)** 125 mL **(b)** 60 mL
(c) 1500 mL or 1.5 L **(d)** 0.5 mL

SC 2

1 30 mm^3
2 0.03 m^3
3 2.5 L
4 2 L

SC 3

1 **(a)** 1000 L or 1 kL **(b)** 1 L
(c) 23.92 kL **(d)** 385.56 mL
2 **(a)** 1078 mL, 1.078 L **(b)** 2.6 cm
3 32 cm
4 2.048 L

Topic 13: Circles (features and circumference)

Recall

I can round decimals to the required place value

1 **(a)** 67.34 **(b)** 4.82 **(c)** 2.26
(d) 9.01 **(e)** 4.31

I can perform calculations involving decimals

1 **(a)** 9.62 **(b)** 5.11 **(c)** 56.943 **(d)** 0.248

I can square integers and decimal numbers

1 **(a)** 9 **(b)** 25 **(c)** 121
(d) 0.04 **(e)** 0.25 **(f)** 1.21

I can convert between metric units of measurement

1 **(a)** 25 mm **(b)** 125 cm **(c)** 30.5cm
(d) 0.255 m **(e)** 5250 mm

I can calculate the perimeter of shapes

1 **(a)** 61m **(b)** 32 mm

I can draw circles of any radius using a compass

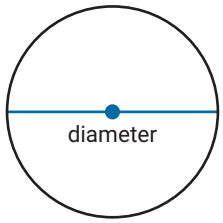
1 **(a)** Open the compass to 4 cm and draw a complete circle.
(b) Open the compass to 6 cm and draw a complete circle.

13.1 Recognise circle features

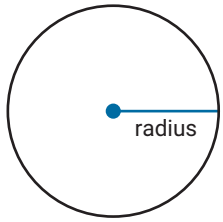
SC 1

1 **(a)** Radius, line segment from the centre to a point on the circumference
(b) Circumference, the perimeter of the circle
(c) Diameter, line segment connecting two points on the circle and passing through the centre
(d) Semicircle, half a circle, formed by cutting the circle along a diameter

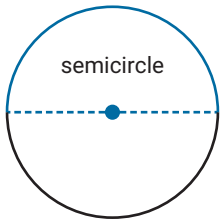
2 **(a)**



(b)

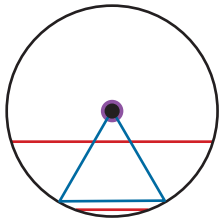


(c)

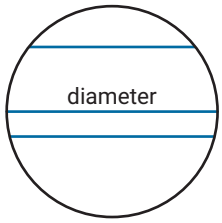


3 The diameter

4 **(a)** Yes. Line segments connected to two points on a circle can be made as small as you want, by making the two endpoints closer and closer together.
For example, drawing an equilateral triangle, which has the radius as its sides, you can see that lines drawn closer to the centre point must be longer than the radius, while lines further away must be smaller than the radius.



(b) No. Drawing parallel lines will show that the lines on each side of a diameter must be smaller than the diameter, because the circle curves inwards at both ends of a diameter.



5 **(a)** Fold the circle in half, matching the curves. Fold it in half again. The centre is where the folds meet. Two diameters will meet only in the centre.
(b) One point of the circumference of the template must be on the centre point.
(c) One point of the circumference of the template must be on the centre point of the first circle as well as on one of the intersections of the first two circles.
(d) If the intersections with the first circle were joined, the triangles would all be equilateral as their sides match the radius of the circle. Hence the angles at the centre of the first circle would be 60°.
$\frac{360°}{60°} = 6$, so six circles fit in the outer layer.

SC 2

1 **(a)** 5 cm **(b)** 7.5 cm
(c) 10 cm **(d)** $(d \div 2)$ cm
(e) Divide the length of the diameter by 2 to get the radius.

2 **(a)** 20 cm **(b)** 30 cm
(c) 40 cm **(d)** $(2 \times r)$ cm
(e) Multiply the length of the radius by 2 to get the diameter.

3 Student A is correct. The diameter of the bigger circle will be 3 times as long as the diameter of the smaller circle.
The diameter of the smaller circle can be written as $2r$ cm, the diameter of the bigger circle can be written as $6r$ cm.
$\frac{6r}{2r} = 3$ times as big.
You can justify your answer with calculations.

Small radius	Small diameter	Big radius	Big diameter
1 cm	2 × 1 cm = 2 cm	3 × 1 cm = 3 cm	2 × 3 cm = 6 cm
10 cm	2 × 10 cm = 20 cm	3 × 10 cm = 30 cm	2 × 30 cm = 60 cm
100 cm	2 × 100 cm = 200 cm	3 × 100 cm = 300 cm	2 × 300 cm = 600 cm

4 Student B is correct. The diameter will be 6 cm bigger.
The diameter of the smaller circle can be written as $2r$ cm, the diameter of the larger circle can be written as $(r + 3) + (r + 3) = (2r + 6)$ cm.
So the diameter of the bigger circle is 6 cm bigger.
You can justify your answer with calculations.

Small radius	Small diameter	Big radius	Big diameter
1 cm	2 × 1 cm = 2 cm	3 + 1 cm = 4 cm	2 × 4 cm = 8 cm
10 cm	2 × 10 cm = 20 cm	3 + 10 cm = 13 cm	2 × 13 cm = 26 cm
100 cm	2 × 100 cm = 200 cm	3 + 100 cm = 103 cm	2 × 103 cm = 206 cm

5 **(a)** $AE = 20$ cm **(b)** $AC = CE = 10$ cm
(c) $AC = CE = 10$ cm
(d) $AB = BC = CD = DE = 5$ cm

6 **(a)** OA, OB, OC, OD, OE, OF
(b) AD, BE, CF

13.2 Understand how circumference is related to radius and diameter

SC 1

1 **(a)** 12.6 cm **(b)** 18.8 cm
(c) 25.1 cm **(d)** 37.7 cm
(e) When the length of the radius of a circle doubles, the circumference of the circle also doubles.
(f) For every increase in radius by 1 cm, the circumference of the circle increases by approximately 6 cm.

2 **(a)**

Radius	Diameter	Circumference	$\frac{C}{d}$	$\frac{C}{r}$
2 cm	4 cm	12.6 cm	3.15	6.3
3 cm	6 cm	18.8 cm	3.13	6.27
4 cm	8 cm	25.1 cm	3.14	6.28
6 cm	12 cm	37.7 cm	3.14	6.28

(b) The circumference is about 3.14 times as long as the diameter.
The circumference is about 6.28 times as long as the radius.

3 **(a)** The side length of the square is equal to the length of the diameter or two times the length of the radius; $2 \times r$.
(b) The side length of the hexagon is equal to the radius r of the circle.
(c) The circumference of the circle is between $3 \times d$ and $4 \times d$, so it is between $6 \times r$ and $8 \times r$.
The perimeter of the square is $4 \times d$ or $8 \times r$.
The perimeter of the hexagon is $6r$.
In order from longest to shortest: square, circle, hexagon.

4 **(a)** Diameter is $2 \times 6 = 12$ m.
Circumference is between 36 and 48 m. It could also be estimated as:
$C = 6.28 \times r$
$= 6.28 \times 6$
$= 37.68$ m
(b) Diameter is $2 \times 6.5 = 13$ m.
Circumference is between 39 and 52 m. It could also be estimated as:
$C = 6.28 \times r$
$= 6.28 \times 6.5$
$= 40.82$ m
(c) Circumference is between 18 and 24 m. It could also be estimated as:
$C = 3.14 \times d$
$= 3.14 \times 6$
$= 18.84$ m
(d) Circumference is between 19.5 and 26 m. It could also be estimated as:
$C = 3.14 \times d$
$= 3.14 \times 6.5$
$= 20.41$ m

5 **(a)** Diameter is between 7.5 and 10 m.
$d = C \div 3.14$
$= 30 \div 3.14$
$= 9.55$ m (2 d.p.)
(b) Diameter is between 8.75 and 11.7 m.
$d = C \div 3.14$
$= 35 \div 3.14$
$= 11.15$ m (2 d.p.)
(c) Diameter is between 8.88 and 11.83 m.
$d = C \div 3.14$
$= 35.5 \div 3.14$
$=11.31$ m (2 d.p.)

SC 2

1 **(a)** 3.1 **(b)** 3.141 592 65

2 **(a)** 3 **(b)** 3.14 **(c)** 3.1416
(d) As the accuracy of the measurements increases, the approximation $\frac{C}{d}$ gets closer to π.

3 The radius r is half the length of the diameter d.
$d = 2r$

4 **(a)** 3 **(b)** 3.14 **(c)** 3.1416
(d) As the accuracy of the measurements increases, the approximation $\frac{C}{d}$ gets closer to π.

5 **(a)**

Diameter	$C = \pi \times d$	$C = \frac{22}{7} \times d$
1 m	$C = 1\pi$ $= 3.14159$ m	$C = \frac{22}{7} \times 1$ $= 3.14286$ m
10 m	$C = 10\pi$ $= 31.4159$ m	$C = \frac{22}{7} \times 10$ $= 31.4286$ m
100 m	$C = 100\pi$ $= 314.159$ m	$C = \frac{22}{7} \times 100$ $= 314.286$ m
1000 m	$C = 1000\pi$ $= 3141.59$ m	$C = \frac{22}{7} \times 1000$ $= 3142.86$ m
10 000 m	$C = 10\,000\pi$ $= 31415.9$ m	$C = \frac{22}{7} \times 10\,000$ $= 31428.6$ m

(b) Answers will vary. You could talk about the margin of error. For every 1 km or 1000 m of diameter, the estimated error will be approximately 3142.86 − 3141.59 = 1.27 m.

13.3 Solve length problems involving circles

SC 1

1 **(a)** 12.57 cm **(b)** 13.35 cm **(c)** 14.14 cm

2 **(a)** 43.98 m **(b)** 46.62 m **(c)** 48.17 m

3 **(a)** $d = 50$ mm, $C = 157.08$ mm
(b) $d = 60$ mm, $C = 188.50$ mm
(c) $d = 70$ mm, $C = 219.91$ mm

4 **(a)** $r = 7.5$ m, $C = 47.12$ m
(b) $r = 9$ m, $C = 56.55$ m
(c) $r = 10.5$ m, $C = 65.97$ m

5 Answers will vary. Some students prefer to use the formula to match the information given. Other students prefer to use only one formula, for example, $C = \pi \times d$, then calculate the missing length, in this case, r, by dividing the diameter by 2.

SC 2

1 **(a)** **(i)** $d = 0.80$ cm **(ii)** $r = 0.40$ cm
(b) **(i)** $d = 7.96$ cm **(ii)** $r = 3.98$ cm
(c) **(i)** $d = 79.58$ cm **(ii)** $r = 39.79$ cm

2 **(a)** **(i)** $r = 0.40$ cm **(ii)** $d = 0.80$ cm
(b) **(i)** $r = 3.98$ cm **(ii)** $d = 7.96$ cm
(c) **(i)** $r = 39.79$ cm **(ii)** $d = 79.58$ cm

3 Given a circumference, if you need to determine the radius you could use $C = 2\pi r$.
Alternatively, you could determine the length of the diameter using $C = \pi d$, then halve d.
Some people find it easier to use a single formula for circumference and determine the missing length with their understanding that the radius is half the length of the diameter.

4 **(a)** 7.96 mm **(b)** 9.55 mm **(c)** 11.14 mm

5 **(a)** 2.39 m **(b)** 2.86 m **(c)** 3.34 m

SC 3

1 **(a)** 5.14 m **(b)** 42.85 cm

2 100.53 cm

3 **(a)** 91.42 cm **(b)** 44.60 cm
(c) 37.70 cm **(d)** 9.23 m

4 **(a)** 143.96 cm **(b)** 42.97 cm

5 368 mm

6 40.0 cm

Topic 14: Angles and transversals

Recall

I can use a protractor to measure the size of angles

1 A = 15°, B = 58°, C = 113°, D = 140°, E = 167°, F = 85°

I can identify angles by their size

1 **(a)** Smallest: A, Largest: C
(b) Smallest: B, Largest: C
(c) Smallest: A, Largest: C

2 Name each pair of angles that are the same size.
D = F, C = H, A = G, B = E

3 A acute, B reflex, C straight, D right, E acute, F reflex, G obtuse, H obtuse

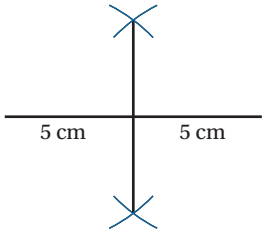
I can name angles

1 **(a)** $\angle LMN$ **(b)** $\angle EIF$ **(c)** $\angle XYZ$ **(d)** $\angle MOP$

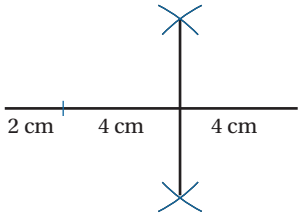
14.1 Construct perpendicular and parallel lines

SC 1

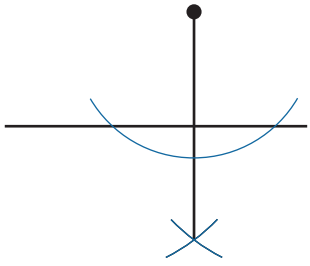
1



2



3



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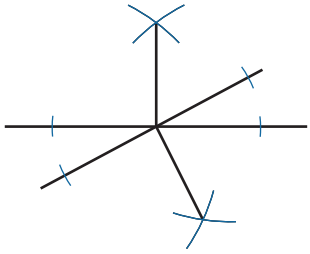
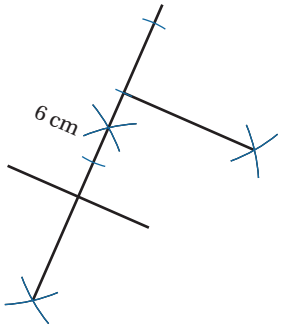


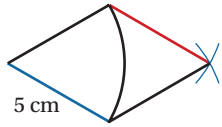
SC 2

1



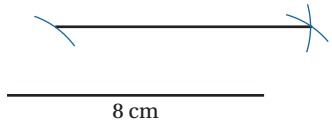
The oblique line sloping down to the right is parallel to the original line, also sloping down to the right, below, as they are both perpendicular to the line sloping down to the left.

2



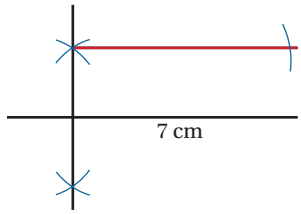
The line opposite the original line is the same length and parallel to the original line. Using the same radius for the compass throughout ensures all lengths of the quadrilateral are equal, hence a rhombus was formed.

3



The second line is the same length and parallel to the original line. Using the same radius for the compass from either end of the original line makes every point on the first two arcs equidistant from the ends, hence the lines joining the ends of the 8 cm lines will be equal. Opening the compass to 8 cm and with compass point anywhere on either arc makes the length this line 8 cm. Since both pairs of opposite sides are equal, the original line is parallel and equal to the line drawn above it.

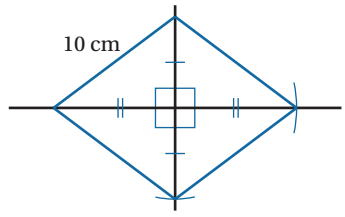
4



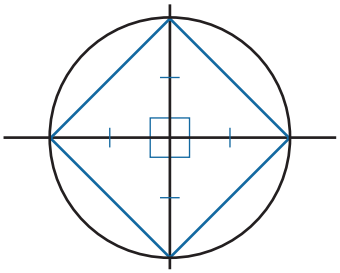
The second line is the same length and parallel to the original line. A perpendicular was constructed near one end. A distance of 7 cm was marked off along the original line. The compass was opened so the radius matched the distance along the perpendicular line to the intersection of arcs. An arc was made above the other end of the 7 cm line segment. With the compass, opened at 7 cm and the compass point on the upper intersection on the left, the arc intersecting the one at the right gives the final vertex for a rectangle.

SC 3

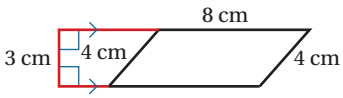
1 The diagonals of a rhombus bisect each other at right angles.



2 The diagonals of a square are equal in length and bisect each other at right angles.

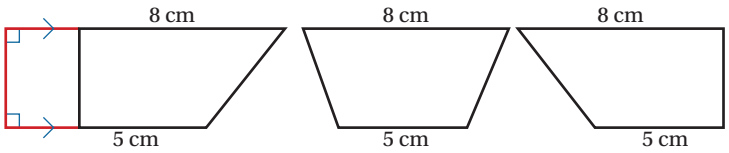


3



The final side length did not need to be measured.

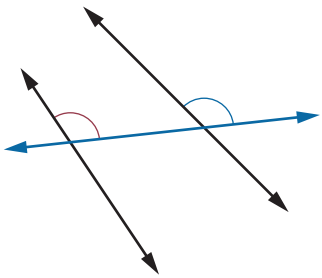
4



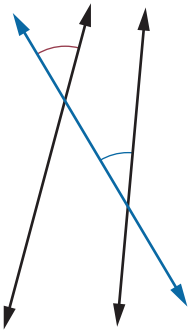
14.2 Identify corresponding, alternate and co-interior angles

SC 1

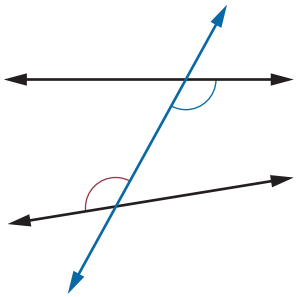
1 (a)



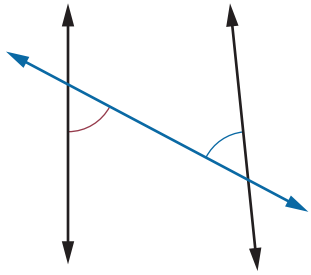
(b)



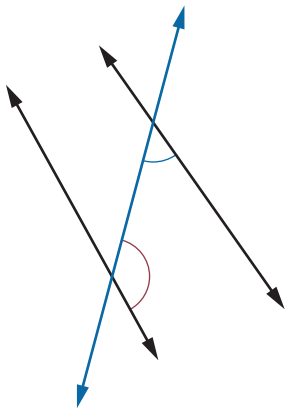
2 (a)



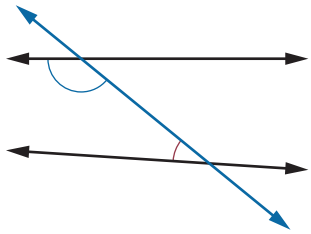
(b)



3 (a)



(b)



4 (a) e and i **(b)** f and j
(c) e and i **(d)** h and j

SC 2

1 (a) Angle b is alternate to angle d.
(b) Angle d is vertically opposite to angle f.
(c) Angle e is co-interior to angle b.
(d) Angle b is corresponding to angle f.

2 Explanations for angle sizes may vary.

- a is 110° because it makes a straight angle with 70°.
- b is 70° because it is vertically opposite to 70°, or because it makes a straight angle with a.
- c is 110° because it is vertically opposite to a, or because it makes a straight angle with 70°.
- d is 70° because it is corresponding to 70°, or because it is co-interior to c.
- e is 110° because it is corresponding to a, or because it is co-interior to e, or because it is vertically opposite to 110°.
- f is 70° because it is vertically opposite to d, or because it is supplementary to a.

3 (a) Corresponding angles on parallel lines are equal.
(b) Co-interior angles on parallel lines are supplementary.

(c) Alternate angles on parallel lines are equal.
(d) Vertically opposite angles are equal.
(e) Adjacent angles on a straight line are supplementary.

4 $p = 147°$; $g = 33°$
5 $p = 31°$
6 $m = 61°$
7 $m = 95°$
8 $b = 124°$
9 The lattice has an acute angle of 58°.

14.3 Determine the exterior angle of a triangle

SC 1

1 $a = 63°$, $b = 67°$, $c = 50°$
Exterior angle: $a + b = 67° + 63°$, hence the exterior angle of a triangle is equal to the sum of the interior opposite angles.

2 $q = 60°$, $p = 45°$, $r = 45°$
Exterior angle: $p + q = r + 60°$, hence the exterior angle of a triangle is equal to the sum of the interior opposite angles.

3 $x = 50°$, $d = 40°$, $c = 90°$, $e = 90°$, $f = 50°$, $a = 40°$, $b = 50°$
Exterior angles: $c + d = 90° + 40°$, $e + f = 90° + x$, $a + b = 40° + x$;
hence the exterior angle of a triangle is equal to the sum of the interior opposite angles.

4 $b = 50°$, $a = 60°$, $c = 70°$, $s = 50°$, $q = 60°$, $p = 70°$, $r = 70°$
Exterior angles: $p + 60° = c + a$, $q + 50° = a + b$, $r + s = c + b$;
hence the exterior angle of a triangle is equal to the sum of the interior opposite angles.

SC 2

1 **(a)** $p = 155°$ **(b)** $p = 126°$ **(c)** $p = 133°$
2 **(a)** $x = 82°$ **(b)** $x = 41°$
3 $x = 46°$, $y = 92°$
4 **(a)** $a = 91°$, exterior angle of a triangle; $b = 65°$ and $c = 51°$, vertically opposite angles
(b) $d = 9°$, adjacent angles forming a line; $e = 99°$ and $f = 131°$, exterior angle of a triangle

14.4 Calculate the internal angle sum of a triangle

SC 1

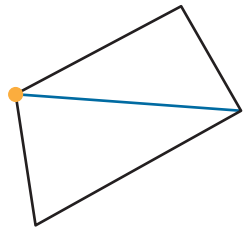
1 $x = 115°$ and $y = 22°$, alternate angles on parallel lines.
$z = 43°$, a straight angle is 180°.
The three angles of the triangle are the same as the three angles forming a line, so the angle sum of a triangle is 180°.

2 **(a)** $a = 35°$ **(b)** $b = 11°$ **(c)** $c = 127°$
3 **(a)** $x = 55°$, vertically opposite angles;
$y = 180 - 2 \times 55 = 70°$, isosceles triangle and angle sum of a triangle.
(b) $y = 180 - 60 = 120°$, equilateral triangle and adjacent angles on a line; $x = 180 - 120 - 28 = 32°$, angle sum of a triangle.
Alternatively, using the exterior angle rule:
$y = 60 + 60 = 120°$ and $60° = x + 28$,
so $x = 60 - 28 = 32°$
(c) $y = 60°$, equilateral triangle; $2x = 180 - 44 = 136°$, so $x = 68°$, isosceles triangle and angle sum of a triangle; $z = 360 - 60 - 68 = 232°$, revolution.

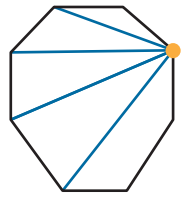
4 $2a + 146 = 180°$, isosceles triangle and angle sum of a triangle, so $a = 17°$.
Similarly: $2g + 50 = 180°$, $g = 65°$;
$b = 90 - 17°$, adjacent angles forming a right angle.
so $b = 73°$
By symmetry: $c = 73°$
Similarly: $f = 90 - 65°$, so $f = 25°$
$d + e = 360 - 146 - 50 = 164°$
By symmetry (congruent triangles): $d = e$,
so $d = e = \frac{164}{2} = 82°$

SC 2

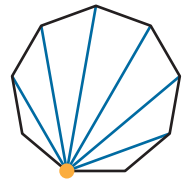
1 **(a)** **(i)**



(ii)

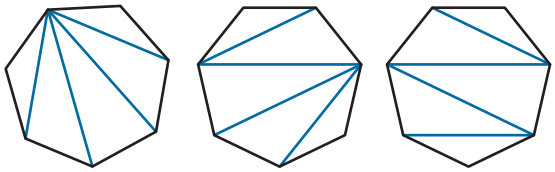


(iii)



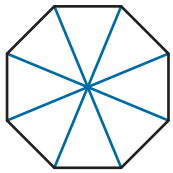
(b) **(i)** Name of polygon: quadrilateral
Number of sides (or angles): 4
Number of triangles: 2
(ii) Name of polygon: octagon
Number of sides (or angles): 8
Number of triangles: 6
(iii) Name of polygon: nonagon
Number of sides (or angles): 9
Number of triangles: 7
(c) The number of triangles is always 2 fewer than the number of sides.

2 **(a)** Possible diagrams:



(b) Always 5. **(c)** $180° \times 5 = 900°$

3 **(a)**



(b) 8 triangles

(c) 360° around the centre point: one revolution.

(d) $180° \times 8 - 360° = 1080°$

(e) $\frac{1080°}{8} = 135°$

4 **(a)** $180° \times 3 = 540°$

(b) $\frac{540°}{5} = 108°$

(c) $180° - 108° = 72°$

SC 3

1 **(a)** Angle sum: 540°, $p = 81°$

(b) Angle sum: 720°, $q = 118°$

2 **(a)** Angle sum: 540°, $x = 108°$

(b) Angle sum: 1080°, $x = 67°$

3 **(a)** $\frac{8 \times 180°}{10} = 144°$

(b) $\frac{5 \times 180°}{7} = 128\frac{4}{7}°$ or 129° to the nearest degree

(c) $\frac{2 \times 180°}{4} = 90°$ (as expected for a square)

4 **(a)** Interior: $\frac{4 \times 180°}{6} = 120°$; exterior: $180 - 120 = 60°$

(b) Interior: $\frac{6 \times 180°}{8} = 135°$; exterior: $180 - 135 = 45°$

(c) Interior: $\frac{2 \times 180°}{4} = 90°$; exterior: $180 - 90 = 90°$ (as expected for a square)

Topic 15: Ratios and measurement

Recall

I can group and count items by common attributes

1

Yellow speckled	Purple speckled	Green striped
5	5	4

I can identify the highest common factor for a set of numbers

1 **(a)** 4 **(b)** 3 **(c)** 4

I can use the HCF to simplify fractions

1 **(a)** $\frac{3}{5}$ **(b)** $\frac{3}{4}$ **(c)** $1\frac{1}{5}$ **(d)** $1\frac{3}{5}$

I can write equivalent fractions

1 **(a)** $\frac{24}{\boxed{104}} = \frac{3}{13}$ **(b)** $\frac{7}{9} = \frac{\boxed{14}}{18}$

(c) $\frac{\boxed{2}}{3} = \frac{22}{33}$ **(d)** $\frac{16}{28} = \frac{4}{\boxed{7}}$

I can express mixed numbers as improper fractions

1 **(a)** $\frac{7}{3}$ **(b)** $\frac{23}{4}$ **(c)** $\frac{30}{7}$ **(d)** $\frac{53}{6}$

15.1 Solve practical problems involving ratios of length

SC 1

1 **(a)** 2 **(b)** 3 **(c)** 5 **(d)** 10

2 **(a)** 1.1 **(b)** 1.2 **(c)** 1.5 **(d)** 1.9

3 **(a)** 0.5 **(b)** 0.1 **(c)** 0.2 **(d)** 0.25

4 **(a)** reduction, scale factor 0.5

(b) reduction, scale factor 0.25

(c) enlargement, scale factor 2

(d) enlargement, scale factor 4

(e) reduction, scale factor 0.5

(f) enlargement, scale factor 2

5 **(a)** 1.29 : 1

(b) The Gascoyne River is 1.29 times as long as the Goulburn River.

6 **(a)** 1808 km : 700 km = 2.58 : 1 (correct to 2 decimal places)

(b) The Murray River in New South Wales is 2.58 times as long as the Murray River in South Australia.

7 **(a)** 3.71 : 1.6 = 2.32 : 1 (2 d.p.)

(b) 2558 : 2640 = 0.97 : 1 (2 d.p.)

8 **(a)** 1.33 **(b)** 1.26 **(c)** 2.6

9 2.38

SC 2

1 **(a)** 3.14 **(b)** 3.142 **(c)** 3.2

2 **(a)** 6.3 **(b)** 6.28 **(c)** 6.283

3 **(a)** 22 : 7 **(b)** $\frac{22}{7}$

4 **(a)** 62.8 cm **(b)** 7.85 m

(c) 81.64 cm **(d)** 144.44 mm

5 **(a)** 220 mm **(b)** 17.6 cm
(c) 660 mm **(d)** 13.2 m

6 **(a)** $D = 15.92$ cm, $r = 7.96$ cm
(b) $D = 44.56$ mm, $r = 22.28$ mm
(c) $D = 0.80$ m, $r = 0.40$ m

SC 3

1 **(a)** **(i)** 80 cm **(ii)** 120 cm
(iii) 160 cm **(iv)** 200 cm
(b) **(i)** 37.5 cm **(ii)** 40 cm
(iii) 42.5 cm **(iv)** 45 cm
(c) As the length of the femur increases by 10 cm, the height of the person increases by $10 \times 4 = 40$ cm. As the height of the person increases by 10 cm, the length of the femur increases by $10 \div 4 = 2.5$ cm.

2 **(a)** **(i)** 46.2 cm **(ii)** 136.29 mm
(iii) 3.696 m
(b) **(i)** 1.39 m **(ii)** 15.58 cm
(iii) 129.87 mm

3 **(a)** **(i)** 25 cm **(ii)** 31 cm
(b) **(i)** 151 cm **(ii)** 178 cm

4 **(a)** 61 cm **(b)** 157 cm

5 **(a)**

Scale diagram	Original
5	8
15 cm	24 cm
18.75 cm	30 cm

(b)

Scale diagram	Original
20	11
10.91 m	6 m
12 cm	6.6 cm

15.2 Explore ratios in measured quantities

SC 1

1 **(a)** 3 times the side lengths of the original prism
(b) $3^2 = 9$ times the area of the face of the original prism
(c) $3^3 = 27$ times the volume of the original prism

2 **(a)** $\frac{1}{3}$ times the side lengths of the original prism
(b) $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ times the area of the face of the original prism
(c) $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$ times the volume of the original prism

3 **(a)** enlargement $\frac{6}{5} = 1.2$
(b) The short side length enlarges to $1.2 \times 5 = 6$ cm, the other side length enlarges to $1.2 \times 10 = 12$ cm.
(c) 50 cm^2 **(d)** 72 cm^2
(e) 250 cm^3 **(f)** 432 cm^3

4 **(a)** reduction: $\frac{4}{5} = 0.8$
(b) The short side length reduces to $0.8 \times 5 = 4$ cm, the other side length reduces to $0.8 \times 10 = 8$ cm.
(c) 32 cm^2 **(d)** 128 cm^3

5 **(a)** Base: 16.8 cm; height: 12.6 cm
(b) Original: 54 cm^2; enlarged: 105.84 cm^2
(c) 1.96
(d) $1.4^2 = 1.96$. The ratio for area is the square of the ratio for length.

6 **(a)** 4.8 cm × 6.4 cm × 9.6 cm
(b) 108 cm^2 **(c)** 276.48 cm^2
(d) 2.56 **(e)** 72 cm^3
(f) 294.912 cm^3 **(g)** 4.096
(h) $1.6^2 = 2.56$. The surface area ratio is the square of the length ratio. $1.6^3 = 4.096$. The volume ratio is the cube of the length ratio.

SC 2

1 **(a)** 200 mL
(b) Any amount less than 200 mL.
Possible amount: 100 mL.

2 **(a)** Red to blue ratios are $5 : 9 = 0.55\ldots$ and $2 : 3 = 0.66\ldots$ so the second one is more maroon.
(b) $\frac{3}{5}$ cup

3 **(a)** $\frac{2}{5}$ **(b)** 400 mL **(c)** 7.5 L

4 **(a)** $6\frac{1}{4}$ cups of self-raising flour, 150 g of butter and $1\frac{7}{8}$ cups of milk
(b) **(i)** 72 g of butter and $\frac{9}{10}$ cup of milk
(ii) 14.4, so 14 scones

Topic 16: Visualising 3D

Recall

I can identify faces, vertices and edges of solids

1 **(a)** Faces: 5 Edges: 9 Vertices: 6
(b) Faces: 6 Edges: 12 Vertices: 8
(c) Faces: 4 Edges: 6 Vertices: 4
(d) Faces: 5 Edges: 8 Vertices: 5

2 **(a)** CD **(b)** JL **(c)** LO
(d) EF **(e)** MN

I can connect the cross-section shape to its matching right-prism

1 **(a)** $1 \rightarrow D$ **(b)** $2 \rightarrow E$ **(c)** $3 \rightarrow A$
(d) $4 \rightarrow C$ **(e)** $5 \rightarrow B$

16.1 Visualise and apply nets

SC 1

1 **(a)** Triangular prism
(b) Cube

2 **(a)** Pentagonal pyramid
(b) Triangular pyramid

3 The two nets will form the same square-based prism. The bases fold in from different directions in the second net.

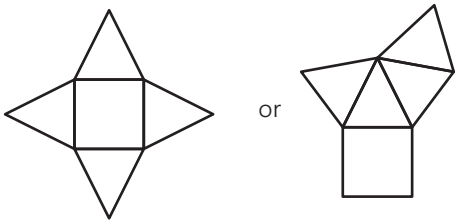
4 **(a)** The ends of the rectangles next to the base must be the same length if they are to meet.
(b) The sides of the triangles that would meet are of different lengths, so they won't match. The little triangles are not high enough to reach above the base.
(c) The square bases fit the same end of the prism.

5 **(a)** Net 2 **(b)** Net 3
(c) Net 1 **(d)** Net 3

6 **(a)** A cylinder **(b)** A cone

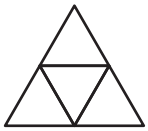
SC 2

1

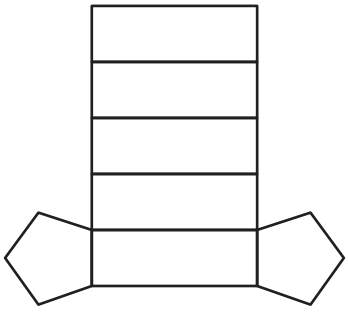


2 The height of the triangles would be greater.

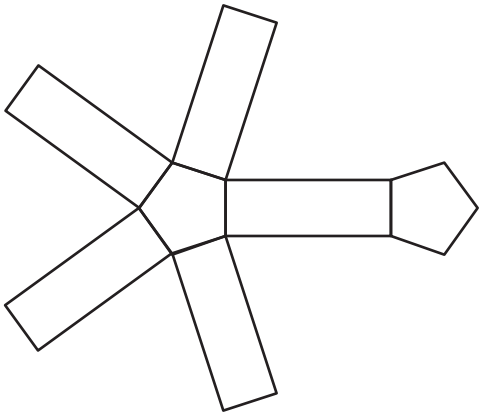
3



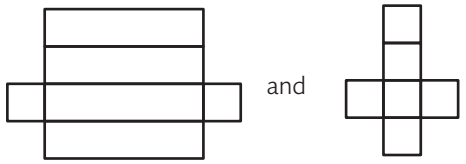
4



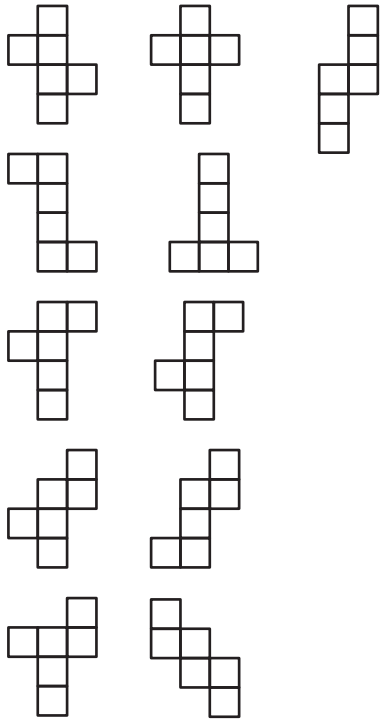
or



5



6 Any three of the following is correct.



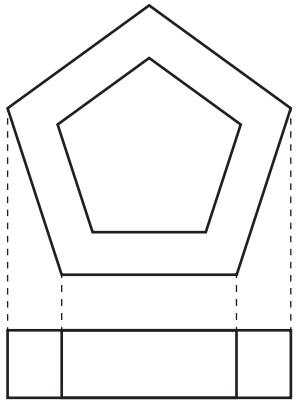
16.2 Visualise and draw the perspective views of a 3D object

SC 1

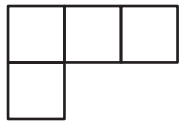
1 Front view: octagon; Footprint: isosceles triangle; Side view: isosceles triangle

2 **(a)** Front view: isosceles triangle; Footprint: equilateral triangle; Side view: isosceles triangle
(b) Front view: rectangle; Footprint: circle; Side view: rectangle

3

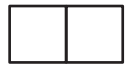


4 **(a)**



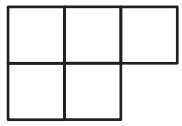
top view

front view

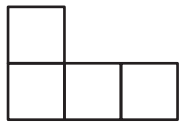


side view

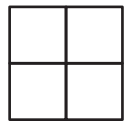
(b)



top view

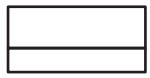


front view



side view

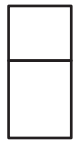
5 **(a)**



front view

side view

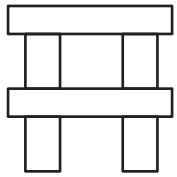
(b)



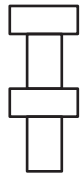
front view

side view

(c)



front view



side view

SC 2

1 **(a)** A tall square-based prism with a narrower rectangular prism in front, same height, not as wide and very little depth. On top of the smaller prism is a much smaller rectangular prism structure.

(b) A rectangular prism at the bottom with modest height. On top is an isosceles-triangular prism, same depth as the bottom, but height is only a fraction of the height of the bottom part.

(c) A pair of triangular prisms on top of rectangular prisms. The rectangular prism on the left has double the height, greater width and depth than the one on the right. Both of the triangular prisms are not very high and both match the depth of the rectangular prism below them.

2 **(a)** Bottom has a rectangular prism, very wide, not very high or deep. Three identical tall, wide rectangular prisms above them spaced out with gaps. Along the top is a horizontal, very wide possibly square-based prism, same depth as the prisms below. This prism is slightly less wide than the bottom prism.

(b) Bottom is a rectangular prism, not very high. Above is an isosceles triangular prism with the base of the triangle matching the depth of the house and depth of the prism matching the width of the bottom prism. The height of the triangle is higher than the height of the bottom prism. There are two extrusions from the roof. The bottom part of each is a right-triangular prism and the top of each is an isosceles triangular prism that has been truncated by the angle of the roof.

3 Bottom: 6 units × 2 units × 1 unit
Top, back: 4 units × 1 unit × 1 unit
Top, front: 2 units × 1 unit × 1 unit

4 **(a)** Two sides: 4 units × 2 units × 1 unit;
Middle: 2 units × 1 unit × 1 unit.

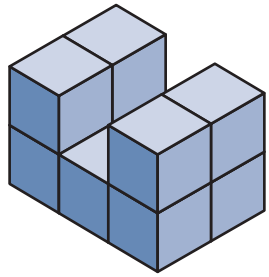
(b) Front: 4 units × 1 unit × 1 unit;
Back: 6 units × 1 unit × 1 unit;
Two sides of 2 units × 2 units × 1 unit.

16.3 Draw and build objects made from cubes

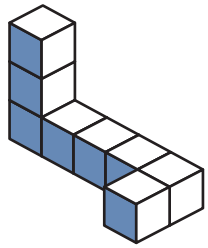
SC 1

1 Student constructions should look the same as the diagrams.

2



3 **(a)**



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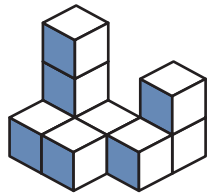

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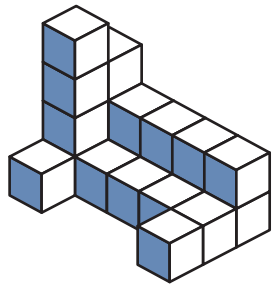
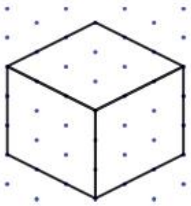


SC 2

1 (a)



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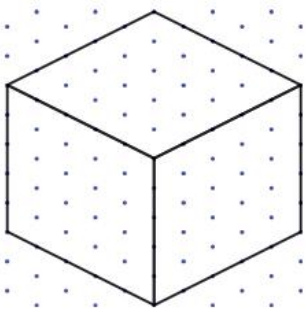
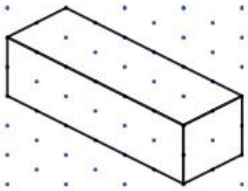


2 (a)



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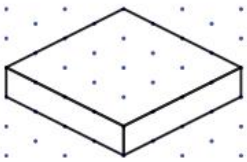
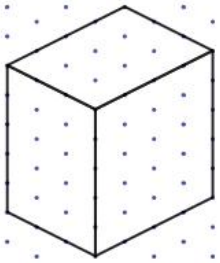
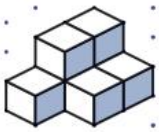


(c)

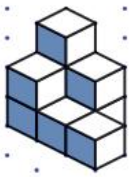


3 (a)

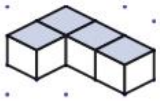
(b)



(c)



4 (a)



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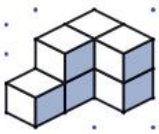
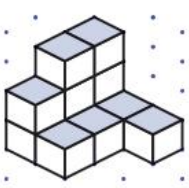


(c)

5 (a)

3	2	2	2	2
4	1	1	1	1
1				1

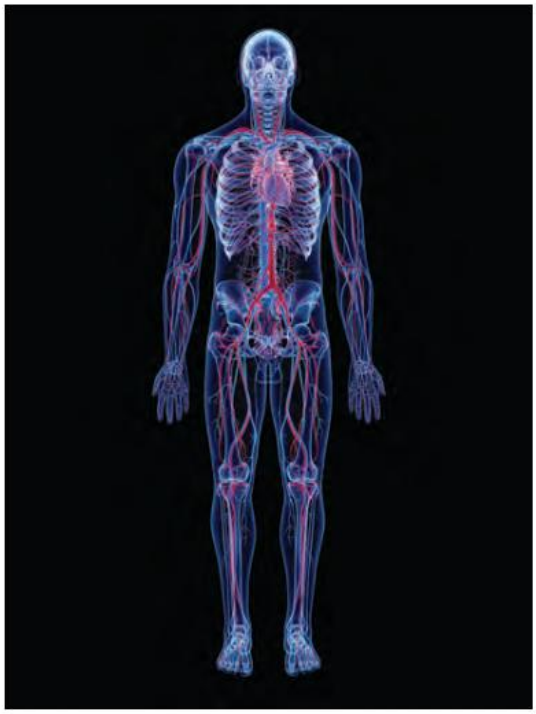
(b)



16.4 Draw and use objects represented in art form

SC 1

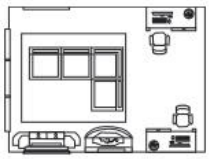
1 (a)



(b)

2

3



4 Student responses will vary.

SC 2

1 (a) The paths are laid out in plan view, but the animals are side or front view.

(b) In real life, neither the paths nor the animals would be so big compared with the enclosures.

(c) Toilets and other facilities.

2 Student answers will vary, but should include symbols used for rivers and waterholes.

3 **(a)** Chatswood, Sydenham, Cabramatta, Warwick Farm, Liverpool, Glenfield, Parramatta, Harris Park, Strathfield.

(b) Take any line from Saint James and change at Wynyard or Central to take the North Shore line to North Sydney.

(c) The Eastern Suburbs line goes under 3 other lines between Martin Place and Kings Cross. (For anyone who's caught a train at Martin Place, they'll remember how deep underground you need to go to the platform.)

Topic 17: Triangles and quadrilaterals

Recall

I can identify triangles and quadrilaterals

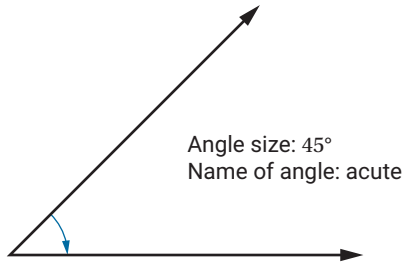
1 **(a)** A triangle is a 2D shape that has 3 sides and 3 internal angles.

(b) A quadrilateral is a 2D shape that has 4 sides and 4 internal angles.

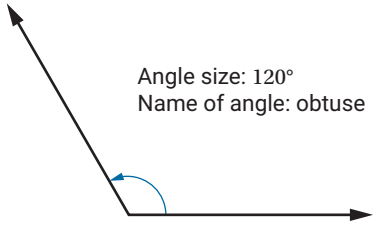
(c) Squares and rectangles are special types of quadrilaterals.

I can use a protractor to measure and name angles

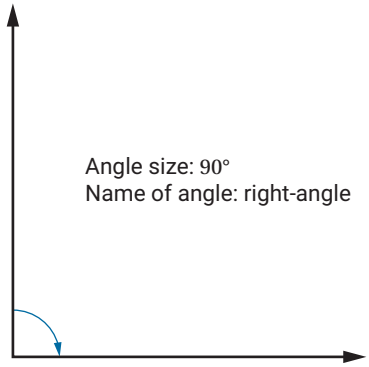
1 **(a)**



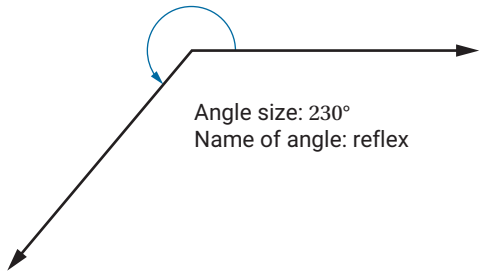
(b)



(c)



(d)



17.1 Construct triangles and quadrilaterals

Exploration Task: Properties of shapes

The statements are: true, false and false.

SC 1

1 **(a)** Rigid equilateral triangle

(b) No triangle possible, because the total of the shorter strips needs to be greater than the longest strip.

(c) Rigid obtuse-angled isosceles triangle

(d) No triangle possible because the total of the shorter strips was less than the longest strip.

(e) Rigid right-angled triangle

(f) Rigid acute-angled isosceles triangle

2 The sum of any two sides of a triangle must be more than the length of the third.

3 **(a)** A triangle can be formed.

(b) A triangle cannot be formed.

(c) A triangle can be formed.

(d) A triangle cannot be formed.

(e) A triangle can be formed.

4 **(a)** A triangle can be formed.

(b) A triangle cannot be formed.

(c) A triangle can be formed.

(d) A triangle cannot be formed.

(e) A triangle cannot be formed.

(f) A triangle can be formed.

(g) A triangle can be formed.

SC 2

1 The following are approximate answers.

(a) 82° **(b)** 90° **(c)** 96°

(d) 60° **(e)** 71°

2 Angle opposite 25 m: 34°; angle opposite 35 m: 51°; angle opposite 45 m: 96°.

The angles may add up to more than 180° due to rounding to the nearest degree.

3 **(a)** 71°, 90°, 112°, respectively.

(b) The three triangles had two sides the same, with the largest side varying. As the largest side increased, the angle opposite increased. The triangles were acute-angled, right-angled and obtuse-angled, respectively.

4 130° or 50°

SC 3

1 **(a)** Flexible rhombus (or square)
(b) No quadrilateral possible because the total of the three shorter strips needs to be more than the longest strip.
(c) Flexible parallelogram (or rectangle)
(d) Flexible kite
(e) Flexible quadrilateral
2 In all cases, the shape was then fixed.
3 **(a)** 38° **(b)** 60° **(c)** 75°
4 10.8 cm
5 11.7 cm
6 114 cm

Exploration Task: Properties of triangles
There are 12 triangles in total.

17.2 Classify types of triangles by their side and angle properties

SC 1

1 **(a)** Scalene triangle **(b)** Equilateral triangle
(c) Isosceles triangle **(d)** Isosceles triangle
(e) Equilateral triangle **(f)** Scalene triangle
(g) Isosceles triangle
2 **(a)** Equilateral triangle **(b)** Scalene triangle
(c) Isosceles triangle **(d)** Scalene triangle
(e) Isosceles triangle
(f) Scalene or isosceles triangle
(g) Scalene or isosceles or equilateral triangle
(h) Scalene or isosceles triangle
3 **(a)** Scalene triangle **(b)** Isosceles triangle
(c) Scalene triangle **(d)** Scalene triangle
(e) Equilateral triangle **(f)** Isosceles triangle
4 **(a)** Isosceles triangle **(b)** Scalene triangle
(c) Isosceles triangle **(d)** Isosceles triangle
5 **(a)** Scalene triangle **(b)** Scalene triangle
(c) Scalene triangle **(d)** Isosceles triangle
6 **(a)** Isosceles triangle **(b)** Scalene triangle
(c) Isosceles triangle

SC 2

1 **(a)** Acute-angled triangle
(b) Acute-angled triangle
(c) Right-angled triangle
(d) Acute-angled triangle
(e) Obtuse-angled triangle
(f) Obtuse-angled triangle
2 **(a)** Acute-angled triangle
(b) Right-angled triangle
(c) Obtuse-angled triangle
(d) Right-angled triangle
(e) Obtuse-angled triangle
(f) Acute-angled triangle
3 **(a)** Both are right-angled triangles
(b) Two obtuse-angled triangles and two acute-angled triangles
4 **(a)** Both are obtuse-angled triangles
(b) Both are acute-angled triangles
(c) All are right-angled triangles

SC 3

1 **(a)** Isosceles right-angled triangle
(b) Scalene obtuse-angled triangle
(c) Isosceles obtuse-angled triangle
(d) Scalene acute-angled triangle
2 **(a)** Equilateral triangle
(b) Scalene right-angled triangle
(c) Scalene acute-angled triangle
(d) Isosceles acute-angled triangle
3 **(a)** Scalene right-angled triangle
(b) Isosceles obtuse-angled triangle
(c) Isosceles acute-angled triangle
(d) Scalene acute-angled triangle
4 **(a)** Isosceles obtuse-angled triangle
(b) Scalene obtuse-angled triangle
(c) Scalene right-angled triangle
5 **(a)** Both isosceles right-angled triangles
(b) All isosceles right-angled triangles
6 **(a)** Both isosceles obtuse-angled triangles
(b) All scalene right-angled triangles
7 **(a)** The largest angle is 128° so this is a scalene obtuse-angled triangle.
(b) The side opposite 50° is 6.2 cm, so this is very nearly isosceles, and visually obvious as acute angled. It is an acute-angled scalene triangle.

17.3 Classify types of quadrilaterals by their side and angle properties

SC 1

1 **(a)** A square **(b)** A trapezium
(c) A rectangle **(d)** A rhombus
2 **(a)** A parallelogram, rectangle, square, rhombus
(b) A rhombus **(c)** A kite
3 **(a)** A square or rhombus
(b) A parallelogram, rhombus, square
(c) A square
(d) A rhombus
(e) A parallelogram or rhombus
4 **(a)** A square
(b) A trapezium (isosceles, or symmetrical)
(c) A kite
(d) A rhombus
(e) No special name

SC 2

1 **(a)** Both pairs of opposite sides equal and parallel.
(b) The rectangle has all angles equal/right angles.
(c) A rectangle is a parallelogram with one angle a right angle.

2 A rhombus has all the properties of a parallelogram, both pairs of opposite sides parallel, but the rhombus has the additional property of all sides being equal. Parallelograms that do not have this property are not rhombuses.

3 Yes. Parallelograms have all of the properties of a trapezium as well as the pair of parallel sides being equal (or the other pair of opposite sides being parallel as well).

4 The property that makes a rhombus special is that all sides are equal. The property that makes a rectangle special is that all angles are right angles. The properties in common with a parallelogram are the same, but the other special properties are different.

5 **(a)** A square, a rectangle, a rhombus, a parallelogram
(b) A parallelogram

6 **(a)** A rhombus **(b)** A square

SC 3

1 **(a)** Convex pentagon **(b)** Concave octagon
(c) Convex hexagon **(d)** Concave dodecagon

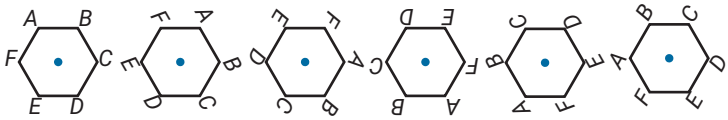
2 **(a)** Triangle: none; Quadrilateral: 2; Pentagon: none; Hexagon: 3; Heptagon: none; Octagon: 4
(b) Odd number of sides: none; even number of sides, n: $\frac{n}{2}$ pairs
(c) **(i)** None **(ii)** 6

3 **(a)** Irregular convex polygon
(b) Irregular convex polygon
(c) Irregular convex polygon
(d) Concave polygon

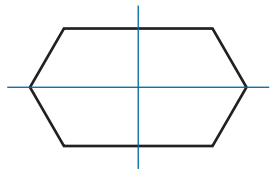
4 For quadrilaterals, pentagons, and so on, it is possible for all the side lengths to be the same, yet the angles can be variable. The rhombus is an example of a quadrilateral with equal sides, but angles of two different sizes.

5 **(a)** Each line of symmetry goes through a vertex and the midpoint of the side opposite, at right angles.
(b) Half of the lines of symmetry go through a pair of opposite vertices. The other half go through midpoints of opposite (parallel) sides at right angles.

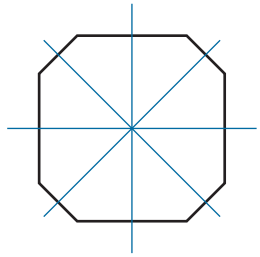
6 **(a)** 6
(b)



7 **(a)**



(b)



8 **(a)** 540° **(b)** 108°
(c) Possible angles (totalling 540°): 200°, four of 85°

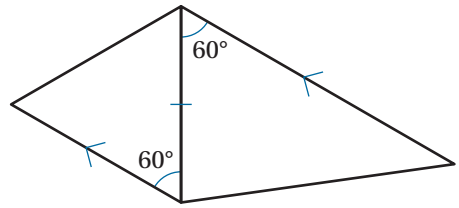
17.4 Use parallel side lengths of quadrilaterals to construct rectangles of equivalent area

SC 1

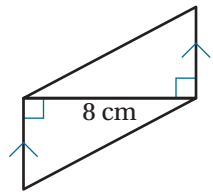
1 The first two, **(a)** and **(b)**, could form a quadrilateral by sharing the 5 cm side as the diagonal. The side orientation for the quadrilateral could be: 3 cm, 4 cm, 7 cm, 8 cm or 3 cm, 4 cm, 8 cm, 7 cm.

2 **(a)** $\angle ABD = \angle CDB$ or $\angle ADB = \angle CBD$
(b) $\angle BAC = \angle DCA$ or $\angle DAC = \angle BCA$
(c) $\angle ABD = \angle CDB$

3 **(a)** Trapezium



(b) Trapezium



4 **(a)** Longest sides together: a square; shorter sides together: a parallelogram
(b) The longest side of the smaller triangle must match a smaller side of the larger triangle. Two 45° angles form a right angle, so the shape is a trapezium with two right-angled vertices.

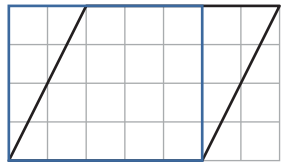
5 Longest sides together: a rectangle; shortest sides together: a long, narrow parallelogram; medium sides together: a parallelogram that has one short pair of parallel sides.

6 **(a)** Parallelograms. If the 8 cm sides are together, then the 6 cm sides must be opposite each other and the 7 cm sides must be opposite each other. The shape has both pairs of opposite sides equal, so it is a parallelogram. Also, the angles opposite the 6 cm sides form the Z shape to prove that opposite sides are parallel. Same explanation if the 6 cm or 7 cm sides are placed together.

(b) Kites. If the 8 cm sides are together, then the 6 cm sides must be next to each other and the 7 cm sides must be next to each other. The shape has two pairs of adjacent sides equal, so it is a kite. Also, the angles opposite the 8 cm sides are equal, confirming the symmetry. Same explanation if the 6 cm or 7 cm sides are placed together.

SC 2

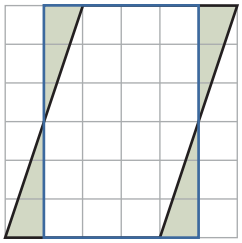
1



A

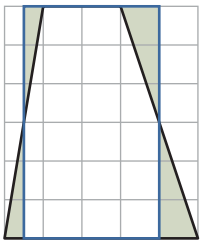
The non-included triangle to the right of the rectangle has been removed from the parallelogram and the new included triangle to the left of the parallelogram is the same size and shape and hence the same area. The areas of the parallelogram and rectangle are the same because equal areas have been added and subtracted.

2 **(a)** **(i)** and **(ii)**



(iii) The width of the rectangle is the same as the base length of the parallelogram and the height of the rectangle is the same as the height of the parallelogram.

(b) **(i)** and **(ii)**



(iii) The width of the rectangle (3.5 units) is the average of the parallel side lengths (2 units and 5 units) of the trapezium since $\frac{2+5}{2} = 3.5$ and the height of the rectangle is the same as the height of the trapezium.

3 Top rectangle: 4 units by 5 units;
Bottom rectangle: 4.5 units by 2 units

4 Approximately 6.3 units by 8 units

SC 3

1 **(a)** 5 cm **(b)** 18 cm **(c)** 48.5 mm

2 **(a)** 2 m × 1.3 m **(b)** 39 cm × 32 cm
(c) 31.5 mm × 16 mm

3 **(a)** 300 mm^2 **(b)** 52 cm^2 **(c)** 2100 mm^2

4 **(a)** 1038 cm^2 **(b)** 30 cm × 34.6 cm

5 83 cm^2

Topic 18: Translations, reflections, rotations

Recall

I can identify transformations of plane shapes

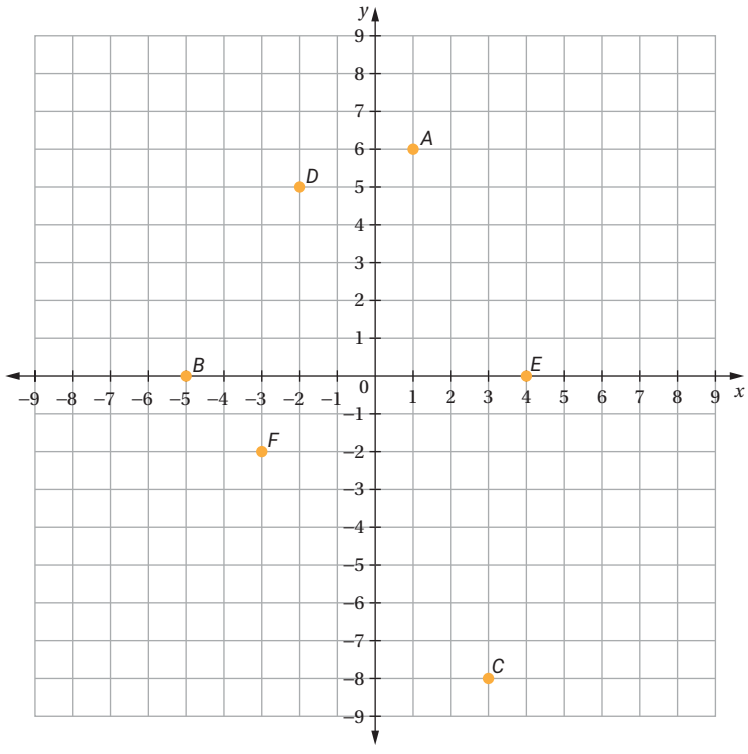
1 **(a)** translation **(b)** rotation **(c)** reflection

2 **(a)** rotation **(b)** reflection **(c)** translation

I can read and plot points on a Cartesian plane

1 $A(3, 1)$, $B(-3, 3)$, $C(-4, -6)$, $D(0, 0)$, $E(6, -3)$, $F(0, 7)$ and $G(-7, 0)$

2



(a) $A(1, 6)$ **(b)** $B(-5, 0)$ **(c)** $C(3, -8)$

(d) $D(-2, 5)$ **(e)** $E(4, 0)$ **(f)** $F(-3, -2)$

18.1 Translate and reflect points and shapes on a Cartesian plane

SC 1

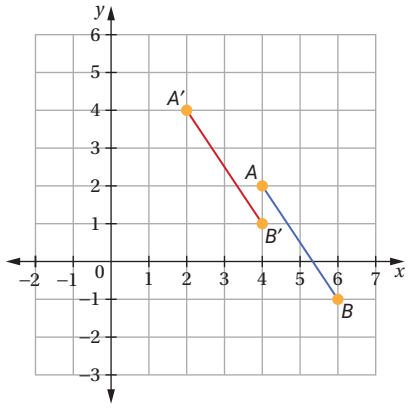
1 **(a)** Add 1 to both the x and y-coordinates.
(b) Add 1 to the x-coordinate and subtract 1 from the y-coordinate.
(c) Subtract 1 from both the x and y coordinates.
(d) Subtract 1 from the x-coordinate and add 1 to the y-coordinate.

2 **(a)** [2, 3] **(b)** [2, −3]
(c) [−2, 3] **(d)** [−2, −3]

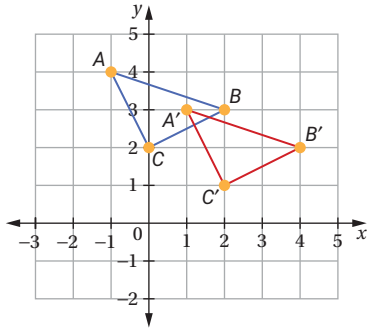
3 **(a)** (5, 7) **(b)** (5, 3)
(c) (−3, 7) **(d)** (−3, 2)

4 **(a)** $A'(6, 5)$ **(b)** $B'(-1, -1)$
(c) $C'(-1, 11)$ **(d)** $D'(-1, 2)$
(e) $E'(-1, 1)$ **(f)** $F'(-7, 0)$

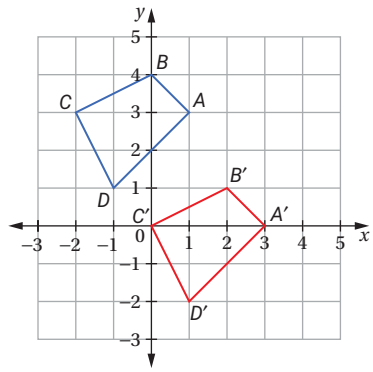
5 **(a)** $A'(2, 4)$, $B'(4, 1)$
(b)



6 **(a)** $A'(1, 3)$, $B'(4, 2)$, $C'(2, 1)$
(b)



7 **(a)** $A'(3, 0)$, $B'(2, 1)$, $C'(0, 0)$, $D'(1, -2)$
(b)



SC 2

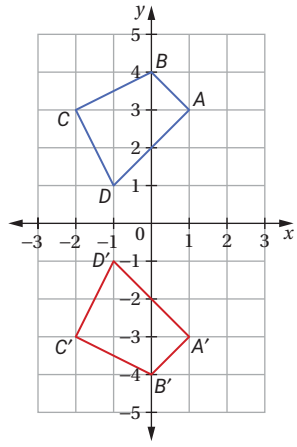
1 The x-coordinate remains the same and the y-coordinate is multiplied by −1.

2 The x-coordinate is multiplied by −1 and the y-coordinate remains the same.

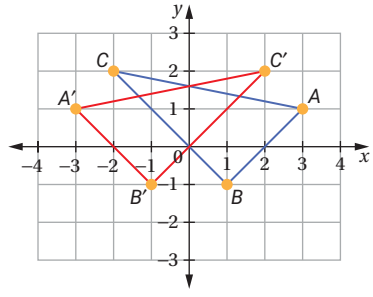
3 **(a)** (4, −5) **(b)** (−4, 5)

4 **(a)** $A'(6, -4)$ **(b)** $B'(-1, -1)$
(c) $C'(2, 3)$ **(d)** $D'(-5, 0)$
(e) $E'(1, -3)$ **(f)** $F'(-7, 0)$

5 **(a)** $A'(1, -3)$, $B'(0, -4)$, $C'(-2, -3)$, $D'(-1, -1)$
(b)



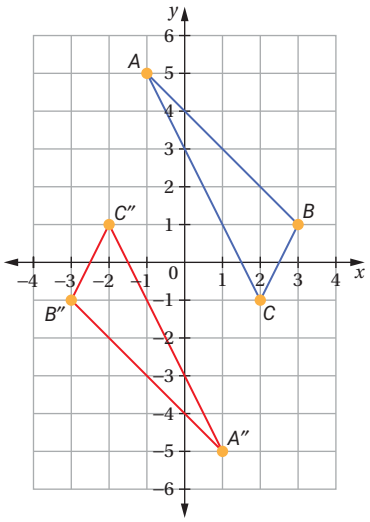
6 **(a)** $A(3, 1) \rightarrow A'(-3, 1)$, $B(1, -1) \rightarrow B'(-1, -1)$, $C(-2, 2) \rightarrow C'(2, 2)$
(b)



7 **(a)** $A'(-1, -5)$, $B'(3, -1)$, $C'(2, 1)$

(b) $A''(1, -5)$, $B''(-3, -1)$, $C''(-2, 1)$

(c)



SC 3

1 Reflection in the y-axis.

2 **(a)** **(i)** $[-7, -2]$

(ii) $B'(-1, -3)$, $C'(-10, -2)$

(b) **(i)** $[3, -1]$

(ii) $E(2, -1)$, $F(-2, 2)$

3 **(a)** **(i)** x-axis

(ii) $A'(3, -3)$, $C'(-1, -5)$

(b) **(i)** y-axis

(ii) $D(3, -1)$, $E(-1, -3)$

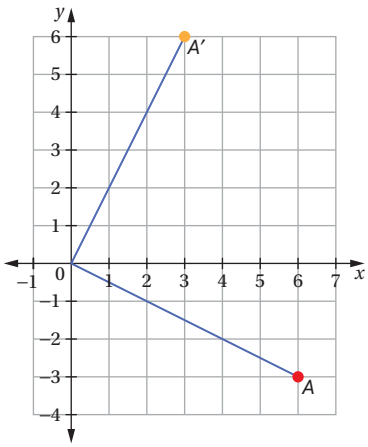
4 **(a)** Translation: $[1, -3]$.

(b) Translation: $[-1, 3]$

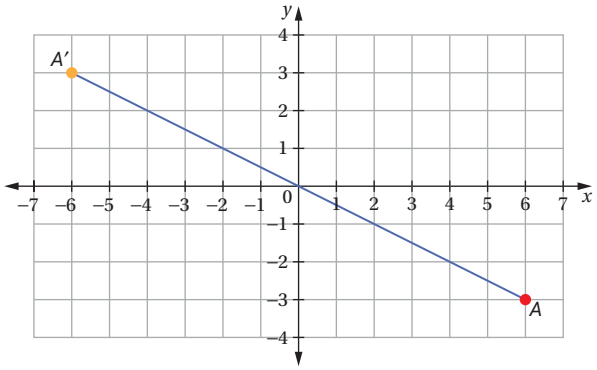
18.2 Rotate shapes on a Cartesian plane

SC 1

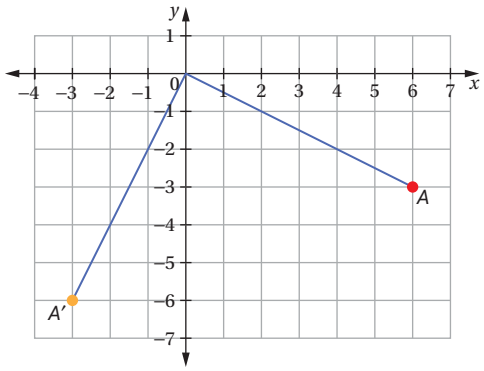
1 **(a)** **(i)** $A(6, -3) \rightarrow A'(3, 6)$



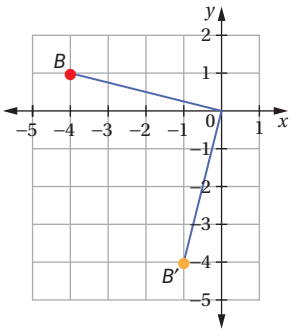
(ii) $A(6, -3) \rightarrow A'(-6, 3)$



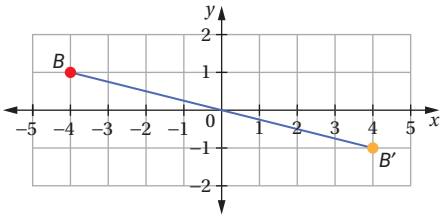
(iii) $A(6, -3) \rightarrow A'(-3, -6)$



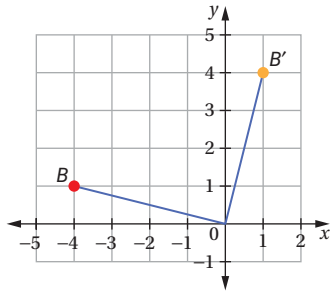
(b) **(i)** $B(-4, 1) \rightarrow B'(-1, -4)$



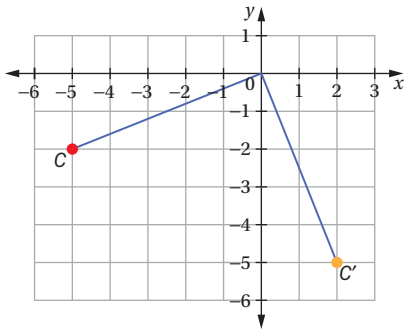
(ii) $B(-4, 1) \rightarrow B'(4, -1)$



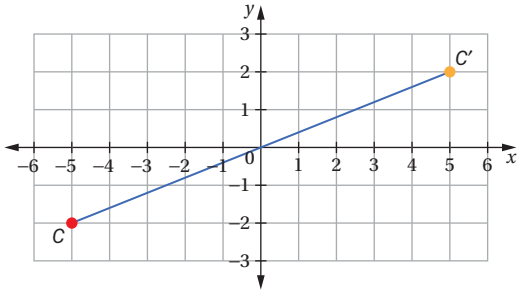
(iii) $B(-4, 1) \rightarrow B'(1, 4)$



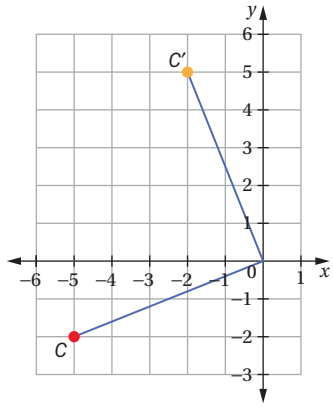
(c) (i) $C(-5, -2) \rightarrow C'(2, -5)$



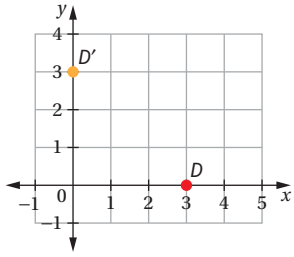
(ii) $C(-5, -2) \rightarrow C'(5, 2)$



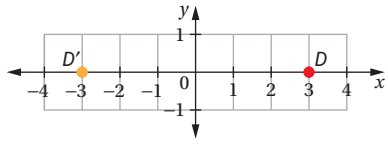
(iii) $C(-5, -2) \rightarrow C'(-2, 5)$



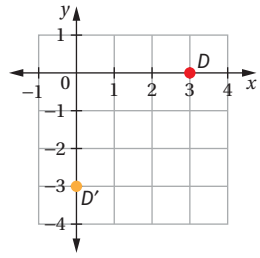
(d) (i) $D(3, 0) \rightarrow D'(0, 3)$



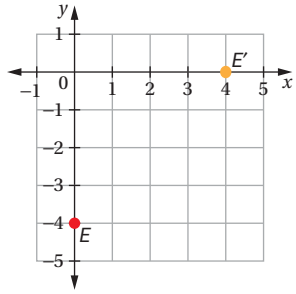
(ii) $D(3, 0) \rightarrow D'(-3, 0)$



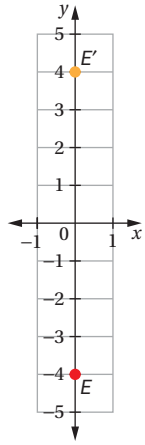
(iii) $D(3, 0) \rightarrow D'(0, -3)$



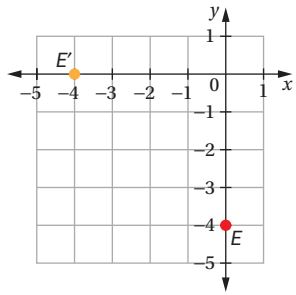
(e) (i) $E(0, -4) \rightarrow E'(4, 0)$



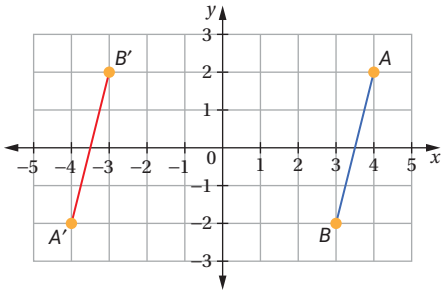
(ii) $E(0, -4) \rightarrow E'(0, 4)$



(iii) $E(0, -4) \rightarrow E'(-4, 0)$



2 $A(4, 2) \rightarrow A'(-4, -2), B(3, -2) \rightarrow B'(-3, 2)$



3 $B(-1, 3), C(-3, -1), D(1, -3)$

4 Answers may vary but should include information such as the following.

(a) When a point is rotated 90° clockwise around the origin, the point is transformed to the next quadrant in the clockwise direction. The x- and y-coordinates of the transformed point swap, and the y-coordinates change sign.

(b) When a point is rotated 90° anticlockwise around the origin, the point is transformed to the next quadrant in the anticlockwise direction. The x- and y-coordinates of the transformed point swap, and the new x-coordinates change sign.

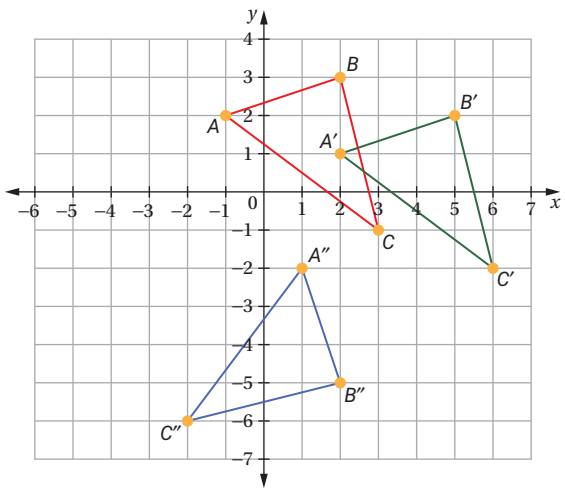
(c) When a point is rotated 180° around the origin in either direction, the point is transformed to the diagonally opposite quadrant. The x- and y-coordinates both change sign.

5 **(a)** $A(3, 5) \rightarrow A'(3, -5)$

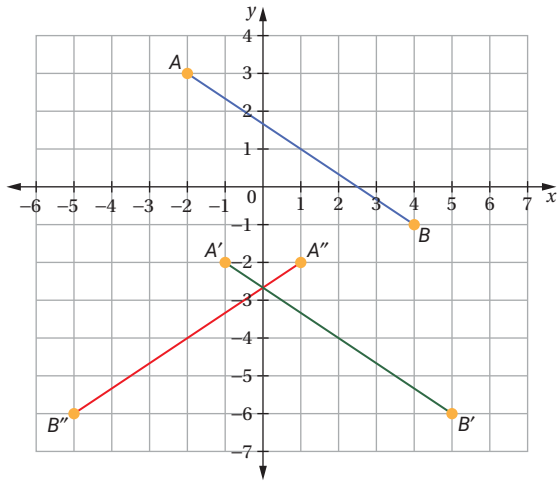
(b) $A(3, 5) \rightarrow A'(-3, -5)$

SC 2

1

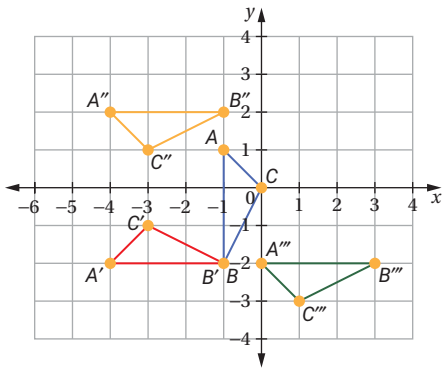


2 **(a)**

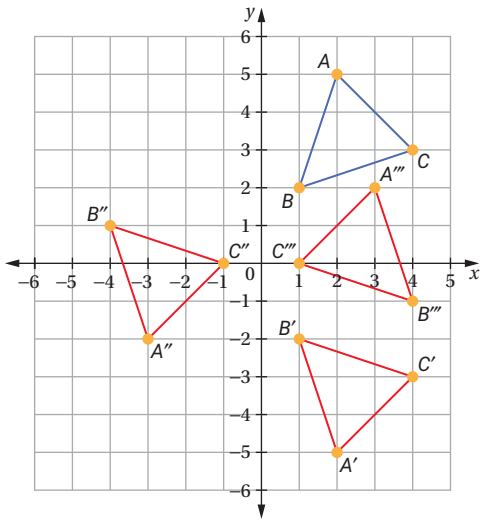


(b) Translation of $[1, -5]$, reflection in the y-axis.

3



4 **(a)**



(b) Rotation of 180°, in either direction, about the origin.

Topic 19: Data and measures of central tendency

Recall

I can order numbers by size

1 **(a)** 3, 6, 9, 12, 18
(b) 1.2, 1.4, 1.8, 2.0, 2.2, 2.4
(c) $1, 1\frac{3}{4}, 2\frac{1}{2}, 3, 4\frac{1}{2}, 5\frac{1}{2}$
(d) $0.5, 1, 1\frac{2}{3}, 2.5, 3, 3\frac{1}{4}, 3\frac{1}{2}, 5, 6.5$

I can determine the range of scores in a data set

1 **(a)** 15 **(b)** 1.2 **(c)** $4\frac{1}{2}$ **(d)** 6

I can determine the score with the highest frequency (mode) in a set of data

1 **(a)** 4 **(b)** 3 **(c)** 8

Exploration Task: Whales and dolphins
(a) Approximately 35 whales were sighted in 2018.
(b) 120 dolphins were sighted in 2019.
(c) In 2020, 45 whales + 160 dolphins = 205 aquatic mammals were sighted.

19.1 Understand different data types

SC 1

1 Secondary
2 **(a)** Primary **(b)** Secondary
(c) Primary **(d)** Primary
3 Primary data will give them an estimate how many bilbies are currently living in the park at this time.
Secondary data will tell them how many used to live in the park, or how many live in similar parks.
Thus primary data would provide the best information, but it will be more difficult to collect.
4 **(a)** Jeri could interview all the other employees in similar roles and ask how much they get paid.
This information could be considered too personal and not account for the different lengths of time people have worked in the role.
(b) Jeri could research the salaries for similar roles online. Jeri may not know whether the information source is reliable. Different companies may have very different pay rates and thus the information might not be useful for this situation.
5 Average 22.3 frogs per area represents primary data.
Average 76.5 frog per area represents secondary data.
6 Data that you collect yourself via a questionnaire, survey or observation is primary data; for example, counting the number of cars in the staff car park after 4 pm.
When data is collected from another source, e.g. the Australian Bureau of Statistics (ABS), the data is secondary data; for example, the number of electric vehicles (EVs) sold in Australia per month.

SC 2

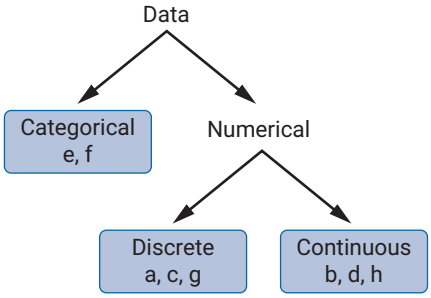
1 Numerical
2 **(a)** Categorical.
Sample explanation: Each item represents a category and the number of each item sold was counted, not measured.
(b) As a bar chart, pie graph or column graph.
3 **(a)** Sample answer. Maxine is incorrect, the data is categorical. The figures have been counted.
(b) Sample answer. The mean of the frequencies is a meaningless figure. The groups are categorical, we cannot find the mean colour.
4 **(a)** Numerical **(b)** Categorical **(c)** Numerical
(d) Numerical **(e)** Numerical **(f)** Categorical
5 Sample answer
When you count, the data is categorical. For example, if the question is "Who is your favourite movie star?" you count the votes for each different actor.
Alternatively, numerical data is when you measure the figures in your data. For example, you measure the height of a group of people.

SC 3

1 **(a)** Discrete **(b)** Continuous **(c)** Discrete
(d) Continuous **(e)** Continuous
2 **(a)** Yes, the data is numerical. The frequency of each numerical outcome is being measured. The value is tallied after each roll and counted.
(b) The results of each die roll are the whole numbers 1, 2, 3, 4, 5 and 6 only.
No fractions or decimal values are being measured, so the data is discrete.
3 **(a)** True. A survey to find my friends' most popular singer.
(b) True. A web search to find out which shoe colour is the most popular.
(c) False. Recording the outcome of rolling two dice is both numerical and discrete.
(d) False. Measuring the height of all your friends is numerical and continuous.
(e) True. Categorical data is divided into separate groups and counted. There are no decimal or fraction values between the groups.
(f) True. Using census data to find the age of the Australian population.

4 **(a)** Sample answer.
Primary data is first classified as categorical or numerical. The numerical data can be classified as discrete or continuous.
(b) Yes. Both primary and secondary data can be categorical or numerical.

5



19.2 Calculate the mean and range of a set of data

SC 1

1 **(a)** **(i)** 8 **(ii)** 13 **(iii)** 95
(b) Set C because it has the greatest range.

2 **(a)** **(i)** 4 **(ii)** $2\frac{3}{4}$ or $\frac{11}{4}$ or 2.75 **(iii)** 8
(b) Set B because it has the smallest range.

3 A set of five scores with a highest value of 42 and a lowest value of $42 - 27 = 15$, and three values in between.
Sample answer: 42, 15, 20, 26, 32

4 −10.5

5 The missing score could be either −4 or 12.
$11 - (-4) = 15$
$12 - (-3) = 15$

SC 2

1 **(a)** 3 **(b)** 4 **(c)** −12.5
(d) 11 **(e)** 242.4 **(f)** 4
(g) $4\frac{7}{8}$ or $\frac{39}{8}$ or 4.875

2 55.4

3 The mean score for the last five games is 17.2, so this scoring rate is higher than expected.

4 1.68 m

5 20

6 38 cm

SC 3

1 **(a)** Jenna should not calculate the mean or range because there is an extreme value in the data set, 210.
(b) Jamila can calculate the mean or range because there are no extreme values in the data set.

2 **(a)** 63.7 **(b)** 52
(c) **(i)** 72 **(ii)** 4
(d) Sample answer. The original data contained an extreme value that affected the calculation of the mean and range.

3 **(a)** False. An extreme value does influence the mean of an otherwise closely grouped data set.
For example, 1, 2, 3, 4, 5000, the total of the first four scores is 10, which is very small compared to the score of 5000.
(b) True. The range is more affected than the mean by an extreme value in the data set. For example, 1, 2, 3, 4, 5000, the mean of the data is 1002 and the range 4999.
(c) False. An extreme value can also be much less than the rest of the data, for example: 2, 4500, 4800, 5000, 5300.
(d) True. When the data is ranked, the extreme value can be either at the highest or lowest value.
For example: 1, 2, 3, 4, 5000 and 2, 4500, 4800, 5000, 5300.

Exploration Task: Problems with problems
Answers will vary.

19.3 Calculate the mean and range for a set of grouped data

SC 1

1 **(a)** **(i)** 30 **(ii)** 20
(b) Set A because it has the greatest range.

2 **(a)** Range = 3; $\bar{x} = 2.42$ **(b)** Range = 6; $\bar{x} = 2.8$
(c) Range = 30; $\bar{x} = 22.66$

3 **(a)** Range of cars parked: 12
(b) 5.27 cars
(c) There are four weekends over a 30-day period, and no-one used the school car parks on weekends.

4 **(a)** Completed table:

x	f	$x \times f$
3	6	18
5	4	20
7	8	56
9	6	54
11	5	55
13	4	52
Total	30	255

$\bar{x} = \frac{255}{30} = 8.5$

(b) Sample answer: The third column simplifies calculating the sum of all the scores.

5 Any answer where the range of the scores is 7 and the total frequency of all the scores is 48.
Sample answer:

x	f
4	8
5	12
6	15
8	11
11	2
Total	48

SC 2

1 (a)

Class	Score (x)	Frequency (f)
0–4	$\frac{0+4}{2}=2$	10
5–9	$\frac{5+9}{2}=7$	14
10–14	$\frac{10+14}{2}=12$	7
15–19	$\frac{15+19}{2}=7$	9
20–24	$\frac{20+24}{2}=22$	3

(b) Range is $24 - 0 = 24$.

2 (a)

Class	Score (x)	Frequency (f)
0–9	$\frac{0+9}{2}=4.5$	10
10–19	$\frac{10+19}{2}=14.5$	14
20–29	$\frac{20+29}{2}=24.5$	7
30–39	$\frac{30+39}{2}=34.5$	9
40–49	$\frac{40+49}{2}=44.5$	3

(b) $10 + 14 + 7 + 9 + 3 = 43$

(c)
$$\frac{4.5\times10+14.5\times14+24.5\times7+34.5\times9+44.5\times3}{43}=20.08$$

3 (a)

Class	Score (x)	Tally	Frequency (f)
0.0–0.4	0.2	𝍸	5
0.5–0.9	0.7	||	2
1.0–1.4	1.2	|||	3
1.5–1.9	1.7	|	1
2.0–2.4	2.2	||||	4

(b) $\bar{x} = \frac{0.2\times5+0.7\times2+1.2\times3+1.7\times1+2.2\times4}{15}$
$= \frac{16.5}{15}$
$= 1.1$

4 (a)

Class	Score (x)	Tally	Frequency (f)
0–9	$\frac{0+9}{2}=4.5$	||	2
10–19	$\frac{10+19}{2}=14.5$	|||	3
20–29	$\frac{20+29}{2}=24.5$	|||	3
30–39	$\frac{30+39}{2}=34.5$	||||	4
40–49	$\frac{40+49}{2}=44.5$	𝍸	5

(b) 28.62 **(c)** 29.82

(d) The answers are different. **(c)** is more accurate. The grouped data uses the mean of each interval, which gives a close estimate.

5 Sample answer.
No, this data cannot be grouped. The data is discrete and there are no values between the three sizes.

19.4 Determine the median of a data set

SC 1

1 (a) 3 **(b)** 4 **(c)** 13 **(d)** 9
(e) 240 **(f)** 5.8 **(g)** 5.1

2 (a) 3.5 **(b)** 4.5 **(c)** 12.5 **(d)** 9.5
(e) 280 **(f)** 5.35 **(g)** 6.25

3 (a) 573.9 **(b)** 20

(c) Sample answer.
The median value of 20 best represents the central tendency of the dataset. There is an outlier score that affects the accuracy of the mean to represent the central tendency.

4 Sample answer. 1, 5, 5, 9, 10, median is 5.

5 (a) 28

(b) The median age stays the same. There are now an even number of family members, so the median value will be the average of the two parents, who are the same age.

(c) Uncle San's arrival means there is an odd number of family members, and the middle value will still be one of the parents. Therefore, without further information, Uncle San's age cannot be determined.

(d) 10 family members means the median value is the average of the fifth and sixth member when arranged in ascending age order:
2, 5, 7, 8, 27, 28, 28, 32, 56, 58
$\frac{27+28}{2} = 27.5$
Median age is 27.5 years.

SC 2

1 (a)

x	f	$c.f.$
5	21	21
6	15	36
7	18	54
8	20	74
9	11	85

(b)

Class	x	f	$c.f.$
10–14	12	38	38
15–19	17	47	85
20–24	22	29	114
25–29	27	36	150

2 (a) 2

(b) 50 scores (even), so the median value is between the 25th and 26th scores.
25th and 26th scores are both in the 10–14 class (score of 12), so the median score is $\frac{12+12}{2} = 12$

3 (a)

x	f	$c.f.$
10	16	16
20	24	40
30	15	55
40	9	64

Median is 20.

(b)

x	f	$c.f.$
5.5	9	9
5.6	8	17
5.7	10	27
5.8	23	50
5.9	5	55

Median is 5.8.

19.5 Determine an appropriate measure of central tendency

SC 1

1 (a) 1 **(b)** 5 **(c)** 12
(d) no mode **(e)** 240

2 (a) 3 **(b)** 20
(c) Modal class is 2–4.

3 (a) 18.9 **(b)** 4
(c) The mean of 18.9 is the better choice. There are no extreme values to affect the mean. The mode for this data set is also the minimum value so it does not represent the central tendency of all the scores well.

4 (a) Bimodal, 2 and 3 have the same highest frequency.
(b) There is no mode because there are now 4 scores with the same highest frequency of 2.

5 1, 2, 3, 5, 9

6 (a) The lowest and highest value tiles in the bag.
(b) The sum of the tile values and the total number of tiles in the bag.
(c) The value of each tile and the total number of tiles in the bag.
(d) The value of each tile and the number of each valued tile that is in the bag.

SC 2

1 (a) Sometimes true **(b)** Always true
(c) Sometimes true **(d)** Sometimes true
(e) Always true **(f)** Sometimes true
(g) Sometimes true

2 (a) 9.2 **(b)** 9 **(c)** 8
(d) Sample answer. The mode is the most frequently sold shoe size and this information can help when restocking the shelves. The mean and median do not help with this decision.

3 (a)

Weight class (g)	Class centre (x)	Frequency (f)
240–<250	245	1
250–<260	255	5
260–<270	265	2
270–<280	275	2

(b) $\frac{245\times1+255\times5+265\times2+275\times2}{10} = 260$

(c) $\left(\frac{10+1}{2}\right) = 5.5$, therefore, the median value is the average of the 5th and 6th scores.
$\frac{255+265}{2} = 260.$

(d) The modal class 250–260.

(e) Because the data is grouped, the mean, median and mode are all good estimates of the central tendency, because the exact raw scores are not used for each calculation.

4 The data contains an outlier, so the mean will not accurately represent a typical amount.
There is no mode, so this is not relevant.
The median is the best option.

152.00, 158.20, 161.75, 174.55, 185.30, 190.80, 193.45, 200.30, 206.35, 381.95

$\left(\frac{10+1}{2}\right) = 5.5$

The median will be the average of the 5th and 6th amounts.

Median value $= \left(\frac{185.3+190.8}{2}\right) = \$188.05$

5 No. You need more information such as the total number of scores, the sum of their values and whether the data contains any outliers before you can determine the mean.

6 Possible answers

3, 6, 6, 7

4, 6, 6, 8

7 Possible answers

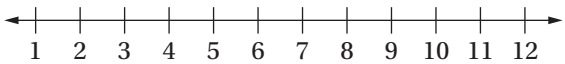
(a) 3, 6, 6, 7 **(b)** 4, 6, 6, 8

Topic 20: Displaying and understanding data

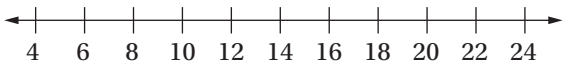
Recall

I can identify the scale of number lines

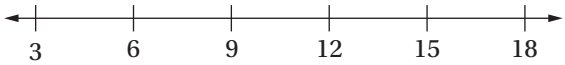
1 **(a)**



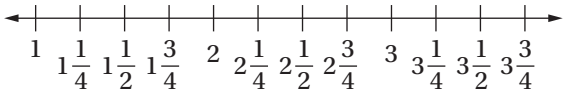
(b)



(c)

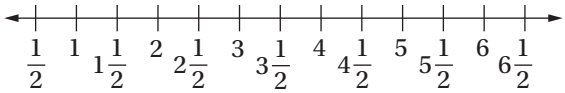


(d)



(e)

(f)



I can tally scores and complete a frequency table to display collected data

1 **(a)** and **(b)**

	Tally	Frequency
a	𝍸 |	6
e	𝍸 𝍸 𝍸	15
i	𝍸 |||	8
o	𝍸 ||	7
u	||||	4
		40

2

Age	Tally	Frequency
14	|	1
15	𝍸	5
16	𝍸 ||	7
17	||	2
Total		15

20.1 Understand stem-and-leaf plots

SC 1

1 **(a)** Square represents the value 1|4 = 14.

(b) Circle represents the value 3|0 = 30.

(c) Triangle represents the value 4|3 = 43.

2 **(a)** Square represents the value 4|8 = 4.8.

(b) Circle represents the value 5|5 = 5.5.

(c) Triangle represents the value 7|0 = 7.0.

3 **(a)** Key that has the stem representing the tens place value and the leaves the ones place value.
Sample answer: Key 1|0 = 10

(b) Key that has the stem representing the ones place value and the leaves the tenths place value.
Sample answer: Key 1|5 = 1.5

(c) Key that has the stem representing the hundred and tens place value and the leaves the ones place value.
Sample answer: Key 10|1 = 101

4 **(a)**

Stem	Leaf
1	1 2 3 4
2	5 7
3	2 4 6

Key 1 | 1 = 11

(b)

Stem	Leaf
1	1 2 3 4
2	5 7
3	2 4 6

Key 1 | 1 = 1.1

(c)

Stem	Leaf
10	6 7
11	0 4
12	0 2 6 7
13	2 4 5

Key 10 | 1 = 110

(d)

Stem	Leaf
6	5 7 8
7	1 3 9
8	1 1 5 9
9	0 2

Key $1 \mid 1 = 11$

5

Stem	Leaf
3	0 3 4 6 6 7 8 9
4	0 2 3 4 4 5 6
5	1 1 3 4 6 6 6 7
6	2 7

Key $3 \mid 0 = 30$

6 All the values are arranged in ascending order and have the same stem. A stem-and-leaf plot would not display the data any differently.

SC 2

1 **(a)** 68 **(b)** 6.8 **(c)** 680

2

Stem	Leaf
11	1 3 4
11	6 8 8
12	1 2 4
12	6 7 9 9
13	1
13	5
14	1 3
14	6 6

Key $10 \mid 2 = 10.2$

3

Stem	Leaf
0	4 4 4
0	6 7 8
1	0 3
1	7 9
2	0 2 2 4
2	5 6 8 9

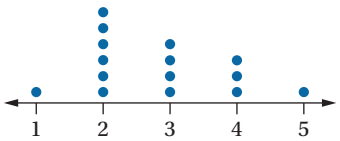
Key $0 \mid 4 = 4$

20.2 Create and interpret statistical plots

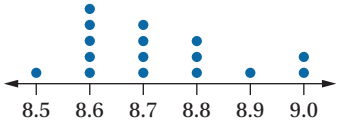
SC 1

1 **(a)** 4.2 **(b)** 42

2 **(a)**



(b)



3 **(a)**

Stem	Leaf
1	1 4 4 5 7 9
2	2 2 3 3 3 4 6 8
3	2

Key $1 \mid 5 = 15$

(b)

Stem	Leaf
7	6 7 8
8	0 1 3 3 4 5 6 6 6 7
9	0 1 3

Key $1 \mid 5 = 1.5$

SC 2

1 **(a)** Mode = 1, range = 4

(b) Mode = 4, range = 5

(c) Mode = 14, range = 4

2 **(a)** Mode = 23, range = 21

(b) Mode = 8.6, range = 1.7

(c) Mode = 4.3, range = 1.0

3 **(a)** Mode: the most common value is 43, with a frequency of 5.
The value 40 is the next most common with a frequency of 4.
Therefore, if the value 40 is added, the dataset will become bimodal.

(b) Mode: the most common value is 103, with a frequency of 5.
The value 91 is the next most common with a frequency of 4.
Therefore, if the value 91 is added, the dataset will become bimodal.

4 The largest value that can be added to give the dataset a range of 1.2 is 2.6.

20.3 Determine statistical measures of centre from data displays

SC 1

1 Mode: 1 pen; Range: 4 pens; Median: 2 pens; Mean: 1.9 pens

2 Mode: 4 books; Range: 5 books; Median: 2.5 books; Mean: 2.5 books

3 Mode: 14 years; Range: 4 years; Median: 14 years; Mean: 13.7 years

4 The two measures that change due to the addition of the new value are the range and mean.

SC 2

1 **(a)** 53.5 **(b)** 5.8

2 **(a)** 49 **(b)** 6.0

3 **(a)** mode = 15, range = 25, median $= \frac{15+25}{2} = 20$, mean $(\bar{x}) = \frac{360}{16} = 22.5$

(b) mode = 2.5, range = 2.5, median $= \frac{2.5+2.5}{2} = 2.5$, mean $(\bar{x}) = \frac{28.4}{12} = 2.4$ (1 d.p.)

(c) bimodal, modes are 65 and 81, range = 30, median = 81, mean $(\bar{x}) = \frac{1332}{17} = 78.4$ (1 d.p.)

SC 3

1 **(a)** 215 cm **(b)** 21 °C

2 Answers may vary.

(a) Sample answer. The height of 215 cm might be an unusually tall person (instead of an error in typing or in measuring).

(b) Sample answer. The 21 °C temperature might be a 'cool change' in the weather on that one day (instead of an error in typing or in measuring).

3 In order: 0, 0, 0, 0, 5, 5, 6, 7, 9, 10, 10, 12, 22, 30, 449

(a) Outlier: $449

(b) Mode is $0; range is $449; mean is $38.47; median is $7.

(c) Mode is $0; range is $30; mean is $8.29; median is $6.50.

(d) The range and mean change the most. The mode is unchanged and the median changes only slightly.

(e) Answers will vary. Sample answer: If a 0 value is removed, the new mode is $0; range is $449; median is $6.50; mean is $40.36.

4 The median is a better representation of the central tendency because the value 62 is an outlier that impacts the calculation of the mean.

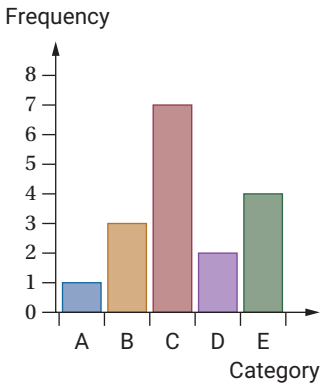
20.4 Interpret and describe numerical data displays

Exploration Task: Interpreting a column graph

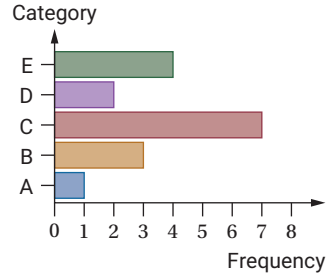
The graph could be made easier to interpret by placing the letters in alphabetical order or grouping the vowels and consonants.

SC 1

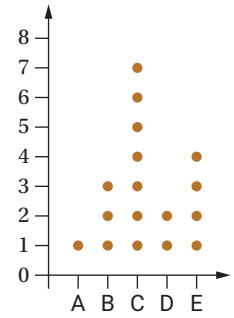
1 **(a)** **(i)**



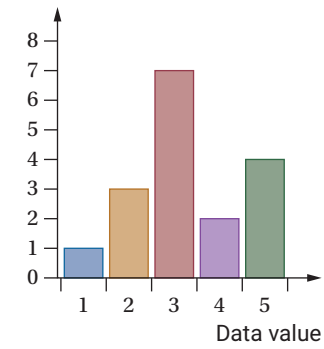
(ii)



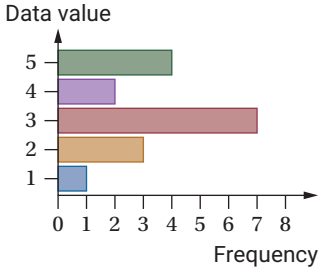
(iii)



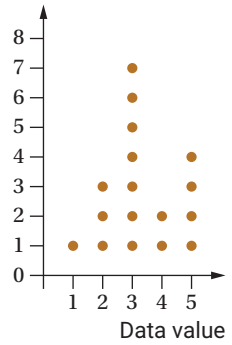
(b) **(i)**



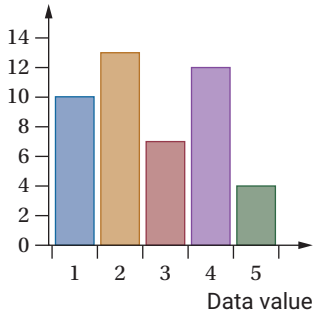
(ii)



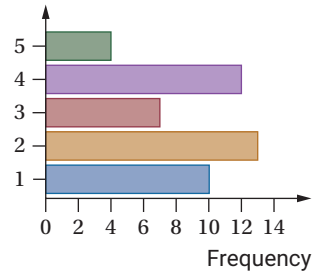
(iii) Frequency



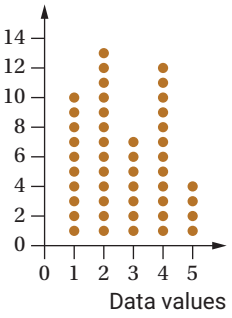
(c) (i) Frequency



(ii) Data value



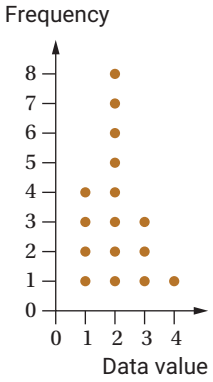
(iii) Frequency



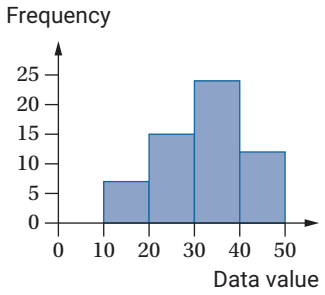
2 Both a bar or column graph and dot plot can be used to display categorical or discrete numerical data.
The advantages of a dot plot for small numbers of data is that every dot represents a data value; the total number of data values recorded can be determined by counting. As the number of data values included in the display increases, a bar or column graph is simpler to construct. In a bar or column graph, the length or height of the bars determine the frequency. The total number of data values is determined by adding up the frequency of each value.

3 The stem-and-leaf plot shows the exact data values. The stem-and-leaf plot, like a dot plot, would be more suitable to use when displaying small numbers of data, because each data value is listed on the graph. The histogram uses the length or height of bars to show frequency, however the raw data values are not shown.

4 (a) A dot plot or bar graph is an appropriate choice because the data is discrete and numerical. There are a small number of data values.



(b) A histogram is an appropriate data display. The data is continuous numerical data.

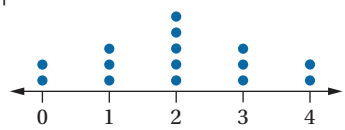


SC 2

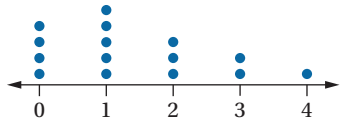
1 (a) Symmetrical **(b)** Clustered
(c) Positively skewed **(d)** Negatively skewed

2 Sample answers.

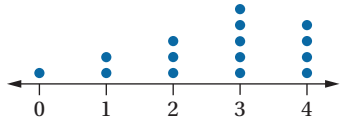
(a)



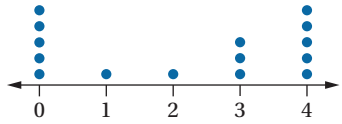
(b)



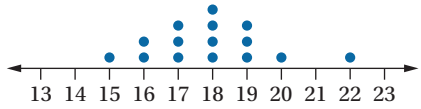
(c)



(d)

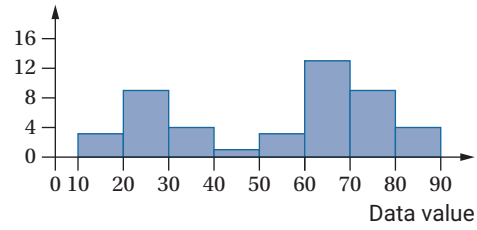


3 (a)



(b) Symmetrical

4 (a) Frequency



(b) Bimodal, clustered

(c) The shapes are similar.

5 Bimodal graphs are usually the result of combining two sets of data; for example, male and female heights, foot traffic during morning and evening peak hours. Each mode represents its cluster or set of data.

Topic 21: Probability (simulations and sample space)

Recall

I can identify and complete patterns of numbers

1 (a) 11, 19, 27, 35, 43, 51, 59

(b) 23, 35, 47, 59, 71, 83, 95

(c) 80, 40, 20, 10, 5, 2.5

(d) 10, 50, 250, 1250, 6250, 31 250

I can write fractions in their simplest form

1 (a) $\frac{4}{5}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{2}{7}$

I can use a scale to represent the chance of an event happening

1 (a) An event that has no chance of happening has a 0% chance of occurring.

(b) An event that will always happen has a 100% chance of occurring.

(c) An event that has the same chance of failure or success has a 50% chance of succeeding.

(d) An event with a 75% chance of success has a greater chance of succeeding than failing.

(e) An event with a 5% chance of success has a very high chance of failure.

I can use a scale of 0 to 1 to estimate the chances of events

1 (a) B (b) E (c) G (d) A

(e) F (f) C (g) D

21.1 List sample spaces and calculate the probability of single-step events

SC 1

1 (a) Event. Only one of many possible outcomes.

(b) Experiment. Describes the activity.

(c) Sample space. Lists all the possible outcomes.

2 (a) Experiment. Describes the activity.

(b) Event. Only one of many possible outcomes.

(c) Sample space. Lists all the possible outcomes.

3 (a) Yes (b) Yes (c) No

(d) Yes (e) Yes (f) No

4 (a) The *sample space* was red, blue, green.

(b) The *event* was spinning green.

(c) The most likely *outcome* was red.

SC 2

1 (a) {King of Hearts, King of Diamonds, King of Clubs, King of Spades}

(b) {Head, Tail} (c) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

2 (a) {1, 2, 3, 4} (b) {A, B, C, D, E}

(c) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

3 (a) {1, 2, 3}

(b) No, there are three 1s, three 2s but only two 3s. The numbers 1 and 2 are more likely to be spun.

4 Sample answers.

(a) Tossing a coin

(b) Spinning a spinner with the numbers 1, 2, 3 and 4 on it

(c) Spinning a spinner with the letters W, X, Y and Z on it

(d) Choosing a vowel from a set of letters

(e) Rolling a die

5 (a) Zero (b) {0, 1, 2, 3, 5}

(c) No. Many things can affect how likely a spinner is to land in a circle; for example, the size and weight of the spinner, the distance from the throwing point to the circles, wind direction and strength, and the skill of the player.

(d) Many variations are possible.

(i) The circles are drawn and then a line is drawn 2 m from the first circle.

(ii) Players take turns to stand behind the line and fly their spinner, trying to land it inside one of the circles.

(iii) Players score points according to the number of rocks in the circle their spinner lands in.

(iv) The first person to score 10 points is the winner.

6 Individual answers, but they should comment on whether they had assigned the points correctly; i.e. was what they thought would be the hardest actually the one with the lowest frequency?

SC 3

1 **(a)** {1, 2, 3} **(b)** $\frac{1}{3}$ **(c)** $\frac{2}{3}$ **(d)** 0

2 **(a)** {1, 2, 3, 4} **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{4}$

(d) $\frac{3}{4}$ **(e)** $\frac{2}{4} = \frac{1}{2}$ **(f)** 0

3 **(a)** $\frac{4}{52} = \frac{1}{13}$ **(b)** $\frac{48}{52} = \frac{12}{13}$ **(c)** $\frac{4}{52} = \frac{1}{13}$

(d) $\frac{13}{52} = \frac{1}{4}$ **(e)** $\frac{26}{52} = \frac{1}{2}$ **(f)** $\frac{1}{52}$

4 **(a)** $\frac{2}{52} = \frac{1}{26}$ **(b)** $\frac{6}{52} = \frac{3}{26}$ **(c)** $\frac{5}{52}$

(d) $\frac{4}{52} = \frac{1}{13}$ **(e)** $\frac{6}{52} = \frac{3}{26}$

5 **(a)** 48 **(b)** 2 **(c)** $\frac{2}{48} = \frac{1}{24}$

(d) $\frac{2}{48} = \frac{1}{24}$ **(e)** $\frac{44}{48} = \frac{11}{12}$ **(f)** $\frac{4}{48} = \frac{1}{12}$

6 **(a)** {M, A, T, H, S}

(b) $\frac{1}{5}$ **(c)** $\frac{4}{5}$ **(d)** $\frac{1}{5}$ **(e)** $\frac{4}{5}$

7 **(a)** {M, A, T, H, E, I, C, S}

(b) $\frac{1}{11}$ **(c)** $\frac{10}{11}$ **(d)** $\frac{4}{11}$ **(e)** $\frac{7}{11}$

8 **(a)** {action, drama, comedy, horror}

(b) Action, because there are more action movies.

(c) $\frac{2}{8} = \frac{1}{4}$ **(d)** $\frac{1}{8}$

21.2 Record outcomes and run trials of chance experiments

SC 1

1 **(a)** 32 **(b)** 38 **(c)** 30

(d) Yes, as each frequency is around one third of the total, which would be 33 (each colour is one third of the circle). Variation is expected due to the number of trials being relatively small.

2 **(a)** 13 **b)** 27 **(c)** 25 **(d)** 35

(e) The proportion of each colour on the spinner is not the same (there are more purple slices than blue, red or yellow).

3 **(a)** 17 **(b)** 23 **(c)** 15

(d) Similar, because the proportions of each colour remain the same for each spinner.

4 **(a)** The most likely responses are that there are 6 even totals and 5 odd totals in the table, so Player 1 has a slight advantage, or it doesn't matter.

(b) The game should be close; however, in the table of results, for a large number of trials, the 7 should occur more frequently, and the 2 and 12 less frequently.

(c) Answers vary, but both seem to have an equal chance.

(d) Player 1 would have a better chance of winning, as there are more even outcomes than odd outcomes.

(e) Answers will vary, but Player 1 should win more often.

(f) Player 1 seems to have an advantage.

(g) **(i)** Addition game

		Number on die 1					
	Sum	**1**	**2**	**3**	**4**	**5**	**6**
Number on die 2	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Number of even totals: 18

Number of odd totals: 18

Both players have an equal chance.

(ii) Multiplication game

		Number on die 1					
	Product	**1**	**2**	**3**	**4**	**5**	**6**
Number on die 2	**1**	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

Number of even totals: 27
Number of odd totals: 9
Player 1 has a much better chance.

5 **(a)** Sample answer. Let a head on the coin represent the light being red. Flip a coin 4 times and count the number of heads. This is one trial. Repeat until you have 10 trials.
(b) Sample answer. The mean number of heads (red light) will be 2 in the long run, meaning Mia takes an average of 17 minutes to get home.
(c) Each trial will be different, but the larger the number of trials, the more consistent the answer becomes.

6 **(a)** $146.40 **(b)** $53.60
(c) Numerous answers. Need to be random.
Sample answers
Option 1: Place 200 equal-sized pieces of paper, each with a different number between 1 and 200. Designate the numbers 1 to 4 as the major prize winners and 5 to 14 as the middle level prizes. Draw 50 pieces of paper.
Option 2: Simplify by dividing the numbers of prizes and number of tickets by 2. Then place 100 equal-sized pieces of paper, each with a different number between 1 and 100. Designate the numbers 1 and 2 as the major prize winners and 3 to 7 as the middle level prizes. Draw 50 pieces of paper.
(d) $\frac{93}{100}$

SC 2

1 **(a)** Various answers.
(b) Various answers.
(c) Various answers, but the percentage should be closer to 50% than **(b)**.
(d) Yes, a fair coin flip gives an equal probability for each outcome. In the long run, this will be true, but for the sample size of 50 it may not be as evident.
(e) The relative frequency of a head should be closer to 50% after 1000 tosses.

2 **(a)** Various answers.
(b) Yes, the bottle cap gives an equal probability for each outcome. In the long run, this will be true, but for the sample size of 40 it may be harder to see.

3 **(a)** Sample answers.
Use 3 different coloured pencils, use 3 slips of folded paper, use a spinner with 3 equal divisions.
(b) There are only 2 possible outcomes.
(c) A 1 or 2 is one outcome, 3 or 4 another and 5 or 6 the third.
(d) Sample answer

Frequencies are likely to be between 50 and 80.

4 **(a)** =RANDBETWEEN(1, 6)
(b) =RANDBETWEEN(1, 2), where 1 is a head and 2 is a tail
(c) Various answers, but there should be a similar spread of figures. More trials are required to confirm this.
(d) Answers will vary.

5 **(a)** Various answers
(b) It is not expected that the frequencies will be even, but they should have a similar spread of results.
(c) The simulator results should have a more even spread of results, since the number of trials is larger.

21.3 Compare theoretical and experimental probability

SC 1

1 **(a)** $E(X) = 56$
(b) No. The 7 out of 10 is an average over the length of her career.

2 **(a)** $E(X) = 54$

(b) Not completely. Sam will score on 90% of the occasions, not 100%.

3 **(a)** $E(X) = 20$

(b) No. The 1 out of 6 is an average.

4 **(a)** $E(X) = 80$ **(b)** $E(X) = 80$ **(c)** $E(X) = 133$

5 **(a)** 120 **(b)** 60

(c) Blue is half the area and red is one quarter.

6 **(a)** 44

(b) They were not evenly spread. Concentrated at the low end.

(c) The die supplied a much more evenly distributed set of figures.

SC 2

1 **(a)** Sample answer.

Number of tosses	Expected number of tails	Number of tails	% of tails
1	0.5	0	0%
5	2.5	3	60%
15	7.5	8	53.3%
21	10.5	11	52.4%
30	15	16	53.3%

(b) The expected percentage is 50% and as the number of trials increases, the percentage gets closer to 50%.

(c) Sample answer. Close to 50%, for example 48.6%.

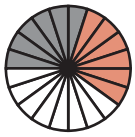
(d) Sample answer. The percentage is closer to 50%.

2 **(a)** No, because white has a markedly higher frequency than the other two colours. It seems that there are two white sections for every black or red section.

(b) Sample answer. Approximately one quarter of the spinner is black, a little bit less than one quarter is red and a little bit more than one half is white.

(c) No, because white still has a markedly higher frequency than the others. However, it now seems that the portion of white sections is less.

(d) Sample answer. Approximately 31% of the spinner is black, 40% of the spinner is white and 29% of the spinner is red.



(e) In part **(b)**, there are many more trials, and this gives a more accurate representation of the colour proportions.

3 **(a)** Individual answers.

(b) It is likely that one or more of the numbers is over-represented and one or more under-represented based upon the expected number of 2 for each outcome.

(c) Individual answers.

(d) It is likely that the frequencies of each number will vary from the expected number of 100, but the percentage difference will be much smaller.

(e) With a larger number of trials, the theoretical and experimental probabilities come closer together.

Topic 1: Number properties

Topic review

Multiple choice

1 B

2 D

3 C

4 C

5 B

6 C

7 B

8 B

9 C

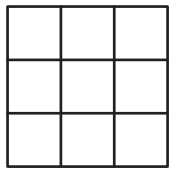
10 D

11 C

12 D

Short answer

1 A square measuring 3 by 3 contains 9 squares.



2 $3^2 = 9$; and $4^2 = 16$ and there are no whole numbers between 3 and 4.

3 25, 36, 49, 64, 81, 100

4 **(a)** 2 **(b)** 8 **(c)** 9 **(d)** 12 **(e)** 17

5 **(a)** $4 \times 4 \times 4$ **(b)** $y \times y \times y \times y$

6 24

7 **(a)** 7 **(b)** 3 **(c)** 16 **(d)** 4

8 **(a)** 2 and 7 **(b)** 2 and 3

(c) 3 and 7 **(d)** 2, 3 and 7

9 **(a)** 10^3 **(b)** 10^6 **(c)** 10^2

10 **(a)** 524 **(b)** 650 **(c)** 704 **(d)** 6352

(e) 6045 **(f)** 6305 **(g)** 6005

Extended response

1 **(a)** 6 **(b)** 4 **(c)** 3

(d) 2. At 12 seconds, both A and B are silent, and at 15 seconds, both A and C are silent.

(e) 60 seconds

2 **(a)** 1×64; 2×32; 4×16; 8×8

(b) $2 \times 4 \times 8$; $2 \times 2 \times 16$; $4 \times 4 \times 4$

(c) $4 \times 4 \times 4$

(d) $2 \times 2 \times 2 \times 2 \times 2 \times 2$

3 24. Because 5, 10 and 15 are the ones with a factor of 5, they must be one half of each pair. 15 must be paired with the smallest of the remaining numbers.

Using logic, you can arrange the numbers in ascending order and multiply the largest and smallest, the second largest with the second smallest, etc.

$5 \times 72 = 360$.

$\frac{360}{15} = 24$

4 **(a)** 26 houses

20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119

(b) 42 houses

20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119

(c) 8 houses

20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99
100	101	102	103	104	105	106	107	108	109
110	111	112	113	114	115	116	117	118	119

5 None. You could write them all and test them, but a quicker way is to notice that $1 + 3 + 5 = 9$. From the divisibility laws, any number whose digits add to 9 is divisible by 9 so cannot be a prime number.

Topic 2: Operations with decimals

Topic review

Multiple choice

1 C
2 B
3 B
4 C
5 A
6 C
7 A, B and C
8 D
9 C
10 B
11 C
12 D
13 B
14 B
15 C

Short answer

1 **(a)** 10.71 **(b)** 16.65
2 **(a)** 37.5 **(b)** 0.0168
3 **(a)** 0.01415 **(b)** 0.05
4 **(a)** 3 **(b)** 5 **(c)** 2 **(d)** 4 **(e)** 4 **(f)** 3
5 **(a)** 1.1875 kg
(b) Probably not; in a domestic kitchen it is unlikely a measurement to this accuracy could be attained.
6 0.25, 0.0625, 0.015625
7 **(a)** 28; 31.82 **(b)** 30; 26.26
(c) 20; 17.556 **(d)** 9; 9.5718
8 8.4 m
9 **(a)** $11.50 **(b)** $11.48
10 **(a)** $570.30 **(b)** $570.31
11 **(a)** 573.75 mL
(b) This is probably too accurate. The measuring cup shows that even the nearest mL might be difficult to achieve, but 573 mL or 575 mL are probably reasonable answers.
12 **(a)** 20.8 **(b)** 15 **(c)** 1170 **(d)** 8837.5
13 $31.10/hour
14 157 bricks will be required.
15 **(a)** 16 566 **(b)** 313
16 **(a)** 23.1 **(b)** 1.6209
17 **(a)** $1.85 **(b)** $3.15

Extended response

1 **(a)** $87 \times 5 = 435$ **(b)** $88 \times 6 = 528$
(c) As both values in the first estimate are smaller than the actual values, and both values in the second estimate are bigger than the actual values, the exact value must lie between them.
(d) 476.52686
(e) It seems that Esther just made a mistake copying out her answer and she swapped the order of the first two digits.
2 **(a)** 7 whole batches **(b)** 84 rolls
(c) 2 extra rolls
(d) There is still 0.0083 kg of flour unused.
3 **(a)** 78 points
(b) $111.8 - 78 = 33.8$ points difference
(c) $30 \times 1.6 = 48$ points
(d) $42.6 \div 1.7 = 25.0588...$

Topic 3: Fractions, decimals and percentages

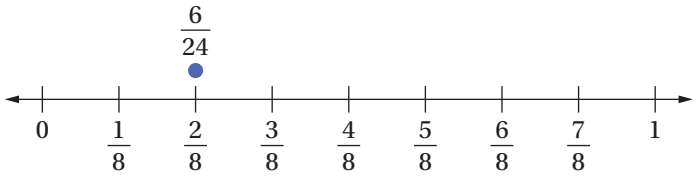
Topic review

Multiple choice

1 C
2 B
3 A
4 B
5 A
6 D
7 B
8 A
9 C
10 D
11 C
12 D
13 D
14 A

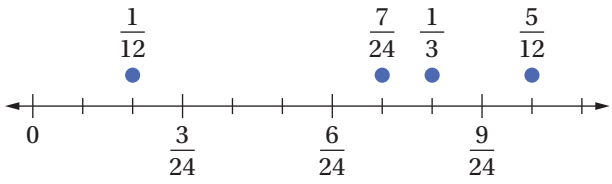
Short answer

1



2 $\frac{6}{10}$

3



4 $\frac{2}{3}, \frac{7}{10}, \frac{23}{30}, \frac{4}{5}, \frac{5}{6}, \frac{13}{15}$

5 $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{40}{100}$

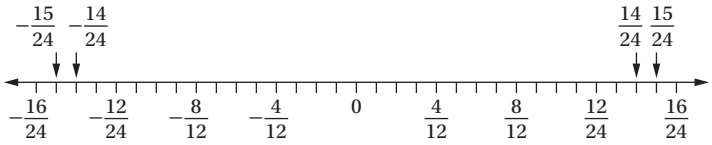
6

Activity	Time spent (hours)	Fraction of day (24 hours)
Sleeping	8	$\frac{1}{3}$
School	6	$\frac{1}{4}$
Sport	1	$\frac{1}{24}$
Work	3	$\frac{1}{8}$
Other (eating, travelling, etc.)	6	$\frac{1}{4}$

7 **(a)** Durant: $\frac{54}{60}$; Lillard: $\frac{55}{60}$; Rubio: $\frac{50}{60}$

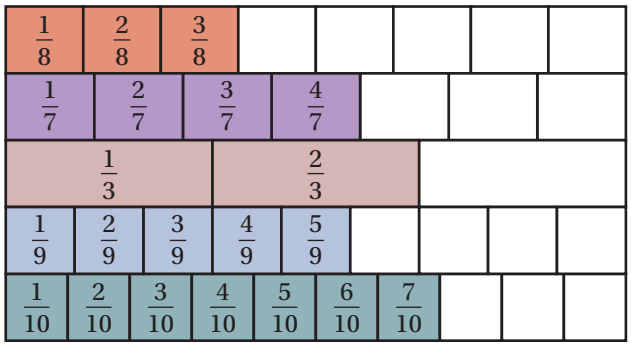
(b) Lillard, Durant, Rubio

8 **(a)**



(b) $-\frac{5}{8}, -\frac{7}{12}, \frac{7}{12}, \frac{5}{8}$

9 **(a)**



(b) $\frac{3}{8}, \frac{5}{9}, \frac{4}{7}, \frac{2}{3}, \frac{7}{10}$

10 $-\frac{2}{5}$

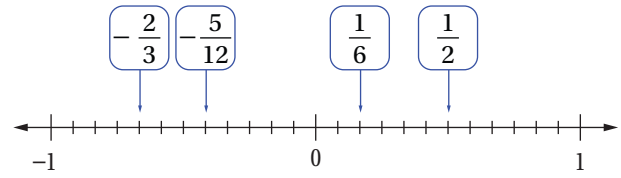
11 **(a)** $\frac{1}{3} < \frac{5}{9}$

(b) $\frac{2}{5} < \frac{2}{3}$

(c) $-\frac{3}{5} > -\frac{3}{4}$

(d) $-\frac{1}{2} < \frac{2}{4}$

12



13 $\frac{4}{5}, \frac{2}{3}, -\frac{5}{12}, -\frac{3}{4}, -\frac{7}{9}$

14 **(a)** $\frac{3}{8} = \frac{60}{160}$ **(b)** $\frac{150}{100} = \frac{6}{4}$

(c) $\frac{180}{24} = \frac{15}{2}$ **(d)** $-\frac{36}{48} = -\frac{3}{4}$

15 **(a)** 3 and 4 **(b)** −1 and 0

(c) 3 and 4 **(d)** −2 and −1

16 **(a)** A: $1\frac{1}{6}$ B: $2\frac{5}{6}$

(b) A: $1\frac{1}{9}$ B: $2\frac{7}{9}$

17 **(a)** $0.23 = \frac{2}{10} + \frac{3}{100}$

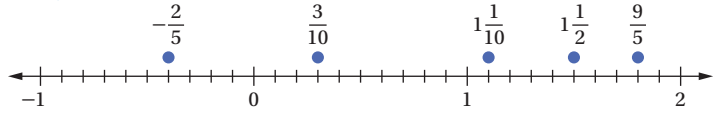
(b) $0.125 = \frac{1}{10} + \frac{2}{100} + \frac{5}{1000}$

(c) $0.0056 = \frac{5}{1000} + \frac{6}{10000}$

Extended response

1 **(a)** −1 to 2 **(b)** tenths

(c)



(d) $1\frac{1}{10}$

(e) Seven whole numbers: 2–8

2 **(a)** **(i)** $\frac{7}{20}$ **(ii)** $\frac{57}{100}$ **(iii)** $\frac{103}{500}$

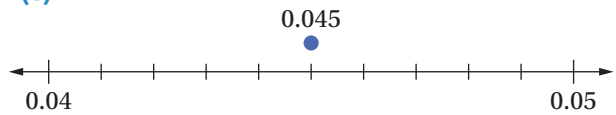
(b) Between $\frac{3}{10}$ and $\frac{4}{10}$

(c) Between $\frac{20}{100}$ and $\frac{21}{100}$

(d) **(i)** A: $0.62 = \frac{31}{50}$ B: $0.66 = \frac{33}{50}$

(ii) C: $0.651 = \frac{651}{1000}$ D: $0.657 = \frac{657}{1000}$

(e)



Topic 4: Operations with fractions

Topic review

Multiple choice

1 C
2 C
3 C
4 D
5 A, B and D
6 D
7 B
8 D
9 D
10 A
11 C
12 D
13 D
14 C

Short answer

1 **(a)** $\frac{14}{55}$ **(b)** $\frac{31}{48}$
2 **(a)** $2\frac{13}{20}$ **(b)** $1\frac{9}{28}$
3 $-1\frac{9}{20}$
4 $\frac{1}{10}$
5 **(a)** $\frac{1}{4}$ **(b)** 6
6 12 pencils remain in the box.
7 $\frac{1}{15}$
8 20 students play a team sport and 12 play soccer.
9 The chair should be 69 cm high.
10 **(a)** $\frac{1}{5}$ **(b)** $5\frac{5}{7}$ **(c)** 4 **(d)** $\frac{8}{9}$
11 **(a)** False **(b)** True **(c)** True **(d)** False
12 **(a)** $\frac{23}{36}$ **(b)** $2\frac{1}{5}$ **(c)** $\frac{7}{10}$
13 $\frac{3}{5}\times\left(2-\frac{7}{3}\right)$; $\frac{7}{16}-\frac{1}{4}$; $20\times\frac{4}{5}$; $1\frac{1}{7}\times 21$; $9\div\frac{1}{3}$
14 $3\frac{1}{24}$ oranges each
15 90 smaller packages
16 **(a)** \$60 **(b)** $12\frac{1}{7}$ **(c)** $12\frac{2}{3}$ **(d)** $\frac{1}{3}$
17 **(a)** $7\frac{5}{14}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{15}$ **(d)** $1\frac{23}{143}$

Extended response

1 4 cups
2 **(a)** 4 whole flowerpots
(b) $\frac{4}{9}$ of a flowerpot
(c) $\frac{8}{45}$ of a packet of clay
3 **(a)** $2\frac{5}{12}$ packets of flour
(b) $19\frac{1}{3}$ cups of flour
(c) No; $4\times 2\frac{1}{3}=9\frac{1}{3}$, and each packet only contains the equivalent of 8 cups of flour.
(d) 8 whole cakes

Topic 5: Modelling with fractions, decimals and percentages

Topic review

Multiple choice

1 B
2 D
3 C
4 D
5 B
6 B
7 C
8 B
9 B
10 D
11 B
12 B
13 A
14 D

Short answer

1 **(a)** 3.021, 3.12, 3.21
(b) 0.3, 33%, $\frac{1}{3}$, 0.35
(c) 19%, $\frac{1}{5}$, 0.20001, $\frac{9}{40}$
2 **(a)** \$18 000
(b) 40% profit based on cost price
3 **(a)** 655.5 kg **(b)** 3450
4 **(a)** \$6.70 **(b)** \$3.30
5 **(a)** 1013.8 **(b)** 11.4 **(c)** 76.26

6

Fraction (simplest form)	Decimal	Percentage
$\frac{1}{10}$	0.1	10%
$\frac{1}{3}$	0.333...	$33\frac{1}{3}\%$
$\frac{9}{25}$	0.36	36%
$1\frac{3}{5}$	1.6	160%
$\frac{1}{8}$	0.125	12.5%
$\frac{3}{40}$	0.075	7.5%
$\frac{17}{40}$	0.425	42.5%

7 **(a)** 26.46 **(b)** 352.08 **(c)** 2.28

8 **(a)** 900.6% **(b)** 0.7%

9 **(a)** 0.23002 **(b)** 0.000004

10 **(a)** 10% **(b)** 5% **(c)** 20% **(d)** 3.125%

11 **(a)** $21\frac{1}{2}$ **(b)** $\frac{217}{2000}$

12 **(a)** $35 **(b)** $0.37 **(c)** $2999

13 Sophie took 15 seconds.

14 **(a)** $98 **(b)** $182

15 **(a)** 2405 mg **(b)** 17.58%

16 **(a)** $43.50 **(b)** $3.70 **(c)** $25 450

17 **(a)** DB: 30.9%; QB: 30.8%; KB: 31.5%; SKB: 30.7%
(b) This was probably a 30% off sale.

18 9.442 kWh

Extended response

1 **(a)** $5092
(b) $29 467 because it is the base tax amount for the next tax bracket. It is stated in the tax table.
(c) Tax payable: $22 967; percentage of income: 22.97%
(d) $2269
(e) On $118 000: $28 817 which is 24.42% of their income. On $121 900: $30 170 which is 24.75% of their income.
(f) The word 'progressive' means that as the income increases, so does the percentage of tax paid. The assumption is that the more a person earns, the more they can contribute in taxation.

2 **(a)** $253 **(b)** $11/litre
(c) Although the second paint has a lower cost per litre, its total cost is higher. Assuming the smaller quantity is sufficient for the job at hand, then Dave would be better off buying the first paint at the lower total cost.

3 **(a)** 40% don't walk to school.
(b) 36 students walk to school.
(c) 22.73% walk to school.
(d) This percentage is $44\frac{3}{13}\%$ or 44.23%.
(e) The increase is $8\frac{1}{3}\%$.

4 **(a)** $4104
(b) This represents a 17.55% increase compared to the previous year. In decimal form, this is 0.18, correct to 2 d.p.
(c) The new price will be $4750.
(d) The total cost will be $200 880.

Topic 6: Integers

Topic review

Multiple choice

1 A
2 C
3 A
4 C
5 A
6 C
7 B
8 A
9 B
10 C
11 D
12 C

Short answer

1 **(a)** −1, 0, 1, 2, 3
(b) −8, −7
(c) −3, −2,−1

2 **(a)** −1, 1, 3
(b) −45, −35, −25
(c) −14, −18, −22

3 $860

4 161 °C

5 $126 debt or −$126

6 115 °C

7 **(a)** 327 m
(b) **(i)** 304 m **(ii)** 2 m

8 Sample answer: Filled the car with petrol $160, buying a new jacket $240, buying a new shirt $50

9 −7

10 −52 °C

11 15 °C

12 $21

13 **(a)** −9 **(b)** −63 **(c)** 13

14 −3

Extended response

1 (a)

Title	Cost	Sale price	Profit/ loss
Fake Fashions	$14.50	$19.99	$5.49
Majestic Mountains	$17.50	$19.99	$2.49
Killer Kingdoms	$23.50	$19.99	$3.51
Curios Quizzing	$22.50	$19.99	$2.51

(b) Fake Fashions, Majestic Mountains

(c)

Title	Number sold	Profit/ loss per book	Profit/ loss
Fake Fashions	5	$5.49	$27.45
Majestic Mountains	6	$2.49	$14.94
Killer Kingdoms	4	$3.51	$14.04
Curios Quizzing	3	$2.51	$7.53

(d) Profit $20.82

2 (a) Denali and Churu
(b) Pilani and Snag
(c) Death Valley and Verkhoyansk
(d) Dallol and Oymyakon

Topic 7: Understanding ratios

Topic review

Multiple choice

1 D
2 B
3 A
4 C
5 B
6 B
7 B
8 D
9 A
10 D
11 C
12 B
13 C

Short answer

1 (a) 6 : 5 **(b)** 5 : 2 **(c)** 3 : 1 **(d)** 2 : 13
2 7 : 16 : 24
3 3 : 8 : 12
4 (a) 8 : 7 **(b)** 8 : 15 **(c)** $\frac{7}{15}$
5 (a) 5 : 11 **(b)** 5 : 1 : 2 **(c)** 1 : 8
6 (a) 6 : 13 **(b)** 7 : 5 **(c)** 15 : 28
7 (a) 2 : 15 **(b)** 15 : 1 **(c)** 4 : 3
8 (a) 39 : 10 **(b)** 144 : 89
9 (a) 225 mL **(b)** 25 mL
10 (a) 2.38 **(b)** 2230 m
11 (a) $4200, $3000 **(b)** 12 kg, 20 kg
12 $297.50
13 25 m^2
14 (a) 1.07 **(b)** 0.99
15 (a) 6 **(b)** 3 : 13
16 (a) 13 : 3 **(b)** $\frac{13}{16}$

Extended response

1 (a) Lyndall, 1 : 1; Penny, 2 : 1; Allister, 1 : 2
(b) 1 : 1 **(c)** 1 : 1
2 (a) (i) 43 : 35 **(ii)** 4 : 1
(iii) The woman is 1.23 times as tall as her son. The woman is 4 times as old as her son.
(b) (i) 172 : 165 **(ii)** 3 : 1
(iii) The woman is now 1.04 times as tall as her son. The woman is now 3 times as old as her son.

3

	A	B	C	Total
	$50	$90	$70	$210
Parts	5	9	7	21
	$10	$18	$14	$42
	$100	$180	$140	$420

(a) 5 : 9 **(b)** $\frac{5}{7}$ **(c)** $\frac{1}{3}$ **(d)** $18 **(e)** $140

Topic 8: Algebra

Topic review

Multiple choice

1 C
2 A
3 C
4 B
5 C
6 D
7 C
8 B
9 C
10 C
11 D
12 D
13 C
14 C
15 C

Short answer

1 **(a)** Always **(b)** Sometimes
(c) Never **(d)** Sometimes

2 **(a)** $5n + 4$ **(b)** $7m + 2d$

3 $90n + 625$

4 **(a)** $10H$ **(b)** \$95

5 The like terms are $-2m, \frac{4m}{5}, -m$.

6 **(a)** $12b - 4a - 2$
(b) $12b - 17a$
(c) $6a - 6b + 7$

7 **(a)** 2 **(b)** 25 **(c)** 27 **(d)** $\frac{1}{3}$

8 $-7, 2, 11, 20, 29$

9 **(a)** 1 **(b)** -1

10 The fraction $\frac{m}{3}$ can be written as $\frac{1}{3} \times m$, so has a coefficient of $\frac{1}{3}$ and a variable part of m. Hence it is a like term with m as they have exactly the same variable part.

On the other hand, the fraction $\frac{3}{m}$ can be written as $3 \times \frac{1}{m}$, so has a coefficient of 3 and a variable part of $\frac{1}{m}$. Since the variable parts are not exactly the same it is not a like term with m.

11 **(a)** 7 **(b)** -2 **(c)** -9 **(d)** 1

12 $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1$

13 **(a)** $m = 10$ **(b)** $p = 3$

14 **(a)** 13 cm **(b)** 14.14 cm **(c)** 7.07 cm

15 **(a)** $a^2 + b^2$ **(b)** $\frac{\sqrt{x}}{2} + 3$

16 **(a)** $a + 6$ **(b)** $2a$ **(c)** $4a + 6$
(d) Mindy has 11 apples and Tara has 10 apples.
(e) 46 apples

17 **(a)** $2\left(a^2 + \frac{b}{3}\right)$ **(b)** 58

18 **(a)** True **(b)** False
(c) False **(d)** True

Extended response

1 **(a)** $\$10 \times 16 + \$25 = \$185$
(b) $\$10 \times 34 + \$25 = \$265$
(c) The total cost of food and cake can be calculated by multiplying the number of people by \$10, then adding \$25 for the cake.
(d) Total cost $= 10 \times n + 25 = 10n + 25$

2 **(a)** $2s + t$ **(b)** $5t$ **(c)** 25 cm **(d)** 30 cm

3 **(a)** $4n + 3$
(b) **(i)** 7 counters
(ii) 23 counters
(iii) 47 counters
(c) 15 counters in each cup

4 **(a)** $3p - 210$
(b) First prize is \$500, second prize is \$400, third prize is \$200, and fourth prize is \$190.
(c) \$240
(d) First prize is \$110, second prize is \$10, third prize is \$5, and fourth prize is $-$5.
(e) The expression suggests the total of the prizes is $3 \times 110 - 210 = 120$. The sum of the four prizes is $110 + 10 + 5 - 5 = 120$, so it matches, but still does not make it possible.
(f) For there to be four prizes of positive value, there needs to be a minimum value set for the first prize. Using \$1 as the fourth prize makes the first prize \$122.

5 **(a)** $x + 8$ **(b)** $5x + 20$ **(c)** \$12

Topic 9: Linear relationships

Topic review

Multiple choice

1 A and D
2 C
3 D
4 B
5 B
6 B
7 D
8 D
9 B
10 C

Short answer

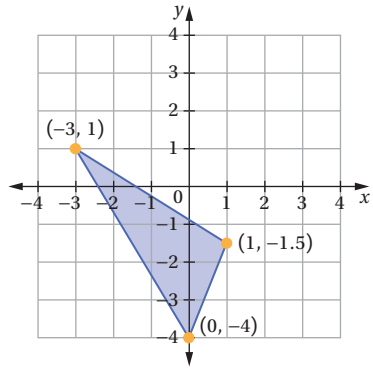
1 **(a)** 10 **(b)** 5 **(c)** 31 **(d)** 39

2

Input: x	0	1	2	3
Output: $3x + 6$	6	9	12	15

3 **(a)** 2 **(b)** -3 **(c)** 4 **(d)** 5

4



5 **(a)** The pattern is not linear. As each x-value increases by 1, the y-value decreases by 4, 4, 3 and then 4. The inconsistency means that the pattern is not linear.

(b) The pattern is linear. As each x-value increases by 1, the y-value always decreases by 11.

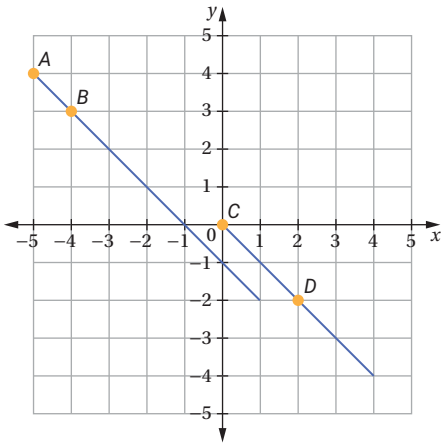
(c) The pattern is linear. As each x-value increases by 1, the y-value always increases by 6.

6 **(a)** The missing value is 7.

(b) The missing value is -2.

(c) The missing value is -2

7



The points do not form a linear pattern.

8

Any three points on a Cartesian plane will make a linear pattern.	False
Any two points of a Cartesian plane can be joined by a straight line.	True

9 **(a)** Point D does not fit the linear pattern.

(b) It should be plotted at (3,5).

(c) It would be exactly $\left(2\frac{1}{3}, 3\right)$. The y-value is exact, but a reasonable approximation for the other value should be accepted.

10 **(a)** $y = \frac{3x}{2} - 1$

x	-3	-2	-1	0	1
y	$-5\frac{1}{2}$	-4	$-2\frac{1}{2}$	-1	$\frac{1}{2}$

(b) $y = \frac{-2(4-x)}{3}$

x	-1	0	2	5	7
y	$-3\frac{1}{3}$	$-2\frac{2}{3}$	$-1\frac{1}{3}$	$\frac{2}{3}$	2

11 **(a)** $y = -3x + 2$ **(b)** $y = 4 - \frac{x}{5}$

12 **(a)** $6x - 5 = 3x + 3x - 5$

(b) $2x + 8 = 3x + 7 + 1 - x$

(c) $5(x - 2) = 3x + 9 + 2x - 19$

(d) $\frac{4x}{3} + 5 = x - 4 + \frac{x}{3} + 9$

13 Sample answers:

(a) $x + 5x + 1$; $2x + 4x + 1$; $3x + 3x + 1$

(b) $4 - 2x - x$; $4 - 4x + x$; $2 + 2 - 3x$

(c) $2x + 2$; $x + x + 2$; $x + x + 1 + 1$

(d) $15 - 6x$; $5 + 10 - 6x$; $15 - 3x - 3x$

14 **(a)** $2x + 3x + 4 - 2 = 5x + 2$

(b) $7ab - 4 - 3b + 6ba + 9b = 13ab + 6b - 4$

(c) $8a + 9b - 6ab + 4$ is already in simplest form.

(d) $2(3x - 1) + 5(2x - 3) = 6x - 2 + 10x - 15 = 16x - 17$

15 **(a)** $12ab - 8ac = 4a(3b - 2c)$

(b) $24 - 12a = -12(a - 2)$

(c) $30a - 10 - 8a^2 + 4ab = 10(3a - 1) - 4a(2a - b)$

(d) $12 - 6a - 30b + 5c = 6(2 - a) - 5(6b - c)$

16 **(a)** 7

(b) -3

(c) $1\frac{1}{3}$

17 **(a)**

x	-5	-1	0	3	5
y	14	6	4	-2	-6

(b) $y = -2x + 4$

18 **(a)** $x = 5$ **(b)** $x = -4$

(c) $x = 6$ **(d)** $x = 4$

Extended response

1 **(a)** $x = 1$ to 1

$x = 2$ to 4

$x = 3$ to 7

The output values increase by 3 for each increase of 1 in the x-value.

(b) $x = 1$ to 2

$x = 2$ to 5

$x = 3$ to 10

The output values increase by a different, greater value each time.

(c) $x = 1 \rightarrow \frac{1}{3}$

$x = 2 \rightarrow \frac{1}{4}$

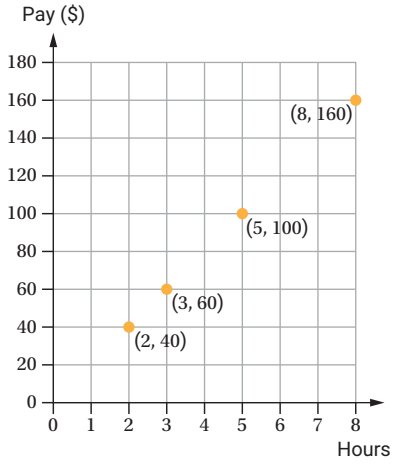
$x = 3 \rightarrow \frac{1}{5}$

The output values become increasingly smaller fractions. The values will tend to zero, but never reach it.

2 **(a)**

Hours	2	3	5	8
Pay ($)	40	60	100	160

(b)



(c) $140

(d) It would be at the origin, (0, 0). This is reasonable as you shouldn't expect any pay if you don't work any hours.

3 **(a)** 12 toothpicks **(b)** 19 toothpicks

(c) 7 toothpicks

(d) number of toothpicks = 7 × number of squares + 5

(e) The three-square pattern uses 26 toothpicks.
$7 \times 3 + 5 = 21 + 5 = 26$
The rule works.

Topic 10: Making and reading linear graphs

Topic review

Multiple choice

1 D
2 C
3 C
4 D
5 C
6 B

Short answer

1 **(a)** Parts A, C and F **(b)** Parts D and G
(c) Parts B and E **(d)** 150 m from the origin
(e) m/min

2 **(a)** $y = 2x + 4$ **(b)** $y = -x + 7$
(c) $y = -3$

3 The trip can be started between midnight and 1:30 am, from 7:30 am to 2 pm and then again after 6 pm.

4 The y-intercept is at $\left(0, \frac{1}{5}\right)$ and the gradient is $-\frac{2}{5}$.

5 **(a)** 1999, 2010, 2011 **(b)** 1997–2012

6 **(a)** 4 hectares **(b)** 17.5 acres
(c) If 15 acres = 6 hectares, then 30 acres = 12 hectares.

7 **(a)** Positive **(b)** Negative **(c)** Zero

8 **(a)** $80
(b) Approximately $275
(c) Approximately $405
(d) Exactly 4 hours
(e) Total cost = $(80 + 65 × number of hours)

Extended response

1 **(a)** Electric vehicles have shown the smallest number of sales.
(b) Petrol/diesel/LPG vehicles have shown the biggest decrease in sales.
(c) Hybrid vehicles have shown the biggest increase in sales.
(d) The petrol/diesel/LPG graph does not start its vertical axis at zero, whereas the other graph does.
(e) Approximately 1 010 000 vehicles.
(f) Approximately 912 000 vehicles.
(g) Sample answer: Electric and hybrid vehicle sales will increase at least as rapidly as shown in the graph and other fuel types will fall away significantly.

Topic 11: Solving linear equations

Topic review

Multiple choice

1 B
2 A
3 B
4 B
5 D
6 B
7 C
8 A
9 B
10 C
11 B
12 C
13 D
14 B

Short answer

1 Answers will vary.
(a) A number added to 5 is the same as 20.
(b) 5 less than a number is equal to 20.
(c) 5 multiplied by a number gives 20.
(d) 20 is one fifth of a number.

2 $12(x - 5) + 32 = 92$

3 $420

4 $\frac{15-4x}{3}=13$

5 $45=x+14+19$

6 $5c=50$

7 $2(2x+10)=92$

8 $8-m=9$

9 $3x+4=-15$

10 $\frac{x}{2}+x+60=180$

11 $\frac{5a}{8}-6=-\frac{2}{3}$

12 $6h+35=150$

13 $2(x+2x)=1500$

$2\times 3x=1500$

$6x=1500$

14 $3\left(\frac{c}{2}\right)+2c=7$

15 $17+2x=-6$

16 $7+21+8+x+2x+2=111$

$3x+38=111$

17 $45+100+x+2x=360$

$3x+145=360$

18 **(a)** False **(b)** True **(c)** True

19 Sample answers.

(a) Two is added to a certain number to give a result of eight.

(b) A certain number divided by five results in two.

(c) Six times a certain number is thirty-six.

20 **(a)** $\times 7, +1$ **(b)** $\div 6, -9$

21 **(a)** $3x+4=22$ **(b)** $x=6$

Extended response

1 In the top left corner, place Domino 1 horizontally. In the top right corner, place Domino 4 vertically. In the bottom left corner, place Domino 2 vertically with the $a-13=7$ side in the actual corner.

2 **(a)** A: $2n+5$ B: $2n+6$ C: $4n+11$

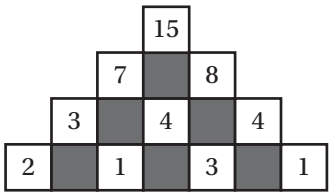
(b) $4n+11=15$

(c) $4n+11=15$

$4n=4$

$n=1$

The pyramid would look like this:



Topic 12: Area and volume

Topic review

Multiple choice

1 A

2 A

3 B

4 B

5 D

6 C

7 A

8 B

9 D

10 B

11 C

12 A

13 C

Short answer

1 **(a)** 308 cm^2 **(b)** $65\frac{11}{32}$ cm^2

2 **(a)** 4.5 m^2 **(b)** 15.41 cm^2

3 9 cm

4 27 cm^2

5 26 cm^2

6 12.25 cm^2

7 7.4 m^2

8 1.56 cm^2

9 19.44 cm^2

10 320 cm^2

11 **(a)** 55 m^2 **(b)** $5390

12 27 cm^3

13 3 cm

14 1125 m^3

15 3872 mL

16 110.656 L

17 477 mL

18 Ahmad's jar holds 423 mL more.

Extended response

1 **(a)** 93.6 m^2 **(b)** 9 m **(c)** 9 m **(d)** 18 m

2 **(a)** 0.16 m^2 **(b)** $5.60 **(c)** $6.70 **(d)** $6.82

3 **(a)** 1000 **(b)** $320 **(c)** 5.75 kg **(d)** 17

4 **(a)** 250 cm^3 **(b)** 8 cents **(c)** $161 **(d)** $3.40

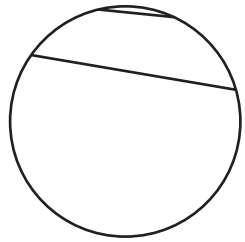
Topic 13: Circles (features and circumference)

Topic review

Multiple choice

1 B
2 D
3 A
4 C
5 A

Short answer

1 

2 **(a)** 28 cm and two of 12 cm
(b) 14 cm and two of 6 cm

3 **(a)** 14.4 cm **(b)** 45.2 cm
(c) $\frac{C}{d} \approx 3.14$, $\frac{C}{r} \approx 6.28$

4 The radius of the circle will a bit be less than 5 cm.

5 14 cm

6 **(a)** 3.15 **(b)** 3.24

7 **(a)** 110 m **(b)** 484 cm or 4.84 m

8 37.88 m

9 120 cm

10 **(a)** 61.7 cm **(b)** 59 cm

Extended response

1 **(a)** 277.08 m **(b)** 7.85 m
(c) 3320.58 m or 3.32058 km

Topic 14: Angles and transversals

Topic review

Multiple choice

1 B
2 D
3 B
4 C
5 A
6 D
7 B
8 A
9 D
10 A
11 D
12 B
13 D
14 B
15 A
16 D
17 C

Short answer

1 **(a)** $x = 68°$, alternate angles on parallel lines are equal.
(b) $x = 56°$, co-interior angles on parallel lines are supplementary.
(c) $a = 52°$, alternate angles on parallel lines are equal; $b = 128°$, co-interior angles on parallel lines (or adjacent angles on a line) are supplementary.

2 $a = 66°$, co-interior angles on parallel lines are supplementary.
$b = 42°$, alternate angles on parallel lines are equal.

3 **(a)** Interior angles 108°, 59°, 90° (adjacent angles on a line). Angle next to x: 103° (angle sum of a quadrilateral). $x = 77°$ (adjacent angles on a line).
(b) $b = 58°$ (corresponding angles on parallel lines are equal). Angle next to a: 58° (alternate angles on parallel lines are equal). $a = 122°$ (adjacent angles on a line).

4 **(a)** $x = 44°$ **(b)** $x = 139°$ **(c)** $x = 131°$

5 $b = 51°$ (adjacent angles on a line). $a = 113°$ (exterior angle of a triangle). $c = 118°$ (adjacent angles on a line).

6 $a = 59°$ (revolution and equilateral triangle). $b = 102°$ (revolution). $c = 19°$ (angle sum of a triangle). $d = 281°$ (revolution and equilateral triangle).

7 The first is suitable for compass construction because the 6 cm side lengths can be marked off with arcs from each end of the 8 cm base. A set square is suitable for the second one because the height can be marked on the perpendicular line. A perpendicular from the second line gives the side parallel to the base.

8 **(a)** 900° **(b)** 1800°

9 **(a)** 120° **(b)** 135°

10 140°

11 $x = 80°$

12 **(a)** 72° **(b)** 60°

Extended response

1 **(a)** h and y **(b)** w **(c)** h
(d) q **(e)** f and m **(f)** g

2 **(a)** angle BJG or angle CJG
(b) angle DAG or angle EAG or angle CAG or angle DAB or angle EAB or angle CAB
(c) angle EBA
(d) angle ECF or angle DCF

3 $a = 142 - 123 = 19°$ (exterior angle of a triangle).
$b = 180 - 142 = 38°$ (adjacent angles on a line).
$c = 180 - 130 - 19 = 31°$ (adjacent angles forming a line).
$d = 180 - 90 - 31 = 59°$ (angle sum of a triangle).

Topic 15: Ratios and measurement

Topic review

Multiple choice

1 A
2 C
3 D
4 D
5 C
6 B
7 B
8 A
9 C
10 A
11 A
12 B
13 C

Short answer

1 **(a)** 3.76 : 1 or 3.76
(b) 0.69 : 1 or 0.69
(c) 1.30 : 1 or 1.30
2 2.14
3 39.25 cm
4 **(a)** 77 cm **(b)** 272.8 cm
(c) 5.25 cm **(d)** 14.7 cm
5 22.2 cm and 55.6 cm
6

2	4	5
8.8 cm	17.6 cm	22 cm
1.5 m	3 m	3.75 m
21 mm	42 mm	52.5 mm

7 **(a)** 13 125
(b) **(i)** 4.096 cm^3 **(ii)** 53 760 cm^3
(c) 24 cm by 40 cm by 56 cm
(d) 53 760
8 **(a)** 40 **(b)** 50
9 $7\frac{1}{2}$ medium potatoes and 450 g of pumpkin
10 **(a)** 9 cups **(b)** 11 : 24

Extended response

1 **(a)** 0.5
(b) **(i)** Shade I: 0.6, darker than baby-blue
(ii) Shade II: 0.45, paler than baby-blue
(iii) Shade III: 0.8 dark blue, but not as dark as navy blue
(c) Average portion of navy blue:
$\left(\frac{6}{16} + \frac{5}{16} + \frac{8}{18}\right) \div 3 = 0.3773\ldots$
Average portion of white: $1 - 0.3773 = 0.6227$
The navy blue to white ratio is about 0.61, so the paint will be slightly darker than shade I.
(d) Shade I: add 2 cups of white
Shade II: add $\frac{1}{2}$ cup of navy blue
Shade III: add 6 cups of white

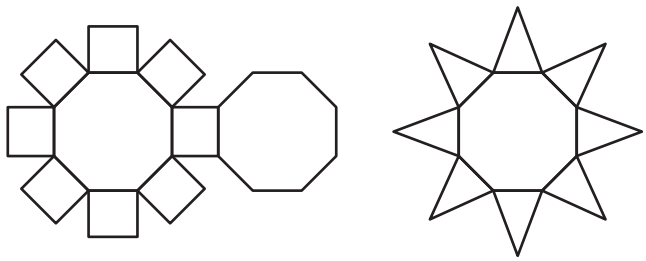
Topic 16: Visualising 3D

Topic review

Multiple choice

1 D
2 A
3 C
4 C
5 B
6 C
7 D
8 B
9 D
10 A

Short answer

1

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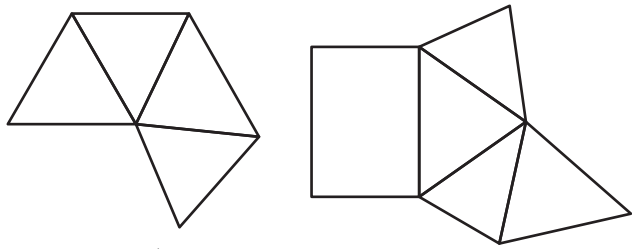
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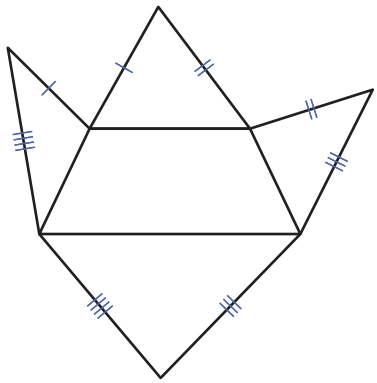

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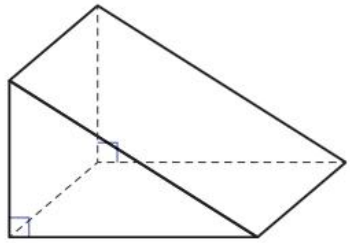
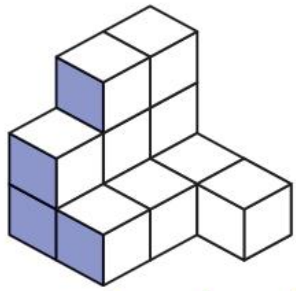
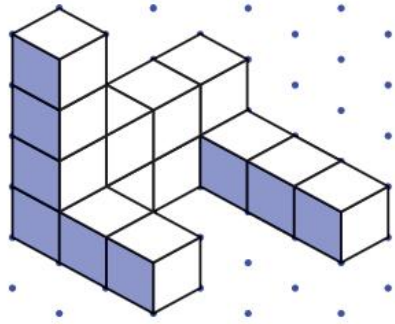


5 19, 20, 21, 22 or 23 cubes

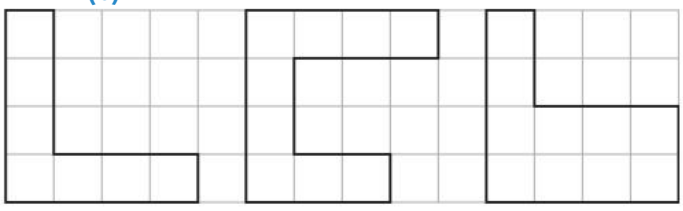
6



7 **(a)**



(b)



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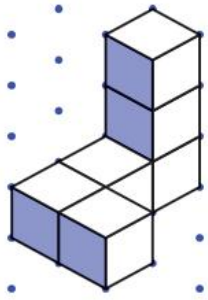
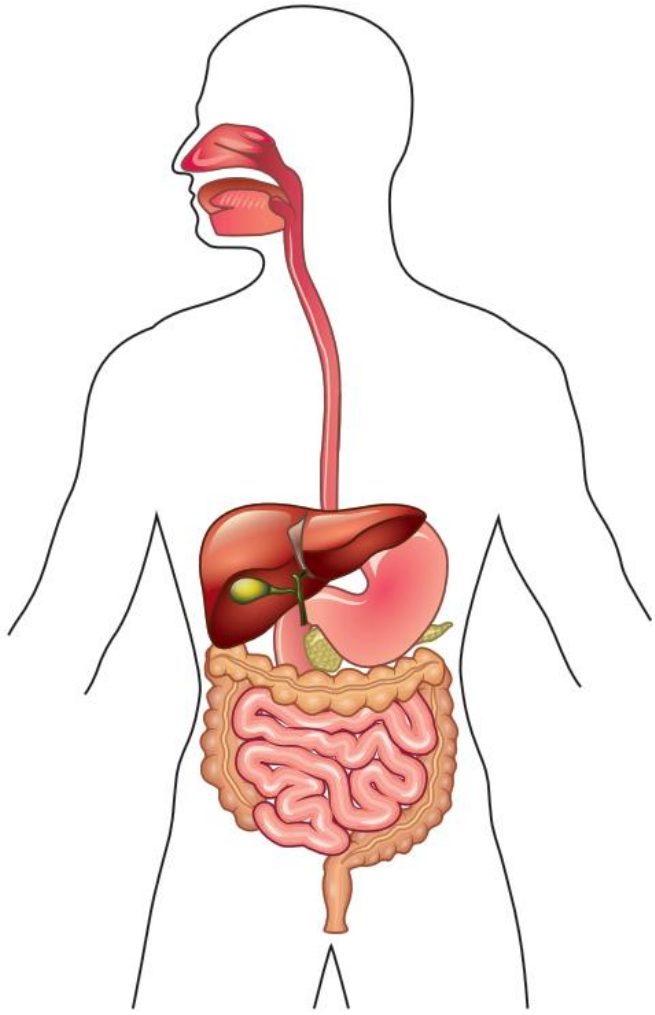
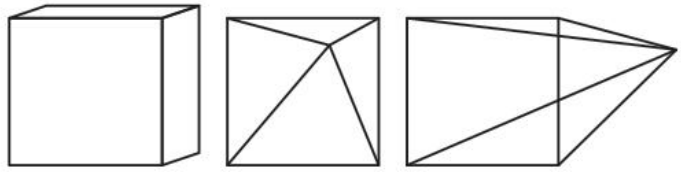


9

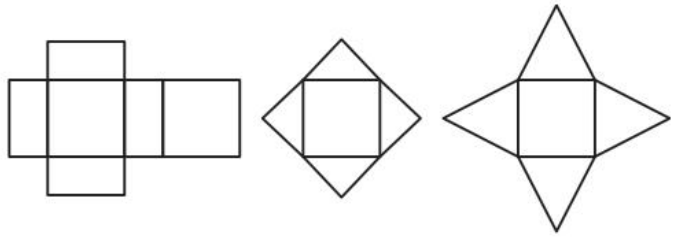


Extended response

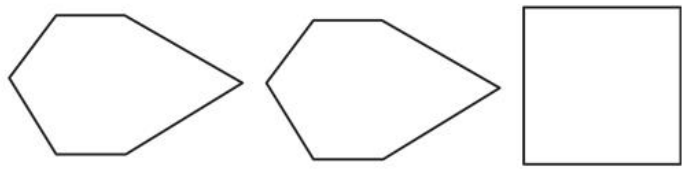
1 **(a)**



(b)



(c)



Topic 17: Triangles and quadrilaterals

Topic review

Multiple choice

1 B
2 C
3 C
4 A
5 D
6 B
7 A
8 B
9 C
10 D
11 D
12 B
13 D
14 B
15 C
16 C
17 C
18 D

Short answer

1 **(a)** No, because 16 + 18 = 34 and 34 < 35
(b) No, because 6 + 6 = 12, so the other side must be less than 12 cm.
(c) The three lengths can form a triangle.
2 87°
3 107°
4 **(a)** An isosceles triangle
(b) A scalene triangle
(c) An equilateral triangle
5 **(a)** An obtuse-angled triangle
(b) An acute-angled triangle
(c) A right-angled triangle
6 **(a)** An acute-angled isosceles triangle
(b) An obtuse-angled scalene triangle
(c) An obtuse-angled isosceles triangle
(d) A scalene right-angled triangle
(e) An equilateral triangle (always acute-angled)
7 **(a)** A kite
(b) A rhombus
(c) A rectangle
(d) A trapezium
8 **(a)** A quadrilateral with all angles right angles
(b) A quadrilateral with all sides equal
(c) A quadrilateral with both pairs of opposite sides parallel
9 The angles of a rhombus are not all the same, a requirement for regular polygons.
10 **(a)** A concave pentagon, irregular
(b) A convex, irregular heptagon
(c) A convex, irregular decagon
(d) A concave dodecagon, irregular
11 **(a)** 24.5 cm × 12 cm
(b) 294 cm^2

Extended response

1 **(a)** **(i)** 20 cm, 20 cm and 39 cm, obtuse-angled
(ii) 18 cm, 22 cm and 39 cm, obtuse-angled
(b) 18 cm, 20 cm and 24 cm, scalene
(c) 30 cm, 20 cm and 27 cm or 18 cm, 32 cm and 27 cm are both scalene, acute-angled triangles.
2 **(a)** Could be both obtuse-angled triangles or both acute-angled triangles
(b) Two obtuse-angled triangles and two acute-angled triangles
3 **(a)** 30 cm × 10 cm **(b)** 300 cm^2
(c) 900 cm^2
(d) 30 cm × 30 cm, actually a square

Topic 18: Translations, reflections, rotations

Topic review

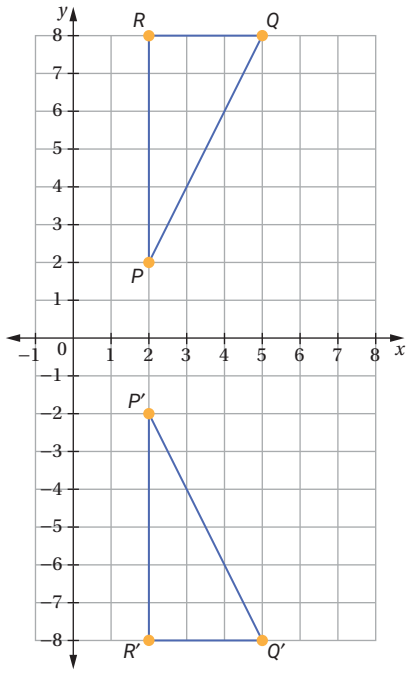
Multiple choice

1 C
2 A
3 D
4 C
5 B and D
6 A
7 B
8 B
9 D
10 C

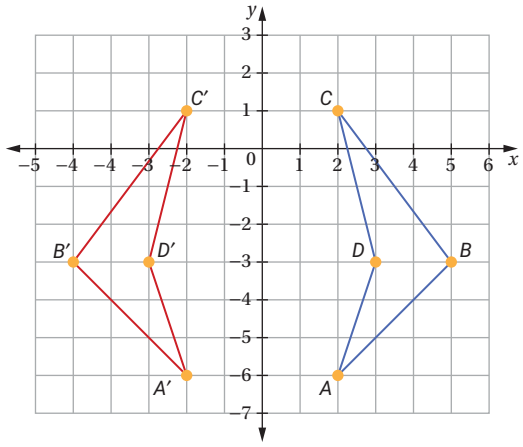
Short answer

1 **(a)** $M'(2, -2)$ **(b)** $A'(-1, 2)$ **(c)** $C'(-6, 0)$
2 **(a)** $G'(4, -2)$ **(b)** $M'(-3, -4)$ **(c)** $F'(-3, -7)$

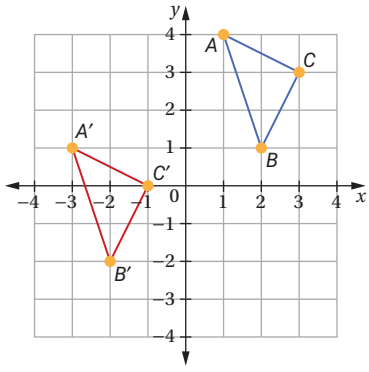
3 **(a)**



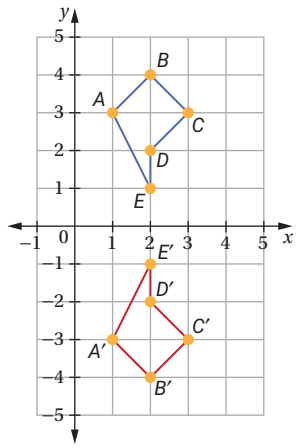
(b)



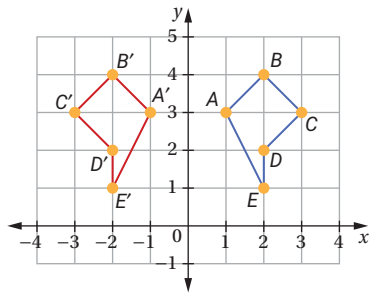
4



5 **(a)**



(b)



6 **(a)** $U'(7, 2)$, $V'(4, 2)$, $W'(5, 6)$

(b) $U'(-2, 7)$, $V'(-2, 4)$, $W'(-6, 5)$

7 **(a)** Reflection in the y-axis

(b) Rotation of 90° clockwise about the origin

(c) Translation $[-5, -2]$

8 **(a)** $[7, 4]$ **(b)** $[-3, -8]$

9 **(a)** 180° clockwise or anticlockwise

(b) 90° clockwise or 270° anticlockwise

10 $X(4,7)$ to $X'(4, -7)$; $X'(4, -7)$ to $X''(-7, -4)$

11 $R(3, 3)$ to $R'(3, 0)$; $R'(3,0)$ to $R''(-3, 0)$

12 $[3, 0]$

13 **(a)** Horizontal axis (x-axis)

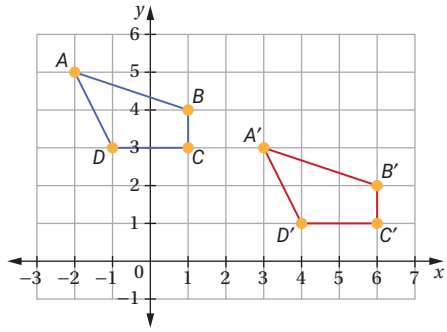
(b) $Q'(-2, -3)$, $R'(-1, -2)$, $S'(-2,0)$, $T'(-3, -2)$

(c) $R''(0, 2)$, $S''(-1, 4)$, $T''(-2, 2)$

Extended response

1 **(a)** **(i)** $A'(3, 3)$, $B'(6, 2)$, $C'(6, 1)$, $D'(4, 1)$

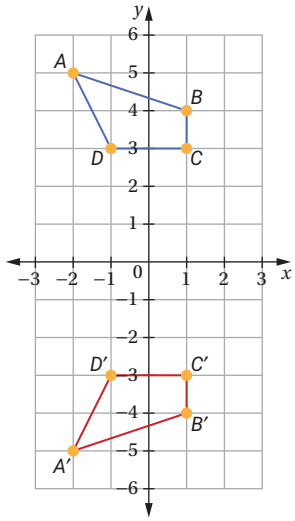
(ii)



(iii) The translation $[-5, 2]$

(b) **(i)** $A'(-2, -5), B'(1, -4), C'(1, -3), D'(-1, -3)$

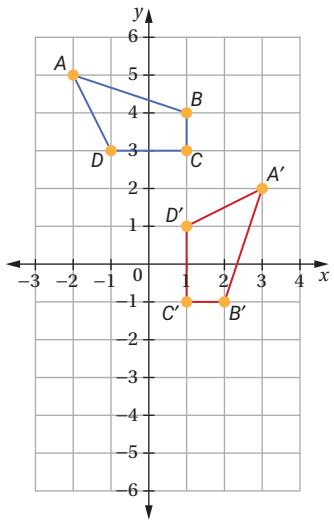
(ii)



(iii) A reflection in the x-axis

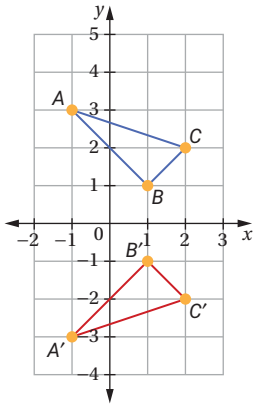
(c) **(i)** $A'(3, 2), B'(2, -1), C'(1, -1), D'(1, 1)$

(ii)

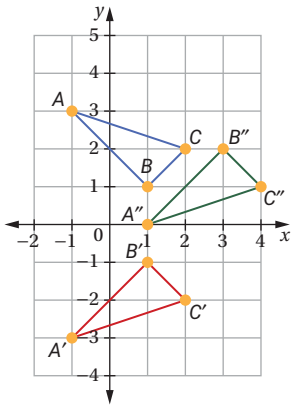


(iii) An anticlockwise rotation of 90° about the point $(-1, 1)$.

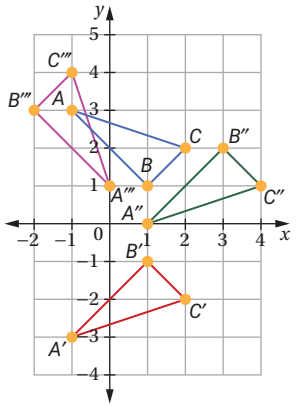
2 **(a)** **(i)**



(ii)



(iii)



(b) Rotate $A'''B'''C'''$ 90° clockwise about the origin to form $A''B''C''$; translate $A''B''C''$ 2 units left and 3 units down to form $A'B'C'$; reflect $A'B'C'$ in the x-axis to form ABC.

Topic 19: Data and measures of central tendency

Topic review

Multiple choice

1 C
2 B
3 C
4 D
5 B
6 B
7 B
8 B
9 B
10 C
11 D
12 B
13 B

Short answer

1 Numerical
2 **(a)** Categorical
(b) Because the data is categorical.
3 Sample answer
Categorical data generally requires answering a survey question with a word or category, and data is analysed by counting the frequency. For example, 'What is your favourite TV series?' – we count the votes for each series. Numerical data generally requires answering a survey question with a number, by either counting or measuring. For example, 'How old are you?' or 'How tall are you?'.
4 **(a)** Numerical **(b)** Categorical **(c)** Numerical
(d) Numerical **(e)** Numerical **(f)** Categorical
5 The mean is 19.6, which is above the long-term mean.
6 1.68 m
7 30
8 **(a)** Set A: 6.56 and 6. Set B: 43 and 9.
(b) Set B contains an extreme value and that impacts the mean and range, so it is not appropriate to use them for Set B.
9 8
10 **(a)** **(i)** 19 **(ii)** 17 **(iii)** 21
(b) Set C, because it has the greatest range.
11 Sample answer:
10, 11, 11, 14, 16
12 –2

Extended response

1 **(a)** **(i)**

Points per game	Mean	Median	Mode
Cassie	18.5	17	14, 18
Dina	16.1	15	14, 15, 16

(ii)

Rebounds per game	Mean	Median	Mode
Cassie	11.6	12	10,16
Dina	10.4	9	9

(b) The mean and median are similar for both players, but the shape of the data is quite different. The data for Cassie is symmetrical but is positively skewed for Dina. Dina has an extreme value of 24, which suggests the mean is not a good estimate of the central tendency.
2 **(a)** **(i)** 23 **(ii)** 18.5 **(iii)** 23
(b) **(i)** 21.5 **(ii)** 6 **(iii)** 41
3 Sample answer:
The difference between categorical and numerical data.
The difference in discrete and continuous data and the way they are displayed.
How to find the mean, median and mode.
Knowing when to use the median instead of the mean.
Calculating the range and knowing when it should not be used.
Describing data sets as symmetric, clustered, positive skew, negative skew or bimodal.

Topic 20: Displaying and understanding data

Topic review

Multiple choice

1 B
2 A
3 B
4 D
5 D
6 C
7 D
8 B
9 A
10 C
11 B
12 D

Short answer

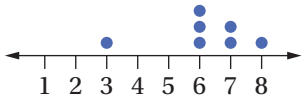
1 **(a)** 19.3 **(b)** 21 **(c)** 12 **(d)** 18
2 **(a)** 2.5 **(b)** 2.5 **(c)** 4 **(d)** 5
3

Stem	Leaf
2	0 3 3 4 8 8 9
3	1 1 2 2 3 4 6
4	2 2 3 3 5 7 8
5	5 6 7
6	1 5

Key 2|0 = 20
4

Stem	Leaf
2	3 3 4
2	5 5 7 7 8 8 8 9
3	1 1 1 1 3 4
3	5 5 6 7 7 8 9
4	2 2 3 3 4 4
4	5 5 6 6 7 9 9
5	0 1 3 4
5	5 6 6 7 8 8 9

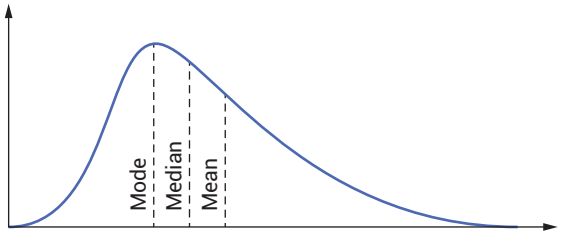
5



6 **(a)** Negative skew
(b) Symmetrical
(c) Positive skew

7 The data has more than one peak, so it has two clusters, each with its own mode.

8



9 **(a)** Continuous, because there are no breaks between the columns.
(b) Discrete, because there are breaks between the columns.
(c) Discrete, because there are breaks between the columns.

Extended response

1 **(a)** The 4 in the bottom row should be in the row above because that contains the numbers from 30–34.
(b) To make more groups
(c) **(i)** 39 **(ii)** 27 **(iii)** 26.05 or 26.1
(d) Mode. This represents the most popular figure and in this case, it is also the maximum, so it does not represent the centre of the data.
(e) 2.8

2 **(a)** Sentence B **(b)** Sentence B
(c) Sentence A
(d) 4, 5, 7 and 8 all have a frequency of 3.
(e) There is more than one correct response.
Sample answers:
Sentence B has more words of nine or more characters, so is probably harder to read.
Sentence A has a higher median, so on average it has longer words.

3 **(a)** 13 **(b)** 41
(c) No mode. There are three values with a frequency of 3.

4 Sample answer:
How to find the mean, median and mode of data displayed in a stem-and-leaf plot or a dot plot.
Knowing when to use the median instead of the mean.
Calculating the range and knowing when it should not be used.
Creating stem-and-leaf plots.
Creating dot plots.

Topic 21: Probability (simulations and sample space)

Topic review

Multiple choice

1 D
2 B
3 D
4 C
5 A
6 B
7 C

Short answer

1 **(a)** {Spades, Clubs, Diamonds, Hearts}
(b) {Red, Black}

2 **(a)** {Blue, Green}
(b) The blue covers more area and should appear more often than the green.
(c) The area of the blue is not changed so its frequency will not change.
(d) The area of the green has been reduced from 4 to 3 wedges, so the frequency will also be lower.
(e) Blue has the larger area, so is expected to have the higher frequency. This means the higher relative frequency.

3 **(a)** Sample answers: Using 5 different coloured pencils; have 5 slips of folded paper with names on them; use a spinner with 5 equal divisions.
(b) There are only 2 possible outcomes.
(c) 1, 2, 3, 4 and 5 could be the outcomes. Ignore any 6 that is rolled and re-roll the die.

4 **(a)** 50 times is too small for an experiment and many more trials would need to be run to answer this.
(b) If the coin is fair, the chance of tossing a head remains the same for every toss. The coin has no memory of what has happened in the past, so it remains unaffected by past events.

5 **(a)** Sample answer. Roll a die 90 times. A 1 or 2 represents travelling with her mum, 3 or 4 is travelling by train and 5 or 6 is catching the school bus.
(b) $\frac{1}{3}$
(c) $E(X) = \frac{1}{3} \times 90 = 30$

6 **(a)** Sample answer. Because it appears that $\frac{2}{3}$ of the circle is yellow, $\frac{2}{3} \times 600 = 400$.

(b) Sample answer. Because it appears that $\frac{4}{10}$ of the circle is yellow, $\frac{4}{10} \times 600 = 240$.

(c) Sample answer. $\frac{1}{3}$ of Spinner 1 is green and $\frac{4}{10}$ of Spinner 2 is green. Because $\frac{1}{3} < \frac{4}{10}$, there will be fewer green wedges spun on Spinner 1.

7 **(a)** 50 red, 120 blue and 30 green

(b) There would be exactly 50 red, 120 blue and 30 green. The estimated figures may vary, but these cannot.

8 **(a)** Using the experimental results:

$$\text{Pr(event)} = \frac{\text{number of favourable trials}}{\text{total number of trials}}$$

$$\text{Pr}(3) = \frac{12}{60}$$

$$\text{Pr}(3) = \frac{1}{5}$$

(b) Answers will vary.
Sample answer:
If the die is rolled many more times, the experimental probability might get closer to the theoretical probability of $\frac{1}{6}$. Another reason could be that the die is not a standard fair die (e.g. it may be heavier on one side).

9 **(a)**

$$\text{Pr(event)} = \frac{\text{number of favourable trials}}{\text{total number of trials}}$$

$$\text{Pr(on time)} = \frac{36}{48}$$

$$\text{Pr(on time)} = 75\%$$

(b) Including these 6 new results, Jacinta's bus is now on time 36 days out of 54.

$$\text{Pr(on time)} = \frac{36}{54}$$

$$= 66.7\%$$

However, being late 6 days in a row might mean that there is some other reason for this lateness, such as new roadworks slowing traffic.

Extended response

1 **(a)** \$2000 **(b)** \$1580
(c) \$420 **(d)** \$200
(e) Answers will vary.
(f) Answers will vary.
(g) Out of \$2000 income, \$1800 must be returned in prizes. Since the current prize pool returns only \$1580, you need to increase the prizes by at least \$220.
Sample suggestion:
3 popped balloons wins a prize valued at \$17.
2 popped balloons wins a prize valued at \$7.
1 popped balloon wins a prize valued at \$2.
0 popped balloons wins a prize valued at \$1.